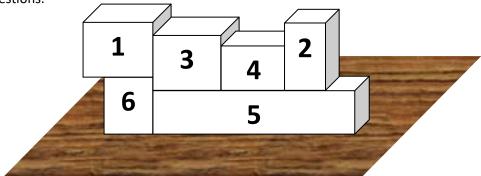


Al-Assignment#4 (CSP) SOLUTION

1) In this question, you will be solving a box-coloring problem. In this domain, there are 6 boxes stacking on each others. Each of the boxes must be colored in one color of three possible colors (Gold, Silver, and Bronze). The boxes must be colored in such a way that

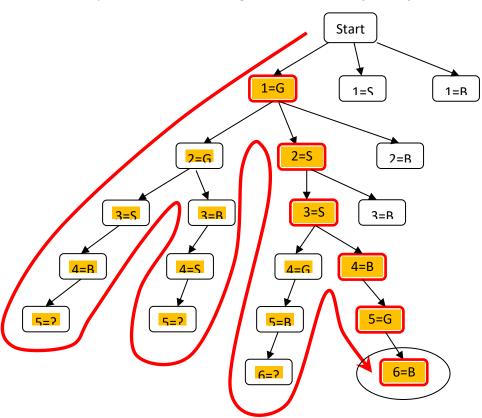
no adjacent boxes are the same color. Two boxes are considered to be adjacent if they touch along an edge. (Two boxes that touch only at a corner are not adjacent.) Here is the picture that you'll be using for all of the questions:



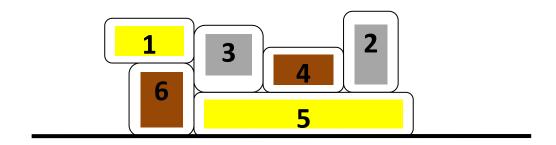
A) Backtracking search

Suppose we decide to use simple backtracking search to find a solution to this constraint satisfaction problem. The variable ordering heuristic we use is simply to instantiate the variables in numbered order. The color value ordering heuristic is to consider the color values in the order Gold, Silver, Bronze. You can use any reasonable shorthand to indicate the instantiations (e.g., "1=G" can mean box 1 is instantiated to the gold color).

(i) Show the complete search tree, circling the solution node, if one is found.



(ii) Show the final coloring, if one is found, on the picture above or on a copy of the picture.



(iii) How many variable instantiations (search steps for both successful and unsuccessful) are tried by this search method?

16 variable instantiations

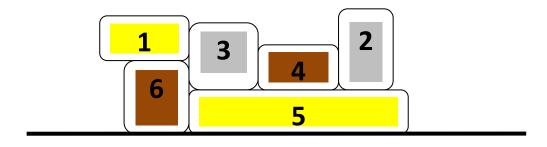
B) Forward checking

Now suppose we use forward checking to eliminate illegal values from the domains of un-instantiated variables. (Recall that in forward checking; only the constraints immediately connected to instantiated variables are checked.) Furthermore, suppose we use a variable ordering heuristic that chooses the variable with the fewest legal instantiations remaining to instantiate next. If more than one such variable exists, the one earlier in the numbered order is selected (for example, choose box 1 before box 3). The same color value ordering heuristic is used as in backtracking search (i.e., consider first gold, then silver, then bronze).

(i) Show the complete search tree for forward checking search. At each node, show the remaining legal values for the un-instantiated variables. For example, at the first node below the root, only box 1 will be colored, so you should indicate the legal values for variables 2, 3, 4, and 5. Continue until your search finds a solution or fails.

Step		Box 1	Box 2	Box 3	Box 4	Box 5	Box 6
Start	Instantiations	GSB	GSB	GSB	GSB	GSB	GSB
1	1=G	G	GSB	SB	GSB	GSB	SB
2	3=S	G	GSB	S	GB	GB	В
3	6=B	G	GSB	S	GB	G	В
4	5=G	G	SB	S	В	G	В
5	4=B	G	S	S	В	G	В
6	2=S	G	S	S	В	G	В

(ii) Show the final coloring, if one is found, on a copy of the picture above.



(iii) How many variable instantiations are tried by this search method?

6 variable instantiations

C) Solution spaces

(i) How large is the search space for this problem? That is, how many different colorings, legal or illegal, are there for the blank picture shown above?

All possible solutions are $3^6 = 729$ cases.

(ii) For this picture, how many different solutions (legal colorings) are there?

You can see that box 1, 3 and 6 are connected to each other, so each box have to get a unique color, therefore there are 6 possible solutions here. Because Box 1 has 3 possible colors in which each case there are 2 possible ways to color box 3 & box 6. Therefore, there are 3x2=6 possible solutions for box 1, 3 and 6.

Then, for each possible solution of box 1, 3 and 6, the color of box 2 have to be the same color as box 3, in which will force box 4 and 5 to have only one possible color for each box.

Finally, there are 3x2x1 = 6 possible solutions.

2) Representing and solving a CSP problem:

Constraints At The Zoo

Last month, five mothers brought their five kids to the zoo. It was a fun time for the kids. Some of the kids dared to touch animal fur and feed the food for some animals. Moreover, every kid found out that each of them had one kind of animals of their own favorite and one kind of hated animals. Surprisingly, all those animals were only five distinctive kinds of animals. In addition, an animal that a kid loved would be the animal that another kid hated, because each kid liked and disliked different kinds of animals. Additionally, in order to reward the kids who well behaved for the whole day, the mothers bought snacks for the kids—one kind for each kid. More clues are listed as follows...

- 1. **Julia** chose **candy**. **Mary** did not choose **apple**. Kid whose last name was **Procter** did not like **monkey**.
- 2. Alan Small's favorite animal was not seal. Beth is the girl who did not choose doughnut.
- 3. No boys chose **doughnut**. One boy chose **corn-chip**. Another boy liked **monkey**. One who did not like **giraffe** chose **popcorn**.
- 4. The kid whose last name was **Brown** liked **lion**. **Beth** liked **seal**. **Mary**'s last name was not **Cook**.
- 5. **Tom**, whose last name was not **Brown** did not choose **apple**. The kid whose last name was **Macgreger** chose **doughnut**.
- 6. The girl who liked **elephant** did not like **seal**. The kid whose last name was **Cook** liked **monkey**. The animal **Alan** hated was not **giraffe**.

A) Define this problem formally as a CSP problem.

Initials Domain

```
First name = {Julia(F), Mary(F), Alan(M), Beth(F), Tom(M)}
```

Set of Variables

First name, Last name, Favorite animal, Hated animal, Snack

A domain of possible value

```
First name = {Julia(F), Mary(F), Alan(M), Beth(F),Tom(M)}
```

Last name = {Procter, Small, Brown, Cook, Macgreger}

Favorite animal = {Monkey, Giraffe, Lion, Seal, Elephant}

Hated animal = {Monkey, Giraffe, Lion, Seal, Elephant}

Snack = {Candy, Caramel apple, Doughnut, Popcorn, Corn chip}

Set of constrains

Alldiff(First name), Alldiff(Last name), Alldiff(Favorite animal), Alldiff(Hated animal), Alldiff(Snack)

Favorite animal ≠ Hated animal

Julia = candy

Mary ≠ caramel apple

Procter ≠ monkey

Alan = Small

Alan ≠ seal

Beth ≠doughnut

Boy ≠ doughnut

One boy = corn chip

Another boy = monkey

One child ≠ giraffe = popcorn

Brown = lion

Beth = seal

Mary ≠ Cook

Tom ≠ Brown

Tom ≠ caramel apple

Macgreger = doughnut

The girl = elephant ≠ seal

Cook = monkey

Alan ≠ giraffe

B) Using the provided information, *your task is to determine each kid's first name and last name, their favorite and hated animals, and snack each kid chose*. Use the MRV heuristic and forward checking. Some of the constraints can be encoded directly in the initial domains. **Show your steps**.

Step 1 Initial

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Procter, Small,				
	Brown, Cook,				
	Macgreger	Macgreger	Macgreger	Macgreger	Macgreger
Favorite animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Hated animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Snack	Candy, Caramel				
	apple, Doughnut,				
	Popcorn, Corn chip				
Sex	Male	Male	Female	Female	Female

Step 2 Julia chose candy.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Procter, Small,				
	Brown, Cook,				
	Macgreger	Macgreger	Macgreger	Macgreger	Macgreger
Favorite animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Hated animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Snack	Candy, Caramel	Candy, Caramel	Candy	Candy, Caramel	Candy, Caramel
	apple, Doughnut,	apple, Doughnut,		apple, Doughnut,	apple, Doughnut,
	Popcorn, Corn chip	Popcorn, Corn chip		Popcorn, Corn chip	Popcorn, Corn chip
Sex	Male	Male	Female	Female	Female

Step 3 Mary did not choose caramel apple.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Procter, Small,				
	Brown, Cook,				
	Macgreger	Macgreger	Macgreger	Macgreger	Macgreger
Favorite animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Hated animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Snack	Caramel apple,	Caramel apple,	Candy	Caramel apple,	Caramel apple,
	Doughnut, Popcorn,	Doughnut, Popcorn,		Doughnut, Popcorn,	Doughnut, Popcorn,
	Corn chip	Corn chip		Corn chip	Corn chip
Sex	Male	Male	Female	Female	Female

Step 4
Alan Small's favorite animal was not seal.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Procter, <u>Small</u> ,	Procter, <u>Small</u> ,	Procter, <u>Small</u> ,	Procter, <u>Small</u> ,
		Brown, Cook,	Brown, Cook,	Brown, Cook,	Brown, Cook,
		Macgreger	Macgreger	Macgreger	Macgreger
Favorite animal	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,
	Lion, <u>Seal</u> , Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Hated animal	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,
	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Snack	Caramel apple,	Caramel apple,	Candy	Doughnut,	Caramel apple,
	Doughnut, Popcorn,	Doughnut, Popcorn,		Popcorn, Corn chip	Doughnut, Popcorn,
	Corn chip	Corn chip			Corn chip
Sex	Male	Male	Female	Female	Female

Step 5
Beth did not choose doughnut.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Procter, Brown,	Procter, Brown,	Procter, Brown,	Procter, Brown,
		Cook, Macgreger	Cook, Macgreger	Cook, Macgreger	Cook, Macgreger
Favorite animal	Monkey, Giraffe,				
	Lion, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Hated animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Snack	Caramel apple,	Caramel apple,	Candy	Doughnut,	Caramel apple,
	Doughnut, Popcorn,	Doughnut, Popcorn,		Popcorn, Corn chip	Doughnut, Popcorn,
	Corn chip	Corn chip			Corn chip
Sex	Male	Male	Female	Female	Female

Step 6 No boys chose doughnut.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Procter, Brown,	Procter, Brown,	Procter, Brown,	Procter, Brown,
		Cook, Macgreger	Cook, Macgreger	Cook, Macgreger	Cook, Macgreger
Favorite animal	Monkey, Giraffe,				
	Lion, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Hated animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Snack	Caramel apple,	Caramel apple,	Candy	Doughnut,	Caramel apple,
	Doughnut, Popcorn,	Doughnut, Popcorn,		Popcorn, Corn chip	Popcorn, Corn chip
	Corn chip	Corn chip			
Sex	Male	Male	Female	Female	Female

Step 7
Mary's last name was not Cook.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Procter, Brown,	Procter, Brown,	Procter, Brown,	Procter, Brown,
		Cook, Macgreger	Cook, Macgreger	Cook, Macgreger	Cook, Macgreger
Favorite animal	Monkey, Giraffe,				
	Lion, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Hated animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Snack	Caramel apple,	Caramel apple,	Candy	Doughnut,	Caramel apple,
	Popcorn, Corn chip	Popcorn, Corn chip		Popcorn, Corn chip	Popcorn, Corn chip
Sex	Male	Male	Female	Female	Female

Step 8
The kid whose last name was Macgreger chose doughnut.
Only Mary has doughnut in table. So Mary's last name is Macgreger.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Procter, Brown,	Procter, Brown,	Procter, Brown,	Procter, Brown,
		Cook, Macgreger	Cook, Macgreger	Macgreger	Cook, Macgreger
Favorite animal	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,
	Lion, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Hated animal	Monkey, Lion, Seal,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,
	Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Snack	Caramel apple,	Popcorn, Corn chip	Candy	Doughnut,	Caramel apple,
	Popcorn, Corn chip			Popcorn, Corn chip	Popcorn, Corn chip
Sex	Male	Male	Female	Female	Female

Step 9 Beth liked seal.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Procter, Brown, Cook	Procter, Brown, Cook	Macgreger	Procter, Brown, Cook
Favorite animal	Monkey, Giraffe, Lion, Elephant	Monkey, Giraffe, Lion, <u>Seal</u> , Elephant	Monkey, Giraffe, Lion, <u>Seal</u> , Elephant	Monkey, Giraffe, Lion, <u>Seal</u> , Elephant	Monkey, Giraffe, Lion, Seal, Elephant
Hated animal	Monkey, Lion, Seal, Elephant	Monkey, Giraffe, Lion, Seal, Elephant	Monkey, Giraffe, Lion, Seal, Elephant	Monkey, Giraffe, Lion, Seal, Elephant	Monkey, Giraffe, Lion, Seal, Elephant
Snack	Caramel apple, Popcorn, Corn chip	Popcorn, Corn chip	Candy	Doughnut	Caramel apple, Popcorn, Corn chip
Sex	Male	Male	Female	Female	Female

Step 10 Tom, whose last name was not Brown did not choose caramel apple.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Procter, <u>Brown</u> ,	Procter, Brown,	Macgreger	Procter, Brown,
		Cook	Cook		Cook
Favorite animal	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Seal
	Lion, Elephant	Lion, Elephant	Lion, Elephant	Lion, Elephant	
Hated animal	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,
	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Snack	Caramel apple,	Caramel apple,	Candy	Doughnut,	Caramel apple,
	Popcorn, Corn chip	Popcorn, Corn chip			Popcorn, Corn chip
Sex	Male	Male	Female	Female	Female

Step 11 The animal Alan hated was not giraffe.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Procter, Cook	Procter, Brown,	Macgreger	Procter, Brown,
			Cook		Cook
Favorite animal	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Seal
	Lion, Elephant	Lion, Elephant	Lion, Elephant	Lion, Elephant	
Hated animal	Monkey, Giraffe,				
	Lion, Seal, Elephant				
Snack	Caramel apple,	Popcorn, Corn chip	Candy	Doughnut,	Caramel apple,
	Popcorn, Corn chip				Popcorn, Corn chip
Sex	Male	Male	Female	Female	Female

Step 12 The kid whose last name was Brown liked lion. (We already know that Beth liked seal.)

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Procter, Cook	<u>Procter</u> , Brown,	Macgreger	Procter <u>, Brown</u> ,
			<u>Cook</u>		Cook
Favorite animal	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Seal
	Lion, Elephant	Lion, Elephant	Lion, <u>Elephant</u>	Lion, Elephant	
Hated animal	Monkey, Lion, Seal,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,
	Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Snack	Caramel apple,	Popcorn, Corn chip	Candy	Doughnut,	Caramel apple,
	Popcorn, Corn chip				Popcorn, Corn chip
Sex	Male	Male	Female Female Female		Female

Step 13
The kid whose last name was Cook liked monkey. (We already know that Beth liked seal.)

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	<u>Procter</u> , Cook	Brown	Macgreger	Procter, <u>Cook</u>
Favorite animal	Monkey, Giraffe,	Monkey, Giraffe,	Lion	Monkey, Giraffe,	Seal
	Elephant	<u>Elephant</u>		Elephant	
Hated animal	Monkey, Lion, Seal,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,
	Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant
Snack	Caramel apple,	Popcorn, Corn chip	Candy	Doughnut,	Caramel apple,
	Popcorn, Corn chip				Popcorn, Corn chip
Sex	Male Male Female Female		Female	Female	

Step 14
One boy chose corn chip. Another boy liked monkey. (We already know that Tom liked monkey.)

	Person 1	Person 2	Person 3 Person 4		Person 5	
First name	Alan	Tom	Julia	Mary	Beth	
Last name	Small	Cook	Brown	Macgreger	Procter	
Favorite animal	Giraffe, Elephant	Monkey	Lion	Giraffe, Elephant	Seal	
Hated animal	Monkey, Lion, Seal,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	
	Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	
Snack	Caramel apple,	Popcorn, Corn chip	Candy Doughnut,		Caramel apple,	
	Popcorn, Corn chip				Popcorn, Corn chip	
Sex	Male	Male	Female	Female	Female	

Step 15 Procter did not like monkey.

	Person 1	Person 2	Person 3	Person 4	Person 5	
First name	Alan	Tom	Julia	Mary	Beth	
Last name	Small	Cook	Brown	Macgreger	Procter	
Favorite animal	Giraffe, Elephant	Monkey	Lion	Giraffe, Elephant	Seal	
Hated animal	Monkey, Lion, Seal,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	Monkey, Giraffe,	
	Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	Lion, Seal, Elephant	
Snack	Corn chip	Popcorn	Candy	Doughnut,	Caramel apple	
Sex	Male	Male	Female	Female	Female	

Step 16
One who did not like giraffe chose popcorn.

	Person 1	Person 2	Person 3	Person 4	Person 5	
First name	Alan	Tom	Julia	Mary	Beth	
Last name	Small	Cook	Brown	Macgreger	Procter	
Favorite animal	Giraffe, Elephant	Monkey	Lion	Giraffe, Elephant	Seal	
Hated animal	Lion, Seal, Elephant	Giraffe, Lion, Seal,	Giraffe, Lion, Seal,	Giraffe, Lion, Seal,	Monkey	
		Elephant	Elephant	Elephant		
Snack	Corn chip	Popcorn	Candy	Doughnut,	Caramel apple	
Sex	Male	Male	Female	Female	Female	

Step 17 Favorite animal must be different with Hated animal.

	Person 1	Person 2	Person 3	Person 4	Person 5
First name	Alan	Tom	Julia	Mary	Beth
Last name	Small	Cook	Brown	Macgreger	Procter
Favorite animal	Giraffe, Elephant	Monkey	Lion	Giraffe, Elephant	Seal
Hated animal	Lion, Seal, Elephant	Giraffe	Lion, Seal, Elephant	Lion, Seal, Elephant	Monkey
Snack	Corn chip	Popcorn	Candy Doughnut,		Caramel apple
Sex	Male	Male	Female	Female Female	

Step 18
The girl who liked elephant did not like seal.

	Person 1	Person 2	Person 3	Person 4	Person 5	
First name	Alan	Tom	Julia	Mary	Beth	
Last name	Small	Cook	Brown	Brown Macgreger I		
Favorite animal	Giraffe, Elephant	Monkey	Lion	Giraffe, Elephant	Seal	
Hated animal	Lion, <u>Seal</u> , <u>Elephant</u>	Giraffe	Seal, Elephant	Lion, Seal, Elephant	Monkey	
Snack	Corn chip	Popcorn	Candy	Doughnut, Caramel apple		
Sex	Male	Male	Female	Female	Female	

Step 19 Solution

	Person 1	Person 2	Person 3 Person 4		Person 5	
First name	Alan	Tom	Julia	Mary	Beth	
Last name	Small	Cook	Brown	Macgreger	Procter	
Favorite animal	Giraffe	Monkey	Lion	Elephant	Seal	
Hated animal	Lion	Giraffe	Elephant	Seal	Monkey	
Snack	Corn chip	Popcorn	Candy	Doughnut,	Caramel apple	
Sex	Male	Male	Female	Female	Female	

3) Consider the game of Sudoku 4x4 below:

	2	
4		
	4	
3		

The objective of this game is to fill a 4×4 grid with digits so that each column, each row, and each of the four 2×2 sub-grids that compose a big box containing all of the digits from 1 to 4. The puzzle initially provides a partially completed grid, which typically has a unique solution.

Find a possible solution. Please use any technique of CSP to solve it. Be sure to define this problem formally as a CSP problem and show all your works.

Define this problem formally as a CSP problem.

Set of Variables

DigitAtPos[1...4, 1...4]

A domain of possible value

 $\{1, 2, 3, 4\}$

Set of constrains

Alldiff(DigitAtPos[1,1], DigitAtPos[1,2], DigitAtPos[1,3], DigitAtPos[1,4])
Alldiff(DigitAtPos[2,1], DigitAtPos[2,2], DigitAtPos[2,3], DigitAtPos[2,4])
Alldiff(DigitAtPos[3,1], DigitAtPos[3,2], DigitAtPos[3,3], DigitAtPos[3,4])
Alldiff(DigitAtPos[4,1], DigitAtPos[4,2], DigitAtPos[4,3], DigitAtPos[4,4])
Alldiff(DigitAtPos[1,1], DigitAtPos[2,1], DigitAtPos[3,1], DigitAtPos[4,1])
Alldiff(DigitAtPos[1,2], DigitAtPos[2,2], DigitAtPos[3,2], DigitAtPos[4,2])
Alldiff(DigitAtPos[1,3], DigitAtPos[2,3], DigitAtPos[3,3], DigitAtPos[4,3])
Alldiff(DigitAtPos[1,4], DigitAtPos[2,4], DigitAtPos[2,1], DigitAtPos[2,2])
Alldiff(DigitAtPos[1,3], DigitAtPos[1,4], DigitAtPos[2,3], DigitAtPos[2,4])
Alldiff(DigitAtPos[3,1], DigitAtPos[3,2], DigitAtPos[4,1], DigitAtPos[4,2])
Alldiff(DigitAtPos[3,3], DigitAtPos[3,4], DigitAtPos[4,3], DigitAtPos[4,4])

 $\mathsf{DigitAtPos}[3,\,1] = \{2\}$

 $\mathsf{DigitAtPos}[2,\,2] = \{4\}$

 $DigitAtPos[3, 3] = \{4\}$

DigitAtPos[2, 4] = {3}

Use the MRV heuristic and forward checking

	D[1,1]	D[2,1]	D[3,1]	D[4,1]	D[1,2]	D[2,2]	D[3,2]	D[4,2]	D[1,3]	D[2,3]	D[3,3]	D[4,3]	D[1,4]	D[2,4]	D[3,4]	D[4,4]
1	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4
2	1,3,4	1,3,4	2	1,3,4	1,2,3,4	1,2,3,4	1,3,4	1,3,4	1,2,3,4	1,2,3,4	1,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,3,4	1,2,3,4
3	1,3	1,3	2	1,3,4	1,2,3	4	1,3	1,3	1,2,3,4	1,2,3	1,3,4	1,2,3,4	1,2,3,4	1,2,3	1,3,4	1,2,3,4
4	1,3	1,3	2	1,3,4	1,2,3	4	1,3	1,3	1,2,3	1,2,3	4	1,2,3	1,2,3,4	1,2,3	1,3	1,2,3
5	1,3	1	2	1,3,4	1,2,3	4	1,3	1,3	1,2,3	1,2	4	1,2,3	1,2,4	3	1	1,2
6	3	1	2	3,4	2,3	4	1,3	1,3	1,2,3	2	4	1,2,3	1,2,4	3	1	1,2
7	3	1	2	4	2	4	1,3	1,3	1,2	2	4	1,2,3	1,2,4	3	1	1,2
8	3	1	2	4	2	4	1,3	1,3	1,2	2	4	1,2,3	1,2,4	3	1	1,2
9	3	1	2	4	2	4	1,3	1,3	1	2	4	1,2,3	1,4	3	1	1,2
10	3	1	2	4	2	4	1,3	1,3	1	2	4	2,3	4	3	1	1,2
11	3	1	2	4	2	4	1,3	1,3	1	2	4	3	4	3	1	1,2
12	3	1	2	4	2	4	1,3	1	1	2	4	3	4	3	1	1,2
13	3	1	2	4	2	4	3	1	1	2	4	3	4	3	1	2
14	3	1	2	4	2	4	3	1	1	2	4	3	4	3	1	2
15	3	1	2	4	2	4	3	1	1	2	4	3	4	3	1	2
16	3	1	2	4	2	4	3	1	1	2	4	3	4	3	1	2
17	3	1	2	4	2	4	3	1	1	2	4	3	4	3	1	2
	3	1	2	4	2	4	3	1	1	2	4	3	4	3	1	2

Solution is ...

3	1	2	4
2	4	3	1
1	2	4	3
4	3	1	2