UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1998

MEng Honours Degrees in Computing Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the Royal College of Science
Associateship of the City and Guilds of London Institute

PAPER 4.99

MODAL AND TEMPORAL LOGIC
Wednesday, April 29th 1998, 10.00 - 12.00

Answer THREE questions

For admin. only: paper contains 4 questions

1 In this question, KT denotes the Hilbert system $K + (\Box A \rightarrow A)$ (that is, the formula $\Box A \rightarrow A$ is added to K as an axiom, for all formulas A); and q denotes a propositional atom (or variable).

a i) List the axioms and rules of KT.

ii) Let C be a class of Kripke frames. Explain what it means to say that KT is sound and complete over C.

iii) Specify a class C of Kripke frames over which KT is sound and complete.

b Draw an annotated diagram of a Kripke model in which the formula $\Box\Box q \rightarrow q$ is *not* valid.

c Show, either by giving a proof in KT, or by using a completeness theorem for KT, that $\vdash_{KT} \Box\Box q \rightarrow q$.

d Using Sahlqvist's algorithm, find a first-order frame condition that holds in a frame iff $\Box\Box q \rightarrow q$ is valid in it.

2a i) Define, by induction on A, the meaning of '$M \vDash A(t)$', for a Kripke model $M = (W,R,h)$, a world t of the frame of M, and a temporal formula A written with Until and Since.

ii) Translate 'It will rain until the sun comes out or it snows' into temporal logic with U.

iii) Can the formula $U(\top,\bot)$ have a model whose frame is $(\mathbb{Q}, <)$, the ordered rational numbers? Explain your answer.

In parts b and c, $M = (W,R,h)$ and $M' = (W',R',h')$ denote Kripke models with accessibility relations R, R', respectively.

b Define:

i) a p-morphism from M to M' (for modal logic)

ii) a temporal p-morphism from M to M' (for the temporal logic FP).

c A *strong temporal p-morphism from M to M'* is a temporal p-morphism f from M to M' with the additional property that for each $x, y \in W$ with $R(x,y)$, f maps the set $\{z \in W : R(x,z) \text{ and } R(z,y)\}$ *onto* the set $\{z' \in W' : R'(f(x), z') \text{ and } R'(z', f(y))\}$.

Show that any strong temporal p-morphism f from M to M' *preserves US-formulas:* that is, for all temporal formulas A written with U and S, and for all $t \in W$, we have $M \vDash A(t)$ iff $M' \vDash A(f(t))$. [Use induction on A.]

The three parts carry, respectively, 40%, 20%, 40% of the marks.

3a Briefly describe and justify additional proof rules which adapt a tableau system for classical propositional logic for proof with a normal modal logic. Illustrate your answer by use of appropriate rules to show that the axiom schema K holds:

$$\text{K:} \quad \Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$$

b Provide a tableau proof of the formula $\Box p \rightarrow \Box\Diamond\Box\Diamond p$ in the logic S4. Justify each step of your argument. (S4 is the same as the reflexive and transitive normal modal logic KT4).

c i) What is meant by a non-rigid designator in modal logic? Give an every day example from computer programming.

ii) Describe an interpretation in a Kripke model which makes the formula $[\lambda x.\Diamond P(x)](a)$ true at a world w.

iii) It has been observed that equality cannot in general be substituted into modal context. Although we know the morning star m and the evening star e are equal (in the sense that each designates the planet Venus), the ancient Greeks did not know this.

Use predicate abstraction to explain this paradox by contrasting the two different modal formalisations of *know* $e=m$.

4 a (i) Briefly describe and justify the global axioms which are accepted for a normal modal logic of knowledge, and indicate the distinction made between knowledge and belief.

(ii) As a formalisation of an agent's belief or knowledge, a normal modal logic forces omniscience. What does this mean? How does omniscience arise with these logics?

(iii) Let $K_i\, p$ express the fact that agent i knows that p holds. Define a modality K^* for the common knowledge of two agents.

b) Let i, j be rational and sequential agents, each with a distinct set of mental attitudes $sees_i$, $believes_i$ and (temporal) $next_i$, represented using normal modalities interpreted on indexed sets of possible worlds W_i, W_j.

(i) Write down (a pair of) axiom schemes for a theory in which each agent successively updates persistent beliefs to accord with what it sees.

Suppose, furthermore, that $tells(i,j)$ and $hears(i,j)$ are communication modalities, common to all agents, where it is informally intended that $tells(i,j)\ p$ hold in a world (or at a time) w_i in W_i at which agent i tells agent j that p holds, and $hears(i,j)\ p$ holds in a world w_i at which agent i hears from j that p holds.

(ii) Assuming that communication is instantaneous, what interaction axiom (scheme) expresses the intended communication? Can it be defined semantically?

(iii) Suppose that communication is near perfect, but that occasionally one agent may not hear what another tells it, or may hear incorrectly. Briefly describe the goal and temporal nature of a finite protocol for reliable communication.

(iv) Briefly explain why the above communications modalities are not normal.

End of Paper