Optical Communication 2015: Solutions

1. All parts have equal value. [20]

a) *Approximately how long does it take for an optical pulse to travel down 200 km of silica-based optical fibre?*

$\Delta t = n'L/c$, estimate $n' = 1.5$, $\Delta t = 1.5 \times 2\times10^5 / (3 \times 10^8) = \underline{1\ \text{ms}}$

b) *A step-index highly multi-mode silica fibre has a numerical aperture NA = 0.30. Estimate the maximum angle away from the axial direction that light can propagate and still be guided by total internal reflection at the core-cladding boundary.*

Critical angle θ with respect to the surface normal is $\sin^{-1}(n_2/n_1)$. and $n_2/n_1 = 1 - \Delta n/n$, with $n \cong 1.5$. Get index difference from $NA \cong \sqrt{(2n\cdot\Delta n)}$, $\Delta n = NA^2/2n$, $\theta = \sin^{-1}(1 - NA^2/2n^2) = 88.5°$, max angle with respect to the axis is the complementary angle $\underline{11.5°}$.

c) *A photodiode produces 10.4 nA of photocurrent for an incident optical power of 15 nW at a nominal wavelength λ_o = 980 nm. Calculate the quantum efficiency η of the diode.*

$I_{ph} = \eta\, e\lambda\Phi/hc$ $\eta = 6.63\times10^{-34} \times 3\times10^8 \times 10.4\times10^{-9}/(1.6\times10^{-16} \times 0.98\times10^{-6} \times 15\times10^{-9}) = \underline{0.879}$

d) *A certain symmetric slab waveguide of thickness d = 4 μm supports exactly 3 TE modes for λ_o = 850 nm. Calculate the range of possible values of NA of this waveguide.*

Useful to sketch the Kd/2 vs $k_{1x}d/2$ diagram. The circular arc must have radius between $3\pi/2$ and 2π to support exactly 3 modes. The arc radius $R = NA\pi d/\lambda_o$, so NA is between $(\lambda_o/\pi d)(3\pi/2)$ and $(\lambda_o/\pi d)(2\pi)$, giving $\underline{0.318 \le NA \le 0.425}$.

e) *Briefly explain why the wavelength of zero dispersion in silica fibre is different from that of bulk silica glass.*

The chromatic dispersion for the fibre is a combination of the material dispersion of the glass, and waveguide dispersion, which is caused by the changing mode shape with wavelength, and consequently the changing distribution of the mode's power between the core and cladding. This latter effect shifts the dispersion zero.

f) *The drift velocity of charge carriers in a semiconductor for low values of applied electric field E equals the product of E and what other quantity? For which type of carrier does this quantity usually have a higher value, electrons or holes?*

This is the <u>mobility</u>. It is usually higher for <u>electrons</u> than holes.

g) *An optical detector receives a steady signal of 10^{11} photons per second at λ_o = 1550 nm. Give the power level of the optical signal in both nW and dBm.*

The power (energy/sec) = $10^{11} \times (hc/\lambda_o) = \underline{12.8\ \text{nW}}$
Converting to dBm: $10\log_{10}(12.8\times10^{-6}\ \text{mW}) = \underline{-49\ \text{dBm}}$

h) *If the refractive index of a material has an imaginary component, what does this imply about a wave propagating in that material?*

This implies attenuation (with amplitude dropping exponentially with distance).

i) What is the main advantage of graded index over step-index multi-mode fibre?

Graded index multi-mode fibre achieves a much lower multi-mode dispersion effect by clustering most of the mode indices together at one end of the range of allowed values.

j) A certain glass absorbs an optical wave propagating in it such that over a distance of 100 m the intensity is reduced by 5%. Calculate the corresponding attenuation coefficient in dB/km.

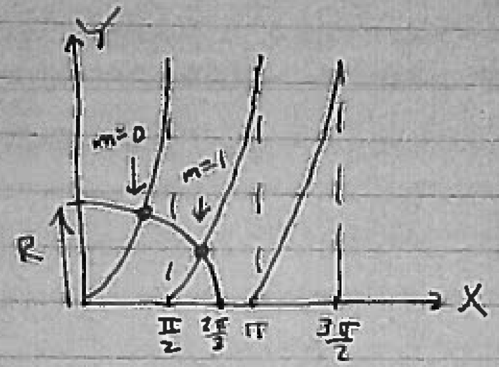
Over 100 m the loss in dB is $10\log_{10}(0.95) = 0.22$ dB, therefore the rate of loss is <u>2.2 dB/km</u>.

2. a) In the core, $E(x) = A\cos(k_{1x}x)$ for $m=0$

so $\cos(k_{1x}d/2) = \frac{1}{2}\cos(0) = \frac{1}{2}$

$k_{1x}d/2 = \cos^{-1}(\frac{1}{2}) = \underline{\pi/3} = X$

b)



From the eigenvalue eqns, $Y = X\tan X = \frac{\pi}{3}\tan\frac{\pi}{3}$

$\therefore Y = \sqrt{3}\,\pi/3$

$R = \sqrt{X^2+Y^2} = \underline{2\pi/3}$ Hence $\underline{2}$ TE modes are supported

c) $\beta^2 = n_1^2k_0^2 - k_{1x}^2$

$n' = \beta/k_0 = \sqrt{n_1^2 - (k_{1x}/k_0)^2}$

$k_{1x} = \frac{2\pi}{3d}$ - need to set d. Use $R = NA\,k_0 d/2$

and using $R = 2\pi/3$, $d = \frac{4\pi}{3}\cdot\frac{1}{k_0 NA}$

Combining:

$k_{1x}/k_0 = \frac{2\pi}{3k_0}\cdot\frac{3k_0 NA}{4\pi} = NA/2$

$n' = \sqrt{n_1^2 - (n_1^2 - n_2^2)/2} = \sqrt{\frac{n_1^2 + n_2^2}{2}}$

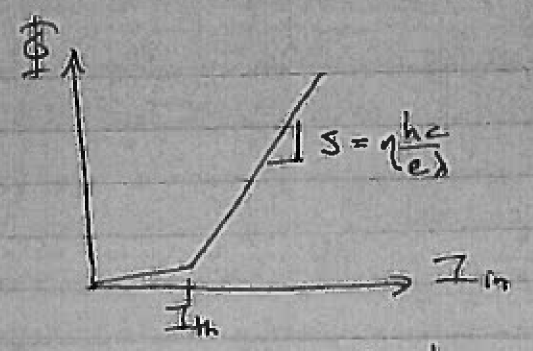
d) In the cladding, $E(x) = B\exp(-Kx)$

and we have $B\exp(-Kd/2) = 1/2$ $\quad (E(0) = 1)$

$\therefore E(d/2) = \frac{\exp(-Kd\cdot 2)}{2\exp(-Kd/2)} = \frac{1}{2}\exp(-3Kd/2) = \frac{1}{2}\exp(-3Y)$

and $Y = \sqrt{3}\,\pi/3$ $\therefore E(d/2) = \frac{1}{2}\exp(-\sqrt{3}\,\pi) = \underline{0.0022}$ V/m

3. a)



For low input current, the laser acts like an LED, with spontaneous emission dominating. I_{th} is the input level beyond which stimulated emission dominates, and output increases rapidly. The slope efficiency S is the differential optical output per unit input current, corresponding to the quantum efficiency η times one photon per electron.

b) We approximate the output at I_{th} as zero, so:

$$\Phi = S(I - I_{th})$$

$$S = \eta hc/e\lambda = 0.85 \times 6.63 \times 10^{-34} \times 3 \times 10^{8} / (1.6 \times 10^{-19} \times 1.3 \times 10^{-6})$$

$$= 0.81 \text{ W/A}. \quad \cancel{I_{th} = \frac{\Phi}{S} - I = \frac{23}{0.81}} -$$

$$I_{th} = I - \Phi/S = 26 - 2.3/0.81 =$$

$$= \underline{23.2 \text{ mA}}$$

c) Now $\Phi = S(26 - 21.2) = 0.81(4.8) = \underline{3.9 \text{ mW}}$

d) The cavity must satisfy $L = m\lambda/2$

$$\lambda_i = 2L/m_i \quad \Delta\lambda = \frac{2L}{m_i} - \frac{2L}{m_i+1} \cong 2L/m_i^2$$

$$\Delta\lambda = 2L\left(\frac{\lambda}{2L}\right)^2 = \frac{\lambda^2}{2L} = \frac{\lambda_0^2}{2Ln'^2} \quad L = \frac{\lambda_0^2}{2n'^2\Delta\lambda} = \frac{1300\text{ nm}^2}{2(3.4)^2\, 0.2\text{ nm}}$$

$$= \underline{0.73 \text{ mm}}$$

e) A single longitudinal mode can be obtained by replacing the mirrors with wavelength selective reflectors in the form of topographic gratings, either at each end or along the length - distributed feedback or distributed Bragg reflector. The grating period should be half λ_0/n'.

4. a) For these links, $SNR = \Phi_R / NEP\sqrt{B/2}$, taking $\Delta f = B/2$ and Φ_R the received power. Link A needs to receive 10/8 times the power of link B to get the same SNR, a difference of 0.97 dB. The difference in attenuation is 0.01 dB/km, therefore to balance out the length $L = 0.97/.01 = \underline{97\text{ km}}$

b) Taking link A, and assuming a transmitted power of 10 mW = 10 dBm, the received power is 10 dBm $- 97(.35)$ dB $= -24.0$ dBm $\cong 4.0\,\mu W$

Taking SNR = 12, $\sqrt{B/2} = \dfrac{4\times10^{-6}}{10^{-4}\times 12} = 0.33\times10^{5}$

$B/2 \cong 10^{9}$, $\underline{B \cong 2\text{ Gbit/s}}$

Then we approximate the responsivity as 1 A/W (very rough, λ is not given). If shot noise dominated then $SNR = \sqrt{\dfrac{I_{ph}}{eB}} = \sqrt{\dfrac{R\Phi_R}{eB}} = \sqrt{\dfrac{4\times10^{-6}}{1.6\times10^{-19}\times2\times10^{9}}}$

$\cong 160$ This is much greater than the actual SNR, so shot noise term is 160/12 smaller.

c) Equate the SNR for the 2 cases receiver or shot noise dominating:

$$SNR = \frac{\Phi_R}{NEP\sqrt{B/2}} = \sqrt{\frac{R\Phi_R}{eB}} \quad \therefore \frac{2\Phi_R}{NEP^2} = \frac{R}{e} = \frac{\lambda}{hc}$$

$$\therefore 2\Phi_R = \frac{NEP^2}{(hc/\lambda)} = \frac{NEP^2}{E_{ph}} \quad ✓$$

d) For shot noise only, $SNR = \dfrac{I_{ph}}{\sqrt{2eI_{ph}}\sqrt{\Delta f}}$ taking $\Delta f = B/2$:

$SNR = \sqrt{\dfrac{I_{ph}}{eB}}$ But I_{ph}/e is the number of electrons/sec, which divided by B gives no. of electrons / b.t.

5. a) $n(\lambda_0) = n_0 + a(\lambda_0 - \lambda_c)$

Material dispersion is proportional to $d^2n/d\lambda^2$

Here $dn/d\lambda_0 = a$, $d^2n/d\lambda_0^2 = 0$, no dispersion.

b) Group delay $\tau_g = L/v_g$ where v_g is the group velocity, $v_g = d\omega/dk$

We have $\omega = k_0 c$, $k = nk_0$

$$d\omega/dk = \frac{d\omega/dk_0}{dk/dk_0} = \frac{c}{n + k_0 dn/dk_0} \qquad \frac{dn}{dk_0} = \frac{dn}{d\lambda_0}\frac{d\lambda_0}{dk_0}$$

$\lambda_0 = 2\pi/k_0 \therefore d\lambda_0/dk_0 = -2\pi/k_0^2$

$dn/d\lambda_0 = a \quad \therefore \quad dn/dk_0 = -2\pi a/k_0^2 = -a\lambda_0/k_0$

$$\frac{d\omega}{dk} = \frac{c}{n - a\lambda_0} = \frac{c}{n_0 - a\lambda_c} = v_g \quad \therefore \quad \underline{\tau_g = \frac{L}{c}(n_0 - a\lambda_c)}$$

c) In the case of a guided mode, $V_g = d\omega/d\beta$, where β is the wave-vector in the direction of propagation. $\beta = n'k_0$. The pulse spreading is proportional to the variation in V_g over the spectral width:

$$\Delta\tau_g = (d\tau_g/d\lambda_0)\Delta\lambda_0 \quad \text{with } \tau_g = L/v_g = Ld\beta/d\omega$$

Taking $\tau_g = L\dfrac{d\beta/dk_0}{d\omega/dk_0} = \dfrac{L}{c}(n' + k_0 dn'/dk_0)$

and $dn'/dk_0 = (dn'/d\lambda_0)d\lambda_0/dk_0 = -(\lambda_0/k_0)dn'/d\lambda_0$

Then $\tau_g = \dfrac{L}{c}(n' - \lambda_0 dn'/d\lambda_0)$

and
$$\Delta\tau_g = \frac{L}{c}\Delta\lambda_0\frac{d}{d\lambda_0}\left(n' - \lambda_0\frac{dn'}{d\lambda_0}\right) = \frac{L}{c}\Delta\lambda_0\left(\frac{dn'}{d\lambda_0} - \frac{dn'}{d\lambda_0} - \lambda_0\frac{d^2n'}{d\lambda_0^2}\right)$$

$$= \underline{-\frac{L}{c}\Delta\lambda_0(d^2n'/d\lambda_0^2)\lambda_0}$$

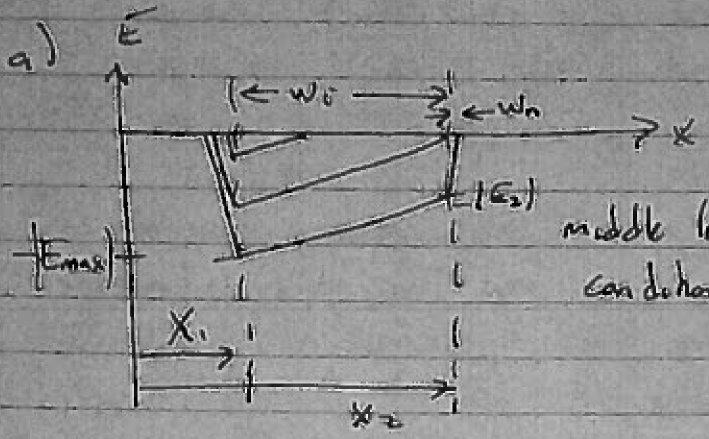
d) From (c) we can see that the pulse spreading $|\Delta \tau_g| = \frac{L}{c} \cdot \Delta\lambda_0 \cdot (d^2 n' / d\lambda_0^2) \lambda_0 / c$

We define the dispersion coefficient $D = \lambda_0 \cdot d^2 n' / d\lambda_0^2$ and replace $\Delta\lambda_0$ by the more exact statistical variations σ_λ. Then $\Delta \tau_g = \frac{L}{c} \cdot \sigma_\lambda \cdot D$.

This can be represented as a power penalty so long as the main effect is reduced pulse height rather than inter-symbol interference. A reasonable maximum spreading without much ISI is $\frac{1}{4}$ b.t., ie. $\Delta \tau_g = 0.25 / B$.

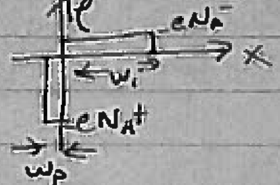
Then we need $\frac{L}{c} \cdot \sigma_\lambda \cdot D \cdot B < 0.25$.

6. a)



middle line satisfies the condition of (a)

When the intrinsic region is just depleted, the charge density looks like :



with $w_p N_A^+ = w_i N_A^-$

and $|E_{max}| = \dfrac{w_i(eN_D^-)}{\varepsilon_r \varepsilon_0} = \dfrac{5\times10^{-6}(1.6\times10^{-19})(10^{19})}{12\times8.85\times10^{-12}}$

$= 75.3$ kV/m

and $w_p = w_i\left(\dfrac{N_D^-}{N_A^+}\right) = 0.1\,\mu m.$ $V = \int E\,dx$ $|V| = \frac{1}{2}\times5.1\times10^{-6}\times75.3$

$= \underline{192\text{ mV}}$

b) Need $|E_2| = \dfrac{5\times10^4}{1400\times10^{-4}} = 357$ kV/m

In this case $E_1 = -357\times10^3 + 75.3\times10^3 = 432$ kV/m

$w_p = \dfrac{12\times8.85\times10^{-12}\times432\times10^3}{1.6\times10^{-19}\times5\times10^{20}} = 0.57\,\mu m$

$w_n = \dfrac{12\times8.85\times10^{-12}\times357\times10^3}{1.6\times10^{-19}\times10^{20}} = 2.37\,\mu m$

Integrating E gives $|V| = \frac{1}{2}(357\times2.37\times10^{-3}) + \frac{1}{2}(432\times.57\times10^{-3})$

$+ \frac{1}{2}((357+432)\times5\times10^{-3})$

$= \underline{2.52\text{V}}$

For holes, need $|E| = \dfrac{5\times10^4}{440\times10^{-4}} = 1136$ V/m

not found <u>anywhere</u>.

6. c) As an approximation, at this level most of the integral $\int E\,dx$ will come from the intrinsic region. Increasing V by 1% $\cong$ 2 mV will increase $|E_{max}|$ by $\approx \frac{2\,mV}{5\,\mu m} = 400$ V/m. This will extend the depletion

region into N_D^+ by $\frac{\varepsilon_r \varepsilon_0 (400)}{e N_D^+} = 2.6$ nm

and increase W_p by $\frac{\varepsilon_r \varepsilon_0 (400)}{e N_A^+} \cong 0.5$ nm

The capacitance per unit area is $C' = \frac{\varepsilon_r \varepsilon_0}{W_i + W_n + W_p}$

Since the total width is increased by $\approx 3.1/5000 = 0.06$%, the capacitance is decreased by 0.06%, ie much less than the 1% voltage change.

d) The maximum applied voltage is limited by the peak field to avoid breakdown. If the intrinsic region is p-doped, the peak field will be at the bottom of the intrinsic region where the velocities will also be maximised, but most photons are absorbed nearer the surface. In the extreme case where the intrinsic layer is not fully depleted, this would shift the depletion layer deeper into the device.