IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2009

EEE/ISE PART II: MEng, BEng and ACGI

Corrected Copy

SIGNALS AND LINEAR SYSTEMS

Monday, 8 June 2:00 pm

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.T. Stathaki, P.T. Stathaki

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1.

Consider the discrete-time system with the following input-output relationship

$$y[n] = 2x[n] - x[n+2] - x[n-2].$$

(i) Is this system linear and time invariant (LTI)? Justify your answer.

[5]

(ii) Is the system described above memoryless? Justify your answer.

[5]

(iii) Is the system described above causal? Justify your answer.

[5]

(iv) Are memoryless systems in general causal? Justify your answer.

[5]

(v) Are causal systems in general memoryless? Justify your answer.

[5]

(vi) Prove that the output of a Linear Time-Invariant system is the convolution between the input of the system and the impulse response of the system.

[5]

(vii) Two continuous-time signals $x_1(t)$ and $x_2(t)$ are multiplied and the product $x(t)$ is sampled by a periodic impulse train. Both $x_1(t)$ and $x_2(t)$ are band-limited so that

$$X_1(\omega) = 0, \omega \geq 2\pi B_1$$
$$X_2(\omega) = 0, \omega \geq 2\pi B_2$$

where $X_i(\omega)\,(i = 1,2)$ is the Fourier transform of $x_i(t)$. Determine the maximum sampling period T_s that will allow perfect reconstruction of $x(t)$ from its samples. B_1 and B_2 are constants.

[5]

(viii) Consider a continuous signal $x(t)$ with Laplace transform $X(s)$. Find the Laplace transform of the signal $x(t - t_0)$ where t_0 is a constant parameter.

[5]

2.

(a) Consider the continuous signal $x(t)$ that is periodic with period T and fundamental frequency $\omega_0 = \frac{2\pi}{T}$. Suppose that the Fourier series coefficients of $x(t)$ are c_k.

(i) Find the Fourier series coefficients of the signal $x^*(t)$.

[4]

(ii) Find the Fourier series coefficients of the signal $x(-t)$.

[4]

The symbol $*$ denotes complex conjugate.

(b) Let $x(t)$ be a periodic signal whose Fourier series coefficients are

$$c_k = \begin{cases} 1, & k = 0 \\ -j\left(\frac{1}{3}\right)^{|k|}, & \text{otherwise} \end{cases}$$

(i) Is $x^*(t)$ real?

[4]

(ii) Is $x^*(t)$ even?

[4]

(iii) Is $x(-t)$ real?

[4]

Justify your answers.

(c) Parseval's relation for a discrete time periodic signal $x[n]$ is given by the following expression

$$\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{n=\langle N \rangle} |c_k|^2$$

where N is the period of the discrete signal $x[n]$, c_k are the Fourier series coefficients of $x[n]$ and $n = \langle N \rangle$ indicates that n varies over a range of N successive integers. Suppose that we are given the following information about $x[n]$:

1. $x[n]$ is a real and odd signal. In that case the Fourier series coefficients c_k of $x[n]$ are purely imaginary and odd.
2. $x[n]$ has period 10.
3. $c_{11} = 3j$.
4. $\frac{1}{10} \sum_{n=\langle 10 \rangle} |x[n]|^2 = 18$.

Using Parseval's theorem with $-1 \le n \le 8$ find the Fourier series coefficients c_k of $x[n]$.

[10]

3.

(a) (i) Consider a continuous signal $x(t)$ with Laplace transform $X(s)$. Show that the Laplace transform of the signal $\delta(t)$ is 1

with $\delta(t)$ the continuous impulse function defined as

$$\delta(t)=\begin{cases}+\infty, & t=0\\ 0, & \text{otherwise.}\end{cases}$$

[5]

(ii) Find the analytical expression and the Region of Convergence (ROC) of the Laplace transform of the causal function $e^{-at}u(t)$, with $u(t)$ the continuous unit step function defined as

$$u(t)=\begin{cases}1, & t\geq 0\\ 0, & \text{otherwise.}\end{cases}$$

[5]

(iii) Find the analytical expression and the Region of Convergence (ROC) of the Laplace transform of the non-causal function $-e^{-at}u(-t)$, with $u(t)$ defined as above.

[5]

(b) Consider a continuous non-causal signal $x(t)$, $x(t)=0$, $t>0$ with Laplace transform $X(s)=\dfrac{s+2}{s-2}$. Find the analytical expression of $x(t)$ using the results obtained in a(i), a(iii) and also find the region of convergence of $X(s)$.

[Hint: you may write the above Laplace transform as $X(s)=1+\dfrac{4}{s-2}$].

[5]

(c) Consider a continuous Linear Time-Invariant system with input the signal $x(t)$ defined in question (b) above and output the signal $y(t)=-\frac{2}{3}e^{2t}u(-t)+\frac{1}{3}e^{-t}u(t)$, with $u(t)$ defined as above.

(i) Determine the transfer function of the system $H(s)$ and its Region of Convergence (ROC) knowing that the ROC of $Y(s)$ has to be the intersection of the ROCs of $X(s)$ and $H(s)$.

[5]

(ii) Determine the impulse response of the system $h(t)$.

[5]

4.

(a) Consider a discrete signal $x[n]$ with z-transform $X(z)$. Find the z-transforms of the signals $x_1[n] = x[n+3]$ and $x_2[n] = x[-n+1]$ as functions of the z-transform $X(z)$.

[6]

(b) Consider a signal $y[n]$ which is related to two signals $x_1[n]$ and $x_2[n]$ by

$$y[n] = x_1[n+3] * x_2[-n+1]$$

where

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n] \text{ and } x_2[n] = \left(\frac{1}{3}\right)^n u[n]$$

with $u[n]$ the discrete unit step function defined as

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Given that

$$a^n u[n] \overset{z}{\leftrightarrow} \frac{1}{1-az^{-1}}, |z| > |a|$$

use the results obtained in question (a) above to determine the z-transform $Y(z)$ of $y[n]$.

[6]

(c) Consider an even sequence $x[n]$ (i.e., $x[n] = x[-n]$) with rational z-transform $X(z)$.

(i) From the definition of the z-transform, show that

$$X(z) = X\left(\frac{1}{z}\right)$$

[6]

(ii) From your results in part c(i), show that if a pole (zero) of $X(z)$ occurs at $z = z_0$, then a pole (zero) must also occur at $z = 1/z_0$.

[6]

(iii) Verify the results in part c(ii) for the following sequence

$$\delta[n+1] + \delta[n-1]$$

with $\delta[n]$ the discrete impulse function defined as

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise.} \end{cases}$$

[6]

E2.5

SOLUTIONS 2009

1.
Consider the discrete-time system with the following input-output relationship

$$y[n] = x[n+1] - x[n-1].$$

(i) Is this system linear and time invariant (LTI)? Justify your answer. **[5]**

Answer

We see that if the input signal $x_1[n]$ produces an output signal $y_1[n]$ and the input signal $x_2[n]$ produces an output signal $y_2[n]$ then the input signal $a_1x_1[n]+a_2x_2[n]$ produces the output $y_3[n]=(a_1x_1[n+1]+a_2x_2[n+1])-(a_1x_1[n-1]+a_2x_2[n-1])$

$=(a_1x_1[n+1]-a_1x_1[n-1])+(a_2x_2[n+1]-a_2x_2[n-1])=a_1y_1[n]+a_2y_2[n]$

Therefore, the system is linear.

If the input signal $x[n]$ produces an output signal $y[n]$ then the input signal $x[n-n_o]$ produces the output $y_1[n]=x[n-n_o]-x[n-1-n_o]$.

We see that $y[n-n_o]=y_1[n]$

Therefore, the system is time invariant.

(ii) Is the system described above memoryless? Justify your answer. **[5]**

Answer

It is not memoryless since the output at time instant n depends on the input at time instants $n+1$ and $n-1$.

(iii) Is the system described above causal? Justify your answer. **[5]**

Answer

It is not causal since the output at time instant n depends on future inputs.

(iv) Are memoryless systems in general causal? Justify your answer. **[5]**

Answer

Obviously yes.

(v) Are causal systems in general memoryless? Justify your answer. **[5]**

Answer

No. If the output at time instant n depends on the input at time instant n **and** past time instants the system is causal but not memoryless.

(vi) Prove that the output of a Linear Time-Invariant system is the convolution between the input of the system and the impulse response of the system. **[5]**

Answer

Consider a discrete LTI system with input $x[n]=\sum_{k=-\infty}^{+\infty}x[k]\delta[n-k]$.

According to the definition of linearity, the response (output) of the system to the signal $x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$ is equal to the sum of the scaled responses of the system to each of the shifted unit impulses. If we denote the response of the system to the shifted unit impulse $\delta[n-k]$ with $h_k[n]$, then the response to the input $x[n]$ will be $y[n] = \sum_{k=-\infty}^{+\infty} x[k]h_k[n]$. Note that $h_0[n]$ will be the response (output) of the system to the unshifted impulse $\delta[n]$. Moreover, according to the definition of time-invariance the condition $h_k[n] = h_0[n-k]$ holds. For simplicity we drop the subscript on $h_0[n]$ and define the so called impulse response (or unit impulse response) $h[n] = h_0[n]$, that is the response of the system when the input is the unit impulse $\delta[n]$. We then conclude that for an LTI system we have

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

(vii) Two continuous-time signals $x_1(t)$ and $x_2(t)$ are multiplied and the product $x(t)$ is sampled by a periodic impulse train. Both $x_1(t)$ and $x_2(t)$ are band-limited so that

$$X_1(\omega) = 0, \omega \geq 2\pi B_1$$
$$X_2(\omega) = 0, \omega \geq 2\pi B_2$$

where $X_i(\omega) = 0\ (i = 1,2)$ is the Fourier transform of $x_i(t)$. Determine the maximum sampling period T_s that will allow perfect reconstruction of $x(t)$ from its samples.

[5]

Answer
Multiplication in time is equivalent to convolution in frequency. In continuous domains the size of the convolution equals the summation of the sizes of the signals that are convolved. Therefore, $x(t)$ will be band-limited with bandwidth limited to $2\pi(B_1 + B_2)$. For the Nyquist criterion to be satisfied the sampling frequency and period must satisfy the following:

$$\Omega_s = 2\pi f_s \geq 2[2\pi(B_1 + B_2)] \Rightarrow f_s \geq 2(B_1 + B_2) \Rightarrow T_s = \frac{1}{f_s} \leq \frac{1}{2(B_1 + B_2)}$$

(vii) Consider a continuous signal $x(t)$ with Laplace transform $X(s)$. Prove that if $X(-s) = -X(s)$ the signal $x(t)$ is odd, i.e., $x(-t) = -x(t)$

[5]

Answer

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt$$

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt$$

$$X(-s) = \int_{-\infty}^{+\infty} x(t)e^{st}dt = -\int_{+\infty}^{-\infty} x(-t)e^{-st}dt = \int_{-\infty}^{+\infty} x(-t)e^{-st}dt$$

$X(-s) = -X(s) \Rightarrow x(-t) = -x(t)$, therefore, $x(t)$ is odd.

2.

(a) Consider the continuous signal $x(t)$ that is periodic with period T and fundamental frequency $\omega_0 = \frac{2\pi}{T}$. Suppose that the Fourier series coefficients of $x(t)$ are c_k.

(i) Find the Fourier series coefficients of the signal $x^*(t)$.

Answer

$x^*(t) = \sum_{k=-\infty}^{+\infty} c_k{}^* e^{-jk\omega_0 t}$. In that case the Fourier series of $x^*(t)$ are c_{-k}^*.

[2]

(ii) Find the Fourier series coefficients of the signal $x(-t)$.

Answer

$x(-t) = \sum_{k=-\infty}^{+\infty} c_k e^{-jk\omega_0 t}$. In that case the Fourier series of $x(-t)$ are c_{-k}.

[2]

(b) Let $x(t)$ be a periodic signal whose Fourier series coefficients are

$$c_k = \begin{cases} 1, & k = 0 \\ -j\left(\frac{1}{3}\right)^{|k|}, & \text{otherwise} \end{cases}$$

(i) Is $x^*(t)$ real?

Answer

The Fourier series of $x^*(t)$ are

$$c_k = \begin{cases} 1, & k = 0 \\ j\left(\frac{1}{3}\right)^{|k|}, & \text{otherwise} \end{cases}$$

In that case $x^*(t) = 1 + \sum_{k=-\infty}^{+\infty} j\left(\frac{1}{3}\right)^{|k|} e^{jk\omega_0 t} = 1 + \sum_{k=-\infty}^{-1} j\left(\frac{1}{3}\right)^{|k|} e^{jk\omega_0 t} + \sum_{1}^{+\infty} j\left(\frac{1}{3}\right)^{|k|} e^{jk\omega_0 t} =$

$= 1 + \sum_{1}^{+\infty} j\left(\frac{1}{3}\right)^{|k|} e^{-jk\omega_0 t} + \sum_{1}^{+\infty} j\left(\frac{1}{3}\right)^{|k|} e^{jk\omega_0 t} =$

$= 1 + \sum_{1}^{+\infty} 2j\left(\frac{1}{3}\right)^{|k|} \cos(jk\omega_0 t)$. It is NOT real.

[3]

(ii) Is $x^*(t)$ even?

Answer

Yes.

[3]

(iii) Is $x(-t)$ real?

Answer

The Fourier series of $x(-t)$ are

$$c_k = \begin{cases} 1, & k = 0 \\ -j\left(\frac{1}{3}\right)^{|k|}, & \text{otherwise} \end{cases}$$

In that case $x(-t) = 1 + \sum_{k=-\infty}^{+\infty} -j\left(\frac{1}{3}\right)^{|k|} e^{jk\omega_0 t} = 1 + \sum_{k=-\infty}^{-1} -j\left(\frac{1}{3}\right)^{|k|} e^{jk\omega_0 t} + \sum_{1}^{+\infty} -j\left(\frac{1}{3}\right)^{|k|} e^{jk\omega_0 t} =$

$= 1 + \sum_{1}^{+\infty} -j\left(\frac{1}{3}\right)^{|k|} e^{-jk\omega_0 t} + \sum_{1}^{+\infty} -j\left(\frac{1}{3}\right)^{|k|} e^{jk\omega_0 t} =$

$= 1 + \sum_{1}^{+\infty} -2j\left(\frac{1}{3}\right)^{|k|} \cos(k\omega_0 t)$. It is NOT real.

[3]

(c) The Parseval's relation for a discrete time periodic signal $x[n]$ is given by the following expression

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{n=\langle N\rangle} |c_k|^2$$

where N is the period of the discrete signal $x[n]$, c_k are the Fourier series coefficients of $x[n]$ and $n = \langle N\rangle$ indicates that n varies over a range of N successive integers. Suppose that we are given the following information about $x[n]$:

1. $x[n]$ is a real and even signal. In that case the Fourier series coefficients c_k of $x[n]$ are purely imaginary and odd.
2. $x[n]$ has period 10 and Fourier coefficients c_k.
3. $c_{11} = 3j$. From this we get $-c_1 = c_{-1} = -3j$.
4. $\frac{1}{10}\sum_{n=\langle 10\rangle} |x[n]|^2 = 18$.

Using the Parseval's theorem with $-1 \le n \le 8$ find the Fourier series coefficients c_k of $x[n]$.

$\sum_{-1}^{8} c_k^2 = 50 \Rightarrow c_{-1}^2 + c_1^2 + c_0^2 + \sum_{2}^{8} c_k^2 = 18 \Rightarrow c_0^2 + \sum_{2}^{8} c_k^2 = 0 \Rightarrow c_0 = 0$ **and** $c_i = 0, i = 2,\ldots,8$

[7]

3.

(a) (i) Consider a continuous signal $x(t)$ with Laplace transform $X(s)$. Show that the Laplace transform of the signal $\delta(t)$ is 1

with $\delta(t)$ the continuous impulse function defined as

$$\delta(t) = \begin{cases} +\infty, & t = 0 \\ 0, & \text{otherwise.} \end{cases}$$

Answer

For the Laplace transform of $\delta(t)$ we get:

$$X(s) = \int_{-\infty}^{+\infty} \delta(t) e^{-st} dt = e^{0t} = 1$$

[3]

(ii) Find the analytical expression and the Region of Convergence (ROC) of the Laplace transform of the causal function $e^{-at}u(t)$, with $u(t)$ the continuous unit step function defined as

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Answer

The Laplace transform of that signal is given by $X(s) = \int_{-\infty}^{+\infty} e^{-at}u(t)e^{-st}dt = \int_{0}^{+\infty} e^{-at}e^{-st}dt$

$$= \int_{0}^{+\infty} e^{-(a+s)t}dt = \left.\frac{e^{-(a+s)t}}{-(a+s)}\right|_{t\to+\infty} - \left.\frac{e^{-(a+s)t}}{-(a+s)}\right|_{t=0} = \frac{1}{s+a} \text{ if } \mathrm{Re}\{s\} > -a.$$

[3]

(iii) Find the analytical expression and the Region of Convergence (ROC) of the Laplace transform of the non-causal function $-e^{-at}u(-t)$, with $u(t)$ defined as above.

Answer

The Laplace transform of that signal is given by $X(s) = \int_{-\infty}^{+\infty} -e^{-at}u(-t)e^{-st}dt = -\int_{-\infty}^{0} e^{-at}e^{-st}dt$

$$= -\int_{-\infty}^{0} e^{-(a+s)t}dt = \left.\frac{e^{-(a+s)t}}{(a+s)}\right|_{t=0} - \left.\frac{e^{-(a+s)t}}{-(a+s)}\right|_{t\to-\infty} = \frac{1}{s+a} \text{ if } \mathrm{Re}\{s\} < -a.$$

[3]

(b) Consider a continuous non-causal signal $x(t)$, $x(t) = 0$, $t > 0$ with Laplace transform $X(s) = \frac{s+2}{s-2}$. Find the analytical expression of $x(t)$ using the results obtained in a(i), a(iii) and also find the region of convergence of $X(s)$.

Answer

$X(s) = \frac{s+2}{s-2} = 1 + 4\frac{1}{s-2}$. The non-causal function with Laplace transform $\frac{s+2}{s-2}$ is $\delta(t) - 4e^{2t}u(-t)$. The region of convergence is $\mathrm{Re}\{s\} < 2$.

[3]

(c) Consider a continuous Linear Time-Invariant system with input the signal $x(t)$ defined in question (b) above and output the signal $y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$.

(i) Determine the transfer function of the system $H(s)$ and its Region of Convergence (ROC) knowing that the ROC of $Y(s)$ has to be the intersection of the ROCs of $X(s)$ and $H(s)$.

Answer

$$Y(s) = \frac{2}{3}\frac{1}{s-2} + \frac{1}{3}\frac{1}{s+1}, \text{ ROC: } (\mathrm{Re}\{s\} < 2) \cap (\mathrm{Re}\{s\} > -1)$$

$$\frac{Y(s)}{X(s)} = \frac{2}{3}\frac{1}{s+2} + \frac{1}{3}\frac{s-2}{(s+1)(s+2)} = H(s)$$

$$H(s) = \frac{2}{3}\frac{1}{s+2} + \frac{1}{3}\frac{s-2}{(s+1)(s+2)} = \frac{2}{3}\frac{1}{s+2} + \frac{1}{3}\frac{-3(s+2)+4(s+1)}{(s+1)(s+2)}$$

$$= \frac{2}{3}\frac{1}{s+2} + \frac{1}{3}\frac{-3(s+2)+4(s+1)}{(s+1)(s+2)} = \frac{2}{3}\frac{1}{s+2} - \frac{1}{(s+1)} + \frac{4}{3}\frac{1}{(s+2)} = -\frac{1}{(s+1)} + 2\frac{1}{(s+2)}$$

For $Y(s)$ the ROC is $(\mathrm{Re}\{s\} < 2) \cap (\mathrm{Re}\{s\} > -1)$

For $X(s)$ the ROC is $\mathrm{Re}\{s\} < 2$. Therefore, for $H(s)$ the ROC is $(\mathrm{Re}\{s\} > -1)$, i.e., the system is causal.

[4]

(ii) Determine the impulse response of the system $h(t)$.

Answer

$$H(s) = -\frac{1}{(s+1)} + 2\frac{1}{(s+2)} \Rightarrow h(t) = (-e^{-t} + 2e^{-2t})u(t) \text{ since the system is causal.}$$

[4]

4.

(a) Consider a discrete signal $x[n]$ with z-transform $X(z)$. Find the z-transforms of the signals $x_1[n] = x[n+3]$ and $x_2[n] = x[-n+1]$ as functions of the z-transform $X(z)$.

Answer

$$X_1(z) = \sum_{-\infty}^{+\infty} x[n+3]z^{-n} = \sum_{-\infty}^{+\infty} x[n+3]z^{-(n+3)}z^3 = z^3 \sum_{-\infty}^{+\infty} x[n+3]z^{-(n+3)} = z^3 X(z)$$

$$X_2(z) = \sum_{-\infty}^{+\infty} x[-n+1]z^{-n} = \sum_{-\infty}^{+\infty} x[-n+1]z^{-n+1}z^{-1} = z^{-1} \sum_{-\infty}^{+\infty} x[-n+1]z^{-n+1} = z^{-1} X(z^{-1})$$

[4]

(b) Consider a signal $y[n]$ which is related to two signals $x_1[n]$ and $x_2[n]$ by

$$y[n] = x_1[n+3] * x_2[-n+1]$$

where

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n] \text{ and } x_2[n] = \left(\frac{1}{3}\right)^n u[n]$$

with $u[n]$ the discrete unit step function defined as

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Given that

$$a^n u[n] \overset{z}{\leftrightarrow} \frac{1}{1 - az^{-1}}, |z| > |a|$$

use the results obtained in question (a) above to determine the z-transform $Y(z)$ of $y[n]$.

Answer

$$Y(z) = z^3 X_1(z) z^{-1} X_2(z^{-1}) = z^2 X_1(z) X_2(z^{-1})$$

$$X_1(z) = \frac{1}{1 - (\frac{1}{2})z^{-1}}, \ X_2(z) = \frac{1}{1 - (\frac{1}{3})z^{-1}}, \text{ therefore}$$

$$Y(z) = z^2 \frac{1}{1 - (\frac{1}{2})z^{-1}} \frac{1}{1 - (\frac{1}{3})z}$$

[4]

(c) Consider an even sequence $x[n]$ (i.e., $x[n] = x[-n]$) with rational z-transform $X(z)$.

(i) From the definition of the z-transform, show that

$$X(z) = X\left(\frac{1}{z}\right)$$

Answer

$$X\left(\frac{1}{z}\right) = \sum_{-\infty}^{+\infty} x[n]z^n = \sum_{-\infty}^{+\infty} x[-n]z^{-n} = \sum_{-\infty}^{+\infty} x[n]z^{-n} = X(z)$$

[4]

(ii) From your results in part c(i), show that if a pole (zero) of $X(z)$ occurs at $z = z_0$, then a pole (zero) must also occur at $z = 1/z_0$.

Answer

(iii) If there is a pole at $z = z_0$, then $X(z) = P(z)\frac{1}{z - z_0}$ where $P(z)$ is a rational polynomial. Therefore, $X(z) = X\left(\frac{1}{z}\right) = P\left(\frac{1}{z}\right)\frac{1}{\frac{1}{z} - z_0}$, i.e., a pole occurs at $z = 1/z_0$. The same proof for zeros holds.

[4]

(iv) Verify the results in part c(ii) for each of the following sequences

$$\delta[n+1] + \delta[n-1]$$

with $\delta[n]$ the discrete impulse function defined as

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise.} \end{cases}$$

Answer

The z-transform of $\delta[n+1] + \delta[n-1]$ is $z + z^{-1}$, which has a pole at 0 and a pole at infinity.

[4]