IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

EEE/ISE PART I: MEng, BEng and ACGI

Corrected Copy

COMMUNICATIONS 1

Wednesday, 11 June 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	P.L. Dragotti, P.L. Dragotti
	Second Marker(s) :	M.K. Gurcan, M.K. Gurcan

Special Information for the Invigilators: none

Information for Candidates

The trigonometric Fourier series of a periodic signal $x(t)$ of period $T_0 = 2\pi/\omega_0$ is

$$x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t),$$

with

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt, \quad a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t dt, \quad b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega_0 t dt.$$

Some Fourier Transforms

$$\begin{aligned}
\cos\omega_0 t &\iff \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)] \\
\mathrm{rect}(\tfrac{t}{\tau}) &\iff \tau\mathrm{sinc}(\tfrac{\omega\tau}{2}) \\
\tfrac{W}{\pi}\mathrm{sinc}(Wt) &\iff \mathrm{rect}(\tfrac{\omega}{2W}) \\
\tfrac{\alpha^2}{2\pi}\mathrm{sinc}^2\left(\tfrac{\alpha t}{2}\right) &\iff \Delta\left(\tfrac{\omega}{\alpha}\right)
\end{aligned}$$

where

$$\Delta(\omega) = \begin{cases} 1-|\omega|, & |\omega| \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

The Scaling Property of the Fourier Transform

$$g(\alpha t) \iff \frac{1}{|\alpha|}G\left(\frac{\omega}{\alpha}\right).$$

Time-Shifting Property of the Fourier Transform

$$g(t-t_0) \iff G(\omega)e^{-j\omega t_0}$$

Some useful trigonometric identities

$$\cos x\cos y = \tfrac{1}{2}\cos(x-y) + \tfrac{1}{2}\cos(x+y).$$

$$\sin x\sin y = \tfrac{1}{2}\cos(x-y) - \tfrac{1}{2}\cos(x+y)$$

Steady-state impedance of a terminated transmission line

$$Z_{in} = Z_0\frac{Z_L + jZ_0\tan(kL)}{Z_0 + jZ_L\tan(kL)}$$

Power Series of e^x:

$$e^x = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + ...$$

Frequency modulation by a sinusoidal signal

$$\varphi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t.$$

where $\beta = \Delta f / B$.
Roots of $J_0(x)$:

x	2.405	5.5201	8.6537	11.7915

The Questions

1. This question is compulsory.

   (a) Consider the following two signals: $x_1(t) = \sin(2\pi t)\text{rect}(t-0.5)$ and $x_2(t) = \sin(2\pi t)\text{rect}(t-1)$, where $\text{rect}(t)$ is the unit gate function defined by

   $$\text{rect}(t) = \begin{cases} 1 & \text{for } \; -1/2 \leq t \leq 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

   Notice that the signals are also sketched in Figure 1.

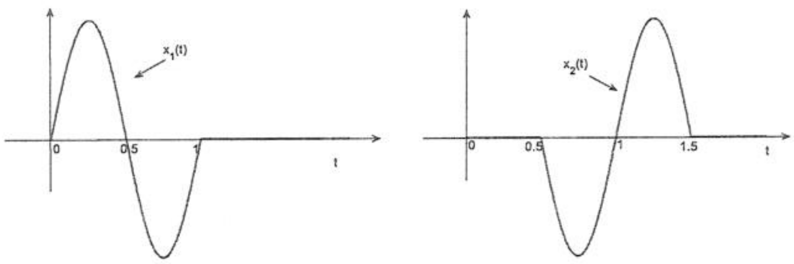

Figure 1: The two signals $x_1(t)$ and $x_2(t)$.

   i. Compute the energy of $x_1(t)$.

   [4]

   ii. Compute the energy of $x_2(t)$.

   [4]

   iii. Compute the energy of $x_1(t) + x_2(t)$.

   [4]

   (b) Consider the periodic signal $x(t)$ shown in Figure 2.

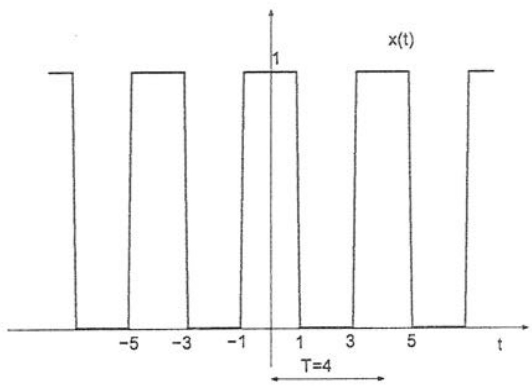

Figure 2: The periodic signal $x(t)$.

i. Compute the trigonometric Fourier series of $x(t)$. That is, compute the Fourier coefficients a_0, a_n and b_n.

[4]

ii. The signal $x(t)$ is fed to a filter $h(t)$ giving output $y(t)$. The frequency response of the filter is

$$H(\omega) = \begin{cases} 1 & \text{for} \quad |\omega| \leq 3 \text{ rad/s} \\ 0 & \text{otherwise} \end{cases}$$

Write the exact expression of the output $y(t)$.

[4]

(c) The Fourier transform of the triangular pulse $x(t)$ in Figure 3(a) is

$$X(\omega) = \frac{1}{\omega^2}(e^{j\omega} - j\omega e^{j\omega} - 1).$$

Using this information, the scaling property and the time-shifting property, find the Fourier transform of the signal $y(t)$ shown in Figure 3(b).

[4]

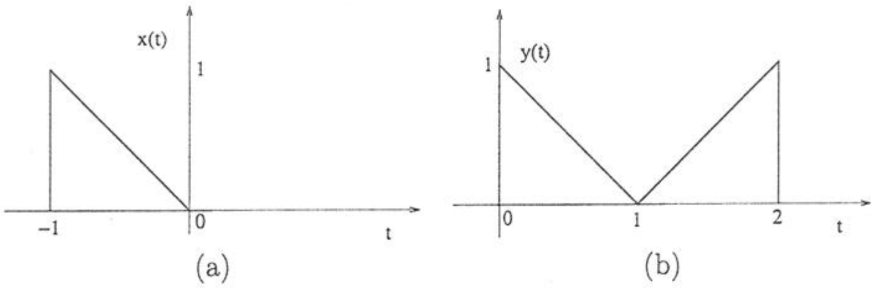

Figure 3: The two signals $x(t)$ and $y(t)$.

(d) Consider the power signal $x(t) = \cos 100t$.

i. Compute the autocorrelation function of $x(t)$ defined as

$$\mathcal{R}_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)dt.$$

[4]

ii. Find the Power Spectral Density of $x(t)$.

[4]

(e) Consider the following two modulating signals:

$$m_1(t) = \begin{cases} a_1 t + a_0 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$m_2(t) = \begin{cases} b_2 t^2 + b_1 t + b_0 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

where the a's and the b's are constant parameters. The signal $m_1(t)$ is applied to a frequency modulator leading to

$$\varphi_{FM}(t) = 10\cos[2\pi f_c t + k_f \int_{-\infty}^{t} m_1(\alpha) d\alpha].$$

The signal $m_2(t)$ is applied to a phase modulator leading to

$$\varphi_{PM}(t) = 10\cos[2\pi f_c t + k_p m_2(t)].$$

Assuming that $k_f = k_p = 1$, determine the conditions for which the two modulated signals are exactly the same.

[4]

(f) A sinusoidal source $v(t) = 10\sin(2\pi f_0 t)$ V with internal resistance R is connected to a transmission line with characteristic impedance $Z_0 = 50\ \Omega$. The transmission line has length $L = 100$ m and is connected to a load Z_L (see Figure 4). Assume $Z_L = 0$ and assume phase velocity $u = 2 \cdot 10^8$ m/s, find the lowest non-zero frequency at which $Z_{in} = 0$. (Recall that $Z_{in} = V(-L)/I(-L)$).

[4]

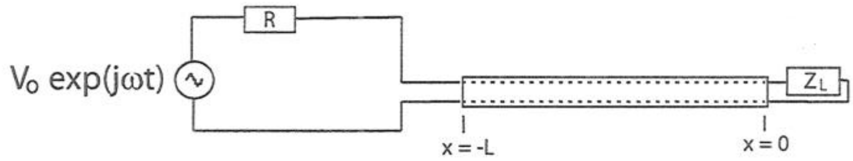

Figure 4: A transmission line connected to a sinusoidal source.

2. Consider the FM signal

$$\varphi(t) = 10\cos[2\pi f_c t + k_f \int_{-\infty}^{t} x(\alpha)d\alpha].$$

(a) Assume that $k_f = \pi$ and that the modulating signal is given by

$$x(t) = \frac{5000}{\pi}\text{sinc}^2(50t) + \frac{10000}{\pi}\text{sinc}^2(50t)\cos 100t.$$

i. Sketch and dimension the Fourier transform of $\frac{5000}{\pi}\text{sinc}^2(50t)$.

[6]

ii. Sketch and dimension the Fourier transform of $x(t)$.

[6]

iii. Using Carson's rule, determine the bandwidth of $\varphi(t)$.

[6]

(b) Assume now that $x(t) = A_m \cos(2\pi f_m t)$. In a certain experiment conducted with $f_m = 1$ kHz and increasing A_m (starting from $A_m = 0$), it is found that the carrier component of the FM signal is reduced to zero for the first time when $A_m = 2$.

i. What is the coefficient k_f of the modulator?

[6]

ii. What is the value of A_m for which the carrier component is reduced to zero for the second time?

[6]

3. Consider a non-ideal diode where the current $i(t)$ through the diode and the voltage $v(t)$ across it are related by:

$$i(t) = I_0[e^{-\frac{v(t)}{V_T}} - 1],$$

where $I_0 = 1$ A and $V_T = 0.026$ V. Let $v(t) = m(t) - c(t)$ be the sum of the modulating message $m(t)$ and the carrier $c(t)$, where $m(t) = V_T \cos(2\pi f_m t)$, $c(t) = V_T \cos(2\pi f_c t)$, $f_m = 1$ kHz and $f_c = 100$ kHz.

(a) Expand $i(t)$ as a power series in $v(t)$, retaining terms up to $v^2(t)$.

[6]

(b) Sketch and dimension the spectrum of the resulting diode current $i(t)$.

[6]

(c) The resulting diode current $i(t)$ is fed to an ideal bandpass filter giving output $y(t)$. The bandpass filter has a bandwidth of $2W$ rad/s and is centered at $\omega_c = 2\pi f_c$ where $f_c = 100$ kHz. Specify the bandwidth W required in order for the output signal to be a full-AM signal with carrier frequency f_c and modulating signal $m(t)$.

[6]

(d) What is the modulation index of the resulting full AM signal?

[6]

(e) Compute the power efficiency η of the resulting full AM signal.

[6]

4. At the junction between two cables of characteristic impedance Z_0, an additional line of characteristic impedance Z_1 is joined in parallel as shown in Figure 5. Signals arrive only from the left-hand side. Both the additional line and the second line have matched terminations as shown in Figure 5.

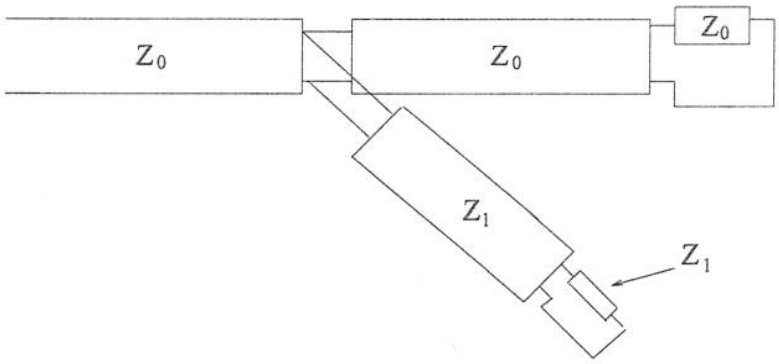

Figure 5: Additional line joined in parallel at the junction between two cables.

(a) Find the minimum value of Z_1 for which no more than 1% of the power is reflected at the junction. Assume $Z_0 = 50\ \Omega$ and $Z_1 > Z_0$.

[10]

(b) For the case $Z_0 = 50\ \Omega$ and $Z_1 = 100\ \Omega$ and an incident sinusoidal signal of amplitude 1 V, find the voltage and current amplitudes of the reflected and transmitted signal.

[10]

(c) For the values of part (b) compute the transmitted and reflected power.

[10]