

Paper Number(s): **E1.6**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE/ISE PART I: M.Eng., B.Eng. and ACGI

COMMUNICATIONS I

Wednesday, 22 May 10:00 am

There are FIVE questions on this paper.

Answer THREE questions.

Corrected Copy

Time allowed: 2:00 hours

Examiners responsible:

First Marker(s): Gurcan, M.K.

Second Marker(s): Yeatman, E.M.

Useful equations

If the signal $g(t)$ is periodic with period $T_0 = \frac{2\pi}{\omega_0}$. The trigonometric Fourier series is

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \quad \text{where } a_0 = \frac{1}{T_0} \int_0^{T_0} g(t) dt, \text{ and}$$

$$a_n = \frac{2}{T_0} \int_0^{T_0} g(t) \cos(n\omega_0 t) dt \quad \text{for } n \geq 1 \quad b_n = \frac{2}{T_0} \int_0^{T_0} g(t) \sin(n\omega_0 t) dt \quad \text{for } n \geq 1$$

The compact Fourier series is given by

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n), \quad C_n = \sqrt{a_n^2 + b_n^2}, \quad C_0 = a_0 \text{ and } \theta_n = \tan^{-1} \frac{-b_n}{a_n}$$

The exponential Fourier series is given by

$$g(t) = \sum_{n=-\infty}^{\infty} D_n \exp(jn\omega_0 t) \quad \text{where}$$

$$D_n = \frac{1}{T_0} \int_0^{T_0} g(t) \exp(-jn\omega_0 t) dt \quad \text{or} \quad D_n = \frac{C_n}{2} \exp(j\theta_n) \quad \text{and} \quad D_{-n} = \frac{C_n}{2} \exp(-j\theta_n).$$

Useful Fourier Transformations

$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) \exp(j\omega t) d\omega$	\Leftrightarrow	$G(\omega) = \int_{-\infty}^{\infty} g(t) \exp(-j\omega t) dt$
$g(-t)$	\Leftrightarrow	$G(-\omega)$
$\exp(-at) u(t)$	\Leftrightarrow	$\frac{1}{a + j\omega}$
$\cos \omega_0 t$	\Leftrightarrow	$\pi [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$
$\sin \omega_0 t$	\Leftrightarrow	$j\pi [\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
$\text{rect}\left(\frac{t}{\tau}\right)$	\Leftrightarrow	$\tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$
$\frac{W}{\pi} \text{sinc}(Wt)$	\Leftrightarrow	$\text{rect}\left(\frac{\omega}{2W}\right)$
$g(t) = \Delta\left(\frac{t}{\tau}\right)$	\Leftrightarrow	$\frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right)$
$g(t \pm T)$	\Leftrightarrow	$G(\omega) \exp(\pm j\omega T)$
$\frac{1}{2j} [g(t+T) - g(t-T)]$	\Leftrightarrow	$G(\omega) \sin(\omega T)$
$g(t) \cos \omega_c t$	\Leftrightarrow	$\frac{1}{2} [G(\omega - \omega_c) + G(\omega + \omega_c)]$

Two useful integrals : $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$ and

$$\int \frac{x^2}{x^2 + a^2} dx = x - a \tan^{-1} \frac{x}{a}$$

- 1 a) Find E_x and E_y , the energies of the signals $x(t)$ and $y(t)$ shown in Fig. 1.1. Sketch the signals $x(t) + y(t)$ and $x(t) - y(t)$ and calculate the energies of either of these two signals. [5]



Figure 1.1.

- b) A periodic signal $g(t)$ is expressed by the following Fourier series:

$$g(t) = 3 \cos(t) + \sin\left(5t - \frac{\pi}{6}\right) - 2 \cos\left(8t - \frac{\pi}{3}\right).$$

- Sketch the amplitude and phase spectra for the compact Fourier series. [2]
- By inspection of spectrum in part (i), write the exponential Fourier series for $g(t)$. [2]
- By inspection of spectra in part (ii), sketch the double-sided magnitude spectrum for $g(t)$. [2]
- By inspection of coefficients in part (iii), plot the double-sided power spectral density for $g(t)$. Using the power spectral density coefficients calculate the signal power. [1]

- c) Consider the time waveform, $g(t) = e^{-at} u(t)$ where $a > 0$, given in Figure 1.2.

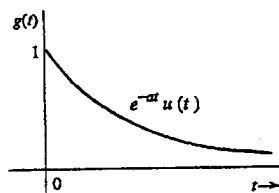


Figure 1.2.

- Find the Fourier transform of $g(t)$. [2]
- Using $g(-t) \Leftrightarrow G(-\omega)$ and the result of part (i), find the Fourier transform of $e^{at} u(-t)$ and $e^{-|at|}$. [2]
- Verify Parseval's theorem for the signal $g(t) = e^{-at} u(t)$. [2]
- Estimate the essential bandwidth W rad/s of the signal $g(t) = e^{-at} u(t)$ if the essential band is required to contain 95% of the signal energy. [2]

- 2 a) The signal in Figure 2.1 is modulated with carrier $\cos(10t)$. Find the Fourier transform of this signal using the appropriate properties of the Fourier transform given in the Fourier transformation table on page 1. Sketch the magnitude spectrum for the signal given in Figure 2.1. *Hint:* This function can be expressed in the form $g(t) \cos \omega_0 t$. [6]

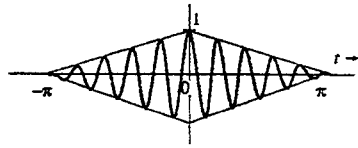


Figure 2.1.

- b) A certain channel has ideal amplitude, but non-ideal phase response (Figure 2.2) [7]
given by

$$|H(\omega)| = 1$$

$$\theta_h(\omega) = -\omega t_0 - k \sin \omega T \quad k \ll 1$$

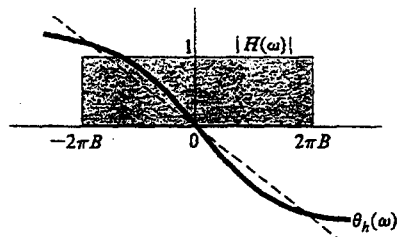


Figure 2.2.

Show that $y(t)$, the channel response to an input pulse $g(t)$ band-limited to B Hz, is

$$y(t) = g(t - t_0) + \frac{k}{2} [g(t - t_0 - T) - g(t - t_0 + T)]$$

Hint: use $\exp(-jk \sin \omega T) \approx 1 - jk \sin \omega T$

- c) Find the mean square value (or power) of the output voltage $y(t)$ of the system [7]

shown in Figure 2.3 if the input voltage power spectral density is $S_x(\omega) = \text{rect}\left(\frac{\omega}{2}\right)$.

Calculate the power (mean square value) of the input signal $x(t)$.

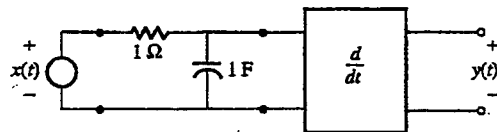


Figure 2.3.

- 3 a) Two signals $m_1(t)$ and $m_2(t)$, both band-limited to 5000 rad/s, are to be transmitted simultaneously over a channel by the multiplexing scheme shown in Figure 3.1. The signal at point b is the multiplexed signal, which now modulates a carrier of frequency 20,000 rad/s. The modulated signal at point c is transmitted over a channel.

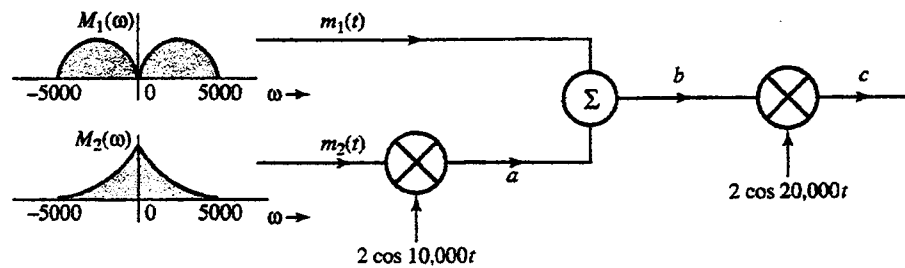


Figure 3.1.

- i) Sketch the amplitude spectra at points a , b , and c . [3]
 - ii) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ from the modulated signal at point c . [4]
- b) For each of the following baseband signals: (I) $m_1(t) = \cos 1000t$; (II) $m_2(t) = 2 \cos 1000t - \cos 2000t$; (III) $m_3(t) = \cos(1000t) \cos(3000t)$:
- i) sketch the amplitude spectra of $m_i(t)$, $i = 1, 2, 3$. [2]
 - ii) sketch the amplitude spectrum of the DSB-SC signals $m_i(t) \cos(10000t)$, $i = 1, 2, 3$. [3]
 - iii) identify the upper sideband (USB) and the lower sideband (LSB) of the spectra. [2]
 - iv) identify the frequencies in the baseband, and the corresponding frequencies in the DSB-SC USB, and LSB spectra. [1]
- c) Figure 3.2 shows a scheme for coherent (synchronous) demodulation. Show that this scheme can demodulate the full AM signal $[A + m(t)] \cos \omega_c t$ regardless of the value of A . Show that a scheme that can demodulate DSB-SC can also demodulate full AM. Is the converse true? Explain. [5]

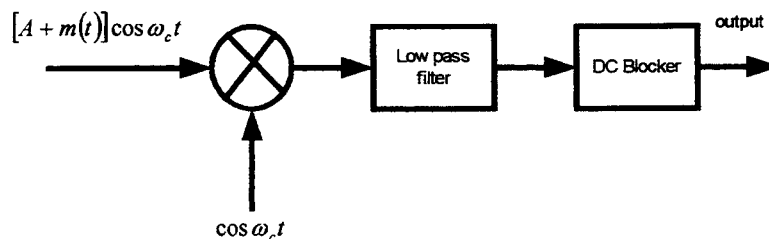


Figure 3.2.

- 4 a) A frequency-modulated signal with carrier frequency $\omega_c = 2\pi \times 10^5$ rad/s is described by the equation $S_{FM} = 10 \cos(\omega_c t + 5 \sin 3000 t + 10 \sin 2000\pi t)$
- Find the power of the modulated signal. [1]
 - Find the frequency deviation Δf and the modulation index β . [2]
 - Estimate the bandwidth of S_{FM} . [2]
- b) Consider the signal $m(t)$, given in Figure 4.1. A modulating signal is generated by passing the signal $m(t)$ through a filter, which lets through the first 3 harmonics of the fundamental frequency and maintains the maximum and minimum signal amplitudes. Estimate the bandwidth for a frequency modulated signal when the filtered modulating signal is used to modulate a carrier of $f_c = 100$ MHz. Assume that the angular frequency deviation constant is $k_f = 2\pi \times 10^5$ rad/Vs. [5]

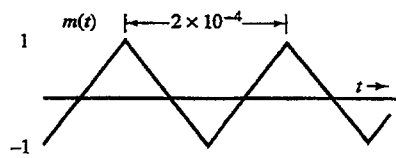


Figure 4.1.

- c) A signal $g(t)$ band-limited to B Hz is sampled by a periodic pulse train $P_{T_s}(t)$ made up of a unity amplitude rectangular pulse of width $1/8B$ seconds (centered at the origin) repeating at the Nyquist rate ($2B$ pulses per second). Show that the sampled signal $\bar{g}(t)$ is given by [5]
- $$\bar{g}(t) = \frac{1}{4} g(t) + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) g(t) \cos(n\omega_s t), \quad \omega_s = 4\pi B.$$
- d) A compact disc (CD) records each audio channel signals digitally by using PCM. Assume the audio signal bandwidth to be 15 kHz.
- What is the Nyquist rate? [1]
 - If the Nyquist samples are quantised into $L = 65,536$ levels and then binary coded, determine the number of binary digits required to encode a sample. [2]
 - Determine the number of binary digits per second (bit/s) required to encode the audio signal. [1]
 - For practical reasons, signals are sampled at a rate well above the Nyquist rate. Practical CDs use 44,100 samples per second. If $L = 65,536$, determine the number of bits per second required to encode the signal, and the minimum bandwidth required to transmit the encoded signal. [1]

- 5 a) In Figure 5.1, the input signal is $\phi(t) = m(t)$ and the amplitude satisfies the relationship $A \gg |\phi(t)|$. The two diodes are identical with a resistance r ohms in the conducting mode and infinite resistance in the cut-off mode. Figure 5.2 shows the diode current versus voltage characteristics.

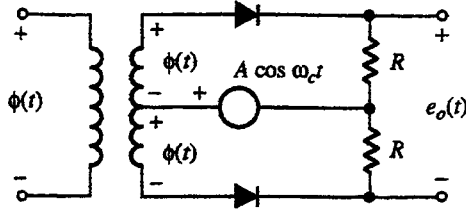


Figure 5.1

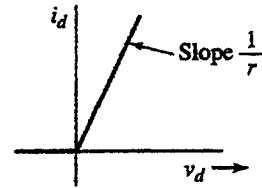


Figure 5.2

- i) Show that the output $e_o(t)$ is given by

[4]

$$e_o(t) = \frac{2R}{R+r} w(t) m(t)$$

where $w(t)$ is the switching periodic signal shown in Figure 5.3 with amplitude spectrum given in Figure 5.4.

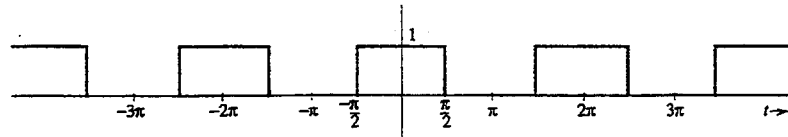


Figure 5.3

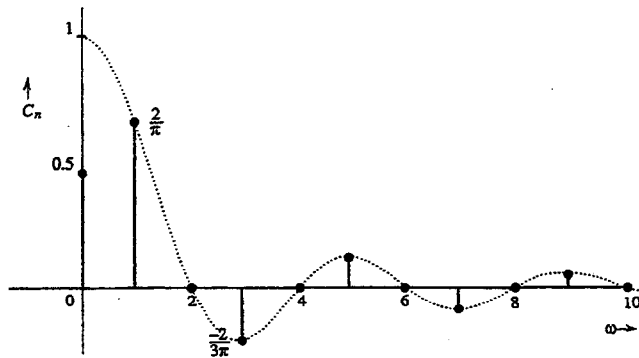


Figure 5.4.

- ii) Show that this circuit can be used as a DSB-SC modulator. [3]
 iii) How would you use this circuit as a synchronous demodulator for DSB-SC signals. [2]
- b) A frequency-modulated signal is given as $u(t) = 100 \cos[2000 \pi t + \phi(t)]$ where $\phi(t) = 5 \sin(20 \pi t)$.

Question continued over

Using the values of $J_n(5)$ for $n=0,1,\dots,5$ given in the following table,

n	0	1	2	3	4	5
$J_n(5)$	-0.178	-0.328	0.047	0.365	0.391	0.261

- i) determine and sketch the amplitude spectrum [4]
 - ii) determine the percentage of the modulated signal power carried by these harmonics [1]
 - iii) From Carlson's rule, determine the approximate bandwidth for the FM signal. [2]
- c) An analogue signal, whose amplitude is uniformly distributed between $-A$ and $+A$ V, is to be quantized using a uniform quantizer, which has the transfer function shown in Figure 5.5. [5]

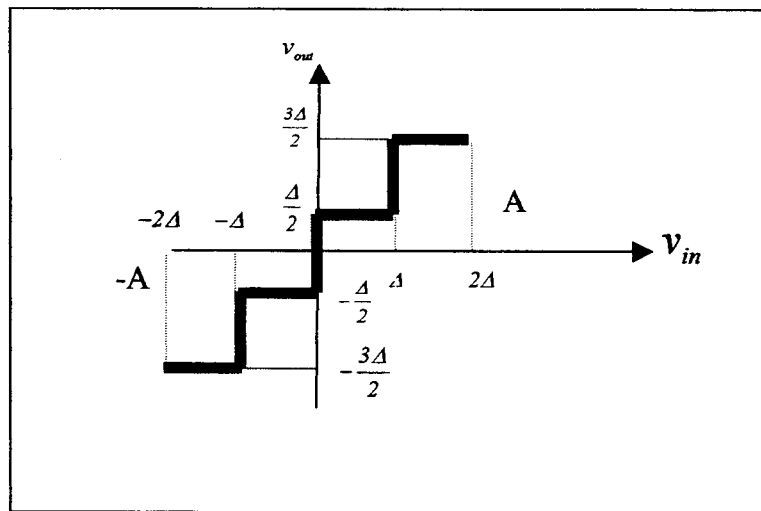
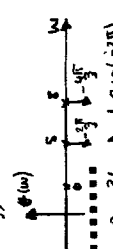
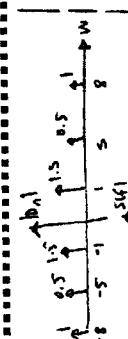



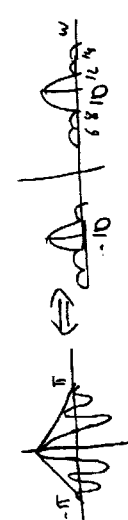
Figure 5.5

Find the quantization noise power in terms of the maximum signal amplitude.

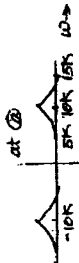
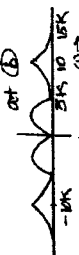
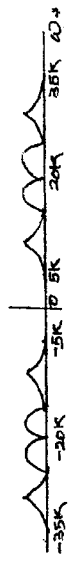
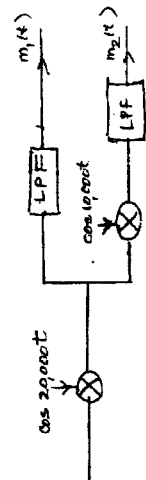
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MODEL ANSWER and MARKING SCHEME			
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Second Examiner Dr. E. M. Yeatman		Question	1-c Page 2 out of 15
Question labels in left margin		Marks allocations in right margin	
1-a	<div>$E_x = \int_{-\infty}^{\infty} (1)^2 dt = 2$$E_y = \int_{-\infty}^{\infty} (1)^2 dt = 2$$E_z = \int_{-\infty}^{\infty} (1)^2 dt = 2$$E_{xy} = E_x E_y = 4$</div>		
1-b	<div>$g(t) = 3 \cos t + \sin (5t - \frac{\pi}{6}) - 2 \cos (8t - \frac{\pi}{6})$<p>For compact trigonometric form all terms must have cosine form and amplitudes must be positive. We re-write $g(t)$. as</p>$g(t) = 3 \cos t + \cos (5t - \frac{\pi}{6} - \frac{\pi}{2}) + 2 \cos (8t - \frac{\pi}{6} - \pi)$$= 3 \cos t + \cos (5t - \frac{2\pi}{3}) + 2 \cos (8t - \frac{7\pi}{6})$</div>		
ii)	<div>$p_0 = p_1 = p_2 = p_3 = p_4 = p_5 = 0, \quad p_1 = 2/a, \quad p_2 = \frac{1}{2} \exp(j \frac{\sqrt{3}}{2})$$p_3 = \frac{1}{2} \exp(j \frac{\sqrt{3}}{2}), \quad p_4 = \exp(j \frac{\sqrt{3}}{2}), \quad p_5 = \exp(j \frac{\sqrt{3}}{2})$$g(t) = \sum_{k=-\infty}^{\infty} p_k \exp(j k t)$</div>		
iii)			
iv)	<div>$P = 2 (2.25 + 0.5) = 2.75 = 7$</div>		

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Question labels in left margin		Marks allocations in right margin	
i-c	$\exp(-at) \cdot u(t) \Leftrightarrow \frac{1}{a+j\omega}$ $g(-t) \Leftrightarrow G(-\omega)$ $G(\omega) = \frac{1}{a-j\omega}$ $G(-\omega) = \frac{1}{a+j\omega}$ $g(t) = \exp(-at) u(t) \quad (a > 0)$ $E_g = \int_{-\infty}^{\infty} g^2(t) dt = \int_{-\infty}^{\infty} \exp(-2at) dt = \frac{1}{2a}$ $G(\omega) = \frac{1}{a+j\omega}$ $E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) ^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\omega^2 + a^2} d\omega$ $= \frac{1}{2\pi a} \tan^{-1} \frac{\omega}{a} \Big _{-\infty}^{\infty} = \frac{1}{2a}$		
ii)			
iii)			
iv)			

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Question labels in left margin			
2-b			
$Y(\omega) = G(\omega) \text{rect}\left(\frac{\omega}{4\pi B}\right) \exp(-j(\omega t_0 + k \sin \omega t))$ $\approx G(\omega) \text{rect}\left(\frac{\omega}{4\pi B}\right) [1 - jk \sin \omega t] \exp(-j\omega t_0)$ <p>This follows from the fact that</p> $\exp(x) \approx 1+x \text{ when } x \ll 1.$ <p>More over, $G(\omega) \text{rect}\left(\frac{\omega}{4\pi B}\right) = G(\omega)$</p> <p>Because $G(\omega)$ is band limited to B Hz.</p> <p>Hence</p> $Y(\omega) = G(\omega) \exp(-j\omega t_0) - jk G(\omega) \sin \omega t \exp(-j\omega t_0)$ <p>from Fourier transform table</p> $\frac{1}{2j} [g(t-t_0) - g(t-t_0-\tau)] \Leftrightarrow G(\omega) \sin \omega \tau$ <p>Hence</p> $y(t) = g(t-t_0) + \frac{k}{2} [g(t-t_0-\tau) - g(t-t_0+\tau)]$			

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Second Examiner Dr. E. M. Yeatman	Question 2-a	Marks allocations in right margin	
Question labels in left margin			
2-a	<p>The signal $g(t)$ is a triangular pulse $\Delta\left(\frac{t}{2\pi}\right)$</p> <p>$g(t) = \Delta\left(\frac{t}{2\pi}\right) \cos(10t)$</p> <p>From Fourier transformation table</p> <p>$\Delta\left(\frac{t}{2\pi}\right) \Leftrightarrow \pi \text{sinc}^2\left(\frac{\pi\omega}{2}\right)$</p> <p>From the modulation property</p> <p>$g(t) = \Delta\left(\frac{t}{2\pi}\right) \cos(10t) \Leftrightarrow \frac{\pi}{2} \left\{ \text{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \text{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right] \right\}$</p> <p></p> <p></p>		

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Question labels in left margin			
2-c	<p>The ideal differentiator transfer function is $j\omega$. Hence the transfer function of the entire system is</p> $H(\omega) = \frac{1}{1+j\omega} \cdot j\omega = \frac{j\omega}{1+j\omega} \quad \text{and}$ $ H(\omega) ^2 = \frac{\omega^2}{\omega^4 + 1}$ $\langle X^2(t) \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} \text{rect}\left(\frac{\omega}{2}\right) d\omega = \frac{1}{\pi} \int_0^{\pi} d\omega = \frac{1}{\pi}$ $\langle Y^2(t) \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} \text{rect}\left(\frac{\omega}{2}\right) \frac{\omega^2}{\omega^4 + 1} d\omega$ $= \frac{1}{\pi} \int_0^1 \frac{\omega^2}{\omega^4 + 1} d\omega = \frac{1}{\pi} \left(1 - \frac{\pi}{4}\right)$ $= 0.0683$		

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Second Examiner Dr. E. M. Yeatman	Question 3-a	Marks allocations in right margin	
Question labels in left margin			
3-a	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>at (a)</p>  </div> <div style="text-align: center;"> <p>at (b)</p>  </div> </div> <div style="text-align: center; margin-top: 20px;"> <p>at (c)</p>  </div> <p>The above figure shows the signals at points a, b and c.</p> <div style="text-align: center; margin-top: 20px;">  </div> <p>The receiver to recover $m_1(t)$ and $m_2(t)$ from the received signal is shown above.</p>		

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Question labels in left margin	Marks allocations in right margin

3-c $g(t) = [A + m(t)] \cos \omega_c t$. Hence,

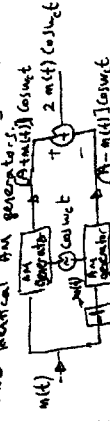
The signal at the output of multiplier is
 $g(t) = [A + m(t)] \cos \omega_c t$
 $= \frac{1}{2} [A + m(t)] + \frac{1}{2} [A + m(t)] \cos 2\omega_c t$

The first term is a baseband signal because its spectrum is centered at $\omega = 0$. The low-pass filter allows this term to pass, but suppresses the second term, whose spectrum is centered at $\pm 2\omega_c$. Hence the output of the low pass filter is

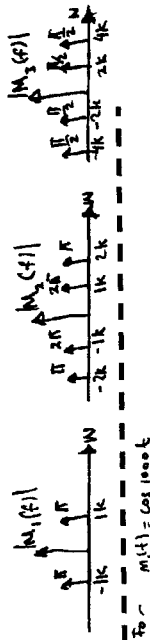
$$y(t) = A + m(t)$$

When this signal is passed through a dc blocker, the dc term A is suppressed yielding the output $m(t)$. This shows that the system can demodulate AM signal regardless of the value of A. This is a synchronous or coherent demodulator.

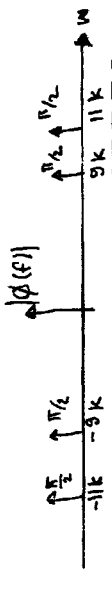
When an input to a DSB-SC generator is $m(t)$, the corresponding output is $m(t) \cos \omega_c t$. Clearly if the input is $A + m(t)$, the corresponding output will be $[A + m(t)] \cos \omega_c t$. This is precisely the AM signal. Thus by adding a dc of value A to the baseband signal $m(t)$, we can generate AM signal using a DSB-SC generator. The converse is generally not true. However we can generate DSB-SC using AM generators if we use two identical AM generators.



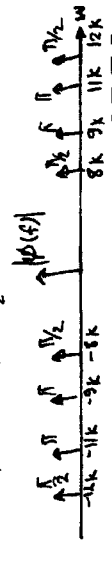
First Examiner: Dr. Mustafa Gurcan	Paper Code: EI.6
Second Examiner: Dr. E. M. Yetman	Question 3-b
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Question labels in left margin	Marks allocations in right margin



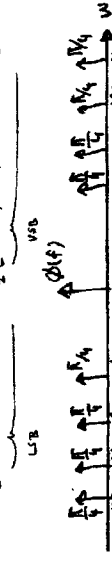
$\phi(t) = m_1(t) \cos 10,000t = \cos 10,000t \cos 10,000t = \frac{1}{2} [\cos 20,000t + \cos 0]$



For $m_2(t) = 2 \cos 1000t + \cos 2000t$
 $\phi(t) = m_2(t) \cos 10,000t = 2 \cos 1000t \cos 10,000t + \cos 2000t \cos 10,000t$
 $= \cos 9,000t + \cos 11,000t + \frac{1}{2} [\cos 12,000t + \cos 8,000t]$



For $m_3(t) = \cos 1000t \cos 3000t$
 $\phi(t) = m_3(t) \cos 10,000t = \cos 10,000t \cos 3,000t \cos 10,000t = \frac{1}{2} [\cos 13,000t + \cos 7,000t]$



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<p> $\Delta f = 10^5 \text{ kHz}$ $T_0 = 2 \cdot 10^{-4} \text{ s}, f_0 = \frac{1}{T_0} = 5 \cdot 10^3$ Signal bandwidth $= B = 3 \cdot f_0 = 15 \cdot 10^3$ Modulated signal bandwidth $= 2(\Delta f + B) = 2(10^5 + 15 \cdot 10^3)$ $= 2(100 + 15) \cdot 10^3 = 230 \cdot 10^3 \text{ Hz}$ $= 230 \text{ kHz}$. </p>		<p> The pulse train is a periodic signal with fundamental frequency 28 kHz. Hence $\omega_s = 2\pi(28) = 4\pi B$. The period is $T_0 = 1/28$. It is an even function of t. Hence the Fourier series for the pulse train can be expressed as $p(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos n\omega_s t$ Using $c_0 = \frac{1}{T_0} \int_{T_0} g(t) dt$ $c_n = \frac{2}{T_0} \int_{T_0} g(t) \cos n\omega_s t dt \quad n=1, 2, \dots$ and $b_n = \frac{2}{T_0} \int_{T_0} g(t) \sin n\omega_s t dt \quad n=1, 2, \dots$ we get $c_0 = c_n = \frac{1}{T_0} \int_{-T_0/4}^{T_0/4} dt = \frac{1}{4}$ $c_n = c_n = \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} \cos n\omega_s t dt = \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right)$ </p>	

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<p> The signal bandwidth is the highest frequency mHz. $B = \frac{2000\pi}{2\pi} = 1000 \text{ Hz}$ The carrier amplitude is 10, and the power is $P = \frac{10^2}{2} = 50$ To find the frequency deviation Δf, we find the instantaneous frequency w_i given by $w_i = \frac{d}{dt} \theta(t) = \omega_c + 15,000 \cos 2000t + 20,000 \sin 2000t$ The carrier deviation is $15,000 \text{ rad/s}$ and the maximum value of this expression is $15,000 + 20,000$. This is the maximum carrier deviation $\Delta \omega$. Hence, $\Delta f = \frac{\Delta \omega}{2\pi} = 12,387.32 \text{ Hz}$ </p>		<p> $B = \frac{\Delta f}{B} = \frac{12,387.32}{1000} = 12.387$ </p>	
<p> Bandwidth $= 2(\Delta f + B) = 2(12.387 + 1) = 26.774 \text{ MHz}$ </p>		<p> $w_i = \omega_c + k_f m(t)$ Applying throughout by 2π, we have the equation in terms of the variable f. The instantaneous frequency is $f_i = f_c + \frac{k_f}{2\pi} m(t)$ $= 10^8 + 10^5 m(t)$ $(f_i)_{\min} = 10^8 + 10^5 (-1) = 99.9 \text{ MHz}$ $(f_i)_{\max} = 10^8 + 10^5 (1) = 100.1 \text{ MHz}$ </p>	

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4-d	<p>The bandwidth is 18 kHz. The Nyquist rate is 30 kHz.</p> <p>$65536 = 2^{16}$, so that 16 binary digits are needed to encode each sample.</p> <p>$30000 \times 16 = 480000 \text{ bits/s}$</p> <p>$48000 \times 16 = 768000 \text{ bits/s}$</p> <p>$R = 2B$</p> <p>$B = \frac{R}{2} = \frac{768000}{2} = 384000 \text{ Hz}$</p>			

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5-a	<p>The resistance of each diode is r ohms while conducting, and ∞ when off. When the carrier $A \cos \omega_c t$ is positive, the diodes conduct (during the entire positive half cycle), and when the carrier is negative the diodes are open (during the entire negative half cycle). Thus during the positive half cycle, the voltage $\frac{R}{R+r} \phi(t)$ appears across each of the resistors R. During the negative half cycle, the output voltage is zero. Therefore the diodes act as a gate in the circuit that is basically a voltage divider with a gain $\frac{R}{R+r}$. The output is therefore</p> $e_o(t) = \frac{2R}{R+r} w(t) m(t)$ <p>The period of $w(t)$ is $T_w = \frac{2\pi}{\omega}$. Using the Fourier series</p> $w(t) = \frac{1}{2} + \frac{2}{\pi} \left[\cos \omega_c t - \frac{1}{3} \cos 3\omega_c t + \frac{1}{5} \cos 5\omega_c t - \dots \right]$ <p>Fourier series</p> $w(t) = \frac{1}{2} + \frac{2}{\pi} \left[\cos \omega_c t - \frac{1}{3} \cos 3\omega_c t + \frac{1}{5} \cos 5\omega_c t - \dots \right]$ <p>The output $e_o(t)$ is</p> $e_o(t) = \frac{2R}{R+r} w(t) m(t)$ $= \frac{2R}{R+r} m(t) \left[\frac{1}{2} + \frac{2}{\pi} \left(\cos \omega_c t - \frac{1}{3} \cos 3\omega_c t + \frac{1}{5} \cos 5\omega_c t - \dots \right) \right]$			

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5-b	<p>The Fourier series expansion of $\exp(j\beta \sin(2\pi f_m t))$ is</p> $C_n = f_m \int_{-f_m}^{f_m} \exp(j\beta \sin(2\pi f_m t)) \exp(-j2\pi n f_m t) dt$ $= \frac{1}{4f_m} \int_{-f_m}^{f_m} \exp(j\beta \cos u - j\pi n u) \exp(j\pi n u) du$ $= \exp(j\frac{2\pi n}{2}) J_n(\beta)$ <p>Hence</p> $u(t) = A_c \operatorname{Re} \left[\sum_{n=-\infty}^{\infty} C_n \exp(j\pi n f_m t) \exp(j2\pi n f_c t) \right]$ $= A_c \operatorname{Re} \left[\sum_{n=-\infty}^{\infty} \exp(j2\pi (f_c + n f_m) t + \frac{n\pi}{2}) \right]$ <p>The magnitude and phase spectra of $u(t)$ for $\beta=5$ and frequencies in the interval $[950, 1050]$ are shown below</p>			

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5-a	<p>If we pass the output $e(t)$ through a low pass filter centered at ω_c, the filter suppresses the signal $m(t)$ and $m(t)\cos\omega_c t$ for all $n \neq 1$, leaving only the modulated term $\frac{A_c}{2} m(t) \cos\omega_c t$ intact. Hence the system acts as a demodulator.</p> <p>The same circuit can be used as a demodulator if we use a low pass filter at the output. In this case, the input is $q(t) = m(t)\cos\omega_c t$ and the output is $\frac{A_c}{2} m(t)$.</p> <p>$q(t) = [A + m(t)] \cos\omega_c t$. Hence,</p> <p>The signal at the output of multiplier is</p> $q(t) = [A + m(t)] \cos\omega_c t$ $= \frac{1}{2} [A + m(t)] + \frac{1}{2} [A + m(t)] \cos 2\omega_c t$ <p>The first term is a baseband signal because its spectrum is centered at $\omega=0$. The low-pass filter allows this term to pass, but suppresses the second term, whose spectrum is centered at $\pm 2\omega_c$. Hence the output of the low pass filter is</p> $y(t) = A + m(t)$ <p>When this signal is passed through a dc blocker, the dc term A is suppressed yielding the output $m(t)$. This shows that the system can demodulate AM signal regardless of the value of A. This is a synchronous or coherent demodulator.</p>			

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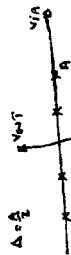
Question labels in left margin	Marks allocations in right margin
1. The following are the marks of 10 students in a class test. Find the mean marks.	10
2. The following are the marks of 10 students in a class test. Find the mean marks.	10
3. The following are the marks of 10 students in a class test. Find the mean marks.	10
4. The following are the marks of 10 students in a class test. Find the mean marks.	10
5. The following are the marks of 10 students in a class test. Find the mean marks.	10
6. The following are the marks of 10 students in a class test. Find the mean marks.	10
7. The following are the marks of 10 students in a class test. Find the mean marks.	10
8. The following are the marks of 10 students in a class test. Find the mean marks.	10
9. The following are the marks of 10 students in a class test. Find the mean marks.	10
10. The following are the marks of 10 students in a class test. Find the mean marks.	10

Marks allocations in right margin

5-c



$\Delta = \frac{1}{2}$



None over one star

$$\int_{-\frac{A}{2}}^{\frac{A}{2}} \left(\frac{A}{2} - x\right)^2 \frac{1}{2A} dx = \frac{b^3}{2A \cdot R_2}$$

even we have four levels

$$\sigma^2 = \text{Total var} = 40 \quad \frac{\Delta^2}{24 \times 12} \quad \text{where } \Delta = \frac{\Delta}{2}$$

$$\sigma^2 = \frac{13}{8.6 \text{ A}} = \frac{1^2}{48}$$

$$A=3$$

9/48
62