UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1997

BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
MSc Degree in Computing Science
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute
Associateship of the Royal College of Science

PAPER M3.39 / I3.18

SIMULATION AND MODELLING

Monday, April 21st 1997, 10.00 - 12.00

Answer THREE questions

For admin. only: paper contains 4 questions

1 A Surrey-based communications firm wishes to evaluate a new one-way local-area network design which they have called the "Eshernet". Each device "thinks" for a while during which time it prepares its next message for transmission. When it is ready to transmit it first looks to see if the Eshernet is busy. If it is the device stalls and backs off for a randomly generated time after which it retries the transmission request. If not, it claims the Eshernet (i.e. makes it busy) and starts transmitting. At the end of the transmission the Eshernet is released (i.e. marked "not busy") and the device restarts. Each retry operates in the same way as the original request: if the Eshernet is still busy when the retry is attempted the device backs off again for a (different) randomly generated time. All devices are capable of receiving messages from the Eshernet at all times, regardless of whether the device is thinking (i.e. preparing its next message for transmission) or backed-off; any time incurred in processing a received message forms part of the measured think times of the deviceand so need not be considered separately.

Design a discrete-event simulator for an Eshernet with N attached devices which estimates the utilisation of the Eshernet and the mean number of retries required to complete each transmission. You may assume the existence of an event scheduler such that `schedule( e, p, t )` arranges for the function call `e( p )` to be made at simulated time `t`. You may also assume a server object which includes access functions for claiming and releasing a server and printing its utilisation at the end of the simulation. You may also assume the following support functions:

`thinktime()`	generates a sample device think time
`transtime()`	generates a sample transmission time
`backofftime()`	generates a sample back-off time

Your solution should identify the (global) state variables which are required to model the system state together with the measurement variables which are required to compute the two performance measures. You should also provide informal pseudocode for each event function and should state how the performance measures can be computed at the end of the simulation. *No other pseudocode is required.*

2 A particular processing plant handles two types of jobs: a proportion 100q% of jobs are *simple* jobs which are processed by a machine M_1 each requiring an average service time of μ_1^{-1} hours; the remaining 100(1-q)% of jobs are *complex* jobs, which are like simple jobs except they require additional processing by a machine M_2 which takes an average time of μ_2^{-1} hours. The arrival rate of jobs to the plant is λ jobs per hour and jobs waiting to be processed are queued up in very large storage areas which are well approximated by infinite capacity queues. The current set up is shown in the form of a queueing network in Figure 1 below.

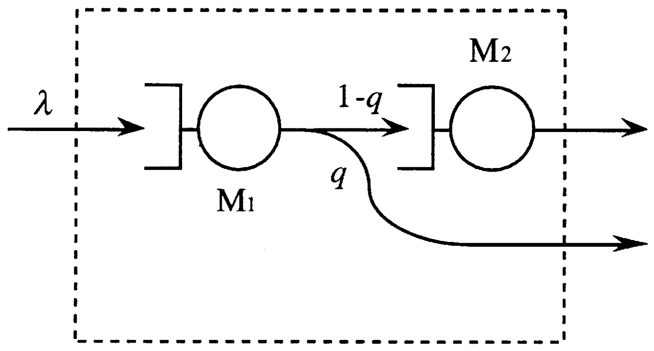


Figure 1

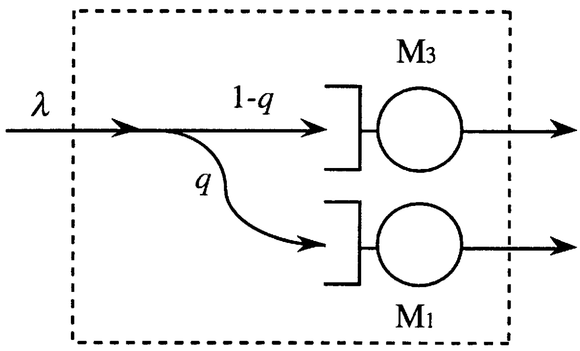


Figure 2

Let the state of the system be the pair (i,j), i,j>0, where i is the population of the queue for M1 and j that of M2, and let the equilibrium state probabilities be $p_{i,j}$, i,j>0.

a Draw the state transition diagram for this system showing the states (i,j), $0 \leq i,j \leq 2$. From this (or by any other means), write down the equilibrium balance equations for the system. What is the solution to these balance equations?

b The owner of the plant wants to reduce the average processing time of jobs and is seeking an alternative set up. She has been offered a new machine M_3 which is capable of processing a complex job in the same mean time as the existing plant (i.e. $\mu_1^{-1} + \mu_2^{-1}$ hours) and the idea is to trade in machine M_2 for machine M_3 and use M_1 and M_3 in combination as shown in Figure 2. The various parameters have the following values:

$$\lambda = 5 \quad \mu_1 = 10 \quad \mu_2 = 5 \quad q=0.5$$

Assuming that the arrival process is Poisson and that the service times of all three machines are exponentially distributed, compute the following equilibrium quantities for both set-ups:

i. The utilisation of each machine
ii. The mean number of jobs queueing for each machine (i.e. not including jobs currently being processed)
iii. The mean processing time for each job (i.e. the mean time each job spends inside the boxed region in the respective diagram).

On the basis of these calculations what recommendation would you make to the owner of the plant?

Turn over...

3a The *Weibull* distribution has a cumulative distribution function (cdf) given by

$$F(x) = 1 - \exp(- (x/\alpha)^{\beta})$$

and density function

$$f(x) = (\beta/\alpha) (x/\alpha)^{\beta-1} \exp(- (x/\alpha)^{\beta}$$

Describe *two* methods by which samples from a Weibull distribution can be computed. As part of your answer you should state any mathematical properties which your methods assume, although you are not required to prove them.

3b The time to failure of a number of electronic devices (in thousands of hours) has been measured and the results are summarised in the table below:

Interval	(0.0, 0.4]	(0.4, 0.8]	(0.8, 1.2]	(1.2, 1.6]	(1.6, 2.0]	(2.0, 2.4]	≥2.4
Frequency	414	1002	889	467	167	48	13

It is suspected that these times have a Weibull distribution and in order to test this the measured data has been used to estimate the parameters α and β of the distribution; these estimates are α =1.03, β =1.97. The null hypothesis:

H_0: The measured data is Weibull-distributed with parameters α =1.03, β =1.97

is thus proposed.

Perform a χ^2 test on the data to test this hypothesis *at the 10% significance level.*

4a Explain what is meant by *equilibrium* (or *steady-state*) behaviour and describe one method for estimating when equilibrium has been reached during a discrete-event simulation.

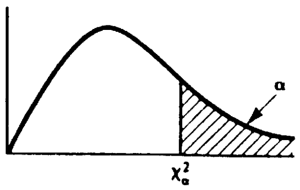
4b Explain how can the method of *batched means* be used to accumulate N *independent* estimates of some unknown quantity during a single simulation run. As part of your answer outline a method for implementing batch means within a discrete-event simulator.

4c Explain how N estimates of some unknown quantity (such as might be produced from a batched means simulation experiment) can be used to compute a *point estimate* and a *90% confidence interval* for the unknown quantity. How, statistically, could you halve the width of the 90% confidence interval?

4d A Post Office queueing system has an arrival process whose rate changes throughout the day. This process has been studied and a function $\lambda(t)$ has been produced which provides an accurate representation of the arrival rate t seconds after the office opens. The Post office opens at 9-00am (t=0) and closes at 5-00pm (t=28800). At both these times, $\lambda(t)=0$. Outline briefly, and in very general terms, how you would use $\lambda(t)$ to simulate this system in order to estimate the mean population of the queue at the end of each hour (i.e. at times 10-00am, 11-00am, ..., 5-00pm). Do not concern yourself with details of the service processes; you are *not* required to detail any simulation code.

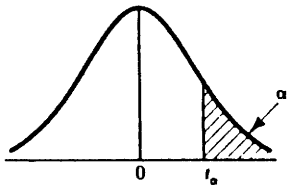
End of paper

PERCENTAGE POINTS OF THE CHI-SQUARE DISTRIBUTION
WITH ν DEGREES OF FREEDOM



ν	$\chi^2_{0.005}$	$\chi^2_{0.01}$	$\chi^2_{0.025}$	$\chi^2_{0.05}$	$t^2_{0.10}$
1	7.88	6.63	5.02	3.84	2.71
2	10.60	9.21	7.38	5.99	4.61
3	12.84	11.34	9.35	7.81	6.25
4	14.96	13.28	11.14	9.49	7.78
5	16.7	15.1	12.8	11.1	9.2
6	18.5	16.8	14.4	12.6	10.6
7	20.3	18.5	16.0	14.1	12.0
8	22.0	20.1	17.5	15.5	13.4
9	23.6	21.7	19.0	16.9	14.7
10	25.2	23.2	20.5	18.3	16.0
11	26.8	24.7	21.9	19.7	17.3
12	28.3	26.2	23.3	21.0	18.5
13	29.8	27.7	24.7	22.4	19.8
14	31.3	29.1	26.1	23.7	21.1
15	32.8	30.6	27.5	25.0	22.3
16	34.3	32.0	28.8	26.3	23.5
17	35.7	33.4	30.2	27.6	24.8
18	37.2	34.8	31.5	28.9	26.0
19	38.6	36.2	32.9	30.1	27.2
20	40.0	37.6	34.2	31.4	28.4
21	41.4	38.9	35.5	32.7	29.6
22	42.8	40.3	36.8	33.9	30.8
23	44.2	41.6	38.1	35.2	32.0
24	45.6	43.0	39.4	36.4	33.2
25	49.6	44.3	40.6	37.7	34.4
26	48.3	45.6	41.9	38.9	35.6
27	49.6	47.0	43.2	40.1	36.7
28	51.0	48.3	44.5	41.3	37.9
29	52.3	49.6	45.7	42.6	39.1
30	53.7	50.9	47.0	43.8	40.3
40	66.8	63.7	59.3	55.8	51.8
50	79.5	76.2	71.4	67.5	63.2
60	92.0	88.4	83.3	79.1	74.4
70	104.2	100.4	95.0	90.5	85.5
80	116.3	112.3	106.6	101.9	96.6
90	128.3	124.1	118.1	113.1	107.6
100	140.2	135.8	129.6	124.3	118.5

PERCENTAGE POINTS OF THE STUDENTS t
DISTRIBUTION WITH ν DEGREES OF FREEDOM



ν	$t_{0.005}$	$t_{0.01}$	$t_{0.025}$	$t_{0.05}$	$t_{0.10}$
1	63.66	31.82	12.71	6.31	3.08
2	9.92	6.92	4.30	2.92	1.89
3	5.84	4.54	3.18	2.35	1.64
4	4.60	3.75	2.78	2.13	1.53
5	4.03	3.36	2.57	2.02	1.48
6	3.71	3.14	2.45	1.94	1.44
7	3.50	3.00	2.36	1.90	1.42
8	3.36	2.90	2.31	1.86	1.40
9	3.25	2.82	2.26	1.83	1.38
10	3.17	2.76	2.23	1.81	1.37
11	3.11	2.72	2.20	1.80	1.36
12	3.06	2.68	2.18	1.78	1.36
13	3.01	2.65	2.16	1.77	1.35
14	2.98	2.62	2.14	1.76	1.34
15	2.95	2.60	2.13	1.75	1.34
16	2.92	2.58	2.12	1.75	1.34
17	2.90	2.57	2.11	1.74	1.33
18	2.88	2.55	2.10	1.73	1.33
19	2.86	2.54	2.09	1.73	1.33
20	2.84	2.53	2.09	1.72	1.32
21	2.83	2.52	2.08	1.72	1.32
22	2.82	2.51	2.07	1.72	1.32
23	2.81	2.50	2.07	1.71	1.32
24	2.80	2.49	2.06	1.71	1.32
25	2.79	2.48	2.06	1.71	1.32
26	2.78	2.48	2.06	1.71	1.32
27	2.77	2.47	2.05	1.70	1.31
28	2.76	2.47	2.05	1.70	1.31
29	2.76	2.46	2.04	1.70	1.31
30	2.75	2.46	2.04	1.70	1.31
40	2.70	2.42	2.02	1.68	1.30
60	2.66	2.39	2.00	1.67	1.30
120	2.62	2.36	1.98	1.66	1.29
∞	2.58	2.33	1.96	1.645	1.28