Information for Candidates

Switch-Mode Power Supplies

Voltage ratio equations for buck SMPS $\frac{V_O}{V_I} = \delta$ or $\frac{V_O}{V_I} = \frac{1}{1+\frac{2fL\,I_O}{V_I\delta^2}}$

Voltage ratio equations for boost SMPS $\frac{V_O}{V_I} = \frac{1}{1-\delta}$ or $\frac{V_O}{V_I} = \frac{1}{1-\frac{V_I\delta^2}{2fL\,I_I}}$

Three-Phase Systems

Line Voltages and Current

Star $V_L = \sqrt{3}\,V_P;\ I_L = I_P$ Delta $V_L = V_P;\ I_L = \sqrt{3}\,I_P$

Power $P_{3\phi} = 3\,V_P I_P cos(\phi) = \sqrt{3}\ V_L I_L cos(\phi)$

Induction Machines

$\omega_S = {}^{\omega_E}/_{p}\ ;\quad s = \frac{\omega_S - \omega_R}{\omega_S}\ ;\quad T_{EM} = \frac{3\,I'^2_R\,R'_R}{\omega_R}\left(\frac{1-s}{s}\right)$

Photovoltaic Systems

$$I_{PV} = I_{Ph} - I_{AK} - I_{Sh} \qquad I_{AK} = I_0\left(e^{\frac{V_{PV}+I_{PV}R_S}{K_I v_T}} - 1\right)$$

Power Flow in Lines and Cables

Cable Parameters

$R'_{LF} = \frac{1}{\sigma_C \pi\, r_C^2} + \frac{1}{\sigma_C 2\pi\, r_O\, t_O}$ $\quad L' = \frac{\mu}{2\pi} ln\left({}^{r_O}/_{r_C}\right)$ $\quad C' = \frac{2\pi\varepsilon_0\varepsilon_{RI}}{ln\left({}^{r_O}/_{r_C}\right)}$ $\quad G' = \frac{2\pi\sigma_I}{ln\left({}^{r_O}/_{r_C}\right)}$

OHL Parameters (approximate form)

$R'_{LF} = \frac{2}{\sigma_C \pi\, r_C^2}$ $\quad L' = \frac{\mu}{\pi} ln\left({}^{d}/_{r_C}\right)$ $\quad C' = \frac{\pi\varepsilon_0\varepsilon_{RI}}{ln\left({}^{d}/_{r_C}\right)}$ $\quad G' = \frac{\pi\sigma_I}{ln\left({}^{d}/_{r_C}\right)}$

Power Flow (full form)

$$P_S = \frac{{V_S}^2}{Z_{SR}} cos(\theta) - \frac{V_R V_S}{Z_{SR}} cos(\theta + \delta) \qquad Q_S = \frac{{V_S}^2}{Z_{SR}} sin(\theta) - \frac{V_R V_S}{Z_{SR}} sin(\theta + \delta)$$

Voltage Drop (approximate form) $\Delta V = |V_S| - |V_R| \approx \frac{R\,P_S + Z\,Q_S}{|V_S|}$

1. This question covers several topics and all parts should be attempted.

a) Consider a national-scale AC electricity system.

i) For an example country, the UK perhaps, describe the changes in proportions of generation from various sources in the last decade and the reasons for the change. [4]

Discussion should note the rise of low-carbon / renewable sources, principally wind and photo-voltaic energy, in many countries of the world. [1 mark]

Rise of renewables is driven by subsidies in the form of feed-in tariffs, green certificates or contracts-for-difference, although subsidy-free invest is now viable in some cases. [1 mark]

Among the fossil fuels, gas has been displacing coal, partly driven by reducing gas prices (or expanding supply) and carbon taxes that increase the costs of coal more than gas. [1 mark]

Various other points can be made regarding position of nuclear, hydro or biomass. In the UK hydro is maintained and little prospect of expansion, biomass has been subsidised and is not substantial and new-build nuclear is subsidised to replace decommissioned plant. [1 mark]

ii) Explain why very high voltages are used for national-scale transmission but lower voltages for local distribution. [4]

The dominant power loss is in the series resistance of the lines so there are operational cost savings to be made by increasing the voltage and decreasing the current hence the use of very high voltages in transmission. [2 marks]

In distribution, where distances between substations is lower, the savings in operational cost do not offset the capital cost of the substation equipment to the same extent and so less expensive lower voltage (10s not 100s kV) is used is optimises the lifetime cost. [2 marks]

iii) Explain why a three-phase system is used. [4]

Distributed (not concentrated) windings are used in generators for the better cooling, waveform shape and machine layout they offer. However, placing all coils in series (single phase) causes voltage cancellation and so breaking the coils into phase groups (poly phase) is preferred but there are diminishing returns in high phase numbers. High phase numbers have high costs in transformers and protection systems. [2 marks]

Poly-phase has the advantage over single-phase of being able to transfer constant instantaneous power and create rotating (not alternating) magnetic fields. 3-phase systems, and above, have zero neutral current underbalanced conditions and neutral conductor can be omitted. [1 mark]

Taking all factors into account, three-phase stands out as the best compromise. [1 mark]

iv) Explain why DC is used in place of AC for some parts of a transmission system. [4]

In AC cables, the capacitance rises with distance and so does the shunt capacitive current. [1 marks]

The capacitive current occupies part of the current rating of the cable and means long cables need to be de-rated which adds to the system cost, or needs compensation which also adds cost. [1 mark]

DC avoids these additional route-costs but adds very significant costs of AC/DC power conversion at the route ends. [1 mark]

Above some distance, in the region of 100 km, the cost of a DC cable plus its terminal equipment is less than that of AC. [1 mark]

(Alternative answer considering inductive voltage drop in OHL is acceptable.)

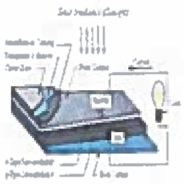
(Use of DC for connecting asynchronous systems can substitute for part of the argument)

b) Consider a typical photovoltaic panel.

i) Describe, with the aid of a sketch, the structure of a photovoltaic panel. [4]

Sketch showing

- wafer with *pn*-junction
- thin stripes of top-side metallisation
- protective and anti-reflective top-side layer



[2 marks for sketch and 2 marks for highlighting salient features]

ii) Explain why a maximum power point tracking algorithm is needed for a photo-voltaic panel **and** briefly explain how it operates. [4]

A PV cell exhibits a peak power output at a particular terminal voltage and it is advantageous to operate at that voltage [1 mark]

The optimal voltage is dependent on the irradiance and temperature (and aging) and so cannot be pre-selected nor readily determined from measurements so must be found through a search algorithm during operation. [2 marks]

The search is a trial-and-error process known as perturb-and-observe. Small step changes are made to the terminal voltage. The direction (increase or decrease) of each step depends on whether the previous step was seen to increase or increase the power output [1 mark]

c) Consider a Boost SMPS.

i) Describe the operating principle of the Boost SMPS [4]

Turning the MOSFET on imposes the full input voltage across the inductor, causing the current to increase and energy to be stored. During this period the diode is reverse biased and the output current is provided by the output capacitor. [2 marks]

Following turn-off of the MOSFET, the inductor current flows via the now forward biased diode to deliver charge to the output capacitor. Under this condition, there is negative voltage across the inductor so its current is reducing and it delivers some its stored energy to the output capacitor. The capacitor voltage will reach a value sufficiently above the input voltage to create sufficient decrease in inductor current to balance the increase caused by the input voltage during the on-time. [2 marks]

ii) **The commonly used expression for output voltage in continuous conduction mode is $\frac{V_O}{V_I} = \frac{1}{1-\delta}$. Explain why the voltage observed in practice would be less than this.** **[4]**

There several reasons why there are voltage drops and/or power losses which mean that a lower output voltage is achieved than the idealised equation predicts.

- During the on-time, voltage is dropped across the MOSFET which causes a lower increase in inductor current than predicted reducing the equilibrium output voltage.
- During the off-time, voltage is dropped across the diode which causes a higher decrease in inductor current than predicted reducing the equilibrium output voltage
- Resistance in the inductor drops voltage.
- Switching power loss in the MOSFET also reduces the energy transfer to the output capacitor.

[Full marks if at least three factors and a coherent argument are given.]

iii) **The graph of efficiency against output power normally exhibits a central plateau with efficiency reducing a low and high output powers. Explain why this is so.** **[4]**

There are three types of power loss:

- Fixed loss, independent output power/current, due to operation of the control circuitry. At lower output powers, this term dominates and caused low efficiency.
- Power loss due to diode voltage drop and MOSFET switching which are proportional to current and so scale with output power giving a constant efficiency where these dominate in mid-range output power.
- Power loss proportional to current squared in the resistance of the MOSFET and inductor. This dominate and high power and grow faster than output power so cause a decrease in efficiency.

[1 mark for each factor and 1 for a coherent overall argument]

d) **For the equivalent circuit of an induction machine shown in figure Q1, identify which components give rise to power loss and what physical process or feature of the machine they represent.** **[4]**

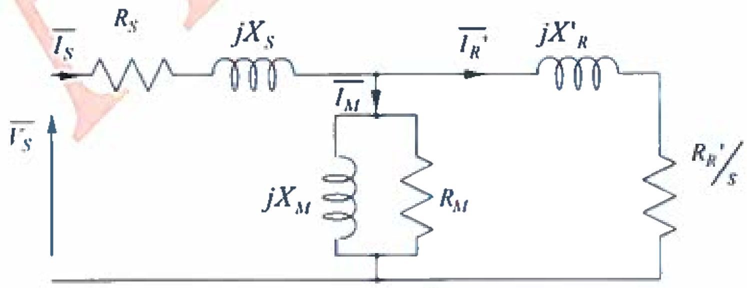


Figure Q1

Only the three resistors contribute to power loss, the inductors do not.
R_S models the resistance of the stator winding and the ohmic heating that results from it. [1 mark]
R_M models, approximately, the power losses due to eddy currents and hysteresis in the steel core. [1 mark]

R_R models the resistance of the rotor winding and the ohmic heating that results from it but it is only a portion of the term R_R /s with remainder, $R_R\,(1/s-1)$ representing conversion to mechanical energy (and not a loss). [2 marks]

[2 marks removed for mis-identifying flux-leakage or inductance as a power loss]

2.

A **buck** switch-mode power supply (SMPS) is to be used to provide a 5 V output from an 18 V input. The SMPS is intended to operate at a maximum output current of 10 A.

The MOSFET to be used has an on-state resistance of $R_{DS(on)}$ = 35 mΩ and turn-on and turn-off energy losses of $E_{Sw\text{-}on}$ = 9 μJ and $E_{Sw\text{-}off}$ = 7 μJ. The diode to be used is a Schottky device with an on-state voltage of $V_{AK(on)}$ = 0.45 V.

The capacitor to be used has a capacitance of 330 μF and has an effective series resistance (ESR) of R_{ESR} = 70 mΩ.

You may assume that the SMPS operates in continuous conduction mode for the calculations that follow.

a)

i) Calculate the duty-cycle with which the SMPS should operate. [2]

Apply standard formula [2 marks for accurate answer].

$$\delta = \frac{V_O}{V_I} = \frac{5}{18} = 0.2778$$

ii) Sketch the shape of the current through the inductor, MOSFET and capacitor at this duty-cycle. [4]

[2 marks for basic shapes the one mark each for the two features note]

- Up-slope of inductor current Mosfet conduction period should be roughly a quarter of period
- Capacitor current should be centred on zero

b)

i) Calculate the average power loss in the diode at maximum output current. [4]

[2 marks for equation and 2 for accurate answer]

$$P_{diode} = (1-\delta)V_{AK}I_O = (1-0.2778) \times 0.45 \times 10 = 3.25\ W$$

ii) Calculate the maximum switching frequency allowed if the total power loss in the MOSFET and diode are to be limited to 5 W at maximum output current.

[6]

[3 marks for correct equations for conduction and switching loss. 3 marks for accurate answer.]

Power loss allowed in Mosfet is 5 – 3.25 = 1.75 W

Calculate first the conduction power loss.

$$P_{Mos\ cond} = \delta\ I_O^2\ R_{DS} = 0.2778 \times 10^2 \times 0.035 = 0.972\ W$$

Power loss allowed for switching is 1.75 – 0.972 =0.778 W

Calculate allowed frequency from loss per cycle.

$$P_{Mos\ Sw} = f \times (E_{on} + E_{off})$$

$$f = \frac{P_{Mos\ Sw}}{E_{on} + E_{off}} = \frac{0.778}{16 \times 10^{-6}} = 48.6\ kHz$$

iii) **State an equation for the sum of the voltage ripples across the output capacitance and its ESR and hence find the inductor current ripple allowed if the output voltage ripple is to be limited to 40 mV. Assume that the frequency found in (ii) is used. [7]**

[3 marks for equation]

$$\Delta v = \frac{\Delta i_L}{8fC} + \Delta i_L R_{ESR}$$

Re-arrange to find current ripple [1 mark for equation and 3 for accurate answer]

$$\Delta i_L = \Delta v \Big/ \left(\frac{1}{8fC} + R_{ESR}\right) = 0.04 \Big/ \left(\frac{1}{8 \times 48.6 \times 10^3 \times 330 \times 10^{-6}} + 0.07\right)$$
$$= 0.514\ \text{A}$$

iv) **Calculate the inductor value that should be used in this design. [3]**

Re-arrange equation for current ripple to find inductance. [2 marks for method; 1 for answer]

$$\Delta i_L = \frac{V_I - V_O}{L} \times \frac{\delta}{f}$$

$$L = \frac{V_I - V_O}{\Delta i_L} \times \frac{\delta}{f} = \frac{18 - 5}{0.514} \times \frac{0.278}{48.6 \times 10^3} = 146\ \mu H$$

v) **If the input voltage rose to 20 V (and duty-cycle adjusted to maintain output voltage), would this SMPS become more of less efficient? (You only need consider the power losses in the semiconductors.) [4]**

[3 marks for logical argument and 1 mark for correct conclusion]

There are three terms in the power losses: diode conduction loss, Mosfet conduction loss and switching loss. The last term is independent of duty-cycle so need not be considered.

$$P_{Cond} = (1 - \delta) I_O V_{AK} + \delta\, I_O^2\, R_{DS} = 4.5(1 - \delta) + 3.5\delta = 4.5 - 1.0\delta$$

At 10 A and with the characteristics given, the losses reduce as duty-cycle increases.

So, for an increase in input voltage, the duty-cycle decreases, the losses increase and the efficiency decreases.

3. This question addresses several aspects of AC systems.

a) A three-phase induction machine with 2 pole-pairs has the following equivalent circuit parameters:

Stator resistance, $R_S = 0.8\ \Omega$.

Stator leakage reactance, $X_S = 2\ \Omega$;

Magnetising resistance, $R_M = 200\ \Omega$;

Magnetising reactance, $X_M = 60\ \Omega$;

Referred rotor leakage reactance, $X_R = 2\ \Omega$;

Referred rotor resistance, $R_R = 0.8\ \Omega$;

The machine is supplied at a phase voltage of 200 V and a frequency of 50 Hz.

When used to drive a particular mechanical load, the machine runs at 1,445 rpm.

i) Give an expression for the total input impedance of the machine and calculate its value in complex form for the conditions given. [8]

[3 marks for correct formulation of impedance, including correct parallel terms and inclusion of s]

Total impedance:

$$Z_T = R_S + jX_S + \left(R_M // jX_M // \left(\frac{R'_R}{s} + jX'_R\right)\right)$$

[2 marks for finding slip accurate to 3 sig. fig.]

Slip needs to be calculated first from: $s = \frac{\omega_S - \omega_R}{\omega_S}$

Synchronous speed calculated by: $\omega_S = {}^{\omega_E}/_{p} = 50 \times {}^{2\pi}/_{2} = 157.08\,\frac{\text{rad}}{\text{s}}\ or\ 1{,}500\ rpm$

Hence slip is: $s = \frac{1500 - 1445}{1500} = 0.0367$

[3 marks for accurate answer]

$$Z_T = 17.734\ +\ j8.992\ =\ 19.883\angle 26.89°\ \Omega$$

ii) Calculate the mechanical power being developed and the electro-magnetic torque. [10]

First, find stator current from the phase voltage $I_S = \frac{V_S}{Z_T} = \frac{200}{19.88\angle 26.9°} = 10.06\angle -26.9°\ \Omega$

[2 marks for accurate answer]

Then rotor current is found from the current divider rule (or otherwise)

$$I_R = I_S \times \left(\frac{R_M // jX_M}{R_M // jX_M + \left(\frac{R'_R}{s} + jX'_R\right)}\right)$$

[2 marks for correct formulation]

$I_R = 8.41\ \angle -9.68°$ [1 mark for accurate answer]

Power from standard formula $P_{EM} = 3\, I'^2_R\, R'_R \left(\frac{1-s}{s}\right) = 3 \times 8.41^2 \times 0.8 \times \left(\frac{1-0.0367}{0.0367}\right) = 4.46\ \text{kW}$
[1 mark for correct formula: 2 marks for accurate answer]

Torque found by dividing by rotor speed

$$T = \frac{P_{EM}}{\omega_R} = \frac{4461}{(1-0.0367) \times 157.08} = 29.5\ \text{Nm}$$

[1 mark for correct formula: 1 mark for accurate answer]

b) An overhead line is found to have an inductive reactance per unit length of 0.1 Ω/km and negligible resistance. The line is 300 km long and is operated with sending and receiving end voltages of 400 kV and 390 kV respectively.

i) Calculate the angle difference between sending and receiving end voltages that will exist if the line transfers 1,200 MW. [4]

Use standard approximation of power flow in reactance dominated line.

[2 marks for method: 2 marks for accurate answer]

$$P = \frac{V_R V_S}{X} sin(\delta)$$

$$sin(\delta) = \frac{X\,P}{V_R V_S} = \frac{300 \times 0.1 \times 1200 \times 10^6}{400 \times 10^3 \times 390 \times 10^3} = 0.2308$$

$$\delta = sin^{-1}(0.2308) = 13.34°$$

ii) If the line current has to be limited to 1,000 A, what is the maximum angle that can be allowed and what power flow does this allow. [4]

Voltage drop along line will be jIX and the other two vectors are known. Use cosine rule of find the angle.

$$(IX)^2 = V_R{}^2 + V_S{}^2 - 2V_R V_S cos(\delta)$$

$$cos(\delta) = \frac{V_R{}^2 + V_S{}^2 - (IX)^2}{2V_R V_S} = \frac{(390 \times 10^3)^2 + (400 \times 10^3)^2 - (1000 \times 300 \times 0.1)^2}{2 \times 400 \times 10^3 \times 390 \times 10^3}$$
$$= 0.9974$$

$$\delta = cos^{-1}(0.9974) = 4.13°$$

$$P = \frac{V_R V_S}{X} sin(\delta) = \frac{400 \times 10^3 \times 390 \times 10^3}{300 \times 0.1} sin(4.13°) = 374\ MW$$

[2 marks for correct use of cosine rule to find load angle: 2 marks for final answer]

iii) Calculate the reactive power flows at the sending and receiving ends under the conditions in (ii) and compare with the reactive power of the reactance of the line itself. [4]

$$Q_S = \frac{V_S{}^2}{X} - \frac{V_R V_S}{X} cos(\delta)$$

$$Q_S = \frac{(400 \times 10^3)^2}{30} - \frac{400 \times 10^3 \times 390 \times 10^3}{30} cos(4.13) = 146\ MVAr$$

$$Q_R = \frac{{V_R}^2}{X} - \frac{V_R V_S}{X} cos(\delta)$$

$$Q_R = \frac{(390 \times 10^3)^2}{30} - \frac{400 \times 10^3 \times 390 \times 10^3}{30} cos(4.13) = -116\ MVAr$$

$$Q_{Line} = I^2\, X = 1000^2 \times 30 = 30\ MVAr$$

Reactive power leaving the receiving end matches the reactive power entering the sending end minus the reactive power absorbed by the line.

[1 mark for each numerical answer plus one mark for comparison]