Imperial College London
BSc/MSci EXAMINATION June 2012

This paper is also taken for the relevant Examination for the Associateship

LASERS, OPTICS AND BIOPHOTONICS
For Third and Fourth Year Physics Students

Monday 28 May 2012: 10:00 to 12:00

The paper consists of two sections: A & B.
Section A contains one question, worth 20 marks total
Section B contains four questions worth 15 marks each

Candidates are required to answer ***ALL*** *parts of Section A and* ***TWO*** *questions from Section B.*
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

Please note the information on the last page that you may find useful when answering some of the questions.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

Section A

Question 1

(i) Briefly explain what you understand by ray transfer matrices and explain any assumptions that limit their applicability

(2 marks)

The ray transfer matrices (RTM) for propagating a distance d in a medium of refractive index n, for refraction at a spherical (convex) surface of radius R with incident and transmitted refractive indices of n_1 and n_2 and for the action of a thin lens are respectively given by:

$$\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ \dfrac{-(n_2 - n_1)}{R} & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

(ii) By using the RTM for refraction at appropriate spherical surfaces, find an expression for the focal length f of a thin symmetric biconvex lens made from glass of refractive index, n, in terms of n and the radius of curvature R of the lens surfaces. Calculate the RTM of a composite lens comprising a thin lens of focal length f_1 followed by a distance d in air and a thin lens of focal length f_2.

(3 marks)

(iii) Consider the equation of a single electron oscillator in a medium driven by an electric field, $E = E_0 e^{i\omega t}$, is given by: $m\dfrac{d^2x}{dt^2} + m\gamma\dfrac{dx}{dt} + Kx = -eE$

Explain the meaning and significance of each term in the above equation and show that the polarisation of a medium made up of such oscillators is given by $P = \varepsilon_0 \chi E$ where $\chi = \dfrac{Ne^2}{m}\left(\dfrac{1}{\omega_o^2 - \omega^2 + i\gamma\omega}\right)$ and N is the number of oscillators/unit volume and ω_0 is the resonant frequency

(4 marks)

(iv) Briefly explain the significance of χ being complex in relation to the propagation of radiation in a medium

(2 marks)

(v) Explain why it is not possible to realise a 2 level laser and outline the relative merits of three and four level lasers

(3 marks)

Question 1 continued

(vi) Briefly explain what is meant by a Q-switched laser and a mode-locked laser, outlining their differences and give an example of an application of each

(3 marks)

(vii) Explain what is meant by the numerical aperture, the point spread function and the optical transfer function of an imaging system.

(3 marks)

(Total 20 marks)

Section B

Question 2

(i) If an electric field of a dipole oscillator losing energy with a decay rate γ is described by $E(t) = E_0 e^{i\omega_0 t}.e^{-\frac{\gamma t}{2}}$ for $t \geq 0$ and $E(t) = 0$ for $t < 0$, write down the formula for the intensity as a function of time and show that the corresponding spectral intensity profile is given by $\dfrac{I_0}{\left((\omega_0 - \omega)^2 + \left(\dfrac{\Delta\omega}{2}\right)^2\right)}$ where $\Delta\omega = \gamma$ is the natural linewidth. Sketch this function.

(4 marks)

Note that the Fourier transform of a function, $f(t)$, is given by: $F(\omega) = \int_{-\infty}^{\infty} f(t).e^{-i\omega t}dt$

(ii) Calculate the natural linewidths of the HeNe, organic dye, Nd:YAG, Nd:glass and Ti:Sapphire laser media using the data in the table at the end of this question book and explain in each case why they differ from the observed gain linewidths.

(6 marks)

(iii) Which of the laser media listed would be most suitable for ultrashort pulse generation? Explain your reasoning.

(1 mark)

(iv) For many applications such as spectroscopy or holography it is desirable to achieve a narrow laser spectral width. Explain how this can be achieved and what can limit the minimum linewidths achievable.

(4 marks)

(Total 15 marks)

Question 3

(i) Explain how the nonlinear response of a medium to an incident electromagnetic wave can lead to an intensity-dependant refractive index given by $n = n_0 + n_2 I$ and show how this arises from χ_3, the third order nonlinear susceptibility.

(2 marks)

(ii) Explain what is meant by self focussing and self phase modulation and how they arise as a consequence of the optical Kerr effect.

(2 marks)

(iii) Explain the role of self focussing and self phase modulation in the generation of ultrashort pulses in a Kerr Lens mode-locked laser. What limits the minimum achievable pulse duration in such a laser?

(4 marks)

(iv) Explain why the optical Kerr effect presents a major challenge when amplifying ultrashort pulses to high peak powers and outline how this is addressed using chirped pulse amplification.

(2 marks)

(v) The nonlinear phase shift acquired by a signal of intensity I on propagation through a medium can be written as $\phi_{NL} = \int_0^L \frac{2\pi}{\lambda} n_2 I(z) dz$. If self focussing is considered to be a significant issue when the beam propagating through a medium acquires a nonlinear phase shift > π, estimate the maximum safe peak laser power for a Ti:Sapphire laser amplifier in which the gain medium is 4 cm long and the laser beam at 800 nm wavelength is of 2 mm radius.
You may assume that $n_2 = 3.45 \times 10^{-20}$ m^2 W^{-1} for Ti:Sapphire.

(1 mark)

(vi) Suppose that you wish to amplify laser pulses of 100 fs duration. Estimate by how much these pulses need to be stretched in order to achieve output pulses from a chirped pulse amplifier with a peak power of 1 TW and explain your reasoning.

(1 mark)

(vii) Suppose a different chirped pulse amplifier system requires the pulses to be stretched to 200 ps. One way to stretch the input pulses is to propagate them through an optical fibre. Assuming that the dispersion parameter for the glass core is given by D = 120 ps/nm.km, estimate what length of fibre would be required to stretch transform limited pulses of 100 fs duration with a centre wavelength of 800 nm to 200 ps if the effect of self phase modulation is ignored. Briefly discuss the difference self phase modulation would make if it was taken into account.

(3 marks)

(Total 15 marks)

Question 4

(i) Outline the design and operating principles of an infinity-corrected wide-field fluorescence microscope, a laser scanning confocal fluorescence microscope and a two photon fluorescence microscope. Illustrate your answer with appropriate diagrams and discuss their relative advantages when imaging live cells on a glass coverslip.

(7 marks)

(ii) Compare the axial and transverse spatial resolution of such microscopes if they each have an objective lens of 0.7 numerical aperture (NA) and are applied to imaging cells with a fluorescent label emitting at 600 nm that can be directly excited at 400 nm. (You may assume that all the microscopes are imaging in air.)

(4 marks)

(iii) Discuss the challenges associated with optical imaging in and through biological tissue and explain how the performance of the three approaches to fluorescence microscopy in part (i) would compare when imaging through biological tissue.

(4 marks)

(Total 15 marks)

Question 5

Write notes on THREE of the following. Use sketches and mathematical expressions to illustrate your notes where appropriate.

All sections carry equal marks.

(i) Laser cooling and optical tweezers

(ii) Development of optical communications systems including the impact of absorption and attenuation

(iii) Therapeutic applications of lasers

(iv) Optical coherence tomography

(v) Fluorophores and contrast in fluorescence microscopy

(Total 15 marks)

The following table presents some key parameters for various laser media.

Gain medium	**Main/peak gain wavelength**	**Gain linewidth** $\Delta\upsilon$ (GHz)	**Upper state lifetime** τ_{21}	**Peak gain** $\sigma(\upsilon)$ (x 10^{-20} cm^2)
HeNe	632.8 nm	1.4	150 ns	3 x10^7
Argon ion	488, 514 nm, ...	~3.5	10 ns (488 nm)	1.9 x10^8 (488 nm)
CO_2	10.6 μm	0.05 to 1	400 μs	~150 to 3900 (low to atm. pressure)
N_2	337 nm	250	40 ns	4 x10^7
ArF, KrF, XeCl excimer	193, 249,308 nm	3000 (KrF)	10 ns (KrF)	50,000 (KrF)
Semiconductor diode (GaN, AlGaInP, GaAlAs, InGaAs, InGaAsP)	375-480, 630-700, 800-900, 960-980, 1300-1600 nm	~800	~3 ns (GaAs)	~15,000 (GaAs)
Nd:YAG	1064 nm	126	230 μs	28
Nd:glass	1064 nm	5040	300 μs	4
Er:YAG	2940 nm	~ 100	100 μs	2.6
Er:glass	1540 nm	3000	8 ms	0.7
Yb:YAG	1030 nm	2600	1.16 ms	1.8
Yb:glass	1030 nm	5000	1.16 ms	0.25
Ti:Sapphire	790 nm	~100,000	3.2 μs	0.8
Cr:Sapphire (Ruby)	694.3 nm	170	4 ms	2.7
Cr:LiSAF	830 nm	~70,000	67 μs	4
Organic dye	320-1500 nm	~10,000	3 ns	~4000