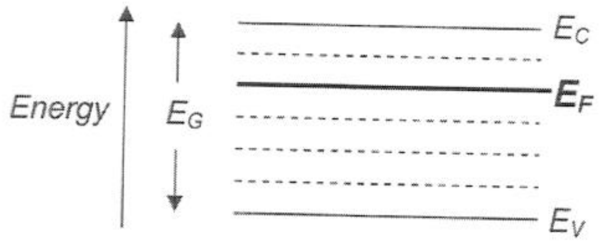
ANSWERS: - E1.3 DEVICES + FIELDS

Q1 (a) 2008

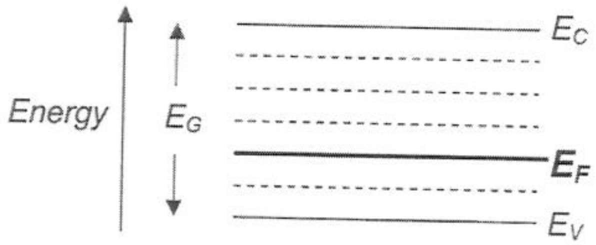
1/12

n-type:



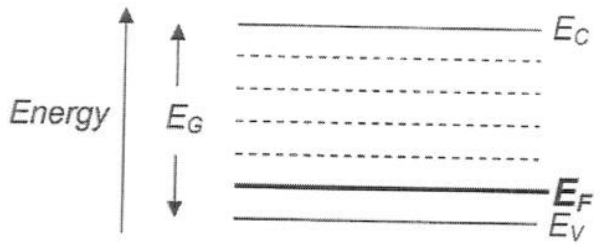
[1/2]

p-type:



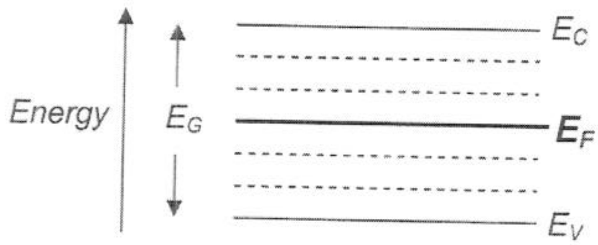
[1/2]

p^+:



[1/2]

Intrinsic:



[1/2]

Q1 (b)

Electrons:

Drift current: left to right [½]
Diffusion current: right to left [½]

Holes:

Drift current: Left to right [½]
Diffusion current: left to right [½]

Q1 (c) Fermi-Dirac distribution:

$$f(E) = \frac{1}{1 + \exp\left[\frac{(E - E_F)}{kT}\right]}$$

where k = Boltzman constant
E = energy
T = temperature (Kelvin)

∴ Relationship between temperatures:

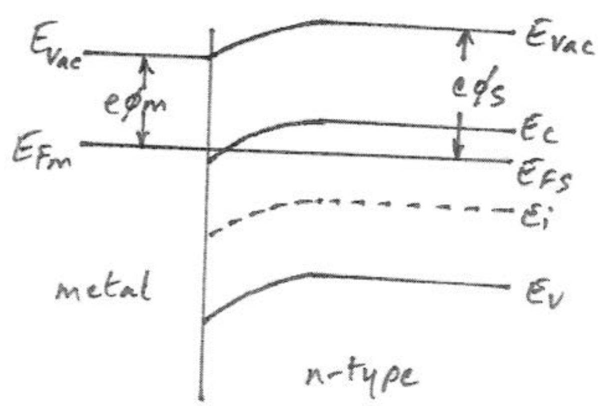
$$T_1 < T_2 < T_3 \qquad [2]$$

Relationship between electron concentrations:

$$n_1 < n_2 < n_3 \qquad [2]$$

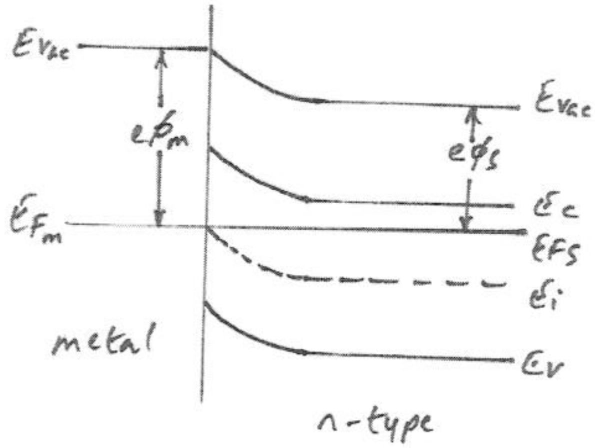
Q1 (d)

(i)


E_{vac}
$e\phi_m$
E_{Fm}
E_{vac}
$e\phi_s$
E_c
E_{Fs}
E_i
E_v
metal
n-type

[3]

(ii)


E_{vac}
$e\phi_m$
E_{F_m}
E_{vac}
$e\phi_s$
E_c
E_{Fs}
E_i
E_v
metal
n-type

[3]

Q1(e)

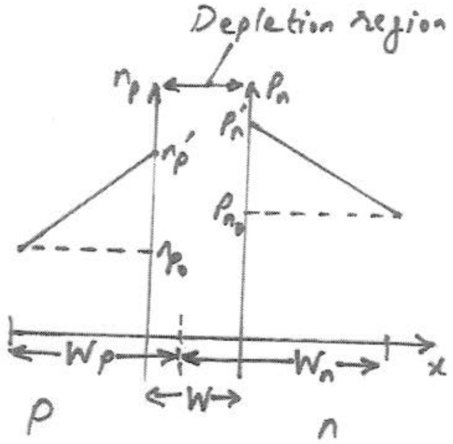
(i) As $p_{n0} = \frac{n_i^2}{N_D}$ and $n_{p0} = \frac{n_i^2}{N_A}$,

and $N_A > N_D$,

$\Rightarrow n_{p0} < p_{n0}$ [2]

(ii)

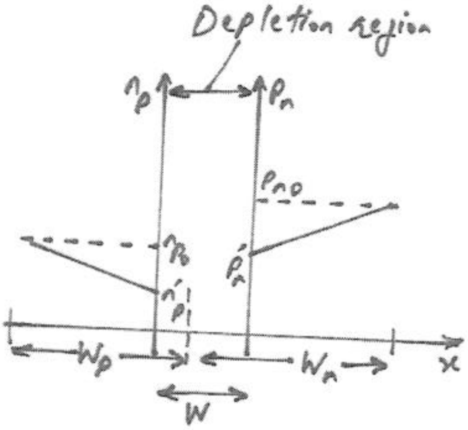
Forward Bias



[2]

(iii)

Reverse Bias

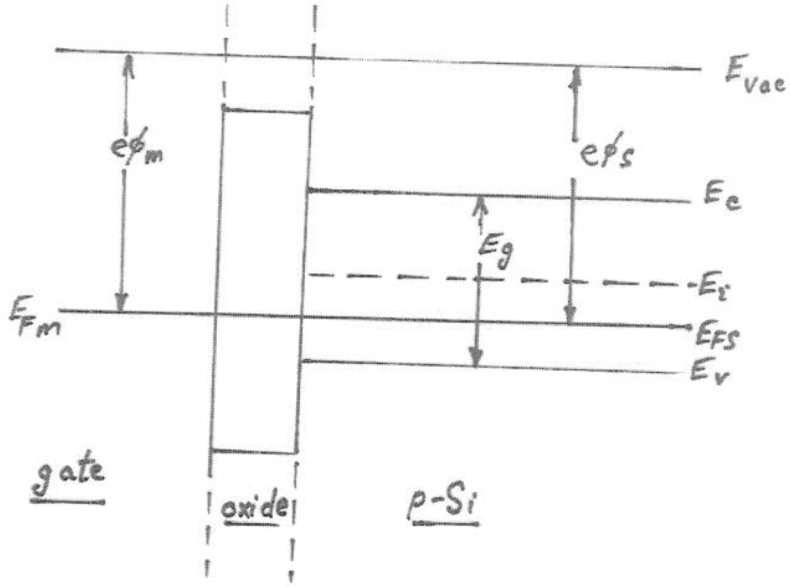


[2]

5

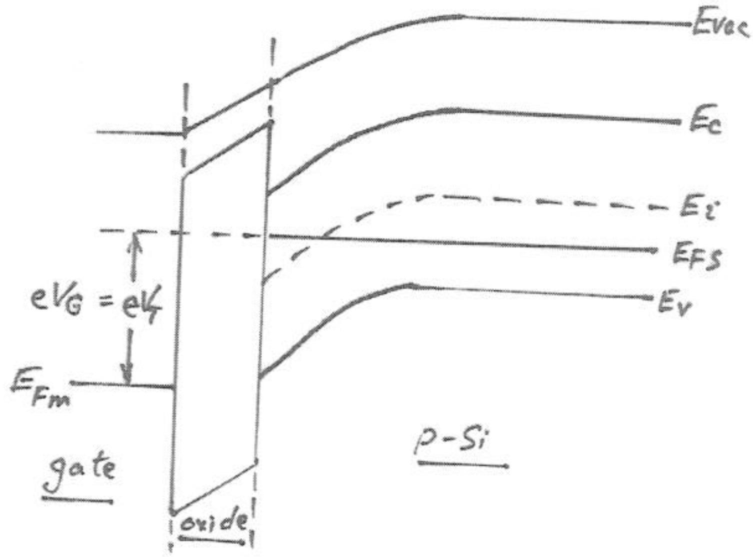
Q2

(a)(i)



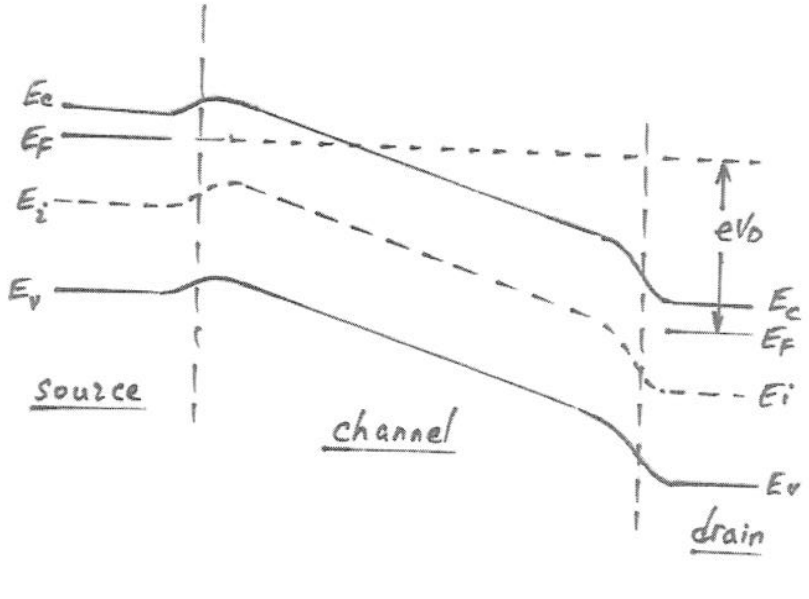
[5]

(a)(ii)



[5]

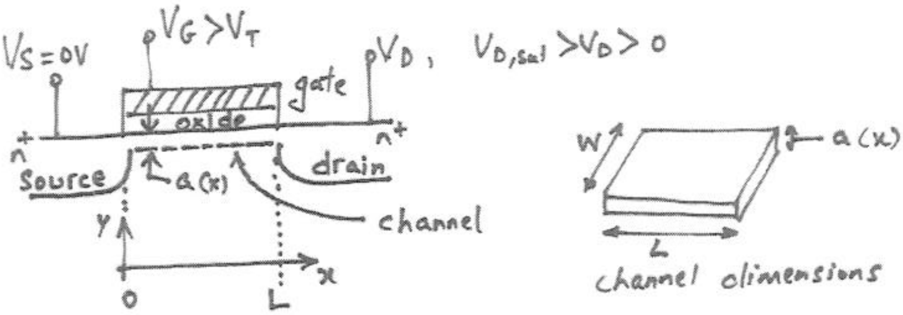
(a)
(iii)



[5]

Q2 (b)

(i)



The drain-source current in a MOSFET in the triode region, $V_G > V_T$ and $V_{D,sat} > V_D > 0$, is dominated by drift current. We may assume the diffusion current is negligible.

$$\therefore \quad I_{DS} = e\mu\, n(x) \frac{dV(x)}{dx} \cdot A(x) \quad -(1)$$

where $n(x)$ = No. of electrons, at a point 'x' in the channel, per volume
$V(x)$ = Potential of the channel at 'x'.
$A(x)$ = Cross-sectional area of the channel at 'x'.

[5]

(ii) In the triode region, $V_{D,sat} > V_{DS} > 0$,
$V_{GS} > V_T$

We assume that the vertical electric field caused by the gate voltage is much greater than the longitudinal electric field caused by the drain voltage. (Long channel approximation).

We also assume that at $V_{GS} = V_T$, the charge per unit area 'WL' in the channel, $Q_n = 0$

Then at a point 'x'

$$Q_n(x) = -C_{ox}(V_{GS} - V(x) - V_T) \quad -(2)$$

$$\text{Also,} \quad Q_n(x) = -e \times \frac{\text{no. of carriers}}{W \times L} \times \frac{a(x)}{a(x)}$$

$$= -e \cdot n(x) \cdot a(x)$$

$$\text{Using Eq. (2)} \Rightarrow \quad n(x) = \frac{C_{ox}(V_{GS} - V(x) - V_T)}{e \cdot a(x)} \quad -(3)$$

Substituting $A(x) = W \cdot a(x)$ & Eq. (3) into Eq. (1)

$$\Rightarrow \quad I_{DS} = e\mu \cdot \frac{C_{ox}(V_{GS} - V(x) - V_T)}{e \cdot a(x)} \cdot \frac{dV}{dx} \cdot W \cdot a(x)$$

$$\Rightarrow \quad I \cdot dx = \mu \cdot W \cdot C_{ox}\,(V_{GS} - V(x) - V_T)\,dV$$

$$\Rightarrow \quad \int_0^L I_{DS} \cdot dx = \mu W C_{ox} \int_0^{V_{DS}} \left[(V_{GS} - V_T) - V(x)\right] dV$$

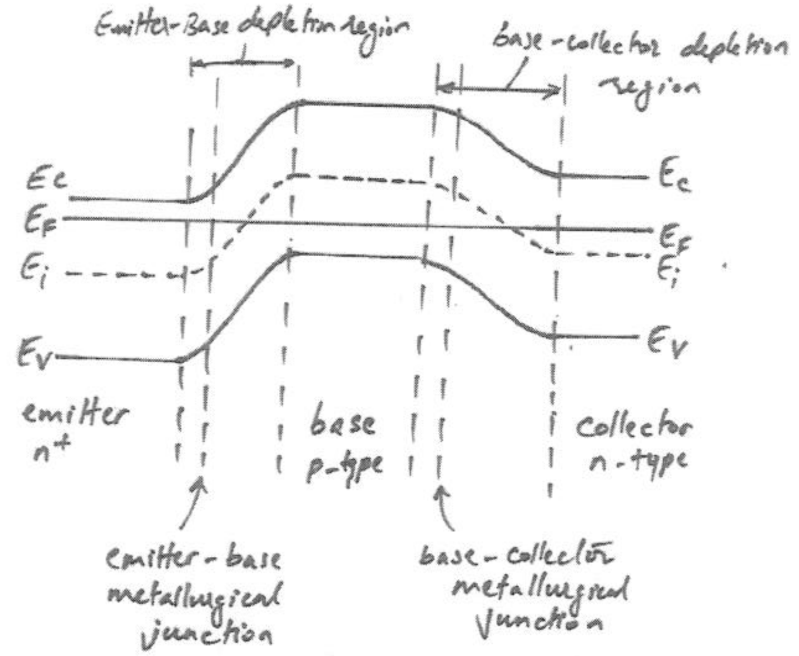
$$\Rightarrow \quad I_{DS} = \frac{\mu W}{L} C_{ox}\left((V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right)$$

//

[10]

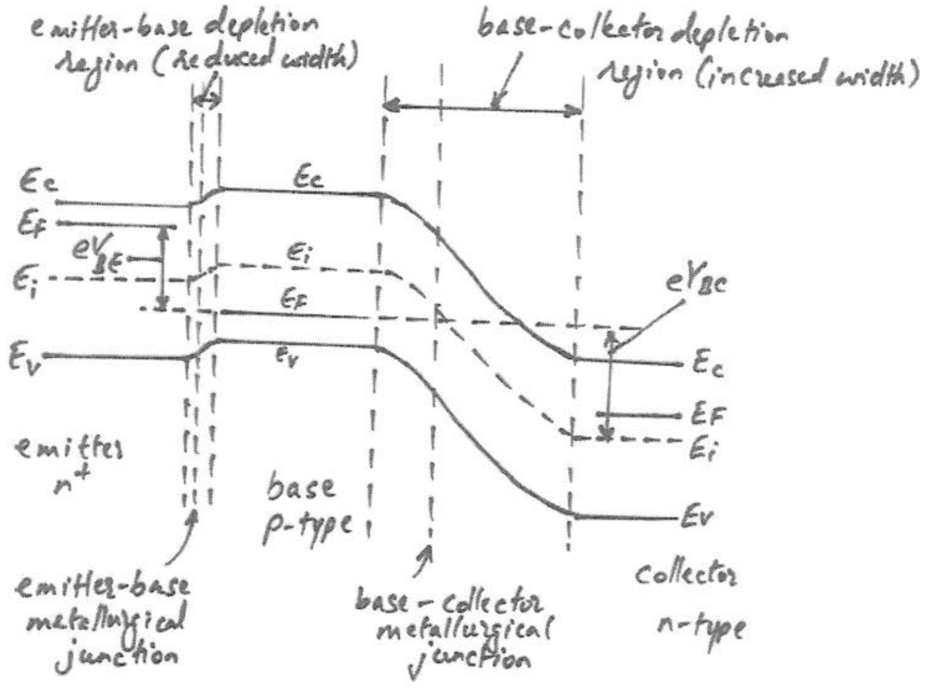
Q3

(a)


Emitter-Base depletion region
base-collector depletion region
Ec
EF
Ei
Ev
Ec
EF
Ei
Ev
emitter
n+
base
p-type
collector
n-type
emitter-base metallurgical junction
base-collector metallurgical junction

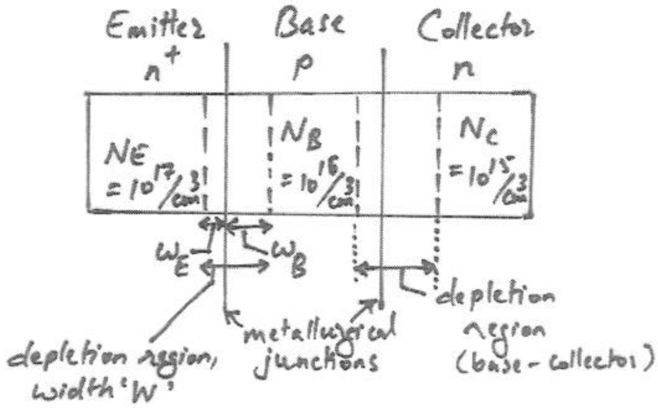
[5]

(b)


emitter-base depletion region (reduced width)
base-collector depletion region (increased width)
Ec
EF
eVBE
Ei
Ev
Ec
Ei
EF
Ev
eVBC
Ec
EF
Ei
Ev
emitter
n+
base
p-type
emitter-base metallurgical junction
base-collector metallurgical junction
collector
n-type

[5]

Q3
(c)(i)



The widths of the depletion regions in the emitter (w_E) and the base (w_B), on either side of the base-emitter metallurgical junction, are marked on the above diagram.

The total width of the base-emitter depletion region is: $W = w_E + w_B$ —(1)

$$\text{Built-in voltage} = V_0 = \frac{kT}{e} \ln \frac{N_A N_D}{n_i^2} \quad \text{(formula sheet)}$$

$$= \frac{kT}{e} \ln \frac{N_E N_B}{n_i^2}$$

$$= 0.026 \ln \frac{10^{17} \cdot 10^{16}}{(1.45 \times 10^{10})^2}$$

$$= 0.76\,V$$

For applied forward bias voltage $V_{BE} = 0.6\,V$,

$$W = \sqrt{\frac{2\,\varepsilon_{Si}\,\varepsilon_0}{e}\left(\frac{N_E + N_B}{N_E N_B}\right)(V_0 - V_{BE})}$$

$$= \sqrt{\frac{2 \times 11 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19}}\left(\frac{10^{17} + 10^{16}}{10^{17} \cdot 10^{16}}\right)(0.76 - 0.6)}$$

$$= 1.46 \times 10^{-5}\,cm$$

$$= 0.146\,\mu m$$

Assuming charge neutrality across the entire depletion region W:-

$$e N_E w_E - e N_B w_B = 0$$

$$\Rightarrow N_E w_E - N_B w_B = 0 \quad \text{—(2)}$$

Solving Eq.(1) and Eq.(2) simultaneously:

$$\cancel{N_E w_E} + N_E w_B = N_E W$$
$$\cancel{N_E w_E} + N_B w_B = 0$$

$$(N_E + N_B) w_B = N_E W$$

$$\Rightarrow w_B = \frac{N_E W}{N_E + N_B}$$

$$\Rightarrow w_B = \frac{10^{17}}{10^{17} + 10^{16}} \times 0.146 \text{ mm}$$

$$\Rightarrow w_B = 0.133 \text{ mm} \quad //$$

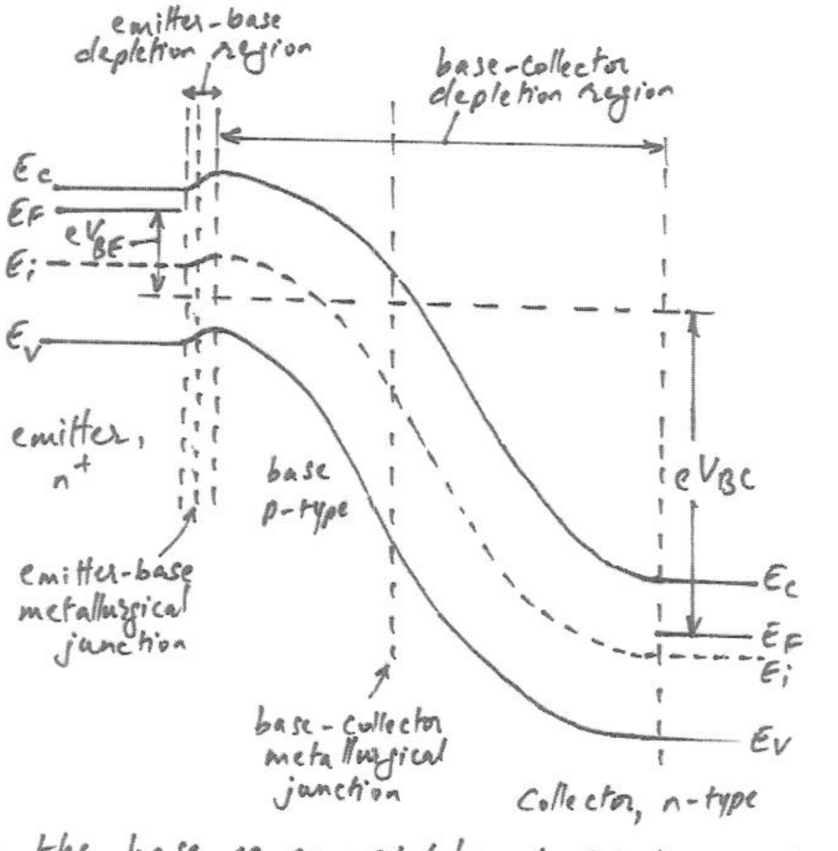
$$\therefore \quad w_E = W - w_B$$

$$\Rightarrow w_E = 0.146 - 0.133$$

$$\Rightarrow w_E = 0.013 \text{ mm}$$
$$= 13 \text{ nm} \quad //$$

[15]

(c) (ii)



As the base is completely depleted, practically all electrons injected from the emitter will reach the collector, as they are swept across the base-collector depletion region. The base cannot, therefore, control the collector current.

[5]

Fields and Devices Part B: SOLUTIONS

4. This question is mandatory. Answer all parts. Each part is worth 4 marks.

a) A uniform static electric field has magnitude 10^3 V/m, and its direction makes a 60° angle with the x axis. Calculate the potential difference between two points on the x axis 2 cm apart.
$E_x = E\cos(60) = 0.5E = 500\ V/m$, $\Delta V = E_x\Delta x = 500(0.02) = 10V$.

b) A conducting sphere of radius 1 cm, in air, has a net surface charge of 10^{-9} C. Calculate the electric field magnitude at its surface.
$D = Q/A = Q/(4\pi r^2)$, $E = Q/(4\pi\varepsilon_o r^2) = 10^{-9}/(4\pi\times 8.85\times 10^{-12}\times 10^{-4}) = 90\ kV/m$.

c) An ideal transformer has a turns ratio N_1:N_2 = 10:20, and a load of 10 Ω is connected to the secondary coil terminals. If an AC voltage of magnitude 5 V is applied to the primary coil, what will be the magnitude of the current I_2 flowing in the secondary coil?
$V_2 = (20/10)V_1$, $I_2 = V_2/R = 2\times 5/10\ 1\ A$

d) A linear ferromagnetic material, with a relative permeability ε_r = 800, contains a magnetic flux density B = 0.5 T. Calculate the magnetic field strength H in the material.
$B = \mu H = \mu_r\mu_o H$, so $H = B/(\mu_r\ \mu_o) = (0.5/(4\pi\times 10^{-7}\times 800) = 497\ A/m$

e) A 50 turn, open circuit, round coil of radius 1 cm lies in a magnetic field with time varying flux density B(t) = $B_o\sin(\omega t)$, with B_o = 2 T and ω = 100 rad/s. The magnetic flux direction is perpendicular to the plane of the coil. Calculate the magnitude of the potential induced in the coil.
$V(t) = NAdB/dt = N\pi r^2\omega B_o\cos(\omega t)$, $V_o = N\pi r^2\omega B_o = 50\pi\times 10^{-4}\times 100\times 2 = 3.14\ V$

5. A certain one-dimensional system has an electric potential of the form:

$$V(x) = -2x^{-2} + 4x^{-1}$$

with x in cm and V in volts.

a) Derive an expression for the magnitude of the electric field E(x). Considering only positive x values, find a position x_o such that $E(x_o) = 0$. Calculate also the potential $V(x_o)$. [6]

$E(x) = -dV/dx = -4x^{-3} + 4x^{-2}$. $E=0$ for $4x^{-3} = 4x^{-2}$, giving $x_o = 1$ cm. $V(1) = 2V$.

b) Find the position x_m (with $x_m > 0$) for which E(x) has its largest positive value. Calculate $E(x_m)$ and $V(x_m)$. [6]

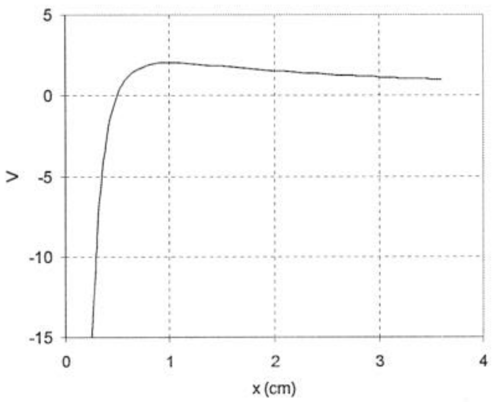
E(x) is largest where dE/dx = 0. $dE/dx = 12x^{-4} - 8x^{-3}$. $dE/dx = 0$ *for* $x_m = 1.5$.

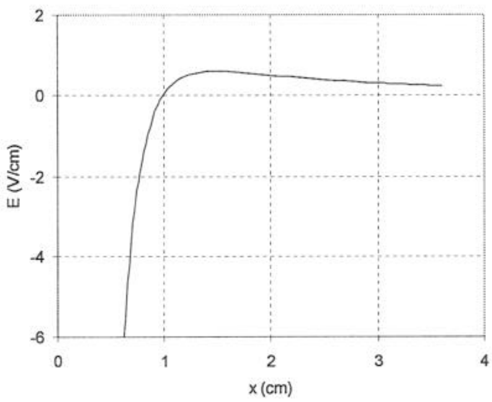
$E(x_m) = 0.592$ V/cm; $V(x_m) = 1.78$ V. Check, e.g. by using d^2E/dx^2, that this is a maximum not a minimum.

c) Give an expression for the additional force F(x) required to hold a charge Q stationary in this electrostatic field. By integrating this force, find the work required to move a charge Q from $x = \infty$ to $x = x_0$. Show that this is equal to the change in electrostatic energy of the charge. [6]

The field applies a force $F = EQ$, so the external force required = $-EQ$. The work done is $W = -\int F dx = -Q\int(-4x^{-3} + 4x^{-2})dx = Q(-2x^{-2} + 4x^{-1})$. Evaluating this from ∞ to x_o gives $W = Q(-2x_o^{-2} + 4x_o^{-1})$ which since $x_o = 1$ gives $W = 2Q$. But this is just the electrostatic potential difference $Q\Delta V$, since $V(\infty) = 0$ and $V(x_o) = 2$. This can also be shown simply by observing that integrating E gives the expression for V.

d) Sketch quantitatively, both E(x) and V(x), for positive values of x. [6]





Sketches should have labelled and scaled axes, and key points such as maxima and zero crossings reasonably accurately placed, along with a few other specific calculated points to place the curves. The axis ranges should be similar to the above in order to clearly show the points of interest.

e) What is the nature of the position $x = x_0$ for positive charges? What is its nature for negative charges? [6]

It is an equilibrium point; for positive charges an unstable one, for negative charges a stable one.

6. Two coils of N_1 and N_2 turns respectively are wound around a cylindrical iron core of relative permeability μ_r, length L and cross-sectional area A, as shown in Fig. 6.1. A current $I_1 = I_o sin(\omega t)$ is introduced in the first coil, while the second coil is open circuit.

a) Using Ampere's Law for the magnetic field strength H, taken along the dotted path as shown, find an expression for $H_i(t)$ in the core. State any approximations or assumptions used. Hence, calculate the magnitude of the magnetic flux density in the core for the following values: $N_1 = 20$, $A = 1\ \text{cm}^2$, $L = 12$ cm, $\mu_r = 4000$, $I_o = 0.5$ A, $\omega = 100$ rad/s. [8]

Ampere's law: $\int H \cdot dl = NI$. *We can neglect H outside core, since the flux spreads over a very large area. Assuming* H_i *is parallel to dl, this gives* $H_i = N_1 I_1/L = N_1 I_o \sin(\omega t)/L$. *Then:* $B_i = \mu_r \mu_o H_i = \mu_r \mu_o N_1 I_o \sin(\omega t)/L$ *, and the magnitude is* $B_o = 4000 \times 4\pi \times 10^{-7} \times 20 \times 0.5/0.12 = 0.419$ *T.*

b) Give an expression for the total magnetic flux $\Phi(t)$ flowing in the core. State any approximations or assumptions used. Hence, find an expression for the voltage $V_2(t)$ induced at the terminals of the second coil. Calculate the magnitude of $V_2(t)$ for the parameter values given in (a), and taking $N_2 = 10$. [8]

We approximate the flux density as uniform, and neglect any flux leakage. Then $\Phi = BA$, *so* $\Phi(t) = \mu_r \mu o N_1 A I_o \sin(\omega t)/L$. *The induced voltage in coil 2 is just* $N_2 d\Phi/dt$, *giving* $V_2(t) = \mu_r \mu o N_1 N_2 A I_o \omega \cos(\omega t)/L$, *and the magnitude:*
$V_2 = 4000 \times 4\pi \times 10^{-7} \times 20 \times 10 \times 10^{-4} \times 0.5 \times 100/0.12 = 41.9$ *mV.*

c) Neglecting the resistance of the coil wires, find an expression for the voltage $V_1(t)$, and hence the ratio $V_2(t)/V_1(t)$. [8]

Again, $\Phi(t) = \mu_r \mu o N_1 A I_o \sin(\omega t)/L$. *The induced voltage in coil 1 is just* $N_1 d\Phi/dt$, *giving* $V_1(t) = \mu_r \mu o N_1^2 A I_o \omega \cos(\omega t)/L$, *and so* $V_2/V_1 = N_2/N_1$.

d) Consider the effect of connecting a finite resistance R_2 across the terminals of the second coil, such that a current I_2 can flow in this coil. If I_1 is unchanged, would the magnetic flux in the core in the presence of R_2 be increased, decreased or unchanged compared to the case where the second coil is open circuit? Explain your reasoning. [6]

The induced voltage V_2 *will cause a current* I_2 *to flow which will have the same direction as* I_1. *If we recalculate* H_i *using Ampere's Law with this additional current, we will get an increased value as the integration path cuts more current in total. Thus the magnetic field, and thus the flux, are increased.*