

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2008

BEng Honours Degree in Computing Part III  
MEng Honours Degree in Electrical Engineering Part IV  
BEng Honours Degree in Information Systems Engineering Part III  
MEng Honours Degree in Information Systems Engineering Part III  
BSc Honours Degree in Mathematics and Computer Science Part III  
MSci Honours Degree in Mathematics and Computer Science Part III  
for Internal Students of the Imperial College of Science, Technology and Medicine

*This paper is also taken for the relevant examinations for the  
Associateship of the City and Guilds of London Institute  
This paper is also taken for the relevant examinations for the  
Associateship of the Royal College of Science*

PAPER C317=I3.16=E4.32

GRAPHICS

Friday 2 May 2008, 14:30  
Duration: 120 minutes

*Answer THREE questions*

Paper contains 4 questions  
Calculators required

1 Clipping to a viewing volume

In a graphics application a polygon scene is viewed from the origin along the positive  $z$  direction. The viewing window for drawing the scene lies in the plane  $z=20$  and has corners defined by the world coordinates  $(-10,-10,20)$ ,  $(10,-10,20)$ ,  $(10,10,20)$  and  $(-10,10,20)$ . The viewing volume is defined by the four planes going through the origin and edges of the viewing window, the front clipping plane which has equation  $z=20$ , and the back clipping plane which has equation  $z=200$ .

- a Find the unit outer surface normal vector for the six bounding planes of the viewing volume.
- b A polygon has the following four vertex coordinates  $[100,100,210]$ ,  $[95,85,180]$ ,  $[70,70,170]$  and  $[70,90,160]$ . Determine which vertices, if any, are inside the viewing volume.
- c Write down the matrix that will project the 3D points of the scene onto the viewing plane ( $z=20$ ). Explain how the resulting homogenous coordinate must be processed to determine the final Cartesian  $(x,y)$  coordinate.
- d A triangular polygon has vertices  $[10,10,40]$ ,  $[10,30,40]$  and  $[30,10,40]$ . Find the 2D coordinates of the projected vertices and sketch what will be seen in the viewing window.
- e Discuss the pros and cons of carrying out clipping in 2D as opposed to 3D

*The five parts carry equal marks.*

## 2 Radiosity

- a The radiosity method for scene illumination involves the solution of the following system of equations:

$$B_i = E_i + R_i \sum_j B_j F_{ij}$$

where, for patch  $i$ ,  $B_i$  is the radiosity,  $R_i$  is its reflectance,  $E_i$  is its emitted energy and  $F_{ij}$  is a form factor linking patch  $i$  to patch  $j$ .  $\sum_j$  indicates the sum taken over all patches.

The form factors are defined by the equation:

$$F_{ij} = \cos \phi_i \cos \phi_j \text{Area}(A_j) / \pi r^2$$

Explain, with a suitable diagram the role of each term in the equation for the form factors.

- b The reflectance function defined in part a is normally restricted to Lambertian shading. Explain why it is not possible to extend it to include specular reflections.
- c The radiosity method is used to render a scene composed of triangular patches. Two patches are defined as follows:  
Patch  $i$  (10,12,8), (10,13,8), (10,12,9)  
Patch  $j$  (5,6,12), (8,6,12), (5,6,13)  
Assuming that these two patches are visible from each other calculate the two form factors  $F_{ij}$  and  $F_{ji}$ . (Use the centroids of the triangle  $(P_1+P_2+P_3)/3$  to estimate the distance)
- d Explain briefly how the hemicube method is used to calculate form factors.
- e A hemicube is defined by the top plane  $z=1$  and side planes  $x=1$ ,  $y=1$ ,  $x=-1$ ,  $y=-1$ . Assuming that the hemicube patches all have area  $\Delta A$ , derive a formula for the delta form factors of the patches on the top plane in terms of the distance  $r$  of their centre to the origin.

*The five parts carry equal marks.*

### 3 Colour Representation

A graphics card is set to a colour representation where 24 bits are used for each pixel. These are configured normally with 8 bits representing the red value, 8 bits representing the green and 8 representing the blue. A polygon in the scene is coloured using:  $r=150$ ,  $g=99$  and  $b=51$ .

- a Calculate the corresponding  $[x,y]$  coordinate in the CIE diagram and plot the point on the diagram provided, labeling it P.
- b By constructing a line on the CIE diagram, estimate the wavelength of the fully saturated colour, which when mixed with white will produce the colour of part a.
- c Using your CIE diagram estimate the saturation of the colour of part a.
- d. Using your CIE diagram estimate the wavelength of the complement colour to that found in part b.
- e A monitor has the following  $[x,y]$  CIE coordinates for its phosphors:  
Red =  $[.6,.35]$   
Green =  $[.27,.6]$   
Blue =  $[.15,.07]$   
Plot the area that includes all possible colours that can be displayed on the CIE diagram.
- f Explain briefly why the shape of the CIE diagram must be convex.

*The six parts carry, respectively, 20%, 15%, 20%, 15%, 15% and 15% of the marks.*

#### 4 Warping and Morphing

A two-dimensional free-form deformation based on linear B-splines is defined by the following equation:

$$u(x, y) = \sum_{l=0}^1 \sum_{m=0}^1 B_l(u) B_m(v) f_{i+l, j+m}$$

The linear basis functions  $B$  are defined as

$$\begin{aligned} B_0(s) &= 1 - s \\ B_1(s) &= s \end{aligned}$$

An image of size 200 x 200 pixels is warped by a 5 x 5 mesh of control points.

- a Describe in detail how to compute the integer lattice coordinates  $i, j$  and the fractional lattice coordinates  $u, v$  for a given pixel.
- b Compute the integer lattice coordinates  $i, j$  and the fractional lattice coordinates  $u, v$  for the following pixels:

(100, 100)  
(75, 60)  
(125, 180)

- c Compute the new location of the pixels above after warping. All control points  $\phi_{i,j}$  are (0, 0) except for the central control point  $\phi_{2,2}$  whose value is (20, -10).
- d What is the computational complexity of warping an image with  $n$  pixels using a free-form deformation based on linear B-splines with  $m$  control points? How does the computational complexity of the warping change if cubic B-splines are used instead of linear B-splines.
- e What are the advantages and disadvantages of warping using the Beier-Neely algorithm compared to warping using free-form deformations?

*The five parts carry equal marks.*