Local Search Algorithms

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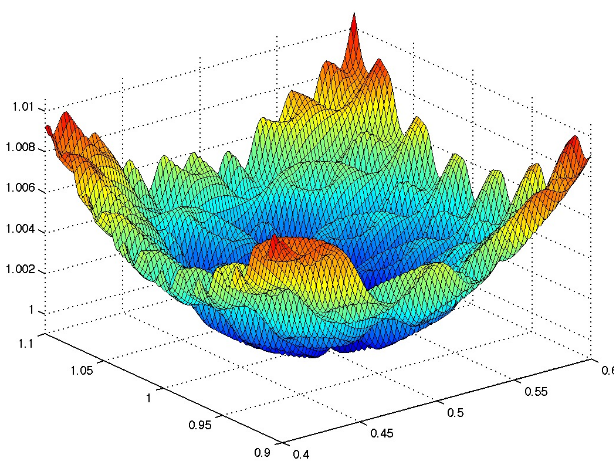
Local search algorithms

- Some types of search problems can be formulated in terms of **optimization**
  – We don't have a start state, don't care about the path to a solution
  – We have an **objective function** that tells us about the quality of a possible solution, and we want to find a good solution by minimizing or maximizing the value of this function



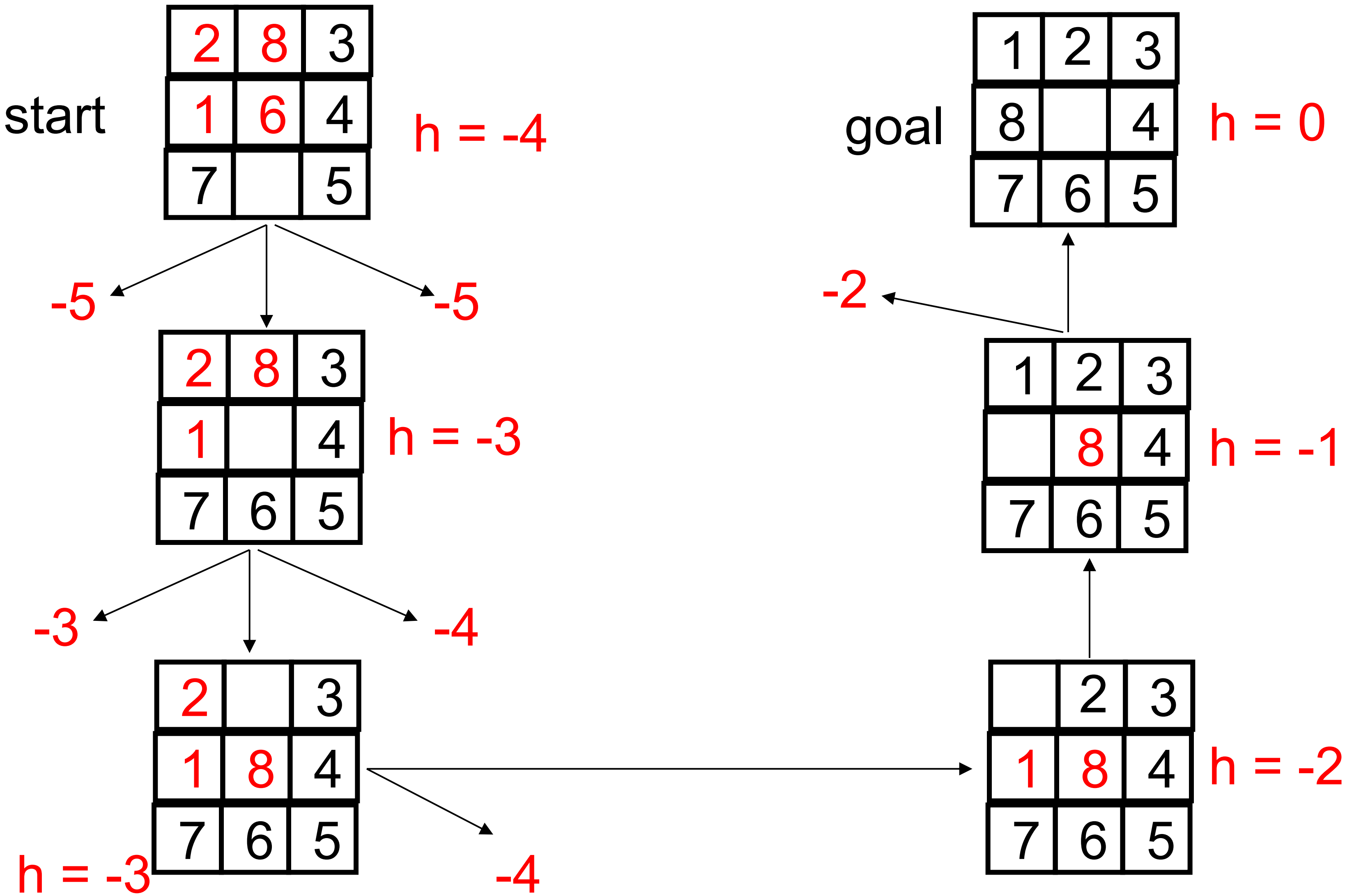
Approaches

- Hill Climbing
- Simulated Annealing
- Genetic Algorithms

Hill-climbing (greedy) search

- Idea: keep a single “current” state and try to locally improve it
- “Like climbing mount Everest in thick fog with amnesia”

Hill Climbing Example



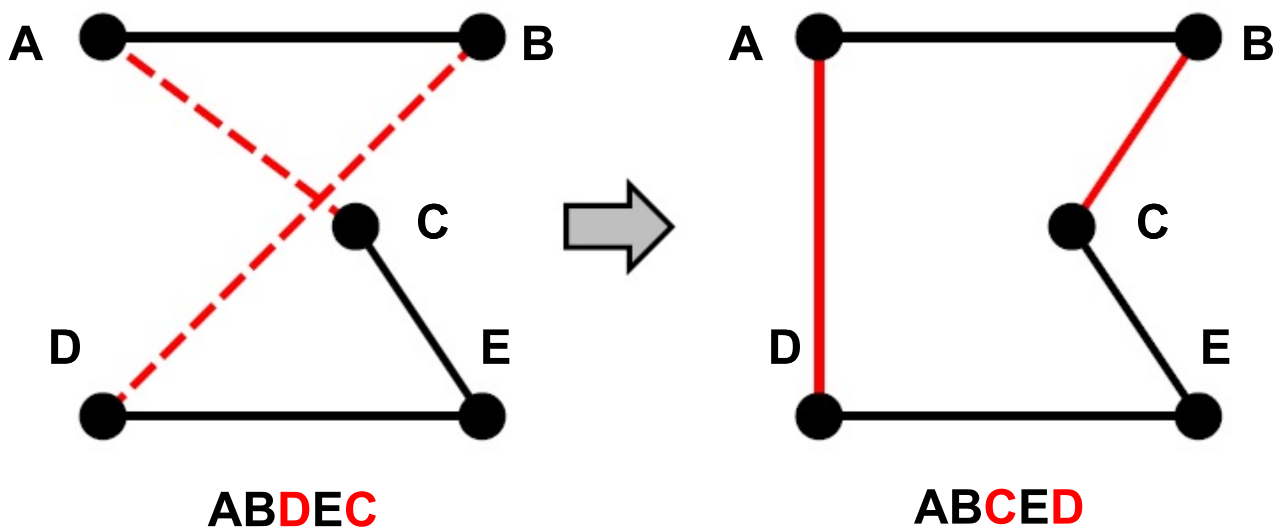
f(n) = -(number of tiles out of place)

Example: Traveling salesman problem

- Find the shortest tour connecting a given set of cities
- **State space:** all possible tours
- **Objective function:** length of tour

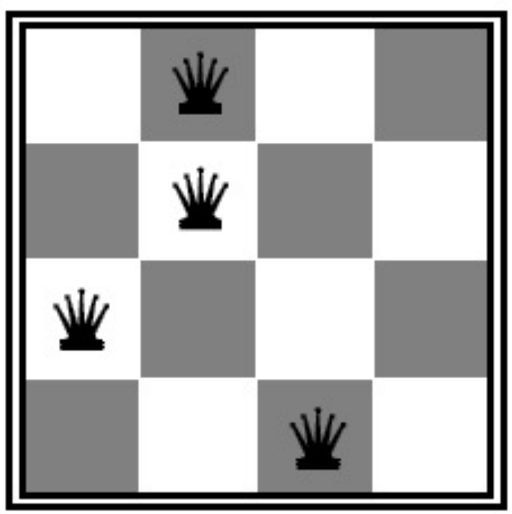
Example: Traveling Salesman Problem

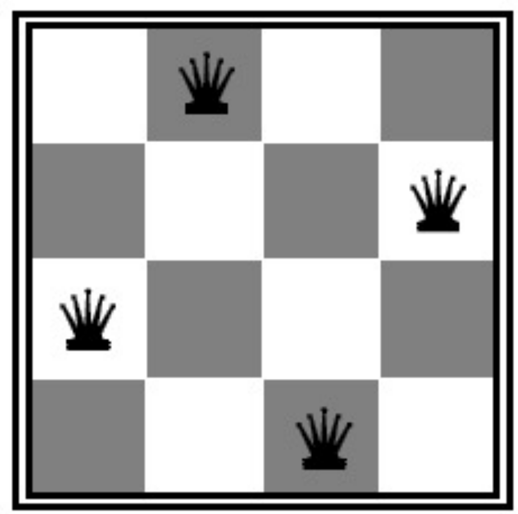
- Find the shortest tour connecting n cities
- **State space:** all possible tours
- **Objective function:** length of tour
- What's a possible local improvement strategy?
  - Start with any complete tour, perform pairwise exchanges



Example: *n*-queens problem

- Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal
- **State space:** all possible *n*-queen configurations
- What's the **objective function**?
  - Number of pairwise conflicts





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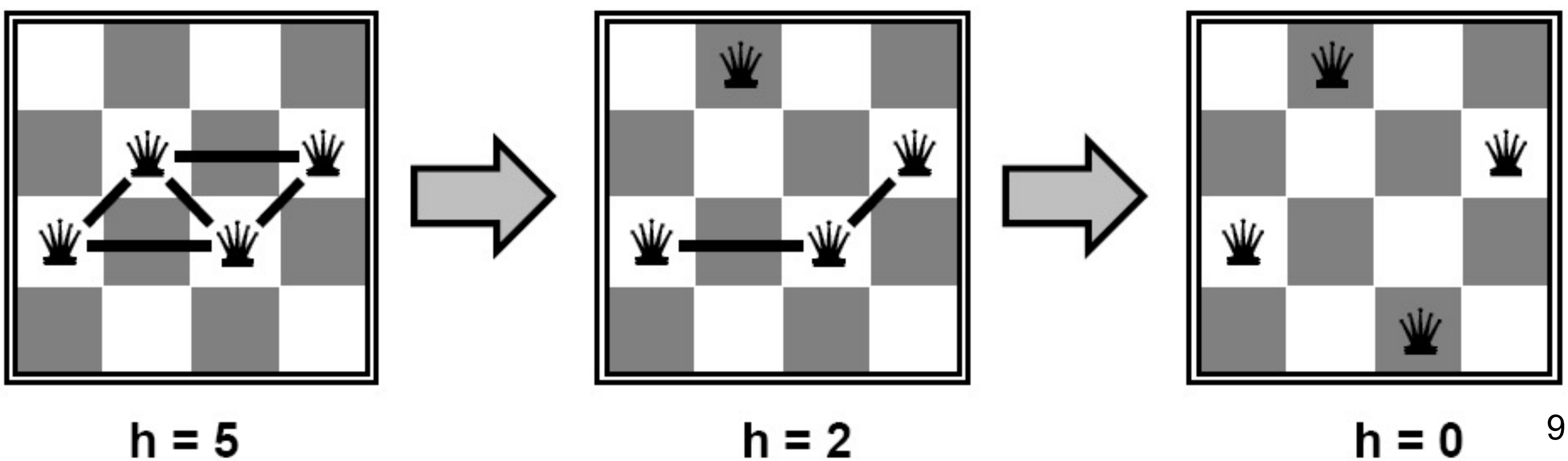
- What's a possible local improvement strategy?
  - Move one queen within its column to reduce conflicts



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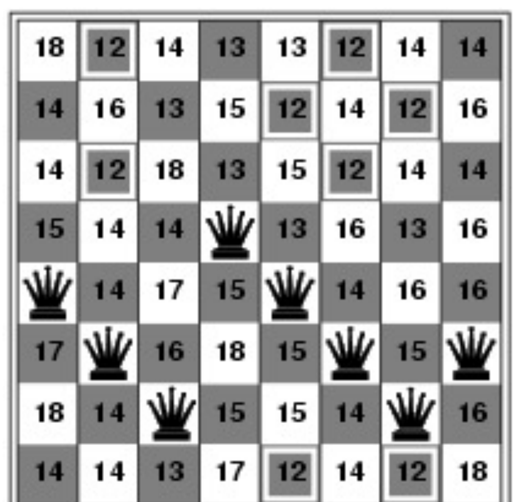
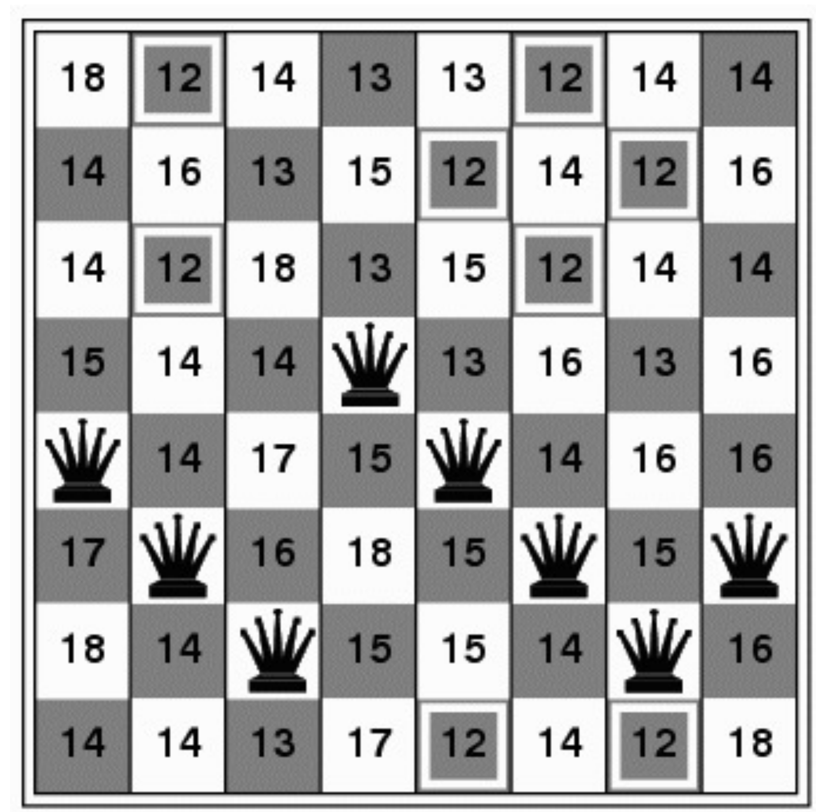


h = 17

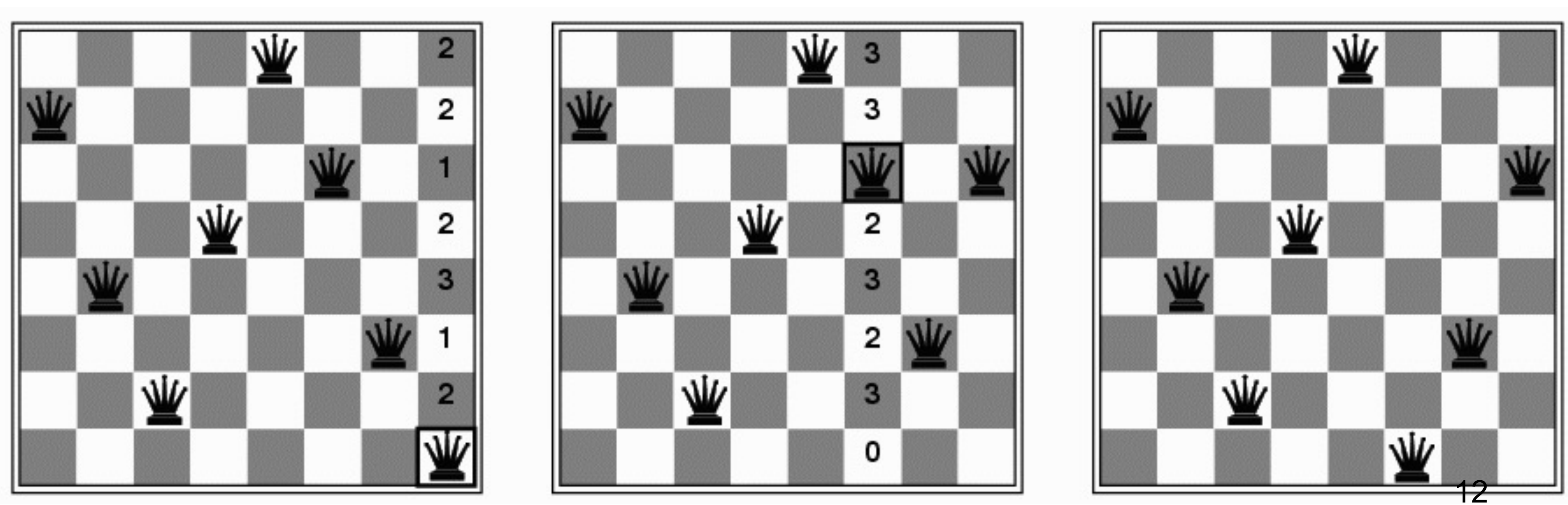
Example (hill-climbing)

- The figure at right has a heuristic cost estimate value of h=17, the number of conflicting pairs of queens.
- The figure gives the estimate of each successor state.
- The successor function returns all possible 8*7=56 states reachable after a given move in a column.



Example Solution

- Queen in lower right of first figure in conflict with 2 others by moving up one row. Moving to a row with 1 conflict would be a local minima.
- In second figure, queen is moved to local minima 0 which turns out to be a global minimum.
- Last figure has total number of conflicts h=0, a global minimum.



Hill-climbing (greedy) search

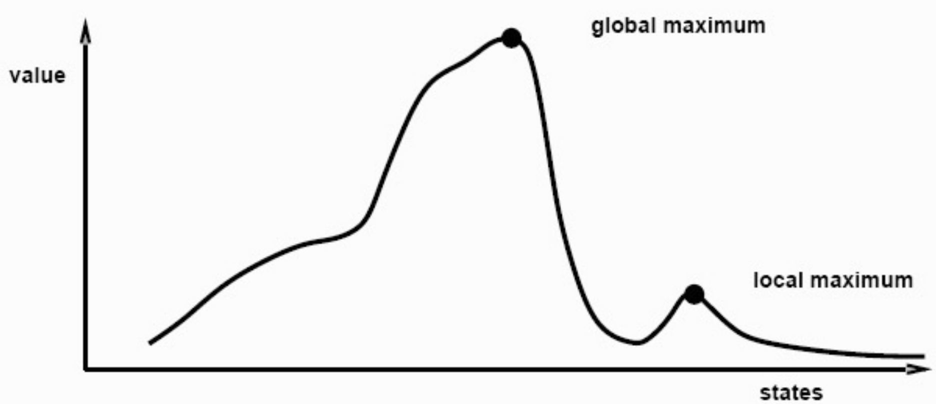
- Initialize *current* to starting state
- Loop:
  - Let *next* = highest-valued successor of *current*
  - If value(*next*) < value(*current*) return *current*
  - Else let *current* = *next*

- Variants: choose first better successor, randomly choose among better successors

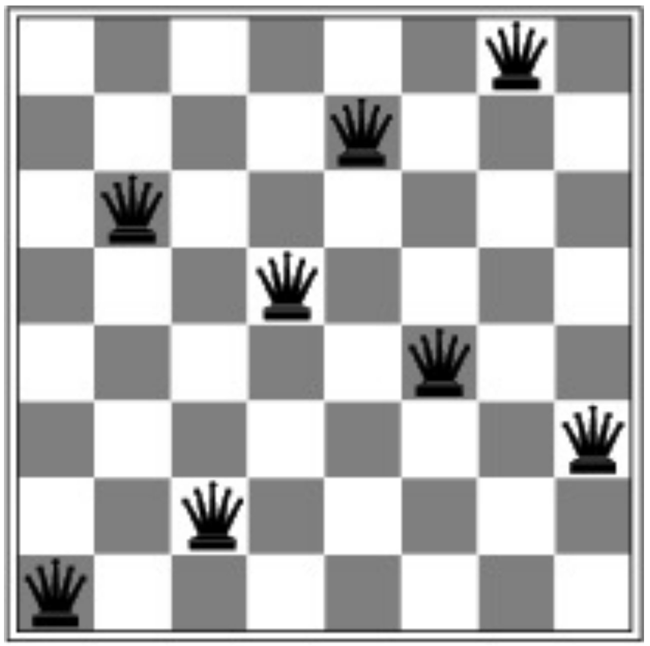
Hill-climbing search

- Is it complete/optimal?
  - No – can get stuck in local optima (peak higher than neighbors but lower than global maximum)
  - Example: local optimum for the 8-queens problem

Problem: depending on initial state, can get stuck on local maxima

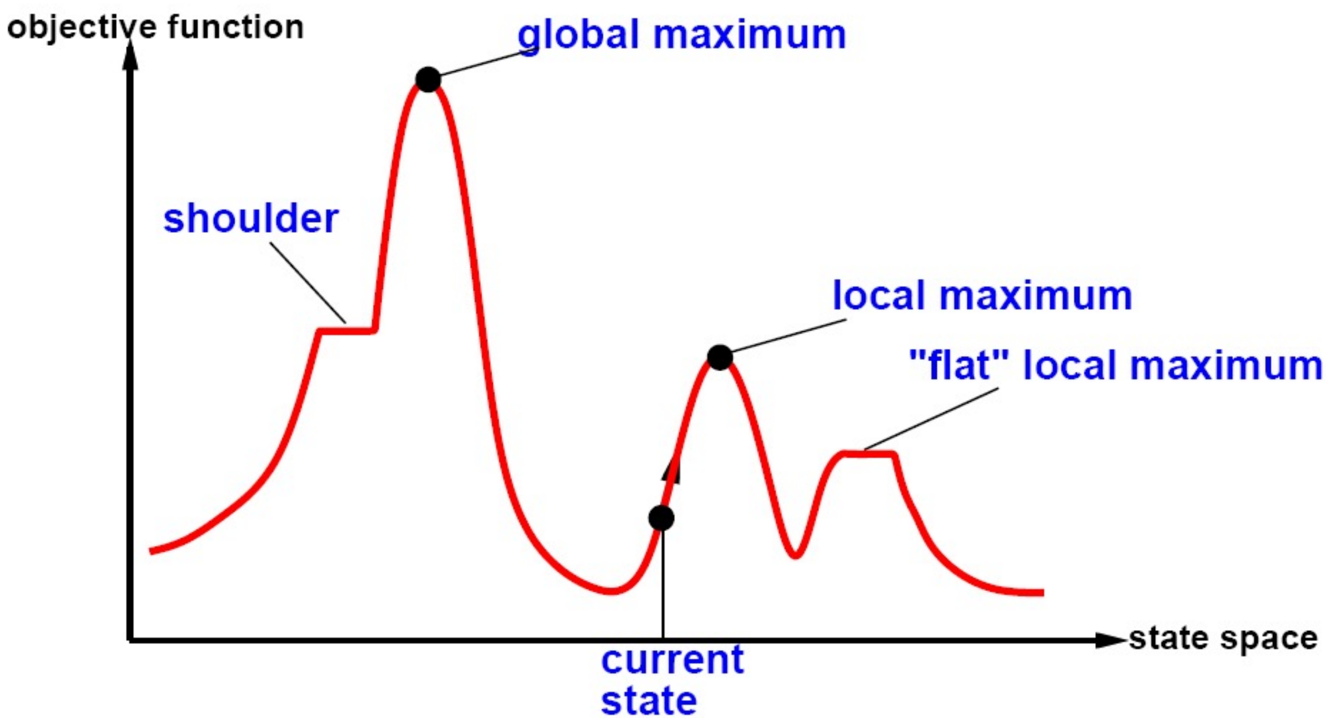


In continuous spaces, problems w/ choosing step size, slow convergence



$h = 1$

The state space “landscape”



- How to escape local maxima?
  – Random restart hill-climbing

    iteratively does hill-climbing, each time with a random initial condition . The best is kept: if a new run of hill climbing produces a better than the stored state, it replaces the stored state.

Simulated annealing search

- Hill-climbing algorithm never makes downhill move - always incomplete
- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency
  - Probability of taking downhill move decreases with number of iterations, steepness of downhill move
  - Controlled by *annealing schedule*
- Inspired by annealing process
  - used to harden metals and glass by heating them to a high temperature and then gradually cooling them, thus allowing the material to reach a low-energy crystalline state

Simulated annealing search

- Initialize *current* to starting state
- for $i = 1$ to ∞

  let *next* = random successor of *current*

  let Δ = value(*next*) – value(*current*)

  if $\Delta > 0$ then let *current* = *next*

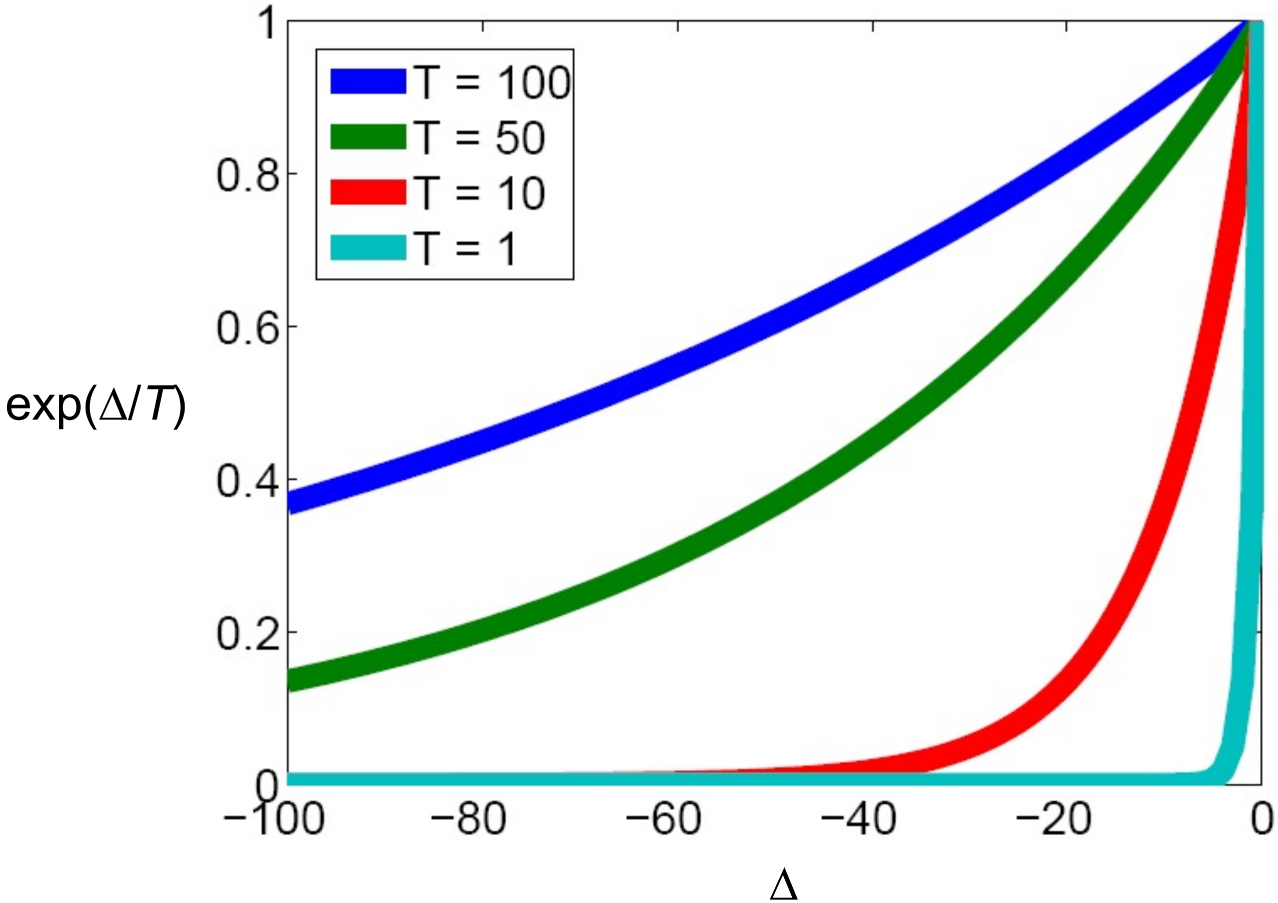
  else let *current* = *next* with probability $\exp(\Delta/T(i))$
- T gradually decreased to 0 over time t.

Picks random rather than best state move as in hill-climbing.

If move is improvement over current state, always accepted, otherwise accepted with probability < 1.

Probability decreases exponentially when move is worse than current and as T decreases over time; bad moves more likely to be accepted early.

Effect of temperature



Simulated annealing search

- One can prove: If temperature decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching one
- However:
  - This usually takes impractically long
  - The more downhill steps you need to escape a local optimum, the less likely you are to make all of them in a row
- More modern techniques: general family of *Markov Chain Monte Carlo* (MCMC) algorithms for exploring complicated state spaces

Genetic Algorithms - History

- Pioneered by John Holland in the 1970's
- Got popular in the late 1980's
- Based on ideas from Darwinian Evolution
- Can be used to solve a variety of problems that are not easy to solve using other techniques

Evolution in the real world

- Each cell of a living thing contains ***chromosomes*** - strings of *DNA*
- Each chromosome contains a set of ***genes*** - blocks of DNA
- Each gene determines some aspect of the organism (like eye colour)
- A collection of genes is sometimes called a ***genotype***
- A collection of aspects (like eye colour) is sometimes called a ***phenotype***
- Reproduction involves **recombination** of genes from parents and then small amounts of ***mutation*** (errors) in copying
- The ***fitness*** of an organism is how much it can reproduce before it dies
- Evolution based on “survival of the fittest”

To start with …

- Suppose you have a problem
- You don’t know how to solve it
- What can you do?

A dumb solution

A “blind generate and test” algorithm:

Repeat
- Generate a random possible solution
- Test the solution and see how good it is

Until solution is good enough

Can we use this dumb idea?

- Sometimes - yes:
  - if there are only a few possible solutions
  - and you have enough time
  - then such a method *could* be used
- For most problems - no:
  - many possible solutions
  - with no time to try them all
  - so this method *can not* be used

A “less-dumb” idea (GA)

Generate a *set* of random solutions

Repeat

  Test each solution in the set (rank them)

  Remove some bad solutions from set

  Duplicate some good solutions

    make small changes to some of them

Until the best solution is good enough

How do you encode a solution?

- Obviously this depends on the problem!
- GA’s *often* encode solutions as fixed length “bitstrings” (e.g. 101110, 111111, 000101)
- Each bit represents some aspect of the proposed solution to the problem
- For GA’s to work, we need to be able to “test” any string and get a “score” indicating how “good” that solution is

Silly Example - Drilling for Oil

- Imagine you had to drill for oil somewhere along a single 1km desert road
- Problem: choose the best place on the road that produces the most oil per day
- We could represent each solution as a position on the road
- E.g., a whole number between [0..1000]

Where to drill for oil?

Solution1 = 300

Solution2 = 900

Road

0

500

1000

Digging for Oil

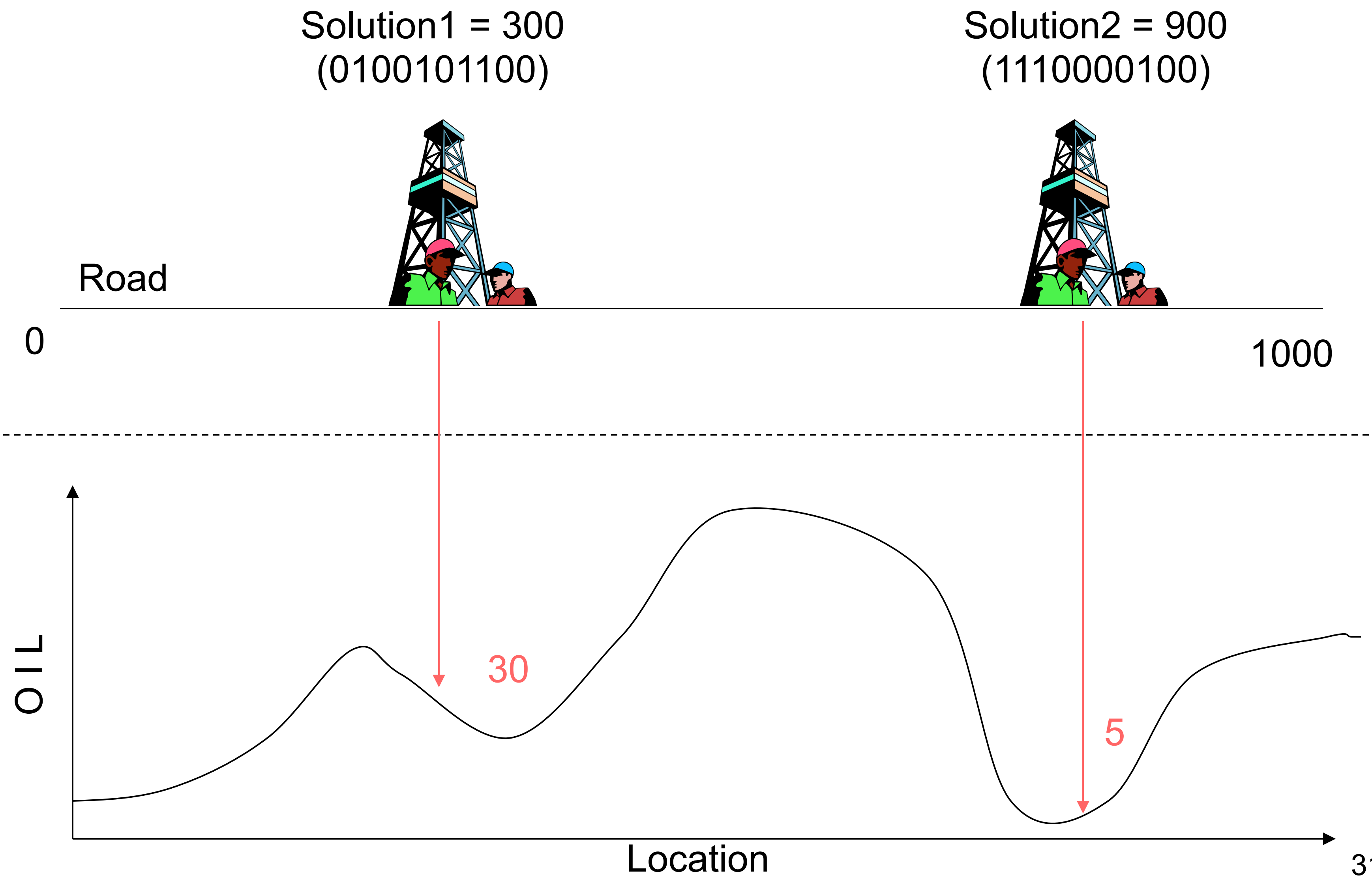
- The set of all possible solutions [0..1000] is called the *search space* or *state space*
- In this case it's just one number but it could be many numbers or symbols
- Often GA's code numbers in binary producing a bitstring representing a solution
- In our example we choose 10 bits which is enough to represent 0..1000

Convert to binary string

	512	256	128	64	32	16	8	4	2	1
900	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**1**	**0**	**0**
300	**0**	**1**	**0**	**0**	**1**	**0**	**1**	**1**	**0**	**0**
1023	1	1	1	1	1	1	1	1	1	1

In GA’s these encoded strings are sometimes called “*genotypes*” or “*chromosomes*” *and the individual bits are sometimes called “genes”*

Drilling for Oil



Summary (so far)

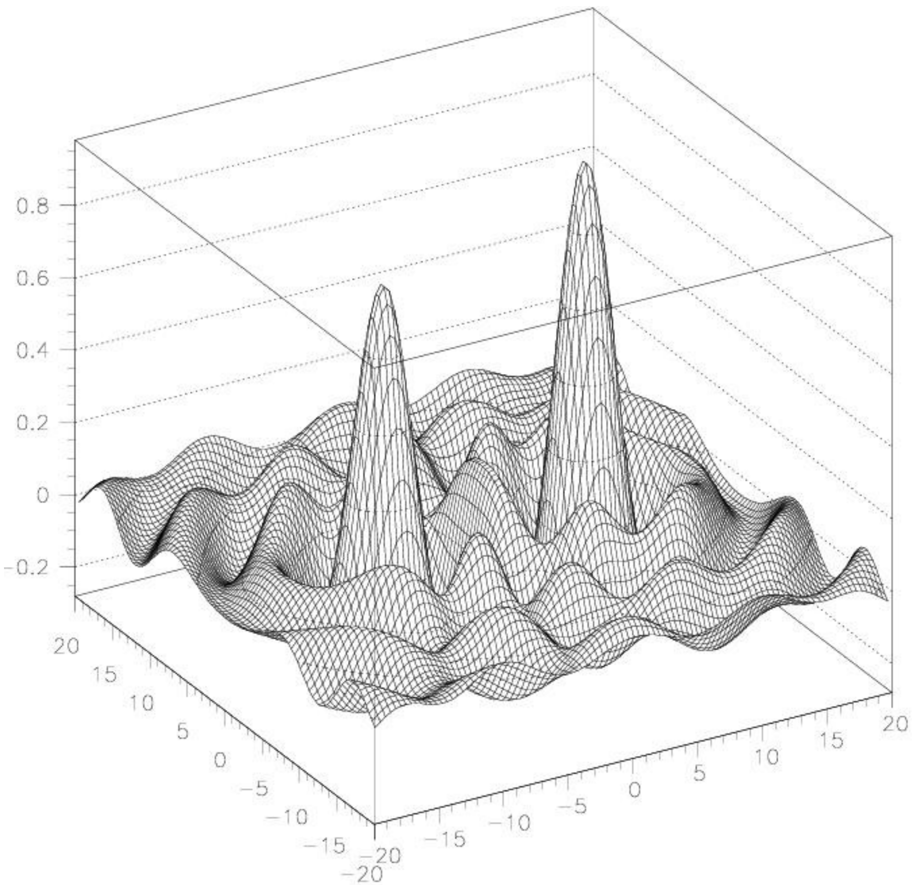
We have seen how to:

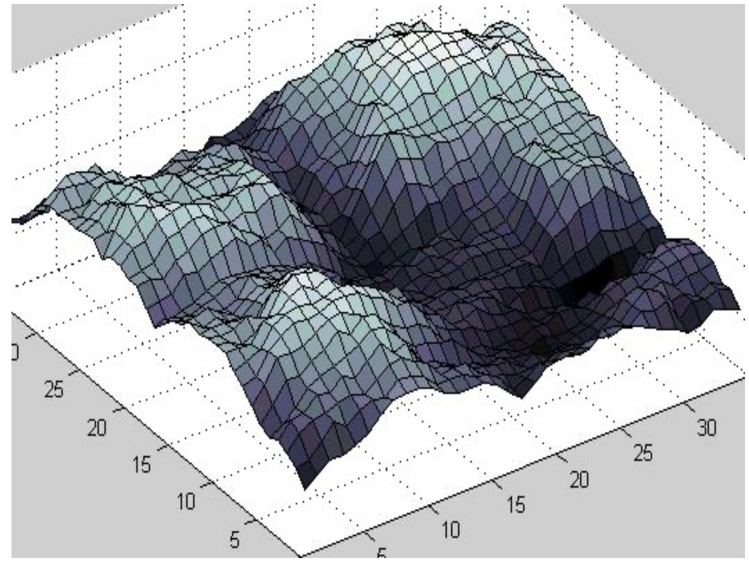
- represent possible solutions as a number
- encode a number into a binary string
- generate a score for each number given a *function* of “how good” each solution is - this is often called a *fitness function*
- Our silly oil example is really optimisation over a function f(x) where we adapt the parameter x

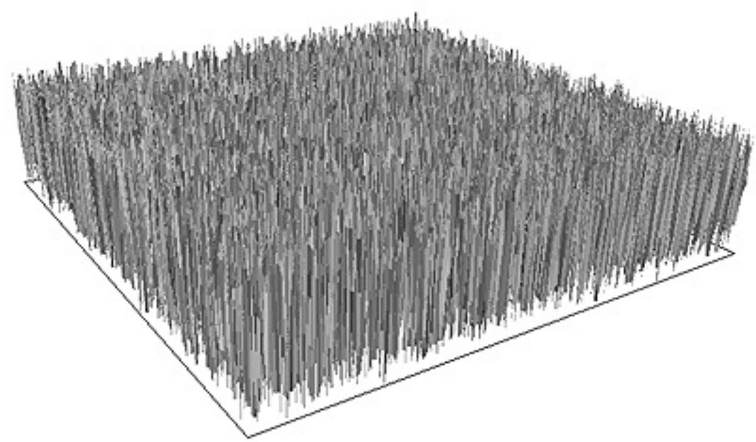
Search Space

- For a simple function f(x) the search space is one dimensional.
- By encoding several values into the chromosome many dimensions can be searched, e.g., two dimensions f(x,y)
- Search space an be visualised as a surface or *fitness landscape* in which fitness is height
- A GA tries to move the points to better places (higher fitness) in the space

Fitness landscapes







Search Space

- The nature of the search space dictates how a GA will perform
- A completely random space - bad for a GA
- Also GA’s can get stuck in local maxima if search spaces contain lots of these
- Generally, spaces in which small improvements get closer to the global optimum are good

Back to the (GA) Algorithm

Generate a *set* of random solutions

Repeat

  Test each solution in the set (rank them)

  Remove some bad solutions from set

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Until best solution is good enough

Adding Reproduction - Crossover

- Although it may work for simple search spaces - our algorithm is still very simple
- Relies on random mutation to find a good solution
- Introducing reproduction - better results

Adding Reproduction - Crossover

- Two high scoring “parent” bit strings (*chromosomes)* are selected and with some probability (crossover rate) combined
- Producing two new *offspring* (bit strings)
- Each offspring may then be changed randomly (*mutation*)

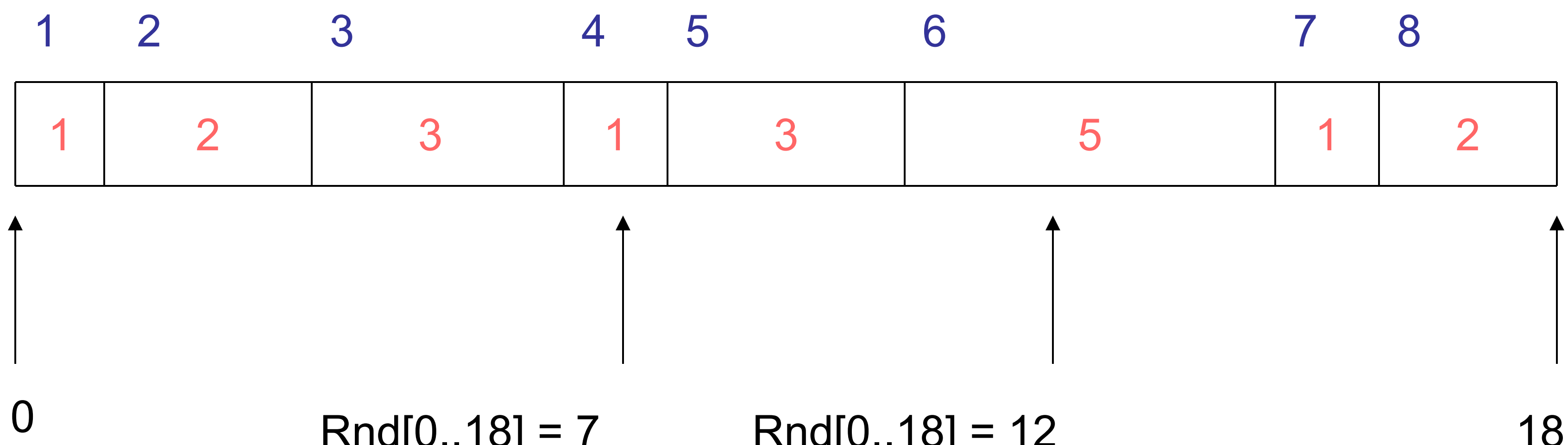
Selecting Parents

- Many schemes possible, goal: better scoring chromosomes more likely to be selected
- Score often termed the *fitness*
- “Roulette Wheel” selection can be used:
  - Add up the fitness's of all chromosomes
  - Generate a random number R in that range
  - Select the first chromosome in the population that - when all previous fitness’s are added - gives you at least the value R

Example population

No.	Chromosome	Fitness
1	1010011010	1
2	1111100001	2
3	1011001100	3
4	1010000000	1
5	0000010000	3
6	1001011111	5
7	0101010101	1
8	1011100111	2

Roulette Wheel Selection



Rnd[0..18] = 7

Chromosome4

Parent1

Rnd[0..18] = 12

Chromosome6

Parent2

Crossover - Recombination

1010000000	Parent1	Offspring1	1011011111
1001011111	Parent2	Offspring2	1000000000

Crossover single point - random

With some high probability (*crossover rate*) apply crossover to the parents. (*typical values are 0.8 to 0.95*)

Mutation

mutate

Offspring1 1011011111

Offspring2 1000000000

Original offspring

Offspring1 1011001111

Offspring2 1010000000

Mutated offspring

With some small probability (the *mutation rate*) flip each bit in the offspring (*typical values between 0.1 and 0.001*)

Back to the (GA) Algorithm

Generate a *population* of random chromosomes

Repeat (each generation)

- Calculate fitness of each chromosome
- Repeat
  - Use roulette selection to select pairs of parents
  - Generate offspring with crossover and mutation
- Until a new population has been produced

Until best solution is good enough

Many Variants of GA

- Different kinds of selection (not roulette)
  - Tournament
  - Elitism, etc.
- Different recombination
  - Multi-point crossover
  - 3 way crossover etc.
- Different kinds of encoding other than bitstring
  - Integer values
  - Ordered set of symbols
- Different kinds of mutation

Many parameters to set

- Any GA implementation needs to decide on a number of parameters: Population size (N), mutation rate (m), crossover rate (c)
- Often these have to be "tuned", based on results obtained - no general theory to deduce good values
- Typical values might be: $N = 50$, $m = 0.05$, $c = 0.9$

Genetic algorithms-summarized

- Start with randomly generated population of valid candidate solutions
- For generation in (0..n)
  - Select 2 solutions (parents) randomly from population for reproduction based on *fitness* of each state. The more fit a solution, the more likely selected to reproduce.
  - Reproduce a new solution (child) by combining random parts of the 2 selected solutions (crossover)
  - Mutate a random number of new solution to allow exploration other possible solutions
  - Population = new solutions
- Best solution is the fittest of population solutions

Summary: Local Search

- **Hill-climbing algorithms** keep only a single solution in memory, but can get stuck on local optima
- **Simulated annealing** escapes local optima, and is optimal given a “long enough” cooling schedule
- **Genetic algorithms** can search a large space by modeling biological evolution