E4.38
C3.3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

MSc and EEE PART IV: MEng and ACGI

MODELLING AND CONTROL IN POWER ENGINEERING

Monday, 17 May 10:00 am

Time allowed: 3:00 hours

Corrected Copy

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

Please use separate answer books for Sections A and B.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	T.C. Green, B.C. Pal
	Second Marker(s) :	D. Popovic, D. Popovic

Section A

1.

(a) Equation 1.1 is the general form of the state-space averaged model of a switch-mode power supply. Describe the basis on which state-space models of the on-state and off-state are combined to form the averaged model. Explain the terms $\tilde{x}, X, \tilde{\delta}, \Delta$ and U. [6]

$$\dot{\tilde{\mathbf{x}}} \cong \left(\Delta \mathbf{A}_{\mathbf{On}} + (1-\Delta)\mathbf{A}_{\mathbf{Off}}\right)\tilde{\mathbf{x}} + \left(\left(\mathbf{A}_{\mathbf{On}} - \mathbf{A}_{\mathbf{Off}}\right)\mathbf{X} + \left(\mathbf{B}_{\mathbf{On}} - \mathbf{B}_{\mathbf{Off}}\right)\mathbf{U}\right)\tilde{\delta}$$

$$\tilde{\mathbf{y}} \cong \left(\Delta \mathbf{C}_{\mathbf{On}} + (1-\Delta)\mathbf{C}_{\mathbf{Off}}\right)\tilde{\mathbf{x}} + \left(\left(\mathbf{C}_{\mathbf{On}} - \mathbf{C}_{\mathbf{Off}}\right)\mathbf{X} + \left(\mathbf{D}_{\mathbf{On}} - \mathbf{D}_{\mathbf{Off}}\right)\mathbf{U}\right)\tilde{\delta}$$

Equation (1.1)

(b) Figure 1.1 shows the circuit of a buck-boost switch-mode power supply and the current paths that exist when the mosfet is on and when the MOSFET is off.

(i) Find the state-space averaged model of this circuit [10]

(ii) Find the transfer function that relates the duty-cycle to the output voltage. [4]

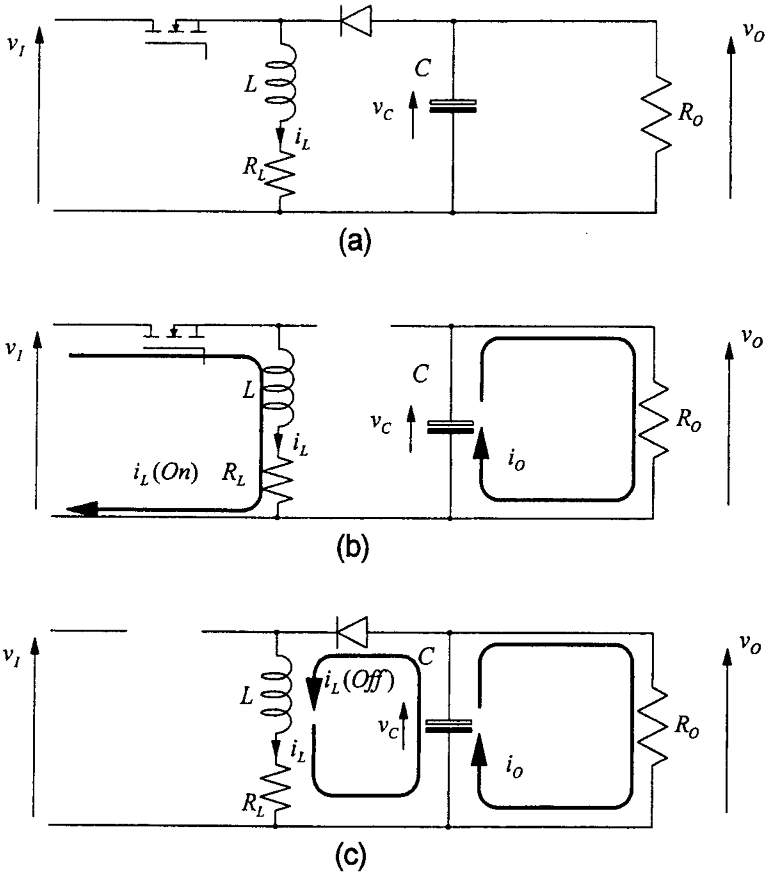


Figure 1.1

2.

(a) Explain how field orientation control of an induction motor achieves a linear relationship between torque and current and explain how a fast response of torque is achieved. [5]

(b) Sketch block diagrams of both direct and indirect field orientation control schemes for an induction machine. Describe each of the blocks and give key equations. [10]

(c) Discuss why indirect field orientation is more common in practice than the direct method. [5]

3.

(a) The transform matrices in equations 3.1 and 3.2 are used to transform three-phase variables to the *αβγ* form and then to the *dqγ* form. Explain the properties of the matrices and the usefulness of the transformations obtained. [7]

$$[T]=\sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad \text{(Equation 3.1)}$$

$$[T_R]=\begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(Equation 3.2)}$$

(b) For the circuit in figure 3.1,

(i) write the circuit equations in matrix form [3]

(ii) transform the equations to *dqγ* form [5]

(iii) sketch the circuit of the transformed system [5]

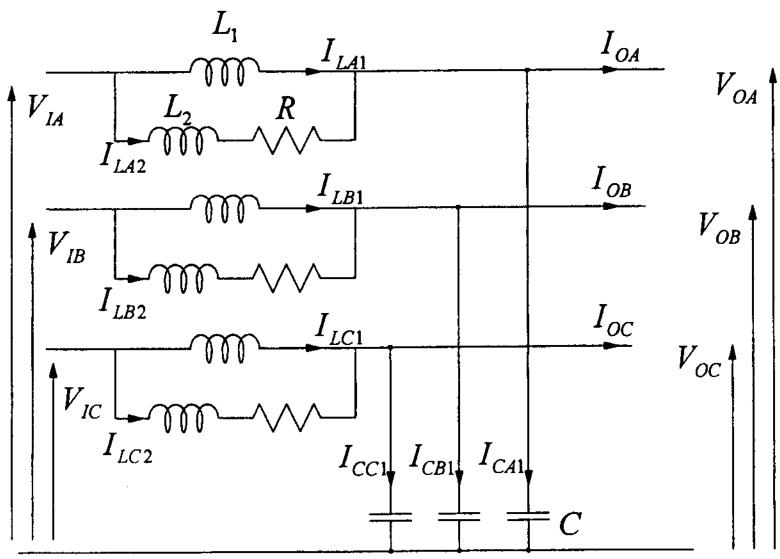


Figure 3.1

Section B

4.

a) What are the various components of an excitation system for an electrical generator? List different types of excitation systems in common use. [6]

b) Fig 4.1 is a block diagram of a simple single machine infinite bus system.

i) Identify the state variables and describe the physical meaning of the blocks. Write down the vector differential equations and express them in state-space form. Take $\Delta T_m, \Delta\omega$ as input and output respectively. [9]

ii) Find an expression for the eigen-values of the system. Discuss how D influences the poles of the system. [5]

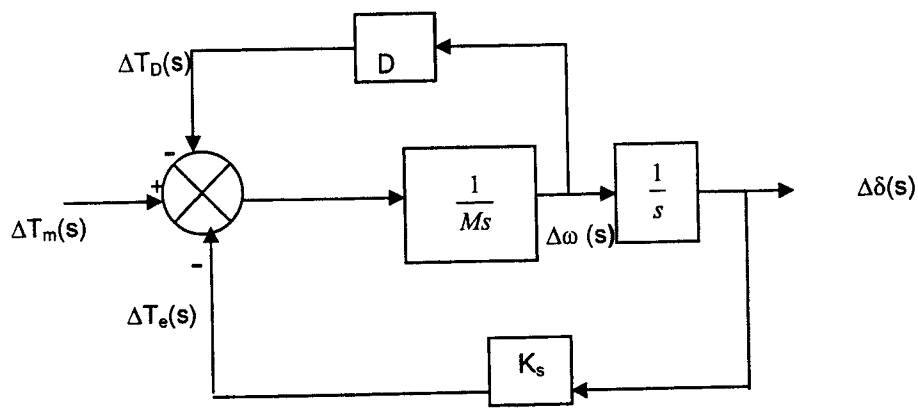


Figure 4.1

5.

a) Write short notes on any **four** from the following topics as they apply to a power system. [4x5=20]

i) Voltage stability
ii) The operating state of power system
iii) Synchronising torque
iv) Static models of loads
v) Static var compensator (SVC)
vi) Power system stabiliser (PSS)

6.

a) What is St. Clair's curve in the context of power transmission? Discuss various regions of the curve and explain the limits encountered in each region. How these limits can be raised? [10]

b) What is the purpose of having multiple stages in a modern steam turbine and how what difference does this make to the dynamic model of the turbine? [3]

c) Figure 6.1 is a block diagram of a simple turbine speed control system.

i) Find the value of R for which the speed control loop is stable. [4]

ii) What value of R corresponds to critically damping of the system? [3]

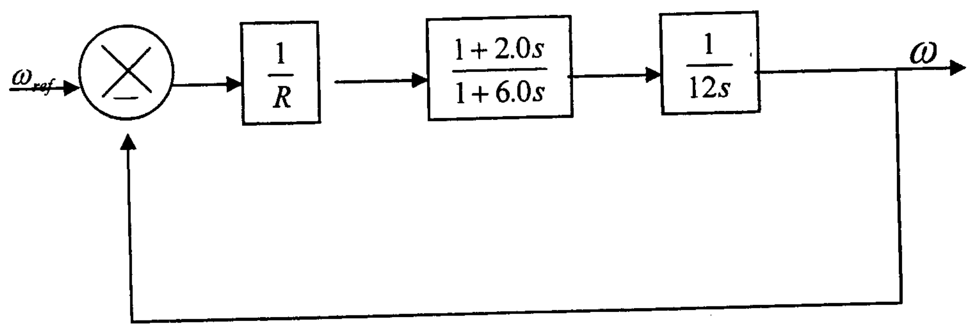


Figure 6.1

Model Answer Master - April 04

Modelling + Control in Power Engineering [4.38]

C7.7

1.

(a) Equation 1.1 is the general form of the state-space averaged model of a switch-mode power supply. Describe the basis on which state-space models of the on-state and off-state are combined to form the averaged model. Explain the terms $\tilde{x}, X, \tilde{\delta}, \Delta$ and U. [6]

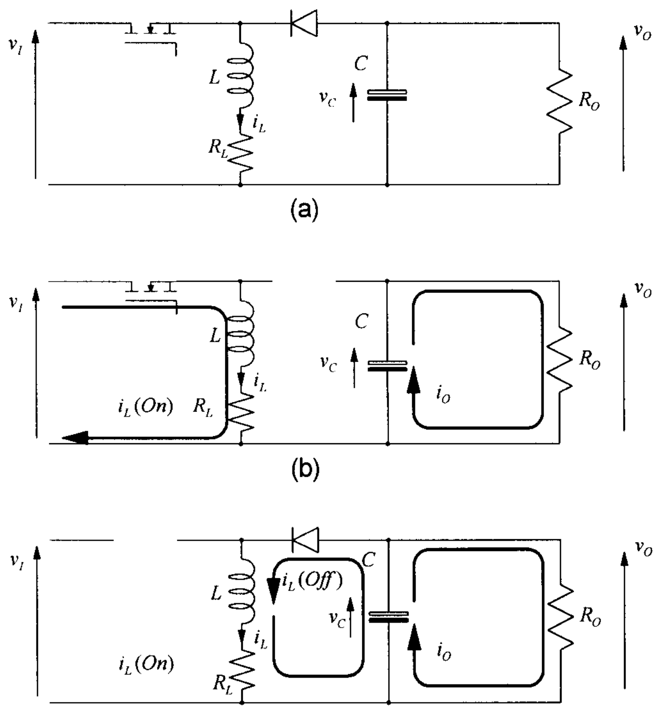
$$\dot{\tilde{\mathbf{x}}} \cong \left(\Delta \mathbf{A}_{\mathbf{On}} + (1-\Delta)\mathbf{A}_{\mathbf{Off}} \right) \tilde{\mathbf{x}} + \left(\left(\mathbf{A}_{\mathbf{On}} - \mathbf{A}_{\mathbf{Off}} \right)\mathbf{X} + \left(\mathbf{B}_{\mathbf{On}} - \mathbf{B}_{\mathbf{Off}} \right)\mathbf{U} \right)\tilde{\delta}$$

$$\tilde{\mathbf{y}} \cong \left(\Delta \mathbf{C}_{\mathbf{On}} + (1-\Delta)\mathbf{C}_{\mathbf{Off}} \right) \tilde{\mathbf{x}} + \left(\left(\mathbf{C}_{\mathbf{On}} - \mathbf{C}_{\mathbf{Off}} \right)\mathbf{X} + \left(\mathbf{D}_{\mathbf{On}} - \mathbf{D}_{\mathbf{Off}} \right)\mathbf{U} \right)\tilde{\delta}$$

Equation (1.1)

- *Switch-mode circuits are piece-wise linear*
- *State-space models of each state can be formed using the same state vector for each*
- *If the switching between states is at a high frequency compared with the rates of change of the state variable, then the change of state vector over a switching cycle can be approximated by the weighted sum of the changes due to the two states*
- *The resulting model contains products of the duty-cycle and the state vector and/or state matrices*
- *The model is then linearised for perturbations around an operating point.*
- *The terms are the perturbation and the operating point values of the state vector; the perturbation and the operating point values of the duty-cycle of the on-state and the operating point of the input vector.*

(b) Figure 1.1 shows the circuit of a buck-boost switch-mode power supply and the current paths that exist when the mosfet is on and when the mosfet is off. Find the state-space averaged model of this circuit [10]



(a)

(b)

(c)

On State

$$v_i = i_L R_L + L\frac{di_L}{dt}$$

$$\frac{dv_C}{dt} = \frac{-i_O}{C}$$

$$i_O = \frac{v_C}{R_O}$$

$$\mathbf{x} = \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$\mathbf{A_{On}} = \begin{bmatrix} -\frac{R_L}{L} & 0 \\ 0 & -\frac{1}{CR_O} \end{bmatrix} \quad \mathbf{B_{On}} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \quad \mathbf{C_{On}} = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \mathbf{D_{On}} = \begin{bmatrix} 0 \end{bmatrix}$$

Off state

[4.38]

$$v_C = L\frac{di_L}{dt} + i_L R_L$$

$$\frac{dv_C}{dt} = \frac{-i_O}{C} + \frac{-i_L}{C}$$

$$i_O = \frac{v_C}{R_O}$$

$$\mathbf{x} = \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$\mathbf{A}_{\mathbf{Off}} = \begin{bmatrix} -\frac{R_L}{L} & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{CR_O} \end{bmatrix}$$

$$\mathbf{B}_{\mathbf{Off}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \mathbf{C}_{\mathbf{Off}} = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \mathbf{D}_{\mathbf{Off}} = \begin{bmatrix} 0 \end{bmatrix}$$

After averaging

$$\begin{aligned}
\mathbf{A} &= \left(\Delta \mathbf{A}_{\mathbf{On}} + (1-\Delta)\mathbf{A}_{\mathbf{Off}}\right) \\
&= \begin{bmatrix} -\frac{\Delta R_L}{L} & 0 \\ 0 & -\frac{\Delta}{CR_O} \end{bmatrix} + \begin{bmatrix} -\frac{(1-\Delta)R_L}{L} & \frac{(1-\Delta)}{L} \\ -\frac{(1-\Delta)}{C} & -\frac{(1-\Delta)}{CR_O} \end{bmatrix} \\
&= \begin{bmatrix} -\frac{R_L}{L} & \frac{(1-\Delta)}{L} \\ -\frac{(1-\Delta)}{C} & -\frac{1}{CR_O} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathbf{C} &= \left(\Delta \mathbf{C}_{\mathbf{On}} + (1-\Delta)\mathbf{C}_{\mathbf{Off}}\right) \\
&= \begin{bmatrix} 0 & 1 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\widetilde{\mathbf{E}} &= \left(\mathbf{A}_{\mathbf{On}} - \mathbf{A}_{\mathbf{Off}}\right)\mathbf{X} + \left(\mathbf{B}_{\mathbf{On}} - \mathbf{B}_{\mathbf{Off}}\right)\mathbf{U} \\
&= \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \begin{bmatrix} V_I \end{bmatrix} \\
&= \begin{bmatrix} \frac{V_I - V_C}{L} \\ \frac{I_L}{C} \end{bmatrix}
\end{aligned}$$

[4.38]

$$\widetilde{\mathbf{F}} = \left(\mathbf{C}_{\mathbf{On}} - \mathbf{C}_{\mathbf{Off}}\right)\mathbf{X} + \left(\mathbf{D}_{\mathbf{On}} - \mathbf{D}_{\mathbf{Off}}\right)\mathbf{U}$$
$$= 0$$

(iii) Find the transfer function that relates the duty-cycle to the output voltage. [4]

$$\frac{v_O(s)}{\delta(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{E}$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s + \frac{R_L}{L} & -\frac{1-\Delta}{L} \\ \frac{1-\Delta}{C} & s + \frac{1}{CR_O} \end{bmatrix}^{-1} \begin{bmatrix} \frac{V_I - V_C}{L} \\ \frac{I_L}{C} \end{bmatrix}$$

$$= \frac{1}{s^2 + s\left(\frac{R_L}{L} + \frac{1}{CR_O}\right) + \frac{R_L}{LCR_O} + \frac{(1-\Delta)^2}{LC}} \left\{ \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s + \frac{1}{CR_O} & \frac{1-\Delta}{L} \\ -\frac{1-\Delta}{C} & s + \frac{R_L}{L} \end{bmatrix} \begin{bmatrix} \frac{V_I - V_C}{L} \\ \frac{I_L}{C} \end{bmatrix} \right\}$$

$$= \frac{LC}{LCs^2 + s\left(CR_L + \frac{L}{R_O}\right) + \frac{R_L}{R_O} + (1-\Delta)^2} \left\{ \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} n/a \\ -\frac{1-\Delta}{L}\frac{V_I - V_C}{L} + \frac{I_L}{C}\left(s + \frac{R_L}{L}\right) \end{bmatrix} \right\}$$

$$= \frac{-(1-\Delta)(V_I - V_C) + I_L(sL + R_L)}{LCs^2 + s\left(CR_L + \frac{L}{R_O}\right) + \frac{R_L}{R_O} + (1-\Delta)^2}$$

[4.38]

2.

(a) Explain how field orientation control of an induction motor achieves a linear relationship between torque and current and explain how a fast response of torque is achieved.

[5]

The torque equation of the machine, when expressed in the dq frame, in terms of flux-linkage and current is:

$$T_{EM} = P\left(i_{RD}\psi_{RQ} - i_{RQ}\psi_{RD}\right)$$

This product is not linear. To linearise, the model is aligned so that the d-axis is aligned to the rotor flux and thus the q-axis component of flux is set to zero. Thus the equation becomes:

$$T_{EM} = P\left(-i_{RQ}\psi_{RD}\right)$$

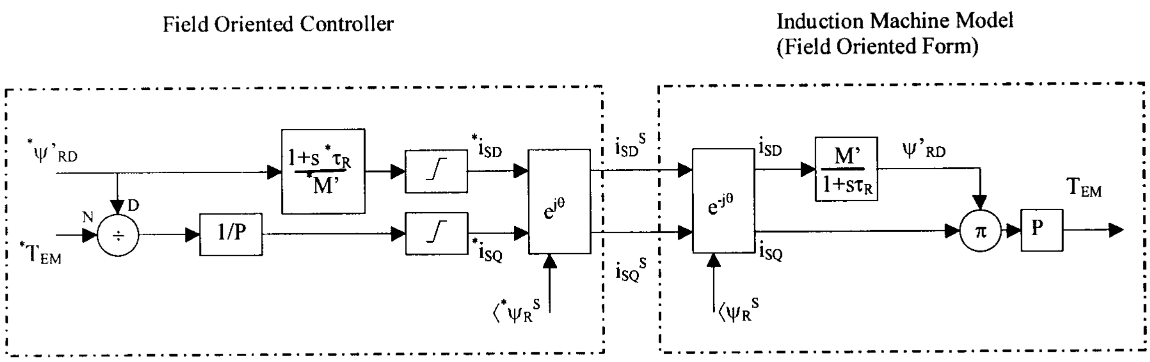
The flux is established in the d-axis circuit magnetising inductance and is thus a slow first order system. For this reason the flux is regulated to be constant. The q-axis current flows in a circuit with only the leakage inductance and so is a short time-constant first order circuit.

(b) Sketch block diagrams of both direct and indirect field orientation control schemes for an induction machine. Describe each of the blocks and give key equations.

[10]

In order to control the machine using the rotor flux reference frame model we need to determine the angular position of the reference frame so that forward and reverse transformations can be applied to converter variables between the control model dq form and physical abc form. There are two methods.

1. *Direct orientation: determine the rotor flux vector and calculate its angle.*



The estimate of the rotor flux is obtained by measuring the air-gap flux and then applying a correction of the rotor leakage.

$$\psi_{AG\alpha\beta} = M\left(i_{S\alpha\beta} + i_{R\alpha\beta}\right)$$

$$\psi_{R\alpha\beta} = M i_{S\alpha\beta} + L_R i_{R\alpha\beta}$$

$$= \psi_{SR\alpha\beta} + L_{lR} i_{R\alpha\beta}$$

Alternatively, the stator flux can be estimated from an integration of the stator voltage equation and then correction made for leakage.

[4.38]

$$v_{S\alpha\beta} = R_S\, i_{S\alpha\beta} + \frac{d\psi_{S\alpha\beta}}{dt}$$

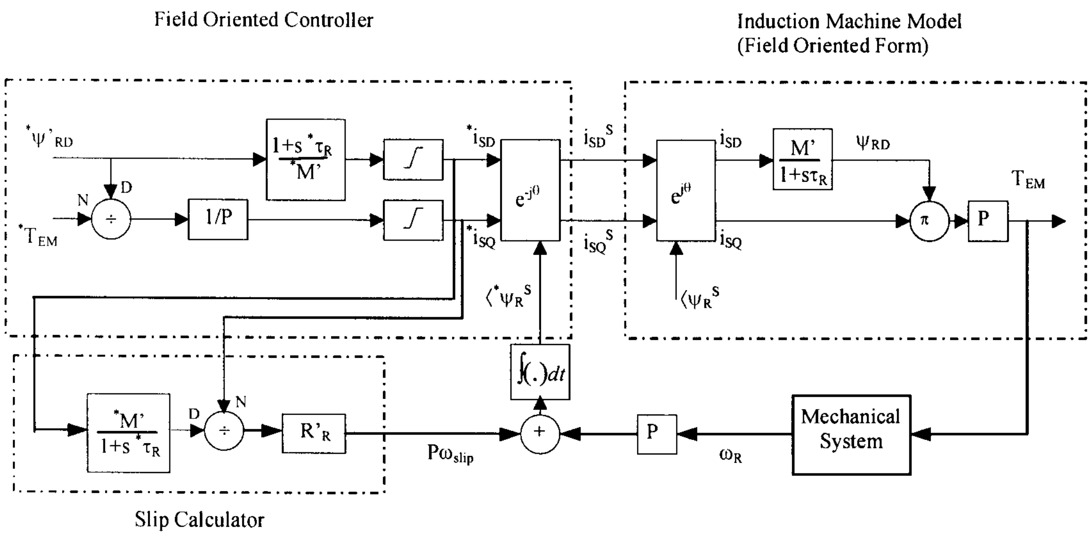
$$^{*}\psi_{S\alpha\beta} = \int_t \left(v_{S\alpha\beta} - {}^{*}R_S i_{S\alpha\beta}\right) dt$$

$$^{*}\psi'_{R\alpha\beta} = {}^{*}\psi_{S\alpha\beta} - {}^{*}L'_{lS}\, i_{S\alpha\beta}$$

$$^{*}\psi_{R\alpha\beta} = \frac{^{*}L_R}{^{*}M_{SR}}\, {}^{*}\psi'_{R\alpha\beta}$$

2. *Indirect orientation: determine the rotor position and the speed with which the rotor flux advances with respect to the rotor (the slip speed).*

A slip calculator is used to set the reference frame angle in feed-forward fashion.



Under each method, stator currents are imposed from either a current source inverter or a voltage source inverter with local current control. The flux magnitude is controlled using an estimate of the flux magnitude and there is q-axis current calculator to set the torque.

(c) Discuss why indirect field orientation is more common in practice than the direct method

[5]

Direct field orientation requires either measurement of air-gap flux or estimation of the flux from stator terminal measurements.
Fitting air-gap flux sensors results in a non-standard machine which has comparatively delicate sensors in a vulnerable part of the machine.
Estimating flux from the stator voltage equations requires the stator resistance to be known to a high degree of accuracy (particularly for low speed operation). Because this resistance is temperature dependent and temperature variation has to be expected, the estimation is not accurate and proper orientation of the control ot the flux is not achieved.

[4.38]

Indirect field orientation relies on instantaneous slip estimation and while this is dependent on rotor parameters it is found to be less sensitive to errors.

3.

(a) The transform matrices in equations Q3.1 and Q3.2 are used to transform three-phase variables to the *αβγ* form and then to the *dqγ*. Explain the properties of the matrices and the usefulness of the transformations obtained. [7]

$$[T]=\sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad \text{(Equation Q3.1)}$$

$$[T_R]=\begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(Equation Q3.2)}$$

T separates zero-sequence components into the γ-term leaving only positive and negative sequence components in the αβ-terms. Balanced three phase sets are transformed into an equivalent two-phase set in αβ. The transform is produces orthogonal terms in order that power can be separately calculated for each term and the transform is power invariant since $T^T = T^{-1}$

T_R does not transform the γ-term. A reverse rotation is applied to the the αβ-terms that transforms a positive sequence set to a station set in dq. The dq terms represent the amplitude and phase of the original three-phase set.

(b) For the circuit in figure 3.1,

(i) write the circuit equations in matrix form [3]

(ii) transform the equations to *dqγ* form [5]

(iii) sketch the circuit of the transformed system [5]

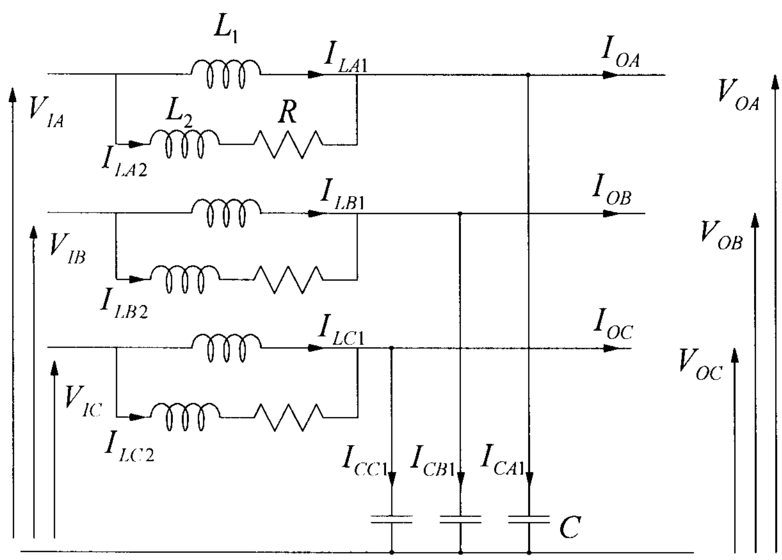


Figure Q3.1

$$v_{I\,abc} = L_1 \frac{di_{1abc}}{dt} + v_{Oabc}$$

$$v_{Iabc} = L_2 \frac{di_{2abc}}{dt} + R\,i_{2abc} + v_{Oabc}$$

$$i_{1abc} + i_{2abc} = C\frac{dv_{2abc}}{dt} + i_{Oabc}$$

$$vI_{dq\gamma} = L_1 \frac{di_{1dq\gamma}}{dt} + X_1\, i_{1dq\gamma} + v_{Odq\gamma}$$

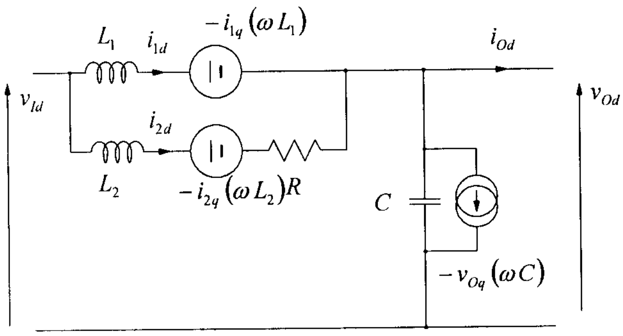
$$\text{where } X_1 = \begin{bmatrix} 0 & -\omega L_1 & 0 \\ \omega L_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

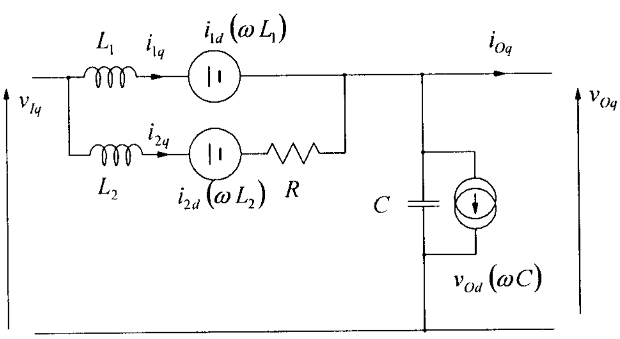
$$v_{Idq\gamma} = L_2 \frac{di_{2dq\gamma}}{dt} + X_2\, i_{2dq\gamma} + R\,i_{2dq\gamma} + v_{Odq\gamma}$$

$$\text{where } X_2 = \begin{bmatrix} 0 & -\omega L_2 & 0 \\ \omega L_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

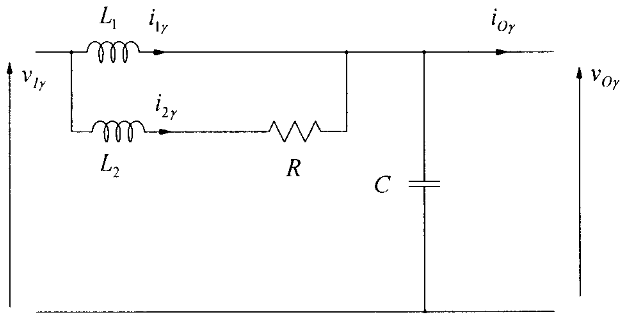
$$i_{1dq\gamma} + i_{2dq\gamma} = C\frac{dv_{2dq\gamma}}{dt} + B_C\, v_{2dq\gamma} + i_{Odq\gamma}$$

$$\text{where } \mathrm{B}_C = \begin{bmatrix} 0 & -\omega C & 0 \\ \omega C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$





[4.38]



4. (a) What are the various components of an excitation system for an electrical generator? List different types of excitation systems in common use. [6]

(b) Fig 4.1 below is block diagram of simple single machine infinite bus system.

(i) Identify the state variables and describe the physical meaning of the blocks. Write down the vector differential equations and express them in state space form.. Take $\Delta T_m, \Delta\omega$ as input and output respectively.

(ii) Find an expression for the eigen-values of the system. Discuss how D influences the poles of the system [14]

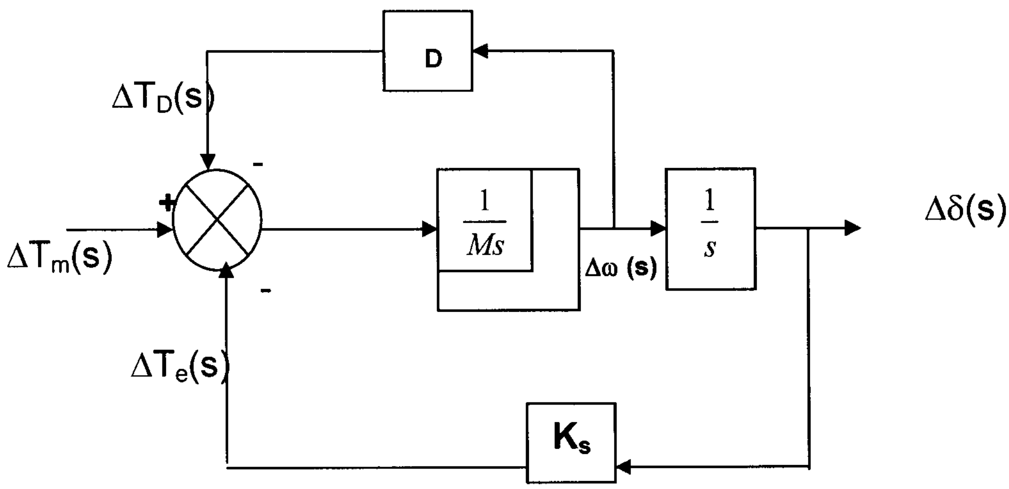


Fig 4.1

Answer 4 (a.). The role of an excitation system is to provide DC field flux in the rotor for generation of voltage. The basic component is the exciter and this could be a DC or AC alternator with a brush and slip-ring arrangement or solid-state bridge circuit that rectifies AC voltage fed through transformer to DC voltage. The rating is decided by the rating of the synchronous generator. Generation voltage is kept constant at different loads. This needs a voltage regulator. This is electronic amplifies the error voltage (i.e. the difference between desired and actual generator output voltage. The rotor and stator have various currents and power limits. These are realised through limiters and comparator circuits that takes the generator output current and rotor output current as inputs. At times an excitation system needs a stabilisation circuit for better voltage response known as excitation system stabiliser (ESS). Sensors are necessary to sense and feed currents and voltage of stator.

Depending on the nature of deriving exciter power, the excitation system is broadly categorised as:

DC (uses DC generator coupled with main alternator shaft)

AC (uses AC alternator mounted on the shaft)

Static or ST type (uses a voltage source fed bridge rectifier system feeding through slip ring. [6 marks]

(b) The state variables are $\Delta\delta, \Delta\omega$. The blocks containing M and D respectively represent inertia constant and damping (synchronous) of the turbine-generator. The block containing K_s is synchronising power co-efficient of machine around an operating condition defined by machine angle δ_0.

The differential equations are : [3 marks]

$$\frac{d\Delta\delta}{dt} = \Delta\omega$$

$$\frac{d\Delta\omega}{dt} = -\frac{K_s}{M}\Delta\delta - \frac{D}{M}\Delta\omega - \frac{\Delta T_m}{M}$$

The state-space model can be constructed as

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K_s}{M} & -\frac{D}{M} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} \Delta T_m$$

The output equation is very simple as $y = \Delta\omega$, it can be written as

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \Delta T_m$$

Hence this is in standard state-space form where

$$X = \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix}, u = \Delta T_m, y = \Delta\omega$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{K_s}{M} & -\frac{D}{M} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \end{bmatrix}$$

[6 marks]

The eigen values of *A* are obtained from the solution of the equation

$$\det\left[sI - A\right] = 0$$

Which in this case is the roots of $s^2 + \frac{D}{M}s + \frac{K_s}{M} = 0$

i.e. $-\frac{D}{2M} \pm \sqrt{\frac{D^2}{4M^2} - \frac{K_s}{M}}$

The influence of D on eigenvalues or poles of the system is as follows:

$D = 0$

$\lambda = \pm j\sqrt{\frac{K_s}{M}}$ poles are purely imaginary and in conjugate, this is the conditionally stable case.

As long as *D>0* poles are in the left half of eigen plane, the system is stable,
D<0 is an unstable case where poles are in the left right half and is known as negative damping situation.

$D = \pm 2\sqrt{KM}$ condition suggests that the system has real poles. [5 marks]

[4.38]

5.

a) Write short notes on any four from the following [4x5=20]

i) Voltage stability
ii) Operating state of power system
iii) Synchronising torque
iv) Static load
v) Static var compensator (SVC)
vi) Power system stabiliser (PSS)

Answers

(i) Voltage stability:

Voltage stability is the ability of power system to maintain steady acceptable voltage at all the busses in the system under normal operating conditions and after busses being subjected to disturbances. The system enters into a state of voltage instability when an increased demand in load or a change in the system condition causes progressive decrease in voltage. The main factor causing the voltage instability is the inability of the power system to meet the demand for reactive power and constant power type of load. Modern electronic load systems operate on constant power mode as a result with decrease in voltage current increases and voltage at the feeding points are lowered further. This leads to cascading effect. There have been few instances of power blackouts through voltage instability. Power blackout in 2003 in Eastern United States is a most recent example. [5 marks]

(ii) Operating states:

For the purpose of analysing power system security, it is helpful to classify the system operating conditions into five states:

- Normal
- Alert
- Emergency
- Extremis
- Restorative

These states are represented in figure and the ways in which the transition takes place from one state to another. In normal state, all the system variables are within normal *range* and no equipment is overloaded. The system operates in a secure manner with sufficient security margin. And with any contingency occurring in this state, the system can withstand it without violating any equality and inequality constraints.

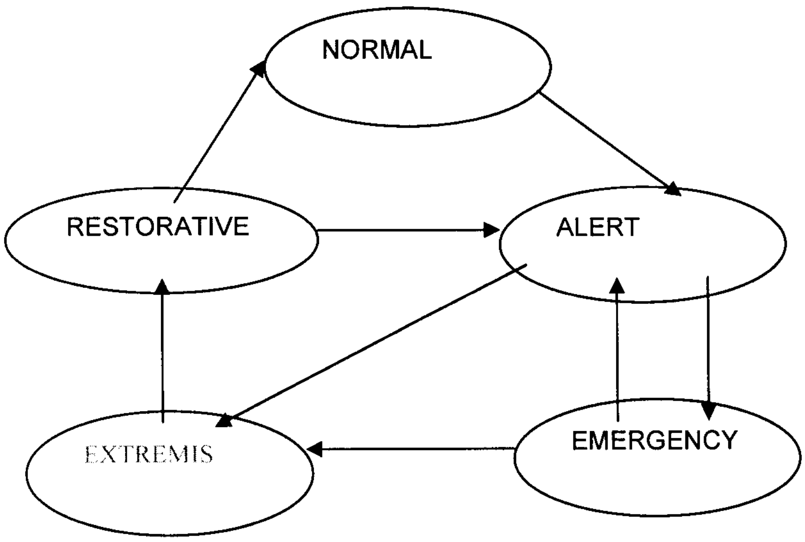


Fig 1.2 Operating States of Power System

If the security margin falls below certain level or if the possibility of disturbance increases because of adverse weather conditions such as approach of severe storms, all system variables are still with in acceptable range and all constraints are satisfied. However, the system has been weakened to a level where the occurrence of contingency can cause an overloading of equipment and system enters into the emergency state. In emergency state, equality constraints are met but inequality constraints are not met. If the disturbance is severe the system may enter into extremis where both equality and inequality constraints are violated.

Preventive action such as generation shifting or increase in reserve, can be taken to restore the system into the normal state. If the restorative steps do not succeed, the system remains in alert state.

In the emergency state, the voltages at many busses are low or equipment loadings exceed short-term emergency loading. The system is still intact and may be restored to alert state by initiating of emergency control actions like fault clearing, excitation control, fast-valving, generation tripping, generation run back, HVDC modulation and load curtailment.

If the above measures are ineffective, the system is in extremis. The result is cascading outage and possibly a shut down of major portion of the system. Control action such as load shedding and controlled system separation are employed to save the system from widespread blackout.

The restorative state represents a condition in which control action is being taken to reconnect all facilities and to restore the load. Here equality constraints are not met but inequality constraints are met. The system can transit to either alert state or normal state depending on the system conditions. [5 marks]

(iii) Synchronising torque:

In interconnected operation, a large number of synchronous machines operate in parallel. The rotor of each of the machines is locked with respect to the rest of the system and remains in equilibrium. The angular separation influences the amount of power transfer. A disturbance, such as a fault, disturbs this equilibrium and as a result the angles change. The machines on the other busses adjust their electrical torque because of the temporary change in load angle and try to restore equilibrium. This is known as synchronising torque that is very important in transient stability. The system can be represented in a transient stability study by non-linear differential equations.

$$\frac{d\delta}{dt} = \omega - \omega_s \text{..} (1.1)$$

$$M\frac{d^2}{dt^2}\delta = P_m - \frac{EV}{X}\sin\delta - D(\omega - \omega_s)\text{.............} (1.2)$$

The behaviour of synchronous machine for stable and unstable situations following large disturbance is shown in Figure 5.2

In case 1, the rotor angle increases to maximum and then decreases and oscillates with decreasing amplitude until it reaches a steady state. In case 2, the rotor angle continues to increase until synchronism is lost. This form of instability is called first swing instability and is due to insufficient synchronising torque. In case 3, system is stable during first swing but becomes unstable as a result of growing oscillation. This form of instability generally occurs when the post fault steady state condition itself is small signal unstable and not necessarily as a result of transient disturbance.

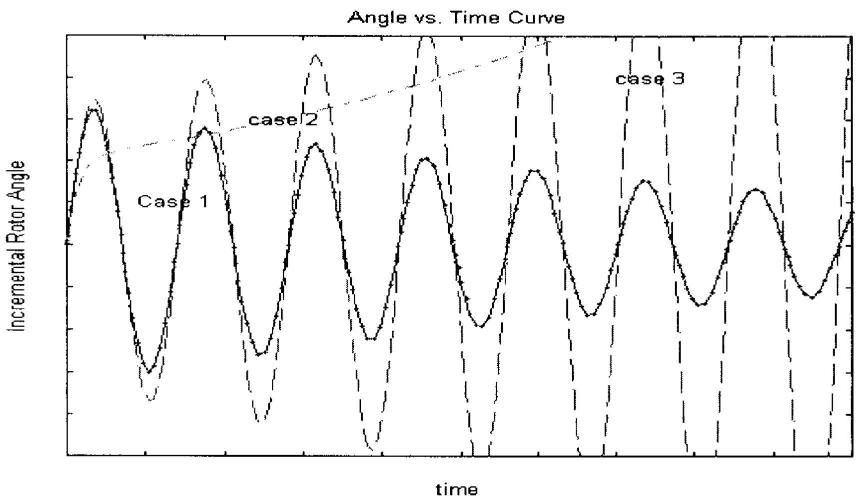


Fig. 5.2 Rotor Angle Vs. Time plots

Small disturbances occur in a system because of small amount of load change, small perturbation in various inputs such as AVR, governor and load reference etc. This leads to small signal stability problem. Power system is never under perfectly steady-state condition rather in quasi-steady state. For small disturbance, non-linear differential equations can be linearised around initial operating condition.

$$P_e = \frac{EV}{X} sin\delta + D(\omega - \omega_s)................(1.3)$$

$$T_e\omega_s = \frac{EV}{X} sin\delta + D(\omega - \omega_s).............(1.4)$$

$$T_e = \frac{1}{\omega_s}\frac{EV}{X} sin\delta + \frac{1}{\omega_s} D(\omega - \omega_s)........(1.5)$$

$$\Delta T_e = \frac{1}{\omega_S}\frac{EV}{X} cos\,\delta_0.\Delta\delta + \frac{D}{\omega_s}\Delta\omega..........(1.6)$$

$$\Delta T_e = K_s\Delta\delta + K_d\Delta\omega............................(1.7)$$

Here $K_s\Delta$ (missing delta?) is the component of torque change in phase with rotor angle perturbation $\Delta\delta$ and is referred to as the synchronising torque component.

K_s is the synchronising torque coefficient.

Lack of sufficient synchronising torque results in aperiodic instability through rotor angle variation.

(iv) Static models of loads:

In power system stability and power flow studies, the common practice is to represent the composite load characteristics as seen from bulk power delivery points. The load models are traditionally classified into two broad categories.

A static load model expresses the characteristics of the load at any instant of time as algebraic functions of the bus voltage magnitude and frequency at that instant. The active power component, *P,* and reactive power component, *Q,* are considered separately. Traditionally, the voltage dependency of the load characteristics has been represented by the exponential model .

$$P = P_0(\overline{V})^a$$

$$Q = Q_0(\overline{V})^b$$

where, $$\overline{V} = \left(\frac{V}{V_0}\right)$$

when P_0, Q_0 and V_0 are the values at the initial operating condition. The parameters of this model are the exponents *'a'* and *'b'*. With these exponents equal to 0,1,2, the model represents load of a constant power (CP), constant current (CC) or constant impedance (CI) type respectively. The exponent *'a'* (or *'b'*) is approximately equal to the slope dp/dv (or dq/dv) at $V = V_0$. For composite system loads, the exponent *'a'* usually lies in the range between 0.5 and 1.8. Exponent *'b'* varies as a non-linear function of the voltage. For Q at higher voltages, *'b'* tends to be significantly higher than *'a'*.

An alternative model that has been widely used to represent the voltage dependency of loads is the polynomial model.

[4.38]

$$P = P_0\left[p_1(\overline{V})^2 + p_2\overline{V} + p_3\right]$$
$$Q = Q_0\left[q_1(\overline{V})^2 + q_2\overline{V} + q_3\right]$$

(This *model is commonly referred to as the ZIP model as it is composed of constant impedance (Z), constant current (I) and constant power (P) components. The parameters of the model are the coefficient 'p1' to 'p3' and 'q1' to 'q3' that denote the proportion of each component. The frequency dependency of the load characteristic is usually represented as the exponential and polynomial model by a factor as follows:*

$$P = P_0(\overline{V})^a\left[1 + K_{pf}\Delta f\right]$$
$$Q = Q_0(\overline{V})^b\left[1 + K_{qf}\Delta f\right]$$
$$P = P_0\left[p_1(\overline{V})^2 + p_2\overline{V} + p_3\right]\left(1 + K_{pf}\Delta f\right)$$

$$Q = Q_0\left[q_1(\overline{V})^2 + q_2\overline{V} + q_3\right]\left(1 + K_{qf}\Delta f\right)$$

Typically, K_{pf}*, ranges from 0 to 3.0 and* K_{qf} *ranges from -2.0 to 0.0.*

A comprehensive static model that offers the flexibility of accommodating several forms of load representation is as follows:

$$P = P_0\left[P_{ZIP} + P_{EX1} + P_{EX2}\right]$$
$$Q = Q_0\left[Q_{ZIP} + Q_{EX1} + Q_{EX2}\right]$$

when,

$$P_{ZIP} = p_1\overline{V}^2 + p_2\overline{V} + p_3$$
$$P_{EX1} = p_4(\overline{V})^{a1}\left[1 + K_{pf1}\Delta f\right]$$
$$P_{EX2} = p_5(\overline{V})^{a2}\left[1 + K_{pf2}\Delta f\right]$$

A similar expression for Q_{ZIP} Q_{EX1} and Q_{EX2} can be written. [5 marks]

(v) Static VAR Compensator (SVC) :

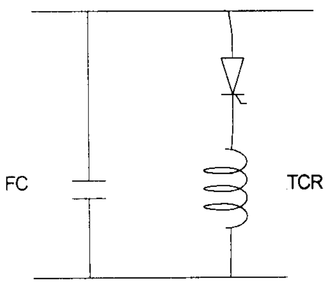
This is a thyristor based controllable devices which appear across transmission line to provide continuously controllable reactive power. As a result the voltage profile of the system is improved and transmission system loss is minimised. The fast and dynamic voltage support feature makes it an important device for steady state voltage and oscillation damping control.

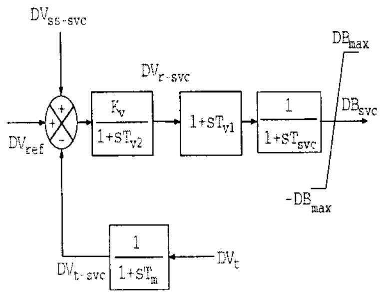
A typical topology of a Static Var Compensator (SVC) shown in Fig 7.8 comprises of a parallel combination of a thyristor controlled reactor and a fixed capacitor. It is basically a shunt connected static var generator/absorber whose output is adjusted to exchange capacitive or inductive current so as to maintain or control specific parameters of the electrical power system, typically bus voltage.

The reactive power injection of a SVC connected to bus k is given by

$$Q_k = V_k^2 B_{svc}$$

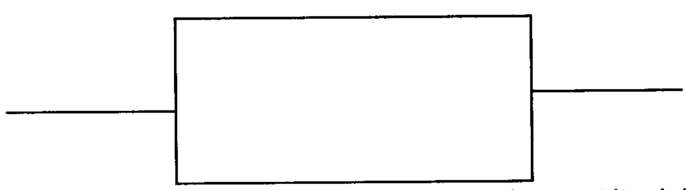
where Bsvc = $B_C - B_L$ and B_C and B_L are the susceptance of the fixed capacitor and thyristor controlled reactor respectively. It is also important to note that SVc does not exchange real power with the system. The small-signal dynamic model of a SVC is given in Fig. 7.9 where Tsvc is the response time of the thyristors, Tm is the time constant representing the delay in measurement and Tv1 and Tv2 are the time constants of the voltage regulator block. ΔBsvc is given ΔB_C-ΔB_L. The differential equations from this block diagram can easily be derived.





(vi) Power system stabiliser (PSS):

Power system stabiliser is an important component practically in all modern synchronous machine control system. It is known that modern excitation systems are usually equipped with high gain fast acting (static) voltage regulator. This is deliberately done to improve transient stability performance of the system i.e. introducing large synchronising torque through fast field forcing. Whilst this improves first swing stability performance, the system often loses stability in the subsequent swings through inadequate damping torque. Moreover the inherent damping torque of the system is reduced by the AVR action in the low frequency range (0.2 to 2 Hz). This is taken care of by a supplementary action that modulates excitation system reference voltage. This control is known as a power system stabiliser (PSS). Normally generator speed deviation or output power deviation or a combination of both are taken as the feedback signal. This signal is passed through a washout filter to ensure inaction of PSS during steady changes of system operating condition. The output of washout filter is connected to phase compensator circuits having adequate gain. The output of compensator is fed to voltage reference junction of excitation system. The amount of phase compensation is calculated to counteract phase lag encountered between generator electrical torque and reference voltage. The objective is to inject a component in electrical torque in phase with speed deviation i.e damping action. The gain is decided based on the overall closed-loop damping requirement. The generic structure of a PSS is of the form



Normally 'n' is 2 but depending on phase compensation requirement it might vary. [5 marks]

[4.38]

6.

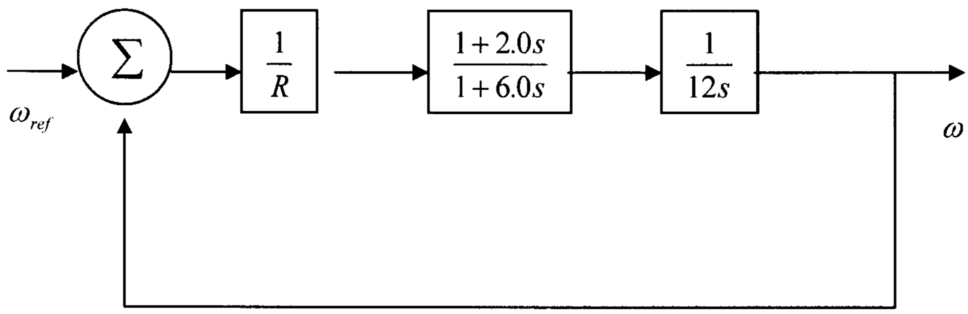
a) What is St. Clair's curve in the context of power transmission? Discuss various regions of the curve and explain the limits encountered in each region. How these limits can be raised? [10]

b) What is the purpose of having multiple stages in a modern steam turbine? [3]

c) Figure 6.1 below is block diagram of a simple turbine speed control system.

i) Find the value of R for which the speed control loop is stable. [4]

ii) What value of R corresponds to critically damping of the system? [3]



Answer (a)

Back in 1950s, H.P St Clair, expressed power transfer capability of transmission line as percentage of surge impedance loading versus line length based on design and practical experience. This curve, shown in Fig 1, is universal and quite useful for transmission planning as well as for operation and is most commonly known as *St. Clair Curve* Three factors limit the loadability of a line:

- Thermal (up to 50 miles or 80 km)
- Voltage drop (dielectric) (between 50 – 200 miles, (80-320 km))
- Stability (beyond 320 km)

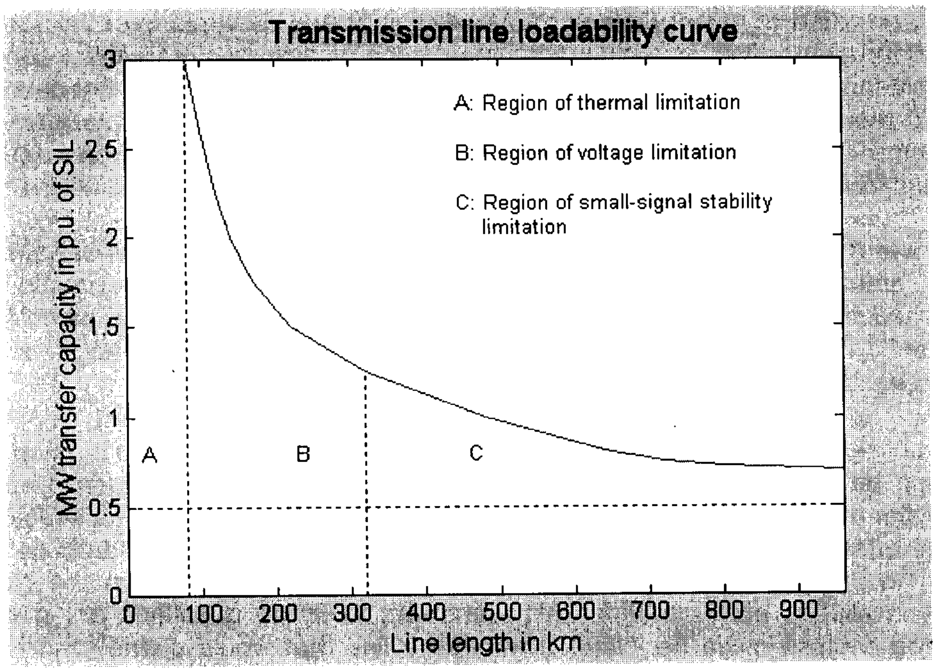


Fig 1: Transmission Line loadability curve

Thermal
The heat produced by current flow in a transmission system has two undesirable effects:

- Annealing and gradual loss of mechanical strength of the aluminium conductor caused by continued exposure to temperature extremes
- Increased sag and decreased clearance to ground due to conductor expansion at higher temperatures

The second of the above two effects is generally the limiting factor in setting the maximum permissible operating temperature. At this limit, the resulting line sag approaches the statutory ground clearance. The maximum allowable conductor temperatures based on annealing consideration is 127 degree Celsius for aluminium conductor. The allowable maximum current (i.e the ampacity) is dependent on ambient temperature, wind velocity. The thermal time constant is of the order of 10 to 20 minutes. Therefore the distinction is always made between short time rating during contingency and normal continuous rating. The nominal rating is generally decided on a conservative basis with the worst case operating scenario considered. The worst case scenario might not occurs in years, means a considerable amount of capacity is not utilised for most of the operating life of the line. Although some utility assign winter and summer loading, still the line remains unutilised for most of the time. It is important to assess the loadability of a line based on ambient conditions and loading history by off line computer program. Given the current state of the art in GPS and fibre optic technology for data handling and communication, online assessment of thermal loading limit on a day to day or even in an hour to hour basis is possible. This information certainly would optimise the usage of the thermal capacity.

Voltage (dielectric): Insulation is the most important factor in EHV/UHV transmission. The lines are designed with conservative margin. A line can be subjected to 10% extra voltage above nominal one, provided transient (lightning and switching) and dynamic over voltage conditions are managed through proper protection equipment such as arresters or thyristor controlled over voltage suppressors. Most of the time an overloaded line (beyond surge impedance) is under compensated resulting in poor voltage. The poor voltage conditions leads to many problems, increased current flow and hence losses. The FACTS technology could be used to ensure improved voltage profile or even acceptable over-voltage and power flow conditions.

Stability: There are number of stability issues that restricts the limit on transfer capability. They are transient, small signal (steady state), voltage stability etc. From the power angle relationship it is established that the maximum power can flow between two systems when the angular separation is 90 degree. With a 30% margin, the maximum permissible angle is 44 degree. This limit can be raised with the introduction FACTS.

It is now established that excessive voltage drop in the line reactance, drop in midpoint voltage and restriction on relative angular separation between two ends of the line do not allow the system operator to utilise the full power transfer capacity of the system. This situation leads to very relevant questions: Is there any way to cease this midpoint voltage drop as power flow across the line increases? Can the effective angular separation between the two ends be ceased with increased power transfer? The answers to these questions are generally yes. Possibly some devices could be installed within the line that would change the transmission characteristic of the line or in other words push the limits upwards creating stronger corridor for power flow. These devices are used in the system for long and were known as compensation devices until recently they were integrated to broader category called FACTS devices.
Let us consider the simplified power angle characteristic in equation of the simplified two-machine system.

$$P = \frac{V^2}{X} \sin \delta \qquad \text{.(2.3)}$$

$$Q_s = -Q_r = \frac{2V^2}{X}(1 - \cos \delta) \qquad \text{(2.4)}$$

The maximum real power $\frac{V^2}{X}$ that can be transferred over a lossless line at a given transmission voltage is $\frac{V^2}{X}$. This is totally determined by the line reactance X and thus sets the theoretical limits for steady state power transmission. This limit is not realistic because line voltage drop, midpoint voltage drop and angular separation limitations as discussed in the previous section. Let us assume a voltage source of magnitude V is installed in the middle of the line. The power angle equations are now modified to

$$P_p = 2\frac{V^2}{X} \sin(\delta / 2) \qquad (2.5)$$

$$Q_p = 4\frac{V^2}{X}(1 - \cos \delta) \qquad (2.6)$$

It is very interesting to see that the maximum power transfer capacity is now doubled but at the cost of rapidly increasing demand for reactive power on the mid-point compensator and end generators.
This concept of mid-point compensation can be seen as segmentation of line. Fig 2.6 shows the case with four line segments. Theoretically, the transmittable power would double with each doubling of the segments for the same overall line length. With increase in segmentation, the line voltage drop would also decrease rapidly, approaching the ideal case of constant voltage profile. Ultimately, with a sufficiently large number of line segments, an ideal distributed

compensation system could theoretically be established, which would have the characteristics of conventional surge impedance loading, but would have no power transmission limitations, and would maintain flat voltage profile at any load.
It will be appreciated that such a distributed compensation hinges on the instantaneous response and unlimited var generation and absorption capability of the shunt compensators employed, which would have to stay in synchronism with the prevailing phase of the segment voltages and maintain the pre-defined amplitude of the transmission voltage, independent of load variation.

(b) The steam power plant operates on Rankine cycle. The cyclic efficiency is proportional to the difference in temperature between inlet steam and exhaust steam. In a single stage turbine much of the energy is lost as discharge temperature is too high. This leads to lower cyclic efficiency. In multistage turbine the discharge from high pressure stage is let through intermediate (IP) stage and further through low pressure (LP) stage. This improves cyclic efficiency. Typically in three-stage turbine about 60 – 70% of total power is developed in intermediate and low pressure stage. The volume of steam expands in these stages. This makes the turbine size larger especially in low pressure stage. For dynamic modelling purpose each stage is modelled as a separate mass. **[3 marks]**

(c) The characteristic equation of the $(1+GH=0)$ of the closed-loop system is
$1+\left(\frac{1+2s}{1+6s}\right)\left(\frac{1}{12s}\right)\left(\frac{1}{R}\right)=0$ This simplifies to $s^2+\left(\frac{12R+2}{72R}\right)s+\left(\frac{1}{72R}\right)=0$
This is in standard form $s^2+2\zeta\omega_n s+\omega_n{}^2=0$

(i) For stability the following condition must be satisfied
$\frac{1}{72R}>0$ and $\frac{12R+2}{72R}>0$ or $R>-\frac{1}{6}$. Therefore any positive value of R will result in a stable response. **[4 marks]**

(ii) For critical damping $2\zeta\omega_n=2*1.0*\frac{1}{\sqrt{72R}}=\frac{12R+2}{72R}$. This simplifies to

$R^2-1.667R+0.0278=0$. Solving for *R* we have *R*= 0.017 or 1.65. For practical reason lower of the above two values are important. Thus critical damping can be obtained with *R* = 1.7% **[3 marks]**