UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2001

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C337=I3.18

SIMULATION AND MODELLING

Tuesday 8 May 2001, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

The following applies to both questions 1 and 2.

The 'completion loop' of an assembly plant takes in partially finished products and completes them by adding two components C1 and C2. A separate worker is responsible for adding each component and C1 is added before C2. Finished products are tested at the end of the line. Those that pass the test are rated 'OK' and are sent on to the shipping department. Others are rated as 'seriously broken' and the product is rejected and subsequently disposed of. The remainder are diagnosed as 'fixable' by replacing *both* C1 and C2 (the test unit cannot distinguish between faults from C1 and C2 and it proves more economical to replace them both). The fixable parts are disassembled by an additional worker who strips out C1 and C2 and passes the job back to the head of the completion loop for re-assembly. The system can be represented as a queueing network as shown in Figure 1 below.

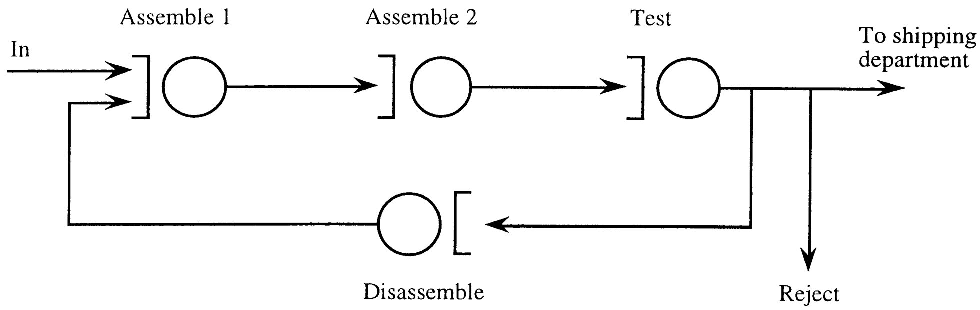


Figure 1

The buffers between the various servers are actually large shelf stacks which are well approximated by first-in-first-out queues with infinite capacity. Observations show that the assembly, disassembly and test times are all approximately exponentially distributed. The means are 22 minutes (assembly 1), 20 minutes (assembly 2), 17 minutes (test) and 83 minutes (disassembly), the latter being large because the disassembly worker works part-time. The arrival process is well approximated by a Poisson process with mean inter-arrival time of 28 minutes. 4% of all completed products are rejected and 18% are sent back for disassembly.

1 Compare and contrast the *event scheduling* and *process-oriented* approaches to discrete-event simulation. You should explain in general terms how they both work and discus their relative merits in terms of ease of use and computational efficiency. As part of your answer, outline how each might be used to model the assembly plant system described above. When discussing the process-oriented approach assume that the jobs (products) in the system are represented as processes and sketch the code for the corresponding '`Job`' class. Outline also how the arrival stream would be modelled in each case. You are NOT required to detail a complete simulation model in either case. You should, however, include the principal functions/methods of the API for supporting each approach and give examples of how these functions/methods would be used in the respective models.

2 Refer back to the assembly plant problem described earlier, and illustrated in Figure 1. Parts a-c apply to this system when it is at equilibrium.

a How many (completed) jobs per hour on average are sent to the shipping department? Justify your answer.

b What is the utilisation of each worker and what is the probability that all four of them are simultaneously idle (i.e. have no jobs to work on)? What mathematical result/property does the latter calculation exploit?

c Considering only products that are ultimately rated 'OK', how long on average do they spend in the completion loop before being sent to the shipping department? Show your working.

d In an attempt to reduce costs it has been decided to invest in better testing equipment that can pinpoint failures in C1 and C2. Unfortunately it is not possible to remove component C1 unless C2 has already been removed (C2 encases C1) but if the fault is known to lie with C2 only it can be replaced without having to remove C1 as well. Both C1 and C2 take the same amount of time on average to remove and two out of every three failures can be fixed by replacing C2 only. Explain how the queueing network shown in Figure 1 would have to be modified in order to represent the new set-up. Draw the modified queueing network and annotate it with all the routing probabilities. Hint: Do not worry about how workers will be assigned to perform disassembly.

The four parts carry 10%, 40%, 20% and 30% of the marks respectively.

3 It is well known that in an IP network packets tend to arrive at the routers in bursts. A router has therefore been monitored in order to measure the number of packets in each burst. During the observation period there were n bursts in total and the observed burst sizes were $B_1, B_2, ..., B_n$. The size of a burst was defined to be the number of packets following the initial packet in each burst, so the B_i, $1 \le i \le n$, can take the values 0,1,2,...

a When simulating a router, one way to model the input process is to simply "replay" the observed trace of burst sizes and to use this to construct a model of individual packet arrivals within the simulator. State THREE disadvantages of this "trace-driven" approach and explain why, in each case, it is preferable to fit the data to, and subsequently sample from, a known mathematical distribution.

b A frequency histogram of the observed burst sizes was constructed from the B_i, $1 \le i \le n$, and the recorded bucket contents were as follows:

Burst size, B:	0	1	2	3	4	5	>5
Frequency:	61192	22321	9368	4159	1834	704	422

The sample mean of the B_i, $1 \le i \le n$, was $\overline{B}=0.67$

As part of a distribution fitting exercise you are asked to try to fit this data to a geometric distribution which has the probability distribution function

$$p(n) = \Pr\{B=n\} = q\,(1-q)^n$$

for some parameter q. A suitable estimator for q is $\frac{1}{\overline{B}+1}$. Perform a χ^2 test on this data at the 5% significance level to test the null hypothesis *H0: the observations are samples from a geometric distribution.*

c. Instead of fitting the burst sizes to a mathematical distribution it has been decided to build an *empirical* distribution sampler, in other words a sampler which generates the values 0, 1, 2, ... in accordance with the *observed* frequencies of burst sizes. So, for example, given the above data an empirical distribution sampler should generate a burst size of two 9368 times for every 100000 samples, on average. Outline how you would build an empirical sampler for this data given the above frequency counts as input. You may use any language/notation you wish when sketching any code.

The three parts carry 30%, 35% and 35% of the marks respectively.

4a Following an initial warm-up period to allow equilibrium to be reached, a simulated single-server queue is observed for a total of T simulated seconds. During this observation period the queue population and the waiting time (the total time spent in the system) of each customer was recorded. At the end of the observation period there were n customer completions and their recorded waiting times were $W_1, W_2, ..., W_n$. At time t, $0 \le t \le T$, the queue population is given by the state variable n(t).

i. How would you estimate the throughput of the system and the mean customer waiting time over the observation period?

ii. Suppose that the (true) mean population of the queue is L. This can be estimated from the simulation by computing the integral:

$$\frac{1}{T}\int_0^T n(t)\,dt$$

Explain why this integral correctly estimates L and describe an *efficient* way of computing it during the simulation.

iii. Using Little's Law derive an alternative estimate for L which avoids the integral and the need to use n(t), $0 \le t \le T$.

b With the aid of an example, outline the factors which determine the time taken for a simulated system to approach (near) equilibrium. As part of your answer explain i. why a heavily-loaded system is likely to take longer to reach equilibrium than one that is lightly loaded, and ii. the significance of the values assigned to the state variables during initialisation.

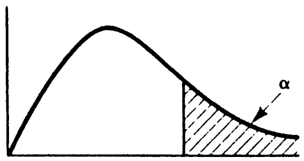
c A simulation is used to produce n *independent* estimates, $X_1, X_2, ..., X_n$, of some unknown quantity, α say. A confidence interval for the sample mean of these observations is given by

$$\overline{X} \pm \frac{zS}{\sqrt{n}}$$

where $\overline{X}$ and S are the sample mean and sample standard deviation of the X_i, $1 \le i \le n$, respectively, and z is obtained from tables of the Student's 't' distribution with n-1 degrees of freedom. In addition to independence, what additional assumption must be made about the X_i, $1 \le i \le n$, for this confidence interval to be correct? If you were to gather the n observations during a single execution of a simulation why might the observations NOT be independent and what precautions would you put in place to reduce any correlations between them?

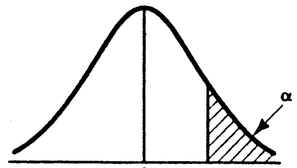
The three parts carry 40%, 30% and 30% of the marks respectively. For part a the three sections are weighted 1:2:1.

PERCENTAGE POINTS OF THE CHI-SQUARE DISTRIBUTION
WITH ν DEGREES OF FREEDOM



ν	$\chi^2_{0.005}$	$\chi^2_{0.01}$	$\chi^2_{0.025}$	$\chi^2_{0.05}$	$t^2_{0.10}$
1	7.88	6.63	5.02	3.84	2.71
2	10.60	9.21	7.38	5.99	4.61
3	12.84	11.34	9.35	7.81	6.25
4	14.96	13.28	11.14	9.49	7.78
5	16.7	15.1	12.8	11.1	9.2
6	18.5	16.8	14.4	12.6	10.6
7	20.3	18.5	16.0	14.1	12.0
8	22.0	20.1	17.5	15.5	13.4
9	23.6	21.7	19.0	16.9	14.7
10	25.2	23.2	20.5	18.3	16.0
11	26.8	24.7	21.9	19.7	17.3
12	28.3	26.2	23.3	21.0	18.5
13	29.8	27.7	24.7	22.4	19.8
14	31.3	29.1	26.1	23.7	21.1
15	32.8	30.6	27.5	25.0	22.3
16	34.3	32.0	28.8	26.3	23.5
17	35.7	33.4	30.2	27.6	24.8
18	37.2	34.8	31.5	28.9	26.0
19	38.6	36.2	32.9	30.1	27.2
20	40.0	37.6	34.2	31.4	28.4
21	41.4	38.9	35.5	32.7	29.6
22	42.8	40.3	36.8	33.9	30.8
23	44.2	41.6	38.1	35.2	32.0
24	45.6	43.0	39.4	36.4	33.2
25	49.6	44.3	40.6	37.7	34.4
26	48.3	45.6	41.9	38.9	35.6
27	49.6	47.0	43.2	40.1	36.7
28	51.0	48.3	44.5	41.3	37.9
29	52.3	49.6	45.7	42.6	39.1
30	53.7	50.9	47.0	43.8	40.3
40	66.8	63.7	59.3	55.8	51.8
50	79.5	76.2	71.4	67.5	63.2
60	92.0	88.4	83.3	79.1	74.4
70	104.2	100.4	95.0	90.5	85.5
80	116.3	112.3	106.6	101.9	96.6
90	128.3	124.1	118.1	113.1	107.6
100	140.2	135.8	129.6	124.3	118.5

PERCENTAGE POINTS OF THE STUDENTS t
DISTRIBUTION WITH ν DEGREES OF FREEDOM



ν	$t_{0.005}$	$t_{0.01}$	$t_{0.025}$	$t_{0.05}$	$t_{0.10}$
1	63.66	31.82	12.71	6.31	3.08
2	9.92	6.92	4.30	2.92	1.89
3	5.84	4.54	3.18	2.35	1.64
4	4.60	3.75	2.78	2.13	1.53
5	4.03	3.36	2.57	2.02	1.48
6	3.71	3.14	2.45	1.94	1.44
7	3.50	3.00	2.36	1.90	1.42
8	3.36	2.90	2.31	1.86	1.40
9	3.25	2.82	2.26	1.83	1.38
10	3.17	2.76	2.23	1.81	1.37
11	3.11	2.72	2.20	1.80	1.36
12	3.06	2.68	2.18	1.78	1.36
13	3.01	2.65	2.16	1.77	1.35
14	2.98	2.62	2.14	1.76	1.34
15	2.95	2.60	2.13	1.75	1.34
16	2.92	2.58	2.12	1.75	1.34
17	2.90	2.57	2.11	1.74	1.33
18	2.88	2.55	2.10	1.73	1.33
19	2.86	2.54	2.09	1.73	1.33
20	2.84	2.53	2.09	1.72	1.32
21	2.83	2.52	2.08	1.72	1.32
22	2.82	2.51	2.07	1.72	1.32
23	2.81	2.50	2.07	1.71	1.32
24	2.80	2.49	2.06	1.71	1.32
25	2.79	2.48	2.06	1.71	1.32
26	2.78	2.48	2.06	1.71	1.32
27	2.77	2.47	2.05	1.70	1.31
28	2.76	2.47	2.05	1.70	1.31
29	2.76	2.46	2.04	1.70	1.31
30	2.75	2.46	2.04	1.70	1.31
40	2.70	2.42	2.02	1.68	1.30
60	2.66	2.39	2.00	1.67	1.30
120	2.62	2.36	1.98	1.66	1.29
∞	2.58	2.33	1.96	1.645	1.28