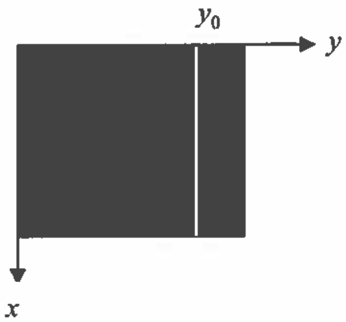
1. a) (i) Plot the image intensity.

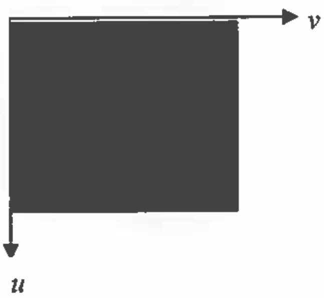



(ii) $F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j(\frac{2\pi}{M}ux+\frac{2\pi}{M}vy)}=\frac{1}{MN}\sum_{x=0}^{M-1}f(x,y_0)e^{-j(\frac{2\pi}{M}ux+\frac{2\pi}{M}vy_0)}$

$=\frac{1}{MN}\sum_{x=0}^{M-1}ce^{-j(\frac{2\pi}{M}ux+\frac{2\pi}{M}vy_0)}=\frac{1}{MN}ce^{-j\frac{2\pi}{M}vy_0}\sum_{x=0}^{M-1}e^{-j\frac{2\pi}{M}ux}$

$\sum_{x=0}^{M-1}e^{-j\frac{2\pi}{M}ux}=\frac{1-e^{-j\frac{2\pi}{M}uM}}{1-e^{-j\frac{2\pi}{M}u}}=\begin{cases}0 & u\neq 0\\ M & u=0\end{cases}$

$|F(u,v)|=\begin{cases}0 & u\neq 0\\ \frac{c}{N} & u=0\end{cases}$




$|F(u,v)|=\begin{cases}cN, & v=0\\ 0, & \text{otherwise}\end{cases}$

(iii) Compare the plots found in (i) and (ii) above.
**As seen a straight line in space implies a straight line perpendicular to the original one in frequency.**

b) Figure (c) is the right answer since it contains edges which are perpendicular to the edges of the original image. As we know, each image in space produces a perpendicular image in the amplitude of the DFT.

c) (i) The first image $f_1(x,y)$ has a solid horizontal edge. Its mean is $\frac{r_1+s_1}{2}$. The zero-mean version of it is $f_1(x,y)=\begin{cases}\frac{r_1-s_1}{2} & 1\leq x\leq M, 1\leq y\leq \frac{M}{2}\\ \frac{s_1-r_1}{2} & 1\leq x\leq M, \frac{M}{2}< y\leq M\end{cases}$. The second image $f_2(x,y)$ has a solid vertical edge. Its mean is $\frac{r_2+s_2}{2}$. The zero-mean version of it is

$$f_2(x,y)=\begin{cases}\frac{r_2-s_2}{2} & 1\le x\le M, 1\le y\le \frac{M}{2}\\ \frac{s_2-r_2}{2} & 1\le x\le M, \frac{M}{2}<y\le M\end{cases}$$. The variance of $f_1(x,y)$ is $\frac{r_1^2+s_1^2}{2}$. The variance of $f_2(x,y)$ is $\frac{r_2^2+s_2^2}{2}$. The covariance between the two images is zero (this is the mean of the product of the two images). This is because $f_1(x,y)$ is of the form $\begin{bmatrix} a \\ \cdots \\ -a \end{bmatrix}$ and $f_2(x,y)$ is of the form $[b \;\vdots\; -b]$ therefore $f_1(x,y)f_2(x,y)=\begin{bmatrix} ab & \vdots & -ab \\ \cdots & \vdots & \cdots \\ -ab & \vdots & ab \end{bmatrix}$. So the mean of $f_1(x,y)f_2(x,y)$ is zero. In that case the covariance matrix of the population is $C=\begin{bmatrix} \frac{r_1^2+s_1^2}{2} & 0 \\ 0 & \frac{r_2^2+s_2^2}{2} \end{bmatrix}$. The eigenvalues of the covariance matrix are $\frac{r_1^2+s_1^2}{2}$ and $\frac{r_2^2+s_2^2}{2}$. The images $g_1(x,y)$ and $g_2(x,y)$ are simply the zero mean versions of the original images.

(ii) There is no point of using the KL transform since it is obvious visually that the images are uncorrelated.

Question 2 – Answer

(i) The intensities of the two inner squares are very similar and therefore the inner pattern is <u>not</u> visible. It basically looks like a single square instead of $\boxed{\square}$

$$P(r_3) = \frac{64 \times 64/2}{256 \times 256} = \frac{1}{32}$$

$$p(r_2) = \frac{1}{32}$$

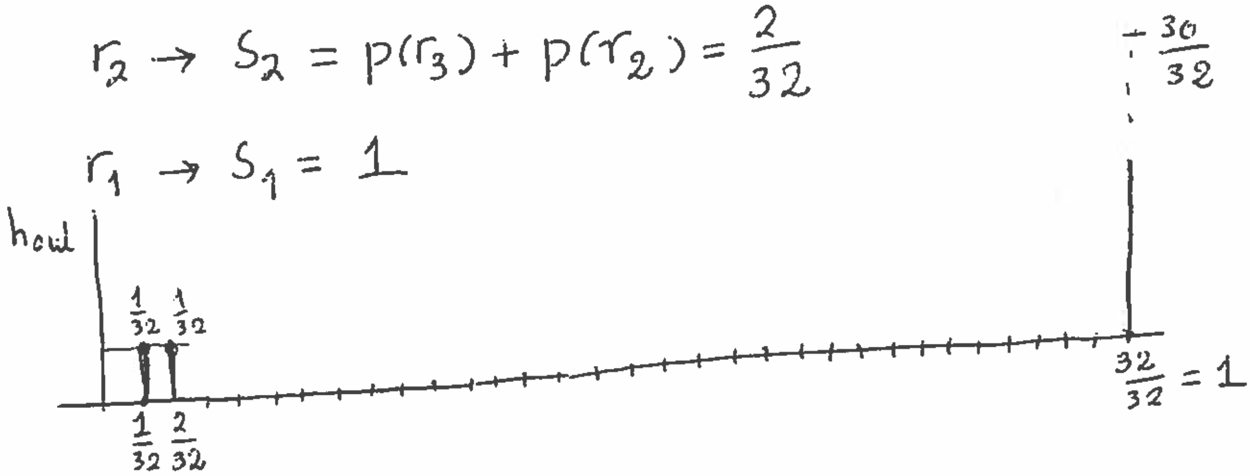
$$\Rightarrow p(r_1) = \frac{30}{32}$$

After histogram equalisation

$$r_3 \rightarrow S_3 = p(r_1) = \frac{1}{32}$$

$$r_2 \rightarrow S_2 = p(r_3) + p(r_2) = \frac{2}{32}$$

$$r_1 \rightarrow S_1 = 1$$



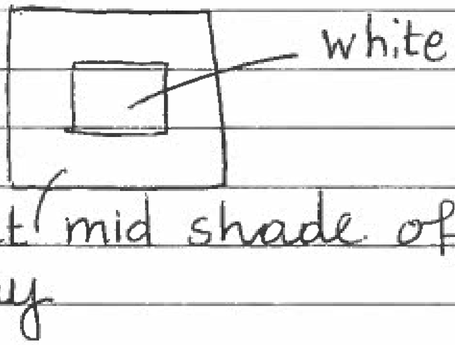

The inner pattern will still not be visible in the histogram equalised image

(ii) If we do local histogram equalisation the patch with the patter will perfectly fit in a scanning patch. For that patch we have

$p(r_3) = \frac{1}{2}$, $p(r_2) = \frac{1}{2}$

$r_3 \to s_3 = \frac{1}{2}$

$r_2 \to s_2 = 1$




The rest of the image will turn white.

Therefore, the inner pattern will be visible $\Rightarrow$

(iii) Adaptive (local) HE is definitely more beneficial

b) We assume that the images are extended

by zeros.

For the left image

borders {
black corners response : 0 (2 on total)
white corners response : $\frac{4}{9}$ (2 on total

white non border pixels next to the edge have response $\frac{6}{9} = \frac{2}{3}$
(6 on total)

black non border pixels next to the edge have response $\frac{3}{9} = \frac{1}{3}$
(6 on total)

top and bottom white pixel next to the edge : $\frac{4}{9}$
(2 on total)

top and bottom ~~white~~ black pixel next to the edge $\frac{2}{9}$
(2 on total)

border white pixels $\frac{2}{3} = \frac{6}{9}$ (10 on total)

border black pixels 0 (10 on total)

rest of white pixels : 12 / rest of black pixels 12

~~Total number of border pixels 28 / rest 64-28 = 36~~
36/2 = 18

response 1 response 0

| Intensities | Number of pixels | Probability |
|---|---|---|
| 0 | 24 | 24/64 |
| 2/9 | 2 | 2/64 |
| 3/9 | 6 | 6/64 |
| 4/9 | 4 | 4/64 |
| 6/9 | 16 | 16/64 |
| 1 | 12 | 12/64 |

For the right image

internal white pixels response $\frac{5}{9}$ (18 pixels)

internal black pixels response $\frac{4}{9}$ (18 pixels)

corner white $\frac{2}{9}$ (2 pixels)

corner black $\frac{2}{9}$ (2 pixels)

border white $\frac{3}{9}$ (12 pixels)

border black $\frac{3}{9}$ (12 pixels)

| Intensities | Number of pixels | Probability |
|---|---|---|
| 2/9 | 4 | 4/64 |
| 3/9 | 24 | 24/64 |
| 4/9 | 18 | 18/64 |
| 5/9 | 18 | 18/64 |

$\Rightarrow$ Histograms are different

Question 3 – Answer

a)

(i) Book work

(ii) $h(x,y) = \begin{cases} 1 & x=0,\ y=0 \\ 2 & x=0\ \ y=1 \\ 1 & x=0\ \ y=2 \end{cases}$

$$H(u,v) = \frac{1}{M^2}\sum\sum h(x,y)e^{-j\frac{2\pi}{M}(ux+vy)}$$

$$= \frac{1}{M^2}\left(\cancel{e^{-j\frac{2\pi}{M}0}}^{\,1} + 2e^{-j\frac{2\pi}{M}v} + e^{-j\frac{2\pi}{M}2v}\right)$$

$$= \frac{1}{M^2}e^{-j\frac{2\pi}{M}v}\left(e^{j\frac{2\pi}{M}v} + 2 + e^{-j\frac{2\pi}{M}v}\right)$$

$$= \frac{1}{M^2}e^{-j\frac{2\pi}{M}v}\left(2\cos\frac{2\pi}{M}v + 2\right)$$

$$H(u,v) = 0 \Rightarrow \cos\frac{2\pi}{M}v = -1$$

$$\frac{2\pi}{M}v = k\pi, \qquad k \text{ odd}$$

$$k=1 \quad \frac{2\pi}{M}v = \pi \Rightarrow v = \frac{M}{2}$$

$$k=3 \quad \frac{2\cancel{\pi}}{M}v = 3\cancel{\pi} \Rightarrow v = \frac{3M}{2} \quad \text{invalid}$$

since $v \in [0, M-1]$

(iii) book work

(iv) $C(u,v) = \frac{1}{M^2}e^{-j\frac{2\pi}{M}v}\left(2\cos\frac{2\pi}{M}v - 2\right)$

$$\frac{2\pi}{M}v = k\pi, \quad k \text{ even} \Rightarrow v = 0$$

Therefore, $H(u,v)$ and $C(u,v)$ are never 0 at the same time and the restored image can always be estimated.

~

# Question 4 – Answer

a) (i) 

(ii)  histogram of $g(x,y)$

(iii) Obviously $g(x,y)$

b) **<u>One solution that does not always work! (extended Huffman)</u>**

All the questions are answered here

| Letter | Probability | Codeword |
|---|---|---|
| $s_1$ | 0.95 | 0 |
| $s_2$ | 0.02 | 11 |
| $s_3$ | 0.03 | 10 |

Table 1: Huffman code for three-letter alphabet; H = 0.335 bits/symbol; $l_{avg}$ = 1.05 bits/symbol; redundancy = 0.715 bits/symbol or 213% of entropy.

| Letter | Probability | Code |
|---|---|---|
| $s_1s_1$ | 0.9025 | 0 |
| $s_1s_2$ | 0.0190 | 111 |
| $s_1s_3$ | 0.0285 | 100 |
| $s_2s_1$ | 0.0190 | 1101 |
| $s_2s_2$ | 0.0004 | 110011 |
| $s_2s_3$ | 0.0006 | 110001 |
| $s_3s_1$ | 0.0285 | 101 |
| $s_3s_2$ | 0.0006 | 110010 |
| $s_3s_3$ | 0.0009 | 110000 |

Table 2: The Huffman code for the extended alphabet; $l_{avg}$ = 1.222 bits/new symbol or $l_{avg}$ = 0.611 bits/original symbol; redundancy = 72% of entropy; redundancy drops to acceptable values for N=8 (alphabet size = 6561).