IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

MSc and EEE/EIE PART IV: MEng and ACGI

ADVANCED COMMUNICATION THEORY

Corrected copy

Wednesday, 27 April 2016, 10:00 am

Time allowed: 3:00 hours

There are 16 questions on this paper.

Answer ALL questions.

The multiple choice questions together account for 40% of the marks.

Answers to multiple choice questions 1-10 should be given on the paper itself.

Students are not permitted to use more than one answer book.

Students are not permitted to take the question paper away.

The following are provided:
A table of Fourier transforms
A Gaussian Tail Function graph

Examiners responsible:

First Marker(s): A. Manikas
Second Marker(s): D. Mandic

PART-I

1. A matched filter is used to detect the signal $s(t)$, given by

$$\text{where } s(t) = 5\,\text{rect}\left\{\frac{t}{10^{-6}\text{ sec}}\right\} \times 10^{-3}$$

which is corrupted by additive white Gaussian noise with double-sided power spectral density 10^{-9} W/Hz. The maximum Signal-to-Noise (SNR) ratio at the filter output is [4 marks]

   (a) 0.5;
   (b) 5;
   (c) 50;
   (d) 500;
   (e) none of the above.

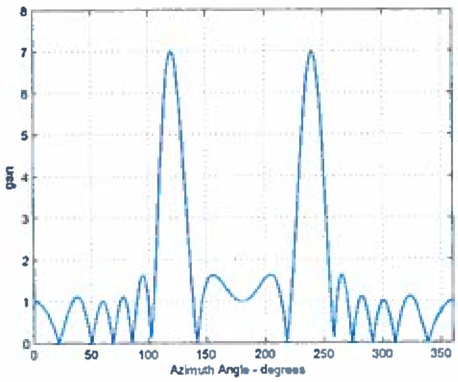
2. The Fredholm integral equation of the first kind, which provides the general equation for a matched filter, is given as follows: [4 marks]

   (a) $\int_0^{T_{cs}} h_{\text{opt}}(\text{z}).R_{nn}(\tau-\text{z}).dz = s(T_{cs}-\tau)$; $0 \leq \tau \leq \infty$;
   (b) $\int_0^{T_{cs}} h_{\text{opt}}(\text{z}).R_{nn}(\tau-\text{z}).dz = s(T_{cs}-\tau)$; $0 \leq \tau \leq T_{cs}$;
   (c) $\int_0^{T_{cs}} h_{\text{opt}}(\text{z}).R_{nn}(\text{z}-\tau).dz = s(T_{cs}-\tau)$; $0 \leq \tau \leq \infty$;
   (d) $\int_0^{T_{cs}} h_{\text{opt}}(\text{z}).R_{nn}(\text{z}-\tau).dz = s(T_{cs}-\tau)$; $0 \leq \tau \leq T_{cs}$;
   (e) none of the above.

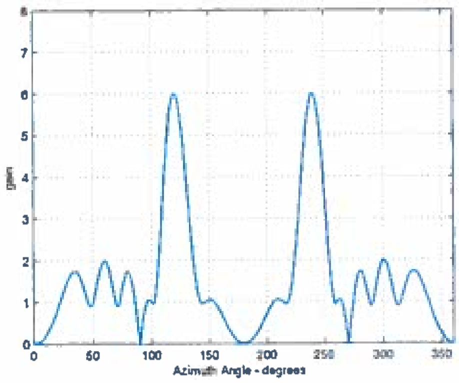
3. Consider a binary communication system which uses the following two equiprobable signals $s_0(t)$ and $s_1(t)$ of equal energy E and cross correlation $\rho_{01} = -1$. The signals are transmitted over a communication channel which adds white Gaussian noise having a double-sided power spectral density of 10^{-6} W/Hz. If the forward transition matrix $\mathbb{F}$ of the equivalent discrete channel is $\mathbb{F}=\begin{bmatrix} 0.994, & 0.006 \\ 0.006, & 0.994 \end{bmatrix}$ then the energy E is [4 marks]

   (a) 2.25×10^{-6};
   (b) 4.26×10^{-6};
   (c) 6.25×10^{-6};
   (d) 8.25×10^{-6};
   (e) none of the above.

4. The two figures below show the array patterns of two different linear arrays.



(1)



(2)

Which of the following statements is correct? [4 marks]

(a) In Figure (1) the array is a uniform linear array of 7 sensors.
(b) In Figure (1) the array has no weights (i.e. weights equal to 1).
(c) In Figure (2) the array is a uniform linear array of 6 sensors.
(d) In Figure (2) the array has no weights (i.e. weights equal to 1).
(e) None of the above.

5. Consider a beamformer which employes a uniform linear array of N antennas. The carrier frequency is 2.4 GHz and the manifold vector for a signal with Direction-of-Arrival $(\theta = 150^0, \phi = 0^0)$ is

$$[-0.5902 + 0.8072i, 0.2089 - 0.9779i, 0.2089 + 0.9779i, -0.5902 - 0.8072i]^T$$

To steer the main lobe of the array towards the direction $(\theta = 150^0, \phi = 0^0)$, the weight vector $\underline{w}$ should be [4 marks]

(a) $\begin{bmatrix}1, & 1, & 1, & 1\end{bmatrix}^T$;
(b) $[-0.5902 - 0.8072i, 0.2089 + 0.9779i, 0.2089 - 0.9779i, -0.5902 + 0.8072i]^T$;
(c) $[+0.5902 - 0.8072i, -0.2089 + 0.9779i, -0.2089 - 0.9779i, +0.5902 + 0.8072i]^T$;
(d) $[-0.5902 + 0.8072i, 0.2089 - 0.9779i, 0.2089 + 0.9779i, -0.5902 - 0.8072i]^T$;
(e) none of the above.

6. Consider a planar array of 4 antennas with Cartesian coordinates given by the following matrix

$$\begin{bmatrix} -2, & 2, & -2, & 2 \\ -0.5, & -0.5, & 0.5, & 0.5 \\ 0, & 0, & 0, & 0 \end{bmatrix} \text{ in units of half-wavelengths}$$

The array aperture is [4 marks]

(a) 4.1231;
(b) 4.0311;
(c) 4;
(d) 1;
(e) none of the above.

7. Consider a uniform linear array of N antennas. The carrier frequency is 2.4 GHz and the manifold vector for a signal with Direction-of-Arrival ($\theta = 30^0, \phi = 0^0$) is

$$[-0.5902 - 0.8072i, 0.2089 + 0.9779i, 0.2089 - 0.9779i, -0.5902 + 0.8072i]^T$$

The origin of the Cartesian coordinates (array reference point) is the [4 marks]

(a) 1st antenna;
(b) 2nd antenna;
(c) 3rd antenna;
(d) 4th antenna;
(e) the centroid of the array geometry.

8. With reference to SISO wireless channels, which of the following statements is correct? [4 marks]

(a) If the displacement of a wireless receiver is less than the "coherence distance" D_{coh} then the channel experiences small-scale fading.
(b) The "coherence distance" D_{coh} is the largest distance that a wireless receiver can move with the channel appearing to be invariable.
(c) If the transfer function of a wireless channel varies with time then "space-selectivity" and "spatial-coherence" are identical concepts.
(d) "Fast fading" implies that the magnitude of the transfer function of a wireless channel does not vary with time in the interval $nT_{cs} < t < (n+1)T_{cs}$ with T_{cs} denoting a channel symbol duration and n is an integer.
(e) None of the above.

9. With reference to a MIMO wireless communication system, consider that both the Tx and Rx antenna arrays are linear arrays with the Cartesian coordinates of their elements given by the columns of the following matrices

$$\text{Tx} : [\underline{r}_1, \underline{r}_2] = \begin{bmatrix} 0, & 0 \\ -0.5, & 0.5 \\ 0, & 0 \end{bmatrix} \text{ in units of half-wavelength.}$$

$$\text{Rx} : [\underline{r}_1, \underline{r}_2] = \begin{bmatrix} -2, & +2 \\ 0, & 0 \\ 0, & 0 \end{bmatrix} \text{ in units of half-wavelength.}$$

Which of the following statements, associated with its virtual Rx antenna array of an equivalent SIMO wireless communication system, is correct? [4 marks]

(a) $\begin{bmatrix} -2, & 2, & -2, & 2 \\ -0.5, & -0.5, & 0.5, & 0.5 \\ 0, & 0, & 0, & 0 \end{bmatrix}$ (i.e.a planar array).

(b) $\begin{bmatrix} -0.5, & -0.5, & 0.5, & 0.5 \\ -2, & 2, & -2, & 2 \\ 0, & 0, & 0, & 0 \end{bmatrix}$ (i.e. a planar array).

(c) $\begin{bmatrix} -2.5, & 1.5, & -1.5, & 2.5 \\ 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0 \end{bmatrix}$ (i.e. a linear array).

(d) $\begin{bmatrix} 0, & 0, & 0, & 0 \\ -2.5, & 1.5, & -1.5, & 2.5 \\ 0, & 0, & 0, & 0 \end{bmatrix}$ (i.e. a linear array).

(e) None of the above.

10. Consider a beamformer which employs a uniform array of $N = 5$ antennas that operates in the presence of a single signal with direction $(\theta = 30^0, \phi = 0^0)$. The carrier frequency is 2.4 GHz and the manifold vector for the Direction-of-Arrival $(\theta = 30^0, \phi = 0^0)$ is

$$[-0.1125 + 0.9936i, 0.6661 + 0.7458i, 1.0000, 0.6661 - 0.7458i, -0.1125 - 0.9936i]^T$$

Consider that the array steers its main lobe towards the direction $(\theta = 30^0, \phi = 0^0)$, the power of the received signal is 1 and the channel noise is additive white Gaussian noise of power 0.1. If at the output of the beamformer P_{out} is the power of the desired signal and SNR_{out} denotes the signal-to-noise ratio, which of the following statements is correct? [4 marks]

(a) P_{out}=5 and SNR_{out}=10.

(b) P_{out}=25 and SNR_{out}=10.

(c) P_{out}=5 and SNR_{out}=50.

(d) P_{out}=25 and SNR_{out}=50.

(e) None of the above.

PART-II

11. Consider an M-ary Communication System for which the signal set is described as follows:

$$s_i(t) = A_i \text{rect}\left\{\frac{t}{T_{cs}}\right\}, \; i = 1, 2, ..., M.$$

$$\text{with} \begin{cases} M = 4 \\ A_i = (2i - 1 - M) \times 10^{-3}\text{Volts} \\ T_{cs} = 4\,\text{sec} \\ \Pr(\text{H}_1) = \Pr(\text{H}_4) = 1/8 \text{ and } \Pr(\text{H}_2) = \Pr(\text{H}_3) = 3/8 \end{cases}$$

The signals are transmitted over a communication channel which adds white Gaussian noise with a double-sided power spectral density of 10^{-6} W/Hz.

(a) Find the energy $E_i, i = 1, 2, 3, 4$. [3 marks]

(b) Calculate the values of the signal-vectors $\underline{w}_{s_i}$, $i = 1, 2, 3, 4$ for the above signal-set. [3 marks]

(c) Draw a labelled block diagram of the MAP receiver, based on the signals vectors $\underline{w}_{s_i}$, $i = 1,2,3,4$. [3 marks]

(d) Plot the constellation diagram and label the decision regions. [3 marks]

12. Consider that one of the paths from the transmitter of a CDMA user arrives at the reference point of an antenna array CDMA receiver from direction (azimuth, elevation)$= (90°, 0°)$. The corresponding PN-sequence, of period N_c, is generated by the polynomial $D^2 + D + 1$ in GF(2) while the discrete path delay (mod-N_c) is equal to 3. For this path, if the Cartesian coordinates of the antenna array elements are given by the columns of the following matrix

$$[r_1, r_2, r_3] = \begin{bmatrix} -1, & 0, & +1 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \end{bmatrix} \text{ in units of half-wavelength,}$$

find

(a) the manifold vector; [5 marks]

(b) the spatio-temporal array manifold vector. [5 marks]

13. Draw a block structure and write a mathematical equation for the impulse response of the following multipath frequency selective channels:

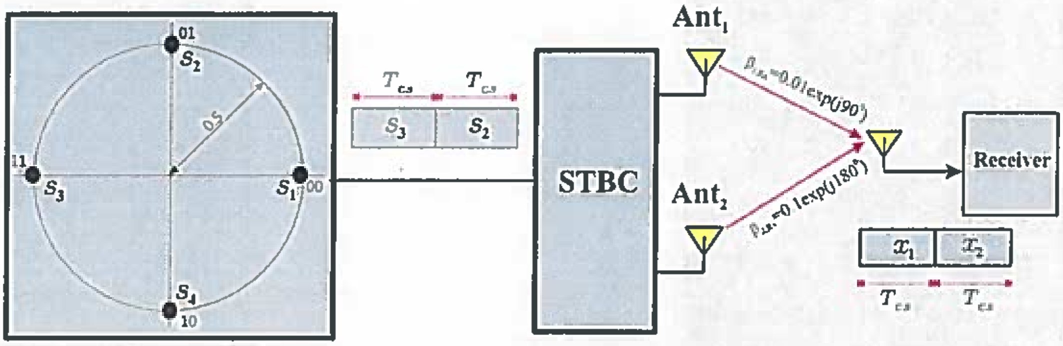
(a) SISO, [4 marks]

(b) SIMO, [4 marks]

(c) MISO, and [4 marks]

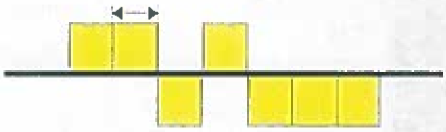
(d) MIMO. [4 marks]

14. Consider the QPSK MISO system of 2 Tx antennas operating in a frequency flat wireless channel as shown the following figure:



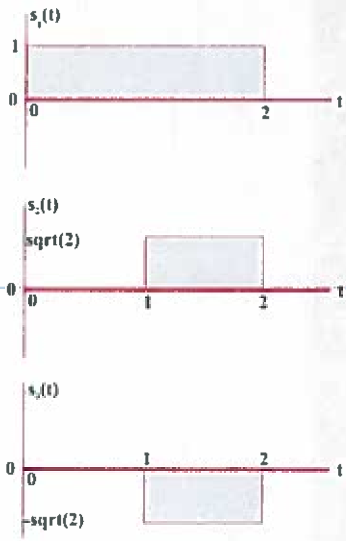
If the QPSK symbols $[s_3, s_2]$ are transmitted using the above "Space-Time Block Coder" (STBC) find the receiver's input $[x_1, x_2]$, ignoring the noise. [10 marks]

15. Sketch the impulse response of a matched filter for the following signal. [4 marks]



16. Starting with $s_1(t)$ and using the Gram-Schmidt procedure,

(a) find an orthonormal basis set for the signal set shown below: [4 marks]

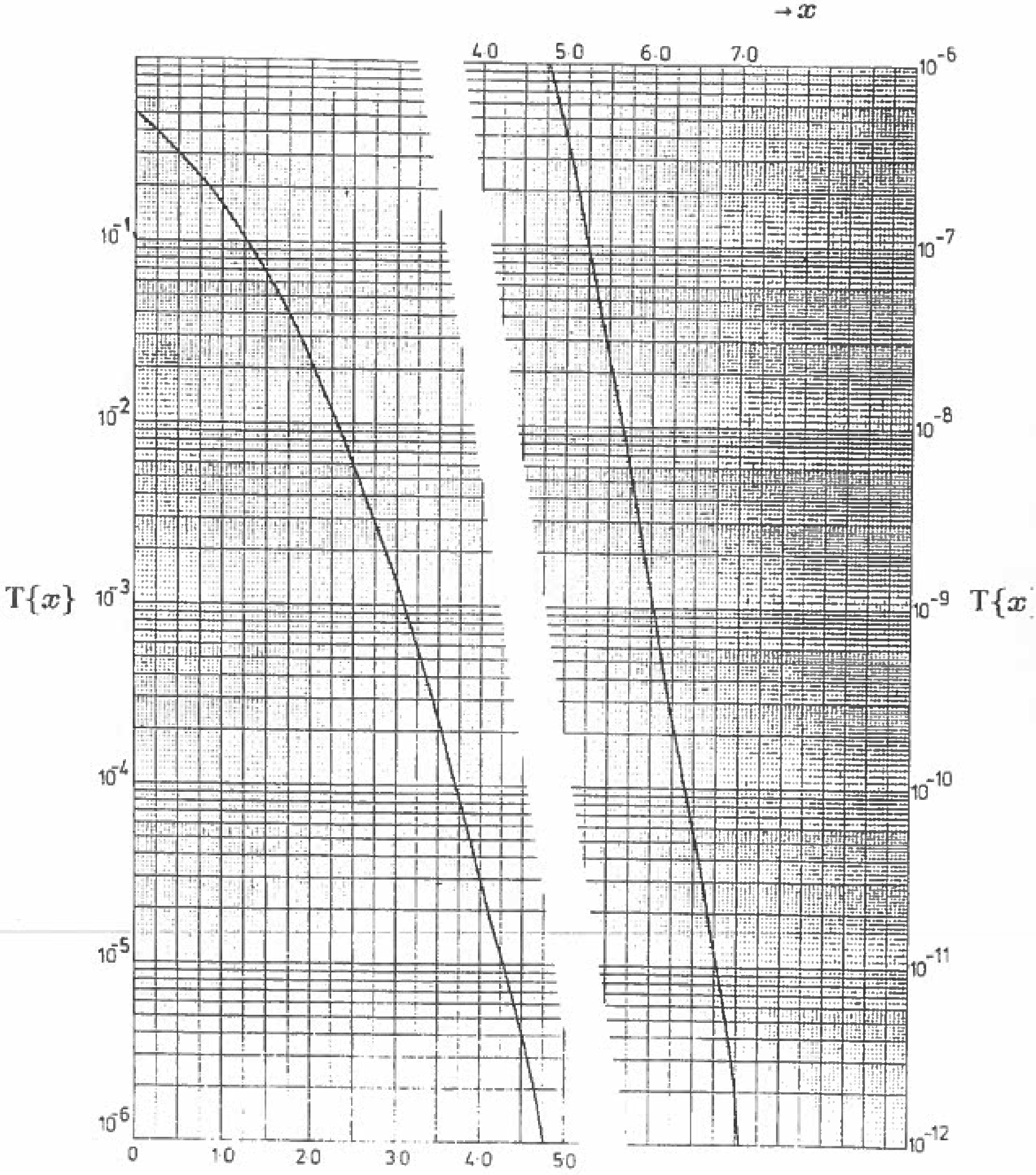


(b) Express the signals as vectors of coefficients of the orthonormal representation. [4 marks]

Fourier Transform Tables			
	Description	Function	Transformation
1	Definition	$g(t)$	$G(f) = \int_{-\infty}^{\infty} g(t).e^{-j2\pi ft}dt$
2	Scaling	$g\left(\frac{t}{T}\right)$	$\lvert T\rvert .G(fT)$
3	Time shift	$g(t-T)$	$G(f).e^{-j2\pi fT}$
4	Frequency shift	$g(t).e^{j2\pi Ft}$	$G(f-F)$
5	Complex conjugate	$g^*(t)$	$G^*(-f)$
6	Temporal derivative	$\frac{d^n}{dt^n}g(t)$	$(j2\pi f)^n.G(f)$
7	Spectral derivative	$(-j2\pi t)^n.g(t)$	$\frac{d^n}{df^n}G(f)$
8	Reciprocity	$G(t)$	$g(-f)$
9	Linearity	$A.g(t)+B.h(t)$	$A.G(f)+B.H(f)$
10	Multiplication	$g(t).h(t)$	$G(f)*H(f)$
11	Convolution	$g(t)*h(t)$	$G(f).H(f)$
12	Delta function	$\delta(t)$	1
13	Constant	1	$\delta(f)$
14	Rectangular function	$\text{rect}\{t\} \triangleq \begin{cases} 1 & \text{if } \lvert t\rvert < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$	$\text{sinc}\{f\} \triangleq \frac{\sin(\pi f)}{\pi f}$
15	Sinc function	$\text{sinc}(t)$	$\text{rect}\{f\}$
16	Unit step function	$u(t) \triangleq \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$	$\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$
17	Signum function	$\text{sgn}(t) \triangleq \begin{cases} 1 & t>0 \\ -1 & t<0 \end{cases}$	$-\frac{j}{\pi f}$
18	decaying exp (two-sided)	$e^{-\lvert t\rvert}$	$\frac{2}{1+(2\pi f)^2}$
19	decaying exp (one-sided)	$e^{-\lvert t\rvert}.u(t)$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
20	Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
21	Lambda function	$\Lambda\{t\} \triangleq \begin{cases} 1-t & \text{if } 0 \le t \le 1 \\ 1+t & \text{if } -1 \le t \le 0 \end{cases}$	$\text{sinc}^2\{f\}$
22	Repeated function	$\text{rep}_T\{g(t)\} = g(t)*\text{rep}_T\{\delta(t)\}$	$\lvert\frac{1}{T}\rvert\text{comb}_{\frac{1}{T}}\{G(f)\}$
23	Sampled function	$\text{comb}_T\{g(t)\} = g(t).\text{rep}_T\{\delta(t)\}$	$\lvert\frac{1}{T}\rvert\text{rep}_{\frac{1}{T}}\{G(f)\}$

The graph below shows the Tail function $\mathbf{T}\{x\}$ which represents the area from x to ∞ of the Gaussian probability density function N(0,1), i.e.

$$\mathbf{T}\{x\} = \int_x^\infty \frac{1}{\sqrt{2\pi}}.\exp\left(-\frac{y^2}{2}\right)dy$$



Note that if $x > 6.5$ then $\mathbf{T}\{x\}$ may be approximated by $\mathbf{T}\{x\} \approx \frac{1}{\sqrt{2\pi}.x}.\exp\left\{-\frac{x^2}{2}\right\}$