# UNIVERSITY OF LONDON
## IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

## EXAMINATIONS 1998

MEng Honours Degrees in Computing Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

*This paper is also taken for the relevant examinations for the*
*Diploma of Membership of Imperial College*
*Associateship of the Royal College of Science*
*Associateship of the City and Guilds of London Institute*

## PAPER 4.80

## AUTOMATED REASONING

Monday, April 27th 1998, 10.00 - 12.00

*Answer THREE questions*

For admin. only: paper contains 4 questions

**(In all questions variables begin with lower case u - z; other names are constants. Predicates use upper case)**

1 a Use the semantic tree method to show that the pair of clauses (1) and (2) is unsatisfiable.

(1) $P(u,u) \vee P(u,d)$

(2) $\neg P(b,v) \vee \neg P(c,w)$

Annotate your answer to explain clearly the reason for unsatisfiability.

b The Davis-Putnam procedure can be used to detect unsatisfiability of a set of general clauses S by applying two steps:

- Find a set G of ground instances of S.
- Apply the Davis-Putnam procedure to refute G.

Hyper-linking may be used for the first of the above steps.

i) Explain how hyper-linking applied to the clauses (1) and (2) above generates the following set of ground instances

| | |
|---|---|
| $P(c,c) \vee P(c,d)$ | $\neg P(b,b) \vee \neg P(c,c)$ |
| $\neg P(b,d) \vee \neg P(c,c)$ | $P(b,b) \vee P(b,d)$ |
| $\neg P(b,b) \vee \neg P(c,d)$ | $\neg P(b,d) \vee \neg P(c,d)$ |

ii) Apply the Davis-Putnam procedure to the ground instances given in part bi) to show the unsatisfiability of the pair of clauses (1) and (2).

c Let S be a set of clauses each having no more than one positive literal (i.e. S is a set of Horn clauses) and at least one negative literal.

Why must S have *at least one* model?

Hence, or otherwise, show that for an *unsatisfiable* set of Horn clauses there is no need for the Davis-Putnam procedure to use the split operation.

*The three parts carry, respectively, 40%, 35% , 25% of the marks.*

2 a

i) Use hyper-resolution to find a refutation from the clauses (1) - (5) below. Make clear the method.

(1) $\neg T(x,y) \lor R(c,y)$

(2) $\neg R(x,f(y,w)) \lor \neg R(x,z) \lor R(x,f(y,z))$

(3) $T(f(a,d), f(a,b))$

(4) $R(x,d)$

(5) $\neg T(x,y) \lor \neg R(c,x)$

ii) The positive literals in a clause are to be ordered so that their selection is restricted. Briefly explain *one* way in which this could be done.

iii) Replace every literal in clauses (1) - (5) by its complement to give (1') - (5'). Why must (1') - (5') have no models ?

iv) Use the ordering method given in part aii) to refute (1') - (5') by ordered hyper-resolution.

b

i) State two potential benefits of using connection graph book-keeping in a refutation.

ii) Draw the initial connection graph for clauses (1), (3), (4) and (5) above.

Show that the graph reduces to the empty graph.

What is the significance of the empty graph?

*The two parts carry, respectively, 65% , 35% of the marks.*

*Turn over ...*

3 a Consider the rewrite rules (1) - (3) below, ordered by a suitable recursive path ordering.

(1) s(f(x,y)) ==> f(s(x), s(y))
(2) f(g(x), g(x)) ==> g(x)
(3) s(g(x)) ==> x

i) Show that it is not possible to rewrite f(s(g(x)), s(g(x))) and s(g(x)) into a common term using the rules.

Show that the terms f(s(g(x)), s(g(x))) and s(g(x)) are equal if (1) - (3) are used as equations.

ii) Explain the discrepancy in part (i) in terms of *critical pairs* and *confluence*.

iii) Apply the Knuth-Bendix algorithm to (1) - (3) above to obtain a confluent set of rewrite rules.

iv) What are the possible outcomes of the Knuth-Bendix algorithm?

b

i) What is paramodulation?

ii) Use paramodulation and resolution to refute clauses (4) - (7) below.

(4) $\neg$ K(a,a)
(5) K(x,y) $\wedge$ K(y,x) $\rightarrow$ x=y
(6) K(h(x),x)
(7) h(h(x)) = x

iii) Briefly state any advantage paramodulation holds over the alternative of using resolution and equality axioms.

*The two parts carry, respectively, 70%, 30% of the marks.*

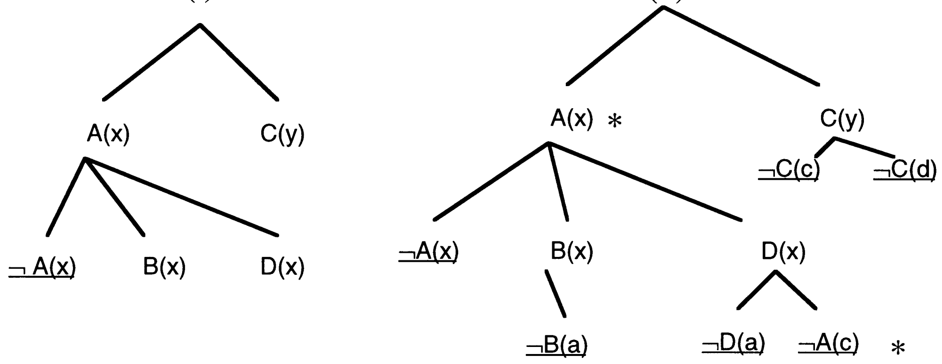
4 a Explain how the method of model elimination tableau, when developed left-right and depth-first, simulates a refinement of linear resolution. Your answer should make clear what the restrictions of the refinement are.

b Use the model elimination tableau method described in part (a) to refute clauses (1) - (6) below using (6) as top clause.

| | |
|---|---|
| (1) | $\neg P(x) \vee \neg Q(x,y)$ |
| (2) | $R(x,f(y)) \vee \neg P(x)$ |
| (3) | $R(u,v) \vee \neg R(u, f(v))$ |
| (4) | $\neg Q(c,b) \vee \neg R(a,c)$ |
| (5) | $Q(x,y) \vee Q(a,x) \vee Q(c,y)$ |
| (6) | $P(a)$ |

c Write down a fully quantified expression that represents the open branches in the tableau (i) below. Hence, or otherwise, explain why it is sound to close the tableau as in (ii).

(i) (ii)




d Formulate a relaxation of the rule for closure by ancestor matching based on part (c) (such as that used between the two starred literals in diagram c(ii) above).

Briefly discuss the advantage of relaxing the rule.

*The four parts carry, respectively, 30%, 20%, 25%, 25% of the marks.*

*End of paper*