IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EEE PART II: MEng, BEng and ACGI

Corrected Copy

MATHEMATICS 2A (E-STREAM AND I-STREAM)

Thursday, 29 May 2:00 pm

Time allowed: 1:30 hours

There are TWO questions on this paper.

Answer TWO questions.

Answer both questions

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D. Angeli
Second Marker(s) : R.R.A. Syms

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Appendix A

Table of Laplace Transforms

$f(t)$	$F(s)$
$\delta(t)$	1
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{\lambda t}\sin(\omega t)$	$\frac{\omega}{(s-\lambda)^2+\omega^2}$
$e^{\lambda t}\cos(\omega t)$	$\frac{(s-\lambda)}{(s-\lambda)^2+\omega^2}$
$e^{\lambda t}$	$\frac{1}{s-\lambda}$
$te^{\lambda t}$	$\frac{1}{(s-\lambda)^2}$
$\frac{t^n e^{\lambda t}}{n!}$	$\frac{1}{(s-\lambda)^{n+1}}$
$t\sin(\omega t)$	$\frac{2s}{(s^2+\omega^2)^2}$
$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$

1. Consider the function $u : \mathbb{R}^2 \to \mathbb{R}$ defined as:

$$u(x,y) = \frac{e^y \cos(x)}{2} + 2xy + \frac{e^{-y}\cos(x)}{2}.$$

a) Show that u is harmonic; [6]

b) Use the Cauchy-Riemann formulae to find the conjugate of u, $v : \mathbb{R}^2 \to \mathbb{R}$; [7]

c) Denote $z = x + iy$; find the expression for a holomorphic function f, such that:

$$f(z) = u(x,y) + iv(x,y)$$

[3]

Consider the holomorphic function f of a complex variable z defined below:

$$f(z) = \frac{z}{(z^2+1)(z^2+4)(z^2+9)}.$$

a) Find the domain of the function f; [1]

b) Compute the residues of the function at its poles; [6]

c) Use Cauchy's formula to compute the improper integral:

$$\int_{-\infty}^{+\infty} \frac{x}{(x^2+1)(x^2+4)(x^2+9)} dx.$$

[5]

d) Could the integral also be evaluated by other methods?
(Hint: argue that f is an odd function). [2]

2. Consider the linear differential equation:

$$x^{(3)}(t) + \ddot{x}(t) + \dot{x}(t) + x(t) = 0$$

where $x : \mathbb{R} \to \mathbb{R}$ is a scalar function of the unknown variable t, and with the initial conditions $\ddot{x}(0) = \dot{x}(0) = 0$, $x(0) = 1$. (Notation: $x^{(3)}(t)$ denotes the third order derivative of $x(t)$).

Use Laplace's transform (and anti-transform) to compute an expression for $x(t)$, solution of the previous equation. [20]

(In the question marks are allocated as follows: 6 marks for correct transformed equation; 4 marks for correct frequency domain solution; 5 marks for correct Heaviside decomposition; 5 marks for correct time-domain expressions).