UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2001

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C343=I3.22

OPERATIONS RESEARCH

Thursday 10 May 2001, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 a Formulate a linear programming problem for finding a vector $(x_1, x_2) \in \mathbb{R}^2$ satisfying

$$4\,x_1 + x_2 \leq 5 \quad \text{and} \quad x_1 \geq 0, x_2 \geq 0$$

and having the maximum of

$$2\,x_1 - x_2 \quad \text{and} \quad -3\,x_1 + 2\,x_2$$

as small as possible.

b Compute an optimal solution of the above linear program using the simplex algorithm.

(All parts carry equal marks)

2 a Fire stations are to be established to serve six different areas of a city. Eight possible locations can be used, providing the area coverages given in the following table:

Location	*Areas Served*
A	1, 2, 6
B	1, 2, 3, 4
C	5, 6
D	3, 4, 5
E	1, 3, 5
F	1, 4, 6
G	2, 3, 5
H	1, 4, 5, 6

Formulate an integer program, with variables taking only the values 0 or 1, whose solution will tell which locations to use so as to cover all areas with the fewest possible fire stations. [Do NOT solve the integer programming problem.]

b Consider the integer program

Maximise $Z = 20\,x_1 + 15\,x_2$

subject to

$$\begin{aligned} 2\,x_1 + x_2 &\leq 12.5 \\ 2\,x_1 &\leq x_2 \\ x_2 &\leq 8 \\ x_j &\geq 0; \text{ and integer }; \; j = 1, 2. \end{aligned}$$

Use the branch-and-bound algorithm to find an optimal integer solution. [Hint: the solution of the above problem as a linear program (i.e. without integrality constraints) is x_1=2.25, x_2=8 with objective function value 165.]

(All parts carry equal marks)

3 a Consider the matrix game with the following payoff matrix

2	4	6
3	1	5

where the elements above correspond to the reward of the row player. Determine whether any of the column or row strategies are dominated. Formulate the linear programming problems for computing the optimal strategies of both players.

b Suppose that we add a constant c to every element in a genaral reward matrix A and let A' denote this new matrix. Show that the optimal strategies are the same for A and A' and that

$$[\text{optimum value of } A'] = c + [\text{optimum value of } A].$$

c Discuss briefly the relationship of the column and row players' linear programs.

(All parts carry equal marks)

4 a Consider the following (primal) linear program

Maximise $Z = -x_1 - 2x_2 - x_3$

subject to

$$\begin{aligned} x_1 + x_2 + 2x_3 &\le 1 \\ 2x_1 - x_3 &\le 1 \\ x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 &\ge 0 \end{aligned}$$

Construct the dual problem, determine its optimum (i.e. the optimum of the dual) by inspection and use duality theory to show that for the optimal solution of the primal problem $Z \le 0$.

b Consider the following (primal) linear program

Maximise $Z = 2x_1 - 4x_2$

subject to

$$\begin{aligned} x_1 - x_2 &\le 1 \\ x_1 \ge 0, \quad x_2 &\ge 0. \end{aligned}$$

Construct the dual problem and find its optimum by inspection. Use the complimentary slackness property and the optimal solution for the dual problem to find the optimal solution for the primal problem.

Suppose that c_1, the coefficient of x_1 in the primal objective function, can have any value. For which values of c_1 does the dual problem have no feasible solutions?

(All parts carry equal marks)