IMPERIAL COLLEGE LONDON

E4.04
SC6
ISE4.9

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

MSc and EEE/ISE PART IV: MEng and ACGI

Corrected Copy

ADVANCED DATA COMMUNICATIONS

Tuesday, 29 April 2:30 pm

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

All questions carry equal marks. The maximum mark for each subquestion is shown in brackets.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : M.K. Gurcan
Second Marker(s) : E. Gelenbe

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Instructions to Candidates
Useful equations

$$For\ T = 1$$

$$\frac{1}{T}\sum_{n=-\infty}^{\infty}\frac{1}{1+a \quad \exp\left(j\left(\omega+\frac{2\pi n}{T}\right)\right)} \overset{Fourier\ Transform}{\Longleftrightarrow} \sum_{k=-\infty}^{0}(-a)^{k}\,\delta\left(t-kT\right)$$

$$\frac{1}{\sqrt{T}}\,\mathrm{sinc}\left(\frac{t}{T}\right) \overset{Fourier\ Transform}{\Longleftrightarrow} \sqrt{T}\,\mathrm{rect}\left(T\ f\right)$$

$$\sum_{k=0}^{\infty}(-a)^{2k} = \frac{1}{1-a^2}$$

For $P_e = 10^{-7}$ the gap value $\Gamma = 9.8dB$

For $P_e = 10^{-6}$ the gap value $\Gamma = 8.8dB$

$$\begin{aligned} \text{For } 2\times\left(1-\frac{1}{4}\right)Q\left(\sqrt{\frac{3SNR}{15}}\right) &= 5\times 10^{-7} \\ SNR &= 123.5 \end{aligned}$$

$$2.4\times Q\left(\sqrt{\frac{10^{1.4}}{1.7}}\right) = 1.45\times 10^{-4}$$

Questions

1. Answer the following subquestions

   (a) Consider the four waveforms shown in Figure 1

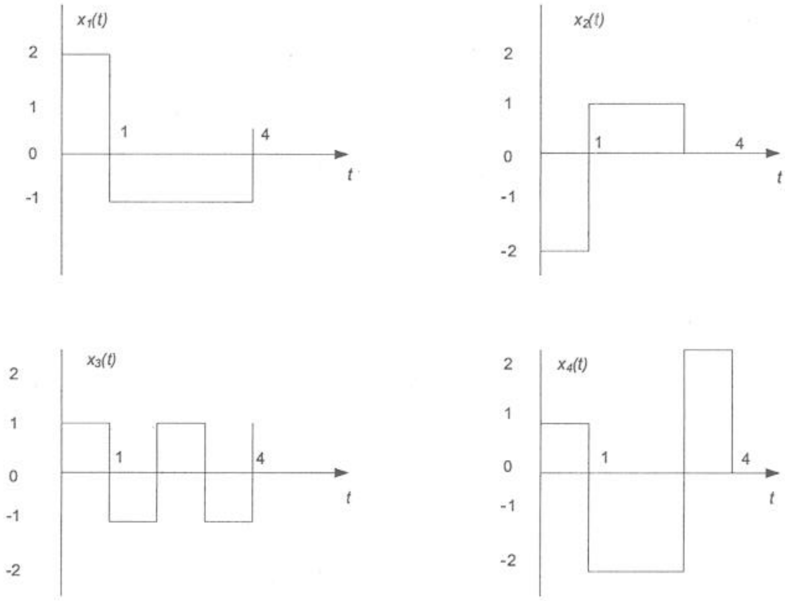

Figure 1. Time waveforms

   i. Determine the dimensionality, N, of the waveforms and the basis functions $\phi_1(t), \cdots, \phi_N(t)$. [3]

   ii. Represent the four waveforms by vectors $\vec{x}_1$, $\vec{x}_2$, $\vec{x}_3$, and $\vec{x}_4$, when using the basis functions. [1] Determine the minimum distance between any pair of vectors. [1]

   (b) Consider the following signal constellation points when transmitting the corresponding time waveforms over an additive-white-Gaussian-noise AWGN channel.

$$
\begin{aligned}
x_0 &= (-1,-1) \\
x_1 &= (1,-1) \\
x_2 &= (-1,1) \\
x_3 &= (1,1) \\
x_4 &= (0,3).
\end{aligned}
$$

   Answer parts b.i and b.ii in terms of the noise variance σ^2.

   i. Find the Union-Bound for the probability of error P_e when using the Maximum Likelihood (ML) detector on this signal constellation. [2]

   ii. Find the Nearest-Neighbour-Union-Bound (NNUB) for the probability of error P_e when using the ML detector with this signal constellation. [2]

   iii. Let the SNR = 14 dB and determine a numerical value for P_e using the NNUB. [1]

   (c) Consider the signal

$$
x(t) = \begin{cases} \dfrac{At}{T}\cos(2\pi f_c t) & \text{for } 0 \le t \le T \\ 0 & \text{otherwise.} \end{cases}
$$

   i. Determine the impulse response of the matched filter for the signal. [2]

ii. Determine the output of the matched filter at $t = T$. [3]

(d) Binary antipodal signals are used to transmit information over an AWGN channel. The prior probabilities for the two input symbols (bits) are $1/2$.

i. Determine the optimum maximum-likelihood decision rule for the detector. [2]

ii. Determine the average probability of error as a function of signal-to-noise-ratio. [3]

2. Answer the following subquestions.

(a) Consider the 64 QAM constellation with the distance $d = 2$ between the adjacent constellation points (see Figure 2). The prior probabilities for the constellation points are equal. The 32 hybrid QAM ($\times$) is obtained by taking one of two points of the constellation

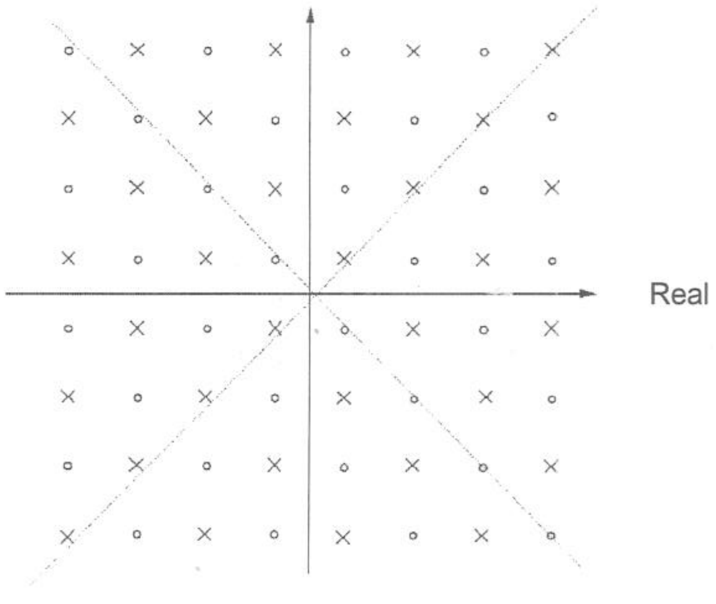

Figure 2 QAM constellation diagram.

i. Compute the average energy ε_x of the 64 QAM and the 32 hybrid QAM constellations. [2]

ii. Find the NNUB for the probability of error for the 64 QAM and 32 hybrid QAM constellations in terms of the noise variance σ^2. [2]

iii. What is the minimum distance d_{min} for a 32 cross QAM constellation having the same energy as the 32 hybrid QAM? [2]

iv. Find the NNUB for the probability of error for the 32 cross QAM constellation. Compare the NNUB with the probability of error for the 32 hybrid QAM constellation. [1]

(b) A three-level PAM system is used to transmit the output of a memoryless ternary source whose rate is 2000 symbols/sec. The prior probabilities for each constellation point is $\frac{1}{3}$. The signal constellation is shown in Figure 3.

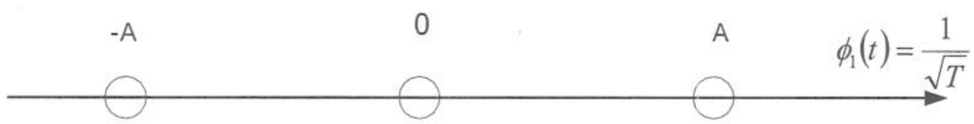

Figure 3 a memoryless ternary system.

Determine

i. the input to the detector, [2]

ii. the optimum threshold which minimizes the average probability of error, and [2]

iii. the average probability of error in terms of the noise variance σ^2. [1]

(c) A QAM system is to be used to transmit over an AWGN channel with SNR=27.5 dB at a symbol rate of $1/T = 5$ M symbol/s. The desired probability of symbol error is $P_e \leq 10^{-6}$. Answer the following parts

i. List two basis functions that you would use for modulation. [2]

ii. Estimate the highest average bit rate $\bar{b}$ per dimension, and the data rate, R, that can be achieved with the QAM system. [2]

iii. Determine which signal constellation is to be used. [2]

iv. Find how much (in dB) the average energy, $\bar{\mathcal{E}}_x$, per dimension would need to be increased to have 5 Mbps more data rate at the same probability of error? [2]

3. Answer the following subquestions.

(a) The Levin-Campello loading algorithm will be used to improve the energy utilization for PAM/QAM signals when transmitting them over the multi-tone modulation channel with $1 + 0.5D^{-1}$. Assume that the system has $N = 8$ dimensions and operates at a bit error rate of $P_e = 10^{-6}$ when the matched filter bound signal-to-noise-ratio $SNR_{MFB} = 10dB$ and the average energy per dimension $\bar{\varepsilon}_x = 1$. Answer the following questions.

i. Create a table of incremental energies $e(n)$ vs. the channel number $n = 0, \cdots, 4$. [2]

ii. Use the EF algorithm to make the average number of bits per dimension $\bar{b} = 1$. [2]

iii. Use the E-Tightening algorithm to find the largest $\bar{b}$. [2]

iv. The total number of bits b obtained in part (a.iii) is to be reduced by 2 bits. Use the EF and B-Tightening algorithms to maximize the margin. What is the maximum margin? [2]

(b) A multi-tone modulation system operates over the channel $H(f) = 1 + 0.5e^{(j2\pi f)}$. The system operational parameters are: the matched filter bound SNR $SNR_{MFB} = 10dB$, the average energy per dimension $\bar{\varepsilon}_x = 1$ and the system dimension $N = 8$. Using the Rate-Adaptive water-filling optimization method answer the following questions.

i. Calculate the optimal distribution of energy for the sub-channels and the maximum bit rate assuming that the gap, $\Gamma = 1$ (0dB). [3]

ii. Calculate the gap for PAM/QAM which produces an argument of the Q−function equal to 9dB. (The gap for $\bar{b} \geq 1$ is the difference between the SNR derived from capacity and the argument of the Q-function for a particular probability of error). [2]

iii. Calculate the optimal distribution of energy for the sub-channels and the maximum bit rate using the gap found in part (b.ii). [2]

(c) For the system in problem (3.b), the system margin will be maximized using the Margin-Adaptive water filling method for a system dimension of $N = 8$. Answer the following questions.

i. Is transmission of uncoded QAM/PAM at $P_e = 10^{-7}$ at a data rate of 1 possible? What will the margin be when operating at the data rate of 1? [3]

ii. For the data rate of 1, what gap provides a margin value equal to zero? [2]

4. Answer the following subquestions.

(a) Show that a pulse having the raised cosine spectrum given by [7]

$$Q_{RC}(f) = \begin{cases} T & 0 \le |f| \le \dfrac{1-\alpha}{2T} \\ \dfrac{T}{2}\left\{1 + \cos\left(\dfrac{\pi T}{\alpha}\left(|f| - \dfrac{1-\alpha}{2T}\right)\right)\right\} & \dfrac{1-\alpha}{2T} \le |f| \le \dfrac{1+\alpha}{2T} \\ 0 & \text{otherwise} \end{cases}$$

satisfies the Nyquist criterion given by equation

$$q(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

for any value of the roll-off factor α, where n is an integer, and T is the symbol period.

(b) A PAM system transmits time waveforms over a filtered AWGN channel when using the basis function $\varphi(t) = \frac{1}{\sqrt{T}} \operatorname{sinc}\left(\frac{t}{T}\right)$ with $T = 1$ over a channel with a frequency response $(|a| < 1)$:

$$H(\omega) = \begin{cases} \frac{1}{1+a\exp(j\omega)} & |\omega| \le \pi \\ 0 & |\omega| > \pi \end{cases}$$

when the SNR$=\frac{\bar{\mathcal{E}}_x}{\sigma^2} = 15dB$.

i. Find the Fourier Transform of the pulse, $P(\omega)$. [2]

ii. Find the pulse energy $|p|^2$. [1]

iii. Find $Q(D)$, the function characterizing ISI for the channel. [1]

iv. Find the equaliser filter coefficients $W(D)$ for the zero forcing equaliser and MMSE linear equaliser on this channel. [3]

v. If $a = 0$, what data rate is achievable when the time waveforms are transmitted over this channel according to the gap approximation at a probability of error $P_e = 10^{-6}$? [1]

(c) Data symbols are transmitted over a 4 kHz voice-band telephone (bandpass) channel. Assuming that the transmitter pulse shape has a raised cosine spectrum with a 50% roll-off, determine the bit rate that can be transmitted through the channel if each of the following modulation methods are used:

i. binary PSK, [1]

ii. four-phase PSK [2]

iii. 8-point QAM. [2]