UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2003

BEng Honours Degree in Computing Part III
MSc in Computing for Industry
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part IV
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C343=I3.22

OPERATIONS RESEARCH

Friday 2 May 2003, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1a Consider the linear programming problem:
minimise

$$-2x_1 + 4x_2 + 7x_3 + x_4 + 5x_5$$

subject to

$$\begin{array}{rcrcrcrcrcr} -x_1 & + & x_2 & + & 2x_3 & + & x_4 & + & 2x_5 & = & 7 \\ -x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & + & x_5 & = & 6 \\ -x_1 & + & x_2 & + & x_3 & + & 2x_4 & + & x_5 & = & 4 \end{array}$$

and

$$x_1 \text{ free}, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0.$$

Find an initial feasible solution to this problem by using thephase I simplex algorithm.

b Let

$$v(b) = \min\{c^T x \mid \mathbf{A}x = b; x \geq 0\}$$

with $c, x \in \mathbb{R}^n, b \in \mathbb{R}^m, \mathbf{A} \in \mathbb{R}^m \times \mathbb{R}^n$. If the vector b is perturbed by a vector $\varepsilon \in \mathbb{R}^m$ we consider the perturbed problem

$$v(b + \varepsilon) = \min\{c^T x \mid \mathbf{A}x = b + \varepsilon; x \geq 0\}.$$

In general the solution to the latter problem does not have the same basic representation, and hence the optimal basis matrix, as the former. Using the optimal basis matrix of the former, define its shadow prices and use the shadow prices to establish a relationship between $v(b)$ and $v(b + \varepsilon)$.

(All parts carry equal marks).

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2a Find the dual of the linear program

$$\min\{c^T x \mid \mathbf{A}x = b; x \geq 0\}.$$

b Formulate the following problem as a standard linear programming problem (with non-negative constraints on all the variables):
minimise

$$x_0 = |x_1| + 2|x_2|$$

subject to

$$\begin{array}{rcrcr} x_1 & + & x_2 & \geq & 2 \\ -x_1 & + & x_2 & \geq & 3 \\ -x_1 & - & 3x_2 & \geq & -12 \end{array}$$

(no non-negativity constraints).

(All parts carry equal marks).

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3a Solve the following integer programming problem by the branch and bound method minimise

$$x_0 = 4x_1 - 6x_2$$

subject to

$$\begin{array}{rcrcr} -x_1 & + & x_2 & \leq & 1 \\ x_1 & + & 3x_2 & \leq & 9 \\ 3x_1 & + & x_2 & \leq & 15 \end{array}$$

and

$$x_1 \geq 0, x_2 \geq 0, x_1, x_2 \text{ integer.}$$

(Hint: The optimal solution of the linear program without the integer constraints is $x_1 = 1.5, x_2 = 2.5, x_0 = -9$.)

b Briefly explain Gomory's cutting plane algorithm and derive the cut for the first constraint of the optimal tableau

	x_1	x_2	x_3	x_4	RHS
x_0	0	0	$\frac{28}{11}$	$\frac{15}{11}$	63
x_1	0	1	$\frac{7}{22}$	$\frac{1}{22}$	$\frac{7}{2}$
x_2	1	0	$-\frac{1}{22}$	$\frac{3}{22}$	$\frac{9}{2}$

(All parts carry equal marks).

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4a Consider the following reward matrix corresponding to the payoffs to the row player in a two person zero-sum game:

	column player	
row	2	4
player	3	1

Formulate each player's optimal strategy.

b Consider the two-person non-constant-sum game. Two airlines are attempting to determine their advertising budgets. Their combined ticket sales is £240 million and each can spend either £6 million or £10 million on advertising. If one airline spends more money then the other, the airline that spends more will have sales of £190 million. If both airlines spend the same amount on advertising, they will have equal sales. For each company:

$$\text{profit} = (10\% \text{ of sales for the company}) - (\text{advertising costs}).$$

Formulate the profit for each airline and each advertising alternative. Suppose each airline is interested in maximising profit. Find an equilibrium point for this game.

(All parts carry equal marks).

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