UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1996

MSc Degree in Foundations of Advanced Information Technology
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Diploma of Membership of Imperial College

PAPER F4.12

LOGIC—MODEL THEORY AND PROOF THEORY
Thursday, May 9th 1996, 10.00 - 12.00

Answer THREE questions
including Questions 1 and 2

For admin. only: paper contains
4 questions
2 pages (excluding cover page)

Section A *(Answer BOTH questions)*

1 Using semantic tableaus, show which of the following formulas are theorems. If a formula is not a theorem, give a countermodel in which the formula is false.

a $((Q \rightarrow P) \wedge (R \rightarrow Q)) \rightarrow (R \rightarrow P)$

b $(P \vee Q) \rightarrow ((P \vee R) \rightarrow (R \vee Q))$

c $(P(a, b) \wedge \forall x P(b, x)) \rightarrow \neg\exists y \neg P(a, y)$

d $(P \rightarrow \exists x Q(x)) \rightarrow \exists x(P \rightarrow Q(x))$

NB. Outside parentheses have been omitted.

The four parts carry, respectively, 25%, 25%, 25% and 25% of the marks.

2a Define the concept of satisfiability in propositional logic.

b Define the concept of a Herbrand model.

c Let X be any formula of a first-order language L and M any Herbrand model for L. Prove that $\exists x Q(x)$ is true in M for an assignment A if and only if X(y/t) is true in M for A, for some term t in the domain of M.

d Let L be a first-order language with only three constant symbols, viz. a, b and c. Prove that if X is a formula in L without any occurrences of negation, then X is satisfiable.

The four parts carry, respectively, 10%, 20%, 30% and 40% of the marks.

Section B *(Answer ONE question)*

3a Let $X_1, \ldots, X_n$ and Y be well-formed formulas of propositional logic. Prove that $\{X_1, \ldots, X_n\} \vDash Y$ if and only if $(Z \rightarrow Y)$ is a tautology, where Z is the formula $X_1 \wedge \ldots \wedge X_n$ (parentheses omitted for simplicity).

b State and prove the *weak completeness* theorem for first-order logic.

c State and prove the *compactness* theorem for first-order logic.

The three parts carry, respectively, 30%, 30% and 40% of the marks.

4a Define the concept of a downward saturated set for first-order logic.

b Let L be a first-order language without function symbols and only two constant symbols, 0 and 1. Let Δ contain only the formula $\exists x \forall y (P(x) \rightarrow Q(y))$. Show how many different downward saturated sets can be constructed from Δ.

c Let L (in part b) be extended to contain one unary function symbol f. Define a Herbrand model in which the formula $\exists x \forall y (P(x) \rightarrow Q(y))$ is true. Show that the model is a Herbrand model.

The three parts carry, respectively, 20%, 40% and 40% of the marks.

End of paper

