Paper Number(s): **E3.02**
AC4

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

MSc and EEE PART III/IV: M.Eng., B.Eng. and ACGI

INSTRUMENTATION

Tuesday, 23 April 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

Time allowed: 3:00 hours

Corrected Copy

Examiners responsible:

First Marker(s): Papavassiliou,C.

Second Marker(s): Lucyszyn,S

Special instructions for invigilators: None

Information for candidates: None

1. A Hall sensor is a rectangular sheet conductor with 4 electrodes. It is subjected to a normal magnetic flux B. The longitudinal (V_L) and transverse (V_T) voltages developed on the sensor current terminals when an RMS current I flows through the sensor are:

$$V_L = V_R^+ - V_R^- = \frac{L}{W}\frac{1}{ne\mu}I$$

and

$$V_T = V_h^+ - V_h^- = \frac{B}{ne}I$$

where n is the sheet carrier density, e the electron charge, μ the carrier mobility, and the other terms are defined in Figure 1.1.
We need to measure both the longitudinal voltage and the transverse voltage.

a) What sources of noise can you identify in the longitudinal measurement? Write an expression for the power spectral density of each one of them. Comment on the frequency dependence of each of the PSD functions. Write an expression for the total RMS noise voltage on V_L.

[6]

b) What sources of noise can you identify in the transverse measurement?

[8]

c) When a quantity X depends on a measurable y, the noise amplitude N_X depends on the noise amplitude N_y of the observable as:

$$N_X = \frac{\partial X}{\partial y}N_y$$

Write an expression for the noise in the determination of the resistance $R = \frac{L}{W}\frac{1}{ne\mu}$ in terms of the total RMS noise in V_L.

[6]

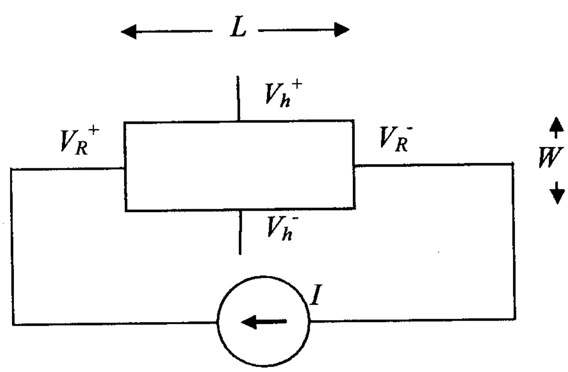

Figure 1.1: A Hall effect sensor with its current bias and terminal voltage definitions

2.

a) Write a definition for the noise factor, noise figure and noise measure of a device.

[6]

b) Prove that the noise figure of an attenuator equals numerically its attenuation.

[4]

c) In a low noise application we need to cascade two amplifiers, with gains and noise factors G_1, F_1 and G_2, F_2 respectively. Derive which one should be placed first for minimum overall noise figure.

[10]

3.

a) Describe the transfer oscillator and its operation, and draw a block diagram for it. State one use for the transfer oscillator.

[6]

b) Write the linearised loop equation for the transfer oscillator in lock and define the loop components (filter and phase detector) so that the loop is unconditionally stable with a specified phase margin. You may assume that the Voltage Controlled Oscillator (VCO) tuning range extends between f_0 and $3/2\,f_0$, with f_0 the VCO free running frequency, and the control range of the VCO is the supply voltage.

[8]

c) Draw a graph of the steady state VCO drive voltage versus input frequency, assuming there is no filter in the feedback path. Suggest a way the transfer oscillator can be used as a wideband signal detector.

[6]

4.

a) What is quantisation noise in an A/D converter? Define the signal to quantisation noise ratio (SQNR) of an A/D converter. Derive the value of SQNR for an N bit converter.

[6]

b) Draw a block diagram for a low pass sigma delta modulator comprising a 2 bit ADC and a 2 bit DAC. Define the oversampling ratio (OSR). Derive the SQNR of the converter in part (a) in terms of OSR in the limit $OSR \gg 1$ assuming a 2nd order loop filter. What is the effective number of bits of this modulator?

[14]

5.

a) Define the Y parameters of a 2-port network and draw an equivalent circuit whose components have for values the entries of the Y-matrix. Define the S-parameters and derive the S-parameters in terms of the Y parameters.

[6]

b) Derive the value of the source impedance Z_S which maximises the signal to noise ratio at the output of the circuit in Figure 5.1. You may assume Z_1, Z_2 and Z_S as well as the controlled current source are noiseless. You may also assume that the noise source V_{NS} is not correlated to either of the noise sources V_{ND} and I_{ND}. However, V_{ND} and I_{ND} are partially correlated so you may write

$$V_{ND}^2 = V_{ND0}^2 + Z_C^2 I_{ND}^2$$

with V_{ND0} and I_{ND} uncorrelated. V_S is the signal in this problem. Show that the minimum noise factor condition differs from the impedance matching condition. What is the minimum noise factor? How does the noise factor depend on Z_S?

[14]

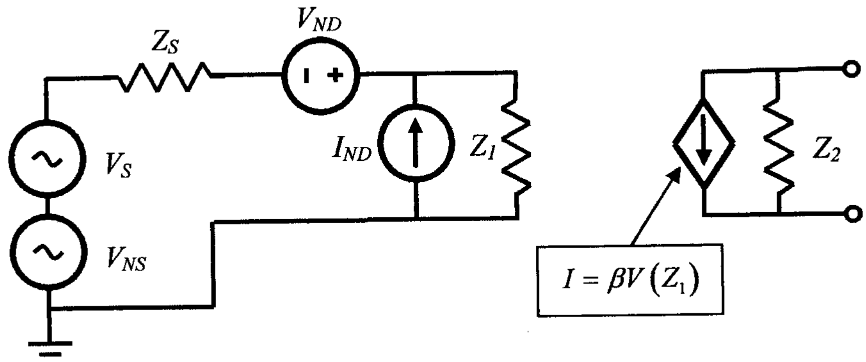

Figure 5.1: A noisy network

6.

a) Describe the Vector Network Analyser (VNA), and draw a block diagram for it. What does it measure? Why is the VNA measurement equivalent to a complete small signal model of the device under test? Why is it preferable to other ways of measuring gain/impedance?

[6]

b) You are given a perfect 1-port VNA with 50 ohm nominal port impedance. This analyser does not require calibration. You need, however, to use a non-ideal cable (one which exhibits both loss and dispersion and cannot be described by a transmission line equation). You are to use this VNA for 1-port impedance measurements. Write the equations describing a measurement of an arbitrary impedance terminating this cable, in terms of the cable's S-parameters. Use this equation to derive, from the measurements of a precision open circuit, a precision short circuit and a precise 50 Ohm resistor, a correction procedure for measurements through this cable. [Hint you do not need to evaluate all entries of the cable's S-matrix, since 2 of them appear as a product in all expressions!]

[14]

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Model Answers and Mark Schemes First Examiner: Christos Papavassiliou 1/6

Paper Code: EE 3.02 Second Examiner: Stepan Lucyszyn

INSTRUMENTATION

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Q1 A

Call: $R = \frac{L}{Wne\mu}$, $R_H = \frac{B}{ne}$

- Shot noise from current source: $\langle I_N^2 \rangle = 2qIB$, white , $\langle V_{N1}^2 \rangle = 2qIBR^2$
- Johnson noise from R: $\langle V_{N2}^2 \rangle = 4kTRB$, white
- Flicker noise due to carriers in the material: $\frac{d\langle V_{n3}^2 \rangle}{df} = \frac{k}{f}$

Voltage noise sources are independent. If measurement bandwidth is between f_1 and f_2, Total noise RMS voltage is:

$$V_N = \sqrt{\langle V_{n1}^2 \rangle + \langle V_{n2}^2 \rangle + \langle V_{n3}^2 \rangle} \text{ with } \langle V_{n3}^2 \rangle = \int_{f_1}^{f_2} \frac{k}{f} df = k \ln\left(\frac{f_2}{f_1}\right)$$ 6

B

- Shot noise from current source: $\langle I^2 \rangle = 2qIB$, white , $\langle V_{N1}^2 \rangle = 2qIBR_H^2$
- There is no Johnson noise from R_H because the voltage and current appear in different ports. There is, however Johnson noise from the transverse resistance $R' = \frac{W}{Lne\mu}$: $\langle V_{N2}^2 \rangle = 4kTR'B$, white

Flicker noise due to carriers in the material: $\frac{d\langle V_{n3}^2 \rangle}{df} = \frac{k'}{f}$ 8

C

To determine R we also need to measure the current, and clearly this is noisy, but with the noise partially correlated with the voltage noise. We note that:

$$R = \frac{V}{I} = \frac{\left((I+\delta I)R + \delta V_{N1} + \delta V_{N2}\right)}{I+\delta I} = R + \frac{\delta V_{N1} + \delta V_{N2}}{I+\delta I}$$

So all the noise sources in the second term are uncorrelated. Then,

$$\delta R = \sqrt{\left(\frac{\partial R}{\partial V}\right)^2 \delta V^2 + \left(\frac{\partial R}{\partial I}\right)^2 \delta I^2} = \sqrt{\frac{\langle V_{n1}^2 \rangle + \langle V_{n2}^2 \rangle}{I^2} + \frac{V^2}{I^4}\delta I^2} = \frac{1}{I}\sqrt{\langle V_{n1}^2 \rangle + \langle V_{n2}^2 \rangle + R^2 \delta I^2}$$ 6

Q2 A

- Noise factor: $F = \frac{S_i}{N_i} \Big/ \frac{S_o}{N_o}$ (S, N in power) 2
- Noise figure: $N = 10\log_{10} F$ 2
- Noise measure: is F for an infinitely long cascade of identical amplifiers:

$$M = F + \frac{F-1}{G} + \frac{F-1}{G^2} + \frac{F-1}{G^3} + \cdots = 1 + \sum_{k=01}^{\infty} (F-1)G^{-k} = 1 + \frac{F-1}{1 - 1/G}$$ 2.

B

The noise factor of an attenuator ($G = \frac{1}{A}$) is :

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$F = \frac{S_i}{N_i}\Big/\frac{S_o}{N_o} = \frac{S_i}{N_i}\Big/\frac{AS_i}{N_i} = \frac{1}{A}$ the numerical equality in dB follows. 4

C Assume $M_1 < M_2$. Then,

$$1+G_1\frac{F_1-1}{G_1-1} < 1+G_2\frac{F_2-1}{G_2-1} \Rightarrow G_1(F_1-1)(G_2-1) < G_2(F_2-1)(G_1-1)$$

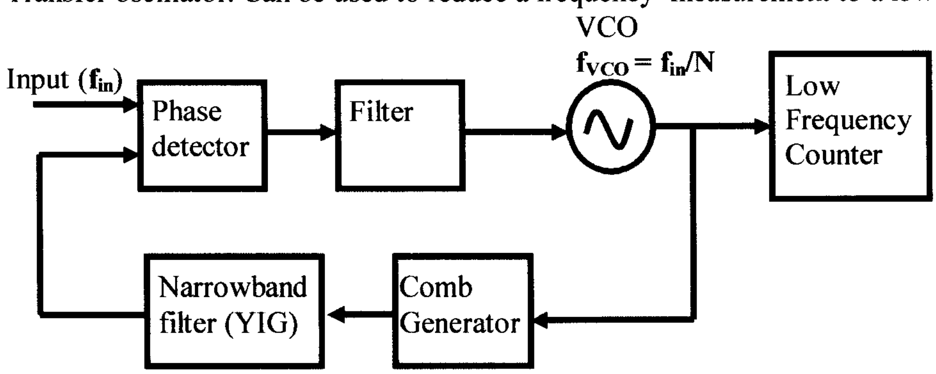
$$\Rightarrow G_1F_1G_2 + F_2G_2 - G_2 < G_2F_2G_1 + F_1G_1 - G_1$$

in the cascade of two amplifiers, assume the 1-2 series has a lower noise factor than the 1-2 series. Then

$$F_1 + \frac{F_2-1}{G_1} < F_2 + \frac{F_1-1}{G_2} \Rightarrow \cdots \Rightarrow F_1G_2G_1 + F_2G_2 - G_2 < F_2G_1G_2 + F_1G_1 - G_1$$

which is the same expression. 6

Q3 A Transfer oscillator. Can be used to reduce a frequency measurement to a lower range.



6

B

$$B(s) = \frac{\varphi_{out}}{\varphi_{in}} = \frac{K_dK_oF(s)}{s+K_dK_oF(s)N} = \frac{KF(s)/N}{s+KF(s)},\quad K = K_dK_oN$$

Can ignore the feedback loop filter, and use a lead-lag forward filter to ensure stability: $F(s) = \frac{1+\tau_2 s}{1+\tau_1 s}$. We can also use the minimum gain detector, type 3:

$K_d = \frac{V_s}{4\pi}$ At large frequency we need the loop gain

$$KF(s) << 1 \Rightarrow K_dK_oNF(s) \ll 1 \Rightarrow \frac{\tau_2}{\tau_1} \ll \frac{1}{K_dK_oN}.$$ 8

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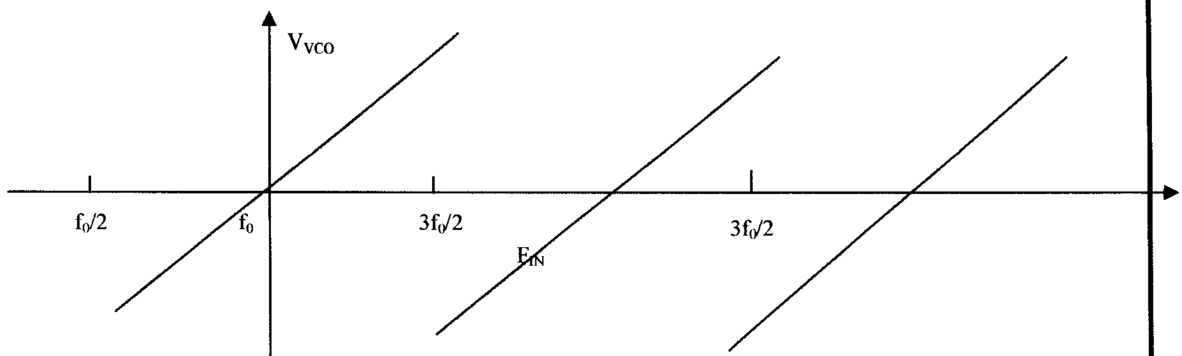
C



A small modulation of the V_{VCO} will be non zero if radiation is present at the input, for a range of multiplier values N. 6.

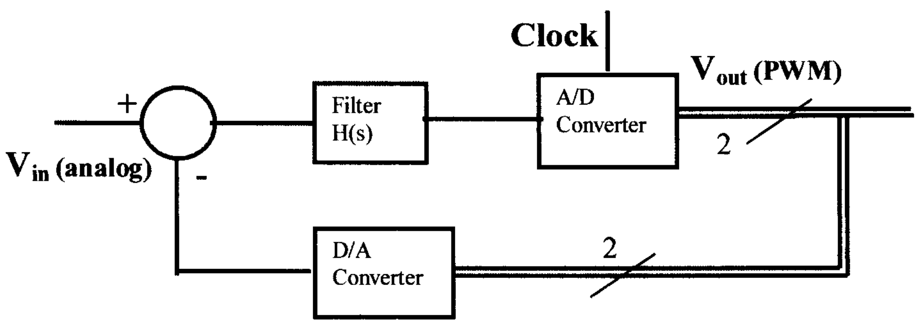
Q4
A

The discrepancy between the signal and its digital representation called quantisation noise. If $2^N q = 2A$, A the signal amplitude,

for N bit converter, $\overline{E}^2(e_q) = \int_{-\infty}^{\infty} e_q{}^2 p(e_q)\ de_q \ = \int_{-q/2}^{q/2} \frac{e_q{}^2}{q}\ de_q = \frac{q^2}{12}$ 2

$$\text{SNQR} = \frac{signal\ \ power}{quantisation\ \ noise\ \ power} = 10\log\left(6A^2/q^2\right) = 10\log\left(3\bullet 2^{2N-1}\right) = \left(1.76 + 6.02N\right)\text{dB}$$ 4

B



6

oversampling ratio $M = f_s / f_N = 2^k$

The SQNR is: 6

$$\frac{S}{E}(f_N) = \left(\frac{3\bullet 2^{2N-1} f_s}{f_N}\right)\frac{H^2(f_N)}{H^2(f_s)} \approx \left(\frac{3\bullet 2^3 f_s}{f_N}\right)\frac{f_s^4}{f_N^4} = 3\bullet 2^3 \bullet \left(2^k\right)^5 = 3\bullet 2^{3+5k}$$

the effective number of bits N is: 2

a. $2N-1 = 3+5k \Rightarrow N = 2+2.5k$

4/6

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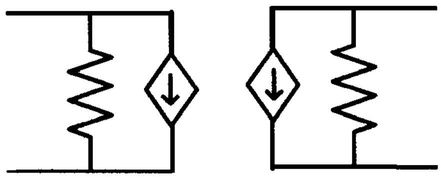
Q5 A

Independent variables: V1, V2
Dependent variables: I1, I2
State equations:

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \qquad \text{or} \qquad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Equivalent circuit: 

2.

We can define the Scattering matrix as the 2-port generalisation of the reflection coefficient:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

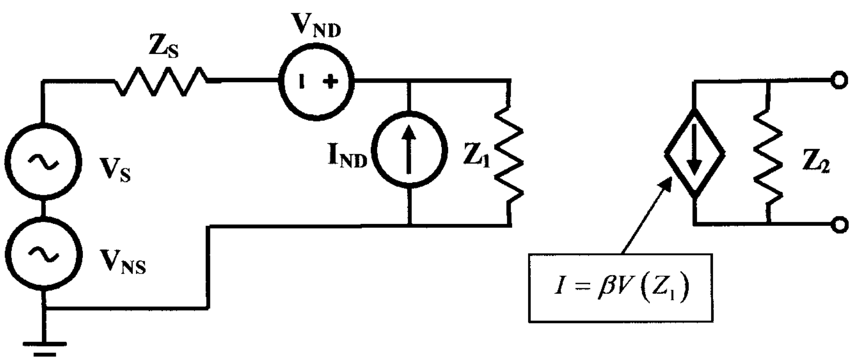
2

with $V = \sqrt{Z_0}(\alpha + \beta)$, $I = \frac{1}{\sqrt{Z_0}}(\alpha - \beta)$ On each of the 2 ports. Conversion S-Y:

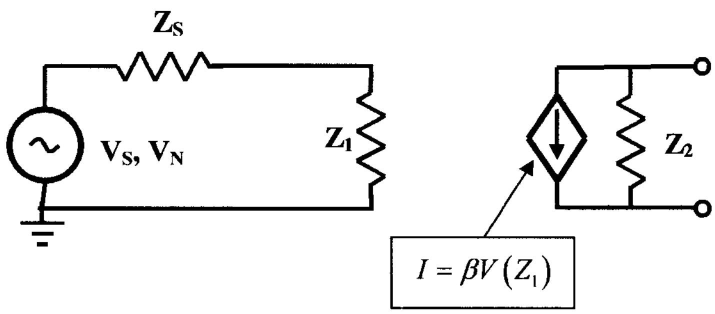
$$\ddot{S} = (1 - \ddot{Y})(1 + \ddot{Y})^{-1}$$

2.

B



from $V_{ND}^2 = V_{NDO}^2 + I_{ND}^2 R_C^2$
the equivalent magnitudes of the signal and noise transfer functions:



4

Reduced schematic

$$V_N = \sqrt{V_{NS}^2 + V_{NDO}^2 + |Z_C + Z_S + Z_1|^2 I_{ND}^2}$$

6

then , the SNR at the output is just the SNR on R_1:

$\text{SNR} = \frac{V_S^2}{V_N^2} = \frac{V_S^2}{V_{NS}^2 + V_{NDO}^2 + |Z_C + Z_S + Z_1|^2 I_{ND}^2}$. The noise factor is $F \propto (\text{SNR})^{-1}$.

the maximum SNR is when the denominator is minimum, i.e. when $Z_S = (Z_C + Z_1)^*$ which differs from the impedance match condition: $Z_S = Z_1^*$

The minimum noise factor is:

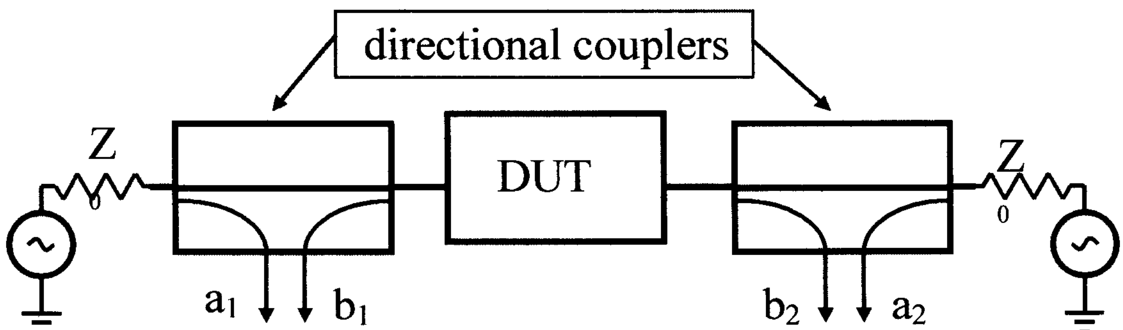
$$F_{MIN} = \frac{V_{NS}^2 + V_{NDO}^2 + (2\,\text{Re}(Z_1 + Z_C))^2 I_{ND}^2}{V_{NS}^2}$$

but $V_{NS}^2 = 4kT(2\,\text{Re}(Z_1 + Z_C))B$, and we can also write $\frac{V_{ND0}^2}{I_{ND}^2} = R_N^2$ then

$$F_{MIN} = 1 + \frac{\left(R_N^2 + (2\,\text{Re}(Z_1 + Z_C))^2\right) I_{ND}^2}{4kT(2\,\text{Re}(Z_1 + Z_C))}$$

4

Q6 A



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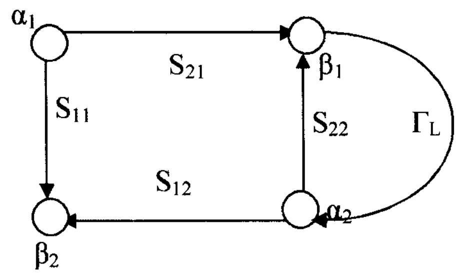
The VNA measures S-parameters, the transformation $\vec{S} = (z-1)(z+1)^{-1}$ shows equivalent between S-parameters and other representations. It is preferable because it does not require open and short terminations. 2.

B



The calibration graph (the 2-port is the cable) gives for the input reflection coefficient:

$\Gamma_{IN} = \dfrac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L}$ standards: $\Gamma_{short} = \dfrac{S_{11}+\Delta}{1+S_{22}}$, $\Gamma_{open} = \dfrac{S_{11}-\Delta}{1-S_{22}}$, $\Gamma_{load} = S_{11}$

These are 3 equations with 4 unknowns, but S_{12}, S_{21} appear only in

$\Delta = S_{11}S_{22} - S_{12}S_{21}$.

Then we can solve,

$$1+S_{22} = \frac{\Gamma_{load}+\Delta}{\Gamma_{short}} \quad , \quad 1-S_{22} = \frac{\Gamma_{load}-\Delta}{\Gamma_{open}}$$

$$\begin{bmatrix} -1 & \Gamma_{short} \\ 1 & -\Gamma_{open} \end{bmatrix}\begin{bmatrix} \Delta \\ S_{22} \end{bmatrix} = \begin{bmatrix} \Gamma_{load}-\Gamma_{short} \\ \Gamma_{load}+\Gamma_{open} \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta \\ S_{22} \end{bmatrix} = \begin{bmatrix} \Gamma_{load}-\Gamma_{short} \\ \Gamma_{load}+\Gamma_{open} \end{bmatrix}\begin{bmatrix} -1 & \Gamma_{short} \\ 1 & -\Gamma_{open} \end{bmatrix}^{-1} =$$

$$= \frac{-1}{\Gamma_{open}-\Gamma_{short}}\begin{bmatrix} \Gamma_{load}-\Gamma_{short} \\ \Gamma_{load}+\Gamma_{open} \end{bmatrix}\begin{bmatrix} \Gamma_{open} & \Gamma_{short} \\ 1 & 1 \end{bmatrix} =$$

$$= \frac{1}{\Gamma_{open}-\Gamma_{short}}\begin{bmatrix} \Gamma_{open}(\Gamma_{load}-\Gamma_{short}) + (\Gamma_{load}+\Gamma_{open}) \\ \Gamma_{short}(\Gamma_{load}+\Gamma_{open}) + (\Gamma_{load}-\Gamma_{short}) \end{bmatrix}$$ 14