IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE/EIE PART III/IV: MEng, BEng and ACGI

11:00

Correction

Fig 4.1

DIGITAL SIGNAL PROCESSING

Corrected Copy

Monday, 15 December 9:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.A. Naylor
Second Marker(s) : W. Dai

Digital Signal Processing

1. Consider an FIR filter of order $M-1$.

   a) Write down the z-domain system function for this filter and the expression for the frequency response of this filter. [2]

   b) Explain the key properties of a linear phase FIR filter and describe the corresponding characteristics of the filter coefficients and the roots of the system function. [4]

   c) Show that the frequency response of a linear phase FIR filter with M being odd can be written

$$H(e^{j\omega}) = e^{j\lambda}\left\{h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\alpha} 2h(n)\cos\left(\omega(n-\beta)\right)\right\}$$

   and give expressions for λ, α and β. [6]

   d) Explain the meaning of group delay and state an expression for the group delay of a linear phase FIR filter. [3]

   e) Find the coefficients of a linear phase FIR filter with $M=5$ satisfying the specification

$$|H(e^{j0})| = 0 \quad \text{dB}$$
$$|H(e^{j\frac{\pi}{3}})| = -6 \quad \text{dB}$$
$$|H(e^{j\pi})| = -\infty \quad \text{dB}.$$

[5]

2. a) Give the definition of the z-transform $X(z)$ of a discrete-time signal $x(n)$. State the meaning of the *region of convergence* of the z-transform. [3]

b) Write the two-sided signal $x(n) = a^n$ as the sum of two one-side functions. By considering the z-transform of each one-sided function, show that the z-transform of $x(n) = a^n$ has no region of convergence. [4]

c) Consider the linear system

$$H(z) = \frac{1 - 1.2z^{-1}}{1 + 0.4z^{-1} - 0.12z^{-2}}.$$

Calculate the first 5 non-zero sample amplitudes of the impulse response of the system by finding the inverse z-transform of $H(z)$ using long division. [7]

d) Construct the signal flow graph for $H(z)$ using the minimum number of delay elements. [6]

3. a) Let $x(n)$ be a discrete time signal and let $X(k)$ be the DFT of $x(n)$. Consider $\hat{x}(n)$ to be the inverse DFT of $X(k)$. By considering the formulae for the forward and inverse DFT, prove that

$$\hat{x}(n) = x(n).$$

[4]

b) Consider two finite duration discrete time signals, $p(n)$ and $q(n)$, given by

$$p(n) = [2, -1, 1, 0]$$
$$q(n) = [-3, -1, 0, -2].$$

i) Give the formula for circular convolution of $p(n) \circledast q(n)$ and hence compute the sample values of

$$r(n) = p(n) \circledast q(n)$$

where $\circledast$ represents circular convolution. Show and explain your method. [6]

ii) Given that $R(k)$, $P(k)$ and $Q(k)$ are the DFTs of $r(n)$, $p(n)$ and $q(n)$ respectively, prove that $R(k) = P(k)Q(k)$. [4]

c) Briefly explain how linear convolution can be implemented using circular convolution. [4]

d) Briefly explain how circular cross-correlation can be implemented using circular convolution. [2]

4. All filters in this question can be assumed ideal.

Consider a signal with sampling frequency $fs = 336$ kHz applied at the input to a downsampling process employing decimation and ideal filtering. The downsampling factor is 21.

a) i) Draw a labelled block diagram of a DSP system that performs the downsampling process given an input signal $x(n)$ and an output signal $y(m)$. Write an expression for $y(m)$ in terms of $x(n)$. [2]

ii) Draw a labelled block diagram of a DSP system that performs the same operation as in (i) but employing two downsampling processes in cascade. [2]

iii) For the DSP system in (ii), give the sampling frequency of all signals in the block diagram. Also state the passband edge frequencies of filters employed. [3]

b) Consider the system of Fig. 4.1. The input and output signals are denoted $x(n)$ and $y(m)$ respectively and two intermediate signals are shown as $p(l)$ and $q(l)$.

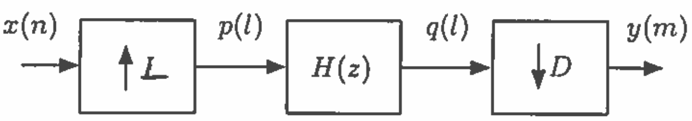

Figure 4.1

i) Write expressions for $p(l)$ and $q(l)$ and $y(m)$. [4]

ii) Find an expression in the frequency domain for the output signal in terms of the input signal. [4]

c) Given an appropriate lowpass filter $H(z)$ with impulse response

$$h(n) \quad n = 0, 1, \ldots, N-1,$$

design an interpolator with interpolation factor $L = 2$ that exploits the Type 2 polyphase decomposition

$$H(z) = E_1(z^2) + z^{-1} E_0(z^2)$$

to achieve computational efficiency.

Show the signal flow diagram for your design and specify fully all processing functions and filter impulse responses. [5]