IMPERIAL COLLEGE LONDON

EE4-05
EE9-CS7-22
EE9-SO7
EE9-FPN2-01

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

MSc and EEE PART IV: MEng and ACGI

Corrected copy

TRAFFIC THEORY & QUEUEING SYSTEMS

Wednesday, 18 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : J.A. Barria
Second Marker(s) : D.P. Mandic

Special instructions for students

1. Erlang Loss formula recursive evaluation:

$$E_N(\rho) = \frac{\rho E_{N-1}(\rho)}{N + \rho E_{N-1}(\rho)}$$
$$E_0(\rho) = 1$$

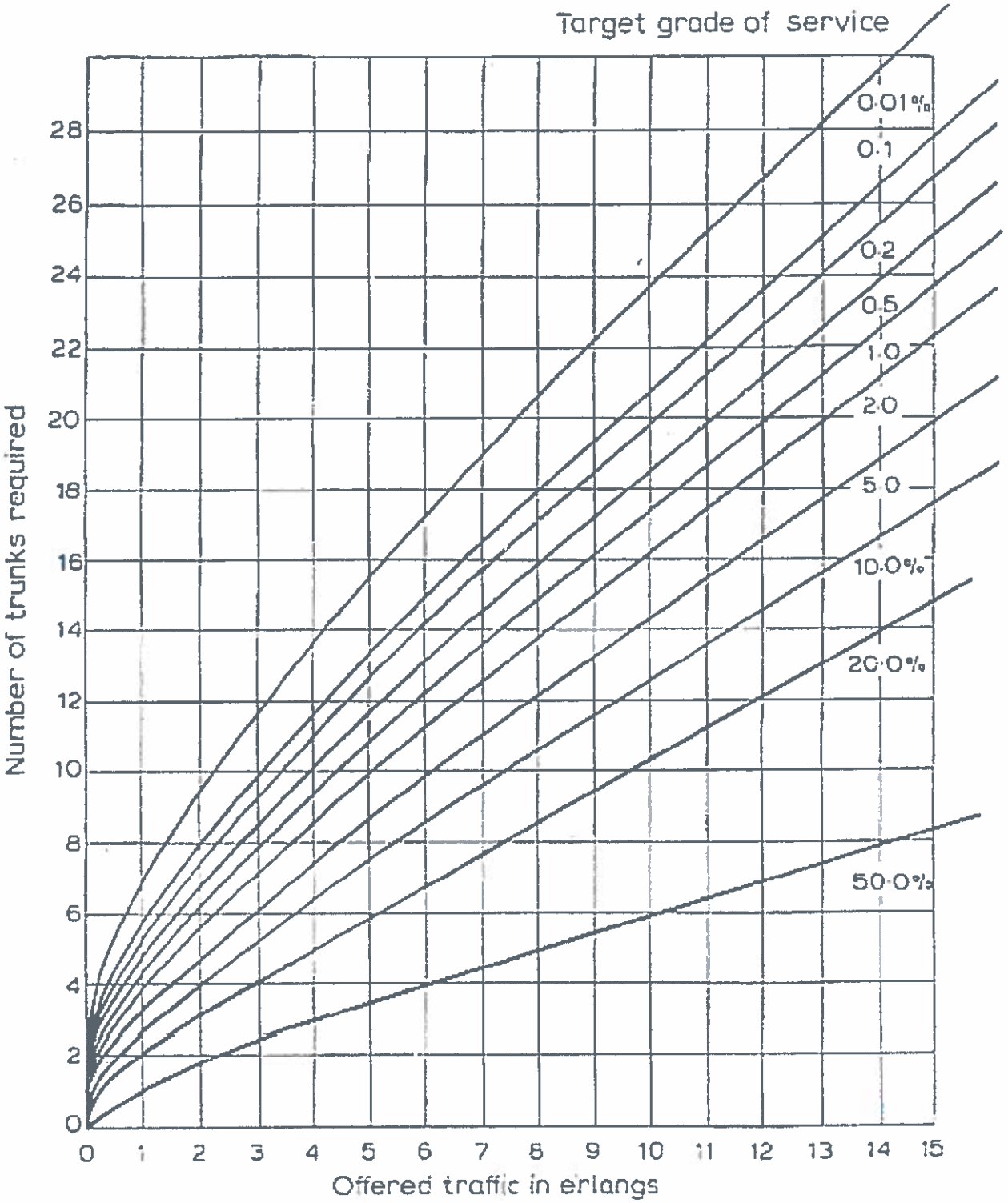
2. Engset Loss formula recursive evaluation (for a fixed M and $p = \alpha/1+\alpha$):

$$e_N = \frac{(M-N+1)\alpha e_{N-1}}{N + (M-N+1)\alpha e_{N-1}}$$
$$e_0 = 1$$
$$\alpha = \lambda/\mu$$

3. Traffic capacity on basis of Erlang B formula (next page).

4. Expected residual time

$$E[R] = \frac{1}{2}\lambda E[S^2] = \frac{1}{2}\sum_{k=1}^{m}\lambda_k E[S_k^2]$$



Traffic capacity on basis of Erlang B. formula.

The Questions

1.

a) For the M/M/K queueing system:

i) Derive the probability that a delayed arrival will have to wait for more than τ seconds before entering service. [6]

The M/M/K system is composed of twenty (20) servers, the rate of call arrival is 10 calls per minute and the average duration of a call is 1 minutes:

ii) Calculate the probability that a delayed arrival will have to wait for more than 30 seconds. [4]

b) ON-OFF source model and Engset model:

i) Define and describe the characteristics of the ON-OFF source model. [2]

ii) Using an ON-OFF source model: Derive the probability distribution of the Engset model and prove that it is binomial. [5]

iii) Clearly state the assumptions underlying the Engset model which make it different from the Erlang model. [3]

2.

a) Describe and discuss how you would use a Markov Modulated Poisson Process to model:

i) A traffic overflow model, [4]

ii) A single packet voice model, [3]

iii) A packet voice N-multiplexed model. [3]

b) The computer system shown in Fig. 2.1. is composed of three (3) statistically identical processors and two (2) statistically identical buses.

The system is operational if at least 1 processor and one bus are in working order. Otherwise is considered a failure. The system components failure rate are:

Processor failure rate = λ failures per year.

Bus failure rate = μ failures per year.

The coverage factor is a function of the number of k operational copies of the component: c(k λ) for processors, and c(k μ) for buses.

i) Define the state space of the system. Clearly identify the operational states. [4]

ii) Construct the Markov chain. [3]

iii) Derive and show the value of all transition rates. [3]

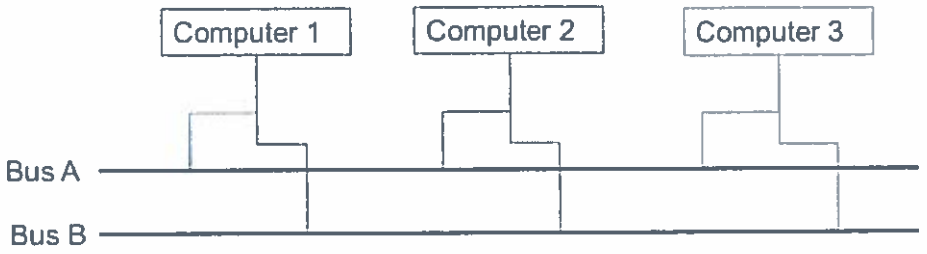


Figure 2.1.

3.

a) Priority queueing systems:

i) Define transit time in a pre-emptive resume priority queue system. [3]

ii) Derive an expression for the expected transit time in a pre-emptive resume priority system. [4]

iii) Explain why the transit time in a pre-emptive restart priority system will result in longer effective service time, and hence in longer transit time than in the transit time derived in part ii). [3]

b) In the context of call admission control and using Fig 3.1 as a reference:

i) Introduce and discuss the concept of equivalent capacity. [3]

ii) Discuss why peak bandwidth assignment and averaged bandwidth assignment would not be appropriate. Use Fig. 3.1. to argue your answer. [2]

iii) Using an ON-OFF source model, derive a simple expression of equivalent capacity. Clearly identify and define all parameters of the equivalent capacity model. [5]

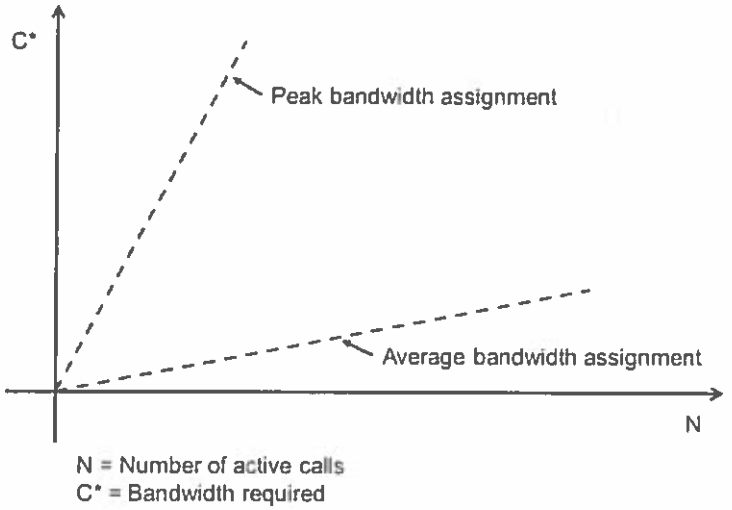


Figure 3.1.

4.

a) Construct a Markov chain model to represent the activity of an overflow link in which the blocked calls are only allowed to overflow to a single channel.

The mean sojourn time when the overflow link would expect new calls has been estimated as 6.9 minutes, and the mean sojourn time when the overflow link would not expect any incoming call has been estimated as being 0.141 minutes.

Also assume that the arrival rate to the first choice link is 3 demands/minute, and that the call holding time is 3 minutes.

i) Construct an IPP model of the overflow link:

Define the state space.

Calculate the transition rates. [5]

ii) State the balance equations of this model. Discuss whether you can solve this model using local balance equations or global balance equations. [5]

b) A call centre handles incoming enquires that can be placed in a waiting queue. The call centre has ten (10) active positions (operators).

Assume that the incoming traffic is 8 Erlangs of pure chance traffic and that the mean call duration of incoming enquires is 75 seconds.

i) Calculate the probability that an incoming call will be delayed. [6]

ii) Calculate the mean delay experienced by calls which are accepted but delayed. [4]