

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2014

### EEE PART I: MEng, BEng and ACGI

**Corrected Copy**

## SEMICONDUCTOR DEVICES

Wednesday, 4 June 10:00 am

**Time allowed: 2:00 hours**

**There are THREE questions on this paper.**

**Answer ALL questions.**

Question One carries 40% of the marks. Questions Two and Three each carry 30%.

**Any special instructions for invigilators and information for candidates are on page 1.**

**Examiners responsible**

<b>First Marker(s) :</b>	K. Fobelets
<b>Second Marker(s) :</b>	S. Lucyszyn



### Constants

permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$
permeability of free space:	$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10} \text{ cm}^{-3} \text{ at } T = 300\text{K}$
dielectric constant of Si:	$\epsilon_{Si} = 11$
dielectric constant of SiO <sub>2</sub> :	$\epsilon_{ox} = 4$
thermal voltage:	$V_T = kT/e = 0.026\text{V at } T = 300\text{K}$
charge of an electron:	$e = 1.6 \times 10^{-19} \text{ C}$
Planck's constant:	$h = 6.63 \times 10^{-34} \text{ Js}$
Bandgap Si:	$E_G = 1.12 \text{ eV at } T = 300\text{K}$
Effective density of states of Si:	$N_C = 3.2 \times 10^{19} \text{ cm}^{-3} \text{ at } T = 300\text{K}$ $N_V = 1.8 \times 10^{19} \text{ cm}^{-3} \text{ at } T = 300\text{K}$

## Formulae

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

Schrödinger's equation  
in one dimension

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_f}{kT}\right)}$$

Fermi distribution

$$n_i = \sqrt{N_V N_C} \exp\left(\frac{-E_G}{2kT}\right)$$

Intrinsic carrier concentration

$$n = N_C e^{\frac{(E_C - E_F)}{kT}}$$

Concentration of electrons

$$p = N_V e^{\frac{(E_V - E_F)}{kT}}$$

Concentration of holes

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon}$$

Poisson equation in 1  
dimension

$$\left. \begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p \frac{dp(x)}{dx} \end{aligned} \right\}$$

Drift and diffusion current  
densities in a semiconductor

$$I_{DS} = \frac{\mu C_{ox} W}{L} \left( (V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Current in a MOSFET

$$\left. \begin{aligned} J_n &= \frac{eD_n n_{p0}}{L_n} \left( e^{\frac{eV}{kT}} - 1 \right) \\ J_p &= \frac{eD_p p_{n0}}{L_p} \left( e^{\frac{eV}{kT}} - 1 \right) \end{aligned} \right\}$$

Current densities for a pn-  
junction with lengths  $L_n$  &  $L_p$

$$V_0 = \frac{kT}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT}\right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$

Minority carrier injection  
under bias  $V$

$$D = \frac{kT}{e} \mu$$

Einstein relation

$$w_n(V) = \left[ \frac{2\epsilon(V_{bi} - V)N_A}{e(N_A + N_D)N_D} \right]^{1/2} \quad \& \quad w_p(V) = \left[ \frac{2\epsilon(V_{bi} - V)N_D}{e(N_A + N_D)N_A} \right]^{1/2}$$

Depletion widths under bias  $V$

$$W_{depl}^{\max} = 2 \left[ \frac{\epsilon kT \ln\left(\frac{N_{\text{substrate}}}{n_i}\right)}{eN_{\text{substrate}}} \right]^{1/2}$$

Maximum depletion width

1.

- a) At room temperature, pure, undoped Si has a certain density of electrons and holes. Give the concentration of holes and electrons in this material. [2]
- b) If Si is doped with an acceptor (B) concentration of  $10^{17} \text{ cm}^{-3}$ , calculate the position of the Fermi Level,  $E_F$ , with respect to the conduction band in this material. [5]
- c) Sketch the energy band diagram ( $E_c$ ,  $E_v$ ,  $E_F$ ,  $E_b$ ,  $E_G$ ) of the material in 1b). [5]
- d) Calculate the resistance of the material in fig. 1.1. The material characteristics are the same as in question 1b). The electron mobility is  $300 \text{ cm}^2/\text{Vs}$  and the hole mobility is  $200 \text{ cm}^2/\text{Vs}$ . [5]

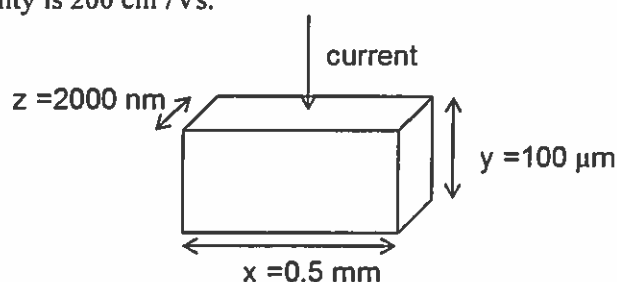


Figure 1.1: Silicon with material parameters from 1b). The current direction and dimensions are given.

- e) Fig. 1.2 gives a sketch of a Si pn junction under bias.

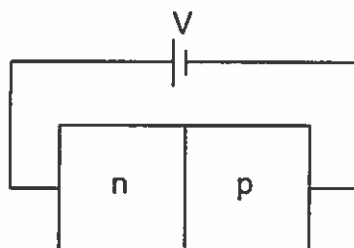


Figure 1.2: pn diode with biasing voltage. The relative difference between the donor and acceptor concentration is:  $N_D > N_A$ .

- i) Sketch the current voltage characteristics of the pn diode in fig. 1.2 for  $0 \text{ V} < |V| < 1 \text{ V}$ . [4]
- ii) Sketch the material cross section and include the depletion width in each region. Make sure the relative magnitudes of the depletion widths are correct. [4]
- iii) Add the variation of the minority carrier concentration to the sketch of 1e)ii). Ensure the relative magnitude of the plots is correct. Label the axis. [5]
- f) Give the relationship between  $V_{eE}$  and  $V_{eB}$  and the relationship between  $V_{eB}$  and  $V_{eC}$ , such that the BJT in fig. 1.3 is biased as indicated by the arrow. [4]

Question 1 continues on next page

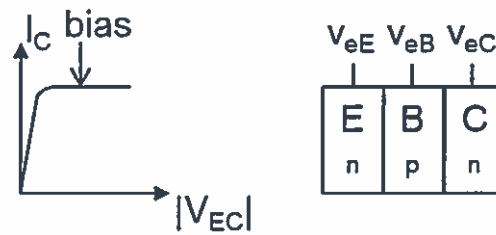


Figure 1.3: Left: the output characteristic of a npn BJT in common emitter mode. The arrow with the word “bias” indicates the point at which the BJT is operating. Right: the material cross section with the nodal voltages on each contact E, B, and C.

- g) Consider two npn BJTs with exactly the same dimensions and material parameters. The only difference is the doping density in the emitter:  $N_{E1} > N_{E2}$  with  $N_{Ei}$  the emitter doping of BJT<sub>*i*</sub> ( $i = 1, 2$ ). Under the same biasing conditions, will  $I_{C1}$  be  $>$ ,  $<$  or  $=$  than  $I_{C2}$ ? Verify your answer.

[6]

2.

- a) Consider a pn diode with homogeneously doped p and n regions. The doping concentrations used are  $N_A = 10^{16} \text{ cm}^{-3}$  and  $N_D = 10^{19} \text{ cm}^{-3}$ . The depletion regions (not shown) are  $w_n$  and  $w_p$ . The section lengths are  $X_n = X_p = 0.035 \text{ cm}$ .

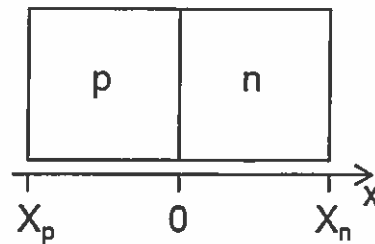


Figure 2.1: un-biased pn diode.

- i) At which position in  $x$  is the absolute value of the electric field maximum? [2]
  - ii) Write the total charge  $\rho(x)$  in the depletion region of the p-type material. You can use the depletion approximation and assume that the majority carrier concentration is equal to the doping concentration. [2]
  - iii) Using the Poisson equation, derive the expression of the electric field in the p-section depletion region as a function of doping concentrations. Depletion widths  $w_n$  and  $w_p$  should not appear in your final expression. Simplify the expression but do not fill in the values. [6]
- b) Assume that the doping concentration in the n-type region is not homogeneous but varies slowly, and is defined by the function:
- $$N(x) = 10^{17} - 5 \times 10^{18} x$$
- i) Sketch the variation of the concentrations of electrons and holes in this layer when no bias is applied. [4]
  - ii) Derive the expression for the internal electric field. Eliminate all material parameters so that the equation is only a function of constants and  $x$ . [4]
  - iii) Draw the direction of the electric field on the graph in 2b)i). [2]
- c) Using charge neutrality in the homogeneously doped p-type region and the law of mass action, show that:
- $$p \approx N_A$$
- if the acceptor doping density,  $N_A$ , is sufficiently larger than the intrinsic carrier concentration. [10]

3.

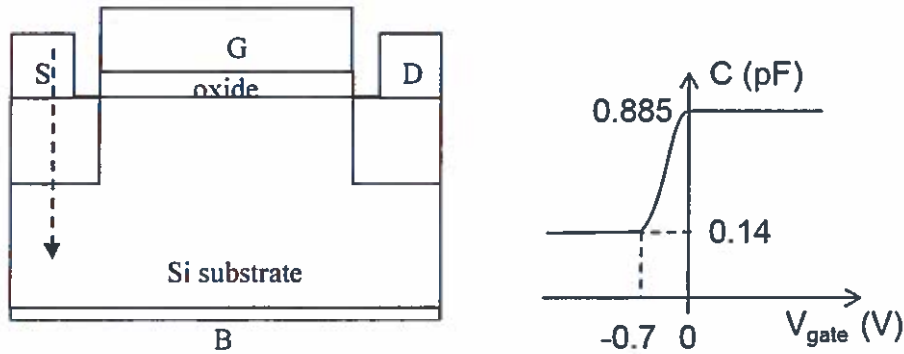


Figure 3.1: Left: material cross-section of an enhancement mode MOSFET. Right: its capacitance-voltage (C-V) characteristics.  $V_{gate}$  is the voltage applied between the gate and bulk with the bulk grounded. The symbols: G, S, D, denote gate, source and drain respectively.

The geometric parameters of the MOSFET are:

Gate width,  $W_G = 100 \mu\text{m}$

Gate length,  $L_G = 5 \mu\text{m}$

a)

- i) Is the MOSFET in fig. 3.1 a p-channel or n-channel MOSFET? [2]
- ii) Give the doping type in the source, drain and substrate regions. [2]
- iii) Sketch the energy band diagram ( $E_c$ ,  $E_v$ ,  $E_F$ ,  $E_G$ ) along the dashed line in fig. 3.1 ensuring that the specific characteristics of the contact are clear. [6]

b)

- i) Extract the thickness of the oxide from the C-V measurements. [2]
- ii) Calculate the doping density in the substrate if you know that at  $V_{GS} = 0$  V, flat band condition (no band bending between gate and substrate) is obtained. The work function of the metal is  $\phi_m = 4.259$  eV and the difference between the local vacuum level and the conduction band in the semiconductor is:  $E_{vac} - E_c = 4.050$  eV. [4]
- iii) Give the expression for the maximum depletion,  $W_{depl_{max}}$  width in function of the measured capacitance in fig. 3.1. Use symbols  $C_{min}$ ,  $C_{max}$ ,  $C_{depl_{max}}$ , etc... and do not fill in the values. [4]

c)

- i) Give the threshold voltage,  $V_{th}$  for the MOSFET in fig. 3.1. [2]
- ii) Estimate the mobility of the carriers in the channel from the data in fig. 3.2, assuming that the doping density in the substrate is  $10^{16} \text{ cm}^{-3}$ . [2]
- iii) Sketch the output characteristic of the MOSFET for  $V_{GS} = -1$  V and  $0 \text{ V} < |V_{DS}| < 1$  V. Indicate the voltage and current at saturation. [6]

Question 3 continues on next page



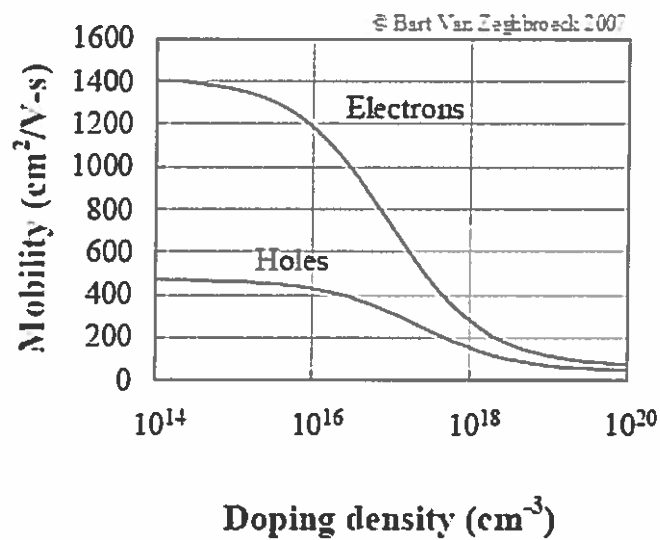


Figure 3.2: The variation of the hole and electron mobility as a function of doping.

