UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1998

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
BSc Honours Degree in Mathematics and Computer Science Part II
MSci Honours Degree in Mathematics and Computer Science Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science Associateship of the City and Guilds of London Institute

PAPER 2.8 / MC2.8

ALGORITHMS, COMPLEXITY AND COMPUTABILITY

Thursday, May 7th 1998, 4.00 - 5.30

Answer THREE questions

For admin. only: paper contains 4 questions

1a A Turing machine M is defined as a 6-tuple:

$$M = (Q, \Sigma, I, q_0, \delta, F)$$

Explain the components $Q, \Sigma, I, q_0, \delta, F$.

The notation 1^n, below, denotes a string of n 1's. You may assume that the position of square 0 of each Turing machine tape is implicitly marked.

b Design a 1-tape Turing machine M with input alphabet {1} such that if the initial contents of the tape are 1^n (for some $n \geq 0$) then M halts and succeeds if and only if n is divisible by 2.

Use a flow chart diagram, and explain your notation.

c Design a 2-tape Turing machine M_1 with input alphabet {1}, such that if the initial contents of tape 1 are 1^n, then M_1 halts and succeeds if and only if n is a power of 2 i.e., if $n = 2^r$ for some integer $r \geq 0$, and halts and fails otherwise.

Use a flow chart diagram.

d By modifying M_1 or otherwise, describe *briefly* how you would design a 3-tape Turing machine M* with input alphabet {1}, such that if $n = 2^r$ for some integer $r \geq 0$ then M* halts and succeeds with output 1^r, and halts and fails otherwise.

The four parts carry, respectively, 20%, 20%, 40% and 20% of the marks.

2a Explain what the halting problem is. What does it mean to say that the halting problem is unsolvable ?

b Explain what the technique of reduction is, and how it can be used to show that a problem is unsolvable.

c Let C be the standard typewriter alphabet and C* be the set of words over C. The symbol $*$ is used as a delimiter. Let the partial function $f : C^* \longrightarrow \{0,1\}$ be given by

$$f(code(S) * w) = \begin{cases} 1, \text{ if S halts and succeeds on input w} \\ \quad \text{and its output contains the symbol 0} \\ 0, \text{ otherwise} \end{cases}$$

for any standard Turing machine S and word w of C.

Prove, either directly or by reduction of the halting problem, that there is no Turing machine M such that $f_M = f$.

The three parts carry, respectively, 30%, 30% and 40% of the marks.

3a Define a *minimal spanning tree* for a connected weighted graph G.

Explain what is meant by the *separation property* of a spanning tree, and how this is used in Prim's Algorithm.

b Use Prim's Algorithm to generate a minimal spanning tree for the graph in Fig. 1, starting at K. Show all your working.

Is the spanning tree unique? Explain your answer.

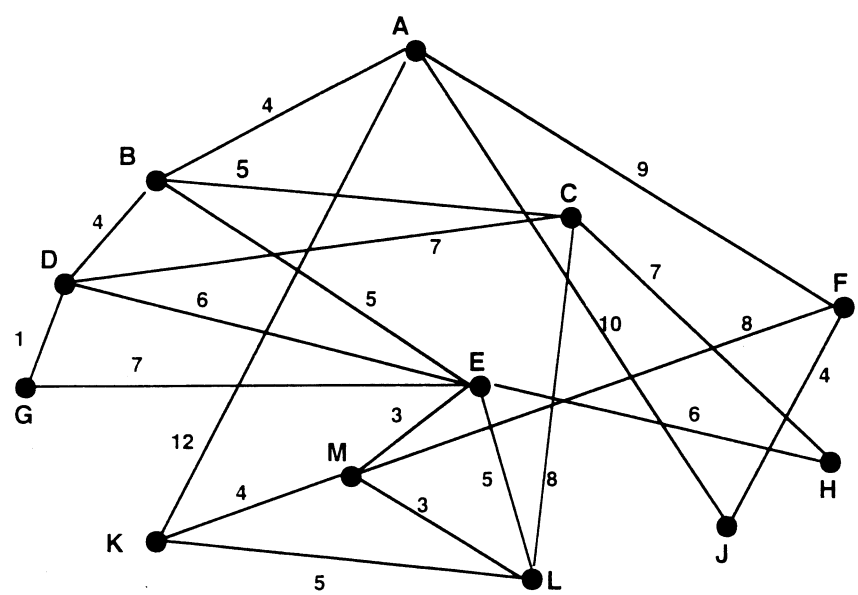

Figure 1.

c Dijkstra's Shortest-path Algorithm is a variation on Prim's Algorithm as follows:

"For the shortest path in a graph G from node x to node y: build a spanning tree starting at node x; nodes in the fringe have weight equal to their minimum path through the partially completed spanning tree from the start node through the tree and the shortest path length has highest priority. When node y is added to the tree, the path from x to y is the shortest path from x to y in G".

Use Dijkstra's Algorithm to find the shortest path from A to L. Show all your working.

The three parts carry, respectively, 30%, 40% and 30% of the marks.

Turn over ...

4a Let A and B be arbitrary *yes-no* problems. Define what it means to say that A *reduces to* B *in p-time* (in symbols, $A \leq B$).

b Let $\leq$ be the relation of part a. Prove that if $A \leq B$ and $B \in NP$ then $A \in NP$.

c Let $\leq$ be the relation of part a.

i) Define the class NPC of NP-complete *yes-no* problems.

ii) Let HCP, PSAT be the Hamiltonian circuit and the propositional satisfaction problems, respectively.

Let A be a *yes-no* problem, and suppose that $HCP \leq A$ and $A \leq PSAT$. Prove directly from your definition in part c(i) that A is NP-complete. [You can assume that HCP and PSAT are NP-complete.]

The three parts carry, respectively, 20%, 50% and 30% of the marks.

End of paper