0.0.4 Example:

$x^2 = 1-(y-1)^2$

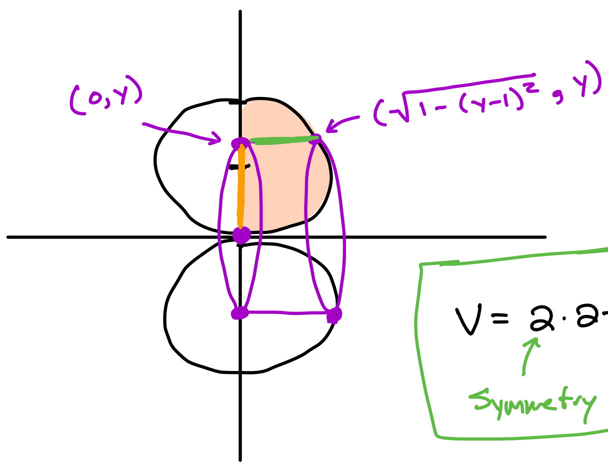
$x = \pm\sqrt{1-(y-1)^2}$

Area inside the circle $x^2 + (y-1)^2 = 1$ rotated around x-axis.

thickness = dy

height = $\sqrt{1-(y-1)^2}$

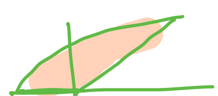
radius = y



$$\boxed{V = \underset{\text{Symmetry}}{2}\cdot 2\pi\int_0^2 y\sqrt{1-(y-1)^2}\,dy}$$

$y^2-2 = x$

0.0.5 Example: $y^2 = x+2$



Area bounded by $y = \sqrt{x+2}$, $y = x$, x-axis; rotated around the line $y = -3$.

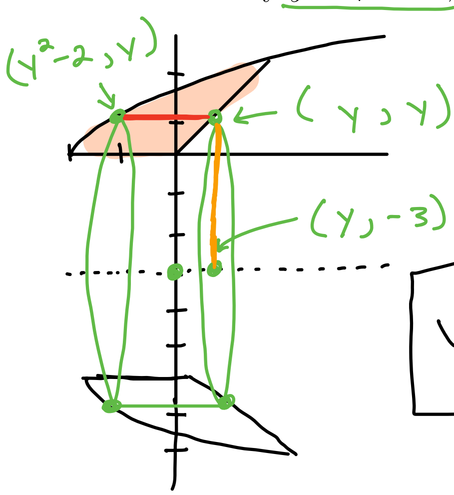
thick = dy

height = $y - (y^2-2)$

$= y - y^2 + 2$

radius = $y-(-3)$

$= y+3$



$$\boxed{V = 2\pi\int_0^2 (y+3)(y-y^2+2)\,dy}$$

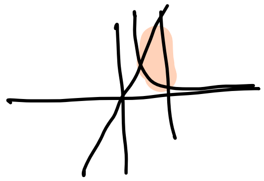
Find upper endpt: set equal to each other

$y^2-2 = y$

$y^2-y-2 = 0$

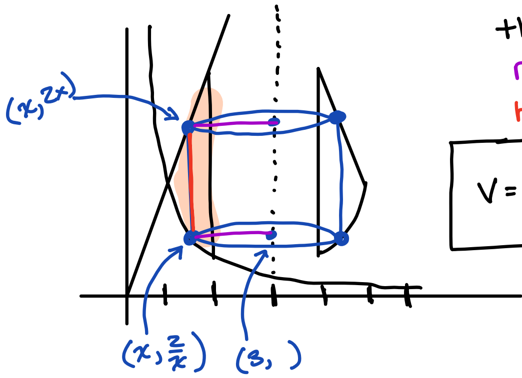
$(y+1)(y-2) = 0$

$y = 2$ or -1



0.0.6 Example:

Area bounded by $y = 2x$, $y = \frac{2}{x}$, and $x = 2$; rotated around the line $x = 3$.



thickness $= dx$

radius $= 3 - x$

height $= 2x - \frac{2}{x}$

$$\boxed{V = 2\pi \int_1^2 (3-x)\left(2x - \frac{2}{x}\right) dx}$$

$$V = 2\pi \int_1^2 6x - \frac{6}{x} - 2x^2 + 2 \, dx = 2\pi \left[3x^2 - 6\ln x - \frac{2}{3}x^3 + 2x \right]_1^2$$

$$= 2\pi \left(\left[12 - 6\ln 2 - \frac{16}{3} + 4 \right] - \left[3 - \underset{0}{6\ln 1} - \frac{2}{3} + 2 \right] \right)$$

$$= 2\pi \left(\left[\frac{32}{3} - 6\ln 2 \right] - \left[\frac{13}{3} \right] \right)$$

$$= \boxed{2\pi \left[\frac{19}{3} - 6\ln 2 \right]} \approx 13.66$$

6.4 Work

1 The Physics we need for this section

Force is a push or pull on an object. Weight is the most common force we will consider. Force is measured in pounds or newtons. To convert kilograms to newtons, multiply by 9.8 (assuming you are on Earth).

Acceleration is the rate of change of velocity. The most common acceleration we will use is the acceleration due to gravity on Earth. This is the constant $g = 9.8$ meters/second^2 $= 32$ feet/second^2.

Newton's Law says that Force = Mass $\times$ Acceleration. (This applies when the force is constant.)

$$F = ma$$

Work is the amount of energy required to do something. Measured in Joules or pound-feet. When force is constant, Work = Force $\times$ distance.

$$W = Fd$$

When force is not constant,

$$W = \int_a^b F(x)dx,$$

where $F(x)$ is the force function and dx is a tiny change in distance.

1.1 Constant Force Examples

These don't require calculus.

1.1.1 Non-metric example:

Suppose you lift a 5 pound rock to a height of 4 feet. How much work do you do?

Work = Force × distance

$= 5 \cdot 4 = 20$ pound-ft.

1.1.2 Metric example:

Suppose you lift a 10 kg rock to a height of 3 meters. How much work do you do?

Work = $\underbrace{\text{Force}}_{\text{mass} \cdot 9.8}$ · distance

$= 10 \cdot 9.8 \cdot 3 = 294$ Nm or 294 Joule

2 Problems involving springs

Hooke's Law: The force required to hold a spring stretched x units beyond its natural length is proportional to x:

$$f(x) = kx$$

The value k is called the **spring constant**; each spring has its own constant. The constant has units of Newtons/meter.

Convert to meters

2.0.1 Example:

A spring has a constant of 5 newtons/meter, and a natural length of 30 cm. How much ~~force~~ work is required to stretch it from 35 cm to 42 cm?

.3 m

.35 .42

$F(x) = 5x$

$$\text{Work} = \int_{.05}^{.12} 5x\,dx = \frac{5}{2}x^2 \Big]_{.05}^{.12} = \frac{5}{2}\left(.12^2 - .05^2\right)$$

$$= .02975 \text{ J}$$

2.0.2 Example:

A spring has a constant of 3 newtons per meter and a natural length of 75 cm. How much work is required to stretch it from 1 meter to 2 meters?

2.0.3 Example:

It takes 25 N to hold a spring stretched to 30 cm. Its natural length is 20 cm. How much work is needed to stretch it from 20cm to 32 cm?