E4.38 Drive Systems

There are 6 questions, answer any four.

No coursework element for this course.

Set by Dr T C Green

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1) (a) Describe the operation of a synchronous machine fed via an inverter under closed-loop control and provide a schematic of the system and a circuit diagram of the power converter. Compare this system to a brushless DC motor (with trapezoidal back EMF).

[10]

(b) A synchronous machine with a permanent-magnet field is operated in closed-loop using a power converter with a 550 V DC-link. The machine has the following properties:

Maximum RMS phase current, $I_{max} = 25$ A;
Inductance of each phase, $L = 12$ mH;
Resistance of each phase, $R = 0.2\ \Omega$;
EMF constant, $k = 1.4\ \mathrm{V_{RMS}}$ .s/rad.

Calculate the maximum speed at which maximum torque can be produced (base speed) and the power developed at that speed.

[6]

(c) Briefly explain how a permanent-magnet synchronous machine is operated beyond base speed and the impact this has on the torque available.

[4]

2) (a) Outline the steps in forming the *dq* model of an induction machine of the format given below.

$$[v_{DQ}] = [R_{DQ}][i_{DQ}] + [G_{DQ}][i_{DQ}] + [L_{DQ}]\frac{d}{dt}[i_{DQ}]$$

$$\text{where } [G_{DQ}] = \begin{bmatrix} 0 & -\omega L_S & 0 & -\omega M \\ \omega L_S & 0 & \omega M & 0 \\ 0 & -P\omega_{slip} M & 0 & -P\omega_{slip} L_R \\ P\omega_{slip} M & 0 & P\omega_{slip} L_R & 0 \end{bmatrix}.$$

[7]

(b) The instantaneous power absorbed by an induction machine can be found by multiplying the voltage equation by the transpose of the current. Identify the type of power associated with each of the terms that result from this multiplication.

[4]

(c) Find an expression for the torque developed by an induction machine from the expression for instantaneous power.

[5]

(d) Show that the torque can be expressed as the product of rotor current and rotor flux linkage.

[4]

3) (a) Describe the advantages of field orientation control of an induction machine over simpler schemes such as V/f control.

[4]

(b) Sketch block diagrams of both direct and indirect field orientation control schemes for an induction machine. Describe each of the blocks and give key equations.

[10]

(c) Discuss why indirect field orientation is more common in practice than the direct method.

[4]

(d) Give some factors that adversely affect the operation of an indirect vector controller.

[2]

4) (a) Discuss the features of the transform matrices T and T_R (defined below) in terms of their usefulness in modelling three-phase systems.

$$[T]=\sqrt{\frac{2}{3}}\cdot\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad [T_R]=\begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[7]

(b) Write a set of voltage and current equations to describe the three-phase circuit shown in Figure 1 and transform them to *dq* form.

[8]

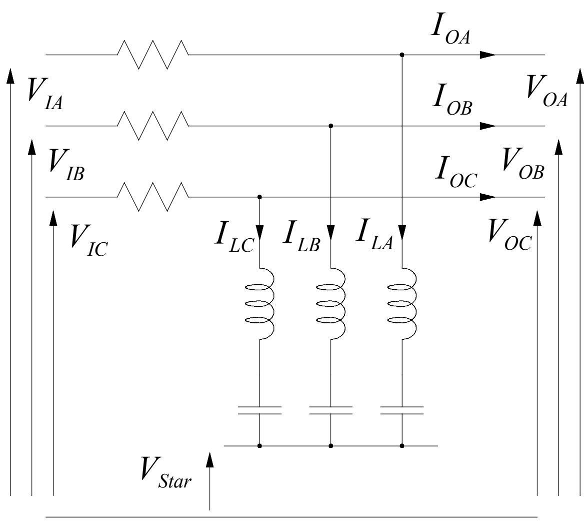


Figure 1, A three-phase passive network

(c) Draw an equivalent circuit of the transformed system.

[5]

5) (a) Sketch a single-stack variable reluctance stepper motor and explain its operation.

[5]

(b) Compare the single-stack and multi-stack forms of variable reluctance motor.

[4]

(c) Choose a machine configuration to give 24 steps per revolution.

[4]

(d) Describe how half-stepping and castellation may be used to increase the number of steps per revolution.

[4]

(e) Describe the advantages of hybrid stepper motors.

[3]

6) (a) Describe (qualitatively) how a three-phase winding supplied with three-phase current produces a rotating magnetic field. Compare the magnetic field produced by a three-phase winding to that of a single-phase winding.

[7]

(b) Discuss the advantages and disadvantages of single-phase induction machines compared to three-phase machines.

[6]

(c) Describe one method by which a single-phase machine can be modified so that it provides torque at standstill.

[7]

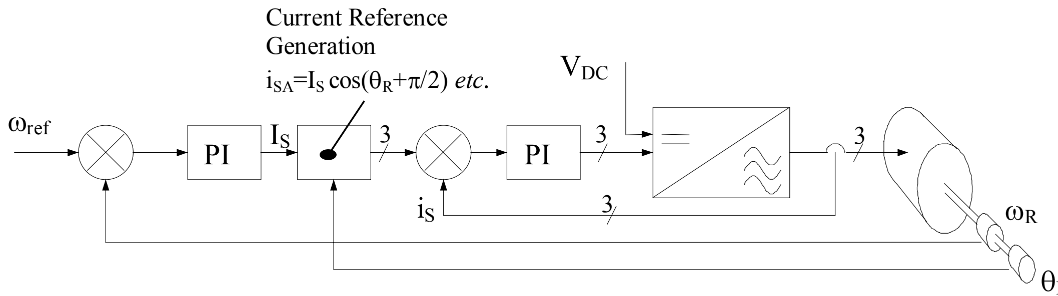
Answers to E4.38 2000/01

1) (a) Describe the operation of a synchronous machine fed via an inverter under closed-loop control and provide a schematic of the system and a circuit diagram of the power converter. Compare this system to a brushless DC motor (with trapezoidal back EMF).

[10]

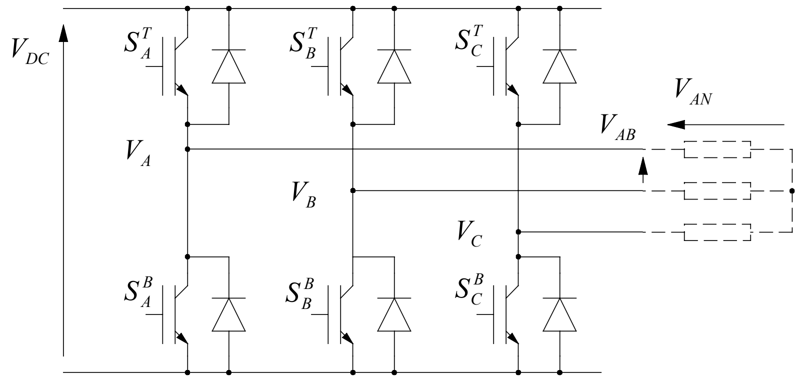
The synchronous machine produces useful torque only when the rotor field and stator field rotate synchronously. Thus, the speed or position of the rotor must be known so that an appropriate set of stator currents can be imposed.

The synchronous machine control system follows the standard layout: the speed error is applied to a PI controller to determine the torque requirement. The torque reference is converted to a current reference. The current must be aligned to E_R which is known to lead ψ_R. The torque production is most effective (i.e., the highest torque per amp is achieved) if the angle between the back EMF and the current is zero. Fortunately (and unlike an induction machine), the position of the rotor flux is known because its axis is fixed to the physical axis of the rotor. Therefore, the phase angle of the current reference can be directly determined from a measurement of the rotor position.



Schematic diagram of a closed-loop torque angle control of a synchronous machine.

The inner current-control loop can be based on three controllers in a stationary frame (as illustrated); two controllers in the $\alpha\beta$ frame or two controllers in the dq *frame with voltage feed-forward. The* dq *frame controllers will provide the best tracking of the current* *reference. The power converter circuit is a standard three-phase inverter.*



As the speed of the machine increases the magnitude of the induced rotor voltage increases because the rate of change of flux linkage increases. There is also an increase in the voltage drop across the synchronous reactance. The current controllers will need to apply greater and greater voltage as the speed increases.

The brushless DC machine is formed by taking the armature and field winding of a permanent magnet DC machine and exchanging their positions on the stator and rotor.

Feature	PM Synchronous	Brushless DC
Rotor	Permanent magnet field	Permanent magnet field
Stator	Three-phase distributed winding	Three-phase concentrated winding
Current waveform	Sinusoidal	Square or trapezoidal
Power converter	6-switch inverter bridge: viewed as DC to AC converter	6-switch inverter bridge: viewed as DC to ±DC converter
Control	Current vector phase-locked to rotor position	Current vector phase-locked to rotor position

Conceptually, these two drives are the same; the terminology reflects the two historic routes to the same point. There are to uses of the terminology.

(b) A 2-pole (1-pole-pair) synchronous machine with a permanent-magnet field is operated in closed-loop using a power converter with a 550 V DC-link. The machine has the following properties:

Maximum RMS phase current, I_{max} = 25 A
Inductance of each phase, L = 12 mH
Resistance of each phase, R = 0.2 Ω
EMF constant, k = 1.4 V_{RMS} .s/rad

Calculate the maximum speed at which maximum torque can be produced (base speed) and the power developed at that speed.

$$V_{RMS}^{Max} = 550 \times \frac{1}{2} \times \frac{1}{\sqrt{2}} = 194.4V$$

$$\begin{aligned} V^{Max} &= E^{Max} + I^{Max}R + j\omega L I^{Max} \\ &= \omega^{Max}k + I^{Max}R + j\omega^{Max}LI^{Max} \end{aligned}$$

$$a\omega^2 + b\omega + c = 0$$

$$\begin{aligned} a &= k^2 + L^2I^2 \\ b &= 2kIR \\ c &= I^2R^2 - V^2 \end{aligned}$$

Taking positive root

$$\begin{aligned} \omega &= 132.4 rad/s \\ &= 1{,}264 rpm \end{aligned}$$

$$T = kI = 1.4 \times 25 = 35Nm$$

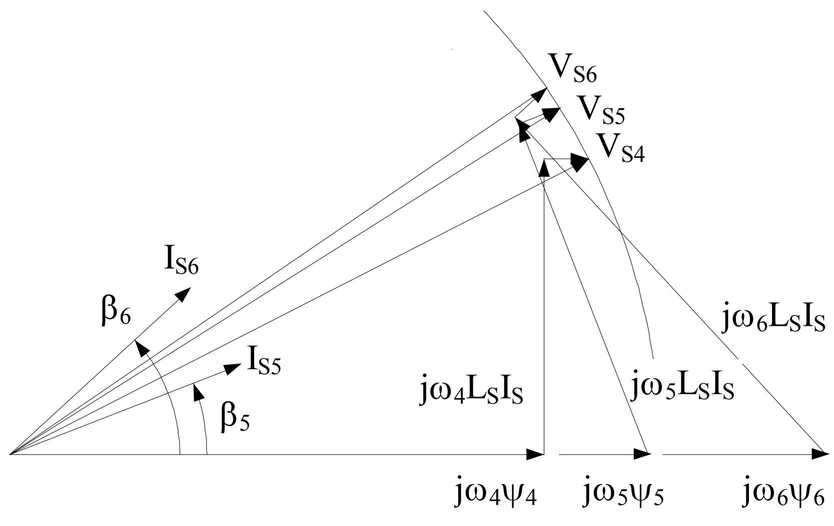
$$P = \omega T = 132.4 \times 35 = 4.634kW$$

[6]

(c) Briefly explain how a permanent-magnet synchronous machine is operated beyond base speed and the impact this has on the torque available.

[4]

For a permanent magnet rotor machine the only option at high speed is to phase advance the current to increase β. This has two effects: it reduces the required magnitude of applied voltage and it reduces the available torque. The effect is very similar to field weakening. In terms of traditional infinite bus operation, it would be termed over-excited operation.



Phase advance of the current vector allows E_R to increase without V_S increasing.

2) (a) Outline the steps in forming the dq model of an induction machine of the format given below.

$$[v_{DQ}] = [R_{DQ}][i_{DQ}] + [G_{DQ}][i_{DQ}] + [L_{DQ}]\frac{d}{dt}[i_{DQ}]$$

$$\text{where } [G_{DQ}] = \begin{bmatrix} 0 & -\omega L_S & 0 & -\omega M \\ \omega L_S & 0 & \omega M & 0 \\ 0 & -P\omega_{slip} M & 0 & -P\omega_{slip} L_R \\ P\omega_{slip} M & 0 & P\omega_{slip} L_R & 0 \end{bmatrix}$$

[7]

A shorter of version of the answer than this is acceptable.
Write voltage equations for stator and rotor circuits of the form:

$$[v_S] = [R_S][i_S] + [L_S]\frac{d}{dt}[i_S] + \frac{d}{dt}([m_{SR}][i_R])$$
$$[v_R] = [R_R][i_R] + \frac{d}{dt}([m_{RS}][i_S]) + [L_R]\frac{d}{dt}[i_R]$$

The mutual inductances are time-varying because of the rotation of the rotor coils with respect to the stator coils. The electrical and mechanical cycles are related through the number of pole pairs. The mutual inductances are sinusoidal functions of $P\omega_R t$

The equations are transformed to orthogonal $\alpha\beta\gamma$ variables using the T matrix

$$[T] = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$[v_{S\alpha\beta\gamma}] = [R_{S\alpha\beta\gamma}][i_{S\alpha\beta\gamma}] + [L_{S\alpha\beta\gamma}]\frac{d}{dt}[i_{S\alpha\beta\gamma}] + \frac{d}{dt}([m_{SR\alpha\beta\gamma}][i_{R\alpha\beta\gamma}])$$
$$[v_{R\alpha\beta\gamma}] = [R_{R\alpha\beta\gamma}][i_{R\alpha\beta\gamma}] + \frac{d}{dt}([m_{RS\alpha\beta\gamma}][i_{S\alpha\beta\gamma}]) + [L_{R\alpha\beta\gamma}]\frac{d}{dt}[i_{R\alpha\beta\gamma}]$$

The stator variables are at a frequency of ω and rotor variables at a frequency of ω - $P\omega_R$. The rotation matrix of appropriate frequency can be applied to the stator and rotor voltage equations.

$$
\begin{aligned}
\left[v_{SDQ}\right] = & \left[T_R(\omega)\right]\left[R_{S\alpha\beta\gamma}\right]\left[T_R(\omega)\right]^{-1}\left[T_R(\omega)\right]\left[i_{S\alpha\beta\gamma}\right] \\
& + \left[T_R(\omega)\right]\left[L_{S\alpha\beta\gamma}\right]\left[T_R(\omega)\right]^{-1}\left[T_R(\omega)\right]\frac{d}{dt}\left[i_{S\alpha\beta\gamma}\right] \\
& + \left[T_R(\omega)\right]\frac{d}{dt}\left(\left[m_{SR\alpha\beta\gamma}\right]\left[i_{R\alpha\beta\gamma}\right]\right)
\end{aligned}
$$

$$
= \left[R_{SDQ\gamma}\right]\left[i_{SDQ\gamma}\right] + \left[L_{SDQ\gamma}\right]\left[T_R(\omega)\right]\frac{d}{dt}\left[i_{S\alpha\beta\gamma}\right] + \left[T_R(\omega)\right]\frac{d}{dt}\left(\left[m_{SR\alpha\beta\gamma}\right]\left[i_{R\alpha\beta\gamma}\right]\right)
$$

The stator/rotor mutual coupling terms need special treatment which involves introducing a rotation and counter rotation pair of transforms of (ω-$P\omega_R$)

$$
\begin{aligned}
\left[v_{SDQ}\right] = & \left[R_{SDQ\gamma}\right]\left[i_{SDQ\gamma}\right] + \left[G_{SDQ\gamma}(\omega)\right]\left[i_{SDQ\gamma}\right] + \left[L_{SDQ\gamma}\right]\frac{d}{dt}\left[i_{SDQ\gamma}\right] \\
& + \left[T_R(\omega)\right]\frac{d}{dt}\left(\left[m_{SR\alpha\beta\gamma}\right]\left[T_R(\omega - P\omega_R)\right]^{-1}\left[T_R(\omega - P\omega_R)\right]\left[i_{R\alpha\beta\gamma}\right]\right)
\end{aligned}
$$

$$
\left[T_R(\omega)\right]\frac{d}{dt}\left(\left[m_{SR\alpha\beta\gamma}\right]\left[i_{R\alpha\beta\gamma}\right]\right) = \left[G_{SRDQ}(\omega)\right]\left[i_{RDQ\gamma}\right] + \left[M_{SRDQ}\right]\frac{d}{dt}\left[i_{RDQ\gamma}\right]
$$

A similar treatment of the cross coupling in the rotor circuit is needed.

(e) The instantaneous power absorbed by an induction machine can be found by by multiplying the voltage equation by the transpose of the current. Identify the type of power associated with each of the terms that result from this multiplication.

[4]

$$
p = \left[i_{DQ}\right]^T\left[v_{DQ}\right]
$$

$$
\begin{aligned}
= & \left[i_{DQ}\right]^T\left[R_{DQ}\right]\left[i_{DQ}\right] \\
& + \left[i_{DQ}\right]^T\left[G_{DQ}\right]\left[i_{DQ}\right] \\
& + \left[i_{DQ}\right]^T\left[L_{DQ}\right]\frac{d}{dt}\left[i_{DQ}\right]
\end{aligned}
$$

These three terms can be identified as:

i. *The power dissipation in the resistances of the windings*

ii. *The power converted by the machine from electrical to mechanical form and reactive power exchange between phases*

iii. *The power that flows to store or release energy in the magnetic field.*

(f) Find an expression for the torque developed by an induction machine from the expression for instantaneous power.

[5]

Converted power is found from the rotational voltage and the current

$$E_{DQ} = [G_{DQ}]i_{DQ} = \begin{bmatrix} 0 & -\omega L_S & 0 & -\omega M \\ \omega L_S & 0 & \omega M & 0 \\ 0 & -P\omega_{slip} M & 0 & -P\omega_{slip} L_R \\ P\omega_{slip} M & 0 & P\omega_{slip} L_R & 0 \end{bmatrix} \begin{bmatrix} i_{SD} \\ i_{SQ} \\ i_{RD} \\ i_{RQ} \end{bmatrix}$$

$$= \begin{bmatrix} -\omega L_S i_{SQ} - \omega M i_{RQ} \\ \omega L_S i_{SD} + \omega M i_{RD} \\ -P\omega_{slip} M i_{SQ} - P\omega_{slip} L_R i_{RQ} \\ P\omega_{slip} M i_{SD} + P\omega_{slip} L_R i_{RD} \end{bmatrix}$$

$$p_{em} = i_{DQ}^T E_{DQ} = \begin{bmatrix} i_{SD} & i_{SQ} & i_{RD} & i_{RQ} \end{bmatrix} \begin{bmatrix} -\omega L_S i_{SQ} - \omega M i_{RQ} \\ \omega L_S i_{SD} + \omega M i_{RD} \\ -P\omega_{slip} M i_{SQ} - P\omega_{slip} L_R i_{RQ} \\ P\omega_{slip} M i_{SD} + P\omega_{slip} L_R i_{RD} \end{bmatrix}$$

$$= \omega\left(-L_S i_{SQ} i_{SD} - M i_{RQ} i_{SD} + L_S i_{SD} i_{SQ} + M i_{RD} i_{SQ}\right)$$
$$+ P\omega_{slip}\left(-M i_{SQ} i_{RD} - L_R i_{RQ} i_{RD} + M i_{SD} i_{RQ} + L_R i_{RD} i_{RQ}\right)$$

$$= \left(M i_{RD} i_{SQ} - M i_{RQ} i_{SD}\right)\left(\omega - P\omega_{slip}\right)$$

$$= P\omega_R M\left(i_{RD} i_{SQ} - i_{RQ} i_{SD}\right)$$

And now dividing the converted power by the rotor speed to find the torque

$$T_{em} = \frac{p_{em}}{\omega_R} = PM\left(i_{RD} i_{SQ} - i_{RQ} i_{SD}\right)$$

(g) Show that the torque can be expressed as the product of rotor current and rotor flux linkage.

[4]

The air-gap flux linkage (assuming stator and rotor to have equal numbers of turns or that one winding is referred) is the total flux linking the rotor and stator and is $M(i_S+i_R)$

$$
\begin{aligned}
T_{em} &= PM\left(i_{RD}i_{SQ} - i_{RQ}i_{SD}\right) \\
&= PM\left(i_{RD}i_{SQ} - i_{RQ}i_{SD} + i_{RD}i_{RQ} - i_{RQ}i_{RD}\right) \\
&= P\left(i_{RD}M\left(i_{SQ} + i_{RQ}\right) - i_{RQ}M\left(i_{SD} + i_{RD}\right)\right) \\
&= P\left(i_{RD}\psi_{AGQ} - i_{RQ}\psi_{AGD}\right)
\end{aligned}
$$

The rotor flux-linkage is the air-gap flux-linkage plus a leakage term. Adding the leakage flux linkage to the torque equation introduces two cancelling terms.

$$
\begin{aligned}
T_{em} &= P\left(i_{RD}\psi_{AGQ} - i_{RQ}\psi_{AGD} + i_{RD}L_{lR}i_{RQ} - i_{RQ}L_{lR}i_{RD}\right) \\
&= P\left(i_{RD}\psi_{RQ} - i_{RQ}\psi_{RD}\right)
\end{aligned}
$$

3) (a) Describe the advantages of field orientation control of an induction machine over simpler schemes such as V/f control.

[4]

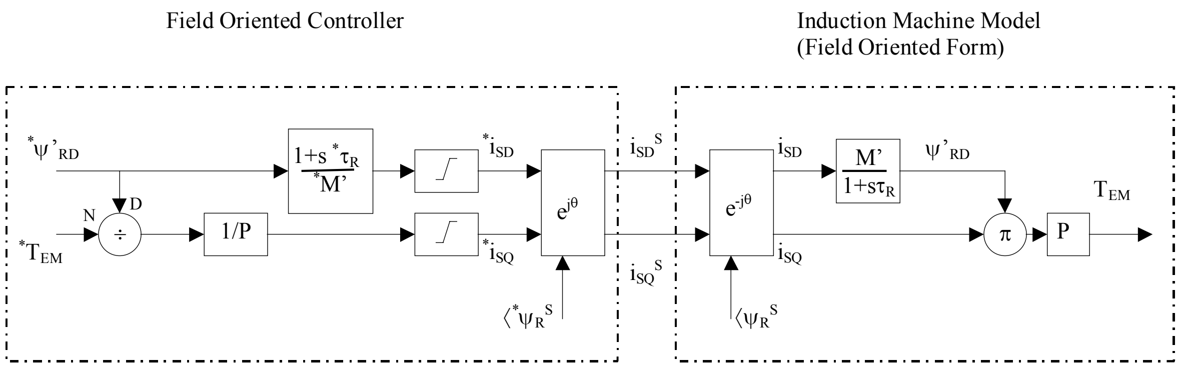
The advantages are related to the transient behaviour of the machine and follow from the fact that only field orientation control uses a control model that is valid under transients. This enables the controller de-couple the flux-producing and torque-producing components of stator current and therefore to maintain the flux magnitude constant and ensure that the torque producing component of current is not perturbed. Further, the torque-producing component of current has no dynamics contributed by the rotor or magnetising inductances and is thus fast response. In contrast, simple schemes are badly perturbed by transients and it can take several seconds for the flux to regain its steady-state value.

(e) Sketch block diagrams of both direct and indirect field orientation control schemes for an induction machine. Describe each of the blocks and give key equations.

[10]

In order to control the machine using the rotor flux reference frame model we need to determine the angular position of the reference frame so that forward and reverse transformations can be applied to converter variables between the control model dq form and physical abc form. There are two methods.

1. *Direct orientation: determine the rotor flux vector and calculate its angle.*



The estimate of the rotor flux is obtained by measuring the air-gap flux and then applying a correction of the rotor leakage.

$$\psi_{AG\alpha\beta} = M\left(i_{S\alpha\beta} + i_{R\alpha\beta}\right)$$

$$\begin{aligned}\psi_{R\alpha\beta} &= M\, i_{S\alpha\beta} + L_R\, i_{R\alpha\beta} \\ &= \psi_{SR\alpha\beta} + L_{lR}\, i_{R\alpha\beta}\end{aligned}$$

Alternatively, the stator flux can be estimated from an integration of the stator voltage equation and then correction made for leakage.

$$v_{S\alpha\beta} = R_S\, i_{S\alpha\beta} + \frac{d\psi_{S\alpha\beta}}{dt}$$

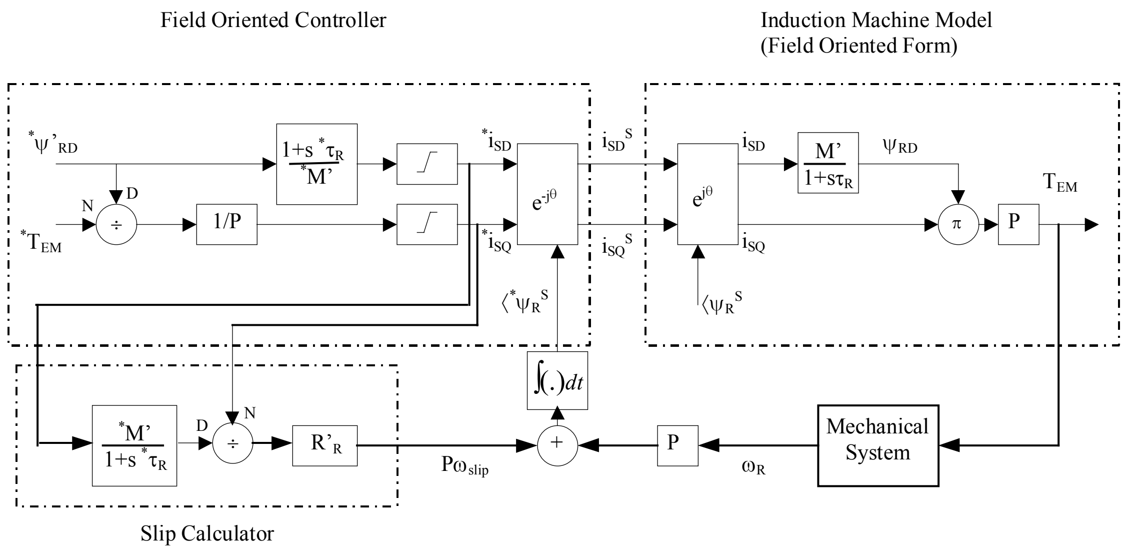
$$^{*}\psi_{S\alpha\beta} = \int_t \left(v_{S\alpha\beta} - {}^{*}R_S i_{S\alpha\beta}\right)dt$$

$$^{*}\psi'_{R\alpha\beta} = {}^{*}\psi_{S\alpha\beta} - {}^{*}L'_{lS}\, i_{S\alpha\beta}$$

$$^{*}\psi_{R\alpha\beta} = \frac{^{*}L_R}{^{*}M_{SR}}\, {}^{*}\psi'_{R\alpha\beta}$$

2. *Indirect orientation: determine the rotor position and the speed with which the rotor flux advances with respect to the rotor (the slip speed).*

A slip calculator is used to set the reference frame angle in feed-forward fashion.



Under each method, stator currents are imposed from either a current source inverter or a voltage source inverter with local current control. The flux magnitude is controlled using an estimate of the flux magnitude and there is q-axis current calculator to set the toruqe.

(f) Discuss why indirect field orientation is more common in practice than the direct method

[4]

Direct field orientation requires either measurement of air-gap flux or estimation of the flux from stator terminal measurements.
Fitting air-gap flux sensors results in a non-standard machine which has comparatively delicate sensors in a vulnerable part of the machine.
Estimating flux from the stator voltage equations requires the stator resistance to be known to a high degree of accuracy (particularly for low speed operation). Because this resistance is temperature

dependent and temperature variation has to be expected, the estimation is not accurate and proper orientation of the control ot the flux is not achieved.
Indirect field orientation relies on instantaneous slip estimation and while this is dependent on rotor parameters it is found to be less sensitive to errors.

(g) Give some factors that adversely affect the operation of an indirect vector controller.

[2]

The orientation of the controller is adversely affected by parameter variations that occur due to temperature variations and put the flux or slip estimates in error. The ideal vector controller assumes that stator currents can be imposed but there are stator circuit dynamics (inductances) to be over come. Delays here will result in some coupling of the two current components. The under lying machine model is a linear model that ignores saturation and winding harmonics.

4) (a) Discuss the features of the transform matrices T and T_R (defined below) in terms of their usefulness in modelling three-phase systems.

$$[T]=\sqrt{\frac{2}{3}}\cdot\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix} \qquad [T_R]=\begin{bmatrix}\cos(\omega t) & \sin(\omega t) & 0\\ -\sin(\omega t) & \cos(\omega t) & 0\\ 0 & 0 & 1\end{bmatrix}$$

[7]

Both T *and* $\mathrm{T_R}$ *have the property that their inverse is equal to the their transpose. This is advantageous when calculating power by multiplying a voltage or current by its transpose. The transform is power invariant and the variables in the transformed system are orthogonal such that now mixed products are necessary to calculate the power.*

$$v_{\alpha\beta\gamma}^{T}v_{\alpha\beta\gamma}=(Tv_{abc})^{T}(Tv_{abc})=v_{abc}^{T}v_{abc}$$
$$p=v_a^2+v_b^2+v_c^2=v_\alpha^2+v_\beta^2+v_\gamma^2=v_d^2+v_q^2+v_\gamma^2$$

T *has the additional property that balanced three-phase sets transform to a two variable system.*

$\mathrm{T_R}$ *has the property of rotation such that rotating AC sets can be transformed to stationary sets and the only remaining time variance of the signal is the time variance of the vector magnitude and phase angle.*

(b) Write a set of voltage and current equations to describe the three-phase circuit shown in Figure 1 and transform them to *dq* form.

[8]

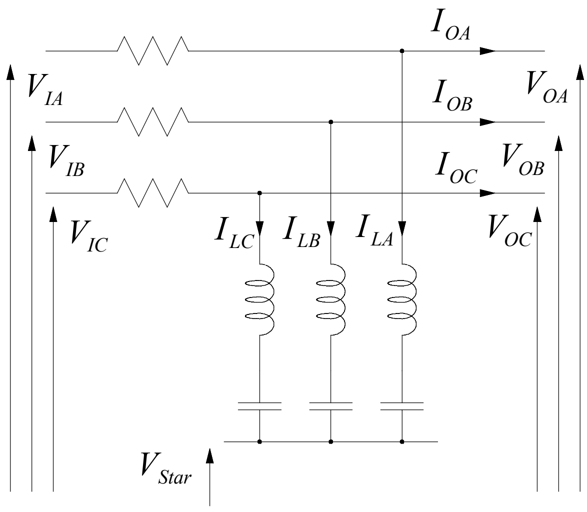


Figure 2, A three-phase passive network

The voltage equation involving the input side of phase-A is:

$$v_{IA} = R(i_{LA} + i_{OA}) + L\frac{di_{LA}}{dt} + v_{CA} + v_{Star}$$

The capacitor voltage of phase-A is given by:

$$C\frac{dv_{CA}}{dt} = i_{LA}$$

In matrix form (combining all phases) these equations are:

$$[v_{IABC}] = [R][i_{LABC}] + [R][i_{OABC}] + [L]\frac{d[i_{LABC}]}{dt} + [v_{CABC}] + \begin{bmatrix} v_{Star} \\ v_{Star} \\ v_{Star} \end{bmatrix}$$

$$[C]\frac{d[v_{CABC}]}{dt} = [i_{LABC}]$$

Where all parameter matrices are diagonal and all variables are column vectors

Transforming to αβγ, by multiplying through by T *and inserting* $T.T^{-1}$ *pairs, yields:*

$$[v_{I\alpha\beta\gamma}] = [R][i_{L\alpha\beta\gamma}] + [R][i_{O\alpha\beta\gamma}] + [L]\frac{d[i_{L\alpha\beta\gamma}]}{dt} + [v_{C\alpha\beta\gamma}] + \begin{bmatrix} 0 \\ 0 \\ v_{Star} \end{bmatrix}$$

$$[C]\frac{d[v_{C\alpha\beta\gamma}]}{dt} = [i_{L\alpha\beta\gamma}]$$

To transform first to an dqγ frame we multiply through by T_R *and insert* $T_R\, T_R^{-1}$ *pairs:*

$$\begin{aligned}
[v_{Idq\gamma}] &= [T_R][v_{I\alpha\beta\gamma}] \\
&= [T_R][R][T_R]^{-1}[T_R][i_{L\alpha\beta\gamma}] + [T_R][R][T_R]^{-1}[T_R][i_{O\alpha\beta\gamma}] \\
&\qquad + [T_R][L][T_R]^{-1}[T_R]\frac{d[i_{L\alpha\beta\gamma}]}{dt} + [T_R][v_{C\alpha\beta\gamma}] + [T_R]\begin{bmatrix} 0 \\ 0 \\ v_{Star} \end{bmatrix} \\
&= [R][i_{Ldq\gamma}] + [R][i_{Odq\gamma}] + [L][T_R]\frac{d[i_{L\alpha\beta\gamma}]}{dt} + [v_{Cdq\gamma}] + \begin{bmatrix} 0 \\ 0 \\ v_{Star} \end{bmatrix}
\end{aligned}$$

The derivative term needs special treatment because T_R *is time-varying. This yields two terms:*

$$[v_{Idq\gamma}] = [R][i_{Ldq\gamma}] + [R][i_{Odq\gamma}] + [G][i_{Ldq\gamma}] + [L]\frac{d[i_{Ldq\gamma}]}{dt} + [v_{Cdq\gamma}] + \begin{bmatrix} 0 \\ 0 \\ v_{Star} \end{bmatrix}$$

$$where\ [G] = \begin{bmatrix} 0 & -\omega L & 0 \\ \omega L & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

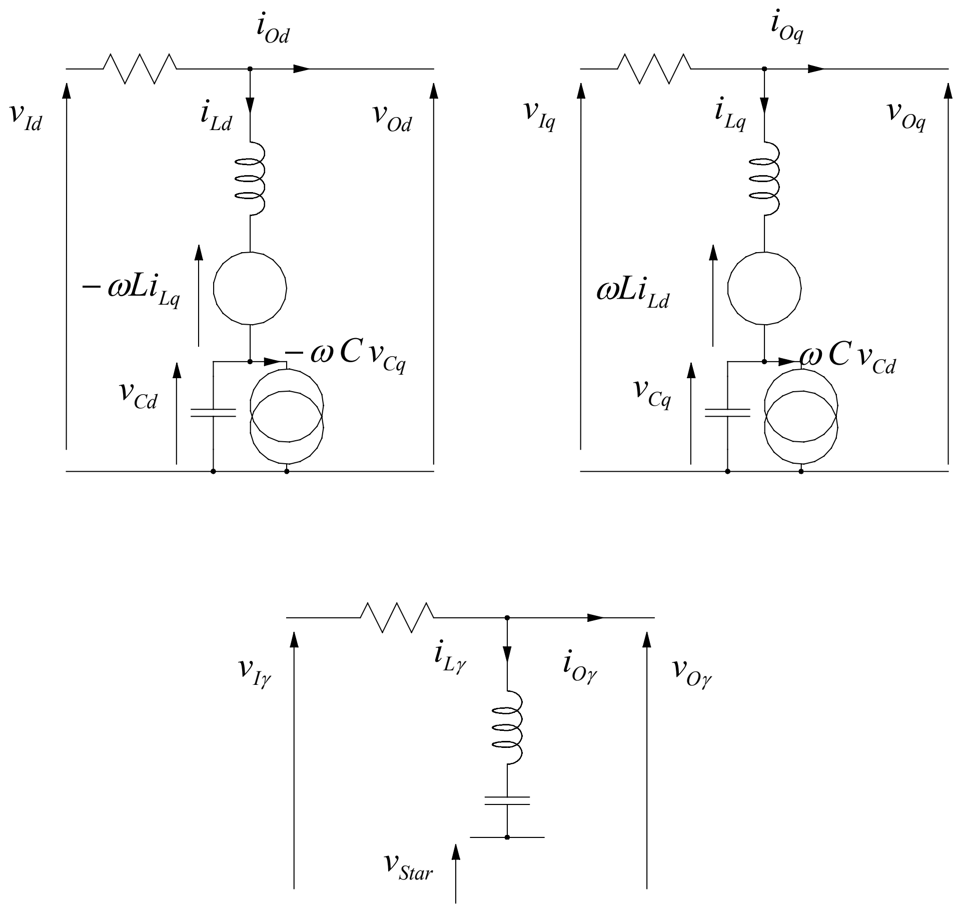
A similar treatment of the capacitor equation yields:

$$[H][v_{Cdq\gamma}] + [C]\frac{d[v_{Cdq\gamma}]}{dt} = [i_{Ldq\gamma}]$$

$$where\ [H] = \begin{bmatrix} 0 & -\omega C & 0 \\ \omega C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

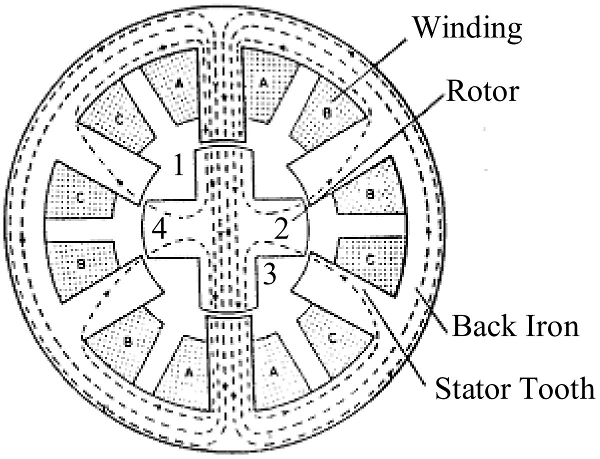
(c) Draw an equivalent circuit of the transformed system.

[5]



5) (a) Sketch a single-stack variable reluctance stepper motor and explain its operation.

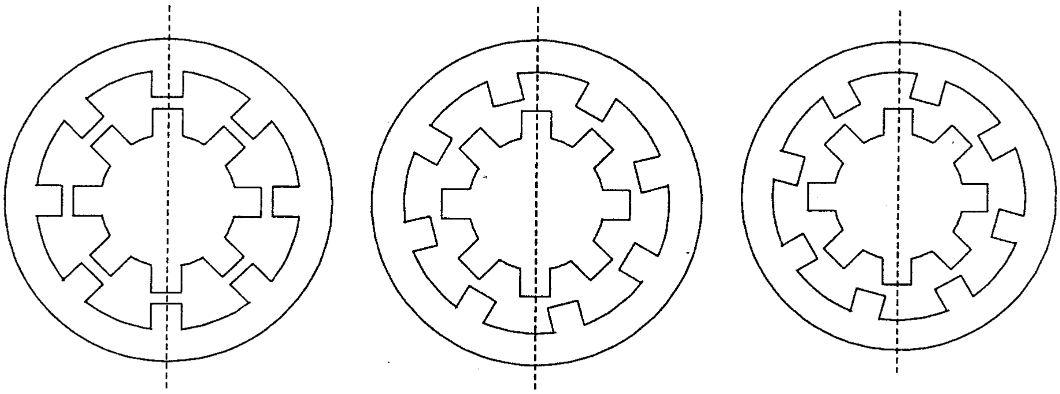
[5]



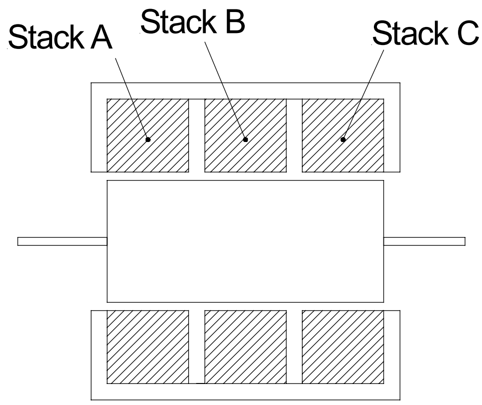
The rotor is an unexcited salient structure. The stator is also salient but has coils wound around pairs of poles. The diagram is shown with one pair of rotor poles in alignment with one pair of stator poles. Exciting this stator phase will keep the rotor aligned. Because the number of rotor poles and number of stator poles are different, none of the other poles align. If the excitation of phase-A is removed and phase-B excited instead then rotor poles 2&4 will pull into alignment and the rotor will move anti-clockwise. Exciting in the sequence ABCA… will give continuous anti-clockwise motion.

(f) Compare the single-stack and multi-stack forms of variable reluctance motor.

[4]



End view of each of three stacks



Side view

The multi-stack arrangement can make better use of the available magnetic material and offers a higher step number for a given number of phases and stator poles.

(g) Choose a machine configuration to give 24 steps per revolution

[4]

This could be a single stack machine for which:

$$\text{Step angle} = \left(\frac{2\pi}{N_R} - \frac{2\pi}{N_S}\right)$$

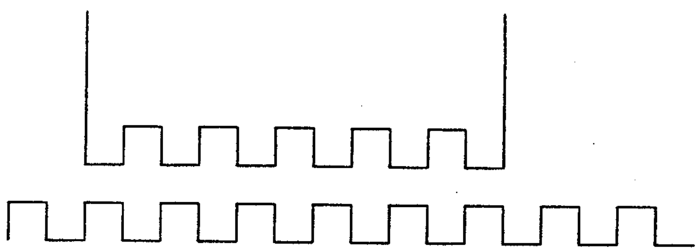
$$N_{Steps} = \frac{N_S \times N_R}{N_S - N_R}$$

For which the choice of 8 stator poles (4 phases) and 6 rotor poles gives 24 steps per revolution.

(h) Describe how half-stepping and castellation may be used to increase the number of steps per revolution

[4]

- *Half-stepping involves exciting two phases together to give an alignment position halfway between the alignment position of the two phases. Thus if we swap the excitation sequence A,B,C to A,AB,B,BC,C,CA then twice as many steps (of half the step angle) are produced per revolution.*
- *Castellation involves forming minor teeth on the surface of the rotor and of the and stator poles (with the same pitch in each case):*



Each change of excitation brings a new alignment of the castellations on the new pole. In a cycle of excitation the rotor moves forward one castellation pitch: Step angle, $\beta = \dfrac{2\pi}{N_{Teeth} \cdot N_{Stacks} \cdot N_{Castellations/Tooth}}$

(i) Describe the advantages of hybrid stepper motors.

[3]

Hybrid stepper motors contain permanent magnet excitation of the rotor in addition to current excitation of the stator. This gives the machine a detent torque, that is a torque is present to maintain the last alignment even after the stator has been switched off. For positioning systems that require long periods in one position this can give very significant energy saving.

6) (a) Describe (qualitatively) how a three-phase winding supplied with three-phase current produces a rotating magnetic field. Compare the magnetic field produced by a three-phase winding to that of a single-phase winding.

[7]

- *A single coil produces a uniform distribution of flux around the air-gap with a change in polarity at each conductor*
- *A distribution of coils in adjacent slots produces a "stair-case" flux that approximates a sinusoidal distribution in space*
- *When the current excitation of the coil is made sinusoidal in time the flux alternates in direction.*
- *A three-phase winding has three such coils separated by angles of 120° and excited by currents phase displaced by 120°*
- *The total flux is the vector sum of the three contributions.*
- *As each phase current peaks in turn the location of the peak flux moves from one phase winding to the next.*
- *A single phase winding creates and alternating flux and a three-phase winding creates a rotating flux*

(b) Discuss the advantages and disadvantages of single-phase induction machines compared to three-phase machines.

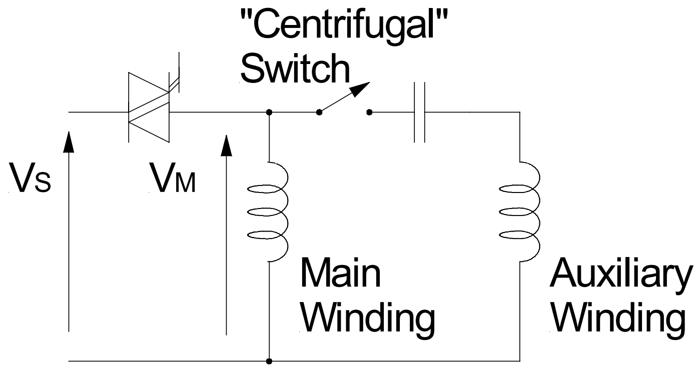
[6]

- *A pure single-phase machine can not develop a torque at standstill and so is not self-starting*
- *Single-phase machines suit low power equipment supplied from the single phase mains but provision of a starting arrangement adds to the expense*
- *A single phase-machine develops a pulsating torque (twice line frequency) whereas a there-phase machine suffers little torque pulsation*
- *Three-phase machine have a better power density than single-phase machines because of the better use made of the stator circumference.*

(c) Describe one method by which a single-phase machine can be modified so that it provides torque at standstill.

[7]

- *The machine must be biased to favour either the forwards or backwards rotating component of the alternating flux*
- *A full second winding can be used to fully suppress the backwards field*
- *A low-rating second winding can be used to simply favour the forward field*
- *Second winding can be placed at 90° and supplied through a large value capacitor to achieve a current at near 90° phase shift*



- *Alternatively, shading (a shorting ring) can be applied to one side of each pole to phase shift the field in that region.*