EE4-57
EE9-CS2-2

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected copy

DISCRETE-EVENT SYSTEMS

Monday, 9 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D. Angeli
Second Marker(s) : E.C. Kerrigan

1. A buffer has a maximum capacity of 6 units. Packets arrive in blocks of 2 or 4 units (events a_2 and a_4, respectively) and depart in blocks of 3 units (event d_3).

a) Build a finite deterministic automaton G that models the arrival and departure of packets as well as buffer occupancy, assuming an empty buffer at initial time. [4]

b) Assume that events a_2 and a_4 are partially observable, viz. only an a event is detected but no information on packet size is available. Model this as a non-deterministic automaton G_N. [2]

c) Denote by an even interval (or an odd interval) any set of integers of type $\{n, n+2, \ldots, n+2k\}$, for some k non-negative and integer and n even (respectively odd). Let n and $n+2k$ be in the range $\{0, \ldots, 6\}$ so as to make physical sense as occupancy levels of the buffer. Show that the state transition map of $f_N(\cdot, a)$ of G_N maps even intervals to even intervals and odd intervals to odd intervals. Instead, $f_N(\cdot, d_3)$ maps even intervals to odd intervals and vice versa. (*Hint: write explicit expressions for the map*). [4]

d) Build an observer automaton G_O for G_N. [7]

e) Remark from the graph of G_O that, regardless of the arrival and departure sequences, one can never be certain to have exactly four units of space taken in the buffer. Can you explain by alternative types of considerations why this is the case ? [3]

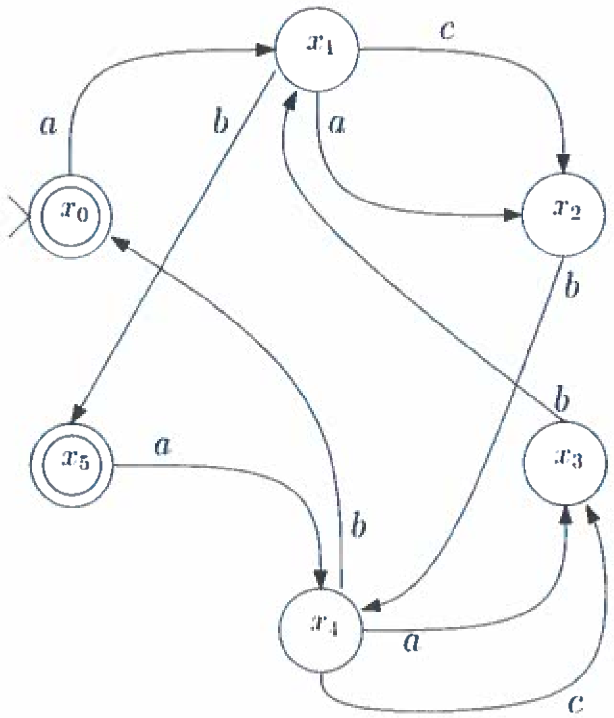

Figure 2.1 The transition diagram of G

2. Consider the Automaton G shown in Fig. 2.1, with state space $X = \{x_0, x_1, x_2, x_3, x_4, x_5\}$, and event set $E = \{a, b, c\}$.

a) Find in X the equivalence classes of $\sim$, where $x_a \sim x_b$ denotes $\mathscr{L}_m(G/x_a) = \mathscr{L}_m(G/x_b)$ and $\mathscr{L}(G/x_a) = \mathscr{L}(G/x_b)$. [5]

b) Find a minimal automaton G_{min} equivalent to G, in the sense that $\mathscr{L}_m(G) = \mathscr{L}_m(G_{\text{min}})$ and $\mathscr{L}(G) = \mathscr{L}(G_{\text{min}})$ while the cardinality of X_{min} (the state space of G_{min}) is as small as possible. [5]

c) Build an automaton generating the language $K \subset \{a, c\}^*$ corresponding to the following verbal specification: "between two a events there should always be at least one c event, and between two c events there should always be at least one a event". [3]

d) Assume that $E_{uc} = \{c\}$. Is the specification modeled by K controllable with respect to $\mathscr{L}(G)$? [2]

e) Design an automaton G_S that can represent a supervisor S so that $\mathscr{L}(S/G_{\text{min}})$ is the supremal controllable sublanguage of K with respect to $\mathscr{L}(G)$. [5]

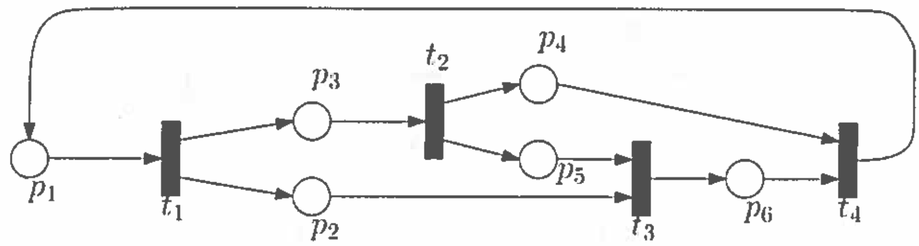

Figure 3.1 The Petri Net N

3. Consider the Petri Net N shown in Fig. 3.1.

a) Compute the incidence matrix C, when ordering places and transitions according to the numbering provided in Fig. 3.1. [3]

b) Compute all the minimal P-semiflows of the net. [5]

c) Is the network structurally bounded? (Justify your answer). [3]

d) Consider the initial marking $M_0 = [1,0,0,0,0,0]'$. Find the reachable set $\mathscr{R}(M_0)$ and sketch the transition diagram of the associated automaton G_N. [3]

e) Consider two automata G_a and G_b that have the same structure of G_N and associated event sets $E_a = \{t_1^a, t_2^a, t_3^a, t_4^a\}$, $E_b = \{t_1^b, t_2^b, t_3^b, t_4^b\}$ (notice that $E_a \cap E_b = \emptyset$). Compute their parallel composition $G_a || G_b$ and draw its transition diagram. [3]

f) Do you expect the previous graph to be isomorphic to the one arising by taking an initial marking $M_0 = [2,0,0,0,0,0]'$? (Justify your answer) [3]

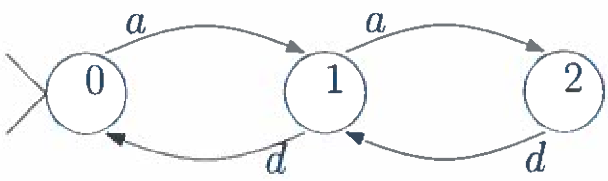

Figure 4.1 Queue automaton G

4. Consider the automaton G shown in Fig. 4.1. Assume both events a and d (arrivals and departures) happen at random times with exponentially distributed clocks of rate λ.

a) Model the situation in which the initial state of the queue is unknown by means of a non-deterministic automaton G_N (*Hint: add an extra state playing the role of initial state and connect it to the remaining states in a suitable way*). [4]

b) Build a deterministic automaton that, by observing a and d events alone and assuming an arbitrary initial state, keeps track of an estimate of all possible current states of the queue. [4]

c) Considering the observer automaton designed at the previous step and the occurrence rates of a and d events sketch the transition diagram and transition matrix Q of the associated Markov Chain. [3]

d) Argue that with probability 1 the state of the system will be correctly estimated (without uncertainty) after a finite time. [3]

e) Compute the average time it takes to have a correct state estimate. [3]

f) Would you expect this to happen for any deterministic automaton G ? (Justify your answer. *Hint: Would minimality of the automaton play a role ?*) [3]