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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

MSc and EEE/ISE PART IV: MEng and ACGI

OPTICAL COMMUNICATION

Corrected Copy

Monday, 14 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 6

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : E.M. Yeatman
Second Marker(s) : A.S. Holmes

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3.0 \times 10^{8}$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x}\tan(k_{1x}d/2) \text{ and } \kappa = -k_{1x}\cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]

a) A certain optical receiver detects 10^{13} photons/s, at a nominal wavelength of 1.3 μm. What is the equivalent received optical power in dBm?

b) A certain symmetric slab waveguide supports a single TE mode. How many TM modes will this guide support?

c) Briefly explain the physical significance of the imaginary part of the refractive index of a material.

d) Why is silicon not a suitable material for photodetectors for long-haul optical communication systems?

e) Which type of signal degradation is more difficult to compensate in an optical link: attenuation, dispersion, or nonlinearity?

f) Briefly explain the main performance advantage for optical communications of distributed feedback lasers over Fabry-Perot lasers.

g) A step-index silica-based optical fibre has an index difference of 0.02. Estimate its numerical aperture

h) An optical point-to-point link has a fibre length of 50 km and an attenuation coefficient of 0.3 dB/km. Which would you expect to have a worse effect on the signal-to-noise ratio: doubling the cable length, or doubling the bit-rate? Assume thermal noise dominates in all cases.

i) A loop of silica fibre is to be used to temporarily store 1000 bits of data, for a system data rate of 2.5 Gbit/s. Estimate the length of fibre needed.

j) What is the principal attenuation mechanism in silica optical fibre at nominal wavelengths less than 1 μm?

2. On pg. 1 the eigenvalue equations are given for TE modes in a symmetric slab waveguide as shown in Fig. 2.1

a) What are the boundary conditions leading to these equations? [4]

b) Derive the eigenvalue equations using a field profile approach. [10]

c) Derive the additional equation from which, along with the eigenvalue equations, the mode indices of a waveguide of this type can be calculated, and describe a process by which this calculation could be done. [6]

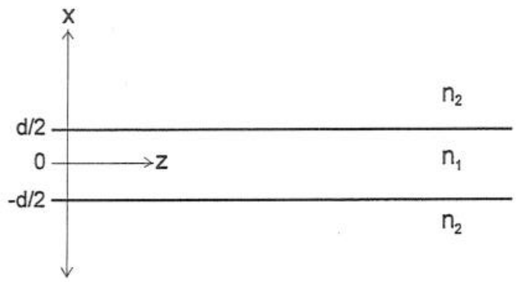


Figure 2.1 Slab waveguide

3. An optical link is constructed for a span of 125 km, to operate at 2.5 Gbit/s. The transmitter to be used couples 10 mW of power into a fibre having 0.3 dB/km attenuation. The signal is detected by a p-i-n photodiode receiver having a responsivity of 0.7 A/W, and a noise equivalent power of 8 pW/√Hz.

a) Calculate the received optical power. [2]

b) Calculate the optical SNR, assuming that receiver noise dominates. Repeat the calculation assuming that shot noise dominates. Hence indicate which is the limiting factor. Is the overall SNR adequate to achieve a bit error rate of 10^{-9}? [6]

c) An optical amplifier is added to the link, having 30 dB gain and a noise figure of 4 dB. Recalculate the SNR for the approximation of receiver noise dominating, and for amplifier noise dominating. Indicate which is now the limiting factor, and whether a bit error rate of 10^{-9} can now be achieved. [6]

d) The source spectral width is $\sigma_\lambda = 1.5\ nm$. Choose a reasonable criterion for the acceptable level of dispersion, and hence calculate the maximum fibre dispersion coefficient D that will not excessively degrade the performance of the link. [6]

4. a) Briefly describe the four factors which reduce the external quantum efficiency in a light emitting diode (LED), and ways in which their effects can be reduced. [6]

b) For an LED emitting into air, find the fraction of emitted photons lost through each of the four mechanisms of part (a), and hence calculate the external quantum efficiency η_{ext}, if none of the special measures to improve it have been used. Assume an attenuation coefficient of 0.5×10^2 cm^{-1}, and that the active region emits photons equally in all directions. The distance from the active region to the surface is 10 μm, and the refractive index of the semiconductor is 3.7. State any other assumptions or approximations made. [6]

c) Calculate the quantum efficiency for this same LED emitting into guided modes of a multi-mode fibre with a numerical aperture of 0.15. [4]

d) Describe the advantages of laser diodes over light emitting diodes for optical communication applications. [4]

5. A certain single mode fibre has a wavelength-dependent effective index n' given by:

$$n' = n_g + \alpha(\lambda_o - \lambda_c)^2 \qquad (5.1)$$

where λ_o is the free-space wavelength and λ_c is the centre wavelength of the operating range.

a) Derive expressions for the phase delay τ_p, and the group delay τ_g, for a fibre length L. Note that the group velocity is given by $v_g = d\omega/d\beta$ where β is the propagation constant. [8]

b) Using your solution to (a), derive an expression for the dispersion coefficient D, using $D = |(d\tau_g/d\lambda_o)/L|$. Hence, show that $D = |\lambda_o(d^2n/d\lambda_o^2)/c|$ for this case. [8]

c) Show that the ratio of phase to group velocity is given by

$$v_p/v_g = 1 - 2\alpha\lambda_o(\lambda_o - \lambda_c)/n' \qquad (5.2)$$

in this case. [4]

6. a) A silicon p-n photodiode (Fig. 6.1) has a depletion layer thickness of w, and p and n doping levels respectively of N_A and N_D, with $N_A = 5N_D = 5\times10^{20}$ m^{-3}. The quantities w_p and w_n are the depleted widths in the p and n regions respectively. A reverse bias voltage V_b is applied. Find an expression for the full depletion width w as a function of V_b, and the value of V_b for which w = 5 μm. [8]

b) Neglecting Fresnel reflection, find an expression for the quantum efficiency η of the photodiode of (a) if the total p region thickness h = 8 μm, and the absorption coefficient $\alpha = 0.8\times10^5$ m^{-1}. Hence find the value of V_b for which η = 0.80. [6]

c) For an electron mobility in the silicon of 0.12 m^2/Vs, find the bias voltage for which the peak electron drift velocity in this device reaches 5×10^4 m/s. [6]

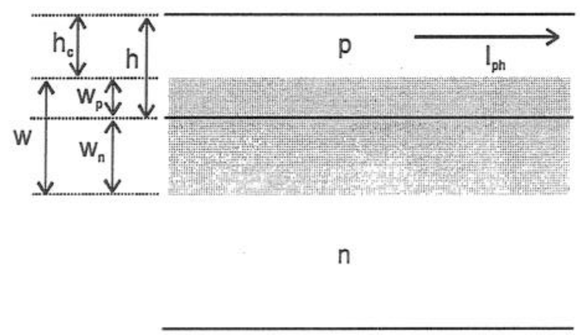


Figure 6.1 p-n photodiode

Optical Communication May 2007
Solutions

(1) a) $P = \frac{hc}{\lambda} \times \dot{N} = \frac{6.63\times10^{-34} \times 3\times10^{8}}{1.3\times10^{-6}} \times 10^{13} = 1.53\,\mu W$

$= 10 \log(1.53\times10^{-3}) = -28$ dBm

b) One (no. of TE = no. of TM modes)

c) $Im\{n\}$ indicates absorption; since $k = n\omega/c$, it results in an imaginary part of k (attenuation).

d) Long haul optical comm. systems use wavelengths around 1.3 & 1.55 μm – at these wavelengths Si is transparent (bandgap too large)

e) Nonlinearity (has no general compensation technique)

f) DFB lasers use frequency selective reflectors which eliminate the multiple wavelength lines associated with longitudinal modes in F-P lasers

g) $NA = \sqrt{n_1^2 - n_2^2} \simeq \sqrt{2n\cdot\Delta n}$

$n \simeq 1.5 \quad \therefore \quad NA \simeq \sqrt{2\times1.5\times.02} = 0.245$

h) Doubling the bit rate doubles the noise power, so reduces SNR by 3 dB. Doubling the length here adds $50 \times 0.3 = 15$ dB attenuation – much worse!

i) $L = \Delta t \times v = \frac{\Delta t \cdot c}{n} = \frac{N\cdot c}{B\cdot n} = \frac{10^3 \times 3\times10^8}{2.5\times10^9\times1.5}$

$= 80$ m

j) Rayleigh scattering

(2) a) Boundary conditions: $E(x)$ and $\frac{dE(x)}{dx}$ must be continuous (match at boundaries).

b) See notes pp 11-12 part A.

c) Additional condition is that β $(\equiv k_z)$ must be the same in all media (phase match at boundaries)

$$\beta^2 = n_1^2 k_0^2 - k_{1x}^2 = n_2^2 k_0^2 + K^2$$

$(k_{2x} = j K)$

Thus $K^2 + k_{1x}^2 = (n_1^2 - n_2^2) k_0^2$ $(k_0 = \omega / c)$

This gives circular arcs on the $K - k_{1x}$ diagram which cross the eigenvalue lines, crossing points give allowed values (modes). Then we can find $n' = \frac{\beta}{k_0}$ from the allowed k_{1x} values with $\beta = \sqrt{n_1^2 k_0^2 - k_{1x}^2}$

3

(3) a) $\alpha = \frac{\alpha_{dB}}{10 \log e} = \frac{0.3}{4.34} = 0.069 \text{ km}^{-1}$

$\exp(-\alpha L) = \exp(-.069 \times 125) = 1.8 \times 10^{-4}$

$\Phi_R = \Phi_T \exp(-\alpha L) = 1.8 \mu W$

b) $SNR_{opt} = \frac{\Phi_R}{NEP\sqrt{\Delta f}} = \frac{1.8 \times 10^{-6}}{8 \times 10^{-12} \times \sqrt{2.5 \times 10^9 / 2}}$

$= 6.35 \quad (= 8.0 \text{ dB})$

for receiver noise dominating.

- too low for $BER = 10^{-9}$

For shot noise dominating:

$SNR_{opt} = \frac{I_{ph}}{\sqrt{2e I_{ph} \Delta f}} = \sqrt{\frac{I_{ph}}{eB}}$

$I_{ph} = R \Phi_R$

$SNR_{opt} = \sqrt{\frac{0.7 \times 1.8 \times 10^{-6}}{1.6 \times 10^{-19} \times 2.5 \times 10^9}}$

$= 55.7 \quad (= 17.5 \text{ dB})$

Receiver noise dominates. BER 10^{-9} cannot be achieved (needs $SNR \geq 12$)

c) For amplified link, receiver dominated SNR is improved by 30 dB, ie to 6350. Shot noise is replaced by ASE, which will give SNR worse than shot noise limit by 4 dB, ie 13.5 dB (22.4).

This is enough to reach BER 10^{-9}

d) Assume a maximum dispersion time σ_D of ¼ bit,

ie $\sigma_D = D \sigma_\lambda L = \frac{0.25}{B}$

$\frac{1}{B} = 400 \text{ ps} \qquad D_{max} = \frac{100 \text{ ps}}{1.5 \text{ nm} \times 125 \text{ km}} = \cancel{0.??}\ 0.53 \frac{\text{ps}}{\text{nm} \cdot \text{km}}$

(4)

a) i) half the light goes downwards. This can be recovered by a heterostructure to reduce absorption outside active region, and mirror at bottom surface.

ii) Light is absorbed between active region and surface. Again, reduce by heterostructure (higher E_g in upper part).

iii) Fresnel Reflection at upper surface from index difference – reduce by AR coating.

iv) Total Int. Reflection – photons not within $\theta_c = \sin^{-1}\left(\frac{1}{n_s}\right)$ of normal will not escape. Can be reduced by hemispherical cap.

b) i) 50% lost downwards.

ii) Calculate TIR next. $\theta_c = \sin^{-1}\left(\frac{1}{3.7}\right) = 15.7^\circ$

fraction emitted $f = \int_0^{\theta_c} d\Omega \Big/ \int_0^{\pi/2} d\Omega \qquad d\Omega = 2\pi \sin\theta \, d\theta$

$\therefore f = 1 - \cos\theta_c = 0.0372$

96.3% of <u>upwards</u> light is lost.

iii) Within θ_c, path length to surface varies little, so fraction re-absorbed $\cong 1 - \exp(-\alpha L)$ with $L = 10 \times 10^{-6}$ m, $\alpha = 0.5 \times 10^4 \text{ m}^{-1}$, $\exp(-\alpha L) = 0.951$

4.9% lost.

iv) We will use normal incidence to approximate Fresnel reflection: $R = \left(\frac{3.7-1}{3.7+1}\right)^2 = .33$

Overall $\eta_{ext} = \frac{1}{2} \times f \times e^{-\alpha L} \times (1-R) = \underline{1.18\%}$

(4) c) $NA = 0.15 \cong \sqrt{2n \cdot \Delta n}$

taking $n = 1.5$ gives

$$\Delta n = \frac{0.15^2}{3} = .0075$$

for guiding $\theta_c = \sin^{-1}\left(\frac{n_0}{n_c}\right) \cong \sin^{-1}\left(\frac{1.5}{1.5075}\right)$

$$= 84.3°$$

So max angle off fibre axis is $90 - 84.3 = 5.7° = \theta_c$

θ_c in semiconductor given by:

$$\sin\theta_{cs} = \frac{n_c}{n_s}\sin\theta_c = \frac{1.5}{3.7}\sin(5.7)$$

$$\theta_{cs} = 2.31°$$

$\therefore f = 1 - \cos(2.31) = 0.00081$

$$R = \left(\frac{3.7-1.5}{3.7+1.5}\right)^2 = 0.179$$

$e^{-\alpha L}$ is unchanged.

$$\eta_{ext} = \frac{1}{2}(.00081)(1-.179)0.951 = .00032$$

$$= 0.032\ \%$$

d) Advantages of laser diodes over LEDs:

- narrow spectrum $\Delta\lambda$
- faster modulation
- much more directional output (high coupling to fibre)
- high external efficiency

(5) a) $\tau_p = L/v_p \quad v_p = \frac{\omega}{\beta} = \frac{\omega}{n' k_0}$

$\tau_p = \frac{n' k_0 L}{\omega} = \frac{n' L}{c} = [n_g + \alpha(\lambda_0 - \lambda_c)^2] L/c$

$\tau_g = \frac{L}{v_g} \quad v_g = \frac{d\omega}{d\beta} \quad \tau_g = L \frac{d\beta}{d\omega}$

$= L \frac{d\beta}{dk_0} \Big/ \frac{d\omega}{dk_0} \quad \omega = ck_0 \therefore \tau_g = \frac{L}{c}\frac{d\beta}{dk_0}$

$= \frac{L}{c}\frac{d\beta}{d\lambda_0} \cdot \frac{d\lambda_0}{dk_0} \quad \lambda_0 = \frac{2\pi}{k_0} \quad \frac{d\lambda_0}{dk_0} = -\frac{2\pi}{k_0^2}$

$\tau_g = -\frac{L}{c}\frac{2\pi}{k_0^2}\frac{d\beta}{d\lambda_0} \quad \beta = n' k_0$

$\frac{d\beta}{d\lambda_0} = k_0 \frac{dn'}{d\lambda_0} + n' \frac{dk_0}{d\lambda_0}$

$\frac{d\beta}{d\lambda_0} = k_0 [2\alpha(\lambda_0 - \lambda_c)] - \frac{2\pi}{\lambda_0^2}[n_g + \alpha(\lambda_0 - \lambda_c)^2]$

$\tau_g = \frac{L\lambda_0}{c}\left[\frac{1}{\lambda_0}(n_g + \alpha(\lambda_0^2 + \lambda_c^2 - 2\lambda_0\lambda_c)) - 2\alpha(\lambda_0 - \lambda_c)\right]$

$= \frac{L}{c}\left[n_g + \alpha\lambda_0^2 + \alpha\lambda_c^2 - 2\alpha\lambda_0^2\right) = \frac{L}{c}\left[n_g + \alpha(\lambda_c^2 - \lambda_0^2)\right]$

b) $\frac{d\tau_g}{d\lambda_0} = \frac{L}{c}(-2\alpha\lambda_0) \quad D = \left|\frac{d\tau_g}{d\lambda_0}\frac{1}{L}\right| = \frac{2\alpha\lambda_0}{c}$

$\frac{\lambda_0}{c}\frac{d^2 n}{d\lambda_0^2} = \frac{\lambda_0}{c}(2\alpha) = D$ ✓ (agrees)

c) $v_p = c/n' \quad v_g = (d\beta/d\omega)^{-1}$

$\beta = \frac{n'\omega}{c} \quad \frac{d\beta}{d\omega} = \frac{n'}{c} + \frac{\omega}{c}\frac{dn'}{d\omega} = \frac{n'}{c} + \frac{\omega}{c}\frac{dn'}{d\lambda_0}\frac{d\lambda_0}{d\omega}$

$d\lambda_0/d\omega = -2\pi c/\omega^2 = -\lambda_0/\omega$

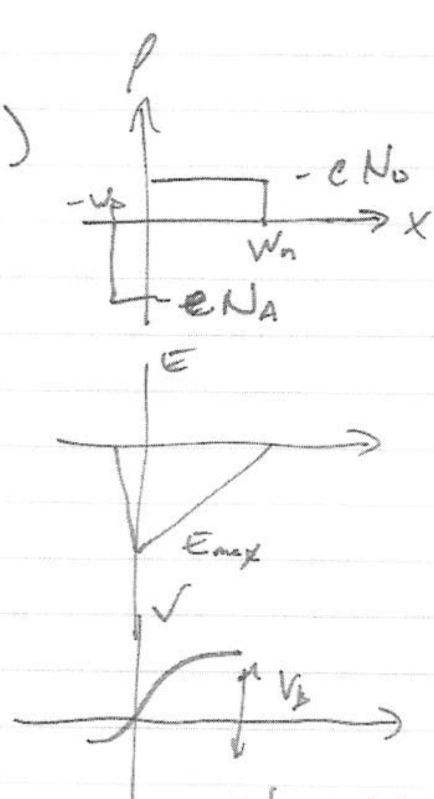
$dn'/d\lambda_0 = 2\alpha(\lambda_0 - \lambda_c) \quad \therefore d\beta/d\omega = \frac{n'}{c} - \frac{\omega}{c}\left(\frac{\lambda_0}{\omega}\right)(2\alpha(\lambda_0 - \lambda_c))$

$d\beta/d\omega = \frac{1}{c}(n' - 2\alpha\lambda_0(\lambda_0 - \lambda_c)) \quad \frac{v_p}{v_g} = 1 - \frac{2\alpha\lambda_0(\lambda_0 - \lambda_c)}{n_g}$

$v_g = c/(n' - 2\alpha\lambda_0(\lambda_0 - \lambda_c))$

assuming in v_g $n' \simeq n_g$

6) a)



$$V_b = |E_{max}|\frac{(w_n + w_p)}{2}$$

$$|E_{max}| = \frac{eN_A w_p}{\varepsilon}$$

$$w_n = \frac{N_A}{N_D} w_p \qquad W = w_n + w_p$$

$$\therefore V_b = \frac{eN_A w_p^2 (1 + N_A/N_D)}{2\varepsilon}$$

$$= \frac{eN_A W^2}{2\varepsilon_r\varepsilon_0 (1 + N_A/N_D)}$$

$$V_b = \frac{1.6\times10^{-19} \times 5\times10^{20} \times (5\times10^{-6})^2}{2\times12\times8.85\times10^{-12}(1+5)}$$

$$= \underline{1.57\ V}$$

b) $\eta = e^{-\alpha x_1} - e^{-\alpha x_2}$

$x_1 = h - w_p \qquad x_2 = h + w_n$

$$\eta = e^{-\alpha h}\left(e^{\alpha w_p} - e^{-5\alpha w_p}\right)$$

$\alpha h = 0.8\times10^5 \times 8\times10^{-6} = 0.64$

$\exp(-\alpha h) = 0.527$

$\frac{0.80}{0.527} = 1.52 = e^{+\theta} - e^{-5\theta} \qquad \theta = \alpha W_p$

Successive approximation: $\theta \simeq 0.48$

$$w_p = \frac{0.48}{0.8\times10^5} = 6\ \mu m$$

$W = 6w_p = 36\ \mu m$

$$V_b = 1.57\left(\frac{36}{5}\right)^2 = \underline{\underline{81 V}}$$

8/8

(6) c) $v_d = \mu E$

$$|E_{max}| = \frac{5\times10^4}{0.12} = 4.17\times10^5 \text{ V/m}$$

$$E_{max} = \frac{eN_A w_p}{\varepsilon} = \frac{2V_b}{W} = \sqrt{\frac{2eN_A V_b}{\varepsilon(1+N_A/N_D)}}$$

$$V_b = |E_{max}|^2 \times \frac{\varepsilon_r \varepsilon_0 (1+N_A/N_D)}{2\times e N_A}$$

$$= \frac{(4.17\times10^5)^2 \times 12 \times 8.85\times10^{-12}(6)}{2\times1.6\times10^{-19}\times5\times10^{20}}$$

$$= \underline{0.69\text{ V}}$$