IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EEE PART II: MEng, BEng and ACGI

Corrected Copy

MATHEMATICS 2B (E-STREAM AND I-STREAM)

Friday, 30 May 2:00 pm

Time allowed: 1:30 hours

There are TWO questions on this paper.

Answer TWO questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : B. Clerckx

Second Marker(s) : R.R.A. Syms

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THE QUESTIONS

[25]

1. a) Consider two discrete random variables X and Y that have joint probability mass function represented by the joint probability table

		Y		
		0	1	2
	0	0.05	0.05	0.15
X	1	0.05	0.05	0.25
	2	0.15	0.20	0.05

i) Compute the probability that X is smaller or equal to Y, i.e. $P(X \leq Y)$ and the probability that X is strictly smaller than Y, i.e. $P(X < Y)$.

[2]

ii) Compute the marginal probability mass function of X and Y.

[2]

iii) Compute the expectation of X, i.e. $E(X)$, and the expectation of Y, i.e. $E(Y)$.

[2]

iv) Compute the variance of X and the variance of Y, i.e. $\text{Var}(X)$ and $\text{Var}(Y)$, the covariance between X and Y, i.e. $\text{Cov}(X,Y)$, and the correlation coefficient between X and Y, i.e. $\text{Corr}(X,Y)$.

[4]

v) Are X and Y uncorrelated? Independent? Provide your reasoning.

[2]

vi) Compute the conditional probability mass function of X given that $Y = 0,1,2$.

[3]

vii) Compute the conditional expectation of X given that $Y = 0,1,2$.

[3]

viii) Relying on your result in vii), compute the expectation of X, i.e. $E(X)$.

[2]

b) By making use of the standard normal table, compute the following integral

$$\int_{-\infty}^{2.35} \sqrt{\frac{2}{\pi}}\, e^{-2(u-2)^2}\, du.$$

Provide your reasoning. [5]

[25]

2. a) Consider a communication system where the transmitter is equipped with one antenna and the receiver with two antennas. The power of the signal received at antenna i is denoted as P_i, $i = 1,2$, and is modeled as an exponentially distributed random variable with parameter $\lambda > 0$. The receiver only uses one antenna at a time and selects the antenna with the largest power. Hence, the power of the signal after selection is given by $P = \max_{i=1,2} P_i$. We assume the receive antennas are deployed such that P_1 and P_2 are independent.

i) Find the probability that the power of the signal after selection, P, falls below a certain level S. Provide your reasoning. [4]

ii) Find the probability density function of P. Provide your reasoning. [4]

iii) We are interested in computing the error probability of this communication system. The error probability is the probability of wrongly decoding the transmitted signal and can be approximated as the moment generating function of P evaluated at the point $t = -d$ for $d > 0$. Making use of such approximation, find the error probability. Provide your reasoning. [4]

iv) From the results in iii), find the expected value of the received power after selection. Provide your reasoning. [4]

b) i) Assume $X \sim N(\mu, \sigma^2)$. Find the moment generating function $m_X(t)$ of X. Provide your reasoning. [4]

ii) Consider the following statement: If $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ and X_1,X_2 are independent random variables, we have $2X_1 - X_2 \sim N(2\mu_1 - \mu_2, 2\sigma_1^2 - \sigma_2^2)$. Is the statement correct? If yes, provide a proof. If not, correct the statement and provide a proof. [5]