Imperial College London

BSc/MSci EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

SPACE PHYSICS

For 3rd and 4th Year Physics Students

Friday, 25th May 2012: 14:00 to 16:00

The paper consists of two sections: A & B. Section A contains one question. Section B contains four questions.
Answer ALL parts of Section A and TWO questions from Section B.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

SECTION A

1. (i) Define plasma β and write down a mathematical expression for it. What values of plasma β are typically found in the Sun's photosphere and in the corona, and what are the physical implications of this? [2 marks]

In the presence of a gradient in the magnetic field, charged particles undergo Gradient-B drift at a velocity of

$$\mathbf{v}_{\nabla \mathbf{B}} = \pm \frac{1}{2} \mathbf{v}_{\perp} r_L \frac{\mathbf{B} \times \nabla B}{B^2}$$

where **B** is the magnetic field, $\mathbf{v}_{\perp}$ is the particle velocity perpendicular to **B** and r_L is the gyroradius. In the equatorial plane the Earth's magnetic field strength at radial distance r is given by $B(r) = B_p(R_p/r)^3$, where B_p=31,000 nT and R_p=6,400 km.

(ii) Write down an expression for the gyroradius r_L. [1 mark]

(iii) Let τ be the time required for a charged particle in the equatorial plane at orbital distance r to drift once around Earth. Show that τ can be expressed as

$$\tau = \frac{4\pi}{6} q B \frac{r^2}{E_{kin}}$$

where q is the charge and E_{kin} the kinetic energy of the particle. [4 marks]

(iv) Calculate τ for a 1 keV proton located at r=5R_p. What is τ for a 1 keV electron at the same location? [2 marks]

The solar wind compresses a planet's magnetic field to within a cavity known as a magnetosphere, which is characterised by a magnetopause at standoff distance r_{MP} from the planet on the Sun-planet axis.

(v) Assuming mass conservation, proton mass m and solar wind speed u_{SW}, derive an expression for the change of solar wind density $n(d)$ with distance d from the Sun relative to a reference density n_0=$n(d_0)$ at a distance d_0. [2 marks]

(vi) A force balance holds in the equatorial plane at the sub-solar point (nose) of the magnetopause. Assuming this force balance, derive an expression for the magnetopause standoff distance r_{MP}. Briefly justify your assumed magnetic field value at the magnetopause. The planetary dipole magnetic field strength at distance r is given by $B(r) = B_p(R_p/r)^3$, where B_p is the field strength at R_p. [5 marks]

(vii) Exoplanet HD80606b has a highly eccentric orbit around its parent star with closest distance (periastron) of 0.03 AU and largest distance (apastron) of 0.9 AU. If you assume a stellar wind similar to our solar wind (n=7×10^6 m^{-3} at 1 AU, u_{SW} ≈400 km/s) and a magnetic field similar to Jupiter's with B_p=500,000 nT, calculate the magnetopause standoff distance between HD80606b and its parent star at periastron and apastron in multiple of R_p. [4 marks]

[Total 20 marks]

SECTION B

2. The MHD induction equation is given by

$$\frac{\partial \mathbf{B}}{\partial \mathbf{t}} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\mathbf{1}}{\sigma \mu_{\mathbf{0}}} \nabla^{\mathbf{2}} \mathbf{B}$$

where **B** is the magnetic field, **v** the plasma velocity, σ the conductivity and μ_0 the permeability constant.

(i) Derive the induction equation from Faraday's Law. Define all variables you use. You may use without proof the identity

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \mathbf{A}) - \nabla^2 \mathbf{A}$$

[4 marks]

(ii) Briefly describe the physical meaning of both terms on the right side of the induction equation. [2 marks]

(iii) Derive an expression for the Reynolds number R_M and describe the physical significance of the ranges of values that R_M can attain. [3 marks]

(iv) When magnetic fields of opposite polarity are brought together a thin current layer forms separating the polarities. What is the approximate value of the Reynolds number in the current sheet, and what physical process will dominate there according to the terms in the induction equation? [1 mark]

(v) The Sweet-Parker model describes the changes of plasma velocity and magnetic field topography during the reconnection process. Draw a simple diagram showing the reconnection geometry in the Sweet-Parker model and clearly label key regions and quantities. [2 marks]

(vi) By assuming energy conservation, derive an expression for the outflow velocity of plasma in the Sweet-Parker model. Comment on the strengths and limitations of the Sweet-Parker model in terms of describing observed reconnection events in space. [3 marks]

[Total 15 marks]

3. To a first approximation the Earth's magnetic field in spherical coordinates can be expressed as

$$B_r = -2B_{eq} \cos\theta \left(\frac{R_p}{r}\right)^3; \qquad B_\theta = -B_{eq} \sin\theta \left(\frac{R_p}{r}\right)^3; \qquad B_\phi = 0$$

where r is the radial distance from the Earth's centre, θ is magnetic colatitude (θ=0 at the northern pole and θ=90^o at the equator), ϕ denotes longitude, B_{eq} is the magnetic field strength at the surface and R_p is the planetary radius.

(i) Plasma in the Earth's inner magnetosphere can exhibit two types of flow which compete with one another, co-rotation and convection. Please explain briefly the origin of each flow. Under which external condition does convection flow primarily occur, and why? [3 marks]

(ii) Draw a simple diagram looking down onto the equatorial plane which shows the Earth, the magnetopause and direction of solar wind flow. Draw the directions of the interplanetary magnetic field and the Earth's magnetic field, assuming southward IMF configuration. Draw the convection electric field in the inner magnetosphere and the directions of convective and co-rotation flows. [3 marks]

(iii) A stagnation point exists in the equatorial inner magnetosphere where convection and co-rotation flows cancel one another, giving a zero net flow velocity of plasma. Derive an expression for the distance R_{sp} at which this stagnation point forms as a function of ω_p, R_p, E_0, B_{eq}, where ω_p is the rotation frequency of Earth and E_0 is the magnitude of the convective electric field. In your diagram of part (ii) indicate the approximate location of this stagnation point. [2 marks]

(iv) Calculate R_{sp} for Earth and Jupiter by assuming at Earth: E_0=4 $\times$ 10^{-4} Vm^{-1}, R_p=6,400 km, B_{eq}=31,000 nT, ω_p= 7$\times$10^{-5} rad s^{-1} and at Jupiter the values: E_0=4 $\times$ 10^{-5} Vm^{-1}, R_p=71,400 km, B_{eq}=500,000 nT, ω_p= 1.7$\times$10^{-4} rad s^{-1}. How do your calculated values for R_{sp} compare with the average magnetopause standoff distances on both planets (10 R_p at Earth and 80 R_p at Jupiter), and what are the physical implications of these comparisons? [3 marks]

(v) In reality plasma in Jupiter's magnetosphere does not co-rotate out to the distance R_{sp} calculated above. Why? Briefly describe how and from where angular momentum is supplied to Jupiter's magnetosphere. Name the principal process that leads to a lag from co-rotation in Jupiter's magnetosphere, and how this is manifested visually. To aid your description, draw a simple diagram in which you clearly show and label key parameters. [4 marks]

[Total 15 marks]

4. When assuming zero mass outflow from the isothermal corona, the change of plasma pressure with distance from the Sun can be expressed by

$$\frac{dp}{dr} = -\rho\frac{GM}{r^2}$$

where p is the plasma pressure, r the radial distance from the Sun's centre, ρ is mass density, G is the gravitational constant and M the Sun's mass.

(i) Derive an expression for the plasma pressure $p(r)$. Give an expression for $p(r)$ at $r \to \infty$ and argue why coronal plasma must therefore have an outflow velocity forming the source for the solar wind. You do not need to calculate numerical values for plasma pressure. [3 marks]

The motion of coronal plasma in the radial direction can be described using the momentum equation

$$\rho\, u\, \frac{du}{dr} = -\rho\, \frac{GM}{r^2} - \frac{dp}{dr}$$

where u is the radial component of plasma velocity.

(ii) Show that the solar wind velocity can then be expressed as

$$\frac{1}{u}\,\frac{du}{dr}\left(u^2 - c_s^2\right) = \frac{4kT}{m\,r} - \frac{GM}{r^2}$$

where $c_s = \sqrt{\frac{2kT}{m}}$ is the sound speed and m the proton mass. You may from mass conservation assume without proof that

$$\frac{1}{n}\,\frac{dn}{dr} = -\frac{2}{r} - \frac{1}{u}\frac{du}{dr}$$

[3 marks]

(iii) State the physical significance of both terms on the right side of the above Parker solar wind equation. Derive an expression for the radius r_c at which both terms balance [2 marks]

(iv) Sketch the classes of mathematically possible solutions for u as a function of r. On your axes mark r_c and c_s. For each class of solution for the above solar wind equation give brief reasoning for the shape of the curve that you draw. Comment on the physical validity of each solution as a description of the actual solar wind velocity. Identify which solution best describes the solar wind and argue why. [6 marks]

(v) For your final chosen solution for the solar wind velocity under (iv) briefly discuss the shape of the curve. What are the implications for the relative balance of acceleration terms in the momentum equation as a function of distance from the Sun? [1 mark]

[Total 15 marks]

5. Planet Mars has an atmosphere composed primarily of CO_2 which on the dayside is ionised by solar EUV radiation to form an ionosphere of primarily O_2^+ ions.

(i) The attenuation of solar radiation is quantified by the optical depth τ which for the case of a single gas and vertical incidence of radiation at a given wavelength is expressed as

$$\tau(z_0) = \int_{z_0}^{\infty} \sigma^{abs}\, n(z) dz$$

where z_0 is the local altitude, σ^{abs} is the total photon absorption cross section and $n(z)$ the gas number density. Assuming the temperature T in Mars' atmosphere above 100 km to be approximately height-independent, derive a simple expression for $\tau(z_0)$ as a function of n, σ^{abs} and density scale height H. Write down an expression for H as a function of T, mean molecular mass m and gravity g. [2 marks]

(ii) Absorption of high energy photons leads to photo-ionisation of the atmosphere. Show that the altitude z_{peak} at which the maximum photo-ionisation occurs is given by

$$z_{peak} = z_0 + H \ln\left(n_0\, \sigma^{abs} H\right)$$

where $n_0 = n(z_0)$. [4 marks]

(iii) Calculate the altitude z_{peak} of maximum solar photo-ionisation on the dayside of Mars. Assume that z_0=100 km, $g \approx 3.5$ m/s^2, $m \approx 44$ atomic mass units, $T \approx 180$ K, $n_{100km} \approx 2 \times 10^{12}$ cm^{-3} and $\sigma^{abs} \approx 31 \times 10^{-18}$ cm^2. As part of your calculation determine an approximate value for H. [2 marks]

(iv) The electron production rate due to photo-ionisation at a single wavelength may be expressed as

$$P_e = n(z)\, \sigma^{ion} I^{\infty} \exp\left(-\tau(z)\right)$$

where σ^{ion} is the photo-ionisation cross section, I^{∞} denotes the solar radiation flux at the top of the atmosphere. Assuming photochemical equilibrium, calculate the dayside peak electron density on Mars. You may assume a coefficient for dissociative recombination of $\alpha \approx 3 \times 10^{-7}$ cm^{3}s^{-1}, $\sigma^{ion} \approx 30 \times 10^{-18}$ cm^2 and $I^{\infty} \approx 2 \times 10^9$ cm^{-2}s^{-1} at Mars. [5 marks]

(v) Mars is on an eccentric orbit around the Sun, with a perihelion distance of 1.38 AU and aphelion distance of 1.67 AU. Furthermore, solar radiation flux in the UV can vary by around a factor of 2 from solar minimum to solar maximum. Calculate the ratio $\gamma = n_{e\,max}/n_{e\,min}$ where $n_{e\,max}$ is the peak dayside electron density at solar maximum and perihelion and $n_{e\,min}$ is the peak dayside electron density at solar minimum and aphelion. [2 marks]

[Total 15 marks]