IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE PART II: MEng, BEng and ACGI

MATHEMATICS 2A (E-STREAM AND I-STREAM)

Thursday, 28 May 2:00 pm

Time allowed: 1:30 hours

Corrected Copy

There are TWO questions on this paper.

Answer TWO questions.

Answer both questions

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D. Angeli
Second Marker(s) : D. Nucinkis

1. Consider the integral of a scalar real variable t given below:

$$I = \int_0^{2\pi} e^{-\sin(t)} \cos(\cos(t))\, dt.$$

a) Show that I equals:

$$\mathrm{Im}\left[\int_{\partial^+ B} \frac{e^{iz}}{z}\, dz\right],$$

where B denotes the unit disc in $\mathbb{C}$ and $\partial^+ B$ its oriented boundary (in the counter-clockwise direction). [4]

b) Consider the function $f(z) = \frac{e^{iz}}{z}$; find its domain of definition in $\mathbb{C}$, argue that it is holomorphic in its domain, and find its poles and multiplicities; [3]

c) Use the Residue formula to evaluate the integral I [3]

d) Consider the scalar function $u(x,y) : \mathbb{R}^2 \to \mathbb{R}$ defined as:

$$u(x,y) = e^x \cos(y) + e^{-y} \sin(x).$$

Show that u is harmonic. [4]

e) Find its harmonic conjugate function $v(x,y)$, by making use of the Cauchy-Riemann's equations. [4]

f) Find a holomorphic function g of the complex variable z such that

$$g(z) = u(x,y) + iv(x,y)$$

when $z = x + iy$. [3]

g) Find the points in $\mathbb{C}$ where the map g is conformal. [4]

2. Consider the linear differential equation:

$$\frac{d^3}{dt^3}x(t)+3\frac{d}{dt}x(t)-4x(t)=u(t)$$

where $x:\mathbb{R}\to\mathbb{R}$ is a scalar function of the unknown variable t, and $u(t)$ denotes the Heaviside function (unit step).

a) Use Laplace's transform (and inverse transform) to compute an expression for $x(t), t \geq 0$, solution of the previous equation for $x(0)=0, \dot{x}(0)=0$ and $\ddot{x}(0)=0$. [20]

b) Find initial conditions for $x, \dot{x}$ and $\ddot{x}$ so that the solution to the considered equation is constant for $t \geq 0$. [5]

(In item a) marks are allocated as follows: 6 marks for correct transformed equation; 4 marks for correct frequency domain solution; 5 marks for correct Heaviside decomposition; 5 marks for correct time-domain expressions).

Appendix A

Table of Laplace Transforms

$f(t)$	$F(s)$
$\delta(t)$	1
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{\lambda t}\sin(\omega t)$	$\frac{\omega}{(s-\lambda)^2+\omega^2}$
$e^{\lambda t}\cos(\omega t)$	$\frac{(s-\lambda)}{(s-\lambda)^2+\omega^2}$
$e^{\lambda t}$	$\frac{1}{s-\lambda}$
$te^{\lambda t}$	$\frac{1}{(s-\lambda)^2}$
$\frac{t^n e^{\lambda t}}{n!}$	$\frac{1}{(s-\lambda)^{n+1}}$
$t\sin(\omega t)$	$\frac{2s}{(s^2+\omega^2)^2}$
$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$