UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1998

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER 2.6

STATISTICS

Friday, May 15th 1998, 4.00 - 5.30

Answer THREE questions

For admin. only: paper contains 4 questions

1 A machine component can fail and does so depending on the operating temperature T of the machine. 94% of components fail if $T > 100$; 30% of components fail if $80 < T \leq 100$; 5% fail if $70 < T \leq 80$ and none fail if $T \leq 70$. During a particular operation it is known that $P(T > 100) = 0.01$, $P(80 < T \leq 100) = 0.11$ and $P(70 < T \leq 80) = 0.38$.

a Calculate the probability the component will not fail during an operation.

b Given that the component fails, calculate the probability that the operating temperature was less than or equal to 80.

c The machine breaks down if the component fails. To reduce the risk of this, two components are put in parallel so the machine will break down only if both components fail. Assuming the components act independently of each other, calculate the probability the machine does not break down during an operation.

d To reduce the risk further, n components are put in parallel so all n components would have to fail for the machine to break down. What is the smallest n can be to ensure that the probability the machine does not break down is greater than 0.9999.

2 The life span of a species of fly, in hours, is known to be a random variable X with probability density function

$$f_X(x) = \theta^2 x e^{-\theta x} I_{(0,\infty)}(x),$$

where the parameter $\theta > 0$ is unknown.

a Show that $f_X(x)$ is a probability density function.

b Find an expression, in terms of θ, for the expectation of X.

c Find an expression for the maximum likelihood estimator of θ based on a random sample $X_1, \ldots, X_n$ of size n from f_X. If this estimator is denoted by $\hat{\theta}$, show that $1/\hat{\theta}$ is an unbiased estimator for $1/\theta$.

d 10 life spans are observed to be, in hours,

$$45, 50, 47, 43, 44, 53, 52, 38, 40, 49.$$

Calculate the method of moment estimator for θ.

The four parts carry, respectively, 15%, 15%, 45% and 25% of the marks.

Turn over ...

3 Suppose $X_1, \ldots, X_n$ are independent normal random variables with mean μ and variance σ^2.

a Write down an expression for

$$P\left(\mu - z\sigma/\sqrt{n} < \bar{X} < \mu + z\sigma/\sqrt{n}\right),$$

where $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i$. Leave your answer in terms of the standard normal distribution function $\Phi(x)$.

b If $n = 10$, $\bar{X} = 3.6$ and σ is known to be 4, find 90% and 95% confidence intervals for μ, given that $\Phi(1.96) = 0.975$ and $\Phi(1.65) = 0.95$.

c Now suppose that σ is unknown. Write down an expression for an unbiased estimator of σ^2 in terms of $X_1, \ldots, X_{10}$.

d If $\sum_{i=1}^{10}(X_i - \bar{X})^2 = 174.24$, find the revised 90% and 95% confidence intervals for μ, given that $F_9(2.26) = 0.975$ and $F_9(1.83) = 0.95$, where $F_9(x)$ is the Student t distribution function with 9 degrees of freedom.

The four parts carry, respectively, 15%, 45%, 10% and 30% of the marks.

4 Suppose 30.0, 26.3, 28.9, 30.6 and 32.3 are five independent normal random variables with unknown mean μ and known variance $\sigma^2 = 16$. The hypothesis to be tested is

$$H_0 : \mu \geq 32 \text{ versus } H_1 : \mu > 32.$$

a Find the critical region for the observed sample mean at the $100\alpha\%$ level.

b Hence find the 1%, 5% and 10% level critical regions, given that $\Phi(2.33) = 0.99$, $\Phi(1.65) = 0.95$ and $\Phi(1.29) = 0.90$.

c Say whether the null hypothesis is rejected or not at each of the levels in part b.

d If σ^2 is not 16 but actually 8, would this result in the critical region found in a becoming smaller or bigger?

Hint to part a: Find $P(\bar{X} < \mu - z\sigma/\sqrt{5})$ in terms of $\Phi(z)$ and use the fact that

$$P(\bar{X} < 32 - z\sigma/\sqrt{5}) \leq P(\bar{X} < \mu - z\sigma/\sqrt{5})$$

if $\mu \geq 32$.

The four parts carry, respectively, 45%, 30%, 15% and 10% of the marks.

End of Paper