Solver of Tricky Triple Puzzles Based on Constraint Programming

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Abstract. In this article, we show how to use constraint programming to solve Tricky Triple Puzzles. We present a consistent method to analyze these puzzles and to obtain a solution to them.

Keywords: tricky-triple · prolog · clpfd · constraint-programming

1 Introduction

This project consists of building a program, in Logic Programming with Restrictions, for solving a combinatorial decision. The problems studied are Tricky Triple Puzzles, which are grid puzzles. To do so, we will analyze these problems and proceed to implement our solver using SICStus Prolog. Afterward, we will be discussing the performance results obtained. This article has the following structure:

- **Problem Description**: tricky triple puzzle detailed description
- **Approach**: problem modulation
  - **Decision Variables**: decision variables' meaning and domain
  - **Constraints**: problem constraints' description
- **Solution Presentation**: explanation for the predicates that provide a user interface
- **Experiments and Results**: results from all the tests
  - **Search Strategies**: result's analysis from different search strategies
  - **Dimensional Analysis**: result's analysis from different puzzle's dimensions
- **Conclusions and Future Work**: conclusions withdraw and limitations of this project
- **References**: books, web pages, and articles used
- **Annex**: result tables and other extras

2 Problem Description

The Tricky Triple puzzles are a type of grid puzzle. The goal of the puzzle is to fill each of the grid's white cells with one of 3 symbols, a square, a circle, or a triangle. The only rule is that each group of 3 adjacent white cells (horizontally, vertically, or diagonally) must contain exactly 2 of one of the symbols. So, each group of 3 white cells will have 2 of 3 symbols. Each puzzle given has a unique solution.

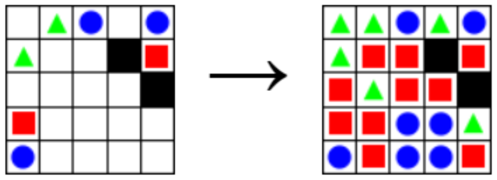


Fig. 1. Example of Tricky Triple Puzzles, before and after solving it

3 Approach

Throughout the development of this project, we used a Constraint Logic Programming approach. Our grid is a list of lists, where each element is a number indicating the symbol placed on that cell. Each one of these numbers represents a different symbol. The number 0 represents a black cell, which cannot be generated by the solver. The number 1 represents a green triangle, while the number 2 represents a red square and the number 3 a blue circle.

3.1 Decision Variables

All tricky triple puzzles take the form of an NxN square grid divided by cells. The grid is represented internally by a list of lists forming a square matrix.

The grid cells can either be black, a cell that the program can't fill, or, more commonly, white.

All the white cells need to be either a triangle (represented by 1), a square (represented by 2), or a circle (represented by 3) to reach the puzzle solution.

The Decision Variables are all the elements of the list of lists, meaning all the puzzle cells. Their domain is 0, 1, 2, 3. However, being the representative of a black cell, the zero is not considered a valid value to fill an empty cell, since that would transform a white cell into a black cell.

3.2 Constraints

The restrictions implemented in our solver are faithful to the ones from the puzzle rules.

Each cell must contain a symbol
In the puzzle solution, all the cells have to be assigned.

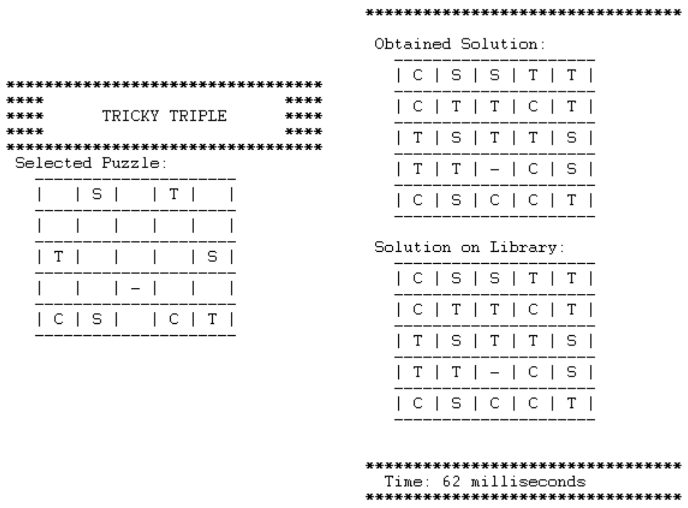
No black cells can be assigned
In every white cell, we need to put a square, a triangle, or a circle. Meaning we can't put a black cell on a white cell.

Each group of 3 adjacent white cells (horizontally, vertically, or diagonally) must contain exactly 2 of one of the symbols
Consider a group of three adjacent, horizontally, vertically, or diagonally, white cells. In this group, two of the cells have the same symbol, and the last cell must have a different one.

4 Solution Presentation

To present the solution, we use two predicates from the file *display.pl*.

- The predicate `display_grid/2` displays the grid in a human-friendly way so that the user can identify the cells and the grid's symbols.
- The predicate `get_readable_symbol/2` translates the grid elements' internal representation into more readable symbols for those to be displayed to the user.



```
**********************************
****                          ****
****      TRICKY TRIPLE       ****
****                          ****
**********************************
 Selected Puzzle:
   ---------------------
   |   | S |   | T |   |
   ---------------------
   |   |   |   |   |   |
   ---------------------
   | T |   |   |   | S |
   ---------------------
   |   |   | - |   |   |
   ---------------------
   | C | S |   | C | T |
   ---------------------
```

```
**********************************
 Obtained Solution:
   ---------------------
   | C | S | S | T | T |
   ---------------------
   | C | T | T | C | T |
   ---------------------
   | T | S | T | T | S |
   ---------------------
   | T | T | - | C | S |
   ---------------------
   | C | S | C | C | T |
   ---------------------

 Solution on Library:
   ---------------------
   | C | S | S | T | T |
   ---------------------
   | C | T | T | C | T |
   ---------------------
   | T | S | T | T | S |
   ---------------------
   | T | T | - | C | S |
   ---------------------
   | C | S | C | C | T |
   ---------------------

**********************************
  Time: 62 milliseconds
**********************************
```

Fig. 2. Output Example for a Puzzle Solution

5 Experiments and Results

The tests performed on this solver aren't as exhaustive as we could have hoped since we could only use the pre-generated puzzles, and there weren't that many of them. However, we can still draw some conclusions from the times measured when solving these puzzles.

5.1 Search Strategies

To compensate for our lack of different puzzles, we did extensive tests with the predefined ones. After that, we grouped the tests by the puzzle's dimension and the labeling options used in that solution.

From all the collected information, we can withdraw some conclusions:

- A small board with bad labeling options can take more time to solve than a bigger one with good labeling options.
- When introducing some partially solved boards, the solver would be consistently faster.
- The solver's execution time increases with the increase in the grid's dimensions.

The graph below illustrates the obtained results. Table 1 contains the average values for each dimension and labeling option.

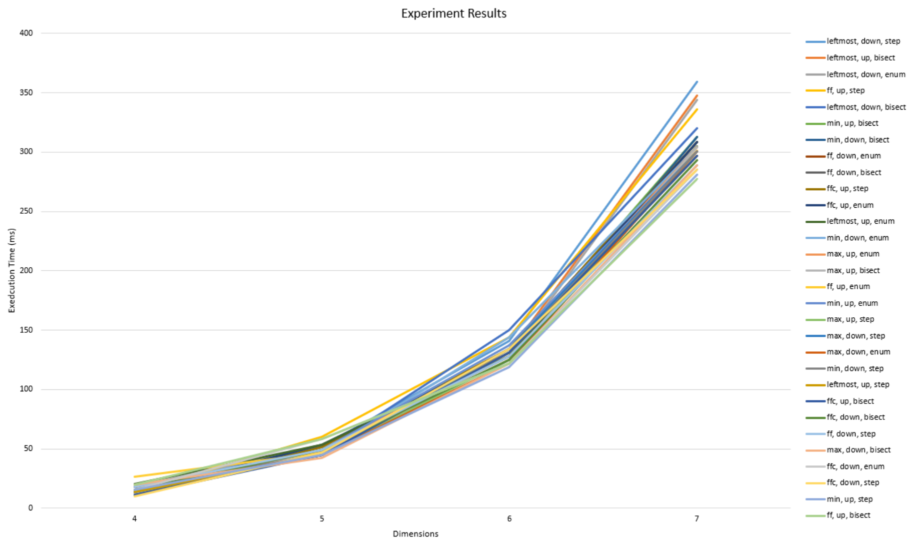


Fig. 3. Experiment Results

Analyzing Figure 3, we can conclude that the fastest labeling option, for grids up to 7x7, is ”ff, up, bisect”.

However, using the same data, we can try to predict the behavior of the labeling options when increasing the grid's dimensions by making a trendline. Using a degree 3 polynomial trendline, we obtained a prediction for execution times when solving bigger grids. The equations for these trendlines are in Table 2. Analyzing Figure 4, we can conclude that perhaps the best options would be "min, down, enum" or "leftmost, down, bisect".

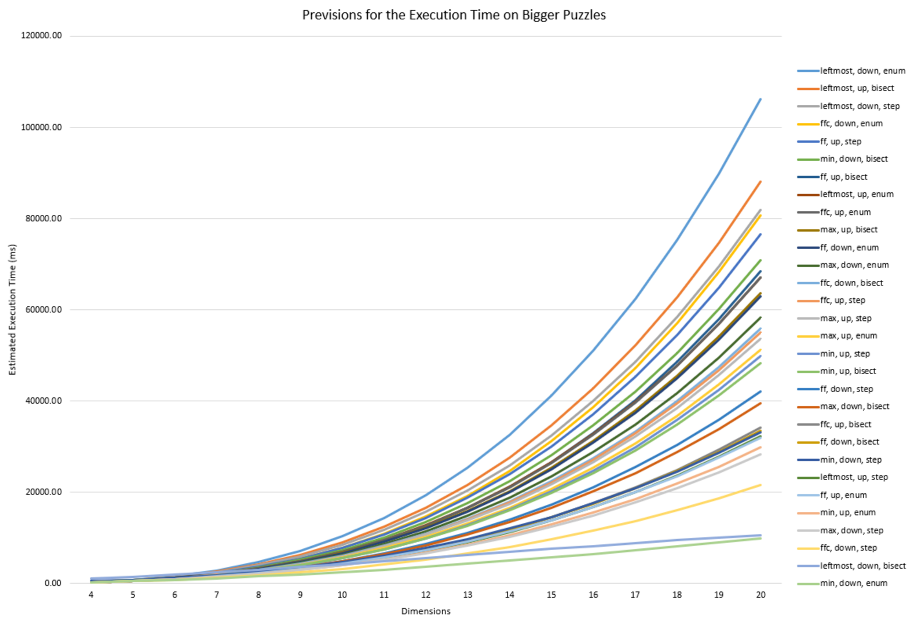


Fig. 4. Execution Times Predictions

5.2 Dimensional Analysis

We will be using ”ff, up, bisect” as a labeling option since it is the better one for grids up to 7x7, as already mentioned.

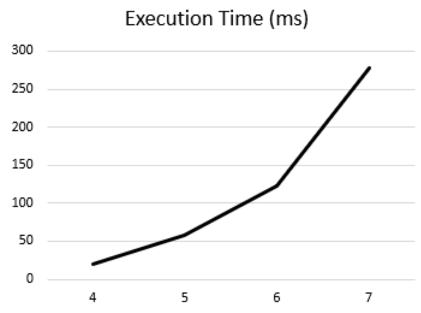


Fig. 5. Execution Times

As we can see in Figure 5, when the grid's dimension increases, the execution time also increases.

To know the rate at which this increase happens, we need to find an appropriate regression function. This function will establish a relation between the grid's dimensions and the execution time. However, with only 4 points, we can't estimate with certainty what will be the behavior of the best regression function.

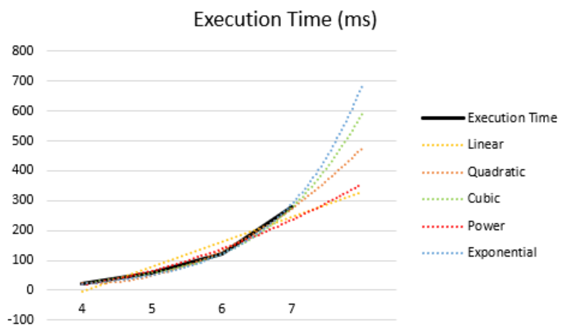


Fig. 6. Execution Times With Trendlines

From all the data we have, the best approximation is a cubic, a third-degree polynomial, as demonstrated in Figure 6.

6 Conclusions and Future Work

Upon starting this project, some assumptions were made about the ideal labeling options for solving these types of puzzles.

Initially, the idea was that the first fail options (”ff” or ”ffc”) were the ideal forms of selecting the next variables to explore, as this would emulate the form that a human would solve these puzzles, focusing on the variables that only have one possibility. There was also the assumption that the type of selection of values was of little importance. Since the domain is so small, with only a total of 3 possible values and no ”greater than” or ”less than” types of restrictions, the difference in the performance of ”step”, ”enum” or ”bisect” and of ”up” or ”down” would average out too negligible.

Upon analyzing the results some of our assumptions seem correct. The choice between ”up” and ”down” seems of little to no importance, with any increased performance of one over the other being incidental. The results for using either ”step”, ”enum” or ”bisect” don’t demonstrate any clear pattern, being too scattered to draw any solid conclusions.

But, unlike what was assumed, when choosing the method of selecting the next variable to explore, the results were a lot different than expected. It was expected that either ”ff” or ”ffc” would be the ideal way of selecting variables, but the results were too scattered to both prove or disprove that theory. The data shows that for 7x7 puzzles the ideal option is ”ff, up, bisect”, but our predictions for greater dimensions show that ”min, down, enum” or ”leftmost, down, bisect” would be better options. One can postulate that what is happening is that is more efficient to start with a somewhat random variable, explore the options of this variable and use the more constrained variables as a way to eliminate invalid values for the starting variable, but further and bigger test cases, meaning puzzles, would be necessary to test this theory.

In conclusion, although some insight can already be obtained by the tests already executed, no solid conclusion can be reached without more puzzles, preferably ones with larger dimensions. Still, it is our belief that this is a good starting point for any further exploration of tricky triple puzzles.

7 References

1. Tricky Triple Puzzle Page: https://erich-friedman.github.io/puzzle/shape/
2. SICStus Prolog Documentation: https://sicstus.sics.se/documentation.html

8 Annex

8.1 Average Execution Times Table

Table 1. Average Times Obtained (ms)

Labeling Options			Grids			
			4x4	5x5	6x6	7x7
leftmost	up	step	13.43	44.57	131.4	296.75
leftmost	up	enum	20.14	53.57	128	304.75
leftmost	up	bisect	13.43	46.86	131.2	347.75
leftmost	down	step	17.71	51.43	140.6	359.25
leftmost	down	enum	17.86	53.57	128.2	343.75
leftmost	down	bisect	17.71	47	150	320.25
ff	up	step	13.42	60.14	143.8	336
ff	up	enum	26.71	46.86	131.2	301
ff	up	bisect	20	58	122	277.25
ff	down	step	18	49.14	128	289
ff	down	enum	15.71	49.14	128	308.5
ff	down	bisect	15.71	44.57	134.4	308.5
ffc	up	step	11.14	51.43	134.4	308.5
ffc	up	enum	13.29	53.57	131.4	308.5
ffc	up	bisect	11.14	44.57	131.4	296.75
ffc	down	step	9.71	46.86	134.4	285
ffc	down	enum	15.57	58.14	121.8	285.25
ffc	down	bisect	15.57	49	125	293
min	up	step	15.71	44.58	118.8	281.25
min	up	enum	15.57	49.14	137.4	300.75
min	up	bisect	13.43	44.71	131.2	312.5
min	down	step	17.86	44.57	131.4	300.5
min	down	enum	15.71	46.86	143.8	304.5
min	down	bisect	13.43	46.86	125	312.5
max	up	step	13.43	44.58	125	300.75
max	up	enum	13.43	49	131.4	304.5
max	up	bisect	13.29	47	125	304.5
max	down	step	13.43	44.71	134.4	300.75
max	down	enum	17.86	44.58	122	300.75
max	down	bisect	20	42.43	122	289

8.2 Trendlines using to Estimate Execution Time for Bigger Grids

In the following equations,

– x = Grid's Dimensions
– y = Execution Time

Table 2. Trendlines

Labeling Options			Equations
leftmost	up	step	$y = 12.337x^3 - 46.293x^2 + 86.235x - 34.564 = 0$
leftmost	up	enum	$y = 13.549x^3 - 55.836x^2 + 106.09x - 50.379 = 0$
leftmost	up	bisect	$y = 17.001x^3 - 82.55x^2 + 164.36x - 80.95 = 0$
leftmost	down	step	$y = 11.933x^3 - 53.128x^2 + 122.57x - 67.943 = 0$
leftmost	down	enum	$y = -1.0774x^3 + 43.321x^2 + 93.137x + 68.607 = 0$
leftmost	down	bisect	$y = 6.6024x^3 - 12.286x^2 + 21.112x - 2.2714 = 0$
ff	up	step	$y = 10.774x^3 - 42.286x^2 + 84.869x - 39.929 = 0$
ff	up	enum	$y = 9.369x^3 - 33.5x^2 + 68.345x - 28.5 = 0$
ff	up	bisect	$y = 3.8833x^3 + 7.1857x^2 - 19.883x + 24.529 = 0$
ff	down	step	$y = 8.0738x^3 - 27.1x^2 + 65.069x - 34.9 = 0$
ff	down	enum	$y = 10.288x^3 - 42.957x^2 + 97.14x - 51.186 = 0$
ff	down	bisect	$y = 10.22x^3 - 40.821x^2 + 84.351x - 33.607 = 0$
ffc	up	step	$y = -0.3405x^3 + 34.943x^2 - 71.302x + 52.414 = 0$
ffc	up	enum	$y = 7.3119x^3 - 20.457x^2 + 45.76x - 19.186 = 0$
ffc	up	bisect	$y = 9.5357x^3 - 35.071x^2 + 72.179x - 33.357 = 0$
ffc	down	step	$y = 3.5429x^3 + 10.843x^2 - 37.186x + 49.514 = 0$
ffc	down	enum	$y = 3.4012x^3 + 6.9357x^2 - 11.044x + 16.279 = 0$
ffc	down	bisect	$y = 7.6726x^3 - 21.393x^2 + 41.613x - 14.464 = 0$
min	up	step	$y = 3.044x^3 + 10.936x^2 - 22.83x + 22.279 = 0$
min	up	enum	$y = 8.4345x^3 - 25.25x^2 + 43.423x - 8.75 = 0$
min	up	bisect	$y = 3.6929x^3 + 7.9x^2 + 22.836x + 29.1 = 0$
min	down	step	$y = 3.806x^3 + 5.0071x^2 - 10.52x + 15.136 = 0$
min	down	enum	$y = 4.1869x^3 + 1.5786x^2 - 0.6155x + 5.9929 = 0$
min	down	bisect	$y = 8.2381x^3 - 28.143x^2 + 60.19x + 24.714 = 0$
max	up	step	$y = 5.7381x^3 - 10.571x^2 + 22.69x + 0.1429 = 0$
max	up	enum	$y = 5.0476x^3 - 1.7143x^2 - 7.7619x + 24.429 = 0$
max	up	bisect	$y = 13.118x^3 - 68.164x^2 + 155.24x - 84.621 = 0$
max	down	step	$y = 2.1095x^3 + 12.543x^2 - 15.252x + 10.314 = 0$
max	down	enum	$y = 7.1417x^3 - 20.164x^2 + 39.358x - 10.621 = 0$
max	down	bisect	$y = 10.875x^3 - 52.25x^2 + 118.62x - 57.25 = 0$