Optoelectronics Solutions 2016

1. a) For time-dependent fields, Faraday's law is:

$\text{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t = -\mu_0\, \partial\underline{\mathbf{H}}/\partial t$, where μ_0 is the permeability of free space.

For time-independent fields at angular frequency ω, $\underline{\mathbf{E}} = \underline{E} \exp(j\omega t)$ and $\underline{\mathbf{H}} = \underline{H} \exp(j\omega t)$.

Faradays' law can then be rewritten in time independent form as:

$\text{curl}(\underline{E}) = -j\omega\underline{B} = -j\omega\mu_0\underline{H}$

[2]

For the particular case of y-polarization, when $\underline{E} = E_y\, \underline{j}$,

$$\text{curl}(\underline{E}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix} = -\partial E_y/\partial z\, \underline{i} + \partial E_y/\partial x\, \underline{k}$$

Consequently, $\underline{H} = (j/\omega\mu_0)\, \{-\partial E_y/\partial z\, \underline{i} + \partial E_y/\partial x\, \underline{k}\}$

[3]

The boundary conditions that must be satisfied at an interface between two media are that the tangential components of $\underline{E}$ and $\underline{H}$ must match. In this case, the tangential components are E_y and H_z. Consequently, E_y and $\partial E_y/\partial x$ must match.

[2]

b) A y-polarized plane wave propagating in the z-direction in a medium with propagation constant k can be written as $\underline{E} = E_0 \exp(-jkz)\, \underline{j}$, where E_0 is the wave amplitude.

[2]

Since E_y is a function of z only, the magnetic field is $\underline{H} = -(j/\omega\mu_0)\, \partial E_y/\partial z\, \underline{i}$, or:

$\underline{H} = -(j/\omega\mu_0)\, -jk\, E_0 \exp(-jkz)\, \underline{i} = -(k/\omega\mu_0)\, E_0 \exp(-jkz)\, \underline{i}$

[2]

If the above is written as $\underline{H} = -H_0 \exp(-jkz)\, \underline{i}$, then $E_0/H_0 = \omega\mu_0/k$.

This is the characteristic impedance of the medium, Z_0.

[2]

c) The irradiance or time averaged power flow $\underline{S}$ is defined as:

$\underline{S} = 1/2\, \text{Re}(\underline{E} \times \underline{H}^*)$

[1]

For the plane wave above, with $\underline{E} = E_0 \exp(-jkz)\ \underline{j}$ and $\underline{H} = -H_0 \exp(-jkz)\ \underline{i}$:

$$\underline{S} = 1/2\ \mathrm{Re}\left(\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & E_y & 0 \\ H_x^* & 0 & H_z^* \end{vmatrix} \right) = 1/2\ \mathrm{Re}(-E_y H_x^*)\ \underline{k}, \text{ or}$$

$S = 1/2\ \mathrm{Re}\{E_0 \exp(-jkz)\ H_0^* \exp(+jkz)\}\ \underline{k} = 1/2\ \mathrm{Re}(E_0 H_0^*)\ \underline{k}$

In terms of the characteristic impedance Z_0, this can be written as $\underline{S} = 1/2\ E_0 E_0^*/Z_0\ \underline{k}$

[2]

The power passing through an area A is $P = \iint_A \underline{S} \cdot d\underline{a}$, where $d\underline{a}$ is a vector whose direction is perpendicular to A, and whose magnitude da represents a small element of A. If S is uniform and A is a plane area, then $P = \underline{S} \cdot \underline{A}$.

[1]

If A is a 1 m^2 area arranged perpendicular to the z-axis, then $\underline{A} = 1\ \underline{k}$. In this case:

$P = \underline{S} \cdot \underline{k} = 1/2\ |E_0|^2/Z_0$

If $E_0 = 1$ V/m and $Z_0 = 377\ \Omega$ (for free space)

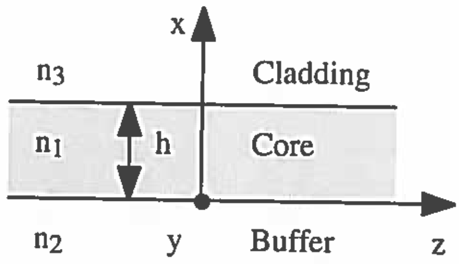
$P = 1/2 \times 1^2/377\ \Omega = 1.326$ mW

[2]

If A is a 1 m^2 area arranged parallel to the z-axis, then $\underline{A} = 1\ \underline{i}$ (say). In this case, $P = 0$.

[1]

2. a) The typical arrangement of a three-layer asymmetric slab dielectric waveguide, drawn on the x, z plane is as shown below:



[2]

Total internal reflection (TIR) must be achieved at each interface if waveguiding is to occur. A necessary condition for TIR is that $n_1 > n_2$ and $n_1 > n_3$. However, TIR also requires that the critical angle is exceeded, and this condition may break down at either interface. In this case, energy will leak out of the guide, which is then described as being 'cut off'.

[3]

b) For TE polarization, the wave equation in layer i is:

$\nabla^2 E_{yi}(x, z) + n_i^2 k_o^2 E_{yi}(x, z) = 0 \qquad (i = 1, 2, 3)$

[1]

Modal solutions are product solutions with the general form:

$E_{yi}(x, z) = E_i(x) \exp(-j\beta z) \qquad (i = 1, 2, 3)$

where $E_i(x)$ is the transverse field and β is the propagation constant.

[1]

Differentiation gives:

$\partial^2 E_{yi}/\partial x^2 = d^2E_i/dx^2 \exp(-j\beta z)$

$\partial^2 E_{yi}/\partial x^2 = -\beta^2 E_i \exp(-j\beta z)$

Substitution into the wave equation then gives:

$d^2E_i/dx^2 \exp(-j\beta z) - \beta^2 E_i \exp(-j\beta z) + n_i^2 k_o^2 E_i \exp(-j\beta z) = 0$, or

$d^2E_i/dx^2 + (n_i^2 k_o^2 - \beta^2) E_i = 0 \qquad (i = 1, 2, 3)$

This is the waveguide equation.

[2]

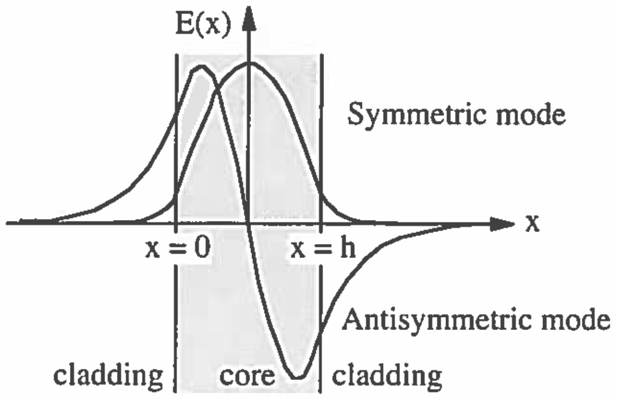
The waveguide equation is a second order differential equation, with the general form:

$d^2E_i/dx^2 + \alpha^2 E_i = 0$, where $\alpha = \sqrt{(n_i^2 k_0^2 - \beta^2)}$

Solutions for the transverse fields are trigonometric functions if $n_i^2 k_0^2 - \beta^2 > 0$ (in the core), and exponential functions if $n_i^2 k_0^2 - \beta^2 < 0$ (in the other two layers).

[1]

c) The transverse fields of the two lowest order modes of a symmetric guide are symmetric (mode 0) and antisymmetric (mode 1), as shown below:



[2]

The boundary conditions that must be satisfied are continuity of the transverse field and its derivative, on $x = 0$ and $x = h$. Clearly, the fields and their derivatives are continuous.

[2]

d) The eigenvalue equation for asymmetric slab waveguide is:

$\tan(\kappa h) = \kappa[\gamma + \delta] / [\kappa^2 - \gamma\delta]$

Here $\kappa = \sqrt{(n_1^2 k_0^2 - \beta^2)}$, $\gamma = \sqrt{(\beta^2 - n_2^2 k_0^2)}$ and $\delta = \sqrt{(\beta^2 - n_3^2 k_0^2)}$

[2]

If the waveguide is symmetric, $n_2 = n_3$ and $\gamma = \delta$. The eigenvalue equation then reduces to:

$\tan(\kappa h) = 2\kappa\gamma / [\kappa^2 - \gamma^2]$

Dividing top and bottom by κ^2, this equation reduces to:

$\tan(\kappa h) = 2(\gamma/\kappa) / [1 - (\gamma/\kappa)^2]$

Bearing in mind that $\tan(2A) = 2\tan(A)/\{1 - \tan^2(A)\}$, this can be simplified to:

$\tan(\kappa h/2) = \gamma/\kappa$ - the eigenvalue equation for symmetric modes

Similarly, dividing top and bottom by γ^2, we obtain:

$\tan(\kappa h) = -2(\kappa/\gamma) / [1 - (\kappa/\gamma)^2]$

Using the same tan expansion we can obtain:

$\tan(\kappa h/2) = -\kappa/\gamma$ - the eigenvalue equation for antisymmetric modes

[2]

Cutoff occurs when γ tends to zero. At this point, β tends to n_2k_0, and κ to $\sqrt{(n_1^2k_0^2 - n_2^2k_0^2)}$.

In this case $\kappa h/2 = 0, \pi, 2\pi \ldots$ (symmetric modes) and $\kappa h/2 = \pi/2, 3\pi/2 \ldots$ (antisymmetric)

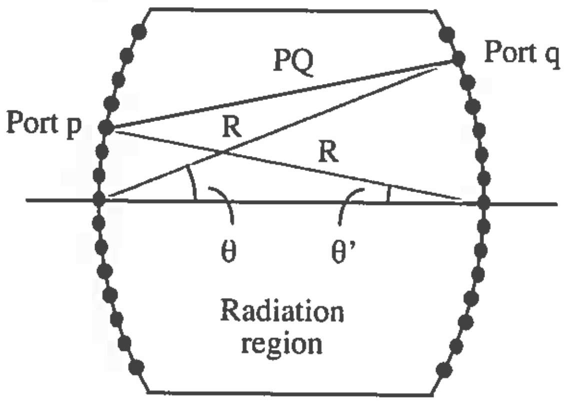
The general cutoff condition is then $k_0\sqrt{(n_1^2 - n_2^2)} = \nu\pi/2$

[2]

3. a) The radiative star consists of an array of N channel guide inputs linked to a similar array of outputs by a region of planar waveguide with circular boundaries. Light from a single input port radiates into the planar region and illuminates the entire array of outputs, with approximately the same performance for each port, carrying out the N-way power-division function with high uniformity and low insertion loss. Radiation from different inputs can cross without interacting, enabling the star function. The advantage over a non-radiative star is design simplicity, which allows scalability to very high port counts.

[3]

To calculate the transfer function of a radiative star, consider the geometry below. Here the distance PQ between an input port p and an output port q in the radiation region can be found as follows.



The x- and y-components of PQ are:

$PQ_x = R - R\{1 - \cos(\theta)\} - R\{1 - \cos(\theta')\}$

$PQ_y = R\{\sin(\theta) - \sin(\theta')\}$

Using small angle approximations:

$PQ_x \approx R(1 - \theta^2/2 - \theta'^2/2)$

$PQ_y \approx R(\theta - \theta')$

Using binomial approximations, $PQ^2 = PQ_x^2 + PQ_y^2$ can be found as:

$PQ^2 \approx R^2(1 - \theta^2 - \theta'^2) + R^2(\theta^2 - 2\theta\theta' + \theta'^2) \approx R^2(1 - 2\theta\theta')$

Again using a binomial approximation, PQ itself can be found as $PQ \approx R(1 - \theta\theta')$

Since the ports are at angles $\Delta\theta$ apart, $\theta = q\Delta\theta$ and $q' = p\Delta\theta$, and $PQ \approx R(1 - pq\Delta\theta^2)$

Now, the amplitude at q is related to the amplitude at p by two factors:

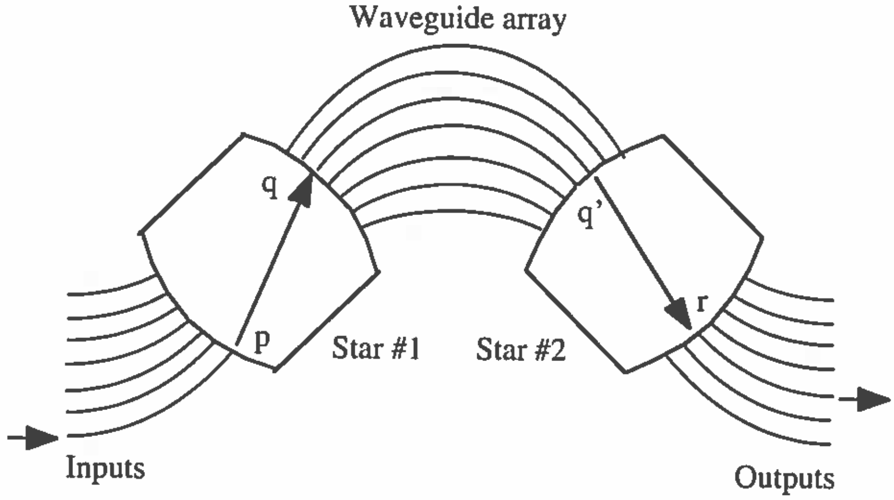
A factor $1/\sqrt{N}$ describing the power division that must occur as the beam radiates

A factor exp(-jβPQ) describing the phase shift between P and Q in a region with propagation constant β.

Consequently $A_q = A_p/\sqrt{N} \exp\{-j\beta R(1 - pq\Delta\theta^2)\}$

[5]

b) An AWG MUX consists of two radiative star couplers linked together by a set of curved waveguides whose path lengths vary linearly across the array, as shown below:



[2]

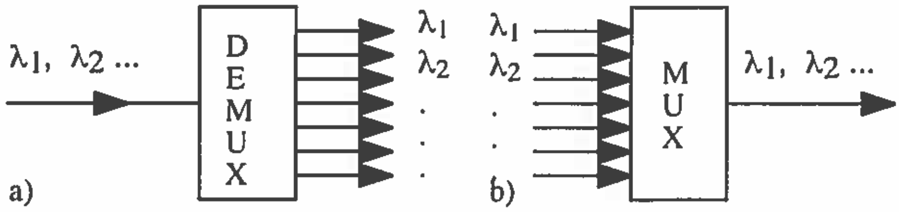
The device effectively combines a set of near-identical filters, with regularly spaced wavelengths for peak transmission. A wavelength-multiplexed input to a single port will therefore emerge with the separate channels spatially separated over the entire set of output ports, hence providing a demultiplexing (DEMUX) function. Operation is reciprocal, allowing the device to combine a separated set of channels onto a single output, hence providing a multiplexing (MUX) function. Operation also obeys permutation rules, so that the use of different inputs merely results in permutation of the outputs.

[3]

The advantage over alternative multiplexer designs based on individual filters is the extreme simplicity, which allows scaling to very large channel counts.

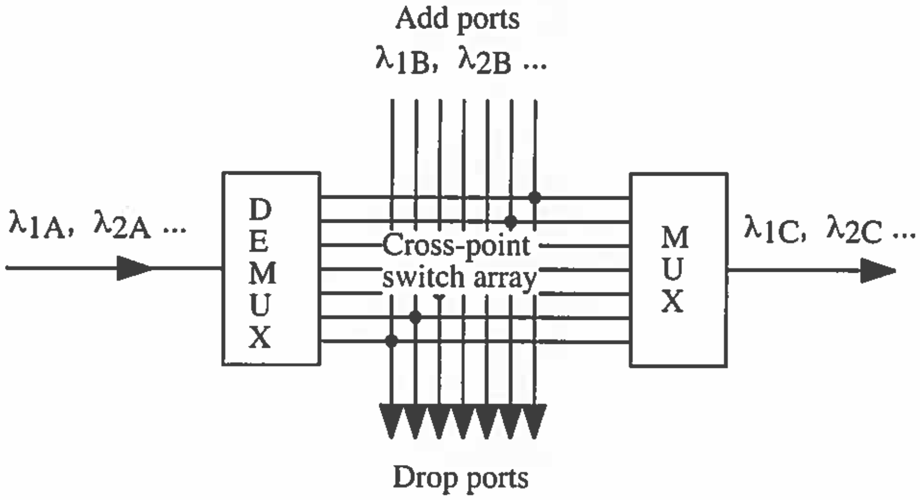
[1]

c) In a block representation, the DEMUX and MUX functions of an AWG MUX can be represented as shown in a) and b) below.



[2]

An ADD-DROP multiplexer can be constructed using two AWG MUX components arranged back-to-back to provide DEMUX and MUX functions, with a cross-point switch array in between to allow channels to be added or dropped, as shown below.



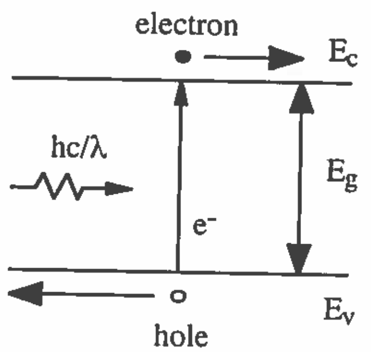
[4]

4. a) Electronic detection of light in semiconductors involves the promotion of electrons from the valence band to the conduction band through the absorption of photons. Necessary conditions are that the photon energy $E = hc/\lambda$ (where h is Planck's constant, c is the velocity of light and λ is the wavelength) is greater than the energy gap E_g. The result is the elimination of the photon and the creation of an electron-hole pair. The additional carriers thus created correspond to a photocurrent, which can then be converted into a voltage.

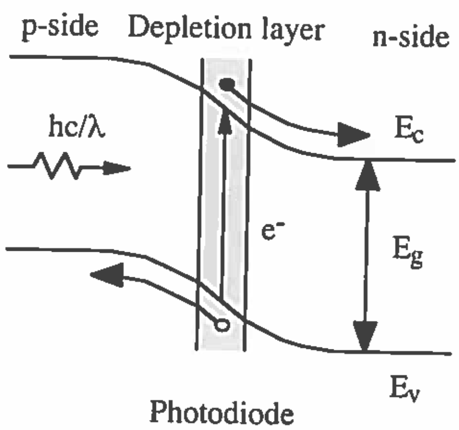
[2]

Band diagrams representing absorption in a photoconductor and a photodiode are as shown below. The main difference is the presence of a depletion layer in the photodiode, which contains an extremely strong built-in electric field. If absorption takes place within the depletion layer, this field will rapidly separate the photogenerated electron and hole, preventing them from recombining and instead expelling them to the regions outside the depletion layer where they diffuse to the contacts. As a result, recombination can be minimised. In contrast, the photoconductor requires an external field for charge collection. Since this field is relatively limited, significant recombination occurs, and photoconductors are relatively inefficient. The external field in any case results in a dark current, which can be avoided in a photodiode using reverse bias.

[3]



Photoconductor



Photodiode

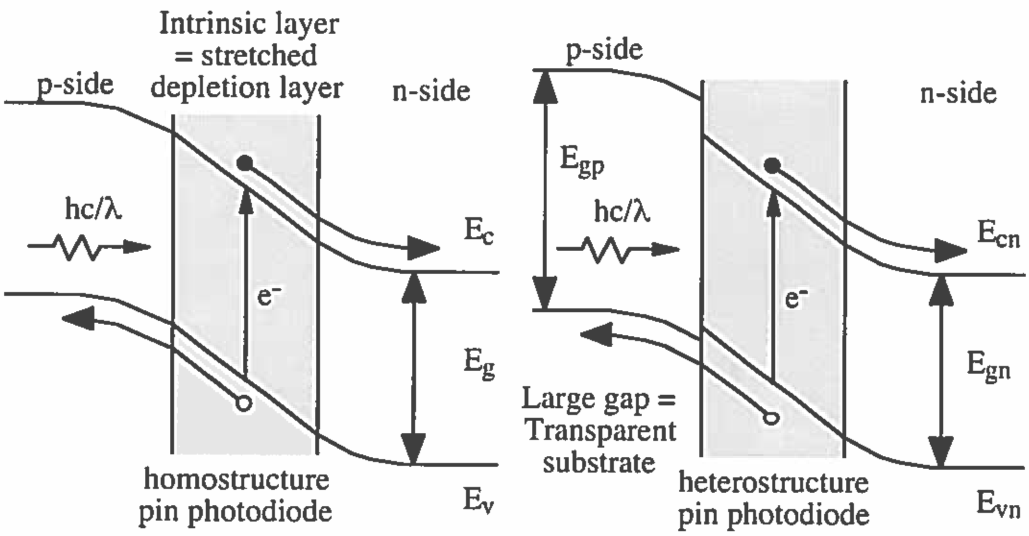
[3]

b) Photogeneration of carriers in a pn junction diode is limited by two main factors, whose combined effect is described by the quantum efficiency. Firstly, photons with high energy (and hence a short wavelength) may be absorbed in the layer of semiconductor immediately in front of the depletion layer. Secondly, photons with low energy (or long wavelength) may pass through the depletion layer without absorption, only being absorbed in the layer behind. In each case, there is no strong electric field to separate the carrier pairs, which may then recombine.

[2]

These problems may be addressed using pin photodiode structures. In a homostructure pin diode, the addition of a central intrinsic layer yields a wider depletion layer, which acts as a more effective collection region for low energy photons. In a heterostructure pin diode, replacement of the first layer with wide bandgap material prevents absorption of high-energy photons before they reach the depletion layer. Representative band diagrams are as shown below.

[2]



[3]

c) An optical beam of power P represents a flux of $P/E = P\lambda/hc$ photons/sec. Assuming that each photon generates a carrier pair, and that these carriers reach the contacts, each absorption event results in one electron charge e transiting the device. The ideal photocurrent is then $I_P = P\lambda e/hc$. The effects above reduce the effectiveness of the detection process. This reduction is represented by the quantum efficiency, leading to a real photocurrent of $I_P = P\eta\lambda e/hc$. The responsivity $R = I_P/P$ is then $R = \eta\lambda e/hc$.

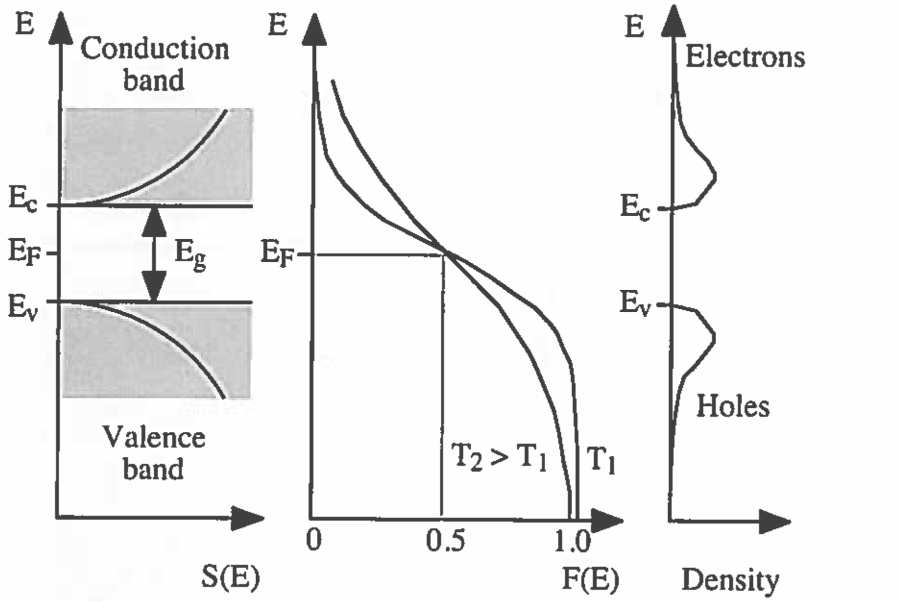
[3]

Quantum efficiency can be estimated from the responsivity by re-arranging the equation above to get $\eta = Rhc/\lambda e$. If $\lambda = 1.55$ µm, and R = 1 mA/mW:

$\eta = 6.62 \times 10^{-34} \times 3 \times 10^{8}/(1.55 \times 10^{-6} \times 1.6 \times 10^{-19}) = 0.8$, or 80%

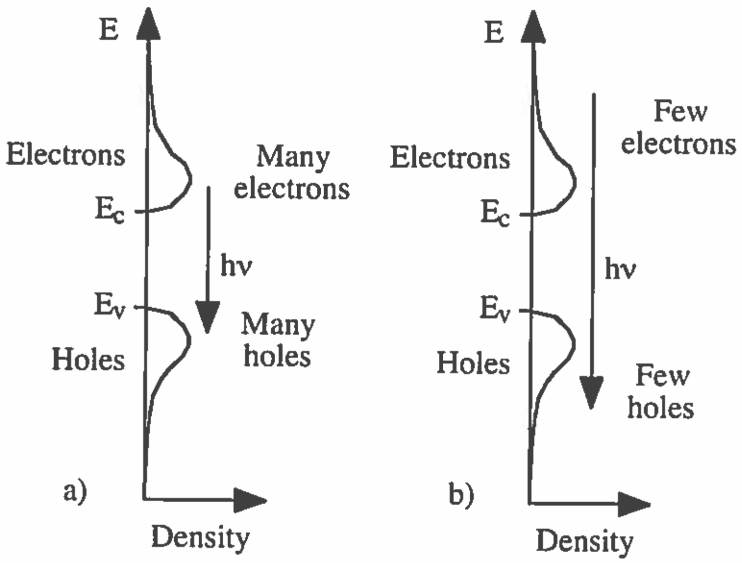
[2]

5. a) The density of occupied states is found as the product of two functions that are each dependent on electron energy E: the density of available states S(E), and the Fermi-Dirac function F(E), which gives the probability of occupation.



[4]

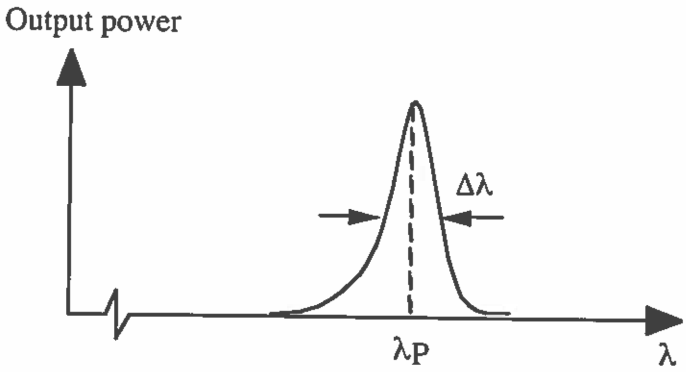
Light emission in LEDs involves the recombination of electrons in the conduction band with holes in the valence band. Peak emission occurs at wavelengths corresponding to transitions between the peaks of the density functions for electrons and holes. Emission is reduced on either side of this peak, because there are fewer electrons and holes available.



[2]

The emission spectrum of a LED is a broad band centred on the wavelength of peak emission λ_p. The bandwidth $\Delta\lambda$ is consequently much too large for use as a carrier in a long-distance communication system, which is typically wavelength-division multiplexed.

[1]



[1]

b) The key breakthrough needed for lighting (which requires white light) was the development of the blue LED, which in turn required an InGaN semiconductor.

[1]

The two most common methods for generating white light using LEDs are:

i) Using LEDs emitting three different wavelengths (blue, green and red)

ii) Using a single blue LED plus a yellow phosphor ($Y_3Al_5O_{12}$: Ce) for wavelength conversion.

[2]

The adaptations needed for use of LEDs in lighting are:

i) Structuring of the LED surface to increase the external efficiency

ii) The use of multiple emitters to achieve sufficient brightness

iii) Additional circuitry to convert AC mains to a low-voltage, high-current DC supply

[3]

c) LEDs emit more light in the visible range than other light sources, so their energy efficiency is much higher. Their operating temperatures are much lower than other sources (especially filament light bulbs), so thermally induced degradation mechanisms such as dislocation migration operate more slowly and their lifetime is longer.

Consequently their energy and servicing costs are much lower. Against this, LED fabrication involves a much more advanced process, so their manufacturing costs are higher. The additional requirement for electronic circuitry makes the capital costs much higher. However, overall, LEDs are considerably cheaper to operate.

[5]

6. The rate equations for a semiconductor laser diode are:

$d\phi/dt = \beta n/\tau_{rr} + G\phi(n - n_0) - \phi/\tau_p$

$dn/dt = I/ev - n/\tau_e - G\phi(n - n_0)$

a) The five different terms on the RHS all describe rates of different processes, thus:

$\beta n/\tau_{rr}$	Creation of photons by radiative recombination
$G\phi(n - n_0)$	Creation/loss of photons/electrons by absorption and stimulated emission
ϕ/τ_p	Loss of photons from the cavity through the end mirrors
I/ev	Injection of electrons via the drive current
n/τ_e	Loss of electrons by all recombination processes, radiative and non-radiative

[5]

The main differences from the LED rate equations are:

i) The addition of the terms $G\phi(n - n_0)$, which are not needed below threshold

ii) The addition of the factor β

[2]

There is no rate equation for holes, because it is substantially the same as the equation for electrons; absorption and radiative emission both lead to creation or loss of a carrier pair.

[1]

b) In the steady state, the LHS of both equations can be set to zero.

Below threshold, we can ignore absorption and stimulated emission. We then have:

$\beta n/\tau_{rr} - \phi/\tau_p = 0$

$I/ev - n/\tau_e = 0$

From the lower equation, the electron density is $n = I\tau_e/ev$.

Consequently n is proportional to I during LED operation.

From the upper equation, the rate of loss of photons per unit volume is:

$\phi/\tau_p = \beta n/\tau_{rr} = \beta\ (\tau_e/\tau_{rr})\ I/ev$

Since v is the active volume, the total photon flux is:

$\Phi = v\ \phi/\tau_p = \beta\ (\tau_e/\tau_{rr})\ I/e$

Now each photon carries an energy hc/λ, where h is Planck's constant, c is the velocity of light and λ is the wavelength. The optical power output is therefore:

$P = \beta\,(\tau_e/\tau_{rr})\,(hc/\lambda)\, I/e$

Consequently P is proportional to I during LED operation.

[3]

This expression implies that the rate of creation of photons is directly proportional to the rate of injection of electrons, with two efficiency factors:

β (the fraction of the spontaneously generated light that is coupled into the laser stripe)

τ_e/τ_{rr} (the quantum efficiency). This can be written as r_{rr}/r_e, where $r_{rr} = 1/\tau_{rr}$ is the rate of radiative recombination and $r_e = 1/\tau_e$ is the total rate of electron recombination, and hence represents the fraction of electrons that recombine to generate light).

[2]

c) Above threshold, we can ignore spontaneous emission. We then have:

$G\phi(n - n_0) - \phi/\tau_p = 0$

$I/ev - n/\tau_e - G\phi(n - n_0) = 0$

From the upper equation;

$G(n - n_0) - 1/\tau_p = 0$, so $n = n_0 + 1/G\tau_p$.

Consequently n is fixed during laser operation.

Combining the upper and lower equations:

$\phi/\tau_p = I/ev - n/\tau_e$

Since n is fixed, we can rewrite this as:

$\phi/\tau_p = (I - I_{th})/ev$ where $I_{th} = nev/\tau_e$ is the threshold current

As before, the photon flux is $\Phi = v\phi/\tau_p$, and the optical power is $P = \Phi hc/\lambda$, so:

$P = (hc/\lambda)(I - I_{th})/e$

Consequently P depends linearly on I during laser operation, but starts from zero at I_{th}.

[2]

From the above, we can see that:

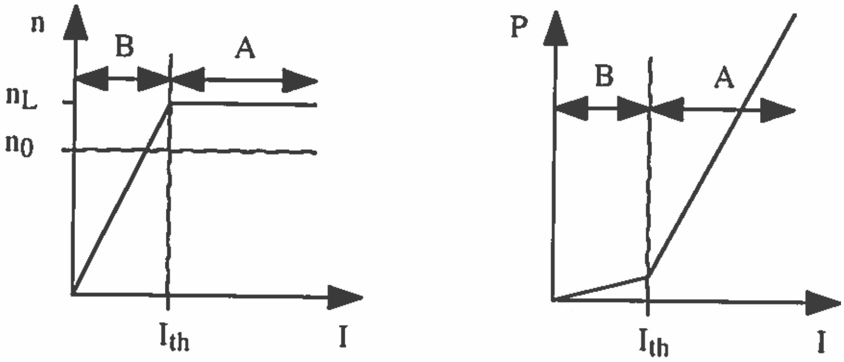
The value of n at transparency (when stimulated emission and absorption balance) is n_0

The value of n during lasing is $n_L = n_0 + 1/G\tau_p$

The threshold current is $I_{th} = nev/\tau_e$

[3]

And the variations of n and P with I are as follows (B = below, A = above threshold):



[2]