1. MyOpenMath
   A. Tries and Versions - you can get 100%, videos
   B. Questions?
2. Roll
3. Section 6.1 Areas between curves

5.5. $\int_0^1 x^3(x^4-2)^6\,dx = \int_{-2}^{-1} \frac{1}{4}u^6\,du = \frac{1}{4}\frac{1}{7}u^7\Big]_{-2}^{-1}$

$u = x^4 - 2$
$du = 4x^3dx$
$\frac{1}{4}du = x^3dx$
Top: $(1)^4 - 2 = -1$
bott: $(0)^4 - 2 = -2$

$= \frac{1}{28}u^7\Big]_{-2}^{-1} = \frac{1}{28}\left((-1)^7 - (-2)^7\right)$

$= \frac{1}{28}(-1 + 128)$

$= \frac{127}{28}$

7. $\int \frac{dx}{(7x+4)^3} = \int \frac{\frac{1}{7}du}{u^3} = \frac{1}{7}\int \frac{1}{u^3}du = \frac{1}{7}\int u^{-3}du$

$u = 7x + 4$
$du = 7dx$
$\frac{1}{7}du = dx$

$= \frac{1}{7}\frac{1}{(-2)}u^{-2} + C = -\frac{1}{14}u^{-2} + C$

$= -\frac{1}{14u^2} + C = \boxed{-\frac{1}{14(7x+4)^2} + C}$

9. $\int \frac{3x}{x^2+1}dx = \int \frac{3x}{u}\frac{du}{2x} = \int \frac{3}{2}\cdot\frac{1}{u}du = \frac{3}{2}\int \frac{1}{u}du$

$u = x^2 + 1$
$du = 2xdx$
$\frac{du}{2x} = dx$

$= \frac{3}{2}\ln u + C$

$= \boxed{\frac{3}{2}\ln(x^2+1) + C}$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C = \tan^{-1}(x) + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

---

6. $$\int \sec(6x)\tan(6x)\,dx = \int \sec u \tan u \cdot \frac{1}{6}\,du$$

$$\boxed{\begin{aligned} u &= 6x \\ du &= 6\,dx \\ \tfrac{1}{6}\,du &= dx \end{aligned}}$$

$$= \frac{1}{6}\sec u + C = \boxed{\frac{1}{6}\sec(6x) + C}$$

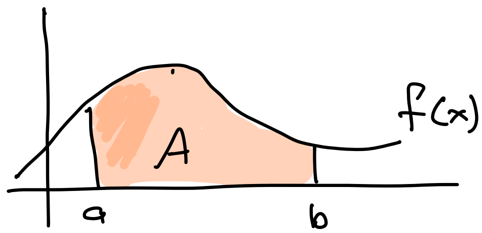
$F(x)$ is the antiderivative of $f(x)$.

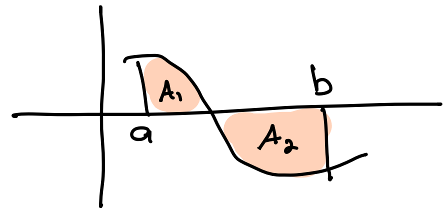
6.1 Area Between Curves

Recall: The area under a curve (i.e. the area between the curve and the x-axis) is given by

$$A = \int_a^b f(x)\,dx.$$

$$= F(b) - F(a)$$





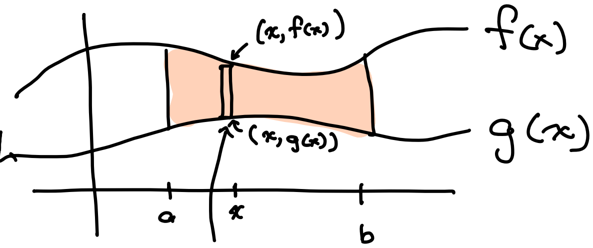
$$\int_a^b f(x)\,dx = A_1 - A_2$$

1 Vertical Slicing

Suppose you want the area between the curves $y = f(x)$ on top and $y = g(x)$ on bottom:

Top - bottom

$$\text{Area} = \int_a^b [f(x) - g(x)]\,dx$$



Rectangle area = height · width

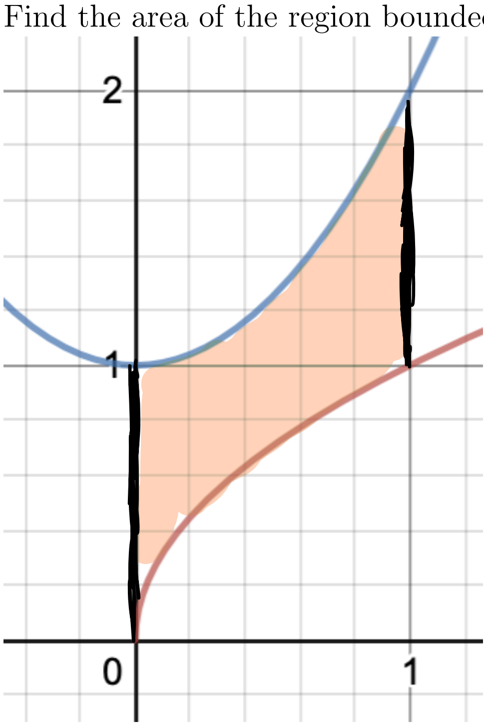
$= [f(x) - g(x)]\,dx$

width = tiny change in x

$= dx$

1.0.1 Example:

Find the area of the region bounded by $y = \sqrt{x}$, $y = x^2 + 1$, $x = 0$, and $x = 1$.



$$\int_0^1 (x^2 + 1) - (\sqrt{x})\,dx$$

Top bottom

$x^{1/2}$

$\frac{2}{3}x^{3/2}$

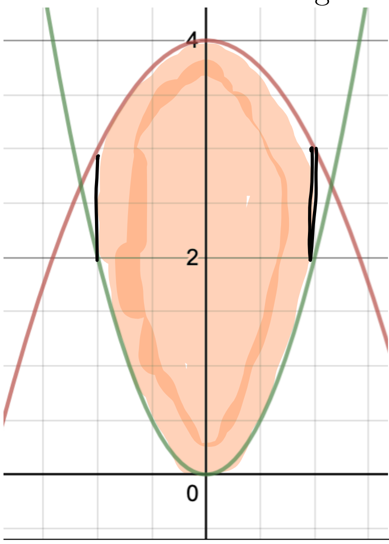
$$= \frac{1}{3}x^3 + x - \frac{2}{3}x^{3/2}\Big]_0^1$$

$$= \left(\frac{1}{3} + 1 - \frac{2}{3}\right) - (0)$$

$$= \boxed{\frac{2}{3}}$$

1.0.2 Example:

Find the area of the region bounded by $y = \overset{\text{top}}{4 - x^2}$, $y = \overset{\text{bot}}{2x^2}$, $x = -1$, and $x = 1$.



$$A = \int_{-1}^{1} (4 - x^2) - (2x^2)\, dx = \int_{-1}^{1} 4 - 3x^2\, dx$$

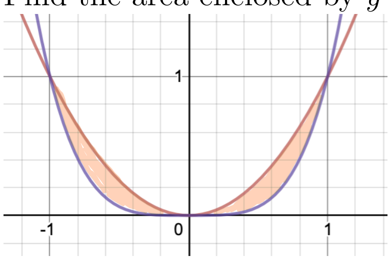
$$= 2\int_0^1 4 - 3x^2\, dx = 2\left[4x - x^3\right]_0^1$$

$$= 2(4-1) - 2(0-0)$$

$$= 2 \cdot 3 = \boxed{6}$$

1.0.3 Example:

Find the area enclosed by $y = x^2$ and $y = x^4$.



$$2\int_0^1 x^2 - x^4\, dx = 2\left[\tfrac{1}{3}x^3 - \tfrac{1}{5}x^5\right]_0^1$$

$$= 2\left[\tfrac{1}{3} - \tfrac{1}{5}\right] - 2\left[\tfrac{1}{3}\cdot 0^3 - \tfrac{1}{5}\cdot 0^5\right]$$

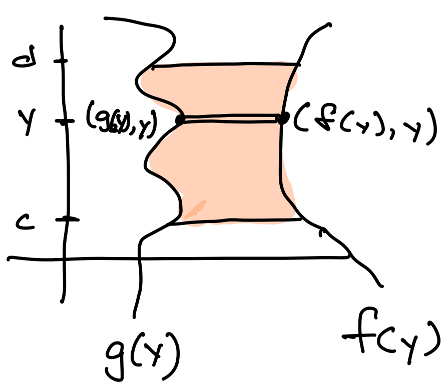
$$= 2\left[\tfrac{5}{15} - \tfrac{3}{15}\right] = 2\left[\tfrac{2}{15}\right]$$

$$= \boxed{\tfrac{4}{15}}$$

2 Horizontal Slicing

Suppose you want the area between the curves $x = f(y)$ on right and $x = g(y)$ on left:

$$\text{Area} = \int_c^d \overset{\text{Right} - \text{Left}}{[f(y) - g(y)]}\, dy$$



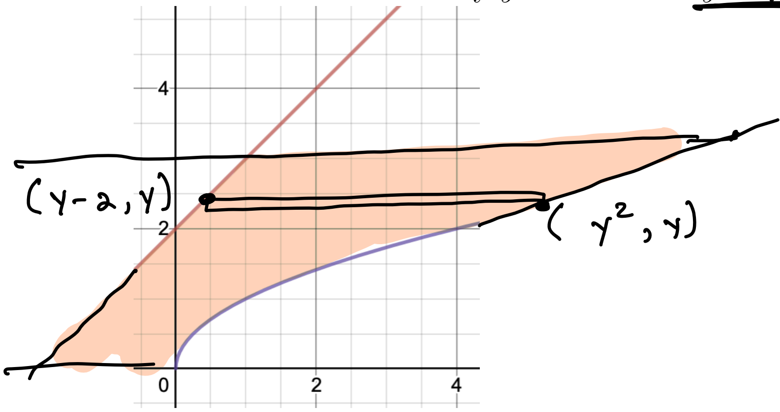
Area of Rect = width · height

$= [f(y) - g(y)] \cdot dy$

2.0.1 Example:

Left: $y-2=x$ Right: $y^2=x$

Find the area bounded by $y = x + 2$ and $y = \sqrt{x}$ between $y = 0$ and $y = 3$.



$$\text{Area} = \int_0^3 y^2 - (y-2)\,dy$$

$$= \int_0^3 y^2 - y + 2\,dy$$

$$= \frac{1}{3}y^3 - \frac{1}{2}y^2 + 2y \Big]_0^3$$

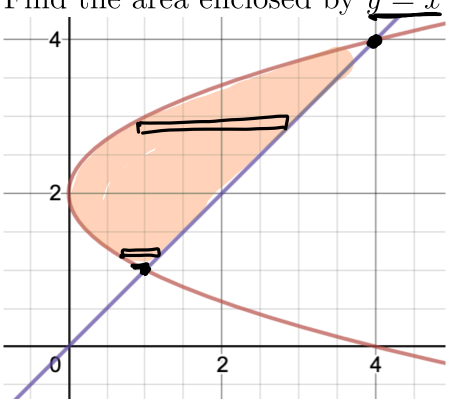
$$= \frac{1}{3}(27) - \frac{1}{2}(9) + 6$$

$$= 9 - \frac{9}{2} + 6 = \boxed{10.5}$$

2.0.2 Example:

Right − left

Find the area enclosed by $y = x$ (Right) and $x = y^2 - 4y + 4$ (left)

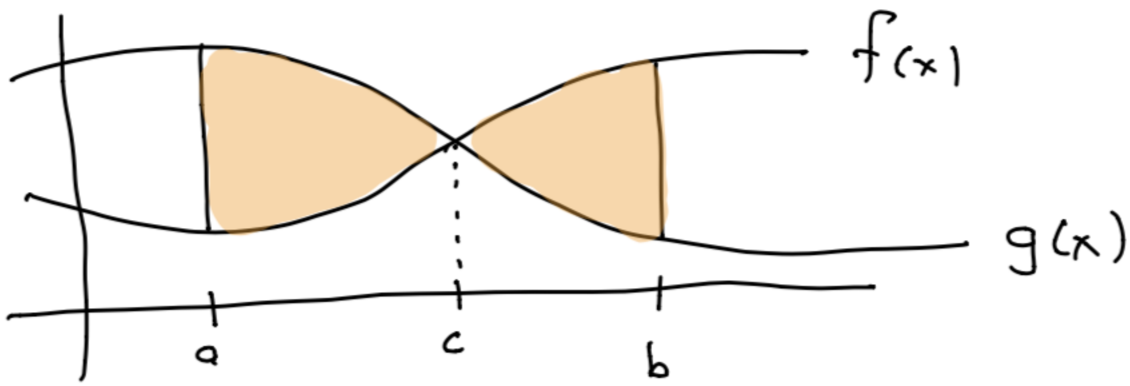


$$A = \int_1^4 y - (y^2 - 4y + 4)\,dy$$

$$= \int_1^4 5y - y^2 - 4\,dy$$

$$= \ldots$$

3 What if the curves cross between a and b?

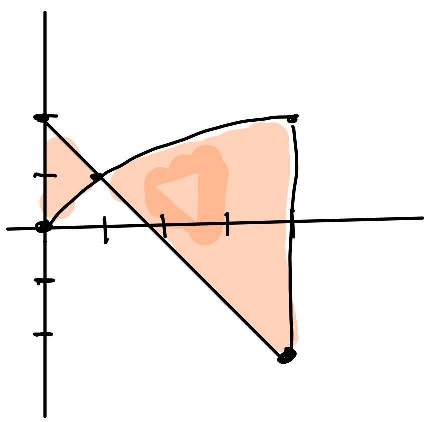


In this case we split the area at each point where the curves cross:

$$\text{Area} = \int_a^c [g(x) - f(x)]\, dx + \int_c^b [f(x) - g(x)]\, dx \quad = \quad \int_a^c \Big| [f(x) - g(x)] \Big|\, dx$$

3.0.1 Example:

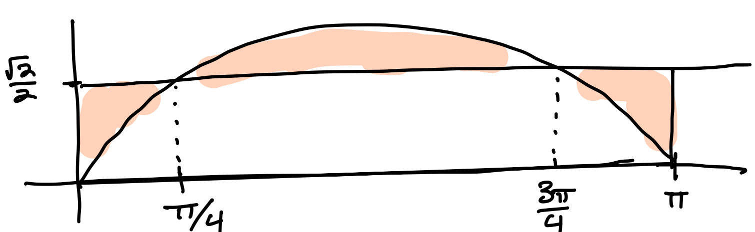
Find the area of the region between $y = \sqrt{x}$ and $y = 2 - x$ from $x = 0$, and $x = 4$.



$$A = \int_0^1 2 - x - \sqrt{x}\, dx + \int_1^4 \sqrt{x} - (2-x)\, dx$$

3.0.2 Example:

Find the area between $y = \sin x$ and $y = \frac{\sqrt{2}}{2}$ from $x = 0$ to $x = \pi$.



$$A = 2\int_0^{\pi/4} \frac{\sqrt{2}}{2} - \sin x\, dx + \int_{\pi/4}^{3\pi/4} \sin x - \frac{\sqrt{2}}{2}\, dx$$