Imperial College London
Department of mathematics
Dr I. Shevchenko

Assignment 4

The project should be submitted as one zip- or tar-file to i.shevchenko@imperial.ac.uk by the due date. The file should contain all codes used to generate your results and a pdf-file of the report. The assignment must include a pledge that this is all your own work, your name and CID. Any marks received for the assignment are only indicative and may be subject to moderation and scaling.

Exercise 1 (Explicit and Implicit LMMs; PECE; Absolute Stability) % of CW mark: 15

a) Develop an explicit and an implicit 3-step absolutely stable method of your own design with the same order of consistency. (**% of CW mark: 3**)

b) Develop a predictor-corrector method based on the explicit (predictor) and the implicit (corrector) method. (**% of CW mark: 2**); **Mastery Component**.

c) Find the region of absolute stability for each method. (**% of CW mark: 1**)

d) Solve the initial value problem

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 998 & -999 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 2\sin(t) \\ 999(\cos(t) - \sin(t)) \end{pmatrix}, \begin{pmatrix} x_1(t_0) \\ x_2(t_0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \ t = (0, 20].$$

with each method. (**% of CW mark: 3**).

e) Compute the global error e_n vs time for each method. Study how e_n depends on the time step size. (**% of CW mark: 1**)

f) Analyse how many time steps each method needs to achieve the global error $e_n = 10^{-6}$. (**% of CW mark: 1**)

g) Solve the the Lorenz system with each method

$$\begin{cases} x' = \sigma(y - x), \\ y' = x(\rho - z) - y, \\ z' = xy - \beta z, \end{cases}$$

where $\sigma = 10$, $\beta = 8/3$, $\rho = 28$ and initial conditions $\mathbf{v}(0) = (1, 1, 1)^{\mathrm{T}}$; $\mathbf{v} = (x, y, z)^{\mathrm{T}}$; $t = [0, 100]$. (**% of CW mark: 3**)

h) Compute $\varepsilon(t) = ||\mathbf{v}(t) - \widetilde{\mathbf{v}}(t)||_2 / ||\mathbf{v}(t)||_2$ for each method; $\widetilde{\mathbf{v}}(0) = \mathbf{v}(0) + 10^{-6}$. Explain your findings. (**% of CW mark: 1**)