Imperial College London

MEng (Engineering) Examination 2016

Year 1

AE1-106 Properties of Materials

Thursday 2nd June 2016: 14.00 to 17.00
[3 hours]

The paper is divided into Section A and Section B
and contains ***THREE*** questions.
All questions carry equal marks.
Candidates may obtain full marks for complete answers to ***ALL*** questions.

You must answer each section in a separate answer booklet.

A data sheet is attached.

The use of lecture notes is NOT allowed.

Section A

1.

(a) A component of initial volume $V_0 = 500\,\text{mm}^3$ is subject to a uniform increase in temperature $\Delta T = +50\,\text{K}$. It is made from an isotropic metal with thermal expansion coefficient $\alpha = 10^{-5}\,\text{K}^{-1}$ and it is unconstrained (therefore stress-free) during the expansion. Calculate the change in volume of the component ΔV.

[15%]

(b) A component is made from an isotropic metal with mechanical properties $E = 100$ GPa, $\nu = 0.3$ and $\varepsilon_Y = 5\times10^{-4}$. The stress state at a point in this component is given, in a certain Cartesian reference system, by

$$[\sigma] = \begin{pmatrix} 10 & 0 & 5 \\ 0 & 20 & 0 \\ 5 & 0 & 0 \end{pmatrix} \text{MPa}.$$

The material obeys the von Mises yield criterion.

i. Check that the given state of stress is not sufficient to initiate yielding at this point. [10%]

ii. Calculate the corresponding elastic strain tensor at this point. [10%]

Hint: the equivalent von Mises stress in an arbitrary Cartesian reference system (x, y, z) is given by

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[\left(\sigma_{xx}-\sigma_{yy}\right)^2 + \left(\sigma_{xx}-\sigma_{zz}\right)^2 + \left(\sigma_{yy}-\sigma_{zz}\right)^2 + 6\left(\tau_{xy}{}^2 + \tau_{xz}{}^2 + \tau_{yz}{}^2\right)\right]}.$$

(c) A bar of cross-section $A_0 = 100\,\text{mm}^2$ and length $L_0 = 1\,\text{m}$ is made from a perfectly plastic ductile metal with $E = 100$ GPa; $\sigma_Y = 350$ MPa; H=0 (assume nominal stress quantities). The bar is forced to elongate by $\Delta L = 0.3\,\text{m}$ and is then unloaded. Calculate (i) the final length of the bar, (ii) the corresponding true plastic strain and (iii) the force necessary to stretch the bar by an amount ΔL.

[20%]

[Question continued on next page]

(d) Estimate the hardness of the metallic material described in Part (c). [10%]

(e) Two bars of equal length $L_0 = 1\,\mathrm{m}$ are connected in series (end-to-end) and loaded by a progressively increasing tensile force. The bars are made from two materials (A and B) with properties

$$E^A = 100\ \mathrm{GPa},\ \sigma_Y{}^A = 350\ \mathrm{MPa};\quad E^B = 50\ \mathrm{GPa},\ \sigma_Y{}^B = 300\ \mathrm{MPa}.$$

The cross-sectional areas of the two bars are $A_0{}^A = 100\ \mathrm{mm}^2$, $A_0{}^B = 1.2\, A_0{}^A$. Determine which of the bars will yield first and calculate the corresponding force and total elongation of the system, ΔL. [15%]

(f) Define the glass transition temperature T_G for a thermoset polymer. Sketch the uniaxial tensile stress versus strain response of a thermoset polymer at temperatures above and below T_G. [10%]

(g) Sketch the uniaxial compressive stress versus strain responses of a ductile material at increasing levels of porosity, showing the evolution from a fully dense material to a foam. Mark the salient phases of the response of a foam in your sketch. [10%]

2.

(a) A metallic-ceramic composite is manufactured by thoroughly mixing Titanium powder (index T) with Alumina powder (index A) and then sintering. The process results in a solid microstructure of negligible porosity and volume fraction of Alumina $\varphi_A = 0.2$. The properties of the constituent materials are

$$E^T = 110 \text{ GPa}, \ \rho^T = 4500 \,\text{kg m}^{-3}; \quad E^A = 250 \text{ GPa}, \ \rho^A = 3500 \,\text{kg m}^{-3}.$$

Calculate the density of the composite and the expected range of its Young's modulus. [15%]

(b) A simply-supported metallic beam of span $L = 1\,\text{m}$ and square cross-section of area $t \times t$ needs to be designed. The beam is required to carry a transverse force $F = 2\,\text{kN}$ at mid-span without failure. The maximum stress in the beam is given by $\sigma_{\max} = 3FL/(2t^3)$.

i. Derive the material merit index which must be maximised to obtain a design of minimum mass. [10%]

ii. Select which of the following two materials is most appropriate for this design: steel $(\sigma_Y = 350\,\text{MPa}, \ \rho = 8000\,\text{kg m}^{-3})$ or Titanium alloy $(\sigma_Y = 750\,\text{MPa}, \ \rho = 4500\,\text{kg m}^{-3})$. Calculate the minimum value of t. [10%]

(c) Define the Mode I strain energy release rate G_I. Define the critical strain energy release rate G_{IC} and illustrate mathematically its relation to surface tension for a ductile material. [10%]

[Question continued on next page]

(d) A long strip of paper of width $w = 200\ \mathrm{mm}$ and thickness $t = 50\ \mu\mathrm{m}$ is loaded in tension perpendicularly to its width. The paper has elastic modulus $E = 3\ \mathrm{GPa}$ and critical strain energy release rate $G_{IC} = 3\ \mathrm{kJ\,m^{-2}}$. A small central crack, of length $2a = 6\ \mathrm{mm}$ and perpendicular to the applied force, is present in the strip. Calculate the force at which the strip will fracture. [15%]

(e) Describe briefly, with the aid of suitable sketches, the phenomenon of progressive micro-void growth and coalescence observed at the tip of a crack in a ductile component upon mechanical loading. [10%]

(f) Discuss briefly, with the aid of suitable sketches, the differences in the uniaxial tensile and compressive responses of ceramics, explaining the macroscopic stress versus strain responses in tension and compression in terms of the corresponding microscopic fracture mechanisms. [10%]

(g) A fully loaded passenger lift cabin has total mass $m = 2500\ \mathrm{kg}$ and is at rest at a height $h = 10\ \mathrm{m}$ above the ground. The lift's cables suddenly break and the cabin undergoes free-fall for a height h. At the bottom of the lift shaft there is a crash absorber of cross-sectional area $A = 2\ \mathrm{m}^2$ and height H in the vertical direction; the crash absorber is made from a foam of nominal densification strain $\varepsilon_D = 0.9$. Calculate the maximum plateau strength of a suitable foam and the height H of the crash absorber to ensure that the cabin deceleration does not exceed $a_{\max} = 10g$. [20%]

Section B

3. The table below lists five different materials and indicates their structure in the solid state, as well as their melting temperature.

Material	**Structure**	**Melting Temperature °C**
A	f.c.c.	660
B	Rock Salt (NaCl)	2900
C	Diamond lattice	3550
D	Zinc Blende	1412
E	h.c.p.	639

(a) The five materials are: Magnesium Oxide (MgO), Magnesium (Mg), Aluminium (Al), Diamond and Silicon Carbide (SiC).

(i) Identify materials A-E and indicate the type of bonding present in their solid state. Briefly describe a key characteristic of each material. [10%]

(ii) For each material, state the number of nearest neighbouring atoms or ions to any individual particle (atoms or ions) in the solid. [5%]

(iii) State which of the five materials you would expect to have: the highest modulus of elasticity; the highest hardness; the highest ductility, and briefly explain your reasoning. [5%]

[Question continued on next page]

(b) With reference to Figure 1, the potential energy U of two atoms separated by a distance r is given by:

$$U = -\frac{A}{r^m} + \frac{B}{r^n} \qquad m = 2; n = 10$$

Given that the atoms form a stable molecule at a separation of $r_0 = 0.30$ nm with an energy $U_0 = -4.0$ eV (N.B. 1 eV = 1.602 × 10^{-19} J), calculate A and B. Also, find the force required to break the molecule, and the critical separation at which the molecule breaks. Sketch a potential energy versus separation distance curve for the molecule, and sketch beneath this curve the appropriate force versus separation distance curve. [18%]

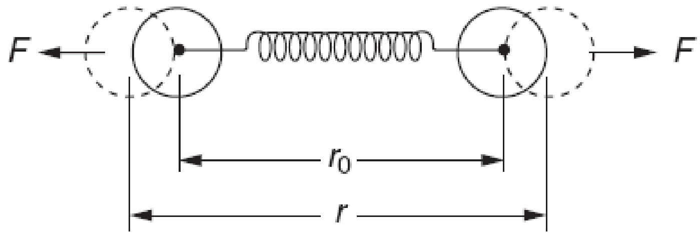


Figure 1: Idealisation of two atoms in a molecule.

(c) (i) Sketch three-dimensional views of the unit cell of a body-centred cubic (bcc) crystal, showing
a (100) plane, a (110) plane, a (111) plane and a (210) plane. [8%]

(ii) The slip planes of bcc iron are the {110} planes; sketch the atomic arrangement in these planes, and mark the <111> slip directions. [4%]

(iii) Sketch the slip plane for the face centred cubic (fcc) crystal structure in three-dimensional view and label the slip directions in the plane. What is the slip system for this structure? [4%]

[Question continued on next page]

(d) A single crystal of aluminium is oriented so that the $[001]$ direction is parallel to an applied tensile stress of 60 MPa. Calculate the resolved shear stress acting on the $[111]$ plane in the $[\bar{1}10]$, $[0\bar{1}1]$ and $[1\bar{1}0]$ directions. Which slip system(s) will become active first? Justify your answer. [16%]

(e) The strength of titanium is 450 MPa when the grain diameter (d) is 17 μm and 565 MPa when the grain diameter is 0.8 μm. Determine the constants in the Hall-Petch equation

$$\sigma_{yield} = \sigma_0 + k_y d^{-0.5}.$$

If the strength of the titanium is required to be 680 MPa, to what should the grain size be reduced? Explain why the grain size affects the strength of the material. [15%]

(f) Name three other strengthening mechanisms that are commonly used in metallic materials. Briefly explain how each mechanism works and give an example for each mechanism. [15%]

Course Code and Title : AE 1-106 Properties of Materials

Setter: VL Tagarielli

1

	Marks

$1(a)$

$\Delta V = V_0 \varepsilon_V;\ \varepsilon_V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\alpha\Delta T;\ \Delta V = 3V_0\alpha\Delta T = 0.75\ \text{mm}^3.$

15

$1(b)$

$G = E/\left[2(1+\nu)\right] = 38.46\,\text{GPa};\ \sigma_Y = E\varepsilon_Y = 50\,\text{MPa}$

(i)

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[(10-20)^2 + (10-0)^2 + (20-0)^2 + 6\left(0+5^2+0\right)\right]} = 19.36 < \sigma_Y$$

10

(ii)

$\varepsilon_x = \frac{\sigma_1}{E} - \frac{\nu}{E}(\sigma_2) = 3.33\ 10^{-5}$ $= \frac{10(1-2\cdot 0.3)}{100\cdot 10^3} = 0.4\cdot 10^{-4} = 4\cdot 10^{-5}$

$\varepsilon_y = \frac{\sigma_2}{E} - \frac{\nu}{E}(\sigma_1) = 16.66\ 10^{-5}$ $= \frac{20}{100\,10^3} - \frac{0.3\cdot 10}{100\cdot 10^3} = 17\cdot 10^{-5}$

$\varepsilon_z = -\frac{\nu}{E}(\sigma_1 + \sigma_2) = -9.00\ 10^{-5}$

$\gamma_{xy} = \tau_{xy}/G = 0$

$\gamma_{xz} = \tau_{xz}/G = 13.00\ 10^{-5}$ ✓

$\gamma_{yz} = \tau_{yz}/G = 0$

10

$1(c)$

$\varepsilon^{el} = \sigma_Y/E = 3.5\ 10^{-3};\ \varepsilon_n^{app} = \Delta L/L_0 = 0.3;\ F = A_0\sigma_Y = 35\,\text{kN};$

$\varepsilon_n^{pl} = \varepsilon_n^{app} - \varepsilon^{el} = 0.2965;\ \Delta L^{pl} = \varepsilon_n^{pl} L_0 = 0.2965\,\text{m};\ L_f = L_0 + \Delta L^{pl} = 1.2965\,\text{m};$

$\varepsilon^{pl} = \ln\left(1+\varepsilon_n^{pl}\right) = 0.2597.$

20

$1(d)$

$H = 3\sigma_Y = 1050\ \text{MPa}.$

10

$1(e)$

$F^A = F^B;\ \ F_Y{}^A = A^A\sigma_Y{}^A < F_Y{}^B = A^B\sigma_Y{}^B \Rightarrow F_Y = F_Y{}^A = 35\ \text{kN}.$

$$\Delta L = \Delta L^A + \Delta L^B = F_Y L\left(\frac{1}{A^A E^A} + \frac{1}{A^B E^B}\right) = \frac{F_Y L}{A^A}\left(\frac{1}{E^A} + \frac{1}{1.2E^B}\right) = 9.33\,\text{mm}.$$

15

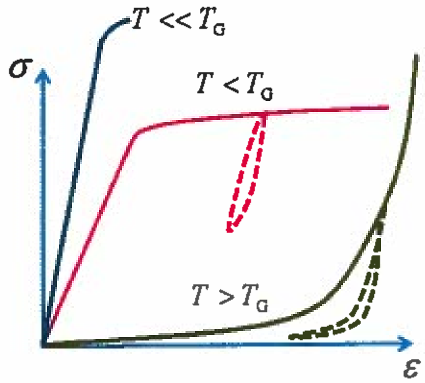
Course Code and Title : AE 1-106 Properties of Materials

Setter: VL Tagarielli

2

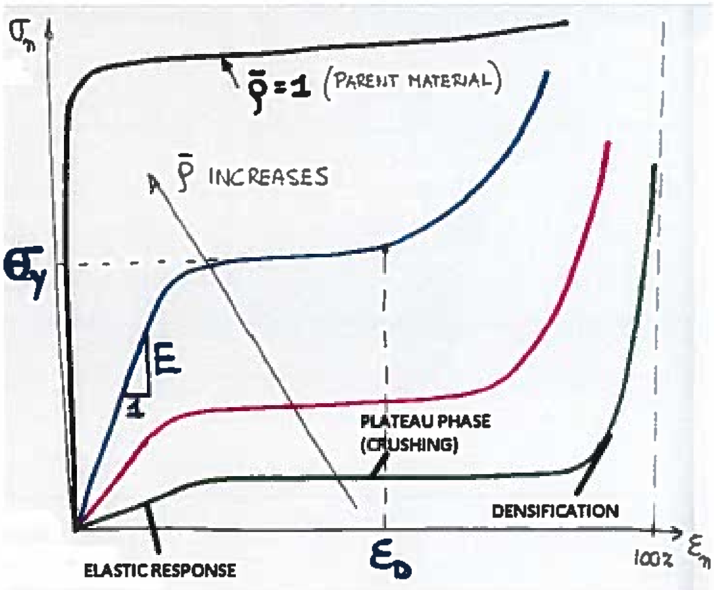
1(*f*)

Temperature at which secondary bonds melt, eliminating mechanical side-ways interactions between polymer chains.



10

1(*g*)



10

Tot 100

Course Code and Title : AE 1-106 Properties of Materials

Setter: VL Tagarielli

3

2(a)

$\rho = \varphi_A \rho^A + (1-\varphi_A)\rho^T = 4300 \text{ kg m}^{-3};$

$E_{\max} = \varphi_A E^A + (1-\varphi_A)E^T = 138\,\text{GPa};$

$E_{\min} = \left(\dfrac{\varphi_A}{E^A} + \dfrac{(1-\varphi_A)}{E^T}\right)^{-1} = 123.9\,\text{GPa};$ **15**

2(b)

$3FL/(2t^3) \le \sigma_y \Rightarrow t \ge \sqrt[3]{\dfrac{3FL}{2\sigma_Y}}$; take $t = \sqrt[3]{\dfrac{3FL}{2\sigma_Y}}$;

(i) $M = t^2 L\rho = L\left(\dfrac{3FL}{2}\right)^{\frac{2}{3}} \dfrac{\rho}{\sigma_Y^{2/3}}; \quad M_I = \dfrac{\sigma_Y^{2/3}}{\rho};$ **10**

(ii) $M_{I\text{steel}} < M_{I\text{Ti}} \Rightarrow \sigma_Y = 750\,\text{MPa}, \quad \rho = \cancel{8000}\ 4500\,\text{kg m}^{-3}.$

$t = \sqrt[3]{\dfrac{3FL}{2\sigma_Y}} = \cancel{20.46\,\text{mm}}.$ ~~= 0.02~~ 0.0159 m **10**

2(c)

$G_I = -\dfrac{dU}{dA}$ quantifies the decrease of elastic strain energy ($-dU$) in a cracked body when the cracked area increases (by dA).

At fracture (unstable crack propagation) in a ductile material it is

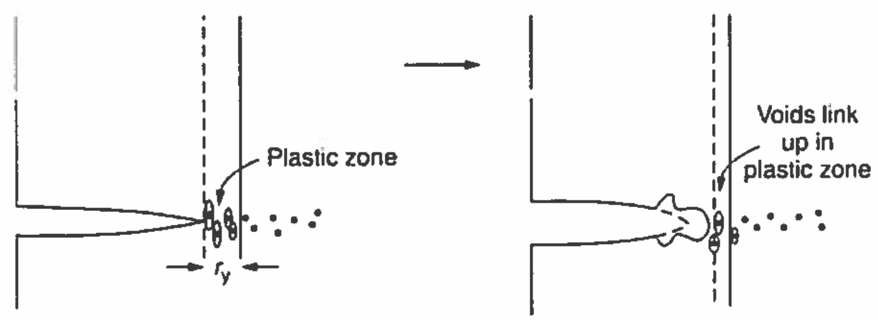
$-dU > 2\gamma dA + (\text{plastic dissipation at crack tip}) = G_{Ic} dA.$ **10**

2(d)

$K_{Ic} = \sqrt{EG_{Ic}} = 3\text{ MPa}\sqrt{m}. \quad \sigma^* \sqrt{\pi a} = K_{Ic} \Rightarrow \sigma^* = 30.902\,\text{MPa};\ F^* = \sigma^* wt = 309.02\,\text{N}.$ **15**

2(e)

Voids originate from impurities, precipitates or grain boundary within plastic zone
Voids expand and elongate (in mode I) until they link-up (coalesce), creating new fracture surfaces and advancing the crack.



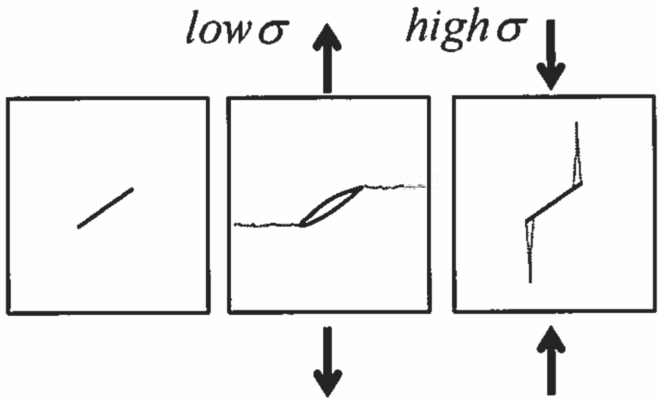
10

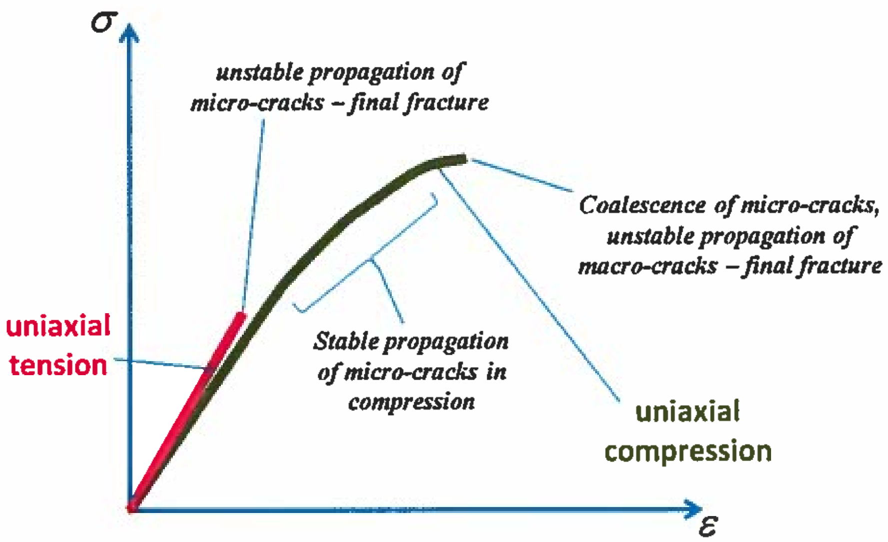
Course Code and Title : AE 1-106 Properties of Materials

Setter: VL Tagarielli

4

2(*f*)





In tension cracks tend to propagate unstably at low stresses, giving sudden failure.
In compression frictions between the crack faces increases the stress necessary for crack growth.
Cracks in compression grow vertically; initially their growth is stable, then microcracks coalesce giving large cracks in the direction of loading, finally propagating unstably (and giving final fracture by 'shuttering'). **10**

2(*g*)

1.1 m

$$\sigma_{pl}^{\max} = ma_{\max} / A = 0.1226\,\text{MPa};\ \sigma_{pl}^{\max}\varepsilon_D AH > mgh \Rightarrow H > \frac{mgh}{\sigma_{pl}^{\max} A\varepsilon_D} = 1.\cancel{223}\,\text{m}.$$ **20**

Tot 100

Course Code and Title : AE 1-106 Property of Materials

Setter: Qianqian Li

Write on this side only (in ink) between the margins, not more than one solution per sheet please. Solutions must be signed and dated by both exam setter and referee.

Marks

3 (a)

Material	Structure	Melting Temperature °C
Aluminium, Al	f.c.c.	660
Magnesium Oxide, MgO	Rock Salt (NaCl)	2900
Diamond	Diamond lattice	3550
Silicon Carbide, SiC	Zinc Blende	1412
Magnesium, Mg	h.c.p.	639

i. Material A: Al … metallic – stiff and strong, high melting point, conductive materials, ductile **2**

Material B: MgO … ionic; good insulator, strong but brittle **2**

Material C: Diamond … covalent; very stiff and very strong, very high melting point, very good insulator. **2**

Material D: SiC … covalent; very stiff and very strong, high melting point, good insulator. **2**

Material E: Mg … metallic. Same as Al.. **2**

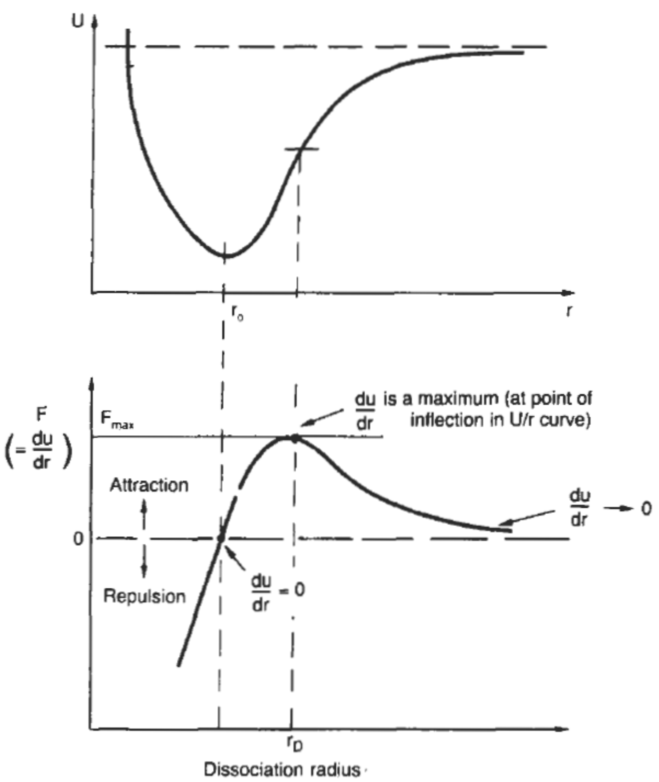
ii. Al … 12; MgO … 6; Diamond … 4; SiC … 4 ; Mg …. 12 **5**

iii. Modulus … Diamond, strongest (covalent) bonding / stiffest "springs" between atoms, **2**

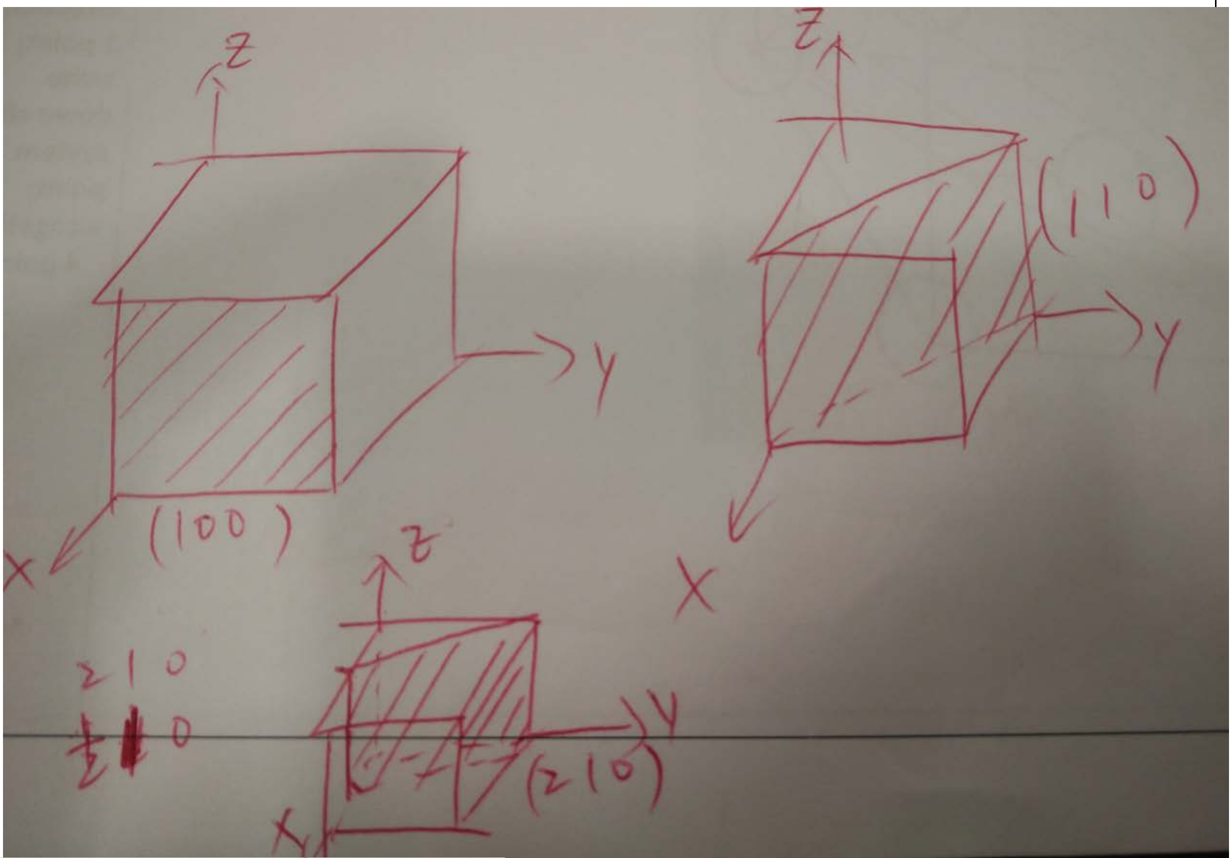
Highest hardness … Diamond, covalent bonding, highest melting point **1**

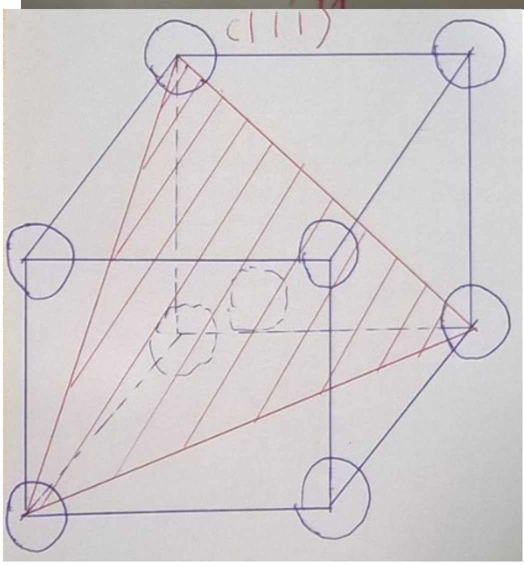
Ductility … Al, metallic bonding (non directional), fcc structure, large number of available slip systems. **2**

20%

3(b) In order to answer this question we need the first, second and third derivative of the given equation	
$U = -\frac{A}{r^2} + \frac{B}{r^{10}}$ (1)	**1**
$\frac{dU}{dr} = 2\frac{A}{r^3} - 10\frac{B}{r^{11}}$ (2)	**1**
$\frac{d^2U}{dr^2} = -6\frac{A}{r^4} + 110\frac{B}{r^{12}}$ (3)	
$\frac{d^3U}{dr^3} = 24\frac{A}{r^5} - 1320\frac{B}{r^{13}}$ (4)	
at $r=r_0$, $dU/dr=0$ as we are at minimum of the potential energy curve; solve dU/dr to get equation for A (or B) expressed by the other variable B (A respectively): $A=5B/(r_0^8)$;	**2**
and insert into Eq. 1 to get A and B	
A= $7.21*10^{-38}$ [J*m^2] or $7.21*10^{-20}$ [J*nm^2] or $4.5*10^{-19}$ [ev*m^2]	**2**
B= $9.46*10^{-115}$ [J*m^{10}] or $9.46*10^{-25}$ [J*nm^2] or $5.9*10^{-96}$ [ev*m^{10}]	**2**
we know at dissociation bond length U function has a turning point so to get the r_D Eq.3 has to be set zero and Eq.4 should give non zero result. From this we find:	
$r_D^8=110B/6A$	**2**
r_D = 0.353 nm or $0.353*10^{-19}$ m; $d^3U/dr^3 \neq 0$	**2**
to get the force required we just need to calculate the force at r_D and accordingly,	
we plug r_D into Eq. 2:	
$F=(2A/r^3)-(10B/r^{11})$	**1**
F=2.38 nN or $2.386*10^{-9}$ N or 14.9 ev*nm^{-1}	**1**
	**2(draw the curve 1, state the points correctly 1)** **2(draw the curve 1, state the points 1)**
	18%

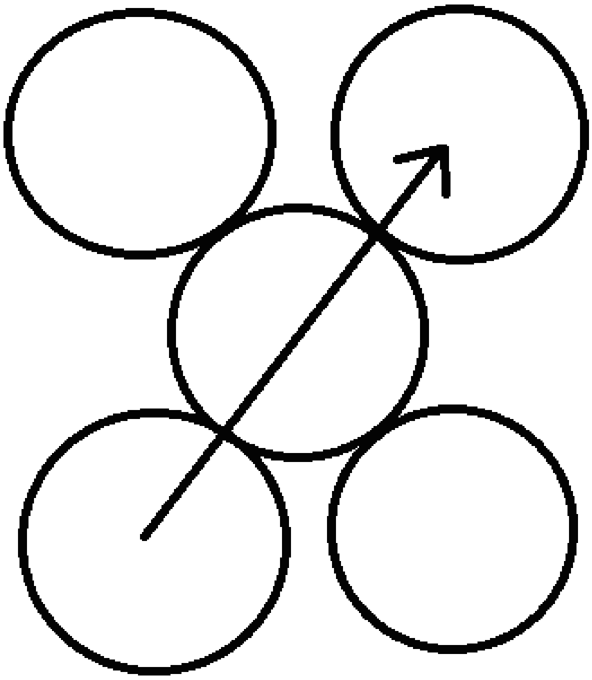
3(c)
(i)





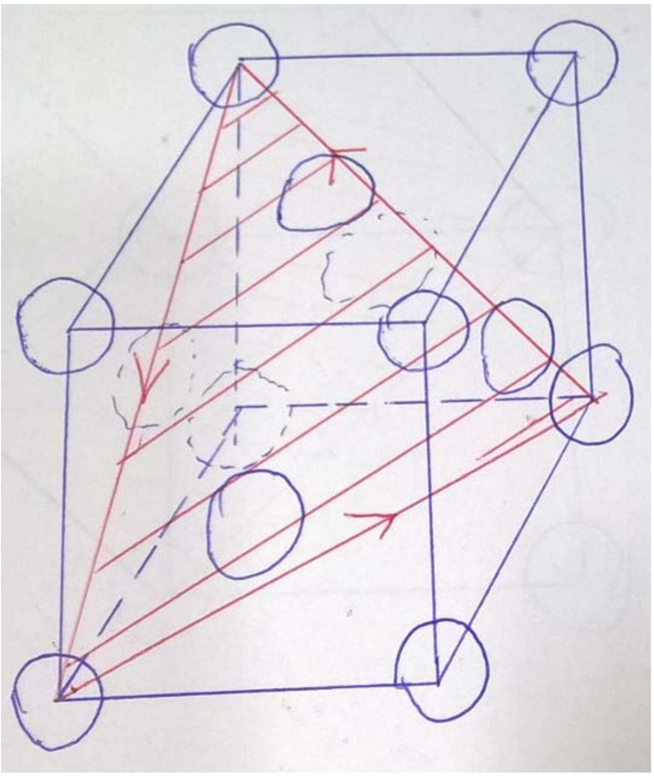
2 points each, altogether 8

3(c)
(ii)



Atom drawing 2 point, direction 2 point, altogether 4 points.

(iii) Slip planes {111}, directions <110>.



Drawing slip plane 1 points, slip direction 1 point; write down slip system 2 point; altogether 4 points

16%

3(d) The resolved shear stress can be calculated by $\tau_y = \sigma \cos\varphi \cos\vartheta$ **4**

Angle between the vectors is given by $\cos\theta = \frac{\vec{a}\vec{b}}{|a||b|}$ **2**

for (111) and [001] cosφ=1/√3 or 54.74° for all instances and for the directions **2**

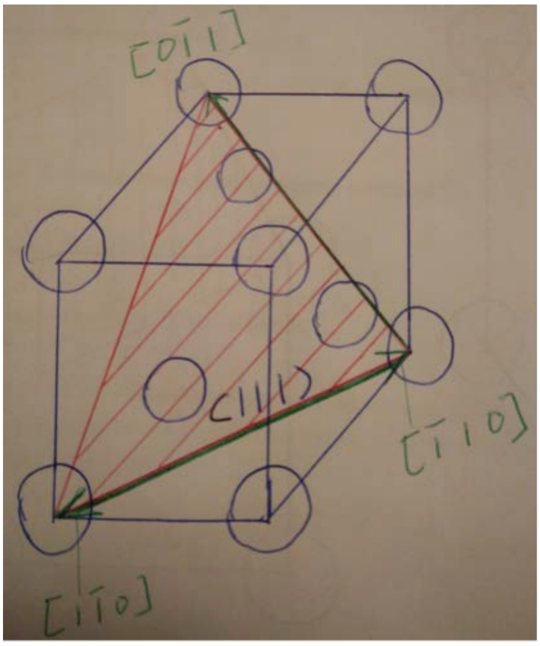
for $[\bar{1}10]$ cosϑ=0 (direction is perpendicular to stress direction) **2**

for $[\mathbf{0}\bar{\mathbf{1}}\mathbf{1}]$ cosϑ=1/√2 **2**

for $[\mathbf{1}\bar{\mathbf{1}}\mathbf{0}]$cosϑ=0 **2**

Slip systems with highest Schmid-factor and thus activated first is (111) $[0\bar{1}1]$. **1**

To illustrate: **1**



16%

3(e) Constants determined by graphic extrapolation σ_0 = 418 MPa, k_y = 0.131 $MPam^{-0.5}$	**3** **3**
and grain size for strength 680 MPa is 0.25um. $680=418+0.131*d^{(-0.5)}$ $d=0.25x10^{-6}$ m=0.25um or $0.25*10^{-6}$m	**5**
Grain boundaries act as obstacles to dislocation motion (Dislocation pile ups at grain boundaries) and the increased obstacle density leads to a strengthening effect.	**4**
	15%

3(f) 1. Precipitation hardening By choice of suitable alloying system and thermal treatment it is possible to form closely spaced precipitates within the crystal host lattice. These precipitates act as obstacles to dislocation motion and have to be overcome either by cutting through them or bowing around them, which leads to a strengthening effect. Such as Mg alloys. 2. Solid solution strengthening By solving atoms of a different species in the host lattice dislocation motion can be hindered. The atomic mismatch between the atoms leads to stress/strain fields that interact with the dislocations and lead to reduced mobility and, hence, strengthening. Such as carbon in steel. 3. Work hardening Plastic deformation leads to the multiplication of dislocations (or lengthening of dislocations) increasing dislocation density in the material. As dislocations interact with each other and can act as obstacles to each others motion this leads to strengthening of the material. Such asforging, rolling, drawing and extrusion, and cold work.	**4 points each; example 1 point each.** **15%**

Datasheet – AE1-106 Properties of Materials

Stress – strain definitions and equations of elasticity.

$$\sigma_n = \frac{F}{A_0}; \quad \sigma_t = \frac{F}{A} = \sigma_n\left(1+\varepsilon_n\right) \quad \text{(for plastically incompressible solids)};$$

$$\varepsilon_n = \frac{\Delta L}{L_0}; \; \varepsilon_t = \ln\left(1+\varepsilon_n\right);$$

$$\tau = G\gamma; \quad G = \frac{E}{2(1+\nu)};$$

$$\varepsilon_{V_n} = \frac{\Delta V}{V_0}; \quad \sigma_H = -p = \frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3};$$

$$\sigma_H = K\varepsilon_V; \quad K = \frac{E}{3(1-2\nu)}$$

$$\varepsilon^{th} = \alpha\,\Delta T$$

$$\begin{cases} \varepsilon_1 = \dfrac{\sigma_1}{E} - \dfrac{\nu}{E}(\sigma_2+\sigma_3) + \alpha\,\Delta T \\ \varepsilon_2 = \dfrac{\sigma_2}{E} - \dfrac{\nu}{E}(\sigma_1+\sigma_3) + \alpha\,\Delta T \\ \varepsilon_3 = \dfrac{\sigma_3}{E} - \dfrac{\nu}{E}(\sigma_1+\sigma_2) + \alpha\,\Delta T \\ \gamma_{12} = \tau_{12} / G \\ \gamma_{13} = \tau_{13} / G \\ \gamma_{23} = \tau_{23} / G \end{cases}$$

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{dl}{l\,dt} = V / l$$

Atomic bond

$$U(r) = -\frac{A}{r^m} + \frac{B}{r^n}, \quad n > m; \quad F(r) = \frac{dU}{dr};$$

$$S(r) = \frac{dF}{dr} = \frac{d^2U}{dr^2}; \quad S_0 = S(r_0)$$

Fracture mechanics

$$K_I = Y\sigma\sqrt{\pi a} \geq K_{IC} \quad \text{at unstable crack propagation (fracture)}$$

$$G_{IC} = \frac{{K_{IC}}^2}{E}$$

Two – phase composites

$$\rho = \varphi_f \rho_f + (1 - \varphi_f)\rho_m;$$

$$E_1 = \varphi_f E_f + (1 - \varphi_f)E_m;$$

$$\frac{1}{E_2} = \frac{\varphi_f}{E_f} + \frac{(1 - \varphi_f)}{E_m}; \quad \frac{1}{G_2} = \frac{\varphi_f}{G_f} + \frac{(1 - \varphi_f)}{G_m}$$

Cellular solids (foams)

$$\bar{\rho} = \frac{\rho_{foam}}{\rho_{parent\ material}}; \quad f = \frac{V_{pores}}{V_{total}} = 1 - \bar{\rho}.$$