EE4-25
EE9-CS1-2
EE9-FPN2-06

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

MSc and EEE/EIE PART IV: MEng and ACGI

DESIGN OF LINEAR MULTIVARIABLE CONTROL SYSTEMS

Tuesday, 16 May 10:00 am

Time allowed: 3:00 hours

Corrected copy

There are FOUR questions on this paper.

Answer ALL questions

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : I.M. Jaimoukha
Second Marker(s) : E.C. Kerrigan

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1. Consider a state-variable model described by the dynamics

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t).\end{aligned}$$

a) Suppose that

$$G(s) \stackrel{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{ccc|cc} -1 & 2 & 0 & 1 & 2 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & -4 & 3 & 4 \\ \hline 2 & 3 & 0 & 0 & 0 \\ 1 & 4 & 0 & 0 & 0 \end{array}\right].$$

i) Find the uncontrollable and/or unobservable modes and determine whether the realisation is detectable and stabilisable. [4]

ii) Obtain a minimum realisation of $G(s)$. [4]

b) i) Suppose there exists a matrix $Q = Q' \succ 0$ such that

$$A'Q + QA \prec 0.$$

Prove that A is stable. [4]

ii) Suppose there exist matrices $Q = Q' \succ 0$ and Y such that

$$A'Q + QA + YC + C'Y' \prec 0.$$

Prove that the pair (A, C) is detectable and find a matrix L (in terms of Q and Y) such that $A + LC$ is stable. [4]

iii) State a corresponding result for the stabilisability of the pair (A, B), together with a matrix K such that $A + BK$ is stable. [4]

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2. a) Consider a state–variable model described by the dynamics

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t) + Du(t),$$

and let $H(s) = D + C(sI - A)^{-1}B$ denote the corresponding transfer matrix.

i) By defining suitable Lyapunov and cost functions and completing a square, derive sufficient conditions, in the form of matrix inequalities, that simultaneously guarantee the stability of $H(s)$ and the condition $\|H\|_\infty < \gamma$, where $\gamma > 0$ is given. [5]

ii) By using a Schur complement argument, express the conditions derived above in a form that is linear in the matrices C and D. [5]

b) Consider the output injection problem shown in Figure 2. Let $w = [\ w_1^T \quad w_2^T\ ]^T$ and let $T_{ew}(s)$ denote the transfer matrix from w to e. An internally stabilizing output injection gain matrix L is to be designed such that, for a given $\gamma > 0$, $\|T_{ew}\|_\infty < \gamma$.

i) Derive a state space realization for $T_{ew}(s)$. [5]

ii) By using the answer to Part (a) above, or otherwise, derive sufficient conditions for the existence of a feasible L. Your conditions should be in the form of the existence of certain solutions to linear matrix inequalities. [5]

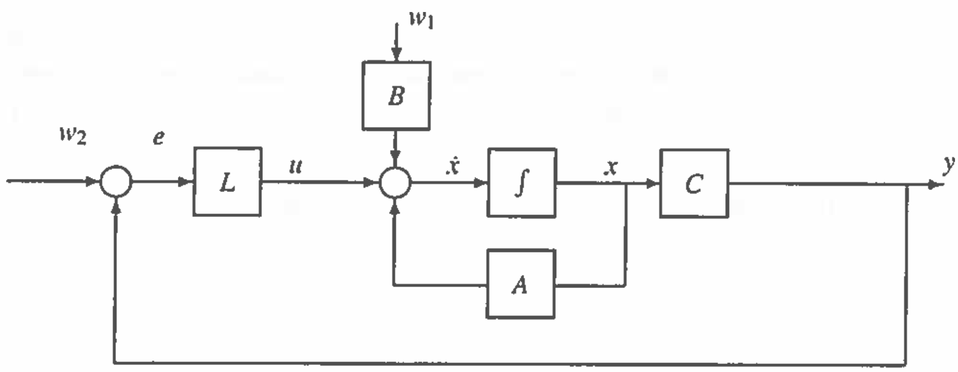

Figure 2

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3. Consider the feedback configuration in Figure 3. Here, $G(s)$ is a nominal plant model and $K(s)$ is a compensator. The stable transfer matrices $\Delta_a(s)$ and $\Delta_m(s)$ represent additive and multiplicative uncertainties, respectively, on the nominal model.

The design specification is to synthesize a compensator $K(s)$ such that the following performance and robustness specifications are satisfied:

(i) when $\Delta_a = 0$ and $\Delta_m = 0$, $\|e(j\omega)\| < |w(j\omega)^{-1}|\|r(j\omega)\|, \forall\omega$,

(ii) when $\Delta_a = 0$, the loop is stable for all Δ_m such that $\|\Delta_m(j\omega)\| < |w_m(j\omega)|, \forall\omega$,

(iii) when $\Delta_m = 0$ the loop is stable for all Δ_a such that $\|\Delta_a(j\omega)\| < |w_a(j\omega)|, \forall\omega$,

where $w(s)$, $w_a(s)$ and $w_m(s)$ are appropriate weighting functions.

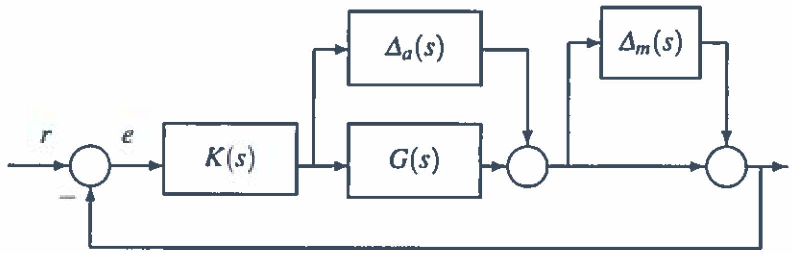

Figure 3

a) Derive $\mathscr{H}_\infty$-norm bounds, in terms of $G(s)$, $K(s)$, $w(s)$, $w_a(s)$ and $w_m(s)$ that are sufficient to achieve the design specifications. [6]

b) Define suitable cost, external, measured and control signals and draw a block diagram, showing all these signals, the nominal model, the compensator, as well as suitable weighting functions. [6]

c) Hence derive a generalised regulator formulation of the design problem that captures the sufficient conditions. [8]

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4. Consider the regulator shown in Figure 4 for which it is assumed that the pair (A, B_2) is controllable, the triple (A, B_1, C_1) is minimal and $x(0) = x_0$. Let $H(s)$ denote the transfer matrix from w to $z = [\, z_1^T \;\; z_2^T \,]^T$ and let $\gamma > 0$ be given.

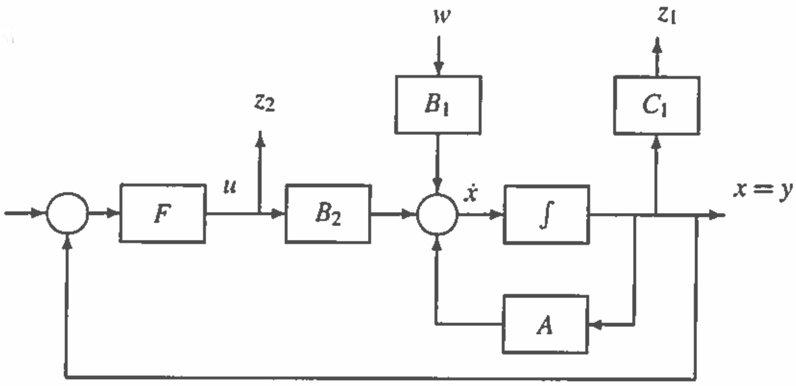

Figure 4

A stabilizing state-feedback gain matrix F is to be designed such that:

(i) $\|H\|_\infty \leq \gamma$ for some $\gamma > 0$, for robustness against disturbances.

(ii) $\|z\|_2^2 \leq \gamma_2^2$ for some $\gamma_2 > 0$, for regulation.

a) Write down the generalized regulator formulation for this design problem.[4]

b) Let $J = \|z\|_2^2 - \gamma^2 \|w\|_2^2$. By defining a suitable Lyapunov function involving an auxiliary variable $X = X'$, and carrying out two completions of squares, derive an expression for J that can be used to solve both design problems. [4]

c) Use the expression for J to formulate the design problem satisfying only condition (i) above and derive sufficient conditions for its solution. Your conditions should be in the form of the existence of a certain solution to an algebraic Riccati equation. It should also include an expression for F and an expression for the worst–case disturbance w^*. [4]

d) Using the solution to Part c) above, derive the tightest bound γ_2^2 that satisfies condition (ii) above. [4]

e) Assume that the solution of the Riccati equation above, here denoted as $X(\gamma)$, is decreasing in γ in the sense that $\gamma_1 \geq \gamma_0$ implies that $X(\gamma_1) \preceq X(\gamma_0)$. Suggest a design algorithm that can achieve a trade–off between the robustness and regulation requirements in (i) and (ii) above. [4]

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