EE 4-08 SOLUTIONS

1. a) i) Bookwork

ii) Bookwork

b) i) Bookwork

ii) 1. A large fraction of the signal energy is packed within very few transform coefficients, the ones near the origin. By keeping the low index transform coefficients and replacing the rest with zero we can achieve image compression. 2. Basis functions consist of 1s and -1s and therefore the transform is more resistant to errors.

iii) We know that $N=2^n$ and therefore, in case of $N=2$ we have $n=1$, x,y 0 or 1 and $b_0(0)=b_0(0)=0$ and $b_0(1)=b_0(1)=1$. For $f(x,y)=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$ we calculate the Walsh transform coefficients as follows.

$$W(u,v)=\frac{1}{N}\sum_{x=0}^{2-1}\sum_{y=0}^{2-1}f(x,y)\left[\prod_{i=0}^{n-1}(-1)^{(b_i(x)b_{n-1-i}(u)+b_i(y)b_{n-1-i}(v))}\right]$$

$$W(u,v)=\frac{1}{N}\sum_{x=0}^{1}\sum_{y=0}^{1}f(x,y)\left[\prod_{i=0}^{1-1}(-1)^{(b_i(x)b_{-i}(u)+b_i(y)b_{-i}(v))}\right]$$

$$W(u,v)=\frac{1}{N}\sum_{x=0}^{1}\sum_{y=0}^{1}f(x,y)(-1)^{(b_0(x)b_0(u)+b_0(y)b_0(v))}$$

$$=\frac{1}{2}f(0,0)(-1)^{(b_0(0)b_0(u)+b_0(0)b_0(v))}+\frac{1}{2}f(0,1)(-1)^{(b_0(0)b_0(u)+b_0(1)b_0(v))}$$

$$+\frac{1}{2}f(1,0)(-1)^{(b_0(1)b_0(u)+b_0(0)b_0(v))}+\frac{1}{2}f(1,1)(-1)^{(b_0(1)b_0(u)+b_0(1)b_0(v))}$$

$$=\frac{1}{2}f(0,0)(-1)^{(0\cdot b_0(u)+0\cdot b_0(v))}+\frac{1}{2}f(0,1)(-1)^{(0\cdot b_0(u)+1\cdot b_0(v))}$$

$$+\frac{1}{2}f(1,0)(-1)^{(1\cdot b_0(u)+0\cdot b_0(v))}+\frac{1}{2}f(1,1)(-1)^{(1\cdot b_0(u)+1\cdot b_0(v))}$$

$$=\frac{1}{2}f(0,0)(-1)^{0}+\frac{1}{2}f(0,1)(-1)^{b_0(v)}+\frac{1}{2}f(1,0)(-1)^{b_0(u)}+\frac{1}{2}f(1,1)(-1)^{b_0(u)+b_0(v)}$$

$$=\frac{1}{2}(-1)^{0}+\frac{1}{2}2(-1)^{b_0(v)}+\frac{1}{2}2(-1)^{b_0(u)}+\frac{1}{2}3(-1)^{b_0(u)+b_0(v)}$$

$$=\frac{1}{2}+(-1)^{b_0(v)}+(-1)^{b_0(u)}+\frac{3}{2}(-1)^{b_0(u)+b_0(v)}$$

$$W(0,0)=\frac{1}{2}+(-1)^{b_0(0)}+(-1)^{b_0(0)}+\frac{3}{2}(-1)^{b_0(0)+b_0(0)}=\frac{1}{2}+(-1)^{0}+(-1)^{0}+\frac{3}{2}(-1)^{0}=\frac{1}{2}+1+1+\frac{3}{2}=4$$

$$W(0,1)=\frac{1}{2}+(-1)^{b_0(0)}+(-1)^{b_0(1)}+\frac{3}{2}(-1)^{b_0(0)+b_0(1)}=\frac{1}{2}+(-1)^{0}+(-1)^{1}+\frac{3}{2}(-1)^{0+1}$$

$$=\frac{1}{2}+1-1-\frac{3}{2}=-1$$

$$W(1,0)=\frac{1}{2}+(-1)^{b_0(1)}+(-1)^{b_0(0)}+\frac{3}{2}(-1)^{b_0(1)+b_0(0)}=\frac{1}{2}+(-1)^{1}+(-1)^{0}+\frac{3}{2}(-1)^{1+0}$$

$$=\frac{1}{2}-1+1-\frac{3}{2}=-1$$

$$W(1,1)=\frac{1}{2}+(-1)^{b_0(1)}+(-1)^{b_0(1)}+\frac{3}{2}(-1)^{b_0(1)+b_0(1)}=\frac{1}{2}+(-1)^{1}+(-1)^{1}+\frac{3}{2}(-1)^{1+1}$$

$$=\frac{1}{2}-1-1+\frac{3}{2}=0$$

Therefore, $W(u,v)=\begin{bmatrix}4 & -1\\ -1 & 0\end{bmatrix}$.

c) i) $$f_1(x,y)=\begin{cases} r_1 & 1\le x\le \frac{M}{4}, 1\le y\le M\\ r_2 & \frac{M}{4}< x\le M, 1\le y\le M\end{cases}$$

Mean value of $f_1(x,y)$ is $m_1=\frac{r_1}{4}+\frac{3r_2}{4}$. Zero-mean version of $f_1(x,y)$ is

$$f_1(x,y)-m_1=\begin{cases}\frac{3r_1}{4}-\frac{3r_2}{4} & 1\le x\le \frac{M}{4}, 1\le y\le M\\ \frac{r_2}{4}-\frac{r_1}{4} & \frac{M}{4}< x\le M, 1\le y\le M\end{cases}$$

Mean value of $f_2(x,y)$ is r_3. Zero-mean version of $f_2(x,y)$ is $f_2(x,y)-m_2=0$.

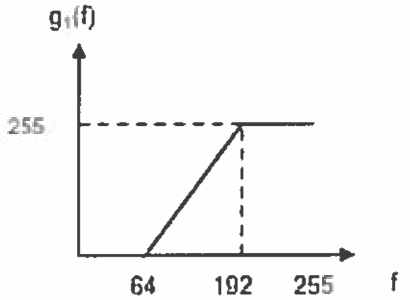
Variance of $f_1(x,y)-m_1$ is $\frac{1}{4}\frac{9}{16}(r_1-r_2)^2+\frac{3}{4}\frac{1}{16}(r_1-r_2)^2=\frac{3}{16}(r_1-r_2)^2$.

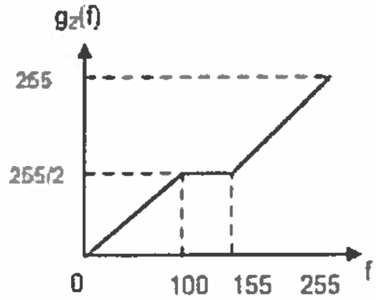
Variance of $f_2(x,y)-m_2$ is 0.

Covariance between $f_1(x,y)-m_1$ and $f_2(x,y)-m_2$ is 0. Therefore, the covariance matrix is $\begin{bmatrix}\frac{3}{16}(r_1-r_2)^2 & 0\\ 0 & 0\end{bmatrix}$ with eigenvalues $\frac{3}{16}(r_1-r_2)^2$ and 0. Therefore, by using the Karhunen Loeve transform we produce two new images, with one of them being 0 and the other being $f_1(x,y)-m_1$.

ii) The above result is expected since one of the given images is constant and therefore it doesn't carry any information. This means that there is only one principal component in the given set.

2. a) The transformation used for histogram equalisation is $s=T(r)=\int_0^r p_r(w)dw$. Based on that we get:





b) $P_F(f)=\int_0^f p_F(f)df=\int_0^f \frac{2}{255}(1-\frac{f}{255})df=\frac{f}{255^2}(255\times 2-f)$

$P_G(g)=\int_0^g p_g(g)dg=\int_0^g \frac{2}{255^2}gdg=\frac{g^2}{255^2}$

$$\frac{g^2}{255^2}=\frac{f}{255^2}(255\times 2-f)\Rightarrow g^2=(510f-f^2)\Rightarrow g=\pm\sqrt{510f-f^2}$$

g should be between 0 and 255 and therefore $g(f) = \sqrt{510f - f^2}$

c) Trivial. The answer is median. This can be demonstrated by simple example, i.e., median(0,1,0)+median(2,0,0)=0+0=0. This is not the same as the median of (2,1,0) which is 1.

d) Book work. The answer is differentiation filter.

3. a) i) Bookwork
ii) Bookwork

b)

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v)$$

$$|H(u,v)|^2 = (-\sqrt{2\pi}\sigma(u^2+v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}) \times (-\sqrt{2\pi}\sigma(u^2+v^2)e^{+2\pi^2\sigma^2(u^2+v^2)}) = (-\sqrt{2\pi}\sigma(u^2+v^2))^2$$

$$H^*(u,v) = -\sqrt{2\pi}\sigma(u^2+v^2)e^{+2\pi^2\sigma^2(u^2+v^2)}$$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v) = \left[\frac{-\sqrt{2\pi}\sigma(u^2+v^2)e^{+2\pi^2\sigma^2(u^2+v^2)}}{(-\sqrt{2\pi}\sigma(u^2+v^2))^2 + K}\right] G(u,v)$$

4. a) i) 1. To save disk space. 2. To decrease transmission time when transferring files over networks. 3. To make some programs work faster (e.g. by decreasing disk access time).
ii) It must deviate from uniform.
iii) After differential coding using the given formula, the resulting image has only 3 values 1,-1,0 and 0 dominates, therefore it is must more efficient to use differential coding.

b) i)

$p(110) = 5/25 = 0.2$
$p(120) = 7/25 = 0.28$
$p(140) = 5/25 = 0.2$
$p(160) = 4/25 = 0.16$
$p(170) = 3/25 = 0.12$
$p(180) = 1/25 = 0.04$

ii) Derive the Huffman code.

Symbol	Prob	Code	Prob	Code	Prob	Code	Prob	Code	Prob	Code
120	0.28	01	0.28	01	0.32	00	0.40	1	0.60	0
110	0.2	10	0.2	10	0.28	01	0.32	00	0.40	1
140	0.2	11	0.2	11	0.2	10	0.28	01		
160	0.16	000	0.16	000	0.2	11				
170	0.12	0010	0.16	001						
180	0.04	0011								

iii) Calculate the average length of the fixed code and that of the derived Huffman code.
Fixed length: 3 bits/symbol
Huffman: Lavg=2.48 bits/symbol

iv) Compression ratio 3/2.48
Redundancy=3-2.48