IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

EEE PART II: MEng, BEng and ACGI

Corrected copy

CONTROL ENGINEERING

Friday, 9 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : I.M. Jaimoukha
Second Marker(s) : S.A. Evangelou

1. Consider the feedback loop shown in Figure 1.1. Here K_p is a constant compensator and $G(s) = G_1(s)G_2(s)$ is the system where each of $G_1(s)$ and $G_2(s)$ is a transfer function representing the circuit shown in Figure 1.2 and where the value of the parameters for $G_1(s)$ are such that $C_i = 0$, $R_iC_f = 1$, $R_fC_f = 1/2$, while those for $G_2(s)$ are such that $C_i = C_f$, $R_iC_i = 1/3$, $R_fC_f = 1$. Assume all the capacitors are initially uncharged.

a) Determine the transfer function $G(s)$. [5]

b) Write down the differential equation relating $u(t)$ to $y(t)$. [5]

c) Obtain a state-space realisation for $G(s)$. [5]

d) Assume that the system is operating in open loop. Let $u(t)$ be a unit step applied at $t = 0$. Use the final value theorem, which should be stated, to find the steady-state value of $y(t)$. [5]

e) Suppose that $r(t)$ be a unit step applied at $t = 0$. Find the minimum value of K_p such that the steady-state value of $e(t)$ is less than 0.01. [5]

f) Suppose that $r(t)$ be a unit ramp applied at $t = 0$. Find the steady-state value of $e(t)$. [5]

g) Draw the Nyquist diagram for $G(s)$. [5]

h) Use the Nyquist criterion, which should be stated, to find the number of unstable closed loop poles for all $-\infty < K_p < \infty$. [5]

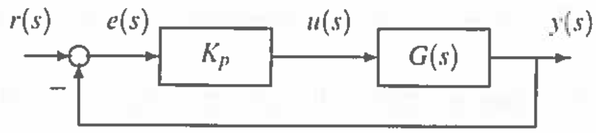

Figure 1.1

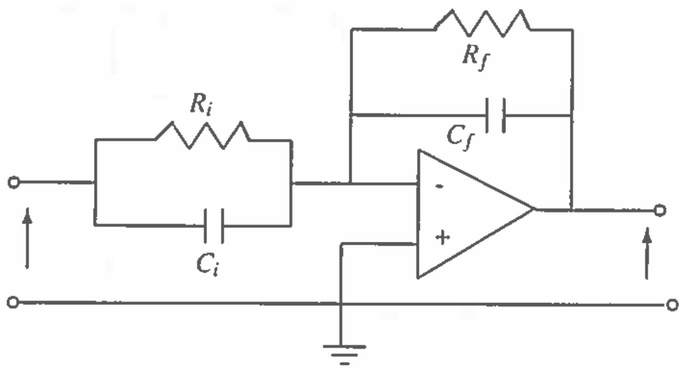

Figure 1.2

2. In the feedback system in Figure 2.1, $G(s) = \frac{4}{(s+1)^3}$ and $K(s)$ is a compensator.

Figure 2.1

a) Sketch the Nyquist diagram of $G(s)$. Use the Routh array to find the real–axis intercepts, together with the corresponding frequencies. [6]

b) Take $K(s) = 1$. Use the Routh array to show that the feedback loop is stable. Find the gain and phase margins and the cross–over frequency. [6]

c) Without doing any design, briefly describe how a phase–lead compensator would affect the gain and phase margins. [6]

d) Suppose that $K(s)$ is a stabilising compensator. Figure 2.2 depicts a stable additive uncertainty $\Delta(s)$.

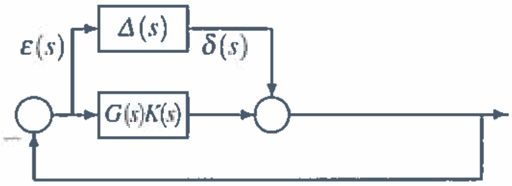

Figure 2.2

i) Show that signals $\varepsilon(s)$ and $\delta(s)$ are related as $\varepsilon(s) = -(\delta(s) + Q(s)\varepsilon(s))$ for some $Q(s)$. Give an expression for $Q(s)$ in terms of $G(s)$ and $K(s)$. [3]

ii) Show that the feedback loop in Figure 2.2 is equivalent to that in Figure 2.3 for some $S(s)$. Give an expression for $S(s)$ in terms of $G(s)$ and $K(s)$. [3]

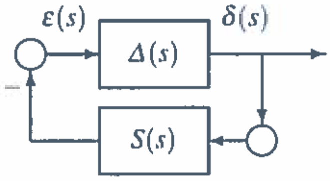

Figure 2.3

iii) Use the Nyquist stability criterion to show that the feedback loop in Figure 2.3 is stable for all $\Delta(s)$ satisfying $|\Delta(j\omega)| < 1/|S(j\omega)|$. [3]

iv) Hence suggest a design requirement on the loop gain $|G(j\omega)K(j\omega)|$ to increase robustness against additive uncertainties. [3]

3. Consider the control system in Figure 3.1 employing rate feedback. Here, K_p and K_v are design parameters.

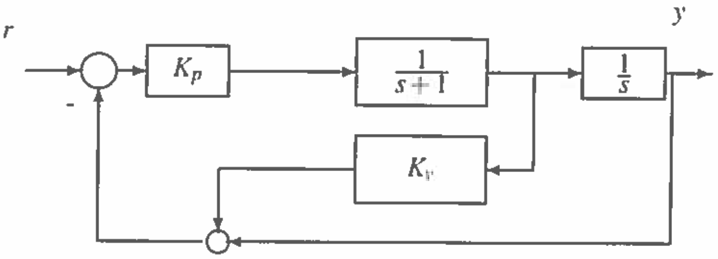

Figure 3.1

The design specifications are:

- The closed loop damping ratio is $1/\sqrt{2}$.
- The closed loop time constant is 0.5 s.

a) Find the location of the closed loop poles that achieves the design specifications. [5]

b) Draw an equivalent block diagram that has a single loop. [5]

c) Show that the characteristic equation has the form

$$1 + K(s+z)G(s) = 0$$

and evaluate z and K in terms of K_p and K_v. Give an expression for $G(s)$. [5]

d) Sketch the root locus of $G(s)$ and show that the design specifications cannot be satisfied when $K_v = 0$. [5]

e) Use the angle and gain criteria to determine the values of K_v and K_p that achieve the design specifications. [5]

f) Draw the root locus of the compensated system $(s+z)G(s)$. [5]