IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE PART II: MEng, BEng and ACGI

CONTROL ENGINEERING

Corrected Copy

Wednesday, 3 June 2:00 pm

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : I.M. Jaimoukha
Second Marker(s) : S.A. Evangelou

1. a) Figure 1.1 below illustrates an RLC circuit with the standard interpretation of symbols. The input is $v_i(t)$ and the output is $q(t)$, the capacitor charge.

i) Derive the differential equation relating q to v_i. [5]

ii) Determine the transfer function relating q to v_i. [5]

iii) Let $L = 1\ H$ and let $v_i(t)$ be a unit step input. The following design specifications are required to be satisfied (S1): The capacitor charges to its steady–state value within approximately 10^{-3} seconds. (S2): The maximum overshoot of $q(t)$ is 5% of its steady–state value.

A. Derive the values of R and C so that the design specifications are satisfied. [5]

B. For these values of R and C, derive the steady state value of $q(t)$. [5]

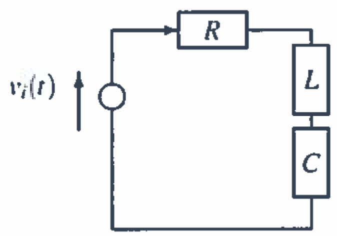

Figure 1.1

b) Figure 1.2 below illustrates an aircraft's autopilot where K_1 and K_2 are design parameters.

i) Derive the transfer function that relates the error signal to the reference signal. [5]

ii) Use the Routh-Hurwitz criterion to find the maximum value of K_1 (in terms of K_2) for closed–loop stability. [5]

iii) Calculate the steady-state error (in terms of K_1 and K_2) when the reference is a unit ramp function. [5]

iv) Let $K_2 = 1$. Use the answers above to find the minimum value of the achievable steady–state error when the reference is a unit ramp. [5]

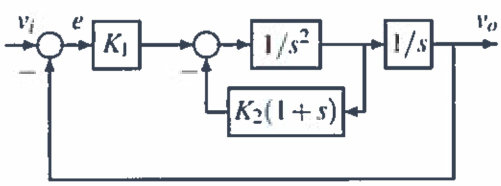

Figure 1.2

2. Consider the feedback control system in Figure 2.1 below.

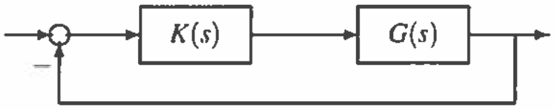

Figure 2.1

Here, $K(s)$ is the transfer function of a compensator while $G(s)$ is a stable transfer function with no finite zeros whose frequency response is shown in Figure 2.2.

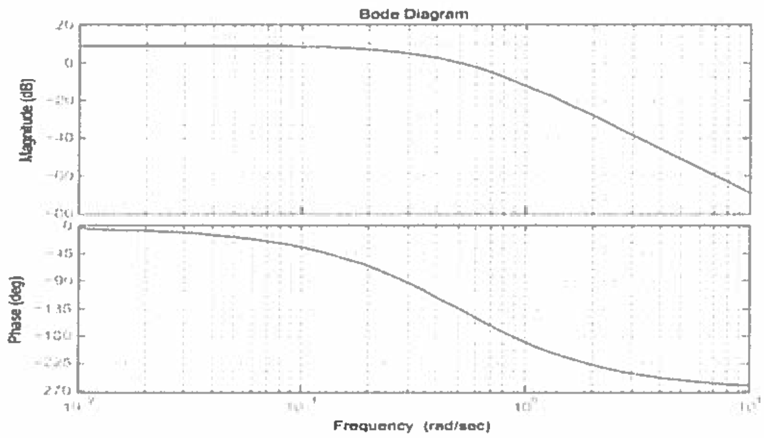

Figure 2.2

a) Use the frequency response to sketch a **rough** Nyquist diagram of $G(s)$, indicating the low and high frequency portions and the real–axis intercepts. [8]

b) Give approximate values for the gain and phase margins. [8]

c) Use the Nyquist stability criterion, which should be stated, to determine the number of unstable closed-loop poles when:

   i) $K(s) = 1$. [4]

   ii) $K(s) = 10$. [4]

d) Let $K(s)$ have the frequency response shown in Figure 2.3 overleaf. Describe $K(s)$ briefly and indicate its effects on the performance and stability of the feedback loop. [6]

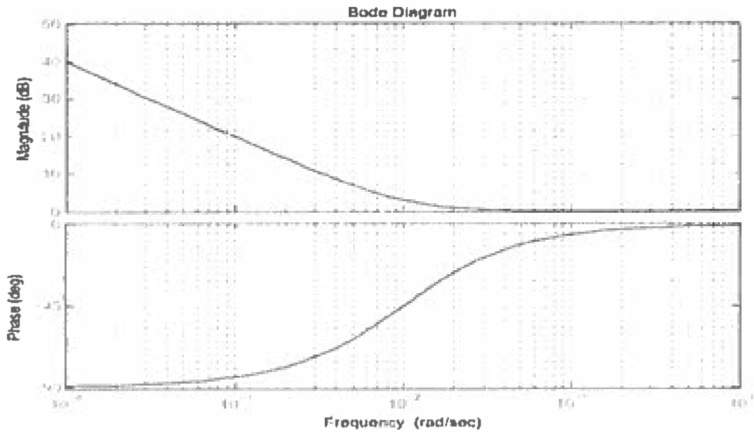

Figure 2.3

3. Consider the feedback loop shown in Figure 3.1 below. Here

$$G(s) = \frac{1}{(s+3)^3} = \frac{1}{s^3+9s^2+27s+27}.$$

It is required design a compensator $K(s)$ such that the following design specifications are satisfied:

- The settling time is approximately 2 seconds.
- The response is oscillatory with a maximum overshoot of approximately 5%.

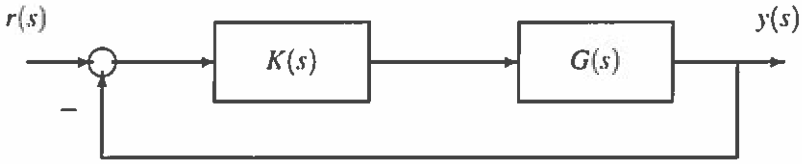

Figure 3.1

a) Find the location of the closed–loop poles that achieves the design specifications above. [3]

b) For this part, take $K(s) = k$ where $k > 0$.

i) Sketch the locus of the closed–loop poles as k varies from 0 to ∞. [5]

ii) Use the Routh–Hurwitz criterion to determine the range of values of k for which the closed–loop is stable. [5]

iii) Show that the design specifications cannot be achieved by any k. (*Hint:* $(1+j2)^3 = -11-j2$) [5]

c) For this part, take $K(s)$ to be a PD compensator.

i) Use the angle criterion to choose the compensator zero so that the compensated root–locus contains the location of the closed–loop poles that achieves the design specifications. (*Hint:* $\tan^{-1}(2) \approx 63.4^\circ$ and $\tan^{-1}(2/11) \approx 10.3^\circ$) [4]

ii) Sketch the root–locus of the compensated system. [4]

iii) Use the gain criterion to design $K(s)$. [4]