# Databases

Lecture 10

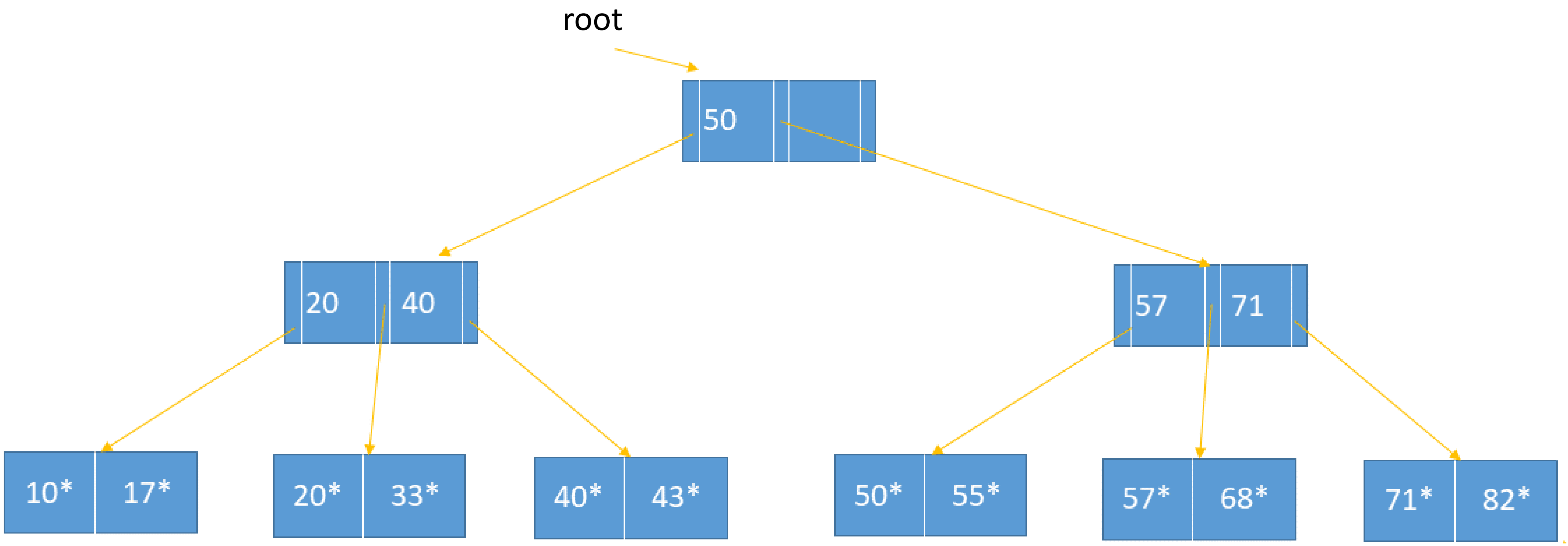
Indexes. Trees

# Indexed Sequential Access Method (ISAM)

- insertion
  - find the corresponding leaf page, add the entry
  - if there is no space on the page, add an overflow page
- deletion
  - find the leaf page that contains the entry, remove the entry
  - if an overflow page is emptied, it can be eliminated
- inserts / deletes
  - only leaf pages are affected (static structure)

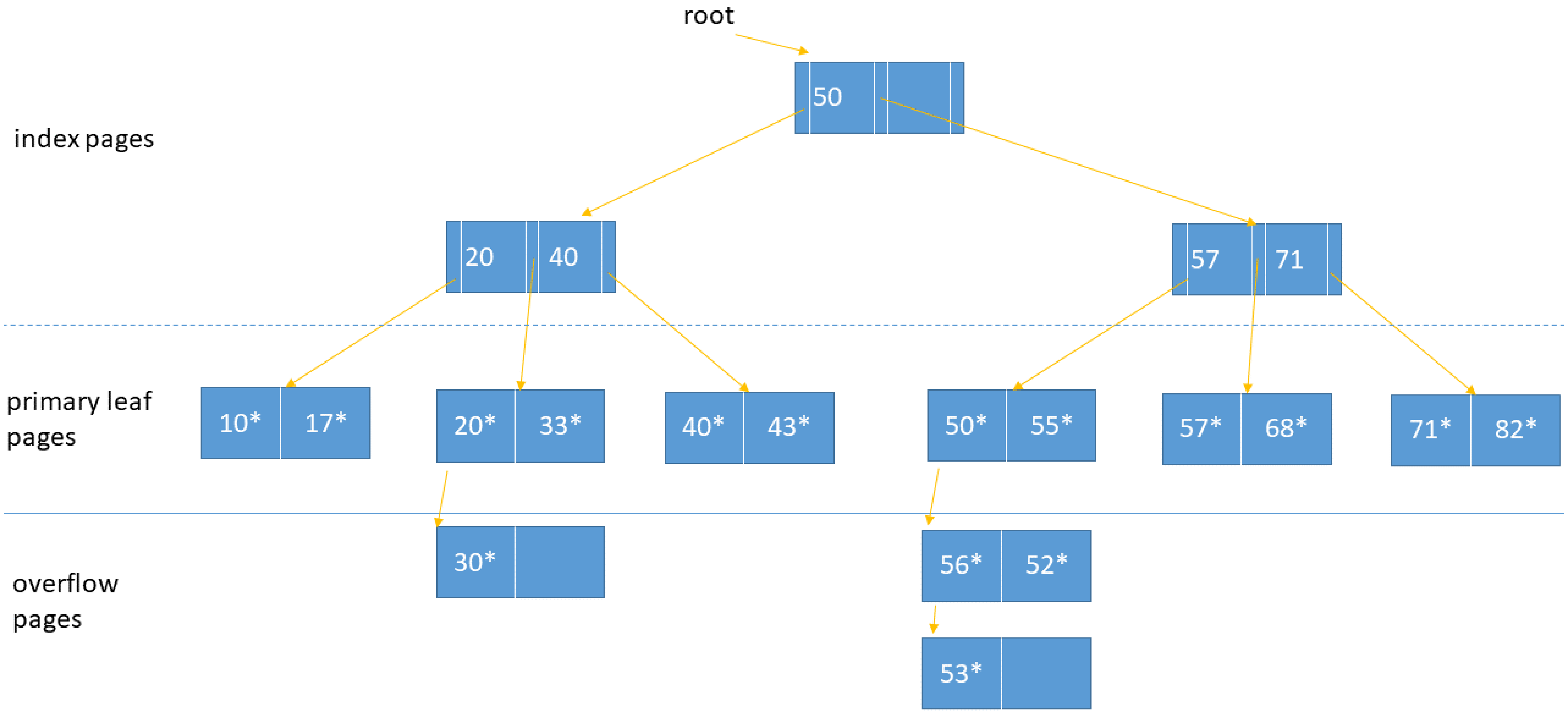
* Example - ISAM tree

- leaf page - 2 entries

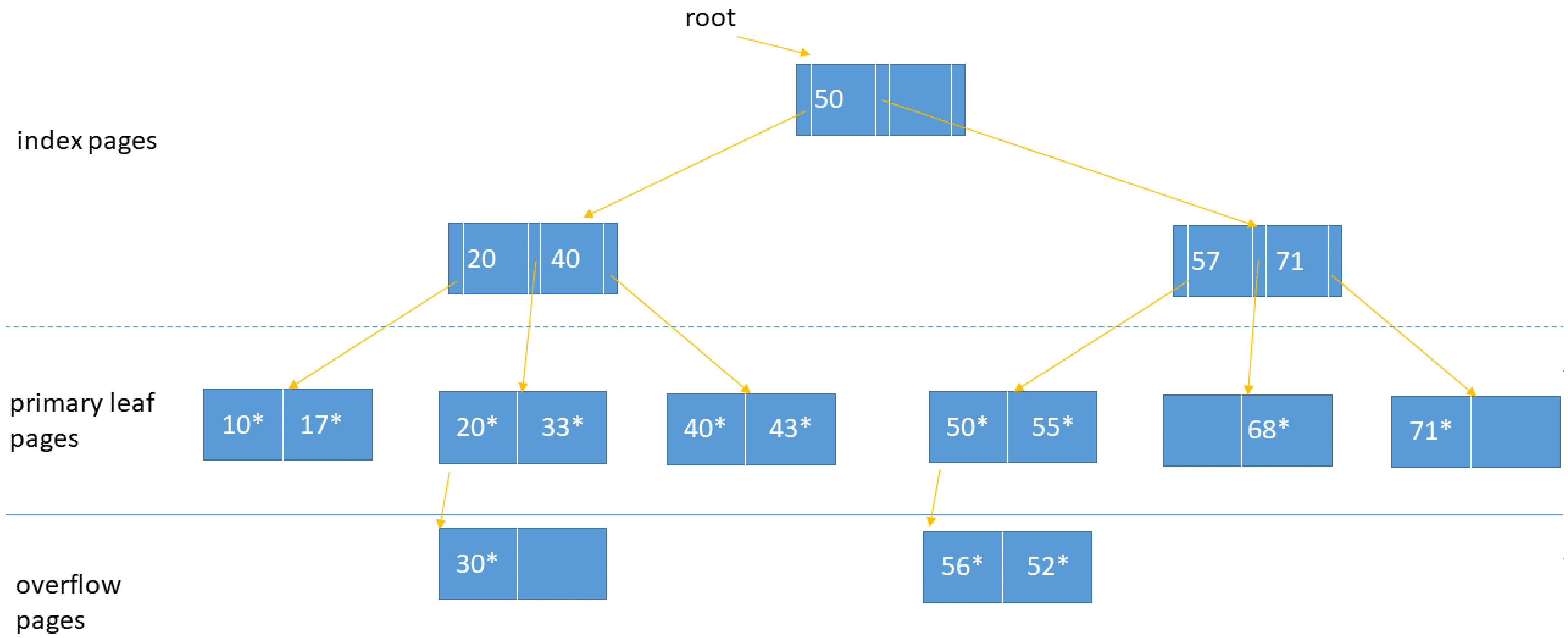



- only key values are shown

- after inserting 30*, 56*, 52*, 53*



- after deleting 53*, 57*, 82*



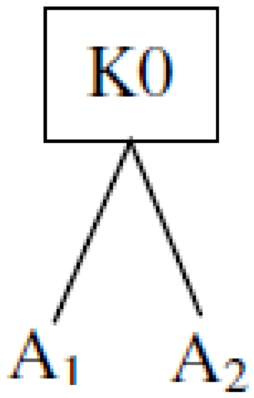

- even if it doesn't appear in the leaves, 57 still appears in an inner node

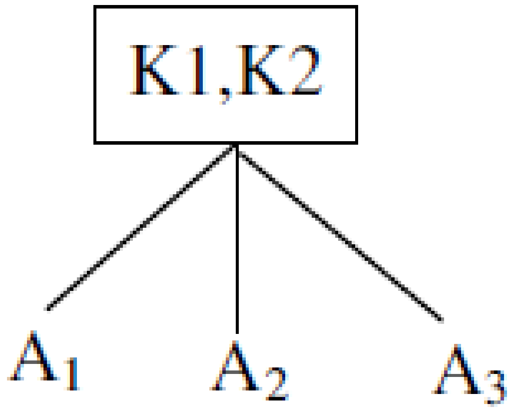
- benefits and drawbacks
  - inserts / deletes - faster
    - no balancing
    - no I/O operations (changes) on inner nodes
  - better concurrent access, since only leaf pages are modified
  - long overflow chains can develop
    - usually not sorted (to optimize inserts)
    - irregular search time if structure not balanced
    - at file creation - 20% of each page is free for future insertions
    - eliminated through deletes / file reorganization
  - ISAM - suitable when data size and distribution is relatively static

# 2-3 tree

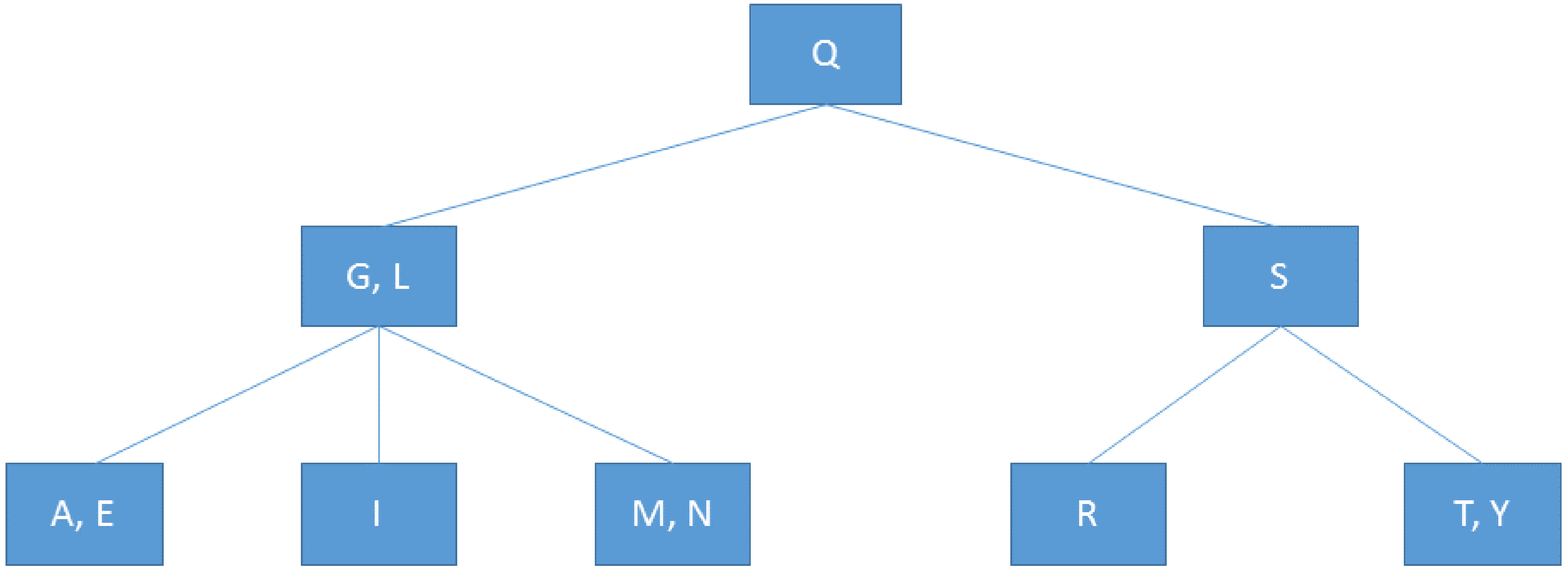
2-3 tree storing key values (collection of distinct values)

- all the terminal nodes are on the same level
- every node has 1 or 2 key values
  - a non-terminal node with one value $K_0$ has 2 subtrees: one with values less than $K_0$, and one with values greater than $K_0$
  - a non-terminal node with 2 values $K_1$ and $K_2$, $K_1 < K_2$, has 3 subtrees: one with values less than $K_1$, a subtree with values between $K_1$ and $K_2$, and a subtree with values greater than $K_2$



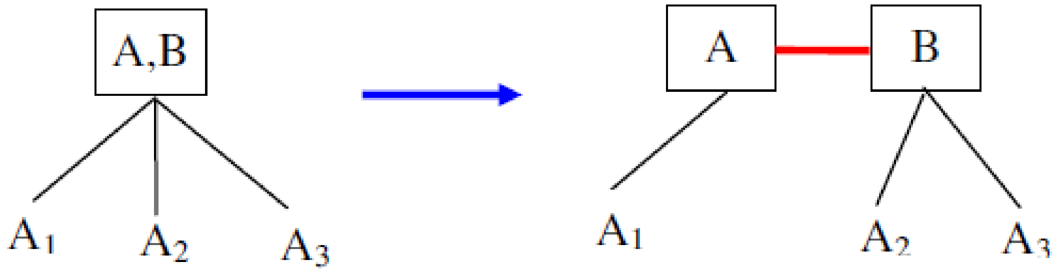



* Example (key values are letters)




- storing a 2-3 tree
  - 2-3 tree index storing the values of a key
  - tree - key value + address of record (file / DB address of record with corresponding key value)

- 2 options
  1. transform 2-3 tree into a binary tree
     - nodes with 2 values are transformed (see figure below)
     - nodes with 1 value - unchanged




     - the structure of a node

| K | ADDR | PointerL | PointerR | IND |
|---|---|---|---|---|

       - K - key value
       - ADDR - address of the record with the current key value (address in the file)
       - PointerL, PointerR - the 2 subtrees' addresses (address in the tree)

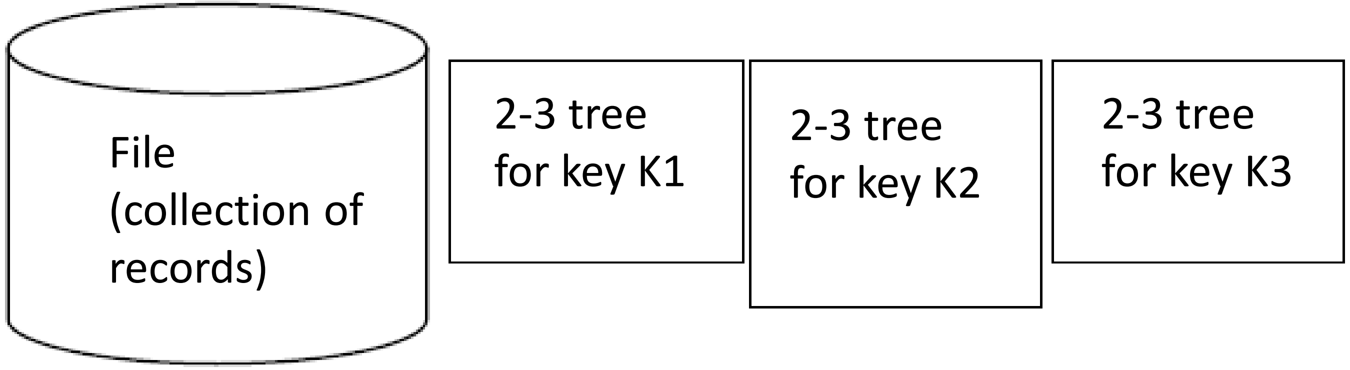
- IND - indicator that specifies the type of the link to the right (the 2 possible values can be seen in the previous figure)

2. the memory area allocated for a node can store 2 values and 3 subtree addresses

| NV | $K_1$ | $ADDR_1$ | $K_2$ | $ADDR_2$ | $Pointer_1$ | $Pointer_2$ | $Pointer_3$ |
|---|---|---|---|---|---|---|---|

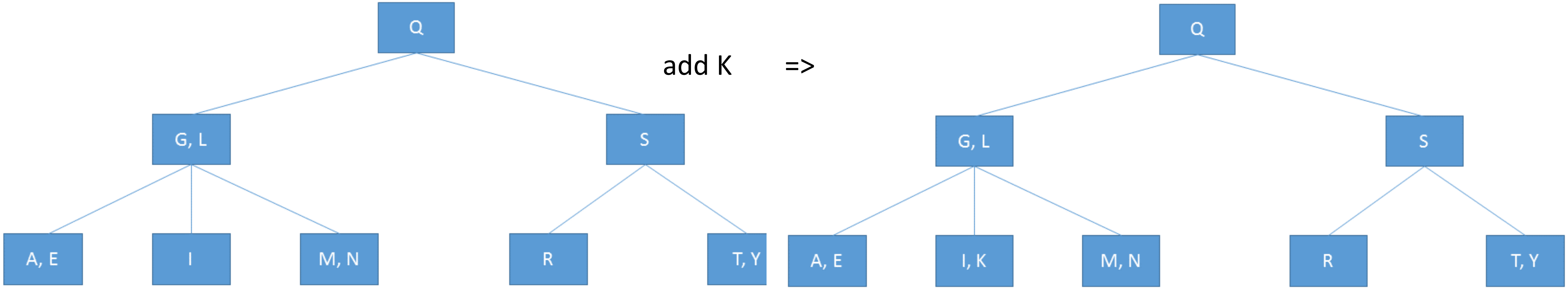
- NV – number of values in the node (1 or 2)
- $K_1$, $K_2$ – key values
- $ADDR_1$, $ADDR_2$ – the records' addresses (corresponding to $K_1$ and $K_2$)
- $Pointer_1$, $Pointer_2$, $Pointer_3$ – the 3 subtrees' addresses

- obs. a file (a relation in a relational DB) can have several associated 2-3 trees (e.g., one tree / key)

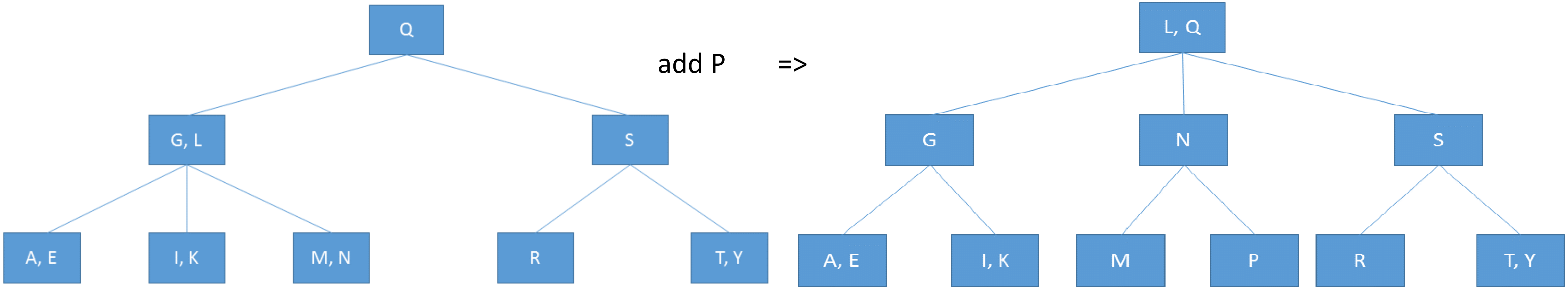


- operations in a 2-3 tree
  - searching for a record with key value $K_0$
  - inserting a record
  - removing a record
  - tree traversal (partial, total)

- add a new value
  - values in the tree must be distinct, i.e., the new value should not exist in the tree
  - perform a test, i.e., search for the value in the tree; if the new value can be added, the search ends in a terminal node
  - if the reached terminal node has 1 value, the new value can be stored in the node



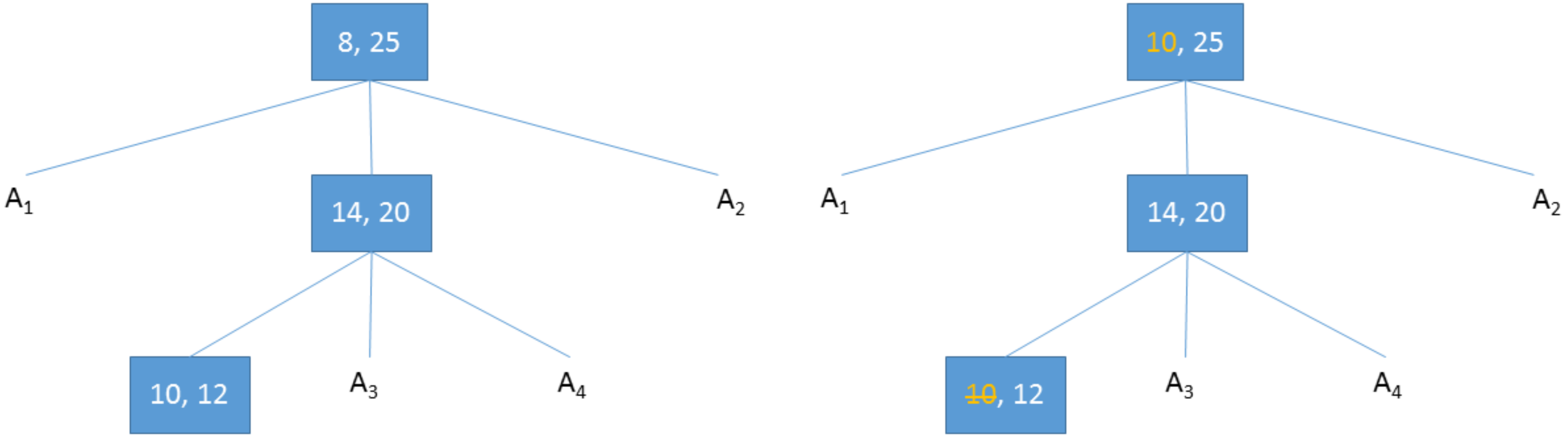

- if the reached terminal node has 2 values, the new value is added to the node, the 3 values are sorted, the node is split into 2 nodes: one node will contain the smallest value, the 2nd node - the largest value, and the middle value is attached to the parent node; the parent is then analyzed in a similar manner



- delete a value $K_0$

1. search for $K_0$; if $K_0$ appears in an inner node, change it with a neighbor value $K_1$ from a terminal node (there is no other value between $K_0$ and $K_1$)
   - $K_1$'s previous position (in the terminal node) is eliminated

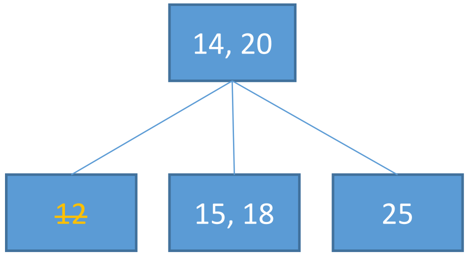
- e.g., remove 8:



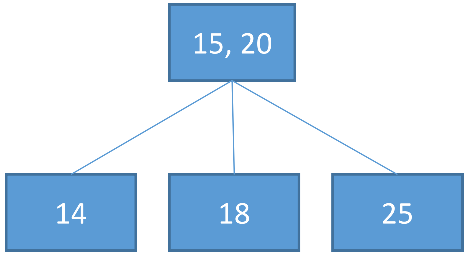

2. perform this step until case a / b occurs

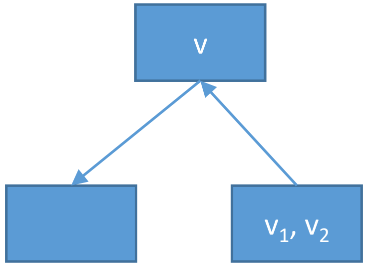
a. if the current node (from which a value is removed) is the root or a node with 1 remaining value, the value is eliminated; the algorithm ends ->

b. if the delete operation empties the current node (it has no values), but 2 values exist in one of the sibling nodes (left / right), 1 of the sibling's values is transferred to the parent, 1 of the parent's values is transferred to the current node; the algorithm ends




delete value 12

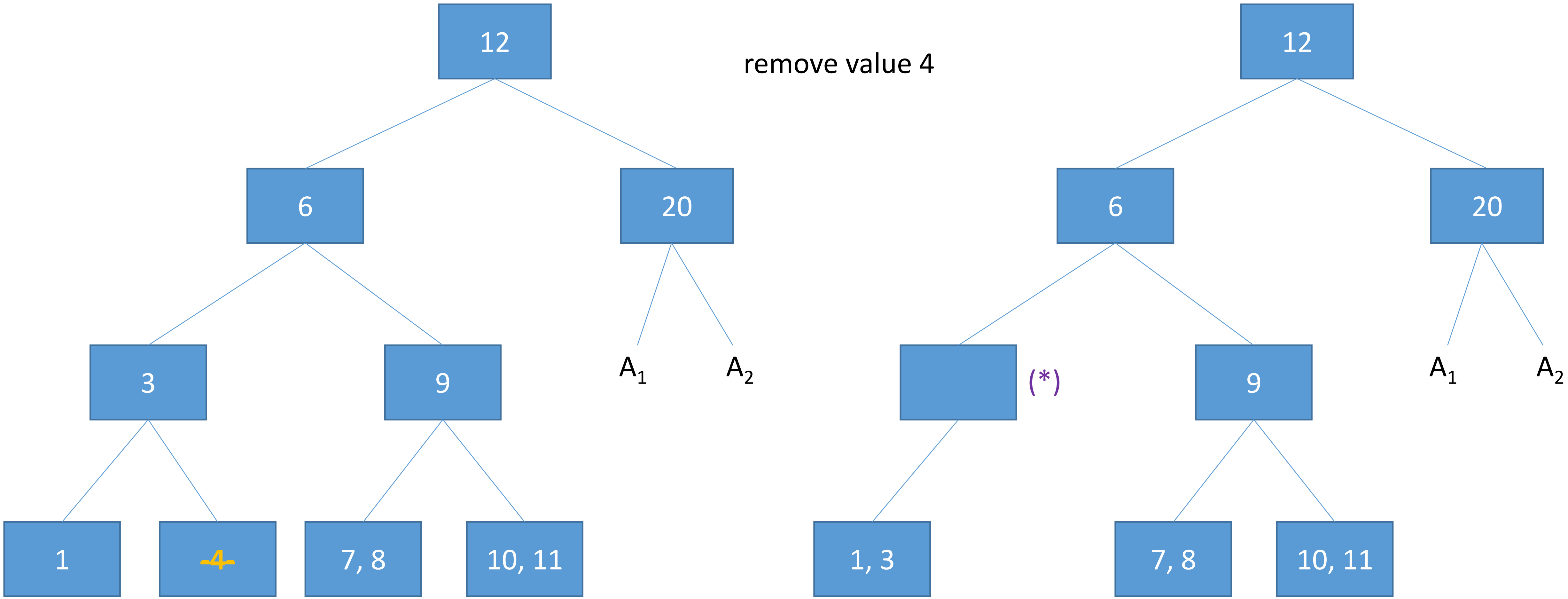


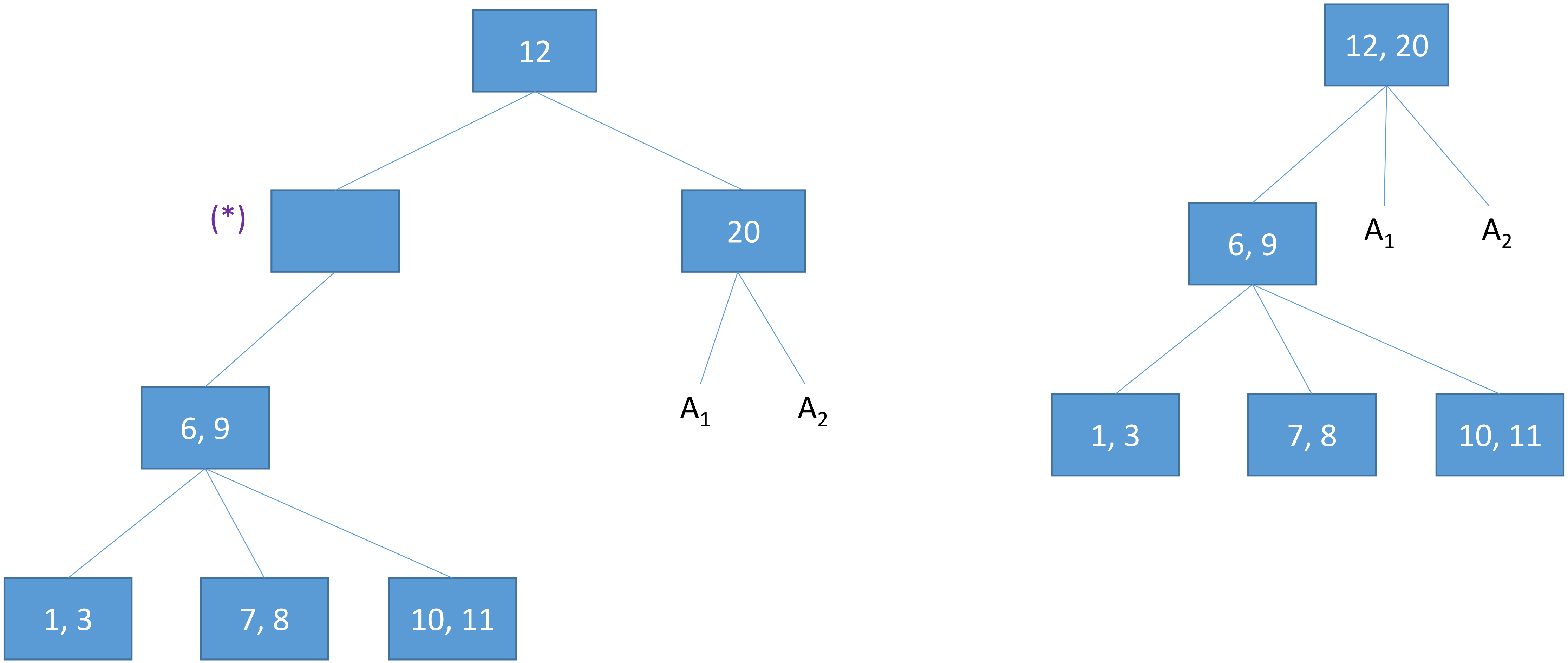




c. if the previous cases do not occur (current node has no values, sibling nodes have 1 value each), then the current node is merged with a sibling and a value from the parent node; case 2 is then analyzed for the parent

- if the root is reached and it has no values, it is eliminated and the current node becomes the root

- e.g., case c for the node marked with (*)

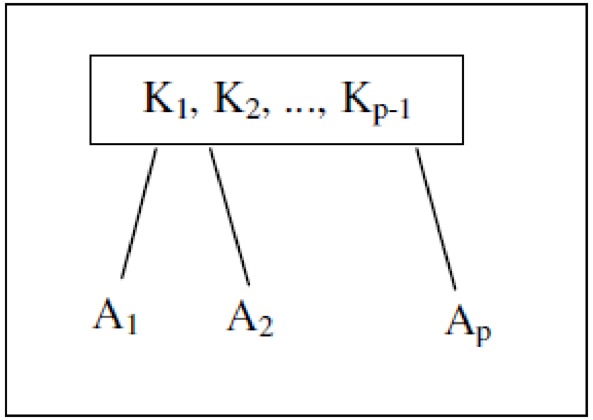



12
(*)
20
A1
A2
6, 9
1, 3
7, 8
10, 11
12, 20
6, 9
A1
A2
1, 3
7, 8
10, 11

# B-tree

- generalization of 2-3 trees
- B-tree of order m

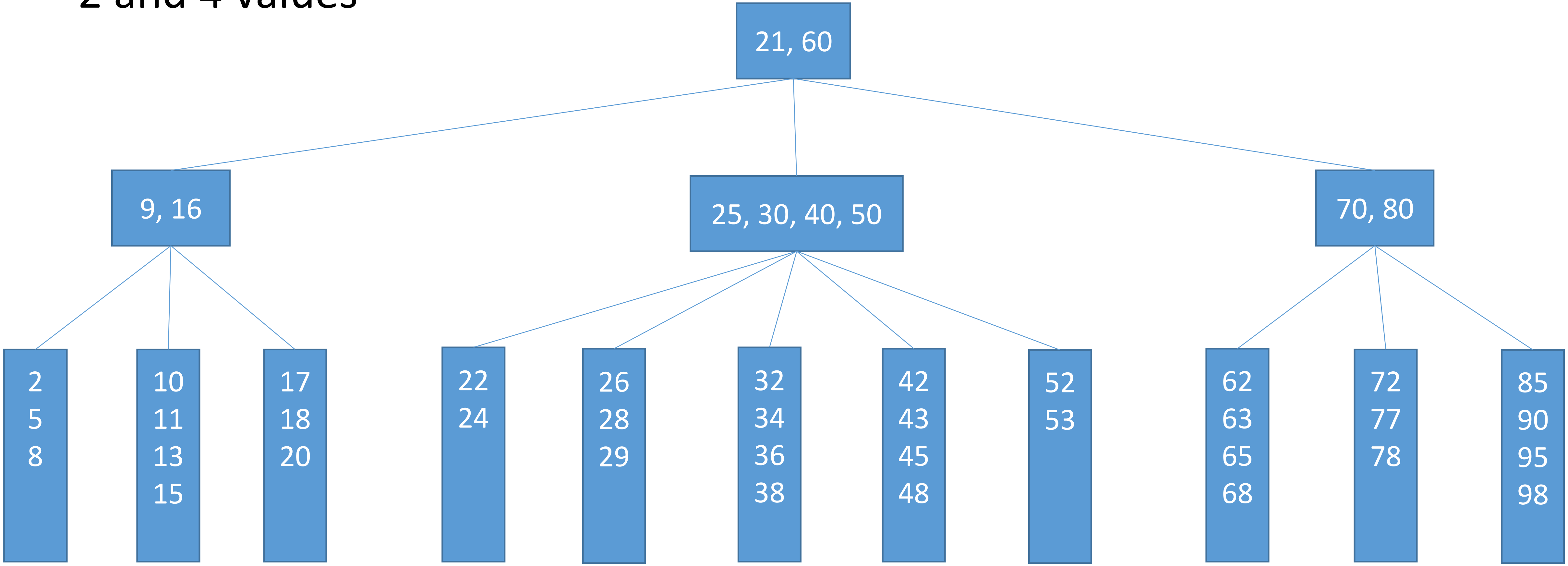
  1. if the root is not a terminal, it has at least 2 subtrees
  2. all terminal nodes – same level
  3. every non-terminal node – at most m subtrees
  4. a node with p subtrees has p-1 ordered values (ascending order), i.e., $K_1<K_2<...<K_{p-1}$
     - $A_1$: values less than $K_1$
     - $A_i$: values between $K_{i-1}$ and $K_i$, i=2,...,p-1
     - $A_p$: values greater than $K_{p-1}$
  5. every non-terminal node – at least $\left\lceil \frac{m}{2} \right\rceil$ subtrees




- obs. limits on number of subtrees (and values) / node result from the manner in which inserts / deletes are performed so that the second requirement in the definition is met

* Example - B-tree of order 5

- non-terminal, non-root node – at most 5, at least 3 subtrees, i.e., between 2 and 4 values



- B-tree of order m
  - storing the values of a key (a database index)
  - tree
    - key value + address of record

1. transformed into a binary tree
    - 2-3 tree method - generalization

2. the memory area allocated for a node can store the maximum number of values and subtree addresses

| NV | $K_1$ | $ADDR_1$ | ... | $K_{m-1}$ | $ADDR_{m-1}$ | $Pointer_1$ | ... | $Pointer_m$ |
|---|---|---|---|---|---|---|---|---|

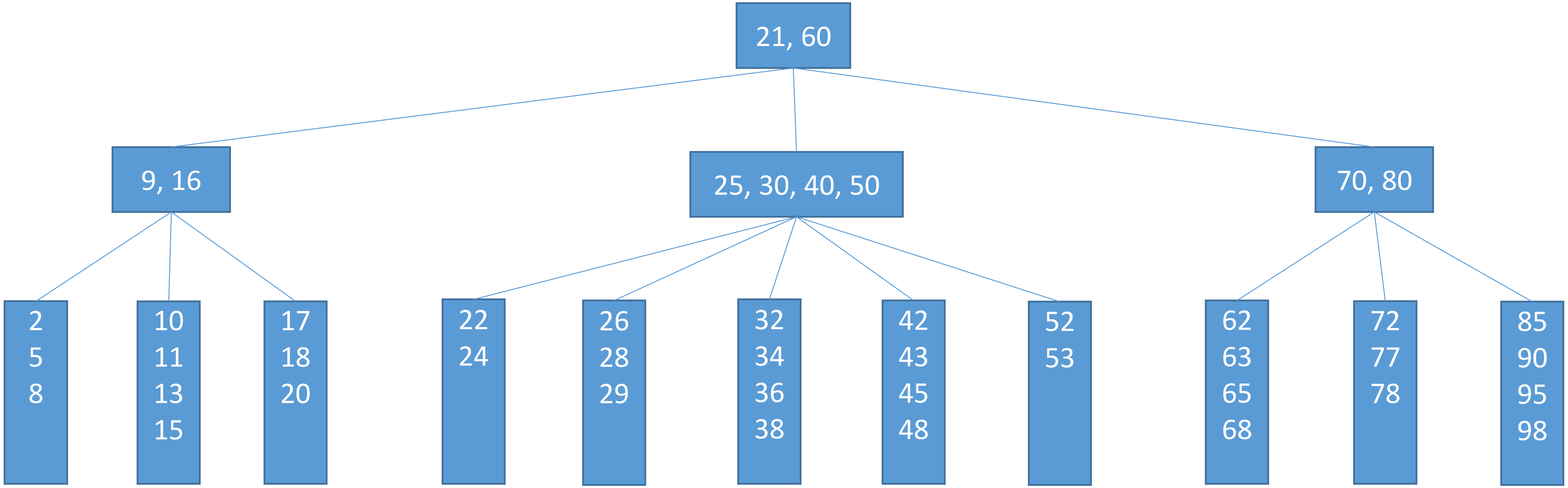
- NV - number of values in the node
- $K_1$, ..., $K_{m-1}$ - key values
- $ADDR_1$, ..., $ADDR_{m-1}$ - the records' addresses (corresponding to the key's values)
- $Pointer_1$, ..., $Pointer_m$ – subtree addresses

- B-tree of order m
  - useful operations in a B-tree
    - searching for a value
    - adding a value
    - removing a value
    - tree traversal (partial, total)

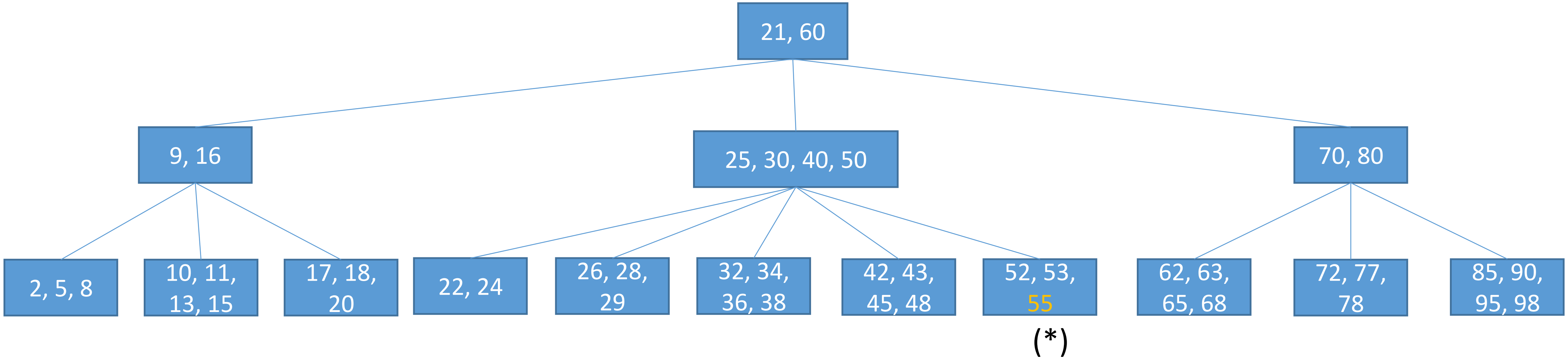
- B-tree of order m
  - adding a new value

    1. values in the tree must be distinct, i.e., the new value should not exist in the tree; perform a test, i.e., search for the value in the tree
       - if the new value can be added, the search ends in a terminal node

    2. if the reached terminal node has less than m-1 values, the new value can be stored in the node, e.g., 55 is added to the tree below:



- B-tree of order m
  - adding a new value
    - the resulting tree is shown below:

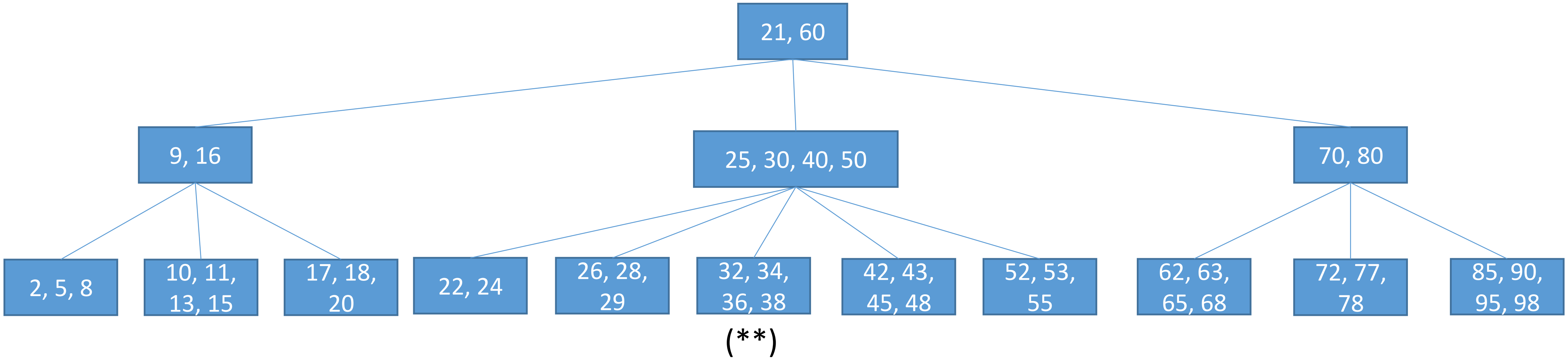



    - 55 belongs to the node marked with (*), which can store at most 4 values

- B-tree of order m
  - adding a new value

    3. if the terminal node already has m-1 values, the new value is attached to the node, the m values are sorted, the node is split into 2 nodes, and the middle value (median) is attached to the parent node

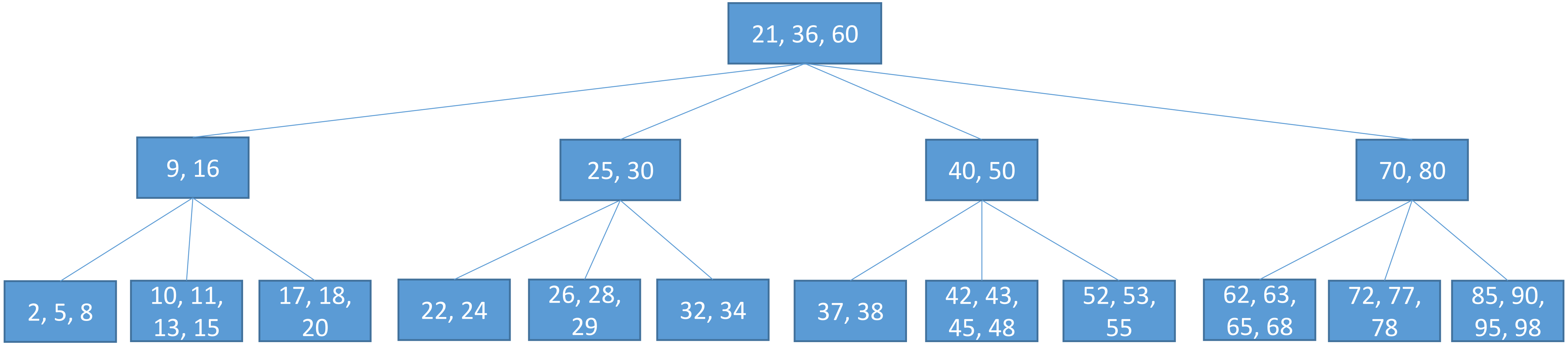
    - e.g., add 37 to the tree below



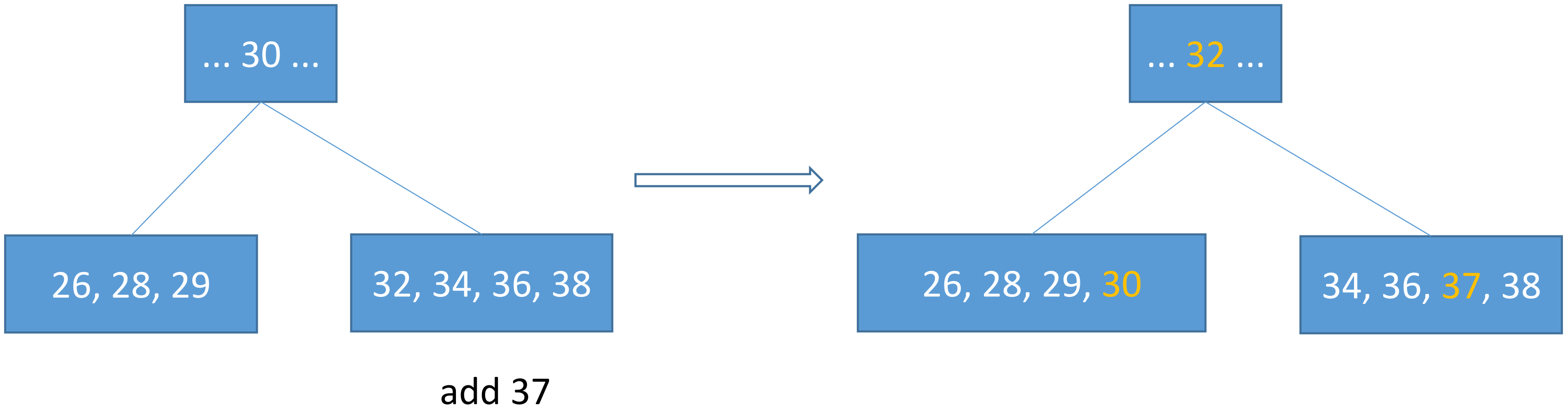

    - the node marked with (**) should contain values *32, 34, 36, 37, 38*

->

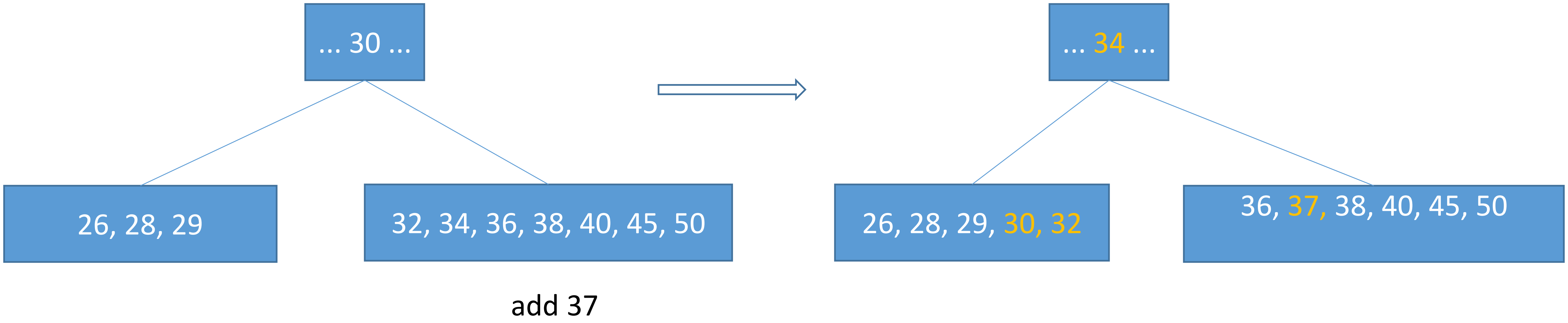
- B-tree of order m
  - adding a new value
    - since the node's capacity is exceeded, it is split into nodes *32, 34*, and *37, 38*, and 36 is attached to the parent node (with values *25, 30, 40, 50*)
    - in turn, the parent must be split into 2 nodes (values *25, 30*, and *40, 50*), and 36 is attached to its parent



- B-tree of order m
  - adding a new value
    - optimizations
      - before performing a split - analyze whether one or more values can be transferred from the current node (with m-1 values) to a sibling node
      - e.g., B-tree of order 5 (non-terminal node - between 2 and 4 values, i.e., between 3 and 5 subtrees):

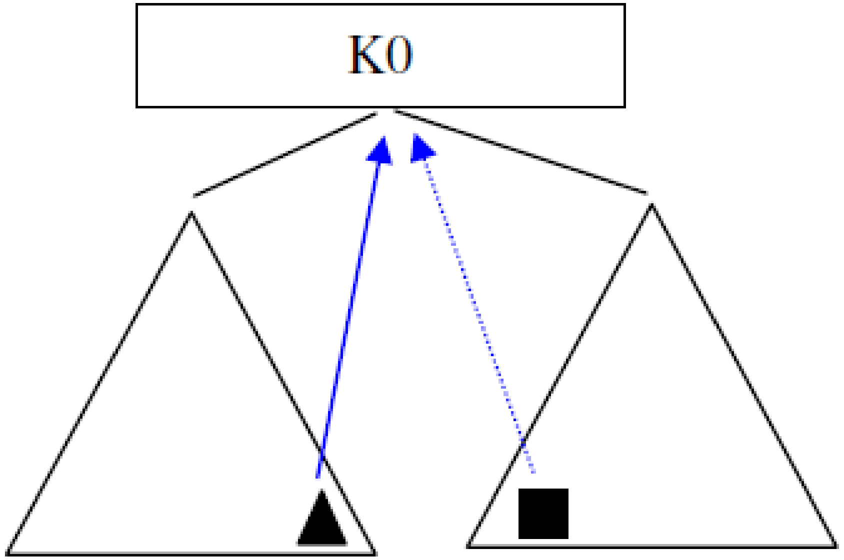


- B-tree of order m
  - adding a new value
    - optimizations
    - e.g., B-tree of order 8 (non-terminal node - between 3 and 7 values, i.e., between 4 and 8 subtrees):



- B-tree of order m
  - removing a value
    - a node can have at most m subtrees, i.e., a maximum of m-1 values, and at least $\left\lceil \frac{m}{2} \right\rceil$ subtrees, i.e., at least $\left\lceil \frac{m}{2} \right\rceil - 1 = \left\lfloor \frac{m-1}{2} \right\rfloor$ values
    - when eliminating a value from a node, an underflow can occur (the node can end up with less values than the required minimum)
  - eliminate value $K_0$

    1. search for $K_0$; if it doesn't exist, the algorithm ends
    2. if $K_0$ is found in a non-terminal node (like in the figure on the right), $K_0$ is replaced with a *neighbor value* from a terminal node (this value can be chosen between 2 values from the trees separated by $K_0$)



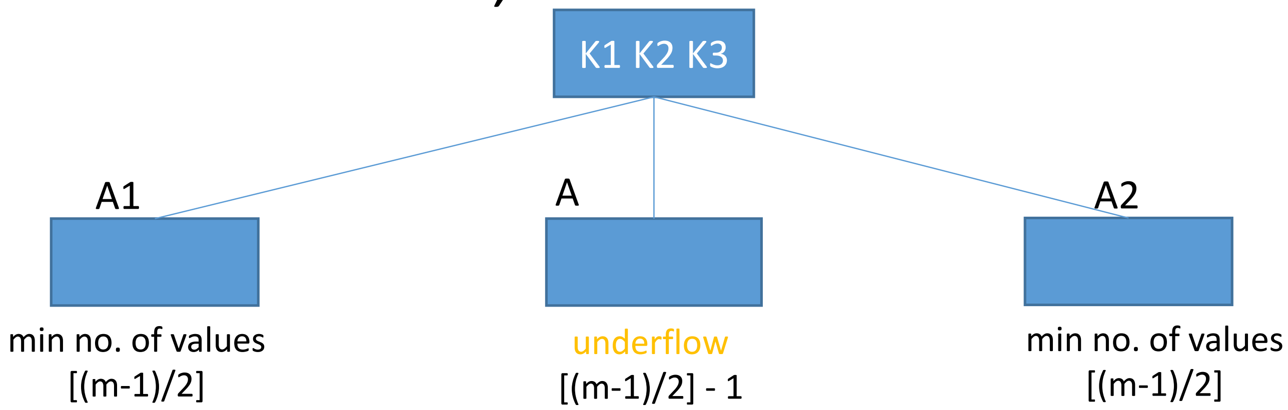
- B-tree of order m
  - removing a value

    3. perform this step until case a / b occurs

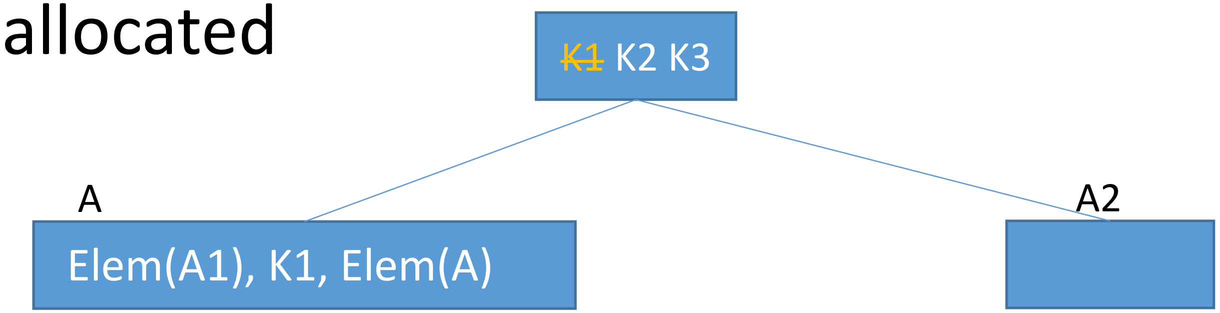
    a. if the current node (from which a value is removed) is the root or underflow doesn't occur, the value is eliminated; the algorithm ends

    b. if the delete operation causes an underflow in the current node (A), but one of the sibling nodes (left / right - B) has at least 1 extra value, values are transferred between A and B via the parent node; the algorithm ends

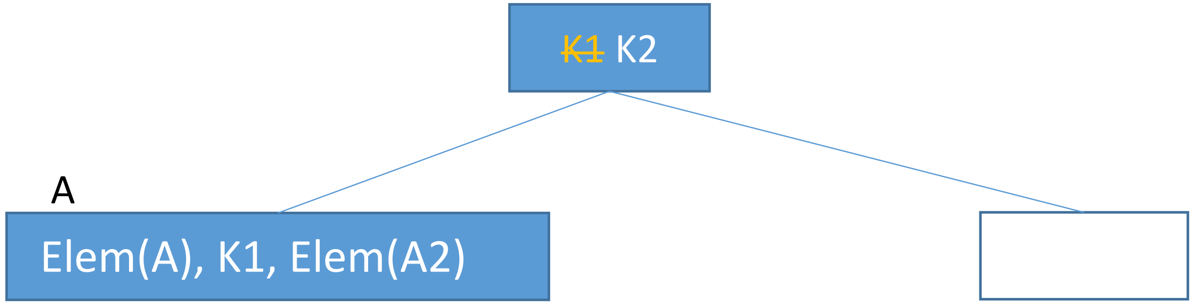
    c. if there is an underflow in A, and sibling nodes A1 and A2 have the minimum number of values, nodes must be concatenated:



- B-tree of order m
  - removing a value
    - if A1 exists, A1 is merged with A and value K1 (separating A1 from A); the node at address A1 is deallocated




    - if there is no A1 (A is the first subtree for its parent), A is merged with A2 and K1 (separating A from A2); the node at address A2 is deallocated




    - case 3 is then analyzed for the parent node
    - if the root is reached and has no values, it is removed and the current node becomes the root

- B-tree of order m
  - optimizations

obs 1. a block stores a node from a B-tree

- e.g.:
  - key size: 10b
  - record address / node address: 10b
  - NV value (number of values in the node): 2b
  - block size: 1024b (10b for the header)
- then: 2+(m-1)*(10+10)+m*10=1024-10 => m=34
- if the size of a block is 2048b and the other values are unchanged, then the order of the tree is m = 68, i.e., a node can have between 33 and 67 values

- B-tree of order m
  - optimizations
- the maximum number of required blocks (from the file that stores the B-tree) when searching for a value - the maximum number of levels in the tree; for m=68, if the number of values is 1.000.000, then:
  - the root node (on level 0) contains at least 1 value (2 subtrees)
  - on the next level (level 1) - at least 2 nodes * 33 values/node = 66 values
  - level 2 – at least 2*34 nodes * 33 values/node = 2.244 values
  - level 3 – at least 2*34*34 nodes * 33 values/node = 76.296 values
  - level 4 – at least 2*34*34*34 nodes * 33 values/node = 2.594.064 values, which is greater than the number of existing values => this level does not appear in the tree

=> at most 4 levels in the tree

- after at most 4 block reads and a number of comparisons in the main memory, it can be determined whether the value exists (the corresponding record's address can then be retrieved) or the search was unsuccessful

- B-tree of order m
  - optimizations

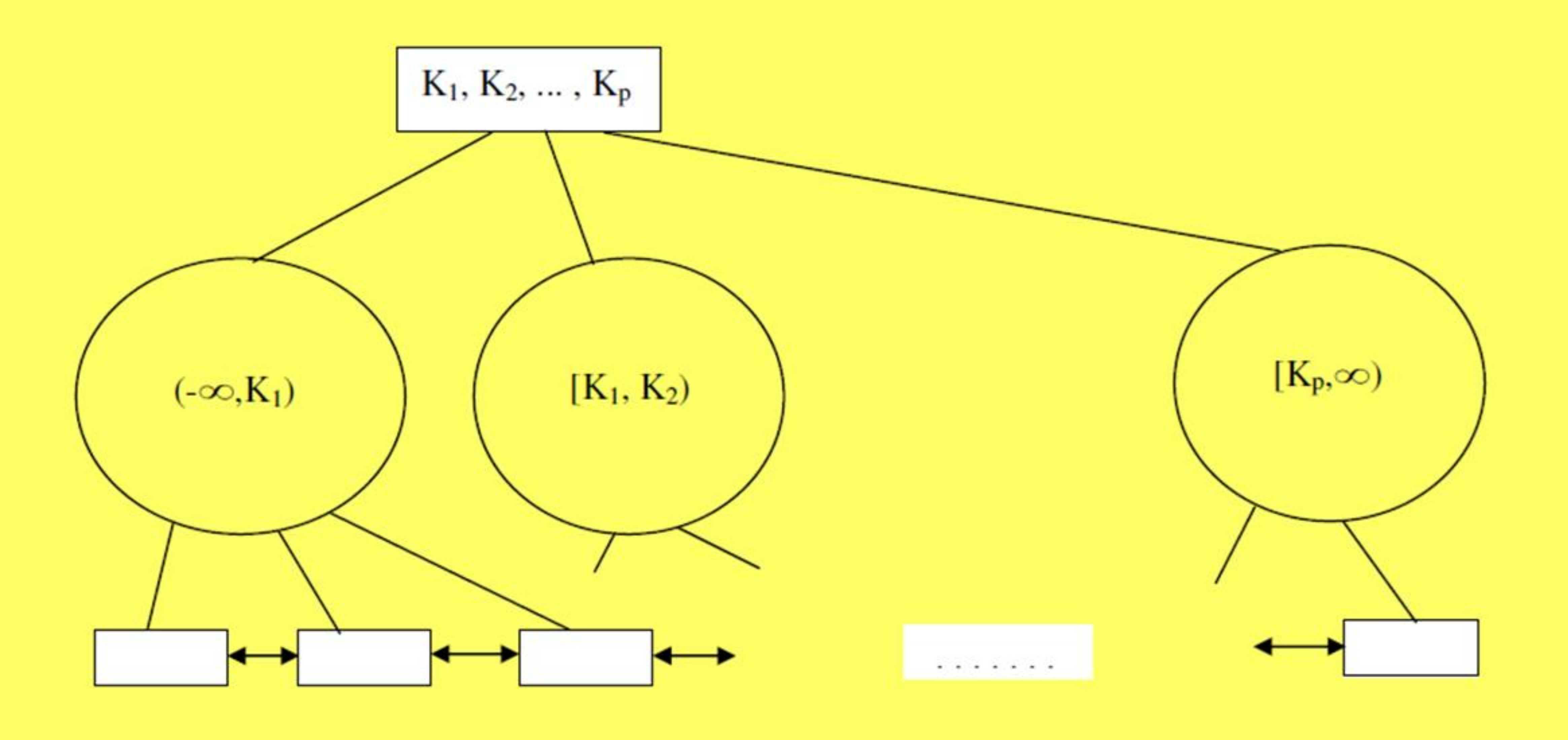
obs 2. in terminal nodes, subtree addresses are null; the space allocated for these addresses could be used to store additional (K, Addr) pairs

# B+ tree

B+ tree

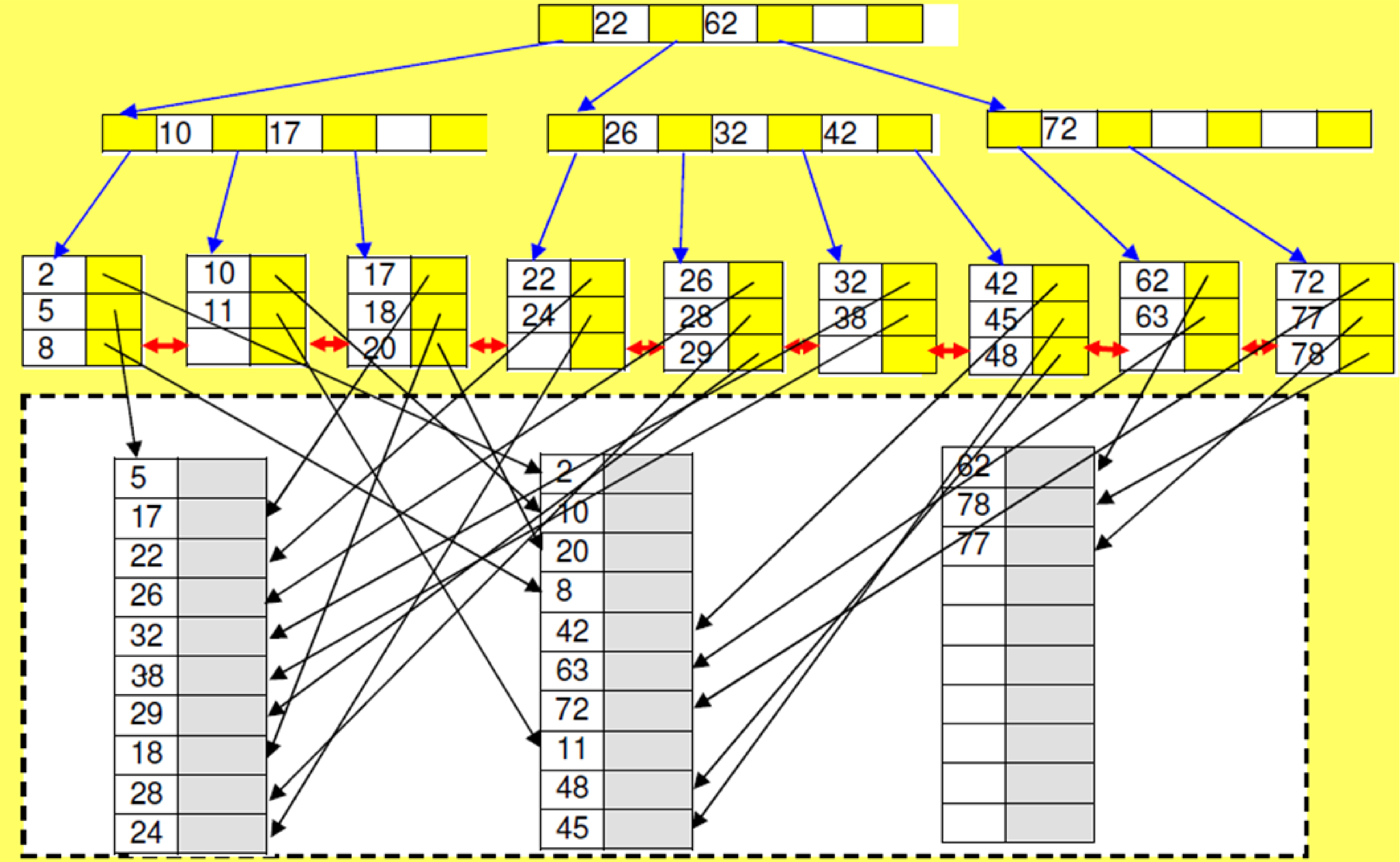
- B-tree variant
- the last level contains all the possible values (key values and the corresponding records' addresses)
- some key values can also appear in non-terminal nodes, without the records' addresses; their purpose is to separate values from terminal nodes (guide the search)
- terminal nodes are maintained in a doubly linked list; the data can be easily scanned via these links
- an inner node has the same min. / max. number of values as a B-tree
- a terminal node has at least [m / 2] values (instead of [(m − 1) / 2]) and at most (m-1) values
- for the min. / max. number of values, check obs 2. related to B-tree optimizations

# B+ tree



# B+ tree

- B+ tree of order 4



# B+ tree

- storing a B+ tree
  - B-tree methods
- operations (algorithms)
  - B-tree

B+ tree - in practice

* prefix key compression

- larger key size => less index entries fit on a page, i.e., less children / index page => larger B+ tree height
- keys in index entries - just direct the search => often, they can be compressed
- adjacent index entries with search key values: *Meteiut*, *Mircqkjt*, *Morqwkj*
- it's enough to store the values *Me, Mi,* etc
- what if the subtree also contains *Micfgjh*? => need to store *Mir* (instead of *Mi*)
- it's not enough to analyze neighbor index entries *Meteiut* and *Morqwkj*; the largest value in *Mircqkjt*'s left subtree and the smallest value in its right subtree also need to be examined
- inserts / deletes - modified correspondingly

B+ tree - in practice

- concept of *order* - relaxed, replaced by a physical space criterion, e.g., nodes should be at least half-full
- terminal / non-terminal nodes - different numbers of entries; usually, inner nodes can store more entries than terminal ones
- variable-length search key => variable-length entries => variable number of entries / page
- if a3 is used (<k, rid_list>) => variable-length entries (in the presence of duplicates), even if attributes are of fixed length

# B+ tree - in practice

- values found in practice
  - order – 200
  - fill factor (node) – 67%
  - fan-out – 133
  - capacity
    - height 4: $133^4$ = 312,900,721 records
    - height 3: $133^3$ = 2,352,637 records
- top levels can often be kept in the BP
  - 1st level – 1 page (8KB)
  - 2nd level – 133 pages (≅ 1MB)
  - 3rd level – 17689 pages (≅ 133 MB)

B+ tree - benefits

- balanced index => uniform search time
- rarely more than 3-5 levels, the top levels can be kept in main memory => only a few I/O operations are needed to search for a record
- widely used in DBMSs, among the most optimized components
- ideal for range selections, good for equality selections as well

# References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