IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE PART II: MEng, BEng and ACGI

FIELDS

Corrected Copy

Monday, 15 June 2:00 pm

Time allowed: 1:30 hours

There are THREE questions on this paper.

Question One carries 40 marks. Question Two and Question Three carry 30 marks each.

Answer ALL questions.

Any special instructions for invigilators and information for candidates are on page 1.

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Electromagnetic Fields 2015 – Formula Sheet

- Vectors (Cartesian co-ordinates):

$\underline{\mathbf{a}} \cdot \underline{\mathbf{b}} = \mathbf{a_x\, b_x + a_y\, b_y + a_z\, b_z}$

$\underline{\mathbf{a}} \times \underline{\mathbf{b}} = \{\mathbf{a_y b_z - a_z b_y}\}\, \underline{\mathrm{i}} + \{\mathbf{a_z b_x - a_x b_z}\}\, \underline{\mathrm{j}} + \{\mathbf{a_x b_y - a_y b_x}\}\, \underline{\mathrm{k}}$

- Differential operators (Cartesian co-ordinates)

$\nabla = \partial/\partial x\, \underline{i} + \partial/\partial y\, \underline{j} + \partial/\partial z\, \underline{k}$

$\nabla\phi = \partial\phi/\partial x\, \underline{i} + \partial\phi/\partial y\, \underline{j} + \partial\phi/\partial z\, \underline{k}$

$\nabla \cdot \underline{\mathbf{F}} = \partial \mathbf{F}_x/\partial x + \partial \mathbf{F}_y/\partial y + \partial \mathbf{F}_z/\partial z$

$\nabla \times \underline{\mathbf{F}} = \{\partial \mathbf{F}_z/\partial y - \partial \mathbf{F}_y/\partial z\}\, \underline{i} + \{\partial \mathbf{F}_x/\partial z - \partial \mathbf{F}_z/\partial x\}\, \underline{j} + \{\partial \mathbf{F}_y/\partial x - \partial \mathbf{F}_x/\partial y\}\, \underline{k}$

$\nabla^2\phi = \partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 + \partial^2\phi/\partial z^2$

- Identities:

$\nabla \cdot (\phi\underline{\mathbf{F}}) = \underline{\mathbf{F}} \cdot \nabla\phi + \phi\, \nabla \cdot \underline{\mathbf{F}}$

$\nabla \cdot (\phi\nabla\psi - \psi\nabla\phi) = \phi\, \nabla^2\psi - \psi\, \nabla^2\phi$

$\nabla \times \nabla \times \underline{\mathbf{F}} = \nabla(\nabla \cdot \underline{\mathbf{F}}) - \nabla^2\underline{\mathbf{F}}$

- Integral theorems:

$\int\int_A \underline{\mathbf{F}} \cdot d\underline{a} = \int\int\int_V \nabla \cdot \underline{\mathbf{F}}\, dv$

$\int_L \underline{\mathbf{F}} \cdot d\underline{L} = \int\int_A (\nabla \times \underline{\mathbf{F}}) \cdot d\underline{a}$

- Maxwell's equations – integral form

$\int\int_A \underline{\mathbf{D}} \cdot d\underline{a} = \int\int\int_V \rho\, dv$

$\int\int_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = -\int\int_A \partial\underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \int\int_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

- Maxwell's equations – differential form

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

- Material equations

$\underline{\mathbf{J}} = \sigma\, \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon\, \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu\, \underline{\mathbf{H}}$

- Electromagnetic waves (pure dielectric media)

Time dependent vector wave equation $\nabla^2\underline{\mathbf{E}} = \mu_0\varepsilon\, \partial^2\underline{\mathbf{E}}/\partial t^2$

Where $\underline{\mathbf{E}}$ is a time-dependent vector field

Time independent scalar wave equation $\nabla^2\underline{E} = -\omega^2\mu_0\varepsilon_0\varepsilon_r\, \underline{E}$

For z-going, x-polarized waves $d^2E_x/dz^2 + \omega^2\mu_0\varepsilon_0\varepsilon_r\, E_x = 0$

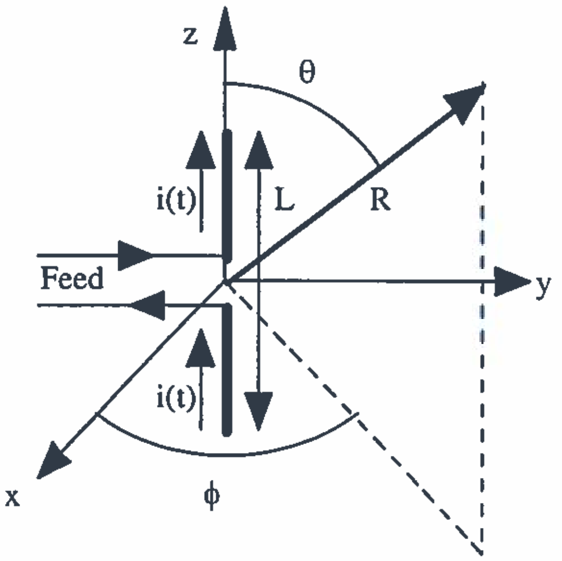
Where $\underline{E}$ is a time-independent vector field

- Antenna formulae

Far-field pattern of half-wave dipole

$E_\theta = j\, 60I_0\, \{\cos[(\pi/2)\cos(\theta)] / \sin(\theta)\}\, \exp(-jkR)/R$; $H_\phi = E_\theta/Z_0$

Here I_0 is peak current, R is range and $k = 2\pi/\lambda$



Power density $\underline{S} = 1/2\ \mathrm{Re}\,(\underline{E} \times \underline{H}^*) = S(R, \theta)$

Normalised radiation pattern $F(\theta, \phi) = S(R, \theta, \phi) / S_{max}$

Directivity $D = 1/\ \{1/4\pi \int\int_{4\pi} F(\theta, \phi) \sin(\theta)\, d\theta\, d\phi\}$

Gain $G = \eta D$ where η is antenna efficiency

Effective area $A_e = \lambda^2 D/4\pi$

Friis transmission formula $P_r = P_t\, (\eta_t\eta_r A_t\, A_r/R^2\lambda^2)$

Electromagnetic Fields 2015 – Questions

1. Using diagrams and developing formulae where appropriate, explain briefly each of the following:

a) The electromagnetic spectrum

[8]

b) The transmission line equations

[8]

c) Group velocity

[8]

d) The imaging equation

[8]

e) The Friis transmission formula

[8]

[Σ = 40]

2. a) Waves propagating in a lumped-element L-C ladder network have the dispersion relation:

$$\omega = 2\omega_0 \sin(ka/2)$$

Where ω is the angular frequency, $\omega_0 = 1/\sqrt{(LC)}$, L and C are the series inductance and shunt capacitance per section, k is the propagation constant and a is the length of each section.

Sketch the dispersion characteristic, and explain the type of waves that can exist in each key frequency range. What is the phase velocity at low frequency?

[6]

b) A transmission line of characteristic impedance Z_0 carries a voltage wave $V = V_0 \exp(-jkz)$ in the positive z-direction. Write down an expression for the associated current wave. What are the corresponding expressions for waves travelling in the negative z-direction?

[5]

c) A first line of characteristic impedance Z_1 is connected to a second line of characteristic impedance Z_2. Derive the voltage reflection and transmission coefficients for incidence from the first line. Comment on the value of the transmission coefficient when $Z_2 > Z_1$.

[10]

d) Write down the incident, reflected and transmitted powers, and hence find the power reflection and transmission coefficients. Show that power is conserved at the discontinuity, and comment on the maximum value of the power transmission coefficient.

[9]

[$\Sigma = 30$]

3. a) The Poynting vector $\underline{\mathbf{S}} = \underline{\mathbf{E}} \times \underline{\mathbf{H}}$, where $\underline{\mathbf{E}}$ and $\underline{\mathbf{H}}$ are the time-varying electric and magnetic fields, defines the instantaneous power flow in an electromagnetic field. Assuming that $\underline{\mathbf{E}} = \mathrm{Re}\{\underline{E} \exp(j\omega t)\}$ and $\underline{\mathbf{H}} = \mathrm{Re}\{\underline{H} \exp(j\omega t)\}$, where $\underline{E}$ and $\underline{H}$ are time-independent fields, show that the time-averaged power flow is $\underline{S} = 1/2\ \mathrm{Re}(\underline{E} \times \underline{H}^*)$.

[6]

b) Assuming TE incidence from medium 1 at an angle θ_1, the amplitude reflection coefficient at an interface between two dielectric media with refractive indices n_1 and n_2 with $n_1 > n_2$ is:

$$\Gamma_E = \{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)\} / \{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)\}$$

Here, θ_2 is the angle of the transmitted wave in medium 2. Show that the power reflectivity is unity once total internal reflection has occurred.

[6]

c) The scalar wave equation for spherically symmetric electric fields $E(r)$ is:

$$d^2E/dr^2 + (2/r)\, dE/dr + \omega^2\mu_0\varepsilon_0 E = 0$$

Find a solution for spherical waves.

[6]

d) A broadside antenna consists of two dipoles, held a distance d apart and driven in phase with the same signal. Sketch the arrangement, and find an expression for the normalised radiation pattern at an angle ϕ from a line joining the dipoles. At what angle is the radiation maximised?

[6]

e) The scalar wave equation for z-propagating plane waves is:

$$d^2E/dz^2 + \omega^2\mu_0\varepsilon_0 E = 0$$

Assuming the material is a conductor, so that the permittivity can be written in terms of the conductivity as $\varepsilon = \sigma/j\omega$, find an expression for the skin depth.

[6]

[$\Sigma = 30$]