Imperial College London
Department of mathematics
Dr I. Shevchenko

Name:
CID:

Tutorial 10

Any marks received for the tutorial are only indicative and may be subject to moderation and scaling.

Exercise 1 (BVP) — % of CW mark: 2.0

Consider the initial boundary value problem

$$
\begin{aligned}
&u_t = u_x^2, \quad t \in [0, t_f], \quad x \in [a, b], \quad u_0, \alpha, \beta \in \mathbb{R}, \\
&u(0, x) = u_0, \\
&u(t, a) + u_x(t, a) = \alpha, \\
&u(t, b) = \beta.
\end{aligned}
$$

Approximate u_x with the central difference scheme, and use the Backward Euler and Newton's methods to solve the equation. Write down in the explicit form the RHS and Jacobian for the Newton method for the grid nodes x_i, $i = 0, 1, 2, 3, 4$.

Exercise 2 (Finite Difference Method) — % of CW mark: 2.0
Mastery Component

Consider the initial value problem

$$u_t = -cu_x, \quad c \in \mathbb{R}.$$

Approximate u_x with the central difference scheme, and use the Backward Euler method as a time integrator. Find the time step h and the space step Δx for which the finite difference discretization is stable.