IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

DEVICES AND FIELDS

Monday, 9 June 10:00 am

Time allowed: 2:00 hours

Corrected Copy

There are FIVE questions on this paper.

There are two sections. Answer THREE questions including at least ONE question from each section.

QUESTION 1 is COMPULSORY

Use a separate answer book for each section.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : K. Fobelets, D. Popovic

Second Marker(s) : K.D. Leaver, B.C. Pal

Section A:
Answer at least ONE question. Use a separate answer book for each section.

Special Information for Invigilators:

1) Hand out sheet with Figure 3.2, which should be tied into the answer book.
2) Inform the candidates that question 1 of section A is obligatory.

Information for Candidates:

Question 1 of section A is obligatory.

Constants:

n_i	=	$1.45\ 10^{10}\ cm^{-3}$ for Si at 300K
kT/e	=	0.026 V at 300 K
e	=	$1.6\ 10^{-19}$ C
ε_0	=	$8.854\ 10^{-12}$ F/m
N_{CSi}	=	$2.8\ 10^{19}\ cm^{-3}$
N_{VSi}	=	$1.04\ 10^{19}\ cm^{-3}$
ε_r	=	4 for SiO_2
ε_r	=	11.7 for Si

Formulae:

$$\frac{n_p'}{n_p} = \frac{p_n'}{p_n} = \exp\left(\frac{eV}{kT}\right)$$

$$w_n^2 = \frac{2\varepsilon}{e}\left[\frac{N_A}{N_A N_D + N_D^2}\right](V_0 - V), \text{ with } \varepsilon = \varepsilon_0\,\varepsilon_r$$

$$w_p^2 = \frac{2\varepsilon}{e}\left[\frac{N_D}{N_A N_D + N_A^2}\right](V_0 - V)$$

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$I_{DS} = C_{OX}\,\mu_e \frac{W}{L}\left[(V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right]$$

1. Obligatory question

a) On a schematic diagram of a pn-diode, draw the direction of hole and electron flows across the junction when the diode is in reverse bias. [2]

b) Name the kind of contact formed when a metal with workfunction ϕ_m is deposited onto a p-type semiconductor with workfunction ϕ_s, when $\phi_m < \phi_s$? Draw the energy band diagram of this contact without external bias voltage. [3]

c) Calculate, at room temperature, the total depletion layer width in a Si pn diode under a reverse bias of -1V with $N_A=10^{16}$ cm^{-3}, $N_D=10^{19}$ cm^{-3} and a diode diameter of 50μm. Make the necessary assumptions to speed up your calculations. [2]

d) Why is the current gain β of a bipolar transistor worse in inverse mode than its performance in normal mode? Give your answer in terms of the emitter injection efficiency γ and the base current. [4]

e) When switching a bipolar transistor, in common emitter configuration, from fully on (i.e. saturation) to off, a delay time t_{sd} occurs before the base-collector junction can go to reverse bias. Explain briefly the reason for this delay. [2]

f) What is the sign of the threshold voltage for a p-channel depletion mode MOSFET? Explain briefly why it has this sign. [2]

g) Derive an expression for the transconductance of a MOSFET in the triode region. [2]

h) Sketch the energy band diagram as a function of distance, from source to drain through the Si channel (ie along the length of the channel) of a p-channel enhancement mode MOSFET, when $V_{GS}>>V_{th}$ and $V_{DS}=0$V. (include source and drain regions). [3]

2. BJT

a) Draw the large signal equivalent circuit of a pnp BJT, based upon the Ebers-Moll equations given below,
$I_E = \alpha_I I_C + I_{EO} (\exp[eV_{EB}/kT] - 1)$
$I_C = \alpha_N I_E - I_{CO} (\exp[eV_{CB}/kT] - 1)$
These equations assume that the emitter current flows into the emitter terminal, the collector current out of its terminal. Include the base terminal and the positive base current direction at its terminal. [6]

b) A certain n^+pn transistor has equal doping concentration in base and collector. Sketch graphs of the minority carrier concentration as a function of distance in each layer of the BJT in:

(i) cut-off

(ii) saturation

Neglect recombination, and include the depletion widths. Make sure that the relative level of the minority carrier concentrations compared to the equilibrium levels, is correct! [6]

c) *Derive an expression* for the collector current per unit area through an ideal n^+pn BJT in terms of base width and doping concentrations, assuming that 1) all electrons injected by the emitter reach the collector and 2) the emitter current consists of electrons only. Ignore the emitter-base depletion region but take the depletion region at the collector side into account to calculate the effective base width.

Calculate the values for the collector current density at room temperature, when $V_{EB} = 0.6$ V forward bias, for both $V_{CB} = -1$V and -10V bias. For transistor parameters:

$D_p = 10$ cm^2/s (diffusion constant for holes)
$D_n = 25$ cm^2/s (diffusion constant for electrons)
$W_B = 1$ μm (base width)
$N_D = 10^{18}$ cm^{-3} (emitter doping)
$N_A = 10^{16}$ cm^{-3} (base doping)
$N_D = 10^{16}$ cm^{-3} (collector doping) [8]

3. MOSFET

a) Sketch the transfer function (I_{DS}-V_{GS}) of an n-channel depletion mode MOSFET in the triode region. Indicate the threshold voltage. [4]

b) Give the energy band diagram in the y direction, from the semiconductor-oxide junction to the substrate *through* the channel for an n-channel MOSFET in inversion at pinch-off, both at the source (x=0) and at the drain (x=L) end of the channel (see Figure 3.1). Indicate the inversion and depletion width. [6]

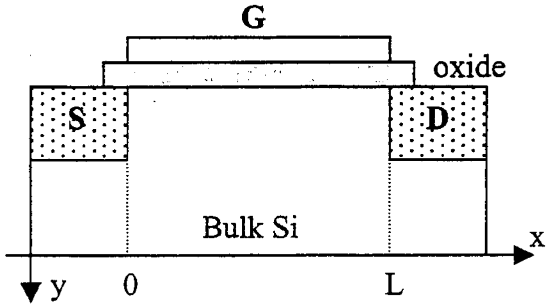


Figure 3.1: Cross section of MOSFET

c) Measurements of a MOSFET, give the saturation transconductance g_{msat} as a function of the gate voltage V_{GS}. This measurement is given in Figure 3.2. The carrier mobility is 10^3 cm^2/Vs, the gate width 0.0141 cm and the gate length 1 μm.

(i) Derive an expression for the transconductance in saturation in terms of mobility μ, gate width W and gate length L.

(ii) Derive the threshold voltage from Figure 3.2 (Answer on the answer sheet)

(iii) Derive the thickness of the oxide from Figure 3.2 (Answer on the answer sheet). [10]

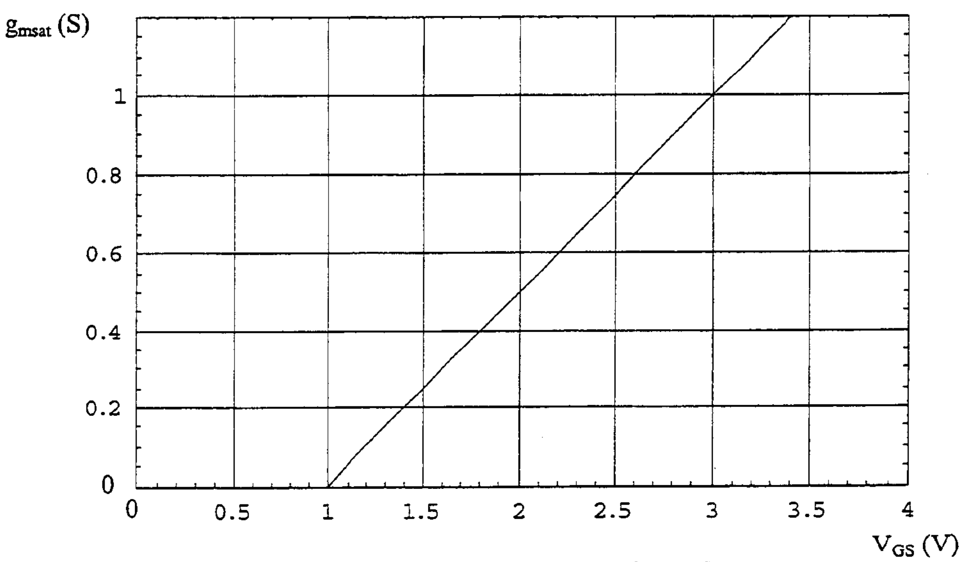


Figure 3.2: g_{msat} as a function of V_{GS}

SECTION B

Answer at least ONE question. Use a separate answer book for each section

4. (a) What is a conservative field? [3]

   (b) Explain why

   $$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = 0$$

   could be regarded as a mathematical statement of Kirchhoff's voltage law? [5]

   (c) Consider a coaxial capacitor filled with insulating material of permittivity ϵ as shown in Figure 4.1. Assume that charge is uniformly distributed along the length of the conductors with line charge density $\rho_l = Q/l$ on the outer conductor and $-\rho_l$ on the inner conductor. Ignore fringing fields near the ends of the coaxial line.

   (i) Obtain an expression for the capacitance of the coaxial capacitor. [4]

   (ii) At what value of r, $a \leq r \leq b$ is $|\boldsymbol{E}\,|$ maximum? [4]

   (iii) What is the breakdown voltage if $a = 1$ cm, $b = 2$ cm, and the dielectric has the relative permittivity of $\epsilon_r = 6$. The permittivity of free space is $\epsilon_o = 8..854 \times 10^{-12}$ F/m. Assume that dielectric breaks down when $|\boldsymbol{E}\,| = 200$ MV/m. [4]

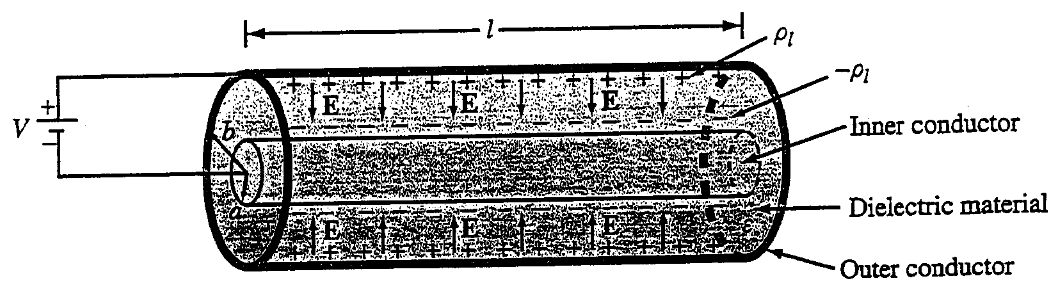


Figure 4.1

5. (a) Answer and explain

   (i) What is the shape of the lines of vector $\boldsymbol{B}$ of a single current element? Is the magnitude of $\boldsymbol{B}$ constant along these lines? Is $\boldsymbol{B}$ constant along these lines? [2]

   (ii) A straight conductor with a current I passes through the center of a sphere of radius R. What is the magnetic flux through the spherical surface? [2]

   (iii) A charge Q is moving along the axis of a circular current-carrying contour normal to the plane of the contour. Discuss the influence of the magnetic field on the motion of the charge. [2]

   (iv) Does the shape of a dc circuit influence the currents in its branches? [2]

(b) The loop of area A is in the $x-y$ plane as shown in Figure 5.1 and $\boldsymbol{B} = \hat{z}B_o \sin wt$ with B_o positive. What is the direction of current I at

   (i) $t = 0$ [6]

   (ii) $wt = \pi/4$ [3]

   (iii) $wt = \pi/2$. [3]

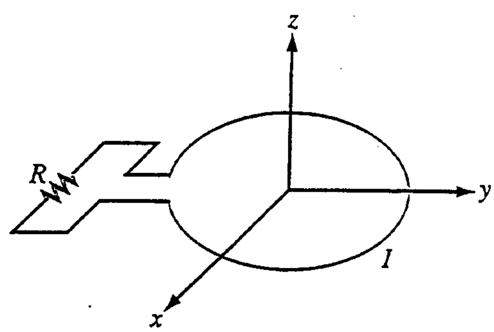
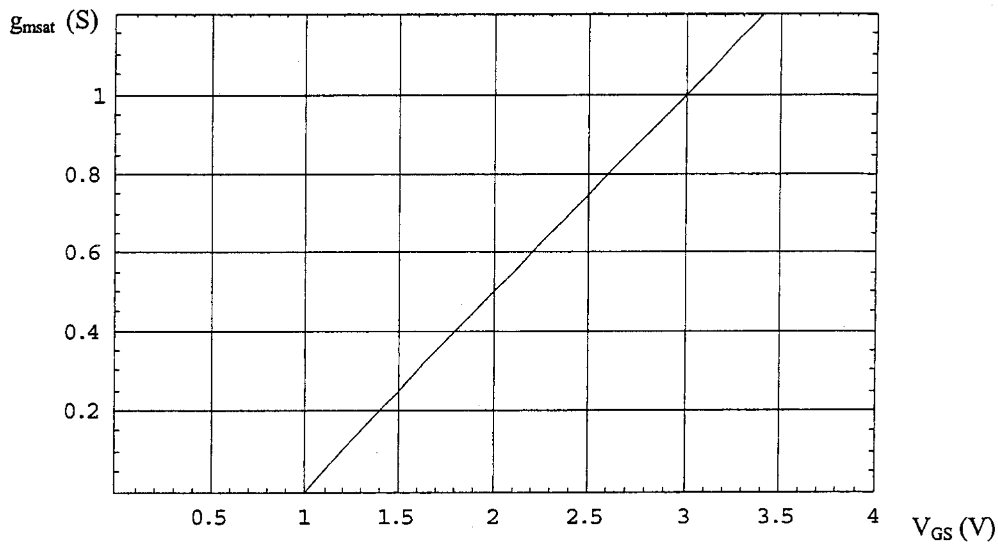


Figure 5.1

Section A Question 3 Figure 3.2

Answer sheet

Figure 3.2: g_{msat} as a function of V_{GS}



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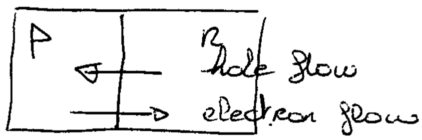
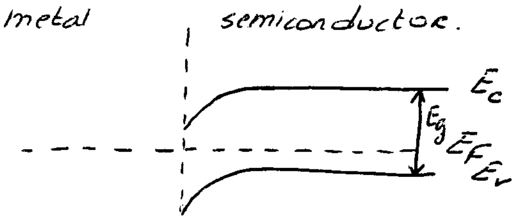
Answers

Q1. a)



b) Schottky contact



[bands bending down]

c) Assumption: since $N_A \ll N_D$ the depletion region in the n Region is negligible : $w_n \ll w_p$

$\Rightarrow w_{tot} \simeq w_p$

$$w_p = \sqrt{\frac{2\varepsilon_0 \varepsilon_r}{e}\left[\frac{N_D}{N_A N_D + N_A^2}\right](V_0 - V)}$$

$$V_0 = \frac{kT}{e} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$V_0 = 0.026\,V \ln\left(\frac{10^{16}\ 10^{19}}{1.45\ 10^{10}}\right) = 0.879\ V$$

$w_p \simeq 4.97\ 10^{-5}$ cm $\qquad (w_n \simeq 4.97\ 10^{-8}$ cm$)$

d) * The emitter injection efficiency γ in normal mode is larger than in reverse mode since the emitter doping is much higher than the collector. Therefore the E-B junction is a better 1-type carrier injector than the CB-junction.

* The base current in inverted mode will be higher for the same biasing conditions since the amount of carriers flowing back from B to C is larger (INVERTED MODE) than the amount of carriers flowing back from B to E (NORMAL MODE)

e) The delay time t_{sd} is caused by discharging (reduction of carriers) the base. This discharging is happening via a carrier flow out of the base and recombination of carriers in the base.

f) V_{th} is positive. Since a channel of holes exists at $V_{GS} = 0$ we have to apply a positive voltage to remove the positively charged carriers from the base.

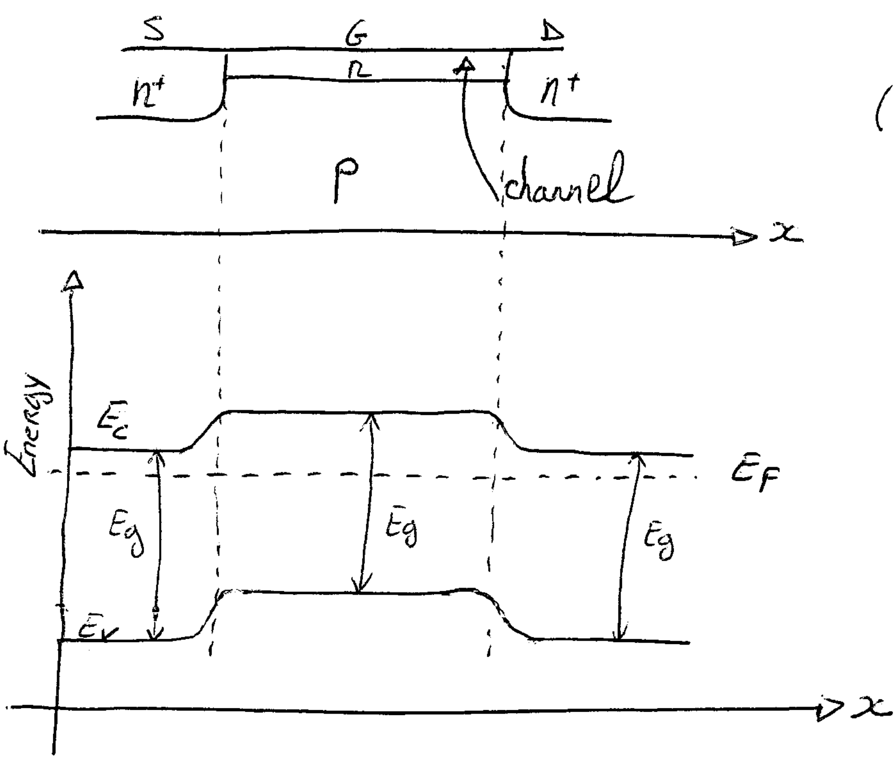
g)
$$I_{DS} = \frac{\mu W C_{ox}}{L}\left((V_{GS} - V_T)\, V_{DS} - \frac{1}{2} V_{DS}^2 \right)$$

saturation: $V_{DS} = V_{GS} - V_T$

$$I_{DS}^{sat} = \frac{\mu W C_{ox}}{2L} (V_{GS} - V_T)^2$$

$$g_m^{sat} = \frac{d I_{DS}^{sat}}{d V_{GS}} = \frac{\mu W C_{ox}}{L} (V_{GS} - V_T)$$

h)



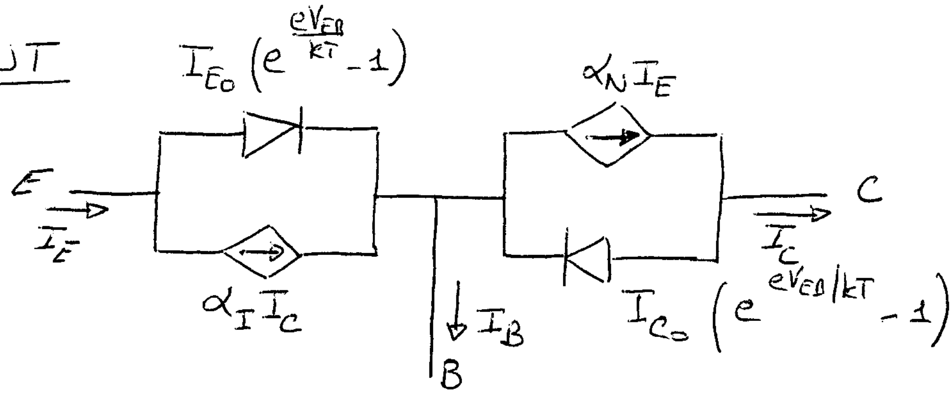
(not necessary, but helpfull)

$V_{DS} = 0 \Rightarrow V_x(x) = 0$

$\Rightarrow E_F$ constant

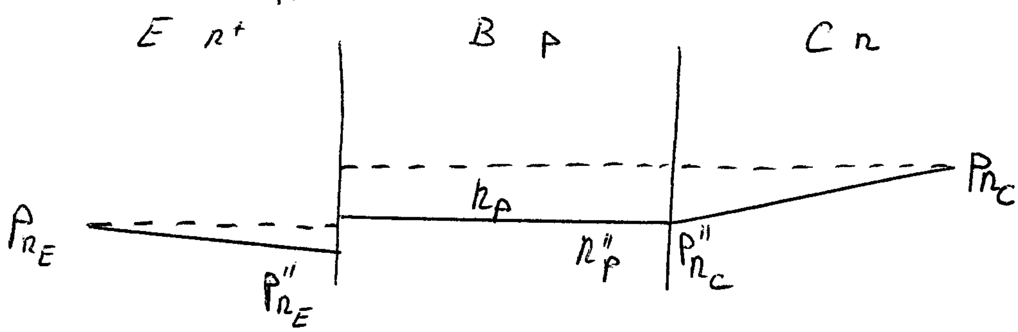
Q2 BJT

a)

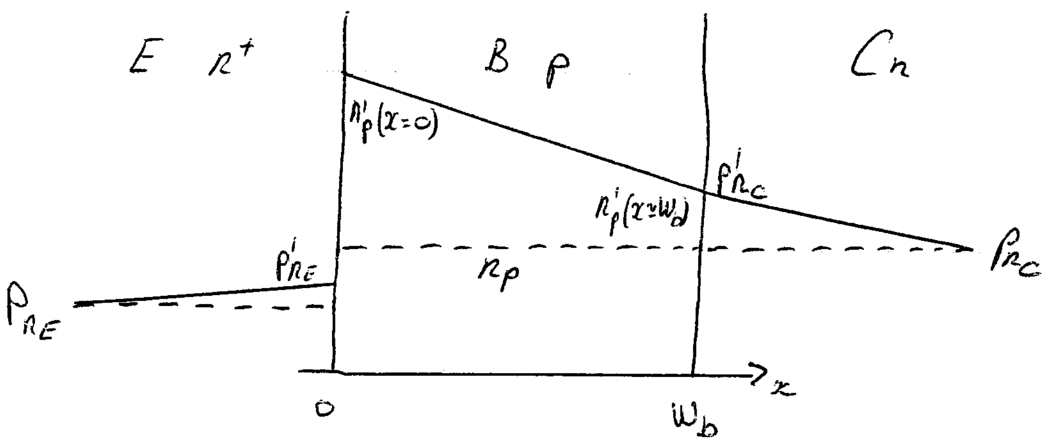


b) (dashed lines indicate equilibrium.

i) cut-off



ii) saturation



c) metallurgic basewidth W_b

when the depletion in the base-collector junction increases, the effective base width reduces; calculate depletion width extending in base w_p between base-collector junction.

$$V_0 = \frac{kT}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 0.026\,V \ln\left(\frac{10^{16}\,10^{16}}{(1.4\,10^{10})^2}\right) = 0.699\,V \simeq 0.7\,V$$

$$w_p = \sqrt{\frac{\varepsilon_0 \varepsilon_r}{2}\left(\frac{N_D}{N_A(N_A+N_D)}\right)(V_0 - V)}$$

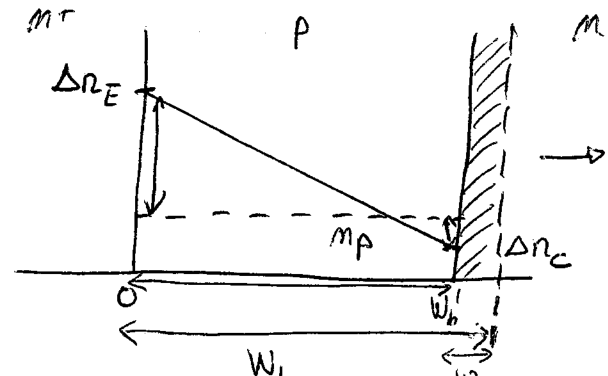
w_p ~~for~~ $V_{CB} = -1V$

$w_p = 3.34\;10^{-5}$ cm

w_p for $V_{CB} = -10V$

$w_p = 8.39\;10^{-5}$ cm

Current when $\gamma = 1$ $\alpha = 1$ (no recombination) in n^+pn bipolar is electron current only.



→ linear excess carrier variation (no recombination)

where $W_b' = W_b - w_p$

$$\Delta n_E = n_p' - n_p = n_p\left(e^{e|V_{EB}|/kT} - 1\right)$$
$$\Delta n_c = n_p'' - n_p = n_p\left(e^{-e|V_{CB}|/kT} - 1\right) \simeq -n_p$$

excess carrier concentration;

$$\delta n_p = ax + b$$

$$\begin{cases} @\ x = 0 & \delta n_p = \Delta n_E \\ @\ x = W_b' & \delta n_p = -n_p \end{cases}$$

$$\Rightarrow b = \Delta n_E$$
$$a = -\frac{(n_p + \Delta n_E)}{W_b'}$$

$$\delta n_p = -\frac{(n_p + \Delta n_E)}{W_b'} x + \Delta n_E$$

$$= -\frac{\left(n_p + \left(n_p \exp\left(\frac{eV_{EB}}{kT}\right) - n_p\right)\right)}{W_b'} x + \Delta n_E$$

$$= -\frac{n_p}{W_b'} \exp\left(\frac{eV_{EB}}{kT}\right) x + \Delta n_E$$

Current: diffusion $\quad I_e = e D_e \frac{d\,\delta n_p}{dx}$

$$I_e = \frac{e D_e n_p}{W_b'} \exp\left(\frac{eV_{EB}}{kT}\right)$$

since $\alpha = 1 \quad I_c = I_e$

$$I_c = \frac{e D_e n_i^2}{W_b' N_A} \exp\left(\frac{0.6}{0.026}\right)$$

for $V_{CB} = -1V$

$$I_c = \frac{1.6\ 10^{-19} C \quad 25 \frac{cm^2}{s} \left(1.45\ 10^{10} cm^{-3}\right)^2}{\left(10^{-4} cm - 3.34\ 10^{-5} cm\right) 10^{16} cm^{-3}} \exp\left(\frac{0.6}{0.026}\right)$$

$$= 13.3 \quad A/cm^2$$

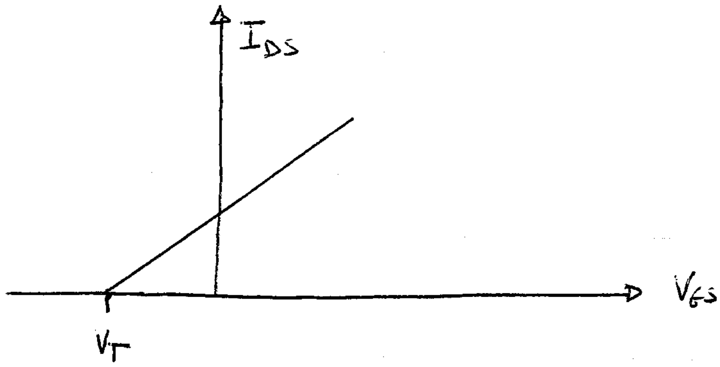
for $V_{CB} = -10V$

$$I_c = 55.04 \quad A/cm^2$$

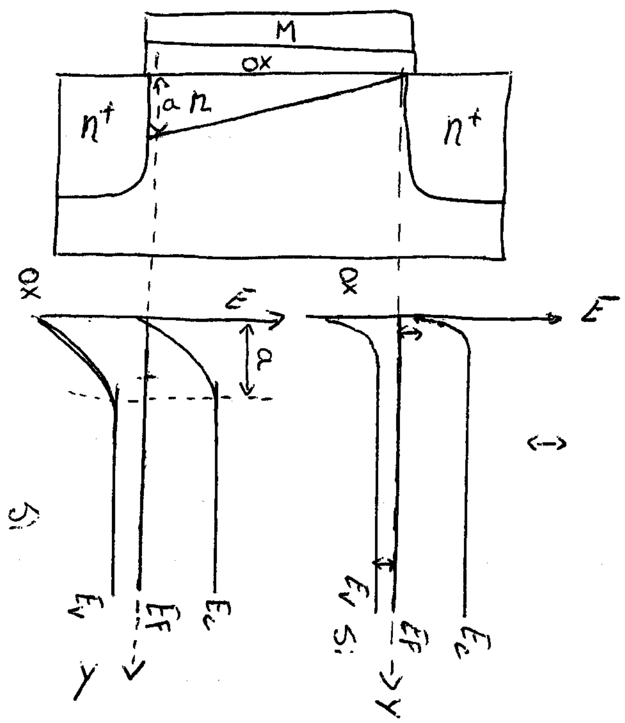
Note if W_b is used instead of W_b' then

$$I_c = 11.2\cancel{7}\ A/cm^2$$

Q3 a)



b)



$\leftrightarrow$ $E_c - E_F$ @ junction $\equiv$ $E_F - E_v$ @ contact

c) i) $I_{DS} = \mu_e \frac{W}{L} C_{ox} \left[(V_{GS} - V_{th}) V_{DS} - \frac{V_{DS}^2}{2} \right]$

$V_{DS} = V_{GS} - V_T$: $I_{DS}^{sat} = \frac{\mu_e W C_{ox}}{2L} (V_{GS} - V_{th})^2$

$g_m^{sat} = \frac{d I_{DS}^{sat}}{d V_{GS}} = \frac{\mu_e W C_{ox}}{L} (V_{GS} - V_{th})$

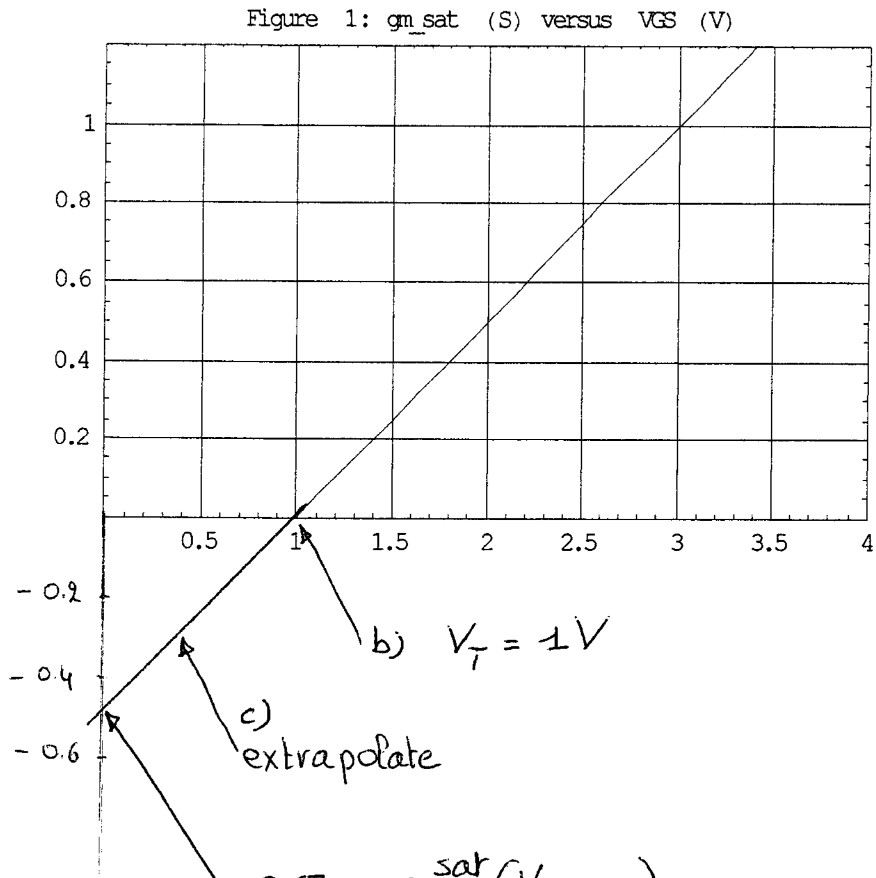
ii) if $g_m^{sat} = 0 \Rightarrow \boxed{V_{GS} = V_T}$

see graph on next page

iii) if $V_{GS} = 0 \Rightarrow g_m^{sat} = - \frac{\mu_e W \varepsilon_0 \varepsilon_R}{L\, t_{ox}} V_{TR}$

thus: $t_{ox} = - \frac{\mu_e W \varepsilon_0 \varepsilon_R V_{TR}}{L\, g_m^{sat}(V_{GS} = 0)}$

EE1.3 Semiconductor Devices



$-0.5 = g_m^{sat}(V_{GS} = 0)$

$$t_{ox} = \frac{\mu_e W \varepsilon_0 \varepsilon_R \, 1V}{L \cdot 0.55} = \frac{10^3 \; 0.0141 \; 8.85 \; 10^{-14} \; 4}{10^{-4} \; 0.5}$$

$$t_{ox} \simeq 10^{-7}\,\text{cm} \quad (9.98 \; 10^{-8}\,\text{cm})$$

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4. a) (BOOKWORK)

A vector field whose line integral along any closed path is zero is called a conservative field. 3

$$\oint_C \underline{E} \cdot d\underline{l} = 0 \quad \text{(electrostatics)}$$

electrostatic field

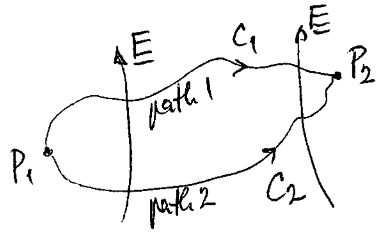
b) (BOOKWORK)

Consider a potential difference between any two points P_1 and P_2

$$\int_{P_2}^{P_1} dV = V_2 - V_1 = V_{21} = -\int_{P_1}^{P_2} \underline{E} \cdot d\underline{l}$$

The voltage difference between two nodes in an electric circuit has the same value regardless of which path in the circuit we follow between the nodes. 5

Moreover, Kirchhoff's voltage law states that the net voltage drop around a closed loop is zero.



path 1 : $P_1 \to P_2$
path 2 : $P_2 \to P_1$ $\Rightarrow$ $\int_{P_1}^{P_2}$ becomes a closed contour

$\Rightarrow V_{21} = 0$.

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4. c) (i) (Bookwork)

Construct a cylindrical Gaussian surface around the inner conductor with radius r, $a<r<b$.

Line charge of the inner conductor is $(-\rho_l)$.

$$\Rightarrow \quad \underline{E} = \hat{\underline{r}} \frac{-\rho_l}{2\pi\varepsilon r} = -\hat{\underline{r}} \frac{Q}{2\pi\varepsilon r l}$$

$$\Rightarrow \quad V = -\int_a^b \underline{E} \cdot d\underline{l} = -\int_a^b \left(-\hat{\underline{r}} \frac{Q}{2\pi\varepsilon r l}\right) (\hat{\underline{r}}\, dr)$$

$$= \frac{Q}{2\pi\varepsilon l} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon l}{\ln(b/a)}$$ 4

(ii) $$\underline{E} = -\hat{\underline{r}} \frac{\rho_l}{2\pi\varepsilon r}$$ 4

$|\underline{E}|$ is maximum at $r=a$

(iii) (computed example)

$$|\underline{E}| = \frac{\rho_l}{2\pi\varepsilon \underset{\overset{\shortparallel}{a}}{r}} = \frac{\rho_l}{2\pi(6\varepsilon_0)10^{-2}} = 200\ [MV/m]$$ 4

$$\Rightarrow \rho_l = 667.6\ [\mu C/m]$$

$$V = \frac{\rho_l}{2\pi\varepsilon} \ln\left(\frac{b}{a}\right) = \frac{667.6 \cdot 10^{-6}}{2\pi \cdot 6 \cdot 8.854 \cdot 10^{-12}} \ln 2 = 1.39\ [MV]$$

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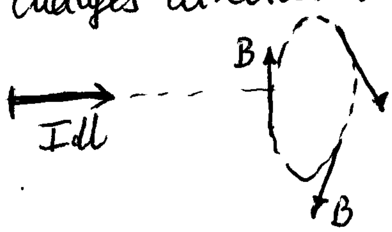
5. (a) (Bookwork)

(i) The lines of $\underline{B}$ are circles centered on the line carrying the element. Due to symmetry, the magnitude of $\underline{B}$ is constant along such a line. The vector $\underline{B}$ is not constant along such a line because it changes direction when going around such a circle. 2



(ii) Zero, because it is always zero. In this case even more so since the lines of $\underline{B}$ are tangential to the spherical surface. 2

(iii) $\underline{B}$ on the circular contour axis is directed along the axis. If the charge is moving along the axis, it is moving parallel to $\underline{B}$ $\Rightarrow$ the magnetic force on the charge is zero. 2

(iv) it does not since there is no induced electric field due to dc currents. 2

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5 (b) (New computed example)

$$I = \frac{V_{emf}}{R}$$

Loop is not moving or changing shape with time

$\Rightarrow V_{emf}^{m} = 0, \quad V_{emf} = V_{emf}^{tr}$

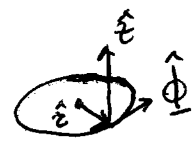
$$V_{emf}^{tr} = -N \int_S \frac{\partial \underline{B}}{\partial t} \cdot d\underline{s}$$

$$I = \frac{V_{emf}^{tr}}{R} = -\frac{1}{R} \int_S \frac{\partial \underline{B}}{\partial t} \cdot d\underline{s}$$ 4

Choose the surface normal as $+\hat{\underline{z}}$. Apply the right hand rule $\Rightarrow$ positive current flowing in the $\hat{\underline{\Phi}}$ direct.

$$I = -\frac{A}{R}\frac{\partial}{\partial t}(B_0 \sin \omega t) = -\frac{AB_0\omega}{R}\cos \omega t$$

A, ω, R – all positive



(i) $t=0 \Rightarrow \omega t = 0$ ie $\cos \omega t = 1$

$\Rightarrow I < 0$ and the current is ~~flowing~~ in the $-\hat{\underline{\Phi}}$ direction so as to produce an induced magnetic field that opposes $\underline{B}$. 2

(ii) $\omega t = \pi/4 \Rightarrow \cos \omega t = \sqrt{2}/2 \Rightarrow I < 0$ and still in $(-\hat{\underline{\Phi}})$ direction 3

(iii) $\omega t = \pi/2 \Rightarrow \cos \omega t = 0 \Rightarrow I = 0$. There is no current flowing in either direction. 3

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