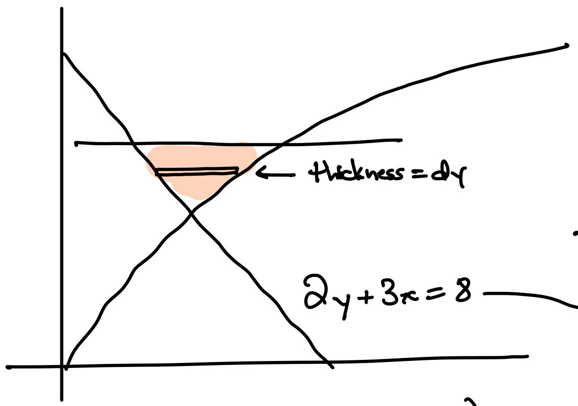
1. HW Questions from 6.1?
3. 6.2 Volumes of revolution-disks and washers



$2y = 5\sqrt{x}$

$\frac{2}{5}y = \sqrt{x}$

$\frac{4}{25}y^2 = x$

$3x = 8 - 2y$

$x = \frac{8}{3} - \frac{2}{3}y$

$$\text{width} = \frac{4}{25}y^2 - \left(\frac{8}{3} - \frac{2}{3}y\right)$$

$$= \frac{4}{25}y^2 - \frac{8}{3} + \frac{2}{3}y$$

$$\int_{2.5}^{3} \frac{4}{25}y^2 - \frac{8}{3} + \frac{2}{3}y \, dy$$

$$\left.\frac{4}{75}y^3 - \frac{8}{3}y + \frac{1}{3}y^2\right]_{2.5}^{3} = -\frac{89}{25} - \left(-\frac{15}{4}\right)$$

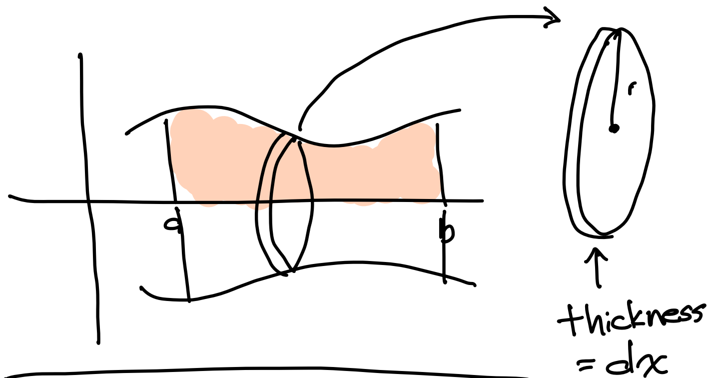
$$= \frac{19}{100}$$

6.2 Volumes of Revolution (Washers and Disks)

Idea: Take an area, rotate it around a line, and find the volume of the solid that it produces.

1 Disks around a horizontal axis

Let $f(x)$ be a function that is positive between $x = a$ and $x = b$. Let's rotate the area under $f(x)$ from a to b around the x-axis and work out the volume of the resulting 3-D shape.



Volume of Disk

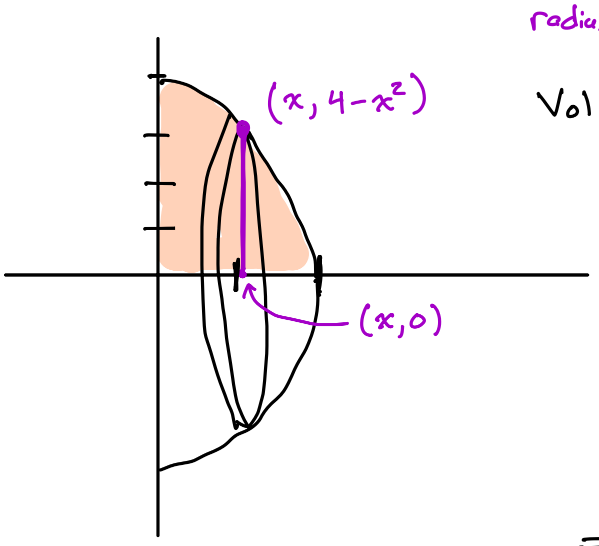
$= \text{Area} \cdot \text{thickness}$

$= \pi(\text{radius})^2 dx$

$$\boxed{\text{Volume of Solid} = \int_a^b \pi(\text{radius})^2 \, dx}$$

1.0.1 Example:

Take the area under $y = 4 - x^2$ in the first quadrant, rotate it around the x-axis, and find the volume.



radius $= 4-x^2$

$$\text{Vol} = \pi\int_0^2 (4-x^2)^2 \, dx$$

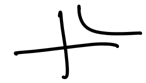
$$= \pi \int_0^2 16 - 8x^2 + x^4 \, dx$$

$$= \pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right]_0^2$$

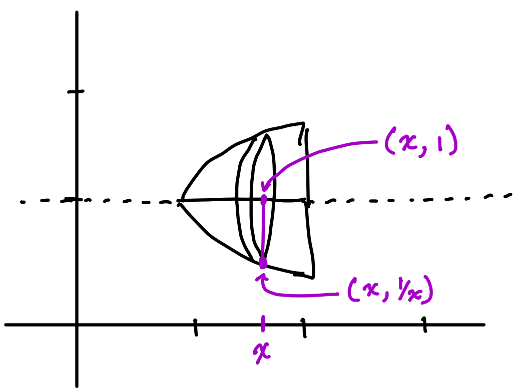
$$= \pi \left[32 - \frac{64}{3} + \frac{32}{5} \right] =$$

$$\boxed{= \frac{256\pi}{15} \approx 53.617}$$

1.0.2 Example:



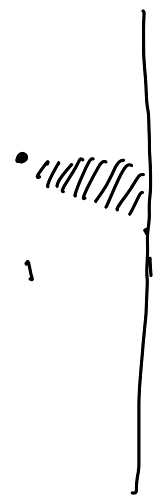
Set up an integral for the area bounded by $y = 1$, $y = \frac{1}{x}$, and $x = 2$, and rotate it around the line $y = 1$. Use a calculator to approximate the area.



radius $= 1 - \frac{1}{x}$

$$\text{Volume} = \pi \int_1^2 \left(1 - \frac{1}{x}\right)^2 dx$$

$$\approx .357$$



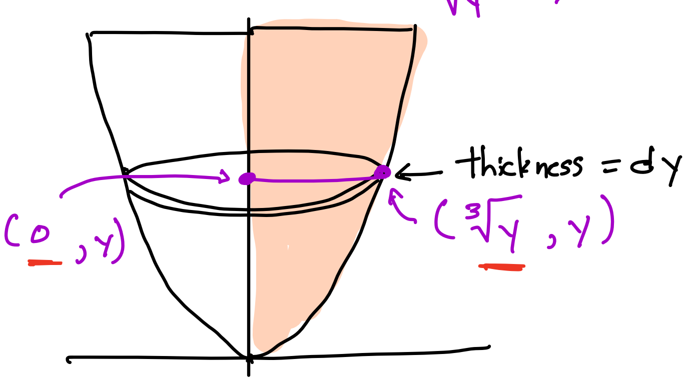
2 Disks around a vertical axis

If we rotate around a vertical axis, the disks lie flat, and the integration is done with respect to y.

2.0.1 Example:

Take the area bounded by $y = x^3$, the y-axis, and $y = 8$ and rotate it around the y-axis, and find the volume.

$\sqrt[3]{y} = x$



radius $= \sqrt[3]{y} = y^{1/3}$

$$\text{Vol} = \pi \int_0^8 (y^{1/3})^2 dy$$

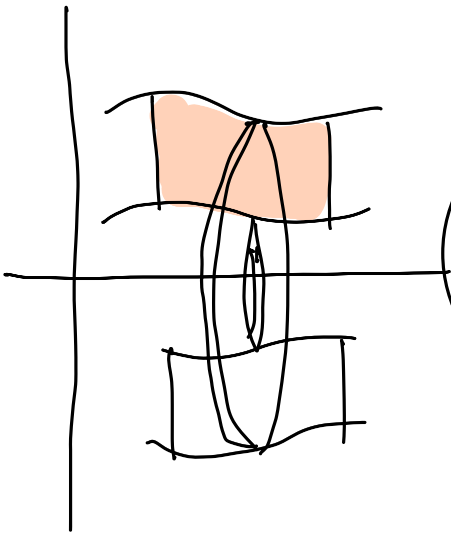
$$= \pi \int_0^8 y^{2/3} dy = \pi \frac{3}{5} y^{5/3} \Big]_0^8$$

$$= \frac{3\pi}{5}\left(8^{5/3} - 0^{5/3}\right)$$

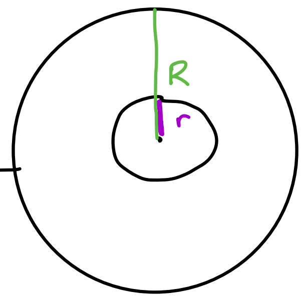
$$= \frac{3\pi}{5}(32) = \boxed{\frac{96\pi}{5}} \approx 60.319$$

3 Washers

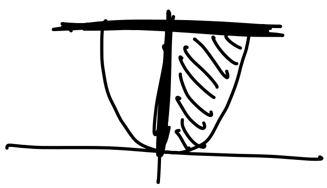
If the area does not touch the axis of rotation, the cross-section is a washer instead of a disk.



Area of Washer $= \pi R^2 - \pi r^2 = \pi (R^2 - r^2)$

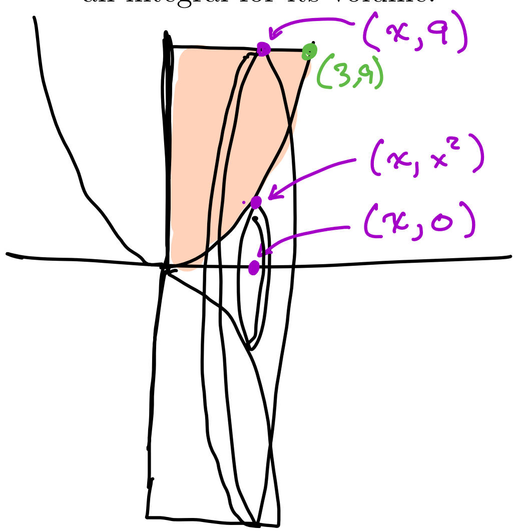


$$\boxed{Vol = \pi \int_a^b R^2 - r^2 \, dx}$$



3.0.1 Example:

Take the area bounded by $y = x^2$, the y-axis, and $y = 9$. Rotate it around the x-axis, and set up an integral for its volume.



$R = 9 \qquad r = x^2$

$$\boxed{V = \pi \int_0^3 9^2 - (x^2)^2 \, dx}$$

3.0.2 Example:

Repeat the previous example, but this time rotate around the y-axis.