3.4 Chain Rule

1 Nested Functions

The Chain Rule allows us to take the derivative of composite, or nested functions.

Suppose we want to take the derivatives of $f(x) = (2x+1)^2$ and $g(x) = (2x+1)^5$

We could multiply them out, and then take the derivative:

2 The Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and F' is given by

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In Leibniz notation,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

The line I say to myself when I use the chain rule is

"times the derivative of the inside function"

3 (Relatively) Easy Example Problems

3.0.1 Example:

Find the derivative of $f(x) = (x^2 + 2x + 1)^3$

3.0.2 Example:

Find the derivative of $g(x) = \sqrt{5x - 7}$

3.0.3 Example:

Find the derivative of $f(x) = \sin(5x)$

3.0.4 Example:

Find the derivative of $g(x) = \sin(3x^5)$

3.0.5 Example:

Find the derivative of $f(x) = e^{\tan x}$

3.0.6 Example:

Find the second derivative of $f(x) = \cos(x^2)$

3.0.7 Example:

$f(x) = \sqrt{g(x)}$. Evaluate $f'(5)$ given that $g(5) = 16$ and $g'(5) = -3$

4 Tougher Example Problems

4.0.1 Example:

Find the derivative of $f(x) = \left(\frac{1-x}{x^2+1}\right)^3$

4.0.2 Example:

Find the derivative of $g(x) = \sin\left[(2x)^{10}\right]$

4.0.3 Example:

Find the derivative of $y = \tan(e^{5x})$

4.0.4 Example:

Find the derivative of $f(t) = (x^2 e^t)^7$

4.0.5 Example:

Find the derivative of $g(x) = \sin((2x + 3)^5)$