E1.4 SOLUTIONS

Question 1

a) The 10 kΩ resistor carries the emitter current, so for a collector voltage of 2.5 V we require $I_E = 0.25$ mA, implying a base current of $I_B = 250/201 = 1.244\ \mu A$. Assuming $V_{BE} = 0.7$ V, the required value of R_B is given by $\mathbf{R_B = (5 - 0.7)/I_B = 3.46\ M\Omega}$. [6]

b) From the simplified Ebers-Moll equation, the collector currents can be expressed as:

$$I_{C1} = I_S \exp[(V_{IN1} - V_E)/V_T] \ ; \ I_{C2} = I_S \exp[(V_{IN2} - V_E)/V_T]$$

where V_E is the common emitter voltage, V_T is the thermal voltage, and I_S is the saturation current (NB assuming identical transistors). It follows that $I_{C1}/I_{C2} = \exp(V_D/V_T)$, and we also know that $I_{C1} + I_{C2} = I$ where I is the tail current. Eliminating I_{C1} or I_{C2} the collector currents are obtained as:

$$I_{C1} = I/[1+\exp(-V_D/V_T] \ ; \ I_{C2} = I/[1+\exp(V_D/V_T]$$

The differential output voltage is then:

$$V_{OUT} = R_C(I_{C1} - I_{C2}) = R_C I\left[\frac{1}{1+\exp(-V_D/V_T)} - \frac{1}{1+\exp(V_D/V_T)}\right] = R_C I \tanh(V_D/2V_T)$$

with $R_C = 5$ kΩ, $I = 1$ mA, and $V_T = 25$ mV, the output voltage becomes $5\tanh(20V_D)$ as required. [6]

The double-ended differential gain can be obtained simply as the derivative of the large-signal relationship at $V_D = 0$:

$$A = \left.\frac{dV_{OUT}}{dV_D}\right|_{V_D=0} = 100\,\mathrm{sech}^2(20V_D)\Big|_{V_D=0} = \mathbf{+100}$$

A solution based on analysis of the SSEC is also acceptable. [4]

c) Since the transistors are matched, only the finite beta and the output resistance will contribute to the current error. Taking both of these contributions into account, the output current I can be expressed as:

$$I \approx I_{ref} - 2I/\beta + \Delta V_{CE}/r_o$$

where I_{ref} is the input current. So, the currents will be equal when $\Delta V_{CE} \approx 2Ir_o/\beta = 2V_A/\beta$, where we have used $r_o = V_A/I$. Putting $V_A = 120$ V, $\beta = 100$ gives $\Delta V_{CE} \approx 2.4$ V, and since the input side transistor has $V_{CE} \approx 0.7$ V (assumption), this implies $\mathbf{V_{OUT} \approx 3.1\ V}$. [6]

d) Since we are calculating for the case where the MOSFET is at pinch-off, we know we can use the active mode drain current equation. With $V_G = 0$, and $V_S = I_D R_S$, we need to solve:

$$I_D = V_S/R_S = K(-V_S - V_t)^2 \Rightarrow 2V_S^2 - 5V_S + 2 = 0$$

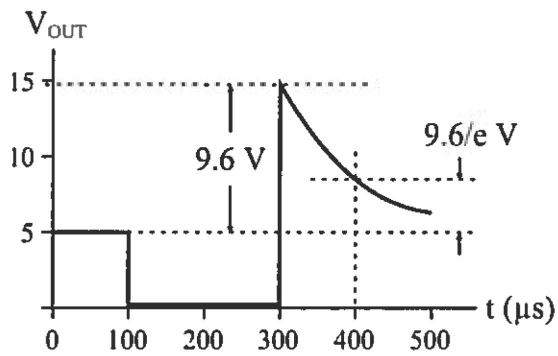
The roots are $V_S = 0.5$, $V_S = 2$, and the valid root is the first one (since second leaves MOSFET sub-threshold). The drain current is $V_S/R_S = 50\ \mu A$.

Since $V_S = 0.5$, and $V_{DS} = V_{GS} - V_t = 0.5$ V, the drain voltage is 1.0 V, and the supply voltage is $V_{DD} = V_D + I_D R_D = 1 + 0.05\text{m}\times 30\text{k} = \mathbf{2.5\ V}$. [6]

e) Before the input pulse, the transistor is off and the circuit is in steady-state with $I_L = 0$ and $V_{OUT} = 5$V. During the input pulse, the base current is $I_B = (5 - 0.7)/5\text{k} = 0.86$ mA, and the collector current is $\beta I_B = 172$ mA if transistor is active, or <172 mA if it is saturated. We have assumed $V_{BE} = 0.7$ V.

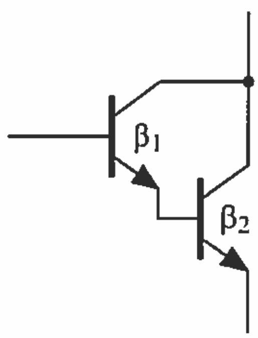
The inductor current is continuous, and the maximum current in the 100 Ω resistor is ≈ 50 mA, so the transistor is initially saturated with $V_{CE} = V_{CEsat} \approx 0.2$ V (assumption) and $I_C \approx 48$ mA. With 4.8 V across the inductor, the inductor current will rise at a rate of $dI_L/dt = V_L/L = 480$ A/sec. By the end of the pulse it will have reached $480 \times 200\mu = 96$ mA. Since the total load current at this point (96 + 48 = 144 mA) is <172 mA we know the transistor does not come out of saturation.

When the transistor switches off, the inductor current will continue to flow in the 100 Ω resistor, and the output voltage will rise suddenly to $V_{CC} + I_L R = 5 + 0.096\times 100 = 14.6$ V. It will then decay exponentially over time, with an asymptotic value of 5 V. The time-constant for this decay will be $L/R = 0.01/100 = 100\ \mu s$. [4]



[4]

f)



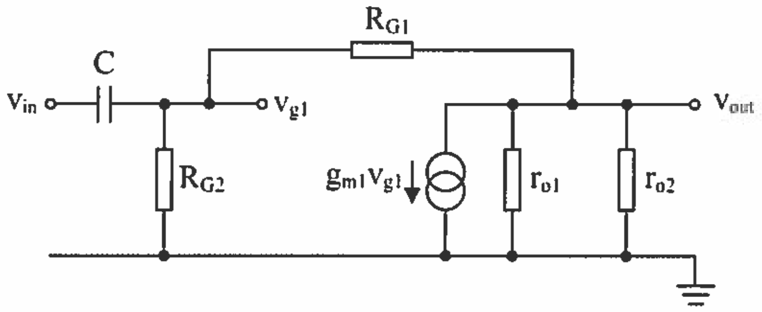
The base current of the RH transistor is $I_{B2} = (1 + \beta_1)I_{B1}$. The total collector current for the Darlington pair is $I_C = I_{C1} + I_{C2} = \beta_1 I_{B1} + \beta_2 I_{B2} = [\beta_1 + (1 + \beta_1)\beta_2]I_{B1}$. The overall current gain is then $I_C/I_{B1} = \beta_1 + \beta_2 + \beta_1\beta_2$ [4]

Question 2

a) The drain current will be set by the depletion load which has $V_{GS} = 0$. Assuming this device is active (as question suggests), we have $\mathbf{I_D = K_2(-V_{t2})^2 = 0.45\ mA}$. The lower MOSFET must be carrying the same current, so we also have $I_D = K_1(V_{GS1} - V_{t1})^2$. Rearranging this we obtain $V_{GS1} = \sqrt{(I_D/K_1)} + V_{t1} = 1.75$ V, where we have taken the +ve square root because Q1 is above threshold. The potential divider forces $V_{OUT} = 3V_{GS1}$, so $\mathbf{V_{OUT} = 5.25\ V}$. [6]

Q1 has $V_{DS} = 5.25$ V, $V_{GS} = 1.75$ V, $V_t = 1$ V, so $V_{DS} > V_{GS} - V_t$ and **active**.
Q2 has $V_{DS} = 10 - 5.25 = 4.75$V, $V_{GS} = 0$, $V_t = -1.5$ V, so also **active**.
Q2 will enter triode when $V_{DS} = -V_t = 1.5$ V, i.e. when $V_{DD} = 5.25 + 1.5 = \mathbf{6.75\ V}$. [3]

b) SSEC (input capacitor may be omitted):



$g_{m1} = 2\sqrt{(K_1 I_D)} = 1.2$ mA/V
$r_{o1} = r_{o2} = V_A/I_D = 222$ kΩ
$R_{G1} = 2$ MΩ
$R_{G2} = 1$ MΩ

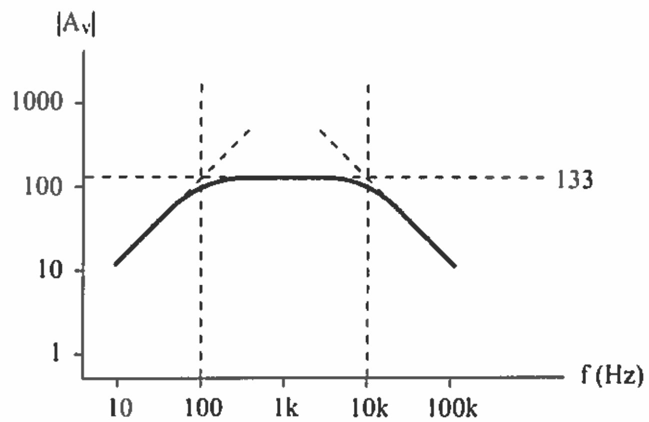
[6]

KCL at output gives: $g_{m1}v_{g1} + v_{out}/r_{o1} + v_{out}/r_{o2} + (v_{out} - v_{g1})/R_{G1} = 0$. Collecting terms in v_{out} and v_{g1}, and noting the $v_{g1} \approx v_{in}$ in the mid-band, the mid-band gain is obtained as:

$$A_v = v_{out}/v_{g1} = -(g_{m1} - 1/R_{G1}) \cdot (r_{o1} // r_{o2} // R_{G1}) = -1.2\text{m} \times 105.3\text{k} = \mathbf{-126.3}$$ [3]

The input resistance is $R_i = R_{G2}//[R_{G1}/(1-A_V)] = 1\text{M}//15.7\text{k} = \mathbf{15.5k}$. The cut-off frequency of the input filter is $f_{ci} = 1/(2\pi R_i C)$, so for $f_{ci} = 100$ Hz we require $C = 1/(2\pi R_i f_{ci}) = \mathbf{103\ nF}$. [5]

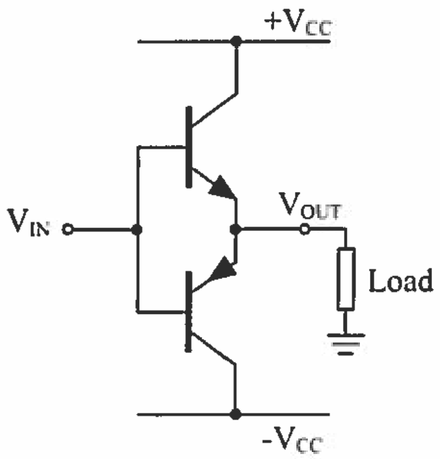
c) The load produces a low-pass response, with cut-off $f_{co} = 1/(2\pi R_o C_L)$ where R_o is the amplifier output resistance and C_L is the load capacitance. With $R_o = r_{o1}//r_{o2}//R_{G1} = 105.3$ kΩ, and $C_L = 150$ pF, the cut-off is at $f_{co} = 10.08$ kHz. [2]

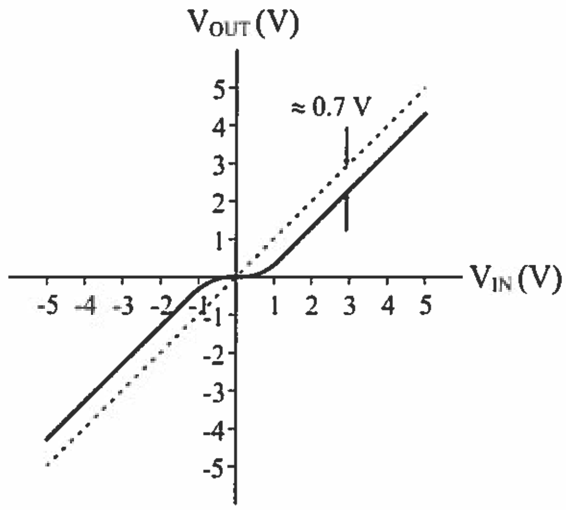


[5]

Question 3

a)





[4 + 4]

The output stage is a unity (voltage) gain amplifier, but there is a V_{BE} offset between input and output, and a region near the origin where it is unresponsive, leading to cross-over distortion. [2]

b) From symmetry, Vout = 0 occurs when Vin = 0. KVL under these conditions gives:

$$V_{BE1} = V_{BE2} + I_2R$$

where we have ignored the base current of Q2. From the simplified Ebers-Moll equation we know that $V_{BE1} = V_T\ln(I_1/I_{S1})$ and $V_{BE2} = V_T\ln(I_2/I_{S2}) = V_T\ln[I_2/(NI_{S1})]$. Substituting for V_{BE1} and V_{BE2} in the top equation we obtain $V_T\ln(I_1/I_{S1}) = V_T\ln[I_2/(NI_{S1})] + I_2R$. Taking the inverse log of both sides gives the desired result. [8]

Rearranging the given equation: $R = (V_T/I_2)\ln(NI_1/I_2)$. With $I_{S1} = 0.05$ pA, $N = 10$, $V_T = 25$ mV, $I_1 = 2$ mA, $I_2 = 10$ mA, we find **R = 1.73 Ω.** [2]

c) With $V_{OUT} = 5$ V and a 50 Ω load, Q2 has an emitter current of $I_{E2} = 100$ mA. The base-emitter voltage of Q2 under these conditions is $\mathbf{V_{BE2} = V_T\ln(\alpha I_{E2}/I_{S2}) = 650}$ **mV**. The emitter current of Q1 is $I_{E1} = I_1 - I_{E2}/(1 + \beta) = 2 - 100/101 = 1.01$ mA, and its base-emitter voltage is $\mathbf{V_{BE1} = V_T\ln(\alpha I_{E1}/I_{S1}) = 593}$ **mV**. The input voltage is $Vin = Vout + I_{E2}R + V_{BE2} - V_{BE1}$, which gives **Vin = 5.23 V**. The voltage gain is therefore **Vout/Vin = 0.956**. [7]

d) With $V_{OUT} = 10$ V, the emitter currents of Q2 and Q1 are $I_{E2} = 200$ mA and $I_{E1} = 0.02$ mA respectively (almost at limit of output range). Repeating the above calculations in this case gives $V_{BE2} = 668$ mV, $V_{BE1} = 495$ mV, Vin = 10.52 V, and **Vout/Vin = 0.951**.

The amplifier appears to be linear to within ≈0.5%, based on the gains at 50% and 100% of full scale output. However, the comparison doesn't tell us anything about the residual cross-over distortion. [3]