EE4-55
EE9-AO11

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

MSc and EEE PART IV: MEng and ACGI

Corrected copy

MEMS AND NANOTECHNOLOGY

Monday, 21 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer Question 1.
Answer Question 2 OR Question 3.
Answer Question 4 OR Question 5.

Question 1 carries 40% of the marks. Remaining questions carry 30% each.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	Z. Durrani, A.S. Holmes, Z. Durrani
	Second Marker(s) :	A.S. Holmes, Z. Durrani, A.S. Holmes

Information for Candidates

The following physical constants may be used:

electron charge:	$e = 1.6 \times 10^{-19}$ C
electron mass:	$m_e = 9.1 \times 10^{-31}$ kg
Planck's constant:	$h = 6.63 \times 10^{-34}$ Js
Permittivity of free space:	$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

The piezoresistive equations for silicon, referred to axes aligned to the <100> directions, are:

$$E_1 / \rho_e = J_1[1 + \pi_{11}\sigma_1 + \pi_{12}(\sigma_2 + \sigma_3)] + J_2\pi_{44}\tau_{12} + J_3\pi_{44}\tau_{13}$$
$$E_2 / \rho_e = J_2[1 + \pi_{11}\sigma_2 + \pi_{12}(\sigma_1 + \sigma_3)] + J_1\pi_{44}\tau_{12} + J_3\pi_{44}\tau_{23}$$
$$E_3 / \rho_e = J_3[1 + \pi_{11}\sigma_3 + \pi_{12}(\sigma_1 + \sigma_2)] + J_1\pi_{44}\tau_{13} + J_2\pi_{44}\tau_{23}$$

The maximum stress σ_{max} in a square membrane of side L and thickness h subject to a uniformly distributed load p is:

$$\sigma_{\max} \approx 0.3 \frac{pL^2}{h^2}$$

This question is compulsory

1\. a) A photon with angular frequency ω_p is absorbed by a Si nanocrystal with band gap E_g, exciting an electron from the top of the valance band to the bottom of the conduction band. Write equations for conservation of energy and momentum for this process. Hence, explain briefly why bulk Si is only weakly optically active, but a Si nanocrystal can be strongly optically active. [5]

b) (i) By using the $|2s\rangle$, $|2p_x\rangle$, $|2p_y\rangle$ and $|2p_z\rangle$ states given below, construct three hybridised sp^2 orbitals for the σ bonds of a carbon atom in graphene.

$$|2s\rangle \sim \left(1 - \frac{Zr}{2a_0}\right) e^{\frac{-Zr}{2a_0}}$$

$$|2p_x\rangle \sim x e^{\frac{-Zr}{2a_0}}$$

$$|2p_y\rangle \sim y e^{\frac{-Zr}{2a_0}}$$

$$|2p_z\rangle \sim z e^{\frac{-Zr}{2a_0}}$$ [3]

(ii) Sketch the shapes of the σ and π orbitals along a hexagonal ring of 6 carbon atoms in graphene. [2]

c) Using suitable diagrams, explain the vapour-liquid-solid growth mechanism for a silicon nanowire. [4]

d) Figure 1.1 shows the lattice of graphene, with translational vectors a_1 and a_2. For three carbon nanotubes constructed from this lattice, with Chiral vectors C_h corresponding to the vectors X, Y and Z, write down the translational numbers (n, m). Hence, determine if the nanotubes are metallic or semiconducting. [6]

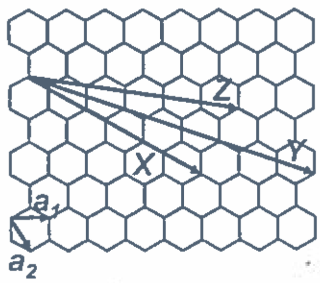

Figure 1.1

Question 1 continues on the next page.

Question 1 continued.

e) Figure 1.2 shows a micromechanical resonator fabricated on a BSOI wafer with a mechanical layer thickness of 15 μm. The device is driven by a pair of electrostatic actuators which are not drawn to scale. Each actuator contains 199 electrode fingers on a 10-μm pitch, the width of each electrode being 8 μm. If the resonator has a mechanical Q of 100, and a suspension stiffness of 10 N/m, estimate the peak-to-peak oscillation amplitude when it is driven at resonance, in differential mode, by a pair of anti-phase square waves derived directly from 5 V CMOS logic. [5]

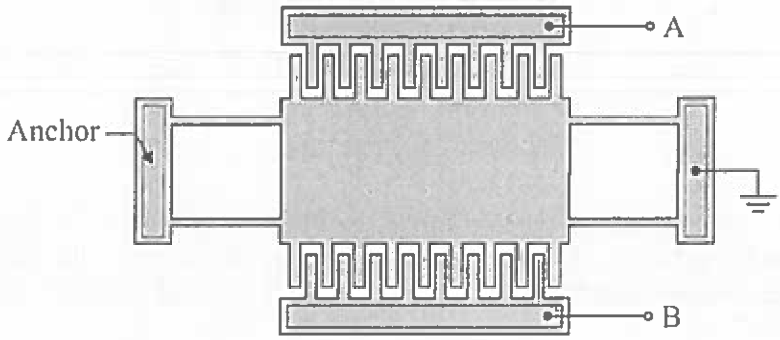

Figure 1.2

f) Sketch a typical shape bimorph electrothermal actuator, showing both the undeflected and deflected states. Assuming any temperature rise is confined to the hot arm, derive approximate expressions in terms of appropriate device parameters for (i) the unloaded deflection and (ii) the actuator force at zero deflection. [5]

g) The resolution of a projection lithography system is typically expressed in the form:

$$R = k_1 \frac{\lambda}{NA}$$

where λ is the optical wavelength, NA is the numerical aperture, and k_1 is a dimensionless parameter. By considering the imaging of a grating, show that a system with on-axis, plane wave illumination is expected to achieve $k_1 = 0.5$. Explain briefly why lower k_1 values can be reached using off-axis illumination. [6]

h) Write down the bending equation for a buckled beam with pinned supports at both ends. By solving the equation subject to appropriate boundary conditions, obtain an expression, in terms of the beam dimensions and Young's modulus, for the critical load at which buckling will occur. [4]

End of Question 1.

2. An n-channel Si MOSFET is shown in Figure 2.1. The device has gate length L, gate width W, and gate effective oxide thickness t_{ox}. The doping concentration in the source and drain regions is N_D, and the bulk doping concentration N_A. The drain and gate voltages are V_d and V_g respectively, and the source is grounded.

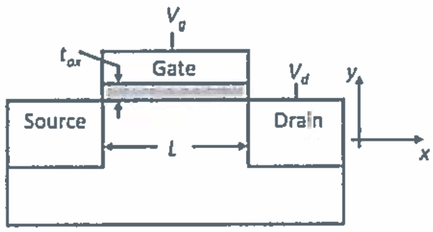

Figure 2.1

a) Under constant electric field scaling conditions, the MOSFET dimensions are scaled down by a factor of k. For the scaled-down device, by what factor do the doping concentrations, applied voltages, gate capacitance, and inversion layer charge density change? [14]

b) By what factors do the currents in the saturation and sub-threshold regions change? [8]

c) It is now found that while the device dimensions can be scaled down by k, device voltages can only be scaled down by a smaller factor, $\sigma < k$, requiring a generalised scaling approach. If the doping concentrations may be scaled up by a larger factor αk, where $\alpha > 1$, determine the factor σ. Also determine the effect on the device electric fields and inversion layer charge density. [8]

3. a) The wave function $\psi(r, \theta, \phi)$, in spherical coordinates (r, θ, ϕ), for an electron in an atomic potential well, is shown below:

$$\psi(r,\theta,\phi) = A\exp\left(-\frac{Zr}{a_0}\right)$$

Here, Z is the atomic number, and a_0 is the Bohr radius. Find the constant A, given that the volume element in spherical coordinates is given by:

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

[10]

b) Consider electrons in a potential well in Cartesian coordinates (x, y, z) where the potential energy $V(x,y,z) = V_x + V_y + V_z$ is given by:

$$\begin{array}{llll} V_x(x) = 0 & \text{for } 0 \le x \le L_x & , & V(x) = \infty \ \text{ elsewhere} \\ V_y(y) = 0 & \text{for } 0 \le y \le L_y & , & V(y) = \infty \ \text{ elsewhere} \\ V_z(z) = 0 & \text{for } 0 \le z \le L_z & , & V(z) = \infty \ \text{ elsewhere} \end{array}$$

i) Show that the time-independent Schrödinger Equation in three-dimensions, $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$, can be separated into three one-dimensional equations.

[8]

ii) Hence, solve these equations for $V(x,y,z)$ and derive expressions for the normalised electron wave functions ψ_{ngl}, and energy levels E_{ngl}, in the well. Here, n, g, and l are integers greater than zero.

[12]

4. Figure 4.1 shows a silicon micromachined accelerometer comprising a proof mass m supported by a pair of cantilevers. The device is fabricated on a BSOI wafer having a mechanical layer thickness of $h = 10\ \mu m$ and a buried oxide thickness of $g = 2\ \mu m$. The cantilevers each have width $w = 20\ \mu m$ and length $l = 300\ \mu m$, and the corresponding parameters for the proof mass are $W = L = 1$ mm. The mass is perforated by a 40×40 array of $5 \times 5\ \mu m^2$ apertures which are required for the fabrication process.

a) Suggest a possible fabrication sequence for the device. You should list the process steps involved but you do not need to describe them in detail. You may assume that a layer of gold will be evaporated at the end of the sequence to provide a suitable surface for attaching bond wires. Why are the perforations in the proof mass required? [6]

b) Write down an expression for the bending moment as a function of position in either cantilever when the accelerometer is subject to an upward acceleration a normal to the wafer. You should assume that the cantilevers are mass-less and neglect the gravitational force on the proof mass. [4]

By solving the bending equation for either cantilever, show that the deflection and slope of the cantilever at the point where it meets the mass are given by:

$$v_l = -\frac{l^2(4l+3L)}{2Ewh^3}ma \quad ; \quad v'_l = -\frac{3l(l+L)}{Ewh^3}ma$$

where E is Young's modulus. [10]

c) The accelerometer is to be read out by measuring the capacitance between the proof mass and the substrate. Estimate the baseline capacitance when $a = 0$. Also estimate the shifts in capacitance due to applied accelerations of $\pm 10\ ms^{-2}$. Comment on the linearity of the device. [10]

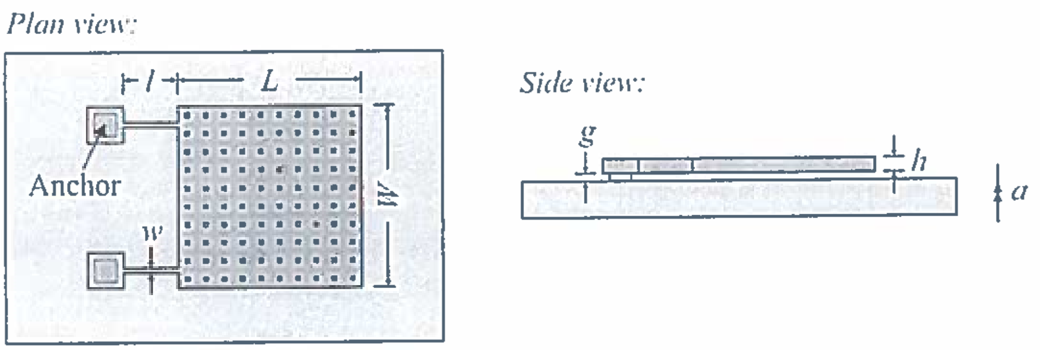

Figure 4.1

The capacitance of a wedged air capacitor is $C = [\varepsilon_0 W \ln(g_2 / g_1)]/\theta$ where W is the width in the direction of constant gap, g_1 and g_2 are the minimum and maximum gaps, and θ is the angle between the plates. You should assume values of $E = 160$ GPa and $\rho = 2330\ kg/m^3$ respectively for the Young's modulus and density of silicon.

5. a) Briefly describe the piezoelectric and piezoresistive effects and illustrate how they are used in MEMS devices. [6]

b) A region of silicon is subject to purely axial stresses σ_x and σ_y along the $[1\bar{1}0]$ and [110] directions respectively. Show that the associated stress components in a coordinate frame aligned to the $<100>$ directions are:

$$\sigma_1 = \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) \quad ; \quad \tau_{12} = \frac{1}{2}(\sigma_y - \sigma_x)$$ [8]

Also show that, when the region of interest lies at a membrane edge which is aligned along the x-direction, the stresses σ_x and σ_y will satisfy:

$$\sigma_x = \upsilon \sigma_y$$

where υ is Poisson's ratio. [4]

c) Figure 5.1 shows a membrane pressure sensor with piezoresistive readout. The membrane is formed in a (100)-oriented wafer by anisotropic etching, and readout is via a single <100>-aligned piezoresistor. A bias voltage V_{BA} applied between terminals A and B generates a longitudinal current which, in turn, gives rise to a transverse voltage V_{CD} between terminals C and D in the presence of membrane stress.

Using the piezoresistive equations for silicon, and neglecting the effect of the membrane stress on the longitudinal resistance of the piezoresistor, show that the sensitivity of the sensor (in Volts per Pascal) is expected to be:

$$S = \frac{V_{CD}}{p} \approx \frac{3\pi_{44}(1-\upsilon)}{20}\left(\frac{w}{l}\right)\left(\frac{L}{h}\right)^2 V_{BA}$$

where w and l are the piezoresistor dimensions, L is the membrane side length and h is the membrane thickness. [8]

What advantages might the sensor in Figure 5.1 have over one with a more traditional bridge-type piezoresistive readout? [4]

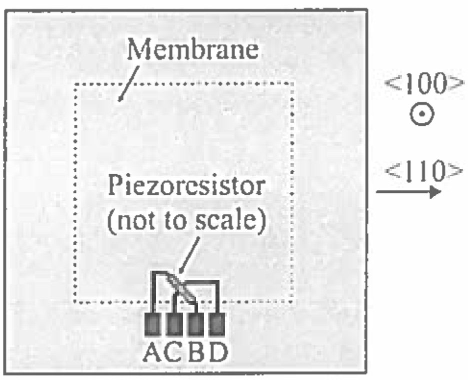

Figure 5.1