2.3 Calculating Limits using the Limit Laws

1 The Limit Laws

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \qquad \text{and} \qquad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$

4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{if } \lim_{x \to a} g(x) \neq 0$

FACT: If $f(x)$ is a polynomial function, rational function, or square root function and a is in the domain of $f(x)$, then

$$\lim_{x \to a} f(x) = f(a)$$

.

This fact is called the **Direct Substitution Rule**.

1.1 really easy examples

1.1.1 Example

$$\lim_{x\to 2} \frac{5 + x - x^2}{3x - 1}$$

1.1.2 Example

$$\lim_{x\to 5^+} \sqrt{3x^2 + 6}$$

1.2 More examples

1.2.1 Example

$$\lim_{x\to 5} \frac{x^2 - 7x + 10}{25 - x^2}$$

1.2.2 Example

$$\lim_{x\to -11^+} \frac{x^2 + 6x - 55}{x^3 + 8x^2 - 33x}$$

1.2.3 Example

$$\lim_{x \to 4} \frac{5 - \sqrt{6x + 1}}{4 - x}$$

1.2.4 Example

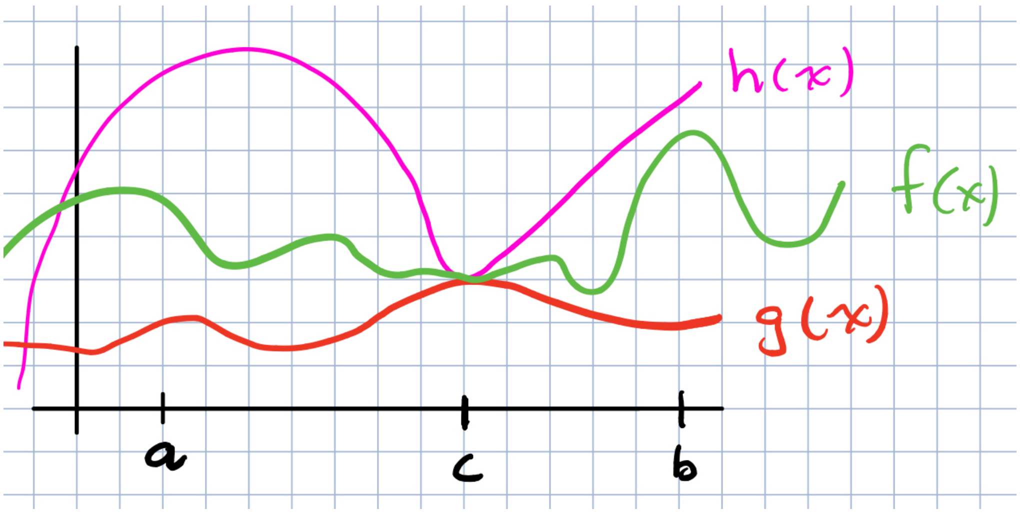
$$\lim_{t \to 0} \frac{t}{(2t + 1)^2 - 1}$$

1.3 Squeeze Theorem

Suppose we know that

- $g(x) \leq f(x) \leq h(x)$ for all values of x near c AND
- $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$

Then it must be true that $\lim_{x \to c} f(x) = L$.



1.3.1 Example

Evaluate the limit

$$\lim_{x \to 0} x^2 \sin(x)$$