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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2012

EEE/ISE PART II: MEng, BEng and ACGI

Corrected Copy

COMMUNICATION SYSTEMS

Wednesday, 13 June 2:00 pm

Time allowed: 1:30 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	C. Ling
	Second Marker(s) :	J.A. Barria

Information for Students

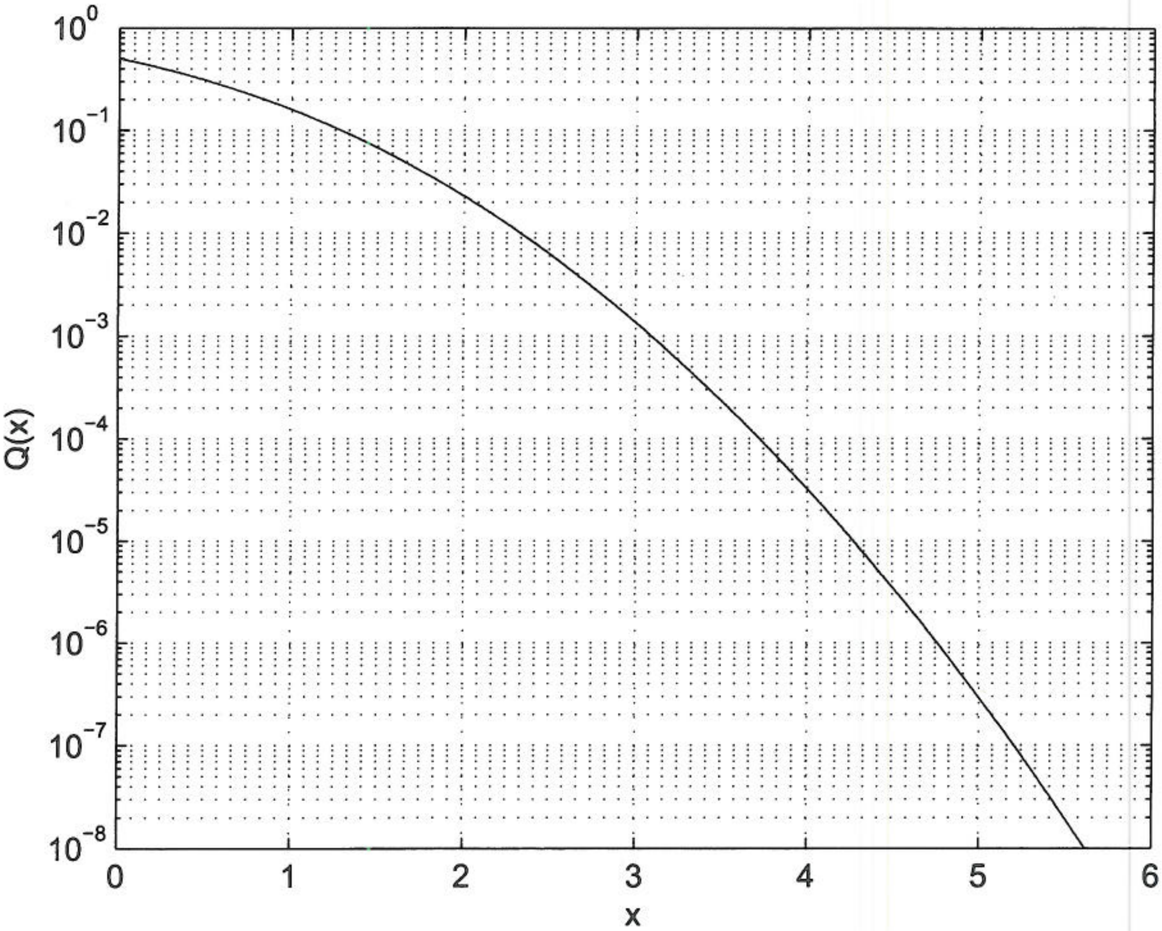


Figure 0.1 The graph of the Q-function.

1. a) i) Write down the canonical form of bandpass noise $n(t)$ in terms of baseband signals $n_c(t)$ and $n_s(t)$, and state the relationship between the powers of $n(t)$, $n_c(t)$ (in-phase component) and $n_s(t)$ (quadrature component). [4]

ii) With the help of a phasor diagram, explain the threshold effect in FM.

[6]

iii) What are cyclic codes? What are their advantages compared to other linear block codes? [6]

b) Consider a discrete memoryless source emitting i.i.d. symbols from an alphabet $S = \{s_1, s_2, \cdots, s_K\}$ where the probability $P(s_k) = p_k$. Typical sequences of length N are defined as those containing the correct portions of symbols. It is known that for large N, all other sequences are negligible (i.e., the probability of all typical sequences is almost 1).

i) State the source coding theorem. [4]

ii) Write down the probability of a typical sequence and then construct a method of source coding that achieves the bound predicted by the source coding theorem. [8]

c) i) Consider the random process

$$X(t) = a\cos(\omega t + \Theta)$$

where a and ω are constants and Θ is a random variable uniformly distributed in $[0, 2\pi)$. Determine whether this is a stationary or nonstationary process, by computing the mean and autocorrelation function.

[6]

ii) Now consider a slightly different random process

$$Y(t) = A\cos(\omega t + \theta)$$

where θ and ω are constants and A is a random variable uniformly distributed in $[-1, 1)$. Determine whether this is a stationary or nonstationary process, by computing the mean and autocorrelation function.

[6]

2. a) Consider the AM signal $s(t) = [A + m(t)]\cos(\omega_c t)$, where A is the carrier amplitude, and $m(t)$ is the message. The output SNR of the synchronous detector is given by

$$\mathrm{SNR_{AM}} = \frac{P_m}{A^2 + P_m}\mathrm{SNR_{baseband}} \tag{2.1}$$

where P_m is the message power. Let $\mathrm{SNR_{baseband}}$ be fixed.

i) In single-tone modulation, the modulating wave is a sinusoidal wave $m(t) = A_m\cos(\omega_m t)$. Using (2.1), compute $\mathrm{SNR_{AM}}$ in terms of the modulation index μ, and determine the optimum μ which maximizes $\mathrm{SNR_{AM}}$ for $0 \le \mu \le 1$. [5]

ii) Let $\mu = 1$. Suppose the message $m(t)$ is a zero-mean Gaussian random process. Derive $\mathrm{SNR_{AM}}$ when the overload probability is 6×10^{-5} (i.e., the probability $P(|m(t)| > m_p) = 6 \times 10^{-5}$). [10]

b) In a correlated binary PSK system, which is slightly different from the convetional one, the waveform

$$s_0(t) = Ak\sin(2\pi f_c t) + A\sqrt{1-k^2}\cos(2\pi f_c t), \quad 0 \le t < T \tag{2.2}$$

is sent to signal a bit "0", and

$$s_1(t) = Ak\sin(2\pi f_c t) - A\sqrt{1-k^2}\cos(2\pi f_c t), \quad 0 \le t < T \tag{2.3}$$

is sent to signal a bit "1". The first term representing a carrier component is included for the synchronization purpose. The parameter k can be chosen as $0 \le k < 1$.

i) Compute the correlation coefficient ρ between signals $s_0(t)$ and $s_1(t)$ as a function of k, where

$$\rho = \frac{\int_0^T s_0(t)s_1(t)dt}{\left[\int_0^T s_0^2(t)dt \int_0^T s_1^2(t)dt\right]^{1/2}}.$$

[5]

ii) The signal is transmitted through an additive white Gaussian noise (AWGN) channel with noise variance σ^2. Given that the optimum detector has the error probability

$$P_e = Q\left(\frac{A}{\sigma}\sqrt{\frac{1-\rho}{2}}\right),$$

derive the error probability as a function of k. Explain the results for $k = 0$ and $k = 1$. [5]

iii) The power efficiency loss is defined as the ratio of the powers required by the correlated PSK and by the conventional PSK to achieve the same error probability. Determine the power efficiency loss as a function of k. For $k = \sqrt{2}/2$, calculate the power efficiency loss in decibel (dB). How do you reduce the power efficiency loss, and what problem will it cause? [5]

3. a) Consider the digital modulation format – differential phase-shift keying (DPSK).

i) Explain why it is impossible to detect PSK with an envelope detector, and consequently, DPSK is needed. [2]

ii) Draw a diagram of DPSK modulation. Given the information symbols

$$a_1 = -1, a_2 = 1, a_3 = 1, a_4 = -1, a_5 = 1$$

and the reference symbol $b_0 = 1$, determine the differentially encoded symbols b_1, b_2, b_3, b_4, b_5. [5]

iii) Sketch a block diagram representing differential demodulation. Given the phases of the modulated signal

$$\theta_0 = 0, \theta_1 = \pi, \theta_2 = 0, \theta_3 = \pi, \theta_4 = \pi, \theta_5 = 0$$

determine the recovered information symbols. [5]

b) Consider a $(6,2)$ linear block code generated by the following matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \quad (3.1)$$

i) Is this a systematic or non-systematic code? [2]

ii) List all the codewords of this code and determine the minimum Hamming distance. [4]

iii) Give the parity-check matrix H of this code. [3]

iv) Compute the syndrome table for a single error. [6]

v) The vector $y = [010101]$ is received. Find the syndrome and hence the most likely data bits. [3]

EE2-4

Communication Systems 2012 Solutions

B – Bookwork
E – New Examples
A – New Applications

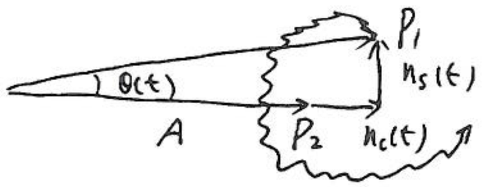
1. a) i) $n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$

[4B]

The powers of $n(t)$, $n_c(t)$, and $n_s(t)$ are equal to each other.

Feedback: generally answered well.

ii)



[6B]

When noise is weak, P_1 wanders around P_2, leading to small phase noise and small frequency noise.

When noise is strong, P_1 may sweep around origin, resulting in phase change of 2π in a short time, hence large frequency noise. In thise case, the performance of the FM receiver will deteriorate substantially. This is the threshold effect.

Feedback: generally answered ok, but some students explained the AM threshold effect, which is not the answer to this question.

iii) Cyclic codes are a class of Linear block codes, where a cyclic shift of a codeword is another codeword.

They have advantages both mathematically and in implementation. One can use polynomial representation in mathematical formulation. This also simplifies hardware realization using shift registers.

[6B]

Feedback: generally answered well.

b) i) Source coding theorem: for any discrete memoryless sources, the average codeword length (per symbol) is bounded by the entropy $H(S)$. [4B]

Generally answered well.

ii) $P_{typical} = p_1^{Np_1} p_2^{Np_2} \dots p_K^{Np_K}$

Since the probability of typical sequences is almost 1, there are $\frac{1}{P_{typical}}$ of them. Therefore, the number of bits required to represent them is

$$L_N = \log_2 \frac{1}{P_{typical}} = -\log_2 (p_1^{Np_1} p_2^{Np_2} \dots p_K^{Np_K})$$

$$= -N(p_1 \log_2 p_1 + p_2 \log_2 p_2 + \dots + p_K \log p_K)$$

$$= N H(S).$$

Average codeword length

$$\bar{L} = \frac{L_N}{N} = H(S) \text{ bits/symbol}$$ [8B]

Some confusions arose here. A common mistake was to quote Huffman coding, a practical scheme of source coding that in general doesn't achieve the optimum bound. Another mistake was to think of this question as channel coding.

c)

i) Mean $E[X(t)] = E[a\cos(wt+\theta)] = 0$

$= a\,E[\cos(wt+\theta)]$ [6B]

Auto correlation

$$R_X(t, t+\tau) = E[X(t)\,X(t+\tau)]$$

$$= E\; a^2[\cos(wt+\theta)\cos(w(t+\tau)+\theta)]$$

$$= \frac{a^2}{2} E[\cos(2wt + w\tau + 2\theta)] + \frac{a^2}{2} E[\cos(w\tau)]$$

$$= \frac{a^2}{2}\cos w\tau$$

This is a function of τ only.

Therefore, it is stationary.

Part i) was generally answered well. A small number of students couldn't remember the definition of stationary though.

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ii) Mean $E[Y(t)] = E[A(\cos wt + \theta)]$ [6E]

$= E[A]\cdot\cos(wt+\theta)$

$= 0$

Autocorrelation

$$R_Y(t, t+\tau) = E[Y(t)\,Y(t+\tau)]$$

$$= E[A^2\cos(wt+\theta)\cos(w(t+\tau)+\theta)]$$

$$= E[A^2]\cdot\cos(wt+\theta)\cos(w(t+\tau)+\theta)$$

$$= E[A^2]\cdot\frac{1}{2}\cdot[\cos(2wt+w\tau+2\theta)+\cos(w\tau)]$$

This is a function of both t and τ.

Therefore, it is non-stationary.

Part ii) was answered less well. A common mistake was to do the expectation with respect to time or phase. In fact, the random variable is A (amplitude) in here.

2 . a)

Part i) was answered well generally.

i) For single tone, $m_p = A_m$, $P_m = \frac{A_m^2}{2}$

$$\mu = \frac{m_p}{A} = \frac{A_m}{A} \Rightarrow A_m = \mu A$$

$$SNR_{AM} = \frac{A_m^2/2}{A^2 + A_m^2/2} \underbrace{\uparrow}_{SNR_{baseband}} = \frac{\mu^2}{2+\mu^2} SNR_{baseband}$$

[5E]

$$SNR_{AM} = \frac{2+\mu^2-2}{2+\mu^2} SNR_{baseband}$$

$$= 1 - \frac{2}{2+\mu^2} SNR_{baseband}$$

When $\mu = 1$, the maximum is $SNR_{AM} = \frac{1}{3} SNR_{baseband}$

ii) Let σ be the standard deviation. Then,

$$P_m = \sigma^2$$

To find m_p, notice that

$$P(|m(t)| > m_p) = 2Q\left(\frac{m_p}{\sigma}\right) = 6 \times 10^{-5}$$

[3 A]

$$\Rightarrow Q\left(\frac{m_p}{\sigma}\right) = 3 \times 10^{-5}$$

From the graph of Q-function,

$$\Rightarrow \frac{m_p}{\sigma} \approx 4$$

$$\Rightarrow m_p \approx 4\sigma$$

[3 A]

Meanwhile,

$$\mu = 1 \Rightarrow m_p = A \Rightarrow 4\sigma = A \Rightarrow \sigma = \frac{A}{4}$$

Thus,

$$SNR_{AM} = \frac{\sigma^2}{A^2+\sigma^2} SNR_{baseband} = \frac{\left(\frac{A}{4}\right)^2}{A^2+\left(\frac{A}{4}\right)^2} SNR_{baseband}$$

[4A]

$$= \frac{1}{17} SNR_{baseband}$$

To answer this question, you need to know how to use the graph of the Q-function. Reasonable errors from reading the graph are tolerated (e.g., 4.2 rather than 4).

4

b) i) $\int_0^T s_0^2(t)\,dt = \int_0^T s_1^2(t)\,dt$

$= \int_0^T A^2k^2 \sin^2(2\pi f_c t) + A^2(1-k^2)\cos^2(2\pi f_c t)\,dt$

$= \frac{A^2}{2}T$

$\int_0^T s_0(t)\, s_1(t)\,dt$

$= \int_0^T A^2k^2 \sin^2(2\pi f_c t) - A^2(1-k^2)\cos^2(2\pi f_c t)\,dt$ [5A]

$= \frac{A^2}{2}T(2k^2-1)$

$$\rho = \frac{\frac{A^2}{2}T(2k^2-1)}{\frac{A^2}{2}T} = 2k^2-1$$

Note that high-frequency terms are ignored. Some students kept these terms (marks given if correctly presented), which made the subsequent works unnecessarily complicate.

ii) $P_e = Q\left(\frac{A}{\sigma}\sqrt{\frac{1-\rho}{2}}\right) = Q\left(\frac{A}{\sigma}\sqrt{\frac{1-(2k^2-1)}{2}}\right)$

$= Q\left(\frac{A}{\sigma}\sqrt{1-k^2}\right)$

When $k=0$, $P_e = Q\left(\frac{A}{\sigma}\right)$, which is conventional PSK.

When $k=1$, $P_e = Q(0) = \frac{1}{2}$. This is because all power is used for synchronization, while no power is used to signal bits. [5A]

iii) Power efficiency loss $= 1-k^2$

For $k = \frac{\sqrt{2}}{2}$, it is equal to $\frac{1}{2}$, which is 3 dB.

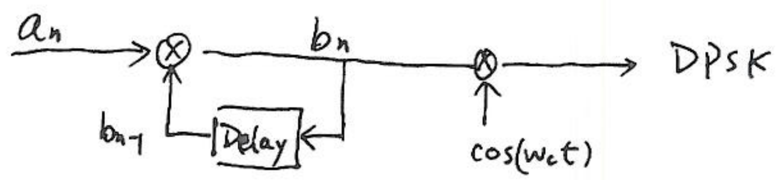
One may reduce the loss by decreasing k, but the carrier for synchronization will get weaker.

[5A]

Answers to Part (b) were generally poor, yet a small number of capable students got very good marks. It was meant to be a hard question to give an edge.

3. a) i) In PSK, the modulated signals have the same amplitude, same frequency, which cannot be distinguished by an envelope detector. [2 B]

ii)



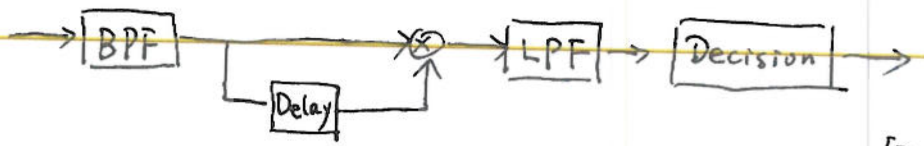
[2 B]

a_n: -1 1 1 -1 1

b_n: 1 -1 -1 -1 1 1

[3 E]

iii)



[2 B]

θ_n: 0 π 0 π π 0

[3 E]

$\hat{a}_n$: -1 -1 -1 1 -1

Answers were ok (it's basically bookwork), but surprisingly some students couldn't remember what's DPSK! Common mistake: draw the modulator and demodulator the other way round.

b) i) Since G can be written in the form

$$G = [I_2 \vdots P] \quad \text{where } I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

it is a systematic code

Just recall the definition of systematic codes.

[2 E]

ii) Codeword $x = u \cdot G$

$$= [u_1 \; u_2] \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Codewords

u	x
00	000000
01	011011
10	101110
11	110101

[4 A]

$$d_{min} = \min_{x \neq 0} W_H(x) = 4$$

d_min may also be obtained by examining the parity-check matrix H, i.e., the smallest number of linearly dependent columns.

iii) $$H = [P^T \vdots I_4] = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

[3 A]

iv) Syndrome table $s = e \cdot H^T$

e	s
100000	1110
010000	1011
001000	1000
000100	0100
000010	0010
000001	0001

[6 A]

v) $S = y \cdot H^T$

$= [0\ 1\ 0\ 1\ 0\ 1]\ H^T$

$= [1\ 1\ 1\ 0]$

$\Rightarrow e = [1\ 0\ 0\ 0\ 0\ 0]$ [3 A]

$\Rightarrow x = y + e = [1\ 1\ 0\ 1\ 0\ 1]$

$\Rightarrow$ data bits $= [1\ 1]$

Part (b) was handled relatively well by candidates, probably due to its familiar format.