## Assignment 5

The project should be submitted as one zip- or tar-file to i.shevchenko@imperial.ac.uk by the due date. The file should contain all codes used to generate your results and a pdf-file of the report. The assignment must include a pledge that this is all your own work, your name and CID. Any marks received for the assignment are only indicative and may be subject to moderation and scaling.

### Exercise 1 (RK methods and BVP) — % of CW mark: 20

**a)** Develop your own 4-stage explicit Runge–Kutta method, find its local truncation error and region of absolute stability (**% of CW mark: 5**)

**b)** Compare the Runge–Kutta method with the best of your methods developed in CW4 by applying them to

**c)** The Lorenz system

$$\begin{cases} x' = \sigma(y-x), \\ y' = x(\rho - z) - y, \\ z' = xy - \beta z, \end{cases}$$

where $\sigma = 10$, $\beta = 8/3$, $\rho = 28$ and initial conditions $\mathbf{v}(0) = (1,1,1)^{\mathrm{T}}$; $\mathbf{v} = (x,y,z)^{\mathrm{T}}$; $t = [0, 100]$. (**% of CW mark: 5**)

**d)** The boundary value problem (**Mastery Component**)

$$u_t = \varepsilon u_{xx} + \gamma u^2(1-u), \quad x \in [0, L], \quad t = (0, T], \tag{1}$$

where $\lambda = \frac{\sqrt{2\gamma/\varepsilon}}{2}$, $\gamma = 1/\varepsilon$, $L = 10$, $T = 10$, $\varepsilon = \{0.01, 0.05, 0.1\}$.
The initial and boundary conditions are given by

$$u(0,x) = (1 + \mathrm{e}^{\lambda(x-1)})^{-1}, \quad u_x(t,0) = u_x(t,L) = 0.$$

Use a uniform grid of 1000 points to approximate equation (1) in space. (**% of CW mark: 10**)