Imperial College London

MEng (Engineering) Examination 2017

Year 1

AE1-102 Aircraft Performance

Friday 2nd June 2017: 14.00 to 15.00
[1 hour]

There are ***TWO*** questions.
Candidates may obtain full marks for complete answers to ***BOTH*** questions.

A table of the standard atmosphere and a data sheet are provided.

The use of lecture notes is NOT allowed.

1. To minimise flight time and fuel consumption, eastward travelling long-haul flights aim to take advantage of the polar or subtropical jet streams. In the presence of head or tail winds, the standard form of the Breguet range equation is no longer valid.

   (a) Show that in the presence of a tailwind of speed V_{tw}, the range R of a jet aircraft, operating at constant velocity and L/D, is given by:

   $$R = -\frac{2V_{mD}(L/D)_{\max}}{c}\left(\frac{\overline{V}+\overline{V}_{tw}}{\overline{V}^2+\overline{V}^{-2}}\right)\ln\left(\frac{W_{fin}}{W_{init}}\right), \tag{1}$$

   where $\overline{V}_{tw} = V_{tw}/V_{mD}$ and all other symbols have their usual meaning. [40%]

   (b) Using equation (1) or otherwise, prove that the range is maximised when

   $$\overline{V}_{tw} = -\frac{\overline{V}}{2}\left(\frac{\overline{V}^4-3}{\overline{V}^4-1}\right). \tag{2}$$

   [35%]

In January 2015, a British Airways Boeing 777-200 operating from New York's JFK to London Heathrow encountered tailwind speeds of the order of 320 km/h, arriving 1.5 hours ahead of schedule.

   (c) How much fuel did the 777-200 above save due to the tailwinds encountered? You may assume that at the start of the cruise segment the aircraft weighs 222×10^3 kg, that $V_{mD} = 673$ km/h and that the cruise segment, with no wind, would require a minimum 38×10^3 kg of fuel. A graphical representation of equation (2) is given in figure 1, overleaf. [25%]

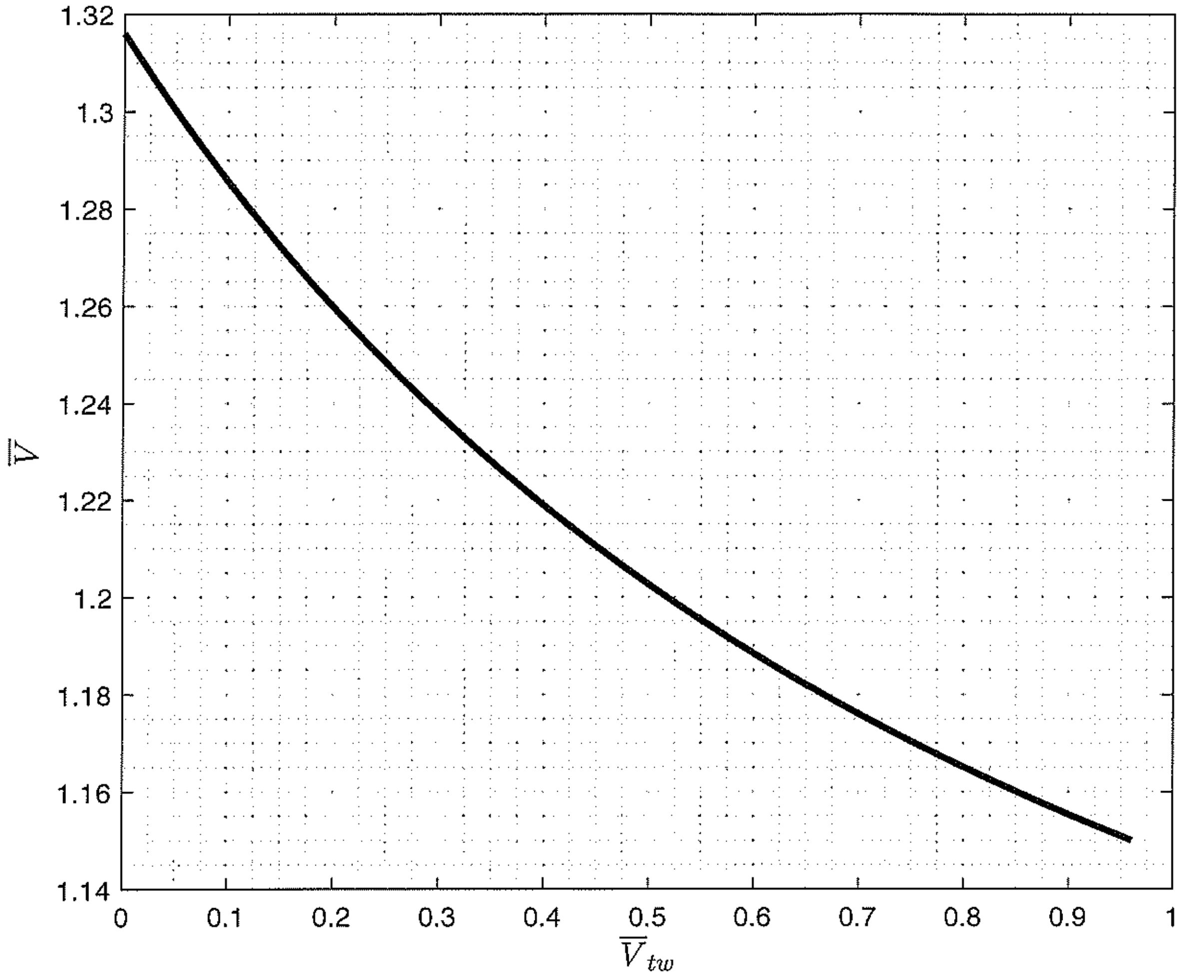

Figure 1: Plot of headwind velocity vs. best cruise speed

2. (a) Explain how leading-edge flaps and trailing-edge flaps alter the lift coefficient curve with respect to angle of attack. Why are these devices often used in combination? Use a sketch to explain your answer. [25%]

(b) During the conceptual design of a small regional jet, you identify that the aircraft will be speed unstable during its approach to land. The characteristics of the aircraft in the approach configuration are:

- maximum landing mass $m_L = 35\,100$ kg,
- wing loading $W_L/S = 4\,450$ N/m^2,
- zero-lift drag coefficient $C_{D_0} = 0.12$,
- wing aspect-ratio $A\!\!\!R = 9.0$,
- span loading efficiency $k = 1.4$ and
- approach maximum lift coefficient $C_{Lmax} = 2.97$.

i. Show that the aircraft is indeed speed unstable during the approach phase. Note that FAR regulations require the approach speed $V_{appr} = 1.3V_S$. [25%]

ii. Why would it be ill advised to leave the aircraft unmodified upon this finding? [25%]

iii. The chief engineer suggests the addition of a speed break to address the problem. What is the minimum frontal area required? You may assume the drag coefficient of a flat plate (non-dimensionalised by frontal area) to be $C_{Dfp} = 1.2$. [25%]

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Setter: E. Levis

1

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Marks

Question 1

(a) $\frac{dW}{dS} = -\frac{\dot{W}}{dS/dt} = \frac{-c \cdot F_N}{dS/dt}$ where S is ground covered. (5)

In the presence of a tailwind $\frac{dS}{dt} = V + V_{tw}$

So: $\frac{dW}{dS} = \frac{-c \cdot F_N}{V + V_{tw}}$ (10)

In steady, level flight $F_N = D$ & $L = W$

$$\frac{dW}{dS} = \frac{-c\,D}{V+V_{tw}} = \frac{-c \cdot \bar{D} \cdot W}{(V+V_{tw})(L/D)_{max}} \qquad D = \frac{D}{D_{min}} \frac{D_{min}}{W} W = \frac{\bar{D}\,W}{(L/D)_{max}}$$

From datasheet $V = \bar{V} \cdot V_{mD}$, $V_{tw} = \bar{V}_{tw} V_{mD}$ & $\bar{D} = \frac{1}{2}(\bar{V}^2 + \bar{V}^{-2})$

$$\frac{dW}{dS} = \frac{-c \cdot W (\bar{V}^2 + \bar{V}^{-2})}{2 V_{mD} (L/D)_{max} (\bar{V} + \bar{V}_{tw})}$$ (20)

Integrating & assuming $(L/D)_{max}$, c, $\bar{V}$, V_{mD} are constant (2)

$$\int_0^1 dS = \frac{-2 V_{mD} (L/D)_{max}}{c} \frac{\bar{V} + \bar{V}_{tw}}{\bar{V}^2 + \bar{V}^{-2}} \int_0^1 \frac{dW}{W}$$ (5)

So

$$R = \frac{-2 V_{mD} (L/D)_{max}}{c} \frac{\bar{V} + \bar{V}_{tw}}{\bar{V}^2 + \bar{V}^{-2}} \ln\left(\frac{W_2}{W_1}\right)$$

(b) To maximise range $dR/d\bar{V} = 0$. Ignoring constants.

$$\frac{d}{d\bar{V}}\left(\frac{\bar{V} + \bar{V}_{tw}}{\bar{V}^2 + \bar{V}^{-2}}\right) = \frac{(\bar{V}^2 + \bar{V}^{-2}) - (\bar{V} + \bar{V}_{tw})(2\bar{V} - 2\bar{V}^{-3})}{(\bar{V}^2 + \bar{V}^{-2})^2}$$ (10)

Working with the numerator only & substituting in that $\bar{V}_{tw} = -\frac{\bar{V}}{2}\left(\frac{\bar{V}^4 - 3}{\bar{V}^4 - 1}\right)$

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2

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Q1(b) continued

$$\bar{V}^2+\bar{V}^{-2}-\left[\bar{V}-\frac{\bar{V}}{2}\left(\frac{\bar{V}^4-3}{\bar{V}^4-1}\right)\right](2\bar{V}-2\bar{V}^{-3}) =$$

$$= \bar{V}^2+\bar{V}^{-2}-\left[2\bar{V}^2-2\bar{V}^{-2}-\bar{V}^2\left(\frac{\bar{V}^4-3}{\bar{V}^4-1}\right)+\bar{V}^{-2}\left(\frac{\bar{V}^4-3}{\bar{V}^4-1}\right)\right] =$$

$$= -\bar{V}^2+3\bar{V}^{-2}+(\bar{V}^2-\bar{V}^{-2})\left(\frac{\bar{V}^4-3}{\bar{V}^4-1}\right) \quad \text{multiply by } \bar{V}^2$$

$$= -\bar{V}^4+3+(\bar{V}^4-1)\frac{(\bar{V}^4-3)}{(\bar{V}^4-1)} = 0$$ => It is indeed the relation that will give max Range. 25

-15 for mistake in math.

(c) $V_{MD}=673\,km/h$, $V_{tw}=320\,km/h$ $\Big\}$ $\bar{V}_{tw}\simeq 0.475$ 5

So from graph $\bar{V}$ best range for given V_{tw} is $\simeq 1.205$ 5

If aerodynamic & propulsive efficiency the same

$$\left[\frac{\bar{V}+\cancelto{0}{\bar{V}_{tw}}}{\bar{V}^2+\bar{V}^{-2}}\ln\left(\frac{W_{fin}}{W_{int}}\right)\right]_{no\ wind} = \left[\frac{\bar{V}+\bar{V}_{tw}}{\bar{V}^2+\bar{V}^{-2}}\ln\left(\frac{W'_{fin}}{W_{init}}\right)\right]_{tw}$$ 10

For no tail wind condition $\bar{V}_{best\,R}=\sqrt[4]{3}$ or from graph ≈ 1.315

$$0.57\ln\left(\frac{W_{fin}}{W_{init}}\right)=0.7848\ln\left(\frac{W'_{fin}}{W_{int}}\right)$$ $W_{fin}=184\times10^3\,kg$, $W_{int}=222\times10^3\,kg$

0.82

$$\Rightarrow \ln\left(\frac{W'_{fin}}{W_{int}}\right) = \cancel{-0.12978}\ \ -0.1363 \Rightarrow W'_{fin}\simeq \cancel{195}\ 193\times10^3\,kg$$ 5

So fuel saved is ~~11×10³~~ kg

9×10³.

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3

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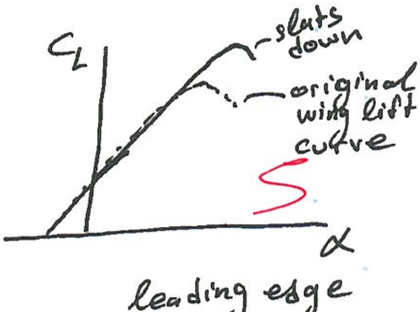
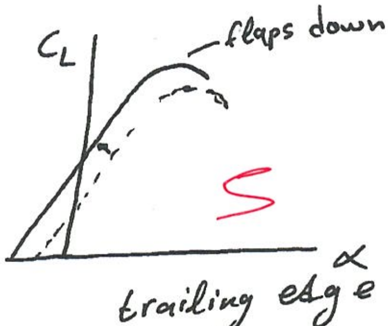
Question 2

(a) Leading-edge devices largely affect the maximum wing C_L by increasing the stall angle of attack 5

Trailing-edge devices impart increases in C_{Lmax} by adding camber thus increasing C_{Lmax} & zero lift ~~AoA~~ AoA. 5





5 5 (20)

Leading edge devices often used to increase stall AoA to reasonable values following effect of flaps & to alleviate nose down pitching moments. (5)

(b)(i) First we find $V_s = \sqrt{\frac{2W_L/S}{\rho_0 C_{Lmax}}} = \sqrt{\frac{2\times 4450}{1.225\times 2.97}} = 49.46\,m/s$ 8 (10)

therefor $V_A = 1.3\,V_s \approx 64.3\,m/s$ 2

Next calculate $V_{imD} = \sqrt{\frac{2\,W/S}{\rho_0}}\left(\frac{k}{\pi AR\,C_{D_0}}\right)^{1/4}$

$= \sqrt{\frac{2\times 4450}{1.225}}\left(\frac{1.4}{\pi\cdot 9\times 0.12}\right)^{1/4} = 68.315\,m/s$ (10)

since $V_A < V_{imD}$ aircraft is speed-unstable. (5)

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LAST PAGE 4

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Question 2(b) continued

b)(ii) Operating in the speed unstable region means that following a reduction in velocity the aircraft's natural responce would be to slow down forther as now $D > F_N$. When operating close to V_s & to the ground that is not a good idea. 15 10

iii) For the aircraft to be speed stable $V_A \geq V_{imp}$

$$V_A \geq \sqrt{\frac{2\, W/S}{\rho_0}} \left[\frac{K}{\pi AR (C_{D_0} + \Delta C_{D_0})}\right]^{1/4}$$

where ΔC_{D_0} is the drag added by the speed breaks.

Solving for ΔC_{D_0} we get

$$\Delta C_{D_0} \geq \frac{K}{\pi AR}\left(\frac{2\, W/S}{\rho_0 V_A^2}\right)^2 - C_{D_0}$$

Substituting in values $\Delta C_{D_0} \geq 0.0329$ 10

Aircraft C_D values non-dimensionalized by wing S_{ref}. Knowing that C_D of flat plate is 1.2,

$$\tfrac{1}{2}\rho_0 V^2 S_{ref} \Delta C_{D_0} = \tfrac{1}{2}\rho_0 V^2 A_{fp} C_{D_{fp}} \quad \& \quad S_{ref} = \frac{m_L \cdot g}{W_L/S}$$

$$\rightarrow A_{fp} = \frac{\Delta C_{D_0} \cdot S_{ref}}{C_{D_{fp}}} = \frac{\Delta C_{D_0} \cdot m_L \cdot g}{C_{D_{fp}} \cdot W_L/S} = 2.123\,m^2$$ 15

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY & MEDICINE

M Eng Part I Mechanics of Flight – Aircraft Performance

Data and Formulae

The total drag coefficient is given by

$$C_D = C_{D_0} + \frac{k}{\pi \mathrm{AR}} C_L^2.$$

The total drag curve is given by

$$D = \left(A \cdot V_i^2 + B \cdot \frac{n^2}{V_i^2} \right),$$

where n is the load factor ($n=1$ for steady level flight) and

$$\mathrm{A} = C_{D_0} \tfrac{1}{2} \rho_0 S$$

$$\mathrm{B} = \frac{\mathrm{kW}^2}{\left(\frac{1}{2}\rho_0 S\right)(\pi \mathrm{AR})}.$$

The minimum drag and EAS for minimum drag are

$$D_{\min} = 2n\sqrt{AB} \quad \text{and} \quad V_{imD} = (B/A)^{1/4} n^{1/2}.$$

V_{imD} and D_{min} are used to define the relative speed and relative drag respectively:

$$\overline{V} = \frac{V_i}{V_{imD}\big|_{n=1}} \quad \text{and} \quad \overline{D} = \frac{D}{D_{\min}\big|_{n=1}}.$$

where $\overline{D}$ satisfies

$$\overline{D} = \frac{1}{2}\left(\overline{V}^2 + \frac{n^2}{\overline{V}^2} \right)$$

The maximum lift-to-drag ratio, which is constant for a given aircraft at a given configuration and weight, is

$$(L/D)_{\max} = \frac{L}{D_{\min}} = \frac{1}{2}\sqrt{\frac{\pi \mathrm{AR}}{kC_{D_0}}} = \frac{1}{2}\frac{W}{\sqrt{AB}}$$

For steady level flight, thrust equals drag and the drag equation becomes

$$V_i^4 - \frac{F_N}{A} V_i^2 + \frac{B}{A} = 0,$$

from which two EAS solutions can be obtained:

$$V_{i1}^2 = \frac{F_N}{2A} + \sqrt{\left(\frac{F_N}{2A}\right)^2 - \frac{B}{A}} \quad \text{and} \quad V_{i1}^2 = \frac{F_N}{2A} - \sqrt{\left(\frac{F_N}{2A}\right)^2 - \frac{B}{A}}.$$

In steady level flight in the stratosphere, ceiling density ratio is given by

$$\sigma_{\max alt} = \frac{2\sqrt{AB}}{1.439 F_{N0}}.$$

The range of an aircraft flying at constant velocity and constant lift coefficient is

$$R_1 = \left(\frac{V_{\mathrm{mD,1}}(L/D)_{\max}}{c}\right)\left[\frac{2}{\overline{\mathrm{V}}+\overline{\mathrm{V}}^{-3}}\right]\ln\left(\frac{W_{init}}{W_{fin}}\right).$$

Range of an aircraft flying at constant density and constant lift coefficient:

$$R_2 = \left(\frac{V_{\mathrm{mD,1}}(L/D)_{\max}}{c}\right)\left(\frac{4\overline{\mathrm{V}}}{\overline{\mathrm{V}}^2+\overline{\mathrm{V}}^{-2}}\right)\left(1-\sqrt{\frac{W_{fin}}{W_{init}}}\right)$$

In both cases, maximum range is obtained by flying at $\overline{V} = \sqrt[4]{3}$.

The gradient of climb is

$$(L/D)_{\max}\sin(\Gamma) = \tau - \frac{1}{2}\left(\overline{V}^2 + \frac{1}{\overline{V}^2}\right).$$

The rate of climb is

$$(L/D)_{\max}\,\overline{v}_c = \tau\cdot\overline{V} - \frac{1}{2}\left(\overline{V}^3 + \frac{1}{\overline{V}}\right)$$

where $\overline{v}_c = \dfrac{v_c}{V_{mD}}$.

List of Symbols:

AR	= wing aspect ratio (= b^2/S)
b	= wing span
c	= specific fuel consumption (Weight of fuel consumption per unit thrust per sec.)
C_D	= drag coefficient
C_{D_0}	= lift-independent drag coefficient
C_L	= lift coefficient
D	= drag
D_{min}	= drag at the minimum drag speed
F_N	= thrust
F_{N0}	= thrust at sea-level
k	= loading efficiency
L	= lift
$(L/D)_{max}$	= maximum lift-to-drag ratio
R	= range, for steady level flight
S	= wing surface area
σ	= air density ratio
τ	= thrust-to-drag ratio (= $F_N/D_{\mathrm{min}}\|_{n=1}$)
V	= true airspeed (TAS)
V_i	= equivalent airspeed (EAS)
V_{imD}	= EAS for minimum drag
$V_{\mathrm{mD,1}}$	= TAS for minimum drag at beginning of cruise (when weight is W_{init})
W_{init}	= aircraft weight at the beginning of the cruise
W_{fin}	= aircraft weight at the end of the cruise

	SI Units	The Standard Atmosphere							
Alt. Feet	Alt. m.	T/K	a	ρ	σ	√σ	p/p0	p (Pa)	μ
0	0	288.2	340.7	1.225	1.0000	1.0000	1.0000	101325	1.795E-05
1640	500	284.9	338.8	1.167	0.9529	0.9762	0.9421	95463	1.779E-05
3281	1000	281.7	336.9	1.112	0.9075	0.9526	0.8870	89878	1.763E-05
4921	1500	278.4	334.9	1.058	0.8638	0.9294	0.8346	84561	1.747E-05
6562	2000	275.2	332.9	1.007	0.8217	0.9065	0.7846	79502	1.731E-05
8202	2500	271.9	331.0	0.957	0.7812	0.8838	0.7371	74690	1.715E-05
9843	3000	268.7	329.0	0.909	0.7422	0.8615	0.6920	70117	1.699E-05
11483	3500	265.4	327.0	0.863	0.7048	0.8395	0.6491	65774	1.683E-05
13123	4000	262.2	325.0	0.819	0.6688	0.8178	0.6084	61651	1.666E-05
14764	4500	258.9	323.0	0.777	0.6342	0.7964	0.5698	57739	1.650E-05
16404	5000	255.7	320.9	0.736	0.6010	0.7753	0.5332	54031	1.633E-05
18045	5500	252.4	318.9	0.697	0.5692	0.7545	0.4986	50519	1.617E-05
19685	6000	249.2	316.8	0.660	0.5387	0.7339	0.4658	47193	1.600E-05
21325	6500	245.9	314.7	0.624	0.5094	0.7137	0.4347	44047	1.583E-05
22966	7000	242.7	312.7	0.590	0.4814	0.6938	0.4054	41073	1.566E-05
24606	7500	239.4	310.6	0.557	0.4545	0.6742	0.3776	38264	1.549E-05
26247	8000	236.2	308.4	0.525	0.4289	0.6549	0.3515	35612	1.532E-05
27887	8500	232.9	306.3	0.495	0.4043	0.6358	0.3268	33112	1.515E-05
29528	9000	229.7	304.2	0.467	0.3808	0.6171	0.3035	30755	1.498E-05
31168	9500	226.4	302.0	0.439	0.3584	0.5987	0.2816	28536	1.480E-05
32808	10000	223.2	299.8	0.413	0.3371	0.5806	0.2610	26448	1.463E-05
34449	10500	219.9	297.6	0.388	0.3167	0.5627	0.2417	24486	1.445E-05
36089	**11000**	**216.66**	**295.4**	**0.364**	**0.2972**	**0.5452**	**0.2235**	**22644**	**1.427E-05**
37730	11500	216.7	295.5	0.336	0.2747	0.5241	0.2066	20931	1.428E-05
39370	12000	216.7	295.5	0.311	0.2539	0.5039	0.1909	19344	1.428E-05
41010	12500	216.7	295.5	0.287	0.2346	0.4844	0.1764	17877	1.428E-05
42651	13000	216.7	295.5	0.266	0.2168	0.4657	0.1631	16522	1.428E-05
44291	13500	216.7	295.5	0.245	0.2004	0.4477	0.1507	15270	1.428E-05
45932	14000	216.7	295.5	0.227	0.1852	0.4303	0.1393	14112	1.428E-05
47572	14500	216.7	295.5	0.210	0.1712	0.4137	0.1287	13042	1.428E-05
49213	15000	216.7	295.5	0.194	0.1582	0.3977	0.1190	12053	1.428E-05
50853	15500	216.7	295.5	0.179	0.1462	0.3824	0.1099	11139	1.428E-05
52493	16000	216.7	295.5	0.166	0.1351	0.3676	0.1016	10295	1.428E-05
54134	16500	216.7	295.5	0.153	0.1249	0.3534	0.0939	9514	1.428E-05
55774	17000	216.7	295.5	0.141	0.1154	0.3397	0.0868	8793	1.428E-05
57415	17500	216.7	295.5	0.131	0.1067	0.3266	0.0802	8127	1.428E-05
59055	18000	216.7	295.5	0.121	0.0986	0.3140	0.0741	7510	1.428E-05
60696	18500	216.7	295.5	0.112	0.0911	0.3018	0.0685	6941	1.428E-05
62336	19000	216.7	295.5	0.103	0.0842	0.2901	0.0633	6415	1.428E-05
63976	19500	216.7	295.5	0.095	0.0778	0.2789	0.0585	5928	1.428E-05
65617	20000	216.7	295.5	0.088	0.0719	0.2682	0.0541	5479	1.428E-05

NB: Conversion factors - 1 knot = 1.689 ft/s = 0.5148 m/s