IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

EEE PART I: MEng, BEng and ACGI

ELECTRONIC MATERIALS

Corrected Copy

Monday, 19 May 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : W.T. Pike

Second Marker(s) : T.J. Tate

Special instructions for students

Fundamental constants

Permittivity of free space, ε_0 = 8.85 x 10^{-12} F/m
Permeability of free space, μ_0 = 4π x 10^{-7} H/m
Planck's constant, h = 6.6 x 10^{-34} Js
Boltzmann's constant, k = 1.38 x 10^{-23} J/K
Electron charge, e = 1.6 x 10^{-19} C
Electron mass, m = 9.1 x 10^{-31} kg
Speed of light, c = 3.0 x 10^{8} ms^{-1}

Schrödinger's equation

General form:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r})+V(\mathbf{r})\psi(\mathbf{r})=E\psi(\mathbf{r})$$

In one dimension:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

In spherical coordinates:

$$\nabla^2=\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}$$

Free-electron theory

Density of states (3D):

$$g(E)=\frac{1}{\pi^2\hbar^3}(m)^{3/2}\sqrt{2E}$$

Fermi energy

$$E_f=\frac{\hbar^2\pi^2}{2m}\left(\frac{3n}{\pi}\right)^{2/3}$$

Fermi distribution

$$f(E)=\frac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$$

Electrons in semiconductors

Effective mass:

$$m_e^*=\frac{\hbar^2}{d^2E(k)/dk^2}$$

Concentration of electrons in a semiconductor of bandgap E_g:

$$n = \frac{1}{\sqrt{2}\hbar^3}\left(\frac{m_e^* kT}{\pi}\right)^{3/2} e^{-\frac{(E_g - E_f)}{kT}}$$

$$= N_c e^{-\frac{(E_g - E_f)}{kT}}$$

Concentration of holes

$$p = \frac{1}{\sqrt{2}\hbar^3}\left(\frac{m_h^* kT}{\pi}\right)^{3/2} e^{-\frac{E_f}{kT}}$$

$$= N_v e^{-\frac{E_f}{kT}}$$

Polarization

Lorentz correction for local field:

$$\mathbf{E}_{loc} = \mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}$$

Electronic polarization:

$$P_0 = \frac{\varepsilon_0 \omega_p^2 E_0}{\omega_m^2 - \omega^2 + j\omega\gamma}$$

where

$$\gamma = \frac{r}{m},$$

$$\omega_m^2 = \omega_0^2 - \frac{\omega_p^2}{3},$$

$$\omega_0^2 = k/m,$$

$$\omega_p^2 = \frac{ne^2}{m\varepsilon_0}.$$

Orientational Polarization:

Static:

$\mathrm{P} = n\mu\,\mathrm{L}(\mu E/kT)$ where $\mathrm{L}(x) = \coth(x) - 1/x$

Dynamic:

$$P_0 = \frac{P_s}{1 + \mathrm{j}\omega\tau},$$

Magnetism

Magnet dipole due to electron angular momentum:

$$\mu_m = -\frac{e\mathbf{L}}{2m}$$

Magnet dipole due to electron spin:

$$\mu_m = -\frac{e\mathbf{S}}{m}$$

Paramagnetism:

$$M = n\mu_m \mathrm{L}\left(\frac{\mu_m \mu_0 H}{kT}\right)$$

The Questions

1. [Compulsory]

a) A device contains a large resistor fabricated from a block of conductive material with two contacts on opposite faces. To miniaturise the resistor, its height, width and length are halved. How does its resistance change? [4]

b) Explain how the Hall effect can produce a voltage whose sign depends on the charge of the current carrier. [4]

c) What is the angle between the (100) and the (111) planes of silicon? [4]

d) For a block of elastic material use diagrams to define Young's modulus, the shear modulus and Poisson's ratio. [8]

e) The equation for the vertical deflection v of a cantilever of length l, at distance x from its support, under a load P at its end, can be expressed by $v = \frac{P}{EI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right)$. Define E and I and derive an expression for the angular deflection from the horizontal of the end of the cantilever. [8]

f) Explain how polarisation at the atomic scale of a dielectric material between the plates of a capacitor can increase the capacitance. [4]

g) Draw a diagram of a magnetic tape write head, illustrating its principle of operation. [8]

2.

a) Show that a solution of the one-dimensional Schrödinger equation for wavefunction $\psi(x)$ for a particle of mass m and total energy E in a constant potential V,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V\psi(x)=E\psi(x)$$

is $\psi(x) = A\exp(jkx)$ (where A is a constant) with the wavenumber, k, of the particle given by:

$$k=\pm\frac{\sqrt{2m(E-V)}}{\hbar}.$$ [10]

b) A particle of energy E is travelling in a positive direction in a 1D potential which is 0 for $x < 0$ and a constant V, where $V > E$ *for* $x > 0$. Show that the probability of finding the particle at any negative value of x is constant but the probability decays exponentially with distance for positive x with the probability of the particle reaching a distance $x = L$ given by

$$\exp\left(-2L\frac{\sqrt{2m(V-E)}}{\hbar}\right).$$ [12]

c) Electrons accelerated by 2 V are incident on a potential energy barrier 2.1 eV high. How thick must the barrier be to ensure that only half the electrons tunnel through? [8]

3.

a) Draw labelled band diagrams for

(i) a typical conductor,

(ii) an intrinsic semiconductor and

(iii) a dielectric.

All have the same Fermi energy. The diagrams should label the bands and indicate what regions of the bands have occupied, unoccupied or partially occupied states at room temperature. [9]

b) In a metal what is the occupancy of an electron state located at the Fermi energy? For an intrinsic semiconductor of bandgap 1 eV, calculate the occupancy of the states at the bottom of the conduction band. Show that a 10 C rise in temperature will increase the concentration of electrons in the conduction band of the semiconductor by a factor of nearly two. [9]

c) Using band diagrams explain how doping can be used to alter the occupancy of states in a semiconductor and therefore affect the concentration of electrons and holes. [12]

4.

a) Sketch the Langevin function L(x), noting that for small x, L(x) ~ x/3. Using the formula given for paramagnetic materials describe the magnetisation of a paramagnet in response to an external magnetic field. Indicate how the orientation of the atomic magnetic dipoles corresponds to the observed response. [12]

b) In Weiss theory the response of a ferromagnet to an external magnetic field can be modelled by replacing H by $(H + \lambda M)$, where λ is known as the Weiss constant. Show by making a suitable substitution that the ferromagnetic response can be rewritten as

$$\alpha x - \beta H = L(x), \qquad (4.1)$$

and find expressions for α and β. [8]

c) Hence by graphically solving 4.1 show that in the absence of a magnetic field, a ferromagnet can exhibit permanent magnetism if the temperature of the magnet is less than a critical temperature given by $T_C = \dfrac{\lambda n \mu_0 \mu_m^2}{3k}$. [10]