IMPERIAL COLLEGE LONDON

EE4-55
EE9-AO11

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

MEMS AND NANOTECHNOLOGY

Wednesday, 15 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer Question 1.
Answer Question 2 OR Question 3.
Answer Question 4 OR Question 5.

Question 1 carries 40% of the marks. Remaining questions carry 30% each.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : Z. Durrani, A.S. Holmes, Z. Durrani
Second Marker(s) : A.S. Holmes, Z. Durrani, A.S. Holmes

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This question is compulsory

1. a) In the Boltzmann approximation in a semiconductor, the Fermi-Dirac distribution may be replaced by the Boltzmann distribution, $f(E) = \exp[-(E - E_F)/k_BT]$. At temperature T = 300K, for what range of energy E with respect to the Fermi energy E_F would such an approximation be valid? You may assume that a less than 10% deviation is acceptable. Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J/K.

[5]

b) (i) By using the $|2s\rangle$, $|2p_x\rangle$, $|2p_y\rangle$ and $|2p_z\rangle$ states given below, construct three hybridised sp^2 orbitals for the σ bonds at a carbon atom in graphene.

$$|2s\rangle \sim \left(1 - \frac{Zr}{2a_0}\right) e^{\frac{-Zr}{2a_0}}$$

$$|2p_x\rangle \sim x e^{\frac{-Zr}{2a_0}}$$

$$|2p_y\rangle \sim y e^{\frac{-Zr}{2a_0}}$$

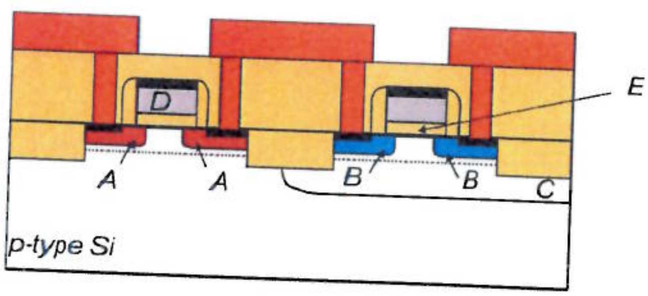
$$|2p_z\rangle \sim z e^{\frac{-Zr}{2a_0}}$$

[3]

(ii) Sketch the shapes of the σ and π orbitals along a hexagonal ring of 6 carbon atoms in graphene.

[2]

c) The diagram below shows a cross-section through a CMOS inverter. What are the regions, A, B, C, D, and E?



[5]

d) Using a suitable diagram, explain the operation of a parallel plate reactive-ion etching system.

[5]

Question 1 continues on the next page.

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Question 1 continued.

e) Starting from the elementary bending equation, derive an expression for the transverse stiffness of a flexure (i.e. a beam constrained to deflect laterally without end rotation) in terms of its dimensions and Young's modulus. Also derive an expression for the maximum tensile strain in such a beam in terms of the end deflection and beam dimensions. [6]

f) List the process steps in a typical silicon surface micromachining sequence for a device requiring two mechanical layers. You should identify the materials and processes used at each stage, but you do not need to describe the processes in detail. [4]

g) Describe briefly the principle of piezoelectric transduction. Also show, with aid of a sketch, how this transduction mechanism might be applied to a cantilever-based silicon micromachined accelerometer. [5]

h) Sketch the structure of a typical electrothermal shape bimorph actuator, and explain its operation. Also derive an approximate expression for the tip deflection in terms of appropriate dimensions and the average temperature rises in the different sections. [5]

End of Question 1.

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2. An electron travelling in the positive x-direction is incident from the region $x < 0$ onto a one-dimensional potential barrier of height V_B and width L. The potential energy $V(x)$ is given by:

$$V(x) = V_B, \text{ for } 0 \leq x \leq L$$
$$V(x) = 0, \text{ elsewhere}$$

a) Write down the general form of the time-independent wavefunctions $\psi(x)$ when $x < 0$, x lies between 0 and L, and when $x > L$. [6]

b) Write down expressions for the wave-vectors when $x < 0$, x lies between 0 and L, and $x > L$. [6]

c) Using your wavefunctions $\psi(x)$ in the different regions, and the boundary conditions at $x = 0$ and $x = L$, write down equations linking these wavefunctions at $x = 0$ and $x = L$. [10]

d) Hence, derive an expression for the amplitude transmission coefficient t for the electron travelling across the barrier. [8]

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3. Consider a p-channel, enhancement mode Si MOSFET, with gate oxide thickness t_{ox}, and donor doping concentration in the bulk N_D. In the bulk Si, far from the gate oxide–Si interface, the energy difference between the Fermi level E_F and the intrinsic level E_i is $e\psi_B$, where e is the electron charge and ψ_B is the corresponding value in volts.

a) Assuming 'flat-band' conditions at zero applied bias, sketch the energy band diagram (show E_C, E_V, E_i, and E_F) along a line perpendicular to the gate oxide–bulk Si interface plane, for accumulation, depletion, and inversion. Your diagram should show the energy bands in the bulk Si, oxide, and gate regions. [6]

b) Use Poisson's Equation, $-\partial^2 V/\partial x^2 = \partial F/\partial x = \rho/\varepsilon_{Si}\varepsilon_0$ to derive expressions for the electric field F, and the potential V in the bulk Si, along a line perpendicular to the gate oxide–bulk Si plane. Here, ρ is the sheet charge density, and ε_0 and ε_{Si} are the vacuum and relative permittivity. Hence, what is the surface potential V_s relative to the charge-neutral region of the bulk Si, where F is zero? [16]

c) Hence, derive an expression for the threshold voltage V_{th} for the device. [8]

4. a) By considering the forces on an elementary section, show that the lateral deflection $v(x, t)$ of a vibrating beam satisfies the wave equation:

$$EI\frac{\partial^4 v}{\partial x^4} + m\frac{\partial^2 v}{\partial t^2} = 0$$

where m is the mass per unit length of the beam, and E and I are respectively its Young's modulus and second moment of area. You should assume that the beam is loaded only by inertial forces due to its own motion i.e. that there is no damping. [8]

b) Verify that the following is a general solution of the wave equation in part a):

$$v(x,t) = [A\cos kx + B\sin kx + C\cosh kx + D\sinh kx]\cdot \exp(j\omega t)$$

Also derive the mathematical relationship between ω and k. [6]

c) State the boundary conditions that will apply to the function $v(x, t)$ in the case of a beam that is built in at both ends. By applying these boundary conditions show that for such a beam k must satisfy the eigenvalue equation:

$$\cos(kL)\cosh(kL) = 1$$

where L is the length of the beam. [8]

d) A chemical sensor is fabricated from a built-in silicon beam that has a selectively absorbing polymer coating of thickness 0.5 μm covering its upper surface. The polymer has a density of 1500 kg/m^3 and its Young's modulus is negligible in comparison to that of silicon. If the cantilever is 10 μm wide, 400 μm long, and 5 μm deep, calculate the frequency of the sensor's lowest order vibrational mode. Also determine the frequency shift that will occur when the polymer absorbs 1 pico-gram of analyte. [8]

Assume values of 170 GPa and 2330 kg/m^3 for the Young's modulus and density of silicon. The smallest positive root of the equation $\cos(x)\cosh(x) = 1$ is $x = 4.73$.

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5. a) Figure 5.1 shows the model for an out-of-plane electrostatic actuator comprising a suspended moveable plate and a fixed plate covered by a dielectric layer. Derive an expression for the total force acting on the moveable plate. Also sketch the variation of this force with the gap g for several values of V and explain, with the aid of your sketch, the origin of *snap-down* instability. [10]

b) State the conditions that apply at the point of snap-down, and hence show that the snap-down voltage is expected to be:

$$V_P = \sqrt{\frac{8k(g_0 + t_d/\varepsilon_r)^3}{27\varepsilon_0 A}}$$ [8]

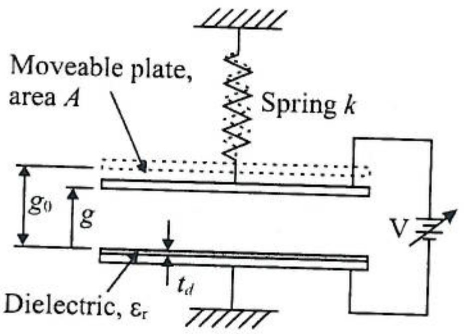
Also derive an expression for the release voltage of the actuator. [4]

c) Figure 5.2 shows an electrostatically actuated, capacitive shunt RF MEMS switch designed for low actuation voltage. It consists of a gold plate suspended over a transmission line by a simple hammock suspension. The suspension beams are each 250 μm long and 4 μm wide, and the gold layer is 2 μm thick.

The plate has a central region which overlaps with the signal line, and two larger actuation regions which overlap with the ground plane. The total effective area for actuation is 105,600 μm^2. The dielectric spacer is a 100 nm-thick layer of silicon nitride, and the initial gap between the plate and the dielectric is 5 μm.

Estimate the snap-down voltage of the switch, stating any assumptions needed for your calculation. What factors might make the actual snap-down voltage of a practical device different from your calculated value? [8]

You should assume E = 80 GPa for gold, and ε_r = 7.6 for silicon nitride. The permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.



$$\text{Capacitance } C = \frac{A\varepsilon_0}{(g + t_d/\varepsilon_r)}$$

Figure 5.1

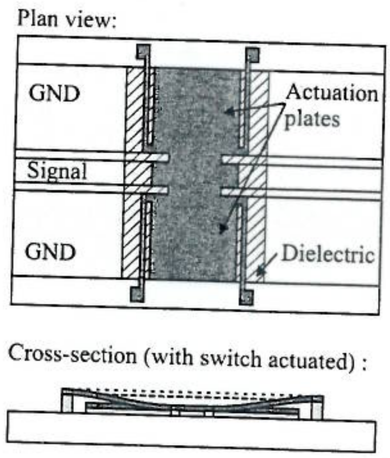


Figure 5.2

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MEMS/NANO Examination 2013: Answers

Question 1

(a) The Fermi-Dirac *f(E)* distribution is given by:

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{k_B T}\right)}$$

In contrast, the Boltzmann *f'(E)* distribution is given by:

$$f'(E) = \frac{1}{\exp\left(\frac{E-E_F}{k_B T}\right)}$$

The difference between the two distributions for <10% error is then:

$$\frac{f'(E)-f(E)}{f(E)} < 0.1$$

[Marks: 2]

This implies that:

$$\frac{f'(E)}{f(E)} - 1 < 0.1$$

$$\Rightarrow \frac{f'(E)}{f(E)} < 1.1$$

$$\Rightarrow \frac{1+\exp\left(\frac{E-E_F}{k_B T}\right)}{\exp\left(\frac{E-E_F}{k_B T}\right)} < 1.1$$

$$\Rightarrow \frac{1}{\exp\left(\frac{E-E_F}{k_B T}\right)} < 0.1$$

$$\Rightarrow \exp\left(\frac{E-E_F}{k_B T}\right) > 10$$

$$\Rightarrow E - E_F > \ln 10 \times 1.38 \times 10^{-23} \times 300$$

$$\Rightarrow E - E_F > 0.059 eV$$

[Marks: 3]

(b) The given 2s state is symmetric, and the three 2p states are aligned along *x*, *y*, and *z* axis

Symmetric:

$$|2s\rangle = \psi_{200} \sim \left(1 - \frac{Zr}{2a_0}\right) e^{-\frac{Zr}{2a_0}}$$

Aligned along x, y, and z axis:

$$|2p_x\rangle \sim xe^{-\frac{Zr}{2a_0}} \qquad |2p_y\rangle \sim ye^{-\frac{Zr}{2a_0}} \qquad |2p_z\rangle \sim ze^{-\frac{Zr}{2a_0}}$$

For each sp2 'σ 'orbital, we require the 2s state, and two 2p states. Furthermore, if one sp2 orbital is aligned with the x axis, then the others need to be 60^0 above and below $-x$ axis. This then gives the required 120^0 separation between the three sp2 orbitals.

In view of this, we may construct linear combinations for the sp2 orbitals as follows:

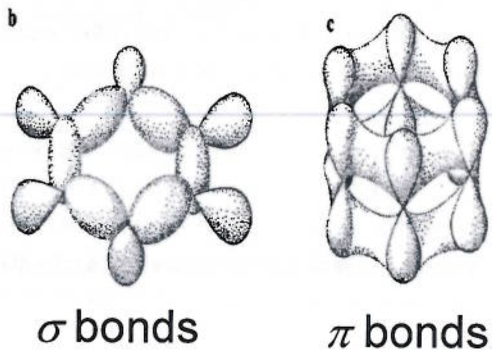
$$|sp_1^2\rangle = |2s\rangle + \sqrt{2}|2p_x\rangle$$

$$|sp_2^2\rangle = |2s\rangle + \sqrt{\frac{3}{2}}|2p_y\rangle - \sqrt{\frac{1}{2}}|2p_x\rangle$$

$$|sp_3^2\rangle = |2s\rangle - \sqrt{\frac{3}{2}}|2p_y\rangle - \sqrt{\frac{1}{2}}|2p_x\rangle$$

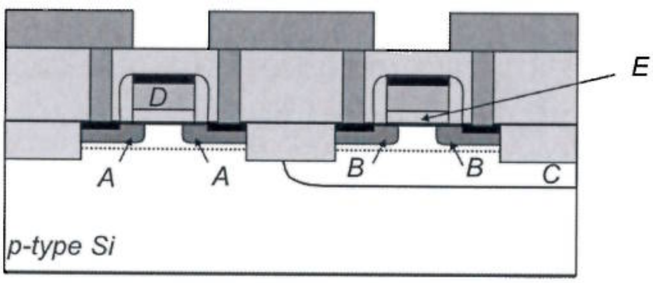
[Marks: 3]

Sketches for σ and π orbitals along a hexagonal ring:



[Marks: 2]

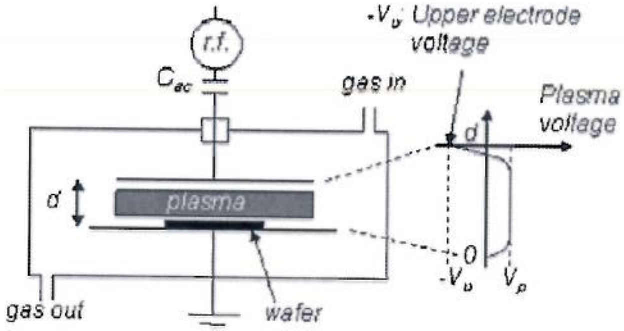
(c) Answer:



A: *n*+ source and drain regions of the *n*-channel MOSFET.
B: *p*+ source and drain regions of the *p*-channel MOSFET.
C: *n*-well, for the fabrication of the *p*-channel MOSFET.
D: Gate, fabricated using metal or heavily-doped polysilicon
E: Gate oxide, thermally grown. A high-k dielectric layer may also lie in this region.

[Marks: 1+1+1+1+1+1]

(d) A parallel plate reactive-ion etching system is shown schematically below:.



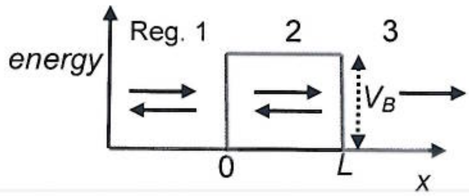
The system uses a glow discharge, i.e. weakly-ionised plasma (>90% neutral particles). An rf. electric field E_{rf} is used to generate the plasma, as follows. Free electrons are accelerated by E_{rf}. These collide with gas atoms, generating more free electrons and ions and increasing the plasma density. However reduction in the number of electrons and ions due to collision with the electrodes lowers the plasma density. The two processes then lead to an equilibrium plasma density. The plasma gas is chosen to generate reactive species, e.g. F or Cl, and these species etch the wafer. Both mechanical (ion sputtering) and chemical etching occurs in the etching process. The ion sputtering is caused by a reduced potential at either plate compared to the plasma potential, associated with charge build-up on the plates. Positive ions then bombard the wafer surface, leading to mechanical, anisotropic etching.

[Marks: 5]

Question 2

(a) Here, the potential energy $V(x)$ given by:

$V(x) = V_B,\ 0 \le x \le L$
$V(x) = 0,\ x < 0,\ x > L$



The general, space-dependent wavefuctions in region 1, 2 and 3 are:

Region 1:
$$\psi_1 = a_{1i}\exp(ik_1x) + a_{1r}\exp(-ik_1x) = \exp(ik_1x) + r\exp(-ik_1x)$$

where we assume $a_{1i} = 1$
and r = amplitude reflection coeff.

Region 2:
$$\psi_2 = a_{2i}\exp(ik_2x) + a_{2r}\exp(-ik_2x)$$

Region 3:
$$\psi_3 = a_{3i}\exp(ik_1x)$$

[Marks: 2 + 2 + 2]

(b) Wave-vectors in regions 1 and 2, for energy E, are as follows:

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}} \qquad k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

As region 3 is identical to region 1, therefore $k_3 = k_1$

[Marks: 2 + 2 + 2]

(c) The boundary conditions at $x = 0$ are:

$$\psi_1 = \psi_2 \quad \text{and} \quad \frac{d\psi_1}{dx} = \frac{d\psi_2}{dx}$$

The boundary conditions at $x = L$ are:

$$\psi_2 = \psi_3 \quad \text{and} \quad \frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$$

[Marks: 2]

Applying these to the general wavefuctions in (a) then gives us:

At $x = 0$:

$$\psi_1 = \psi_2 \quad \Rightarrow \quad 1 + r = a_{2i} + a_{2r} \qquad (1)$$

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \quad \Rightarrow \quad k_1(1-r) = k_2(a_{2i} - a_{2r}) \qquad (2)$$

[Marks: 4]

Similarly, at $x = L$:

$$a_{2i}e^{ik_2L} + a_{2r}e^{-ik_2L} = a_{3i}e^{ik_1L} = t \qquad (3)$$

$$k_2\left(a_{2i}e^{ik_2L} - a_{2r}e^{-ik_2L}\right) = k_1t \qquad (4)$$

[Marks: 4]

(d) Dividing Eq. 4 by k_2 and then adding to Eq. 3 gives:

$$a_{2i} = \left(1 + \frac{k_1}{k_2}\right)\frac{te^{-ik_2L}}{2} \qquad (5)$$

Substituting Eq. 5 into Eq. 3 gives:

$$a_{2r} = \left(1 - \frac{k_1}{k_2}\right)\frac{te^{ik_2L}}{2} \qquad (6)$$

[Marks: 4]

Dividing Eq. 2 by k_1, and then adding to Eq. 1 gives:

$$2 = a_{2i}\left(1 + \frac{k_2}{k_1}\right) + a_{2r}\left(1 - \frac{k_2}{k_1}\right) \qquad (7)$$

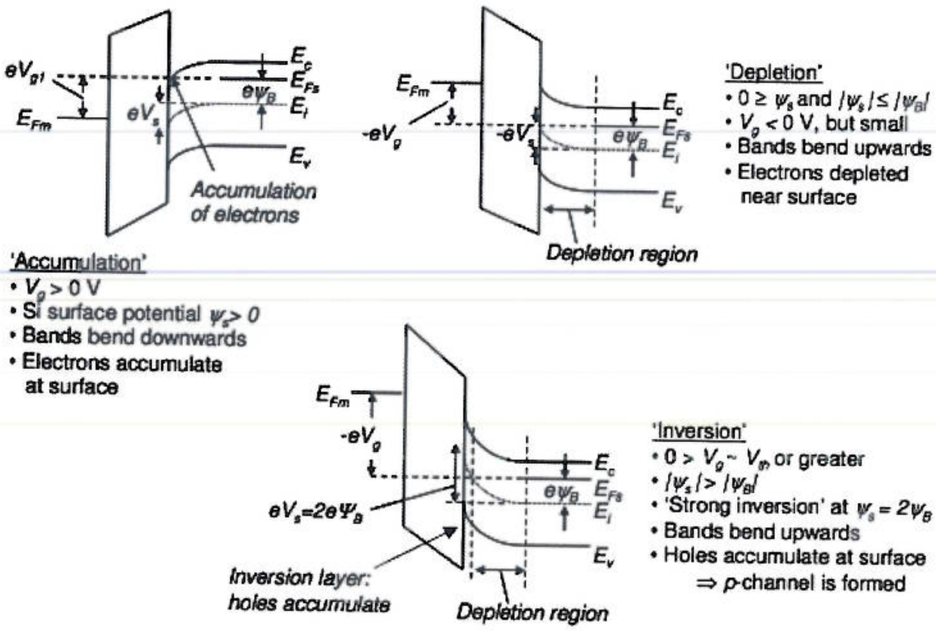
Finally, substituting Eq. 5 and 6 into Eq. 7 and simplifying gives the amplitude transmission coefficient t:

$$\boxed{t = \frac{4k_1k_2}{(k_1+k_2)^2e^{-ik_2L} - (k_1-k_2)^2e^{ik_2L}}} \qquad (8)$$

[Marks: 4]

Question 3

(a) The band diagrams in a *p*-channel, enhancement-mode MOSFET, for accumulation, depletion and inversion are shown below:



The answer should show the energy bands using E_c, E_F, E_v and E_i.

[Marks: 2 + 2 + 2]

(b) Poisson's equation can be solved to find electric field F and potential V, perpendicular to the gate-oxide – Si bulk plane, as follows:

In depletion region of width W_d, for $0 < x < W_d$, Poisson's Eq. gives:

$$\frac{\partial F}{\partial x} = \frac{\rho}{\varepsilon_{Si}\varepsilon_0} = \frac{eN_D}{\varepsilon_{Si}\varepsilon_0}$$ [2]

Integration ⇒

$$F = \int \frac{eN_D}{\varepsilon_{Si}\varepsilon_0} dx = \frac{eN_D x}{\varepsilon_{Si}\varepsilon_0} + c$$

Boundary condition: $F = 0$ at $x = W_d$ ⇒ $\boxed{F = \frac{-eN_D W_d}{\varepsilon_{Si}\varepsilon_0}\left(1 - \frac{x}{W_d}\right)}$ [4]

Next, using $\frac{\partial V}{\partial x} = -F$ ⇒

$$V = -\int F dx = -\int \frac{-eN_D W_d}{\varepsilon_{Si}\varepsilon_0}\left(1 - \frac{x}{W_d}\right) dx = \frac{eN_D W_d}{\varepsilon_{Si}\varepsilon_0}\left(x - \frac{x^2}{2W_d}\right) + c$$ [4]

Boundary condition: $V = 0$ at $x = W_d$ ⇒ $\boxed{V = \frac{-eN_D W_d^2}{2\varepsilon_{Si}\varepsilon_0}\left(1 - \frac{x}{W_d}\right)^2}$ [4]

To find the surface field V_s, simply substitute $x = 0$ in the equation for V above. This gves the following expression for V_s, note the negative sign:

$$V_s = \frac{-eN_D W_d^2}{2\varepsilon_{si}\varepsilon_0} \qquad [2]$$

(c) The threshold voltage V_{th} may be calculated as follows:

__Onset of strong inversion:__

V_g = *Threshold voltage* V_{th}
and $V_s = 2\psi_B$

$\Rightarrow V_{th} = V_g = V_{ox} + V_{Si} = -Q_s/C_{ox} - 2\psi_B$
Note that for a p-channel MOSFET, $V_g < 0$

$\Rightarrow V_{th} \approx -(eN_D W_d/C_{ox} + 2\psi_B)$

(Neglecting inv. layer charge Q_n) [4]

From the expression for V_s derived in (b):

$$W_d = \sqrt{\frac{2\varepsilon_{Si}\varepsilon_0(2\psi_B)}{eN_D}} \quad \Rightarrow \quad \boxed{V_{th} = -\left(\frac{\sqrt{2e\varepsilon_{Si}\varepsilon_0 N_D(2\psi_B)}}{C_{ox}} + 2\psi_B\right)}$$

where: $C_{ox} = \dfrac{t_{ox}}{\varepsilon_{ox}\varepsilon_0}$ $\quad t_{ox}$ = oxide thickness

[4]

Question 1

e) The bending equation for a flexure with a transverse end load P is:

$$M = EI\frac{d^2v}{dx^2} = P(L-x) - C$$

where $v(x)$ is the deflection profile, x is distance along the cantilever measured from the root, E is Young's modulus, I is the second moment of area, L is the cantilever length, and C is the couple required at each support to prevent end rotation. Assuming a rectangular cross-section, $I = wh^3/12$ where w and h are the cross-sectional dimensions, with h being in the direction of the load P. Taking moments for the entire beam we find $C = PL/2$.

Integrating twice, and applying the boundary conditions $v = 0$, $v' = 0$ at $x = 0$, the deflection profile is obtained as $v(x) = (PL^3/12\mathrm{EI})\cdot(3u^2 - 2u^3)$ where $u = x/L$.

Putting $u = 1$, the end deflection is $v_L = v(L) = PL^3/12EI$, so the stiffness is:

$$k = P/v(L) = 12\mathrm{EI}/\mathrm{L}^3$$

The axial strain varies as:

$$\varepsilon = -\frac{y}{R} \approx -y\frac{d^2v}{dx^2} = -\frac{yP(L/2-x)}{EI}$$

where y is the distance from the neutral axis in the direction of the applied load. The maximum tensile strain occurs when $y = -h/2$ and $x = 0$, and is given by $\varepsilon_{max} = PLh/4EI$. Combining this with the earlier result for the end deflection gives:

$$\varepsilon_{max} = \frac{3v_L h}{L^2}$$ [6]

f) Process steps for a surface micromachining process with 2 mechanical layers:

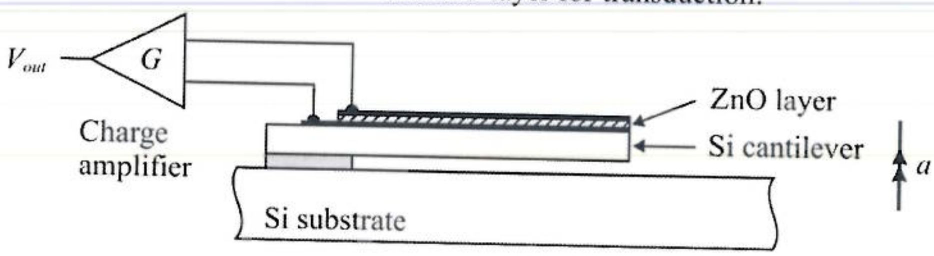
Deposit and pattern first sacrificial layer
Deposit and pattern first mechanical layer
Deposit and pattern second sacrificial layer
Deposit and pattern second mechanical layer
Remove sacrificial layer

Sacrificial layers: doped silicon dioxide
Mechanical layers: polysilicon
Depositions by LPCVD
Patterning by RIE
Sacrificial layer removal by wet chemical or vapour phase etching [4]

g) When a piezoelectric material is subject to mechanical stress it becomes electrically polarized, and charge is induced on its free surfaces. The relationship between applied stress and polarization is of the form: $P_i = d_{ij}\sigma_j$ where P_i are the three components of polarization, σ_j are the six components of stress, and d_{ij} are a the elements of a 3×6 matrix of piezoelectric coupling coefficients. Typical piezoelectric materials in MEMS are PZT, ZnO and quartz (silicon is not piezoelectric).

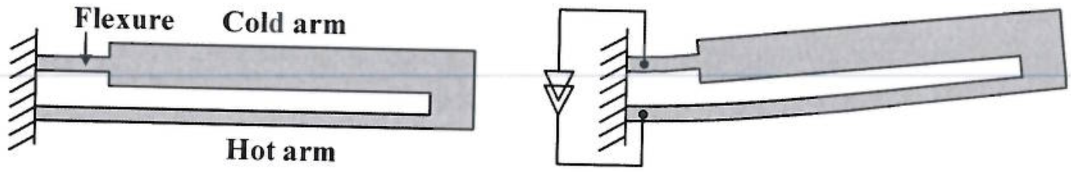
In a typical transduction scenario, a film of piezoelectric material is deposited on a silicon beam or membrane, and the charge induced on the top and bottom surfaces of the film is taken as a measure of the in-plane stress. The piezoelectric film is sandwiched between electrodes, and the readout electronics can either measure the open-circuit voltage on the electrodes or the free charge required on the electrodes to null out this voltage.

Silicon micromachined accelerometer with ZnO layer for transduction:

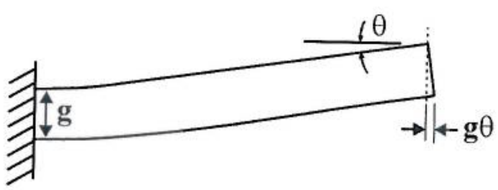


[5]

h) Typical structure of an electrothermal actuator:



Current passed through structure causes differential heating because cold arm has lower resistance (and hence less heat generated) per unit length, and better heat conduction. The structure deflects to allow differential expansion of the hot and cold arms. The difference in length between the hot and cold sides is approximately $\alpha(LhTh - LcTc - LfTf)$ where Lh, Lc and Lf are the lengths of the hot arm, cold arm and flexure respectively, and Th, Tc and Tf are the corresponding average temp rises. This must be equal to $g\theta$, where θ is the (small) angular deflection and g is the gap between the beams:



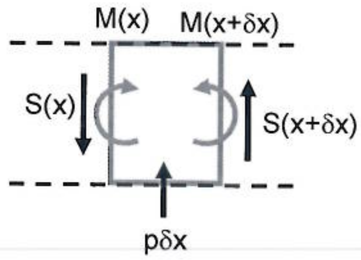
Assuming the flexure bends as a circular arc, and the cold arm remains straight, the tip deflection is then:

$$v = (Lc + Lf/2).\theta \approx \alpha(Lc + Lf/2).(LhTh - LcTc - LfTf)/g$$

[5]

Question 4

a)



Resolving forces: $S(x+\delta x) - S(x) + p\delta x = 0$ which, in the limit $\delta x \to 0$, leads to $p = -\partial S/\partial x$. Similarly, taking moments gives $S = -\partial M/\partial x$. Combining these results with the bending equation $M = EI\partial^2 v/\partial x^2$ we obtain $EI\partial^4 v/\partial x^4 = p$. If the load p is purely inertial then $p = -m\partial^2 v/\partial t^2$ where m is mass per unit length.
⇒ required result. [8]

b) By direct evaluation, we find that the given form of v satisfies $\partial^4 v/\partial x^4 = k^4 v$ and $\partial^2 v/\partial t^2 = -\omega^2 v$, so it is a solution of the wave equation provided:

$$EIk^4 = m\omega^2$$ [6]

c) The boundary conditions in the case of a beam that is built in at both ends are $v = 0$ and $\partial v/\partial x = 0$ at both $x = 0$ and $x = L$. Applying these to the general solution:

$$v = 0 \text{ at } x = 0 \Rightarrow A + C = 0$$
$$v' = 0 \text{ at } x = 0 \Rightarrow B + D = 0$$

$$v=0 \text{ at } x=L \Rightarrow A(\cos kL - \cosh kL) + B(\sin kL - \sinh kL)=0$$
$$v'=0 \text{ at } x=L \Rightarrow -A(\sin kL + \sinh kL) + B(\cos kL - \cosh kL)=0$$

Eliminating A and B from the last two equations gives the required result. [8]

d) From part b), and the given root of the eigenvalue equation, we know that the frequency of the lowest mode is:

$$f_0 = (1/2\pi)(4.73/L)^2\sqrt{(EI/m)}$$

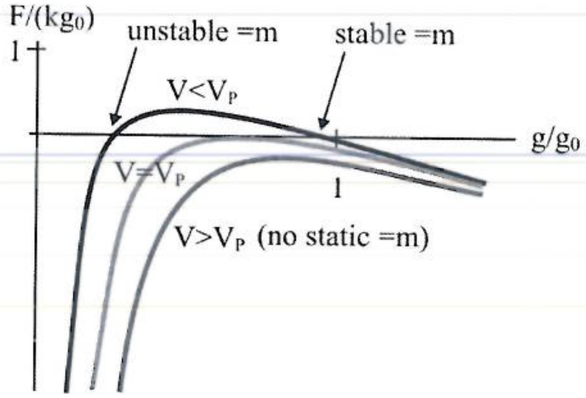
$I = bd^3/12 = 10 \times 5^3 \times 10^{-24}/12 = 1.042 \times 10^{-22}\ m^4$
$m = 10 \times 10^{-6} \times (5 \times 10^{-6} \times 2330 + 0.5 \times 10^{-6} \times 1500) = 1.24 \times 10^{-7}$ kg/m
With E = 170 GPa and L = 400 μm this gives $f_0 = 266$ kHz. [5]

Since $f_0 \propto 1/\sqrt{m}$ we know that $\delta f_0/f_0 = -\frac{1}{2}\delta m/m$. Putting $\delta m = 1 \times 10^{-15}/L = 2.5 \times 10^{-12}$ kg/m, with m and f_0 as above, gives $\delta f_0 = 2.7$ Hz. [3]

Question 5

a) The electrostatic force on the moveable plate is $F_e = -A\varepsilon_0 E^2/2$ where $E = V/(g + t_d/\square_r)$ is the electric field in the gap. The only other force acting on the moveable plate is the spring force $F_k = k(g_0 - g)$, so the total force is:

$$F = F_e + F_k = k(g_0 - g) - \frac{\varepsilon_0 A V^2}{2(g + t_d/\varepsilon_r)^2} \qquad (1) \qquad [5]$$



LH graph shows variation of force with gap for different values of V. When $V < V_P$ there are two equilibrium points (i.e. points where $F = 0$). One is stable, and the other is unstable, but the moveable plate naturally settles at the stable one (and never reaches the unstable one) if the applied voltage is increased from zero. For $V > V_P$ there is no equilibrium point, and the total force is always negative so the plate snaps down. [5]

b) The conditions at the point of snap-down are $F = 0$ and $\partial F/\partial g = 0$. Differentiating the force equation gives:

$$\frac{\partial F}{\partial g} = -k + \frac{\varepsilon_0 A V^2}{(g + t_d/\varepsilon_r)^3} \qquad (2)$$

So, when $\partial F/\partial g = 0$ we have: $\quad \varepsilon_0 A V^2 = k(g + t_d/\varepsilon_r)^3 \qquad (3)$

Substituting (3) into (1), and setting $F = 0$, we find that $g = \dfrac{2g_0}{3} - \dfrac{t_d}{3\varepsilon_r}$ at the point of snap-down. Substituting this value of g into (3) gives the quoted result. [8]

c) The release voltage is obtained by setting the total force to zero at g = 0. From (1) this gives:

$$0 = kg_0 - \frac{\varepsilon_0 A V^2}{2(t_d/\varepsilon_r)^2} \quad \Rightarrow \quad V = \sqrt{\frac{2kg_0}{\varepsilon_0 A}}\frac{t_d}{\varepsilon_r} \qquad [4]$$

d) The stiffness of each flexure is $12EI/L^3$, where L is the flexure length and $I = wt^3/12$ is the second moment of area, with w being the flexure width and t being the thickness of the gold mechanical layer. The hammock suspension comprises four flexures in parallel, so the total stiffness is $k = 48EI/L^3 = 4Ewt^3/L^3$. Putting $L = 250$ µm, $w = 4$ µm, $t = 2$ µm, $E = 80$ GPa we obtain $k = 0.655$ N/m.

With $k = 0.655$N/m, $g_0 = 5$ µm, and A = 105,600 µm^2, the snap-down voltage is obtained as $V_P = 5.1$ V. This ignores the term t_d/ε_r which makes negligible difference to the answer. [6]

Any residual stress in the gold layer will modify the suspension stiffness and hence the pull-down voltage (compressive stress will tend to lower it; tensile residual stress will have the opposite effect). The plate may also be subject to bowing if there is a stress gradient in the gold, and this will modify the initial gap. Other possible answers include: fabrication tolerance on initial gap; trapped charge in dielectric. [2]