EE4-08
EE9-SO14

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected Copy

DIGITAL IMAGE PROCESSING

Friday, 8 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	P.T. Stathaki
	Second Marker(s) :	T-K. Kim

1. a) i) Explain why the two dimensional Discrete Cosine Transform is separable.

[2]

ii) Show that the two dimensional Discrete Cosine Transform can be implemented using the one dimensional Discrete Cosine Transform. [2]

b) Let $f(x,y)$ denote an $M \times N$-point 2-D sequence that is zero outside $0 \le x \le M-1$, $0 \le y \le N-1$, where M and N are integers and powers of 2. In implementing the standard Discrete Walsh Transform of $f(x,y)$, we relate $f(x,y)$ to a new $M \times N$-point sequence $W(u,v)$.

i) Define the sequence $W(u,v)$ in terms of $f(x,y)$. [2]

ii) State the advantages of the Discrete Walsh Transform. [2]

iii) In the case of $M = N = 2$ and $f(x,y) = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ calculate the Walsh transform coefficients.

[2]

c) Consider the population of vectors $\underline{f}$ of the form

$$\underline{f}(x,y) = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \end{bmatrix}.$$

Each component $f_i(x,y)$, $i = 1,2$ represents an image of size $M \times M$ where M is even. The population arises from the formation of the vectors $\underline{f}$ across the entire collection of pixels (x,y).

The two images are defined as follows:

$$f_1(x,y) = \begin{cases} r_1 & 1 \le x \le \frac{M}{4}, 1 \le y \le M \\ r_2 & \frac{M}{4} < x \le M, 1 \le y \le M \end{cases}$$

$$f_2(x,y) = r_3,\ 1 \le x \le M,\ 1 \le y \le M$$

Consider now a population of random vectors of the form

$$\underline{g}(x,y) = \begin{bmatrix} g_1(x,y) \\ g_2(x,y) \end{bmatrix}$$

where the vectors $\underline{g}$ are the Karhunen-Loeve (KL) transforms of the vectors $\underline{f}$.

i) Find the images $g_1(x,y)$ and $g_2(x,y)$ using the Karhunen-Loeve (KL) transform.

[8]

ii) Comment on whether you could obtain the result of c)-i) above using intuition rather than by explicit calculation.

[2]

2. a) The probability density functions of two grey level images $f(x,y)$ and $g(x,y)$ with intensities r and s respectively, are illustrated in Figure 2a below. For each image $f(x,y)$ and $g(x,y)$ find and sketch the transformation function that produces a histogram equalised image.

[5]

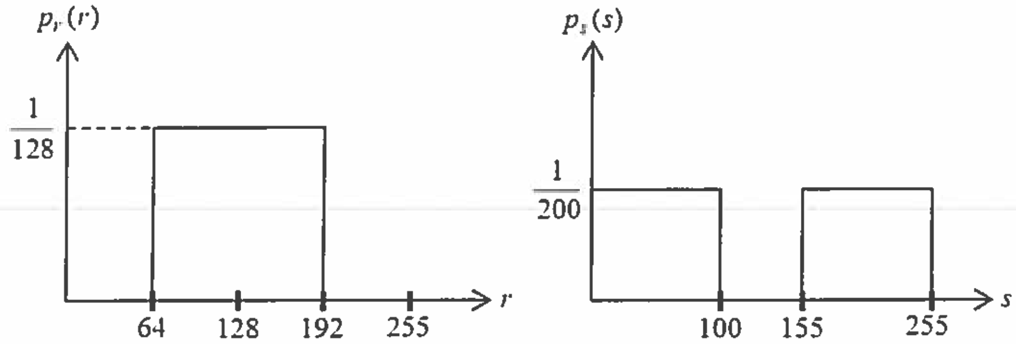


Figure 2a

b) Suppose that a grey level image $f(x,y)$ with intensity r has the probability density function shown on the left in Figure 2b below. We would like to modify the image intensities so that the new image has the probability density function given on the right of Figure 2b. Derive a transformation function that will accomplish this.

[5]

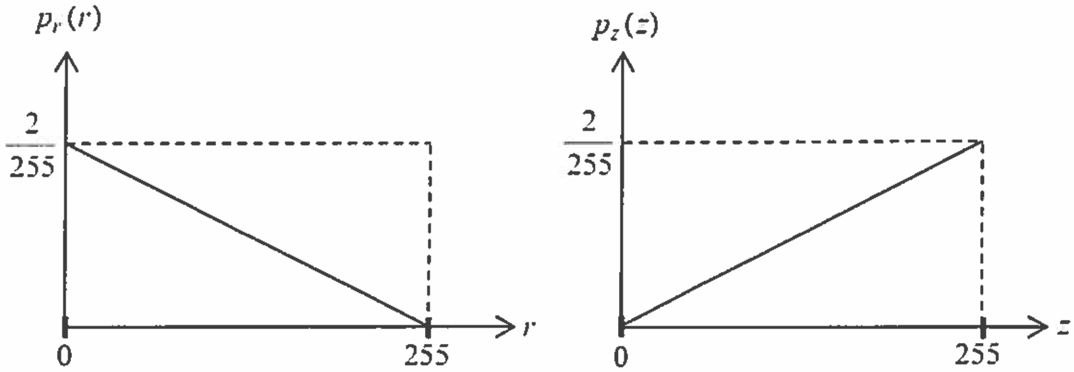


Figure 2b

c) State which one of the following filters is nonlinear: High Boost Filter, Weighted Averaging Filter, Sobel Filter, Median Filter. Justify your answer with an example.

[5]

d) Explain which one of the following filters is commonly used for sharpening images: Averaging Filter, Differentiation Filter, Weighted Averaging Filter, Median Filter.

[5]

3. a) We are given the degraded version g of an image f such that in lexicographic ordering

$$g = Hf + n$$

where H is the degradation matrix which is assumed to be block-circulant and n is the noise term which is assumed to be zero-mean, white and independent of the image f. The images have size $N \times N$.

i) Consider the Wiener Filtering image restoration technique. Prove the general expressions for both the Wiener filter estimator and the restored image in both spatial and frequency domains and explain all symbols used. **[10]**

ii) Discuss the disadvantages of the Wiener Filtering image restoration technique. **[5]**

b) In a particular scenario, the image under consideration is degraded by a transfer function which, in the frequency domain, is given by the function below:

$$H(u,v) = \sqrt{2\pi}\sigma(u^2 + v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}$$

In the above formulation σ is a constant parameter. Generate the expression of the Wiener filter in frequency domain by assuming that the ratio of power spectra of the noise and un-degraded (original) image is constant.

[5]

4. a) i) Name three reasons why it might be a good idea to compress data. [2]

ii) Discuss the characteristics of the histogram that an image must possess in order to be amenable to compression using a Huffman code. [2]

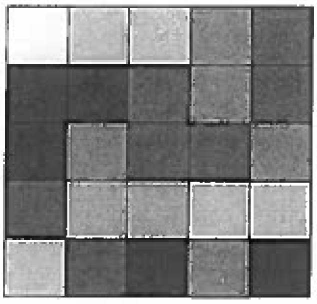
iii) Consider the 8×8 image $f(x, y), x, y = 0, \ldots, 7$ shown in Figure 4a below. The top left corner is the point $(x, y) = (0,0)$ and x is the horizontal dimension. Explain how differential coding can be used to compress this image if the prediction formula is $f(x, y) - f(x+1, y)$ for $x < 7$ and 0 for $x = 7$.

7	6	5	4	3	2	1	0
7	6	5	4	3	2	1	0
0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7
7	7	7	7	7	7	7	7
5	5	5	5	5	5	5	5
3	3	3	3	3	3	3	3
1	1	1	1	1	1	1	1

Figure 4a

[4]

b) The following Figure 4b shows a 5×5 image with 6 different grey levels with values shown on the right figure.



180	160	160	140	120
110	110	120	140	120
110	140	120	120	140
120	160	160	170	170
170	120	110	140	110

Figure 4b

i) Derive the probability of appearance (that forms the histogram) for each intensity (grey) level. Calculate the entropy of this image. [3]

ii) Derive a Huffman code. [3]

iii) Calculate the average length of the fixed length code and that of the derived Huffman code. [3]

iv) Calculate the compression ratio and the relative coding redundancy. [3]