EE4-24
EE9-CS1-3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected Copy

DISCRETE-TIME SYSTEMS AND COMPUTER CONTROL

Q4 part c)

Friday, 16 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T. Parisini
Second Marker(s) : E.C. Kerrigan

Discrete-time Systems and Computer Control

Information for candidates:

- $\mathcal{Z}\left(\frac{1}{s}\right) = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$

- $\mathcal{Z}\left(\frac{1}{s+a}\right) = \frac{z}{z-e^{-aT}} = \frac{1}{1-z^{-1}e^{-aT}}$

- $\mathcal{Z}\left(\frac{1}{s^2}\right) = T\frac{z}{(z-1)^2} = T\frac{z^{-1}}{(1-z^{-1})^2}$

- $\mathcal{Z}\left(\frac{1}{s^3}\right) = \frac{T^2}{2}\frac{z(z+1)}{(z-1)^3} = \frac{T^2}{2}\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$

- $\mathcal{Z}(\sin(\omega t)) = \frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1} = \frac{z^{-1}\sin(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$

- Transfer function of the ZOH: $H_0(s) = \frac{1-e^{-sT}}{s}$

- Tustin transformation: $s = \frac{2}{T}\frac{z-1}{z+1}$

- Note that, for a given signal r, or $r(t)$, $R(z)$ denotes its $\mathcal{Z}$-transform.

$$\mathcal{Z}\left(\frac{1}{(s+a)^2}\right) = T\frac{z e^{-aT}}{(z-e^{-aT})^2}$$ (given @ 12:20)

1. Consider the mass-spring-damper accelerometer depicted in Fig. 1.1.

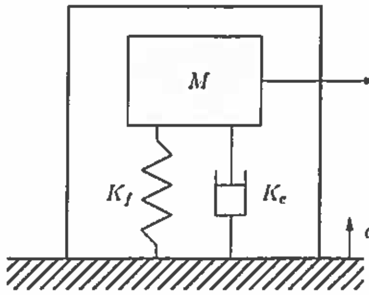

Figure 1.1 Spring-mass-damper accelerometer.

The armature is moving with vertical acceleration $a(t)$ and the measured acceleration $a_m(t)$ is proportional to the vertical displacement of the mass M, that is $a_m(t) = Kp(t)$. The dynamic description of the accelerometer is given by

$V_a(t) = K p(t)$

$$\frac{d^2}{dt^2}p(t) + \frac{K_f}{M}\frac{d}{dt}p(t) + \frac{K_e}{M}p(t) = a(t)\,;\; a_m(t) = Kp(t)$$

where K_f, K_e, and M are given constants.

a) Determine the continuous-time transfer function $G(s)$ from the input $a(t)$ and the output $a_m(t)$.

[3 marks]

b) Determine the Laplace transform of the measured acceleration $a_m(t)$ when the input acceleration takes on the form given in Fig. 1.2 and with the following values for the constants: $M = 2$, $K = 6$, $K_f = 10$, and $K_e = 12$.

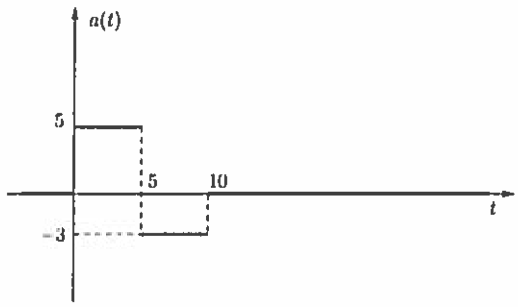

Figure 1.2 Time-profile of the input $a(t)$ to the accelerometer.

[5 marks]

c) Now, assume that a device embedded in the sensor records the measured acceleration $a_m(t)$ with a sampling period $T = 1$s. Determine the $\mathscr{Z}$-transform of the sampled measured acceleration.

[8 marks]

d) Is it possible to determine a discrete-time equivalent model of the accelerometer? Justify your answer.

[4 marks]

2. Consider the discrete-time dynamic system shown in Fig. 2.1

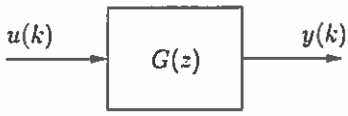

Figure 2.1 Discrete-time system.

where

$$G(z) = \frac{10z^2 - 8z}{5z^2 - 8z + 3}$$

a) Compute (if possible) the position constant k_p and the velocity constant k_v of $G(z)$.

[3 marks]

b) Suppose that $u(k) = \delta(k)$, where $\delta(k)$ is the discrete-time unit impulse function, that is $\delta(0) = 1$; $\delta(k) = 0, \forall k \neq 0$. Compute the first three values of the impulse response of the system, that is compute $h(0), h(1), h(2)$, where $h(k) = \mathscr{Z}^{-1}[G(z)]$.

[5 marks [

c) The "*steady-state*" output sequence is defined as

$$y_{ss}(k) = y(k) - y_{trans}(k)$$

where $y_{trans}(k)$ denotes the transient output terms *(if any)* that become negligible for sufficiently large values of k (that is, $y_{trans}(k) \to 0$ for $k \to \infty$).

Suppose that the input sequence $u(k)$ is given by

$$u(k) = \begin{cases} [4 + 2\sin(2k)], & k \geq 0 \\ 0, & k < 0 \end{cases}$$

Determine the analytical expression of the "*steady-state*" output sequence $y_{ss}(k)$.

[9 marks]

d) To answer Question 2c), would it be possible to exploit the discrete-time frequency response theorem? Justify your answer.

[3 marks]

3. Consider the digital control system shown in Figure 3.1.

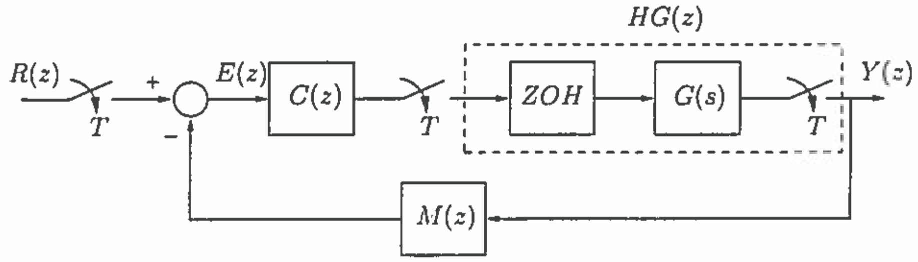

Figure 3.1 Block diagram for Question 3.

where $T = 0.1$s is the sampling time, "ZOH" stands for "zero-order hold" and

$$G(s) = \frac{1}{(1+s)^2} \quad \text{and} \quad M(z) = \frac{1}{2}$$

a) Determine the equivalent discrete-time model $HG(z)$ for the plant $G(s)$ connected to a ZOH and a sampler.

Discrete version of $\frac{1}{(s+a)^2}$ given @ 12:20

[4 marks]

b) Consider a continuous-time controller $C(s)$ made by a low-pass filter

$$C(s) = \frac{K}{s+K}$$

where $K > 0$ is a positive parameter to be suitably chosen. Determine explicitly the discrete-time approximations $C_{Tu}(z)$ and $C_{pz}(z)$ of the controller $C(s)$ through the Tustin transformation and through the pole-zero correspondence, respectively. Compare the two approximations $C_{Tu}(z)$ and $C_{pz}(z)$ and comment on your findings.

[4 marks]

c) Considering the discrete-time approximation of the controller $C_{Tu}(z)$ (that is, the Tustin approximation) obtained in your answer to Question 3b) above and exploiting your answer to Question 3a), compute (if possible) the transfer function from the reference input $R(z)$ and the output $Y(z)$.

[6 marks]

d) Determine (if possible) a value $\tilde{K}$ of K so as to obtain a second-order closed-loop transfer function $G_{cl}(z)$. In case this specific value $\tilde{K}$ of the parameter K does exist, check whether the resulting closed-loop control system is asymptotically stable.

[6 marks]

4. Consider the digital control system shown in Figure 4.1

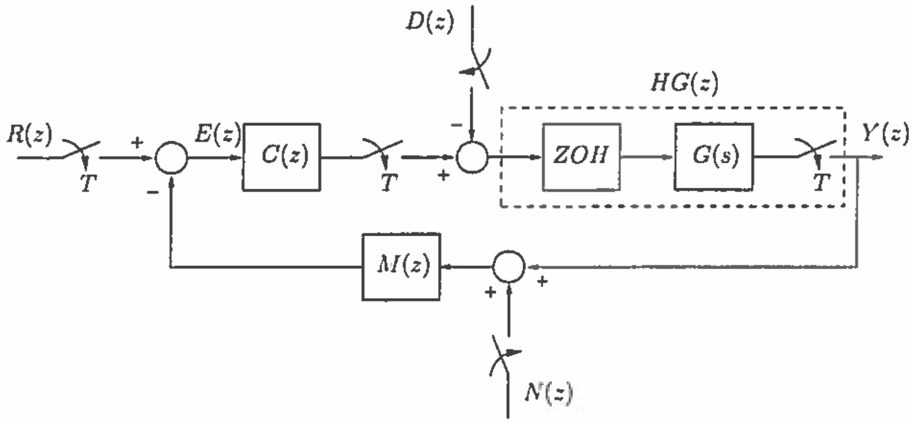

Figure 4.1 Block diagram for Question 4.

where T is the sampling time, "ZOH" stands for "zero-order hold", and $HG(z)$ denotes the equivalent discrete-time model for the plant $G(s)$ connected to the ZOH and the sampler.

a) Determine the closed-loop discrete-time transfer function $G_{cl}^{(ry)}(z)$ from the reference input variable $R(z)$ and the output $Y(z)$ expressed in terms of the generic discrete-time transfer functions $C(z)$, $HG(z)$ and $M(z)$.

[4 marks]

b) Determine the closed-loop discrete-time transfer functions $G_{cl}^{(dy)}(z)$ and $G_{cl}^{(ny)}(z)$ from the disturbance input variables $D(z)$ and $N(z)$, respectively, expressed in terms of the generic discrete-time transfer functions $C(z)$, $HG(z)$ and $M(z)$.

[6 marks]

c) Set

$$HG(z) = \frac{1-e^{-T}}{z-e^{-T}}, \quad M(z) = 1, \quad \text{and} \quad \overset{C(z)}{\cancel{R(z)}} = K_P + \frac{K_I z}{z-1}$$

where $K_P > 0$ and $K_I > 0$ are constant parameters of a discrete-time PI controller. Setting the sampling time as $T = 0.1$s, compute (if possible) a pair of constants $K_P > 0$ and $K_I > 0$ such that

- The velocity error e_v satisfies $|e_v| \le 0.01$
- The discrete-time closed-loop system is asymptotically stable.

[10 marks]