Final copy

EE3-12

EE9 SO13

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

MSc and EEE PART III/IV: MEng, BEng.and ACGI

OPTOELECTRONICS

Monday, 16 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	R.R.A. Syms
	Second Marker(s) :	W.T. Pike

Fundamental constants

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ m kg/C^2

$c = 3 \times 10^8$ m/s

$h = 6.62 \times 10^{-34}$ Js

Maxwell's equations – integral form

$\int \int_A \underline{\mathbf{D}} \cdot d\underline{a} = \int \int \int_V \rho \, dv$

$\int \int_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = - \int \int_A \partial \underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \int \int_A [\underline{\mathbf{J}} + \partial \underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

Maxwell's equations – differential form

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial \underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial \underline{\mathbf{D}}/\partial t$

Material equations

$\underline{\mathbf{J}} = \sigma \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu \underline{\mathbf{H}}$

Vector calculus (Cartesian co-ordinates)

$\mathrm{grad}(\phi) = \partial\phi/\partial x \, \underline{i} + \partial\phi/\partial y \, \underline{j} + \partial\phi/\partial z \, \underline{k}$

$\mathrm{div}(\underline{\mathbf{F}}) = \partial \mathbf{F}_x/\partial x + \partial \mathbf{F}_y/\partial y + \partial \mathbf{F}_z/\partial z$

$\mathrm{curl}(\underline{\mathbf{F}}) = \underline{i} \, \{\partial \mathbf{F}_z/\partial y - \partial \mathbf{F}_y/\partial z\} + \underline{j} \, \{\partial \mathbf{F}_x/\partial z - \partial \mathbf{F}_z/\partial x\} + \underline{k} \, \{\partial \mathbf{F}_y/\partial x - \partial \mathbf{F}_x/\partial y\}$

$\mathrm{curl}\,\{\mathrm{curl}(\underline{\mathbf{F}})\} = \mathrm{grad}\,\{\mathrm{div}(\underline{\mathbf{F}})\} - \nabla^2 \underline{\mathbf{F}}$

$\int \int_A \underline{\mathbf{F}} \cdot d\underline{a} = \int \int \int_V \mathrm{div}(\underline{\mathbf{F}}) \, dv$

$\int_L \underline{\mathbf{F}} \cdot d\underline{L} = \int \int_A \mathrm{curl}(\underline{\mathbf{F}}) \cdot d\underline{a}$

1. A y-polarized optical beam of free-space wavelength λ is incident on an interface between two dielectric media at an angle θ_1 as shown in Figure 1. The two media have refractive indices n_1 and n_2, and $n_1 < n_2$.

a) Modify the sketch to show any additional waves that may be generated. Write down assumed solutions for the time-independent electric fields in each medium. By assuming an appropriate boundary condition for the electric field, prove Alhazen's law and Snell's law, and find a relation between the amplitudes of the waves.

[9]

b) Explain how the time-dependent magnetic field can be found from the electric fields you have assumed. By assuming an appropriate boundary condition for the magnetic field, find a second relation between the amplitudes of the waves. Hence find the transmission and reflection coefficients.

[11]

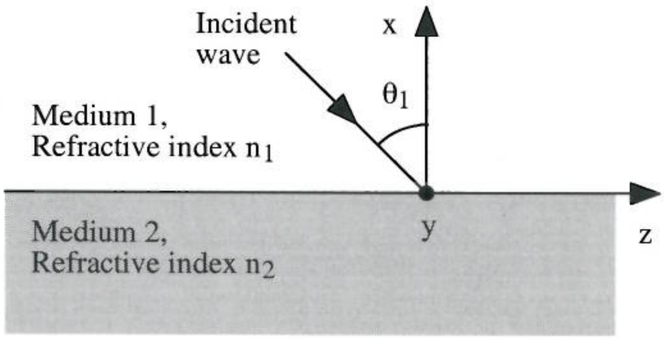


Figure 1.

2. Figure 2 shows a symmetric slab dielectric waveguide, which consists of a layer of refractive index n_1 and thickness h sandwiched between two semi-infinite layers each of refractive index n_2. For TE modes, solutions can be written as:

$$E_{yi}(x, z) = E_i(x) \exp(-j\beta z) \text{ (for i = 1, 2, 3)}$$

Where the transverse fields are given by:

$$\begin{array}{ll} \text{In layer 1:} & E_1 = E \cos(\kappa x - \phi) \\ \text{In layer 2:} & E_2 = E' \exp(\gamma x) \\ \text{In layer 3:} & E_3 = E'' \exp\{-\gamma(x - h)\} \end{array}$$

a) Assuming that the governing equation is the scalar equation:

$$\nabla^2 E_{yi}(x, z) + n_i^2 k_o^2 E_{yi}(x, z) = 0$$

Show that the modal solutions satisfy the waveguide equation:

$$d^2E_i/dx^2 + \{n_i^2 k_o^2 - \beta^2\} E_i = 0$$

[4]

b) Explaining your reasoning, sketch the transverse fields of the two lowest-order guided modes.

[6]

c) Find the values of the constants κ and γ, show that $\tan(\phi) = \gamma/\kappa$, derive the eigenvalue equation $\tan(\kappa h/2) = \pm\kappa/\gamma$ and briefly explain its meaning.

[10]

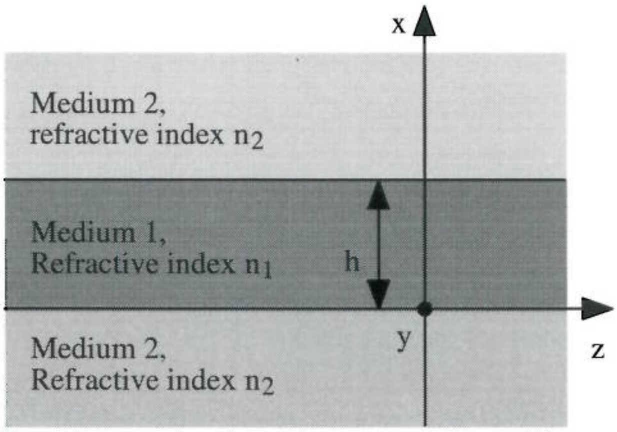


Figure 2.

3. a) Explain qualitatively why bends in a dielectric waveguide must give rise to radiation loss.

[6]

b) Sketch the construction of a titanium diffused lithium niobate channel waveguide Mach-Zehnder interferometric modulator based on Y-junctions, and explain its operation.

[8]

c) A modulator as described above is equipped with phase modulators for which the voltage giving π radians phase change is $V_\pi = 10$ V. The phase modulators are driven in opposite directions with the voltage waveform shown in Figure 3 below. Explaining your reasoning, sketch the time variation of the output power.

[6]

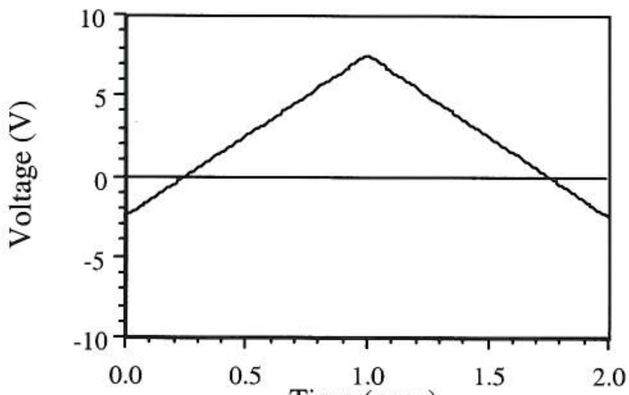


Figure 3.

4. Illustrating your answer with diagrams where appropriate, explain the meaning and significance of the following terms:

a) Direct and indirect gap semiconductors

[5]

b) Lattice matching in heteroepitaxy

[5]

c) Stimulated emission

[5]

d) Carrier injection phase modulator

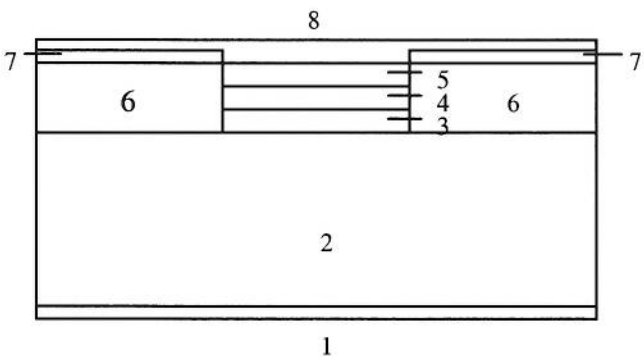
[5]

5. Optoelectronic devices are often constructed using III-V materials such as GaAs and GaAlAs. The dependence of the refractive index n of GaAs on the free carrier concentration follows the linear relation $\Delta n = -Ne^2\lambda^2/(8\pi^2c^2m^*\varepsilon_0 n_\infty)$. Here N is the carrier density, m* is the effective mass, n_∞ is the refractive index at short wavelength and e, λ, c and ε_0 have their usual meanings. The refractive index of $Ga_{1-x}Al_xAs$ varies linearly from n = 3.57 for x = 0 to 3.36 for x = 0.35.

a) Why is the starting material almost always a binary substrate? Illustrating your answer with sketches of the variation in refractive index across the layer structure, explain the difference between homostructure and heterostructure waveguides. What are the key advantages of the latter?

[12]

b) Figure 4 shows the cross-section of a GaAlAs/GaAs laser, together with a list of materials from which it is constructed. Explaining your reasoning, assign the different materials to the layers. Identify the double heterostructure.



n-$Ga_{0.7}Al_{0.3}As$
SiO_2
p-$Ga_{0.7}Al_{0.3}As$
Al metal
GaAs
n^+-GaAs
$Ga_{0.85}Al_{0.15}As$

Figure 4.

[8]

6. The lumped-element rate equations for a semiconductor laser are:

$$dn/dt = I/ev - n/\tau_e - G\phi(n - n_0)$$
$$d\phi/dt = \beta n/\tau_{rr} + G\phi(n - n_0) - \phi/\tau_p$$

Here n and ϕ are the electron and photon densities, I is the injected current, v is the active volume, τ_e, τ_{rr} and τ_p are the electron, radiative recombination and photon lifetimes and β is the coupling factor for spontaneously-emitted light.

a) Explain the meaning of the constants β, G and n_0. Assuming an emission wavelength of λ, explain how the optical power output may be found if the equations can be solved. Why is an equation for the hole density not needed?

[6]

b) In the steady state, what approximations can be made to these equations i) below and ii) above threshold? Estimate the slope efficiency dP/dI in each case.

[8]

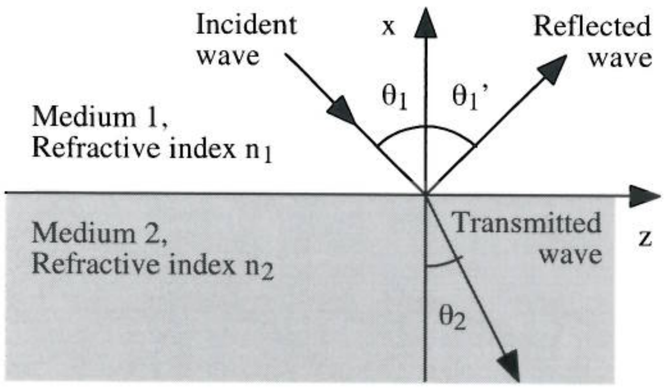
c) An InGaAsP Fabry-Perot laser has a cavity length, width and depth of L = 350 μm, w = 2 μm and d = 0.1 μm and an effective index of $n_{eff} = 3.5$. Estimate the photon lifetime. Assuming that the electron lifetime is $\tau_e = 10^{-9}$ sec, the threshold current is 20 mA and the gain coefficient is $G = 10^{-12}$ m^3/sec, calculate the electron density at transparency.

[6]

EE3-12

Optoelectronics 2011 – Solutions

1 a) The electromagnetic interface problem involves incident, reflected and transmitted waves as shown below.



[2]

Solutions can be written down as incident and reflected waves in Medium 1, and a transmitted wave in Medium 2, so that:

$E_{y1} = E_I \exp[-jk_0n_1(z \sin(\theta_1) - x \cos(\theta_1))] + E_R \exp[-jk_0n_1(z \sin(\theta_1') + x \cos(\theta_1'))]$

$E_{y2} = E_T \exp[-jk_0n_2(z \sin(\theta_2) - x \cos(\theta_2))]$

[2]

Here E_I, E_R and E_T are the amplitudes of the incident, reflected and transmitted waves and $k_0 = 2\pi/\lambda$ is the propagation constant in free space.

The boundary condition that must be satisfied is that tangential components of $\underline{E}$ must match across a boundary. Since $\underline{E}$ is wholly tangential, we must have $E_{y1} = E_{y2}$ on $x = 0$, or:

$E_I \exp[-jk_0n_1z \sin(\theta_1)] + E_R \exp[-jk_0n_1z \sin(\theta_1')] = E_T \exp[-jk_0n_2z \sin(\theta_2)]$

[2]

The only way this equation can be satisfied for all z is if:

$n_1 \sin(\theta_1) = n_1\sin(\theta_1') = n_2\sin(\theta_2)$

Hence $\theta_1 = \theta_1'$ (Alhazen's law)

$n_1 \sin(\theta_1) = n_2\sin(\theta_2)$ (Snell's law)

[2]

Hence $E_I + E_R = E_T$

[1]

b) The time-dependent electric and magnetic fields are related by curl($\underline{\mathbf{E}}$) = $-\partial\underline{\mathbf{B}}/\partial t$

Hence the time-independent fields are related by curl($\underline{E}$) = $-j\omega\underline{B} = -j\omega\mu_0\underline{H}$

Hence $\underline{H} = (j/\omega\mu_0)$ curl($\underline{E}$)

$$\text{Now curl(E)} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

In this case, only E_y is non-zero,

While R.R.A. Syms

$$\text{so curl(E)} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix}$$

Hence:
$H_x = (-j/\omega\mu_0)\ \partial E_y/\partial z$
$H_y = 0$
$H_z = (+j/\omega\mu_0)\ \partial E_y/\partial x$

[2]

The boundary condition that must be satisfied is that tangential components of $\underline{H}$ must match across a boundary. Only H_z is tangential, so we must have $\partial E_{y1}/\partial x = \partial E_{y2}/\partial x$ on $x = 0$.

[2]

Doing the differentiation, we get:

$$\partial E_{y1}/\partial x = jk_0n_1\cos(\theta_1)\ E_I \exp[-jk_0n_1(z\sin(\theta_1) - x\cos(\theta_1))] - jk_0n_1\cos(\theta_1)\ E_R \exp[-jk_0n_1(z\sin(\theta_1) + x\cos(\theta_1))]$$

$$\partial E_{y2}/\partial x = jk_0n_2\cos(\theta_2)\ E_T \exp[-jk_0n_2(z\sin(\theta_2) - x\cos(\theta_2))]$$

Hence on $x = 0$ we get:

$$\partial E_{y1}/\partial x = jk_0n_1\cos(\theta_1)\ E_I \exp[-jk_0n_1z\sin(\theta_1)] - jk_0n_1\cos(\theta_1)\ E_R \exp[-jk_0n_1z\sin(\theta_1)]$$

$$\partial E_{y2}/\partial x = jk_0n_2\cos(\theta_2)\ E_T \exp[-jk_0n_2z\sin(\theta_2)]$$

Hence:

$$n_1\cos(\theta_1)\ E_I \exp[-jk_0n_1z\sin(\theta_1)] - n_1\cos(\theta_1)\ E_R \exp[-jk_0n_1z\sin(\theta_1)] = n_2\cos(\theta_2)\ E_T \exp[-jk_0n_2z\sin(\theta_2)]$$

From previous results, the exponential terms must cancel, so that:
$n_1\cos(\theta_1)\ E_I - n_1\cos(\theta_1)\ E_R = n_2\cos(\theta_2)\ E_T$

[3]

The two equations that must be solved to find the transmission and reflection coefficients are:
$E_I + E_R = E_T$
$n_1\cos(\theta_1)\ E_I - n_1\cos(\theta_1)\ E_R = n_2\cos(\theta_2)\ E_T$

Hence:
$n_1\cos(\theta_1)\ E_I - n_1\cos(\theta_1)\ E_R = n_2\cos(\theta_2)\ \{E_I + E_R\}$
$\{n_1\cos(\theta_1) - n_2\cos(\theta_2)\}E_I = \{n_1\cos(\theta_1) + n_2\cos(\theta_2)\}E_R$
So the reflection coefficient is:
$\Gamma = E_R/E_I = \{n_1\cos(\theta_1) - n_2\cos(\theta_2)\}/\{n_1\cos(\theta_1) + n_2\cos(\theta_2)\}$

[2]

And:
$n_1\cos(\theta_1)\ E_I - n_1\cos(\theta_1)\ \{E_T - E_I\} = n_2\cos(\theta_2)\ E_T$
$2n_1\cos(\theta_1)E_I = \{n_1\cos(\theta_1) + n_2\cos(\theta_2)\}E_T$
So the transmission coefficient is:
$T = E_T/E_I = 2n_1\cos(\theta_1)/\{n_1\cos(\theta_1) + n_2\cos(\theta_2)\}$

[2]

Uhile R.R.A.Syms

2. a) The governing equation is the scalar wave equation:
$\nabla^2 E_{yi}(x, z) + n_i^2 k_o^2 E_{yi}(x, z) = 0$ for $i = 1, 2, 3$, or:
$\partial^2 E_{yi}/\partial x^2 + \partial^2 E_{yi}/\partial z^2 + n_i^2 k_o^2 E_{yi}(x, z) = 0$

Assuming that $E_{yi}(x, z) = E_i(x) \exp(-j\beta z)$, the derivatives are:
$\partial E_{yi}/\partial x = dE_i/dx \exp(-j\beta z)$
$\partial^2 E_{yi}/\partial x^2 = d^2E_i/dx^2 \exp(-j\beta z)$

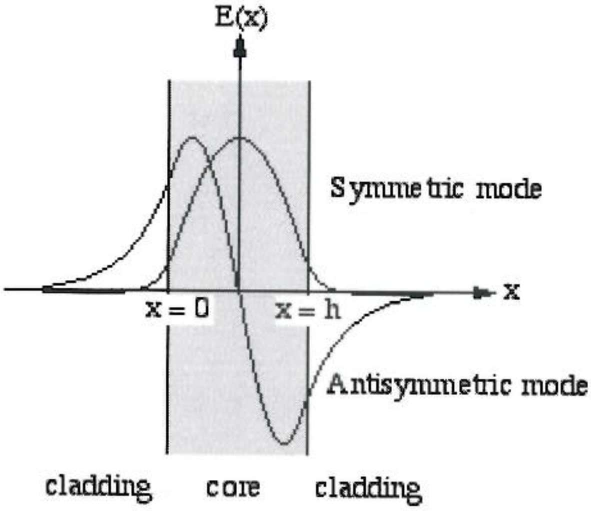
$\partial E_{yi}/\partial z = -j\beta E \exp(-j\beta z)$
$\partial^2 E_{yi}/\partial z^2 = -\beta^2 E_i \exp(-j\beta z)$

Substituting into the scalar wave equation and cancelling the exponential terms, we get:
$d^2E_i/dx^2 + [n_i^2 k_o^2 - \beta^2] E_i = 0$

[4]

b) For any guide, the transverse fields are constructed from the field functions, matching the fields and their first derivatives with respect to x at the boundaries. For a symmetric guide, the modes must be symmetric or anti-symmetric. The transverse field patterns of the two lowest order modes must therefore be as shown below.

[2]



[4]

c) In layer 1, the transverse field is given by $E_1 = E \cos(\kappa x - \phi)$
This satisfies the differential equation $d^2E_1/dx^2 + \kappa^2 E_1 = 0$
Comparison with the waveguide equation then implies that $\kappa = \sqrt{(n_1^2 k_0^2 - \beta^2)}$

In layer 2, the transverse field is $E_2 = E' \exp(\gamma x)$
This solution satisfies the differential equation $d^2E_2/dx^2 - \gamma^2 E_2 = 0$
Comparison with the waveguide equation then implies that $\gamma = \sqrt{(\beta^2 - n_2^2 k_0^2)}$

[2]

Matching the transverse electric fields on x = 0 and x = h, we get:
$E \cos(\phi) = E'$ (1)
$E \cos(\kappa h - \phi) = E''$ (2)
The derivatives of the transverse electric fields are:

R.R.A. Syms

$dE_1/dx = -\kappa E \sin(\kappa x - \phi)$
$dE_2/dx = \gamma E' \exp(\gamma x)$
$dE_3/dx = -\gamma E'' \exp[-\gamma(x - h)]$
Matching the derivatives on $x = 0$ and $x = h$ gives:
$\kappa E \sin(\phi) = \gamma E'$ (3)
$\kappa E \sin(\kappa h - \phi) = \gamma E''$ (4)

Dividing Equation 3 by Equation 1, we get:
$\tan(\phi) = \gamma/\kappa$
Similarly, dividing Equation 4 by Equation 2, we get:
$\tan(\kappa h - \phi) = \gamma/\kappa$

[3]

Using the standard identity $\tan(A - B) = \{\tan(A) - \tan(B)\} / \{1 + \tan(A) \tan(B)\}$ we get:
$\tan(\kappa h - \phi) = \{\tan(\kappa h) - \tan(\phi)\} / \{1 + \tan(\kappa h) \tan(\phi)\}$

Substituting for $\tan(\phi)$, we then get:
$\{\tan(\kappa h) - \gamma/\kappa\} / \{1 + \tan(\kappa h) \gamma/\kappa\} = \gamma/\kappa$
Re-arranging, we then get:
$\tan(\kappa h) - \gamma/\kappa = \gamma/\kappa\{1 + \tan(\kappa h) \gamma/\kappa\}$
Collecting together terms we then get:
$\tan(\kappa h)\{1 - \gamma^2/\kappa^2\} = 2\gamma/\kappa$
Or $\tan(\kappa h) = (2\gamma/\kappa)/\{1 - (\gamma/\kappa)^2\}$

Since $\tan(2A) = 2\tan(A) / \{1 - \tan^2(A)\}$
We obtain the eigenvalue equation $\tan(\kappa/2) = \gamma/\kappa$
However, multiplying the earlier equation by κ^2/γ^2 we obtain:
$\tan(\kappa h) = (2\kappa/\gamma)/\{(\kappa/\gamma)^2 - 1\} = (-2\kappa/\gamma)/\{1 - (\kappa/\gamma)^2\}$
Hence we obtain the alternative eigenvalue equation $\tan(\kappa/2) = -\kappa/\gamma$

[4]

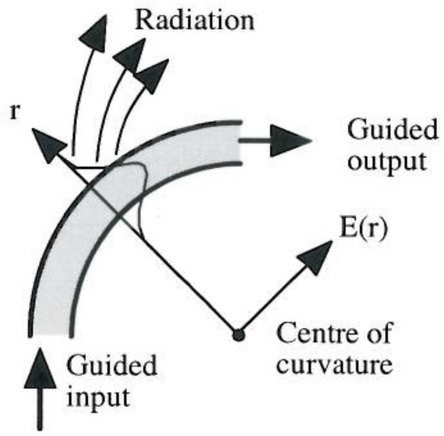
Since κ and γ are both functions of β, the eigenvalue equation has a single unknown, and may be solved to find the propagation constant.

[1]

R.R.A. Syms

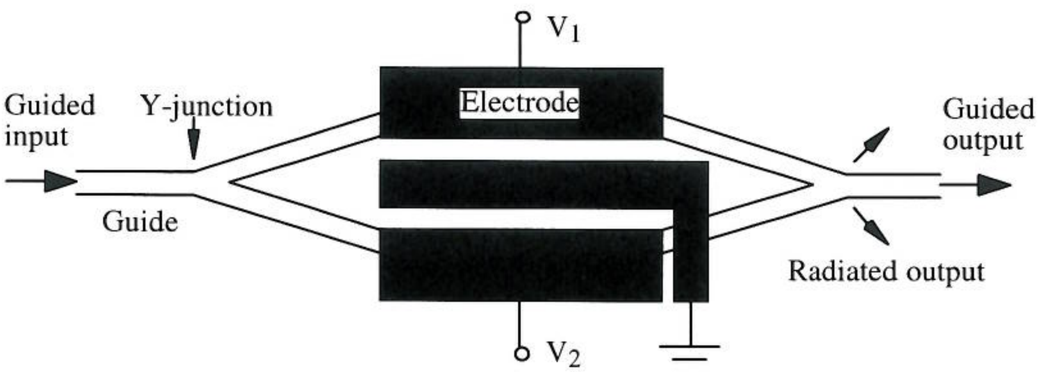
3.a) Any curvature of a waveguide results in a loss to radiation additional to the normal propagation loss. The cause of the radiation can be understood by considering the profile of the transverse field as the guided mode travels round the bend. To keep in step, this pattern must rotate around the centre of curvature of the guide, like a spoke in a rotating wheel. Consequently, the further from the centre, the faster the field must move. However, the outermost evanescent part of the field extends to infinite radius, so at some distance from the core the field would have to exceed the speed of light in the substrate material to keep up. The portion of the field outside this point must be radiated, reducing the power in the guided mode. Bend loss is therefore very dependent on the confinement offered by the guide.

[3]

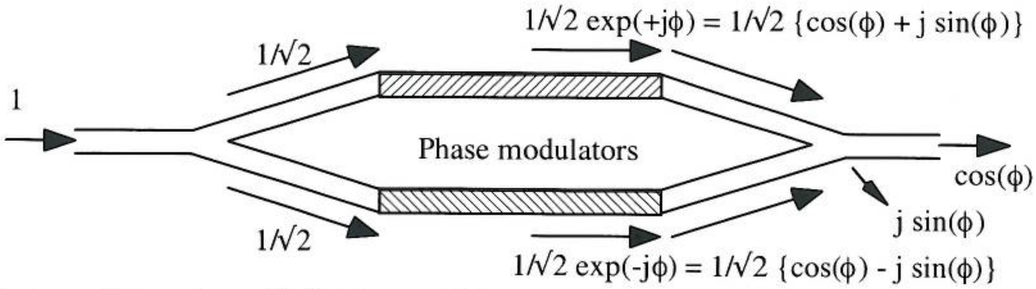


[3]

b) A Y-junction-based Mach-Zehnder interferometer consists of two back-to-back Y-junctions linked by sections of waveguide carrying phase modulators as shown below.



[3]

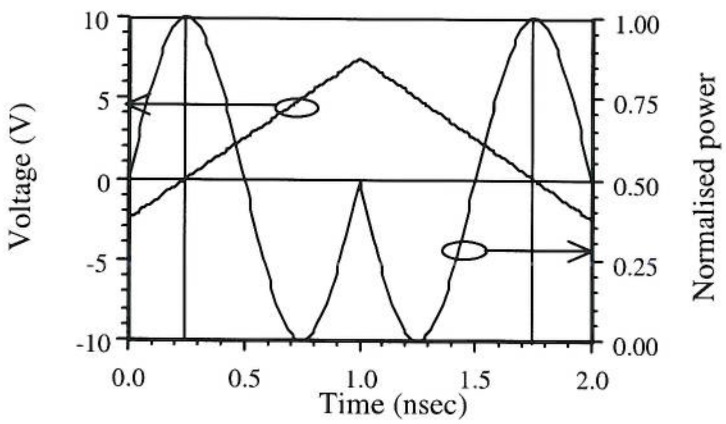


The input Y-junction will divide a guided mode of unity amplitude into two separated modes each of amplitude $1/\sqrt{2}$ in the two output ports. After passing through the phase modulators, these components are phase-shifted by equal and opposite amounts ϕ, so that the mode amplitudes become $1/\sqrt{2}\ \exp(\pm j\phi) = 1/\sqrt{2}\ \{\cos(\phi) \pm j\sin(\phi)\}$. The output Y-junction will combine two in-phase modes of amplitude $1/\sqrt{2}$ into a single output of unity amplitude, and radiate any anti-phase components. The guided output amplitude is therefore $\cos(\phi)$, corresponding to a guided output power of $\cos^2(\phi)$. The radiated output power is $\sin^2(\phi)$.

[5]

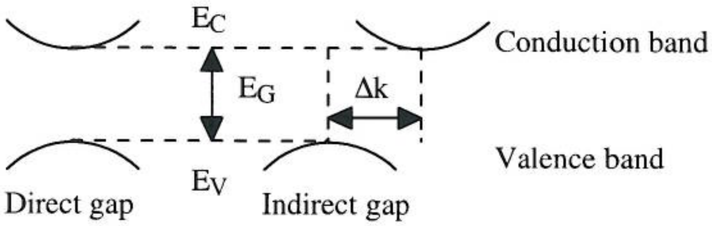
c) For the phase modulators shown, $V_\pi = 10$ V.
When t = 0.00 nsec, V = -2.5 V, so that $\phi = -\pi/4$ and $P = \cos^2(\pi/4) = 0.5$
When t = 0.25 nsec, V = 0 V, so that $\phi = 0$ and P = 1.0
When t = 0.50 nsec, V = +2.5 V, so that $\phi = +\pi/4$ and P = 0.5
When t = 0.75 nsec, V = +5.0 V, so that $\phi = +\pi/2$ and $P = \cos^2(\pi/2) = 0$
When t = 1.00 nsec, V =+7.5V, so that $\phi = +3\pi/4$ and P = 0.5
When t = 1.25 nsec, V = +5V, so that $\phi = +\pi/2$ and P = 0
When t = 1.50 nsec, V = +2.5 V, so that $\phi = +\pi/4$ and P = 0.5
When t = 1.75 nsec, V = 0 V, so that $\phi = 0$ and P = 1.0
When t = 2.00 nsec, V = -2.5 V, so that $\phi = -\pi/4$ and P = 0.5
The time variation of the guided output power is then as shown below.

[3]



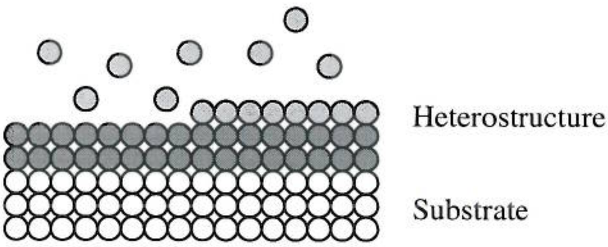
[3]

4a) The energy gap is the energy difference between the conduction band and the valence band in a semiconductor. Direct gap materials have the conduction band minimum in the E-k diagram aligned with the valence band maximum. In this case, band-to-band transitions may be achieved simply using the energy of a photon, without the requirement for large additional momentum (which the photon does not posess). Indirect gap materials have the conduction band minimum in the E-k diagram displaced laterally from the valence band maximum. Because momentum is proportional to k, there must now be a momentum difference between the two bands, in addition to an energy difference. Band to band transitions are then only possible if an additional source of the necessary momentum is available, such as a phonon (which has large momentum but little energy). The need for a three-body interaction greatly limits the rate of optoelectronic interactions in indirect gap materials. Consequently, direct gap materials are needed for light emission.



[5]

b) Epitaxy is a process used to grow crystalline materials in an ordered manner that preserves the regular arrangement of the crystal structure. There are two main variants, liquid phase epitaxy (older) and vapour phase epitaxy. In each case a seed crystal or substrate provides a template. The deposited material may have exactly the same composition (homoepitaxy) or different composition (heteroepitaxy). Heteroepitaxy is used to grow multilayer structures that provide confinement for carriers and photons, and hence allow efficient optoelectronic interactions. Strain-free heteroepitaxy is only possible if the two compounds have matching lattice spacings; otherwise large numbers of defects are created. The range of possible heterostructures that can be grown is therefore restricted to those that are lattice matched to the substrate, usually a binary compound. Direct gap lattice-matched materials that can be used to form emitters include GaAlAs/GaAs and InGaAsP/InP.

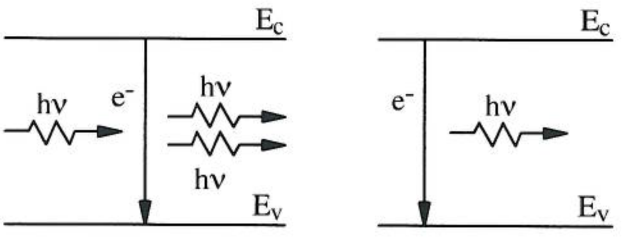


[5]

c) Stimulated emission is a mechanism for light production involving a downward transition of an electron from the conduction band to the valence band that is triggered by a photon. The energy thus liberated is released as a second photon that is identical in frequency, phase, direction and polarisation to the first. Consequently, stimulated emission can be used to amplify a light beam, or (in a resonant structure) generate laser action. The output light is very monochromatic

R.R.A. Syms

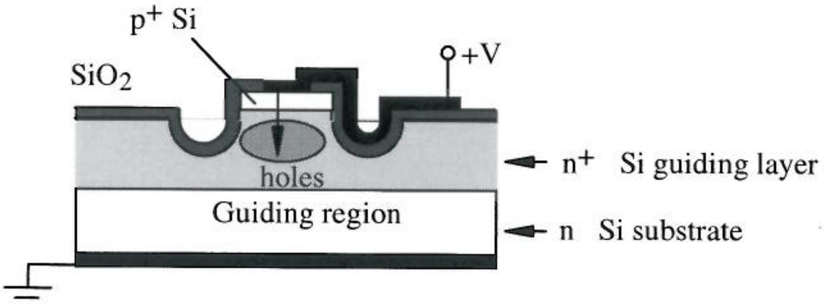
(and hence can be used as a carrier in a long-distance optical telecommunications system) and very directional (and hence can be coupled efficiently into an optical fibre). In contrast, spontaneous emission involves a random transition from the conduction band to the valence band, and is exploited in light emitting diodes. Photons are emitted across a broad spectral band, with random directions and polarizations. Consequently, the output light has poor properties and is suitable only for short-distance telecommunications.



Stimulated emission   Spontaneous emission

[5]

d) A carrier injection phase modulator is a guided-wave optical semiconductor device, which uses injection of minority carriers in a forward biased p-n junction to achieve a phase change via the free carrier contribution to the refractive index. The achievable index change is $\Delta n \approx -Ne^2/2m\varepsilon_0 n_\infty \omega^2$, where N is the density of injected carriers. Carrier injection modulators are relatively slow, since the carriers must disappear by recombination. Consequently, this mechanism is normally used only in materials such as silicon that are non centro-symmetric and hence lack an electro-optic effect.



[5]

Ulle R.R.A.Syms

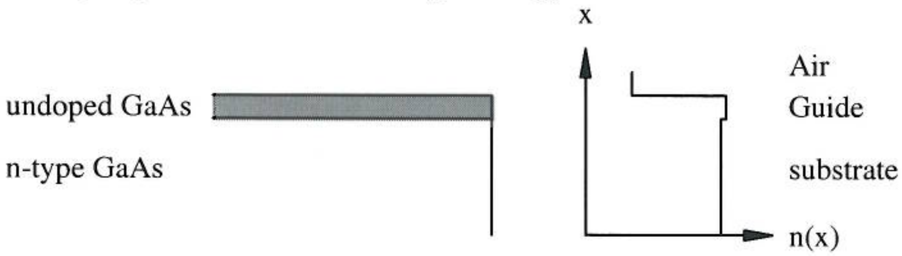
5. a) The starting material is always a binary substrate because it is too difficult to grow a large boules of a ternary compound. The fundamental difficulty is to maintain stoichiometry, due to differences in the vapour pressure of the different atomic species.

[2]

Homostructure waveguides are constructed using the materials of the same intrinsic composition, but with refractive index differences established through differences in carrier concentration.

[1]

Because the refractive index falls when the carrier concentration rises, the substrate must be more heavily doped than the core of the guide. Typical construction is then as shown below.

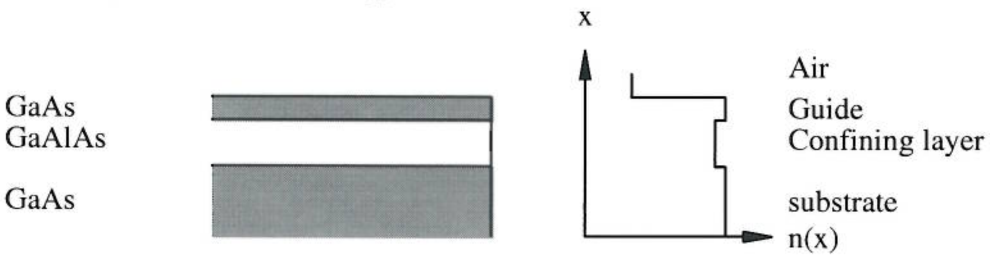


[3]

Heterostructure waveguides are constructed using materials of different intrinsic composition, with refractive index differences established through differences in material composition.

[1]

Because the refractive index falls as the Al concentration rises, a GaAlAs layer can typically be used only for confinement. Typical construction is then as shown below.



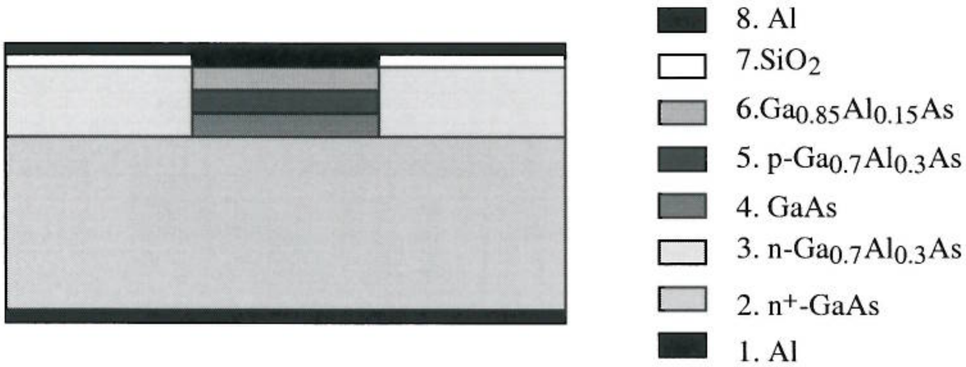
[3]

The key advantages of a heterostructure are the higher refractive index difference that may be obtained and the ability to confine carriers (through differences in the bandgap) as well as photons (through differences in refractive index).

[2]

b) Assuming the compositions given:

Layer 1 must be the bottom contact (Al metal)
Layer 2 must be the substrate (GaAs; heavily doped n+ to make an ohmic contact)
Layers 3, 4 and 5 must be a PIN double heterostructure (n-GaAlAs,GaAs, GaAs, p-GaAlAs)
Layer 6 must be a GaAlAs burying layer to provide lateral confinement
Layer 7 must be a silica insulating later to block current flow except through the heterostructure
Layer 8 must be the top contact (Al metal)



[7]+[1] for heterostructure

6a) The lumped-element rate equations for a semiconductor laser are:
$dn/dt = I/ev - n/\tau_e - G\phi(n - n_0)$
$d\phi/dt = \beta n/\tau_{rr} + G\phi(n - n_0) - \phi/\tau_p$

Here:
β is a factor that describes coupling of spontaneously emitted light into the laser cavity
G is the gain coefficient for stimulated emission
n_0 is the electron concentration at transparency

[3]

The rate of loss of photons per unit volume is ϕ/τ_p.
Hence the total photon flux is $\Phi = \phi v/\tau_p$
If each photon carries energy hc/λ, the power output is $P = (\phi v/\tau_p)(hc/\lambda)$

[2]

There is no need for an additional equation for holes because electrons and holes are created and destroyed in pairs in optoelectronic interactions, so their dynamics must be identical.

[1]

b) In the steady state, the rate equations become:
$I/ev - n/\tau_e - G\phi(n - n_0) = 0$
$\beta n/\tau_{rr} + G\phi(n - n_0) - \phi/\tau_p = 0$

Below threshold, we may neglect stimulated emission, so that:
$I/ev - n/\tau_e = 0$
$\beta n/\tau_{rr} - \phi/\tau_p = 0$

[2]

Above threshold, we may neglect spontaneous emission, so that:
$I/ev - n/\tau_e - G\phi(n - n_0) = 0$
$G\phi(n - n_0) - \phi/\tau_p = 0$ and hence $n = n_0 + 1/G\tau_p$

[2]

Below threshold, the light output is:
$P = (\phi v/\tau_p)(hc/\lambda) = (\beta n v/\tau_{rr})(hc/\lambda) = \beta(I/e)(\tau_e/\tau_{rr})(hc/\lambda)$
Hence the slope efficiency is:
$dP/dI = \beta(\tau_e/\tau_{rr})(hc/e\lambda)$

[2]

Above threshold, the light output is:
$P = (\phi v/\tau_p)(hc/\lambda) = (I/e - nv/\tau_e)(hc/\lambda)$, or
$P = \{(I - I_{th})/e\}(hc/\lambda)$ where $I_{th} = nev/t_e$ is the threshold current
Hence the slope efficiency is:
$dP/dI = (hc/e\lambda)$

[2]

c) The photon lifetime is $\tau_p = L/\{v_g \log_e(1/R_1R_2)\}$
Here $v_g = c/n$ is the group velocity
$R_1 = R_2 = (3.5 - 1)/(3.5 + 1) = 0.555$ is the mirror reflectivity
Hence $\tau_p = 350 \times 10^{-6} \times 3.5 / \{3 \times 10^8 \log_e(1/0.555^2)\} = 3.473 \times 10^{-12}$ s

R.R.A

[2]

As previously found, the threshold current is $I_{th} = nev/t_e$

Hence, the electron density during lasing is $n = I_{th}\tau_e/ev$

However, the electron density during lasing is also given by $n = n_0 + 1/G\tau_p$

Hence, the electron density at transparency is $n_0 = I_{th}\tau_e/ev - 1/G\tau_p$

[2]

The active volume is $v = 350 \times 2 \times 0.1 \times 10^{-18} = 70 \times 10^{-18}$ m^3

Hence, the electron density at transparency is

$n_0 = 20 \times 10^{-3} \times 10^{-9} / (1.6 \times 10^{-19} \times 70 \times 10^{-18}) - 1/(10^{-12} \times 3.473 \times 10^{-12})$

Or $n_0 = 1.5 \times 10^{24}$ m^{-3}

[2]

R.R.A. [signatures]