Section A – Answer any 2 out of 3 questions in section A

Question 1

a) What are the key objectives for operation and design of electrical energy systems?

[3]

The key objectives are i) security of supply (fuel security, network security), ii) cost effectiveness (support of economic growth), and iii) acceptable environmental impact (local and global).

b) Explain how synchronous generators control flows in the transmission network and balance demand and supply in real time.

[3]

Demand and supply balance must be maintained at all times. Thus, any change in demand is met by almost instantaneous change in generation. Control of flows in the transmission network is maintained by generation outputs (that match demand) and changes in these outputs at various parts of the network changes the flows through the network.

c) Measurement demonstrates that diversified peak demand of 100 households is 400kW, while the peak demand of each individual household is 8kW. Estimate the diversified peak demand of 1000 households. Briefly explain the benefits of diversification.

[4]

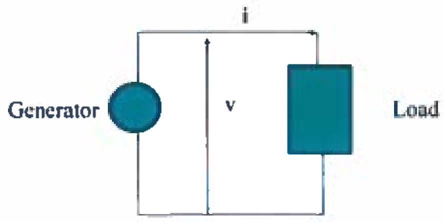
The total undiversified peak demand for n=100 households is 8*100 = 800kW, while the diversified peak demand is P_{100} =50%*800= 400kW. Thus according to formula $P_n = j_n n P_1$, where $P_1 = 8$ we have $400\,kW = j_{100} * 100 * 8 \rightarrow j_{100} = 0.5$. Also According to formula $j_n = j_\infty + \frac{1-j_\infty}{\sqrt{n}} \rightarrow j_n\sqrt{n} = j_\infty\sqrt{n} + (1-j_\infty) \rightarrow j_n\sqrt{n} = j_\infty\sqrt{n} + 1 - j_\infty \rightarrow$ $j_\infty = \frac{j_n\sqrt{n}-1}{\sqrt{n}-1}$, giving $= \frac{0.5*\sqrt{100}-1}{\sqrt{100}-1} = \frac{0.5*10-1}{9} = \frac{4}{9} = 0.4$

Thus for 1000 households, it is $P_{1000} = j_{1000} * 1000 * 8 = j_{1000} * 8000$. Also, $j_{1000} = j_\infty + \frac{1-j_\infty}{\sqrt{1000}} = 0.4 + \frac{1-0.4}{\sqrt{1000}} = 0.4 + \frac{0.6}{\sqrt{1000}} = 0.418$. Thus $P_{1000} = 0.418 * 8000 = 3344\,kW$.

d) For the network presented in diagram below, derive the expression for the instantaneous active power and demonstrate that the power oscillates with double frequency. Sketch time diagrams of voltage, current and power.

[4]

It is $p(t) = v(t)i(t) = 2VIsin(\omega t)\sin(\omega t - \phi)$ and we know $\cos(a-b) - \cos(a+b) = 2sina\ sinb$. Thus $sin(\omega t)\sin(\omega t - \phi) = \frac{\cos(\phi)-\cos(2\omega t-\phi)}{2}$. Hence $(t) = VI[\cos(\phi) - \cos(2\omega t - \phi)]$.



$v(t) = \sqrt{2}V\sin\omega t$ $\qquad p(t) = v(t)\cdot i(t)$

$i(t) = \sqrt{2}I\sin(\omega t - \phi)$

Figure 1: Schematic diagram AC circuit composed of generator and load

e) A 3-phase synchronous generator (synchronous reactance X=1 pu and negligible armature resistance) feeds a strong network and maintains its output terminal voltage V=1pu while the supply current is I= 0.7pu at 0.9 power factor.

(i) Quantify the internal generator excitation voltage (E) in terms of magnitude and angle

[2]

Power factor angle $\theta = \cos^{-1} 0.9 = 25.84\ lagging$. Thus $E = |E|(cos\delta + jsin\delta) = V_t + jXI_a$ =1+j*1*0.7*$(cos25.84 - jsin25.84) = 1.3051 + j0.63 = 1.4492(\cos 25.77 + jsin25.77)$

(ii) Compute the active and reactive output delivered by the generator

[2]

It is $P = \frac{|V||E|}{X} sin\delta = \frac{1*1.4492}{1} sin25.77 = 0.63\ pu$ and $Q = \frac{|V|}{X}(Ecos\delta - |V|) = \frac{1}{1}(1.3051 - 1) = 0.3051\ pu$

(iii) Assuming that the active output of the generator remains constant, while excitation increased by 10%, compute the change in angle δ between internal and terminal voltages.

[2]

A 10% rise in excitation leads to: $\frac{|V||E_{new}|}{X} sin\delta_{new} = 0.63$, ie $\frac{1*1.1*1.4492}{1} sin\delta = 0.63$ ie $sin\delta = \frac{0.63}{1.1*1.4492} = 0.395 \rightarrow \delta_{new} = 23.26^o$.

Question 2

a) Diagram below shows phase voltage and current vectors of a three phase system

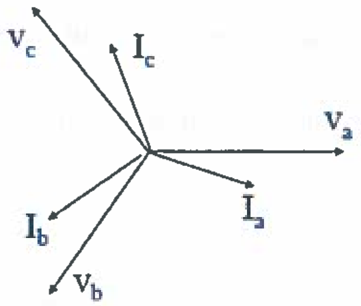


Figure 2: Schematic diagram of phase voltage and current vectors of a three phase system

(i) Write the time expressions for currents and voltages. [2]

The expressions are as follows.
$v_a(t) = \sqrt{2}V sin\omega t\,,\ i_a(t) = \sqrt{2}I sin(\omega t - \phi)$
$v_b(t) = \sqrt{2}V sin(\omega t - 120^o)\,,\ i_b(t) = \sqrt{2}I sin(\omega t - \phi - 120^o)$
$v_b(t) = \sqrt{2}V sin(\omega t - 240^o)\,,\ i_c(t) = \sqrt{2}I sin(\omega t - \phi - 240^o)$

(ii) Demonstrate that the three-phase active power is time-independent. [4]

The expressions are as follows.
$v_a(t) = \sqrt{2}V sin\omega t\,,\ i_a(t) = \sqrt{2}I sin(\omega t - \phi)$
$v_b(t) = \sqrt{2}V sin(\omega t - 120^o)\,,\ i_b(t) = \sqrt{2}I sin(\omega t - \phi - 120^o)$
$v_b(t) = \sqrt{2}V sin(\omega t - 240^o)\,,\ i_c(t) = \sqrt{2}I sin(\omega t - \phi - 240^o)$
$p_3(t) = v_a i_a + \cdots + v_c i_c = 3VIcos\phi$

$p_3(t) = v_a i_a + v_b i_b + v_c i_c$
$= 2VIsin\omega t \cdot sin(\omega t - \phi) + 2VIsin(\omega t - 120^o) sin(\omega t - 120^o - \phi)$
$+2VIsin(\omega t - 240^o) sin(\omega t - 240^o - \phi).$

Also $sina\ sinb = \frac{1}{2}(cos(a-b) - cos(a+b))$. Thus:

$2VI\left(\frac{1}{2}(cos(\phi) - cos(2\omega t - \phi))\right)$

$+2VI\left(\frac{1}{2}(cos(\phi) - cos(2\omega t - 240 - \phi))\right)$

$+2VI\left(\frac{1}{2}(cos(\phi) - cos(2\omega t - 480 - \phi))\right) =$

$VI(cos(\phi) - cos(2\omega t - \phi)) + VI(cos(\phi) - cos(2\omega t - 240 - \phi)) +$
$+VI(cos(\phi) - cos(2\omega t - 480 - \phi)) = 3VIcos\phi - cos(2\omega t - \phi) -$
$cos(2\omega t - 240 - \phi) - cos(2\omega t - 480 - \phi)$.
Where:

$cos(2\omega t - \phi) = cos(2\omega t)cos(\phi) + sin(2\omega t)sin(\phi)$

$\cos(2\omega t - 240 - \phi) = \cos(2\omega t - 240)\cos(\phi) + \sin(2\omega t - 240)\sin(\phi)$
$\cos(2\omega t - 480 - \phi) = \cos(2\omega t - 480)\cos(\phi) + \sin(2\omega t - 480)\sin(\phi)$
Their sum gives $\cos(\phi)\left(\cos(2\omega t) + \cos(2\omega t - 240 - \phi) + \cos(2\omega t - 480 - \phi)\right) + \sin(\phi)(\sin(2\omega t) + \sin(2\omega t - 240) + \sin(2\omega t - 480)) = 0$

Thus the result is zero. Thus the three phase reactive power is zero.

(iii) Calculate the three-phase reactive power

[3]

As shown above.

b) Assume that12 tonnes of water need to be lifted on the roof of the Electrical and Electronic Engineering Department, total height 36 meters. Quantify the energy involved in carrying out this task in Joules and kWh, and estimate the cost if an electricity driven elevator is used.

[5]

To move 12 tonnes ie 12,000kg will require (energy in joules): 12,000kg * 9.81 m/s^2 * 36m = 4,237,920 Joule. We know that 1 kWh = 3.6 * 10^6 J. This is approximately 1 kWh ie costing 5pence.

c) Compute the probability that a system of 4 identical generating units, each of 50MW capacity and failure rate of 5%, will not meet peak demand of 140MW.

[6]

Availability of generator is 95%.

Capacity (MW)	Probability	Cumulative probability
200	$95\%^4 = 81.45063\%$	100.0000%
150	$4 \times 95\%^3 \times 5\%^1 = 17.14750\%$	18.54938%
100	$6 \times 95\%^2 \times 5\%^2 = 1.35375\%$	1.401875%
50	$4 \times 95\%^1 \times 5\%^3 = 0.0475\%$	0.048125%
0	$5\%^4 = 0.000625\%$	0.000625%

Probability that demand will not be met is 1.4018%.

Question 3

A three-bus power network is presented in Figure below. Data relevant for the load flow analysis on this system are given in per unit values.

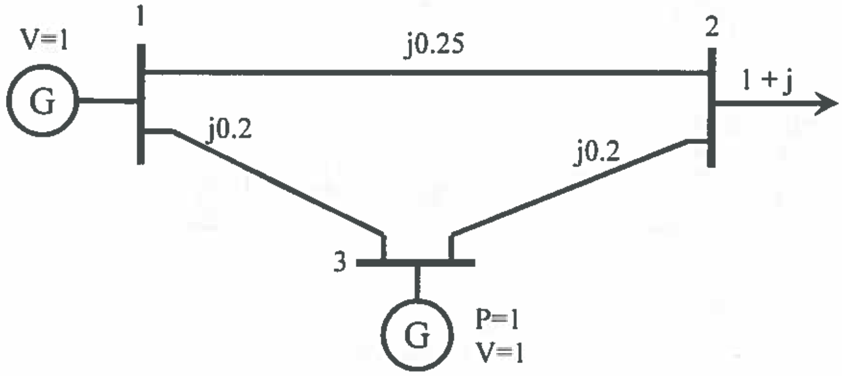


Figure 3. Three bus network showing per unit data for voltages,

active and reactive loads and line reactances

a) Explain the concept of the slack bus in the load flow calculations

[2]

It serves as the voltage angle reference and since Plosses are unknown, it is used to balance load and generation.

b) Form the Ybus matrix and explain why it is singular (i.e. not invertible)

[3]

The Ybus is $\begin{bmatrix} -9 & 4 & 5 \\ 4 & -9 & 5 \\ 5 & 5 & -10 \end{bmatrix}$

c) Perform 2 iteration of the Gauss-Seidel load flow method

[9]

The <u>first iteration</u> of Gauss Siedel is as follows.

It is $V_1^{(0)} = V_2^{(0)} = V_3^{(0)} = 1 + j0$ where 1: slack bus, 2: PQ bus , 3: PV bus.

$$\tilde{V}_2^{(1)} = \frac{1}{Y_{22}} \cdot \left(\frac{S_2^{(0)*}}{\tilde{V}_2^{(0)*}} - Y_{21} \cdot \tilde{V}_1^{(1)} - Y_{23} \cdot \tilde{V}_3^{(0)} \right) = \frac{1}{-j9} \cdot \left(\frac{-1+j}{1} - j4 \cdot 1 - j5 \cdot 1 \right)$$
$$= (0.8889 - j\,0.1111)$$

At this point we calculate Q_3 in order to update it with the new value for V_2.

$Q_3^{(1)} = -Im\left(\tilde{V}_3^{(0)*}\left[\tilde{V}_3^{(0)}(y_{31} + y_{32}) - y_{31}\tilde{V}_1^{(0)} - y_{32}\tilde{V}_2^{(1)}\right]\right) = -Im(1[1(-10j) + 5j + 5j * (0.8889 - j0.1111)])=$

$-Im[-10j + 5j + 5j(0.8889 - j0.1111)] = 0.5555\ pu.$

$$\tilde{V}_3^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{(1)}}{\tilde{V}_3^{(0)}} + y_{31}\tilde{V}_1^{(0)} + y_{32}\tilde{V}_2^{(1)}}{y_{13} + y_{23}} = \frac{\frac{1 - j(0.5555)}{1} - 5j + (-5j)(0.8889 - j\,0.1111)}{-10j}$$
$$= 1.0000 + j0.0445$$

$$\tilde{V}_3^{(1)} = V_3^{spec} \cdot \frac{\tilde{V}_3^{(1)}}{\left|\tilde{V}_3^{(1)}\right|} = 1.00 \cdot \frac{1.0000 + j0.0445}{|1.0000 + j0.0445|} = 0.999 + 0.0445j$$

<u>Second iteration</u>

$$\tilde{V}_2^{(2)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{\tilde{V}_2^{(1)*}} + y_{21}\tilde{V}_1^{(1)} + y_{32}\tilde{V}_3^{(1)}}{y_{12} + y_{23}} = \frac{\frac{-1 + j(1)}{0.8889 + j0.1111} - 4j + (-5j)(0.999 + j0.0445)}{-9j}$$
$$= 0.861 - j0.083$$

$$Q_3^{(2)} = -Im\left(\tilde{V}_3^{(1)*}\left[\tilde{V}_3^{(1)}(y_{31} + y_{32}) - y_{31}\tilde{V}_1^{(1)} - y_{32}\tilde{V}_2^{(2)}\right]\right)$$
$$= -Im((0.999 - 0.0445j)[(0.999 + 0.0445j)(-10j) + 5j(1) + 5j(0.861 - j0.083)])$$
$$= 0.7226pu$$

$$\tilde{V}_3^{(2)} = \frac{\frac{P_3^{sch} - jQ_3^{(2)}}{\tilde{V}_3^{(1)*}} + y_{31}\tilde{V}_1^{(1)} + y_{32}\tilde{V}_2^{(2)}}{y_{13} + y_{23}} = 0.9982 + j0.059\,pu$$

$$\tilde{V}_3^{(2)} = V_3^{spec} \cdot \frac{\tilde{V}_3^{(2)}}{\left|\tilde{V}_3^{(2)}\right|} = 1.00 \cdot \frac{0.9982 + j0.059}{|0.9982 + j0.059|} = 0.9983 + j0.059\,pu.$$

d) Using the results from c), calculate the power mismatch at the PQ bus

[6]

The mismatch is found as follows:
Flows from PQ bus

$$S_{21} = V_2^{(2)} \cdot \left(\left(V_2^{(2)} - V_1^{(0)}\right) \cdot y_{21}\right)^* = -0.332 - j0.4512\,pu$$
$$S_{23} = V_2^{(2)} \cdot \left(\left(V_2^{(2)} - V_3^{(2)}\right) \cdot y_{23}\right)^* = -0.6683 - j0.5321\,pu$$

Power mismatch in PQ bus

$$S_2 = S_{21} + S_{23} = -1.0003 - j0.9833\,pu$$
$$M_2 = S_2^{spec} - S_2 = (-1 - j) - (S2) = 0.0003 - j0.0167pu$$

Section B – Answer any 2 out of 3 questions in section B

Question 4

a) Explain how the system strength at a given busbar is related to the short circuit level there.

[4]

Short circuit level (SCL) at a given busbar is defined as:

$$\text{SCL (in MVA)} = \sqrt{3} V_{TH} I_F$$

where V_L: pre-fault voltage (kV) and I_F: fault-current (kA)

$$\text{SCL (in pu)} = \frac{\text{SCL (in MVA)}}{S_{\text{base}}} = \frac{I_F}{I_{\text{base}}} = I_F\,(\text{pu}) = \frac{V_{TH}}{Z_{TH}}$$

Short circuit level in inversely proportional to the Thevenin impedance at a particular bus bar.

System strength at a particular bus bar is high if the Thevenin impedance is low as it implies any voltage disturbance in the system will have less of an impact at that busbar.

Therefore, a strong (weak) system corresponds to a low (high) Thevenin impedance which is equivalent to a high (low) short circuit level.

b) Explain how the short circuit level would be affected due to the following developments:

(i) An AC interconnection is set up between two different power systems.

[2]

Before interconnection, a fault at any busbar would be fed by only the generators in the system where the fault has occurred. After the two power systems are interconnected, a fault at any busbar would be fed by all the generators in both the systems. So the short circuit level would increase due to the interconnection. Another way to explain is reduction in Thevenin equivalent impedance after the two systems are interconnected.

(ii) Some of the conventional thermal power plants are replaced by non-synchronous generation such as wind power.

[2]

Conventional thermal power plants use synchronous generators which have large fault current capability. However, converter-interfaced non-synchronous generators such as wind power cannot supply large fault currents due to restricted overcurrent capability of the power electronic converters. So the short circuit level would reduce.

c) For a closed electric circuit with finite resistances, flux linkages due to any reason and sources of electromotive force but no series capacitance, state and prove the theorem of constant flux linkage.

[4]

Theorem of constant flux linkage - Flux linkage of any closed circuit with finite resistance and electromotive force cannot change instantaneously

Proof:

Consider a closed electrical network with finite resistances, flux linkages due to any reason and sources of electromotive force but no series capacitance. Sum of the voltages around such a circuit is:

$$\sum Ri + \frac{d\psi}{dt} = \sum e$$

Integrating w.r.t time over 0 to Δt

$$\sum R \int_0^{\Delta t} idt + \Delta\psi = \sum \int_0^{\Delta t} edt$$

For finite R and e, as $\Delta t \rightarrow 0$, $\Delta \Psi \rightarrow 0$

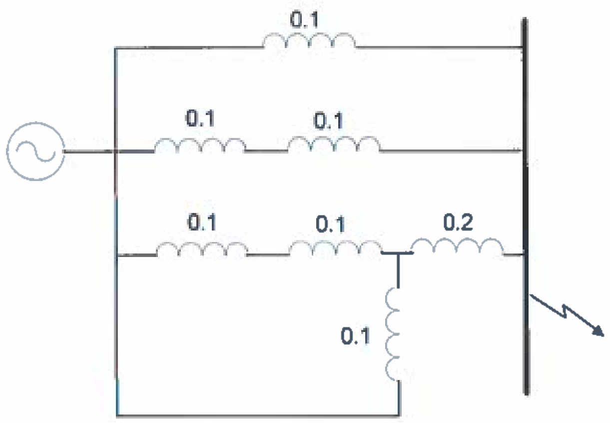
d) Four 11 kV, 50 MVA three-phase generators designated as A, B, C, and D are connected as shown in Figure 4.1. The sub-transient reactance of each generator is 0.1 p.u. The generators are connected by means of three 100 MVA reactors which join A to B, B to C, and C to D as shown in Figure 4.1. The reactance of these reactors is 0.2 p.u., 0.4 p.u., and 0.2 p.u., respectively. Using a 50 MVA base calculate the following for a three-phase symmetrical fault on the terminals of generator B:

(i) Short circuit level (MVA)

[5]

On 50 MVA base, the reactances of line AB, BC and CD are 0.1 p.u., 0.2 p.u. and 0.1 p.u., respectively.

The equivalent circuit with reactances in p.u. on a 50 MVA base is shown below:



Thevenin equivalent impedance at the fault point is:
$Z_{eq} = 0.0533$ p.u..

Short circuit level is

$$S_{SC} = \frac{50}{0.0533} = 937.5 \text{ MVA}$$

(ii) Fault current (in kA)

[3]

Fault current is

$$I_f = 1.667 \times \frac{937.5 \times 10^3}{\sqrt{3} \times 11 \times 10^3} = 49.21 \text{ kA}$$

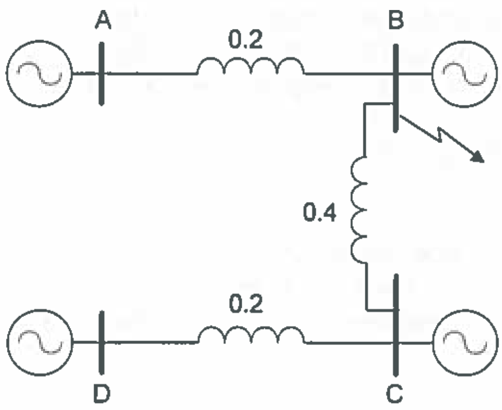


Figure 4.1: Single-line diagram of the generator arrangement for Question 4(d)

Question 5

a) For an unbalanced three-phase star connected load with a neutral connection, derive the following using the symmetrical component transformation:

(i) the neutral current has only got the zero sequence component.

[3]

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & 1 \\ 1 & 1 & \alpha \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

The neutral current is given by:

$$I_n = I_a + I_b + I_c = 3I_{a0} + I_{a1}(1 + \alpha + \alpha^2) + I_{a2}(1 + \alpha + \alpha^2) = 3I_{a0}$$

(ii) there is no zero sequence component in the line voltages.

[2]

Sum of the line voltages is given by:

$$V_{ab} + V_{bc} + V_{ca} = V_{an} - V_{bn} + V_{bn} - V_{cn} + V_{cn} - V_{an} = 0$$

Zero sequence component of line voltage is:

$$V_{L0} = \frac{1}{3}(V_{ab} + V_{bc} + V_{ca}) = 0$$

b) Explain with physical reasons (not mathematically) why a line to ground fault near the generator terminal is more severe than a three-phase fault while the opposite is true for such faults on the transmission line far from the generator.

[4]

Zero sequence reactance of a generator is much smaller than its positive and negative sequence reactances. For transmission lines, zero sequence reactance is larger than the positive and negative sequence reactances.

Three-phase fault current is limited by the positive sequence reactance only. Line to ground fault current is limited by positive, negative and zero sequence reactance if there is a path for zero sequence current to flow.

A line to ground fault near the generator terminal would encounter relatively small zero sequence reactance of the generator while much larger zero sequence reactance of the transmission lines are involved for line to ground faults far from the generator terminals. Hence, line to ground fault close to the generator terminals is usually more severe than three phase faults.

c) An industrial customer is supplied from a three-phase 132kV system with a short circuit level of 4000 MVA at the supply point. Three 15 MVA transformers, connected in parallel, are used to step down the 132 kV supply to an 11 kV busbar from which six 5 MVA, 11 kV motors are fed. The transformers are delta-star connected with the star point of each 11 kV winding, solidly earthed. Each transformer has a reactance of 0.1 p.u. (on their rating). The fault contribution of each motor is equal to five times their rated current with 1.0 p.u. terminal voltage. Using a base of 100 MVA, calculate the following:

(i) the fault current (in kA) for a line-to-ground (LG) fault on the 11 kV busbar when no motors are connected

[6]

On 100 MVA base, fault level at 132 kV busbar is 4000/100 = 40 p.u.

Therefore, equivalent reactance is

$X_{eq} = \frac{1}{40} = 0.025$ p.u.

Sequence reactance of the 132 kV busbar and each transformer on 100 MVA base is calculated on the following table:

	Positive	Negative	Zero
132 kV busbar	0.025 p.u.	0.025 p.u.	No contribution as 132 kV side is delta connected
Each transformer	0.1x100/15 =0.667 p.u.	0.667 p.u.	0.667 p.u.
Equivalent reactance	0.025+0.667/3 = 0.247 p.u.	0.247 p.u.	0.667/3 = 0.222 p.u.

Line to ground fault current is given by:

$$I_{f-LG} = \frac{3}{0.247 + 0.247 + 0.222} = 4.19 \text{ p.u.}$$

Base current is:

$$I_{\text{base}} = \frac{100 \times 10^3}{\sqrt{3} \times 11 \times 10^3} = 5.25 \text{ kA}$$

Fault current in kA is:

$I_{f-LG} = 4.19 \times 5.25 = 22$ kA

(ii) three-phase short circuit level (in MVA) at the 11 kV busbar if all the motors are operating and the voltage at the 11 kV busbar is 1.0 p.u.

[5]

All the motors are connected.
Fault level provided by each motor = 5 x 5 MVA = 25 MVA = 0.25 p.u.
Equivalent reactance = 1/0.25 = 4 p.u.

Since 6 motors are connected in parallel, equivalent reactance = 4/6 = 0.667 p.u.

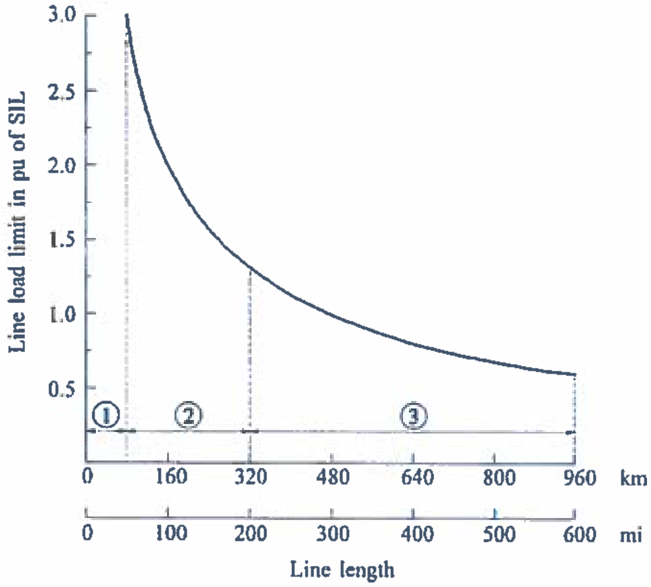
Positive sequence reactance at the 11 kV bus bar = 0.247 || 0.667 = 0.18 p.u.

Three phase short circuit level = 100/0.18 = 554.8 MVA

Question 6

a) Describe with a diagram how the transmission capacity (or loadability) of uncompensated AC overhead transmission lines is limited by various considerations depending on the transmission distance.

[4]



For short distances up to about 80 km, the transmission capacity of an overhead line is limited by the thermal capacity of the conductors.

For medium lengths between 80 and 320 km, the transmission capacity is limited by voltage drop (or stability) considerations.

For longer lines above 320 km, the transmission capacity is limited by angle stability or small-signal stability consideration.

b) Using the sinusoidal power-angle characteristics and the swing equation, derive an expression for the natural frequency of oscillation of a round-rotor synchronous generator in terms of the synchronising coefficient and inertia constant.

[5]

Sinusoidal power angle characteristics:

$$P_e = \frac{EV}{X}\sin\delta$$

For incremental changes in electrical power output:

$$\Delta P_e = \left(\frac{\partial P_e}{\partial \delta}\right)_0 \Delta\delta$$

Swing equation:

$$\frac{2H}{\omega_S}\frac{d^2\delta}{dt^2} = P_m - P_e$$

Considering incremental changes

$$\frac{2H}{\omega_S}\frac{d^2\Delta\delta}{dt^2} = -\Delta P_e \rightarrow \frac{2H}{\omega_S}\frac{d^2\Delta\delta}{dt^2} + \left(\frac{\partial P_e}{\partial \delta}\right)_0 \Delta\delta = 0 \rightarrow \frac{2H}{\omega_S}s^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_0 = 0$$

$$s = \pm\left[-\frac{\left(\frac{\partial P_e}{\partial \delta}\right)_0 \omega_S}{2H}\right]^{\frac{1}{2}}$$

c) A 500 MVA, 50 Hz round-rotor synchronous generator has synchronous reactance ogf 0.2 p.u. and inertia constant of 4 MW-s/MVA. The generator is connected to a large power system through a transformer and overhead line which have a combined reactance of 0.3 p.u. on a base of 500 MVA. The magnitude of the voltage at both the generator terminals and at the connection point with the large power system is 1.0 p.u. The generator delivers 450 MW to the power system. Neglecting resistances, calculate the following:

(i) the internal voltage of the generator behind the synchronous reactance. [3]

$$P\,(\text{p.u}) = \frac{450}{500} = 0.9\text{ p.u.}$$

$$P = \frac{V_t V_{inf}}{X}\sin\theta \rightarrow \theta = \sin^{-1}\left(\frac{0.9\times 0.3}{1}\right) = 15.66°$$

Current I is given by:

$$I = \frac{1\angle\theta - 1}{j0.3}$$

Internal voltage of the generator is:

$$E = 1\angle\theta + j0.2\times\frac{1\angle\theta - 1}{j0.3}$$

$|E| = 1.04$ p.u.

(ii) the natural frequency of oscillation (in Hz) of the generator under the above condition [4]

Power angle of the generator is given by:

$$\sin\delta_0 = \frac{P}{P_{max}} = \frac{0.9\times(0.2+0.3)}{1.04\times 1} \rightarrow \delta_0 = 25.64°$$

Natural frequency of oscillation is

$$\omega_n = \left[\frac{\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0} \omega_S}{2H}\right]^{0.5} = \left[\frac{\frac{1.04 \times 1}{(0.2 + 0.3)} \cos 25.64° \times 50}{2 \times 4}\right]^{0.5} = 1.37 \text{ Hz}$$

(iii) the critical clearing angle for a three-phase fault at the generator terminals. Use equal area criterion and assume the generator power output to be zero during the fault.

[4]

Maximum allowable power angle is

$$\delta_{\max} = 180° - \delta_0 = 180° - 25.64° = 154.36°$$

Accelerating area is

$$\int_{\delta_0}^{\delta_{\text{cl}}} (P - 0) d\delta = 0.9 \times (\delta_{\text{cl}} - 25.64°) \text{ p.u.-deg}$$

Decelerating area is

$$\int_{\delta_{\text{cl}}}^{\delta_{\max}} (P_{\max} \sin \delta - P) d\delta = \frac{1.04}{(0.2 + 0.3)} \times (\cos \delta_{\text{cl}} - \cos 154.36°) + 0.9 \times (\delta_{\text{cl}} - 154.36°) \text{ p.u.-deg}$$

Using equal-area criteria

$$\delta_{\text{cl}} = \cos^{-1}\left(\frac{2.08 \cos 154.36° + 0.9 \times 154.36 \times {}^{\pi}/_{180} - 0.9 \times 25.64 \times {}^{\pi}/_{180}}{2.08}\right)$$
$$= 85.95°$$