2017 Model answers — Introduction to Signals and Communications

Exam

1. a. i. $F(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt$

$$= \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \frac{-1}{a+j\omega} \cdot e^{-(a+j\omega)t} \Big|_{0}^{\infty}$$

Marks

$$\Rightarrow \quad F(\omega) = \frac{1}{a+j\omega}.$$

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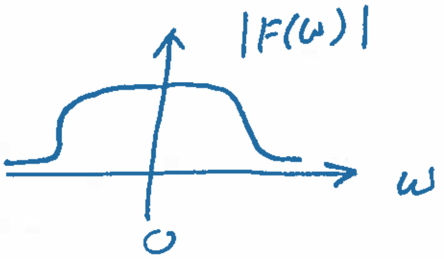
$$|F(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \qquad \theta_F(\omega) = (-1)\tan^{-1}\left(\frac{\omega}{a}\right)$$

ii.



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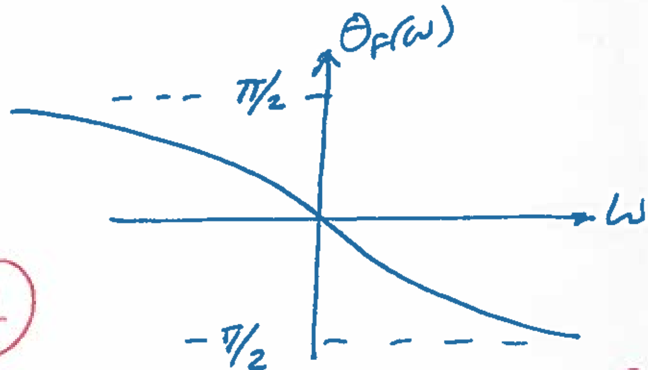

iii. The system behaves as a low-pass filter. 2

iv. $\mathcal{F}[f(t-t_0)] = \int_{-\infty}^{\infty} f(t-t_0) e^{-j\omega t} dt$

$$= \int_{-\infty}^{\infty} f(t-t_0)\, e^{-j\omega(t-t_0)} e^{-j\omega t_0} dt$$

$$= e^{-j\omega t_0} \int_{-\infty}^{\infty} f(t-t_0) e^{-j\omega(t-t_0)} dt$$

$$\Rightarrow \quad \hat{F}(\omega) = e^{-j\omega t_0} F(\omega).$$

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1. b. i. $\omega_0 = \frac{2\pi}{T}$ 1

ii. $a_0 \neq 0$ means that $g(t)$ has non-zero dc component during time period. 1

iii. The Fourier series is accurate for $g(t)$ because (a) $\cos(n\omega_0 t)$ and $\sin(n\omega_0 t)$ are all mutually orthogonal, and (b) the components (sinusoidal) are complete. 2

iv. If $g(t)$ changes very rapidly in time, the magnitudes of a_n and b_n can be non-negligible for very large n values. 2

v. If $g(t)$ is an even function of t,

$$g(t) = g(-t)$$

Then, $a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$

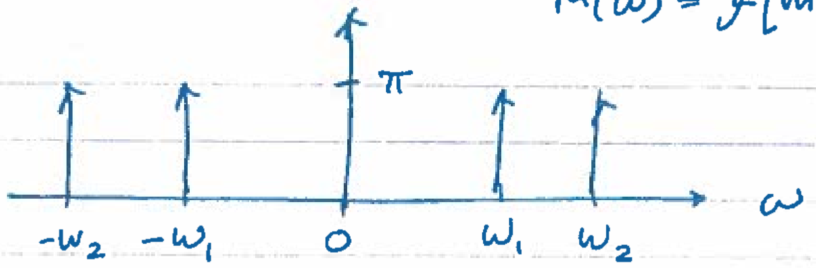
$$= a_0 + \sum_{n=1}^{\infty} a_n \cos(-n\omega_0 t) + b_n \sin(-n\omega_0 t)$$

$$\Rightarrow \quad 2\sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) = 0$$ 3

Since all $\sin(n\omega_0 t)$ and $\sin(m\omega_0 t)$ are orthogonal for $m \neq n$, it must be the case where $b_n = 0 \ \forall\, n = 1, 2, \ldots, \infty$

a_0 and a_n's can be of any value.

1.c. i. $M(\omega) = \mathcal{F}[m(t)]$



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ii. $\phi(t) = m(t)\cos(\omega_c t)$

$$= \left[\cos(\omega_1 t) + \cos(\omega_2 t)\right]\cos(\omega_c t)$$

$$= \frac{1}{2}\left[\cos(\omega_c-\omega_1)t + \cos(\omega_c+\omega_1)t + \cos(\omega_c-\omega_2)t + \cos(\omega_c+\omega_2)t\right]$$

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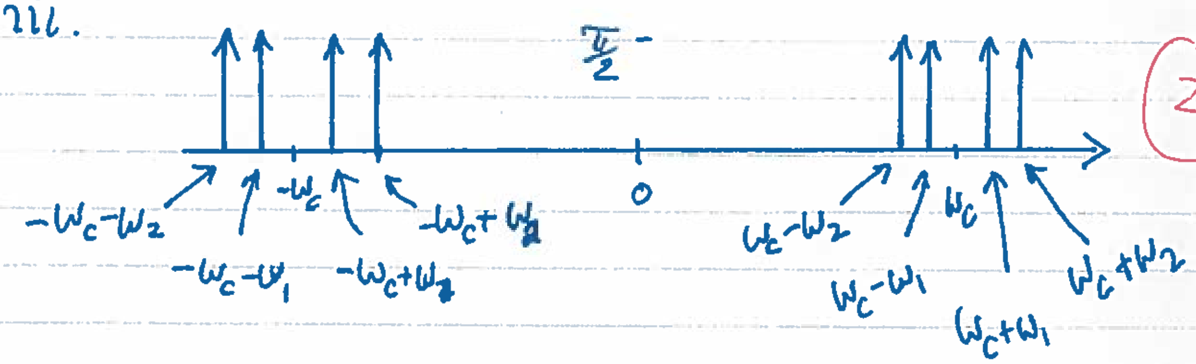
iii.



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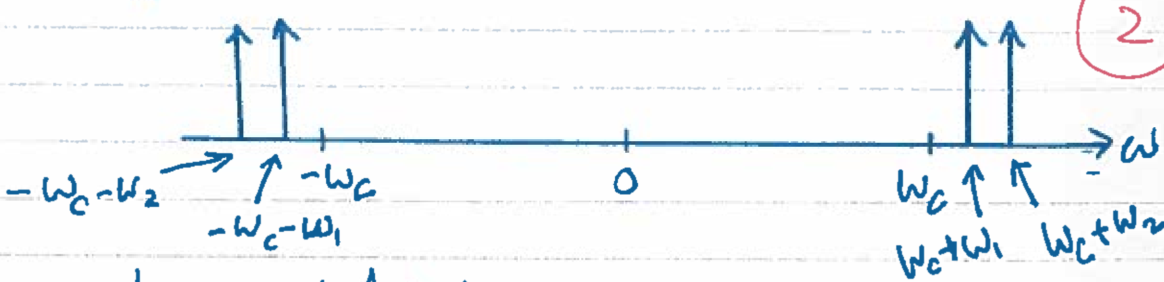
iv. The bandwidth of $\phi(t)$ is $\frac{W_2}{2\pi}$ Hz. 1

v. Upper-side band:



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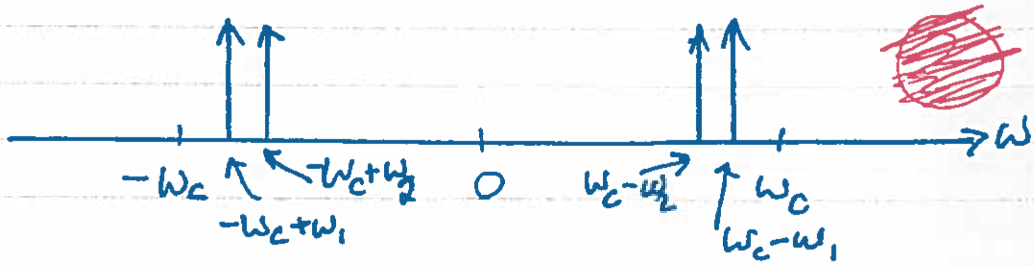
Lower-side band:



1.c. vi. $\varphi_{USB}(t) = \frac{1}{2}\left[\cos(\omega_c + \omega_1)t + \cos(\omega_c + \omega_2)t\right]$ 2

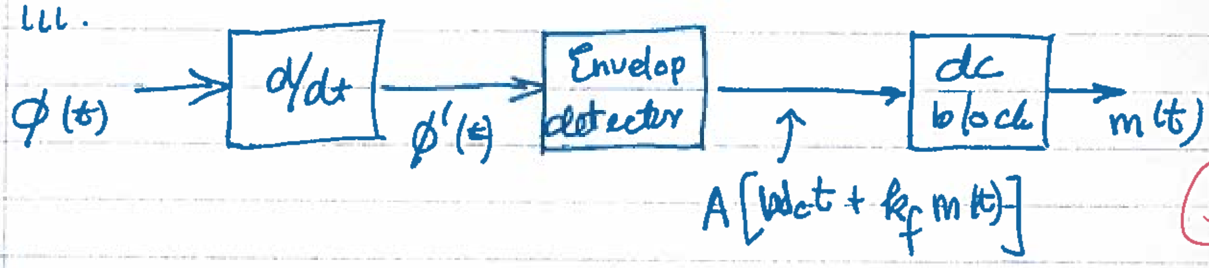
d. i. $\phi(t) = A\cos\left(\omega_c t + k_f \int m(\alpha)\,d\alpha\right)$ 2

ii. $\phi'(t) = d\phi(t)/dt$

$= A\left[\omega_c + k_f m(t)\right]\cdot$

$\sin\left[\omega_c t + k_f \int m(\alpha)\,d\alpha\right]$ 2

iii.



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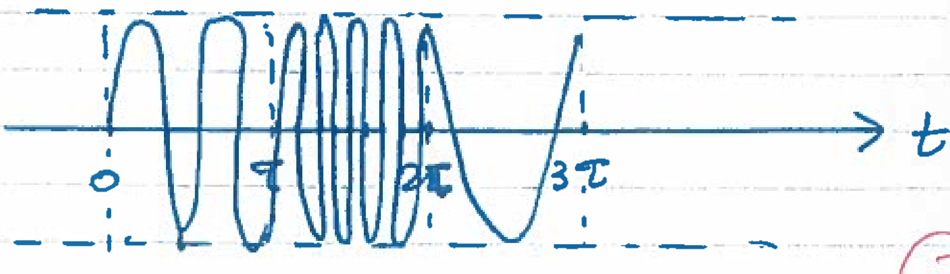
iv.



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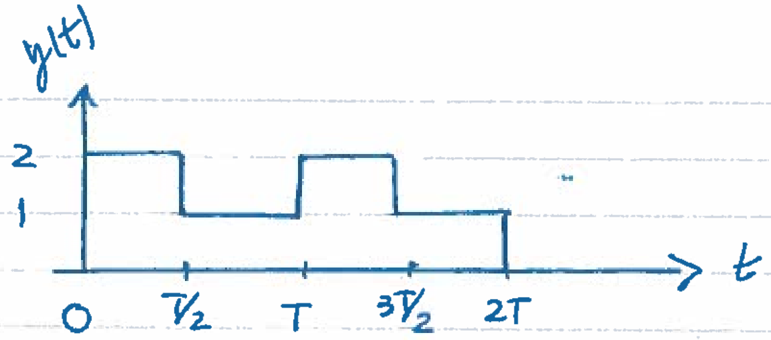
$\omega_i = \underbrace{\quad}_{\omega_c + k_f} \quad \underbrace{\quad}_{\omega_c + 2k_f} \quad \underbrace{\quad}_{\omega_c}$

2.a. i.



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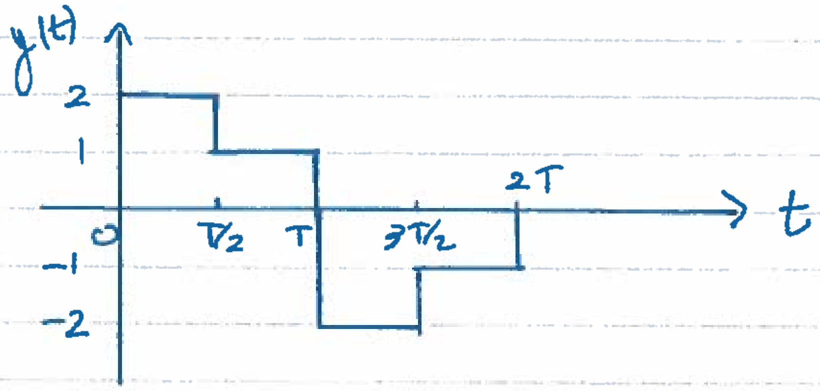
ii.



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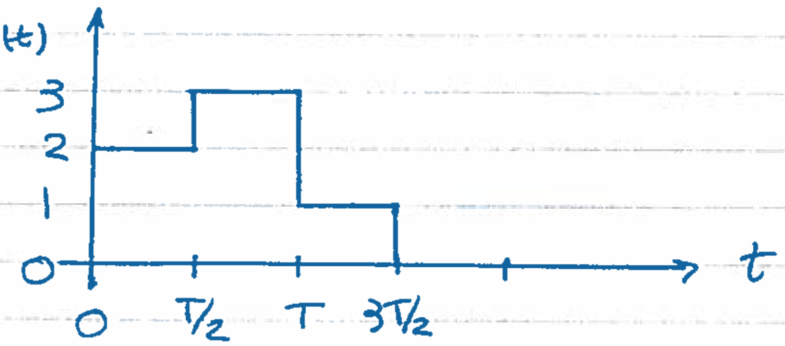
iii.



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iv. Maximum signal rate :

$$f = 1/T$$

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This is so because each impulse (signal) has a response time (duration) of T time units (sec). To avoid use of complicated technique for receiving, keep the signal rate of $1/T$ samples (impulses) per second will be supported by the channel. Signal received and integrated over a time period T will indicate whether a 0 or 1 is sent.

2.b. i. By definition, with $\omega_0 = 2\pi/T$

$$D_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\cdot e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \sum_{m=-\infty}^{\infty} \delta(t-mT)\, e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-jn\omega_0 t}\,dt$$

$$\Rightarrow D_n = \frac{1}{T} e^{-jn\omega_0 \cdot 0} = \frac{1}{T} \quad \forall n$$

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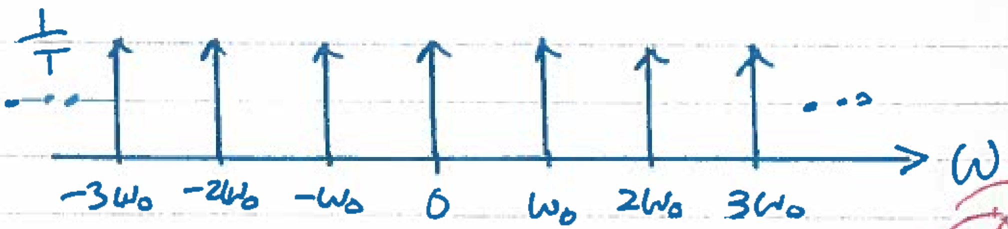
ii. Since $f(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t}$

$$\Rightarrow f(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}$$

Spectrum of $f(t)$:



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iii) By Parseval's Theorem,

$$P = \sum_{n=-\infty}^{\infty} |D_n|^2 = \sum_{n=-\infty}^{\infty} \left(\frac{1}{T}\right)^2$$

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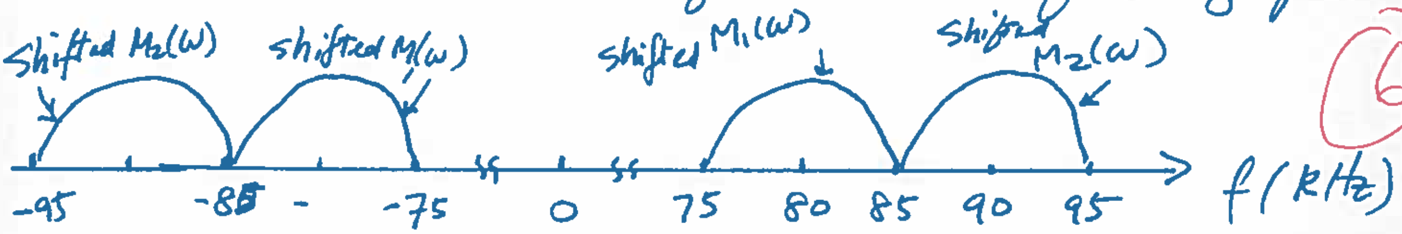
3.a. i. The modulation method:

A. Modulate $m_2(t)$ by multiplying it with $\cos(20{,}000\pi t)$ i.e., sinusoidal at 10 kHz

B. Add the baseband $m_1(t)$ to the result in Step A.

C. Modulate the resultant signal in Step B by multiplying it with $\cos(160{,}000\pi t)$ i.e., sinusoidal at 80 kHz

The resultant AM signal has the following spectrum:



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ii. The transmitted signal is

$$\phi(t) = m_1(t)\cos(160{,}000\pi t) + m_2(t)\cos(180{,}000\pi t)$$

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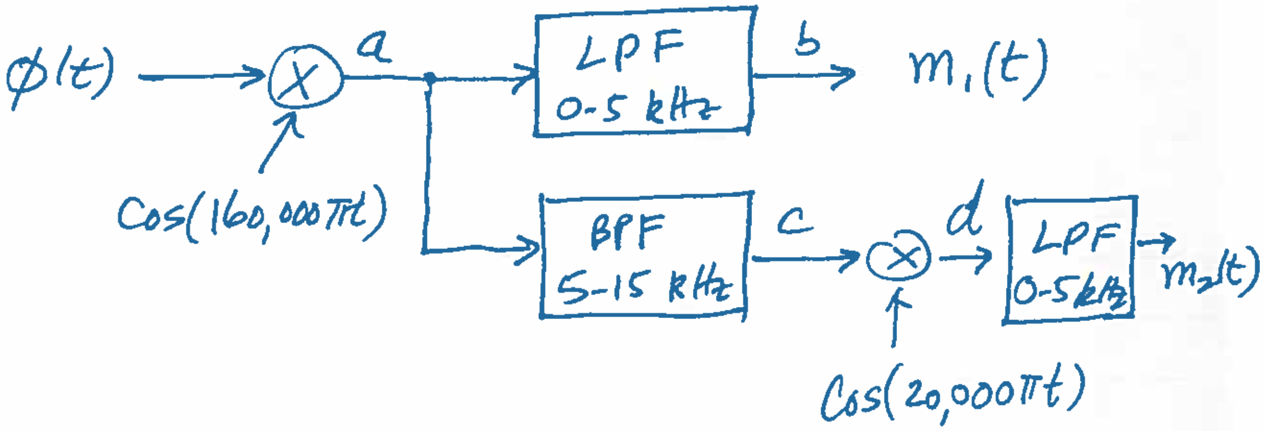
iii.



Signal at point a:

$$S_a(t) = \phi(t)\cos(160{,}000\pi t)$$
$$= \left[m_1(t)\cos(\omega_{80k}t) + m_2(t)\cos(\omega_{90k}t)\right] \cdot \cos(\omega_{80k}t)$$

$$S_a(t) = \frac{1}{2} m_1(t) \left[\cos\left(2\omega_{80k} t\right) + 1 \right]$$
$$+ m_2(t) \cos\left(\omega_{90k} t\right) \cdot \cos\left(\omega_{80k} t\right)$$
$$= \frac{1}{2} \left[m_1(t) + m_1(t) \cos\left(2\omega_{80k} t\right)\right]$$
$$+ \frac{1}{2} m_2(t) \left[\cos\left((\omega_{90k} - \omega_{80k}) t\right) \right.$$
$$\left. + \cos\left((\omega_{90k} + \omega_{80k}) t\right) \right]$$

At point b,

$S_b(t)$ is the output of the LPF (0-5 KHz) with input $S_a(t)$. So, $S_b(t) = \frac{1}{2} m_1(t)$

At point c, the output of the BPF (5-15 kHz) is

$$S_c(t) = \frac{1}{2} m_2(t) \cos(\omega_{10k} t)$$

Signal at point d:

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$$S_d(t) = S_c(t) \cdot \cos(\omega_{10k} t)$$
$$= \frac{1}{2} m_2(t) \cos^2(\omega_{10k} t)$$
$$= \frac{1}{4} m_2(t) \cdot \left[\cos 2\omega_{10k} t + 1 \right]$$
$$= \frac{1}{4} m_2(t) \cdot \left[1 + \underbrace{\cos(2\omega_{10k} t)}_{\uparrow \text{ filtered out by LPF.}} \right]$$

Thus, the final output from the LPF is $\frac{1}{4} m_2(t)$.

3.a, iv. The preferred frequency to be included in the transmitted signal is 80 kHz sinusoidal signal. This is so because the receiver needs both signals of 10 kHz and 80 kHz. However, once 80 kHz signal is received, it is relatively easy to use it to generate a sinusoidal of 10 kHz. (e.g., every 8 cycles in the 80 kHz signal is one cycle in the 10 kHz signal; just a counting process.

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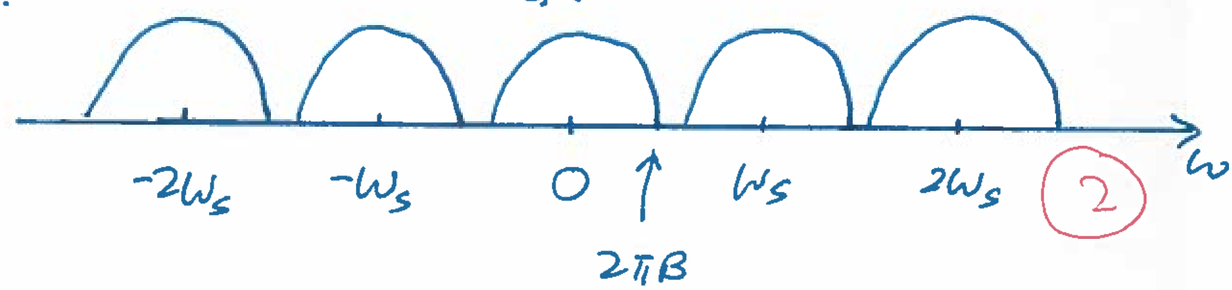
3. b. i. $\tilde{g}(t) = g(t) \cdot S(t)$ 2

$$= \frac{1}{T_s} \cdot \left[g(t) + 2g(t)\cos(\omega_s t) + 2g(t)\cos(2\omega_s t) + 2g(t)\cos(3\omega_s t) + \cdots \right]$$

ii. Each term in $\tilde{g}(t)$ is basically $g(t)\cos(n\omega_s t)$.

From the frequency domain perspective, $g(t)\cos(n\omega_s t)$ corresponds to shift $g(t)$ to a frequency band centered at $n\omega_s$, similar to AM operation. 2

iii. $\tilde{G}(\omega)$



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iv. $\omega_s \geq 2(2\pi B)$ to avoid overlap of spectra

$\Rightarrow 2\pi f_s \geq 2(\cancel{2\pi} B)$

$\Rightarrow f_s \geq 2B$ 2