UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2000

MEng Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C481

MODELS OF CONCURRENT COMPUTATION

Thursday 4 May 2000, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions

1a i) State the three τ-laws of CCS.

ii) State the Expansion Theorem for the parallel composition of a number of CCS processes.

b Let P,Q,R be CCS processes. Show by equational reasoning that

$$a.R + a.(Q + \tau.(P+\tau.R)) = a.(Q + \tau.(P+\tau.R))$$

Mention any laws you use.

c A scheduler S is specified by $S = a_1.a_2.a_3.S$. It is desired to implement S using a 'cycler' C, defined by

$$C(b,a,d) = b.C'\langle b,a,d\rangle$$
$$C'(b,a,d) = a.C''\langle b,a,d\rangle$$
$$C''(b,a,d) = \overline{d}.C\langle b,a,d\rangle$$

i) Show how to implement S by a process I built from three appropriately renamed cyclers. Include a flow diagram.

ii) Prove that I = S using the Expansion Theorem.

The three parts carry, respectively, 30%, 25%, 45% of the marks.

2a Define *weak bisimulation* and *weak equivalence* ($\approx$) for CCS processes.

b Let P, Q be CCS processes.

i) Show that $P + \tau.(Q+\tau.P) \approx Q + \tau.P$.

ii) State whether it is necessarily the case that $P + \tau.(Q+\tau.P) = Q + \tau.P$. Justify your answer.

c i) Define the satisfaction relation $\models$ for formulas of weak Hennessy-Milner Logic ('weak' means that τ-actions are hidden).

ii) What does it mean to say that weak Hennessy-Milner Logic *respects* weak equivalence? Sketch a proof that weak Hennessy-Milner Logic respects weak equivalence.

d Let $R = \tau.a + \tau.b$ and $S = \tau.a + \tau.b + \tau.(a + \tau.b)$. Find a Hennessy-Milner formula which shows that R and S are not weakly equivalent. Give a brief explanation with a transition diagram.

The four parts carry, respectively, 20%, 25%, 35%, 20% of the marks.

3a Define the operators $\Box$, $\sqcap$ and \ of CSP in the Failures Model, paying attention to alphabets, but ignoring divergence.

b Let P and Q be processes, and let a be an event. For each of the following two equational laws, state whether it is true or false in general. If it is true, prove it in the failures model; if it is false give a counterexample. We abbreviate P\{a} to P\a.

i) $(P \sqcap Q)\backslash a = P\backslash a \sqcap Q\backslash a$

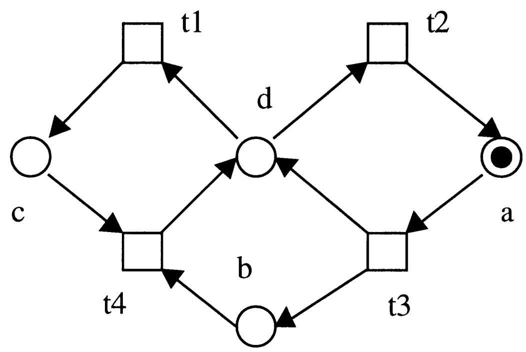
ii) $(P \Box Q)\backslash a = P\backslash a \Box Q\backslash a$

c A vending machine will take 10p coins (event *coin*) and will vend tea (event *tea*) for 30p or coffee (event *coffee*) for 40p. The machine continues indefinitely. As usual, drinks are not vended until sufficient coins have been inserted. Up to four coins can be inserted before a drink is vended. If tea is vended, if there is 10p credit remaining then this is immediately returned (event *return*) before more coins can be inserted.

Give a failures-style specification for the machine.

The three parts carry, respectively, 25%, 40%, 35% of the marks.

4a Consider the following marked Petri net M:



i) Draw a (possibly partial) transition diagram for M, following M's behaviour at least far enough to show the possible transitions.

Use your diagram to answer the following questions, with brief justification:

ii) Is M safe?

iii) Is M deterministic?

iv) Is M sequential?

v) Is M live?

vi) What is the set of reachable markings of M?

b i) Define an *event structure*.

ii) Sketch how to convert an event structure into a safe net. Explain why the net you construct is safe.

The two parts carry, respectively, 55%, 45% of the marks.

