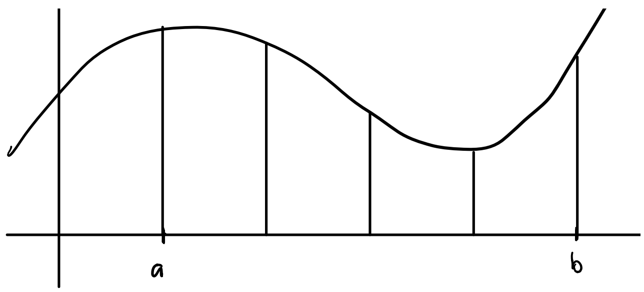
5.2 The Definite Integral

1 Definition of Definite Integral

Consider the area under the curve $f(x)$, between $x = a$ and $x = b$. (For now we'll assume that $f(x)$ is positive between a and b.)



$$\text{Area} \approx \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

This limit gives the exact area, and is called the **Definite Integral of** $f(x)$ **from** a **to** b; it is denoted by

$$\int_a^b f(x)\,dx$$

1.0.1 Example:

Approximate $\int_0^2 x^2\,dx$ using left endpoints and 4 rectangles.

1.0.2 Example:

Use the online calculator to approximate $\int_0^\pi \sin x\,dx$ using left endpoints and 100 rectangles.

2 Using Geometry to evaluate $\int_a^b f(x)dx$

If $f(x)$ is a "nice" function, we can use geometry to get the exact value of $\int_a^b f(x)\,dx$. Evaluate the following integrals:

2.0.1 Example:

$$\int_0^4 4 - x\; dx$$

2.0.2 Example:

$$\int_1^4 2x + 1\; dx$$

2.0.3 Example:

$$\int_{-3}^3 9 - x^2\; dx$$

Note: If $f(x)$ goes below the x-axis, the area underneath counts negative toward the integral.

2.0.4 Example:

$$\int_0^{10} 4 - x \, dx$$

2.0.5 Example:

$$\int_0^{2\pi} \sin x \, dx$$

2.0.6 Example:

$$\int_2^4 -x \, dx$$

3 Properties of Definite Integrals

- $\displaystyle\int_a^b c\ dx = c(b-a)$

- $\displaystyle\int_a^b f(x) + g(x)\ dx = \int_a^b f(x)\ dx + \int_a^b g(x)\ dx$

- $\displaystyle\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$

- $\displaystyle\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$

- $\displaystyle\int_a^a f(x)\ dx = 0$

- $\displaystyle\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$