

## SOLUTIONS

1

(a) bookwork

(i) breadth first, expand a node at deepest level, otherwise back up and try node at next deepest level

depth first, expand all nodes at level  $n$ , then all nodes at level  $n+1$ , then ...

(ii) pick a path to expand (the frontier node thereof), note other-paths

find all the places to go from frontier node, make new-paths

for

breadth first, form a queue (append new-paths to other-paths)

depth first, form a stack (append other-paths to new-paths)

(iii) let  $b$  be branching factor,  $d$  be depth of solution,  $m$  maximum depth of treebreadth first: optimal yes, complete yes, time complexity  $O(b^d)$ , space  $O(b^d)$ depth first: optimal no, complete no, time complexity  $O(b^m)$ , space  $O(b \cdot m)$ 

(iv) ID search uses limited depth-first search to successive depths.

Thus it completes exhaustive search to  $d=0$ ,  $d=1$ ,  $d=2$ , etc

Because of the branching factor, the need to expand a few nodes at low depths several time is a relatively small overhead compared to expanding many nodes a few times at high depths.

It thus combines completeness/optimality of breadth first with time/space complexity of depth first

(b) application

(i) grid list of facts

2-tuple (Location, Colour)

location is 2-tuple (Integer,Integer) giving (X,Y) coordinate

Colour is atom, one of {red, green, blue, yellow, magenta, etc}

(ii) 2 tuple (Location, ChipList)

Location is 2 tuple (Integer,Integer) giving (X,Y) coordinate

ChipList is list of 2 tuple (Colour,Integer)

(iii)

start state e.g. ( (2,3), [ (red,5), (blue,2), (green,3), ... ] )

goal state e.g. ( (27,54), \_ )

(iv)

state\_change( east, (Loc,Chips), (NewLoc, NewChips) ) :-

Loc = (X,Y),

X1 is X + 1,

grid( (X1,Y), Colour ),

append( Front, [ (Colour,N1) | Back ], Chips ),

N1 &gt; 0,

N is N1 - 1,

append( Front, [ (Colour,N) | Back ], NewChips ),

NewLoc = (X1,Y).

Same for west, north and south.

2

(a) [bookwork]

admissible heuristic: never overestimates actual cost of getting to goal from frontier node, important because used in ensuring optimality

pathmax equation restores optimality in heuristics  $f(\text{succ}(n)) = \max(f(n), g(\text{succ}(n)) + h(\text{succ}(n)))$ , important as it ensures path cost along any path never decreasing, used to ensure optimality

(b) [bookwork/application]

$G' = \langle \text{start\_node}, \text{Op} \rangle$  where

start\_node is the root node

op is set of state transformers  $\text{op}: \text{node} \rightarrow (\text{edge}, \text{node})$

Op has to compute every element of the incidence relation.

What we then need to store with each path we create is  $f$ , the path cost function  $g$  plus the estimated path cost to the goal given by the heuristic function  $h$ .

Then our inductive definition of the graph is given by:

$$N'_G = \bigcup_{i=0}^{\infty} N_i$$

where

$$N_0 = \{ \langle \text{start} \rangle \}$$

$$N_{i+1} = \{ n_{i+1} \mid \exists \text{op} \in \text{Op} . \exists n_i \in N_i . n_{i+1} = \text{op}(n_i, e) \}$$

$$R'_G = \bigcup_{i=1}^{\infty} R_i$$

where

$$R_1 = \{ (n_0, n_1) \mid \exists \text{op} \in \text{Op} . n_1 = \text{op}(n_0) \}$$

$$R_{i+1} = \{ (n_i, n_{i+1}) \mid \exists \text{op} \in \text{Op} . \exists n_i \in N_i . n_{i+1} = \text{op}(n_i, e) \}$$

$$P'_G = \bigcup_{i=0}^{\infty} P'_i$$

where

$$P'_0 = \{ (\langle \text{start} \rangle, 0) \}$$

$$P'_{i+1} = \{ (p_i \mathbin{++} \langle n_{i+1} \rangle, f) \mid \exists \text{op} \in \text{Op} . \exists (p_i, g) \in P'_i . (n_{i+1}, e) = \text{op}(\text{frontier}(p_i)) \text{ AND } f = g + e + h(n_{i+1}) \}$$

Let  $f^*$  be the actual cost of getting from the start state to the optimal goal state. Then the  $A^*$  algorithm constructs:

All the paths  $(p, f)$  in whichever  $P'_i$  such that  $f < f^*$

Some of the paths  $(p, f)$  in whichever  $P'_i$  such that  $f = f^*$

None of the paths  $(p, f)$  in any  $P'_i$  such that  $f > f^*$

At each step,  $A^*$  selects the path in  $(p, f)$  from whichever  $P'_i$  such that this  $f$  is least so far, and expands that.

(c) [bookwork/application]

Optimality:

Optimal solution has cost  $f^*$  to get to optimal goal  $G$

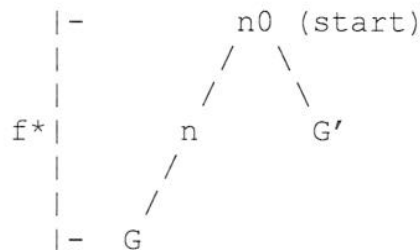
Suppose A\* search returns path to sub-optimal goal G'

We show that this is impossible

$$\begin{aligned} f(G') &= g(G') + h(G') \\ &= g(G') + 0 \quad G' \text{ is a goal state, we require } h \text{ to be } 0 \\ &= g(G') \end{aligned}$$

If G' is sub-optimal then  $g(G') > f^*$

Now consider a node n on path to optimal solution G



$$\begin{array}{llll} \text{Then:} & f^* & \geq & f(n) & \text{monotonicity} \\ & f(n) & \geq & f(G') & \text{otherwise A* expands n first} \\ & f^* & \geq & f(G') & \text{transitivity of } \geq \\ & f^* & \geq & g(G') & \text{a contradiction} \end{array}$$

So either G' was optimal or A\* does not return a sub-optimal solution.

Let  $f^*$  be cost of optimal node.

A\* expands all nodes with f-cost less than  $f^*$ .

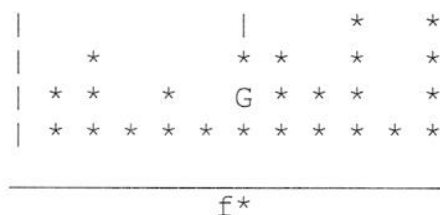
A\* expands some nodes with f-cost =  $f^*$ .

A\* expands no nodes with f-cost >  $f^*$ .

Since  $f(n) = g(n) + h(n)$ , this means that A\* expands all those nodes such that  $h(n) < f^* - g(n)$ .

In other words, the more nodes for which this relation holds, the more nodes will be expanded by A\* using this heuristic, and the less efficiently will the search space be explored.

Alternatively, consider histogram of nodes according to actual f-cost, whereby  $f\text{-actual}(n) = g(n) + h\text{-actual}(n)$ .



Only those nodes to the left of the  $f^*$  bar will be expanded. In practice of course, we don't have  $h\text{-actual}$  we just have  $h$ . Thus we could have a redistribution of the histogram which pushes more of the nodes to the left of the  $f^*$  bar.

In other words, we want to ensure  $f^* < f(n) + h(n) < f\text{-actual}(n)$

(d) [understanding]

use happy and gloomy to compute an interval rather than a point

then use ordering on interval size to decide order in which to expand nodes

3

(a) [bookwork]

max is player trying to win, or MAXimize advantage

min is opponent who attempts to MINimize max's score. Assume that min uses the same information as max and attempts always to move to a state that is worst for max  
each leaf node is given a score of 1 or 0, depending on whether the state is a win for max or min respectively

exhaustively generate the graph

propagate leaf values up the graph according to the rules:

if the parent is a MAX, give it the minimum value of its children

if the parent is a MIN, give it the maximum value of its children

(b)[bookwork/application]

exhaustive search is not always possible

therefore use alpha-beta search

➡ Associate one of two values with each node

—Alpha value, associated with MAX nodes, which can never decrease

• Alpha is the 'least' MAX can get, given MIN will do its best to minimise MAX's value

-- Beta value, associated with MIN nodes, which can never increase

• Beta is the 'most' MAX can get, given MIN will do its best to minimise MAX's value

### ➤ Algorithm

➡ Search to full ply using depth first

➡ Apply heuristic evaluation to all siblings at ply

—Assume these are MIN nodes

➡ Propagate value of siblings to parent using Minimax rules

—If MIN nodes, back up the maximum value

➡ Offer this value to **grandparent** MIN node as possible beta cutoff

➡ Descend to other grandchildren

➡ Terminate (prune) exploration of parent if any of their values is greater than or equal to the beta cutoff

➡ Do the same for MAX nodes

➡ Two rules for terminating search

—Search stopped below any MIN node having a beta value

**less** than or equal to alpha value of any of its MAX ancestors

Search stopped below any MAX node having an alpha value

**greater** than or equal to beta value of any of its MIN ancestors

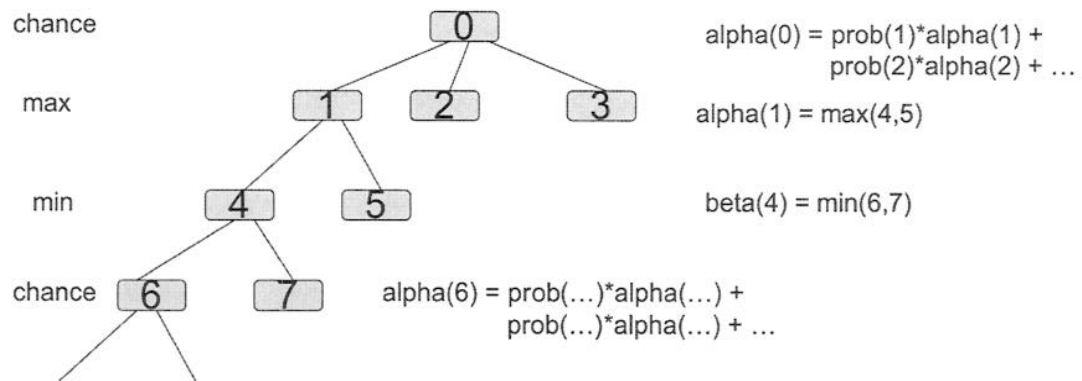
(c) [application/understanding]

have three rounds one for chance, one for max and one for min

max returns as before

min returns as before

chance returns sum of chance \* alpha score for each child



(d) [understanding]

ID search can be used to order nodes in estimate of utility

Effectiveness of pruning is greater when nodes are searched in order of their estimated value.

4

(a) [bookwork]

eliminate implication and equivalence

reduce scope of negation

rename variables

move quantifiers left without changing order

skolemize

drop universal quantifiers

convert to conjunctive normal form

make each conjunct a separate clause

give variables with same name in different clauses, different names

(b) [understanding]

(i) fact

(ii) rule

(iii) goal

(c) [application]

$\forall a \forall b. \text{parent}(a, b) \rightarrow \text{ancestor}(a, b)$

$\forall a \forall b \forall c. \text{parent}(a, c) \wedge \text{ancestor}(c, b) \rightarrow \text{ancestor}(a, b)$

$\text{parent}(\text{Bob}, \text{Alice})$

$\text{parent}(\text{Alice}, \text{Eve})$

$f1 \quad \neg \text{parent}(a, b) \vee \text{ancestor}(a, b)$

$f2 \quad \neg \text{parent}(a1, c) \vee \neg \text{ancestor}(c, b1) \vee \text{ancestor}(a1, b1)$

$f3 \quad \text{parent}(\text{Bob}, \text{Alice})$

$f4 \quad \text{parent}(\text{Alice}, \text{Eve})$

$\neg \text{ancestor}(\text{Bob}, \text{Eve})$

resolve with  $f2$ , unifier  $\{a1 = \text{Bob}, b1 = \text{Eve}\}$

$\neg \text{parent}(\text{Bob}, c) \vee \neg \text{ancestor}(c, \text{Eve})$

resolve with  $f3$ , unifier  $\{c = \text{Alice}\}$

$\neg \text{ancestor}(\text{Alice}, \text{Eve})$

resolve with  $f1$ , unifier  $\{a = \text{Alice}, b = \text{Eve}\}$

$\neg \text{parent}(\text{Alice}, \text{Eve})$

resolve with  $f4$ ,

contradiction

(c) [understanding]

Some of the models which make the premises true do not necessarily make the conclusion true, specifically those models where it is a different 'x' or person that makes each of the existential statements true.

So, for example, a model with  $\text{Sko1}$  and  $\text{Sko2}$  will satisfy the two statements

$\exists x. \text{parent}(\text{Bob}, x1)$

$\exists x. \text{parent}(x2, \text{Eve})$

with  $\text{parent}(\text{Bob}, \text{Sko1})$  and  $\text{parent}(\text{Sko2}, \text{Eve})$

but  $\text{ancestor}(\text{Bob}, \text{Eve})$  is not a true statement in this model.

This is why the inference rule for existential elimination must introduce a new constant:

$\exists x. \text{parent}(\text{Bob}, x)$   
 $\exists x. \text{parent}(x, \text{Eve})$   
 $\text{parent}(\text{Bob}, \text{Skol})$   
 $\text{parent}(\text{Skol}, \text{Eve})$

not

$\exists x. \text{parent}(\text{Bob}, x)$   
 $\exists x. \text{parent}(x, \text{Eve})$   
 $\text{parent}(\text{Bob}, \text{Skol})$   
 $\text{parent}(\text{Skol}, \text{Eve})$

5

(a) [bookwork]

sound:  $\vdash$  implies  $\models$

a proof in the system is an entailment in the language  
essential

complete  $\models$  implies  $\vdash$

an entailment in the language can be proved by the system  
desirable

decidable

theorem algorithm terminates with yes

non-theorem algorithm terminates with no

desirable but can't do much about the language (e.g. predicate logic semi-decidable)

efficient

deal with exponential complexity

(b) [bookwork/understanding]

➡ *KE proof procedure*

— to prove  $Q$ , begin with  $\neg Q$  and search for a refutation

— expand  $\neg Q$  according to, but clearing away, logical structure of  $Q$

— expansion takes form of tree, called a tableau, with formulas labelling nodes

— tableau is representation of disjunctive normal form

• each branch represents the conjunction of formulas on the branch

• the tableau is a disjunction of its branches

c) [application]

query =  $(a \wedge b \wedge \dots) \wedge (c \vee d \vee \dots) \wedge (\neg e \wedge \neg f \wedge \dots)$

d) [application/understanding]

All the search terms are premises

The query is the conclusion

To kick off a KE tableau, at the root we can add all the premises, and the negation of the conclusion

$\vdash (a \wedge b \wedge \dots) \wedge (c \vee d \vee \dots) \wedge (\neg e \wedge \neg f \wedge \dots)$

$\neg((a \wedge b \wedge \dots) \wedge (c \vee d \vee \dots) \wedge (\neg e \wedge \neg f \wedge \dots)) \vdash \text{conc}$

branch 1

2  $\neg(a \wedge b \wedge \dots)$

PB 1/1

branch 1.1

3  $\neg a$

PB 2/1

4 closes if search term  $a$  is in the premises

branch 1.2

5  $a$

PB 2/2

6  $\neg(b \wedge \dots)$

b 2, 5

branch 1.2.1

7  $\neg b$

PB 3/1

8 closes if search term  $a$  is in the premises

9  $b$

PB 3/2 etc



<i>Branch 2</i>		
11	$(a \wedge b \wedge \dots)$	<i>PB 1/2</i>
12	$\neg((c \vee d \vee \dots) \wedge (\neg e \wedge \neg f \wedge \dots))$	<i>b 11, conc</i>
<i>Branch 2.1</i>		
13	$\neg(c \vee d \vee \dots)$	<i>PB 4/1</i>
14	$\neg c$	<i>a 13</i>
15	$\neg d$	<i>a 13</i>
16	$\dots$	<i>a 13</i>
	<i>closes if either c or d or ... is in the premises</i>	
<i>Branch 2.2</i>		
17	$(c \vee d \vee \dots)$	<i>PB 4/2</i>
18	$\neg(\neg e \wedge \neg f \wedge \dots)$	<i>b 12 17</i>
<i>Branch 2.2.1</i>		
19	$\neg\neg e$	<i>PB 5/1</i>
20	$e$	<i>--</i>
	<i>closes if e is not in the premises</i>	
<i>Branch 2.2.2</i>		
21	$\neg e$	<i>PB5/2</i>
22	$\neg(\neg f \wedge \dots)$	<i>b, 18, 21</i>
<i>Branch 2.2.2.1</i>		
23	$\neg\neg f$	<i>PB 6/1</i>
24	$f$	<i>--</i>
	<i>closes if f is not in the premises</i>	
<i>Branch 2.2.2.2</i>		
25	$\neg f$	<i>PB 6/2</i>
26	$\neg(\dots)$	<i>b, 22, 25</i>
<i>etc</i>		

*Assumption is closed world assumption used to close branches, 2.2.2.x, i.e. if the search term is not present in the premises, then its negation must also be present.*

6

(a)

$wff ::= \Box wff \mid \Diamond wff$

(b) [application]

$M = \langle W, R, \Vdash \rangle$

$W = \{\alpha, \beta, \gamma, \delta, \zeta\}$

$R = \{ \alpha R \beta, \beta R \delta, \gamma R \alpha, \gamma R \beta, \gamma R \delta, \gamma R \gamma, \delta R \zeta \}$

$\Vdash p = \{ \alpha, \beta, \gamma \}$

$\Vdash q = \{ \beta, \gamma, \delta \}$

(c) [application]

(i) true

p and q is true everywhere from  $\gamma$

(ii) false

box q is true since q is true in every (the only) world accessible from b (ie d)  
but dia p is false since there is no world accessible from d in which p is true

(iii) true

p is true in a

dia q is true in b, since q is true in d and d is accessible from b

so dia dia q is true in a since dia q is true in b and b is accessible from a

(iv) true

anything is necessarily true in a world with no successor

(c) [application]

$M = \langle W, R, P \rangle$

$W = \{a, b\}$

$R = \{aRb\}$

$\Vdash p = \{a\}$

p is true in a

by MP box dia p true

so dia p true in b

but dia p false in b

assume p, show box dia p

suppose p is true in some a

suppose a R b, any b

then b R a by symmetry

so dia p at b

since dia p is true at any (every) b accessible from a

then box dia p is true at a, as required

(d) [application]

1         $\neg(\text{dia } p \rightarrow \text{box dia } p)$

1        dia p

1         $\neg\text{box dia } p$

2        p

3         $\neg\text{dia } p$

2         $\neg p$

close