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E2.3B

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2009

EEE PART II: MEng, BEng and ACGI

Corrected
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POWER

Tuesday, 16 June 2:00 pm

Time allowed: 1:30 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T.C. Green, T.C. Green

Second Marker(s) : B. Chaudhuri, B. Chaudhuri

1.

(a) A single-phase voltage source of 5 kV is connected to a load via a cable. The load has an impedance of 100+j40 Ω and the cable has an impedance of 2+j3 Ω. Calculate the voltage appearing across the load and the real and reactive powers of the load. [10]

(b) Briefly describe the short-circuit and open-circuit tests of a transformer and explain why they are used. [8]

(c) The following equations form a derivation of the voltage transfer ratio of a buck switch-mode power supply. Explain the basis of each line of the derivation and explain any assumptions that have been made. [6]

$$\Delta I(on) + \Delta I(diode) = 0$$

$$\frac{V_I - V_O}{L} t_{On} + \frac{-V_O}{L} t_{diode} = 0$$

$$\frac{V_O}{V_I} = \frac{t_{On}}{t_{On} + t_{diode}}$$

$$= \frac{t_{On}}{T}$$

(d) The induction machine has no external connection to the rotor windings. Explain how current is caused to flow in the rotor winding and also explain why the current that flows is affected by the speed of the machine. State the condition under which no current flows in the rotor winding. [6]

(e) Explain why an electricity system uses a variety of voltage magnitudes for distribution and transmission. [5]

(f) The nominal frequency of supply in the UK is 50 Hz. Describe why the actual operating frequency may deviate slightly from this value. [5]

2. Figure Q2 show the circuit of a boost switch-mode power supply (SMPS). It is to be operated in continuous with an input voltage of 5 V and output voltage of 15 V. It will be required to process a maximum throughput power of 10 W and have an output voltage ripple which is no greater than 75 mV. The turn-on and turn-off energy losses of the MOSFET operated at 2 A are 4 μJ and 6 μJ. The on-state resistance of the MOSFET is 0.2 Ω.

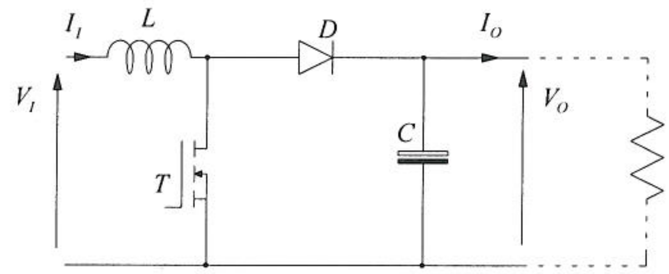


Figure Q2

(i) Calculate the value of input current when the circuit operates at maximum power and sketch graphs of the current through the inductor, the diode and the capacitor. [7]

(ii) Calculate the value of duty-cycle at which the SMPS should be operated assuming continuous conduction. [4]

(iii) The switching frequency of the circuit has been set at 30 kHz. Calculate the value of inductor required such that the inductor current ripple is 5% of the input current. [5]

(iv) Calculate the values of the capacitance and effective series resistance (ESR) of the capacitor that ensures that the voltage ripple due to these two terms are 20% and 80% (respectively) of the maximum output voltage ripple. [8]

(v) The chosen switching frequency is re-examined after a heat-sink design has been completed. This indicates that the maximum power that can be dissipated in the MOSFET is 0.75 W. Calculate the maximum switching frequency that can be used. [3]

(vi) Explain the expected effect on the inductor current ripple and the output voltage ripple of the change in switching frequency from the value in (iii) to that found in (v). [3]

3. A 1-pole-pair induction machine is used on a three-phase supply of 50 Hz. It has the equivalent circuit shown in figure Q3 with the parameter values noted below.

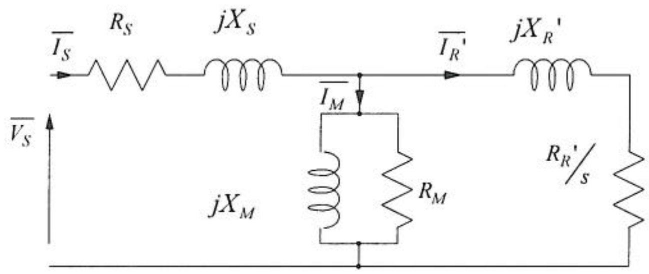


Figure Q3

R_S	$= 1.1\ \Omega$
R'_R	$= 0.75\ \Omega$
X_S	$= 2.0\ \Omega$
X'_R	$= 0.8\ \Omega$
X_M	$= 50\ \Omega$
R_M	$= 200\ \Omega$

(a) Describe the physical effect that gives rise to each of the components of the equivalent circuit. [8]

(b) The machine is operated at 2,900 r.p.m. and the stator current is measured as 12.0 A. Calculate the following:

(i) the slip, [4]

(ii) the rotor current, [6]

(iii) the output power and torque, [6]

(iv) the total power loss. [6]

4.

(a) Explain why there is a maximum in the graph of transformer efficiency against secondary current. Identify the point at which this peak is expected to occur. [6]

(b) A transformer with a turns-ratio of 2.6:1 is represented by the equivalent circuit shown in figure Q4 with the parameter values noted below.

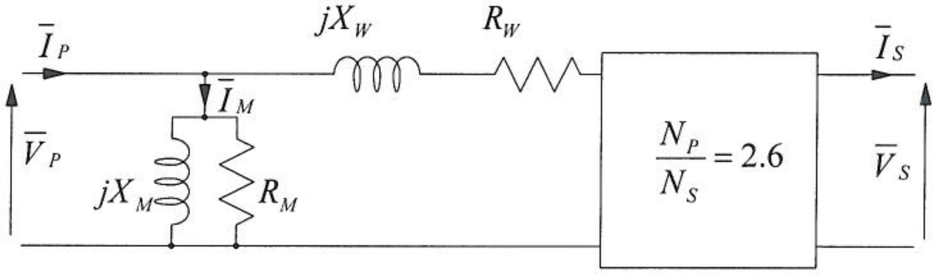


Figure Q4

R_W $= 9.0\ \Omega$
X_W $= 12.0\ \Omega$
X_M $= 1250\ \Omega$
R_M $= 4000\ \Omega$

The transformer secondary is connected to a load that draws 1.0 kW at a lagging power factor of 0.8 when supplied at 230 V.

(i) Calculate the magnitude and angle of the secondary current (with respect to the secondary voltage) [4]

(ii) Calculate the voltage regulation (as a percentage) of this transformer for this load. [10]

(iii) Calculate the efficiency at this load condition. [8]

(iv) Find an approximate value of load power (at the same power factor) that would achieve maximum efficiency. [2]

E2.7B

Solutions 2009

Question 1 is compulsory.
Answer question 1 and any two of questions 2-4
Question 1 caries 40% and the other questions carry equal weighting of 30%.

1.

(a) A single-phase voltage source of 5 kV is connected to a load via a cable. The load has an impedance of 100+j40 Ω and the cable has an impedance of 2+j3 Ω. Calculate the voltage appearing across the load, the real and reactive powers of the load. [10]

$$V_L = V_S \frac{Z_L}{Z_L + Z_C}$$
$$= 5\times10^3 \times \frac{100 + j40}{100 + j40 + 2 + j3} = 5\times10^3 \times \frac{107.70\angle 21.80°}{110.69\angle 22.86°} = 4.86\angle -1.06°\ kV$$

[2 for method, 2 for numerical answer and unit]

$$I_L = \frac{V_S}{Z_L + Z_C}$$
$$= \frac{5\times10^3}{110.69\angle 22.86°} = 45.17\angle -22.86°\ A$$
$$P_L = {I_L}^2 \operatorname{Re}\{Z_L\}$$
$$= 45.17^2 \times 100 = 204\ kW$$
$$Q_L = {I_L}^2 \operatorname{Im}\{Z_L\}$$
$$= 45.17^2 \times 40 = 81.6\ kVAr$$

[2 for method, 2 for each numerical answer and unit]

(b) Briefly describe the short-circuit and open-circuit tests of a transformer and explain why they are used. [8]

Short-circuit test has secondary short-circuited and primary supplied at reduced voltage (such that primary current is approximated its rated value). The dominant current flow is through the secondary branch of the equiv. circuit and magnetising current can be ignored. Thus the impedance of the (combined) windings can be identified from the measured voltage and current and resistance split from reactance with the measured real power.

Open-circuit test is conducted with an open-circuit secondary at nominal primary voltage. The only current flow is through the magnetising branch. Thus the impedance of the magnetising branch can be identified from the measured voltage and current and resistance split from reactance with the measured real power.

(c) The following equations form a derivation of the voltage transfer ratio of a buck switch-mode power supply. Explain the basis of each line of the derivation and explain any assumptions that have been made. [6]

$$\Delta I(on) + \Delta I(diode) = 0$$

$$\frac{V_I - V_O}{L} t_{On} + \frac{-V_O}{L} t_{diode} = 0$$

$$\frac{V_O}{V_I} = \frac{t_{On}}{t_{On} + t_{diode}}$$

$$= \frac{t_{On}}{T}$$

First line is a statement that the inductor current is in steady-state, i.e., no net **change** in inductor current over one complete cycle.
The second line is based on **rates-of-change** of current from the voltage imposed across the inductor. It assumes that the inductor has no resistance and that there are no voltage drops across the semiconductors when conducting.
Third line is a simple re-arrangement
Fourth line introduces the period, T of the switching and this is on the assumption that the diode conductors for all of the off-time, i.e. continuous conduction.

(d) The induction machine has no external connection to the rotor windings. Explain how current is caused to flow in the rotor winding and also explain why the current that flows is affected by the speed of the machine. State the condition under which no current flows in the rotor winding. [6]

A three-phase stator winding, when supplied with a balanced three-phase current, established a flux across the air-gap that **rotates** around the air-gap but has a constant amplitude. This rotating field cuts the rotor winding and the rate-of-change of flux linkage induced voltages in the rotor. Because the rotor winding is closed, currents then flow. The value of the **rate-of-change** of flux-linkage depends on the velocity of the flux wave **relative** to the rotor winding and so depends on how fast the physical rotor rotates. Thus the voltage and current amplitudes also depend on the rotor speed. If the rotor rotates in synchronism wit the field then now voltages are induced and no current flows.

(f) Explain why an electricity system uses a variety of voltage for distribution and transmission. [5]

Power losses contribute to the variable costs of running a network. They can be reduced by using a higher voltage and a lower current to carry a given power. However, high voltage equipment (circuit breakers and transformers) have a high capital cost. Only in bulk transfer between a limited number of nodes is very high voltage used. As the power transfer reduces and the number of nodes increases closer to the consumers, lower voltages are used to strike a better balance between capital costs and running costs.

(g) The nominal frequency of supply in the UK is 50 Hz. Describe why the actual operating frequency deviates slightly from this value.. [5]

Needs rewording. A sudden mismatch between the prime-mover power applied to generators and the power consumed by loads disturbs the power-balance (or torque-balance) of the generators and accelerates/decelerates them according to the moments-of-inertia. A sudden reduction of power consumption results in mechanical power being added to the inertia of the generators. Their collective speed increases and the frequency of the generated AC increases.

2. Figure Q2 show the circuit of a boost switch-mode power supply (SMPS). It is to be operated with an input voltage of 5 V and output voltage of 15 V. It will be required to process a maximum throughput power of 10 W and have an output voltage ripple which is no greater than 75 mV. The turn-on and turn-off energy losses of the MOSFET operated at 2 A are 4 μJ and 6 μJ. The on-state resistance of the MOSFET is 0.2 Ω.

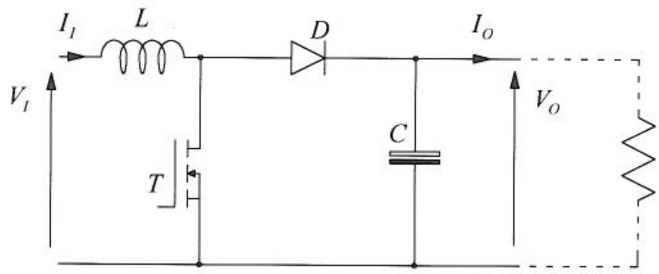
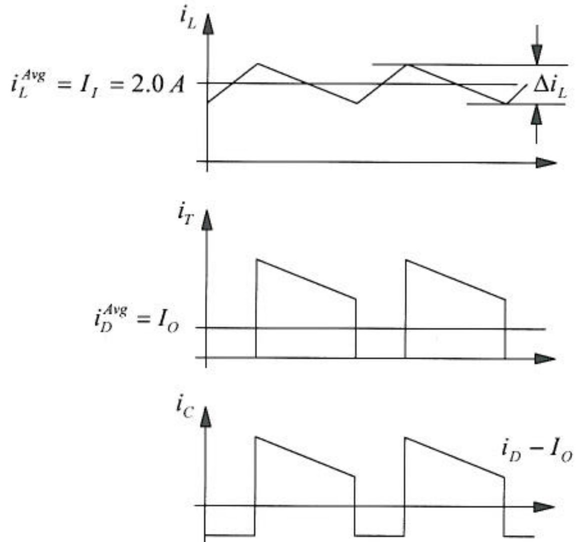


Figure Q2

(i) Calculate the value of input current when the circuit operates at maximum power and sketch graphs the current through the inductor, the diode and the capacitor. [7]

[Calculation and Bookwork]
Input current is found from throughput power.

$$I_I = \frac{P}{V_I}\frac{10}{5} = 2.0\,A$$



(ii) Calculate the value of duty-cycle at which the SMPS should be operated. [4]

[Calculation example]
Re-arrange voltage gain equation

$$\frac{V_O}{V_I} = \frac{1}{1-\delta}$$

$$\delta = \frac{V_O - V_I}{V_O} = \frac{15-5}{15} = 0.667$$

(iii) The switching frequency of the circuit has been set at 30 kHz. Calculate the value of inductor required such that the inductor current ripple is 5% of the input current. [5]

[Calculation example]

Consider rate of rise of current during on-time

$$\Delta i_L = \frac{di_L}{dt} t_{On} = \frac{V_I}{L} \cdot \frac{\delta}{f}$$

$$L = \frac{V_I}{\Delta i_L} \cdot \frac{\delta}{f} = \frac{5 \times 0.667}{0.05 \times 2 \times 30 \times 10^3} = 1.11\,mH$$

(iv) Calculate the value of effective series resistance (ESR) of the capacitor that ensures that the voltage ripple due to this resistance is less than 80% maximum output voltage ripple and a value for the capacitance so that is contributes less than 20%. [8]

[Calculation example]

Voltage ripple is proportional to peak-to-peak current in capacitor. This peak-to-peak current is equal to the diode current peak and, in turn the inductor current peak.

$$\Delta v_{ESR} = i_C^{ptp} R_{ESR}$$

$$i_C^{ptp} = \hat{i}_L = I_I + \tfrac{1}{2}\Delta i_L = I_L(1 + 0.025) = 2.05$$

$$R_{ESR} = \frac{\Delta v_{ESR}}{i_C^{ptp}} = \frac{0.8 \times 0.075}{2.05} = 29.2\,m\Omega$$

Ripple due to capacitance can be assessed during the time the diode is off (and the MOSFET is on) which is when the output current alone flows in the capacitor.

$$\Delta v_C = \frac{\Delta q}{C}$$

$$\Delta q^{On} = I_O t_{On} = \frac{P}{V_O} \cdot \frac{\delta}{f} = \frac{10 \times 0.667}{15 \times 30 \times 10^3} = 14.8\,\mu C$$

$$C = \frac{\Delta q}{\Delta v_C} = \frac{14.8\mu}{0.2 \times 75m} = 987\,\mu F$$

Choose 470 µF as preferred value on the safe side.

(v) The chosen switching frequency is re-examined after a heat-sink design has been completed. This indicates that the maximum power that can be dissipated in the MOSFET is 0.75 W. Calculate the maximum switching frequency that can be used. [3]

[Calculation example]
Re-arrange equation for total power loss

$$P_{Loss} = \delta I_{DS}^2 R_{DS(On)} + f\left(E_{On} + E_{Off}\right)$$

$$f = \frac{P_{Loss} - \delta I_{DS}^2 R_{DS(On)}}{E_{On} + E_{Off}} = \frac{0.75 - 0.667 \times \left(\frac{10}{5}\right)^2 \times 0.2}{4\mu + 6\mu} = 21.7kHz$$

(vi) Explain the expected effect on the inductor current ripple and the output voltage ripple of the change in switching frequency from the value in (iii) to that found in (v). [3]

The inductor current ripple reduces because the on time has reduced (duty-cycle is unchanged by frequency increased.
The output voltage ripple due to the capacitance is decreases because the on-time has decreased and so too has the charge removed by the output current. This is the minor term in the output voltage ripple.
The output voltage ripple due to the ESR is essentially unchanged since it is dominated by the input current amplitude. This is the dominant term in the output voltage ripple.

3. A 1-pole-pair induction machine is used on a three-phase supply of 50 Hz. It has the equivalent circuit shown in figure Q3 with the parameter values noted below.

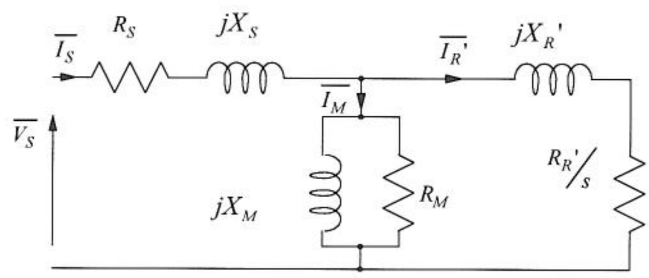


Figure Q3

R_S	$= 1.1\ \Omega$
R'_R	$= 0.75\ \Omega$
X_S	$= 2.0\ \Omega$
X'_R	$= 0.8\ \Omega$
X_M	$= 50\ \Omega$
R_M	$= 200\ \Omega$

(a) Describe the physical effect that gives rise to each of the components of the equivalent circuit. [8]

[Bookwork]
RS and RR arise from the physical resistance of the stator and rotor windings respectively. RR is then referred to the stator. In the equiv. cct. RR appears divided by s which generally increases its value. This extra resistance models the conversion of electrical power to mechanical power. [2]
XM arises from the flux that links stator and rotor across the air-gap; it models the stator current flow to magnetise the machine. [2]
XS and XR arise from the fluxes that link only one winding and create additional inductive voltage drop in stator and rotor respectively. Again, the rotor leakage reactance has been referred to the stator. [2]
RM arises from the power losses that occur in magnetising the machine such as hysteresis and eddy current losses. These loss cause additional magnetising current to flow that is approximately proportional to voltage and so a parallel resistance suffices. [2]

(b) The machine is operated at 2,900 r.p.m. and the stator current is measured as 12.0 A. Calculate the following:

(i) the slip [4]

[Calculation] [2 for eqn; 2 for value]
Use standard definition but apply to speeds in r.p.m. Note P=1.

$$s = \frac{\omega_S - \omega_R}{\omega_S} = \frac{n_S - n_R}{n_S} = \frac{50 \times 60 - 2{,}900}{50 \times 60} = 1/30$$

(ii) the rotor current [6]

[Calculation] [2 for ZR and ZM; 2 for current divider; 2 for final value]

Method is to apply current divider rule. First find the impedances of the rotor and magnetizing branches.

$$\frac{R'_R}{s} = 30 \times 0.75 = 22.5\,\Omega$$

$$Z_R = 22.5 + j0.8\,\Omega$$

$$Z_M = \frac{jX_M R_M}{R_M + jX_M} = \frac{j10{,}000}{206.16\angle 14.04^\circ} = 48.51\angle 75.96^\circ = 11.77 + j47.06$$

$$I'_R = I_S \frac{Z_M}{Z_M + Z_R} = 12 \times \frac{48.51\angle 75.96^\circ}{34.27 + j47.85} = 12 \times \frac{48.51\angle 75.96^\circ}{58.85\angle 54.39^\circ} = 12 \times 0.8243\angle 21.57^\circ$$

$$|I'_R| = 9.892\,A$$

(iii) the output power and torque [6]

[Calculation] [2 for both eqns; 3 for both values; 1 for both units]

Standard equations apply

$$P_{Mech} = 3I'^2_R R'_R \left(\tfrac{1}{s} - 1\right) = 3 \times 9.892^2 \times 0.75 \times (30 - 1) = 6.38\,kW$$

$$T = \frac{P_{Mech}}{\omega_R} = \frac{6384}{2900 \times \frac{2\pi}{60}} = 21.02\,Nm$$

(iv) the total power loss [6]

[Calculation]

Winding power loss found from relevant currents [3]

$$P_{RS} = 3I_S^2 R_S = 3 \times 12^2 \times 1.1 = 475.2\,W$$

$$P_{RS} = 3I'^2_R R'_R = 3 \times 9.892^2 \times 0.75 = 220.2\,W$$

Magnetising loss requires calculation first of the voltage across rotor branch [3]

$$|V_{AG}| = |I'_R||Z_S| = 9.892 \times 22.51 = 222.71\,V$$

$$P_{RM} = 3\frac{V_{AG}^2}{R_M} = 3\frac{222.71^2}{200} = 744\,W$$

4.

(a) Explain why there is a maximum in the graph of transformer efficiency against secondary current. Identify the point at which this peak is expected to occur. [7]

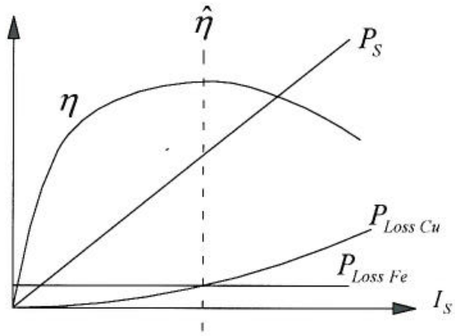
Efficiency can be found by subtracting loss terms to the input power or by adding back losses to the output power. [1]

$$\eta = \frac{P_S}{P_P}$$

$$= \frac{P_S}{P_S + P_{Loss\,Cu} + P_{Loss\,Fe}}$$

$$= \frac{P_P - P_{Loss\,Cu} - P_{Loss\,Fe}}{P_P}$$

The output power is proportional to the secondary current. Copper loss is load dependent and varies as the square of the secondary current and the iron loss largely load independent (unless the load becomes excessive). Efficiency is low at low secondary current because the fixed losses are large in comparison to the output power. Efficiency is also low at high secondary current because the variable losses are disproportionately large. Between these extremes lies a peak efficiency (which is in the normally operating range of a well designed transformer). [3]



The peak efficiency occurs when the copper loss equals the iron loss. [2]

(b) A transformer with a turns ratio of 2.6:1 is represented by the equivalent circuit shown in figure Q4 with the parameter values shown below.

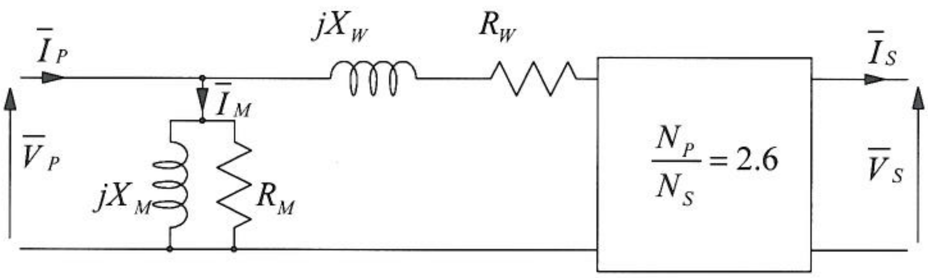


Figure Q4

R_W = 9.0 Ω
X_W = 12.0 Ω
X_M = 1250 Ω
R_M = 4000 Ω

The transformer secondary is connected to a load that draws 1.0 kW at a lagging power factor of 0.8 when supplied at 230 V.

(i) Calculate the magnitude and angle of the secondary current (with respect to the secondary voltage) [4]

[Calculation] [2 for eqn; 2 for value]

$$I_S = \frac{P_S}{V_S \cos(\phi)} = \frac{1{,}000}{230 \times 0.8} = 5.43\ A$$

$$\phi = \cos^{-1}(0.8) = \pm 36.87°$$

A lagging power factor load implies the negative angle for the current with respect to the voltage.

(ii) Calculate the voltage regulation (as a percentage) of this transformer for this load. [10]

[Calculation]

Find the referred secondary current [2]

$$I'_S = I_S \frac{N_S}{N_P} = \frac{5.43}{2.6} = 2.088\angle -36.87°\ A$$

Find voltage drop across winding impedance [2]

$$V_W = I'_S\left(R_W + jX_W\right) = 2.088\angle -36.87° \times 15\angle 53.13° A = 31.32\angle 16.26° \quad V$$

Find the primary voltage [2]

$$V_P = V'_S + V_W = V_S \frac{N_P}{N_S} + V_W = 230 \times 2.6 + 30.07 + j = 628.1 + j8.76\,V$$

$$\left|V_P\right| = 628.1\,V$$

Determine no-load secondary voltage direct from primary voltage [2]

$$V_{S(NL)} = V_P \frac{N_S}{N_P} = \frac{628.1}{2.6} = 241.6\,V$$

Regulation found as percentage change over no-load condition [2]

$$\text{Reg} = \frac{V_{S(NL)} - V_S}{V_{S(NL)}} = \frac{241.6 - 230.0}{241.6} = 4.80\%$$

Approx regulation can be found as

$$\text{Reg} \approx \frac{I'_S\, R_W \cos(\phi) + I'_S\, X_W \sin(\phi)}{V_P} = \frac{2.088 \times 9 \times 0.8 + 2.088 \times 12 \times \sin(36.87)}{628.1} = \frac{30.0}{628.1} = 4.78\%$$

Care needed in applying angle of reactance term (the power factor angle is needed (positive in this case))

(iii) Calculate the efficiency at this load condition. [8]

[Calculation]
Find losses [2 for eqns; 2 for values]

$$P_{RW} = I'^2_S\, R_W = 2.088^2 \times 12.0 = 25.1\, W$$

$$P_{RM} = \frac{V_P^2}{R_M} = \frac{628.1^2}{4000} = 98.6\, W$$

Calculate efficiency by adding back losses to output power [2 for eqn ; 2 for value]

$$\eta = \frac{P_O}{P_O + P_{RW} + P_{RM}} = \frac{1000}{1000 + 25.1 + 98.6} = 88.99\ \%$$

(iv) Find an approximate value of load power (at the same power factor) that would achieve maximum efficiency. [2]

[Interpretation and extension of calculation]
Max efficiency when fixed and variable losses are equal so find secondary current that achieves this.

set $P_{RW} = P_{RM} = 98.6\, W$

$$I'_S = \sqrt{\frac{P_{RW}}{R_W}} = \sqrt{\frac{98.6}{9}} = 3.31\, A$$

$$I_S = I'_S \frac{N_p}{N_S} = 3.31 \times 2.6 = 8.61\, A$$

$$P_S = V_S I_S \cos(\phi) = 230 \times 8.61 \times 0.8 = 1.58\, kW$$

(In fact, the efficiency curve is quite flat around the maximum and there is little real difference in efficiency which is good for a transformer having to operate at a range of different loads.)

$$\eta^{Max} = \frac{P_O}{P_O + P_{RW} + P_{RM}} = \frac{1580}{1580 + 98.6 + 98.6} = 88.9\ \%$$

End