The Derivative as a Function

1 The Definition of a Derivative

Instead of plugging in a number for a, we can evaluate the derivative at the variable x, and get the derivative as a function, as a function $f'(x)$, defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Recall that the derivative, evaluates at x, tells the slope of $f(x)$ at x.

1.1 Finding $f'(x)$ using the definition

1.1.1 Example

$f(x) = \sqrt{2x}$ Find $f'(x)$.

1.1.2 Example

$f(x) = 3x + 4$ Find $f'(x)$.

1.1.3 Example

$g(x) = \dfrac{x+3}{2x+5}$ Find $f'(x)$.

1.1.4 Example

$f(x) = x^2 + 6x - 1$ Find $f'(x)$.

1.1.6 Example

$f(x) = \dfrac{1}{2x+3}$ Find $f'(x)$.

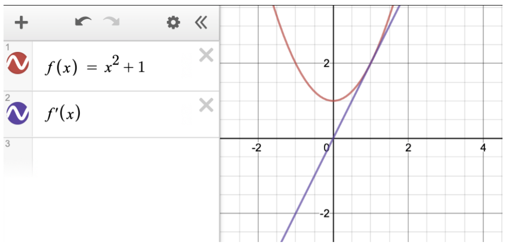
1.1.7 Example

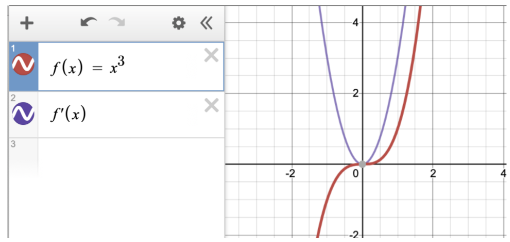
$f(x) = \sqrt{x+5}$ Find $f'(x)$.

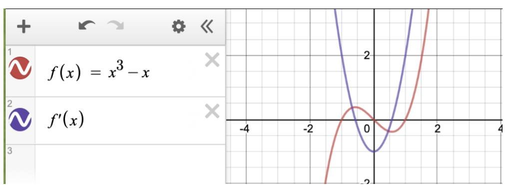
2 The Graph of $f'(x)$

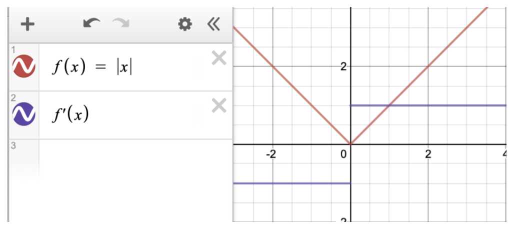
If we graph $f(x)$ and $f'(x)$ on the same set of axes, we can see that the value of $f'(x)$ tells the slope of $f(x)$.

2.0.1 Examples



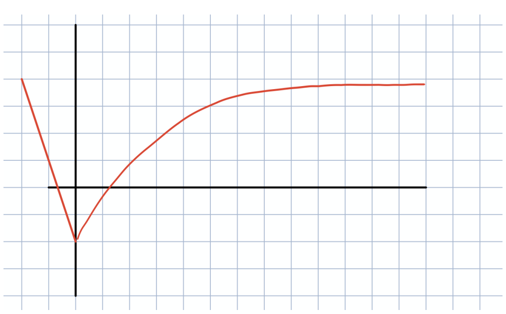


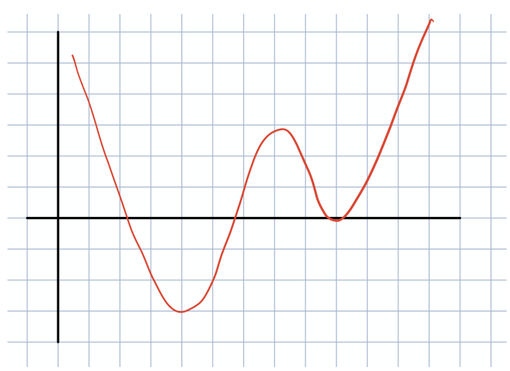




2.0.2 Examples

Draw a rough sketch of the derivative of the functions shown here:





3 Differentiability

Definition: $f(x)$ is **differentiable** at a if $f'(a)$ exists; $f(x)$ is differentiable on the open interval (a,b) if it is differentiable at every number in the interval.

Graphically, there are three reasons that $f(x)$ might not be differentiable at a.

- $f(x)$ is discontinuous at a,
- $f(x)$ has a sharp corner (aka a *cusp*) at a, or
- $f(x)$ has a vertical tangent at a,

3.0.1 Examples

Look at the graphs to determine where the following functions are not differentiable.

$f(x) = \sqrt[3]{x}$ $g(x) = |x+3|$ $h(x) = \tan x$ $y = \lfloor x \rfloor$

4 Notation

There are several different notations used to denote derivatives. Suppose $y = f(x)$. We can denote its derivative in several ways:

- Prime notation: y', $f'(x)$
- Leibniz notation: $\dfrac{dy}{dx}, \dfrac{df}{dx}$
- Differential operator: $\dfrac{d}{dx}(y)$, $\dfrac{d}{dx}(f)$, $Df(x)$, $D_x(f(x))$

The process of taking a derivative is called **differentiation**. Don't call it "deriving" or I will throw things at you.

4.0.1 Example

Evaluate $\dfrac{d}{dx}(3x+4)$

4.0.2 Example

Evaluate $\dfrac{d}{dr}(2\pi r)$

4.0.3 Example

Evaluate $\dfrac{d}{dt}\left(\dfrac{2}{3} - \dfrac{3}{7}t\right)$

5 Higher Derivatives; Velocity and Acceleration

If we take the derivative of $f'(x)$, we get the **second derivative**, which is denoted by either $f''(x)$, or $\frac{d^2y}{dx^2}$