IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

EEE PART III/IV: MEng, BEng and ACGI

ELECTRICAL ENERGY SYSTEMS

Friday, 18 January 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer 2 questions from Section A and 2 questions from Section B. Use a separate answer book for each section.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : G. Strbac, B. Chaudhuri
Second Marker(s) : B. Chaudhuri, G. Strbac

Part A – Answer any 2 out of 3 questions in Part A

1)

a) For a transmission circuit given in Figure 1.1 and the corresponding phasor diagram shown below, show:

i) $\overline{V_s} = V_r + \left(\frac{RP_r + XQ_r}{Vr}\right) + j\left(\frac{XP_r - RQ_r}{Vr}\right)$ [2]

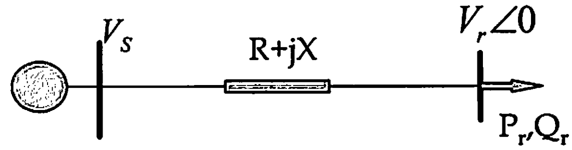


Figure 1.1 Transmission circuit with voltage and power specified at the receiving end

Assume that that the resistance can be neglected and that voltages (magnitudes and phase angles) are known at both sending and receiving ends.

i) Write down the expressions for sending end active and reactive powers as functions of voltage magnitudes and phase angles. [3]

ii) Based on these expressions, show that the active power flow requires a difference in phase angle and that the reactive power flow requires a difference in voltage magnitude. [2]

iv) What is the maximum amount of active power that can be transported via this transmission line? [2]

b) Consider a system supplied with 5 identical generators of 500MW capacity and availability of 0.9. The states in which this generation system can find itself are given in Table 1.1.

Table 1.1. System state probabilities

STATE	State Probability	Probability that the Generation is equal to or greater than the State
2,500 MW		
2,000 MW		
1,500 MW		
1,000 MW		
500 MW		
0 MW		

i) Calculate the state probabilities for this system and the probability that the generation will be greater than the given state. [4]

ii) If the system peak load is 1,800 MW, find the probability that generation will not be able to meet it. [2]

c)

(i) Describe the purpose and the effects of turbine governor control and excitation system control of a synchronous generator when it supplies isolated demand [3]

(ii) How these change when the generator is connected to a very large power system. [2]

2)

A generator is supplying a load over a 400 kV overhead line with a length of 300 km. The equivalent circuit of the line presented in Figure 2.1.

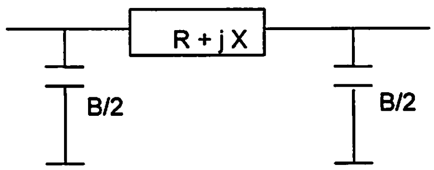


Figure 2.1: equivalent circuit of the transmission line

Transmission line parameters are given in Table below:

Series resistance r' (Ω/km)	Series reactance x' (Ω/km)	Shunt susceptance b' (μS/km)
0.02	0.25	5

Load varies between minimum value of 300 MW and maximum value of 1,200 MW.

Assuming that the voltage at the load end is kept at its nominal value of 400 kV, for minimum and maximum loading condition calculate:

i) The voltage at the sending end, [7]

ii) The active and reactive power generated and losses, [7]

iii) Discuss the differences in reactive power generated by the generator under minimum and maximum demand conditions. [6]

3)

a) Explain briefly why it is more difficult to transport reactive power than active power over high-voltage AC transmission systems. [3]

b) A three bus power network is presented in Figure 3.1. Data relevant for the load flow analysis on this system are given in per unit.

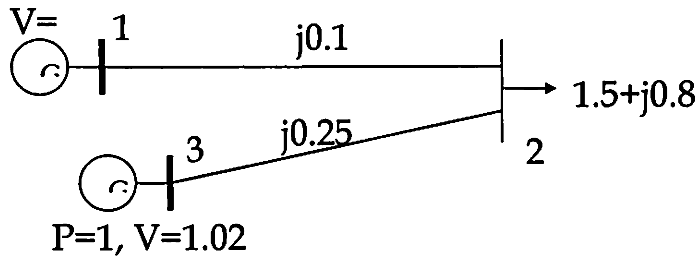


Figure 3.1 Three bus network showing per unit data for voltages, active and reactive load, and lines reactance

i) Form the Y_{bus} matrix for this system. [3]

ii) Perform two iterations of the Gauss-Seidel load flow. [14]

Part B – Answer any 2 out of 3 questions in part B

4. a) State three reasons to justify why short-circuit study is an important element in power systems analysis.

[5]

b) A segment of a power system comprising of four generators (G_1, G_2, G_3 and G_4) and two reactors (X_1 and X_2) is shown in Fig. 4.1. The MVA rating and reactance (in per unit of respective rating) of each component is shown on the figure. Calculate the fault MVA for a 3-phase short circuit at point A on the system. Neglect pre-fault current and assume 1.0 pu pre-fault voltage everywhere in the system. Choose 10 MVA as the common system base.

[8]

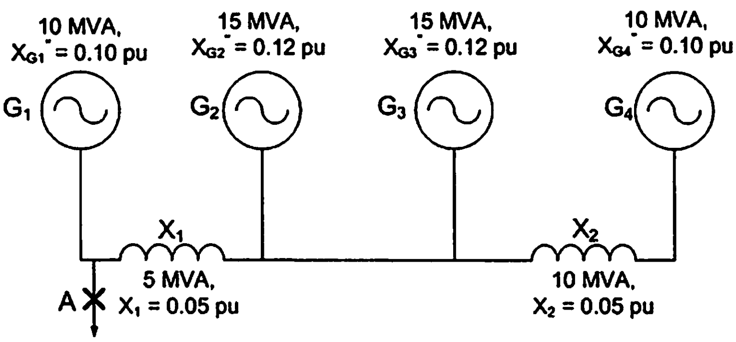


Fig 4.1: A segment of a power system comprising of four generators (G_1, G_2, G_3 and G_4) and two reactors (X_1 and X_2)

c) The source end of a power system comprising of two 11 kV generators (G_1 and G_2), one 33/11 kV transformer (T_1) and a current limiting reactor (X_1) is shown in Fig. 4.2. The MVA rating and reactance (in per unit of the respective rating) of each component is shown on the figure. Find the absolute value of the reactance of X_1 in ohms such that the short circuit level for a three-phase fault on the outgoing line at point F does not exceed 300 MVA. Neglect pre-fault current and assume 1.0 pu pre-fault voltage everywhere in the system. Choose 15 MVA as the common system base.

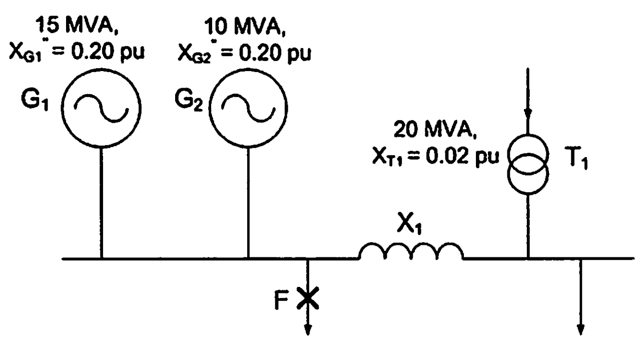


Fig 4.2: source end of a power system comprising of two generators (G_1 and G_2), one transformer (T_1) and a current limiting reactor (X_1)

[12]

5. a) Explain why the steady-state value of the positive sequence reactance of a synchronous machine is greater than the corresponding negative and zero sequence reactances.

[5]

b) A three-phase unbalanced delta connected load draws 100 A of line current from a balanced three-phase supply. Using the method of symmetrical components calculate the magnitude and phase angle of the sequence components of the currents in phases B and C if the current in phase A drops down to zero due to an open-circuit fault. Assume ABC phase sequence.

[8]

c) The positive, negative and zero sequence sub-transient reactance of a 5 MVA, 13.8 kV, 50 Hz, three-phase star connected synchronous generator is 0.1 pu, 0.1 pu and 0.01 pu, respectively. The neutral of the generator is grounded through a reactor. Calculate the minimum value of the inductance (in Henry) of the reactor which is necessary to limit the single line to ground (LG) fault level to less than the three-phase fault level at the terminal of the generator. Neglect pre-fault current and assume 1.0 pu pre-fault voltage. Neglect fault impedance.

[12]

6. a) Describe how different forms of stability constraints limit the power transmission capacity of lines for a range of transmission distances.

[5]

b) A round-rotor synchronous generator is operating at 25% of its maximum power capacity in steady-state. Under this condition the mechanical power input to the generator is suddenly doubled. Use equal area criteria to calculate the maximum excursion of rotor angle in degrees. Assume constant generator terminal voltage throughout and neglect resistances.

[10]

c) A round-rotor synchronous generator supplies its rated power to an infinite bus with the voltages at both ends maintained at 1.0 pu. The equivalent reactance of the transmission lines connecting the generator and the infinite bus is 0.3 pu under normal condition. Due to occurrence of a fault on the transmission corridor the power output of the synchronous generator drops down to zero. After the fault is cleared the equivalent reactance between the generator and the infinite bus becomes 0.5 pu. Calculate the critical clearing angle in degrees. Neglect resistances.

[10]

Part A – Answer any 2 out of 3 questions in Part A

Solution to Question 1

(a) For a transmission circuit given in Figure 1.1, we show:

$$\overline{S_r} = P_r + jQ_r = \overline{V_r} \cdot \overline{I^*}$$
$$\overline{I} = \frac{P_r - jQ_r}{\overline{V_r^*}}$$
$$\overline{V_s} = \overline{V_r} + (R + jX) \cdot \overline{I}$$

(i)
$$\overline{V_s} = \overline{V_r} + (R + jX) \cdot (\frac{P_r - jQ_r}{\overline{V_r^*}})$$ [2]

$$\overline{V_r} = \overline{V_r^*} = V_r \angle 0^0 = V_r$$
$$\overline{V_s} = V_r + (\frac{RP_r + XQ_r}{V_r}) + j(\frac{XP_r - RQ_r}{V_r})$$

For the network shown in Figure 1.1 we have:

$$\overline{I} = \frac{\overline{V_s} - \overline{V_r}}{jX}$$

(i)
$$S = \overline{V_s} \cdot \overline{I}^* = \overline{V_s} \cdot \frac{\overline{V_s^*} - \overline{V_r^*}}{-jX} = \frac{V_s^2 - V_s V_r e^{j(\delta_s - \delta_r)}}{-jX} = \frac{jV_s^2 - V_s V_r e^{j(\delta_s - \delta_r + 90^\circ)}}{X}$$ [3]
$$= \frac{jV_s^2 - V_s V_r [(\cos(\delta_s - \delta_r + 90^\circ) + j\sin(\delta_s - \delta_r + 90^\circ)]}{X}$$

(ii)

$$S = \frac{V_r \cdot V_s}{X} \sin(\delta_s - \delta_r) + j\frac{V_s^2 - V_r V_s \cos(\delta_s - \delta_r)}{X}$$

$$P = \frac{V_s \cdot V_r}{X} \sin(\delta_s - \delta_r)$$ P flow requires a difference in phase angle [2]

$$Q = \frac{V_s^2 - V_s \cdot V_r \cos(\delta_s - \delta_r)}{X}$$ Q flows requires a difference in voltage magnitude

If both angles are the same then the sine will be zero, which implies that active power will be zero as well. For a small difference in angles the sine is the same as the difference (in radians), and therefore the active power is proportional to the difference in phase angles.

For a small difference in phase angles the cosine is close to one, and if voltage at the receiving end is the same as voltage at the sending end, the reactive power will be zero.

(iii) $P = \frac{V_s \cdot V_r}{X} \sin(\delta_1 - \delta_2) = P_{max} \sin(\delta_1 - \delta_2)$ [2]

For the angle difference of 90°, the sine becomes one.

(b) The states of the generation system with the appropriate probabilities are given in the following table.

STATE	State Probability	Probability that Generation is equal to or greater than State
2500 MW	0.59049	0.59049
2000 MW	0.32805	0.91854
1500 MW	0.0729	0.99144
1000 MW	0.0081	0.99954
500 MW	0.00045	0.99999
0 MW	0.00001	1

[4]

If the system load is 1,800 MW, at least 2,000 MW of available capacity is necessary to fully cover that demand. This means the probability of not being able to cover the demand is equal to $1 - P(\text{Gen} \geq 2{,}000 \text{ MW})$, which is 0.082, or 8.2%. [2]

(c) The maximum reactive power that the generator can absorb is determined by the condition with the internal voltage $E = 0$. In that case, the theoretical maximum capacitive reactive power (in per unit values) can be obtained from the generator capability chart:

$$Q_{cap,max} = V_t^2 / X_d = 0.58 \text{ pu}$$

In absolute terms, this amounts to 368.3 MVAr (cap). [3]

The active power export in this case is equal to 0.

Solution to Question 2

First, we need to calculate the base quantities.

$$S_b = 1{,}000\ MVA$$

$$V_b = 400\ kV$$

$$Z_b = \frac{V_b^2}{S_b} = 160\ \Omega$$

Following, we proceed towards calculating the per-unit quantities.

$$r = \frac{r' \cdot L}{Z_b} = 0.0375\ p.u.$$

$$x = \frac{x' \cdot L}{Z_b} = 0.4688\ p.u.$$

$$z = r + jx = 0.0375 + j0.4688\ p.u.$$

$$b = b' \cdot L \cdot Z_b = 0.24\ p.u.$$

(i),(ii)

For **minimum** loading conditions (300MW):

$$s_2 = \frac{S_2}{S_b} = 0.3\ p.u.$$

$$v_2 = \frac{V_2}{V_b} = 1\ p.u.$$

$$i_2 = \frac{s_2^*}{v_2^*} = 0.3\ p.u.$$

$i_{20} = -j\frac{b}{2} \cdot v_2 = -j\ 0.12\ p.u.$

$$i_{12} = i_2 - i_{20} = 0.3 + j0.12\ p.u.$$

$$\Delta v_{12} = i_{12} \cdot z = -0.0450 + j0.1451\ p.u.$$

$$v_1 = \Delta v_{12} + v_2 = 0.9550 + j0.1451\ p.u.$$

$\boldsymbol{V_1 = v_1 \cdot V_b = 382 + j58\ kV}$ [7]

$$i_{10} = -j\frac{b}{2} \cdot v_1 = 0.0174 - j0.1146\ p.u.$$

$$i_1 = i_{12} - i_{10} = 0.2826 + j0.2346\ p.u.$$

$$s_1 = v_1 \cdot i_1^* = 0.3039 - j0.1830\ p.u.$$

$$\boldsymbol{S_1 = s_1 \cdot S_b = 303.91 - j183.03\ MVA}$$

(i),(ii)

For **maximum** loading conditions (1,200MW):

$$s_2 = \frac{S_2}{S_b} = 1.2\,p.u.$$

$$v_2 = \frac{V_2}{V_b} = 1\,p.u.$$

$$i_2 = \frac{s_2^*}{v_2^*} = 1.2\,p.u.$$

$i_{20} = -j\frac{b}{2}\cdot v_2 = -j\,0.12\,p.u.$

$$i_{12} = i_2 - i_{20} = 1.2 + j0.12\,p.u.$$

$$\Delta v_{12} = i_{12}\cdot z = -0.0112 + j0.5670\,p.u.$$

$$v_1 = \Delta v_{12} + v_2 = 0.9888 + j0.5670\,p.u.$$

$$\boldsymbol{V_1 = v_1\cdot V_b = 395.5 + j226.8\,kV}$$

$$i_{10} = -j\frac{b}{2}\cdot v_1 = 0.0680 - j0.1186\,p.u.$$

$$i_1 = i_{12} - i_{10} = 1.1320 + j0.2386\,p.u.$$

$$s_1 = v_1\cdot i_1^* = 1.2545 + j0.4059\,p.u.$$

$$\boldsymbol{S_1 = s_1\cdot S_b = 1{,}254.5 + j405.86\;MVA}$$

[7]

(iii) A transmission line generates almost constant reactive power represented by its susceptance. A transmission line absorbs reactive power in its reactance. The absorption depends on the loading condition. If the line is more heavily loaded, it will absorb more reactive power. Therefore, in this example, in a low load condition the generator should absorb reactive power (–183 MVAr) while in a higher load condition generator needs to produce reactive power (405 MVAr). [6]

Solution to Question 3

(a) Unlike active power, reactive power cannot be transmitted across long distances, for the following reasons:

- Transmitting reactive power requires a voltage drop that would become unacceptable for long distances.
- Since $X >> R$, the reactive losses are much larger than the active losses and the transmission of reactive power would be inefficient.
- Therefore, we need sources of reactive power around the network.

[3]

(b)

(i) Y_{bus} matrix is obtained as follows:

$z_{12} = 0 + j0.1$ p.u

$$y_{12} = \frac{1}{z_{12}} = -j10 \text{ p.u.}$$

$z_{23} = 0 + j0.25 p.u$

$$y_{23} = \frac{1}{z_{23}} = -j4 p.u.$$

$$Y_{11} = y_{12} = -j10 \text{ p.u.}$$
$$Y_{22} = y_{12} + y_{23} = -j14 \text{ p.u.}$$
$$Y_{33} = y_{23} = -j4 \text{ p.u.}$$
$$Y_{12} = Y_{21} = -y_{12} = j10 \text{ p.u.}$$
$$Y_{13} = Y_{31} = 0$$
$$Y_{23} = Y_{32} = -y_{23} = j4 \text{ p.u.}$$
$$Y = j\begin{bmatrix} -10 & 10 & 0 \\ 10 & -14 & 4 \\ 0 & 4 & -4 \end{bmatrix}$$

[3]

(ii)

$s_2 = 1.5 + j0.8 p.u$

$V_1 = ?$

$V_2 = ?$

$P_1 = ?$

$Q_1 = ?$

$P_2 = ?$

$Q_2 = ?$

$$V_1^{(0)} = 1 + j0; \text{ Slack bus}$$
$$V_2^{(0)} = 1 + j0; \text{ PQ bus}$$
$$V_3^{(0)} = 1.02 + j0; \text{ PV bus}$$

FIRST ITERATION:

$$V_2^{(1)} = \frac{1}{Y_{22}} \cdot (\frac{S_2^*}{V_2^{(0)*}} - Y_{21} \cdot V_1^{(0)} - Y_{23} \cdot V_3^{(0)})$$
$$= \frac{1}{-j14} \cdot \left(\frac{-1.5 + j0.8}{1 - j0} - j10 \cdot (1 + j0) - j4 \cdot (1 + j0) \right)$$
$$= 0.9486 \text{ - j}0.1071 \quad \text{p.u.}$$

$$\widetilde{V}_3^{(1)} = \frac{1}{Y_{33}} \cdot (\frac{S_3^{(0)*}}{V_3^{(0)*}} - Y_{31} \cdot V_1^{(1)} - Y_{32} \cdot V_2^{(0)})$$
$$= \frac{1}{-j4} \cdot (\frac{1}{1 - j0} - j0 \cdot (1 + j0.12) - j4 \cdot (1 + j0)) = \ 0.9686 + j0.1380 \, p.u$$

$$V_3^{(1)} = \left| V_3^{spec} \right| \frac{\widetilde{V}_3^{(1)}}{\left| \widetilde{V}_3^{(1)} \right|} = 1.02 \cdot \frac{0.9686 + j0.1380}{\left| 0.9686 + j0.1380 \right|} = 1.0098 + \text{j}0.1438 \text{ p.u.}$$

$$Q_3^{(1)} = -\text{Im}(V_3^{(1)*} \cdot (Y_{31} \cdot V_1^{(1)} + Y_{32} \cdot V_2^{(0)} + Y_{33} \cdot V_3^{(1)})) = 0.3917 \text{p.u.}$$
$$S_3^{(1)} = \text{Re}(S_3^{(0)}) + jQ_3^{(1)} = 1.0 + \text{j}\,0.3917 \text{ p.u}$$

Buses record

[Index]	[Re{Vol}]	[Im{Vol}]	[P]	[Q]	[DV]	[P Mism]	[Q Mism]
1.0000	1.0000	0	1.0714	0.5143	0	0	-0.0000
2.0000	0.9486	-0.1071	-1.5000	-0.8000	0.1188	0.5499	-0.3022
3.0000	1.0098	0.1438	1.0000	0.3917	0.1442	0.0215	0.0000

Branches record

[Index]	[From]	[To]	[P from]	[P to]	[P loss]	[Q from]	[Q to]	[Q loss]
1.0000	1.0000	2.0000	1.0714	-1.0714	0	0.5143	-0.3730	0.1412
2.0000	2.0000	3.0000	-0.9785	0.9785	0	-0.1248	0.3917	0.2669

SECOND ITERATION:

Similarly as the first iteration including flow and mismatch calculation

Buses record

[Index]	[Re{Vol}]	[Im{Vol}]	[P]	[Q]	[DV]	[P Mism]	[Q Mism]
1.0000	1.0000	0	0.6372	0.6928	0	0	-0.0000
2.0000	0.9307	-0.0637	-1.5000	-0.8000	0.0470	0.1162	0.0011
3.0000	1.0013	0.1944	1.0000	0.4834	0.0513	0.0210	-0.0000

$$S_{12} = V_1^{(2)} \cdot ((V_1^{(2)} - V_2^{(2)}) \cdot y_{12})^* = 0.6372 + j0.5928 \text{ p.u.}$$
$$S_{21} = V_2^{(2)} \cdot ((V_2^{(2)} - V_1^{(2)}) \cdot y_{12})^* = -0.6372 - j0.6042 \text{ p.u.}$$
$$S_{23} = V_2^{(2)} \cdot ((V_2^{(2)} - V_3^{(2)}) \cdot y_{23})^* = \text{-}0.9790 - \text{j}0.1970 \text{ p.u}$$
$$S_{32} = V_3^{(2)} \cdot ((V_3^{(2)} - V_2^{(2)}) \cdot y_{23})^* = 0.9790 + \text{j}0.4834 \text{ p.u}$$
$$\Delta S_2^{(2)} = S_2^{(2)} - S_{23} - S_{21} = 0.1162 + j0.0011 \text{ p.u}$$
$$\Delta S_3^{(2)} = S_3^{(2)} - S_{31} - S_{32} = 0.0210 + \text{j}0 \text{ p.u}$$

Branches record

[Index]	[From]	[To]	[P from]	[P to]	[P loss]	[Q from]	[Q to]	[Q loss]
1.0000	1.0000	2.0000	0.6372	-0.6372	0	0.6928	-0.6042	0.0886
2.0000	2.0000	3.0000	-0.9790	0.9790	-0.0000	-0.1970	0.4834	0.2865

Slack bus generation:

$$S_1 = -S_2 - S_3 = 1.5 + j0.8 - 1 - j0.4834 = 0.5 + j0.3166 \text{ p.u.}$$

Convergence parameters:

$$\Delta V_2^{(2)} = \left|V_2^{(2)} - V_2^{(1)}\right| = \left|(0.9307 + j0.0637) - (0.9486 - j0.1071)\right| = 0.1717 \text{ p.u.}$$

$$\Delta P_{max}^{(2)} = \Delta P_2^{(2)} = 0.1162$$

$$\Delta Q_{max}^{(2)} = \Delta Q_2^{(2)} = 0.0011$$

[14]

Part B – Answer any 2 out of 3 questions in part B

4. a) State three reasons to justify why short-circuit study is an important element in power systems analysis.

[5]

Answer:

1. Calculate circuit breaker ratings -Must be able to withstand and make/break the fault current
2. Design protection (tripping) system -Is the fault current large enough to be detected?
3. System stability analysis- Type and duration of the faults affects system stability
4. Power quality - Faults cause voltage sag in parts of the network

b) A segment of a power system comprising of four generators (G_1, G_2, G_3 and G_4) and two reactors (X_1 and X_2) is shown in Fig. 4.1. The MVA rating and reactance (in per unit of respective rating) of each component is shown on the figure. Calculate the fault MVA for a 3-phase short circuit at point A on the system. Neglect pre-fault current and assume 1.0 pu pre-fault voltage everywhere in the system. Choose 10 MVA as the common system base.

[8]

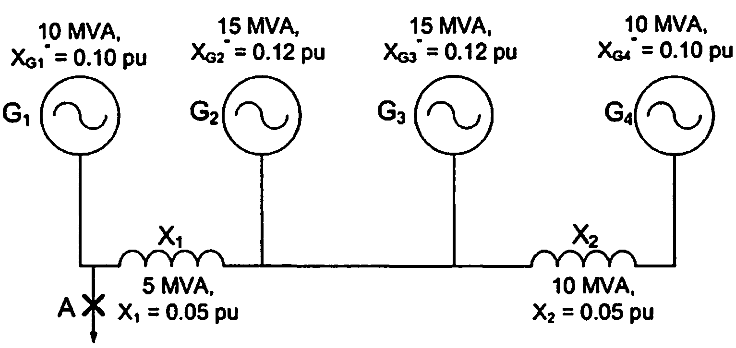


Fig 4.1: A segment of a power system comprising of four generators (G_1, G_2, G_3 and G_4) and two reactors (X_1 and X_2)

Solution:

$S_{Base} = 10$ MVA

$X''_{G1} = j0.1$ pu, $X''_{G2} = j0.12 \times \dfrac{10}{15} = j0.08$ pu $= X''_{G3}, X''_{G4} = j0.1$ pu

$X_1 = j0.05 \times \dfrac{10}{5} = j0.10$ pu, $\quad X_2 = j0.05$ pu

Equivalent impedance of the system looking at the fault point A is

$X_{eq-A} = X''_{G1}||[X_1 + \{(X''_{G2}||X''_{G3})||(X_2 + X''_{G4})\}] = j0.0568 \text{ pu}$

Fault MVA

$$S_{Fault-A} = \frac{S_{Base}}{|X_{eq-A} \text{ (in pu)}|} = \frac{10}{0.0568} = 176 \text{ MVA}$$

c) The source end of a power system comprising of two 11 kV generators (G_1 and G_2), one 33/11 kV transformer (T_1) and a current limiting reactor (X_1) is shown in Fig. 4.2. The MVA rating and reactance (in per unit of respective rating) of each component is shown on the figure. Find the absolute value of the reactance of X_1 in ohms such that the short circuit level for a three-phase fault on the outgoing line at point F does not exceed 300 MVA. Choose 15 MVA as the common system base. Neglect pre-fault current and assume 1.0 pu pre-fault voltage everywhere in the system.

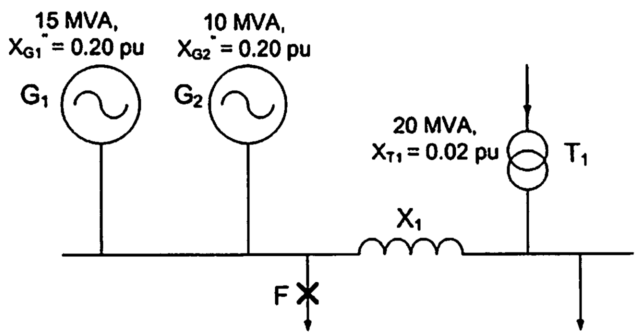


Fig 4.2: source end of a power system comprising of two generators (G_1 and G_2), one transformer (T_1) and a current limiting reactor (X_1)

[12]

Solution:

$S_{Base} = 15$ MVA

$X''_{G1} = j0.2 \text{ pu}, X''_{G2} = j0.2 \times \frac{15}{10} = j0.3 \text{ pu}$

$X_{T1} = j0.02 \times \frac{15}{20} = j0.015 \text{ pu}$

Suppose $X_1 = jx$

Equivalent impedance of the system looking at the fault point F is

$$X_{eq-F} = (X''_{G1}||X''_{G2})\ ||(X_1 + X_{T1}) = \frac{j0.12(j0.015 + jx)}{j(0.135 + x)}$$

Maximum allowable short circuit level at point F is 300 MVA

$S_{max-F} = 300$ MVA

$$S_{Fault-F} = \frac{S_{Base}}{|X_{eq-F}\text{ (in pu)}|} = S_{max-F}$$

$$|X_{eq-F}\text{ (in pu)}| = \frac{S_{Base}}{S_{max-F}} = 0.05\text{ pu}$$

Equating the expressions for X_{eq-F} we get

$$\frac{j0.12(j0.015 + jx)}{j(0.135 + x)} = j0.05 \;\Rightarrow x = 0.0707\text{ pu}$$

Base impedance for the reactor X_1 is

$$Z_{Base} = \frac{kV_{Base}^2}{S_{Base}} = \frac{11^2}{15} = 8.067\ \Omega$$

Reactance of X_1 in absolute units is

$$X_1(\text{in }\Omega) = X_1(\text{in pu}) \times Z_{Base} = 0.0707 \times 8.067 = 0.5705\ \Omega$$

5. a) Explain why the steady-state value of the positive sequence reactance of a synchronous machine is greater than the corresponding negative and zero sequence reactances.

[5]

Answer:

- Magnetic field produced by the positive sequence current rotates at synchronous speed in the same direction as the rotor which implies high flux linkage and hence relatively high positive sequence reactance (X_1) in steady state

- Magnetic field produced by negative sequence current rotates at synchronous speed but opposite to the direction of rotation of the rotor. Hence the induced current prevents flux from penetrating the rotor resulting in less flux linkage and hence $X_2 < X_1$

- Net magnetic field produced by the zero sequence currents in theoretically zero i.e. X_0 should ideally be zero. However, due to leakage flux, end turns harmonic flux etc. X_0 has a small value which is usually much less than X_1 and X_2

b) A three-phase unbalanced delta connected load draws 100 A of line current from a balanced three-phase supply. Using the method of symmetrical components calculate the magnitude and phase angle of the sequence components of the currents in phases B and C if the current in phase A drops down to zero due to an open-circuit fault. Assume ABC phase sequence.

[8]

Solution:

Due to the open circuit fault on phase A
$I_A = 0, I_B = -I_C = 100$ A

Zero sequence components

$$I_{A0} = I_{B0} = I_{C0} = 0$$

Positive sequence component
$$I_{A1} = \frac{1}{3}(\alpha - \alpha^2)I_B = \frac{j}{\sqrt{3}}100 \text{ A} = 57.74\angle 90^\circ \text{ A}$$

For ABC phase sequence

$$I_{B1} = I_{A1} \times \alpha^2 = 57.74\angle{-30^\circ} \text{ A}$$

$$I_{C1} = I_{A1} \times \alpha = 57.74\angle 210^\circ \text{ A}$$

Negative sequence component

$$I_{A2} = \frac{1}{3}(\alpha^2 - \alpha)I_B = -\frac{j}{\sqrt{3}}100 \text{ A} = 57.74\angle{-90^\circ} \text{ A}$$

$I_{B2} = I_{A2} \times \alpha^2 = 57.74\angle{-210^\circ}$ A

$I_{C2} = I_{A2} \times \alpha = 57.74\angle{30^\circ}$ A

c) The positive, negative and zero sequence sub-transient reactance of a 5 MVA, 13.8 kV, 50 Hz, three-phase star connected synchronous generator is 0.1 pu, 0.1 pu and 0.01 pu, respectively. The neutral of the generator is grounded through a reactor. Calculate the minimum value of the inductance (in Henry) of the reactor which is necessary to limit the single line to ground (LG) fault level to less than the three-phase fault level at the terminal of the generator. Neglect pre-fault current and assume 1.0 pu pre-fault voltage. Neglect fault impedance.

[12]

Solution:

$X_1 = X_2 = j0.1$ pu, $X_0 = j0.01$ pu

LG fault

For LG fault the three sequence networks are connected in series across three times the neutral grounding reactance. Hence the sequence component of the current is given by:

$$I_{A0} = I_{A1} = I_{A1} = \frac{1.0}{2X_1 + X_0 + 3X_n}$$

Fault current in phase A is

$$I_{f-LG} = 3I_{A1} = \frac{3.0}{2X_1 + X_0 + 3X_n}$$

Three-phase fault

For a three-phase fault the negative and zero sequence networks does not play a part as the system remains balanced during fault. Hence the fault current is given by:

$$I_{f-3\phi} = \frac{1.0}{X_1}$$

For $I_{f-LG} < I_{f-3\phi}$

$$\frac{3.0}{2X_1 + X_0 + 3X_n} < \frac{1.0}{X_1} \Rightarrow X_n > \frac{1}{3}(X_1 - X_0)$$

Minimum value of the reactance required is

$$X_n = \frac{1}{3}(X_1 - X_0) = 0.03 \text{ pu}$$

$$X_n(\text{in } \Omega) = 0.03 \times \frac{13.8^2}{5.0} = 0.2857\ \Omega$$

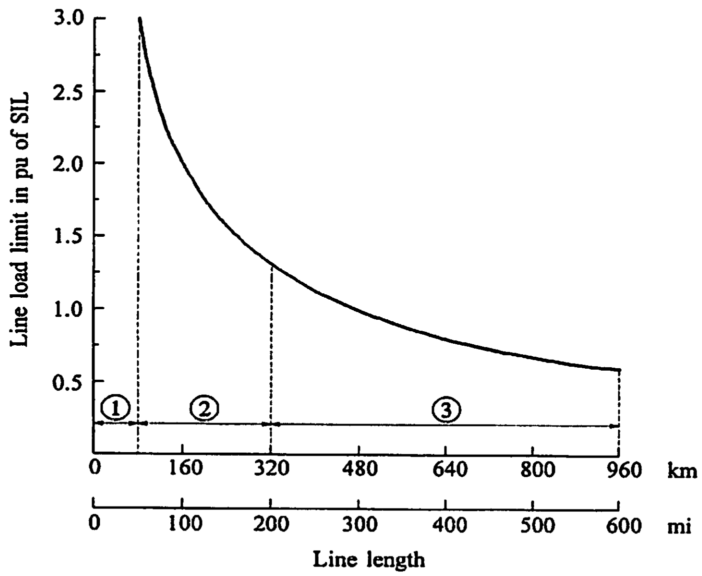
Hence the minimum required inductance is

$$L_n = \frac{X_n}{2\pi f} = 3.6\ \text{mH}$$

6. a) Describe how different forms of stability constraints limit the power transmission capacity of lines for a range of transmission distances.

[5]

Answer:



① 0–80 km: Region of thermal limitation
② 80–320 km: Region of voltage drop limitation
③ 320–960 km: Region of small-signal (steady-state) stability limitation

Up to about 80-100 kms the transmission lines can be used up to their thermal limits. Beyond 80-100 kms up to about 300-320 kms (medium lines) it is the voltage stability limitations and beyond 320 kms (long lines) it is the angle stability constraints that limits the loadability or transmission capacity.

Line loadability curve (below) expressed in pu of surge impedance loading (SIL)/natural loading
So that it is applicable to lines of all voltage classes.

b) A round-rotor synchronous generator is operating at 25% of its maximum power capacity in steady-state. Under this condition the mechanical power input to the generator is suddenly doubled. Use equal area criteria to calculate the maximum excursion of rotor angle in degrees. Assume constant generator terminal voltage throughout and neglect resistances.

[10]

Solution:

Neglecting resistance the power-angle characteristics for a round-rotor synchronous generator is
$P = P_{max} \sin\delta$

In steady state
$$\delta_0 = \sin^{-1}\frac{P_0}{P_{max}} = \sin^{-1}(0.25) = 14.48^{\circ}$$

After sudden doubling of mechanical input power

$P_1 = 2P_0$

$$\delta_1 = \sin^{-1}\frac{2P_0}{P_{max}} = \sin^{-1}(0.5) = 30^\circ$$

Accelerating area is given by

$$\int_{\delta_0}^{\delta_1} (P_1 - P_{max}\sin\delta)\, d\delta$$

Decelerating area is given by

$$\int_{\delta_1}^{\delta_{max}} (P_{max}\, sin\,\delta - P_1)\, d\delta$$

Using equal-area criterion

$$\int_{\delta_0}^{\delta_1} (P_1 - P_{max}\sin\delta)\, d\delta = \int_{\delta_1}^{\delta_{max}} (P_{max}\sin\delta - P_1)\, d\delta$$

$$\Rightarrow\ \cos\delta_0 - \cos\delta_{max} = \frac{P_1}{P_{max}}(\delta_{max} - \delta_0)\ \Rightarrow \delta_{max} = 45^\circ$$

c) A round-rotor synchronous generator supplies its rated power to an infinite bus with the voltages at both ends maintained at 1.0 pu. The equivalent reactance of the transmission lines connecting the generator and the infinite bus is 0.3 pu under normal condition. Due to occurrence of a fault on the transmission corridor the power output of the synchronous generator drops down to zero. After the fault is cleared the equivalent reactance between the generator and the infinite bus becomes 0.5 pu. Calculate the critical clearing angle in degrees. Neglect resistances.

[10]

Solution:

Neglecting resistance the power-angle characteristics for a round-rotor synchronous generator is

$$P = \frac{V_1 V_2}{X}\sin\delta$$

Under pre-fault condition

$$1.0 = \frac{1.0}{0.3}\sin\delta_0\ \Rightarrow\ \delta_0\ = 17.46^\circ$$

During fault $P = 0$

Under post-fault condition

$$1.0 = \frac{1.0}{0.5}\sin\delta_1 \Rightarrow \delta_1 = 30°, \ P_{max1} = \frac{1.0}{0.5} = 2.0$$

Suppose δ_{cr} is the critical clearing angle

Accelerating area is given by

$$\int_{\delta_0}^{\delta_{cr}} (P_m - 0)\, d\delta = P_m(\delta_{cr} - \delta_0)$$

Maximum available decelerating area is given by

$$\int_{\delta_{cr}}^{\pi-\delta_1} (P_{max1}\, sin\, \delta - P_m)\, d\delta = 2.0(\cos\delta_{cr} + \cos\delta_1) - P_m(\pi - \delta_1 - \delta_{cr})$$

The critical clearing angle is given by:

$$P_m(\delta_{cr} - \delta_0) = 2.0(\cos\delta_{cr} + \cos\delta_1) - P_m(\pi - \delta_1 - \delta_{cr})$$

$$\Rightarrow \delta_{cr} = \cos^{-1}[0.5P_m(\pi - \delta_1 - \delta_{cr}) - \cos\delta_1] = 70.3°$$