## Sustainable Electrical Systems 2017-2018

**Important notes:**

There were five questions in this exam paper. Students were expected to answer only four of them.

The model solutions are presented in this document. Notes about how the students solved the questions were added during marking where relevant. These *a-posteriori* notes are highlighted in green letters.

### Q1. A question about wind power generation

a) The plots in figures Q1.1a and Q1.1b show the output power and the rotational speed of a wind turbine for a range of wind speeds. The diameter of the turbine is $D$=50m and the density of the air is approximately $\rho$ =1.225kg·m$^{-3}$. Answer the questions described below.

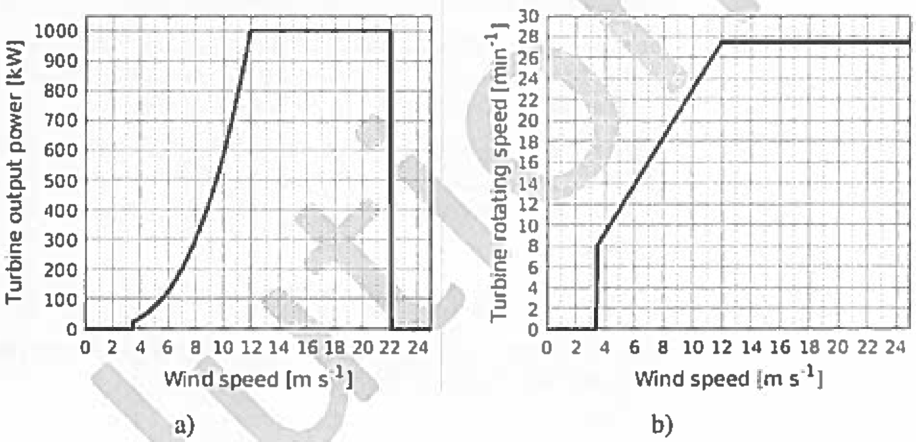



Figure Q1.1- Characteristic power and speed curves of a wind turbine.

i) Calculate the power coefficient of operation, $C_P$, for the wind speeds, $v_w$, of 2 m·s$^{-1}$, 6 m·s$^{-1}$, 10 m·s$^{-1}$, 14 m·s$^{-1}$ and 18 m·s$^{-1}$. Sketch the curve of power coefficient against wind speed compared to the maximum $C_P$ predicted by Betz. [4]

ii) Sketch the curve of turbine torque against wind speed. [2]

iii) Calculate the tip-speed ratio of operation for a wind speed of $v_w$=8 m·s$^{-1}$. [1]

iv) Calculate the transformation ratio of the gearbox of the turbine if the electrical generator is designed to turn at 1,500 min$^{-1}$ when operating at full power. [1]

b) The graph in Figure Q1.2 shows the cumulative probability distribution of the wind speed for a site where a wind turbine is installed. Answer the questions described below.

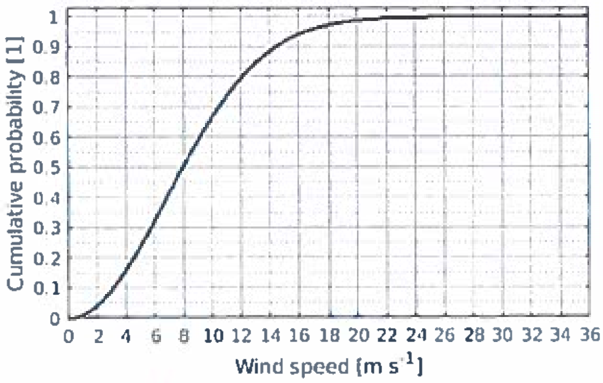


Figure Q1.2- Cumulative probability distribution of the wind speed.

i) Calculate the probability of the wind speed being within the MPPT region of the wind turbine, which in this case is between 4 m·s$^{-1}$ and 10 m·s$^{-1}$. [2]

ii) Calculate the average wind speed of this site. [1]

iii) Generate three simulated wind speed data samples using the following random samples obtained from a uniform probability distribution between 0 and 1: $s_1$=0.7, $s_2$=0.1, $s_3$=0.8. [1]

iv) Wind turbines have an MPPT region of operation and a power reduction region. Describe these two modes of operation briefly and explain why wind turbines are designed to have these two modes of operation. [4]

c) Answer the following question regarding wind generation technologies:

i) Describe (briefly) the DFIG wind turbine topology and the PMSG wind turbine. Highlight their pros and cons. [4]

**Solution to Q1**

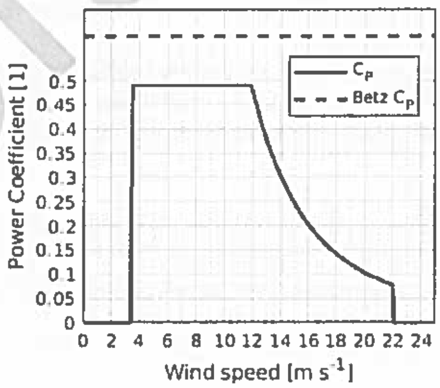
a.i) The power coefficient, $C_P$, is defined as the ratio between the power extracted by the turbine and the available power, $P_0$, which in turn is defined as:

$$P_0 = \frac{1}{2}\rho A v_w^3$$

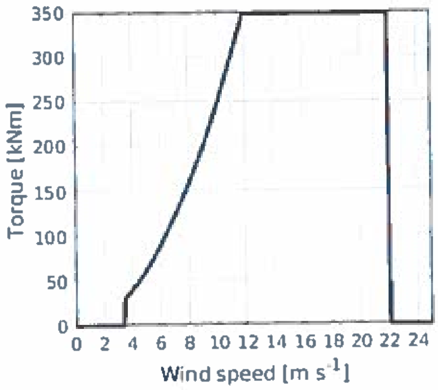
where $\rho$ is the density of the air and A is the area swept by the turbine blades. Substituting for the wind speeds in the question we obtain the following:

| Wind speed [m/s] | Power coefficient [1] |
|---|---|
| 2 | 0 |
| 6 | 0.49 |
| 10 | 0.49 |
| 14 | 0.3 |
| 18 | 0.14 |

On the other hand, the maximum possible power coefficient according to Betz is 16/27 which gives about 60% (0.593). Students should identify that the wind turbine in the example case has the typical two regions of operation: an MPPT region, where $C_P$ is maximised, and a power reduction region where $C_P$ decreases at a rate of $1/v_w^3$ in order to keep the power yield equal to the maximum rated power. This enables to sketch the curve of $C_P$ against wind speed:




a.ii) The torque can be calculated by dividing the turbine power by the turbine speed. The torque will be zero in the regions where no power is generated, it will follow a quadratic function in the MPPT region and it will be constant in the power reduction region:




a.iii) The tip speed ratio, $\lambda$, is defined as:

$$\lambda = \frac{\omega R}{v_w}$$

where is $\omega$ the rotational speed of the turbine and $R$ is the radius of the turbine. The turbine in the exercise has a radius of 25 m and according to the plot of the rotational speed, it will operate at 18 min$^{-1}$ for a wind speed of 8 m·s$^{-1}$. Substituting these values into the equation gives a tip speed ratio of 5.89.

a.iv) The rotational speed of the turbine at full power is about 27.5 min$^{-1}$, therefore a gearbox with a transformation ratio of 55 (1,500 divided by 27) is needed in order to couple the turbine to the generator.

b.i) The cumulative probability distribution of the wind speed, $\Phi(v_w)$ , gives us the probability of the wind speed being lower than $v_w$. The probability of the wind speed being between 10 m·s$^{-1}$ and 4 m·s$^{-1}$ will be the probability of the wind speed being lower than 10 m·s$^{-1}$minus the probability of the wind speed being lower than 4 m·s$^{-1}$, this gives:

$$p = \Phi(10\ \mathrm{m\ s^{-1}}) - \Phi(4\ \ \mathrm{m\ s^{-1}}) = 0.65 - 0.15 = 0.5 = 50\%$$

b.ii) The average of the wind speed can be found by looking at the wind speed that gives a 50% of probability of the wind speed being lower (and therefore 50% of probability of being higher). According to the curve of the cumulative distribution function this happens for a wind speed of $v_w$=8 m·s$^{-1}$.

b.iii) In this question students are asked to use the inverse transform sampling, where random samples of a given distribution are generated by evaluating the inverse of the cumulative distribution function on a series of samples obtained from a uniform distribution between 0 and 1. Here the values given in the question have to be checked in the vertical axis of the plot of the cumulative distribution in order to find the corresponding values of wind speed. This gives:

| Sample from the uniform distribution [1] | Wind speed [m/s] |
|---|---|
| 0.7 | 10 |
| 0.1 | 3 |

| | |
|---|---|
| 0.8 | 12 |

b.iv) BOOKWORK. The kinetic energy of an airflow depends on the cube of the wind speed. Therefore, high wind speed leads to high kinetic power available. However, high wind speeds are rare and the energy available over long term due to high wind speeds is small. Being able to obtain optimal power at high wind speeds requires sizing the turbine to withstand high torque and high rotational speed. This proves uneconomical and wind turbines are designed to limit the power they extract from the wind above a certain wind speed threshold. A trade-off between the cost of the turbine and its efficiency to generate power at high wind speeds must be found. Most wind turbine manufacturers offer different types of wind turbines optimised for sites with different wind speed characteristics.

Under the MPPT region, the turbine keeps its blade pitch angle at a fixed position and it controls the torque of the generator in order to make the turbine stabilise around the maximum power point of operation where the yield is maximised. At high wind speeds, when the yield that would correspond to the MPP would exceed the ratings of the turbine, the turbine is kept operating at its maximum rotational speed and torque and the blade pitch angle is adjusted in order to reduce the power coefficient and limit the power intake. Different methods of power reduction can be used alternatively: pitch the feather enables precise regulation of the power while pitch to stall enables a rapid reduction of the yield during sudden wind gusts.

c.i) BOOKWORK. The DFIG consists of a wound-rotor induction generator connected to the turbine shaft through a gearbox with a large transformation ratio. The stator of the machine is connected to the collection grid through a step-up transformer whereas the rotor is connected to a back-to-back converter that is used to control the torque of the generator. The converter is rated ~30% of the rated power of the turbine and the direction of its power flow depends on whether the DFIG operates at sub-synchronous or super-synchronous speed.

The PMSG uses a synchronous generator with permanent magnets as the source of excitation. The generator can be designed to have a greater number of pairs of poles than the DFIG in order to reduce its operating speed and enable its connection to the turbine shaft with a smaller gearbox or even without a gearbox. The stator of the machine is connected to the network through a fully-rated back-to-back converter.

The DFIG enables variable speed operation using a power converter that handles only a fraction of the power. This reduces the cost of the converter and its energy losses. On the other hand it makes the machine more vulnerable to disturbances in the network (voltage unbalance, voltage dips, etc). The PMSG enables a generator with a simpler gearbox or no gearbox; gearboxes were identified as a concern in terms of maintenance cost and availability (the gearbox is hard to replace in situ). Synchronous machines can have higher efficiency than induction machines and by using magnets rather than a wound rotor configuration with slip rings, maintenance can be reduced. Also, a full-rated power converter gives greater freedom to provide services to the network to mitigate grid-integration issues caused by renewables.

**Q2. A question about solar photovoltaic (PV) generation**

a) The characteristic curve of output power against terminal voltage of a PV cell exposed to an irradiation of 13.6 W is shown in Figure Q2.1. Answer the questions listed below.

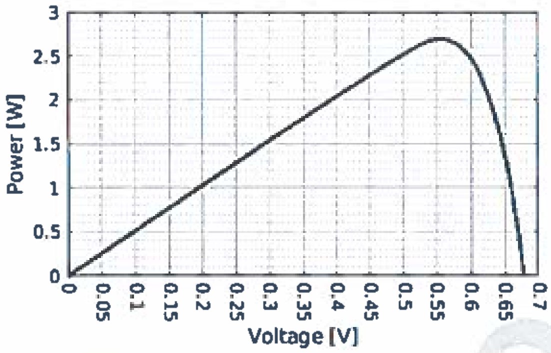



Figure Q2.1 – Output power against terminal voltage of a PV cell under an irradiation of 13.6 W.

i) Calculate the efficiency of the PV panel when operating at the Maximum Power Point (MPP). [1]

ii) Calculate the load resistance we would need to connect to the PV cell in order to extract the maximum energy at the given irradiation. [1]

iii) Estimate the short circuit current of the PV cell when operating at the given irradiation. [1]

iv) Describe briefly how a basic perturb-and-observe MPPT algorithm for PV works. [2]

b) The illustrations in Figure Q2.2 show the shadow cast by an identical vertical pole in two different locations at the same time of the day and on the same day of the year. Answer the questions listed below.


Case 1) 11 Sept, 12 noon, Brussels (Belgium)

Case 2) 11 Sept, 12 noon, Lagos (Nigeria)

Figure Q2.2- Shadow cast by a vertical pole in two different locations (data: $a$=2.5m, $b$=2.7m).

i) Estimate the approximate elevation angle of the sun in both cases. [1]

ii) Estimate the approximate AM number in both cases. Explain the meaning of the AM number briefly. [3]

iii) We are told that the direct irradiance over a surface that is perfectly perpendicular to the sun rays in Case 1 is of 200 $W \cdot m^{-2}$. Calculate the total direct irradiance (W) a PV panel of 2 $m^2$ would receive if it was held horizontally on the ground [2]

iv) Explain briefly what an equinox is. [1]

c) Answer the following questions related to PV technology:

i) A PV generation array with 16 PV panels is to be built. Give an overview of how these would be connected if two different array concepts were used: (a) an array of 4 strings of 4 PV panels with a centralised inverter and (b) an array of 16 PV panel with individual microinverters per each panel. Highlight the potential pros and cons of each of the two configurations. [3]

ii) Draw an approximate characteristic curve of voltage against current of a PV cell and explain how the curve would change if the irradiation of the panel was halved (50%). Give an approximate estimate of how the power yield would change if we know that in both cases the PV cell was operating at the same output voltage, which was lower than the MPP voltage. [3]

iii) Explain briefly what the problem of having two PV cells connected in parallel is and how the problem can be mitigated. [2]

## Solution to Q2

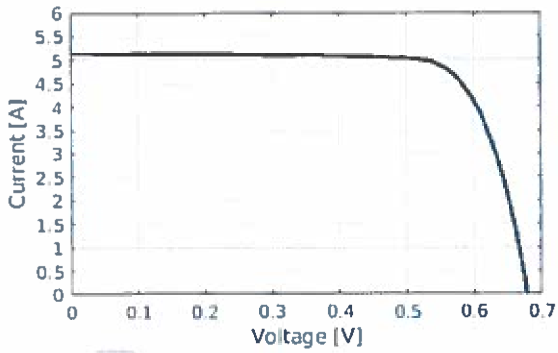
a.i) The coordinates of the MPP can be found by inspecting the graph and they give about $P_{PV}$=2.7 W, the irradiation is $P_{SUN}$=13.6 W, therefore the efficiency is:

$$\eta = \frac{P_{PV}}{P_{SUN}} = 19.9\ \%$$

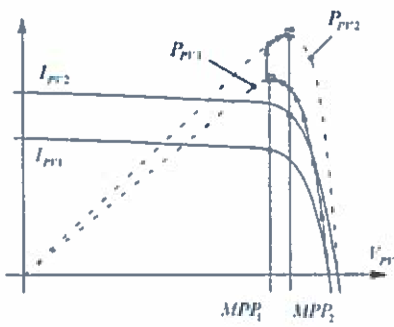
a.ii) The voltage at the MPP is about $V$=0.55 V. Therefore the resistance required would be:

$$R = \frac{V^2}{P_{PV}} = 0.11\ \Omega$$

a.iii) The output power of the PV panel is the product between the terminal voltage, $V$, and the output current, $I$. A plot of $V$ against $I$ could be obtained from the power curve given in the description of the question and from there the short circuit current would be found as the crossing point of the curve with the vertical axis at $V$=0. Alternatively, by checking a couple of points close to the V=0 it is easy to see that the current follows an asymptote towards $I$=5 A.




a.iv) BOOKWORK. A basic perturb-and-observe for MPPT works by changing the voltage applied to the PV curve iteratively at following incremental or decremental steps depending on the variation of the power observed when the change takes place. The process typically starts at the open circuit voltage and the voltage of the cell is reduced step by step until a further decrement of voltage causes the power to decrease rather than causing it to increase. When this situation take place, the direction of variation of voltage is changed and the voltage is then increased until a further increasing step causes the power to decrement. This behaviour will typically cause a sustained oscillation around the MPP.



b.i) The elevation angle is the angle between a horizontal surface parallel to the ground and the incoming sunrays. For case 1) the tangent of the angle can be found by dividing the height of the stick by the length of its shadow; by calculating the arctangent it is found that the angle is of $\gamma=43°$. For case 2), No shadow is cast because sunrays are perpendicular to the ground, which implies that the elevation angle is of $\gamma=90°$.

b.ii) BOOKWORK. The AM or Air Mass number quantifies the distance sunrays have to travel across the atmosphere before hitting the ground. The number is normally expressed as a multiple of the thickness of the atmosphere; an AM of 1 implies that the sunrays are perpendicular to the ground. The atmosphere greatly affects the characteristics of the irradiation that reaches the ground, a greater AM number generally implies less irradiation seen at ground level.

The AM number can be calculated approximately if the elevation angle, $\gamma$, is known. The equation is:

$$AM \approx \frac{1}{\sin(\gamma)}$$

For case 1, the AM is 1.47, whereas the AM for case 2 is of 1.

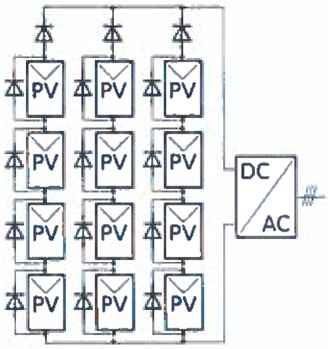
b.iii) Direct irradiation is dependent on the relative angle, $\alpha$, between the surface's normal vector and the sunrays. The actual irradiation can be calculated as a projection that depends on the cosine of the angle:

$$P_{irrad} = P \cdot \sin(\alpha)$$

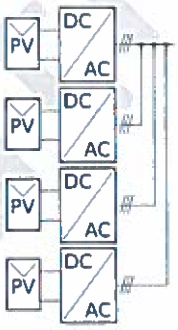
In this case $\alpha$ is equal to the elevation angle, therefore the irradiation per unit of exposed surface gives $P_{irrad}$=136 W·m$^{-2}$. We are told that the PV panel has a surface of 2 m$^2$, therefore the total irradiation is of 272 W.

b.iv) BOOKWORK. An equinox is a time of the year when the sun is located on the plane of the equator of the earth. This situation happens two times a year, which gives the autumnal equinox (late September) and the vernal equinox (late March). In the equinoxes, the duration of the day and the night are equal (12 hours) everywhere in the world.

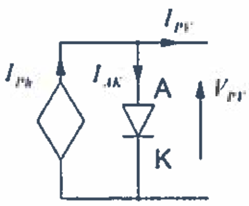
c.i) BOOKWORK. The centralised inverter arrangement would involve connecting groups of 4 PV panels in series to form strings that would then be connected in parallel and fed to a large inverter. String diodes would have to be connected in series with each string in order to avoid current being fed back from the strings with the highest voltage to the rest, a situation that can happen if there's mismatch between the temperatures of the cells in the different strings. The potential advantage of this arrangement is that a single large inverter can be comparatively cheaper and of better efficiency than separate smaller inverters. A simplified diagram with 3 strings of 4 panels (rather than 4 strings) is shown below:



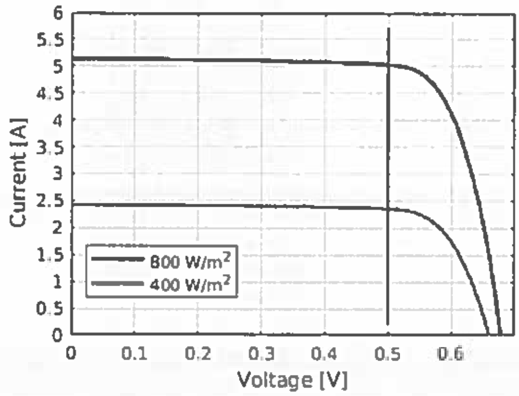

The microinverter arrangement implies that each individual PV panel would have its own inverter and the outputs of all inverters would be connected in parallel. This arrangement has the benefit that individual MPPT control can be done in each panel, which has the potential to maximise the yield of the PV panels when the panels are exposed to different irradiation (e.g. nearby shadows, clouds, etc) or when the amount of dust over the PV panels is different. On the other hand small inverters are comparatively more expensive and potentially more lossy. Further, with many more components, failures are likely to happen more often. The implications of this are not that obvious because failure rates will depend on multiple factors and their impact will be different. For instance, the failure of a single microinverter will imply the loss of 1/16 of generation, whereas the failure of the big inverter means the loss of the entire array. A simplified diagram of a microinverter arrangement (with 4 PV panels rather than 16) is shown below:




c.ii) BOOKWORK. The characteristic curve of a PV cell can be explained using a simplified equivalent model which consists of a current source that is proportional to the irradiation over the cell and a diode in positive bias connected in parallel with the output terminals of the cell. The voltage across the cell depends on the fraction of the current that flows through the diode:

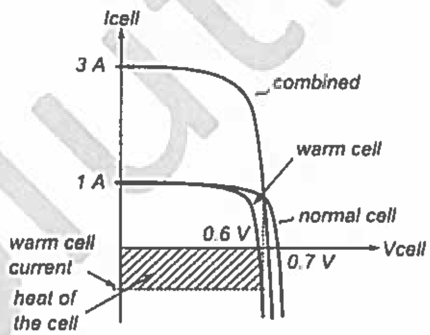



The effect of changing the irradiation of the panel produces approximately a vertical scaling of the $V$ versus $I$ curve as shown below:

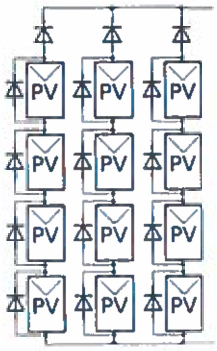



The region of the V-I curve on the left of the MPP point is of approximately constant current that is approximately equal to the short circuit current. If the cell receives half of the previous irradiation, its short circuit current will be approximately half of what it used to be. Therefore, if the cell is operated at the same voltage, the output power will be halved approximately.

c.iii) BOOKWORK. When two cells are connected in parallel, their voltages will be the same. If two identical PV cells are connected together and both cells are exposed to the same irradiation and temperature, their output currents will be the same. If their irradiation is different, their output currents will differ. This is not a major problem; however, if the temperature of the cells is different, it can happen that one of the cells is operated at a region where its output current is negative. This would effectively cause one of the PV panels (the one at the lowest temperature) to feed current to the other PV panel:




This problem isn't a great concern for two individual cells because the power the cell would sink would be small, but it can become a greater problem when a large string of cells is connected in parallel with another string of cells and more power is available. The way to prevent this problem when multiple strings are needed in parallel is to connect a string diode in series with the string, which would prevent the current of the string from changing direction as shown below:




The main drawback of this solution is that string diodes will cause a voltage drop and waste energy, which causes a small drop of the overall efficiency of the PV panel or the collection of PV panels.

Q3. **A question about distributed generation, voltage control and security of supply**

a) A network planner is considering the connection of a 10 MW wind farm. Two possible connection options are investigated: $\boxed{1}$ the connection of the wind farm to an 11 kV feeder supplied by a 33/11 kV substation and $\boxed{2}$ the connection to a 33 kV feeder supplied by a 132/33 kV substation. In both cases, the On Load Tap Changing (OLTC) transformer is set to maintain the voltage at busbar A at constant 1.03 p.u.(see Figure Q3.1). The allowed voltage fluctuations in both cases are ±6%. The wind farm power factor at full power output is 0.95.

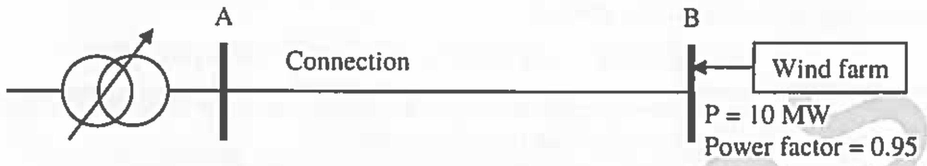



Figure Q3.1- Wind farm connection (same diagram for both options $\boxed{1}$ and $\boxed{2}$).

i) Considering the connection scheme presented in Figure 3.1, determine the maximum possible connection length if the wind farm for both option ($\boxed{1}$) and ($\boxed{2}$). The capacity of the 11 kV circuit is of 810 A and both resistivity and reactivity are of 0.06 Ω per km. The capacity of the 33 kV circuit is of 1,208 A, the resistivity is of 0.046 Ω per km and the reactivity is of 0.101 Ω per km. [6]

ii) Explain the advantages and disadvantages of connecting this wind farm to 11kV ($\boxed{1}$) or 33kV ($\boxed{2}$). [4]

b) Answer the following questions related to the security of supply:

i) Explain the importance of considering the contribution of non-network solutions to network security within network security standards. [2]

ii) Explain briefly the fundamental approach used to quantify the contribution of distributed generation to network security. State one strength and one weakness of the present approach used to quantify the security contribution of distributed generation. [3]

iii) The contribution to security of supply of two identical distributed generation units is 75%. Determine the availability of the distributed generation units considering the normalised load duration curve shown in Figure Q3.2. [5]

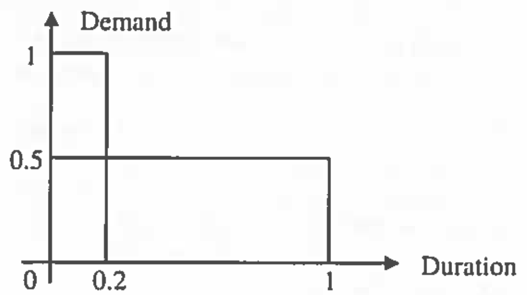
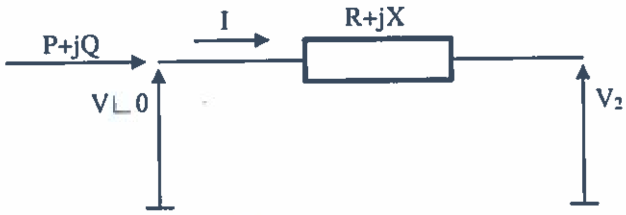



Figure Q3.2- Normalised load duration curve.

**Solution to Q3**

a.i) The following diagram shows the equivalent circuit of the cable:




The voltage across $R$ and $X$ is given by:

$$\overrightarrow{\Delta V} = \sqrt{3} \cdot I \cdot (R + jX) = \sqrt{3} \cdot \frac{P - jQ}{\sqrt{3} \cdot V} \cdot (R + jX)$$

Given that $V$ is used as the reference for angles (its angle is zero), the conjugate value is also $V$. Multiplying the apparent power and impedance gives:

$$\overrightarrow{\Delta V} = \frac{P \cdot R + Q \cdot X + j(P \cdot X - Q \cdot R)}{V}$$

The voltage rise is approximately given by the formula:

$$\Delta V = |\vec{V}| - |\vec{V_2}| \approx \frac{R \cdot P + X \cdot Q}{V}$$

It should be noted that typically:

$$\Delta V = |\vec{V}| - |\vec{V_2}| \neq |\overrightarrow{\Delta V}| = |\vec{V} - \vec{V_2}|$$

In the case of the 11kV connection, the maximum allowed voltage at busbar B is 1.06 and the formula can be written as a function of the length of the cable as:

$$\Delta V = V_B - V_A \approx \frac{r \cdot L \cdot P + x \cdot L \cdot P \cdot \tan(\cos^{-1} 0.95)}{V_B}$$

$$\mathrm{L} = \frac{V_B \cdot (V_B - V_A)}{P \cdot (r + x \cdot \tan(\cos^{-1} 0.95))}$$

$$\mathrm{L_{max}} = \frac{1.06 \cdot 11 \cdot (1.06 \cdot 11 - 1.03 \cdot 11)}{10 \cdot (0.06 + 0.06 \cdot \tan(\cos^{-1} 0.95))} = 4.83\ km$$

The rating of the cable is greater than the installed wind farm capacity:

$$S_r = \sqrt{3} \cdot 11 \cdot \frac{810}{1000} = 15.4\ MVA > \frac{10}{0.95} = 10.5\ MVA$$

The maximum possible length of the wind farm connection for the 11 kV network, $\boxed{1}$, is 4.83 km.

In the case of 33kV, $\boxed{2}$:

$$\mathrm{L_{max}} = \frac{1.1 \cdot 33 \cdot (1.1 \cdot 33 - 1.03 \cdot 33)}{10 \cdot (0.046 + 0.101 \cdot \tan(\cos^{-1} 0.95))} = 105.88\ km$$

The maximum length of wind farm connection on 33 kV network is almost 106 km. Furthermore, it is obvious that the rating of the cable is satisfactory.

a.ii) [BOOKWORK] Connecting to 11 kV network is generally less costly than connecting to higher voltages. Furthermore, potential for losses reduction in upstream network, by supplying local demand, is greater. However, potential size of 11 kV connected wind farm, limited by voltage rise effect and thermal capacity of 11 kV circuit, is lower than if connected at 33 kV. Hence, benefit of 33 kV connected wind farm is potential greater wind farm capacity and/or longer distance connection.

b.i) [BOOKWORK] Non-network technologies may provide economic solution in providing security of supply and eliminate network reinforcement. Network investment could be deferred until demand development becomes more certain potentially avoiding network overinvestment.

b.ii) [BOOKWORK] The contribution of distributed generation to network security, at present, is calculated as if it was connected to an equivalent ideal circuit that never failed. Relevant demand is scaled such that peak demand is the same as the capacity of the distributed generation. Equivalent single busbars approach is used. Given that the network is not taken into account, the strength is the simplicity of the approach, which makes the contribution of distributed generators of the same reliability performance be the same. The weakness is that in fact, the security of supply is function also of network reliability performance; which means that the contribution of distributed generation is lower in better performing networks and greater in worse performing networks. Hence, the present approach could potentially overestimate or underestimate the contribution of distributed generation leading to under- or overinvestment.

b.iii) The capacity outage probability table of two identical generating units is:

| Capacity in | Probability |
|---|---|
| $2C$ | $A^2$ |
| $C$ | $2A(1-A)$ |
| $0$ | $(1-A)^2$ |

where $A$ is the probability of a unit being available and $C$ is the capacity of a single unit.

The Expected Energy Not Supplied (EENS) for the two generating units when peak demand is equal to $2C$ is

| Capacity in | Probability | Energy not supplied |
|---|---|---|
| $2C$ | $A^2$ | 0 |
| $C$ | $2A(1-A)$ | $(2C-C)\,0.2\,T\,2A(1-A)$ |
| 0 | $(1-A)^2$ | $2C\,T(1-A)^2$ |

where $T$ is the period of time. Adding these energy not supplied for the different combinations of capacity available we get:

$$EENS_G = (2C - C) \cdot 0.2 \cdot T \cdot 2 \cdot A \cdot (1 - A) + 2 \cdot C \cdot T \cdot (1 - A)^2$$
$$= 2 \cdot C \cdot T \cdot (1 - A) \cdot (1 - 0.8 \cdot A)$$

The EENS for firm capacity is

$$EENS_{FC} = (1 - F) \cdot 2 \cdot C \cdot 0.2 \cdot T = (1 - 0.75) \cdot 2 \cdot C \cdot 0.2 \cdot T = 0.1 \cdot C \cdot T$$

The availability can be found as

$$EENS_G = EENS_{FC}$$
$$2 \cdot C \cdot T \cdot (1 - A) \cdot (1 - 0.8 \cdot A) = 0.1 \cdot C \cdot T$$
$$0.8 \cdot A^2 - 1.8 \cdot A + 0.95 = 0$$
$$A = 0.845 = 84.5\%$$

**Q4.** **A question about low carbon heat**

a) Give a very brief description of the following types of low carbon heating systems:

i) Air source heat pump (ASHP). [2]

ii) District heating. [2]

iii) Hydrogen heating. [2]

iv) Resistive heating (Direct electric). [2]

b) A comparison of each of the aforementioned types can be made by using a "spider gram" as shown in Figure Q4.1 where "1" is ranked "Best" and 4 is the "Worst". Answer the question described below.

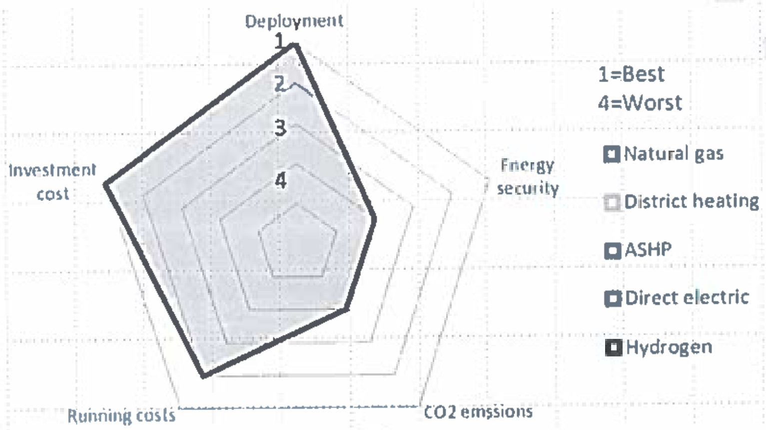



Figure Q4.1- Diagram comparing the features of different heating systems

i) Plot each of the low carbon heating systems on a spider gram (natural gas is shown as an illustration) and comment very briefly on the result. [6]

c) A householder is trying to decide on whether to purchase a low cost resistive heating system or a more expensive air source heat pump system. The data are shown in Table Q4.1. Answer the question described below.

| | Heating system fixed costs | Running costs | Efficiency |
|---|---|---|---|
| Resistive heating | £150/year | £0.15/kWh | 100% |
| Air source heat pump | £750/year | £0.15p/kWh | 300% |

Table Q4.1- Characteristics of different heating systems.

i) What is the annual heat demand whereby the total annual costs of both heating systems are the same? [3]

d) The householder's been advised that they can have a hybrid heat pump for the same fixed cost as standard ordinary air source heat pump. Explain why they might have similar fixed costs and what would be the system related benefits of a hybrid heat pump. [3]

**Solution to Q4**

a.i) [BOOKWORK] ASHP transfers thermal energy from the outside air to the area to be heated. There are 4 stages to the cycle and these comprise:

- Evaporation - The refrigerant passes through a heat exchanger where it evaporates absorbing heat in the process.
- Compression - The heated refrigerant is then compressed to higher pressure using an electrically driven compressor.
- Condensation - The compressed refrigerant passes through another heat exchanger located in the space to be heated and condenses, changing state back to a liquid and rejecting heat in the process.
- Expansion – The liquid refrigerant passes through an expansion device lowering its pressure prior to returning to the evaporator.

a.ii) [BOOKWORK] District heating is a system where heat for more than one building or an area is produced at a central location and distributed through a network of insulated pipes. Heat sources include:

- Combined heat and power units
- Network heat pumps
- Waste(d) heat recovery
- Solar thermal
- Geothermal

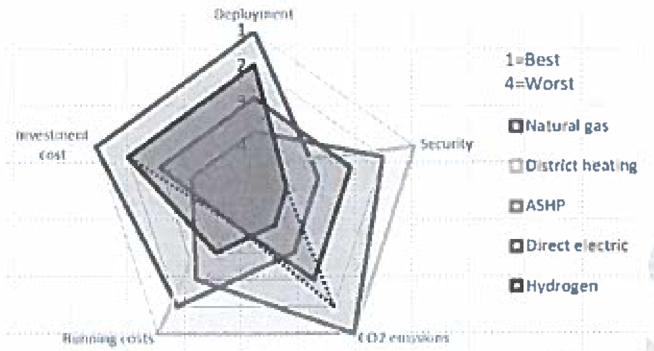
The main components of a district heating system are:

- Heat storage - to provide additional capacity and backup, etc.
- Heat network – connecting heat sources with buildings using supply and return pipes.
- Pumps – to circulate the hot water
- Heat substations – for pressure and temperature changes
- Heat metering – to monitor production and consumption
- Heat exchangers/ heat interface units - for connection of the heat network to buildings

a.iii) [BOOKWORK] Hydrogen heating is based on the repurposing of the existing gas distribution network from natural gas to hydrogen. Hydrogen can be produced using a number of processes but at present the most suited for large scale production is steam methane reforming using natural gas with carbon capture and storage and electrolysis using low carbon electricity.

a.iv) [BOOKWORK] There are a variety of different types of resistive heating systems. These include: radiant, convector, storage, direct electric, fan and panel heaters. However, they can only be regarded as low carbon if they use low carbon electricity.

b.i) [BOOKWORK] The diagram would look like this:




District heating scores well on running costs and $CO_2$ emissions as heat production can be very low cost with low levels of $CO_2$ emissions, e.g. waste heat, CHP. It also scores well at energy security as it reduces the UK's dependence on energy import. On the other hand, it scores badly on investment and deployment due to the cost and disruption associated with constructing the heat network and associated infrastructure.

ASHP heating scores well on $CO_2$ emissions due to its high efficiency but is dependent on low carbon electricity. It scores badly on investment and deployment due to the cost and disruption associated with reinforcing the electricity network and other associated infrastructure including upgrades to buildings to ensure their suitability for ASHP heating.

Direct electric does not score well on any specific feature but is generally better than some or worse than others. It scores badly on running costs as it is dependent on low carbon electricity (which is likely to be more expensive than gas) and has a much lower efficiency than ASHP. Note: direct electric may be more suited to buildings with low heat demand where the higher running costs are less important.

Hydrogen scores well on investment cost and deployment as it uses existing network infrastructure but badly on $CO_2$ emissions unless CCS for hydrogen production from steam methane reforming is employed or low carbon electricity is used for hydrogen production from electrolysis.

c.i) We need to find the heat demand, $X$ (kWh/year), when the cost of resistive heating is equal to the cost of ASHP heating. This means:

$$£150/\text{year} + £0.15/\text{kWh}\ X = £750/\text{year} + £0.15/\text{kWh}\ X/3$$

$$X\ (£0.15/\text{kWh} - £0.05/\text{kWh}) = £600/\text{year}$$

$$X = 600/0.1\ \text{kWh/year} = 6{,}000\ \text{kWh/year}$$

Hence, the total annual costs are the same when the heat demand is 6,000kWh/year.

d) [BOOKWORK] A hybrid heat pump comprises an ASHP combined with a gas boiler. Typically the heat output rating of the heat pump can be lower, e.g. $5kWh_{th}$ instead of $10kW_{th}$. During periods of high demand the gas boiler will operate to boost heat output. Hence the cost of the heat pump will be less and can offset the cost of the gas boiler.

The main benefit of the hybrid heat pump is that the electricity demand is lower and this may mean that costly network reinforcement can be avoided. In addition, there is less need for peaking electricity plant to meet peak heat demand as this is met by the gas boiler.

**Q5. A combined question about demand-side response, hydro power generation and offshore power transmission**

**Demand-side response**

a) Answer the following questions:

i) Explain why energy supply companies may be interested in changing demand profiles of their customers. [3]

ii) Explain the concept of demand response and give an example of a demand response activity. [4]

iii) Explain the concept of EU appliance labelling scheme, what is the policy objective and how it may be met. [3]

**Hydraulic power generation**

b) A micro hydraulic generation system consists of an upper water reservoir that feeds a penstock that goes to a turbine that generates power (see Figure Q5.2). The characteristics of the system and its operating point are summarised in Table Q5.1. Answer the questions listed below.

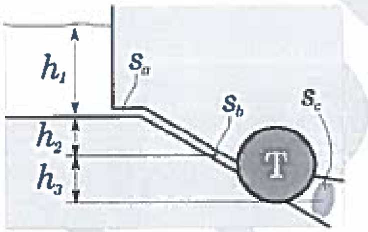



Figure Q5.2- A micro hydraulic generation system.

| Parameter | Value | Description |
|---|---|---|
| $h_1$ | 6 m | Water depth upper reservoir. |
| $h_2$ | 5 m | Distance #2 |
| $h3$ | 1 m | Distance #3 |
| $S_a, S_b$ | 0.1 m$^2$ | Cross-section of the inlet |
| $S_c$ | 0.3 m$^2$ | Cross-section of the outlet |
| $\rho$ | 1,000 kg/m$^3$ | Density of water |
| $\dot{m}$ | 400 kg/s | Mass flow under normal operation |

Table Q5.1-Characteristic parameters of the micro-hydro system

i) Consider the situation where the intake valve of the turbine is shut and no water is flowing. Calculate the gauge pressure (the difference with the atmospheric pressure) in $S_a$ and $S_b$. [1]

ii) Consider a normal operation point where the mass flow is $\dot{m}$ = 400 kg/s. Calculate the speed of the water in $S_a$, $S_b$ and $S_c$. [1]

iii) Calculate the gauge pressure in $S_a$ and $S_b$ under normal operation. [1]

iv) Calculate the power extracted from the water if we know that the pressure in $S_c$ under normal operation is of $p_c$=30 kPa. [2]

v) Would a Kaplan turbine be a suitable choice for this application? Justify your answer briefly. [1]

**Offshore power transmission**

c) Answer the following questions related to HVDC transmission:

i) Explain briefly why the critical distance at which HVDC transmission becomes a better solution than HVAC is different when using overhead lines and when using underground cables. [2]

ii) List the potential advantages of VSC-HVDC when compared to LCC-HVDC. [2]

**Solution to Q5**

**Demand-side response**

a.i) [BOOKWORK] Energy companies are interested in the demand profile of customers because it may affect their ability to provide supply based on their generation portfolio.

Modifying demand may reduce their imbalance related costs and their revenue streams can be improved by better matching the demand with their generation portfolio. This may be particularly relevant for suppliers that operate inflexible nuclear generation or variable renewable generation.

a.ii) [BOOKWORK] *Demand response – change in demand driven by electricity system or market conditions*. Demand response can be participatory or automated. Time-of-use (ToU) tariffs are a participatory form of demand response and require consumers to change their activity related to energy demand. ToU tariff require the means to measure and record energy demand at the different time periods covered by the tariff. In a largely fossil fuel based system, prices would typically be high at the peak period where less efficient plant is active and losses are high.

The points and periods of such tariffs have historically been set on a seasonal basis and were therefore predictable. In a scenario of increased variable renewable generation, it would be very beneficial to have dynamic tariffs whereby high and low price periods can be adjusted day to day based on weather conditions, driving the energy production by RES.

a.iii) [BOOKWORK] One of the most successful demand side policy relating to residential electricity demand is the EU appliance labelling scheme. The scheme provides information on the relative efficiency of similar appliances. Appliances are graded A+++ to D with the later being the least efficient. The scheme works by measuring the energy demand of appliances under test conditions and a graded by a predetermined scale. The letter grading allows rapid comparison of energy use at the point of sale and are intended to improve purchasing decisions. Other similar schemes such as the energy star are simply endorsement with no specific information on the label. While scheme works on the ability to compare the data, the scheme has also allowed the identification and subsequent banning of the poorest performing devices.

**Hydraulic power generation**

b.i) If no water is flowing, the kinetic terms in Bernoulli's equation are zero. The following relationship can be found between any point the surface of the water in the upper reservoir ($S_0$, taken as the reference for height) and $S_a$ or $S_b$:

$$\underbrace{\frac{p_0}{\rho \cdot g}}_{=0} + \underbrace{z_0}_{=0} = \frac{p_i}{\rho \cdot g} + z_i$$

with $i=a,b$.

Substituting the numerical values for the two points of interest we obtain: $p_a$=59 kPa, $p_b$=108 kPa.

b.ii) The speed of the water can be found by dividing the volumetric flow by the cross-section of the pipe. The volumetric flow can be found by dividing the mass flow by the density of the water, this gives $Q$=0.4 $m^3 \cdot s^{-1}$. Therefore: $v_a=v_b$=4 $m \cdot s^{-1}$ and $v_c$=1.33 $m \cdot s^{-1}$.

b.iii) The water pressure under normal operation will be lower than it was when the valve was shut. The solution of b.i) has to be adapted by adding the speed term on the right side of the equation:

$$\underbrace{\frac{p_0}{\rho \cdot g}}_{=0} + \underbrace{z_0}_{=0} = \frac{p_i}{\rho \cdot g} + \frac{v_i^2}{2 \cdot g} + z_i$$

with $i=a,b$.

Substituting the numerical values for the two points of interest, including the speed, which was found in b.ii) we obtain: $p_a$=51 kPa, $p_b$=100 kPa.

b.iv) The power extracted from the water by the turbine can be found by performing a power balance between the input and the output of the turbine:

$$P_{out} = P_b - P_c = \left( \underbrace{\frac{1}{2} Q\rho v_b^2}_{\substack{kinetic \\ power}} + \underbrace{Q\rho g z_b}_{\substack{potential \\ power}} + \underbrace{p_b Q}_{\substack{pressure \\ power}} \right) - \left( \frac{1}{2} Q\rho v_c^2 + Q\rho g z_c + p_c Q \right)$$

At this point we have all information about height, pressure and speed at the input and at the output of the turbine. If we substitute the numerical values above, we obtain: $P_{out}$=35 kW.

b.v) [BOOKWORK] According to the information available, the head in this location is about 12 m, which is very low and falls in the range where reaction turbines, and more specifically Kaplan turbines, are normally used (between 5 and 70 m).

**Offshore power transmission**

c.i) [BOOKWORK] Both underground cables and overhead lines have unavoidable parasitic capacitance between the conductor and the ground. The capacitance becomes greater the longer the line is. When the conductor is used to carry AC current, stray currents flow through this capacitance. This current does not contribute to power transmission but causes heating of the conductors and leaves less margin for the current that carries active power. The problem can't be mitigated by increasing the voltage of the line, which in fact causes the stray currents to be greater. The problem does not occur when the line is used for DC because the parasitic capacitance stays charged and no current flows through it in steady state. HVDC requires conversion stations, which incur high costs, but there's a critical distance where it becomes a more cost-effective solution than HVAC. The reason why the critical distance is different in overhead lines and in underground cables is because cable capacitance is much higher, which makes stray AC currents become a problem at much shorter distances.

c.ii) [BOOKWORK] VSC-HVDC has potential for greater power density to enable compact substations to be used onshore and in constrained spaces onshore. It also offers greater controllability and it can be used in weaker AC networks. The design of the converter is easy to adapt to different voltage or power levels for their use in locations with different characteristics, whereas LCC requires greater case-by-case customization. The voltage-source arrangement facilitates the connection of multiple terminals, making it more appropriate to build future HVDC grids.