Imperial College London

MSci EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

ADVANCED PARTICLE PHYSICS

For 4th-Year Physics Students

Wednesday, 30th May 2012: 10:00 to 12:00

Answer THREE questions.
All questions carry equal marks.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

1. (i) Fermi's Golden Rule for a particle X decaying at rest is

$$\Gamma = |M|^2 \frac{\rho}{2m_X}$$

Carefully explain the different terms in this equation.

[2 marks]

(ii) A muon decays via the weak force to an electron.

Draw and carefully label the lowest order Feynman diagram for this decay.

[3 marks]

(iii) If the mass of the electron is neglected the width of this decay is given by:

$$\Gamma = \frac{G_F^2 m_\mu^5}{24(2\pi)^3}$$

where m_μ is the mass of the muon. Show that the lifetime of the muon is $\approx 2.2\ \mu s$

[1 mark]

(iv) Estimate the lifetime of the τ lepton. Carefully note any assumptions that you make. *Note that the τ can decay to u and d quarks as well as other leptons*

[3 marks]

(v) The π^- can also decay via the weak force to either a muon or an electron. Assuming the neutrino is massless show that the energy and momentum of the of the lepton, l, to which it decays are given by

$$E_l = \frac{m_\pi^2 + m_l^2}{2m_\pi} \quad ; \quad p_l = \frac{m_\pi^2 - m_l^2}{2m_\pi}$$

where m_π is the mass of the pion and m_l is the mass of the lepton.

[2 marks]

(vi) With the muon having a greater mass than the electron one might naïvely expect the branching fraction to electrons to be higher than that to muons. In fact the reverse is the case and

$$\frac{\Gamma(\pi^- \to e^- + \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- + \overline{\nu}_\mu)} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2}$$

This is referred to as "helicity surpression".

By calculating the velocities of the muons and the electrons coming from charged pions decaying at rest and by using your knowledge of the structure of the interaction term in the Lagrangian of the charged weak current, explain the physical origin of helicity suppression in the decay of thr π^-. In your answer be careful to distinguish between helicity and handedness (where appropriate)

[4 marks]

(vii) You are presented with a source of π^+ mesons created exactly at rest. Wishing to study their decay products you place the source in a very well constructed time projection chamber (TPC). The density of the gas in the TPC is 1.79 x 10^{-4}g cm^{-3}

[This question continues on the next page . . .]

and the average flight distance is 20 cm. The $\frac{\mathrm{dE}}{\mathrm{d}x}$ behaviour of the gas is shown in the figure below. The TPC has a momentum resolution of $\frac{\Delta p}{p^2} = 0.0006\mathrm{MeV}^{-1}$ when p is measured in MeV, and measures the average energy lost by the charged particles per unit length with a resolution $\frac{\Delta E}{E}$ of 4.5%. In separate figures sketch and label the momentum and energy loss per unit length distributions observed after 1,000,000 π^+ have decayed in your TPC. You may find it useful to use a log scale on your $y-$axis. Explain how your TPC could be used to identify the decay products of the π^+ had they not been known in advance.

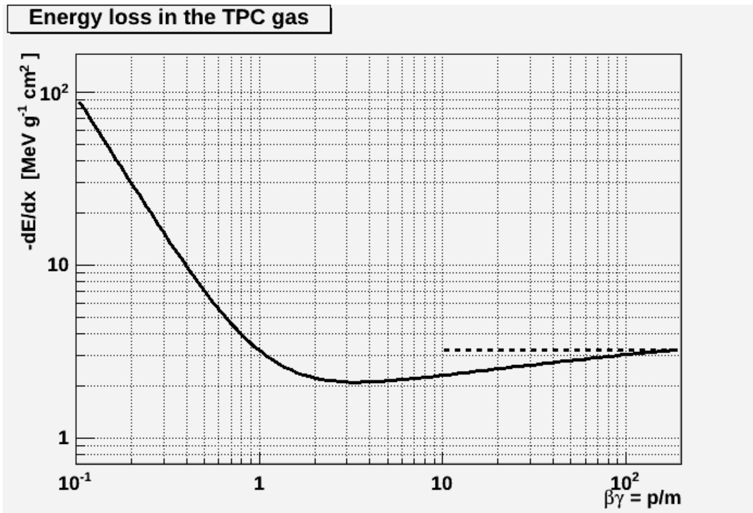

Figure 1: The $\frac{\mathrm{dE}}{\mathrm{d}x}$ properties of the gas in the TPC. The solid curve shows the energy loss for charged muons (and other heavy charged particles) while the dashed curve shows the energy loss for electrons. The lower energy region for the electron curve is deliberately omitted.

[5 marks]

[Total 20 marks]

Information and formulas that you may find useful for this question
Particle Masses and Constants:

$$m_{\pi^-} = 139.6\,MeV$$
$$m_{\tau^-} = 1776\,MeV$$
$$m_{\mu^-} = 105.7\,MeV$$
$$m_{e^-} = 0.511\,MeV$$
$$G_F = 1.166 \times 10^{-5} GeV^{-2}$$
$$\hbar = 6.582 \times 10^{-16} eV.s$$

A wavefunction ψ for a particle with helicity h and velocity β can be decomposed into handedness states using the following equations:

$$\psi_{h=+\frac{1}{2}} = \sqrt{\frac{1+\beta}{2}}\psi_R - \sqrt{\frac{1-\beta}{2}}\psi_L$$

[This question continues on the next page ...]

$$\psi_{h=-\frac{1}{2}} = \sqrt{\frac{1-\beta}{2}}\,\psi_R - \sqrt{\frac{1+\beta}{2}}\,\psi_L \tag{1}$$

2. (i) The covariant form of the Dirac equation is:

$$i\gamma^{\mu}\partial_{\mu}\psi = m\psi$$

Dirac's motivation was to produce a Hamiltonian which was linear and of the form:

$$H = \boldsymbol{\alpha}.\boldsymbol{p} + \beta m$$

By considering the spatial and temporal parts of the Dirac equation separately show that this was achieved and express α and β in terms of the γ matrices.

[3 marks]

(ii) By assuming that total angular moment $\boldsymbol{J}$ is conserved and that $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ where $\mathbf{L}$ is orbital angular momentum and $\mathbf{S}$ is spin show that

$$[\hat{H}, \hat{\boldsymbol{S}}] = \gamma^0 \boldsymbol{\gamma} \times \boldsymbol{\nabla}$$

Only consider the x component when answering this question

[7 marks]

(iii) Again, only considering the x component show that $\hat{\boldsymbol{S}} = \frac{1}{2}\gamma^5\boldsymbol{\gamma}$ satisfies the relationship for $[\hat{H}, \hat{\boldsymbol{S}}]$, where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

[6 marks]

(iv) By considering the operator $\hat{S^2}$ show that the Dirac equation describes particles with spin $= \frac{1}{2}$.

[4 marks]

[Total 20 marks]

Formulas that you may find useful for this question

The gamma matrices γ^{μ} $\mu = 0,1,2,3$ and γ^5 satisfy the following relationships:

$$\{\gamma^{\nu}, \gamma^{\mu}\} = 2g^{\nu\mu}$$
$$\{\gamma^5, \gamma^{\mu}\} = 0$$
$$(\gamma^5)^2 = 1$$

3. (i) If the angle between the direction of spin of a muon and the momentum of the electron into which it decays is θ then

$$\frac{d\Gamma}{d(\cos\theta)} \propto 1 - \frac{1}{3}\cos\theta$$

Explain how this decay violates parity (P) conservation. [3 marks]

(ii) The neutral kaon, K^0, $(d\bar{s})$ can mix into its antiparticle $\overline{K}^0$ $(\bar{d}s)$. Draw the two lowest order Feynman diagrams for this process.

[4 marks]

(iii) The K^0 and $\overline{K}^0$ are $J^p = 0^-$ states so

$$\hat{P}|K^0 >= -|K^0 >$$
$$\hat{P}|\overline{K}^0 >= -|\overline{K}^0 >$$

If you adopt the convention

$$\hat{C}|K^0 >= |\overline{K}^0 >$$

Show that neither the $\overline{K}^0$ nor K^0 are $\hat{C}\hat{P}$ eigenstates.

[2 marks]

(iv) Construct two $\hat{C}\hat{P}$ eigenstates $|K_1 >$ and $|K_2 >$ which have eigenvalues of +1 and -1 respectively from the $|\overline{K}^0 >$ and $|K^0 >$ wavefunctions. Demonstrate that they are $\hat{C}\hat{P}$ eigenstates with these eigenvalues.

[3 marks]

(v) One of these eigenstates decays to two pions and the other to three. Identify which decays to two pions and explain your answer.

[2 marks]

(vi) You would expect the $|K_2 >$ to live significantly longer than the $|K_1 >$. Explain why this is.

[1 mark]

(vii) In nature we do actually see two neutral kaons with very different lifetimes. These are called the K-long (K_L^0) which has a lifetime of approximately 5×10^{-8} s and the K-short (K_S^0) which has a lifetime of approximately 9×10^{-11} s. However, it turns out that the weak eigenstates are not exactly $\hat{C}\hat{P}$ eigenstates and so

$$|K_L^0 >= \frac{1}{\sqrt{1+\varepsilon^2}}(|K_2 > +\varepsilon|K_1 >)$$

where $\varepsilon \approx 2 \times 10^{-3}$. How would you measure ε experimentally to a statistical error of 10%? You should consider everything from and including the source of your neutral kaons.

[5 marks]

[Total 20 marks]

4. We consider a simple "toy" model which includes only three types of particle, a, B and C, each with spin 0 and masses m_a, m_B and m_C, where $m_a \ll m_B \ll m_C$, and all particles are their own antiparticles. The fields for each particle type are written ϕ_a, ϕ_B and ϕ_C. The Lagrangian density for this model is believed to take the form

$$\begin{aligned} \mathscr{L} &\propto \phi_a\phi_B\phi_a + \phi_a\phi_C\phi_a + \phi_a\phi_B\phi_C & (1) \\ &+ \text{ free particle terms.} & (2) \end{aligned}$$

(i) Draw the possible vertices that can contribute to Feynman diagrams in this toy model, using the common factor g for the coupling constant. [3 marks]

We assume throughout this problem that the magnitude of g is very small, i.e., that perturbation theory can be applied through the use of Feynman diagrams, and higher-order contributions are small.

(ii) Both the B and C are unstable. Give the first-order diagrams for the decay of the C particle.

[3 marks]

(iii) Comment on the relative rates for the decay modes given in (ii). [3 marks]

(iv) What are the final states that can be expected, given an initial state with a stationary C? [3 marks]

(v) In an a-a collider, running with variable centre-of-mass energies below m_B, what are the final states that are possible, and how does the cross-section change with energy? [3 marks]

(vi) How would the properties of the B and C be studied at such a collider, if the largest attainable centre-of-mass energy is E_{max}, where $E_{\text{max}} \sim 3 \times m_B \ll m_C$?

[5 marks]

[Total 20 marks]

5. An e^+-e^- collider can be used to discover new particles and study their properties. Here we focus on the paper "Evidence for Anomalous Lepton Production in e^+-e^- Annihilation" by M. Perl et al.

The event signature that is being reported is

$$e^+ + e^- \to e^\pm + \mu^\mp + \geq 2 \text{ undetected particles.} \tag{1}$$

We first consider the possible explanations for the observed signal within a two-generational Standard Model (missing the third-generation quarks and leptons).

(i) What are the candidates for Standard Model particles which could be the undetected particles in equation (refeqn:signal)? [3 marks]

(ii) At the time of the experiment, the weak vector bosons in the Standard Model had not been discovered. Show how these could produce the signal final state as given in equation (1). [3 marks]

(iii) Give the lowest-order Feynman diagrams for the process which can produce an e^+e^- pair and $\mu^+\mu^-$ pair in the final state, as described in the paper. [2 marks]

(iv) The solid angle coverage of the detector is given as 2.6π steradians in the paper. Assuming azimuthal and polar symmetries, calculate the coverage of the detector in terms of $\cos\theta$, where θ is the polar angle. [2 marks]

The process in (iii) can give rise to the signal process (1) if two of the particles (an electron and muon of either charge) are not detected, which is quite possible given the relatively small solid angle coverage as shown above.

(v) It is stated in the paper that calculations indicate that the rate of this process is small. What experimental observation proves this point unambiguously? [2 marks]

(vi) A system of lead-scintillator shower counters, followed by 20 cm of iron and wire spark chambers is used to help identify particle types.

What are the characteristic signals that help discriminate between the following: electrons, muons, and hadrons?

[3 marks]

(vii) Given a sample of two-track ('prong') events, a "coplanarity" angle θ_{copl} is defined such that:

$$\cos\theta_{\text{copl}} = -(\mathbf{n}_1 \times \mathbf{n}_{e^+}) \cdot (\mathbf{n}_2 \times \mathbf{n}_{e^+}) / |\mathbf{n}_2 \times \mathbf{n}_{e^+}||\mathbf{n}_2 \times \mathbf{n}_{e^+}|, \tag{2}$$

where $\mathbf{n}_1$, $\mathbf{n}_2$ and $\mathbf{n}_{e^+}$ are the unit vectors along the directions of particles 1, 2 and the positron beam.

Describe what the coplanarity angle represents, and how it is used to help find events caused by the signal process. [5 marks]

[Total 20 marks]