UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2000

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BEng Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C343=I3.22

OPERATIONS RESEARCH

Wednesday 3 May 2000, 14:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions

1 A tool manufacturing company TOOL.COM produces three types of tools, T1, T2, and T3. These tools use two raw materials, M1 and M2. In particular, each tool of type T1 requires 3 units of M1 and 5 units of M2, each tool of type T2 requires 5 units of M1 and 3 units of M2, and each tool of type T3 requires 6 units of M1 and 4 units of M2.

The daily availability of raw materials is 1000 units for M1 and 1200 units for M2. The manager in charge of production was informed by the marketing department that according to their research, the daily (combined) demand for all three tools (together) must be at least 500 units.

a Model the production and marketing constraints by using linear programming.

b Formulate (but do not solve) a standard-form linear program whose optimum gives the largest possible combined daily production of the three tools satisfying the resource constraints on the raw materials.

c Would the manufacturing department be able to satisfy the demand (of the marketing department), and why? [Hint: If you knew the optimum for the previous linear program, you could immediately decide on this. However, in order to answer the question, you don't have to solve the linear program - and you are not expected to do so - there is a much simpler way to find the answer.]

d State the Fundamental Theorem of Linear Programming.

(The four parts carry, respectively, 30%, 30%, 20% and 20% of the marks).

2 Consider the following linear program:

$$\begin{array}{rl} \text{Min} & 3x_1 + 2x_2 \\ \text{s.t.} & 2x_1 + \ x_2 \ \leq \ 2, \\ & 3x_1 + 4x_2 \ \geq \ 12, \\ & x_1 \ \geq \ 0, \ x_2 \ \geq \ 0. \end{array}$$

a Express this linear program in standard form.

b Solve it by using the simplex method and interpret the result.

c Formulate the corresponding dual linear program and discuss if it can have a finite optimum.

(The three parts carry, respectively, 20%, 40% and 40% of the marks).

3 a Consider the optimisation problem

$$\underset{x}{\text{minimise}} \left\{ c^T x \;\middle|\; a_i^T x \leq b_i \; ; \; i = 1, \ldots, m; \right.$$

$$\text{any k } (< \text{m}) \text{ of these m constraints must be satisfied;}$$

$$\left. \alpha^T x = \beta \; ; \; \beta = \beta_1 \text{ or } \beta_2 \text{ or } \beta_3 \; ; \; x \geq 0 \right\},$$

where c, a_i, α , x are vectors of appropriate dimension and b_i, β are scalars. Formulate this problem as a mixed integer program.

b Consider the following optimal tableau of the continuous relaxation of a integer maximisation problem with variables x_i , i = 1, ..., 6 required to be nonnegative integer variables:

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_0	0	0	0	2	2	2	30
x_1	1	0	0	3/10	1/5	0	5/2
x_2	0	1	0	1/20	1/5	0	5/4
x_3	0	0	1	1/4	0	1	25/4

Derive a Gomory cut based on the x_1 - row and state fully (but do not solve) the next problem that needs to be solved by the cutting plane algorithm. Discuss the termination condition for the cutting plane algorithm.

c If the Branch-and-Bound algorithm is applied to solve the problem in (b) above, describe the next step of the algorithm. Do not solve the problem. Discuss the termination condition(s) for the search down each branch in this algorithm.

(All parts carry equal marks)

4 Consider the following game: two armies (A and B) are advancing on two cities. A has four regiments and B has three regiments. At each city, the army that sends more regiments to the city captures both the city and the opposing army's regiments. If both A and B send the same number of regiments to a city, the battle at the city is a draw. Each army scores 1 point per city captured and 1 point per captured regiment. Each army wishes to maximise the difference between their reward and their opponent's reward.

a Formulate the reward matrix for this game and check (giving reasons) for the existence of a saddle point. [Hint: let strategy (i, j) for each player indicate that player sends i regiments to city 1 and j regiments to city 2.]

b Formulate (but do not solve) the problem each player needs to solve to determine the optimal strategy for each army.

(All parts carry equal marks)