UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

MEng Honours Degree in Information Systems Engineering Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C417=I4.46

ADVANCED GRAPHICS AND VISUALISATION

Friday 14 May 2004, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 Surface Modelling using the Coon's Patch

Sixteen points on a terrain map are given in the table below. The tabulated values are heights (y co-ordinates) taken over an x-z grid of unit spacing.

z coordinate \ x coordinate	0	1	2	3
0	10	11	12	13
1	10	**12**	**14**	16
2	11	**13**	**15**	19
3	12	13	14	15

A set of Coon's patches are to be used to draw the terrain. This question refers to the patch that will be used to interpolate the four points shown in boldface in the above table having co-ordinates:

$\mathbf{P}(0,0) = (1,12,1)$, $\mathbf{P}(1,0) = (2,14,1)$, $\mathbf{P}(0,1) = (1,13,2)$ and $\mathbf{P}(1,1) = (2,15,2)$

Note that the parameters defining the patch μ and ν are both in the range [0..1]. The μ parameter follows the direction of the x axis and the ν that of the z axis in the above table.

a Determine the eight gradients with respect to the parameters at the four corners of the patch.

b Given that a cubic spline curve patch has coefficients defined by the following vector equation:

$$\begin{bmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{P}_0 \\ \mathbf{P}_0' \\ \mathbf{P}_1 \\ \mathbf{P}_1' \end{bmatrix}$$

determine the equations of the four curves that bound the patch.

c Given that the Coon's patch has the following equation:

$$\mathbf{P}(\mu,\nu) = \mathbf{P}(\mu,0)(1-\nu) + \mathbf{P}(\mu,1)\nu + \mathbf{P}(0,\nu)(1-\mu) + \mathbf{P}(1,\nu)\mu - \mathbf{P}(0,0)(1-\nu)(1-\mu) - \mathbf{P}(0,1)\nu(1-\mu) - \mathbf{P}(1,0)(1-\nu)\mu - \mathbf{P}(1,1)\nu\mu$$

find the co-ordinate of the patch at its centre ($\mu=\nu=0.5$)

d Determine the gradient of the surface with respect to the parameter μ at the centre of the patch ($\mu=\nu=0.5$).

The four parts carry, respectively, 20%, 30%, 30%, and 20% of the marks

2 Camera Geometry

a Write down in matrix form the equations for camera projection, describing which parts represent intrinsic and extrinsic properties.

b A camera has a square CCD matrix of 512 x 512 pixels and dimensions of 15 x 15mm. The CCD is 35mm from the lens. The optics are aligned so that the principal point is in the centre of the CCD.

i) Write down the intrinsic projection matrix for this case.
ii) What might affect the accuracy of this projection model for a real camera?

c Two such cameras are placed in the y=0 plane with their optical axes along the z-axis, but displaced by ±60mm in the x-direction. A point of interest, **P**, is at (0,0,500). Calculate the projection of this point in each camera.

d The cameras are to be used to reconstruct positions in 3D. Assuming we can resolve a landmark at **P** with an accuracy of 1 pixel in both cameras, calculate the accuracy with which the point **P** would be reconstructed

i) in the x-direction
ii) in the z-direction.

(Hint - consider one camera and estimate how far along each axis you would need to move for the projected point to change by 1 pixel.)

e If these two cameras were used to create a stereo video-based augmented reality system, would the answer to d (ii) be the accuracy of depth seen by the observer? Give reasons for your answer.

The five parts carry, respectively, 15%, 25%, 15%, 30% and 15% of the marks.

3 Visualisation of flow data

Consider the following simple 2D flow visualization problem. You are given the velocity V(x,y) at the four corners of the unit square as follows:

V(0,0) = (1,1); V(1,0) = (3,1); V(1,1) = (1,2); V(0,1) = (2,1)

where we express the velocity as a vector of two components, the first giving the velocity component in the x-direction, and the second the velocity component in the y-direction. Suppose a particle is released at time t = 0 from seed point (0.7,0.0).

a Calculate an estimate of where the particle will travel to after a single time step of 0.1 seconds.

b State the two major sources of error in this calculation, and explain how these arise.

c Calculate an estimate of the velocity at the new position of the particle.

d When visualising 3D flow fields using glyphs, such as with the "hedgehog" approach, we can reduce visual clutter by showing fewer glyphs. Describe two different strategies for deciding where to put glyphs in order to reduce visual clutter.

e For 3D flow visualization, discuss the difficulties in rendering streamlines and how these difficulties can be overcome.

The five parts carry equal marks.

4 Surface Decimation and Triangulation

a Briefly describe the vertex-clustering algorithm for surface decimation. What are its advantages and disadvantages?

b The marching cubes algorithm can be used to generate isosurfaces from volume data. However, for very large volumes the marching cubes algorithm can generate a very large number of polygons. Discuss two different strategies for reducing a number of polygons in such isosurfaces and compare both approaches in terms of the quality of the resulting isosurfaces and their computational costs.

c Explain what is meant by the following terms in connection with Delaunay triangulation:

i) Illegal Edge
ii) Empty circle property

d Given three points $\mathbf{P}_0 = (0,0)$, $\mathbf{P}_1 = (10,0)$ and $\mathbf{P}_2 = (5,3)$, construct a Voronoi diagram.

e A fourth point $\mathbf{P}_4 = (8, -2)$ is now added to the data set. Explain how the Voronoi diagram of part d is adapted to incorporate this new point.

f Sketch the Delaunay triangulation corresponding to the Voronoi diagram of part e.

The six parts carry, respectively, 30%, 20%, 10%, 10%, 20% and 10% of the marks