2.2 Limits

1 Definition

The notation

$$\lim_{x \to a} f(x) = L$$

means that we can make the value of $f(x)$ as close as we want to L by choosing x sufficiently close to a. It's read as "the limit of $f(x)$ as x approachaes a is equal to L".

Suppose that you have $y = f(x)$. We use limits to answer questions like "What happens to y when x gets really close to some pre-chosen value."

1.1 Example

Evaluate the limit below by looking at its graph.

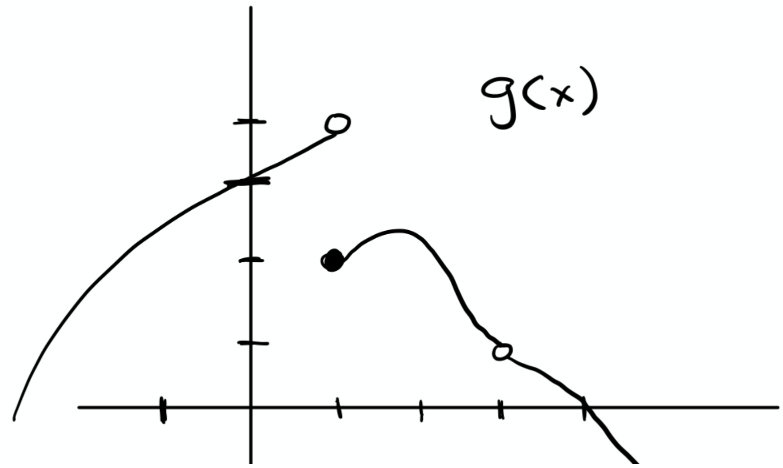
$\lim\limits_{x \to -6} \dfrac{x^2 + 4x - 12}{x + 6}$

2 One-sided and two-sided limits

The limits we did above are two-sided. A one-sided limit asks "What happens to $f(x)$ when x approaches a **from the left (or right)**"

$\lim\limits_{x \to a^-} f(x) = L$	$\lim\limits_{x \to a^+} f(x) = L$
approach from the left	approach from the right

2.1 Example



(a) $\lim_{x\to 1^-} g(x) =$ ______ (b) $\lim_{x\to 1^+} g(x) =$ ______ (c) $\lim_{x\to 1} g(x) =$ ______

(d) $\lim_{x\to 3^-} g(x) =$ ______ (e) $\lim_{x\to 3^+} g(x) =$ ______ (h) $\lim_{x\to 3} g(x) =$ ______

(d) $\lim_{x\to 0^-} g(x) =$ ______ (e) $\lim_{x\to -1} g(x) =$ ______ (h) $\lim_{x\to 4^+} g(x) =$ ______

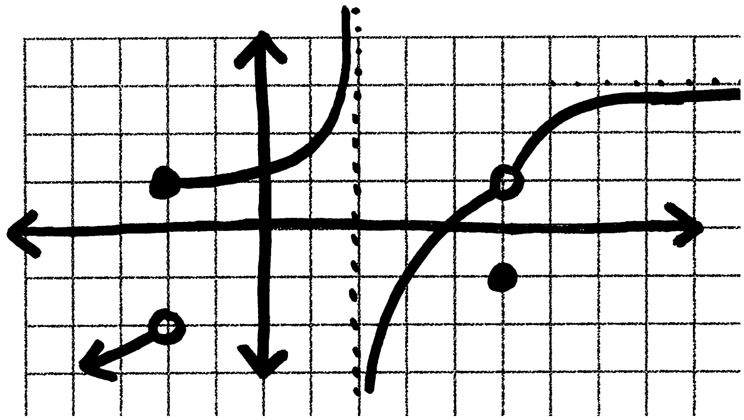
Note:

- If $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ then $\lim_{x\to a} f(x)$ **does not exist (DNE)**.

- If $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ are equal, then $\lim_{x\to a} f(x)$ is equal to them.

2.2 Example

Use the graph to evaluate the following:



(a) $\lim_{x \to -2^-} f(x) =$ ______ (b) $\lim_{x \to -2^+} f(x) =$ ______ (c) $\lim_{x \to -2} f(x) =$ ______

(d) $\lim_{x \to 2} f(x) =$ ______ (e) $\lim_{x \to 5} f(x) =$ ______ (h) $\lim_{x \to \infty} f(x) =$ ______

3 Limits with piecewise functions

3.1 Example

$$f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 6 - x & \text{if } x \geq 2 \end{cases}$$

$\lim_{x \to 2^-} f(x)$

$\lim_{x \to 2^+} f(x)$

$\lim_{x \to 2} f(x)$

3.2 Example

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$$

$$\lim_{x \to 1^-} f(x)$$

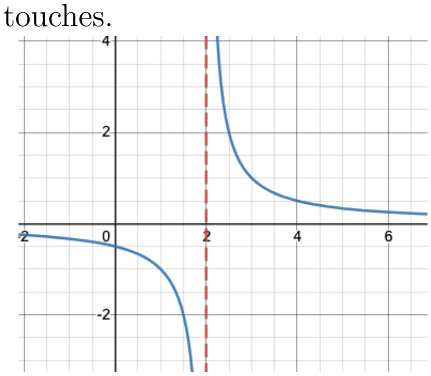
$$\lim_{x \to 1^+} f(x)$$

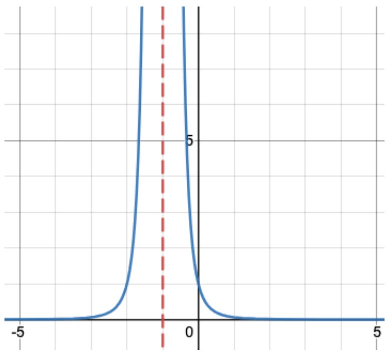
$$\lim_{x \to 1} f(x)$$

4 Vertical asymptotes

4.1 Definition, finding VA using graphs

A Vertical Asymptote is a vertical line that a graph approaches but never actually touches.





$$f(x) = \frac{1}{x-2}$$

$$f(x) = \frac{1}{(x+1)^4}$$

4.1.1 Example

Find any VA of this function by looking at its graph.

$$f(x) = \frac{x^2 - 2x - 3}{x^4 - 9x^2}$$

4.2 Finding Infinite limits / VA without using the graph.

4.2.1 Example

$$\lim_{x \to 3^-} \frac{(x^2 + 4)}{(x + 2)(x - 3)}$$

4.2.2 Example

$$\lim_{x \to 5^+} \frac{(x-4)(x+5)}{(x-5)^2(x+1)}$$

4.2.3 Example

$$\lim_{x \to 5^+} \frac{x^2+x}{x^2-25}$$

4.2.4 Example

$$\lim_{x \to 0^+} \frac{2x^2+4x}{x^4+3x^2}$$