4.2 Mean Value Theorem

1 Rolle's Theorem

Suppose you have a function that has two points with the same y-coordinate. If the function is continuous (no breaks) and differentiable (no sharp corners) between those points, then somewhere between them, the tangent is horizontal.

Rolle's Theorem:

Let f be a function that satisfies all of these conditions:

1. f is continuous on $[a, b]$
2. f is differentiable on (a, b)
3. $f(a) = f(b)$

Then there is at least one number c in (a, b) for which $f'(c) = 0$.

1.0.1 Example:

Show that the function $f(x) = x^3 - 5x^2 - 8x + 10$ satisfies the conditions of Rolle's theorem on $[-2, 6]$, and find all the values of c that satisfy the conclusion.

2 The Mean Value Theorem

Suppose you have a function that is continuous (no breaks) and differentiable (no sharp corners). Pick two points along the function and draw the line segment connecting them. Somewhere between these two points there is guaranteed to be at least one place where the tangent line is parallel to the segment.

The Mean Value Theorem:

Let f be a function that is

1. f is continuous on $[a, b]$, and
2. f is differentiable on (a, b)

Then there is at least one number c in the interval (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

2.0.1 Example:

Find the values that satisfy the Mean Value Theorem for the function $f(x) = x^2 - 3x - 2$ on the interval $[-2, 3]$.