Using and extending the Tonal Diffusion Model to study the harmonic language of Gabriel Fauré.

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Abstract

In this paper, we apply the Tonal Diffusion Model (Lieck, Moss & Rohrmeier, 2020) to a new corpus of 98 musical works by Gabriel Fauré, a composer known for his refined and evolving harmonic language. We provide new evidence for the finesse of the Tonal Diffusion Model compared to baseline models learning static harmonic profiles. We then investigate the evolution of the harmonic language based on the interpretation of the weights learned by the model, associated with transitions on the Tonnetz. Our results reveal a greater harmonic complexity of Fauré's "third manner" compared to his first, which is in line with generally accepted musicological theories on his music. We also introduce and test two extensions of the Tonal Diffusion Model relevant to studying Fauré's music: a model based on a Tonnetz extended to tritone intervals, and a model allowing a multiplicity of tonal origin. However, we have not been to take full advantage of this latter model in the context of this study.

1 Introduction

The Tonal Diffusion Model (TDM) was introduced by Martin Lieck, Fabian Moss, and Martin Rohrmeier in 2020 as a promising probabilistic tool for computational musicology and Music Information Retrieval (MIR). One of the central challenges in this field has long been the automatic identification of musical key using algorithmic methods (Krumhansl, 2001; Temperley, 2007; Hu & Saul, 2009). Many existing approaches rely on the assumption of prototypical pitch-class distributions associated with fixed harmonic profiles, such as the major or minor mode. While effective in constrained settings, these models often fail to capture the stylistic and harmonic variability observed across individual musical works. The Tonal Diffusion Model goes beyond this paradigm by allowing the inference of piece-specific harmonic representations (or *profiles*) learned directly from the data following a logic close to that of a topic model. Before presenting how we employ and extend the Tonal Diffusion Model in this study, we will first introduce its underlying principles for the unfamiliar reader.

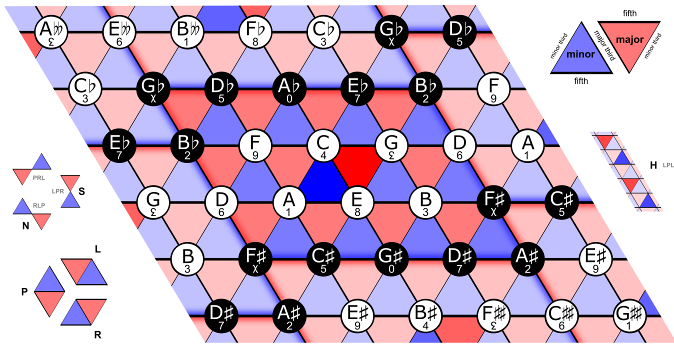

Figure 1: Neo-Riemannian Tonnetz (author : T. Piesk)

1.1 An attempt at popularization

As input, the TDM takes the tonal pitch-class distribution of a musical piece. Coined by Temperley (2000), this term refers to the counting of notes in the piece, assuming octave equivalence (e.g., E4 = E3), while preserving enharmonic distinctions (e.g., B# ≠ C). Tonal pitch-class distributions can also be represented spatially, for instance along a line of fifths. Such distributions already convey partial information about the harmonic content of a piece, but they remain limited in expressiveness and can be difficult to interpret directly.

The TDM relies on a richer representation of tonal space known as the *Tonnetz* (**Figure 1**), whose underlying principle was first introduced by Euler (1729) and has continued to attract the interest of mathematicians and musicologists since its rediscovery in the nineteenth century (Hauptmann, 1853; Riemann, 1896). The Tonnetz can be understood as an infinite graph whose vertices correspond to tonal pitch classes and whose edges represent so-called *primary intervals*. Horizontal edges correspond to successive perfect fifths, while relations of major and minor thirds are represented by diagonals running from bottom left to top right and from bottom right to top left, respectively.

This representation has long been regarded as particularly insightful for the study of harmonic relations, and its cognitive relevance has also been noted (Krumhansl, 1998), although other empirical research suggests that it may be less intuitive for listeners without formal training in music theory (Besada *et al.*, 2024). One way to appreciate the relevance of the Tonnetz for harmonic analysis is to observe that its vertices and edges form a lattice of triangles, each corresponding to a possible major or minor triad. Furthermore, triads that are musically "close"(in the sense that one can be obtained from the other by altering a single pitch through chromatic motion, as described in neo-Riemannian transformational theory — Cohn, 1997 ; Lewin, 1987) are located near each other on the Tonnetz.

More generally, the multiple possible paths connecting two pitch classes on the Tonnetz reflect the different ways their relationship may be interpreted depending on tonal context. For instance, a C# may be understood as a major third above A in the key of A major, but it can also be interpreted as the fifth of the fifth of the fifth of A within a cycle-of-fifths harmonic progression.

We hope to have shown that representing the pitch-class distribution of a musical piece on the Tonnetz allows for a more fine-grained analysis of its harmonic structure than what can be achieved with a simple bar chart.

The central idea behind the Tonal Diffusion Model is to assign a specific weight to each type of transition on the Tonnetz, that is, to each primary interval. In the model, each pitch class is generated by a random walk process starting from a unique initial pitch, referred to as the *tonal origin*, and governed by a probability distribution over possible path lengths. This process is referred to as diffusion. Consequently, the model is defined by three piece-level parameters: a tonal origin, a set of interval transition weights, and a distribution over path lengths. These parameters are typically initialized either randomly or uniformly, and are subsequently optimized so as to generate a pitch-class distribution that best matches the empirical distribution observed in the musical data. This optimization is performed via Bayesian inference; the full mathematical formulation of the model, including its extension to multiple tonal origins, is detailed in the original paper.

An important feature of the TDM is that these piece-level parameters are musically interpretable, as they correspond directly to structural properties of the Tonnetz. For instance, consider the tonal pitch classes F and E♭♭. Moving from one to the other along the axis of minor thirds requires a path of length three, whereas traversing the same relation along the line of fifths requires a much longer path. As a result, if the TDM assigns a relatively high weight to third-based transitions, this suggests a harmonic language characterized by frequent modulations between distant keys, a trait often associated with Romantic harmony.

In this perspective, the authors demonstrate that a one-dimensional TDM assigning zero weight to third-based transitions performs significantly worse on works by Liszt than on music by Bach. They further compare the TDM to several baseline models and show that the variant assuming a Binomial distribution over path lengths achieves the best overall performance. Beyond quantitative evaluation, they illustrate the model's ability to capture salient harmonic features in individual works by Bach, Beethoven, and Liszt, thereby highlighting its descriptive power at the level of single musical pieces.

1.2 Objectives of this study

The first objective of this study is to replicate the original evaluation of the Tonal Diffusion Model by applying it to a new corpus consisting of 98 works by Gabriel Fauré. His music was indeed underrepresented in previously used datasets, with only nine *mélodies* included in the TP3C corpus. BThe originality of our approach also ies in its intermediate scale: it is neither a large-scale diachronic survey nor a purely qualitative analysis of a small number of individual works. In this respect, our methodology is closer in spirit to that of Laneve *et al.* (2023), who employed methods based on the Discrete Fourrier Transform to study the evolution of Debussy's harmonic language (in particular his use of scales and modes) over time.

Although Fauré has not attracted as much scholarly attention as his younger compatriot Debussy, the refinement and subtlety of his harmonic language were already noted by Copland in 1924. Several musicologists have proposed periodizations of Fauré's output, thereby motivating the relevance of a diachronic perspective. In this study, we adopt Tait's (1986) division into three major periods (1861–1884, 1884–1906, and 1906–1924), as it is the most minute and analytically convenient among those we encountered. Modal colour and chromaticism are frequently cited as prominent characteristics of Fauré's second and third stylistic periods (Greer, 1991), and they arguably account for much of the harmonic complexity and modernity of his later works. By contrast, his use of chordal extensions and alterations remains comparatively restrained when compared to that of many of his contemporaries.

In this context, White (2018) advances the idea that such harmonic devices allow Fauré to superimpose two or three distinct "tonal layers" within a single passage, without fully entering the realm of polytonality. The musical examples used to support this claim are drawn primarily from Fauré's second and third periods. However, it would be misleading to suggest a radical stylistic rupture in his output. For example, on the one hand, Greer emphasizes that many traits commonly associated with Fauré's later works are already present in his early compositions, becoming more salient only after the 1880s. On the other hand, Copland and others have noted the relative conventionality of certain early works, such as the First Violin Sonata, as well as the immediately appealing and ear-flattering harmonic language of Fauré's early *mélodies*, which may account for their enduring popularity.

Taken together, these characteristics make Fauré's music an ideal candidate for a challenging evaluation of the Tonal Diffusion Model. At the same time, they suggest the potential need for model extensions when applied to this repertoire. Accordingly, the second objective of this study is to implement two such extensions already mentioned by Lieck, Moss, and Rohrmeier in their original paper: first, the addition of an extra dimension to the Tonnetz through the inclusion of the tritone interval; and second, the allowance for multiple tonal origins, corresponding to the most general formulation of the model.

Eventually, the questions at hand for us are the following : **Can we replicate the performances of TDM on a new body of works by Gabriel Fauré? Could specific extensions of the TDM provide a better modeling of Fauré's harmony and its evolution?**

1.3 Predictions

To answer these questions, we will test the following predictions. Firstly, with regard to the performance of models, we predict that :

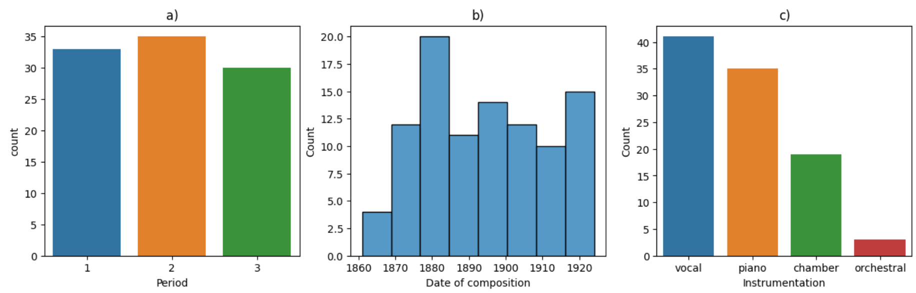

Figure 2: Distribution of the works according to **a)** period, **b)** date of composition, **c)** instrumentation

P1 – The TDM Binomial 2D (fifths + thirds) achieves better average performances than the baseline models on the Fauré corpus.

P2 – The TDM Binomial 2D is more efficient on the second and third periods than on the first (idea of a link between the finesse of the model and the performance on complex harmonic languages).

Then using our newly implemented extensions, we expect that :

P3 – With the TDM Binomial 3D same model (fifths + thirds + tritones), relationships of thirds and of tritones are more important for the works of the second and third periods (this would reflect bolder modulations, a harmony that does not only revolve around the circle of fifths).

P4 – The TDM with multiple tonal origins achieves better performance on works from the second and third periods; in particular, the tonal origins involved are further apart for the third period.

2 Data

We used *Corpusfaurecomplete*, a dataset of tonal pitch class distributions created from the scores of 98 pieces composed by Gabriel Fauré. (Note that our definition of 'piece' may refer to movements or numbers from complete works, taken individually.) Scores were first gathered in XML-based file formats, either extracted from the PDMX (Long et al.,2024; a large corpus of scores scraped from MuseScore), downloaded directly on MuseScore, or transcribed by us. In the latter case, we only took piano reductions for orchestral pieces; we favoured isolated movements over entire works, and transcribed nothing other than the notes (no tempo or expression indications, etc., to save us unnecessary wasted time). All these works were selected by choosing an opus number at random, so as to obtain 20 pieces for each period. Those *mxl* scores were then parsed and their pitch class distributions retrived using the python library *ms3*. 9 additional distributions were added for the pieces already contained in the *TP3C* corpus

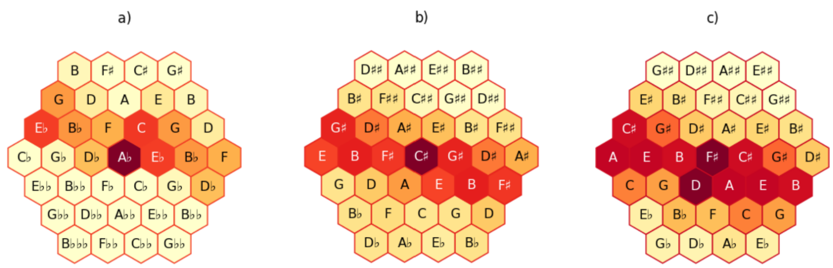

Figure 3: Tonal pitch-class distributions plotted as heatmaps on the Tonnetz, of three pieces from the dataset : **a)** Dans les ruines d'une abbaye Op.2, n°1 (1865, first period), **b)** Nocturne n°7, Op.74 (1898, second period), **c)** Impromptu n°5, Op.102 (1909, third period). The plots were generated with the Python library *pitchplots* (Moss et al., 2019) slightly revised and updated by ourselves.

used by Lieck, Moss & Rohrmeier. We further added information about each piece's (eventual) title, date of composition, (and thus period in our nomenclature), work group, opus number, movement number, etc., drawing on what was precised in the International Music Score Library Project (IMSLP). We assigned to each work its corresponding *period* in Fauré's output, following Tait's nomenclature.

The dataset contains works composed between 1860 and 1924 (**Figure 2.b**), that is encompassing all of Fauré's career as a composer. One familiar wit the composer's music will not be surprised to find mostly *mélodies* and pieces for piano solo, although there are also a few movements from his chamber or orchestral works (**Figure 2.c**). Our dataset thus seems representative enough of Fauré's , even though it is not exhaustive.

3 Methods

We used, adapted and slightly reworked the code provided in the supplementary github repository to Lieck, Moss & Rohrmeier (2020). In particular, we had to change the implementation of the Kullback-Leibler Divergence, and of the log-binomial probability to make it work on our machine. We also simplified the loops for model evaluation, plots, and saving of the results.

To implement the "tritone" extension, we simply added up and down tritone transitions to the existing interval steps parameter (+6 and -6 in terms of fifths), and gave larger initial weights to fifths and thirds in the transition matrix. In our implementation of the multi-center TDMD, tonal origins were treated explicitly as initial states of the latent Tonnetz diffusion process, rather than being handled *via* circular shifts of a single reference distribution. The marginal likelihood was computed by running the diffusion independently from each possible tonal origin and marginalising over path length, using the same dynamic-programming scheme as in the original model article, to which we re-

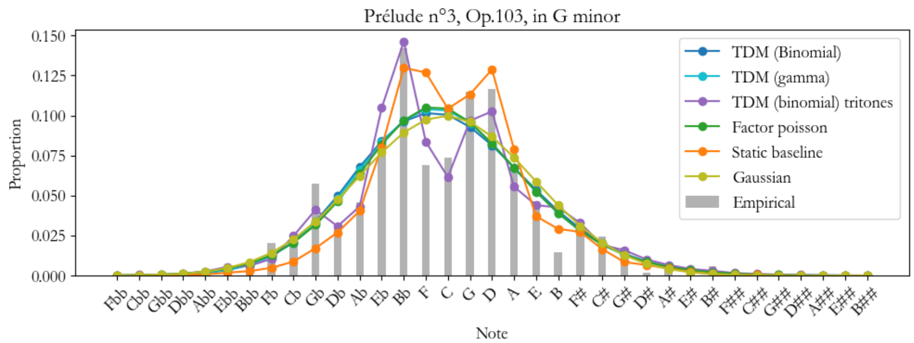

Figure 4: Barplot of the profiles learned by the different models versus the actual distribution of scores, for a piece taken from the 3rd period. The static model seems a bit lucky here. On the other hand, we observe the accuracy of our Triton Binomial TDM, which perfectly matches the distribution, whereas the other TDMs merely imitate a Gaussian distribution and seem unsuitable.

fer for further details. For the initial Dirichlet process, we drew on its existing implementation for interval weights only.

After normalizing the score counts, we successively launched 4 instances of the TDM (varying path-length distributions), on the entire corpus: Poisson 2D, Binomial 2D, Gamma 2D, and Binomial 3D (i.e with tritone transitions added on the Tonnetz). We also evaluated three baseline models from among those implemented and introduced by Lieck, Moss & Rohrmeier. The GaussianModel simply fits a Gaussian distribution to the tonal pitch-class distribution. The SimpleStaticModel learns a single profile across the entire corpus and applies it to each distribution by transposing it. The FactorModel (Poisson) is a simplified version of TDM: it is a diffusion model based on Tonnetz, but instead of explicitly simulating all the steps of the diffusion process, it assumes that the final result can be written as a direct product of several independent "factors," one for each type of interval.

For each piece and each model, the selected "loss" corresponds to the Kullback-Leibler divergence between the actual rating distribution and the profile generated by the model after 700 iterations, including any early stopping (**Figure 4**. (For the Gaussian Model, however, only one pass through the data is required.) We selected the learned parameters (i.e interval weights) associated with the best loss. Each of our hypotheses was subjected to an adequate statistical test.

4 Results

4.1 Model comparison

On average, across the entire corpus, the TDM Binomial 2D performs significantly better than all other 2D TDM models (Poisson, Gamma) or baseline

Model	p	μ
TDM Binomial		0.05670
TDM Gamma	< 0.0000	0.07528
FactoModel Poisson	0.0001	0.06405
StaticModel (2 profiles)	< 0.0000	0.13594
GaussianModel	< 0.0000	0.09592

Table 1: Results of a series of Wilcoxon tests comparing the average loss (Kullbback-Leibler divergence) for each model with that of the TDM Binomial.

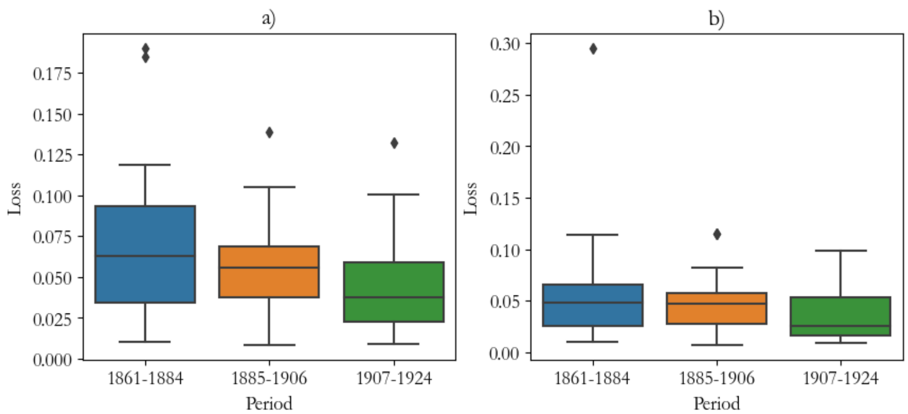

Figure 5: Boxplots of the loss according to each period, for models **a)** TDM Binomial 2D, **b)** TDM Binomial 3D (with tritones).

models (**Table 1**). Thus, **P1** is verified.

Besides, the new TDM Binomial 3D achieves even better average ($\mu = 0.04683$), but also per-period performances than its 2D equivalent ($p < 0.0000$; Wilcoxon test).

As for our new multi-center TDM, ...

At last (**Figure 5**), one can observe that these two best models (binomial 2D and 3D, respectively) perform significantly better on works from Fauré's third manner than on works from the first period ($p_1 = 0.0111$; $p_2 = 0.0197$; Mann-Whitney U tests). The results are no longer significant when we compare the first and second manners. Therefore, **P2** is only half-verified.

4.2 Interval weights analyses

The (normalised) weights of fifths transitions on the Tonnetz remain constantly high across the different periods, and there is no signifcant difference between the first and the others. As opposed to our expectations, weights given to thirds transitions (both minor and major) are not significantly different accross

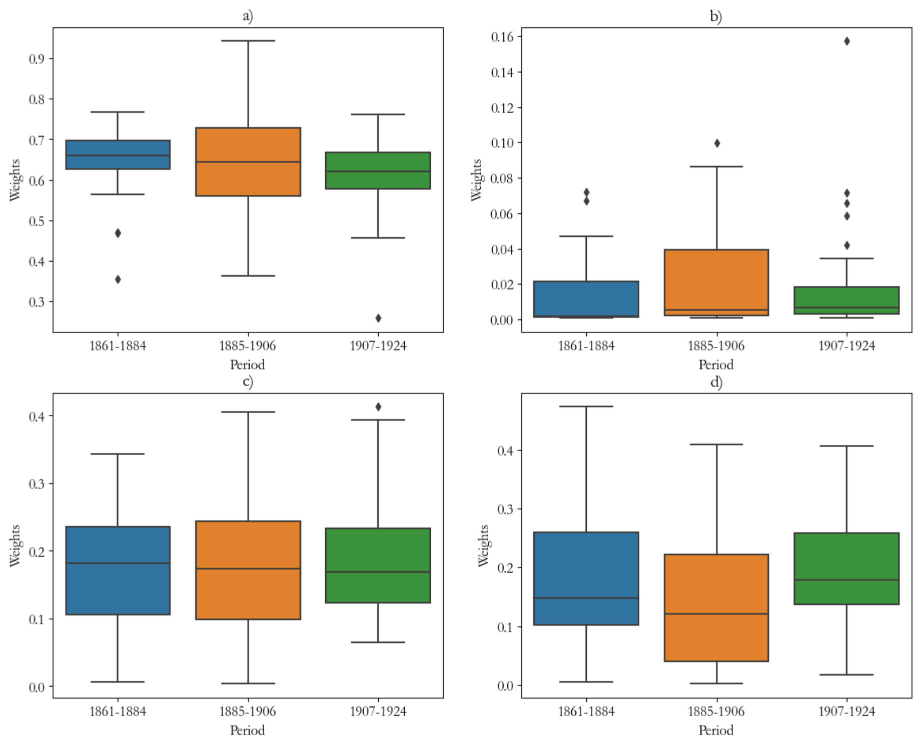

Figure 6: Boxplots of the TDM Binomial 3D normalised weights according to each period, in the following order **a)** fifths, **b)** tritones, **c)** minor thirds, **d)** major thirds.

periods. In particular, we observe that these weights tend already to be high for the first period. However, tritone weights are significantly higher for both second and third period works compared to that of the first period ($p_2 = 0.0083$ and $p_2 = 0.0219$ respectively). We observe further that those tritone weights are very small (more than 10 times smaller than the weights of thirds as a matter of fact).

5 Discussion

The results presented above provide partial but informative support for the hypotheses formulated in this study, while also highlighting several methodological and conceptual limitations of the Tonal Diffusion Model when applied to Fauré's music.

The first hypothesis (**P1**) is clearly supported by our results: among all two-dimensional models, the Binomial TDM consistently outperforms both alternative path-length distributions (Poisson and Gamma) and the baseline models. This finding corroborates the conclusions of Lieck, Moss, and Rohrmeier, who already identified the Binomial distribution as particularly well suited for modeling tonal diffusion processes. One possible explanation is that the Binomial distribution implicitly constrains path lengths more strongly than the Poisson or Gamma alternatives, thereby preventing excessively long random walks on the Tonnetz that would result in overly diffuse pitch-class distributions.

Extending the model to three dimensions by including the tritone interval further improves performance, both on average and within each stylistic period. This result suggests that the additional Tonnetz dimension captures harmonic relations that are not adequately modeled by fifths and thirds alone. In this respect, the superiority of the Binomial 3D model supports the idea that Fauré's harmonic language—especially in its later stages—cannot be reduced to movements along the circle of fifths and simple third relations.

This yielded such unconvincing results that we did not include them, and we cannot verify hypothesis 4. The model tends to learn paths of excessively long lengths (more than 40 steps), which completely smooths the distributions. This may be due to an implementation problem (although we followed the mathematical and algorithmic formulation of Lieck, Moss, & Rohrmeier to the letter) that we unfortunately did not have time to resolve.

6 Conclusion

This work successfully replicated the results of the original article on the Tonal Diffusion Model (TDM), applying it to a new corpus of works by Gabriel Fauré. As in the initial study of Liszt's works, the classical Binomial variant of the TDM (based on the relationships of fifths and thirds) significantly outperforms baseline models with static harmonic profiles. This confirms the value of this approach for representing complex and evolving harmonic structures. Furthermore, the introduction of a tritone extension of the model (which includes tritone intervals in the Tonnetz) further improves performance on the Fauré corpus, particularly for works from the composer's later period. These results suggest that this "3D" variant of the TDM effectively captures more subtle and unstable harmonic relationships, characteristic of Fauré's late musical language, but less so in Bach or Beethoven. We have shown that the TDM is a useful tool for studying the diachronic evolution of a composer's musical language. This work also suggests that even minimal extensions of the model can capture interesting characteristics of late 19th century and early 20th century extended harmony, such as the presence of bold modulations and different tonal layers in Fauré's work.

However, extensions far more ambitious than those we have introduced could be the subject of future study and allow for a more detailed analysis of the music of Fauré, or of composers with even more complex and evolving harmonic languages, such as Scriabin. Such models could be based on extended versions of the Tonnetz, such as the *Generalised Tonnetz* (Tymoczko, 2012) or the *Golden Tonnetz* (Imai, 2025), which could have been interesting for studying modal colours in late Fauré, something we had to abandon here. Eventually a model involving a time component would allow for an even more refined reconstruction and summarisation of harmonic processes and modulation sequences within a piece. For this is perhaps the most debatable assumption made by TDM : the independence of tones from one another.

Supplementary material

The code and data to reproduce our results are available on this adress : https://github.com/LeVieuxRoidAllemonde/Projet-de-recherche-encadr- Our hypotheses were preregistered on OSF (https://osf.io/m9g2f/overview?view$_only$ = $6cd9958086284623b498b12c03970709$).

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