[The exam as a whole had a low average mark of 54.7 with more failure grades than is normal. The standard moderation process was applied to reduce the number of fail grades to 10%. The averages for each question (before moderation) were Q1: 22.9/40; Q2 17.9/30; Q3 13.8/30. It was clear from the rushed and incomplete answers to question 3 that candidates found it harder than usual to complete the exam in the time allowed.]

1. This question covers several topics and all parts should be attempted.

a) Explain why electricity supply systems use three phases with voltages offset by 120°. Explain why three-phase supply is useful in induction machines. [5]

[Bookwork; 3 marks for reasons for three-phase and 2 for rotating field and smooth torque in induction machine]

- Three phase voltages at 120° applied to a balanced load produce zero return current and so no return conductor is needed. Thus a three-phase system can be a 3-wire system. Similar argument applies to higher phase numbers but a two-phase system requires a return conductor and therefore 3 wires also.
- A three-phase generator has a winding in which the first and last coils have 60° phase difference in voltage and some cancellation of voltage magnitude as a consequence. For smaller phase numbers this gets worse and for larger phase numbers this gets better
- Three phases is a good compromise between the penalties of too many transmission wires at high phase number and too much voltage cancellation at low phase number
- With a set of three phase windings with three-phase supply, a rotating magnetic field can be produced. With a single phase winding one gets only an alternating field. The rotating field can create torques that act on the rotor of a motor and give smooth rotation.

[Several candidates just mentioned 'improved efficiency' without elaborating on the reduced voltage cancellation effect for higher phase numbers. Many of them forgot to mention that three phase voltages at 120° applied to a balanced load produce zero return current and so no return conductor is needed.]

b) Two nodes in a network are connected by a line with an impedance of 0.5 + j1.2 Ω. The voltage at sending end is 33 kV and at receiving end is 32.5 kV with a lag of 1.5º compared to the sending end.

i) Calculate the current in the line and the real and reactive power consumed by the line itself. [4]

ii) Calculate the real and reactive power at the sending end. [3]

[Standard Calculations. 1 mark for current, 2 marks for accurate line powers, 1 mark for proper units]

$$Z_{SR} = 0.5 + j1.2 = 1.300\angle 67.38°$$
$$I_{SR} = \frac{V_S - V_R}{Z_{SR}} = \frac{33k - 32.5k\angle -1.5°}{0.5 + j1.2}$$
$$= 755.3 - j111.2 \quad A$$
$$\left|I_{SR}\right|^2 = 5.83\times10^5 A^2$$
$$P_{Line} = \left|I_{SR}\right|^2 \times R_{Line} = 5.83\times10^5 \times 0.5$$
$$= 0.29 \quad MW$$
$$Q_{Line} = \left|I_{SR}\right|^2 \times X_{Line} = 5.83\times10^5 \times 0.5$$
$$= 0.70 \quad MVAr$$

[A common mistake was in the calculation of current. Some candidates just considered the sending end voltage while some did not take into account the phase angle of the receiving end voltage properly. A few used the impedance Z (rather than R or X) to calculate Pline and Qline and came up with complex expressions for these.]

[Standard Calculations. 2 mark for accurate powers and 1 mark for proper units.]

$$P_S = \frac{V_S^2}{Z_{SR}}\cos(\theta) - \frac{V_S V_R}{Z_{SR}}\cos(\theta + \delta)$$
$$= \frac{33k^2}{1.3}\cos(67.38) - \frac{33k \times 32.5k}{1.3}\cos(67.38 + 1.5)$$
$$= 24.9 \quad MW$$
$$Q_S = \frac{V_S^2}{Z_{SR}}\sin(\theta) - \frac{V_S V_R}{Z_{SR}}\sin(\theta + \delta)$$
$$\frac{33k^2}{1.3}\sin(67.38) - \frac{33k \times 32.5k}{1.3}\sin(67.38 + 1.5)$$
$$= 3.67 \quad MVAr$$

c) Consider an AC electricity system operating with a balance between generated power and the consumed power. Explain what happens to the frequency of the system if a large generator is suddenly disconnected. Explain the role of the governors of the remaining generators in controlling the frequency. [7]

[Bookwork and interpretation. 4 marks for describing effect of torque or power mismatch on inertia and consequence. 3 marks for governor action]

- With a match between generated power and consumed power, there is also a match between turbine torque and generator reaction torque on the generator shaft. With no net applied torque there is no acceleration and a constant speed and frequency.
- With the loss of a generator, the load served by that generator is taken from the other generators. Thus there is more reaction torque now applied that turbine torque and this gives a net decelerating torque. Equivalently, there is more power extracted from the remaining generators than delivered to them and the deficit is supplied by removing kinetic energy through deceleration of the generators. In either view, the generators decelerate and the frequency reduces so long as the imbalance exists.
- The governors on generators so equipped will detect the reduction in speed and (proportionately) increase the power input to the generator by, for instance, opening a steam valve further. The additional power reduces the power/torque deficit and arrests the drop in

speed/frequency. The speed/frequency will eventually stabilise at a new lower frequency at which the governors introduce enough additional power to balance the system.

[Most candidates got this one right although some forgot to explain why the frequency would decrease and how the governors would restore the frequency.]

d) It is expected that an underground power cable will have a higher capacitance per unit length and a lower inductance per unit length than an equivalent overhead line. Explain why this is expected. [7]

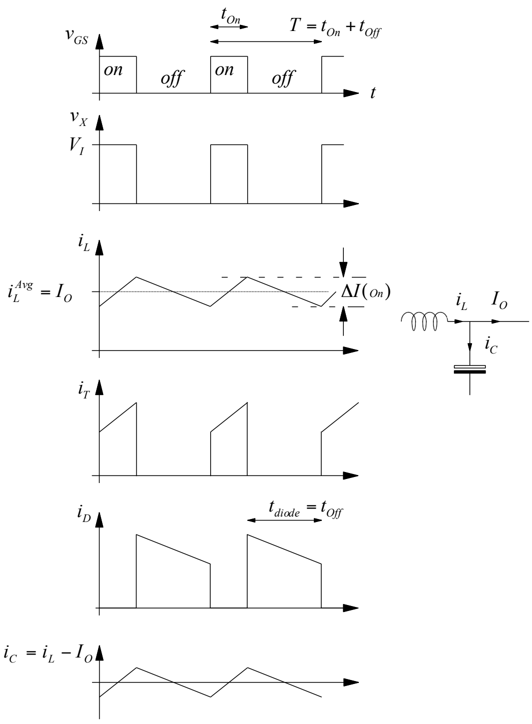
[Bookwork and interpretation. 3 marks for describing physical structure differences and 4 marks for relating this to inductance and capacitance]

- A cable has go and return conductors in close proximity (a few centimetres) separated by, typically, a plastic insulator.
- An overhead line has go and return conductors well separated (a few metres) in air.
- Capacitance increases with relative permittivity (dielectric constant) so plastic increases capacitance by a factor of typically 3 or 4 over air.
- Capacitance decreases with separation, linearly in a parallel plate capacitor but logarithmically for cables and lines as indicated by the standard equations for cables and lines
  $C' = \frac{2\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(r_O/r_C\right)}$, $C' = \frac{\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(d/r_C\right)}$
- Thus thin plastic insulation of cables gives much higher capacitance than OHL
- Inductance of a circuit loop depends on the area of the loop. In the case of a go and return conductor this depends linearly on the length and logarithmically on the separation distance
  $L' = \frac{\mu_0}{2\pi}\ln\left(r_O/r_C\right)$, $L' = \frac{\mu_0}{\pi}\ln\left(d/r_C\right)$
- Thus OHL, with their larger separation have higher inductance per unit length.

[Most candidates mentioned the difference in distance of separation between conductors for cables and OHL but forgot to include the effect of high permittivity of cable insulation.]

e) Sketch the shape of the current through the inductor, transistor, diode and capacitor for a buck switch-mode power supply (SMPS). Assuming the ripple is small and the average inductor current is 10 A, calculate the conduction loss in the transistor and diode given that the transistor has an on-state resistance of 25 mΩ and the diode has an on-state voltage of 0.7 V and the SMPS is running with a duty cycle of 0.55. [7]

[Bookwork. 4 Marks for accurate sketches]



[Most candidates got this right]

[Standard Calculation. 1.5 Marks for each answer]

$$P_T = \delta I_{DS}^2 R_{DS} = 0.55 \times 10^2 \times 0.025$$
$$= 1.375 \quad W$$
$$P_D = (1-\delta) I_{AK} V_{AK} = 0.45 \times 10 \times 0.7$$
$$= 3.150 \quad W$$

[A common mistake on the diode loss part was to use δ instead of (1-δ). Some students used 'J' as the unit of power loss.]

f) Describe how a maximum power point tracking algorithm for a PV panel works and why it is needed. Explain what factors may cause the output from a PV panel to vary during the course of a single day. [7]

[Bookwork. 3 marks for MPPT; 4 marks for factors causing variation]
MPPT works by making small perturbations in the operating point of the PV panel and observing the power yield (a so called perturb-and-observe method). If the perturbation resulted in an increase in power then the change is retained and a change in the same direction is attempted in the next iteration. If the perturbation resulted in a reduced power then a step back in the opposite direction will be taken at the next iteration. The perturbation is typically a small step in the reference voltage for the DC/DC converter attached to the PV panel or a small step in the duty-cycle of the converter.

MPPT is needed firstly because this is not a tracking problem where the objective is to regulate the power to a given value but is instead an optimisation problem trying to find the maximum power. Second, the system is difficult to characterise accurately in a model for open-loop / off -line optimisation because the light intensity and temperature etc. are not known or easily measured.

During a day the sun's passage across the sky cause the angle of incidence of the light falling on the panel to change and therefore the radiation intensity on the panel to change. As the sun moves the distance light travels through the air to reach the panel also changes (the air-mass number changes) which affects the degree of absorption. Cloud cover may also appear and change in position and character which will change the amounts of direct and indirect sunlight. Objects such as tress and buildings may also cast shows on the panel at some times of day.

[Some candidates wrongly associated MPPT with the orientation of the PV panels. Some forgot to answer the 'why MPPT?' part while some forgot to include the 'how MPPT works?' part.]

2.

a) The rotor of an induction machine has no electrical connection to an external supply. Explain why this is a desirable feature and also explain how it is that torque is exerted on the rotor. [6]

[Bookwork; 2 marks for desirable features and 4 for torque mechanism]

- Electrical connection to rotor requires brushes and slip rings which require frequent maintenance and give rises to arcing which precludes use in some environments

[Several candidates said an advantage was lower power loss if there is no direct connection to the rotor. This is not necessarily true, the magnetic filed still needs to be established by the flow of magnetising current and there is no obvious reason for saying the I^2R loss will be less. This would apply to a brushless machine that uses permanent magnets on the rotor but that is not an induction machine.]

- 3-phase stator winding with 3-phase current creates a rotating magnetic field
- If the rotor rotates at any speed other than synchronous speed then there is a rate of change of flux linkage with the rotor winding
- Voltages are induced along the rotor bars which drive AC currents through the short-circuited bars.
- Rotor flux set up that rotates with respect rotor bars and is synchronous with stator flux. Rotor flux lags the stator flux and this angle difference between the two fields exerts a torque on the rotor.

[Answers that omitted any mention of rate-of-change of flux and relative motion of field and rotor lost marks.]

b) A 3-phase, 2 pole-pair, three-phase induction machine is star-connected and provided with a supply with a phase voltage of 2,000 V, 50 Hz. The parameters of the machine's equivalent circuit, referred to the stator, are:

stator resistance	5.0 Ω,
stator leakage reactance	15.0 Ω,
magnetising resistance	8,000 Ω.
magnetising reactance	900 Ω,
referred rotor resistance	3.5 Ω,
referred rotor leakage reactance	8.0 Ω,

When driving a particular mechanical load, the machine is observed to spin at 1,455 rpm.

i) Calculate the slip. [3]

[Standard calculation]

$$\omega_S = \frac{2\pi f}{p} = 50\pi \quad rad/s$$

$$n_S = \frac{f \times 60}{p} = 1{,}500 \quad rpm$$

$$s = \frac{1500 - 1455}{1500} = 0.03$$

[Generally answered correctly. A typical machine operates with a slip of less than 0.05. An error in calculating the synchronous speed, such as not putting the right value of p in the equation can give a slip way outside this range. You should spot this as an error and go back and check your working carefully.]

ii) Give an expression for the total input impedance of the machine and calculate its value in complex form. [7]

[Standard calculation but with involved manipulation. 3 marks for expression; 4 marks for accurate numerical answer]
[Some candidates did not give an expression in symbolic form. Some did not show the imaginary operator *j* and some used *L* for reactance rather than *X*. The first two lost marks, the last mistake didn't (this year). Common mistakes were numerical mistakes with the calculation of impedances in parallel. This year, only one candidate attempted to do the whole calculation by ignoring complex arithmetic but 5 candidates ignored *s* which is a *crucial* variable.]

$$Z_T = R_S + jX_S + \left(R_M // jX_M\right) // \left({R'_R}/{s} + jX'_R \right)$$

$$Z_T = 5.0 + j15 + \left(8000 // j900\right) // \left({3.5}/{0.03} + j8 \right)$$

$$Z_T = 5.0 + j15 + \left(\frac{j7.2M}{8000 + j900} \right) // \left(116.7 + j8\right)$$

$$Z_T = 5.0 + j15 + \left(\frac{7.2M\angle 90^\circ}{8050\angle 6.42^\circ} \right) // \left(116.7 + j8\right)$$
$$= 5.0 + j15 + \left(894.4\angle 83.58^\circ\right) // \left(116.7 + j8\right)$$
$$= 5.0 + j15 + \left(894.4\angle 83.58^\circ\right) // \left(116.7 + j8\right)$$
$$= 116.25 + j36.80 = 121.9\angle 17.56^\circ \quad \Omega$$

iii) Calculate the electro-magnetic torque. [6]

[Standard calculation across several steps]
First find stator current [2 marks]

$$I_S = \frac{V_S}{Z_T} = \frac{2000}{121.9\angle 17.56^\circ} = 16.40\angle -17.56^\circ \quad A$$

Next find rotor current [2 marks]

$$I_R = I_S \frac{Z_M}{Z_R + Z_M}$$
$$= 16.40\angle -17.56^\circ \times \frac{99.99 + j888.75}{99.99 + j888.75 + 116.7 + j8}$$
$$= 15.90\angle -10.40^\circ \quad A$$

Next find torque [2 marks]
[A small number of candidates squared the current in complex form and got a complex torque, which is meaningless. The I^2 comes from a dot product of current with itself and so is scalar and is obtained by squaring the magnitude of the current.]

$$T = 3\left|I'_R\right|^2 R'_R \frac{\left(\frac{1}{s}-1\right)}{\omega_R}$$

$$= 3\times 15.90^2 \times 3.5\times \frac{\left(\frac{1}{0.03}-1\right)}{1455\times \frac{2\pi}{60}}$$

$$= 563.4 \quad Nm$$

iv) Calculate the efficiency. [4]

[Standard calculation possible via several routes]
Input power [1 mark]

$$P_{In} = 3V_S I_S \cos(\phi) = 3\times 2000\times 16.40\times \cos(17.56°)$$

$$= 93.82 \quad kW$$

Output power [1 mark]

$$P_{Mech} = 3\left|I'_R\right|^2 R'_R\left(\frac{1}{s}-1\right)$$

$$= 3\times 15.90^2 \times 3.5\times\left(\frac{1}{0.03}-1\right)$$

$$= 85.84 \quad kW$$

Efficiency [1 mark plus 1 mark for good accuracy]

$$\eta = \frac{P_{Mech}}{P_{In}} = \frac{85.84k}{93.82k} = 91.5\%$$

[A large proportion of answers had an error in one or more of the terms but there was not really a common theme to the errors. It is better to acknowledge unusual answers than ignore them: a comment can show some understanding that allows the examiner to be more forgiving of a numerical error. If the output power is greater than the input power then say that this clearly indicates an error (rather than give the impression that you believe in perpetual motion machines and “over unity” power devices). If the efficiency comes out at 1% then say that it indicates an error because a reasonable machine operating under a reasonable loading condition should achieve well above 75%.]

c) Explain what is expected to happen to the rotor current and torque if the slip is doubled. There is no need to calculate numerical answers but approximations used in the argument must be stated. [4]

[Interpretation of unfamiliar situation]
If the slip is originally small (normal operation) then the R_R/s term dominates the impedance of the rotor branch. If the slip is doubled then the impedance (almost) halves and the current doubles..

Current is a squared term in the mechanical power equation but slip also appears in the denominator meaning that doubled slip leads to doubled mechanical power.

[Both factors needed to be discussed: effect of s on current and the s in the denominator. Discussing only one leads to an incorrect conclusion. Several answers got confused between increases and decreases as the argument developed. Also note that when impedances are in parallel it is the lower impedance that dominates the result – some argued (incorrectly) that the change in rotor impedance is unimportant since the magnetising impedance is larger.]

3.

[Question 3 was intended to be the most difficult of the three but was perhaps a little too difficult. It also appeared from the rushed and incomplete answers to this question from several candidates that the exam as a whole was over long. The average mark on Q3 was low at 13.8. Almost all students completed parts (a)(i) and (ii) correctly. When attempting to find equations for an unfamiliar calculation on SMPS, remember that equations containing $j\omega$ terms apply to sinusoidal voltages and currents and have no place in an answer to an SMPS question.]

A **boost** SMPS has the following design properties.

Input voltage	$V_I = 6.0$ V
Output voltage	$V_O = 18.0$ V
Maximum output current	$I_O^{max} = 0.5$ A
Switching frequency	$f = 100$ kHz
Inductor	$L = 500$ µH
Capacitor	$C = 1{,}000$ µF with ESR = 25 mΩ

a) Consider the boost SMPS operating in continuous mode.

i) Calculate the duty-cycle at which the circuit should be operated to achieve an output voltage of V_O =18.0 V. [3]

[Manipulation of standard equation]

$$\frac{V_O}{V_I} = \frac{1}{1-\delta}$$

$$V_O - V_O\delta = V_I$$

$$\delta = \frac{V_O - V_I}{V_O} = \frac{18-6}{18}$$

$$= \frac{2}{3}$$

ii) Calculate the ripple component of the inductor current. [3]

[Standard Equation]

$$\Delta I_L = \frac{V_I}{L}\frac{\delta}{f} = \frac{6 \times \frac{2}{3}}{500\mu \times 100k}$$

$$= 0.08 \quad A$$

iii) Find the average value of the current through the diode and the average current through the inductor when the load draws the maximum output current. [4]

[Application of basic principle]

Average diode current must equal average output current [2 marks]

$$I_D = I_O = 0.5 \quad A$$

Average inductor current must equal the input current which can be found from conservation of power if SMPS is assumed lossless. [2 marks]

$$I_L = I_I = I_O\frac{V_O}{V_I} = 0.5 \times \frac{18}{6} = 1.5 \quad A$$

iv) Calculate the voltage ripple across the capacitor when operating at the maximum output current. [8]

[Application of standard method]
There are two components of voltage ripple: from capacitance and from ESR. Worst case is found by adding their magnitudes. [1 mark]
Capacitor discharged by output current when the diode is off (when the transistor is on). Peak to peak ripple is found from change in charge. [4 marks]

$$\Delta V_C = \frac{Q}{C} = \frac{I_O \delta}{Cf} = \frac{0.5 \times \frac{2}{3}}{1000\mu \times 100k}$$
$$= 3.3 \quad mV$$

[Despite warnings in class and in comments on previous exams, some students still apply the equations for the voltage ripple in the buck converter.]

Resistive term is peak-to-peak capacitor current (equal to ptp inductor current) multiplied by ESR [3 marks]

$$\Delta V_R = R_{ESR}\left(I_L + \tfrac{1}{2}\Delta I_L\right) = 0.025\left(1.5 + \tfrac{1}{2} \times 0.08\right)$$
$$= 38.5 \quad mV$$
$$\Delta V_O = \Delta V_C + \Delta V_R = 3.3m + 38.5m = 41.8 \quad mV$$

v) Calculate the value of inductor current at which the SMPS enters discontinuous conduction mode and the value of output current to which this corresponds. [4]

[Variation of seen problem]
Conduction mode is determined by inductor current which in the Boost SMPS case is equivalent to the input current. Critical conduction occurs when inductor current returns to zero just as the next on-tie is about to begin. Inductor current is a triangle wave between zero and peak of ripple current. The average value is half of the peak.

$$I_I^{Crit} = \tfrac{1}{2}\Delta_L = \tfrac{1}{2} \times 0.08 = 0.04 \quad A$$

This critical input current flows in the input connection; the value of output current that would flow under this condition can be found by assuming that power is preserved.

$$I_O^{Crit} = I_L^{Crit}\frac{V_I}{V_O} = 0.04 \times \frac{6}{18} = 0.0133 \quad A$$

b) Consider now the boost SMPS operating in discontinuous mode with a duty cycle of 0.5.

i) Calculate the energy stored in the inductor at the end of the on-time. [4]

[Unfamiliar problem]
Stored energy in an inductor is

$$E = \tfrac{1}{2}LI^2$$

The current in discontinuous mode starts from zero at the beginning of the on-time and so at the end of the on-time is simply (note duty cycle value)

$$I\big|_{\text{End on time}} = \Delta I_L = \frac{V_I}{L}\frac{\delta}{f} = \frac{6\times 0.5}{500\mu\times 100k}$$
$$= 0.06 \quad A$$

So the energy is

$$E = \tfrac{1}{2}\times 500\mu\times 0.06^2 = 0.9 \quad \mu J$$

[Although unfamiliar, most students who made a serious attempt at this part got it right. Some used values of currents from previous answers which did not apply (because this is discontinuous conduction with a different duty cycle). An alternative route to the same answer is to integrate the input power (the constant input voltage multiplied by the rising input current) up to the end of the on-time. Some candidates got this method right. Some candidates offered some pretty strange equations for stored energy in an inductor.]

ii) Find the value of output voltage that the SMPS will adopt under these conditions if the output is connected to a load resistor of a 600 Ω. [4]

[Unfamiliar problem]

In discontinuous mode, each switching cycle will completely discharge the inductor into the capacitor. The power flow into the capacitor is thus

$$P_{In} = f\times E = 100k\times 0.9\mu = 90 \quad mW$$

The capacitor voltage will settle to a value that causes the power delivered to the resistor to match this value.

$$P_{Out} = \frac{V_O^2}{R}$$
$$V_O^2 = P_{Out}R = P_{In}R$$
$$V_O = \sqrt{0.09\times 600}$$
$$= 7.35 \quad V$$

[Only a very few candidates got this correct. This section probably needed a hint at the route to the answers (such as "consider the energy balance between the input and the output"). Some candidates attempted to use the voltage ratio equation for discontinuous operation (from the formula sheet) but this requires knowledge of the input current which is unknown unless the diode conduction time is known.]

