EE4-40
EE9-SO20

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected Copy

INFORMATION THEORY

Tuesday, 13 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) : C. Ling
Second Marker(s) : D. Gunduz

The Questions

1. Basics of information theory.

a) X and Y are correlated binary random variables with $p(X=Y=0)=0$ and all other joint probabilities equal to 1/3. Calculate $H(X)$, $H(Y)$, $H(X \mid Y)$, $H(Y \mid X)$, $H(X,Y)$, $I(X;Y)$.

[6]

b) Suppose X_1 and X_2 are i.i.d. Bernoulli random variables taking values of 0 and 1 with equal probabilities ($p = 0.5$). Let $Y_1 = X_2$, $Y_2 = X_1$, and $Y_3 = X_1 \oplus X_2$. Compute the following mutual information:

i) $I(X_1 ; Y_1)$

ii) $I(X_2 ; Y_2)$

iii) $I(X_{1:2} ; Y_{1:2})$

iv) $I(X_1 ; X_2 \mid Y_3)$

[8]

c) Consider a Markov process with two states, 0 and 1, and transition matrix

$$T = \begin{pmatrix} 1-p & p \\ q & 1-q \end{pmatrix}.$$

i) Determine the stationary distribution.

ii) Calculate the entropy rate, $H(\mathcal{X})$.

iii) Find the values of p and q that maximize $H(\mathcal{X})$.

[11]

b) Upper bound on the rate-distortion function. For the case of a continuous random variable X with mean zero and variance σ^2 and squared-error distortion, show that the Gaussian distribution has the largest rate-distortion function, i.e., the rate-distortion function for X is bounded as follows:

$$R(D) \leq \frac{1}{2}\log\frac{\sigma^2}{D}.$$

Hint: use the following joint distribution of X and $\hat{X}$ in Fig. 2.2.

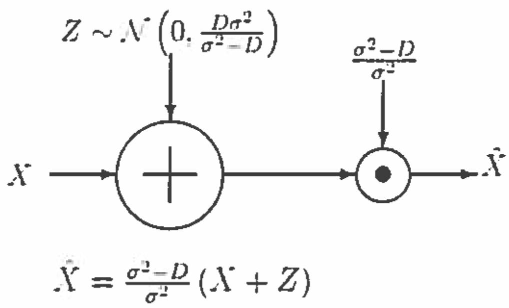

Fig. 2.2. Joint distribution of X and $\hat{X}$. X and Z are independent.

[10]

4. Network information theory.

a) Consider the inference channel in Fig. 4.1. There are two senders with equal power P, two receivers, with crosstalk coefficient a. The noise is Gaussian with zero mean and variance N. Show that the capacity under very strong interference (i.e., $a^2 \geq 1 + P/N$) is equal to the capacity under no interference at all.

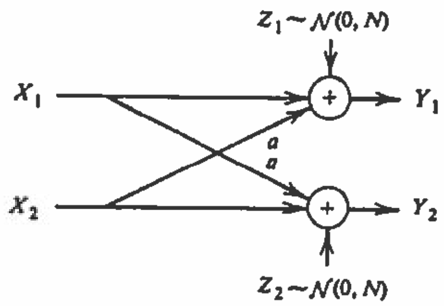

Fig. 4.1. Interference channel.

[10]

b) Slepian-Wolf coding. Two senders know random variables U_1 and U_2 respectively. Let the random variables (U_1, U_2) have the following joint distribution:

$U_1 \backslash U_2$	0	1	2	$\cdots$	$m-1$
0	α	$\frac{\beta}{m-1}$	$\frac{\beta}{m-1}$	$\cdots$	$\frac{\beta}{m-1}$
1	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0
2	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$m-1$	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0

where $\alpha+\beta+\gamma = 1$. Find the region of rates (R_1, R_2) that allow a common receiver to decode both random variables reliably.

[15]