5.4 Indefinite Integrals & Net Change

1 Indefinite Integrals

Indefinite Integral is another name for general antiderivative.

1.0.1 Teaching Example:

$$\int x^2 + x + \sin x \; dx$$

1.0.2 Example:

$$\int \frac{1}{1+x^2} \; dx$$

1.1 Indefinite integrals vs integrals:

Definite $\int_a^b f(x)dx$	**Indefinite** $\int f(x)dx$
has endpoints	no endpoints
Evaluates to give a number	Evalautes to give a family of functions ending in $+C$.
Most graphing calculators and Desmos can do them.	Can be evaluated using a computer algebra system.

1.2 Verifying Indefinite Integrals

Evaluating integrals is objectively more difficult than computing derivatives. But if we are given a value of an indefinite integral, we can check it by taking a derivative.

1.2.1 Example:

Verify the integral $\int 2x\cos x - x^2 \sin x \, dx = x^2 \cos x + C$

1.2.2 Example:

Verify the integral $\int \ln x \, dx = x \ln x - x + C$

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2 The Net Change Theorem

The Net Change Theorem

The integral of a rate of change is the net change:

$$\int_a^b F'(x)\, dx = F(b) - F(a)$$

We can use this for any application involving rates of change, but we will focus on velocity and position.

2.1 Net Change in Position and Total Distance Traveled

Recall: Velocity tells both the speed and direction of movement.

Fact: Integrating velocity will give the net change in position.

Suppose a ball is launched straight upward from ground level with an initial velocity of 48 feet per second. The velocity of the ball t seconds after it is launched is given by the equation $v(t) = 48 - 32t$. Let's do three things:
1. Find the height function.

2. Find the net change in height between two times.
Find the net change from $t = 0$ to $t = 1$

Find the net change from $t = 1$ to $t = 3$

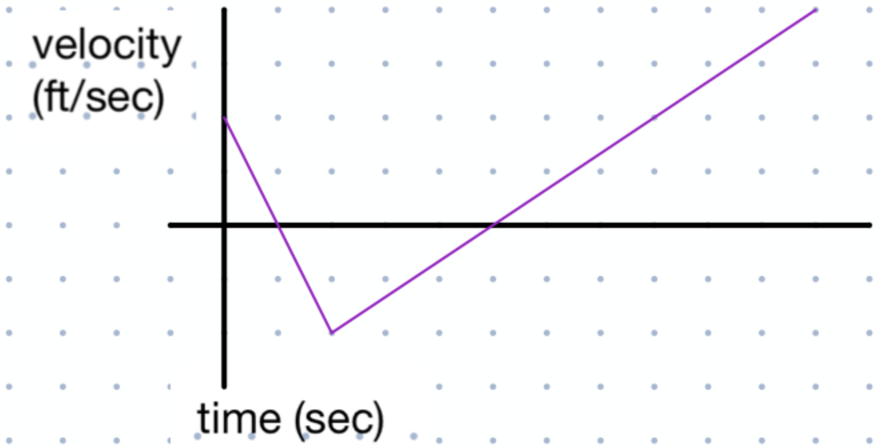
3. Find the total distance traveled by the ball from $t = 0$ to $t = 3$.

2.1.1 Example:

A particle moves along the x-axis has velocity given by the equation $v(t) = t^2 - 8t + 15$. (a) Find the net change in position from $t = 0$ to $t = 10$. (b) Find the total distance traveled during that period by hand and check it using the riemann sum calculator.

2.1.2 Example:

A particle moves along the y-axis, with velocity given by the function shown here.



Describe its motion, and find the net distance and total distance traveled between $t = 0$ and $t = 10$.

2.1.3 Example:

A particle moves along a straight line. Its velocity given by $v(t) = t^2 - 8t + 12$. Find the net and total distance traveled from $t = 0$ to $t = 9$