EE1-05

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE PART I: MEng, BEng and ACGI

ENERGY CONVERSION

Corrected Copy

Monday, 8 June 10:00 am

Time allowed: 2:00 hours

THREE

There are ~~TWO~~ questions on this paper.

Answer All questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : O. Sydoruk
Second Marker(s) : B.C. Pal

ENERGY CONVERSION

1. A uniformly charged slab has thickness a and is infinitely large in the other two directions. The volume charge density is ρ, see Figure 1.1.

Figure 1.1

a) What properties of the electric field can be deduced using symmetry considerations? Plot the electric field lines *outside* the slab. [5]

b) Find the electric field inside and outside the slab. Plot the variation of the field with the distance from the middle of the slab. [15]

c) Derive and analyse an expression for the voltage U_{AB} between point A and point B as shown in Figure 1.2. Point A is outside the slab, at a distance a from its surface. Point B is in the middle of the slab. The vertical distance between points A and B is a. [20]

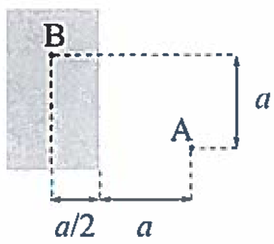

Figure 1.2

2. An infinitely long, thin, and straight conducting wire carries a dc current I.

a) Using Biot-Savart's law and symmetry considerations, discuss the form and the direction of the magnetic field lines. Sketch the field lines. [5]

b) Using Ampere's law, find the magnetic flux density B created by the current [10]

c) Using the Biot-Savart law, find the magnetic flux density B created by the current, i.e. derive the same result as in b). [15]

3. Figure 3.1 shows an equivalent circuit of an ideal transformer. The primary and the secondary coils have N_1 and N_2 turns, respectively.

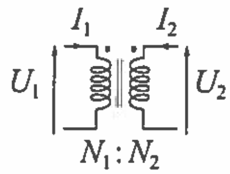

Figure 3.1

a) What simplifying assumptions are made about the flux and the magnetic field strength in the core of an ideal transformer? [5]

b) Derive the relationships between the voltages, U_1 and U_2, and the currents I_1 and I_2 of the ideal transformer of Figure 3.1 [10]

c) Figure 3.2 shows an equivalent circuit of a real transformer. Briefly explain the physical meaning of the components R_t, X_t, R_i, and X_m and the reasons why they are connected in series and in parallel with the transformer coil. [15]

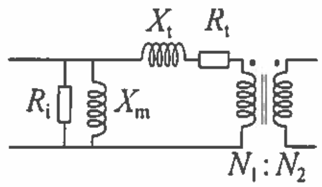

Figure 3.2

Formula sheet

Maxwell's equations in integral form

$$\oint_l (\mathbf{E} \cdot \mathrm{d}\mathbf{l}) = -\frac{\mathrm{d}}{\mathrm{d}t} \int_S (\mathbf{B} \cdot \mathrm{d}\mathbf{S})$$
$$\oint_l (\mathbf{H} \cdot \mathrm{d}\mathbf{l}) = \int_S (\mathbf{J} \cdot \mathrm{d}\mathbf{S}) + \frac{\mathrm{d}}{\mathrm{d}t} \int_S (\mathbf{D} \cdot \mathrm{d}\mathbf{S})$$
$$\oint_S (\mathbf{D} \cdot \mathrm{d}\mathbf{S}) = \int_V \rho \mathrm{d}V$$
$$\oint_S (\mathbf{B} \cdot \mathrm{d}\mathbf{S}) = 0$$

Gauss's law for electric fields in differential form, Cartesian coordinates

$$\varepsilon_0 \left(\frac{\partial E_x(x,y,z)}{\partial x} + \frac{\partial E_y(x,y,z)}{\partial y} + \frac{\partial E_z(x,y,z)}{\partial z} \right) = \rho(x,y,z)$$

Gauss's law for electric fields in differential form, centrosymmetric distributions, spherical coordinates

$$\frac{1}{r^2} \frac{\mathrm{d}}{\mathrm{d}r} (r^2 E(r)) = \frac{\rho(r)}{\varepsilon_0}$$

Electric flux density and field strength: $\mathbf{D} = \varepsilon_0 \varepsilon_d \mathbf{E}$. Magnetic flux density and field strength: $\mathbf{B} = \mu_0 \mu_r \mathbf{H}$.

Coulomb's law

$$\mathbf{F} = \frac{q_1 q_2}{4\pi \varepsilon_0 \varepsilon_d r^3} \mathbf{r}$$

The Biot-Savart law

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{[\mathrm{d}\mathbf{l} \times \mathbf{r}]}{r^3}$$

Voltage, potential

$$U_{\mathrm{AB}} = \varphi(\mathrm{A}) - \varphi(\mathrm{B}) = \int_{\mathrm{A}}^{\mathrm{B}} (\mathbf{E} \cdot \mathrm{d}\mathbf{l})$$

Electrostatic energy

$$W = \frac{1}{8\pi \varepsilon_0} \sum_{i \neq j} \frac{q_i q_j}{r_{ij}}$$

Capacitance: $C = q/U$. Inductance: $L = \Phi/I$. Force on a charge in electric field: $\mathbf{F} = q\mathbf{E}$; in magnetic field: $\mathbf{F} = q[\mathbf{v} \times \mathbf{B}]$.

Rotating machines. Torque, definition (force perpendicular to arm): $T = Fa$. Torque for a motor with N coils: $T = K\Phi I$, where $K = 2N/\pi$. Back-emf: $e = K\Phi\omega$.

Useful integrals

$$\int \frac{\mathrm{d}x}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{a^2 + x^2}) \qquad \int \frac{\mathrm{d}x}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}} \qquad \int \frac{x\mathrm{d}x}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$