1.

a) ii) A positive mobile charge in a semiconductor [2]

b) because the electron mass is smaller than the hole mass [2]

c) from the formulae sheet:. [4]

$n = N_C exp\left(-\frac{E_C-E_F}{kT}\right)$

$p = N_V exp\left(\frac{E_V-E_F}{kT}\right)$

$n \times p = N_C \times N_V exp\left(-\frac{E_C-E_V}{kT}\right)$ (1)

Definition of intrinsic level E_i

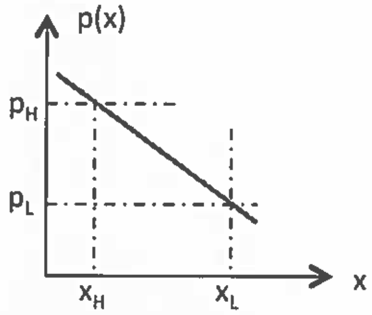
$n_i = N_C exp\left(-\frac{E_C-E_i}{kT}\right)$

$p_i = N_V exp\left(\frac{E_V-E_i}{kT}\right)$

$n_i \times p_i = n_i^2 = N_C \times N_V exp\left(-\frac{E_C-E_V}{kT}\right)$ (2)

(1) = (2)

d) Assume the variation of the hole concentration is given by the following figure: [4]



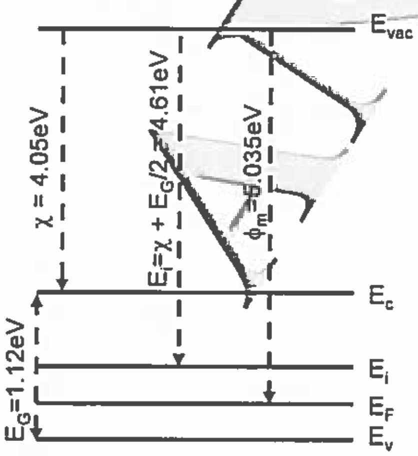
hole flux is from high to low concentration, thus in the +x direction
hole current is proportional to hole flux multiplied by charge, hole charge is positive, thus current is also in the +x direction
$\frac{dp}{dx} = \frac{p_H-p_L}{x_H-x_L} < 0$ (negative gradient) thus $\frac{dp}{dx}$ needs to be multiplied by -1 to get the correct direction.

e)

i) Holes (free carriers in valence band are holes) [2]

ii) b) (potential energy increases when going deeper into the band) [2]

f) Make a sketch of the energy band diagram and fill in all parameters. [6]



this sketch shows that it is a p-type semiconductor.

Different ways to calculate hole concentration:

$p = n_i \times exp\left(\frac{E_i-E_F}{kT}\right)$ (derive from energy band diagram or know by heart)

$p = 1.45 \times 10^{10} \times exp\left(\frac{\phi_m-E_i}{kT}\right) = 1.45 \times 10^{10} \times exp\left(\frac{5.035-4.61}{0.026}\right) \approx 1.8 \times 10^{17}\ cm^{-3}$

or

$p = N_V \times exp\left(\frac{E_V-E_F}{kT}\right)$ (from equation sheet)

$$p = 3.2 \times 10^{19} \times exp\left(\frac{\phi_m-(\chi+E_G)}{kT}\right) = 1.45 \times 10^{10} \times exp\left(\frac{5.035-4.05-1.12}{0.026}\right) \approx 1.8 \times 10^{17}\ cm^{-3}$$

g) Since p+n junction we assume that hole current is larger than electron current [6]

$I_{tot} \approx I_p$ [3]

using amplitudes: $|J_p| = eD_p \frac{dp}{dx} = eD_p \frac{p'_n - p_{n_0}}{L_p}$

excess minority carriers in n-region at depletion region edge are holes:

$p'_n - p_{n_0} = \frac{L_p|J_p|}{eD_p} = \frac{5\ 10^{-4} \times 6.11\ 10^{-4}}{1.6\ 10^{-19} \times 2} = 76375\ cm^{-3}$ [3]

h)

i) Two doping types of more or less the same order of magnitude -> compensation doping. [8]
Derive formulae from equations that need to be know by heart (charge neutrality and law of mass action)

$-N_A + N_D - n + p = 0$ (1)

$n \times p = n_i^2$ (2) [4]

from (2) $p = \frac{n_i^2}{n}$ insert in (1)

$-N_A + N_D - n + \frac{n_i^2}{n} = 0$ rewrite to get quadratic equation in n:

$n^2 - (N_D - N_A)n - n_i^2 = 0$ solve quadratic equation and ensure positive result [4]

$$n = \frac{(N_D-N_A)+\sqrt{(N_D-N_A)^2+4n_i^2}}{2} = \frac{(10^{18}-10^{17})+\sqrt{(10^{18}-10^{17})^2+4\times(1.45\times10^{10})^2}}{2} = 9\ 10^{17}\ cm^{-3}$$

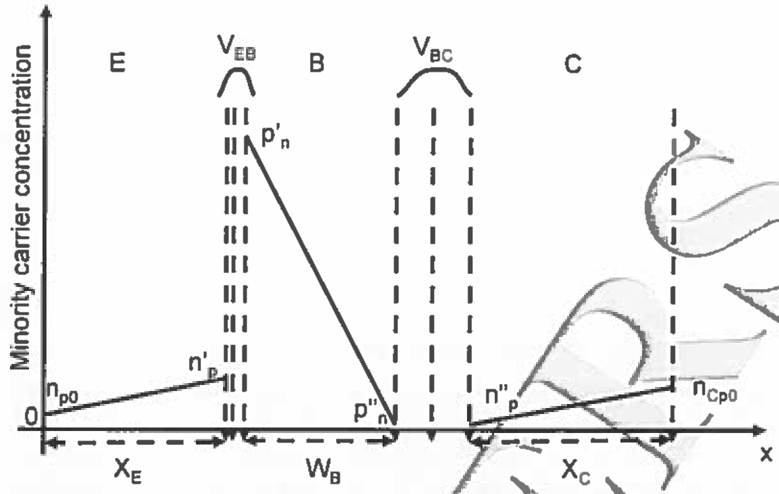
$$p = \frac{n_i^2}{n} = \frac{(1.45\times10^{10})^2}{9\times10^{17}} \approx 234\ cm^{-3}$$

j) $V_{th} < 0$ [2]

k) the base region [2]

2.

a) Based on the equivalent circuit we derive that this is a pnp BJT. In region II the BJT is in forward active mode, thus EB junction forward biased and BC junction reverse biased. -> $V_{EB}>0$ and $V_{CB}<0$ in the plot below. [6]



The material widths defined in the plot above are junction widths minus the depletion regions. Thus these are the effective width of the regions.

b) $I_E = A\frac{eD_n^E(n'_p - n_{p0})}{X_e} + A\frac{eD_p^B(p'_n - p''_n)}{W_B}$. [6]

since $n'_p >> n_{p0}$ and $p'_n >> p''_n$ this equation simplifies to:

$$I_E \approx A\frac{eD_n^E n'_p}{X_e} + A\frac{eD_p^B p'_n}{W_B}$$

$$I_E \approx A\frac{eD_n^E n_{p0} exp\left({}^{V_{EB}}/_{V_T}\right)}{X_e} + A\frac{eD_p^B p_{n0} exp\left({}^{V_{EB}}/_{V_T}\right)}{W_B}$$

with n_{p0} minority carrier concentration in emitter and p_{n0} minority carrier concentration in base.

This equation neglects leakage currents from collector into base. Then:

$$I_C \approx A\frac{eD_p^B p_{n0} exp\left({}^{V_{EB}}/_{V_T}\right)}{W_B}$$

$$I_B \approx A\frac{eD_n^E n_{p0} exp\left({}^{V_{EB}}/_{V_T}\right)}{X_e}$$

c) Calculate the value for I_E and I_B for $I_C = 1$ mA, ignoring base width modulation. [3]

Current gain β can be extracted from fig. 2.1a region II.

$$I_C = \beta I_B$$

$$\beta = \frac{I_C}{I_B} = \frac{10^{-2}}{10^{-5}} = 1000$$

for $I_C = 1$ mA $= 10^{-3}$ A I_B is then 10^{-6} A or 1 μA.

$I_E = I_C + I_B = 1$ mA + 1 μA ≈ 1 mA (0.001001 A)

d) . [6]

Input resistance: $R_\pi = \left(\frac{dI_B}{dV_{EB}}\right)^{-1}$

$$\frac{dI_B}{dV_{EB}} = \frac{Ad\left(\frac{eD_n^E n_{p0} exp\left({}^{V_{EB}}/_{V_T}\right)}{X_e}\right)}{dV_{EB}} = A\frac{eD_n^E n_{p0} exp\left({}^{V_{EB}}/_{V_T}\right)}{V_T X_e} = \frac{I_B}{V_T}$$

$$R_\pi = \left(\frac{dI_B}{dV_{EB}}\right)^{-1} = \frac{V_T}{I_B}$$

Output resistance: $R_0 = \frac{\partial V_{ce}}{\partial I_C} = \frac{|V_A|}{I_C}$ assuming that $V_{CE} \approx V_{BC}$ with V_A the early voltage that is a parameter describing base with modulation. For Fig. 2.1a $R_o = 0\ \Omega$.

$$i_{BJT} = I_C = A\frac{eD_p^B p_{no} exp\left({}^{V_{EB}}/_{V_T}\right)}{W_B}$$

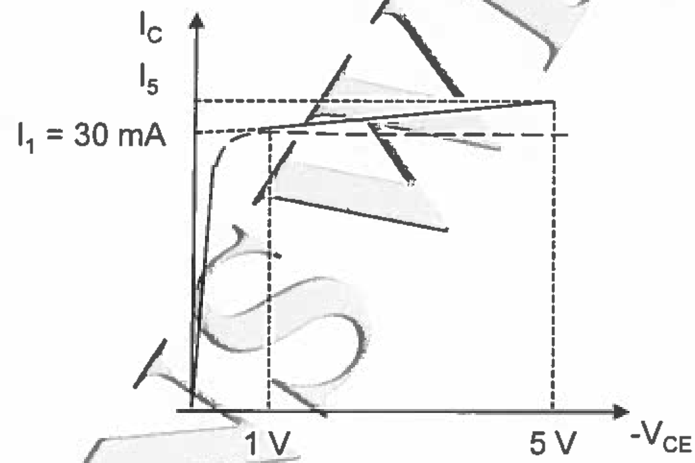
e) The datasheet for the pnp BJT in fig. 2.1 gives an input resistance R_π = 8 kΩ and the output conductance is g_o= 40 µS. Calculate the value of the transconductance g_m. [3]
Derive the equation that relates R_π to g_m

$$R_\pi = \frac{dV_{EB}}{dI_B} = \frac{dV_{EB}}{dI_C} \times \frac{dI_C}{dI_B} = \frac{1}{g_m} \times \beta$$

$$g_m = \frac{\beta}{R_\pi} = \frac{10^3}{8\ 10^3} = 0.125\ S$$

f) Using the result of e) re-plot the output characteristic for I_B = -20 µA taking base width modulation into account and give the values of the collector current I_C for V_{CE} = -5 V. [6]

Sketching the output characteristic with base width modulation gives a slope in the current in region II. This slope is represented in the output conductance given in e)



Doing some simple algebra:
Take $R_o = g_o^{-1}$

$$R_o = \frac{dV}{dI} = \frac{\Delta V}{\Delta I} = \frac{\Delta V}{I_5 - I_1}$$

$$I_5 - I_1 = \frac{\Delta V}{R_o} = g_o \Delta V$$

$$I_5 = g_o \Delta V + I_1$$

$$I_5 = 4\ 10^{-3} \times 4 + 0.03 = 0.046\ \text{A}$$

3. p-channel depletion mode MOSFET

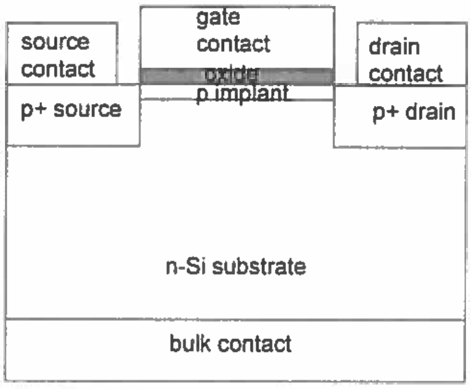
a) . [6]

A p-channel depletion mode MOSFET is a MOSFET with p-type source and drain region, n-type bulk and with holes in the channel region at $V_{GS} = 0V$. A positive voltage will need to be applied to deplete the channel and switch the MOSFET off.

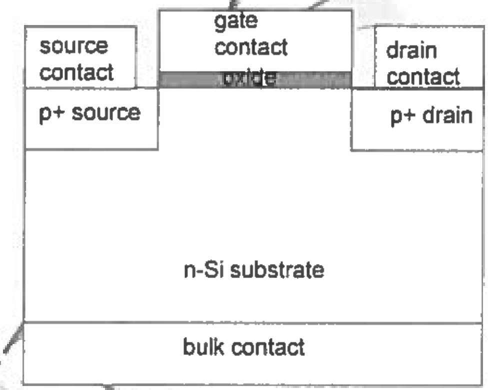
Method 1: Implanting the top surface of the Si substrate with acceptor doping atoms, N_A such that the majority carrier concentration in that region are holes, thus taking into account compensation doping: $N_A > N_D$ (ND is doping in substrate).

Method 2: choosing a suitable gate contact such that the workfunction difference between the n-type substrate and the gate material creates an inversion layer of holes: $\phi_{Si} < \phi_{gate}$

b) [6]



method 1



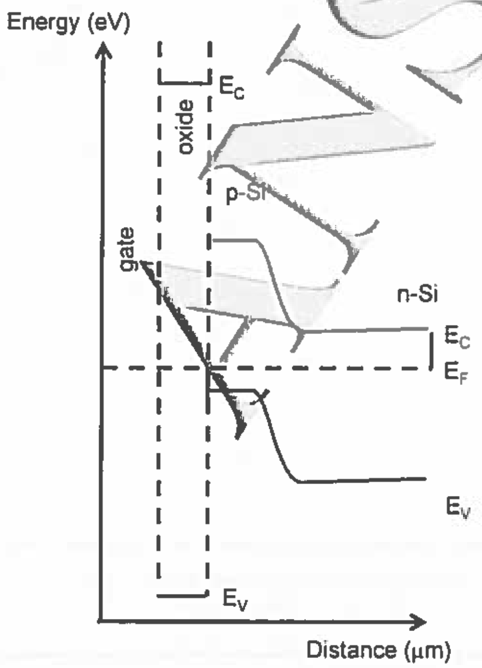
method 2

c) Method 1

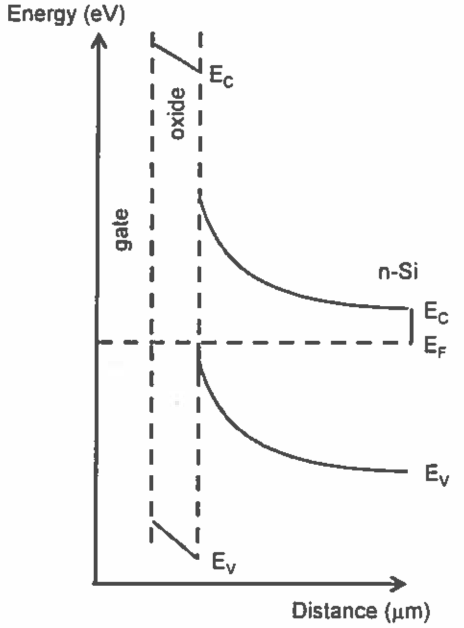
Assumptions:

1. no workfunction difference between the gate material and the Si substrate underneath therefore the oxide energy bands are flat
2. graded junction between the implanted p and n substrate region

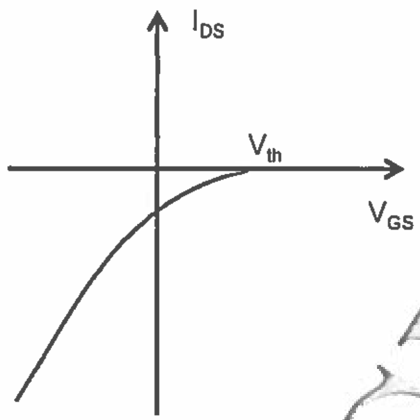
Note that the bands bend further than required for reaching threshold in order to have a true depletion mode device.



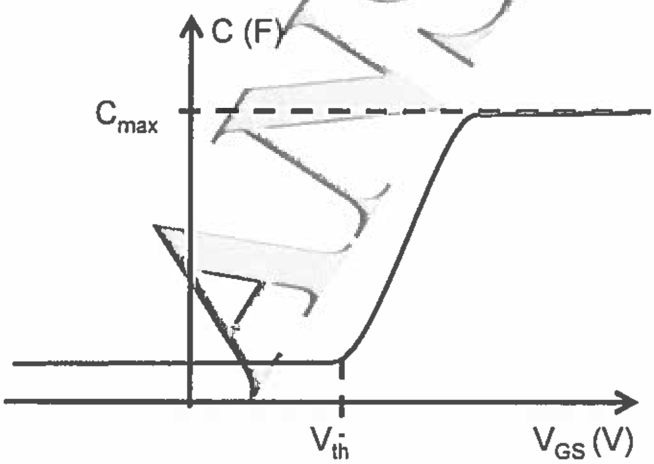
Method 2:
No assumptions (unless we want to assume that a suitable metal for the gate exist)
Note that the bands bend further than required for reaching threshold in order to have a true depletion mode device.



d) For both methods the IV is equivalent and must be quadratic in saturation [2] and Vth must be positive [2] and the currents negative for the p-MOS [2]. Currents are positive for the nMOS (convention). [6]



e) The CV characteristic of method 1 and 2 are similar to good approximation. [6]



$C_{max} = C_{ox} A$
$C_{ox} = \varepsilon_o \varepsilon_{ox}/t_{ox}$