5.4 Indefinite Integrals & Net Change

1 Indefinite Integrals

Indefinite Integral is another name for general antiderivative.

1.0.1 Teaching Example:

$\int x^2 + x + \sin x \, dx$

1.1 Definite vs Indefinite integrals:

Definite $\int_a^b f(x)\,dx$	**Indefinite** $\int f(x)\,dx$
has endpoints	no endpoints
Evaluates to give a number	Evalautes to give a family of functions ending in $+C$.
Most graphing calculators, Desmos, and the online calculator can estimate them.	Can be evaluated by a computer algebra system (e.g. wolfram alpha, mathematica)
	Another name for antiderivative.

1.1.1 Example:

Evaluate the indefinite integrals:

$\int \sin(x) + x^2 \, dx$

$\int \frac{1}{1+x^2} \, dx$

$\int \sqrt{x} \, dx$

$\int e^x \, dx$

2 The Net Change Theorem

The Net Change Theorem

The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

We can use this for any application involving rates of change, but we will focus on velocity and position.

2.1 Net Change in Position and Total Distance Traveled

Recall: Velocity tells both the speed and direction of movement.

Fact: Integrating velocity will give the net change in position.

Facts:

- The **net change** in position from time a to time b is $\displaystyle\int_a^b v(t)\,dt$
- The **total distance traveled** from time a to time b is $\displaystyle\int_a^b \big|v(t)\big|\,dt$. to find this
  1. Find all times between a and b when the velocity is 0.
  2. Cut the time up at the values you found in step 1, and make separate integrals for each interval.
  3. Find the net change in each interval, make them all positive, and add them up.

2.1.1 Teaching Example

Suppose a ball is launched straight upward from ground level with an initial velocity of 48 feet per second. The velocity of the ball t seconds after it is launched is given by the equation $v(t) = 48 - 32t$.

(a) Find the net change in the ball's height from $t = 0$ to $t = 2$.

(b) Find when the velocity is zero.

(c) Find the total distance traveled by the ball from $t = 0$ to $t = 2$.

2.1.2 Example:

A particle moveing along the x-axis has velocity given by the equation $v(t) = t^2 - 8t + 15$. (a) Find the net change in position from $t = 0$ to $t = 10$. (b) Find the total distance traveled during that period. (c) check your answers using desmos.

2.1.3 Example:

A particle moves along a straight line. Its velocity given by $v(t) = t^2 - 8t + 12$. Find the net and total distance traveled from $t = 0$ to $t = 9$