IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

EEE/ISE PART I: MEng, BEng and ACGI

Corrected Copy

COMMUNICATIONS 1

Friday, 11 June 10:00 am

correction to
Q1(b) 10.20am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.L. Dragotti, P.L. Dragotti
Second Marker(s) : M.K. Gurcan, M.K. Gurcan

Special Information for the Invigilators: none

Information for Candidates

Some Fourier Transforms

$$\cos\omega_0 t \iff \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$$

$$\mathrm{rect}(\tfrac{t}{\tau}) \iff \tau\mathrm{sinc}(\tfrac{\omega\tau}{2})$$

$$\tfrac{W}{\pi}\mathrm{sinc}(Wt) \iff \mathrm{rect}(\tfrac{\omega}{2W})$$

A useful integral

$$\int \frac{1}{1+x^2}dx = \tan^{-1} x.$$

Time-Shifting Property of the Fourier Transform

$$g(t-t_0) \iff G(\omega)e^{-j\omega t_0}$$

Time differentiation

$$\frac{d^n g}{dt^n} \iff (j\omega)^n G(\omega)$$

Some useful trigonometric identities

$$\cos x\cos y = \tfrac{1}{2}\cos(x-y)+\tfrac{1}{2}\cos(x+y)$$

$$\sin(x-y) = \sin x\cos y - \sin y\cos x$$

$$a\cos x + b\sin x = c\cos(x+\theta),$$

where $c=\sqrt{a^2+b^2}$, $\theta = tan^{-1}(-b/a)$.

Euler's formula

$$\sin x = \frac{e^{jx}-e^{-jx}}{2j}$$

Steady-state impedance of a terminated transmission line

$$Z_{in} = Z_0\frac{Z_L + jZ_0\tan(kL)}{Z_0 + jZ_L\tan(kL)}$$

The Questions

1. This question is compulsory.

   (a) Consider the following two signals: $x_1(t) = \text{rect}(t-0.5)$ and $x_2(t) = \text{rect}(t-1)$. Notice that the signals are also sketched in Figure 1a.

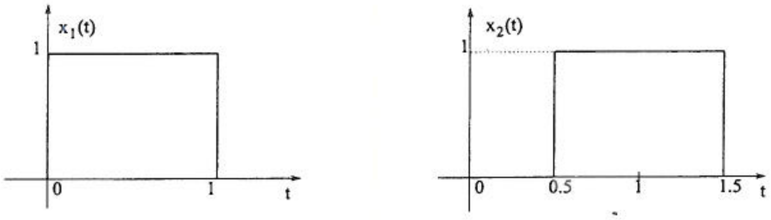


Figure 1a: The two signals $x_1(t)$ and $x_2(t)$.

   i. Compute the energy of $x_1(t)$.

[4]

   ii. Compute the correlation coefficient

$$c_{x_1x_2} = \frac{1}{\sqrt{E_{x_1}E_{x_2}}}\int_{-\infty}^{\infty} x_1(t)x_2(t)dt.$$

   Then using the computed correlation coefficient, determine whether

$$E_{x_1+x_2} = E_{x_1} + E_{x_2}.$$

[4]

Question 1 continues on next page

(b) Using the definition of the Fourier transform, compute the Fourier transform of

$$x(t) = e^{-4t}u(t),$$

where $u(t)$ is the unit step function given by

$$u(t) = \begin{cases} 1 & \text{for } \cancel{t \geq 1,}\ t \geqslant 0 \\ 0 & \text{otherwise.} \end{cases}$$

[4]

(c) Using Parseval's theorem, find the energy of $x(t) = e^{-4t}u(t)$.

[4]

(d) Consider the RC circuit shown in Figure 1b. Assume that the Power Spectral Density (PSD) of the input is $S_x(\omega) = \text{rect}(\omega/2)$. Compute the power of $y(t)$.

[4]

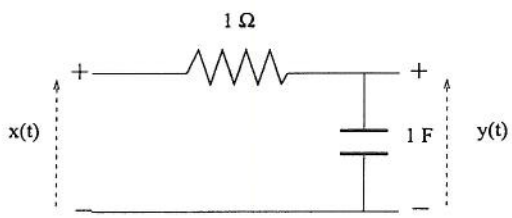


Figure 1b: The RC circuit.

Question 1 continues on next page

(e) Consider the following DSB-SC signal:

$$\varphi(t) = m(t)\cos 2000t,$$

where $m(t) = 400\text{sinc}(400t)$.

i. Sketch and dimension the Fourier transform of $\varphi(t)$.

[4]

ii. Sketch the block diagram of the corresponding synchronous receiver.

[4]

(f) Consider the following full-AM signal:

$$\varphi(t) = (A + m(t))\cos\omega_c t,$$

where $m(t) = \frac{1-t}{e^t}u(t)$ and $u(t)$ is the unit step function. Find the minimum value of A that allows the use of an envelope detector.

[4]

(g) Sketch the PM and FM waves produced by the modulating signal $m(t)$ shown in Figure 1c. Assume $\omega_c = 2\pi \times 10^6$rad/s, $k_f = 2000\pi$ and $k_p = \pi/2$.

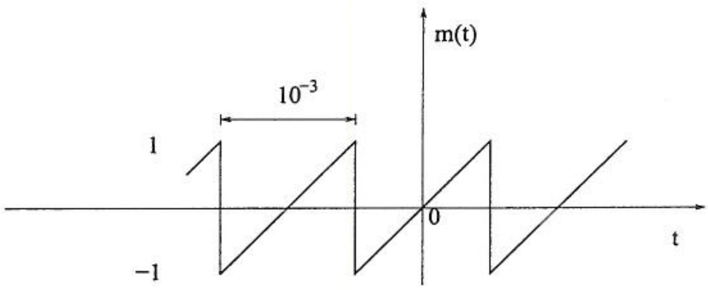


Figure 1c: The modulating signal $m(t)$.

[4]

(h) Determine the Nyquist sampling rate of the following signal

$$g(t) = \text{sinc}\ (400\pi t).$$

[4]

2. A signal transmitted over a channel is distorted because of various channel imperfections. We assume that the channel is linear and time-invariant, so that it can be treated as a linear time-invariant system.

(a) Assume that the linear-time invariant channel has the following amplitude response

$$|H(\omega)| = \begin{cases} 1 & \text{for } \omega \in [-2\pi B, 2\pi B] \\ 0 & \text{otherwise} \end{cases}$$

and the following phase response:

$$\theta_h(\omega) = -\omega t_0 - k \sin \omega T \text{ with } k \ll 1.$$

The input signal $x(t)$ is band-limited to B Hz, show that the output is

$$y(t) = x(t - t_0) + \frac{k}{2}[x(t - t_0 - T) - x(t - t_0 + T)].$$

[Hint: Use the fact that for small k, we have that $\exp(-jk \sin \omega T) \approx 1 - jk \sin \omega T$. Use also the fact that $Y(\omega) = H(\omega)X(\omega)$].

[6]

(b) In an ideal communication channel, the output $y(t)$ should be a delayed version of the input x(t). Namely $y(t) = x(t - t_0)$. Unfortunately your communication channel has the following transfer function:

$$H(\omega) = (1 + j3\omega)e^{-j5\omega}.$$

i. Determine the exact time domain expression of the output $y(t)$ if the input is $x(t) = \text{sinc } t$.

[6]

Question 2 continues on the next page

ii. Your aim is to correct the distortion introduced by the channel. The output signal $y(t)$ is therefore fed to the RC circuit as shown in Figure 2. Find the value of the product RC so that the output voltage $g(t)$ is equal to $x(t-5)$. Justify your answer.

[6]

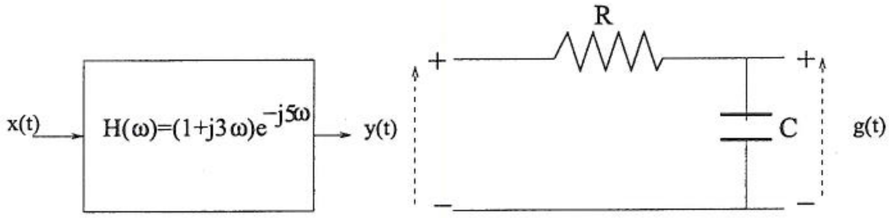


Figure 2: Correcting the channel distortion with an RC circuit.

(c) A multipath transmission takes place when a transmitted signal arrives at the receiver by two or more paths of different delays. For example, in radio links, the signal can be received by direct path between the transmitter and the receiver and also by reflection from other objects.

i. Consider the case of only two paths. In this case the received signal is $y(t) = x(t-t_0) + \alpha x(t - t_0 - \Delta t)$ where $x(t)$ is the transmitted signal. Write the transfer function of this multipath channel.

[6]

ii. The multi-path distortion is partially corrected by combining delayed versions of the received signal as follows:

$$x_{est}(t) = \sum_{n=0}^{2} a_n y(t - n\Delta t).$$

Find the values of a_n so that $x_{est}(t) \approx x(t-t_0)$. Assume that $\alpha \ll 1$. [Hint: To justify your answer you may need to use the fact that $1/(1-x) \approx 1 + x + x^2 + x^3$, if $x \ll 1$].

[6]

3. Consider the FM receiver shown in Figure 3 and assume that both the differentiator and the envelope detector are ideal. The FM signal is

$$\varphi(t) = 10\cos[\omega_c t + k_f \int_0^t m(x)dx].$$

The modulating signal is $m(t) = \cos 100t$ and $k_f = 100$.

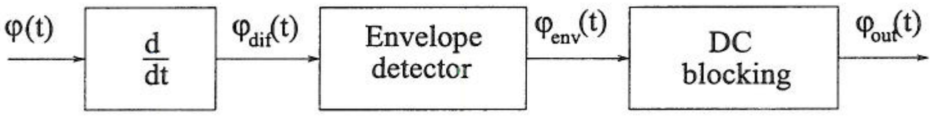


Figure 3: The FM receiver.

(a) Write the exact expression of $\varphi_{dif}(t)$.

[4]

(b) Write the exact expression of $\varphi_{env}(t)$.

[4]

(c) Determine the minimum value of ω_c such that $\varphi_{out}(t) = k_f m(t)$.

[4]

(d) Assume now that the differentiator has an ideal behaviour only for input signals in the frequency range $f \in [f_1, f_2]$, with $f_1 = 10$MHz and $f_2 = 30$MHz. If $f_c = 15$MHz and $m(t) = \cos 100t$, find the maximum value of k_f that still ensure that $\varphi(t)$ is in the frequency range where the differentiator behaves correctly. Use Carson's rule to calculate the bandwidth of the FM signal.

[6]

(e) The differentiator is now replaced with a delay line that produces a delay ΔT. The delay-line output is subtracted from $\varphi(t)$. The delay is such that $\omega_c \Delta T = \pi/2$ and $\omega_m \Delta T \ll 1$. The resulting composite wave

$$\varphi_{dif}(t) = \varphi(t) - \varphi(t - \Delta t)$$

is then envelope detected. Assuming that $\varphi(t) = A\cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$ with $\beta \ll 1$, derive the expression of the envelope detected signal $\varphi_{env}(t)$. Make the following approximations: $\cos(x) \approx 1$, $\sin(x) \approx x$ when $x \ll 1$.

[12]

4. A sinusoidal source $v(t) = 10\sin(2\pi f_0 t)$ Volts with $f_0 = 1\text{MHz}$ and with internal resistance $R = 50\ \Omega$ is connected to a transmission line of length $L = 25\text{m}$. The transmission line has characteristic impedance $Z_0 = 50\ \Omega$, phase velocity $u = 2 \cdot 10^8\text{m/sec}$ and is connected to a load Z_L (see Figure 4).

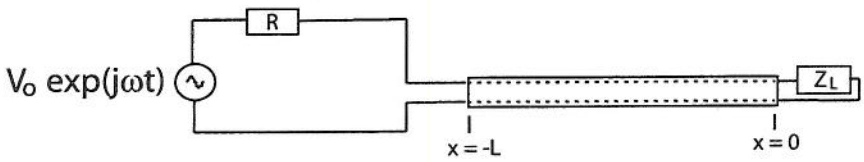


Figure 4: A transmission line connected to a sinusoidal source.

(a) Choose Z_L so that there is no reflection in the line.

[2]

(b) Assume $Z_L = 0$, write the expression of the steady-state voltage $v(x,t)$ in the transmission line.

[8]

(c) For the voltage expression found in part (b), calculate the value of the largest voltage amplitude and indicate where in the line this is achieved.

[10]

(d) Find the minimum value of Z_L for which no more than 4% of the power is reflected. Assume $Z_L > Z_0$.

[10]

QUESTION 1 E1.6 COMMUNICATIONS I SOLUTIONS 2010

1

(a)

i. $E_{x_1} = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_0^1 (1)^2 dt = \mathbf{1}.$

ii. $E_{x_1} = E_{x_2} = 1$. THUS $C_{x_1 x_2} = \int_{0.5}^{1} (1)^2 dt = 0.5$

SINCE $C_{x_1x_2} \neq 0$ SIGNALS ARE NOT ORTHOGONAL AND $E_{x_1+x_2} \neq E_{x_1} + E_{x_2}$. IN PARTICULAR

$$E_{x_1+x_2} = E_{x_1} + E_{x_2} + 2C_{x_1x_2} \cdot \sqrt{E_{x_1} E_{x_2}} = 3$$

(b)

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_0^{\infty} e^{-4t} e^{-j\omega t} dt = \left. \frac{e^{-(4+j\omega)t}}{-(4+j\omega)} \right|_0^{\infty} =$$

$$= \frac{1}{4+j\omega}$$

(c) $E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{(4)^2 + \omega^2} d\omega = \left. \frac{1}{2\pi \cdot 4} \tan^{-1} \frac{\omega}{4} \right|_{-\infty}^{\infty} = \frac{1}{8}$

(d)

$$H(w) = \frac{Y(w)}{X(w)} = \frac{1}{1+jwRC} = \frac{1}{1+jw}$$

$$S_y(w) = |H(w)|^2 \cdot S_x(w)$$, THEREFORE

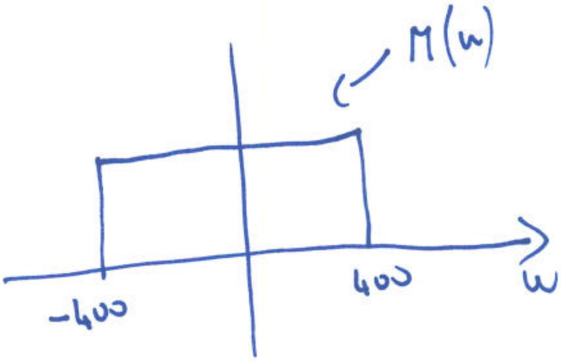
$$P_y = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_y(w)\,dw = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(w)|^2 S_x(w)\,dw$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\mathrm{RECT}(w/2)}{1+w^2}\,dw$$

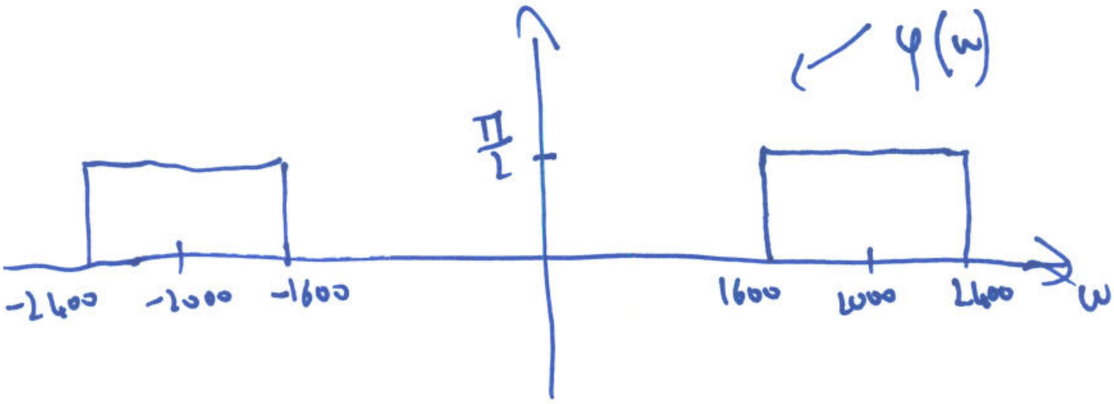
$$= \frac{1}{2\pi}\int_{-1}^{1} \frac{1}{1+w^2}\,dw = \frac{1}{2\pi}\,\mathrm{TAN}^{-1} w \Big|_{-1}^{1} = \frac{1}{4}$$

(e)

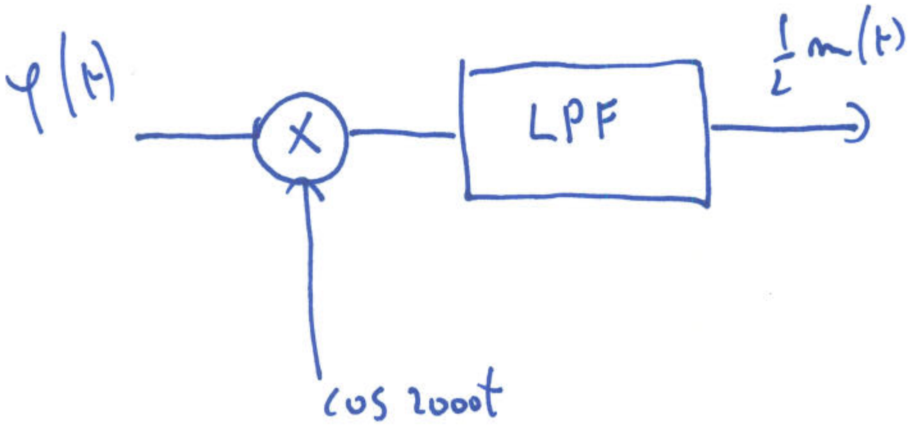
i.) $400\,\mathrm{SINC}\,400t \iff \pi\,\mathrm{RECT}\left(\frac{w}{800}\right)$



$$\hat{\varphi}(\omega) = \frac{1}{2}\Pi(\omega - 2000) + \frac{1}{2}\Pi(\omega + 2000)$$



ii.



THE IDEAL LOW-PASS FILTER HAS CUT OFF FREQUENCY $\omega_c = 400$ RAD/S

(b) WE WANT $A + m(t) \geq 0$

$$m(t) = \frac{1-t}{e^t} u(t)$$

$$\frac{dm}{dt} = \frac{-e^t - e^t(1-t)}{e^{2t}} = 0 \Rightarrow 1 - t = -1 \Rightarrow t = 2,$$

FOR $t \geq 0$

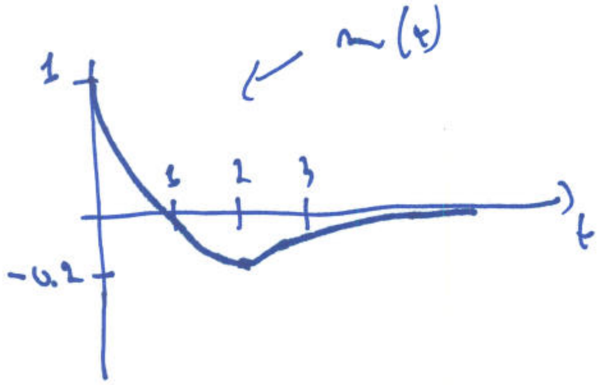
T=2 CORRESPONDS TO A MINIMUM

$$m_{min} = -\frac{1}{e^2} = -0.1354$$

THUS

$$A \geq |m_{min}| = +\frac{1}{e^2}$$

GRAPHICALLY $m(t)$ HAS THE FOLLOWING SHAPE



(g) → ANSWER TO THIS QUESTION ON THE NEXT PAGE

~~$x(t) = \begin{cases} 10\cos 200\pi t & \text{FOR } t<0 \\ 10\cos(200\pi t + 2\pi t^2) & \text{FOR } t \geq 0 \end{cases}$~~

(h) $\quad \mathrm{SINC}(400\pi t) \iff \frac{1}{400}\,\mathrm{RECT}\left(\frac{\omega}{800\pi}\right)$

THEREFORE $B_g = 200\,\mathrm{Hz} \Rightarrow f_s = 2B_g = 400\,\mathrm{Hz}$

(g) FOR FM

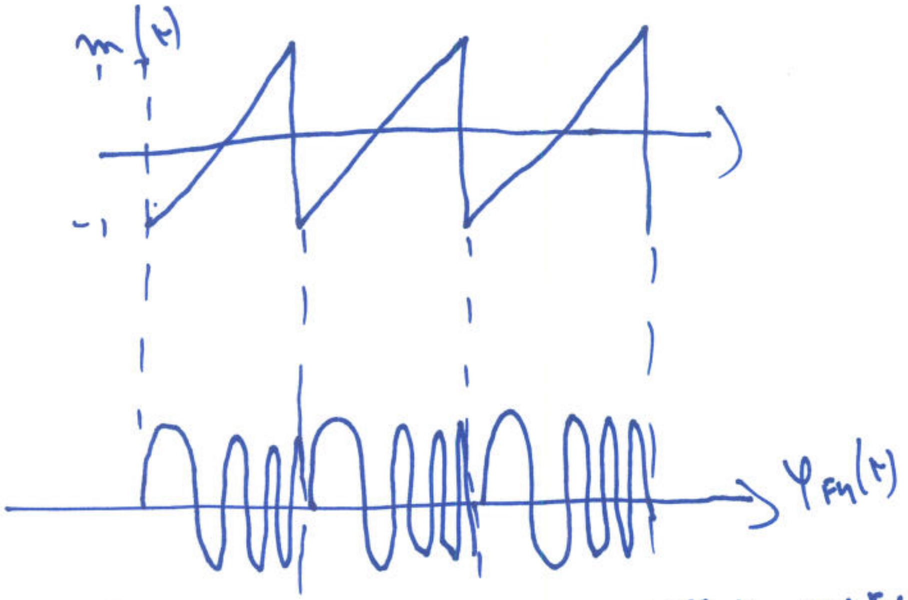
$$\Delta f = \frac{k_f m_p}{2\pi} = \frac{20000\pi}{2\pi} = 10^4 \text{ Hz}.$$

HENCE

$$(f_i)_{max} = 10^6 + 10^4 = 1.01 \text{ MHz}$$

$$(f_i)_{min} = 10^6 - 10^4 = 0.99 \text{ MHz}$$

THE CARRIER FREQUENCY INCREASES LINEARLY FROM 0.99 MHz TO 1.01 MHz OVER ONE CYCLE:



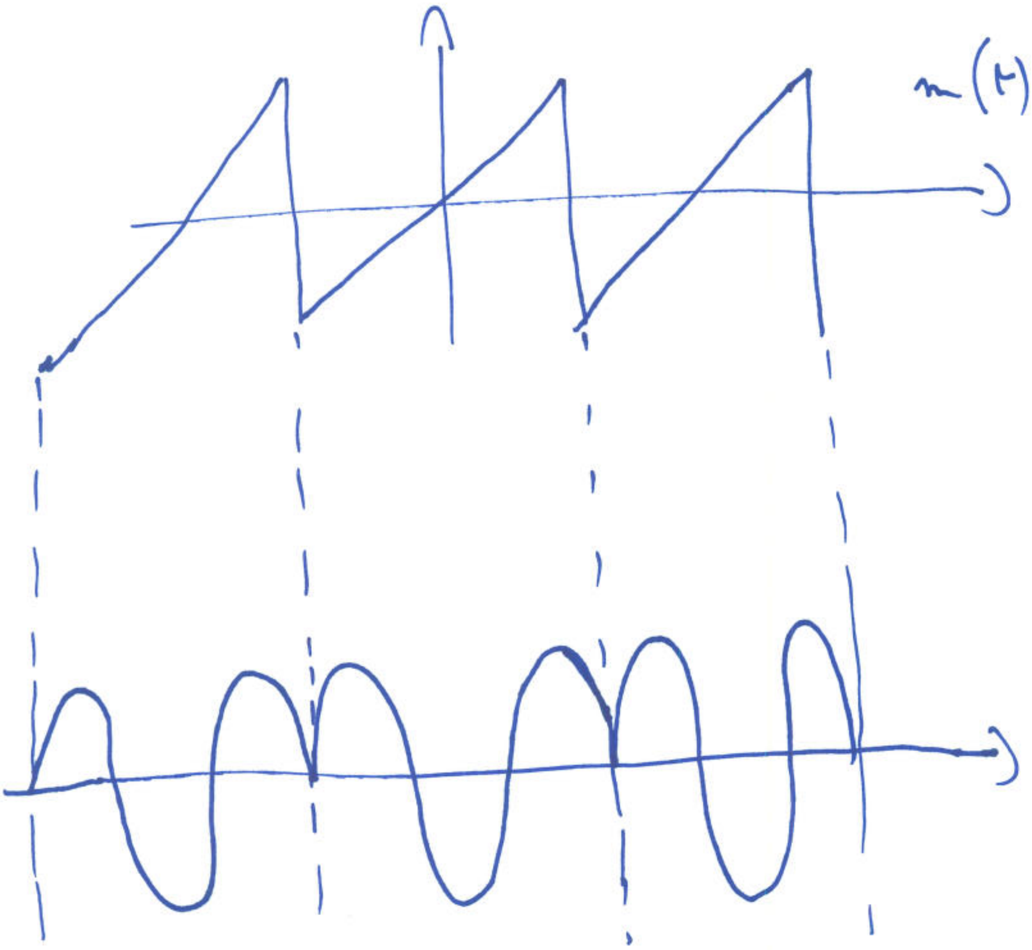
FOR PM: THE SIGNAL IS COMPUTED DIRECTLY SINCE THE DERIVATIVE OF m(t) HAS JUMPS. OVER THE INTERVAL $t \in \left[\frac{10^{-3}}{2}, \frac{10^{-3}}{2}\right]$ $m(t) = 2000t$. THUS

$$\varphi_{PM}(t) = \cos\left[2\pi(10^6)t + \frac{\pi}{2} m(t)\right] = \cos\left[2\pi(10)^6 t + 1000\pi t\right]$$

AT THE DISCONTINUITY THE JUMP IS $m_d = 2$. HENCE, $k_p m_p = \pi$. THIS MEANS THAT THE CARRIER FREQUENCY IS CONSTANT, BUT AT DISCONTINUITIES

4 C

THERE IS A PHASE DISCONTINUITY OF π RADIANTS.



QUESTION 2

5

~~(a)~~

$$\cancel{H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{X(\omega)e^{-j\omega t_0}}{X(\omega)} = e^{-j\omega t_0}}$$

~~[illegible]~~

(a)

$$Y(\omega) = H(\omega)\,X(\omega).$$

SINCE $X(\omega)$ IS BAND-LIMITED TO B Hz
AND $|H(\omega)| = 1$ FOR $\omega \in [-2\pi B, 2\pi B]$
WE CAN WRITE

$$Y(\omega) = |H(\omega)|\,e^{+j\theta_H} \cdot X(\omega) = X(\omega)\,e^{j\theta_H}$$

$$= X(\omega)\left(e^{-j\omega t_0}\,e^{-jK\sin\omega T}\right)$$

WE USE THE FACT THAT $e^{-jK\sin\omega T} \simeq 1 - jK\sin\omega T$

AND OBTAIN:

$$Y(\omega) = X(\omega)\,e^{-j\omega t_0}\left(1 - jK\sin\omega T\,\cdot\,\cancel{X(\omega)}\right)$$

$$= X(\omega)\,e^{-j\omega t_0} - \frac{K}{2}\left(X(\omega)\,e^{j\omega T} - X(\omega)\,e^{-j\omega T}\right)e^{-j\omega t_0}$$

USING THE TIME-SHIFTING PROPERTY WE THEN OBTAIN:

$$y(t) = x(t-t_0) + \frac{K}{2}\left(x(t-t_0-T) - x(t-t_0+T) \right)$$

(b)

i. $$Y(\omega) = X(\omega)e^{-j5\omega} + 3j\omega e^{-j5\omega} X(\omega)$$

$$\Downarrow$$

$$y(t) = x(t-5) + \frac{d\, x(t-5)}{dt}$$

$$x(t) = \text{sinc}\, t \qquad \frac{dx}{dt} = \frac{\cos t}{t} - \frac{\text{sinc}\, t}{t},$$

$$y(t) = \text{sinc}(t-5) + \frac{3\cos(t-5)}{(t-5)} - 3\,\frac{\text{sinc}\,(t-5)}{(t-5)}$$

ii. $$H_{RC}(\omega) = \frac{1}{1+j\omega RC},$$

YOU WANT $$H(\omega)\cdot H_{RC}(\omega) = \frac{1+j3\omega}{1+j\omega RC}\cdot e^{-j5\omega} = e^{-j5\omega}$$

WHICH MEANS $RC = 3$.

(c)

i.

$$H(w) = \frac{Y(w)}{X(w)} = e^{-jwt_0} + \alpha e^{-jw(t_0+\Delta t)}$$

$$= e^{-jwt_0}\left(1 + \alpha e^{-jw\Delta t}\right)$$

ii.

TO CORRECT $H(w)$ YOU WANT A NEW FILTER $H_{EQ}(w)$ SUCH THAT

$$H_{EQ}(w) \cdot H(w) = e^{-jwt_0} \Rightarrow H_{EQ}(w) = \frac{1}{1 + \alpha e^{-jw\Delta t}}$$

IF $\alpha \ll 1$ THEN

$$H_{EQ}(w) \simeq 1 - \alpha e^{-jw\Delta T} + \alpha^2 e^{-j2w\Delta T} - \alpha^3 e^{-j3w\Delta t} + \dots$$

THUS

$$a_0 = 1, \quad a_1 = -\alpha, \quad a_2 = \alpha^2, \quad a_3 = -\alpha^3 \dots$$

QUESTION 3

$$\varphi(t) = 10\cos(w_c t + \sin 100t)$$

(a)

$$\varphi_{dif}(t) = \dot\varphi(t) = -10(w_c + k_f \cos 100t)\sin(w_c t + \sin 100t)$$

$$= -10(w_c + k_f\, m(t))\sin(w_c t + \sin 100t)$$

(b)

$$\varphi_{ENV}(t) = 10(w_c + k_f\, m(t))$$

(c) WE NEED $w_c + k_f\, m(t) \geq 0$

$$w_c \geq 100\, m_p = 100$$

(d) USING CARSON'S RULE $(B_{FM} = 2(\Delta f + B))$

WE HAVE THAT $\varphi(w)$ IS NON-ZERO

IN THE INTERVAL $f \in [f_c - (\Delta f + B), f_c + (\Delta f + B)]$.

WE NEED THIS INTERVAL BE INCLUDED IN

$[f_1, f_2)$.

WE THUS NEED $f_c - (\Delta f + B) \geq f_1$

$$\Rightarrow \Delta f \leq f_c - f_1 - B = 15\,10^6 - 10\cdot 10^6 - \frac{100}{2\pi}$$

$$\Delta f = 5\cdot 10^6 - \frac{100}{2\pi}. \qquad \Delta f = \frac{k_f m_p}{2\pi} \Rightarrow$$

$$k_f = \frac{2\pi}{m_p}\left(5\cdot 10^6 - \frac{100}{2\pi}\right) = 10^7\pi - 100.$$

(a) FIRST NOTICE THAT THE OUTPUT OF AN IDEAL RECEIVER

IS $\varphi_{ENV}(t) \approx A\left(w_c + K_f m(t)\right)$

AND AFTER DC-BLOCKING WE SHOULD GET $\varphi_{OUT}(t) \approx K_f m(t)$.

THE HOPE IS THAT THE DELAY-LINE CAN ACHIEVE SOMETHING SIMILAR.

$$\varphi(t) = A\cos\left(w_c t + \beta \sin 2\pi f_m t\right)$$

$$\varphi_{dif}(t) = \varphi(t) - \varphi(t - \Delta T)$$

$$= A\cos\left(w_c t + \beta \sin 2\pi f_m t\right) - A\cos\left(w_c t - w_c \Delta T + \beta \sin 2\pi f_m (t-\Delta T)\right)$$

$$= A\cos\left(w_c t + \beta \sin 2\pi f_m t\right) - A\sin\left(w_c t + \beta \sin 2\pi f_m (t - \Delta T)\right)$$

WHERE WE HAVE USED THE FACT THAT $w_c \Delta T = \frac{\pi}{2}$

AND $\cos\left(x - \frac{\pi}{2}\right) = \sin x$.

NOW WE USE THE IDENTITY

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

AND MAKE THE FOLLOWING APPROXIMATIONS:

$$\cos(2\pi f_m \Delta T) \approx 1, \quad \sin 2\pi f_m \Delta T \approx 2\pi f_m \Delta T$$

THIS YIELDS TO

$$\varphi_{DIF}(t) = A\cos\left(w_c t + \beta \sin 2\pi f_m t\right) - A\sin\left(w_c t + \beta \sin 2\pi f_m t - 2\pi f_m \Delta T \beta \cos 2\pi f_m t\right).$$

WE USE THE SAME TRIGONOMETRIC IDENTITY AGAIN AND MAKE THE FOLLOWING APPROXIMATIONS

$$\cos\left(2\pi f_m \Delta T \beta \cos 2\pi f_m t\right) \simeq 1$$

$$\sin\left(2\pi f_m \Delta T \beta \cos 2\pi f_m t\right) \simeq 2\pi f_m \Delta T \beta \cos 2\pi f_m t$$

THIS LEADS TO:

$$\varphi_{DIF}(t) = A\cos\left(w_c t + \beta \sin 2\pi f_m t\right) - A\sin\left(w_c t + \beta \sin 2\pi f_m t\right) + A\, 2\pi f_m \Delta T \beta \cos 2\pi f_m t \cos\left(w_c t + \beta \sin 2\pi f_m t\right)$$

WE NOW USE THE IDENTITY:

$$a \cos w_0 t + b \sin w_0 t = \sqrt{a^2+b^2} \cos\left(w_0 t + \tan^{-1}\left(\frac{-b}{a}\right)\right)$$

AND OBTAIN

$$\varphi_{DIF} = A\left(1 + \left(1 + 2\pi f_m \Delta T \beta \cos 2\pi f_m t\right)^2\right)^{1/2} \cos\left(w_c t + \beta \sin 2\pi f_m t + \theta\right).$$

φ_{ENV} IS THUS EQUAL TO $\varphi_{ENV}(t) = A\left[1 + \left(1 + 2\pi f_m \Delta T \beta \cos 2\pi f_m t\right)^2\right]^{1/2}.$

SINCE $2\pi f_m \Delta T \beta \cos 2\pi f_m t << 1$

WE HAVE THAT

$$y_{ENV}(t) \simeq \sqrt{2}A\left(1 + 2\pi f_m \frac{\Delta T}{2} \beta \cos 2\pi f_m t\right)$$

AND AFTER THE DC BLOCKING WE OBTAIN

$$y_{OUT}(t) \simeq 2\pi f_m \frac{\Delta T}{2} \beta \cos 2\pi f_m t \simeq k_f m(t)$$

HERE $m(t) = \cancel{2\pi f_m t} \cos 2\pi f_m t$

QUESTION 4

11

(a) REFLECTION IS AVOIDED WHEN THE TERMINATION IS MATCHED: $Z_L = Z_0 = 50\,\Omega$

(b) WHEN $Z_L = 0$ $\quad K_V = \frac{V_-}{V_+} = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$

$$V(x,t) = V_+ e^{j\omega t}\left(e^{-jkx} + \frac{V_-}{V_+} e^{jkx}\right)$$

$$= V_+ e^{j\omega t}\left(e^{-jkx} - e^{jkx}\right)$$

$$= -2jV_+ e^{j\omega t} \sin kx$$

(c) $V(-L,t) = \frac{Z_{IN}}{Z_{IN} + Z_0} V_0 e^{j\omega_0 t}$;

$$+2jV_+ \cancel{e^{j\omega_0 t}} \sin kL = \frac{Z_{IN}}{Z_{IN} + Z_0} V_0 \cancel{e^{j\omega_0 t}}$$

$Z_{IN} = jZ_0 \tan kL \quad \Rightarrow$

$$2\cancel{j}V_+ \cdot \sin kL = \frac{\cancel{j} \tan kL}{(1 + j\tan kL)} V_0$$

$$kL = \frac{2\pi \cdot \cancel{10^8}}{3 \cdot \cancel{10^8}} \cdot \frac{\cancel{100}}{4} = \frac{\pi}{4};$$

$$\tan\frac{\pi}{4} = 1 \quad \text{AND} \quad \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

WE THUS HAVE

$$\sqrt{2}\, V_+ = \frac{V_0}{1+j} \implies V_+ = \frac{10.\sqrt{2}}{\sqrt{2}(1+j)} \text{ VOLTS}$$

NOW

$$V(x,t) = -2j V_+ e^{j\omega t} \sin kx$$

AND SINCE $L = 25$ m, THE MAXIMUM IS ACHIEVED AT $x = L = 25$ m AND

$$|V(L,t)| = 2 \sin\frac{\pi}{4} \cdot \frac{10}{2} = \frac{10.\sqrt{2}}{2} \text{ VOLT}$$

(d)

$K_p = |K_v|^2$ WE WANT

$$K_p = 0.04$$

$K_v = \dfrac{Z_L - Z_0}{Z_L + Z_0}$ AND SINCE WE ARE ASSUMING $Z_L > Z_0 \implies K_v > 0$

THUS $|K_v|^2 = 0.04 \implies K_v = 0.2$

$$K_V = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.2 \Rightarrow Z_L - Z_0 = 0.2\left(Z_L + Z_0\right)$$

$$\Rightarrow 0.8 Z_L = 1.2 Z_0 \Rightarrow Z_L = \frac{1.2}{0.8} 50 = 75\,\Omega$$