EE4-24
EE9-CS1-3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE/EIE PART IV: MEng and ACGI

DISCRETE-TIME SYSTEMS AND COMPUTER CONTROL

Tuesday, 12 May 10:00 am

Time allowed: 3:00 hours

Corrected Copy

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T. Parisini
Second Marker(s) : E.C. Kerrigan

DISCRETE-TIME SYSTEMS AND COMPUTER CONTROL

Information for candidates:

In the following, $\delta(k)$ denotes the discrete-time unit impulse sequence, $u(t)$ denotes the continuous-time unit step function, and T denotes the sampling time.

- $\mathscr{Z}[\delta(k)] = 1$
- $\mathscr{Z}[u(t)] = \mathscr{Z}\left[\mathscr{L}^{-1}\left(\frac{1}{s}\right)\right] = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$
- $\mathscr{Z}\left[e^{-at}\cdot u(t)\right] = \mathscr{Z}\left[\mathscr{L}^{-1}\left(\frac{1}{s+a}\right)\right] = \frac{z}{z-e^{-aT}} = \frac{1}{1-z^{-1}e^{-aT}}$
- $\mathscr{Z}[t\cdot u(t)] = \mathscr{Z}\left[\mathscr{L}^{-1}\left(\frac{1}{s^2}\right)\right] = T\frac{z}{(z-1)^2} = T\frac{z^{-1}}{(1-z^{-1})^2}$
- $\mathscr{Z}\left[te^{-at}\cdot u(t)\right] = \mathscr{Z}\left[\mathscr{L}^{-1}\left(\frac{1}{(s+a)^2}\right)\right] = Te^{-aT}\frac{z}{(z-e^{-aT})^2} = Te^{-aT}\frac{z^{-1}}{(1-e^{-aT}z^{-1})^2}$
- Transfer function of the ZOH: $H_0(s) = \frac{1-e^{-sT}}{s}$
- Expression of a second-order polynomial with complex-conjugate roots in terms of the damping ratio ξ and the natural angular frequency ω_n: $s^2+2\xi\omega_n s+\omega_n^2$.
- Note that, for a given signal r, or $r(t)$, $R(z)$ denotes its $\mathscr{Z}$-transform.

1. Consider the impulse-sampled schemes depicted in Fig. 1.1 and 1.2 below with sampling-time T:

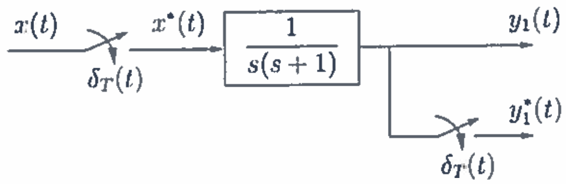

Figure 1.1

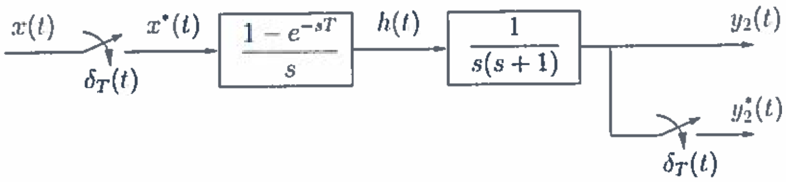

Figure 1.2

where $x(t) = e^{-t} \cdot u(t)$, with $u(t)$ denoting the unit step-function, and $\delta_T(t) = \sum_{k=0}^{\infty} \delta(t - kT)$, where $\delta(t)$ is the Dirac impulse function. In Fig. 1.2, the input $h(t)$ to the block with transfer function $G(s) = \dfrac{1}{s(s+1)}$ is generated by a ZOH with transfer function $\dfrac{1-e^{-sT}}{s}$.

a) With reference to Fig. 1.1, determine the closed-form expression of the discrete-time sequence $y_1(kT)$, $k = 0, 1, \ldots$.

[8 marks]

b) With reference to Fig. 1.2, determine the closed-form expression of the discrete-time sequence $y_2(kT)$, $k = 0, 1, \ldots$.

[8 marks]

c) Set $T = 1$ sec and plot the first 5 samples of $y_1(kT)$ and $y_2(kT)$, that is, $y_1(kT)$, $k = 0, 1, \ldots, 4$ and $y_2(kT)$, $k = 0, 1, \ldots, 4$. Compare the two discrete-time sequences and comment on your findings.

[4 marks]

2. Consider the discrete-time dynamic system described by the block scheme depicted in Fig. 2.1:

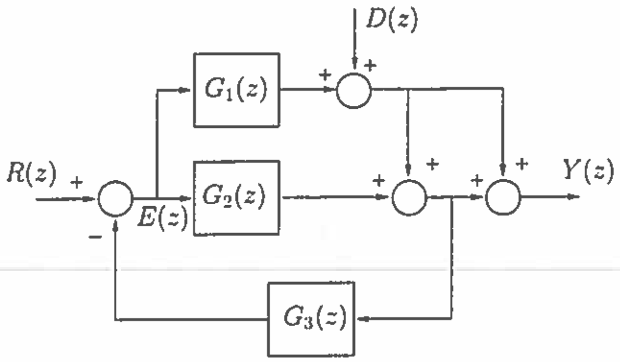

Figure 2.1 Block scheme of a discrete-time system.

where the discrete-time transfer functions $G_1(z), G_2(z), G_3(z)$ are given by

$$G_1(z) = \frac{z-1}{3z+2}; \quad G_2(z) = \frac{K}{z}, \text{ with } K \in \Re; \quad G_3(z) = \frac{z}{z-1},$$

and where $R(z), E(z), D(z), Y(z)$ denote the $\mathscr{Z}$ transforms of the discrete-time variables $r(k), e(k), d(k), y(k)$, respectively.

a) Determine the transfer function $H_{ry}(z)$ from the input $r(k)$ to the output $y(k)$, that is, $H_{ry}(z) = \dfrac{Y(z)}{R(z)}$.

[5 marks]

b) Determine the transfer function $H_{dy}(z)$ from the input $d(k)$ to the output $y(k)$, that is, $H_{dy}(z) = \dfrac{Y(z)}{D(z)}$.

[5 marks]

c) Determine all values of $K \in \Re$ (if any) such that the overall discrete-time dynamic system shown in Fig. 2.1 is asymptotically stable.

[5 marks]

d) Letting $K = 1$, $r(k) = 0, \forall k \geq 0$, and $d(k) = \delta(k)$ (where $\delta(k)$ is the discrete-time unit impulse function), determine the closed-form expression of the output sequence $y(k), \forall k \geq 0$.

[5 marks]

3. Consider the digital system shown in Figure 3.1:

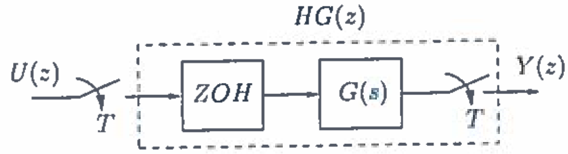

Figure 3.1

where T is the sampling time, "ZOH" stands for "zero-order hold", and $HG(z)$ denotes the equivalent discrete-time model for the plant $G(s) = \dfrac{1/2}{(s^2+1/4)(s^2+9/4)}$ connected to the ZOH and the sampler, where $H(s) = \dfrac{1-e^{-sT}}{s}$ denotes the transfer function of the ZOH.

a) Determine the poles of $G(s)$ and discuss the stability of the continuous-time system described by $G(s)$.

[3 marks]

b) For the generic sampling time T, determine $HG(z)$.

[8 marks]

c) Set $T = \pi$ sec and compute $HG(z)$ for this specific choice of T. Moreover, the choice $T = \pi$ sec, which is possible in theory, is not implementable in exact way in a practical digital control system. Discuss why.

[3 marks]

d) Provide a justification for the fact that the number of poles of the equivalent discrete-time model $HG(z)$ determined in your answer to Question 3c) is lower than the number of poles of the continuous-time transfer function $G(s)$.

[6 marks]

4. Consider the antenna tracking a satellite depicted in Fig. 4.1(a) and the block diagram of the continuous-time tracking control system shown in Fig. 4.1(b):

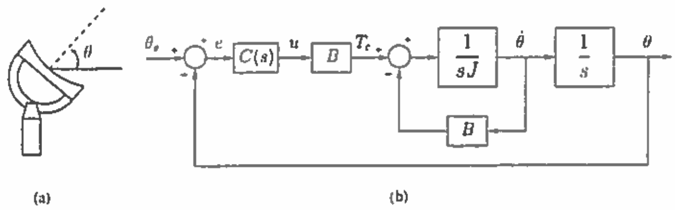

Figure 4.1

where θ denotes the pointing angle, T_c denotes the net torque from the drive, J the inertia moment of the antenna and B the friction coefficient.

a) Setting $J/B = 10$, determine the open-loop transfer function $G(s)$ from the input $u(t)$ to the output $\theta(t)$, that is, $G(s) = \Theta(s)/U(s)$, where $\Theta(s)$ and $U(s)$ denote the Laplace transforms of $\theta(t)$ and $u(t)$, respectively.

[3 marks]

b) Consider a reference angle $\theta_s(t) = 0.01 \cdot t,\ t \geq 0$ to be tracked. Determine the parameters $K \in \Re, K > 0$ and $a \in \Re, a > 0$ of a tracking controller $C(s) = K \cdot (1+10s)/(1+as)$ such that the closed-loop system is asymptotically stable, $\lim_{t\to\infty} |e(t)| \leq 1/100$, and $\xi \geq 0.5$ and $\omega_n \simeq 1$rad/sec, where ξ is the damping ratio and ω_n is the natural angular frequency of the closed-loop poles.

[5 marks]

c) Consider the digital control system shown in Figure 4.2:

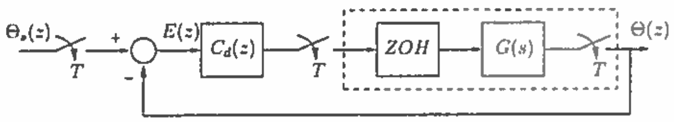

Figure 4.2

where $G(s)$ is the transfer function obtained in your answer to Question 4a) and $HG(z)$ denotes the equivalent discrete-time model for the plant $G(s)$ connected to the ZOH and the sampler. $H(s) = (1-e^{-sT})/s$ is the transfer function of the ZOH. Setting $T = 0.2$sec, compute the "pole-zero correspondence" discrete-time approximation $C_d(z)$ of the controller $C(s)$ obtained in your answer to Question 4b) and compute $HG(z)$.

[6 marks]

d) Determine the closed-loop transfer function $G_{cl}(z) = \Theta(z)/\Theta_s(z)$ and check whether the digital closed-loop control system is asymptotically stable. Compute the corresponding poles in the s-plane and determine the associated damping-ratio $\tilde{\xi}$ and natural angular frequency $\tilde{\omega}_n$. Compare $\tilde{\xi}$ and $\tilde{\omega}_n$ with ξ and ω_n determined in your answer to Question 4b). Comment on your findings.

[6 marks]