IMPERIAL COLLEGE LONDON

E4.06
AO9
SO9
ISE4.36

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

MSc and EEE/ISE PART IV: MEng and ACGI

OPTICAL COMMUNICATION

Tuesday, 11 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 6

All questions carry equal marks.

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	E.M. Yeatman
	Second Marker(s) :	K.D. Leaver

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3.0 \times 10^{8}$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x}\tan(k_{1x}d/2) \quad \text{and} \quad \kappa = -\, k_{1x}\cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]

a) Two types of step index silica fibre are labelled A and B. Type A has an index difference of 3×10^{-3} and a core diameter of 6 μm. Type B has an index difference of 4×10^{-3} and a core diameter of 4 μm. Estimate which type has the largest cutoff wavelength for single mode operation.

b) A certain fibre supports only a single mode for both 1510 and 1512 nm wavelength. Which of these two wavelengths has the higher phase velocity, and why?

c) What loss mechanism in an optical fibre causes a polarisation dependent attenuation?

d) A cooled light emitting diode emits light predominantly within a wavelength range of approximately 1300 to 1330 nm. Estimate its operating temperature.

e) Which of silicon and gallium arsenide has the steeper edge in its absorption spectrum? Which particular characteristic of the band structure mainly determines this?

f) An optical point-to-point link has a fibre length of 50 km and an attenuation coefficient of 0.3 dB/km. Which would you expect to have a worse effect on the signal-to-noise ratio: doubling the cable length, or doubling the bit-rate? Assume thermal noise dominates in all cases.

g) An anti-reflection coating is desired to be placed at a planar semiconductor-to-plastic interface, for operation at normal incidence with a free-space wavelength of 1310 nm. Calculate a suitable thickness and refractive index for this layer. The index of the semiconductor is 3.50 and that of the plastic is 1.51.

h) A semiconductor laser operating at a nominal wavelength of 780 nm has a slope efficiency of 1.1 A/W. Calculate the quantum efficiency.

i) An unamplified optical link has a signal-to-noise ratio (SNR) dominated by shot noise. What will be the effect on the SNR if an optical amplifier is added within the link, and why?

j) Dispersion in a certain fibre causes a doubling of the temporal width of a Gaussian, transform-limited pulse propagating through it. What effect does the same dispersion have on the spectral width of this pulse?

2. A symmetric slab waveguide as shown in Fig. 1 has a core thickness $d = 20\ \mu m$, and a numerical aperture NA = 0.105. The free-space wavelength is 1.50 μm.

a) Determine the total number of TE modes supported by this guide. [4]

b) Estimate the 1/e decay distance of the evanescent field for the <u>lowest</u> order TE mode supported, to 3 significant digits. [8]

c) Sketch and dimension the field profile E(x) for the <u>highest</u> order TE mode supported by this guide. Estimate the position of all nulls and peaks in this profile, to 3 significant digits. [8]

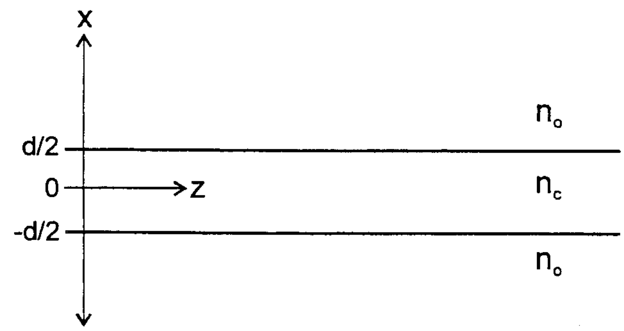


Figure 2.1

3. A certain single mode fibre has a wavelength-dependent effective index n' given by:

$$n' = n_g + \alpha(\lambda_o - \lambda_c)^2 \qquad (3.1)$$

where λ_o is the free-space wavelength and λ_c is the centre wavelength of the operating range.

a) Derive expressions for the phase delay τ_p, and the group delay τ_g, for a fibre length L. Note that the group velocity is given by $v_g = d\omega/d\beta$ where β is the propagation constant. [12]

b) Using your solution to (a), derive an expression for the dispersion coefficient D, using $D = |(d\tau_g / d\lambda_o)/L|$. Hence, show that $D = |\lambda_o(d^2n/d\lambda_o^2)/c|$ for this case.

Since dispersion causes a frequency (or wavelength) dependent phase shift, it can be modelled as an all-pass filter. Find an expression for a filter function $H(\omega)$ equivalent to the dispersion effect of this fibre for a length L. [8]

4. a) A Fabry-Perot laser diode emitting at 780 nm has a threshold current $I_{th} = 20$ mA at 300K, dropping by 0.5mA/K as the temperature rises. The overall quantum efficiency above threshold is 0.4 (independent of temperature). Calculate the temperature dependence $d\Phi/dT$ of output optical power above threshold. [6]

b) The laser diode in (a) has mirror reflectivities $R = 0.98$ at both ends, and a waveguide absorption coefficient of $0.2\ cm^{-1}$. The cavity length is 0.5 mm. Neglecting other attenuation mechanisms, calculate the fraction of photons generated which are emitted through the front mirror. [6]

c) If the laser in (b) has a waveguide effective index of 3.51, and an operating temperature of 300K, calculate the spacing in nm of its spectral modes, and estimate the number of these modes which contain non-negligible power for a current just above I_{th}. [8]

5. a) A laser of output power 3 dBm at $\lambda_o = 1550$ nm emits into a fibre of length L = 100 km and attenuation coefficient (for power) $\alpha = 5\times10^{-5}\ m^{-1}$. The receiver has quantum efficiency $\eta = 0.8$ and noise equivalent power (NEP) of 20 pW/√Hz. Show that receiver noise dominates over shot noise in this case. [6]

b) An optical transmitter generates Gaussian pulses of spectral width $\sigma_\lambda = 0.1$ nm and temporal width $\sigma_o = 50$ ps. These are propagated through a fibre type having a dispersion coefficient D = 10 ps/nm·km. Assume that a sampling receiver is used, such that the signal-to-noise ratio is determined by the peak optical power of the received pulse. Hence, derive an expression for the power penalty, in dB, resulting from dispersion, and find the fibre length L for which this penalty is 3 dB.

Determine whether the transmitted pulses above are transform limited. [14]

6. A silicon avalanche photodiode has intrinsic and avalanche layer thicknesses of 8 and 3 μm respectively (Fig. 6.1). The acceptor doping levels are $N_{A+} = 10^{21}$ m^{-3} and $N_{AA} = 4\times10^{21}$ m^{-3} in the p$^+$ and avalanche layers respectively, and $N_{Ai} = 10^{20}$ m^{-3} in the intrinsic layer, and the donor doping level is $N_D = 6\times10^{21}$ m^{-3} in the n layer.

a) A bias voltage V_b is applied to the device which is just sufficient to fully deplete both the avalanche and intrinsic layers. For this voltage, calculate the electric field magnitude at the i-A and A-n interfaces, and hence calculate V_b. Plot and dimension the field profile E(x). Why is it important to fully deplete the intrinsic layer? [8]

b) Calculate the applied bias voltage V_b needed to raise the electric field magnitude above 10^6 V/m in the whole of the intrinsic region. Plot and dimension the charge concentration profile ρ(x) and the field profile E(x) for this case. For this voltage, over what fraction of the avalanche region will E(x) be above the breakdown level of 10^7 V/m? [8]

c) For an absorption coefficient of 10^5 m^{-1}, and neglecting Fresnel reflection at the surface, calculate the fraction of incident photons absorbed in the depletion region for the applied voltage calculated in (b). Briefly indicate the main performance trade-offs involved in choosing the intrinsic layer thickness. [4]

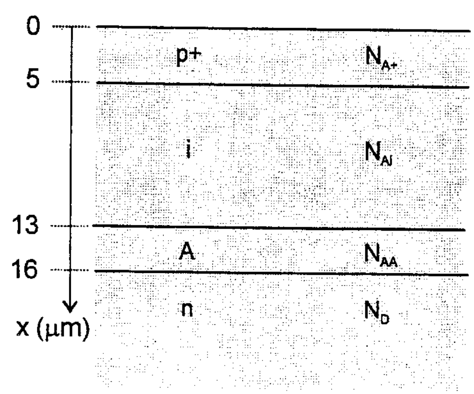


Figure 6.1

2004 Solutions

E4.06 Optical Comms

A09 | S09 | ISE 4.36

A = bookwork C = new theor. application
B = new theory D = new computed example

(1)

D a) $V = r_o k_o \sqrt{n_c^2 - n_o^2} \cong (2\pi r_o/\lambda_o)\sqrt{3 \cdot \Delta n}$

cutoff at $V = 2.405$

$\lambda_o \cong 2\pi r_o \sqrt{3 \cdot \Delta n}/2.405$

$\dfrac{\lambda_{oA}}{\lambda_{oB}} = \dfrac{6}{4}\sqrt{\dfrac{3}{4}} > 1$, $\therefore$ $\underline{\lambda_{oA} > \lambda_{oB}}$ [2]

C b) The larger wavelength has more light in the cladding (since m=0 for both), so n' is lower, so V_p is <u>higher for 1512 nm</u>. [2]

A c) Bending loss.

d) $\Delta E \cong 2kT \cong hc\left(\dfrac{1}{1.3\times10^{-6}} - \dfrac{1}{1.33\times10^{-6}}\right)$ [2]

D $T = \dfrac{6.63\times10^{-34}\times3\times10^{8}}{2\times1.38\times10^{-23}\times10^{-6}}\left(\dfrac{1}{1.3} - \dfrac{1}{1.33}\right) = \underline{125\ K}$ $(= -148°C)$

A e) GaAs is direct bandgap, therefore has steeper absorption edge than Si, which is indirect bandgap. [2]

C f) Doubling the bit-rate will normally require doubling the receiver bandwidth, which doubles the additive noise power, decreasing electrical SNR by 3dB. Adding 50 km × .3dB/km lowers optical SNR by 15dB, or electrical SNR by 30 dB, so this is worse. [2]

1

D g) We need $n_{AR} = \sqrt{n_1 n_2} = \sqrt{3.5 \times 1.5} = \underline{2.30}$ [2]

and $t = \frac{\lambda}{4} = \frac{\lambda_o}{4 n_{AR}} = \frac{1310}{4 \cdot 2.3} = \underline{142\ nm}.$

D h) $S = \frac{\eta h c}{e \lambda} = \eta \frac{\times 6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 780 \times 10^{-9}} = \eta \times 1.59\ A/W$ [2]

$1.1 = 1.59 \eta \quad , \eta = 1.1/1.59 = \underline{0.69}$

C i) If shot noise dominates, adding an optical amplifier makes the final SNR <u>worse</u> because of ASE (by the noise figure) [2]

C j) Dispersion is a linear effect - there is no effect on spectral width. [2]

(2) As well as the eigenvalue equations (given), we have (from phase matching):

$$K^2 + k_{1x}^2 = R^2 = NA^2 k_0^2$$

a) A mode m is supported if:

$$R > m\pi/d$$

D

ie $m < \frac{d}{\pi}\cdot NA\cdot k_0 = \frac{2d\,NA}{\lambda_0} = \frac{2(20).105}{1.5}$

$= 2.8$

$\therefore$ 3 modes are supported, $m = 0, 1$ and 2 [4]

b) For the lowest ($m=0$) order mode:

$$K^2 = k_{1x}^2 \tan^2\left(\frac{k_{1x}d}{2}\right)$$

and $K^2 + k_{1x}^2 = R^2 = k_{1x}^2\left(1+\tan^2(k_{1x}d/2)\right)$

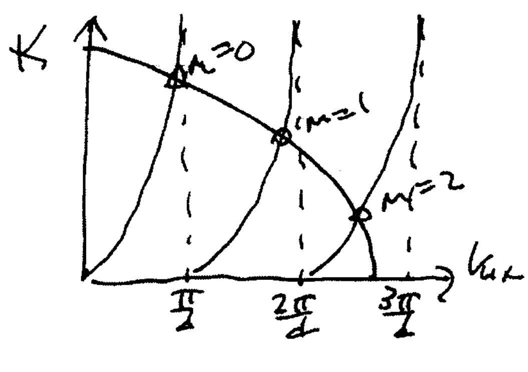
taking $X = k_{1x}d/2$:

D

$$X^2(1+\tan^2 X) = X^2/\cos^2 X = R^2(d/2)^2$$

$$\left(\frac{\cos X}{X}\right)^{-1} = \pm R = \pm \frac{2\pi NA d}{2\lambda_0} = \pm \frac{\pi(.105)(20)}{1.5} = 4.398$$

$$\frac{\cos X}{X} = \pm 0.2274$$



We can see that $m=0$ should give k_{1x} just below π/d, so X just below $\pi/2$.
By successive approximation, we find $X = 1.276$.

$$K^2 = R^2 - k_{1x}^2 = R^2 - (2X/d)^2 = (.105\cdot 2\pi/1.5)^2 - \left(\frac{2\times 1.276}{20}\right)^2$$

$K = 0.421\ \mu m^{-1}$

So the decay distance $= 1/K = 2.38\ \mu m$. [8]

c) Highest mode is $m=2$. Since this is even, $\frac{\cos X}{X} = \pm 0.2274$ still applies,

but from graph we expect $k_{1x} \cong 2.5\pi/d$, or $X \cong 1.25\pi = 3.93$. Using this as starting point, we can find $X = 3.71$.

then $k_{1x} = \frac{2X}{d} = 0.371\ \mu m^{-1}$

C

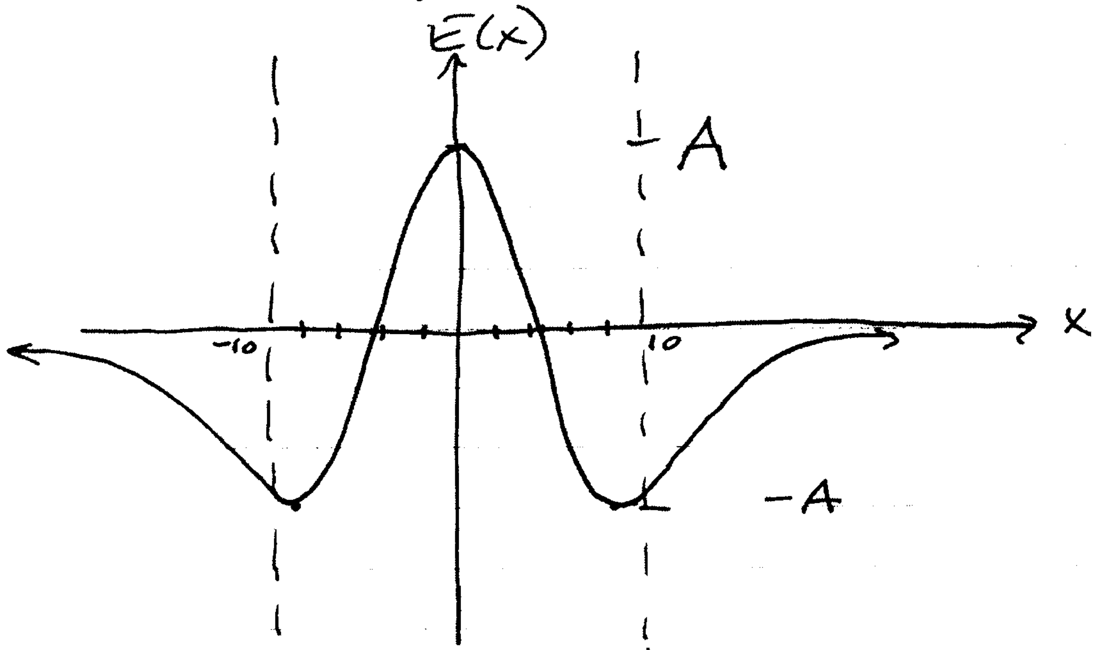
$E(x) = A\cos(k_{1x}x)$ for $|x| \le d/2$

zeros at $k_{1x}x = \pm \pi/2$

$x = \pm \frac{\pi}{2}\left(\frac{1}{.371}\right) = \pm 4.23\ \mu m$

Maxima/minima at $x=0$, and $k_{1x}x = \pm\pi$,

or $x = \pm \frac{\pi}{.371} = \pm 8.46\ \mu m$.



[8]

(3)

a) $\tau_p = L/v_p \qquad v_p = \omega/\beta = \omega/n'k_0$

C

$$\tau_p = \frac{n'k_0 L}{\omega} = \frac{n'L}{c} = \left[n_g + \alpha(\lambda_0-\lambda_c)^2\right]\frac{L}{c}$$

b) $\tau_g = \frac{L}{v_g} \qquad v_g = \frac{d\omega}{d\beta} \qquad \tau_g = L\,d\beta/d\omega$

C

$$= \frac{L\,d\beta}{dk_0}\Big/\frac{d\omega}{dk_0} \qquad \omega = ck_0 \quad \therefore \tau_g = \frac{L}{c}\frac{d\beta}{dk_0}$$

$$= \frac{L}{c}\frac{d\beta}{d\lambda_0}\cdot\frac{d\lambda_0}{dk_0} \qquad \lambda_0 = \frac{2\pi}{k_0} \quad \frac{d\lambda_0}{dk_0} = -\frac{2\pi}{k_0^2}$$

$$\tau_g = -\frac{L}{c}\frac{2\pi}{k_0^2}\frac{d\beta}{d\lambda_0} \qquad \beta = n'k_0 \quad \frac{d\beta}{d\lambda_0} = k_0\frac{dn'}{d\lambda_0} + n'\frac{dk_0}{d\lambda_0}$$

$$\frac{d\beta}{d\lambda_0} = k_0\left(2\alpha(\lambda_0-\lambda_c)\right) - \frac{2\pi}{\lambda_0^2}\left(n_g + \alpha(\lambda_0-\lambda_c)^2\right)$$

$$\tau_g = \frac{L\lambda_0}{c}\left[\frac{1}{\lambda_0}\left(n_g + \alpha(\lambda_0^2+\lambda_c^2-2\lambda_0\lambda_c)\right) - 2\alpha(\lambda_0-\lambda_c)\right]$$

$$= \frac{L}{c}\left(n_g + \alpha\lambda_0^2 + \alpha\lambda_c^2 - 2\alpha\lambda_0^2\right) = \frac{L}{c}\left(n_g + \alpha(\lambda_c^2-\lambda_0^2)\right) \quad [12]$$

c) $\frac{d\tau_g}{d\lambda_0} = \frac{L}{c}(-2\alpha\lambda_0)$

$$D = \left|\frac{d\tau_g/d\lambda_0}{L}\right| = \frac{2\alpha\lambda_0}{c}$$

$$D = \left|\frac{\lambda_0}{c}\,d^2n/d\lambda_0^2\right| = \left|\frac{\lambda_0}{c}\cdot 2\alpha\right| \quad \checkmark \text{ agrees.}$$

d) The phase shift due to propagation $\phi(\omega) = \omega\tau_p$

$\phi(\omega) = \frac{\omega L}{c}(n_g - \alpha(\lambda_0-\lambda_c)^2)$ where $\lambda_0 = 2\pi c/\omega$

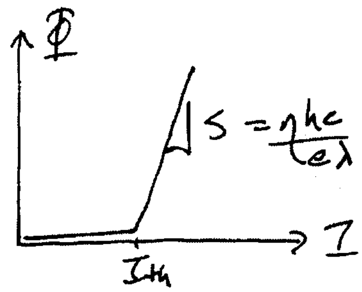
B

The first term is the mean phase shift and the second is the dispersion, so:

$$H(\omega) = \exp\left[+j\frac{\omega L}{c}\alpha\left(\frac{2\pi c}{\omega}-\lambda_c\right)^2\right] \quad [8]$$

(4)

a)



$d\Phi/dT = \frac{d\Phi}{dI'} \cdot \frac{dI'}{dt}$ where $I' = I - I_{th}$

$\frac{d\Phi}{dI'} = S = \frac{\eta hc}{e\lambda} = \frac{0.4\times 6.63\times10^{-34}\times 3\times10^{8}}{1.6\times10^{-19}\times 780\times10^{-9}} = 0.64$ W/A

$\frac{dI'}{dT} = -\frac{dI_{th}}{dT} = -\left(-\frac{0.5\times10^{-3}}{K}\right) = 0.5\times10^{-3}$ A/K

$d\Phi/dT = 0.64\times.5\times10^{-3} = 0.32$ mW/K [6]

b) $R_1 = R_2 = 0.98$, $\alpha = \;$ cm^{-1}, $L = 0.5$ mm

$2\alpha L = 0.04$

fraction of photons remaining after a round trip is $R^2\exp(-2\alpha L)$, so total fraction lost is $1 - R^2\exp(-2\alpha L)$.

Fraction lost through mirrors (ie emitted) is $1 - R^2$

so $\eta_e = \frac{1 - R^2}{1 - R^2\exp(-2\alpha L)} = \frac{1 - .98^2}{1 - .98^2\exp(-.005)} = 0.892$
$= 0.446$ per mirror

It is also acceptable to use $\eta_e = \frac{\ln(1/R_1R_2)}{\ln(1/R_1R_2) + 2\alpha L} = 0.89$
giving 0.445 for one mirror. [6]

c) $L = m\lambda/2 = m\lambda_0/2n'$ $\therefore \lambda_0 = 2n'L/m$

$\Delta\lambda_0 = 2n'L\left(\frac{1}{m} - \frac{1}{m+1}\right) = \frac{2n'L}{m(m+1)} \cong \frac{2n'L}{m^2}$

$\therefore \Delta\lambda_0 \cong \lambda_0^2/2n'L = 780^2/2(3.51)(0.5\times10^{6}) = \underline{0.17\text{ nm}}$

Total spread in photon energies near I_{th} is $\sim 2kT$ as for LED, so $\frac{2kT}{hc} \cong \left(\frac{1}{\lambda_{min}} - \frac{1}{\lambda_{max}}\right)$

$\frac{1}{\lambda_{min}} = \frac{1}{780\times10^{-9}} + \frac{2(1.38\times10^{-23})(300)}{6.63\times10^{-34}\times3\times10^{8}} = \frac{1}{755\text{ nm}}$ $\quad \lambda_{max} - \lambda_{min} \cong 24.5$ nm

\# modes $\cong \frac{24.5}{0.17} \cong \underline{144}$ [8]

(5)

a) $3\,dBm = 2\,mW = P_T$

$P_R = P_T \exp(-\alpha L) = 2\times10^{-3}\exp(-5) = 13.5\,\mu W$

Need to convert $(I^*_{sh})^2 = 2eI_{ph}$ to an equivalent optical power:

$$I^* = R\Phi^* \qquad \Phi^*_{sh} = \sqrt{2eI_{ph}}/(\eta e\lambda/hc)$$

$$I_{ph} = \frac{\eta e \Phi_R \lambda}{hc} \quad \therefore \quad \Phi^*_{sh} = \sqrt{\frac{2hc\Phi_R}{\eta\lambda}}$$

$$\Phi^*_{sh} = \sqrt{\frac{2\times6.63\times10^{-34}\times3\times10^{8}\times13.5\times10^{-6}}{0.8\times1.55\times10^{-6}}} = 2\times10^{-12}\,W/\sqrt{Hz}$$

$= 2\,pW/\sqrt{Hz} \ll NEP$ so receiver noise dominates. [6]

D

b) $\sigma(L) = \sqrt{\sigma_0^2 + \sigma_D^2}$

where $\sigma_D = D\cdot L\cdot\sigma_\lambda$

Conservation of pulse energy gives:

$P_0\sigma_0 = P(L)\sigma(L)$ where $P(L)$ is peak power after propagation length L.

then $\dfrac{P(L)}{P_0} = \dfrac{\sigma_0}{\sigma(L)} = \dfrac{1}{\sqrt{1+(DL\sigma_\lambda/\sigma_0)^2}}$

C

power penalty in dB =

$$10\log\left(1/\sqrt{(1+(DL\sigma_\lambda/\sigma_0)^2)}\right)$$

$$= -20\log(1+(DL\sigma_\lambda/\sigma_0)^2)$$

$= -3\,dB$ for $(1+(DL\sigma_\lambda/\sigma_0)^2) = \sqrt{2}$

$DL\sigma_\lambda/\sigma_0 = 0.643 \qquad L = \dfrac{0.643\times50}{10\times.1} = 32.1\,km$ [10]

c) transform limited if $\sigma_\omega\sigma_0 \cong \frac{1}{2}$

$\dfrac{\sigma_\omega}{\omega} \cong \dfrac{\sigma_\lambda}{\lambda} \qquad \sigma_\omega \cong \sigma_\lambda\left(\dfrac{2\pi c}{\lambda_0^2}\right) = 10^{-10}\left(\dfrac{2\pi\cdot3\times10^8}{(1.55\times10^{-6})^2}\right)$

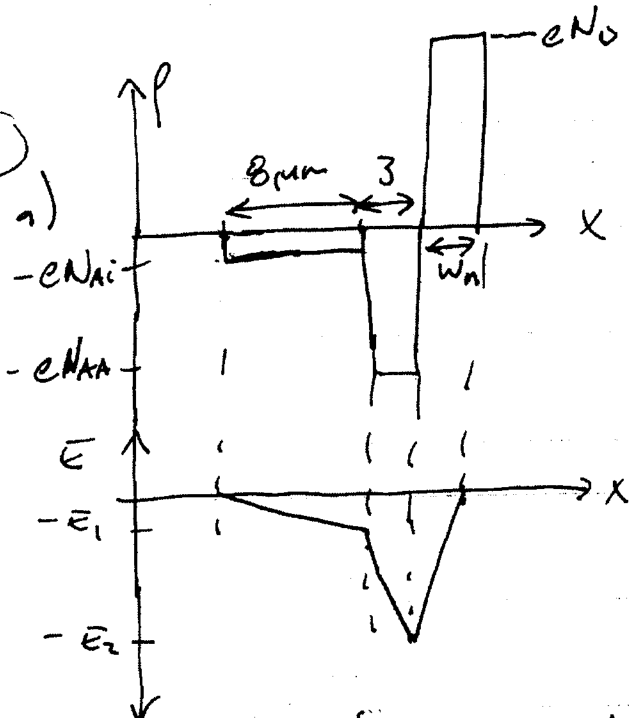
C

$$= 7.8\times10^{10}$$

$\sigma_\omega\sigma_0 = 7.8\times10^{10}\times50\times10^{-12} = 3.9$

Pulses are not transform limited. [4]

6 a)



Charge conservation: $8\mu m \times N_{Ai} + 3\mu m \times N_{AA} = W_n N_D$

$$W_n = 8\mu m\left(\frac{10^{20}}{6\times10^{21}}\right) + 3\mu m\left(\frac{4\times10^{21}}{6\times10^{21}}\right) = 2.13\,\mu m$$

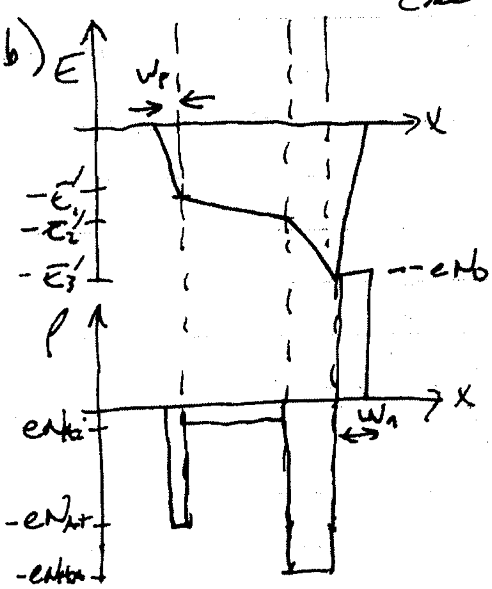
$$E_1 = \frac{eN_{Ai}W_i}{\varepsilon_r\varepsilon_0} = \frac{1.6\times10^{-19}\times10^{20}\times 8\mu m}{12\times 8.85\times10^{-12}} = 1.21\times10^6\ V/m$$ (from $E = \frac{1}{\varepsilon}\int \rho\,dx$)

$$E_2 = E_1 + \frac{eN_{AA}W_A}{\varepsilon_r\varepsilon_0} = 19.3\times10^6\ V/m$$

$$V_b = -\int E\,dx = \tfrac{1}{2}E_1W_i + \tfrac{1}{2}(E_1+E_2)W_A + \tfrac{1}{2}(E_2)W_n$$
$$= \tfrac{1}{2}(1.21\times8 + 20.5\times3 + 19.3\times2.1) = 55.9\,V$$ [8]

(see * after c)

b)



In this case $E_1' = 10^6\ V/m$

$$W_p = \frac{10^6\varepsilon_r\varepsilon_0}{eN_{A+}} = \frac{10^6\times12\times8.85\times10^{-12}}{1.6\times10^{-19}\times10^{21}} = 0.66\ \mu m$$

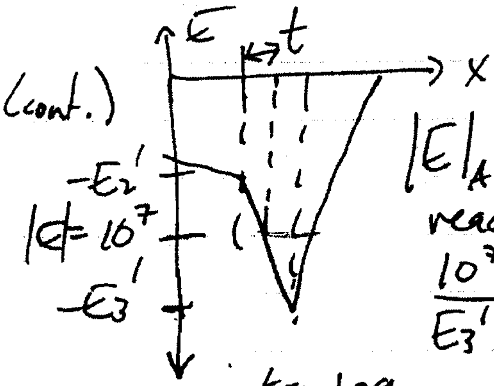
$$E_2' = E_1' + E_1 = 2.21\times10^6\ V/m$$

$$E_3' = E_2' + (E_2 - E_1) = 20.3\times10^6\ V/m$$

$$W_n = \frac{E_3'\varepsilon_r\varepsilon_0}{eN_D} = 2.24\ \mu m$$

$$V_b = \tfrac{1}{2}\left(W_pE_1' + W_i(E_1'+E_2') + W_A(E_2'+E_3') + W_nE_3'\right)$$
$$= 69.3\,V$$ [8]

b) (cont.)



$|E|_A$ varies linearly from E_2' to E_3'
reaches 10^7 V/m at t

$$\frac{10^7 - E_2'}{E_3' - E_2'} = \frac{t}{3\mu m} = \frac{10-2.2}{20.5-2.2} = 0.43$$

C

$t = 1.29\,\mu m$. 56% of avalanche region has $|E| > 10^7$ V/m.

[4]

c) $\eta = e^{-\alpha x_1} - e^{-\alpha x_2}$

D

$x_1 = 5\mu m - W_p = 4.33\,\mu m$
$x_2 = 16\,\mu m + W_n = 18.24\,\mu m$
$\eta = \exp(-0.433) - \exp(-1.82) = 0.487$

Tradeoffs: thick intrinsic layer increases η but ~~de~~increases transit time and thus reduces bandwidth.

d) * Depleting intrinsic layer helps provide a predictable depletion layer thickness (low dependence on V_b)