IMPERIAL COLLEGE LONDON

BEng, MEng and ACGI Examinations 2015-16

Part 1

Biomedical Engineering
BE1-MECH 1
Mechanics 1, Main Examination

14/06/2016, 14.30 – 16.30
Duration: 120 minutes

The paper has 4 (FOUR) questions.
Answer all 4 questions.
Each question is worth 100 marks.

Marks for questions and parts of questions are shown next to the question. The marks for questions (and parts thereof) are indicative, and they may be slightly moderated at the discretion of the examiner.

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Question 1

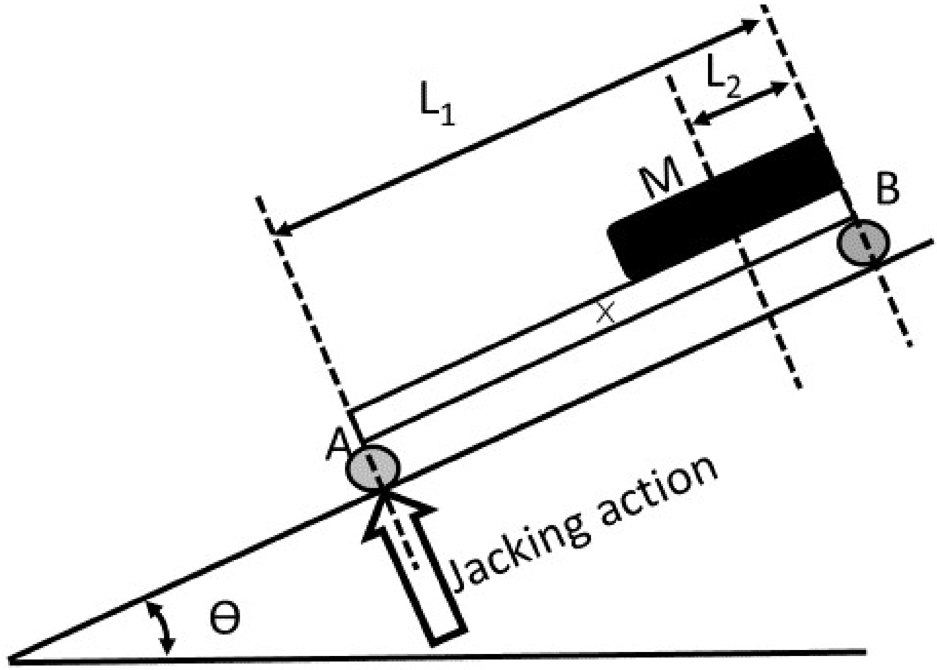

Figure 1

Your trailer (of mass m_t) has suffered a flat tyre (the front wheel at A) on a slope, and you need to jack it up to change the wheel. The brakes are locked on all wheels. The load M is located L_2 forward of the back axle B.

(a) Draw the Free Body Diagram for the Trailer Bed (AB) **10 marks**

(b) Derive an expression for the force in the jack required, in terms of the parameters shown **25 marks**

(c) If the slope of the road Θ is 15°, the mass of the trailer m_t is 15 tonnes, the load M is 75 tonnes, the trailer length L_1 is 25 metres and the load is positioned 5 m forward of the back axle (L_2), what is the force required to lift the trailer off the wheel at A. **15 marks**

(the question continues over the page)

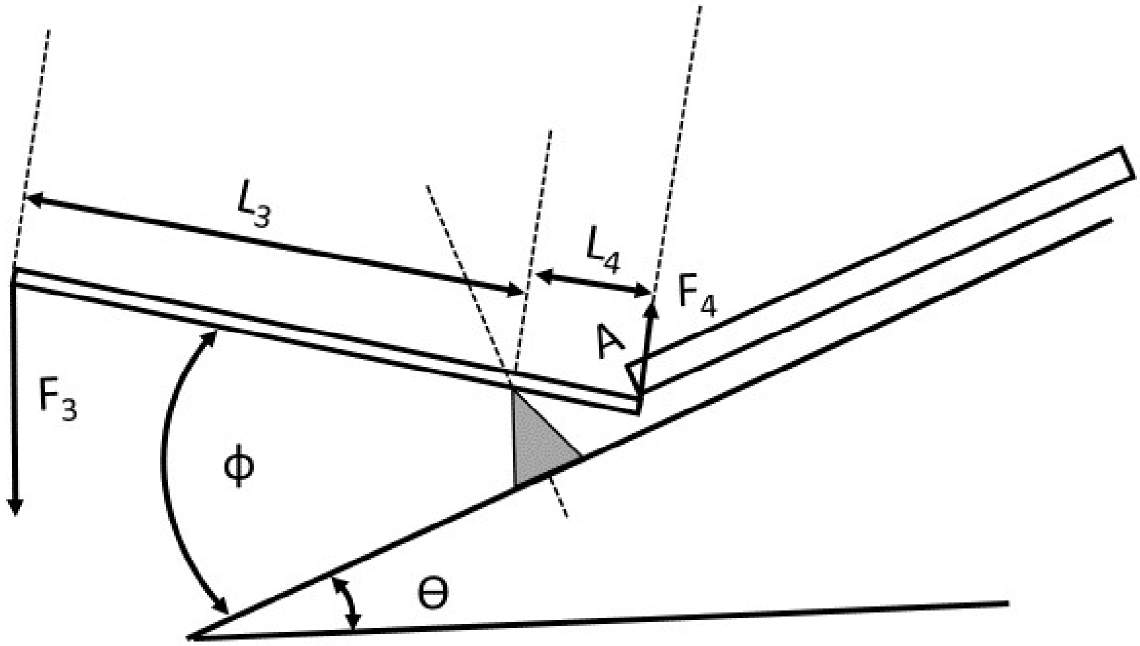

Figure 2

(d) If your jack is broken, and you have to improvise with the lever shown in Figure 2, derive an expression for the force F3 to be applied to the end of the lever (vertically downwards) in terms of the parameters shown/given. **25 marks**

(e) If the ratio L3:L4 is 12:1 , the angle ϕ is 45° , and assume now the jacking force required is 50 kN (ignore your answer to part (b)), what force will be required at F_3. **25 marks**

Question 1.

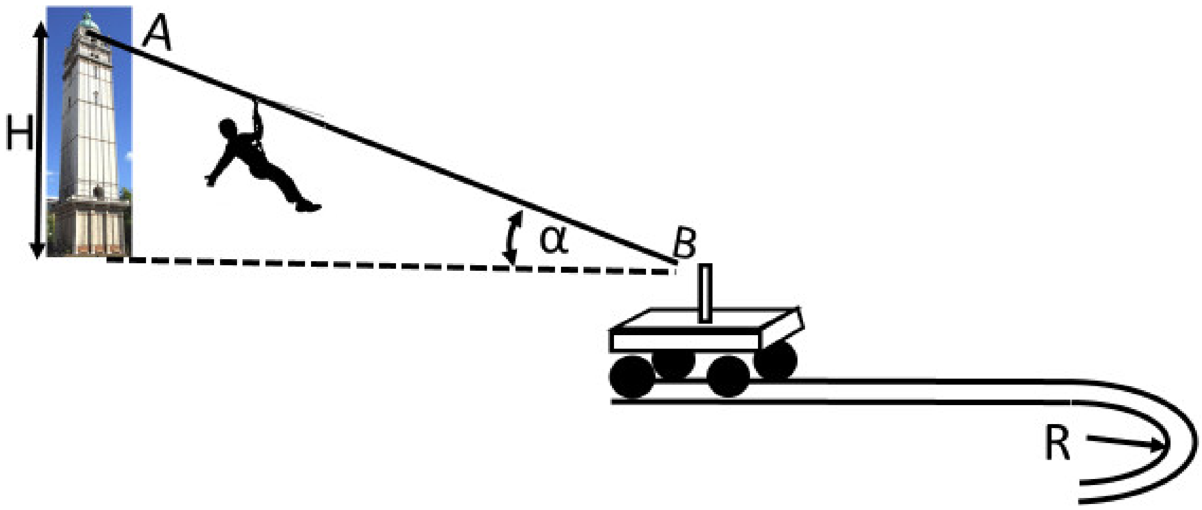

Figure 3

Imperial College has been transformed into a theme park ("The Thrill of Learning"). One of the extreme rides is a zip line from the Queen's Tower which delivers the participant to a cart which runs on a track through the rest of the South Kensington campus. A participant of mass m slides along the zip line AB at angle α, landing on the cart (of mass M), on which he/she will sit by holding on to the post and the combined mass (cart plus participant) will then travel along the track to a series of curves.

(a) If the vertical height descended by the passenger along the zip line is H, and friction and air resistance can be ignored, derive an expression for the velocity of the participant when he/she arrives at the cart (at B), in terms of the masses and distances given. **10 marks**

(b) He/she then releases the zip line, clings onto the cart, and cart + participant move off (consider this an inelastic collision). Derive an expression for the velocity of the cart + participant after the participant lands and grasps onto the cart. **15 marks**

(c) The cart + participant now continue on to the first curve , which has a radius of R. Still ignoring friction, derive an expression for the force with which he/she must hold on to stay on the cart (in terms of the masses and distances given). **15 marks**

(d) If air resistance cannot be neglected whilst descending along the zip line, but is defined by

$$F_d = kv,$$

where v is their velocity along the zip line, derive an expression for the participant's velocity at time t. **60 marks**

Question 3

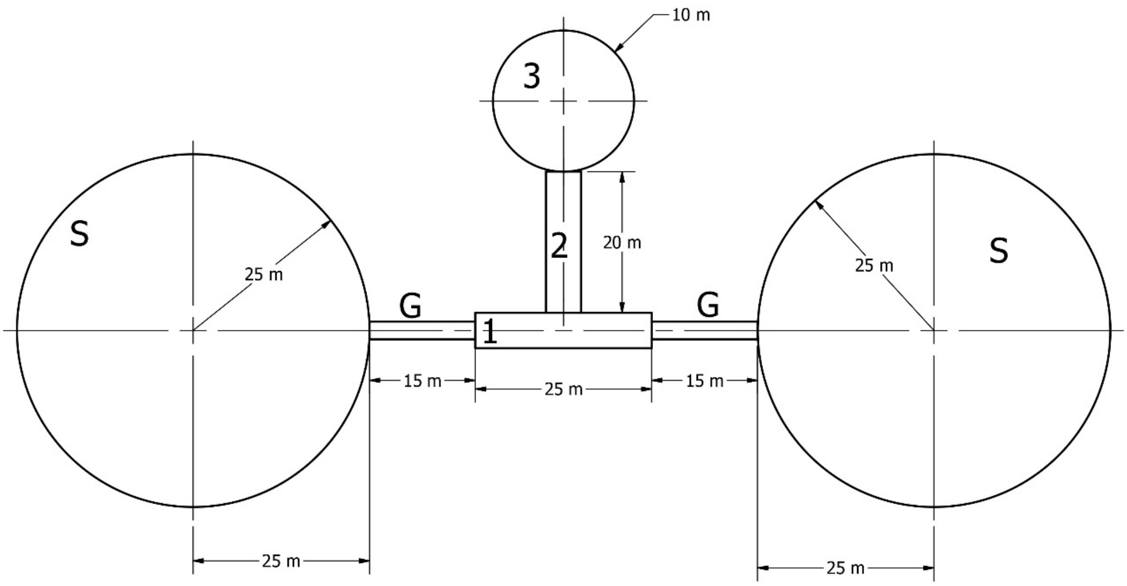

Figure 4

The International Space Station (Figure 4 and 5) consists of the following modules, joined together as shown:

- Housing 1 – a Cylindrical shell of diameter 5 m, length 25 m and mass 1000 kg
- Housing 2 – another cylindrical shell of diameter 5 m, length 20 m and mass 750 kg welded perpendicular to the edge of housing 1, at exactly half the length of housing 1 (assume the end of Housing 2 is 2.5 m from the centerline of Housing 1).
- Housing 3 – a Spherical Shell of diameter 20 m and mass 800 kg, welded to Housing 2 such that its Centre is 8 m from the end of Housing 2
- Two gantries G – cylindrical shells of length 15 m, diameter 2.5 m and mass 500 kg each, welded to either end of Housing 1.
- Two Solar panels (S) mounted on the gantries, which can be represented by discs of 50 m diameter mounted at the end of the gantries, co-axial with module 1, of 1500 kg each.

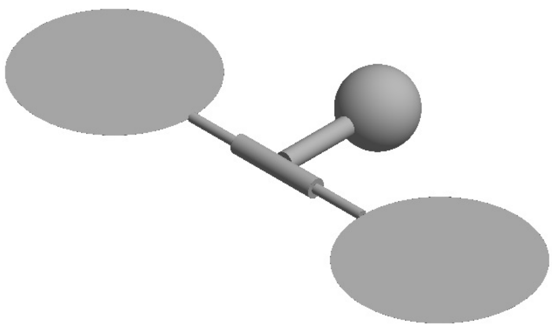

Figure 5

a) Calculate the location of the Centre of Mass of the Space Station in the X and Y directions. **30 marks**

b) Calculate the Moment of Inertia of the ISS about its Centre of Mass, in the X-Y plane. (A table of Moments of Inertia is appended on the back page of this exam paper) **70 marks**

Question 4.

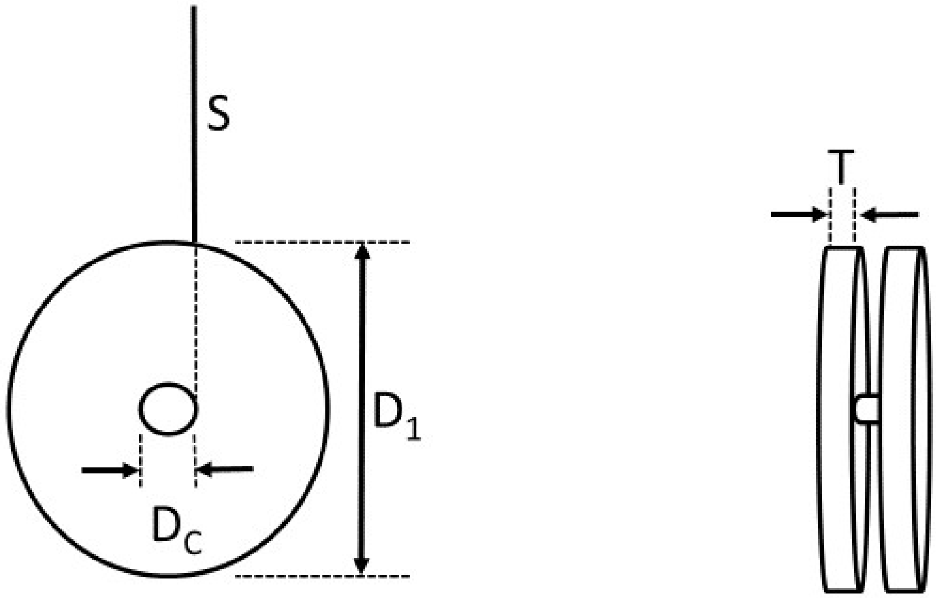

Figure 6

You have a yo-yo which consists of two discs of diameter D_1 and thickness T joined by an axle of negligible mass, diameter D_c and length T. The total mass of the assembly is M. It runs on a string S of length L, which has a loop at the end to allow virtually frictionless spinning once it unwinds to the bottom of the string length.

(a) For your new trick, starting with the string fully wound around the axle, you first release the yo-yo, allowing it to roll down to the bottom of the string acquiring rotational velocity; when it unwinds to the bottom it then spins at constant angular velocity ω. If the height dropped whilst unwinding was L, and friction and air resistance can be ignored, derive an expression for ω in terms of D_1, D_c, M, & L.
20 marks

(b) You then lower your hand so that the yo-yo just touches the ground, and you adjust your hand vertically so as to maintain the tension in the string at 0.9 x Mg. If the coefficient of friction between the yo-yo and the floor is μ_d, draw a free body diagram for the yo-yo in this state (you can assume it will attain linear equilibrium whilst it keeps spinning) and show the relationships between the various forces acting. State your assumptions. **40 marks**

(c) Derive the mathematical expressions fully describing the motion of the yo-yo under these conditions. **40 marks**

Moments of Inertia of different shapes

Shape	**Axis/direction**	**I =**
Point mass at distance R from axis		mR^2
Sphere of radius R	about centre	$2/5\ mR^2$
Spherical shell of radius R	about centre	$2/3\ mR^2$
Thin rod of radius R and height H	about axis	$½\ mR^2$
“	normal to its axis	$1/12\ mH^2$
Cylindrical shell of radius R and height H	about axis	mR^2
“	normal to its axis	$1/12\ mH^2$
Hoop	about its diameter	$½\ mR^2$
Cylinder or radius R and height H	about its axis	$½\ mR^2$
“	normal to its axis	$1/12\ m(H^2 + 3R^2)$