4.1 Maximum and Minimum Values

1 Absolute Extrema

1.1 Definitions; Finding Extrema Using Graphs

Absolute Max and Min: Let c be a number in the domain of a function f. Then $f(c)$ is the

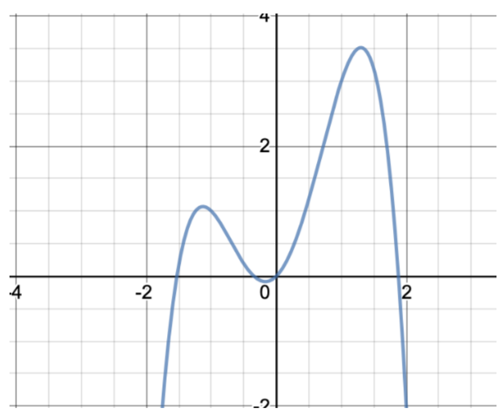
- **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D.
- **absolute minumum** value of f if $f(c) \leq f(x)$ for all x in D.

Local Max and Min: Let c be a number in the domain of a function f. Then $f(c)$ is a

- **local maximum** value of f if $f(c) \geq f(x)$ for all x near c.
- **local minumum** value of f if $f(c) \leq f(x)$ for all x near c.

1.1.1 Example:

The function shown here has ...



1.1.2 Example:

The function $f(x) = x^2$ has ...

1.1.3 Example:

The function $y = -x^3 + 4x$ has ...

1.2 Max and Mins on a Closed Interval

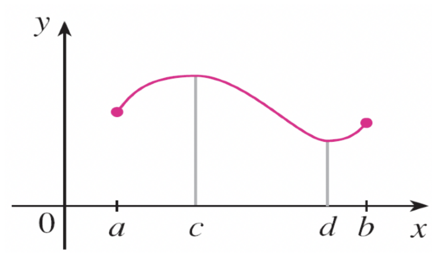
Extreme Value Theorem:

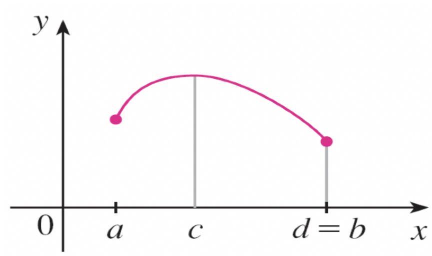
If f is continuous on a closed interval $[a,b]$, then the greatest and least values attained by the function on are the **absolute maximum and absolute minimum value on that interval**. Furthermore, the absolute max and min on the interval $[a,b]$ must occur at either

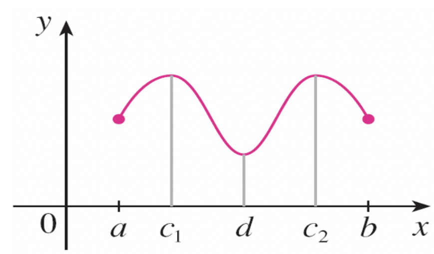
- a critical number, or
- an endpoint of the interval.

Fact: The absolute max and absolute min occur either at an endpoint or at a critical number.

1.2.1 Examples:







1.2.2 Example:

Find the absolute max and absolute min of the function $f(x) = 7 - x - x^2$ on the interval $[-2, 1]$.

1.2.3 Example:

Find the absolute max and absolute min of the function $f(x) = \sqrt[5]{x}(10 - x)$ on the interval $[0, 32]$.

2 Deeper look at the extreme value theorem

2.1 Why the interval must be closed

2.2 Why the function must be continuous on $[a, b]$