UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2002

BEng Honours Degree in Computing Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C344

DECISION ANALYSIS

Thursday 2 May 2002, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 A merchant bank is to choose one of three possible investments:

- a ***cautious*** investment that would perform well in an improving economy and only make a small loss in a deteriorating economy,
- a ***risky*** investment that would perform exceptionally well in an improving economy , but would do very badly indeed in a deteriorating economy, and
- a ***counter-cyclical*** investment that would do poorly in an improving economy, but well in a deteriorating economy.

The bank's assessed prior probabilities for the alternative scenarios: ***improving***, ***stable*** and ***deteriorating*** economy, together with its estimated profits from the three investments under these scenarios, is given in the following table:

Profit (in £million)	improving economy	stable economy	deteriorating economy
cautious	30	5	-10
risky	40	10	-30
counter-cyclical	-10	0	15
Prior probability	0.1	0.5	0.4

a On the basis of the expected monetary value criterion, which investment should the bank choose?

b Some at the bank recommend developing a model to forecast the future state of the economy, in order to make a sensible investment decision. What is the most the bank should be prepared to pay in development costs, no matter how reliable the model might prove to be?

c On further consideration, though, whilst the bank is comfortable with its assessed prior probability of 0.1 for an improving economy, it feels considerably less secure regarding its assignment of the remaining 0.9 prior probability between a stable and a deteriorating economy.

i) Sketch a graph showing how the expected profit for each of the three investments varies with the prior probability of a stable economy.

ii) Give a decision rule for the optimal choice of investment for different values of the prior probability of a stable economy.

The three parts carry, respectively, 10%, 20%, 70% of the marks.

2 A follower of the SMARTER method of multi-attribute decision making is in the process of buying a new car. She has reduced the set of possible purchases to four models - the Quaver, the Crotchet, the Minim and the Semibreve - costing £26K, £15K, £30K and £20K, respectively.

The cars she is considering differ in engine power, reliability, body strength and styling. Following the SMARTER method she comes up with values of each car for the four attributes, where in all cases 100 is best and 0 is worst:

	Attributes			
Cars	**Power**	**Reliability**	**Strength**	**Styling**
Quaver	100	90	0	0
Crotchet	0	100	90	70
Minim	70	40	100	40
Semibreve	50	0	40	100

a Do you think that there was any essential difference in the way that the values for the four attributes were obtained?

b The decision maker believes that an additive model can be employed for obtaining aggregate values for the cars. How do you think that the decision maker was able to convince herself of this?

c The decision maker ranked the attributes in descending order of swing weight as follows: power, reliability, strength, styling. How do you think she went about this?

d On the basis of the above information, calculate the aggregate value of each car.

e Faced with buying a model with the lowest level of reliability, the decision maker reckons she would be willing to pay an extra £8K to bring its reliability up to the highest level. Which car should she choose?

The five parts carry, respectively, 15%, 15%, 15%, 30%, 25% of the marks.

3 A company is considering various short-term projects that would deliver cash flows for two years, each totalling between £50K and £200K per annum. The company regards the cash flows in years 1 and 2 as separate attributes X and Y, and considers that X and Y are mutually utility independent.

a What is the general form of the multiple utility function $U_{X,Y}$? Explain how the various coefficients in the formula would normally be obtained.

b The company has constructed individual utility functions U_X and U_Y. Some of the values of U_X and U_Y are given in the following table:

c	$U_X(c)$	$U_Y(c)$
50K	0.00	0.00
75K	0.41	0.29
150K	0.91	0.82
180K	0.97	0.94
200K	1.00	1.00

Although it has not followed the normal procedures for finding the coefficients in the formula for $U_{X,Y}$, the company has made the following assessments:

- It would be indifferent between

  (1) a sure outcome of £75K each year for two years and

  (2) a risky investment with a 50% chance of £200K each year and a 50% chance of £50K each year.

- It would be indifferent between getting:

  (1) £180K for the first year and £50K for the second, and

  (2) £50K for the first year and £200K for the second.

Express the multiple utility function $U_{X,Y}$ in terms of U_X *and* U_Y.

c Use this multiple utility function to choose between the following pair of two-year projects:

- ***Project A:*** 50% chance of yielding £150K each year and 50% chance of yielding £75K each year.
- ***Project B:*** 50% chance of yielding £150K in the first year and £75K in the second, and 50% chance of yielding £75K in the first year and £150K in the second.

The three parts carry, respectively, 25%, 60%, 15% of the marks.

4 A student has seven days remaining before tests begin in his three courses, and wants to allocate revision time as effectively as possible. He needs to spend at least one day on each course and wants to concentrate on just one course on any day. He can allocate between 1 and 5 days revision time to any course and estimates that the marks (out of 10) that he can achieve in the tests depend on his allocation of time, in the following way:

	Estimated Marks		
Revision Days	Course 1	Course 2	Course 3
1	3	5	2
2	5	5	4
3	7	6	7
4	7	9	8
5	8	9	9

a Give the details (the stages, states, recursive equations, etc.) of how the problem of maximising the total marks obtained in the three tests can be expressed as a dynamic programming problem.

b Solve the problem using dynamic programming.

The two parts carry equal marks.