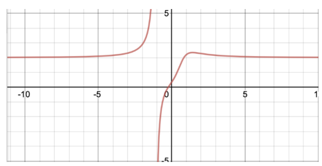
Limits at Infinity; Horizontal Asymptotes

1 Limits at Infinity and Horizontal Asymptotes from Graphs

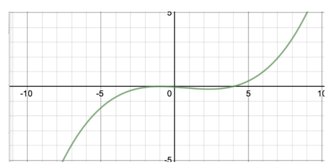
- The limit $\lim_{x\to\infty} f(x)$ describes the behavior of the function as x grows large.
- The limit $\lim_{x\to-\infty} f(x)$ describes the behavior of the function as x grows large in the negative direction.
- We can determine this from the graph by looking at the graph behavior as x goes to the right or left.
- If either $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$ is equal to a finite number L, then $y = L$ is a horizontal asymptote of $f(x)$.

1.0.1 Examples



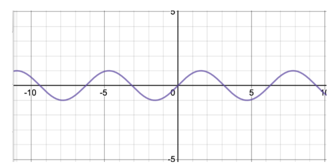
$\lim_{x\to\infty} f(x)$

$\lim_{x\to-\infty} f(x)$



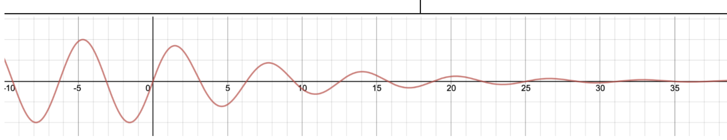
$\lim_{x\to\infty} g(x)$

$\lim_{x\to-\infty} g(x)$



$\lim_{x\to\infty} h(x)$

$\lim_{x\to-\infty} h(x)$



$\lim_{x\to\infty} s(x)$

2 Limits at Infinity and H.A. from Equations

2.1 Polynomials

Polynomials have no HA; their limits at ∞ and $-\infty$ are infinite, and depend only on the leading term.

2.1.1 Examples

$$\lim_{x\to\infty} x^2 + 3x - 1$$

$$\lim_{x\to-\infty} x^4 - 4x^2 + x - 3$$

$$\lim_{x\to-\infty} -3x^3 + x^2 + -2x + 2$$

2.2 Rational Functions

Recall: A rational function has the form $f(x) = \frac{T(x)}{B(x)}$, where $T(x)$ and $B(x)$ are both polynomials. A rational function may or may not have a H.A:

- If $T(x)$ has greater degree than $B(x)$, then there is no H.A., and the limits at ∞ and $-\infty$ are infinite.
- If $B(x)$ has greater degree than $T(x)$, then the line $y = 0$ is a H.A.
- If $T(x)$ and $B(x)$ have the same degree, then $y = \frac{lc_T}{lc_B}$ is a H.A., where lc_T and lc_B are the leading coefficients of $T(x)$ and $B(x)$.

WARNING: These rules only apply for limits as $x \to \infty$ or $x \to -\infty$. Don't try to use them if x approaches a finite number.

2.2.1 Examples

a. $\displaystyle\lim_{x\to\infty} \frac{2x^2 + 6x - 9}{-5x + 4}$

b. $\displaystyle\lim_{x\to-\infty} \frac{9x^2 - 4x + 2}{-4x^3 + x^3 + 3}$

c. $\displaystyle\lim_{x\to-\infty} \frac{x^3 + x^2 + x + 1}{5x^3 + x^2}$

d. $\displaystyle\lim_{x\to-\infty} \frac{3x + 4}{5x + 7}$

e. $\lim_{x\to-\infty} \frac{3x^2+4}{5x+7}$

2.2.2 Formal Method

2.2.3 Examples

Multiply top and bottom by $\frac{1}{x^d}$, where d is the degree of the denominator. This also works for functions that have a square root in them.

a. $\lim_{x\to-\infty} \frac{3x+4}{5x+7}$

b. $\lim_{x\to\infty} \frac{2x^2+6x-9}{-5x+4}$

c. $\lim_{x\to\infty} \frac{\sqrt{9x^4+3x^2+x+1}}{x^2+x+1}$