EE4-47
EE9-CS6-1

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE PART IV: MEng and ACGI

MODELLING AND CONTROL OF MULTI-BODY MECHANICAL SYSTEMS

Friday, 1 May 10:00 am

Time allowed: 3:00 hours

Corrected Copy

There are FOUR questions on this paper.

Answer ALL questions

All questions carry equal marks.

This is an OPEN BOOK examination.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : S.A. Evangelou
Second Marker(s) : A. Astolfi

1. A uniform wheel of mass m is held by and rotates about a massless axle of length l passing through the wheel centre of mass, as shown in Figure 1.1. One end of the axle (point A) is fixed on Earth by a joint that enables two axle rotational degrees of freedom; general rotations about point A except spinning of the axle. The wheel is axisymmetric with spin moment of inertia I_{yy} and radial moment of inertia, passing through the centre of mass, I_{xx}.

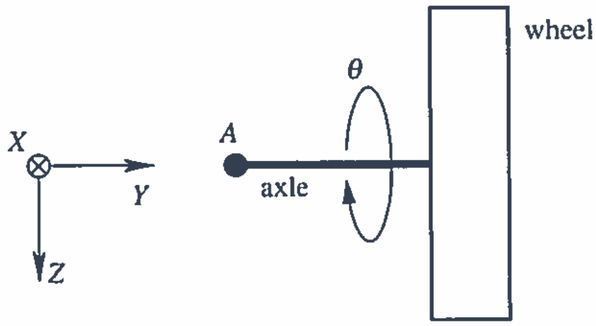

Figure 1.1 Wheel on an axle.

An axle-fixed Cartesian coordinate system with unit vectors $\boldsymbol{i}'$, $\boldsymbol{j}'$ and $\boldsymbol{k}'$ is used to analyse the motion of the system. This coordinate system has a fixed origin A, it rotates with the axle and its unit vectors are initially respectively aligned with the earth-fixed axes X, Y and Z, in which X is into the page, as shown in Figure 1.1.

a) The rotation of the axle is represented by two Euler angles ψ and ϕ in the yaw-roll convention. By making use of the standard single-axis-rotation transformation matrices, determine the axle angular velocity vector, $\boldsymbol{\Omega_a}$, in the axle-fixed coordinate system. [3]

b) Calculate the inertia matrix of the wheel with respect to the axle-fixed axes (origin at point A). [3]

c) The rotation of the wheel is represented by three Euler angles ψ, ϕ and θ in the yaw-roll-pitch convention, in which ψ and ϕ are common with the axle rotation. By making use of the standard single-axis-rotation transformation matrices, determine the wheel angular velocity vector, $\boldsymbol{\Omega_w}$, in the axle-fixed coordinate system. [3]

d) Compute the angular momentum vector, $\boldsymbol{H}$, of the wheel for its motion about point A, using the axle-fixed rectangular coordinate system. [3]

e) Assume that the axle is kept horizontal and it is rotating at a constant yaw velocity $-\omega_{yaw}$, and the wheel is spinning at a constant speed ω_{spin}.

i) Write the angular momentum vector of the wheel. [2]

ii) Determine the external torque vector required to maintain this motion. [4]

iii) Compute the length l of the axle required to obtain the required torque by the influence of gravity. [2]

2. A uniform cylindrical tube of inner radius R_1, outer radius R_2, height h, mass m and density ρ, is shown in Figure 2.1. The origin O of the Cartesian coordinate system with axes X, Y, and Z is at the centre of mass of the tube. The Z axis is the axis of symmetry of the tube.

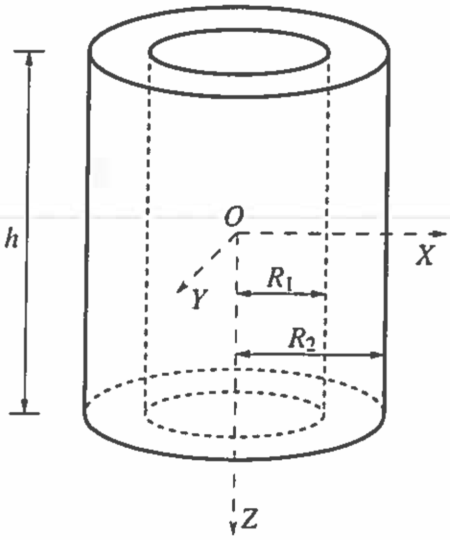

Figure 2.1 Cylindrical tube.

a) Compute the moment of inertia of the tube about:

i) its axis of symmetry; [6]

ii) the radial axis X; [6]

iii) the radial axis Y. [2]

b) Assume that $R_2 = 3R_1$ such that the tube wall thickness is $2R_1$. The tube is then machined from the inside and its wall thickness is halved. Calculate the fraction by which the tube Z-axis moment of inertia is reduced. [6]

3. A uniform wheel of radius a, mass M and spin inertia I rolls without slipping on a horizontal surface, as shown in Figure 3.1. The wheel centre moves horizontally by a distance x and the wheel rotates by an angle ϕ. A pendulum is attached on the wheel at its centre. The pendulum consists of a mass m connected to the wheel via a massless rod of length l, as shown in Figure 3.1. Assume that the pendulum is free to move in a vertical plane under the influence of gravity and the interaction with the wheel at the frictionless wheel centre joint. A control torque T_d is applied on the wheel and reacts on Earth.

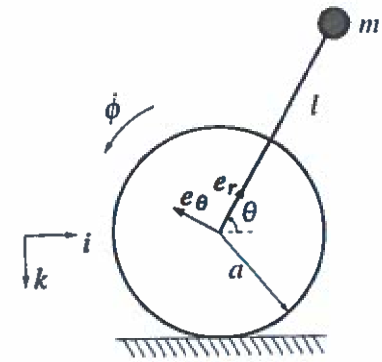

Figure 3.1 Rolling wheel and pendulum.

A fixed Cartesian coordinate system with unit vectors $\boldsymbol{i}$ and $\boldsymbol{k}$, and a moving Cartesian coordinate system with unit vectors $\boldsymbol{e_r}$ and $\boldsymbol{e_\theta}$ (see Figure 3.1) are used to analyse the motion of the two masses. The origin and rotation angle of the moving coordinate system are the wheel centre and θ respectively.

a) Write

i) the position vector, [1]

ii) the velocity vector, and [1]

iii) the acceleration vector [3]

of both the wheel centre of mass and the pendulum mass.

b) Write the equation of motion of mass m in vector form and hence:

i) derive one of the equations of motion of the system; [3]

ii) compute the force in the rod which holds the mass m. [3]

c) Derive the equation of the rolling constraint. [1]

d) Write the equation of translational motion of the mass M in vector form and hence compute the friction force between the wheel and horizontal surface that maintains the rolling constraint. [4]

e) Write the second equation of motion describing the rotation of the wheel. [4]

4. A beam rotates in a vertical plane about its centre at point O (fixed on Earth), as shown in Figure 4.1. A uniform ball of radius a rolls without slipping on the beam under the influence of gravity; the ball is always touching the beam. The moment of inertia of the beam about its axis of rotation is I_{be}, and the mass and spin inertia of the ball is m and I_{ba} respectively.

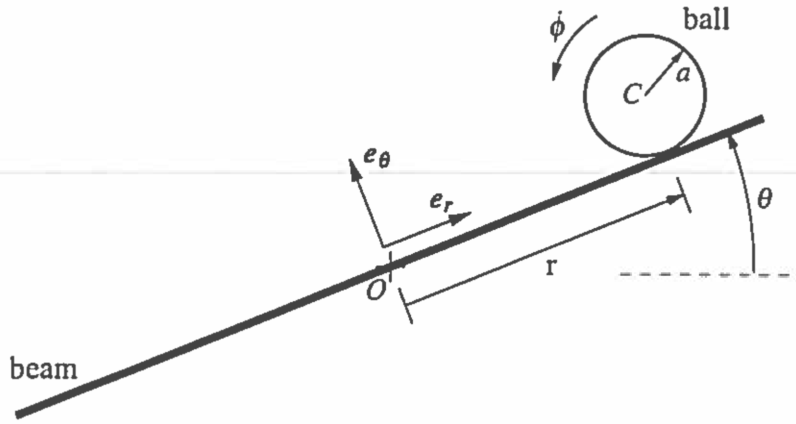

Figure 4.1 Ball and beam.

Polar unit vectors $\boldsymbol{e_r}$ and $\boldsymbol{e_\theta}$ are used to analyse the motion of the ball and beam. This coordinate system has a fixed origin at point O but it rotates with the beam by an angle θ. The ball rotates by an angle ϕ and the displacement from O of the ball contact point with the beam is r.

a) Write the position vector, $\boldsymbol{r_C}$, of the centre of mass of the wheel in the moving coordinate system. [2]

b) Determine the velocity vector of the centre of mass of the wheel. [2]

c) Compute the total kinetic energy and potential energy of the ball and beam, and hence determine the Lagrangian function of the system. [6]

d) Derive the equation of the rolling constraint and state if it is holonomic or non-holonomic. [2]

e) Use the Lagrangian approach to derive the equations of motion of the ball and beam in terms of the generalised coordinates θ, ϕ and r. Also compute the friction force between the ball and beam that maintains the rolling constraint. [8]