IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

ENGINEERING MATERIALS

Wednesday, 11 June 10:00 am

Time allowed: 2:00 hours

There are FIVE questions on this paper.

Answer THREE questions.

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

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Second Marker(s) : T.J. Tate

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Special Information for Invigilators: None

Information for Candidates:

Fundamental constants

Permittivity of free space, ε_0 = 8.85 x 10^{-12} F/m
Permeability of free space, m_0 = 4p x 10^{-7} H/m
Planck's constant, h = 6.62 x 10^{-34} Js
Boltzmann's constant, k = 1.38 x 10^{-23} J/K
Electron charge, e = 1.6 x 10^{-19} C
Electron mass, m = 9.1 x 10^{-31} kg
Speed of light, c = 3.0 x 10^{8} ms^{-1}

Schrödinger's equation

General form:

$$-\frac{h^2}{8\pi^2 m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

In one dimension:

$$-\frac{h^2}{8\pi^2 m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

In spherical coordinates:

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$$

Free-electron theory

Density of states (3D):

$$g(E) = \frac{8\pi}{h^3}(m)^{3/2}\sqrt{2E}$$

Fermi energy

$$E_f = \frac{h^2}{8m}\left(\frac{3N_e}{\pi}\right)^{2/3}$$

Fermi distribution

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$$

Electrons in semiconductors

Effective mass:

$$m_e^* = \frac{h^2}{4\pi^2}\frac{1}{d^2E(k)/dk^2}$$

Number density of electrons in a semiconductor of bandgap E_g:

$$n \equiv N_e = \frac{4\pi}{h^3}\sqrt{2\pi}\left(m_e^* kT\right)^{3/2} e^{-\frac{(E_g-E_f)}{kT}}$$

$$= N_c e^{-\frac{(E_g-E_f)}{kT}}$$

Number density of holes

$$p \equiv N_h = \frac{4\pi}{h^3}\sqrt{2\pi}\left(m_h^* kT\right)^{3/2} e^{\frac{-E_f}{kT}}$$
$$= N_v e^{\frac{-E_f}{kT}}$$

Polarization

Lorentz correction for local field:

$$\mathbf{E}_{loc} = \mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}$$

Electronic polarization:

$$P_0 = \frac{\varepsilon_0 \omega_p^2 E_0}{\omega_m^2 - \omega^2 + j\omega\gamma}$$

where

$$\gamma = \frac{r}{m},$$
$$\omega_m^2 = \omega_0^2 - \frac{\omega_p^2}{3},$$
$$\omega_0^2 = k/m,$$
$$\omega_p^2 = \frac{ne^2}{m\varepsilon_0}.$$

Orientational Polarization:

Static:

$$\mathrm{P} = nm\,\mathrm{L}(mE/kT)$$

where

$$\mathrm{L}(x) = \coth(x) - 1/x$$

Dynamic:

$$P_0 = \frac{P_s}{1 + j\omega\tau},$$

Magnetism

Magnet dipole due to electron angular momentum:

$$\boldsymbol{\mu}_m = -\frac{e\mathbf{L}}{2m}$$

Magnet dipole due to electron spin:

$$\boldsymbol{\mu}_m = -\frac{e\mathbf{S}}{m}$$

Paramagnetism:

$$M = nm_m\,\mathrm{L}(m_0 m_m H/kT)$$

The Questions

1. (a) Define the following terms:
   i. intrinsic semiconductor,
   ii. extrinsic semiconductor,
   iii. acceptor,
   iv. donor,
   v. Fermi energy. [5]

(b)
   i. A pure semiconductor has a resistivity of $2.3 \times 10^5\ \Omega cm$. If the electron mobility is $1500\ cm^2V^{-1}s^{-1}$ and the hole mobility is $450\ cm^2V^{-1}s^{-1}$, show that the hole and electron carrier concentrations are both $1.4 \times 10^{10}\ cm^{-3}$. [4]

   ii. The semiconductor is doped with $10^{14}\ cm^{-3}$ of donors. What is the concentration of the carriers now, assuming extrinsic conditions? What is the resistivity? [4]

   iii. Finally, the semiconductor is additionally doped with a concentration of $10^{14}\ cm^{-3}$ acceptors. What is the concentration of carriers and the resistivity of the semiconductor in this case? [4]

   iv. Where will be the Fermi energy for the semiconductor for each of these three cases, (b) i, ii, and iii? [3]

2. (a) Consider the electric field component of a propagating electromagnetic wave:

$$E = E_0 \exp\{j(\omega t - kx)\} \qquad (2.1)$$

Show that a complex propagation constant $k = k' - jk''$ implies an exponentially decaying wave and sketch a snapshot of the wave starting at x = 0 in the positive x direction. [4]

(b) Given that the speed of light in a material is given by

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}, \qquad (2.2)$$

show for a weakly absorbing non-magnetic dielectric with dielectric constant $\varepsilon = \varepsilon' - j\varepsilon'', \varepsilon'' << \varepsilon'$, the complex refractive index is given by

$$n' - jn'' \approx \sqrt{\varepsilon_r'} - j\frac{\varepsilon_r''}{2\sqrt{\varepsilon_r'}} \qquad (2.2)$$

[4]

Show that a propagating wave incident on a weakly absorbing dielectric will drop to 1/e of its initial amplitude after travelling a distance $\frac{2c\sqrt{\varepsilon_r'}}{\omega \varepsilon_r''}$. (2.3) [6]

(c) Find the reduction in power in dB for an electromagnetic wave of frequency 2 MHz travelling 20 m in a coaxial cable with a PVC dielectric; the refractive index of PVC is 1.9 and its loss tangent is 0.06. [6]

3. (a) Define the following terms

   i. electronic wavefunction,
   ii. quantum number,
   iii. Pauli's exclusion principle
   iv. density of states. [4]

(b) Sketch the three lowest-energy electronic wavefunctions and probability distributions for the states in a one-dimensional box of length *L*. [4]

Given that the energy of each state is given by

$$E = \frac{h^2 k^2}{8\pi^2 m} \qquad (3.1)$$

where *k* is the wavenumber of the state, show that the density of states for a one-dimensional box is given by

$$g(E) = \frac{2}{h}\sqrt{\frac{2m}{E}} \qquad (3.2)$$ [7]

(c) A density-of-states diagram is shown in fig. 3.1 for a one-dimensional semiconducting carbon nanotube:

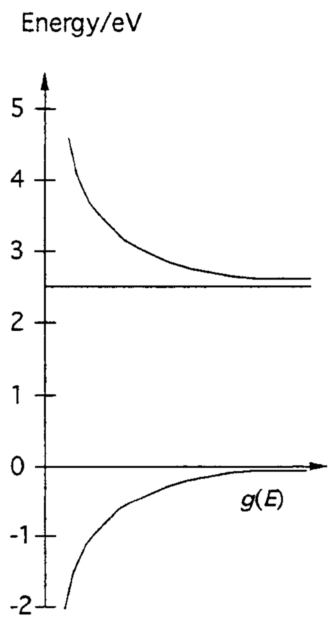


Figure 3.1

Explain the shape of the diagram, comparing and contrasting with the usual three-dimensional case. Copy the diagram and label the valence band, conduction band and energy gap and determine the approximate Fermi energy for an undoped carbon nanotube. [5]

4. (a) Explain the design and operation of magnetic write and read heads. [10]

(b) Derive an expression for the write current in terms of the coercive field of the magnetic tape material, the effective gap of the magnetic circuit and the number of write coils, stating any approximations made. [3]

(c) A 3.25-inch-diameter hard drive can rotate up to 5400 rpm. What is the maximum data write rate if the head has a gap of 10 μm, stating any assumptions you make? [3]

(d) What are the desirable properties of recordable magnetic media? [4]

5. (a) Identify the terms in the following expression for the interatomic potential, V, as a function of the interatomic spacing r:

$$V(r) = \frac{A}{r^p} - \frac{B}{r^q} \tag{5.1}$$

[4]

(b) Derive and sketch the corresponding interatomic force as a function of r identifying the equilibrium position and the yield point of the bond. Derive an expression for the equilibrium spacing.

[6]

(c) Show that that the yield strain is given by

$$\left(\frac{p+1}{q+1}\right)^{1/(p-q)} - 1 \tag{5.2}$$

and hence derive the theoretical yield strain for an ionic solid where the attractive force can be assumed to be coulombic and the repulsive force varies as $1/r^{12}$.

[10]

Electronic Materials 1.5 **2003**
Answers

1. (a)

i. Intrinsic semiconductor: carriers predominantly provided by thermal excitation across the bandgap rather than from thermalised dopants. [1]

ii. Extrinsic semiconductor: carriers predominantly provided by thermalised dopants. [1]

iii. Acceptor: dopant able to accept electron from valence band producing a hole in the valence band. [1]

iv. Donor: dopant able to donate an electron to the conduction band. [1]

v. Fermi energy: energy at which the occupancy of any state would be 0.5. (*book)* [1]

(b) i. Conductivity is given by

$$\frac{1}{\rho} = \sigma = e(\mu_e N_e + \mu_n N_h)$$ [2]

By charge conservation, in an intrinsic material the number of holes will equal the number of electrons

$$N_i = N_e = N_h = \frac{1}{\rho(\mu_e + \mu_h)e} = 1.4 \times 10^{10} \text{cm}^{-3}$$ [3]

ii. In the extrinsic case the concentration of electrons equals the concentration of donors By mass balance,

$$N_e N_h = N_i^2$$

[1]

Hence

$$N_e \equiv n = 10^{14} \text{cm}^{-3}$$

[1]

and $N_h \equiv p = 2.1 \times 10^6 \text{cm}^{-3}$

[1]

The resistivity will be given by:

$$\rho = \frac{1}{e(\mu_e N_e + \mu_n N_h)}$$

$$= \frac{1}{1.60 \times 10^{-19}(1500 \times 10^{14} + 450 \times 2.1 \times 10^6)}$$

$$= 42\Omega\text{cm}$$

[1]

iii. The acceptor states are filled by electrons from the donors.

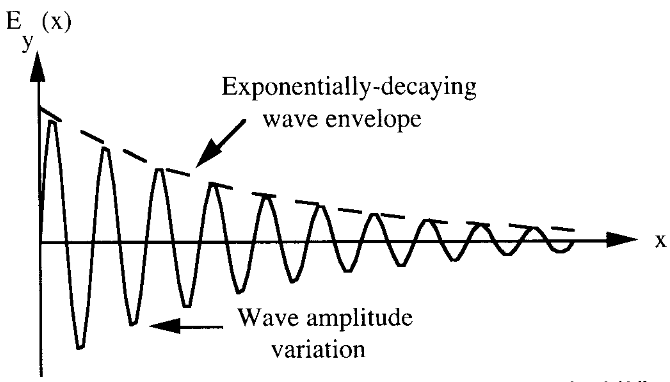
The carrier concentrations are the same as the intrinsic case, as is the resistivity. (*application of theory*) [3]

Fermi level starts in the middle of the bandgap, moves towards the donor level with the first doping, and returns to the middle of the bandgap with the additional doping. (*application of theory*) [3]

2. Putting $k = k' - jk''$, we obtain

$$E = E_0 \exp\{j(\omega t - k'x)\}\exp(-k''x)$$

which is an exponentially decaying wave:



[4]

with wavenumber *k*' and decay length 1/*k*" (*book*)

(b) Refractive index in a nonmagnetic material is given by [2]

$$n = \frac{c}{v} = \frac{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\varepsilon_r}$$

Hence

$$n' - jn'' = \sqrt{\varepsilon_r' - j\varepsilon''}$$

Expanding binomially, for $\varepsilon'' << \varepsilon'$ [2]

$$n' - jn'' \approx \sqrt{\varepsilon_r'} - j\frac{\varepsilon_r''}{2\sqrt{\varepsilon_r'}}$$

(Alternatively, squaring the expression $n' - jn'' = \sqrt{\varepsilon_r' - j\varepsilon''}$ and dropping terms in n" gives another derivation)

From $v = \dfrac{c}{n} = \dfrac{\omega}{k}$ [2]

$$k = \frac{\omega}{c} n = \frac{\omega}{c}(n' - jn'')$$

[2]

and so $k'' = \dfrac{\omega \varepsilon_r''}{2c\sqrt{\varepsilon_r'}}$.

[2]

Hence the 1/e amplitude reduction occurs after $\frac{1}{k''} = \frac{2c\sqrt{\varepsilon_r'}}{\omega\varepsilon_r''}$.(*new theory*) [2]

(c) The reduction in amplitude will be given by $\exp(-k''x)$. In this case the loss tangent, $\tan\delta = \frac{\varepsilon''}{\varepsilon_r'}$ and $n' = \sqrt{\varepsilon_r'}$ and hence [1]

$$k'' = \frac{\omega\varepsilon_r''}{2c\sqrt{\varepsilon_r'}} = \frac{\omega n \tan\delta}{2c}$$ [2]

Inserting the given values, the amplitude reduction is given by [1]

$$\exp(-k''x) = \exp\left(-\frac{2\pi \times 20 \times 10^6 \times 0.06 \times 20}{2 \times 3 \times 10^8}\right) = 0.77$$

Hence the loss is 20log0.77 = -2.2dB (*new computed example*)

3.

i. Electronic wavefunction: a solution to Schödinger's wave equation.

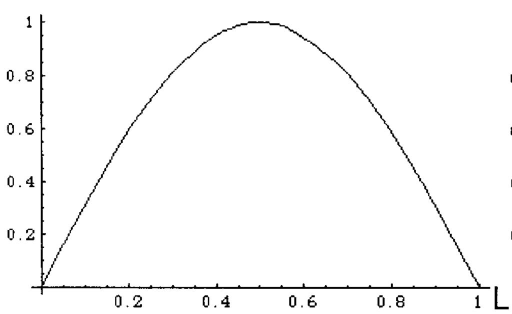
ii. Quantum number: a number that labels different wavefunctions; there is a separate quantum number for each parameter of a wavefunction.

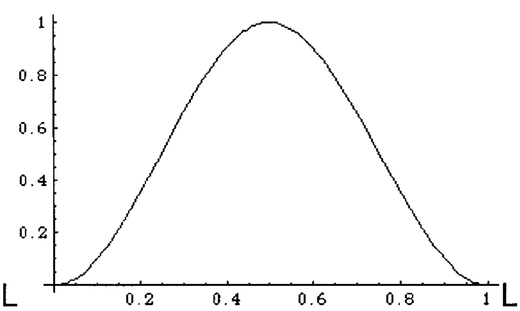
iii. Pauli's exclusion principle: each electron must have a different wavefunction, or each electron should be labelled with a unique set of quantum numbers [4]

iv. Density of states: the number of electronic states per unit energy per unit volume)(*book)*

Wavefunction   Probability distribution

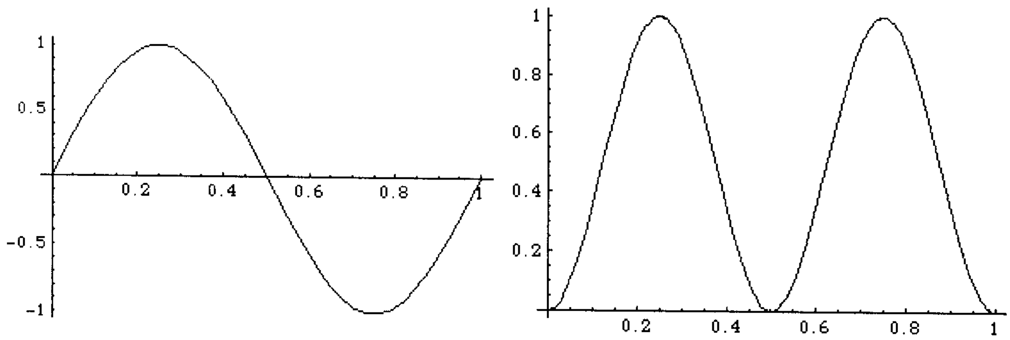
First





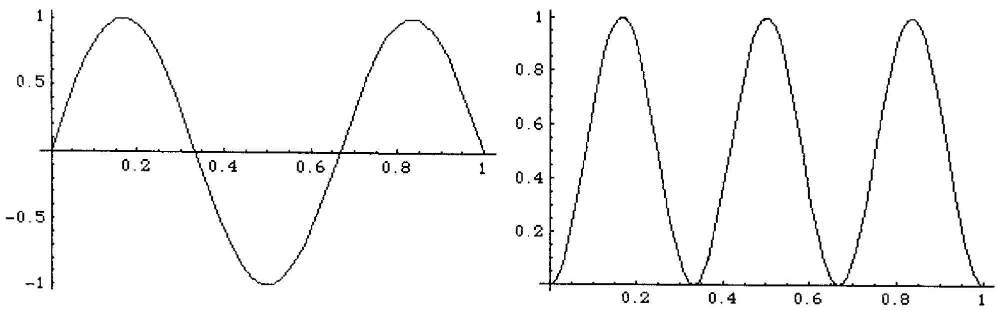
[2]

Second



[1]

Third



[1]

The energy is given by $E = \dfrac{h^2k^2}{8\pi^2 m}$ [2]

From the boundary conditions,

$$k = \frac{2\pi}{\lambda} = \frac{\pi N_k}{L},\ N_k = 1,2,3\ldots$$ [1]

If the states are occupied to a quantum number N_k, the number of states,

$$N = 2N_k$$

for the two spin states per Nk quantum number. Given the energy of each state is given by [2]

$$E = \frac{h^2k^2}{8\pi^2 m}$$

$$N_k = \frac{2L}{h}\sqrt{2mE}$$

and hence

$$N = \frac{4L}{h}\sqrt{2mE}$$ [2]

and so

$$g(E) = \frac{1}{L}\frac{dN}{dE}$$

$$= \frac{2}{h}\sqrt{\frac{2m}{E}}$$

[Alternatively, states will be evenly spaced in wavenumber, two electrons for each state.with the number of electrons per wavenumber given by: [2]

$$\frac{dN}{dk} = 2 \times \frac{L}{\pi}$$

The density of states, which is the number of states per unit energy per unit length in this case is given by [1]

$$g(E) = \frac{1}{L}\frac{dN}{dE}$$
$$= \frac{1}{L}\frac{dN}{dk}\frac{dk}{dE}$$
$$= \frac{1}{L}\frac{2L}{\pi}\frac{dk}{dE}$$

From $E = \dfrac{h^2k^2}{8\pi^2 m}$ [2]

$$k = 2\pi\frac{\sqrt{2mE}}{h}$$

and so

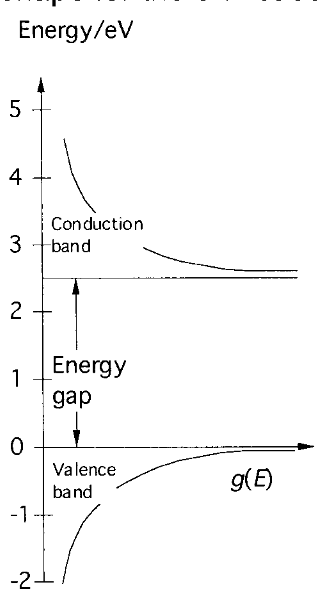
$$\frac{dk}{dE} = \frac{\pi}{h}\sqrt{\frac{2m}{E}}$$

giving

$$g(E) = \frac{1}{L}\frac{2L}{\pi}\frac{\pi}{h}\sqrt{\frac{2m}{E}}$$ [2]
$$= \frac{2}{h}\sqrt{\frac{2m}{E}}$$

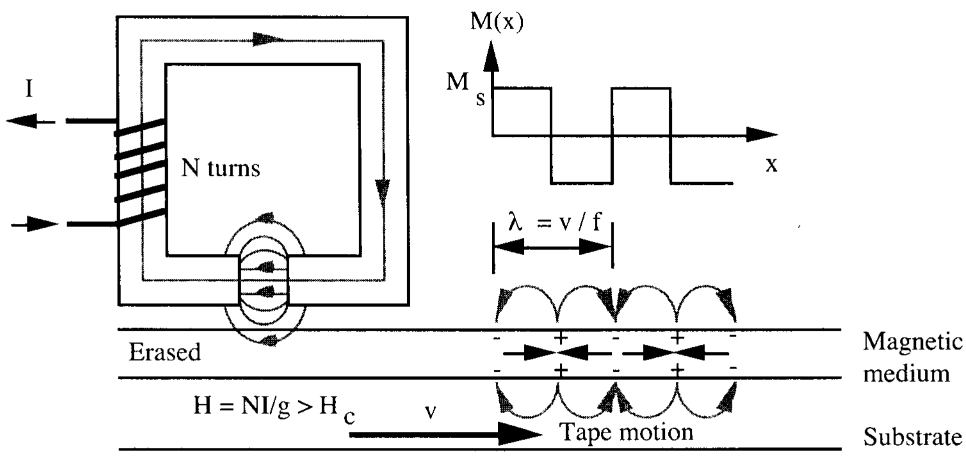
as required.](*new theory*)

(c) The DOS for the carbon nanotube has the same form as the 3-D DOS, but with the inverse-square-root shape replacing the parabolic shape for the 3-D case. [2]



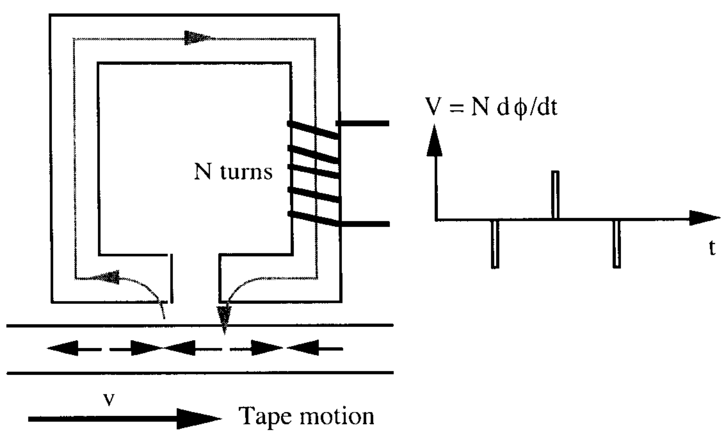
(*new application*) [3]

4. The figure below shows the operation of a magnetic recording head. A plastic tape coated with small magnetic particles is moved at constant speed past a magnetic core, which has a short air gap between two pole pieces. The current *I* flowing through the *N*-turn coil creates a magnetic field in the core, which in turn creates a field in the gap. The tape is magnetised by the fringing field near the gap. Assuming that this field is greater than the coercive field of the tape, the tape is magnetized to saturation with a dipole direction that depends on the sign of the write current. [4]



[3]

Magnetic tape is read out by a head of similar construction. The magnetic flux of the pattern stored on the tape links into the magnetic circuit of the core. Time variations of the linked flux *f* then result in an induced EMF of *Ndf*/*dt* in the read coil. The read voltage is then proportional to the *derivative* of the write current. [4]



[3]

(book)

(b) For the write head, Ampere's law for the magnetic circuit gives [1]

$$NI = H_m l_m + H_g g$$

The magnetic material has a much larger permeability ($>10^4$) than the air while the gap will be a few thousandths of the total magnetic circuit.

Hence $H_m l_m << H_g g$ and we can write [1]

$$I = \frac{H_g g}{N}$$

As the gap field must be greater than the coercive field of the magnetic material to permanently write to the medium, we obtain:

$$I > \frac{H_c g}{N}$$ *(book)* [1]

(c) Each bit will be the length of the gap, and so the maximum write rate will be [1]

$$\frac{3.14 \times 3.25 \times 0.0254 \times 5400}{60 \times 10^{-5}} = 2.3M\text{bit/s}$$
$$= 290\text{kbytes/s}$$
(new theory) [2]

(c) Magnetic storage media consist of plastic layers coated with small particles of magnetic material. Important aspects:

• **Size**: particles smaller than a domain wall thickness can only contain a single domain, and hence cannot change their magnetisation by domain wall motion. They behave as predicted by the simple Weiss model. This improves stability. However, particles should not be so small that the energy needed to change their magnetization is comparable to *kT*, or thermal effects may remove the stored data.

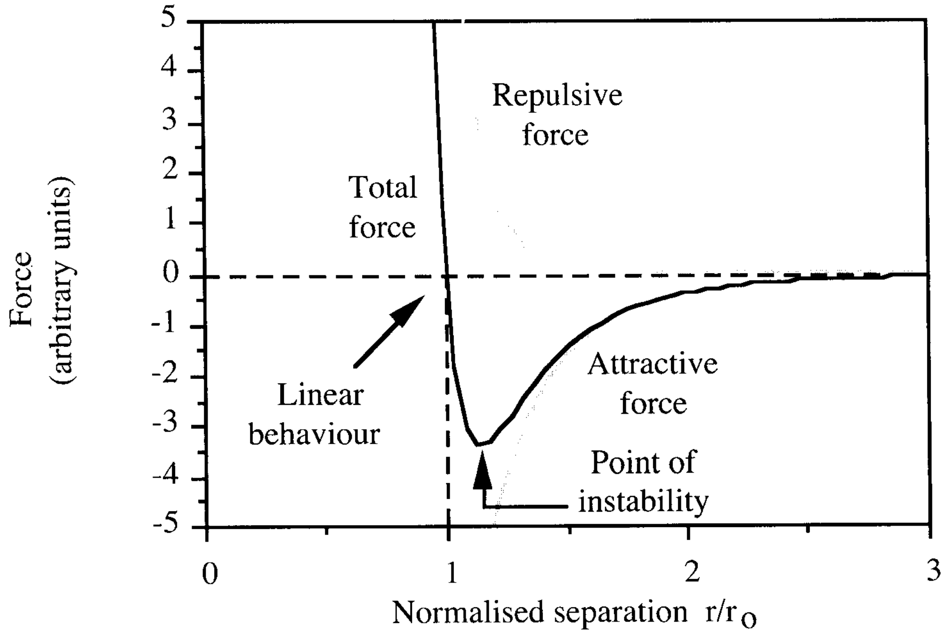
• **Shape**: needle-shaped particles are easy to magnetise parallel to their long axis, and hard to magnetise in the orthogonal direction. Magnetic anisotropy again helps the stored pattern remain stable.

• **Orientation**: particles should be roughly parallel, and aligned with the write field to exploit shape anisotropy.

• **Packing density**: a large particle density ensures a large magnetic flux at readout, and thus a large read voltage. *(book)* [4]

5. (a) A: coefficient of the repulsive force [4]
p: power of repulsive force
B:coefficient of attractive force
q: power of attractive force (book)

(b) $F(r) = -\frac{dV(r)}{dr} = \frac{pA}{r^{p+1}} - \frac{qB}{r^{q+1}}$



[3]

At the equilibrium position the attractive and repulsive forces are opposite and equal: [

$$\frac{dV(r)}{dr} = -\frac{pA}{r^{p+1}} + \frac{qB}{r^{q+1}} = 0,$$ [1]

Hence at $r = r_0$

$$r_0 = \left(\frac{pA}{qB}\right)^{\frac{1}{p-q}}$$ (book) [2]

(c) The yield point is given by $\frac{d^2V}{dr^2} = 0$. [1]

Hence

$$\frac{p(p+1)A}{r^{p+2}} = \frac{q(q+1)B}{r^{q+2}}$$

$$r^{p-q} = \frac{p(p+1)A}{q(q+1)B}$$ [3]

Substituting in $r_0 = \left(\frac{pA}{qB}\right)^{\frac{1}{p-q}}$ gives

$$r = \left(\frac{p+1}{q+1}\right)^{\frac{1}{p-q}} r_0$$ [2]

Hence the yield strain is $\left(\frac{p+1}{q+1}\right)^{\frac{1}{p-q}} - 1$ (new theory)

For a coulombic potential,

$$V(r) \propto \frac{1}{r}$$ [1]

and so q = 1, p = 12..

Hence the yield strain is 0.18 (new computed example) [2]