UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2003

MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C438=I4.54

COMPLEXITY

Wednesday 30 April 2003, 15:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1a i) What does it mean for a language to be decidable in (deterministic) logspace?

ii) Let EQUAL = $\{x \in \{0,1\}^* : x$ has the same number of 0s and 1s$\}$. Explain why EQUAL is in L (=LOGSPACE).

b i) What does it mean for a language to be *complete* for the class P of polynomial-time decidable languages?

ii) Show that if a language $L \in \{0,1\}^*$ is P-complete then the complementary language $\overline{L}$ is also P-complete.

c i) Give the definition of the problem 2SAT.

ii) A *bipartite* graph is an undirected graph whose nodes can be partitioned into two subsets X and Y, in such a way that X and Y both form independent sets, i.e. no two nodes of X are adjacent, and no two nodes of Y are adjacent. Show that the following problem is in P:

BIPARTITE: Given a graph G, is G bipartite?

You may assume that 2SAT is in P.

The three parts carry, respectively, 30%, 35%, 35% of the marks.

2 In what follows, NL abbreviates NLOGSPACE. Also, * is the Kleene star, meaning zero or more occurrences concatenated together.

a Let $L_1, L_2 \subseteq \Sigma^*$ both belong to NL. Let L_1L_2 be defined by
$L_1L_2 = \{xy \in \Sigma^* : x \in L_1 \wedge y \in L_2\}$
where xy denotes the concatenation of words x and y.
Show that L_1L_2 is in NL.

b The reachability problem RCH is:

RCH: Given a directed graph G and nodes x,y, is there a path from x to y in G?

i) Show that RCH is in NL.

ii) Show that the following problem is NL-complete:
EVENRCH: Given a directed graph G and nodes x,y, is there a path of even length from x to y in G?
You may assume that RCH is NL-complete.

iii) Show that the following problem is NL-complete:
ODDRCH: Given a directed graph G and nodes x,y, is there a path of odd length from x to y in G?

c A *one-way machine* is a single-tape nondeterministic Turing Machine which is read-only and only moves to the right. It has an "accept" state. Instructions for one-way machines may be written as (q,s,q') where q, q' are states and s is a symbol. A language is *one-way* if it is accepted by a one-way machine.

i) Show that the following language is one-way:
$L = \{w \in \{0,1\}^* : w$ is of the form either 0^* or $(01)^*\}$
Give your answer as a state-transition diagram.

ii) Show that the following problem is in NL:

ONEWAY: Given a word w and a one-way machine M, is w accepted by M?

The three parts carry, respectively, 30%, 40%, 30% of the marks.

3a i) State Brent's Principle.

ii) In a directed graph, a *cycle* is a sequence of adjacent nodes which starts and finishes at the same node. Call a node *cyclic* if it is part of a cycle of length > 0. Show how to compute the number of cyclic nodes in a directed graph with n nodes in parallel time $O(\log^2 n)$.
How many processors does the computation require by Brent's Principle?
[Hint: consider transitive closure and matrix multiplication.]

b i) Give the definitions of the complexity classes NC and NC_j.

ii) What is the Parallel Computation Thesis? Briefly state the evidence for it.

c The language ZeroOne consists of all words belonging to $\{0,1\}^*$ where there is no occurrence of 0 after an occurrence of 1, i.e., all words of ZeroOne are of the form 0^*1^*.
Show that ZeroOne is in NC_1.
What is the approximate size of your circuit?

The three parts carry, respectively, 40%, 30%, 30% of the marks.

4a State the definitions of the following classes:

i) FNP

ii) FP

iii) FP^{NP}

b i) Is the Travelling Salesman Problem (TSP) in FNP? Explain your answer briefly.

ii) Show that TSP is in FP^{NP}.
[Hint: show how to calculate TSP using the decision version of TSP.]

c An *independent set I* in a graph *G* is a set of nodes of *G* such that no two nodes of *I* are joined by an edge. Let three related problems be defined as follows:

IND: Given a graph *G*, and a number *k*, does G have an independent set of size *k*?
FINDSIZE: Given a graph *G*, find the size of a maximal independent set.
FIND: Given a graph *G*, find a maximal independent set.

i) Show that IND is in NP.

ii) Show that FINDSIZE is in FP^{NP}.

iii) Show that FIND is in FP^{NP}.

The three parts carry, respectively, 20%, 35%, 45% of the marks.