E2.JA DEVICES & FIELDS E2.JA - sec. A

2008 -

(1)

Answers EE2 Semiconductor Devices (2008)

1 a) Roles

b) $n_n = 5\ 10^{17}\ cm^{-3}$ majority carrier concentration

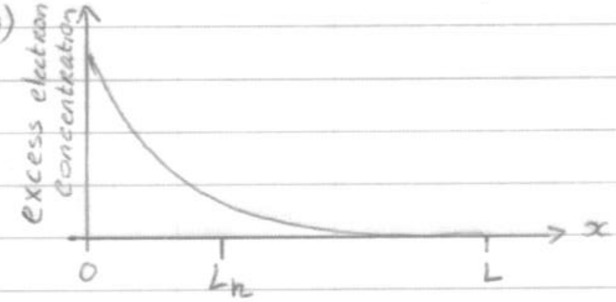
$p_n = \frac{n_i^2}{N_D} = \frac{(1.45\ 10^{10}\ cm^{-3})^2}{5\ 10^{17}\ cm^{-3}} = 420.5\ cm^{-3}$

= minority carrier concentration.

c) Drift is caused by an electric field

Diffusion is a result of carrier gradients

d)

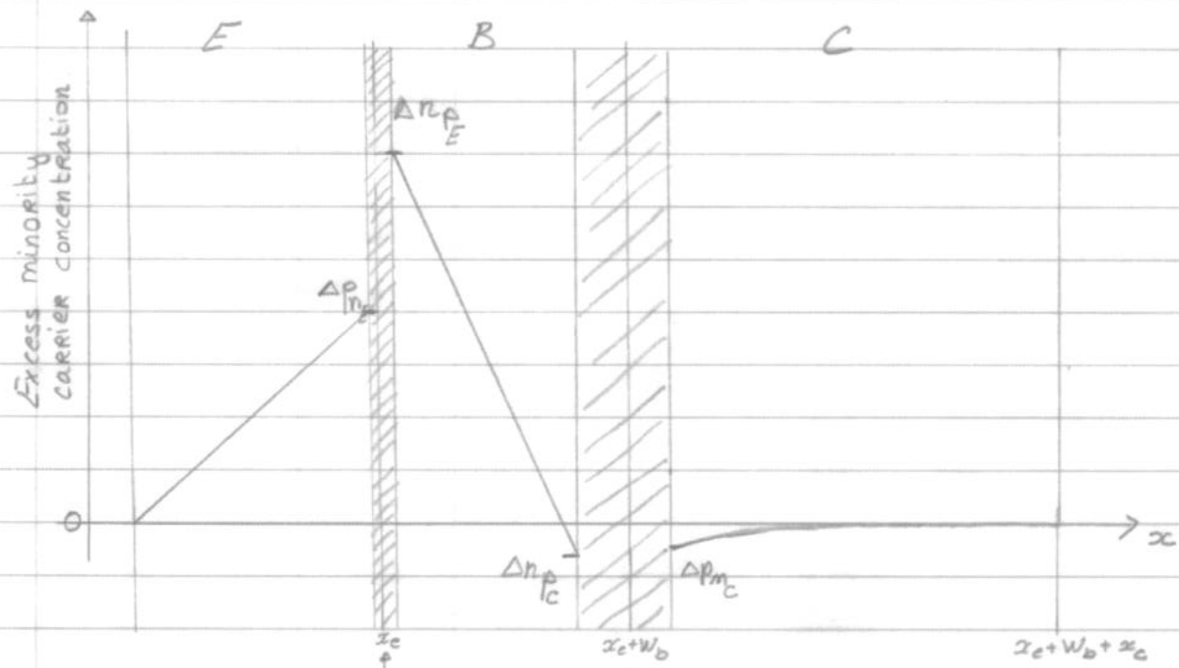


Should be an exponentially decreasing curve that goes to zero @ $x = L$.

e) The main cause of delay in a BJT is the charge in the base region. This charge needs to be completely removed before the collector current can become zero.

2 a.) Energy band diagram

b) Excess minority carrier concentration



$\Delta n_{p_E} > \Delta p_{n_E}$

$$n_p'' = n_{p_0} \exp\left(-\frac{V_{BC}}{V_T}\right)$$

$$\left.\begin{aligned} \Delta n_p &= n_p'' - n_{p_0} \simeq -n_{p_0} = -\frac{n_i^2}{N_A} \\ \Delta p_n &\simeq -p_{n_0} = -\frac{n_i^2}{N_D} \end{aligned}\right\} \quad \begin{aligned} & n_{p_0} > p_{n_0} \\ & \Rightarrow -n_{p_0} < -p_{n_0} \end{aligned}$$

Notes:
- Linear variation in E & B
- Exponential variation in C
- EB depletion width small
- BC depletion width large

c) No recombination in the base ⇒
base current is the resupply of holes that
are injected into the emitter
Collector current is the electron component
of the base - emitter current.

thus $$I_B = \frac{\Delta p_{n_E}}{x_e} e D_{p_E}$$

$$= (p'_{n_E} - p_{n_{E_0}}) \frac{e D_{p_E}}{x_e}$$

$p'_{n_E} >> p_{n_{0_E}}$

$$\simeq p'_{n_E} \frac{e D_{p_E}}{x_e}$$

$$= \frac{e p_{n_{0_E}} D_{p_E}}{x_e} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$= \frac{e n_i^2 D_{p_E}}{x_e N_{D_E}} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$I_C = -\left(\frac{\Delta n_{p_E} - \Delta n_{p_C}}{W_b}\right) e D_{n_B}$$

$|\Delta n_{p_E}| >> |\Delta n_{p_C}|$

$$\simeq -\frac{\Delta n_{p_E}}{W_b} e D_{n_B}$$

$$= -(n'_{p_B} - n_{p_{0_B}}) \frac{e D_{n_B}}{W_b}$$

$n'_{p_B} >> n_{p_{0_B}}$

$$\simeq -\frac{e D_{n_B}}{W_b} n'_{p_B}$$

$$= -\frac{e D_{n_B}}{W_b} n_{p_{0_B}} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$= -\frac{e D_{n_B}}{W_b} \frac{n_i^2}{N_{A_B}} \exp\left(\frac{V_{EB}}{V_T}\right)$$

d) $\beta = \dfrac{|I_C|}{I_B}$

$$= \frac{\dfrac{\cancel{e}\, D_{n_B}\, \cancel{n_i^2}\, \cancel{\exp\left(\frac{V_{EB}}{V_T}\right)}}{W_b\, N_{A_B}}}{\dfrac{\cancel{e}\, D_{p_E}\, \cancel{n_i^2}\, \cancel{\exp\left(\frac{V_{EB}}{V_T}\right)}}{x_e\, N_{D_E}}}$$

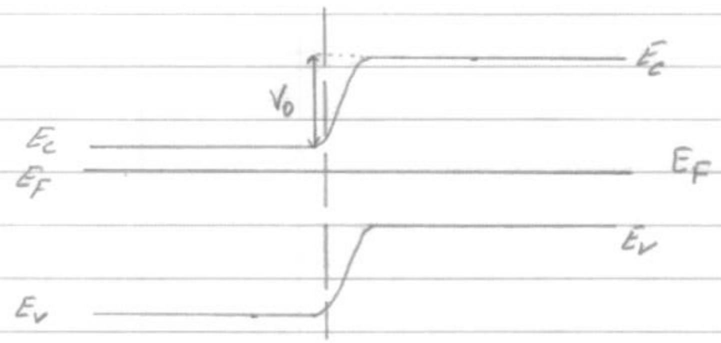
$$= \frac{D_{n_B}\, N_{D_E}\, x_e}{D_{p_E}\, N_{A_B}\, W_b}$$

$$= \frac{20\ \frac{cm^2}{s}\ 10^{19}\ cm^{-3}\ 200\ 10^{-7}\ cm}{12\ \frac{cm^2}{s}\ 10^{16}\ cm^{-3}\ 200\ 10^{-7}\ cm}$$

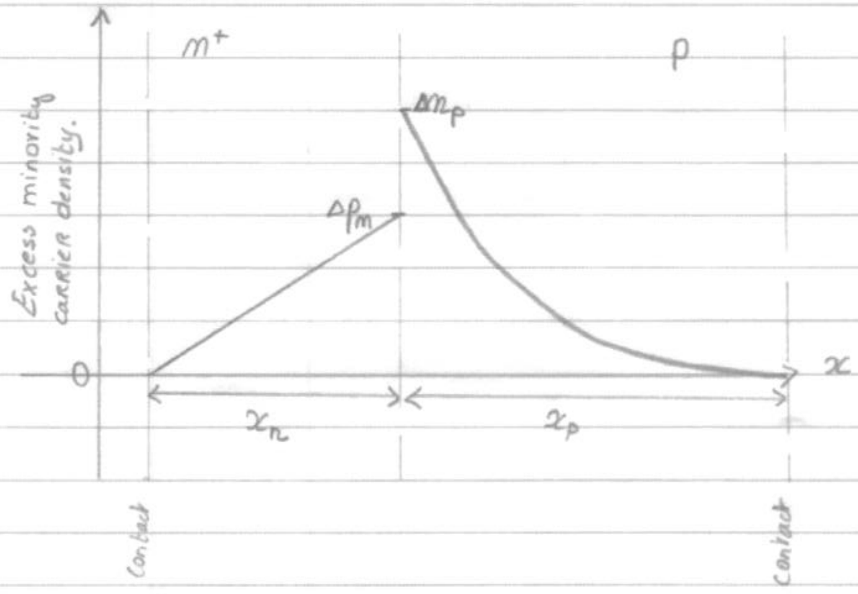
$$= 1.67\ 10^3$$

e) Spice uses the Ebers-Moll equations because they are valid for operation in both the saturation and active regions of operation - and therefore separate sets of equations for the different regions are not required. In addition, these equations only require the use of other existing SPICE models - the current controlled source and the pn diode.

3 a) Energy band diagram



b)



Note: • variation in n^+ layer linear
• variation in p layer exponential.

c) $$Q_p = \frac{e \times \Delta p_n \times x_n \times A}{2} = \frac{e\,(p_n' - p_{n0}) \times x_n \times A}{2} = A\,\frac{e\,x_n}{2}\,p_{n0}\left(\exp\left(\frac{V_{EB}}{V_T}\right) - 1\right)$$

$$= \frac{e\,x_n\,n_i^2}{2\,N_D}\left(\exp\left(\frac{V_{EB}}{V_T}\right) - 1\right) \times A$$

$$Q_n = -e \times A \times \int_0^{x_p} \Delta n_p \exp\left(-\frac{x}{L_n}\right) dx$$

$$= e \times A \times \Delta n_p\, L_n \exp\left(\frac{-x}{L_n}\right)\Bigg|_0^{x_p}$$

$$Q_n \approx +e\,A\,(n_p' - n_{p0})\,L_n\,(0-1)$$

$$= -\frac{e\,A\,n_i^2}{N_A}\,L_n\left(\exp\left(\frac{V_{EB}}{V_T}\right)-1\right)$$

d) since n^+p junction $I_{tot} \approx I_n$

$$I_n = \frac{Q_n}{\tau_n} \quad \text{Recombination current.}$$

$$I_n = -\frac{e\,A\,\Delta n_p\,L_n}{\tau_n}$$

$$J_n = \frac{I_n}{A} = \frac{-1.6\;10^{-19}\,C \quad 2\;10^{12}\,cm^{-3} \quad 500\;10^{-7}\,cm}{0.1\;10^{-6}\,s}$$

$$= -16\;10^{-5}\,\frac{A}{cm^2}$$

e) i) $I = 0A$ open circuit.

ii) $V_d > 0V$ forward bias voltage remains until excess charge is removed.

E2.7A
Sec. B

2E Electromagnetic Fields 2008 – Solutions

4. a) Key contributions were as follows:

i) Wilhelm Roentgen: Discovered X-rays

Heinrich Hertz: Discovered radio waves

[2]

ii) Alexander Graham Bell: Invented the telephone

Guglielmo Marconi: Invented the radio

John Logie Baird: Invented the television

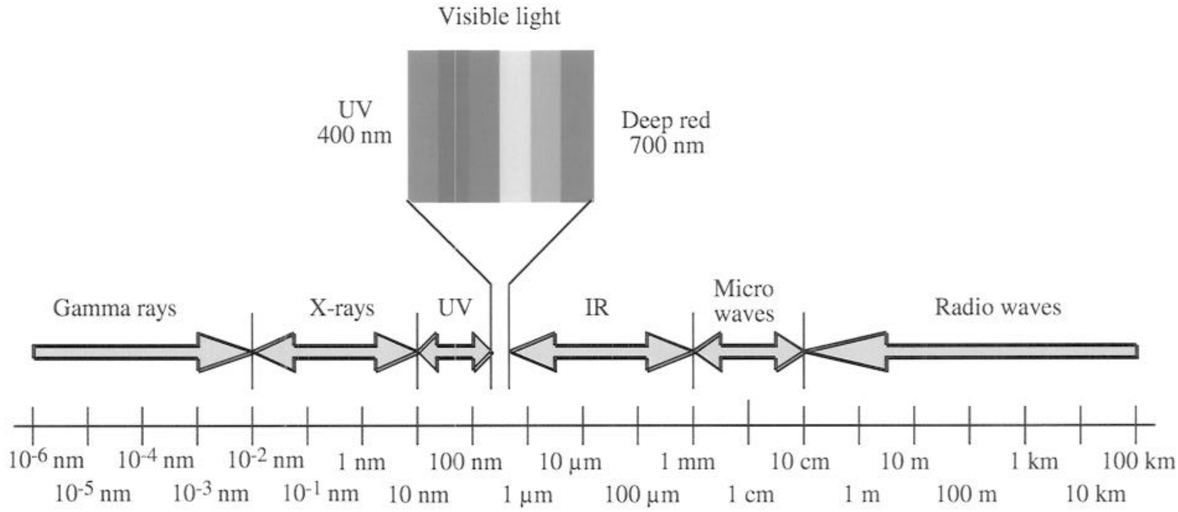
[3]

iii) John Tyndall: Demonstrated light guidance by total internal reflection in a water jet

Charles Kao and George Hockham: Proposed low loss optical fibre communications

[2]

b)



[1 mark for each band = 7]

c) Radio waves – main difficulty is diffraction; overcome using phased arrays

[2]

Microwaves – main difficulty is attenuation; overcome by avoiding water absorption frequency.

[2]

Light waves – main difficulty is scattering; overcome by confinement inside an optical fibre.

[2]

5. a) The phase velocity is the velocity of a single travelling wave.

The group velocity is the velocity of a group of waves, which can represent a modulated carrier and hence describe information propagation.

[3]

The two constituent waves with frequencies $\omega + \Delta\omega$ and $\omega - \Delta\omega$ must have corresponding propagation constants $k + \Delta k$ and $k - \Delta k$. The combined signal can therefore be written as:

$A(z, t) = A_0\{\exp\{j[(\omega + \Delta\omega)t - (k + \Delta k)z\} + \exp\{j[(\omega - \Delta\omega)t - (k - \Delta k)z\}\}$ or

$A(z, t) = A_0\{\exp[j(\Delta\omega t - \Delta kz)] + \exp[-j(\Delta\omega t - \Delta kz)]\} \exp[j(\omega t - kz)]$ or

$A(z, t) = 2A_0 \cos(\Delta\omega t - \Delta kz) \exp[j(\omega t - kz)]$

This result describes a carrier with a modulating envelope in the form of a travelling cosinusoidal wave. The velocity of the envelope is $v_g = \Delta\omega/\Delta k$.

[3]

b) If the dispersion characteristic of the ionosphere is $\omega = \sqrt{[\omega_p^2 + c^2k^2]}$, then:

$v_{ph} = \omega/k = \sqrt{[(\omega_p^2/k^2) + c^2]}$ and

[3]

$v_g = d\omega/dk = c^2/\sqrt{[(\omega_p^2/k^2) + c^2]}$

[3]

For the atmosphere, which has $\omega_p = 0$, $\omega = ck$, so:

$v_{ph} = \omega/k = c$

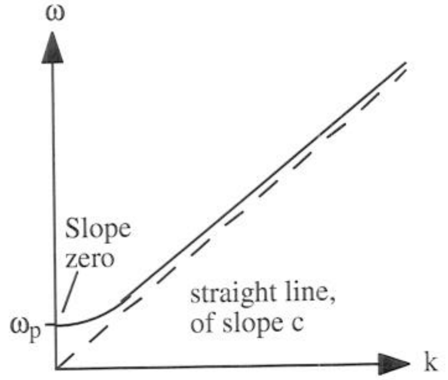
[3]

$v_g = d\omega/dk = c$

[3]

c) The dispersion diagram is as shown below. Since v_g tends to zero when ω tends to ω_p, there can be no transmission of information in this frequency range.

[3]



[3]

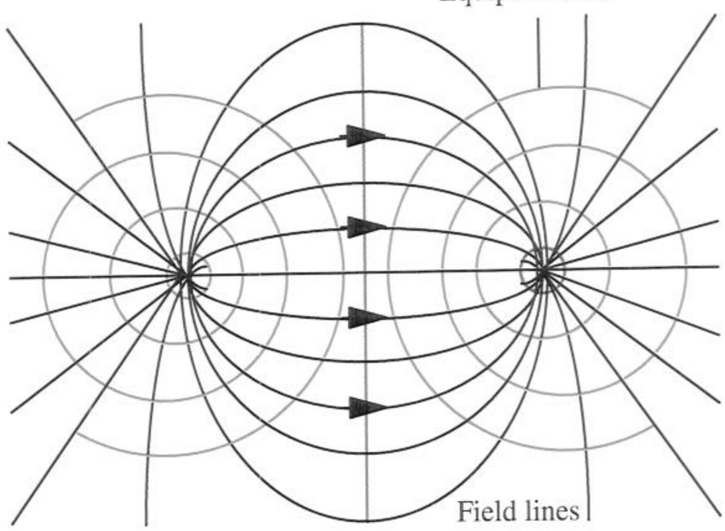
d) Rewriting the equation for the dispersion characteristic, we get: $\omega^2 = \omega_p^{\ 2} + c^2k^2$
Hence $k = (1/c)\ \sqrt{(\omega^2 - \omega_p^{\ 2})}$.
If $\omega < \omega_p$, this result can be written as $k = \pm\, j(1/c)\ \sqrt{(\omega_p^{\ 2} - \omega^2)} = \pm\, jk'$

[3]

A wave solution $A(z, t) = A_0 \exp[j(\omega t - kz)]$ then becomes $A(z, t) = A_0 \exp(j\omega t) \exp(\pm k'z)$, i.e. an exponentially decaying wave.
The significance of this result is that waves with angular frequencies less than ω_p will not be able to penetrate the ionosphere; instead, they will be reflected (as Marconi found).

[3]

6. a) Because $a \ll d$, the equipotentials are circles, centred approximately on $x = \pm d/2$ (in a more detailed analysis, the centres depend on the radius). The field lines form an orthogonal set.



[9]

The electric flux density at a radius r from a cylindrical line charge can be found from Gauss' law as $\underline{D} = q/2\pi r\ \underline{r}$

For the left-hand wire, the electric flux density at P is $\underline{D}_1 = q/2\pi r\ \underline{i}$

For the right hand wire, the corresponding value is $\underline{D}_2 = q/2\pi(d - r)\ \underline{i}$

The total electric flux density is therefore $\underline{D} = (q/2\pi)\ \{1/r + 1/(d - r)\}\ \underline{i}$

[3]

The electric field is $\underline{E} = (q/2\pi\varepsilon_0)\ \{1/r + 1/(d - r)\}\ \underline{i}$,

The voltage between the wires is then $V = {}_a\int^{d-a} E\ dr = {}_a\int^{d-a} (q/2\pi\varepsilon_0)\ \{1/r + 1/(d - r)\}\ dr$, or

$V = (q/2\pi\varepsilon_0)\ [\log_e\{r/(d - r)\}]_a^{d-a} = (q/\pi\varepsilon_0)\ \log_e\{(d - a)/a\}$

The capacitance per unit length is therefore $C = q/V = \pi\varepsilon_0/\log_e\{(d - a)/a\}$.

[6]

b) The magnetic field at a radius r from a wire carrying a current I can be found from Ampere's law as $\underline{H} = I/2\pi r\ \underline{\theta}$.

For the left-hand wire, the magnetic field at P is $\underline{H}_1 = I/2\pi r\ \underline{j}$

For the right-hand wire, the corresponding value P is $\underline{H}_2 = I/2\pi(d - r)\ \underline{j}$

The total magnetic field is then $\underline{H} = (I/2\pi)\{1/r + 1/(d - r)\}\ \underline{j}$

So the total flux density at P is $\underline{B} = (\mu_0 I/2\pi)\ \{1/r + 1/(d - r)\}\ \underline{j}$

[3]

So the flux per unit length crossing between the two wires is

$\Phi = (\mu_0 I/2\pi)\ {}_{-a}\int^{d-a}\ \{1/r + 1/(d - r)\}\ dr$

Integrating, we get

$\Phi = (\mu_0 I/2\pi)\ [\log_e\{r/(d - r)\}]_a^{d-a} = (\mu_0 I/\pi)\ \log_e\{(d - a)/a\}$

The inductance $L = \Phi/I$ is then $L = (\mu_0/\pi)\ \log_e\{(d - a)/a\}$ **[6]**

The phase velocity is $v_{ph} = 1/(LC)^{1/2} = 1/(\mu_0\varepsilon_0)1^{/2}$ **[3]**