PERFORMANCE TASK

PRE-FINALS MODELING AND SIMULATION

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EXPONENTIAL DISTRIBUTION

In a certain game, a boss-spawns on average every 5 minutes. What is the probability that no boss will spawn within the next 3 minutes?

$$P(X > x) = e^{-\lambda x}$$

SOLUTION

$$P(X > 3) = e^{-1/5} * 3$$

$$e^{-0.6}$$

$$e = 2.71828$$

$$e^{-0.6} = 1/e^{0.6} = 1/2.71828^{0.6}$$

$$2.71828^{0.6} = 1.8221$$

1/1.8221 = 0.5488 or 54.88% chance that no boss will spawn within the next 3 minutes

NORMAL DISTRIBUTION

You conducted an experiment growing 100 plants of the same species. You found that their heights follow a normal distribution with:

- Mean (μ) = 45 cm
- Standard deviation (σ) = 5 cm

What is the probability that a randomly selected plant is taller than 50cm?

SOLUTION

Where:

X = 50 cm

μ = 45 cm

σ = 5 cm

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{50 - 45}{5}$$

$$Z = \frac{5}{5}$$

$$Z = 1$$

$$P(Z<1)=0.8413$$

$$P(X>50)=1-P(Z<1)=1-0.8413=0.1587$$

POISSON DISTRIBUTION

During rush hour, an average of 8 cars pass through a toll gate every 5 minutes.

What is the probability that exactly 10 cars will pass through the toll gate in a 5-minute interval?

$$P(X = k) = \frac{e^{-\lambda} \cdot \lambda k}{k!}$$

SOLUTION

Where:

$\lambda = 8$

$k = 10$

$e = 2.71828$

$$P(X = 10) = \frac{e^{-8} \cdot 8^{10}}{10!}$$

$e^{-8} = 0.000335$

$8^{10} = 1,073,741,824$

$10! = 3,628,800$

$$P(X = 10) = \frac{0.000335 \cdot 1,073,741,824}{3,628,800}$$

$P(X = 10) = 0.1126$ or 11.26% chance that exactly 10 cars will pass through the toll gate during a 5-minute interval

BINOMIAL DISTRIBUTION

You are taking a 10-question true or false quiz without reviewing, so you guess randomly on each question.

What is the probability that you answer exactly 6 questions correctly?

$$P(X = k) = \binom{n}{k} * p^k * (1 - p)^{n-k}$$

SOLUTION

Where:
n = 10
k = 6
p = 0.5

$$P(X = 6) = \binom{10}{6} * (0.5)^6 * (0.5)^4$$

$$\binom{10}{6} = 210,\ (0.5)^{10} = 1/1024$$

$$P(X = 6) = 210 * 1/1024 = 0.2051$$

20.51% is the approximate chance that you will get 6 out of 10 questions correct by guessing

TRIANGULAR DISTRIBUTION

You’re estimating how long it will take to finish designing a crypto trading dashboard. Based on your experience:

- Minimum time: 2 days
- Maximum time: 6 days
- Most likely time: 4 days

What is the expected time to complete the project?

$$\mu = \frac{a + b + c}{3}$$

SOLUTION

$$\mu = \frac{2 + 6 + 4}{3}$$

$$= \frac{12}{3}$$

$$= 4$$

The expected time to finish designing the crypto dashboard is 4 days

LOGNORMAL DISTRIBUTION

You're analyzing the monthly income of freelancers in a tech industry.
Income is known to follow a lognormal distribution with:

- Mean of the logarithm (μ) = 10
- Standard deviation of the logarithm (σ) = 0.5

What is the probability that a freelancer earns less than $30,000 in a month?

SOLUTION

Solve for P(X<30000)

Natural Logarithm:

ln(30000) =10.31

$$Z = \frac{\ln(X) - \mu}{\sigma}$$

$$Z = \frac{10.31 - 10}{0.5} \qquad = \frac{0.31}{0.5}$$

$$= 0.62$$

P(Z<0.62)= 0.7324, there is a 73.24% chance that a freelancer earns less than $30,000 in a month

GAMMA DISTRIBUTION

You're tracking how long it takes to complete multiple assignments. The time follows a Gamma distribution with:

- Shape parameter (k) = 3
- Rate parameter (λ) = 0.5
- (this means the scale = 1 / λ = 2)

What is the probability that you finish all assignments in less than 6 hours?

$$P(X<x) = \text{GammaCDF}(x; k, \theta)$$

SOLUTION

Where:

x = 6 hours

k = 3 (shape)

θ = 2(scale)

$P(X<x) = \text{GammaCDF}(x; k, \theta)$

$P(X < 6) = 0.5768$, there is an approximately 57.68% chance of finishing all assignments in less than 6 hours

BETA DISTRIBUTION

On Shark Tank, you pitch your product to investors.
You want to model the probability of success in the market using a Beta distribution with:

- $\alpha = 3$ (level of confidence or optimism)
- $\beta = 2$ (level of uncertainty or hesitation)

What is the expected probability that your product will succeed?

$$\mu = \frac{\alpha}{\alpha + \beta}$$

SOLUTION

$$\mu = \frac{3}{3 + 2}$$

$$\mu = \frac{3}{5}$$

$\mu = 0.6$, the expected probability of success is 60%

WEIBULL DISTRIBUTION

The battery life of a phone follows a Weibull distribution with:

- Shape (k) = 2
- Scale (λ) = 3

What is the probability that the battery lasts less than 2 hours?

$$P(X<x)=1-e^{-(x/\lambda)k}$$

SOLUTION

Where:

$x = 2$

$\lambda = 3$

$k = 2$

$$P(X<2) = 1 - e^{-(2/3)^2} = 1 - e^{-0.4444}$$

$$e^{-0.4444} = 0.6412$$

$$P(X<2) = 1 - 0.6412 = 0.3588$$

There is approximately a 35.88% chance that the battery lasts less than 2 hours.

UNIFORM DISTRIBUTION

You flip a fair coin. What is the probability that it lands on heads?

$$P(\text{outcome}) = n1$$

$$P(\text{Heads}) = 1/2 = 0.5$$

The probability of flipping heads is 0.5 or 50%.

THANK YOU