E3.12
SO13

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

MSc and EEE PART III/IV: MEng, BEng.and ACGI

Corrected Copy

OPTOELECTRONICS

Thursday, 5 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : R.R.A. Syms
Second Marker(s) : W.T. Pike

Fundamental constants

$e = 1.6 \times 10^{-19}\ C$

$m_e = 9.1 \times 10^{-31}\ kg$

$\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$

$\mu_0 = 4\pi \times 10^{-7}\ m\ kg/C^2$

$c = 3 \times 10^{8}\ m/s$

$h = 6.62 \times 10^{-34}\ Js$

1a) Figure 1 shows a ray representation of two coherent, interfering optical waves. Assuming that both waves have equal amplitude, are polarized in the y-direction, have wavelength λ and are propagating in free space, write down a time-independent expression for the combined field as i) the sum of two plane waves, and ii) a z-propagating mode. What is the transverse field variation of the mode? What is its propagation constant? Explain how the solution you have found relates to the modal solutions for guided waves propagating in a slab waveguide.

[8]

b) Calculate the corresponding magnetic field of the modal solution above. Derive an expression for the z-component of the irradiance. Sketch the variation, and explain how it depends on θ and λ.

[12]

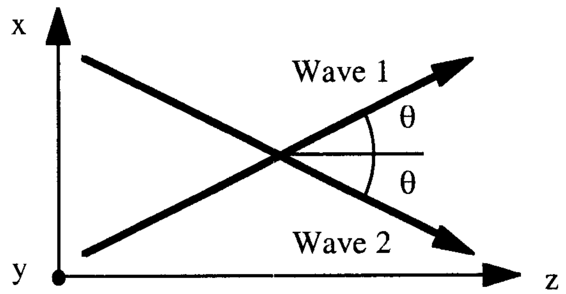

Figure 1.

2a) For a symmetric dielectric waveguide formed from a core of thickness h and refractive index n_1 surrounded by a cladding of refractive index n_2, the eigenvalue equation is:

$\tan(\kappa h/2) = \gamma/\kappa$ for all modes with symmetric field patterns

$\tan(\kappa h/2) = -\kappa/\gamma$ for all antisymmetric modes.

Where $\kappa = \sqrt{(n_1^2 k_0^2 - \beta^2)}$ and $\gamma = \sqrt{(\beta^2 - n_2^2 k_0^2)}$.

And k_0 and β are the propagation constants of a free-space wave and the guided mode

Derive the cutoff condition for a guided mode of order v. Plot a graph showing the maximum value of h for single-mode operation at a wavelength of 0.85 μm as a function of the index step Δn up to $\Delta n = 0.3$, assuming that $n_1 \approx n_2 \approx 3.57$.

[8]

b) Explain briefly the process of epitaxy as used in III-V optoelectronics. The refractive index of $Ga_{1-x}Al_xAs$ varies as n = 3.57 - 0.6286x. Sketch the construction of a symmetric double heterostructure waveguide formed using this material system. Indicate the layering, the refractive index variation, the location and the transverse field variation of the guided mode and any critical dimensions, assuming that the guide is to have a Δn of 0.1 and be single-moded at a wavelength of 0.85 μm.

[12]

3a) Describe the construction of an electro-optic phase modulator in the Ti: $LiNbO_3$ integrated optics system. What limits the modulation bandwidth that can be obtained with lumped element electrodes, and how can this limitation be overcome?

[8]

b) Figure 2 shows an unbalanced Y-junction-based Mach-Zehnder inferometric modulator. Explain its operation. With no volts applied to the phase modulator, the transmission is 50%; when a voltage of -2.5 V is applied, the transmission rises to 85.3%. Find the voltage needed for 100% transmission, and the voltage-length product for π radians phase shift in the phase modulator.

[12]

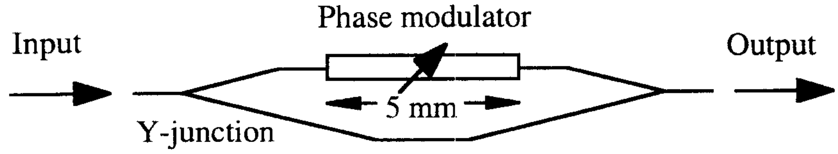

Figure 2.

4. Figure 3 shows an arrayed waveguide grating multiplexer. The device is constructed from two identical radiative star couplers, which are linked by an array of channel guides. Each star contains $N = 2M + 1$ input and output ports, arranged at regular angular intervals α on a planar waveguide region with curved boundaries of radius R. In the array, the guide lengths vary as $L_q = L_0 + q\Delta L$, where q is the guide number.

a) Assuming that the device is lossless, derive an expression for the amplitude A_q at Port q when a mode of amplitude A_p is incident on Port p. Use it to obtain expressions for the amplitude A_{pqr} at Port r due to the component travelling via guide q, and for the amplitude A_{pr} due to all contributions.

[10]

b) Show that the output power obtained when $p = r = 0$ is $(P_0/N^2)\,|\sin(N\gamma)/\sin(\gamma)|^2$, where P_0 is the input power and $\gamma = \beta q\Delta L/2$.
You may assume that $\sum_{q=-M}^{M} x^q = (x^{N/2} - x^{-N/2})/(x^{1/2} - x^{-1/2})$.

[10]

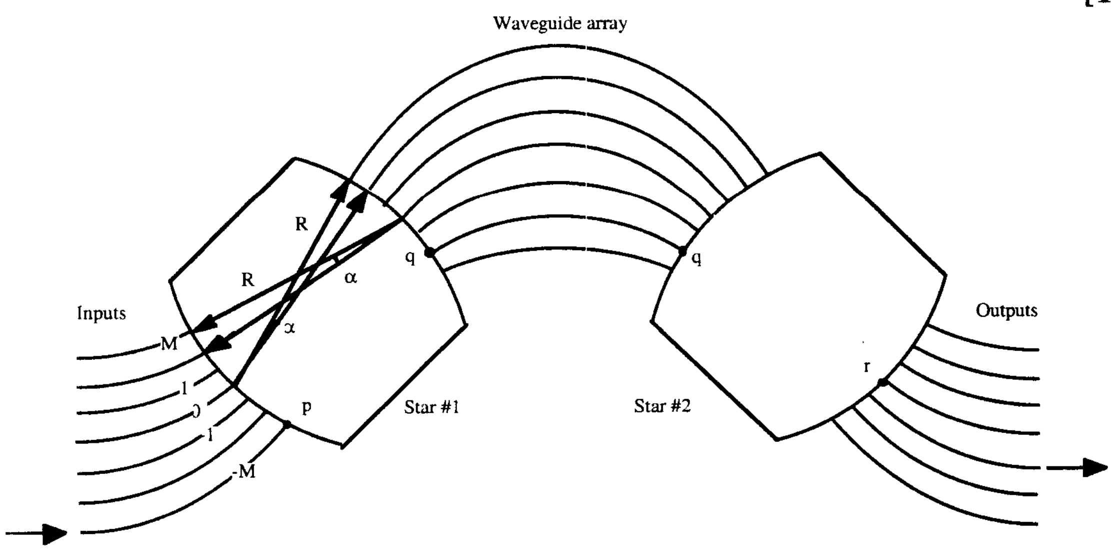

Figure 3.

5a) Describe the process of optical absorption in a semiconductor. Explain the operation of a photoconductive detector. Why is this device of limited use in a high-speed optical communications system? How does a photodiode offer improved performance? What limits the quantum efficiency in a p-n photodiode? What are the further advantages of a p-i-n photodiode?

[13]

b) When tested with a variety of different laser sources, an $In_{0.53}Ga_{0.47}As$ photodiode is found to have the performance characteristics shown in Table I. Plot a graph of quantum efficiency versus wavelength. If the energy gap of the photodiode material is 0.74 eV, estimate the wavelength at which its efficiency will fall to zero.

[7]

Laser type	Wavelength (μm)	Power (mW)	Photocurrent (mA)
Argon	0.5145	100	21.97
HeNe	0.633	1	0.306
GaAs	0.87	5	2.45
Nd: YAG	1.06	50	32.02
InGaAs	1.33	10	8.25
InGaAs	1.55	5	4.56

Table I.

6a) Explain why light-emitting diodes (LEDs) are not the preferred sources in optical communications systems. Sketch the construction of double heterostructure surface–emitting and edge emitting LEDs, and explain any performance advantages offered by the latter.

[14]

b) In a two-state, lumped-element model, the rate equations governing the internal operation of an LED are:

$dn/dt = I/ev - n/\tau_e$
$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$

Here n and ϕ are the electron and photon densities, I is the current, v is the active volume, τ_e is the electron lifetime, τ_{rr} is the radiative recombination lifetime and τ_p is the photon lifetime.

A GaAs LED emitting at 0.85 μm wavelength has an active volume measuring 100 μm x 100 μm x 1 μm. A step-change in current of 10 mA is applied to the LED at t = 0. Calculate and sketch the time variation of the injected electron density and the internally generated optical power, assuming that $\tau_e \approx 1$ nsec and $\tau_{rr} \approx 2$ nsec. Hence, estimate the maximum data rate that the LED may be used to transmit, assuming that signals should reach 80% of full power to define a bit.

[6]

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Optoelectronics 2005 - Solutions

1a) If both waves are polarised in the y-direction, travel in free space, and have equal amplitude E_y, their time-independent electric fields are given by:

$\underline{E}_1 = \underline{j}\ E_y \exp[-jk_0(z \cos(\theta) + x \sin(\theta))]$ For wave 1, and:
$\underline{E}_2 = \underline{j}\ E_y \exp[-jk_0(z \cos(\theta) - x \sin(\theta))]$ For wave 2, where $k_0 = 2\pi/\lambda$

The total field can be written as a linear superposition of these two fields, in the form:

$$\underline{E} = \underline{j}\ E_y \{\exp[-jk_0(z \cos(\theta) + x \sin(\theta))] + \exp[-jk_0(z \cos(\theta) - x \sin(\theta))]\}$$

[2]

Grouping together common exponential terms, the solution may be rearranged as:

$$\underline{E} = \underline{j}\ 2E_y \exp[-jk_0 z \cos(\theta)] \cos[k_0 x \sin(\theta)]$$

[2]

This can be written in the alternative form:

$$\underline{E} = \underline{j}\ E(x) \exp(-j\beta z)$$

Here $E(x) = 2E_y \cos[(k_0 x \sin(\theta)]$ is the transverse field of the mode and $\beta = k_0\cos(\theta)$ is the propagation constant.

[2]

The field solutions in the core of a waveguide are typically constructed from upward and downward going waves, generated by total reflection at the boundaries. The transverse field variation in the core is therefore always a portion of a cosinuisoidal function, with the phase origin and the periodicity determined by the boundary conditions and the mode number. Outside the core, the field typically decays away exponentially. Matching of the fields inside and outside the core is carried out, by applying boundary conditions.

[2]

b) The time independent magnetic field $\underline{H}$ may be found using $\nabla \times \underline{E} = -j\omega\mu_0 \underline{H}$

Since $\underline{E}$ only has a y-component, we may write:

$$\nabla \times \underline{E} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix} = -\underline{i}\ \partial E_y/\partial z + \underline{k}\ \partial E_y/\partial x$$

Carrying out the necessary manipulations, we get:

$$H_x = 2E_y (-\beta/\mu_0\omega) \exp(-j\beta z) \cos[k_0 x \sin(\theta)]$$
$$H_z = 2E_y (-jk_0 \sin\theta /\mu_0\omega) \exp(-j\beta z) \sin[k_0 x \sin(\theta)]$$

[4]

For time independent fields, the irradiance is given by $\underline{S} = 1/2 \text{ Re } [\ \underline{E} \times \underline{H}^*\ ]$
For the fields above, the vector cross product term is:

$$\underline{E} \times \underline{H}^* = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & E_y & 0 \\ H_x^* & 0 & H_z^* \end{vmatrix} = -\underline{k}\ E_y H_x^*$$

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[signature]

Hence, the z-component of the irradiance is:
$S_z = -1/2 \ \mathrm{Re}\ [E_y H_x^*] = 4E_y^2\ (\beta/2\mu_0\omega)\ \cos^2[k_0 x\ \sin(\theta)]$

[4]

The irradiance would therefore be visible as a pattern of straight fringes, oriented parallel to the z-axis, as shown in Figure1. The spatial frequency of the pattern is $K = k_0\sin(\theta)$, so the separation between fringes is $\Lambda = 2\pi/K$. Λ decreases as the angle between the two beams increases, and is of the same order as the wavelength of the light.

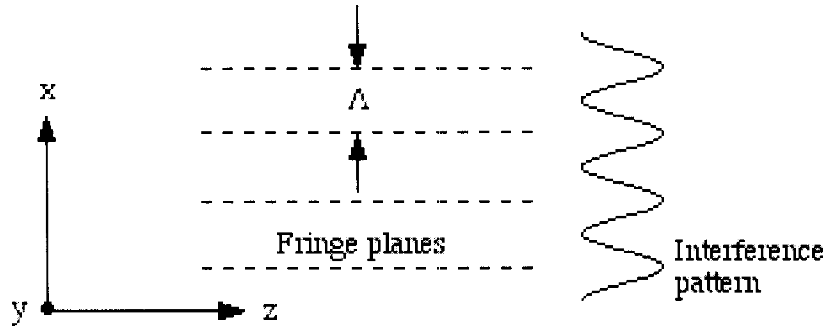


Figure 1.

[4]

2a) For a symmetric dielectric guide formed from a core of thickness *h* and refractive index n_1 surrounded by a cladding of refractive index n_2, the eigenvalue equation is:
$\tan(\kappa h/2) = \gamma/\kappa$ for all modes with symmetric field patterns
$\tan(\kappa h/2) = -\kappa/\gamma$ for all antisymmetric modes.
Where $\kappa = \sqrt{(n_1^2 k_0^2 - \beta^2)}$ and $\gamma = \sqrt{(\beta^2 - n_2^2 k_0^2)}$.

At cutoff, total internal reflection ceases to occur and the guided mode is no longer confined to the core. When this occurs, $\beta \to n_2^2 k_0^2$. Hence, at cutoff
$\tan(\kappa h/2) \to 0$ for all modes with symmetric field patterns
$\tan(\kappa h/2) \to -\infty$ for all antisymmetric modes.

In this case, we have:
$\kappa h/2 \to 0, \pi, 2\pi, 3\pi \ldots$ for all modes with symmetric field patterns
$\kappa h/2 \to \pi/2, 3\pi/2, 5\pi/2 \ldots$ for all antisymmetric modes.

Thus, in general the cutoff condition for a mode of order ν is $\kappa h/2 = \nu\pi/2$, and hence:
$(k_0 h/2)\sqrt{(n_1^2 - n_2^2)} = \nu\pi/2$

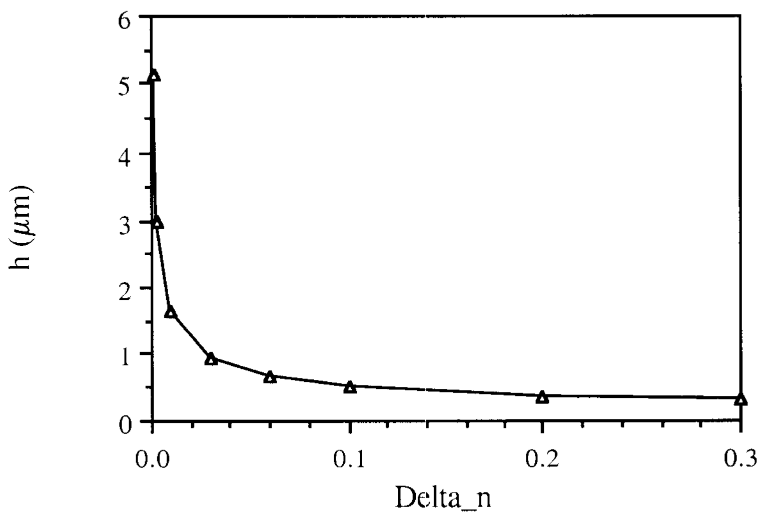
[3]

Re-arranging the above, we have $(\pi h/\lambda)\sqrt{(n_1^2 - n_2^2)} = \nu\pi/2$. For $\nu = 1$, we then obtain:
$h = \lambda/\{2\sqrt{(n_1^2 - n_2^2)}\}$ or:
$h = \lambda/\{2\sqrt{[(n_1 + n_2)(n_1 - n_2)]}\} = \lambda/\{2\sqrt{(2n\Delta n)}\}$

[2]

Assuming that $\lambda = 0.87\ \mu$m, we get (for example):

Δn	h (μm)
0.001	5.14802
0.003	2.97221
0.01	1.62795
0.03	0.93989
0.06	0.66461
0.1	0.51480
0.2	0.36402
0.3	0.29722



[3]

b) Epitaxy - should mention:

- Ordered growth of crystalline material
- III-V materials important in optoelectronics are GaAs/GaAlAs and InP/InGaAsP
- Substrates are always binary materials (e.g. GaAs or InP)
- Growth must be lattice matched, so composition of e.g. $In_{1-x}Ga_xAs_{1-y}P_y$ is restricted
- Material must be direct gap for emission, so composition of e.g. $Ga_{1-x}Al_xAs$ is restricted

[4]

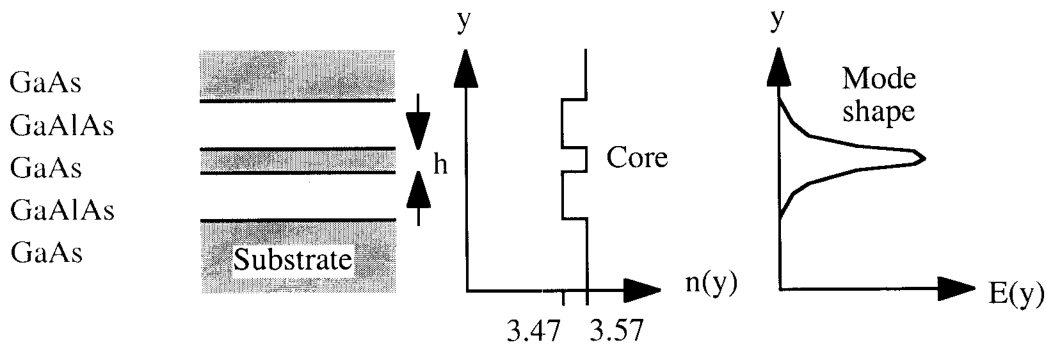
Because the refractive index of $Ga_{1-x}Al_xAs$ falls with x following n = 3.57 - 0.6286x, a waveguide cannot be made directly on a GaAs substrate. Instead a buffer layer of GaAlAs must first be grown, and a GaAs waveguide then grown on top. To obtain an index step of - 0.1, the necessary value of x is x = 0.1/0.6286 = 0.15908.

[2]

The maximum value of h for single mode operation can be obtained part a) as 0.51480 μm

[2]

If a symmetric double heterostructure guide is required, the result is then as shown below.



[4]

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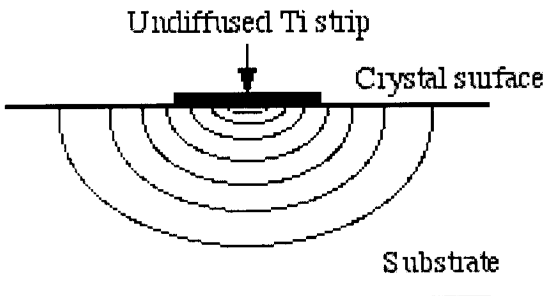
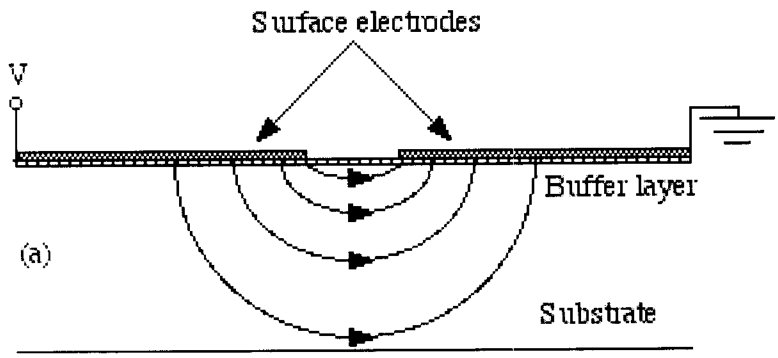
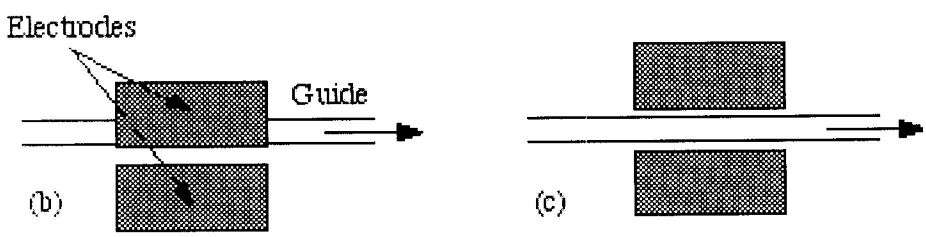
3a) Ti: $LiNbO_3$ waveguides are fabricated by diffusion. Titanium metal can be diffused into lithium niobate substrates, by first depositing the metal in patterned strips of ≈ 1000 Å thickness and then carrying out an in-diffusion at a high temperature (≈ 1000°C) for 3 to 9 hours. The additional impurities cause a change in refractive index that is approximately proportional to their concentration, with a typical maximum value of $\Delta n \approx 0.01$. The figure below shows contours of constant refractive index obtained by diffusion of Ti into $LiNbO_3$.



[3]

Figure a) below shows a cross-section of a Ti:$LiNbO_3$ phase shifter, which uses a static electric field to alter the refractive index of a waveguide via the electro-optic effect. The field is applied through a pair of electrodes deposited on the surface of the crystal. Figures b) and c) show alternative geometries, which are used with different polarizations. A thin dielectric buffer layer is used to space the electrodes away from the guides, to reduce absorption. The static field alters the refractive index in the region of the guide, which in turn alters the phase of a guided optical wave. Due to the strong confinement of the mode, the electrode gap can be small (of the order of a waveguide width), so that a high field can be created using a small voltage (≈ 10 V).





[3]

The parallel plate electrodes present a capacitative load, and combine with the output impedance R (typically, 50 Ω) of the source to create a first-order R-C time constant, limiting the maximum operating frequency. The solution is to replace the electrodes with a coplanar electrical transmission line, which has real impedance.

[2]

R.R.A. Syms
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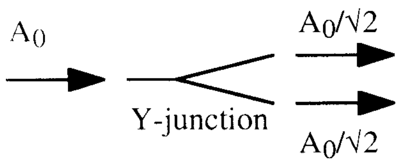
b) The MZI modulator works as follows. The first Y-junction will split an input of amplitude A_0 into two beams, each of amplitude $A_0/\surd 2$, as shown below. Here, the denominator accounts for power conservation.

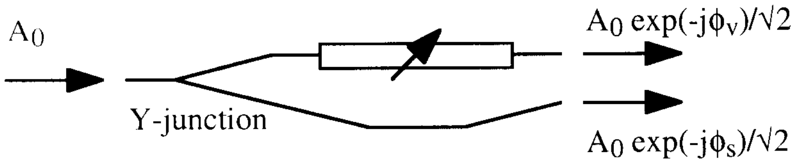


After passing through the body of the device, the two beams will have incurred an electro-optically-induced phase shift ϕ_v and a static phase shift ϕ_s, as shown below.
The two intermediate amplitudes are:

Upper: $A_0 \exp(-j\phi_v)/\surd 2$
Lower: $A_0 \exp(-j\phi_s)/\surd 2$

These two amplitudes may be rewritten as:

Upper: $A_0 \exp\{-j(\phi_v + \phi_s)/2\} \exp\{-j(\phi_v - \phi_s)/2\}$
Lower: $A_0 \exp\{-j(\phi_v + \phi_s)/2\} \exp\{+j(\phi_v - \phi_s)/2\}$

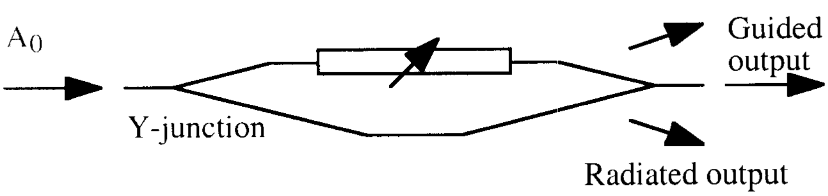


The second Y-junction will pass the symmetric component of this combined field, and radiate the anti-symmetric component, as shown below. After accounting for power conservation once again, the output amplitudes are:

Guided: $A_0 \exp\{-j(\phi_v + \phi_s)/2\} \cos\{(\phi_v - \phi_s)/2\}$
Radiated: $-jA_0 \exp\{-j(\phi_v + \phi_s)/2\} \sin\{(\phi_v - \phi_s)/2\}$

The output powers may be found from the modulus squared of the output amplitudes, as:

Guided: $A_0^2 \cos^2\{(\phi_v - \phi_s)/2\} = A_0^2\{1 + \cos(\phi_v - \phi_s)\}/2$
Radiated: $A_0^2 \sin^2\{(\phi_v - \phi_s)/2\} = A_0^2\{1 - \cos(\phi_v - \phi_s)\}/2$



The guided output therefore varies cosinusoidally with ϕ_v, with a phase offset of ϕ_s.

[6]

With no voltage applied to the phase modulator, the transmission is 50%.
Hence, $\{1 + \cos(-\phi_s)\}/2 = 0.5$, and the static phase shift is $\phi_s = \pi/2$.
With -2.5V applied to the modulator, the transmission rises to 85.3%.
Hence $\{1 + \cos(\phi_{-2.5} - \pi/2)\}/2 = 0.853$ and $\cos(\phi_{-2.5} - \pi/2) = 0.706$
The solution corresponding to the smallest electrooptic phase shift is $\phi_{-2.5} = \pi/4$
For 100% transmission, we need a phase shift ϕ_v such that $\phi_v - \pi/2 = 0$, namely $\phi_v = \pi/2$.
Since the electrooptic effect is linear, this requires a voltage of 2 x -2.5V, or -5V.

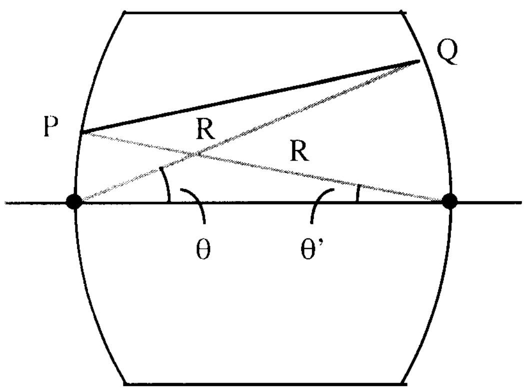
[4]

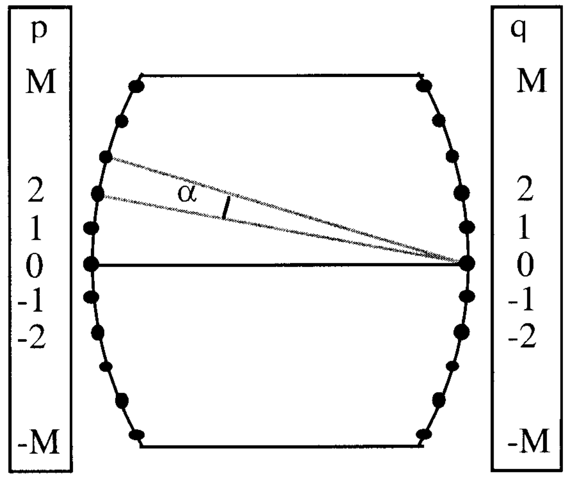
$\phi_v = \pi$ then requires a voltage of -10V and the voltage-length product is 10 x 5 = 50 V mm.

[2]

4a) Referring to the figure below, the distance PQ between points P and Q is:

$$PQ^2 = R^2\{\{\sin(\theta) - \sin(\theta')\}^2 + \{1 - [1 - \cos(\theta)] - [1 - \cos(\theta')]\}^2\}$$





Assuming small angle approximations, we get:

$$PQ^2 \approx R^2\{(\theta - \theta')^2 + (1 - \theta^2/2 - \theta'^2/2)^2\}$$

Using the binomial approximation, we then get:

$$PQ \approx R(1 - \theta\theta')$$

When P and Q are chosen from a set of N equally separated and symmetrically located points, we may write $\theta' = \alpha p$ and $\theta = \alpha q$, where α is the angular separation of adjacent points and q is the port number. In this case, the path length PQ is:

$$PQ \approx R(1 - \alpha^2 pq)$$

When a single amplitude A_p is presented at point P on the input surface, the amplitude at Q may be found by making allowances for the change in phase and amplitude incurred in propagating the distance PQ. If this is done, we obtain:

$$A_q = (A_p/\sqrt{N}) \exp\{-j\beta R(1 - \alpha^2 pq)\}$$

Here β is the propagation constant, while the factor of $\sqrt{N}$ accounts for power conservation, since the input power is split N ways without loss.

[5]

After passing through an additional delay of length $L_q = L_0 + q\Delta L$ following guide Q in the waveguide array, the amplitude becomes:

$$A_q' = (A_p/\sqrt{N}) \exp(-j\beta\{R[1 - pq\alpha^2] + L_0 + q\Delta L\}$$

This amplitude is then the q^{th} input of the right-hand star, and finally emerges from (say) the r^{th} output waveguide. Taking into account the effect on both amplitude and phase of the second star, we may obtain the output as:

$$A_{pqr} = (A_p/N) \exp(-j\beta\{R[1 - pq\alpha^2] + R[1 - rq\alpha^2] + L_0 + q\Delta L\})$$

R.R.A. Syms

Combining terms and summing over all possible values of q, the output amplitude is then:

$A_{pr} = (A_p\exp\{-j\beta[2R + L_0]\} /N) \, {}_{q=-M}\Sigma^M \exp(-j\beta q[\Delta L - (p + r)R\alpha^2])$

[5]

b) The output power $P_{pr} = |A_{pr}|^2$is:

$P_{pr} = (P_p/N^2) \, | {}_{q=-M}\Sigma^M \exp(-j\beta q[\Delta L - (p + r)R\alpha^2])|^2$ Where $P_p = |A_p|^2$.

[4]

If the input is to the central guide (p = 0), and the output is also taken from the central guide (r = 0), this reduces to:

$P_{00} = (P_0/N^2) \, | {}_{q=-M}\Sigma^M \exp(-j\beta q\Delta L)|^2$

The summation above has the form ${}_{q=-M}\Sigma^M x^q$, where $x = \exp(-j\beta q\Delta L)$.
This can be written as:

${}_{q=-M}\Sigma^M x^q = (x^{N/2} - x^{-N/2}) /(x^{1/2} - x^{-1/2})$

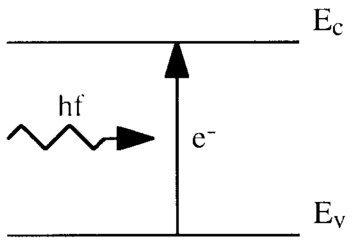
Hence:

$P_{00} = (P_{in}/N^2) \, | \sin(N\beta q\Delta L/2) / \sin(\beta q\Delta L/2)|^2$

[6]

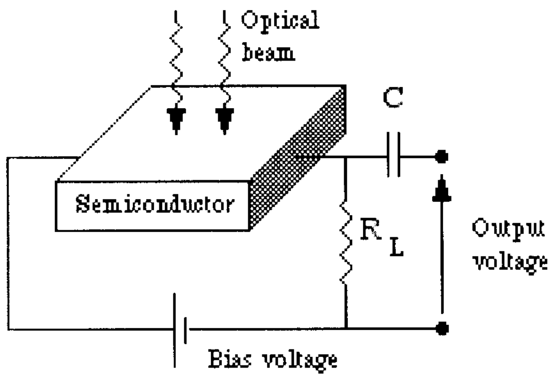
R.R.A. Syms

5a) The figure below illustrates the process of optical absorption in a semiconductor. A photon of energy hf is incident on a material with an energy gap $E_g = E_c - E_v$, where E_c and E_v are the conduction and valence band energies. Provided $hf > E_g$, an electron may be promoted from the valence band to the conduction band, absorbing the photon and generating an electron-hole pair in the process, so that the conductivity is increased.



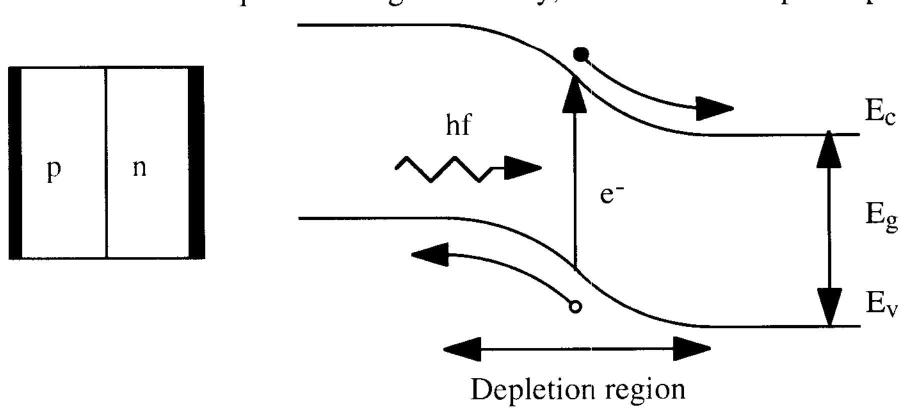
[2]

In a photoconductive detector, a voltage is applied across a slab of semiconductor, and the rise in conductivity obtained on illumination is detected as an increased flow of current and an increase in voltage across the load R_L. There are two limitations. First, the dark conductivity will not be zero, due to the inevitable presence of thermally generated carriers, so there will be a DC voltage across R_L. Although this may be blocked by the capacitor C, thermal carriers generate noise. Second, a long electron lifetime is needed for high sensitivity. However, large τ_e implies that photo-generated carriers will persist after the beam is switched off. It is therefore difficult to combine high sensitivity with rapid response.



[3]

In a photodiode, absorption is arranged to take place in the depletion region of a p-n junction. This region is devoid of thermal carriers, and contains a high electric field that can sweep the photo-generated carriers apart before they can recombine. A long recombination time is then not required for high efficiency, and a far more rapid response may be obtained.



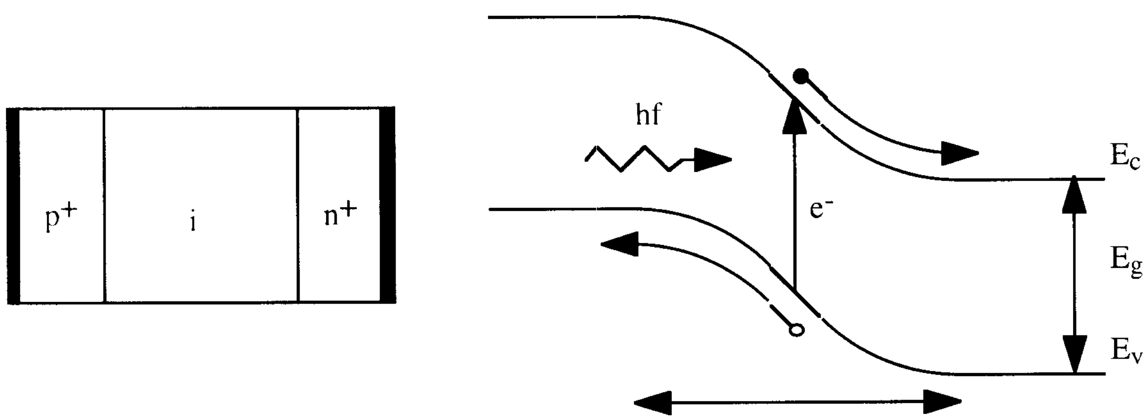
[3]

RR [signature]

A surface entry photodiode suffers from a reduction in quantum efficiency at both short and long wavelengths. In the former case, the photons are so energetic that they are absorbed before they reach the depletion layer. In the latter case, the photons have so little energy that they pass through the depletion layer before being absorbed.

[2]

The basic limitation of a p-n junction photodiode - that the depletion layer is so thin that radiation of long wavelength is only weakly absorbed - is overcome in the p-i-n structure shown in the figure below. Here, a region of intrinsic or lightly doped material is introduced between two heavily doped p- and n-type regions. Because the doping is so low in this region, the depletion layer can then be arranged to extend right through it under a modest reverse bias. The effective depletion layer width may therefore be fixed at a value far greater than the 'natural' one, approximately the width of the intrinsic region.



[3]

b) If the optical power is P, the number of photons per second falling on the detector is:

$P/hf = P\lambda/hc$

If each of these photons generated an electron-hole pair that reached the contacts, the photocurrent would be:

$I_p = Pe\lambda/hc$

In practice, not all the carriers reach the contacts. This effect is described by the quantum efficiency h, so that the actual photocurrent is:

$I_p = \eta Pe\lambda/hc$

Re-arranging the above, the quantum efficiency may be extracted as:

$\eta = I_p hc/Pe\lambda$

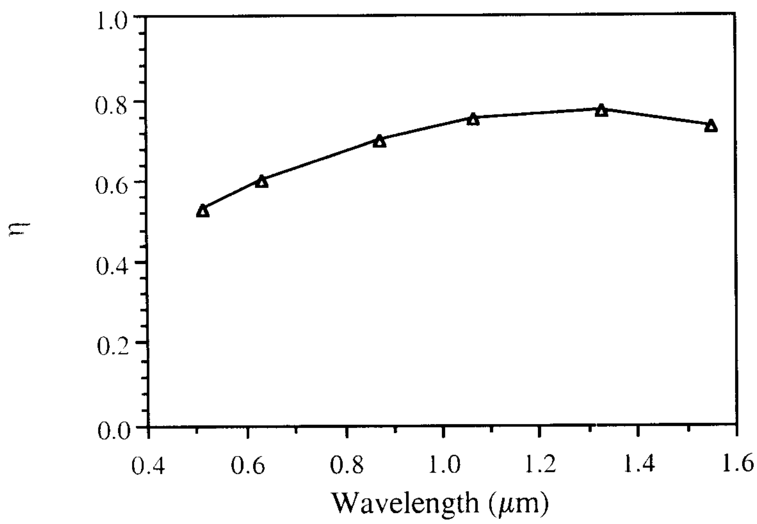
For the data in Table I, we obtain:

Laser type	Wavelength (μm)	Power (mW)	Photocurrent (mA)	η
Argon	0.5145	100	21.97	0.53
He Ne	0.633	1	0.306	0.60

R.R.A. Syms

GaAs	0.87	5	2.45	0.70
Nd: YAG	1.06	50	32.02	0.75
InGaAs	1.33	10	8.25	0.77
InGaAs	1.55	5	4.56	0.73

The results lead to the following spectral variation of quantum efficiency:



[5]

The photodiode will respond provided $hf = hc/\lambda > eE_g$ (if E_g is measured in eV). The longest wavelength that can be detected is then:

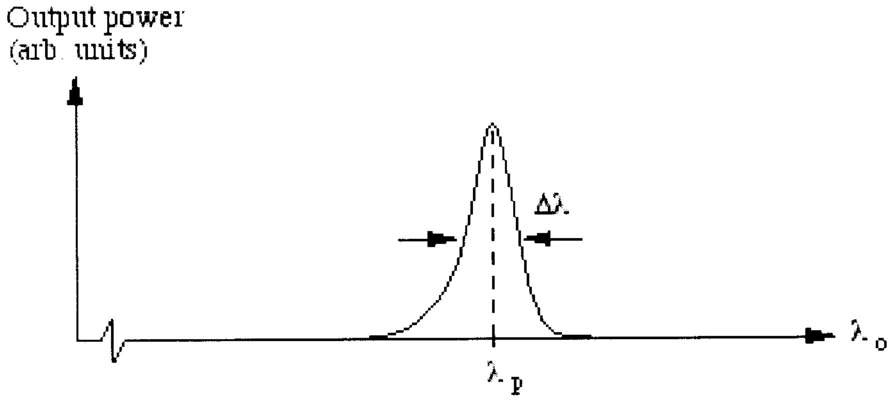
$\lambda = hc/eE_g = (6.62 \times 10^{-34} \times 3 \times 10^8) / (1.6 \times 10^{-19} \times 0.74) = 1.677 \times 10^{-6}$ m, or 1.677 μm.

[2]

R.R.A. Syms

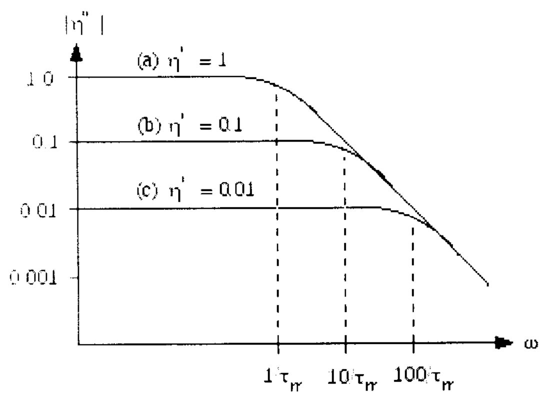
6.a) The LED suffers from a number of key disadvantages, which limit its use as a source in an optical communications:

i) Extended bandwidth: $\Delta\lambda \approx 3.1\ kT/E_g^2$



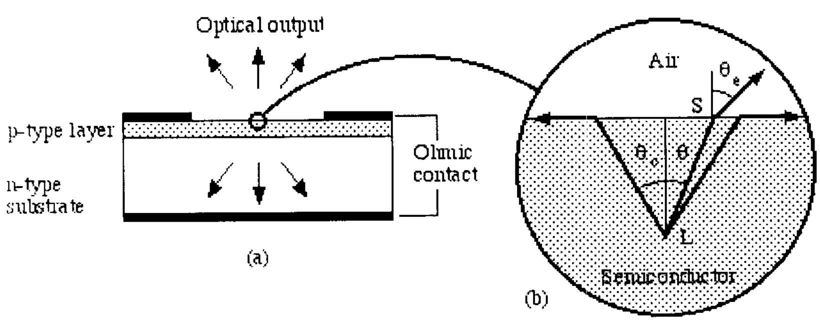
[2]

ii) Limited frequency response: LEDs have a first order response with a break point at $\omega_c = 1/\eta'\tau_{rr}$, where $\eta' = \tau_e/\tau_{rr}$ is the DC internal efficiency. There is a trade off between speed and efficiency.



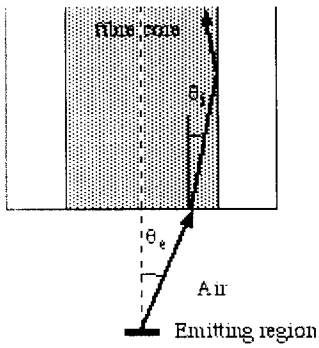
[2]

iii) Poor external efficiency – much of the emission is totally reflected at the interface between the semiconductor and air. The external efficiency is $\eta_e \approx 1/n(n + 1)^2$. For GaAs, (n = 3.5), $\eta_e \approx 1.4\%$.



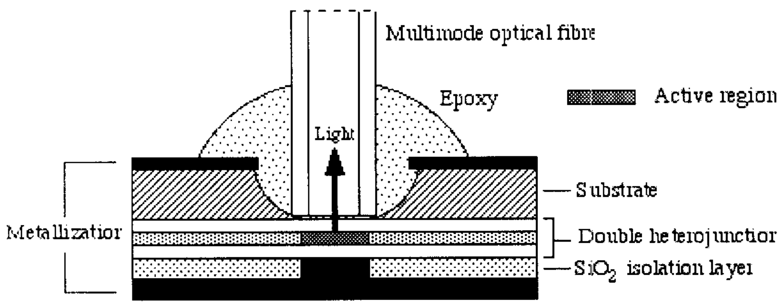
[2]

iv) Poor coupling efficiency to an optical fibre. The external efficiency is $\eta_c = T'(0)\, NA^2$, where $T'(0) = 4n'/(n'+1)^2$ is the transmission into the fibre core, which has index n' and a numerical aperture of NA.



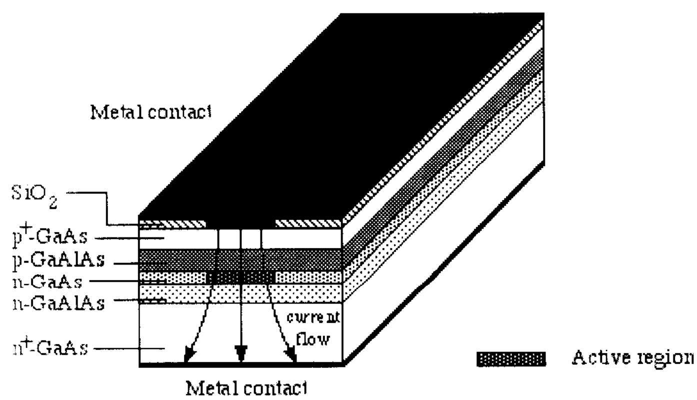
[2]

Surface emitting DH LED:



[2]

Edge emitting LED:



[2]

The edge emitting LED uses the double heterostructure as a waveguide, hence channeling a larger fraction of the emission into the forward direction. This light is a better match to the input requirements of an optical fibre. Consequently, the external efficiency is increased. The internal efficiency may therefore be reduced, allowing the modulation bandwidth to be increased. ELEDs are therefore faster and more efficient than surface emitting LEDs.

[2]

b) The rate equations governing the internal operation of an LED are:

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$dn/dt = I/ev - n/\tau_e$
$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$

Here n and ϕ are the electron and photon densities, I is the current, v is the active volume, τ_e is the electron lifetime, τ_{rr} is the radiative recombination lifetime and τ_p is the photon lifetime.

The first equation may be rewritten as:

$dn/dt + n/\tau_e = I/ev$

This solution may be found by the PI and CF method. The complementary function is:

$n = n_0 \exp(-t/\tau_e)$

While the particular integral is:

$n = I\tau_e/ev$

The general solution is then a sum of the PI and CF, with n_0 chosen to satisfy the boundary conditions. In this case, assuming n = 0 at t = 0 gives:

$n = I\tau_e/ev \{1 - \exp(-t/\tau_e)\} = n_{max}\{1 - \exp(-t/\tau_e)\}$ where $n_{max} = I\tau_e/ev$

Because τ_p is so short, we may neglect $d\phi/dt$ in the second equation. The emitted light per unit volume is ϕ/τ_p, and the total light flux is therefore $\Phi = \phi v/\tau_p = nv/\tau_{rr}$. In this case, we get:

$\Phi = (I/e)(\tau_e/\tau_{rr})\{1 - \exp(-t/\tau_e)\}$

Each photon carried an energy $hf = hc/\lambda$, where h is Plank's constant, c is the velocity of light and λ is the wavelength, so the internal power is:

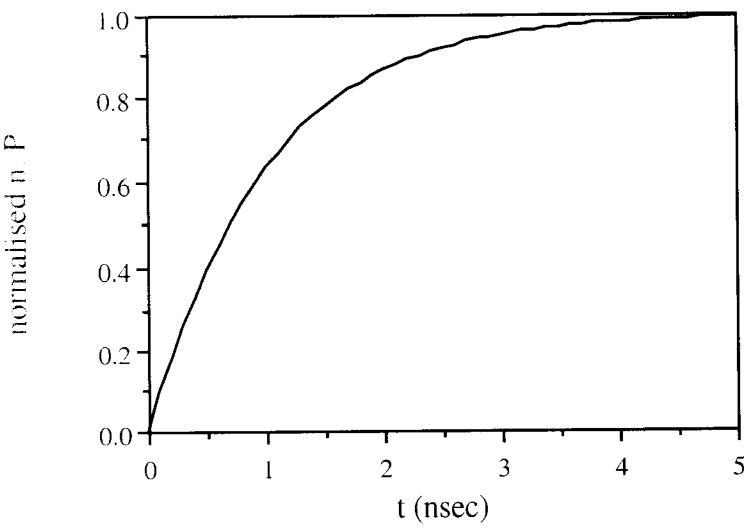
$P = (I/e)(\tau_e/\tau_{rr})(hc/\lambda)\{1 - \exp(-t/\tau_e)\} = P_{max}\{1 - \exp(-t/\tau_e)\}$ where $P_{max} = (I/e)(\tau_e/\tau_{rr})(hc/\lambda)$ **[3]**

In this case:

$I = 10^{-2}$ A
$v = 100\ \mu m \times 100\ \mu m \times 1\ \mu m = 10^{-14}\ m^{-3}$
$\tau_e \approx 1$ nsec and $\tau_{rr} \approx 2$ nsec, so $(\tau_e/\tau_{rr}) = 0.5$
$e = 1.6 \times 10^{-19}$ C
$h = 6.62 \times 10^{-34}$ Js
$c = 3 \times 10^{8}$ m/s
$\lambda = 0.85 \times 10^{-6}$ m

So:
$n_{max} = 6.25 \times 10^{21}\ m^{-3}$
$P_{max} = 7.301 \times 10^{-3}$ W

For each of n and P, the response to a step change is therefore an exponential rise to a maximum n_{max} or P_{max}, thus:



[2]

The maximum data rate is determined by the time needed to settle each bit. From the above, we require around 2 nsec to reach 80% of full power, making the maximum data rate 500 Mbit/sec.

[1]