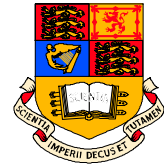


**IMPERIAL COLLEGE  
LONDON**

**[E303/ISE3.3]**



**DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING  
EXAMINATIONS 2003**

**EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI**

# **SOLUTIONS 2003**

## **COMMUNICATION SYSTEMS**

## ANSWER to Q1

- 1)    A      B      C      D      E
- 2)    A      B      C      D      E
- 3)    A      B      C      D      E
- 4)    A      B      C      D      E
- 5)    A      B      C      D      E
- 6)    A      B      C      D      E
- 7)    A      B      C      D      E
- 8)    A      B      C      D      E
- 9)    A      B      C      D      E
- 10)   A      B      C      D      E
- 11)   A      B      C      D      E
- 12)   A      B      C      D      E
- 13)   A      B      C      D      E
- 14)   A      B      C      D      E
- 15)   A      B      C      D      E
- 16)   A      B      C      D      E
- 17)   A      B      C      D      E
- 18)   A      B      C      D      E
- 19)   A      B      C      D      E
- 20)   A      B      C      D      E

## ANSWER to Q2

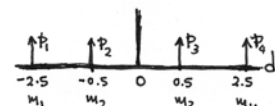
**a)**  $d(kT_s)$  is a Gaussian signal with mean  $\mu_d$  and std  $\sigma_d$  i.e.  $\text{pdf}_d = \mathcal{N}(\mu_d, \sigma_d)$

mean  $= \mu_d = \mathcal{E}\{d(kT_s)\} = \mathcal{E}\{g(kT_s) - g((k-3)T_s)\} = 0 - 0 = 0$   
std  $= \sigma_d = \sqrt{P_d}$

$$\begin{aligned}
 T_s &= \frac{1}{12 \times 10^6} \\
 P_d &= \mathcal{E}\{d^2(kT_s)\} \\
 &= \mathcal{E}\{(g(kT_s) - g((k-3)T_s))^2\} \\
 &= \underbrace{\mathcal{E}\{g^2(kT_s)\}}_{= R_{gg}(0)} + \underbrace{\mathcal{E}\{g^2((k-3)T_s)\}}_{= R_{gg}(0)} - 2 \underbrace{\mathcal{E}\{g(kT_s) \cdot g((k-3)T_s)\}}_{= R_{gg}(3T_s)} \\
 &= 2 R_{gg}(0) - 2 R_{gg}(3T_s) \\
 &= 2 \exp\{-6000 \times 0\} - 2 \exp\{-6000 \times 3 \times \frac{1}{12 \times 10^6}\} \\
 &= 2 - 2 \exp\{-1.5\} = 1.5537
 \end{aligned}$$

**b)**  $\text{pdf}_d = \mathcal{N}(0, \sqrt{1.5537}) = \mathcal{N}(0, 1.2465)$

pdf of  $d_q(kT_s)$  :



$$p_1 = \int_{-\infty}^{-1.2465} \text{pdf}_d(d) dd = \mathcal{T}\left\{\frac{1.2465}{1.2465}\right\} = \mathcal{T}\{1\} = 0.16$$

$$p_2 = 0.5 - p_1 = 0.34$$

$$p_3 = p_2 = 0.34$$

$$p_4 = p_1 = 0.16$$

**c)**

$m_2$	0.34	0.34	0.66	1
$m_3$	0.34	0.34	0.34	
$m_1$	0.16	0.32		
$m_4$	0.16			

i.e.

Source Coder	
$m_1 \mapsto 010$	$l_1 = 3$
$m_2 \mapsto 1$	$l_2 = 1$
$m_3 \mapsto 00$	$l_3 = 2$
$m_4 \mapsto 011$	$l_4 = 3$

**d)**

$$p = \begin{bmatrix} p_1 = 0.16 \\ p_2 = 0.34 \\ p_3 = 0.34 \\ p_4 = 0.16 \end{bmatrix} \quad l = \begin{bmatrix} l_1 = 3 \\ l_2 = 1 \\ l_3 = 2 \\ l_4 = 3 \end{bmatrix}$$

$$\bar{\ell} = p^T l = \sum_{i=1}^4 p_i l_i = 0.16 \times 3 + 0.34 \times 1 + 0.34 \times 2 + 0.16 \times 3 = 1.98 \frac{\text{bits}}{\text{level}}$$

$$r_{\text{data}} = \bar{\ell} \times 12k = 1.98 \times 12k = 23.7 \text{ kbits/s}$$

$$r_{\text{inf}} = \mathbb{H} \times 12k = \underbrace{p^T \log_2(p)}_{\mathbb{H}=\text{entropy}=1.9044} \times 12k = 22.8526 \text{ kbits/s}$$

## ANSWER to Q3

a) Transmitter:

I/P A-law encoder:  $g(kT_s) = -3.7V$

maximum input:  $g_{max} = 5V$

$$\frac{1}{A} = 0.0114 \Rightarrow \frac{1}{A} < x < 1$$

$$\|x\| = \left\| \frac{g(kT_s)}{g_{max}} \right\| = 0.74$$

Therefore, O/P of A-law encoder:  $g_c(kT_s) = \frac{1+\ln(A\|x\|)}{1+\ln(A)} \times g_{max}$

$$\Rightarrow g_c(kT_s) = -4.72496V$$

$$\Rightarrow b_0 < -4.7249V < b_1$$

Therefore, O/P of quantizer =  $m_1 = -4.37V$

Receiver:

I/P A-law decoder:  $m_1 = -4.37V$  (or  $m_1 = -4.375$ )

O/P:  $g_{out}(kT_s) = \frac{1}{A} \exp\left\{\frac{m_1}{g_{max}}(1 + \ln(A)) - 1\right\} \times g_{max}$

$$\Rightarrow g_{out}(kT_s) = -2.509V \text{ (or } -2.5227)$$

$$n_q = -3.7V - (-2.509V) = 1.191V \text{ (or } -3.7 + 2.5227 = 1.1773)$$

b)

$$Q = 8$$

$$\gamma = \log_2 Q = \log_2 8 = 3$$

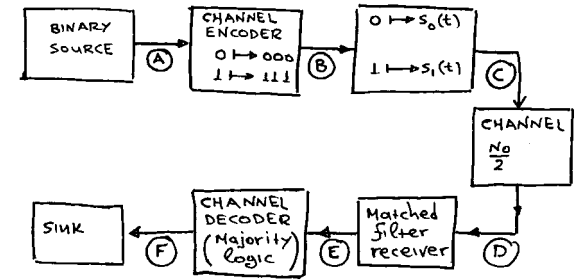
$$SNR_{out} = 4.77 + 6\gamma - 20\log_{10}(1 + \ln A) = 8.0058 \text{ dB}$$

c)

$$n_b = \gamma F_s = 3 \times 18k = 54 \text{ kbits/s (or } 54/8 = 6.75 \text{ kBytes/sec)}$$

$$6.75 \text{ k} \frac{\text{Bytes}}{\text{sec}} \times t = 2\text{GB} \Rightarrow t = \frac{2 \times 10^9 \text{ Bytes}}{6.75 \times 10^3 \text{ Bytes/sec}} = \frac{2}{6.75 \times 3600} \times 10^6 \text{ hours} = 82.3 \text{ hours}$$

## ANSWER to Q4



a)

$$\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2 \times 10^{-12}$$

point A: bit rate = 166.6667 kbits/sec

point B: bit rate =  $3 \times 166.667 \text{ k} = 500 \text{ kbits/sec}$

point C:

channel symbol rate ((point C) = bit rate (point B))

i.e.  $r_{cs} = 500 \text{ k} \frac{\text{channel-symbols}}{\text{sec}}$

$$\Rightarrow T_{cs} = \frac{1}{r_{cs}} = 2 \times 10^{-6} \text{ sec per channel symbol}$$

b)

point D:

$$E_0 = A^2 \frac{T_s}{2} \times 2 = A^2 T_{cs}$$

$$E_1 = A^2 \frac{T_s}{4} \times 4 = A^2 T_{cs}$$

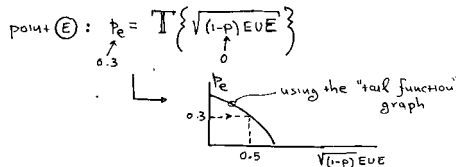
$$E_b = \frac{1}{2}(E_0 + E_1) = A^2 T_{cs}$$

Therefore

$$EUE = \frac{E_b}{N_0} = \frac{A^2 T_{cs}}{2 \times 10^{-12}}$$

Furthermore  $p = \frac{1}{E_b} \int_0^{T_{cs}} s_0(t) \cdot s_1(t) dt = \dots$

$$\Rightarrow p = 0$$



$$\therefore \sqrt{EUE} = 0.5 \Rightarrow EUE = 0.25$$

Therefore

$$\frac{A^2 T_{cs}}{2 \times 10^{-12}} = 0.25 \Rightarrow A^2 = \frac{0.5 \times 10^{-12}}{2 \times 10^{-6}} \Rightarrow A^2 = 0.25 \times 10^{-6} \Rightarrow A = 0.5 \text{ mV}$$

c)

point F:

$$\begin{aligned} \Pr(\text{correct}) &= 1 - \underbrace{\Pr(2 \text{ errors in a 3bit sequ.})} - \underbrace{\Pr(3 \text{ errors in a 3bit sequ.})} \\ &= 1 - \binom{3}{2} p_e^2 (1 - p_e) - \binom{3}{3} p_e^3 (1 - p_e)^0 \\ &= 1 - 3p_e^2(1 - p_e) - p_e^3 \quad (\text{where } p_e = 0.3) \\ &= 0.784 \end{aligned}$$

d)

$$\mathbb{F} = \begin{bmatrix} \Pr(D_0|H_0) & \Pr(D_0|H_1) \\ \Pr(D_1|H_0) & \Pr(D_1|H_1) \end{bmatrix} = \begin{bmatrix} 0.784 & 0.216 \\ 0.216 & 0.784 \end{bmatrix}$$

e)

$$\mathbb{J} = \mathbb{F} \cdot \text{diag}(\underline{p}) = \mathbb{F} \cdot \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} = \begin{bmatrix} \Pr(D_0, H_0) = 0.392 & \Pr(D_0, H_1) = 0.108 \\ \Pr(D_1, H_0) = 0.108 & \Pr(D_1, H_1) = 0.392 \end{bmatrix}$$