

# Databases

Lecture 9

Indexes. Trees

Indexes

# Indexes

- problems
  - what does a data entry contain?
  - how are the entries of an index organized?

## Indexes - Data Entries

- let  $k^*$  be a data entry in an index; the data entry:
  - alternative 1
    - is an actual data record with search key value =  $k$
  - alternative 2
    - is a pair  $\langle k, \text{rid} \rangle$  ( $\text{rid}$  – id of a data record with search key value =  $k$ )
  - alternative 3
    - is a pair  $\langle k, \text{rid\_list} \rangle$  ( $\text{rid\_list}$  – list of ids of data records with search key value =  $k$ )

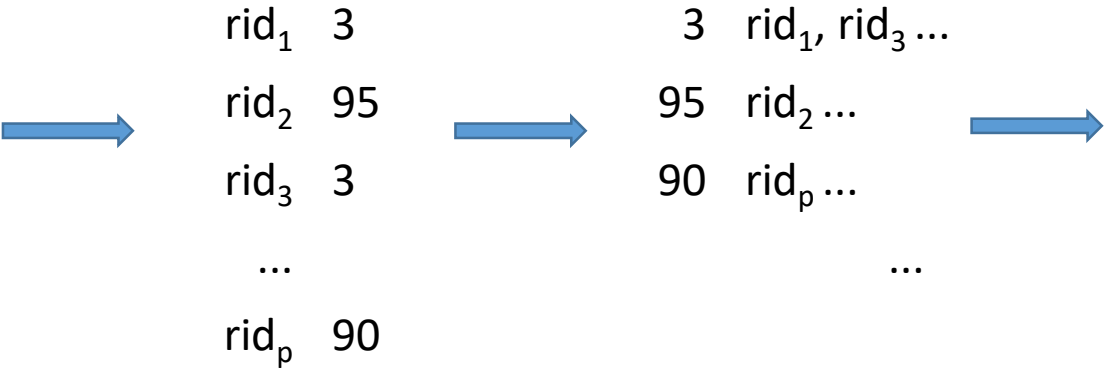
## Indexes - Data Entries

- a1
  - the file of data records needn't be stored in addition to the index
  - the index is seen as a special file organization
  - at most 1 index / collection of records should use alternative a1 (to avoid redundancy)
- a2, a3
  - data entries point to corresponding data records
  - in general, the size of an entry is much smaller than the size of a data record
  - a3 is more compact than a2, but can contain variable-length records
  - can be used by several indexes on a collection of records
  - independent of the file organization

# Creating an index - example

	Name	Score	Age
rid <sub>1</sub>	Popescu	3	44
rid <sub>2</sub>	Ionescu	95	80
rid <sub>3</sub>	Vladescu	3	45
...		...	
rid <sub>p</sub>	Xulescu	90	14

search key



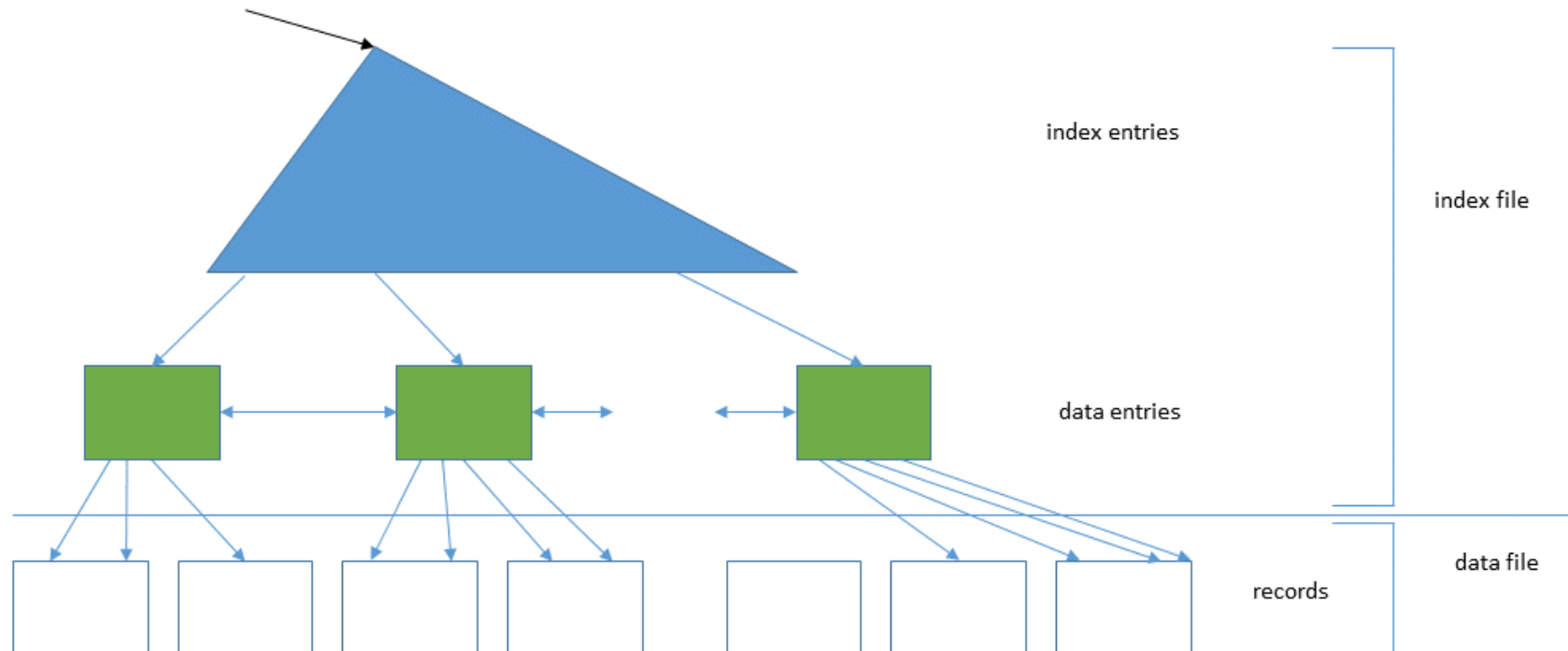
data entry

...	
3	rid <sub>1</sub> , rid <sub>3</sub> ...
90	rid <sub>p</sub> ...
95	rid <sub>2</sub> ...
...	

index file

# Indexes - Properties

- clustered vs. unclustered
  - clustered index
    - the order of the data records is close to / the same as the order of the data entries
    - index entries guide the search for data entries



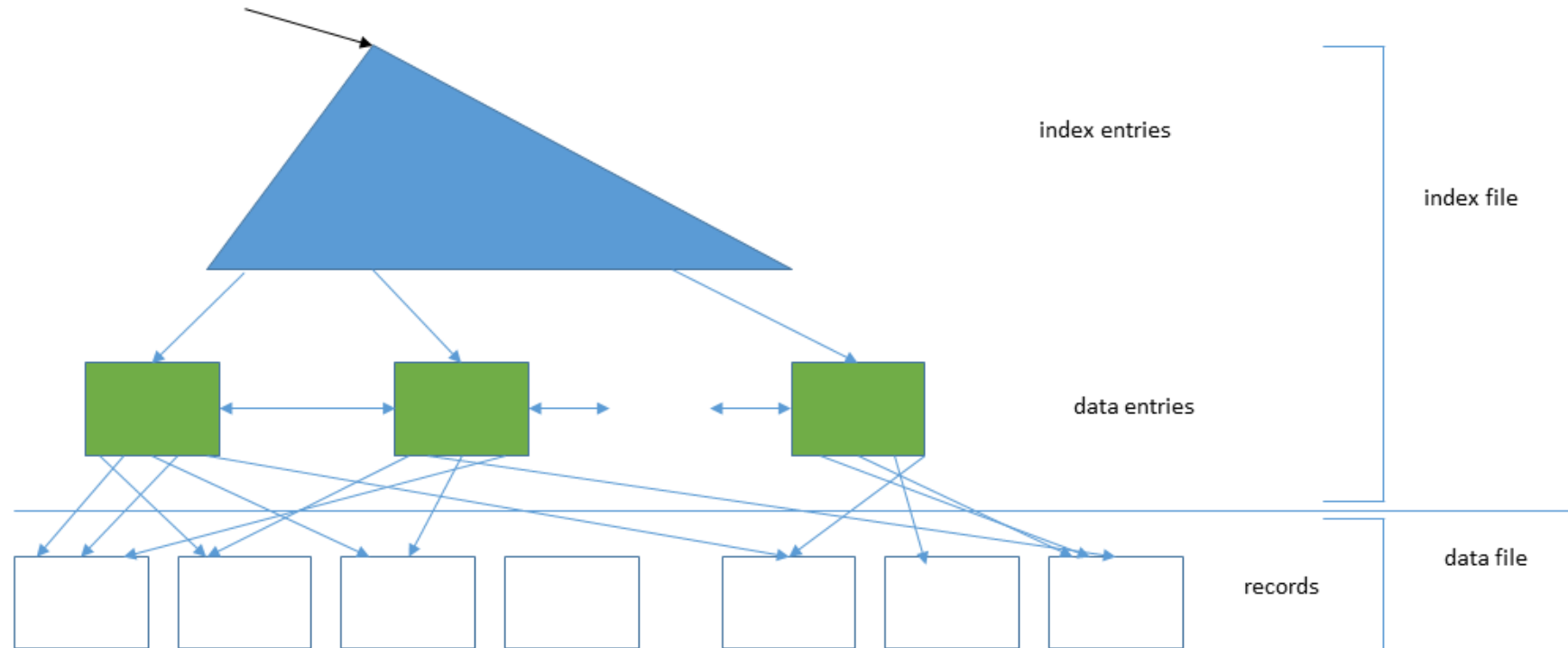
## Indexes - Properties

- clustered vs. unclustered
  - a1 → clustered index
  - indexes using a2 / a3 are clustered only if the data records are ordered on the search key
  - at most 1 clustered index / data file
  - maintaining a clustered index
    - costly operation



# Indexes - Properties

- clustered vs. unclustered
  - unclustered index
    - index that is not clustered



## Indexes - Properties

- clustered vs. unclustered
    - unclustered index
      - several unclustered indexes could be built on a file
  - range search query
    - unclustered index usage cost
      - each data entry that meets the condition in a query could contain a rid pointing to a distinct page
- => the number of I/O operations could be equal to the number of data entries that satisfy the query's condition

## Indexes - Properties

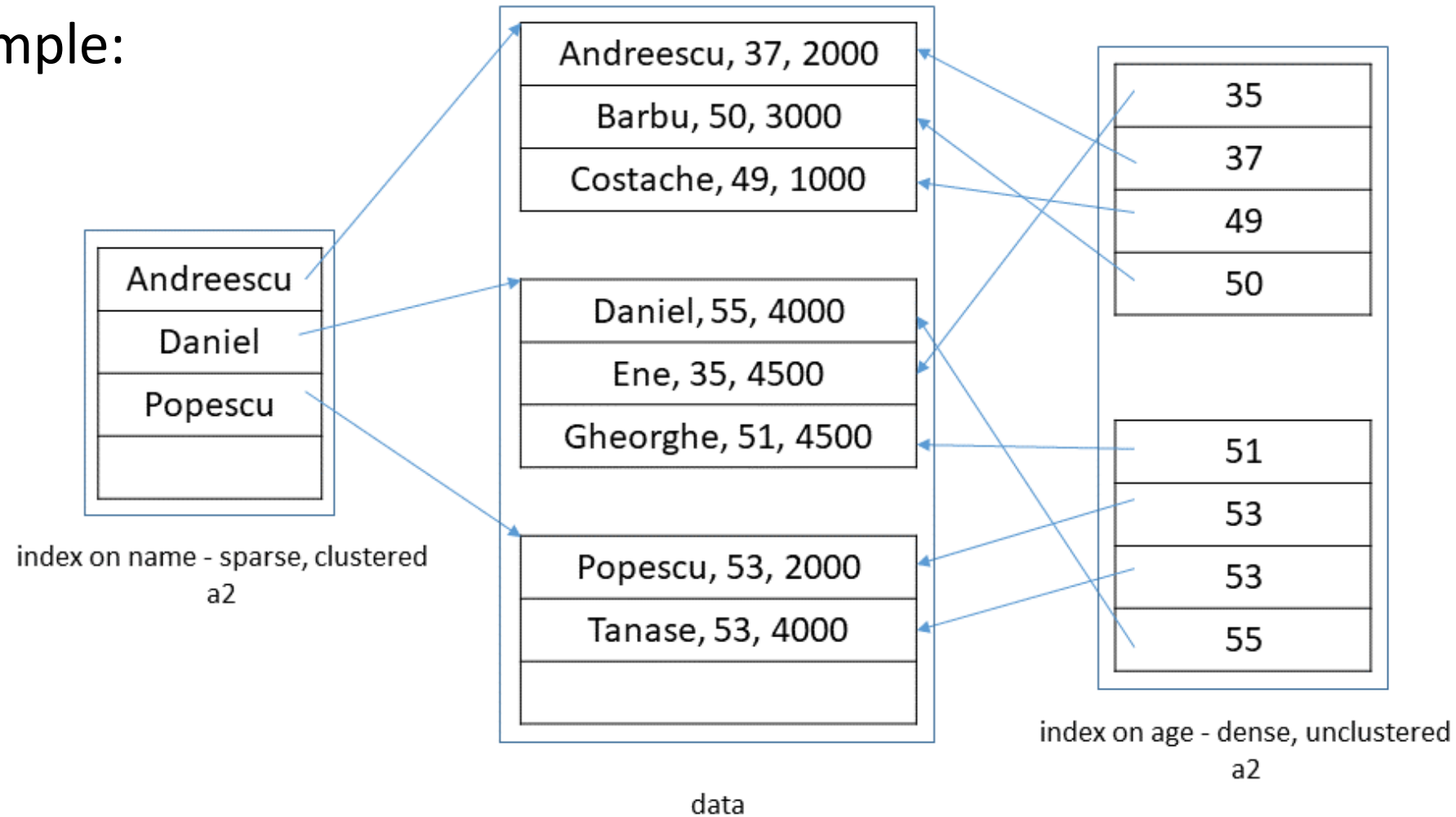
- primary and secondary
  - primary index
    - the search key includes the primary key
  - secondary index
    - index that is not primary
- unique index
  - the search key contains a candidate key
- duplicates
  - data entries with the same search key value
- primary indexes, unique indexes cannot contain duplicates
- secondary indexes can contain duplicates

## Indexes - Properties

- sparse vs. dense
- dense index
  - at least one data entry for every search key value that appears in a data record
  - in the presence of duplicates and a2, multiple data entries can have the same search key value
- sparse index
  - one entry for each data page
  - more compact
- a1 => dense index
- sparse index => clustered

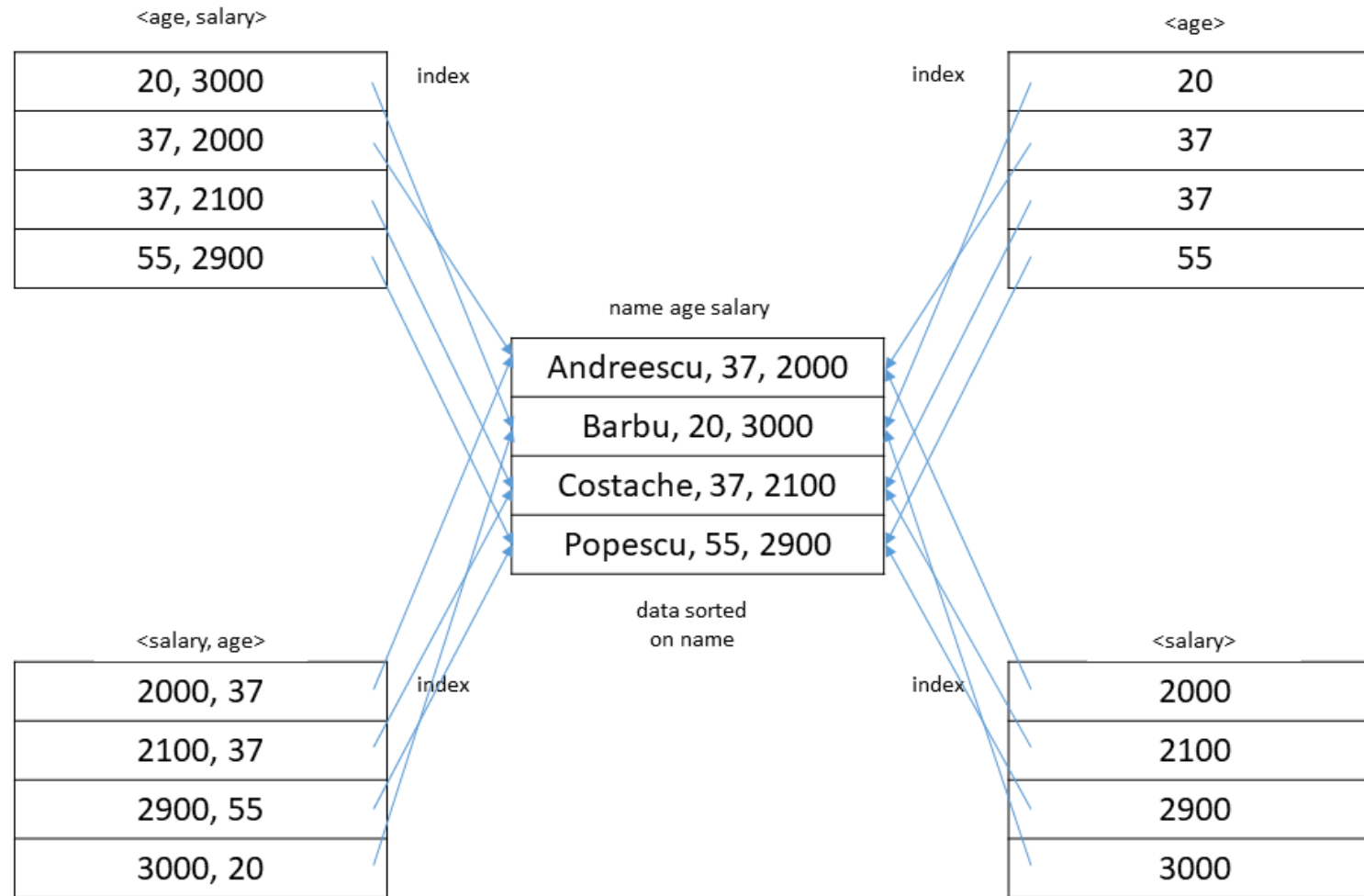
# Indexes - Properties

- sparse vs. dense
- example:



# Indexes - Properties

- composite (concatenated) search key - search key that contains several fields
  - example:



\* example

- the *Professors* relation sorted by age
- a page stores at most 3 records: the 1<sup>st</sup> tuple is in page 1 slot 1, the second - in page 1 slot 2, etc

prof_id	name	mail	age	impact_factor
4563	Popescu	<a href="mailto:pu@cs.edu">pu@cs.edu</a>	20	2.5
4570	Antonescu	<a href="mailto:au@cs.edu">au@cs.edu</a>	25	3
4500	Ionescu	<a href="mailto:iu@by.edu">iu@by.edu</a>	30	3.5
4555	Tanasescu	<a href="mailto:tu@cs.edu">tu@cs.edu</a>	65	3.1
4450	Tanasescu	<a href="mailto:tu@mc.edu">tu@mc.edu</a>	65	4

- dense index on *age* with a1 – the entire file
- dense index on *age* with a2 – data entries (20, <1, 1>), (25, <1, 2>), (30, <1, 3>), (65, <2, 1>), (65, <2, 2>)\*

\*recall that a record can be identified by <page id, slot number>

\* example

prof_id	name	mail	age	impact_factor
4563	Popescu	<a href="mailto:pu@cs.edu">pu@cs.edu</a>	20	2.5
4570	Antonescu	<a href="mailto:au@cs.edu">au@cs.edu</a>	25	3
4500	Ionescu	<a href="mailto:iu@by.edu">iu@by.edu</a>	30	3.5
4555	Tanasescu	<a href="mailto:tu@cs.edu">tu@cs.edu</a>	65	3.1
4450	Tanasescu	<a href="mailto:tu@mc.edu">tu@mc.edu</a>	65	4

- dense index on *age* with a3 – entries (20, <1, 1>), (25, <1, 2>), (30, <1, 3>), (65, <2, 1>, <2, 2>)
- sparse index on *age* with a1 – cannot be built (by definition)
- sparse index on *age* with a2 – (20, <1, 1>), (65, <2, 1>)
- sparse index on *age* with a3 – (20, <1, 1>), (65, <2, 1>, <2, 2>)



\* example

prof_id	name	mail	age	impact_factor
4563	Popescu	<a href="mailto:pu@cs.edu">pu@cs.edu</a>	20	2.5
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4450	Tanasescu	<a href="mailto:tu@mc.edu">tu@mc.edu</a>	65	4

- dense index on *impact\_factor* with a2 – (2.5, <1, 1>), (3, <1, 2>), (3.1, <2, 1>), (3.5, <1, 3>), (4, <2, 2>)
- dense index on *impact\_factor* with a3 – (2.5, <1, 1>), (3, <1, 2>), (3.1, <2, 1>), (3.5, <1, 3>), (4, <2, 2>)
- sparse index on *impact\_factor* with a1 – cannot be built (by definition)
- sparse index on *impact\_factor* with a2 or a3 – the values in the search key are not ordered

Data Collection Stored in a Binary Tree

- binary search algorithm on ordered collections
  - very fast; reduced search time (+)
  - maintaining the records' sort order - costly (especially for dynamic collections with many INSERT, UPDATE, DELETE operations) (-)
- solution
  - store the collection using a *binary tree*
  - record - stored in a node
  - node with key value  $v$ 
    - left subtree
      - records with key values  $< v$
    - right subtree
      - records with key values  $> v$

- memory structure for a binary tree node
  - record's values stored in K, INF

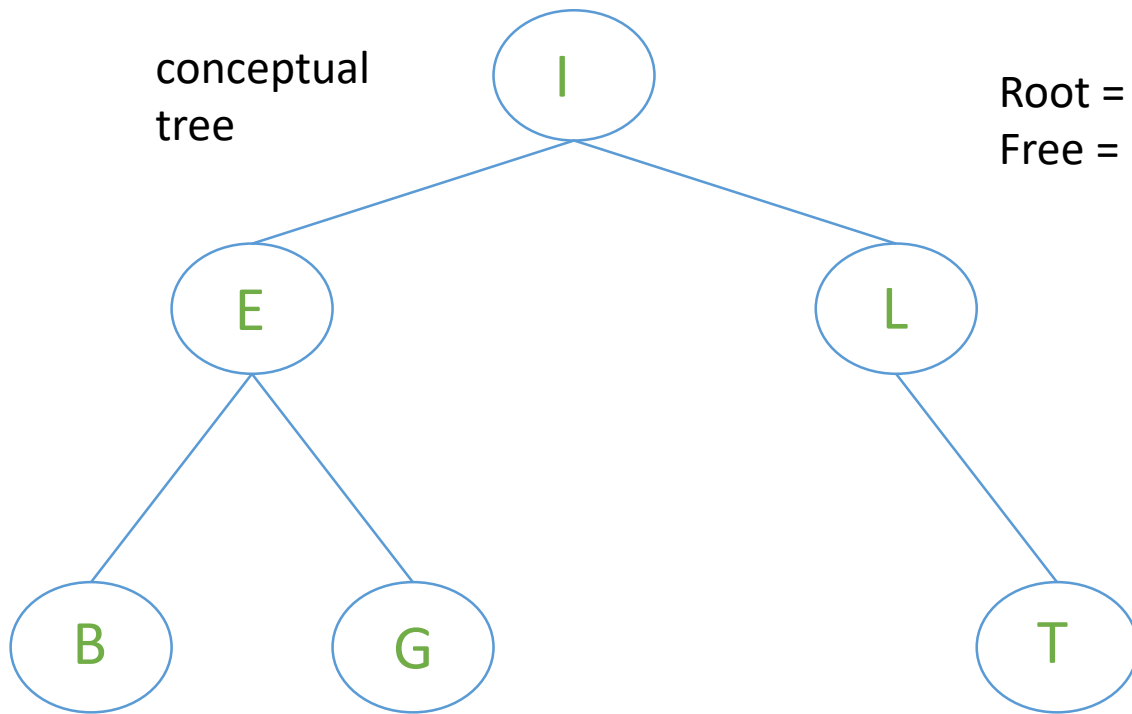
r



- memory structure for a binary tree file
  - collection of nodes
  - pointer to the root
  - list of empty nodes
    - linked by PointerL

## \* Example

- Root - pointer to the root
- Free - pointer to the head of the empty nodes list



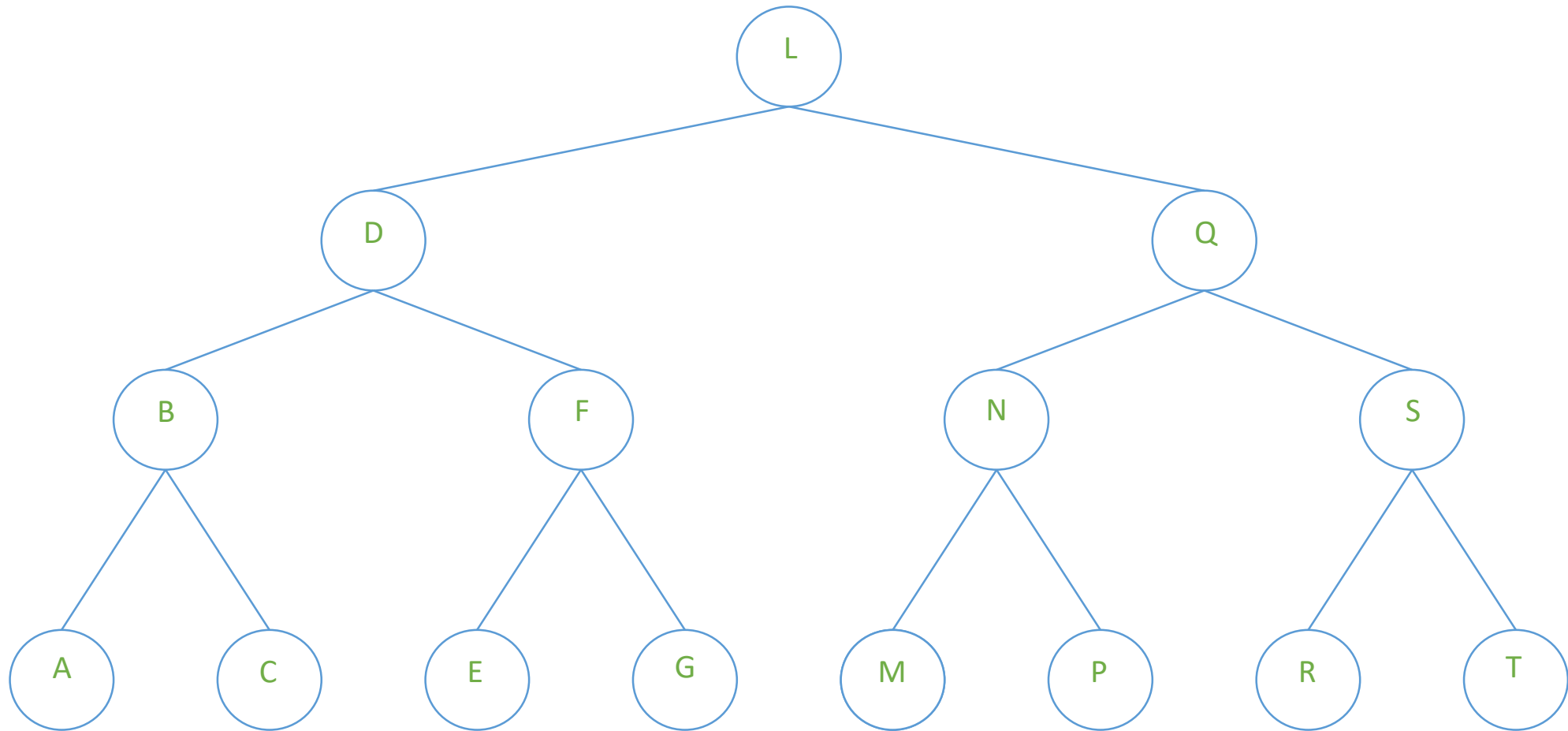
Root = 1  
Free = 3

1	2	4	I	INF <sub>I</sub>
2	8	7	E	INF <sub>E</sub>
3	-6	NULL		
4	NULL	5	L	INF <sub>L</sub>
5	NULL	NULL	T	INF <sub>T</sub>
6	-9	NULL		
7	NULL	NULL	G	INF <sub>G</sub>
8	NULL	NULL	B	INF <sub>B</sub>
9	NULL	NULL		

- operations in a binary tree (BT)
  - searching for a record with key value  $K_0$
  - inserting a record
  - removing a record
  - traversal - partial (between 2 values) / total

\* Example - binary tree (only key values are shown)

- key values: {L, D, B, Q, N, F, S, R, T, M, E, G, P, A, C}

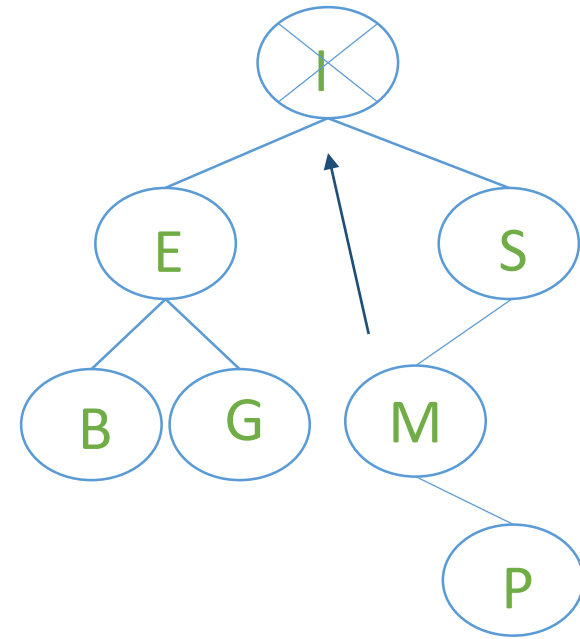


- searching for a record
  - search starts at the root, follows PointerL / PointerR - based on a comparison result, a subtree is not considered anymore
  - how many comparisons to find record with key value M in the previous tree?
  - analysis, e.g.:
    - collection of English texts; tuples: (L, 50k), (D, 100k), etc
    - analyzing the most common letters in English; how many times does a letter appear in the collection?



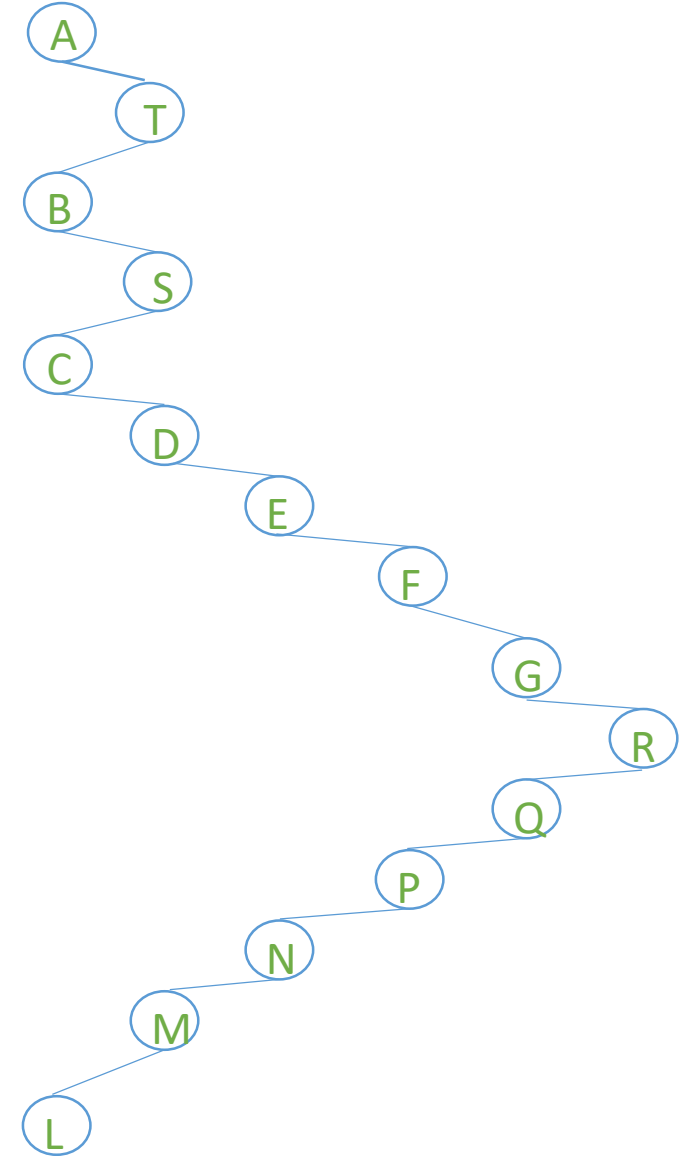
- inserting a record
  - detect the position of the record
  - store the record in a free node
  - link the node to its parent

- removing a record
  - search for the record
  - 3 cases
    - it doesn't have children
      - the parent's pointer := NULL
    - it has 1 child
      - the child is attached to the parent
    - it has 2 children
      - it's replaced with the closest value (e.g., in-order successor)
  - the node is added to the empty nodes list



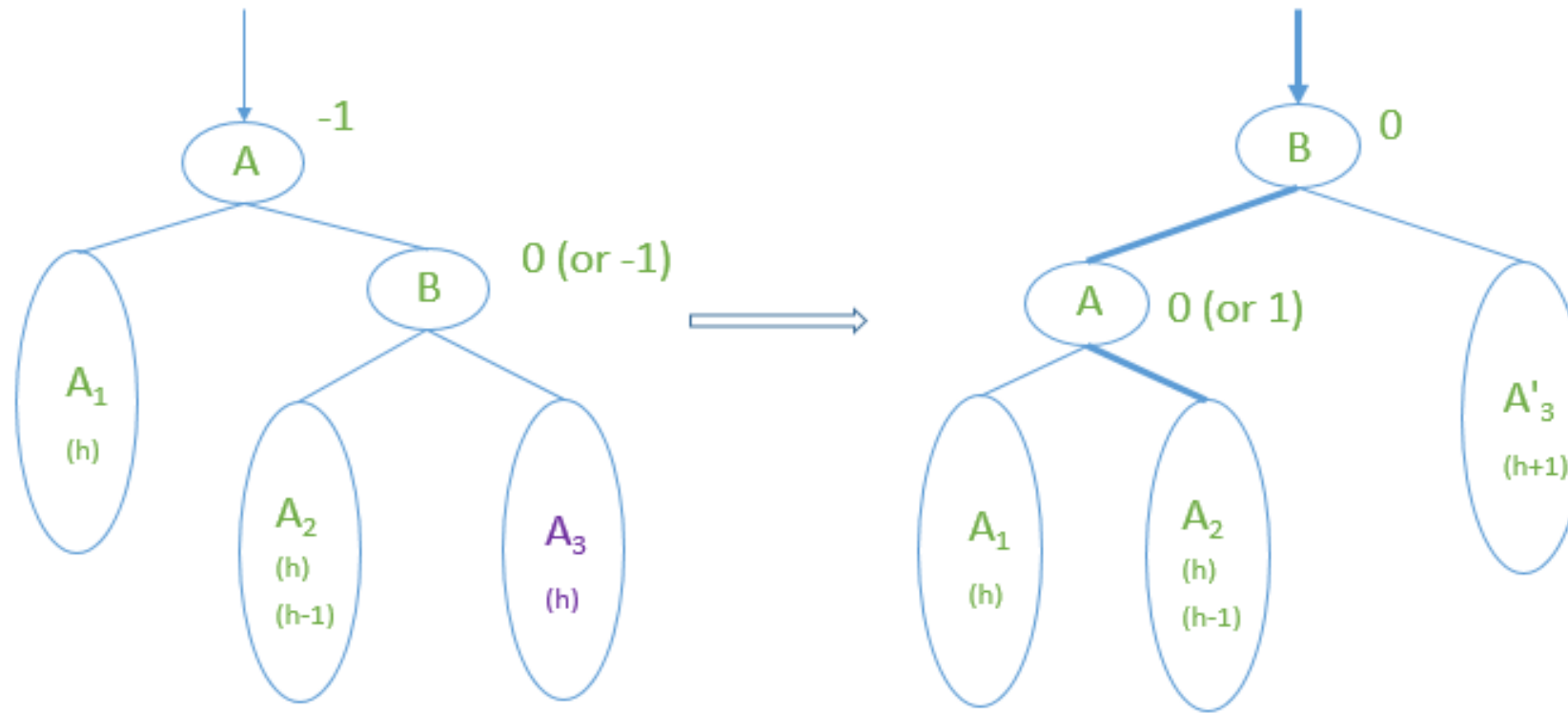
## \* Example

- key values {L, D, B, Q, N, F, S, R, T, M, E, G, P, A, C} are now provided in a different order: {A, T, B, S, C, D, E, F, G, R, Q, P, N, M, L}
- the tree on the right is obtained
- how many comparisons to find record with key value M?
- *degenerate tree*
  - sequential search
- the shape of the tree, and hence the search time, depends on the order in which data is added to the collection
- minimum search time - an *optimal tree*, i.e., one in which terminals are stored on at most 2 consecutive levels



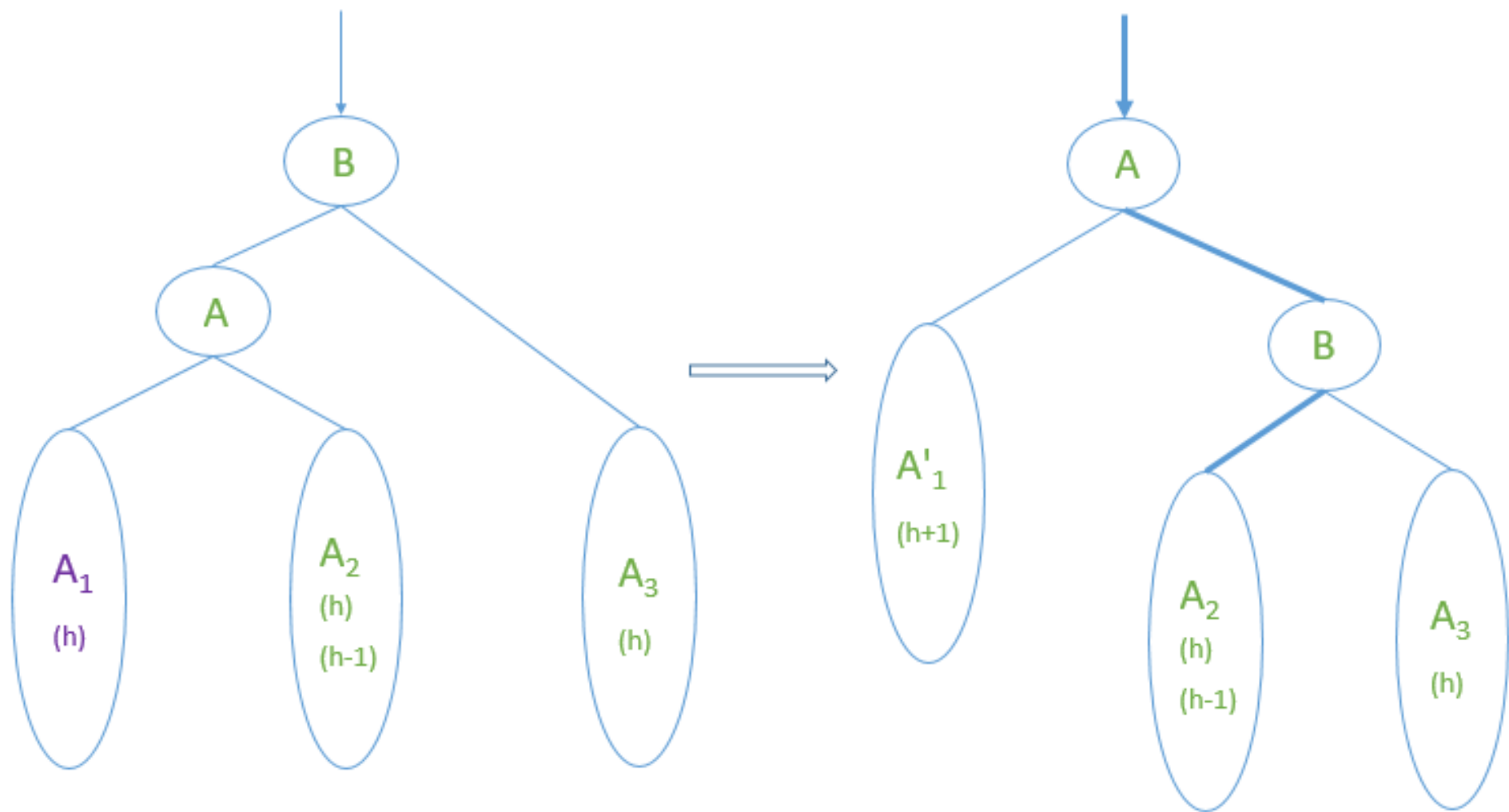
- definitions
  - *height* of the tree
    - length of longest path from root to terminals
  - a binary tree is *balanced* if, for every node N, the difference between the heights of N's subtrees is 0, 1 or -1
- obs. operations on a balanced tree can unbalance it; the tree can be rebalanced through a small number of changes
- a value is added to the  $A_3$  tree on the next page (v1), causing its height to change (it increases by 1)
  - the subtrees' heights are shown between parentheses; for nodes A and B, the difference between the heights of their subtrees is also shown
  - after the insertion, the tree becomes unbalanced
  - the right-hand side of the figure shows a transformation that rebalances the tree with the root in A

v1

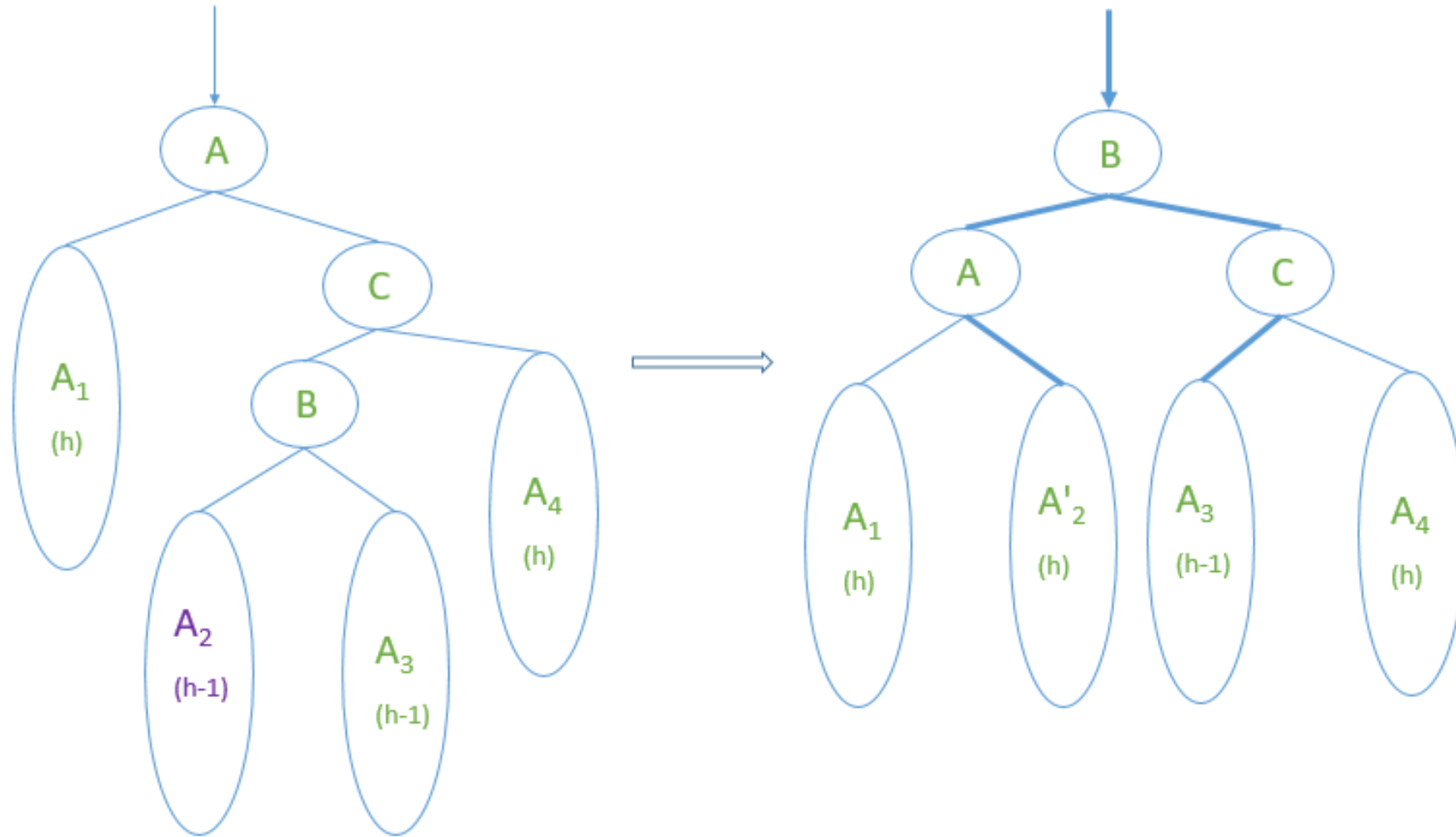


- some assignments are necessary for this rebalancing in the memory area that stores the tree (PointerR for A, PointerL for B, pointer referring to A will refer to B)
- [Kn76] enumerates all 6 rebalancing transformations

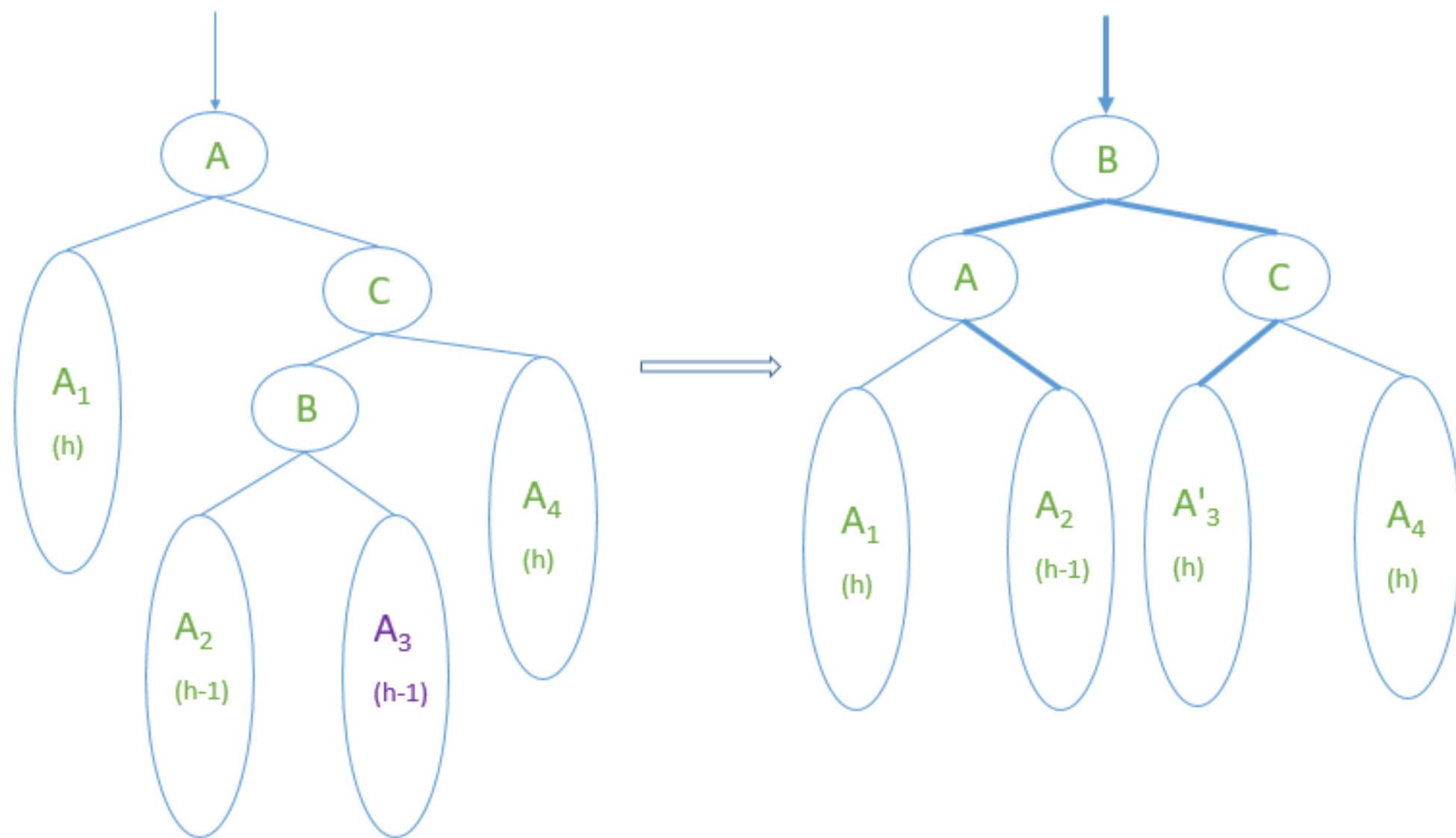
v2



v3

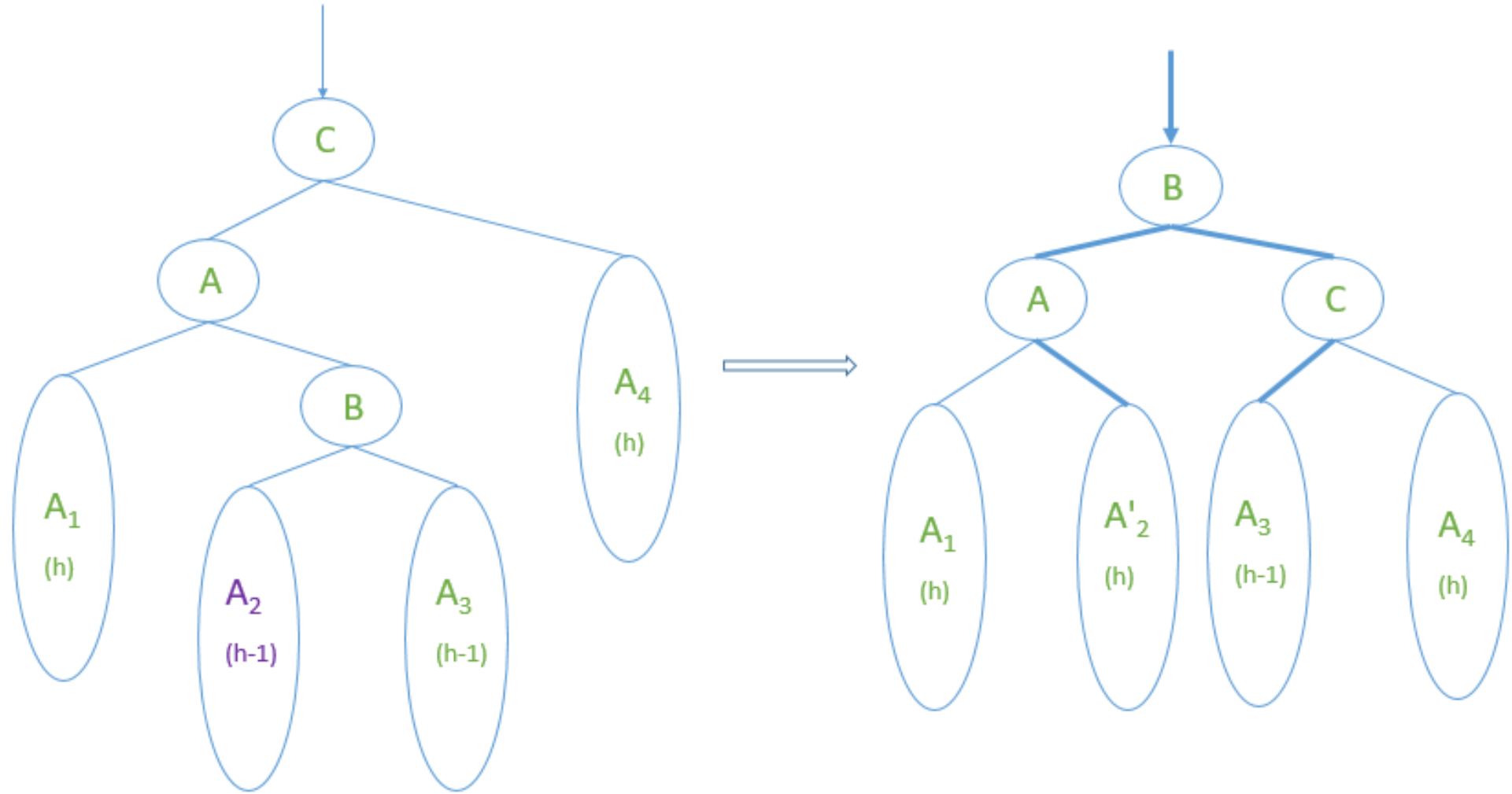


v4

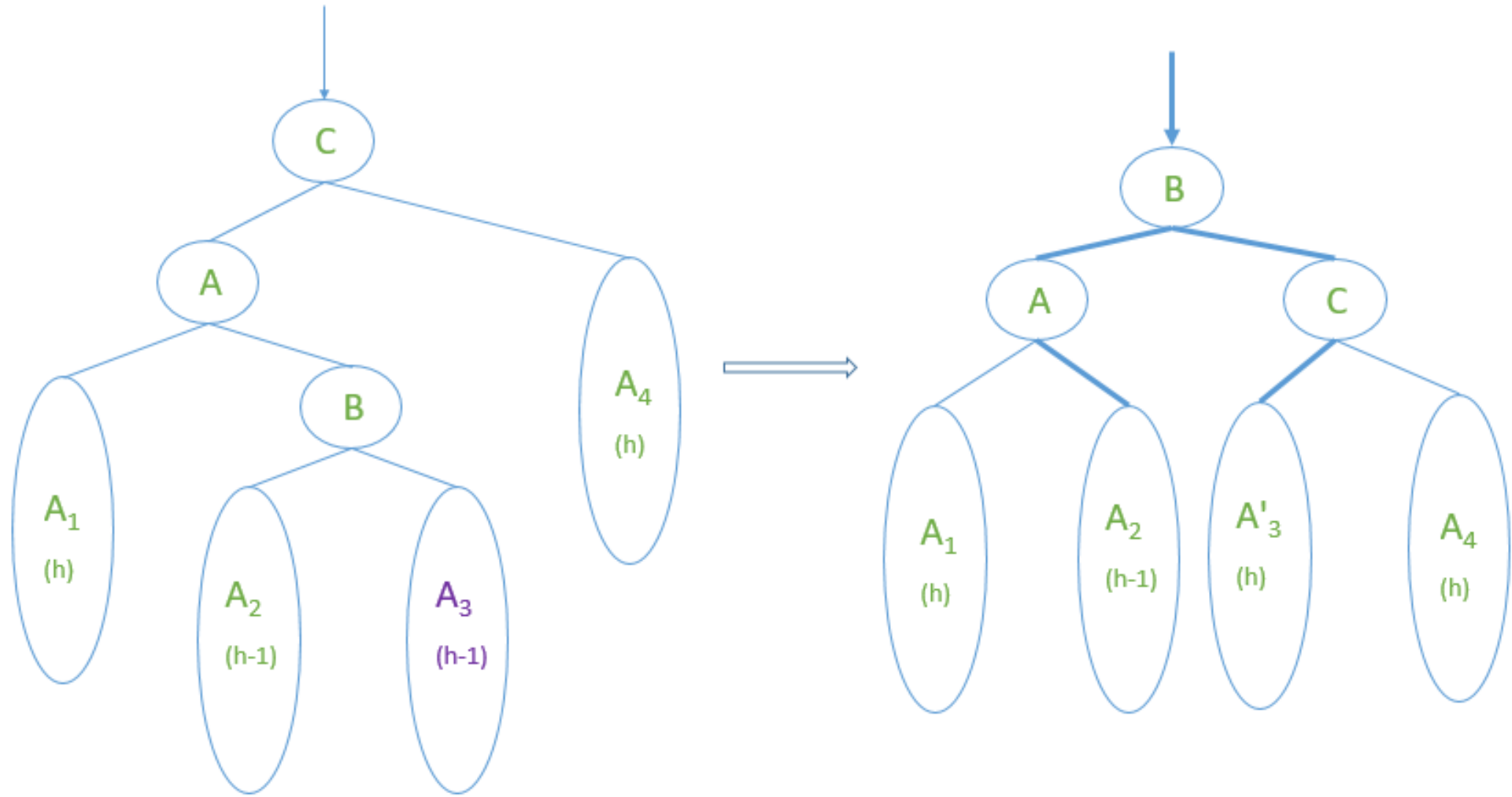




v5



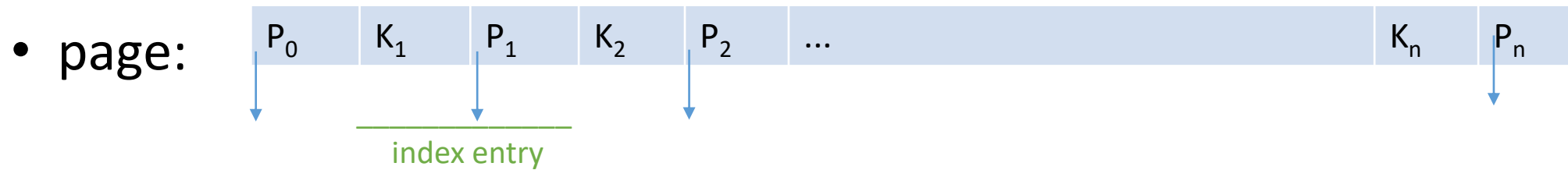
v6



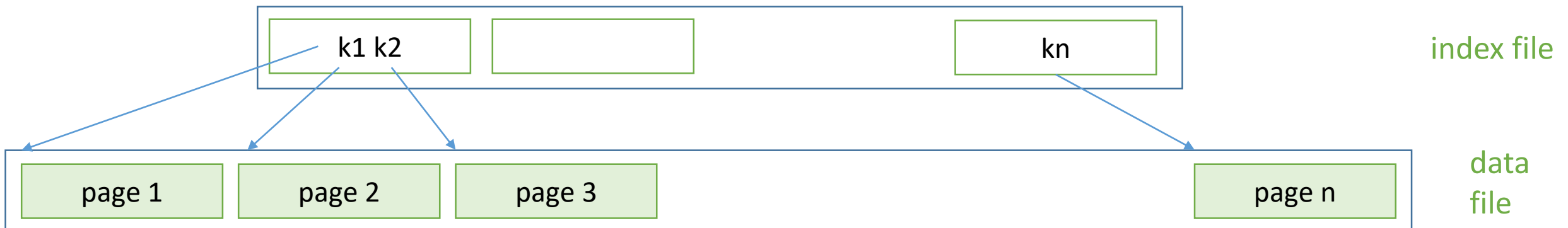
# Indexed Sequential Access Method (ISAM)

\* Example. Q: Find all phones with *rating* > 9 - range selection query

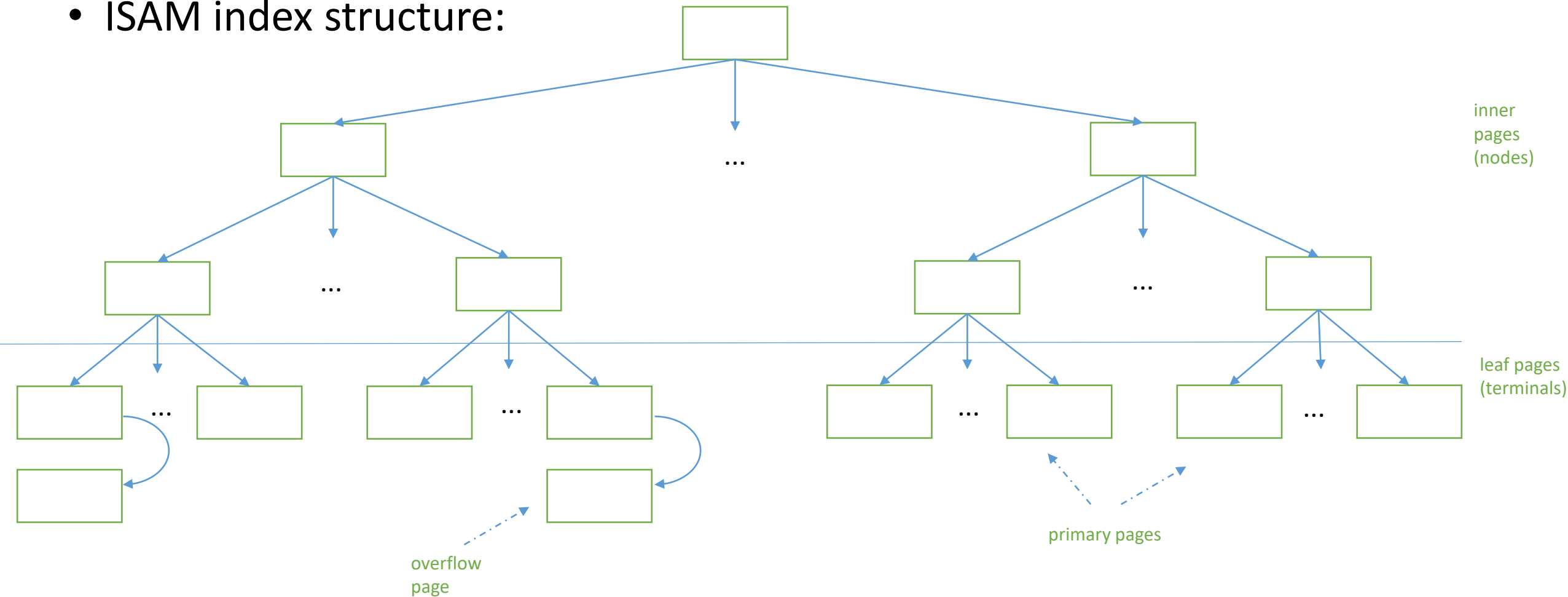
- data stored in sorted file - identify 1<sup>st</sup> phone using binary search; scan file to get the rest of the phones
- large file => potentially expensive binary search
- create another file - one record / page in the data file
- records of the form <1<sup>st</sup> key on the page, pointer to the page>, sorted on the key (*rating* in the example)



- one-level index file:



- size of index file - much smaller than size of data file => faster binary search
- to further reduce the cost of binary search, index files are repeatedly created on top of previously created ones, until one such file fits on a single page
- ISAM index structure:



- file creation
  - allocate leaf pages - sequentially allocated, sorted on the key
  - allocate inner pages
  - inserts that exceed a page's capacity – allocate overflow pages
- search
  - starts at the root
  - comparisons with the key to find the leaf page
  - cost – disk I/O
    - $\log_f n$ , where  $n$  – number of primary leaf pages,  $f$  – number of children / index page (*fan-out*)
  - binary search cost:  $\log_2 n$
  - one-level index cost:  $\log_2(n/f)$

data pages

index pages

overflow pages

# References

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