Distributed
Spring 2005

E2.2 SAMPLE Examination

PLEASE ANSWER QUESTION 1 AND ANY **TWO** of QUESTIONS 2,3 and 4.

QUESTION 1 IS **MANDATORY**

1

a) A transformer with a 1:10 turns ratio has a voltage gain of 10. What is its maximum possible current gain? Is such a transformer an amplifier?

[5 marks]

b) What is the value of the input impedance of an ideal transimpedance amplifier? What is the value of the output impedance of an ideal current amplifier?

[5 marks]

c) You need to use realistic current amplifier as a voltage amplifier . To do this you decide to use negative feedback to improve its input-output impedance. What feedback connection will you choose, and why?

[5 marks]

d) What is the input impedance of the circuit in figure 1.1? The voltage gain of the amplifier is G=19, as indicated. The amplifier is an otherwise ideal voltage amplifier.

[5 marks]

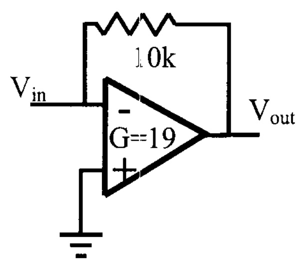


Figure 1.1

e) Write an expression for the general form of the second order band-pass filter transfer function. Identify any parameters appearing in this equation.

[5 marks]

f) The FET Cascode amplifier is a two-stage amplifier. Name the two stages, and describe the terminal characteristiocs (input-output impedance) and function of each one. Why does the cascode amplifier have a wider bandwidth than its first stage used alone at the same voltage gain?

[5 marks]

g) Which family of 2-port parameters is the most suitable to represent a realistic operational amplifier?

[5 marks]

h) Prove that a unilateral voltage amplifier is also a unilateral current amplifier, and conversely, a unilateral current amplifier is also a unilateral voltage amplifier.

[5 marks]

ANSWERS Q1:

(a) Max current gain is 0.1 since the transformer is passive and its power gain needs to be less than or equal to unity. This is NOT an amplifier.
(b) 0 and ∞ respectively.
(c) series-shunt, to raise the input impedance and lower the output impedance
(d) 0.5k due to the Miller effect (the feedback element divided by G+1).
(e) $H(s)=\dfrac{H_0 2\zeta s\omega_0}{s^2+2\zeta s\omega_0+\omega_0^2}$ (H_0 is max gain at resonance, ω_0 the resonant frequency , ζ the damping factor.
(f) Common Source (transconductance amplifier) – Common gate (transimpedance amplifier. The Voltage gain of the common source is -1, hence the miller effect is minimized.
(g) G parameters, also known as voltage gain or inverse hybrid parameters.
(h) Need to show $\left.\dfrac{\partial V_1}{\partial V_2}\right|_{i_1=0}=0$ But we know the voltage amplifier is unilateral. Therefore, in G representation: $I_1=G_{11}V_1 \Rightarrow V_1=\dfrac{1}{G_{11}}I_1$ This implies we can write: $\left.\dfrac{\partial V_1}{\partial V_2}\right|_{i_1=0}=\left.\dfrac{\partial V_1}{\partial V_2}\right|_{V_1=0}=0$ QED.

2.

(a) Calculate the voltage gain of the amplifier in figure 2.1 if the op-amp is ideal. What is this transfer function?

[10 marks]

(b) Now consider a finite gain op-amp, with $G = 1000$ otherwise ideal. If $1/2\pi\sqrt{LC} = 1kHz$, $\sqrt{L/C} = 50\Omega$ calculate at which frequency the gain will deviate from its ideal value by more than 10%

[10 marks]

(c) Now assume the op-amp is a real dominant pole amplifier, with a low frequency open loop gain of G=1000 and a gain bandwidth product of GBW=10^4 Hz. Write an expression, and make a magnitude and phase bode plot for the closed loop amplifier.

[10 marks]

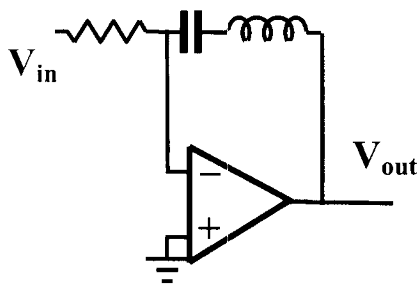


Figure 2.1: An amplifier.

Answer Question 2:

a) This is an inverting amplifier, therefore

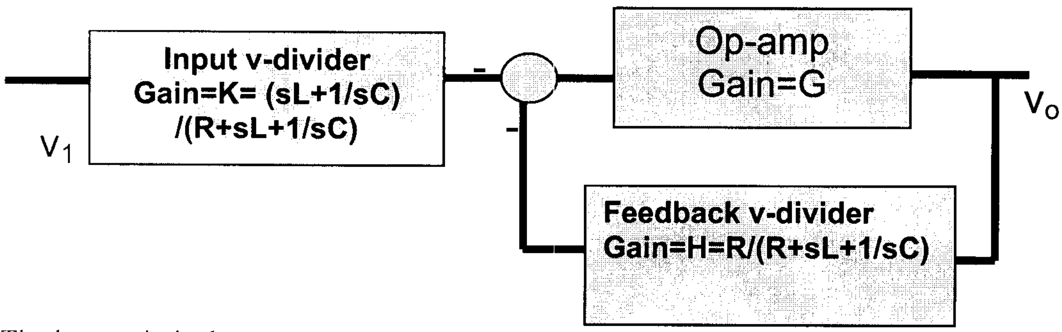
$$A_V = -\frac{sL + 1/sC}{R} = -\frac{s^2LC + 1}{R}$$

Tis is a band reject function

[10 marks]

b) The correction comes from the formula

$A_V = G_0 \dfrac{G}{1+GH}$ the question is to make GH>100. We can view the amplifier as shown in class:



The loop gain is then

$G_{LOOP} = \dfrac{A_V R}{R + sL + 1/sC} = \dfrac{A_V RsC}{s^2LC + sRC + 1}$ This is a bandpass function, with a peak gain of $H_0 = A_V RC / RC = A_V$. Since A_V=1000, it follows that the loop gain will be less than 10 at 0.01 and 100 times the centre frequency, i.e. 10Hz and 100kHz.

[10 marks]

c) the dominant pole amplifier has a response:

$G(s) = G_0 / (1 + s\tau)$ and the gain bandwidth product is :

$GBW = G_0 / \tau$. Then $G_0 = 10^3$ and $\tau = 1/2\pi \cdot 10Hz = 15.9 m\sec$ or, simply, the breakpoint is at 10Hz.

We can now write for the transfer function:

$$A = \frac{sL + 1/sC}{R + sL + 1/sC} \frac{\dfrac{G_0}{1+s\tau}}{1 + \dfrac{R}{R + sL + 1/sC} \dfrac{G_0}{1+s\tau}} =$$

$$= \frac{(s^2LC + 1)G_0}{(s^2LC + sRC + 1)(1 + s\tau) + sRCG_0}$$

This has a breakpoint at f=1kHz,

[10 marks]

3 Filters can be synthesized by transforming the function of circuit elements using feedback circuits. In this problem we synthesise a 2^{nd} order LC filter by using a Generalised impedance converter.

(a) Analyse the generalized impedance converter in figure 3.1, and derive an equation expressing the terminal impedance at A in terms of the components used.

[10 marks]

(b) Choose components to implement a grounded inductor using only resistors and capacitors. What is the inductance value?

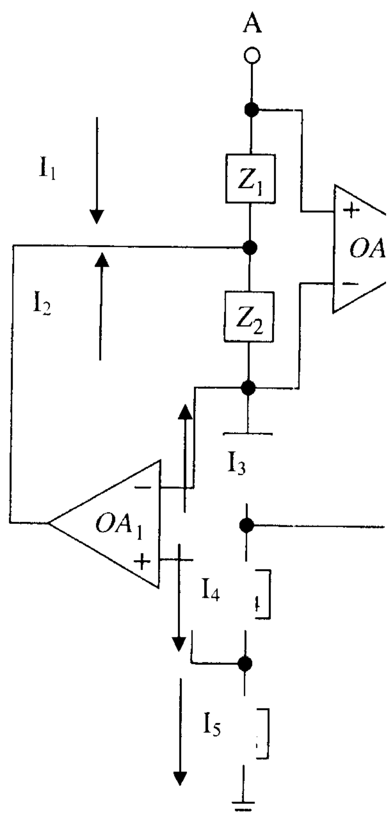
[5 marks]

(c) Choose components that allow you to create a floating capacitance of greater value than the available components.

[5 marks]

(d) Using only resistors (any value), op-amps and 100nF capacitors design a 2^{nd} order LC voltage low pass filter with a break frequency at 15.9Hz. Since this is an audio filter choose your components so that $\sqrt{\frac{L}{C}} = 600\ \Omega$ in your filter. (this choice of components impedance-matches the filter to a 600 Ohm source, 600 ohms being quite standard in audio work)

[10 marks]



$\frac{Z_1Z_3Z_5}{Z_2Z_4}$

Figure 3.1

Answer Q3:

ANSWER:

a) The golden rules $V_+ = V_-$ and $i_+ = i_- = 0$ for each op-amp lead to:

$$V_{in} = V_{2+} = V_{2-} = V_{1-} = V_{1+} \quad \text{and} \quad i_3 Z_3 = i_4 Z_4 \Rightarrow i_3 = i_4 \frac{Z_4}{Z_3} = i_2 \;\; , \;\; i_1 Z_1 = i_2 Z_2 \Rightarrow i_1 = i_2 \frac{Z_2}{Z_1} \;.$$
$$\Rightarrow i_5 = V_A / Z_5 = i_4$$

putting all together,

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

[10 marks]

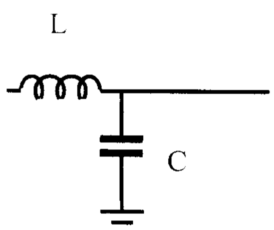
b) make Z_2 or Z_4 a capacitor, keep everything else a resistor of suitable magnitude.

[5 marks]

c) make Z_1 , Z_3 or Z_5 a capacitor, keep everything else a resistor of suitable magnitude. The gain will be the ratio. If a floating element is required simply use the ground connection as a second terminal.

[5 marks]

d) For the filter required we need to implement:



$$\text{with} \quad \left. \begin{array}{l} \dfrac{1}{\sqrt{LC}} = 15.9\text{Hz} = 100\text{rad/s} \\ \sqrt{\dfrac{L}{C}} = 600\ \Omega \end{array} \right\} \Rightarrow C = 16.67 \mu F, L = 6H$$

Starting with 100nF capacitor we need to magnify the capacitance by a factor of 166.7, and invert the 100nF capacitance to 6H:

For the capacitor, say

$$Z_3 = \frac{1}{j\omega C} \Rightarrow Z_1 = \frac{1}{j\omega C \frac{R_2 R_4}{R_1 R_5}} \Rightarrow$$

$$\frac{R_2 R_4}{R_1 R_5} = 16.67\mu F / 100nF = 166.7$$

For the inductor:

$$Z_2 = \frac{1}{j\omega C} \Rightarrow Z_L = j\omega C \frac{R_1 R_3 R_5}{R_4} \Rightarrow$$

$$\frac{R_1 R_3 R_5}{R_4} = 6/100n = 6 \times 10^7$$

[10 marks]

4 Analyse the filter in figure 4.1 What function does it perform? What is its Q and centre frequency? What is its maximum gain?

[30 marks]

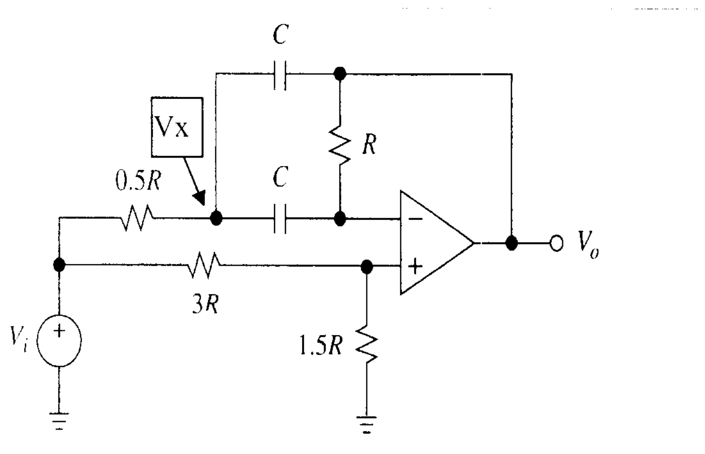


Figure 4.1

Answer, Question 4

(as some may remember this is a Deliyannis-Friend MFB filter)

This problem is included here because the solution is not (by error) in the distributed sets.
Let V_x the unknown node, G=1/R, $\tau = RC$

$$V_- = \frac{1}{3}V_i = V_+$$

$$\text{KCL on } V_- : \begin{cases} \left(V_o - \frac{V_i}{3}\right)G = sC\left(\frac{V_i}{3} - V_x\right) \Rightarrow \\ \left(V_o - \frac{V_i}{3}\right)\frac{1}{s\tau} = \frac{V_i}{3} - V_x \Rightarrow V_x = \frac{V_i}{3} - \left(V_o - \frac{V_i}{3}\right)\frac{1}{s\tau} \\ V_x = \frac{V_i}{3s\tau}(s\tau + 1) - \frac{V_o}{s\tau} \end{cases}$$

$$\text{KCL on } V_x : \begin{cases} sC\left(\frac{V_i}{3} - V_x\right) + 2G(V_i - V_x) + sC(V_o - V_x) = 0 \Rightarrow \\ s\tau\left(\frac{V_i}{3} - V_x\right) + 2(V_i - V_x) + s\tau(V_o - V_x) = 0 \Rightarrow \\ V_i\left(\frac{s\tau}{3} + 2\right) + s\tau V_o = 2V_x(1 + s\tau) \end{cases}$$

Combine the two,

$$V_i\left(\frac{s\tau}{3} + 2\right) + s\tau V_o = 2\left(\frac{V_i}{3s\tau}(s\tau + 1) - \frac{V_o}{s\tau}\right)(1 + s\tau) \Rightarrow$$

$$V_i s\tau(s\tau + 6) + 3s^2\tau^2 V_o = 2V_i(1 + s\tau)^2 - 6V_o(1 + s\tau) \Rightarrow$$

$$V_i(s^2\tau^2 + 6s\tau - 2 - 2s^2\tau^2 - 4s\tau) = -V_o(6 + 6s\tau + 3s^2\tau^2) \Rightarrow$$

$$\frac{V_o}{V_i} = \frac{-s^2\tau^2 + 2s\tau - 2}{6(0.5s^2\tau^2 + s\tau + 1)} = -\frac{1}{3}\frac{0.5s^2\tau^2 - s\tau + 1}{0.5s^2\tau^2 + s\tau + 1}$$

This is an (inverting) all pass filter of gain=1/3, centre frequency:
$\frac{1}{\omega_0^2} = 0.5\tau^2 \Rightarrow \omega_0 = \frac{\sqrt{2}}{\tau}$ and quality factor:

$$2Q/\omega_0 = \tau \Rightarrow 2Q = \sqrt{2} \Rightarrow Q = \frac{1}{\sqrt{2}}$$