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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

MSc and EEE PART IV: M.Eng. and ACGI

INFORMATION THEORY

Friday, 2 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : L.F. Turner
Second Marker(s) : J.A. Barria

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1. An image source can be considered as being represented by a sequence of black and white dots (pixels), with 1 denoting a black dot and 0 a white dot. A statistical analysis of the image shows that it can be fully represented as a two-state Markov process whose transition probabilities are described by the matrix

$$\begin{array}{cc} & \text{Next Pixel} \\ & \begin{array}{cc} 0 & 1 \end{array} \\ \text{Preceding Pixel} \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix} \end{array}$$

Assuming that the process starts in the 0 state, examine by calculation how the entropy of the source changes as the first six pixels are generated.

What is the long-term (steady-state) entropy of the source?

If, as an approximation, the source was considered to be memoryless, what would its entropy be?

Examine how a Shannon/Fano code could be used to encode the source data in preparation for transmission over a binary communication channel and comment on the efficiency of the encoding process with respect to:

(i) the entropy of the assumed memoryless source, and
(ii) the true steady-state entropy of the source.

2. Information is to be transmitted using an 8-phase modem system the constellation diagram of which is shown in Figure 1. The figure also shows the decision boundaries used at the modem receiver. During the course of transmission the phase of the transmitted signal is subject to a variation whose probability density function, $P(\theta^o)$, is:

$$P(\theta^o) = \frac{1}{90} - \frac{1}{8100}\theta^o; \; 0 \leq \theta^o \leq 90$$

$$= \frac{1}{90} + \frac{1}{8100}\theta^o; \; -90 \leq \theta^o \leq 0$$

Determine the capacity of the channel.

If the information that is to be transmitted is from a memoryless source that has four possible outputs whose respective probabilities are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{8}$ explain what has to be done in order for the channel to be used at capacity.

You may neglect the effects of noise.

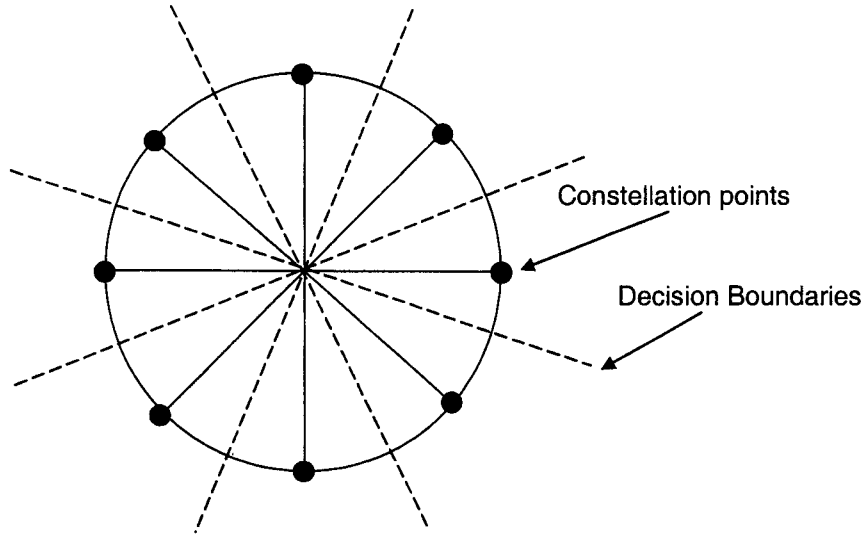

Figure 1

3. Consider a noisy communication channel in which X denotes the input ensemble and Y and YZ possible output ensembles.

Explain the meaning, and significance, and the limitations, of the following expressions:

(i) $I = H(X) - H(X/Y)$
(ii) $I = H(X) - H(X/YZ)$.

Prove that $I = H(X) - H(X/Y) = H(Y) - H(Y/X)$ and explain why the alternative forms might be useful.

Compare and contrast the Shannon/Fano and Huffman methods of source coding and describe a method that could be used to implement the encoding schemes.

Prove, other than by way of example, that for an information source with $K \geq 2$ output symbols, an optimum prefix binary code exists in which the two least likely codewords X_k and X_{k-1} have the same length and differ only in the last digit.

4. Discuss the statement; "A restriction on the rate at which date pulses can be transmitted over a communication channel does not in itself place a fundamental restriction on the rate at which data can be transmitted over the channel".

A binary symmetric channel has a cross-over probability of P. Determine from first principles the capacity of the channel.

The binary symmetric channel is to be used to transmit information from an information source whose outputs are the integers 0, 1 and 2, which occur with equal probabilities. Explain what has to be done in order to transmit data at, or close to, the channel capacity.

5. An information source selects its outputs, one at a time, from a set of n symbols. The source has memory which extends over M successive outputs generated by the source.

If blocks, S_i, consisting of N successive source symbols are found to have associated probabilities $P(S_i)$; $i = 1, ..., n^N$ prove that, provided $N > M$,

$$H = -\sum_{i=1}^{n^N} P(S_i) log\, P(S_i) = (N - M)H(x) + \delta,$$

where $H(x)$ is the true entropy of the source, and δ is a positive constant that is independent of N.

Use your result to prove Shannon's noiseless coding theorems for the memory source, and comment on the implementation of your proposed encoder.

6. Data is transmitted over a discrete memoryless noisy binary channel using pulses of amplitude $\pm V$ volts. The channel is corrupted by zero-mean additive white Gaussian noise and attentuation can be neglected. If two decision thresholds at $\pm kV$ are employed in the channel decision making system and received signal levels between the decision thresholds are considered to be 'ambiguous'; determine the capacity of the channel as a function of the various decision probabilities when

(i) $k < 1$
(ii) $k \gg 1$

Explain the practical significance of the two approaches to decoding and discuss the associated advantages and disadvantages.

If, in deriving your results, you employ any special arguments, then these should be proved.