5.1 Areas and Distances

There are two major parts to calculus:

Differential Calculus	Integral Calculus
∘ slope of a tangent line	∘ area under a curve
∘ derivatives	∘ antiderivatives, aka integrals
∘ rates of change	∘ accumulation

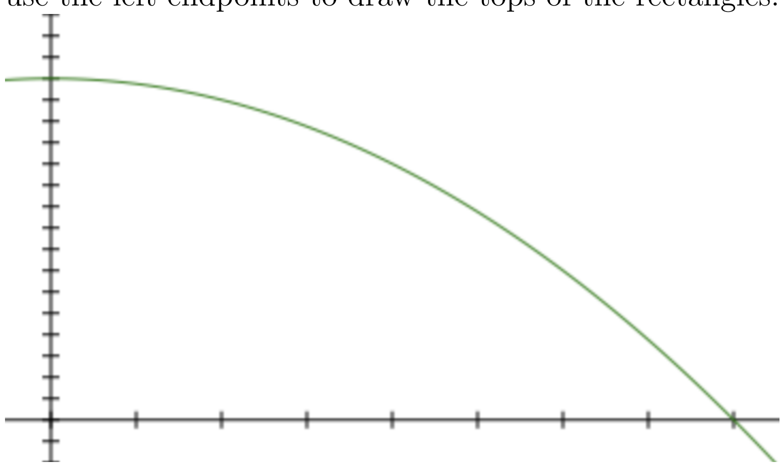
And the **Fundamental Theorem of Calculus** ties the two parts together.

Chapter 5 begins our study of Integral Calculus.

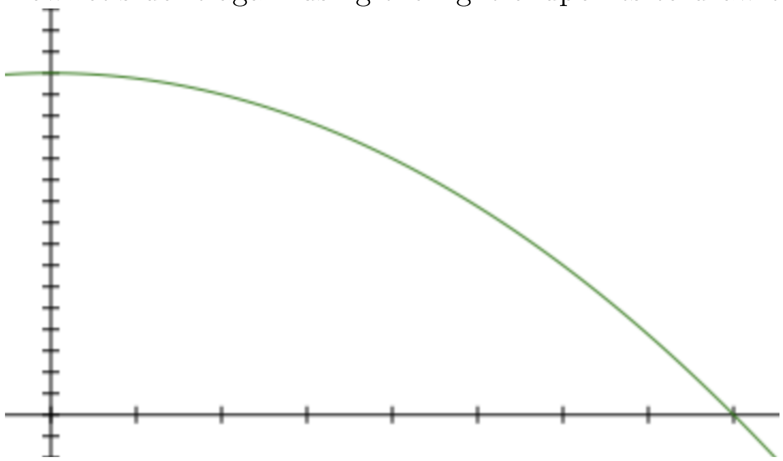
1 Approximating the Area Under a Curve

Finding the area under a curve is an important problem in the sciences. We will start by approximating the area. In later section we'll develop some tools (integral calculus) to make it exact.

Let's approximate the area under $f(x) = 16 - \frac{1}{4}x^2$ in the first quadrant, using 4 rectangles. We will use the left endpoints to draw the tops of the rectangles.



Now let's do it again using the right endpoints to draw the tops of the rectangles:



Ways to improve our estimate:

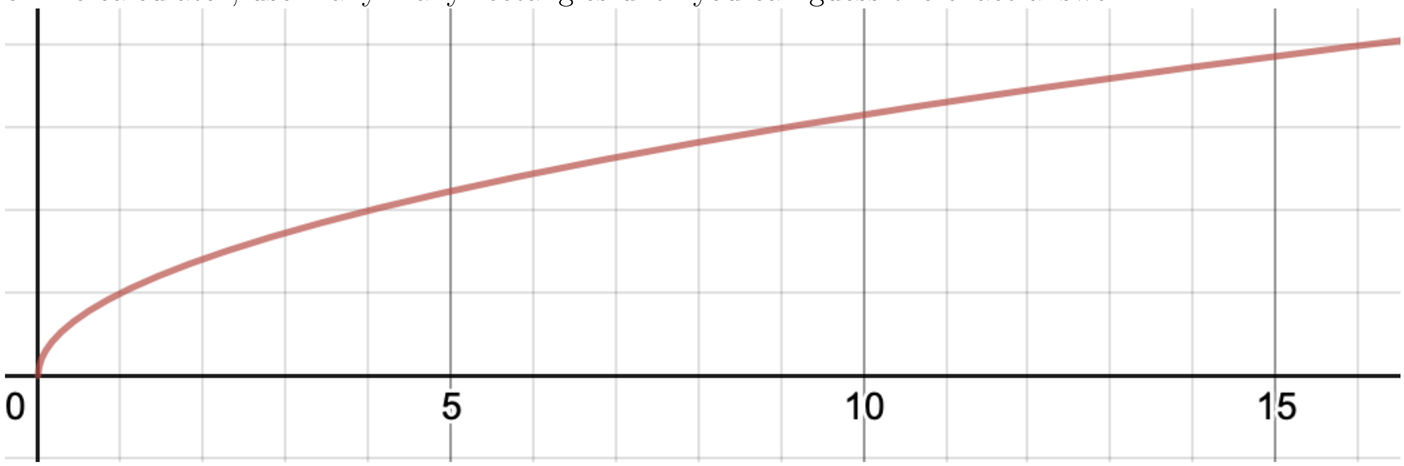
- Use the midpoint to draw the top of the rectangle.
- Instead of using a flat top, use a slant (trapezoidal rule).
- Instead of using a flat top, use a curve (Simpson's rule).
- **Use more rectangles.** (As $n \to \infty$, the approximation becomes exact.)

Tips for helping with the calculations:

- Use a spreadsheet to do all the arithmetic.
- Use my online Riemann Sum calculator `https://www.westga.edu/~cleach/graphing/riemann.html`

1.0.1 Example:

(a) Approximate the area under $f(x) = \sqrt{x}$ from $x = 0$ to $x = 16$ by hand/spreadsheet using 8 rectangles and left endpoints. (b) Use the online calculator to check the answers, and (c) using the online calculator, use many many rectangles until you can guess the exact answer.



2 Distances

The speed of a particle is measured every five seconds. The speeds are given in the table below:

t (sec)	0	5	10	15	20	25	30
v (ft/sec)	0	7	8	12	15	21	25

We want to estimate how far the particle traveled from $t = 0$ to $t = 30$.

One way to do this is to use the velocity at the start of each 5-second interval as the velocity during that entire interval:

Another way is to use the velocity at the end of each 5-second interval as the velocity during that entire interval:

Ways to improve this estimate:

- Take the average of the starting and ending velocities at each interval.
- If the velocity were given using a function instead of a table, we could use shorter intervals. (This is analogous to using more rectangles in the area problem.)