Imperial College London

BSc/MSci EXAMINATION June 2012

This paper is also taken for the relevant Examination for the Associateship

SUN, STARS AND PLANETS

For 2nd, 3rd and 4th-Year Physics Students

Wednesday, 6th June 2012: 10:00 to 12:00

Answer ALL parts of Section A and TWO questions from Section B.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

Fundamental physical constants

a	radiation density constant	7.6×10^{-16} J m^{-1} K^{-4}
c	speed of light	3.0×10^{8} m s^{-1}
G	gravitational constant	6.7×10^{-11} N m^{2} kg^{-2}
h	Planck's constant	6.6×10^{-34} J s
k	Boltzmann's constant	1.4×10^{-23} J K^{-1}
e	electron charge	1.6×10^{-19} C
m_e	mass of electron	9.1×10^{-31} kg
m_H	mass of hydrogen atom	1.7×10^{-27} kg
N_A	Avogadro's number	6.0×10^{23} mol^{-1}
σ	Stefan-Boltzmann constant	5.7×10^{-8} W m^{-2} K^{-4}
ϵ_0	permittivity of free space	8.9×10^{-12} F m^{-1}
μ_0	permeability of free space	$4\pi \times 10^{-7}$ H m^{-1}
$\mathcal{R}$	gas constant	8.3×10^{3} J K^{-1} kg^{-1}

Astrophysical quantities

$L_\odot$	solar luminosity	3.8×10^{26} W
$M_\odot$	solar mass	2.0×10^{30} kg
$R_\odot$	solar radius	7.0×10^{8} m
$T_{\mathrm{eff}\odot}$	effective temperature of Sun	5780 K
AU	astronomical unit	1.5×10^{11} m
pc	parsec	3.1×10^{16} m

Equations of Stellar Structure

$$\frac{dr}{dm} = \frac{1}{4\pi \rho r^2}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

$$\frac{dT}{dm} = -\frac{3\kappa L}{64\pi^2 a c r^4 T^3} \qquad \text{if heat transport is radiative}$$

$$\frac{dT}{dm} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dm} \qquad \text{if heat transport is convective}$$

$$\frac{dL}{dm} = \epsilon$$

SECTION A

1. (i) In a Hertzsprung-Russell diagram, the main sequence of a stellar cluster only comprises stars with masses below 2.8 $M_\odot$.

Estimate the age of the cluster. [You may take the main-sequence life time of the Sun to be about 10 Gyr.] [3 marks]

(ii) Consider two objects that are separated by a distance a and orbit their common centre of mass on circular orbits with period P. Show that $P^2 \propto a^3$.

Explain briefly why the 'Kirkwood gaps' arise in the asteroid belt. Find the location of one of the Kirkwood gaps.

[Jupiter's semi-major axis is 5.2 AU; the asteroid belt is located between about 2 and 4 AU]. [4 marks]

(iii) Explain the term *bolometric correction*.

Star X has an effective temperature of 10 000 K, an apparent visual magnitude $m_V = 5.59$ and is at a distance of 70 parsec from Earth. Its bolometric correction is -0.32. Derive the absolute bolometric magnitude $M_{\rm bol}$ of star X and determine its luminosity.

[You may take the Sun's absolute bolometric magnitude to be $M_{\rm bol,\odot} = 4.8$.]

[4 marks]

(iv) A planet with albedo 0.35 orbits its host star on a circular orbit at a distance of 0.6 AU. The luminosity of the host star is 0.4 $L_\odot$. Calculate the no-atmosphere temperature of the planet.

Explain what is meant by the *habitable zone* and decide whether the planet may be in the habitable zone. [5 marks]

(v) A planet with a mass $M_p = 4 \times 10^{27}$ kg, radius $R_p = 84\,000$ km and effective temperature $T_p = 400$ K orbits a low-mass star with effective temperature $T_* = 3800$ K.

(a) Calculate the escape velocity for the planet. Would you expect the planet to be able to maintain a hydrogen atmosphere?

(b) Calculate the wavelengths where you would expect the emission of the planet and the emission of the star to peak [you may take the wavelength of the Sun's emission peak to be 500 nm].
Out of transit, the total spectrum of the planet shows two peaks; explain briefly why this is the case.

[5 marks]

(vi) Explain what is meant by the terms *(nuclear) binding energy*.

Complete the reaction for the *pp* chain given below

$$4\,{}^1_1\mathrm{H} \longrightarrow {}^4_2\mathrm{He} + ...$$

Given that the mass of helium is 3.97 times the mass of hydrogen, estimate the energy released in this reaction. [3 marks]

[Total 24 marks]

SECTION B

2. (i) Consider a homologous group of fully radiative stars of the same homogenous chemical composition, but different masses M and radii R.

(a) Taking the stellar structure equations together with the ideal gas equation, derive scaling for the stars' central density ρ_c and central pressure P_c. Show also that the central temperature T_c scales as $T_c \propto M/R$.

(b) Derive the expected mass-luminosity relation for the case where the opacity κ is due to electron scattering only, so that $\kappa = \kappa_0$.

[4 marks]

(ii) In the following, you may assume that on the main sequence

$$L \propto M^a \qquad \text{and} \qquad R \propto M^b,$$

with $a = 3.2$, $b = 0.6$ for $M \geq 3M_\odot$, and $a = 4.0$, $b = 1.0$ for $M \leq 3M_\odot$.

(a) Write down an expression for the luminosity L in terms of the radius R and the *effective* temperature $T_{\rm eff}$. Show that it is possible to write

$$L \propto T_{\rm eff}^{\alpha},$$

and derive α for stars with masses above and below 3 $M_\odot$.

(b) Draw a Hertzsprung-Russell diagram (HRD) and include a main-sequence that follows the scalings derived in part (a). Assume that a 3-$M_\odot$ star has an effective temperature of 10,000 K and a luminosity of 63 $L_\odot$.

[Hint: you might find it convenient to use logarithmic axes with temperature and luminosity ranges of $\log_{10} T_{\rm eff} = [3.7, 4.2]$, and $\log_{10}(L/L_\odot) = [-1, 3]$.

[6 marks]

(iii) The Pleiades cluster is at a distance of 135 pc. Two of its members, Taygeta and Sterope, have apparent bolometric magnitudes of $m_{\rm Tay} = 2.93$ and $m_{\rm Ste} = 5.0$.

(a) Calculate the absolute bolometric magnitude and the luminosity (in units of the solar luminosity) for Taygeta and Sterope. Take the Sun's absolute bolometric magnitude to be 4.72.

(b) Assume that Taygeta and Sterope follow the scalings set out in part (ii). Place both stars on the HRD and estimate their effective temperatures (to within 1000 K).

[6 marks]

(iv) Another star belonging to the cluster has a luminosity of 80 $L_\odot$ and an effective temperature of 5100 K. Place this star on the HRD and speculate on what kind of star it might be. [2 marks]

[Total 18 marks]

3. (i) Explain the term *solar constant* and calculate its value. [2 marks]

(ii) Briefly describe the characteristics of sunspots.

Explain qualitatively why sunspots appear dark, presenting some order-of-magnitude estimates to show that the suggested mechanism is plausible energetically.

[Quantities you might find useful are the photospheric density, $\rho \simeq 10^{-3}$ kg m^{-3}, the typical surface convection velocity, $v_c \simeq 2$ km s^{-1}, and typical sunspot magnetic fields of $B \simeq 0.3$ T.] [4 marks]

In the following sections you may neglect solar limb darkening.

(iii) (a) The largest single sunspot recorded covered 0.6 % of the visible solar disk. Assuming an average sunspot temperature of 5100 K, calculate the expected relative change in the solar constant as the sunspot crosses the solar disk. [5 marks]

(b) During solar maximum when the number of sunspots is largest, the solar constant is on average slightly higher than during solar minimum. Briefly explain why this is the case. [2 marks]

(iv) This morning you might have witnessed the Venus transit (this is a very rare event, the next transit will be in the year 2117). Estimate the relative decrease in solar flux during the transit and compare it to the decrease expected for a large sunspot.

[Venus has a radius of 6000 km and is at a distance of approximately 0.7 AU from the Sun]. [5 marks]

[Total 18 marks]

4. Star A with mass M_A and star B with mass M_B are part of a binary system at a distance of 150 pc from Earth. The stars orbit their common centre of mass on circular orbits with orbital radii of r_A and r_B. The system is seen edge-on. It has a binary period, P, of 8 years, and the maximum angular separation between stars A and B was measured to be 0.04 arcsec.

(i) The total mass M of the system is related to the orbital period P and the binary separation $a = r_A + r_B$ through

$$M = \frac{4\pi^2}{G}\frac{a^3}{P^2}. \qquad \text{DO NOT SHOW THIS!}$$

Calculate the distance between star A and star B in AU (astronomical units) and determine the total mass of the system. [3 marks]

(ii) An observing team is trying to measure the individual angular separations of the components from their centre-of-mass, but is unable to get high enough precision. Their measurements suggest angular orbital radii of $\alpha_A = 0.015 \pm 0.008$ arcsec and $\alpha_B = 0.025 \pm 0.008$ arcsec for star A and B, respectively.

They obtain spectroscopic data and detect a maximum radial velocity of $v_B = 17$ km s^{-1} for star B.

(a) Express the orbital velocity as a function of the orbital radius and period. State the relationship between the masses and the orbital radii.

(b) Calculate the orbital radius r_B inferred from the spectroscopic measurements and check whether it is consistent with the direct astrometric measurements.

(c) Derive the individual masses for star A and B. [7 marks]

(iii) Consider an adiabatically rising element of gas that maintains pressure balance as it rises. It is embedded in a plane-parallel atmosphere where the pressure $P(z)$ and density $\rho(z)$ depend only on the height of the atmosphere z; the gravitational acceleration g is constant. In this case, the atmosphere is stable to convection when the following inequality holds

$$\frac{P}{\rho}\frac{d\rho}{dP} > \frac{1}{\gamma}. \qquad \text{DO NOT SHOW THIS!}$$

Here γ is the adiabatic coefficient.

(a) Use the ideal gas equation to express this inequality in terms of the temperature and pressure gradients.

(b) Explain briefly why a change in the adiabatic coefficient or the temperature gradient might trigger convection and where this might happen on a star.

[4 marks]

(iv) State in which layers convection plays a role in the energy transport for main-sequence stars of different masses.

Comment on whether stars A and B are expected to have convection zones and, if so, where they would be located. For star A, use the stability criterion derived in (iii) to qualitatively explain your choice. [4 marks]

[Total 18 marks]

5. Write concisely (1-2 pages each) on TWO of the topics below. In what you write, pay attention to the main features of the phenomena involved, and identify the relevant physics underpinning your understanding of them. Illustrate your answer with sketch diagrams where this is helpful.

(i) Saturn's ring system;

(ii) Jupiter's Galilean Moons;

(iii) Kuiper-belt objects;

(iv) The different layers of the solar atmosphere;

(v) Heating and cooling processes of terrestrial planets (including a discussion of the relevance of planetary size).

[18 marks]

[Total 18 marks]