IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE/EIE PART II: MEng, BEng and ACGI

Corrected Copy

COMMUNICATION SYSTEMS

Tuesday, 26 May 2:00 pm

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D. Gunduz

Second Marker(s) : J.A. Barria

Information for Students

Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} \tau f$
2.	$2W \operatorname{sinc} 2Wt$	$\Pi\left(\frac{f}{2W}\right)$
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 \tau f$
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$
6.	$\exp(-\alpha\|t\|),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$
7.	$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f\tau)^2}$
8.	$\delta(t)$	1
9.	1	$\delta(f)$
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$
15.	$\operatorname{sgn} t$	$(j\pi f)^{-1}$
16.	$\frac{1}{\pi t}$	$-j \operatorname{sgn}(f)$
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda}\, d\lambda$	$-j \operatorname{sgn}(f) X(f)$
18.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s)$, $f_s = T_s^{-1}$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\tan(\frac{\pi}{2} - x) = \cot x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x} = \frac{2\cot x}{\cot^2 x - 1} = \frac{2}{\cot x - \tan x}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$$

$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$$

$$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

Fourier Transform Theorems[a]

Name of Theorem		
1. Superposition (a_1 and a_2 arbitrary constants)	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(f) + a_2X_2(f)$
2. Time delay	$x(t - t_0)$	$X(f)e^{-j2\pi f t_0}$
3a. Scale change	$x(at)$	$\lvert a\rvert^{-1}X\left(\frac{f}{a}\right)$
b. Time reversal	$x(-t)$	$X(-f) = X^*(f)$
4. Duality	$X(t)$	$x(-f)$
5a. Frequency translation	$x(t)e^{j\omega_0 t}$	$X(f - f_0)$
b. Modulation	$x(t)\cos\omega_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
6. Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
7. Integration	$\int_{-\infty}^{t} x(t')\,dt'$	$(j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
8. Convolution	$\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\,dt' = \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\,dt'$	$X_1(f)X_2(f)$
9. Multiplication	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\,df' = \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\,df'$

Differentiation Rule of Leibnitz

Let $F(z) = \int_{a(z)}^{b(z)} f(x,z)dx$. Then we have

$$\frac{dF(z)}{dz} = \frac{db(z)}{dz}f(b(z),z) - \frac{da(z)}{dz}f(a(z),z) + \int_{a(z)}^{b(z)} \frac{\partial f(x,z)}{\partial z}dx$$

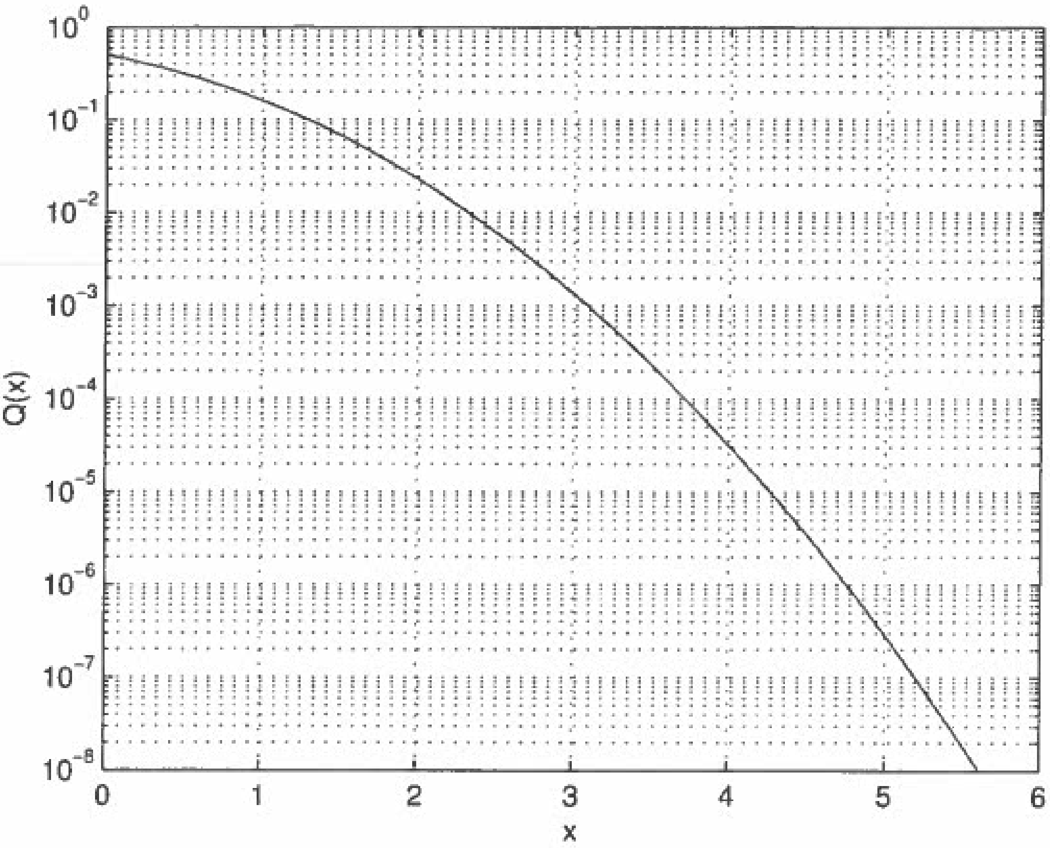


Figure 0.1 The graph of the Q-function, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$.

For large x, we have $Q(x) \approx \frac{1}{\sqrt{2\pi}x} e^{-x^2/2}$

1. a) i) Explain the differences between a coherent and a noncoherent receiver. [2]

ii) Consider a narrow-band signal $n(t)$ centred around frequency f_c. Write down the canonical representation of $n(t)$ in terms of its in-phase component $n_I(t)$ and quadrature component $n_Q(t)$. Draw a diagram that shows how you can obtain $n_I(t)$ and $n_Q(t)$ from $n(t)$. [3]

iii) Compare the signal-to-noise ratio (SNR) performance of the double sideband-suppressed carrier (DSB-SC) modulation scheme with that of baseband transmission. [2]

iv) Compare frequency modulation (FM) with DSB-SC modulation in terms of its SNR performance and the bandwidth of the transmitted signal. [3]

b) For each of the following functions below, state if it can represent the autocorrelation function of a real wide sense stationary random process. If your answer is no, explain why not.

i)

$$R_X(t_1,t_2) = \cos(\alpha t_1) - \sin(\beta t_1), \text{ for } \alpha, \beta \in \mathbb{R}^+.$$

[3]

ii)

$$R_X(t_1,t_2) = \begin{cases} Ae^{-2(t_1-t_2)}, & \text{if } t_1 \geq t_2, \\ Ae^{-3(t_2-t_1)}, & \text{if } t_1 < t_2 \end{cases}$$

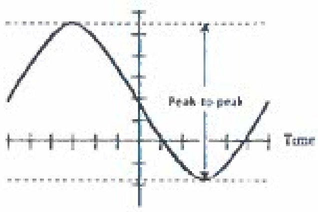
[3]

iii)

$$R_X(t_1,t_2) = |t_1 - t_2|\cos(|t_1 - t_2|\pi).$$

[4]

c) The amplitude of a bandlimited analog signal is quantized with a uniform scalar quantizer. The bandwidth of the signal is 4 KHz. The quantization error should not exceed the 0.4% of the peak-to-peak amplitude of the analog signal.



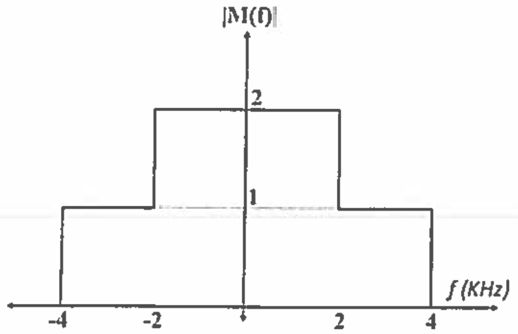
i) What is the minimum required sampling rate? [2]

ii) What is the minimum number of bits per sample? [3]

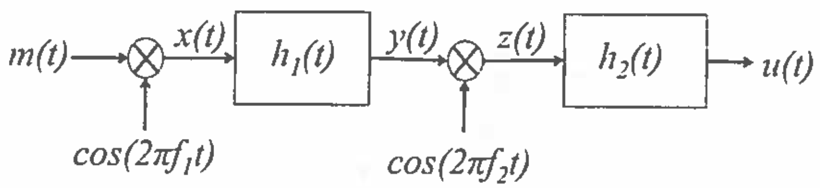
iii) What is the corresponding bit rate? [2]

iv) If these bits are to be transmitted using 16-ary pulse amplitude modulation (PAM), what is the required symbol duration? [3]

d) Consider a real-valued signal $m(t)$ whose Fourier transform is denoted by $M(f)$. The *magnitude spectrum* $|M(f)|$ is given below.



The signal $m(t)$ is passed through the following system, where $f_1 = 20$ KHz and $f_2 = 24$ KHz:



The frequency response of the two filters are given below.

$$H_1(f) = \begin{cases} 2, & \text{if } 20 \text{ KHz} \le |f| \le 24 \text{ KHz}, \\ 0, & \text{otherwise.} \end{cases} \qquad H_2(f) = \begin{cases} 2, & \text{if } |f| \le 4 \text{ KHz}, \\ 0, & \text{otherwise.} \end{cases}$$

i) Sketch the magnitude spectrum of the signals $x(t)$, $y(t)$, $z(t)$ and $u(t)$. [5]

ii) Assume that the output signal $u(t)$ is passed through the same system again to obtain a signal $w(t)$ at the output. Find the magnitude spectrum of $w(t)$. [3]

iii) Express $w(t)$ in terms of $m(t)$? [2]

2. a) A discrete source produces symbols from the alphabet $\{a,b,c,d,e,f,g\}$. Each symbol is generated independently from the others, and the probabilities of different symbols are given as follows: $P(a) = 1/16$, $P(b) = 5/16$, $P(c) = 3/32$, $P(d) = 1/16$, $P(e) = 3/16$, $P(f) = 1/4$, $P(g) = 1/32$.

i) Use the Huffman coding procedure to generate the codewords for the symbols. [8]

ii) What is the average codeword length for the Huffman code obtained above? [2]

iii) What is the entropy of this source? [4]

iv) Discuss whether the Huffman code obtained above is optimal or not (i.e., does it meet the Shannon bound)? [2]

b) Consider the random process

$$X(t) = N_1 \cos(2\pi f_c t) - N_2 \sin(2\pi f_c t),$$

where $f_c > 0$ is a given fixed frequency, and N_1 and N_2 are independent zero-mean Gaussian random variables with variance σ^2.

i) Find the mean and autocorrelation function of $X(t)$. [5]

ii) Is $X(t)$ a wide sense stationary (WSS) process? [2]

iii) Find and plot the power spectral density (PSD) of $X(t)$. [3]

iv) Let $Y(t) = X^{(n)}(t)$ denote the n-th derivative of $X(t)$. Find the PSD of $Y(t)$. [4]

3. a) A binary message source generates bit "0" with probability p_0 and "1" with probability p_1. These bits are transmitted over a binary digital communication system. Bit "0" is transmitted with a pulse of amplitude -1, and bit "1" is transmitted with a pulse of amplitude 1. The noise in the channel is zero-mean additive white Gaussian with variance 0.5. The receiver uses a matched filter followed by threshold detection.

i) Determine the optimum detection threshold if $p_1 = 0.5$. [2]

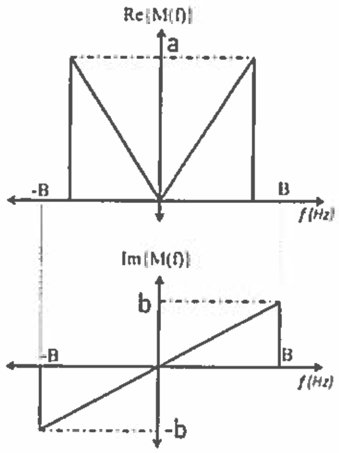
ii) Determine the optimum detection threshold if $p_1 = 0.2$. What is the corresponding probability of error? [6]

iii) Assume that the receiver sets the optimum threshold as derived in question ii). However; the message source generates bits with $p_1 = 0.7$. What is the probability of error? How does this compare with the probability of error you found above? [6]

b) The Hilbert transform of a signal $m(t)$ is given by:

$$\hat{m}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau} d\tau.$$

Let $M(f)$ denote the spectrum of the finite energy real message signal $m(t)$. The real and imaginary components of $M(f)$ are shown below.



i) Write $\hat{m}(t)$ in the form of a convolution of $m(t)$ with another signal. [2]

ii) Let $\hat{M}(f)$ denote the Fourier transform of $\hat{m}(t)$. Write down $\hat{M}(f)$ in terms of $M(f)$. Plot the real and imaginary components of $\hat{M}(f)$. [4]

iii) Consider the single-sideband (SSB) modulated signal given below:

$$s(t) = m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t).$$

Write down the spectrum of $s(t)$ in terms of $M(f)$ and $\hat{M}(f)$, and plot its real and imaginary components. [4]

iv) Assume that $s(t)$ is transmitted over an additive white Gaussian noise channel. Draw a diagram illustrating a coherent receiver for this modulation scheme. Write down the demodulated signal in the time domain. [4]

v) Compare the signal-to-noise ratio (SNR) performance of the SSB modulation scheme with that of the DSB-SC modulation scheme. What is the advantage of using SSB modulation? [2]