UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2002

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C337=I3.18

SIMULATION AND MODELLING

Thursday 25 April 2002, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

The following description applies to both questions 1 and 2.

At a telephone call centre customers dial in according to a Poisson process (i.e. the times between calls are exponentially distributed). An average of 54 calls arrive per hour. A pre-recorded voice asks each caller to press 1, 2, or 3 on their phone keypad depending on the nature of the service they require. This connects the customer to a pool of receptionists who are trained to deal specifically with the given request type. The probabilities of each request type and the approximate distributions of the various request service times (in seconds), i.e. the time taken to service the customer once connected to a receptionist, has been determined using historical data. The results are as follows:

Type	**Probability**	**Approximate service time distribution**
1	0.33	Exponential, parameter 1/123
2	0.27	Gamma, parameters 1/196 and 2
3	0.40	Exponential, parameter 1/77

If all receptionists are busy when a customer has made their selection the customer enters a FIFO (first-in-first-out) queue and the system plays them some nice music whilst they wait to be connected.

1a Design a simulation model of the call centre using the event scheduling approach which estimates the average connection time (i.e. time between making a selection and commencing service with a receptionist) for each of the three service types. You may assume the existence of library classes for managing events and for sampling the required distributions. You may also assume the existence of a class `Queue` for modelling FIFO queues with methods that include the mean time customers have spent in the queue since it was created.

b Naturally, customers will not wait forever to be connected and after a while they will give up and put the phone down (a customer *loss*). If a customer's allotted wait time expires before they are connected the simulation should record a loss and take the appropriate action. Discuss how you would go about adding this functionality to your simulation model, highlighting any required extensions to the event manager and queue classes that are not necessary for part a. You are NOT required to detail the new model code.

When sketching model code you may use any notation you wish, so long as it is readable and easily understood.

The two parts carry 60% and 40% of the marks respectively.

2a State briefly the relative advantages and disadvantages of discrete event simulation and queueing theory when modelling the performance of queueing systems.

2b For the call centre model described above *without* customer losses, suppose now that there is exactly one receptionist for each service type and that the service times are *all* exponentially distributed with parameters 1/123, 1/196 and 1/77 for types 1, 2 and 3 respectively. Assuming the arrival process as specified,

i) Draw a queueing network for this modified system, and annotate the diagram with the external arrival rate, routing probabilities and service rates.

ii) Assuming that the queues all have infinite capacity, calculate the mean number of customers either being serviced or waiting for service, at each queue at equilibrium.

iii) State Little's Law and use it, together with your results from part ii, to compute the mean waiting time (queueing time plus service time) for the call centre as a whole.

2c What is meant by simulation model *verification* and how does this differ from model *validation*? As part of your answer explain how your results from part b could be used to help verify a simulation model of the call centre, for example that suggested by question 1a.

The three parts carry 30%, 40% (10+20+10) and 30% of the marks respectively.

3a Explain what is meant by a *point estimate* and a *confidence interval*, in the analysis of output data from a discrete-event simulation. As part of your answer, state the mathematical interpretation of a confidence interval and show how to calculate the point estimate and 95% confidence interval for the following output data

```
6.69 6.82 6.53 6.21 6.70 6.88 6.42 6.61 6.39 6.78
```

You can assume the data are samples from a normal distribution.

3b Why it is important that simulation output data are (approximately) normally distributed when calculating a confidence interval? Suggest one reason why simulation output is often observed to be approximately normally distributed. *Hint*: a measurement taken during a simulation is often an average of a sequence of low-level observations, e.g. the sample mean of the waiting times of individual customers in a queue.

3c Consider a discrete-event simulation of a departmental intranet comprising a stated number of user terminals, servers, internal routers etc. Suppose the load on the system is generated entirely by the users (i.e. there is no external input traffic) and that the simulation output is an estimate of the mean IP packet delay through one of the routers.

i) If the simulation is executed for a fixed number of (simulated) user requests, what would you expect to happen to the width of the confidence interval as the load increases, e.g. as a result of higher request rate or more users? Explain your answer.

ii) Suppose you wanted to use the simulation to generate estimates of a predetermined "accuracy". State how you would quantify "accuracy" in this context in terms of a confidence interval. What would be the effect on *total* simulation execution time (possibly over several runs) of both increasing and decreasing the required level of accuracy, and why?

The three parts carry 40%, 30% and 30% (15+15) of the marks respectively.

4a Describe the *inverse transform* method for sampling continuous distributions and justify, mathematically, why it works. As part of your answer show how it can be used to sample the Weibull distribution, which has density function

$$f(x) = (\beta x^{\beta-1} \exp(- (x/\alpha)^{\beta})) / \alpha^{\beta} \qquad x=0$$

where α and β are both positive parameters, and cumulative distribution function (cdf)

$$F(x) = 1 - \exp(- (x/\alpha)^{\beta}) \qquad x=0$$

4b A post office has been monitored over an extensive period of time to determine the distribution of service types, as requested by users of the system. There are n service types and the observed number of customers requesting service i was $f(i)$, $i=1,2,..,n$. Design a class (using a Java-like notation) for sampling this distribution, i.e. for generating on request the service type samples with the correct relative frequencies. Assume that the observed frequencies, $f(i)$, $i=1,2,..,n$, are presented (e.g. as a parameter to the class constructor) in the form of an array of integers. *Both style and efficiency will be taken into account when awarding credit.*

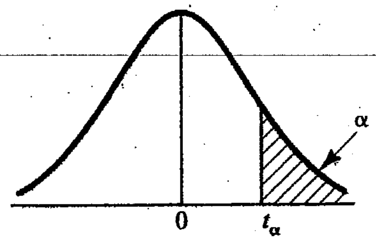
4c i) Give *three* reasons why, for the purposes of discrete-event simulation, sampling a mathematical distribution is preferable to using data measured from the operation of a real system (i.e. a trace file).

ii) Describe *briefly one* method for testing goodness of fit between observed data and a known mathematical distribution.

iii) Suppose the post office data (part b) were fitted to a known mathematical distribution. How would you expect this sampler to differ from that of part b in terms of the (long-run) distribution of service types they generate?

The three parts carry 30%, 35% and 35% (10+15+10) of the marks respectively.

Table A.5. PERCENTAGE POINTS OF THE STUDENTS t DISTRIBUTION WITH ν DEGREES OF FREEDOM



ν	$t_{0.005}$	$t_{0.01}$	$t_{0.025}$	$t_{0.05}$	$t_{0.10}$
1	63.66	31.82	12.71	6.31	3.08
2	9.92	6.92	4.30	2.92	1.89
3	5.84	4.54	3.18	2.35	1.64
4	4.60	3.75	2.78	2.13	1.53
5	4.03	3.36	2.57	2.02	1.48
6	3.71	3.14	2.45	1.94	1.44
7	3.50	3.00	2.36	1.90	1.42
8	3.36	2.90	2.31	1.86	1.40
9	3.25	2.82	2.26	1.83	1.38
10	3.17	2.76	2.23	1.81	1.37
11	3.11	2.72	2.20	1.80	1.36
12	3.06	2.68	2.18	1.78	1.36
13	3.01	2.65	2.16	1.77	1.35
14	2.98	2.62	2.14	1.76	1.34
15	2.95	2.60	2.13	1.75	1.34
16	2.92	2.58	2.12	1.75	1.34
17	2.90	2.57	2.11	1.74	1.33
18	2.88	2.55	2.10	1.73	1.33
19	2.86	2.54	2.09	1.73	1.33
20	2.84	2.53	2.09	1.72	1.32
21	2.83	2.52	2.08	1.72	1.32
22	2.82	2.51	2.07	1.72	1.32
23	2.81	2.50	2.07	1.71	1.32
24	2.80	2.49	2.06	1.71	1.32
25	2.79	2.48	2.06	1.71	1.32
26	2.78	2.48	2.06	1.71	1.32
27	2.77	2.47	2.05	1.70	1.31
28	2.76	2.47	2.05	1.70	1.31
29	2.76	2.46	2.04	1.70	1.31
30	2.75	2.46	2.04	1.70	1.31
40	2.70	2.42	2.02	1.68	1.30
60	2.66	2.39	2.00	1.67	1.30
120	2.62	2.36	1.98	1.66	1.29
∞	2.58	2.33	1.96	1.645	1.28