3.5 Implicit Derivatives

1 Warm-up

Let's start by taking some derivatives.

1.0.1 Example:

$y = 3x^3 + 6x + f(x)$

1.0.2 Example:

$y = x^3 f(x)$

1.0.3 Example:

$y = x^3 + \sin(f(x)) + [f(x)]^5$

2 Implicit Derivatives

If we have an equation that relates x and y, but it's difficult to solve for y, we can treat y as an unknown function of x.

- treat y as an unknown function of x,
- the derivative of y is y' (or $\frac{dy}{dx}$ if we use Leibniz notation),

- the derivative of $g(y)$ is $g'(y) \cdot y'$ (i.e. don't forget the chain rule),
- we usually have to solve for y' at the end,
- the expression for y' will usually have x and y in it.

2.0.1 Example:

$x^2 + y^2 = 9$ Find y'.

2.0.2 Example:

Find y' given that $\quad y^2 + 2x^2 - 3y + yx = 4$

2.0.3 Example:

Find y' given that $\quad y^3 = 5x - 7y^2$

.

2.0.4 Example:

Find the slope of the line tangent to the curve $y^2 + y - \sqrt{x} = 5x = 10$ at the point $(4, 3)$

2.0.5 Example:

Find y' given that $\quad x^2 + y^2 = xy$

2.0.6 Example:

Find y' given that $\quad x \sin y = y$

2.0.7 Example:

Find all the places where the graph of $x^3 + y^2 = a - 2y$ has a horizontal or vertical tangent.

2.1 Finding $\frac{dx}{dy}$

So far we have been finding y', or $\frac{dy}{dx}$. If we prefer, we can regard y as the independent variable and find the rate of change of x with respect to y. You can call this x', but it's better to use the Leibniz notation and call it $\frac{dx}{dy}$.
In doing this, the derivative of y is 1

2.1.1 Example:

Find $\frac{dx}{dy}$ given that $\quad x^3 + y^3 = 6xy$

3 Derivatives of Inverse Trig Functions

We can use the concept of implicit differentiation to find formulas for derivatives of the inverse trig functions.

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}, x \neq \pm 1$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}, x \neq \pm 1$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}, x \neq \pm 1, 0$$

$$\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}}, x \neq \pm 1, 0$$

3.0.1 Example:

Find the derivative of $f(x) = \arccos(x^2 + 5x)$

3.1 Derivation of $\frac{d}{dx}(\arctan(x))$

3.2 Derivation of $\frac{d}{dx}(\arcsin(x))$