Master

E2.3B

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

EEE PART II: MEng, BEng and ACGI

POWER

Tuesday, 15 June 2:00 pm

Time allowed: 1:30 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	T.C. Green, T.C. Green
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The Questions

Section A

1. This question covers several topics and all parts should be attempted.

a) A three-phase source with a line voltage of 11 kV is connected to three impedances, each with a resistance of 200 Ω and inductive reactance of 50 Ω, in star configuration. Calculate the line current, real power and reactive power drawn by the load and the power factor. [6]

b) i) Explain the operation of a boost switch-mode power supply during the **on** and **off** states of the transistor. [4]

ii) A boost SMPS operating in continuous mode has an output voltage of 15 V. The switching frequency is 150 kHz and the diode is known to conduct for 4 μs. Calculate the duty-cycle and input voltage. [6]

c) i) Describe the principle of operation of an induction machine. [5]

ii) A four pole-pair induction machine is connected to a 50 Hz supply and runs at a slip of 0.04. Calculate the rotor speed in rpm. [5]

d) Describe the causes of power loss in a real transformer. [6]

e) i) Describe the difference between the functions of generation, transmission, distribution and supply in an electricity supply system. [5]

ii) Explain the advantages of using very high voltages for transmission and why lower voltages are used for distribution and end use. [3]

Section B

2. A three-phase load for a 50 Hz system is composed of three impedances of $10+j5\ \Omega$. It is connected to a supply composed of three voltage sources of 220 V.

   a) What combination of basic components, that is, resistance, inductance and capacitance, could be used to create the load impedance in the simplest form? [3]

   b) The three voltage sources are connected in star and the three load impedances are also connected in star.

      i) Draw a circuit diagram of the sources connected to the loads marking the phase and line voltages. [3]

      ii) Draw a labelled phasor diagram showing the phase and line voltages and line currents. [4]

      iii) State the magnitude of the line voltage. [3]

      iv) State the power factor of the load. [3]

      v) Calculate the real power drawn by the three-phase load. [6]

   c) The sources are switched to a delta connection but the load remains in star connection.

      i) State the line voltage of the system. [3]

      ii) Calculate the real power drawn by the three-phase load. [5]

3.

a) Define the following terms and explain how they affect the operation of a transformer.

i) Flux leakage. [2]

ii) Magnetic saturation. [2]

iii) Hysteresis. [2]

iv) Eddy currents. [2]

b) A transformer nameplate indicates that it is intended for use as a 6 kV to 400 V transformer at a rated apparent power of 200 kVA. It will be used to supply a secondary load of $0.3 + j0.1\ \Omega$ from a primary voltage supply of 6 kV. The impedance properties of the transformer were measured as follows.

Combined winding resistance referred to primary	$R_W = 0.40\ \Omega$
Combined leakage reactance referred to primary	$X_W = 2.35\ \Omega$
Magnetising resistance	$R_M = 9.45\ \text{k}\Omega$
Magnetising reactance	$X_M = 1.15\ \text{k}\Omega$

In calculating the impedances, it has been assumed that the equivalent circuit has the magnetising components directly connected to the primary terminals.

i) Estimate the turns-ratio of the transformer from the quoted operating voltages. [2]

ii) Draw an equivalent circuit of the transformer and load. [2]

iii) Calculate the combined impedance of the winding impedance and the load impedance referred to the primary side. [3]

iv) Calculate the secondary voltage and the regulation that this represents (assuming a no-load voltage of 400 V). [4]

v) Calculate the real power and reactive power of the load. [4]

vii) Calculate the efficiency of the transformer with this load. [4]

c) The primary side of the transformer is connected to a substation via a 30 m long cable which has a resistance (per unit length) of 0.05 Ω/m and an inductive reactance (per unit length) of 0.07 Ω/m. Calculate the substation voltage (in magnitude and angle form) in order to maintain a voltage of 6 kV at the transformer primary side. Assume transformer primary voltage to be at an angle of zero. [3]

4. A power supply for a laptop is to be designed to deliver 90 W. The input voltage of the power supply is 50 V and the output voltage is 24 V.

a) A buck SMPS is designed using an inductor of 225 μH, a switching frequency of 100 kHz and a MOSFET with the following properties:

On-state resistance, $R_{DS(on)} = 0.215\ \Omega$

Turn-on loss (at circuit conditions), $E_{on} = 40\ \mu J$

Turn-off loss (at circuit conditions), $E_{off} = 35\ \mu J$

Thermal resistance (junction to heatsink), $R_{th(JS)} = 4$ K/W

Maximum temperature, $T_{J(max)} = 125°C$

i) Sketch the circuit and calculate the duty-cycle. [4]

ii) Calculate the current ripple. [3]

iii) Calculate the value of inductor current for the onset of discontinuous conduction mode. [4]

iv) For operation at full output power, calculate the power loss in the MOSFET and the efficiency of the power supply. [5]

v) Calculate the required thermal resistance between the heatsink and air assuming an ambient air temperature of $T_A = 25°C$. [3]

vi) The power supply is required to have an output voltage ripple of 5 mV or less. Discuss the choice of components that would need to be made to achieve this ripple specification and support the discussion with calculation of values. [5]

b) A linear regulator is proposed as an alternative design.

i) Calculate the expected efficiency of the linear regulator for the same conditions. [3]

ii) Assuming the use of the same transistor temperature limits and junction-to-heatsink thermal impedance, would it be feasible to use a linear regulator? [3]

The Answers

Section A

Comment. Question 1 is intended to allow candidates to demonstrate competence across the breadth of the module. The average mark was 25 (out of 40) which is lower than expected and seemed to reflect some selective revision in that some candidates could not attempt all parts.

1. This question covers several topics and all parts should be attempted

a) A three-phase source with a line voltage of 11 kV is connected to three impedances, each with a resistance of 200 Ω and inductive reactance of 50 Ω, in star configuration. Calculate the line current, real power and reactive power drawn by the load and power factor. [6]

[Calculation][1 for equ; 1 for ans and unit] Line current

$$I_A = \frac{V_L}{\sqrt{3}Z_A} = \frac{11 \times 10^3}{\sqrt{3}(200 + j50)} = 29.89 - j7.47\ A = 30.81 - 14.04°$$

[Calculation][2 for equ; 1 for ans and unit] Real and reactive power

$S = \sqrt{3}V_L I_L^* = \sqrt{3} \times 11 \times 10^3(29.89 + j7.47) = 0.569 + j0.142\ MVA$
$P = Re(S) = 0.569\ MW$
$Q = Im(S) = 0.142\ MVar$

[Calculation] [1 for ans] Power factor

$pf = cos(\angle V - \angle I) = cos(0—14.04) = 0.97$

Comment. For such a straightforward question there were a surprising number of incorrect answers. A common mistake was to miss he point that this was 3-phase and give only the power of a single-phase load. Some candidates who treated this as three-phase did not calculate the phase voltage correctly and simply assumed 11kV appeared across the quoted impedance. Some treated the resistive and inductive elements as if they were in parallel (e.g. $P=V^2/R$) which, of course, leads to an incorrect answer. A small but important mistake by some candidates was to use the angle of the current as the argument of the sine and cosine functions. It should the angle of the voltage minus the angle of the current. Here the angle of the voltage is zero so this error leads to a change in the sign of the reactive power.

b)

i) Explain the operation of a boost SMPS during the on and off states of the transistor. [4]

[Bookwork][2]
When the transistor is switched on
The inductor is connected across the input voltage, the inductor current rises and energy is stored in the inductor's magnetic field.

[Bookwork][2]
When the transistor is switched off
The diode is brought into conduction; the inductor current flows to the output side capacitor which is at a higher voltage than the input. A negative voltage is imposed across the inductor and the current reduces. Energy is released from the inductor to the capacitor.

Comment. Most answers were good. It was essential to discuss either the rate-of-change of current or the storage and release of energy; it was not sufficient to only describe the current paths..

ii) A boost SMPS has an output voltage of 15 V. The circuit operates in continuous conduction mode and the diode conducts for 4 μs and the switching frequency is 150 kHz. Calculate the duty-cycle and input voltage. [6]

[Calculation][2 for equ; 1 for ans and unit] Duty Cycle

$$\frac{T}{t_{diode}} = \frac{1}{1-\delta}$$
$$1-\delta = \frac{t_{diode}}{T}$$
$$\delta = 1 - \frac{t_{diode}}{T} = 1 - (4 \times 10^{-6} \times 150 \times 10^{3}) = 40\%$$

[Calculation][2 for equ; 1 for ans and unit] Input voltage

$$\frac{V_O}{V_I} = \frac{1}{1-\delta}$$
$$V_I = V_O(1-\delta) = 15(1-0.4) = 9V$$

Comment. Most common error was to not read this question carefully and treat the 4 μs specification as being the on-time of the switch. It is always worth checking that the duty-cycle you obtain lies between 0 and 1. Any other results indicates and error.

c)

i) Describe the principle of operation of an induction machine. [5]

[Bookwork][5]

- *The stator is connected to a three phase supply and the rotor windings are not (normally) supplied, but either shorted or connected to resistors.*
- *The supply currents flowing in the stator create a rotating magnetic field.*
- *The stator flux density rotates at synchronous speed.*
- *This field is linked to the rotor winding and its movement relative to winding induces voltages in the rotor windings.*
- *Induced rotor voltages cause currents to flow in the windings.*
- *The consequent flow or rotor current establishes a rotor field, at synchronous speed, that interacts with the stator field to develop torque that opposes the relative motion.*
- *This developed torque accelerates the rotor until synchronous speed is reached.*
- *At which there is no relative motion between the conductors, causing no voltage to be induced, hence no current will flow and no torque is developed.*

Comment. A logical sequence of statements was essential here. A collection of keywords in a chaotic arrangement does not constitute a description of a principle. There are two key points that had to be present in a good answer: the fact that the stator field is *rotating* and the importance of there being *relative* movement between the rotor bars and the stator field. There were some strange mentions of electric fields playing a role in the induction machine operation; this was penalised.

ii) A four pole-pair induction machine is connected to a 50 Hz supply and runs at a slip of 0.04. Calculate the rotor speed in rpm. [5]

[Calculation][1 for equ; 1 for ans and unit] Stator speed

$$\omega_S = \frac{\omega_E}{P} = \frac{2\pi 50}{4} = 25\pi \text{ □□□/□}$$

[Calculation][1 for equ; 1 for ans and unit] Rotor speed

$$s = \frac{\omega_S - \omega_R}{\omega_S}$$

$$\omega_R = \omega_S - s\omega_S = 25\pi - 0.04 \times 25\pi = 24\pi \text{ □□□/□}$$

[Calculation][1 for equ; 1 for ans and unit] Angular speed to rpm

$$n\square R = \frac{24\pi \times 60}{2\pi} = 720\ rpm$$

Comment. Most answers were correct; there were some silly mistakes in converting between rad/s and rpm.

d) Describe the causes of power loss in a real transformer. [6]

[Bookwork][6]

- *Resistive loss*
  The primary and secondary windings (typically of copper wire) have resistance which give rise to an I^2R power loss. [2]
- *Magnetization currents*
  The core has reluctance and requires magnetization currents. These magnetising currents are caused by sinusoidal changes in the supply voltage causing cyclic re-magnetization of the transformer core as determined by the hysteresis curve of the steel core. There are direct power losses from hysteresis. This represented as additional magnetising current flowing in a resistive element. [2]
- *Eddy currents*
  Cyclic changes in the core flux induce EMFs in the steel of the core which result in eddy currents. These currents dissipate heat due to the core's resistance. This represented as additional magnetising current flowing in a resistive element. [2]

- *Magnetostriction*
  (Not required) Ferromagnetic materials change shape or dimension when a magnetic flux is passed through them known as magnetostriction. The magnetic flux in a transformer is cyclic and hence causes the core to expand and contract with each electrical cycle. This deformation causes elastic strain energy to be dissipated within the core. Also, these vibrations cause a small amount of power consumption producing a 100 Hz hum.

  [Marks subtracted if power loss not properly interpreted, for instance inclusion of flux leakage which is not dissipative]

Comment. A large number of candidates wrote about flux leakage in a way that suggested it caused power loss. There was no need to mention flux leakage or magentising flux; 2 or 3 were deducted depending on how incorrect the candidates thinking seemed to be. If it was clearly stated that the inductances gave rise to reactive power loss then this was not penalised (but the question asked about power loss, when power is not qualified it is taken to be real power.) A list of power losses was not sufficient; the question asked for a description. Some candidates added a discussion of fixed and variable losses; this was rewarded.

e)

i) Describe the difference between the functions of generation, transmission, distribution and supply in an electricity supply system. [5]

[1 for proper distinction between four sectors; other marks for descriptions]
[Bookwork][1]

- *Generation*

The function of generation is to produce electrical power by means of energy conversion. Typically in the UK this is done by a small number of large thermal or hydro plants. Each converts the mechanical energy at the output of a turbine into electrical energy.

Electrical energy produced is sold in a forward competitive market, typically in half hour delivery slots. The market closes one hour or one day ahead of delivery. Counter parties in market are supply businesses (retailers) or large end-use customers.

[Bookwork][1]

- *Transmission*

  The function of transmission is ship large amounts of power over long distances from generation sources to load centres. Transmission systems are generally interconnected in a mesh structure to allow economically efficient provision of reserve services, balancing services, security of supply and stability. Voltages are typically at 100 kV to 400 kV to reduce the I^2R losses of transmission lines of which are generally overhead pylons. The transmission network is a highly interacting and complicated mechanism in which any component action may influences any or may other system components. Transmission is normally a regulated (local) monopoly that applies use-of-system charges.

[Bookwork][1]

- *Distribution*

  The function of distribution is to take electrical energy from a bulk supply point and supply individual consumers. Distribution networks are usually a radial structure using underground cables with unidirectional flow. The growth of distributed generation and renewable generation is changing this topology. Distribution voltages are typically 33kV, 11kV and 400V. For the most part, the DNO has consumers but no generation. Distribution is again usually a regulated (local) monopoly that applies use-of-system charges.

[Bookwork][1]

- *Supply*

  Trading business that have bought energy wholesale in generation market and sell retail to domestic and other small consumers. Sometimes regionally based but in the UK there is a competitive market in supply.

Comment. Answers were not very good on the whole because of poor separation between the regulated and competitive parts of the industry and between the energy production/use aspect and the transport aspect. The most common error was to discuss supply in terms of transport of energy (rather like distribution) rather than as a retail business selling energy.

ii) Explain the advantages of using very high voltages for transmission and why lower voltages are used for distribution and end-use. [3]

[Bookwork][2]
Transmission
Higher voltages allow greater power transfer over longer distances. Higher voltages mean lower currents which reduce the I^2R loss due to the series ohmic resistance within the overhead line. Lower loss equates to higher efficiency in both electrical and economic terms.

The current capacity of a wire is governed by its diameter. By using higher voltages and lower currents the amount of overhead line needed is greatly reduced lowering capital costs. The disadvantage being, the high voltage requires greater insulation and voltage withstand of equipment (circuit breakers, transformers and towers) which have a high capital cost.

[Bookwork][1]
Distribution
Distribution networks deal with lower power transfers. A lower voltage is used to reduce the capital cost of distribution equipment (circuit breakers, switch gear, transformers and cables) and strike a balance between capital and running costs.

Also, distribution in urban areas typically uses underground cables to supply consumer demand. Underground cables have a high capacitance than overhead lines due to the close proximity of earth potential and the dielectric constant of the insulation. By using lower voltages in distribution, the capacitive current is reduced.

Comment. Second aspect of this question needed a clear statement of a trade-off between capital cost and running cost in choosing an appropriate voltage level. In the first aspect some candidates discussed losses in terms of $P=V^2/R$ which is inappropriate for a series resistance and, strikingly, would lead to the conclusion that low voltages should be used.

Section B

2. A three-phase load is composed of three impedances of 10 + j5 Ω. It will be supplied from three voltage sources of 220 V.

a) What combination of basic components (that is, resistance, inductance and capacitance) could be used to form the load impedance in the simplest form? [3]

The impedance has an angle in the first quadrant and so when current floes through, it will produce a voltage the leads the current. The impedance contains an inductive element and resistive element in series and no capacitance is required.

[Calculation][2 for equ; 1 for ans and unit]

$Z = R + j\omega L$

$R = 10\Omega$

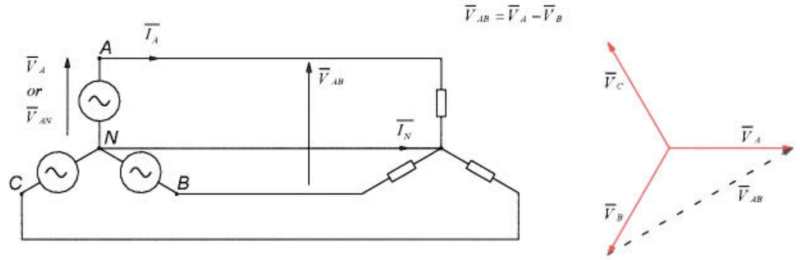
$L = \frac{5}{2\pi 50} = 15.92\, mH$

Comment: Several students left it in R+jX form without calculating the value of L in mH. The question asked for inductance not reactance. Some jus mentioned a combination of R and L and forgot to mention that R and L should be in series

b) The three voltage sources are connected in star and the three load impedances are also connected in star.

i) Draw a circuit diagram of the sources connected to the loads marking the phase and line voltages. [3]

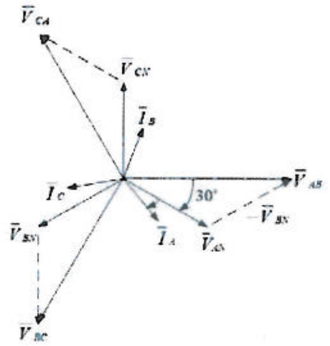
[Bookwork][2 structure; 1 for labels] Circuit diagram (and phasor diagram not needed here)



Comment: Most students have drawn this correctly. However, some forgot to mark the phase and line voltages

ii) Draw a labelled phasor diagram showing the phase and line voltages and line currents. [4]

[Application][Must be three phase: 1 for phase voltages; 1 for line voltages; 1 for line currents; 1 for accuracy]
Phase and line voltages and line currents



Comment: Almost no one showed the 30 deg phase angle between the phase and line voltages. Some students had the line current in phase with either the line or the phase voltages. Some of them failed to label the phasors properly

iii) State the magnitude of the line voltage. [3]

[Calculation][1 for equ; 2 for ans and unit] Line voltage
$V_L = \sqrt{3}V_{AN} = 381\ V$

Comment: Most of the students got this right.

iv) State the power factor of the load. [3]

[Calculation][1 for equ; 2 for ans and unit] Power factor
$$\phi = tan^{-1}\left(\frac{5}{10}\right) = 26.57°$$
$$pf = cos(\phi) = cos(26.57) = 0.89$$

Comment: Most of the students got this right although some arrived at the correct answers through indirect and lengthy routes e.g. calculating P and Q explicitly for the load and then getting the pf from there.

iv) Calculate the real power drawn by the three-phase load. [6]

[Calculation][2 for overall method; 2 for equ; 2 for ans and unit]
Calculate current, then apparent power then real power
$$I_A = \frac{V_{AN}}{Z_A} = \frac{220}{(10+j5)} = 17.6 - j8.8\ A = 19.68\ A\ -26.57°$$
$$S = 3V_{AN}I_L^* = 3 \times 220 \times (17.6 + j8.8) = 11.62 + j5.81\ kVA$$
$$P = Re(S) = 11.62\ kW$$

Comment: The common mistake was calculating the current by dividing the line (instead of phase) voltage by the phase impedance. Some students forgot the conjugate in S=3VI*.

c) The sources are switched to a delta connection but the load remains in star connection.

i) State the line voltage of the system. [3]

[Application – mixed star-delta not explicitly taught][2 for correct interpretation of problem; 1 for ans and unit]
Line voltage
$V_{Line} = 220\ V$
Comment: Most people got this right with only a few exceptions who divided 220 by √3.

ii) Calculate the real power drawn by the three-phase load. [5]

[Calculation][2 for overall method; 2 for equ; 2 for ans and unit]
Calculate current, then apparent power then real power

$$I_A = \frac{V_{Line}}{\sqrt{3}Z_A} = \frac{220}{\sqrt{3}(10 + j5)} = 10.16 - j5.08\,A = 11.36\,A\ -26.57°$$
$$S = \sqrt{3}V_{Line}I_L^* = \sqrt{3} \times 220 \times (10.16 + j5.08) = 3.87 + j1.94\,kVA$$
$$P = Re(S) = 3.87\,kW$$

Comment: The common mistake incorrect consideration of phase and line quantities while calculating the current and also using the expression $S = \sqrt{3}V_L I_L = 3\,V_p\,I_p$

3.

a) Explain the following terms and how they affect the operation of a transformer.

i) Flux leakage. [2]

[Bookwork][2]
Leakage flux is magnetic flux created by a primary or secondary current that is in a local path that does not link the other winding and therefore does not contribute to mutual coupling (transformer action). It does still cause an induced voltage in the original winding (as a self inductance) and can be represented as a simple uncoupled inductance in series with the transformer. It therefore causes inductive voltage drops in the primary and secondary windings and adds to the reduction of voltage under load (regulation)

ii) Magnetic saturation. [2]

[Bookwork][2]
Transformers must use a core of ferromagnetic material (to reduce leakage and reduce magnetising current) but ferromagnetic material saturate (1.2T for electrical grade steel). This limits the flux density but because onset of saturation is gradual transformers operate in partial saturation. Sinusoidal flux is impressed by the source voltage, the non-linar B/H curve means the magnetising current becomes "peaky". This does not directly cause power loss but it does increase the I^2R *loss in the primary winding.*

Comment: Many students forgot to mention the effect of magnetic saturation.

iii) Hysteresis. [2]

[Bookwork][2]
Ferromagnetic core material also exhibits hysteresis in its B/H curve: the flux current trajectory is different for increasing and decreasing current and a loop is formed. Each cycle of this loop represents a loss of energy so there is a power loss proportional to loop area and frequency.

iv) Eddy currents. [2]

[Bookwork][2]
The alternating magnetic field causes induced voltages in the iron of the core. Eddy currents are small circulating currents within the core due to the induced voltages. Eddy currents cause power dissipation in the resistance of the core material. By laminating the core, these currents and their dissipation of power can be reduced.

Comment: Students either forgot to state the reason for the flow of Eddy current or its effect and how it could be minimized..

b) A transformer nameplate indicates that it is intended for use as 6 kV to 400 V transformer at a rated apparent power of 200 kVA. It will be used to supply a load of $0.3 + j0.1\ \Omega$ from a 6 kV supply. The impedance properties of the transformer were measured as follows.

Combined winding resistance referred to primary	$R_W = 0.4\ \Omega$
Combined leakage reactance referred to primary	$X_W = 2.35\Omega$

Magnetising resistance	$X_M = 9.45\ \text{k}\Omega$
Magnetising reactance	$R_M = 1.15\ \text{k}\Omega$

In calculating the impedances it has been assumed that the equivalent circuit has the magnetising components directly connected to the primary terminals.

i) The turns-ratio of the transformer is unknown. Estimate the turns-ratio from the quoted operating voltages. [2]

[Calculation][1 for equ; 1 for ans and unit]

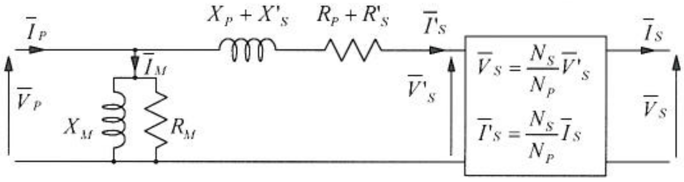
$$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{400}{6 \times 10^3} = \frac{1}{15}$$

$$N_P : N_S = 15:1$$

Comment: Most students gave the correct answer.

ii) Draw an equivalent circuit of the transformer and load. [2]

[Bookwork] [2][need to add load on secondary termainals!]



iii) Calculate the combine impedance of the winding impedance and the load impedance referred to the primary side. [3]

[Calculation][2 for equ; 1 for ans and unit]

$$Z' = Z\left(\frac{N_P}{N_S}\right)^2 = (0.3 + j0.1)(15)^2 = 67.5 + j22.5$$

$$Z' + R_W + jXW = 67.5 + j22.5 + 0.4 + j2.35 = 67.9 + j24.85$$

Comment: A few students missed the square of the turns ratio. Some messed up trying to combine the impedance of the magnetising branch while it was clearly stated that they needed to consider only the winding impedance.

iv) Calculate the secondary voltage and the regulation that this represents (assuming a no-load voltage of 400 V). [4]

[Calculation][1 for equ; 1 for ans and unit]
Referred secondary voltage

$$V_S' = V_P \frac{Z'}{Z' + R_W + jXW} = 6 \times 10^3 \times \frac{67.5 + j22.5}{67.9 + j24.85} = 5.9 - j0.17\ kV = 5.9\ kV\ \angle -1.67°$$

[Calculation][1 for ans and unit]
Secondary Voltage

$$V_S = V_S'\left(\frac{N_S}{N_P}\right) = 5.9 - 0.17\left(\frac{1}{15}\right) = 393.45 - j11.45\ V = 393.62\ V\ \angle -1.67°$$

[Calculation][1 for ans and unit]
Voltage Regulation

$$regulation = \frac{V_S^{NL} - V_S}{V_S^{NL}} \times 100 = \frac{400 - 393.62}{400} \times 100 = 1.59\%$$

v) Calculate the real power and reactive power of the load. [4]

[Calculation][1 for equ; 1 for ans and unit] Secondary current

$$I_S = \frac{V_S}{Z} = \frac{393.45 - j11.45}{0.3 + j0.1} = 1168.91 - j427.8\ A = 1244.74\ A\ -20.10°$$

[Calculation][1 for equ; 1 for ans and unit] Real and reactive power

$$S_S = V_S I_S^* = (393.45 - j11.45)(1168.91 - j427.8) = 464.81 + j154.94\ kVA$$
$$P_S = Re(S) = 464.81\ kW$$
$$Q_S = Im(S) = 154.94\ kVar$$

Comment: Missing conjugate was a problem. Some students started from the primary side and messed it up on somewhere along the way to the secondary side.

vii) Calculate the efficiency of the transformer with this load. [4]

[Calculation][1 for ans and unit] Primary Current

$$I_P = I_M + I_S^{'} = \frac{V_P}{R_M} + \frac{V_P}{jX_M} + \frac{V_P}{Z^{'} + R_W + jRWW} = 83.14 - j29.15\ A = 88.12\ A - 19.32°$$

[Calculation][1 for ans and unit] Real and reactive power

$$S_P = V_S I_S^* = (6 \times 10^3)(83.14 + j29.15) = 498.87 + j174.93\ kVA$$
$$P_P = Re(S_P) = 498.87\ kW$$
$$Q_P = Im(S_P) = 174.93\ kVar$$

[Calculation][1 for equ; 1 for ans and unit] Efficiency

$$\eta = \frac{P_S}{P_P} \times 100 = \frac{464.81}{498.87} \times 100 = 93.17\%$$

[Other approaches are possible including calculation of each loss term [2] and addition to output power to obtain input power and efficiency [2]]

Comment: Similar mistakes as in the calculation of real power of the load were seen for the calculation of the source power in this part. Iron and copper loss calculations were mostly correct.

c) The primary side of the transformer is connected to a substation via a cable of 30 m which has a resistance (per unit length) of 0.05 Ω/m and an inductive reactance (per unit length) of 0.07 Ω/m. Calculate the substation voltage (in magnitude and angle form) in order to maintain an 6 kV supply at the transformer primary side. (Assume transformer primary voltage to be at an angle of zero.) [3]

[Calculation][1 for equ; 1 for ans and unit] Substation voltage

$$V_{SS} = V_P + I_P Z_L = 6 \times 10^3 + (83.14 - j29.15)(30 \times 0.05 + j30\ \times 0.07) = 6.19 + j0.13\ kV = 6.19\ kV\ 1.21°$$

[Calculation][1 for ans and unit both magnitude and angle]
Substation voltage magnitude
$|V_{SS}| = 6.19\ kV$

Substation voltage angle with respect to the transformer angle
$angle = 1.21°$
Comment: Most students got the expression right but surprisingly very few managed to work out the final answer correctly.

4. A laptop power supply is to be designed to deliver 90W. The power supply input voltage is 50V and the output voltage is 24V.

a) A buck SMPS is designed using an inductor of 225 μH, a switching frequency of 100 kHz and a MOSFET with the following properties:

On-state resistance, $R_{DS(on)} = 0.215\ \Omega$

Turn-on loss (at circuit conditions), $E_{on} = 40\ \mu J$

Turn-off loss (at circuit conditions), $E_{off} = 35\ \mu J$

Thermal resistance (junction to heatsink), $R_{th(JS)} = 4$ K/W

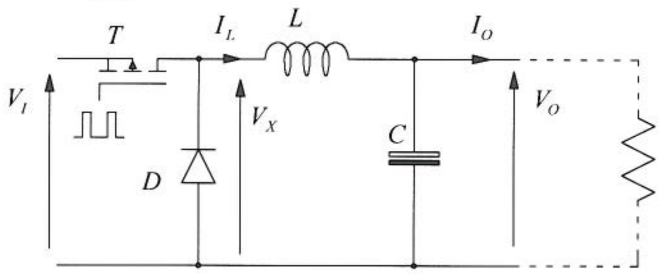
Maximum temperature, $T_{J(max)} = 125°C$

i) Sketch the circuit and calculate the duty cycle [4]

[Calculation][1 for equ; 1 for ans] Duty cycle

$$\delta = \frac{V_O}{V_I} = \frac{24}{50} = 48\%$$

[Bookwork][2] Circuit sketch



ii) Calculate the current ripple. [3]

[Calculation][2 for equ; 1 for ans and unit] Current ripple

$$\Delta i_L = \frac{V_O}{Lf_S}(1-\delta) = \frac{24}{225 \times 10^{-6} \times 100 \times 10^3}(1-0.48) = 0.554A$$

iii) Calculate the minimum inductor current at which discontinuous mode is entered. [4]

[Calculation][2 for equ; 2 for ans and unit] Minimum inductor current

$$i_{oB} = \frac{\delta}{2Lf_S}(V_I - V_O) = \frac{0.48}{2 \times 225 \times 10^{-6} \times 100 \times 10^3}(50-24) = 0.277\ A$$

Comment. The three early parts of this question were handled well by almost all candidates.

iv) For operation at full output power, calculate the power loss in the MOSFET and the efficiency of the power supply. [5]

[1 for overall method]
Current through MOSFET when on is the inductor current which has an average value equal to output current. Assume, $I_{DS} = I_O$

Calculate efficiency at full load. Output current at full load

$$I_O = \frac{P_O}{V_O} = \frac{90}{24} = 3.75\ A$$

[Calculation][2 for equ; 1 for ans and unit] Power loss

$$P_{Loss} = \delta I_{DS}^2 R_{DS(on)} + \left(E_{on} + E_{off}\right) f_S$$

$$P_{Loss} = 0.48 \times 3.75^2 \times 0.215 + (40 \times 10^{-6} + 35 \times 10^{-6}) \times 100 \times 10^3 = 8.95\ W$$

[Calculation][1 for ans and unit] Efficiency

$$\eta = \frac{P_O}{P_O + P_{Loss}} \times 100 = \frac{90}{90 + 8.95} \times 100 = 90.95\%$$

Comment. Most candidates knew the equation for power loss; the most common problem was over calculating an appropriate value for the drain-source current of the MOSFET. Use the ripple component of the inductor current was clearly incorrect and was penalised. Using the average value of the input current is also incorrect since the MOSFT never actually carries a current of this magnitude. The appropriate choice is the average inductor current (since this flows in the MOSFET when it is on) which is equal to the output current. The issue of what value of output current is determined by the worst-case. The worst-case power will occur at maximum output power.

v) Calculate the required thermal resistance between the heatsink and air assuming an ambient air temperature of $T_A = 25$°C. [3]

[Calculation][2 for equ; 1 for ans and unit]

$$R_{t\square(SA)} = \frac{T_{J(max)} - T_A}{P_{Loss}} - R_{t\square(JS)} = \frac{125 - 25}{8.95} - 4 = 7.17\ K/W$$

vi) The power supply is required to have an output voltage ripple of 5 mV or less. Discuss the choice of components that would need to be made to achieve this and support this with calculation of values. [5]

[Calculation and interpretation][2 for calculation of minimum capacitance; 1 for maximum resistance value and 2 for discussion of issue]

For the given voltage ripple, the minimum ideal capacitor (no ESR or ESL) can be calculated.

$$C = \frac{\Delta i_L}{8 f_S \Delta V} = \frac{0.554}{8 \times 5 \times 10^{-3} \times 100 \times 10^3} = 139\ \mu F$$

The ESR adds a voltage drop of $\Delta I.R_{ESR}$. *If only the ESR is considered, it needs to be less than*

ESR $< 0.005/0.55 < 9m\Omega$

The value of the capacitance and ESR of a capacitor are linked and both depend on choices made by the manufacturer so cannot be independently specified. However, ESR is known to be dominant in practice so a guideline is to allow about 90% of ripple across ESR and 10% across C. Thus a capacitor of 1,500 μF or greater *and an ESR of 8 mΩ or less is required, although a trade off is possible. This has ignored the ESL which might also be important.*

Comment. The calculations here were pretty standard; what made it difficult was not being given much guidance over how to proceed and showing sufficient understanding to find a way to approach this problem was what was being looked for.

b) A linear regulator is proposed as an alternative design.

i) Calculate the expected efficiency of the linear regulator for the same conditions. [3]

Assuming $I_I = I_O$

Output current and input current

$$I_I = I_O = \frac{P_O}{V_O} = \frac{90}{24} = 3.75\ A$$

[Calculation][1 for ans and unit] Input power

$$P_I = V_I I_I = 50 \times 3.75 = 187.5\ W$$

[Calculation][1 for ans and unit] Power loss

$$P_{Loss} = P_I - P_O = 187.5 - 90 = 97.5\ W$$

[Calculation][1 for ans and unit] Efficiency

$$\eta = \frac{P_O}{P_O + P_{Loss}} \times 100 = \frac{90}{90 + 97.5} \times 100 = 48\%$$

ii) Assuming the use of the same transistor temperature limits and junction-to-heatsink thermal impedance, would it be feasible to use a linear regulator? [3]

[Application][3]

The thermal resistance between the heat sink and air would need to be.

$$R_{t\square(SA)} = \frac{T_{J(max)} - T_A}{P_{Loss}} - R_{t\square(JS)} = \frac{125 - 25}{97.5} - 4 = -2.97\ K/W$$

Which would require active cooling since the thermal resistance is negative.

From the above calculation, it can be seen that using a liner regulator would not be feasible as active cooling would be needed where as this isn't needed for the buck power supply.