Paper Number(s): **E4.38**

master copy - August 02

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE PART IV: M.Eng. and ACGI

DRIVE SYSTEMS

Friday, 10 May 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

Time allowed: 3:00 hours

Examiners responsible:

First Marker(s): Green,T.C.

Second Marker(s): Popovic,D.

1. (a) Signals u_1, u_2 and u_3, and transformation matrices T and T_R are specified below. For each of the signals u_1, u_2 and u_3, state whether the signal contains positive, negative or zero sequence components. Describe the expected form of transformed signals when the matrices T and T_R are applied.

[6]

$$u_1 = \begin{bmatrix} U_1 \cos\left(\omega t + \frac{\pi}{4}\right) \\ U_1 \cos\left(\omega t - \frac{2\pi}{3} + \frac{\pi}{4}\right) \\ U_1 \cos\left(\omega t + \frac{2\pi}{3} + \frac{\pi}{4}\right) \end{bmatrix}$$

$$u_2 = \begin{bmatrix} U_0 + U_2 \cos(\omega t) \\ U_0 + U_2 \cos\left(\omega t - \frac{2\pi}{3}\right) \\ U_0 + U_2 \cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$u_3 = \begin{bmatrix} U_1 \cos(\omega t) \\ U_1 \cos\left(\omega t + \frac{2\pi}{3}\right) \\ U_1 \cos\left(\omega t - \frac{2\pi}{3}\right) \end{bmatrix}$$

$$T = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad T_R = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) (i) Write a set of voltage and current equations to describe the circuit in Figure 1.1 and transform them to dq form. [8]

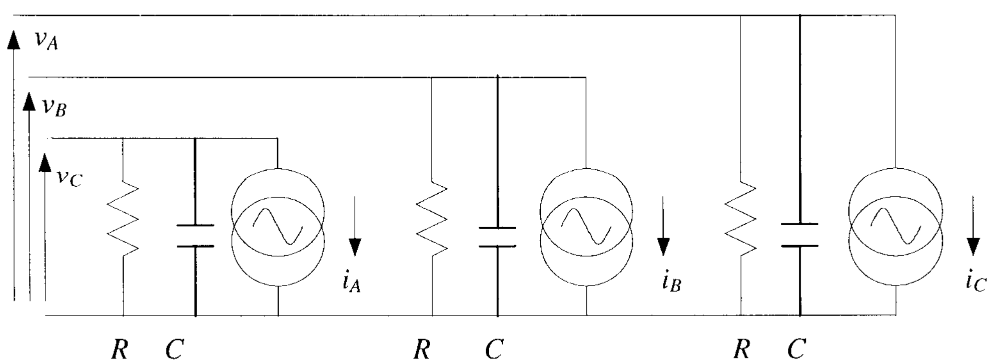

Figure 1.1

(ii) From the transformed equations find an expression for the power associated with each component and describe the type of power concerned. [6]

2. (a) Describe how it is possible that a universal motor can produce useful torque on both a DC and an AC supply. Sketch the graph of torque against speed for such a motor. Describe how the speed of a universal motor may be controlled where the available supply system is AC. [6]

(b) Explain why a single-phase induction machine commonly incorporates an auxiliary winding in addition to its main winding in order to produce starting torque. Outline the connection arrangement of such a winding in a "capacitor start" machine. [6]

(c) Figure 2.1 shows the equivalent circuit of the main winding of a single-phase induction machine in which s_F is the slip with respect to the forward field. The parameters of the machine are:

supply frequency = 50Hz,
number of pole-pairs = 1,
stator resistance, $R_S = 2\ \Omega$,
stator leakage reactance, $X_S = 10\ \Omega$,
magnetising reactance, $X_M = 100\ \Omega$,
referred rotor leakage reactance, $X_R' = 10\ \Omega$,
referred rotor resistance, $R_R' = 3\ \Omega$.

For a certain operating condition, it is found that the stator current is 2 A and the speed is 2950 rev./min. Calculate the mechanical power developed in the forward and backward rotating branches of the circuit. [8]

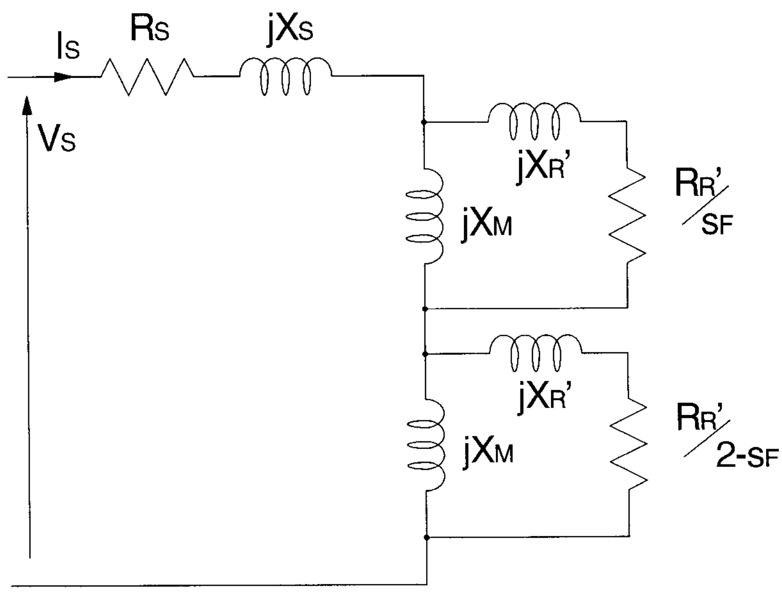

Figure 2.1

3. The voltage equation for a dq model of an induction machine with the rotor leakage inductance referred to the stator side and the rotor short-circuited is:

$$[v'_{DQ}] = [R'_{DQ}][i'_{DQ}] + [G'_{DQ}][i'_{DQ}] + [L'_{DQ}]\frac{d}{dt}[i'_{DQ}]$$

where

$$[v'_{DQ}] = \begin{bmatrix} v_{SD} \\ v_{SQ} \\ 0 \\ 0 \end{bmatrix}, \quad [i_{DQ}] = \begin{bmatrix} i_{SD} \\ i_{SQ} \\ i'_{RD} \\ i'_{RQ} \end{bmatrix},$$

$$[R'_{DQ}] = \begin{bmatrix} R_S & 0 & 0 & 0 \\ 0 & R_S & 0 & 0 \\ 0 & 0 & R_R' & 0 \\ 0 & 0 & 0 & R_R' \end{bmatrix},$$

$$[G'_{DQ}] = \begin{bmatrix} 0 & -\omega L_S & 0 & -\omega M' \\ \omega L_S & 0 & \omega M' & 0 \\ 0 & -P\omega_{slip} M' & 0 & -P\omega_{slip} L'_R \\ P\omega_{slip} M' & 0 & P\omega_{slip} L'_R & 0 \end{bmatrix},$$

$$[L_{DQ}] = \begin{bmatrix} L_S & 0 & M' & 0 \\ 0 & L_S & 0 & M' \\ M' & 0 & L'_R & 0 \\ 0 & M' & 0 & L'_R \end{bmatrix},$$

and

$$i'_{RD} = i_{RD}\left(\frac{L_R}{M}\right) \quad etc.,$$

$$R'_R = R_R\left(\frac{M}{L_R}\right)^2,$$

$$L'_R = L_R\left(\frac{M}{L_R}\right)^2,$$

$$M' = M\left(\frac{M}{L_R}\right).$$

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3. Continued

(i) The rotor leakage inductance in the referred model is defined by $L'_{LR} = L'_R - M'$. Show that this is zero under the referral given here. [2]

(ii) Multiply the voltage equation by the transpose of the current vector to obtain an equation for power. Identify the term that expresses the energy converted to mechanical form and from this show that the torque produced is equivalent to that calculated from the un-referred model:

$$T = PM\left(i_{SQ}\, i_{RD} - i_{SD} i_{RQ}\right)$$ [8]

(iii) Figure 3.1 shows an equivalent circuit of the referred equation using flux linkages such as $\psi'_{RD} = M'\ (i_{SD} + i'_{RD})$. Re-draw the equivalent circuit of Figure 3.1 for a case where orientation at $\psi'_{RQ} = 0$ has been achieved and the stator is supplied from a current source not a voltage source. Explain the significance of orienting the model such that $\psi'_{RQ} = 0$ in terms of the torque equation, the equivalent circuit. and the dynamics governing the relationships between stator current, flux and torque. [10]

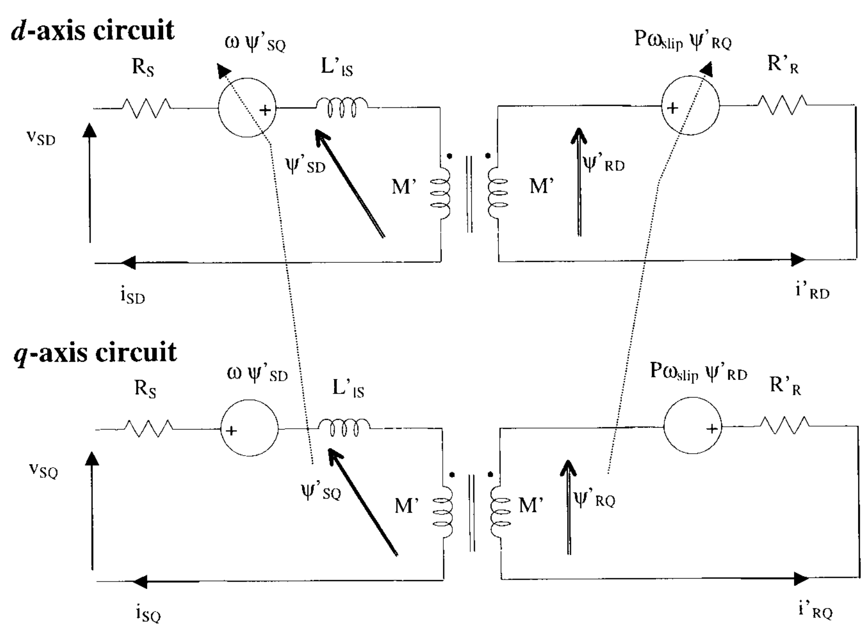


Figure 3.1

4. (a) Outline the steps necessary to converter a 6-coil, stationary-reference-frame model of an induction machine into a form suitable for field orientation. [8]

(b) Figure 4.1 shows a form of field orientation controller.

(i) Explain the difference between a direct and an indirect controller and state which is represented in figure 4.1. [4]

(ii) Given the equations for the d- and q-axis rotor voltages (below), justify the form of the slip calculator shown in Figure 4.1.

d-axis

$$0 = R'_R\, i'_{RD} + M'\frac{d(i_{SD} + i'_{RD})}{dt}$$

q-axis

$$0 = R'_R\, i'_{RQ} + P\omega_{slip} M'(i_{SD} + i'_{RD})$$

[4]

(iii) The machine is shown as current fed. Describe what extra control elements are necessary if a voltage source inverter is to be used. [4]

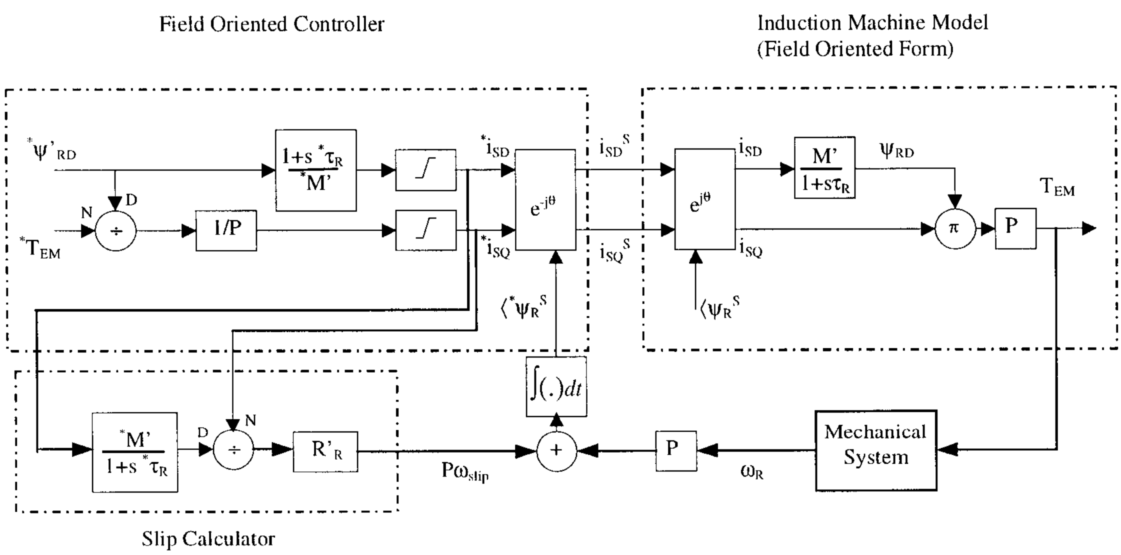


Figure 4.1

5. (a) For a three-phase 6/4-pole variable reluctance motor:

(i) sketch the shapes of the laminations for both stator and rotor and mark where the windings lie for each pole for a series connection. [5]

(ii) determine the step angle for the motor. [3]

(iii) mark the three phase-windings in the sketch produced for (i) with the letters A, B and C. Sketch the shape of the current waveforms required for phases A, B and C so that the rotor in your sketch rotates in the anti-clockwise direction. [4]

(iv) draw a diagram of a circuit suitable for supplying a three-phase machine. [3]

(b) Describe the main advantages and disadvantages of a switched reluctance motor in comparison to an induction machine. Your comments should include (but need not be restricted to) reliability, machine manufacturing costs, total cost of the drive and the instantaneous torque. [5]

6. (a) Justify the following statements through brief explanation.

(i) It is necessary to have many poles in synchronous machines driven by hydro-turbines. [2]

(ii) The rotor of a cylindrical-pole machine is made of solid iron whereas the stator is made of laminated steel. [2]

(iii) A synchronous machine develops torque only at synchronous speed. [2]

(iv) A salient-pole synchronous machine continues to produce power even when its excitation is lost. [2]

(b) A cylindrical-rotor synchronous machine is connected to an infinite bus of $V\angle 0$ volts through a resistance of R_a and synchronous reactance X_s. The excitation voltage is $E\angle\delta$. Develop an expression for real and reactive power delivered by the machine. [6]

(c) A 3-phase, 2-pole, Y-connected cylindrical-pole synchronous generator is connected to a 50 Hz grid system. The rated apparent power of the machine is 247 MVA; the rated voltage is 15.75 kV (line-line) and the per phase synchronous reactance is 1.0 Ω.

(i) Determine the excitation voltage and the power angle when the machine is delivering rated MVA at a power factor of 0.85 lagging. [3]

(ii) The excitation is held constant at the value obtained in part (i) while the prime mover power is slowly increased. Determine the steady-state stability limit and the corresponding values of stator current and power factor under the maximum power transfer condition. [3]

Answers for E4.38 Drive Systems - 2002

master solutions - Aug. 02

Paper set by Drs T C Green and B C Pal.

The second examiner is un~~allocated~~. – D. Popovic

1) (a) For each of the signals u_1, u_2 and u_3, state whether they contain positive, negative or zero sequence components. Describe the expected form of transformed signals when the matrices T and T_R are applied.

[6]

$$u_1 = \begin{bmatrix} U_1\cos\left(\omega t + \frac{\pi}{4}\right) \\ U_1\cos\left(\omega t - \frac{2\pi}{3} + \frac{\pi}{4}\right) \\ U_1\cos\left(\omega t + \frac{2\pi}{3} + \frac{\pi}{4}\right) \end{bmatrix}$$

$$u_2 = \begin{bmatrix} U_0 + U_2\cos(\omega t) \\ U_0 + U_2\cos\left(\omega t - \frac{2\pi}{3}\right) \\ U_0 + U_2\cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$u_3 = \begin{bmatrix} U_1\cos(\omega t) \\ U_1\cos\left(\omega t + -\frac{2\pi}{3}\right) \\ U_1\cos\left(\omega t - \frac{2\pi}{3}\right) \end{bmatrix}$$

$$T = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad T_R = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

***u1** is a positive sequence set at an angle of $\pi/4$.*
When transformed by T, two orthogonal components will result with the third term equal to zero.
When transformed by T_R, the time variation is removed leaving

$$u_{1D} = \sqrt{\tfrac{3}{2}}\,u_1\cos\left(\tfrac{\pi}{4}\right) \quad u_{1Q} = \sqrt{\tfrac{3}{2}}\,u_1\sin\left(\tfrac{\pi}{4}\right) \quad u_{1\gamma} = 0$$

***u2** is a sum of a DC zero sequence set and positive sequence set at an angle of 0.*
When transformed by T, two orthogonal components will result with the third term equal to the zero sequence component.
When transformed by T_R, the time variation is removed and because the positive sequence set is at an angle of zero the quadrature term is equal to zero.

$$u_{2D} = \sqrt{\tfrac{3}{2}}\,u_2 \quad u_{2Q} = 0 \quad u_{1\gamma} = \sqrt{3}\,u_0$$

***u3** is a negative sequence set at an angle of 0.*
When transformed by T, two orthogonal components with negative sequence will result and the third term will be equal to zero.
*When transformed by T_R, the time variation is **not** removed. In fact the rotation of the set is doubled because the transform matrix rotates in the same direction as a negative sequence set.*

$$u_{3D} = \sqrt{\tfrac{3}{2}}\,u_3\cos(2\omega t) \quad u_{3Q} = \sqrt{\tfrac{3}{2}}\,u_3\sin(2\omega t) \quad u_{1\gamma} = 0$$

(b) (i) Write a set of current equations to describe the circuit in figure 1 and transform them to *dq* form.

[8]

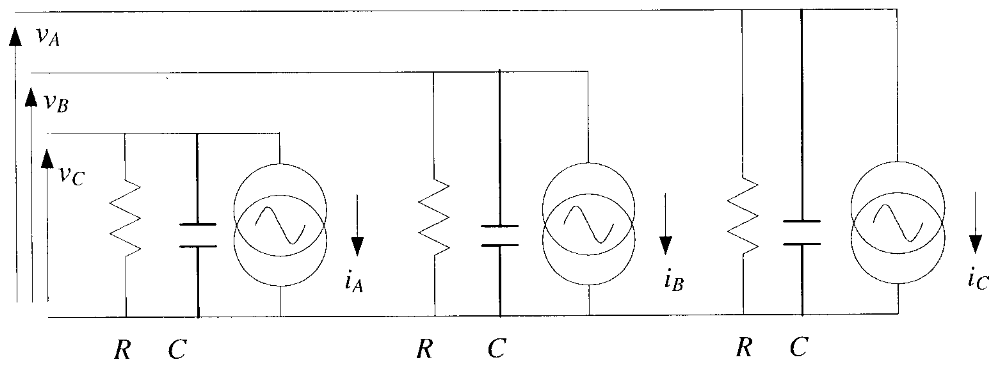


Figure 1

The total input current for each phase is governed by an equation such as:

$$i_{iA} = \frac{1}{R}v_A + C\frac{dv_A}{dt} + i_A$$

Overall we have:

$$i_i = Y_R\, v + C\frac{dv}{dt} + i$$

$$Y_R = \begin{bmatrix} 1/R & 0 & 0 \\ 0 & 1/R & 0 \\ 0 & 0 & 1/R \end{bmatrix} \quad C = \begin{bmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{bmatrix}$$

To transform first to an $\alpha\beta\gamma$-frame we multiply through by T and insert $T\,T^{-1}$ pairs:

$$i_{i\alpha\beta\gamma} = Ti_i = TY_RT^{-1}\; TCT^{-1}\frac{dv}{dt} + Ti$$

$$= Y_R v_{\alpha\beta\gamma} + C\frac{dv_{\alpha\beta\gamma}}{dt} + i_{\alpha\beta\gamma}$$

where diagonal matrices are unchanged by the transform and T is time-invariant and can be moved through derivative operators.

To transform to a $dq\gamma$-frame we multiply through by T_R and insert $T_R\, T_R^{-1}$ pairs. Because T_R is time varying, it must be included in the derivative operation.

$$
\begin{aligned}
i_{idq\gamma} &= T_R i_{i\alpha\beta\gamma} = T_R Y_R T_R^{-1}\, T_R\, v_{\alpha\beta\gamma} + T_R C \frac{d}{dt} i_{L\alpha\beta\gamma} + T_R i_{\alpha\beta\gamma} \\
&= Y_R v_{dq\gamma} + T_R C \frac{d}{dt}\left(T_R^{-1} i_{Ldq\gamma}\right) + i_{dq\gamma} \\
&= Y_R v_{dq\gamma} + T_R C \frac{d}{dt}\left(T_R^{-1}\right) v_{dq\gamma} + T_R C T_R^{-1} \frac{d}{dt} v_{dq\gamma} + i_{dq\gamma} \\
&= Y_R v_{dq\gamma} + \begin{bmatrix} 0 & -\omega C & 0 \\ \omega C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v_{dq\gamma} + C \frac{d}{dt} v_{dq\gamma} + i_{dq\gamma}
\end{aligned}
$$

(ii) From the transformed equations find an expression for the power associated with each component and describe the type of power concerned.

[6]

Power is found by multiplying through by the transpose of the voltage vector:

$$
p = v_{dq\gamma}^T \cdot i_{idq\gamma} = v_{dq\gamma}^T Y_R v_{dq\gamma} + v_{dq\gamma}^T \begin{bmatrix} 0 & -\omega C & 0 \\ \omega C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v_{dq\gamma} + v_{dq\gamma}^T C \frac{d}{dt} v_{dq\gamma} + v_{dq\gamma}^T i_{dq\gamma}
$$

The first term is the V^2/R term expected of a resistor.

$$
v_{dq\gamma}^T Y_R v_{dq\gamma} = \frac{1}{R}\left(v_d^2 + v_q^2 + v_\gamma^2\right)
$$

The second term is the reactive power exchanged with the capacitors and sums to zero.

$$
v_{dq\gamma}^T \begin{bmatrix} 0 & -\omega C & 0 \\ \omega C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v_{dq\gamma} = -v_d \omega C v_q + v_q \omega C v_d = 0
$$

The third term is the change in stored energy in the capacitors during transient operation and will be zero in steady state.

The fourth term is the exchange of real power with the current sink.

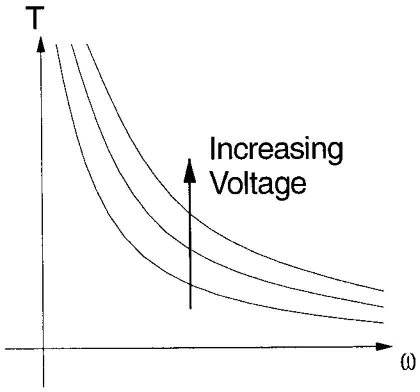
$$
v_{dq\gamma}^T i_{dq\gamma} = v_d i_d + v_q i_q + v_\gamma i_\gamma
$$

2) (a) Describe how it is possible that a universal motor can produce useful torque on both a DC and an AC supply. Sketch the torque against speed graph of such a motor. Describe how the speed of a universal motor may be controlled where the available supply system is AC.

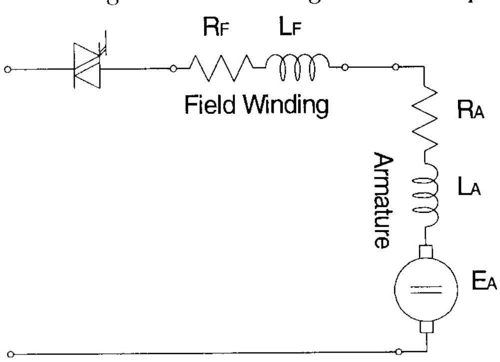
[6]

In a universal motor the field flux is set up by the same current that flows in the armature (because these windings are connected in series). Therefore, when the armature current reverses, between one half-cycle and another of AC, the field also reverses and the direction of the torque is unchanged. When connected to an AC supply the torque produced pulsates at twice line frequency but remains in one direction.

The torque-speed curve of a universal motor is the same shape as a DC series motor.



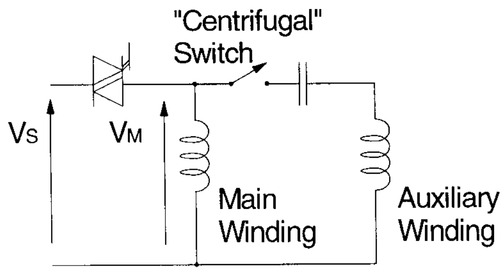
The speed, for a given torque, can be increased by increasing the applied voltage. For low power application (typical of the universal motor), the average voltage can be varied by phase-angle control using a Triac or pair of thyristors.



(b) Explain why a single-phase induction machine frequently incorporates an auxiliary winding in addition to its main winding in order to produce starting torque. Outline the connection arrangement of such a winding in a "capacitor start" machine.

[6]

A single winding supplied with AC produces a pulsating magnetic field, not a rotating field. This can be viewed as the sum of two counter rotating fields. However, at standstill, these counter rotating fields will produce no net torque on the rotor and the machine will not start. An auxiliary winding 90° physically displaced from the main winding and supplied at a suitable phase difference can be used to increase, say, the forward field at the expense of the *backwards field. The ideal phase difference is 90° but a reasonable approximation can be achieved with a large valued capacitor. If the capacitor is only needed for starting it can be switched out with a centrifugal switch.*



(c) Figure 2 shows the equivalent circuit of the main winding of a single-phase induction machine in which s_F is the slip with respect to the forward field. The parameters of the machine are:

supply frequency = 50Hz,
number of pole-pairs = 1,
stator resistance, RS = 2 Ω,
stator leakage reactance, XS = 10 Ω,
magnetising reactance, XM = 100 Ω,
referred rotor leakage reactance, XR' = 10 Ω,
referred rotor resistance, RR' = 3 Ω.

For a certain operating condition, it is found that the stator current is 2 A and the speed is 2950 rev./min. Calculate the mechanical power developed in the forward and backward rotating branches of the circuit.

[8]

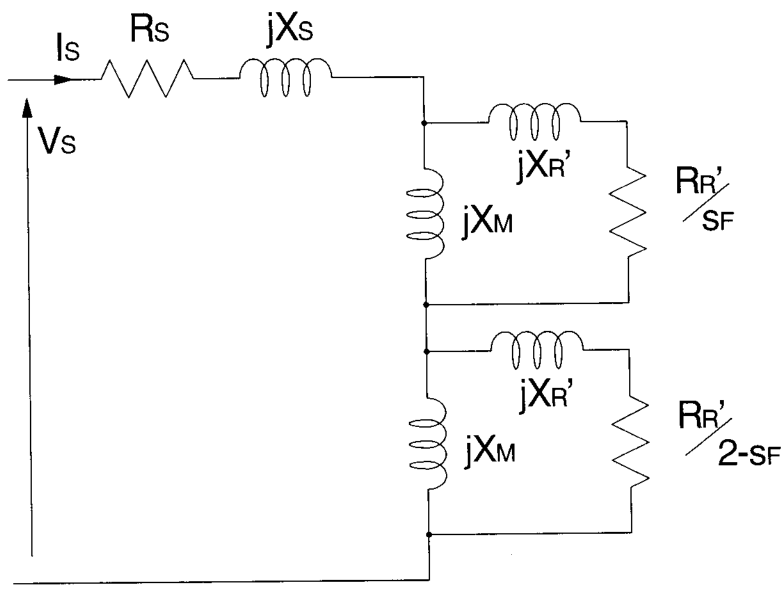


Figure 2

The rotor branch impedances are calculated first, then the portion of current flowing in the rotor and then the converted power.

Forward branch:

$$Z_{RF} = \left(\frac{R_R}{s_F} + jX_R\right) = 1.80 + j10.00 \quad \Omega$$

$$I_{RF} = I_S \frac{Z_M}{Z_M + Z_{RF}} = 0.948 \quad A$$

$$P_F = I_{RF}{}^2 R_R \frac{1 - s_F}{s_F} = 159\, W$$

$$T_F = \frac{P_F}{\omega_R} = \frac{159}{309} = 0.515\, Nm$$

Reverse branch:

$$Z_{RB} = jX_M \,//\left(\frac{R_R}{2 - s_F} + jX_R\right) = 1.51 + j10.00 \quad \Omega$$

$$I_{RB} = I_S \frac{Z_M}{Z_M + Z_{RB}} = 1.82 \quad A$$

$$P_B = I_{RB}{}^2 R_R \frac{-1 + s_F}{2 - s_F} = -4.9\, W$$

$$T_B = \frac{P_B}{\omega_R} = \frac{-4.9}{309} = -0.016\, Nm$$

3) The voltage equation for a dq model of an induction machine with the rotor leakage inductance referred to the stator side and the rotor short circuit is:

$$[v'_{DQ}] = [R'_{DQ}][i'_{DQ}] + [G'_{DQ}][i'_{DQ}] + [L'_{DQ}]\frac{d}{dt}[i'_{DQ}]$$

where:

$$[v'_{DQ}] = \begin{bmatrix} v_{SD} \\ v_{SQ} \\ 0 \\ 0 \end{bmatrix} \qquad [i_{DQ}] = \begin{bmatrix} i_{SD} \\ i_{SQ} \\ i'_{RD} \\ i'_{RQ} \end{bmatrix}$$

$$[R'_{DQ}] = \begin{bmatrix} R_S & 0 & 0 & 0 \\ 0 & R_S & 0 & 0 \\ 0 & 0 & R_R' & 0 \\ 0 & 0 & 0 & R_R' \end{bmatrix}$$

$$[G'_{DQ}] = \begin{bmatrix} 0 & -\omega L_S & 0 & -\omega M' \\ \omega L_S & 0 & \omega M' & 0 \\ 0 & -P\omega_{slip} M' & 0 & -P\omega_{slip} L'_R \\ P\omega_{slip} M' & 0 & P\omega_{slip} L'_R & 0 \end{bmatrix}$$

$$[L_{DQ}] = \begin{bmatrix} L_S & 0 & M' & 0 \\ 0 & L_S & 0 & M' \\ M' & 0 & L'_R & 0 \\ 0 & M' & 0 & L'_R \end{bmatrix}$$

and:

$$R'_R = R_R\left(\frac{M}{L_R}\right)^2$$

$$L'_R = L_R\left(\frac{M}{L_R}\right)^2$$

$$M' = M\left(\frac{M}{L_R}\right)$$

(i) The rotor leakage inductance is defined as $L'_{LR} = L'_R - M'$. Show that this is zero under the referral given here.

[2]

$$L'_R = \frac{M^2}{L_R} \quad M' = \frac{M^2}{L_R} \quad L'_{lR} = 0$$

(ii) Multiply the equation by the transpose of the current vector to obtain an equation for power. Identify the term that expresses the energy converted to mechanical form and from this show that the torque produced is equivalent to that calculated from the un-referred model, that is,

$$T = PM'\left(i_{SQ}\, i'_{RD} - i_{SD} i'_{RQ}\right)$$

[8]

$$\begin{aligned}
p &= i'_{DQ}{}^{T}\ v'_{DQ} \\
&= i'_{DQ}{}^{T}\ R'_{DQ}\, i'_{DQ} \\
&\quad + i'_{DQ}{}^{T}\ G'_{DQ}\, i'_{DQ} \\
&\quad + i'_{DQ}{}^{T}\ L'_{DQ} \frac{d\, i'_{DQ}}{dt}
\end{aligned}$$

The power involving G includes both convertion to mechanical form and reactive power. The reactive terms will sum to zero.

$$\begin{aligned}
p_{EM} &= i'_{DQ}{}^{T}\ G'_{DQ}\, i'_{DQ} \\
&= \begin{bmatrix} i_{SD} & i_{SQ} & i'_{RD} & i'_{RQ} \end{bmatrix} \begin{bmatrix} -\omega L_S\, i_{SQ} - \omega M' i'_{RQ} \\ +\omega L_S\, i_{SD} + \omega M' i'_{RD} \\ -P\omega_{slip}\, M' i_{SQ} - P\omega_{slip}\, L'_R\, i'_{RQ} \\ +P\omega_{slip}\, M' i_{SD} + P\omega_{slip}\, L'_R\, i'_{RD} \end{bmatrix} \\
&= \omega L_S \left(-i_{SD}\, i_{SQ} + i_{SQ}\, i_{SD}\right) \\
&\quad + \omega M'\left(-i_{SD}\, i'_{RQ} + i_{SQ}\, i'_{RD}\right) \\
&\quad + P\omega_{slip}\, M'\left(-i'_{RD}\, i_{SQ} + i'_{RQ}\, i'_{SD}\right) \\
&\quad + P\omega_{slip}\, L'_R \left(-i'_{RD}\, i'_{RQ} + i'_{RQ}\, i'_{RD}\right) \\
&= \left(\omega - P\omega_{slip}\right) M'\left(i_{SQ}\, i'_{RD} - i_{SD} i'_{RQ}\right)
\end{aligned}$$

$$\begin{aligned}
T &= \frac{p_{EM}}{\omega_R} = \frac{\left(\omega - P\omega_{slip}\right)}{\left(\dfrac{\omega}{P} - \omega_{slip}\right)} M'\left(i_{SQ}\, i'_{RD} - i_{SD} i'_{RQ}\right) \\
&= PM'\left(i_{SQ}\, i'_{RD} - i_{SD} i'_{RQ}\right)
\end{aligned}$$

Applying the referral yields:

$$\begin{aligned}
T &= PM\left(\frac{M}{L_R}\right)\left(i_{SQ}\, i_{RD}\left(\frac{L_R}{M}\right) - i_{SD} i_{RQ}\left(\frac{L_R}{M}\right)\right) \\
&= PM\left(i_{SQ}\, i_{RD} - i_{SD} i_{RQ}\right)
\end{aligned}$$

(iii) Figure 3 shows an equivalent circuit of the referred equation using flux linkages such as $\psi'_{RD} = M'\,(i_{SD} + i'_{RD})$. Re-draw the equivalent circuit of figure

3 for a case where orientation at $\psi'_{RQ} = 0$ has been achieved and the stator is supplied from a current source not a voltage source. Explain the significance of orienting the model such that $\psi'_{RQ} = 0$ in terms of the torque equation, the equivalent circuit. and the dynamics governing the relationships between stator current, flux and torque.

[10]

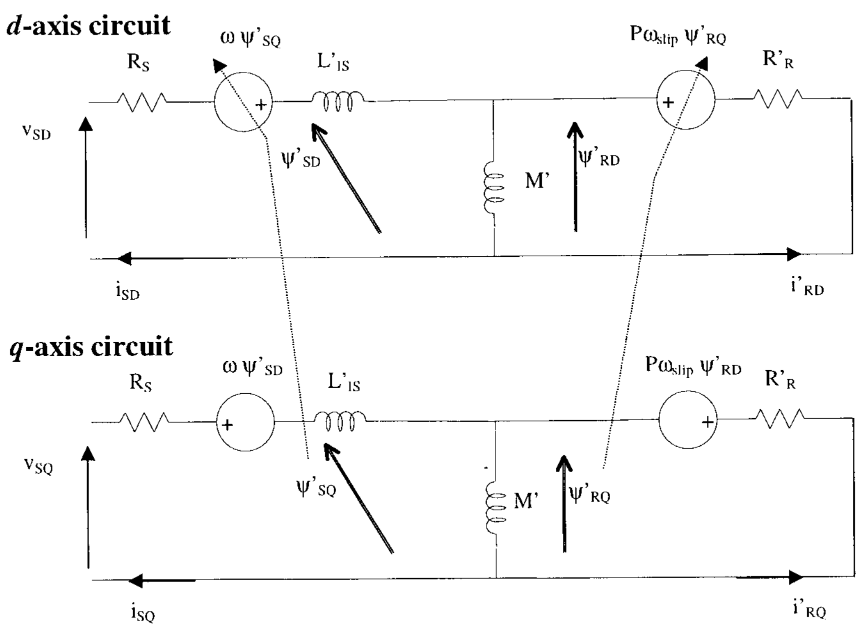
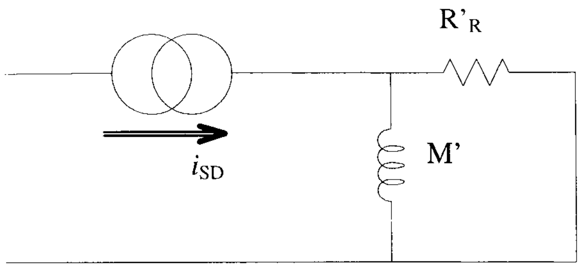


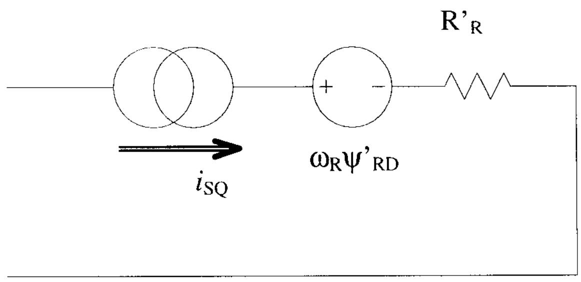
Figure 3

Orienting so that $\psi'_{RQ} = 0$

- *sets to zero the rotational voltage from the rotor of the d-axis circuit*
- *sets to zero the current through the magnetising inductance in the q-axis circuit (i.e. makes it open circuit)*

Supplying the stator from a current source removes the need to model the stator impedance





The d-axis is now a simple R L current divider and it and the d-axis flux has first order dynamics. The time constant of the circuit is long since it involves the magnetising inductance which is large.

$$i_{MD} = i_{SD} \frac{R'_R}{R'_R + sM'} = i_{SD} \frac{1}{1 + s\tau_R}$$

The q-axis has no dynamics and the rotor current responds instantly to changes in stator current. Thus, if the d-axis current, and hence the flux magnitude, is regulated to be constant, the torque will respond instantly to changes in q-axis stator current.

4) (a) Outline the steps necessary to converter a 6-coil, stationary-reference-frame model of an induction machine into a form suitable for field orientation.

[8]

Form voltage equations: Kirchoff's law is applied to each of the six coils and the equations assembled into a 6-element vector equation. Each coil has an induced voltage due to flux linkage with flux produced by every other coil and itself. That is, the mutual inductance matrix is a fully populated 6x6 matrix. Half of the terms in the mutual inductance matrix are time-varying because of rotation of the rotor. The electrical and mechanical cycles are related through the number of pole pairs. The mutual inductances are sinusoidal functions of $P\omega_R t$

$$[v_S] = [R_S][i_S] + [L_S]\frac{d}{dt}[i_S] + \frac{d}{dt}([m_{SR}][i_R])$$

$$[v_R] = [R_R][i_R] + \frac{d}{dt}([m_{RS}][i_S]) + [L_R]\frac{d}{dt}[i_R]$$

Separation of zero sequence term: apply Clark transform to create two orthogonal phases and zero-sequence term. This also diagonalises the stator and rotor self inductance sub matrices. No zero sequence current in a 3-wire system so model reduces to 4-element vector equation.

$$[T] = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$[v_{S\alpha\beta\gamma}] = [R_{S\alpha\beta\gamma}][i_{S\alpha\beta\gamma}] + [L_{S\alpha\beta\gamma}]\frac{d}{dt}[i_{S\alpha\beta\gamma}] + \frac{d}{dt}([m_{SR\alpha\beta\gamma}][i_{R\alpha\beta\gamma}])$$

$$[v_{R\alpha\beta\gamma}] = [R_{R\alpha\beta\gamma}][i_{R\alpha\beta\gamma}] + \frac{d}{dt}([m_{RS\alpha\beta\gamma}][i_{S\alpha\beta\gamma}]) + [L_{R\alpha\beta\gamma}]\frac{d}{dt}[i_{R\alpha\beta\gamma}]$$

Removal of rotation: a Park transform can be applied to remove the time variance of the inductance matrix and sinusoidal variations in voltage and current. The rotation of the Park transform should be of supply frequency.

For the stator:

$$\begin{aligned}[v_{SDQ}] = {} & [T_R(\omega)][R_{S\alpha\beta\gamma}][T_R(\omega)]^{-1}[T_R(\omega)][i_{S\alpha\beta\gamma}] \\ & + [T_R(\omega)][L_{S\alpha\beta\gamma}][T_R(\omega)]^{-1}[T_R(\omega)]\frac{d}{dt}[i_{S\alpha\beta\gamma}] \\ & + [T_R(\omega)]\frac{d}{dt}([m_{SR\alpha\beta\gamma}][i_{R\alpha\beta\gamma}])\end{aligned}$$

$$= [R_{SDQ\gamma}][i_{SDQ\gamma}] + [L_{SDQ\gamma}][T_R(\omega)]\frac{d}{dt}[i_{S\alpha\beta\gamma}] + [T_R(\omega)]\frac{d}{dt}([m_{SR\alpha\beta\gamma}][i_{R\alpha\beta\gamma}])$$

Transform and Inverse Pairs need to be inserted as for any circuit but for the mutual inductance terms the rotations should be (supply-slip) frequency for the stator voltage equations and slip frequency for the rotor. The stator variables are at a frequency of ω *and* *rotor variables at a frequency of $P\omega_R$. These pairs achieve stationary rotor currents and non-time-varying mutual inductances between stator and rotor.*

$$[v_{SDQ}] = [R_{SDQ\gamma}][i_{SDQ\gamma}] + [G_{SDQ\gamma}(\omega)][i_{SDQ\gamma}] + [L_{SDQ\gamma}]\frac{d}{dt}[i_{SDQ\gamma}]$$

$$+[T_R(\omega)]\frac{d}{dt}\left([m_{SR\alpha\beta\gamma}][T_R(\omega - P\omega_R)]^{-1}\,[T_R(\omega - P\omega_R)][i_{R\alpha\beta\gamma}]\right)$$

$$[T_R(\omega)]\frac{d}{dt}\left([m_{SR\alpha\beta\gamma}][i_{R\alpha\beta\gamma}]\right) = [G_{SRDQ}(\omega)][i_{RDQ\gamma}] + [M_{SRDQ}]\frac{d}{dt}[i_{RDQ\gamma}]$$

(b) Figure 4 shows a form of field orientation controller.

(i) Explain the difference between a direct and an indirect controller and state which is represented in figure 4.

[4]

There are two methods of determining the angular position of the rotor flux in order to orient the controller model.

1. *Direct orientation: determine the rotor flux vector and calculate its angle.*

2. *Indirect orientation: determine the rotor position and the speed with which the rotor flux advances with respect to the rotor (the slip speed).*

Although direct orientation may appear to be the natural choice, it is in fact the less attractive alternative. It is not possible to measure the flux linked with the rotor and even measurement of the total flux in the air-gap is difficult. Estimating the flux position is possible but prone to errors.

The indirect method requires the slip speed to be integrated to give the slip angle. The integration introduces an unknown constant of integration (the starting position of the flux). The indirect method relies on the system being self-correcting. If the initial determination of the flux position is in error then the imposed stator current is at the wrong angle with respect to the real rotor flux. The system adjusts over time to bring the assumed flux position and the real flux position into alignment.

The presence of the slip calculator in Figure 4 identifies this control as an indirect controller.

(ii) Given the equations for the d- and q-axis rotor voltages (below), justify the form of the slip calculator.

d-axis

$$0 = R'_R\, i'_{RD} + M'\frac{d\left(i_{SD} + i'_{RD}\right)}{dt}$$

q-axis

$$0 = R'_R\, i'_{RQ} + P\omega_{slip} M'(i_{SD} + i'_{RD})$$

[4]

From the q-axis equation, and by indroducing the d-axis flux linkage, we obtain the slip:

$$P\omega_{slip} = -\frac{R'_R\, i'_{RQ}}{M'(i_{SD} + i'_{RD})} = -\frac{R'_R\, i'_{RQ}}{\psi'_D}$$

We need to work with stator side variables. There is no q-axis magnetising current because of the orientation of the model to the d-axis flux therefore $i'_{RQ} = i_{SQ}$

$$P\omega_{slip} = \frac{R'_R\, i'_{SQ}}{\psi'_D}$$

This accounts for the division operation and scaling by R'_R

The d-axis flux linkage can be found from the d-axis voltage equation: $0 = R'_R\, i'_{RD} + \frac{d\psi'_{RD}}{dt}$

Note that $i'_{RD} = \psi_{RD}/M' - i_{SD}$*:*

$$0 = R'_R\left(\frac{\psi'_D}{M'} - i_{SD}\right) + \frac{d\psi'_D}{dt}$$

$$\frac{\psi'_D}{i_{SD}}(s) = \frac{R'_R}{\frac{R'_R}{M'} + s} = \frac{M'}{1 + \tau_R s}$$

Thus a first order transfer function is applied to the d-axis stator current to find the d-axis flux magnetude.

(iii) The machine is shown as current fed. Describe what extra control elements are necessary if a voltage source inverter is to be used.

[4]

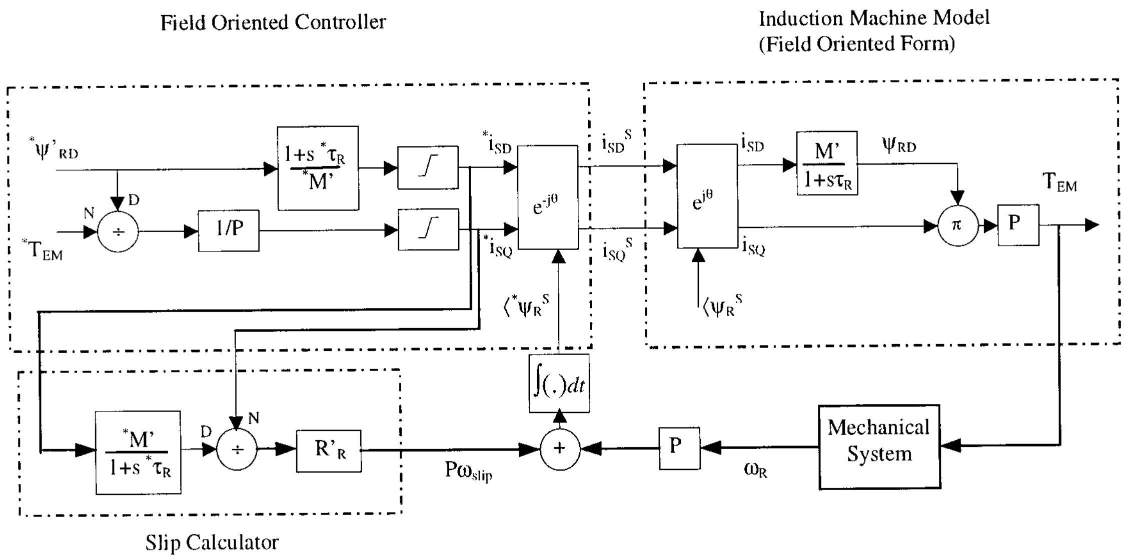
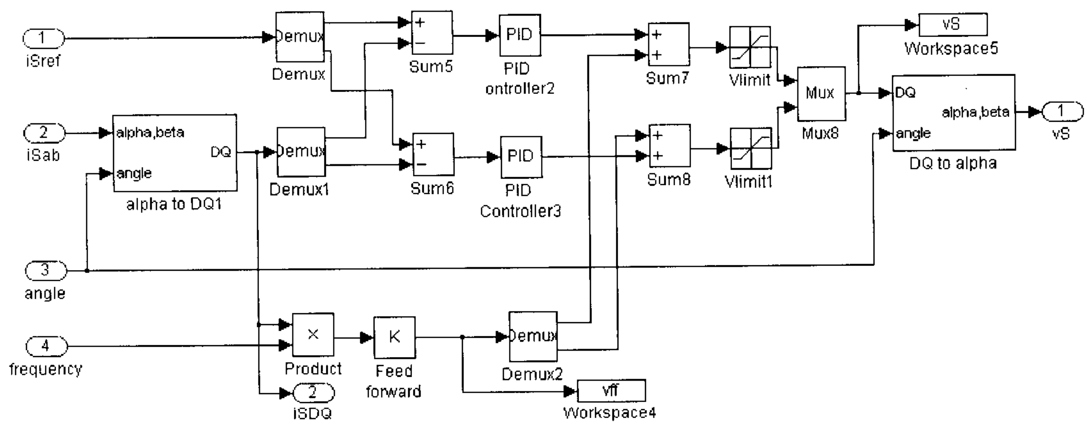


Figure 4

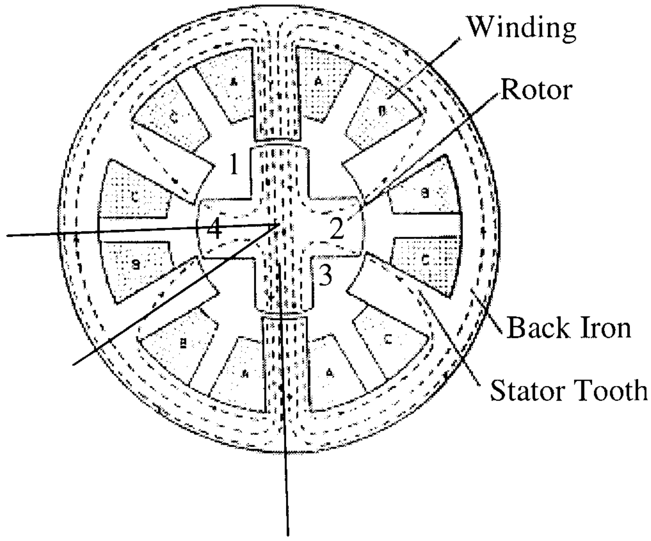
Closed loop control of the stator current can be achieved in the dq-frame. Stator currents are transformed from abc to dq using standard trasforms. The error is pplied to a PI control which, working with stationary variables, will produce zero steay-state error. During a transient there is coupling between the d- and q-axes. This can be removed by feed-forward of the coupling terms calculated from the applied frequency and the stator self-inductance.



5) (a) For a three-phase 6/4-pole variable reluctance motor:

(i) Sketch the shapes of the laminations for both stator and rotor and mark where the windings lie for each pole for a series connection.

[5]



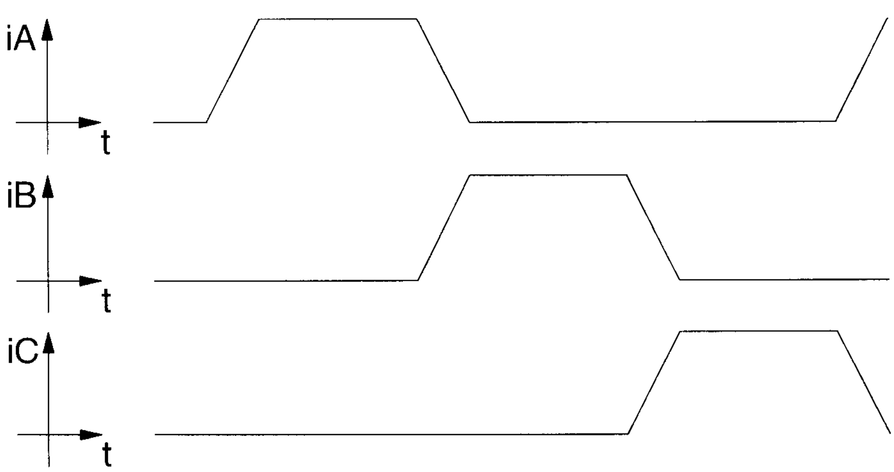
(ii) Determine the step angle for the motor.

[3]

$$\beta_{\text{Step}} = \beta_{Rotor} - \beta_{Stator} = \left(\frac{360^\circ}{N_R} - \frac{360^\circ}{N_S}\right)$$
$$= 90^\circ - 60^\circ = 30^\circ$$

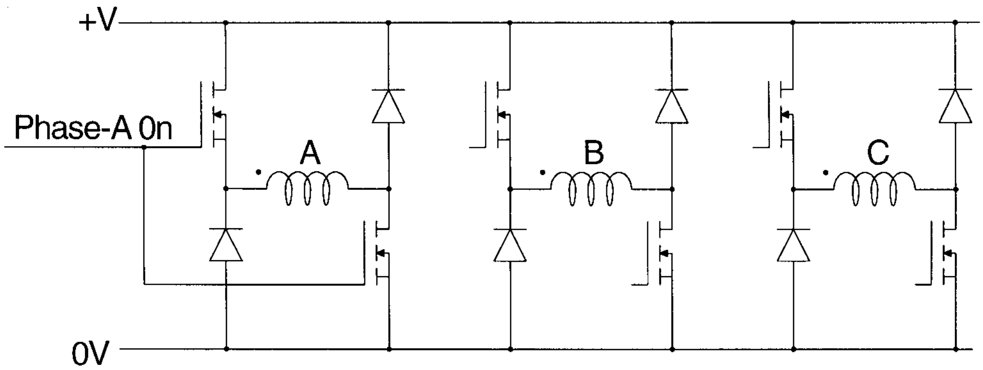
(iii) Mark the three phase-windings in the sketch produced for (i) with the letters A, B and C. Sketch the shape of the current waveforms required for phases A, B and C so that the rotor in your sketch rotates in the anti-clockwise direction.

[4]



(iv) Draw a diagram of a circuit suitable for supplying a three-phase machine.

[3]



(b) Describe the main advantages and disadvantages of a variable reluctance motor in comparison to an induction machine. Your comments should include (but need not be restricted to) reliability, machine manufacturing costs, total cost of the drive and the instantaneous torque.

[5]

A variable reluctance machine has a simple, unexcited rotor. It is therefore cheap to construct and robust. The stator has simple concentrated windings which are relatively easy to wind and cheap to install. The drive electronics does not have a shoot through path and has an inherent slow rise of fault current. The circuit shown above has as many low- and high-side transistors as a 3-phase inverter but simpler, cheaper circuits (with performance compromises) exist.

6) (a) Justify the following statements through brief explanation.

(i) It is necessary to have many poles in synchronous machines driven by hydro-turbines.

[2]

The speed at which the water wheels rotate are very low (of the order few hundred rpm). In order for the generator connected to hydro turbine to get connected to the grid of frequency 'f' the following relation has to be satisfied $f = \frac{PN}{120}$ *where 'P' is number of poles, N is the speed in rpm. At low speed large number of poles are necessary to achieve a fixed grid frequency.*

(ii) The rotor of a cylindrical-pole machine is made of solid iron whereas the stator is made of laminated steel.

[2]

The rotor is excited from a DC source. This does not induce current on rotor body. The EMF generated on the stator winding is AC (usually of 50/60 Hz) and so the stator body is subjected eddy currents circulation causing heating of the stator body. The lamination on stator is to minimise eddy currents circulation and hence keeping temperature under control. Solid rotor body also produces damping action when subjected to oscillations.

(iii) A synchronous machine develops torque only at synchronous speed.

[2]

The unidirectional torque results from the interaction of two magnetic fields having fixed angular separation amongst them with time. MMF produced by stator current in the gap rotates at synchronous speed. The interaction with rotor magnetic field will only result unidirectional torque if the rotor field rotates at that speed. The rotor excitation, being DC in nature, can only rotate with the same speed of the stator field when the rotor rotates at the synchronous speed.

(iv) A salient-pole synchronous machine continues to produce power even when its excitation is lost.

[2]

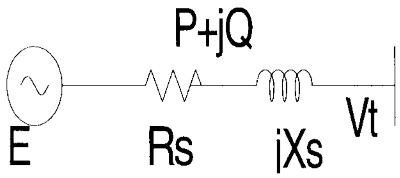
The expression for power developed by a salient pole synchronous machine is:

$$P = \frac{EV_t}{X_d}\sin\delta + \frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

where various symbols have their usual meaning.
The second term is free from excitation E. *Hence when* $X_d \neq X_q$, *which is the case with salient-pole machines, power is developed.*

(b) A cylindrical-rotor synchronous machine is connected to an infinite bus of $V\angle 0$ volts through a resistance of R_a and synchronous reactance X_s. The excitation voltage is $E\angle\delta$. Develop an expression for real and reactive power delivered by the machine.

[6]



Induced voltage $\overline{E} = E\angle\delta$,

Terminal voltage or infinite bus voltage: $\overline{V_t} = V_t\angle 0$

Stator impedance: $\overline{Z}_s = R_s + jX_s = Z_s\angle\theta_s$

The complex power delivered to infinite bus:

$$S = V_t I_s^*$$

$$I_s^* = \left|\frac{E - V_t}{Z_s}\right|^* = \frac{E}{Z_s}\angle\theta_s - \delta - \frac{V_t}{Z_s}\angle\theta_s$$

$$S = \frac{EV_t}{Z_s}\angle\theta_s - \delta - \frac{V_t^2}{Z_s}\cos\angle\theta_s$$

$$S = P + jQ$$

$$P = \frac{EV_t}{Z_s}\cos(\theta_s - \delta) - \frac{V_t^2}{Z_s}\cos\theta_s \quad \textit{watt/phase}$$

$$Q = \frac{EV_t}{Z_s}\sin(\theta_s - \delta) - \frac{V_t^2}{Z_s}\sin\theta_s \quad \textit{VAR/phase}$$

(c) A 3-phase, 2-pole, Y-connected cylindrical-pole synchronous generator is connected to a 50 Hz grid system. The rated apparent power of the machine is 247 MVA; the rated voltage is 15.75 kV (line-line) and the per phase synchronous reactance is 1.0 Ω.

(i) Determine the excitation voltage and the power angle when the machine is delivering rated MVA at a power factor of 0.85 lagging.

[3]

Per phase line voltage 15.75/√3= 9.093 kV
Line current can be computed from the relation
$\sqrt{3}V_L I_L = 247(kA)$ $I_L = 9.054$ *kA,*
Phase angle of the current $-\cos^{-1}0.85 = -31.78$ *degree* *(1 mark)*

Calculate $E\angle\delta$ *from the expression:*
$E\angle\delta = V_t + j\bar{I}X_s$
E = 15.85 kV per phase or 27.46 kV line to line, power angle δ *=29 degree (2 mark)*

(ii) The excitation is held constant at the value obtained in part (i) while the prime mover power is slowly increased. Determine the steady-state stability limit and the corresponding values of stator current and power factor under the maximum power transfer condition?

[3]

At maximum power transfer condition $\delta = 90$ *(1 mark)*

*Pmax = 3*15.854*9.093/1 = 432 MW*

Solve for current from the expression

$$I = \frac{E\angle 90 - V\angle 0}{jXs}$$

I = 18.276 KA, pf 0.86 leading. (2 marks)