Imperial College London

MEng (Engineering) Examination 2016

Year 1

AE1-109 Mechanics

Tuesday 31st May 2016: 14.00 to 16.00
[2 hours]

The paper is divided into Section A and Section B
and contains ***FOUR*** questions.
All questions carry equal marks.
Candidates may obtain full marks for complete answers to ***ALL*** questions.

You must answer each section in a separate answer booklet.

A data sheet is attached.

The use of lecture notes is NOT allowed.

Section A

1) (a) A uniform 80 kg bar *AB* with small frictionless end rollers is supported by the horizontal and vertical surfaces as illustrated in figure 1. A wire connects point *A* and point *C* with frictionless pin joints at point *A* and point *C*.

(i) Draw the free body diagram.

(ii) Calculate the tension T in the wire and the forces at point *A* and at point *B*.

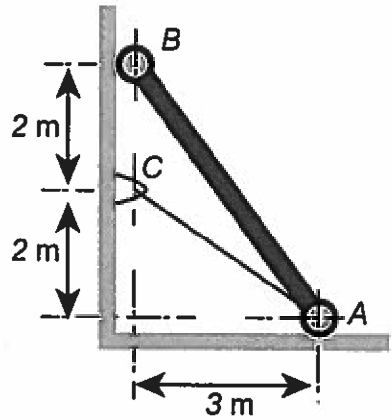

Figure 1

[30%]

b) A tension *T* of magnitude 10 kN is applied to the cable attached to the top *A* of a rigid mast and secured to the ground at B as shown in figure 2. The mast is rigidly glued to the ground at point *O*. The cable is attached with frictionless pin joints at points *A* and *B*.

(i) Express the force *T* as a vector.

(ii) Using a vector-based solution, determine the moment M_z created by *T* about the z-axis passing through the base *O*.

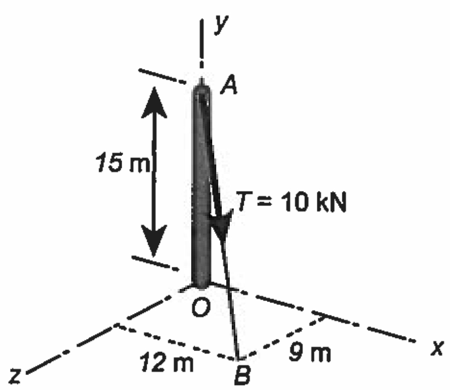

Figure 2

[70%]

2) a) A uniform 60 kg bar AB is subjected to force P with smooth guides at B as illustrated in figure 3. The rod rests on a flat surface at a single point of contact. The static friction coefficient at A is 0.8.

(i) If $P = 400$ N, find the friction force at A.

(ii) Find the force P required to cause slippage at A.

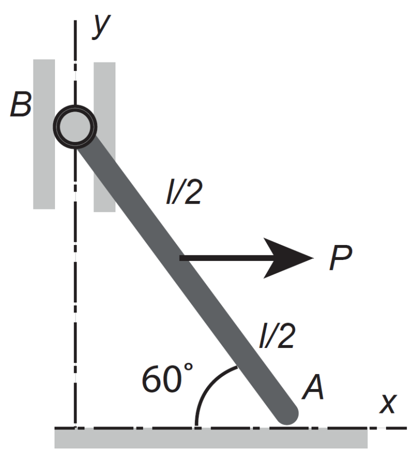

Figure 3

[40%]

b) A small object is released from rest at A and slides down the smooth circular surface of radius R to a conveyor B as illustrated in figure 4. Determine an expression for the normal contact force N between the smooth circular surface and the object in terms of θ. Specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the object transfers to the conveyor. [60%]

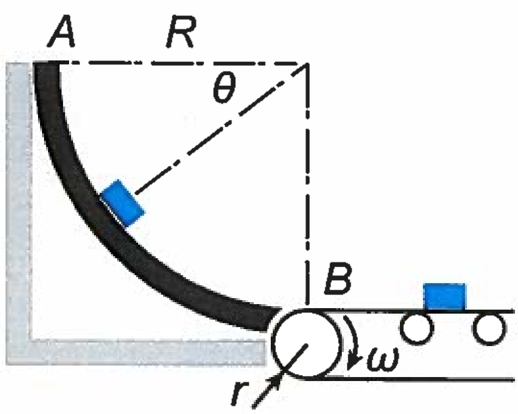

Figure 4

Section B

3. The left hand side of Figure 5 shows a massless rigid plate which is attached to a rigid base by means of a spring of stiffness $k = 10$ N/m and a damper with damping constant c. A ball of mass $m = 0.1$ kg is firmly glued to the rigid plate. When the combined ball and rigid plate are displaced and then released, it is observed that the amplitude of displacement is reduced to 90% of its initial value over a single period of oscillation. Contact between all surfaces is frictionless and effects of gravity may be neglected throughout.

   (a) By deriving an expression for the logarithmic decrement from first principles, show that the value of the damping constant must be $c = 0.0335$ Ns/m. [50%]

   The combined ball and rigid plate are then displaced by 10 mm towards the rigid base before being released.

   (b) Determine an expression for the resulting displacement of the mass with respect to time. [30%]

   At a point during the oscillation the ball becomes detached causing it to travel away from the rigid plate at a velocity of 50 m/s. After travelling a distance the ball strikes a frictionless surface oriented as shown at $45°$ to the horizontal.

   (c) Given the coefficient of restitution of the impact is $\epsilon = 0.5$ determine the angle of the ball's rebound with respect to the contacting surface. [20%]

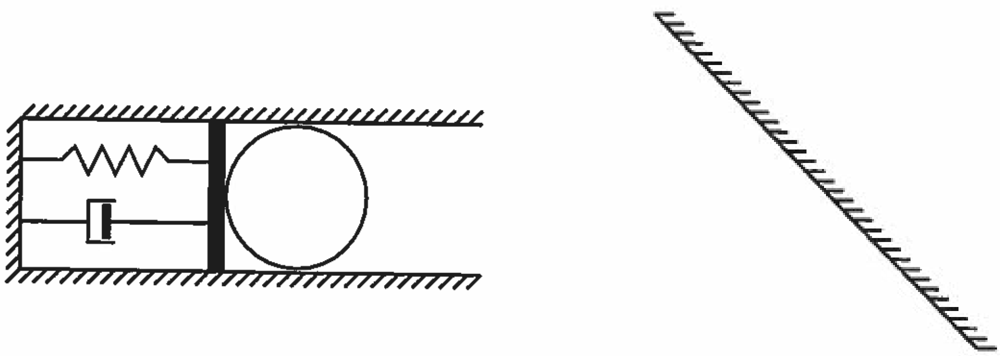

Figure 5

4. There is an increasing problem with space debris around the Earth. At the end of a satellite's life it is now a requirement that it be removed from orbit to prevent further accumulation of debris. This may be carried out, as shown in Figure 6, either by bringing the satellite back to Earth in an elliptical crash trajectory causing it to burn up in the atmosphere, or by transitioning it into an escape trajectory.

Consider the general case when the satellite is initially in a circular orbit of radius r_c.

(a) Determine an expression for the absolute value of the velocity impulse Δv_{escape} required to transition to an escape trajectory in terms of r_c. [20%]

(b) Show that the absolute value of the velocity impulse required to place the satellite into the crash trajectory is

$$\Delta v_{crash} = \sqrt{\frac{GM}{r_c}} - \sqrt{\frac{GM}{r_c}\frac{2R_E}{R_E + r_c}}$$

in which R_E is the radius of the Earth and GM is the product of the universal gravitational constant and the Earth's mass. [40%]

(c) Determine the radius of initial orbit as a multiple of R_E at which the most efficient means of debris removal changes from the crash trajectory to the escape trajectory. [30%]

(d) Finally, starting from Kepler's 2nd law, determine an expression for the time taken for the satellite to complete one full orbit in its original station. [10%]

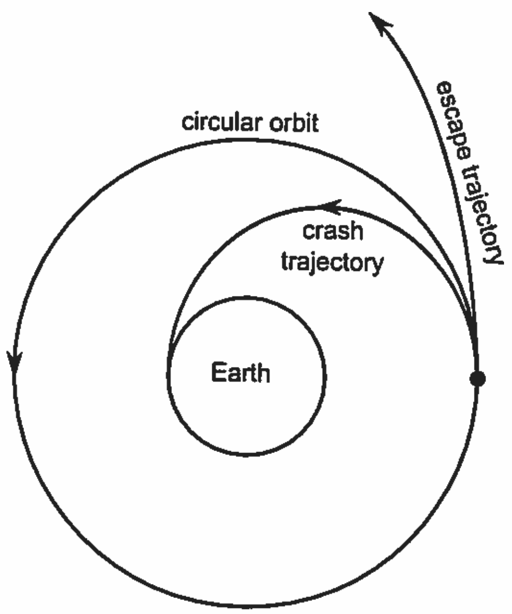

Figure 6

AE1-109 – Mechanics Data Sheet

Imperial College of Science, Technology & Medicine

Department of Aeronautics

Kepler's Laws of Planetary Motion

Kepler's 1st Law:

$$r = \frac{h^2}{GM(1 + e\cos\phi)}$$

Kepler's 2nd Law:

$$\frac{dA}{dt} = \frac{h}{2}$$

Kepler's 3rd Law:

$$GM = \left(\frac{2\pi}{T}\right)^2 a^3$$

In the above, h is specific angular momentum, G is the universal gravitational constant, M is the mass of the object at the focus, e is trajectory eccentricity, r is radial distance from the focus, and ϕ is the angle from the position at periapsis.

Conservation of Specific Energy

Specific energy is conserved in any given trajectory.

$$\frac{1}{2}v^2 - \frac{GM}{r} = E$$

In the above, v is the speed of the particle, and E is the specific energy. All other variables are as defined above.

Constants

Radius of Earth	$R_E = 6371$ km
Radius of Mars	$R_M = 3396$ km
Radius of Venus	$R_V = 6052$ km
Distance between the Earth and the Sun	$R_{E-S} = 149.6 \times 10^6$ km
Mass of Earth	$M_E = 5.972 \times 10^{24}$ kg
Mass of Sun	$M_S = 1.989 \times 10^{30}$ kg

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Setter: Mirko Kovac

1

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1)

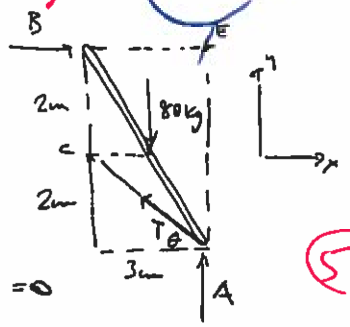
a.

294.3 N

10%

10

$\theta = \tan^{-1}\frac{2}{3} = 33.7°$



$\circlearrowleft \sum M_A = 0 : \quad 4B - 80\,(3/2) = 0$

$B = 30\ kg$

294

353

88

$\circlearrowleft + \sum M_C = 0 : \quad (T\cos 33.7°)4 - 80\,(3/2) = 0$

$T = 36.1\ kg$

5% for kg if was correct

20%

20

$\sum F_y = 0 : \quad A + 36.1 \sin 33.7° - 80 = 0$

$A = 60\ kg$

$T = 36.1\ kg = 354.1\ N$

588.6 N

b.

Box 40%

−10 if not whole

$T = T n_{AB} = 10\left[\frac{12i - 15j + 9k}{\sqrt{(12)^2 + (-15)^2 + 9^2}}\right] = 10\,(0.566i - 0.707j + 0.424k)\ kN$

40

$\begin{pmatrix} 5656.9 \\ -7071.1 \\ 4242.6 \end{pmatrix}$

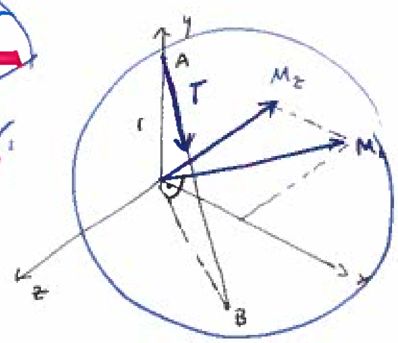
$\boxed{M_0 = r \times F}$ $\quad M_0 = 15j \times 10\,(0.566i - 0.707j + 0.424k)$

$= 150\,(-0.566k + 0.424i)\ kN\cdot m$

$\rightarrow M_z = M_0 \cdot k = 150\,(-0.566k + 0.424i)\cdot k = -84.9\ kNm$

$\rightarrow M_z = -84.9k\ kNm$

30



Box 40%

$\frac{1}{75\sqrt{2}}\begin{pmatrix} 12 \\ -15 \\ 5 \end{pmatrix}$ $= \checkmark$ $\quad -60\sqrt{2}$

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2)

a. 40 pts

$W = mg = 60(9.81) = 589\ N$

(i) $P = 400N$. Assume equil.

$\Sigma F_y = 0$; $N_A - 589 = 0$, $N_A = 589\ N$

$\Sigma M_c = 0$; $400 \frac{l}{2} \sin 60^\circ + 589 \frac{l}{2} \cos 60 - F l \sin 60^\circ = 0$

$F = 370 N$ 10%.

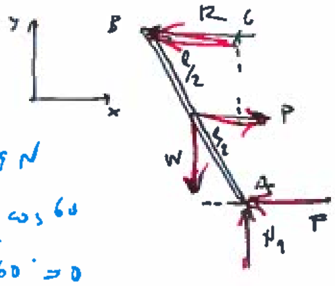
(ii) $F = \mu_s N_A = 471\ N$

$\Sigma M_c = 0$; $P \frac{l}{2} \sin 60^\circ + 589 \frac{l}{2} \cos 60^\circ - 471 (l \sin 60^\circ) = 0$

$P = 602\ N$ 10%.

20%

20%



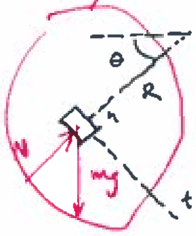
b) 60 pts.

The free body diagram of the object is shown together with the coordinate directions n & t. The normal force N depends on the n-component of the acceleration, which in turn, depends on the velocity. The velocity will be cumulative according to the tangential acceleration a_t. Hence, we will find at first for any general position. 20%

$[\Sigma F_t = m a_t]$ $mg \cos\theta = m a_t$ $a_t = g \cos\theta$

$[v\,dv = a_t\,ds]$ $\int_0^v v\,dv = \int_0^\theta g \cos\theta\, d(R\theta)$

$v^2 = 2gR\sin\theta$



We obtain the normal force by summing forces in the positive n-direction, which is the direction of the n-component of acceleration

$[\Sigma F_n = m a_n]$ $N - mg \sin\theta = m \frac{v^2}{R}$ $N = 3mg \sin\theta$

conveyor pulley must turn at rate $v = r\omega$ for $\theta = \frac{\pi}{2}$ so that

$\omega = \sqrt{2gR/r}$ 20%

$\sqrt{\frac{2gR}{r}}$

$\omega_c = \sqrt{\frac{Rg}{r}}$

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Setter: Matthew Santer   Solutions corrected. MS

3 i

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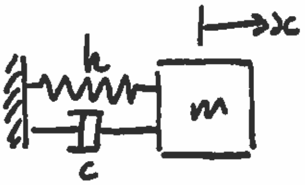
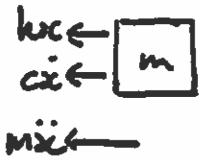
Marks

Q3.



f.b.d



$$m\ddot{x} + c\dot{x} + kx = 0$$ 10

$$\ddot{x} + 2\gamma\dot{x} + \omega^2 x = 0$$

try $x = Ae^{\alpha t}$

$\dot{x} = A\alpha e^{\alpha t}$

$\ddot{x} = A\alpha^2 e^{\alpha t}$

$$Ae^{\alpha t}(\alpha^2 + 2\gamma\alpha + \omega^2) = 0$$ 10

$$\alpha = -\gamma \pm \sqrt{\gamma^2 - \omega^2}$$

$$x = e^{-\gamma t}\left(A_1 e^{\sqrt{\gamma^2-\omega^2}\,t} + A_2 e^{-\sqrt{\gamma^2-\omega^2}\,t}\right)$$

Question implies an underdamped system

$$x(t) = e^{-\omega\zeta t}\left(A_1 e^{i\Omega t} + A_2 e^{-i\Omega t}\right)$$ 10

$$\Omega = \omega\sqrt{1-\zeta^2} \qquad \omega = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{c}{2\sqrt{km}}$$

$$x(t) = e^{-\omega\zeta t}\left(C_1 \cos\Omega t + C_2 \sin\Omega t\right)$$

$$x(t+T) = e^{-\omega\zeta(t+T)}\left(C_1 \cos\Omega t + C_2 \sin\Omega t\right)$$

$$\frac{x(t)}{x(t+T)} = e^{\omega\zeta T} \qquad \ln\left(\frac{x(t)}{x(t+T)}\right) = \omega\zeta T = \Delta$$

$$\Delta = \frac{\omega\zeta 2\pi}{\omega\sqrt{1-\zeta^2}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$ 10

Bookwork

~~Derivation is straightforward, but remembered formulae is acceptable.~~ change.

cont.

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Marks

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Q3 cont.

a) $x(t+T) = 0.9\, x(t)$ $\qquad \dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 0.1054$

$$\zeta = 0.017$$

$m = 0.1$ kg
$k = 10$ N/m $\qquad \zeta = \dfrac{c}{2\sqrt{km}} \Rightarrow c = 0.034$ Ns/m — 10 (50)
$\omega = 10$ rad/s

b)
$$x(t) = e^{-\omega\zeta t}(C_1 \cos\Omega t + C_2 \sin\Omega t)$$

at $t = 0$, $x = -10$ mm
$\dot{x} = 0$

$\zeta\omega = 0.17$
$\Omega \approx 10$

$$\dot{x}(t) = e^{-\omega\zeta t}(C_2\Omega\cos\Omega t - C_1\Omega\sin\Omega t) - \omega\zeta e^{-\omega\zeta t}(C_1\cos\Omega t + C_2\sin\Omega t)$$ — 10

$0 = C_2\Omega - \omega\zeta C_1$

$-10 = C_1$

$C_2 = \dfrac{\omega\zeta C_1}{\Omega} = \cancel{-1.7}\ -0.17$ — 10

change.

$$x(t) = e^{-0.17t}(-10\cos 10t - \cancel{1.7}\ 0.17 \sin 10t)$$ — 10

cont

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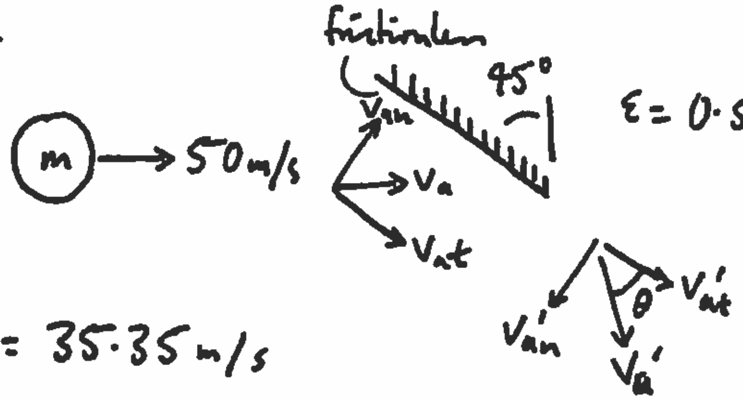
Marks

Q3 cont



$\varepsilon = 0.5$

$V_{an} = 35.35\ m/s$

$V_{at} = 35.35\ m/s$

$V'_{at} = 35.35\ m/s$

$V'_{an} = -\varepsilon V_{an} = -17.67\ m/s$

$\theta = 26.56°$

20

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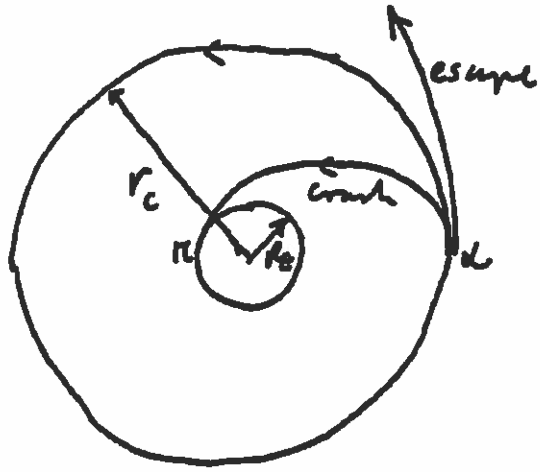
	Marks

Q4.



$$r = \left(\frac{h^2}{GM}\right)\frac{1}{1+e\cos\phi}$$

a) circular orbit $\quad r_c = \dfrac{h^2}{GM} = \dfrac{r_c^2 V_c^2}{GM} \quad V_c = \sqrt{\dfrac{GM}{r_c}}$ — 5

escape trajectory $e = 1$ $\quad r_e = \dfrac{h^2}{2GM} = \dfrac{r_c^2 V_e^2}{2GM} \quad V_e = \sqrt{\dfrac{2GM}{r_c}}$ — 5

$$\Delta V_{escape} = \sqrt{\frac{2GM}{r_c}} - \sqrt{\frac{GM}{r_c}}$$ — 10

b) $r_\pi = R_E \quad \phi = 0$

$r_\alpha = r_c \quad \phi = 180°$

$$r_\pi = \left(\frac{h^2}{GM}\right)\frac{1}{1-e} = r_c$$

$$r_\alpha = \left(\frac{h^2}{GM}\right)\frac{1}{1+e} = R_E$$ — 10

$$\frac{1+e}{1-e} = \frac{r_c}{R_E} \qquad e = \frac{r_c - R_E}{R_E + r_c}$$ — 10

cont.

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Q4 cont.

* change.

$$r_c = \left(\frac{h^2}{GM}\right)\frac{1}{1-e} = \frac{V_{crash}^2 r_c^2}{GM}\,\frac{1}{1-\frac{r_c - R_E}{R_E + r_c}}$$

$$r_c = \frac{V_{crash}^2 r_c}{GM}\,\frac{R_E + r_c}{2R_E}$$

$$V_{crash} = \sqrt{\frac{GM}{r_c}\left(\frac{2R_E}{R_E + r_c}\right)}$$ 10

$$\Delta V_{crash} = \sqrt{\frac{GM}{r_c}\left(\frac{2R_E}{R_E + r_c}\right)} - \sqrt{\frac{GM}{r_c}}$$ negative as requires a slow down 5

$$|\Delta V_{crash}| = \sqrt{\frac{GM}{r_c}} - \sqrt{\frac{GM}{r_c}\left(\frac{2R_E}{R_E + r_c}\right)}$$ 5

c) $$\sqrt{\frac{GM}{r_c}} - \sqrt{\frac{GM}{r_c}\left(\frac{2R_E}{R_E + r_c}\right)} = \sqrt{\frac{2GM}{r_c}} - \sqrt{\frac{GM}{r_c}}$$ 10

$$\sqrt{\frac{GM}{r_c}\left(\frac{2R_E}{R_E + r_c}\right)} = (2-\sqrt{2})\sqrt{\frac{GM}{r_c}}$$

$$\frac{R_E}{R_E + r_c} = \frac{(2-\sqrt{2})^2}{2}$$

$$r_c = 4.828\,R_E$$ 20

cont.

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	Marks
Q4 cont	
d) $\frac{dA}{dt} = \frac{h}{2} = \frac{A}{T} = \frac{\pi r_c^2}{T} = \frac{V_c r_c}{2}$	5
$T = \frac{2\pi r_c}{V_c} \qquad V_c = \left(\frac{GM}{r_c}\right)^{\frac{1}{2}}$	
$T = \frac{2\pi r_c^{3/2}}{\sqrt{GM}}$	5

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