Master copy
Aug 02
Sections A + B
only

Paper Number(s): **E2.3**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE PART II: M.Eng., B.Eng. and ACGI

POWER, FIELDS (AND DEVICES) — Sample questions to be provided.

Wednesday, 12 June 2:00 pm

There are NINE questions on this paper.

There are three sections. Answer FIVE questions including at least ONE question from each of sections A, B and C.

Use a separate answer book for each section.

Corrected Copy

Time allowed: 3:00 hours

Examiners responsible:

First Marker(s): Green,T.C. Leaver,K.D. Papavassiliou,C.

Second Marker(s): Popovic,D. Holmes,A.S. Juhasz,C.

Information for Candidates

Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho \quad ; \quad \mathbf{D} = \varepsilon \mathbf{E} \qquad\qquad \nabla \cdot \mathbf{B} = 0 \quad ; \quad \mathbf{B} = \mu \mathbf{H}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

In a MOSFET of channel length L and width W: $g_m = \frac{eWV_{sat}}{V_T L^2} f\left(V_{GS}\right)$ where $f(V_{GS})$ depends on doping levels, etc.

Depletion depth: $L_D = A\sqrt{\Phi - V}$.

Total current density due to electrons: $I_n = enE + De\frac{dn}{dx}$

Einstein's relationship: $\frac{D}{kT} = \frac{\mu}{e}$

When labelling diagrams please use the following standard notation:

- N^-, P^- for lightly doped N or P type material
- N, P for moderately doped N or P type material
- N^+, P^+ for heavily doped N or P type material
- ***e*, *h*** in circles for mobile electrons and holes
- "+" and "-" symbols for ionised donors and acceptor respectively

Section A

1. (a) Draw an equivalent circuit of an induction machine and give a physical interpretation of the components of the circuit.

[5]

(b) A 3-phase induction machine has been designed for a 50 Hz supply and a maximum referred rotor current of 25 A. Its equivalent circuit parameters are as follows:

Number of pole-pairs, $P = 2$;
Stator resistance, $R_S = 0.1\ \Omega$;
Rotor resistance, $R'_R = 0.1\ \Omega$;
Magnetising reactance, $X_M = 15\ \Omega$;
Magnetising loss resistance, $R_M = 30\ \Omega$;
Stator leakage reactance, $X_S = 0.5\ \Omega$;
Rotor leakage reactance, $X'_R = 0.5\ \Omega$;

(i) State the synchronous speed of the machine.

[2]

(ii) Calculate the slip the machine would require to develop 5 kW of mechanical power with the maximum referred rotor current flowing. You may assume windage and friction to be negligible.

[4]

(iii) Calculate the torque developed by the machine under the conditions in (ii).

[2]

(iv) Calculate the voltage with which the stator would need to be supplied for conditions in (ii).

[4]

(v) Calculate the efficiency of the machine under the conditions in (iv)

[3]

2. (a) (i) Explain why the power dissipation in a transistor used in a switch-mode power supply circuit is dependent on both the switching frequency and the duty-cycle.

[4]

(ii) Explain the benefits of using a high switching frequency in a switch-mode power supply

[3]

(b) A MOSFET has been attached to a heatsink which allows a heat dissipation of 15 W without exceeding the maximum temperature of the transistor. The MOSFET has drain-source on-state resistance of $R_{DS}(on) = 0.1\ \Omega$. The MOSFET is to be used in a boost converter with an input of 24 V and an output of 100 V. When switching an inductor current of 10 A in this circuit, the MOSFET dissipates 125 μJ for each turn-on and turn-off pair of events.

(i) Calculate the duty-cycle required in this circuit assuming the circuit to be in continuous conduction.

[2]

(ii) Calculate the maximum switching frequency that can be used if the maximum inductor current is to be 10 A.

[4]

(iii) Calculate the inductor value required to keep the circuit in continuous conduction with an input power of 5 W.

[4]

(iv) Specify a capacitor ESR to keep the voltage ripple below 500 mV with an input power of 300 W.

[3]

3. (a) Explain why steel is used extensively in the construction of generators and why the air-gap between rotor and stator is small.

[3]

(b) Draw a circuit diagram of the 2-wattmeter method of measuring the power consumed by a 3-phase load. Give an expression for the power read by each meter and indicate how the total power is found.

[4]

(c) Three impedances of 10+j5 Ω are available for connection to a 3-phase, 50 Hz voltage supply.

(i) State what resistance, inductance and capacitance should be used to provide such impedances in the simplest way.

[2]

(ii) Explain whether the impedances would consume more power when connected in star or in delta.

[3]

(iii) Calculate the apparent, real and reactive power when these three impedances are connected in delta to a 400 V supply.

[6]

(iv) Calculate the simplest resistance, inductance and capacitance required to consume 10% more real power while maintaining the same current magnitude.

[2]

Section B

4. A long thin conducting wire carrying a positive charge q per unit length lies between and parallel to a pair of parallel grounded conducting planes. The planes are separated by a distance a, and the wire lies at a distance b from one of them. The following equation is suggested as the solution for the potential $V(x,y)$ between the plates:

$$\tanh\left(\frac{2\pi\varepsilon V}{q}\right)=\frac{\sin\frac{\pi b}{a}\sin\frac{\pi y}{a}}{\cosh\frac{\pi x}{a}-\cos\frac{\pi b}{a}\cos\frac{\pi y}{a}}$$

(i) Show that the planes $y = 0$ and $y = a$ are indeed at zero potential as required, and deduce the (x,y) coordinates of the thin wire.

[4]

(ii) Find the electric field strength as a function of x at $y = 0$, and hence make a sketch graph of the charge per unit area on the conductor there. Assuming that $b = a/4$, calculate and show on your sketch the maximum value of the charge per unit area and its location, and also the value, or values, of x/a at which it is half the maximum.
Do not spend time calculating other values.

[13]

(iii) Use Laplace's equation to show that, at the surface of each flat conductor, $\frac{\partial E_y}{\partial y}=0$, where E_y is the y-component of the electric field strength.

[3]

5. (a) State a relationship between the inductance of a circuit and the flux linking it. [3]

(b) Figure 5 shows parts of two long parallel wires **P** and **Q**, with separation b and negligible thickness, lying in a plane parallel to and at a distance a from an identical second pair of long conductors **R** and **S**, symmetrically located. The surrounding medium is air.

Each pair of wires forms a go-and-return circuit, and the **P-Q** circuit alone carries a current I as shown.

(i) Explain why the magnetic flux linking the circuit **R-S** that is generated by wire **P** is equal to that generated by wire **Q**.

(ii) Find the magnetic flux per unit length linking the circuit **R-S** due to the current I.

(iii) Hence evaluate the mutual inductance between the two circuits, per unit length, when $b = 1$ mm and $a = 3$ mm. [11]

(c) The voltage induced in the circuit **R-S** by switching on or off the current I is required to be as near zero as possible, the wires remaining parallel. Suggest how the wire locations might be rearranged to achieve this, without changing either the mean distance a between the wire pairs or the self-inductance of either circuit. [6]

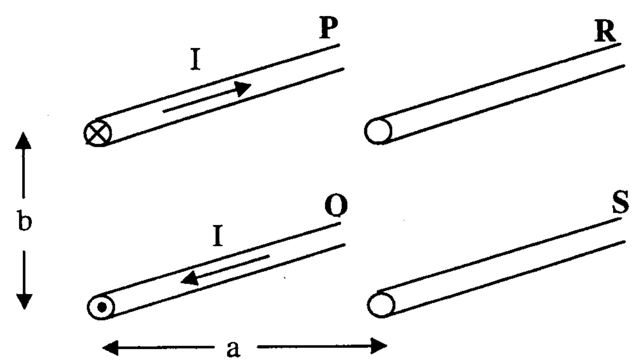


Figure 5

6. (a) State a vector equation for the electric field of a plane transverse electromagnetic wave of frequency f and wavelength λ, travelling parallel to the z-axis, and polarised parallel to the x-axis.

[3]

(b) Use one of Maxwell's equations to derive the associated magnetic field, and find an expression which involves neither f nor λ for the *intrinsic impedance* of the medium in which the wave moves. You may assume that the wave's velocity is $1/(\mu\varepsilon)^{1/2}$, where μ and ε have the usual meanings.

[6]

(c) This wave meets at normal incidence the plane surface of a dielectric having relative permittivity ε_r and a relative permeability of unity.

Assumptions necessary : announced 17.10

Deduce the amplitude reflection coefficient, and show that the fraction of the incident power that is reflected is equal to

$$\frac{1+\varepsilon_r - 2\sqrt{\varepsilon_r}}{1+\varepsilon_r + 2\sqrt{\varepsilon_r}}$$

By deducing in the same way the fractional power transmitted through the surface, prove that no power is lost.

[8]

(d) With the aid of the expression you gave in answer to (a) above, deduce why a polarised wave results when an unpolarised wave is passed at normal incidence through a grid of long, fine conducting wires.

[3]

Sample questions to be provided for Sec. C (Jenkes)

E2.3 Power, Fields and Devices 2001/2002

master solutions

There are 9 questions in 3 sections.
Answer 5 questions, with at least one from each section.
Use a separate answer book for each section.

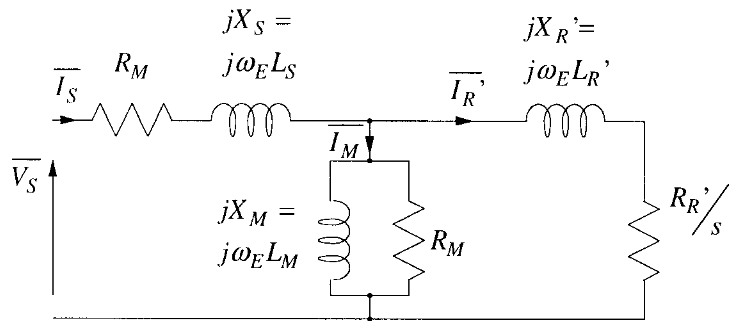
E2.3

Section A – Power
Examiner: Dr T C Green
2nd Marker: Dr D Popovic

August 02

1. (a) Draw an equivalent circuit of an induction machine and give a physical interpretation of the components of the circuit.

[5]



All current flow at ω_E

- *Stator resistance, R_S models the resistance of wire from which the stator is wound*
- *Rotor resistance, R'_R models the resistance of the stator bars and has been referred to the stator*
- *Magnetising reactance, X_M models the flux produced by the stator and rotor currents that links both the stator and rotor.*
- *Magnetising loss resistance, R_M models the hysteresis and eddy current losses in the core of the machine.*
- *Stator leakage reactance, X_S models the flux created by the flow of stator current that does not link the rotor.*
- *Rotor leakage reactance, X'_R models the flux created by the flow of rotor current that does not link the stator.*

(b) A 3-phase induction machine has been designed for a 50 Hz supply and a maximum referred rotor current of 25 A. Its equivalent circuit parameters as follows:

Number of pole-pairs, $P = 2$;
Stator resistance, $R_S = 0.1\ \Omega$;
Rotor resistance, $R'_R = 0.1\ \Omega$;
Magnetising reactance, $X_M = 15\ \Omega$;
Magnetising loss resistance, $R_M = 30\ \Omega$;
Stator leakage reactance, $X_S = 0.5\ \Omega$;
Rotor leakage reactance, $X'_R = 0.5\ \Omega$;

(i) State the synchronous speed of the machine.

[2]

$$\omega_S = \frac{\omega_E}{P} = \frac{2\pi 50}{2} = 157.1\ rad/s$$

$$n_S = \frac{60 f_E}{P} = \frac{60 \times 50}{2} = 1500\ rpm$$

(ii) Calculate the slip the machine would require to develop 5 kW of mechanical power with the maximum referred rotor current flowing. You may assume windage and friction to be negligible.

[4]

$$P_{EM} = 3I'^{\,2}_R R'_R \left(\frac{1-s}{s} \right)$$

$$s\left(P_{EM} + 3I'^{\,2}_R R'_R\right) = 3I'^{\,2}_R R'_R$$

$$s = \frac{3I'^{\,2}_R R'_R}{P_{EM} + 3I'^{\,2}_R R'_R}$$

$$s = \frac{3 \times 25^2 \times 0.1}{5000 + 3 \times 25^2 \times 0.1} = 0.0361$$

(iii) Calculate the torque developed by the machine under the conditions in (ii).

[2]

$$T_{EM} = \frac{P_{EM}}{\omega_S (1-s)} = \frac{5000}{50\pi(1-0.0361)} = 33.0\ Nm$$

(iv) Calculate the voltage with which the stator would need to be supplied for conditions in (ii).

[4]

$$V_S = I_S (R_S + jX_S) + V_{AG}$$

$$I_S = I'_R + I_{X_M} + I_{R_M}$$

$$I_{X_M} = \frac{V_{AG}}{jX_M}$$

$$I_{R_M} = \frac{V_{AG}}{R_M}$$

$$V_{AG} = I'_R \left(\frac{R'_R}{s} + jX'_R \right) = 25\left(\frac{0.1}{0.0361} + j0.5 \right) = 69.25 + j12.5\ V$$

$$V_S = I_S(R_S + jX_S) + V_{AG}$$
$$I_S = I'_R + I_{X_M} + I_{R_M}$$
$$I_{X_M} = \frac{V_{AG}}{jX_M}$$
$$I_{X_M} = \frac{69.25 + j12.5}{j15} = 0.833 - j4.62\ A$$
$$I_S = 25 + 0.833 - j4.32 + 2.31 + j0.417 = 28.143 - j4.203\ A$$

$$V_S = (28.143 - j4.203) \times (0.1 + j0.5) + 69.25 + j12.5$$
$$= (2.814 + 2.101 + 69.25) + j(14.07 - 0.42 + 12.5) = 74.32 + j26.19$$
$$= 78.8\ V$$

(v) Calculate the efficiency of the machine under the conditions in (iv) [3]

$$P_R = 3 \times 25^2 \times 0.1 = 187.5\ W$$
$$P_M = 3 \times \frac{(69.25^2 + 12.5^2)}{30} = 495.2\ W$$
$$P_S = 3 \times (28.14^2 + 4.20^2) \times 0.1 = 242.5\ W$$
$$\eta = \frac{P_{EM}}{P_{EM} + P_S + P_M + P_R} = \frac{5000}{5000 + 242.5 + 495.2 + 187.5}$$
$$= 84.4\%$$

2) (a) (i) Explain why the power dissipation in a transistor used in a switch-mode power supply circuit is dependent on both the switching frequency and the duty-cycle.

[4]

There are two contributions to power loss. Conduction loss occurs because the transistor has a small voltage drop when conducting. The average conduction loss depends on the proportion of time for which the device is conducting and hence is dependent on duty-cycle. The other contribution is the switching loss. This occurs because, when switching an inductive, the off-state voltage is maintained across the device as the current changes and thus for the transition time the power dissipation is high. There is a certain energy loss per operation and the average power loss is proportional to the number of operations per second, i.e., the frequency.

(ii) Explain the benefits of using a high switching frequency in a switch-mode power supply

[3]

A key design criteria for SMPS is the voltage ripple at the output. This depends on the capacitance and series resistance of the capacitor and on the current ripple. For a given amplitude of current ripple, a higher operating frequency will require a lower capacitance to keep the voltage ripple within specification. The resistive voltage ripple will depend on the current amplitude only. However, the current ripple amplitude is also inversely proportional to frequency (for a buck SMPS). Thus, a high frequency allows the design to use smaller valued, smaller volume and cheaper passive components.

(b) A MOSFET has been attached to a heatsink which allows a heat dissipation of 15 W without exceeding the maximum temperature of the transistor. The MOSFET has drain-source on-state resistance of $R_{DS}(on) = 0.1\ \Omega$. The MOSFET is to be used in a boost converter with an input of 24 V and an output of 100 V. When switching an inductor current of 10 A in this circuit, the MOSFET dissipates 125 μJ for each turn-on and turn-off pair of events.

(i) Calculate the duty-cycle required in this circuit assuming the circuit to be in continuous conduction.

[2]

$$\frac{V_O}{V_I} = \frac{1}{1-\delta}$$

$$\delta = \frac{V_O - V_I}{V_O} = \frac{100-24}{100} = 0.76$$

(ii) Calculate the maximum switching frequency that can be used if the maximum inductor current is to be 10 A.

[4]

$$P_{Loss}^{\ Max} = \delta\, I^2 R_{DS}(on) + f\, E_{Sw}$$

$$f = \frac{P_{Loss}^{\ Max} - \delta\, I^2 R_{DS}(on)}{E_{Sw}} = \frac{15 - 0.76\times10^2\times0.1}{125\times10^{-6}} = 59.2\ kHz$$

(iii) Calculate the inductor value required to keep the circuit in continuous conduction with an input power of 5 W.

[4]

For borderline continuous conduction:

$$P_I = I_L^{Avg} V_I = \tfrac{1}{2} i_L^{Peak} V_I$$

$$i_L^{Peak} = \frac{V_I}{L} \times \frac{\delta}{f}$$

$$L = \frac{V_I^2 \delta}{2 f P_I}$$

$$L = \frac{24^2 \times 0.76}{2 \times 59.2 \times 10^3 \times 5} = 740\ \mu H$$

(iv) Specify a capacitor ESR to keep the voltage ripple below 500 mV with an input power of 300 W.

[3]

$$R_{ESR} = \frac{v_O^{pk-pk}}{i_C^{pk-pk}}$$

$$i_C^{pk-pk} = i_L^{Avg} + \tfrac{1}{2}\Delta i_L = \frac{P_I}{V_I} + \frac{\delta V_I}{2fL} = \frac{300}{24} + \frac{0.76 \times 24}{2 \times 59.2 \times 10^3 \times 0.74 \times 10^{-3}} = 12.7\ A$$

$$R_{ESR} = \frac{0.5}{12.7} = 39\ m\Omega$$

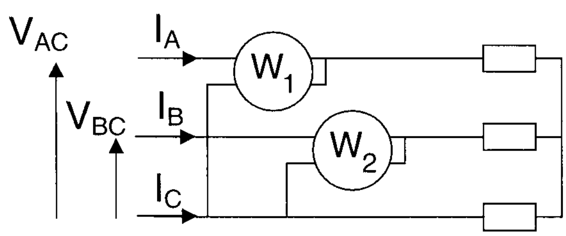
3) (a) Explain why steel is used extensively in the construction of generators and why the air-gap between rotor and stator is small.

[3]

It is desirable to create the magnetic flux in a generator with as small a field winding as possible. Both the magnetising/field current and the number of turns (and hence resistance) should be minimised to minimise conduction power loss in the field circuit. Thus, the magnetic circuit should have as low a reluctance as possible and a high permeability material such as steel should be chosen. Steel also has a high saturation flux density compared to other materials, is mechanically strong and can be machined. A clearance gap, known as the air-gap, must be allowed between the rotating and stationary parts but because the magnetic flux must cross this gap (twice) it should be kept as narrow as possible to keep its reluctance low.

(b) Draw a circuit diagram of the 2-wattmeter method of measuring the power consumed by a 3-phase load. Give an expression for the power read by each meter and indicate how the total power is found.

[4]



$$W_1 = |V_{AC}||I_A|\cos(\angle V_{AC} - \angle I_A)$$
$$W_2 = |V_{BC}||I_B|\cos(\angle V_{BC} - \angle I_B)$$
$$P_{Total} = W_1 + W_2$$

(c) Three impedances of 10+j5 Ω are available for connection to a 3-phase, 50 Hz voltage supply.

(i) State what resistance, inductance and capacitance should be used to provide such impedances in the simplest way.

[2]

The impedance has an angle in the first quadrant and so when a current flows through it will produce a voltage that leads the current. The impedance contains an inductive element and resistive element in series and no capacitance is required: $Z=R+j\omega L$,. $R=10\Omega$, $L=5/(2\pi 50)=15.9$ mH

(ii) Explain whether the impedances would consume more power when connected in star or in delta.

[3]

When connected in delta, the line voltage, which is √3 greater than the phase voltage, will be imposed across the impedance. This will give the larger current of the two arrangements and hence the larger I^2R power consumption.

(iii) Calculate the apparent, real and reactive powers when these three impedances are connected in delta to a 400 V supply.

[6]

$$I_{Ph} = \frac{V_{ph}}{Z} = \frac{400}{10 + j5} = \frac{400}{11.18\angle 26.6°} = 35.8\angle -26.6°\ A$$

$$I_{Ln} = \sqrt{3} I_{Ph} = 62.0\ A$$

$$S = 3V_{Ph} I_{Ph} = 3 \times 400 \times 35.8 = 42.96\ kVA$$

$$P = S\cos(\phi) = 38.42\ kW$$

$$Q = S\sin(\phi) = 19.21\ kVAr$$

(iv) Calculate the simplest resistance, inductance and capacitance required to consume 10% more real power while maintaining the same current magnitude.

[2]

The same current flow requires the same magnitude of impedance, i.e., 11.18 Ω. The extra power must come from a change in impedance angle.

$$\cos(\phi') = 1.1\cos(\phi) = 0.983$$

$$R = Z\cos(\phi') = 11.18 \times 0.983 = 11.0\ \Omega$$

$$\omega L = Z\sin(\phi') = 11.18\sin\left(\cos^{-1}(0.983)\right) = 2.0\ \Omega$$

$$L = \frac{2.0}{2\pi 50} = 6.36\ mH$$

Solutions - Fields - (Sec B) 2002

E 2.3 2001-02 Question 4 : SOLUTION Page 1 of 5

(a) When $y=0$, $\tanh KV = \dfrac{0}{\cosh\frac{\pi x}{a} - 1} = 0$ $\left(K = \dfrac{2\pi\varepsilon}{q}\right)$.

$y=a$, $\tanh KV = \dfrac{0}{\cosh\frac{\pi x}{a}+1} = 0$

When $x=0, y=b$, $\tanh KV = 1$ i.e. $V \to \infty$ $\Rightarrow$ wire located here.

(b) $K \operatorname{sech}^2 KV \dfrac{\partial V}{\partial y} =$

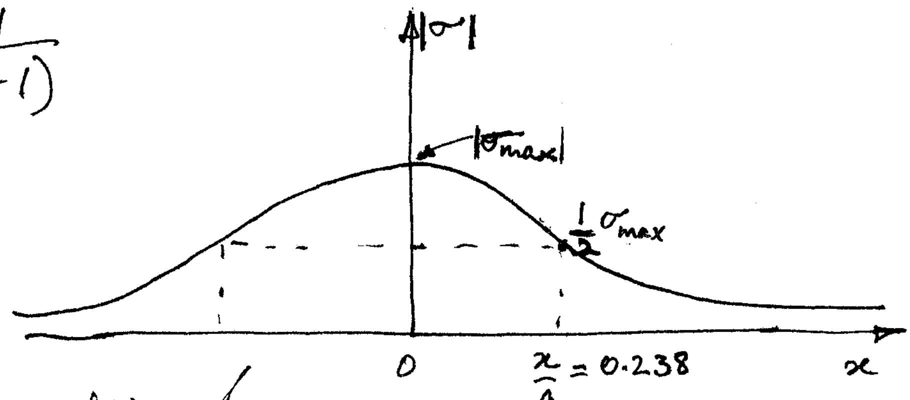
$$= \frac{\frac{\pi}{a}\left[\operatorname{ch}\frac{\pi x}{a} - \cos\frac{\pi b}{a}\cos\frac{\pi y}{a}\right]\sin\frac{\pi b}{a}\cos\frac{\pi y}{a} - \frac{\pi}{a}\sin\frac{\pi b}{a}\sin\frac{\pi y}{a}\left[\cos\frac{\pi b}{a}\sin\frac{\pi y}{a}\right]}{\left[\operatorname{ch}(\pi x/a) - \cos(\pi b/a)\cos(\pi y/a)\right]^2}$$

and $\partial V/\partial x = 0$ since $V=0$ (constant).
At $y=0$, $\operatorname{sech}^2 KV = 1/\operatorname{ch}^2 KV = 1$

$$\therefore K\left.\frac{\partial V}{\partial y}\right|_{y=0} = \frac{\frac{\pi}{a}\left\{\left[\operatorname{ch}\frac{\pi x}{a} - \cos\frac{\pi b}{a}\right]\sin\frac{\pi b}{a}\right\}}{\left[\operatorname{ch}\frac{\pi x}{a} - \cos\frac{\pi b}{a}\right]^2} = \frac{\frac{\pi}{a}\sin\frac{\pi b}{a}}{\left(\operatorname{ch}\frac{\pi x}{a} - \cos\frac{\pi b}{a}\right)} \quad \text{---}(1)$$

For $b/a = 1/4$, $\sin\frac{\pi b}{a} = \cos\frac{\pi b}{a} = 1/\sqrt{2}$ & hence by use of Gauss' law, charge density $\sigma = -\varepsilon\dfrac{\partial V}{\partial y}$ which at $x=0$ is

$$\sigma_{max} = \frac{-q}{2a} \times \frac{1}{(\sqrt{2}-1)}$$



continued ...

Keith [signature] Andrew Holmes [signature]

(b) continued

Field falls to $\frac{1}{2}$ peak value when denominator of (1) doubles,

i.e. when $\quad \operatorname{ch}\frac{\pi x}{a} - \frac{1}{\sqrt{2}} = 2\left(1 - \frac{1}{\sqrt{2}}\right)$

i.e. when $\quad \operatorname{ch}\frac{\pi x}{a} = 2 - \frac{1}{\sqrt{2}} = 1.293$

Hence $\frac{\pi x}{a} = \pm 0.7478$, i.e. $\boxed{x = \pm 0.238a}$

(c) Because both $\partial V/\partial z = 0$ and $\partial V/\partial x = 0$ at $y = 0$,

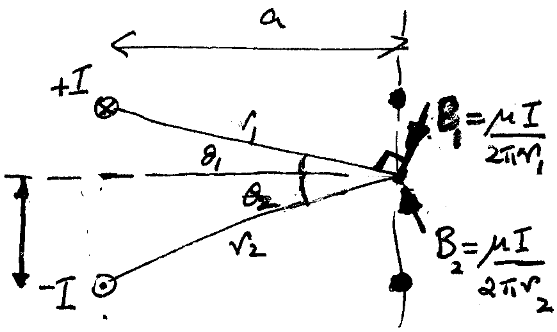
then $\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial z^2} = 0$. Hence $\nabla^2 V = \frac{\partial^2 V}{\partial y^2}$, and

since Laplaces eqn requires $\nabla^2 V = 0$ the result follows.

It implies that $\partial E_y/\partial y = 0$ at both $y = 0$ and $y = a$,

(a) $L = \Phi / I$

(b) At point shown, x-component of flux density is



$$B_x = \frac{\mu I}{2\pi}\left(\frac{\sin\theta_1}{r_1} + \frac{\sin\theta_2}{r_2}\right)$$

$$= \frac{\mu I}{2\pi}\left(\frac{y}{r_1^2} + \left(\frac{b-y}{r_2^2}\right)\right) = \frac{\mu I}{2\pi}\left(\frac{y}{y^2+a^2} + \frac{b-y}{(b-y)^2+a^2}\right)$$

Integrate one term (the second integral is equal by symmetry)

$$\therefore \Phi_x = \frac{\mu I}{\pi}\int_0^b \frac{y\,dy}{y^2+a^2} = \frac{\mu I}{2\pi}\int_{y=0}^{y=b} \frac{d(y^2+a^2)}{y^2+a^2} = \frac{\mu I}{2\pi}\left[\log(y^2+a^2)\right]_0^b$$

$$= \frac{\mu I}{2\pi}\log\left(\frac{b^2+a^2}{a^2}\right)$$

Hence $M = \dfrac{\Phi_x}{I} = \dfrac{\mu}{2\pi}\ln\left(\dfrac{b^2}{a^2}+1\right) = 2\times10^{-7}\ln 1.11$
$= 2.1\times10^{-8}\ H/m$ } when $\frac{b}{a} = \frac{1}{3}$

~~If $a = a_0$, $\Phi_x = \frac{\mu I}{2\pi}\log\frac{b}{a^2}$ $\therefore L_1 = L_2 = \frac{\mu}{2\pi}\ln\frac{b^2}{a^2}$~~

~~and $M_{12} = \frac{\mu}{2\pi}\ln\left(\frac{b^2}{a^2}+1\right)$~~

(c) The mutual inductance is reduced to zero if e.g. (a) one pair of wires is rotated 180° at regular intervals along the axis (b) one pair is rotated 90° to make the pairs lie in orthogonal planes. (c) or any other satisfactory solution.

(a) e.g. $\underline{E} = E_0 \hat{x} \exp 2\pi j \left(ft - \frac{z}{\lambda}\right)$

(b) $\nabla \times \underline{H} = \frac{\partial \underline{D}}{\partial t} \Rightarrow \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{x} = \varepsilon E_0 . 2\pi j f \exp 2\pi j \left(ft - \frac{z}{\lambda}\right)$

But $\frac{\partial H}{\partial y} = 0$ so only H_y is finite:

$$H_y = \frac{-\varepsilon E_0 . 2\pi j f \hat{y} \exp 2\pi j \left(ft - \frac{z}{\lambda}\right)}{-j 2\pi / \lambda}$$

$$= +\varepsilon E_0 f \lambda \hat{y} \exp 2\pi j \left(ft - \frac{z}{\lambda}\right) = \frac{E}{\eta}$$

Where $\left|\frac{E}{H}\right| = \eta = (\varepsilon f \lambda)^{-1}$

Now $f\lambda = c = 1/\sqrt{\mu\varepsilon}$, so $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

(c) At surface of dielectric $E_{t_1} = E_{t_2}$ i.e. if at $z = 0$:

$$\underbrace{E_r \exp 2\pi j f_1 t}_{\text{(reflected)}} + \underbrace{E_i \exp 2\pi j (f_1 t)}_{\text{(incident)}} = \underbrace{\vec{E}_2 \exp 2\pi j (f_2 t)}_{\text{(transmitted)}}$$

$\therefore f_2 = f_1$ and $E_i + E_r = E_2$

Also at $z = 0$ $H_{t_1} = H_{t_2}$ i.e.:

$$-\frac{E_r}{\eta_1} + \frac{E_i}{\eta_1} = \frac{E_2}{\eta_2}$$ where $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ and $\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} = \frac{\eta_1}{\sqrt{\varepsilon_r}}$

Hence $E_r = -E_i + E_2 = -E_i + \frac{\eta_2}{\eta_1}(E_i - E_r)$

$$\therefore \overleftarrow{E}_r \left(1 + \frac{\eta_2}{\eta_1}\right) = \vec{E}_i \left(\frac{\eta_2}{\eta_1} - 1\right)$$

Powers are proportional to $\frac{E^2}{\eta}$. Thus reflected power $\div$ incident power is

$$= \frac{|E_r^2|}{|E_i^2|} = \frac{(1 - \eta_2/\eta_1)^2}{(1 + \eta_2/\eta_1)^2} = \left(\frac{1 - 1/\sqrt{\varepsilon_r}}{1 + 1/\sqrt{\varepsilon_r}}\right)^2 = \frac{1 + \varepsilon_r - 2\sqrt{\varepsilon_r}}{1 + \varepsilon_r + 2\sqrt{\varepsilon_r}}$$

continued —

(c) (continued)

$$\frac{\text{Transmitted power}}{\text{Incident power}} = \frac{E_2^2}{\eta_2} \cdot \frac{\eta_1}{E^2}$$

$$= \frac{\eta_1 \eta_2}{E_i^2}\left(\frac{E_i}{\eta_1} - \frac{E_i}{\eta_1}\frac{(\eta_2-\eta_1)}{\eta_2+\eta_1}\right)^2 = \frac{\eta_1\eta_2}{\eta_1^2}\left(\frac{\eta_2+\eta_1-\eta_2+\eta_1}{\eta_2+\eta_1}\right)^2$$

$$= \frac{4\cdot\eta_2\eta_1}{(\eta_2+\eta_1)^2} = \frac{4\,\eta_2/\eta_1}{\left(\frac{\eta_2}{\eta_1}+1\right)^2} = \frac{4/\sqrt{\varepsilon_r}}{\left(1+\frac{1}{\sqrt{\varepsilon_r}}\right)^2}$$

Thus $\dfrac{\text{total power}}{\text{incident power}} = \dfrac{(1-1/\sqrt{\varepsilon_r})^2 + 4/\sqrt{\varepsilon_r}}{(1+1/\sqrt{\varepsilon_r})^2} = \dfrac{(1+1/\sqrt{\varepsilon_r})^2}{(1+1/\sqrt{\varepsilon_r})^2} = 1.$

(d) A current is induced along the wires resulting in power absorption of the electric field parallel to the length of the wires. The perpendicular component of E is not affected.