IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE PART I: MEng, BEng and ACGI

MATHEMATICS 1B (E-STREAM AND I-STREAM)

Corrected Copy

Friday, 29 May 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Answer ALL questions. All questions carry equal marks (25% each)

Please answer questions from Section A and Section B in separate answer books.

NO CALCULATORS ALLOWED

Any special instructions for invigilators and information for candidates are on page 1.

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EE1-10B MATHEMATICS II

Section A

1. a) Given the equations of three planes

$$\underline{\mathbf{r}}\cdot(1,-1,2)=2\,,\qquad \underline{\mathbf{r}}\cdot(0,1,-3)=\alpha\,,\qquad \underline{\mathbf{r}}\cdot(2,1,-5)=1$$

show that when $\alpha=1$ the three planes do not intersect, but form the sides of a prism. Find the value of α so that the three planes intersect, and obtain the intersection. [6]

b) Show that for any three vectors $\underline{\mathbf{u}},\underline{\mathbf{v}},\underline{\mathbf{w}}$,

$$[(\underline{\mathbf{u}}+\underline{\mathbf{v}})\times(\underline{\mathbf{v}}-\underline{\mathbf{w}})]\cdot(\underline{\mathbf{u}}+\underline{\mathbf{w}})=0\,. \qquad [\ 5\ ]$$

c) Consider the matrix

$$A=\begin{pmatrix} 1 & -2 & 2 \\ 0 & -1 & 2 \\ -6 & 5 & -4 \end{pmatrix}.$$

(i) Show that $\lambda=1$ is an eigenvalue of A, and find the other eigenvalues. [4]

(ii) Find an eigenvector of A, corresponding to $\lambda=1$. [3]

d) Let A be an invertible matrix, and λ an eigenvalue of A.

(i) Show that $\lambda\neq 0$; [3]

(ii) Hence, or otherwise, show that $1/\lambda$ is an eigenvalue of A^{-1}. [4]

2. a) i) Evaluate the determinant of the matrix

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ \alpha & 1 & -5 \end{pmatrix}$$

and state the value of α for which A is singular. [3]

ii) Let $\alpha = 3$. Use Gauss-Jordan elimination (row operations) to find A^{-1}. [5]

iii) Use the inverse found in (ii) to solve the set of linear equations

$$\begin{aligned} -x + 2y + z &= 3 \\ -x + y + 2z &= 0 \\ x + 3y - 5z &= -8 \end{aligned}$$ [4]

b) Given the function $f(x) = \dfrac{1}{\sqrt{1-x}}$,

i) Obtain the Maclaurin series for $f(x)$ up to the term in x^3 and state the remainder term; [5]

ii) Find the maximum error incurred in using the series up to the term in x^3 to estimate $\dfrac{1}{\sqrt{0.9}}$.

[*You may leave the answer in terms of a fraction.*] [4]

c) Given the power series $\sum_{n=1}^{\infty} \dfrac{x^n}{2^n - 1}$, find all values of x for which the series converges. [4]

Section B

3. a) Derive the second order linear ordinary differential equation with constant co-efficients whose general solution is

$$y(x) = c_1 e^x + c_2 e^{2x} + e^{3x}.$$ [6]

b) Find the general solution of the first order ordinary differential equation

$$\frac{dy}{dx} = xy^{-1} e^{x-y}.$$ [6]

c) Consider the following differential equation:

$$(2xy + e^x)\,dx + \left(x^2 + \cos y\right)dy = 0.$$

i) Show that the differential equation is exact. [3]

ii) Hence find the solution of the differential equation. [3]

d) Consider the following differential equation:

$$\frac{d^2z}{dx^2} - \frac{3}{x}\frac{dz}{dx} = -3x \cdot$$

i) Define a suitable transformation to turn the equation to a linear first order differential equation. [3]

ii) Hence or otherwise, derive the general solution to the equation. [4]

4. a) Evaluate $\dfrac{\partial z}{\partial x}$ when

i) $z(x,y) = (x+y)e^{xy}$. [2]

ii) $z(x,y)$ is defined implicitly by $F(x,y,z) = e^{z} + z\cos x + \sin y = 0$. [3]

iii) $z(x,y) = \displaystyle\int \frac{xy+1}{x^2+y^3}\,dx$. [3]

b) Consider the partial differential equation (PDE)

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \frac{x^2}{(x^2+y^2)^2}.$$

i) By considering the change of coordinates

$$\rho = \sqrt{x^2+y^2}, \qquad \phi = \tan^{-1}\frac{y}{x},$$

and using the chain rule

$$\begin{bmatrix} \dfrac{\partial f}{\partial x} \\[2ex] \dfrac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \rho}{\partial x} & \dfrac{\partial \phi}{\partial x} \\[2ex] \dfrac{\partial \rho}{\partial y} & \dfrac{\partial \phi}{\partial y} \end{bmatrix} \begin{bmatrix} \dfrac{\partial f}{\partial \rho} \\[2ex] \dfrac{\partial f}{\partial \phi} \end{bmatrix},$$

write the PDE in terms of the variables ρ and ϕ. [4]

ii) Assume that $\dfrac{\partial f}{\partial \rho} = 0$. Find the solution of the PDE. [4]

c) Let $f(x,y) = 2x^3 - 2x^2 - 2xy + y^2$.

i) Evaluate the gradient of f and derive its stationary points [4]

ii) Evaluate the Hessian of f and use it to classify the stationary points of f. Justify your classification. [5]