Optoelectronics Solutions 2018

1. a) The time-independent scalar wave equation $\partial^2 E_y/\partial x^2 + \partial^2 E_y/\partial z^2 + n^2 k_0^2 E_y = 0$ describes by y-polarized waves propagating in the x-z plane, i.e. TE waves.

[2]

The constants n and $k_0 = 2\pi/\lambda$ are the refractive index of the medium and the propagation constant of free space, respectively.

[2]

b) Assuming a solution $E_{y1} = E_0 \exp\{-jnk_0(z \cos(\theta) - x \sin(\theta))\}$ we can obtain:

$\partial E_{y1}/\partial x = -jnk_0 \sin(\theta) E_{y1}$

$\partial^2 E_{y1}/\partial x^2 = -n^2 k_0^2 \sin^2(\theta) E_{y1}$

$\partial E_{y1}/\partial z = -jnk_0 \cos(\theta) E_{y1}$

$\partial^2 E_{y1}/\partial z^2 = -n^2 k_0^2 \cos^2(\theta) E_{y1}$

Hence:

$\partial^2 E_{y1}/\partial x^2 + \partial^2 E_{y1}/\partial z^2 + n^2 k_0^2 E_{y1} = \{-n^2 k_0^2 \sin^2(\theta) - n^2 k_0^2 \cos^2(\theta) + n^2 k_0^2\} E_{y1} = 0$

Consequently the solution satisfies the wave equation.

[4]

The solution represents an oblique plane wave travelling at an angle θ to the z-axis.

[1]

c) For linear materials, Maxwell's equations and any wave equation that can be derived from them are inherently linear. Consequently, if a plane wave is a solution, a sum of plane waves is also a solution.

[1]

Modal solutions are solutions written in the form $E_y = E(x) \exp(-j\beta z)$, where E(x) is the transverse field and β is the propagation constant.

If $E_{y2} = E_0 [\exp\{-jnk_0(z \cos(\theta) + x \sin(\theta))\} + \exp\{-jnk_0(z \cos(\theta) - x \sin(\theta))\}]$ then:

$E_{y2} = E_0 [\exp\{-jnk_0 x \sin(\theta)\} + \exp\{+jnk_0 x \sin(\theta))\}] \exp\{-jnk_0 z \cos(\theta)$ or:

$E_{y2} = 2E_0 \cos\{nk_0 x \sin(\theta)\} \exp\{-jnk_0 z \cos(\theta)\}$

Hence E_{y2} is equivalent to an alternative field E_{y2}' given by:

$E_{y2} = E(x) \exp(-j\beta z)$ with $E(x) = 2E_0 \cos\{nk_0 x \sin(\theta)\}$ and $\beta = nk_0 \cos(\theta)$

[4]

The field description E_{y2} is clearly a sum of two plane waves, propagating at angles $\pm\theta$. The alternative description E_{y2}' is a standing wave pattern.

[2]

d) An inhomogeneous wave is a field whose amplitude is not the same everywhere. An example is the exponentially-decaying field $E_{y3} = E_0 \exp(\alpha x) \exp(-j\beta z)$. With this assumption, we can obtain:

$\partial E_{y3}/\partial x = \alpha E_{y3}$

$\partial^2 E_{y3}/\partial x^2 = \alpha^2 E_{y3}$

$\partial E_{y3}/\partial z = -j\beta E_{y3}$

$\partial^2 E_{y3}/\partial z^2 = -\beta^2 E_{y3}$

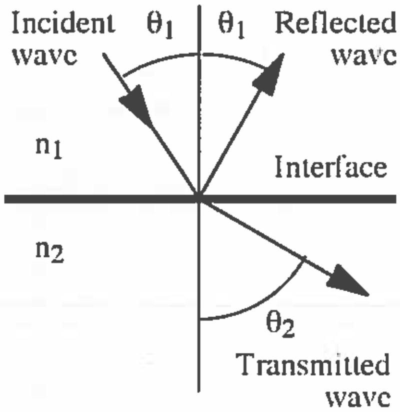
Hence:

$\partial^2 E_{y3}/\partial x^2 + \partial^2 E_{y3}/\partial z^2 + n^2 k_0^2 E_{y3} = (\alpha^2 - \beta^2 + n^2 k_0^2) E_{y3}$

The solution is clearly valid if $\alpha^2 - \beta^2 + n^2 k_0^2 = 0$

[4]

2. a) Snell's law $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ relates the angle of incidence θ_1 to the angle of refraction θ_2 when an electromagnetic wave is incident on a boundary between two dielectric media with refractive indices n_1 and n_2 as shown below.



[3]

Inverting the equation allows θ_2 to be found as $\theta_2 = \sin^{-1}\{(n_1/n_2) \sin(\theta_1)\}$.

Clearly, a real value can only obtained when $(n_1/n_2) \sin(\theta_1) \leq 1$.

The maximum value for θ_1 (the critical angle) is therefore $\theta_c = \sin^{-1}(n_2/n_1)$.

[2]

Assuming $n_1 = 1.5$, $n_2 = 1$ (a glass-air interface) we obtain $\theta_c = \sin^{-1}(1/1.5) = 41.81°$

[2]

b) The reflection coefficient for TE incidence is

$\Gamma_E = \{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)\} / \{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)\}$

Rearranging Snell's law, we get $\sin(\theta_2) = (n_1/n_2) \sin(\theta_1)$

Consequently, $\cos(\theta_2) = \sqrt{\{1 - (n_1/n_2)^2 \sin^2(\theta_1)\}}$

For incidence beyond the critical angle, $1 - (n_1/n_2)^2 \sin^2(\theta_1) < 0$

Consequently $\cos(\theta_2)$ is imaginary, and can be written as $\cos(\theta_2) = j\alpha$ (say)

Under these circumstances, $\Gamma_E = \{n_1 \cos(\theta_1) - jn_2\alpha_2\} / \{n_1 \cos(\theta_1) + jn_2\alpha_2\}$

The expression for Γ_E is in the form of a ratio of complex conjugates.

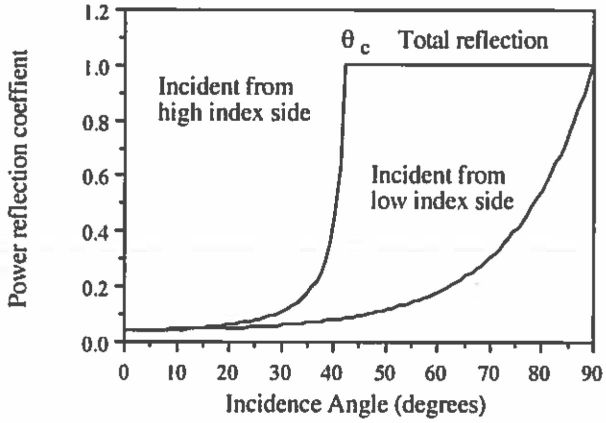
Consequently the power reflectivity is $|\Gamma_E|^2 = \Gamma_E\Gamma_E^* = 1$

[3]

At normal incidence on a glass-air interface:

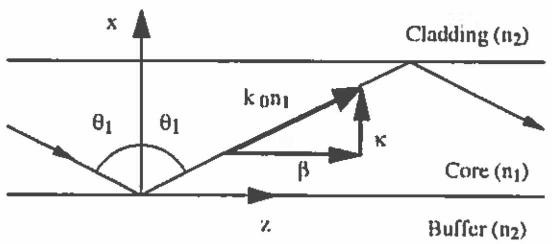
$\Gamma_E = (n_1 - n_2) / (n_1 + n_2) = (1.5 - 1) /(1.5 + 1) = 0.2$ and $|\Gamma_E|^2 = 0.04$.

The angular variation in reflectivity for incidence on either side of the interface is as shown below. Both have $|\Gamma_E|^2 = 0.04$ at $\theta_1 = 0$, and power reflectivity rises as θ_1 increases. However, for incidence from the high index side, the reflectivity rises more sharply, and reaches unity at $\theta_1 = \theta_c$, where it remains at higher angles.



[4]

c) Referring to the ray diagram below, the propagation constant is $\beta = k_0 n_1 \sin(\theta_1)$
When total internal reflection breaks down, θ_2 tends to 90° and Snell's law reduces to $n_1 \sin(\theta_1) = n_2$. Consequently β tends to $n_2 k_0$, $\kappa = \sqrt{(n_1^2 k_0^2 - \beta^2)}$ tends to $\kappa = \sqrt{(n_1^2 k_0^2 - n_2^2 k_0^2)}$ and $\gamma = \sqrt{(\beta^2 - n_2^2 k_0^2)}$ tends to zero. The evanescent field in the cladding therefore starts to be replaced by a propagating wave.



[4]

Under these conditions, the eigenvalue equations reduce to:

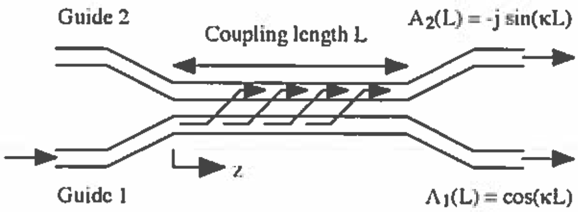
$\tan(\kappa h/2) = 0$ so $\kappa h/2 = 0, \pi, 2\pi \ldots$ (symmetric modes)

$\tan(\kappa h/2) = -\infty$ so $\kappa h/2 = \pi/2, 3\pi/2, 5\pi/2 \ldots$ (antisymmetric modes)

Hence the general cutoff condition is $\kappa h/2 = \nu\pi/2$, where ν is the mode number.

[2]

3. a) A directional coupler is a lossless 4-port device based on a pair of coupled dielectric waveguides. Passive devices are used for power splitting and electro-optic devices for switching. Couplers have three main sections, linked by S-bends: input and output sections, where the two guides are relatively far apart, and a central coupling section, where they are close enough that their evanescent fields overlap and allow an exchange of power.



[4]

When the waveguides are identical, the guided modes travel at the same speed, and the coupler is said to be synchronous

[1]

b) In this case, the equations governing the variation of the amplitudes A_1 and A_2 of the modes in the two guides with distance z are:

$dA_1/dz = -j\kappa A_2$ (1)

$dA_2/dz = -j\kappa A_1$ (2)

Here κ is the coupling coefficient. Power conservation can be proved as follows

Multiplying (1) by A_1^*, we get $A_1^* dA_1/dz = -j\kappa\, A_1^* A_2$ (3)

Taking the complex conjugate of (3), we get $A_1 dA_1^*/dz = +j\kappa\, A_1 A_2^*$ (4)

Multiplying (2) by A_2^*, we get $A_2^* dA_2/dz = -j\kappa\, A_2^* A_1$ (5)

Taking the complex conjugate of (5), we get $A_2 dA_2^*/dz = +j\kappa\, A_2 A_1^*$ (6)

Adding together equations (3)-(6) we then get

$A_1^* dA_1/dz + A_1 dA_1^*/dz + A_2^* dA_2/dz + A_2 dA_2^*/dz = 0$

Or $d(A_1A_1^* + A_2A_2^*)/dz = 0$

Now $A_1A_1^* = P_1$ and $A_2A_2^* = P_2$, where P_1 and P_2 are the powers in the two guides.

Consequently $d(P_1 + P_2)/dz = 0$, so $P_1 + P_2 = \text{const}$

[4]

c) The equations can be decoupled, by differentiating (1) and substituting (2), as follows:

$d^2A_1/dz^2 = -j\kappa dA_2/dz = -\kappa^2 A_1$

The general solution to this second order differential equation is:

$A_1 = A \sin(\kappa z) + B \cos(\kappa z)$ so

$dA_1/dz = \kappa A \cos(\kappa z) - \kappa B \sin(\kappa z)$

Assuming the BCs $A_1 = 1$, $A_2 = 0$ (so $dA_1/dz = 0$) on $z = 0$, we must have

$A = 0$ and $B = 1$

$A_1(z) = \cos(\kappa z)$

$A_2(z) = -(1/j\kappa)\, dA_1/dz = -j \sin(\kappa z)$

At $z = L$, we then obtain $A_1 = \cos(\kappa L)$ and $A_2 = -j \sin(\kappa L)$.

The corresponding powers are $P_1 = |A_1|^2 = \cos^2(\kappa L)$ and $P_2 = |A_2|^2 = \sin^2(\kappa L)$.

[3]

For a 3 dB split, we require $P_1 = P_2 = 1/2$, which in turn requires $\kappa L = \pi/4$

If $\kappa = 0.25\ \text{mm}^{-1}$, we require a length of $L = \pi/(0.25 \times 4)$ mm = 3.14 mm

[1]

d) The output can be found by summing contributions from paths 1 and 2 below. Ignoring common phase delays:

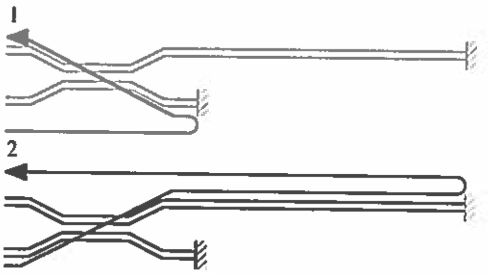
Path 1 gives: $\cos(\kappa L) \,.\, -j \sin(\kappa L)$

Path 2 gives: $-j \sin(\kappa L) \,.\, \exp(-j2\beta\Delta L) \,.\, \cos(\kappa L)$

Now, for a 3 dB coupler, $\sin(\kappa L) = \cos(\kappa L) = 1/\sqrt{2}$, so summing paths 1 and 2 gives:

$A_{out} = -(j/2)\{1 + \exp(-j2\beta\Delta L)\} = -j \exp(-j\beta\Delta L) \cos(\beta\Delta L)$

Consequently, $P_{out} = |A_{out}|^2 = \cos^2(\beta\Delta L)$



[5]

The output therefore fluctuates cosinusoidally with $\beta\Delta L$. Since ΔL depends on temperature, the device can be used as a thermometer.

[2]

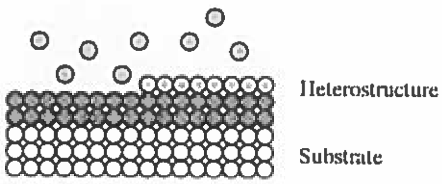
4 a) Si is centro-symmetric, and consequently cannot be electro-optic. However, both GaAs and InP are non-centro-symmetric; both are therefore electro-optic.

[2]

The binary compound InP is used in optoelectronics because it can provide a lattice-matched substrate for the quaternary material $In_{1-x}G_xAs_{1-y}P_y$. InGaAs is direct-gap, and has an energy gap E_g that can be tuned by varying x and y. Importantly, the range of E_g allows light emission at 1.55 μm, the low loss wavelength of silica-based optic fibre.

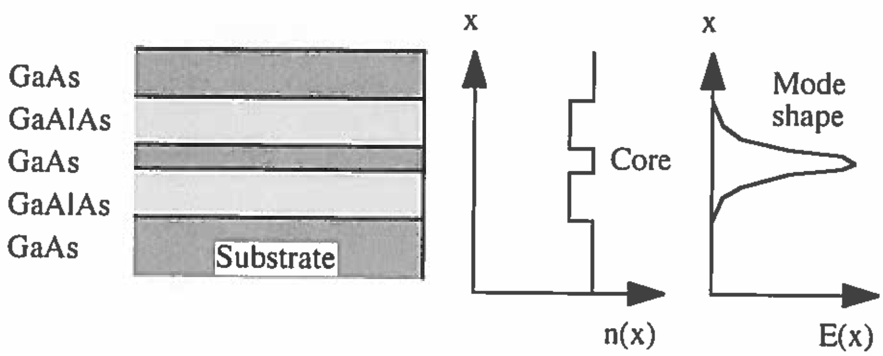
[3]

b) Epitaxy is a method of ordered growth, used to form layers of lattice-matched, strain-free semiconductor. The starting point is normally a binary substrate (GaAs or InP), on which layers of ternary or quaternary compounds ($Ga_{1-x}Al_xAs$ or $In_{1-x}G_xAs_{1-y}P_y$. respectively) are grown. Standard methods include liquid and vapour phase epitaxy. In each case, E_g and the doping can be varied to form a double heterostructure, the basis of laser diodes.



[3]

c) Doping GaAs with Al to form $Ga_{1-x}Al_xAs$ lowers the refractive index. Consequently, a waveguide cannot be formed by depositing GaAlAs on GaAs (the starting substrate). Instead, the GaAlAs must act as the buffer layer for a further deposited GaAs layer, which may be made symmetric by the addition of GaAlAs and GaAs layers. The layer structure, refractive index profile and transverse field are then as shown below.

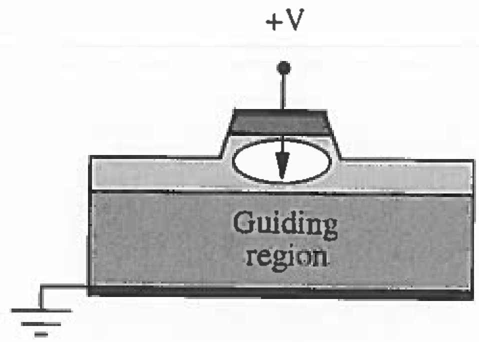


[4]

d) Doping a semiconductor lowers its refractive index, due to the (negative) carrier contribution to the refractive index. The vertical layer structure therefore forms a planar waveguide; lateral confinement is then obtained by etching the upper two layers into a rib. The p^+ and n-type layers also form a p^+-n diode. The function of the heavy doping is to form ohmic (rather than rectifying) contacts between the metal and semiconductor.

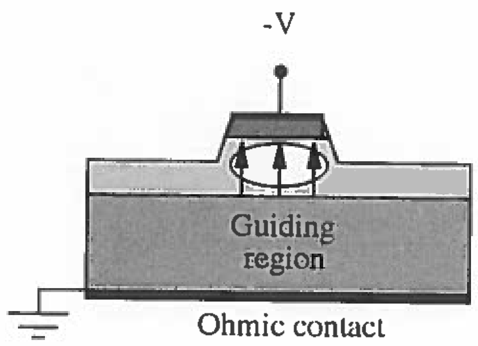
[3]

Under forward bias, holes are injected from the p^+ layer into the guiding region. A phase change is then achieved by the carrier contribution to the refractive index.



[2]

Under reverse bias, the depletion layer expands to fill the whole of the guiding region. Since InP is electro-optic, a refractive index change is then caused by the strong electric field.



[2]

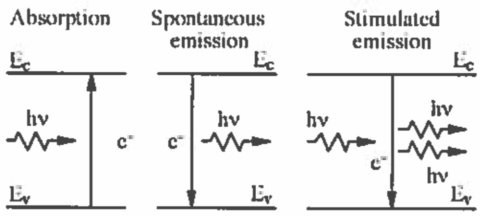
The electro-optic phase modulator operates much faster than the carrier injection device, because carriers can only be removed in the latter case when the forward bias is switched off via the relatively slow process of recombination.

[1]

5. a) The three physical processes involving electrons, holes and photons are:

- Absorption, which involves the promotion of an electron to a higher energy level through the absorption of the energy of a photon.
- Spontaneous emission, which involves the generation of a photon when an electron falls randomly to a lower energy level. The photon is emitted with random phase, polarization and direction.
- Stimulated emission, which involves the generation of a second, identical photon when an electron falls to a lower energy level, through the stimulating effect of a first photon.

For semiconductors, the processes can be represented in terms of energy band diagrams as shown below.

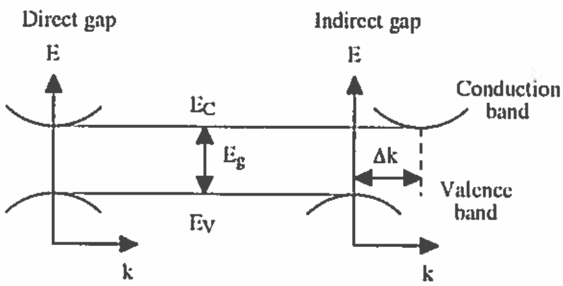


[3 x 2 = 6]

Absorption is dominant in photodiodes, spontaneous emission is dominant in LEDs and stimulated emission is dominant in laser diodes.

[2]

b) Direct gap materials have the minimum of the E-k diagram for the conduction band immediately above the maximum for the valence band. Indirect gap materials have an offset Δk between these points as shown below.

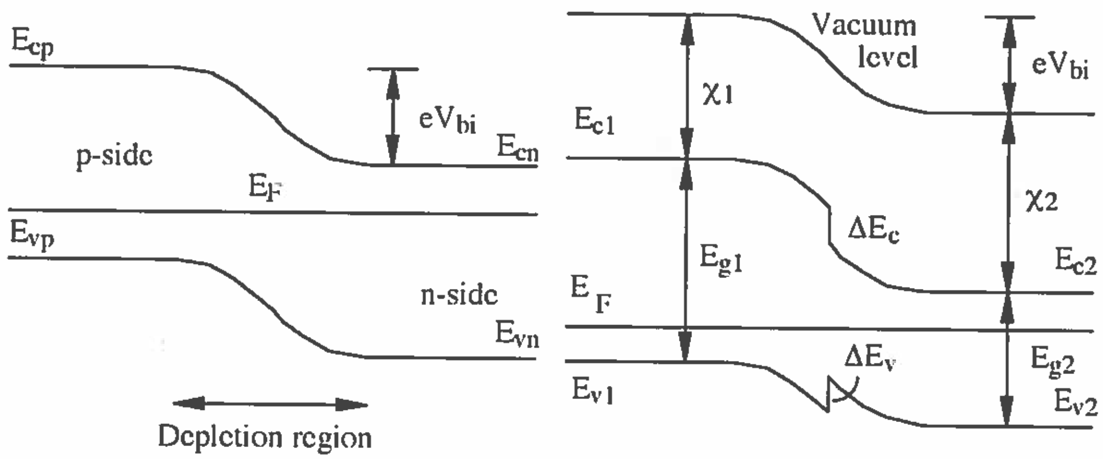


[4]

The significance of this difference is as follows. In a direct gap material, band-to-band transitions only require only energy E_g, which can be supplied by a photon. In an indirect gap material, additional momentum Δk is needed. Since a photon has very little momentum, it cannot supply this, and additional interaction with a phonon (which has low energy but high momentum is required). Since the probability of a three-body interaction is much lower, transition rates are much lower in indirect-gap materials. Because silicon is an indirect-gap material, its light emission is very low, and consequently it cannot be used to fabricate an effective light-emitting diode or laser diode.

[2]

c) Homojunctions are formed from materials with the same energy gap on either side of the junction, while heterojunctions are formed from materials with different energy gaps. In a homojunction, the variation in energy across the junction is smooth, and similar for the conduction and valence band. In a heterojunction, the variation is discontinuous, and the discontinuity is different. Representative band diagrams in equilibrium are as shown below.



Homojunction

Heterojunction

[4]

The discontinuities in the band diagram of a heterojunction allows a much larger barrier for one type of carrier, which may then be localised much more effectively. The use of a double heterojunction then allows the construction of spatially separated barriers for electrons and holes, which may then be confined in an active volume where they may interact effectively.

[2]

6. a) The rate equations for a light emitting diode can be written in the form:

$dn/dt = I/ev - n/\tau_e$

$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$

where:

I/ev is the rate of injection of electrons, per unit volume

n/τ_e is the rate of removal of electrons by recombination, per unit volume

n/τ_{rr} is the rate of photon generation by radiative recombination, per unit volume

ϕ/τ_p is the rate of loss of photons by escape, from the active volume, per unit volume

[4]

For a laser diode, the equations are modified by the addition of terms $\pm G\phi(n - n_0)$ that describe absorption and stimulation, and a cavity coupling factor β, as follows:

$dn/dt = I/ev - n/\tau_e - \mathbf{G\phi(n - n_0)}$

$d\phi/dt = \boldsymbol{\beta} n/\tau_{rr} + \mathbf{G\phi(n - n_0)} - \phi/\tau_p$

Here G is the gain constant and n_0 is the electron density at transparency.

[3]

b) The active volume of a laser diode is the volume in which radiative recombination occurs, typically the central layer of a double heterostructure etched into a stripe guide geometry.

[1]

The difference between electron and photon lifetime is that electrons are eliminated by recombination with holes, while photons simply escape from the active volume.

[1]

For a laser with cavity length L, end-mirrors with amplitude reflectivities R_1 and R_2 and group velocity v_g, the photon lifetime τ_p is found as follows:

Each cavity transit takes time $T = 2L/v_g$

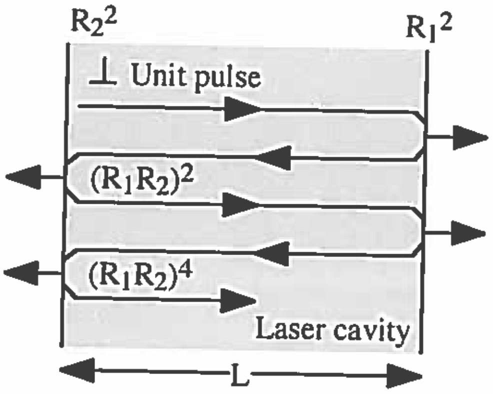
Ignoring gain, the power in a unit pulse after one transit is $(R_1R_2)^2$

The power remaining after N transits is then $(R_1R_2)^{2N}$

The number of transits needed to reduce the power to 1/e can be found from

$(R_1R_2)^{2N} = 1/e$, or $N = 1/\{2 \log_e(1/R_1R_2)\}$

The photon lifetime is then $\tau_p = NT = L/\{v_g \log_e(1/R_1R_2)\}$



[4]

c) In the steady state, the laser rate equations reduce to:

$0 = I/ev - n/\tau_e - G\phi(n - n_0)$

$0 = \beta n/\tau_{rr} + G\phi(n - n_0) - \phi/\tau_p$

During lasing, spontaneous emission ($\beta n/\tau_{rr}$) may be neglected by comparison with stimulated emission. The photon rate equation then reduces to:

$G\phi(n - n_0) - \phi/\tau_p = 0$ so

$G(n - n_0) - 1/\tau_p = 0$ and

$n = n_0 + 1/G\tau_p$

[3]

Under these circumstances the electron rate equation reduces to:

$I/ev - n/\tau_e - G\phi(n - n_0) = I/ev - n/\tau_e - \phi/\tau_p = 0$

The photon output per unit volume is then $\phi/\tau_p = I/ev - n/\tau_e$

The total photon flux is $\Phi = \phi v/\tau_p = I/e - nv/\tau_e$

Defining the threshold current as $I_t = nev/\tau_e$ this can be written as $\Phi = (I - I_t)/e$

If each photon carries energy hc/λ, where h is Planck's constant, c is the velocity of light and λ is the wavelength, the output power is:

$P = (hc/\lambda e)\,(I - I_t)$

[3]

At 1.5 μm wavelength, the slope efficiency $dP/dI = hc/\lambda e$ is:

$dP/dI = 6.62 \times 10^{-34} \times 3 \times 10^{8} / (1.5 \times 10^{-6} \times 1.6 \times 10^{-19}) = 0.8275$ W/A

[1]