IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

EEE PART II: MEng, BEng and ACGI

CONTROL ENGINEERING

Corrected copy

Wednesday, 6 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : I.M. Jaimoukha
Second Marker(s) : S.A. Evangelou

1. a) Consider the system illustrated in Figure 1.1 where all the symbols have the standard interpretation. The input is the applied force $f(t)$ and the output is the displacement of the box $z(t)$. Take $M = K_2 = D = 1$ in appropriate units.

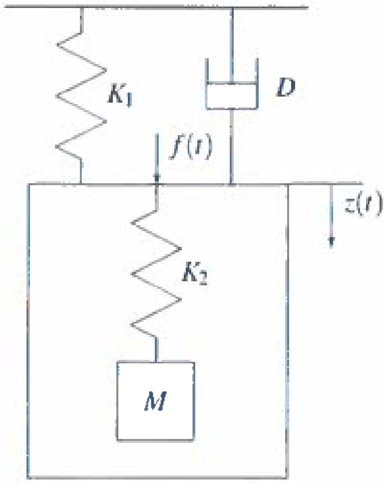

Figure 1.1

i) Show that the transfer function relating z to f is given by

$$G(s) = \frac{n(s)}{s^3 + (1 + K_1)s^2 + s + K_1}$$

where $n(s)$ is a polynomial in s. What is $n(s)$? [5]

ii) Use the Routh array to find the range of values of K_1 for stability. [5]

iii) Find the value of K_1 for which $G(s)$ is marginally stable. For this value of K_1, what are the poles of $G(s)$? [5]

iv) Let $f(t)$ be a unit step applied at $t = 0$. Use the final value theorem, which should be stated, to find the steady–state value z_{ss}, of $z(t)$ in terms of K_1. What is the value of K_1 for which $z_{ss} = 2$? [5]

b) In Figure 1.2 below, $G(s) = 2/(s^2 - 1)$ and $K(s)$ is a compensator.

i) Use the Routh-Hurwitz stability criterion to determine if the closed-loop can be stabilised using:

I. a proportional compensator [5]

II. a PI compensator [5]

III. a PD compensator [5]

ii) Design a PD so that the closed–loop is critically damped and has a pole at $s = -1$. [5]

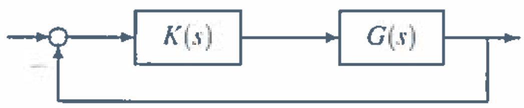

Figure 1.2

2. Consider the feedback control system in Figure 2.1 below. Here, $K(s)$ is the transfer function of a compensator while $G(s)$ is a stable transfer function with no finite zeros whose frequency response is shown in Figure 2.2.

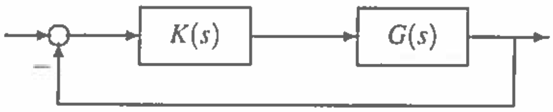

Figure 2.1

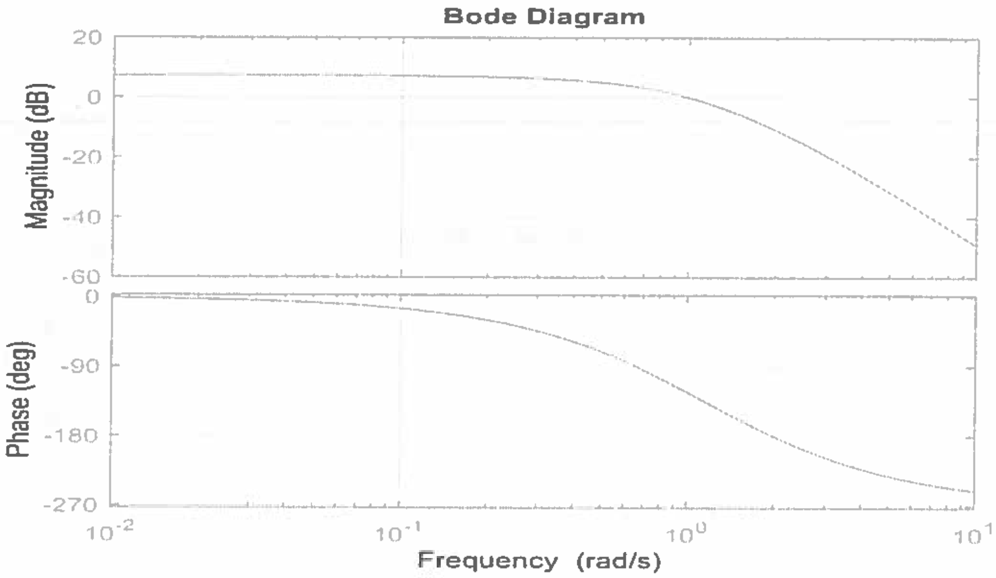

Figure 2.2

a) Use the frequency response to sketch a **rough** Nyquist diagram of $G(s)$, indicating the low and high frequency portions and the real–axis intercepts. [5]

b) Give approximate values for the crossover frequency and the gain and phase margins. Comment on the adequacy of the stability margins. [5]

c) i) State the Nyquist stability criterion. [5]

ii) Use the Nyquist stability criterion to determine the number of unstable closed-loop poles when:

I. $K(s) = 1$, [5]

II. $K(s) = 10$. [5]

d) Let $K(s)$ have the frequency response shown in Figure 2.3 overleaf. Describe $K(s)$ briefly and indicate its effects on the performance and stability of the feedback loop. [5]

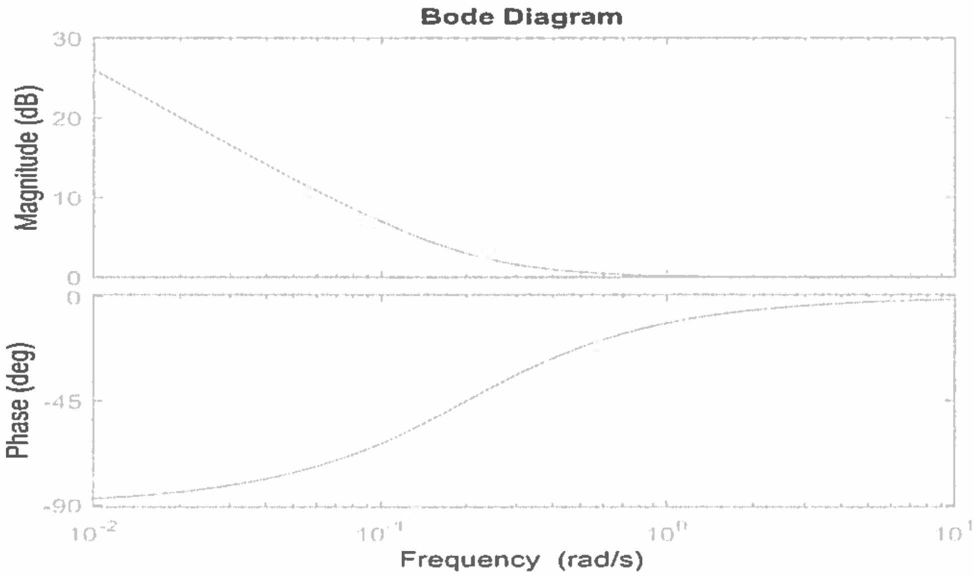
Bode Diagram
Magnitude (dB)
30
20
10
0
Phase (deg)
0
-45
-90
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/s)

Figure 2.3

3. Consider the feedback loop shown in Figure 3.1 below. Here

$$G(s) = \frac{1}{s-1}.$$

It is required to design a compensator $K(s)$ such that the closed–loop has a double pole at $s = -2$.

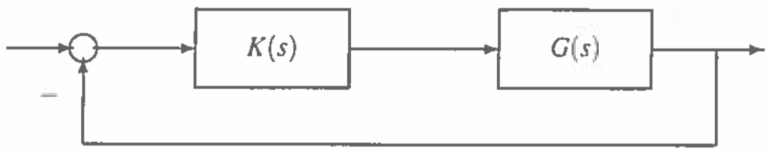

Figure 3.1

a) Draw the root–locus of $G(s)$. [6]

b) Design a PI compensator $K(s) = K_P + K_I s^{-1}$, where $K_P > 0$ and $K_I > 0$, that achieves the design specification as follows:

i) Write $K(s)$ as $K(s) = K_P(s+z)/s$ where $z = K_I/K_P$. Draw the root–locus of the compensated system $\hat{G}(s) = \frac{s+z}{s(s-1)}$ for some arbitrary $z > 0$. [6]

ii) Demonstrate using the root–locus that the design specifications can be achieved in principle for some value of z. [6]

iii) Find the value of z that achieves the design specification. [6]

iv) Use the gain criterion to find the value of K_P. Hence, deduce the value of K_I. [6]