4.1 Critical Numbers

Chapter 4 is titled *Applications of Differentiation*. We will learn a few ways to use derivatives to do useful stuff.

1 Critical Numbers

Definition: A **critical number** of a function f is a number c in the domain of f such that either

$$f'(c) = 0 \text{ or } f'(c) \text{ is undefined.}$$

Q: Why do we care about critical numbers?

A: Because they can help us find the peaks and valleys (aka relative maxima and minima) of a graph.

Finding the critical numbers of $f(x)$:

1. First find $f'(x)$ (i.e. take the derivative)
2. Set $f'(x)$ equal to zero and solve. The solutions are your critical numbers.
3. Check for any numbers c for which $f'(c)$ is undefined. This can happen if ...
   - $f'(x)$ has a denominator or
   - $f'(x)$ is a trig function other than sine or cosine

1.1 Examples:

1.1.1 Example:

Find the critical numbers of $f(x) = 3x^2 + 5x - 9$

1.1.2 Example:

Find the critical numbers of $y = x^3 + 4x^2 + 4x - 1$

1.1.3 Example:

Find the critical numbers of $y = \dfrac{x^2+1}{1-x^2}$

1.1.4 Example:

Find the critical numbers of $y = x^{\frac{2}{3}}(x+5)$

1.1.5 Example:

Find the critical numbers of $f(x) = x^3 + 5$

Maximum and Minimum Values

2 Absolute Extrema

2.1 Definitions; Finding Extrema Using Graphs

Absolute Max and Min: Let c be a number in the domain of a function f. Then $f(c)$ is the

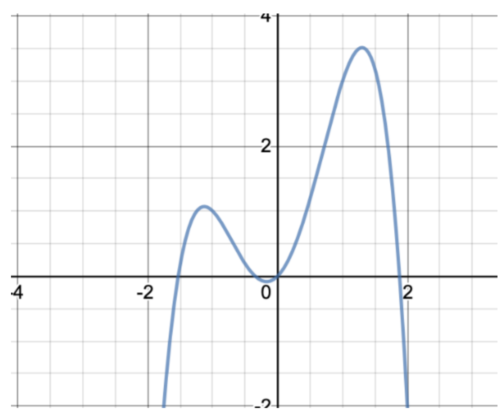
- **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D.
- **absolute minumum** value of f if $f(c) \leq f(x)$ for all x in D.

Local Max and Min: Let c be a number in the domain of a function f. Then $f(c)$ is a

- **local maximum** value of f if $f(c) \geq f(x)$ for all x near c.
- **local minumum** value of f if $f(c) \leq f(x)$ for all x near c.

2.1.1 Example:

The function shown here has ...



2.1.2 Example:

The function $f(x) = x^2$ has ...

2.1.3 Example:

The function $y = -x^3 + 4x$ has ...

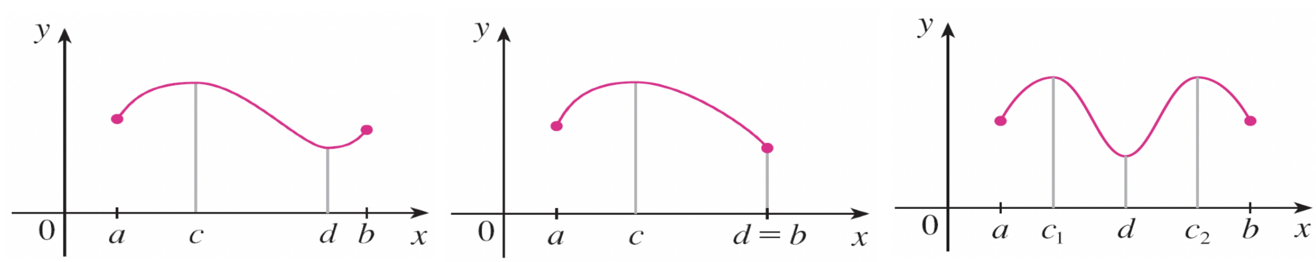
2.2 Max and Mins on a Closed Interval

Extreme Value Theorem:
If f is continuous on a closed interval $[a, b]$, then the greatest and least values attained by the function on are the **absolute maximum and absolute minimum value on that interval**. Furthermore, the absolute max and min on the interval $[a, b]$ must occur at either

- a critical number, or
- an endpoint of the interval.

Fact: The absolute max and absolute min occur either at an endpoint or at a critical number.

2.2.1 Examples:



2.2.2 Example:

Find the absolute max and absolute min of the function $f(x) = 7 - x - x^2$ on the interval $[-2, 1]$.

2.2.3 Example:

Find the absolute max and absolute min of the function $f(x) = \sqrt[5]{x}(10 - x)$ on the interval $[0, 32]$.