Imperial College London

M4/5A10

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)

May-June 2017

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

Fluid Dynamics II

Date: Friday 19 May 2017

Time: 10:00 - 12:30

Time Allowed: 2.5 Hours

This paper has 5 Questions.

Candidates should use ONE main answer book.

Supplementary books may only be used after the relevant main book(s) are full.

All required additional material will be provided.

- DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO.
- Affix one of the labels provided to each answer book that you use, but DO NOT USE THE LABEL WITH YOUR NAME ON IT.
- Credit will be given for all questions attempted, but extra credit will be given for complete or nearly complete answers to each question as per the table below.

Raw Mark	Up to 12	13	14	15	16	17	18	19	20
Extra Credit	0	½	1	1 ½	2	2 ½	3	3 ½	4

- Each question carries equal weight.
- Calculators may not be used.

1. Under what conditions do the Stokes equations

$$\nabla p = \mu \nabla^2 \mathrm{u}, \ \nabla \cdot \mathrm{u} = 0,$$

describe a good approximation to the equations of motion for an incompressible fluid of viscosity μ?

Suppose that the fluid occupies a volume V bounded by a surface S with the velocity of the flow prescribed on S. Show that the Stokes equations cannot have more than one solution satisfying the required boundary condition.

Now consider a two-dimensional Stokes flow in the $x-y$ plane. Define a stream function Ψ for the flow and show that it satisfies the biharmonic equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \Psi = 0.$$

2. A viscous incompressible fluid of kinematic viscosity ν occupies the region $y \geq 0$. At the rigid wall $y = 0$ fluid enters with constant speed V in the y direction and a long way from the wall the x-velocity component is $U \cos \omega t$ where t denotes time and ω, U are constant frequencies and speeds respectively.

Show that if the y velocity is taken to be $-V$ everywhere, the equations of motion for the flow admit a solution for the velocity field which is independent of x.

By writing the flow quantities such as u in the form $u(y,t)$ = Real part of $[U(y)e^{i\omega t}]$ determine the x velocity component and the pressure gradient in the x direction.

Discuss how the velocity field for the flow simplifies in the limits:

$$(a)\, V \to \infty, \ \ (b)\, \omega \to 0, \ \ (c)\, \omega \to \infty.$$

3. Consider the steady flow of a viscous incompressible fluid of kinematic viscosity ν in the two-dimensional channel

$$-hF(x/L) \leq y \leq hF(x/L), \quad -L \leq x \leq L.$$

The flow is driven by a pressure drop M_0 between the ends of the channel. If a typical fluid speed in the x direction is U_0 determine the appropriate non-dimensional form of the Navier Stokes equations involving the parameter $\delta = h/L$ and the modified Reynolds number $R_m = U_0h^2/(L\nu)$.

What is meant by the lubrication limit associated with the flow?

Now suppose that $\delta \to 0$ with R_m held fixed. Show that the leading order terms in the expansions of the scaled velocity and pressure fields satisfy

$$R_m(u_0u_{0X} + v_0u_{0Y}) = -p_{0X} + u_{0YY},$$
$$p_{0Y} = 0,$$
$$u_{0X} + v_{0Y} = 0.$$

Here X, Y are suitably scaled variables in the x, y directions. What boundary conditions are to be applied at the walls?

Now take the further limit $R_m \to 0$ and expand u_0 in the form

$$u_0 = u_{00} + R_m u_{01} + \ldots,$$

together with similar expansions for v_0, p_0. Find the differential equation satisfied by p_0 and hence by integrating it determine the pressure and velocity fields throughout the channel.

Write down the equations to determine the order R_m terms in the expansion of the velocity and pressure fields. What conditions are to be satisfied at the wall by the velocity field at order R_m?

4. A viscous fluid of kinematic viscosity ν flows over the flat plate $x \geq 0, y = 0$. A long way from the wall the flow in the x direction tends to $U(x)$. Under what circumstances can the stream function Ψ for the flow be approximated by the boundary layer equation

$$\Psi_y \Psi_{xy} - \Psi_x \Psi_{yy} = UU_x + \nu \Psi_{yyy} \ ? \tag{1}$$

Explain the circumstances under which these equations with $U = 0$ also describe a two-dimensional jet flow emanating from $x = y = 0$.

Show that the equations with $U = 0$ have a similarity solution of the form

$$\Psi = k_1 x^m f(\eta), \eta = \frac{y}{x^n k_2},$$

with k_1, k_2, m, n constants and $m + n = 1$.

What conditions are to be imposed on Ψ when $y \to \pm\infty$?

Deduce from (1) that for a jet flow the integral

$$\int_{-\infty}^{\infty} (\Psi_y)^2 \, dy$$

is independent of x. Hence show that $2m - n = 0$ and that if $k_1 k_2 = 6\nu$ the function f satisfies

$$f''' + 2ff'' + 2f'^2 = 0.$$

Integrate this equation twice to show that

$$f' + f^2 = \lambda^2,$$

where λ is a constant. Integrate once again to show that $f = \lambda \tanh \lambda\eta$.

5. A viscous incompressible fluid of viscosity ν occupies the region $y \geq 0$.The equations of motion are taken in the nondimensional form

$$u_x + v_y + w_z = 0,$$
$$[\partial_t + u\partial_x + v\partial_y + w\partial_z]\mathbf{u} = -\nabla p + \frac{1}{R}\nabla^2\mathbf{u}.$$

The Reynolds number R is assumed to be large.

At the wall the velocity field satisfies $\mathbf{u} = (0, -\frac{1}{R}, 0)$ whilst $\mathbf{u} \to (1, -\frac{1}{R}, 0)$ when $y \to \infty$. Show that the equations of motion allow a solution of the form

$$\mathbf{u} = (u, v, w) = \mathbf{u}_b = (1 - e^{-y}, -\frac{1}{R}, 0), \ \ p = 0.$$

Now consider that solution in an $O(1)$ layer centred a long way from the wall at $y = \log R$. Define the new variable

$$Y = y - \log R,$$

and show that in this layer

$$\mathbf{u}_b = (1, 0, 0) + \frac{1}{R}(-e^{-Y}, 0, 0), \ \ p = 0.$$

Within the layer now look for a more general solution of the form

$$\mathbf{u} = \mathbf{u}_s = (1, 0, 0) + \frac{1}{R}\mathbf{U}(X, Y, Z) + \cdots, \ \ p = P(X, Y, Z) + \cdots,$$

where

$$X = x - ct, \ \ Z = z,$$

with the wavespeed

$$c = 1 - \frac{c_1}{R} + \cdots.$$

(Note that $\cdots$ denotes 'smaller terms').

Deduce that in the layer the convective operator transforms as

$$\partial_t + u\partial_x + v\partial_y + w\partial_z \to \frac{1}{R}\mathcal{L} + \cdots, \ \ \mathcal{L} = ([U + c_1]\partial_X, V\partial_Y, W\partial_Z).$$

Hence show that $\mathbf{U}, P$ satisfy

$$\mathcal{L}\mathbf{U} = -\nabla P + \nabla^2\mathbf{U}, \ \ \nabla \cdot \mathbf{U} = 0,$$

where ∇ is the gradient operator in (X, Y, Z) variables.

What boundary conditions must be satisfied when $Y \to \pm\infty$ if we demand that $\mathbf{u}_s \to \mathbf{u}_b$ away from the layer?

Write down the conditions to be satisfied if $\mathbf{u}_s$ is to be periodic in X, Z with wavenumbers α and β respectively.

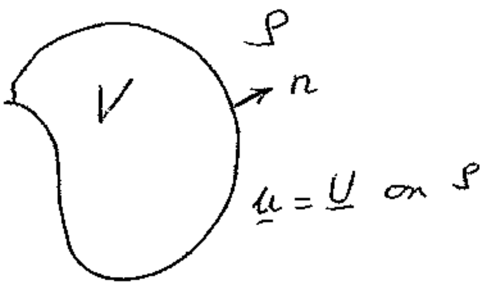
A10 Fluids II (2017)

EXAMINATION SOLUTIONS 2016-17

Question 1 (All Bookwork)

Parts

The Stokes equations apply to low Reynolds number flows, so the nonlinear terms are negligible. (3)



Suppose $(p_1, \underline{u}_1)$ and $(p_2, \underline{u}_2)$ both satisfy

$$\nabla p = \mu \nabla^2 \underline{u}, \quad \nabla \cdot \underline{u} = 0$$
$$\underline{u} = \underline{U} \text{ on } S.$$

Consider the function $\overline{\underline{u}} = \underline{u}_1 - \underline{u}_2$, $\overline{p} = p_1 - p_2$ (6)

then $\overline{\underline{u}}$ satisfies $\overline{\underline{u}} = 0$ on S and

$$0 = -\overline{p}_{x_i} + \mu (\overline{u}_i)_{x_j x_j}$$

Now multiply by $\overline{u}_i$ and integrate over the volume V:

$$0 = -\int_V \frac{\partial}{\partial x_i} \left(\overline{p}\, \overline{u}_i \right) dV + \mu \int_V \overline{u}_i \frac{\partial^2 \overline{u}_i}{\partial x_j^2} dV$$

(where we used continuity $\dfrac{\partial \overline{u}_i}{\partial x_i} = 0$)

(2)

So that using the divergence theorem on the first integral

EXAMINATION SOLUTIONS 2016-17

Question 1

Parts

$$0 = -\int_S \bar{p}\,\bar{u}_i n_i \, dS + \mu \int_V \left[\frac{\partial}{\partial x_j}\left(\bar{u}_i \frac{\partial \bar{u}_i}{\partial x_j} \right) - \left(\frac{\partial \bar{u}_i}{\partial x_j} \right)^2 \right] dV$$

and since $\bar{u}_i = 0$ on S and the first volume integral can be replaced by

$$\int_S \bar{u}_i \frac{\partial \bar{u}_i}{\partial x_j} n_i \, dS$$

which again vanishes. Hence we have

$$\int_V \left(\frac{\partial \bar{u}_i}{\partial x_j} \right)^2 dV = 0.$$

(4)

Thus $\underline{\bar{u}} =$ constant, but $\underline{\bar{u}} = 0$ on S so the required result follows. (2)

For a 2D flow we define $u = \psi_y$, $v = -\psi_x$

and then
$$p_x = \mu(\partial_x^2 + \partial_y^2)\psi_y \quad \text{— (1)}$$
$$p_y = \mu(\partial_x^2 + \partial_y^2)(-\psi_x) \quad \text{— (2)}$$

so $\frac{\partial}{\partial y}(1) - \frac{\partial}{\partial x}(2)$ gives

$$(\partial_x^2 + \partial_y^2)^2 \psi = 0.$$

(3)

EXAMINATION SOLUTIONS 2016-17 C

Question 2

All unseen

M se

Parts

The equations of motion are

$$u_t + uu_x + vu_y = -\frac{1}{\rho} p_x + \nu(u_{xx} + u_{yy})$$

$$v_t + uv_x + vv_y = -\frac{1}{\rho} p_y + \nu(v_{xx} + v_{yy})$$

$$u_x + v_y = 0.$$

From continuity if $v = -V$ then $u_x = 0$ and u is a function of y, t only.

The y momentum gives $0 = p_y$ and so the pressure can only be a function of x, t.

The x momentum gives

$$u_t + 0 - Vu_y = -\frac{1}{\rho} p_x + \nu u_{yy}$$

If we let $y \to \infty$ then we obtain

$$-\omega U \sin \omega t = -\frac{1}{\rho} p_x$$

(8)

and so we have that $p_x = \left[-i\omega\rho U e^{i\omega t} \right]_{\text{real part}}$ (2)

Hence if we let $u = \left(\tilde{u}(y) e^{i\omega t} \right)_{\text{real part}}$

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	EXAMINATION SOLUTIONS 2016-17	C
Question 2		M se
Parts		

then

$$\nu \tilde{u}'' + V\tilde{u}' - i\omega \tilde{u} = -i\omega U$$

Solution of the homogeneous equation $\sim e^{\lambda y}$

where $\lambda = \frac{+1}{2\nu}\left\{-V \pm \sqrt{V^2 + 4\nu i\omega}\right\}$ (2)

and for exponential decay take the $-$ root.

The required boundary conditions are

$$\tilde{u} = 0, \; y = 0,$$
$$\tilde{u} \to U, \; y \to \infty,$$

Then $\tilde{u} = U\left\{1 - e^{-\frac{1}{2\nu}\left[V + \sqrt{V^2 + 4i\omega\nu}\right]y}\right\}$ (2)

and then $u = \left(\tilde{u}e^{i\omega t}\right)_{\text{real part}}$.

a) When $V \to \infty$ $\frac{-1}{2\nu}\left[V + \sqrt{}\right] \to \frac{-V}{2\nu}$

so that $\tilde{u} \to U\left\{1 - e^{-Vy/\nu}\right\}$ (2)

which is asymptotic suction flow. Thus flow responds in a quasi steady manner.

b) When $\omega \to 0$ again $\frac{-1}{2\nu}\left[V + \sqrt{}\right] \to \frac{-V}{2\nu}$ (2)

and get asymptotic suction flow and a quasi steady response.

c) When $\omega \to \infty$ $\frac{-1}{2\nu}\left[V + \sqrt{}\right] \to \sqrt{\frac{i\omega}{\nu}}$

so $\tilde{u} \to U\left\{1 - e^{-\left(\frac{i\omega}{\nu}\right)^{1/2}}\right\}$ which is a Stokes layer driven by a pressure gradient (2)

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EXAMINATION SOLUTIONS 2016-17

Question 3

Parts

Take new variables $X = x/L$, $Y = y/h$

and then from continuity $v \sim \frac{h}{L} u = \frac{h}{L} U_0$.

Hence write $(u, v) = U_0 (U, \delta V)$ where

$\delta = \frac{h}{L}$.

Now from the continuity equation we obtain

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0.$$

Bookwork

(2)

If the streamwise pressure gradient is in balance with the largest viscous term then $\frac{p}{L} \sim \frac{\mu U_0}{h^2}$

and so we define

$$p = \frac{L \nu U_0}{h^2} P$$

(2)

The X momentum then becomes

$$R_M (U U_X + V U_Y) = -P_X + U_{YY} + \delta^2 U_{XX}$$

(2)

and the Y momentum is

$$\delta^2 R_M (U V_X + V V_Y) = -P_Y + \delta^2 V_{YY} + \delta^4 V_{XX}$$

(2)

where $R_M = \frac{U_0 h^2}{L \nu}$.

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EXAMINATION SOLUTIONS 2016-17

Question 3.

Parts

The lubrication limit is $\delta \to 0$, $R_m \to 0$

or equivalently $\delta \to$ with $\frac{U_0 h}{\nu}$ fixed.

With $\delta \to 0$ and R_M fixed we write

$$
\begin{aligned}
U &= u_0(X, Y, R_M) + O(\delta), \\
V &= v_0(X, Y, R_m) + O(\delta), \\
P &= P_0(X, Y, R_M) + O(\delta).
\end{aligned}
$$

and the leading order approximations to the scaled X, Y momentum equations and continuity gives

$$R_m(u_0 u_{0X} + v_0 u_{0Y}) = -P_{0X} + u_{0YY},$$

$$0 = -P_{0Y},$$

$$u_{0X} + v_{0Y} = 0,$$

which must be solved subject to

$$u_0 = v_0 = 0, \quad Y = \pm F(X).$$ 1

If we now take the further limit $R_m \to 0$

and let

$$u_0 = u_{00} + R_m u_{01} + \cdot \quad , \text{etc}$$

then at leading order we obtain

3

2

Unseen

EXAMINATION SOLUTIONS 2016-17

C

Question 3

M
se

Parts

$u_{00YY} = p_{0X}$, $u_{00X} + v_{00Y} = 0$, $u_{00} = v_{00} = 0$, $Y = \pm F$

Thus $u_{00} = \frac{p_{0X}}{2}(Y^2 - F^2)$ (1)

Similar to problem sheet

and then from continuity

$$\int_{-F}^{F} \frac{\partial v_{00}}{\partial Y} dY = -\int_{-F}^{F} \frac{\partial u_{00}}{\partial X} dY$$ (1)

$$\Rightarrow \quad \frac{d}{dX}\left(F^3 p_{00X}\right) = 0$$

$$\Rightarrow \quad p_{00X} = \frac{B}{F^3}, \quad B \text{ a constant}$$

and then $p_{00}(X=1) - p_{00}(X=-1) = B\int_{-1}^{1} \frac{dX}{F^3}$ (1)

But from the scalings we have that

$$p_{00}(X=1) - p_{00}(X=-1) = \frac{-M_0 h^2}{L \nu U_0}$$

so that $\frac{M_0 h^2}{L \nu U_0} = -B\int_{-1}^{1} \frac{dX}{F^3}$, so u_{00} and p_{00} fixed. (2)

At order R_M the equations to be solved are

$u_{00} u_{00X} + v_{00} u_{00Y} = -p_{01X} + u_{01YY}$ (1)

$u_{01X} + v_{01Y} = 0$

$p_{01Y} = 0$

with boundary conditions $u_{01} = v_{01} = 0$, $Y = \pm F$ (1)

EXAMINATION SOLUTIONS 2016-17		C
Question 4		M se
Parts	The boundary layer equations are valid if the Reynolds number R associated with the flow is large and the boundary layer has thickness $\sim R^{-1/2}L$ where L is a typical length in the x direction. For a jet emanating from $x=y=0$ we assume a thin layer of fluid near $y=0$ is set into motion. Once again the Reynolds number must be assumed large and the layer must be of thickness $R^{-1/2}$. Given $\Psi = k_1 x^m f(\eta)$, $\eta = \frac{y}{x^n k_2}$ where m, n, k_1, k_2 are constants then $\partial_y \to \frac{1}{k_2 x^n}\partial_\eta$, $\partial_x \to \partial_x - \frac{n\eta}{x}\partial_\eta$	2 Bookwork 2 1

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	EXAMINATION SOLUTIONS 2016-17	C
Question 4		M se
Parts	The terms on the left hand side of (1) $\sim \frac{x^{2m-1}}{x^{2n}}$ whilst the viscous term on the right hand side $\sim x^{m-3n}$. Thus for a similarity solution we require that $m-3n = 2m-2n-1$ $\Rightarrow m+n=1.$ When $y \to \pm\infty$ we require that $u=\Psi_y \to 0$ Hence $f' \to 0$, $\eta \to \pm\infty$. On the left hand side we add the term $\Psi_y \Psi_{xy} - \Psi_y \Psi_{xy}$ so that the left hand side becomes $\frac{\partial}{\partial x}(\Psi_y^2) - \frac{\partial}{\partial y}(\Psi_x \Psi_y) = \nu \Psi_{yyy}$ Now integrate both sides from $-\infty$ to ∞ $\frac{\partial}{\partial x}\int_{-\infty}^{\infty} \Psi_y^2 dy - 0 = 0$ Hence the result follows.	(1) (1) Unseen (3)

	EXAMINATION SOLUTIONS 2016-17	C
Question 4		M se
Parts	With the assumed similarity form	

$$\int_{-\infty}^{\infty} \Psi_y^2 dy \sim x^n \cdot x^{2m} x^{-2n}$$

so that $2m - n = 0$ (1)

$\therefore \quad n = 2/3, \; m = 1/3$

Now take $\Psi = k_1 x^{1/3} f(\eta), \; \eta = \frac{y}{k_2 x^{2/3}}$

Now $\Psi_y = \frac{k_1 x^{1/3} f'}{k_2 x^{2/3}} = \frac{k_1 f'}{k_2 x^{1/3}}, \; \Psi_x = \frac{x^{-2/3}}{3} \langle f - 2\eta f' \rangle k_1$ (3)

$\Psi_{yy} = \frac{k_1}{k_2^2} \frac{f''}{x}, \; \Psi_{yyy} = \frac{k_1}{k_2^3} \frac{f'''}{x^{4/3}}$

$\Psi_{xy} = \frac{x^{-2/3} k_1}{3} \frac{1}{k_2 x^{2/3}} \left(-f' - 2\eta f''\right)$ linear

$= \frac{-k_1}{3k_2 x^{4/3}} \langle f' + 2\eta f'' \rangle$

Then substituting into (1) gives

$$\frac{k_1^2}{k_2^2} \frac{1}{x^{4/3}} \left\{ -\frac{f'}{3} (f' + 2\eta f'') - \frac{1}{3} (f - 2\eta f') f'' \right\}$$

$$= \frac{k_1}{k_2^3} \nu \frac{f'''}{x^{4/3}}$$

so that if $k_1 k_2 = 6\nu$ then (2)

$$f''' + 2ff'' + 2f'^2 = 0$$

EXAMINATION SOLUTIONS 2016-17		C
Question		M se
Parts	Integrating once gives $$f'' + 2ff' = \text{constant} = 0$$ from condition at infinity. Integrating again gives $$f' + f^2 = \lambda^2$$, since $f' > 0$ as expected. So $$\frac{df}{d\eta} = \lambda^2 - f^2$$ $$\int_0^\eta \frac{df}{\lambda^2 - f^2} = \int_0^\eta d\eta$$ $$\therefore \tanh^{-1} f/\lambda = \lambda\eta$$ $$f = \lambda \tanh \lambda\eta .$$	1 [illegible] 1 2

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	EXAMINATION SOLUTIONS 2016-17	C
Question 5		M se
Parts	First part substitute into momentum and Continuity equations to verify. Also the solution given satisfies the given boundary conditions.	All linear
	Now $1-e^{-y} = 1 - e^{-\log R - Y}$ $= 1 - \frac{1}{R} e^{-Y}$	4
	so $\underline{u}_B = (1,0,0) + \frac{1}{R}(-e^{-Y}, 0, 0)$.	
	With the change of variables $\partial_x \to \partial_X$, $\partial_y \to \partial_Y$	2
	$\partial_z \to \partial_Z$, $\partial_t \to -c\,\partial_X = -\langle 1 - \frac{c_1}{R} + \cdots \rangle \partial_X$	1
	Hence $\partial_t + u\partial_x + v\partial_y + w\partial_z$ $\to -\langle 1 - \frac{c_1}{R} + \cdots \rangle \partial_X + \partial_X + \frac{U}{R}\partial_X + \frac{V}{R}\partial_Y + \frac{W}{R}\partial_Z$ $= \frac{1}{R}\left([U + c_1]\partial_X + V\partial_Y + W\partial_Z \right)$	2

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EXAMINATION SOLUTIONS 2016-17

Question 5

Parts

Now consider x momentum.

$u_t + uu_x + vu_y + wu_z \to Lu = LU$ (2)

so that $LU = -P_X + \nabla^2 U$,

Similarly $LV = -P_Y + \nabla^2 V$,

and $LW = -P_Z + \nabla^2 W$. (1)

Continuity then gives $U_X + V_Y + W_Z = 0$

If $\underline{u}_s \to \underline{u}_b$ we require that

$U \to e^{-Y}$, $Y \to \pm\infty$ (2)

$V \to -1$, $Y \to \pm\infty$ (2)

$W \to 0$, $Y \to \pm\infty$. (2)

Finally periodicity conditions would require

$$U\left(X + \frac{2\pi}{\alpha}, Y, Z\right) = U\left(X, Y, Z + \frac{2\pi}{\beta}\right) = U(X, Y, Z)$$ (2)

together with similar boundary conditions on V, W, P.

Imperial College London
Department of Mathematics

Examiner's Comments

Exam: A-10 Session: 2016-2107

Question 1

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

A straightforward bookwork question. Some reasonable attempts but should have been an easy question to score high marks. Appears many students had not expected such a question.

Marker: PHALL

Signature: PHall Date:

Please return with exam marks (one report per marker)

Imperial College London
Department of Mathematics

Examiner's Comments

Exam: A10 Session: 2016-2107

Question 2

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

A combination of a steady problem and a time periodic one which were both discussed in class. Reasonable attempts on the whole but few got to the end of the question.

Marker: PHALL

Signature: [illegible] Date:

Please return with exam marks (one report per marker)

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Department of Mathematics

Examiner's Comments

Exam: A10 Session: 2016-2107

Question 3

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

On the whole this was well done.

Marker: PHALL

Signature: PHall Date:

Please return with exam marks (one report per marker)

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Department of Mathematics

Examiner's Comments

Exam: A10 Session: 2016-2107

Question 4

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

Students tended to derive the equation rather than simply state when the equation were valid. The messy calculation caused many ~~[illegible]~~ mistakes, a similar problem had been done in class

Marker: PHALL

Signature: PHall Date: ____

Please return with exam marks (one report per marker)