Exam Answer #1

Model answer to Q 1(a): Computed Example (normal/abnormal)

i) Intrinsic impedance.

$$\varepsilon_r'' = \frac{\sigma}{\omega\varepsilon_o} = 3.706/23.234$$

$$\eta = \frac{377}{\sqrt{\varepsilon_r' - j\varepsilon_r''}} = 88.65 + j9.28\Omega / 47.80 + j9.71\Omega$$

[3]

ii) Propagation constant.

$$\gamma = \frac{j\omega\mu_o}{\eta} = 7.84 + j74.88/27.38 + j134.83$$

[2]

iii) Skin depth.

$$\delta = \frac{1}{\mathrm{Re}(\gamma)} = 128mm/36.5mm$$

[2]

iv) Power attenuation in dB per unit wavelength.

$$\alpha = 7.84/27.38$$

$$\beta = 74.88/134.83$$

$$\lambda = \frac{2\pi}{\beta} = 83.9mm/46.6mm$$

$$Power\ Attenuation = e^{-2\alpha\lambda}\ Np/\lambda$$

$$Power\ Attenuation = 8.686\alpha\lambda\ dB/\lambda = 5.715/11.083 dB/\lambda$$

[3]

v) Power flux density given a RMS electric field intensity is 2 V/m.

$$P_D = \frac{|E|^2}{\mathrm{Re}(\eta)} = 45/84 mW/m^2$$

[2]

Model answer to Q 1(b): Computed Example

$$\rho = \frac{\eta_{abnormal} - \eta_{normal}}{\eta_{abnormal} + \eta_{normal}} = -0.293 + j0.044$$

$$\Gamma = |\rho|^2 = 0.088$$

[4]

Model answer to Q 1(c): Deductive Reasoning

It can be seen from the results in 4(a) that the two different tissue types have very different microwave properties. The tumour is almost twice as effective at attenuating/absorbing microwave power, when compared to normal tissue. Therefore, it may be possible to detect the tumour by transmitting microwave pulsed energy into the breast and observing the transmission and reflected properties. With transmission measurements, high levels of attenuation and longer delays will be found with large tumours. With reflection measurements, large reflected waves will be observed at the interface between the different tissue types.

[4]

Exam Answer #2

Model answer to Q 2(a): Computed Example

$$\varepsilon'_{reff} = 12.86 \quad \text{and} \quad tan\delta = \varepsilon''_{reff}/\varepsilon'_{reff} = 6x10^{-4} \quad \text{so} \quad \varepsilon''_{reff} = 0.007716 \text{ at } 300\text{ GHz}$$

$$\varepsilon_{eff} = \varepsilon - j\sigma/\omega \quad \text{where } \varepsilon = \varepsilon_o\,(\varepsilon'_r - j\varepsilon''_r) \text{ and } \sigma = \sigma' - j\,\sigma'' \rightarrow \sigma_o \quad \text{and} \therefore \varepsilon'_{reff} \rightarrow \varepsilon'_r$$

$$\varepsilon_{reff} \rightarrow \varepsilon'_r - j\left(\varepsilon''_r + \sigma_o\,/\omega\varepsilon_o\right) \text{ and } \varepsilon''_r \rightarrow \varepsilon''_{reff} - \sigma_o\,/\omega\varepsilon_o \;= 0.006967$$

$$\sigma_{eff} = j\omega\varepsilon_{reff} = \sigma + j\omega\varepsilon \rightarrow \left(\sigma_o \; + \omega\varepsilon_o\,\varepsilon''_r\right) + j\omega\varepsilon_o\,\varepsilon'_r = 0.1288 + j214.6$$

[5]

Model answer to Q 2(b): Computed Example

$$\rho_o = 8\,k\Omega\cdot cm = 80\,\Omega\cdot m$$

$$\therefore \sigma_o = \frac{1}{\rho_o} = 0.0125\,S/m$$

Therefore, it can be seen that at 300 GHz the conductivity is 10.3 times greater than the originally quoted measured value at DC.

[3]

Model answer to Q 2(c): Computed Example

$$\rho = \frac{\eta - \eta_o}{\eta - \eta_o} \quad where \quad \eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \quad and \quad \eta = \sqrt{\frac{\mu_o\mu_r}{\varepsilon_o\varepsilon_r}} \rightarrow \sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r{}'}}$$

$$\therefore \rho = \frac{1-\sqrt{\varepsilon_r{}'}}{1+\sqrt{\varepsilon_r{}'}} = -0.564$$

$$\Gamma = |\rho|^2 = 31.8\%$$

[5]

Model answer to Q 2(d): New Derivation

$$H(z) = H(0)e^{-\gamma z} + H(0)e^{+\gamma z}$$

$$P_{ABSORBED} = |H(z)|^2_{z=0}\,R_s = 4|H(0)|^2\,R_s$$

$$P_{INCIDENCE} = |H(0)|^2\,\eta_o$$

$$\Gamma = \frac{P_{REFLECTED}}{P_{INCIDENCE}} \quad where \quad P_{REFLECTED} = P_{INCIDENCE} - P_{ABSORBED}$$

$$\therefore \Gamma = 1 - \frac{P_{ABSORBED}}{P_{INCIDENCE}} = 1 - 4\frac{R_s}{\eta_o}$$

[5]

Model answer to Q 2(e): Computed Example

$$\Gamma = 1 - 4\frac{R_s}{\eta_o} = 1 - 4\frac{0.1}{120\pi} = 99.89\%$$

[2]

Exam Answer #3

Model answer to Q 3(a): New Derivation

$$\eta_l = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{(\omega\mu_o\mu_r'') + j\omega(\mu_o\mu_r')}{(\sigma' + \omega\varepsilon_o\varepsilon_r'') + j\omega\left(\frac{-\sigma''}{\omega} + \varepsilon_o\varepsilon_r'\right)}} \quad [\Omega]$$

$$\gamma \equiv \frac{j\omega\mu}{\eta_l} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \sqrt{[(\omega\mu_o\mu_r'') + j\omega(\mu_o\mu_r')]\left[(\sigma' + \omega\varepsilon_o\varepsilon_r'') + j\omega\left(\frac{-\sigma''}{\omega} + \varepsilon_o\varepsilon_r'\right)\right]} \quad [m^{-1}]$$

[2]

Model answer to Q 3(b): Bookwork

$$Zo = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad [\Omega]$$

$$\gamma \equiv \frac{R + j\omega L}{Zo} = \sqrt{(R + j\omega L)(G + j\omega C)} \quad [m^{-1}]$$

[2]

Model answer to Q 1(c): New Derivation

The equivalent primary line (i.e. distributed-element) parameters for this generic case are given by the following:

$$R \equiv \omega\mu_o\mu_r'' \quad [\Omega/m]; \; L \equiv \mu_o\mu_r' \quad [H/m]; \; G \equiv \sigma' + \omega\varepsilon_o\varepsilon_r'' \quad [S/m]; \; C \equiv \frac{-\sigma''}{\omega} + \varepsilon_o\varepsilon_r' \quad [F/m]$$

For the elementary lumped-element circuits to be valid, the distance of propagation Δz must be very much shorter than the wavelength λ of the electromagnetic wave within the homogeneous material.

[2]

Model answer to Q 3(d): New Derivation

(i) The constitutive parameters are represented by the following:

$\mu \to \mu_o; \quad \varepsilon \to \varepsilon_o \cong 0; \quad \sigma \to \sigma_o$

[1]

(ii)

$$Zo_o = \sqrt{\frac{R_o + j\omega L_o}{G_o + j\omega C_o}}$$

$$Zo_o \Rightarrow \eta_{lo} = \sqrt{\frac{j\omega\mu_o}{\sigma_o}} \Rightarrow Z_{So} \equiv (R_{So} + jX_{So}) \, [\Omega / square]$$

where $X_{So} \equiv R_{So}$; $R_{So} \equiv \Re\{Z_{So}\} = \sqrt{\frac{\omega\mu_o}{2\sigma_o}} \, [\Omega / square]$; $L_{So} \equiv \frac{\Im\{Z_{So}\}}{\omega} = \sqrt{\frac{\mu_o}{2\omega\sigma_o}} \, [H / square]$

[2]

(iii)

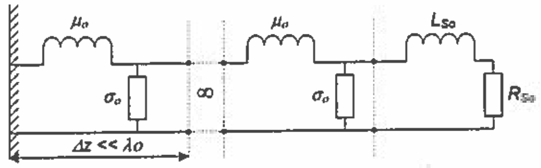
$$\gamma_o \equiv \frac{R_o + j\omega L_o}{Zo_o} = \sqrt{(R_o + j\omega L_o)(G_o + j\omega C_o)}$$

[1]

(iv) Therefore, the distributed-element parameters for the classical skin-effect model become:

$$R_o = 0 \quad ; \quad L_o = \mu_o \quad ; \quad G_o = \sigma_o \quad ; \quad C_o = \varepsilon_o \cong 0$$

[1]



Equivalent transmission line model for the classical skin-effect model

[2]

It is interesting to note that these parameters are frequency invariant, but with $\sigma_o \propto \tau$ there is a strong temperature dependence.

[1]

Model answer to Q 3(e): Calculated Example

For gold at room temperature, $\sigma_o = 4.517*10^7$ [S/m] and $\mu_o = 4\pi \times 10^{-7}\ H/m$. Calculate the:

(i) characteristic impedance Zo = 0.296 (1 + j) [Ω] [1]

(ii) surface inductance $Ls = \Im\{Zo\}/\omega = 47$ [fH/square] [1]

(iii) propagation constant $\gamma = \sqrt{\omega\mu_o\sigma_o/2} = 13.35$ (1 + j) [μm⁻¹] [1]

(iv) wavelength $\lambda = 2\pi/\Im\{\gamma\} = 0.47$ [μm] [1]

(v) maximum recommended value of Δz = 4.7 nm [1]

(vi) Elementary lumped-element circuit values using Δz given in (v)

$\mu_o \cdot \Delta z = 5.9\,[fH]$ and $\sigma_o \cdot \Delta z = 212\,[mS]$

[1]

Exam Answer #4

Model answer to Q 4(a): Bookwork

(i) The velocity factor is the ratio of the speed of waves on the transmission line to the speed of light, c, in vacuum, where $c = 3x10^8$ m/s. The velocity factor is a dimensionless number less that unity. It basically represents how much the wave is slowed down inside the transmission line, when compared to propagation in free space.

[2]

(ii) The characteristic impedance is the intrinsic wave impedance for an infinitely long length of transmission line, or of a finite length of line at times before any reflections have arrived back at the measuring source. It is important to match two lengths of different propagating media to the same impedance, in order to avoid reflected waves and associated reflected power. Conversely, two specific impedances are required to generate a specific reflected wave.

[2]

(iii) The return loss is the number of dB by which the return wave power is less than the forward wave power. This is important because it gives a quantitative measure of how much reflected power is generated and thus indicates the level of impedance mismatch.

[2]

(iv) The complex reflection coefficient is the complex ratio of return wave amplitude to forward wave amplitude. This is important, as it allows the exact impedance at a discontinuity to be calculated.

[2]

(v) The phase delay is the amount of phase shift experienced by the wave in travelling along a certain length of transmission line. It is important to be able to calculate this delay so that any reflected or transmitted waves can be combined in-phase or 180° out-of-phase.

[2]

Model answer to Q 4(b): Computed exercise

$$Zo = \sqrt{\frac{L}{C}}; \quad v_p = \frac{1}{\sqrt{LC}} \quad and \quad VF = \frac{v_p}{c}$$

If $Zo = 50\ \Omega$ and $VF = 0.6$, $L = 278$ nH and $C = 111$ pF for a length of 1 m transmission line.

[4]

Model answer to Q 4(c): Computed exercise

$$z_L = \frac{Z_L}{Zo} = 1.5 + j0.26\ \Omega$$

$$\rho = \frac{z_L - 1}{z_L + 1} = 0.21 + j0.08 = 0.22e^{+j21.54}$$

$$Return\ Loss = 13\ dB$$

As the reference plane is moved towards the generator, the modulus of the complex reflection coefficient stays the same (assuming the line is lossless), but the phase angle is reduced by 360° for each λg/2 of displacement of the reference plane from the load.

[6]

Exam Answer #5

Model answer to Q 5(a): Bookwork Derivation

The voltage and current on a transmission line can be represented as:

$$V(z) = V_+ e^{-\gamma z} + V_- e^{+\gamma z}$$

$$I(z) = I_+ e^{-\gamma z} + I_- e^{+\gamma z}$$

$where,\ V_\pm (I_\pm)$ represents voltage (current) waves at $z = 0$

$and,\quad e^{\mp\gamma z}$ represents wave propagation in the $\pm z$ direction

and the propagation constant, $\gamma = \alpha + j\beta$

The voltage and current on a transmission line can now be represented as:

$$V(z) = V_+\left(e^{-\gamma z} + \rho(0)e^{+\gamma z}\right)$$

$$I(z) = I_+\left(e^{-\gamma z} - \rho(0)e^{+\gamma z}\right)$$

The voltage (and current) on the line is composed of a superposition of the incident and reflected waves, which create a "standing wave", due to the mismatched load termination (even if the generator is matched to the line). Here, the incident and reflected wave magnitudes alternately cancel and reinforce one another. This standing wave disappears when the line is said to be "matched", i.e. $Z_T = Zo$, and we are left with just a single wave travelling in the +z direction.

[2]

Model answer to Q 5(b): Textbook derivation

Now, the impedance looking into a transmission line that is terminated with a load Z_T is:

$$Zin = \frac{V(l)}{I(l)} = Zo\frac{\left(e^{+\gamma l} + \rho(0)e^{-\gamma l}\right)}{\left(e^{+\gamma l} - \rho(0)e^{-\gamma l}\right)} = \frac{\left((Z_T + Zo)e^{+\gamma l} + (Z_T - Zo)e^{-\gamma l}\right)}{\left((Z_T + Zo)e^{+\gamma l} - (Z_T - Zo)e^{-\gamma l}\right)}$$

$$Zin = Zo\frac{\left(Z_T(e^{+\gamma l} + e^{-\gamma l}) + Zo(e^{+\gamma l} - e^{-\gamma l})\right)}{\left(Zo(e^{+\gamma l} + e^{-\gamma l}) + Z_T(e^{+\gamma l} - e^{-\gamma l})\right)}$$

$$Therefore,\ zin = \frac{Zin}{Zo} = \frac{z_T + \tanh(\gamma l)}{1 + z_T \tanh(\gamma l)} \Rightarrow \frac{z_T + j\tan\theta}{1 + jz_T\tan\theta} \text{ for a lossless line}$$

[6]

Model answer to Q 5(c): Textbook derivation

(a) If $l = \lambda g/2$ then $Zin = Z_T$, therefore, no impedance transformation – useful for realising interconnects over a narrow bandwidth.

[2]

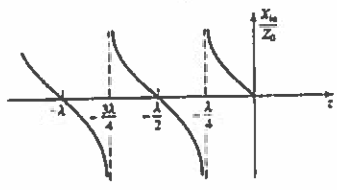
(b) If $l = \lambda g/4$ then $Zin = Zo^2/Z_T$, therefore, this is a quarter-wavelength impedance transformer – acts as an impedance inverter over a narrow bandwidth.

[2]

(c) If $Z_T = Zo$ then $Zin = Zo$, therefore, no impedance transformation – useful for realising interconnects over a very wide bandwidth.

[2]

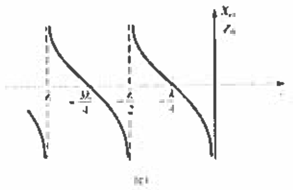
(d) If $Z_T = 0$ then $Zin = jZo\tan\theta$ and the impedance is always reactive and periodic along the line, which takes a value from 0 to +j∞ and -j∞ to 0 as l increases from 0 to $\lambda g/4$ and $\lambda g/4$ to $\lambda g/2$. This is useful for realising any value of "effective" inductance or capacitance over a narrow bandwidth.



(a) Voltage, (b) current, and (c) impedance (R_{in} = 0 or ∞) variation along a short-circuited transmission line.

[3]

(e) If $Z_T = \infty$ then $Zin = -jZo\cot\theta$ and the impedance is always reactive and periodic along the line, which takes a value from -j∞ to 0 and 0 to +j∞ as l increases from 0 to $\lambda g/4$ and $\lambda g/4$ to $\lambda g/2$. This is useful for realising any value of "effective" capacitance or inductance over a narrow bandwidth.



(a) Voltage, (b) current, and (c) impedance ($R_{in} = 0$ or ∞) variation along an open circuited transmission line.

[3]

Exam Answer #6

Model answer to Q 6(a): Bookwork

The main problem with conventional microstrip circuits, when trying to ground the source connection of a FET, is that the through-substrate metal-plated via connection has both parasitic inductance and resistance. This can prevent an oscillator from oscillating or cause amplifiers to oscillate. Both these examples represent catastrophic circuit failure.

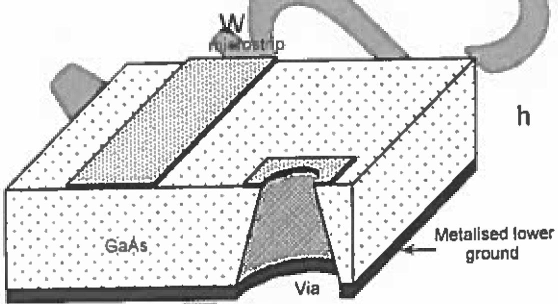
A conventional microstrip circuit can be modified to reduce this problem by turning it into thin-film microstrip.

[3]

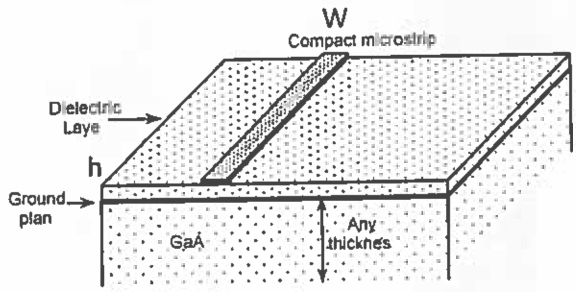
Model answer to Q 6(b): Bookwork

When compared to conventional microstrip, the ground plane is brought up to the top of the substrate. A thin layer of non-conductor defines the dielectric that separates the ground plane from the main signal line. The dielectric layer can be much thinner for TFMS than with conventional microstrip, and easily deposited using a spin-on or lamination bonding process

Conventional Microstrip



Thin-Film Microstrip (TFMS)



[4]

Model answer to Q 6(c): Bookwork

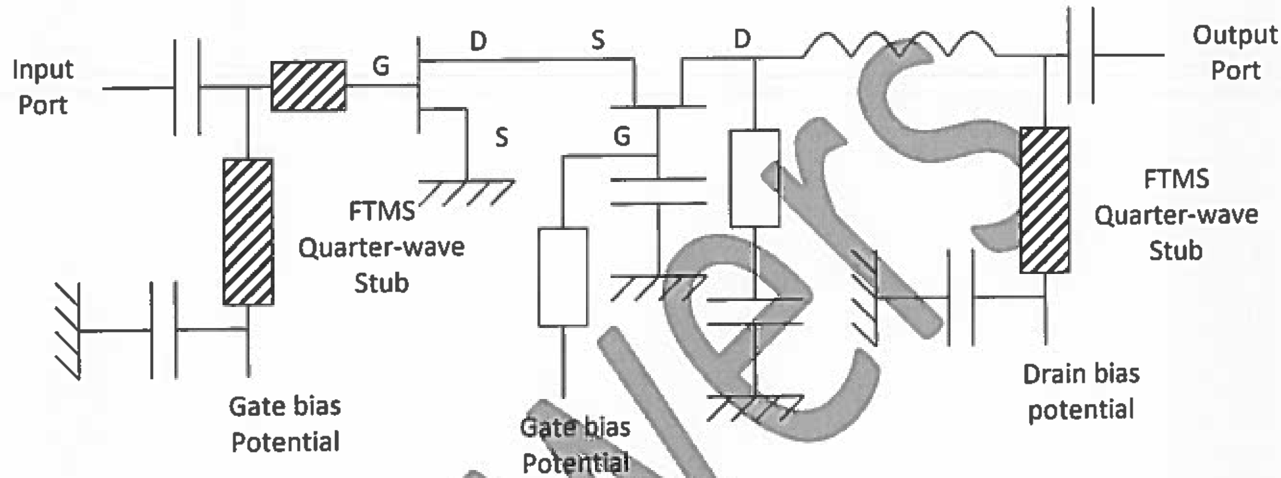
i) The characteristic impedances Zo of a TFMS and conventional microstrip line can be the same (hint: use the variables defined in 6(b)). It will be found that $Zo \propto \left(\frac{h}{W_{effective}}\right)$. Therefore, as long as this ratio stays the same then characteristic impedance can stay the same. This means that W will be much narrower with TFMS than with conventional microstrip.

[3]

ii) losses in a TFMS is higher than a corresponding conventional microstrip line. The reason is that power dissipated as heat is defined by: $P_{DISSIPATED} = |Js|^2 Rs$, where Js is the surface current and Rs is the surface resistance. Since Js is directly proportional to the conduction current density $Jc(0)$ [A/m^2], it follows that as the cross-sectional area of the microstrip's signal track is much smaller with TFMS then $Jc(0)$, Js and $P_{DISSIPATED}$ are all much greater.

[4]

Model answer to Q 6(d): Bookwork



The two largest TFMS transmission lines are used as quarter-wave stubs, which act as DC bias chokes. The amplifier has a cascode topology.

[6]