# Model Answers for Radio Frequency Electronics (E4.18)

2003

1.(a) **New computed example**

(i) $Characteristic\ impedance, Zo = \sqrt{\frac{L}{C}} = 75\Omega$

(ii)

$$propagation\ constant, \gamma = \frac{j\omega L}{Zo} = j42.4$$
$$attenuation\ constant, \alpha = 0\ Np/m$$
$$phase\ constant, \beta = 42.4\ rad/m$$

(iii) $guided\ wavelength, \lambda g = \frac{2\pi}{\beta} = 148.2\ mm$

(iv)

$$phase\ velocity, v_p = f\ \lambda g = 2.67x10^8\ m/s$$
$$\frac{v_p}{c} = 0.889$$

[7]

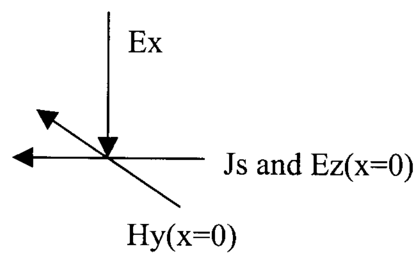
(b) **Tutorial question**

*Power Flux Density,*

$$P_D(x) = \text{Re}\{E_z(x)\ H_y^*(x)\}$$
$$where, H_y(x) = E_z(x)/Zs$$
$$P_D(x) = |H_y(x)|^2\ \text{Re}(Zs)$$
$$H_y(0) = Js$$
$$P_D(0) = |Js|^2\ Rs \equiv Ps$$

*alternatively,*

$$P_D(x) = \left|\frac{E_z(x)}{Zs}\right|^2\ \text{Re}(Zs)$$
$$E_z(0) = ZsJs$$
$$P_D(0) = |Js|^2\ Rs \equiv Ps$$




[6]

(c) **New computed example**

(i)

$$H_y = \frac{E_x}{Zo} = \frac{1}{75} = 13.3\ mA/m$$
$$Js = H_y = 13.3\ mA/m$$

(ii)

$$Zs = \sqrt{\frac{j\omega\mu_o}{\sigma_o}} = 11.1\,(1+j)\ m\Omega/square$$

(iii) $Ps = |Js|^2\ Rs = 1.97\ \mu W/m^2$

[7]

2(a) **Turorial question**

$$z_{IN} = \frac{z + jz_{TX}\tan\vartheta}{z_{TX} + jz\tan\theta} \equiv z_o$$

$$\therefore Z_{TX}\left(Z + jZ_{TX}\tan\vartheta\right) = Zo(Z_{TX} + jZ\tan\vartheta)$$

$$\mathrm{Re}\{LHS\} \equiv \mathrm{Re}\{RHS\}$$

$$\therefore \theta = \tan^{-1}\left\{\frac{Z_{TX}(Zo - R)}{XZo}\right\}$$

$$\mathrm{Im}\{LHS\} \equiv \mathrm{Im}\{RHS\}$$

$$\tan\theta = \frac{Z_{TX}X}{ZoR - Z_{TX}^2} \equiv \frac{Z_{TX}(Zo - R)}{XZo}$$

$$\therefore Z_{TX} = \sqrt{ZoR - \frac{X^2 Zo}{Zo - R}}$$

[10]

2(b) **Tutorial question**

From the last expression in 2(a), the limits are:

$$R \neq Zo$$

$$X < \sqrt{R(Zo - R)}$$

[3]

2(c) **New computed example**

For a 20 pF capacitance in series with a 2 Ω resistance at 900 MHz, the termination load impedance is Z = 2 – j8.842 Ω.

Using the expressions from 2(b), R is not equal to 50 Ω and X < 4.3 Ω, so both values are within the acceptable mathematical limits.

Using the expressions from 2(a), $Z_{TX}$ = 4.3 Ω and $\vartheta$ = 25°.

[5]

2(d) **Bookwork**

The value of $Z_{TX}$ calculated in 2(c) would be considered very low in general. In practice, a conventional microstrip line could not be used to implement such a low impedance because the width of the signal line would be too wide. However, thin-film microstrip technology may be suitable as the widths of the lines are much narrower.

[2]

3. (a) **Lecture discussion**
From the conservation of energy, for a lossless tow-port network:

$$|S21|^2 = 1 - |S11|^2$$

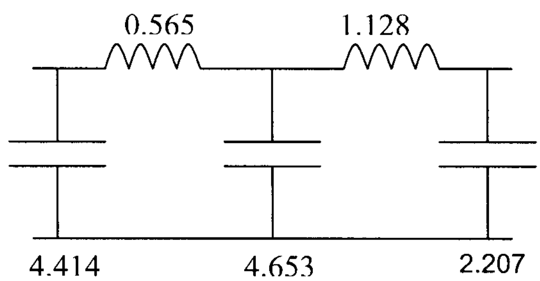
At the –3 dB cut-off frequency, $|S21|^2 = 0.5$ and, therefore, $|S11|^2 = 0.5$
In other words both the insertion loss and return loss levels are -3 dB.

For a maximum return loss level or –6.868 dB, the worst-case insertion loss level is 1 dB.

[5]

(b) **New application of theory**
-3 dB cut-off frequencies are at $f_{P1}$ = 540 MHz and $f_{P2}$ = 660 MHz
centre frequency, fo = $\sqrt{(f_{P1} f_{P2})}$
pass band bandwidth, Bp = $f_{P1}$- $f_{P2}$ = 120 MHz
stop band bandwidth, Bs = 60 MHz
f/fc = Bp/Bs = 2

From the attenuation curves, the 5th order 1 dB ripple Chebyshev filter with $Rs/R_L$ = 0.5 meets the specification with a stop band attenuation margin of 3 dB. The normalised values for the low-pass prototype is given below:
For a band stop filter, the capacitors are replaced by a series L-C

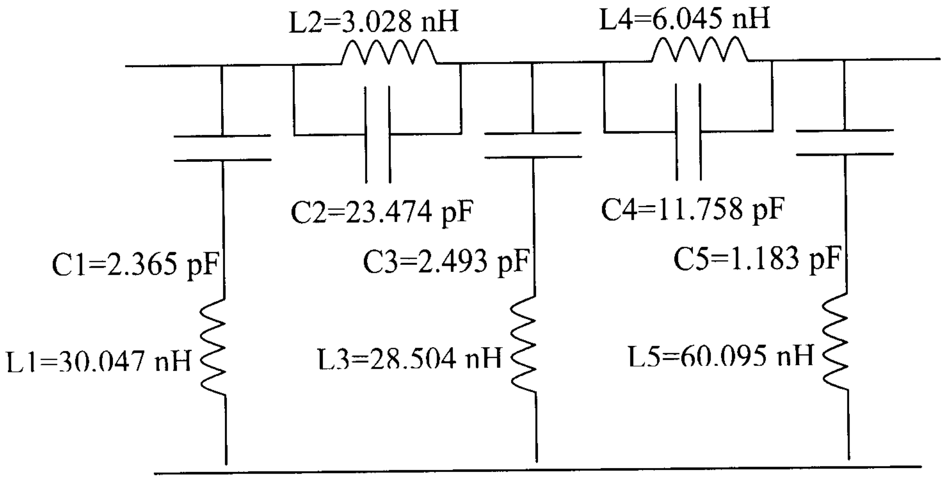



resonators and the inductors are replaced by parallel L-C resonators. The un-normalized shunt connected series tuned circuit element values are:

$$Cs = \frac{BpCn}{2\pi f_o^2 R_L} \quad and \quad Ls = \frac{R_L}{2\pi BpL_n}$$

The un-normalized series connected parallel tuned circuit element values are:

$$Cp = \frac{1}{2\pi BpCnR_L} \quad and \quad Lp = \frac{BpLnR_L}{2\pi f_o^2}$$

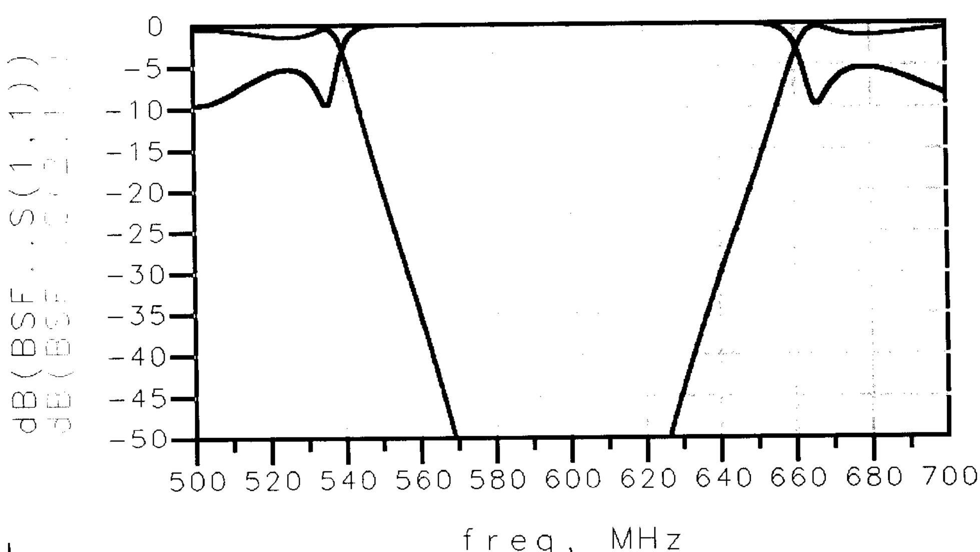



[10]

(**b**) **Lecture discussion**

Group delay is defined as $\tau = -\partial\angle S21/\partial\omega$

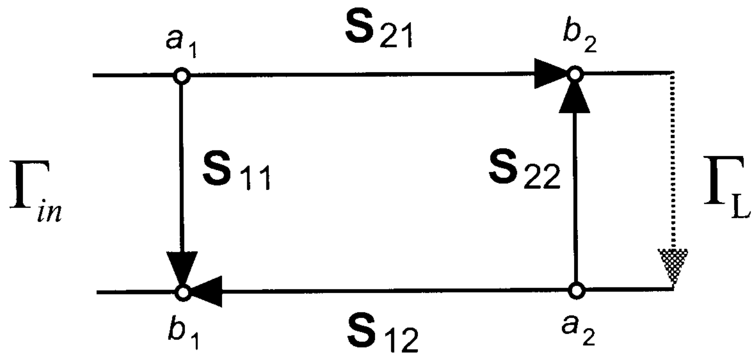
With sharp cut-off frequencies, high order filters are needed. This means that there are large numbers of passive filter components, where electromagnetic energy is exchanged between them. As a result the signal stays within the filter longer than for filters with a less sharp insertion loss cut off and, thus, the group delay is higher. Also, more energy is dissipated in the components and, therefore, insertion loss is higher unless larger components are employed.




The slight deviations in pass band levels are due to rounding errors in the component values (3 decimal places).

[5]

4. (a) **Tutorial question**




**Flow graph representation of a device with output load termination**

Using Mason's non-touching loop rule it can easily be shown that:

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$where,\ \Gamma_L = \frac{Z_L - Zo}{Z_L + Zo}$$

[5]

(b) **Lecture notes**

Conditional stability occurs when the input voltage wave reflection is equal or greater than unity. Therefore, the stability circle can be determined by equating the expression in 4(a) for the reflection coefficient to unity. When the circle encompasses the matched impedance, zo, point then a load's voltage wave reflection coefficient is inside the circle this corresponds to unconditional stability and conditional stability outside the circle. Conversely, the opposite is true when the stability circle does not encompass the zo point.

[5]

(c) **New application of theory**

$\Gamma_L$ = -1.222 and therefore $\Gamma_{IN}$ = -3.3

[5]

(d) **New application of theory**

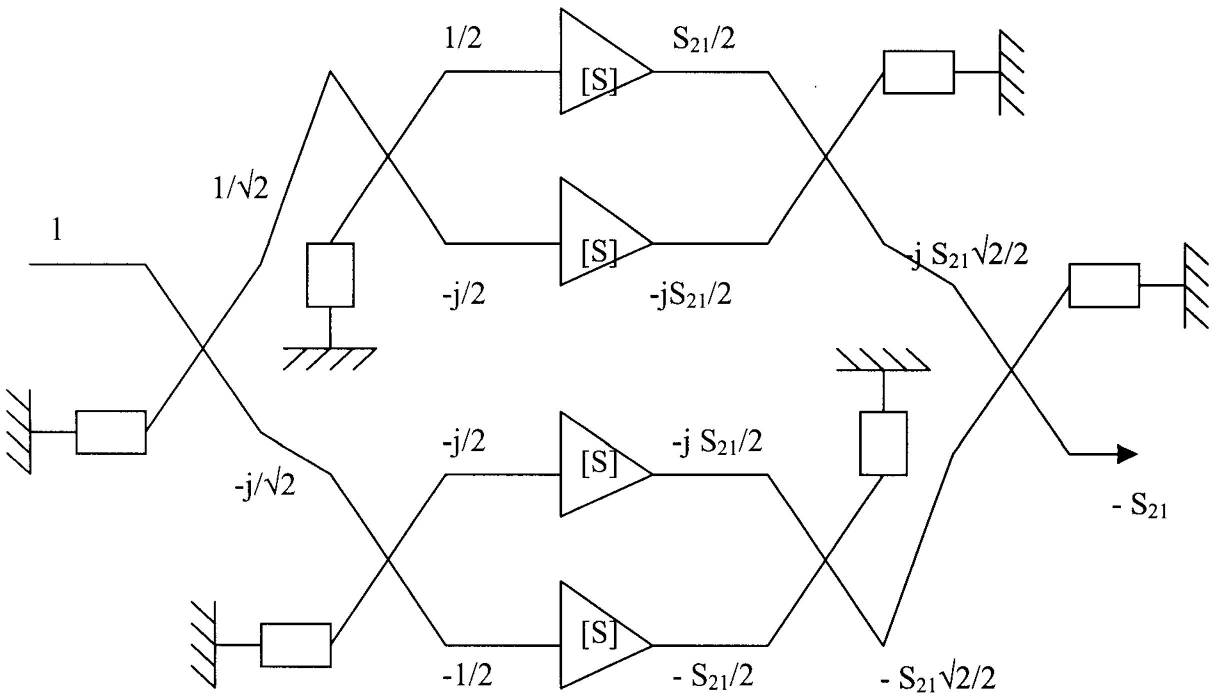
A Rollett's stability factor of $K = 2.3$ is greater than unity and, therefore, it would normally represent an unconditionally stable circuit. However, since the reflection coefficient of the load is greater than unity then the circuit is now conditionally stable.

When $K$ is greater than unity (and the magnitudes of $S_{11}$ and $S_{22}$ are both less than unity), the device is unconditionally stable and the maximum gain that can be achieved is called the maximum available gain (MAG), given by:

$$\text{MAG} = \left|\frac{S_{21}}{S_{12}}\right|\left(K - \sqrt{K^2 - 1}\right) = 4.575 = 6.6 \text{ dB}$$

[5]

5. (a) **New application of theory**




$$S_{21}\big|_{overall} = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)S_{21}\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right)S_{21}\left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right) +$$
$$\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)S_{21}\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{-j}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right)S_{21}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = S_{21}$$

$$Insertion\ Loss = 10\log\left\{ \left| S_{21} \right|^2 \right\}$$

$$S_{11}\big|_{overall} = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)S_{11}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right)S_{11}\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) +$$
$$\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)S_{11}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right) + \left(\frac{-j}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right)S_{11}\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right) = 0$$

$$Return\ Loss = 10\log\left\{ \left| 0 \right|^2 \right\} \to -\infty$$

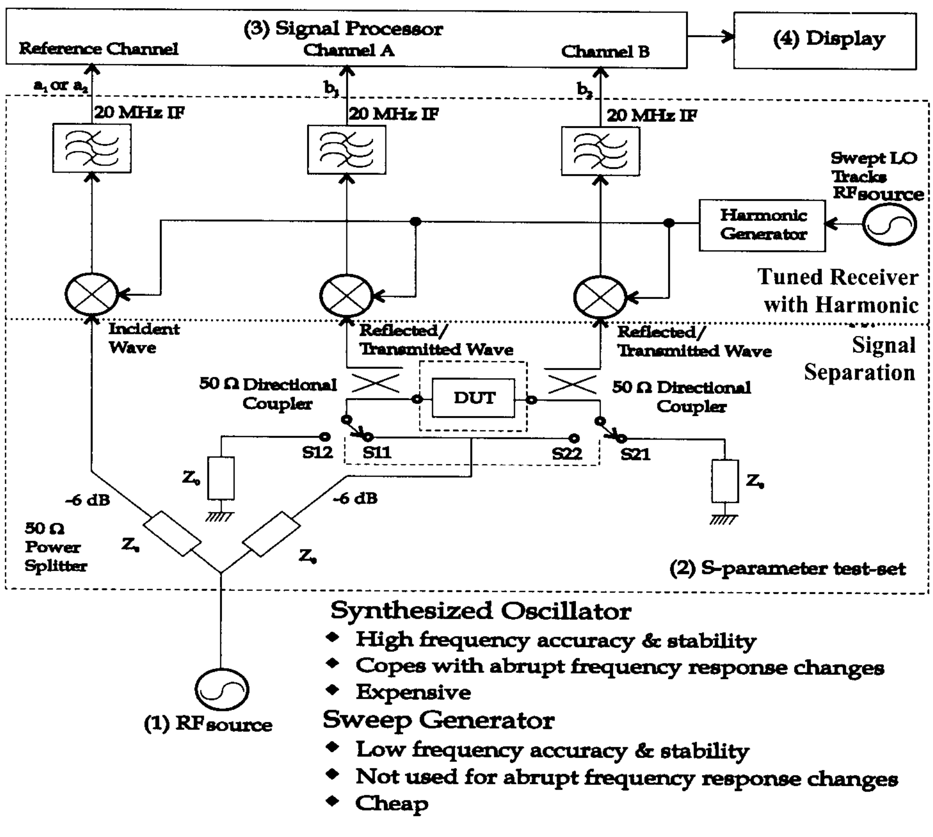
(b) **Lecture discussion**

The main application of this amplifier is power combining, since the output power is ideally a factor of 4 greater than that of the single-ended amplifier. If one of the single-ended amplifiers fails then $S_{21}|_{overall} = 3S_{21}/4$ and Insertion loss is $10\log\{(3|S_{21}|/4)^2\} = 17.5$ dB, i.e. a drop of 2.5 dB from a fully working amplifier. The input return loss should not change if the impedance of the failed amplifier doesn't change and is, therefore, still minus infinity. Therefore, this type of power combining amplifier is useful because it provides redundancy in the case of failure and also ideal port impedance matching. The disadvantages of this topology is that it requires 4 identical single-ended amplifiers. Also, practical losses in the couplers result in a direct loss in power gain and output efficiency will be significantly reduced.

6(a) **Lecture notes**

A cheap scalar network analyser consists of an $RF_{SOURCE}$, a 2-resistor power splitter (which has excellent frequency response across an ultra-wide bandwidth), a wideband diode detector (at low RF power it has a square-law response, i.e. the output DC voltage is proportional to the absorbed RF power; this response then goes linear at higher RF power levels) and processor/display. *[The student can draw this].* Apart from delivering fast measurements, a very useful application of the scalar network analyser is its ability to characterise the transmission properties of mixers, where the incident signal will be at a different frequency to the output signal. Only the magnitude of transmission measurements are possible. Here, the source and detector are connected together and then the device under test is inserted. The difference is measurements gives the insertion loss.

The equipment most commonly used is called a vector network analyser (VNA), because this instrument can also measure the phase angle of the S-parameters. As a result, the DUT can now be characterised using complete S-parameter measurements (along with DC measurements). Compared to direct diode detection, tuned receivers have a higher dynamic range, are immune from harmonic and spurious responses, can measure phase and, therefore, facilitate better calibration techniques. Amplitude measurements are taken from the relative ratio of an unknown signal (test channel A or B) to a reference signal (reference channel). Phase measurements are taken from the relative phase difference between the test and reference channels. *[The student only needs to have the outline of the following block diagram].*



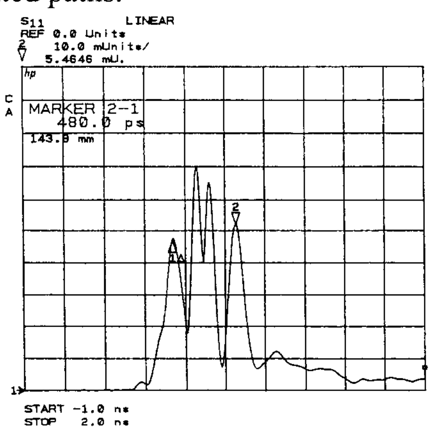

[8]

6(b) **Lecture notes**
When compared with test fixtures, commercial probe station measurements:

(1) Are available in a single-sweep system from DC to 120 GHz, whereas commercial test fixtures can operate from DC to 60 GHz.
(2) Are more accurate and much more repeatable, since they introduce much smaller systematic errors.
(3) Have a simpler calibration procedure, which can be automated with on-wafer calibration and verification standards.
(4) Enable the VNA measurement reference planes to be located at the probe tips or at some distance along the MMIC's transmission line. In the latter case, transition effects can be removed all together.
(5) Provide a fast, non-destructive means of testing the MMIC, thus allowing chip selection prior to dicing and packaging.
(6) Overall, the microwave probe station can provide the most cost effective way of measuring MMICs when all costs are taken into account.

[4]

6(c) **Lecture notes**
Some VNAs can perform synthetic-pulse time-domain reflectometry (TDR). Here, the discrete form of the inverse Fourier transform (IFT) is applied to a real sequence of harmonically related frequency-domain (F-D) measurements. This is directly equivalent to mathematically generating synthetic unity-amplitude impulses (or unity-amplitude steps), which are then 'applied' to the embedded DUT. The resulting time-domain (T-D) reflection and transmission responses can then be analysed to provide information about the DUT and test fixture discontinuities. In reflection measurements, it is possible to remove the effects of unwanted impedance mismatches or else isolate & view the response of an individual feature. With a multiple port test-fixture, transmission measurements can give the propagation delay & insertion loss of signals travelling through a particular path by removing the responses from the unwanted paths.




**Time-domain response for the input voltage reflection coefficient of an embedded mismatched MMIC through-line**

The frequency response of $|S_{11}|$ can be emphasised for the embedded MMIC by gating out the two outer peaks in the associated time-domain response. This 'de-embedding' process is not needed with on-wafer probing, as the reference planes are located at the probe tips. In other words, any connectors have already been calibrated out.

[8]