UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

MEng Honours Degree in Information Systems Engineering Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C480=I4.42

AUTOMATED REASONING

Wednesday 12 May 2004, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

NOTE: **In all four questions:**

- **variables begin with lower case u - z;**
- **names beginning with other lower case letters are functors;**
- **predicates begin with an upper case letter.**

1 a i) Why is "q(u) ==> p(u,v)" an incorrectly formed rewrite rule?

ii) What does it mean for a set of rewrite rules R to be confluent and terminating?

iii) Let R be a terminating and confluent set of rewrite rules and t and s be two ground terms. Explain how to decide whether, or not, t=s in the context of R. Justify your answer.

b i) Find a suitable ordering to show that equations (1) and (2) below can be oriented (to form new rules called (3) and (4)) so that they are terminating.

(1) f(g(x),z) = g(g(x))
(2) g(x) = f(x,x)

ii) Explain why rules (3) and (4) are also confluent.

c i) Apply the Knuth Bendix algorithm to generate a confluent set of rewrite rules from (3), (4) (found in part b(i)) and (5) (below).

(5) g(g(x)) = e

ii) What can you say about the result of rewriting any ground term using your final set of confluent rules obtained in part c(i) in the case that the signature of R consists only of {f, g, e}?

The three parts carry, respectively, 35%, 25%, 40% of the marks.

2 a i) State how the hyper-resolution refinement restricts resolution steps.

ii) Use hyper-resolution to derive the empty clause from the clauses (6) – (11). Explain the steps of your answer.

(6) ¬Q(w,z) v ¬P(w)
(7) S(y,a)
(8) ¬Q(c,b) v ¬R(a,c)
(9) Q(x,y) v Q(a,x) v Q(c,y)
(10) R(u,v) v ¬S(v,u) v ¬S(u,u)
(11) P(a)

iii) Explain why the Otter theorem prover would report "sos-empty", when presented with the following data. Your answer should show the effect on the *sos-list* and *usable-list* following each of the step(s) that Otter would be expected to take.

sos-list.	usable-list.	set(hyper_res).
¬P(x) v ¬Q(x).	P(a) v ¬Q(b).	clear(order_hyper).
Q(a).	Q(b).	
end_of_list.	end_of_list.	

In what way is the obtained result surprising?

b Suppose that steps are made in the connection graph proof procedure according to the prioritised criteria (I) – (V) (i.e. select according to (I) if possible, then (II), etc.).

(I) Select a link between 2 unit clauses.
(II) Remove an *inconsistent link.*
(III) Select a link between a unit clause and another literal L such that L is incident to the selected link only.
(IV) Select a link between literals L and M in 2 different clauses such that L and M are the only literals incident to the link.
(V) Select any link.

i) Briefly justify the above order of link selection. Your answer should also explain the italicised phrase in (II).

ii) Apply the procedure to clauses (6) – (11) above in order to derive the empty clause, choosing links according to the priorities expressed in (I) – (III). (There should be no need for (IV) or (V).)

The two parts carry, respectively, 55%, 45% of the marks.

3 a A new refinement for ground clausal tableau uses the following strategy. The closure rule is the same as for Model Elimination.

Prepare clauses: One literal in each clause is marked as the "selected literal".

Add-G: Let G be a ground atom not mentioned in the given clauses. For any two clauses C and D that have complementary respective selected literals L and ¬L, form at least one of G v C or G v D. G then becomes the selected literal in any clause in which it appears. The top clause will always be ¬G.

Extend leftmost open branch (B):The *selected literal* in the clause chosen for extension must close B, but may complement any literal in B, not just the leaf (as would be required in Model Elimination).

i) Suppose *only* the steps Prepare clauses and Extend leftmost open branch are to be used. Using the clauses (12) – (15) show that this strategy is not complete. (That is, for some top clause and some literal selection, a closed tableau cannot be found.) Explain your answer. **Hint**: begin with top clause ¬Q(b,b).

(12) ¬Q(b,b)
(13) ¬Q(b, g(b)) v ¬Q(g(b), b)
(14) Q(g(b),b) v Q(b, b)
(15) Q(b, g(b)) v Q(b,b)

ii) The strategy is complete if all three steps are enabled. Using the same selected literals as in part ai), apply the add-G rule and find a closed tableau.

iii) By considering the completeness of the locking resolution strategy, explain why the add-G rule will always apply to at least one pair of clauses.

b Give all the results of paramodulating f(f(x))=x (from left to right) into P(f(u),u).

c i) Give two equality axioms, which, together with clauses (16) – (18), are unsatisfiable and use the semantic tree method to show this. Explain your answer.

(16) a=b
(17) P(f(a))
(18) ¬ P(f(b))

ii) Use the tree found in part ci) to derive a resolution refutation of the equality axioms and clauses (16)-(18). Explain your answer.

The three parts carry, respectively, 40%, 15%, 45% of the marks.

4 a State how the application of the following rules differ between the standard and free-variable tableaux methods:

i) the (γ)-rule for universal quantifiers

ii) the closure rule.

b Use the basic Model Elimination (ME) tableau method to show that the clauses (19) – (21) are unsatisfiable. Make clear the steps of ME in your answer.

(19) $\neg P(y, x) \vee P(a,x)$
(20) $P(a, x) \vee P(b,x)$
(21) $\neg P(a, b) \vee \neg P(y, y)$

c i) Write down the set of all ground (Herbrand) instances of clauses (19) – (21).

ii) Use the Davis Putnam (DP) procedure to show the unsatisfiability of the instances given in ci). Your answer should make clear the steps of DP.

d Give all the factors of $P(x,y) \vee P(a,y) \vee P(z,z)$. Which (if any) are safe factors?

The four parts carry, respectively, 10%, 35%, 40%, 15% of the marks.