E3.02
AC4

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

MSc and EEE PART III/IV: MEng, BEng.and ACGI

INSTRUMENTATION

Monday, 10 May 10:00 am

Corrected Copy

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : C. Papavassiliou
Second Marker(s) : S. Lucyszyn

1.

(a) Briefly describe the sensitivity, resolution, threshold and dynamic range of a sensor

[8]

(b) Compute the sensitivity, resolution, threshold and dynamic range of a pressure sensor which:

- Its output is proportional to the supply voltage
- Its supply voltage is 5V DC
- It shows 100mV RMS electrical noise at its output
- At 1 atmosphere its output is 2V
- It cannot detect pressures smaller than 0.1 atmospheres

[8]

(c) Define the cross sensitivity of a sensor. Propose a way to increase the number of bits of the pressure sensor in this question by exploiting its cross sensitivity.

[4]

2.

(a) Draw the Thevenin and Norton equivalent circuits for a noisy resistor.

[5]

(b) An amplifier is connected to a source of Thevenin impedance Z_s and to a load of impedance Z_L. Write an expression relating the noise factor of the amplifier to Z_s and Z_L. Identify any parameters you need to use.

[5]

(c) Define an ideal voltage amplifier. What are the input and output impedance of an ideal voltage amplifier? Define the noise figure. Compute the noise figure of an ideal voltage amplifier driven by an ideal voltage source.

[5]

(d) Define an ideal transformer. Compute the noise figure of an ideal transformer.

[5]

3.

(a) With the aid of a diagram describe the Colpitts oscillator. Write an expression for its resonant frequency. Suggest a way to convert the Colpitts oscillator into a voltage controlled oscillator.

[5]

(b) Consider an analogue multiplier operating from a voltage supply $\pm 5V$. Assuming the multiplier's voltage transfer function is given by:

$$V_{out} = V_S \tanh\left(\frac{V_1 V_2}{4(kT/e)^2}\right)$$

Where kT/e is the thermal voltage. Calculate the gain and range of this multiplier when used as a phase detector, for square signals between $\pm V_s, V_s >> kT/e$.

[7]

(c) Draw a diagram for an FM demodulator designed around the VCO and phase detector discussed in parts (a) and (b) above. Assume the VCO has been designed for a free running frequency of 100 MHz and a gain of 10 MHz/V. What should the loop filter be so that the demodulator should have zero steady state phase error on input frequency ramps?

[8]

4.

(a) Define the Oversampling Ratio for a bandpass sampling system, when the signals occupy a small bandwidth B about a carrier frequency f_c

[2]

(b) Derive an expression for the Quantisation noise of a ΣΔ Analogue to Digital converter, as a function of oversampling ratio and order of the loop filter

[10]

(c) Design a bandpass ΣΔ modulator to be used as an FM receiver for a remote control system in the 27 MHz band. The signal bandwidth is 20 KHz. What should the minimum sampling frequency be so that the receiver has 10 bits signal to noise ratio using the minimum order band pass loop filter?

(Note that a bandpass system is obtained by the mapping $\omega' \rightarrow \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$, i.e. the usual low pass loop filter maps into a band pass filter).

[8]

5.

(a) Describe the method of spline interpolation in general and for polynomials of order N in particular. Assume a measurement has generated data pairs $\{x_i, y_i\}$. Write equations describing what happens at the measured points. Assume for the interpolation you are using a set of functions $\{y = f_n(x)\}$. Assume that the first and second derivatives of the functions at the measured points can be computed.

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(b) A set of (x,y) experimental data pairs are given, and a quadratic curve must be fitted to them by the method of least squares. Explain why it is preferable to use Orthogonal rather than Algebraic polynomials for this operation. Write the equations to determine the coefficients for least squares fitting the following model to the experimental data:

$$y_i = \sum_0^2 a_j P_j(x_i),$$

where P_j are the first 3 Legendre polynomials:

$$P_0 = 1$$
$$P_1 = x$$
$$P_2 = \frac{1}{2}\left(3x^2 - 1\right)$$

[15]

6. Figure 6.1 is a schematic diagram of an on-wafer y-matrix measurement at high frequency. The following is known about the measurement:

- Two identical coaxial cables connect the instrument to the Device Under Test (DUT). The admittance matrix of each cable is not symmetric.
- The instrument is capable of measuring the 2-port Y matrix of what is connected to its terminals.
- When no device is measured a null admittance matrix is measured.

(a) Write an expression for the input admittance of a 2-port network with an admittance Y_L connected to its output port.

[5]

(b) Show how the Y_{11} entry of the cable matrix can be computed by measurements with the probes shorted to ground.

[5]

(c) Devise a sequence of admittance measurements on known standards connected to the ends of the cables (i.e. resistors, open circuits, short circuits, through connections from one cable to the other) that will allow you to compute the other 3 entries of the admittance matrix of 1 cable. Assume $Y_{21} = Y_{12}$ for each cable. Setup these equations but do not solve them.

[5]

(d) Assume that you know the admittance matrices for the two identical cables used for the experiment. Describe, with a set of matrix equations and transformations, how you can remove the effect of cables from a device measurement. Do not carry the computation through.

[5]

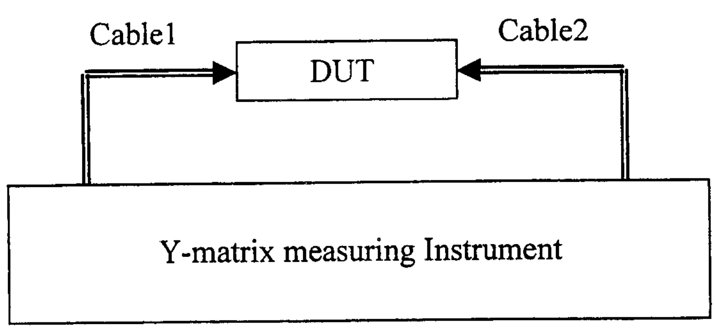

Figure 6.1: Schematic of measurement setup for Question 6.

23/4/04

1.

(a) List and briefly describe the sensitivity, resolution, threshold and dynamic range of a sensor

[8]

(b) Compute the the sensitivity, resolution, threshold and dynamic range of a pressure sensor which:

- Its output is proportional to the supply voltage
- Its supply voltage is 5V DC
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(c) Define the cross sensitivity of a sensor. Propose a way to increase the number of bits of the pressure sensor in this question by exploiting its cross sensitivity.

[4]

Answer 1a. Bookwork

Sensitivity is the ratio of electrical output to signal input (input transducer), or physical output to electrical input (output transducer). e.g., a temperature sensor may be quoted as 50 μV/K. However, the term sensitivity ay also be used in its usual electronic sense, i.e. the logarithmic derivative of the change of some property of a device with respect to a parameter, such as ambient temperature, or the value of one of the sensor's resistors. For clarity, we will refer to this as the cross-sensitivity of x on y. The sensitivity is also called the Gain of the sensor or instrument.

The term sensitivity is occasionally misused to refer to the *minimum detectable signal,* i.e. the sensor's *detectivity* or *threshold*, which , incidentally, equals the noise floor of the sensor.

Detectivity or Threshold is the minimum signal level that can be detected. I.e. the signal level which when detected by the sensor results to an electrical signal equal to the noise floor of the device.

Resolution is the smallest change of input detectable at the output. In analogue systems the resolution is usually limited by noise. In digital systems resolution is 1 LSB (least significant bit). A high resolution does not necessarily imply a high accuracy (a watch may resolve to the nearest second, while it may be a few minutes off).

Dynamic Range is the ratio between the maximum and minimum signals the transducer may handle. The maximum is usually limited by compression or distortion, while the minimum by noise.

Answer 1b. Computed example

Sensitivity=2V/atm
Detectivity = 0.1 Atm
Resolution = 0.05 Atm (0.1 V at the stated sensitivity), about 5.5 bits
Dynamic range: Is the ratio of saturation to threshold, i.e. = 50

Answer 1c.

Bookwork and Theory interpretation
A sensor's output can be proportional to unintended stimuli, such as temperature, power supply, etc. This is called cross sensitivity. The resolution of this sensor can be enhanced by modulating the power supply and using a lock-in amplifier at the output.

2.

(a) Draw the Thevenin and Norton equivalent circuits for a noisy resistor.

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[5]

Answer 2a. Bookwork, interpretation

$R_T=R_N=R$
$V_T=4kTRB$
$I_N=4ktB/R$

[5]

Answer 2b. Bookwork + interpretation
The noise factor is independent of load impedance. Its dependence on source admittance is:

$$F = F_{\min} + \frac{R_n}{G_S}\left|Y_S - Y_{OPT}\right|^2$$

F_{min} is the minimum noise factor for the amplifier, when $Y_S=Y_{opt.}$ R_N the noise resistance is the rate the noise factor increases as one deviates from R_{opt}.
$G_S=Re(Y_S)$.

[5]

Answer 2c. Bookwork
The noise figure is defined as
A voltage amplifier has $Y_{in}=0$ and $Z_{out}=0$. It is driven by a voltage source with
$G_s, Y_s \to \infty \Rightarrow F \to \infty$

Alternative proof:

$$S_{in} = \mathrm{Re}\left(vi^*\right) = \varepsilon \to 0, N_{in} = \delta \to 0 \Rightarrow$$

$$F = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} = \frac{S_{in}/N_{in}}{GS_{in}/\left(GS_{in}+N_A\right)} = \frac{\varepsilon}{\delta}\frac{N_A}{G\varepsilon} \to O\left(\frac{1}{\delta}\right) \to \infty$$

Answer 2d. Bookwork

An ideal transformer exhibits no losses (i.e. no added noise) and unity power gain. Then,

$$F = \frac{S_{in} / N_{in}}{S_{out} / N_{out}} = \frac{S_{in} / N_{in}}{GS_{in} / \left(GS_{in}\right)} = 1$$

[5]

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(b) Consider an analogue multiplier operating from a voltage supply $\pm 5V$. Assuming the multiplier's voltage transfer function is given by:

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Where kT/e is the thermal voltage. Calculate the gain and range of this multiplier when used as a phase detector, for square signals between $\pm V_s, V_s >> kT/e$.

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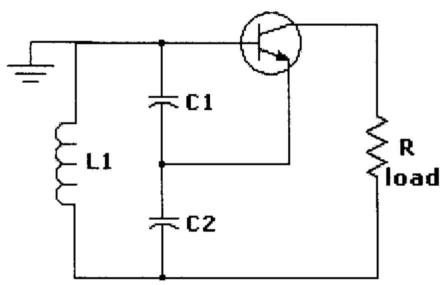
(c) Draw a diagram for an FM demodulator designed around the VCO and phase detector discussed in parts (a) and (b) above. Assume the VCO has been designed for a free running frequency of 100 MHz and a gain of 10 MHz/V. What should the loop filter be so that the demodulator should have zero steady state phase error on input frequency ramps?

[8]

Answer 3a. Bookwork+ interpretation



The current transfer ratio of the resonant circuit viewed as a current divider between C2 and L is:

$$\frac{i_{out}}{i_{in}} = \frac{j\omega C_2}{j\omega C_2 + \frac{1}{j\omega L}} = \frac{-\omega^2 LC_2}{1-\omega^2 LC_2}$$

At resonance , i.e. at a frequency of $\sqrt{\frac{C_1+C_2}{LC_1C_2}}$ the current gain of the feedback network is :

$$\frac{i_{out}}{i_{in}} = \frac{-\omega^2 LC_2}{1-\omega^2 LC_2} = \frac{C_1+C_2}{C_2} = 1+\frac{C_1}{C_2} > 1$$

suggesting that there is more than unity real positive feedback and the Barkhausen criterion is satisfied.

The Colpitts oscillator can be turned into a VCO by using a varactor, voltage controlled capacitor.

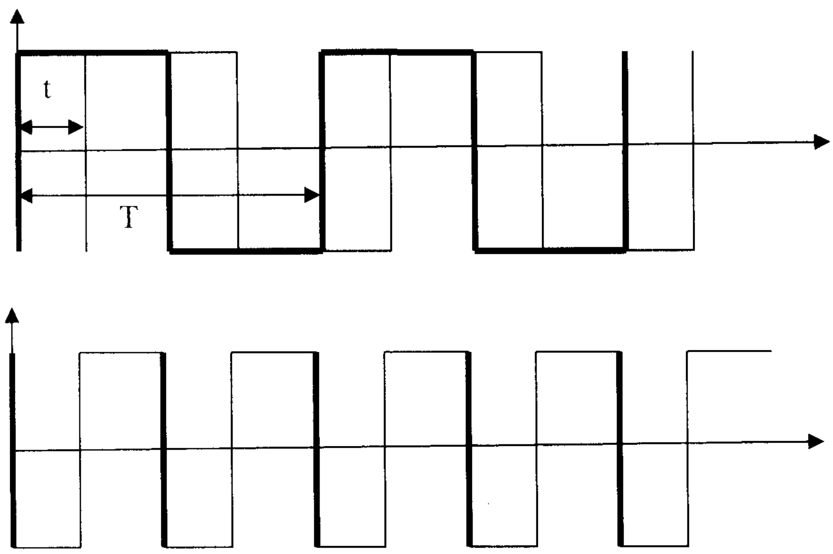
[5]

Answer 3b. Computed example

A multiplier with a response given by:

$$V_{out} = V_S \tanh\left(\frac{V_1 V_2}{4(kT)^2}\right)$$

As a phase detector for bipolar square signals , we note that $V_S >> kT$



From the diagram, the detector is a XOR detector between the 2 logic levels. The average of the output:
At $\varphi = 0, V_{out} = V_s$, at $\varphi = \pi/2, V_{out} = 0$, and at $\varphi = \pi, V_{out} = -V_S$.
The response is therefore monotonic between $0 \le \varphi \le \pi$ and the gain is:

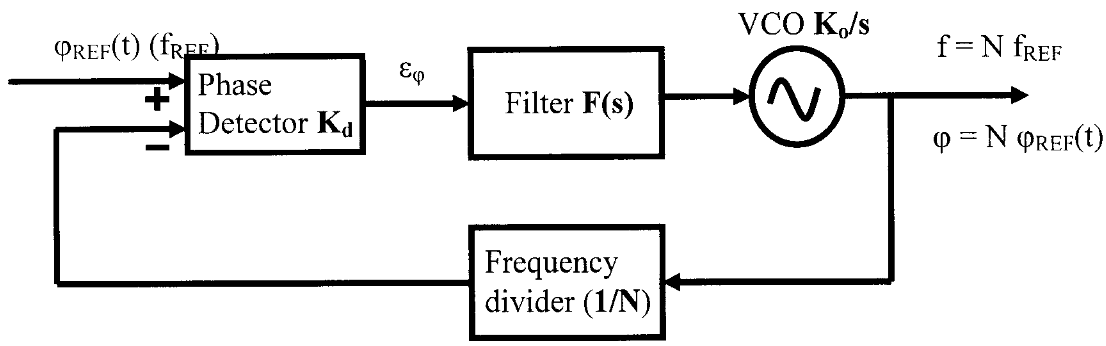
$$\frac{dV_{out}}{d\varphi} = \frac{-2V_S}{\pi}$$

The detector can also show the negative of this response between $\pi \le \varphi \le 2\pi$

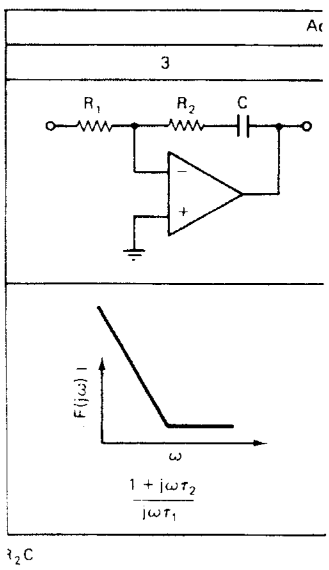
[7]

Answer 3a. Computed example

A simple PLL loop will do. The gain info for the VCO is superfluous.



For zero phase error one needs a pole at zero frequency in the loop filter, so an active filter must be used. A leadlag filter is best because the input is frequency agile, i.e. a large phase margin is desirable. The most suitable filter would be:



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(a) Define the Oversampling Ratio for a bandpass sampling system, when the signals occupy a small bandwidth B about a carrier frequency f_c

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(c) Design a bandpass ΣΔ modulator to be used as an FM receiver for a remote control system in the 27 MHz band. The signal bandwidth is 20 KHz. What should the minimum sampling frequency be so that the receiver has 10 bits signal to noise ratio using the minimum order band pass loop filter? (Note that a bandpass system is obtained by the mapping $\omega' \to \omega + \frac{1}{\omega}$, i.e. the usual low pass loop filter maps into a band pass filter).

[8]

Answer 4.a Bookwork

The ratio of the sampling rate to the Nyquist rate. Since for narrowband bandpass signals f_N->2B, the OSR is f_s/2B

[2]

Answer 4.b Bookwork and extension of theory

The total quantisation noise energy is :

$$\bar{E}^2(e_q) = \int_{-\infty}^{\infty} e_q^{\ 2} p(e_q)\ de_q \ = \int_{-q/2}^{q/2} \frac{e_q^{\ 2}}{q}\ de_q = \frac{q^2}{12}$$

over a bandwidth of 2 f_N, with q the quantisation step, $2A = 2^N q$ (A is half the peak to peak amplitude of the signal). The SNQR is , for a signal sampled at the Nyquist rate:

$$SQNR = 10\log\left(6A^2/q^2\right) = 10\log\left(3\bullet 2^{2N-1}\right) = \left(1.76 + 6.02N\right)\text{dB}$$

For a ΣΔ converter, the (amplitude) signal transfer function is: $G_s(s) = \dfrac{H(s)}{1+H(s)}$

and the noise transfer function is: $G_E(s) = \dfrac{1}{1+H(s)}$

so for a filter of order k,

$$SNQR = SNQR_{Nyquist} \left.\frac{G_s^2}{G_N^2}\right|_f \left.\frac{f_N}{f}\frac{G_N^2}{G_S^2}\right|_{f_N} = SNQR_N \frac{H(s)}{H(s_N)} \simeq SNQR_N \frac{s_{2B}^{2k+1}}{s^{2k+1}} = SNQR_N \bullet OSR^{2k+1}$$

[10]

Answer 4.c Extension of theory

The answer to part b suggests 2k+1 bits extra SNQR for each bit of oversampling. If we want 10 bits, then we need a 1-bit loop, 3 bits oversampling, i.e. f_s>=160kHz and a 1st order filter, or f_s>=80kHz for a second order filter. Note for a bandpass system we are considering a two pole, 1 zero filter as a first order filter. **[8]**

5.

(a) Describe the method of spline interpolation in general and for polynomials of order N in particular. Assume a measurement has generated data pairs $\{x_i, y_i\}$. Write equations describing what happens at the measured points. Assume for the interpolation you are using a set of functions $\{y = f_n(x)\}$. Assume that the first and second derivatives of the functions at the measured points can be computed.

[5]

(b) A set of (x,y) experimental data pairs are given, and a quadratic curve must be fitted to them by the method of least squares. Explain why it is preferable to use Orthogonal rather than algebraic polynomials for this operation. Write the equations to determine the coefficients for least squares fitting the following model to the experimental data:

$$y_i = \sum_0^2 a_j P_j(x_i),$$

where P_j are the first 3 Legendre polynomials:

$$P_0 = 1$$

$$P_1 = x$$

$$P_2 = \frac{1}{2}(3x^2 - 1)$$

[15]

Answer 5.a Bookwork+ interpretation

In spline interpolation curves are passed through pairs of adjacent points. The curves are adjusted to agree by value and the first M derivatives at the experimental points. If polynomials of order N are used, they are required to agree by value and the N-1 derivatives at the points.

[5]

Answer 5b. Computed example

The error in the measurements will introduce large errors in the coefficients if algebraic polynomials are used, because powers of x are not orthogonal in the interval of the measurement.

First the x variable must be scaled to the interval of domain of the orthogonal polynomials:

$$(x_{\min}, x_{\max}) \to (-1,1) \Rightarrow x' = -1 + 2\frac{x - x_{\min}}{x_{\max} - x_{\min}}$$

the equations are derived from minimising

$$\sum_i \left(y_i - \sum_0^2 a_j P_j(x_i) \right)^2$$

i.e. requiring:

$$\frac{\partial}{\partial a_j} \sum_i \left(y_i - \sum_{j=0}^{2} a_j P_j(x_i) \right)^2 = 0 \Rightarrow$$

$$\sum_i \left(y_i P_j(x_i) \right) = \sum_k a_k \sum_i \left(P_k(x_i) P_j(x_i) \right)$$

this is a linear system: (Note that: $\sum P_0 = N, \sum P_1 = \sum x', \sum P_2 = 3/2 \sum x'^2 - 1/2N$)

$$\begin{bmatrix} \left(\sum P_2\right)^2 & \sum P_2 \sum P_1 & N\sum P_2 \\ \sum P_2 \sum P_1 & \left(\sum P_1\right)^2 & N\sum P_1 \\ N\sum P_2 & N\sum P_1 & N^2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} \sum y P_2 \\ \sum yx \\ \sum y \end{bmatrix}$$

[15]

6. Figure 6.1 is a schematic diagram of an on-wafer y-matrix measurement at high frequency. The following is known about the measurement:

- Two identical coaxial cables connect the instrument to the Device Under Test (DUT). The admittance matrix of each cable is not symmetric.
- The instrument is capable of measuring the 2-port Y matrix of what is connected to its terminals.
- When no device is measured a null admittance matrix is measured.

(a) Write an expression for the input admittance of a 2-port network with an admittance Y_L connected to its output port.

[5]

(b) Show how the Y_{11} entry of the cable matrix can be computed by measurements on the probes shorted to ground.

[5]

(c) Devise a sequence of admittance measurements on known standards connected to the ends of the cables (i.e. resistors, open circuits, short circuits, through connections from one cable to the other) that will allow you to compute the other 3 entries of the admittance matrix of 1 cable. Assume $Y_{21} = Y_{12}$ for each cable.

[5]

(d) Assume that you know the admittance matrices for the two identical cables used for the experiment. Describe with a set of matrix equations and transformations how you can remove the effect of cables from a device measurement. Do not carry the computation through.

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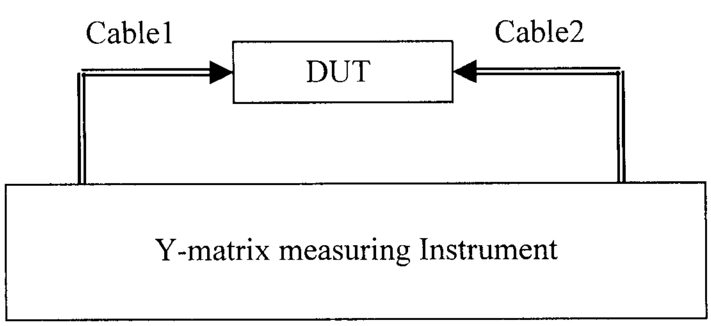

Figure 6.1: Schematic of measurement setup for Question 6.

Answer 6.a: Application of theory

A 2 port network loaded with Y_L has an input admittance:

$$\left.\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= -V_2Y_L = Y_{21}V_1 + Y_{22}V_2 \Rightarrow V_2 = -\frac{Y_{21}}{Y_L + Y_{22}}V_1 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow Y_{in} = \frac{I_1}{V_1} = \frac{Y_{11}Y_L + Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_L + Y_{22}}$$

Answer 6.b Bookwork

By definition of the Y matix, Y_{11} is the input admittance with the output shorted to ground.

[5]

Answer 6.c Computed example

Once Y_{11} is known we need 3 measurements to compute the 3 remaining unknowns. However, Y_{21} and Y_{12} are equal, and appear in a product. Therefore only 2 further measurements are required:

Open circuit: Letting $Y_L = 0 \Rightarrow Y_{in} = Y_{11} - \frac{Y_{21}Y_{12}}{Y_{22}} \Rightarrow \frac{Y_{21}Y_{12}}{Y_{22}} = Y_{11} - Y_{in,open}$

One more equation is for any known load solves the problem.

[5]

Answer 6.d Computed example

The sequence cable1, DYT, cable2 are in cascade. So the transmission matrix of the 3 together is the product of the transmission matrices. However, Cable 2 is invereted, so the ports of cable 2 must be swapped.

[5]