7. $\int (3x+4)\ln(x)\,dx = \left(\frac{3}{2}x^2+4x\right)\ln x - \int \left(\frac{3}{2}x^2+4x\right)\frac{1}{x}\,dx$

$$\boxed{\begin{array}{ll} u=\ln x & dv=(3x+4)\,dx \\ du=\frac{1}{x}\,dx & v=\frac{3}{2}x^2+4x \end{array}}$$

$= \frac{3}{2}x^2\ln x + 4x\ln x - \int \frac{3}{2}x + 4\,dx$

$= \frac{3}{2}x^2\ln x + 4x\ln x - \frac{3}{4}x^2 - 4x + C$

8. $\int 3x^2 \sin\left(\frac{1}{5}x\right)\,dx$ Tabular

$+3x^2$	$\sin\left(\frac{1}{5}x\right)$
$-6x$	$-5\cos\left(\frac{1}{5}x\right)$
$+6$	$-25\sin\left(\frac{1}{5}x\right)$
0	$125\cos\left(\frac{1}{5}x\right)$

$-15x^2\cos\left(\frac{1}{5}x\right) + 150x\sin\left(\frac{1}{5}x\right) + 750\cos\left(\frac{1}{5}x\right) + C$

7.2 Trig Integrals

Our two main tools in this section are u-substitution and trig identities.

1 Integrals of the form $\displaystyle\int \sin^m x \cos^n x\, dx$

1.0.1 Starter Example:

Suppose we want to integrate $\displaystyle\int \underline{\sin^2 x}\, \underline{\cos x\, dx}$. We can do this with u-substitution.

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\, dx \end{aligned}}$$

$$= \int u^2\, du = \frac{1}{3}u^3 + C$$

$$= \boxed{\frac{1}{3}\sin^3 x + C}$$

1.1 At least one exponent is odd

Suppose that cosine has an odd exponent. Then let $u = \sin x$, $du = \cos x dx$, and use $\sin^2 x + \cos^2 x = 1$ to rewrite everything in terms of $\sin x$.

1.1.1 Example:

$$\int \sin^3 x \cos^4 x\, dx = \int \sin^2 x\, \underbrace{\cos^4 x}_{u^4} \underbrace{(\sin x\, dx)}_{(-du)} = \int (1-\cos^2 x)\cos^4 x\,(\sin x\, dx)$$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x\, dx \end{aligned}}$$

$$= \int (1-u^2)\,u^4(-du) = \int -u^4 + u^6\, du$$

$$= -\frac{1}{5}u^5 + \frac{1}{7}u^7 + C$$

$$= \boxed{-\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C}$$

1.1.2 Example:

$$\int \sin^7 x \cos^5 x \, dx = \int \overset{u^7}{\sin^7 x} \underset{\substack{\uparrow \\ (\cos^2 x)^2 \\ (1 - \sin^2 x)^2}}{\cos^4 x} \overset{du}{(\cos x \, dx)}$$

Tip: If both exponents are odd, letting u be the function with the higher exponent will make for less work.

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x \, dx \end{aligned}}$$

$$= \int \sin^7 x \, (1 - \sin^2 x)^2 \cdot (du) = \int u^7 (1 - u^2)^2 \, du$$

$$= \int u^7 (1 - 2u^2 + u^4) \, du = \int u^7 - 2u^9 + u^{11} \, du$$

$$= \frac{1}{8} \sin^8 x - \frac{2}{10} \sin^{10} + \frac{1}{12} \sin^{12} x + C$$

1.2 Both exponents are even

If both exponents are even, we rewrite the integral using the double-angle formulas

$$\sin^2 x = \tfrac{1}{2}\Big[1 - \cos(2x)\Big] \quad \text{and} \quad \cos^2 x = \tfrac{1}{2}\Big[1 + \cos(2x)\Big]$$