Paper Number(s): **E2.6**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

EEE PART II: M.Eng., B.Eng. and ACGI

CONTROL ENGINEERING

Wednesday, 20 June 2:00 pm

There are FIVE questions on this paper.

Answer THREE questions.

Time allowed: 2:00 hours

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Examiners: Limebeer,D.J.N. and Astolfi,A.

1. Consider the vibration isolation arrangement illustrated in Figure 1. below:

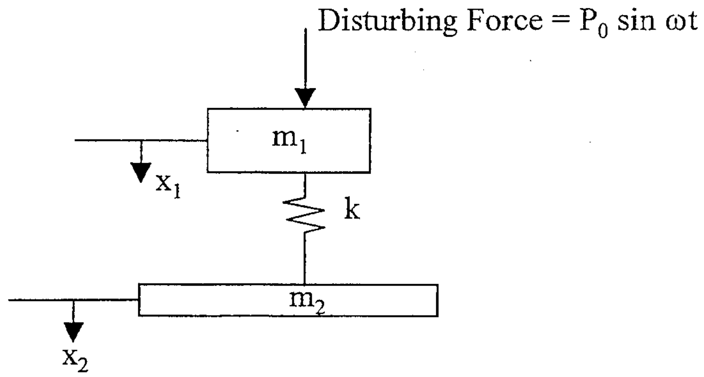

Figure 1

a) Derive the equations of motion for this system. [5]

b) Show that they have a solution of the form :

$$x_1(t) = x_{1m} \sin \omega t$$

$$x_2(t) = x_{2m} \sin \omega t$$

[5]

c) By finding an expression for x_{2m}, show that the system has a resonant frequency given by:

$$\omega_n^2 = \frac{k(m_1 + m_2)}{m_1 m_2}$$

[5]

d) Show that the force transmitted to m_2 by the spring is

$$F = \frac{m_2}{m_1 + m_2} \cdot \frac{P_0 \sin \omega t}{\frac{\omega^2}{\omega_n^2} - 1}.$$

[5]

2. Consider the system with open-loop transfer function:

$$G(s) = \frac{500}{s(s+15)}.$$

Suppose, this system is placed in a unity-gain feedback closed-loop with a variable gain k in the forward path.

a) Find the closed-loop transfer function and the associated closed-loop characteristic polynomial. [5]

b) Find expressions, in terms of k, for ξ and ω_n. [5]

c) Find the value of k that results in $\omega_n = 30$. What is the corresponding value of ξ? [5]

d) If the system is subjected to a ramp input of 0.5 rad/s, what is the steady-state tracking error? [5]

3. a) Carefully explain the differences between a Nyquist diagram of G(s) and a Nyquist diagram of -G(s). [3]

Consider the system in Figure 2. below:

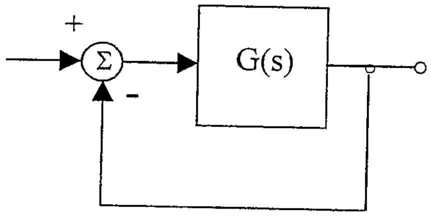

Figure 2.

in which

$$G(s)=\frac{3}{s(s+1)^2}$$

b) Sketch the Nyquist diagram of G(s). [5]

c) Use the Nyquist diagram in b) to determine the number of closed-loop poles in the right-half plane. [3]

d) Check the result in c) using the Routh-Hurwitz criterion. [3]

e) Suppose a variable gain k is introduced into the feedback loop. Find the value of k that produces a marginally stable system and find the associated frequency of oscillation. [6]

4. Figure 3 shows a feedback control systems with internal rate compensation.

a) Sketch the root-locus for the case that $\beta = 0$, that is the zero rate feedback case. [6]

b) Set $K_1 = 2$ and $K_2 = 5$, and plot another root-locus with β the varied parameter. [7]

c) Determine the value of β such that the closed-loop system is critically damped. [7]

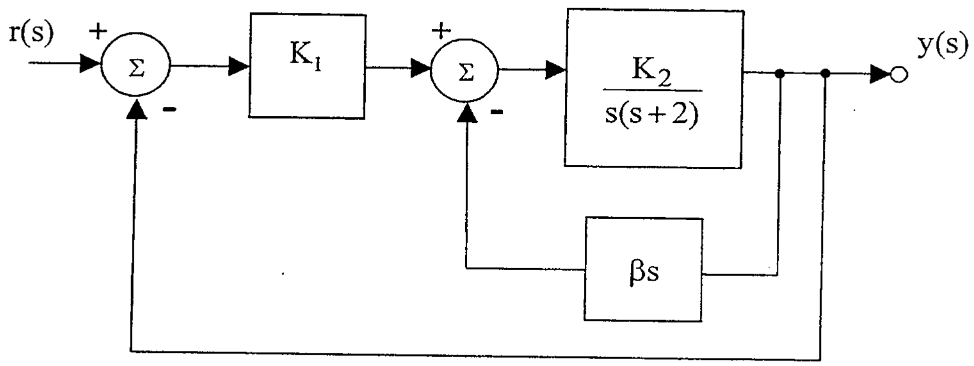

Figure 3.

5. Figures 4 and 5 show lateral and axial views of a point mass m rigidly mounted on a rotating flexible shaft. The angular velocity of the shaft is constant at ω. The eccentricity of the mass and the rotation combine to produce a deflection of the point of attachment S away from the point B on the axis of the bearings.

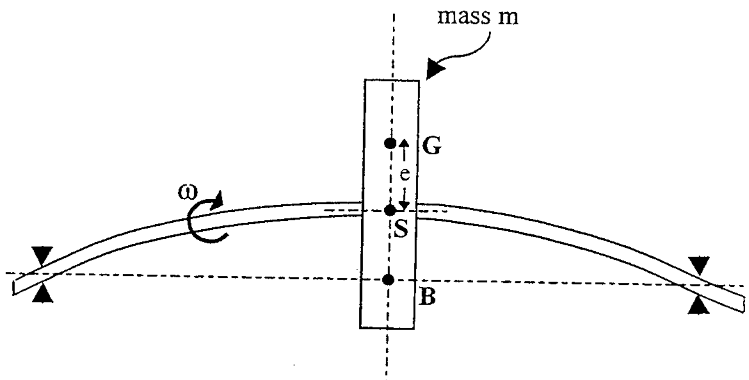

Figure 4.

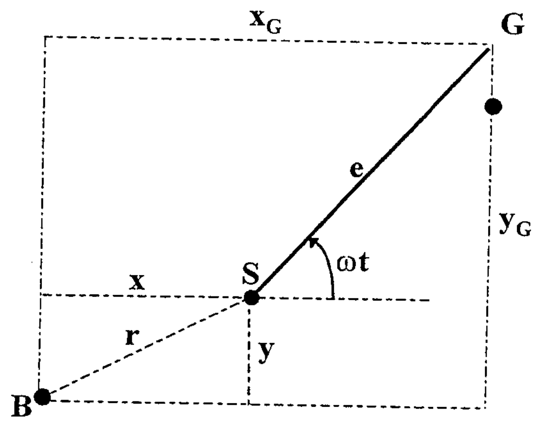

Figure 5.

Suppose that the spring constant of the deflection of the shaft is k and that, relative to the axis point B, the x-y coordinates of the centre of gravity G of the mass are given by

$$x_G = x + e \cos \omega t$$

$$y_G = y + e \sin \omega t$$

where x and y are the coordinates of the attachment point S.

Question 5 continues on the next page

a) Using Newton's second law, show that

$$m\ddot{x} + kx = m\omega^2 e \cos \omega t$$
$$m\ddot{y} + ky = m\omega^2 e \sin \omega t.$$ [5]

b) Prove that the equations in a) have solutions

$$x(t) = A \cos \omega t$$
$$y(t) = A \sin \omega t$$

where

$$A = \frac{e}{\left(\frac{k}{m\omega^2} - 1\right)}.$$ [5]

c) Explain the sense in which $\omega_c = \sqrt{\frac{k}{m}}$ is a "critical speed". [4]

d) Sketch the shaft deflection r as a function of the angular velocity ω. [6]

Question 1

a) Balancing forces on the two masses gives:

$$m_1\ddot{x}_1 + k(x_1 - x_2) = P_0 \sin\omega t$$

$$m_2\ddot{x}_2 = k(x_1 - x_2) \qquad [5]$$

b) $$\ddot{x}_2 = -\omega^2 x_{2m} \sin\omega t$$

$$\Rightarrow -m_2\omega^2 x_{2m} \sin\omega t = k(x_{1m} - x_{2m}) \sin\omega t$$

$$\therefore \quad x_{2m}(k - m_2\omega^2) = k\, x_{1m}$$

$$\ddot{x}_1 = -\omega^2 x_{1m} \sin\omega t$$

$$\Rightarrow -m_1\omega^2 x_{1m} + k_2(x_{1m} - x_{2m}) = P_0$$

$$x_{2m}\left[(k - m_1\omega^2)\left(1 - \frac{m_2\omega^2}{k}\right) - k\right] = P_0$$

$$\therefore \quad x_{2m}\left[m_1 m_2 \omega^4 - (m_1 + m_2)\omega^2 k\right] = k P_0$$

$$\therefore \quad x_{2m} = \frac{P_0}{(m_1+m_2)\omega^2\left[\frac{m_1 m_2 \omega^2}{(m_1+m_2)k} - 1\right]} \qquad [5]$$

c) $$\therefore \quad \omega_n^2 = \frac{k(m_1+m_2)}{m_1 m_2}$$

gives

$$x_{2m} = \frac{P_0}{(m_1+m_2)\omega^2\left[\omega^2/\omega_n^2 - 1\right]} \qquad [5]$$

d) $F = m_2 \ddot{x}_2$

$= -m_2 \omega^2 x_{2m} \sin \omega t$

$= \dfrac{-m_2 \omega^2 P_0 \sin \omega t}{(m_1+m_2)\omega^2 \left[\omega^2/\omega_n^2 - 1\right]}$ [5]

Question 2

1) a) $$H(s) = \frac{k\,G(s)}{1 + k\,G(s)}$$

$$= \frac{500k}{s^2 + 15s + 500k}$$ [5]

1) b) $$\omega_n = \sqrt{500k}$$

$$\xi = 15/2\omega_n$$

$$= 3/4\sqrt{5k}$$ [5]

1) c) $900 = 500k \Rightarrow$ $\underline{k = 9/5}$

$$\xi = 3/4\sqrt{9}$$

$$= \underline{0.25}$$ [5]

1) d) $$e(s) = \frac{r(s)}{1 + k\,G(s)}$$

$$= \frac{s(s+15)}{s^2+15s+500k} \cdot \frac{1}{2s^2}$$

$$e(\infty) = \lim_{s \to 0} s\,e(s) = \left.\frac{s+15}{s^2+15s+500k}\right|_{s=0} \cdot \frac{1}{2}$$ [5]

$$= \frac{15}{500k} \cdot \frac{1}{2} = \frac{15 \times 5}{500 \times 9 \times 2} = \underline{8.3 \times 10^{-}}$$

$= 1/120$

Question 3

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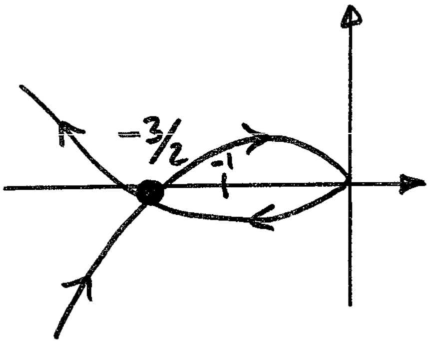
a) points $a + jb$ become points $-a - jb$. [3]

b)



$$g(j\omega) = \frac{3}{j\omega(-\omega^2 + 2j\omega + 1)}$$

@ $\omega = 1$

$$g(j) = 3/j(2j) = -3/2$$

[5]

c) There are two encirclements of the -1 point, and so there must be two poles of the closed-loop in the rhp. [3]

d) $clcp = s^3 + 2s^2 + s + 3$

Hence the Routh array is:

1	1
2	3
-1	0
1	

[3]

Two sign changes $\Rightarrow$ two rhp roots in the clcp

e) $$\left.\begin{aligned} clcp &= s^3 + 2s^2 + s + 3k \\ &= (s^2 + \omega^2)(s + \alpha) \\ &= s^3 + \alpha s^2 + \omega^2 s + \omega^2 \alpha \end{aligned}\right\} \Rightarrow \quad \begin{aligned} \alpha &= 2 \\ \omega &= 1 \\ k &= 2/3 \end{aligned}$$ [6]

Question 4

$\frac{5}{8}$

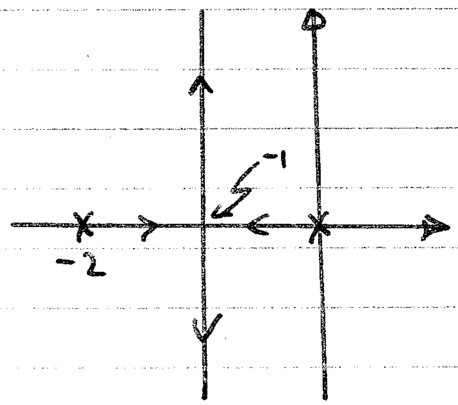
a) $$y(s)/v(s) = \frac{K_1\, g(s)}{1 + K_1\, g(s)}$$

where

$$g(s) = \frac{K_2 / s(s+2)}{1 + \beta s K_2 / s(s+2)}$$

$$= \frac{K_2}{s^2 + 2s + \beta s K_2}$$

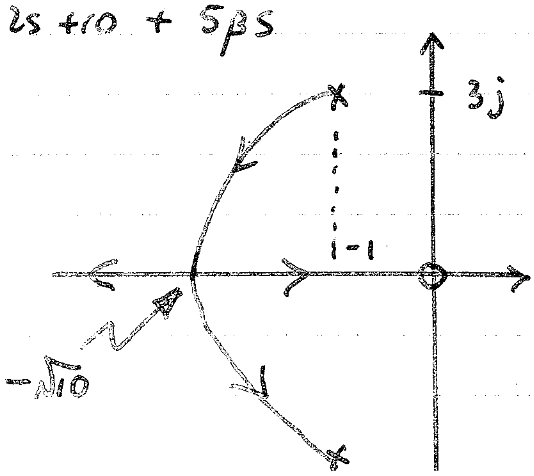
$$\therefore \quad y(s)/v(s) = \frac{\overbrace{K_1 K_2}^{k}}{s^2 + s(2 + \beta K_2) + \underbrace{K_1 K_2}_{k}}$$



[6]

b) $$\text{clcp} = s^2 + s(2 + 5\beta) + 10$$

$$= s^2 + 2s + 10 + 5\beta s$$



[7]

c) $s^2 + s(2+5\beta) + 10 = (s+\alpha)^2$

$$= s^2 + 2\alpha s + \alpha^2$$

$\therefore \quad \alpha = \sqrt{10}$

$$2 + 5\beta = 2\sqrt{10}$$

$$\beta = \frac{2(\sqrt{10}-1)}{5}$$

$$= \underline{0.8649} \qquad [7]$$

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Question 5

a)
$$\left.\begin{aligned} m\ddot{x}_G + kx &= 0 \\ m\ddot{y}_G + ky &= 0 \end{aligned}\right\} \quad \text{Newton II}$$

Now
$$\ddot{x}_G = \ddot{x} - e\omega^2 \cos\omega t$$
$$\ddot{y}_G = \ddot{y} - e\omega^2 \sin\omega t$$

and so
$$m\ddot{x} + kx = me\omega^2 \cos\omega t$$
$$m\ddot{y} + ky = me\omega^2 \sin\omega t$$
[5]

b)
$$x(t) = A\cos\omega t \Rightarrow \ddot{x} = -A\omega^2 \cos\omega t$$
$$y(t) = A\sin\omega t \Rightarrow \ddot{y} = -A\omega^2 \sin\omega t$$
$$\therefore -Am\omega^2\cos\omega t + kA\cos\omega t = me\omega^2\cos\omega t$$
$$\therefore A(k - m\omega^2) = me\omega^2$$
$$\therefore A = e\Big/\left(\frac{k}{m\omega^2} - 1\right)$$
[5]

c) @ $\omega = \sqrt{k/m}$, the value of A "explodes" and so, therefore, does the shaft deflection. [4]

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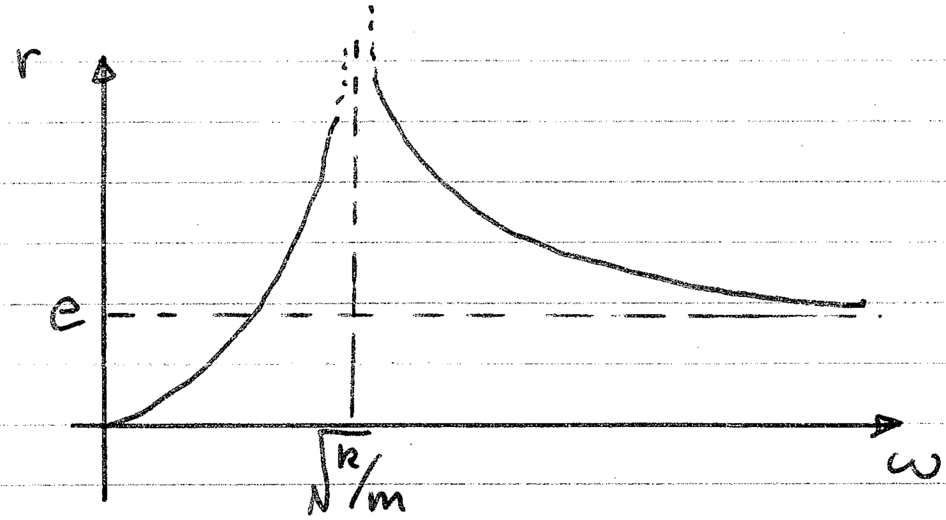
d)



[6]