2.7 Derivatives and Rates of Change

1 Tangents

The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope m, where m is given by using either

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad \text{OR} \qquad m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

2 Derivatives

The **derivative** is the most important concept in calculus.

Definition: The **derivative of a function** f **at a number** a, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

The value of $f'(a)$ tells us ...

- the slope of the line tangent to $f(x)$ at the point with x-coordinate a.
- the slope of the curve $f(x)$ at the point with x-coordinate a.
- the instantaneous rate of change of $f(x)$ at the point with x-coordinate a.

2.0.1 Example

These two instructions are asking the exact same thing:

- Find the slope of the line tangent to $f(x) = 4x - x^2 + 3$ at $a = 4$.
- Calculate the derivative of $f(x) = 4x - x^2 + 3$ at $a = 4$.

2.0.2 Example

Find the equation of the line tangent to $g(x) = \sqrt{x}$ at $a = 4$.

3 Instantaneous Rates of Change

Derivatives can be used to compute instantaneous rates of change. The most common example of this is computing the velocity of an object given a function that tells its position. In fact, the physics of motion was one of the major motivations for Isaac Newton when he developed Calculus.

3.0.1 Example

If a rock is dropped off of a building that is 256 feet tall, the height of the rock after t seconds is given by $f(t) = 256 - 16t^2$. Find the instantaneous velocity of the rock at $t = 3$.