IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

EEE/EIE PART I: MEng, BEng and ACGI

Corrected copy

INTRODUCTION TO SIGNALS AND COMMUNICATIONS

Friday, 2 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : K.K. Leung
Second Marker(s) : P.T. Stathaki

Special Instructions for Invigilator: **None**

Information for Students:

Fourier Transforms

$$\cos\omega_o t \quad \Leftrightarrow \quad \pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$$

Some useful trigonometric identities

$$\cos x\cos y = \tfrac{1}{2}\cos(x-y)+\tfrac{1}{2}\cos(x+y)$$
$$\sin(x-y) = \sin x\cos y - \sin y\cos x$$
$$a\cos x + b\sin x = c\cos(x+\theta)$$

where $c=\sqrt{a^2+b^2}$ and $\theta=\tan^{-1}(-b/a)$

Complex exponential

$$e^{jx} = \cos x + j\sin x$$

1. This is a general question. (40%)

a. Consider a time-domain signal $f(t) = e^{-at}u(t)$, where $u(t) = 1$ for $t \geq 0$ and 0 otherwise, and a is positive.

i. Derive the Fourier transform $F(\omega)$ of $f(t)$. [3]

ii. Sketch the frequency spectrum (both the magnitude and phase) of $f(t)$. [2]

iii. From the frequency-spectrum magnitude in part ii, what can be said about the function of the linear system if $f(t)$ represents the unit impulse response of the system? [2]

iv. Let $\hat{f}(t) = f(t - t_o)$ and $\hat{F}(\omega)$ denote the Fourier transform of $\hat{f}(t)$. Derive the relationship between and $\hat{F}(\omega)$ and $F(\omega)$. [3]

b. The trigonometric Fourier series representing a signal $g(t)$ over a time period $t_1 \leq t \leq t_1 + T$ for some fixed t_1 and T is given by

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t).$$

i. Express ω_0 in terms of T. [1]

ii. What can be interpreted physically if the coefficient a_0 is non-zero? [1]

iii. Give two reasons why the above Fourier series can fully represent any arbitrary signal $g(t)$ over the time period of $t_1 \leq t \leq t_1 + T$. [2]

iv. If $g(t)$ changes very rapidly during the time period, what is the implication on the magnitude of the coefficients a_n and b_n for $n = 1, 2, \ldots, \infty$. [2]

v. If $g(t)$ is an even function of t, what can be said about any of the coefficients, a_0, a_n and b_n for $n = 1, 2, \ldots, \infty$, and why? [3]

c. Consider the modulating signal $m(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$ where $\omega_2 > \omega_1$.

i. Sketch the spectrum of $m(t)$. [2]

ii. Give an expression $\phi(t)$ for the amplitude-modulated signal, Double-Side-Band with Suppressed Carrier (DSB-SC), with the modulating signal $m(t)$ and the carrier angular frequency ω_c radians per second, where $\omega_c > \omega_2$. [2]

iii. Sketch the spectrum of the DSB-SC signal in part ii. [2]

iv. What is the bandwidth of the DSB-SC signal? (Consider only positive frequencies.) [1]

v. From the spectrum for $\phi(t)$ in part iii, identify two disjoint sets of frequency components from which the modulating signal $m(t)$ can be recovered by demodulating either one of the sets? [2]

vi. Write an expression for the time-domain signal corresponding to the set of higher frequency components (both in positive and negative frequency) obtained in part v. [2]

1. This is a general question. (Continued)

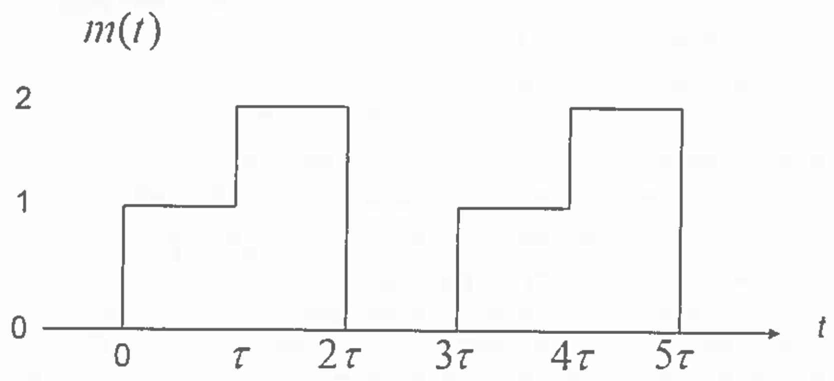
d. Let $\phi(t)$ denote the frequency-modulation (FM) waveform of the modulating signal $m(t)$ with k_f being the proportionality constant for the frequency deviation, and A and ω_C being the amplitude and angular frequency of the carrier (in radians per second), respectively.

i. Give an expression for $\phi(t)$. [2]

ii. The waveform $\phi(t)$ is input to an ideal differentiation circuit. Give an expression (denoted by $\phi'(t)$) for the output of the differentiator. [2]

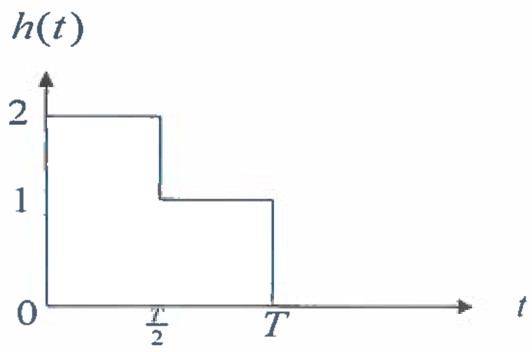
iii. Using the result in part ii, provide a block diagram and explain how the FM signal can be demodulated. [3]

iv. For the modulating signal $m(t)$ given below, sketch the corresponding FM waveform $\phi(t)$. Provide the frequency values associated with the waveform in your diagram. [3]



2. Signals and their transforms. (30%)

a. Consider a linear time-invariant (LTI) system for which the unit impulse response function is given by $h(t) = 2$ for $0 < t \le T/2$, $h(t) = 1$ for $T/2 < t \le T$ $h(t) = 0$ otherwise as shown below, where T is a positive constant. Let $\delta(t)$ denote the unit impulse at $t = 0$.



i. Assume that a signal $x(t) = \delta(t) + \delta(t - T)$ is input to the system. Determine and sketch the output signal $y(t)$ of the system. [3]

ii. Repeat part i for an input signal of $x(t) = \delta(t) - \delta(t - T)$. [3]

iii. Repeat part i for an input signal of $x(t) = \delta(t) + \delta(t - T/2)$. [3]

iv. Now, consider a communication channel that is represented by the above linear system. That is, when a unit impulse $\delta(t)$ is transmitted over the channel, the signal at the receiving end of the channel is $h(t)$. Assume that only periodic unit impulses with positive or negative magnitude can be transmitted to represent 1's or 0's, respectively, as in part ii. By examining results in parts i to iii, suggest the maximum signal rate in terms of the number of unit impulses per second for transmission over the channel and proper decoding by a simple receiver? Explain your result. [7]

b. The exponential Fourier series for a periodic signal $f(t)$ with period T is given by

$$f(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_o t} \quad \text{where } D_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jn\omega_o t} dt \text{ and } \omega_o = \frac{2\pi}{T}.$$

Now, consider the following signal that represents a train of unit impulses with a period T:

$$f(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT).$$

i. Find the D_n's for $n = -\infty$ to ∞ in the exponential Fourier series for $f(t)$. [6]

ii. Sketch the frequency spectrum of $f(t)$. [4]

iii. Express the power of $f(t)$ in terms of the D_n's and explain why. [4]

3. Communications techniques. (30%)

a. Design a communications system that uses a frequency band from 75 to 95 kHz to simultaneously transmit two signals, $m_1(t)$ and $m_2(t)$, each of which has a bandwidth of 5 kHz. Two sinusoidal signals of 10 and 80 kHz are available for the system.

   i. Describe a method of amplitude modulation (AM) by using the sinusoidal signals of 10 and 80 kHz to transmit the signals $m_1(t)$ and $m_2(t)$ over the 75-95 kHz band so that the signals can be recovered by the same sinusoidal signals at the receiver. Sketch the frequency spectrum of the transmitted signal. [6]
   ii. Give an expression for the transmitted AM signal. [5]
   iii. Assuming that ideal filters are available, draw a block diagram of the receiver and show mathematically how the signals $m_1(t)$ and $m_2(t)$ are recovered by using the sinusoidal signals of 10 and 80 kHz at the receiver. [7]
   iv. If one sinusoidal signal at a particular frequency can be included as part of the transmitted signal to simplify the receiver design, what is the preferred frequency of the tone and why can that help? [4]

b. Consider that a signal $g(t)$ with a bandwidth of B Hz is sampled at a frequency of f_S Hz to obtain the sampled signal $\tilde{g}(t)$. Let the Fourier transforms of $g(t)$ and $\tilde{g}(t)$ be denoted by $G(\omega)$ and $\tilde{G}(\omega)$, respectively. Further, let the sampling be represented by applying a train of periodic impulses $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_S)$ to $g(t)$ where $T_S = \frac{1}{f_S}$. As the Fourier series of a periodic signal, $s(t)$ can be expressed as

$$s(t) = \frac{1}{T_S}[1 + 2\cos(\omega_S t) + 2\cos(2\omega_S t) + 2\cos(3\omega_S t) + \ldots] \quad \text{where } \omega_S = 2\pi f_S = \frac{2\pi}{T_s}.$$

   i. Express $\tilde{g}(t)$ in terms of $g(t)$ and various terms of $s(t)$. [2]
   ii. From the frequency-domain perspective, what is the physical interpretation of each of the terms in $\tilde{g}(t)$ obtained in part i? [2]
   iii. Based on part ii above, draw the frequency spectrum for $\tilde{g}(t)$. [2]
   iv. From the result in part iii, determine the relationship between B and f_S such that the signal $g(t)$ can be fully recovered from $\tilde{g}(t)$. [2]