$$\int \sec^3 x \tan^3 x\,dx$$

$u = \tan x \quad du = \sec^2 x$

$$\int \underset{\uparrow}{\sec x}\ \underset{u^3}{\tan^3 x}\ \underset{du}{(\sec^2 x\,dx)}$$

problem

$u = \tan x \quad du = \underline{\sec^2 x\,dx}$

OR

$u = \sec x \quad du = \underline{\sec x \tan x\,dx}$

$u = \sec x \quad du = \sec x \tan x\,dx$

OR

$$\int \underset{u^2}{\sec^2 x}\ \underset{\substack{(\sec^2 x - 1)\\ u^2 - 1}}{\tan^2 x}\ \underset{du}{(\sec x \tan x\,dx)}$$

$$\int u^2 (u^2 - 1)\,du$$

Worksheet 7.2: Trig Integrals

I. Pick the Correct u

The trig integrals below can all be done using u-substitution and a pythagorean identity. For the integrals below, determine what u should be.

Steven + Todd

1. $\int \sin^3 x \cos^4 x \, dx$

2. $\int \sin^5 x \cos^7 x \, dx$ $u = \cos x$, $du = -\sin x dx$

$\int \sin^4 x \cos^7 x \, (\sin x dx)$

$(\sin^2 x)^2 \quad u^7 \quad (-du)$

$(1-\cos^2 x)^2$

$(1-u^2)^2$

$\boxed{\int (1-u^2)^2 u^7 (-du)}$

3. $\int \sec^3 x \tan^3 x \, dx$ $u = \sec x$, $du = \sec x \tan x$

$\int \sec^2 x \tan^2 x \, (\sec x \tan x dx)$

$u^2 \quad (\sec^2 x - 1) \quad du$

$\int u^2 (u^2-1) du$

4. $\int \sec^2 x \tan^2 x \, dx$

5. $\int \sec^4 x \tan^3 x \, dx$

$u = \sec x \quad du = \sec x \tan x dx$

$\int \sec^3 x \tan^2 x (\sec x \tan x dx)$

$u^3 \quad (\sec^2 x - 1) \quad du$

$u^2 - 1$

$\int u^3 (u^2-1) du$

6. $\int \cos^5 x \, dx$ $u = \sin x$, $du = \cos x dx$

$\int \cos^4 x \, \underbrace{\cos x dx}_{du}$

$(\cos^2 x)^2$

$\int (1-\sin^2 x)^2 \, du$

$\int (1-u^2)^2 du$

II. Work these out completely. (these are 6 and 3 again)

7. $\int \cos^5 x \, dx$

8. $\int \sec^3 x \tan^3 x \, dx$ (Do this on the back)

1. $u = \cos x$
2. $u = \cos x$ is faster
   $u = \sin x$ also works
3. $u = \sec x$
4. $u = \tan x$
5. $u = \sec x$ or $u = \tan x$
6. $u = \sin x$
7. $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$
8. $\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

7.3 Trigonometric Substitution

1 Nice square roots

Trig substitution (sometimes called rationalizing substitution) is good for certain integrals that contain square roots.

Trigonometric Substitutions:

$$\sqrt{a^2 - u^2} \quad \Rightarrow \quad u = a\sin\theta$$

$$\sqrt{a^2 + u^2} \quad \Rightarrow \quad u = a\tan\theta$$

$$\sqrt{u^2 - a^2} \quad \Rightarrow \quad u = a\sec\theta$$

Where a is a constant and u is a variable or variable expression.

1.0.1 Example:

$$\int \frac{\sqrt{4-x^2}}{x^2}dx = \int \frac{2\cos\theta \cdot 2\cos\theta\, d\theta}{4\sin^2\theta} = \int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$x = 2\sin\theta$$

$$dx = 2\cos\theta\, d\theta$$

$$\sqrt{4 - 4\sin^2\theta} = \sqrt{4(1-\sin^2\theta)}$$

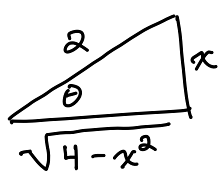
$$= \sqrt{4\cos^2\theta} = 2\cos\theta$$

$$= \int \cot^2\theta\, d\theta = \int \csc^2\theta - 1\, d\theta$$

$$= -\cot\theta - \theta + C$$

require Δ arcsin

$$\sin\theta = \frac{x}{2} = \frac{\text{opp}}{\text{hyp}}$$



$$\cot\theta = \frac{\text{adj}}{\text{opp}}$$

$$= \frac{\sqrt{4-x^2}}{x}$$

$$\theta = \arcsin\left(\frac{x}{2}\right)$$

$$\boxed{= -\frac{\sqrt{4-x^2}}{x} - \arcsin\left(\frac{x}{2}\right) + C}$$

1.0.2 Example:

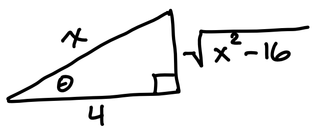
$$\int \frac{dx}{x^2\sqrt{x^2-16}} = \int \frac{\cancel{4}\sec\theta\,\cancel{\tan\theta}\,d\theta}{16\sec^2\theta\cdot\cancel{4}\,\cancel{\tan\theta}} = \frac{1}{16}\int\frac{1}{\sec\theta}\,d\theta$$

$$\boxed{\begin{aligned} x &= 4\sec\theta \\ dx &= 4\sec\theta\tan\theta\,d\theta \\ \sqrt{16\sec^2\theta-16} &= \sqrt{16(\sec^2\theta-1)} \\ &= \sqrt{16\tan^2\theta} = 4\tan\theta \end{aligned}}$$

$$= \frac{1}{16}\int\cos\theta\,d\theta = \frac{1}{16}\sin\theta + C$$

$$= \boxed{\frac{1}{16}\,\frac{\sqrt{x^2-16}}{x} + C}$$

$$=$$



$$\sec\theta = \frac{x}{4} = \frac{\text{hyp}}{\text{adj}} \qquad \sin\theta = \frac{\sqrt{x^2-16}}{x}$$

1.0.3 Example:

$$\int \frac{3x}{\sqrt{9-x^2}}\,dx$$

$x = 3\sin\theta$ will work, but u-sub using $u = 9-x^2$ is faster.

$$\boxed{\begin{aligned} u &= 9-x^2 \\ du &= -2x\,dx \end{aligned}}$$

$$= \int \frac{3x}{\sqrt{u}}\,\frac{du}{-2x} = -\frac{3}{2}\int u^{-\frac{1}{2}}\,du = -\frac{3}{2}\cdot 2\cdot u^{\frac{1}{2}} + C$$

$$= -3u^{1/2} + C = \boxed{-3\sqrt{9-x^2} + C}$$

1.0.4 Example:

$$\int \frac{dx}{\sqrt{16x^2-9}} \longrightarrow \int \frac{\frac{3}{4}\sec\theta\tan\theta\,d\theta}{3\tan\theta} = \int \frac{3\sec\theta\tan\theta\,d\theta}{4\cdot 3\tan\theta}$$

$(4x)^2-9$

$4x = 3\sec\theta$

$4dx = 3\sec\theta\tan\theta\,d\theta$

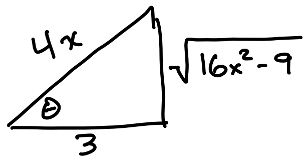
$\sqrt{(4x)^2-9} = \sqrt{(3\sec\theta)^2-9}$

$= \sqrt{9\sec^2\theta-9} = \sqrt{9(\sec^2\theta-1)}$

$= \sqrt{9\tan^2\theta}$

$= 3\tan\theta$

$$= \frac{1}{4}\int \sec\theta\,d\theta = \frac{1}{4}\ln\left|\sec\theta+\tan\theta\right| + C$$



$\sec\theta = \frac{4x}{3} = \frac{\text{hyp}}{\text{adj}}$

$$= \boxed{\frac{1}{4}\ln\left|\frac{4x}{3}+\frac{\sqrt{16x^2-9}}{3}\right| + C}$$

$$= \frac{1}{4}\ln\left|\frac{4x+\sqrt{16x^2-9}}{3}\right| + C$$

$$= \frac{1}{4}\left[\ln\left|4x+\sqrt{16x^2-9}\right| - \ln 3\right] + C$$

$$= \frac{1}{4}\ln\left|4x+\sqrt{16x^2-9}\right| \underbrace{- \frac{1}{4}\ln 3 + C}_{}$$

constant

$$= \boxed{\frac{1}{4}\ln\left|4x+\sqrt{16x^2-9}\right| + C_2}$$