IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

EEE PART II: MEng, BEng and ACGI

CONTROL ENGINEERING

Corrected copy

Friday, 10 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : I.M. Jaimoukha
Second Marker(s) : S.A. Evangelou

1. a) Consider the feedback loop shown in Figure 1.1 on the next page. Here K_p is a proportional compensator and $G(s) = G_1(s)G_2(s)$ is the system where each of $G_1(s)$ and $G_2(s)$ is a transfer function representing the circuit shown in Figure 1.2 on the next page and where the value of the parameters for $G_1(s)$ are such that

$$C_i = 0, \qquad R_iC_f = 1, \qquad R_fC_f = 1,$$

and those for $G_2(s)$ are such that

$$C_i = 0, \qquad R_iC_f = 1, \qquad R_fC_f = 0.5.$$

Assume all the capacitors are initially uncharged.

i) Determine the transfer function $G(s)$. [5]

ii) Determine the DC gain of $G(s)$. [5]

iii) Assume that the system is operating in open loop. Let $u(t)$ be a unit step applied at $t = 0$. Use the final value theorem, which should be stated, to find the steady–state value of $y(t)$. [5]

iv) Let $r(t)$ be a unit step applied at $t = 0$. Find the minimum value of K_p such that the steady–state value of $e(t)$ is less than 0.01. [5]

b) In Figure 1.1 on the next page, $G(s) = 4/(s^2 + s - 2)$ and K_p is a proportional compensator.

i) Draw the Nyquist diagram for $G(s)$. [5]

ii) Use the Nyquist criterion, which should be stated, to find the number of unstable closed loop poles for all $-\infty < K_p < \infty$. [5]

iii) Comment on the gain margin when $K_p = 1$. [5]

iv) In terms of K_p, find expressions for the closed loop transfer function, DC gain and damping ratio. For a unit step reference signal, comment on the difficulty of designing a proportional compensator to simultaneously achieve good steady–state response (in terms of the DC gain) and good transient response (in terms of the damping ratio). [5]

(Figures for Question 1)

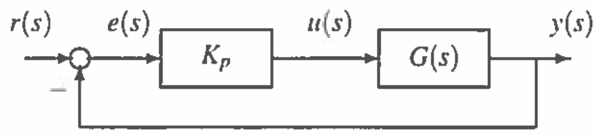

Figure 1.1

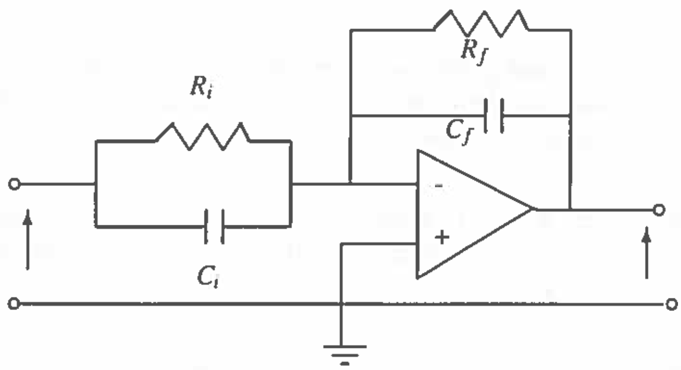

Figure 1.2

2. Consider the feedback control system in Figure 2.1 below.

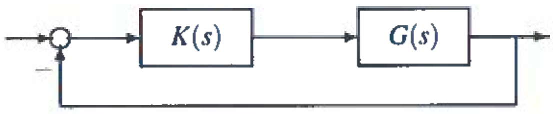

Figure 2.1

Here, $K(s)$ is the transfer function of a compensator and

$$G(s) = \frac{3-s}{(s+1)(s+2)}.$$

a) Sketch a Nyquist diagram of $G(s)$, indicating the low and high frequency portions. Use the Routh array to find the real–axis intercepts, together with the corresponding frequencies. [6]

b) Find the gain and phase margins and the cross–over frequency. Comment on the robustness and expected transient performance of the closed loop when using the compensator $K(s) = 1$. [6]

c) Suppose that $K(s) = 1$. What is the steady–state value of the output for a unit step reference signal? Comment on the expected steady–state performance of the closed loop. [6]

d) Suppose that $K(s) = K_p$ is a proportional compensator. Use the Nyquist stability criterion to determine the number of unstable closed-loop poles for all $-\infty < K_p < \infty$. [6]

e) A dynamic compensator $K(s)$ is to be designed. In view of the answer to Parts b and c above, state whether you would recommend a phase–lead or a phase–lag compensator. Give a brief justification of your recommendation. [6]

3. Let $G(s) = \frac{1}{s(s+2)}$ and consider the feedback loop shown in Figure 3.1 below.

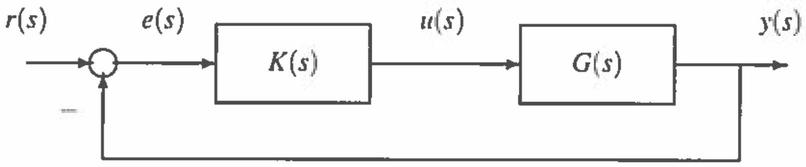

Figure 3.1

A PD compensator is required such that the following specifications in response to a step reference signal are satisfied:

(S_1) The response is critically damped.

(S_2) The settling time is 1 s. (*Assume the settling time is 4 times the time constant.*)

a) Sketch the root locus of $G(s)$. [4]

b) Suppose that $K(s) = K_p$ is a proportional compensator.

i) Show that there exists a $K_p > 0$ such that the first specification (S_1) is satisfied and derive the value of K_p. [4]

ii) For this value of K_p, what is the approximate settling time? [4]

c) Find the location of the closed-loop poles that achieves the design specifications above. Can these specifications be achieved by a proportional compensator? Justify your answer. [4]

d) Design a PD compensator $K(s) = K(s+z)$, where $K > 0$ and $z > 0$, that achieves the design specifications. [6]

e) Draw the root locus of the compensated system $G(s)K(s)$. [4]

f) Suppose that $r(t) = t$. Find the steady-state error $\lim_{t \to \infty} e(t)$. [4]