8) $$\underbrace{\int e^{7x}\sin(2x)\,dx}_{I} = -\frac{1}{2}e^{7x}\cos(2x) + \frac{7}{2}\int e^{7x}\cos 2x\,dx$$

$$\boxed{\begin{array}{ll} u = e^{7x} & dv = \sin(2x)dx \\ du = 7e^{7x}dx & v = -\frac{1}{2}\cos(2x) \end{array}}$$

$$= -\frac{1}{2}e^{7x}\cos(2x) + \frac{7}{2}\left[\frac{1}{2}e^{7x}\sin(2x) - \frac{7}{2}\underbrace{\int e^{7x}\sin(2x)}_{I}\right.$$

$$\boxed{\begin{array}{ll} u = e^{7x} & dv = \cos(2x)dx \\ du = 7e^{7x}dx & v = \frac{1}{2}\sin(2x) \end{array}}$$

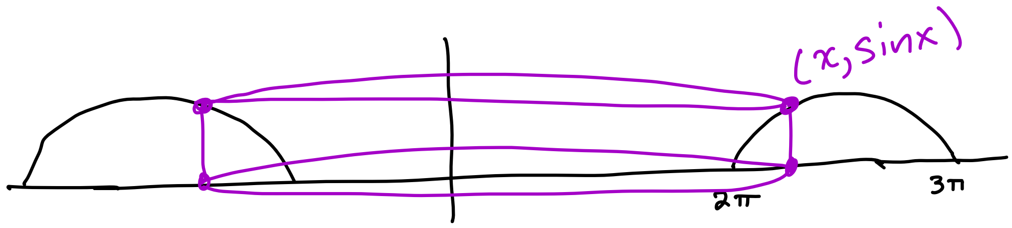
$$\frac{4}{4}I = -\frac{1}{2}e^{7x}\cos(2x) + \frac{7}{4}e^{7x}\sin 2x - \frac{49}{4}I$$

$$\frac{49}{4}I \qquad\qquad + \frac{49}{4}I$$

$$\frac{4}{53}\cdot\frac{53}{4}I = \left[-\frac{1}{2}e^{7x}\cos(2x) + \frac{7}{4}e^{7x}\sin 2x\right]\frac{4}{53}$$

$$I = -\frac{2}{53}e^{7x}\cos(2x) + \frac{7}{53}e^{7x}\sin(2x) + C$$

9.



radius = x

height = $\sin x$

$$V = 2\pi\int_{2\pi}^{3\pi} x\sin x\,dx$$

$+\ x$	$\sin x$
$-\ 1$	$-\cos x$
0	$-\sin x$

$$\stackrel{2\pi}{=}\Big[-x\cos x + \sin x\Big]_{2\pi}^{3\pi}$$

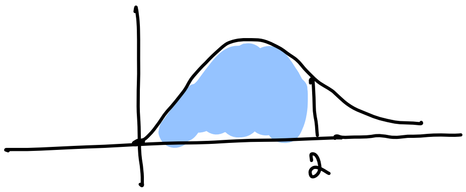
$$2\pi\left[\left[-3\pi\underbrace{\cos(3\pi)}_{(-1)} + \underbrace{\sin(3\pi)}_{0}\right] - \left[-2\pi\underbrace{\cos(2\pi)}_{(1)} + \underbrace{\sin(2\pi)}_{0}\right]\right]$$

$$2\pi\Big([3\pi] - [-2\pi]\Big) = \boxed{5\pi}\cdot 2\pi = \boxed{10\pi^2}$$

10. $\int x \sin(9x)\,dx = -\frac{1}{9}x\cos(9x) + \frac{1}{81}\sin(9x) + C$

$+$	x	$\sin(9x)$
$-$	1	$-\frac{1}{9}\cos(9x)$
	0	$-\frac{1}{81}\sin(9x)$

11. $y = 3xe^{-x}$



$+$	$3x$	e^{-x}
$-$	3	$-e^{-x}$
	0	e^{-x}

$$A = \int_0^2 3xe^{-x}\,dx$$

$$= -e^{-x}\cdot 3x - 3e^{-x}\Big]_0^2$$

$$= \left(-e^{-2}\cdot 6 - 3e^{-2}\right) - \left(\underbrace{-e^{0}\cdot 3\cdot 0}_{0} - \underbrace{3e^{0}}_{-3}\right)$$

$$\left(-\frac{6}{e^2} - \frac{3}{e^2}\right) - (-3)$$

$$-\frac{9}{e^2} + 3 \approx 1.782$$

1.2.1 Example:

$$\int \cos^4 x\,dx = \int \left[\cos^2 x\right]^2 dx = \int \left[\tfrac{1}{2}\left(1+\cos 2x\right)\right]^2 dx = \frac{1}{4}\int \left(1+\cos 2x\right)^2 dx$$

$$= \frac{1}{4}\int 1 + 2\cos(2x) + \cos^2(2x)\,dx = \frac{1}{4}\int \underline{1} + 2\cos(2x) + \underline{\tfrac{1}{2}} + \tfrac{1}{2}\cos(4x)\,dx$$

$$= \frac{1}{4}\int \frac{3}{2} + 2\cos(2x) + \frac{1}{2}\cos(4x)\,dx = \frac{1}{4}\left[\frac{3}{2}x + 2\,\frac{\sin(2x)}{2} + \frac{1}{2}\cdot\frac{\sin(4x)}{4}\right]$$

$$= \frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C,$$

2 Integrals of the form $\int \sec^m x \tan^n x\,dx$

These are similar to the sine/cosine integrals:

- If power of secant is even, let $u = \tan x$,
- If power of tangent is odd, let $u = \sec x$,
- otherwise, it's more complicated ....

2.0.1 Example:

$$\int \sec^2 x \tan^3 x\,dx = \int \tan^3 x\,(\sec^2 x\,dx) = \int u^3\,du = \frac{1}{4}u^4 + C = \boxed{\frac{1}{4}\tan^4 x + C}$$

$$\boxed{\begin{aligned} u &= \tan x \\ du &= \sec^2 x\,dx \end{aligned}}$$

OR

$$\int \underbrace{\sec x}_{u}\,\underbrace{\tan^2 x}_{\substack{\sec^2 x - 1 \\ u^2 - 1}}\,\underbrace{(\sec x \tan x\,dx)}_{du} = \int u(u^2-1)\,du$$

$$\boxed{\begin{aligned} u &= \sec x \\ du &= \sec x \tan x\,dx \end{aligned}}$$

$$= \int u^3 - u\,du = \frac{1}{4}u^4 - \frac{1}{2}u^2 + C$$

$$= \frac{1}{4}\sec^4 x - \frac{1}{2}\sec^2 x + C$$

2.0.2 Example:

$$\int \tan^5 x\, dx = \int \tan^2 x \tan^2 x \tan x\, dx = \int (\sec^2 x - 1)^2 \tan x\, dx$$

$$= \int (\sec^4 x - 2\sec^2 x + 1) \tan x\, dx$$

$$= \int \sec^4 x \tan x\, dx - 2\int \sec^2 x \tan x\, dx + \int \tan x\, dx$$

$\boxed{\begin{array}{l} u = \sec x \\ du = \sec x \tan x\, dx \end{array}}$

$\int u^3\, du$

$\frac{1}{4} u^4$

$\frac{1}{4}\sec^4 x$

$\boxed{\begin{array}{l} u = \tan x \\ du = \sec^2 x\, dx \end{array}}$

$-2\int u\, du$

$-2\frac{u^2}{2}$

$-u^2$

$-\tan^2 x$

$\int \frac{\sin x}{\cos x}\, dx$

$u = \cos x$

$du = \sin x\, dx$

$\int \frac{1}{u}\, du$

$= \ln u$

$= \ln|\cos x|$

$$\boxed{\frac{1}{4}\sec^4 x - \tan^2 x + \ln|\cos x| + C}$$

2.0.3 Example:

$$\int \sec^4 x \tan^4 x\, dx$$

$\boxed{\begin{array}{l} u = \tan x \\ du = \sec^2 x\, dx \end{array}}$

$$\int \underbrace{\sec^2 x}_{\substack{\tan^2 x + 1 \\ u^2+1}} \underbrace{\tan^4 x}_{u^4} \underbrace{(\sec^2 x\, dx)}_{du}$$

$$= \int (u^2+1) u^4\, du = \int u^6 + u^4\, du$$

$\vdots$