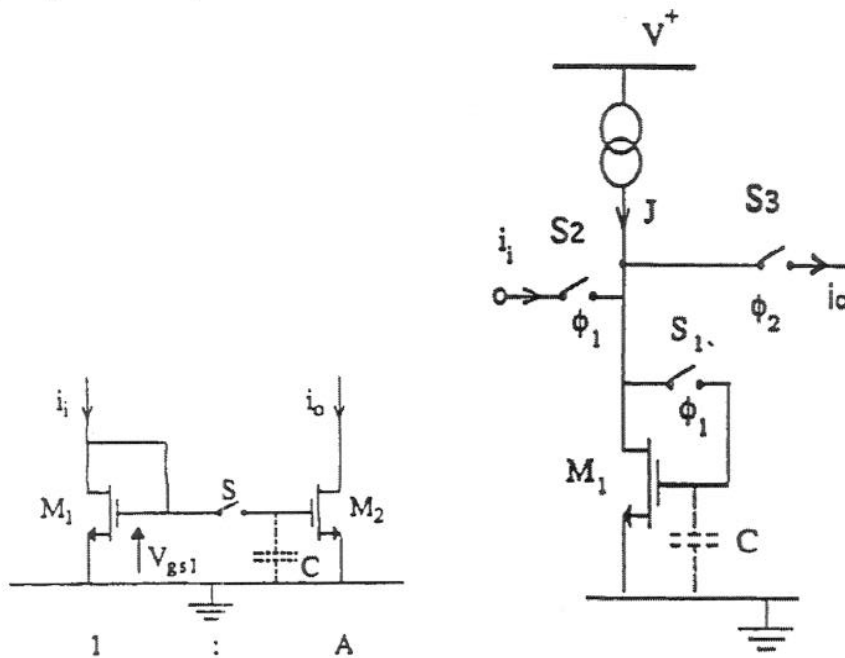


The Answers

1.

(a) [Bookwork]



First generation cell on the left. Single phase clock on S, it is a track and hold circuit. Major limitation is transistor mismatch (gain, size, temperature due to different power dissipation)

Second generation on the right. Two phase clock, during ϕ_1 it acts like a track circuit, during ϕ_2 it acts like a hold circuit.

(b) [Interpretation] The step response is derived from the continuous time response of the current mirror. This is a single pole system with a pole at

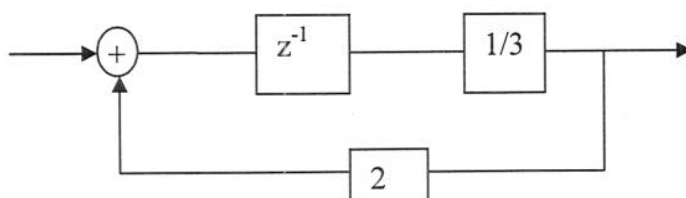
$$\omega_0 = g_m / 2C_{gs} = \omega_T / 2, \text{ so that the step response is } y(t) = 1 - e^{-\omega_0 t} = 1 - e^{-\omega_T t / 2}$$

(c) [Numerical example] Settling to 0.01% requires 9.2 time constants, so that the maximum sampling frequency is $f_s < f_T / 19 \approx 52 \text{ MHz}$. Finally, the sampling theorem dictates that $f_B < f_s / 2$. It follows that the signal frequency must be less than $\max f_B = 26 \text{ MHz}$.

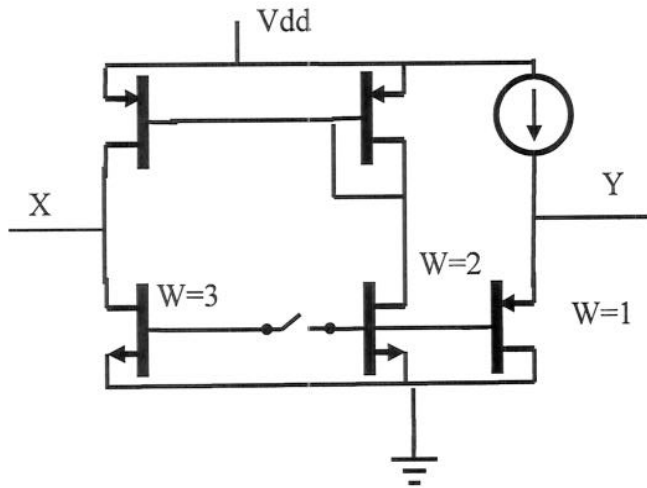
(d) [Synthesis example] We can factor this as

$$Y_N = \frac{1}{3}((2Y_{N-1} + X_{N-1}) + X_N) \Rightarrow y = \frac{1}{3}(z^{-1}(2y + x) + x)$$

A diagram for this is



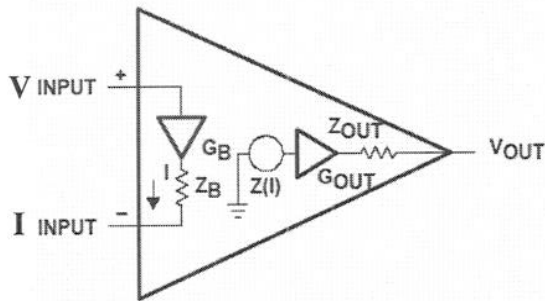
This can be implemented with a two output 1st gen.mirror with $W(Q1)=2$, and the y output $W=1$, the feedback output $=2$.



[5]

2.

- (a) [bookwork] A CFOA comprises a voltage follower from the non-inverting to the inverting input, and a high gain transimpedance amplifier from the inverting input to the (voltage) output. A block diagram follows:



The slew rate is unlimited since it is proportional to the input current, which is only limited by the power handling of the device. In a conventional, voltage mode, op-amp, the slew rate is limited by the integrating node current supply.

[5]

- (b) [Interpretation] There are two transfer functions: The voltage buffer which has a response:

$$H(s) = \frac{1}{1 + s\tau_B}, \text{ and the transimpedance amplifier which consists of a current mirror, an}$$

impedance and a voltage buffer, so:

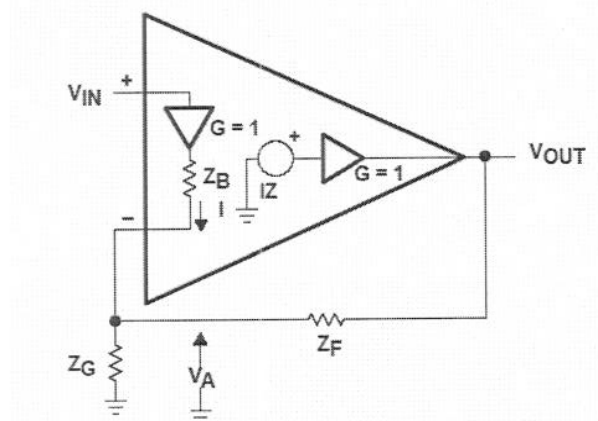
$$T(s) = \frac{Z(s)}{(1 + s\tau_M)(1 + s\tau_B)}. \text{ Usually the transimpedance } Z(s) \text{ is a shunt RC network, so}$$

$$T(s) = \frac{Z_0}{(1 + s\tau_M)(1 + s\tau_B)(1 + sZ_0C_0)} \approx \frac{1}{sC_0(1 + s\tau_M)(1 + s\tau_B)}$$

At low frequencies this is an integrator. (or a dominant pole transimpedance amplifier)

[5]

- (c) [Extension of bookwork] The non-inverting amplifier can be analysed as



$$V_{out} = -Z(s)I_-$$

$$\begin{aligned} I_- &= \frac{1}{Z_B}(V_- - V_{in}) = \frac{1}{Z_B} \left(V_{in} \left(\frac{Z_G // Z_F}{Z_B + Z_G // Z_F} - 1 \right) + V_{out} \frac{Z_B // Z_G}{Z_B // Z_G + Z_F} \right) = \\ &= \frac{1}{Z_B} \left(\frac{-V_{in}(Z_B Z_G + Z_F Z_B)}{Z_B Z_G + Z_F Z_B + Z_F Z_G} + V_{out} \frac{Z_B Z_G}{Z_B Z_G + Z_F Z_B + Z_F Z_G} \right) \Rightarrow \\ V_{out} &= \frac{-Z(s)}{Z_B} \left(\frac{-V_{in}(Z_B Z_G + Z_F Z_B)}{Z_B Z_G + Z_F Z_B + Z_F Z_G} + V_{out} \frac{Z_B Z_G}{Z_B Z_G + Z_F Z_B + Z_F Z_G} \right) \Rightarrow \\ \frac{V_{out}}{V_{in}} &= \frac{Z(s)(Z_G + Z_F)}{Z_B Z_G + Z_F Z_B + Z_F Z_G + Z(s)Z_G} \end{aligned}$$

[5]

Assuming a dominant pole $Z(s) = \frac{Z_0}{1+s\tau}$ we get

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{Z_0(Z_G + Z_F)}{(1+s\tau)(Z_B Z_G + Z_F Z_B + Z_F Z_G) + Z_0 Z_G} = \\ &= \frac{Z_0(Z_G + Z_F)}{Z_B Z_G + Z_F Z_B + Z_F Z_G + Z_0 Z_G} \frac{1}{1 + \frac{s\tau(Z_B Z_G + Z_F Z_B + Z_F Z_G)}{Z_B Z_G + Z_F Z_B + Z_F Z_G + Z_0 Z_G}} \end{aligned}$$

If $Z_0 \gg Z_B, Z_G, Z_F$ and $Z_B \ll Z_G, Z_F$

$$\frac{V_{out}}{V_{in}} \approx \left(1 + \frac{Z_F}{Z_G} \right) \frac{1}{1 + \frac{s\tau(Z_B + Z_F)}{Z_0}} \text{ so that the gain depends on both } Z_G \text{ and } Z_F \text{ while the}$$

bandwidth $\omega_B = \frac{Z_0}{\tau(Z_B + Z_F)}$ only depends on Z_F , Z_B and Z_0

[5]

3.

(a) [bookwork]

i. a CCII is a voltage and current controlled current source. It can be described as:

$$\begin{bmatrix} i_y \\ v_x \\ i_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \pm 1 & 0 \end{bmatrix} \begin{bmatrix} v_y \\ i_x \\ v_z \end{bmatrix}$$

ii. The 1st generation conveyor mirrors i_x to i_y . $i_y=0$ in the 2nd generation conveyor.

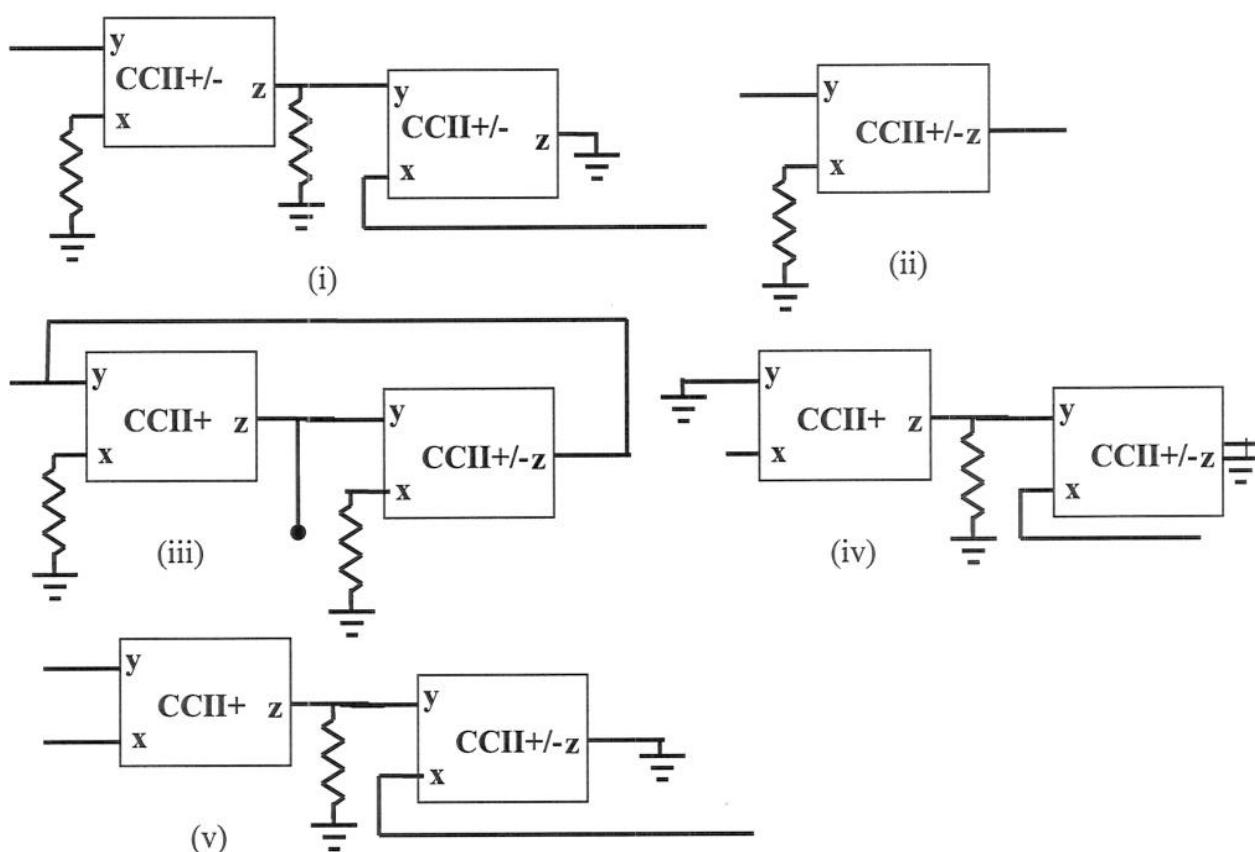
iii. an op-amp has 2 voltage inputs and a voltage output. A cc has a voltage input, a current input and a current output.

iv. A CFOA has a voltage output

v. A FET.

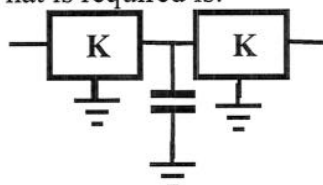
[5]

(b) [Mostly bookwork, some interpretation]



[10]

(c) [Synthesis Example] A grounded inductor between two gyrators is equivalent to a floating inductor. So What is required is:



Where each of the gyrators is the answer to biii

[5]

4.

a) [bookwork] On a closed loop of p-n junctions $\prod_{CW} J_C = \prod_{CCW} J_C$ (J are collector current densities)

For this to be true we need:

- number of CW junctions = number of CCW junctions
- number of CW npn = number of CCW npn
- number of CW pnp = number of CCW pnp
- all junctions at the same temperature

[5]

b) [interpretation] Valid at subthreshold, same expressions and conditions as bipolar principle.

Advantages: low power, drain or source current can be used indiscriminantly

Disadvantage: Threshold voltage comes into play.

[5]

c) [computed example]

$$I_2 = I_{out} + I_6$$

$$I_1 I_2 I_3 I_4 I_5 = I_6 I_7 I_8 I_9 I_{10} \Rightarrow z^2 I_1 I_{out}^2 = z^2 I_3 I_4^2 \Rightarrow I_{out} = I_4 \sqrt{\frac{I_3}{I_1}}$$

$$\text{if } |I_3| = \left| \frac{d}{dt} I_{in} \right| = \omega |I_{in}| \text{ and } |I_1| = \left| \int I_{in} dt \right| = \frac{1}{\omega} |I_{in}| \Rightarrow I_{out} \propto \omega I_{in}$$

[10]

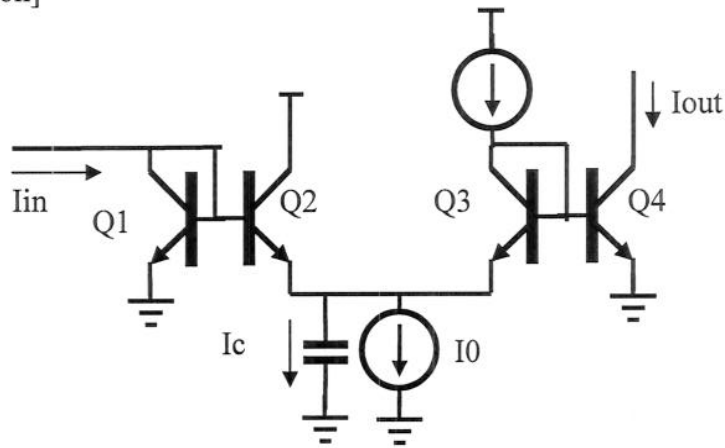
5.

a) [bookwork]

$$\begin{aligned}
 I_{in} &= I_s e^{\beta V_i} \Rightarrow V_i = V_T \ln(I_{in} / I_s) \\
 I_C + I_0 &= C \frac{dV_C}{dt} + I_0 = I_s e^{\beta(V_i - V_C)} = I_{in} e^{-\beta V_C} \\
 I_{out} &= I_s e^{\beta V_i} = I_s e^{\beta V_C + \ln(I_0 / I_s)} = I_s e^{\beta V_C} I_0 / I_s = I_0 e^{\beta V_C} = I_0 x \\
 x &= e^{\beta V_C} \\
 \frac{dx}{dt} &= \beta \frac{dV_C}{dt} x \\
 C \frac{dV_C}{dt} + I_0 - I_{in} e^{-\beta V_C} &= 0 \Rightarrow \frac{C}{\beta x} \frac{dx}{dt} + I_0 - \frac{I_{in}}{x} = 0 \Rightarrow \frac{C}{\beta} \frac{dx}{dt} + I_0 x = I_{in}
 \end{aligned}
 \left. \vphantom{\begin{aligned} I_{in} &= I_s e^{\beta V_i} \\ I_C + I_0 &= C \frac{dV_C}{dt} + I_0 \\ I_{out} &= I_s e^{\beta V_i} \end{aligned}} \right\} \text{with } \beta = q / kT = 1 / V_T$$

[5]

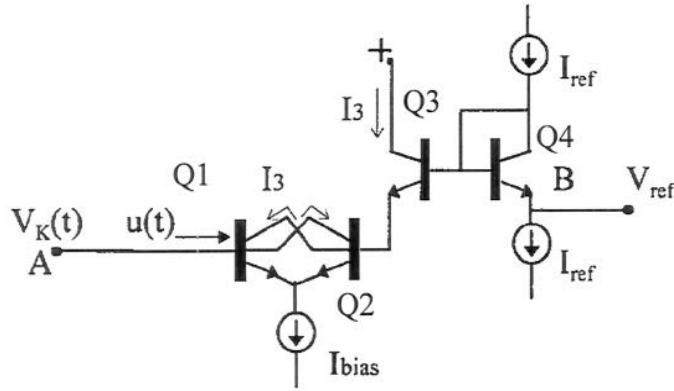
b) [interpretation]



$$\begin{aligned}
 I_{C2} &= I_C - I_U = I_s e^{\beta V_{in}} / e^{\beta V_C} \Rightarrow I_{C2}' = \beta I_{C2} [V_{in}' - V_C'] \\
 V_C' &= \frac{1}{C} I_C = \frac{1}{C} [I_{C2} - I_U] \\
 I_{C2}' &= \beta I_{C2} [V_{in}' - V_C'] = \beta I_{C2} \left[V_{in}' - \frac{1}{C} [I_{C2} - I_U] \right] \Rightarrow \\
 I_{C2}' - \frac{\beta}{C} (C V_{in}' + I_U) I_{C2} + \frac{\beta}{C} I_{C2}^2 &= 0 \Rightarrow I_{C2}' - \frac{\beta}{C} (C V_{in}' + I_U) I_{C2} + \frac{\beta}{C} I_{C2}^2 = 0
 \end{aligned}$$

[5]

c) [computed example, interpretation]



$$\left. \begin{aligned} I_3 &= I_s e^{\beta V_{BE3}} \\ V_{B3} &= V_{B4} = V_{ref} + \frac{1}{\beta} \ln(I_{ref} / I_s) \\ V_{E3} &= V_K \end{aligned} \right\} \Rightarrow I_3 = u = I_s \exp \beta \left(V_{ref} + \frac{1}{\beta} \ln(I_{ref} / I_s) - V_K \right) \Rightarrow$$

$$u = I_s \exp \beta (V_{ref} - V_K) \exp \left(\ln(I_{ref} / I_s) \right) = I_{ref} \exp \beta (V_{ref} - V_K)$$

This is useful for 2 reasons:

- it implements the exponential mapping $x = e^{\beta V_c}$ i.e. generates a current signal proportional to the exponential of a voltage somewhere else in the circuit.
- It implements the $z = 1/x$ function required to linearise the ODE describing the log domain cell.

[5]

d) [computed example] Normalised notation:

$$t \rightarrow \omega_0 t$$

$$\frac{d^2 y}{dt^2} + \omega_0^2 y = \omega_0^2 U \Rightarrow \ddot{y} + y = x$$

$$\left. \begin{aligned} \dot{y} &= z \\ \dot{z} + y &= x \end{aligned} \right\} \Rightarrow \frac{d}{d\tau} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ x \end{bmatrix}$$

[5]