EE1-05

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

EEE PART I: MEng, BEng and ACGI

Corrected copy

ENERGY CONVERSION

Wednesday, 13 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer All questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	O. Sydoruk
	Second Marker(s) :	B.C. Pal

Formula sheet

Maxwell's equations in integral form

$$\oint_l (\mathbf{E}\cdot \mathrm{d}\mathbf{l}) = \frac{\mathrm{d}}{\mathrm{d}t}\iint_S (\mathbf{B}\cdot \mathrm{d}\mathbf{S})$$

$$\oint_l (\mathbf{H}\cdot \mathrm{d}\mathbf{l}) = \iint_S (\mathbf{J}\cdot \mathrm{d}\mathbf{S}) + \frac{\mathrm{d}}{\mathrm{d}t}\iint_S (\mathbf{D}\cdot \mathrm{d}\mathbf{S})$$

$$\oiint_S (\mathbf{D}\cdot \mathrm{d}\mathbf{S}) = \iiint_V \rho \mathrm{d}V$$

$$\oiint_S (\mathbf{B}\cdot \mathrm{d}\mathbf{S}) = 0$$

Gauss's law for electric fields in differential form, Cartesian coordinates

$$\frac{\partial D_x(x,y,z)}{\partial x} + \frac{\partial D_y(x,y,z)}{\partial y} + \frac{\partial D_z(x,y,z)}{\partial z} = \rho(x,y,z)$$

Gauss's law for electric fields in differential form, centrosymmetric distributions, spherical coordinates

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2 D(r)) = \rho(r)$$

Electric flux density and field strength: $\mathbf{D} = \varepsilon_0\varepsilon_{\mathrm{d}}\mathbf{E}$. Magnetic flux density and field strength: $\mathbf{B} = \mu_0\mu_{\mathrm{r}}\mathbf{H}$.

Coulomb's law

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0\varepsilon_{\mathrm{d}} r^3}\mathbf{r}$$

The Biot-Savart law

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{[\mathrm{d}\mathbf{l}\times\mathbf{r}]}{r^3}$$

Voltage, potential

$$U_{\mathrm{AB}} = \varphi(\mathrm{A}) \quad \varphi(\mathrm{B}) = \int_{\mathrm{A}}^{\mathrm{B}} (\mathbf{E}\cdot \mathrm{d}\mathbf{l})$$

Electrostatic energy

$$W = \frac{1}{8\pi\varepsilon_0}\sum_{i\neq j}\frac{q_i q_j}{r_{ij}}$$

Capacitance: $C = q/U$. Inductance: $L = \Phi/I$. Force on a charge in electric field: $\mathbf{F} = q\mathbf{E}$; in magnetic field: $\mathbf{F} = q[\mathbf{v}\times\mathbf{B}]$.

Rotating machines. Torque, definition (force perpendicular to arm): $T = Fa$. Torque for a motor with N coils: $T = K\Phi I$, where $K = 2N/\pi$. Back-emf: $e = K\Phi\omega$.

Useful integrals

$$\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{a^2+x^2}) \qquad \int \frac{\mathrm{d}x}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}} \qquad \int \frac{x\mathrm{d}x}{(x^2+a^2)^{3/2}} = \frac{1}{\sqrt{x^2+a^2}}$$

ENERGY CONVERSION

1. a) For a positive point charge, sketch electric field lines and equipotential lines, briefly explaining the relationship between the two. [5]

   b) A charge distribution has the form of a full circle of a radius R. The linear charge density varies as $\lambda = \lambda_0 \theta$, where λ_0 is a constant, and the angle θ is defined in Figure 1.1. Find the total charge of the distribution. [5]

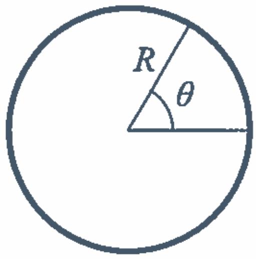

Figure 1.1

   c) How does the capacitance of a capacitor depend on its charge? Briefly justify your answer. [5]

   d) Discuss how charging a capacitor increases the energy it stores. Assuming a capacitance C and a capacitor voltage U, express the energy stored in terms of (i) the voltage and the capacitance; and (ii) the charge and the capacitance. [10]

   e) Three equal positive point charges q are placed at the corners of an equilateral triangle with the length of a side equal to l. Find the electrostatic energy of the charge distribution. [7]

   f) List the usual assumptions for perfect conductors and use them to show that the electric field inside a perfect conductor is zero. [8]

2. a) An ion of a charge q and a mass m enters, with a zero velocity, a region with a homogeneous dc electric field of a strength E. This region is kept in vacuum. Find the velocity of the ion after it travels a distance L. [10]

   b) The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 S/d$, where S is the area of the plates and d is the distance between them. Suggest two methods for using the capacitor as a sensor of mechanical displacements. [10]

   c) Plot the normal magnetisation curve (from zero initial magnetisation to saturation) of a ferromagnet on a BH-diagram and explain briefly the main features of the curve. [10]

3. a) A dc current is distributed in space with the surface current density $\mathbf{J}(\mathbf{r})$. Derive a mathematical expression that shows that the current is conserved. [15]

b) Three infinitely long thin conductors are arranged as shown in Figure 3.1. All conductors are in the same plane. The angle between conductor 2 and 3 is 90°. The angle between conductors 1 and 2 is equal to the angle between conductors 1 and 3. Conductor 1 is carrying a current equal to I; conductors 2 and 3 are both carrying currents equal to $I/2$. The currents in all conductors are flowing from bottom to top. Point P lies vertically above point O, in the same plane as the conductors. The length of OP is d. Find the magnetic flux density B at point P. [15]

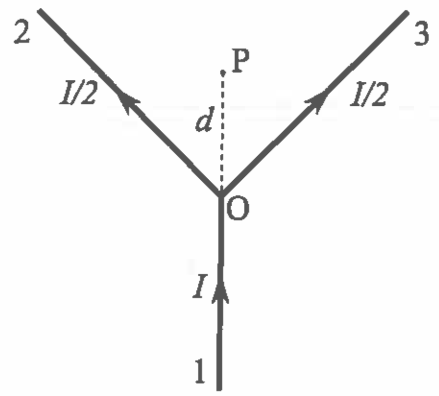

Figure 3.1