

### 3.3.4.2 Khai triển Maclaurin đối các hàm số cơ bản

Sau đây là khai triển xấp xỉ một số hàm cơ bản tại lân cận  $x = 0$ .

$$1) \quad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots, \quad \forall x \in \mathbb{R} \quad (3.51)$$

$$2) \quad \ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots, \quad -1 < x \leq 1 \quad (3.52)$$

$$3) \quad \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots, \quad \forall x \in \mathbb{R} \quad (3.53)$$

$$4) \quad \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots, \quad \forall x \in \mathbb{R} \quad (3.54)$$

$$5) \quad \sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots, \quad \forall x \in \mathbb{R} \quad (3.55)$$

$$6) \quad \cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots, \quad \forall x \in \mathbb{R} \quad (3.56)$$

$$7) \quad \arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots, \quad -1 \leq x \leq 1 \quad (3.57)$$

$$8) \quad \arcsin x = \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{2n+1} = x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots, \quad -1 < x < 1 \quad (3.58)$$

$$9) \quad (1+x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n = 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^2 + \cdots, \quad |x| < 1, \quad (3.59)$$

$$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)}{n!}$$

$$10) \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots, |x| < 1 \quad (\text{một trường hợp riêng của 9}) \quad (3.60)$$