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Model Answers and Mark Schemes First Examiner: POPOVIĆ ; PAL

Paper Code: E3.13 Second Examiner: PAL ; POPOVIĆ

Electrical Energy Systems – Solutions 2003

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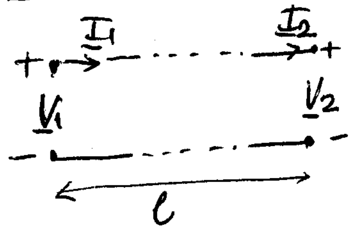
1. (bookwork + application of theory)

a) Line terminated in $\underline{Z}_c \Rightarrow \underline{V}_2 = \underline{Z}_c \underline{I}_2$



$$\underline{V}_1 = \underline{V}_2 \cosh \gamma l + \underline{Z}_c \underline{I}_2 \sinh \gamma l$$
$$\underline{I}_1 = \underline{I}_2 \cosh \gamma l + \frac{\underline{V}_2}{\underline{Z}_c} \sinh \gamma l$$ 2

$$\Rightarrow \underline{V}_1 = \underline{V}_2 (\cosh \gamma l + \sinh \gamma l) = \underline{V}_2 e^{\gamma l} = \underline{V}_2 e^{\alpha l} e^{j\beta l}$$
$$\underline{I}_1 = \underline{I}_2 (\cosh \gamma l + \sinh \gamma l) = \underline{I}_2 e^{\gamma l} = \underline{I}_2 e^{\alpha l} e^{j\beta l}$$ 2

$$\Rightarrow \frac{\underline{V}_1}{\underline{I}_1} = \frac{\underline{V}_2}{\underline{I}_2} = \underline{Z}_c$$ 1

$$|\underline{V}_1| = |\underline{V}_2| e^{\alpha l} \Rightarrow \frac{|\underline{V}_2|}{|\underline{V}_1|} = e^{-\alpha l}$$ 1

$$\frac{|\underline{I}_2|}{|\underline{I}_1|} = e^{-\alpha l}$$ 2

Due to reference direction for $\underline{I}_2$,

$$-\underline{S}_{21} = \underline{V}_2 \underline{I}_2^* = \underline{V}_1 e^{-\alpha l} e^{-j\beta l} \underline{I}_1^* e^{-\alpha l} e^{j\beta l}$$
$$= \underline{S}_{12} e^{-2\alpha l}$$

$$\Rightarrow \frac{-\underline{S}_{21}}{\underline{S}_{12}} = e^{-2\alpha l}$$ 2

alternatively:

$\underline{V}_1 = \underline{Z}_c \underline{I}_1 \Rightarrow$

$\underline{V}_1 \underline{I}_1^* = \underline{Z}_c |\underline{I}_1|^2$

$\underline{V}_2 \underline{I}_2^* = \underline{Z}_c |\underline{I}_2|^2$

$\Rightarrow \frac{-\underline{S}_{21}}{\underline{S}_{12}} = \frac{|\underline{I}_2|^2}{|\underline{I}_1|^2} = e^{-2\alpha l}$

Since α is real,

$$\frac{-P_{21}}{P_{12}} = e^{-2\alpha l}$$ 2

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1\. (cont)

b) lossless line $R = G = 0$

$Z_c = \sqrt{\frac{\ell}{C}}$ - real

$\gamma = \sqrt{zy} = \sqrt{j\omega\ell j\omega C} = j\omega\sqrt{\ell C}$ 2

inductance per unit length

$\Rightarrow \alpha = 0, \quad \beta = \omega\sqrt{\ell C}$

$\Rightarrow \frac{|\underline{V}_2|}{|\underline{V}_1|} = \frac{|\underline{I}_2|}{|\underline{I}_1|} = \frac{-\underline{S}_{21}}{\underline{S}_{12}} = \frac{-P_{21}}{P_{12}} = 1$ 6

$P_{12} = Re\{\underline{V}_1 \underline{I}_1^*\} = Re\{Z_c |\underline{I}_1|^2\} = Z_c |\underline{I}_1|^2 = \frac{|\underline{V}_1|^2}{Z_c}$

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2. (New computed example)

Base of 100 MVA and 161 kV in the TL.

Base voltage in the generator circuit = 13.2 kV 2

Base voltage in the motor circuit = 13.8 kV 2

Generator: $X = 0.15\ \frac{100}{15} \cdot \left(\frac{13.8}{13.2}\right)^2 = 1.093\ pu$ 2

TL: $Z_{base} = \frac{161^2}{100} = 259.21\ \Omega$

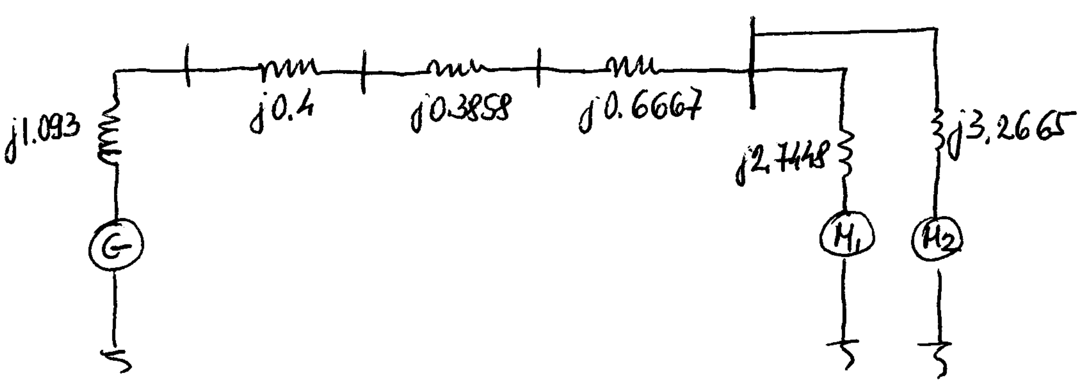
$X_{line} = \frac{100}{259.21} = 0.3858\ pu$ 2

T1: $X = 0.1\ \frac{100}{25} = 0.4\ pu$ 2

T2: $X = 0.1\ \frac{100}{15} = 0.6667\ pu$ 2

M1: $X = 0.15\ \frac{100}{5}\left(\frac{13.2}{13.8}\right)^2 = 2.7448\ pu$ 2

M2: $X = 0.15\ \frac{100}{5}\left(\frac{14.4}{13.8}\right)^2 = 3.2665\ pu$ 2



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3. (New computed example)

a) Bus 1 – slack bus Unknowns: $\theta_2, \theta_3, |V_3|$ 1

Bus 2 – PV bus

Bus 3 – PQ bus

$$P_2(x) = |V_2||V_1| B_{21} \sin(\theta_2-\theta_1) + |V_2||V_3| B_{23} \sin(\theta_2-\theta_3)$$ 2

$$= 10.5 \sin\theta_2 + 10.5|V_3| \sin(\theta_2-\theta_3)$$

$$P_3(x) = |V_3||V_1| B_{31} \sin(\theta_3-\theta_1) + |V_3||V_2| B_{32} \sin(\theta_3-\theta_2)$$ 2

$$= 10|V_3| \sin\theta_3 + 10.5|V_3| \sin(\theta_3-\theta_2)$$

$|V_2|$ is known $\Rightarrow$ eliminate equation Q_2

$$Q_3(x) = -\left[|V_3||V_1| B_{31} \cos(\theta_3-\theta_1) + |V_3||V_2| B_{32} \cos(\theta_3-\theta_2) + |V_3|^2 B_{33}\right]$$ 2

$$= -\left[10|V_3| \cos\theta_3 + 10.5|V_3| \cos(\theta_3-\theta_2) - 19.98|V_3|^2\right]$$

where

$$x = \begin{bmatrix} \theta_2 \\ \theta_3 \\ |V_3| \end{bmatrix}$$ 1

b) Jacobian

$$J(x) = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \theta_2} & \frac{\partial Q_3}{\partial \theta_3} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix}$$ 1

$$\frac{\partial P_2}{\partial \theta_2} = |V_2||V_1| B_{21} \cos(\theta_2-\theta_1) + |V_2||V_3| B_{23} \cos(\theta_2-\theta_3)$$

$$= 10.5 \cos\theta_2 + 10.5|V_3| \cos(\theta_2-\theta_3)$$

$$\frac{\partial P_2}{\partial \theta_3} = -|V_2||V_3| B_{23} \cos(\theta_2-\theta_3) = -10.5|V_3| \cos(\theta_2-\theta_3)$$

$$\frac{\partial P_2}{\partial |V_3|} = |V_2| B_{23} \sin(\theta_2-\theta_3) = 10.5 \sin(\theta_2-\theta_3)$$

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3 (cont)

$$\frac{\partial P_3}{\partial \theta_2} = -10.5\,|V_3| \cos(\theta_3 - \theta_2)$$

$$\frac{\partial P_3}{\partial \theta_3} = 10|V_3| \cos\theta_3 + 10.5|V_3| \cos(\theta_3 - \theta_2)$$

$$\frac{\partial P_3}{\partial |V_3|} = 10 \sin\theta_3 + 10.5 \sin(\theta_3 - \theta_2)$$ 2

$$\frac{\partial Q_3}{\partial \theta_2} = -10|V_3||V_2| \sin(\theta_3 - \theta_2) = -10.5|V_3| \sin(\theta_3 - \theta_2)$$

$$\frac{\partial Q_3}{\partial \theta_3} = 10|V_3| \sin\theta_3 + 10|V_3||V_2| \sin(\theta_3 - \theta_2)$$
$$= 10|V_3| \sin\theta_3 + 10.5|V_3| \sin(\theta_3 - \theta_2)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -\left[10\cos\theta_3 + 10.5\cos(\theta_3 - \theta_2) - 39.96|V_3|\right]$$

$P_2 = P_{G2} = 0.6661$

$P_3 = -P_{D3} = -2.8653$ 1

$Q_3 = -Q_{D3} = -1.2244$

Initial guess : $\theta_2^0 = \theta_3^0 = 0$ (flat start)

$|V_3| = 1$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix}^0 = \begin{bmatrix} P_2 \\ P_3 \\ Q_3 \end{bmatrix} - \begin{bmatrix} P_2(x^0) \\ P_3(x^0) \\ Q_3(x^0) \end{bmatrix} = \begin{bmatrix} 0.6661 \\ -2.8653 \\ -1.2244 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -0.52 \end{bmatrix} = \begin{bmatrix} 0.6661 \\ -2.8653 \\ -0.7044 \end{bmatrix}$$ 1

quite big mismatch

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3 (cont)

$$J^0 = \left[\begin{array}{cc|c} 21 & -10.5 & 0 \\ -10.5 & 20.5 & 0 \\ \hline 0 & 0 & 19.46 \end{array}\right]$$ 1

$$J^{-1} = \begin{bmatrix} J_{11} & 0 \\ 0 & J_{22} \end{bmatrix}^{-1} = \begin{bmatrix} J_{11}^{-1} & 0 \\ 0 & J_{22}^{-1} \end{bmatrix} = \begin{bmatrix} 0.0640 & 0.0328 & 0 \\ 0.0328 & 0.0656 & 0 \\ 0 & 0 & 0.0514 \end{bmatrix}$$

$$\Delta X^0 = \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta|V_3| \end{bmatrix} = \begin{bmatrix} -0.0513 \text{ rad} \\ -0.1660 \text{ rad} \\ -0.0362 \end{bmatrix} = \begin{bmatrix} -2.9393^\circ \\ -9.5111^\circ \\ -0.0362 \end{bmatrix}$$ 1

$$X^1 = X^0 + \Delta X = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2.9395^\circ \\ -9.5111^\circ \\ -0.0362 \end{bmatrix} = \begin{bmatrix} -2.9395^\circ \\ -9.5111^\circ \\ 0.9638 \end{bmatrix}$$ 1

Next iteration : using new values $\theta_2' = -2.9395^\circ$

$\theta_3' = -9.5111^\circ$

$|V_3|' = 0.9638$

$\Rightarrow P_2(x') = 0.6198$

$\Delta P_2' = 0.6661 - 0.6198 = 0.0463$

Similarly, $\Delta P_3'$ and $\Delta Q_3'$

$$\Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix}^1 = \begin{bmatrix} -0.0463 \\ -0.1145 \\ -0.2251 \end{bmatrix}$$

(note that in one iteration the mismatch vector has been reduced by a factor of about 10) 1

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3. (cont.)

$$J^1 = \begin{bmatrix} 20.5396 & -10.0534 & 1.2017 \\ -10.0534 & 19.5589 & -2.8541 \\ 1.1582 & -2.7508 & +18.2199 \end{bmatrix}$$ 1

$$J_1^{-1} = \begin{bmatrix} 0.0651 & 0.0336 & 0.0010 \\ 0.0336 & 0.0696 & 0.0087 \\ 0.0009 & 0.0084 & 0.0561 \end{bmatrix}$$

$$X^{(2)} = \begin{bmatrix} \theta_2 \\ \theta_3 \\ |V_3| \end{bmatrix}^{(2)} = \begin{bmatrix} -3.0023^\circ \\ -9.9924^\circ \\ 0.9502 \end{bmatrix}$$ 1

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix}^{(2)} = \begin{bmatrix} +0.0019 \\ -0.0023 \\ -0.0031 \end{bmatrix}$$ ← mismatch is reduced 200 times & small enough.

$\Rightarrow$

$$P_{G1} = P_1 = |V_1||V_2| B_{12} \sin(\theta_1-\theta_2) + |V_1||V_3| B_{13} \sin(\theta_1-\theta_3)$$
$$= 10.5 \sin 3.0023^\circ + 9.502 \sin 9.9924^\circ = 2.1987$$

$$Q_{G1} = Q_1 = -\Big[|V_1||V_2| B_{12}\cos(\theta_1-\theta_2) + |V_1||V_3| B_{13}\cos(\theta_1-\theta_3) + |V_1|^2 B_{11}\Big]$$ 1
$$= -\big[10.5\cos 3.0023^\circ + 9.502\cos 9.9924^\circ - 19.98\big]$$
$$= 0.1365$$

$$Q_{G2} = Q_2 = -\Big[|V_2||V_1| B_{21}\cos(\theta_2-\theta_1) + |V_2||V_3| B_{23}\cos(\theta_2-\theta_3) + |V_2|^2 B_{22}\Big]$$
$$= -1.6395$$

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Model Answers (Dr. B. Pal)

4 Solution

(a) **(Bookwork)** The minimum load or output limitation of a thermal unit is influenced by the steam generator (boiler) and the regenerative cycle rather than the turbine and synchronous generator operation constraints. The stability of the fuel combustion process can not be maintained at less than 30 % of unit output. For example most supercritical units cannot operate below 30 % of design capability as a minimum flow of 30 % is required to cool the tubes in the furnace of the steam generator adequately.
[4marks]

(b) **(New computed example)**When both of the units are in operation, the condition of optimal operating cost is reached when incremental fuel characteristic costs are equal. In the absence of transmission loss, the total load equals total generation. i.e. at optimal point the following has to satisfy.

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} \tag{4.1}$$

$$P_{load} = P_1 + P_2 \tag{4.2}$$

Now,

$$\frac{dC_1}{dP_1} = 10 + 0.016P_1 \quad \$/\text{MWhr} \tag{4.3}$$

$$\frac{dC_2}{dP_2} = 8 + 0.018P_2 \quad \$/\text{MWhr} \tag{4.4}$$

Upon substitution of (4.3) and (4.4) into (4.1) and (4.2) and carrying out necessary manipulation, the following final form is obtained

$$0.016P_1 - 0.018P_2 = -2.0 \tag{4.5}$$

$$P_1 + P_2 = P_{load} \tag{4.6}$$

[5marks]

I use two values of loads one at at time.
For $P_{load} = 1000$ MW
the solution is

$$P_1 = 470.58 \quad \text{MW} \tag{4.7}$$

$$P_2 = 529.42 \quad \text{MW} \tag{4.8}$$

It is seen there is no violation of limits. Hence they share a common incremental cost of generation.The results are

$$\frac{dC_1}{dP_1} = 17.52 \quad \$/\text{MW-hr} \tag{4.9}$$

$$\frac{dC_2}{dP_2} = 17.52 \quad \$/\text{MW-hr} \tag{4.10}$$

$$C_1 = 6477 \quad \$/\text{hr} \tag{4.11}$$

$$C_2 = 6758 \quad \$/\text{hr} \tag{4.12}$$

$$C_T = 13235 \quad \$/\text{hr} \tag{4.13}$$

[4marks]

(ii) **For $P_{load} = 1400$ MW**
the solution is

$$P_1 = 682.36 \quad \text{MW} \tag{4.14}$$

$$P_2 = 717.64 \quad \text{MW} \tag{4.15}$$

It is seen that unit #1 is delivering power more than its minimum limit which is not feasible. Hence it is set at 600 MW and the rest comes from unit #2. With this constraint, the incremental costs($\frac{dC_i}{dP_i}$) and total cost of operation $C_T(C_1 + C_2)$ for this value of load is as follows

$$\frac{dC_1}{dP_1} = 19.6 \quad \$/\text{MW-hr} \tag{4.16}$$

$$\frac{dC_2}{dP_2} = 22.4 \quad \$/\text{MW-hr} \tag{4.17}$$

$$C_1 = 8880 \quad \$/\text{hr} \tag{4.18}$$

$$C_2 = 12160 \quad \$/\text{hr} \tag{4.19}$$

$$C_T = 21040 \quad \$/\text{hr} \tag{4.20}$$

[7marks]

5 solution

(a) (**Bookwork**)A significant components of industrial loads comprise of induction motors. The speed at which these motor run depends on supply frequency. At times it is desired to have constancy in drive speed in some process industries. The supply frequency in these cases should be as constant as possible.
In a power plant, reduction in frequency by 1 Hz, significantly affects the performance of the system. The auxiliary loads comprise of large feed pumps to maintain desired pressure of the steam circulating in boiler and turbines. The drop in supply frequency to boiler feed pump motor will cause drop in steam pressure and hence the power output of the generator.

Low frequency would also result in high magnetisation or over fluxing in transformers and motors. The consequence of this is larger magnetic losses leading to increased temperature rise and shortening of life of these costly components.
Constancy in frequency is very important for time keeping devices like clocks and timers. **[5marks]**

(b) (**New computed example**)With reference to the figure in this question, let me assume that the current consumed by the load $P + jQ$ is $I\angle-\phi$. The sending end voltage E can be expressed as

$$E\angle\delta = V\angle 0 + I\angle-\phi \times (R + jX) \tag{5.1}$$

The simplification into real and imaginary components produces

$$Ecos\delta + jEsin\delta = V + IRcos\phi + IXsin\phi + j(IXcos\phi - IRsin\phi) \tag{5.2}$$

$$Ecos\delta = V + \Delta V_p \tag{5.3}$$
$$Esin\delta = \Delta V_q \tag{5.4}$$

This can alternatively be expressed [as in the question] as

$$E^2 = (V + \Delta V_p)^2 + (\Delta V_q^2) \tag{5.5}$$

where,

$$\Delta V_p = IRcos\phi + IXsin\phi \tag{5.6}$$
$$\Delta V_q = IXcos\phi - IRsin\phi \tag{5.7}$$

Multiplying and dividing (5.6) and (5.7) by V, and substituting $P = VIcos\phi$ and $Q = VIsin\phi$ the following sequence of expression would result

$$\Delta V_p = \frac{VIRcos\phi + VIXsin\phi}{V} \tag{5.8}$$
$$\Delta V_q = \frac{VIXcos\phi - VIRsin\phi}{V} \tag{5.9}$$
$$\Delta V_p = \frac{RP + XQ}{V} \tag{5.10}$$
$$\Delta V_q = \frac{XP - RP}{V} \tag{5.11}$$

Hence the desired result is established. **[9marks]**
When $X >> R$, $RP << XQ$ and this approximation in (5.10) directly leads to $Q \propto \Delta V_p$.
ΔV_q in (5.11) is substituted by $Esin\delta$ from (5.4). This leads to

$$Esin\delta = \frac{XP}{V} - \frac{RQ}{V} \tag{5.12}$$

[2marks]

For $X >> R$, $sin\delta \propto \delta$ when δ is in the range $0 < \delta < \frac{\pi}{4}$ and other quantities such as E, V and X remaining constant, (5.12) can be approximated to $P \propto \delta$. **[4marks]**

6 solution

(a) (**Bookwork**)The method of symmetrical components was introduced around 100 years back to power system analysis. It is an analytical method that resolves one balanced/unbalanced network into three balanced system known as positive, negative and zero sequence. Hence it is a very powerful analytical tool to compute voltage and current in different sections of a network irrespective of the network condition. For balanced situation, three sequence components, positive, negative and zero are independent. In unbalanced cases through fault they are connected at fault points in a fashion influenced by the nature of fault. These sequence components are balanced in themselves.
The unbalanced system is difficult to analyse through three-phase circuit theory. The technique of symmetrical components on the other hand is very handy in calculating fault current especially for faults those are unsymmetrical in nature. **[5marks]**

(b) (**New computed example**)
(i)Given the line-to-ground voltages $V_{ag} = 280\angle 0^0, V_{bg} = 250\angle -110^0$ and $V_{cg} = 290\angle 130^0$ volts, the line-to-line voltages are

$$\begin{aligned} V_{ab} &= V_{ag} - V_{bg} = 280\angle 0^0 - 250\angle - 110^0 \\ &= 434.49\angle 32.73^0 \quad Volts \end{aligned} \tag{6.1}$$

$$\begin{aligned} V_{bc} &= V_{bg} - V_{cg} = 250\angle - 110^0 - 290\angle 130^0 \\ &= 468.08\angle - 77.55^0 \quad Volts \end{aligned} \tag{6.2}$$

$$\begin{aligned} V_{ca} &= V_{cg} - V_{ag} = 290\angle 130^0 - 280\angle 0^0 \\ &= 516.61\angle 154.53^0 \quad Volts \end{aligned} \tag{6.3}$$

[3marks]

(ii)The phase and sequence components are related through transformation matrix T as $V_s = [T] * [V_{abc}]$, where

$$T = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1\angle 120^0 & 1\angle 240^0 \\ 1 & 1\angle 240^0 & 1\angle 120^0 \end{bmatrix} \tag{6.4}$$

Note V_{abc} can be either line to ground voltage or line to line voltage. The sequence components would result accordingly. Hence,

$$\begin{bmatrix} V_{Lg0} \\ V_{Lg1} \\ V_{Lg2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1\angle 120^0 & 1\angle 240^0 \\ 1 & 1\angle 240^0 & 1\angle 120^0 \end{bmatrix} \begin{bmatrix} 280\angle 0^0 \\ 250\angle -110^0 \\ V_{cg} = 290\angle 130^0 \end{bmatrix}$$

$$= \begin{bmatrix} 5.03\angle -57.65^0 \\ 272.40\angle +06.59^0 \\ V_{cg} = 27.82\angle -76.05^0 \end{bmatrix} Volts \qquad (6.5)$$

[5marks]

(iii) Similarly for Line to Line sequence voltage:

$$\begin{bmatrix} V_{LL0} \\ V_{LL1} \\ V_{LL2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1\angle 120^0 & 1\angle 240^0 \\ 1 & 1\angle 240^0 & 1\angle 120^0 \end{bmatrix} \begin{bmatrix} 434.49\angle 32.73^0 \\ 468.08\angle -77.55^0 \\ 516.61\angle 154.53^0 \end{bmatrix}$$

$$= \begin{bmatrix} 0\angle 0.0^0 \\ 471.81\angle +36.58^0 \\ 48.18\angle -106.05^0 \end{bmatrix} Volts \qquad (6.6)$$

[5 marks]

From the results in in (6.5) and (6.6) the following can be written:

$$\frac{V_{LL1}}{V_{Lg1}} = \frac{471.81\angle 36.589^0}{272.4\angle 6.589^0} \qquad = 1.732\angle 30^0 \qquad (6.7)$$

$$\frac{V_{LL2}}{V_{Lg2}} = \frac{48.186\angle -106.05^0}{27.82\angle -76.05^0} \qquad = 1.732\angle -30^0 \qquad (6.8)$$

1.732 is very nearly $\sqrt{3}$. Hence verified. **[2marks]**