Information for Candidates

Switch-Mode Power Supplies

Voltage ratio equations for buck SMPS $\frac{V_o}{V_i} = \delta$ or $\frac{V_O}{V_I} = \frac{1}{1 + \frac{2fLI_O}{V_I \delta^2}}$

Voltage ratio equations for boost SMPS $\frac{V_o}{V_i} = \frac{1}{1-\delta}$ or $\frac{V_o}{V_i} = \frac{1}{1 - \frac{V_i \delta^2}{2fLI_i}}$

Three-Phase Systems

Line Voltages and Current

Star $V_L = \sqrt{3} V_P \quad I_L = I_P$ Delta $V_L = V_P \quad I_L = \sqrt{3} I_P$

Power
$$P_{3\Phi} = 3 V_P I_P \cos(\phi) = \sqrt{3} V_L I_L \cos(\phi)$$

Induction Machines

$$\omega_S = \frac{\omega_E}{P} \qquad s = \frac{\omega_S - \omega_R}{\omega_S} \qquad T_{em} = \frac{3 I_R'^2 R_R'}{\omega_R}\left(\frac{1-s}{s}\right)$$

Photovoltaic Systems

$$I_{PV} = I_{Ph} - I_{AK} - I_{Sh} \qquad I_{AK} = I_0 \left[e^{\frac{V_{PV} + I_{PV} R_S}{K_I v_T}} - 1 \right]$$

Power Flow in Lines and Cables

Cable Parameters

$$R'_{LF} = \frac{1}{\sigma_C \pi r_C^2} + \frac{1}{\sigma_C 2\pi r_O t_O} \qquad L' = \frac{\mu_0}{2\pi} \ln\left(r_O / r_C\right) \qquad C' = \frac{2\pi \varepsilon_0 \varepsilon_{RI}}{\ln\left(r_O / r_C\right)} \qquad G' = \frac{2\pi \sigma_I}{\ln\left(r_O / r_C\right)}$$

OHL Parameters (approximate form)

$$R'_{LF} = \frac{2}{\sigma_C \pi r_C^2} \qquad L' = \frac{\mu_0}{\pi} \ln\left(d / r_C\right) \qquad C' = \frac{\pi \varepsilon_0 \varepsilon_{RI}}{\ln\left(d / r_C\right)} \qquad G' = \frac{\pi \sigma_I}{\ln\left(d / r_C\right)}$$

Power Flow (full form)

$$P_S = \frac{V_S^{\,2}}{Z_{SR}} \cos(\theta) - \frac{V_S V_R}{Z_{SR}} \cos(\theta + \delta)$$

$$Q_S = \frac{V_S^{\,2}}{Z_{SR}} \sin(\theta) - \frac{V_S V_R}{Z_{SR}} \sin(\theta + \delta)$$

Voltage Drop (approximate form)

$$\Delta V = |V_S| - |V_R| \approx \frac{RP_S + XQ_S}{|V_S|}$$

[Overall candidates performed well on this paper and achieved an average mark of 68.4%. The average marks on each question were: Q1 28.2 (71%), Q2 22.7 (76%) and Q3 17.5 (58%). Q2 is on a topic that most students find approachable and the good score is a consequence of good preparation. Q3 covered several sub-topics and the final part was deliberately difficult so the lower score is as expected.]

1. This question covers several topics and all parts should be attempted.

a) The electricity supply industry in European countries has been separated (or unbundled) into four functions: generation, transmission, distribution and supply. Describe what each of the functions is, stating clearly the key differences, and why the four functions have different market and regulatory arrangements. [6]

[Bookwork]
[1 mark for clear description of each sector; 2 marks for clarity on differences and market/regulations]

Generation: The function is generation is to produce electrical power by means of energy conversion. Typically in the UK this is done by a small number of large thermal or hydro plants. Each converts the mechanical energy at the output of a turbine into electrical energy. This conversion is achieved by use of a synchronous generator. The output of a synchronous generator is typically 10-20 kV which is stepped up via a transformer to transmission voltages typically 100 kV to 400 kV.

Electrical energy produced is sold in a forwards market, typically in half hour delivery slots. The market closes one hour or one day ahead of delivery. Counter parties in market are supply businesses (wholesalers) or large end-use customers.

Transmission: The function of transmission is ship large amounts of power over long distances from generation sources to load centres. Transmission systems are generally interconnected in a mesh structure to allow economically efficient provision of reserve services, balancing services, security of supply and stability. Voltages are typically at 100 kV to 400 kV to reduce the I^2R losses of transmission lines of which are generally overhead pylons. The transmission network is normally highly meshed (nodes connect to several others) and routes are composed of double circuits to give a high degree of resilience of circuit outages. Transmission is normally a regulated (local) monopoly because of the high barriers to entry. It applies use-of-system charges to generate revenue subject to controls by the regulator.

Distribution: The function of distribution is to take electrical energy from a bulk supply point and supply individual consumers. Distribution networks are usually a radial structure using underground cables and overhead lines with a normally unidirectional flow of power. The growth of distributed generation and renewable generation is changing this topology. Distribution voltages are typically 33kV, 11kV and 400V. For the most part most part, the DN has consumers but little generation. Distribution is again usually a regulated (local) monopoly that applies use-of-system charges.

Supply: Wholesalers who have bought in the main generation market sell onto domestic and other small consumers. Sometimes regionally based but in the UK there is a competitive market in supply.

[This questions was generally completed well by all candidates in terms of describing the technical functions of Generation, Transmission and Distribution. Some were not clear enough on Supply: this part of the industry has no responsibility for operating equipment; it is purely an energy trading business. Lack of clarity over which sectors are competitive markets and which are regulated monopoly services was a significant shortcoming.]

b) Electricity supply systems are based on alternating current, AC.

i) Explain why all countries of the world have settled on either 50 or 60 Hz rather than higher or lower frequencies. [4]

[Bookwork.]

A high frequency would enable transformers to be smaller but would cause very high voltage drops in inductive lines and high shunt currents in capacitive cables which would give operational difficulties. It would also require generators to rotate at higher speeds which in the early days of power systems would have put undue strain on bearings etc. A low frequency eases the voltage drop and shunt current problems but requires larger transformers and can cause lighting flicker. The best compromise between the factors (historically) is 50/60 Hz. [2 marks if at least one reason in each direction given, further 2 marks for fully developed answer].

ii) There are some cases where DC is preferred to AC for transmission of electrical power. Explain what these cases are. [4]

[Bookwork]

Long cables can have high shunt capacitive current that consumes all of the current rating of the conductors. Long OHL can have high series inductance that limits power flow and causes voltage drop [1 mark]. Operation at lower frequency helps reduce the capacitive current and the series voltage drops. The logical end point of that line of argument is to use DC [1 mark]. However, the cost is high of the AC/DC conversion required to operate a DC line in an otherwise AC system. This cost is only justified with sufficient cost saving in the cable/OHL. For cable this is true for distances greater than about 60 km, such as for wind farms in the North Sea; for OHL 800 km, such as between western and eastern China. [2 marks].

[Some gave answers about sectors other than transmission, such as local DC networks in buildings or ships. These got partial marks. The high cost of AC/DC converters needs to be mentioned otherwise why not use DC on short lines.]

c) An overhead line has a series impedance $3 + j\,8\ \Omega$ and is used to supply a single-phase load. The source voltage is 20 kV and the load is 10 MW at a power factor of 0.90 lagging. Calculate the change of voltage magnitude between sending and receiving ends of the line. You may assume that the power drawn from the source is almost the same as the power delivered to the load (i.e. the line losses are small). [6]

[Standard Calculation]

The standard approximation for voltage drop should be used with the further approximation that sending end power is almost the same are receiving end power.
First the reactive power needs to be found. [2 marks]

$$Q = \frac{P\sin(\cos^{-1}(PF))}{PF} = \frac{10\times10^{6}\sin(\cos^{-1}(0.9))}{0.9} = +4.84\ MVAr$$

Q will be positive for a lagging power factor

[4 marks for accurate answer]

$$\Delta V \approx \frac{RP_S + XQ_S}{|V_S|}$$

$$\Delta V \approx \frac{3\times 10M + 8\times 4.84M}{20k} = 3.44kV$$

[Those who knew this approximation (which is in the formula sheet) gave good answers. Occasionally, candidates treated the 10 MW as the apparent power not the real power; the units confirm what was intended. Some candidates obtained Q as negative. A lagging power factor means that I lags V. That makes the angles positive since $\Phi=\angle V-\angle I$ and with a positive angle, $\sin(\Phi)$ is positive also.

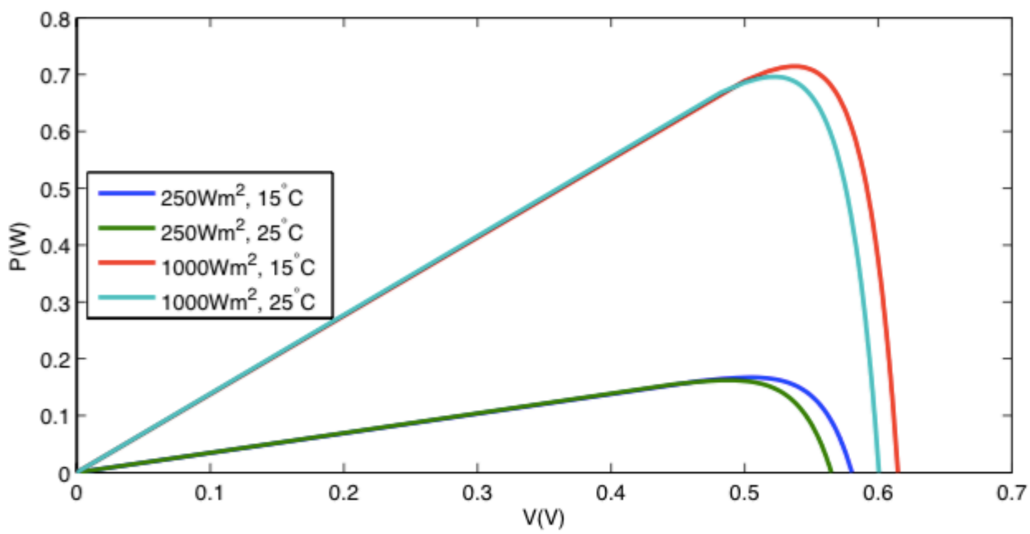
It is possible to take other approaches, such as estimating the current and finding the line voltage drop but care is then needed in finding the difference in voltage magnitudes that the question requests.]

d) Consider a typical photovoltaic panel.

i) Sketch the shape of the power versus voltage curve of a typical photovoltaic panel. [3]

[Bookwork]
Blue curve in figure below is a suitable example. Key features: starts from the origin; increases approximately linearly with voltage until turning point and creases rapidly to zero [1 mark for each figure].

[Shapes were often poor, especially to the right of the peak.]



ii) Add to the sketch two additional curves for an increase in illumination and for an increase in temperature. [4]

[Bookwork]
Red curve in figure above is a suitable example of increased illumination. Key features: greater initial slope; higher peak power at greater voltage. [2 marks]
Green curve in figure above is a suitable example of increased temperature. Key features: little change in slope; slightly lower peak power at lower voltage. [2 mark]

[Most answers were very good, occasionally the temperature effect was incorrectly reversed.]

iii) Explain why a perturb-and-observe maximum power point tracking algorithm is used with photovoltaic panels. [3]

[Bookwork]
Operating at the peak of the P v. V curve is important for making best use of the panel. Identifying the correct voltage in an open-loop fashion is not realistic because determining the temperature and insolation experienced by the panel is difficult (and aging, dirt and shadows complicate matters further). Closed-loop operation is not a regulation task since one does not know the reference power. It is instead a real-time search for the maximum of an unknown curve. Perturb-and-observe is a simple iterative process that moves the operating point a small distance and determines from observation if this was a helpful or unhelpful step and thus can locate the maximum of the curve.

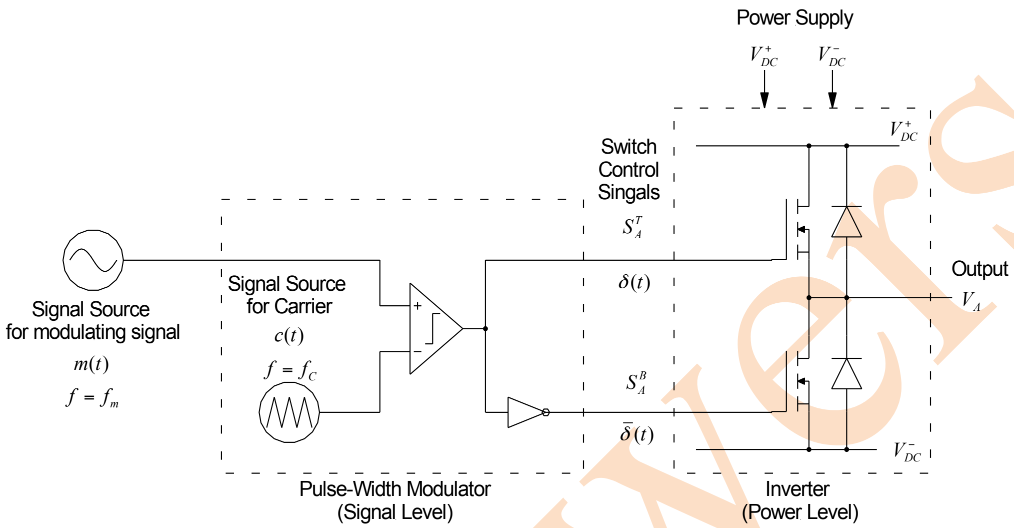
[Some answers concentrated on "how" and failed to explain "why".]

e) Consider a DC to AC converter for single-phase use.

i) Sketch a diagram of the DC to AC converter (either the 2-switch or 4-switch circuit). [3]

[Bookwork]

Key features: pair of transistors in series across DC supply and provided with anti-parallel diodes. AC output is taken from mid-point of transistors. Transistors are turned on and off by a modulator/control circuit.



[Common mistake was to reverse direction of one of the two diodes. They should both point toward the positive rail. At least some of the circuitry on the gate-side of the Mosfets was needed.]

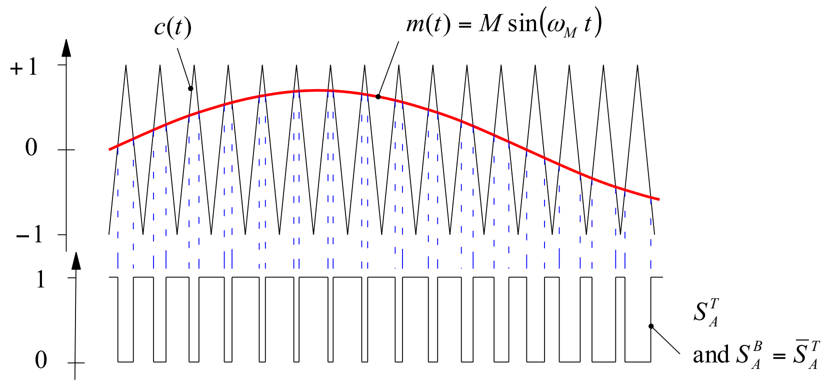
ii) Describe its operation. [5]

[Bookwork]

Transistors are turned on and off in anti-phase to connect AC output node to positive or negative end of the DC supply alternately. Modulating the proportion of time connected to each voltage varies the (shot-term) average voltage at the output.

$$\begin{aligned}\bar{\bar{v}}_A(t) &= \delta(t)V_{DC} - (1-\delta(t))V_{DC}\\ &= m(t)V_{DC}\end{aligned}$$

This is normally done with a pulse-width modulator working with a sinusoidal reference waveform and a triangle-wave carrier.



iii) Why might the basic circuit sometimes be supplemented by an LC filter? [2]

[Bookwork]

The AC output voltage will have a dominant frequency component that matches the reference waveform but will have further components at or close to the switching frequency and multiples of that switching frequency. If the load is inductive, these high frequency voltages drive very little current and are of no consequence for loads like motors. For more sensitive loads and non-inductive loads, it might be necessary to remove these components with a filter. This needs to be a passive filter (because of the power processing required) and not resistive (to avoid losses) so an LC filter is chosen.

2.
A **buck** switch-mode power supply, SMPS, has been designed to provide a 5 V output voltage from an input voltage that is nominally 9 V but can vary by ±1.5 V. The output current will be in the range 0.1 A to 2.0 A. A 220 µF capacitor (with an ESR of 200 mΩ) and a 100 µH inductor have been used. The switching frequency is 250 kHz.

[This questions was generally completed well by candidates, it had some unusual elements but most candidate could think them through.]

a) Calculate the range of duty-cycle required. [3]

[Standard calculation]

$$\delta = \frac{V_O}{V_I}$$

$$\delta_{Max} = \frac{5}{7.5} = 0.667$$

$$\delta_{Nom} = \frac{5}{9} = 0.555$$

$$\delta_{Min} = \frac{5}{10.5} = 0.476$$

b) For operation at the nominal input voltage, estimate the output voltage ripple. [8]

[Standard calculation]

Voltage ripple is (approximately) the sum of two terms; one from the capacitance and one from the resistance of the capacitor. Both depend on the current ripple.

Standard formulae for two voltage terms [3 marks each]

$$i_C = i_L = \frac{V_{I-Nom} - V_O}{L} \frac{\delta}{f} = \frac{9-5}{100 \times 10^{-6}} \frac{0.555}{250 \times 10^3} = 0.0889 A$$

$$V_R = \Delta i_L R_{ESR} = 0.0889 \times 0.555 = 17.8 mV$$

$$\Delta v_C = \frac{\Delta i_L}{8 f C} = \frac{0.0889}{8 \times 250 \times 10^3 \times 200 \times 10^{-6}} = 0.20\ mV$$

[2 mark for final accuracy]

Total ripple voltage is approx. the sum of these two, i.e., 18 mV.

[The most common error was to calculate only the capacitive ripple.]

c) Determine whether the SMPS remains in continuous conduction across the full operating range. [7]

[Standard equation with added difficulty of dealing with variation of input voltage]

More than one approach possible. Could separately check both ends of voltage range or first find the worst case. Worst-case approach shown here.

Ripple must be less than twice lowest output current to avoid discontinuous operation. [2 marks]

Need to find worst case input voltage or check both extremes.

$$\Delta i_L = \frac{V_I - V_O}{L} \cdot \frac{\delta}{f} = \frac{V_I - V_O}{L} \cdot \frac{V_O / V_I}{f} = \frac{V_O}{f\,L}\left(1 - \frac{V_O}{V_I}\right)$$

Largest ripple occurs with largest input voltage. [2 marks]

Check the 10.5 V case.

$$\Delta i_L = \frac{V_O}{f\,L}\left(1 - \frac{V_O}{V_I}\right) = \frac{5}{250\times10^3 \times 100\times10^{-6}}\left(1 - \frac{5}{10.5}\right) = 0.105\,A$$

This less than twice the minimum current and so continuous conduction is ensured in all cases. [3 marks]

[Some candidates only considered the range of output current and not the range of input voltage which lost 3 marks]

d) Consider now that the semiconductor devices used in the Buck SMPS are not perfect. The diode has a forward voltage drop during conduction, $V_{AK(on)}$ of 0.7 V. The switch used is a MOSFET with an on-state resistance, $R_{DS(on)}$ of 200 mΩ. Estimate the power loss in the SMPS at maximum output current and nominal input voltage. [6]

[Standard Calculations]

$\delta = 5/9 = 0.556 \quad 1 - \delta = 0.444$

Diode loss

$P_{Diode} = I_{AK} V_{AK(on)} (1 - \delta) = 2 \times 0.7 \times 0.444 = 0.622\ W$

Mosfet loss

$P_{Mosfet} = I_{DS}{}^2 R_{DS(on)} \delta = 2^2 \times 0.2 \times 0.556 = 0.445\ W$

[The most common error was to use an incorrect current, such as the input current or to calculate an average current. Both devices carry the inductor current while they are on. In terms of averaging, these equations calculate the power loss while the device is on and then average the power loss over the cycle by multiplying by the duty-cycle. For that reason the current does not need to be averaged.

The question did not make explicit that switch loss was to be ignored (and should have done). Some candidates tried to find ways to calculate switching loss but there was no data to enable this.]

e) By noting that the change of inductor current in the on-time plus that in the off-time sum to zero in steady state, derive an expression for the output voltage as a function of input voltage and duty cycle that accounts for the semiconductor voltage drops. You may assume continuous inductor current conduction. [6]

[Non-standard derivation]

$$\Delta i_L{}^{On} + \Delta i_L{}^{Off} = 0$$

$$\Delta i_L{}^{On} = \delta T \frac{V_I - V_{DS(On)} - V_O}{L}$$

$$\Delta i_L{}^{Off} = (1 - \delta) T \frac{-V_{AK(On)} - V_O}{L}$$

$$V_O = V_I \delta - \left(I_L R_{DS(On)} \delta + V_{AK(On)} (1 - \delta)\right)$$

[Some candidates included the voltage drops and then said they were negligible and set them to zero to obtain the standard formula. This was given 2 marks. A common error was to include V_{AK} as a positive term. Consider where the anode and cathode of the diode are in the circuit to see why this is incorrect or consider that the diode voltage drop would not be acting to increase the current.]

3. This questions addresses several aspects of 3-phase systems and induction machines.

a) A typical generator used in a power station has the following features. In each case explain why these features are chosen.

i) The stator and rotor are formed of laminated sheets of steel with a small air-gap between stator and rotor [3]

[Bookwork – 1 mark for each of the following points]

- Desire to produce field flux with as small a winding or magnet as practical and therefore need good magnetic path and steel provides this (and is a workable material).
- Laminations are used to make the flow of eddy currents more difficult and reduce the power loss associated with eddy currents.
- Air-gap is need for clearance between rotating rotor and stationary stator but is a significant reluctance and so is kept short.

ii) A large number of coils are arranged around the inside circumference of the stator and connected into three phase windings [4]

[Bookwork – 1 mark for each of the following points]

- A distributed winding (coils in many slots) makes good use of machine space
- Distributed coils are easier to cool than concentrated windings.
- Individual connections of each coil to a load would be too complex in terms of switches and trnasformers so number of external connections to loads is kepts small (but not too small so as to avoid voltage ancaleation between coilos of a phase).
- Series connection also (by careful design) can cancel harmonic distortion in phase shifted coil voltages to leave phase voltages near sinusoidal.

b) A 3-phase supply has a line voltage of 400 V. It is connected to a load composed of three impedances in star connection, each with a value of 25 + *j* 10 Ω.

i) Calculate line current that flows and the real and reactive power consumed by the load. [4]

[Standard calculation]
Note that phase voltage = line voltage/ √3 = 230.9 V for star connection
Calculate the line current (as identical to phase current [1 mark]

$$Z = 25 + j10 = 26.926.2\angle 21.80°$$

$$I_L = I_P = \frac{V_P}{Z} = \frac{230.9}{26.926\angle 21.80°} = 8.577\angle -21.80° \quad A$$

$$or \quad 7.96 = j3.18 \quad A$$

[Occasional error was to line voltage not phase voltage here. This is a start connection.]

Calcuate power [1 mark for method; 1 mark for accurte answers; 1 mark for correct units]

$$P = \sqrt{3} V_L I_L \cos(\phi) = \sqrt{3} \times 400 \times 8.577 \times \cos(21.80°) = 5.52\ kW$$

$$Q = \sqrt{3} V_L I_L \sin(\phi) = \sqrt{3} \times 400 \times 8.577 \times \sin(21.80°) = 2.20\ kVAr$$

[Occasional error was to use -21.80°; it should be + because it is the angle of *V* with respect to *I*, which is also the angle of *Z*.]

ii) The load impedances are subject to some manufacturing tolerance. For the case of one impedance being 10% below its nominal value and another being

10% above, calculate the magnitude of the current that would flow in the neutral connection. [4]

[Uncommon calculation]

Several approaches possible, also several assumptions about which load is on which phase but neutral current magnitude should be the same

Calculate the three phase currents [2 marks]

$$I_A = \frac{V_P}{Z} = \frac{230.9}{26.926\angle 21.80°} = 8.577\angle -21.80° = 7.96 - j3.19 \quad A$$

$$I_B = \frac{V_P}{Z} = \frac{1}{0.9} \times \frac{230.9\angle -120°}{26.926\angle 21.80°} = \frac{1\angle -120°}{0.9} 8.577\angle -21.80° = 9.530\angle -141.80° = -7.49 - j5.89 \quad A$$

$$I_C = \frac{V_P}{Z} = \frac{1}{1.1} \times \frac{230.9\angle +120°}{26.926\angle 21.80°} = \frac{1\angle +120°}{0.9} 8.577\angle -21.80° = 7.797\angle +98.80° = -1.11 + j7.72 \quad A$$

Sum to find neutral current [2 marks]

$$I_N = -\left(I_A + I_B + I_C\right) = 0.638 + j1.361 \quad A$$

$$\left|I_N\right| = 1.50 \quad A$$

[Most common error was to not include the 120° phase shift between the phase voltages and then not have the correct vector sum of phase currents.]

c) A 3-phase, 2 pole-pair, induction machine is star-connected and provided with a supply with a line voltage of 400 V, 50 Hz.

A series of measurements were made while the machine was driving a particular mechanical load:

rotational speed	1,450 r.p.m.
stator current (magnitude)	135.1 A
power factor	0.82 lagging
referred rotor current (magnitude)	117.7 A

The winding parameters of the equivalent circuit of the machine, referred to the stator, are:

stator resistance	0.04 Ω,
stator leakage reactance	0.4 Ω,
referred rotor resistance	0.055 Ω,
referred rotor leakage reactance	0.4 Ω.
magnetising resistance	20 Ω.
magnetising reactance	8 Ω.

For the particular mechanical load under investigation, calculate the following:

i) the slip [2]

[Standard calculation]

$$n_S = \frac{50 \times 60}{2} = 1{,}500 \quad rpm$$

$$s = \frac{1500 - 1450}{1500} = \frac{1}{30} = 0.0333$$

[Declaring the answer to be 0.03 is not accurate enough, that is an error of 10% carried forward into the subsequent stages of calculation]

ii) the mechanical output power and electro-magnetic torque [5]

[Standard calculation]

[3 marks for accurate calculation of power]

$$P_{EM} = 3|I'_R|^2 R'_R\left(1/s - 1\right)$$
$$= 3 \times 117.7^2 \times 0.055 \times \left(1/0.0333 - 1\right)$$
$$= 66.29 kW$$

[2 marks for accurate final answer and proper unit]

$$T = \frac{P_{EM}}{\omega_R} = \frac{66.29k}{1450 \times 2\pi/60} = 436.5 \quad Nm$$

iii) the efficiency [3]

[Standard calculation]

Output power already calculated; input power needs to be calculated here.

[1 mark for method; 1 for answer]

$$P_{In} = 3V_S I_S \cos(\phi) = 3 \times 400 / \sqrt{3} \times 135.1 \times 0.82$$
$$= 76.75 \quad kW$$

[1 mark for accurate final answer]

$$\eta = \frac{P_{Mech}}{P_{In}} = \frac{66.29k}{76.75k} = 86.4\%$$

iv) the iron loss [5]

[Unfamiliar route to answer required, more than one route possible.]

Route 1 – find voltage across R_M required for calculation of iron loss.

This will have to be calculated from stator voltage minus drop across stator impedance.

[1 mark for method; 1 for accurate answer]

$$V_{AG} = V_S - (R_S + jX_S) = 230.9 - (0.04 - j0.4) \times 135.1 < cos^{-1}(0.82) = 195.6 - j41.2 \;\; V$$

$$|V_{AG}| = 199.9\ V$$

Power loss in R_M from standard equation.

[1 mark or method; 2 for accurate answer]

$$P_{Fe} = 3\frac{V_{AG}^2}{R_M} = 3\frac{199.9^2}{20} = 5.99 \;\; kW$$

Route 2 – as route 1 but find air-gap voltage by multiplying rotor current by rotor branch impedance

Route 3 – use current divider rule between 3 branches to find fraction of I_S that flows in R_M.

Route 4 – find total power loss and subtract all other known losses

$$P_{Fe} = P_{In} - P_{Mech} - 3{I'}_R^2 {R'}_R - 3I_S^2 R_S$$

[Candidates were inventive and several good answers given. One approach tried was to find the magnetising current by subtracting the rotor current from the stator current. The question stated these in magnitude form. Subtracting these without accounting for the angles is a mistake.]