4.1 Critical Numbers

Chapter 4 is titled *Applications of Differentiation*. We will learn a few ways to use derivatives to do useful stuff.

1 Critical Numbers

Definition: A **critical number** of a function f is a number c in the domain of f such that either

$$f'(c) = 0 \text{ or } f'(c) \text{ is undefined.}$$

Q: Why do we care about critical numbers?

A: Because they can help us find the peaks and valleys (aka relative maxima and minima) of a graph.

Finding the critical numbers of $f(x)$:

1. First find $f'(x)$ (i.e. take the derivative)
2. Set $f'(x)$ equal to zero and solve. The solutions are your critical numbers.
3. Check for any numbers c for which $f'(c)$ is undefined. This can happen if ...
   - $f'(x)$ has a denominator or
   - $f'(x)$ is a trig function other than sine or cosine

1.1 Examples:

1.1.1 Example:

Find the critical numbers of $f(x) = 3x^2 + 5x - 9$

1.1.2 Example:

Find the critical numbers of $y = x^3 + 4x^2 + 4x - 1$

1.1.3 Example:

Find the critical numbers of $y = \dfrac{x^2 + 1}{1 - x^2}$

1.1.4 Example:

Find the critical numbers of $y = x^{\frac{2}{3}}(x + 5)$

1.1.5 Example:

Find the critical numbers of $f(x) = x^3 + 5$