UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2003

BEng Honours Degree in Computing Part III
MSc in Advanced Computing
PhD
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C329

COMPUTATIONAL LOGIC

Monday 12 May 2003, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 This question makes use of just two relations:

likes(X, Y)	"X likes Y"
U=V	"U and V are identical"

a Using *only* these two relations, express each of the following by a single first-order logic (FOL) sentence:

i "chris likes logic and algebra, and nothing else"
ii "bob likes logic, and nothing else"
iii "everything likes something that nothing else likes"

b Translate each FOL sentence from part a into a clause-set, giving *some* indication of the steps involved. Do not try to eliminate "=" predicates from any clauses.

c Let C denote the set of all clauses obtained in part b. Explain carefully why C is logically consistent.

d Apply general resolution to show that C ∪ {¬bob=chris} is logically inconsistent, taking care to show the parents used for each step. *Note* —no other axioms about "=" are needed.

The four parts carry, respectively 25%, 25%, 10% and 40% of the marks.

2a Define a Herbrand domain, a Herbrand base and a Herbrand interpretation.

b Use the T_P function to determine the minimal **H**-model **MM(P)** of the following programs (taking **H** as {0, s(0), s(s(0)), ... etc.}), in each case showing at least the first four iterates and giving an informal meaning for the model determined:

i
```
p(0)
p(s(s(X))) if p(X)
p(X) if p(s(X))
```

ii
```
q(Z, 0, Z)
q(X, s(Y), Z) if q(s(X), Y, Z)
```

c Use the T_P function to determine the fair-finite failure set **FF(P)** of the following program **P** (taking **H** as {a, b}), showing the iterates obtained:

```
arc(a, b)
arc(b, b)
path(X, Z) if arc(X, Z)
path(X, Z) if arc(X, Y) ∧ path(Y, Z)
```

d Prove that, for any definite program **P** and **H**-interpretation I,

if I is an **H**-model for **P** then $T_P(I) \subseteq I$

The four parts carry, respectively 15%, 40%, 25% and 20% of the marks.

3a Describe the circumstance in which a branch in an SLDNF evaluation is labeled as floundered, and explain the purpose of this label.

b Given the following normal program **P**

wants(dov, Z) if has(Y, Z)
wants(derek, Z) if has(Y, Z) ∧ **fail** wants(Y, Z)

has(chris, meccano)
has(derek, trainset)

i describe its sensitivity to floundering according to the choice of computation rule employed;

ii using the computation rule which always selects the first query call, draw the SLDNF evaluation of each of the queries:

? wants(derek, meccano)
and ? wants(derek, trainset)

c Show that the above program **P** is not locally stratified and explain what, if anything, this tells us about its semantics.

d Construct the completion **comp(P)** of the above program **P** and determine whether this completion is logically satisfiable. (You need not give the fine details of the equality theory or of your reasoning, but you should offer an adequate argument.)
Hint — analyse the completion at the ground level.

The four parts carry, respectively 15%, 30%, 20% and 35% of the marks.

4a Taking **H** as {chris, bob, logic}, identify all of the 12 anti-symmetric **H**-models of the following program:

likes(chris, X) if likes(X, logic)
likes(bob, logic)

Note —here, anti-symmetry means that, for any X, Y, if a model contains both likes(X, Y) and likes(Y, X) then it must be the case that X=Y. You may use abbreviations as convenient, such as writing CB instead of likes(chris, bob).
Hint — consider how the minimal model can be extended by adding atoms from the Herbrand base whilst preserving satisfiability and anti-symmetry.

b Prove that, for any definite program **P**, the intersection of any two **H**-models of **P** is itself an **H**-model of **P**.
Hint — analyse the cases for any ground clause (q if body) in **G(P)**.

c Give an example of an indefinite program having two distinct **H**-models whose intersection is not an **H**-model, and give an example of another indefinite program having two distinct **H**-models whose intersection is an **H**-model.
Note — in both cases it is sufficient to give propositional examples.

The three parts carry, respectively 60%, 25% and 15% of the marks.

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