

## 3.9 Related Rates

### 1 Introduction and simple geometric examples

Whenever two related quantities change, the rates of change are related to each other.

A square on a computer screen is growing at a rate of 2mm per second. How fast is the area growing when the area is  $25\text{mm}^2$ ?

**Idea:** Treat the length and the area both as functions of time,  $t$ , and use the chain rule.

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#### 1.0.1 Example:

A rock is dropped in a pond, creating a circular ripple. The radius of the ripple increases at 2 meters per second. How fast is the area increasing when the radius is 10 meters?

### 1.0.2 Example:

A spherical balloon is being inflated at a constant rate of one cubic inch per second. How fast is the radius of the balloon increasing when the balloon's volume is 90 cubic inches? Give a decimal answer.

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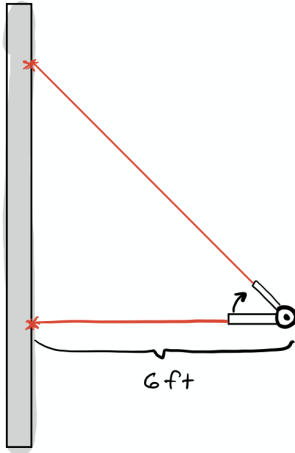
### 1.0.3 Example:

A triangle is changing its size in an animation. The base is shrinking at 3 mm per minute, and the height is growing at 2 mm per minute. At one instant, the base is 10mm and the height is 20mm. How fast is the area growing at that instant?

## 2 More Complicated Problems

### 2.0.1 Example:

A spinning laser pointer is 6 feet from a wall, and it is spinning clockwise at 10 RPM. How fast is the dot moving when the angle made with the wall is  $60^\circ$ ? Give answer in feet per second. (Note: everything must be converted to radians for the numbers to work correctly.)



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### 2.0.2 Example:

At noon, a car and a motorcycle leave the same point in the desert. The car goes north at 60 mph and the motorcycle goes east at 120 mph. How fast is the distance between them changing at 12:30?

### 2.0.3 Example:

A conical water tank (with vertex down) has a height of 12 feet and a radius of 3 feet. Water is being pumped in at a rate of 4 cubic feet per minute. How fast is the water level rising when the water level is 6 feet?