

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2007

EEE PART III/IV: MEng, BEng and ACGI

Corrected Copy

**ELECTRICAL ENERGY SYSTEMS**

Thursday, 10 May 10:00 am

Time allowed: 3:00 hours

**There are SIX questions on this paper.**

**Answer Question ONE and THREE other questions.**

*All questions carry equal marks.*

Correction by  
GS  
• Q4, p 5

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible

First Marker(s) : G. Strbac

Second Marker(s) : C.A. Hernandez-Aramburo

Question 1 [Compulsory]

- (a) Explain the usefulness of using a Per-Unit (PU) system in the analysis of power systems [2]
- (b) A 33/11kV, 15MVA transformer has a leakage reactance of  $4\Omega$  as seen from the HV side
- (i) Calculate the leakage reactance as seen from the LV side. [2]
- (ii) Calculate the PU impedance at the HV side and show that it is the same as the PU impedance as seen from the LV side. [2]
- (c) For the transmission circuit and its phasor diagram given in figure 1.1:
- (i) Show that  $\bar{V}_r = V_s - \left( \frac{RP_s + XQ_s}{V_s} \right) - j \left( \frac{XP_s - RQ_s}{V_s} \right)$  [2]
- (ii) Write the expressions for  $\Delta V$  and  $\delta V$ . [2]
- (iii) How does the expression given in (c)(i) change if the active and reactive powers are specified at the receiving rather than at the sending end? [2]

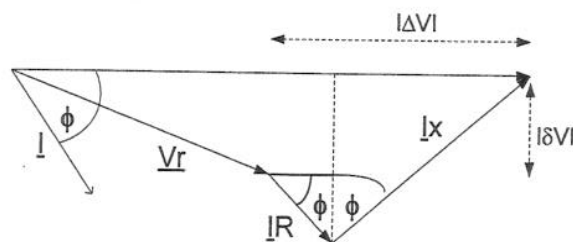
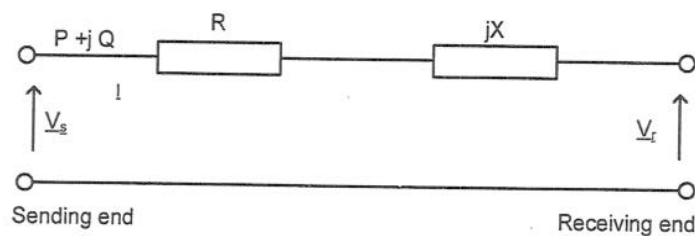


Figure 1.1 A transmission circuit and its corresponding phasor diagram

– The question continues on the following page –

(d)

(i) List three purposes of load flow calculations in a power system. [2]

(ii) What makes the load flow problem non-linear? Explain the main steps of the Gauss-Seidel method. [2]

(e)

(i) Under unsymmetrical fault conditions on a three phase power system, explain why positive and negative sequence currents do not flow through the neutral circuit. [2]

(ii) For a 3-phase fault on the 33 kV busbar shown in figure 1.2, calculate the short circuit current (in amperes) in the fault and the symmetrical fault level (in MVA) at the 33 kV busbar. [2]

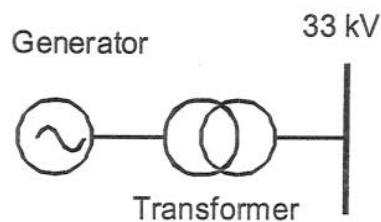


Figure 1.2 A generator and its transformer

The ratings and p.u. reactances of the each element of the plant are given in Table 1.1. In this table, the p.u. values are based on the individual ratings of each element of the plant.

Table 1.1

	Generator	Transformer
Rating (MVA)	125	150
Rating (kV)	11	11/33
X1 (positive phase sequence) (per unit)	0.25	0.12

## Question 2

- (a) Explain how the active and reactive power outputs of a synchronous generator are controlled. [3]
- (b) What determines the maximum reactive power that a generator can export and import? [3]
- (c) A turbo-generator feeds into a very strong network that maintains the terminal voltage of the generator at 1 p.u. The synchronous reactance of the generator is equal to 1 p.u. Initially, the generator runs overexcited with  $E=1.5$  p.u. with real power output of 0.25 p.u. Calculate:
- (i) The power angle and reactive power output for this initial operating condition. [4]
- (ii) The active and reactive power delivered to the system when the turbine torque doubles. [4]
- (iii) The active and reactive power delivered to the system for an increase in the field current of 20% (from the initial condition). [4]
- (iv) Based on your results of parts (i), (ii) and (iii), discuss the extent of coupling between the two basic control inputs. [2]

Question 3

- (a) Explain why in very high voltage ac power systems active power demand can be provided by remote sources while reactive power demand is usually met more locally. [4]
- (b) A simple power system is composed of a generator unit supplying a load over a 100km 400kV overhead transmission line as shown in Figure 3.1. Line parameters are given in Table 3.1.

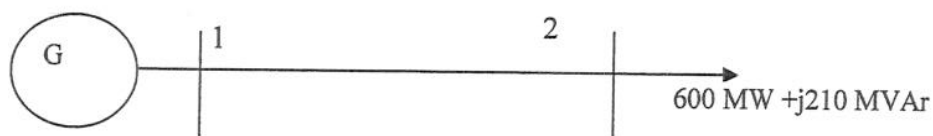


Figure 3.1. A 400 kV system

Table 3.1. Line parameters per km

Series resistance $r$ [ $\Omega/\text{km}$ ]	Series reactance $x$ [ $\Omega/\text{km}$ ]	Shunt susceptance $b$ [ $\mu\text{S}/\text{km}$ ]
0.027	0.304	3.00

If the desired voltage at the load busbar is 400 kV, calculate the following (using base values of  $V_b=400$  kV and  $S_b=100$  MVA):

- (i) Voltage magnitude and phase angle at busbar 1. [6]
- (ii) Active and reactive losses in the line. [5]
- (iii) Active and reactive output of the generator at bus 1. [5]

#### Question 4

A generator connected to a 33 kV circuit supplies a load ~~is~~ connected through a 33 kV distribution line ~~to a 3-phase load~~, as illustrated in figure 4.1. The system is a balanced 3-phase system and its frequency is 50 Hz.

Distribution line details:

$$R = 0.3267 \, \Omega/\text{km}$$

$$X = 0.4356 \, \Omega/\text{km}$$

$$B = 0.0 \, \text{S/km/phase}$$

$$\text{Length} = 10 \, \text{km}$$

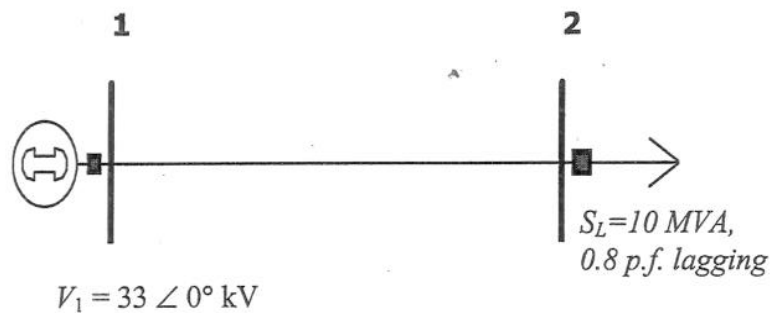


Figure 4.1. The simple power system

Determine:

- (a) The per unit value of distribution line impedance, voltage at generator terminal node and load. (Use a 33 kV, 100 MVA base). [4]
- (b) Calculate the magnitude of the voltage at node 2 using Gauss Seidel load flow algorithm. Perform 3 iterations. [12]
- (c) Compute the losses and voltage drop across the distribution line. [4]

### Question 5

- (a) Explain briefly how the method of symmetrical components may be used to represent a 3-ph system of unbalanced voltages. [4]
- (b) Determine all three symmetrical components of a balanced set of currents with a positive phase sequence. [4]
- (c) Show that there is no positive or negative sequence current in the neutral connection of a three phase system, that the neutral and zero sequence currents are proportional to each other, and that there cannot be a zero sequence current if there is no neutral connection. [4]
- (d) Determine the phase voltages corresponding to the following sequence components: [4]

$$\overline{V}^0 = 20 \angle 80^\circ pu$$

$$\overline{V}^1 = 100 \angle 0^\circ pu$$

$$\overline{V}^2 = 30 \angle 180^\circ pu$$

- (e) For the system shown in figure 5.1, sketch the zero sequence network. [4]

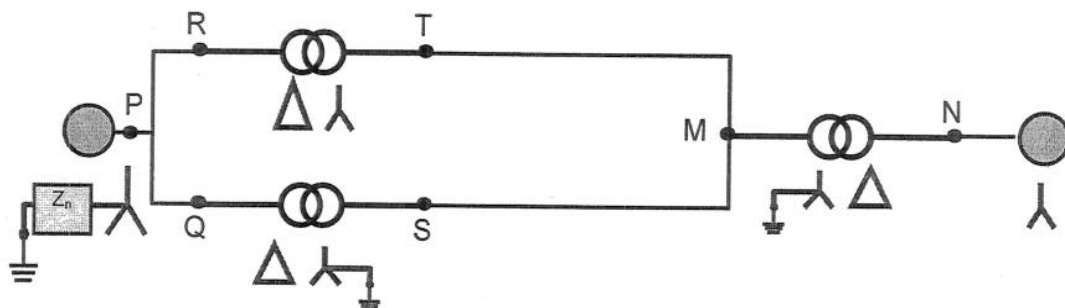


Figure 5.1 A simple power system

### Question 6

Consider the simple three phase power system presented in Figure 6.1.

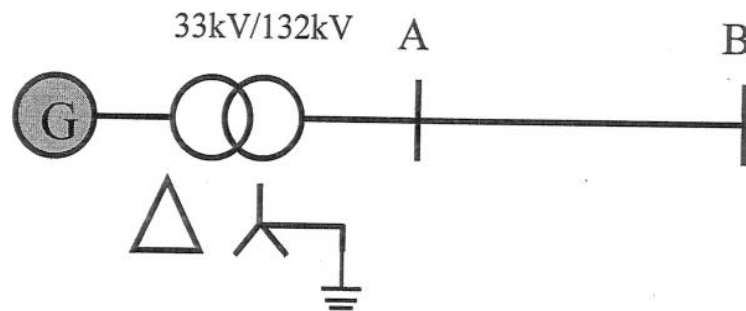


Figure 6.1 Three phase power system

The system's sequence impedances, for a 100MVA base, are given in Table 6.1

Table 6.1

	Positive	Negative	Zero
Generator	0.5	0.666	0.8
Transformer	0.3	0.3	0.3
Line	0.115	0.115	0.172

- Sketch positive, negative and zero sequence circuits and place appropriate sequence impedances in these circuits. [3]
- Show how these circuits should be linked for the evaluation of a single line to ground fault at bus bar B. [4]
- Determine the fault current. [4]
- Determine the phase currents through the transmission line A-B. [3]
- Determine the phase currents through the generator (ignore phase shift across the transformer windings, i.e. assume that the phase shift is zero in this case). [3]
- Determine the fault level at bus B. [3]



## SOLUTIONS 2007

### Question 1: [compulsory]

(a) For full mark the following points need to be elaborated

- Multiple voltage levels: 400kV, 275 kV, 132 kV
- Impedance of transformers depends on side
- Normalised quantities help understanding

[2]

(b) A 33/11kV 15MVA transformer has a leakage reactance of  $4\Omega$  as seen from the HV side

$$(i) \quad Z_{base\ hv} = \frac{U_{hv}^2}{S_b} = \frac{33^2 \cdot 10^6}{15 \cdot 10^6} = 72.6\Omega \text{ , in per unit}$$

$$x = \frac{X_{hv}}{Z_{base\ hv}} = \frac{4}{72.6} = 0.0551 p.u$$

[2]

$$(ii) \quad X_{lv} = X_{hv} \cdot \frac{U_{lv}^2}{U_{hv}^2} = 0.4444\Omega, Z_{base\ lv} = \frac{U_{lv}^2}{S_b} = \frac{11^2 \cdot 10^6}{15 \cdot 10^6} = 8.06\Omega$$

in per unit

$$x = \frac{0.4444}{8.06} = 0.0551 p.u$$

[2]

(c) For a transmission circuit given in Figure 1.1 and the corresponding phasor diagram below, show:

$$\overline{S_s} = P_s + jQ_s = \overline{V_s} \cdot \overline{I}^*$$

$$\overline{I} = \frac{P_s - jQ_s}{\overline{V_s}^*}$$

$$\overline{V_r} = \overline{V_s} - (R + jX) \cdot \overline{I}$$

$$(i) \quad \overline{V_r} = \overline{V_s} - (R + jX) \cdot \left( \frac{P_s - jQ_s}{\overline{V_s}^*} \right)$$

$$\overline{V_s} = \overline{V_s}^* = V_s = \angle 0^\circ = V_s$$

$$\overline{V_r} = \overline{V_s} - \left( \frac{RP_s + XQ_s}{V_s} \right) - j \left( \frac{XP_s - RQ_s}{V_s} \right)$$

[2]

$$(ii) \quad \Delta V = \frac{RP_s + XQ_s}{V_s}; \delta V = \frac{XP_s - RQ_s}{V_s}$$

[2]

$$(iii) \quad \overline{V}_s = V_r + \left( \frac{RP_r + XQ_r}{V_r} \right) + j \left( \frac{XP_r - RQ_r}{V_r} \right) \quad [2]$$

(d)

(i) A brief discussion of three of following topics will give a full mark:

- Given loads and generations in the system, calculate the voltage at each bus
- Using these voltages, calculate the flows in each branch
- Applications
  - Analyse the behaviour of the system for hypothetical situations
  - Initialise other calculations
- Central problem of power system analysis
- Effects of rearranging circuits and incorporating new circuits on system loading
- Effects of temporary loss of generation and transmission circuits on system loading
- Effects of injecting in-phase and quadrature voltages on system loading
- Optimum system running conditions and load distribution
- Improvement from change of conductor size and system voltages

[2]

(ii) In power system analysis we choose to describe load in terms of power injected (or absorbed) rather than in terms of impedances. So voltage is non linear function of power.

The Gauss-Siedel method

Step 1: Guess initial values of voltage  $\overline{V}_L^0$  for load buses. If no specific information is available we generally assume that  $\overline{V}_L^0 = 1.0$  p.u.

Step 2: Compute  $\overline{V}_L^{i+1} = \overline{V}_s - (R + jX) \cdot \left( \frac{P_L - jQ_L}{\overline{V}_L^{i*}} \right)$

Step 3: If  $\left| \overline{V}_L^{i+1} - \overline{V}_L^i \right| > \varepsilon$ , where  $\varepsilon$  is predefined tolerance, the iterative procedure has not yet converged. Let  $i = i + 1$  and go back to step 2.

Step 4: If the convergence condition is satisfied the computation stops. Given voltage phase angles and magnitudes, power flows can be computed.

[2]

(e)

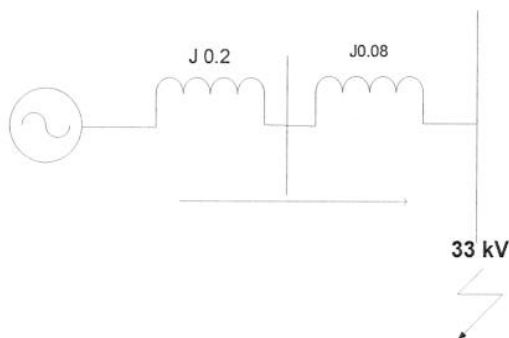
(i) Positive and negative sequence currents do not flow in the neutral circuit because they are balanced

[2]

(ii) Per unit conversion

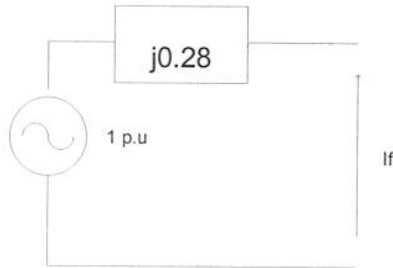
	Generator	Transformer
Rating (MVA)	125	150
Rating (kV)	11	11/33
X1 (positive phase sequence) (per unit)	0.25	0.12
System base (100 MVA)	$(0.25) \cdot \frac{100 \cdot 10^6}{125 \cdot 10^6} = 0.2 p.u.$	$(0.12) \cdot \frac{100 \cdot 10^6}{125 \cdot 10^6} = 0.08 p.u.$

Three- phase fault at 33 kV



$$x = x_{g1} + x_{t1} = 0.28 p.u.$$

$$Z_{th} = \frac{1}{j\frac{1}{x}} = j0.28 \text{ p.u.}$$



$$I_f = \frac{V_{th}}{Z_{th}} = -j3.5714 \text{ p.u.}$$

$$I_b = \frac{S_b}{\sqrt{3} \cdot V_b} = 1749.5 \text{ A}$$

$$I_F = I_f \cdot I_b = -j6248 \text{ A}$$

**Fault level (MVA)**

$$FL = \sqrt{3} \cdot I_F \cdot V_B = -357 \text{ MVA} \quad [2]$$

**SOLUTION to Question 2:**

(a) Increasing a torque on the rotor shaft increases the rotor angle ( $\delta$ ) and results in more active power exported to the network. Increasing the field current and hence increasing the magnitude of  $\overline{E_f}$  results in export of reactive power. [3]

(b) The reactive power is limited by:

- Excitation limit
- MVA limit
- Primer mover limit
- Under excitation limit

[3]

(c)

$$(i) \quad \delta = \sin^{-1} \left( \frac{P_{initial} \cdot X_s}{V_t \cdot E} \right) = 9.5941^\circ$$

$$Q_{initial} = \frac{V_t}{X_s} \cdot (E \cdot \cos \delta - V_t) = 0.4790 \text{ p.u.}$$

[4]

$$(ii) \quad \delta_2 = \sin^{-1}(2 \cdot \sin \delta) = 19.47^\circ$$

$$P = \frac{V_t \cdot E}{X_s} \cdot \sin \delta_2 = 0.5000 \text{ p.u.}$$

$$Q = \left( \frac{V_t}{X_s} \right) \cdot (E \cdot \cos \delta_2 - V_t) = 0.4142 \text{ p.u.}$$

[4]

$$(iii) \quad E_{new} = 1.2 \cdot E$$

$$P = \frac{V_t \cdot E_{new}}{X_s} \cdot \sin \delta = 0.3000 \text{ p.u.}$$

$$Q = \frac{V_t}{X_s} \cdot (E_{new} \cdot \cos \delta - V_t) = 0.7748 \text{ p.u.}$$

[4]

(iv)

Active/Reactive power	P (p.u.)	Q(p.u.)
Case (i)	0.25	0.4790
Case (ii)	0.5	0.4142
Case (iii)	0.3	0.7748

From the results shown above it is evident that in case when turbine torque doubles the active power increase while reactive power is slightly decreases compared with initial working condition. In case when a field current is increased for 20 % the active and reactive power are increased. Therefore, it can be concluded that these two controls are almost independent.

[2]

### SOLUTION Question 3:

- (a) Bringing reactive power from a longer distance increase the current in transmission lines, which leads to an increase in reactive losses ( $I^2 X$ ), that directly contributes to a voltage drop, as in HV networks X is significantly larger than R. In other words reactive power can be transported only at the expense of increased voltage drops. However, the magnitude of voltage drops need to be limited.

[4]

(b)

$$R = r' L = 0.027 \cdot 100 = 2.7 \text{ Ohm}$$

$$X = x' \cdot L = 0.304 \cdot 100 = 30.4 \text{ Ohm}$$

$$B = b' \cdot L = 3.00 \cdot 100 = 300 \text{ e-6 1/Ohm}$$

$$V_b = 400 \text{ kV}, S_b = 100 \text{ MVA} \Rightarrow Z_b = \frac{V_b^2}{S_b} = \frac{400^2}{100} = 1600 \Omega$$

Per unit values:

$$r = \frac{R}{Z_b} = \frac{2.7}{1600} = 0.016875$$

$$x = \frac{X}{Z_b} = \frac{30.4}{1600} = 0.019$$

$$b = B \cdot Z_b = 300 \cdot 10^{-6} \cdot 1600 = 0.48$$

$$P_2 = 6$$

$$Q_2 = 2.1$$

$$Q_{20} = V_2^2 \cdot \frac{b}{2} = 0.24$$

$$Q_{20} = 24 \text{ MVar}$$

$$P_2' = P_2 = 6$$

$$Q_2' = Q_2 - Q_{20} = 2.1 - 0.24 = 1.86$$

$$V_1 - V_2 = \Delta V = \frac{P_2' r + Q_2' x}{V_2} + j \frac{P_2' x - Q_2' r}{V_2} =$$

$$\Delta V = (6 \cdot 0.0016875 + 1.86 \cdot 0.019) + j(6 \cdot 0.019 - 1.86 \cdot 0.0016875)$$

$$\Delta V = 0.04546 + j0.11086$$

$$(i) \quad V_1 = 1.04546 + j0.11086$$

$$|V_1| = 1.05132$$

$$|V_1| = 420.53 \text{ kV}$$

[6]

(ii) **Series losses:**

$$S_{sc} = \Delta V \cdot I_{sc}^* = \Delta V \cdot \left( \frac{\Delta V}{Z_{sc}} \right)^* = \frac{|\Delta V|^2}{Z_{sc}} = \frac{|0.04546 + j0.11086|^2}{0.0016875 + j0.019}$$

$$S_{sc} = 0.0666 + j0.749$$

**Reactive shunt 'gains':**

$$\text{Node 2: } Q_{20} = 0.24$$

$$\text{Node 1: } Q_{10} = V_1^2 \cdot \frac{b}{2} = 1.1053 \cdot 0.24 = 0.2653$$

Total line losses:

$$S_T = S_{sc} - jQ_{10} - jQ_{20} = 0.0666 + j0.749 - j0.2653 - j0.24 = 0.0666 + j0.243$$

[5]

(iii) **Injection at bus 1**

$$S_1 = S_2 + S_T = 6 + j2.1 + 0.0666 + j0.243 = 6.0666 + j2.343$$

$$S_1 = 606.66\text{MW} + j234.3\text{MVar}$$

[5]

#### SOLUTION Question 4

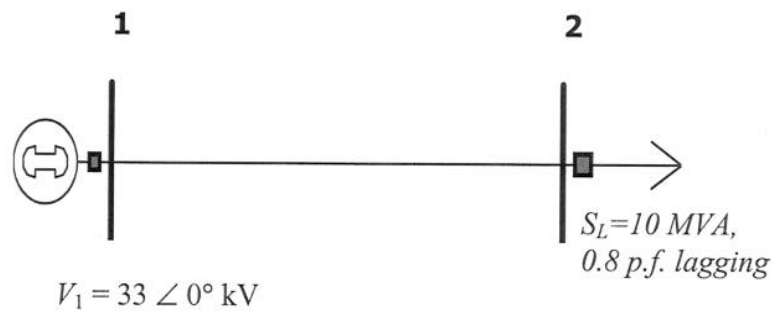


Figure 4.1. The simple power system

(a) **Use a single representation of the system**

$$\text{MVA Base} = 100 \text{ MVA}$$

$$\text{VBase} = 33 \text{ kV}$$

$$Z_{base} = (33 \cdot 10^3)^2 / 10^8 = 10.89 \Omega / \text{p.u.}$$

**Impedance of distribution line**

$$R = 0.3267 \cdot 10 = 3.267 \Omega$$

$$R = 3.267 / 10.89 = 0.3 \text{ p.u.}$$

$$X = 0.4356 \cdot 10 = 4.356 \Omega$$

$$X = 4.356 / 10.89 = 0.4 \text{ p.u.}$$

**Load**

$$P_{Load} = 10 \times 0.8 = 8 \text{ MW}$$

$$P_{Load} = 8/100 = 0.08 \text{ p.u.}$$

$$Q_{\text{Load}} = 10 \cdot (1 - 0.8^2)^{0.5} = 6 \text{ MVar}$$

$$Q_{\text{Load}} = 6/100 = 0.06 \text{ p.u.}$$

[4]

**(b) Iteration 1**

$$\begin{aligned}\bar{V}_2 &= V_1 - (R + jX) \cdot \frac{(P_L - jQ_L)}{\bar{V}_2^*} \\ &= 1 - (0.3 + j0.4) \cdot \frac{(0.08 - j0.06)}{1} \\ &= 1 - (0.048 + j0.014) \\ &= 0.952 - j0.014 \text{ p.u.}\end{aligned}$$

Iteration 2

$$\begin{aligned}\bar{V}_2 &= V_1 - (R + jX) \cdot \frac{(P_L - jQ_L)}{\bar{V}_2^*} \\ &= 1 - (0.3 + j0.4) \cdot \frac{(0.08 - j0.06)}{0.952 + j0.014} \\ &= 0.9494 - j0.01396 \text{ p.u.}\end{aligned}$$

Iteration 3

$$\begin{aligned}\bar{V}_2 &= V_1 - (R + jX) \cdot \frac{(P_L - jQ_L)}{\bar{V}_2^*} \\ &= 1 - (0.3 + j0.4) \cdot \frac{(0.08 - j0.06)}{0.9494 + j0.01396} \\ &= 0.9492 - j0.014 \text{ p.u.}\end{aligned}$$

$$\bar{V}_2 = 0.9492 - j0.014$$

$$= 0.9493 \angle -0.845 \text{ deg}$$

$$= 31.3269 \text{ kV} \angle -0.845 \text{ deg}$$

[12]

**(c)**

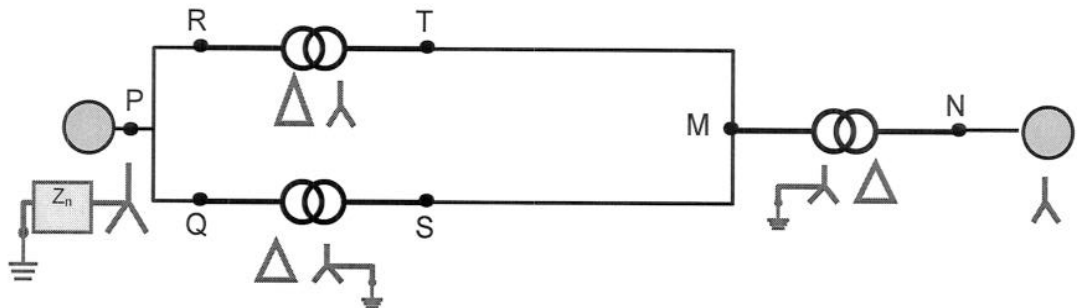
$$\begin{aligned}\text{Voltage drop} &= \bar{V}_1 - \bar{V}_2 \\ &= 1 - (0.9492 - j0.014) \\ &= 0.0508 + j0.014 \text{ p.u.} \\ &= 1.738 \text{ kV} \angle 15.4 \text{ deg}\end{aligned}$$



$$\begin{aligned}
\text{Losses} &= I^2 \cdot (R + jX) \\
&= \frac{|S_L|^2}{|V_2|^2} \cdot (R + jX) \\
&= \frac{|0.1|^2}{|0.9493|^2} \cdot (0.3 + j0.4) \\
&= 0.0033 + j0.0044 \text{ p.u.} \\
&= 0.33 \text{ MW} + 0.44 \text{ MVar}
\end{aligned}$$

[4]

### SOLUTION Question 5



- a) Any set of three-phase voltages (or currents) can be decomposed into the sum of three components
- **A positive sequence component**  
-Set of three voltages of equal magnitude, separated by  $120^\circ$  in the *positive* phase sequence
  - **A negative sequence component**  
-Set of three voltages of equal magnitude, separated by  $120^\circ$  in the *negative* phase sequence
  - **A zero sequence component**  
-Set of three voltages of equal magnitude and phase

[4]

b)

$$\begin{pmatrix} \overline{I^0} \\ \overline{I^1} \\ \overline{I^2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \cdot \begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix}$$

Positive sequence current:

$$\begin{aligned} \overline{I^1} &= \frac{1}{3} (\overline{I_a} + a \cdot \overline{I_b} + a^2 \cdot \overline{I_c}) \\ &= \frac{1}{3} (\overline{I_a} + a \cdot a^2 \cdot \overline{I_a} + a^2 \cdot a \cdot \overline{I_a}) \\ &= \frac{1}{3} (1 + a^3 + a^3) \cdot \overline{I_a} \\ &= \frac{1}{3} (1 + 1 + 1) \cdot \overline{I_a} \\ &= \overline{I_a} \end{aligned}$$

Negative sequence current:

$$\begin{aligned}
 \overline{I^2} &= \frac{1}{3} (\overline{I_a} + a^2 \cdot \overline{I_b} + a \cdot \overline{I_c}) \\
 &= \frac{1}{3} (\overline{I_a} + a^2 \cdot a^2 \cdot \overline{I_a} + a \cdot a \cdot \overline{I_a}) \\
 &= \frac{1}{3} (1 + a + a^2) \cdot \overline{I_a} \\
 &= 0
 \end{aligned}$$

Zero sequence current:

$$\begin{aligned}
 \overline{I^0} &= \frac{1}{3} (\overline{I_a} + \overline{I_b} + \overline{I_c}) \\
 &= \frac{1}{3} (\overline{I_a} + a^2 \cdot \overline{I_a} + a \cdot \overline{I_a}) \\
 &= \frac{1}{3} (1 + a^2 + a) \cdot \overline{I_a} \\
 &= 0
 \end{aligned}$$

[4]

c)

$$\begin{aligned}
 \overline{I_n} &= \overline{I_a} + \overline{I_b} + \overline{I_c} \\
 &= \left( \overline{I_a^0} + \overline{I_a^1} + \overline{I_a^2} \right) + \left( \overline{I_b^0} + \overline{I_b^1} + \overline{I_b^2} \right) + \left( \overline{I_c^0} + \overline{I_c^1} + \overline{I_c^2} \right) \\
 &= \left( \overline{I_a^0} + \overline{I_b^0} + \overline{I_c^0} \right) + \underbrace{\left( \overline{I_a^1} + \overline{I_b^1} + \overline{I_c^1} \right)}_0 + \underbrace{\left( \overline{I_a^2} + \overline{I_b^2} + \overline{I_c^2} \right)}_0 \\
 &= 3 \cdot \overline{I^0}
 \end{aligned}$$

- There is no positive or negative sequence current in the neutral connection
- The neutral and zero sequence currents are proportional
- There can not be a zero sequence current if there is no neutral connection

[4]

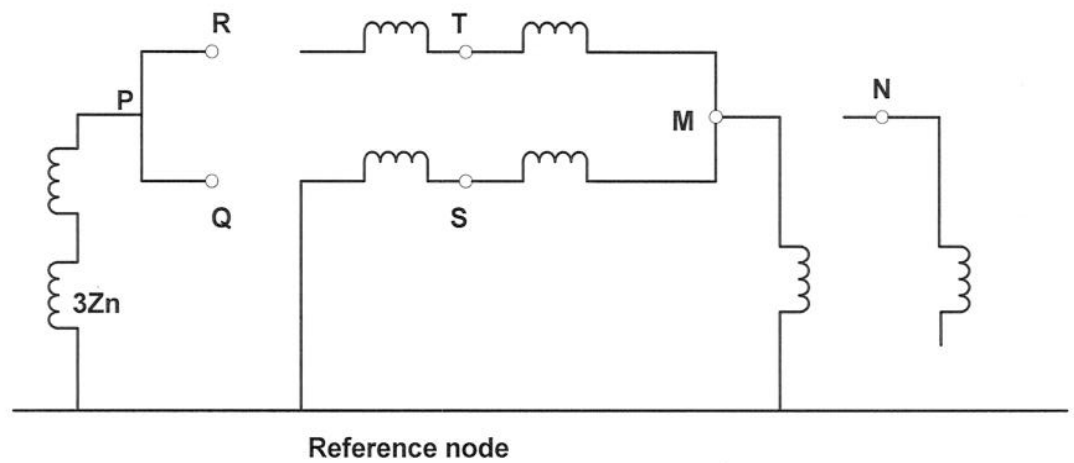
d)

$$\begin{pmatrix} \overline{V_{an}} \\ \overline{V_{bn}} \\ \overline{V_{cn}} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \cdot \begin{pmatrix} \overline{V^0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix}$$

$$\begin{aligned}
\begin{pmatrix} \overline{V_{an}} \\ \overline{V_{bn}} \\ \overline{V_{cn}} \end{pmatrix} &= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \cdot \begin{pmatrix} 20\angle 80^\circ \\ 100\angle 0^\circ \\ 30\angle 180^\circ \end{pmatrix} = \begin{pmatrix} 20\angle 80^\circ + 100\angle 0^\circ + 30\angle 180^\circ \\ 20\angle 80^\circ + 100\angle 240^\circ + 30\angle 30^\circ \\ 20\angle 80^\circ + 100\angle 120^\circ + 30\angle 60^\circ \end{pmatrix} = \begin{pmatrix} 73.47 + j19.70 \\ -31.53 - j92.89 \\ -31.53 + j132.3 \end{pmatrix} \\
&= \begin{pmatrix} 76.07\angle 15.01^\circ \\ 98.09\angle 251.3^\circ \\ 135.98\angle 103.4^\circ \end{pmatrix}
\end{aligned}$$

[4]

e)



[4]

# Solution of Question 6

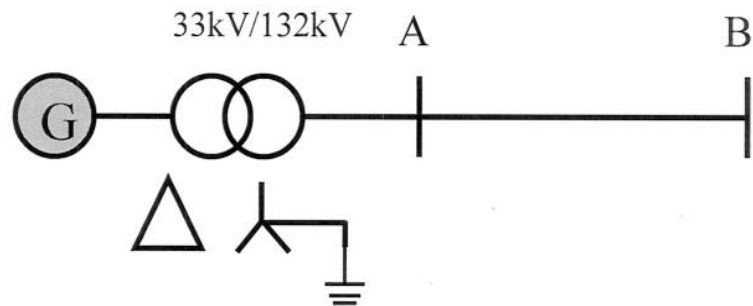
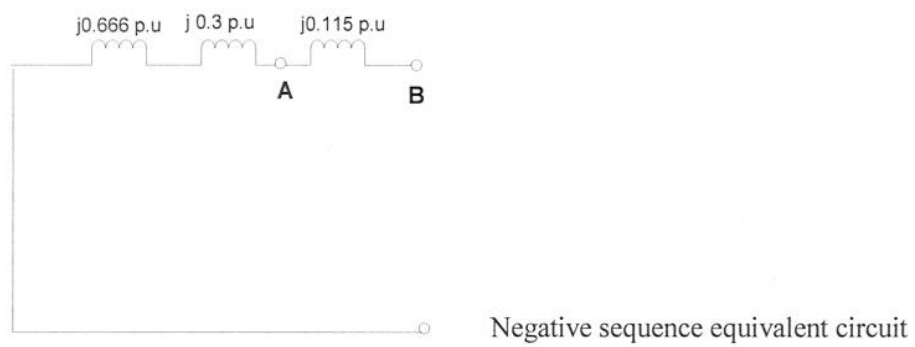
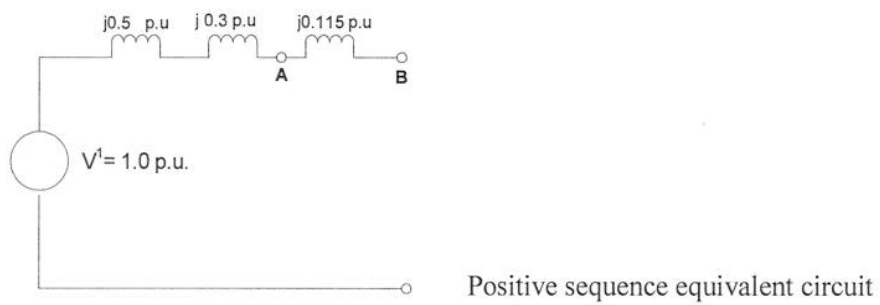
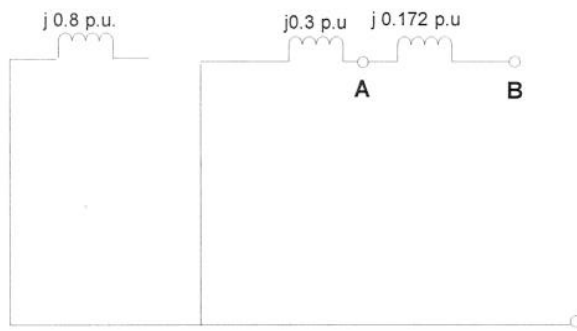


Figure 6.1 Three phase power system

a)

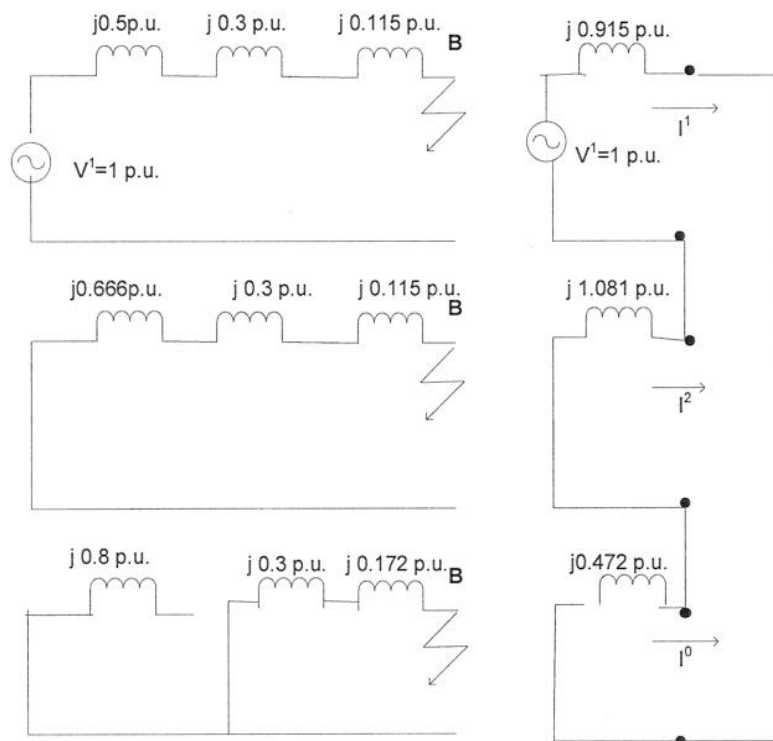




Zero sequence equivalent circuit

[3]

b)



[4]

c)

$$|\overline{I_1}| = |\overline{I_2}| = |\overline{I_0}| = \frac{1.0}{0.915 + 1.081 + 0.472} = 0.405 \text{ p.u.}$$

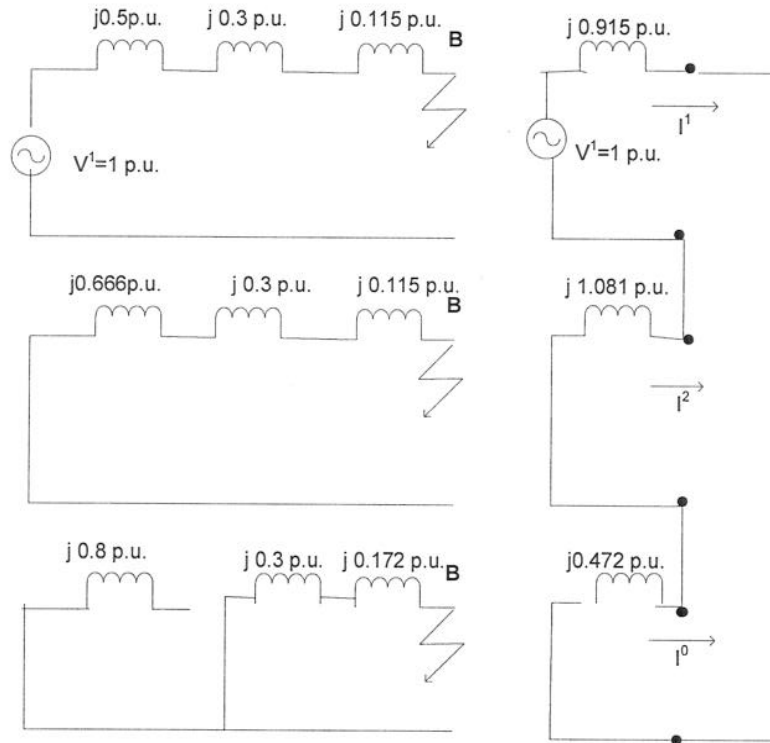
$$I_B = \frac{S_B}{\sqrt{3} \cdot V_B} = \frac{100 \cdot 10^6}{\sqrt{3} \cdot 132 \cdot 10^3} = 437 \text{ A}$$

$$|\overline{i_f}| = 3 \times 0.405 = 1.215 \text{ p.u.}$$

$$|\overline{I_f}| = 1.215 \cdot 437 = 531 \text{ A}$$

[4]

d)

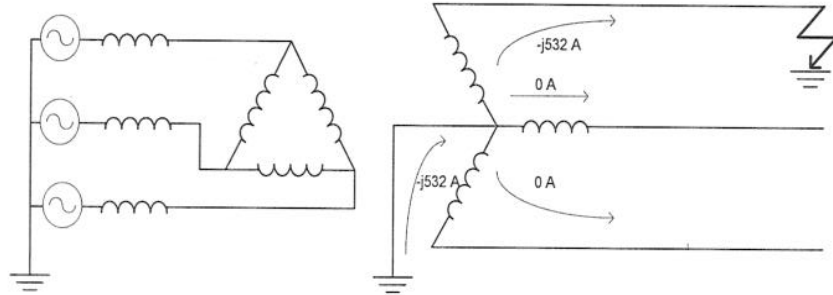


$$I_0 = I_1 = I_2 = \frac{V_1}{Z_{th}} = \frac{1}{j \cdot (0.915 + 1.081 + 0.472)} = -j0.405 \text{ p.u.}$$

$$I_{bl} = \frac{S_b}{\sqrt{3} \cdot V_{bl}} = \frac{100 \cdot 10^6}{\sqrt{3} \cdot 132 \cdot 10^3} = 437 \text{ A}$$

$$I_0 = I_1 = I_2 = -j0.405 \cdot 437 = -j177.2$$

$$\begin{bmatrix} I_{la} \\ I_{lb} \\ I_{lc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} -j177.2 \\ -j177.2 \\ -j177.2 \end{bmatrix} = \begin{bmatrix} -j532 \\ 0 \\ 0 \end{bmatrix}$$



[3]

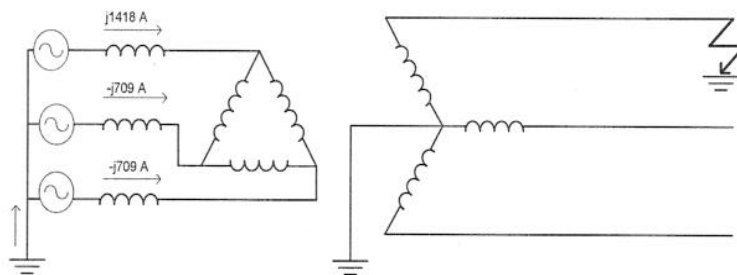
e) **Assumption: Transformer phase shift across transformer is ignored.**

$$I_{Bg} = \frac{S_B}{\sqrt{3}V_B} = \frac{100 \cdot 10^6}{\sqrt{3} \cdot 33 \cdot 10^3} = 1750 \text{ A}$$

$$I_0 = 0$$

$$I_1 = I_2 = -j0.405 \cdot 1750 = -j709 \text{ A}$$

$$\begin{bmatrix} I_{ga} \\ I_{gb} \\ I_{gc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -j709 \\ -j709 \end{bmatrix} = \begin{bmatrix} j1418 \\ -j709 \\ -j709 \end{bmatrix}$$



[3]

f)

$$FL = \sqrt{3} \cdot V_{no \text{ min al}} \cdot I_f = \sqrt{3} \cdot 132 \cdot 531 = 121.4 \text{ MVA}$$

[3]