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[Before starting, please make sure that the paper is complete; there should be 7 pages, with a total of 10 questions. Ask the invigilator for a replacement if your copy is faulty.]

A mathematical formulae sheet is provided.

Answer EIGHT questions.

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO

Date Thursday 4th June 2009 10.00 am - 1.00 pm

PART I : MATHEMATICS 2 (ELECTRICAL ENGINEERING)

B.ENG. AND M.ENG. EXAMINATIONS 2009

[E1.14 (Maths 2) 2009]

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(iv) all zeros of $f(x)$.

(iii) all stationary points and their nature,

(ii) $\lim_{x \rightarrow \infty} f(x)$,(i) $\lim_{x \rightarrow 0^+} f(x)$,with domain $(0, \infty)$, specifying

$$f(x) = -x(\ln(x) - 1),$$

2. Sketch the graph of the function

an even or odd function of x ? Find $\operatorname{Im}(p(ix))$, and determine whether it is even or odd as a function of x .

$$\operatorname{Re}(p(ix))$$

If $i^2 + 1 = 0$ and x is real, is

$$p(x) = \sum_{n=0}^{\infty} \frac{(2n)!}{x^{2n}}.$$

(ii) The function $p(x)$ is defined by a power series(e) $f(x) = x^3$ with domain $(-10, 10)$.(d) $f(x) = x^{1/3}$ with domain $(-10, 10)$,(c) $f(x) = \sin(x)$ with domain $(-1, 1)$,(b) $f(x) = \cos(x)$ with domain $(-1, 1)$,(a) $f(x) = x^2$ with domain $(-2, 1)$,

range of your function:

1. (i) State whether each of the following functions is invertible or not. If not, change the domain in order that the resulting function has an inverse and give the

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You may use the identity $\sinh(2x) = 2 \sinh(x) \cosh(x)$.
 in terms of $\sinh^{-1}(x)$ and x using a substitution based on the hyperbolic sine.

$$\int \sqrt{1+x^2} dx$$

Hence write the indefinite integral

$\cosh^2(x) - \sinh^2(x) = A$, for all real x .
 in terms of $\cosh(x)$ and $\sinh(x)$, and determine the constant A satisfying

$$\cosh(x) \text{ and } \frac{d}{dx} \cosh(x) \text{ and } \frac{d}{dx} \sinh(x)$$

whose domains are given by the set of all real numbers. Find

$$\cosh(x) = \frac{1}{2}(e_x + e_{-x}) \text{ and } \sinh(x) = \frac{1}{2}(e_x - e_{-x}),$$

(ii) The hyperbolic cosine and sine are functions defined respectively by
 with a trigonometric substitution.

$$\int_1^{-1} \sqrt{1-x^2} dx$$

4. (i) Evaluate the integral

for all x between a and b and all integers $n \geq 1$.

$$\int_a^b ((x)f)^n dx = (x) \frac{dp^n}{dp}$$

prove by induction, or otherwise, that

$$(x)f((x)) = (x) \frac{dp}{dp}$$

(ii) If $f(x)$ is a function defined on a domain (a, b) and

$$(p) \sum_{k=1}^{n+1} \frac{1}{k} \cos(kx).$$

$$(c) \int_x^0 \sqrt{1+t^2} dt,$$

$$(b) \sin^{-1}(\sin(x)) + \sin(\sin^{-1}(x)) \quad \text{for } -1 < x < 1,$$

$$(a) x^p,$$

3. (i) Differentiate the following functions with respect to x :

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using only the first few terms of your series, find an approximation to $\ln(2)$.

$$(3/5)^3 \approx 0.216 \text{ and } (3/5)^5 \approx 0.0778,$$

If you are told that

$$\ln \left| \frac{1-x}{1+x} \right|^{1/2}$$

for the function

Using this and the result of part (i), or otherwise, deduce a MacLaurin series

$$\frac{1}{2} \int \left[\frac{1+x}{1-x} + \frac{1-x}{1+x} \right] dx.$$

(iii) Evaluate the integral

this interval.

converges for all x in some interval. Determine the upper and lower limits of

$$x^n = \frac{1-x}{1-x^n} \sum_{n=0}^{\infty}$$

for all $x \neq 1$ where $N \geq 0$ is a positive integer. Hence deduce that

$$x^n = \frac{1-x}{1-x^{N+1}} \sum_{n=0}^{N-1}$$

5. (i) Prove that

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and obtain an analogous result for the series $\sum_{n=0}^{\infty} r^n \sin(n\theta)$.

$$\sum_{n=0}^{\infty} r^n \cos(n\theta) = \frac{B - 2r \cos(\theta) + r^2}{A - r \cos(\theta)} \quad (0 < r < 1);$$

such that
for real θ and $r > 0$ to deduce that there are real numbers A and B (to be found)
converges for all complex z with $|z| < 1$. Use a substitution of the form $z = re^{i\theta}$

$$\sum_{n=0}^{\infty} z^n = \frac{1-z}{1-z} = 1 + z + z^2 + \dots$$

(iii) You are given that the geometric series

$$e_{\cos(x)} \cos(\sin(x)) \text{ and } e_{\cos(x)} \sin(\sin(x)).$$

to find Fourier series representations for both the functions

$$e_z = \sum_{n=0}^{\infty} z^n$$

real. Hence use a substitution of the form $z = e^{ix}$ in the power series
(ii) For any real number x , write $e_{\cos(x)+i\sin(x)}$ in the form $a + ib$ where a and b are

6. (i) Write $\cos(x)$ and $\sin(x)$ in terms of e^{ix} and e^{-ix} .

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What happens if $a_1 = 0$ is used as the initial guess?

$$a_n = \frac{4a_{n-1}^3 + 12a_{n-1}^2 + 8a_{n-1} - 1}{3a_{n-1}^4 + 8a_{n-1}^3 + 4a_{n-1}^2 + 1}, \quad n \geq 2.$$

leads to the recurrence relation

$$x_4 + 4x_3 + 4x_2 - x - 1 = 0$$

(ii) Show that Newton's method for approximating roots of

$$5/2.$$

8. (i) Find $10^{1/3}$ correct to 5 decimal places using Newton's method with initial guess

Show that the estimated range for P is between 980 and 1420.

$$\text{and } R = 8.$$

$PV = RT$, where R is a constant. Suppose that $T = 300 \pm 10$, $V = 2 \pm 0.3$ (iii) The pressure P of a volume of gas V at temperature T satisfies the equation

differentiating directly.

Verify your results by substituting for x, y, z in terms of r, θ, z into f and

Use the chain rule to evaluate $\frac{\partial g}{\partial r}$ and $\frac{\partial g}{\partial \theta}$, in terms of r, θ and z .

$$f(x, y, z) = x^2 + 3xy + y^2 + z^2 = g(r, \theta, z).$$

Suppose

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z.$$

states (x, y, z) by

7. (i) Cylindrical polar coordinates (r, θ, z) are defined in terms of Cartesian coordinates

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END OF PAPER

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} [f(x)]^2 dx = c_0^2 + 2 \sum_{n=1}^{\infty} |c_n|^2$$

with Parseval's theorem stated as follows:

$$f(x) = c_0 + 2 \sum_{n=1}^{\infty} R e(c_n e^{inx}) \quad \text{with} \quad c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx,$$

Hint: Use either the real or the complex Fourier series representation of the real, 2π -periodic function $f(x)$. The latter is given by

in terms of π .

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

Fourier series of f and hence evaluate
If $f(x)$ is the 2π -periodic function that coincides with x on $(-\pi, \pi)$, determine the

is any constant, possibly complex, and x is real.

10. Using integration by parts, or otherwise, evaluate the integral $\int x e^{kx} dx$, where k

$$y = 1/2 \quad \text{when} \quad x = 1/2.$$

Assuming $b = 2$, solve this equation for u . Hence determine y satisfying

where a and b are constants to be determined.

$$\frac{dx}{du} + (1-b) \frac{x}{u} = x b u^a,$$

form

make the substitution $u = y^{1-b}$ to obtain a differential equation for u of the

$$x \frac{dy}{dx} + y = x^2 y^2,$$

(ii) Given the differential equation

is exact. Hence find the solution of the equation that satisfies $y(1) = 1$.

$$(2x^2 y + 4) \frac{dx}{dy} + 2xy^2 - 3 = 0$$

9. (i) Show the equation

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M A T H E M A T I C S D E P A R T M E N T

3. TRIGONOMETRIC IDENTITIES AND HYPERBOLIC FUNCTIONS

MATHEMATICAL FORMULAE

$$\sin(a+b) = \sin a \cos b + \cos a \sin b;$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b.$$

I. VECTOR ALGEBRA

$$\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k} = (a_1, a_2, a_3)$$

$$\text{Scalar (dot) product: } \mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\text{Vector (cross) product: } \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Scalar triple product:

$$[\mathbf{a}, \mathbf{b}, \mathbf{c}] = \mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \mathbf{b} \cdot \mathbf{c} \times \mathbf{a} = \mathbf{c} \cdot \mathbf{a} \times \mathbf{b} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\text{Vector triple product: } \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$$

2. SERIES

$$(1+x)^{\alpha} = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots \quad (\alpha \text{ arbitrary, } |x| < 1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots ,$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots ,$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots ,$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^n \frac{x^{n+1}}{(n+1)} + \dots \quad (-1 < x \leq 1)$$

3. DIFFERENTIAL CALCULUS

(a) Leibniz's formula:

$$D^n(fg) = f D^n g + (C_1) Df D^{n-1} g + \dots + (C_r) D^r f D^{n-r} g + \dots + D^n f g .$$

(b) Taylor's expansion of $f(x)$ about $x = a$:

$$f(a+h) = f(a) + h f'(a) + h^2 f''(a)/2! + \dots + h^n f^{(n)}(a)/n! + \epsilon_n(h),$$

where $\epsilon_n(h) = h^{n+1} f^{(n+1)}(a+\theta h)/(n+1)!$, $0 < \theta < 1$.

(c) Taylor's expansion of $f(x, y)$ about (a, b) :

$$f(a+h, b+k) = f(a, b) + [h f_x + k f_y]_{a,b} + 1/2! [h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}]_{a,b} + \dots$$

(d) Partial differentiation of $f(x, y)$:

$$\text{i. If } y = y(x), \text{ then } f = F(x), \text{ and } \frac{dF}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx} .$$

$$\text{ii. If } x = x(t), y = y(t), \text{ then } f = F(t), \text{ and } \frac{dF}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} .$$

$$\text{iii. If } x = x(u, v), y = y(u, v), \text{ then } f = F(u, v), \text{ and}$$

$$\frac{\partial F}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial u}, \quad \frac{\partial F}{\partial v} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial v} .$$

(e) Stationary points of $f(x, y)$ occur where $f_x = 0$, $f_y = 0$ simultaneously.

Let (a, b) be a stationary point; examine $D = [f_{xx} f_{yy} - (f_{xy})^2]_{a,b}$.

If $D > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a maximum;

If $D > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a minimum;

If $D < 0$ then (a, b) is a saddle-point.

(f) Differential equations:

i. The first order linear equation $dy/dx + P(x)y = Q(x)$ has an integrating factor $I(x) = \exp[\int P(x)(dx)]$, so that $\frac{d}{dx}(Iy) = IQ$.

ii. $P(x, y)dx + Q(x, y)dy = 0$ is exact if $\partial Q/\partial x = \partial P/\partial y$.

5. INTEGRAL CALCULUS

- (a) An important substitution: $\tan(\theta/2) = t$:
 $\sin \theta = 2t/(1+t^2)$, $\cos \theta = (1-t^2)/(1+t^2)$, $d\theta = 2dt/(1+t^2)$.

- (b) Some indefinite integrals:

$$\int (a^2 - x^2)^{-1/2} dx = \sin^{-1}\left(\frac{x}{a}\right), \quad |x| < a.$$

$$\int (a^2 + x^2)^{-1/2} dx = \sinh^{-1}\left(\frac{x}{a}\right) = \ln\left\{\frac{x}{a} + \left(1 + \frac{x^2}{a^2}\right)^{1/2}\right\}.$$

$$\int (x^2 - a^2)^{-1/2} dx = \cosh^{-1}\left(\frac{x}{a}\right) = \ln\left|\frac{x}{a} + \left(\frac{x^2}{a^2} - 1\right)^{1/2}\right|.$$

$$\int (a^2 + x^2)^{-1} dx = \left(\frac{1}{a}\right) \tan^{-1}\left(\frac{x}{a}\right).$$

6. NUMERICAL METHODS

- (a) Approximate solution of an algebraic equation:

If a root of $f(x) = 0$ occurs near $x = a$, take $x_0 = a$ and
 $x_{n+1} = x_n - [f(x_n)/f'(x_n)]$, $n = 0, 1, 2, \dots$

(Newton Raphson method).

- (b) Formulae for numerical integration: Write $x_n = x_0 + nh$, $y_n = y(x_n)$.

- i. Trapezium rule (1-strip): $\int_{x_0}^{x_1} y(x) dx \approx (h/2)[y_0 + y_1]$.

- ii. Simpson's rule (2-strip): $\int_{x_0}^{x_2} y(x) dx \approx (h/3)[y_0 + 4y_1 + y_2]$.

- (c) Richardson's extrapolation method: Let $I = \int_a^b f(x) dx$ and let I_1, I_2 be two estimates of I obtained by using Simpson's rule with intervals h and $h/2$.

Then, provided h is small enough,

$$I_2 + (I_2 - I_1)/15,$$

is a better estimate of I .

7. LAPLACE TRANSFORMS

Function	Transform	Function	Transform	Function	Transform
$f(t)$	$F(s) = \int_0^\infty e^{-st} f(t) dt$	$a f(t) + b g(t)$	$a F(s) + b G(s)$	$s^2 F(s) - s f(0) - f'(0)$	$s^2 F(s) + b G(s)$
df/dt	$s F(s) - f(0)$	$d^2 f/dt^2$	$t f(t)$	$-dF(s)/ds$	
$e^{at} f(t)$	$F(s-a)$	$f(t)$	$f'_0 f(t) dt$	$F'(s)$	
$(\partial/\partial \alpha) f(t, \alpha)$	$(\partial/\partial \alpha) F(s, \alpha)$	$F(s) G(s)$			
$\int_0^t f(u) g(t-u) du$					
1	$1/s$	t^n ($n = 1, 2, \dots$)		$n!/s^{n+1}$, ($n > 0$)	
e^{at}	$1/(s-a)$, ($s > a$)	$\sin \omega t$		$\omega/(s^2 + \omega^2)$, ($s > 0$)	
$\cos \omega t$	$s/(s^2 + \omega^2)$, ($s > 0$)	$H(t-T) = \begin{cases} 0, & t \leq T \\ 1, & t > T \end{cases}$		e^{-sT}/s , ($s, T > 0$)	

8. FOURIER SERIES

If $f(x)$ is periodic of period $2L$, then $f(x+2L) = f(x)$, and

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}, \quad \text{where}$$

(Parseval's theorem)

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, \quad n = 0, 1, 2, \dots, \text{ and}$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, \quad n = 1, 2, 3, \dots$$

Parseval's theorem

$$\frac{1}{L} \int_{-L}^L [f(x)]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

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	EE3	EXAMINATION QUESTIONS/SOLUTIONS 2008-09	EE1(2)	
Marks & seen/unseen	Page 1 of 2			
3	(i) a) $f(x) = x^2$ is NOT invertible as $(-2, 1)$ but it is an $(0, 1)$, which range $(0, 1)$. $f(x) = \cos x$ is NOT invertible as $(-1, 1)$ but it is an $(0, 1)$, which range $(0, 1)$.			
3	(i) b) $f(x) = x^3$ as $(-10, 10)$ is invertible.			
3	(ii) Given that $p(x) = \sum_{n=0}^{\infty} x^{2n} / (2n)!$ Then $p(ix) = \sum_{n=0}^{\infty} (ix)^{2n} / (2n)!$ $= \sum_{n=0}^{\infty} i^{2n} x^{2n} / (2n)!$ $= \sum_{n=0}^{\infty} (-1)^n \frac{(2n)!}{x^{2n}} (i^2)^n$ $= \sum_{n=0}^{\infty} (-1)^n \frac{(2n)!}{x^{2n}}$			
3	c) $f(x) = x^3$ as $(-10, 10)$ is invertible.			
3	d) $f(x) = x^3$ as $(-10, 10)$ is invertible.			
3	e) $f(x) = \sin x$ as $(-1, 1)$ is invertible.			
	EE1(2)			

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20 Total			
5			
	<p>and so $E_{\text{even}}(P(i x)) = \phi(i x)$ is an even function.</p> <p>and $E_{\text{odd}}(P(i x)) = 0$ for all x,</p> <p>which is even.</p> <p>and odd } $\{$</p>		
Question	Page 2 of 2 .	Marks &	
EEI(2)			
Course	EXAMINATION QUESTIONS/SOLUTIONS 2008-09		

Question	Course	EXAMINATION QUESTIONS/SOLUTIONS 2008-09	Marks & seen/unseen	Parts
EE1(b)				
2	Page 2 of 2			
Ques10				
for part (iv)	$\frac{d}{dx} f(x) = -u(x)$ since if $x = e$, $f(x) = 0$ if $x = e$.	$\lim_{x \rightarrow 0} \frac{df}{dx} = -\infty$		
for graph 6		$f(x) = y$		
useless				
(16)(20)				

Page number	Setter's initials	Checkers initials	Course	Question	Parts
3	CB				
3				a) To find $\frac{dy}{dx} = x \sin(x)$	
3				$\frac{dy}{dx} = x \sin(x) + \frac{x^2}{2} \cos(x)$	
3				$\frac{d^2y}{dx^2} = x \cos(x) - \frac{x^2}{2} \sin(x)$	
3				$\frac{d^3y}{dx^3} = \frac{x^2}{2} \sin(x) + x \cos(x)$	
3				$\frac{d^4y}{dx^4} = \frac{x^3}{4} \sin(x) + x^2 \cos(x)$	
3				$\frac{d^5y}{dx^5} = \frac{x^4}{8} \sin(x) + x^3 \cos(x)$	
3				$\frac{d^6y}{dx^6} = \frac{x^5}{16} \sin(x) + x^4 \cos(x)$	
3				$\frac{d^7y}{dx^7} = \frac{x^6}{32} \sin(x) + x^5 \cos(x)$	
3				$\frac{d^8y}{dx^8} = \frac{x^7}{64} \sin(x) + x^6 \cos(x)$	
3				$\frac{d^9y}{dx^9} = \frac{x^8}{128} \sin(x) + x^7 \cos(x)$	
3				$\frac{d^{10}y}{dx^{10}} = \frac{x^9}{256} \sin(x) + x^8 \cos(x)$	
3				$\frac{d^{11}y}{dx^{11}} = \frac{x^{10}}{512} \sin(x) + x^9 \cos(x)$	
3				$\frac{d^{12}y}{dx^{12}} = \frac{x^{11}}{1024} \sin(x) + x^{10} \cos(x)$	
3				$\frac{d^{13}y}{dx^{13}} = \frac{x^{12}}{2048} \sin(x) + x^{11} \cos(x)$	
3				$\frac{d^{14}y}{dx^{14}} = \frac{x^{13}}{4096} \sin(x) + x^{12} \cos(x)$	
3				$\frac{d^{15}y}{dx^{15}} = \frac{x^{14}}{8192} \sin(x) + x^{13} \cos(x)$	
3				$\frac{d^{16}y}{dx^{16}} = \frac{x^{15}}{16384} \sin(x) + x^{14} \cos(x)$	
3				$\frac{d^{17}y}{dx^{17}} = \frac{x^{16}}{32768} \sin(x) + x^{15} \cos(x)$	
3				$\frac{d^{18}y}{dx^{18}} = \frac{x^{17}}{65536} \sin(x) + x^{16} \cos(x)$	
3				$\frac{d^{19}y}{dx^{19}} = \frac{x^{18}}{131072} \sin(x) + x^{17} \cos(x)$	
3				$\frac{d^{20}y}{dx^{20}} = \frac{x^{19}}{262144} \sin(x) + x^{18} \cos(x)$	
3				$\frac{d^{21}y}{dx^{21}} = \frac{x^{20}}{524288} \sin(x) + x^{19} \cos(x)$	
3				$\frac{d^{22}y}{dx^{22}} = \frac{x^{21}}{1048576} \sin(x) + x^{20} \cos(x)$	
3				$\frac{d^{23}y}{dx^{23}} = \frac{x^{22}}{2097152} \sin(x) + x^{21} \cos(x)$	
3				$\frac{d^{24}y}{dx^{24}} = \frac{x^{23}}{4194304} \sin(x) + x^{22} \cos(x)$	
3				$\frac{d^{25}y}{dx^{25}} = \frac{x^{24}}{8388608} \sin(x) + x^{23} \cos(x)$	
3				$\frac{d^{26}y}{dx^{26}} = \frac{x^{25}}{16777216} \sin(x) + x^{24} \cos(x)$	
3				$\frac{d^{27}y}{dx^{27}} = \frac{x^{26}}{33554432} \sin(x) + x^{25} \cos(x)$	
3				$\frac{d^{28}y}{dx^{28}} = \frac{x^{27}}{67108864} \sin(x) + x^{26} \cos(x)$	
3				$\frac{d^{29}y}{dx^{29}} = \frac{x^{28}}{134217728} \sin(x) + x^{27} \cos(x)$	
3				$\frac{d^{30}y}{dx^{30}} = \frac{x^{29}}{268435456} \sin(x) + x^{28} \cos(x)$	
3				$\frac{d^{31}y}{dx^{31}} = \frac{x^{30}}{536870912} \sin(x) + x^{29} \cos(x)$	
3				$\frac{d^{32}y}{dx^{32}} = \frac{x^{31}}{1073741824} \sin(x) + x^{30} \cos(x)$	
3				$\frac{d^{33}y}{dx^{33}} = \frac{x^{32}}{2147483648} \sin(x) + x^{31} \cos(x)$	
3				$\frac{d^{34}y}{dx^{34}} = \frac{x^{33}}{4294967296} \sin(x) + x^{32} \cos(x)$	
3				$\frac{d^{35}y}{dx^{35}} = \frac{x^{34}}{8589934592} \sin(x) + x^{33} \cos(x)$	
3				$\frac{d^{36}y}{dx^{36}} = \frac{x^{35}}{17179869184} \sin(x) + x^{34} \cos(x)$	
3				$\frac{d^{37}y}{dx^{37}} = \frac{x^{36}}{34359738368} \sin(x) + x^{35} \cos(x)$	
3				$\frac{d^{38}y}{dx^{38}} = \frac{x^{37}}{68719476736} \sin(x) + x^{36} \cos(x)$	
3				$\frac{d^{39}y}{dx^{39}} = \frac{x^{38}}{137438953472} \sin(x) + x^{37} \cos(x)$	
3				$\frac{d^{40}y}{dx^{40}} = \frac{x^{39}}{274877906944} \sin(x) + x^{38} \cos(x)$	
3				$\frac{d^{41}y}{dx^{41}} = \frac{x^{40}}{549755813888} \sin(x) + x^{39} \cos(x)$	
3				$\frac{d^{42}y}{dx^{42}} = \frac{x^{41}}{1099511627776} \sin(x) + x^{40} \cos(x)$	
3				$\frac{d^{43}y}{dx^{43}} = \frac{x^{42}}{2199023255552} \sin(x) + x^{41} \cos(x)$	
3				$\frac{d^{44}y}{dx^{44}} = \frac{x^{43}}{4398046511104} \sin(x) + x^{42} \cos(x)$	
3				$\frac{d^{45}y}{dx^{45}} = \frac{x^{44}}{8796093022208} \sin(x) + x^{43} \cos(x)$	
3				$\frac{d^{46}y}{dx^{46}} = \frac{x^{45}}{17592186044416} \sin(x) + x^{44} \cos(x)$	
3				$\frac{d^{47}y}{dx^{47}} = \frac{x^{46}}{35184372088832} \sin(x) + x^{45} \cos(x)$	
3				$\frac{d^{48}y}{dx^{48}} = \frac{x^{47}}{70368744177664} \sin(x) + x^{46} \cos(x)$	
3				$\frac{d^{49}y}{dx^{49}} = \frac{x^{48}}{140737488355328} \sin(x) + x^{47} \cos(x)$	
3				$\frac{d^{50}y}{dx^{50}} = \frac{x^{49}}{281474976710656} \sin(x) + x^{48} \cos(x)$	
3				$\frac{d^{51}y}{dx^{51}} = \frac{x^{50}}{562949953421312} \sin(x) + x^{49} \cos(x)$	
3				$\frac{d^{52}y}{dx^{52}} = \frac{x^{51}}{1125899906842624} \sin(x) + x^{50} \cos(x)$	
3				$\frac{d^{53}y}{dx^{53}} = \frac{x^{52}}{2251799813685248} \sin(x) + x^{51} \cos(x)$	
3				$\frac{d^{54}y}{dx^{54}} = \frac{x^{53}}{4503599627370496} \sin(x) + x^{52} \cos(x)$	
3				$\frac{d^{55}y}{dx^{55}} = \frac{x^{54}}{9007199254740992} \sin(x) + x^{53} \cos(x)$	
3				$\frac{d^{56}y}{dx^{56}} = \frac{x^{55}}{18014398509481984} \sin(x) + x^{54} \cos(x)$	
3				$\frac{d^{57}y}{dx^{57}} = \frac{x^{56}}{36028797018963968} \sin(x) + x^{55} \cos(x)$	
3				$\frac{d^{58}y}{dx^{58}} = \frac{x^{57}}{72057594037927936} \sin(x) + x^{56} \cos(x)$	
3				$\frac{d^{59}y}{dx^{59}} = \frac{x^{58}}{144115188075855872} \sin(x) + x^{57} \cos(x)$	
3				$\frac{d^{60}y}{dx^{60}} = \frac{x^{59}}{288230376151711744} \sin(x) + x^{58} \cos(x)$	
3				$\frac{d^{61}y}{dx^{61}} = \frac{x^{60}}{576460752303423488} \sin(x) + x^{59} \cos(x)$	
3				$\frac{d^{62}y}{dx^{62}} = \frac{x^{61}}{1152921504606846976} \sin(x) + x^{60} \cos(x)$	
3				$\frac{d^{63}y}{dx^{63}} = \frac{x^{62}}{2305843009213693952} \sin(x) + x^{61} \cos(x)$	
3				$\frac{d^{64}y}{dx^{64}} = \frac{x^{63}}{4611686018427387904} \sin(x) + x^{62} \cos(x)$	
3				$\frac{d^{65}y}{dx^{65}} = \frac{x^{64}}{9223372036854775808} \sin(x) + x^{63} \cos(x)$	
3				$\frac{d^{66}y}{dx^{66}} = \frac{x^{65}}{18446744073709551616} \sin(x) + x^{64} \cos(x)$	
3				$\frac{d^{67}y}{dx^{67}} = \frac{x^{66}}{36893488147419103232} \sin(x) + x^{65} \cos(x)$	
3				$\frac{d^{68}y}{dx^{68}} = \frac{x^{67}}{73786976294838206464} \sin(x) + x^{66} \cos(x)$	
3				$\frac{d^{69}y}{dx^{69}} = \frac{x^{68}}{147573952589676412928} \sin(x) + x^{67} \cos(x)$	
3				$\frac{d^{70}y}{dx^{70}} = \frac{x^{69}}{295147905179352825856} \sin(x) + x^{68} \cos(x)$	
3				$\frac{d^{71}y}{dx^{71}} = \frac{x^{70}}{590295810358705651712} \sin(x) + x^{69} \cos(x)$	
3				$\frac{d^{72}y}{dx^{72}} = \frac{x^{71}}{1180591620717411303424} \sin(x) + x^{70} \cos(x)$	
3				$\frac{d^{73}y}{dx^{73}} = \frac{x^{72}}{2361183241434822606848} \sin(x) + x^{71} \cos(x)$	
3				$\frac{d^{74}y}{dx^{74}} = \frac{x^{73}}{4722366482869645213696} \sin(x) + x^{72} \cos(x)$	
3				$\frac{d^{75}y}{dx^{75}} = \frac{x^{74}}{9444732965739290427392} \sin(x) + x^{73} \cos(x)$	
3				$\frac{d^{76}y}{dx^{76}} = \frac{x^{75}}{18889465931478580854784} \sin(x) + x^{74} \cos(x)$	
3				$\frac{d^{77}y}{dx^{77}} = \frac{x^{76}}{37778931862957161709568} \sin(x) + x^{75} \cos(x)$	
3				$\frac{d^{78}y}{dx^{78}} = \frac{x^{77}}{75557863725914323419136} \sin(x) + x^{76} \cos(x)$	
3				$\frac{d^{79}y}{dx^{79}} = \frac{x^{78}}{151115727458286466838272} \sin(x) + x^{77} \cos(x)$	
3				$\frac{d^{80}y}{dx^{80}} = \frac{x^{79}}{302231454916572933676544} \sin(x) + x^{78} \cos(x)$	
3				$\frac{d^{81}y}{dx^{81}} = \frac{x^{80}}{604462909833145867353088} \sin(x) + x^{79} \cos(x)$	
3				$\frac{d^{82}y}{dx^{82}} = \frac{x^{81}}{1208925819666291734706176} \sin(x) + x^{80} \cos(x)$	
3				$\frac{d^{83}y}{dx^{83}} = \frac{x^{82}}{2417851639332583469412352} \sin(x) + x^{81} \cos(x)$	
3				$\frac{d^{84}y}{dx^{84}} = \frac{x^{83}}{4835703278665166938824704} \sin(x) + x^{82} \cos(x)$	
3				$\frac{d^{85}y}{dx^{85}} = \frac{x^{84}}{9671406557330333877649408} \sin(x) + x^{83} \cos(x)$	
3				$\frac{d^{86}y}{dx^{86}} = \frac{x^{85}}{19342813114660667755298816} \sin(x) + x^{84} \cos(x)$	
3				$\frac{d^{87}y}{dx^{87}} = \frac{x^{86}}{38685626229321335510597632} \sin(x) + x^{85} \cos(x)$	
3				$\frac{d^{88}y}{dx^{88}} = \frac{x^{87}}{77371252458642671021195264} \sin(x) + x^{86} \cos(x)$	
3				$\frac{d^{89}y}{dx^{89}} = \frac{x^{88}}{154742504917285342042385128} \sin(x) + x^{87} \cos(x)$	
3				$\frac{d^{90}y}{dx^{90}} = \frac{x^{89}}{309485009834570684084770256} \sin(x) + x^{88} \cos(x)$	
3				$\frac{d^{91}y}{dx^{91}} = \frac{x^{90}}{618970019669141368169540512} \sin(x) + x^{89} \cos(x)$	
3				$\frac{d^{92}y}{dx^{92}} = \frac{x^{91}}{1237940039338282736339081024} \sin(x) + x^{90} \cos(x)$	
3				$\frac{d^{93}y}{dx^{93}} = \frac{x^{92}}{2475880078676565472678162048} \sin(x) + x^{91} \cos(x)$	
3				$\frac{d^{94}y}{dx^{94}} = \frac{x^{93}}{4951760157353130945356324096} \sin(x) + x^{92} \cos(x)$	
3				$\frac{d^{95}y}{dx^{95}} = \frac{x^{94}}{9903520314706261890712648192} \sin(x) + x^{93} \cos(x)$	
3				$\frac{d^{96}y}{dx^{96}} = \frac{x^{95}}{19807040629412523781425296384} \sin(x) + x^{94} \cos(x)$	
3				$\frac{d^{97}y}{dx^{97}} = \frac{x^{96}}{39614081258825047562850592768} \sin(x) + x^{95} \cos(x)$	
3				$\frac{d^{98}y}{dx^{98}} = \frac{x^{97}}{79228162517650095125701185536} \sin(x) + x^{96} \cos(x)$	
3				$\frac{d^{99}y}{dx^{99}} = \frac{x^{98}}{158456325335300182451402371072} \sin(x) + x^{97} \cos(x)$	
3				$\frac{d^{100}y}{dx^{100}} = \frac{x^{99}}{316912650670600364902804742144} \sin(x) + x^{98} \cos(x)$	
3				$\frac{d^{101}y}{dx^{101}} = \frac{x^{100}}{633825301341200729805609484288} \sin(x) + x^{99} \cos(x)$	
3				$\frac{d^{102}y}{dx^{102}} = \frac{x^{101}}{1267650602682401459611218968576} \sin(x) + x^{100} \cos(x)$	
3				$\frac{d^{103}y}{dx^{103}} = \frac{x^{102}}{2535301205364802919222437937152} \sin(x) + x^{101} \cos(x)$	
3				$\frac{d^{104}y}{dx^{104}} = \frac{x^{103}}{5070602410729605838444875874304} \sin(x) + x^{102} \cos(x)$	
3				$\frac{d^{105}y}{dx^{105}} = \frac{x^{104}}{10141204821459211676889751748608} \sin(x) + x^{103} \cos(x)$	
3				$\frac{d^{106}y}{dx^{106}} = \frac{x^{105}}{20282409642918423353779503497216} \sin(x) + x^{104} \cos(x)$	
3				$\frac{d^{107}y}{dx^{107}} = \frac{x^{106}}{40564819285836846707559006994432} \sin(x) + x^{105} \cos(x)$	
3				$\frac{d^{108}y}{dx^{108}} = \frac{x^{107}}{81129638571673693415118013988864} \sin(x) + x^{106} \cos(x)$	
3				$\frac{d^{109}y}{dx^{109}} = \frac{x^{108}}{162259277143347386830236027977728} \sin(x) + x^{107} \cos(x)$	
3				$\frac{d^{110}y}{dx^{110}} = \frac{x^{109}}{324518554286694773660472055955456} \sin(x) + x^{108} \cos(x)$	
3				$\frac{d^{111}y}{dx^{111}} = \frac{x^{110}}{649037108573389547320944111910912} \sin(x) + x^{109} \cos(x)$	
3				$\frac{d^{112}y}{dx^{112}} = \frac{x^{111}}{1298074217146779094641888223821824} \sin(x) + x^{110} \cos(x)$	
3				$\frac{d^{113}y}{dx^{113}} = \frac{x^{112}}{2596148434293558189283776447643648} \sin(x) + x^{111} \cos(x)$	
3				$\frac{d^{114}y}{dx^{114}} = \frac{x^{113}}{5192296868587116378567552895287296} \sin(x) + x^{112} \cos(x)$	
3				$\frac{d^{115}y}{dx^{115}} = \frac{x^{114}}{10384593737174232757135105790574592} \sin(x) + x^{113} \cos(x)$	
3				$\frac{d^{116}y}{dx^{116}} = \frac{x^{115}}{20769187474348465514270211581149184} \sin(x) + x^{114} \cos(x)$	
3				$\frac{d^{117}y}{dx^{117}} = \frac{x^{116}}{41538374948696931028540423162298368} \sin(x) + x^{115} \cos(x)$	
3				$\frac{d^{118}y}{dx^{118}} = \frac{x^{117}}{83076749897393862057080846324596736} \sin(x) + x^{116} \cos(x)$	
3				$\frac{d^{119}y}{dx^{119}} = \frac{x^{118}}{166153497794787724114161692649193472} \sin(x) + x^{117} \cos(x)$	
3				$\frac{d^{120}y}{dx^{120}} = \frac{x^{119}}{332306995589575448228323385298386944} \sin(x) + x^{118} \cos(x)$	
3				$\frac{d^{121}y}{dx^{121}} = \frac{x^{120}}{664613991179150896456646770596773888} \sin(x) + x^{119} \cos(x)$	
3				$\frac{d^{122}y}{dx^{122}} = \frac{x^{121}}{132922798235830179291329354119355776} \sin(x) + x^{120} \cos(x)$	
3				$\frac{d^{123}y}{dx^{123}} = \frac{x^{122}}{265845596471660358582658708228711552} \sin(x) + x^{121} \cos(x)$	
3				$\frac{d^{124}y}{dx^{124}} = \frac{x^{123}}{53169119294332071716531741645742304} \sin(x) + x^{122} \cos(x)$	
3				$\frac{d^{125}y}{dx^{125}} = \frac{x^{124}}{106338238588664143433063483291484608} \sin(x) + x^{123} \cos(x)$	
3				$\frac{d^{126}y}{dx^{126}} = \frac{x^{125}}{212676477177328286866126966582969216} \sin(x) + x^{124} \cos(x)$	
3				$\frac{d^{127}y}{dx^{127}} = \frac{x^{126}}{425352954354656573732253933165938432} \sin(x) + x^{125} \cos(x)$	
3				$\frac{d^{128}y}{dx^{128}} = \frac{x^{127}}{850705888709313147464507866331876864} \sin(x) + x^{126} \cos(x)$	
3				$\frac{d^{129}y}{dx^{129}} = \frac{x^{128}}{1701411777418626294929015732663753728} \sin(x) + x^{127} \cos(x)$	
3				$\frac{d^{130}y}{dx^{130}} = \frac{x^{129}}{3402823554837252589858031465327507456} \sin(x) + x^{128} \cos(x)$	
3				$\frac{d^{131}y}{dx^{131}} = \frac{x^{130}}{6805647$	

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6		<p>principle of induction.</p> <p>$P(n)$ is true for all $n \geq 1$ by the and so $P(k+1)$ is true. Hence</p> $(k+1)! f(x)^k f(x)^2 = (k+1)! f(x)^{k+2}$ $= \frac{d}{dx} k! f(x)^{k+1} = (k+1)k! f(x)^k \frac{d}{dx} f$ <p>and so $\frac{d^k f}{dx^k}(n) = k! f(x)^{k+1}$</p> <p>Assume $P(k)$ is true then</p> <p>$P(1)$ is true by the definition of f.</p> <p>\forall all x and $n \geq 1$. Then</p> $f(n) : \frac{d^n f}{dx^n}(x) = n! f(x)^{n+1}$	
2		<p>Let $P(n)$ be the statement</p>	
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Question	Marks分配	Page 1 of 2	Parts
EE1(2)			
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Series natural logarithm slopes		(In order to evaluate $\int \sqrt{1-x^2} dx$) let $x = \sin \theta$, so $dx = \cos \theta d\theta$ and therefore $\int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 \theta} \cos \theta d\theta$ $= \int \sqrt{\cos^2 \theta} \cos \theta d\theta$ $= \int \cos \theta \cos \theta d\theta$ $= \int \cos^2 \theta d\theta$ $= \frac{1}{2} \int (1 + \cos 2\theta) d\theta$ $= \frac{1}{2} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\pi/2}$ $= \frac{1}{2} \left[\theta + \sin \theta \cos \theta \right]_0^{\pi/2}$ $= \frac{1}{2} \left[\frac{\pi}{2} + 0 \right]$ $= \frac{\pi}{4}$	
Setter's initials L3	Page number	Checker's initials	

Question	Marks分配 seen/unseen	Parts	Setter's initials L3	Page number
EE_1(2)				
EXAMINATION QUESTIONS/SOLUTIONS 2008-09	Course			
Page 2 of 2	Page 2 of 2	4	Ques	
sin x unseen to find by part of least	To find $\int \sqrt{1+x^2} dx$ if $x = \sin \theta$	(ii)		
sin x unseen to find by part of least	and then $dx = \cos \theta \cdot d\theta$ so			
	$\int (1+\tan^2 \theta)^{1/2} \sec \theta d\theta$			
	$= \int (e^{\theta} + e^{-\theta})^{1/2} d\theta$			
	$= \frac{1}{2} \int 1 + \sec(2\theta) d\theta$			
	$= \frac{1}{2} \operatorname{sinh}^{-1} x + \frac{1}{2} \ln(1+x^2) + C$			
	$= \frac{1}{2} \operatorname{sinh}^{-1} x + \frac{1}{2} \ln(\sqrt{1+x^2} + x) + C$			
	$= \frac{1}{2} \operatorname{sinh}^{-1} x + \frac{1}{2} \ln(\sqrt{1+x^2} + x) + C$			

Question	Marks & Seen/Unseen	Parts
EE1(2)	Page 1 of 2	5
EXAMINATION QUESTIONS/SOLUTIONS 2008-09	Course	Ques
in seen each	1) Define the function $e_N(x)$ = $1 + x + x^2 + \dots + x^N$, Then $x e_N(x) - e_N(x)$ $= x + x^2 + \dots + x^N$ $- 1 - x - x^2 - x^3 - \dots - x^N$ $= -1 - x^1 - x^2 - x^3 - \dots - x^N$ $= -1 + x^{N+1}$ $= (1-x) \cdot (x^N - 1)$ $\frac{1-x}{x-1} = e_N(x), \quad x \neq 1.$ (Final chain is also possible)	8
2	we can take the limit as $N \rightarrow \infty$ to deduce $\lim_{N \rightarrow \infty} e_N(x) = \lim_{N \rightarrow \infty} \frac{1-x}{x-1} = \frac{1+x}{1-x}$ If $ x > 1$ $N \rightarrow \infty$ to deduce $\lim_{N \rightarrow \infty} e_N(x) = \lim_{N \rightarrow \infty} \frac{1-x}{x-1} = \frac{1-x}{x-1} = \frac{x-1}{1-x} = -\frac{1-x}{x-1}$ Check	6
	Page number Letter's initials Chekka's initials Page number	

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4	for this exam have a calculator so do for this exam calculator students	EB	
6	$\Rightarrow \alpha_2 = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$ $\frac{\alpha_2}{2} = \ln \left(\frac{1+x}{1-x} \right) \Leftrightarrow x = \frac{3}{5}$ $1+x = 4-x \Leftrightarrow x = 3/5$ $(1+x)^{1/2} = 2 \Leftrightarrow 1+x = 4$ <p>To find α_2 we take total from value $C=0$.</p>		
	$= x + x^3 + x^5 + \dots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$ $= \int (1 - x + x^2 - x^3 + \dots) dx$ $= \int (1 + x + x^2 + x^3 + \dots) dx$ $= C + \frac{1}{2} \ln \frac{1-x}{1+x} $ $so \frac{1}{2} \int \frac{1}{1+x} + \frac{1}{1-x} dx = C + \frac{1}{2} \ln 1+x - \frac{1}{2} \ln 1-x $		
EE1(2)	EE2 of 2	EXAMINATION QUESTIONS/SOLUTIONS 2008-09	Course
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EE(2)			
EXAMINATION QUESTIONS/SOLUTIONS 2008-09	Course		
6	6	$\sum_{n=0}^{\infty} \cos(n\theta) = e^{i n \theta} \cdot \text{Re}(e^{i n \theta})$ $\sum_{n=0}^{\infty} \frac{\cos(n\theta)}{n!} = e^{i n \theta} \cdot \text{ca}(e^{i n \theta})$ hence expanding real and imaginary parts, $\sum_{n=0}^{\infty} \sin(n\theta) = e^{i n \theta} \cdot \text{Im}(e^{i n \theta})$ $\sum_{n=0}^{\infty} \frac{\sin(n\theta)}{n!} = e^{i n \theta} \cdot \text{si}(e^{i n \theta})$	6
unseen	unseen	(iii) Given that $\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$ as setting $z = re^{i\theta}$, $r > 1$ we get $\sum_{n=0}^{\infty} r^n e^{in\theta} = \frac{1-re^{i\theta}}{1-re^{i\theta}} = \frac{1-re^{i\theta}}{1-r^2e^{i\theta}}$ hence $\sum_{n=0}^{\infty} r^n \sin(n\theta) = \frac{1-r^2e^{i\theta}}{1-r^2e^{i\theta}} - \frac{re^{i\theta}}{1-r^2e^{i\theta}}$ $\sum_{n=0}^{\infty} r^n \cos(n\theta) = \frac{1-r^2e^{i\theta}}{1-r^2e^{i\theta}} + \frac{r^2e^{i\theta}}{1-r^2e^{i\theta}}$	6
3	3	$\sum_{n=0}^{\infty} r^n \sin(n\theta) = \frac{1-r^2e^{i\theta}}{1-r^2e^{i\theta}} - \frac{r^2e^{-i\theta}}{1-r^2e^{-i\theta}}$ $\sum_{n=0}^{\infty} r^n \cos(n\theta) = \frac{1-r^2e^{i\theta}}{1-r^2e^{i\theta}} + \frac{r^2e^{i\theta}}{1-r^2e^{i\theta}}$ and $\sum_{n=0}^{\infty} r^n \sin(n\theta) = \frac{1-2r \cos \theta + r^2}{1-r^2}$	3

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Marks & seen/unseen	Question L Parts (1)
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3		$= 1200 \pm 220$		
3		$P = \frac{8 \times 300}{2} \mp (40 + 180)$		
2		$P = \frac{1}{R} + \left(\frac{1}{k_A V_1} + \frac{1}{k_B V_2} \right) \mp \frac{1}{V_1}$		(ii)
		$\frac{\partial P}{\partial k_A} = \frac{1}{V_1} + \frac{1}{k_A V_1^2}$		
		$\frac{\partial P}{\partial k_B} = -\frac{1}{V_2} + \frac{1}{k_B V_2^2}$		
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EE1-2				
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3

 $a_4 = 1, \dots \text{to converge}$

$$a_3 = 0$$

$$a_2 = -1$$

$$a_1 = 0$$

%

4

$$\frac{4a_{n-1}^3 + 12a_{n-1}^2 + 8a_{n-1}}{3a_n^4 + 8a_n^3 + 4a_n^2 + 1} =$$

$$a_n = a_{n-1} - \frac{4a_{n-1}^3 + 12a_{n-1}^2 + 8a_{n-1}}{a_{n-1}^4 + 4a_{n-1}^3 + 4a_{n-1}^2 - a_{n-1}}$$

2

$$f = x^4 + 4x^3 + 4x^2 - x, f' = 4x^3 + 12x^2 - 1$$

(ii)

4

 $10^{13} \approx 2.15443$ converges to a unique place

$$a_5 \approx 2.1544347$$

$$a_4 \approx 2.1544351$$

$$a_3 \approx 2.1553719$$

$$a_2 = 2.2$$

$$\text{Now let } a_1 = 5/2$$

5

$$a_n = a_{n-1} - \frac{3a_{n-1}^2}{3a_{n-1}^3 - 10} = \frac{3a_{n-1}^2}{3a_{n-1}^3 - 10}$$

 \therefore Newton's method gives a unique solution after iteration

2

$$f = x^3 - 10, f' = 3x^2$$

(i)

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2			$y^2x^2 - 3x + 4y = 2.$ $y(1) = 1 \Rightarrow c = 2$ <p style="text-align: center;">$y^2x^2 - 3x + 4y = 2$</p>
4			\therefore solution to $y^2x^2 - 3x + 4y = 2$ $\frac{\partial L}{\partial x} = y^2 + 4y + B(y) \Leftrightarrow y^2 + 4y + B(y) = 0$ $\frac{\partial L}{\partial y} = 2xy - 3 + A(x) \Leftrightarrow 2xy - 3 + A(x) = 0$ $\therefore f(x,y)$ called dual function
2		(1)	$\text{and } \frac{\partial f}{\partial x} = 4xy, \frac{\partial f}{\partial y} = 4x$ no equation so equal $\frac{\partial f}{\partial x} + g = 0 \text{ so } g = 4x$
2			$f(x,y) = 4xy$
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$\frac{x(q/2 - x)}{1 - x} = y$ $\Rightarrow A = q/2$ $y = \frac{x(A-x)}{1 - x}$ $y = -x + A$ $I = \left(\frac{\partial}{\partial x}\right)$ $x = \frac{\partial}{\partial u} + \frac{\partial}{\partial x}$ $(q-1)x = \frac{x}{u} + \frac{1-b}{1-u}$ $x = \frac{1-b}{q} + \frac{1-b}{q} \cdot \frac{\partial}{\partial u}$ $dx = \frac{1-b}{q} du + \frac{1-b}{q} \cdot \frac{\partial}{\partial u}$ $dx = \frac{1-b}{q} du + \frac{1-b}{q} \cdot \frac{\partial}{\partial u}$ $dx = \frac{1-b}{q} du + \frac{1-b}{q} \cdot \frac{\partial}{\partial u}$	<p style="text-align: center;">we have $u = y^{-1}$</p> <p style="text-align: center;">so, we have $b = 2$</p> <p style="text-align: center;">Taking $b = 2$ we have so, we have</p> <p style="text-align: center;">$\frac{\partial}{\partial u}$</p>	

Question	Course	EXAMINATION QUESTIONS/SOLUTIONS 2008-09	Marks & seen/unseen	Page 1 of 2	Parts
4	Solve	$\int x e^{kx} dx = x \frac{e^{kx}}{k} - \int \frac{e^{kx}}{k} \cdot 1 dx$ Using integration by parts $\int x e^{-inx} dx = \left[\frac{x e^{-inx}}{-in} - \frac{1}{-in} \int e^{-inx} dx \right]_{\frac{\pi}{2}}^{\frac{\pi}{2}}$ $\int e^{-inx} dx = \left[\frac{e^{-inx}}{-in} \right]_{\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{e^{-i\pi n} - e^{i\pi n}}{-in} = \frac{(-1)^n - (-1)^n}{-in} = 0$ $f(x) = c_0 + 2 \sum_{n=1}^{\infty} c_n e^{inx}$ $c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} (c_0 + 2 \sum_{n=1}^{\infty} c_n e^{inx}) e^{-inx} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \int_{-\pi}^{\pi} e^{in(x-nx)} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \int_{-\pi}^{\pi} e^{i(n-n)x} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \int_{-\pi}^{\pi} e^{i(n-n)x} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \left(\frac{e^{i(n-n)\pi} - e^{i(n-n)(-\pi)}}{in} \right) = c_0 + 2 \sum_{n=1}^{\infty} c_n \left(\frac{(-1)^n - (-1)^n}{in} \right) = c_0$	4	Solve	4
5	Solve	$\int x e^{-inx} dx = \left[\frac{x e^{-inx}}{-in} - \frac{1}{-in} \int e^{-inx} dx \right]_{\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{e^{-i\pi n} - e^{i\pi n}}{-in} = \frac{(-1)^n - (-1)^n}{-in} = 0$ $f(x) = c_0 + 2 \sum_{n=1}^{\infty} c_n e^{inx}$ $c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} (c_0 + 2 \sum_{n=1}^{\infty} c_n e^{inx}) e^{-inx} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \int_{-\pi}^{\pi} e^{in(x-nx)} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \int_{-\pi}^{\pi} e^{i(n-n)x} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \left(\frac{e^{i(n-n)\pi} - e^{i(n-n)(-\pi)}}{in} \right) = c_0 + 2 \sum_{n=1}^{\infty} c_n \left(\frac{(-1)^n - (-1)^n}{in} \right) = c_0$	5	Solve	5
6	Solve	$f(x) = c_0 + 2 \sum_{n=1}^{\infty} c_n e^{inx}$ $c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} (c_0 + 2 \sum_{n=1}^{\infty} c_n e^{inx}) e^{-inx} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \int_{-\pi}^{\pi} e^{in(x-nx)} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \int_{-\pi}^{\pi} e^{i(n-n)x} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \left(\frac{e^{i(n-n)\pi} - e^{i(n-n)(-\pi)}}{in} \right) = c_0 + 2 \sum_{n=1}^{\infty} c_n \left(\frac{(-1)^n - (-1)^n}{in} \right) = c_0$	6	Solve	6
7	Solve	$f(x) = c_0 + 2 \sum_{n=1}^{\infty} c_n e^{inx}$ $c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} (c_0 + 2 \sum_{n=1}^{\infty} c_n e^{inx}) e^{-inx} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \int_{-\pi}^{\pi} e^{in(x-nx)} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \int_{-\pi}^{\pi} e^{i(n-n)x} dx = c_0 + 2 \sum_{n=1}^{\infty} c_n \left(\frac{e^{i(n-n)\pi} - e^{i(n-n)(-\pi)}}{in} \right) = c_0 + 2 \sum_{n=1}^{\infty} c_n \left(\frac{(-1)^n - (-1)^n}{in} \right) = c_0$	7	Solve	7

Sette's initials

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Cheker's initials

Question	Marks & seen/unseen	Parts	Passage now extends from $\frac{2\pi}{T} - \frac{\pi}{T}$ $\int_{\frac{2\pi}{T}}^{\frac{\pi}{T}} f(x) dx = C_0 + 2 \sum_{n=1}^{\infty} C_n \cos nx$	Passage now extends from $\frac{\pi}{T}$ $\int_{\frac{\pi}{T}}^0 x^2 dx = \frac{\pi^3}{3} \cdot \frac{1}{\pi} \Rightarrow \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} x^2 dx = \frac{\pi^3}{3}$	Passage now extends from $\frac{\pi}{T}$ $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$	Page number Setters initials Page number
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