E3.02
AC4

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

MSc and EEE PART III/IV: MEng, BEng.and ACGI

INSTRUMENTATION

Friday, 6 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : C. Papavassiliou
Second Marker(s) : S. Lucyszyn

THE QUESTIONS

1. In this question we will examine a mechanism that gives rise to correlated noise sources in amplifiers. Assume the only noise sources in the problem are the Johnson noise sources associated with the input and load admittances as well as the output and source impedances. Also assume that the voltage amplifier has a G-matrix:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} 0.001\ \text{S} & 0.1 \\ 10 & 100\ \Omega \end{bmatrix}$$

a) Draw an equivalent circuit for this amplifier, driven by a $R_S = 100\ \Omega$ source and driving a $R_L = 1\ \text{k}\Omega$ load and identify all circuit elements. Do not show any noise contributions.

[5]

b) Re-draw your diagram to show explicitly all the Johnson Noise sources in this circuit. Assume that any controlled sources associated with the amplifier are noiseless. What are the values of the noise sources?

[5]

c) Observe that since the amplifier is not unilateral, the output voltage noise sources give rise to 2 equivalent input noise contributions, through the forward and reverse gain respectively. Use circuit transformations to refer the output port noise sources to the input. What are the values of the noise sources? Are they correlated?

[5]

d) For this part treat the amplifier as unilateral. State the definition of the noise factor. Transform the input current noise sources to voltage noise sources through Norton-Thevenin transformations. What is the total excess noise contribution? Make sure to clearly indicate which terms of the noise contribution add algebraically and which vectorially. How does the total excess noise power depend on R_S and R_L? Write an expression for the noise figure of the amplifier in terms of the excess noise.

[5]

2. Derive a block diagram for a counter - based instrument to measure frequency. The required accuracy is 1% or better in the range of 500 Hz to 10 kHz using a 1 kHz reference oscillator. Choose a solution that leads to the shortest, worst - case, measurement time. The instrument should only have one measurement range (i.e. any frequency dividers used in your design should have a fixed modulus). Show clearly any calculations or assumptions made.

[20]

3.

a) Draw a general diagram for a fractional PLL synthesiser and write an expression for its output frequency.

[5]

b) Design a fractional PLL frequency synthesiser in the range of 100-500 MHz. The synthesiser should use a 1 MHz reference oscillator and any prescalers, dividers and filters required to obtain a 10 kHz frequency step.

Specify:

i) Divider values and period of the dual modulus counter.

ii) The minimum VCO gain required for the application. The synthesiser is to be powered off a single 5V supply.

iii) The filter type and filter parameters consistent with the required resolution. Write an expression for the step response time of the synthesiser. (note that a divider change is equivalent to a step reference frequency change!)

[15]

4. A thermometer has been proposed, consisting of 2 identical transistors Q_1 and Q_2 (fig. 4.1). Q_2 is the unknown temperature sensor at temperatures near 300 K while Q_1 is used as a reference and is kept at the ambient temperature, 300 K. Also, as biased, $V_{BE}(Q_1) = 0.7$ V.

   Assume that the transistor saturation current depends on temperature as:

$$I_{ss} = I_0 e^{-qV_G/kT}$$

   where $V_G = 1.2$ V is the silicon bandgap, $q = 1.6 \times 10^{-19}$ C is the electron charge, T is the absolute temperature and $k = 1.38 \times 10^{-23}$ J is the Boltzman constant. The transistors have a 1GHz bandwidth.

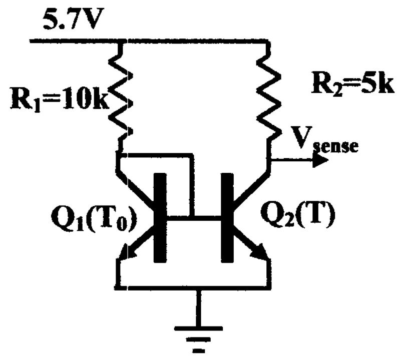


Figure 4.1: The thermometer of question 3.

a) Derive a function describing the sensitivity of this instrument, and express it in terms of the difference of the test temperature from the ambient temperature. Approximate to the leading linear term and the next significant one.

[10]

b) Calculate the noise floor if noise is generated only by the 2 resistors. (Hint: if you regard the circuit as an amplifier, and if there is a voltage V_1 on R_1, then there is $V_1/2$ on R_2)

[5]

c) Calculate the resolution of the instrument. How much measurement integration time is required for the instrument to resolve 0.1K? (the effective noise bandwidth is inverse to the measurement integration time)

[5]

5. The capacitance of a capacitive pressure sensor depends on pressure as follows:

$$C(P) = C_0 + AP + BP^2$$

where:

$$C_0 = 10 \text{ pF}$$
$$A = 100 \text{ pF/Atm}$$
$$B = -10 \text{ pF/Atm}^2$$

It has been decided to use this sensor as if it were a linear sensor with a response of the form $C = aP + b$.

a) Perform an exact least squares fit to determine the best linear model that describes the quadratic sensor over the pressure interval

$$0.5Atm < P < 1.5Atm$$

Hint: Sums become integrations!

[14 Marks]

b) Determine approximate average values for the following quantities:

i) The zero offset of this sensor.
ii) The sensitivity of the sensor.
iii) The differential non-linearity of the sensor.
iv) The integral non-linearity of the sensor.

[6 Marks]

6. Design a bridge instrument to measure the capacitance of a capacitive sensor based on the circuit of figure 6.1. A measurement is made by adjusting V_2, $0 < V_2 < V_1$ (V_2 is in phase with V_1) so that the bridge is nulled. The voltage V_2 is recorded and interpreted.

a) Choose the type of components on the bridge branches. Do not use any variable resistors, variable capacitors or variable inductors. Which branch does the unknown C_x occupy?

[5]

b) Write an equation expressing the unknown capacitance C_x in terms of V_2.

[5]

c) What is the C_x range of this instrument for $0 < V_2 < V_1$? What is the dynamic range for a noiseless measurement? If a big dynamic range is required can you suggest requirements on the relative size of components on the bridge?

[5]

d) Design a circuit so that an op-amp can be used to supply V_2 so the bridge is automatically nulled. It is then easy to just measure V_2 to obtain C_x.

[5]

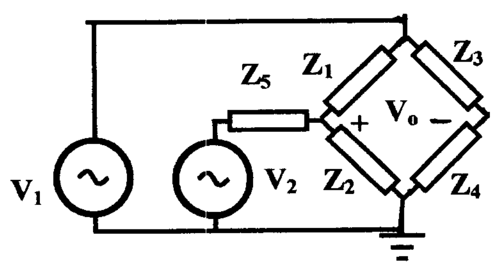
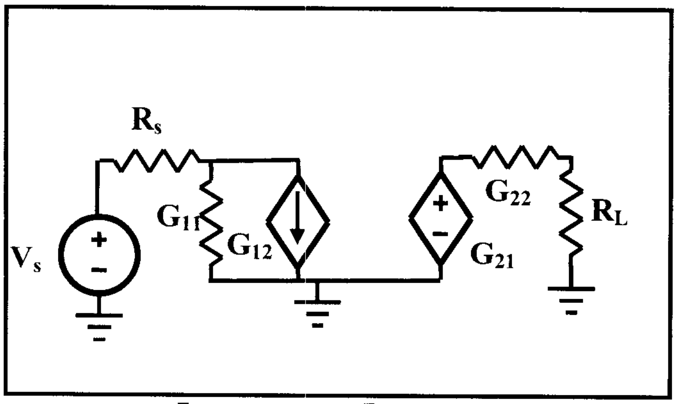


Figure 6.1: Capacitance measurement bridge for Question 6

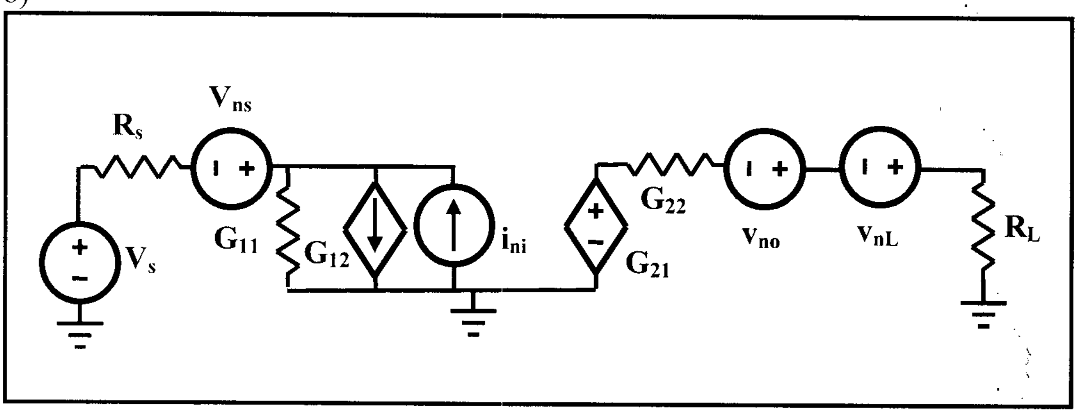
ANSWER Q1: [computed example, extension of theory]
a)



$$G = \begin{bmatrix} 10^{-3}S & .1 \\ 10 & 100\Omega \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

[5]

b)

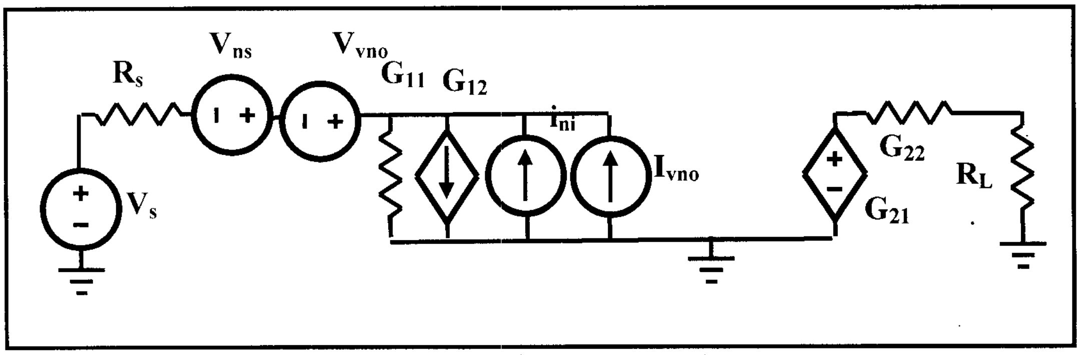


$$i_{ni}^2 = 4kTG_{11}B$$
$$v_{no}^2 = 4kTG_{22}B$$
$$v_{ns}^2 = 4kTR_sB$$
$$v_{nL}^2 = 4kTR_LB$$

The four sources are uncorrelated.

[5]

c)
Each of the output sources gives rise to two input referred contributions, a voltage source through G_{21} and a current source through G_{12}.



$$i_{vno}^2 = 4kTB / \left(G_{12}^2 \left(G_{22} + R_L\right)\right)$$

$$v_{vno}^2 = 4kT\left(G_{22} + R_L\right) B / G_{21}^2$$

The two input referred sources noise sources are correlated:

$$v_{vno}^2 = i_{vno}^2 \left(R_L + G_{22}\right)^2 \frac{G_{12}^2}{G_{21}^2}$$

[5]

d) The amplifier is unilateral, so we only need to examine the input circuit.

By definition , the noise factor is:

$$F = \frac{S/N|_i}{S/N|_o}$$

SNR at the source:

$$\left.\frac{S}{N}\right|_i = \frac{v_s^2}{4kTR_s B}$$

SNR at the output:

$$\left.\frac{S}{N}\right|_o = \frac{v_s^2}{4kTR_s B + v_{g11}^2 + \left(i_{no}/G_{11} + v_{no}\right)^2}$$

This expression clearly depends on the source and load impedances. A more careful consideration of the amplifier as non-unilateral at this stage, eliminates the load dependence.

[5]

ANSWER Q2: [Computed Example]

In principle this can be done either with a frequency or with a period counter Let fx the unknown frequency, and f0 the ref. frequency.

Then a frequency counter gives:

$(N-1)T_x < DT_0 < NT_x$

and the measurement requires a time of DT0. The accuracy required implies that N=100. The lowest count in this type of measurement is at the lowest unknown frequency, so at

$99 \bullet 2x10^{-3} < D \bullet 10^{-3} < 100 \bullet 2x10^{-3}$

so that D=200, and the measurement takes 200msec regardless of the input frequency.

A period counter reverses the roles of fx and f0 so that:

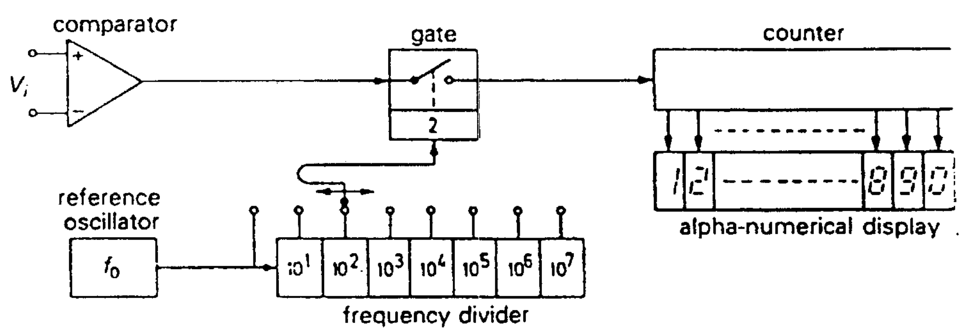
$(N-1)T_0 < DT_x < NT_0$ again the 1% requirement implies $N \geq 100$ so this measurement can be performed in DTx. Then, the lowest count is at the maximum input frequency, so that

$99 \bullet 10^{-3} < D \bullet 10^{-4} < 100 \bullet 10^{-3} \Rightarrow D = 1000$

The measurement will then take 1sec regardless of input frequency.

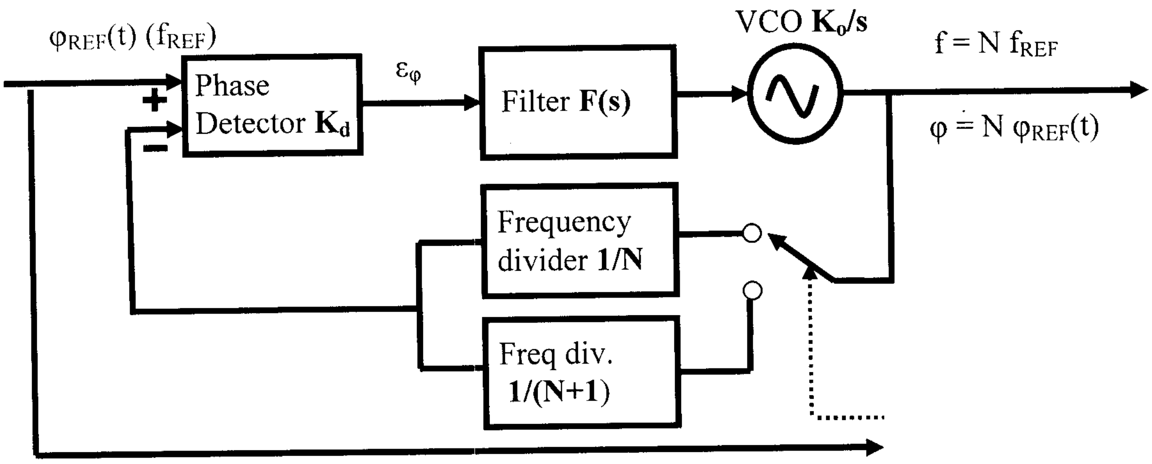
Therefore, a frequency counter, with D=200 is selected, as in the diagram below.

Frequency Counter



ANSWER Q3: [Computed Example]

a)



$$B(s) = \frac{\varphi_{out}}{\varphi_{in}} = \frac{K_d K_o F(s)}{s + K_d K_o F(s)/N} = \frac{NKF(s)}{s + KF(s)} \tag{0.1}$$

where $K = K_d K_o / N$

if out of M cycles m are counted to N+1 and M-m to N,
the effective multiplier is :

$$N_{eff} = \left(\frac{m}{M}(N+1) + \frac{M-m}{M}N\right) = \frac{mN + m + MN - mN}{M} = N + \frac{m}{M}$$

So the fractional PLL can resonve 1/M of f_{ref}.

[5]

b) i)
We need to resolve $f_{ref}/100$ so that M=100.

[5]

b ii) VCO gain:
$K_{vco} \geq 400MHz/5V = 80V/MHz$

[5]

iii) The filter needs to be a lead-lag with a pole below the 10kHz step of the synthesizer. Any detector will do. The settling time is $\tau = 1/2\zeta\omega_n = \tau_1/(1+\tau_2 K)$

since $\omega_n = \sqrt{\frac{K}{\tau_1}}$, $\zeta = \frac{1}{2}\sqrt{\frac{1}{\tau_1 K}}(1+\tau_2 K)$.(will accept as correct also $1/\omega_n, \tau_1$).

[5]

ANSWER Q4: [Computed Example]
Since Q_1 is kept at a constant temperature, Q_2 is biased at a constant V_{BE} =0.7V. Then the collector current of Q_2 is given by:

$$I_{c2} = I_o e^{-qV_G/kT} e^{qV_{BE}/kT}$$

and the output voltage is simply:

$$V_{c2} = V_{CC} - I_{C2}R_2 = V_{CC} - RI_o e^{-q(V_G - V_{BE})/kT_0(1+\delta T/T_0)}$$

The sensitivity is:

$$\frac{dV_{c2}}{dT} = -R\frac{q\left(V_G - V_{BE}\right)}{k\left(T_0 + \delta T\right)^2} I_o e^{-q(V_G - V_{BE})/k(T_0+\delta T)} \simeq$$

$$\simeq \frac{2.5}{.026} e^{-\frac{\delta T}{T_0}} \cdot \frac{0.55}{0.026}\left(1 - \frac{2\delta T}{T_0}\right) \simeq$$

$$\simeq 2304 \cdot \left(1 - \frac{2\delta T}{T_0}\right) \cdot \left(1 - \frac{\delta T}{T_0}\right) e^{-.55/.026} \simeq 1.32 \cdot 10^{-6}\left(1 + \delta T/100\right)$$

Volts/degree.

[10]

b) The Q_1 resistor contributes a noise voltage of :

$V_{n1} = \sqrt{4kTR_1B} = 407uV$

This is effectively added to the 5.7 V supply to give a bias current fluctuation which in turn will map to half as much noise voltage developed on R_2:

$V_{R2n1} = \sqrt{4kTR_1B} / 2 = 203uV$

Similarly, the noise voltage of R_2 is given by:

$V_{n1} = \sqrt{4kTR_2B} = 288uV$

and the total output noise voltage is (since the two sources are independent)

$V_n = \sqrt{V_{n1}^2 + V_{n2}^2} = \sqrt{203^2 + 288^2} = 352\mu VRMS$

[5]

Since the instrument has a sensitivity of 1.32uV/K such an instrument can resolve 266K, which is unacceptable.
We can increase its SNR so it can resolve 0.1 K by reducing its bandwidth by a factor of $(2660)^2$=7.07x10^6 so that a measurement will take 7 msec minimum.

[5]

Answer Q5: [Extension of Theory]

a) we want to approximate the quadratic response with a linear one, so we can do an exact least squares fit:

$$\min_x \Sigma\left(aP+b-C\left(P\right)\right)^2 = \min \int_{P_{\min}}^{P_{\max}} \left(aP+b-C_0-AP-BP^2\right)^2 dP =$$

$$= \min \int_{P_{\min}}^{P_{\max}} \left(\left(a-A\right)P+\left(b-C_0\right)-BP^2\right)^2 dP \Rightarrow$$

$$\frac{\partial}{\partial a}\int_{P_{\min}}^{P_{\max}} \left(\left(a-A\right)P+\left(b-C_0\right)-BP^2\right)^2 dP = \int_{P_{\min}}^{P_{\max}} 2P\left(\left(a-A\right)P+\left(b-C_0\right)-BP^2\right)dP = 0 \Rightarrow$$

$$\frac{2}{3}\left(a-A\right)P^3+P^2\left(b-C_0\right)-\frac{1}{2}BP^4\Big|_{P_{\min}}^{P_{\max}} = 0$$

$$\frac{\partial}{\partial b}\int_{P_{\min}}^{P_{\max}} \left(\left(a-A\right)P+\left(b-C_0\right)-BP^2\right)^2 dP = \int_{P_{\min}}^{P_{\max}} 2\left(\left(a-A\right)P+\left(b-C_0\right)-BP^2\right)dP = 0 \Rightarrow$$

$$\left(a-A\right)P^2+2\left(b-C_0\right)P-\frac{2}{3}BP^3\Big|_{P_{\min}}^{P_{\max}} = 0$$

let $x=a-A$, $y=b-C_0$ with the numbers given,

$$\frac{2}{3}\left(a-A\right)P^3+P^2\left(b-C_0\right)-\frac{1}{2}BP^4\Big|_{P_{\min}}^{P_{\max}} = \frac{2}{3}x\left(1.5^3-.5^3\right)+2y-\frac{1}{2}10\left(1.5^4-.5^4\right)=0 \Rightarrow$$

$$2.166x+2y+25=0$$

and

$$\left(a-A\right)P^2+2\left(b-C_0\right)P-\frac{2}{3}BP^3\Big|_{P_{\min}}^{P_{\max}} = 0 \Rightarrow$$

$$2x+2y+21.667=0$$

This gives rise to the system:

$$\begin{bmatrix} 2.166 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -25 \\ -21.667 \end{bmatrix}$$

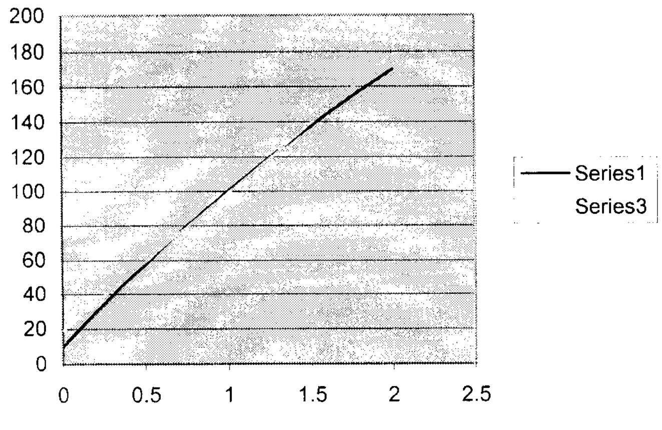
which solves to

$x=-20 \Rightarrow a=80$

$y=9.172 \Rightarrow b=19.172$

[14 Marks]

b)



We can now answer the questions.
Zero offset is b=19.172
Sensitivity=a=80
Differential nonlinearity is the max deviation of the slope from its best value,

$$\max\left(A+BP-a\right)=\max\left(100-10P-80\right)=\max\left(20-10P\right)=15$$

at P=.5
the integral non-linearity is:

$$\max\left|C_0+AP+BP^2-aP-b\right|=\max\left|10+100P-10P^2-80P-19\right|=$$

$$\max\left|-9+20P-10P^2\right|$$

This occurs at P=1 and the integral linearity error is 1pF (actually it is 1.5 pf at the interval endpoints, but the student will probably miss this)

[6 Marks]

Answer Q6: [Computed Example]

a) Assume we want to use a null detection method, with an auxiliary excitation to null the bridge. Since at null $Z_1 Z_4 = Z_2 Z_3$, we can chose, e.g Z_4 for the unknown, and Z_3 a known capacitance. The other components can be resistors.

[5]

b-c) We can also observe, by superposition, that the modified bridge will balance for V_2=0 when

$$Z_1 Z_4 = Z'_2 Z_3 = \left(\frac{Z_2 Z_5}{Z_2 + Z_5}\right) Z_3 \Rightarrow Z_x = \left(\frac{Z_2 Z_5}{Z_1 (Z_2 + Z_5)}\right) Z_3$$

and for $V_2 = V_1$ when

$$Z_1' Z_4 = \left(\frac{Z_1 Z_5}{Z_1 + Z_5}\right) Z_4 = Z_2 Z_3 \Rightarrow Z_X = Z_3 \frac{Z_2 (Z_1 + Z_5)}{Z_1 Z_5}$$

or in general,

$$Z_x = \frac{Z_2 Z_3}{Z_1}\left(\frac{V_2}{V_1}\frac{Z_1 + Z_5}{Z_5} + \left(1 - \frac{V_2}{V_1}\right)\frac{Z_5}{Z_2 - Z_5}\right)$$

Clearly,

$$\frac{Z_2 Z_3}{Z_1}\frac{(Z_1 + Z_5)}{Z_5} > Z_x > \frac{Z_2 Z_3}{Z_1}\frac{Z_5}{(Z_2 + Z_5)} \Rightarrow$$

$$sC_0 \frac{Z_2}{Z_1}\frac{Z_5}{Z_1 + Z_5} < sC_x < sC_0 \frac{Z_2}{Z_1}\frac{Z_2 + Z_5}{Z_5}$$

The dynamic range is:

$$\frac{C_{max}}{C_{min}} = \frac{(Z_2 + Z_5)(Z_1 + Z_5)}{Z_5^2}$$

so we wish to choose $Z_1, Z_2 >> Z_5$ for a big dynamic range.

d) feed the null test voltage to an opamp at inverted polarity, and use the op-amp output to drive v_2.

[5]