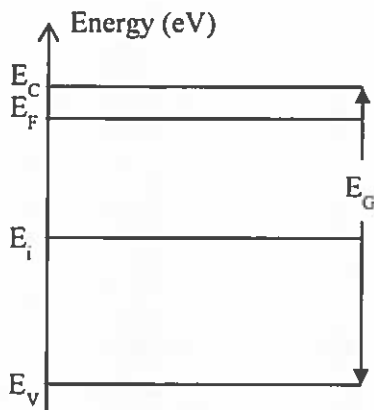


1.

a) BOOKWORK (typical easy question)

i)

[4]



ii) BOOKWORK (with a little twist)

Use equation from formulae list:

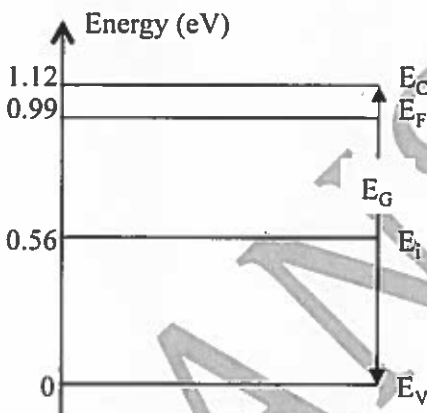
$$n = N_C e^{\frac{(E_C - E_F)}{kT}}$$

$$E_C - E_F = kT \ln \left( \frac{N_C}{n} \right) = kT \ln \left( \frac{N_C}{N_D} \right) = 0.026 \times \ln \left( \frac{3.2 \times 10^{19}}{2 \times 10^{17}} \right) \approx 0.13 \text{ eV} [2]$$

$$E_C - E_i = kT \ln \left( \frac{N_C}{n_i} \right) = 0.026 \times \ln \left( \frac{3.2 \times 10^{19}}{1.45 \times 10^{10}} \right) \approx 0.56 \text{ eV} \text{ (this is midway the bandgap, as expected for Si)} [2]$$

Since  $E_V = 0 \text{ eV}$ ,  $E_C = E_G = 1.12 \text{ eV}$ . [2]

Thus:



[6]

iii) BOOKWORK (never asked before)

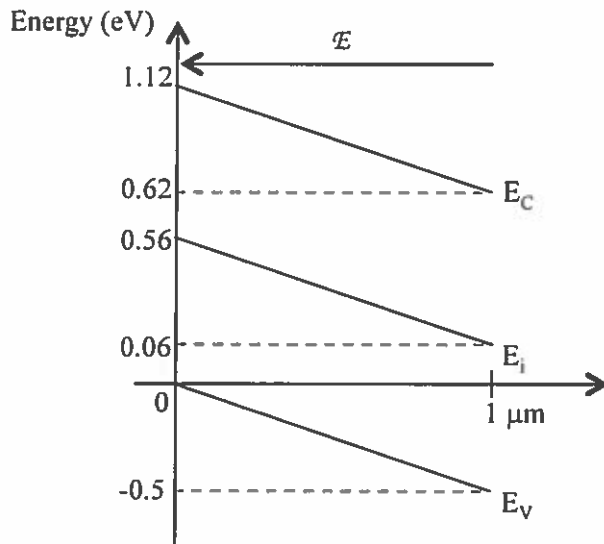
Use equation (by heart):

PE = q V (potential energy = charge × electrostatic potential). [2]

Because: when an electron moves due to an electrostatic potential difference of 1 V, it loses 1 eV of potential energy (definition of an eV).

Thus  $E_C(x=1 \mu\text{m}) = E_C(x=0 \mu\text{m}) - 1 \text{ eV} \times 0.5 \text{ V} [2]$ 

[6]



[2]

iv) BOOKWORK

The electric field must point from + to – and thus point “uphill”: electrons lose potential energy whilst drifting through the device.

Use equation:

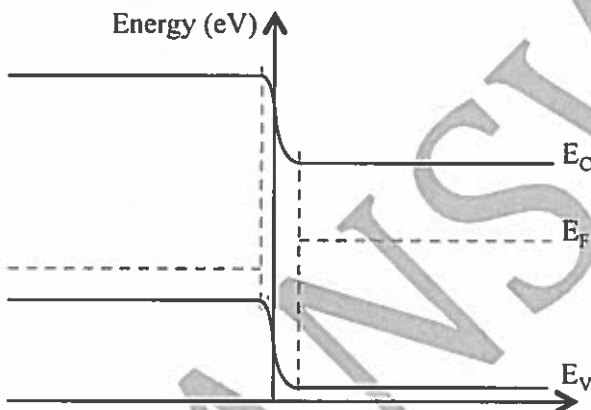
$$\mathcal{E} = -\frac{dV}{dx} \rightarrow |\mathcal{E}| = \frac{\Delta V}{\Delta x} = \frac{0.5V}{1\mu m} = 5 \times 10^5 V/m$$

[4]

b)

i) BOOKWORK

[6]



[2] flat PE in neutral regions

[3] correct  $E_F$  positions +  $\Delta E_F$

[1] nonlinear EP in depletion region

ii) BOOKWORK

ii1) The pn diode in Fig. 1.2 is FORWARD biased.

ii2) Electrons DIFFUSE from RIGHT TO LEFT across the depletion region.

ii3) Holes DIFFUSE from LEFT TO RIGHT across the n-type region.

ii4) The depletion width in the p-type region is 2 × LARGER than that in the p-type region.

[4]

c) BJT - BOOKWORK

i) Bias point (1): EB forward and BC reverse  
Bias point (2): EB forward and BC forward.

[4]

ii) Holes

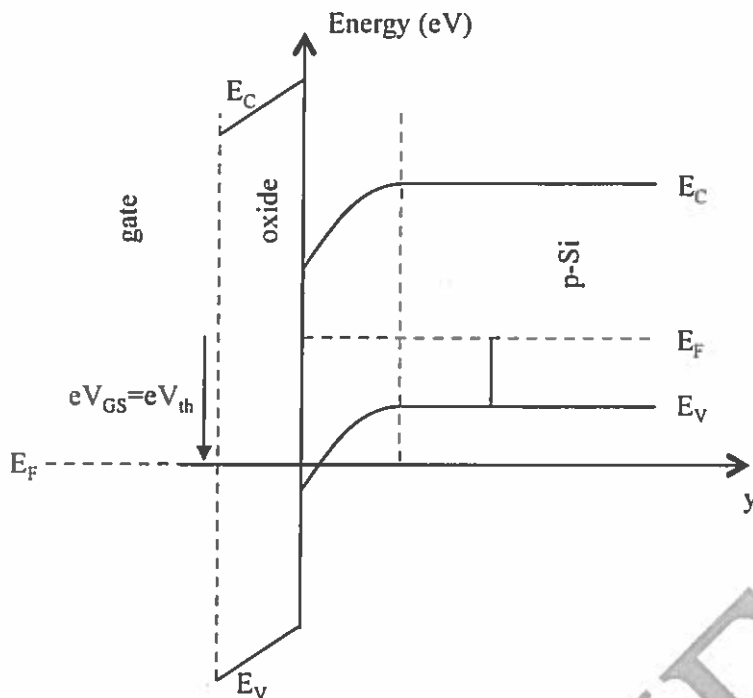
[2]

iii)  $\beta = \frac{(C)}{(B)-(D)}$

[4]

- d) MOSFET - BOOKWORK  
i) .

[6]



[2] correct distance EP to EF at interface

[2] correct banding

[2] correct EF shift

ii) From formulae list.

$$I_{DS} = \frac{\mu C_{ox} W_g}{L_g} \left[ (V_{GS} - V_{th}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

The voltage conditions given mean that the MOSFET is in saturation, thus the current becomes:

$$I_{DS} = \frac{\mu C_{ox} W_g}{2 L_g} (V_{GS} - V_{th})^2$$

The transconductance is:

$$g_m = \frac{dI_{DS}}{dV_{GS}} = \frac{\mu C_{ox} W_g}{L_g} (V_{GS} - V_{th})$$

[4]

## ANSWERS – 2016

2. BJT – different from what was done in lectures and study groups due to varying areas of the different BJT layers. However, due to the statement: “The carrier concentrations and currents are homogeneous in the y-z plane in each layer” this does not make a difference for the majority of the question and only needs to be taken into account when calculating the currents.

a)

This is the average distance a minority carrier diffuses before it recombines with a majority carrier.

[4]

b) From the formulae sheet:  $c = c_0 \exp\left(\frac{V}{V_T}\right)$  and  $c_0$  must be known by heart:  $c_0 = \frac{n_i^2}{N}$  with N doping, all minority carrier concentrations at the boundaries can be calculated.

In emitter:  $p_{n0E} = \frac{n_i^2}{N_D} = \frac{(1.45 \times 10^{10})^2}{5 \times 10^{18}} = 42 \text{ cm}^{-3}$  and  $p'_n = 42 \times \exp\left(\frac{0.26}{0.026}\right) \approx 9 \times 10^5 \text{ cm}^{-3}$  [2]

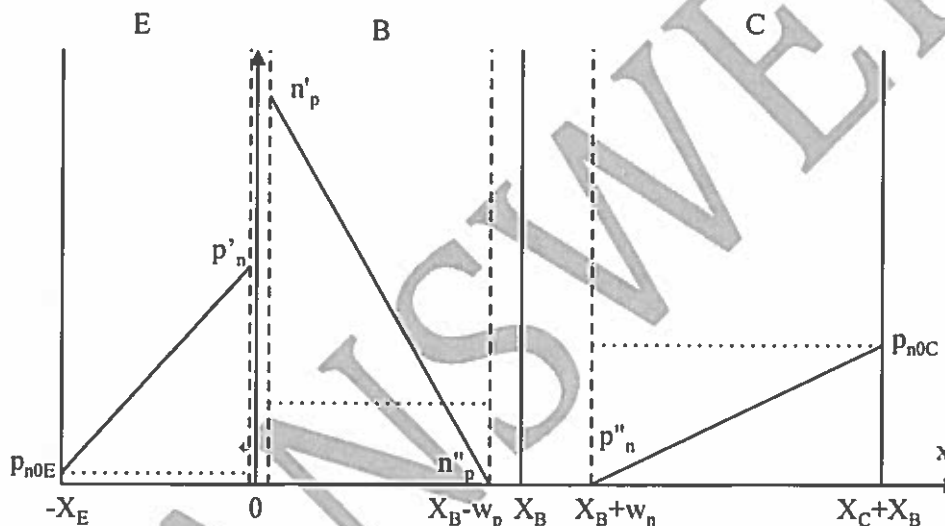
In base:  $n_{p0} = \frac{n_i^2}{N_A} = \frac{(1.45 \times 10^{10})^2}{5 \times 10^{16}} = 4205 \text{ cm}^{-3}$  and  $n'_p = 4205 \times \exp\left(\frac{0.26}{0.026}\right) \approx 9 \times 10^7 \text{ cm}^{-3}$  and  $n''_p = 4205 \times \exp\left(\frac{-1}{0.026}\right) \approx 8 \times 10^{-14} \text{ cm}^{-3}$  [2]

In collector:  $p_{n0C} = \frac{n_i^2}{N_D} = \frac{(1.45 \times 10^{10})^2}{1 \times 10^{16}} \approx 2.1 \times 10^4 \text{ cm}^{-3}$  and  $p''_n = 2.1 \times 10^4 \times \exp\left(\frac{-1}{0.026}\right) \approx 4 \times 10^{-13} \text{ cm}^{-3}$  [2]

[6]

c)

[6]



Dashed lines: depletion region edges (both depletion widths taken into account).

Dotted lines: equilibrium minority carrier concentration in each region

$w_p$  and  $w_n$  depletion widths of the reverse biased BC junction

[3] correct differences in carrier concentration position

[2] to depletion region edges

[1] linear variations

d) The easiest way is to use the above figure to calculate the currents. We will neglect the leakage current across the BC junction as the BJT is operating in forward active mode. The emitter current is the EB diode current, the collector current is the electron current component in the emitter current. Units for the calculations will be cm.

First however we need to calculate the depletion width  $w_p$ . We do not need  $w_n$  because we are neglecting the reverse bias BC junction leakage. We can use the equations given in the formulae sheet.

The built-in voltage

[9]

$$V_{bi} = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.026 \ln\left(\frac{5 \times 10^{16} \times 10^{16}}{(1.45 \times 10^{10})^2}\right) = 0.74 \text{ V} [1]$$

p-region depletion width:

# ANSWERS – 2016

$$w_p(V) = \left[ \frac{2\epsilon_0 \epsilon_{Si} (V_{bi} - V) N_D}{e(N_A + N_D) N_A} \right]^{\frac{1}{2}} = \left[ \frac{2 \times 8.85 \times 10^{-14} \times 11 \times (0.74 + 1) 10^{16}}{1.6 \times 10^{-19} \times (5 \times 10^{16} + 10^{16}) \times 5 \times 10^{16}} \right]^{\frac{1}{2}} = 8.4 \times 10^{-6} \text{ cm} = 8.4 \times 10^{-6} (10^4 \mu\text{m}) = 8.4 \times 10^{-2} \mu\text{m} \quad [2]$$

Magnitude of emitter current: [4]

$$\begin{aligned} I_E = I_p + I_n &= eA_E D_E \frac{\Delta p_{nE}}{X_E} + eA_B D_B \frac{\Delta n_{pB}}{X_B - w_p} \\ &= 1.6 \times 10^{-19} (40 \times 40) (\mu\text{m})^2 4 \frac{9 \times 10^5}{0.4 \mu\text{m}} + 1.6 \\ &\quad \times 10^{-19} (50 \times 50) (\mu\text{m})^2 20 \frac{9 \times 10^7}{(0.8 - 8.4 \times 10^{-2}) \mu\text{m}} \end{aligned}$$

convert  $\mu\text{m}$  into cm

$$\begin{aligned} I_E &= 1.6 \times 10^{-19} (40 \times 40) (10^{-4} \text{ cm})^2 4 \frac{9 \times 10^5}{0.4 (10^{-4} \text{ cm})} + 1.6 \\ &\quad \times 10^{-19} (50 \times 50) (10^{-4} \text{ cm})^2 20 \frac{9 \times 10^7}{(0.8 - 8.4 \times 10^{-2}) (10^{-4} \text{ cm})} \end{aligned}$$

$$I_E = 2.3 \times 10^{-13} \text{ A} + 10^{-10} \text{ A} \quad [2]$$

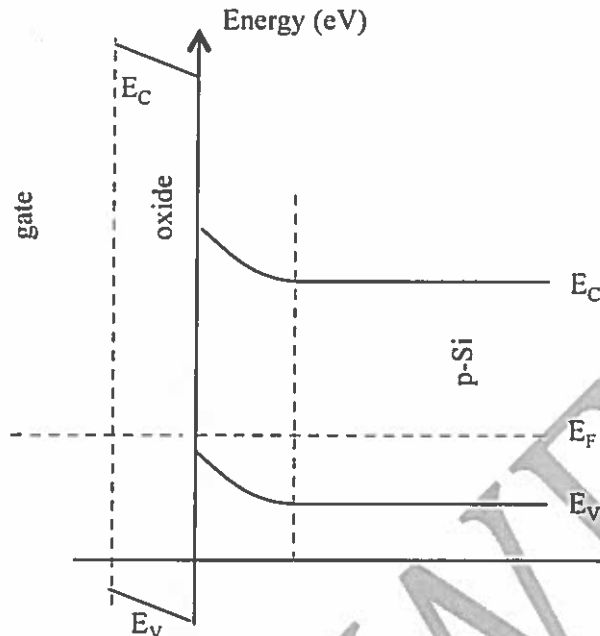
$$I_C = 10^{-10} \text{ A}$$

$$I_B = 2.3 \times 10^{-13} \text{ A}$$

3. MOSFET – different from what was done in the lectures due to existence of a workfunction difference. The influence of workfunction difference was explained for the energy band diagram in the study groups but was not connected to a change in the capacitance voltage characteristics.

a) The mobility is inversely proportional to the mass of the carriers. The mass of electrons in Si is lower than that of holes and thus electron mobility is higher. [4]

b) When  $\phi_G > \phi_p$  then the MOSFET is in accumulation. [6]



[2] correct distance EP to EF at interface

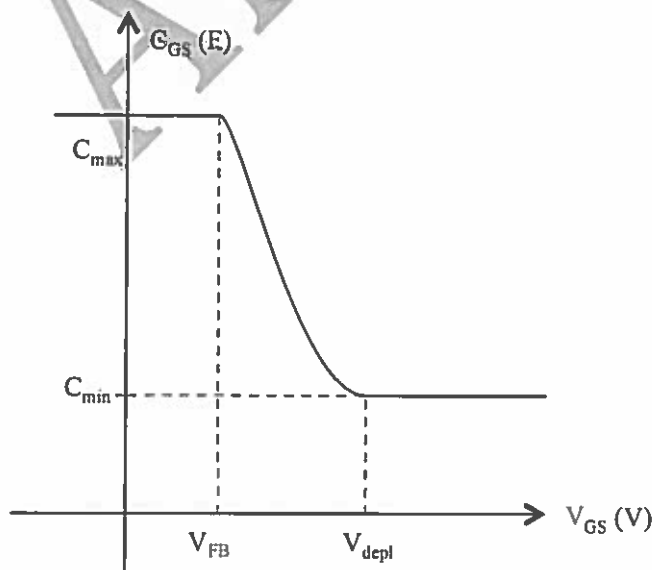
[2] correct banding

[2] correct EF shift

c)  $V_{GS} < 0$  the MOSFET is in accumulation (increase hole density underneath oxide). [2]  
 $C_{GS}$  is maximum and its value is determined by the oxide thickness. [9]

$0 < V_{GS} < V_{FB}$  the MOSFET is still in accumulation until the energy bands flatten at  $V_{GS} = V_{FB} = \phi_G > \phi_p$ . [2]

$V_{FB} < V_{GS} < V_{th}$  a depletion region is building up underneath the gate oxide.  $C_{GS}$  is decreasing until the depletion width has reached a maximum value at  $V_{GS} = V_{th}$  when an inversion channel is created.  $C_{GS}$  is minimum for  $V_{GS} \geq V_{th}$ . [2]



[3]

d)  $C_{\max} = \frac{1}{2} C_{\text{ox}} L_g W_g = \frac{1}{2} \epsilon_{\text{ox}} \epsilon_0 L_g W_g / t_{\text{ox}}$

All parameters expressed in cm

$$C_{\max} = \frac{1}{2} 4 \times 8 \times 10^{-14} \times 2 \times 10^{-4} \times 5 \times 10^{-4} / (10 \times 10^{-7}) = 1.77 \times 10^{-14} \text{ F} \quad [2]$$

[6]

$$C_{\min} = C_{\max} / C_{\text{depl}} \text{ (parallel connection)}$$

The depletion capacitance is associated with the maximum depletion width. The formula is given in the formulae list.

$$W_{\text{depl}}^{\max} = 2 \left[ \frac{\epsilon k T \ln \left( \frac{N_{\text{substrate}}}{n_i} \right)}{e^2 N_{\text{substrate}}} \right]^{1/2}$$

$$W_{\text{depl}}^{\max} = 2 \left[ \frac{11 \times 8.85 \times 10^{-14} \times 0.026 \times \ln \left( \frac{5 \times 10^{16}}{1.45 \times 10^{10}} \right)}{1.6 \times 10^{-19} \times 5 \times 10^{16}} \right]^{1/2} = 1.38 \times 10^{-5} \text{ cm} \quad [2]$$

$$C_{\text{depl}} = \frac{\epsilon_0 \epsilon_{\text{Si}} A}{W_{\text{depl}}^{\max}} = \frac{11 \times 8.85 \times 10^{-14} \times 2 \times 10^{-4} \times 5 \times 10^{-4}}{1.38 \times 10^{-5}} = 5 \times 10^{-15} \text{ F}$$

$$\frac{1}{C_{\min}} = \frac{1}{C_{\max}} + \frac{1}{C_{\text{depl}}} = \frac{C_{\text{depl}} + C_{\max}}{C_{\max} C_{\text{depl}}} \quad [2]$$

$$C_{\min} = \frac{C_{\max} C_{\text{depl}}}{C_{\text{depl}} + C_{\max}} = \frac{1.77 \times 10^{-14} \times 5 \times 10^{-15}}{1.77 \times 10^{-14} + 5 \times 10^{-15}} = 3.9 \times 10^{-15} \text{ F}$$