EE1-10A

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

EEE PART I: MEng, BEng and ACGI

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MATHEMATICS 1A (E-STREAM AND I-STREAM)

Thursday, 24 May 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Answer ALL questions. All questions carry equal marks (25% each).

NO CALCULATORS ALLOWED.
Mathematical Formulae sheet provided

Any special instructions for invigilators and information for candidates are on page 1.

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Second Marker(s) : D.M. Brookes, D.M. Brookes

EE1-10A MATHEMATICS I

1. a) Express in the form $x+iy$: (i) $\dfrac{3i+2}{2i-3}$, (ii) $\left(\dfrac{\sqrt{3}-i}{2}\right)^{2018}$. [4]

b) Obtain all complex z such that $\sin(z^2)$ is purely imaginary. [5]

c) (i) Show that if complex z satisfies $\cot z = k$, where k is real, then

$$e^{2iz} = \frac{k^2-1+2ki}{k^2+1}.$$ [4]

(ii) Hence, or otherwise, find all solutions of $\cot z = -1$. [3]

d) Obtain the limits [5]

$$(i)\ \lim_{x\to\pi/2}\left(x-\frac{\pi}{2}\right)\sin(\sec x),\qquad (ii)\ \lim_{x\to\pi/4}\frac{1-\tan x}{2\sin x-\sqrt{2}}.$$

e) A function is defined as

$$f(x) = \begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & \text{for } x \neq 2; \\ k & \text{for } x = 2, \end{cases}$$

for $x \geq -2.5$. What value of k makes $f(x)$ a continuous function? [4]

2. a) Differentiate to obtain $\dfrac{dy}{dx}$: [6]

$$(i)\ \ y = x^{\ln x},\qquad (ii)\ \ y^2\sqrt{x}-\ln(x+y) = 1.$$

b) Differentiate from first principles to show that the derivative of $\sin x$ is $\cos x$. [You may quote the result for $\lim_{x\to 0}\frac{\sin x}{x}$.] [5]

c) (i) Show that if $y = \left(\cos^{-1}x\right)^2$, then $\sqrt{1-x^2}\dfrac{dy}{dx} = -2\cos^{-1}x$. [3]

(ii) Hence, or otherwise, deduce that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 2 = 0$. [3]

[Recall that $\frac{d}{dx}\left(\cos^{-1}x\right) = -\frac{1}{\sqrt{1-x^2}}$.]

d) For the function $f(x) = (x^2-3)e^x$, determine all stationary points and classify these using the second derivative test. Obtain all asymptotes. Sketch the graph of the function. You do not need to find points of inflection, but should indicate on your graph where other information allows you to deduce them. [8]

3. a) Evaluate the following integrals.

i) $$\int \frac{18x+12}{\sqrt{3x^2+4x-7}}\,dx,$$ [3]

ii) $$\int_3^4 \frac{2x+3}{x^2-x-2}\,dx,$$ [3]

iii) $$\int \frac{\cosh^{-1}x}{(x^2-1)^{1/2}}\,dx,$$ [3]

iv) $$\int_{-2}^{2} \sqrt{4-x^2}\,dx,$$ using a substitution. [4]

b) Let

$$I_n = \int_0^\infty x^{2n} e^{-x}\,dx,$$

where n is a positive integer. Find I_0 and show that $I_n = (2n)!I_0$. Hence, or otherwise, obtain I_6 in terms of a factorial. [7]

c) Show that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

Hence, or otherwise, show that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ also diverges. [5]

4. a) Find the radius and interval of convergence of the infinite series [5]

$$\sum_{n=0}^{\infty} (-1)^n (n+1) x^n,$$

b) Obtain the n^{th} derivative of $f(x) = (1+x)^{-2}$, and hence show that the Maclaurin series of $f(x)$ is given by the series in (a). [5]

c) i) Find the real Fourier Series for each of the functions with period 2, defined on $[-1,1]$ as $f(x) = |x|$ and $g(x) = x$. [7]

ii) Choosing an appropriate value for x in the Fourier series for $g(x)$ above, obtain the value of the infinite sum [4]

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$

iii) Using Parseval's theorem on the Fourier series for $f(x)$ above, obtain the value of the infinite sum [4]

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \frac{1}{6^4} + \cdots$$