

## 3.2 Product Rule and Quotient Rule

### 1 Intro

So far we've seen that for any **constant**  $c$  and function  $f(x)$

$$\frac{d}{dx} \left( cf(x) \right) = cf'(x),$$

but what if we multiply  $f(x)$  by something that it **not** a constant? In that case we have to use the Product Rule.

### 2 Product Rule

The Product Rule states that

$$\frac{d}{dx} \left[ g(x)f(x) \right] = g(x)f'(x) + f(x)g'(x).$$

Memorize it as

**“First times derivative of the second plus second times derivative of the first,”**

#### 2.1 Examples

##### 2.1.1 Example:

Find the derivative of  $f(x) = x^3e^x$ .

##### 2.1.2 Example:

Find the derivative of  $f(x) = (2x + 5)(x^2 + 3x + 4)$ .

### 2.1.3 Example:

Find the slope of the line tangent to  $g(x) = x^3e^x$  at the point with  $x$ -coordinate  $-2$

### 2.1.4 Example:

$f(x) = x^2 \cdot g(x)$ ,  $g(3) = 4$ ,  $g'(3) = -1$ . Evaluate  $f'(3)$ .

## 3 Quotient Rule

The Quotient Rule states that

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

To memorize this, call the function in the numerator “hi” and the function in the denominator “lo”, and remember it as

$$\text{“lo d hi - hi d lo over lolo”,} \quad \text{or} \quad \frac{\text{lo d hi - hi d lo}}{\text{lolo}}$$

**NOTE:** The order matters! It starts with “lo d hi”. Getting the subtraction backwards in the numerator is by far the most common mistake people make when using the quotient rule.

## 3.1 Examples

### 3.1.1 Example:

Find the derivative of  $f(x) = \frac{3x}{2x+5}$ .

### 3.1.2 Example:

Find the derivative of  $f(x) = \frac{x^2 + 2x + 1}{x^3}$  using the QR .

### 3.1.3 Example:

Find the derivative of  $f(x) = \frac{x^2 + 2x + 1}{x^3}$  without using the QR .

### 3.1.4 Example:

$h(x) = \frac{g(x)}{f(x)}$  Evaluate  $h'(2)$

$x$	0	1	2	3
$f(x)$	3	3	1	1
$f'(x)$	3	-2	4	0
$g(x)$	2	3	9	5
$g'(x)$	8	7	7	4

### 3.1.5 Example:

For which values of  $x$  is the tangent of does the graph of  $f(x) = f(x) = \frac{x^2+1}{x+3}$  have a horizontal tangent? Give exact answers, and approximations to 3 decimal places.

## 4 Using Rules More Than Once

### 4.0.1 Example:

Find the derivative of  $f(x) = \frac{x^2 e^x}{3x + 5}$ .

### 4.0.2 Example:

Find the derivative of  $y = e^x x^2 \sin(x)$       Hint:  $\frac{d}{dx}(\sin x) = \cos(x)$ .