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June 06

E2.3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2006

EEE PART II: MEng, BEng and ACGI

not shown, as material is different in 0607.

POWER, FIELDS AND DEVICES

Corrected Copy

Thursday, 1 June 2:00 pm

Time allowed: 3:00 hours

There are NINE questions on this paper.

There are three sections. Answer FIVE questions including at least ONE question from each of sections A, B and C.

Use a separate answer book for each section.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T.C. Green, R.R.A. Syms, A.S. Holmes

Second Marker(s) : C.A. Hernandez-Aramburo, C.A. Hernandez-Aramburo, W.T. Pike

Information for Candidates

Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho \quad ; \quad \mathbf{D} = \varepsilon \mathbf{E} \qquad\qquad \nabla \cdot \mathbf{B} = 0 \quad ; \quad \mathbf{B} = \mu \mathbf{H}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Physical constants:

$$\varepsilon_0 = 8.854 \text{ x } 10^{-12} \text{ F/m}$$

$$\mu_0 = 4\pi \text{ x } 10^{-7} \text{ H/m}$$

Section A

1. This question covers several topics and all parts should be attempted.

(a) A three-phase load draws a line current of 250 A from an 33 kV supply. The power factor is 0.85 lagging. Calculate the real and reactive power of the load. [3]

(b) A single-phase voltage source of 11 kV is connected to a load via a cable. The load has an impedance of 100+j35 Ω and the cable has an impedance of 2+j5 Ω. Calculate the voltage appearing across the load, the real and reactive powers of the load and the efficiency with which power is delivered to the load. [4]

(c) Derive the equation relating output voltage to input voltage and duty-cycle for a buck SMPS in continuous conduction. State any assumptions made. [3]

(d) Explain why closed-loop rather than open-loop control is used to regulate the output voltage of an SMPS. [2]

(e) A 6-pole induction machine connected to a supply with a frequency of 60 Hz runs at a speed of 1150 r.p.m. Calculate the slip. [2]

(f) Explain why the stator and rotor an electrical machine are made of steel and why the air-gap between them is kept small. [2]

(g) Explain why an electricity system uses a several different voltages within a distribution and transmission network. [2]

(h) Explain why a sudden reduction of the power consumed by the loads of an AC electricity system causes a change in the frequency and explain whether the frequency is expected to rise or fall. [2]

2. A boost switch-mode power supply, SMPS, is to be used to convert an input voltage of 5 V to an output voltage of 24 V. The inductor to be used has a value of 500 μH.

(a) Draw a circuit diagram of a boost SMPS. [2]

(b) Calculate the duty-cycle at which the circuit should be operated assuming continuous conduction. [2]

(c) Calculate the input and output currents, assuming a perfectly efficient circuit when the SMPS processes 1 W. [2]

(d) Sketch the waveform of the current through the diode and through the capacitor and express the current flow through the capacitor during the on- and off-times in terms of the output current and inductor current. [3]

(e) Calculate the maximum effective series resistance of the output capacitor if the output voltage ripple is to be kept below 20 mV when processing a power of 1 W. You may assume that the capacitance of the capacitor does not contribute to the voltage ripple and that the ripple of the inductor current is small compared to its average value. [2]

(f) Give an equation for the inductor current ripple as a function of the duty-cycle and frequency. [2]

(g) Calculate the operating frequency required to ensure that the circuit remains in continuous conduction when processing a power of 250 mW. [4]

(h) Find the relationship between the time for which the diode conducts and the on-time of the switch for discontinuous conduction mode and hence find the on-time required to process an input power of 100 mW if the switching frequency is 80 kHz. [3]

3. A 2-pole (1-pole-pair), three-phase induction is under design for a 50 Hz supply. The intention is to produce a machine capable of developing a torque of 20 Nm at a slip of 0.05 and an efficiency of 85%.

The design calculations suggest the machine has the following reactance and resistance parameters.

X_M = 70.0 Ω
X'_R = 5.0 Ω
R'_R = 1.0 Ω
X_S = 5.0 Ω
R_S = 1.5 Ω

The calculations that follow are intended to reveal whether the design has achieved its goal.

(a) Draw the per-phase equivalent circuit of a three-phase induction machine. [3]

(b) The machine has been designed for a current flow of 3.0 A in the magnetising reactance and a rated rotor current of 10.0 A.
  (i) Calculate the voltage across the magnetising reactance needed to achieve the desired magnetising current. [1]
  (ii) Assuming that the voltage calculated in (b)(i) is present, calculate the value of slip at which the rated rotor current will be drawn. [4]

(c) Calculate the electro-mechanical power and the torque developed by the machine under the conditions of (b). [4]

(d) Estimate the resistive power losses ("copper losses") and the efficiency of the machine when operating at maximum current. The iron loss can be ignored and the stator current for small slip can be approximated by $I_S \approx \sqrt{I_M^{\,2} + I'^{\,2}_R}$. [4]

(e) Once built, the machine was tested to establish the actual value of magnetising inductance and the magnetising resistance (which had been ignored in the design). With the rotor spinning at synchronous speed, phase voltages of 220 V were applied. The stator current was recorded as 3.2 A and the total power read on a three-phase wattmeter was 620 W.
  (i) Estimate the value of magnetising resistance [2]
  (ii) Estimate the value of the magnetising inductance [2]

Section B

Use a separate answer book for this section

4. a) What is the ionosphere? How does it affect the propagation of radio waves? [2]

   b) What three factors govern the choice of wavelengths used for night vision equipment? [3]

   c) What three factors limit the possibilities for free space communication? [3]

   d) What was Maxwell's crucial modification to Ampere's law? [2]

   e) The Poynting vector $\underline{\boldsymbol{S}} = \underline{\boldsymbol{E}} \times \underline{\boldsymbol{H}}$ describes the power density of an electromagnetic wave. Explain why this is not a useful concept for high frequency waves. [2]

   f) The phase velocity is of a travelling wave is ω/k. Show that the velocity of a group of waves is $d\omega/dk$. [3]

   g) The characteristic impedance Z_0 of an infinite, lossless transmission line is $\sqrt{(L/C)}$. Give a physical explanation for why Z_0 should be a real number. [2]

   h) What is an antireflection coating used for? How should the refractive index and thickness of the coating relate to the materials involved? [3]

5. a) Figure 5 shows an equivalent circuit for a section of a transmission line. Derive the transmission line equation, and show that the phase velocity of waves on the line is $1/\sqrt{(LC)}$. [8]

b) A first line of characteristic impedance Z_1 is connected to a second line of characteristic impedance Z_2. Find the reflection coefficient for voltage waves incident from the first line. [6]

c) A 50 Ω transmission line is connected in a splitter topology to two parallel 75 Ω lines. Devise a circuit that will suppress reflections for signals incident from the 50 Ω line. What is the reflection coefficient for signals incident from one or other of the 75 Ω lines? [6]

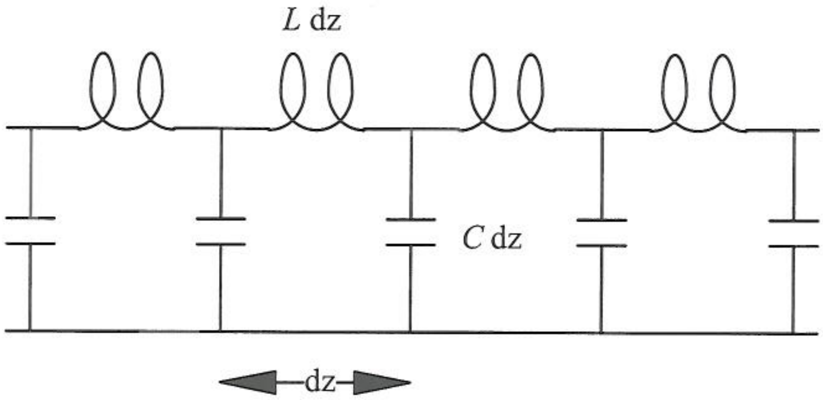


Figure 5

6. a) Give three reasons why it is advantageous to transmit signals using a high-frequency electromagnetic carrier. Explain briefly the operation of a broadside antenna array. [5]

b) Prove the Friis formula for the received power in a radio transmission link

$$P_r = P_t(\eta_t \eta_r A_t A_r / R^2 \lambda^2)$$

Here P_r and P_t are the received and transmitted powers, A_r and A_t are the effective areas of the receiving and transmitting antennae, η_r and η_t are the efficiencies of the receiving and transmitting antennae, R is the range and λ is the wavelength. [5]

c) The time-independent wave equation for a travelling electrical field (say, E_x) in a good conductor is

$$d^2E_x/dz^2 = j\omega\mu_0\sigma E_x$$

Derive an analytic expression for the skin depth. Describe two consequences of the skin effect. [5]

d) Figure 6 shows a cross-section of a stripline waveguide. Making suitable approximations, derive an analytic expression for its characteristic impedance. [5]

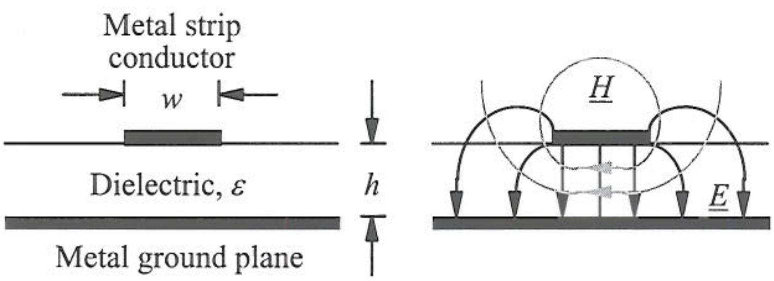


Figure 6

Section A

1. (a) A three-phase load draws a line current of 250 A from an 33 kV supply. The power factor is 0.85 lagging. Calculate the real and reactive power of the load. [3]

A positive lagging power factor means current lags voltage by between 0 and 90°. Both P and Q will be positive.

$$P = \sqrt{3} V_L I_L \cos(\phi)$$
$$= \sqrt{3} \times 33 \times 10^3 \times 250 \times 0.85 = 12.1\ MW$$
$$Q = \sqrt{3} V_L I_L \cos(\phi)$$
$$= \sqrt{3} \times 33 \times 10^3 \times 250 \times \sin\left(\cos^{-1}(0.85)\right) = 7.5\ MVAr$$

(b) A single-phase voltage source of 11 kV is connected to a load via a cable. The load has an impedance of 100+j35 Ω and the cable has an impedance of 2+j5 Ω. Calculate the voltage appearing across the load, the real and reactive powers of the load and the efficiency with which power is delivered to the load. [4]

$$V_L = V_S \frac{Z_L}{Z_L + Z_C}$$
$$= 11 \times 10^3 \times \frac{100 + j35}{100 + j35 + 2 + j5} = 11 \times 10^3 \times \frac{105.95\angle 19.29°}{109.56\angle 21.41°} = 10.36\angle -2.12°\ kV$$
$$I_L = \frac{V_S}{Z_L + Z_C}$$
$$= \frac{11 \times 10^3}{109.56\angle 21.41°} = 100.40\angle -21.41°\ A$$
$$P_L = I_L^{\,2} \operatorname{Re}\{Z_L\}$$
$$= 100.4^2 \times 100 = 1.008\ MW$$
$$Q_L = I_L^{\,2} \operatorname{Im}\{Z_L\}$$
$$= 100.4^2 \times 35 = 0.352\ MVAr$$
$$\eta = \frac{P_L}{P_L + P_C} = \frac{I_L^{\,2} \operatorname{Re}\{Z_L\}}{I_L^{\,2} \operatorname{Re}\{Z_L\} + I_L^{\,2} \operatorname{Re}\{Z_C\}} = \frac{100}{100 + 2} = 98.0\ \%$$

(c) Derive the equation relating output voltage to input voltage and duty-cycle for a buck SMPS in continuous conduction stating any assumptions made. [3]

Assume that the circuit is in steady-state so that there is no net change of inductor current over one switching cycle

$$\Delta i_L^{On} + \Delta i_L^{Off} = 0$$

Assume that the voltage drops across the semiconductors are negligible, the inductor has no resistance and that the capacitor is large valued and its voltage remains constant over a switching cycle. The changes in current during the on- and off-times can then be calculated from the linear rate-of-change of current.

$$\Delta i_L^{On} = \frac{V_I - V_O}{L} t_{On}$$

$$\Delta i_L^{Off} = \frac{-V_O}{L} t_{Off}$$

$$\frac{V_O}{V_I} = \frac{t_{On}}{t_{On} + t_{Off}} = \delta$$

(d) Explain why closed-loop rather than open-loop control is used regulate the output voltage of an SMPS. [2]

Open-loop control would assume that output voltage is a simple function of duty-cycle and input voltage. The appropriate duty-cycle would be calculated and used. Because there are current dependent voltage drops in the circuit not included in this equation the output voltage will decrease as current is drawn. Also, because discontinuous operation will occur at low output currents, the assumed equation will not always apply. To hold the output voltage constant in the light of these two effects requires closed loop control.

(e) A 6-pole induction machine connected to a supply with a frequency of 60 Hz runs at a speed of 1150 r.p.m. Calculate the slip. [2]

$$s = \frac{\omega_S - \omega_R}{\omega_S} = \frac{n_S - n_R}{n_S} = \frac{\frac{60f}{P} - n_R}{60f/P} = \frac{1200 - 1150}{1200} = 4.17\%$$

(f) Explain why the stator and rotor of an electrical machine are made of steel and why the air-gap between them is kept small. [2]

Steel provides a low reluctance path for magnetic flux and so the MMF required (from field winding or PM) is lower for a given flux than with air. Steel also has a relatively high saturation flux density (compared to ferrites) and is mechanically strong and workable. The air-gap should be short because flux has to cross the gap and the gap is a significant part of the reluctance of the magnetic path.

(g) Explain why an electricity system uses a variety of voltage for distribution and transmission. [2]

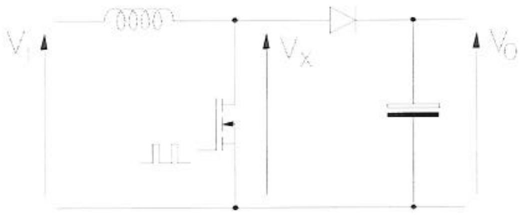
Power losses contribute to the variable costs of running a network. They can be reduced by using a higher voltage and a lower current to carry a given power. However, high voltage equipment (circuit breakers and transformers) have a high capital cost. Only in bulk transfer between a limited number of nodes is very high voltage used. As the power transfer reduces and the number of nodes increases closer to the consumers, lower voltages are used to strike a better balance between capital costs and running costs.

(h) Explain why a sudden reduction of the power consumed by the loads of an AC electricity system causes a change in the frequency and explain whether the frequency is expected to rise or fall. [2]

A sudden mismatch between the prime-mover power applied to generators and the power consumed by loads disturbs the power-balance (or torque-balance) of the generators and accelerates/decelerates them according to the moments-of-inertia. A sudden reduction of power consumption results in mechanical power being added to the inertia of the generators. Their collective speed increases and the frequency of the generated AC increases.

2. A boost switch-mode power supply, SMPS, is to be used to convert an input voltage of 5 V to an output voltage of 24 V. The inductor to be used has a value of 500 μH.

(a) Draw a circuit diagram of a boost SMPS. [2]



(b) Calculate the duty-cycle at which the circuit should be operated assuming continuous conduction. [2]

$$\frac{V_O}{V_I} = \frac{1}{1-\delta}$$

$$\delta = \frac{V_O - V_I}{V_O} = \frac{24-5}{24} = 0.7916$$

(c) Calculate the input and output currents, assuming a perfectly efficient circuit when the SMPS processes 1 W. [2]

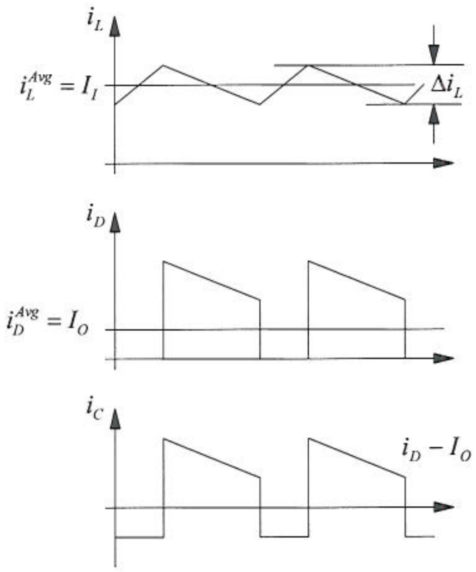
The input current, and hence the inductor current, will be:

$$I_L = \frac{P}{V_I} = \frac{1}{5} = 200\ mA$$

The output current is

$$I_O = \frac{P}{V_O} = \frac{1}{24} = 41.6\ mA$$

(d) Sketch the waveform of the current through the diode and through the capacitor and express the current flow through the capacitor during the on- and off-times in terms of the output current and inductor current. [3]



The current through the capacitor during the off-time will be I_O. During the on-time it will be $i_L - I_O$. The peak to peak amplitude of the current ripple in the capacitor is thus i_L.

(e) Calculate the maximum effective series resistance of the output capacitor if the output voltage ripple is to be kept below 20 mV when processing a power of 1 W. You may assume that the capacitance of the capacitor does not contribute to the voltage ripple and the ripple of the inductor current is small compared to its average value. [2]

The limit on the ESR is:

$$R_{ESR} = \frac{\Delta v_O}{\Delta i_C} = \frac{20m}{200m} = 100\ m\Omega$$

(f) Give an equation for the inductor current ripple as a function of the duty-cycle and frequency. [2]

Considering the change of current during the on-time:

$$\Delta i_L = \frac{V_I}{L} \cdot \frac{\delta}{f}$$

(g) Calculate the operating frequency required to ensure that the circuit remains in continuous conduction when processing a power of 250 mW. [4]

Discontinuous operation will occur if the average inductor current becomes less than half the ripple.

$$I_L^{critical} = \tfrac{1}{2}\Delta i_L$$

$$I_L = I_I = \frac{P}{V_I} = \frac{250m}{5} = 50\ mA$$

$$f = \frac{V_I\,\delta}{\Delta i_L\,L} = \frac{V_I\,\delta}{2 I_L^{critical}\,L} = \frac{5 \times 0.7916}{2 \times 0.05 \times 500 \times 10^{-6}} = 79.16\ kHz$$

(h) Find the relationship between the time for which the diode conducts and the on-time of the switch for discontinuous conduction mode and hence find the on-time required to process an input power of 100 mW if the switching frequency is 80 kHz. [3]

The diode will conduct until the inductor current reduces to zero.

$$\Delta i_L^{\ On} = \frac{V_I}{L} \cdot t_{On}$$

$$\Delta i_L^{\ Off} = \frac{V_I - V_O}{L} \cdot t_{diode}$$

$$\Delta i_L^{\ On} + \Delta i_L^{\ Off} = 0$$

$$t_{diode} = t_{On} \frac{V_I}{V_O - V_I}$$

The average input current is found by integrating the triangular current pulse over one period.

$$\begin{aligned}
i_L^{average} &= \frac{\frac{1}{2}\left(t_{On} + t_{diode}\right)\Delta i_L^{\ On}}{T} \\
&= \tfrac{1}{2} t_{On}\left(1 + \frac{V_I}{V_O - V_I}\right)\frac{V_I}{L} \cdot t_{On} \cdot f \\
t_{On}^{\ 2} &= \frac{i_L^{average}}{\frac{1}{2}\left(1 + \frac{V_I}{V_O - V_I}\right)\frac{V_I}{L} \cdot f} \\
&= \frac{\frac{0.1}{5}}{\frac{1}{2} \times \left(1 + \frac{5}{24 - 5}\right) \times \frac{5}{0.0005} \times 80 \times 10^3} \\
&= 39.6 \times 10^{-12} \\
t_{On} &= 6.3\ \mu s
\end{aligned}$$

(Period is 12.5 μs so answer is reasonable)

3. A 2-pole (1-pole-pair), three-phase induction is under design for a 50 Hz supply. The intention is to produce a machine capable of developing a torque of 20 Nm at slip of 0.05 and an efficiency of 85%.

The design calculations suggest the machine has the following reactance and resistance parameters. The calculations that follow are intended to reveal whether the design has achieved its goal.

X_M = 70.0 Ω
X'_R = 5.0 Ω
R'_R = 1.0 Ω
X_S = 5.0 Ω
R_S = 1.5 Ω

(a) Draw the per-phase equivalent circuit of a three-phase induction machine. [3]

(b) The machine has been designed for a current flow of 3.0 A in the magnetising reactance and a rated rotor current of 10.0 A.

(i) Calculate the voltage across the magnetising reactance need to achieve the desired magnetising current. [1]

$$V_M = jX_M I_M$$
$$|V_M| = 70 \times 3 = 210\ V$$

(ii) Assuming that the voltage calculated in (b)(i) is present, calculate the value of slip at which the rated rotor current will be drawn. [4]

$$V_M = I_R\left(\frac{R'_R}{s} + jX'_R\right)$$
$$\frac{R'_R}{s} = \frac{V_M}{I_R} - jX'_R$$
$$\frac{R'^2_R}{s^2} = \left|\frac{V_M}{I_R}\right|^2 - X'^2_R$$
$$s^2 = \frac{R'^2_R}{\left|\frac{V_M}{I_R}\right|^2 - X'^2_R} = \frac{1}{\left(\frac{210}{10}\right)^2 - 5^2} = \frac{1}{416}$$
$$s = 0.049$$

(c) Calculate the electro-mechanical power and the torque developed by the machine under the conditions of (b) [4]

$$P_{EM} = 3I'^2_R R'_R\left(\frac{1}{s} - 1\right)$$
$$= 3 \times 10^2 \times 1 \times \left(\frac{1}{0.049} - 1\right)$$
$$= 5.82\ kW$$

$$T_{EM} = \frac{P_{EM}}{(1-s)\omega_S} = \frac{P_{EM}}{(1-s)P\,2\pi f} = \frac{5818}{(1-0.049)\times 1\times 2\times \pi \times 50} = 19.5\ Nm$$

(d) Estimate the resistive power losses ("copper losses") and the efficiency of the machine when operating at maximum current. The iron loss can be ignored and the stator current for small slip can be approximated by $I_S \approx \sqrt{I_M{}^2 + I'_R{}^2}$. [4]

$$P_{R'_R} = 3I'_R{}^2\,R'_R = 3\times 10^2 \times 1 = 300\ W$$

$$I_S \approx \sqrt{I'_R{}^2 + I_M{}^2} = \sqrt{100+9} = 10.44\ A$$

$$P_{R_S} = 3I_S{}^2 R_S = 3\times 10.44^2 \times 1.5 = 490.5\ W$$

$$\eta = \frac{P_{EM}}{P_{EM} + P_{R_S} + P_{R'_R}} = \frac{5818}{5818+300+490.5} = 88.0\%$$

(e) Once built, the machine was tested to establish the actual value of magnetising inductance and the magnetising resistance (which had been ignored in the design). With the rotor spinning at synchronous speed, phase voltages of 220 V were applied. The stator current was recorded as 3.2 A and the total power read on a three-phase wattmeter was 620 W.

(i) Estimate the value of magnetising resistance [2]

No rotor current at synchronous speed

$$P_M = P_{Test} - P_{R_S} = P_{Test} - 3I_S{}^2 R_S = 620 - 3\times 3.2^2 \times 1 = 589.3\ W$$

$$P_M \approx \frac{3V_S{}^2}{R_M}$$

$$R_M \approx \frac{3V_S{}^2}{P_M} = \frac{3\times 220^2}{589.3} = 246.4\ \Omega$$

(ii) Estimate the value of the magnetising inductance [2]

Components of magnetising current are in quadrature.

$$I_{R_M} = \frac{V_S}{R_M} = \frac{220}{246.4} = 0.893\ A$$

$$I_{X_M} = \sqrt{I_M{}^2 - I_{R_M}{}^2} = \sqrt{3.2^2 - 0.893^2} = 3.07\ A$$

$$X_M \approx \frac{V_S}{I_{X_M}} = \frac{220}{3.07} = 71.6\ \Omega$$

2E Electromagnetic Fields 2006 – Answers

4. ~~1.~~ a) The ionosphere is a spherical shell (more accurately, a set of shells) in the upper atmosphere containing ions that are charged by the impact of high energy particles from outer space. Since the ionosphere is conducting it affects the propagation of electromagnetic waves. Below the plasma frequency, it is reflective, and above the plasma frequency (when the electromagnetic field varies too rapidly for the charges to respond) it is transparent.

[2]

b) Factors governing the choice of wavelength for night vision equipment are:
- The black body emission spectrum of warm bodies – this peaks at relatively long wavelengths if the bodies are at normal human temperature, so the operating wavelength should be longer than visible wavelengths
- Noise in the detection system – if the detector is photoconductive the thermal noise rises rapidly as the bandgap of the semiconductor is lowered
- Molecular absorbtion due to e.g. H_2O and CO_2 in the atmosphere

[3]

c) The three factors limiting the possibilities for free space communications are
- Absorption – the loss of energy due to the exitation of electronic or molecular vibrational transitions in the medium in which the wave is travelling
- Scattering – re-radiation of the energy from small inhomogeneities or trapped particles in the medium in which the wave is travelling. Scattering increases as $1/\lambda^4$.
- Diffraction – spreading of the energy in a bounded beam due to its finite aperture. Diffraction increases rapidly as the size of the beam approaches a wavelength.

[3]

d) Maxwell's modification to Ampere's law was the addition of the displacement current, which effectively accounts for energy flow via time-varying electric fields rather than via the normal current. Thus he wrote

$$\int_L \underline{\mathbf{H}} \,.\, d\underline{L} = \iint_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \,.\, d\underline{a}$$

Without this addition, it would have been impossible to account for phenomena that occur in the absence of metals, e.g. the propagation of electromagnetic waves.

[2]

e) The Poynting vector $\underline{\mathbf{S}} = \underline{\mathbf{E}} \times \underline{\mathbf{H}}$ gives the instantaneous power density. $\underline{\mathbf{S}}$ is actually impossible to measure, because it contains components at 2ω, where ω is the angular frequency of the field. Much more useful is the time-averaged power, given by:

$$\underline{S} = (1/T) \int_T \underline{\mathbf{S}}\, dt$$

For time varying fields in the form $\underline{\mathbf{E}} = \underline{E} \exp(j\omega t)$ and $\underline{\mathbf{H}} = \underline{H} \exp(j\omega t)$ we obtain

$$\underline{S} = 1/2 \text{ Re } [\, \underline{E} \times \underline{H}^* \,]$$

The expression for $\underline{S}$ contains no time variation, and it turns out that S is the quantity detected by all practical measuring devices (e.g. the eye, photographic film and photodiodes, for light waves).

[2]

f) Consider a group of waves with just two components, at angular frequencies $\omega + d\omega$ and $\omega - d\omega$. The corresponding propagation constants are $k + dk$ and $k - dk$. If the two components have equal amplitudes the combined electric field (if an EM wave) may be written as:

$E = E_0 \exp\{j[(\omega + d\omega)t - (k + dk)z]\} + E_0 \exp\{j[(\omega - d\omega)t - (k - dk)z]\}$, or
$E = E_0 \exp\{j(\omega t - kz)\} [\exp\{+j(d\omega t - dkz)\} + \exp\{-j(d\omega t - dkz)\}]$, or
$E = 2E_0 \exp\{j(\omega t - kz)\} \cos\{(d\omega t - dkz)\}$
Here the amplitude envelope is $\cos\{(d\omega t - dkz)\}$ and travels at a speed $d\omega/dk$.

[3]

g) The characteristic impedance of an infinite line is real, despite the fact that it is constructed from non-dissipative components, because such a line will effectively consume energy. A wave excited at one end of a semi-inifinite line can never emerge. The characteristic impedance of a finite line is not real, for a similar reason.

[2]

h) An anti-reflection coating is an intermediate layer used to suppress reflection of electromagnetic waves at the interface between two different dielectric media. If the media have refractive indices n_1 and n_2, respectively, the index n_3 of AR coating should be the geometric mean of these values, i.e. $n_3 = \sqrt{(n_1 n_2)}$. The thickness d of the coating should be one quarter of a wavelength in that material, i.e. $d = \lambda/4n_3$, so that reflections from the two interfaces add together in antiphase and hence cancel.

[3]

5 ~~2~~a. Referring to the figure below, and assigning voltages V, V' and currents I, I' we get:

$V' = V - j\omega L dz\, I$ and $I' = I - j\omega C dz\, V$

If $V' = V + (dV/dz)\, dz$ and $I' = I + (dI/dz)\, dz$

Then $dV/dz = -j\omega L I$ and $dI/dz = -j\omega C V$

Differentiating, we then get:

$d^2V/dz^2 = - j\omega L dI/dz = -\omega^2 LCV$ and $d^2I/dz^2 = -\omega^2 LCI$

These are the transmission line equations.

[5]

For travelling waves, we guess solutions $V(z) = V_0 \exp(-jkz)$, $I(z) = I_0 \exp(-jkz)$.

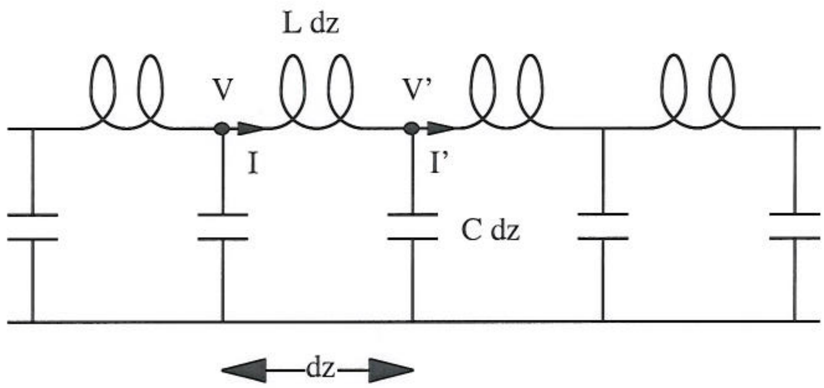
Hence $dV/dz = -jkV$ and $d^2V/dz^2 = (-jk)^2V = -k^2V$.

The assumed solution is then valid if $-k^2V = -\omega^2 LCV$ or if $k = \omega\, (LC)^{1/2}$

Including time variations, the full solution is $\mathbf{V}(z, t) = V(z) \exp(j\omega t) = V_0 \exp\{j(\omega t - kz)\}$

The phase velocity is therefore $v_{ph} = \omega/k = 1/(LC)^{1/2}$

[3]



b) Referring to the figure below: assume incident and reflected waves as:

$V_1 = V_I \exp(-jk_1z) + V_R \exp(+jk_1z)$

$I_1 = (V_I/Z_1) \exp(-jk_1z) - (V_R/Z_1) \exp(+jk_1z)$

Assume transmitted waves on the RHS of the discontinuity:

$V_2 = V_T(\exp(-jk_2z)$; $I_2 = (V_T/Z_2) \exp(-jk_2z)$

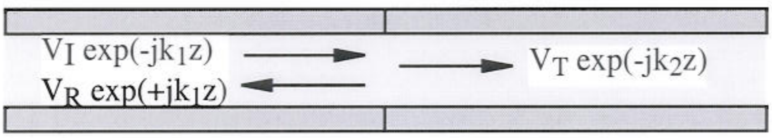
Match voltages and currents on boundary ($z = 0$) to get

$V_I + V_R = V_T$ and $(V_I - V_R)/Z_1 = V_T/Z_2$

Solve the resulting simultaneous equations to find the voltage reflection coefficient as:

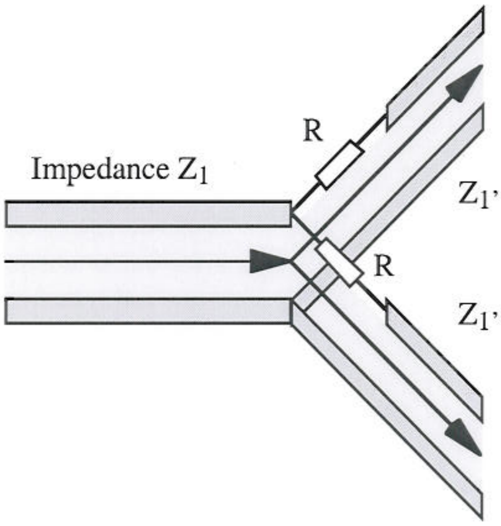
$R_V = V_R/V_I = (Z_2 - Z_1)/(Z_2 + Z_1)$

[6]



Line 1, Characteristic impedance Z_1 $z = 0$ Line 2, Characteristic impedance Z_2

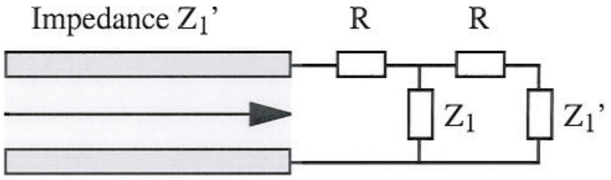
c) Referring to the figure below, resistors R must be inserted in series with the lines Z_1' so that the parallel combination has impedance Z_1. This can be achieved if $(R + Z_1')/2 = Z_1$. If $Z_1' = 75\ \Omega$ and $Z_1 = 50\ \Omega$, we require $R = 25\ \Omega$.



A wave incident from one of the lines with impedance Z_1' will then see the load shown below. The load has an impedance of $Z_L = R + 1/\{1/Z_1 + 1/(R + Z_1')\} = 58.33\ \Omega$.

The voltage reflection coefficient is then
$R_V = (Z_L - Z_1')/(Z_L + Z_1') = (58.33 - 75)/(58.33 + 75) = -0.125$

[6]



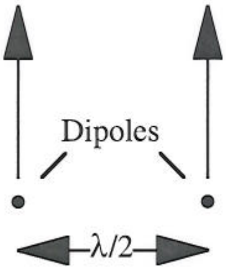
6 3a) Three advantages of upshifting on to a high frequency carrier:

- The wavelength is reduced, so the wave can be launched from a small antenna
- The frequency variation is reduced, so the antenna can be narrow-band
- Different carriers can be used, so the system can be frequency multiplexed

[3]

A broadside array uses an array of two or more dipoles to obtain increased directivity. Referring to the figure below, an array of two dipoles separated by $\lambda/2$ will have a high gain in the broadside direction, but zero gain along the array axis because signals originating from the two elements will cancel.

[2]



b) Assuming that the transmitting antenna is isotropic and loss-less, the intercepted power density is $S_{iso} = P_t/4\pi R^2$, where P_t is the transmit power.

Real antennae are not 100% efficient. If the radiation efficiency is η, the antenna gain is $G = \eta D$, where D is the directivity. The real intercepted power density is therefore $S_{real} = \eta_t D_t S_{iso}$

The effective area A_e is the equivalent area from which antenna can gather power, and deliver it to matched load. The effective area is related to the directivity by $A_e = \lambda^2 D/4\pi$.

The real intercepted power density is therefore $S_{real} = P_t(\eta_t A_t/R^2\lambda^2)$
The power intercepted by the receiving antenna is then $P_{int} = S_{real} A_r = P_t(\eta_t A_t A_r/R^2\lambda^2)$
Finally, the power at the receiver is $P_r = \eta_r P_{int} = P_t(\eta_t \eta_r A_t A_r/R^2\lambda^2)$

[5]

c) Assuming that $d^2E_x/dz^2 = j(\omega\mu_0\sigma) E_x$ and that $E_x = E_0 \exp(-jkz)$
The propagation constant is $k^2 = -j\omega\mu_0\sigma$ so $k = (1 - j)(\omega\mu_0\sigma/2)^{1/2} = k' - jk''$
Because k is complex, the wave decays as it propagates.
The field decays to 1/e of its original amplitude when $z = 1/k'' = (1/\pi f\mu_0\sigma)^{1/2}$
This distance is known as the skin depth δ.

[3]

Two consequences of the skin effect:

- EM wave decays as it travels into metal
- Current is confined near conductor surface at RF frequency, so that resistance increases, from $R_{pul} = 1/\sigma\pi r^2$ to $R_{pul} = 1/\sigma 2\pi r\delta$

[2]

d) The characteristic impedance of a microstrip line can be estimated as follows:

The capacitance per unit length can be estimated by assuming there is a parallel plate capacitor between the strip and ground, so that $C_{pul} \approx \varepsilon w/h$
Including the fringing field (which spreads h on either side), $C_{pul} \approx \varepsilon(w + 2h)/h$

[2]

The inductance per unit length can be found by assuming the magnetic field is uniformly concentrated between the strip and ground.
By Ampere's law, the magnetic field is $Hw \approx I$, so $B = \mu_0 I/w$

The linked flux is $\Phi_{pul} = Bw = \mu_0 I$ so the inductance is $L_{pul} = \Phi_{pul}/I = \mu_0$

[2]

Hence the characteristic impedance is $Z_0 = (L_{pul}/C_{pul})^{1/2} = \{\mu_0/\varepsilon(w/h + 2)\}^{1/2}$

[1]