UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the
Associateship of the Royal College of Science

PAPER C343=I3.22

OPERATIONS RESEARCH

Friday 30 April 2004, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 Consider the problem

$$\max x_0 = x_1 + 9x_2 + x_3$$

subject to

$$\begin{aligned} -x_1 - 2x_2 - 3x_3 &\geq -9 \\ 3x_1 + 2x_2 + 2x_3 &\leq 15 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

a Write this problem as a standard linear programming problem and solve it using the simplex method.

b Write the dual of this problem.

c Given the linear programming problem:

$$\max v$$

subject to:

$$\begin{aligned} v &\leq \sum_{i=1}^{n} a_i x_i \\ 1 &= \sum_{i=1}^{n} x_i \\ x_i &\geq 0 \text{ for } i = 1, \ldots, n \end{aligned}$$

Show that adding a constant c to every element a_i does not change the optimal value of x_i for $i = 1, \ldots, n$ and that the optimal value of v is shifted as a result of this addition.

The three parts carry, respectively, 35%, 30%, and 35% of the marks.

2 a Consider the linear programming problem:

$$v(b) = \min\{c^T x \mid Ax = b, x \geq 0\}$$

where $x, b, c \in \mathbb{R}^n$, $A \in \mathbb{R}^m \times \mathbb{R}^n$ and $v(b)$ denotes the solution given the right hand side b. Define the shadow price vector Π for this problem.

Let the matrix $B \in \mathbb{R}^m \times \mathbb{R}^m$ be the optimal basis matrix for the above problem. If instead of b we are given the vector r such that $B^{-1}r \geq 0$, show that

$$v(r) = v(b) + \Pi^T(r - b).$$

b Given the primal linear program

$$\max\{c^T x \mid Ax \leq b, x \geq 0\}$$

write its dual with the dual variable denoted by y,

Show that if x is feasible with respect to the primal problem and y is feasible with respect to the dual then

$$c^T x \leq b^T y.$$

The two parts carry, respectively, 50% and 50% of the marks.

3 Consider the following reward matrix for a two-person zero-sum game:

Row Player	Column Player S_1	S_2	S_3
S_1	0	-1	$+1$
S_2	$+1$	0	-1
S_3	-1	$+1$	0

Assume that each player chooses a strategy that enables him to do the best he can, given that the opponent knows the strategy he is following.

a Determine the strategies leading to the row minima and column maxima and determine whether or not the game has a saddle point.

b Introduce randomised mixed strategies and formulate the row player's decision problem as an LP and the column player's decision problem as an LP.

c Discuss briefly the relationship between the column and row players' LPs.

The three parts carry, respectively, 35%, 35%, and 30% of the marks.

4 a Consider the integer programming problem

$$\max x_0 = 9x_1 + 6x_2 + 5x_3$$

subject to:

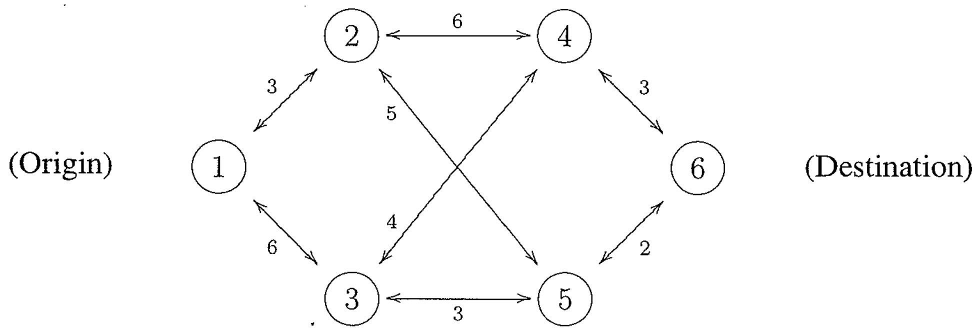
$$\begin{aligned} 2x_1 + 3x_2 + 7x_3 &\leq \tfrac{35}{2} \\ 4x_1 \qquad\quad + 9x_3 &\leq 15 \end{aligned}$$

$$x_1, x_2, x_3 \geq 0 \text{ and } x_1 \text{ integer}$$

Solve this problem using the branch–and–bound technique of integer programming.

[Hint: If the above problem is solved, without the integer restriction on x_1, as a linear program, we have the solution $x_1 = 15/4, x_2 = 10/2, x_3 = 0$ and $x_0 = 215/4$.]

b Consider the shortest route problem from 1 to 6 in the diagram below. The figures above the arcs are the distances and the only routes considered always move one column at a time.



Formulate the shortest route problem as a 0-1 integer programming problem by using mutually exclusive alternatives and contingent decisions. (You are not required to solve the integer programming problem.)

[Hint: Introduce the variables

$$x_{ij} = \begin{cases} 1 & \text{if arc from } i \text{ to } j \text{ is used in the shortest route} \\ 0 & \text{otherwise.]} \end{cases}$$

The two parts carry, respectively, 50% and 50% of the marks.