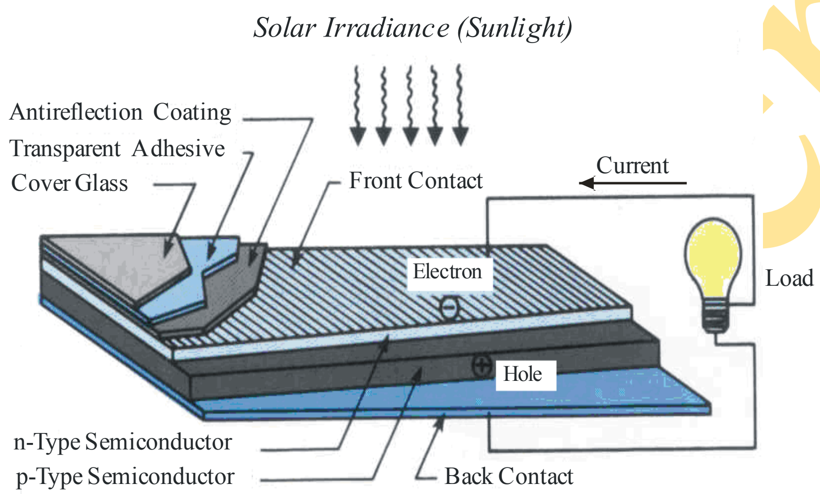
[Overall mark for the EE2-03 exam was 62.6% (standard deviation of 17.6) before the adjustment formula was considered. That is a little lower that in recent years but not by much. It does not seem that any question was unexpectedly difficult but it does seem that the exam was a little longer than was intended so some answers were rushed or incomplete. The average marks per question were 26.0/40; 18.9/30 and 17.8/30. Question 1 is intended to be broad but straightforward and it is perhaps this question that has a lower mark than expected.]

1. This question covers several topics and all parts should be attempted.

a) Consider a typical photovoltaic panel.

i) With the aid of a sketch (showing clearly the intended direction of incident light), describe the construction of a photovoltaic panel. [5]

[Bookwork]
Sketch similar to this required



Key points are:

- *pn* junction
- metalisation in stripes on top surface and solid on bottom
- anti-reflective coating and glass cover

[3×1 mark plus 2 for quality of sketch]

[The common mistakes in this part were an incorrect order of different layers and lack of Cover Glass, Transparent Adhesive and Antireflection Coating.]

ii) The choice of width of the metallisation tracks on the top surface of a panel affects the efficiency of the panel. Describe why this is so. [3]

[Bookwork]
Stripes needed to allow light to enter crystal. Thin stripes avoid large fraction of shadow which prevent some light being captured. However, thick stripes needed to reduce resistance of the current collection path.
[1 mark for each reason and 1 for identifying trade-off exists]

[Most candidates recognised that wide stripes may prevent light being captured but few of them realize the resistance of the current path is also related to the width of stripes.]

iii) List the factors that affect the intensity of light that is incident on a panel. [4]
[Bookwork]

- Length of day given season and location
- Elevation and azimuth of the sun relative to those of the panel
- Air-mass absorbing light energy at certain wavelengths (dependent on path length through atmosphere and therefore position of sun relative to panel).
- Cloud cover causing scattering or shadow
- Buildings, trees, etc. causing shadowing

[4 marks for four well-described reasons]

b) An impedance connected to a single-phase supply of 230 V draws a current of 12 A that leads the voltage by 15°.

i) Calculate the resistance and reactance of the impedance. [2]

[Standard Calculation]

$$Z = \frac{V}{I} = \frac{230}{12\angle^{+}15°} = 19.167\angle^{-}15° = 18.51 - j4.96\ \Omega$$

So, the resistance is 19.51 Ω and the reactance -4.96 Ω. [2 marks for accurate answers]

ii) Calculate the real and reactive powers consumed by the impedance [2]

[Standard Calculation]

$\mathrm{P} = \mathrm{VIcos}(\phi) = 230\times 12\times\cos(-15°) = 2.67$ kW [1 mark for accurate answer and unit]

$Q = VIsin(\phi) = 230\times 12\times\sin(-15°) = -0.71$ kVAr [1 mark for accurate answer and unit]

[Almost all candidates produced an accurate answer in this part but some of them used an incorrect unit for reactive power.]

c) Three impedances, each of 10 + j3 Ω, are connected to a 400 V three-phase supply in delta configuration. Calculate the real and reactive powers consumed. [4]

[Standard Calculation]

For delta connection, phase voltage and line voltage are equivalent. Both are therefore 400 V.

Find phase current $I_P = \frac{V_P}{Z} = \frac{400}{10+j3} = \frac{400}{10.44\angle 16.70°} = 38.31\angle^{-}16.70°$ A

[2 marks for correct current in delta]

Find real power $P = 3V_P I_P cos(\phi) = 3\times 400\times 38.31\times cos(^{-}16.70°) = 44.0$ kW

And reactive power $Q = 3V_P I_P sin(\phi) = 3\times 400\times 38.31\times sin(^{-}16.70°) = 13.2$ kVAr

[2 marks for accurate answers and units]

[A common mistake in this part was a confusion between phase current and line current in a three-phase power calculation.]

d) Consider a national-scale AC electricity system.

i) Describe the principal differences between the transmission and distribution networks and include an explanation of why distribution networks use lower voltages than transmission networks. [4]

[Bookwork]

Transmission networks exist to effect bulk transfer of power from geographic regions with more generation than load to regions with more load than generation. They are long distance networks which are built to achieve high availability by having duplicate circuits on all routes and having a high degree of meshing between the nodes. Distribution networks a relatively short distance networks for distributing power within a load centre. They use a variety of voltage levels determined by the volume of load being served. There are double circuit routes at the higher voltages and single circuit routes at the lower voltages but little or no meshing.

[2 marks for mentioning degree of redundancy and availability, difference in function and difference in geographic scale]

The choice of operating voltage is based on achieving the lowest total cost considering operating costs and capital costs. Higher voltages reduce power losses and hence operating costs but have higher capital costs. For long distance routes with relatively few nodes and substations, high voltage achieves a large advantage in power loss for reasonable capital cost. Where distances are short, and connections are many, the lower voltages achieve a better balance between the costs.
[2 marks]

ii) Explain the disadvantages and advantages of using a frequency higher than 60 Hz and suggest why 400 Hz is the standard for aircraft power systems system. [4]

[Bookwork]
Advantages of >60Hz

- Smaller transformers and generators for a given power rating. [1 mark]

Disadvantages of >60Hz

- Large series voltage drops across inductances of lines and larger shunt-currents in capacitance of cables leading to lower power transfers for a given line/cable.
- Need for greater engineering effort in bearings and turbines to accommodate higher rotational speeds. [2 marks]

Use 400 Hz in aircraft because reduction in weight of transformers and generators is a significant saving in aircraft operations and overcomes the disadvantages (and cable/line runs are short). [1 mark]

[Most candidates realised the benefits of a smaller transformer and the drawbacks of a larger voltage drop in OHL and larger shunt currents in cables, but few recognised the disadvantages of higher rotational speeds.]

e) Consider the characteristics of overhead lines and cables.

i) Explain why the $X{:}R$ ratio of an overhead line is different for different voltage ratings. [2]

[Bookwork]
High voltage overhead lines have large separation distance between conductors in order to provide sufficient insulation.
Inductance per unit length increases with separation distance, d, according to $L' = \frac{\mu_0}{\pi}\ln\left(d/r_C\right)$ (from formula sheet) [1 mark]

Resistance per unit length is approximately independent of voltage rating because currents are kept in the region of 1 kA and so wire cross-section remains constant. Combining these two factors means that $X{:}R$ ratio increases with voltage rating.
[1 mark]

ii) Explain why an underground cable has a higher capacitance per unit length than an overhead line. [2]

[Bookwork and interpretation]
Comparing the equations for capacitance of OHL and cables;

$$C' = \frac{\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(d/r_C\right)} \quad \text{and} \quad C' = \frac{2\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(r_O/r_C\right)}$$

reveals that relative permittivity is important. Cables use plastics of $\varepsilon_R \approx 3$ and OHL use air of $\varepsilon_R \approx 1$ leading to higher capacitance in cables. Also, separation distance in cables (r_O) is smaller than in OHL, d, and both are denominator terms thus also leading to higher capacitance in cables.
[2 marks]

[Many candidates lost marks because they did not mention the permittivity difference.]

iii) An overhead line is found to have an inductance per unit length of 125 μH/km and is operated at 132 kV and 50 Hz. What is the longest length of line that could be operated with a power transfer of 1,500 MW if the load angle is not to exceed 30°. [4]

[Variation of standard calculation.]

Use standard equation and replace reactance by function of length, l, $P = \frac{V_R V_S}{2\pi\ f\ L'\ l} sin(\delta)$

$$l = \frac{V_R V_S}{P\ 2\pi\ f\ L'} sin(\delta) = \frac{132\times10^3\times132\times10^3}{1{,}500\times10^6\times2\pi\times50\times125\times10^{-6}}\times0.5 = 147.9\ km$$

[2 marks for correct manipulation of equation; 2 marks for accurate answer]

[This question was not completed well but the errors and omissions were varied and followed no pattern.]

iv) An overhead line has a series impedance of 0.35 + j 0.40 Ω. Measurements at the sending end are that the voltage is 11.0 kV and real power of +8.0 MW with zero reactive power flows from sending end. Estimate the voltage magnitude change across the line. Then estimate the reactive power that would be needed (as seen at the sending end) to bring the voltage change to zero. [4]

[Calculation using standard approximation.]

Use $\Delta V \approx \frac{R\ P_S + Z\ Q_S}{|V_S|}$

With Q=0, $\Delta V \approx \frac{R\ P_S}{|V_S|} = \frac{0.35\times8\times10^6}{11\times10^3} = 254.5$ V [2 marks for accurate answer]

To achieve drop of zero set numerator term to zero by setting $R\ P_S = -Z\ Q_S$

$Q_S = -\frac{Z}{R} P_S = -\frac{0.35}{0.40}\times8M = -7.0$ MVAr [2 marks for accurate answer]

2.
[This was a familiar topic and a fairly straight forward question. Those who had prepared well scored highly but a surprising number of candidates did not and average mark was lower than expected. There was no obvious pattern to this.]

A boost switch-mode power supply (SMPS) is to be used to provide a 20 V output from a 4 V input. The inductor has a value of 80 μH and the capacitor has a capacitance of 100 μF and a series resistance of 20 mΩ.

a) Describe the operating principle of the boost SMPS including an explanation of how an output voltage higher than the input voltage is achieved. [5]

[Bookwork]

- Turn-on of the Mosfet imposes the full input voltage across the inductor
- Current increases linearly in the inductor (at *di/dt=VI/L*), storing energy as ½LI^2.
- The diode is reverse biased and the capacitor alone supplies energy to the load
- The capacitor has a large stored energy and its voltage changes little during this period.
  [2 marks]
- Turn-off of the Mosfet causes the inductor current to divert to the diode.
- The inductor has a negative voltage imposed across it – the output voltage minus a diode drop applied at the RHS and the input voltage at the LHS
- The inductor current reduces and stored energy is released to the capacitor and load.
- Charge and energy are added to the capacitor.
  [2 marks]
- With no switching the output voltage settles at the input voltage; each switching cycle adds charge to the output capacitor until the output voltage has risen sufficiently to bring the inductor current into steady-state.
  [1 mark]

[It was important to describe the energy storage in the inductor; a key part of the operation. It was also important to note the increase and decrease of inductor current as a result of the applied voltages. Not making these points lost marks.]

b)

i) Calculate the value of duty-cycle required assuming continuous conduction. [2]

[Standard calculation. 1 mark for equation; 1 mark for accurate answer.]

$$\frac{V_O}{V_I} = \frac{1}{1-\delta}$$

$$\delta = 1 - \frac{V_I}{V_O} = 1 - \frac{4}{20} = 0.8$$

ii) Calculate the minimum switching frequency for which the SMPS will stay in continuous conduction for an input power of 2 W. [6]

[Variation of standard calculation.]

The input current is $I_I = \frac{P_I}{V_I} = \frac{2}{4} = 0.5$ A [1 mark]

Inductor ripple current must be less than twice this; $\Delta i_L = 1.0\ A$ [2 marks]

The ripple current is defined by $\Delta i_L = \frac{V_I}{L} \times \frac{\delta}{f} = \frac{V_I}{fL} \times \left(1 - \frac{V_I}{V_O}\right)$ [1 mark]

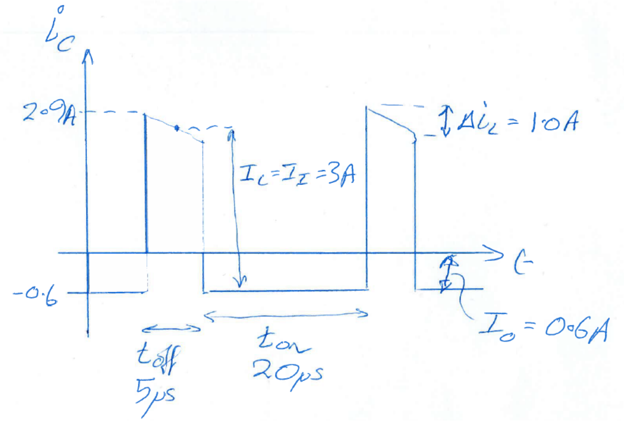
Re-arranging gives expression for frequency, $f = \frac{\delta\, V_I}{\Delta i_L\, L} = \frac{0.8\times 4}{1.0\times 80\times 10^{-6}} = 40$ kHz

[1 mark for re-arranged equation, 2 for accuracy]

[A common error was to set the current ripple in relation to the output current not the input current. It must be the input current since this is directly equivalent to the inductor current in the *boost* circuit.]

iii) Sketch the shape of the current through the capacitor during continuous operation and label the sketch with scales for an input power of 12 W and the switching frequency found in (ii). [5]

[Book work plus calculation]



[2 marks for sketch with correct slope]

Period is 25 μs; on-time is 20 μs and off-time 5 μs. [1 mark for correct time scale]
$\Delta i_L = 1.0$ A
$I_I = \frac{P_I}{V_I} = \frac{12}{4} = 3.0$ A [2 marks for correct current scale]

[Most common reason for loosing marks was to not show the current scaling accurately.]

iv) Calculate the output voltage ripple under the conditions used in (iii). [5]

[Standard Calculation]
There are two components to the ripple: resistive and capacitive. Peak-to-peak resistive voltage proportion depends on peak-to-peak current through capacitor:
$i_C^{ptp} = i_L^{pk} = I_I + ½\Delta i_L = 3.0 + ½\times 1 = 3.5\ A$ [1 mark]
$\Delta v_{ESR} = i_C^{ptp} \times R_{ESR} = 3.5\times 0.02 = 70\ mV$ [1 mark]

The charge delivered to the capacitor is most easily found during the transistor on-time (during which the capacitor is discharged by the load current).

$\Delta q = I_O t_{on} = \frac{P_I}{V_O} t_{on} = \frac{12}{20}\times 20\times 10^{-6} = 12\ \mu\text{C}$ [2 marks]
$\Delta v_C = \frac{\Delta q}{C} = \frac{5\times 10^{-6}}{100\times 10^{-6}} = 120$ mV [1 mark]

The output voltage ripple is the sum of these = 120 + 70 = 190 mV

[Note that the capacitance stated in the question is probably a bit low compared to the ESR for a realistic capacitor.]

v) The MOSFET used in the SMPS has a channel resistance of $_{RDS(on)}$) = 20 mΩ and turn-on and turn-off energy losses, E_{on} and E_{off} of 5 and 7 μJ respectively, and diode on-state voltage is $V_{AK(on)}$ = 0.6 V. Calculate the total power lost in the semiconductors when the SMPS operates at 15 W. [3]

[Standard calculation]

$$P_{loss} = \delta I_{DS}^2 R_{DS(on)} + (1-\delta) I_{AK} V_{AK(on)} + f\left(E_{on} + E_{off}\right)$$

$$P_{loss} = 0.8 \times 3.75^2 \times 0.02 + (1-0.8) \times 3.75 \times 0.6 + 40 \times 10^3 \times (5+7) \times 10^{-6}$$

$$= 0.225 + 0.450 + 0.480 = 1.155\ W$$

[1 marks for equation and 2 for accuracy]

[Most common error was to use an incorrect current amplitude. The Mosfet and diode carry the inductor current (alternately) and this has an average value equal to the input current. Some candidates used the output current for the diode current and this is an error in this version of the equation. One can alternatively use I_O in place of $(1-\delta)I_{AK}$]

vi) One way to reduce the output voltage ripple would be to increase the switching frequency. Without detailed calculation, comment on how effective a doubling of switching frequency would be and the impact on efficiency. [4]

[New interpretation.]

The capacitance term is directly dependent on t_{on} and therefore f and so would halve. For this capacitor, with a quite small capacitance, the capacitive term is large compared with ESR so this reduction is significant. [2 mark]

The ESR terms is itself formed of a frequency independent term related to the input current and a term dependent on the inductor ripple current. The ripple current term is around a tenth of the total so halving it is not very effective. [1 marks]

The switching power loss is approximately half the total power loss so a doubling of frequency adds approximately 50% to power losses. [1 mark]

[Generally, this question was completed well by candidates.]

3.

a) Explain in outline the operating principle of a three-phase induction machine. [7]

[Book work]

- The stator has three windings displaced (in space) by 120° from each other around the stator.
- The three supply currents are displaced (in time) by 120° from each other.
- Three-phase currents in the three-phase winding create a rotating magnetic field at a speed determined by the supply frequency (and pole-pair number).
  [3 marks]
- The magnetic flux from the stator crosses the air-gap and links with the windings on the rotor.
- The relative motions between rotating flux and rotating rotor winding causes a rate-of-change of flux linkage that voltages in the rotor winding.
  [2 marks]
- The induced voltages drive currents around the closed rotor windings.
- The rotor currents establish a rotor field that interacts with the stator field to develop torque that opposes the relative motion (i.e. causes the rotor to accelerate toward the speed of the stator field).
  [2 marks]

[Important points that were sometimes not sufficiently clearly stated were (i) stator creates rotating field; (ii) rotor voltage comes from rate-of-change of flux and (iii) rate-of-change of flux is set by relative speed between rotor bars and stator field.]

b) A three-phase induction machine with one pole-pair has the following equivalent circuit parameters:

Stator resistance, $R_S = 0.8\ \Omega$.

Stator leakage reactance, $X_S = 2\ \Omega$;

Iron loss resistance, $R_I = 200\ \Omega$;

Magnetising reactance, $X_M = 60\ \Omega$;

Referred rotor leakage reactance, $X_R = 2\ \Omega$;

Referred rotor resistance, $R_R = 0.8\ \Omega$;

The machine is supplied at a phase voltage of 200 V and a frequency of 50 Hz.

i) Initially, the machine runs at 2,930 r.p.m. Calculate the stator current and the real power drawn from the stator supply. [8]

[Standard calculation]

Slip needs to be calculated first from: $s = \frac{\omega_S - \omega_R}{\omega_S}$

Synchronous speed calculated by: $\omega_S = {}^{\omega_E}/_{p} = {}^{50\times 2\pi}/_{1} = 314.1\ \mathrm{rad/s}$

Rotor speed is $\omega_R = \frac{2{,}930\times 2\pi}{60} = 306.8\ \mathrm{rad/s}$

Hence slip is: $s = \frac{314.1-306.8}{314.1} = 0.0233$ [2 marks for accurate answer]

[A small number of candidates made an error by a factor of 10 and obtained s=0.233]

Total impedance must be found next:

$$Z_T = R_S + jX_S + \left(R_M//jX_M//\left(\frac{R'_R}{s} + jX'_R\right)\right)$$

[2 marks for correct formulation of impedance]

[Some candidates omitted R_M. It was quoted in the question and had to be included. There were occasional mistakes in the arithmetic of the parallel calculation. Four candidates failed to apply vector arithmetic and treated all impedances as real numbers.]

$Z_T = 23.54 + j14.26 = 27.53\angle 31.2°\ \Omega$

Current is found from the phase voltage $I_S = \frac{V_S}{Z_T} = \frac{230}{27.53\angle 31.2°} = 7.27\angle -31.2°\ \Omega$
[2 marks for accurate answer]

And power calculated for a 3-Ph load $P = 3V_S I_S cos(\phi) = 3\times 230\times 7.27\times \cos(31.2°) = 3.73$ kW
[2 marks for accurate answer and units]

ii) For the conditions in (i), calculate the rotor current and power converted to mechanical form. [8]

[Standard calculation]

Rotor current is found from the current divider rule (or otherwise) $I_R = I_S\times\left(\frac{R_M//jX_M}{R_M//jX_M+\left(\frac{R'_R}{s}+jX'_R\right)}\right)$

[2 marks for correct formulation]

$I_R = 5.47\angle -6.21°$ [2 marks for accurate answer]

Power from standard formula $P_{EM} = 3\, I'^2_R\, R'_R\left(\frac{1-s}{s}\right) = 3\times 5.47^2\times 0.8\times\left(\frac{1-0.0233}{0.0233}\right) = 3.00$ kW
[2 mark for correct formula; 2 marks for accurate answer]

iii) Consider now that the mechanical load speeds up to 3,050 r.p.m and recalculate the power exchanged with the supply and comment on this value. [7]

[Variation of standard calculation plus fresh interpretation]
Rotor speed is $\omega_R = \frac{3{,}050\times 2\pi}{60} = 319.4$ rad/s

Slip is: $s = \frac{314.1-319.4}{314.1} = -0.0167$ [1 mark for accurate answer]

[Most candidates correctly noted the negative slip]

Total impedance is: $Z_T = -27.56 + j33.48 = 43.37\angle 129.5°\ \Omega$ [1 mark for correctly dealing with negative resistance and 1 mark for accurate overall answer]

Current is found from the phase voltage $I_S = \frac{V_S}{Z_T} = \frac{230}{43.37\angle 129.5°} = 4.61\angle -129.5°\ \Omega$
[1 marks for accurate answer]

Power is $P = 3V_S I_S cos(\phi) = 3\times 230\times 4.61\times\cos(-129.5°) = -1.75$ kW
[1 marks for accurate answer]

[Some candidates calculated the mechanical power here and were not penalised for that misunderstanding unless they also failed to properly insert the negative value of slip.]

The machine is running super-synchronous and generating electrical power. The magnitude of the power is smaller than in motoring case because slip is -50 rpm versus +70 rpm and also the losses in the machine subtract from the input mechanical power to yield the stator power (rather than subtracting from the stator power to give the mechanical power).
[2 marks]

