UNIVERSITY OF LONDON

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1996

MEng Honours Degrees in Computing Part IV
MSc Degree in Foundations of Advanced Information Technology
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute

PAPER 4.77

COMPUTING FOR OPTIMAL DECISIONS

Friday, May 10th 1996, 10.00 - 12.00

Answer THREE questions

For admin. only: paper contains
4 questions
3 pages (excluding cover page)

1 a Consider the quadratic programming problem

$$\min \left\{ a^T x + \tfrac{1}{2} x^T \mathcal{Q} x \;\middle|\; \mathcal{H}^T x = h \right\}$$

where $a \in \mathbb{R}^n$; $\mathcal{H} \in \mathbb{R}^{n \times m}$, $\mathcal{Q} \in \mathbb{R}^{n \times n}$, $h \in \mathbb{R}^m$ are given values and $x \in \mathbb{R}^n$ is to be determined. Assuming the $\mathcal{Q}$ is not strictly positive definite, develop a quadratic programming approach that might solve this problem.

b Consider an inequality constrained quadratic programming problem. Suppose the optimum in the intersection of a set of active constraints has been attained. Discuss how it may be possible to improve the current optimum. [You are only required to show that a descent direction may exist. You do not need to establish feasibility.]

2 a Consider the problem

$$\min \left\{ x_1^3 + 4\, x_2^2 + 16\, x_3 \;\middle|\; x_1 + x_2 + x_3 = 5;\ x_1 \geq 1;\ x_2 \geq 1;\ x_3 \geq 1 \right\}.$$

If the sequential unconstrained minimisation technique (SUMT) were to be applied to solve this problem, what would be the unconstrained function to be minimised at each iteration? Summarise the overall technique. Do not solve the problem.

b Let $x \in \mathbb{R}^n$ and $c(x)$, $g_i(x)$, $i = 1, \ldots, m$ be arbitrary real valued differentiable functions. Let $0 \leq \hat{\lambda} \in \mathbb{R}^m$ be specified values. Suppose that $\hat{x}$ is an optimal solution to

$$\max \left\{ c(x) - \sum_{i=1}^{m} g_i(x)\, \hat{\lambda}_i \right\}.$$

Establish the relationship between $\hat{x}$ and an optimal solution to

$$\max \left\{ c(x) \;\middle|\; g_i(x) - g_i(\hat{x}) \leq 0\ ;\ i = 1, \ldots, m \right\}.$$

3 a A computer company manufactures three different computers: A, B and C, and needs to determine its weekly production plan. The available production capacity is as follows:

Activity	Available man-hours/week
Assembly 1	500
Assembly 2	350
Testing	150

The number of production hours required to manufacture each computer are as follows:

Computer	A	B	C
Assembly 1	9	3	5
Assembly 2	5	4	2
Testing	3	2	2

The estimated demand for A and B exceed the production capacity while the demand for C is 20 units/week. Unit manufacturing costs for A, B and C are £250, £100 and £150 respectively. The unit prices required in order to sell x_A units of A, x_B units of B and x_C units of C are £$(350 + 100x_A^{-1/3})$, £$(160 + 40x_B^{-1/4})$ and £$(200 + 50x_C^{-1/2})$ respectively. Formulate an optimisation model for determining x_A, x_B, x_C to maximise profit.

b Consider the problem

$$\max \left\{ \ln (x_1 + x_2) \;\middle|\; x_1 + 2\,x_2 \leq 5,\, x_1 \geq 0;\, x_2 \geq 0 \right\}$$

Formulate the Karush-Kuhn-Tucker first order necessary conditions of optimality and use these conditions to derive an optimal solution.

/Turn over ...

4 a Two stocks are being considered for inclusion in an investment portfolio. The estimated mean and variance of the return on each share is given by

Stock	mean	variance
1	5	4
2	10	100

while the covariance of the returns is 5. The price per share of stocks 1 and 2 are £20 and £30 respectively. The amount budgeted for the portfolio is £100. Formulate the quadratic programming model for the problem of maximising return while minimising portfolio risk.

b Formulate the Frank-Wolfe algorithm to solve this problem. [You should not attempt to solve the problem you have formulated.]

c Discuss the solutions that would be preferred by a risk-averse investor and those that would be preferred by a risk-seeking investor.