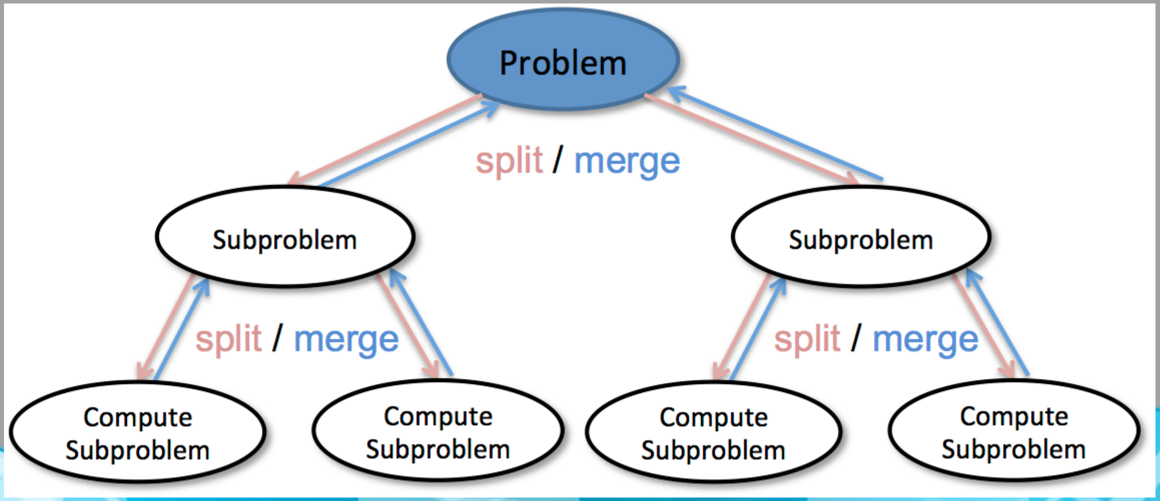
DATA STUCTURE

ALGORITHMS

DIVIDE-AND-CONQUER

- Divide: If the input size is smaller than a certain threshold (say, one or two elements), solve the problem directly using a straightforward method and return the solution so obtained. Otherwise, divide the input data into two or more disjoint subsets.
- Conquer: Recursively solve the sub problems associated with the subsets.
- Combine: Take the solutions to the sub problems and merge them into a solution to the original problem



SORTING ALGORITHMS

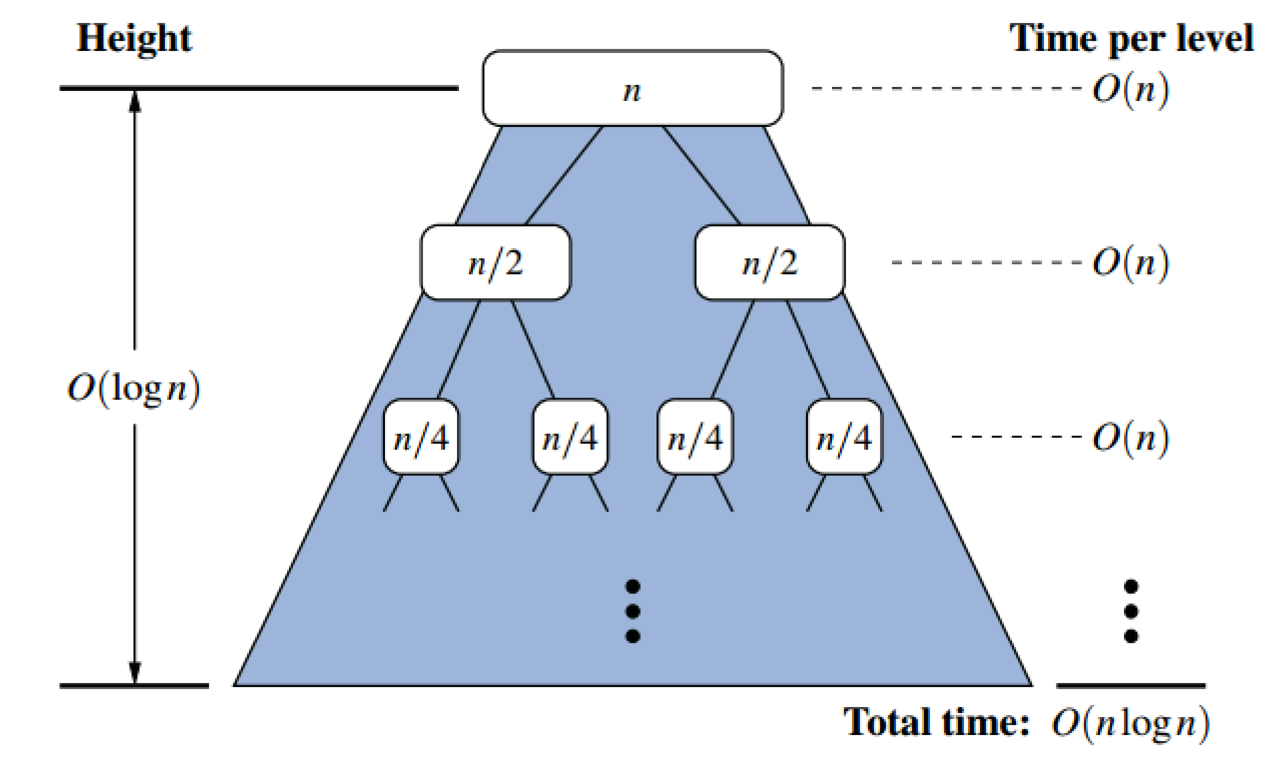
MERGE SORT

DEFINITION

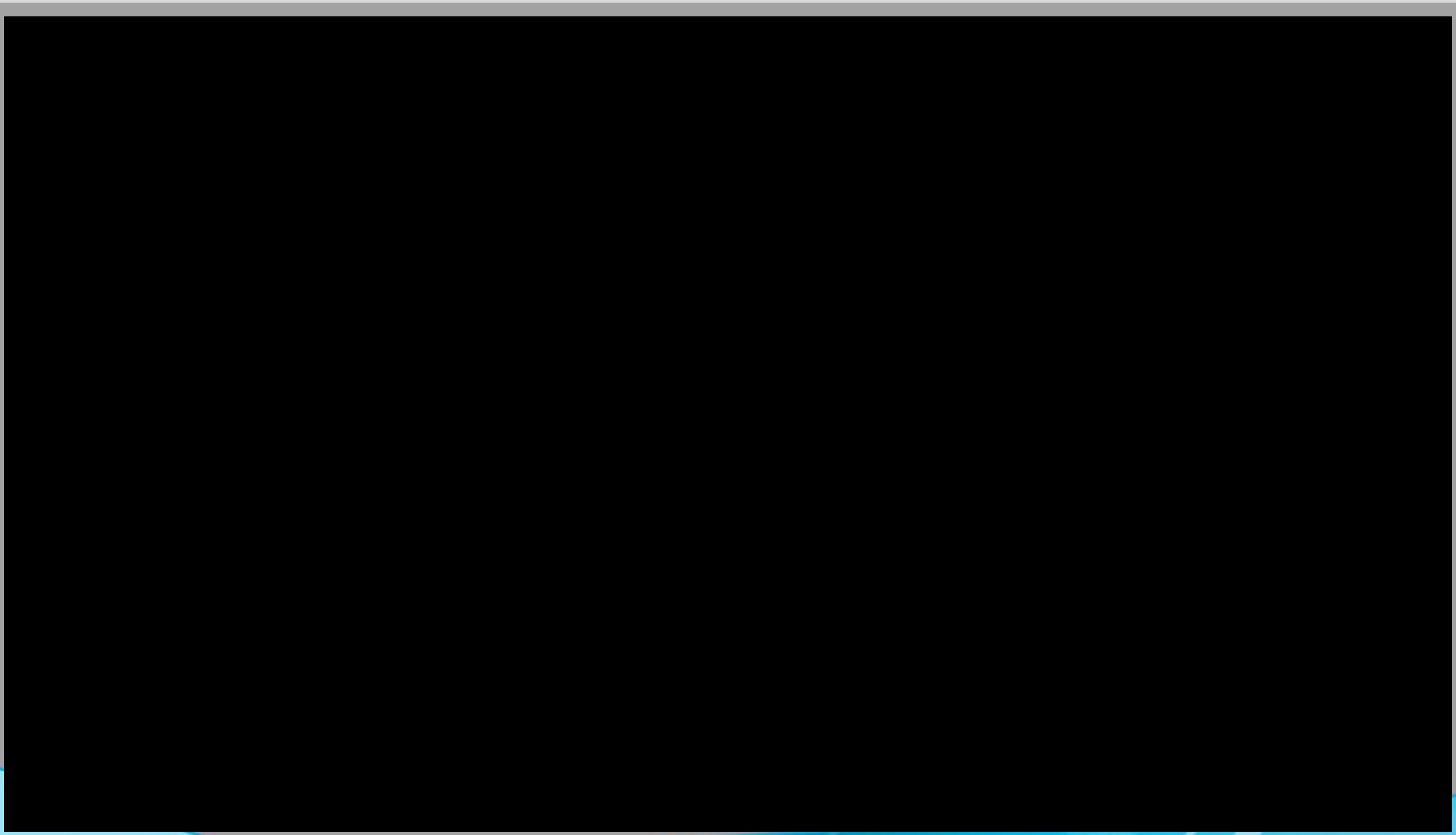
- **Divide** : First divides the n element sequence into two sub sequences having size n / 2 elements each.
- **Conquer** : Then sort the each sub sequences recursively using merge sort.
- **Combine** : Then merge the two sub sequences which are sorted to produce the sorted answer

RUNNING TIME

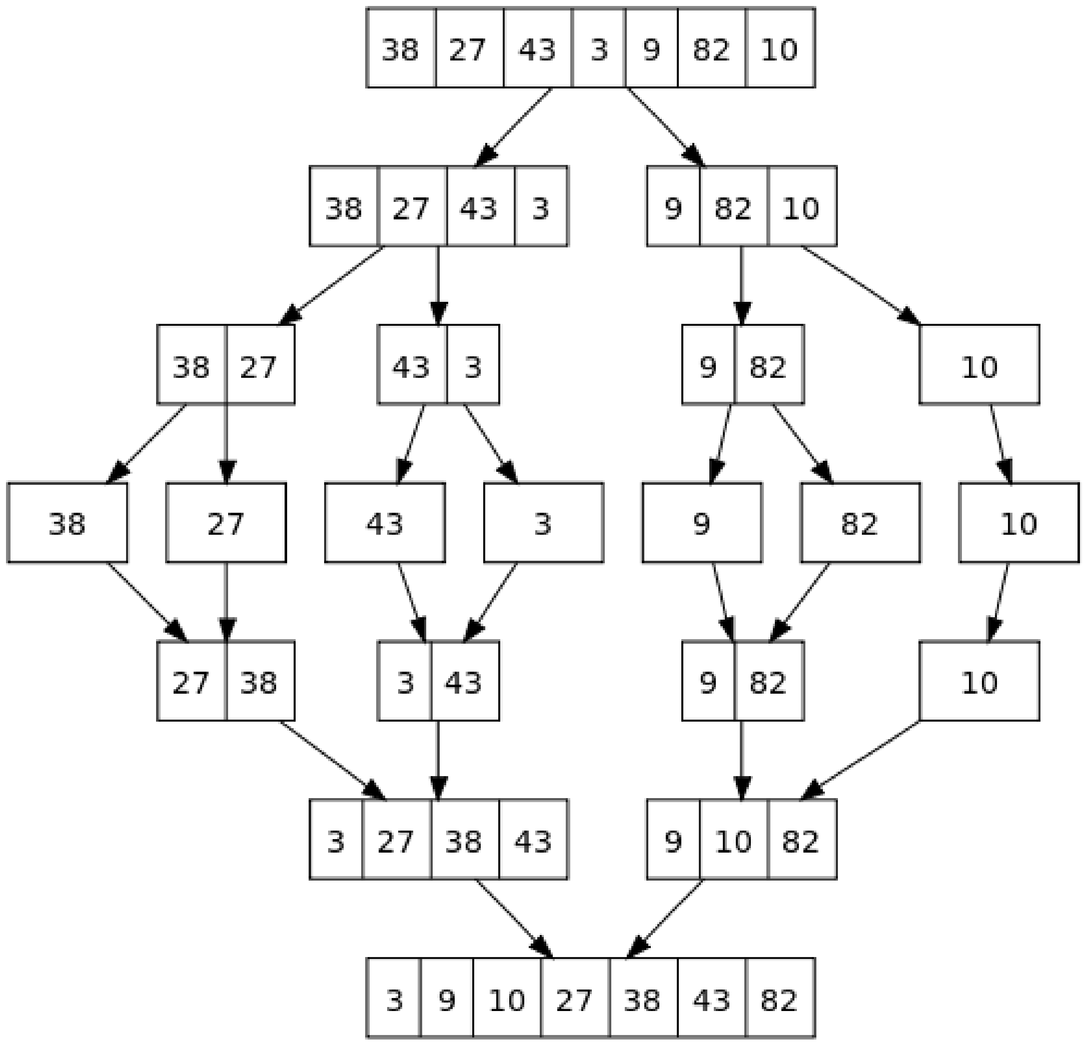
- The run time is: O(n log n).
- Reason: This algorithm splits the items to be sorted into 2 groups, recursively sorts each group, and merges them into a final sorted array. The Run time is O(n log n).



Video



MERGE SORT EXAMPLE



PSEUDO-CODE

```
function mergesort(m)
    var list left, right, result
    if length(m) ≤ 1
    return m
    else
    var middle = length(m) / 2
    for each x in m up to middle - 1
        add x to left
    for each x in m at and after middle
        add x to right
    left = mergesort(left)
    right = mergesort(right)
    if last(left) ≤ first(right)
    append right to left
    return left
    result = merge(left, right)
return result
```

```
function merge(left,right)
    var list result
    while length(left) > 0 and length(right) > 0
        if first(left) ≤ first(right)
            append first(left) to result
            left = rest(left)
        else
            append first(right) to result
            right = rest(right)
        if length(left) > 0
            append rest(left) to result
        if length(right) > 0
    append rest(right) to result
return result
```

SORTING ALGORITHMS

QUICK SORT

DEFINITION

- **Divide**: If S has at least two elements (nothing needs to be done if S has zero or one element), select a specific element x from S, which is called the pivot. As is common practice, choose the pivot x to be the last element in S.

  Remove all the elements from S and put them into three sequences:
  - L, storing the elements in S less than x
  - E, storing the elements in S equal to x
  - G, storing the elements in S greater than x

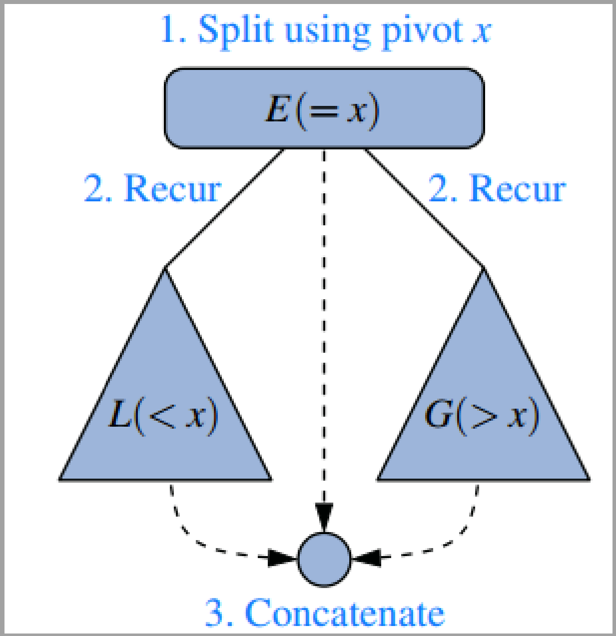
Of course, if the elements of S are distinct, then E holds just one element—the pivot itself.

- **Conquer**: Recursively sort sequences L and G.
- **Combine** : Put back the elements into S in order by first inserting the elements of L, then those of E, and finally those of G.

PSEUDO-CODE

```
function quicksort(array)
    var list less, equal, greater
    if length(array) ≤ 1
    return array

    select a pivot value pivot from array
    for each x in array
    if x < pivot then append x to less
    if x = pivot then append x to equal
    if x > pivot then append x to greater
    return concatenate(quicksort(less), equal, quicksort(greater))
```



Video

HOW TO PICK PIVOT

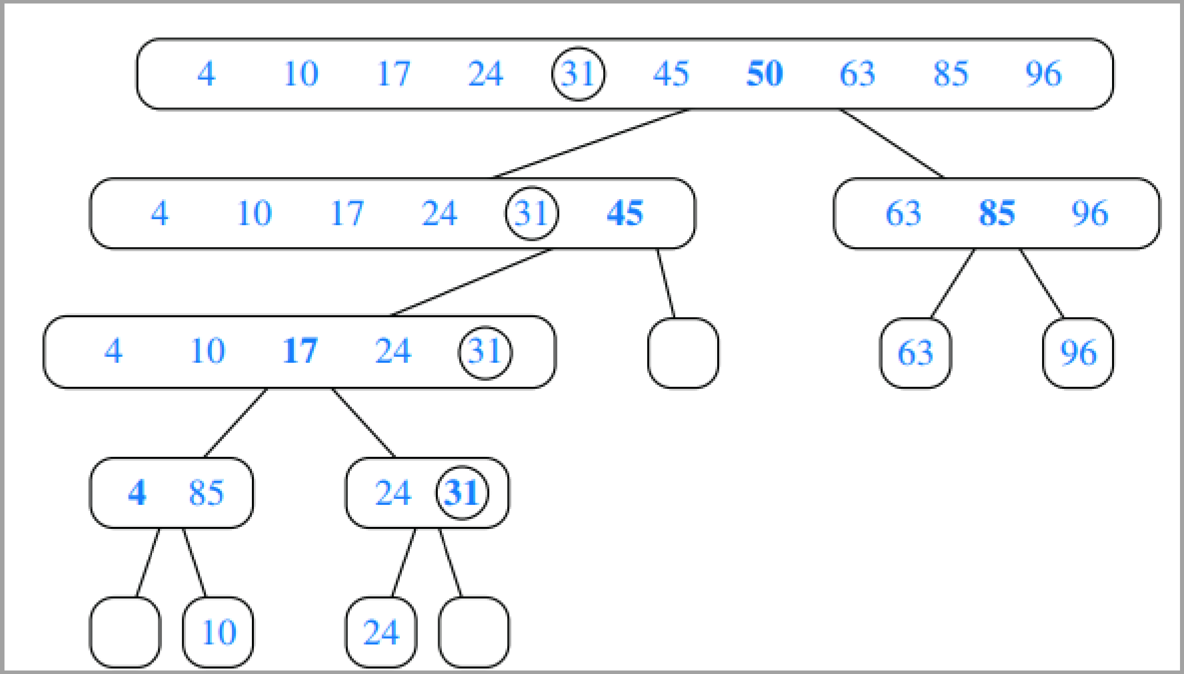
- Picking the first element as pivot
- Picking Pivots at Random
- Picking Median-of-three As pivot
- Using partition algorithm

PICK THE FIRST ELEMENT AS PIVOT

Θ(n2)-time worst case, most notably when the original sequence is already sorted, reverse sorted, or nearly sorted

PICKING PIVOTS AT RANDOM

The expected running time of randomized quick-sort on a sequence S of size n is O(n logn).



PICKING MEDIAN-OF-THREE AS PIVOT

- This median-of-three heuristic will more often choose a good pivot and computing a median of three may require lower overhead than selecting a pivot with a random number generator.
- For larger data sets, the median of more than three potential pivots might be computed.

Example: Median-of-three Partitioning

- Let input `S = {6, 1, 4, 9, 0, 3, 5, 2, 7, 8}`
- `left=0` and `S[left] = 6`
- `right=9` and `S[right] = 8`
- `center = (left+right)/2 = 4` and `S[center] = 0`
- Pivot
  - = Median of `S[left]`, `S[right]`, and `S[center]`
  - = median of 6, 8, and 0
  - `= S[left] = 6`

PARTITION ALGORITHM

Original input : `S = {6, 1, 4, 9, 0, 3, 5, 2, 7, 8}`

Get the pivot out of the way by swapping it with the last element

8 1 4 9 0 3 5 2 7 $\boxed{6}$

pivot

Have two 'iterators' – `i` and `j`

- `i` starts at first element and moves forward
- `j` starts at last element and moves backwards

$\boxed{8}$ 1 4 9 0 3 5 2 $\boxed{7}$ 6

i j pivot

PARTITION ALGORITHM

`While (i < j)`

- Move `i` to the right till we find a number greater than `pivot`
- Move `j` to the left till we find a number smaller than `pivot`
- `If (i < j) swap(S[i], S[j])`
- (The effect is to push larger elements to the right and smaller elements to the left)

Swap the `pivot` with `S[i]`

QUICK SORT RECURSIVE

```
function quicksort(array, 'left', 'right')
// If the list has 2 or more items
if 'left' < 'right'
// See "Choice of pivot" section below for possible choices
choose any 'pivotIndex' such that 'left' ≤ 'pivotIndex' ≤ 'right'
// Get lists of bigger and smaller items and final position of pivot
'pivotNewIndex':= partition(array, 'left', 'right', 'pivotIndex')
// Recursively sort elements smaller than the pivot
quicksort(array, 'left', 'pivotNewIndex' - 1)
// Recursively sort elements at least as big as the pivot
quicksort(array, 'pivotNewIndex' + 1, 'right')
```

QUICK SORT NON_RECURSIVE

```
Procedure QuickSort(a[1..n]) {
    Var list S, E; Int m:=1
    S(m):=1; E(m):= n;
    While m>0 {
        k=S(m); l=E(m)
        m:=m-1;
        if l<k then {
                i=Part(k,l);
                m=m+1;
        S(m):=i+1
        E(m):=l
} }
}
```