UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1996

BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSc Degree in Computing Science
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute
Associateship of the Royal College of Science

PAPER I3.18 / M339

SIMULATION AND MODELLING
Thursday, May 16th 1996, 10.00 - 12.00

Answer THREE questions

1   In a *priority* queueing system customers are allocated a priority on arrival, which is a
      number between 1 and n inclusive, the highest priority being n. There is a single server
      and after a service completion the customer with the highest priority is allocated the server
      next. If more than one such customer exists the one who arrived first is selected. If the
      server is idle on arrival the arriving customer goes straight into service. A customer in
      service is *not* interrupted by the arrival of a higher priority customer.

      For the system in question the arrival process in Poisson with rate r and the service times
      are exponentially distributed with mean $S(p)$ for priority p, $1 \leq p \leq n$.

      A simulator for this system models the stream of arriving customers by the following
      event procedure:

```
PROCEDURE Arrival() ;
   ProcessArrival( GetPri() ) ;
   Schedule( Arrival, Now() + ExpSample( 1/r ) )
END Arrival ;
```

      where `GetPri()` returns a sample priority and `ExpSample( x )` returns a sample
      from an exponential distribution with mean x.

a   Identify the state variables required by the simulator and write down pseudocode for the
      procedure `ProcessArrival` used above and the event procedure `Departure` for
      modelling service completion events. You may assume the existence of an object
      `Server` with access procedures `ClaimServer` and `ReleaseServer`.

b   The probability that an arriving customer is of priority i is given by $p(i)$, $1 \leq i \leq n$, so that

$$\sum_{i=1}^{n} p(i) = 1$$

      Explain how the single Poisson arrival stream above can alternatively be modelled by n
      separate Poisson streams in the simulator, one for each customer class. As part of your
      answer explain how the procedures `ExpSample` and `ProcessArrival` will be called
      and outline the initialisation code required to initiate the arrival streams.

      Hint: if two Poisson streams with rates r_1 and r_2 are merged, the resulting stream is also
      Poisson, but with rate $r_1 + r_2$.

c   Given the probability distribution function p(i) from Part b, define the procedure `GetPri`
      used above for allocating customer priorities.

      *You may use any program notation you wish so long as your solution is clear and
      readable. You need not define any support procedures you use provided you use suitably
      obvious names. Avoid all unnecessary detail.*

      Note: 50% of the marks are allocated to part a.

2 Consider the following tandem queueing network:
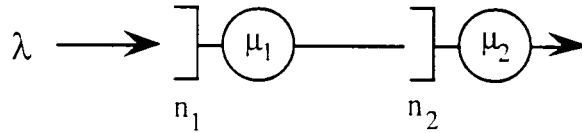
The arrival process is Poisson with rate λ and the service time distributions are both exponential with respective rates μ_1 and μ_2.

Let the state of the system be the pair (n_1, n_2) where n_1 is the population of the first queue and n_2 the population of the second, and let P_{n_1,n_2} be the probability that the system is in the state (n_1, n_2), $n_1, n_2 \geq 0$. For convenience, write $\rho_1 = \lambda/\mu_1$ and $\rho_2 = \lambda/\mu_2$.

2a Draw the Markov diagram showing the states (i,j), $i+j \leq 2$, and the associated transition rates between them.

2b Using the diagram, or any other means, write down the equilibrium balance equations for the system.

2c Show that the general solution $P_{n_1,n_2} = \rho_1^{n_1}(1-\rho_1) \, \rho_2^{n_2}(1-\rho_2)$, $n_1, n_2 \geq 0$, is a solution to your balance equation

2d From the general solution given in part 2c show that equilibrium probability distribution for the number of customers at the second queue, Q_n $n \geq 0$ say, is given by

$$Q_n = \rho_2^{n}(1-\rho_2)$$

2e Derive an expression for the expected total population of the system (i.e. the sum of the populations of each queue)

3a A random number generator has been written and has been called n times to produce the samples $X_1, ..., X_n$, $0 \leq X_i \leq 1$, $1 \leq i \leq n$, $n > 0$. To assess the quality of the generator, it is necessary to test for independence of the generated samples.

The *serial test* is a technique for testing the independence of pairs of samples in the sequence; it works as follows. Sample X_i is paired with sample X_{i+k} for some given $k < n$, $1 \leq i \leq n-k$. The resulting pairs are then used to update a *two-dimensional* histogram consisting of b^2 buckets, $B_{u,v}$, $1 \leq u, v \leq b$. For the pair (X_i, X_{i+k}) the index u is given by $\lfloor X_i/b \rfloor$ and v by $\lfloor X_{i+k}/b \rfloor$; the contents of bucket $B_{u,v}$ is incremented accordingly. The notation $\lfloor r \rfloor$ denotes the largest integer less than or equal to r.

If the samples are independent the expected contents of $B_{u,v}$ after all pairs have been processed is $(n-k)/b^2$ for all $1 \leq u, v \leq b$. The χ^2 test can therefore be used to compare the observed and expected bucket contents.

Apply the serial test to the following data at the 10% significance level, using k=1 and b=2. What conclusions can you draw from the test?

```
.560 .113 .889 .053 .573 .708 .340 .421 .713 .236 .247 .108
.683 .865 .298 .755 .086 .474 .313 .370 .644 .279 .137 .058
.353 .344 .136 .472 .860 .665 .499 .753 .282 .656 .644 .711
.726 .703 .926 .977 .775 .610 .940 .896 .158 .635 .648 .911
.892 .893 .577 .177 .118 .863 .359 .146
```

3b The *Weibull* distribution is often used to model "time to failure" for machinery or electronic components. Its cumulative distribution function (cdf) is given by

$$F(x) = 1 - \exp(-(x/\alpha)^\beta)$$

Show how the *transform* method can be used to generate samples from the Weibull distribution with parameters $\alpha = 1$ and $\beta = 2$. As part of your answer you should state any mathematical properties which the method assumes, although you are not required to prove those properties.

Note: 60% of the marks area allocated to part a.

4   A new packet-switched telecommunications network has been proposed which consists
    of a set of nodes, each with three I/O ports, interconnected in a mesh:
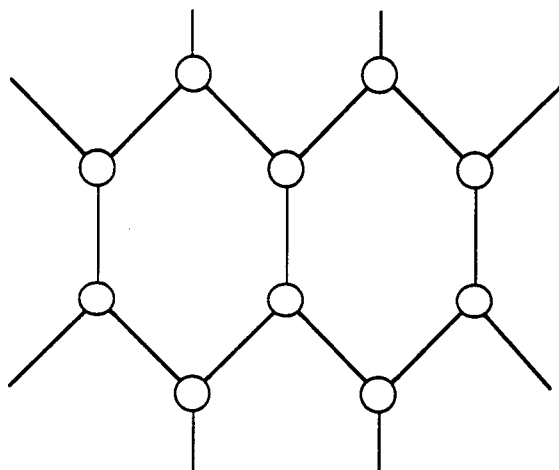
At certain times, a node (the *transmitter*) will initiate a communication with another node
(the *receiver*). This takes the form of a variable-sized packet which is routed to the
receiving node in accordance with a routing algorithm. The routing algorithm tells the
transmitter on which of its three output links it should send the packet first. When a
packet arrives on an input link the algorithm determines whether the packet is destined for
that node (i.e. if it is the receiver of the packet) and, if it is not, which output link to
forward it to in order to reach the destination. Each output link has a FIFO (first-in-first-
out) buffer containing any outstanding packets waiting to use the link.

A stochastic discrete-event simulator has been written to model this system for
experimental purposes, but no measurement code has yet been incorporated. You are
asked to advise the simulation designer on various aspects of the measurement process.

a   Explain *briefly* how the designer might incorporate code to estimate the following
    quantities (do *not* attempt to detail the required code):

    i.   The mean total packet transmission time from transmitter to receiver

    ii.   The mean number of packets queued at a given output buffer

    iii. The distribution of the number of packets received at a given node per unit time

b   It is required to take the above measurements when the system is at *equilibrium*. Explain
    what this means and how you would detect when equilibrium has been reached. How
    would you ensure that the measurements taken exclude any warm-up transient?

c   Having incorporated the measurement code the simulator is executed eight times, with
    each run being based on a different random number sequence. The mean packet
    transmission times in milliseconds recorded on each run are as follows:

        26.2   29.4   25.5   27.9   24.8   26.1   28.0   25.1

    Compute the 90% confidence interval for the mean transmission time.

End of paper