IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

EEE PART I: MEng, BEng and ACGI

Corrected Copy

SEMICONDUCTOR DEVICES

Friday, 17 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.

Question One carries 40% of the marks. Questions Two and Three each carry 30%.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	K. Fobelets
	Second Marker(s) :	W.T. Pike

Constants

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10}$ cm^{-3} at $T = 300$K
dielectric constant of Si:	$\varepsilon_{Si} = 11$
dielectric constant of SiO_2:	$\varepsilon_{ox} = 4$
thermal voltage:	$kT/e = 0.026$V at $T = 300$K
charge of an electron:	$e = 1.6 \times 10^{-19}$ C
Planck's constant:	$h = 6.63 \times 10^{-34}$ Js

Formulae

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$ Schrödinger's equation in one dimension

$$f(E)=\frac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$$ Fermi distribution

$$m_e^*=\frac{\hbar^2}{d^2E(k)/dk^2}$$ Effective mass

$$n=\frac{1}{\sqrt{2}\hbar^3}\left(\frac{m_e^*kT}{\pi}\right)^{3/2}e^{-\frac{(E_g-E_f)}{kT}}=N_c e^{-\frac{(E_g-E_f)}{kT}}$$ Concentration of electrons

$$p=\frac{1}{\sqrt{2}\hbar^3}\left(\frac{m_h^*kT}{\pi}\right)^{3/2}e^{-\frac{E_f}{kT}}=N_v e^{-\frac{E_f}{kT}}$$ Concentration of holes

$$\left.\begin{aligned}J_n(x)&=e\mu_n n(x)E(x)+eD_n\frac{dn(x)}{dx}\\ J_p(x)&=e\mu_p p(x)E(x)-eD_p\frac{dp(x)}{dx}\end{aligned}\right\}$$ Drift and diffusion current densities in a semiconductor

$$I_{DS}=\frac{\mu C_{ox}W}{L}\left((V_{GS}-V_{th})V_{DS}-\frac{V_{DS}^2}{2}\right)$$ Current in a MOSFET

$$\left.\begin{aligned}J_n&=\frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}}-1\right)\\ J_p&=\frac{eD_p p_{n_0}}{L_p}\left(e^{\frac{eV}{kT}}-1\right)\end{aligned}\right\}$$ Diffusion current densities in a pn-junction

$$V_0=\frac{kT}{e}\ln\left(\frac{N_AN_D}{n_i^2}\right)$$ Built-in voltage

$$c=c_0\exp\left(\frac{eV}{kT}\right)\text{ with }\begin{cases}c=p_n\text{ or }n_p\\ c_0\text{ bulk minority carrier concentration}\end{cases}$$ Minority carrier injection under bias V

$$D=\frac{kT}{e}\mu$$ Einstein relation

$$w_n=\sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}\frac{N_A}{N_AN_D+N_D^2}(V_0-V)}$$
$$w_p=\sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}\frac{N_D}{N_AN_D+N_A^2}(V_0-V)}$$ Depletion width in the n and p-type region, under bias V

1.

a) A clean metal surface in a vacuum starts to emit electrons when illuminated by light of a wavelength less than 300 nm. What is the workfunction of the metal? [4]

b) Show that $\psi = \psi_o e^{jkx}$ is a solution of Schrödinger's equation in one dimension when V = 0, and show that in this case $E = \frac{\hbar k^2}{2m}$.

Schrödinger's equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$ [4]

c) What is the concentration of free electrons and free holes in n-type silicon that is doped to a level of $N_D=10^{18}$ cm^{-3}, at room temperature? [4]

d) Fig.1a gives a sketch of the energy band diagram (E_c, E_v, E_G) of intrinsic Si under bias.

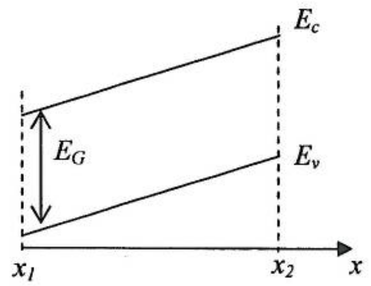


Figure 1a. Sketch of the energy band diagram of intrinsic Si under bias. Bias is applied between $x = x_1$ and x_2

i) Give the direction of the electric field across the structure between x_1 and x_2 with respect to the given x-axis. [2]

ii) Give the direction of the electron current I_n and hole current I_p current with respect to the given x-axis. [4]

e) In fig. 1b, a pn diode is given. The line in the middle between the p and n type region is the junction. Ideal Ohmic contacts are at $x = -X_p$ and X_n.

i) What is the direction of the electron and hole diffusion flux under forward bias with respect to the defined positive x direction? [4]

ii) What is the direction of the electron and hole diffusion flux under reverse bias with respect to the defined positive x direction? [4]

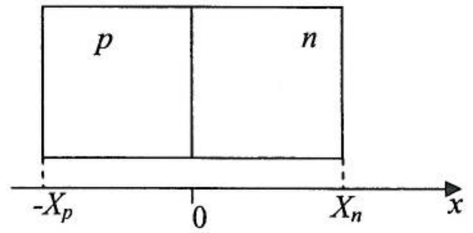


Figure 1b. Sketch of a pn diode with short material lengths.

f) Give the definition of the threshold voltage, V_{th} of an n-channel MOSFET using an energy band diagram including E_c, E_v, E_F. Explain your sketch briefly.

[8]

g) Fig.1c gives a sketch of the circuit around a pnp bipolar junction transistor (BJT) in forward active mode.

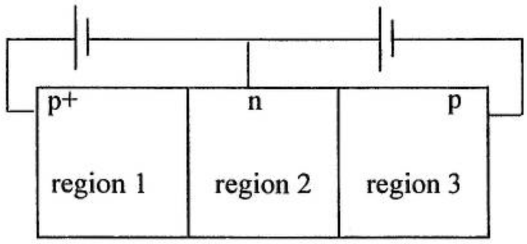


Figure 1c. Sketch of the common base bias configuration of a pnp BJT in forward active mode.

i) Give the name of each region 1, 2 and 3 in the sketch.

[3]

ii) Which junction in the *short* bipolar junction transistor (BJT) determines the current gain completely?

[3]

2.

a) Draw the band diagrams for an insulator, semiconductor and metal, labelling and showing the occupancy of the bands in each case. Use these band diagrams to explain the variation of electrical conductivity for these types of materials.

[10]

b) The general expression of the Poisson equation in a semiconductor is given by:

$$\varepsilon \frac{dE}{dx} = \rho(x)$$
$$\rho(x) = e \times \left(p(x) - n(x) + N_D^+(x) - N_A^-(x)\right) \qquad (1)$$

Where E is electric field, $\rho(x)$ is the charge as a function of position, e is unit charge, ε is dielectric constant, p is concentration of free holes, n is concentration of free electrons, N_D^+ is donor doping density and N_A^- is acceptor doping density.

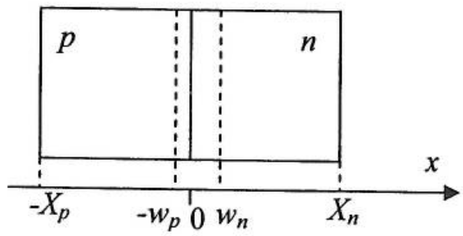


Figure 2a. A sketch of a pn diode indicating the depletion widths w_p and w_n in resp. the p-type and n-type regions and indicating the length X_p and X_n of each region.

i) Rewrite the Poisson equation (1) using the correct value of $\rho(x)$ in the following four regions of the diode:

- for $-X_p < x < -w_p$
- for $-w_p < x < 0$
- for $0 < x < w_n$
- for $w_n < x < X_n$

[8]

ii) At which point x will the electric field be minimum?

[4]

iii) Estimate the donor doping, N_D of a p^+n-diode with $N_A = 10^{18}$ cm^{-3} where the maximum absolute value of the electric field in the diode is $\left|E_{\max}\right| = \dfrac{eN_D^+ w_n}{\varepsilon} = 2 \times 10^5$ V/m for a reverse bias of 5V at room temperature ($T = 300$K). You will need to make appropriate approximations for a p^+n-diode.

[6]

iv) How can the maximum absolute value of the electric field in the diode be decreased?

[2]

3.

a) Fig. 3a shows a calculation of the available states, shown as the shaded region, of the outer electrons of the group IV elements as a function of their atomic spacing, r. The atomic spacing of silicon (Si) and three unknown elements is shown.

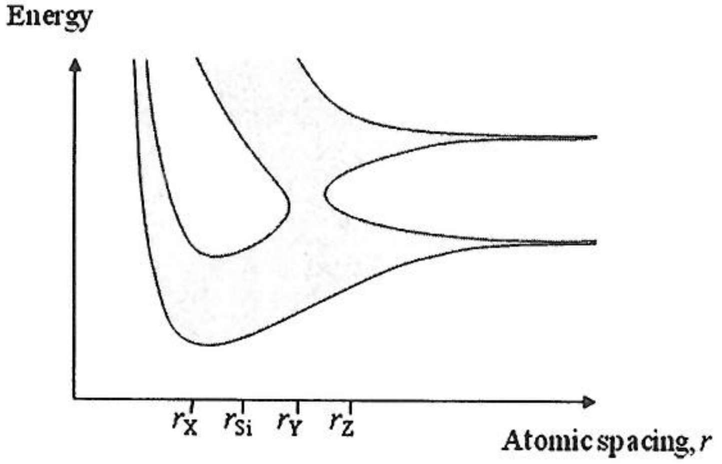


Figure 3a: A calculation of the energy of all the available states of the group IV elements as a function of their atomic spacing.

i) Assuming that for the largest atomic spacing shown in fig. 3a, the electrons are in their atomic orbitals, what are the labels (s, p, d or f) of the two types of states available and how many electrons per atom are found in each of these states? [2]

ii) Reproduce fig. 3a and shade the portion of the available states that will be occupied by electrons assuming there is no thermal excitation. [2]

iii) Indicate on your diagram the band gap, valence band and conduction band for silicon. [3]

iv) For elements X, Y and Z, indicate whether they are insulators, semiconductors or conductors. [3]

b) Fig. 3b gives a sketch of an n-channel enhancement mode metal-oxide-semiconductor field effect transistor (MOSFET) with bias circuits across each junction.

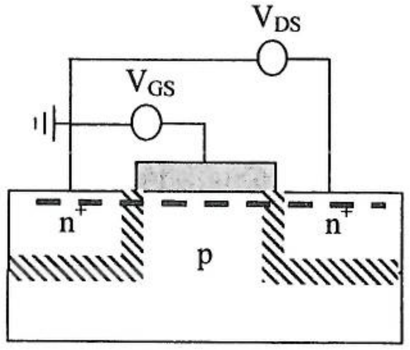


Figure 3b: Sketch of a MOSFET with bias voltages V_{GS} and V_{DS}. The grey area is SiO_2. The hatched regions are depletion regions (not to scale). The dashed line indicates the position where the energy band diagram needs to be drawn.

Sketch the energy band diagram (E_c, E_v, E_F, E_G) through the channel of the MOSFET between source and drain along the dashed line for the case where the MOSFET is in saturation. Make sure that the relative distances between energy levels are correct.

Hint: first draw a material cross section of the channel with the expected free carrier concentration in the different regions indicated for the given bias conditions.

[10]

c) Derive the approximate expressions for the differential resistance and the diffusion capacitance of a forward biased n^+p diode, as a function of applied voltage and current. The length of each region of the diode is equal to the minority carrier diffusion length.

The definition of differential resistance is:

$R_d = \dfrac{dV}{dI}$ with V voltage across the diode and I current through the diode.

The definition of diffusion capacitance is:

$C_{diff} = \dfrac{dQ}{dV}$ with V voltage across the diode and Q stored charge in the neutral region.

Explain all approximations you make. [10]

E1.3 Semiconductor devices - answers 2011

Question 1.

a) At the onset of photoemission:

$$eV_{\text{workfunction}} = \frac{hc}{\lambda}.$$ [2]

Hence for a wavelength of 300 nm,

$$V_{\text{workfunction}} = \frac{hc}{e\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 300 \times 10^{-9}} = 4.14\text{eV}$$ [2]

b) If $V(x) = 0$, Schrödinger's equation,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x),$$

becomes

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$ [1]

Substituting for $\psi = \psi_o e^{jkx}$

$$\frac{d^2\psi(x)}{dx^2} = -k^2\psi_o e^{jkx}$$
$$= -k^2\psi$$ [1]

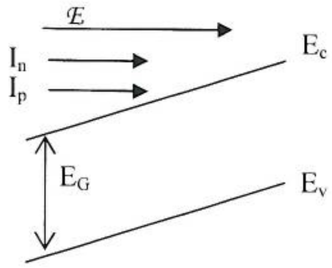
giving

$$E\psi = \frac{hk^2}{2m}\psi \text{ or } E = \frac{hk^2}{2m}$$ [2]

c) Electron concentration n=10^{18} cm^{-3}

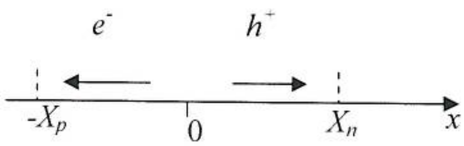
Hole concentration p=n_i^2/N_D=(1.45 x 10^{10} cm^{-3})^2/10^{18} cm^{-3}=2.1 10^2 cm^{-3} [4]

d)



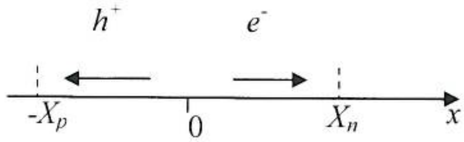
[2+4]

e) i)



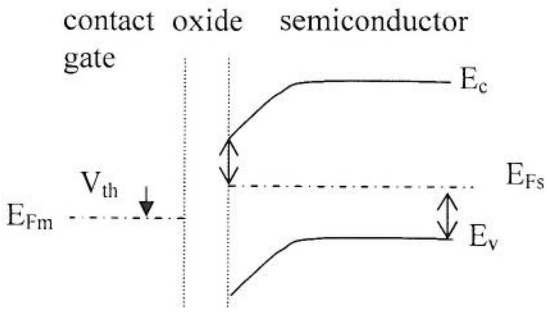
[4]

ii)



[4]

f)



The threshold voltage is the gate voltage that needs to be applied to ensure that the number of inverted carriers (electrons here) at the SiO_2/Si interface is equal to the number of majority carriers at the contact (holes). This equality is indicated by the arrows between band and Fermi level in the semiconductor. [8]

g)

i) region 1: emitter, 2: base, and 3 collector.

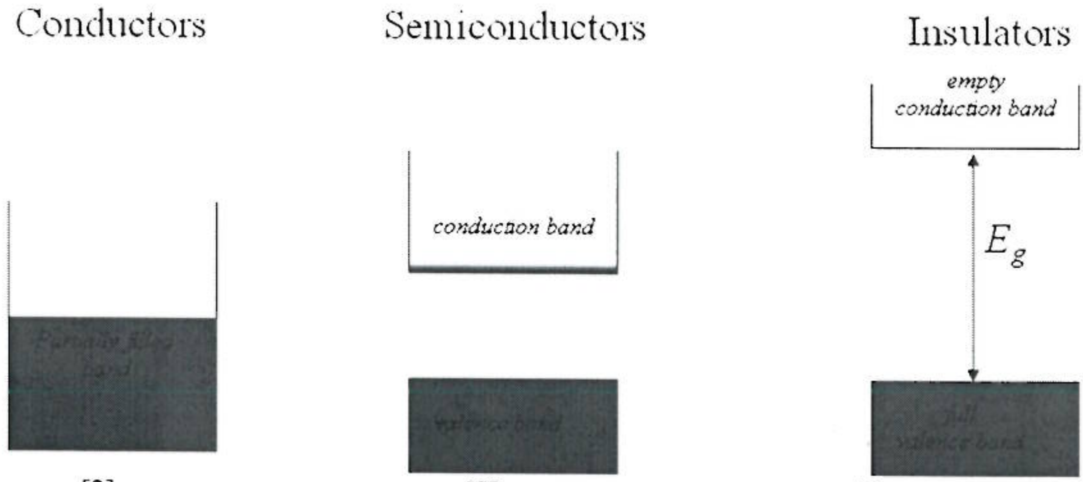
[3]

ii) emitter-base junction

[3]

Question 2.

a)



[2] [2] [2]

Conductivity is possible at low temperatures on metals as there are empty states allowing transport just above the Fermi level. For semiconductors, there needs to be sufficient thermal energy to excite the electrons above the band gap, so conduction increases exponentially with temperature. For dielectrics the band gap is too large for thermal excitation and so there are no electrons in the conduction band and hence no conductivity. [4]

b)

i) Write the Poisson equation (1) in all four regions of the diode:

- p-region for $-X_p<x<-w_p$: $\dfrac{dE}{dx}=0$

- p-region for $-w_p<x<0$: $\dfrac{dE}{dx}=\dfrac{-eN_A^-}{\varepsilon}$

- n-region for $0<x<w_n$: $\dfrac{dE}{dx}=\dfrac{eN_D^+}{\varepsilon}$

- n-region for $w_n<x<X_n$: $\dfrac{dE}{dx}=0$

[8]

ii) $x=0$

[2]

iii)

From the formulae sheet we have the expression of the depletion width in both n and p regions:

$$w_n=\sqrt{\frac{2\varepsilon_0\varepsilon_r}{e}\frac{N_A}{N_AN_D+N_D^2}(V_0-V)}$$
$$w_p=\sqrt{\frac{2\varepsilon_0\varepsilon_r}{e}\frac{N_D}{N_AN_D+N_A^2}(V_0-V)}$$

Since the acceptor doping in the diode is higher than the donor doping $N_A >> N_D \rightarrow w_n >> w_p$. Thus we ignore w_p and we simplify the expression of w_n:

$$w_n=\sqrt{\frac{2\varepsilon_0\varepsilon_r}{e}\frac{N_A}{N_AN_D+N_D^2}(V_0-V)}=\sqrt{\frac{2\varepsilon_0\varepsilon_r}{e}\frac{N_A}{N_D(N_A+N_D)}(V_0-V)}$$
$$w_n\approx\sqrt{\frac{2\varepsilon_0\varepsilon_r}{e}\frac{N_A}{N_D(N_A)}(V_0-V)}=\sqrt{\frac{2\varepsilon_0\varepsilon_r}{e}\frac{1}{N_D}(V_0-V)}$$

The expression for the built-in voltage also comes from the formulae

$$V_0=\frac{kT}{e}\ln\left(\frac{N_AN_D}{n_i^2}\right)$$

list:

Estimating the magnitude of V_0

We know that $N_A >> N_D$ thus $N_D << 10^{18}$ cm^{-3}

Take one extreme: $N_D = 10^{18}$ cm$^{-3} \rightarrow V_0=0.94$V

$$V_0=\frac{kT}{e}\ln\left(\frac{N_AN_D}{n_i^2}\right)=0.026V\ \ln\left(\frac{10^{18}N_D}{(1.45\,10^{10})^2}\right)=0.026V\ \ln\left(4.76\,10^{-3}N_D\right)$$

Take a lower value: $N_D = 10^{16}$ cm^{-3} → V_0=0.82V

Since $V_0 < 1$ V < -V = 5 V, the value of V_0 will be ignored compared to the reverse bias voltage.

$$w_n \approx \sqrt{\frac{2\varepsilon_0\varepsilon_r}{e}\frac{-V}{N_D}}$$

$$|E_{\max}| = \frac{eN_D w_n}{\varepsilon} \approx \frac{eN_D\sqrt{\frac{2\varepsilon_0\varepsilon_r}{e}\frac{-V}{N_D}}}{\varepsilon_0\varepsilon_r} = \sqrt{\frac{e^2N_D^2 2\varepsilon_0\varepsilon_r}{(\varepsilon_0\varepsilon_r)^2 e}\frac{-V}{N_D}}$$

$$|E_{\max}| \approx \sqrt{\frac{-V\, e\, N_D 2}{(\varepsilon_0\varepsilon_r)}}$$

$$|E_{\max}|^2 \approx \frac{-V\, e\, N_D 2}{(\varepsilon_0\varepsilon_r)}$$

$$N_D \approx \frac{\varepsilon_0\varepsilon_r|E_{\max}|^2}{-V2e} = \frac{8.85\,10^{-12}\times 11\times(2\,10^5)^2}{5\times 2\times 1.6\,10^{-19}} = \frac{389.4\,10^{-2}}{16\,10^{-19}} = 24.34\,10^{17}\ cm^{-3}$$

[8]

iv) By lowering the donor doping density
or by making a pin junction (p-intrinsic-n diode)

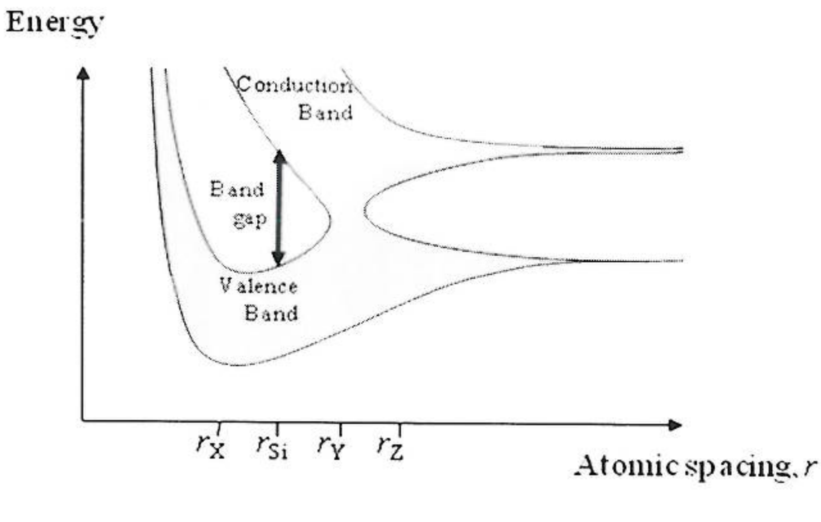
[2]

Question 3

a)

i) s-type, 2; p-type, 6.

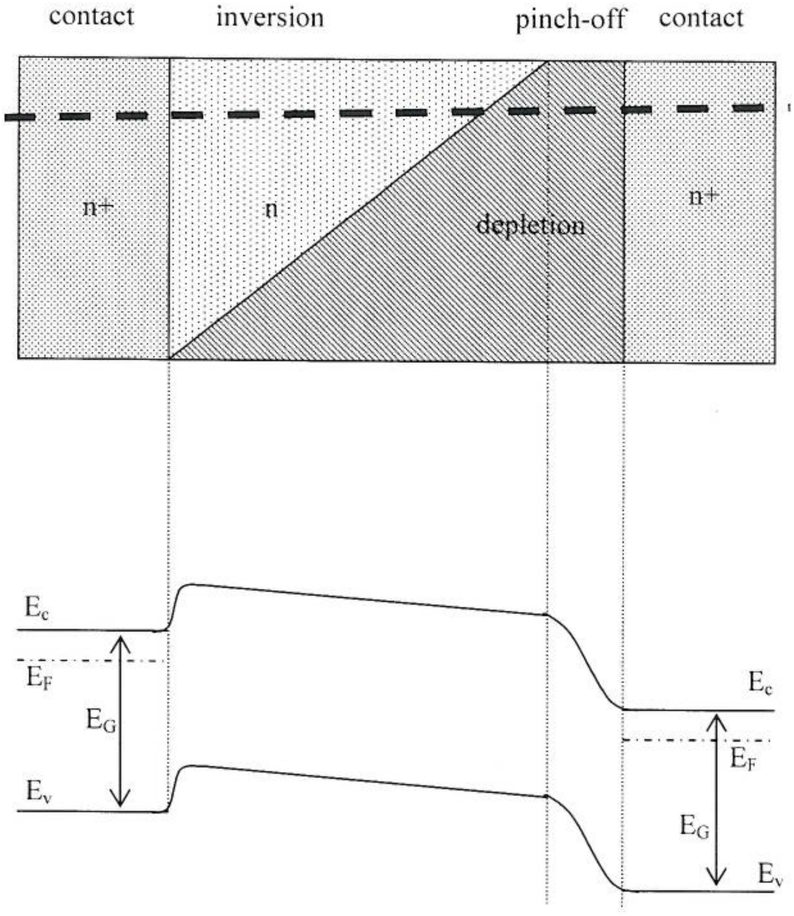
ii)



iii) See above.

iv) X: Semiconductor – has band gap; Y: Metal: empty states just above occupied states; Z Metal: empty states just above occupied states – band gap is in valence band and does not affect conductivity.

b) The best way to draw the correct energy band diagram (E_c, E_v, E_F, E_G) is by sketching a material cross section first. Under the condition of saturation we know that 1: the channel is inverted and thus n-type but 2: in saturation at the drain side the channel is pinched-off this means that near the drain the channel is depleted. This is drawn below. We then draw the energy band diagram to fit with the material cross section.



[10]

c)

From the formulae list we can copy the expression for hole and electron diffusion current density in the diode. This expression is correct because the lengths of the neutral regions are equal to the minority carrier diffusion length. Since we are working with a n+p junction we can ignore the hole current compared to the electron current. Finally, in forward bias the -1 term can be neglected compared to the exponential.

$$I_{tot} \approx I_n = \frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)A \approx \frac{eD_n n_{p_0}}{L_n} e^{\frac{eV}{kT}} A$$

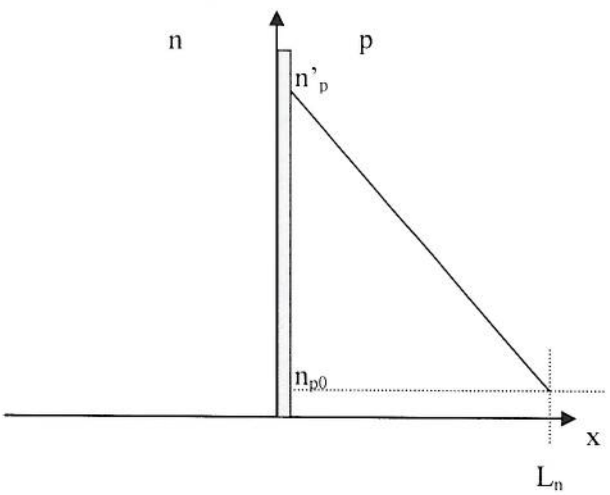
The differential conductance can then be calculated:

$$S_d = \frac{dI}{dV} = \frac{d\left(\frac{eD_n n_{p_0}}{L_n} e^{\frac{eV}{kT}} A\right)}{dV} = \frac{e^2 D_n n_{p_0} e^{\frac{eV}{kT}}}{kTL_n} A = \frac{e\ I}{kT}$$

Thus the differential resistance is:

$$R_d = \frac{kT}{eI}$$

The stored charge in the n+p junction is mainly determined by the electron minority carrier charge stored in the p-region, due to the doping difference between n and p region. Since the regions can be considered short the variation of the minority carrier concentration is linear. This is plotted in the graph below.



It is sufficient to take the excess charge into account as the rest will not vary with applied voltage. In forward bias $n'_p >> n_{p0}$

$$Q_n = \frac{e\left(n'_p - n_{p_0}\right)L_n}{2} A \approx \frac{en'_p L_n}{2} A$$

From the formulae sheet we have:

$$n'_p = n_{p_0} \exp\left(\frac{eV}{kT}\right)$$

$$Q_n \approx \frac{en_{p_0} L_n \exp\left(\frac{eV}{kT}\right)}{2} A$$

The diffusion capacitance is then:

$$C_{diff} \approx \frac{d\left(\frac{en_{p_0} L_n \exp\left(\frac{eV}{kT}\right)}{2} A\right)}{dV} = \frac{e^2 n_{p_0} L_n \exp\left(\frac{eV}{kT}\right)}{2kT} A$$

Taking the expression of the current extracted before:

$$I \approx \frac{eD_n n_{p_0}}{L_n} e^{\frac{eV}{kT}} A$$

$$en_{p_0} e^{\frac{eV}{kT}} A = \frac{L_n}{D_n} I$$

And introducing this expression in C_{diff} gives: [10]

$$C_{diff} \approx \frac{eL_n^2}{2kTD_n} I$$