5.5 Review of u-Substitution

u-Substitution

If u is some function of x, we can use u-substitution to do integrals of the form

$$\int cf(u)u'\,dx.$$

1. Find u. (u is nested inside another function)
2. Take the differential of u to get $du = u'dx$.
3. Replace $u'dx$ with du in the integrand OR Solve the differential for dx: $dx = \frac{du}{u'}$.
4. Substitute into the integral
5. Cancel any remaining x's
6. Integrate with respect to u
7. Put the answer back in terms of x.

0.0.1 Example:

$$\int 2x^3(1-x^4)^2\,dx = \int 2\cancel{x^3}\,u^2\frac{du}{-4\cancel{x^3}} = \int -\frac{1}{2}u^2\,du$$

$$\boxed{\begin{aligned} u &= 1-x^4 \\ du &= -4x^3dx \\ \frac{du}{-4x^3} &= dx \end{aligned}}$$

$$= -\frac{1}{2}\frac{u^3}{3} + C = -\frac{1}{6}u^3 + C$$

$$= \boxed{-\frac{1}{6}(1-x^4)^3 + C}$$

0.0.2 Example:

$$\int \frac{\sqrt{\ln x}}{x}\,dx = \int \frac{\sqrt{u}}{\cancel{x}}\,\cancel{x}\,du = \int \sqrt{u}\,du$$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}dx \\ x\,du &= dx \end{aligned}}$$

$$= \int u^{1/2}du = \frac{u^{3/2}}{\frac{3}{2}} + C = \frac{2}{3}u^{3/2} + C$$

$$= \boxed{\frac{2}{3}(\ln x)^{3/2} + C}$$

Check: $\frac{d}{dx}\left(\frac{2}{3}(\ln x)^{3/2}\right) = \cancel{\frac{3}{2}}\cancel{\frac{2}{3}}(\ln x)^{1/2}\cdot\frac{1}{x} = \frac{\sqrt{\ln x}}{x}$

0.0.3 Example:

$$\int \sin(4x)\,dx = \int \sin u \,\frac{du}{4} = \frac{1}{4}\int \sin u\,du$$

$u = 4x$
$du = 4\,dx$
$\frac{1}{4}du = dx$

$$= \frac{1}{4}(-\cos u) + C$$

$$= \boxed{-\frac{1}{4}\cos(4x) + C}$$

$$\int \cos(5x)\,dx = \frac{1}{5}\sin(5x) + C$$

0.0.4 Example:

$$\int_{0=x}^{1=x} \frac{8x}{(2x^2+1)^3}\,dx = \int_{u=1}^{u=3} \frac{2}{u^3}\,du = \int_1^3 2u^{-3}\,du$$

$u = 2x^2+1$
$du = 4x\,dx$
Top: $u = 2(1)^2+1$
bot: $u = 2(0)^2+1$

$$= \frac{2u^{-2}}{-2}\Big]_1^3 = -\frac{1}{u^2}\Big]_1^3$$

$$= \left(-\frac{1}{9}\right) - \left(-\frac{1}{1}\right) = -\frac{1}{9} + 1 = \boxed{\frac{8}{9}}$$

0.0.5 Example:

$$\int_{-2}^{2} 3x(4-x^2)^7\,dx = \int_0^0 3\cancel{x}\,u^7 \frac{du}{-2\cancel{x}} = 0$$

$u = 4-x^2$
$du = -2x\,dx$
Top: $u = 4-(2)^2 = 0$
bot: $u = 4-(-2)^2 = 0$