EE4-47
EE9-CS6-1

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

MSc and EEE PART IV: MEng and ACGI

Corrected copy

MODELLING AND CONTROL OF MULTI-BODY MECHANICAL SYSTEMS

Friday, 29 April 10:00 am

Time allowed: 3:00 hours

Correction Q. 3
@ ≈ 11:45

There are FOUR questions on this paper.

Answer ALL questions

All questions carry equal marks.

This is an OPEN BOOK examination.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	S.A. Evangelou
	Second Marker(s) :	A. Astolfi

MODELLING AND CONTROL OF MULTIBODY MECHANICAL SYSTEMS

1. An inverted uniform right circular cone of radius R, height h, mass m and density ρ, is shown in Figure 1.1. The origin O of the Cartesian coordinate system with axes X, Y, and Z is at the vertex of the cone. The Z axis is the axis of symmetry of the cone. The volume of the cone is $\frac{1}{3}\pi R^2 h$.

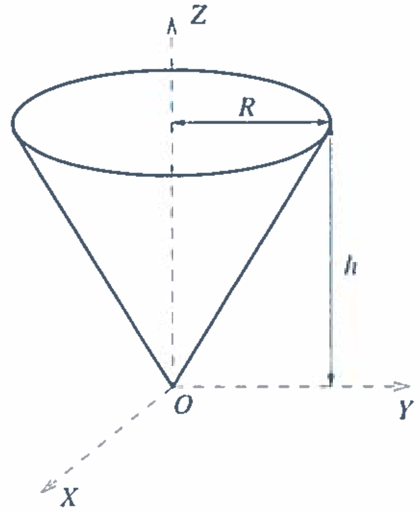

Figure 1.1 Right circular cone.

a) Write in cylindrical coordinates the infinitesimal volume element of the volume integration used to find the moment of inertia. [3]

b) By using cylindrical coordinates, compute the moment of inertia of the cone in terms of m, R and h about the following axes (Hint: one of the limits of the relevant volume integration is a function of one of the coordinates rather than a constant):

i) the axis of symmetry of the cone; [8]

ii) the axis X; [7]

iii) the axis Y. [2]

2. A particle of mass m is sliding without friction, under the influence of gravity, along a wire which is fixed in space in a vertical plane, as shown in Figure 2.1.

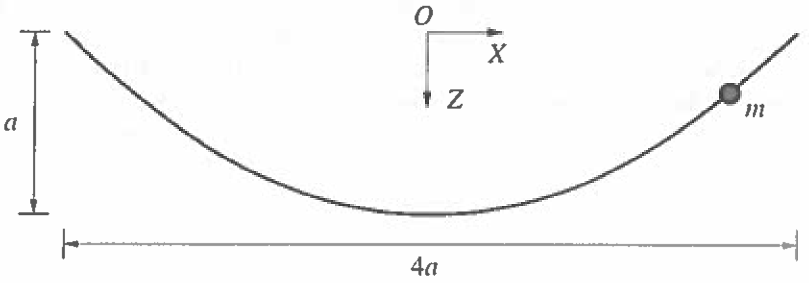

Figure 2.1 Mass particle on a wire.

The wire enforces the following constraint on the (x, z) coordinates of the mass

$$z = -\frac{x^2}{4a} + a,$$

given in terms of a Cartesian coordinate system with origin O and axes X, Z. This Cartesian coordinate system is used to analyse the motion of the mass.

a) Compute the total kinetic energy and potential energy of the mass, and hence determine the Lagrangian function. [5]

b) Use the Lagrangian approach to derive the equation of motion of the mass in terms of x and its derivatives. [8]

c) Calculate the magnitude of the total force that the wire is exerting on the mass. What can be concluded about the direction of this force? [4]

d) Determine the equation of motion of the mass when x is small and hence write the angular frequency of the oscillations of the motion. [3]

3. A beam rotates in a vertical plane about its centre at point O (fixed on Earth), as shown in Figure 3.1. A uniform ball of radius a ~~rolls~~ rotates on the beam under the influence of gravity and a viscous damping force at the contact between the ball and the beam. The viscous damping force is given by $-\mu v_{co}$, in which μ is the damping coefficient and v_{co} is the velocity of the instantaneous contact point between the ball and the beam. The ball is always touching the beam. The moment of inertia of the beam about its axis of rotation is I_{be}, and the mass and spin inertia of the ball is m and I_{ba}, respectively.

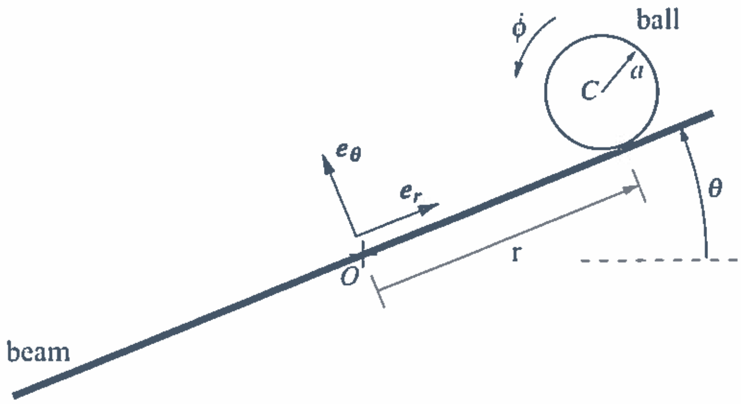

Figure 3.1 Ball and beam.

Polar unit vectors $\boldsymbol{e_r}$ and $\boldsymbol{e_\theta}$ are used to analyse the motion of the ball and beam. This coordinate system has a fixed origin at point O but it rotates with the beam by an angle θ. The ball rotates by an angle ϕ and the displacement from O of the ball contact point with the beam is r.

a) Write the number of degrees of freedom and the generalised coordinates of the system. [2]

b) Write the acceleration vector, $\ddot{r}_C$, of the centre of mass of the ball in the moving coordinate system. [3]

c) Determine the viscous damping force vector. [3]

d) Use the vectorial approach to derive the equations of motion of the ball and beam in terms of the generalised coordinates. [9]

e) Compute the force that keeps the ball always touching the beam. [3]

4. A '3D' double pendulum consists of a mass m_1 connected to a fixed axis via a massless rod of length l_1 and a second mass m_2 connected to the first mass via a massless rod of length l_2, as shown in Figure 4.1. The mass m_1 of the pendulum is free to swing in a vertical plane by an angle θ_1 about the fixed axis, under the influence of gravity and the interaction with the second mass of the pendulum. The mass m_2 is free to swing by an angle θ_2 in a plane which is perpendicular to the rod joining m_1 to the fixed axis, under the influence of gravity and the interaction with the first mass of the pendulum.

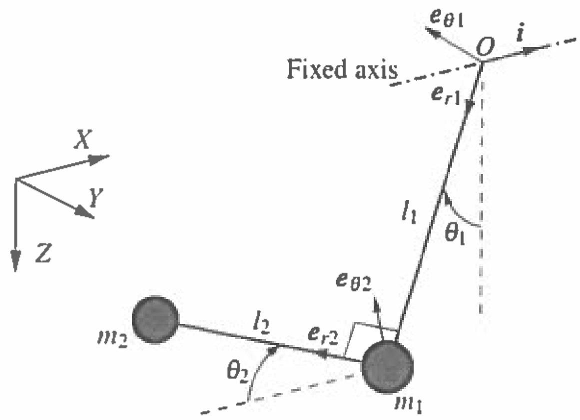

Figure 4.1 3D double pendulum.

Two moving Cartesian coordinate systems with unit vectors $\boldsymbol{e}_{r1}$, $\boldsymbol{e}_{\theta 1}$ and $\boldsymbol{i}$, and origin O for the first, and unit vectors $\boldsymbol{e}_{r2}$, $\boldsymbol{e}_{\theta 2}$ and $\boldsymbol{e}_{r1}$, and origin the position of the first mass m_1 for the second, are used to analyse the motion of the two masses of the pendulum. Note that $\boldsymbol{i}$ does not move and it is in the direction of the fixed axis.

a) Write

- i) the position vector, and [2]
- ii) the velocity vector [4]

of each of the masses.

b) Compute

- i) the kinetic energy, [3]
- ii) the potential energy, and [4]
- iii) the Lagrangian function [1]

of the system.

c) Use the Lagrangian approach to derive the equations of motion of the pendulum system. [6]