

1.

a)

Hole concentration  $p = p_i = 1.45 \times 10^{10} \text{ cm}^{-3}$   
 Electron concentration  $n = n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$

[2]

b)

There are two possible approaches.

1)

Use the equation for the list on p.2:

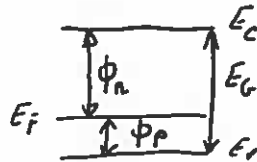
$$p = N_v e^{\frac{(E_v - E_F)}{kT}}$$

Rewrite to  $E_F - E_v = kT \ln\left(\frac{N_v}{p}\right)$  use  $p = N_A = 10^{17} \text{ cm}^{-3}$

$$E_F - E_v = 0.026 \text{ eV} \ln\left(\frac{1.8 \times 10^{19} \text{ cm}^{-3}}{10^{17} \text{ cm}^{-3}}\right) = 0.135 \text{ eV}$$

Then use the band gap  $E_G = 1.12 \text{ eV}$

Based on the following energy band diagram:



We can derived that  $E_G = \phi_n + \phi_p$  and thus  $\phi_n = E_c - E_F = E_G - \phi_p = 1.12 \text{ eV} - 0.135 \text{ eV} = 0.985 \text{ eV}$

2) Start from the electron concentration:

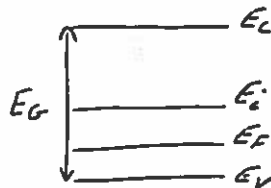
$$n = N_c e^{\frac{(E_c - E_F)}{kT}} \text{ and } n = \frac{n_i^2}{N_A}$$

$$\text{Rewrite to } E_c - E_F = kT \ln\left(\frac{N_c N_A}{n_i^2}\right)$$

$$\text{Fill in numbers } E_c - E_F = 0.026 \text{ eV} \ln\left(\frac{3.2 \times 10^{19} \times 10^{17}}{(1.45 \times 10^{10})^2}\right) = 0.969 \text{ eV} \quad [5]$$

c)

[5]



d)

Formula to know by heart (A-level physics).

[5]

$$R = \frac{\rho \times y}{x \times z} \text{ and } \rho = \frac{1}{\sigma} = \frac{1}{en\mu_n + ep\mu_p} \approx \frac{1}{ep\mu_p}$$

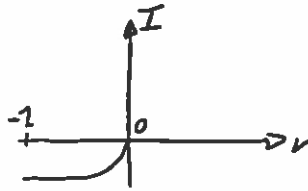
$$R = \frac{1}{ep\mu_p} \times \frac{y}{x \times z} = \frac{100 \times 10^{-2} \text{ cm}}{1.6 \times 10^{-19} \text{ C} \times 10^{17} \text{ cm}^{-3} \times 200 \text{ cm}^2 / \text{Vs} \times 2000 \times 10^{-7} \text{ cm} \times 0.5 \times 10^{-1} \text{ cm}}$$

$$R = 31250 \Omega$$

e)

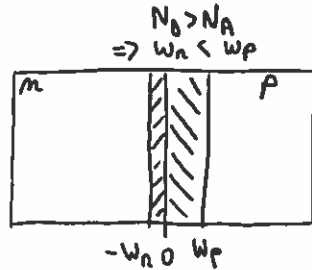
i)

[4]



ii)

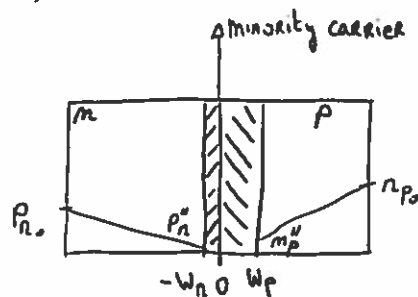
[4]



cross hatched region are the depletion regions.

iii)

[5]



$$N_D > N_A$$

$$p_{n0} = \frac{n_i^2}{N_D} < n_{p0} = \frac{n_i^2}{N_A}$$

- f)  $V_{eE} < V_{eB}$  (forward biased pn diode).  
 $V_{eB} < V_{eC}$  (reverse biased pn diode)

[4]

- g)  $I_{C1} = I_{C2}$

The current density across the E-B pn diode is given by the formula:

$$J_{tot} = J_n + J_p = \frac{eD_n n_{p0}}{W_B} \left( e^{\frac{eV}{kT}} - 1 \right) + \frac{eD_p p_{n0}}{X_n} \left( e^{\frac{eV}{kT}} - 1 \right)$$

With  $W_B$  the base width and  $X_n$  the emitter width. Rewriting in function of doping:

$$J_{tot} = J_n + J_p = \frac{eD_n n_i^2}{N_A W_B} \left( e^{\frac{eV}{kT}} - 1 \right) + \frac{eD_p n_i^2}{N_D X_n} \left( e^{\frac{eV}{kT}} - 1 \right)$$

In an npn BJT the collector current is determined by the minority carrier diffusion current in the base, thus by  $J_n$ . Since the doping in the base is not changing between BJT 1 and BJT 2,  $J_n$  does not change and thus the collector current does not change. What changes is the base current  $I_B$  and thus the current gain  $\beta$ .

[6]

2.

a)

i)  $x = 0$  [2]

ii) Under the depletion approximation we assume that the free carrier concentration in the depletion region is zero. Thus only the ionised charge remains.

$$\rho(x) = -eN_A \quad [2]$$

iii) [6]

Poisson equation from formulae list:

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon}$$

Solving in the depletion region in the p-section.

$$\frac{dE}{dx} = \frac{-eN_A}{\epsilon}$$

Integrate once:

$$E = \frac{-eN_A}{\epsilon}x + C_1 \text{ with } C_1 \text{ an integration constant. } C_1 \text{ can be found from the boundary}$$

condition at the edge of the p-section depletion region. In  $x = -w_p$ ,  $E(x) = 0$ .

$$0 = \frac{eN_A}{\epsilon}w_p + C_1 \text{ thus } C_1 = \frac{-eN_A}{\epsilon}w_p$$

Thus  $E(x) = \frac{-eN_A}{\epsilon}(x + w_p)$ . From the formulae list we have:

$$w_p = \left[ \frac{2\epsilon V_{bi} N_D}{e(N_A + N_D)N_A} \right]^{1/2} \text{ since } N_D \gg N_A \text{ we can simplify this expression to:}$$

$$w_p \approx \left[ \frac{2\epsilon V_{bi}}{eN_A} \right]^{1/2} \text{ thus}$$

$$E(x) = \frac{-eN_A}{\epsilon} \left( x + \left[ \frac{2\epsilon V_{bi}}{eN_A} \right]^{1/2} \right)$$

b)

i)  $n(x) = 10^{17} - 2.5 \times 10^{18}x$

$$p(x) = \frac{n_i^2}{N(x)} = \frac{(1.45 \times 10^{10})^2}{10^{17} - 2.5 \times 10^{18}x} = \frac{2.1 \times 10^{20}}{10^{17} - 2.5 \times 10^{18}x} = \frac{2.1 \times 10^3}{1 - 25x} \quad [4]$$

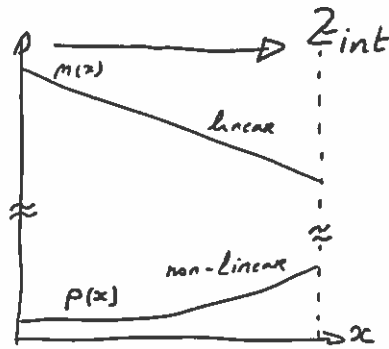
$$n(0) = 10^{17} \text{ cm}^{-3}$$

$$n(0.035) = 1.25 \times 10^{16} \text{ cm}^{-3}$$

$$p(0) = 2100 \text{ cm}^{-3}$$

$$p(0.02) = 4200 \text{ cm}^{-3}$$

$$p(0.035) = 16800 \text{ cm}^{-3}$$



- ii) Start from the drift-diffusion equation for electrons in the formulae sheet:

$$J_n(x) = e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx}$$

Since the voltage is zero, the total current density has to be zero.

$$0 = e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx}$$

$$E(x) = -\frac{D_n}{\mu_n n(x)} \frac{dn(x)}{dx}$$

Using Einstein's equation and differentiating  $n(x) = 10^{17} - 2.5 \times 10^{18} x$ :

$$E(x) = \frac{2.5 \times 10^{18} \times kT}{e[10^{17} - 2.5 \times 10^{18} x]} = \frac{2.5 \times 10^{18} \times 0.026}{[10^{17} - 2.5 \times 10^{18} x]}$$

[4]

$$E(x) = \frac{6.5 \times 10^{16}}{[10^{17} - 2.5 \times 10^{18} x]} = \frac{6.5}{[10 - 250 x]}$$

- iii) The internal electric field points to +x to cause drift opposite to diffusion.  $\xrightarrow{x_{int}}$  [2]

- c) Charge in p-region:  $\rho = e(p - n - N_A)$  [2]

Charge neutrality is required in the p-region.  $0 = (p - n - N_A)$  [1]

Law of mass action:  $n \times p = n_i^2$  [2]

[10]

$$\begin{cases} n \times p = n_i^2 \\ p - n - N_A = 0 \end{cases}$$

$$p - \frac{n_i^2}{p} - N_A = 0 \quad (p \neq 0)$$

$$p^2 - N_A p - n_i^2 = 0 \quad [5]$$

$$p = \frac{N_A \pm \sqrt{N_A^2 - 4n_i^2}}{2}$$

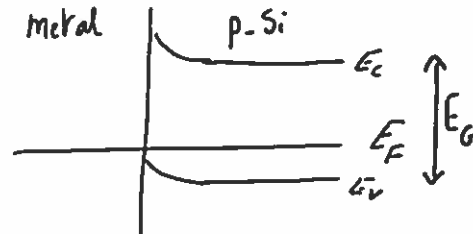
$$p = \frac{N_A + \sqrt{N_A^2 - 4n_i^2}}{2}$$

$$\text{For } N_A \gg n_i \rightarrow \begin{aligned} N_A^2 - 4n_i^2 &\approx N_A^2 \\ p &\approx \frac{N_A + N_A}{2} = N_A \end{aligned}$$

3.

a)

- i) p-channel or pMOS [2]
- ii) source & drain p-type (heavily doped)  
substrate region n-type [2]
- iii) Should be an Ohmic contact on a heavily doped p-type region. Thus bend bending upwards towards metal. [6]



b)

- i) The oxide capacitance can be extracted from the maximum measured capacitance:

$$C_{\max} = C_{ox} \times W_G \times L_G$$

$$C_{ox} = \frac{C_{\max}}{W_G \times L_G} = \frac{0.885 \times 10^{-12} \text{ F}}{100 \times 10^{-4} \times 5 \times 10^{-4}} = 1.77 \times 10^{-7} \text{ F/cm}^2$$

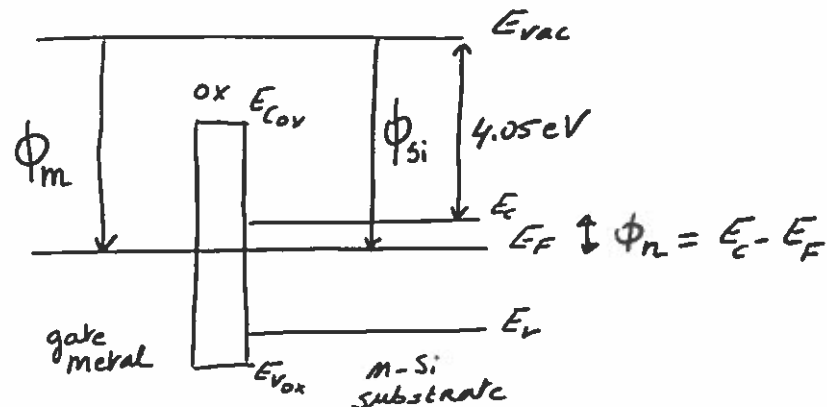
The oxide thickness comes from:

$$C_{ox} = \frac{\epsilon_0 \epsilon_{ox}}{t_{ox}} \rightarrow t_{ox} = \frac{\epsilon_0 \epsilon_{ox}}{C_{ox}}$$

$$t_{ox} = \frac{8.85 \times 10^{-14} \text{ F/cm} \times 4}{1.77 \times 10^{-7} \text{ F/cm}^2} = 2 \times 10^{-6} \text{ cm} = 20 \text{ nm}$$

Extract the thickness of the oxide from the CV measurements. [2]

- ii) A sketch of the flat band situation. [4]



From  $E_c - E_F$  we can find the doping.

$$E_c - E_F = \phi_m - 4.05 \text{ eV} = 4.259 \text{ eV} - 4.05 \text{ eV} = 0.209 \text{ eV}$$

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

$$N_D = 3.2 \times 10^{19} \text{ cm}^{-3} \exp\left(-\frac{0.209}{0.026}\right) = 1.03 \times 10^{16} \text{ cm}^{-3}$$

- iii)  $C_{\min}$  is the series connection of the oxide related capacitance and the depletion capacitance. Extract the maximum depletion width from the C-V measurements. [4]

$$\frac{1}{C_{\min}} = \frac{1}{C_{\max}} + \frac{1}{C_{\text{depl}_{\max}}}$$

$$\frac{1}{C_{\text{depl}_{\max}}} = \frac{1}{C_{\min}} - \frac{1}{C_{\max}}$$

Depletion capacitance per gate area:

$$C'_{\text{depl}_{\max}} = \frac{C_{\text{depl}_{\max}}}{L_G \times W_G}$$

The relationship between depletion width and depletion capacitance:

$$C'_{\text{depl}_{\max}} = eN_D W_{\text{depl}_{\max}}$$

$$\frac{C_{\text{depl}_{\max}}}{L_G \times W_G} = eN_D W_{\text{depl}_{\max}}$$

$$W_{\text{depl}_{\max}} = \frac{C_{\text{depl}_{\max}}}{eN_D \times L_G \times W_G}$$

c)

i)  $V_{th} = -0.7 \text{ V}$ . [2]

ii) Majority carriers in the channel are holes, thus mobility is approximately:  $\mu_p = 410 \text{ cm}^2/\text{Vs}$ . [2]

iii) Output characteristic is  $I_{DS}$  versus  $V_{DS}$ . [1] [6]

A p-channel MOSFET has a negative bias on the drain and “negative” current if S is at  $x=0$  and D is at  $+x$ . [1]

Pinch-off is found for  $V_{DS} = V_{GS} - V_{th} = -1 - (-0.7) = -0.3 \text{ V}$  [1]

Thus the IV characteristic is linear up to  $\sim -0.3 \text{ V}$  and then saturates.

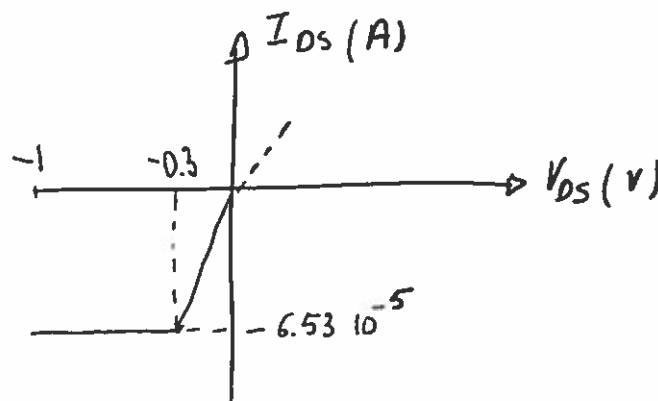
The saturation current is derived from the expression for the drain current found in the formulae list:

$$I_{DS} = \frac{\mu C_{ox} W}{L} \left( (V_{GS} - V_{th}) V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Re-written for saturation

$$I_{DS}^{sat} = \frac{\mu C_{ox} W}{2L} ((V_{GS} - V_{th})^2)$$

$$I_{DS}^{sat} = \frac{410 \times 1.77 \times 10^{-7} \times 100}{2 \times 5} ((-0.3)^2) = 6.53 \times 10^{-5} \text{ A} \quad [1]$$



[2]