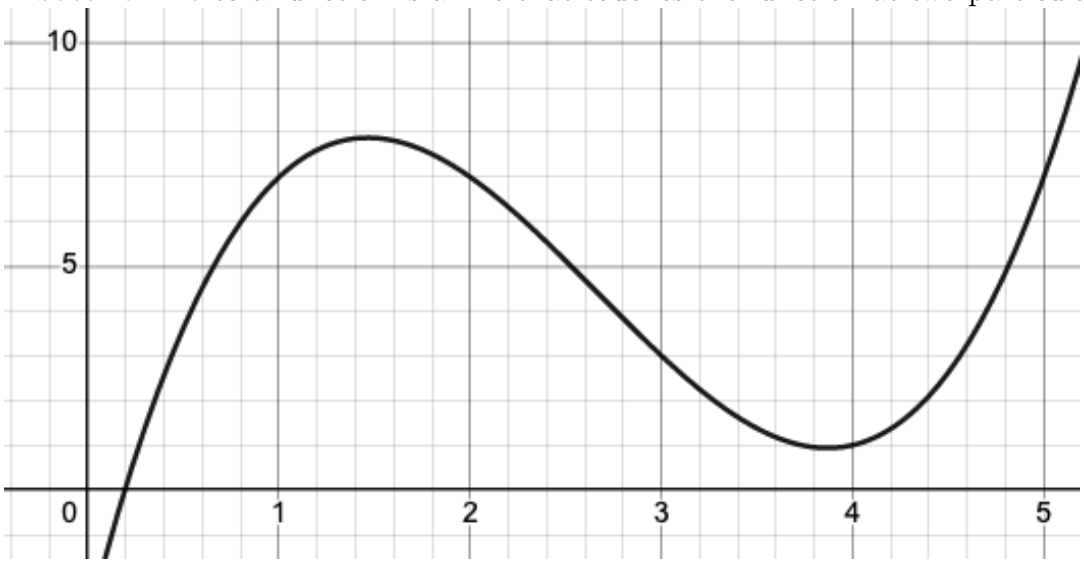
2.1 Tangent and Secant Lines

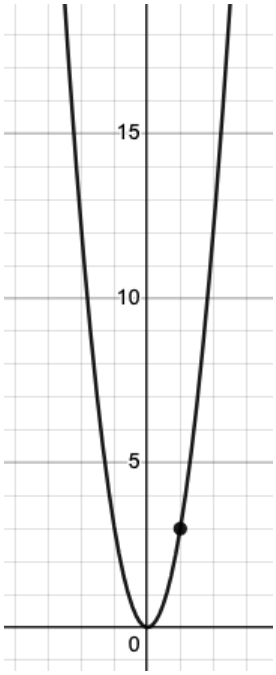
1 Secant Lines

A **secant line** to a function is a line that touches the function at two particular points.



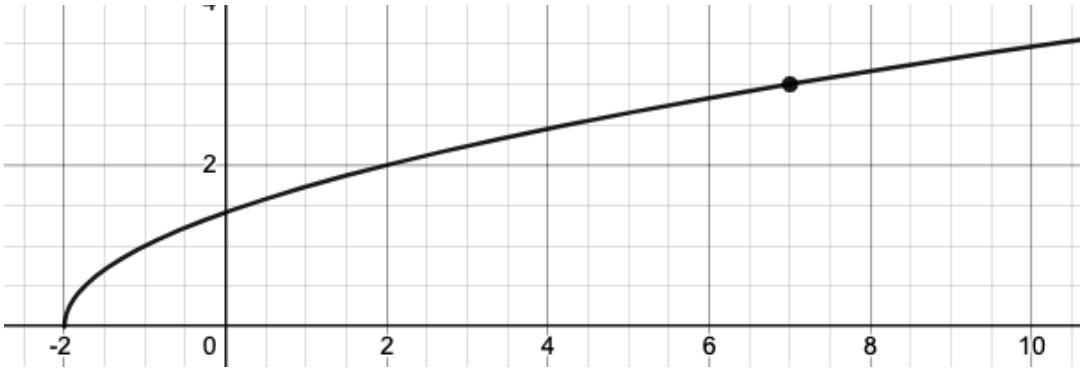
1.0.1 Example:

Find the slopes of the secant lines to $f(x) = 3x^2$ using the point $(1, 3)$ and the points with x-coordinate 2, 1.1, and 1.01. Use this to guess the slope of the tangent line at $(1, 3)$.



1.0.2 Example:

Find the equation of the line tangent to $f(x) = \sqrt{x+2}$ at the point $(7, 3)$. Use Desmos to do the calculations.



1.0.3 Example:

Find the equation of the line tangent to $y = \frac{1}{10}(x^3 - 8x^2 + 15)$ at the point $(1, 0.8)$ Use Desmos to do the calculations.

2 Velocity

2.1 Average Velocity

$$\text{Average velocity} = \frac{\text{distance}}{\text{time}}$$

2.1.1 Example:

Suppose you're driving on the interstate. At 10:03 you're at mile marker 20 and at 10.17 you're at mile marker 48. What's your average velocity?

2.1.2 Example:

A cyclist starts at mile marker 0 and rides along the road. The following table tells his location at various times in minutes.

Elapsed time (minutes)	10	20	30	40	50
Mile Marker of cyclist	3	7	11	16	20

Find the cyclist's average velocity from $t = 10$ to $t = 50$.

2.1.3 Example:

A rock is dropped off of a building 100 meters tall. The height of the rock after t seconds is given by the function below. Find the average velocity of the rock from t=3 to t=3.001.
$f(t) = 100 - 4.9t^2$

2.2 Instantaneous Velocity

Instantaneous Velocity is the velocity at one **instant** in time. Think of an instant as a zero-length interval of time. For example, your car's speedometer tells you your instantaneous velocity.

We can determine the instantaneous velocity at an instant t by taking the average velocity over a very small interval starting (or ending) at t. This is analogous to finding the slope of a tangent line by finding the slope of a secant line using a nearby point.

2.2.1 Example:

A rock is thrown straight up. It's height in feet after t seconds is given by

$$h(t) = 128t - 16t^2 + 4$$

Find the average velocity during the following intervals. Use Desmos or a spreadsheet to do the calculations.

1. from $t = 6$ to $t = 6.1$

2. from $t = 6$ to $t = 6.01$

3. from $t = 6$ to $t = 6.001$

4. from $t = 5.999$ to $t = 6$

What is the instantaneous velocity of the rock at $t = 6$?