UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1998

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Associateship of the City and Guilds of London Institute
Associateship of the Royal College of Science

PAPER 3.43 / I3.22

OPERATIONS RESEARCH

Tuesday, April 28th 1998, 2.00 - 4.00

Answer THREE questions

For admin. only: paper contains 4 questions

1a A compact disk company is able to print three different versions of a popular recording. They receive an order for a minimum of ($\equiv$ at least) 100 cardboards of this recording without specifying the composition of the supply. (One cardboard is a package containing 1000 disks.) The company wants to maximise its profit. The following data are available.

The profit on one cardboard (in £1000):

Type-1: 3
Type-2: 4
Type-3: 6

Producing one load of cardboard of each of them requires 2, 3, and 4 time units with a specific technology. The technology is available for 300 time units in the given planning period. Due to technical circumstances, the ratio of quantities of Type-2 and Type-1 products must be at least 2.

Formulate a linear programming model for this problem that maximises the profit of the delivery.

b Solve the problem formulated in the previous part using the Two-Phase simplex algorithm.

c At what rate would profit go up if the availability of the technology were increased?

(The three parts carry, respectively, 40%, 40% and 20% of the marks).

2a Here are three statements about linear programming (LP). Label them as true or false and justify your answer.

(i) Only basic feasible solutions (BFSs) can be optimal, therefore, the number of optimal solutions cannot exceed the number of BFSs.

(ii) If multiple optimal solutions exist, then an optimal basic feasible solution may have an adjacent BFS which is also optimal.

(iii) If the objective function, evaluated at a vertex (corresponding to a BFS), is better than its value at every adjacent vertex then the solution is optimal.

b Solve the following LP problem using the Two-Phase simplex algorithm:

$$
\begin{array}{rrcrcrcr}
\min\ x_0 = & 5x_1 & + & 2x_2 & + & 4x_3 & & \\
\text{s.t.} & 2x_1 & + & x_2 & + & 2x_3 & \leq & 5 \\
 & 3x_1 & - & 5x_2 & - & 2x_3 & \geq & 4 \\
 & x_1 & - & 2x_2 & - & x_3 & = & 1 \\
 & \multicolumn{7}{l}{x_1 \geq 0,\ x_2 \leq 0,\ x_3 \geq 0.}
\end{array}
$$

(The parts carry, respectively, 50% and 50% of the marks).

Turn over

3 a A computer manufacturer makes two types of computers: A and B. Each unit of computer A requires 3 hours of assembly and 2 hours of testing. Each unit of computer B requires 2 hours of assembly and 3 hours of testing. There are 8 assembly hours and 7 testing hours available every day. The profit per unit sold is £1600 for computer A and £1000 for computer B. The amount of each product produced must be an (nonnegative) integer multiple of 0.25. The objective is to determine the mix of production quantities to maximise profit. Formulate an integer programming model for this problem. [Hint: Production should be defined in .25 units of each product.]

b Solve the problem using the branch and bound algorithm.[Graphical solution is allowed, as well as using the simplex algorithm.]

c Consider the following problem

$$\text{minimise } f_1(x_1) + f_2(x_2)$$

subject to:

$x_1, x_2 \geq 0$;
either $x_1 \geq 3$ or $x_2 \geq 3$;
$|x_1 - x_2| = 0, 3, \text{ or } 6$
[i.e. $|x_1 - x_2|$ may take three values: either 0, or 3, or 6. Hint: 5 binary variables are needed to handle this constraint.]

$$f_1(x_1) = \begin{cases} 7 + 5x_1 & \text{if } x_1 > 0, \\ 0 & \text{if } x_1 = 0, \end{cases} \qquad f_2(x_2) = \begin{cases} 5 + 6x_2 & \text{if } x_2 > 0, \\ 0 & \text{if } x_2 = 0. \end{cases}$$

[Hint: Assume $x_1 \leq M y_1$, $x_2 \leq M y_2$ for M a large positive constant and $y_1, y_2 = 0$ or 1].

4 a Let A be the $n \times n$ reward matrix of a two-person game and let $A^T = -A$. Let C be the $n \times n$ matrix with every element equal to a constant c, chosen such that the matrix $\hat{A} = A + C$ has no negative elements. Using the fact that the value of the game is the same for both players, derive the value of the game for the matrix A and $\hat{A}$.

b If $[\bar{x}_1, \ldots, \bar{x}_n]$ is the row player's optimal strategy for A in (a) above, derive the row player's strategy for $\hat{A}$ and also the column player's strategy for A and $\hat{A}$.

c Let B be the $m \times n$ reward matrix of a two-person, zero-sum game. Let b_{ij} denote the reward to player 1 if she plays strategy i ($i = 1, \ldots, m$) and player 2 plays her strategy j ($j = 1, \ldots, n$). Strategy 1 (say) of player 1 is said to be *weakly dominated* by strategy 2 (say) if $b_{1j} \leq b_{2j}$ for $j = 1, \ldots, n$. Assume that the reward matrix does not have any saddle points and the optimal strategies are mixed strategies. Show that eliminating weakly dominated pure strategies from the reward matrix cannot eliminate all optimal mixed strategies and cannot introduce new ones.