EE4-65
EE9-SO27

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

MSc and EEE PART IV: MEng and ACGI

Corrected copy

WIRELESS COMMUNICATIONS

Monday, 14 May 10:00 am

Time allowed: 2:00 hours

There are TWO questions on this paper.

Answer ALL questions

Both questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : B. Clerckx
Second Marker(s) : K.K. Leung

Important information for students

Notations:

(a) A $n_r \times n_t$ MIMO channel consists in n_r receive antennas and n_t transmit antennas.

(b) a, $\mathbf{a}$, $\mathbf{A}$ denote a scalar, vector and matrix respectively.

(c) $\mathbf{A}^H$ denotes conjugate transpose (Hermitian).

(d) $\mathbf{A}^*$ denotes conjugate.

(e) $\mathbf{A}^T$ denotes transpose.

(f) $|a|$ denotes the absolute value of scalar a.

(g) $\|\mathbf{a}\|$ denotes the (Euclidean) norm of vector $\mathbf{a}$.

(h) "i.i.d." means "independent and identically distributed".

(i) "CSI" means "Channel State Information".

(j) "CSIT" means "Channel State Information at the Transmitter".

(k) "CDIT" means "Channel Distribution Information at the Transmitter".

(l) $\mathcal{E}\{.\}$ denotes Expectation.

(m) $\text{Tr}\{.\}$ denotes the Trace of a matrix.

Assumptions:

(a) The CSI is assumed to be always perfectly known to the receiver.

(b) The receiver noise is a $n_r \times 1$ vector with i.i.d. entries modeled as zero mean complex additive white Gaussian noise with variance σ_n^2.

Some useful relationships:

(a) $\|\mathbf{A}\|_F^2 = \text{Tr}\{\mathbf{A}\mathbf{A}^H\} = \text{Tr}\{\mathbf{A}^H\mathbf{A}\}$

(b) $\text{Tr}\{\mathbf{AB}\} = \text{Tr}\{\mathbf{BA}\}$

(c) $\det(\mathbf{I}+\mathbf{AB}) = \det(\mathbf{I}+\mathbf{BA})$

(d) $\text{Tr}\{\mathbf{A}\mathbf{B}\mathbf{B}^H\mathbf{A}^H\} = \text{vec}(\mathbf{A}^H)^H(\mathbf{I}\otimes\mathbf{B}\mathbf{B}^H)\,\text{vec}(\mathbf{A}^H)$

(e) Gaussian Q-function

$$Q(x) \triangleq P(y \geq x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{y^2}{2}\right)dy$$

(f) Craig's formula

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2(\beta)}\right)d\beta$$

(g) Chernoff bound

$$Q(x) \leq \exp\left(-\frac{x^2}{2}\right)$$

(h) Assume n i.i.d. zero mean complex Gaussian variables $h_1, \ldots, h_n$ (real and imaginary parts with variance σ^2). Defining $u = \sum_{k=1}^{n} |h_k|^2$, the MGF of u is given by

$$\mathcal{M}_u(\tau) = \mathcal{E}\{e^{\tau u}\} = \left[\frac{1}{1 - 2\sigma^2 \tau}\right]^n$$

THE QUESTIONS

1. [50]

a) Consider the transmission of 2 independent streams using Spatial Multiplexing over a 4×2 MIMO channel $\mathbf{H}$. The Channel State Information (CSI) is unknown to the transmitter. The received signal is written as $\mathbf{y} = \mathbf{Hc} + \mathbf{n}$ where $\mathbf{c} = [c_1, c_2]^T$ is the vector of transmitted symbols. The channel matrix is given by

$$\mathbf{H} = \begin{bmatrix} 1 & 1 \\ 1+j & 0 \\ 1 & -1 \\ 1-j & 0 \end{bmatrix}.$$

We would like to apply a combiner $\mathbf{G}$ at the receiver. Suggest such a receive combiner and derive its expression. What kind of combiner is this? Explain your result.

[10]

b) Figure 1.1 displays the average Symbol Error Rate (SER), also called Symbol Error Probability, of three schemes vs. SNR for point-to-point channels with i.i.d. Rayleigh slow fading. The three schemes are (1) Alamouti scheme without CSIT in a two-transmit one-receive MISO channel with BPSK, (2) matched beamforming (also called transmit MRC or MRT) with perfect CSIT in a two-transmit one-receive MISO channel with BPSK, (3) uncoded SISO transmission with BPSK.

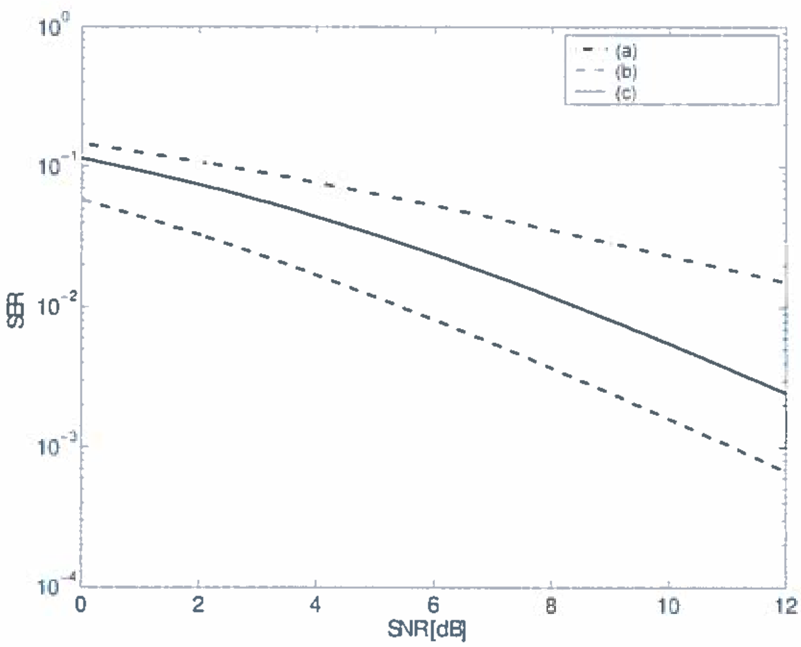

Figure 1.1 Average Symbol Error Probability vs. SNR.

i) Map schemes (1), (2) and (3) to curves (a), (b) and (c) in Figure 1.1 and provide your reasoning.

[6]

ii) From the analytical expressions of the error probability of (b) and (c), explain the gap between those two curves.

[4]

c) Consider a MISO channel $\mathbf{h} = [\, h_1 \quad h_2 \,]$ with two transmit antennas and one receive antenna, constant over two consecutive symbol durations. Symbols c_1 and c_2 are transmitted over the channel during two symbol durations using an Alamouti space-time code.

i) Give an expression for the received signals at the receiver. [3]

ii) Provide a simple receiver strategy to detect c_1 and c_2. [4]

iii) What will happen if the same transmission strategy is applied to a channel that is not constant over two consecutive symbol durations? Provide your reasoning. [3]

d) Figure 1.2 displays the ergodic capacity of point-to-point i.i.d. Rayleigh fast fading channels with Channel Distribution Information at the Transmitter (CDIT) for five antenna ($n_r \times n_t$) configurations (denoted as (a) to (e)) with $n_t + n_r = 9$.

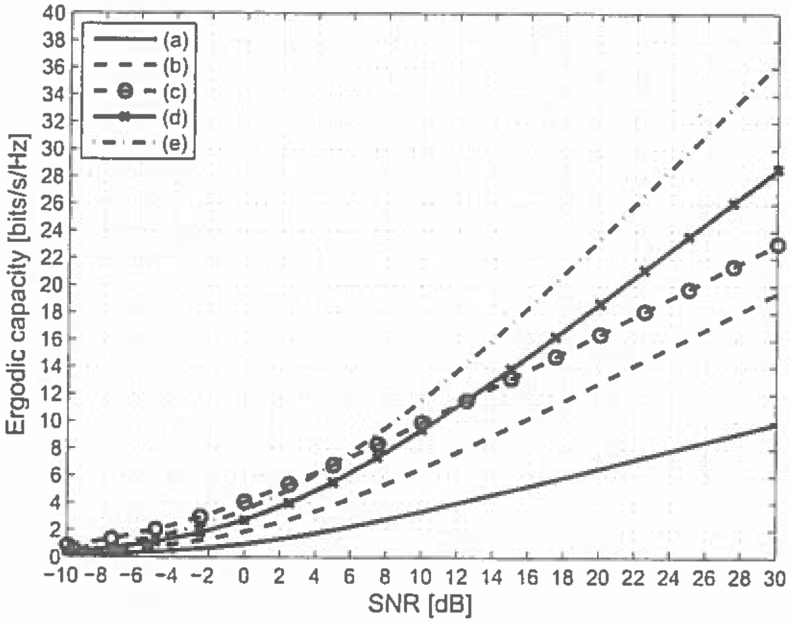

Figure 1.2 Ergodic capacity vs. SNR.

i) What is the achievable (spatial) multiplexing gain (at high SNR) for each of cases (a) to (e)? Provide your reasoning.

[5]

ii) For each of cases (a) to (e), identify the antenna configuration, i.e., n_t and n_r, satisfying $n_t + n_r = 9$ that achieves such multiplexing gain. Provide your reasoning.

[5]

e) Consider the two-user Gaussian SISO Multiple Access Channel over the deterministic channels h_1 and h_2. The system model is written as $y = h_1 c_1 + h_2 c_2 + n$ where the transmit power constraint at transmitter i is given by $\mathcal{E}\left\{ |c_i|^2 \right\} \leq P_i$ for $i = 1, 2$.

i) What is the capacity region of such a channel? Explain its meaning.

[5]

ii) Determine the rates achievable at the corner points of the capacity region and explain the reception strategy to achieve those rates.

[5]

2. [50]

a) Consider the transmission $\mathbf{y} = \mathbf{H}\mathbf{c}' + \mathbf{n}$ with perfect CSIT over a deterministic point-to-point MIMO channel with two transmit and two receive antennas whose matrix is given by

$$\mathbf{H} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

where a and b are complex scalars with $|a| \geq |b|$. The receiver is subject to AWGN noise such that the noise variances on receive antenna 1 and 2 are given by $\sigma_{n,1}^2$ and $\sigma_{n,2}^2$, respectively. Assume $\sigma_{n,1}^2 \leq \sigma_{n,2}^2$. The input covariance matrix is given by $\mathbf{Q} = \mathcal{E}\left\{\mathbf{c}'\mathbf{c}'^H\right\}$ and is subject to the transmit power constraint $\mathrm{Tr}\{\mathbf{Q}\} \leq P$. Compute the capacity with perfect CSIT of that deterministic channel. Explain your reasoning.

[10]

b) Consider the 3×3 point-to-point MIMO channel matrix given by

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}.$$

Assume $\mathbf{H}$ is known at the transmitter. What transmission and reception strategies would you use to maximize the transmission rate over such a channel? Explain your reasoning.

[10]

c) Discuss the validity of the following statements. Detail your argument.

i) To get a diversity gain of three in a MISO point-to-point channel with three transmit antennas, the transmitter needs to know the CSI.

[10]

ii) In a two-user SISO Broadcast Channel, for any ordering of the channels, the capacity region can be achieved with superposition coding with Successive Interference Cancellation (SIC).

[10]

iii) In a MIMO point-to-point communication system based on Spatial Multiplexing, a Zero-Forcing receiver outperforms a Matched Filter because it completely suppresses the interference between streams.

[10]