E2.5

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

EEE/ISE PART II: MEng, BEng and ACGI

SIGNALS AND LINEAR SYSTEMS

Wednesday, 11 June 2:00 pm

Time allowed: 2:00 hours

Correction to Q1 (b)

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.Y.K. Cheung, P.Y.K. Cheung

Second Marker(s) : M.M. Draief, M.M. Draief

Special instructions for invigilators: None

Information for candidates: None

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[Question 1 is compulsory]

1. a) With a single equation, define the characteristic of a linear system.

[2]

$x(\theta)$

b) Find the even and odd components of the signal $\cancel{x(t)} = e^{j\theta}$.

[2]

c) A continuous-time signal $x(t)$ is shown in Figure 1.1. Sketch the signals

i) $x(t)[u(t) - u(t-1)]$

[3]

ii) $x(t)\,\delta(t - \frac{3}{2})$.

[3]

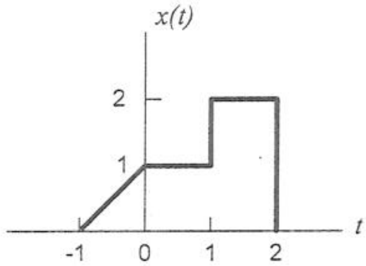

Figure 1.1

d) Consider the RC circuit shown in Figure 1.2. Find the relationship between the input $x(t) = v_s(t)$ and the output $y(t) = i(t)$ in the form of:

i) a differential equation;

[3]

ii) a transfer function.

[3]

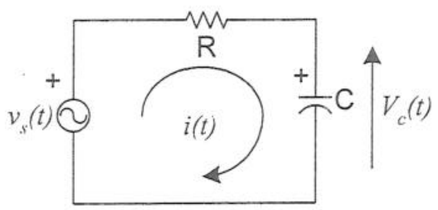

Figure 1.2

e) The unit impulse response of an LTI system is $h(t) = \left[2e^{-3t} - e^{-2t}\right]u(t)$. Find the system's zero-state response $y(t)$ if the input $x(t) = e^{-t}u(t)$. Note that

$$e^{\lambda_1 t}u(t) * e^{\lambda_2 t}u(t) = \frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_1 - \lambda_2}u(t) \qquad \text{for } \lambda_1 \neq \lambda_2.$$

[4]

f) Using the graphical method, find $y(t) = x(t) * h(t)$ where $x(t)$ and $h(t)$ are shown in Figure 1.3.

[4]

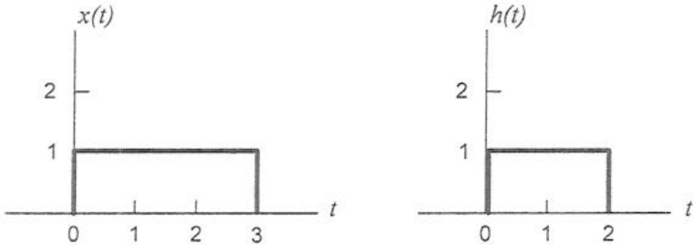

Figure 1.3

g) Find the pole and zero locations for a system with the transfer function

$$H(s) = \frac{s^2 - s + 5/2}{s^2 + 5s + 4}.$$

[4]

h) Given that the Fourier transform of the signal $x(t)$ is $X(\omega)$, i.e. $x(t) \Leftrightarrow X(\omega)$, prove from first principle that

$$x(t - t_0) \Leftrightarrow e^{-j\omega t_0} X(\omega).$$

[4]

i) Using the z-transform pairs $u[k] \Leftrightarrow \dfrac{z}{z-1}$ and $\gamma^k u[k] \Leftrightarrow \dfrac{z}{z-\gamma}$, or otherwise, find the inverse z-transform of

$$F[z] = \frac{z(z-7)}{z^2 - 5z + 4}.$$

[4]

j) A TV signal has a bandwidth of 4.5 MHz. This signal is sampled and quantized with an analogue-to-digital converter.

i) Determine the sampling rate if the signal is to be sampled at a rate 20% above the Nyquist rate.

[2]

ii) If the samples are quantized into 1024 levels, determine the bit-rate (i.e. bits/second) of the binary coded signal.

[2]

2. a) Given the initial conditions $y_0(0) = 0$ and $\dot{y}_0(0) = 1$, find the unit impulse response of an LTI system specified by the equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y(t) = 2\frac{dx}{dt} + 9x(t).$$

[15]

b) An input signal $f(t)$ is expressed in terms of step components as shown in Figure 2.1. The step component at time $t = \tau$ has a height of Δf which can be expressed as

$$\Delta f = \frac{\Delta f}{\Delta \tau}\Delta \tau = \dot{f}(\tau)\Delta \tau.$$

If $g(t)$ is the unit step response of an LTI system to the step input $u(t)$, show that the zero-state response $y(t)$ of the system to the input $f(t)$ can be expressed as

$$y(t) = \int_{-\infty}^{\infty} \dot{f}(\tau)g(t-\tau)d\tau = \dot{f}(t) * g(t).$$

[15]

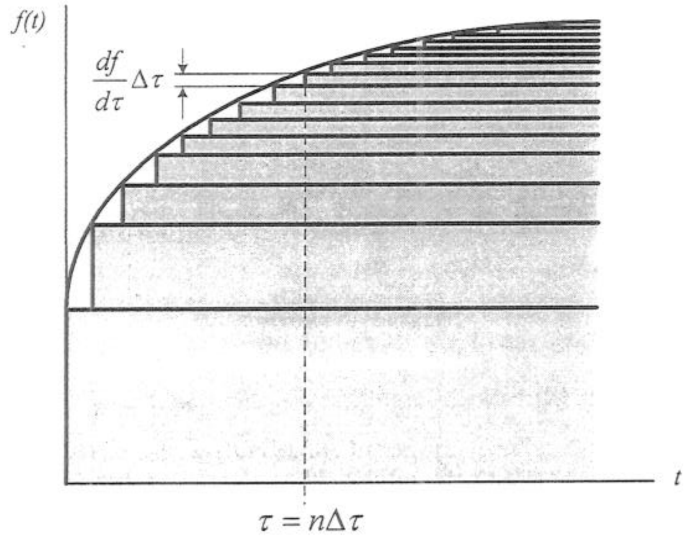

Figure 2.1

3. a) Find the Fourier transform of the signal shown in Figure 3.1 using two different methods:

   i) By direct integration using the definition of the Fourier transform

   [10]

   ii) Using only the time-shifting property and the Fourier transform pair

$$rect\left(\frac{t}{\tau}\right) \Leftrightarrow \tau\ sinc\left(\frac{\omega\tau}{2}\right).$$

   [10]

b) Given that $\int_{-\infty}^{\infty} e^{-x^2/2}dx = \sqrt{2\pi}$, show that the energy E_f of a Gaussian pulse

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}$$

is given by

$$E_f = \frac{1}{2\sigma\sqrt{\pi}}.$$

You should derive the energy E_f from $F(\omega)$ using the Parseval's theorem and the following Fourier transform pair

$$e^{-t^2/2\sigma^2} \Leftrightarrow \sigma\sqrt{2\pi}\ e^{-\sigma^2\omega^2/2}.$$

[10]

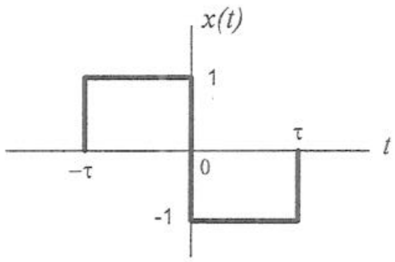

Figure 3.1

4. A discrete-time LTI system is specified by the difference equation

$$y[k+1]-0.5y[k]=f[k+1]+0.8f[k].$$

a) Derive its transfer function in the z-domain.

[6]

b) Find the amplitude and phase response of the system.

[14]

c) Find the system response $y[k]$ for the input $f[k]=\cos(0.5k-\frac{\pi}{3})$.

[10]

[THE END]