Artificial Intelligence
Ex1

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Proof of the optimality of the heuristic function

I chose to work with the Chebychev distance. Indeed, in our case, it is an optimal heuristic function. It is admissible and consistent.

$h(node) = max(|x_{goal} - x_{node}|, |y_{goal} - y_{node}|)$

Admissibility

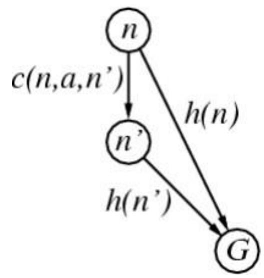
We need to prove that the heuristic function h(n) is always less or equal to the true cheapest cost from n to the goal.
h(n) $\leq$ h*(n)
when h*(n) is the true cheapest cost from n to the goal.

h calculates the minimum distance, without taking into account the cliffs in the grid (-1) that can be on our way to the goal. If all the nodes have cost value of 1, h gives us the real cheapest cost. If the nodes have different cost values, we have h(n) $\leq$ h*(n).
Indeed, if there is a cliff (or several cliffs) on our way to the goal, we'll need to change the way (we maybe can't take the cheapest way), we'll get that or it will be the same cost (because the cliff didn't influence), or we'll have to pass through other nodes and then, h(n) $\leq$ h*(n).

Thus, h(n) $\leq$ h*(n).
We also have, h(goal) = 0

Consistence

We need to prove that for every node n, every successor n' of n generated by an action a,
h(n) $\leq$ c(n, a, n') + h(n')
when c(n, a, n') is the cost from n to n' after an action a.



Let's define :

h(n) : the minimum distance from n to goal.

h(n') : the minimum distance from n' to goal.

path(n, goal) : the path from n to goal.

Let's prove consistence by contradiction.

Assume we have, h(n) > c(n, a, n') + h(n')

Let's say that n passes through n' on its way to goal.

$path : n \rightarrow n' \rightarrow ... \rightarrow goal$

$|path| = |n \rightarrow n'| + |n' \rightarrow ... \rightarrow goal|$

$h(n) = c(n, a, n') + h(n') > h(n)$

We got h(n) > h(n) which is a contradiction. Thus, we have h(n) $\leq$ c(n, a, n') + h(n').

h is consistent.

h is admissible and consistent, we can then say, that h is optimal.