DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2004

M.Sc and EEE/ISE PART IV: M.Eng. and ACGI

Solutions 2004
ADVANCED COMMUNICATION THEORY

— ***There are FOUR questions (Q1 to Q4)***
— ***Answer Question ONE plus TWO other questions.***

Comments for Question Q1:
— ***Question Q1 has 20 multiple choice questions numbered 1 to 20 .***
— ***Circle the answers you think are correct on the answer sheet provided.***
— ***There is only one correct answer per question.***

Distribution of marks
Question-1: 40 marks
Question-2: 30 marks
Question-3: 30 marks
Question-4: 30 marks

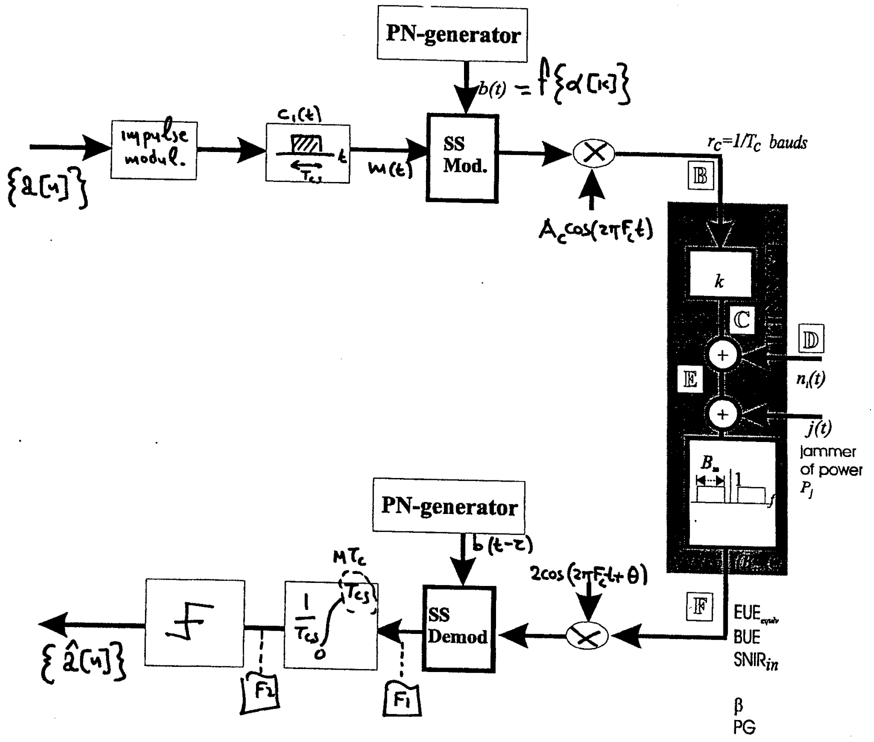
The following are provided:
- ***A table of Fourier Transforms***
- ***A "Gaussian Tail Function" grap***

Examiners responsible: Dr. A. Manikas

ANSWER to Q1

1)	A	B	C	D	E
2)	A	B	C	D	E
3)	A	B	C	D	E
4)	A	B	C	D	E
5)	A	B	C	D	E
6)	A	B	C	D	E
7)	A	B	C	D	E
8)	A	B	C	D	E
9)	A	B	C	D	E
10)	A	B	C	D	E
11)	A	B	C	D	E
12)	A	B	C	D	E
13)	A	B	C	D	E
14)	A	B	C	D	E
15)	A	B	C	D	E
16)	A	B	C	D	E
17)	A	B	C	D	E
18)	A	B	C	D	E
19)	A	B	C	D	E
20)	A	B	C	D	E

ANSWER to Q2 (aim: to examine 'Spread Spectrum Theory')



$\boxed{B}$: $s(t) = A_c m(t) . b(t) \cos(2\pi F_c t)$

$\boxed{F}$: desired signal term $= k s(t) = \underbrace{kA_c}_{\sqrt{2P_s}} m(t)\, b(t) \cos(2\pi F_c t)$

$\boxed{F1}$: desired signal term $= k s(t) . b(t-\tau)\, 2\cos(2\pi F_c t + \theta)$

$$= \sqrt{2P_s}\, m(t)\, b(t) . b(t-\tau)\ 2\cos(2\pi F_c t)\cos(2\pi F_c t + \theta)$$

$\boxed{F2}$: desired signal term $= m_o(t) =$

$$= \frac{\sqrt{2P_s}}{T_{cs}} \int_0^{T_{cs}} \underbrace{m(t)}_{\pm 1} . b(t) . b(t-\tau) \cos\theta . dt$$

$$= \pm \frac{\sqrt{2P_s}}{T_{cs}} \cos\theta \int_0^{MT_c} b(t)\, b(t-\tau)\, dt$$

$$= \pm \sqrt{2P_s} \cos\theta\ R_{b,M}(\tau)$$

Power of $m_o(t) = \mathcal{E}\{m_o^2(t)\} = 2P_s \cos^2\theta . \mathcal{E}\{R_{b,M}^2(\tau)\}$

$$= 2P_s\cos^2\theta\left(\mathrm{Var}\{R_{b,M}(\tau)\} + \mathcal{E}^2\{R_{b,M}(\tau)\}\right)$$

$$= \underbrace{2P_s\cos^2\theta\ \mathrm{Var}\{R_{b,M}(\tau)\}}_{\text{code noise power}} + \underbrace{2P_s\cos^2\theta\ \mathcal{E}^2\{R_{b,M}(\tau)\}}_{\text{desired term}}$$

1. If $0 \leq \tau \leq T_c$ then power of code noise $= 2P_s \cos^2\theta \, \mathrm{var}\{R_{b,M}(\tau)\}$

$$= 2P_s \cos^2\theta \cdot \frac{1}{M}\left(\frac{\tau}{T_c}\right)^2$$

$$= 2P_s \cos^2\theta \, \frac{1}{M} \frac{\tau^2}{T_c^2} \qquad \left(\text{note} : M = \frac{T_{cs}}{T_c}\right)$$

$$= 2P_s \cos^2\theta \frac{1}{T_{cs}T_c} \cdot \tau^2 \qquad \boxed{1}$$

2. If $\tau > T_c$ then power of code noise $= 2P_s \cos^2\theta \, \mathrm{var}\{R_{b,M}(\tau)\}$

$$= 2P_s \cos^2\theta \, \frac{1}{M}$$

$$= 2P_s \cos^2\theta \, \frac{T_c}{T_{cs}} \qquad \boxed{2}$$

$r_c = 10\text{M} \frac{\text{chips}}{\text{sec}} \Rightarrow T_c = 10^{-7}$

$r_b = 1000 \frac{\text{bits}}{\text{sec}} \Rightarrow T_{cs} = 10^{-3}$

$\frac{N_0}{2} = 0.5 \times 10^{-8} \Rightarrow N_0 = 10^{-8}$

$\text{EUE} = 100 \Rightarrow \frac{E_b}{N_0} = 10^2 \Rightarrow \frac{P_s T_{cs}}{N_0} = 10^2 \Rightarrow P_s = 10^2 \frac{\overbrace{N_0}^{10^{-8}}}{\underbrace{T_{cs}}_{10^{-3}}} \Rightarrow \boxed{P_s = 10^{-3}}$

$P_{\substack{\text{code noise} \\ \tau > T_c}} = 1.5 \times 10^{-7} \overset{②}{\Longrightarrow} 1.5 \times 10^{-7} = 2P_s \cos^2\theta \, \frac{T_c}{T_{cs}}$

$\Rightarrow \cos^2\theta = \frac{1.5 \times 10^{-7} \cdot \overbrace{T_{cs}}^{10^{-3}}}{2\underbrace{P_s}_{10^{-3}} \underbrace{T_c}_{10^{-7}}} \Rightarrow$

$\Rightarrow \cos^2\theta = \frac{1.5}{2} = 0.75 \Rightarrow \cos\theta = 0.866$

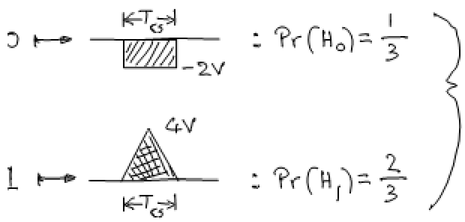
$\Rightarrow \boxed{\theta = 30^\circ}$

$P_{\substack{\text{code noise} \\ \tau}} = 3.75 \times 10^{-8} \overset{①}{\Longrightarrow} 3.75 \times 10^{-8} = 2\underbrace{P_s}_{10^{-3}} \underbrace{\cos^2\theta}_{0.75} \frac{1}{\underbrace{T_{cs}}_{10^{-3}} \underbrace{T_c}_{10^{-7}}} \tau^2$

$\Rightarrow \tau^2 = \frac{3.75 \times 10^{-15}}{2 \times 0.75} \Rightarrow \boxed{\tau = 0.5 \times 10^{-7}}$ ie $\boxed{\tau = 0.5 T_c}$

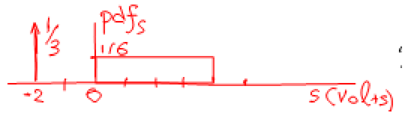
ANSWER to Q3 (aim: to examine 'decision rules')

a)



$\Rightarrow \text{pdf}_s(s) = \frac{1}{3}\delta(s+2) + \frac{2}{3}\times\frac{1}{4}\times\text{rect}\frac{s-2}{4}$

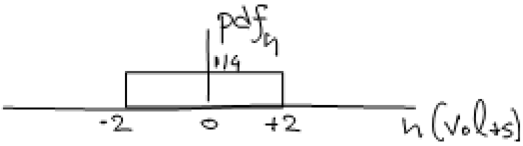
i.e.



b)

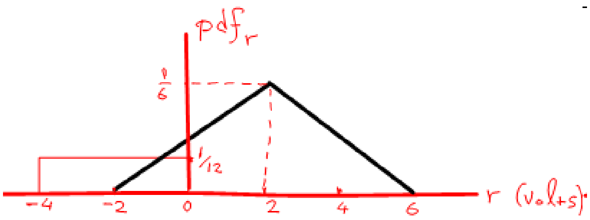
$r(t) = s(t) + n(t) \Rightarrow \text{pdf}_r = \text{pdf}_s * \text{pdf}_n$

where $\text{pdf}_n = \frac{1}{4}\text{rect}\frac{n}{4}$ i.e.

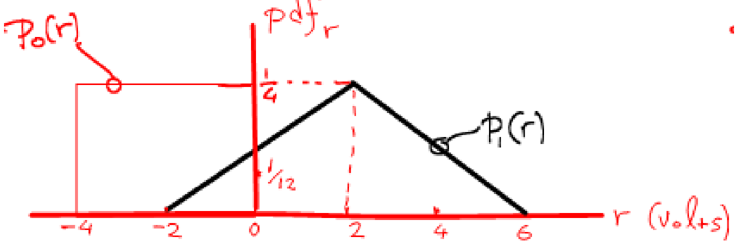


$$\Rightarrow \text{pdf}_r = \overbrace{\tfrac{1}{3}}^{\Pr(\mathrm{H}_0)} \times \underbrace{\tfrac{1}{4}\text{rect}\tfrac{r+2}{4}}_{=p_0(r)} + \overbrace{\tfrac{2}{3}}^{\Pr(\mathrm{H}_1)} \times \underbrace{\tfrac{1}{4}\times\tfrac{1}{4}\times 4\Lambda\left(\tfrac{r-2}{4}\right)}_{=p_1(r)}$$

i.e.



c) likehood functions (placed together)

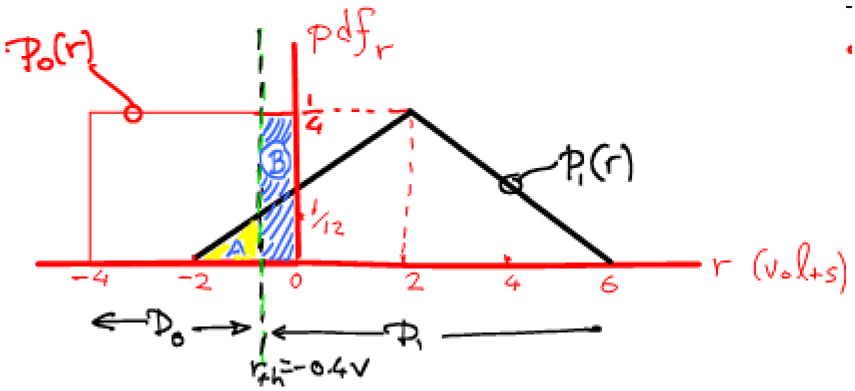


d)

- $p_0(r) = \frac{1}{4}\text{rect}\frac{r+2}{4}$ and $p_1(r) = \frac{1}{4}\Lambda\left(\frac{r-2}{4}\right)$.
- likelihood ratio $= \lambda(r) = \frac{\Lambda\left(\frac{r-2}{4}\right)}{\text{rect}\frac{r+2}{4}}$
- $\lambda_0 = \frac{\Pr(\mathrm{H}_0)}{\Pr(\mathrm{H}_1)} \cdot \frac{c_{10}-c_{00}}{c_{01}-c_{11}} = \frac{1/3}{2/3} \cdot \frac{0.8}{1} = 0.4$

- Therefore, choose H_1 <u>iff</u> $\lambda(r) > \lambda_0$

$$\Rightarrow \Lambda\left(\frac{r-2}{4}\right) > 0.4\text{rect}\frac{r+2}{4}$$
$$\Rightarrow \frac{r+2}{4} > 0.4$$
$$\Rightarrow r > -0.4\text{Volts}$$



e)

i) $P_{FA} = \Pr(D_1|H_0) = \text{area B} = \frac{1}{4} \times 0.4 = 0.1$

$P_{miss} = \Pr(D_0|H_1) = \text{area A} = \int_{-2}^{-0.4} \frac{1}{4}\frac{r+2}{4}dr = 0.08$

$$\mathbb{F} = \begin{bmatrix} 0.9, & 0.08 \\ 0.1, & 0.92 \end{bmatrix}; \ \underline{p} = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$$

ii) $p_e = \Pr(D_1|H_0) \times \Pr(H_0) + \Pr(D_0|H_1) \times \Pr(H_1) = 0.0867$

iii) $\mathbb{J} = \mathbb{F}.\text{diag}(\underline{p}) = \begin{bmatrix} 0.9, & 0.08 \\ 0.1, & 0.92 \end{bmatrix} \begin{bmatrix} 1/3, & 0 \\ 0, & 2/3 \end{bmatrix} = \begin{bmatrix} 0.3, & 0.0533 \\ 0.0333, & 0.6133 \end{bmatrix}$

ANSWER to Q4 (aim: to examine 'DS-CDMA')

$P = 10mW$
$r_b = 500\,\text{kbits/sec} \Rightarrow T_{cs} = \frac{1}{500}$ msec

K = 201 users
No = 2×10^{-9}

$p_e = 3 \times 10^{-5}$

$a = 0.375$
$s = 1/3$

$$p_e = \mathrm{T}\{\sqrt{2\,\mathrm{EUE}_{equ}}\} \Rightarrow 3 \times 10^{-5} = \mathrm{T}\{\sqrt{2\,\mathrm{EUE}_{equ}}\}$$

$\Rightarrow$ (using 'tail' graph' supplied)

$$4 = \sqrt{2\,\mathrm{EUE}_{equ}}$$

$$\mathrm{EUE}_{equ}=8$$

However,

$$\mathrm{EUE}_{equ}=\frac{E_b}{\mathrm{N_o}+\mathrm{N_j}}$$

where $E_b = PT_{cs}$ and $\mathrm{N_j} = \frac{(\mathrm{K}-1).P.a.s}{B_{ss}} = \frac{(\mathrm{K}-1).P.a.s}{\mathrm{PG}/T_{cs}}$

Therefore,

$$\mathrm{EUE}_{equ}=\frac{PT_{cs}}{\mathrm{N_o}+\frac{(\mathrm{K}-1).P.a.s}{\mathrm{PG}/T_{cs}}} \Rightarrow \ldots\ldots\ldots$$

$$\Rightarrow \mathrm{PG} = \frac{(\mathrm{K}-1).P.a.s.T_{cs}}{\frac{PT_{cs}}{\mathrm{EUE}_{equ}}-\mathrm{N_o}}$$

$$\Rightarrow \ldots \Rightarrow \mathrm{PG} = 1000$$