

3.2 Product Rule and Quotient Rule

1 Intro

So far we've seen that for any **constant** c and function $f(x)$

$$\frac{d}{dx} \left(cf(x) \right) = cf'(x),$$

but what if we multiply $f(x)$ by something that is **not** a constant? In that case we have to use the Product Rule.

2 Product Rule

The Product Rule states that

$$\frac{d}{dx} \left[g(x)f(x) \right] = g(x)f'(x) + f(x)g'(x).$$

Memorize it as

“First times derivative of the second plus second times derivative of the first,”

2.1 Examples

2.1.1 Example:

Find the derivative of $f(x) = x^3e^x$.

2.1.2 Example:

Find the derivative of $f(x) = (2x + 5)(x^2 + 3x + 4)$.

2.1.3 Example:

Find the slope of the line tangent to $g(x) = x^3 e^x$ at the point with x -coordinate -2

2.1.4 Example:

$f(x) = x^2 \cdot g(x)$, $g(3) = 4$, $g'(3) = -1$. Evaluate $f'(3)$.

3 Quotient Rule

The Quotient Rule states that

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

To memorize this, call the function in the numerator “hi” and the function in the denominator “lo”, and remember it as

$$\text{“lo d hi - hi d lo over lolo”, or } \frac{\text{lo d hi - hi d lo}}{\text{lolo}}$$

NOTE: The order matters! It starts with “lo d hi”. Getting the subtraction backwards in the numerator is by far the most common mistake people make when using the quotient rule.

3.1 Examples

3.1.1 Example:

Find the derivative of $f(x) = \frac{3x}{2x+5}$.

3.1.2 Example:

Find the derivative of $f(x) = \frac{x^2 + 2x + 1}{x^3}$ using the QR .

3.1.3 Example:

Find the derivative of $f(x) = \frac{x^2 + 2x + 1}{x^3}$ without using the QR .

3.1.4 Example:

$h(x) = \frac{g(x)}{f(x)}$ Evaluate $h'(2)$

x	0	1	2	3
$f(x)$	3	3	1	1
$f'(x)$	3	-2	4	0
$g(x)$	2	3	9	5
$g'(x)$	8	7	7	4

3.1.5 Example:

For which values of x is the tangent of does the graph of $f(x) = f'(x) = \frac{x^2+1}{x+3}$ have a horizontal tangent? Give exact answers, and approximations to 3 decimal places.

4 Using Rules More Than Once

4.0.1 Example:

Find the derivative of $f(x) = \frac{x^2 e^x}{3x + 5}$.

4.0.2 Example:

Find the derivative of $y = e^x x^2 \sin(x)$ Hint: $\frac{d}{dx}(\sin x) = \cos(x)$.