

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2015

MSc and EEE PART III/IV: MEng, BEng and ACGI

**MICROWAVE TECHNOLOGY**

Corrected Copy

Friday, 11 December 9:00 am

Time allowed: 3:00 hours

**There are SIX questions on this paper.**

**Answer FOUR questions.**

*All questions carry equal marks*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      S. Lucyszyn  
Second Marker(s) :      O. Sydoruk



**Special instructions for invigilators**

*This is a closed book examination.*

**Special instructions for students**

*All variables and abbreviations have their usual meaning.*

*Permeability of free space,  $\mu_0 = 4\pi \times 10^{-7} [H / m]$*

*Permittivity of free space,  $\epsilon_0 \approx 8.854 [pF / m]$*

## The Questions

1.

- a) Clearly defining all variables and their SI units, state the following (in relation to the differential form of Maxwell's equations) and give a brief description of their physical interpretation:
- i) Faraday's Law of Electromagnetic Induction and any associated constitutive relationship. [3]
  - ii) Ampere's Law of Magnetomotive Force and any associated constitutive relationship. [3]
  - iii) Electric form of Gauss's Law. [2]
  - iv) Magnetic form of Gauss's Law. [2]
- b) From 1(a), derive an expression for the intrinsic impedance of a homogenous and isotropic material, in terms of the propagation constant of a TEM wave propagating through it. [3]
- c) As an extension on 1(b), derive expressions for the skin depth and wavelength. [2]
- d) Using 1(c), calculate the approximate skin depth and wavelength for a TEM wave propagating through distilled water, having dielectric constant of 53 and bulk DC conductivity of 31 S/m, at 15 GHz. State any assumptions made. [5]

2.

- a) A TEM wave propagates within a lossless dielectric, above a perfectly flat metal conducting sheet, in the  $+z$  direction and with its electric field in the  $-x$  direction and magnetic field in the  $+y$  direction. Give the mathematical relationship between the induced electric field vector at the surface of a normal conductor and its incident tangential magnetic field. Define all variables used. [2]
- b) With the scenario in 2(a), and the help of a diagram, explain the directions of the resulting electric field vector and explain why the wave is not pure TEM just above the surface of the normal conductor. [5]
- c) With the scenario in 2(a), define the Poynting vectors in the dielectric medium well above the surface of the normal conductor and within the conductor just below its surface. Define the conduction current density as a function of conductivity and also with respect to its depth inside the metal. Define all variables used. [5]
- d) State the mathematical relationship between the conduction current density, as a function of depth, and surface current density. The time dependence can be ignored. Hence, from first principles with the classical skin depth model, show that the propagation constant acts as a constant of proportionality. [4]
- e) Using first principles, prove that the conduction current density leads the surface current density by  $45^\circ$  at microwave frequencies. [4]

3.

- a) Sketch the transverse cross-sections for the guided-wave structures listed below. State two positive attributes, two drawbacks and two common applications for each:

i) microstrip line [2]

ii) coplanar waveguide [2]

iii) slotline [2]

iv) dielectric-filled metal-pipe rectangular waveguide [2]

v) image line [2]

vi) dielectric waveguide [2]

- b) With the aid of diagrams that show the electric field in the dominant modes of propagation, illustrate an efficient transition between the following guided-wave structures:

i) microstrip to coplanar waveguide [1]

ii) coplanar waveguide to slotline [1]

iii) slotline to metal-pipe rectangular waveguide [1]

iv) dielectric-filled metal-pipe rectangular waveguide to image line [1]

v) image line to dielectric waveguide [1]

- c) Explain the general mechanisms that limit performance in transitions between dissimilar guided-wave structures. What rules-of-thumb can be adopted to improve efficiency? [3]

4. a) From first-principles, show that the time-average power flow along a lossless transmission line is independent of the line length and is equal to the incident wave power minus the reflected wave power at the termination impedance. Assume the characteristic impedance is purely real. [5]
- b) Define frequency dispersion in a transmission line. If a transmission line is represented by a lumped-element model, show what happens to frequency dispersion when  $RC = GL$ . All variables have their usual meaning. [5]
- c) Lossless transmission lines can be represented by an infinite number of lumped series- $L$ /shunt- $C$  sections. From first principles, derive a self-consistent equation for characteristic impedance, in terms of  $L$  and  $C$ , and use this to derive the corresponding bandwidth for this solution. [5]
- d) Calculate the input impedance, return loss and transmission loss at the cut-off frequency determined in 4(c). [5]

5. Consider a metal-pipe rectangular waveguide with internal dimensions shown in Figure 5.1

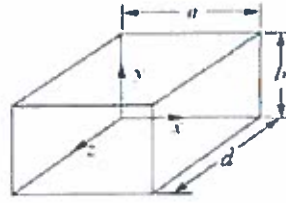


Figure 5.1

- Give an expression for the ideal (i.e. lossless)  $TE_{10}$  guided wavelength for the structure in Figure 5.1, in terms of free-space wavelength and lossless cut-off frequency. Define all variables used. You may assume that the dimensions in Figure 5.1 have  $d > a > b$ . [3]
- Using the resulting equation in 5(a), calculate the ideal cut-off frequency and guided wavelength when  $a = 200 \mu\text{m}$  and the operating frequency is 918 GHz. [2]
- Derive an expression, from first principles, for the ideal (i.e. lossless)  $TE_{mnl}$  resonant mode frequencies for the structure given in Figure 5.1 [4]
- Reduce the equation derived in 5(c) for the  $TE_{101}$  mode with  $a = 2b = d/\sqrt{2}$ . Also, in this specific case, how does the  $TE_{101}$  resonant frequency relate to the  $TE_{10}$  cut-off frequency? [2]
- Using the equations in 5(d), calculate the ideal resonant frequency for a  $200 \mu\text{m}$  wide waveguide. [2]
- Define the unloaded quality factor for a general resonator, in terms of energy and power. [1]
- Derive a simple expression for the free-space wavelength of a  $TE_{101}$  mode,  $\lambda_{101\_ideal}$ , in terms of its width, given that  $a = 2b = d/\sqrt{2}$ . [2]
- Calculate the skin depth  $\delta$  of gold at the ideal resonant frequency found in 5(e), given the value of bulk conductivity of  $4.517 \times 10^7 \text{ S/m}$ . [2]
- Using the results from 5(g) and 5(h), calculate the approximate unloaded Q-factor at resonance, for  $a = 2b = d/\sqrt{2} = 200 \mu\text{m}$ , given the following equation:

$$Q_U = \frac{\lambda_{101\_ideal}}{4\delta} \left\{ \frac{2b(a^2 + d^2)^{\frac{3}{2}}}{[2b(a^3 + d^3) + ad(a^2 + d^2)]} \right\} \quad (5.1)$$

[2]



6.

- a) Give examples of thermionic valves that could still be found within today's home, operating in the kHz, MHz and GHz frequency regions. [3]
- b) With the aid of a diagram, describe in detail the construction and operation of a magnetron and comment on its domestic applications. [10]
- c) Explain the  $\pi$ -mode of operation and how other modes can be suppressed. [5]
- d) Briefly explain a simple method for increasing the output power level of a magnetron valve and give examples of typical power levels for different types of applications. [2]

