UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1997

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER 2.6

STATISTICS

Thursday, April 24th 1997, 2.00 - 3.30

Answer THREE questions

For admin. only: paper contains 4 questions

1 A communication channel is subject to random noise disturbance, ε, where ε is an unobservable $N(0,\ 1)$ random variable. As a result a numerical input message x is received as Y, where

$$Y = x + \varepsilon \quad .$$

a Write down the distributions of Y when $x = 2$ and $x = -2$.

The channel is to be used to transmit a binary message, 0 or 1. In order to reduce the possibility of error, the following scheme for transmission and reception is proposed.

To transmit 0, input $x = -2$.

To transmit 1, input $x = 2$.

On receipt, decode the message as

$$1 \text{ if } Y \geq 0.5$$

$$0 \text{ if } Y < 0.5 \quad .$$

b Find the probability that the receiver makes an error if

(i) the original message was 0;

(ii) the original message was 1.

c If the probability of transmitting a 1 is $p(0 < p < 1)$,

(i) find the probability that the receiver makes an error;

(ii) show that, whatever the value of p, this error is less than 0.07.

Q.1 continued...

Q.1 continued...

d Suppose that a sequence of four independent binary messages are transmitted, each input and reception being independent.

(i) If (1, 0, 0, 0) is transmitted, show that the probability of at most one error in the received message is 0.999.

(ii) Suppose that (1, 0, 0, 0) and (1, 1, 0, 0) are transmitted with probabilities 0.8 and 0.2, respectively. Find the probability of at most one error in the received message.

The four parts carry, respectively, 5%, 25%, 15%, 55% of the marks.

Turn over...

2a The lifetime, Y, of a component, in thousands of hours, has probability density function

$$f(y) = \begin{cases} ye^{-y} & y > 0 \\ 0 & \text{otherwise} \end{cases} .$$

(i) Show that its distribution function, $F(y)$, is given by

$$F(y) = \begin{cases} 1 - (y + 1)e^{-y} & y > 0 \\ 0 & \text{otherwise} \end{cases} .$$

(ii) Derive its reliability and hazard functions.

(iii) Find the probability that a component will still be operating at 1000 hours, given that it has operated without failure for 350 hours.

(iv) Compare your answer in (iii) with the probability that the lifetime of a component exceeds 650 hours.

(v) Comment on your answer in (iv) in relation to results in (ii).

b A system consists of two components of the type described in (a), operating independently and connected in parallel. What is the reliability of the system at 1000 hours?

c Determine the minimum number of components of the type described in (a), operating independently and in parallel, necessary to ensure that the reliability of the system at 1000 hours is at least 0.99?

The three parts carry, respectively, 60%, 20%, 20% of the marks.

3 The total load applied to the column of a building is the sum of the dead load (due to the weight of the structure), the live load (due to human occupancy, movable furniture and the like) and the wind load. The loads are independent, normal random variables whose means and standard deviations are given below.

	Mean (in tonnes)	Standard deviation (in tonnes)
Dead Load	1.91	0.14
Live Load	2.95	0.36
Wind Load	1.54	0.32

a Determine the mean, μ_T, and the standard deviation, σ_T, of the total load applied to a column.

b Independently of the total applied load, the strength of a column is normally distributed with mean μ and a standard deviation which is 15% of the mean μ. The column is designed so that its mean strength μ is k times the mean, μ_T, of the total load ($k > 1$). Derive an expression in terms of k for the probability of failure of the column.

c By what factor must the mean strength μ exceed the mean total applied load μ_T to ensure that the probability of a column failing is only 0.01?

The three parts carry, respectively, 25%, 50%, 25% of the marks.

Turn over...

4a Service times in seconds of computer jobs at a facility's central processing unit are known, from past experience, to be normally distributed with standard deviation 1.5, but unknown mean μ.

(i) Describe how to derive a 95% confidence interval for μ based on a random sample of n service times.

(ii) Determine the size of sample, n, to ensure that the 95% confidence interval for μ is of a width less than 0.5 seconds.

b Battery capacity in ampere-hours is assumed to have a normal distribution with unknown mean μ and unknown variance σ^2. The capacities (in ampere-hours) of 10 independent batteries were recorded as follows:

140, 136, 150, 144, 148, 152, 138, 141, 143, 151.

The sample mean and sample variance for these data are 144.3 and 32.23, respectively.

(i) Find a 99% confidence interval for σ^2.

(ii) Derive a 90% confidence interval for σ^2 of the form

$$\sigma^2 > v \quad ,$$

where v is to be defined and evaluated precisely.

The two parts carry, respectively, 40%, 60% of the marks.

End of Paper

