IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

EEE/EIE PART I: MEng, BEng and ACGI

Corrected copy

ANALYSIS OF CIRCUITS

Friday, 1 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D.M. Brookes
Second Marker(s) : P. Georgiou

ANALYSIS OF CIRCUITS

Information for Candidates:

- Numerical answers must be given as fully evaluated decimal values and not as unevaluated arithmetic expressions.

Notation

The following notation is used in this paper:

1. The voltage waveform at node X in a circuit is denoted by $x(t)$, the phasor voltage by X and the root-mean-square (or RMS) phasor voltage by $\widetilde{X} = \frac{X}{\sqrt{2}}$. The complex conjugate of X is X^*.
2. Component and source values in a circuit are normally given in Ohms, Farads, Henrys, Volts or Amps with the unit symbol omitted. Where an imaginary number is specified, it represents the complex impedance or phasor value.
3. Times are given in seconds unless otherwise stated.
4. Unless otherwise indicated, frequency response graphs should use a linear axis for phase and logarithmic axes for frequency and magnitude.
5. The real and imaginary parts of a complex number, X, are written $\Re(X)$ and $\Im(X)$ respectively.

1. a) Using nodal analysis, calculate the voltages at nodes X and Y of Figure 1.1. [5]

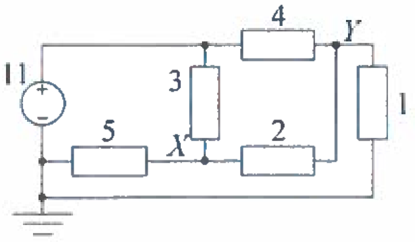

Figure 1.1

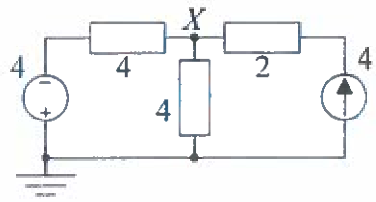

Figure 1.2

b) Use the principle of superposition to find the voltage X in Figure 1.2. [5]

c) Draw the Thévenin equivalent circuit of the two-terminal network in Figure 1.3 and find the values of its components. [5]

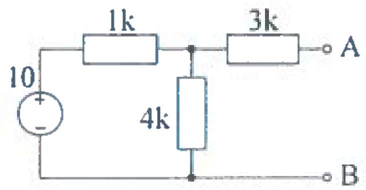

Figure 1.3

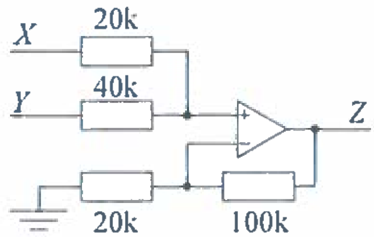

Figure 1.4

d) Assuming the opamp in the circuit of Figure 1.4 is ideal, give an expression for Z in terms of X and Y. [5]

e) Determine R_1 and R_2 in Figure 1.5 so that $Y = 0.25X$ and the parallel combination of R_1 and R_2 has an impedance of $75\,\Omega$. [5]

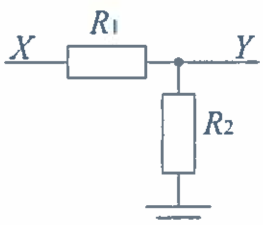

Figure 1.5

f) The circuit of Figure 1.6 shows a 50 Hz voltage source, with RMS voltage phasor $\widetilde{V} = 230$, driving a load of impedance $Z_L = 20 + 10j\,\Omega$ through a line of impedance $Z_T = 0.2 + 0.8j\,\Omega$. Calculate the complex power, $\widetilde{V} \times \widetilde{I}^*$, absorbed by (i) Z_T and (ii) Z_L. [5]

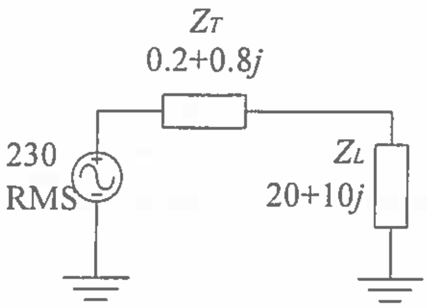

Figure 1.6

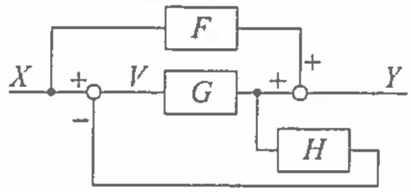

Figure 1.7

g) Determine the gain, $\frac{Y}{X}$, for the block diagram shown in Figure 1.7. The rectangular blocks are drawn with inputs at the left and outputs at the right and have gains of F, G and H respectively. The open circles represents adder/subtractors whose inputs have the signs indicated on the diagram and whose outputs are V and Y respectively. [5]

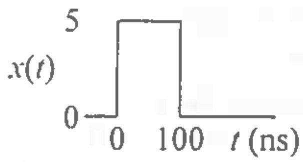

Figure 1.8

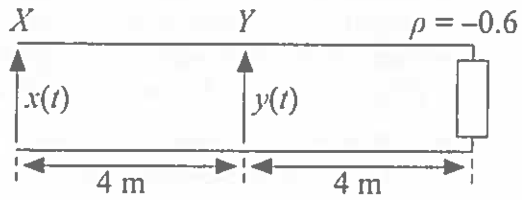

Figure 1.9

h) Figure 1.9 shows a transmission line of length 8 m that is terminated in a resistive load with reflection coefficient $\rho = -0.6$. The line has a propagation velocity of $u = 2 \times 10^8$ m/s. At time $t = 0$, a forward-travelling (i.e. left-to-right) pulse arrives at X with amplitude 5 V and duration 100 ns, as shown in Figure 1.8.

Draw a dimensioned sketch of the waveform at Y, a point 4 m from the end of the line, for $0 \leq t \leq 200$ ns. Assume that no reflections occur at point X. [5]

2. The frequency response of a highpass filter circuit is given by

$$H(j\omega) = \frac{k(j\omega)^2}{(j\omega)^2 + 2\zeta\omega_0 j\omega + \omega_0^2}$$

where k, ζ and ω_0 are positive real numbers and ω is in rad/s.

a) i) Give a simplified expression for the value of $H(j\omega)$ at the frequency $\omega = \omega_0$. [2]

ii) Determine the low and high frequency asymptotes of $H(j\omega)$. [2]

iii) By finding the squared magnitudes of the numerator and denominator expressions in $H(j\omega)$, show that [5]

$$|H(j\omega)|^2 = \frac{k^2}{\left(\frac{\omega_0^2}{\omega^2} - 1\right)^2 + 4\zeta^2\frac{\omega_0^2}{\omega^2}}.$$

iv) By writing the denominator of the previous expression in terms of $x = \frac{\omega_0^2}{\omega^2}$, show that the denominator has a minimum when $x = 1 - 2\zeta^2$.

Hence determine the value of ω at which $|H(j\omega)|$ is maximum and the value of $|H(j\omega)|$ at this frequency. [5]

b) Assuming $\omega_0 = k = 1$, draw a dimensioned sketch showing the magnitude response, $|H(j\omega)|$, in dB for the two cases: (A) $\zeta = 0.1$ and (B) $\zeta = 0.5$. Show both lines on the same set of axes. For each case, calculate the maximum value of $|H(j\omega)|$ in dB and the frequency, ω_p, at which it occurs. [6]

c) In the highpass filter circuit of Figure 2.1, the opamp is ideal, the capacitors have value C and the resistors have values P, Q, R and $(k-1)R$ respectively.

i) Explain why $Y = kV$. [1]

ii) By applying Kirchoff's current law at nodes U and V show that the transfer function $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$ is given by [6]

$$H(j\omega) = \frac{kPQC^2(j\omega)^2}{PQC^2(j\omega)^2 + (2P + (1-k)Q)Cj\omega + 1}.$$

iii) Determine simplified expressions for ζ and ω_0 when $H(j\omega)$ is written in the form given at the start of the question. [3]

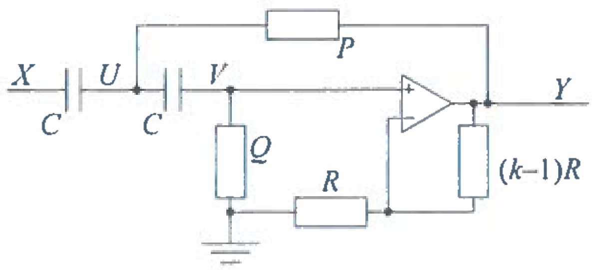

Figure 2.1

3. The diode in Figure 3.1 has a forward voltage of 0.7V when it is conducting. The voltage waveforms at nodes X and Y are $x(t)$ and $y(t)$ respectively and the diode current is $i(t)$ as shown.

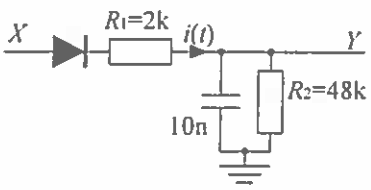

Figure 3.1

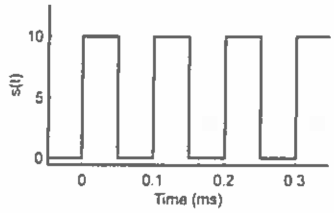

Figure 3.2

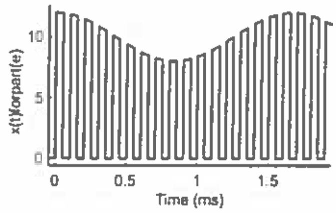

Figure 3.3

a) Assuming that node X is connected to a voltage source, calculate the time constant of the circuit when (a) the diode is conducting and (b) the diode is non-conducting. [4]

b) If $x(t)$ has a constant voltage of 10V, determine the steady-state values of $i(t)$ and $y(t)$. [3]

c) Suppose $x(t) = \begin{cases} 0 & t < 0 \\ 10 & t \geq 0 \end{cases}$. Determine an expression for $y(t)$ for $t \geq 0$. [4]

d) Suppose now that $x(t) = s(t)$ as shown in Figure 3.2 where $s(t)$ is a positive-valued squarewave of period $T = 100\,\mu$s and amplitude 10V.

i) Determine an expression for $y(t)$ for $0 \leq t < 0.5T$ assuming that the diode is conducting throughout this interval and that the value of y at the start of the interval is $y(0) = A$. Hence, derive and simplify an equation relating A and B where $B = y(0.5T)$ is the value of y at the end of the interval. [4]

ii) Determine an expression for $y(t)$ for $0.5T \leq t < T$ assuming that the diode is non-conducting throughout this interval and that the value of y at the start of the interval is $y(0.5T) = B$. Hence derive and simplify a second equation relating A and B assuming that the value of y at the end of the interval is $y(T) = A$. [3]

iii) By combining the equations determined in parts i) and ii), determine the numerical values of both A and B. [2]

iv) Sketch a dimensioned graph of $y(t)$ for $t \in [0, 200\,\mu\text{s}]$. [3]

e) Suppose now that $R_2 = 500\,\text{k}\Omega$ and that $x(t) = (1 + 0.2\cos(2\pi f t))s(t)$ is a modulated squarewave as illustrated in Figure 3.3 for $f = 600$ Hz.

i) Assuming that $y(t) \leq 11.3$, determine an upper bound on the current through R_2. [1]

ii) Explain why the average value of $i(t)$ must equal the average current through R_2. Hence find an upper bound on the average voltage across R_1 during the times that the diode is conducting. [3]

iii) Sketch the waveform $y(t)$ for a modulating frequency of $f = 20$ Hz. It is not necessary to calculate the value of $y(t)$ precisely. [3]