4.9 Antiderivatives

1 Definition

The function $F(x)$ is **an antiderivative** of $f(x)$ if $F'(x) = f(x)$.

1.0.1 Teaching Example:

Find some antiderivatives of the function $f(x) = 2x$.

1.0.2 Examples:

Find **an** antiderivative of the following functions:

$3x^2$	$\sec^2 x$	5
$2x$	x^2	$\cos x$
$\sin x$	$2x + 4$	$\sec x \tan x$

2 Antiderivative Rules

Power Rule for Antiderivatives

The general antiderivative of x^n is

$$\left(\frac{1}{n+1}\right) x^{n+1} + C \qquad \text{if } n \neq -1,$$

$$\ln|x| + C \qquad \text{if } n = -1$$

2.0.1 Examples

Find the general antiderivative of the following functions:

1. $f(x) = x^3 + 3x - 4$

2. $g(x) = \sqrt{x}$

3. $h(x) = \dfrac{3x^2 + 4x - 5}{12x}$

More Antiderivative Rules

function	general antiderivative
e^x	$e^x + C$
$\cos x$	$\sin x + C$
$\sin x$	$-\cos x + C$
$\frac{1}{1+x^2}$	$\arctan x + C$

function	general antiderivative
$\frac{1}{x}$	$\ln x + C$
c	$cx + C$
$g(x) \pm f(x)$	$G(x) \pm F(x) + C$
$c \cdot f(x)$	$c \cdot F(x) + C$

3 Initial Value Problems

In an initial value problem, you're given a function's derivative and the value of the function at one point, and you're asked to find the function.

3.0.1 Teaching Example:

$f'(x) = \frac{1}{3}$, and $f(1) = 5$. Find $f(x)$.

3.0.2 Teaching Example:

$g'(x) = 8x^3 + 12x + 3$, and $g(1) = 6$. Find $g(x)$.

3.0.3 Example:

$f'(x) = 5 - x$, and $f(2) = -1$. Find $f(x)$.

3.0.4 Example:

$f'(x) = \dfrac{3}{1+x^2}$, and $f(0) = 1$. Find $f(x)$.

If an initial value problem gives you $f''(x)$, it has to give you *two* pieces of information in order to recover the function. It can either give you

- $f(x_1) = y_1$ and $f(x_2) = y_2$ (i.e. two points on the function), or
- $f(x_1) = y_1$ and $f'(x_2) = m$ (i.e. one point on the function and the slope at one point).

3.0.5 Example:

$f''(x) = 6x, \qquad f'(1) = 3$, and $f(2) = -5$. Find $f(x)$.

3.0.6 Example:

$f''(x) = 3e^x + 5\sin x, \qquad f'(0) = 2$, and $f(0) = 1$. Find $f(x)$.

3.0.7 Example:

$f''(x) = 4x^2 + 2x + 1, \qquad f(1) = 0$, and $f(-1) = -2$. Find $f(x)$.

3.0.8 Example:

$f''(x) = 2 - \cos x, \qquad f(0) = 1$, and $f(\frac{\pi}{2}) = \frac{\pi^2}{4}$. Find $f(x)$.