UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2002

BEng Honours Degree in Computing Part III
MSc in Computing Science
MEng Honours Degree in Electrical Engineering Part IV
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C317=I3.16=E4.32

GRAPHICS

Friday 3 May 2002, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1. A graphics scene is defined with the normal Cartesian origin at its centre. It is to be drawn from a viewing coordinate system whose centre is at point C=(Cx,Cy,Cz) and whose axes are defined by the unit vectors (***u***,***v***,***w***), such that ***w*** is the direction of view, ***u*** corresponds to the horizontal direction on the screen and ***v*** to the vertical axis of the screen. **C**,***u***,***v*** and ***w*** are all defined in the same Cartesian axis system as the scene.

a. Given that P is a point of the graphics scene in the original coordinate system explain, with the use of a suitable diagram, why the coordinates of point P expressed in the (***u***,***v***,***w***) coordinate system are ((**P-C**).***u***,(**P-C**).***v***,(**P-C**).***w***).

b. Use the result of part (a) to derive the viewing transformation matrix, which transforms all the points of the scene to the (***u***,***v***,***w***) coordinate system.

c. In an animation sequence an aircraft is to start at a point **S**=(Sx,Sy,Sz), spiral out of control and crash into the point which is the origin of the Cartesian system in which the scene is defined. Over a series of 12 frames it is to travel from **S** to the origin with the view direction ***w*** being towards the origin and complete one clockwise revolution about the view direction. Show how to derive the transformation that will calculate the new values of the vectors ***u*** and ***v*** for each successive frame. You should show the primitive transformation matrices that are needed, but you do not need to calculate the combined transformation.

d. If the start point is **S** = (0,100,100) and the initial coordinate system has ***u***=(1,0,0), calculate the position and directions of the viewing coordinate system (**C**,***u***,***v***,***w***) after 6 frames and hence deduce the viewing transformation matrix for that frame.

The four parts carry, respectively, 20%, 20%, 35% and 25% of the marks.

2. In a polygon rendering system a scene is to be viewed from the origin, with the viewplane at z=10 in the world coordinate system. A polygon has 3D coordinates [-20,-20,20], [20,-20,20], [-20,0,60] and [20,0,60], and an internal point has coordinate [-15,-5,50].

a. What are the projected coordinates of the five points onto the viewplane.

b. Assuming that a texture defined in the (α,β) space with the restriction that $0<=\alpha<=1$ and $0<=\beta<=1$ is to be mapped exactly to the polygon, derive a vector expression for an internal point of the polygon in terms of the texture coordinates, the position of the bottom left hand corner and the edge vectors.

c. Calculate the value of the texture coordinates of the internal point defined above in both the three dimensional case and the projected two dimensional case.

d. Given that the scene is drawn in a window defined by (-100, -100, 100, 100) and that the window is mapped to a raster area of 400 by 400 pixels, calculate the pixel coordinates of the five points defined above, assuming that the pixel origin is at the bottom left hand corner.

e. Explain, with the aid of a suitable diagram how a differential algorithm could be used to determine the (α,β) coordinates of the internal point at the raster level. What numerical result do you get for the internal point defined above?

The five parts carry equal marks.

3. Ray tracing

a Explain in detail how to calculate the local illumination at a surface point **P** as seen from a viewpoint **V**. Assume a single light source.

b Describe how images are generated using recursive ray tracing. Give the outline code for casting a ray.

c A ray originates at point **V** and is parallel with direction vector **d**. A right-angled triangle is given by three points $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$ (the right angle is at point $\mathbf{P_1}$). Show in detail how you can calculate the intersection between the ray and the face defined by the triangle.

d In a concrete example, a ray starts at $\mathbf{v} = (9, 9, 0)$ and has a direction vector $\mathbf{d} = (0, 0, 1)$. The points of the triangle are given as $\mathbf{P_1} = (8, 8, 10)$, $\mathbf{P_2} = (12, 8, 10)$, and $\mathbf{P_3} = (8, 10, 10)$. Calculate the intersection point between the ray and the face defined by the triangle.

The four parts carry, respectively, 20%, 30%, 30% and 20% of the marks.

4 Warping and Morphing

a Briefly describe the following terms:

i warping
ii morphing
iii forward mapping
iv backward mapping

b Briefly describe four different blending techniques and how they can be used in morphing to generate new images.

c A two-dimensional free-form deformation based on linear B-splines is defined by a 4 x 3 control point matrix, which is given below. For an image of size 120 x 100 pixels show in detail how to calculate the new location of a pixel (x,y) = (50, 75) after warping.

(-1, 4)	(-3, 7)	(4, 7)	(-4, 2)
(-2, 7)	(-2, 9)	(3, 1)	(3, 6)
(5, 3)	(3, 8)	(7, 2)	(2, 7)

d Estimate the number of evaluations of the spline basis function required to warp a 3D polygonal surface of m vertices defined by a control point mesh of n x n x n control points using

i free-form deformations based on linear B-splines,
ii free-form deformations based on cubic B-splines,
iii thin-plate splines.

Discuss which of the warping techniques above would be most suitable for real-time warping.

The four parts carry, respectively, 20%, 20%, 30% and 30% of the marks.