

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2010

EEE PART II: MEng, BEng and ACGI

Corrected Copy

Q1

**ANALOGUE ELECTRONICS 2**

Monday, 14 June 2:00 pm

Time allowed: 2:00 hours

**There are FOUR questions on this paper.**

**Q1 is compulsory.**

**Answer Q1 and any two of questions 2-4.**

**Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).**

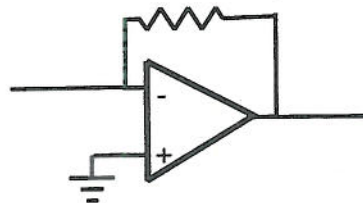
**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      C. Papavassiliou, C. Papavassiliou  
Second Marker(s) :      K.D. Harris, K.D. Harris

## The Questions

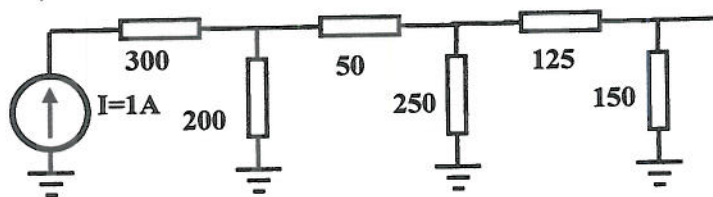
1. (Compulsory)

- a) Calculate the input impedance of the following circuit. The amplifier has a voltage gain of  $G=15$  and is otherwise ideal. The resistor has a value of  $32\text{ k}\Omega$ . The op-amp gain does not depend on frequency.



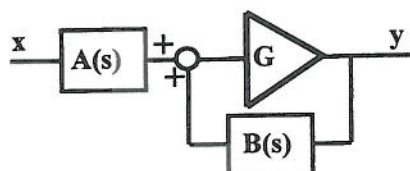
[4]

- b) Draw a small signal equivalent circuit for a MOSFET which has the body terminal connected to the source. How does this model differ from the small signal model of a bipolar transistor?
- c) Explain what is a dominant pole amplifier. Write an expression for the open loop gain as a function of frequency of a dominant pole amplifier.
- d) Calculate the Thevenin equivalent of the following network: (the components are resistors)



[4]

- e) Calculate the transfer function of the filter in the figure below, if both  $A(s)$  and  $B(s)$  are first order low pass filters with poles at  $\omega_A$  and  $\omega_B$  respectively. The amplifier gain  $G$  is constant with frequency. What type of filter is this? What is the maximum value of  $G$  allowed, and why?



[4]

- f) What kind of negative feedback connection can be used to decrease the input impedance and increase the output impedance of an amplifier by a factor of  $X$ ? What is the character of this amplifier (voltage, current, transimpedance or transconductance)? Will the gain increase or decrease with this feedback connection? By what factor will the gain change?

[4]

- g) Write an expression for the transfer function of a second order bandpass filter of peak gain  $G = 10$ , quality factor  $Q=2$  and natural frequency  $f_0 = 1 \text{ kHz}$

[4]

- h) A filter has the following transfer function. State the function and calculate the centre frequency, maximum gain and quality factor of this filter.

$$G(f) = \frac{3 - 3 \cdot 10^{-8} f^2}{10^{-8} f^2 - 2 \cdot 10^{-5} jf - 1}$$

[4]

- i) An op-amp is specified to have a gain-bandwidth product of 1 MHz. This op-amp is used to construct an inverting amplifier. Calculate the maximum gain that can be achieved at a frequency of 100 kHz.

[4]

- j) Which of the three single transistor amplifiers is most suitable for obtaining a large voltage gain at high frequencies? Explain your answer.

[4]

2. For the filter in figure 2.1:
- Assume that the op-amp is ideal. Calculate the transfer function of this filter. [10]
  - Identify the type of the filter. Sketch the magnitude and phase Bode plots for this filter. [5]
  - Assume that the op-amp has a finite gain  $G$ . Calculate the transfer function of this filter. Does the filter still perform the function you identified in part (b)? What has changed? [10]
  - Assume that the op-amp is a dominant pole amplifier with DC gain  $G_0$  and dominant pole at  $\omega_0$ . Calculate the transfer function of the filter. Estimate the maximum value you can give the RC product in terms of  $G_0$  and  $\omega_0$  without significantly losing the functionality you identified in parts (a) and (b). [5]

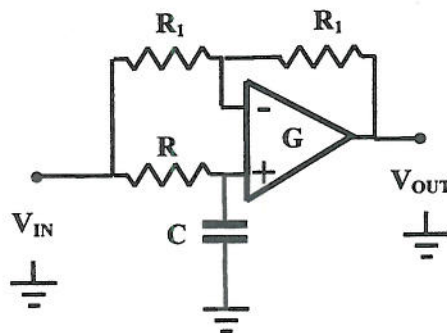


Figure 2.1: Circuit for Question 2.

3. For the filter in figure 3.1, assume that the op-amps are ideal, except in part (e).
- Calculate the transfer function of the circuit in the dashed box, i.e the relation between  $V_{OUT}$  and  $V_1$ . [5]
  - Derive an expression for the input admittance of the circuit in the dashed box. Show that this circuit behaves like a large grounded capacitor. [10]
  - Draw a simplified equivalent circuit for the filter in figure 3.1. Calculate its transfer function. What function does the filter perform? Calculate the pole frequencies and gain of this filter if :  
 $C = 1\text{pF}$  ,  $R = R_2 = R_3 = 10\text{k}\Omega$  ,  $R_1 = R_4 = 10\Omega$  [5]
  - Show that if  $C$  and  $R_2$  are exchanged the circuit in the dashed box behaves like a grounded inductor. What are the limitations of such an active inductor implementation?  
 HINT: in an actual application the op-amp will have a finite gain-bandwidth product. [5]
  - Prove that if  $C$  and  $R_3$  are exchanged the circuit in the dashed box behaves like a grounded inductor. Why is this implementation of an inductor unlikely to work? [5]

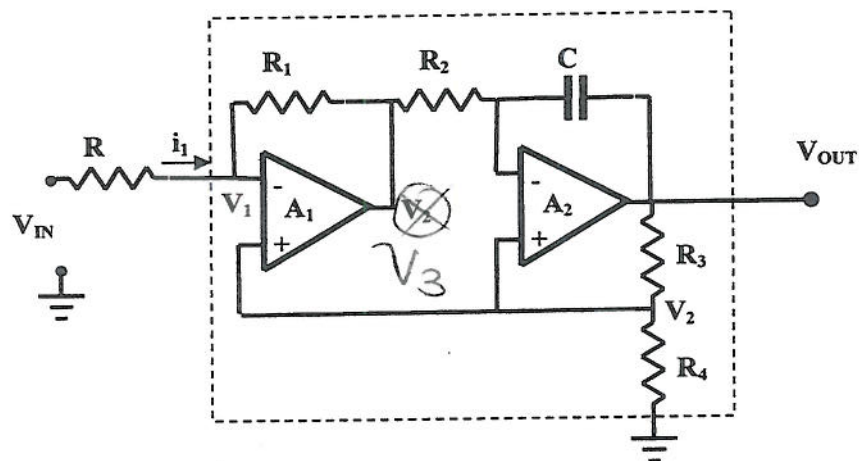


Figure 3.1: Circuit for Question 3.



4. Consider the emitter follower amplifier shown in figure 4.1, together with a signal source and a load. You may assume that the capacitor is infinitely large.

The bipolar transistor Q has  $f_T = 10 \text{ MHz}$ ,  $V_A = 50 \text{ V}$ . Its DC current gain is  $\beta_0 = 100$

The current source has a magnitude  $I_0 = 100 \text{ mA}$  and the power supply is  $V_{CC} = 10 \text{ V}$ .

- Identify the role of each of the dashed boxes in figure 4.1. What is the role of the current source?  
[5]
- Draw a small signal equivalent circuit for the transistor. Calculate values for all elements on the equivalent circuit except  $R_{BB}$ ,  $C_{BC}$  and  $C_{CE}$  which you may assume are negligible.  
[5]
- Write an expression for the current gain of the transistor as a function of frequency.  
[5]
- Using the Miller theorem, or otherwise, derive an expression for the frequency dependence of the input impedance of the amplifier. Assume that  $Z_L$  is real, and  $\beta_0 Z_L \gg R_\pi$ .  
[5]
- Assume that  $Z_L$  is a capacitor. Derive an expression for the input impedance of the emitter follower amplifier. Show that in this case the input impedance has a negative real part. Evaluate the DC limit of the input impedance of the capacitively loaded emitter follower. What is the minimum value the source resistance  $R_S$  can take?  
[5]
- Identify the type of feedback present in this amplifier. Assume the source is resistive and the load a capacitor. Derive an expression for the voltage gain  $G(s) = \frac{V_L}{V_S}$  as a function of frequency. Comment on your result.  
[5]

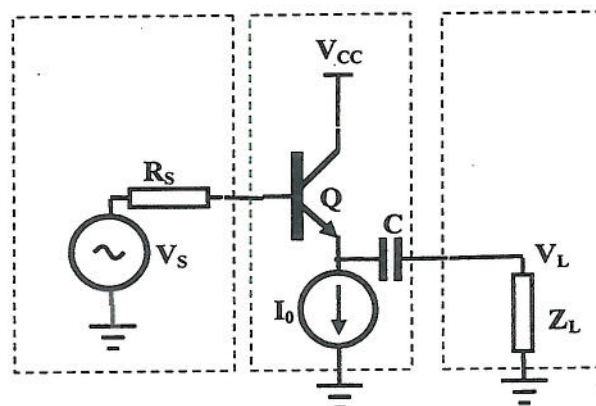


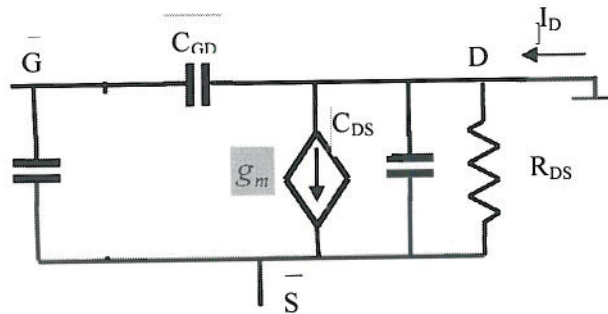
Figure 4.1: Circuit for Question 4.

## The Answers 2010

ANSWER QUESTION 1: (4 marks each) ([B] bookwork, [C] computed example)

a) [C] 2kOhms by application of the Miller theorem.

b) [B] The model is the same as of a BJT without R-pi



c) [B] One pole at a much lower frequency than any other poles – zeroes in the response, or rather, when other characteristic frequencies are at frequencies higher than the unity gain frequency. The gain dependence on frequency is approximately:

$$G(s) = \frac{G_{DC}}{1 + s\tau}$$

d) [C] The 300 Ohm resistor is irrelevant. Step by step:

$$V_T = 200V, R_T = 200\Omega$$

$$V_T = 100V, R_T = 125\Omega$$

$$V_T = 37.5V, R_T = 93.75\Omega$$

$$e) [C] \quad H(s) = \frac{AG}{1 - BG} = \frac{\frac{G}{1 + s\tau_A}}{1 - \frac{G}{1 + s\tau_B}} = \frac{G(1 + s\tau_B)}{(1 + s\tau_B)(1 + s\tau_A) - G(1 + s\tau_A)} =$$

$$= \frac{G(1 + s\tau_B)}{s^2\tau_A\tau_B + s(\tau_A + \tau_B - G\tau_A) + (1 - G)} = \frac{G}{1 - G} \frac{1 + s\tau_B}{s^2 / \omega_0^2 + 2\zeta s / \omega_0 + 1}$$

This is a low pass filter and G cannot be greater than 1, or Q will be negative

f) [B] Shunt-series negative feedback. The gain (current gain!) will reduce by a factor X.

$$g) [C] \text{ let } \omega_0 = 2\pi \text{ kHz then } G(s) = \frac{10}{-\omega^2 / \omega_0^2 + j\omega / 2\omega_0 + 1}$$

h) [C] It is a band stop filter centred at  $10^4$  Hz, peak gain of -3 and  $Q=5$ i) [B] The maximum possible gain is  $10 / \sqrt{2}$  (I will accept 10 as a correct answer)

j) [B] Common base. Because it has unity current gain and positive Miller feedback.

ANSWER QUESTION 2: [New exercise]

- a) If the op-amp is ideal, then

$$V_+ = V_- \Rightarrow (V_{in} + V_{out}) / 2 = \frac{V_{in}}{1 + sRC} \Rightarrow V_{out} = V_{in} \left( \frac{2}{1 + sRC} - 1 \right) \Rightarrow$$

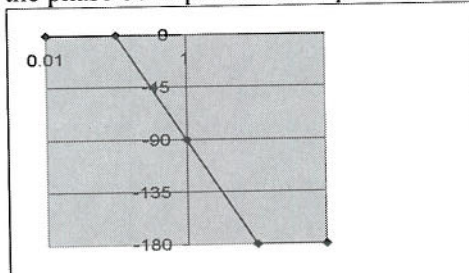
$$\frac{V_{out}}{V_{in}} = \frac{1 - sRC}{1 + sRC}$$

[10]

- b) This is a first order all pass filter

The magnitude Bode plot is trivial (constant with frequency)

the phase bode plot has a slope of 180 degrees per decade



[5]

- c) If the amplifier has a finite gain  $G$  then

$$V_{out} = G(v_+ - v_-) = G \left( \frac{V_{in}}{1 + sRC} - (V_{in} + V_{out}) / 2 \right) \Rightarrow$$

$$(2 + G)V_{out} = GV_{in} \frac{1 - sRC}{1 + sRC} \Rightarrow V_{out} = \frac{G}{2 + G} V_{in} \frac{1 - sRC}{1 + sRC}$$

It performs, therefore, the same function but at a smaller amplitude.

[10]

- d)

$$\frac{V_{out}}{V_{in}} = \frac{G}{2 + G} \frac{1 - sRC}{1 + sRC} = \frac{G_0}{2 + G_0 + 2s\tau_0} \frac{1 - sRC}{1 + sRC} = \frac{G_0}{2 + G_0} \frac{1}{1 + s\tau'} \frac{1 - sRC}{1 + sRC}$$

$$\tau' = \frac{2\tau_0}{2 + G_0}$$

as long as  $\tau' \ll RC$  the function of the filter is not significantly altered.

[5]



ANSWER Question 3. [New exercise]

a) Let  $H = \frac{R_4}{R_3 + R_4}$ , By the golden rule applied on  $A_1$ , the box gain is  $1/H$  !

[5]

b)  $i_{in} = G_1(V_1 - V_2)$ ,  $V_1 = HV_{out}$  but

$$(V_2 - HV_{out})G_2 = (H - 1)V_{out}sC \Rightarrow$$

$$V_2 = V_{out}[(H - 1)sR_2C + H] = V_{in}[(1 - 1/H)sR_2C + 1] \Rightarrow$$

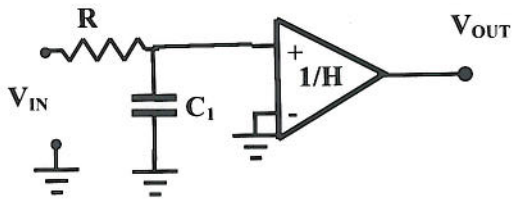
$$i_{in} = G_1(V_1 - V_2) = G_1\left(1 - [(1 - 1/H)sR_2C + 1]\right) \Rightarrow$$

$$Y_{in} = i_{in}/V_1 = G_1((1 - H)/H)sR_2C = sC \frac{R_2}{R_1} \frac{R_3}{R_4}$$

Therefore C can be amplified by the ratio of the resistor products.

[10]

c)



This is an RC low pass filter with

$$C_1 = C \frac{R_2}{R_1} \frac{R_3}{R_4} \text{ followed by a gain } 1/H \text{ (H < 1 since it is a voltage divider!)}$$

The pole is at  $\tau = RC \frac{R_2 R_3}{R_1 R_4}$ . With the numbers given,  $H = 10^{-3}$ ,  $\tau = 10^{-2} \text{ sec}^{-1} \Rightarrow f_p = 6 \text{ Hz}$  (!)

d) The dashed circuit is really a GIC. Therefore  $Y = Y_F \frac{R_2}{R_1} \frac{R_3}{R_4}$ , an inductor can be emulated by making  $Y_F$  a resistor and  $R_2$  or  $R_3$  a capacitor.

If a capacitor is used in the place of  $R_2$  there is an embedded "ideal" differentiator in the circuit which will ring if the op-amp has a finite gain bandwidth product. (note that the students are not required to know what a GIC is, only to observe they can give this admittance inductive character by properly placing the capacitor)

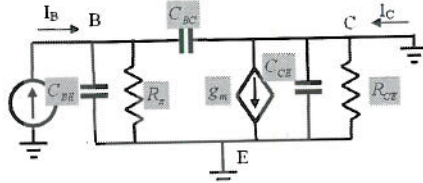
[5]

e) If a capacitor is used in the place of  $R_3$  the positive feedback path gain around  $A_2$  will be increasing with frequency and will cause  $A_2$  to oscillate.

[5]

ANSWER Question 4. Each part is 5 marks. [a,b and c are bookwork. The rest is new computed example]

- a) From left to right source, amplifier, load. The current source establishes the operating point.  
b) The model is the usual  $\pi$  model,



With

$$g_m = \frac{100mA}{25mV} = 4S, R_{CE} = \frac{50V}{100mA} = 0.5k\Omega, R_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25\Omega,$$

$$\frac{g_m}{2\pi C_{BE}} = f_T = 10MHz \Rightarrow C_{BE} = \frac{g_m}{2\pi f_T} = 63.7nF$$

- c) The current gain of the transistor is:

$$h_{fe}(s) = \frac{\beta_0}{1 + j\beta_0 f / f_T}$$

- d) By application of the Miller theorem, the input impedance is

$$Z_{in} = R_\pi // C_{BE} + (h_{fe} + 1)Z_L = \frac{R_\pi}{1 + sC_{BE}R_\pi} + \left( \frac{\beta_0}{1 + s\beta_0 / \omega_T} + 1 \right) Z_L =$$

$$= \frac{R_\pi}{1 + s\beta_0 / \omega_T} + \left( \frac{\beta_0 + 1 + s\beta_0 / \omega_T}{1 + s\beta_0 / \omega_T} \right) Z_L \approx Z_L (\beta_0 + 1) \frac{1 + s\beta_0 / (\beta_0 + 1) \omega_T}{1 + s\beta_0 / \omega_T} \approx \frac{Z_L (\beta_0 + 1)}{1 + s\beta_0 / \omega_T}$$

- e) if  $Z_L = 1 / sC$  then

$$Z_{in} \approx \frac{1}{sC} \frac{\beta_0 + 1}{1 + s\beta_0 / \omega_T} = \frac{\beta_0 + 1}{j\omega C - \omega^2 \beta_0 C / \omega_T} \Rightarrow \text{Re } Z_{in} = \frac{-\omega^2 \beta_0 / \omega_T C (\beta_0 + 1)}{\omega^2 C^2 + \omega^4 \beta_0^2 C^2 / \omega_T^2}$$

$$\lim_{\omega \rightarrow 0} \text{Re } Z_{in} = \frac{-\beta_0}{C\omega_T (\beta_0 + 1)} \approx \frac{-1}{C\omega_T} \left( = \frac{-C_{BE}}{g_m C} \right)$$

The source resistance must be bigger than this value.

- f) This is series-series feedback. The voltage gain of the amplifier is

$$G = \frac{g_m Z_L}{1 + g_m Z_L}. \text{ The source divider is } G_S = \frac{Z_{in}}{Z_S + Z_{in}} \text{ so that the overall gain is}$$

$$G = \frac{g_m Z_L}{1 + g_m Z_L} \frac{Z_{in}}{R_S + Z_{in}} = \frac{g_m}{sC + g_m} \frac{\beta_0 + 1}{R_S (sC + s^2 \beta_0 C / \omega_T) + \beta_0 + 1} =$$

$$= \frac{g_m}{sC + g_m} \frac{1}{s^2 R_S \beta_0 C / \omega_T (\beta_0 + 1) + R_S sC / (\beta_0 + 1) + 1}$$

This is an LPF with  $\omega_0 = \sqrt{\omega_T (\beta_0 + 1) / \beta C R_S} \approx \sqrt{\omega_T / C R_S}$  and  $Q = (\beta_0 + 1) / \sqrt{\omega_T C R_S}$

For the emitter follower to be overdamped, the condition on  $R_S$  is further restricted to

$$R_S > (\beta + 1)^2 / \omega_T C$$