Informed Search

Sanja Lazarova-Molnar

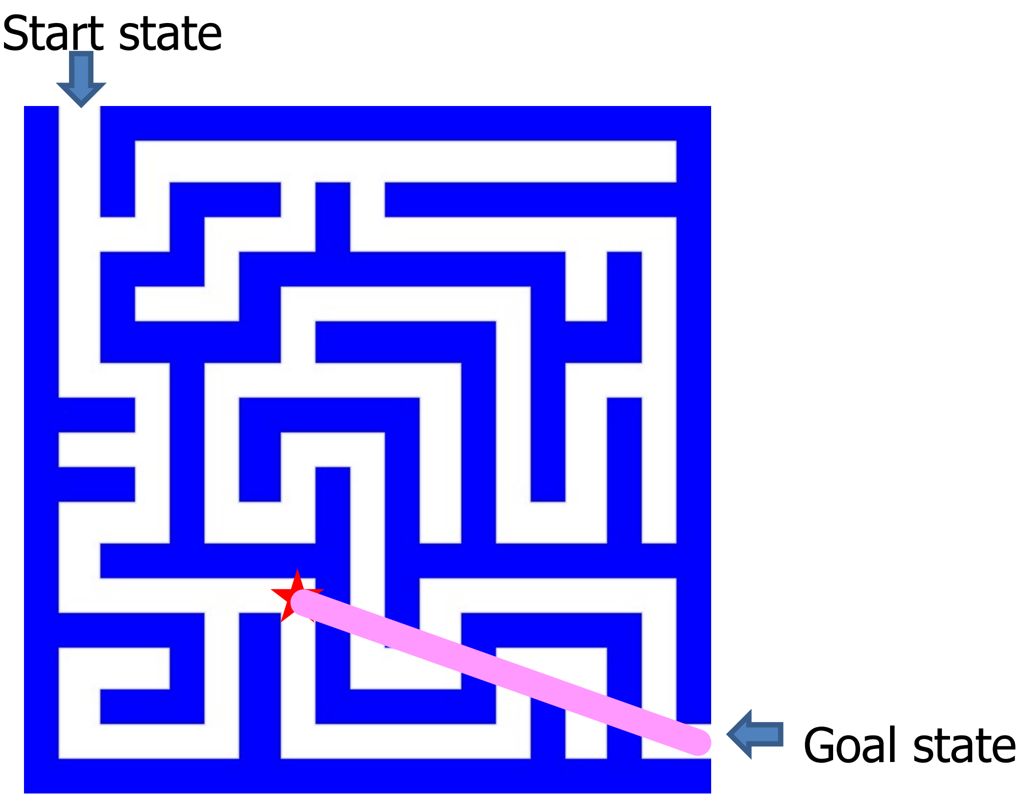
Informed search

- Idea: give the algorithm "hints" about the desirability of different states
  - Use an *evaluation function* to rank nodes and select the most promising one for expansion

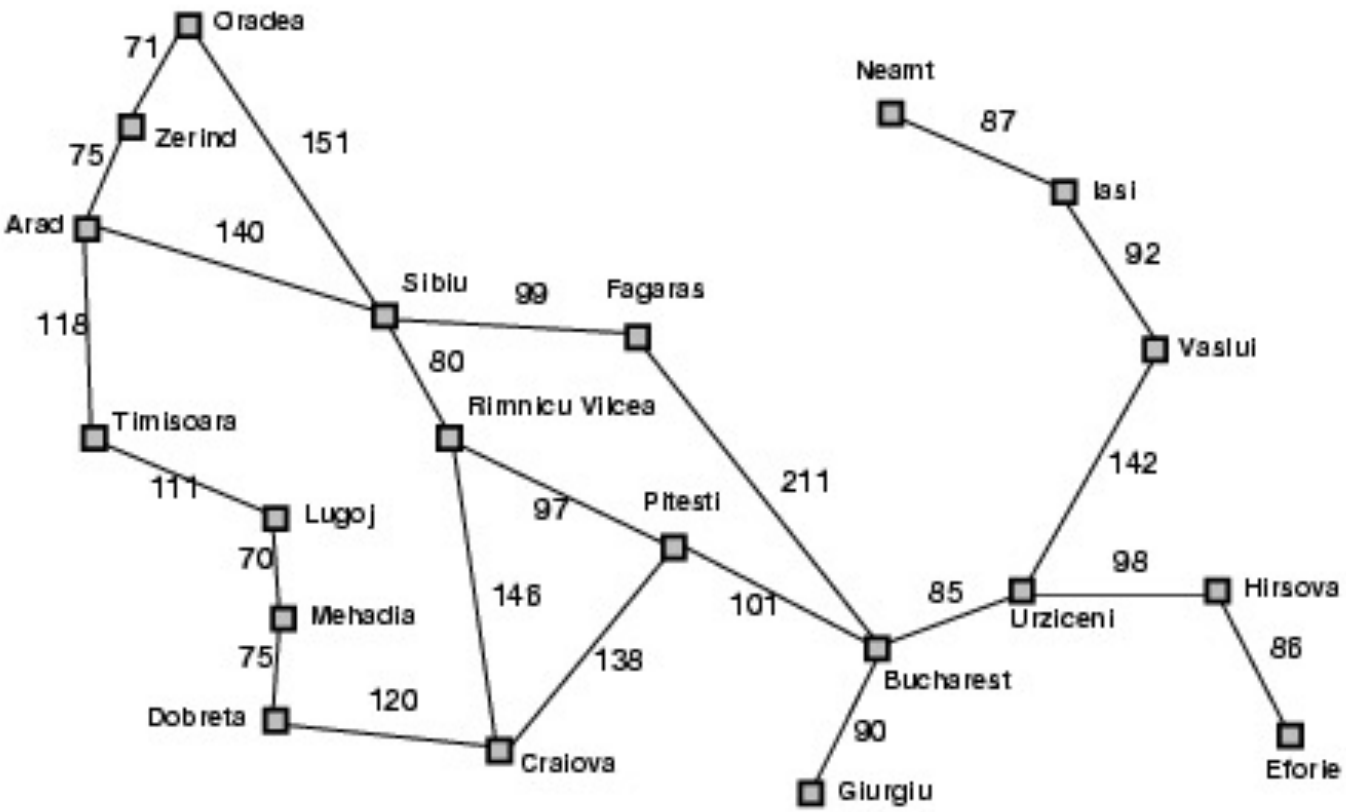
- Greedy best-first search
- A* search

Heuristic function

- Heuristic function *h*(*n*) estimates the cost of reaching goal from node *n*
- Example:



Heuristic for the Romania problem



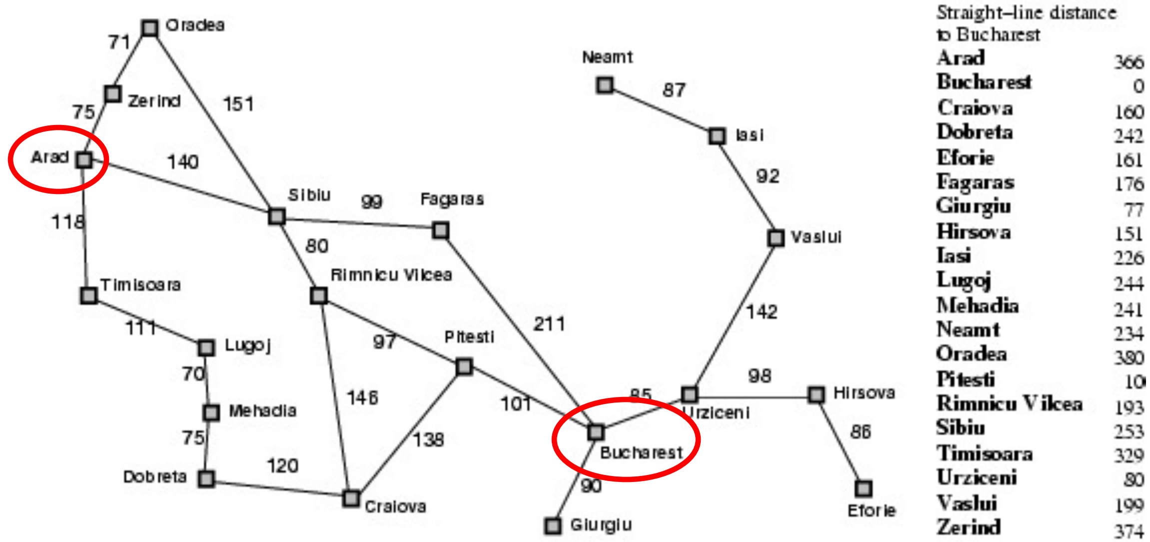
Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy best-first search

- Expand the node that has the lowest value of the heuristic function $h(n)$

Greedy best-first search example



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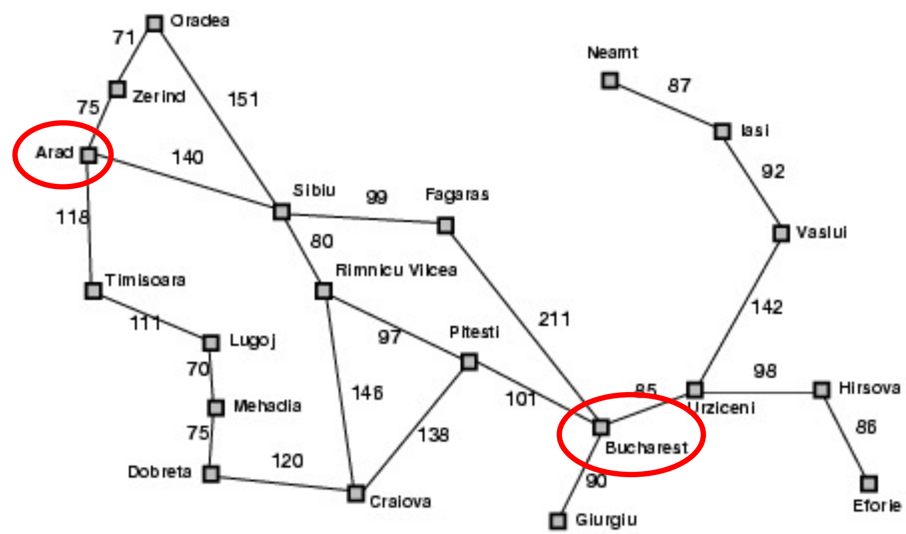

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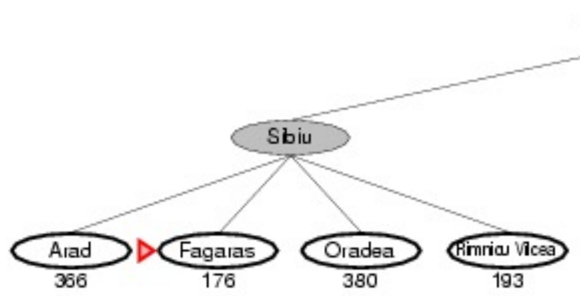

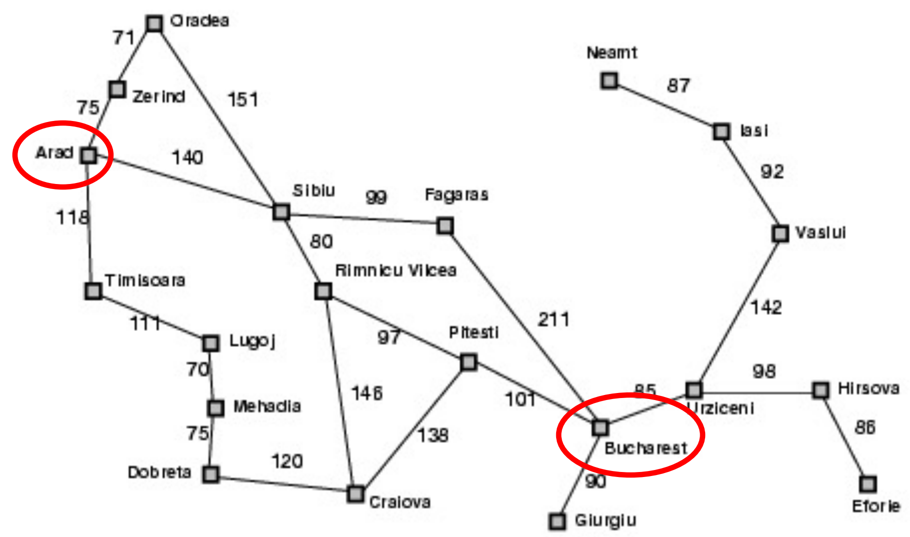

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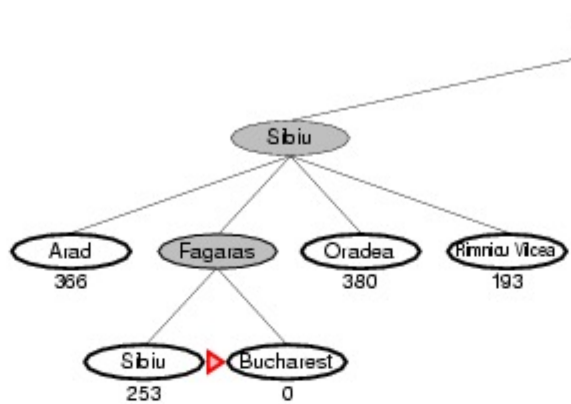
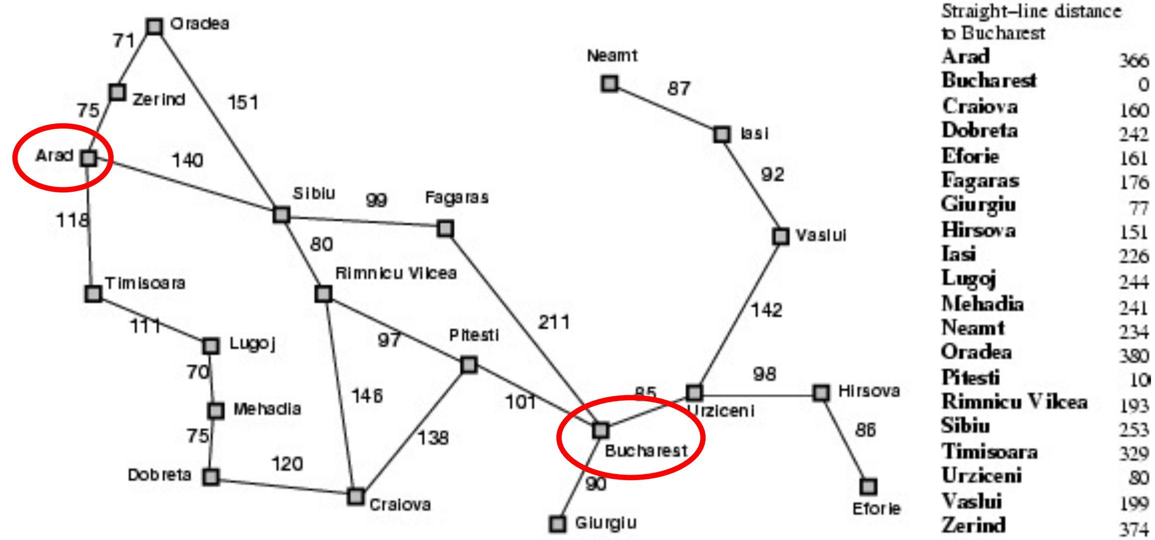
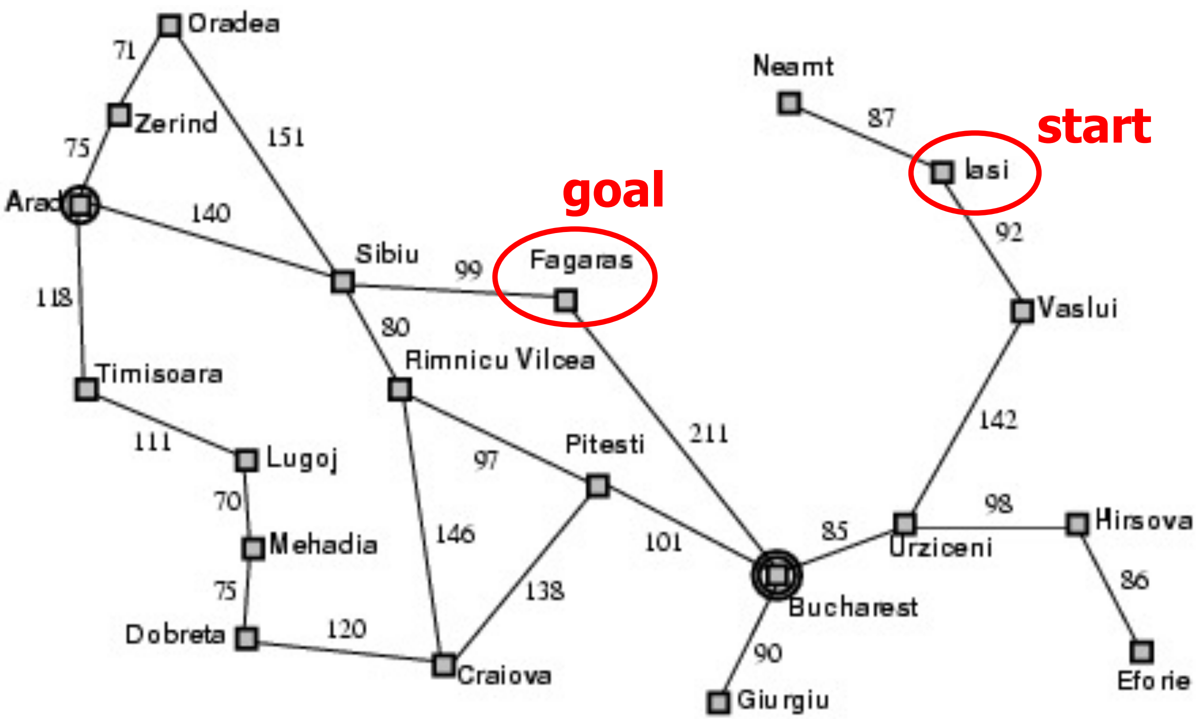




Properties of greedy best-first search

- **Complete?**

  No – can get stuck in loops



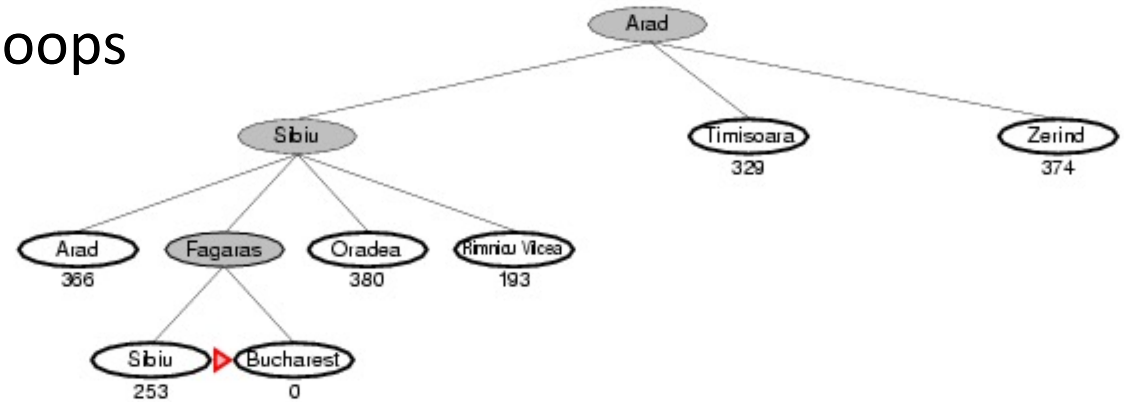
Properties of greedy best-first search

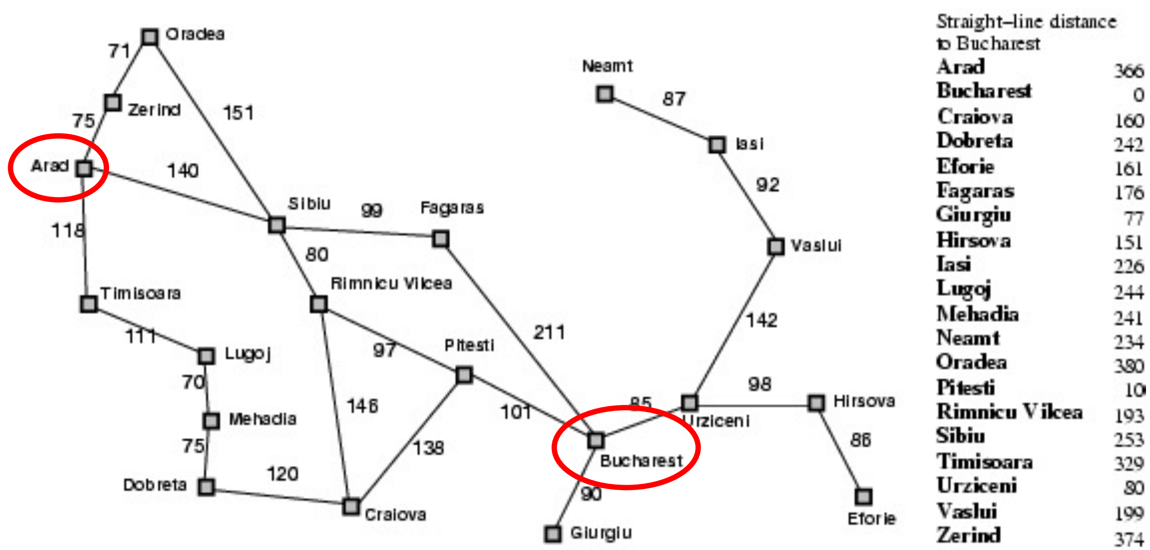
- **Complete?**

  No – can get stuck in loops

- **Optimal?**

  No





Properties of greedy best-first search

- **Complete?**

  No – can get stuck in loops

- **Optimal?**

  No

- **Time?**

  Worst case: $O(b^m)$

  Best case: $O(bd)$ – If $h(n)$ is 100% accurate

- **Space?**

  Worst case: $O(b^m)$

How can we fix the greedy problem?

- Add another parameter to evaluate nodes!?

A^* search

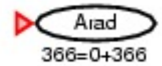
- Idea: avoid expanding paths that are already expensive
- The evaluation function $f(n)$ is the estimated total cost of the path through node n to the goal:

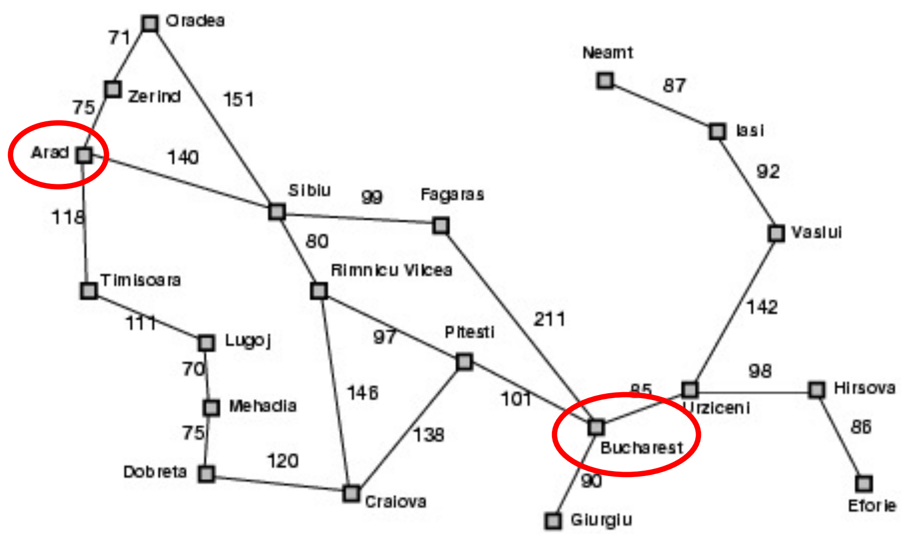
$$f(n) = g(n) + h(n)$$

$g(n)$: cost so far to reach n (path cost)

$h(n)$: estimated cost from n to goal (heuristic)

A* search example





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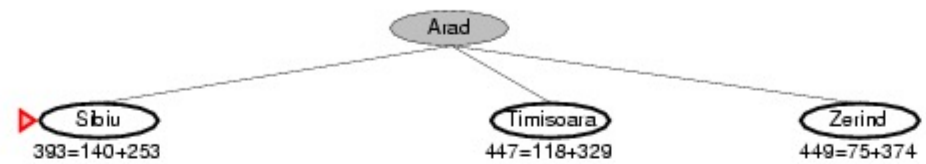

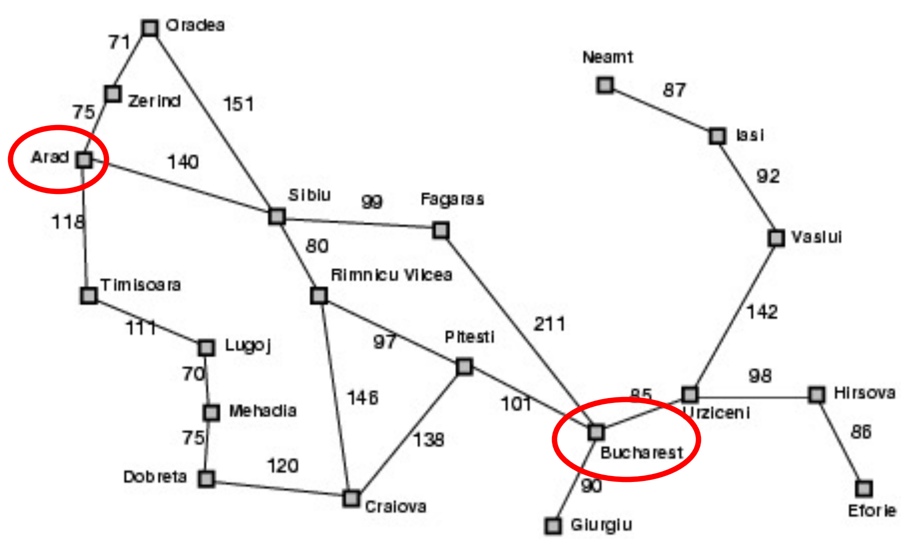

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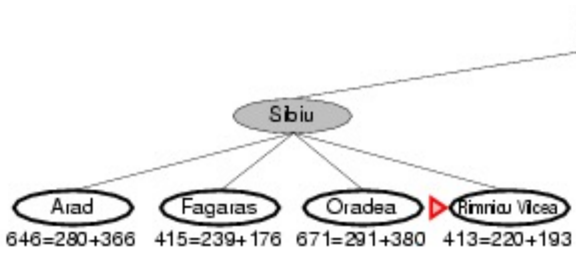

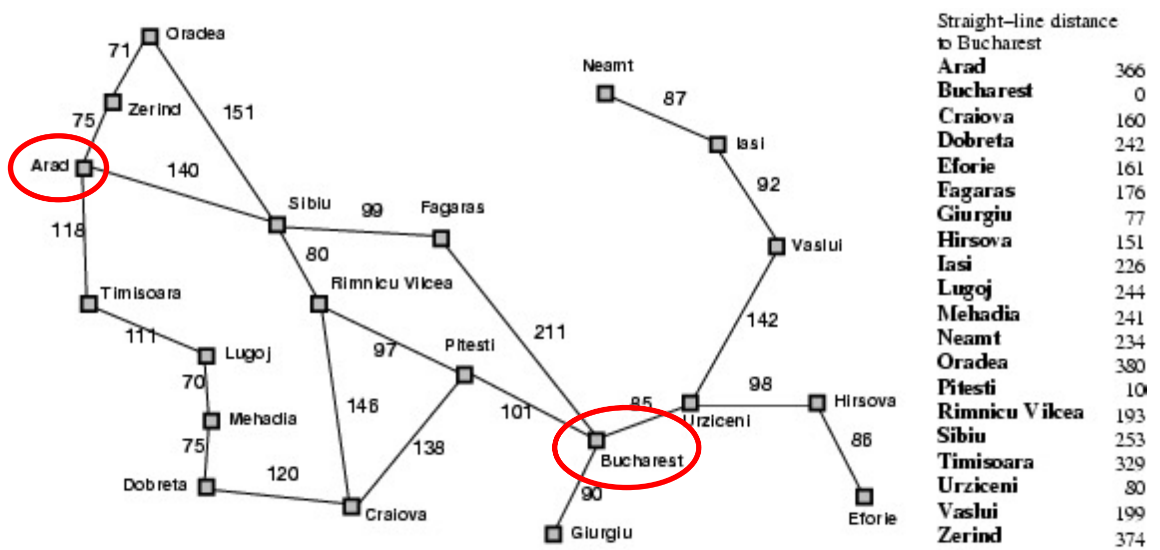

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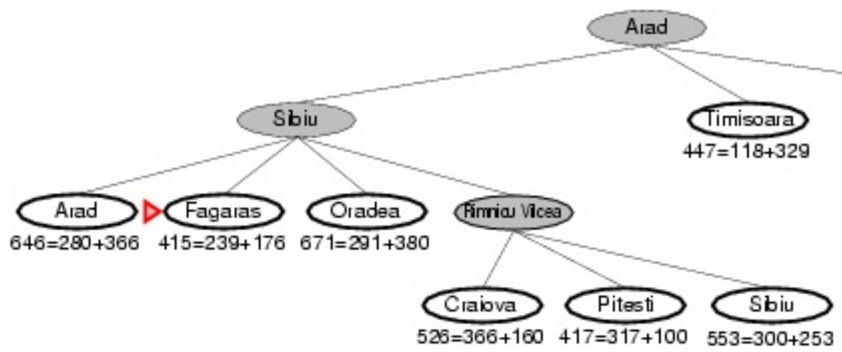

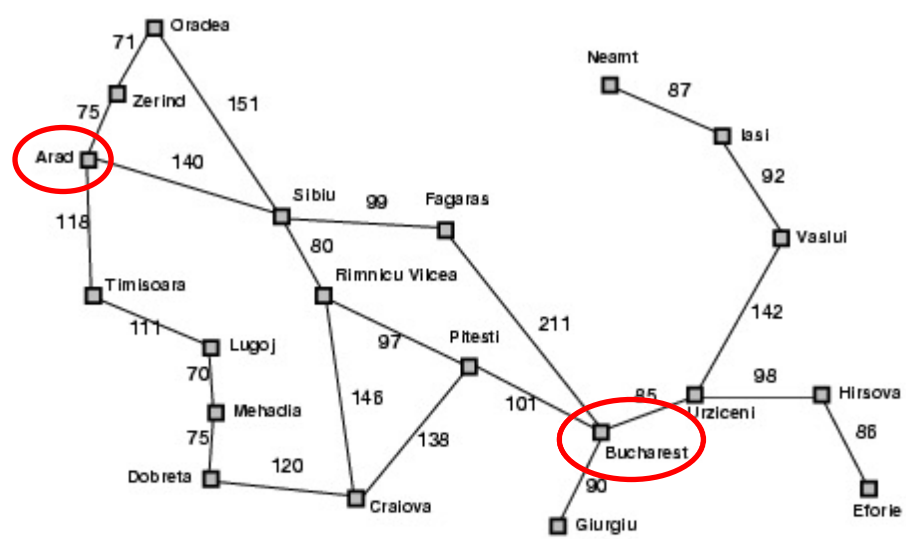

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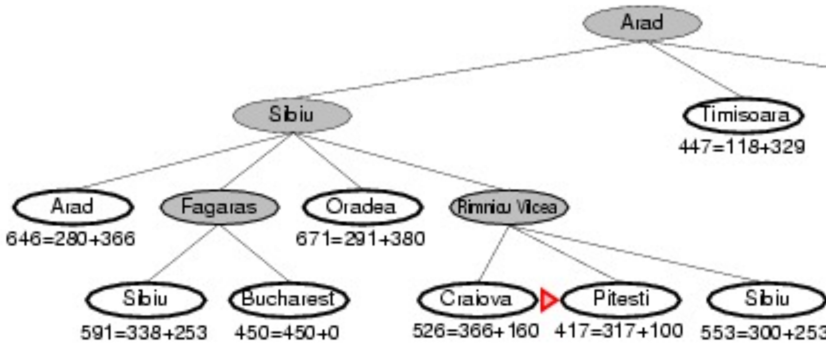
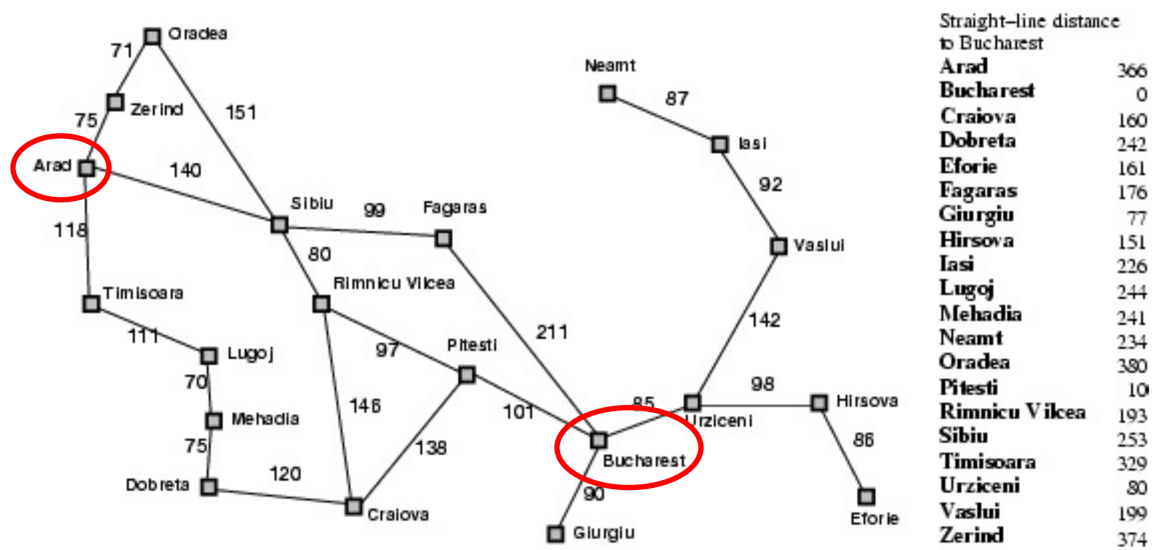
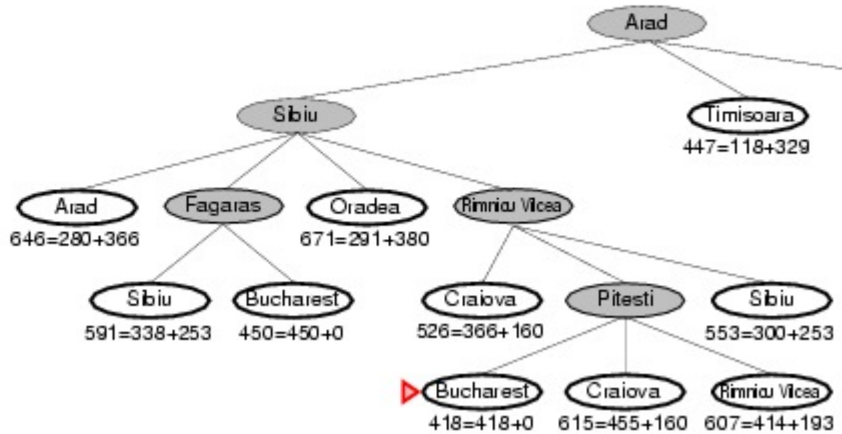
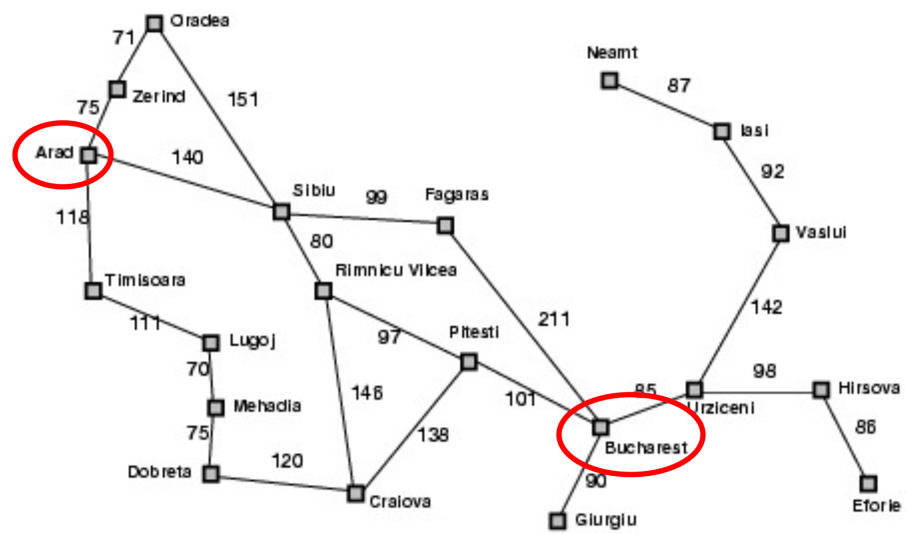
A^* search example





A* search example





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Admissible heuristics

- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n
- Example: straight line distance never overestimates the actual road distance
- **Theorem:** If $h(n)$ is admissible, A^* is optimal

Optimality of A*

- A* is *optimally efficient* – no other tree-based algorithm that uses the same heuristic can expand fewer nodes and still be guaranteed to find the optimal solution
  – any algorithm that does not expand all nodes in the contours between the root and the goal contour runs the risk of missing the optimal solution

Properties of A*

- **Complete?**

  Yes – unless there are infinitely many nodes with $f(n) \leq C^*$

- **Optimal?**

  Yes

- **Time?**

  Number of nodes for which $f(n) \leq C^*$ (exponential)

- **Space?**

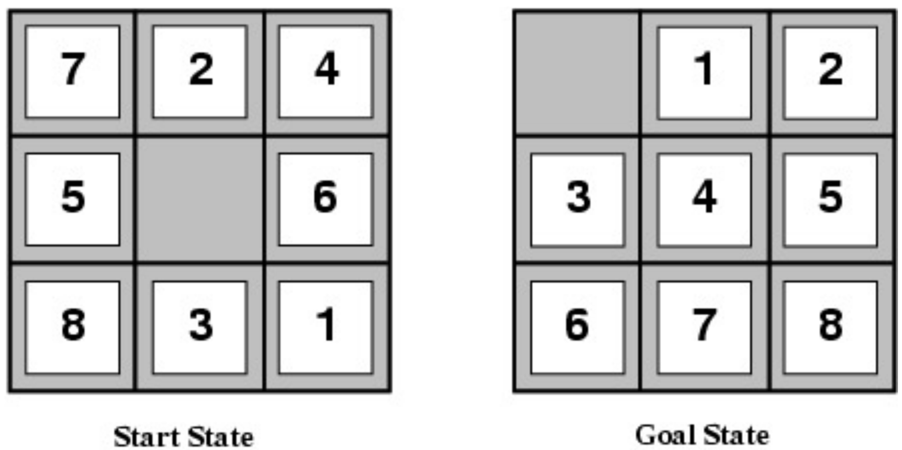
  Exponential

Designing heuristic functions

- Heuristics for the 8-puzzle

  $h_1(n)$ = number of misplaced tiles

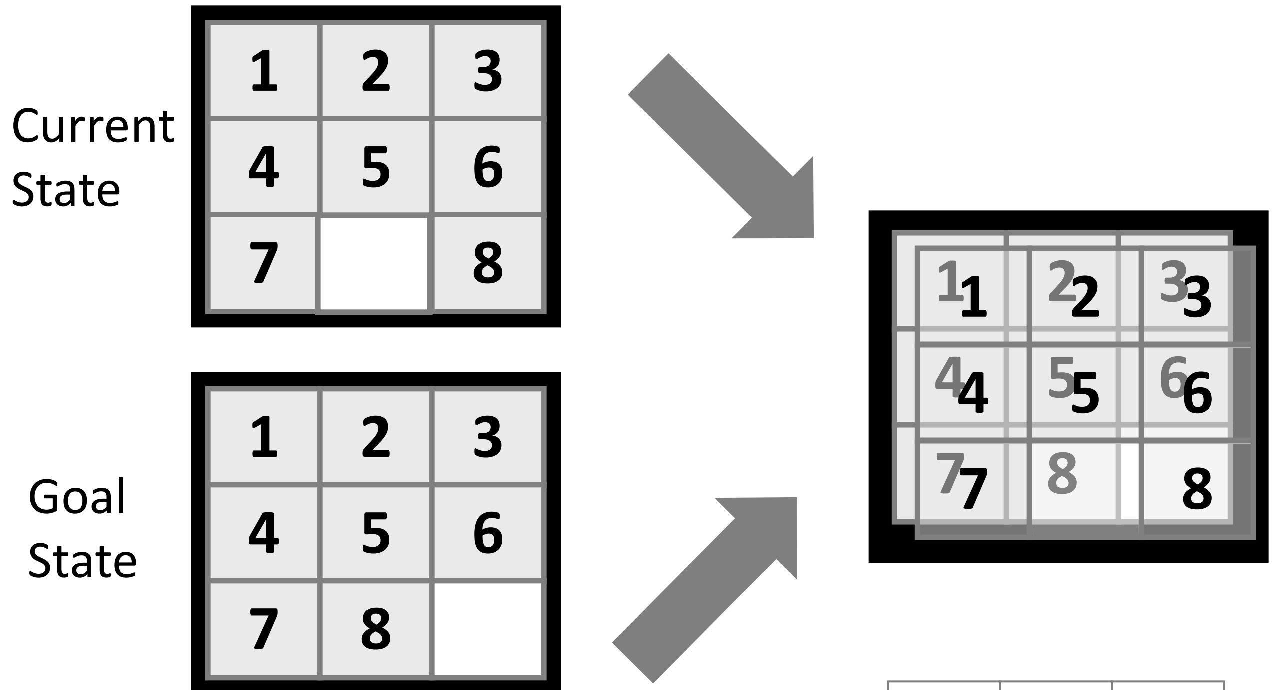
  $h_2(n)$ = total Manhattan distance (number of squares from desired location of each tile)



- Are h_1 and h_2 admissible?

$h_1(n)$ = number of misplaced tiles

The number of ***misplaced tiles*** *(not including the blank)*



N	**N**	**N**
N	**N**	**N**
N	**Y**	

In this case, only “**8**” is misplaced, so heuristic function evaluates to 1

In other words, the heuristic *says* that it *thinks* a solution may be available in just 1 more move

$h_2(n)$ = total Manhattan distance

Manhattan Distance (not including the blank)

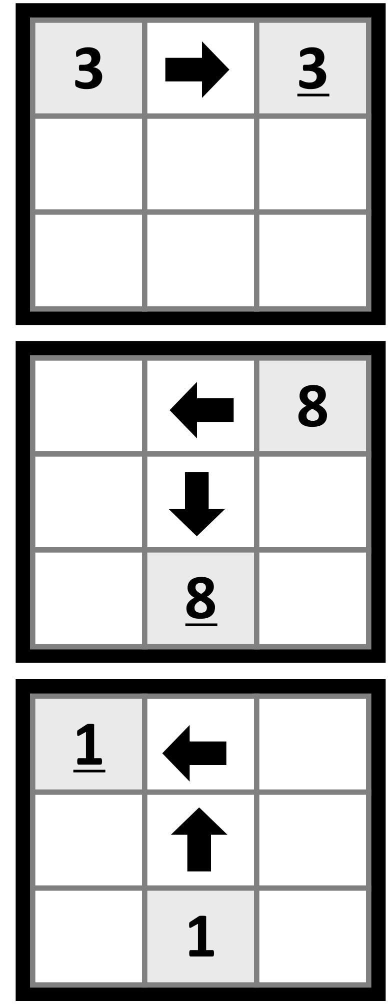
Current State

3	2	8
4	5	6
7	1	

Goal State

1	2	3
4	5	6
7	8	

- The **3**, **8** and **1** tiles are misplaced (by 2, 3, and 3 steps) so the heuristic function evaluates to 8
- Heuristic says that it *thinks* a solution may be available in just 8 more moves.



2 spaces

3 spaces

3 spaces

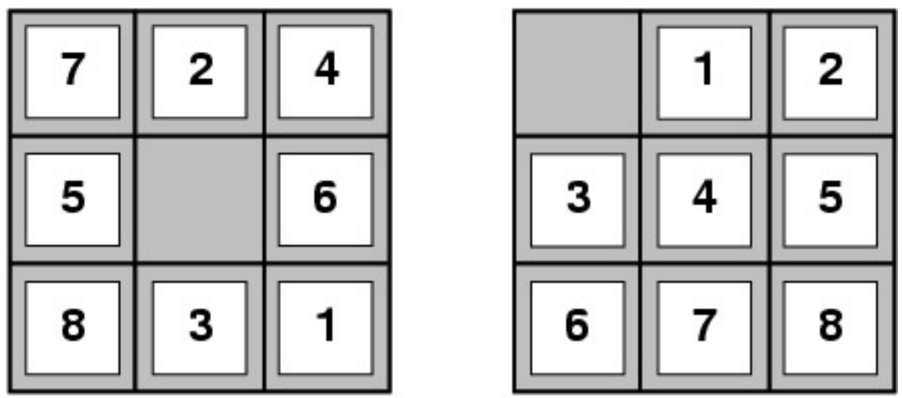
Total 8

Designing heuristic functions

- Heuristics for the 8-puzzle

  $h_1(n)$ = number of misplaced tiles

  $h_2(n)$ = total Manhattan distance (number of squares from desired location of each tile)



Start State Goal State

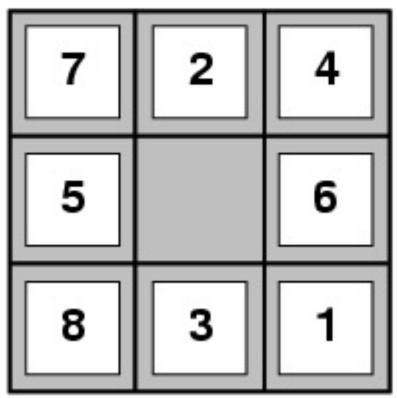
- What are the values for h_1 and h_2 for the start state?
- Are h_1 and h_2 admissible?

Designing heuristic functions

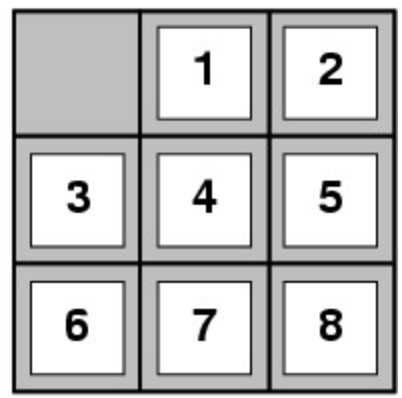
- Heuristics for the 8-puzzle

  $h_1(n)$ = number of misplaced tiles

  $h_2(n)$ = total Manhattan distance (number of squares from desired location of each tile)



Start State



Goal State

$$h_1(\text{start}) = 8$$

$$h_2(\text{start}) = 3+1+2+2+2+3+3+2 = 18$$

- Both h_1 and h_2 are admissible

Dominance

- If h_1 and h_2 are both admissible heuristics and $h_2(n) \geq h_1(n)$ for all n, (both admissible) then h_2 dominates h_1
- Which one is better for search?
  - A* search expands every node with $f(n) < C^*$ or $h(n) < C^* - g(n)$
  - Therefore, A* search with h_1 will expand more nodes, so h_2 is better

C^* - optimal cost

Heuristics from relaxed problems

- A problem with fewer restrictions on the actions - relaxed problem
- The cost of an optimal solution to a relaxed problem - admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Dominance

- Typical search costs for the 8-puzzle (average number of nodes expanded for different solution depths):

- *d*=12 IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes

- *d*=24 IDS ≈ 54,000,000,000 nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Combining heuristics

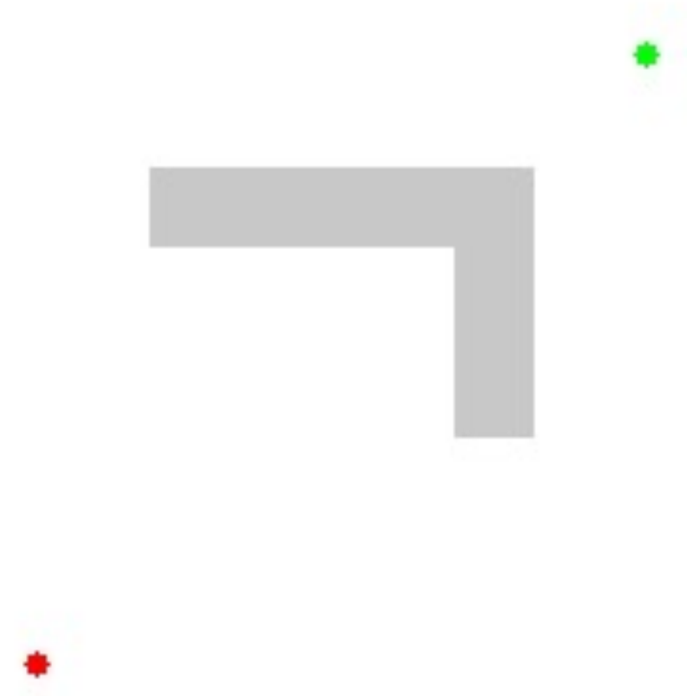
- Suppose we have a collection of admissible heuristics $h_1(n)$, $h_2(n)$, …, $h_m(n)$, but none of them dominates the others
- How can we combine them?

$$h(n) = \max\{h_1(n), h_2(n), \ldots, h_m(n)\}$$

Weighted A* search

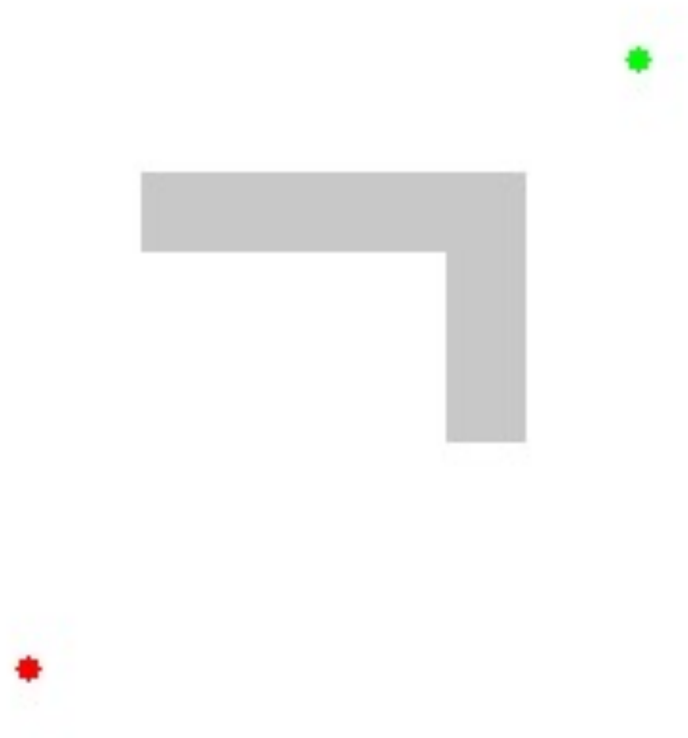
- Idea: speed up search at the expense of optimality
- Take an admissible heuristic, “inflate” it by a multiple $\alpha > 1$, and then perform A* search as usual
- Fewer nodes tend to get expanded, but the resulting solution may be suboptimal (its cost will be at most α times the cost of the optimal solution)

Example of weighted A* search



Heuristic: 5 * Euclidean distance from goal
Source: Wikipedia

Example of weighted A* search



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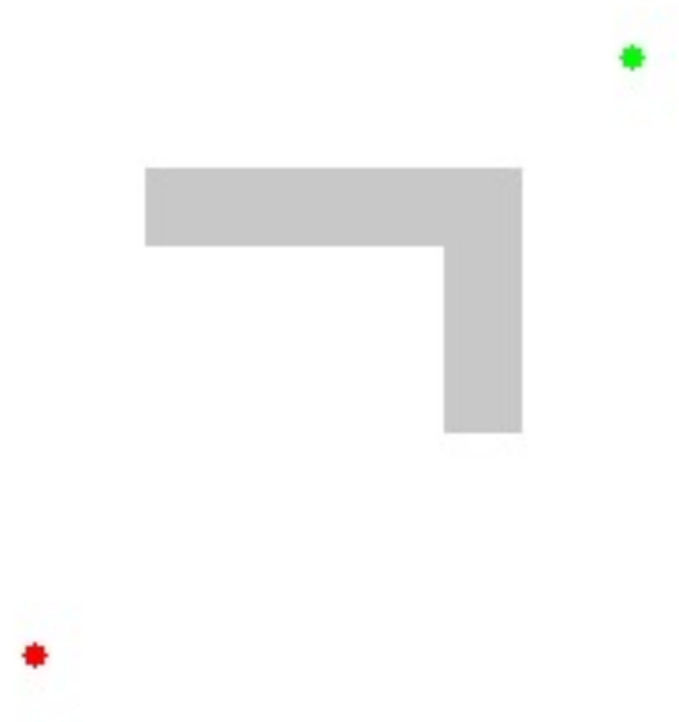


Compare: Exact A*

Dealing with hard problems

- For large problems, A* may require too much space
- Variations conserve memory: IDA* and SMA*
- IDA*, iterative deepening A*, uses successive iteration with growing limits on f, e.g.
  - A* but don’t consider a node n where f(n) >10
  - A* but don’t consider a node n where f(n) >20
  - A* but don’t consider a node n where f(n) >30, ...
- SMA* -- Simplified Memory-Bounded A*
  - Uses queue of restricted size to limit memory use

Uninformed search strategies

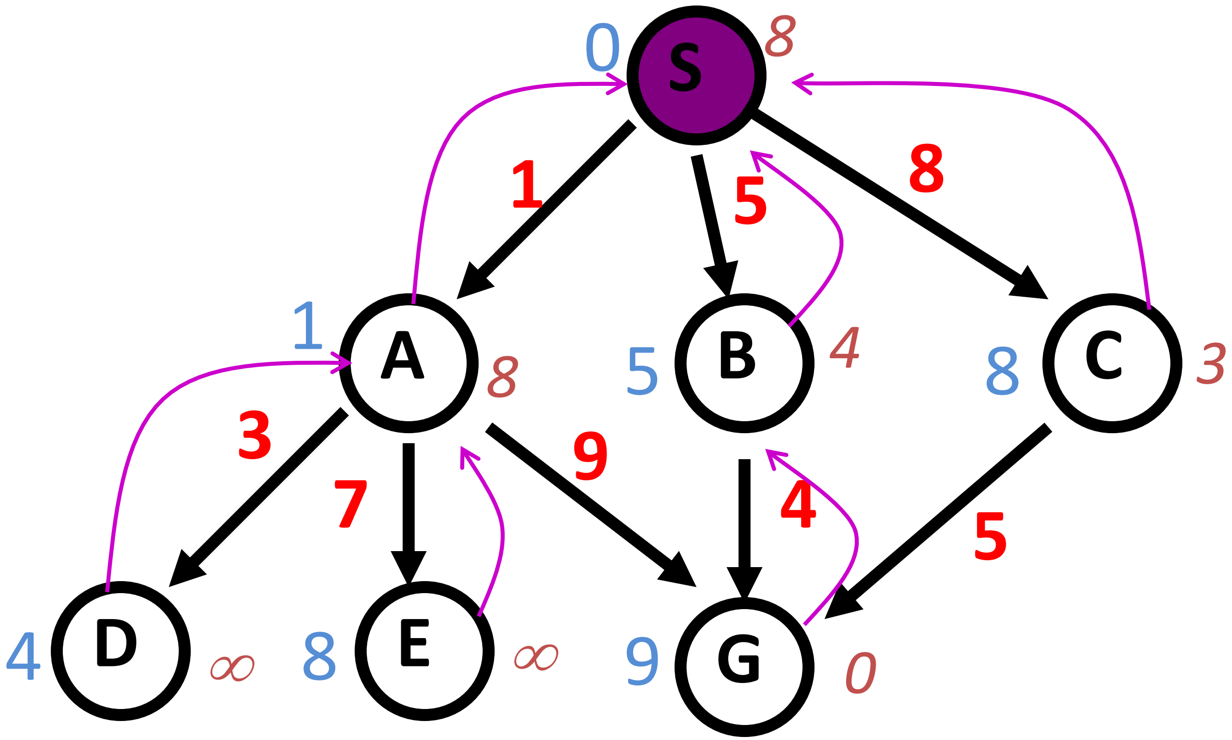
Algorithm	Complete?	Optimal?	Time complexity	Space complexity
BFS	Yes	If all step costs are equal	$O(b^d)$	$O(b^d)$
UCS	Yes	Yes	Number of nodes with $g(n) \leq C^*$	
DFS	No	No	$O(b^m)$	$O(bm)$
IDS	Yes	If all step costs are equal	$O(b^d)$	$O(bd)$

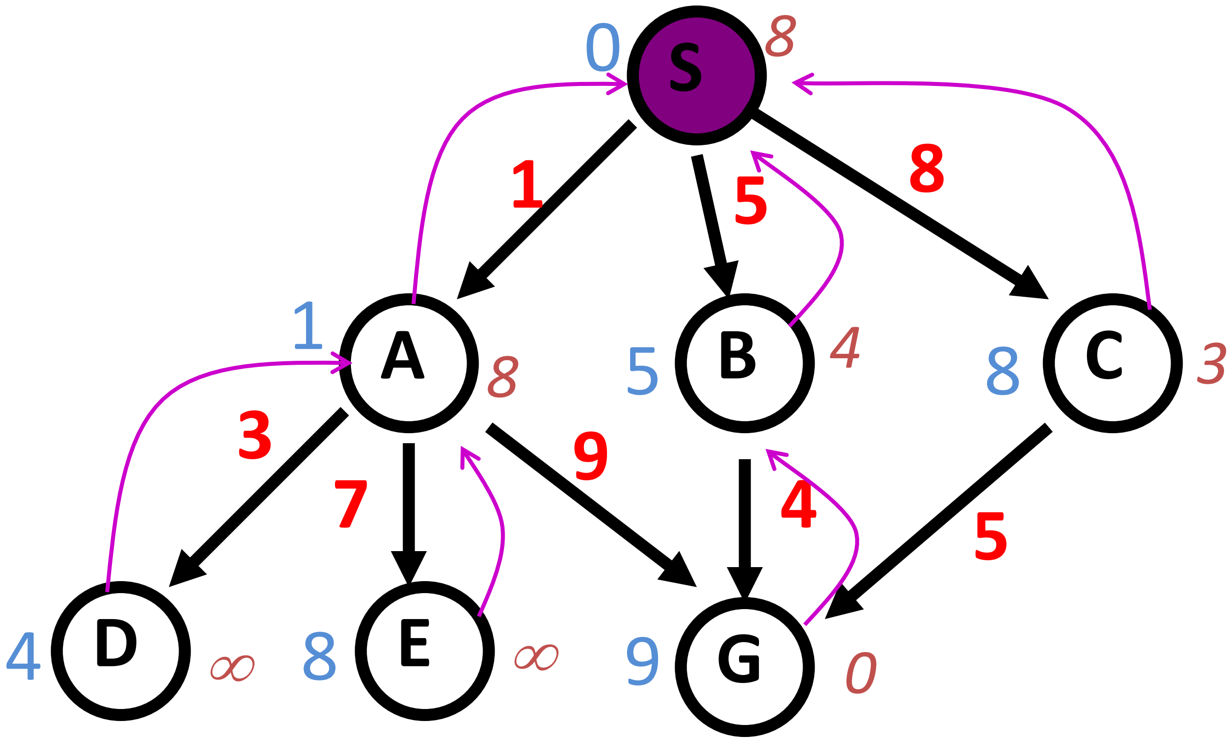
b: maximum branching factor of the search tree
d: depth of the optimal solution
m: maximum length of any path in the state space
C*: cost of optimal solution

All search strategies

Algorithm	Complete?	Optimal?	Time complexity	Space complexity
BFS	Yes	If all step costs are equal	$O(b^d)$	$O(b^d)$
UCS	Yes	Yes	Number of nodes with $g(n) \leq C^*$	
DFS	No	No	$O(b^m)$	$O(bm)$
IDS	Yes	If all step costs are equal	$O(b^d)$	$O(bd)$
Greedy	No	No	Worst case: $O(b^m)$ Best case: $O(bd)$	
A*	Yes	Yes	Number of nodes with $g(n)+h(n) \leq C^*$	

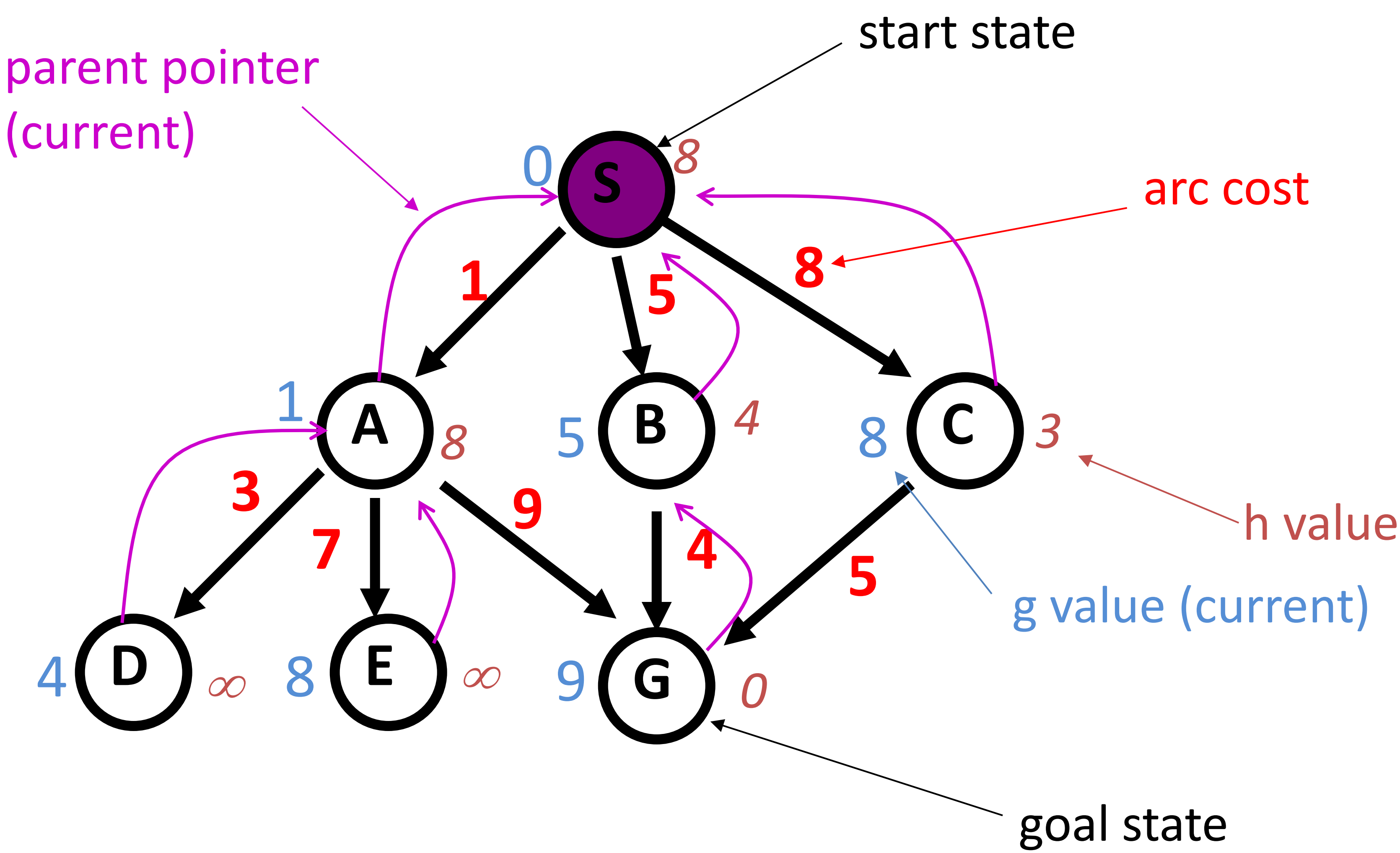
Example search space



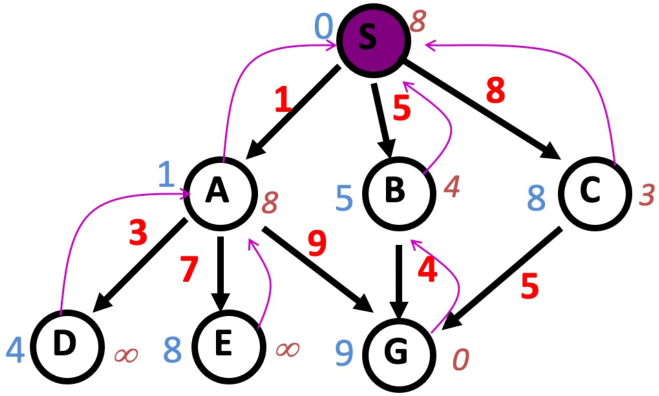


S
0
8
1
5
8
A
1
8
B
5
4
C
8
3
3
7
9
4
5
D
4
∞
E
8
∞
G
9
0

Example search space



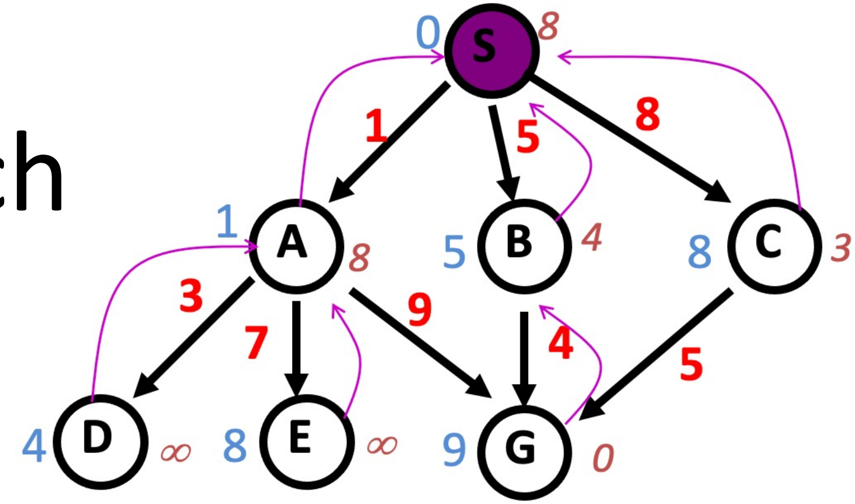
Example



n	g(n)	h(n)	f(n)	h*(n)
S	0	8	8	9
A	1	8	9	9
B	5	4	9	4
C	8	3	11	5
D	4	inf	inf	inf
E	8	inf	inf	inf
G	9	0	9	0

- h*(n) is (hypothetical) perfect heuristic (an oracle)
- Since h(n) <= h*(n) for all n, h is admissible (optimal)
- Optimal path = *S B G* with cost 9

Greedy search

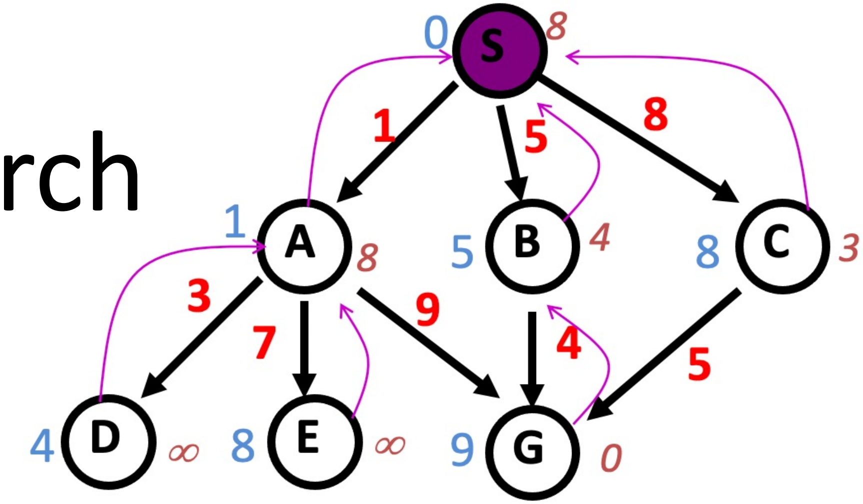


f(n) = h(n)

node expanded	nodes list
	{ S(8) }
S	{ C(3) B(4) A(8) }
C	{ G(0) B(4) A(8) }
G	{ B(4) A(8) }

- Solution path found is S C G, 3 nodes expanded.
- See how fast the search is!! But it is NOT optimal.

A* search

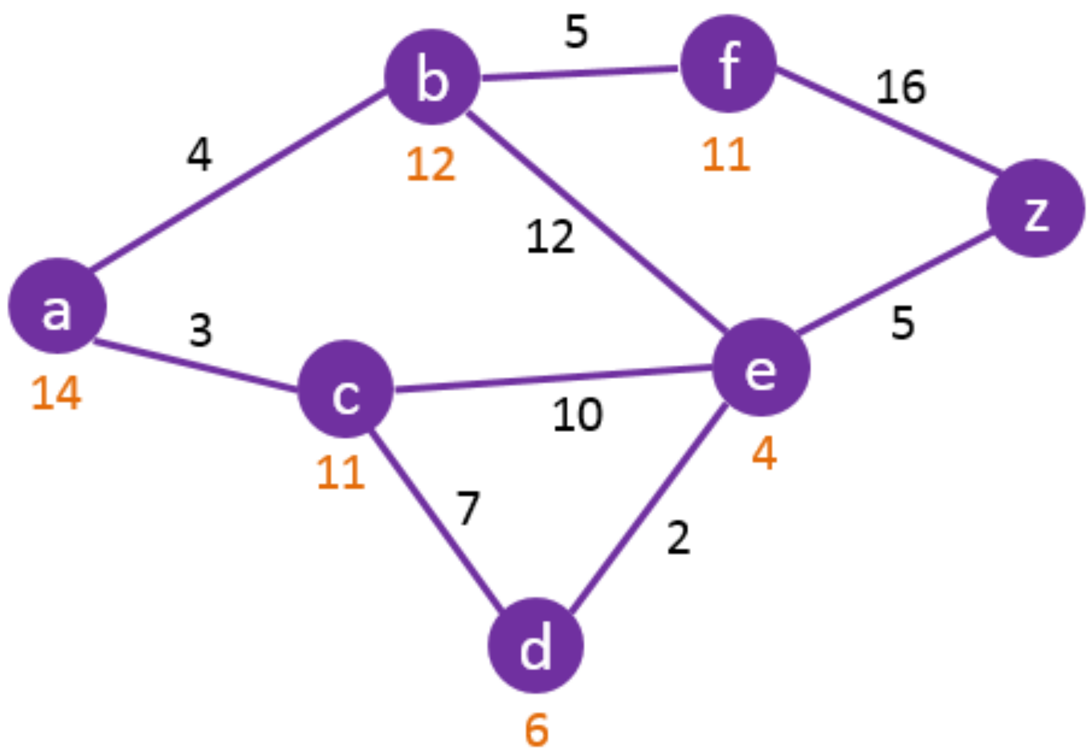


f(n) = g(n) + h(n)

```
node exp.      nodes list
            { S(8) }
 S          { A(9) B(9) C(11) }
 A          { B(9) G(10) C(11) D(inf) E(inf) }
 B          { G(9) G(10) C(11) D(inf) E(inf) }
 G          { C(11) D(inf) E(inf) }
```

- Solution path found is S B G, 4 nodes expanded..
- Still pretty fast. And optimal, too.

Exercise (if there is time, else on your own)



A* Search Algorithm

What is the shortest path to travel from A to Z?

Numbers in orange are the heuristic values, distances in a straight line (as the crow flies) from a node to node Z.

Summary: Informed search

- **Greedy best-first search** uses minimal estimated cost h(n) to goal state as measure; reduces search time, but is neither complete nor optimal
- **A* search** combines uniform-cost search & greedy best-first search: f(n) = g(n) + h(n). Handles state repetitions & h(n) never overestimates
  - A* is complete & optimal, but space complexity high
  - Time complexity depends on quality of heuristic function
  - IDA* and SMA* reduce the memory requirements of A*