Question 1(A) – Nim211

Game Setup: Three piles (2, 1, 1). The last player to remove a stick loses.

Initial State: [2 1 1 A] (A goes first).

Game Tree/Minimax:

- Terminal states labeled +1 if B took the last stick (A wins), or −1 if A took the last stick (B wins).
- Backing up the values shows the initial position is winning for A (value +1).

Alpha-Beta:

- In a left-to-right depth-first search, after exploring the first two branches, we prune the rest when $\alpha \geq \beta$.

Question 1(B) – Nim6

Game Setup: One pile of 6 sticks, remove 1 or 2 sticks each turn, last player to remove a stick loses.

Analysis (winning/losing states):

- n=0 → opponent just took last stick → you win.
- n=1 → forced to take last stick → you lose.
- n=2 → remove 1 (avoid removing last) → win.
- n=3 → remove 2 → leave 1 → opponent loses → win.
- n=4 → whichever you remove, you leave n=3 or n=2 for opponent (both winning for them) → lose.
- n=5 → remove 1 → leave 4 → opponent loses → win.
- n=6 → remove 2 → leave 4 → opponent loses → win.

Conclusion: With 6 sticks, the first player wins by removing 2 sticks initially.

Question 2

Minimax:

- Label leaf utilities (from max's perspective).
- At each min node, take the minimum of children; at each max node, take the maximum of children.
- The root's value is the final minimax value.
- The optimal move for the first player (max) is whichever child leads to that root value.

Alpha-Beta (Left-to-Right):

- Update α, β as you go depth-first.
- Prune subtrees when $\alpha \geq \beta$.

Alpha-Beta (Right-to-Left):

- Same process, but explore children in reverse order.

- Prunings can differ, but the final minimax value is the same.

Question 3

Non-Zero-Sum Game & Nash Equilibrium:

- A Nash equilibrium is a strategy profile where no player can unilaterally improve their payoff by switching strategies.
- Check each possible outcome for unilateral deviations.
- If no pure-strategy NE exists, solve for mixed strategies by making each player indifferent.