EE2-05

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EEE/EIE PART II: MEng, Beng and ACGI

Corrected Copy

SIGNALS AND LINEAR SYSTEMS

Friday, 6 June 2:00 pm

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.

Question One carries 40% of the marks. The other 2 questions each carry 30%.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.L. Dragotti
Second Marker(s) : P.T. Stathaki

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Special Information for the Invigilators: none

Information for Candidates

Some Fourier Transforms

$$\mathrm{rect}(\tfrac{t}{\tau}) \iff \tau \mathrm{sinc}(\tfrac{\omega\tau}{2})$$

$$\tfrac{W}{\pi}\mathrm{sinc}(Wt) \iff \mathrm{rect}(\tfrac{\omega}{2W})$$

Time-shifting property of the Fourier transform

$$x(t-t_d) \iff X(\omega)e^{-j\omega t_d}$$

Scaling property of the Fourier transform

$$x(at) \iff \frac{1}{|a|}X(\frac{\omega}{a})$$

The unit step function $u(t)$ is defined as:

$$u(t) = \begin{cases} 1 & \text{for } \; t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Some Laplace transforms

$$e^{\lambda t}u(t) \iff \tfrac{1}{s-\lambda}$$

$$e^{-at}\cos(bt)u(t) \iff \tfrac{s+a}{(s+a)^2+b^2}$$

A useful z-transform

$$\gamma^n u[n] \iff \frac{z}{z-\gamma} \qquad |z| > |\gamma|$$

The Questions

1. This question carries 40% of the mark.

   (a) Given the signal:

   $$x(t) = \begin{cases} t & \text{for } \; 0 \leq t \leq 1 \\ 0 & \text{otherwise,} \end{cases}$$

   sketch and dimension each of the following signals:

   i. $x_1(t) = x(t-2)$ [2]

   ii. $x_2(t) = x(-2t+4)$ [2]

   iii. $x_3(t) = x(2t) + x(-2t)$ [2]

   (b) State with a brief explanation if the systems with the following input/output relationships are linear/non-linear, time-invariant/time varying.

   i. $y(t) = x(t-2) + x(2-t)$ [4]

   ii. $y(t) = x(t)\cos(3t)$ [4]

   (c) Given the following signal

   $$x(t) = \begin{cases} e^{-t}, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

   compute the convolution $c(t) = x(t) * x(t)$. [5]

Question 1 continues on next page

(d) The Fourier transform of the triangular pulse $x(t)$ in Figure 1(a) is

$$X(\omega) = \frac{1}{\omega^2}(e^{j\omega} - j\omega e^{j\omega} - 1).$$

Using this information, the scaling property and the time-shifting property, find the Fourier transform of the signal $y(t)$ shown in Figure 1(b). Notice that $y(t)$ is real and even, so you expect $Y(\omega)$ to be real and even as well. [5]

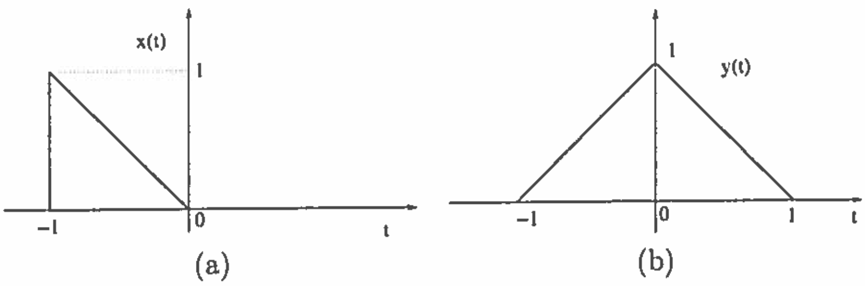

Figure 1: The two signals $x(t)$ and $y(t)$.

Question 1 continues on next page

(e) A linear time-invariant system is specified by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = x(t).$$

i. Find the characteristic polynomial, characteristic roots and characteristic modes of this system. [2]

ii. Find the zero-input component of the response $y(t)$ for $t \geq 0$, if the initial conditions are $y(0) = 0$ and $\dot{y}(0) = 2$. [2]

iii. Find the zero-state response assuming $x(t) = u(t)$ where $u(t)$ is the unit step function [Hint: use the Laplace transform]. [2]

iv. Finally find the total response of the system when the input is $x(t) = u(t)$ and the initial conditions are $y(0) = 0$ and $\dot{y}(0) = 2$. [2]

(f) Consider the signal $x(t) = 6000\text{sinc}(6000\pi t)$

i. Sketch and dimension the Fourier transform of $x(t)$. [2]

ii. Determine the Nyquist sampling rate for $x(t)$. [2]

(g) Find the causal inverse z-transform of

$$X[z] = \frac{z(z+1)}{(z^2 - 5z + 4)}.$$

[4]

2. For the RLC circuit in Fig. 2, the switch is at position A for a long time before $t = 0$, when it is instantaneously moved from position A to position B.

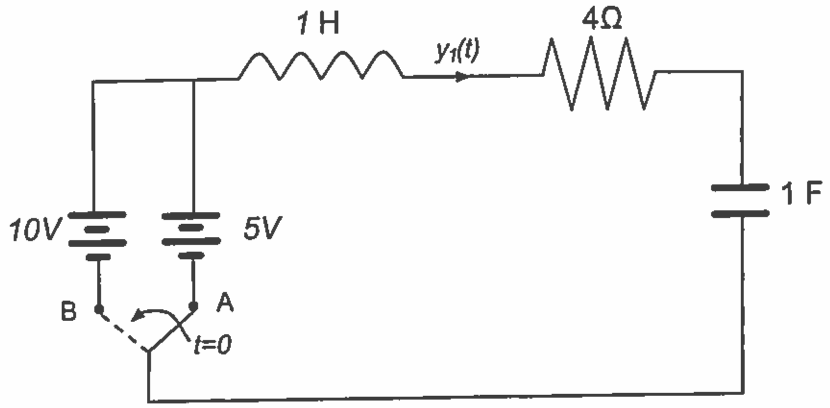

Figure 2: A RLC electric circuit where the switch moves from position A to B at $t = 0$.

(a) Determine the initial condition $y_1(0^-)$ and $v_C(0^-)$, where $v_C(t)$ is the voltage across the capacitor and $y_1(t)$ is the current across the circuit. [10]

(b) Write the loop equation in the Laplace domain. [10]

(c) Find the exact expression of the current $y_1(t)$. [10]

3. Consider the system depicted in Figure 3.

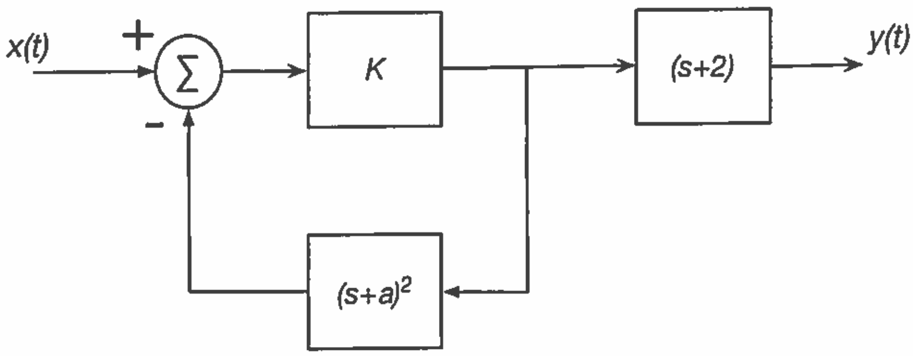

Figure 3: Block diagram of a linear system.

(a) Find the transfer function of the system, that is, find $H(s) = Y(s)/X(s)$. [8]

(b) Assume that $K = 1$ and that $x(t) = u(t)$ is the unit step function, find the value a>0 such that $y(\infty) = \lim_{t\to\infty} y(t) = 0.4$. [7]

(c) Assume now that a=1, $K = 1$ and that $x(t) = u(t)$.

   i. Determine the exact expression of the output $y(t)$. [8]

   ii. Find the value of t at which $y(t)$ is maximum. [7]