

# Traffic Theory & Queueing Systems

Examinations :      Session      2014      Confidential

## MODEL ANSWER and MARKING SCHEME

First Examiner

Paper Code

Second Examiner

Question

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Q1  
a)

i) Each of the channels can be viewed as a Bernoulli trial with success probability

$$P(\text{Busy} \rightarrow \text{Free}) = \mu \Delta t$$

ii) Since the channels are active independently the probability that exactly  $k$  channels will become idle in  $(t, t + \Delta t)$  is binomial

$$P(k \text{ channels} \rightarrow \text{idle}) = \binom{i}{k} (\mu \Delta t)^k (1 - \mu \Delta t)^{i-k}$$

iii) Therefore

$$\begin{aligned} P(1 \text{ channel} \rightarrow \text{idle}) &= \binom{i}{1} \mu \Delta t (1 - \mu \Delta t)^{i-1} \\ &= i \mu \Delta t + o(\Delta t) \end{aligned}$$

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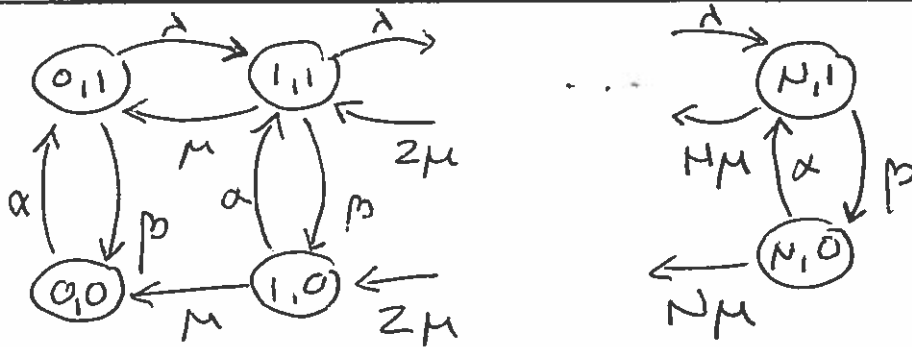
Second Examiner

Question

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Q1  
n)

offered traffic,  $\rho = \frac{\lambda}{\mu} = 9.4$  Erlangs

$$\frac{1}{\mu} = 12.5 \text{ s} \quad - \quad \mu = 0.08$$

$$P[\text{link 1 in saturation}] = E_{12}(9.4) = 0.1$$

$$P(\text{link in saturation}) = \frac{T_{\text{on}}}{T_{\text{on}} + T_{\text{off}}} = 0.1$$

$$\Rightarrow T_{\text{on}}/T_{\text{off}} = 1/9$$

$$E[T_{\text{on}}] = E[\text{given time in saturation}]$$

$$= \frac{1}{N\mu} = 2.08 \text{ s}$$

$$E[T_{\text{off}}] = 18.76 \text{ s}$$

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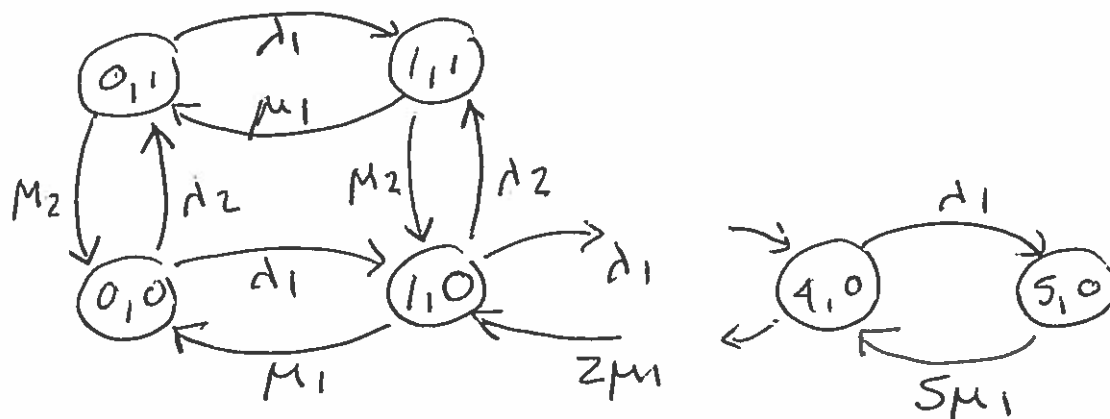
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Q2  
a)



$(N_t, B_t) =$   $N_t =$  Nr of Type 1 calls in prog.  
 $B_t =$  Nr of Type 2 calls in prog.

If the Nr of channel were infinite the processes  $\{N_t\}$  and  $\{B_t\}$  would not interfere i.e. would be independent B/D processes

Truncation of the state space of a reversible process does not destroy reversibility  
 $\Rightarrow$  equilibrium distribution is of product form with Poisson-like factors:

iii)  $\pi_{ij} = K \left( \frac{\rho_1^i}{i!} \right) \left( \frac{\rho_2^j}{j!} \right) \quad i + 4j \leq 5$

$$K = \left[ \sum_{i=0}^5 \frac{\rho_1^i}{i!} + \rho_2 \sum_{i=0}^1 \frac{\rho_1^i}{i!} \right]^{-1} = 0.269$$

iv)  $B_{\text{Type 1}} = \pi_{5,0} + \pi_{1,1} = 0.137$

$$B_{\text{Type 2}} = 1 - (\pi_{0,0} + \pi_{1,0}) = 0.462$$

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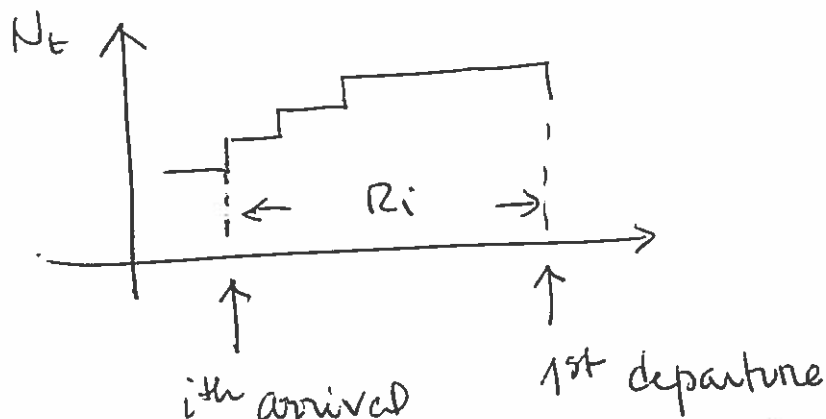
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Q 2  
(5)

i) The effect of re-attempts is important under heavy-traffic conditions. If re-attempts occur soon after the initial blocking the resulting arrival stream will not be a Poisson stream.

ii)  $s_i$  = service time  
 $w_i$  = waiting time  
 $Q_i$  = queue length found on arrival  
 $R_i$  = Residual service time =  
 time until first departure  
 seen by  $i$ th arrival



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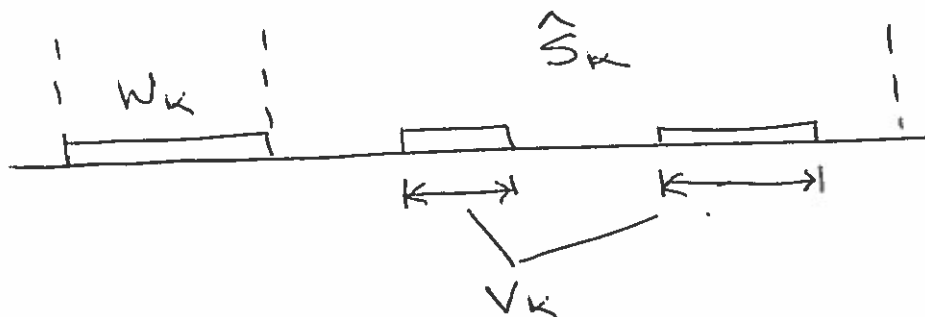
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Q3 a)



$V_k$  = work brought into the system, dump  $\hat{S}_k$  by higher-priority arrivals

$$\begin{aligned}
 E[V_k] &= \sum_{i=1}^{k-1} (\rho_i E[\hat{S}_k]) E[S_i] \\
 &= \left( \sum_{i=1}^{k-1} \rho_i \right) E[\hat{S}_k] \quad , \quad \rho_i = \rho_i E[S_i] \\
 &= \sigma_{k-1} E[\hat{S}_k]
 \end{aligned}$$

Then

$$\begin{aligned}
 E[\hat{S}_k] &= E[\text{true service time}] + E[\text{Interrupt time}] \\
 &= E[S_k] + E[V_k] \\
 &= E[S_k] + \sigma_{k-1} E[\hat{S}_k]
 \end{aligned}$$

$$E[\hat{S}_k] = \frac{E[S_k]}{1 - \sigma_{k-1}}$$

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Q3

i) system capacity  $VC$  cells/s. $x$  incremented by  $\frac{V}{\alpha}$  cells during a talk spanEquivalent capacity  $\frac{VC}{\frac{V}{\alpha}} = \alpha C$  "unit of information"ii If  $i$  sources on the  $iV$  cells/s $\frac{iV}{\frac{V}{\alpha}} = \alpha i$  "unit of information"

iii

$$F_i(t+\Delta t, x) = [N - (i-1)]\alpha \Delta t F_{i-1}(t, x)$$

$$+ (i+1)\alpha \Delta t F_{i+1}(t, x)$$

$$+ \{1 - [(N-i)\alpha + i\alpha]\Delta t\} F_i[t, x - \underbrace{(i-\alpha)\alpha \Delta t}_{\Delta t}]$$

$$+ o(\Delta t)$$

explanation and discussion

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Q4 i)  $M/M/K/N$  system  
a)

$$N = K + B \quad B = N - K$$

$N$  = system capacity,  $K$  = NR of servers

ii)

$$P(\text{delay}) = P(\text{all } K \text{ servers busy} \mid \text{buffer not full})$$

$$= P(K \leq N_t < K+B)$$

$$= \pi_K \left( \frac{1 - \rho^B}{1 - \rho} \right) \dots (\text{derivations})$$

$$\text{iii) } P(\text{loss}) = P(\text{buffer full})$$

$$= P(N_t = K+B)$$

$$= \pi_K \rho^B$$

where

$$\pi_K = \left( \frac{A^K}{K!} \right) \pi_0$$

Q4  
ii)

The state space of the system

$$S = \{0, S, L, 1, 2, 3\}$$

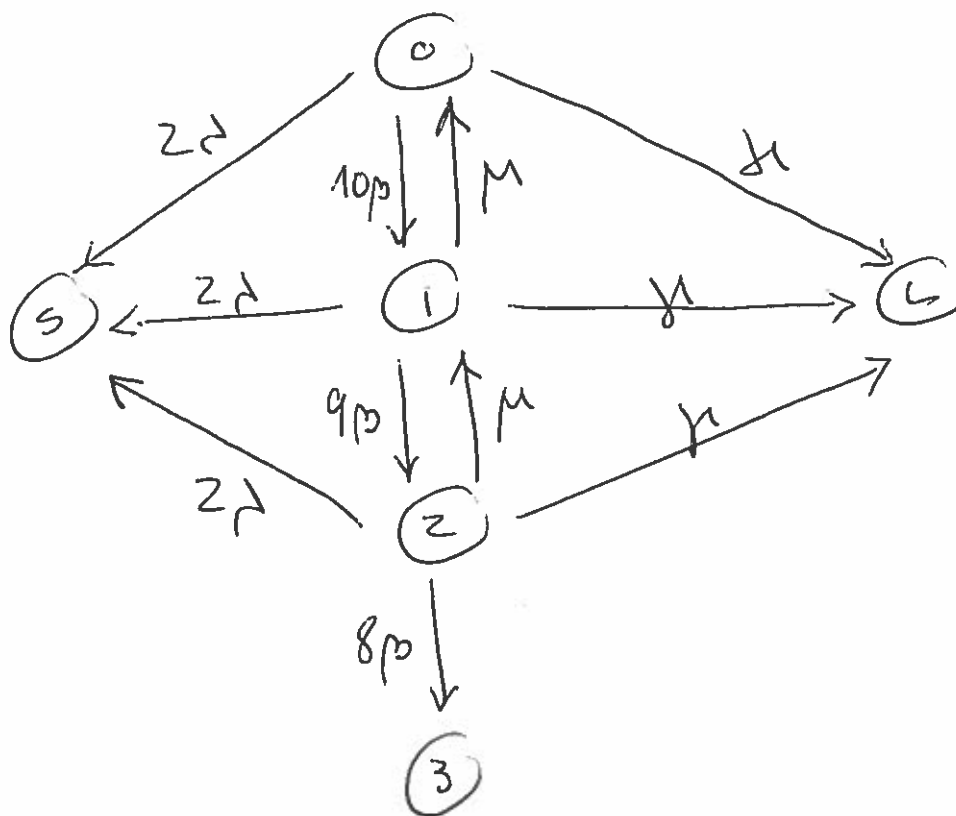
0 = fully operational state

S = switch failure

L = link failure

1, 2, 3 = NR of access node in failure (1, 2, 3)

ii)





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$$\begin{array}{c}
 \begin{array}{ccccccc}
 \hline
 & 0 & 1 & 2 & 3 & 5 & \hline
 & 0 & \mu & 0 & 0 & 22 & \times \\
 & \mu & -(9\mu + 2\lambda + \mu + \mu) & 0 & 0 & 22 & \times \\
 & 0 & \mu & -(8\mu + 2\lambda + \mu + \mu) & 8\mu & 22 & \times \\
 & 0 & - & - & - & . & 0 \\
 & 0 & - & - & - & . & 0 \\
 & 0 & - & - & - & . & 0 \\
 \hline
 \end{array}
 \end{array}$$

Q =