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Examiner: Dr A. Manikas	**Paper: Communication Systems**

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[E303/ISE3.3]

DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2003

EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI

SOLUTIONS 2003
COMMUNICATION SYSTEMS

ANSWER to Q1

1) **A** **B** **C** **D** **E**

2) **A** **B** **C** **D** **E**

3) **A** **B** **C** **D** **E**

4) **A** **B** **C** **D** **E**

5) **A** **B** **C** **D** **E**

6) **A** **B** **C** **D** **E**

7) **A** **B** **C** **D** **E**

8) **A** **B** **C** **D** **E**

9) **A** **B** **C** **D** **E**

10) **A** **B** **C** **D** **E**

11) **A** **B** **C** **D** **E**

12) **A** **B** **C** **D** **E**

13) **A** **B** **C** **D** **E**

14) **A** **B** **C** **D** **E**

15) **A** **B** **C** **D** **E**

16) **A** **B** **C** **D** **E**

17) **A** **B** **C** **D** **E**

18) **A** **B** **C** **D** **E**

19) **A** **B** **C** **D** **E**

20) **A** **B** **C** **D** **E**

ANSWER to Q2

a) $d(kT_s)$ is a Gaussian signal with mean μ_d and std σ_d i.e. $\text{pdf}_d = \text{N}(\mu_d, \sigma_d)$

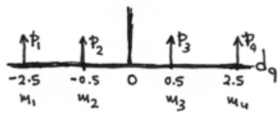
mean $= \mu_d = \mathcal{E}\{d(kT_s)\} = \mathcal{E}\{g(kT_s) - g((k-3)T_s)\} = 0 - 0 = 0$
$std = \sigma_d = \sqrt{P_d}$

$$T_s = \frac{1}{12\times 10^3}$$

$$\begin{aligned}
P_d &= \mathcal{E}\{d^2(kT_s)\} \\
&= \mathcal{E}\{(g(kT_s) - g((k-3)T_s))^2\} \\
&= \underbrace{\mathcal{E}\{g^2(kT_s)\}}_{=R_{gg}(0)} + \underbrace{\mathcal{E}\{g^2((k-3)T_s)\}}_{=R_{gg}(0)} - 2\,\underbrace{\mathcal{E}\{g(kT_s).g((k-3)T_s)\}}_{=R_{gg}(3T_s)} \\
&= 2\,R_{gg}(0) \quad - 2\,R_{gg}(3T_s) \\
&= 2\exp\{-6000\times 0\} - 2\exp\{-6000\times 3\times \tfrac{1}{12\times 10^3}\} \\
&= 2 - 2\exp\{-1.5\} = 1.5537
\end{aligned}$$

b) $\text{pdf}_d = \text{N}(0, \sqrt{1.5537}) = \text{N}(0, 1.2465)$

pdf of $d_q(kT_s)$:



$$p_1 = \int_{-\infty}^{-1.2465} \text{pdf}_d(\text{d})\text{dd} = \text{T}\{\tfrac{1.2465}{1.2465}\} = \text{T}\{1\} = 0.16$$

$p_2 = 0.5 - p_1 = 0.34$

$p_3 = p_2 = 0.34$

$p_4 = p_1 = 0.16$

c)

m_2	0.34	0.34	0.66	1
m_3	0.34	0.34	0.34	
m_1	0.16	0.32		
m_4	0.16			

i.e.

Source Coder	
$m_1 \mapsto 010$	$l_1 = 3$
$m_2 \mapsto 1$	$l_2 = 1$
$m_3 \mapsto 00$	$l_3 = 2$
$m_4 \mapsto 011$	$l_4 = 3$

d)

$$\underline{p} = \begin{bmatrix} p_1 = 0.16 \\ p_2 = 0.34 \\ p_3 = 0.34 \\ p_4 = 0.16 \end{bmatrix} \underline{l} = \begin{bmatrix} l_1 = 3 \\ l_2 = 1 \\ l_3 = 2 \\ l_4 = 3 \end{bmatrix}$$

$$\overline{\ell} \;= \underline{p}^T\underline{l}. = \sum_{i=1}^{4} p_i l_i = 0.16\times 3 + 0.34\times 1 + 0.34\times 2 + 0.16\times 3 = 1.98\,\frac{\text{bits}}{\text{level}}$$

$r_{\text{data}} = \overline{\ell}\; \times 12k = 1.98\times 12k = 23.7$ kbits/s

$r_{\text{inf}} = \;\mathbb{H}\times 12k = \underbrace{\underline{p}^T\log_2(\underline{p})}_{\mathbb{H}=\text{enropy}=1.9044} \quad \times 12k = 22.8526$ kbits/s

ANSWER to Q3

a) Transmitter:

I/P A-law encoder: $g(kT_s) = -3.7\,V$

maximum input: $g_{max} = 5V$

$$\boxed{\begin{array}{l} \frac{1}{A} = 0.0114 \\ \|x\| = \left\| \frac{g(kT_s)}{g_{max}} \right\| = 0.74 \end{array}} \Rightarrow \frac{1}{A} < x < 1$$

Therefore, O/P of A-law encoder: $g_c(kT_s) = \frac{1+\ln(A\|x\|)}{1+\ln(A)} \times g_{max}$

$$\Rightarrow g_c(kT_s) = -4.72496V$$

$$\Rightarrow b_0 < -4.7249V < b_1$$

Therefore, O/P of quantizer $= m_1 = -4.37V$

Receiver:

I/P A-law decoder: $m_1 = -4.37\,V$ (or $m_1 = -4.375$)

O/P: $g_{\text{out}}(kT_s) = \frac{1}{A}\exp\left\{\frac{m_1}{g_{max}}(1+\ln(A)) - 1\right\} \times g_{max}$

$$\Rightarrow g_{out}(kT_s)[= -2.509V \;\; (or\; -2.5227)$$

$$n_q = -3.7\,V - (-2.509V) = 1.1910V \;\; (or\; -3.7 + 2.5227 = 1.1773)$$

b)

$Q = 8$
$\gamma = \log_2 Q = \log_2 8 = 3$

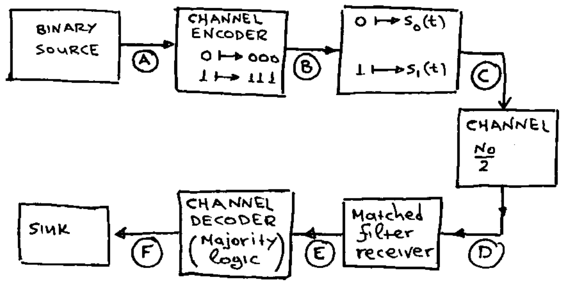
$\text{SNR}_{\text{out}} = 4.77 + 6\gamma - 20\log_{10}(1+\ln A) = 8.0058\,\text{dB}$

c)

$r_b = \gamma\, F_s = 3 \times 18k = 54\; k$bits/s (or 54/8 = 6.75 kBytes/sec)

$6.75\,k\frac{\text{Bytes}}{\text{sec}} \times t = 2\text{GB} \Rightarrow t = \frac{2\times10^9\,\text{Bytes}}{6.75\times10^3\,\text{Bytes}}\,\text{sec} = \frac{2}{6.75\times3600} \times 10^6$ hours $= 82.3$ hours

ANSWER to Q4



a)
$\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2 \times 10^{-12}$

point A: bit rate $= 166.6667\; k$bits/sec

point B: bit rate $= 3 \times 166.667\;k = 500\;k$bits/sec

point C:
channel symbol rate ((point C) = bit rate (point B)

i.e. $r_{cs} = 500\,k\frac{\text{channel-symbols}}{\text{sec}}$

$\Rightarrow \quad T_{cs} = \frac{1}{r_{cs}} = 2 \times 10^{-6}$sec per channel symbol

b)
point D:

$E_0 = A^2\frac{T_{cs}}{2} \times 2 = A^2 T_{cs}$

$E_1 = A^2\frac{T_{cs}}{4} \times 4 = A^2 T_{cs}$

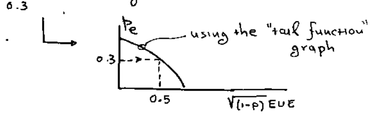
$E_b = \frac{1}{2}(E_0 + E_1) = A^2 T_{cs}$

Therefore
$$\text{EUE} = \frac{E_b}{N_0} = \frac{A^2 T_{cs}}{2\times10^{-12}}$$

Furthermore $\rho = \frac{1}{E_b}\int_0^{T_{cs}} s_0(t)\cdot s_1(t)\,dt = \cdots$

$\Rightarrow \rho = 0$

point E: $\underset{0.3}{p_e} = T\left\{\sqrt{(1-\underset{0}{\rho})\,\text{EUE}}\right\}$



$\therefore \sqrt{\text{EUE}} = 0.5 \Rightarrow \text{EUE} = 0.25$

Therefore

$$\frac{A^2 T_{cs}}{2\times10^{-12}} = 0.25 \Rightarrow A^2 = \frac{0.5\times10^{-12}}{2\times10^{-6}} \Rightarrow A^2 = 0.25\times10^{-6} \Rightarrow \boxed{A = 0.5\,mV}$$

c)
point F:

$$\begin{aligned} \Pr(\text{correct}) &= 1 - \underbrace{\Pr(2\,errors\,in\,a\,3bit\,sequ.)}_{=\binom{3}{2}p_e^2(1-p_e)} - \underbrace{\Pr(3\,errors\,in\,a\,3bit\,sequ.)}_{=\binom{3}{3}p_e^3(1-p_e)^0} \\ &= 1 - 3p_e^2(1-p_e) - p_e^3 \qquad \text{(where } p_e = 0.3) \\ &= 0.784 \end{aligned}$$

d) $\mathbb{F} = \begin{bmatrix} \Pr(D_0|H_0), & \Pr(D_0|H_1) \\ \Pr(D_1|H_0), & \Pr(D_1|H_1) \end{bmatrix} = \begin{bmatrix} 0.784, & 0.216 \\ 0.216, & 0.784 \end{bmatrix}$

e) $\mathbb{J} = \mathbb{F}.\text{diag}(\underline{p}) = \mathbb{F}.\begin{bmatrix} 0.5, & 0 \\ 0, & 0.5 \end{bmatrix} = \begin{bmatrix} \Pr(D_0, H_0) = 0.392, & \Pr(D_0, H_1) = 0.108 \\ \Pr(D_1, H_0) = 0.108, & \Pr(D_1, H_1) = 0.392 \end{bmatrix}$