IMPERIAL COLLEGE LONDON

Examinations 2017-18

BEng MEng Biomedical Engineering
Part 1
BE1-HMECH1
MECHANICS 1, Main Examination

26/05/2018, 10.00-11.30
Duration: 90 minutes

The paper has 3 COMPULSORY questions.
Answer all 3 questions.

Each question is worth 100 marks.

Please answer each question in separate answer book.

Marks for questions and parts of questions are shown next to the question. The marks for questions (and parts thereof) are indicative, and they may be slightly moderated at the discretion of the examiner.

A list of Moments of Inertia formulae is provided separately.

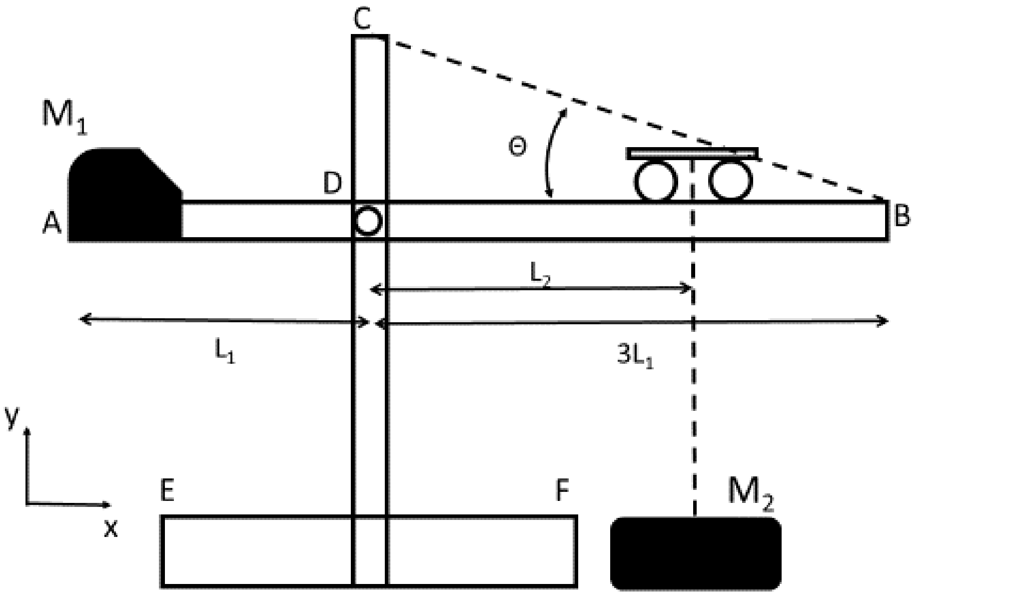
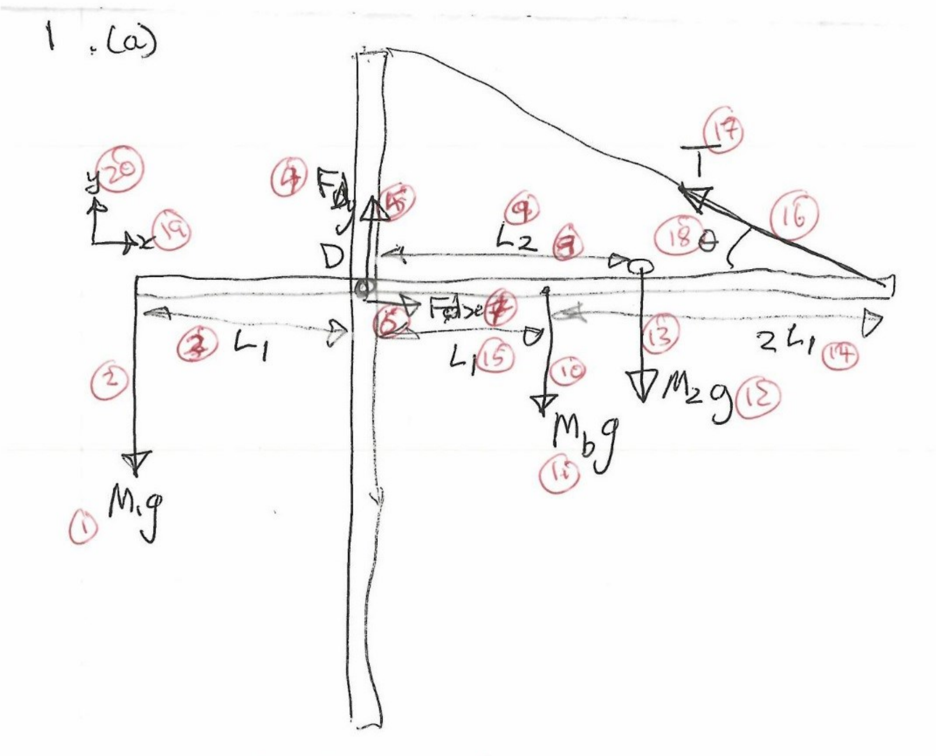


Figure 1

Question 1. A container crane (Figure 1) consists of a horizontal beam AB of length $4L_1$ and mass M_b which is supported by a hinged connection to its support tower at D a distance L_1 from A and a cable tie BC at an angle θ to the horizontal. A counterbalance weight of mass M_1 is mounted at end A, and the loads (containers) are lifted by a vertical winch mechanism running on a trolley as shown at a distance L_2 from the hinged support

a) Draw a free body diagram of the crane beam

30 marks



1.5 points per element

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b) Derive an expression for the tension T in the cable tie BC when the load M2 is being lifted a distance L2 from the tower as shown. You should give the expression in terms of M1, M2, L1, L2, ɵ and any other constants you may need

15 marks

(b) Take moments about D 2

$\Sigma M_D \quad M_1 g L_1 - M_b g L_1 - M_2 g L_2 + T 3 L_1 \sin\theta = 0$ 2 2 2 2 1

$\therefore 3T L_1 \sin\theta = M_2 g L_2 - L_1 g(M_1 - M_b)$ 2

$\therefore T = \dfrac{M_2 g L_2 - L_1 g (M_1 - M_b)}{3 L_1 \sin\theta}$ 2

c) If, L_1 is 5 m, the angle ϴ is 30°, L_2 is 10 m, the beam mass M_b is 1 tonne, the counterbalance mass M_1 is 10 tonnes and the load mass M_2 is 20 tonnes, what are the reaction forces at D (note that it is a hinge)

30 marks

(c) Also $\Sigma F_y \quad F_{dy} - M_1 g - M_b g - M_2 g + T\sin\theta = 0$ 5

$\therefore F_{dy} = (M_1 + M_b + M_2)g - T\sin\theta$ 3

$\Sigma F_x \quad F_{dx} - T\cos\theta = 0$ 3

$\therefore F_{dx} = T\cos\theta$ 3

so $T = \dfrac{20\times10^3 \times 9.8\times10 - 5\times9.8(10-1)\times10^3}{15\sin30}$ 3

$= \dfrac{10^3(20\times9.8\times10 - 5\times9.8\times9)}{15\sin30}$ 3

$= \dfrac{10^3(1960 - 441)}{15\sin30^\circ} = \dfrac{1519\times10^3}{15\sin30}$ 2

$\boxed{T = 202.5\times10^3\ N}$

$\Rightarrow F_{dy} = (10+1+20)g\times10^3 - T\sin\theta$ 3

$= 31g\times10^3 - 202.5\times10^3 = 101.3\times10^3 N$

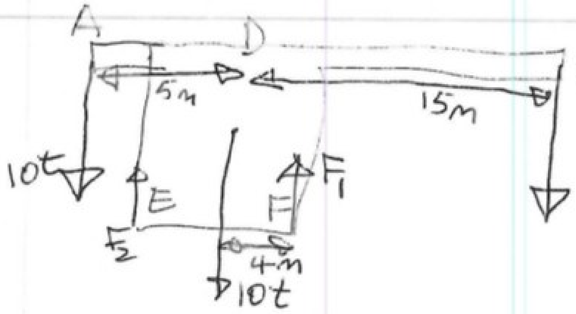
$F_{dx} = T\cos\theta = 202.5\times10^3\cos30 = 175.4\times10^3 N$

2

d) If the base of the crane (EF) is 8 m wide, the base and tower have a mass of 10 tonnes distributed symmetrically, L_1 is 5 m and the counterbalance weight M_1 is 10 tonnes, what is the maximum weight that can be lifted at the end of the boom (L2 = 3L1) without overturning the crane?

25 marks

(d) New body - total crane



10pts for new FBD

For overturning at F, need total moment at F is -ve (clockwise), at limit $F_2 = 0$ 3

take moments about F 2

$$\Sigma M_F \quad 10^4 g\times(5+4) + 10^4 g\times 4 - 11\times M_{max} = 0 \quad 2 \quad 2 \quad 2$$

$$\therefore 11\times g M_{max} = 10^4 g\,(9+4)$$

$$= 130\times g\times 10^3 \quad 2$$

$$\therefore M_{max} = \frac{130g}{11g}\times \text{tonnes}$$

$$= 11.8 \text{ tonnes} \quad 2$$

Question total: 100 marks

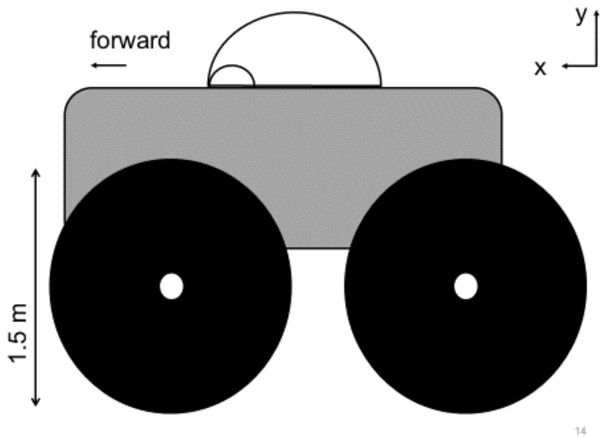
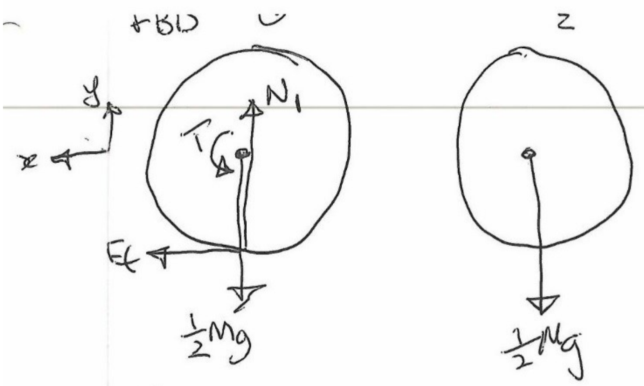


Figure 2

Question 2. The moon rover as shown consists of four wheels (two on either side) which can be represented as thin discs of diameter D and mass m, and a rectangular body of mass M, which is evenly distributed between the two axles.

a) If the electric motors which drive the rover can deliver a maximum torque T (turning moment) to the front axle of the rover, and the coefficient of friction of the wheels on the moon's surface is given by μ_m derive an expression for the maximum acceleration that can be induced in the rover. (Assume the rear axle is frictionless).

40 marks



10 marks for FBD

18 mks

Wheel ①

$\sum F_y \quad N_1 - \frac{1}{2}Mg = 0$ 2

$\therefore N_1 = \frac{1}{2}Mg$ . . . 3 . . . 5 (1)

$\sum F_x \quad F_f = ma$ 2 by Coulomb's Law

$F_f = \mu_s N = \frac{1}{2}\mu_s Mg$ 2 3

$\therefore Ma = \frac{1}{2}\mu_s Mg$ 2

$a = \frac{\mu_s g}{2}$ 3 . . . . (2) 12

Moments on wheel $-F_f D/2 + T = 0$ 3 at equilibrium

$\therefore \frac{F_f D}{2} = T$ 2

or $F_f = \frac{2T}{D}$ at the limit . . . 3 (3) 8

b) If the mass of the whole rover M is 1.2 kg, the acceleration due to gravity on the moon is 1.62 ms^{-2}, the wheels are 1.5 m in diameter and the coefficient of friction between the rover wheels and the moon's surface is 0.2, show that the maximum acceleration that can be delivered is 0.162 $m.s^{-2}$, whatever the power of the motors.

20 marks

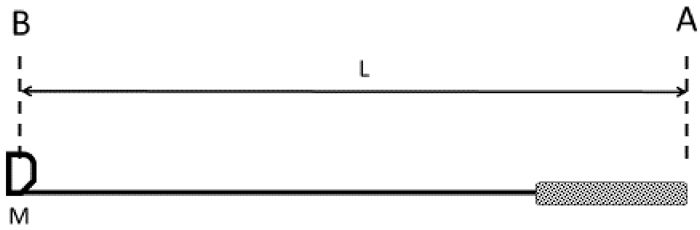
(b) according to (2) above $a = \frac{\mu_s g}{2}$ 5

$\therefore a = \frac{0.2 \times 1.62}{2} = 0.162 \; ms^{-2}$ 5

10

c) As part of the same mission, an astronaut will walk on the moon. If the astronaut's mass is 60 kg and they can push off with a force of 600N for 0.25 s, what is the farthest distance the astronaut can jump on the moon's surface, where the acceleration due to gravity is 1.62 ms^{-2}

Question total: 100 marks



Question 3. A golf club, as shown in Figure 3, consists of a steel tube of diameter d, mass m and length L (AB), on the end of which (B) is welded a striking head which can be considered as a point mass of mass M.

a) Derive an expression for the Moment of Inertia of the club about an axis perpendicular to the long axis at the handle end A

20 marks

3

(a) MoI of a thin rod or cylinder perpendicular to its long axis is $\frac{1}{12} m H^2$ 2

so $M_oI_{\text{handle A}} = \frac{1}{12} m_1 H^2 + m_1 d^2$ (parallel axis theorem) B

$= \frac{1}{12} m_1 L^2 + m_1 \left(\frac{L}{2}\right)^2 = \frac{m_1 L^2}{12} + \frac{m_1 L^2}{4}$ 3 2

$= \frac{M_1 L^2}{3}$ 2 10

MoI due to head (point mass)

$= MR^2 = M_2 L^2$ 2

$\therefore$ MoI of club $= M_oI_{\text{handle}}' + M_oI_{\text{head}}$ 2

$= \frac{1}{3} M_1 L^2 + M_2 L^2 = L^2\left(\frac{M_1}{3} + M_2\right)$ 3 3

b) When it is swung, it starts from zero velocity and traverses a half circle in a horizontal arc as shown, such that it is effectively rotating about A. If the player can apply a maximum torque T, derive an expression for the final velocity of the club head (its tangential velocity) after such a half circle V_c.

45 marks

(b) $\sum M \quad T = I\alpha$ ② $\quad \alpha = \ddot{\theta}$

$\therefore \alpha = \frac{T}{I}$ 3 . . . . . (i

integrate $\therefore \omega = \frac{Tt}{I} + \omega_0$ 3 $\omega_0 = 0$

$= \frac{Tt}{I}$ 3 $\omega = \dot{\theta}$ . . . (

integrate

$\therefore \theta = \frac{T.t^2}{2I} + \theta_0$ 3 assume $\theta_0 =$

$\therefore \pi = \frac{T.t^2}{2I}$ 3

$\therefore t^2 = \frac{2\pi I}{T}$ 3

$\therefore t = \sqrt{\frac{2\pi I}{T}}$ 3

substitute back into (2)

d

$\omega = \frac{T.t}{I}$ subst in for t

$= \frac{T}{I}\left(\sqrt{\frac{2\pi I}{T}}\right)$ 3

$V_T = \omega.R$

$V_T = \frac{T}{I}\left(\sqrt{\frac{2\pi I}{T}}\right).L$ 5

$= \frac{\sqrt{2\pi T}L}{\sqrt{I}}$ 5

$= \frac{\left(\sqrt{2\pi T}\right)L}{\sqrt{L^2\left(\frac{m}{3}+M\right)}}$ 5

$= \frac{\sqrt{2\pi T}}{\sqrt{m/3 + M}}$ 5

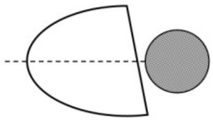


Figure 4

c) The club then strikes a golf ball, which is sat on a tee such that the centres of mass of the club head and the ball are level (Figure 4). The mass of the club head is four times the mass of the ball, and the collision is a perfectly elastic collision. Derive an expression for the final velocity of the golf ball, given the initial velocity of the club head V_c. Assume that the driving torque stops just before the collision.

35 marks

(c) Before collision clubhead V_c 2
ball is stationary.
after collision V_b & V_{c_2} 3
Momentum is conserved 2

$$P_1 = P_2$$

$$\therefore 4mV_{c_1} = 4mV_{c_2} + mV_b \quad 3$$

$$\therefore mV_b = 4m(V_{c_1} - V_{c_2}) \quad 3$$

$$\therefore V_b = 4(V_{c_1} - V_{c_2}) \quad \ldots 2 \ldots (1)$$

Energy is also conserved.

$$E_1 = E_2 \quad \text{(all kinetic)} \quad 1$$

$$\therefore \tfrac{1}{2}4mV_{c_1}^2 = \tfrac{1}{2}4mV_{c_2}^2 + \tfrac{1}{2}mV_b^2 \quad 3$$

$$\therefore V_b^2 = 4V_{c_1}^2 - 4V_{c_2}^2 \quad \ldots\ldots (2) \quad 2$$

$$V_b^2 = 4(V_{c_1}^2 - V_{c_2}^2) \quad \ldots\ldots (3) \quad 2$$

divide by 1 $\quad = 4(V_{c_1} + V_{c_2})(V_{c_1} - V_{c_2})$

$$\therefore V_b = V_{c_1} + V_{c_2}$$

$$V_b = 4(V_{c_1} - V_{c_2}) \quad \ldots\ldots (1) \quad 3$$

$$4V_b = 4(V_{c_1} + V_{c_2}) \quad 3 \quad \text{Sum}$$

$$\therefore 5V_b = 8V_{c_1} \quad 3$$

$$\therefore V_b = \tfrac{8}{5}V_{c_1} \quad 3$$

Question total: 100 marks