E3.12
SO13

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

MSc and EEE PART III/IV: M.Eng., B.Eng. and ACGI

10:15

Fig 1.1 - swap n1 & n2

**OPTOELECTRONICS**

Friday, 9 May 10:00 am

Time allowed: 3:00 hours

**There are SIX questions on this paper.**

**Answer FOUR questions.**

**Corrected Copy**

**Any special instructions for invigilators and information for candidates are on page 1.**

| | | |
|---|---|---|
| Examiners responsible | First Marker(s) : | R.R.A. Syms |
| | Second Marker(s) : | W.T. Pike |

Fundamental constants

$e = 1.6 \times 10^{-19}\ C$

$m_e = 9.1 \times 10^{-31}\ kg$

$\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$

$m_0 = 4\pi \times 10^{-7}\ m\ kg/C^2$

$c = 3 \times 10^8\ m/s$

$h = 6.62 \times 10^{-34}\ Js$

1. An asymmetric dielectric waveguide is constructed from three layers of material, with refractive indices $n_1$, $n_2$ and $n_3$ as shown in Figure 1.1. The core thickness is $d$. The eigenvalue equation for TE modes in such a guide can be obtained as:

$$\tan(\kappa d) = \kappa[\gamma + \delta] / [\kappa^2 - \gamma\delta] \tag{1.1}$$

Here $\kappa = \sqrt{\{n_1^2 k_0^2 - \beta^2\}}$, $\gamma = \sqrt{\{\beta^2 - n_2^2 k_0^2\}}$ and $\delta = \sqrt{\{\beta^2 - n_3^2 k_0^2\}}$, $\beta$ is the propagation constant and $k_0 = 2\pi/\lambda$, where $\lambda$ is the wavelength.

a) Show that when $n_2 = n_3$ (i.e., when the guide is symmetric) the eigenvalue equation reduces to:

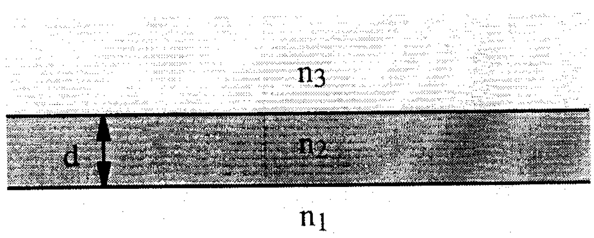
$\tan(\kappa d/2) = \gamma/\kappa$ for all modes with symmetric field patterns
$\tan(\kappa d/2) = -\kappa/\gamma$ for all antisymmetric modes.

(1.2)

[4]

b) Describe the physical phenomena occurring at cutoff. What values do $\beta$, $\gamma$ and $\kappa$ tend to at cutoff? Obtain an expression for the cutoff condition of all guided modes.
[10]

c) A disc drive failure has unfortunately resulted in the partial destruction of records concerning recently fabricated symmetric slab waveguides. The only information that can be salvaged is shown in Table 1.1 below. Complete the table.
[6]




n1

n2

*Figure 1.1.*

| Sample | $n_1$ | $n_2$ | $d$ (μm) | $\lambda_{cutoff}$ (μm)* |
|---|---|---|---|---|
| #1 | 1.500 | | 5.8 | 0.6330 |
| #2 | 1.510 | 1.500 | | 0.6330 |
| #3 | 1.505 | 1.500 | 3.8 | |

*Cutoff wavelength for the second lowest-order mode.

*Table 1.1.*

2. The function $n(r) = n_0 \sqrt{\{1 - (r/r_0)^2\}}$ is used to model an optical fibre with a refractive index that varies parabolically in the x-y plane.

a) Sketch the variation $n(r)$, and also the refractive index variation that might be expected in a real fibre. [4]

b) Ignoring the y-variation of the refractive index, show that all ray trajectories in the $(x, z)$ plane have the form $x = A \sin(Bz + C)$, and find $A$ and $B$. [10]

c) Using the results of part b), explain the operation of a quarter-pitch GRIN rod lens. [6]

3. Figure 3.1 below shows several different Y-junction devices. The guided ports are each numbered, and in the questions below, $A_0$ will be the modal amplitude input to (or output from) Port 0, and so on. $P_0$ will be the corresponding modal power.

a) In the Y-junction shown in Figure 3.1a, find the outputs $A_1$ and $A_2$, and powers $P_1$ and $P_2$, assuming that the input is $A_0 = 1$. [4]

b) In Figure 3.1b, find $A_0$, assuming that $A_1$ and $A_2$ have arbitrary values $a_1$ and $a_2$. Find $P_0$, and also the radiated power $P_R$. [6]

c) In the two-stage Y-junction tree shown in Figure 3.1c find $A_0$, assuming that $A_1$, $A_2$, $A_3$ and $A_4$ have arbitrary values $a_1$, $a_2$, $a_3$ and $a_4$. What would be the corresponding result for an $M$-stage tree, with $N = 2^M$ input guides? For the two-stage Y-junction tree, find $P_0$, and also the radiated powers $P_{RA}$, $P_{RB}$, and $P_{RC}$. Show that these powers sum to the input power. [10]

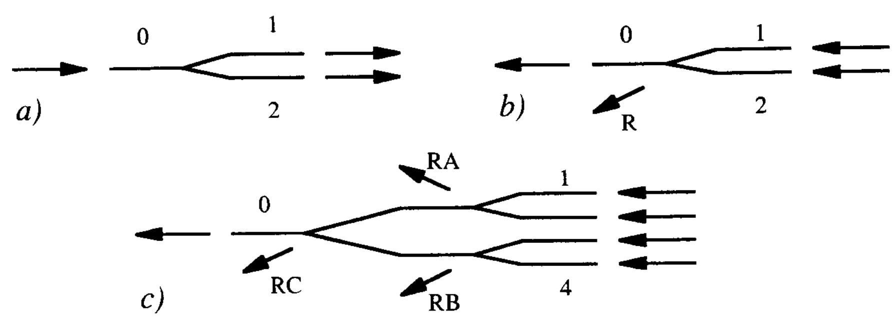



*Figure 3.1*

4. Figure 4.1 below shows an integrated optic *radiative star coupler*. The device has $(2N+1)$ channel guide inputs and outputs, linked by a central region formed from a planar guide whose boundaries are arcs with centres of curvature and radii as shown. The inputs and outputs lie on a regular spacing at the boundary of this region, so that the $m^{th}$ output subtends an angle $m\Delta\theta$ at the centre of curvature.

a) Explain the *broadcast* function of the radiative star coupler. Sketch the layout of a broadcast star coupler formed using directional couplers, and explain the advantages of the radiative star layout.

[8]

b) Ignoring any path difference between the channel guide sections, calculate the distance between the $m^{th}$ input and $n^{th}$ output in the radiative star. Assuming that the propagation constant of the planar guide is $\beta$, show that there is a discrete Fourier transform relation between the input and output amplitude distributions.

[12]

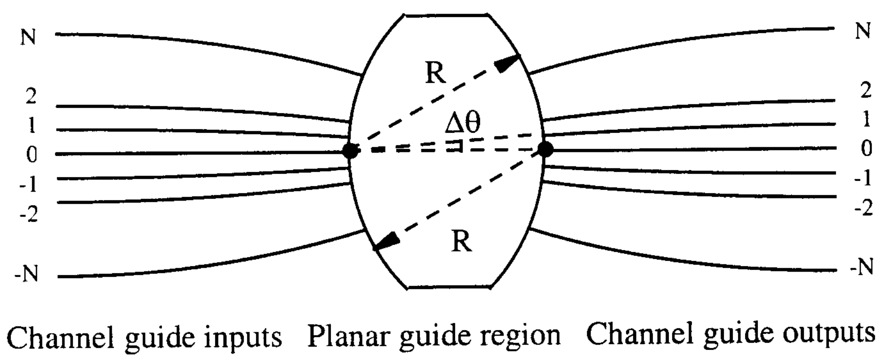



*Figure 4.1*

5. Figure 5.1 below shows the cross-section of a GaAs/GaAlAs double heterostructure semiconductor laser, together with a key to the material composition.

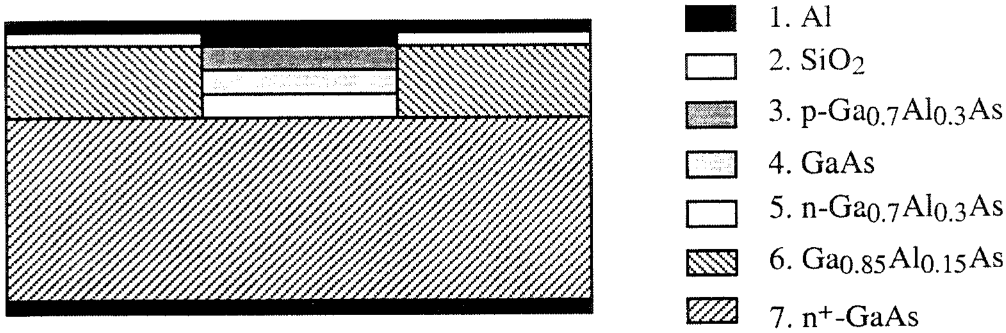



*Figure 5.1*

a) Which materials form the double heterostructure? What are the functions of materials 6 and 7? Sketch and explain the distribution of current flow through the device under forward bias.

[8]

b) Sketch and dimension the energy band diagram under un-biased conditions through the double heterostructure, together with the corresponding variation of refractive index. What is the wavelength of the emission?

[8]

c) How does the energy band diagram modify under forward bias? Explain how the double heterostructure provides an efficient environment for stimulated emission.

[4]

(You may assume that the refractive index and energy gap of $Ga_{1-x}Al_xAs$ vary linearly, as $n = 3.57 - 0.6285\,x$, and $E_g = 1.42 + 1.24\,x$, with $E_g$ measured in eV)

6. The lumped-element rate equations for a semiconductor laser are:

$$dn/dt = I/ev - n/\tau_e - G\phi(n - n_0)$$
$$d\phi/dt = \beta n/\tau_{rr} + G\phi(n - n_0) - \phi/\tau_p \qquad (6.1)$$

a) Identify the terms corresponding to the physical processes of *carrier injection*, *recombination*, *absorption*, *spontaneous emission*, *stimulated emission*, and *radiation from the cavity*. What is the significance of the parameter $n_0$?

[4]

b) Write down two separate steady-state approximations to the equations that are valid i) below threshold, and ii) above threshold. Use these equations to obtain expressions for the electron density below and above threshold. At what current does lasing first start to occur? How should the threshold current be minimised?

[10]

c) Sketch the variations of i) electron density and ii) optical power with injection current. Indicate the values of any important gradients on your figures.

[6]