4.7 Applied Optimization

Optimization problems are basically word problems that involve coming up with a function and finding its maximum or minimum value. We will start with a classic example, building a box from a flat sheet of metal or cardboard.

0.0.1 Example:

A box with an open top is constructed by cutting squares out of the corners of a piece of copper and folding the sides up. If the piece of copper is 3×3 feet, what is the maximum volume possible?

Steps for solving an optimization problem

1. Read the problem.
2. Draw a picture if necessary.
3. Introduce variables and determine the quantity Q to be minimized or maximized.
4. Express Q in terms of the other variables.
5. If Q is a function of more than one variable, eliminate all but one of them.
6. Find the maximum or minimum value of Q.
7. Answer the question.

0.0.2 Example:

Find two positive numbers with product 1600 whose sum is a minimum.

0.0.3 Example:

A farmer has 2400 feet of fencing. He wants to fence off a rectangle next to a river. What dimensions will maximize the area?

0.0.4 Example:

We need to build a cylindrical can that holds one liter. What dimensions will minimize the amount of metal needed to make the can? (Note that 1 liter = 1000 cm^3.)

0.0.5 Example:

Find the point on the line $y = 2x + 5$ that is closest to the point $(3, 4)$.

0.0.6 Example:

We need to design a box with a square base and open top, and volume of 32000 cm^3. Find the dimensions of the box that will minimize the amount of material used.