1.

a) ii) A positive mobile charge in a semiconductor [2]

Many also answered a void but that is wrong because an empty space cannot have a mass nor a charge.

b) because the electron mass is smaller than the hole mass [2]

No problems here

c) from the formulae sheet:. [4]

$$n = N_C exp\left(-\frac{E_C - E_F}{kT}\right)$$

$$p = N_V exp\left(\frac{E_V - E_F}{kT}\right)$$

$$n \times p = N_C \times N_V exp\left(-\frac{E_C - E_V}{kT}\right) \quad (1)$$

Definition of intrinsic level E_i

$$n_i = N_C exp\left(-\frac{E_C - E_i}{kT}\right)$$

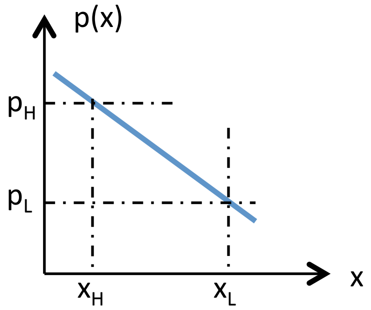
$$p_i = N_V exp\left(\frac{E_V - E_i}{kT}\right)$$

$$n_i \times p_i = n_i^2 = N_C \times N_V exp\left(-\frac{E_C - E_V}{kT}\right) \quad (2)$$

(1) = (2)

Explanation based on generation and recombination rate is also accepted, but all the reasoning steps need to be included. Many knew the expression for n_i as a function of E_G by heart. That is OK of course but not necessary as long as you remember the definition of intrinsic energy level.

d) Assume the variation of the hole concentration is given by the following figure: [4]



hole flux is from high to low concentration, thus in the +x direction

hole current is proportional to hole flux multiplied by charge, hole charge is positive, thus current is also in the +x direction

$\frac{dp}{dx} = \frac{p_H - p_L}{x_H - x_L} < 0$ (negative gradient) thus $\frac{dp}{dx}$ needs to be multiplied by -1 to get the correct direction.

Surprisingly many started with the drift-diffusion equation. Unfortunately this is not the correct approach as this equation already has the sign implicitly included, making it a circular reasoning.

e)

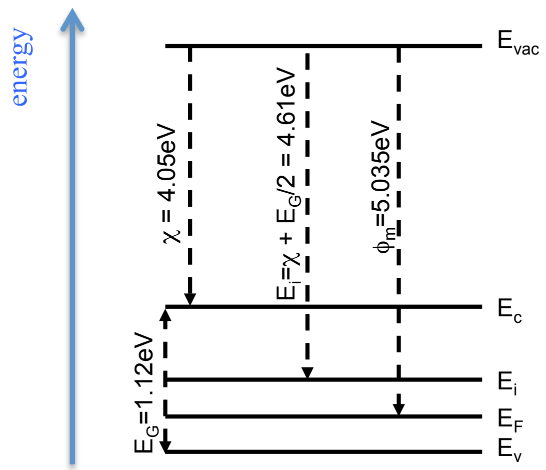
i) Holes (free carriers in valence band are holes) [2]

When answered wrong was probably because the words "free carriers" in the question and figure caption were not read.

ii) b) (potential energy increases when going deeper into the band) [2]

Deeper in the valence band means more energy is needed to excite the electrons into the conduction band.

f) Make a sketch of the energy band diagram and fill in all parameters. [6]



this sketch shows that it is a p-type semiconductor.

Different ways to calculate hole concentration:

$p = N_V \times exp\left(\frac{E_V - E_F}{kT}\right)$ (from equation sheet)

$p = 3.2\times10^{19} \times exp\left(\frac{\phi_m-(\chi+E_G)}{kT}\right) = 3.2\times10^{19}\times exp\left(\frac{5.035-4.05-1.12}{0.026}\right) \approx 1.0\times10^{17}\ cm^{-3}$

or

$p = n_i \times exp\left(\frac{E_i - E_F}{kT}\right)$ (derived from energy band diagram or know by heart)

$p = 1.45\times10^{10}\times exp\left(\frac{\phi_m - E_i}{kT}\right) = 1.45\times10^{10}\times exp\left(\frac{5.035-4.61}{0.026}\right) \approx 1.8\times10^{17}\ cm^{-3}$

error is due to using the value for $n_i = 1.45\times10^{10}\ cm^{-3}$

if calculating n_i based on the given N_c and N_v then:

$n_i = \sqrt{p\times n} = \sqrt{N_c N_v} exp\left(\frac{-E_G}{2\times kT}\right) = 1.06\times10^{10} cm^{-3}$

then $p = 1.3\times10^{17}\ cm^{-3}$

All answers are accepted as long as approach taken in correct.

Many sign errors were made in this question. E_v-E_F < 0 was often taken positive. This can be resolved by looking at the sketch above that shows that E_v has the lowest energy.

g) Since p^+n junction we assume that hole current is larger than electron current [6]

$I_{tot} \approx I_p$ [3]

using amplitudes: $|J_p| = eD_p \frac{dp}{dx} = eD_p \frac{p'_n - p_{n_0}}{L_p}$

excess minority carriers in n-region at depletion region edge are holes:

$p'_n - p_{n_0} = \frac{L_p|J_p|}{eD_p} = \frac{5\ 10^{-4}\times6.11\ 10^{-11}}{1.6\ 10^{-19}\times 2} = 95409\ cm^{-3}$ [3]

Mistakes made in this questions are:

1) many did not make the simplification that Jtot = Jp since it is a p+n diode. This was essential as otherwise not enough parameters were available for the calculations.
2) Very often Ln was used instead of Lp. Remember that the Lp is the diffusion length of the holes in the n-type region. That is the same as saying Lp is the minority carrier diffusion length in the n-type region.

h) Two doping types of more or less the same order of magnitude -> compensation doping. [8]

Derive formulae from equations that need to be know by heart (charge neutrality and law of mass action)

$-N_A + N_D - n + p = 0$ (1)

$n\times p = n_i^2$ (2) [4]

from (2) $p = \frac{n_i^2}{n}$ insert in (1)

$-N_A + N_D - n + \frac{n_i^2}{n} = 0$ rewrite to get quadratic equation in n:

$n^2 - (N_D - N_A)n - n_i^2 = 0$ solve quadratic equation and ensure positive result [4]

$n = \frac{(N_D-N_A)+\sqrt{(N_D-N_A)^2+4n_i^2}}{2} = \frac{(10^{18}-10^{17})+\sqrt{(10^{18}-10^{17})^2+4\times(1.45\times10^{10})^2}}{2} = 9\ 10^{17}\ cm^{-3}$

$p = \frac{n_i^2}{n} = \frac{(1.45\times10^{10})^2}{9\times10^{17}} \approx 234\ cm^{-3}$

This question is about compensation doping. This is a topic that was discussed explicitly in class this year and is also one of the questions in the study groups. There were only few students who did this question correct. Some wrote down the simplified version $n = N_D = 10^{18}$ cm^{-3} and $p = n_i^2/N_D$. In order for this to be acceptable reasoning needed to be given of why this is a good approximation. Surprisingly many gave $n = 1.1\ 10^{18}$ cm^{-3}. This is wrong because one cannot have more electrons than the doping added (for doped material).

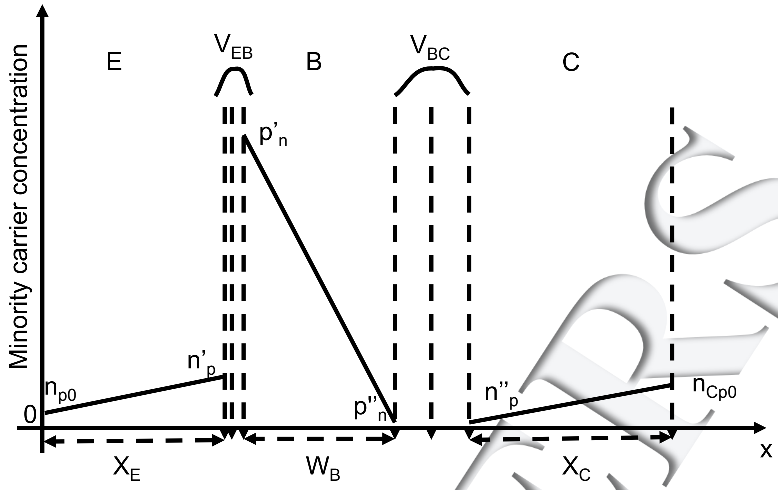
i) $V_{th} < 0$ [2]

j) the base region [2]

It is the minority carrier concentration variation **in the base** that determines the collector current in the situation where all material lengths are short.

2.

a) In region II the BJT is in forward active mode, thus EB junction forward biased and BC junction reverse biased. -> V_{EB}>0 and V_{CB}<0 in the plot below. [6]



The material widths defined in the plot above are junction widths minus the depletion regions. Thus these are the effective width of the regions.

Mostly correct, the main error was not defining all parameters. This is essential in order to ensure the parameters used in b) are clearly defined as copy and past from the formulae sheet can lead to errors when not well thought out.

b) $I_E = A\frac{eD_n^E(n'_p - n_{po})}{X_e} + A\frac{eD_p^B(p'_n - p''_n)}{W_B}$. [6]

since $n'_p \gg n_{p0}$ and $p'_n \gg p''_n$ this equation simplifies to:

$$I_E \approx A\frac{eD_n^E n'_p}{X_e} + A\frac{eD_p^B p'_n}{W_B}$$

$$I_E \approx A\frac{eD_n^E n_{p0} exp\left({}^{V_{EB}}/_{V_T}\right)}{X_e} + A\frac{eD_p^B p_{n0} exp\left({}^{V_{EB}}/_{V_T}\right)}{W_B}$$

with n_{po} minority carrier concentration in emitter and p_{n0} minority carrier concentration in base.

This equation neglects leakage currents from collector into base. Then:

$$I_C \approx A\frac{eD_p^B p_{n0} exp\left({}^{V_{EB}}/_{V_T}\right)}{W_B}$$

$$I_B \approx A\frac{eD_n^E n_{p0} exp\left({}^{V_{EB}}/_{V_T}\right)}{X_e}$$

A common mistake was writing:

$$I_E = A\frac{eD_n^E n_{po}\left(exp\left(\frac{V}{V_T}\right)-1\right)}{X_e} + A\frac{eD_p^B p_{no}\left(exp\left(\frac{V}{V_T}\right)-1\right)}{W_B}$$

The errors in this equation are:

1. V is left undefined
2. The second error is -1 in the second term. The second term actually reads:

$$A\frac{eD_p^B p_{no}\left[\left(exp\left(\frac{V_{EB}}{V_T}\right)\right)-\left(exp\left(-\frac{|V_{CB}|}{V_T}\right)\right)\right]}{W_B} \approx A\frac{eD_p^B p_{no}\left[\left(exp\left(\frac{V_{EB}}{V_T}\right)\right)-0\right]}{W_B}$$ a negative exponential gives a very small value < 1.

3. simplification were specifically asked for but were not always implemented as requested

c) Calculate the value for I_E and I_B for I_C = 1 mA, ignoring base width modulation. [3]

Current gain β can be extracted from fig. 2.1a region II.

$$I_C = \beta I_B$$

$$\beta = \frac{I_C}{I_B} = \frac{10^{-2}}{10^{-5}} = 1000$$

for I_C = 1 mA = 10^{-3} A I_B is then 10^{-6} A or 1 μA.

$I_E = I_C + I_B$ = 1 mA + 1 10^{-6} A ≈ 1 mA (0.001001 A)

Surprisingly, not all students realised that the values could be read off from the IC-VCE characteristics given in the question. In those cases theoretical calculations were tried out but often without the simplifications.
Another mistake was to have the signs wrong in the equation $I_E = I_C + I_B$ and then ending up with $I_E < I_C$ which is incorrect.

d) . [6]

Input resistance: $R_\pi = \left(\frac{dI_B}{dV_{EB}}\right)^{-1}$

$$\frac{dI_B}{dV_{EB}} = \frac{Ad\left(\frac{eD_n^E n_{p0} exp\left({V_{EB}}/{V_T}\right)}{X_e}\right)}{dV_{EB}} = A\frac{eD_n^E n_{p0} exp\left({V_{EB}}/{V_T}\right)}{V_T X_e} = \frac{I_B}{V_T}$$ or by taking all the constants together in I_s:

$$\frac{dI_B}{dV_{EB}} = \frac{d\left(I_s exp\left(\frac{V_{EB}}{V_T}\right)\right)}{dV_{EB}} = \frac{I_s exp\left({V_{EB}}/{V_T}\right)}{V_T} = \frac{I_B}{V_T}$$

$$R_\pi = \left(\frac{dI_B}{dV_{EB}}\right)^{-1} = \frac{V_T}{I_B}$$

Output resistance: $R_0 = \frac{\partial V_{ce}}{\partial I_C} = \frac{|V_A|}{I_C}$ assuming that $V_{CE} \approx V_{BC}$ with V_A the early voltage that is a parameter describing base width modulation. For Fig. 2.1a $R_o = 0\ \Omega$. This can also be derived by taking the formulae: $I_c = I_0\left(1 + \frac{V_{CE}}{V_A}\right)$

$$i_{BJT} = I_C = A\frac{eD_p^B p_{n0} exp\left({V_{EB}}/{V_T}\right)}{W_B}$$

Important is that these equations needed to be derived from their definition which need to be known by heart for devices and analogue electronics. Equations just learned by heart are not a sufficient answer to this question.

e) The datasheet for the pnp BJT in fig. 2.1 gives an input resistance R_π = 8 kΩ and the output conductance is g_o= 40 µS. Calculate the value of the transconductance g_m. [3]

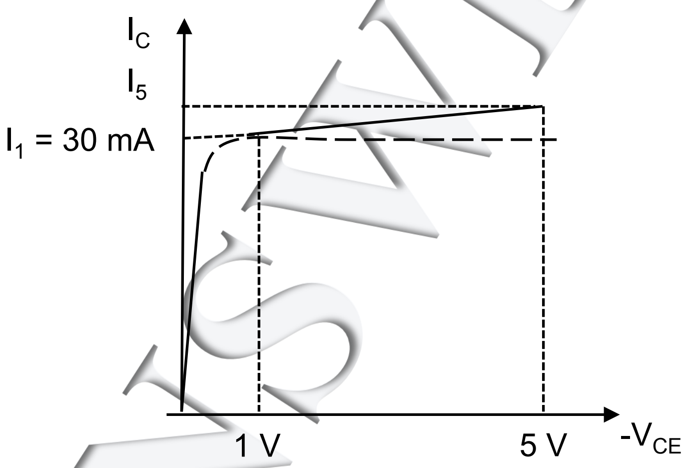
Derive the equation that relates R_π to g_m

$$R_\pi = \frac{dV_{EB}}{dI_B} = \frac{dV_{EB}}{dI_C} \times \frac{dI_C}{dI_B} = \frac{1}{g_m} \times \beta$$

$$g_m = \frac{\beta}{R_\pi} = \frac{10^3}{8\ 10^3} = 0.125\ S$$

f) Using the result of e) re-plot the output characteristic for I_B = -30 µA taking base width modulation into account and give the values of the collector current I_C for V_{CE} = -5 V. [6]

Sketching the output characteristic with base width modulation gives a slope in the current in region II. This slope is represented in the output conductance given in e)



Doing some simple algebra:
Take $R_o = g_o^{-1}$
$R_o = \frac{dV}{dI} = \frac{\Delta V}{\Delta I} = \frac{\Delta V}{I_5 - I_1}$
$I_5 - I_1 = \frac{\Delta V}{R_o} = g_o \Delta V$
$I_5 = g_o \Delta V + I_1$
$I_5 = 4\ 10^{-3} \times 4 + 0.03 = 0.046$ A

The sketch of the output characteristic was actually simple to make and can be made independent of whether the other questions were answered correctly. Just copy one of the characteristics given in the question figure and add a small slope to the characteristic in the constant current mode.
Once this is done, the calculations are just based on simple algebra using a linear equation. The correct numerical answer did indeed rely on e) but the method can always be shown first with parameters before putting in the numbers. Numbers should only be used in the last step and where simplifications need to be made.

3. p-channel depletion mode MOSFET

This was an easy question, therefore accuracy and consistency were of utmost importance.

a) . [6]

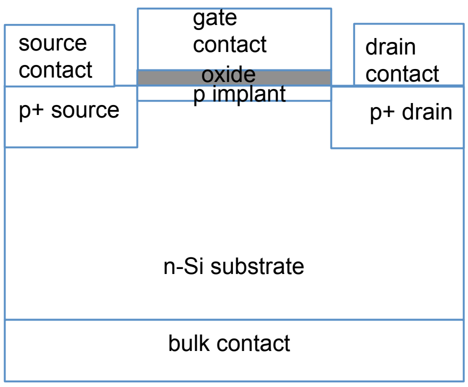
A p-channel depletion mode MOSFET is a MOSFET with p-type source and drain region, n-type bulk and **with holes in the channel region at $V_{GS} = 0$V**. A positive voltage will need to be applied to deplete the channel and switch the MOSFET off, $V_{th} > 0$.

Method 1: Implanting the top surface of the Si substrate with acceptor doping atoms, N_A such that the majority carrier concentration in that region are holes, thus taking into account compensation doping: $N_A > N_D$ (N_D is doping in substrate).

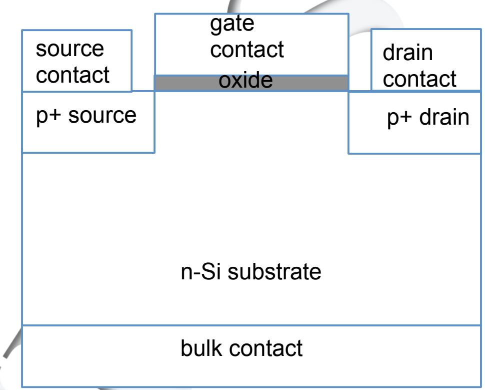
Method 2: choosing a suitable gate contact such that the workfunction difference between the n-type substrate and the gate material creates an inversion layer of holes: $\phi_{Si} < \phi_{gate}$

Many students tried to give a full process, concentration on the description of the answer given in b). That was not necessary as the material cross section is already given. The important part here is to identify **two** methods related to gate or substrate choice to have holes in the channel before a voltage is supplied. Very often only one method was given.

b) [6]



method 1



method 2

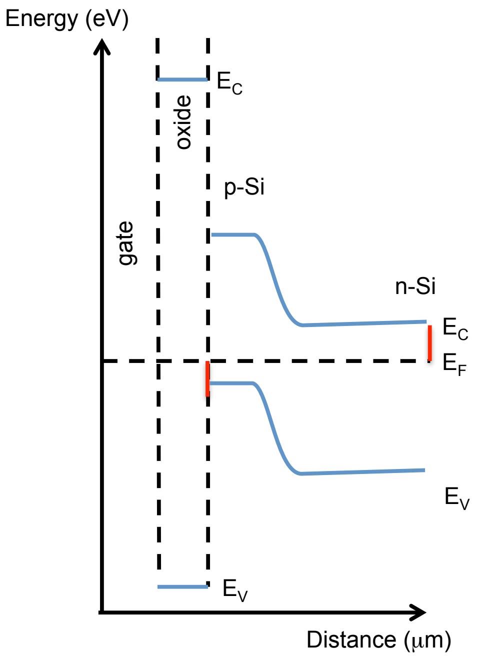
Very often drawings were not consistent with the method give in a).

c) Method 1

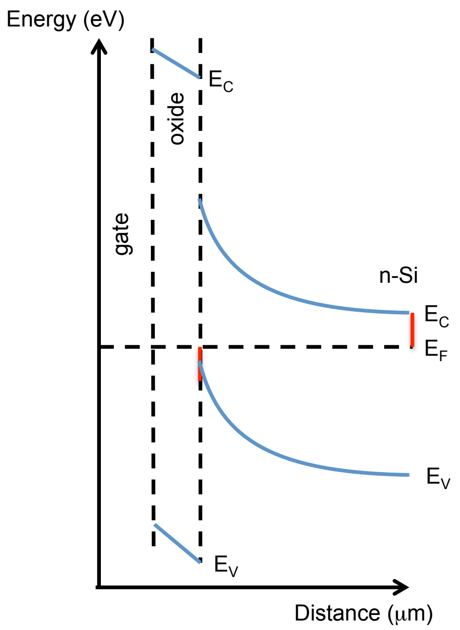
Assumptions:

1. no workfunction difference between the gate material and the Si substrate underneath therefore the oxide energy bands are flat
2. graded junction between the implanted p and n substrate region

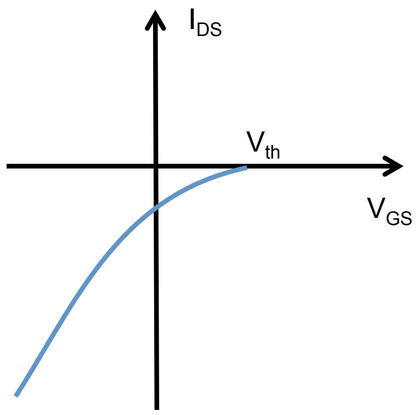
Note that the bands bend further than required for reaching threshold in order to have a true depletion mode device.



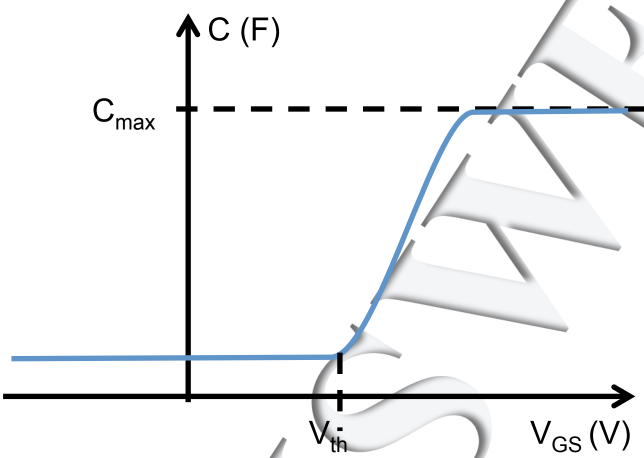
Method 2:
No assumptions (unless we want to assume that a suitable metal for the gate exist)
Note that the bands bend further than required for reaching threshold in order to have a true depletion mode device.



d) For both methods the IV is equivalent and must be quadratic in saturation [2] and V_{th} must be positive [2] and the currents negative for the p-MOS [2]. Currents are positive for the nMOS (convention). [6]



e) The CV characteristic of method 1 and 2 are similar to good approximation. [6]



C_{max} = ½ C_{ox} A for triode region or V_{DS} = 0V. $C_{max} \propto C_{ox}$ A also acceptable
$C_{ox} = \varepsilon_o \varepsilon_{ox}/t_{ox}$
The presence of absence of factor ½ was not taken up into the marking scheme.