MEng (Engineering) Examination 2016

Year 1

AE1-110 Introduction to Structural Analysis

Thursday 26th May 2016: 14.00 to 16.00
[2 hours]

The paper is divided into Section A and Section B
and contains ***FOUR*** questions.

In each section, the FIRST question has
HALF the weight of the SECOND question.

Candidates may obtain full marks for complete answers to ***ALL*** questions.

You must answer each section in a separate answer booklet.

A data sheet is provided

The use of lecture notes is NOT allowed.

Section A

Note that Question 1 is worth half the marks of Question 2.

1. (a) For each of the three pin-jointed frameworks shown in Figure 1a determine whether it is statically determinate, statically indeterminate, a mechanism, or a combination of these. If the framework is a mechanism sketch a possible deformed strain-free configuration, and state what assumptions must be made for this deformed configuration to occur. [50%]

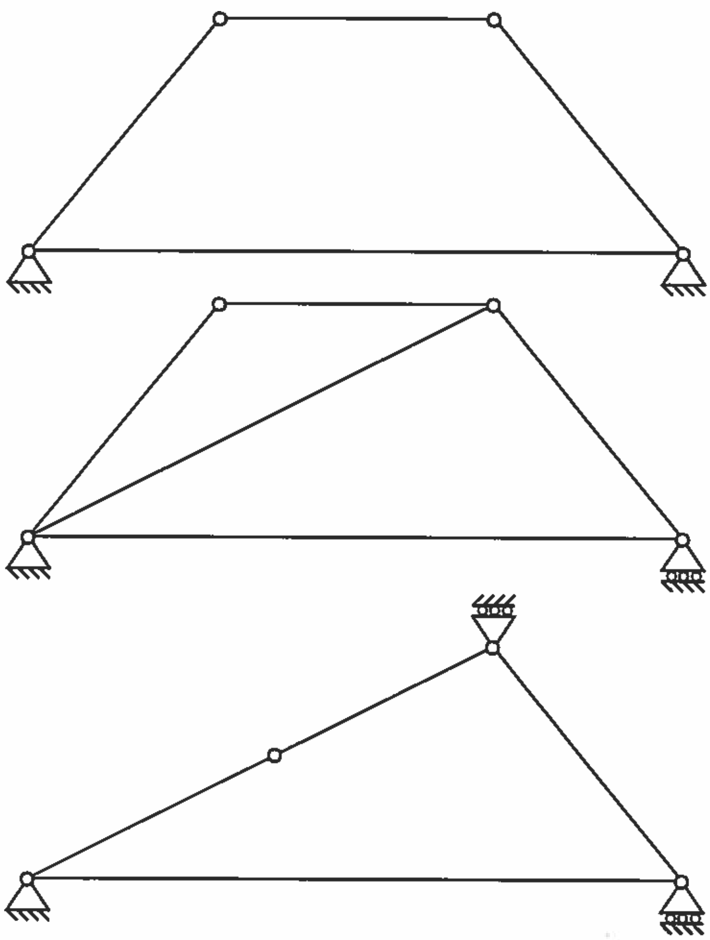


Figure 1a

(Question continued overleaf)

(b) In the pin-jointed framework shown in Figure 1b, all members have Young's modulus E, constant cross section area A, and coefficient of thermal expansion α. The horizontal and vertical members all have length L and the diagonal members are oriented at $45°$ to the horizontal. The nodes are numbered as shown. Bar 13 is subjected to a temperature change of ΔT which results in the displacement u of node 3 which by symmetry is oriented along the same direction as bar 13. Geometric relationships between the bar extensions and the displacement u are tabulated below.

BAR	Length $(\times L)$	Extension $(\times u)$
13	$\sqrt{2}$	1
23	1	$1/\sqrt{2}$
34	1	$1/\sqrt{2}$

i. Write expressions for the strains in the bars in terms of the displacement u. [10%]

ii. Using Hooke's Law in one dimension write expressions for the forces in the bars in terms of u and ΔT. [15%]

iii. Using expressions of nodal equilibrium and your previous answers determine the displacement u resulting from ΔT. [25%]

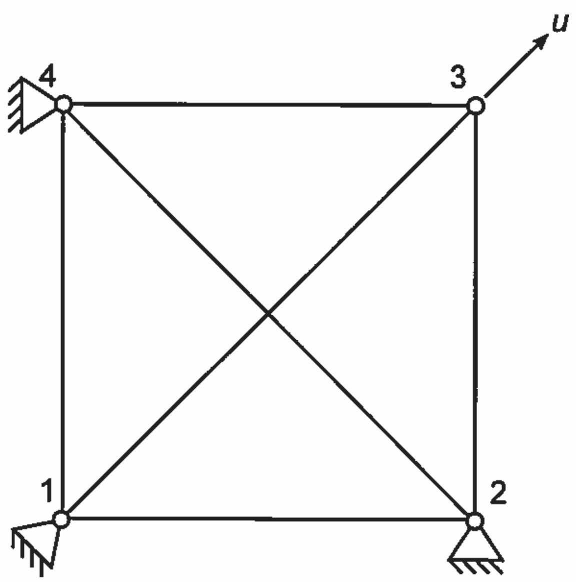


Figure 1b

2. Figure 2 shows a pin-jointed framework with members having Young's modulus E, cross-sectional area A, and coefficient of thermal expansion α. All the horizontal members have initial length L. All diagonal members are oriented at 45° to the horizontal. Boundary conditions are applied to nodes 1 and 3 as shown.

   (a) Confirm that the structure is statically determinate. [5%]

   (b) Using the virtual work method determine the vertical deflection of node 2 resulting from a vertically-downward applied force P applied at node 5. [55%]

   (c) The force P is removed and a temperature increase ΔT is applied to bar 25. Determine the resulting vertical deflection of node 2. [20%]

   (d) The temperature change ΔT is removed and actuators cause the lengths of bars 15 and 35 to contract by an amount δ. Determine the resulting vertical deflection of node 2. [20%]

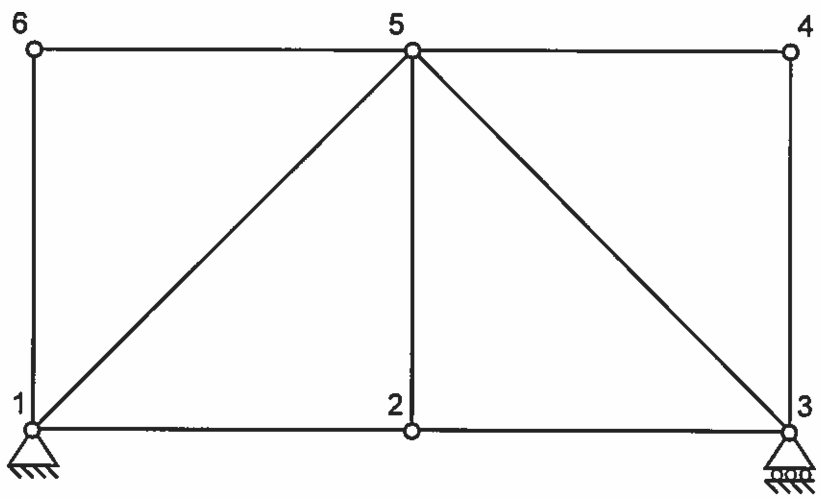


Figure 2

Section B

Note that Question 3 is worth half the marks of question 4.

3.

(a) Figure 3a shows a simply supported beam subjected to two point loads.

i. Determine the support reactions. [15%]

ii. Determine the bending moments at $z = L/4$ and $z = L/2$. [20%]

iii. Without any further calculation sketch the bending moment diagram for the beam. Mark clearly on the diagram the direction of curvature. [15%]

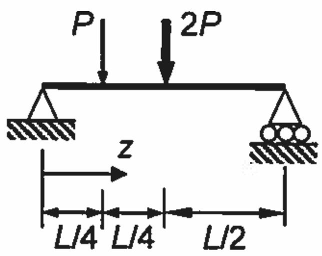


Figure 3a

[Question continued overleaf]

(b) Figure 3b shows a cantilevered bar subjected to a torque of 30 Nm at z = 1 m and includes the resulting twist at the free end (z = 4 m).

Figure 3c shows the same cantilevered bar subjected to a torque of 1 Nm at the free end and again gives the resulting twist.

The same bar is now supported and loaded as shown in Figure 3d. For this case

i. determine the reaction torque at z = 4 m; [20%]

ii. determine and sketch the distribution of the internal torque in the bar. [30%]

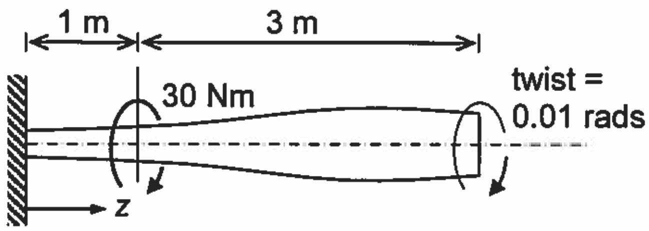


Figure 3b

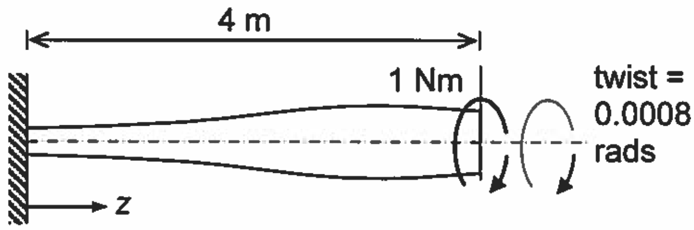


Figure 3c

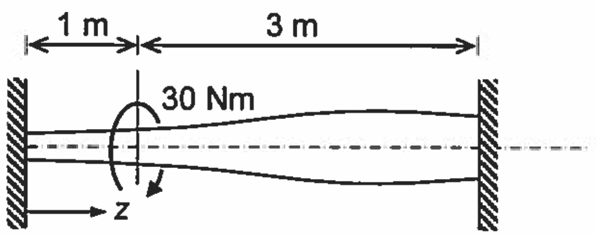


Figure 3d

4.

(a) Figure 4a shows a simply supported beam which projects beyond the right-hand support. The beam is subjected to a uniformly distributed load over $0 < z < L$ and a point load at $z = 2L$.
Derive equations for the shear force and bending moment distributions for the beam. Sketch the corresponding shear force and bending moment diagrams. On the bending moment diagram, indicate the values of any maxima or minima and clearly mark the direction of curvature. [32%]

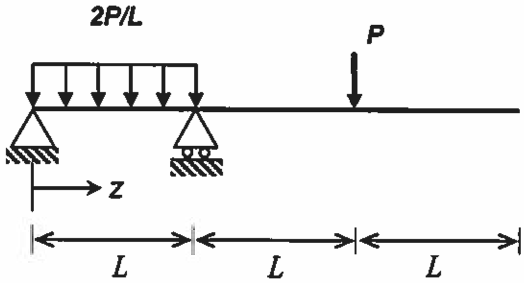


Figure 4a

(b) Figure 4b shows a simply supported beam subjected to a downward linearly varying distributed load and includes the equation for the bending moment.

Derive an equation for the vertical displacement of the beam as a function of z. [20%]

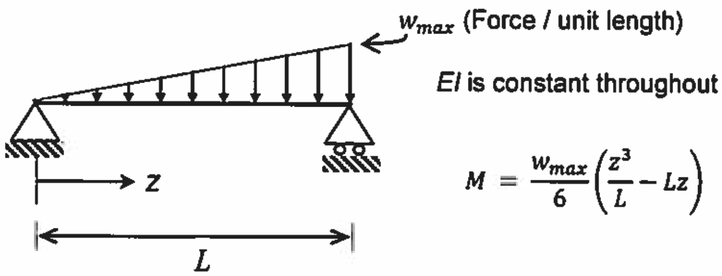


Figure 4b

[Question continued overleaf]

(c) Figure 4c shows a simply supported beam which has a flexural stiffness of $2EI$ over $0 \leq z < L$ and EI over $L \leq z < 2L$. The beam is subjected to a concentrated moment M^* at $z = 2L$ and the resulting vertical deflection at this position is given in the figure.

Figure 4d shows the same beam subjected to a unit vertical force at $z = 2L$ and gives the bending moment distribution.

The same beam is now additionally supported at $z = 2L$ and subjected to a concentrated moment M^* at $z = 2L$ as shown in Figure 4e. Determine the reaction force at $z = 2L$ and the bending moment at $z = 3L/2$. [48%]

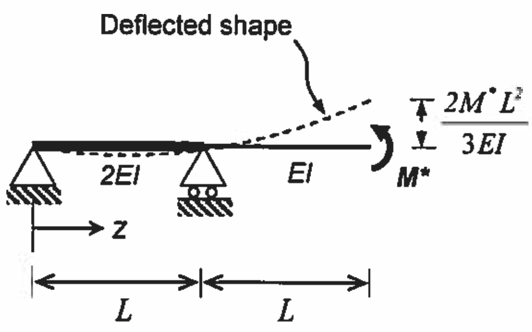


Figure 4c

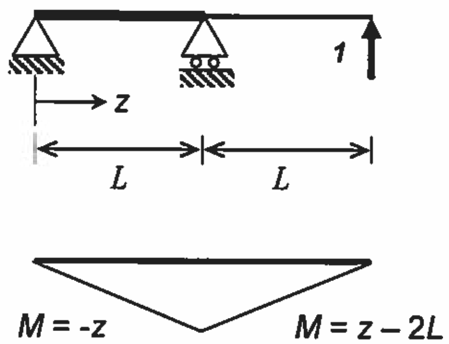


Figure 4d

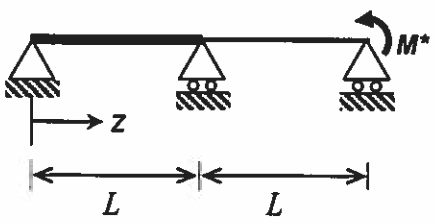


Figure 4e

Introduction to Structural Analysis Data Sheet

Constitutive Stress/strain Law (Hooke's Law):

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \upsilon\sigma_{yy} - \upsilon\sigma_{zz}\right) \text{ etc.}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \text{ etc.}$$

Compatibility:

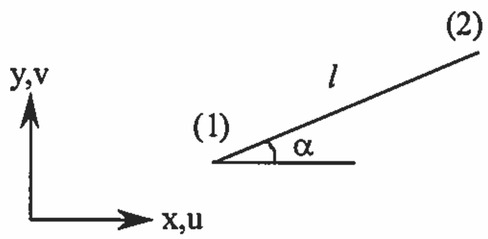
$$\varepsilon_{xx} = \frac{du}{dx} \text{ etc.}$$

Elastic constants:

$$\text{Shear modulus, } G = \frac{E}{2(1+\upsilon)}$$

$$\text{Bulk modulus, } K = \frac{E}{3(1-2\upsilon)}$$

Stretch of a pin-jointed bar in terms of end displacements:



$$\Delta l = (u_2 - u_1)\cos\alpha + (v_2 - v_1)\sin\alpha$$

Virtual Work (unit load) theorem for pin-jointed frameworks:

$$\overline{1}.\delta = \sum \overline{T}_{ij} . e_{ij}$$

Virtual Work (unit load) theorem for beams:

$$\overline{1}.\delta = \int \frac{\overline{M}\,M}{EI}\,dz$$

Stress–moment–curvature relationships for beams:

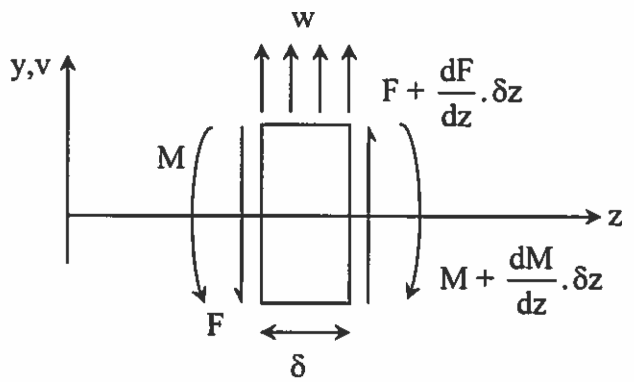
$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} \quad \text{where} \frac{1}{R} = -\frac{d^2v}{dz^2}$$

Stress-torque-twist relationship for circular section tubes:

$$\frac{\tau}{r} = \frac{T}{J} = G.\frac{d\theta}{dz}$$

Load–shear–moment relationship:

$$-w = \frac{dF}{dz}; \; F = \frac{dM}{dz}$$



The torsion constant for a thick circular tube of outer and inner radii R_0 and R_1 is

$$J = \frac{\pi}{2}\left(R_0{}^4 - R_1{}^4\right)$$

and for a thin-walled tube of mid-line radius R and wall thickness t, $J = 2\pi R^3 t$.

Unit load method for singly redundant beam:

$$X.\delta_1 + \delta_0 = 0$$

$$\text{where } \delta_0 = \int \frac{M_0 . \overline{M}}{EI} dz; \; \delta_1 = \int \frac{\overline{M}^2 \, dz}{EI}$$

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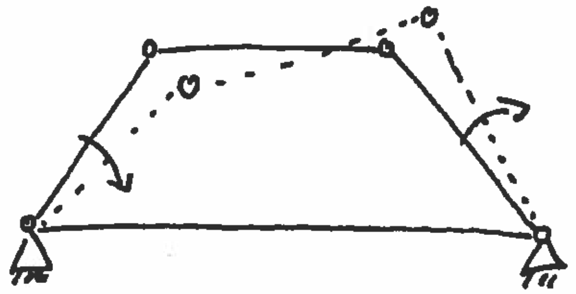
Marks

Q1. a

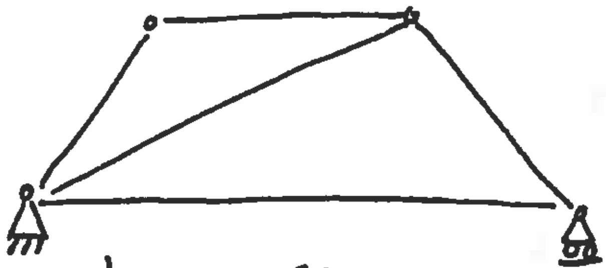


sketch mech. 10

$b + r - 2j$

$4 + 4 - 8 = 0$ 10

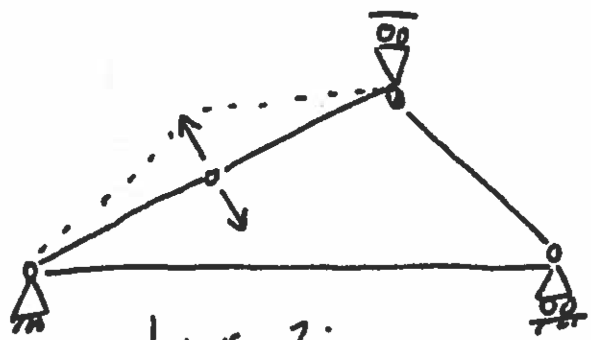
by inspection, one mechanism & one redundancy 20



$b + r - 2j$

$5 + 3 - 8 = 0$ 10

and by inspection statically determinate 10



sketch mech. 10

$b + r - 2j$

$4 + 4 - 8 = 0$ 10

by inspection an infinitessimal mechanism 20

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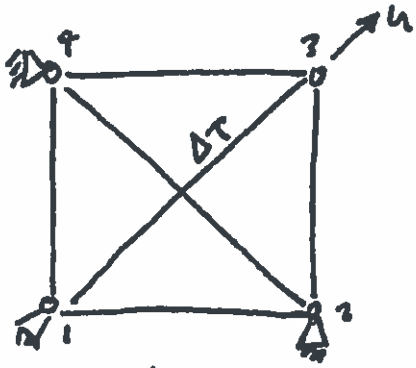
Marks

Q1. b



Bar	Length ($\times L$)	Extension ($\times u$)
13	$\sqrt{2}$	1
23	1	$1/\sqrt{2}$
34	1	$1/\sqrt{2}$

All other bars have zero extension

i)
$$\varepsilon_{13} = u/\sqrt{2}L = \frac{T_{13}}{AE} + \alpha\Delta T$$
$$\varepsilon_{23} = u/\sqrt{2}L = T_{23}/AE$$
$$\varepsilon_{34} = u/\sqrt{2}L = T_{34}/AE$$

20

ii)
$$T_{13} = \frac{uAE}{\sqrt{2}L} - AE\alpha\Delta T$$
$$T_{23} = \frac{uAE}{\sqrt{2}L}$$
$$T_{34} = \frac{uAE}{\sqrt{2}L}$$

30

change solutions $\frac{\sqrt{2}}{\sqrt{2}+1}$

iii) T_{34}, T_{13}, T_{23}

$$\frac{T_{13}}{\sqrt{2}} + T_{23} = 0$$

20

0.586

$$\frac{uAE}{2L} - \frac{AE}{\sqrt{2}}\alpha\Delta T + \frac{uAE}{\sqrt{2}L} = 0 \qquad u = \left(\frac{2\cancel{\sqrt{2}}}{2+\sqrt{2}}\right)L\alpha\Delta T$$

30

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(i)

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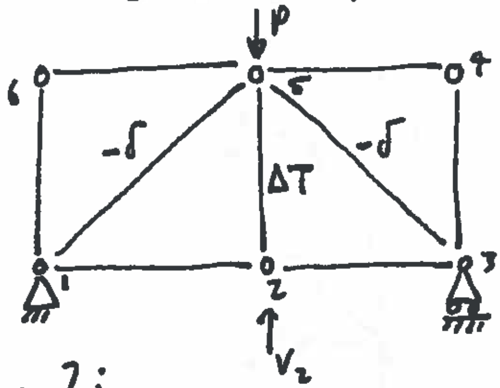
2)



a) $b + r - 2j$

$9 + 3 - 12 = 0$ and by inspection stat. det. — **5**

BAR	Length ($\times L$)	T ($\times P$)	e ($\times \frac{PL}{AE}$)	$T^*_{V_2}$	$e_{\Delta T}$	e_δ	
12	1	$1/2$	$1/2$	$-1/2$	0	0	
15	$\sqrt{2}$	$-1/\sqrt{2}$	-1	$1/\sqrt{2}$	0	$-\delta$	
16	1	0	0	0	0	0	
23	1	$1/2$	$1/2$	$-1/2$	0	0	
25	1	0	0	-1	$L\alpha\Delta T$	0	
34	1	0	0	0	0	0	
35	$\sqrt{2}$	$-1/\sqrt{2}$	$-1\cancel{/2}$	$1/\sqrt{2}$	0	$-\delta$	
45	1	0	0	0	0	0	
56	1	0	0	0	0	0	
$\underline{P}.\underline{\delta} = \underline{T}.\underline{e}$		(20)	(5)	(15)	(10)	(10)	→

b) $1.V_2 = \left(-\frac{1}{4} - \frac{1}{\sqrt{2}} - \frac{1}{4} - \frac{1}{\sqrt{2}}\right)\frac{PL}{AE} = \left(-\frac{1}{2} - \frac{2}{\sqrt{2}}\right)\frac{PL}{AE} = -\left(\sqrt{2} + \frac{1}{2}\right)\frac{PL}{AE}$ — **15**

~~approx~~ 1.914.

c) $1.V_2 = -L\alpha\Delta T$ — **10**

d) $1.V_2 = -\frac{\delta}{\sqrt{2}} - \frac{\delta}{\sqrt{2}} = -\sqrt{2}\,\delta$ — **10**

cont.

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(ii)

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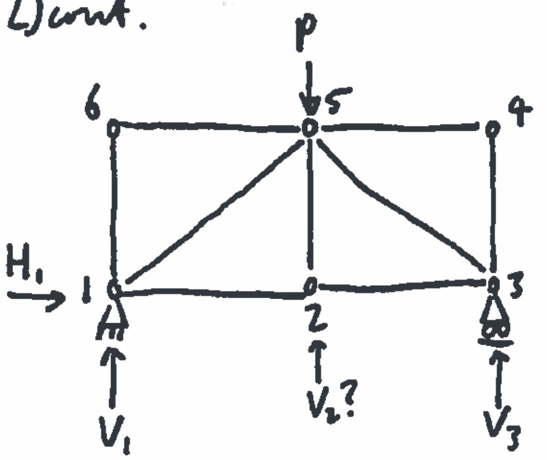
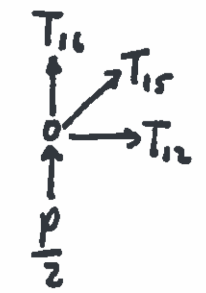
2) cont.



$$H_1 = 0$$
$$V_1 = V_3 = \frac{P}{2}$$

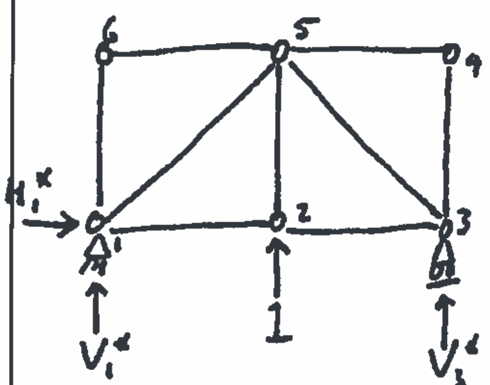


$$T_{16} = T_{34} = T_{45} = T_{56} = 0$$
$$T_{25} = 0$$

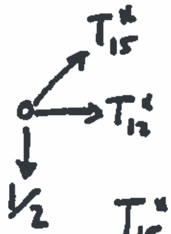
$$\frac{P}{2} + \frac{T_{15}}{\sqrt{2}} = 0 \qquad T_{15} = -\frac{P}{\sqrt{2}} = T_{35}$$

$$\frac{T_{15}}{\sqrt{2}} + T_{12} = 0 \qquad T_{12} = P/2 = T_{23}$$

Fictitious Loading



$$H_1^* = 0$$
$$V_1^* = V_3^* = -\frac{1}{2}$$



$$\frac{T_{15}^*}{\sqrt{2}} = \frac{1}{2}$$
$$T_{15}^* = \frac{1}{\sqrt{2}} = T_{35}^*$$

$$T_{16}^* = T_{34}^* = T_{45}^* = T_{56}^* = 0$$
$$T_{25}^* = -1$$

$$\frac{T_{15}^*}{\sqrt{2}} + T_{12}^* = 0$$
$$T_{12}^* = -\frac{1}{2} = T_{23}^*$$

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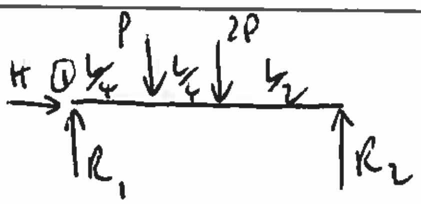
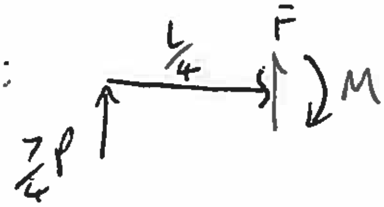
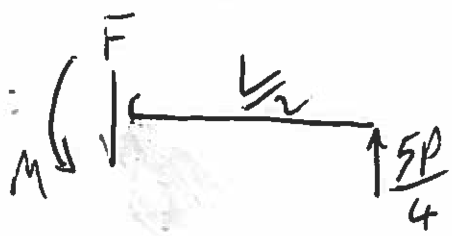
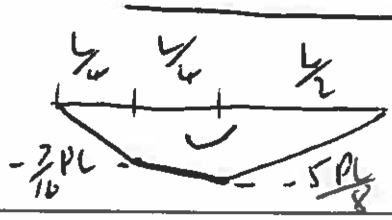
	Marks
3a  i) Horiz eq $\Rightarrow$ $\underline{H=0}$. Moments eq ①↺ $\Rightarrow$ $\frac{PL}{4} + PL - R_2 L = 0$ $\therefore$ $\underline{R_2 = \frac{5P}{4}}$. Vert eq $\Rightarrow R_1 + R_2 - 3P = 0$ $\therefore \underline{R_1 = \frac{7}{4}P}$	3
ii) at $z = \frac{L}{4}$ fbd :  Moments eq about cut ↺ $\Rightarrow \frac{7}{4}P \cdot \frac{L}{4} + M = 0$ $\therefore \underline{M = -\frac{7}{16}PL}$	2
at $z = \frac{L}{2}$ fbd :  Moments equilibrium about cut ↺ $\Rightarrow -\frac{5P}{4} \cdot \frac{L}{2} - M = 0$ $\therefore \underline{M = -\frac{5PL}{8}}$	2
iii) BM diag 	3

$\Sigma 10$

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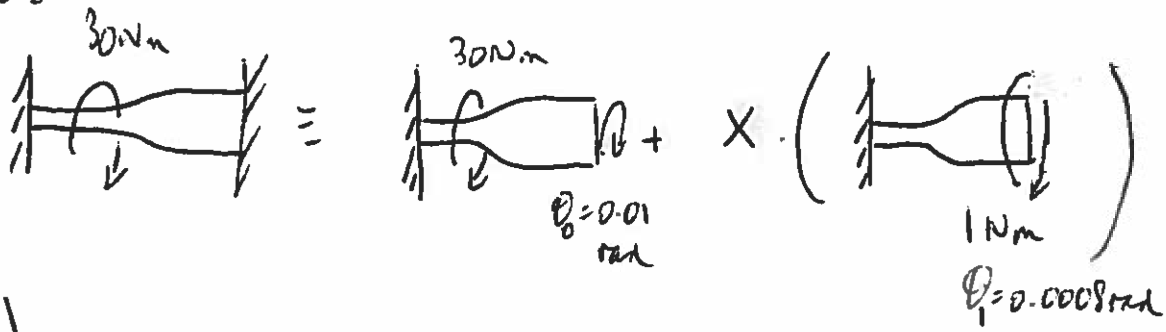
3b



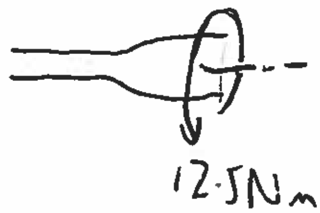
(i)

Require $\theta_0 + x.\theta_1 = 0$

$\therefore \; 0.01 + x \,.\, 0.0008 = 0$

$\therefore \; x = \dfrac{-0.01}{0.0008} = -12.5$

$\therefore$ reaction torque is -12.5 Nm



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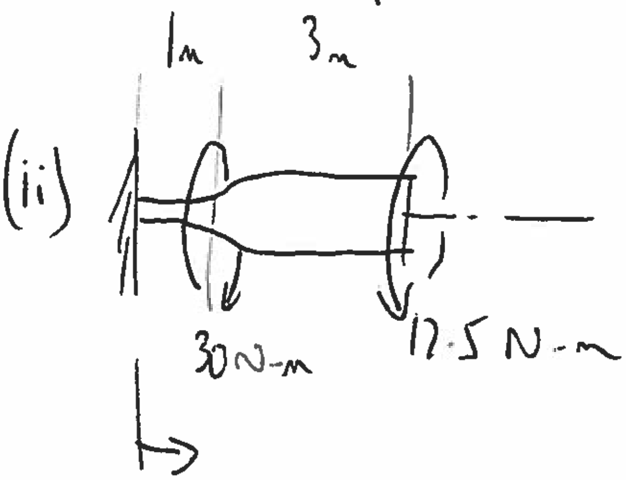
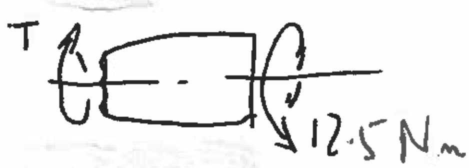
(ii)



$1 < 3 < 4$ fbd:



Eqm: $T = 12.5$ Nm

2

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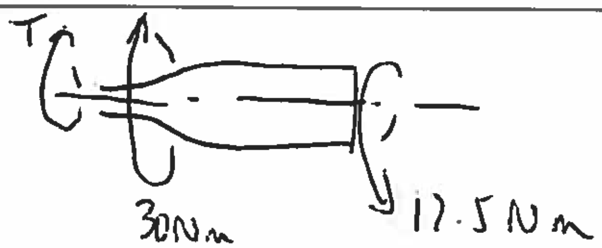
Marks

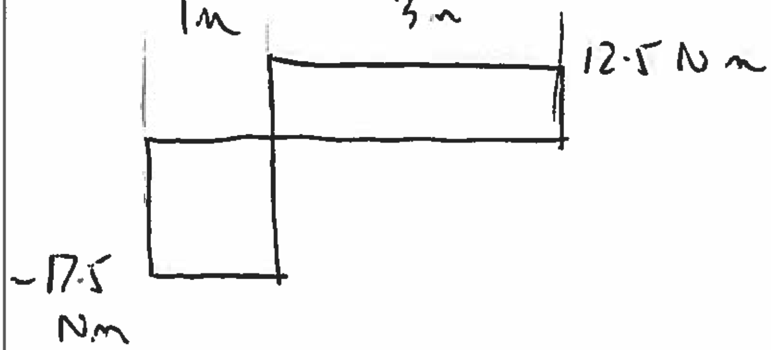
$0 < z < 1$ fbd:



$\Sigma_y \Rightarrow T + 30 - 12.5 = 0$

$\therefore T = -17.5$ Nm 2

$\therefore$ Internal torque distrib:



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$\Sigma 10$

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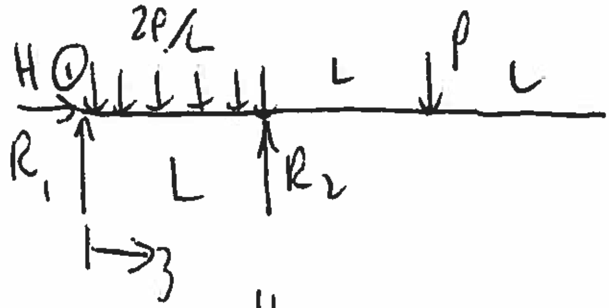
4a



Horiz eq $\Rightarrow$ $H = 0$

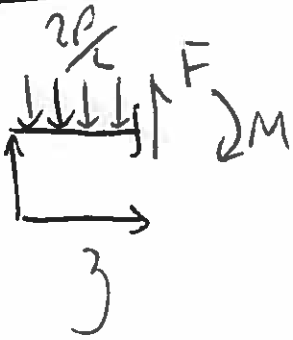
Moments eq 1 $\Rightarrow$ $\frac{2P}{L} L \cdot \frac{L}{2} + P \cdot 2L - R_2 L = 0$

$\therefore$ $R_2 = 3P$ 4

Vert eq $\Rightarrow$ $R_1 + 3P - \frac{2P}{L} \cdot L - P = 0$

$\therefore$ $R_1 = 0$ 4

$0 < z < L$ fbd



Vert eq $\Rightarrow$ $\frac{2P}{L} \cdot z - F = 0$ $\therefore$ $F = \frac{2P}{L} z$ 3

Moments eq about cut: $M - \frac{2P}{L} \cdot \frac{z^2}{2} = 0$

$\therefore$ $M = \frac{2P}{L} \frac{z^2}{2} = \frac{Pz^2}{L}$ 4

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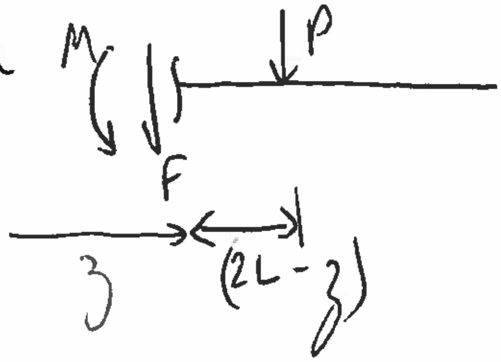
(9)

	Marks
$L < z < 2L$ fbd	

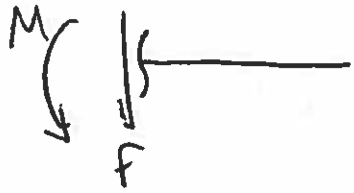


Vert eq $\Rightarrow$ $F + P = 0$ $\therefore$ $\underline{F = -P}$ — 3 3

Moments eq about cut $\Rightarrow$ $P(2L - z) - M = 0$ — 4 4

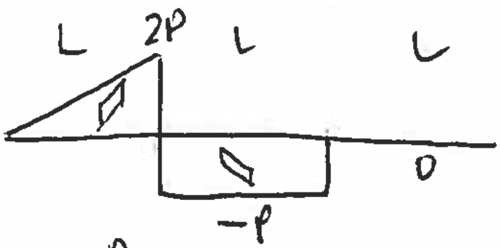
$\therefore$ $\underline{M = P(2L - z)}$

$2L < z < 3L$



Vert & Moments eq $\Rightarrow$ $\underline{F = 0}$, $\underline{M = 0}$ — 2 2

F diag — 4 3



M diag — 4 3

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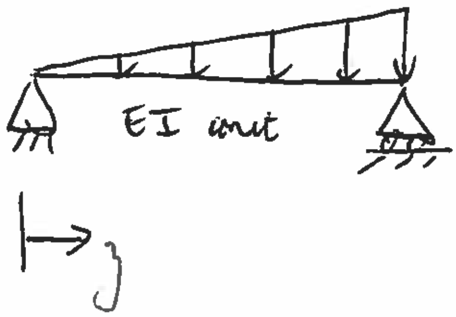
4b



$$M = \frac{w_{max}}{6}\left(\frac{z^3}{L} - Lz\right)$$

(i)

$$\frac{d^2v}{dz^2} = -\frac{M}{EI} = \frac{-w_{max}}{6EI}\left(\frac{z^3}{L} - Lz\right) \qquad 3 \quad 3$$

$$\therefore \frac{dv}{dz} = \frac{-w_{max}}{6EI}\left(\frac{z^4}{4L} - \frac{Lz^2}{2}\right) + C_1 \qquad 4 \quad 4$$

$$\therefore v = \frac{-w_{max}}{6EI}\left(\frac{z^5}{20L} - \frac{Lz^3}{6}\right) + C_1 z + C_2 \qquad 4 \quad 4$$

at $z = 0 \quad v = 0 \quad \therefore 0 = 0 + 0 + C_2$ $\qquad 3 \quad 3$

$\therefore C_2 = 0$

at $z = L, \ v = 0$

$$0 = \frac{-w_{max}}{6EI}\left(\frac{L^4}{20} - \frac{L^4}{6}\right) + C_1 L \qquad 4 \quad 4$$

$$\therefore C_1 = -\frac{w_{max}}{6EI} \cdot \frac{7L^3}{60}$$

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	Marks
$\therefore\ v = \dfrac{-w_{max}}{6EI}\left(\dfrac{z^5}{20L} - \dfrac{Lz^3}{6} + \dfrac{7L^3}{60}z\right)$	2
	20
(ii) for v_{max} find where $\dfrac{dv}{dz} = 0$	
$\dfrac{dv}{dz} = -\dfrac{w_{max}}{6EI}\left(\dfrac{z^4}{4L} - \dfrac{Lz^2}{2} + \dfrac{7L^3}{60}\right) = 0$	
NOT	
$\therefore\ \dfrac{z^4}{4L} - \dfrac{Lz^2}{2} + \dfrac{7L^3}{60} = 0$	
$z^4 - 2L^2z^2 + \dfrac{28L^4}{60} = 0$	
$\therefore\ z^2 = \dfrac{2L^2 \pm \sqrt{4L^4 - \dfrac{4\times 28}{60}L^4}}{2}$	
$= \dfrac{2L^2 \pm 1.46L^2}{2}$	
$= 1.73L^2$ and $0.27L^2$	
$\therefore\ z = 0.52L$	

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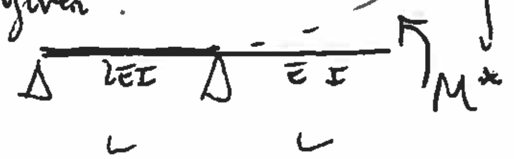
∴ max deflection is

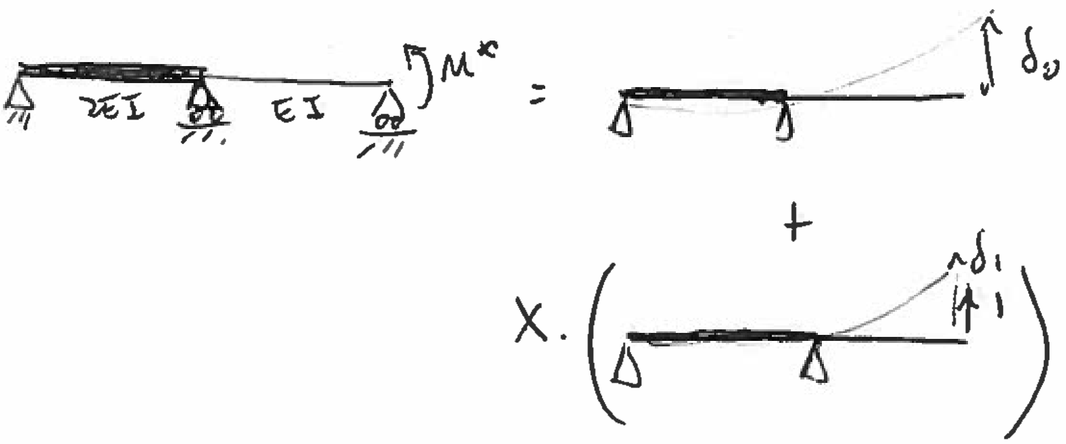
$$V_{max} = \frac{-W_{max}}{6EI}\left(\frac{0.52^5 L^4}{20} - \frac{0.5L^3 L^4}{6} + \frac{7\times 0.52L^4}{60}\right)$$

$$= -0.0065\,\frac{W_{max}L^4}{EI}$$

4c Given:



$$\delta_0 = \frac{2M^*L^2}{3EI}$$



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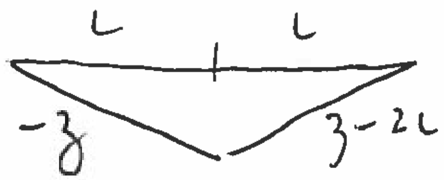
Require $\delta_0 + X\delta_1 = 0$

To determine δ_1

given M diag M_1:



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$\delta_1 = \int_{\text{beam length}} \frac{M_1^2}{EI}\,dz$	3
$= \int_0^L \frac{z^2}{2EI}\,dz + \int_L^{2L} \frac{(z-2L)^2}{EI}\,dz$	4, 4
$= \left[\frac{z^3}{6EI}\right]_0^L + \left[\frac{(z-2L)^3}{3EI}\right]_L^{2L}$	
$= \frac{L^3}{6EI} + \frac{L^3}{3EI}$	4, 4
$= \frac{3L^3}{6EI} = \underline{\frac{L^3}{2EI}}$	4
Substituting δ_0 & δ_1 into $\delta_0 + X\delta_1 = 0 \Rightarrow$	
$\frac{2ML^2}{3EI} + X\frac{L^3}{2EI} = 0$	4

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$$X = -\frac{4}{3}\frac{M^*}{L}$$ 4

This is the reaction at $z = 2L$

BM at $z = \frac{3L}{2}$ in Fig 4c :

due to X

Moments Equilib about cut $\Rightarrow$ $M = \frac{2}{3}M^*$ 4

due to M^* fbd :

Moments equilib. about cut $\Rightarrow$ $M = -M^*$ 4

$\therefore$ M at $z = \frac{3L}{2}$ in 4c

$$= -M^* + \frac{2}{3}M^*$$

$$= \frac{-M^*}{3}$$ 4

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