SECTION A: SEMICONDUCTOR DEVICES - **ANSWERS**

1. a)

Heisenberg's uncertainty principle gives the relationship:

$$\Delta E \Delta t \geq \hbar / 2$$

Show that for a photon whose wavepacket extends over 1 ns the range of frequencies in the wavepacket will be at least 80 MHz.

As

$E = hf$,

$\Delta E = h\Delta f$.

Substituting into the uncertainty relationship:

$$\Delta E \geq \frac{\hbar}{2\Delta t}$$

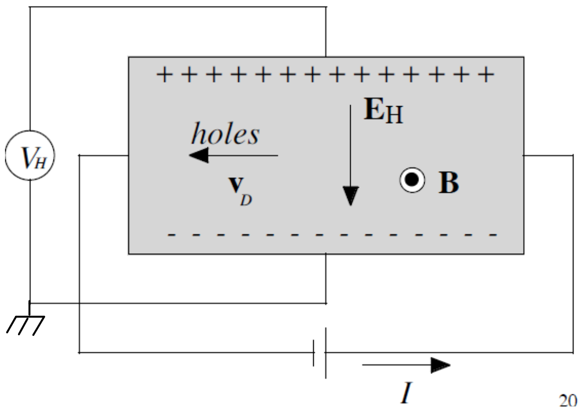
$$h\Delta f \geq \frac{\hbar}{2\Delta t}$$

$$\Delta f \geq \frac{1}{4\pi\Delta t}$$

Hence if Δt = 1 ns, ΔF >/= 80 MHz [new application of theory]

[4]

b). V_H>0V, Hall voltage is positive. Due to the applied voltage V, the majority positive charges carrier holes flow from right to left (in same direction as conventional current). Due to the direction of the magnetic field B they are deflected to the top of the sample, charging it positively, thus an electric field occurs pointing from top to bottom and thus the positive voltage is from bottom to top of the sample. With the given ground node this means that V_H is positive.[4]



c)

The Fermi level defines the position in energy where the probability of finding an electron is equal to 1/2. [2]

d)

i. ii. iii. [6]

Note that all relative distance in energy need to be correct for these answers to be correct.

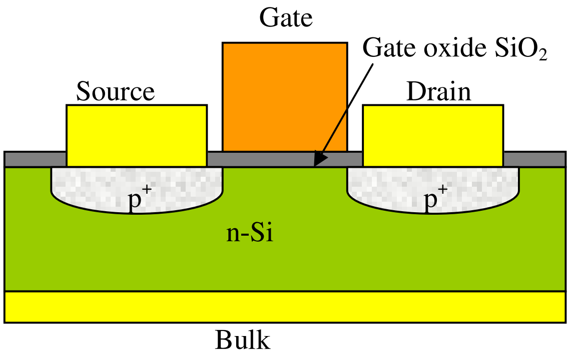
e) Acceleration of electrons under an electric field $m\frac{dv}{dt} = qE$ is compensated by a deceleration caused by scattering processes: $\frac{dv}{dt} = \frac{-v}{\tau}$

as a consequence the average velocity of the electron remains constant. [4]

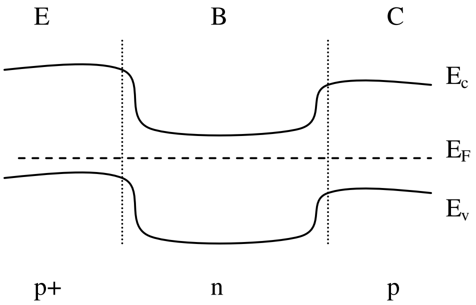
f) L [2]

g) The reverse bias current in a pn diode is due to the concentration of minority carriers available at each side of the junction that can be injected across the junction in reverse bias. Since this number is small (minority carriers) this limits the current that can flow rather than the applied voltage. As the number is small the current is small. [4]

h) pMOS. [4]



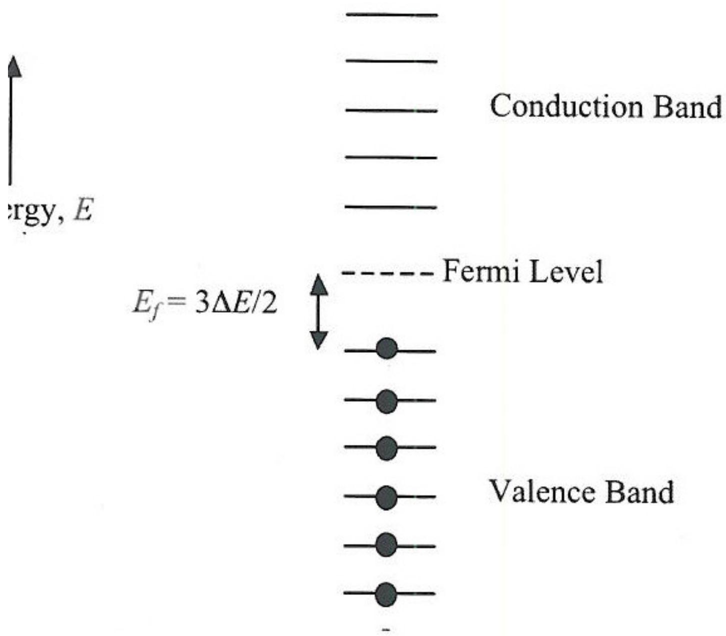
i)



In active mode the emitter-base junction is forward biased and injects carriers (mainly holes in pnp BJT) into the base. The base-collector junction is reverse biased and thus collects the minority carriers (holes) that are injected in to the base. [4]

2.

a)



[5]

b) Charge neutrality: $eN_D w_n A = eN_A w_p A$ [5]

with: e charge of 1 electron

A cross sectional area of the diode

N_D: donor doping concentration

N_A: acceptor doping concentration

w_n: depletion region extending in the n-doped region

w_p: depletion region extending in the p-doped region

since $N_D >> N_A$, $w_n << w_p$

b) From formulae sheet: [10]

$$J_n = \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)$$

$$J_p = \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1\right)$$

thus $I = eA\left(\frac{D_n n_p}{L_n} + \frac{D_p p_n}{L_p}\right)\left(e^{\frac{eV}{kT}} - 1\right)$

Approximations that one can make are:

1) since N_D>>N_A the diode current will be approximately equal to the electron current I_n thus $I \approx I_n = eA\frac{D_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)$

2) since forward bias the "-1" term with the exponential is negligible thus

$$I \approx I_n = eA\frac{D_n n_p}{L_n} e^{\frac{eV}{kT}}$$

Replacing the minority carrier concentration by its expression as a function of doping density: $n_p = \frac{n_i^2}{N_A}$ gives $I \approx eA\frac{n_i^2 D_n}{N_A L_n} e^{\frac{eV}{kT}}$. Thus the p-side doping concentration can be extracted via: $N_A \approx eA\frac{n_i^2 D_n}{I L_n} e^{\frac{eV}{kT}}$

$$N_A \approx 1.6\,10^{-19}C \times 10^{-2}cm^{-2}\frac{\left(1.45\,10^{10}cm^{-3}\right)^2 20\,cm^2/s}{10\,10^{-6}A\,10^{-3}cm} e^{\frac{0.5}{0.026}} = 1.5\,10^{17}cm^{-3}$$

$N_D = 10^3 \text{ x } N_A = 1.5\ 10^{20}\ cm^{-3}$.

d) Current gain $\beta = \frac{I_C}{I_B}$ by definition, with I_C collector current and I_B base current.

In an n^+pn BJT the electrons are the carriers collected by the collector. Thus I_C is the electron current of the emitter-base forward biased junction. I_B is equal to the hole current that is escaping from the base into the emitter.
Thus using the pn junction diffusion current definition in the formulae sheet:

$$I_C = I_n = \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)A$$

$$I_B = I_p = \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1\right)A$$

$$\beta = \frac{I_C}{I_B} = \frac{I_n}{I_p} = \frac{\frac{D_n n_p}{L_n}}{\frac{D_p p_n}{L_p}} = \frac{D_n n_p L_p}{D_p p_n L_n} = \frac{D_n N_D L_p}{D_p N_A L_n}$$ [8]

If the lengths of the emitter and the base are different from the diffusion lengths, then L_n (= diffusion length of the electrons in the p-type region = base) becomes W_b (base width) and L_p (=diffusion length of the holes in the n-type region = emitter) becomes x_e and thus the gain formulae should be rephrased as:

$$\beta = \frac{D_n N_D x_e}{D_p N_A W_B}$$

e) For a higher current gain. [2]

3.

a) The threshold voltage is the voltage that needs to be applied on the gate in order to invert the channel. The definition of inversion is that the number of free carriers in the inverted channel must be the same as the number of free carriers in the bulk. If the number of free carriers in the bulk is increased by increasing the doping concentration of the substrate then a larger amount of free carriers must be generated in the channel. Thus a higher gate voltage will be to be applied to do this, thus the threshold voltage increases. (Note: threshold voltage of an n-type enhancement mode MOSFET is positive) [4]

b) i) The threshold voltage can be derived from the C-V characteristic when depletion is reached. Full depletion is reached when the capacitance is minimum. Thus V_{th}=1V. [4]

ii) The measurement is done in the triode region (low V_{DS}, current linear). [2]

iii) From the capacitance-voltage characteristics we can derive the oxide capacitance:

$$C_{ox} = \frac{C(V_{GS} = 0V)}{A} = \frac{C(V_{GS} = 0V)}{L_g W_g} = \frac{3.54\,10^{-13}\,F}{1000 \times 10^{-12}\,m^2} = 3.54\,10^{-4}\,F/m^2$$ [5]

The current I_{DS} in the triode region is given by:

$$I_{DS} = \frac{\mu C_{ox} W}{L}(V_{GS} - V_{th})V_{DS}$$

At V_{DS}=0.5V and V_{GS}=2V we estimate the current $I_{DS} \approx 5\ 10^{-5}$ A [5]

$$\mu = \frac{I_{DS} L_g}{C_{ox} W_g (V_{GS} - V_{th}) V_{DS}} = \frac{5\,10^{-5}\,A \times 10\mu m}{3.54\,10^{-4}\,F/m^2 \times 100\mu m \times 1V \times 0.5V}$$

$$\mu = 0.0283 \frac{m^2}{Vs} \approx 0.03 \frac{m^2}{Vs} = 300 \frac{cm^2}{Vs}$$

c) Inside the quantum well, $V(x)$ is zero, so S.E. becomes:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

Substituting for $\psi = A\sin k_n x$ gives

$$\frac{\hbar^2 k^2}{2m} A\sin kx = EA\sin kx$$

$$\Rightarrow E = \frac{\hbar^2 k^2}{2m}$$

The boundary conditions automatically match at $x = 0$ through the choice of a sine for the wavefunction. At $x = L$, we have

$$\boxed{\begin{array}{l}\sin kL = 0 \\ \Rightarrow kL = n\pi\end{array}}$$ [2]

for integer n. $n = 0$ is not a solution as this would imply ψ is zero everywhere, and hence there is no electron in the state. Hence we obtain: $k = \frac{n\pi}{L}, \quad n = 1,2,3...$

and $E = \frac{n^2\hbar^2\pi^2}{2mL^2}, \quad n = 1,2,3...$