IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

EEE PART III/IV: MEng, BEng and ACGI

ELECTRICAL ENERGY SYSTEMS

Friday, 20 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer 2 questions from Section A and 2 questions from Section B. Use a separate answer book for each section.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	G. Strbac, B. Chaudhuri
	Second Marker(s) :	B. Chaudhuri, G. Strbac

Part A – Answer any 2 out of 3 questions in part A

1. a) What is the purpose of power flow calculations?

[2]

b) What are the consequences of transporting significant amounts of reactive power over long overhead transmission lines?

[2]

c) How are power flows controlled in a transmission system?

[2]

d) Consider the simple two bus system with a transmission line with resistance R and reactance X, as depicted in Figure 1. Indices G and L indicate Generation and Load respectively.

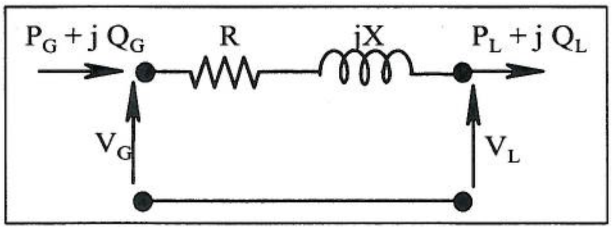

Figure 1: Simple two bus transmission system

i) Show that

$$\overline{V_L} = V_G - \frac{RP_G + XQ_G}{V_G} - j\frac{XP_G - RQ_G}{V_G}$$

[4]

For X = 0.2 pu and R = 0 answer the following questions:

ii) Estimate the maximum reactive power that be transported if the maximum allowed voltage drop should not exceed 10%.

[4]

iii) Calculate the voltage magnitude at the load end when the generator operates at the unity power factor and generates active power of 1pu. What is the active power at the load end? For this situation calculate reactive power losses in the line.

[3]

iv) Explain why the voltage magnitude at the receiving end is greater than the voltage at the sending end in (iii). Hint: calculate the reactive power at the load end.

[3]

2. a) Explain briefly why it is more difficult to transport reactive power than active power over high-voltage AC transmission systems.

[3]

b) A three bus power network is presented in Figure 2. Data relevant for the load flow analysis on this system are given in per unit.

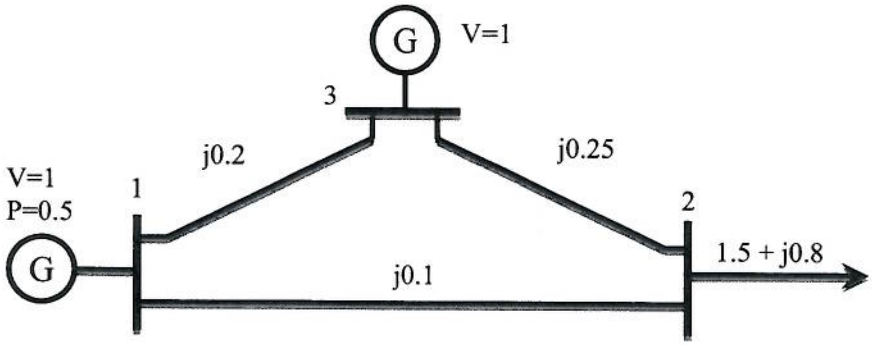

Figure 2: Three bus network showing per unit data for voltages, active and reactive load, and lines reactance

i) Form the Ybus matrix for this system.

[3]

ii) Perform two iterations of the Gauss-Seidel load flow.

[14]

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3. a) Consider a system supplied with three generators with given capacities and availabilities as in Figure 3.

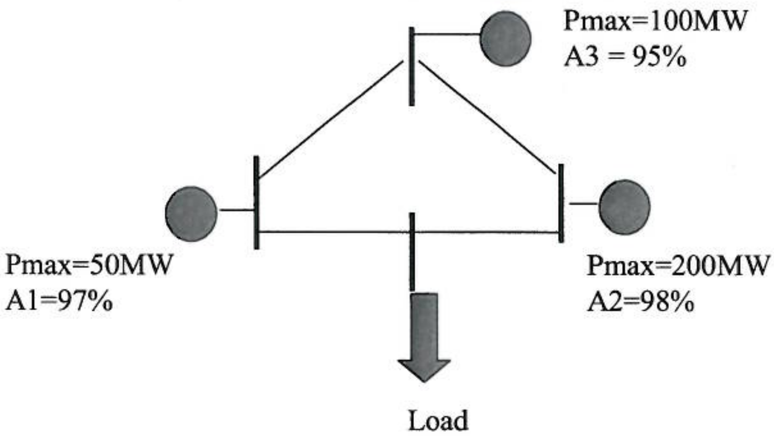

Figure 3: A simple power system

The states in which this generation system can find itself are given in Table 1.

Table 1: System state probabilities

STATE	State Probability	Probability that Generation is equal to or greater than State
350 MW		
300 MW		
250 MW		
200 MW		
150 MW		
100 MW		
50 MW		
0 MW		

i) Calculate state probabilities for this system and the probability that the generation will be greater than the given state.

[4]

ii) If the system peak load is 260 MW, find the probability that generation will not be able to meet it.

[3]

b) Estimate the maximum length of a 400 kV transmission circuit that can transmit 800 MW. Assume that the reactance of the circuit is $0.4\ \Omega/km$ and that resistance and capacitance can be ignored.

[6]

c) Find the peak load of an 11/0.4 kV substation supplying 400 households not using electricity for heating purposes (Type A), and 100 households with electric heating (Type B). Peak demands of individual households are 10 kW and 20 kW, respectively. Coincidence coefficient for Type A households is $j_{A\infty} = 0.2$, and for Type B $j_{B\infty} = 0.5$. Assume that peaks of both groups of consumers coincide.

[7]

Part B – Answer any 2 out of 3 questions in part B

4. a) Explain three reasons why studying faults in power systems is important.

[3]

b) State and justify the two basic assumptions that are often used to simplify fault current calculations.

[4]

c) Describe the components of the fault current in power transmission systems and show that under appropriate assumptions the maximum momentary (instantaneous) value of the fault current could be twice its symmetrical part.

[5]

d) A 25 MVA, 11 kV synchronous generator with sub-transient reactance $X_d'' = 0.2$ pu is supplying three identical motors through a step-up and step-down transformer as shown in the figure below. Each motor has a sub-transient reactance $X_d'' = 0.25$ pu on its 5 MVA, 6.6 kV base. Both the 11/66 kV step-up and the 66/6.6 kV step-down transformers are rated at 25 MVA and have the same leakage reactance of 0.1 pu on their respective base. Assuming the voltage at the motor terminals to be 6.6 kV when a 3-phase fault occurs at point F, calculate the magnitude of the fault current in kA. Choose the system base as 25 MVA.

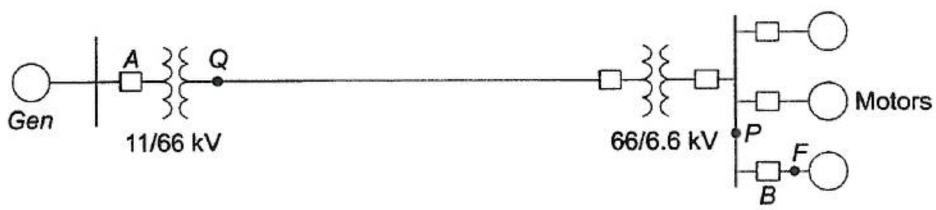

Figure 4: Synchronous generator supplying three motors

[8]

5. a) Describe how loadability (power transmission capacity) of transmission lines is limited due to different forms of stability problems.

[4]

b) Starting from the swing equation explain the concept of equal area criteria and explain how it is useful for stability analysis.

[4]

c) Using the power angle characteristics explain the excursion of rotor angle following a step increase in mechanical input power a synchronous generator.

[4]

d) A synchronous generator is supplying 50 MW to an infinite bus. The steady-state stability limit (or maximum possible power transfer) of the system is 100 MW. Neglecting losses determine whether the system would remain stable if the mechanical power input to the generator is suddenly increased by 30 MW.

[8]

6. a) For a synchronous generator with solidly grounded neutral explain why a line-to-ground (LG) fault near the generator terminal is usually more severe than a three-phase fault.

[5]

b) Show that the positive and negative sequence networks are connected in parallel across the fault impedance for line-to-line (LL) faults.

[5]

c) A 3-phase 10 MVA, 11 kV (line-to-line) synchronous generator with a solidly grounded neutral point supplies a feeder at the same voltage level. The positive, negative and zero sequence impedances of the generator and the feeder are shown in the following table.

	Generator	Feeder
Postive sequence, Z1	j 1.2 Ω	j 1.0 Ω
Negative sequence, Z2	j 0.9 Ω	j 1.0 Ω
Zero sequence, Z0	j 0.4 Ω	j 3.0 Ω

i) For a line-to-ground (LG) fault at the far end of the feeder calculate the fault current in kA. Consider the phase voltage for calculations.

[6]

ii) Calculate the voltage to neutral of the faulty phase at the terminals of the generator.

[4]

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Part A – Model Answers 2011

1. a) What is the purpose of power flow calculations?

[2]

b) What are the consequences of transporting significant amounts of reactive power over long overhead transmission lines?

[2]

c) How are power flows controlled in a transmission system?

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d) Consider the simple two bus system with a transmission line with resistance R and reactance X, as depicted in Figure 1. Indices G and L indicate Generation and Load respectively.

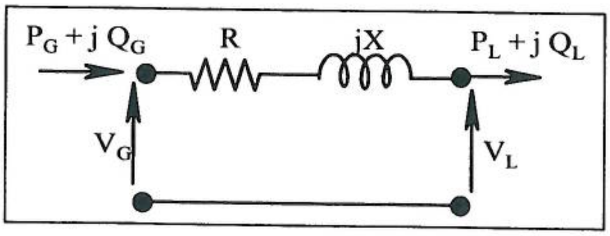


Figure 1: Simple two bus transmission system

i) Show that

$$\overline{V_L} = V_G - \frac{RP_G + XQ_G}{V_G} - j\frac{XP_G - RQ_G}{V_G}$$

[4]

For X = 0.2 pu and R = 0 answer the following questions:

ii) Estimate the maximum reactive power that be transported if the maximum allowed voltage drop should not exceed 10%.

[4]

iii) Calculate the voltage magnitude at the load end when the generator operates at the unity power factor and generates active power of 1pu. What is the active power at the load end? For this situation calculate reactive power losses in the line.

[3]

iv) Explain why the voltage magnitude at the receiving end is greater than the voltage at the sending end in (iii). Hint: calculate the reactive power at the load end.

[3]

Answers 1

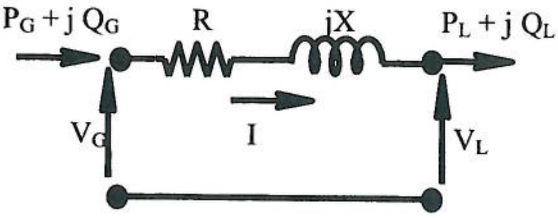
(a) Purpose of power flow: determine if power flows in transmission and distribution circuits is below or above their thermal capacity and voltages at busbars within allowable limits, examine implications of line or generator outages on power flows and voltage profiles, examine feasibility and performance of alternative system development programmes, examine the effects of control devices such as OLTCs and QBs on voltages and floes

(b) Significant amount of reactive power can only be transported over long overhead transmission lines at the expense of significant voltage difference between sending and receiving end.

(c) Power flows are controlled by changing outputs of generators

(d)

(i)



$$\overline{S}_G = P_G + jQ_G = V_G I^* \quad \Rightarrow \quad \overline{I} = \frac{P_G - jQ_G}{\overline{V}_G^*}$$

$$\overline{V}_L = \overline{V}_G - (R + jX)\overline{I}$$

$$\overline{V}_L = \overline{V}_G - (R + jX)\frac{P_G - jQ_G}{\overline{V}_G^*}$$

$$\overline{V}_G = \overline{V}_G^* = V_G \underline{/0^\circ} = V_G$$

$$\overline{V}_L = V_G - \frac{RP_G + XQ_G}{V_G} - j\frac{XP_G - RQ_G}{V_G}$$

(ii)

$$\begin{matrix} R = 0 \\ X = 0.2\text{p.u.} \end{matrix} \quad \Rightarrow \quad \overline{V}_L = V_G - \frac{XQ_G}{V_G} - j\frac{XP_G}{V_G} \quad \Rightarrow \quad \frac{(V_G - \overline{V}_L)}{V_G} = \frac{XQ_G}{V_G^2} + j\frac{XP_G}{V_G^2}$$

Assumption: $V_G = 1\text{p.u.}$

Maximum Q_G is when P_G is zero.

$$V_L^m = V_G - \frac{XQ_G^M}{V_G} \quad \Rightarrow \quad Q_G^M = \frac{(V_G - V_L^m)V_G}{X} = \frac{(1-0.9)1}{0.2} = 0.5\text{p.u.}$$

(iii)

$P_G = 1\text{p.u.}$
$Q_G = 0$

$$\overline{V}_L = V_G - j\frac{XP_G}{V_G} = V_G - j\frac{0.2\times 1}{V_G} \overset{V_G=1}{\approx} 1 - j0.2 = 1.02\underline{/-11.3^\circ}$$

No active losses, so
$P_L = 1$

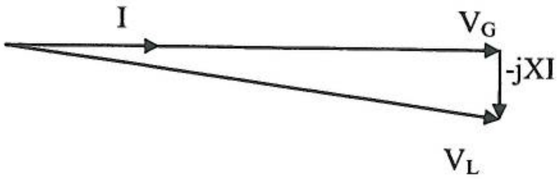
$$\bar{I} = \frac{P_G - jQ_G}{\overline{V}_G^*} = \frac{1-j0}{1} = 1\text{p.u.}$$

$Q_\gamma = I^2X = 0.2\text{p.u.}$

(c)

$Q_L = Q_G - Q_\gamma = 0 - 0.2 = -0.2\text{p.u.}$

At the load busbar reactive power is injected which will tend to raise the voltage at that busbar.



2. a) Explain briefly why it is more difficult to transport reactive power than active power over high-voltage AC transmission systems.

[3]

b) A three bus power network is presented in Figure 2. Data relevant for the load flow analysis on this system are given in per unit.

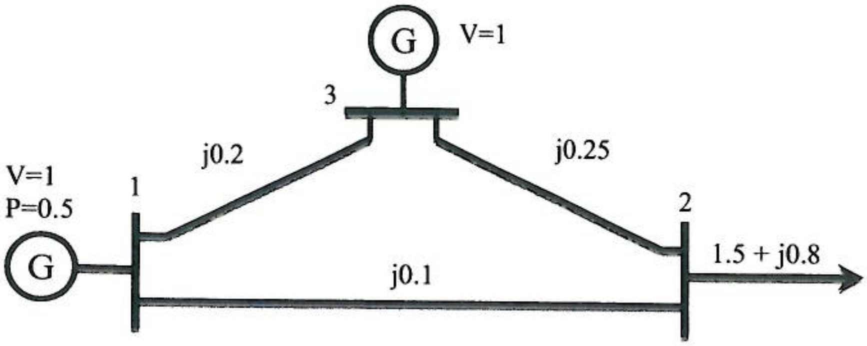


Figure 2: Three bus network showing per unit data for voltages, active and reactive load, and lines reactance

i) Form the Ybus matrix for this system.

[3]

ii) Perform two iterations of the Gauss-Seidel load flow.

[14]

Answers 2

(a) Unlike active power, reactive power cannot be transmitted across long distances, for the following reasons:

- Transmitting reactive power requires a voltage drop that would become unacceptable for long distances.
- Since $X >> R$, the reactive losses are much larger than the active losses and the transmission of reactive power would be inefficient.
- Therefore, we need sources of reactive power around the network.

[3]

(b)

i) Y_{bus} matrix is obtained as follows:

$$z_{12} = 0 + j0.1 \text{ p.u}$$

$$y_{12} = \frac{1}{z_{12}} = -j10 \text{ p.u.}$$

$$z_{13} = 0 + j0.2 p.u$$

$$y_{13} = \frac{1}{z_{13}} = -j5 p.u.$$

$z_{23} = 0 + j0.25 p.u$

$y_{23} = \frac{1}{z_{23}} = -j4 p.u.$

$Y_{11} = y_{12} + y_{13} = -j15$ p.u.

$Y_{22} = y_{12} + y_{23} = -j14$ p.u.

$Y_{33} = y_{13} + y_{23} = -j9$ p.u.

$Y_{12} = Y_{21} = -y_{12} = j10$p.u.

$Y_{13} = Y_{31} = -y_{13} = j5$p.u.

$Y_{23} = Y_{32} = -y_{23} = j4$p.u.

$$Y = j\begin{bmatrix} -15 & 10 & 5 \\ 10 & -14 & 4 \\ 5 & 4 & -9 \end{bmatrix}$$

[3]

(iv)

$s_2 = 1.5 + j0.8 p.u$

$V_1 = ?$

$V_2 = ?$

$P_1 = ?$

$Q_1 = ?$

$P_2 = ?$

$Q_2 = ?$

$V_1^{(0)} = 1 + j0$ Slack bus

$V_2^{(0)} = 1 + j0$ PQ bus

$V_3^{(0)} = 1 + j0$ PV bus

<u>***FIRST ITERATION:***</u>

$V_2^{(1)} = \frac{1}{Y_{22}} \cdot (\frac{S_2^*}{V_2^{(0)*}} - Y_{21} \cdot V_1^{(0)} - Y_{23} \cdot V_3^{(0)}) = 0.9429\text{- j}0.1071\text{p.u.}$

$\Delta V_2^{(1)} = \left|V_2^{(1)} - V_2^{(0)}\right| = 0.1214\ p.u$

$V_3^{(\tilde{1})} = \frac{1}{Y_{33}} \cdot (\frac{S_3^{(0)*}}{V_3^{(0)*}} - Y_{31} \cdot V_1^{(0)} - Y_{32} \cdot V_2^{(1)}) = 0.9746\text{- j}0.0476\text{p.u}$

$V_3^{(1)} = \frac{V_3^{(\tilde{1})}}{\left|V_3^{(\tilde{1})}\right|} =$ 0.9988 - j 0.0488 p.u

$\Delta V_3^{(1)} = \left|V_3^{(1)} - V_3^{(0)}\right| = 0.0488\text{p.u}$

$S_2^{(1)} = V_2^{(1)} \cdot \left(Y_{21} \cdot V_1^{(0)} + Y_{22} \cdot V_2^{(1)} + Y_{23} \cdot V_3^{(1)}\right)^* =$ -1.3048 - j 0.7952 p.u.

$$S_1^{(1)} = V_1^{(0)} \cdot \left(Y_{12} \cdot V_2^{(1)} + Y_{11} \cdot V_1^{(0)} + Y_{13} \cdot V_3^{(1)}\right)^* = 1.3154 + j\ 0.5774\, p.u.$$

$$Q_3^{(1)} = -imag(V_3^{(1)}(\mathrm{Y}_{31} \cdot \mathrm{V}_1^{(0)} + \mathrm{Y}_{32} \cdot \mathrm{V}_2^{(1)} + \mathrm{Y}_{33} \cdot \mathrm{V}_3^{(1)})) = j\ \ 0.2181 p.u.$$

SECOND ITERATION:

$$V_2^{(2)} = \frac{1}{Y_{22}} \cdot \left(\frac{S_1^*}{V_2^{(1)*}} - Y_{21} \cdot V_1^{(0)} - Y_{13} \cdot V_3^{(1)}\right) = 0.9271 - j\ 0.1193\ \text{p.u}$$

$$\Delta V_2^{(2)} = \left|V_2^{(2)} - V_2^{(1)}\right| = 0.0199 \text{p.u.}$$

$$V_3^{(\tilde{2})} = \frac{1}{Y_{33}} \cdot \left(\frac{S_3^{(1)*}}{V_3^{(1)*}} - Y_{31} \cdot V_1^{(0)} - Y_{32} \cdot V_2^{(2)}\right) = 0.9746 - j\ 0.0476\ \text{p.u}$$

$$V_3^{(2)} = \frac{V_3^{(\tilde{2})}}{\left|V_3^{(\tilde{2})}\right|} = 0.9988 - j\ 0.0488\ \text{p.u.}$$

$$\Delta V_3^{(2)} = \left|V_3^{(2)} - V_3^{(1)}\right| = 0\ \text{p.u.}$$

$$S_2^{(2)} = V_2^{(2)} \cdot \left(Y_{21} \cdot V_1^{(0)} + Y_{22} \cdot V_2^{(2)} + Y_{23} \cdot V_3^{(2)}\right)^* = .-1.4754 - j\ 1.0161\ \text{p.u.}$$

$$S_1^{(2)} = V_1^{(0)} \cdot \left(Y_{12} \cdot V_2^{(2)} + Y_{11} \cdot V_1^{(0)} + Y_{13} \cdot V_3^{(2)}\right)^* = 1.4373 + j0.7352 \text{p.u.}$$

$$Q_3^{(2)} = -imag(V_3^{(2)}(\mathrm{Y}_{31} \cdot \mathrm{V}_1^{(0)} + \mathrm{Y}_{32} \cdot \mathrm{V}_2^{(2)} + \mathrm{Y}_{33} \cdot \mathrm{V}_3^{(2)})) = j\ 0.2788 \text{p.u.}$$

[14]

3. a) Consider a system supplied with three generators with given capacities and availabilities as in Figure 3.

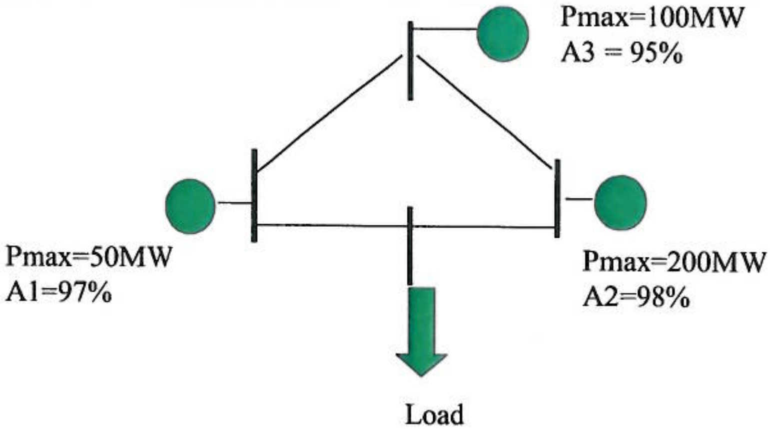


Figure 3: A simple power system

The states in which this generation system can find itself are given in Table 1.

Table 1: System state probabilities

STATE	State Probability	Probability that Generation is equal to or greater than State
350 MW		
300 MW		
250 MW		
200 MW		
150 MW		
100 MW		
50 MW		
0 MW		

i) Calculate state probabilities for this system and the probability that the generation will be greater than the given state.

[4]

ii) If the system peak load is 260 MW, find the probability that generation will not be able to meet it.

[3]

b) Estimate the maximum length of a 400 kV transmission circuit that can transmit 800 MW. Assume that the reactance of the circuit is $0.4\ \Omega/km$ and that resistance and capacitance can be ignored.

[6]

c) Find the peak load of an 11/0.4 kV substation supplying 400 households not using electricity for heating purposes (Type A), and 100 households with electric heating (Type B). Peak demands of individual households are 10 kW and 20 kW, respectively. Coincidence coefficient for Type A households is $j_{A\infty} = 0.2$, and for Type B $j_{B\infty} = 0.5$. Assume that peaks of both groups of consumers coincide.

[7]

Answers 3

(a) The states of the generation system with the appropriate probabilities are given in the following table.

STATE	State Probability	Probability that Generation is equal to or greater than State
350 MW	0.90307	0.90307
300 MW	0.02793	0.93100
250 MW	0.04753	0.97853
200 MW	0.00147	0.98000
150 MW	0.01843	0.99843
100 MW	0.00057	0.99900
50 MW	0.00097	0.99997
0 MW	0.00003	1.00000

[4]

If the system load is 260 MW, at least 300 MW of available capacity is necessary to fully cover that demand. This means the probability of not being able to cover the demand is equal to 1 – P(Gen ≥ 300 MW), which is 0.069, or 6.9%. [3]

(b) The maximum length of a transmission line is as follows:

$$L_{max} = \frac{V_1 \cdot V_2}{x \cdot P_{max}} = \frac{400 \cdot 400}{0.4 \cdot 800} = 500km$$

[6]

(c) The coincidence coefficients for the two customer groups are determined as follows:

$$j_A = j_{A\infty} + \frac{1-j_{A\infty}}{\sqrt{n_A}} \qquad j_B = j_{B\infty} + \frac{1-j_{B\infty}}{\sqrt{n_B}}$$

which yields: $j_A = 0.24, j_B = 0.55$. Total group peak can be found by multiplying the single household peak with the number of households and the corresponding coincidence coefficient:

$$P_{Amax} = P_A n_A j_A \qquad P_{Bmax} = P_B n_B j_B$$

which gives: $P_{Amax} = 960\ \mathrm{kW}$, $P_{Bmax} = 1{,}100\ \mathrm{kW}$. This makes the total peak load of the substation (under the assumption that peaks for both A and B are simultaneous): $P_{total} = 2{,}060\ \mathrm{kW}$. [7]

Part B – Model Answers

4. a) Explain three reasons why studying faults in power systems is important.

[3]

- Calculate circuit breaker ratings
  - Must be able to withstand and make/break the fault current
- Design protection (tripping) system
  - Is the fault current large enough to be detected?
- System stability analysis
  - Type and duration of the faults affects system stability
- Power quality
  - Faults cause voltage sag in parts of the network

b) State and justify the two basic assumptions that are often used to simplify fault current calculations.

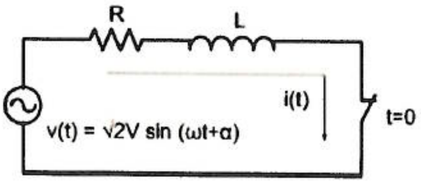
[4]

- Assumption 1: All pre-fault voltage magnitudes are 1.0 pu
  - In practice under normal operation all voltages are nearly 1.0 pu

- Assumption 2: All pre-fault currents are zero
  - change in current due to fault is quite large, typically 10-20 pu
  - sub-transient current is mostly reactive while pre-fault current is predominantly resistive
  - hence total current (sub-transient + pre-fault) magnitude can be assumed to be the larger of the two

c) Describe the components of the fault current in power transmission systems and show that under appropriate assumptions the maximum momentary (instantaneous) value of the fault current could be twice its symmetrical part.

[5]

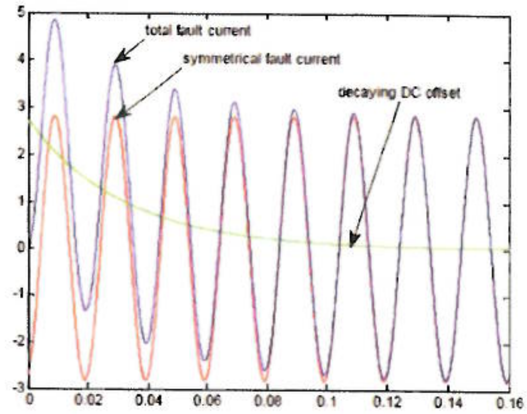


$$\sqrt{2}V\sin(\omega t+\alpha)=L\frac{di(t)}{dt}+Ri(t), t\geq 0$$

Assumptions

System fed from a constant voltage source
Fault occurs under unloaded condition
Line capacitance is neglected

$$i(t)=i_s(t)+i_t(t)$$
$$=\underbrace{\boxed{\frac{\sqrt{2}V}{|Z|}\sin(\omega t+\alpha-\theta)}}_{\text{symmetrical fault current}}+\underbrace{\boxed{\frac{\sqrt{2}V}{|Z|}\sin(\theta-\alpha)e^{-(\frac{R}{L})t}}}_{\text{decaying DC offset}}$$



For transmission lines R << X → θ ≈ 90 deg

Maximum momentary fault current (i_{mm}):

$$i_{mm}=\frac{\sqrt{2}V}{|Z|}\cos\alpha+\frac{\sqrt{2}V}{|Z|}$$

i_{mm} has maximum possible value for α=0 i.e. fault at zero crossing of voltage

$$i_{mm\ \ maxpossible}=2\times\frac{\sqrt{2}V}{|Z|}$$

d) A 25 MVA, 11 kV synchronous generator with sub-transient reactance $X_d'' = 0.2$ pu is supplying three identical motors through a step-up and step-down transformer as shown in the figure below. Each motor has a sub-transient reactance $X_d'' = 0.25$ pu on its 5 MVA, 6.6 kV base. Both the 11/66 kV step-up and the 66/6.6 kV step-down transformers are rated at 25 MVA and have the same leakage reactance of 0.1 pu on their respective base. Assuming the voltage at the motor terminals to be 6.6 kV when a 3-phase fault occurs at point F, calculate the magnitude of the fault current in kA. Choose the system base as 25 MVA.

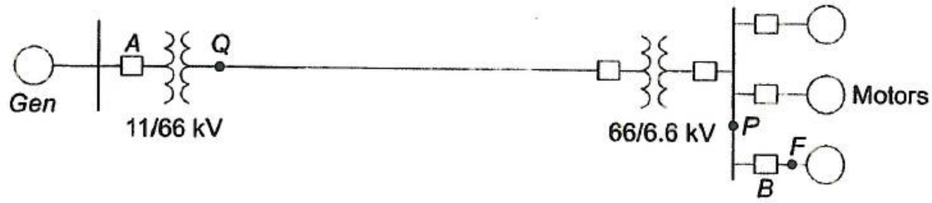


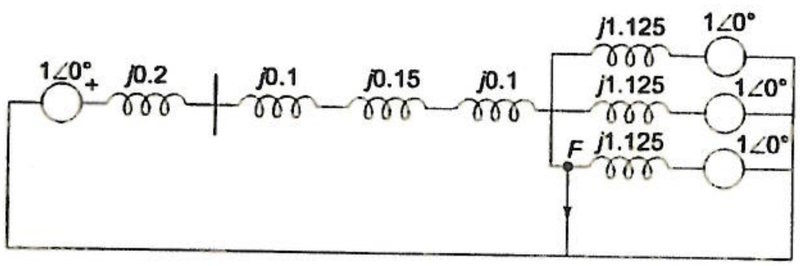
Figure 8: Synchronous generators supplying three motors

[8]

Let's choose a system base of 25 MVA. For a generator voltage base of 11 kV, the line voltage base is 66 kV and the motor voltage base is 6.6 kV.

For each motor $X_{dm}'' = j0.25 \times (25/5) = j1.25$ pu

Line, transformers and generator reactances are already given on their base values. The circuit model for fault calculation at F is shown below:



Equivalent impedance looking into the fault point F is:

$Z_{eq} = j0.55 \,||\, j1.25 \,||\, j1.25 \,||\, j1.25 = j0.237$ pu

Fault current $I_f = 1/Z_{eq} = -j4.22$ pu

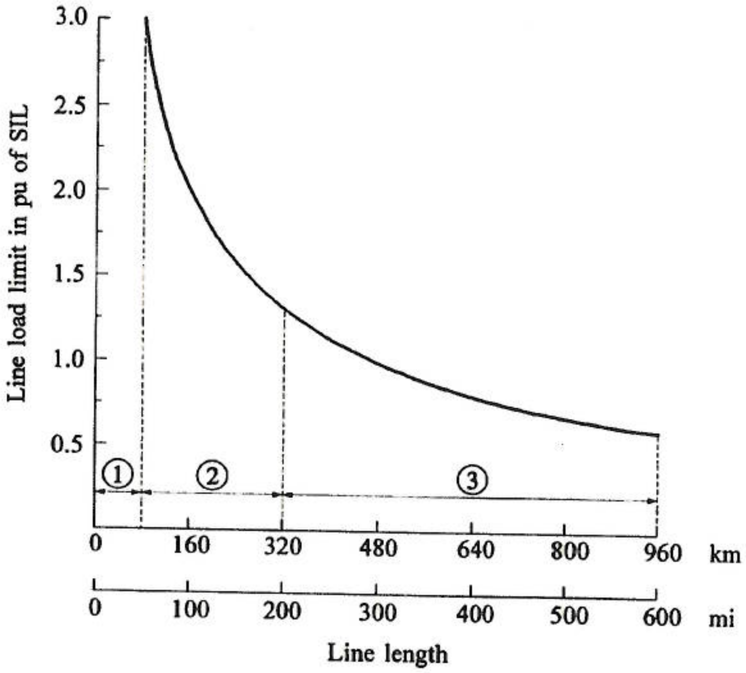
Base current for the motors = 25 / ($\sqrt{3} \times 6.6$) = 2.187 kA

Fault current = 4.22 × 2.187 = 9.229 kA

5. a) Describe how loadability (power transmission capacity) of transmission lines is limited due to different forms of stability problems.

[4]

Up to about 80-100 kms the transmission lines can be used up to their thermal limits. Beyond 80-100 kms up to about 300-320 kms (medium lines) it is the voltage stability limitations and beyond 320 kms (long lines) it is the angle stability constraints that limits the loadability or transmission capacity.
Line loadability curve (below) expressed in pu of surge impedance loading (SIL)/natural loading So that it is applicable to lines of all voltage classes.



①	0–80 km:	Region of thermal limitation
②	80–320 km:	Region of voltage drop limitation
③	320–960 km:	Region of small-signal (steady-state) stability limitation

b) Starting from the swing equation explain the concept of equal area criteria and explain how it is useful for stability analysis.

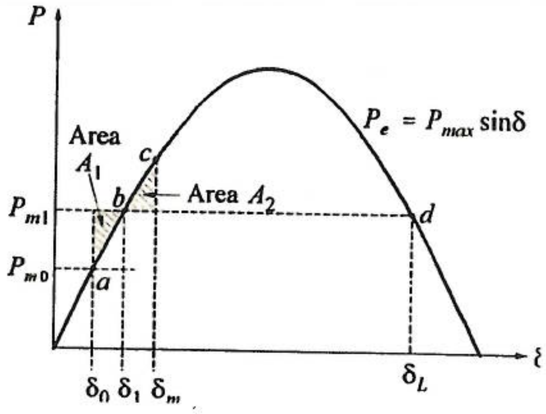
[4]

$$\frac{d^2\delta}{dt^2} = \frac{\omega_0}{2H}(P_m - P_e)$$

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{\omega_0(P_m - P_e)}{H}\frac{d\delta}{dt}$$

$$\frac{d}{dt}\left[\frac{d\delta}{dt}\right]^2 = \frac{\omega_0(P_m - P_e)}{H}\frac{d\delta}{dt}$$

$$\left[\frac{d\delta}{dt}\right]^2 = \int\frac{\omega_0(P_m - P_e)}{H}d\delta$$



$$E_1 = \int_{\delta_0}^{\delta_1}(P_m - P_e)d\delta = \text{area } A_1$$

$$E_2 = \int_{\delta_1}^{\delta_m}(P_e - P_m)d\delta = \text{area } A_2$$

$$\frac{d\delta}{dt} = \int_{\delta_0}^{\delta_m}\frac{\omega_0}{H}(P_m - P_e)d\delta = 0$$

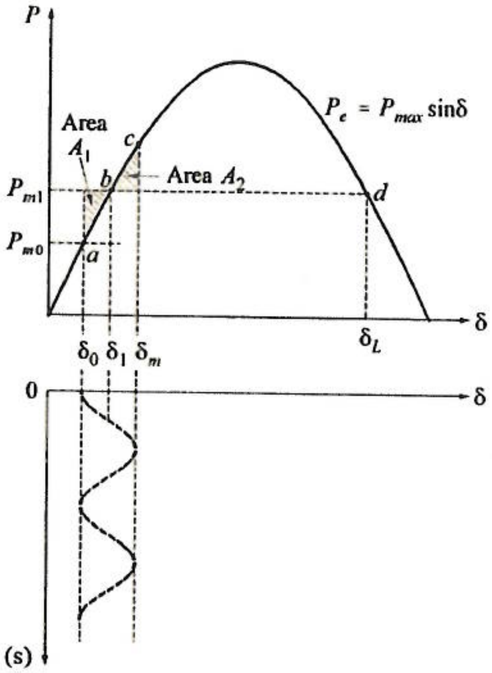
$$E_1 = \int_{\delta_0}^{\delta_1}(P_m - P_e)d\delta = E_2 = \int_{\delta_1}^{\delta_m}(P_e - P_m)d\delta$$

E_1: energy gained during acceleration
E_2: energy lost during deceleration
Neglecting losses, $E_1 = E_2$

Equal area criteria allows us to determine the maximum swing of δ and hence the stability without explicitly computing the time response by solving swing equation

c) Using the power angle characteristics explain the excursion of rotor angle following a step increase in mechanical input power a synchronous generator.

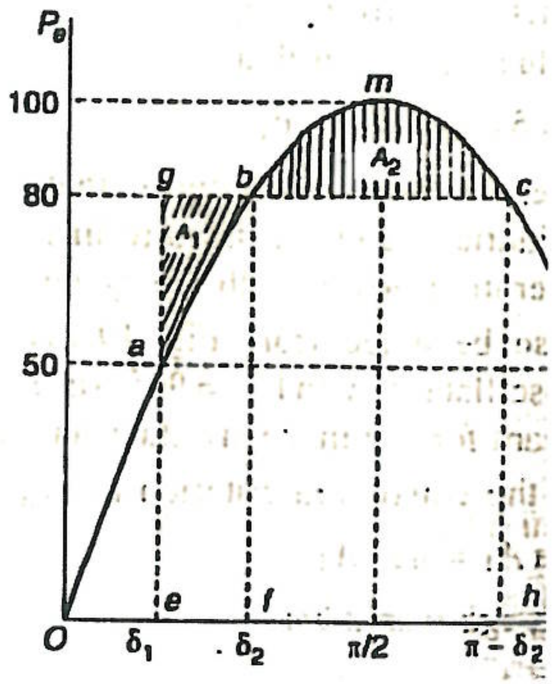
[4]



- Rotor accelerates towards 'b' due to excess mech. power
- At 'b' accelerating power is zero, but rotor speed > synch. speed (ω_0)
- Rotor angle continue to increase, $\delta_1 \rightarrow \delta_m$ but the rotor decelerates as $P_m<P_e$
- At 'c' rotor reaches sync. speed
- As $P_m<P_e$, rotor speed drops below ω_0
- Operating point retraces 'c' $\rightarrow$ 'b' $\rightarrow$ 'a' and the same cycle continues in absence of any damping

d) A synchronous generator is supplying 50 MW to an infinite bus. The steady-state stability limit (or maximum possible power transfer) of the system is 100 MW. Neglecting losses determine whether the system would remain stable if the mechanical power input to the generator is suddenly increased by 30 MW.

[8]



$P_e = P_{max} \sin \delta$

At initial operating point 'a' $P_e = 50$ MW

$50 = 100 \sin \delta_1 \Rightarrow \delta_1 = 0.523$ rad

After 30 MW increase in mechanical power the steady state operating point would be 'b'.

At point 'b' $80 = 100 \sin \delta_2 \Rightarrow \delta 2 = 0.927$ rad

Maximum permissible swing of rotor is up to point 'c'

System would be stable if the shaded accelerating area 'agb' is less than the decelerating area 'bmc'

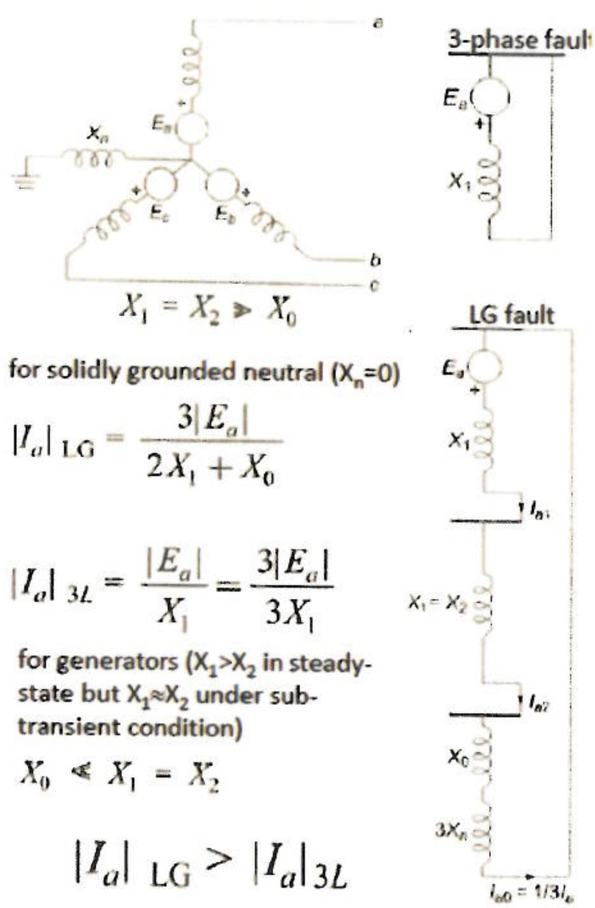
Area 'agb' $= 80 \times (0.927 - 0.523) + 100 \times (\cos \delta_2 - \cos \delta_1) = 5.72$ MW-rad

Area 'bmc' $= -100 \times [\cos (\pi-\delta_2) - \cos \delta_2] - 80 \times (\pi - 2\delta_2) = 117$ MW-rad

Since area 'agb' is less than area 'bmc' the system would remain stable

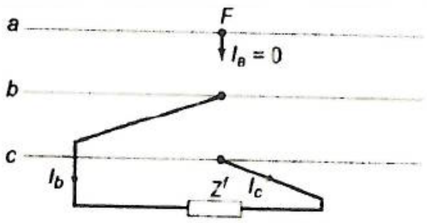
6. a) For a synchronous generator with solidly grounded neutral explain why a line-to-ground (LG) fault near the generator terminal is usually more severe than a three-phase fault.

[5]



b) Show that the positive and negative sequence networks are connected in parallel across the fault impedance for line-to-line (LL) faults.

[5]



Fault condition (currents and voltages) in phase domain

$$I_a = 0$$

$$I_c = -I_b$$

$$V_b - V_c = Z^f I_b$$

Fault condition (currents and voltages) in sequence domain

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 \\ (\alpha - \alpha^2) I_b \\ -(\alpha - \alpha^2) I_b \end{bmatrix}$$

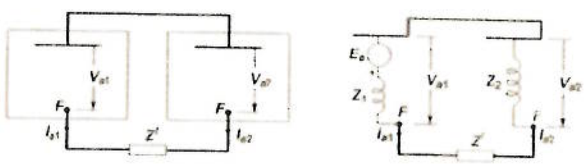
$$I_{a1} = -I_{a2}, I_{a0} = 0$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b - Z^f I_b \end{bmatrix}$$

$$3(V_{a1} - V_{a2}) = (\alpha - \alpha^2) Z^f I_b = 3 Z^f I_{a1}$$

$$V_{a1} - V_{a2} = Z^f I_{a1}$$

Connection between sequence networks at the fault point



c) A 3-phase 10 MVA, 11 kV (line-to-line) synchronous generator with a solidly grounded neutral point supplies a feeder at the same voltage level. The positive, negative and zero sequence impedances of the generator and the feeder are shown in the following table.

	Generator	Feeder
Postive sequence, Z1	j 1.2 Ω	j 1.0 Ω
Negative sequence, Z2	j 0.9 Ω	j 1.0 Ω
Zero sequence, Z0	j 0.4 Ω	j 3.0 Ω

i) For a line-to-ground (LG) fault at the far end of the feeder calculate the fault current in kA. Consider the phase voltage for calculations.

[6]

ii) Calculate the voltage to neutral of the faulty phase at the terminals of the generator.

[4]

i) Rated phase voltage $= 11/\sqrt{3} = 6.35$ kV

Total sequence impedances up to the far end of the feeder are:

$Z_{a0} = j\ 0.4 + j\ 3.0 = j\ 3.4\ \Omega$

$Z_{a1} = j\ 1.2 + j\ 1.0 = j\ 2.2\ \Omega$

$Z_{a2} = j\ 0.9 + j\ 1.0 = j\ 1.9\ \Omega$

For LG fault, the sequence networks are connected in series with the same sequence component of current flowing through each network

Hence, $I_{a0} = I_{a1} = I_{a2} = Vf / (Z_{a0} + Z_{a1} + Z_{a2}) = 6.35/(j\ 3.4 + j\ 2.2 + j\ 1.9) = -j\ 0.847$ kA

Fault current $I_f = 3I_{a0} = -j\ 2.54$ kA

ii) Voltage to neutral of the faulty phase at the generator terminal is:

$V_a = E_a - I_{a0} \times (Z_{ga0} + Z_{ga1} + Z_{ga2}) = 6.35 - (-j\ 0.847)\ (j\ 0.4 + j\ 1.2 + j\ 0.9) = 4.23$ kV