

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

EEE/ISE PART I: MEng, BEng and ACGI

COMMUNICATIONS 1

None

Friday, 25 May 10:00 am

CORRECTED COPY

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.

Answer Q1 and any two of questions 2-4.

Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible

First Marker(s) : P.L. Dragotti, P.L. Dragotti

Second Marker(s) : M.K. Gurcan, M.K. Gurcan

Special Information for the Invigilators: none

Information for Candidates

The trigonometric Fourier series of a periodic signal $x(t)$ of period $T_0 = 2\pi/\omega_0$ is

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t),$$

with

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt, \quad a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t dt, \quad b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t dt.$$

The exponential Fourier series is given by

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \quad \text{with} \quad D_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt$$

Some Fourier Transforms

$$\cos \omega_0 t \quad \Longleftrightarrow \quad \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$$\text{rect}\left(\frac{t}{\tau}\right) \quad \Longleftrightarrow \quad \tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$$

$$\frac{W}{\pi} \text{sinc}(Wt) \quad \Longleftrightarrow \quad \text{rect}\left(\frac{\omega}{2W}\right)$$

$$\frac{\alpha^2}{2\pi} \text{sinc}^2\left(\frac{\alpha t}{2}\right) \quad \Longleftrightarrow \quad \Delta\left(\frac{\omega}{\alpha}\right)$$

Some useful trigonometric identities

$$\cos x \cos y = \frac{1}{2} \cos(x - y) + \frac{1}{2} \cos(x + y).$$

Euler's formula

$$e^{jx} = \cos x + j \sin x.$$

Frequency modulation by a sinusoidal signal

$$\varphi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t.$$

where $\beta = \Delta f/B$.

Table of Bessel Function values (recall that $|J_n(\beta)| = |J_{-n}(\beta)|$):

n	$\beta = 1$	$\beta = 2$	$\beta = 5$	$\beta = 10$
0	0.765	0.224	-0.178	-0.246
1	0.440	0.577	-0.328	0.043
2	0.115	0.353	0.047	0.255

The Questions

1. This question is compulsory.

(a) Consider the signal $x(t) = e^{-5t}u(t)$, where $u(t)$ is the unit step function defined by

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0. \end{cases}$$

i. Compute the energy of $x(t)$.

[4]

ii. Compute the Fourier transform of $x(t)$.

[4]

iii. Compute the energy of $x(t)$ using Parseval's theorem.

[4]

(b) The output signal from a full AM modulator is

$$x(t) = 10 \cos(8000t) + 20 \cos(10000t) + 10 \cos(12000t).$$

i. Sketch and dimension the Fourier transform of $x(t)$.

[4]

ii. Determine the modulating signal $m(t)$ and the carrier $c(t)$.

[4]

iii. Compute the power efficiency η .

[4]

- (c) Consider the signal $x(t) = 10 \cos 100t$.
- Sketch and dimension the spectrum of the DSB-SC modulated signal $s(t) = 2x(t) \cos 1000t$. [4]
 - From the spectrum of $s(t)$, identify the upper sideband (USB) and the lower sideband (LSB) spectra. [2]
 - From the USB spectrum, write the exact expression of the USB modulated signal $\varphi_{USB}(t)$. [2]

- (d) Consider the FM signal

$$\varphi(t) = \cos[2\pi f_0 t + k_f \int_{-\infty}^t m(\alpha) d\alpha]$$

where $m(t) = 10 \cos 2\pi f_m t$ and $k_f = 200\pi$. Using Carson's rule, the bandwidth of $\varphi(t)$ is 4 kHz. Determine the frequency f_m of $m(t)$. [4]

- (e) A 50Ω transmission line is connected to a 100Ω line with a matched termination. A sine wave propagating in the former is incident on the junction. Find
- The voltage reflection coefficient k_v . [2]
 - Show how a resistor R connected in parallel at the junction can eliminate this reflection and find its value. [2]

2. Consider the FM signal

$$\varphi(t) = 10 \cos[1000t + k_f \int_{-\infty}^t m(\alpha) d\alpha]$$

where $m(t) = \cos 100t$.

- (a) Assume that $k_f = 100$. Determine the modulation index β of the frequency modulated signal.

[7]

- (b) Using Carson's rule, determine the bandwidth of the frequency modulated signal.

[7]

- (c) Sketch the spectrum of the modulated signal (plot only the frequency components that lie within the bandwidth derived using Carson's rule).

[8]

- (d) Consider now the phase modulated signal

$$\varphi(t) = 10 \cos[1000t + k_p m(t)]$$

where $m(t) = \cos 100t$. Find the value of k_p that leads to the same instantaneous maximum frequency of the modulated signal of Part (a).

[8]

3. Consider the periodic signal $x(t)$ shown in Figure 1.

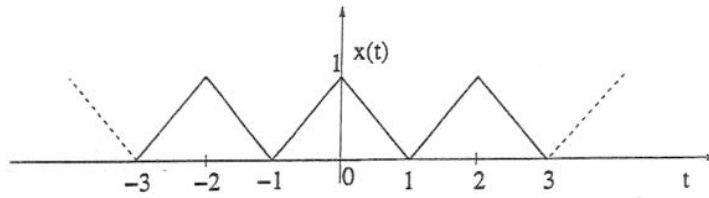


Figure 1: The periodic signal $x(t)$.

- (a) Find the power of $x(t)$. [6]
- (b) Compute the coefficients a_0 , a_n and b_n , of the trigonometric Fourier series of $x(t)$. [6]
- (c) Compute the coefficients D_n of the exponential Fourier series of $x(t)$. [6]
- (d) The signal $x(t)$ is fed to a filter $h(t)$ giving output $y(t)$. The frequency response of the filter is

$$H(\omega) = \begin{cases} 1 & \text{for } |\omega| \leq \alpha \text{ rad/s} \\ 0 & \text{otherwise} \end{cases}$$
 where $\alpha = 3.1\pi$ rad/s. Compute the power of the output $y(t)$. [6]
- (e) Find the range of possible values of α that lead to $P_y = 0.75P_x$ where P_y and P_x are the power of $y(t)$ and $x(t)$ respectively. [6]

4. Suppose the signal $x(t) = m(t) + \cos 1000t$ is applied to a non-linear system whose output is $y(t) = x(t) + \frac{1}{2}x^2(t)$ and assume $m(t) = \frac{1}{\pi}\text{sinc}(t)$.

(a) Sketch and dimension the Fourier transform of $m(t)$.

[6]

(b) Sketch and dimension the Fourier transform of $y(t)$.

[6]

(c) The signal $y(t)$ is fed to an ideal band-pass filter $h(t)$ giving output $z(t)$. The frequency response of the filter is

$$H(\omega) = \begin{cases} 1 & \text{for } 950 \leq \omega \leq 1050 \text{ rad/s} \\ 0 & \text{otherwise} \end{cases}$$

Write the exact expression of the output $z(t)$.

[6]

(d) Can you retrieve $m(t)$ from $z(t)$ using an envelop detector? Justify your answer.

[6]

(e) Now assume the message $m(t)$ has the Fourier transform shown in Figure 2. Find the exact expression of $m(t)$.

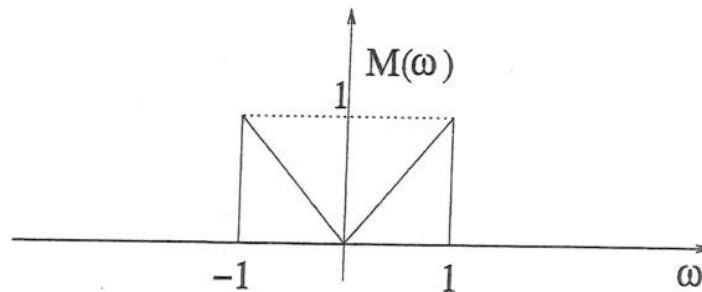


Figure 2: Fourier transform of $m(t)$.

[6]

QUESTION 1

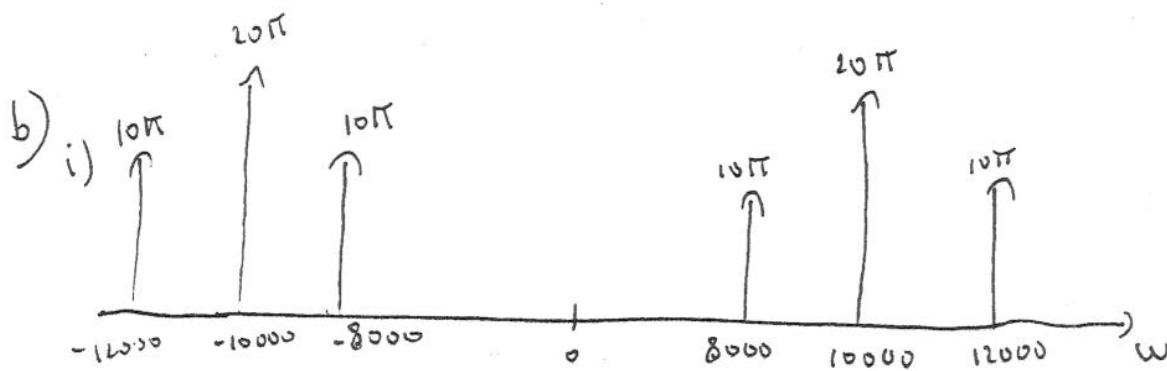
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i. $E_x = \int_{-\infty}^{\infty} x^2(t) dt = \int_0^{\infty} e^{-10t} dt = \frac{1}{10}$

ii. $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_0^{\infty} e^{-(s+j\omega)t} dt = \frac{1}{s+j\omega}$

iii. $E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{25 + \omega^2} d\omega = \frac{1}{10}$

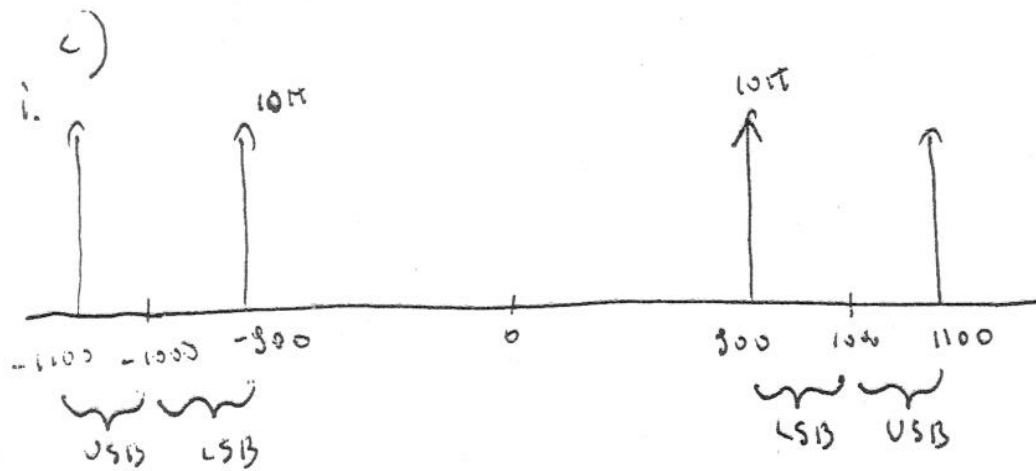


ii.

$m(t) = 20 \cos 20000t$

$c(t) = 20 \cos 10000t$

iii. $\eta = \frac{P_s}{P_s + P_c} = \frac{\frac{(20)^2}{4}}{\frac{(20)^2}{4} + \frac{(20)^2}{4}} = \frac{1}{3}$



iii. $\varphi_{USB} = \cos 1100t$

d) $B_{FH} = 2 \left(f_m + \frac{12 f_{mp}}{2\pi} \right) = 4000 \text{ Hz}$

$2(f_m + 1000) = 4000 \text{ Hz} \Rightarrow f_m = 1000 \text{ Hz}$

(e) i. $K_V = \frac{t_2 - t_0}{t_2 + t_0} = \frac{100 - 50}{150} = \frac{1}{3}$

ii. $\frac{R \cdot 100}{R + 100} = 50 \Rightarrow R = 100 \Omega$

QUESTION 2

3

a) INSTANTANEOUS MAX FREQUENCIES

$$\omega_{\max}^{\text{PH}} = \omega_c + k_p m_p = \omega_c + 100 k_p$$

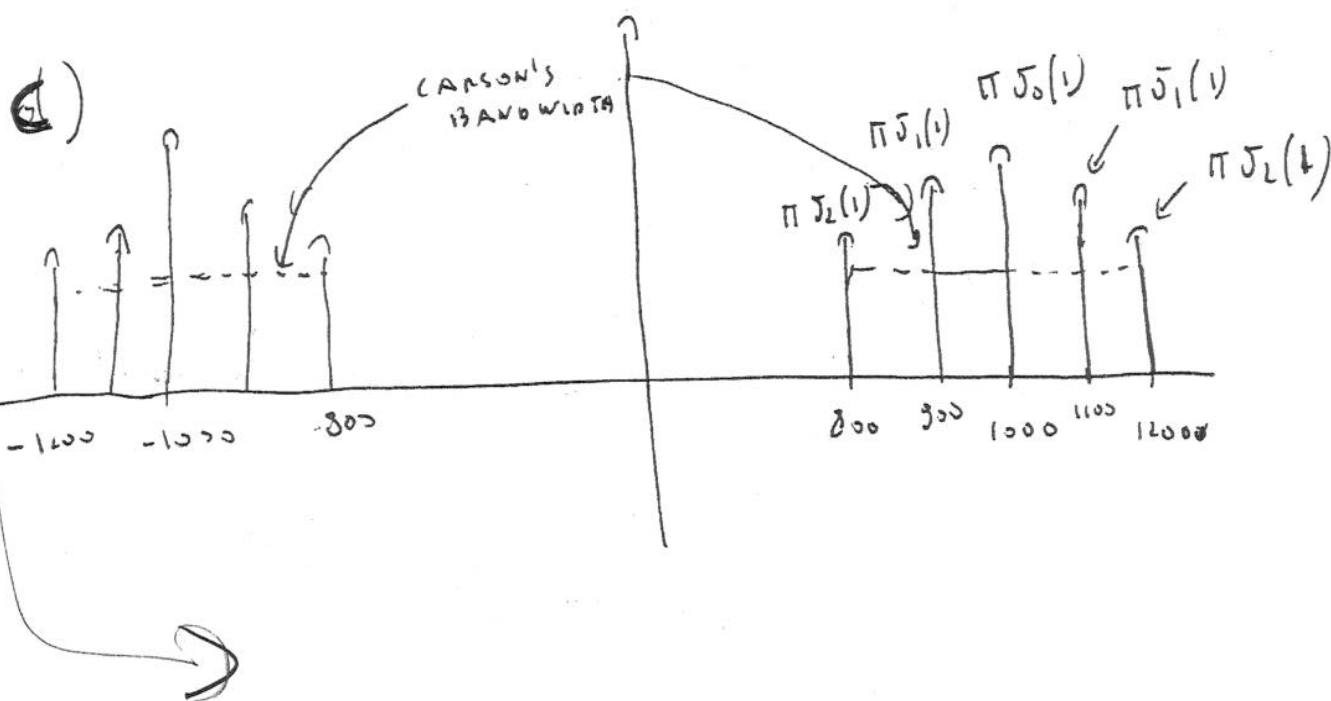
$$\omega_{\max}^{\text{FH}} = \omega_c + k_f m_p = \omega_c + k_f$$

$$\Rightarrow k_f = 100 k_p \Rightarrow$$

$$k_p = 1,$$

$$b) \beta = \frac{\Delta f}{B} = \frac{k_f m_p}{2\pi \cdot B} = \frac{100 \cdot 1}{100} = 1$$

$$B = 2(\beta + 1)f_m = \frac{400}{1\pi} \text{ Hz}$$



QUESTION 3

a) $T_0 = 2$ $\omega_0 = \frac{2\pi}{T_0} = \pi$

$$P_x = \frac{1}{T_0} \int_{-1}^1 x(t) dt = \frac{2}{T_0} \int_0^1 (1-t)^2 dt =$$

$$= \int_0^1 (1+t^2-2t) dt = 1 + \frac{1}{3} - 1 = \frac{1}{3}$$

b) $b_n = 0$ SINCE $x(t)$ IS EVEN

$$a_0 = \frac{1}{T_0} \int_{-1}^1 x(t) dt = \int_0^1 (1-t) dt = 1 - \frac{1}{2} = \frac{1}{2}$$

$$a_m = \frac{2}{T_0} \int_{-1}^1 x(t) \cos m\omega_0 t dt = \frac{4}{T_0} \int_0^1 (1-t) \cos m\omega_0 t dt$$

$$= 2 \left[\int_0^1 \cos m\omega_0 t dt - \int_0^1 t \cos m\omega_0 t dt \right] =$$

$$= \frac{2}{\pi^2 m^2} (1 - \cos m\pi) = \frac{2}{\pi^2 m^2} (1 - (-1)^m)$$

$$x(t) = \frac{1}{2} + \frac{2}{\pi^2} \sum_{m=1}^{\infty} \frac{(1 - \cos m\pi)}{m^2} \cos m\pi t =$$

$$= \frac{1}{2} + \frac{2}{\pi^2} \sum_{m=1}^{\infty} \frac{(1 - (-1)^m)}{m^2} \cos m\pi t,$$

$$= \frac{1}{2} + \frac{4}{\pi^2} \left[\cos \pi t + \frac{1}{9} \cos 3\pi t + \frac{1}{25} \cos 5\pi t \dots \right]$$

-(c)

$$|U_m| = \frac{C_m}{2} = \frac{a_m}{2}$$

$$D_0 = a_0$$

5

$$|D_m| = 0$$

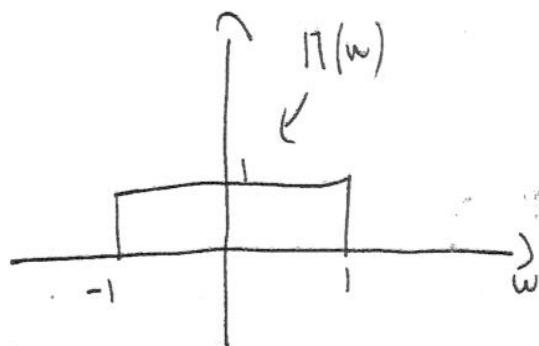
$$d) \frac{1}{4} + \frac{1}{2} \left(\frac{16}{\pi^4} + \frac{16}{81\pi^4} \right) = \frac{1}{4} + \frac{20}{81\pi^4} \approx 0.238$$

$$P_y = \frac{1}{4} + \frac{1}{2} \left(\frac{16}{\pi^4} + \frac{16}{81\pi^4} \right) = \frac{1}{4} + \frac{8}{\pi^4} \left(1 + \frac{1}{81} \right) \approx 0.32$$

$$e) \quad 0 < 2 < \pi$$

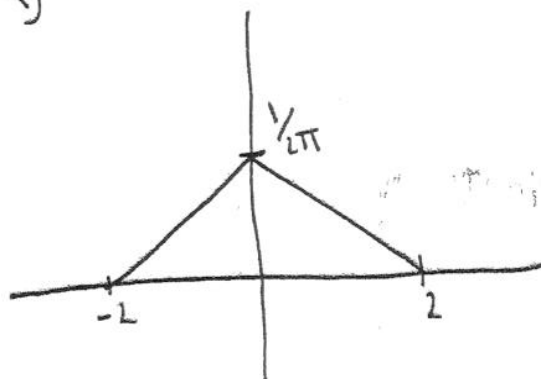
QUESTION 4

a) $\Pi(\omega) = \text{RECT}\left(\frac{\omega}{2}\right)$

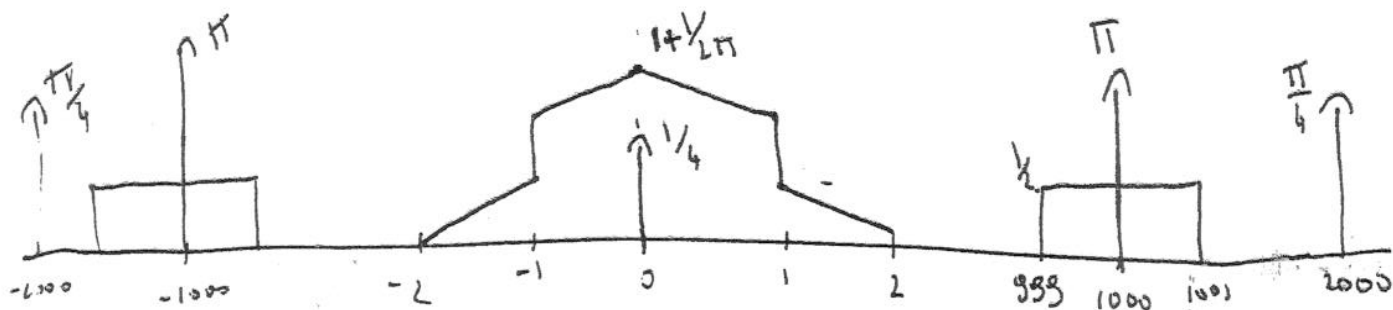


b) $y(t) = m(t) + \cos 1000t + \frac{1}{2} m^2(t) + \frac{1}{4} + \frac{1}{4} \cos 1000t + m(t) \cos 1000t.$

~~$\frac{1}{2} m^2(t) \Leftrightarrow \frac{1}{4\pi} \Pi(\omega) * \Pi(\omega)$~~



$Y(\omega)$

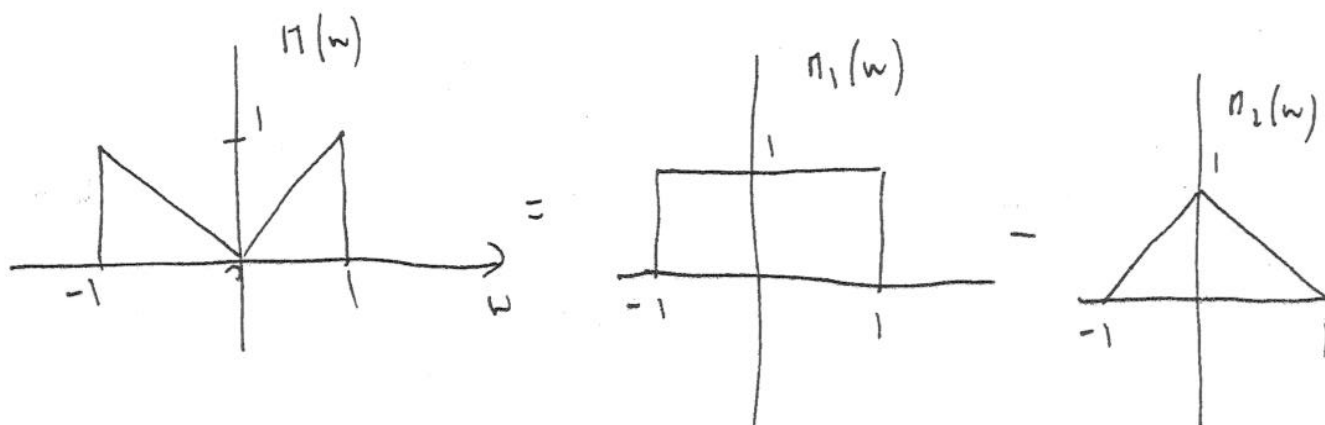


c)

$$f(t) = (1 + m(t)) \cos 1000t$$

d) YES, since $m_p = 1$ AND THEREFORE $1 + m(t) \geq 0 \quad \forall t$

e)



$$M(w) = M_1(w) - M_2(w)$$

Thus

$$m(t) = m_1(t) - m_2(t) = \frac{1}{\pi} \text{sinc } t - \frac{1}{2\pi} \text{sinc}^2\left(\frac{t}{2}\right)$$

