EE4-27
EE9-CS5-2

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected Copy

SYSTEMS IDENTIFICATION

Wednesday, 7 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T. Parisini
Second Marker(s) : S. Evangelou

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1. Consider the discrete-time stochastic dynamic system described by the block-scheme shown in Fig. 1.1.

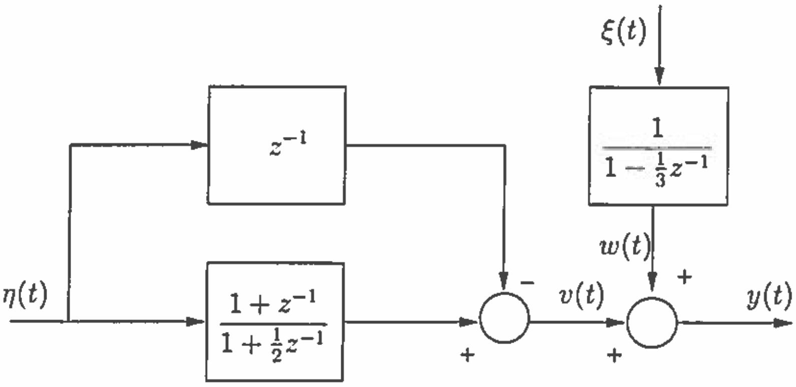

Figure 1.1 Block-scheme of the stochastic dynamic system.

where $\eta(\cdot) \sim WN(0,1)$ and $\xi(\cdot) \sim WN(0,4)$ are white uncorrelated stochastic processes.

a) Determine the transfer function from the input $\eta(t)$ to the output $y(t)$ and the transfer function from the input $\xi(t)$ to the output $y(t)$.

[4 Marks]

b) Determine the analytical expressions of the spectrum $\Gamma_y(\omega)$ as a function of the angular frequency ω of the stochastic process $y(\cdot)$.

[8 Marks]

c) Sketch the behaviour of the spectrum determined in your answer to Question 1b) in the interval $\omega \in [-\pi, \pi]$.

[8 Marks]

2. Consider the stochastic system expressed by the following state equations:

$$\begin{cases} x_1(t+1) = \frac{1}{2}x_1(t) + 3x_2(t) + v_1(t) \\ x_2(t+1) = -\frac{1}{2}x_2(t) + v_2(t) \\ y(t) = x_2(t) \end{cases}$$

where $v1(t)$ and $v2(t)$ are uncorrelated stochastic processes, with $v_1(\cdot) \sim WN(0,1)$ and $v_2(\cdot) \sim WN(1,1)$, respectively.

a) Discuss the stationarity of the steady-state stochastic processes $x_1(\cdot)$ and $x_2(\cdot)$.

[5 Marks]

b) Compute the expected value $m_2 = \mathbb{E}(x_2)$ of the steady-state stochastic process $x_2(\cdot)$.

[4 Marks]

c) Compute the expected value $m_1 = \mathbb{E}(x_1)$ of the steady-state stochastic process $x_1(\cdot)$.

[4 Marks]

d) Compute the covariance matrix of the stochastic processes $x_1(\cdot)$ and $x_2(\cdot)$, given by

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{12} & \lambda_{22} \end{bmatrix}$$

where

$$\lambda_{11} = \text{var}[x_1(t)] = \mathbb{E}[(x_1(t) - m_1)^2]$$
$$\lambda_{22} = \text{var}[x_2(t)] = \mathbb{E}[(x_2(t) - m_2)^2]$$
$$\lambda_{12} = \text{cov}[x_1(t), x_2(t)] = \mathbb{E}[(x_1(t) - m_1)(x_2(t) - m_2)]$$

[7 Marks]

3. Consider the stochastic process $v(\cdot)$ generated by the ARMA(1,1) model

$$v(t) = \frac{1}{4}v(t-1) + \eta(t) + 2\eta(t-1) \tag{3.1}$$

where $\eta(\cdot) \sim WN(0, 1/4)$.

a) First, show that the stochastic process $v(\cdot)$ is stationary. Afterwards, for the steady-state stationary process $v(\cdot)$ compute the expected value $\mathbb{E}[v(t)]$, the variance $\mathrm{var}[v(t)]$ and the sample of the correlation function $\gamma_v(\tau)$ for $\tau = 1$.

[4 Marks]

b) Determine the difference equation yielding the optimal one-step ahead prediction $\hat{v}(t+1|t)$ of $v(t+1)$ on the basis of past values $v(t), v(t-1), v(t-2), \ldots$ of the process $v(\cdot)$.

[5 Marks]

c) Compute the variance of the one-step ahead prediction error $\mathrm{var}[\varepsilon_1(t)] = \mathrm{var}[v(t+1) - \hat{v}(t+1|t)]$.

[3 Marks]

d) Determine the difference equation yielding the optimal two-steps ahead prediction $\hat{v}(t+2|t)$ of $v(t+2)$ on the basis of past values $v(t), v(t-1), v(t-2), \ldots$ of the process $v(\cdot)$.

[4 Marks]

e) Compute the variance of the two-steps ahead prediction error $\mathrm{var}[\varepsilon_2(t)] = \mathrm{var}[v(t+2) - \hat{v}(t+2|t)]$. Compare $\mathrm{var}[\varepsilon_2(t)]$ with $\mathrm{var}[\varepsilon_1(t)]$ computed in your answer to Question 3c). Comment on your findings.

[4 Marks]

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4. Consider the eight samples $v(t)$, $t = 0, 1, \ldots, 7$ shown in Fig. 4.1 of a stationary stochastic process $v(\cdot)$.

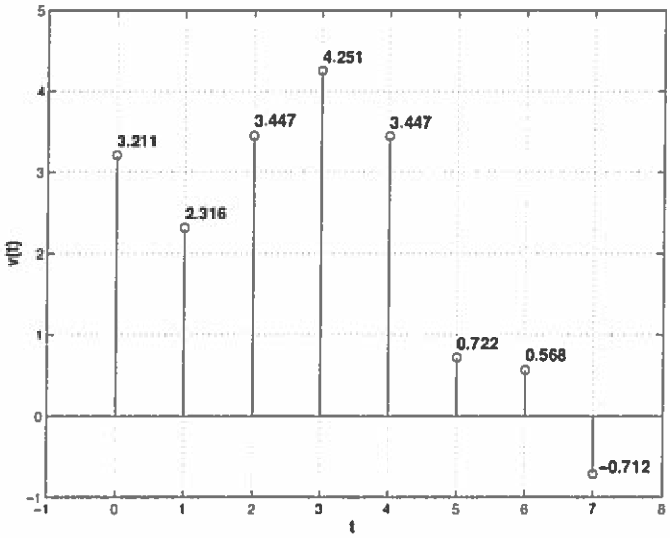

Figure 4.1 Behaviour $v(t), t = 0, 1, \ldots, 7$ of a realisation of the process $v(\cdot)$. The values taken on by $v(t), t = 0, 1, \ldots, 7$ are shown.

a) Compute the empirical estimates $\hat{\gamma}_v(\tau)$ of the first three values of the correlation function $\gamma_v(\tau)$, $\tau = 0, 1, 2$ of the stochastic process $v(\cdot)$.

[7 Marks]

b) On the basis of your answer to Question 4a), establish which of the models below describes the process $v(\cdot)$:

$$
\begin{aligned}
AR(1) &: \quad v(t) = av(t-1) + \xi(t) \\
MA(1) &: \quad v(t) = \xi(t) + c\xi(t-1).
\end{aligned}
$$

where $\xi(\cdot)$ is a zero-mean white process, that is, $\xi(\cdot) \sim WN(0, \lambda^2)$. Justify your answer.

[3 Marks]

c) Sketch a numerical procedure that exploits directly the original samples $v(t), t = 0, 1, \ldots, 7$ to determine an estimate of the unknown parameter of the model chosen in your answer to Question 4b). Using this procedure, compute such an estimate.

[5 Marks [

d) Sketch a numerical procedure (different from the one given in your answer to Question 4c)) that exploits the estimates $\hat{\gamma}_v(\tau)$, $\tau = 0, 1, 2$ computed in your answer to Question 4a), to determine an estimate of the unknown parameter of the model chosen in your answer to Question 4b). Using this procedure, compute such an estimate and compare it with the one obtained in your answer to Question 4c). Comment on your findings.

[5 Marks [