MODELLING AND CONTROL OF MULTIBODY MECHANICAL SYSTEMS

1. A pendulum system consists of two separate pendulums, each with a mass m suspended from a massless rod of length l, as shown in Figure 1.1. The masses m are free to swing in a vertical plane by an angle θ_1 and θ_2 respectively, under the influence of gravity and a torsional spring (not shown in Figure 1.1) connected between the two pendulums. The torsional spring exerts a moment between the two pendulums, about an axis perpendicular to the page, given by the torsional stiffness k times the angle between the two pendulums.

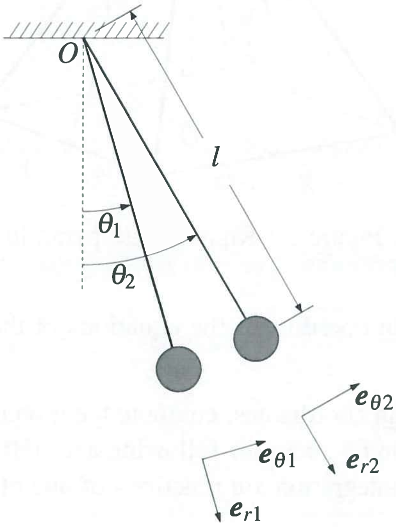

Figure 1.1 Pendulum system.

Two moving Cartesian coordinate systems with unit vectors $\boldsymbol{e}_{r1}$, $\boldsymbol{e}_{\theta 1}$ for the first (attached to the first pendulum at origin O), and unit vectors $\boldsymbol{e}_{r2}$, $\boldsymbol{e}_{\theta 2}$ for the second (attached to the second pendulum at origin O) are used to analyse the motion of the two masses of the pendulum system.

a) Write

   i) the position vector, and [2]

   ii) the velocity vector [2]

   of each of the masses.

almost everyone did these correctly

b) Compute

   i) the kinetic energy, [2]

   ii) the potential energy, and [3]

   iii) the Lagrangian function [1]

   of the system.

mostly done correctly, some forgot to include the $\frac{1}{2}k(\theta_1-\theta_2)^2$ term and some thought P.E. is a vector!

c) Use the Lagrangian approach to derive

   i) the equations of motion of the pendulum system, and [4]

   generally done well, some didn't take the torsional moment into account properly

   ii) the total external force of constraint acting on the system at point O. [6]

many ~~no one~~ did not do this part

2. A right square pyramid of height h, square base side length b, mass m and density ρ, is shown in Figure 2.1. The origin O of the Cartesian coordinate system with axes X, Y, and Z is at the geometric centre of the square base of the pyramid. The Z axis is the axis of symmetry of the pyramid. The volume of the pyramid is $\frac{1}{3}b^2h$.

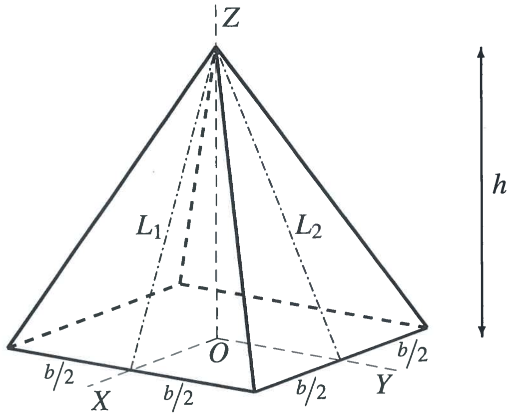

Figure 2.1 Right square pyramid.

a) Write in Cartesian coordinates the equations of the lines L_1 and L_2 shown in Figure 2.1. [3]

Surprisingly many students got this wrong

b) By using Cartesian coordinates, compute the moment of inertia of the pyramid in terms of m, h and b about the following axes (Hint: some of the limits of the relevant volume integration are functions of one of the coordinates rather than constants):

i) the axis Z; [8]

ii) the axis X; [7]

iii) the axis Y. [2]

almost everyone got it right that $I_{yy} = I_{xx}$ even if I_{xx} was wrong

The main issue here was to get the integration limits correctly as most student wrote down the correct integral. Many students got the limits wrong.

3. Two identical uniform wheels of mass m and radius R each are held at their centre of mass respectively at the left and right ends of a uniform axle of length l, as shown in Figure 3.1. The axle has only one rotational degree of freedom about a vertical axis through its middle point, A, represented by an angle ψ. The axle moment of inertia about its axis of rotation is I_{axle}. The left and right wheels spin relative to the axle by angles θ_l and θ_r respectively. Each of the wheels is axisymmetric with spin moment of inertia I_{yy} and radial moment of inertia, passing through the wheel centre of mass, I_{xx}.

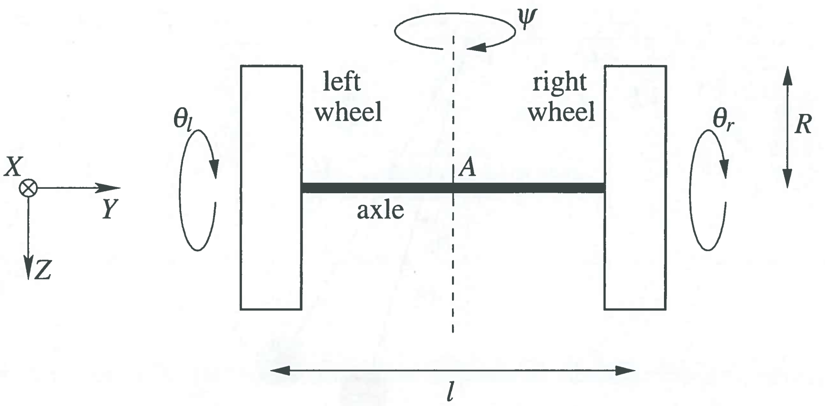

Figure 3.1 Two wheels on an axle.

An axle-fixed Cartesian coordinate system with unit vectors $\boldsymbol{i}'$, $\boldsymbol{j}'$ and $\boldsymbol{k}'$ is used to analyse the motion of the system. This coordinate system has its origin at point A, it rotates with the axle and its unit vectors are initially respectively aligned with the earth-fixed axes X, Y and Z, in which X is into the page, as shown in Figure 3.1.

a) Write the axle angular velocity vector, $\boldsymbol{\Omega_a}$, in the axle-fixed coordinate system. [1]

b) Determine the angular velocity vector of the left and right wheels, respectively $\boldsymbol{\Omega_{wl}}$ and $\boldsymbol{\Omega_{wr}}$, in the axle-fixed coordinate system. [2]

c) Assume that the wheels are locked on the axle, such that the whole system rotates as a single rigid body about the vertical axis through point A. Calculate the moment of inertia of the system about the axis of rotation. [3]

these were performed reasonably well.

d) Assume that the wheels are again free to rotate with respect to the axle and that they rest on a rough horizontal road such that their contact points with the road are instantaneously at rest, as the axle rotates.

i) Write the constraint equation for each wheel due to their road contact point being at rest. Are these constraints holonomic or nonholonomic? [3]

mostly done correctly, some got the signs wrong on some didn't realise that the equations are integrable hence holonomic

generally well attempted

ii) Calculate the inertia matrix of each wheel with respect to the axle-fixed axes (origin at point A). [2]

most used the correct method

iii) Compute the angular momentum vector of the left and right wheels, respectively $\boldsymbol{H_{wl}}$ and $\boldsymbol{H_{wr}}$, for their motion about point A, using the axle-fixed rectangular coordinate system. [3]

iv) Determine the change in the apparent moment of inertia of the system with respect to the vertical axis of rotation through point A, as compared with the moment of inertia found in part c). [6]

no one managed to solve this

4. A pendulum system consists of two separate pendulums, each with a mass m suspended from a massless rod of length l, as shown in Figure 4.1. The masses m are free to swing in a vertical plane by an angle θ_1 and θ_2 respectively, under the influence of gravity and a torsional spring (not shown in Figure 4.1) connected between the two pendulums. The torsional spring exerts a moment between the two pendulums, about an axis perpendicular to the page, given by the torsional stiffness k times the angle between the two pendulums.

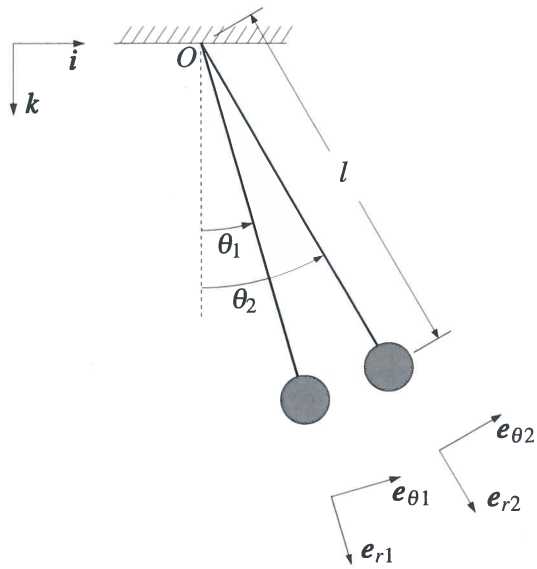

Figure 4.1 Pendulum system.

A fixed Cartesian coordinate system with unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$, and two moving Cartesian coordinate systems with unit vectors $\boldsymbol{e}_{r1}$, $\boldsymbol{e}_{\theta 1}$ for the first (attached to the first pendulum at origin O) and unit vectors $\boldsymbol{e}_{r2}$, $\boldsymbol{e}_{\theta 2}$ for the second (attached to the second pendulum at origin O) are used to analyse the motion of the two masses of the pendulum system.

mostly done correctly

a) Write the number of degrees of freedom and the generalised coordinates of the system. [2]

mixed success, some confused forces and moments.

b) Calculate the total moment vector acting on each mass with respect to the origin O. [3]

well attempted

c) Compute the angular momentum vector of each mass with respect to the origin O. [4]

d) Use the <u>vectorial</u> approach to derive the equations of motion of the pendulum system in terms of the generalised coordinates. [4]

e) Write the acceleration vector of each of the masses. [3]

f) Use the <u>vectorial</u> approach to determine the total external force of constraint acting on the system at point O. [4]

reasonably well attempted.

mixed success, some got the individual forces along the two pendulums correctly but then added them as scalars.