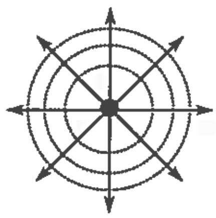
ENERGY CONVERSION: ANSWERS

Question 1. [a) = 5], [b) = 5], [c) = 5], [d) = 10], [e) = 7], [f) = 8]

a) (bookwork) The field lines are straight lines pointing radially away from the charge [1]. The equipotential lines are concentric circles with the charge at their centre [2]. On an equipotential line, the electric field strength is always perpendicular to the line element dl, so that $(\mathbf{E}\cdot\mathbf{dl}) = 0$ [2].



b) (calculated example). The total charge is $q = \int \lambda \mathrm{d}l$ [1]. Since we have a full circle, $dl = R\mathrm{d}\theta$ [2] and

$$q = \int_0^{2\pi} \lambda_0 R\theta \mathrm{d}\theta = 2\pi^2 R\lambda_0 \qquad [2]$$

c) (reasoning) The capacitance does not depend on the charge [2]. The definition of capacitance is $C = q/U$, where q is the charge and U is the voltage [1]. However, q and U are proportional to each other due to the superposition principle (linearity of Maxwell's equations) [1]; increasing q by, say, twice, will increase the field between the conductors constituting the capacitor (and, hence, the voltage between them) also twice. The ratio between the charge and the voltage will remain constant [1].

d) (bookwork) If the capacitor is not charged, it stores no energy. To charge the capacitor, we remove a small positive charge dq from one of the conductors and move it to the other [2]. Once we remove the positive charge, the conductor (initially neutral) becomes charged negatively. This negative charge will create a field and attract the positive charge we've just removed, thus opposing our attempt to move the charge [2]. So, to place the charge dq on the second conductor, we have to do work equal to $\mathrm{d}W = U\mathrm{d}q$, where U is the voltage between the conductors [2].

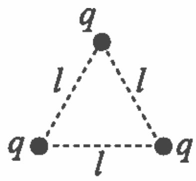
The total work needed to charge the capacitor to a charge q is

$$W = \int_0^q U\mathrm{d}q = \frac{1}{C}\int_0^q q\mathrm{d}q = \frac{q^2}{2C} = \frac{CU^2}{2} \qquad [3]$$

By doing the work, we have increased the potential energy of the capacitor. The energy stored in the capacitor equals W [1].

e) (calculated example) The total energy will be equal to the sum of the energies of individual pairs of charges [2]. We have three pairs here, and due to the geometry they all have the same energy. The energy of a pair is $q^2/(4\pi\varepsilon_0 l)$ [3]. The total energy is thus $3q^2/(4\pi\varepsilon_0 l)$ [2].

f) (bookwork) The usual assumptions are that a conductor has (i) infinite number [1] of (ii) infinitely mobile [2] charge carriers. If a (external) field existed inside the conductor, the mobile charges in it would



move [1]. This would result in redistribution of charge that creates its own (internal) field counteracting the external one [2]. Because the supply of carriers is infinitely large, the charges will redistribute themselves until the internal field cancels out the external one. The total field will be zero. [2]

Question 2. [a) = 10], [b) = 10], [c) = 10]

a) (bookwork) The equation of motion for the ion in vacuum is

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = qE \quad [2]$$

integrating once gives for the velocity

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{qE}{m}t + A \quad [1]$$

where A is a constant. However, the ion is initially immobile, so that $A = 0$ [1]. Integrating one more time gives

$$x = \frac{qEt^2}{2m} + B \quad [1]$$

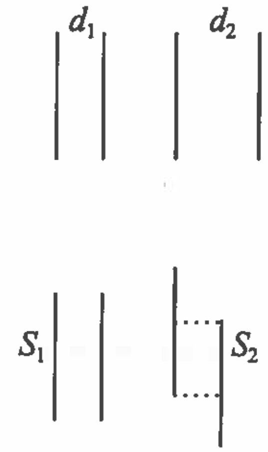
However, we can assume that the initial coordinate is $x = 0$, from which $B = 0$ [1]. The time required to reach $x = L$ is therefore

$$t = \sqrt{\frac{2mL}{qE}} \quad [2]$$

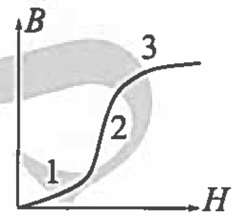
and the corresponding velocity is

$$v = \frac{qE}{m}t = \sqrt{\frac{2LqE}{m}} \quad [2]$$

b) (reasoning/assimilation of bookwork) The capacitance depends on the geometry [2]. Therefore, mechanical displacements will lead to a change of the capacitance and may be detected electronically [2]. The equation for the capacitance shows that it depends on two geometric factors: the distance between the plates and their area. Two possibilities for displacement sensors present themselves. First, a displacement may lead to a change of the distance between the plates [3]. Second, a displacement may lead to the change of the effective area of the capacitor [3]. Both possibilities are shown schematically below.

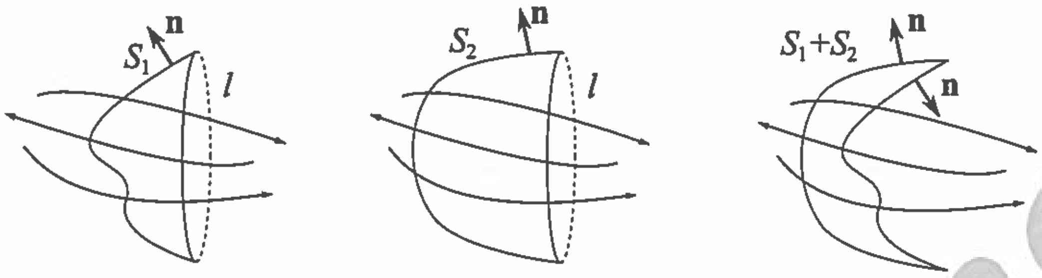


c) The normal magnetisation curve is shown in the figure below [3]. It has three distinct regions [1]:
1. The ferromagnet starts from being non-magnetised and the field is zero ($B = 0$, $H = 0$). An initial increase in H will lead to an (approximately linear) increase in B as the magnetic domains orient themselves along the field. [2]
2. Due to the interaction between the domains, the magnetisation grows strongly with increase of H, leading to a steep increase in B [2].
3. As more and more domains orient themselves along the field, the magnetisation saturates $M = const$, and B changes with H as $B = \mu_0(H + M) = \mu_0(H + const)$ [2].



Question 3. [(a) = 15], [(b) = 15]

a) (bookwork) We first need to find current from the current density. To do so, choose an arbitrary open surface S_1 and denote its border (which is a closed path) by l. Also choose the direction of the normal to this surface (see the Figure below) [2].



The total current flowing through S_1 is then given by

$$I(S_1) = \iint_{S_1} (\mathbf{J}\cdot \mathbf{dS}) \quad [3]$$

We can choose a different open surface S_2 that will have the same border l and have the same direction of the normal as S_1. Then the current through this surface is given by

$$I(S_2) = \iint_{S_2} (\mathbf{J}\cdot \mathbf{dS}) \quad [1]$$

The current will be conserved if for two arbitrary open surfaces S_1 and S_2 sharing the same border the following expression holds

$$\iint_{S_1} (\mathbf{J}\cdot \mathbf{dS}) = \iint_{S_2} (\mathbf{J}\cdot \mathbf{dS}) \quad [3]$$

The surfaces S_1 and S_2 together can form a closed surface, which we will denote by $S_1 + S_2$. However, the normal to the closed surface must always point outwards, so that the normal to S_1 must change its direction, leading to the change of the sign in the corresponding integral. We will then have

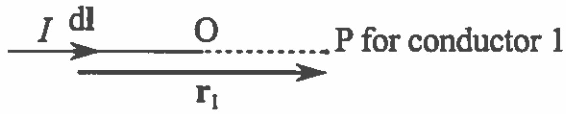
$$-\iint_{-S_1} (\mathbf{J}\cdot \mathbf{dS}) = \iint_{S_2} (\mathbf{J}\cdot \mathbf{dS}) \quad [3]$$

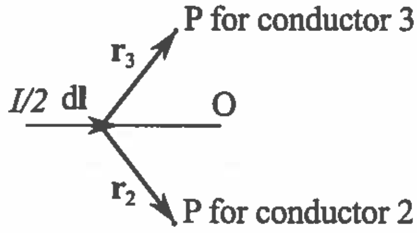
or

$$\oiint_{S_1+S_2} (\mathbf{J}\cdot \mathbf{dS}) = 0 \quad [3]$$

b) (calculated example) The total value of B will be the sum of the values of B created separately by each conductor. For each conductor, the value can be found by applying Biot-Savart's law

$$\mathbf{dB} = \frac{\mu_0 I}{4\pi}\frac{[\mathbf{dl}\times\mathbf{r}]}{r^3} \quad [2]$$





For conductor 1, any element $\mathbf{dl}$ will be parallel to the vector $\mathbf{r}_1$ correcting this element to point P. Therefore $[\mathbf{dl}\times\mathbf{r}_1] = 0$ [3].

For conductors 2 and 3, we have, assuming the same element $\mathbf{dl}$,

$$\mathbf{dB}_{2,3} = \frac{\mu_0 I}{4\pi}\frac{[\mathbf{dl}\times\mathbf{r}_{2,3}]}{r_{2,3}^3}$$

However, due to the symmetry $r_2 = r_3$ [2]. Therefore

$$\mathbf{dB}_2 + \mathbf{dB}_3 = \frac{\mu_0 I}{4\pi}\frac{[\mathbf{dl}\times(\mathbf{r}_2+\mathbf{r}_3)]}{r_2^3} \quad [3]$$

Due to the symmetry (see Figure above) $\mathbf{r}_2+\mathbf{r}_3$ is parallel to $\mathbf{dl}$ [2], so that $\mathbf{dB}_2+\mathbf{dB}_3 = 0$ for any, and all, elements $\mathbf{dl}$ [1]. It follows that the fields B_2 and B_3 due to the conductors will cancel each other. [1]

Therefore $B_{\text{total}} = B_1 + B_2 + B_3 = 0$ [1]