EE2-04

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

EEE/EIE PART II: MEng, BEng and ACGI

COMMUNICATION SYSTEMS

Corrected copy

Thursday, 31 May 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D. Gunduz
Second Marker(s) : J.A. Barria

Information for Students

Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} \tau f$
2.	$2W \operatorname{sinc} 2Wt$	$\Pi\left(\frac{f}{2W}\right)$
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 \tau f$
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$
6.	$\exp(-\alpha\lvert t\rvert),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$
7.	$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f\tau)^2}$
8.	$\delta(t)$	1
9.	1	$\delta(f)$
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$
15.	$\operatorname{sgn} t$	$(j\pi f)^{-1}$
16.	$\frac{1}{\pi t}$	$-j\operatorname{sgn}(f)$
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda}\, d\lambda$	$-j\operatorname{sgn}(f)X(f)$
18.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s),\ f_s = T_s^{-1}$

Useful Relations and Formulas

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\tan(\frac{\pi}{2} - x) = \cot x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cot x}{\cot^2 x - 1} = \frac{2}{\cot x - \tan x}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$$

$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$$

$$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

Differentiation Rule of Leibnitz

Let $F(z) = \int_{a(z)}^{b(z)} f(x,z)dx$. Then we have

$$\frac{dF(z)}{dz} = \frac{db(z)}{dz} f(b(z),z) - \frac{da(z)}{dz} f(a(z),z) + \int_{a(z)}^{b(z)} \frac{\partial f(x,z)}{\partial z} dx$$

Fourier Transform Theorems[a]

Name of Theorem		
1. Superposition (a_1 and a_2 arbitrary constants)	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(f) + a_2X_2(f)$
2. Time delay	$x(t - t_0)$	$X(f)e^{-j2\pi f t_0}$
3a. Scale change	$x(at)$	$\|a\|^{-1}X\left(\frac{f}{a}\right)$
b. Time reversal	$x(-t)$	$X(-f) = X^*(f)$
4. Duality	$X(t)$	$x(-f)$
5a. Frequency translation	$x(t)e^{j\omega_0 t}$	$X(f - f_0)$
b. Modulation	$x(t)\cos\omega_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
6. Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
7. Integration	$\int_{-\infty}^{t} x(t')\,dt'$	$(j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
8. Convolution	$\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\,dt' = \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\,dt'$	$X_1(f)X_2(f)$
9. Multiplication	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\,df' = \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\,df'$

Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}dt$$

Joint Gaussian density

The joint probability density function (pdf) of two correlated Gaussian random variables X and Y is given by

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_X)^2}{\sigma_X^2} - \frac{(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right]}.$$

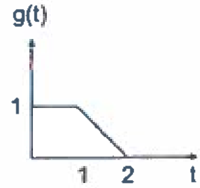
where $\mu_X = E[X]$, $\mu_Y = E[Y]$ are the mean values, σ_X and σ_Y are the standard deviation of X and Y, respectively, and ρ is the correlation coefficient defined as

$$\rho = \frac{\mathrm{Cov}(X,Y)}{\sigma_X\sigma_Y}.$$

1. a) i) Why are digital signals more immune to channel noise compared to analogue signals? [2]

ii) What is the minimum sampling rate such that the function $f(t) = \text{sinc}^2(3t)$ can be reconstructed exactly from its samples? [2]

iii) Let $g(t)$ be the following pulse shape:



Assume that the receiver receives $y(t) = a \cdot g(t) + w(t)$, where $w(t)$ is white Gaussian noise, and a is an unknown constant that the receiver wishes to detect. $y(t)$ is passed through a filter and then sampled at $t = 2$. Assuming that $g(t)$ is known at the receiver, what is the best filter that minimizes the effect of noise? [3]

iv) Consider a QPSK system, where the transmitted signal is denoted by

$$x(t) = A_c \cos(2\pi f_c t + \phi), \qquad \phi \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}.$$

Draw the diagram of a coherent QPSK receiver, and explain the function of each component of the receiver. [5]

b) Let $X(t)$ and $Y(t)$ be two random processes. State whether each of the following statements are true or false, and discuss your answer:

i) If $X(t)$ is strict sense stationary (SSS), it is also wide sense stationary (WSS). [2]

ii) If $X(t)$ is WSS, it is also SSS. [2]

iii) If $X(t)$ is a white process, then it is Gaussian. [2]

iv) If $X(t)$ and $Y(t)$ are WSS, so is $X(t) + Y(t)$. [3]

c) Assume that $X(t)$ is a zero-mean WSS random process with power spectral density $S_X(f) = \Pi(\frac{f}{3000})$, where $\Pi(x)$ is the rectangular function defined as follows:

$$\Pi(x) \triangleq \begin{cases} 1 & \text{if } |x| < 1/2, \\ 1/2 & \text{if } |x| = 1/2, \\ 0 & \text{otherwise .} \end{cases}$$

i) What is the maximum sampling rate that will lead to uncorrelated samples? [4]

ii) If the sampling rate is 1KHz, and each sample is quantized by a 10-bit quantizer, how much storage is needed to store a 5 second time-frame of signal $X(t)$? [2]

d) Assume that $X(t)$ is a real zero-mean WSS Gaussian random process. $X(t)$ is passed through a linear time invariant (LTI) system, and the output is denoted by $Y(t)$. Let $h(t) = \text{sinc}(t)$ denote the impulse response of the LTI system.

i) Which of the following three statements are true?

1. $Y(t)$ is Gaussian.
2. $S_Y(f)$ is bandlimited.
3. $Y(t)$ is WSS, but not necessarily SSS.

[6]

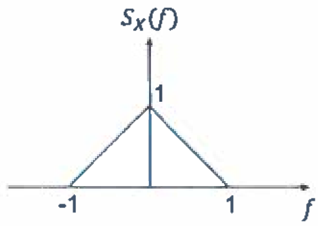
ii) Find $\Pr\{Y(1) \geq 0\}$. [3]

iii) If $\Pr\{Y(1) + Y(2) \leq 2\} = 0.3$, find

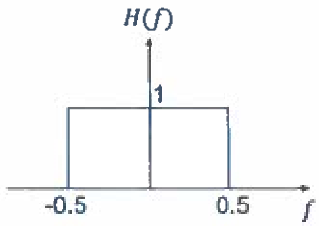
$$\Pr\{Y(-1) + Y(-2) > 2\}.$$

[4]

2. a) A WSS Gaussian random process $X(t)$ has the following power spectral density (PSD):



Assume that $\mathbb{E}[X(t)] = \frac{1}{2}$ for all t, and $X(t)$ is passed through a linear time invariant (LTI) filter with the following frequency response, and the output is denoted by $Y(t)$.



i) Find $\mathbb{E}[Y(t)]$. [3]

ii) Find the PSD of $Y(t)$, i.e., $S_Y(f)$. [4]

iii) Find $\Pr\{Y(0) \geq 2\}$. [5]

iv) Find $\Pr\{Y(1)+Y(2)+Y(3) \geq 3/2\}$. [3]

b) A binary message source generates bit "0" with probability p_0 and "1" with probability p_1. These bits are transmitted over a binary digital communication system. Bit "0" is transmitted with a pulse of amplitude -1, and bit "1" is transmitted with a pulse of amplitude 1. The noise in the channel is zero-mean additive white Gaussian with variance 0.5. The receiver uses a matched filter followed by threshold detection.

i) Determine the optimum detection threshold if $p_1 = 0.5$. [3]

ii) Determine the optimum detection threshold if $p_1 = 0.2$. What is the corresponding probability of error? [6]

iii) Assume that the receiver sets the optimum threshold as derived in question ii). However; the message source generates bits with $p_1 = 0.7$. What is the probability of error? How does this compare with the probability of error you found above? [6]

3. a) Let $m(t) = \sqrt{6}\cos(4\pi t)$ denote a baseband message signal. The signal undergoes standard amplitude modulation (AM). Assume that the additive noise is Gaussian and white, with the autocorrelation function $R_N(\tau) = \frac{1}{2}\delta(\tau)$.

i) Draw the diagram of a coherent AM detector, and explain the function of each component. [5]

ii) Calculate the signal to noise ratio (SNR) at the receiver output. [6]

iii) How does the performance of the above system compare with that of a baseband system with the same transmitted power? [4]

b) Assume that a memoryless source outputs symbols A, B and C with the corresponding probabilities 0.5, 0.3 and 0.2, respectively.

i) Design a Huffman code for compressing this source output. [3]

ii) What is the average codeword length of this code? [3]

iii) What is Shannon's theoretical bound on the minimum average codeword length for this source? [3]

iv) Assume that we want to compress two-letter words independently generated by this source, i.e., *AA*, *AB*, *AC*, ... Design a Huffman code for compressing these two-letter words. What is the average codeword length per symbol for this code? [6]