IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

COMMUNICATIONS 1

Wednesday, 28 May 10:00 am

Time allowed: 2:00 hours

There are FIVE questions on this paper.

Answer THREE questions.

Corrected Copy

jlk

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.L. Dragotti

Second Marker(s) : E.M. Yeatman

Special Information for the Invigilators: none

Information for Candidates

Some Fourier Trasforms

$$\cos\omega_0 t \iff \pi[\delta(\omega-\omega_0)+\delta(\omega-\omega_0)]$$

$$\mathrm{rect}(\tfrac{t}{\tau}) \iff \tau\mathrm{sinc}(\tfrac{\omega\tau}{2})$$

$$\tfrac{W}{\pi}\mathrm{sinc}(Wt) \iff \mathrm{rect}(\tfrac{\omega}{2W})$$

Time-Shifting Property of the Fourier Transform

$$g(t-t_0) \iff G(\omega)e^{-j\omega t_0}$$

Frequency-Shifting Property of the Fourier Transform

$$g(t)e^{j\omega_0 t} \iff G(\omega-\omega_0).$$

Some useful trigonometric identities

$$\cos^2 x + \sin^2 x = 1$$

$$\cos x \cos y = \tfrac{1}{2}\cos(x-y) + \tfrac{1}{2}\cos(x+y).$$

Euler's formula

$$e^{jx} = \cos x + j\sin x.$$

The Questions

1. Consider the following two waveforms $x_1(t) = \sin(4\pi t)\text{rect}(t - 0.5)$ and $x_2(t) = \text{rect}(t-1)$ (See also Figure 1).

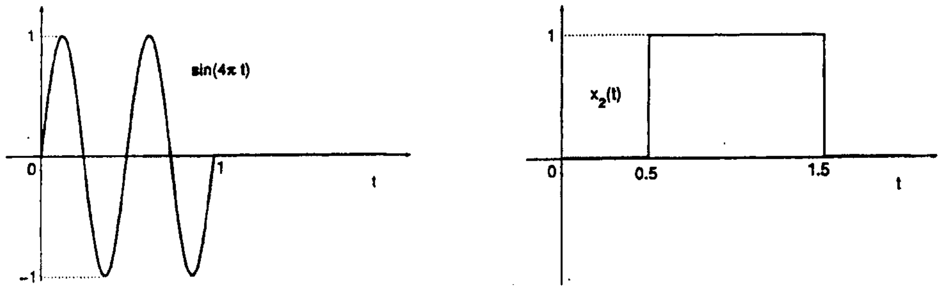


Figure 1: The two signals $x_1(t)$ and $x_2(t)$.

(a) Find the energy of $x_1(t)$.

[4]

(b) Find the energy of $x_2(t)$.

[4]

(c) Find the energy of $x(t) = x_1(t) + x_2(t)$.

[4]

(d) Find the Fourier transform of $x_2(t)$.

[8]

2. The signal $s(t) = \frac{50}{\pi}\text{sinc}(50t)\cos(100t)$ is multiplied by $\cos(100t)$ and the result $x(t)$ is fed to a filter with a frequency response

$$H(\omega) = \begin{cases} 1 & for \;\; |\omega| \leq 30 \text{ rad/s} \\ 0 & otherwise \end{cases}$$

giving output $y(t)$ as shown in Figure 2.

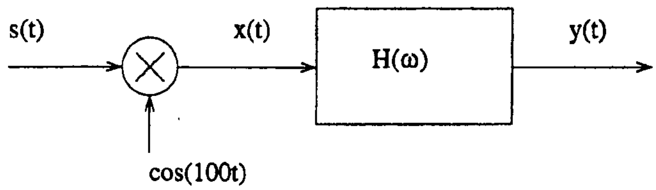


Figure 2: Receiver.

(a) Sketch the Fourier transform of $s(t)$.

[4]

(b) Sketch the Fourier transform of $x(t)$.

[4]

(c) Sketch the Fourier transform of $y(t)$.

[4]

(d) Write the exact expression of the output $y(t)$.

[8]

3. The output signal from an AM modulator is

$$x(t) = 5\cos 9900t + A\cos 10000t + 5\cos 10100t$$

Determine

(a) the modulating signal $m(t)$, [4]

(b) the minimum value of A that allows us to use an envelope detector, [4]

(c) the power efficiency η if $A = 20$, [6]

(d) compare the power efficiency you obtain for $A = 20$ with the one you obtain for the value of A you found in (b). Which case is more efficient? [6]

4. Consider the frequency modulated signal

$$\phi_{FM}(t) = A\cos\left[2\pi f_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right],$$

where the message signal is $m(t) = 10\pi\cos(2\pi f_0 t)$ with $f_0 = 4$ kHz and the carrier is given by $c(t) = 10\cos(2\pi f_c t)$ with $f_c = 10$ MHz. The bandwidth of $\phi_{FM}(t)$, using Carson's rule, is 10 kHz. Determine

(a) the bandwidth of the baseband signal $m(t)$,

[2]

(b) the peak value m_p of the baseband signal,

[2]

(c) the correct value of the coefficient k_f,

[10]

(d) the average transmitted power.

[6]

5. A transmission line having $L_0 = 0.25\ \mu\text{H/m}$ and $C_0 = 100\ \text{pF/m}$ is connected to a $100\ \Omega$ line with a matched termination. A sine wave of 15 V amplitude propagating in the former is incident on the junction. Find

   (a) the voltage amplitude of the reflected wave,

   [7]

   (b) the current amplitude of the reflected wave,

   [7]

   (c) the fraction of the incident power which is transmitted into the second line.

   [6]

E1.6 Communications I
Exam Solutions

1. (a)

$$E_{x_1} = \int_0^1 \sin^2(4\pi t)dt = \int_0^1 \frac{1}{2}dt - \frac{1}{2}\int_0^1 \cos(8\pi t)dt = 1/2$$

(b)

$$E_{x_2} = \int_{0.5}^{1.5} dt = 1$$

(c) The two signals are orthogonal therefore $E_x = 1/2 + 1 = 3/2$.

(d) Using time shifting property, we get

$$G(\omega) = \mathrm{sinc}(\omega/2)e^{-j\omega}.$$

2. (a) Since

$$\frac{50}{\pi}\mathrm{sinc}(50t) \Longleftrightarrow \mathrm{rect}(\omega/100),$$

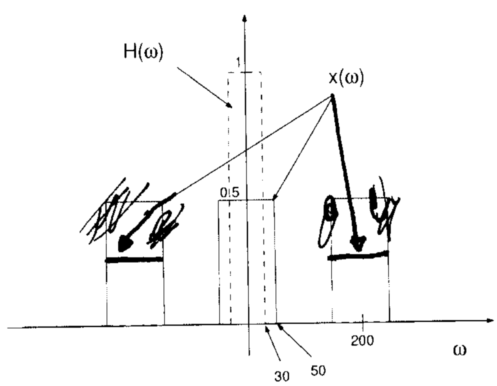
then

$$S(\omega) = \frac{1}{2}\mathrm{rect}(\frac{\omega - 100}{100}) + \frac{1}{2}\mathrm{rect}(\frac{\omega + 100}{100}).$$

(b) In the time domain $x(t)$ is given by

$$x(t) = \frac{25}{\pi}\mathrm{sinc}(50t) + \frac{25}{\pi}\mathrm{sinc}(50t)\cos 200t$$

Its Fourier transform is shown below. In the Figure we also show the effect of the filter $H(\omega)$



(c)

$$Y(\omega) = \frac{1}{2}\mathrm{rect}(\frac{\omega}{60})$$

(d)

$$y(t) = \frac{15}{\pi}\mathrm{sinc}(30t).$$

3. (a) The full AM signal is $(A + m(t))\cos\omega_c t$. In our case, the AM modulated signal is $(A+10\cos(100t))\cos 10000t$. Therefore $m(t) = 10\cos 100t$.

   (b) To use an envelop detector, we need $(A + 10\cos(100t)) \geq 0$ for all t. Therefore the minimum value of A is $A = 10$.

   (c) The efficiency is defined as $\eta = \frac{P_s}{P_c+P_s}$. $P_c = A^2/2 = 20^2/2 = 200$. $P_s = P_{m(t)}/2 = 50/2 = 25$. Therefore $\eta = 25/225 = 1/9$.

   (d) In the case $A = 10$, the efficiency is $\eta = 1/3$. This is the maximum achievable efficiency.

4. (a) The bandwidth of the baseband signal is $B = 4000$Hz.

   (b) The peak value of $m(t)$ is 10π.

   (c) Using Carson's rule the effective bandwidth is given by

$$B_{FM} = 2(\Delta f + B) = 2(\frac{k_f m_p}{2\pi} + B).$$

   The bandwidth of the baseband signal is $B = 4000$Hz. $B_{FM} = 10000$Hz and $m_p = 10\pi$. Therefore

$$B_{FM} = 2(\Delta f + B) = 2(\frac{k_f m_p}{2\pi} + B) = 2(5k_f + 4000) = 10000$$

   and it follows $k_f = 200$.

   (d) Since an angle modulated signal is essentially a sinusoidal signal with constant amplitude, we have

$$P_{FM} = A^2/2 = 100/2 = 50.$$

5. The characteristic impedance of the line is

$$Z_0 = \sqrt{L_0/C_0} = \sqrt{0.25 \cdot 10^{-6}/100 \cdot 10^{-12}} = 50\Omega.$$

The voltage reflection coefficient is $K_v = (Z_L - Z_0)/(Z_L + Z_0) = 1/3$.

   (a) $V_- = K_v V_+ = 5$V.

   (b) $I_- = -K_v I_+ = -K_v V_+/Z_0 = 0.1$A

   (c) $T_p = 1 - |K_v|^2 = 1 - 1/9 = 8/9$