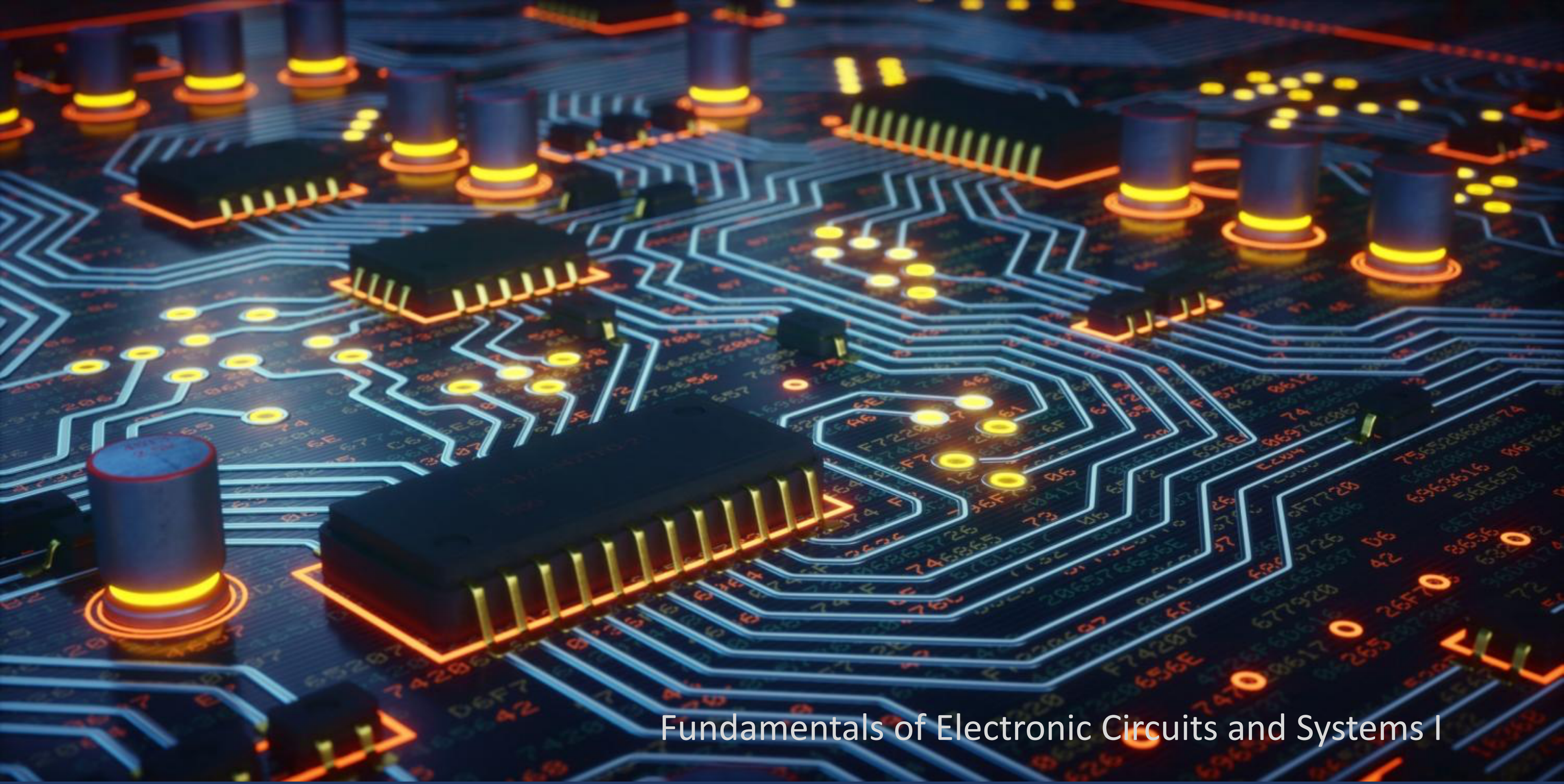

Fundamentals of Electronic Circuits and Systems I

Capacitor & Inductor

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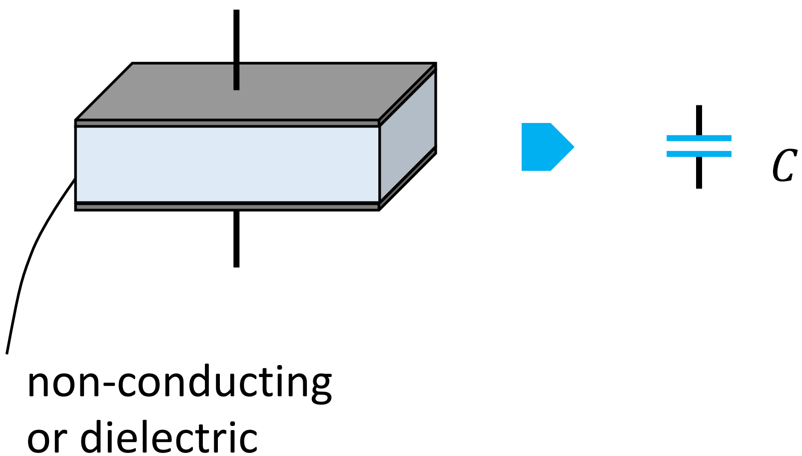
Outlines

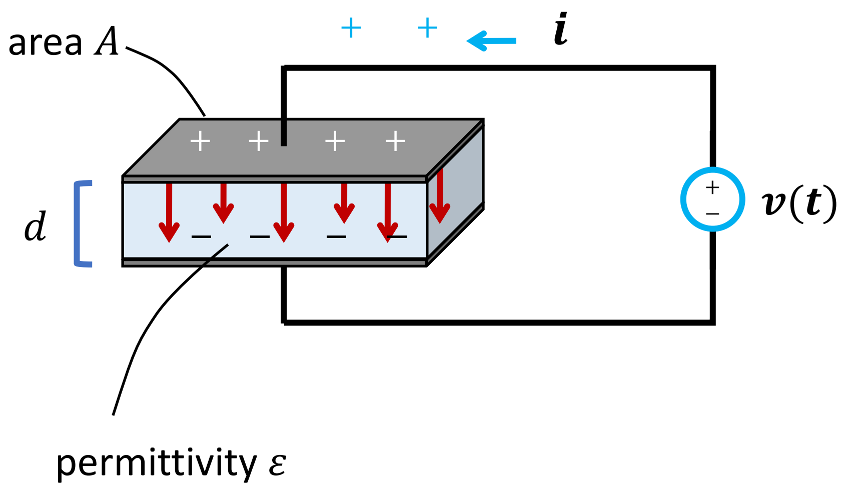
- Capacitor
- Inductor
- Magnetically coupled networks

What is a capacitor?

A CAPACITOR is a circuit element that consists of two conducting surface separated by a non-conducting, or dielectric, material.



What is a capacitor?

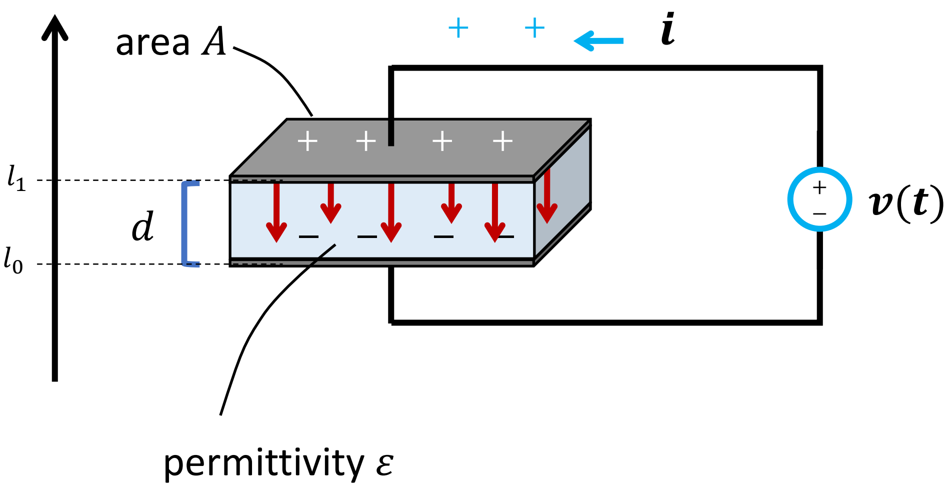


- Step 1: Voltage difference $v(t)$ applied between the two conductors
- Step 2: Charge $q(t)$ is transferred from the source to the capacitor
- Step 3: an electric field across dielectric is created, with a strength of

$$E(t) = \frac{q(t)}{\varepsilon A}$$

electric field $E(t)$ is proportional to charges $q(t)$ on conductors

What is a capacitor?



The voltage between two surfaces

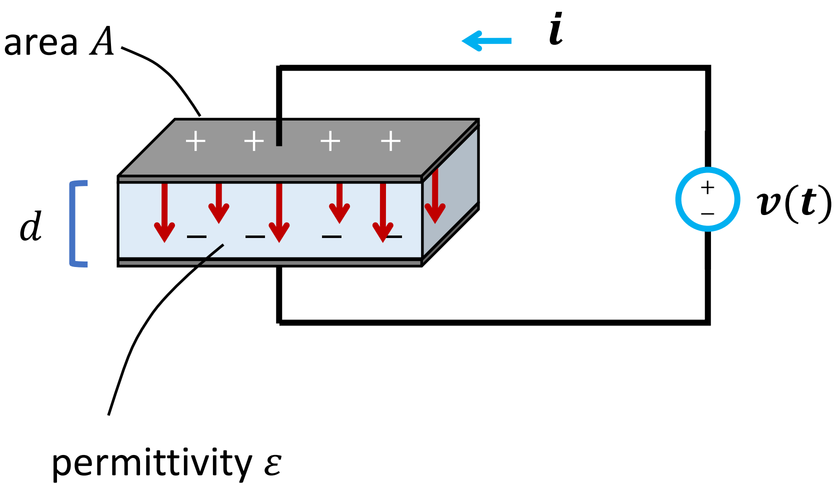
$$v(t) = \int_{l_0}^{l_1} E(t)dl = E(t)l(t)$$

The charge on the capacitor

$$q(t) = E(t)\varepsilon A = \frac{v(t)}{l(t)}\varepsilon A = \underbrace{\boxed{\frac{\varepsilon A}{l(t)}}}_{C(t)} v(t)$$

Potential difference $v(t)$ between conductors is proportional to charges $q(t)$ on conductors

What is a capacitor?

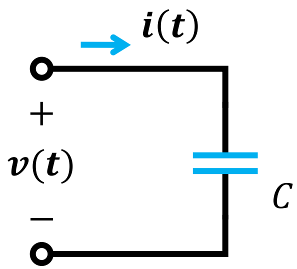


CAPACITANCE of two parallel plates separated by air

$$C = \varepsilon_0 \frac{A}{d}$$

- The unit of capacitance: ***farad*** **(F)**
- More frequently used: μF, pF
- $\varepsilon_0 = 8.85 \times 10^{-12} F/m$, the permittivity of free space

What is a capacitor?



The charge on the capacitor $q = Cv(t)$

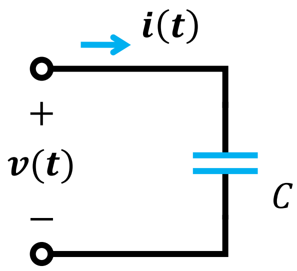
CAPACITANCE is defined as

ratio of charge, $q(t)$, "stored" on conductors to

potential difference ,$v(t)$, between them

Current CANNOT actually flow through a capacitor

What is a capacitor?



$$q = Cv(t)$$

The charge on the capacitor $q(t) = Cv(t)$

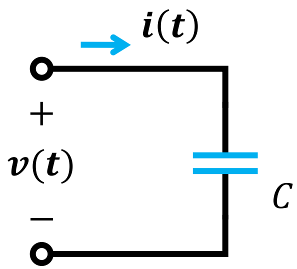
According to the define of current $i(t) = \frac{dq(t)}{dt}$

The current-voltage terminal characteristics of capacitor

$$i(t) = C\frac{dv(t)}{dt}$$

A capacitor is an open circuit at DC

What is a capacitor?



$$q = Cv(t)$$

$$i(t) = C\frac{dv(t)}{dt}$$

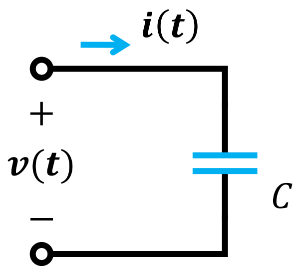
$$dv(t) = \frac{1}{C} idi$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(x)dx \quad = \frac{1}{C}\int_{-\infty}^{t_0} i(x)dx + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

$$= v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

The voltage due to the charge accumulated on C from time $t = -\infty$ to time $t = t_0$

What is a capacitor?



$$q = Cv(t)$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

$$v(t+\Delta t) - v(t) = \frac{1}{C}\int_{t}^{t+\Delta t} i(\tau)d\tau$$

$$|v(t+\Delta t) - v(t)| \geq 0$$

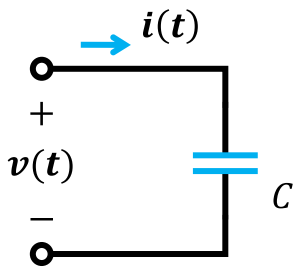
$$\left|\frac{1}{C}\int_{t}^{t+\Delta t} i(\tau)d\tau\right| \leq \frac{1}{C}\int_{t}^{t+\Delta t} |i(\tau)|d\tau$$

$$\leq \frac{1}{C}\int_{t}^{t+\Delta t} I_{max}d\tau \quad = \frac{1}{C}I_{max}\Delta t \xrightarrow{\Delta t\to 0} 0$$

$$0 \leq |v(t+\Delta t) - v(t)| \leq 0$$

Voltage on capacitor CANNOT change abruptly

What is a capacitor?



POWER delivered to a capacitor

$$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$$

ENERGY stored in the electric field

$$\boldsymbol{q = Cv(t)}$$

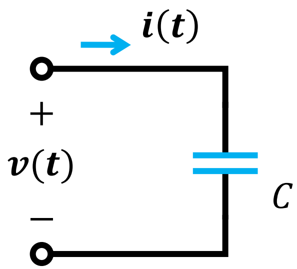
$$\boldsymbol{i(t) = C\frac{dv(t)}{dt}}$$

$$\boldsymbol{v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx}$$

$$w(t) = \int_{-\infty}^{t} Cv(\tau)\frac{dv(\tau)}{d t}d\tau = \frac{1}{2}Cv^2(\tau)\Big|_{v(-\infty)}^{v(t)} = \boldsymbol{\frac{1}{2}Cv^2(t)}$$

Example 1

QUESTION: Find the voltage on the capacitor, $v(t)$, with $i(t)$ changes as a step function

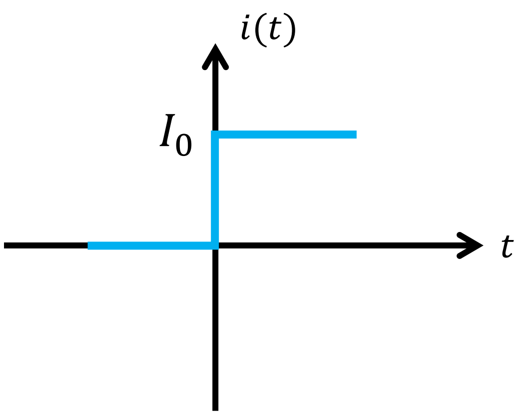


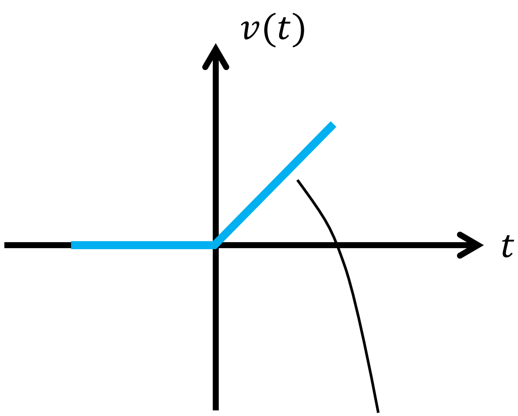
$$i(t) = \begin{cases} 0 & t \leq 0 \\ I_0 & t > 0 \end{cases}$$

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(x)dx$$

If $v(0) = 0$

$$v(t) = \begin{cases} 0 & t \leq 0 \\ \dfrac{I_0 t}{C} & t > 0 \end{cases}$$

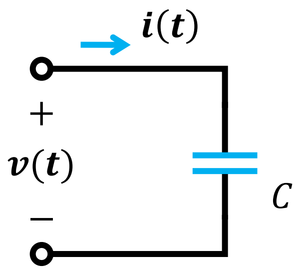




Slope of the line is $\frac{I_0}{C}$

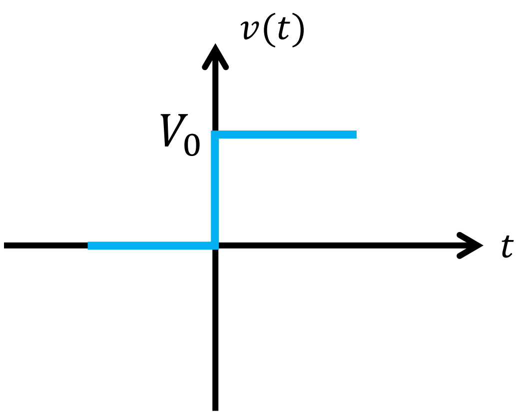
Example 2

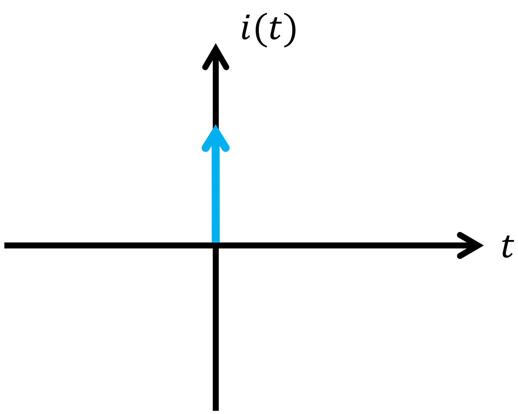
QUESTION: Find the current across the capacitor, $i(t)$, with $v(t)$ changes as a step function



$$v(t) = \begin{cases} 0 & t \le 0 \\ V_0 & t > 0 \end{cases}$$

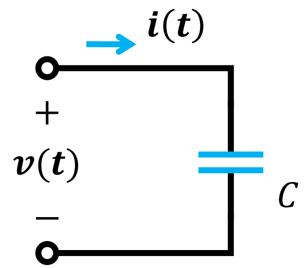
$$i(t) = \frac{dv(t)}{dt} = V_0\delta(t)$$





Outlines

- Capacitor
  - What is a capacitor

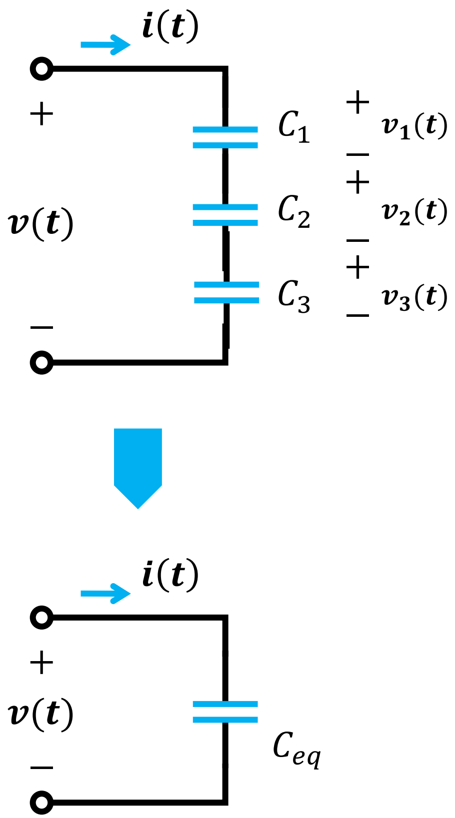


$$q = Cv(t)$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

Capacitors in series



According i-v characteristic of a capacitor

$$i(t) = C_1 \frac{dv_1(t)}{dt} = C_2 \frac{dv_2(t)}{dt} = C_3 \frac{dv_3(t)}{dt}$$

$$i(t) = C_{eq} \frac{dv\,(t)}{dt}$$

According to KVL

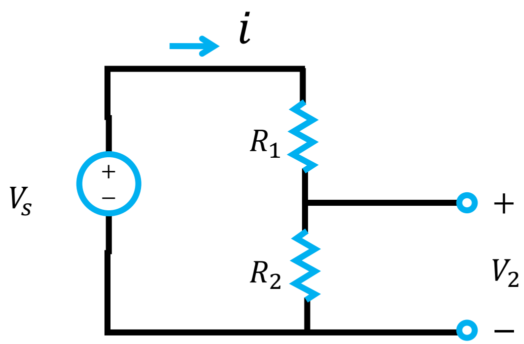
$$i(t) = C_{eq} \frac{d}{dt}\left(v_1(t) + v_2(t) + v_3(t)\right)$$

$$= C_{eq}\, i(t) \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\boldsymbol{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Review: resistors in series

VOLTAGE DIVIDER



According to KVL $\quad V_s = iR_1 + iR_2$

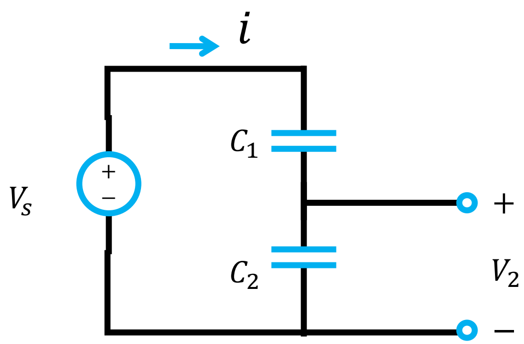
➔ $i = \frac{V_s}{R_1+R_2}$

$$V_2 = iR_2 = \frac{R_2}{R_1 + R_2} V_s$$

Voltage divided over resistors

- **Equivalent resistance, R_{eq}, (series) is sum of resistances** $\quad R_{eq} = \sum_{i=1}^{N} R_i$
- **Voltage difference across a single resistance of resistors in series** $\quad V_i = \frac{R_i}{R_{eq}} V_s$

Voltage divider for capacitors



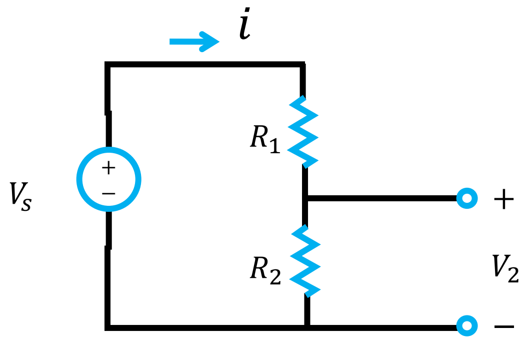
According to KVL

$$V_s = v_1(t) + v_2(t)$$

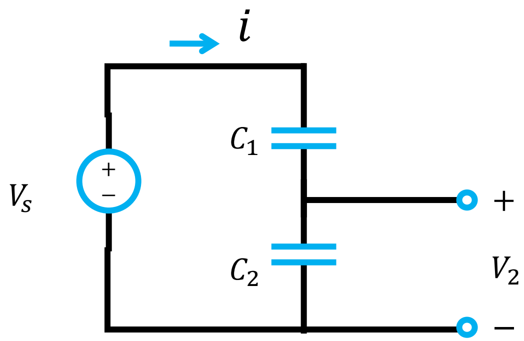
$$= \frac{1}{C_1}\int i(t)dt + \frac{1}{C_2}\int i(t)dt$$

$$\Rightarrow \begin{cases} \boldsymbol{v_1(t) = \dfrac{C_2}{C_1 + C_2} v(t)} \\ \boldsymbol{v_2(t) = \dfrac{C_1}{C_1 + C_2} v(t)} \end{cases}$$

Voltage divider

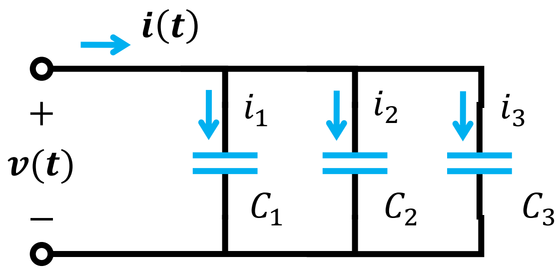


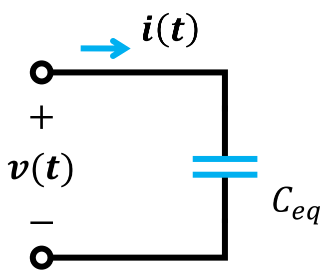
$$
\begin{cases}
v_1(t) = \dfrac{R_1}{R_1 + R_2} v(t) \\
v_2(t) = \dfrac{R_2}{R_1 + R_2} v(t)
\end{cases}
$$



$$
\begin{cases}
v_1(t) = \dfrac{C_2}{C_1 + C_2} v(t) \\
v_2(t) = \dfrac{C_1}{C_1 + C_2} v(t)
\end{cases}
$$

Capacitors in parallel





According to KCL

$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

According i-v characteristic of a capacitor

$$i(t) = C_1 \frac{dv\,(t)}{dt} + C_2 \frac{dv(t)}{dt} + C_3 \frac{dv(t)}{dt}$$

$$= (C_1 + C_2 + C_3) \frac{dv\,(t)}{dt}$$

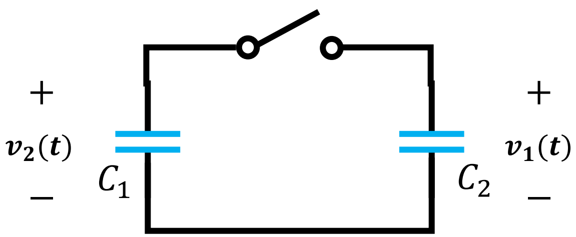
For the equivalent circuit

$$i(t) = C_{eq} \frac{dv\,(t)}{dt}$$

$$\boldsymbol{C_{eq} = C_1 + C_2 + C_3}$$

Example 3

QUESTION: Find the total energy stored in the two capacitors before & after the switch is closed, if the charges on the two capacitors at $t = 0$ are $q_1(0) = Q_1$ and $q_2(0) = Q_2$, respectively.



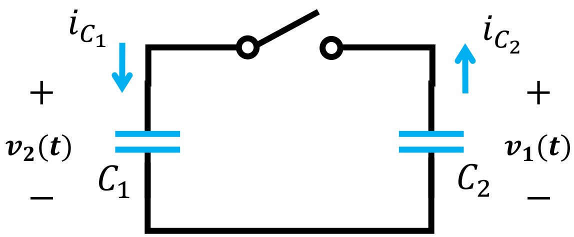
- BEFORE the switch is turned on ($t < 0$)

  total energy stored in the two capacitors

$$w(t < 0) = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$$

Example 3

QUESTION: Find the total energy stored in the two capacitors before & after the switch is turned on, if the charges on the two capacitors at $t = 0$ are $q_1(0) = Q_1$ and $q_2(0) = Q_2$, respectively.



$$w(t < 0) = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$$

- AFTER the switch is turned on ($t \geq 0$)

  Assume the circuit is in steady state @ $t = t_1$

  According to KVL

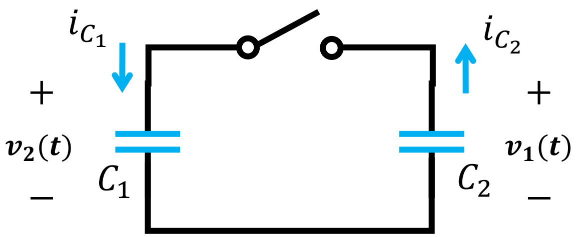
$$v_1(t) = v_2(t) \quad \Rightarrow \quad \frac{q_1(t)}{C_1} = \frac{q_2(t)}{C_2}$$

  According to conservation of charge

$$q_1(t) + q_2(t) = Q_1 + Q_2$$

Example 3

QUESTION: Find the total energy stored in the two capacitors before & after the switch is turned on, if the charges on the two capacitors at $t = 0$ are $q_1(0) = Q_1$ and $q_2(0) = Q_2$, respectively.



$$w(t < 0) = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$$

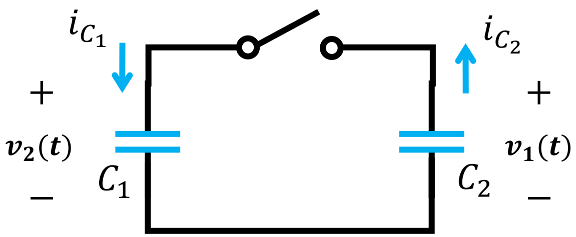
- AFTER the switch is turned on ($t \geq 0$). Assume the circuit is in steady state @ $t = t_1$

$$\begin{cases} \dfrac{q_1(t)}{C_1} = \dfrac{q_2(t)}{C_2} \\ q_1(t) + q_2(t) = Q_1 + Q_2 \end{cases}$$

$$\Rightarrow \begin{cases} q_1(t_1) = \dfrac{C_1}{C_1 + C_2}(Q_1 + Q_2) \\ q_2(t_1) = \dfrac{C_2}{C_1 + C_2}(Q_1 + Q_2) \end{cases}$$

Example 3

QUESTION: Find the total energy stored in the two capacitors before & after the switch is turned on, if the charges on the two capacitors at $t = 0$ are $q_1(0) = Q_1$ and $q_2(0) = Q_2$, respectively.



$$w(t<0) = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$$

- AFTER the switch is turned on ($t \geq 0$). Assume the circuit is in steady state @ $t = t_1$

$$w(t_1) = \frac{q_1^2(t)}{2C_1} + \frac{q_2^2(t)}{2C_2}$$

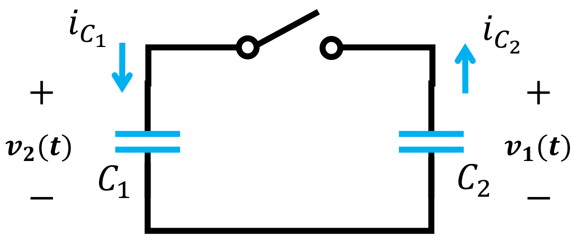
$$= \frac{1}{2C_1}\left(\frac{C_1}{C_1+C_2}(Q_1+Q_2)\right)^2 +$$

$$\frac{1}{2C_2}\left(\frac{C_2}{C_1+C_2}(Q_1+Q_2)\right)^2$$

$$= \frac{(Q_1+Q_2)^2}{2(C_1+C_2)}$$

Example 3

QUESTION: Find the total energy stored in the two capacitors before & after the switch is turned on, if the charges on the two capacitors at $t = 0$ are $q_1(0) = Q_1$ and $q_2(0) = Q_2$, respectively.



- BEFORE the switch is turned on ($t < 0$)

$$w(t < 0) = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$$

- AFTER the switch is turned on ($t \geq 0$). Assume the circuit is in steady state @ $t = t_1$

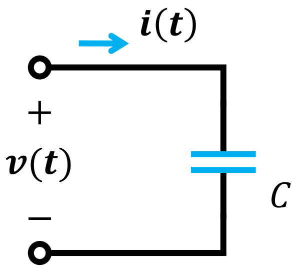
$$w(t_1) = \frac{(Q_1 + Q_2)^2}{2(C_1 + C_2)}$$

WHY $w(t < 0) \neq w(t_1)$?

Outlines

- Capacitor
  - What is a capacitor
  - Capacitors in series/parallel
  - Capacitors voltage divider

- **Inductor**



$$q = Cv(t)$$

$$i(t) = C\frac{dv(t)}{dt}$$

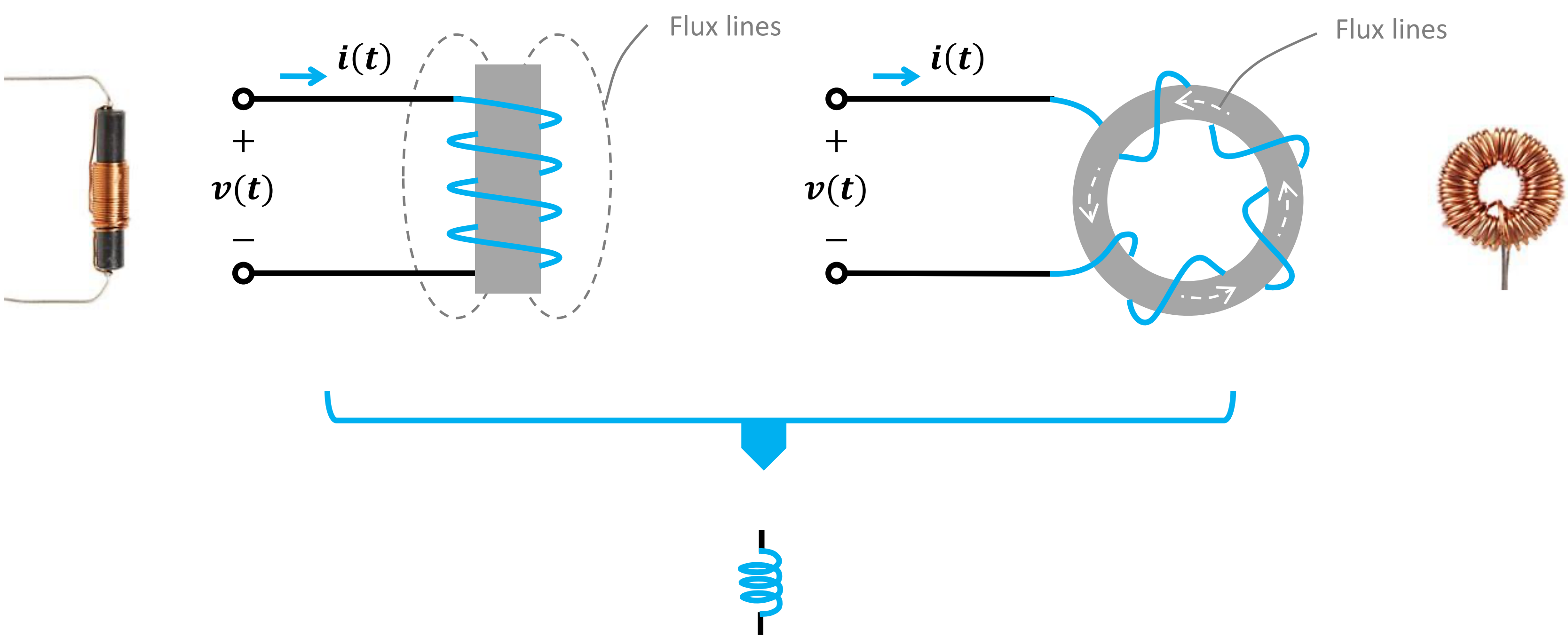
$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

What is an inductor

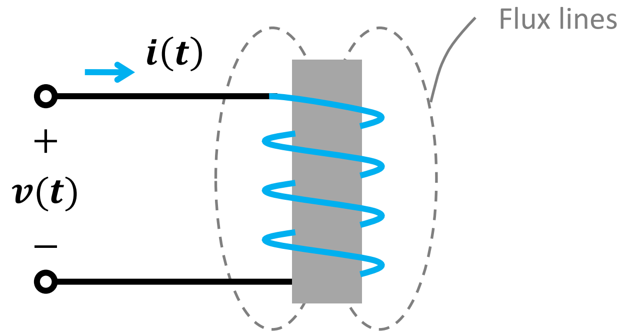
An INDUCTOR is a circuit element that consists of a conducting wire usually in the form of a coil.

Typical inductor I

Typical inductor II



Recall: high school physics



AMPÈRE'S LAW

The magnetic field, B, created by an electric current, I, is proportional to the size of that electric current with a constant of proportionality equal to the permeability of free space, μ_0.

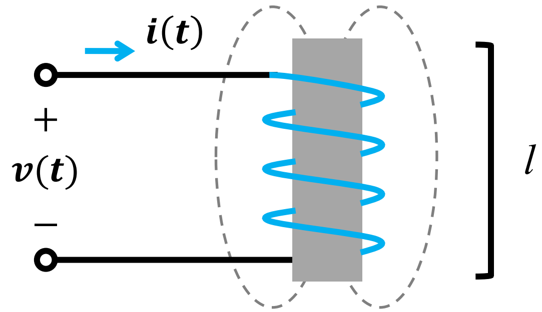
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

FARADAY'S LAW

The electromotive force, ε, or EMF is proportional to the rate of change of magnetic flux, Φ, through a loop to the magnitude of the electro-motive force induced in the loop.

$$\varepsilon = \frac{d\Phi}{dt}$$

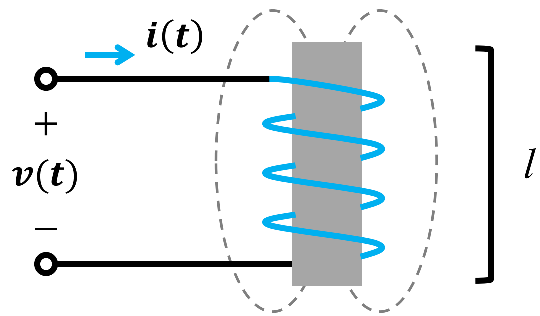
What is an inductor



- Step 1: current $\boldsymbol{i}(\boldsymbol{t})$ pass through the wire
- Step 2: magnetic flux ϕ is generated in the wire
- Step 3: if the wire is coiled, the flux linkage for the coil is

$$\lambda(t) = N\phi$$

What is an inductor



For linear system

$$\lambda(t) = Li(t)$$

According to Faraday's Law

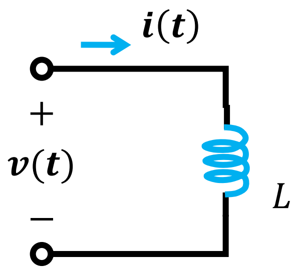
$$v(t) = \frac{d\lambda(t)}{dt} = \frac{dLi(t)}{dt}$$

The current-voltage terminal characteristics of inductor

$$\boldsymbol{v(t) = L\frac{di(t)}{dt}}$$

An inductor is a short circuit at DC

What is an inductor



INDUCTANCE is defined as

ratio of voltage, $v(t)$, across the coils to time

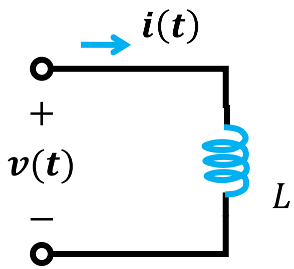
rate of change of current flowing, $i(t)$, through it

$$\lambda(t) = Li(t)$$

$$v(t) = L\frac{di(t)}{dt}$$

- The unit of inductance: ***henry*** **(H)**
- More frequently used: nH

What is an inductor



$$\boldsymbol{\lambda(t) = Li(t)}$$

$$\boldsymbol{v(t) = L\frac{di(t)}{dt}}$$

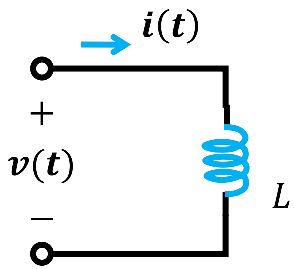
$$di(t) = \frac{1}{L}v(t)dt$$

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(x)dx \quad = \frac{1}{L}\int_{-\infty}^{t_0} v(x)dx + \frac{1}{L}\int_{t_0}^{t} v(x)dx$$

$$= \frac{1}{L}\lambda(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx$$

The flux linkage due to the charge accumulated on L from time $t = -\infty$ to time $t = t_0$

What is an inductor



$$\lambda(t) = Li(t)$$

$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx$$

$$i(t+\Delta t) - i(t) = \frac{1}{L}\int_{t}^{t+\Delta t} v(\tau)d\tau$$

$$|i(t+\Delta t) - i(t)| \geq 0$$

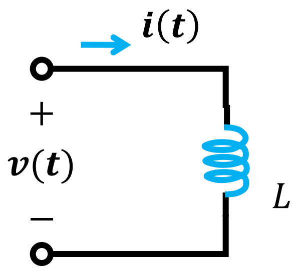
$$\left|\frac{1}{L}\int_{t}^{t+\Delta t} v(\tau)d\tau\right| \leq \frac{1}{L}\int_{t}^{t+\Delta t} |v(\tau)|d\tau$$

$$\leq \frac{1}{L}\int_{t}^{t+\Delta t} V_{max}d\tau \quad = \frac{1}{L}V_{max}\Delta t \xrightarrow{\Delta t \to 0} 0$$

$$\mathbf{0 \leq |i(t+\Delta t) - i(t)| \leq 0}$$

Current through inductor CANNOT change instantaneously

What is an inductor



POWER delivered to an inductor

$$p(t) = v(t)i(t) = Li(t)\frac{di(t)}{dt}$$

$$\boldsymbol{\lambda(t) = Li(t)}$$

ENERGY stored in the electric field

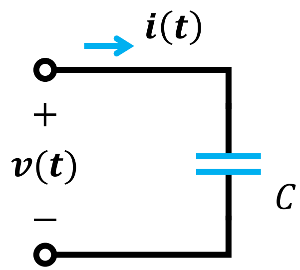
$$\boldsymbol{v(t) = L\frac{di(t)}{dt}}$$

$$w(t) = \int_{-\infty}^{t} Li(\tau)\frac{di(\tau)}{d t}d\tau \quad = \frac{1}{2}Li^2(\tau)\Big|_{i(-\infty)}^{i(t)} \quad \boldsymbol{= \frac{1}{2}Li^2(t)}$$

$$\boldsymbol{i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx}$$

Outlines

- Capacitor
  - What is a capacitor
  - Capacitors in series/parallel
  - Capacitors voltage divider

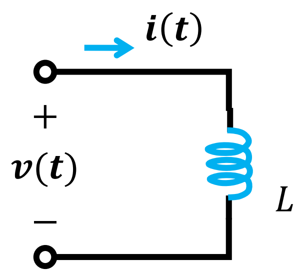


$$q = Cv(t)$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

- Inductor
  - What is an inductor



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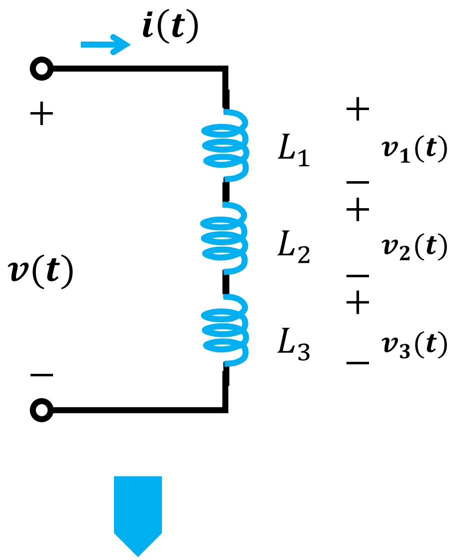
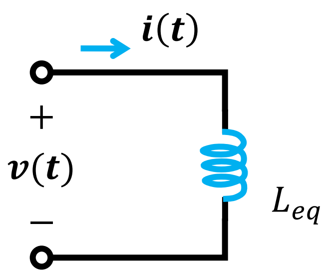
$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx$$

Inductor in series





According to KVL

$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

According i-v characteristic of an inductor

$$v(t) = L_1 \frac{di\,(t)}{dt} + L_2 \frac{di(t)}{dt} + L_3 \frac{di(t)}{dt}$$

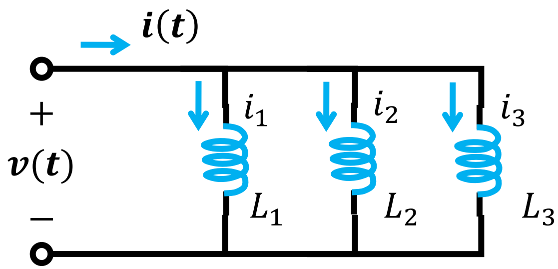
$$= (L_1 + L_2 + L_3) \frac{di\,(t)}{dt}$$

For the equivalent circuit

$$v(t) = L_{eq} \frac{di\,(t)}{dt}$$

$$\boldsymbol{L_{eq} = L_1 + L_2 + L_3}$$

Inductors in parallel

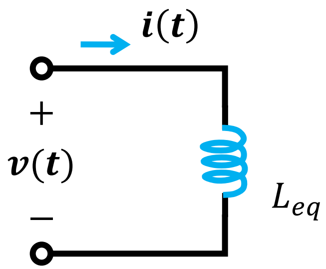


According i-v charasteristic of an inductor

$$v(t) = L_1 \frac{di_1(t)}{dt} = L_2 \frac{di_2(t)}{dt} = L_3 \frac{di_3(t)}{dt}$$

$$v(t) = L_{eq} \frac{di\ (t)}{dt}$$

According to KVL



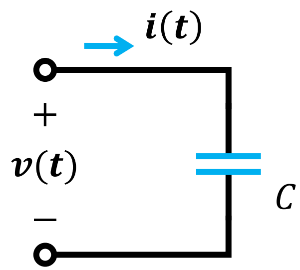
$$v(t) = L_{eq} \frac{d}{dt}(i_1(t) + i_2(t) + i_3(t))$$

$$= L_{eq} i(t) \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right)$$

$$\boldsymbol{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

Outlines

- Capacitor
  - What is a capacitor
  - Capacitors in series/parallel
  - Capacitors voltage divider



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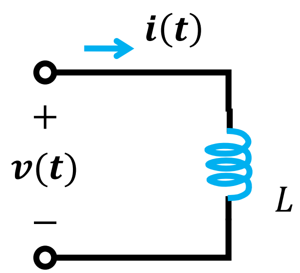
$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

- Inductor
  - What is an inductor
  - Inductors in series/parallel



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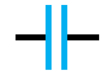
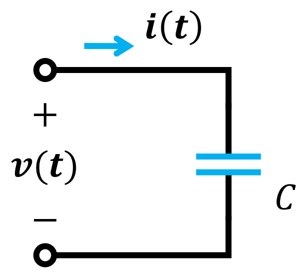
$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx$$

Resistor v.s. Capacitor v.s. Inductor

			
i-v charasteristic	$i = \frac{v}{R}$	$i(t) = C\frac{dv(t)}{dt}$	$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx$
v-i charasteristic	$v = iR$	$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$	$v(t) = L\frac{di(t)}{dt}$
p (power transferred in)	$p = vi$	$p = vi$	$p = vi$
w (stored energy)	0	$w = \frac{1}{2}Cv^2(t)$	$w = \frac{1}{2}Li^2(t)$
Series combination	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel combination	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
DC behavior	NO	open circuit	short circuit
Instantaneous change of v	√	×	√
Instantaneous change of i	√	√	×

Outlines

- Capacitor
  - What is a capacitor
  - Capacitors in series/parallel
  - Capacitors voltage divider



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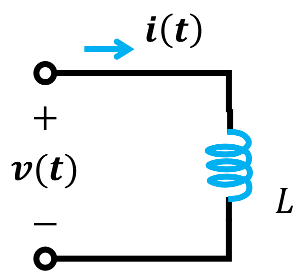
$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

- Inductor
  - What is an inductor
  - Inductors in series/parallel
  - Resistor v.s. capacitor v.s. inductor



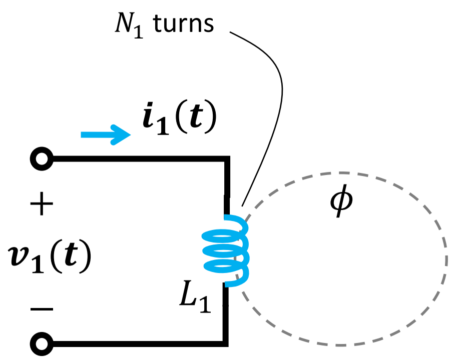
$$\lambda(t) = Li(t)$$

$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx$$

- **Magnetically coupled networks**

What if there is one coil

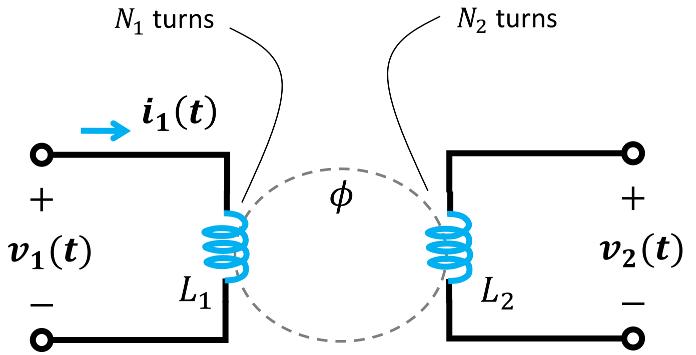


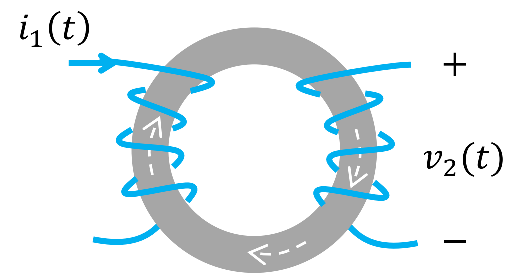
According to AMPÈRE'S LAW, consider the ideal situation

- Flow of electric current will create a magnetic field
- The flux linkage for the coil is $\lambda_1(t) = N_1\phi = L_1 i_1$

For the ideal inductor $v_1(t) = L_1 \dfrac{di_1(t)}{dt}$

What if two coils closed to each other





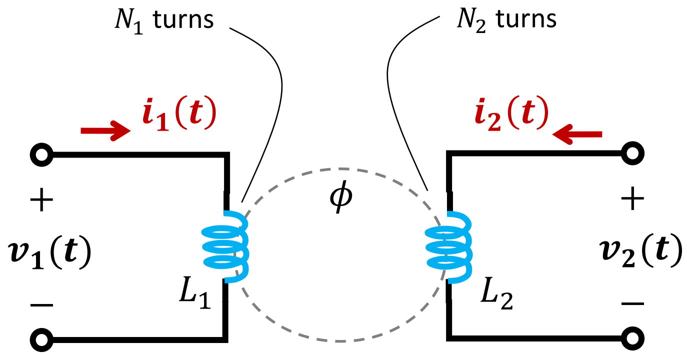
According to FARADAY'S LAW,

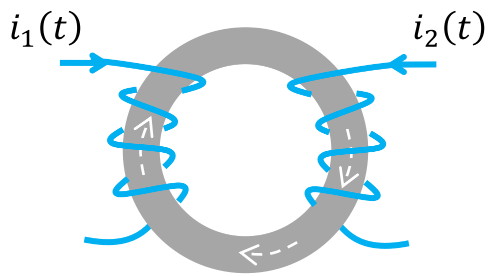
- The flux linkage for the coil N_2 is $\lambda_2(t) = N_2\phi = N_2\dfrac{\lambda_1}{N_1} = N_2\dfrac{L_1 i_1}{N_1}$
- Voltage will be induced

$$v_2(t) = \frac{d\lambda_2(t)}{dt} = \frac{d}{dt}\left(N_2\frac{L_1 i_1}{N_1}\right) = \boxed{\frac{N_2}{N_1}L_1}\frac{di_1}{dt}$$

L_{21}, mutual inductance

Mutual inductance





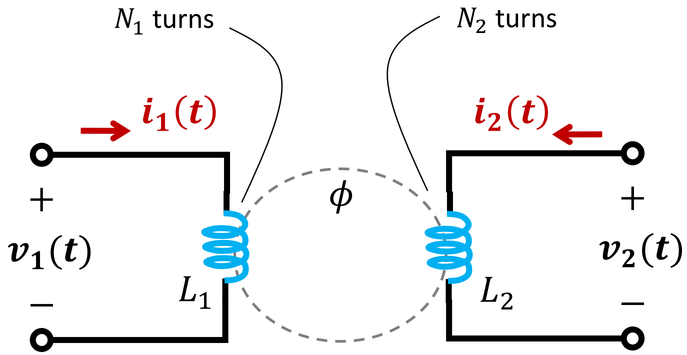
- The flux linkages for each coil

$$\begin{cases}\lambda_1 = L_1 i_1 + L_{12} i_2 \\ \lambda_2 = L_{21} i_1 + L_2 i_2\end{cases}$$

- The current-voltage relationship

$$\begin{cases}v_1 = \dfrac{d\lambda_1}{dt} = L_1 \dfrac{di_1}{dt} + L_{12} \dfrac{di_2}{dt} \\ v_2 = \dfrac{d\lambda_2}{dt} = L_{21} \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt}\end{cases}$$

Mutual inductance



$$\begin{cases} v_1 = \dfrac{d\lambda_1}{dt} = L_1 \dfrac{di_1}{dt} + L_{12}\dfrac{di_2}{dt} \\ v_2 = \dfrac{d\lambda_2}{dt} = L_{21} \dfrac{di_1}{dt} + L_2\dfrac{di_2}{dt} \end{cases}$$

- Self-inductances L_1, L_2, and mutual inductance L_{12}, L_{21}
- For LINEAR SYSTEM $L_{12} = L_{21} = M$

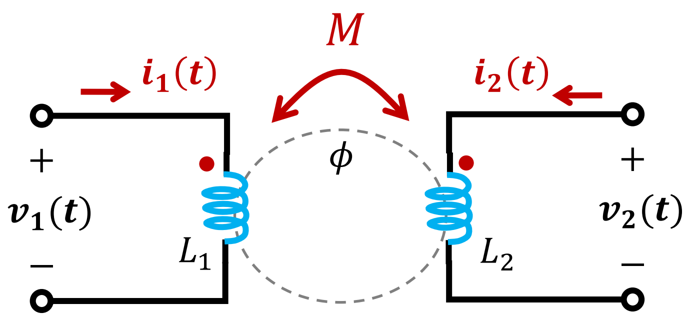
$$\begin{cases} v_1 = \boxed{L_1 \dfrac{di_1}{dt}} + \boxed{M\dfrac{di_2}{dt}} \\ v_2 = M \dfrac{di_1}{dt} + L_2\dfrac{di_2}{dt} \end{cases}$$

a self term due to $i_1(t)$

a mutual term due to $i_2(t)$

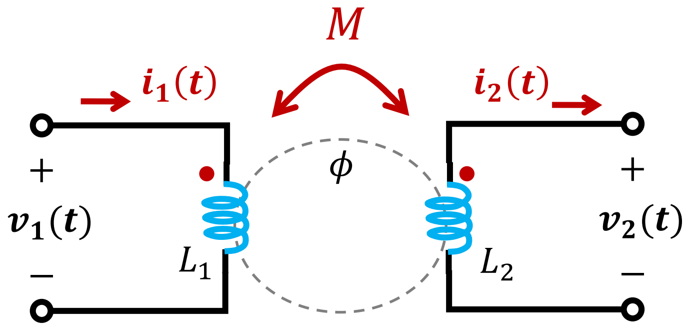
Mutual inductance

- Case 1: both currents enter the dots



$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt} \\ v_2 = M \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} \end{cases}$$

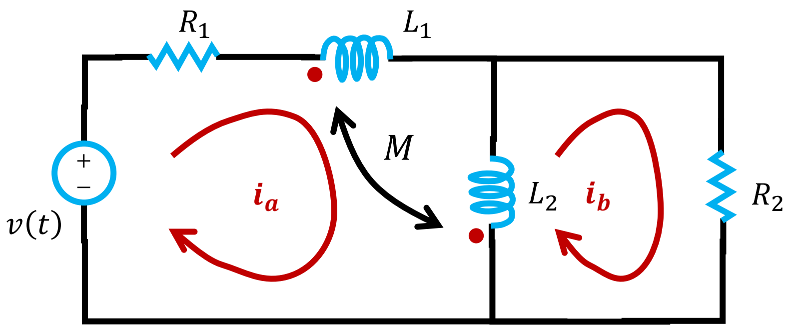
- Case 2: one current enters the dot, the other leaves the dot



$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} - M \dfrac{di_2}{dt} \\ v_2 = -M \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} \end{cases}$$

Example 4

QUESTION: write the equations for the circuit

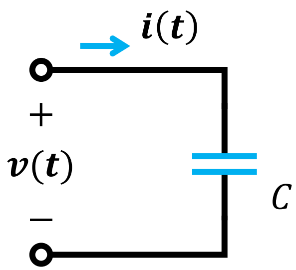


- According to KVL

$$\begin{cases} v(t) = R_1 i_a + \underbrace{L_1 \dfrac{di_a}{dt}}_{\text{self term of } L_1} + \underbrace{M \dfrac{d}{dt}(i_b - i_a)}_{\text{mutual term due to } L_2} + \underbrace{L_2 \dfrac{d}{dt}(i_a - i_b)}_{\text{self term of } L_2} \underbrace{- M \dfrac{di_a}{dt}}_{\text{mutual term due to } L_1} \\ L_2 \dfrac{d}{dt}(i_b - i_a) + M \dfrac{di_a}{dt} + R_2 i_b = 0 \end{cases}$$

Outlines

- Capacitor
  - What is a capacitor
  - Capacitors in series/parallel
  - Capacitors voltage divider

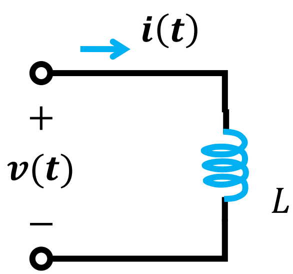


$$q = Cv(t)$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

- Inductor
  - What is an inductor
  - Inductors in series/parallel
  - Resistor v.s. capacitor v.s. inductor

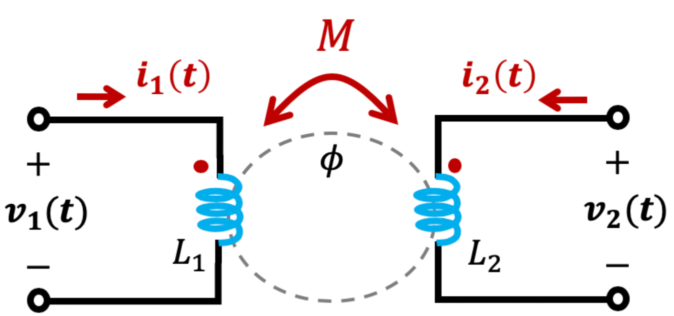


$$\lambda(t) = Li(t)$$

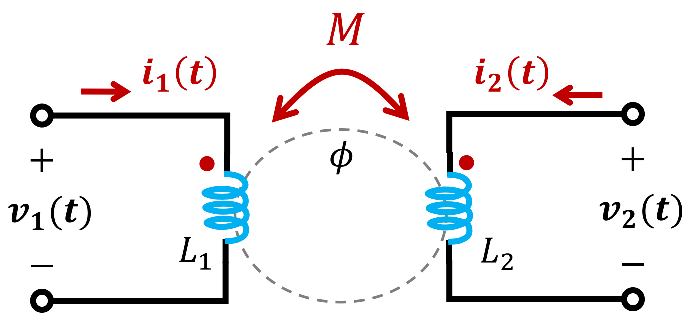
$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx$$

- Magnetically coupled networks
  - What is mutual inductance
  - **Power & Energy**



Power & Energy



$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt} \\ v_2 = M \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} \end{cases}$$

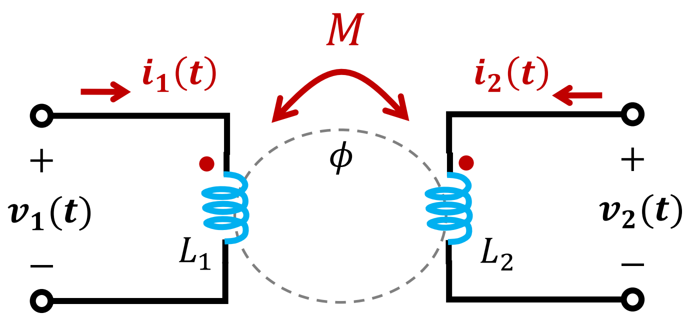
- The total power generated by the network

$$p(t) = v_1(t)i_1(t) + v_2(t)i_2(t)$$

$$= \left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}\right) i_1(t) + \left(M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}\right) i_2(t)$$

$$= \frac{1}{2} L_1 \frac{d}{dt} i_1^2(t) + \frac{1}{2} L_2 \frac{d}{dt} i_2^2(t) + M \frac{d}{dt}\left(i_1(t)i_2(t)\right)$$

Power & Energy



$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt} \\ v_2 = M \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} \end{cases}$$

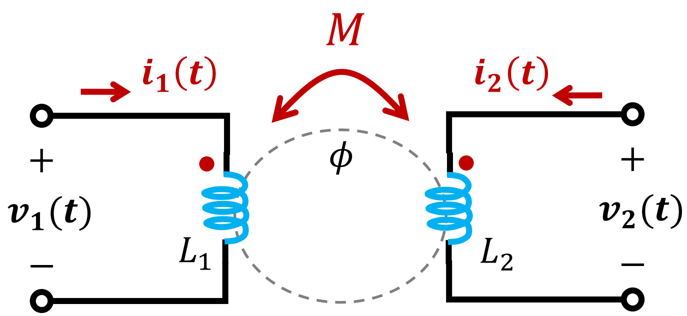
- The total power generated by the network

$$p(t) = \frac{1}{2} L_1 \frac{d}{dt} i_1^2(t) + \frac{1}{2} L_2 \frac{d}{dt} i_2^2(t) + M \frac{d}{dt}\big(i_1(t) i_2(t)\big)$$

- The total energy stored in the network

$$w(t) = \int_{-\infty}^{t} p(\tau) d\tau \quad = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)$$

Power & Energy



$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt} \\ v_2 = M \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} \end{cases}$$

$$w(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)$$

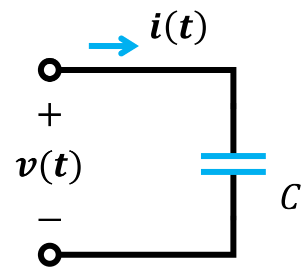
$$= \frac{1}{2}\left(L_1 - \frac{M^2}{L_2}\right) i_1^2 + \frac{1}{2} L_2 \left(i_2 + \frac{M}{L_2} i_1\right)^2$$

Since $w(t) \geq 0$ must be guaranteed ➡ $M \leq \sqrt{L_1 L_2}$

DEFINE the coefficient of coupling between 2 inductors $k = \dfrac{M}{\sqrt{L_1 L_2}}$

Outlines

- Capacitor
  - What is a capacitor
  - Capacitors in series/parallel
  - Capacitors voltage divider



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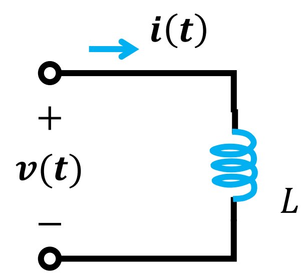
  - What is an inductor
  - Inductors in series/parallel
  - Resistor v.s. capacitor v.s. inductor



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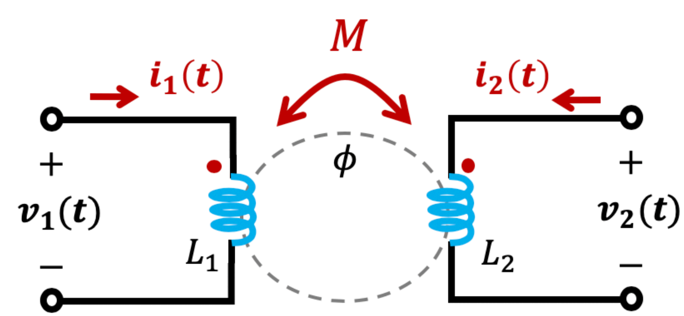
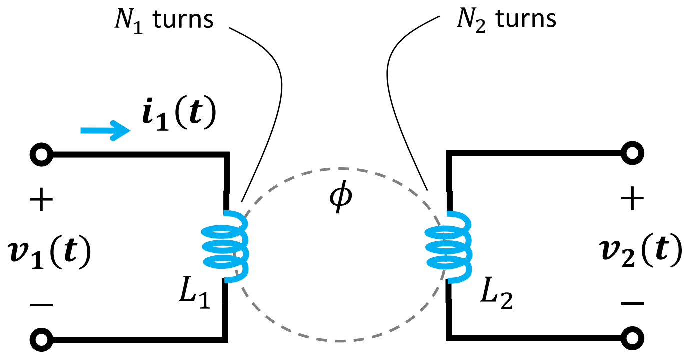
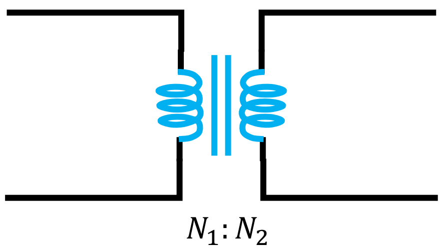
- Magnetically coupled networks
  - What is mutual inductance
  - Power & Energy
  - **Ideal transformer**



Ideal Transformer



Symbol of IDEAL TRANSFORMER



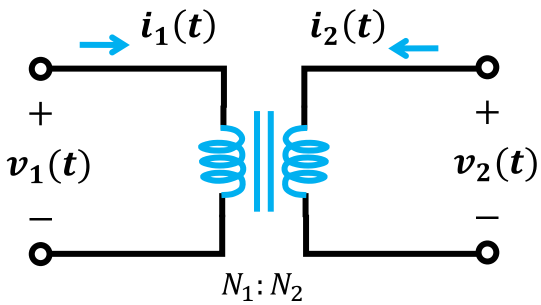
Ideal case

- The core flux, ϕ, links **ALL** the turns of both coils
- There is **NO** wire resistance

$$\begin{cases} v_1 = N_1 \dfrac{d\phi}{dt} \\ v_2 = N_2 \dfrac{d\phi}{dt} \end{cases} \Rightarrow \frac{v_1}{v_2} = \frac{N_1}{N_2}$$

is DEFINED as turns ratio

Ideal Transformer

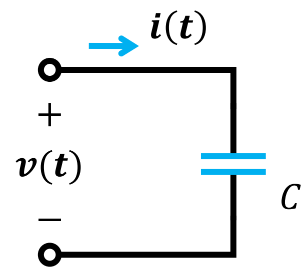


$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

- $i_2(t)$ is generated from coupling
- According to Ampère's LAW $\quad N_1 i_1 + N_2 i_2 = 0$

Outlines

- Capacitor
  - What is a capacitor
  - Capacitors in series/parallel
  - Capacitors voltage divider

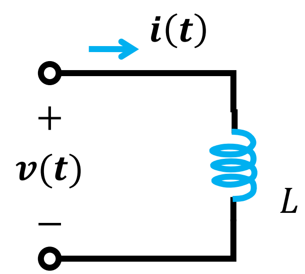


$$q = Cv(t)$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)dx$$

- Inductor
  - What is an inductor
  - Inductors in series/parallel
  - Resistor v.s. capacitor v.s. inductor

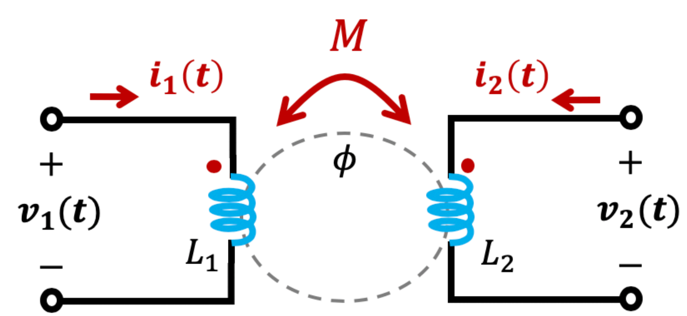


$$\lambda(t) = Li(t)$$

$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx$$

- Magnetically coupled networks
  - What is mutual inductance
  - Power & Energy
  - Ideal transformer



Reading tasks & learning goals

- Reading tasks
  - Basic Engineering Circuit Analysis, 10^{th} edition
    - Chapter 6 and 10
- Learning goals
  - Be able to calculate **V/I** for capacitor/inductor
  - Be able to calculate **stored energy** for capacitor/inductor
  - Be able to combine cap./ind. in series/parallel
  - Understand the concepts of mutual inductance, coefficient of coupling, and turns ratio