**EE4-05**
**EE9-SO7**

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

MSc and EEE PART IV: MEng and ACGI

**TRAFFIC THEORY & QUEUEING SYSTEMS**

Thursday, 9 May 10:00 am

Time allowed: 3:00 hours

**There are FOUR questions on this paper.**

**Answer ALL questions.**

*All questions carry equal marks*

**Any special instructions for invigilators and information for candidates are on page 1.**

| | | |
|---|---|---|
| Examiners responsible | First Marker(s) : | J.A. Barria |
| | Second Marker(s) : | D.P. Mandic |

**Special instructions for students**

1. Erlang Loss formula recursive evaluation:

$$E_N(\rho) = \frac{\rho E_{N-1}(\rho)}{N + \rho E_{N-1}(\rho)}$$
$$E_0(\rho) = 1$$

2. Engset Loss formula recursive evaluation (for a fixed $M$ and $p = \alpha/1+\alpha$ ):

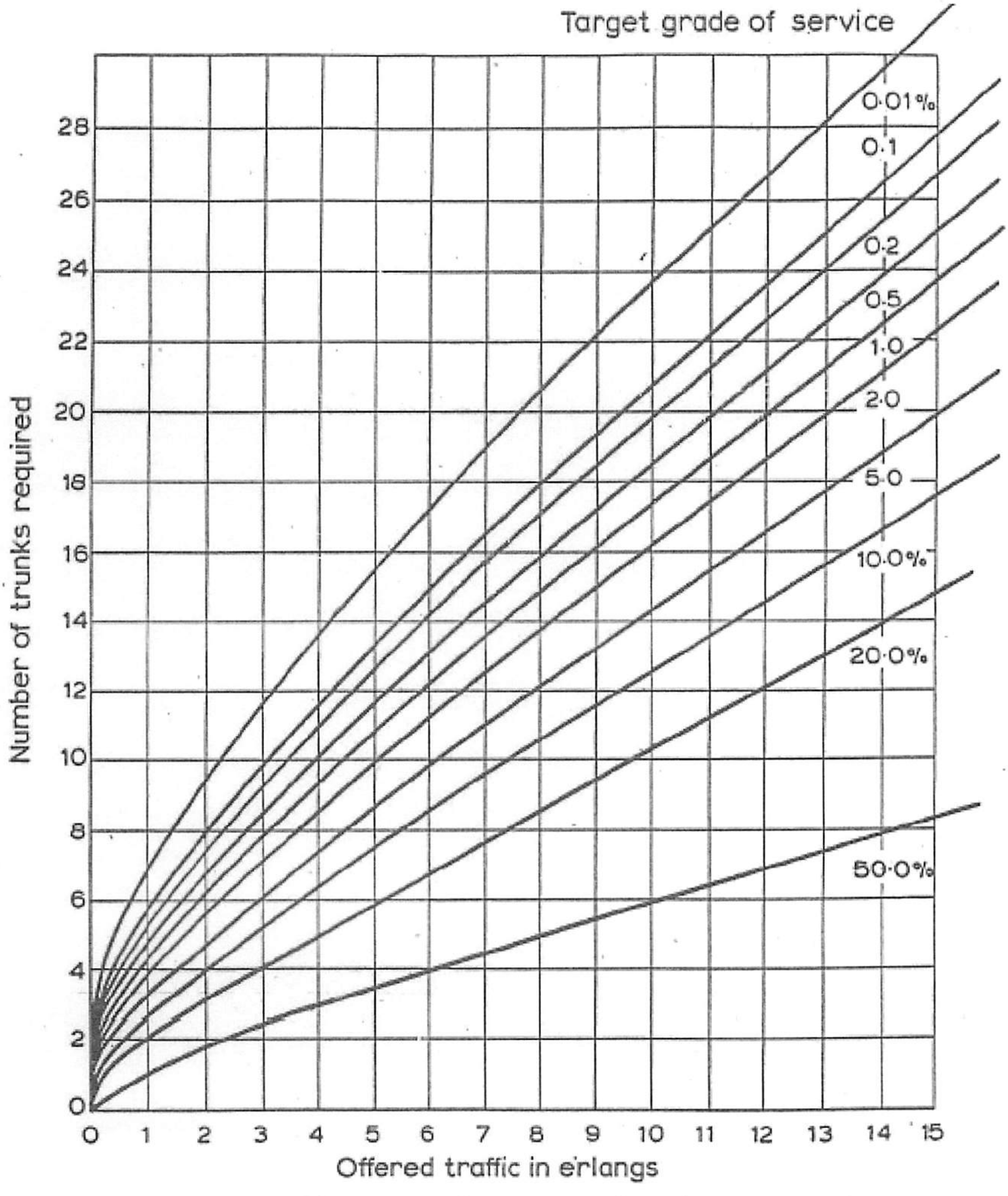
$$e_N = \frac{(M - N + 1)\alpha e_{N-1}}{N + (M - N + 1)\alpha e_{N-1}}$$
$$e_0 = 1$$
$$\alpha = \lambda/\mu$$

3. Traffic capacity on basis of Erlang B formula (next page).

Note: for large $\rho$ , $N$ is approximately linear: $N \approx 1.33\rho + 5$

4. Expected residual time

$$E[R] = \frac{1}{2}\lambda E[S^2]$$




*Traffic capacity on basis of Erlang B. formula.*

## The Questions

1.

a)

i) Describe in your own words a Poisson traffic source in terms of its probability of generating demand.

[3]

ii) The system shown in Fig. 1.1 is composed of M identical Poisson sources (mean arrival rate $\lambda$ ) which are offering traffic to an N channel link.

Derive the equilibrium traffic distribution if M >> N. Assume full availability. State clearly all the assumptions made.

[5]

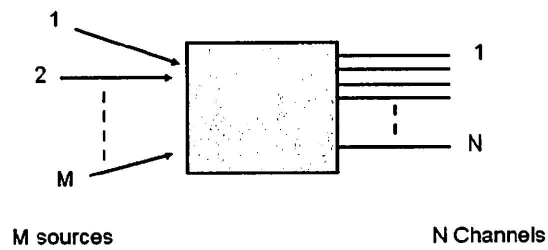



*Figure 1.1*

b) Two buildings are connected to the same public switching exchange via a shared 4-channel access link.

One building has 200 telephone lines installed which offer a traffic level of 0.2 Erlangs per non-busy telephone.

The other building has only 10 telephone lines installed which offer a traffic level of 0.01 Erlangs per non-busy telephone.

i) Derive the Birth/Death (B/D) model for the system described. Carefully state all the assumptions in your derivations.

[4]

ii) Derive the equilibrium standard equivalent finite-source model. Clearly show all the steps of your derivations.

[5]

iii) How many equivalent sources $\overline{M}$ have the standard equivalent finite-source model ?

[3]

2.

a) Pure chance traffic is offered to an M-channel communications link, denoted by Link 1. When Link 1 is saturated the overflow traffic is fed to an N-channel link, denoted by Link 2; as shown in Figure 2.1.

i) Derive the state transition diagram of the 2-dimensional Markov chain representing the system described above.

*Note:* Assume that the offered traffic is pure chance traffic with parameters ($\lambda, \mu$).

[4]

ii) If you know the following parameters of the system: $\lambda = 180$ (calls/hour); $1/\mu = 4$ (m); $M = 12$, and $N = 8$.

Determine:

- the mean traffic carried on Link 1. [2]
- the mean traffic carried on Link 2. [2]
- the call congestion for Link 2 (i.e. the fraction of calls offered to Link 2 which get blocked) [2]

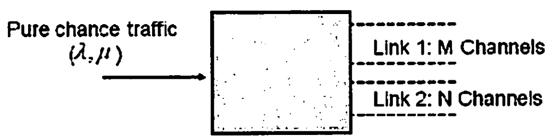



*Figure 2.1*

b) Consider a single-channel packet transmission link with a FIFO input buffer. The transmission rate of the link is 64 (kbits/s).

The arrival stream of packets is Poisson with mean arrival rate of 80 (packets/s) and the length of the packet is geometrically distributed with mean packet length of 700 bits.

i) Determine the probability that the packet will not have to wait for transmission.

[5]

ii) Determine the probability that a packet will have to wait for more than 20 (ms) before its transmission starts.

[5]

3.

a) For the non-preemtive priority queue shown in Fig. 3.1 derive the expected waiting time of a class $j$ arrival.

[5]

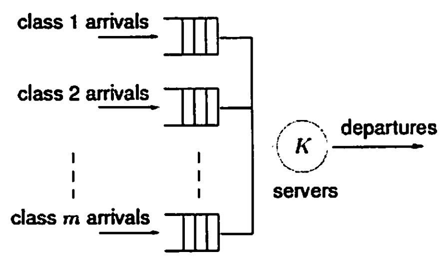
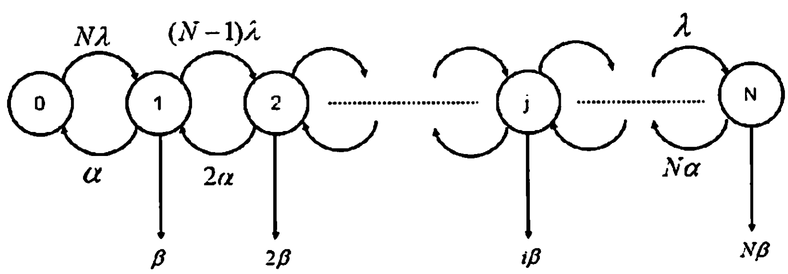



***Figure 3.1.***

b) For the N Independent ON-OFF voice multiplexer model shown in Fig. 3.2:

i) Identify the underlying ON-OFF single voice model. [3]

ii) Using the identified ON-OFF single voice model derive the probability that $j$ out of $N$ sources are active. [4]




*Note:* $\beta$ (packets/s) is the rate at which voice packets are being offered to a multiplexer link.

***Figure 3.2.***

c) In the context of a Broadband network answer the following questions.

i) Define the equivalent capacity of a set of *connections* multiplexed on a link. [3]

ii) Describe two equivalent capacity approximations known to you if the measure of link load is loss of packet or buffer overflow probability. Discuss what characteristics of the *connections* are being captured by each one of the identified approximations. [5]

4.

a) A 24-channel link is being offered on average 420 (calls/hour). The calls are on average 2 (m) long.

i) Determine the call blocking probability $B_c$ of the link described. [4]

ii) Calculate the mean carried traffic. [3]

b) Consider a multiprocessor system consisting of ***n*** processors whose base model is shown in Fig 4.1. Each processor has a constant rate of failure = $\lambda$ (failures/hour) and a coverage factor c.

i) Explain what the state space {0,1, ...3} is representing. [2]

ii) Is this system repairable or non-repairable ? Clearly argue your answer. [2]

iii) Define a suitable reward structure to analyse the system availability. [3]

iv) Modify the base model of Fig. 4.1. to consider the following two maintenance policies:

- If the system is in any state *j* in which one or more processors are in faulty conditions, reconfigure the systems to state *j-1*. Continue to reconfigure the system until all processors of the system are fully operational. [3]

- If the system fails (i.e. no processor is operational) restart the system to its full capacity. [3]

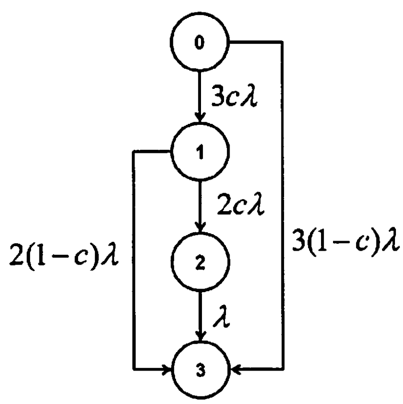



***Figure 4.1***

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Model Answers and Mark Schemes — First Examiner: J. Barria

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a) i) Poisson source: a source which, when idle, generates a demand in $(t, t+\Delta t)$ with probability $\lambda \Delta t$. 3

ii) Traffic distribution for M sources, N channels

$$(i\mu)\pi_i = (M-i+1)\lambda \pi_{i-1}$$

$$\pi_i = \binom{M}{i}\alpha^i \pi_0 \qquad \alpha = \lambda/\mu$$

$$\pi_0 = \sum_{j=0}^{i_{max}} \binom{M}{j}\alpha^j \qquad \alpha = \frac{p}{1-p}$$

for $M \le N$ $\quad \pi_i = \binom{M}{i} p^i (1-p)^{M-i} \quad 0 \le i \le M$

for $M > N$ $\quad \pi_i = K\binom{M}{i} p^i (1-p)^{M-i} \quad 0 \le i \le N$

and K = normalising constant

For $M \gg N$

$$\binom{M}{i} p^i (1-p)^{M-i} \sim \frac{Mp}{i!} e^{-Mp}$$

for $i \ll M$

therefore the distribution is truncated Poisson 5

2

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Q1 b)

$\alpha$ = offered traffic per non busy source

Assumptions

- Exponential holding times
- identical holding time distribution
- full-availability access

B/D model $\quad 0 < i \le 4$

$$(i\mu)\pi_i = [(M_1 - i + 1)\lambda_1 + (M_2 - i + 1)\lambda_2]\,\pi_{i-1}$$ 4

where $M_1 = 200 \quad (\lambda_1/\mu) = 0.01$

$M_2 = 10 \quad (\lambda_2/\mu) = 0.2$

$$(i\mu)\pi_i = [(M_1\lambda_1 + M_2\lambda_2) - (i-1)(\lambda_1+\lambda_2)]\,\pi_{i-1}$$

$$= [(M' - i + 1)\lambda']\,\pi_{i-1} \qquad 0 < i \le 4$$ 5

where $\lambda' = \lambda_1 + \lambda_2$

$$M' = M_1\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right) + M_2\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)$$

$$M' = \frac{400}{21} \sim 20$$ 3

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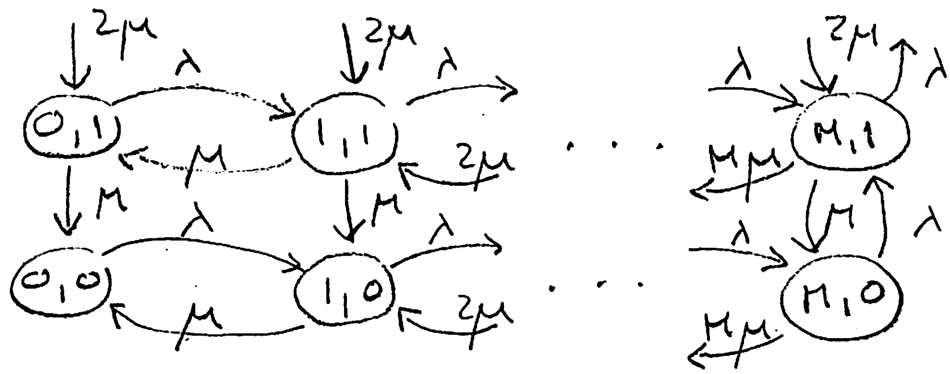
Q2 a) i)




4

ii)

offered Traffic to link 1

$$\rho = \frac{180}{60} \times 4 = 12 \text{ Erlangs}$$

call congestion on link 1

$$B_1 = E_{12}(12) = 0.2$$

carried traffic on link 1

$$\rho_1 = \rho(1 - B_1) = 9.6 \text{ Erlangs}$$ 2

call congestion for links 1 and 2

$$B_2 = E_{20}(12) = 0.01$$ 2

carried traffic on link 2

[Traffic on 1 + 2] − [Traffic on 1]

$$= \rho(B_1 - B_2) = 2.3 \text{ Erlangs}$$ 2

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Q2 b) Geometric message length distribution can be approximated by an exponential distribution

mean service time $= \frac{700}{64000}$ [s]

service rate $\mu = 91.4$ [$s^{-1}$]

Arrival rate $\lambda = 80$ [$s^{-1}$]

Offered traffic $\rho = \frac{\lambda}{\mu} = 0.875$ Erlang

i) $P[W=0] = P[\text{channel idle}] = 1-\rho = 0.125$ 5

ii) $P[W>0.02] = P[W>0]\, P[W>0.02 \mid W>0]$

$$= \rho\, e^{-\mu(1-\rho)0.02}$$

$$= 0.875\, e^{-0.23}$$

$$= 0.695$$

5

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Q3 a

$\lambda_K$ = arrival rate

$S_K$ = service time

$W_K$ = waiting time

$Q_K$ = queue-length seen on arrival

offered traffic in class $K$ : $\rho_K = \lambda_K E[S_K]$

Total offered traffic : $\rho = \sum_{k=1}^{m} \rho_K$

stability condition $\sum_{k=1}^{m} \rho_K < 1$

Using mean value analysis, let

$R$ = residual service time seen by an arbitrary arrival

For class 1 arrivals

$$E[W_1] = E[R] + E[Q_1]E[S_1]$$
$$= E[R] + \lambda_1 E[W_1] E[S_1]$$

$$E[W_1] = \frac{E[R]}{1-\rho_1}$$

For class 2 arrivals

$$E[W_2] = E[R] + (E[Q_1]E[S_1] + E[Q_2]E[S_2]) + (\lambda_1 E[W_2]) E[S_1]$$

$$E[W_2] = \frac{E[R]}{(1-\rho_1)(1-\rho_1-\rho_2)}$$

...

$$E[W_K] = \frac{E[R]}{(1-\sigma_{K-1})(1-\sigma_K)} \; ; \; \sigma_{K-1} = \sum_{i=1}^{K-1} \rho_i$$ 5

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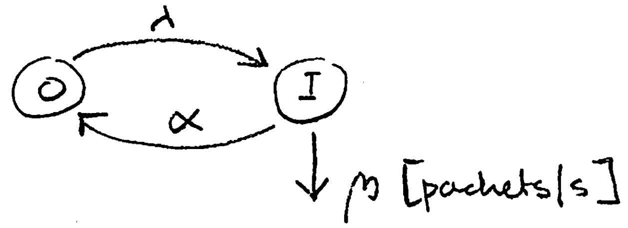
Q3 b)




3

For an N independent ON-OFF voice multiplexer the probability of j out of N source active:

$$\Pi_j = \binom{N}{j}\left(\frac{\lambda}{\lambda+\alpha}\right)^{j}\left(\frac{\alpha}{\lambda+\alpha}\right)^{N-j}$$

4

c) i) The equivalent capacity of a number of connections multiplexed on a link is defined as the amount of bandwidth required to achieve a desired grade of service, for example, buffer overflow probability. 3

ii) - The fluid flow approximation estimates the equivalent capacity when the impact of individual connections characteristic is critical. 2

- The stationary approximation estimates the bandwidth requirements when the effect of statistical multiplexing is of significance. 3

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Q4 a) i) calling rate 420 [calls/h] = 7 [calls/m]

mean call duration = 2 [m]

offered traffic = 7 × 2 = 14 Erlangs

$B_C = E_{24}(14) = 0.005$ (from chart) 4

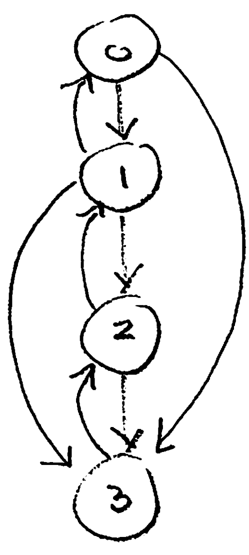
ii) carried traffic = $14(1 - B_C) = 13.93$ Erlang 3

Q4 (b) i) The state space represents the number of faulty processors 2

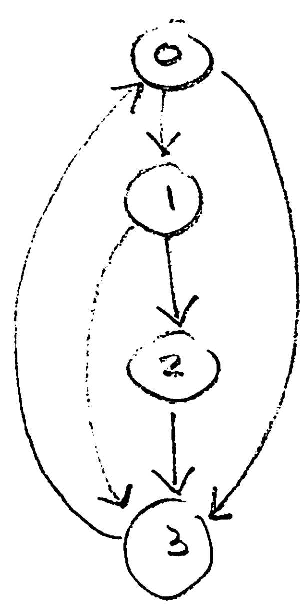
ii) The system is non-repairable as there is no rate of transition between a state N and N-i (i ≤ N). 2

iii) A suitable reward structure when analysing system availability can be to assign a reward 1 to all operational states and reward 0 to the system faulty state (state = 3) 3

iv)




(3)




(3)