

# ADVANCED DATA COMMUNICATION SOLUTIONS - 2008

E4.04 / I & 4.9/

SC 6

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Problem 1.a.

i) As an orthonormal set of basis functions we consider the set

$$\begin{aligned}\psi_1(t) &= \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{o.w.} \end{cases} & \psi_2(t) &= \begin{cases} 1 & 1 \leq t < 2 \\ 0 & \text{o.w.} \end{cases} \\ \psi_3(t) &= \begin{cases} 1 & 2 \leq t < 3 \\ 0 & \text{o.w.} \end{cases} & \psi_4(t) &= \begin{cases} 1 & 3 \leq t < 4 \\ 0 & \text{o.w.} \end{cases}\end{aligned}$$

In matrix notation, the four waveforms can be represented as

$$\begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \\ s_4(t) \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 & -1 \\ -2 & 1 & 1 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & -2 & -2 & 2 \end{bmatrix} \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \\ \psi_4(t) \end{bmatrix}$$

Note that the rank of the transformation matrix is 4 and therefore, the dimensionality of the waveforms is 4

ii) The representation vectors are

$$\begin{aligned}s_1 &= [2 \ -1 \ -1 \ -1] \\ s_2 &= [-2 \ 1 \ 1 \ 0] \\ s_3 &= [1 \ -1 \ 1 \ -1] \\ s_4 &= [1 \ -2 \ -2 \ 2]\end{aligned}$$

iii) The distance between the first and the second vector is

$$d_{1,2} = \sqrt{|s_1 - s_2|^2} = \sqrt{\left| \begin{bmatrix} 4 & -2 & -2 & -1 \end{bmatrix} \right|^2} = \sqrt{25}$$

Similarly we find that

$$\begin{aligned}d_{1,3} &= \sqrt{|s_1 - s_3|^2} = \sqrt{\left| \begin{bmatrix} 1 & 0 & -2 & 0 \end{bmatrix} \right|^2} = \sqrt{5} \\ d_{1,4} &= \sqrt{|s_1 - s_4|^2} = \sqrt{\left| \begin{bmatrix} 1 & 1 & 1 & -3 \end{bmatrix} \right|^2} = \sqrt{12} \\ d_{2,3} &= \sqrt{|s_2 - s_3|^2} = \sqrt{\left| \begin{bmatrix} -3 & 2 & 0 & 1 \end{bmatrix} \right|^2} = \sqrt{14} \\ d_{2,4} &= \sqrt{|s_2 - s_4|^2} = \sqrt{\left| \begin{bmatrix} -3 & 3 & 3 & -2 \end{bmatrix} \right|^2} = \sqrt{31} \\ d_{3,4} &= \sqrt{|s_3 - s_4|^2} = \sqrt{\left| \begin{bmatrix} 0 & 1 & 3 & -3 \end{bmatrix} \right|^2} = \sqrt{19}\end{aligned}$$

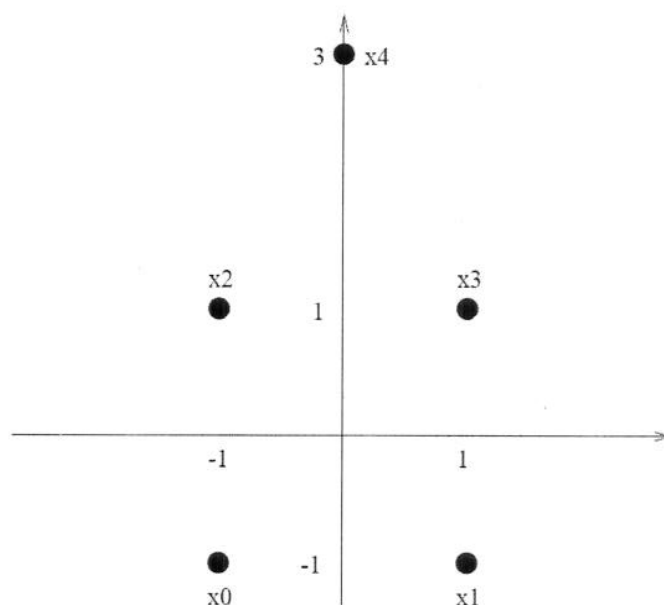
Thus, the minimum distance between any pair of vectors is  $d_{\min} = \sqrt{5}$ .

### Problem 1.b

i) The signal constellation is shown in the figure below.

From the signal constellation, we get  $d_{min} = 2$ . Since we have 5 signals, the Union Bound is given by,

$$\begin{aligned} P_e &\leq 4 Q\left(\frac{d_{min}}{2\sigma}\right) \\ &= 4 Q\left(\frac{1}{\sigma}\right). \end{aligned}$$



ii)

The number of Nearest Neighborhood  $N_e = 1/5(2+2+3+3+2) = 2.4$ . Therefore, the Nearest Neighborhood Union Bound is given by,

$$P_e = 2.4 Q\left(\frac{1}{\sigma}\right).$$

iii)

Since  $\mathcal{E}_x = \frac{1}{5}(2 \times 4 + 3^2) = 3.4$ ,

$$\sigma = \sqrt{\frac{\mathcal{E}_x}{SNR}} = \sqrt{\frac{\frac{1}{2}\mathcal{E}_x}{SNR}} = \sqrt{\frac{1.7}{10^{1.4}}}$$

Therefore, we get

$$P_e(\text{NNUB}) = 2.4 \times Q\left(\sqrt{\frac{10^{1.4}}{1.7}}\right) = 1.45 \times 10^{-4}.$$

Problem 1.c

i)

The impulse response of the matched filter is

$$s(t) = u(T-t) = \begin{cases} \frac{A}{T}(T-t) \cos(2\pi f_c(T-t)) & 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$

ii)

The output of the matched filter at  $t = T$  is

$$\begin{aligned} g(T) &= u(t) \star s(t)|_{t=T} = \int_0^T u(T-\tau) s(\tau) d\tau \\ &= \frac{A^2}{T^2} \int_0^T (T-\tau)^2 \cos^2(2\pi f_c(T-\tau)) d\tau \\ &\stackrel{v=T-\tau}{=} \frac{A^2}{T^2} \int_0^T v^2 \cos^2(2\pi f_c v) dv \\ &= \frac{A^2}{T^2} \left[ \frac{v^3}{6} + \left( \frac{v^2}{4 \times 2\pi f_c} - \frac{1}{8 \times (2\pi f_c)^3} \right) \sin(4\pi f_c v) + \frac{v \cos(4\pi f_c v)}{4(2\pi f_c)^2} \right] \Bigg|_0^T \\ &= \frac{A^2}{T^2} \left[ \frac{T^3}{6} + \left( \frac{T^2}{4 \times 2\pi f_c} - \frac{1}{8 \times (2\pi f_c)^3} \right) \sin(4\pi f_c T) + \frac{T \cos(4\pi f_c T)}{4(2\pi f_c)^2} \right] \end{aligned}$$

# Problem 1.d

The maximum likelihood criterion selects the maximum of  $f(\mathbf{r}|\mathbf{s}_m)$  over the  $M$  possible transmitted signals. When  $M = 2$ , the ML criterion takes the form

$$\frac{f(\mathbf{r}|\mathbf{s}_1)}{f(\mathbf{r}|\mathbf{s}_2)} \underset{s_2}{\overset{s_1}{\geq}} 1$$

or, since

$$\begin{aligned} f(\mathbf{r}|\mathbf{s}_1) &= \frac{1}{\sqrt{\pi N_0}} e^{-(r - \sqrt{\mathcal{E}_b})^2 / N_0} \\ f(\mathbf{r}|\mathbf{s}_2) &= \frac{1}{\sqrt{\pi N_0}} e^{-(r + \sqrt{\mathcal{E}_b})^2 / N_0} \end{aligned}$$

the optimum maximum-likelihood decision rule is

$$\underset{s_2}{\overset{s_1}{r \geq}} 0$$

The average probability of error is given by

$$\begin{aligned} P(e) &= p \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-(r + \sqrt{\mathcal{E}_b})^2 / N_0} dr + (1 - p) \int_{-\infty}^0 \frac{1}{\sqrt{\pi N_0}} e^{-(r - \sqrt{\mathcal{E}_b})^2 / N_0} dr \\ &= p \int_{\sqrt{2\mathcal{E}_b/N_0}}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx + (1 - p) \int_{-\infty}^{-\sqrt{2\mathcal{E}_b/N_0}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= pQ\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] + (1 - p)Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] \\ &= Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] \end{aligned}$$

### Problem 2.a

i

The energy of the 64 QAM is given by,

$$\mathcal{E}_x(64) = \frac{d^2}{6}(M-1) = 42$$

The energy of the hybrid 32 QAM constellation is found as follows. The 64 QAM can be decomposed into two 32 hybrid QAM constellations: The first one represented by dots, and the other one represented by  $\times$ . Since these two constellations are rotations of one another, then their energies must be equal. The energy of the 64 QAM constellation is then,

$$\mathcal{E}_x(64) = \frac{1}{2}\mathcal{E}_x(32) + \frac{1}{2}\mathcal{E}_x(32)$$

(The factor of 1/2 is present because we have 32 points in each hybrid constellation, and 64 points in the square QAM constellation). Therefore,  $\mathcal{E}_x(32) = 42$ . The energies are the same.

ii

The NNUB probability of error for the 64 QAM constellation is,

$$\begin{aligned} P_e &\leq 4\left(1 - \frac{1}{\sqrt{M}}\right)Q\left(\frac{d}{2\sigma}\right) \\ \Leftrightarrow P_e &\leq 3.5Q\left(\frac{1}{\sigma}\right) \end{aligned}$$

To find the NNUB for the 32 hybrid QAM the number of nearest neighbors  $N_e$  has to be computed. The constellation points are divided as follows,

- Inner Points (26) have 4 neighbors.
- Side Points (6) have 3 neighbors.

Therefore,  $N_e = \frac{1}{32}[26 \times 4 + 6 \times 3] = 3\frac{13}{16}$ . Since  $d_{min} = 2\sqrt{2}$ , then,

iii

$$P_e \leq 3.8125Q\left(\frac{\sqrt{2}}{\sigma}\right)$$

The distance  $d^2$  is given by,

$$\begin{aligned} d^2 &= \frac{6\mathcal{E}_x}{\frac{31}{32}M-1} = 8.4 \\ \Leftrightarrow d &= \sqrt{8.4} \simeq 2.898 \end{aligned}$$

iv

The probability of error for the 32 Cross QAM is,

$$P_e \leq 3.5Q\left(\frac{\sqrt{2.1}}{\sigma}\right)$$

For the 32 hybrid QAM constellation,  $P_e \leq 3.8125Q\left(\frac{\sqrt{2}}{\sigma}\right)$ . Since the argument of the Q function for the 32 Cross QAM is bigger than the one for the 32 hybrid constellation - equivalently  $d(cross) > d(hybrid)$ , 32 Cross performs better.

v

The constellation figure of merit for both constellations is,

$$\begin{aligned} CFM(hybrid) &= \frac{d^2/4}{\mathcal{E}_x} = \frac{2}{21} = 0.0952, \\ CFM(cross) &= \frac{\frac{8.4}{4}}{21} = 0.1 \end{aligned}$$

Obviously,  $CFM(cross) > CFM(hybrid)$ , a result that is consistent with (d).

## Problem 2.b

The three symbols  $A$ ,  $0$  and  $-A$  are used with equal probability. Hence, the optimal detector uses two thresholds, which are  $\frac{A}{2}$  and  $-\frac{A}{2}$ , and it bases its decisions on the criterion

$$\begin{aligned} A: & \quad r > \frac{A}{2} \\ 0: & \quad -\frac{A}{2} < r < \frac{A}{2} \\ -A: & \quad r < -\frac{A}{2} \end{aligned}$$

If the variance of the AWG noise is  $\sigma_n^2$ , then the average probability of error is

$$\begin{aligned} P(e) &= \frac{1}{3} \int_{-\infty}^{\frac{A}{2}} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(r-A)^2}{2\sigma_n^2}} dr + \frac{1}{3} \left( 1 - \int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{r^2}{2\sigma_n^2}} dr \right) \\ &\quad + \frac{1}{3} \int_{-\frac{A}{2}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(r+A)^2}{2\sigma_n^2}} dr \\ &= \frac{1}{3} Q\left[\frac{A}{2\sigma_n}\right] + \frac{1}{3} 2Q\left[\frac{A}{2\sigma_n}\right] + \frac{1}{3} Q\left[\frac{A}{2\sigma_n}\right] \\ &= \frac{4}{3} Q\left[\frac{A}{2\sigma_n}\right] \end{aligned}$$

Problem 2.c

(i)

Since QAM transmission is used,

$$\begin{aligned}\phi 1 &= -\sqrt{\frac{2}{T}} \text{sinc}\left(\frac{t}{T}\right) \sin \omega_c t \\ \phi 2 &= \sqrt{\frac{2}{T}} \text{sinc}\left(\frac{t}{T}\right) \cos \omega_c t\end{aligned}$$

(ii)

where  $T = 2 \cdot 10^{-7}$ .

For QAM and  $P_e = 10^{-6}$  the gap is  $\Gamma = 8.8 \text{ dB}$ . Hence,

$$\bar{b} = \frac{1}{2} \log_2 \left( 1 + \frac{SNR}{\Gamma} \right) \simeq 3$$

(iii)

Then,  $R = \frac{b}{T} = \frac{2.3}{2 \cdot 10^{-7}} = 30 \text{ Mbps}$ .

Since  $\bar{b} = 3, b = 6$ , we have  $M = 2^b = 2^6 = 64$ . Therefore this is a 64-square QAM signal constellation.

(iv)

In order to achieve  $R = 35 \text{ Mbps}$ , we should have  $b = 7$ , thus employing 128-cross QAM. We require the same probability of error, so:

$$\begin{aligned}\left( \frac{d_{min}}{2\sigma} \right)^2 &= \frac{3SNR}{\frac{31}{32}M - 1} = 13.5 \text{ dB} \Rightarrow \\ SNR &= 917.9 = 29.6 \text{ dB}\end{aligned}$$

8

### Problem 3.a

i

We will proceed using the gap approximation: i.e.  $E_n(b_n) = \frac{\Gamma}{g_n}(2^{2b_n} - 1) * k$ , where  $k=1$  if PAM and  $k=2$  if QAM.

So, we first need to find  $g_n = \frac{|H_n|^2}{\sigma_n^2}$ . From the system parameters  $\sigma_n^2 = .125$ . So, we have the following table (the center 3 are QAM channels)

subchannel	0	1	2	3	4
$g_n$	18	15.6569	10	4.3431	2

Now, using the above formula, we get:

subchannel	0	1	2	3	4
$e_n(1)$	1.2463	.9690	1.5172	3.4932	11.37
$e_n(2)$	5.07	1.938	3.043	6.9860	45.5233
$e_n(3)$	19.2287	3.8760	6.0686	13.97	182.0567
$e_n(4)$	81.9150	7.7521	12.1372	27.94	728.2345

Note for the QAM channels we could have used  $e_n(b_n) = \frac{\Gamma}{g_n} 2^{2b_n}$

ii

With the above table, it is obvious that the bit allocations are as follows:

subchannel	0	1	2	3	4
$b_n$	2	3	2	1	0
$E_n(b_n)$	6.3215	6.7830	4.5515	3.4932	0

Bits were chosen in the following order: 1,0,2,1,2,3,1,0

iii

$N * \bar{E}_x = 8$ , so we are way over budget. Working backwards, we get

subchannel	0	1	2	3	4
$b_n$	1	2	1	0	0
$E_n(b_n)$	1.2463	2.9070	1.5172	0	0

iv

Again, we just work backwards

subchannel	0	1	2	3	4
$b_n$	1	1	0	0	0
$E_n(b_n)$	1.2463	.9690	0	0	0

The margin in this case is  $10 * \log_{10}(\frac{8}{1.2463+.9690}) = 5.577dB$



(i)

## Problem 3.b

g =

4.3431 10.0000 15.6569 18.0000 15.6569 10.0000 4.3431 2.000

e =

1.1124 1.1041 1.0680 0.9377 0.6680

b =

2.1970 2.0964 1.7730 1.1714 0.6120

N =

8

b\_bar =

1.6113

(ii)

$$\text{gap} = 10^{-9}/3 = 2.6478 = 4.23\text{dB}$$

(iii)

Check the following results  
R = 1.0622

g =

4.3431 10.0000 15.6569 18.0000 15.6569 10.0000 4.3431 2.00

e =

1.2977 1.2757 1.1800 0.8351 0.1206

b =

1.6478 1.5472 1.2239 0.6223 0.0629

N =

8

b\_bar =

1.0622

3.c

①

From Problem 3.b, a rate of 1.06 is achieved with a gap of 4.23 dB. Since  $\Gamma = 9.8 \text{ dB}$  for  $P_e = 10^{-7}$ , about 4.5 dB more energy is needed. So, the  $P_e$  goal cannot be achieved. We can reach the same conclusion by doing MA, which will give us a negative margin.

②

Since we get  $R = 1.0622$  for  $\Gamma = 4.23 \text{ dB}$  from 3.b, an easy way to solve this one would be to try a slightly larger gap than 4.23 dB. To get the exact gap value, we need to solve MA again. The same approach as in problem 3. will give us  $\Gamma = 4.79 \text{ dB}$ , and the corresponding  $P_e = 1.3 \cdot 10^{-3}$ .

③

Part ii gives us  $\Gamma = 4.79 \text{ dB}$  with no margin. So, with  $\Gamma = 0$ , margin = 4.79 dB.

Problem 4.a

Substituting the expression of  $X_{rc}(f)$  in the desired integral, we obtain

$$\begin{aligned}
 \int_{-\infty}^{\infty} X_{rc}(f) df &= \int_{-\frac{1+\alpha}{2T}}^{-\frac{1-\alpha}{2T}} \frac{T}{2} \left[ 1 + \cos \frac{\pi T}{\alpha} \left( -f - \frac{1-\alpha}{2T} \right) \right] df + \int_{-\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} T df \\
 &\quad + \int_{\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} \frac{T}{2} \left[ 1 + \cos \frac{\pi T}{\alpha} \left( f - \frac{1-\alpha}{2T} \right) \right] df \\
 &= \int_{-\frac{1+\alpha}{2T}}^{-\frac{1-\alpha}{2T}} \frac{T}{2} df + T \left( \frac{1-\alpha}{T} \right) + \int_{\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} \frac{T}{2} df \\
 &\quad + \int_{-\frac{1+\alpha}{2T}}^{-\frac{1-\alpha}{2T}} \cos \frac{\pi T}{\alpha} \left( f + \frac{1-\alpha}{2T} \right) df + \int_{\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} \cos \frac{\pi T}{\alpha} \left( f - \frac{1-\alpha}{2T} \right) df \\
 &= 1 + \int_{-\frac{\alpha}{T}}^0 \cos \frac{\pi T}{\alpha} x dx + \int_0^{\frac{\alpha}{T}} \cos \frac{\pi T}{\alpha} x dx \\
 &= 1 + \int_{-\frac{\alpha}{T}}^{\frac{\alpha}{T}} \cos \frac{\pi T}{\alpha} x dx = 1 + 0 = 1
 \end{aligned}$$

Problem 4.b

i

The pulse response is  $p(t) = \phi(t) * h(t)$ . In frequency domain,

$$\begin{aligned} P(f) &= \Phi(f)H(f) \\ &= (\sqrt{T} \Pi(Tf)) \left( \frac{1}{1 + ae^{j2\pi f}} \Pi(f) \right) \\ &= \frac{1}{1 + ae^{j2\pi f}} \Pi(f) \quad (\text{since } T=1) \end{aligned}$$

In terms of  $\omega$ ,

$$P(\omega) = \begin{cases} \frac{1}{1+ae^{j\omega}} & |\omega| \leq \pi \\ 0 & |\omega| > \pi \end{cases}$$

ii

First, let's find  $P(e^{-j\omega T})$ .

$$P(e^{-j\omega T}) = \frac{1}{T} \sum_{n=-\infty}^{\infty} P(\omega + \frac{2\pi n}{T})$$

Since  $T=1$  and  $P(\omega) = 0$  for  $|\omega| > \pi$ ,  $P(e^{-j\omega T}) = \frac{1}{1+ae^{j\omega}}$ . Then, by the inverse Fourier transform,

$$p_k = (-a)^{-k} u[-k].$$

Therefore,

$$\begin{aligned} \|p\|^2 &= T \sum_{k=-\infty}^{\infty} |p_k|^2 \\ &= \sum_{k=-\infty}^0 (-a)^{2k} \\ &= \sum_{k=0}^{\infty} (-a)^{2k} \\ &= \frac{1}{1-a^2}. \end{aligned}$$

iii

By substituting  $e^{-j\omega T} = D$  into  $P(e^{-j\omega T})$ , we get  $P(D) = \frac{1}{1+aD^{-1}}$ .

Therefore,

$$Q(D) = \frac{T}{\|p\|^2} P(D) P^*(D^{-*}) = \frac{1-a^2}{(1+aD)(1+aD^{-1})}.$$

iv

For the zero-forcing equalizer,

$$W_{ZFE}(D) = \frac{1}{\|p\|Q(D)} = \frac{(1+aD)(1+aD^{-1})}{\sqrt{1-a^2}}.$$

To calculate  $W_{MMSE-LE}(D)$ , we need  $SNR_{MFB}$ :

$$SNR_{MFB} = \frac{\|p\|^2 \bar{\mathcal{E}}_x}{\sigma^2} = \frac{10^{1.5}}{1-a^2}.$$

Then,

$$\begin{aligned} W_{MMSE-LE}(D) &= \frac{1}{\|p\|(Q(D) + 1/SNR_{MFB})} \\ &= \frac{\sqrt{1-a^2}}{\frac{1-a^2}{(1+aD)(1+aD^{-1})} + \frac{1+a^2}{10^{1.5}}} \\ &= \frac{(1+aD)(1+aD^{-1})}{\sqrt{1-a^2}[1 + (1+aD)(1+aD^{-1})10^{-1.5}]}. \end{aligned}$$

v

When  $a = 0$ ,  $Q(D) = 1$  and  $\|p\|^2 = 1$ . Since  $SNR = 15\text{dB}$  and  $\Gamma = 8.8\text{dB}$  at  $P_e = 10^{-6}$ ,

$$\bar{b} = \frac{1}{2} \log_2 \left( 1 + \frac{10^{1.5}}{10^{0.88}} \right) = 1.18$$

Then, the maximum data rate achievable is

$$R = \frac{b}{T} = \frac{1.18}{1} = 1.18 \text{ bits/sec}$$

#### Problem 4.c

The bandwidth of the bandpass channel is  $W = 4$  KHz. Hence, the rate of transmission should be less or equal to 4000 symbols/sec. If a 8-QAM constellation is employed, then the required symbol rate is  $R = 9600/3 = 3200$ . If a signal pulse with raised cosine spectrum is used for shaping, the maximum allowable roll-off factor is determined by

$$1600(1 + \alpha) = 2000$$

which yields  $\alpha = 0.25$ . Since  $\alpha$  is less than 50%, we consider a larger constellation. With a 16-QAM constellation we obtain

$$R = \frac{9600}{4} = 2400$$

and

$$1200(1 + \alpha) = 2000$$

Or  $\alpha = 2/3$ , which satisfies the required conditions. The probability of error for an  $M$ -QAM constellation is given by

$$P_M = 1 - (1 - P_{\sqrt{M}})^2$$

where

$$P_{\sqrt{M}} = 2 \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left[ \sqrt{\frac{3\mathcal{E}_{av}}{(M-1)N_0}} \right]$$

With  $P_M = 10^{-6}$  we obtain  $P_{\sqrt{M}} = 5 \times 10^{-7}$  and therefore

$$2 \times \left( 1 - \frac{1}{4} \right) Q \left[ \sqrt{\frac{3\mathcal{E}_{av}}{15 \times 2 \times 10^{-10}}} \right] = 5 \times 10^{-7}$$

Using the last equation and the tabulation of the  $Q[\cdot]$  function, we find that the average transmitted energy is

$$\mathcal{E}_{av} = 24.70 \times 10^{-9}$$

Note that if the desired spectral characteristic  $X_{rc}(f)$  is split evenly between the transmitting and receiving filter, then the energy of the transmitting pulse is

$$\int_{-\infty}^{\infty} g_T^2(t) dt = \int_{-\infty}^{\infty} |G_T(f)|^2 df = \int_{-\infty}^{\infty} X_{rc}(f) df = 1$$

Hence, the energy  $\mathcal{E}_{av} = P_{av}T$  depends only on the amplitude of the transmitted points and the symbol interval  $T$ . Since  $T = \frac{1}{2400}$ , the average transmitted power is

$$P_{av} = \frac{\mathcal{E}_{av}}{T} = 24.70 \times 10^{-9} \times 2400 = 592.8 \times 10^{-7}$$

If the points of the 16-QAM constellation are evenly spaced with minimum distance between them equal to  $d$ , then there are four points with coordinates  $(\pm \frac{d}{2}, \pm \frac{d}{2})$ , four points with coordinates  $(\pm \frac{3d}{2}, \pm \frac{3d}{2})$ , four points with coordinates  $(\pm \frac{5d}{2}, \pm \frac{5d}{2})$ , and four points with coordinates  $(\pm \frac{7d}{2}, \pm \frac{7d}{2})$ . Thus, the average transmitted power is

$$P_{av} = \frac{1}{2 \times 16} \sum_{i=1}^{16} (A_{mc}^2 + A_{ms}^2) = \frac{1}{2} \left[ 4 \times \frac{d^2}{2} + 4 \times \frac{9d^2}{2} + 8 \times \frac{10d^2}{4} \right] = 20d^2$$

Since  $P_{av} = 592.8 \times 10^{-7}$ , we obtain

$$d = \sqrt{\frac{P_{av}}{20}} = 0.00172$$