

Special instructions for students

permittivity of free space:	$\epsilon_o = 8.85 \times 10^{-12} \text{ F/m}$
permeability of free space:	$\mu_o = 4\pi \times 10^{-7} \text{ H/m}$
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10} \text{ cm}^{-3} \text{ at } T = 300\text{K}$
dielectric constant of Si:	$\epsilon_{Si} = 11$
dielectric constant of SiO ₂ :	$\epsilon_{ox} = 4$
thermal voltage:	$kT/e = 0.026\text{V at } T = 300\text{K}$
charge of an electron:	$e = 1.6 \times 10^{-19} \text{ C}$

Formulae

$\left. \begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p \frac{dp(x)}{dx} \end{aligned} \right\}$	Drift and diffusion currents in a semiconductor
$I_{DS} = \frac{\mu C_{ox} W}{L} \left((V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2} \right)$	Current in a MOSFET
$\left. \begin{aligned} J_n &= \frac{eD_n n_p}{L_n} \left(e^{\frac{eV}{kT}} - 1 \right) \\ J_p &= \frac{eD_p p_n}{L_p} \left(e^{\frac{eV}{kT}} - 1 \right) \end{aligned} \right\}$	Diffusion currents in a pn-junction
$V_0 = \frac{kT}{e} \ln \left(\frac{N_A N_D}{n_i^2} \right)$	Built-in voltage
$c = c_0 \exp \left(\frac{eV}{kT} \right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$	Minority carrier injection under bias V
$D = \frac{kT}{e} \mu$	Einstein relation

1. a)

Heisenberg's uncertainty principle gives the relationship:

$$\Delta E \Delta t \geq \hbar/2$$

Show that for a photon whose wavepacket extends over 1 ns the range of frequencies in the wavepacket will be at least 80 MHz.

[4]

- b) Fig.1.1 gives the set-up of the Hall measurement on a semiconductor doped with acceptor atoms. The external voltage V gives rise to a current I . A magnetic field B is applied as indicated in fig. 1.1. Is the Hall voltage V_H positive or negative? Explain your answer briefly.

[4]

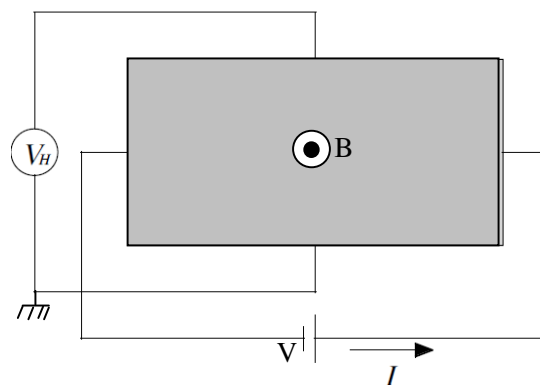


Figure 1.1: Hall measurement set-up for a semiconductor doped with acceptor atoms.

- c) Give the definition of the Fermi level. [2]
- d) Sketch the energy band diagram (E_c , E_v , E_G , E_F) of
- i. a donor doped Si semiconductor [2]
 - ii. an acceptor doped Si semiconductor [2]
 - iii. an intrinsic Si semiconductor [2]
- e) Explain why the velocity of the electrons in a semiconductor under a constant external electric field is not increasing. [4]
- f) Given a slab of n-type Si with an electric field as given in Fig.1.2, what is the direction of the electron flow? (Answer L or R) [2]

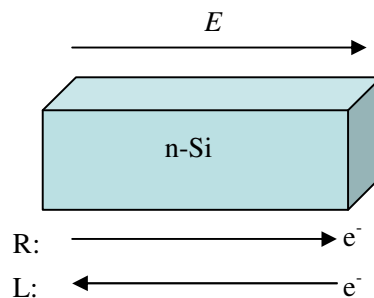


Figure 1.2: The electron flow in a slab of n-Si under an electric field E .

- g) Give the physical reason for the constant but very low reverse bias (saturation) current in a pn diode. [4]
- h) Sketch the material cross section, including doping and contacts, of an un-biased enhancement mode MOSFET that under inversion will have holes in the channel. Give the doping type in each necessary region and the correct annotations to the contacts. [4]
- i) Sketch the energy band diagram (E_c , E_v , E_F) of an un-biased p^+np BJT and explain briefly the function of both the emitter-base and the base-collector junction in the BJT in active mode. [4]

2. a)

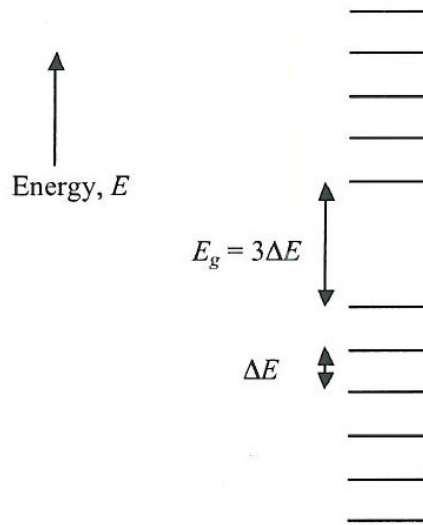


Figure 2: Single electron states of the model system.

Figure 2 shows a simplified model of the single-electron states in a material. The states are uniformly spaced in energy, ΔE except for an energy gap of $3\Delta E$. The spin of the electrons can be ignored.

a) Given the material is an intrinsic semiconductor, draw the occupancies of these states at 0 K, identifying the valence band, conduction band, and Fermi level.

[5]

b) Given is an n^+p diode under zero bias where the doping in the n-region is 10^3 times larger than in the p-region. Give the charge neutrality equation across the depletion region and derive the relation between the extend of the depletion width in the n and the p type regions. Define all parameters you use.

[5]

c) Determine the doping density in each region for the Si diode given in a) such that at an applied forward bias of $V=0.5V$, a current of $I=10\mu A$ is flowing. The parameters of the diode are the following:

The diffusion length of the minority carriers: $L_p=10^{-2}$ cm.

$L_n=10^{-3}$ cm.

The diffusion constant of the minority carriers: $D_p=10$ cm²/s.

$D_n=20$ cm²/s.

The area of the diode is: $A=10^{-2}$ cm².

The temperature is: $T=300K$.

[10]

Indicate clearly any approximations you make.

d) Derive the expression for current gain β in a short n^+pn bipolar transistor as a function of material and geometrical parameters. Ignore I_{CB0} . Define all the parameters you use.

[8]

e) Based on your result in d), explain briefly why the emitter is doped higher than the base.

[2]

3.

- a) Give the definition of threshold voltage (in words) and explain how an increase in doping in the substrate of an enhancement mode n-channel MOSFET will influence the threshold voltage? [4]
- b) A Si MOSFET is measured and the characteristics given in Fig.3.1.
 i) Give the value of the threshold voltage and explain how you derive it. [4]
 ii) Determine in which region the MOSFET is measured (triode or saturation region.) [2]
 iii) Estimate the value of the mobility knowing that the workfunction of the gate contact is equal to the workfunction of the Si substrate, and that the gate length and width are respectively $W_g = 100 \mu\text{m}$ and $L_g = 10 \mu\text{m}$. [10]

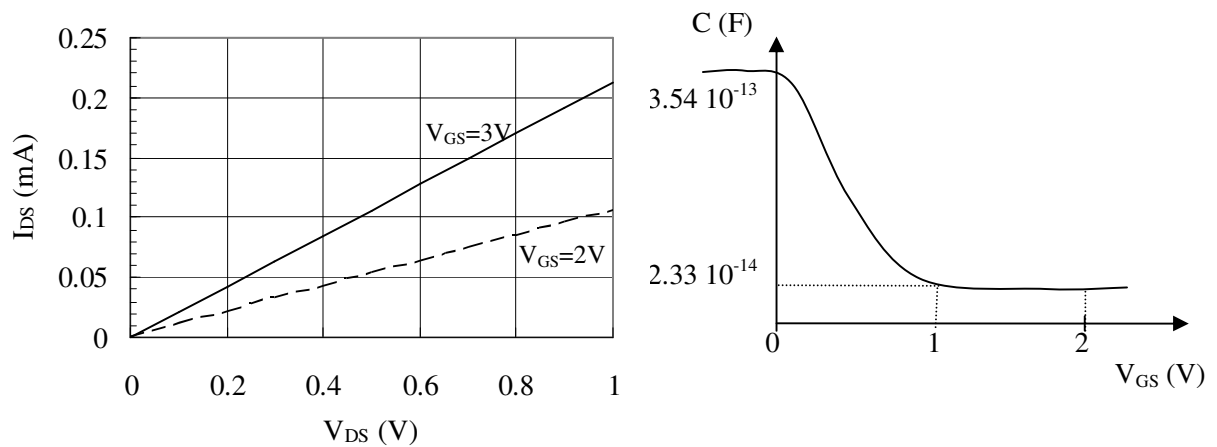


Figure 3.1: Left: output characteristics I_{DS} - V_{DS} of the MOSFET for 2 gate voltages as shown. Right: MOS capacitance C as a function of gate voltage. Note: C takes into account the gate area.

- c) The Schödinger equation is given by: [10]

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$

In the channel of the MOSFET the carriers are trapped in a triangular quantum well. Take the x-direction to be from the gate to the substrate, then assume that the quantum well channel can be approximated by an infinitely deep one-dimensional potential well given by:

$$V(x) = \begin{cases} \infty, & x < 0 \\ 0, & 0 \leq x \leq L \\ \infty, & x > L \end{cases}$$

Show that the wavefunction of the electrons (in the x-direction) in the quantum well channel is given by:

$$\psi(x) = \begin{cases} 0, & x < 0 \\ A \sin kx, & 0 \leq x \leq L \\ 0, & x > L \end{cases}$$

Give the expression of k and E .