Paper Number(s): **E4.39**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE PART IV: M.Eng. and ACGI

ENVIRONMENTAL & ECONOMIC ISSUES IN POWER SYSTEMS

Friday, 26 April 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

Corrected Copy

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Time allowed: 3:00 hours

Examiners responsible:

First Marker(s): Pal,B.C.

Second Marker(s): Popovic,D.

Special instructions for invigilators:	None
Information for candidates:	None

1. (a) Write a one-page essay on `The UK Energy of Today' including insights into production of primary fuels, energy consumption, electricity production and consumption, and use of renewable energy sources. [8]

(b) Outline the principle of operation of a steam turbine. What are the ways to increase steam-plant efficiency? [6]

(c) A large hydropower station has a head of 324m and an average flow of 1370 m^3/s. The reservoir of water behind the dams and dikes is composed of a series of lakes covering an area of 6400km^2. Calculate (i) the available hydraulic power (ii) the number of days this power could be sustained if the level of the impounded water were allowed to drop by 1m. Assume no precipitation or evaporation and neglect water brought in by surrounding rivers and streams. [6]

2. (a) Describe the principle of operation, main characteristics and use, advantages and research needs of microturbines. [8]

(b) Calculate the number of wind generators required to produce the equivalent of a 600MW combined cycle gas-turbine (CCGT). Assume average wind speed is 27.8m/s, circular blade diameter is 20m, and conversion efficiency is 45%. Air density is 1.201 kg/m^3. Under what assumption(s) would the calculated number of wind generators be adequate to replace a 600MW CCGT. [5]

(c) Explain the difference between a base-load and a peak-load generating plant. [3]

(d) The demand of an area regularly varies between 60MW and 110MW in the course of one day, the average power being 80MW. To produce the required energy, consider two options:

(i) Install a base-power generating unit and a diesel-engine peaking plant,

(ii) Install a base-power generating unit and a pumped-storage unit.

What are the respective capacities of the base power and peaking power plants in each case? [4]

3. (a) The Nord Pool is regarded as the most successful example of electricity industry re-regulation to date. Describe its main characteristics including market structure and operation, pricing mechanism and congestion management. [8]

(b) The example system shown in Figure 3.1 is used to illustrate the pricing mechanism in the England and Wales Pool. Generators G1 and G2 have marginal cost functions $MC(P_{G1})=3+0.02P_{G1}$ *[£/MWh]* and $MC(P_{G2})=8+0.07P_{G2}$ *[£/MWh]* respectively. Their maximum generation is 200MW. The perfectly inelastic loads L1 and L2 are 40MW and 150MW respectively.

(i) Illustrate the ideal (unconstrained) dispatch and calculate the system marginal price (SMP).

(ii) Assuming there is a transfer limit of 100MW/line, explain the re-dispatch needed in order to preserve system security.

(iii) Calculate price adjustments, new demand charges and generating payments from the Pool in [£/h] corresponding to the change in dispatch. Assume that pool purchase price is equal to the system marginal price.

[12]

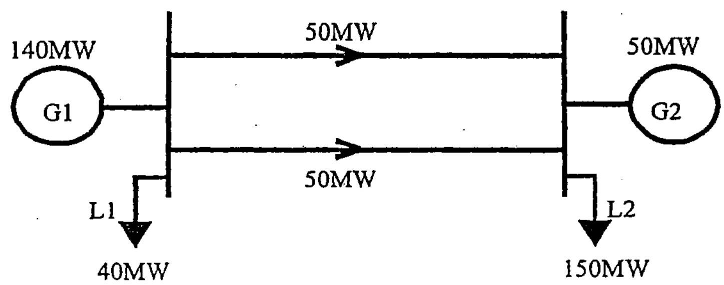

Figure 3.1

4. (a) Discuss various technical benefits of FACTS technology? [4]

(b) What are the basic differences between thyristor based FACTS controllers and converter based FACTS controllers? [2]

(c) Items in column I have one matching item in column II. Associate with item in column I the relevant item from column II. e.g. if item (A) in column I relates to item (C) in column II, then write A→ C [6]

I	II
A: STATCOM	A: Series facts device
B: Subsynchronous resonance (SSR)	B: Turbine shaft condition with series compensated line
C: Loop flows	C: Provides constant current at widely varying ac system voltage
D: Thyristor Controlled Phase Shifter (TCPS)	D Controls of Impedance, angle and power flow simultaneously
E: Unified Power Flow Controller (UPFC)	E: Ensures flat voltage profile through out the length of the line
F: Surge Impedance Loading (SIL)	F: Inherent problem in interconnected system without any control

(d) What is the usefulness of a *St. Clair Curve* in the context of power transmission? Briefly discuss the factors that influence power transfer capability of transmission lines of various lengths. [5]

(e) What are the structural and functional differences between a *Thyristor Controlled Series Capacitor* (*TCSC*) and a *Static Synchronous Series Compensator* (*SSSC*)? [3]

5. (a) What is meant by redundancy factor in state estimations? In an 'N' bus, 'B' branch power system, what would be the redundancy factors if voltage magnitude, real and reactive injection at every bus and line real and reactive power in both directions are measured? [4]

(b) In a state estimation system, '**z**' is the vector of measured variables. The vector of state variables is '**x**'. '**e**' is the error involved with the measurement and the error is assumed to have Gaussian random variation with zero expectation. Using the weighted least squares estimate approach, establish that

$$\mathbf{x} = G^{-1} H^{T} W \mathbf{z}, \qquad E(\hat{\mathbf{x}}) = \mathbf{x},$$

where,

$$\mathbf{z} = H\mathbf{x} + \mathbf{e}, \quad G = H^{T} W H$$

$W = diag(w_1, w_2, \ldots w_n)$ weighting factors w_i

[10]

(c) The simple DC network in Fig. 5.1 shows measurements (**z**) and unknown (**x**) where,

$$\mathbf{x} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} I_1 \\ I_2 \\ V_m \end{bmatrix}$$

The weight matrix W is diag (1000,1000,1000). Evaluate H, and G. [6]

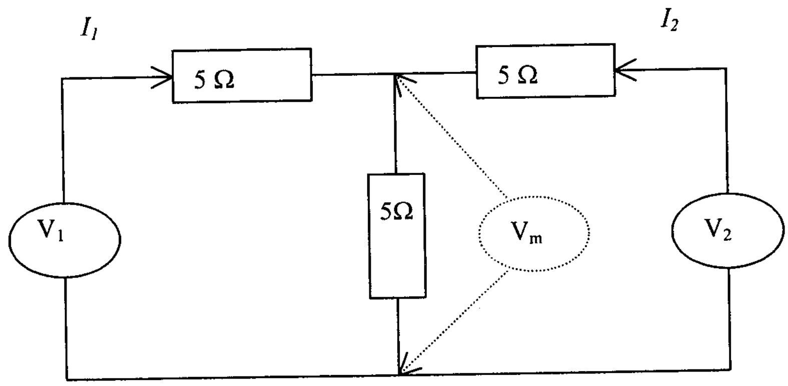

Fig 5.1: A simple DC power network

6. (a) What is Optimal Power Flow (OPF)? [3]

(b) Describe the difference between the control (u) and state variables (x) in the context of OPF. [3]

(c) For the three-bus power system shown in Fig. 6.1, the following are known:

$$x_{12} = 0.05\ pu\ , x_{13} = x_{23} = 0.1\ pu$$
$$\theta_1 = 0.0, P_3 = 3.0\ pu\ , |V_1| = |V_2| = |V_3| = 1.0\ p.u$$
$$F_1(P_1), F_2(P_2)$$

where x_{ij} is the reactance of line connecting bus 'i' and 'j', θ_i and $|V_i|$ are the angle and magnitude of the voltage at bus 'i', $F_1(P_1)$ and $F_2(P_2)$ are the generator fuel cost characteristic functions for units 1 and 2 respectively.

Using the concept of DC load flow, establish the Lagrange function L involving the objective function $f(\mathbf{x,u,p})$ and the constraints $\mathbf{g}(\mathbf{x,u,p})$. Identify the control (u), state (x) and fixed (p) parameters. Obtain the relevant expressions for $\frac{\partial f}{\partial \mathbf{x}}, \frac{\partial f}{\partial \mathbf{u}}, \frac{\partial \mathbf{g}}{\partial \mathbf{x}}, \frac{\partial \mathbf{g}}{\partial \mathbf{u}}$.

Show that:

$\nabla L_{\mathbf{u}} = 0 \Rightarrow \frac{\partial F(P_1)}{\partial P_1} = \frac{\partial F(P_2)}{\partial P_2}$ within the limit of the power output of the generators.

[14]

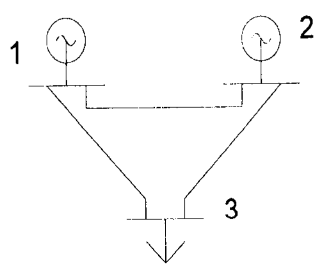

Fig. 6.1: 3-bus simple power system

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Model Answers and Mark Schemes First Examiner: D. POPOVIC (Ques 1, 2, 3)

Paper Code: E4.39 Second Examiner: B. PAL (Ques 1, 2, 3)

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1 a) Production of primary fuels:

petroleum dominates (48%), followed by natural gas (37%) coal (6%) and primary electricity (nuclear & hydro 7%). 2
Renewables & waste account for the remaining production.

Energy consumption

Since 1970, consumption of natural gas & primary electricity has risen considerably, whilst consumption of oil & coal has fallen. 2
Total final energy consumption 160 MToe, primarily in transport (significant rise comparing to 1970).

Electricity

Product.: Gas dominates (39%), but in 2000 output from coal-fired stations has risen by 13%. Nuclear ≈ 21%.

Consumption: Over the last 5 years has grown by ≈ 10%.
CHP – growth continues (in 2000, an increase in capacity of 9%) also, 6% of the total electricity generated in UK came from CHP schemes). 2

Nuclear: record output in 1998; in 2000 Provides ≈ 25% of the total volume of electricity generated in UK

Use of Renewable Energy sources

Biofuels account for 82% of renewable energy sources. Most of the remainder comes from large scale hydro. Hydro accounts for 15% and 2.5%.
75% of renewable energy is used to generate electricity, remaining 25% to generate heat. 2
Renewable energy use grew by 8% in 2000, and has doubled in the last 7 years.
Renewable electricity accounted for 2.8% of electricity generated in the UK in 2000.

Source: WWW2.dti.gov.uk/energy

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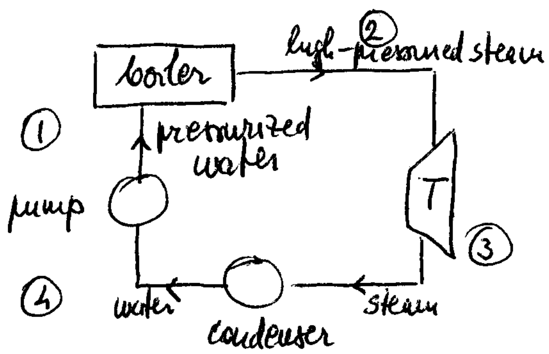
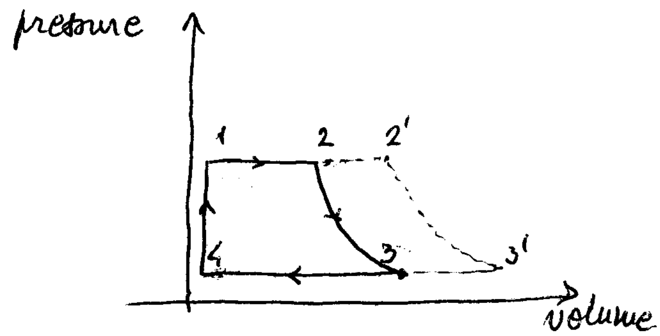
1 b) Steam turbine converts thermal energy into mechanical by steam expansion when it moves through turbine blades.

Usually operates in closed thermodynamic cycle (Rankine cycle) (working substance is constantly recycled without any exchange of its mass)





1

① water under pressure fed into boiler → changes to steam

② boiler output : high-pres., high-temp. steam → fed to turbine → steam expands through turbine → turbine rotates → useful work output

③ temp & pressure drop from expansion steam exits turbine → fed into condenser (exhaust from turbine is cooled to form condensate by the passage through the condenser of large quantities of water (or use cooling towers) 2

But, no exchange of mass from working fluid to cooling water.

④ working fluid turns to water in condenser (approx. atmospheric pressure)
Pump raises water pressure to that in boiler → cycle repeats again from point 1

①-② : heat input path ; constant-pressure ; liquid to vapour
③-④ : exhaust path ; constant pressure
④-① : compression ; at constant volume through the action of pump

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1 b) (cont)

Continually adding heat beyond point where water turns to steam (via super-heater coils in boiler) ⇒ heat input path is extended ⇒ (2')-superheated state 1

(2)-(2') : superheated extension at constant pressure

Higher temp. of steam at 2' ⇒ increase in thermal efficiency

Further increase in thermal efficiency is possible through reheating (some steam is diverted back to boiler - reheated and then sent back to turbine) 1

Even higher efficiency improvements, eg. combined-cycle and cogeneration. 1

c) (i) $P = \rho g W H = 1000 \cdot 9.81\, WH\ [W] = 9.81\, WH\, [kW]$

$= 9.81 \cdot 1370 \cdot 324 = 4354.5\ MW$ 3

(ii) A drop of 1 m in the water level corresponds to $6400 \cdot 10^6\ m^3$ of water.

$W = 1370\ m^3/s$

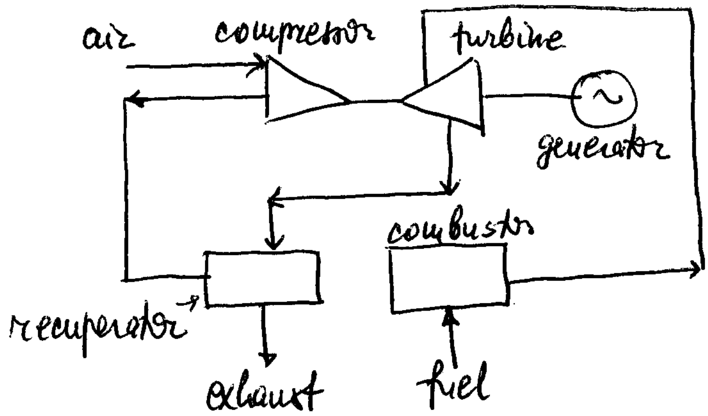
Time for all this water to flow through the turbines is 3

$$t = \frac{6400 \cdot 10^6}{1370} = 4.67 \cdot 10^6\ sec \simeq 54\ days$$

20

2. a) Microturbines

Small gas turbines with a few moving parts (typically only one rotating shaft)



2

Size: 30-75 kW; modular 1
Suitable for customers with loads ≥ 30kW (commercial & industrial)

For sizes ≤ 30kW, losses do not decrease as rapidly as output, so efficiency falls significantly + component costs do not decrease as rapidly as size decreases ⇒ cost per kW increases (⇒ not suitable for small residential use) 2

Installed at \$750 - 1000/kW (target is \$350/kW)
Microturbine of 75kW: $\eta = 30\%$, speed 90000-100000 rpm; approx. 40000 hours of reliable operation 1

Strong points: high reliability, capable of unattended operation at a customer side; heat recovery can be incorporated 1

Needs: - increase efficiency (single-stage compressor & uncooled turbine blades are the largest limitations)
- reduce cost (power electronics, compressor) 1
- further development of equipment that uses waste heat to provide heating & cooling

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2. b) $P_{wind} = \frac{1}{2} \rho A V^3$ [W]

$= \frac{1}{2} \cdot 1.201 \; \pi \left(\frac{20}{2}\right)^2 \cdot 27.8^3$

$= 4053 \cdot 10^3$ [W] 2

$P_{generated} = 0.45 \cdot 4053 = 1823$ kW 1

No of wind generators for 600 MW

$\frac{600 \cdot 10^3}{1823} = 330$ 1

↑ quite a large no

Assumption: the wind is always blowing! 1

c) Base power stations deliver full power at all times. 1
For example, nuclear stations and coal-fired stations.

Peak-load generating stations deliver power for brief intervals during the day (to supply short-term peak power). Must be put into service very quickly. They are equipped with prime movers such as diesel engines, gas turbines, or pumped-storage turbines that can be started up in a few minutes. 2

d) (i) 60 MW base + 50 MW diesel 2

(ii) 80 MW base + 30 MW pumped-storage 2

20

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3. a) NordPool - a successful example of electricity deregulation - primarily because market power has not been an issue (as it was in the E&W Pool). Most liberal electricity market.

Market participants: generators, utilities with distrib/gen. capacity, end user, SOs 1

Market operator: NordPool (responsible for market clearing, accounting and invoicing) 1

Energy markets consist of:

1) the short-term spot market and regulating market

Spot market: day-ahead. NordPool accepts generator offers and demand bids for each hour of the following day. System clearing price = EP (where aggregate demand curve meets the aggregate supply curve

ex ante pricing

+

Regulating market: compensates power imbalances. Generators can submit buyback bids after the day-ahead market trading is finished. In real-time operation, SO selects the cheapest generator (regulator) from a merit list. 3

ex post mechanism

↓

final clearing price

Problem/area of concern: selection of regulating bids using merit order does not necessarily result in the lowest cost to alleviate congestion.

2) the longer-term futures and options financial markets weekly time resolution; settled against uncongested spot market clearing price

3) bilateral contract markets - very significant (90%)

NordPool does not include a centralized scheduling, pricing and dispatch; instead scheduling is responsibility of individual generating companies on the basis of individual profit maximization. (⇒ closer to a real free market) 1

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3 a) (cont)

There are 3 different SOs (each associated with a national grid company - Norway, Sweden and Finland). Also, different but coordinated solutions to tariffs, congestion mangement etc. 1

Problems: different transm. tariffs can give a competitive adv. to generators in one country compared to another. Establishment of ISO (an institution independent of the transm. owners) has been discussed.

Congestion management: (sharing of congestion cost in ex ante market through separation of price areas)

market participants who help alleviate congestion are credited the congestion charge, those who worsen congestion are debited the charge. The charge called Capacity Fee is calculated according to supply and demand characteristics of the areas affected by the bottleneck. 1

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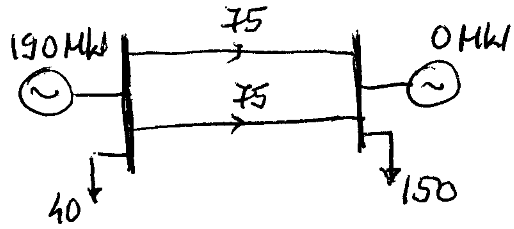
3 b) (i) Without line constraints: Cheaper generator will supply all load.

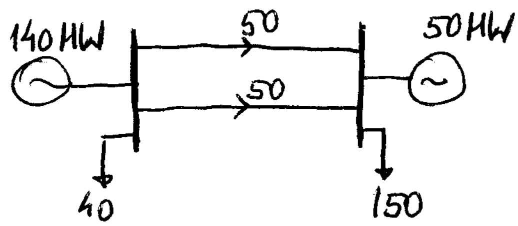


EP: 6.8 [£/MWh], 190 MW 2

SMP: 6.8 [£/MWh] (= 3 + 0.02 · 190)

G1 is marginal generator

(ii) With 100 MW transfer limit: re-dispatch



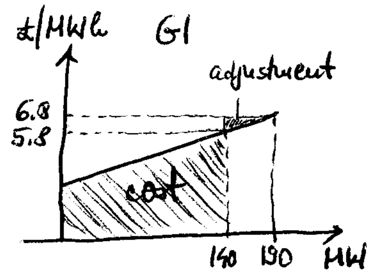
2

(iii) $P_{G1} = 140$ MW (constrained case)

price $= 3 + 0.02 \cdot 140 = 5.8$ [£/MWh]

PPP = SMP = 6.8 1

Generator G1 adjustment: $\frac{50\,(6.8 - 5.8)}{2} = 25$ [£/h]

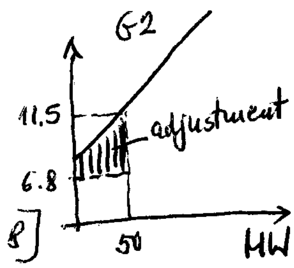


$P_{G2} = 50$ MW (constrained case)

MP $= 8 + 0.07 \cdot 50 = 11.5$ [£/MWh]

Adjustment: area under MC(G2) above [£/MWh 6.8]

$50\,(8 - 6.8) + \frac{1}{2}\, 50\,(11.5 - 8) = 147.5$ [£/h] 1



Generating costs: 1

G1: $140 \cdot 3 + \frac{1}{2}(5.8 - 3) \cdot 140 = 616$ [£/h]

G2: $50 \cdot 6.8$ + adjustment for G2 $= 340 + 147.5 = 487.5$ [£/h] 1

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3 b) (cont)

Generating payments from Pool:

G1: $140 \cdot 6.8 = 952$ [£/h] uniform price 1

G2: $50 \cdot 6.8 = 340$ [£/h] 1

Total payment = gener. payments + adjustments

G1 + G2: $(952+340) + (\underbrace{25+147.5}_{\text{total uplift}}) = 1464.5$ [£/h]

Demand payments ie. PSP = PPP + Uplift) paid by L1, L2

L1: $40 \cdot 6.8 + (\underbrace{\text{total uplift}}_{25+147.5})\frac{40}{190} = 308.3$ [£/h] 1

L2: $150 \cdot 6.8 + (25+147.5)\frac{150}{190} = 1156.2$ [£/h] 1

Total charge = 308.3 + 1156.2 = 1464.5 [£/h] 20

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Model answers (Question 4, 5 and 6)
First Examiner : Dr. Bikash Pal
Second Examiner Dr. D. Popovic

4 (a): Various benefits of FACTS technology include the following:

- Control of power flow as ordered and suit to follow a contract. This is very important from the present day operating scenario of the power system. Contracts are agreed between the suppliers (generation companies) and the distribution companies based on the bidding on the market. This does not take care of transmission constraint that is the responsibility of the transmission operators. FACTS can ensure contracted service by properly controlling power flow in the lines.
- Increase the loading capability of lines to their thermal capabilities including short term and seasonal
- Increase the system security through raising large and small signal stability margin and limiting short circuit current.
- Provide secure tie line connections to neighbouring utilities and reasons thereby decreasing overall generation reserve requirements on both sides. Phase angle regulator at times become handy to create power flow conditions amongst neighbouring utilities through tie lines.
- Providing greater flexibility in siting new generation
- Upgrade of lines
- Reduce reactive power flows thus allowing lines to carry more active power. FACTS devices can generate reactive power to supply local requirement thus eliminating the need of var generation by the generators
- Reduce loop flows
- Increased utilisation of lowest cost generation [4]

(b) Thyristor based controllers such as TCSC and SVC can introduce variable series capacitance and shunt capacitance respectively within the control range. The reactive power compensation is dependent on line current and bus voltage respectively for TCSC and SVC. At high current and low voltage network conditions, they are not very useful; rather they are taken out of service. The convertor based facts controllers operates on the principle of synchronous voltage source and hence their actions are virtually independent of the network operating condition. They can be used during contingency conditions also with proper control. [2]

(c) A→C; B→B; C→F; D→A; E→D; F→E [6]

(d) The *St. Clairs Curve* provides loadability limits of lines at various transmission distances. This is very useful to planner and system operators. These curves are drawn on the basis single conductor line with 60 Hz operating frequency. For 50 Hz, a correction factor of 1.2 is to be multiplied. For bundle conductors the actual loadability would be little higher than the value produced by the curve as the surge impedance loading tends to increase because of increased surge impedance. [1]

Various factors that affect the line loadability are thermal, voltage regulation, and stability. For short lines of length less than 50 miles thermal characteristic is the limitation. This is restricted by sag and minimum ground clearance due to expansion of the conductors at high current and also irreversible change in mechanical strength at annealing temperature. Approximately a line of 50 miles can be loaded to 3 times the surge impedance loading of the lines. The thermal loading is not fixed and is dependent on atmospheric condition such ambient temperature, wind velocity, condition of the conductors and recent loading history. Accordingly the transfer capability varies and there can be short-term (of the order of few minutes) overload capacity and normal load capacity.

For lines of length between 50 to 200 miles, the reactive power generation and absorption capacity of the line dictates the amount of loading. Reactive power management of the line is the important issue and maintaining an acceptable voltage profile is important. At higher loading with no shunt compensation voltage at the receiving end tends to be unacceptably low. The problem can only be overcome with controlled compensation.

For long distance transmission (length beyond 200 miles), the relative angular separation introduced by the equivalent machines at both end is higher. The distributed inductance and shunt capacitance of the line introduces 11-12 degree phase shift per 100 miles of transmission length. This means longer the line length, higher is the angular shift introduced between sending and receiving end voltage and hence lesser is the relative angular separation available for power transfer. It is not possible to load line even close to its SIL values for line longer than 300 miles. Compensation is necessary to enhance loadability of the line. [4]

(e) TCSC is a thyristor based controller series compensation device. It can be realised as part fixed and part controllable through thyristor. Some time it can be part fixed capacitor and thyristor controlled reactor and capacitor type. The commutation is natural. SSSC is voltage sourced based converters. The voltage is a balanced and of system frequency. The magnitude is controllable. Forced commutation (GTO) is used. It is placed in series with the line.

The amount voltage drop across a TCSC is function of line current and hence the reactive power generated by TCSC is function of line current. The power transfer is increased by a fixed percentage of the value of the uncompensated line for the same angle. The injection of voltage of SSSC is independent and controllable and does not depend on line flow but it can be operated in constant impedance mode if required. The angle of the voltage to line current is fixed either at +90 degree or –90 degree. When at +90 degree it can behave as controllable inductor as opposed to capacitor at –90 degree. So compensation in both directions is possible. The amount of power transferred is increased by a fixed percentage of the maximum power transfer value of the uncompensated line. The short term overloading capability is very useful during high current condition and comes as a very useful benefit in first swing stability performance of the network. At fault, the voltage developed across the series capacitor is dangerously high and can be damaging to the costly thyristor switches if over-voltage protection arrangement does not act. The TCSC cannot be useful during contingency (faults) conditions. [3]

5 (a) The ratio between the number of measured variables to number of state variables or unknown variables to be estimated is known as redundancy factor in the context of power system state estimation.
Angle at one of the N buses has to be taken as reference for all other angles and magnitudes. This means in an N-bus system number of state variables to be estimated is *2N-1*. If $|V_i|, P_i, Q_i$ are measured at every bus and the flows $P_{ij}, P_{ji}, Q_{ij}, Q_{ji}$ are measured in every line, the measurement set is full and the number of measurement $N_m = 3N + 4B$ rows, where B is the number of branches. The redundancy factor is $\frac{3N + 4B}{2N - 1}$ or approximately 4.5 in a large fully monitored power system with an average of 1.5 branches per bus. [4]

(b) Lets take a system with 'm' measurement and n state (unknown) variables.
The error

$$\mathbf{e} = \mathbf{z} - \mathbf{z}_{\mathbf{true}} = \mathbf{z} - \mathbf{Hx} \quad (4b.1)$$

The elements of '**H**' come from circuit parameters.
The true values $x_1,..x_n$ can never be determined through measurement. We can determine the best estimate $\hat{x}_1,..\hat{x}_n$ as we shall see soon. The errors are also estimated ones, having the relationship:

$$\hat{\mathbf{e}} = \mathbf{z} - \mathbf{H}\hat{\mathbf{x}} \quad (4b.2)$$

Quantities with hats are denoted as estimate of those without hats.

Equation (4b.2) can be manipulated to

$$\hat{\mathbf{e}} = \mathbf{z} - \hat{\mathbf{z}} = \mathbf{e} - \mathbf{H}(\hat{\mathbf{x}} - \mathbf{x}) \quad (4b.3)$$

In least square weighted estimate, the objective function is:

$$f = \sum_{j=1}^{n} w_j e_j^{\,2} = w_1 e_1^2 + w_2 e_2^2 + \ldots + w_{m-1} e_{m-1}^{\,2} + w_m e_m^2 \quad \text{(4b.4)}$$

The best estimates of the state variables are those values that cause the objective function f to take on its minimum value. According to the usual necessary conditions for minimising f, the estimates $\hat{x}_1,\ldots\hat{x}_n$ are those values of $x_1 \ldots x_n$ which satisfy the equations:

$$\left.\frac{\partial f}{\partial x_1}\right|_{\hat{\mathbf{x}}} = 2\left.\left[w_1 e_1 \frac{\partial e_1}{\partial x_1} + w_2 e_2 \frac{\partial e_2}{\partial x_1} + \ldots + w_{m-1} e_{m-1} \frac{\partial e_{m-1}}{\partial x_1} + w_m e_m \frac{\partial e_m}{\partial x_1} \right]\right|_{\hat{\mathbf{x}}} = 0$$

...

$$\left.\frac{\partial f}{\partial x_k}\right|_{\hat{\mathbf{x}}} = 2\left.\left[w_1 e_1 \frac{\partial e_1}{\partial x_k} + w_2 e_2 \frac{\partial e_2}{\partial x_k} + \ldots + w_{m-1} e_{m-1} \frac{\partial e_{m-1}}{\partial x_k} + w_m e_m \frac{\partial e_m}{\partial x_k} \right]\right|_{\hat{\mathbf{x}}} = 0$$

...

$$\left.\frac{\partial f}{\partial x_n}\right|_{\hat{\mathbf{x}}} = 2\left.\left[w_1 e_1 \frac{\partial e_1}{\partial x_n} + w_2 e_2 \frac{\partial e_2}{\partial x_n} + \ldots + w_{m-1} e_{m-1} \frac{\partial e_{m-1}}{\partial x_n} + w_m e_m \frac{\partial e_m}{\partial x_n} \right]\right|_{\hat{\mathbf{x}}} = 0$$

(4b.5)

The true values are never known, the unknown actual error can then be replaced by the estimated error. In vector matrix form, the set of equations in (4b.5) become

$$[\mathbf{H}]^T [\mathbf{W}][\hat{\mathbf{e}}] = [\mathbf{0}] \quad \text{(4b.6)}$$

The equation (4b.6) can be expressed with the help of $\hat{\mathbf{e}} = \mathbf{z} - H\hat{\mathbf{x}}$

$$\mathbf{H}^T \mathbf{W}\, \hat{\mathbf{e}} = \mathbf{H}^T \mathbf{W}\, (\mathbf{z} - \mathbf{H}\,\hat{\mathbf{x}}) = \mathbf{0} \quad \text{(4b.7)}$$

This can be expressed as: $\hat{\mathbf{x}} = \mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}\mathbf{z}$

Where $\mathbf{G} = \mathbf{H}^T\mathbf{W}\mathbf{H}$ [7]

An expression for the difference

$$\hat{\mathbf{x}} = \mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}\,(\mathbf{H}\mathbf{x} + \mathbf{e})$$
$$\hat{\mathbf{x}} = \mathbf{G}^{-1}\mathbf{G}\mathbf{x} + \mathbf{G}^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{e} \quad \text{(4b.8)}$$
$$\hat{\mathbf{x}} - \mathbf{x} = \mathbf{G}^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{e}$$

Taking expectation on both side in (4b.8)

$$E(\hat{\mathbf{x}} - \mathbf{x}) = E(\mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}\mathbf{e})$$
$$E(\hat{\mathbf{x}}) - \mathbf{x} = \mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}\,E(\mathbf{e}) = \mathbf{0}$$
$$E(\hat{\mathbf{x}}) = \mathbf{x}$$

The standard relation of $E(ax+b) = aE(x) + b$ is made use of where a, b are constant. The error being random gaussian variable with zero expectations, $E(\mathbf{e}) = 0$

Hence, the expected value of the estimate is the true value of the unknown [3]

(c) The Kirchhoff's voltage law can be applied to establish relationship between the measurement and the unknown with error for the system shown in Fig. 5.1. Leaving the error aside, assuming true measurement and true value of unknown, the following equation can be written:

$$
\begin{aligned}
5I_1 + V_m &= V_1 \\
5I_2 + V_m &= V_2 \qquad (4c.1) \\
V_m &= 5(I_1 + I_2)
\end{aligned}
$$

Manipulating and expressing in matrix form

$$
\begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}
$$

$$
\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{2}{15} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (4c.2)
$$

Expressing V_m as function of V_1, V_2, one obtains,

$$
V_m = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (4c.3)
$$

Augmenting this to (4c.2) and replacing $\mathbf{z} = \begin{bmatrix} I_1 & I_2 & V_m \end{bmatrix}^T, \mathbf{x} = \begin{bmatrix} V_1 & V_2 \end{bmatrix}^T$

$$
\mathbf{z} = \begin{bmatrix} \frac{2}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{2}{15} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \mathbf{x} \Rightarrow \mathbf{z} = \mathbf{H}\mathbf{x} \qquad (4c.4)
$$

$$
W = \begin{bmatrix} 1000 & & \\ & 1000 & \\ & & 1000 \end{bmatrix}, \ \mathbf{G} = \mathbf{H}^T \mathbf{W} \mathbf{H} = \begin{bmatrix} 133.33 & 93.33 \\ 93.33 & 133.33 \end{bmatrix} \qquad [6]
$$

6 (a) In economic dispatch, we formulate a single function that holds total generation equals total load plus losses. This indirectly says about the generation must obey the power flow constraint. The formulation of the economic despatch as minimising generation cost as objective function with power flow equation as constraints is known as OPF. The objective of OPF need not be restricted to minimising generation costs, rather can be one different from this such as minimising transmissions losses, minimum shift of generations and transformer tap movements from optimum operating point. In view of deregulated nature of power system in recent days it can even include minimising congestion in the system or maximising available transfer capability (ATC). Irrespective of the objective functions the OPF must produce a solution that would ensure entire set of power flow constraints at the solution point. This makes economic despatch calculation as an OPF. [3]

The generator power output (other than reference generator) , generator voltage, transformer tap position and phase shifter phase angle, switched capacitor setting, reactive power injection from a static var system, DC line flow etc. are control variables in OPF. Generator phase angle (other than reference generator), load bus voltage magnitude and angles are state variables. [3]

(b) In respect of sample power system in Fig 6.1,

The Lagrange function is

$$
L = f(\mathbf{x}, \mathbf{u}, \mathbf{p}) + \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{p})
$$

Where $f(\mathbf{x}, \mathbf{u}, \mathbf{p}) = F_2(P_2) + F_1(P_1(|V_1|, |V_2|, |V_3|, \theta_1, \theta_2, \theta_3))$;

$\boldsymbol{\lambda}^T = [\lambda_1 \quad \lambda_2]$

$$
\mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{p}) = \begin{bmatrix} P_2(|V_1|, |V_2|, |V_3|, \theta_1, \theta_2, \theta_3) - P_2 \\ P_3(|V_1|, |V_2|, |V_3|, \theta_1, \theta_2, \theta_3) - P_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

$$P_1(|V_1|,|V_2|,|V_3|,\theta_1,\theta_2,\theta_3) = -20\,\theta_2 - 10\,\theta_3$$

$$\mathbf{g}(\mathbf{x},\mathbf{u},\mathbf{p}) = \begin{bmatrix} 30\,\theta_2 - 10\,\theta_3 - P_2 \\ -10\,\theta_2 + 20\,\theta_3 - P_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

[4]

In this formulation,
$\mathbf{u} = P_2, \mathbf{x} = [\theta_2, \theta_3]^T, \mathbf{p} = [|V_1|,|V_2|,|V_3|,\theta_1,\theta_2,\theta_3,P_3]^T$ are control, state and fixed parameters respectively. [1]

$$\frac{\partial f}{\partial \mathbf{x}} = -\begin{bmatrix} 20 \\ 10 \end{bmatrix} \frac{\partial F_1}{\partial P_1};\ \frac{\partial f}{\partial u} = \frac{\partial F_2}{\partial P_2};$$

$$\frac{\partial \mathbf{g}}{\partial \mathbf{x}} = \begin{bmatrix} 30 & -10 \\ -10 & 20 \end{bmatrix},\ \frac{\partial \mathbf{g}}{\partial \mathbf{u}} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$ [3]

$$\nabla L_{\mathbf{u}} = \frac{\partial f}{\partial \mathbf{u}} - \left(\frac{\partial \mathbf{g}}{\partial \mathbf{u}}\right)^T \left\{\left(\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right)^T\right\}^{-1} \left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)$$

Substituting the value in the expression for ∇L_u and simplification would lead to $\nabla L_{\mathbf{u}} = \frac{\partial F(P_1)}{\partial P_1} - \frac{\partial F(P_2)}{\partial P_2}$ and equating to zero yields $\frac{\partial F(P_1)}{\partial P_1} = \frac{\partial F(P_2)}{\partial P_2}$ [6]