E3.12
SO13

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

MSc and EEE PART III/IV: MEng, BEng.and ACGI

OPTOELECTRONICS

Monday, 17 May 10:00 am

Time allowed: 3:00 hours

Corrected Copy

There are SIX questions on this paper.

Answer FOUR questions.

(4)

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	R.R.A. Syms
	Second Marker(s) :	W.T. Pike

Fundamental constants

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ m kg/C^2

$c = 3 \times 10^8$ m/s

$h = 6.62 \times 10^{-34}$ Js

1a) The time-dependent vector form of Maxwell's equations is:

$$\text{div}\,\underline{\mathbf{D}} = \rho$$
$$\text{div}\,\underline{\mathbf{B}} = 0$$
$$\text{curl}\,\underline{\mathbf{E}} = -\partial\underline{\mathbf{B}}/\partial t$$
$$\text{curl}\,\underline{\mathbf{H}} = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$$

What are $\underline{\mathbf{D}}$, $\underline{\mathbf{B}}$, $\underline{\mathbf{E}}$, $\underline{\mathbf{H}}$ and $\underline{\mathbf{J}}$? What additional relations are required to introduce the properties of any materials involved? Indicating your assumptions, derive a wave equation for transverse electromagnetic waves in a uniform dielectric medium. What is the phase velocity? How is the refractive index defined?

[10]

b) A number of candidate solutions to the time-independent wave equation are listed below; Here, you may assume that $\underline{\mathbf{E}} = \underline{\mathrm{E}}\exp(j\omega t)$, and so on. In each case, state whether the solution is a viable one or not. For the viable solutions, describe the main features of the wave.

legitimate

i) $\underline{\mathrm{E}} = \mathrm{E}_y \exp(-jk_0 z)\,\underline{\mathrm{j}}$
ii) $\underline{\mathrm{E}} = \mathrm{E}_z \exp(-jk_0 z)\,\underline{\mathrm{k}}$
iii) $\underline{\mathrm{H}} = \mathrm{H}_y \exp(+jk_0 z)\,\underline{\mathrm{j}}$
iv) $\underline{\mathrm{E}} = \mathrm{E}_y \exp\{-jk_0[x + z\sqrt{3}]\}\,\underline{\mathrm{j}}$
v) $\underline{\mathrm{E}} = \mathrm{E}_y \exp(-j\beta x)\exp(-\gamma z)\,\underline{\mathrm{j}}$

[10]

2a) Figure 1 shows a plane wave incident at an angle θ_1 on the boundary between two dielectric media of refractive indices n_1 and n_2. Write down the laws relating the angles of the incident, reflected and refracted waves. Derive an expression for the critical angle θ_c, and calculate θ_c for the case when $n_1 = 1.51$ and $n_2 = 1.5$.

[6]

b) Write down expressions for the electric fields in the two media, assuming firstly that the wave in medium 2 is plane. How do these expressions modify after total internal reflection has occurred? Sketch the transverse field distribution for this case.

[6]

c) The amplitude reflection coefficient Γ_E for TE incidence is given by:

$$\Gamma_E = \{n_1\cos(\theta_1) - n_2\cos(\theta_2)\} \,/\, \{n_1\cos(\theta_1) + n_2\cos(\theta_2)\}$$

Show that the power reflection coefficient is unity when $\theta_1 \geq \theta_c$. Sketch the variation of the power reflection coefficient with incidence angle, when the two media have indices 1.5 and 1, for incidence from i) the high index side and ii) the low index side.

[8]

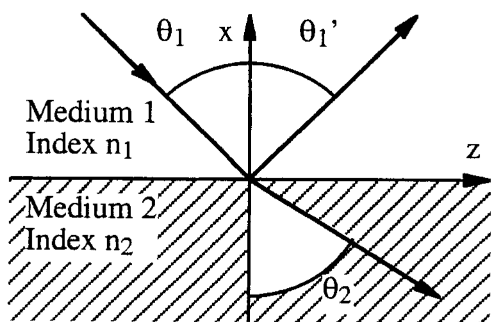


Figure 1.

3a) A particular graded index optical fibre has the refractive index variation:

$$n^2 = n_0^2\{1 - (r/r_0)^2\}, \text{ where } r^2 = x^2 + y^2 \text{ and } r_0 \text{ is a constant}$$

The fibre supports a set of guided modes, whose transverse field variations are:

$$E_{\mu,\nu}(x, y) = H_\mu(\sqrt{2}x/a)\, H_\nu(\sqrt{2}y/a)\, \exp\{-(x^2 + y^2)/a^2\}$$

Here, $H_\mu(\zeta)$ is the Hermite polynomial of order μ, defined as satisfying the differential equation:

$$d^2H_\mu/d\zeta^2 - 2\zeta\, dH_\mu/d\zeta + 2\mu\, H_\mu = 0$$

The first three Hermite polynomials are given in Table I. Sketch the variation of the transverse fields $E_{0,0}(x, y)$ and $E_{2,0}(x, y)$ along the line $y = 0$. Sketch contour maps of the two-dimensional fields $E_{0,1}(x, y)$ and $E_{1,1}(x, y)$.

[8]

b) Write down a scalar waveguide equation governing the propagation of modal fields along the fibre. Show that the transverse fields $E_{\mu,\nu}(x, y)$ are indeed solutions to your equation, and find the value of the constant a and the propagation constant of the guided mode.

[12]

Table I.

μ	0	1	2
$H_\mu(\zeta)$	1	2ζ	$-2 + 4\zeta^2$

4a) Describe the formation of channel guide devices in i) Ti : $LiNbO_3$ and ii) silica-on-silicon. In each case, sketch the waveguide cross-section.

[6]

b) Figure 2 below shows an integrated optic directional coupler. Describe its operation. What additional features would be required to allow the structure to be used as a switch in the Ti : $LiNbO_3$ materials system?

[6]

c) The coupled mode differential equations governing operation in a synchronous directional coupler are:

$$dA_1/dz + j\kappa A_2 = 0$$
$$dA_2/dz + j\kappa A_1 = 0$$

Here, A_1 and A_2 are the amplitudes of the modes in guides 1 and 2, and κ is the coupling coefficient. Calculate and sketch the variations of the powers in the two guides. Show that power is conserved. When is 100% power transfer obtained?

[8]

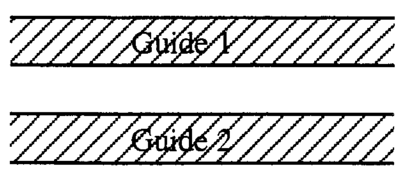


Figure 2.

5. The sandwich student responsible for pre-calibration of components in an opto-electronics company has returned to his university for the start of term, leaving an incomplete set of test data.

a) For the two types of stripe guide Fabry-Perot laser that the company is most interested in, the data shown in Table II are extracted from the student's logbook. Sketch the two light-current characteristics. Which laser has the lower threshold? Which has the higher quantum efficiency when lasing?

[10]

Table II.

Laser type: LD34Z		Laser type: LD72L	
Wavelength: 1.3 μm		Wavelength: 1.5 μm	
Current (mA)	Power (mW)	Current (mA)	Power (mW)
20	0.09	20	0.12
40	0.19	30	0.19
60	5.01	50	4.97
70	8.36	60	8.28
80	11.70	70	11.59

b) The student has apparently used an LD72L laser in a point-to-point communications link, as shown in Figure 3. The detector amplifier uses a 1 kΩ feedback resistor. With no fibre in the link, and with the laser emission coupled directly to the detector, the voltage developed at the output of the amplifier is 3.6 V when the laser drive current is 50 mA. With 50 km of fibre inserted in the link, the output is 114 mV, and with 150 km of fibre it is 1.14 mV. Estimate the quantum efficiency and responsivity of the detector, the coupling loss into the fibre, and the propagation loss.

[10]

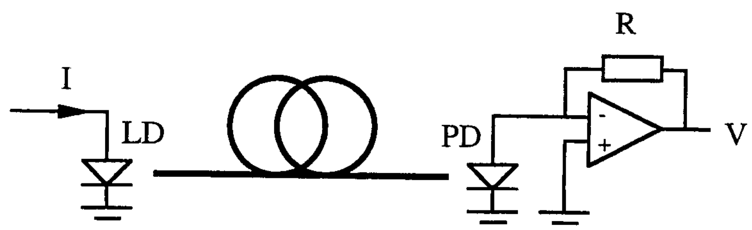


Figure 3.

6a) Explain the processes of optical absorption, spontaneous emission and stimulated emission in a semiconductor. Which processes are dominant in i) light emitting diodes and ii) semiconductor lasers? What differences are there in the properties of the photons emitted by each device?

[8]

b) The rate equations for a light emitting diode (LED) may be taken in the form:

$$dn/dt = I/ev - n/\tau_e$$
$$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$$

Identify the terms and physical processes involved in the equations. What factors determine τ_p in a surface emitting LED?

[4]

c) Estimate the emission wavelength, the optical power generated per milliamp and the DC internal efficiency of an LED, assuming that it is formed from a GaAs (which has an energy gap of 1.42 eV) and that $\tau_e = 1$ nsec and $\tau_{rr} = 2$ nsec. What other factors influence the amount of useful light that the LED actually emits?

[8]

16/2/04 E3.12
SO13

Optoelectronics 2004 - Solutions

1a) The field quantities are:

D – electric flux density
B – magnetic flux density
E – electric field strength
H – magnetic field strength
J – current density

[2]

The additional relations required to introduce material properties are:

$\mathbf{B} = \mu\mathbf{H}$ where μ is the permeability
$\mathbf{D} = \varepsilon\mathbf{E}$ where ε is the permittivity
$\mathbf{J} = \sigma\mathbf{E}$ where σ is the conductivity

[2]

Derivation of a wave equation for dielectric media:

Start with Maxwell's equations:
$\text{div}\,\mathbf{D} = \rho$
$\text{div}\,\mathbf{B} = 0$
$\text{curl}\,\mathbf{E} = -\partial\mathbf{B}/\partial t$
$\text{curl}\,\mathbf{H} = \mathbf{J} + \partial\mathbf{D}/\partial t$

Assume that $\mu = \mu_0$, where μ_0 is the permeability of free space
Assume that there are no currents flowing, and no charges present, so **J** and ρ are both zero
Assume that ε is both uniform and constant

Substitute $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{B} = \mu_0\mathbf{H}$ to get:
$\text{div}\,\mathbf{E} = 0$
$\text{div}\,\mathbf{D} = 0$
$\text{curl}\,\mathbf{E} = -\mu_0\,\partial\mathbf{H}/\partial t$
$\text{curl}\,\mathbf{H} = +\varepsilon\,\partial\mathbf{E}/\partial t$

Take the curl of the 3rd equation above, to get:
$\text{curl}\,[\text{curl}\,\mathbf{E}] = -\mu_0\,\partial/\partial t\,[\text{curl}\,\mathbf{H}]$

Subsitute using the 4th equation above, to get:
$\text{curl}\,[\text{curl}\,\mathbf{E}] = -\mu_0\varepsilon\,\partial^2\mathbf{E}/\partial t^2$

Simplify the result using the standard vector identity:
$\text{curl}\,[\text{curl}\,\mathbf{F}] = \text{grad}\,[\text{div}\,\mathbf{F}] - \nabla^2\mathbf{F}$
$\text{grad}\,[\text{div}\,\mathbf{E}] - \nabla^2\mathbf{E} = -\mu_0\varepsilon\,\partial^2\mathbf{E}/\partial t^2$

Since $\text{div}\,\mathbf{E} = 0$, we finally obtain the time-dependent vector wave equation:
$\nabla^2\mathbf{E} = \mu_0\varepsilon\,\partial^2\mathbf{E}/\partial t^2$

Assuming that the electric field varies harmonically as $\mathbf{E}(x, y, z, t) = E(x, y, z)\exp(j\omega t)$, we obtain the time-independent wave equation:
$\nabla^2\underline{E} = -\omega^2\mu_0\varepsilon\,\underline{E}$

[4]

R.R.A. Syms
[illegible]

Assuming that $\underline{E} = E_y\, \underline{j}$ (for a y-polarized wave, for example), plane wave solutions travelling in the z-direction (again, for example) may be assumed in the form $E_y = E_0 \exp(-jkz)$.

This solution satisfies the wave equation, provided $k^2 = \omega^2\mu_0\varepsilon$, i.e. if $k = \omega\sqrt{\mu_0\varepsilon}$

The phase velocity is then $v_{ph} = \omega/k = 1/\sqrt{\mu_0\varepsilon}$. Defining the permittivity as $\varepsilon = \varepsilon_0\varepsilon_r$, the phase velocity may be written as $v_{ph} = c/n$, where $c = 1/\sqrt{\mu_0\varepsilon_0}$ is the velocity of light in vacuo and $n = \sqrt{\varepsilon_r}$ is the refractive index.

[2]

b) For the wave solutions given:

i) $\underline{E} = E_y \exp(-jk_0z)\, \underline{j}$

This solution represents a plane transverse electromagnetic wave, travelling in vacuum in the +z-direction, and with the electric field polarized in the y-direction. The term $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ is the propagation constant of free space.

[1]

ii) $\underline{E} = E_z \exp(-jk_0z)\, \underline{k}$

This solution does not represent a valid EM wave, as the wave equation does not allow plane waves with a longitudinal field component.

[1]

iii) $\underline{H} = H_y \exp(+jk_0z)\, \underline{j}$

This solution represents a valid TEM wave, travelling in vacuum in the -z-direction, and with the magnetic field polarized in the y-direction and the electric field polarized in the x-direction.

[2]

iv) $\underline{E} = E_y \exp\{-jk_0[x + z\sqrt{3}]\}\, \underline{j}$

This solution represents a valid TEM wave, travelling in a medium of refractive index $n = \sqrt{\{1^2 + (\sqrt{3})^2\}} = 2$, at an angle $\theta = \cos^{-1}(\sqrt{3}/2) = 30°$ to the z-axis, and with the electric field polarized in the y-direction.

[3]

v) $\underline{E} = E_y \exp(-\gamma x) \exp(-j\beta z)\, \underline{j}$

This solution represents an inhomogenous, evanescent or boundary wave, which has an amplitude that decays exponentially in the x-direction, i.e. in a direction perpendicular to the direction of propagation (the z-direction). The wave is polarized in the y-direction.

The decay constant is γ and the propagation constant is β. The two are linked by the relation $\gamma^2 - \beta^2 + k^2 = 0$.

[3]

R.R.A. Syms

[illegible]

2a) The laws governing reflection and transmission are:

Alhazen's law $\theta_1 = \theta_1'$
Snell's law $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

[2]

The refracted wave direction is found as:
$\theta_2 = \sin^{-1}\{(n_1/n_2) \sin(\theta_1)\}$

If $n_1 > n_2$, the argument in the inverse sine can exceed unity if θ_1 is greater than the critical angle θ_c. A real solution for θ_2 is then no longer possible. The critical angle is found from:
$(n_1/n_2) \sin(\theta_c) = 1$ i.e. $\theta_c = \sin^{-1}(n_2/n_1)$

[2]

If $n_1 = 1.51$ and $n_2 = 1.5$, $\theta_c = \sin^{-1}(1.5/1.51) = 83.40°$.

[2]

b) The electric fields are:

$E_1 = E_0 \{\exp\{-jk_0n_1 [z \sin(\theta_1) - x \cos(\theta_1)] + \Gamma_E \exp\{-jk_0n_1 [z \sin(\theta_1') + x \cos(\theta_1')]\}$
$E_2 = E_0 T_E \exp\{-jk_0n_2 [z \sin(\theta_2) - x \cos(\theta_2)]$
Here Γ_E and T_E are the reflection and transmission coefficients.

[2]

Using Alhazen's and Snell's laws and grouping terms together we can write these as:
$E_1 = E_0 \exp\{-jk_0n_1z \sin(\theta_1)\} \{\exp\{+jk_0n_1x \cos(\theta_1)] + \Gamma_E \exp\{-jk_0n_1x \cos(\theta_1)\}$
$E_2 = E_0 T_E \exp\{-jk_0n_1z \sin(\theta_1)\} \exp\{+jk_0n_2x \cos(\theta_2)\}$

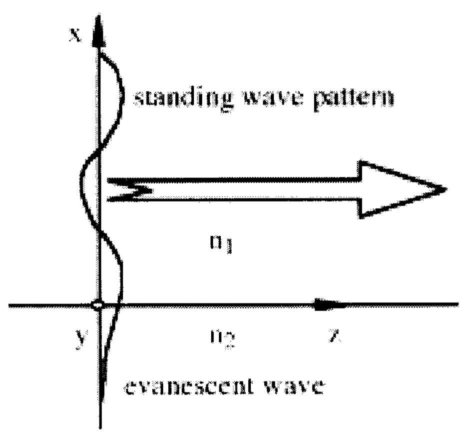
The field in medium 1 is a standing wave, travelling parallel to the interface. After total internal reflection has occurred, θ_2 is no longer real. However, we can express $\cos(\theta_2)$ as:
$\cos(\theta_2) = \sqrt{\{1 - \sin^2(\theta_2)\}} = \sqrt{\{1 - (n_1/n_2)^2 \sin^2(\theta_1)\}}$

Since $(n_1/n_2) \sin(\theta_1) > 1$, we may rewrite the above as:
$\cos(\theta_2) = \pm j \sqrt{\{(n_1/n_2)^2 \sin^2(\theta_1) - 1\}} = \pm j\alpha$

The field in medium 2 then becomes:
$E_2 = E_0 T_E \exp\{-jk_0n_1z \sin(\theta_1)\} \exp\{k_0n_2\alpha x\}$
Here we have chosen the sign of $\cos(\theta_2)$ to obtain an solution that decays in the –x-dir'n.

The complete transverse field variation is then:

[2]



[2]

R.R.A. Syms
A. White

c) The amplitude reflection coefficient Γ_E for TE incidence on the interface is given by:

$$\Gamma_E = \{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)\} / \{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)\}$$

After total internal reflection has occurred, $\cos(\theta_2) = \pm j\alpha$. We can therefore write:
$\Gamma_E = \{n_1 \cos(\theta_1) - j\alpha\} / \{n_1 \cos(\theta_1) + j\alpha\} = z/z^*$

The power reflection coefficient is $|\Gamma_E|^2 = \Gamma_E \Gamma_E^* = (z/z^*)(z^*/z) = 1$

[2]

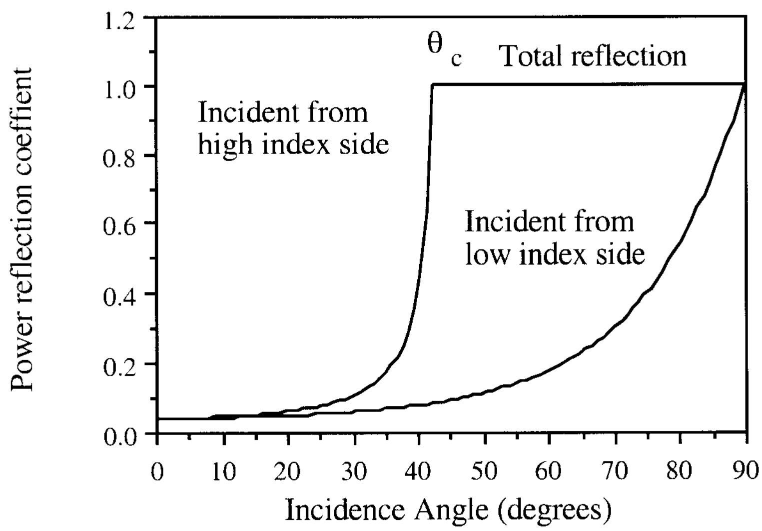
For the data given, $|\Gamma_E|^2 = \{(1.5 - 1) / (1.5 + 1)\}^2 = (1/5)^2 = 0.04$, for normal incidence from either side of the interface.

For incidence from the high index side, the critical angle is $\theta_c = \sin^{-1}(1/1.5) = 41.81^\circ$. After this angle has been reached, the power reflection coefficient is unity.

For incidence from the low-index side, there is no critical angle, but the reflectivity rises smoothly to unity at 90° incidence.

[3]

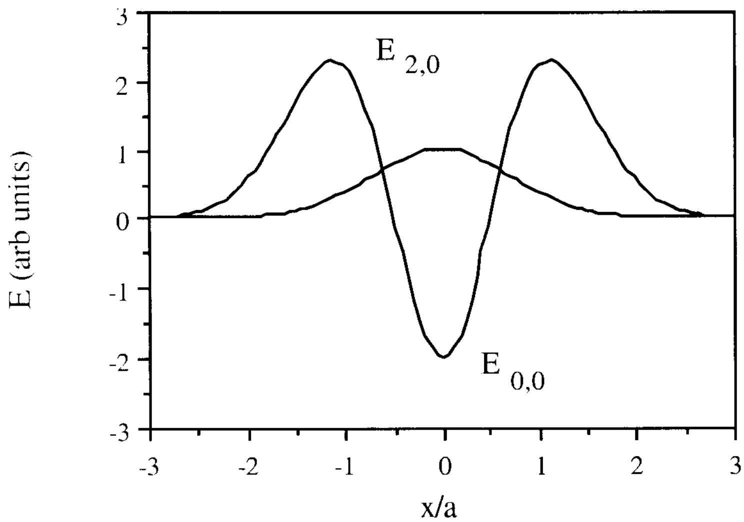
The overall variation in reflectivity is then:



[3]

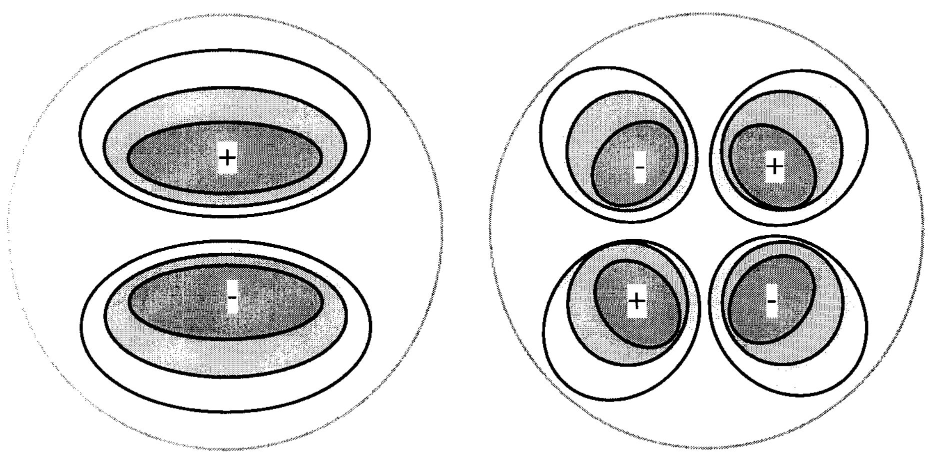
R. R. A. Syms
N. White

3a) One-dimensional modal field variations:



[4]

Two-dimensional modal field variations:



$E_{0,1}$ $E_{1,1}$

[4]

2b) The scalar wave equation is $\nabla^2 E + n^2 k_0^2 E = 0$, where n is the refractive index and $k_0 = 2\pi/\lambda$ is the propagation constant of free space.

If the refractive index is a function of x and y alone, and hence describes a guide oriented in the z-direction, we may assume a solution in the form $E(x, y, z) = E_T(x, y) \exp(-j\beta z)$.

Here $E_T(x, y)$ is the transverse field and β is the propagation constant. By substituting into the wave equation, we obtain the waveguide equation:
$\partial^2 E_T/\partial x^2 + \partial^2 E_T/\partial y^2 + \{n^2 k_0^2 - \beta^2\} E_T = 0$

If the refractive index variation is $n^2 = n_0^2[1 - (r/r_0)^2]$, the waveguide equation reduces to:
$\partial^2 E_T/\partial x^2 + \partial^2 E_T/\partial y^2 + \{n_0^2 k_0^2 [1 - (r/r_0)^2] - \beta^2\} E_T = 0$

[4]

If the transverse field variation is of the form
$E_T = H_\mu(\sqrt{2}x/a)\, H_\nu(\sqrt{2}y/a) \exp\{-(x^2 + y^2)/a^2\}$

Differentiating, we obtain:
$\partial E_T/\partial x = \{(\sqrt{2}/a)\, H_\mu'(\sqrt{2}x/a) - (2x/a^2)\, H_\mu(\sqrt{2}x/a)\}\, H_\nu(\sqrt{2}y/a) \exp\{-(x^2 + y^2)/a^2\}$

Where $H_\mu' = \partial H_\mu(\zeta)/\partial\zeta$, and so on. Similarly:
$\partial^2 E_T/\partial x^2 = \{(2/a^2)\, H_\mu''(\sqrt{2}x/a) - (4\sqrt{2}x/a^3)\, H_\mu'(\sqrt{2}x/a)\} + [(4x^2/a^4) - (2/a^2)]\, H_\mu(\sqrt{2}x/a)\} \, .$
$. \, H_\nu(\sqrt{2}y/a) \exp\{-(x^2 + y^2)/a^2\}$

Similarly:
$\partial^2 E_T/\partial y^2 = \{(2/a^2)\, H_\nu''(\sqrt{2}y/a) - (4\sqrt{2}y/a^3)\, H_\nu'(\sqrt{2}y/a)\} + [(4y^2/a^4) - (2/a^2)]\, H_\nu(\sqrt{2}y/a)\} \, .$
$. \, H_\mu(\sqrt{2}y/a) \exp\{-(x^2 + y^2)/a^2\}$

Substituting these expressions into the waveguide equation, and cancelling out the exponential terms, we get:
$\{(2/a^2)\, H_\mu''(\sqrt{2}x/a) - (4\sqrt{2}x/a^3)\, H_\mu'(\sqrt{2}x/a)\} + [(4x^2/a^4) - (2/a^2)]\, H_\mu(\sqrt{2}x/a)\}\, H_\nu(\sqrt{2}y/a) +$
$\{(2/a^2)\, H_\nu''(\sqrt{2}y/a) - (4\sqrt{2}y/a^3)\, H_\nu'(\sqrt{2}y/a)\} + [(4y^2/a^4) - (2/a^2)]\, H_\nu(\sqrt{2}y/a)\}\, H_\mu(\sqrt{2}x/a) +$
$\{n_0^2 k_0^2[1 - ((x^2 + y^2)/r_0)^2] - \beta^2\}\, H_\mu(\sqrt{2}x/a)\, H_\nu(\sqrt{2}y/a) = 0$

Defining $\xi = \sqrt{2}x/a$, and $\eta = \sqrt{2}y/a$, we can write this as:
$\{(2/a^2)\, H_\mu''(\xi) - (4\xi/a^2)\, H_\mu'(\xi)\} + [(4/a^4 - n^2k_0^2/r_0^2)x^2 - (2/a^2)]\, H_\mu(\xi)\}\, H_\nu(\eta) +$
$\{(2/a^2)\, H_\nu''(\eta) - (4\eta/a^2)\, H_\nu'(\eta)\} + [(4/a^4 - n^2k_0^2/r_0^2)y^2 - (2/a^2)]\, H_\nu(\eta)\}\, H_\mu(\xi) +$
$\{n_0^2 k_0^2 - \beta^2\}\, H_\mu(\xi)\, H_\nu(\eta) = 0$

We now note that the terms in x^2 and y^2 will vanish, if $a^4 = 4r_0^2/n_0^2k_0^2$, so $a = \sqrt{\{2r_0/n_0k_0\}}$
[4]

This gives:
$\{H_\mu''(\xi) - 2\xi\, H_\mu'(\xi)\} - H_\mu(\xi)\}\, H_\nu(\eta) +$
$\{H_\nu''(\eta) - 2\eta\, H_\nu'(\eta)\} - H_\nu(\eta)\}\, H_\mu(\xi) +$
$(a^2/2)\{n_0^2k_0^2 - \beta^2\}\, H_\mu(\xi)\, H_\nu(\eta) = 0$

Dividing through by $H_\mu(\xi)\, H_\nu(\eta)$ we get:
$\{H_\mu''(\xi) - 2\xi\, H_\mu'(\xi)\} - H_\mu(\xi)\} \,/\, H_\nu(\xi) +$
$\{H_\nu''(\eta) - 2\eta\, H_\nu'(\eta)\} - H_\nu(\eta)\} \,/\, H_\mu(\eta) +$
$(a^2/2)\{n_0^2k_0^2 - \beta^2\} = 0$

We now note that the first line is a function of ξ only, the second is a function of η only, and the third is a constant. The equation can only be satisfied for all (ξ, η) if all three lines in the equation above are constants, i.e. if
$\{H_\mu''(\xi) - 2\xi\, H_\mu'(\xi)\} - H_\mu(\xi)\} \,/\, H_\nu(\xi) = A$ (say)
so $H_\mu''(\xi) - 2\xi\, H_\mu'(\xi)\} - (1+A)\, H_\mu(\xi) = 0$
$\{H_\nu''(\eta) - 2\eta\, H_\nu'(\eta)\} - H_\nu(\eta)\} \,/\, H_\nu(\eta) = B$ (say)
so $H_\nu''(\eta) - 2\eta\, H_\nu'(\eta)\} - (1+B)\, H_\nu(\eta) = 0$

Now, Hermite polynomials satisfy the differential equation:
$d^2H_\mu(\zeta)/d\zeta^2 - 2\zeta\, dH_\mu(\zeta)/d\zeta + 2\mu H_\mu(\zeta) = 0$
Hence, we require $-(1+A) = 2\mu$, or $A = -(2\mu + 1)$, and similarly $B = -(2\nu + 1)$.
[4]

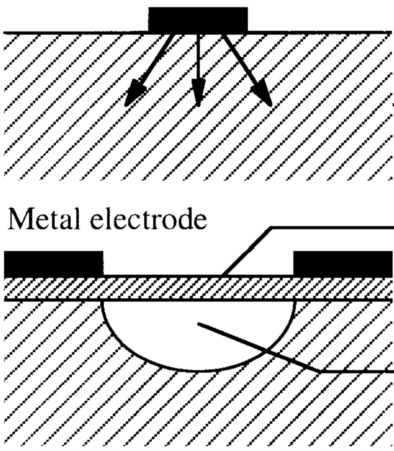
At this point we note that:
$(a^2/2)\{n_0^2k_0^2 - \beta^2\} + A + B = 0$, so that $\beta^2 = n_0^2k_0^2 + (2/a^2)(A + B)$

Substituting for A and B, we then obtain:
$\beta_{\mu,\nu}^2 = n_0^2k_0^2 - (4/a^2)(\mu + \nu + 1)$

And finally substituting for a, we get:
$\beta_{\mu,\nu} = n_0k_0 \sqrt{\{1 - 2(\mu + \nu + 1)/n_0k_0r_0\}}$

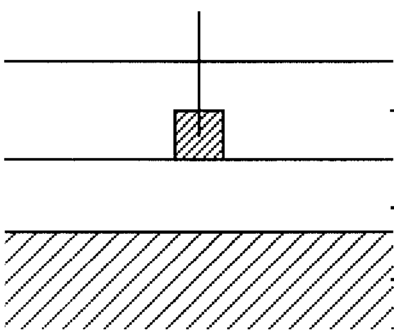
[4]

4a) Ti: $LiNbO_3$ channel guides are formed by indiffusion of titanium metal into the electro-optic crystal lithium niobate, at an elevated temperature. A buffer layer of SiO_2 is then deposited, to separate the guided mode from the effects of metal electrodes, which are used to apply electric fields to the crystal. These fields are used to alter the local refractive index, and hence the phase of the guided mode.



[3]

ii) Silica-on-silicon channel guides are formed in doped silica glasses on a silicon substrate. A thick layer of glass (usually pure SiO_2) is first deposited, to space the guided mode from the substrate. A layer of doped SiO_2 (often GeO_2: SiO_2) is then deposited, and etched into a strip. A layer of low melting point SiO_2 (often P_2O_5: SiO_2) is then deposited over the strip, and reflowed to form a smooth cladding.



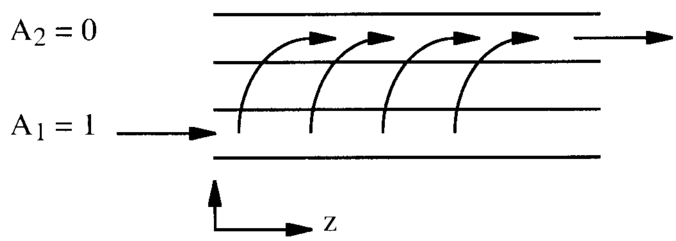
[3]

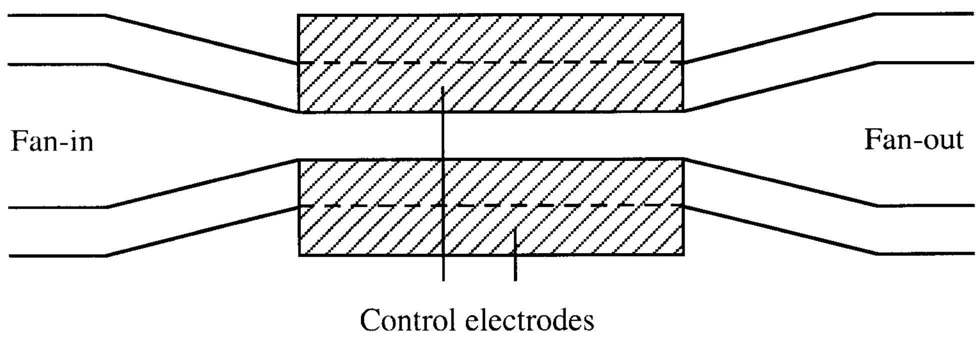
b) The *directional coupler* consists of two identical, parallel guides, which are so closely spaced that the modal fields in the guides overlap. There is then a mechanism for power in one guide to be coupled into the other guide. The power transfer occurs gradually, so that power launched into (for example) guide 1 will slowly couple into guide 2. After a fixed length – the coupling length – the coupling will reach 100%. Power will then start to couple back into guide 1, and so on in a periodic manner.

In a *directional coupler switch*, the length is chosen for 100% power transfer, so that an input to guide 1 results in an output from guide 2. The transfer process is only effective if the two guides are identical. They may be desynchronised with the electro-optic effect in (for example) a Ti: $LiNbO_3$ device. A pair of electrodes is required, one over each guide. Application of a voltage will create an electric field, which increases the refractive index of one guide and decreases the index of the other. The two guides no longer appear identical,

R.R.A. [signature]

and the coupling process is destroyed. The power then emerges from guide 1. Additional fan-in and fan-out sections are also needed.





[6]

c) The governing first order coupled mode differential equations are:

$$dA_1/dz + j\kappa A_2 = 0$$
$$dA_2/dz + j\kappa A_1 = 0$$

Differentiating the upper equation, we get:

$$d^2A_1/dz^2 + j\kappa dA_2/dz = 0$$

Substituting using the lower equation, we then get:

$$d^2A_1/dz^2 + \kappa^2 A_1 = 0$$

This second order equation has the general solution:

$$A_1 = C_1 \cos(\kappa z) + C_2 \sin(\kappa z)$$

For an input of unity amplitude into (say) guide 1, the boundary conditions are that $A_1 = 1$ and $A_2 = 0$ on $z = 0$. From the upper coupled mode equation, the second condition is equivalent to $dA_1/dz = 0$. The solution must then be:

$$A_1 = \cos(\kappa z)$$
$$A_2 = -j \sin(\kappa z)$$

[4]

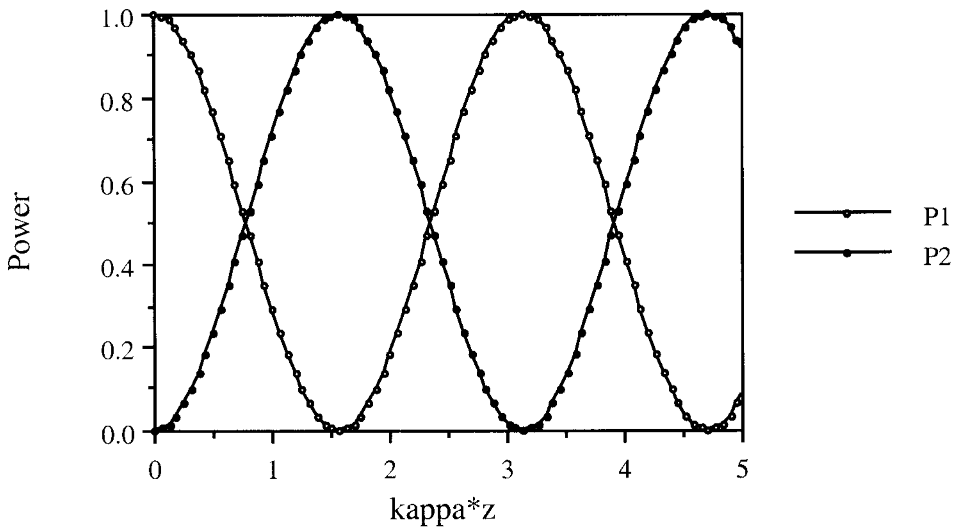
The power carried by a guided mode is proportional to the modulus squared of the field amplitude. The powers in the two guided modes are therefore:

$$P_1 = |A_1|^2 = \cos^2(\kappa z)$$

$$P_2 = |A_2|^2 = \sin^2(\kappa z)$$

Clearly, power is conserved, since $P_1 + P_2 = 1$ throughout.

100% power transfer is obtained when $\kappa z = \pi/2 + \nu\pi$, where ν is an integer. The variation of power with distance is as shown below.



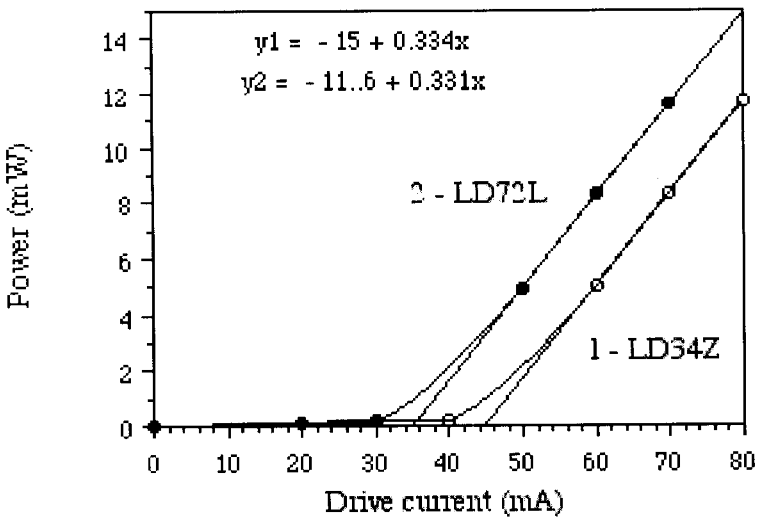
[4]

R.R.A. Syms
W. White

5a) Below threshold, the output of a laser is $P = I \times \eta_1 hc/e\lambda$
Above threshold, the output is $P = \{I - I_t\} \times \eta_2\, hc/e\lambda$

Here I is the drive current, I_t the threshold current, $h = 6.62 \times 10^{-34}$ Js, $c = 3 \times 10^8$ m/s, $e = 1.6 \times 10^{-19}$ C, λ is the emission wavelength and η_1 and η_2 are quantum efficiencies below and above threshold. The laser light-current characteristic is therefore a discontinuous function of current. Above threshold, the slope of the characteristic is $dP/dI = \eta\, hc/e\lambda$. However, half the power emerges from each of the two laser facets, so that the expressions for power and slope efficiency should both be divided by two.

The data in the table may be plotted as shown below.



[4]

From straight-line fits to the data in the lasing regime, the thresholds may be estimated as:
Laser 1 – LD34Z: 45 mA
Laser 2 – LD72L: 35 mA

[2]

The slope efficiencies are:
Laser 1: dP/dI = 0.334 mW/mA
Laser 2: dP/dI = 0.331 mW/mA

The theoretical slope efficiencies are:
Laser 1:
$dP/dI = \eta_1 \times 6.62 \times 10^{-34} \times 3 \times 10^8 / (2 \times 1.6 \times 10^{-19} \times 1.3 \times 10^{-6}) = \eta_1 \times 0.4774$ mW/mA
Laser 2:
$dP/dI = \eta_2 \times 6.62 \times 10^{-34} \times 3 \times 10^8 / (2 \times 1.6 \times 10^{-19} \times 1.5 \times 10^{-6}) = \eta_2 \times 0.41375$ mW/mA

The quantum efficiencies are therefore:
$\eta_1 = 0.334 / 0.4774 = 0.7$
$\eta_2 = 0.331 / 0.41375 = 0.8$

[4]

Laser 2 therefore has the lower threshold and higher quantum efficiency.

b) At 50 mA drive current, the output of LD72L is 4.97 mW.

The photocurrent generated by a photodiode is $I_P = (\eta\, e\lambda/hc)\, P$

Here η is the quantum efficiency, and the responsivity is $R = I_P/P$

In this case, $I_P = \eta \times (1.6 \times 10^{-19} \times 1.5 \times 10^{-6}/(6.62 \times 10^{-34} \times 3 \times 10^{8}) \times P = \eta \times 1.2085 \times P$

Assuming that a 1 kΩ feedback resistor is used, the photocurrent measured with no fibre in the link is $3.6/10^3$ A = 3.6 mA.

The quantum efficiency is therefore $\eta = 3.6 / (1.2085 \times 4.97) = 0.6$

[2]

The responsivity is therefore $R = 3.6/4.97 = 0.724$ mA/mW.

[2]

The voltage output of the detector amplifier is proportional to the detected power.The following voltages are obtained:

With no fibre:	$V_1 = 3.6$ V
With 50 km of fibre:	$V_2 = 114$ mV
With 150 km of fibre:	$V_3 = 1.14$ mV

The loss caused by inserting 50 km of fibre is therefore $L_{50} = -10 \log_{10}(V_2/V_1) = 15$ dB
The loss caused by inserting 150 km of fibre is therefore $L_{150} = -10 \log_{10}(V_3/V_1) = 35$ dB

There are two sources of loss: coupling loss and propagation loss.

Assuming that the coupling loss is the same in each case, the effect of adding (150 – 50) = 100 km of fibre is to increase the loss by (35 – 15) = 20 dB. The fibre propagation loss is therefore 20/100 = 0.2 dB/km.

[3]

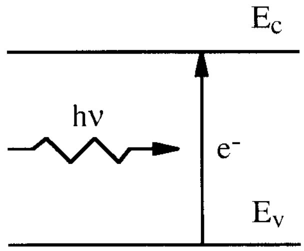
The loss budget of the 50 km link is then comprised of 50 x 0.2 = 10 dB propagation loss plus coupling loss. If the total is to equal 15 dB, the coupling loss must be 5 dB.

[3]

R.R.A. Syms
White

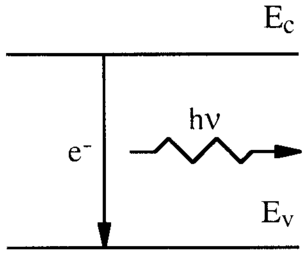
6a) Absorption, spontaneous emission and stimulated emission:

Absorption – a process whereby an electron is promoted from the conduction band to the valence band through interaction with a photon, the photon being destroyed as a result. The photon must have an energy $h\nu > E_g$, where h is Planck's constant, ν is the optical frequency and $E_g = E_c - E_v$ is the energy gap.



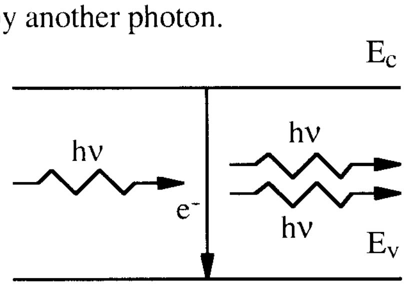
[2]

Spontaneous emission – a process whereby a photon is created by the random recombination of an electron from the conduction band with a hole in the valence band.



[2]

Stimulated emission – a process whereby a photon is created by the recombination of an electron from the conduction band with a hole in the valence band, when stimulated to do so by another photon.



[2]

Spontaneous emission is dominant in LEDs. Absorption, spontaneous emission and stimulated emission all occur in lasers. At low drive currents, the first two processes dominate and stimulated emission is negligible. However, at high currents, the rate of stimulated can rise until it is the dominant emission process, and can actually overcome absorption. Travelling wave gain may then arise, leading to oscillation in optical cavities.

[2]

LED: the photons are all generated randomly, so the emission is broad-band, incoherent omidirectional, and unpolarised. Lasers: each photon generated by stimulated emission is a "carbon copy" of the one that triggered its creation, i.e. identical in phase, frequency, direction and polarization. The emission is narrow-band, coherent, unidirectional, and polarised.

[2]

R.C.A.S.

b) The rate equations for a light emitting diode are:
$dn/dt = I/ev - n/\tau_e$
$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$

Individual terms:

n is the electron density
ϕ is the photon density
I is the injection current
v is the active volume
τ_e is electron lifetime
τ_{rr} is the radiative recombination lifetime
τ_p is the photon lifetime

Processes:

dn/dt	Rate of change of electron density
I/ev	Rate of injection of electrons
$-n/\tau_e$	Rate of loss of electron density by all forms of recombination
$d\phi/dt$	Rate of change of photon density
n/τ_{rr}	Rate of increase in photon density by radiative recombination
$-\phi/\tau_p$	Rate of loss of photon density through escape from the LED surface

[2]

The photon lifetime is roughly $\tau_p \approx L/c$, where L is the transit distance from the active volume to the surface. In a surface emitting LED, L might be ≈ 1 μm, so $\tau_p \approx 10^{-6}/3 \times 10^8 = 3 \times 10^{-15}$ sec, very short indeed.

[2]

c) Obviously, the output spectrum of an LED is broad, and there is a spread in emission wavelength. However, in a 2-state model the wavelength may be estimated as follows. Each emitted photon has energy $h\nu = hc/\lambda$. If the energy gap is E_g, then $hc/\lambda = eE_g$, and $\lambda = hc/eE_g = 6.62 \times 10^{-34} \times 3 \times 10^8/(1.6 \times 10^{-19} \times 1.42)$ m $= 0.874$ μm.

[2]

In the steady state, there is no time variation and the rate equations reduce to:
$I/ev - n/\tau_e = 0$ so $n = I\tau_e/ev$
$n/\tau_{rr} - \phi/\tau_p = 0$ so $\phi/\tau_p = (I/ev)(\tau_e/\tau_{rr})$

Now ϕ/τ_p represents the rate of change of photon <u>density</u>, so the photon output flux is:
$\Phi = v\phi/\tau_p = (I/e)(\tau_e/\tau_{rr})$

This equation suggests that every electron generates a photon, apart from an efficiency factor $\eta = (\tau_e/\tau_{rr})$. In this case, $\eta = 10^{-9}/2\times 10^{-9} = 0.5$.

[2]

If each photon carries an energy hc/λ the optical power output is:
$P = \Phi\, hc/\lambda = (I/e)(hc/\lambda)\,\eta = IE_g\eta$

The output power per amp is then $P/I = E_g\eta$, so
$P/I = 1.42 \times 0.5 = 0.71$ W/A or 0.71 mW/mA

[2]

The useful efficiency of an LED is considerably lower than the 50% internal efficiency found above. Spontaneous emission is isotropic, so half the light will be travelling away

R.R.A. Syms

from the emissive surface. Of the remainder, the vast fraction is internally reflected, because of the large difference in refractive index between the semiconductor and air. The external efficiency may be estimated as $\eta_e \approx 1/\{n(n+1)\}^2$; for n = 3.5, $\eta_e \approx 1.4\%$. Much of the light emerging into air is then wasted for other applications (e.g. coupling into an optical fibre) because the range of emission angle is so large.

[2]

R. R. A. Syms