IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

EEE PART I: MEng, BEng and ACGI

Corrected Copy

MATHEMATICS 1A (E-STREAM AND I-STREAM)

Monday, 23 May 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Answer ALL questions. Question 1 is worth 40%. Questions 2-4 are each worth 20%.

NO CALCULATORS ALLOWED

Any special instructions for invigilators and information for candidates are on page 1.

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EE1-10A MATHEMATICS I

Information for Candidates:

Calculators are not permitted in this exam.

1. a) Express the following complex numbers in the form $x+iy$: [4]

$$\text{(i)}\, z = i^i, \quad \text{(ii)}\, z = \frac{3-i}{2+i}$$

b) Describe and sketch the locus of the complex number which satisifies
$|z+i| > |z-1|$. [4]

c) Obtain the limit: [4]

$$\lim_{x \to 0} \frac{x^2}{1-\cos x}.$$

[Do not use l'Hopital's rule.]

d) Obtain the limit by any method: [4]

$$\lim_{x \to \pi/3} \frac{\sqrt{3}-\tan x}{1+\cos(3x)}.$$

e) Differentiate: $y = (\ln x)^x$. [4]

f) Find $\frac{dy}{dx}$ when $xy + \ln(xy) = 2$. [4]

g) Integrate by substitution: $\int_0^1 \sqrt{1-x^2}\, dx$. [4]

h) Integrate: $\int_1^e \frac{(\ln x)^2}{x}\, dx$. [4]

i) Do the below series converge or diverge? Briefly justify your answer.

$$\text{(I)} \sum_{n=1}^{\infty} \frac{n}{3^n}, \qquad \text{(II)} \sum_{n=1}^{\infty} \frac{n!}{3^n}.$$ [4]

j) Define the odd function $f(t) = \begin{cases} -1 & (-\pi < t \leq 0) \\ 1 & (0 < t \leq \pi) \end{cases}$ with $f(t+2\pi) = f(t)$ for all t.

Show that the Fourier Series for $f(t)$ is $\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)t]$.
[4]

2. a) Find the value of q for which the limit

$$\lim_{x\to\infty} x^q(\sqrt{x-1}-\sqrt{x})$$

exists and is non-zero. Find the limit. [4]

b) Evaluate the integral $\int \frac{\cosh^{-1}(x)}{\sqrt{x^2-1}}\,dx$. [4]

c) Let $I_n = \int_0^{\pi} \cos^n(x)\,dx, \quad n = 0, 1, 2, \ldots$.

i) Integrate by parts to show that $I_n = \frac{n-1}{n} I_{n-2}, \quad$ for $n \geq 2$. [6]

ii) Using the formula from (i), or otherwise, obtain I_6. [2]

d) Differentiate y to obtain the derivative $\frac{dy}{dx}$ for the function

$$y = \ln\left(x + \sqrt{x^2+1}\right)$$

and simplify as much as possible. [4]

3. a) Given the function $f(x) = \frac{1}{1+\sin x}$

i) Find $\int f(x)\,dx$ [4]

ii) Using (i) or otherwise, find the area between $f(x)$ and the x–axis from $x = 0$ to $x = \pi$, and sketch $f(x)$ on the interval $[0, \pi]$. [4]

b) Use De Moivre's Theorem to show that

$$\sin 5\theta = A\sin\theta + B\sin^3\theta + C\sin^5\theta\,,$$

giving the values of A, B, and C. [4]

c) i) Show that $\ln(1+x)$ has Maclaurin series

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + R_4$$

and find the Lagrange remainder R_4 [5]

ii) Show that the radius of convergence for the Maclaurin series in (i) is 1. [3]

4. a) The function $f(x)$ is periodic, with period $T = 2$, and is an even function of x. In the interval $0 < x \leq 1$ the function has the value

$$f(x) = x - 1\,, \qquad 0 \leq x < 1$$

.

i) Sketch the graph of $f(x)$ for $-3 < x < 3$. [2]

ii) Show that the Fourier Series for $f(x)$ is

$$f(x) = -\frac{1}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2m-1)^2}\cos[(2m-1)\pi x]\,.$$

[6]

iii) Using the results of (ii) and Parseval's theorem, show that

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4}\cdots = \frac{\pi^4}{90}\,.$$

[6]

b) Obtain the n^{th} derivative of the functions

$$(i)\; e^{-2x}\,, \qquad (ii)\; x\ln(x)\,,$$

where $n > 1$. [Simplify the answer as much as possible.] [6]