IMPERIAL COLLEGE LONDON

EE4-55
EE9-AO11

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE PART IV: MEng and ACGI

MEMS AND NANOTECHNOLOGY

Corrected Copy

Wednesday, 13 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer Question 1.
Answer Question 2 OR Question 3.
Answer Question 4 OR Question 5.

Question 1 carries 40% of the marks. Remaining questions carry 30% each.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible | First Marker(s) : Z. Durrani, A.S. Holmes, Z. Durrani
Second Marker(s) : A.S. Holmes, Z. Durrani, A.S. Holmes

Information for Candidates

The steady state, 1-dimensional heat flow equation for a beam with distributed electrical heating is:

$$\kappa A \frac{d^2T}{dx^2} + I^2 R = 0$$

where $T(x)$ is the temperature variation along the beam, κ is the thermal conductivity of the beam material, A is the beam cross-sectional area, I is the applied current and R is the resistance per unit length of the heater.

The piezoresistive equations for silicon, referred to axes aligned to the <100> directions, are:

$$E_1 / \rho_e = J_1[1 + \pi_{11}\sigma_1 + \pi_{12}(\sigma_2 + \sigma_3)] + J_2\pi_{44}\tau_{12} + J_3\pi_{44}\tau_{13}$$
$$E_2 / \rho_e = J_2[1 + \pi_{11}\sigma_2 + \pi_{12}(\sigma_1 + \sigma_3)] + J_1\pi_{44}\tau_{12} + J_3\pi_{44}\tau_{23}$$
$$E_3 / \rho_e = J_3[1 + \pi_{11}\sigma_3 + \pi_{12}(\sigma_1 + \sigma_2)] + J_1\pi_{44}\tau_{13} + J_2\pi_{44}\tau_{23}$$

The Boltzmann constant is $k_B = 1.38 \times 10^{-23}$ J/K

This question is compulsory

1. a) A p-channel, depletion mode Si MOSFET is shown in Figure 1.1.

i) Assuming 'flat-band' conditions, sketch the energy band diagram (show E_C, E_V, E_i, and E_F) at $V_{gs} = V_{ds} = 0$ V, along the line y-y' from the gate to the substrate region.

[2]

ii) Sketch the energy band diagram (show E_C, E_V, E_i, and E_F) at the threshold voltage, $V_{gs} = V_{th}$, along the line y-y' from the gate to the substrate region.

[3]

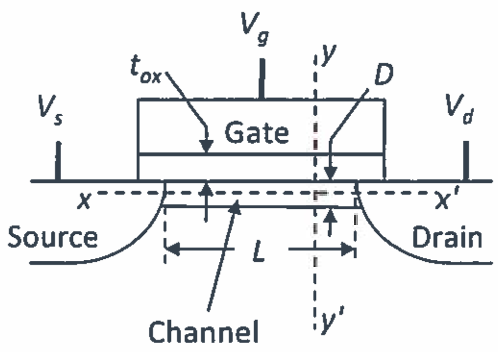

Figure 1.1

b) The unit cell in a Si crystal, where side $a = 0.54$ nm, is shown in Figure 1.2. Using this diagram, calculate the concentration of atoms in crystalline Si, per cubic metre.

[5]

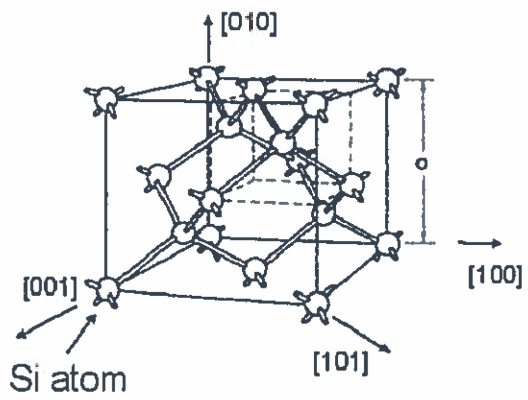

Figure 1.2

c) Using suitable diagrams, explain briefly the operation of an Esaki tunnelling diode.

[5]

Question 1 continues on the next page.

Question 1 continued.

d) A Si (100) surface with an oxide layer of thickness X_i = 50 nm is to be oxidised further at 1000 °C. At this temperature, the oxidation rate constants in the Deal-Grove equation for oxide growth:

$$X^2 + AX = B(t + \tau)$$

are $A = 0.33 \times 10^4$ Å and $B = 2.86 \times 10^4$ Å^2/min, with X the thickness of the oxide layer. Calculate the total thickness of the oxide grown under these conditions after 120 minutes.

[5]

e) The resolution limit R of a projection lithography system is typically expressed in the form:

$$R = \frac{k_1 \lambda}{NA} \quad \text{where} \quad NA = n \sin \theta_m$$

Explain briefly the physical meanings of the terms on the right-hand side of this equation and, by assigning representative values to them, estimate the value of R for a modern lithography system. Does the resolution limit you have calculated represent the minimum feature size that can be realised? Explain your answer.

[6]

f) A 10 μm-thick silicon membrane, 1 mm^2 in area, is to be fabricated by anisotropically etching from the back side of a (100)-oriented, thermally oxidised silicon wafer. The wafer is 500 μm-thick, and the etchant to be used has an anisotropy of 30±5. What size of mask aperture should be defined on the back side, assuming the nominal anisotropy value? Also, if the etch process is terminated so as to yield the correct membrane thickness, what will be the uncertainty in the membrane dimensions?

[5]

g) Starting with the expression for the force on a current-carrying conductor in a magnetic field, derive a scaling law for the force produced by a permanent magnet electromagnetic actuator under the following assumptions: (i) constant current density; (ii) constant heat dissipation per unit length. Which of these assumptions do you think is more reasonable for a MEMS device?

[5]

h) Sketch the structure of a typical material bimorph electrothermal actuator and briefly explain its operation. You should include in your answer examples of typical material combinations.

[4]

End of Question 1.

2. Figure 2.1 shows a one-dimensional potential well, where the potential energy $V(x)$ is given by:

$$V(x) = -V_0, \quad -a/2 \le x \le a/2$$
$$V(x) = 0, \quad \text{elsewhere}$$

a) For $-V_0 < E < 0$, write down the general forms of the wave-functions ψ_1, ψ_2 and ψ_3 in the regions $x < -a/2$, $-a/2 \le x \le a/2$, and $x > +a/2$ respectively.

[6]

b) For $-V_0 < E < 0$, write down the wave-vectors k_1 and k_2 within and outside the well respectively.

[4]

c) Hence show that, for $-V_0 < E < 0$, solutions to the Schrödinger equation must satisfy the conditions:

$$\alpha \tan\alpha = \beta \quad \text{or} \quad -\alpha \cot\alpha = \beta$$

where α and β are functions of k_1 and k_2 respectively.

[12]

d) What are the restrictions on these solutions? Hence, show that the wave-functions within the potential well are either odd or even.

[8]

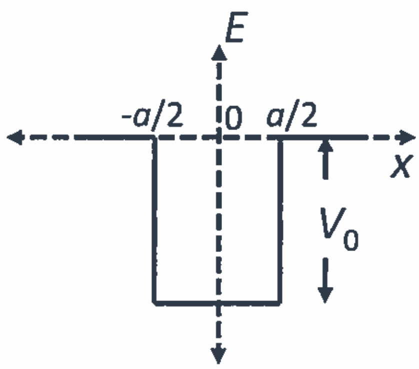

Figure 2.1

3. a) In a one-dimensional Si quantum well of width $D = 10$ nm, the electron energy levels may be approximated by:

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mD^2}$$

Where $\hbar = 1.055 \times 10^{-34}$ J.s is the reduced Planck constant and $m = 0.98 \times 10^{-30}$ kg is the electron effective mass. If there is a 1% probability that energy level E_1 is occupied at temperature $T = 4.2$ K, find:

i) The position of the Fermi energy E_F relative to the energy of the bottom of the potential well E_W.

[6]

ii) The probability of occupation of energy levels E_2 and E_3.

[4]

b) The $|1s\rangle$ state in the hydrogenic atom has the following form in spherical coordinates (r, θ, ϕ):

$$\psi = A \exp\left(-\frac{Zr}{a_0}\right)$$

where Z is the atomic number, and a_0 is the Bohr radius. Find the normalisation constant A. You may use the volume element in spherical coordinates:

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi.$$

[10]

c) In the hydrogenic atom, the $|2s\rangle$, $|2p_0\rangle$ and $|2p_{\pm 1}\rangle$ states can be approximated by:

$$|2s\rangle = \psi_{200} \sim \left(1 - \frac{Zr}{2a_0}\right) e^{-\frac{Zr}{2a_0}}$$

$$|2p_0\rangle = \psi_{210} \sim e^{-\frac{Zr}{2a_0}} r \cos\theta$$

$$|2p_{\pm 1}\rangle = \psi_{21,\pm 1} \sim e^{-\frac{Zr}{2a_0}} e^{\pm i\varphi} r \sin\theta$$

where quantum numbers n, l and m define the wave-function ψ_{nlm}.

i) Using these states, construct the hybridised states $|2p_x\rangle$, $|2p_y\rangle$ and $|2p_z\rangle$.

[6]

ii) Hence, sketch the $|2s\rangle$, $|2p_x\rangle$, $|2p_y\rangle$ and $|2p_z\rangle$ states.

[4]

4. a) Write down the bending equation for a buckled beam that has pinned joints at both ends. By solving the equation subject to appropriate boundary conditions, derive an expression for the first Euler buckling load F_0, i.e. the critical axial load at which buckling occurs.

[6]

b) The transverse stiffness of a flexure is known to decrease roughly linearly with compressive axial load F, reaching zero when the axial load reaches the critical load calculated in part a). Given this information, show that the transverse stiffness in the presence of an axial load may be expressed approximately as:

$$k \approx \frac{12}{\pi^2 L}\left[F_0 - F\right] \quad , \quad F \leq F_0$$

where L is the flexure length. You may quote the standard result for the transverse stiffness of a flexure at zero axial load.

[6]

c) A silicon micromachined resonator, comprising a rectangular proof mass with a hammock suspension, is to be tuned by passing currents through its four suspension beams. The mechanical layer is 20 μm thick, the beams are 500 μm long and 10 μm wide, and the resonator is driven in-plane. Assuming the proof mass is kept cool by conduction across the air gap to the substrate, calculate the average temperature rise required in the beams to produce a 5% shift in the resonant frequency. How would your result differ if the device were operated in vacuum rather than in air? (You do not need to repeat the calculation.)

[10]

d) By solving the 1-dimensional heat flow equation, with appropriate boundary conditions and assumptions, estimate the drive current required to produce the temperature rise you calculated in part c).

[8]

Assume values of $\alpha = 2.5 \times 10^{-6}\ \mathrm{K^{-1}}$, $\kappa = 150\ \mathrm{Wm^{-1}K^{-1}}$ and $\rho = 10\ \Omega\mathrm{cm}$ respectively for the thermal expansion coefficient, thermal conductivity and electrical resistivity of silicon.

5. a) Describe briefly the piezoelectric and piezoresistive effects and illustrate how they are used in MEMS devices.

[6]

b) A region of silicon is subject to purely axial stresses σ_L and σ_T along the [110] and $[1\bar{1}0]$ directions. By considering the equilibrium of a small prismatic element, or otherwise, show that the associated stress components in a coordinate frame aligned to the $<100>$ directions are:

$$\sigma_1 = \sigma_2 = \tfrac{1}{2}(\sigma_L + \sigma_T) \quad ; \quad \tau_{12} = \tfrac{1}{2}(\sigma_L - \sigma_T)$$

Hence show that the resistance change of a [110]-aligned piezoresistor subject to longitudinal and transverse stresses σ_L and σ_T may be written as:

$$\frac{\Delta R}{R} = \sigma_L \pi_L + \sigma_T \pi_T$$

where $\pi_L = \tfrac{1}{2}\{\pi_{11} + \pi_{12} + \pi_{44}\}$ and $\pi_T = \tfrac{1}{2}\{\pi_{11} + \pi_{12} - \pi_{44}\}$, and π_{ij} are the standard piezoresistive coefficients.

[12]

c) A bulk micromachined pressure sensor has a square silicon membrane of side length 2 mm and thickness 20 μm. The membrane stress is monitored using a small piezo-resistive bridge positioned at the mid-point of one of the membrane edges. The resistors in the bridge are matched and are aligned either parallel or perpendicular to the membrane edge. What will be the sensitivity of the device (in Volts per Pascal) if it is operated with the bridge connected to a 5 V supply?

[12]

Assume the following values for the piezoresistive coefficients of the silicon (all in units of 10^{-11} Pa^{-1}): $\pi_{11} = 6.6$, $\pi_{12} = -1.1$, $\pi_{44} = 138.1$. Poisson's ratio for the relevant directions is $\nu = 0.064$. You may use the following approximate expression for the maximum stress σ_{max} in a square membrane of side L and thickness h subject to a uniformly distributed load p:

$$\sigma_{\max} \approx 0.3 \frac{pL^2}{h^2}$$

