Answers for "Intro. to Signals & Comm. 2015

1.a. i) $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

$$= \int_{-a}^{a} e^{-j\omega t} dt$$

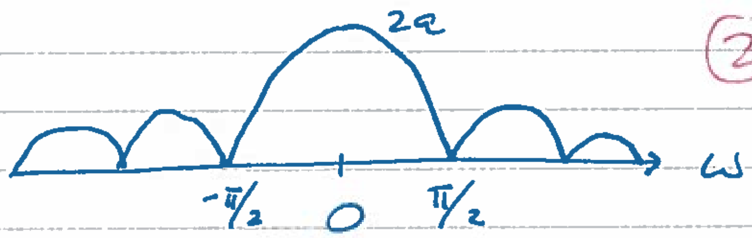
$$= \frac{e^{-j\omega a} - e^{j\omega a}}{-j\omega}$$

total

$$= \frac{2 \sin(\omega a)}{\omega} \quad 2 \qquad [2]$$

$$= 2a \cdot \text{sinc}(\omega a)$$

ii) $|F(\omega)|$



2 [2]

iii) The system is not realizable (1) in practice because of $f(t) \neq 0$ for $t < 0$. That is, it violates the causality property; The system has a response even before the unit impulse is applied to the system. (1)

[2]

1. b. i. $\omega_0 = \frac{2\pi}{T_0}$ (1) [1]

ii.

iii) $\int_0^{T_0} e^{jm\omega_0 t} \cdot e^{-jn\omega_0 t} dt$

$= \int_0^{T_0} e^{j(m-n)\omega_0 t} dt$ (1)

$= \int_0^{2\pi/\omega_0} e^{j(m-n)\omega_0 t} dt$

$= \begin{cases} 0 & m \neq n \\ 2\pi/\omega_0 & m = n \end{cases}$ (2) (1) [3] [1]

iv. $g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$

$\Rightarrow \int_0^{T_0} g(t) e^{-jm\omega_0 t} dt = \int_0^{T_0} \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \cdot e^{-jm\omega_0 t} dt$ (1)

$\Rightarrow \int_0^{T_0} g(t) e^{-jm\omega_0 t} dt = \sum_{n=-\infty}^{\infty} D_n \cdot \int_0^{T_0} e^{j(n-m)\omega_0 t} dt$

$\Rightarrow T_0 D_m = \int_0^{T_0} g(t) e^{-jm\omega_0 t} dt$

$\Rightarrow D_m = \frac{1}{T_0} \int_0^{T_0} g(t) e^{-jm\omega_0 t} dt$ (2)

for $\forall m$ [3]

1. b. v. Each signal component $e^{jn\omega_0 t}$ represents a sinusoidal carrier at frequency $n\omega_0$. [2]

vi. The Fourier series coefficient D_n represents how closely the signal $g(t)$ resembles the component $e^{jn\omega_0 t}$. [2]

1. c. i. $\phi_{AM}(t) = A\cos\omega_c t + A\,m(t)\cos\omega_c t$ [2]

ii. The AM essentially translates the signal $m(t)$ at baseband to the frequency band surrounding the carrier frequency ω_c. [2]

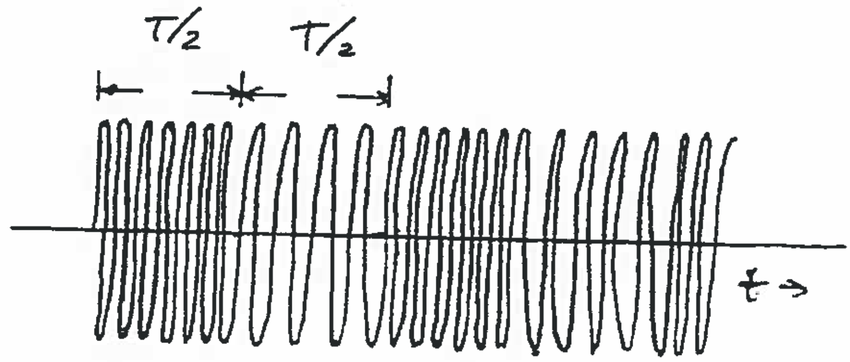
iii. The bandwidth of the AM signal is $2B$ Hz. [2]

iv. $P_c = \frac{A^2}{2}$ [2]

v. $P_s = \frac{E[m^2(t)]}{2} = \lim_{T\to\infty} \int_{-T/2}^{T/2} m^2(t)\cos^2\omega_c t \, dt$ [2]

vi. $\eta = \frac{P_s}{P_s + P_c} = \frac{\frac{E[m^2(t)]}{2}}{\frac{E[m^2(t)]}{2} + A^2/2}$

$\eta = \frac{E[m^2(t)]}{E[m^2(t)] + A^2}$ [2]



Point a

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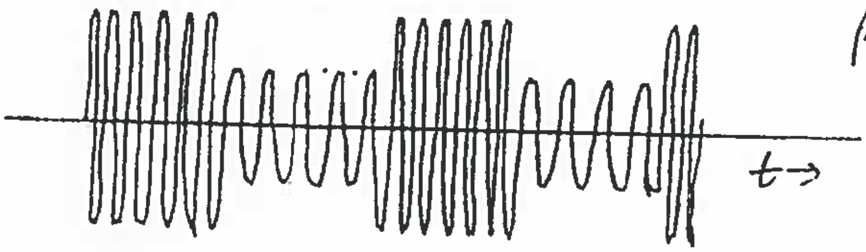

Point b

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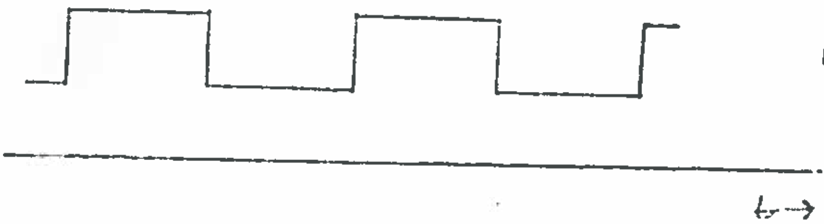

Point c

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Point d

2

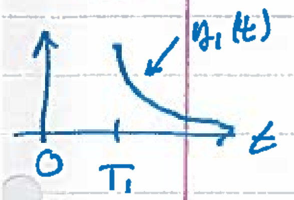
[10]

2. a. i.

$$y_1(t) = \int_{-\infty}^{\infty} x(u)\, h(t-$$

$$y_1(t) = \int_{u=-\infty}^{\infty} h_1(u)\, x(t-u)\, du \quad [3]$$

(2) ii. $$y_1(t) = \int_{u=-\infty}^{\infty} \delta(u-T_1)\, x(t-u)\, du$$



$$y_1(t) = x(t-T_1)$$ (2) due to the property of $\delta(\cdot)$ [4]

iii. The physical effects due to $h_1(t)$ and $h_2(t)$ on the input $x(t)$ are to delay the input by T_1 and T_2 time units, respectively. [2]

iv. $$h(t) = h_1(t) + h_2(t) \quad [2]$$

v.

$$y(t) = y_1(t) + y_2(t)$$

$$= x(t) * [h_1(t) + h_2(t)] \quad (2)$$

$$\Rightarrow Y(\omega) = X(\omega) \cdot [H_1(\omega) + H_2(\omega)] \quad (2)$$

[4]

2. b. i. $\mathcal{F}[f(t-T)] = \int_{t=-\infty}^{\infty} f(t-T) e^{-j\omega t} dt$ (2)

$\Rightarrow \mathcal{F}[f(t-T)] = e^{-j\omega T} \cdot \int_{t=-\infty}^{\infty} f(t-T) \cdot e^{-j\omega(t-T)} dt$

$\Rightarrow \mathcal{F}[f(t-T)] = e^{-j\omega T} F(\omega)$ (2) [4]

ii. The magnitude of the spectrum for $f(t-T)$ does not change, but the phase of the transform does change according to $e^{-j\omega T}$. [2]

iii. $\mathcal{F}[f(at)] = \int_{t=-\infty}^{\infty} f(at) e^{-j\omega t} dt$ (2)

$= \frac{1}{a} \int_{t=-\infty}^{\infty} f(at)\, e^{-j\left(\frac{\omega}{a}\right) at} d(at)$

$= \frac{1}{a} \int_{t'=-\infty}^{\infty} f(t')\, e^{-j\left(\frac{\omega}{a}\right) t'} dt'$

with $t' = at$

$\Rightarrow \mathcal{F}[f(at)] = \frac{1}{a} F\left(\frac{\omega}{a}\right)$. (2)

[4]

2.b. iv. For $a>1$, $f(at)$ is a "compressed" version of $f(t)$ in time. That is, $f(at)$ changes more rapidly when compared with $f(t)$. ~~tht~~ In ~~that~~ case, $F(\frac{\omega}{a})$ is "stretched" in ω.

That is, one can stretch $F(\omega)$ in ω in order to obtain $F(\omega/a)$. As a result, more and more energy of $f(at)$ is located in higher frequency regions.

(2)

For $a<1$, the opposite is true. That is, $f(at)$ can ~~ob~~ be obtained by stretching $f(t)$ in time t. Then $F(\omega/a)$ is "squeezed" in ω when compared with $F(\omega)$. That is, $F(\omega/a)$ has more and more energy located at lower frequency regions when compared with $F(\omega)$.

(1) [3]

v. When a signal has rapid changes in time, the ~~tim~~ transform has higher and higher components in high frequency regions. When the signal changes slowly in time, most of its energy will be associated with low frequencies.

[2]

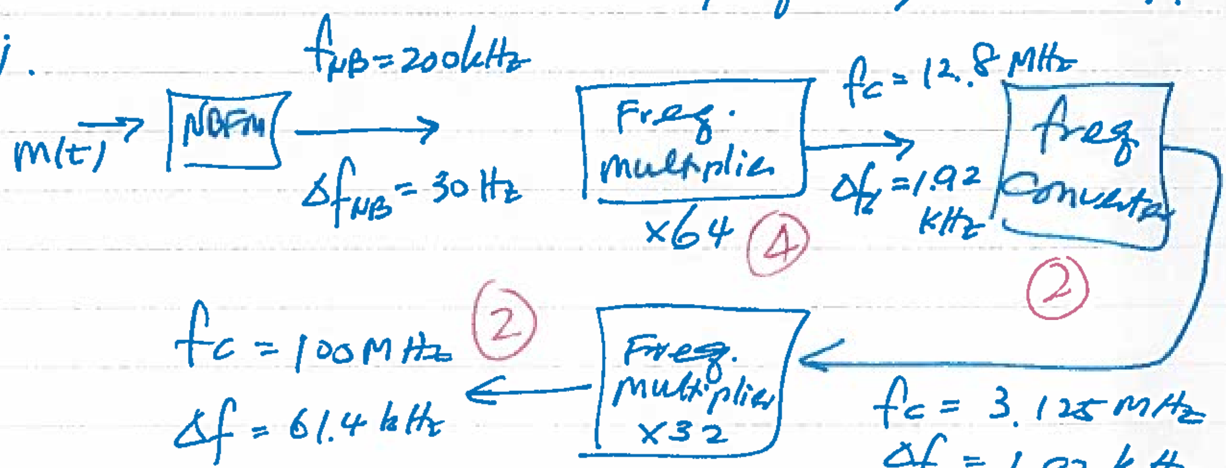
3.a. i. $y(t) = a\phi(t) + b\phi^2(t)$

$$\Rightarrow y(t) = aA\cos[2\pi f_{NB}t + \Delta f_{NB}] + bA^2\cos^2[2\pi f_{NB}t + \Delta f_{NB}]$$ (2)

$$\Rightarrow y(t) = a\cdot A\cos[2\pi f_{NB}t + \Delta f_{NB}] + \frac{bA^2}{2} + \underbrace{\frac{bA^2}{2}\cdot\cos[2\cdot 2\pi f_{NB}t + 2\Delta f_{NB}]}_{\text{double the carrier freq. and frequency deviation.}}$$ (2)

[4]

ii.



[8]

iii. Oscillator frequency for converter

$$9.675\ MHz = 12.8\ MHz - 3.125\ MHz$$ (2)

Converter is to multiply a sinusoidal carrier so that the frequency shift can be effected

$$\cos(\omega_1 t)\cos(\omega_2 t)$$ (4) [6]

$$= \frac{1}{2}[\cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t]$$

where $\omega_1 = 12.8 \times 2\pi$ rad/sec

$\omega_1 - \omega_2 = 3.125 \times 2\pi$ rad/sec

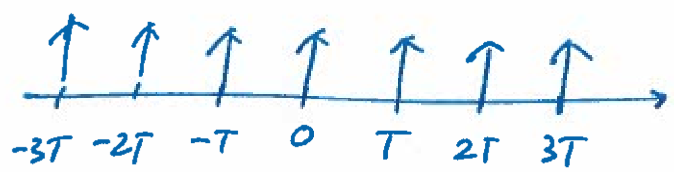
$\Rightarrow$ oscillator freq $\omega_2 = 12.8 - 3.125$

$= 9.675$ MHz

3. b. i. The signaling rate

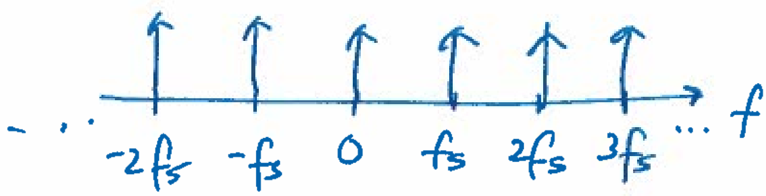
$$f_s \leq 2B$$ [2]

ii. The samples of signal can be interpreted as applying the sampling of pulse train onto a given signal:



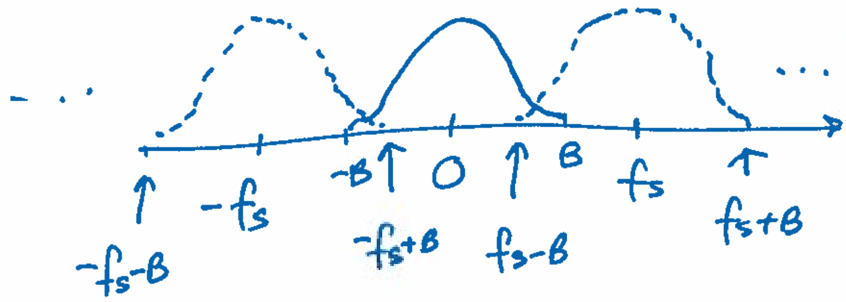
(2)

which has a sprectrum of Fourier transform



(2)

As a result, The signal samples of the signal represent a spectrum of



(3)

[7]

From the above diagram,

if $B \geq f_s - B$,

The replica of the baseband spectrum of the original signal do not overlap. That means the original signal has a bandwidth of B Hz or less, which can be supported for transmission by a channel of B Hz. Otherwise, the original signal has components beyond B Hz, which cannot all pass through the channel of B Hz — ~~correcti~~

Correct reception is impossible.

Overall, the signal rate $f_s \leq 2B$.

iii. $C = 2B \log_2 M$ [3]