UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1998

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER 1.9

MATHEMATICAL METHODS AND GRAPHICS

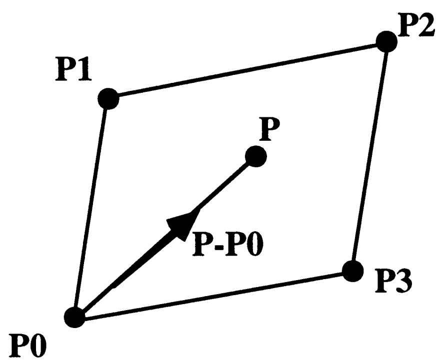
Wednesday, May 13th 1998, 10.00 - 12.00

Answer FOUR questions

For admin. only: paper contains 6 questions

1 In a graphical animation, a projected polygon is to be drawn with a texture. The polygon projects into a parallelogram with vertices **P0**, **P1**, **P2** and **P3** as shown below. The texture is to be applied to the projected polygon after normalisation.

The texture is defined in terms of parameters (α,β) where $0<=\alpha<1$ and $0<=\beta<1$.



P0-P1 = P3-P2

P0-P3 = P1-P2

a Derive a vector expression which could be solved for the texture coordinates (α,β) of the point **P** inside the parallelogram.

b Given the following pixel coordinates **P0**=(10,10), **P1**=(20,60), **P2**= (100,70) and **P3**=(90,20), and **P** =(50,50) use your answer to part (i) to calculate the values of (α,β).

c In a general perspective projection, a rectangular polygon will not necessarily project into parallelogram. Given that the edge vectors of the polygon are **a** = **P1** - **P0**, **b** = **P3** -**P0** and **c** = **P2** - **P3** and that **b**≠**c**, derive a vector equation from which the values of (α,β) can be found.

d Comment on the visual problems that may occur when texture is applied in two dimensions, after projection rather than before projection in three dimensions.

e A texture is procedurally defined as a ten vertical stripes using the following pseudocode:

if (round(α*10) is even) **then** Pixel is white
else Pixel is Black

Explain what visual defect could occur when the size of projected rectangle is small (less than 20 pixels accross).

2 A graphics scene is to be viewed from the origin with the direction of view being along the z axis. The user window has coordinate boundaries [-10,-10,20], [10, -10,20], [10,10,20] and [-10,10,20], and the scene is to be drawn in perspective projection. It is assumed that any polygons for which z>1000 will not be visible and can be ignored.

a Determine the equations of the six planes that bound the scene.

b For each plane write down the condition that a point (Px,Py,Pz) is on the same side of the plane as a point inside the volume as a contained point, for example [0,0,20].

c Write down the condition that a point (Px,Py,Pz) will be visible.

d What is the condition that a projected point (Px', Py') will be inside the user window?

e Is there any computational advantage in determining visibility determination in three dimensions (before projection) rather than in two dimensions?

Turn over

3a By considering the scalar and vector products of

$$\mathbf{a} = (Cos\,\theta, Sin\,\theta, 0)$$

and

$$\mathbf{b} = (Cos\,\phi, Sin\,\phi, 0)$$

derive the trigonometric results

$$Cos\,(\theta - \phi) = Cos\,\theta\, Cos\,\phi + Sin\,\theta\, Sin\,\phi$$

$$Sin\,(\theta - \phi) = Sin\,\theta\, Cos\,\phi - Sin\,\phi\, Cos\,\theta$$

3b For what values of α and β are there unique, no and infinitely many solutions of

$$\begin{aligned} &x + 2y + z = 1 \\ &2x + \alpha y + 2z = \beta \\ &9x + (2 + 4\alpha)y + 3\alpha z = 2 \end{aligned}$$

Where there are infinitely many solutions identify all of them.

Parts (a) and (b) carry 33% and 66% of the marks respectively.

4a Find all first and second partial derivatives of

$$f(x,y)=Cos\left(\frac{x+1}{y+1}\right)$$

4b Find the stationary point of the function

$$f(x,y,z)=e^{-\left(x^2+y^2+z^2\right)/2}$$

and determine whether it is a maximum, minimum or saddle point.

4c The area of an ellipse is $A=\pi ab$ where a,b are the semi-major and semi-minor axes respectively. By considering the total derivative of A find the approximate percentage change in area if a and b are increased by 2% and 3% respectively. How does this compare with the exact result?

Parts (a) and (b) and (c) carry 25%, 50% and 25% of the marks respectively.

Turn Over...

5a Find the cube root(s) of $8i$ expressed in standard form $z = x + iy$.

Illustrate where the solutions lie in an Argand diagram.

5b Show that

$$Cos\,3\theta = 4Cos^3\theta - 3\,Cos\,\theta$$

5c Show that

$$Cos\,\theta + Cos\,2\theta + \ldots + Cos\,n\theta =$$

$$\frac{1}{2}\left[\frac{Sin\;n\theta\;Sin\,\theta}{1 - Cos\,\theta} + Cos\;n\theta - 1\right]$$

Parts (a), (b) and (c) carry 50%, 25% and 25% of the marks respectively.

6a Find the general solutions of the recurrence relations

(i) $u_n - 2u_{n-1} = 3n$

(ii) $u_n - 4u_{n-1} + 4u_{n-2} = 2^n$

6b Determine the Maclaurin series for the function

$$f(x) = \ell n\left(\frac{1+x}{1-x}\right)$$

up to and including terms quartic in x.

Parts (a) and (b) carry 70% and 30% of the marks respectively.

End of paper

