IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

EEE/EIE PART III/IV: MEng, BEng and ACGI

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Corrected copy

DIGITAL SIGNAL PROCESSING

Wednesday, 14 December 9:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.A. Naylor
Second Marker(s) : W. Dai

Questions

DIGITAL SIGNAL PROCESSING

1. a) Sketch the block diagram of a 2-band maximally decimated filter bank structure showing the analysis and synthesis filter banks connected directly in cascade. [3]

Discuss the effects of aliasing in a QMF maximally decimated 2-band filter bank. Include an explanation for aliasing in terms of the aliasing component matrix $\mathbf{H}(z)$. Include supporting analysis. Include a comparison to the case of oversampled filter banks. [5]

Derive the aliasing cancelling conditions. [2]

b) Consider a linear time-invariant system output signal $y(n)$, input signal $x(n)$ and impulse response $\{h_k\}$ for $k = 0, \ldots, N$ given by

k	h_k
0	-0.0087
1	0.0000
2	0.2518
3	0.5138
4	0.2518
5	0.0000
6	-0.0087

and

$$y(n) = \sum_{k=0}^{N} h_k x(n-k).$$

The frequency response of this system is shown in Fig. 1.1.

Use this system to design aliasing cancelling 2-band maximally decimated QMF analysis and synthesis filter banks. Your design should include the coefficients of all filters. [7]

Describe fully the reconstruction properties of this filter bank and state the group delay of the analysis and synthesis filter banks. [3]

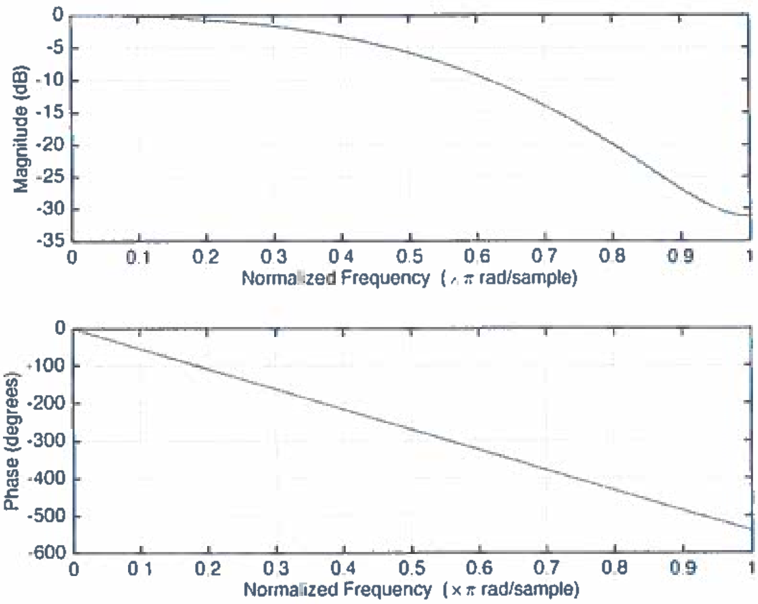

Figure 1.1 Frequency Response

2. a) Show that a 4-point DFT can be expressed in terms of two 2-point DFTs using the Decimation-in-Time FFT approach and write down the recombination equations. [6]

Illustrate your solution using a signal flow graph. [4]

b) Determine the percentage reduction in the number of real multiply operations when a 1024-point DFT is computed using the Decimation-in-Time FFT algorithm instead of a direct implementation of the DFT. [4]

c) Find the magnitude and phase spectra of the discrete-time signal

$$p(n) = \left[\begin{array}{cccc} 0.1 & -0.1 & 0.0 & -0.1 \end{array} \right].$$

[6]

3. Consider a discrete-time sequence $x(n)$ having z-transform $X(z)$.

a) State the expression for $X(z)$ in terms of $x(n)$ and explain what is meant by the *Region of Convergence* in this context. [4]

b) Describe the similarities and differences between the z-transform and the Laplace transform. Illustrate your answer using relevant sketches. Comment on the way in which the spectrum of discrete-time signals is represented in the z-domain. [5]

c) i) A particular linear time-invariant discrete-time system is described by the difference equation

$$y(n) = 0.2y(n-1) + 2x(n).$$

Using z-transform relationships, find expressions for the system function $H(z)$ of this system, and the unit impulse response $h(n)$ of this system. [4]

ii) Consider the signal

$$x(n) = \begin{cases} a^n, & 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases}$$

where M is an integer and $0 < a < 1$.

Determine the z-transform $X(z)$ of the signal $x(n)$ for any integer value of M.

Sketch in the z-plane a representation of $X(z)$ for the particular case of $M = 8$. [4]

iii) Find the inverse z-transform of

$$G(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

for the 3 cases of the ROC:

$$|z| > 1,$$
$$|z| < 0.5,$$
$$0.5 < |z| < 1.$$

[3]

4. A single stage of a lattice structure is shown in Fig. 4.1.

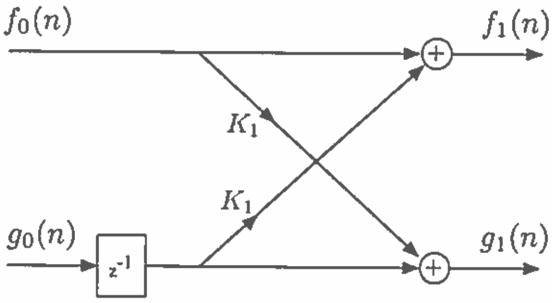

Figure 4.1 Single Stage Lattice Structure

a) Write expressions for $f_1(n)$ and $g_1(n)$ in terms of $f_0(n)$ and $g_0(n)$. [2]

b) Defining $x(n)$ as an input signal that is connected to both $f_0(n)$ and $g_0(n)$, and taking the output $y(n)$ as the signal $f_1(n)$, show that the structure implements a first order FIR filter and write down the difference equation for the filter in terms of $x(n)$, $y(n)$ and the filter coefficients denoted a_k, for integer k. Include formulae for a_k. [4]

c) Sketch a block diagram of two single stage lattice structures in cascade. The first stage is as given in Fig. 4.1 with $x(n)$ as an input signal that is connected to both $f_0(n)$ and $g_0(n)$, as in part b). The second stage uses K_2 in place of K_1 in the first stage. Denote the f and g outputs of the second stage $f_2(n)$ and $g_2(n)$ respectively.

Give expressions for $f_2(n)$ and $g_2(n)$ in terms of $f_1(n)$ and $g_1(n)$. [3]

d) Hence show that the cascaded structure implements a second order FIR filter and write down the difference equation for the filter in terms of $x(n)$, $y(n)$, K_1 and K_2.

Give expressions for the corresponding filter coefficients a_1 and a_2 in terms of K_1 and K_2. [5]

e) Now consider a cascade of m single stage lattice filters for any integer m. Denote this filter's transfer function

$$A_m(z) = \frac{Y(z)}{X(z)} = \frac{F_m(z)}{F_0(z)}.$$

The filter with coefficients b_k and transfer function $B_m(z)$ is defined as

$$B_m(z) = \frac{G_m(z)}{X(z)}.$$

Write down a general expression for b_k in terms of a_k and determine $B_m(z)$ in terms of $A_m(z)$. [6]