Optoelectronics Solutions 2015

1. Propagation of TE polarized waves in the (x, z) plane may be described by the scalar wave equation $\partial^2E_y/\partial x^2 + \partial^2E_y/\partial z^2 + n^2k_0^2E_y = 0$

a) If $E_y(x, z) = E_0 \exp\{-jnk_0[x \sin(\theta) + z \cos(\theta)]\}$ then;

$\partial^2E_y/\partial x^2 = \{-jnk_0 \sin(\theta)\}^2 E_y = -n^2k_0^2 \sin^2(\theta) E_y$

$\partial^2E_y/\partial z^2 = \{-jnk_0 \cos(\theta)\}^2 E_y = -n^2k_0^2 \cos^2(\theta) E_y$

Hence, $\partial^2E_y/\partial x^2 + \partial^2E_y/\partial z^2 + n^2k_0^2E_y = n^2k_0^2\{- \sin^2(\theta) - \cos^2(\theta) + 1\}E_y = 0$

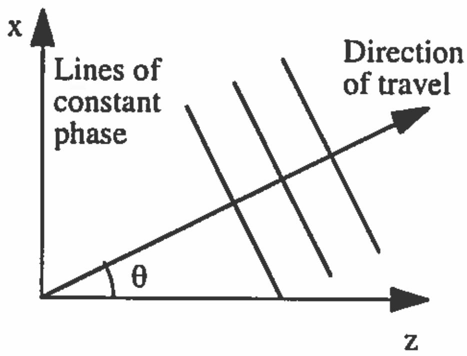
Consequently, the assumed field distribution satisfies the wave equation for all θ.

[3]

This solution represents a plane wave, travelling at an angle θ to the z-axis.

[1]

The wave amplitude is constant; lines of constant phase can be drawn thus:



[2]

a) If $E_y(x, z) = E_0 \exp(\gamma x) \exp(-j\beta z)$ then;

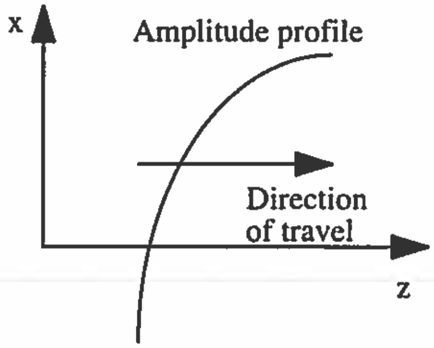
$\partial^2E_y/\partial x^2 = \gamma^2E_y$ $\qquad$ $\partial^2E_y/\partial z^2 = (-j\beta)^2 E_y = -\beta^2E_y$

Hence, $\partial^2E_y/\partial x^2 + \partial^2E_y/\partial z^2 + n^2k_0^2E_y = (\gamma^2 - \beta^2 + n^2k_0^2)E_y$

Consequently, the assumed field distribution satisfies the wave equation, if $\gamma^2 = \beta^2 - n^2k_0^2$.

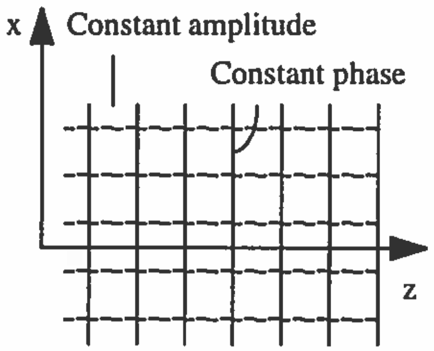
[2]

This solution represents an inhomogeneous wave (i.e. a wave whose amplitude is not constant), travelling in the z-direction, thus:



[2]

Lines of constant amplitude and phase can be drawn thus:



[2]

This solution is unrealistic, since the amplitude tends to infinity as x tends to infinity. However, it can still represent a real field if x is finite, for example in the half-space below an interface. In this case it can represent the evanescent field of a guided mode.

[2]

c) If $E_y = E(x)\exp(-j\beta z)$ then: $\partial^2E_y/\partial x^2 = d^2E/dx^2 \exp(-j\beta z)$ $\partial^2E_y/\partial z^2 = -\beta^2 E \exp(-j\beta z)$

Hence $\partial^2E_y/\partial x^2 + \partial^2E_y/\partial z^2 + n^2k_0^2E_y = \{d^2E/dx^2 + (n^2k_0^2 - \beta^2)E\} \exp(-j\beta z) = 0$

And the waveguide equation must be $d^2E/dx^2 + (n^2k_0^2 - \beta^2)E = 0$

[2]

If $E(x) = E_0 \exp(\gamma x)$ then $d^2E/dx^2 + (n^2k_0^2 - \beta^2)E = \{\gamma^2 + n^2k_0^2 - \beta^2\}E$

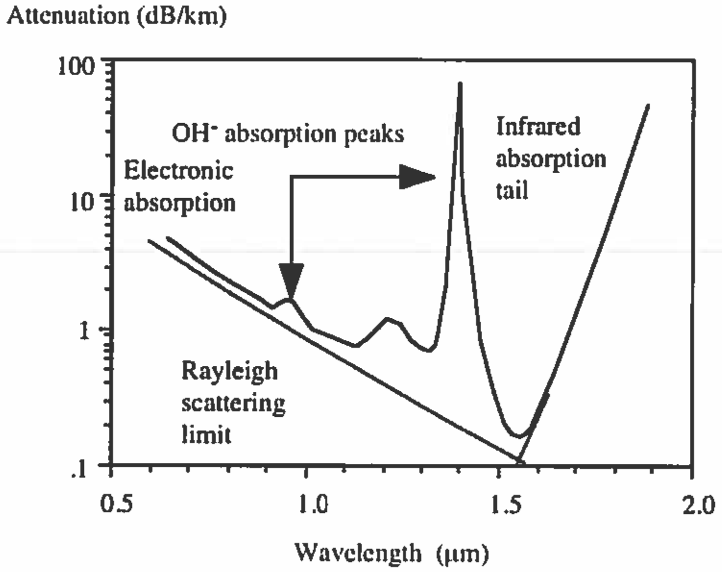
Consequently, the solution is valid provided $\gamma^2 = \beta^2 - n^2k_0^2$, which requires $\beta > nk_0$.

[2]

Solutions for $nk_0 > \beta$ are $E(x) = E_0 \sin(\kappa x)$ and $E_0 \cos(\kappa x)$, where $k^2 = (n^2k_0^2 - \beta^2)$.

[2]

2.a) Spectral variation of loss for silica fibre:

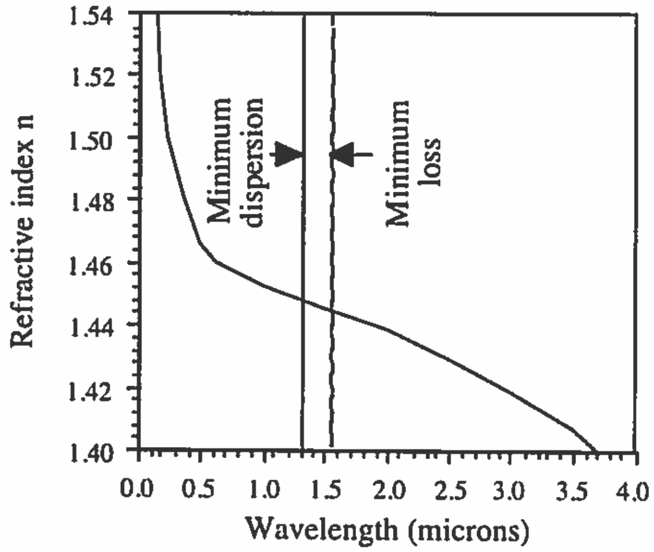


[4]

The optimum operating wavelength for long-distance communications is the one that gives minimum loss. For silica fibre, this wavelength lies at the intersection between the limiting envelopes set by Rayleigh scattering at short wavelength and infrared absorption at long wavelength (around 1.55 μm).

[2]

b) Spectral variation of refractive index for silica fibre:



[2]

The optimum wavelength for high bit-rate communications is the one that gives minimum dispersion, which occurs when $d^2n/d\lambda^2 = 0$. For silica fibre, this inflection lies at 1.3 μm.

[2]

Clearly, the wavelengths for minimum loss and minimum dispersion are different for silica fibre. However, there is an additional contribution to dispersion arising from the shape of the fibre core. By carefully engineering this shape, the dispersion minimum may be shifted to 1.55 μm.

[2]

c) The broadening of a pulse of bandwidth $\Delta\omega$ in a distance L is $\Delta T = L\, d(1/v_g)/d\omega\, \Delta\omega$
Since the group velocity is $v_g = d\omega/dk$, we must have $\Delta T = L\, \Delta\omega\, d^2k/d\omega^2$

[1]

Since the phase velocity is $v_{ph} = \omega/k = c/n$, we must also have $k = n\omega/c$
Consequently $dk/d\omega = \{n + \omega\, dn/d\omega\}/c$

[2]

In free space, $k_0 = 2\pi/\lambda_0 = \omega/c$ so $\lambda_0 = 2\pi c/\omega$
Differentiating, $d\lambda_0/d\omega = -2\pi c/\omega^2 = -\lambda_0/\omega$

[1]

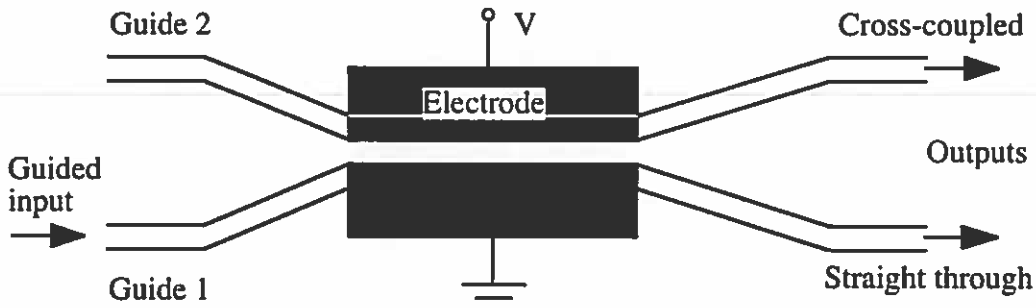
Now $dk/d\omega = \{n + \omega\, (dn/d\lambda_0)\, (d\lambda_0/d\omega)\}/c$
Using the above, we can obtain $dk/d\omega = \{n - \lambda_0\, dn/d\lambda_0\}/c$
And $d(dk/d\omega)/d\lambda_0 = \{dn/d\lambda_0 - \lambda_0\, dn/d\lambda_0 - \lambda_0\, d^2n/d\lambda_0^2\}/c = -(\lambda_0/c)\, d^2n/d\lambda_0^2$

[2]

Now $d^2k/d\omega^2 = d(dk/d\omega)/d\lambda_0\, d\lambda_0/d\omega$
We can approximate this as $d^2k/d\omega^2 = d(dk/d\omega)/d\lambda_0\, (\Delta\lambda_0/\Delta\omega)$
Hence $\Delta T = L\, \Delta\omega\, d^2k/d\omega^2 = -\, L\, \Delta\omega\, (\lambda_0/c)\, d^2n/d\lambda_0^2\, (\Delta\lambda_0/\Delta\omega)$
Or $\Delta T = -\, L\, (\lambda_0/c)\, d^2n/d\lambda_0^2\, \Delta\lambda_0$

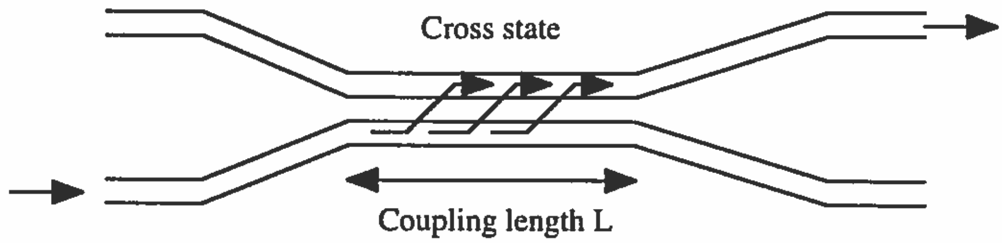
[2]

3a) A directional coupler switch is an arrangement of two channel guides, with a central region where they run parallel and close together. Surface electrodes are used to create an electric field that can alter the effective indices of the two guides in opposite directions via the electro-optic effect in a suitable material.



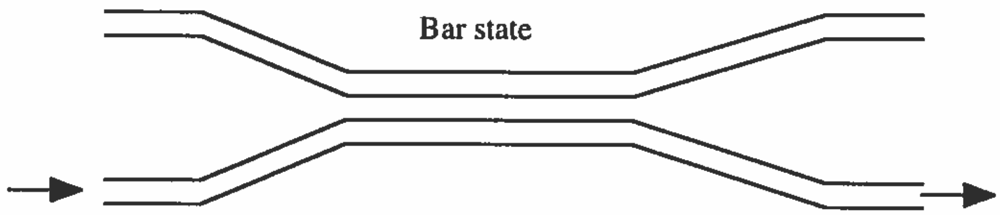
[2]

In the absence of an electric field, light is coupled coherently from one guide to the other. If input is to guide 1 and the coupling length is suitable, the light emerges from guide 2.



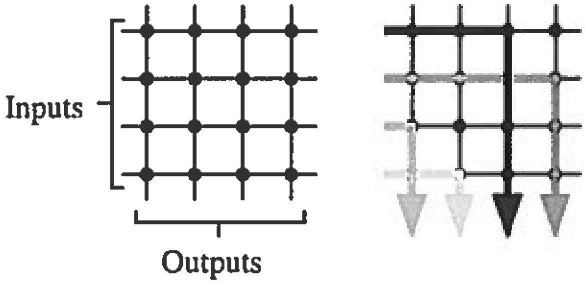
[2]

With an electric field, the coherent coupling is destroyed, and light emerges from guide 1.



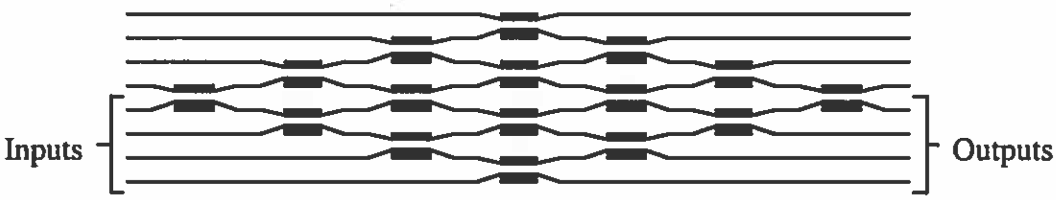
[2]

b) Schematic of a 4 x 4 non-blocking switch:



[1]

Physical arrangement of a 4 x 4 non-blocking switch:



[2]

The number of switches required in a N x N array is N^2.

[1]

c) The general coupled mode equations of an asynchronous directional coupler are:

$dA_1/dz + j\kappa\, A_2 \exp(-j\Delta\beta z) = 0$ (1)

$dA_2/dz + j\kappa\, A_1 \exp(+j\Delta\beta z) = 0$ (2)

Here A_1 and A_2 are the amplitudes of the modes in the two guides, κ is the coupling coefficient and $\Delta\beta$ is the dephasing parameter. To prove power conservation:

Multiply (1) by A_1^*: $A_1^* dA_1/dz + j\kappa\, A_1^* A_2 \exp(-j\Delta\beta z) = 0$ (3)

Multiply the conjugate of (1) by A_1: $A_1 dA_1^*/dz - j\kappa\, A_1 A_2^* \exp(+j\Delta\beta z) = 0$ (4)

Multiply (2) by A_2^*: $A_2^* dA_2/dz + j\kappa\, A_1 A_2^* \exp(+j\Delta\beta z) = 0$ (5)

Multiply the conjugate of (2) by A_2: $A_2 dA_2^*/dz - j\kappa\, A_1^* A_2 \exp(-j\Delta\beta z) = 0$ (6)

Summing (3) – (6) we get $A_1^* dA_1/dz + A_1 dA_1^*/dz + A_2^* dA_2/dz + A_2 dA_2^*/dz = 0$ or:

$d(A_1A_1^*)/dz + d(A_2A_2^*)/dz = 0$ so that $d(P_1 + P_2)/dz = 0$ and $P_1 + P_2 =$ constant

[5]

d) If $\Delta\beta = 0$, the equations simplify to:

$dA_1/dz + j\kappa\, A_2 = 0$ (1)

$dA_2/dz + j\kappa\, A_1 = 0$ (2)

Differentiate (1) to get: $d^2A_1/dz + j\kappa\, dA_2/dz = 0$

Substitute using (2) to get: $d^2A_1/dz + \kappa^2 A_1 = 0$

The general solution is: $A_1 = A \cos(\kappa z) + B \sin(\kappa z)$

So that: $dA_1/dz = -\kappa A \sin(\kappa z) + \kappa B \cos(\kappa z)$

If $A_2 = 0$ at $z = 0$, dA_1/dz must be zero at this point, so $B = 0$ and $A_1 = A \cos(\kappa z)$

If $A_1 = 1$ at $z = 0$, $A = 1$ and $A_1 = \cos(\kappa z)$ and $P_1 = A_1A_1^* = \cos^2(\kappa z)$

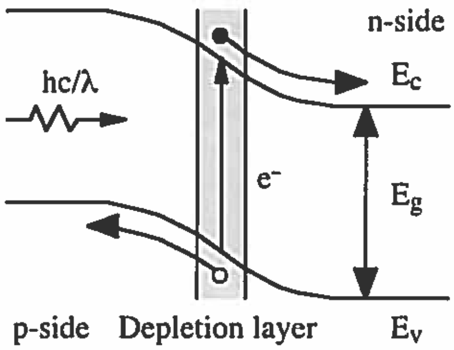
Hence $A_2 = (j/\kappa)\, dA_1/dz = -j \sin(\kappa z)$ and $P_2 = A_2A_2^* = \sin^2(\kappa z)$

Hence $P_1 + P_2 = \cos^2(\kappa z) + \sin^2(\kappa z) = \text{constant}$

[5]

4. a) A photodiode is a p-n homojunction in a semiconductor of bandgap E_g. Photons of energy $hc/\lambda > E_g$ can promote electrons from the valence band to the conduction band. If absorption occurs in the depletion layer, the electrons and holes will be swept apart by the large electric field to the field-free region on either side, and then diffuse to the contacts.

[3]

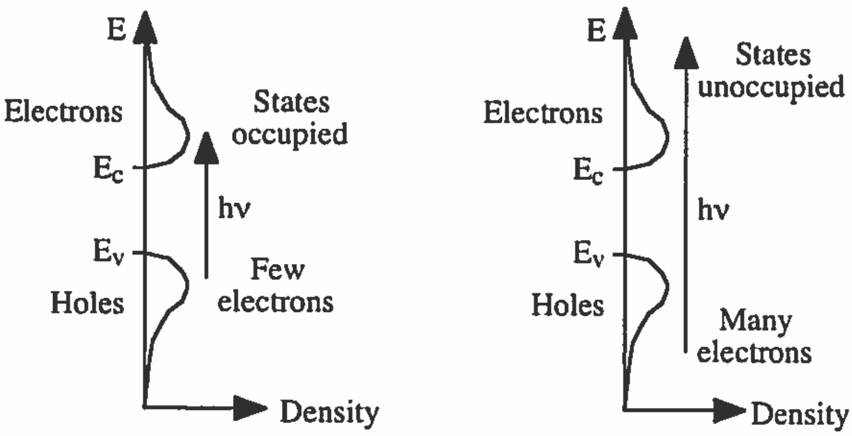


[3]

A photodiode is more efficient than a photoconductive detector because the separation of electrons and holes in the depletion layer reduces recombination.

[1]

b) The distribution of electrons is found from the product of the distribution of states S(E) and the Fermi-Dirac function F(E). The distribution of holes is found from S(E){1 - F(E)}. At room temperature the carriers are clustered near the conduction and valence band edges.



[3]

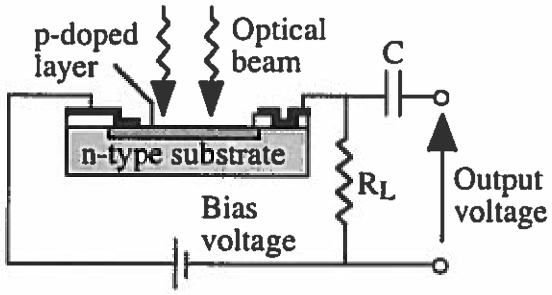
i) A photon with relatively low energy will be able to induce only a relatively small increase in electron energy. Such a transition can only take place from levels containing few electrons to levels containing many. Few empty states will be available for the promoted electron, so the transition rate must be low.

[2]

ii) A photon with high energy will be able to induce a much larger increase in electron energy. Many empty states will be available, so the transition rate will be higher.

[2]

c) The construction of a typical surface entry photodiode is as follows:



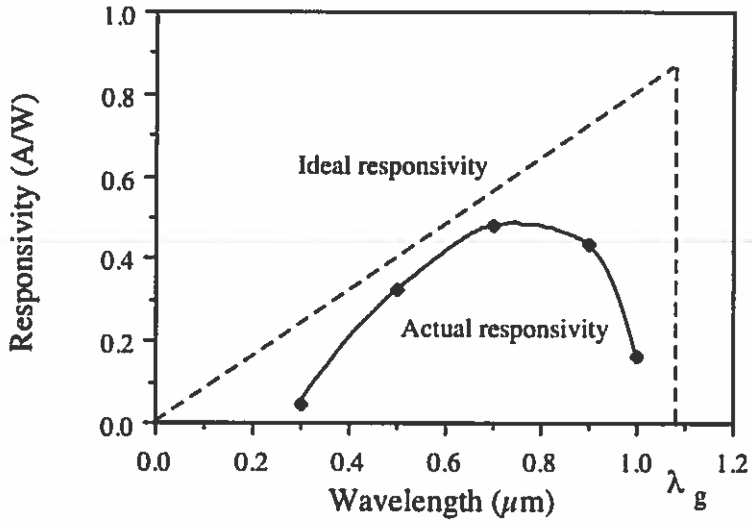
To reach the depletion layer, photons must pass through one layer of semiconductor. High-energy photons are therefore likely to be absorbed before they reach the depletion layer, while low-energy photons are likely to be absorbed only when they have passed through it. Consequently, the quantum efficiency must fall at short and long wavelengths.

[2]

For an incident optical power P, the photon rate is $P\lambda/hc$. Each absorbed photon creates a carrier pair. Since the carriers move in opposite directions, the net effect is the transit of one electron, so the photocurrent is $I_P = Pe\lambda/hc$. The ideal responsivity $R = I_p/P = e\lambda/hc$ then varies linearly with λ up to a critical value $\lambda_g = hc/E_g$ when band-to-band transitions cease.

[2]

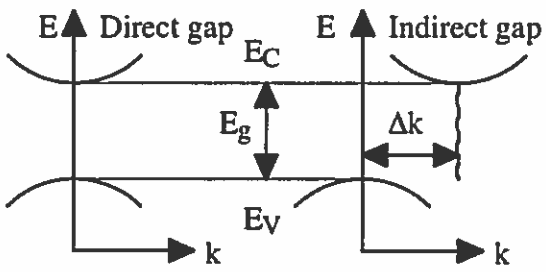
Including the quantum efficiency, $R = \eta e\lambda/hc$ so the actual responsivity will vary as below.



Responsivity (A/W)
1.0
0.8
0.6
0.4
0.2
0.0
Ideal responsivity
Actual responsivity
0.0
0.2
0.4
0.6
0.8
1.0
1.2
λ g
Wavelength (μm)

[2]

5. a) Optoelectronic interactions involve both energy and momentum. Since the wavenumber k of an electron is proportional to its momentum, the relation between energy and momentum can be expressed as an E-k diagram. The variations are parabolic near the band edges. However, in direct-gap material, the conduction band minimum lies vertically above the valence band maximum, while in an indirect-gap material there is an offset Δk.

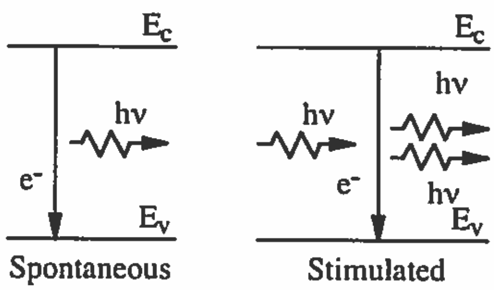


[4]

In the direct gap materials, a photon (which has significant energy but little momentum) can easily induce a band-to-band transition. However, in indirect-gap materials, an additional source of momentum such as a phonon is required. The requirement for a three-body interaction reduces the rate of transitions, so direct-gap materials are universally required.

[2]

b) Spontaneous and stimulated emission both involve transition of an electron from the conduction band to the valence band, generating a photon of energy h. $\geq E_C - E_V$. The former process occurs at random, while the latter requires a photon to stimulate the transition. The additional photon is then identical to the stimulating photon in phase, frequency, direction and polarization.



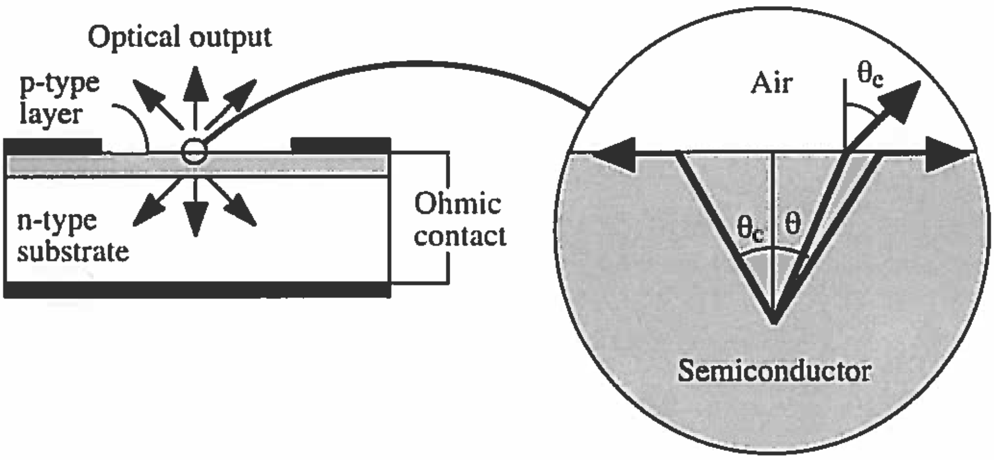
[4]

LEDs use spontaneous emission; their output is polychromatic, undirected and unpolarized.

Lasers use stimulated emission; their output is monochromatic, directed and polarized.

[2]

c) The external efficiency of a LED is limited by total internal reflection. Spontaneously generated light is emitted in all directions. However, because the refractive index of the semiconductor is so high, the critical angle at the semiconductor-air interface is small. Consequently, only a small fraction of this light can escape.



[2]

The external efficiency can be estimated as follows. Snell's law implies that:

$n \sin(\theta_S) = \sin(\theta_A)$, where θ_S and θ_A are the angles in the semiconductor and in air.

The critical angle is then found from $\sin(\theta_C) = 1/n$. Since n is large, $\theta_C \approx 1/n$.

The fraction of light in the cone defined by the critical angle is then $F = \pi\theta_C^2/4\pi = 1/4n^2$

The power reflection coefficient at the semiconductor-air interface is $P_R = (n - 1)^2/(n + 1)^2$

Hence, the power transmission coefficient is $P_T = 1 - P_R = 4n/(n + 1)^2$

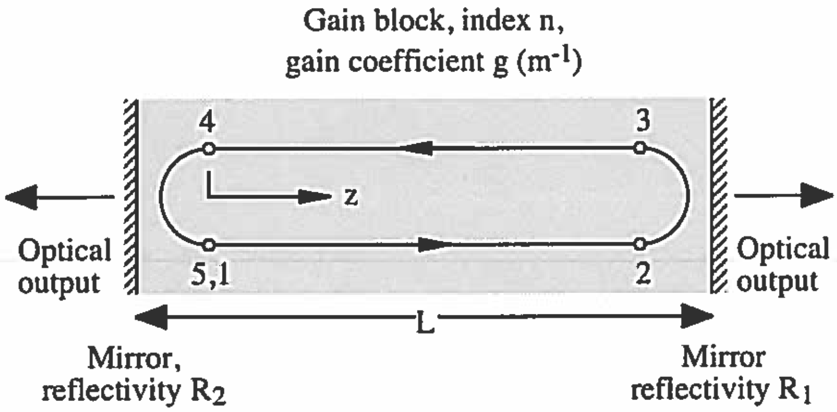
The external efficiency is then $\eta_E = FP_T = (1/4n^2)\, 4n/(n + 1)^2 = 1/\{n(n + 1)^2\}$

[5]

If $n = 3.5$, $\eta_E = 1/(3.5 \times 4.5^2) = 0.014$, or 1.4% - a very small figure.

[1]

6. a) Assume that a cavity round trip can be represented as shown below:



If β is the propagation constant, and g is the gain coefficient:

At point 1, the field is E_0

At point 2, the field is $E_0 \exp(-j\beta L) \exp(gL)$,

At point 3, the field is $E_0 \exp(-j\beta L) \exp(gL) R_1$

At point 4, the field is $E_0 \exp(-j2\beta L) \exp(2gL) R_1$

At point 5, the field is $E_0 \exp(-j2\beta L) \exp(2gL) R_1R_2$

If the laser is resonating, the fields at 1 and 5 will be the same.

Hence, the general resonance condition is $\exp(-j2\beta L) \exp(2gL) R_1R_2 = 1$

[4]

Since the RHS in the above is real, the LHS must also be real.

Consequently the phase condition $2\beta L = 2\nu\pi$ must hold, where ν is an integer

Since $\beta = 2\pi n_{eff}/\lambda$, the laser can only emit the wavelengths $\lambda = 2n_{eff}L/\nu$

[2]

The gain condition $\exp(2gL) R_1R_2 = 1$ must then hold

Consequently, the gain required for lasing is $g = (1/2L) \log_e(1/R_1R_2)$

[2]

b) The gain condition can be re-arranged to read $L = (1/2g) \log_e(1/R_1R_2)$

In a semiconductor laser with cleaved end-facets, $R_1 = R_2 = R = (n - 1)/(n + 1)$

In this case, $R = 2.5/4.5 = 0.555$.

[2]

Consequently $L = \{1/(2 \times 2355)\} \log_e(1/0.555^2) = 2.5 \times 10^{-4}$ m, or 250 μm

[1]

If the laser emits the wavelengths $\lambda_\nu = 2n_{eff}L/\nu$, the separation between two modes is:

$\Delta\lambda = \lambda_\nu - \lambda_{\nu+1} = 2n_{eff}L\{1/\nu - 1/(\nu+1)\} = 2n_{eff}L\{(\nu+1) - \nu\}/\{\nu(\nu+1)\}$

We can approximate this as $\Delta\lambda = 2n_{eff}L/\nu^2 = \lambda_\nu^2/2n_{eff}L$

[2]

In this case, we obtain $\Delta\lambda = (1.5 \times 10^{-6})^2/(2 \times 3.5 \times 250 \times 10^{-6}) = 1.28 \times 10^{-9}$, or 1.286 nm.

[1]

c) The separation between the longitudinal modes is inversely dependent on the cavity length. Consequently, if $\Delta\lambda$ has doubled, L must have halved.

The gain g required for lasing is also inversely dependent on L. If L has halved, the necessary gain coefficient must have doubled, to $g = 2 \times 2355 = 4710\ m^{-1}$.

[4]

The example above shows that the longitudinal modes can only be forced further apart using a shorter cavity, which then requires larger gain. Since this is difficult to provide, an alternative method of improving the spectral output is required, such as an intra-cavity filter.

[2]