Imperial College London

BSc/MSci EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

PRINCIPLES OF INSTRUMENTATION

For 3rd/4th-Year Physics Students

Tuesday, 22nd May 2012: 14:00 to 16:00

The paper consists of two sections: A & B.
Section A contains one question, comprising small parts. [20 marks total]
Section B contains four questions on selected parts of the course. [15 marks each]
Candidates are required to:
*Answer **ALL** parts of Section A and **TWO** questions from Section B.*
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

SECTION A

1. (i) Give the integral form of the Laplace transform and use it to show that

$$\mathcal{L}\left\{e^{-at}\right\} = \frac{1}{s+a}$$

[2 marks]

(ii) Find a solution to the equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \delta(t)$$

Assume zero initial conditions. You may use

$$\mathcal{L}\left\{\frac{d^2}{dt^2}f(t)\right\} = s^2F(s)$$

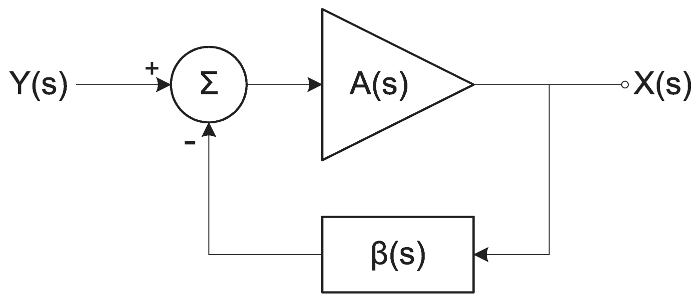
$$\mathcal{L}^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\} = \frac{1}{b-a}\left(e^{-at} - e^{-bt}\right)$$

[2 marks]

(iii) For the system represented by the s-domain block diagram below, show that the transfer function is given by

$$G(s) = \frac{A(s)}{1 + \beta(s)A(s)}$$

[3 marks]



(iv) A system has a transfer function

$$G(s) = \frac{1}{(s-5)(s+2)}$$

Is it stable with respect to a small perturbation at the input? Explain your answer

[2 marks]

[This question continues on the next page . . .]

(v) A 12-bit analogue to digital converter is used to measure a positive voltage signal with a digital resolution of 1 mV. Determine

(a) The maximum digitisable input signal in volts

(b) The dynamic range, in dB, for the digitised signal

(c) The maximum quantisation error

(d) The RMS quantisation noise added

Note that the standard deviation of values uniformly distributed between $\pm R$ is $R/\sqrt{12}$ [4 marks]

(vi) A signal analyser is set to operate at 10^6 samples per second

(a) What is the Nyquist frequency of the digitised signal?

(b) Which of the following input frequencies can be properly sampled?
266 kHz, 498 kHz, 502 kHz, 1.6 MHz

(c) For those improperly sampled, give the alias frequency

[3 marks]

(vii) a co-axial cable of length 105 m is manufactured with capacitance and inductance per metre given by

$$C' = 9.5 \times 10^{-11}\ \mathrm{Fm^{-1}} \quad L' = 2.375 \times 10^{-7}\ \mathrm{Hm^{-1}}$$

The far end of the cable is connected short-circuit. A short pulse of amplitude 1 V is driven onto the near end

(a) How long does it take for the pulse to return to the source?

(b) What is the returning amplitude?

(c) Explain the term 'impedance matching' and describe, in general terms, how you might match this type of cable to a 100 Ω impedance antenna, for a given frequency ν
You do **not** need to perform a detailed calculation [4 marks]

[Total 20 marks]

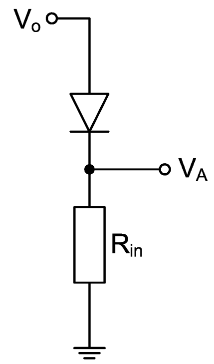
SECTION B

2. (i) Describe the frequency characteristics of flicker noise. In an experiment to measure flicker noise, current from a highly stable source is passed through a resistor and the resultant voltage is measured with an instrument with a bandwidth from DC to 10 kHz. Show that the ratio of flicker noise power in two measurements made over times of 10 s and 1000 s is approximately

$$\frac{P_{1000}}{P_{10}} = 1.4$$

[3 marks]

(ii) The circuit shown below is used to generate white noise. The diode used has a forward-biased voltage drop of 0.6 V



(a) What two processes are responsible for generating a noise voltage at V_A?

(b) Why should we expect the noise voltage at V_A to be white?

(c) Show that the temperature at which the RMS noise voltage due to these two processes is equal is independent of R_{in} [5 marks]

(iii) Sketch a plot of the overall noise power versus frequency which we may expect to see in a typical measurement, labelling the contribution from all the important sources [1 mark]

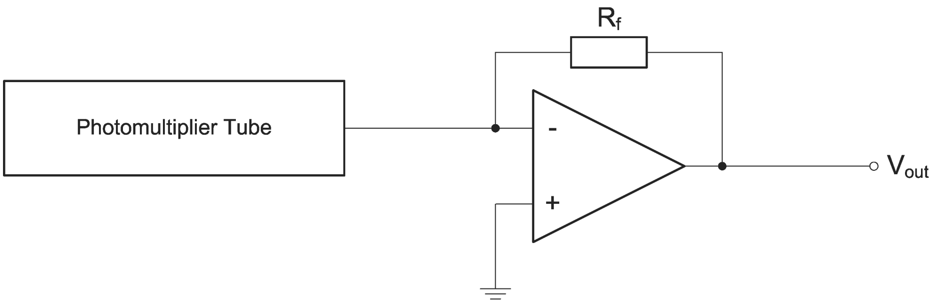
(iv) In an experiment to measure fluorescence from a sample illuminated by a continuous-wave laser, the very weak signal from the detector is dominated by flicker noise. With the aid of a suitable sketch, describe the experimental setup required to apply synchronous detection to this problem. Explain how the signal may be recovered from the noise using this technique. Note that it is **not** necessary to give the detailed circuit diagram for the synchronous detector, however you should briefly explain its function [6 marks]

[Total 15 marks]

3. (i) Explain, with the aid of a sketch, the principle of operation for the photomultiplier tube. Under what circumstances will the tube saturate? [4 marks]

(ii) A tube with 12 dynodes has a photo-cathode quantum efficiency of 25% and each dynode liberates an additional 7 electrons for each incident electron. In dark conditions, the cathode liberates 2×10^4 electrons per second due to thermionic emission. Calculate the charge deposited at the anode for each electron liberated at the cathode, and hence determine the dark-current. [2 marks]

(iii) The tube is connected to a current-to-voltage converter as shown in the figure.



Assuming an ideal op-amp, show that

$$V_{out} = IR_f$$

where I is the tube current. [1 mark]

(iv) This arrangement is used to detect pulses of light from an experiment. Each pulse has an approximately Gaussian intensity profile

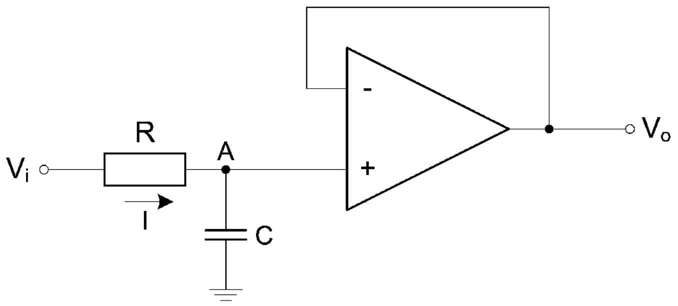
$$P(t) = P_0 e^{-(t-t_0)^2/2\sigma^2}$$

where t_0 is the time of the pulse, σ is 800μs and P_0 is the peak-power, which may be taken to be 0.5 pW at a wavelength of 0.55μm. For R_f = 1000Ω make a sketch of the output voltage profile for each pulse. [4 marks]

(v) A pulse-counter will be used in the experiment. It requires +5V to 'count' a pulse and the voltage must fall to zero volts again before the next pulse can be counted. Design a circuit which will allow the pulse to be counted when it reaches half its maximum value. You may use as many components as you need, such as resistors, op-amps and diodes, and you may assume you have a supply to generate any voltages needed [4 marks]

[Total 15 marks]

4. (i) The circuit shown in the figure below consists of 2 blocks connected together at point A. Explain, in general terms, the function of each block. Assume that the op-amp used is ideal. [2 marks]



(ii) Show that the transfer function for the circuit is given by

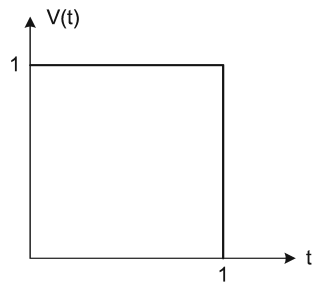
$$G(s) = \frac{1}{RCs + 1}$$

[4 marks]

(iii) Two such circuits are connected together in series such that the output of one is connected to the input of the second. For both, R = 500kΩ and C = 1μF

(a) Find the transfer function of the complete system and evaluate the system's response to a unit-step at the input [5 marks]

(b) Hence deduce the response of the system to a pulse of the form shown in the figure below [3 marks]



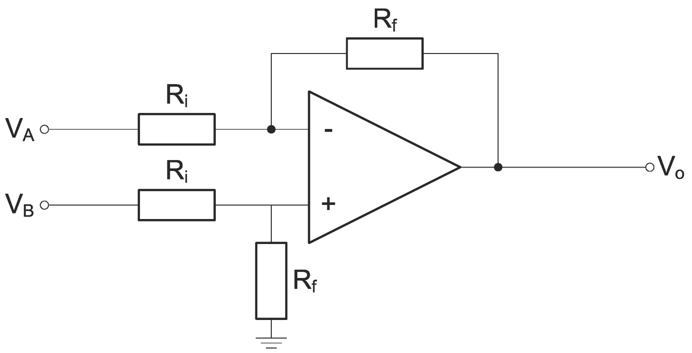
(c) Which two properties of the system allow us to reach this solution? Explain your answer [1 mark]

[Total 15 marks]

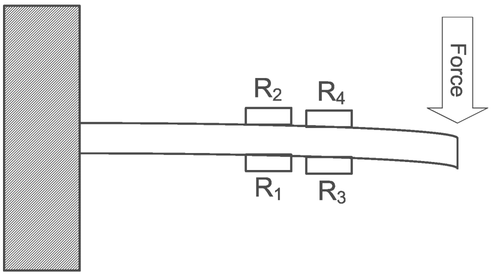
Note:

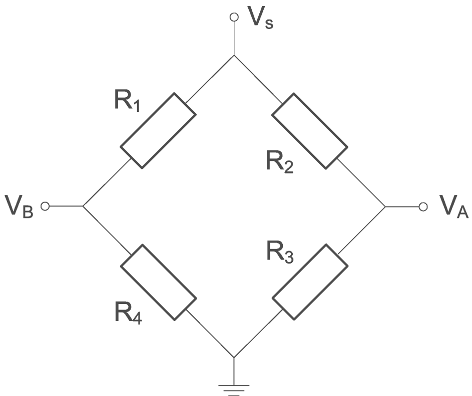
$$\mathcal{L}\left\{\frac{t^{n-1}e^{-at}}{(n-1)!}\right\} = \frac{1}{(s+a)^n}$$

5. (i) The difference amplifier is a circuit to amplify the potential difference between two points in a circuit, neither of which is connected to ground. Assuming an ideal amplifier, derive the relationship between V_o and two input voltages V_A and V_B [4 marks]



(ii) A force transducer is constructed from a flexible beam with four strain gauges, as shown in the figure. The gauges are connected in the form of a Wheatstone bridge. The resistance for each resistor under zero-strain is R and for a given strain (assumed to be proportional to force) the gauges on top of the beam increase by δR while those on the bottom decrease by δR.





V_S is the bridge voltage. Considering the voltages V_A and V_B to be the sensor output, show that

(a) The common-mode voltage is constant

(b) The differential voltage is a linear function of δR [4 marks]

(iii) The difference amplifier could be used to amplify the sensor output. What potential problem is there with this arrangement? Explain why an instrumentation amplifier would be better [2 marks]

(iv) A commercial instrumentation amplifier with a differential gain of 100 and a common-mode rejection ratio of 80 dB is connected across the output of the sensor. The bridge voltage is 10 V and the zero-strain resistance R = 100 Ω. For an output voltage of 2.55 V, find δR [3 marks]

[This question continues on the next page . . .]

(v) Explain why, especially when transmitted over long distances, differential voltages have good immunity from interference [2 marks]

[Total 15 marks]