EE1-06

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EEE/EIE PART I: MEng, Beng and ACGI

Corrected Copy

INTRODUCTION TO SIGNALS AND COMMUNICATIONS

Tuesday, 27 May 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : K.K. Leung

Second Marker(s) : C. Papavassiliou

Special Instructions for Invigilator: **None**

Information for Students:

Some Fourier Transforms

$$\cos\omega_o t \quad <=> \quad \pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$$

Some useful trigonometric identities

$$\cos x\cos y = \tfrac{1}{2}\cos(x-y)+\tfrac{1}{2}\cos(x+y)$$
$$\sin(x-y) = \sin x\cos y - \sin y\cos x$$
$$a\cos x + b\sin x = c\cos(x+\theta)$$

where $c=\sqrt{a^2+b^2}$ and $\theta=\tan^{-1}(-b/a)$

Complex exponential

$$e^{jx} = \cos x + j\sin x$$

1. This is a general question. (40%)

   a. Consider a periodic signal $f(t)$ with period T_0, which can be expressed as a Fourier series: $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$.

      i. How is ω_0 related to T_0? [1]

      ii. Show that the components, $\cos(m\omega_0 t)$ and $\cos(n\omega_0 t)$, are orthogonal to each other for $m \neq n$. [2]

      iii. If the signal has a property that $f(t) = f(-t)$ at all time t, what can be said about any of the series coefficients? Explain why. [2]

      iv. Given the property in part iii, obtain the power of the signal $f(t)$ and explain why. [3]

   b. For a baseband signal $g(t)$, let $\phi(t) = g(t)e^{j\omega_0 t}$ where ω_0 is a fixed angular frequency. Let $G(\omega)$ and $\Phi(\omega)$ be Fourier transforms for $g(t)$ and $\phi(t)$, respectively.

      i. Give the Fourier transform of the complex signal $e^{j\omega_0 t}$. [2]

      ii. Sketch the spectrum diagram for $e^{j\omega_0 t}$ from your result in part i. [2]

      iii. By finding the Fourier transform of $\phi(t)$, express $\Phi(\omega)$ in terms of $G(\omega)$. [2]

      iv. Sketch the spectra for both $G(\omega)$ and $\Phi(\omega)$ in the same diagram. [2]

      v. By examining the spectra in parts ii and iv (of part b), what can be said about the effect of $e^{j\omega_0 t}$ in $\phi(t)$ related to $g(t)$ and why? [2]

   c. A periodic signal $s(t)$ of an infinite sequence of unit impulses is given as follows.

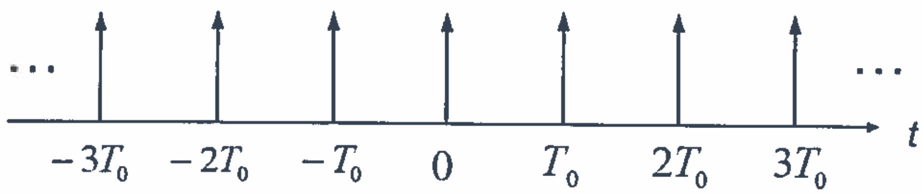

Figure 1. The periodic signal $s(t)$.

   Our objective here is to make use of this signal $s(t)$ to generate an amplitude-modulated (AM) signal $\phi(t)$ for a given modulating signal $m(t)$ and a sinusoidal carrier $\cos(\omega_c t)$. Let the Fourier transform of $m(t)$ be denoted by $M(\omega)$.

      i. Express $s(t)$ as an exponential Fourier series with coefficients D_n for integer n from $-\infty$ to ∞. [2]

      ii. Derive the coefficients D_n for all n from $-\infty$ to ∞. [2]

      iii. Using result in part ii, give an expression for $s(t)m(t)$. [2]

      iv. Sketch the spectrum diagram of $s(t)m(t)$. [2]

      v. From result in part iv, suggest a way to obtain the AM signal $\phi(t)$ and explain why. What is the relationship between T_0 and ω_c in your suggestion? [2]

1. This is a general question. (Continued)

d. Consider a frequency modulation (FM) signal, $\phi_{FM}(t)$, where $m(t)$ is the modulating signal, the carrier frequency is f_c =10 kHz, the carrier amplitude is A and k_f denotes the proportionality constant.

i. Give an expression for $\phi_{FM}(t)$. [2]

ii. Determine the instantaneous frequency for the FM signal as a function of time. [2]

iii. Assume that $m(t)$ is given by the following diagram. For this $m(t)$, the maximum deviation of the FM signal from the carrier frequency is Δf = 1 kHz. Determine k_f. [2]

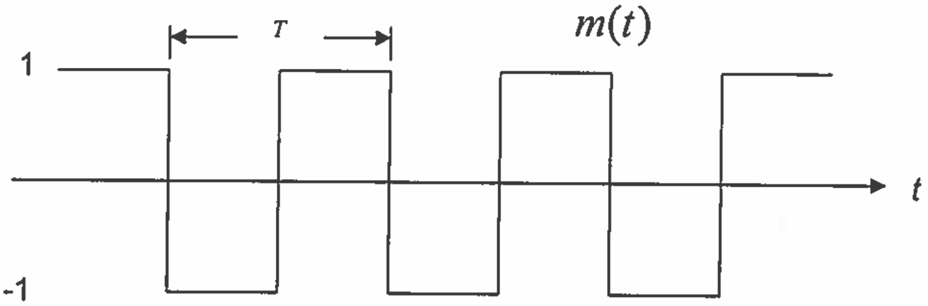

Figure 2. The modulating signal $m(t)$.

iv. Sketch the FM signal $\phi_{FM}(t)$ for the above modulating signal $m(t)$. [2]

v. Provide a block diagram of a receiver design to show how the FM signal can be demodulated by envelope detection and explain the physical meaning of each step in your design. [4]

2. Signals. (30%)

a. Let us obtain the self-convolution $y(t)$ of the signal $x(t)$ in Figure 3 where $y(t) = x(t) * x(t)$.

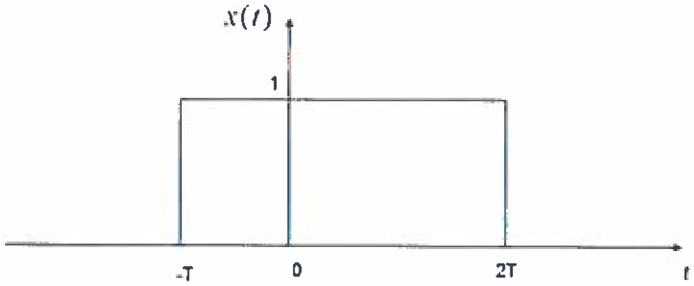

Figure 3. The signal $x(t)$.

i. Express $y(t)$ as a convolution integral of $x(t)$. [3]

ii. Identify several time intervals of t and carry out the convolution integration for $y(t)$ for each of these time intervals. [10]

iii. Sketch a diagram for $y(t)$ as a function of t. [2]

b. Consider a real signal $g(t)$ and derive the autocorrelation function and the corresponding power spectral density (PSD) as follows.

i. Express the autocorrelation function $R_g(\tau)$ for $g(t)$ as an integral of $g(t)$ where τ denotes the time lag. [2]

ii. Since $g(t)$ is real, prove that $R_g(\tau) = R_g(-\tau)$. [3]

iii. Let $G(\omega)$ and $S_g(\omega)$ be the Fourier transforms for $g(t)$ and $R_g(\tau)$, respectively. Prove that $S_g(\omega) = |G(\omega)|^2$. [5]

iv. Let $g(t)$ be an input to a linear, time-invariant (LTI) system with a transfer function denoted by $H(\omega)$. Suppose that $y(t)$ and $Y(\omega)$ are the system output and its Fourier transform. We further use $R_y(t)$ and $S_y(\omega)$ to denote the autocorrelation function for $y(t)$ and its Fourier transform, respectively. Prove that $S_y(\omega) = |H(\omega)|^2 S_g(\omega)$. [5]

3. Communications techniques. (30%)

a. A continuous-time signal $g(t)$ with bandwidth B Hz is periodically sampled, quantized and encoded into a sequence of 0 or 1 bits. Specifically, the sampling period is T_s seconds and each sample is encoded into K bits. The information bits are then transmitted by a communication link using the amplitude shift keying (ASK). Assume that the link can support the maximum data rate of R bits per second (bps). Let ω_c be the carrier frequency in radians/second and the amplitude of the modulated signal be 0 and A to represent the 0 and 1 bit, respectively.

   i. Give an expression for the transmitted signal using ASK on the link. [2]
   ii. Provide a block diagram and explain how to demodulate the ASK signal. [4]
   iii. In order to enable correct recovery of the transmitted signal, obtain the maximum bandwidth of the signal $g(t)$ in terms of R and K. Use a spectrum diagram to explain your reasoning. [6]

b. Design a wideband frequency modulation (WBFM) system using frequency multipliers as follows. Let $m(t)$ be the modulating signal. Assume that a narrow-band FM (NBFM) generator is available to take $m(t)$ as input and generates a narrow-band FM signal with a carrier frequency f_{NB} of 200 kHz and the maximum frequency deviation Δf_{NB} of 30 Hz. As the output of the whole system, the final carrier frequency f_c and the maximum frequency deviation Δf of the transmitting FM signal should be 100 MHz and 61.44 kHz, respectively. Assume that beside the oscillator at 200 kHz for the narrow-band FM generator, a second oscillator of another frequency is available. Furthermore, only multipliers that double the carrier frequency and frequency deviation are available.

   i. Using the NBFM generator, frequency multipliers and frequency converter as building blocks, draw a block diagram for the whole WBFM system. Indicate the carrier frequency and frequency deviation at each step and explain your design. [8]
   ii. If the second oscillator is used, what is its frequency? [2]
   iii. What purpose does the second oscillator serve? Provide a mathematical justification for how such use achieves its purpose. [5]
   iv. Is the design of the WBFM system unique? Can the second oscillator be of a different frequency? Explain. [3]