IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

EEE PART I: MEng, BEng and ACGI

DEVICES AND FIELDS

Corrected Copy

Q4

Friday, 30 May 10:00 am

Time allowed: 2:00 hours

There are SIX questions on this paper.

Question ONE and Question FOUR are compulsory. Answer Question One and Question Four, plus one additional question from Section A and one additional question from Section B.

Questions One and Four each carry 20% of the marks; remaining questions each carry 30% of the marks.

Use a separate answer book for each section.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : Z. Durrani, E.M. Yeatman

Second Marker(s) : K. Fobelets, B.C. Pal

Special instructions for students:

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10}$ cm^{-3} at $T = 300$K
dielectric constant of Si:	$\varepsilon_{Si} = 11$
dielectric constant of SiO_2:	$\varepsilon_{ox} = 4$
thermal voltage:	$kT/e = 0.026$V at $T = 300$K
charge of an electron:	$e = 1.6 \times 10^{-19}$ C

Formulae:

$$\left.\begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n\frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p\frac{dp(x)}{dx} \end{aligned}\right\}$$

Drift and diffusion currents in a semiconductor

$$I_{DS} = \frac{\mu C_{ox}W}{L}\left((V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Current in a MOSFET

$$\left.\begin{aligned} J_n &= \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right) \\ J_p &= \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1\right) \end{aligned}\right\}$$

Diffusion currents in a pn-junction

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT}\right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$

Minority carrier injection under bias V

$$L = \sqrt{D\tau}$$

Diffusion length

$$D = \frac{kT}{e}\mu$$

Einstein relation

Special instructions for students and invigilators:

For Question 1(a), please use the separate answer sheet provided.

Write your CID number on this sheet.

Ensure that the sheet is tied securely to your answer book.

SECTION A: SEMICONDUCTOR DEVICES

1. This question is **mandatory**

a) Mark, on diagrams similar to Fig. 1.1, the position of the Fermi level E_F for *n*-type, *p*-type, *p*+ and intrinsic semiconductor materials. (A separate answer sheet is provided).

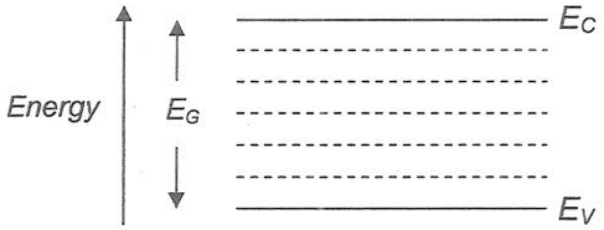


Figure 1.1: Energy band diagram.

[2]

b) An electric field $\mathcal{E}$ is applied across a semiconductor from left to right, as shown in Fig. 1.2. In addition, an excess electron concentration Δn and an excess hole concentration Δp exists in the semiconductor, of the form shown in the diagram below. What is the direction (left to right or right to left) of the drift and diffusion currents for electrons, and for holes?

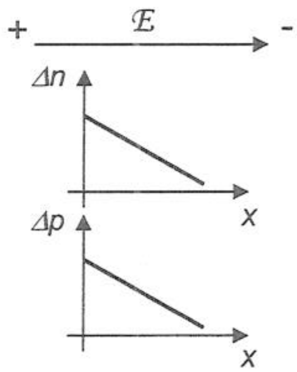


Figure 1.2: Direction of the electric field $\mathcal{E}$, and variation of the excess carrier concentrations Δn and Δp

[2]

c) The Fermi-Dirac distribution $f(E)$, is shown in Fig. 1.3 for three values of temperature T_1, T_2 and T_3 (solid, dotted and dashed lines respectively).

i) What is the relationship (>, <, =) between T_1, T_2 and T_3?

ii) The conduction, E_C, and valence band edge E_V, and the Fermi level E_F are also given in Fig. 1.3. What is the relationship (>, <, =) between the conduction band electron concentrations n_1, n_2 and n_3 seen at the respective temperatures T_1, T_2 and T_3?

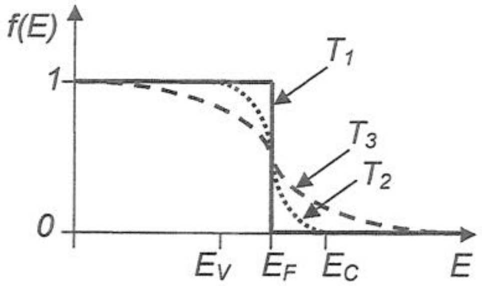


Figure 1.3: Fermi-Dirac distribution function f(E) as a function of energy E for three different temperatures.

[4]

d) Draw the energy band diagram (E_C, E_V, E_F, E_{vac}) for a metal–n-type semiconductor junction, when:

i) $\phi_m < \phi_s$

ii) $\phi_m > \phi_s$

where ϕ_m is the metal workfunction and ϕ_s is the semiconductor workfunction

[6]

e) The cross-section of a short *pn* junction is shown in Fig. 1.4. N_A is the acceptor doping concentration, N_D is the donor doping concentration, and $N_A > N_D$. The widths of the *p* and *n* semiconductor regions are W_p and W_n respectively, and the width of the depletion region at zero bias is W.

   i) What is the relationship (>, <, =) between the equilibrium minority carrier concentrations p_{n0} and n_{p0}?
   ii) Sketch the minority carrier concentrations p_n and n_p, with respect to the *x*-axis and outside the depletion region, for the *pn* junction in forward bias. Mark p_{n0} and n_{p0} on your diagram.
   iii) Sketch p_n and n_p, with respect to the *x*-axis and outside the depletion region, for the *pn* junction in reverse bias. Mark p_{n0} and n_{p0} on your diagram.

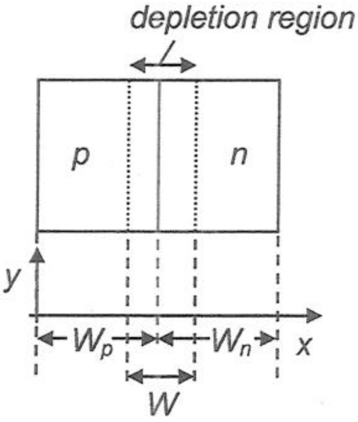


Figure 1.4: Cross section of a pn junction

[6]

2.

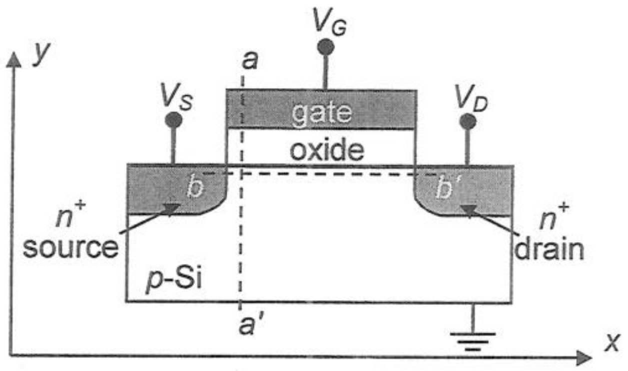


Figure 2.1: Cross section of a Si MOSFET

a) For the enhancement-mode MOSFET shown in Fig. 2.1, sketch the energy band diagram (E_C, E_V, E_F) along the following lines.

i) The line *a-a'*, with $V_S = V_D = V_G = 0$ V [5]
ii) The line *a-a'*, with $V_S = V_D = 0$ V, and $V_G = V_T$. [5]
iii) The line *b-b'*, with $V_S = 0$ V, $V_{D,sat} > V_D > 0$ V, and $V_G > V_T$. [5]

where V_T is the threshold voltage and $V_{D,sat}$ is the saturation voltage. Assume a metal gate with a workfunction equal to the semiconductor workfunction.

b) Consider an *n*-channel enhancement mode MOSFET, with gate width W, channel length L, gate oxide capacitance/area C_{ox}, and carrier mobility μ.

i) For a threshold voltage V_T, gate voltage $V_{GS} > V_T$ and $V_{DS} > 0$V write an equation for the channel current I_{DS}, as a function of the longitudinal electric field, $\mathcal{E}$ along the channel, and the carrier concentration $n(x)$ in the channel. State any assumptions you make. [5]

ii) Hence, using this derive equation (1) below for I_{DS}, at a drain-source voltage V_{DS}, for the MOSFET. State any assumptions you make.

$$I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right) \qquad (1)$$

[10]

3.

a) Draw the energy band diagram (E_C, E_V, E_F) for an n^+pn bipolar junction transistor (BJT), with base-emitter voltage $V_{BE} = 0$ V and base-collector voltage $V_{BC} = 0$ V. Mark on your diagram the edges of the depletion regions in the emitter, base, and collector of the BJT. Also mark the position of the metallurgical junctions.

[5]

b) Draw the energy band diagram (E_C, E_V, E_F) for an n^+pn BJT in the forward active mode. Mark on your diagram the edges of the depletion regions in the emitter, base, and collector of the BJT. Also mark the position of the metallurgical junctions.

[5]

c) Consider an n^+pn BJT with emitter doping concentration $N_E = 10^{17}$ cm^{-3}, base doping concentration $N_B = 10^{16}$ cm^{-3} and collector doping concentration $N_C = 10^{15}$ cm^{-3}. Calculate, at a temperature of 300 K and an applied base-emitter forward bias $V_{BE} = 0.6$ V,

i) The width of the depletion region in the emitter, w_E, and in the base, w_B, on either side of the metallurgical junction. You may assume that the total charge associated with ionised doping atoms across the entire depletion region is zero. The width W of the entire depletion region is given by:

$$W = w_E + w_B = \sqrt{\frac{2\varepsilon_{si}\varepsilon_0}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_0 - V)}$$

where N_A and N_D are the acceptor and donor doping concentrations in a p-n junction, V is the forward-bias voltage across the junction and V_0 is the built-in voltage.

[15]

ii) The BJT of part (i) has a reverse bias voltage applied across the base-collector junction such that the base region is completely depleted of holes. Draw the energy band diagram for this condition. Comment briefly on whether in this case, the base can control the collector current.

[5]

SECTION B: FIELDS

4. This question is mandatory. Answer all parts. Each part is worth 4 marks.

a) A uniform static electric field has magnitude 10^3 V/m, and its direction makes a 60° angle with the x axis. Calculate the potential difference between two points on the x axis 2 cm apart.

b) A conducting sphere of radius 1 cm, in air, has a net surface charge of 10^{-9} C. Calculate the electric field magnitude at its surface.

c) An ideal transformer has a turns ratio $N_1:N_2 = 10:20$, and a load of 10 Ω is connected to the secondary coil terminals. If an AC voltage of magnitude 5 V is applied to the primary coil, what will be the magnitude of the current I_2 flowing in the secondary coil?

d) A linear ferromagnetic material, with a relative permeability ε_r = 800, contains a magnetic flux density B = 0.5 T. Calculate the magnetic field strength H in the material. μ_r (10.16)

e) A 50 turn, open circuit, round coil of radius 1 cm lies in a magnetic field with time varying flux density $B(t) = B_o\sin(\omega t)$, with $B_o = 2$ T and $\omega = 100$ rad/s. The magnetic flux direction is perpendicular to the plane of the coil. Calculate the magnitude of the potential induced in the coil.

5. A certain one-dimensional system has an electric potential of the form:

$$V(x) = -2x^{-2} + 4x^{-1}$$

with x in cm and V in volts.

a) Derive an expression for the magnitude of the electric field E(x). Considering only positive x values, find a position x_o such that $E(x_o) = 0$. Calculate also the potential $V(x_o)$. [6]

b) Find the position x_m (with $x_m > 0$) for which E(x) has its largest positive value. Calculate $E(x_m)$ and $V(x_m)$. [6]

c) Give an expression for the force F(x) required to hold a charge Q stationary in this electrostatic field. By integrating this force, find the work required to move a charge Q from $x = \infty$ to $x = x_o$. Show that this is equal to the change in electrostatic energy of the charge. [6]

d) Sketch, quantitatively, both E(x) and V(x), for positive values of x. [6]

e) What is unique about the position $x = x_o$ for a charge positioned there? How would the sign of the charge affect its behaviour at this position? [6]

6. Two coils of N_1 and N_2 turns respectively are wound around a cylindrical iron core of relative permeability μ_r, length L and cross-sectional area A, as shown in Fig. 6.1. A current $I_1 = I_o sin(\omega t)$ is introduced in the first coil, while the second coil is open circuit.

a) Using Ampere's Law for the magnetic field strength H, taken along the dotted path as shown, find an expression for $H_i(t)$ in the core. State any approximations or assumptions used. Hence, calculate the magnitude of the magnetic flux density in the core for the following values: $N_1 = 20$, $A = 1$ cm^2, $L = 12$ cm, $\mu_r = 4000$, $I_o = 0.5$ A, $\omega = 100$ rad/s. [8]

b) Give an expression for the total magnetic flux $\Phi(t)$ flowing in the core. State any approximations or assumptions used. Hence, find an expression for the voltage $V_2(t)$ induced at the terminals of the second coil. Calculate the magnitude of $V_2(t)$ for the parameter values given in (a), and taking $N_2 = 10$. [8]

c) Neglecting the resistance of the coil wires, find an expression for the voltage $V_1(t)$, and hence the ratio $V_2(t)/V_1(t)$. [8]

d) Consider the effect of connecting a finite resistance R_2 across the terminals of the second coil, such that a current I_2 can flow in this coil. If I_1 is unchanged, would the magnetic flux in the core in the presence of R_2 be increased, decreased or unchanged compared to the case where the second coil is open circuit? Explain your reasoning. [6]

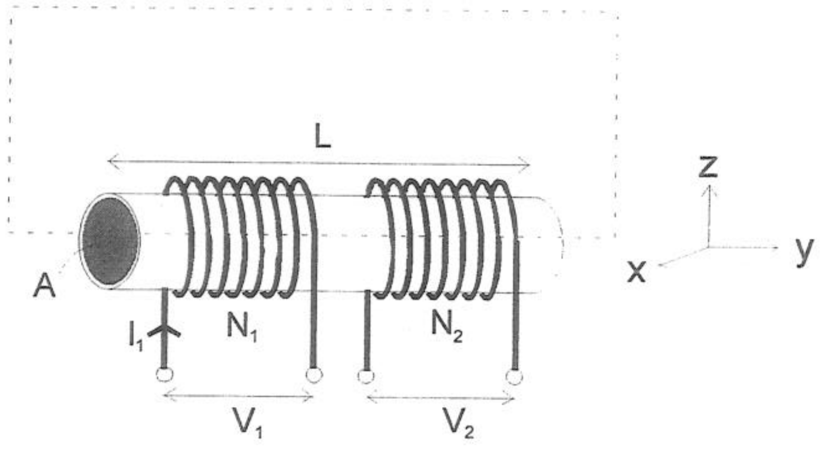
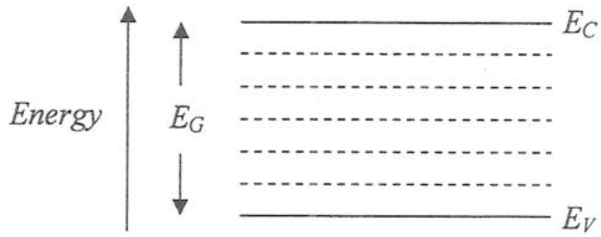


Figure 6.1

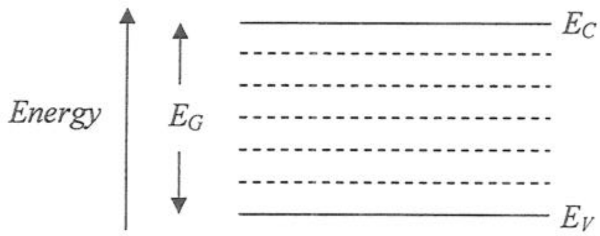
Answer Sheet For Question 1(a)

*CID NO:*___________________________________

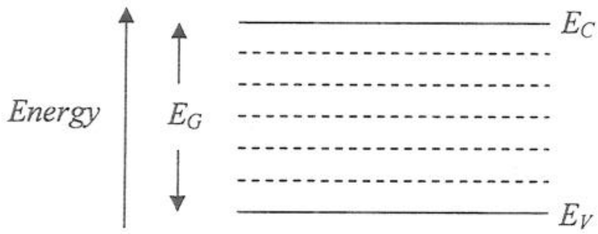
n-type:



p-type:



p^+:



Intrinsic