IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

EEE PART II: MEng, BEng and ACGI

FIELDS

Corrected copy

Monday, 6 June 2:00 pm

Time allowed: 1:30 hours

There are THREE questions on this paper.

Question One carries 40 marks. Question Two and Question Three carry 30 marks each.

Answer ALL questions.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : R.R.A. Syms
Second Marker(s) : S. Lucyszyn

Electromagnetic Fields 2016 – Formula Sheet

- **Differential operators (Cartesian co-ordinates)**

$\nabla = \partial/\partial x\ \underline{i} + \partial/\partial y\ \underline{j} + \partial/\partial z\ \underline{k}$

$\nabla\phi = \partial\phi/\partial x\ \underline{i} + \partial\phi/\partial y\ \underline{j} + \partial\phi/\partial z\ \underline{k}$

$\nabla \cdot \underline{\mathbf{F}} = \partial \mathbf{F}_x/\partial x + \partial \mathbf{F}_y/\partial y + \partial \mathbf{F}_z/\partial z$

$\nabla \times \underline{\mathbf{F}} = \{\partial \mathbf{F}_z/\partial y - \partial \mathbf{F}_y/\partial z\}\ \underline{i} + \{\partial \mathbf{F}_x/\partial z - \partial \mathbf{F}_z/\partial x\}\ \underline{j} + \{\partial \mathbf{F}_y/\partial x - \partial \mathbf{F}_x/\partial y\}\ \underline{k}$

$\nabla^2\phi = \partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 + \partial^2\phi/\partial z^2$

- **Identities**

$\nabla \times \nabla \times \underline{\mathbf{F}} = \nabla(\nabla \cdot \underline{\mathbf{F}}) - \nabla^2\underline{\mathbf{F}}$

- **Integral theorems**

$\iint_A \underline{\mathbf{F}} \cdot d\underline{a} = \iiint_V \nabla \cdot \underline{\mathbf{F}}\ dv$ (Gauss' theorem)

$\oint_L \underline{\mathbf{F}} \cdot d\underline{L} = \iint_A (\nabla \times \underline{\mathbf{F}}) \cdot d\underline{a}$ (Stoke's theorem)

- **Maxwell's equations – integral form**

$\iint_A \underline{\mathbf{D}} \cdot d\underline{a} = \iiint_V \rho\ dv$

$\iint_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\oint_L \underline{\mathbf{E}} \cdot d\underline{L} = -\iint_A \partial\underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\oint_L \underline{\mathbf{H}} \cdot d\underline{L} = \iint_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

- **Maxwell's equations – differential form**

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

- **Material equations**

$\underline{\mathbf{J}} = \sigma\ \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon\ \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu\ \underline{\mathbf{H}}$

- **Electromagnetic waves (pure dielectric media)**

Time dependent vector wave equation $\nabla^2\underline{\mathbf{E}} = \mu_0\varepsilon\ \partial^2\underline{\mathbf{E}}/\partial t^2$

Time independent scalar wave equation $\nabla^2\underline{E} = -\omega^2\mu_0\varepsilon_0\varepsilon_r\ \underline{E}$

For z-going, x-polarized plane waves $d^2E_x/dz^2 + \omega^2\mu_0\varepsilon_0\varepsilon_r\ E_x = 0$

Where $\underline{E}$ is a time-independent vector field

- **Power**

Instantaneous power flow $\underline{\mathbf{S}} = \underline{\mathbf{E}} \times \underline{\mathbf{H}}$

Time-averaged power flow $\underline{S} = 1/2\ \mathrm{Re}(\underline{E} \times \underline{H}^*)$

- **Transmission line formulae**

Transmission line equations for line with per unit length inductance and capacitance L_p and C_p

$dV/dz = -j\omega L_p I$

$dI/dz = -j\omega C_p V$

Phase velocity and impedance of line with per unit length inductance L_p and capacitance C_p

$v_{ph} = 1/\sqrt{(L_p C_p)}$

$Z_0 = \sqrt{(L_p/C_p)}$

Reflection and transmission coefficients at junction between lines of impedance Z_1 and Z_2

$R_V = (Z_2 - Z_1)/(Z_2 + Z_1)$

$T_V = 2Z_2/(Z_2 + Z_1)$

Input impedance for length d of line with properties (Z_0, k) terminated by load Z_L

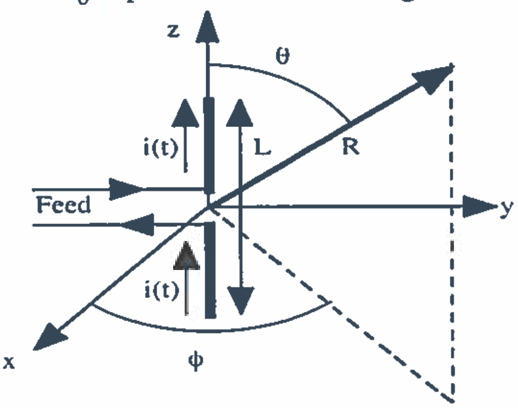
$Z_{in} = Z_0\{Z_L + jZ_0 \tan(kd)\} / \{Z_0 + jZ_L \tan(kd)\}$

- **Antenna formulae**

Far-field pattern of half-wave dipole

$E_\theta = j\, 60 I_0 \{\cos[(\pi/2)\cos(\theta)] / \sin(\theta)\} \exp(-jkR)/R$; $H_\phi = E_\theta/Z_0$

Here I_0 is peak current, R is range and $k = 2\pi/\lambda$



Time averaged power flow $\underline{S} = 1/2\ \mathrm{Re}\,(\underline{E} \times \underline{H}^*) = S(R, \theta)\ \underline{r}$

Normalised radiation pattern $F(\theta, \phi) = S(R, \theta, \phi) / S_{max}$

Directivity $D = 1/\ \{1/4\pi \iint_{4\pi} F(\theta, \phi) \sin(\theta)\, d\theta\, d\phi\}$

Gain $G = \eta D$ where η is antenna efficiency

Effective area $A_e = \lambda^2 D/4\pi$

Friis transmission formula $P_R = P_T(\eta_T \eta_R A_T A_R/R^2\lambda^2) = P_T G_T G_R(\lambda/4\pi R)^2$

Electromagnetic Fields 2016 – Questions

1. Using diagrams and developing formulae where appropriate, explain briefly each of the following:

a) Atmospheric transmission and the human eye

[8]

b) Marconi's achievement

[8]

c) Night vision systems

[8]

d) The radar range equation

[8]

e) The microwave oven

[8]

[Σ = 40]

2. The ladder network shown in Figure 2.1 is constructed from inductors of value L = 10 nH and capacitors of value C = 100 pF. Each section is of length a.

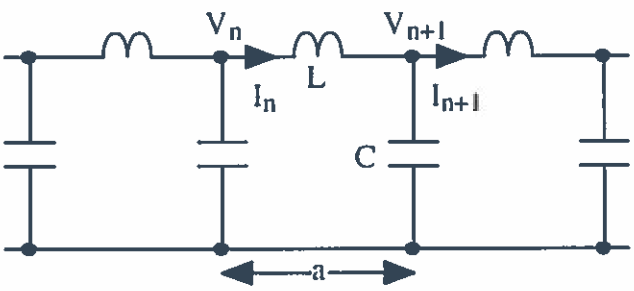


Figure 2.1

a) Write down a pair of equations linking the voltages and currents V_n and I_n in the n^{th} section at angular frequency ω to the corresponding values in the $n+1^{th}$ section. Assuming wave solutions for the currents and voltages, derive the dispersion relation for travelling waves.

[10]

b) Sketch the dispersion characteristic over the range $-4\pi \leq ka \leq 4\pi$, where k is the propagation constant. Explain the relation between solutions lying in the ranges i) $0 \leq ka \leq 2\pi$, ii) $2\pi \leq ka \leq 4\pi$, iii) $0 \geq ka \geq -2\pi$ and iv) $-2\pi \geq ka \geq -4\pi$.

[10]

c) Derive the cutoff frequency of the network, and explain the solutions found above cutoff.

[10]

[Σ = 30]

3. a) In integral form, Gauss' Law and Faraday's law are:

$$\iint_A \underline{\mathbf{D}} \cdot d\underline{a} = \iiint_V \rho \, dv$$

$$\oint_L \underline{\mathbf{E}} \cdot d\underline{L} = - \iint_A \partial \underline{\mathbf{B}}/\partial t \cdot d\underline{a}$$

Convert both equations into differential form.

[6]

b) The time-independent scalar wave equation is:

$$\nabla^2 \underline{E} = -\omega^2 \mu_0 \varepsilon_0 \varepsilon_r \underline{E}$$

i) For z-going, x-polarized plane waves, show that $d^2E_x/dz^2 + \omega^2 \mu_0 \varepsilon_0 \varepsilon_r E_x = 0$.

[3]

ii) In free space, a wave has its electric field described by $\underline{E} = E_0 \exp(-jk_0 z) \, \underline{i}$. What is k_0? What is the corresponding magnetic field? What is the impedance of free space?

[7]

c) Figure 3.1 shows an electromagnetic wave incident on the interface between two dielectric media. Assuming TE incidence:

i) Write down expressions for the electric field in the two materials.

[3]

ii) Explain the boundary condition must be satisfied for the electric field.

[2]

iii) Using this condition, derive Alhazen's law and Snell's law.

[3]

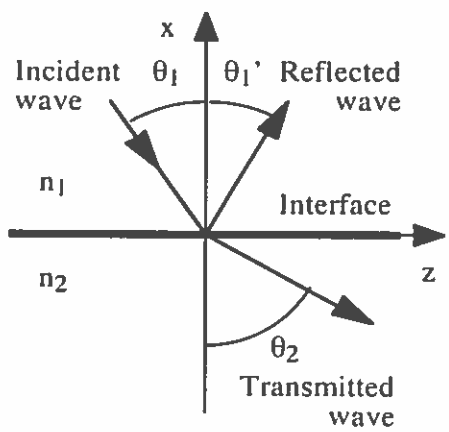


Figure 3.1

d) Show that the wave amplitude decays exponentially in Medium 2 when θ_1 is greater than the critical angle.

[6]

[Σ = 30]