Imperial College London
MSci EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

HYDRODYNAMICS AND SHOCKS

For Third and Fourth-Year Physics Students
Thursday, 24th May 2012: 14.00 to 16.00

Answer THREE questions.
All questions carry equal marks.

Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that the Examiners attach great importance to legibility, accuracy and clarity of expression.

1.

(i) Draw a flow-net for the potential flow region of a steady fluid flow over a sphere. Illustrate the velocity vectors as well as the equi-potential lines of the scalar velocity potential ψ. Clearly state what assumptions are necessary in order to construct the flow-net.

[4 marks]

(ii) Describe d'Alembert's paradox.

[3 marks]

(iii) Describe Prandtl's solution to d'Alembert's paradox.

[3 marks]

(iv) From the continuity relation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underline{v} \right) = 0$$

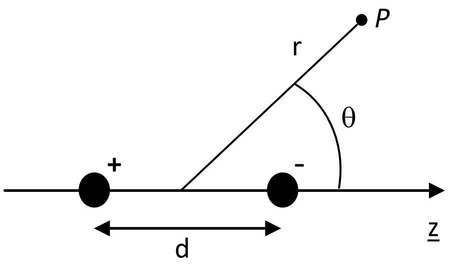
Show that for incompressible, irrotational flow, flow solutions can be determined from Laplace's equation using the scalar velocity potential.

[3 marks]

(v) The velocity potential for flow around a sphere is analogous to an electric dipole in an applied electric field E_0, whose potential at a point *P* has the form

$$V = -E_0 z + const. \frac{\cos\theta}{r^2}$$

Where z = r cos(θ) and *P*, z, r, and θ are defined by the following diagram



Show, by analogy, that if a flow with uniform velocity v_0 in the $\underline{z}$ direction encounters a sphere of radius *a* then the flow velocity around the sphere is given by;

$$v_r = -v_0 z \frac{3a^2}{r^4} \quad \text{and} \quad v_z = v_0 \left(1 + \frac{a^3}{2r^3} \right)$$

Clearly state any assumptions made.

[7 marks]

Total [20 marks]

2.

(i) For a wave in shallow water of average depth *d* use Bernoulli's principle to calculate the phase velocity of the wave, stating any assumptions made.

[5 marks]

(ii) An interface separates a fluid of density ρ_1 and horizontal velocity v_1 which sits above another fluid of density ρ_2 and horizontal velocity v_2. The generalised dispersion relation for fluid waves at this interface is;

$$\omega = k_x \frac{\rho_1 v_1 + \rho_2 v_2}{\rho_1 + \rho_2} \pm \sqrt{\frac{\rho_2 - \rho_1}{\rho_1 + \rho_2} k_h g - k_x^2 \frac{\rho_1 \rho_2 (v_1 - v_2)^2}{(\rho_1 + \rho_2)^2}}$$

where k_x and k_h are the wave numbers in the horizontal and vertical directions.

From this expression determine the phase velocity for surface waves on water. Clearly state and justify any approximations made.

[4 marks]

(iii) Under what conditions is a fluid interface unstable to the Rayleigh-Taylor instability ?

If v_1 and v_2 are zero, derive an expression for the growth rate of the Rayleigh-Taylor instability in terms of the Atwood number.

[4 marks]

(iv) Under what conditions is a fluid interface unstable to the Kelvin Helmholtz instability ?

If the densities of the two fluids are equal, show that the growth rate for the Kelvin Helmholtz instability is $k_x/2\ \Delta v$

[3 marks]

(v) Draw the evolution of an interface between a heavy fluid on top of a light fluid, in both the linear and non-linear stages of the Rayleigh-Taylor instability. Use this to illustrate why the non-linear Rayleigh-Taylor instability is often accompanied by the development of the Kelvin-Helmholtz instability.

[4 marks]

Total [20 marks]

3.

(i) The change in specific internal energy E, across a shock is given by

$$E - E_0 = \tfrac{1}{2}\left(P + P_0\right)\left(\frac{1}{\rho_0} - \frac{1}{\rho}\right)$$

Here $E = U / \rho$ where U is the internal energy density
and $\rho = 1 / V$ where V is the specific volume

Use this to derive an expression for the density ratio across the shock in terms of the pressures before the shock (P_0) and after the shock (P).

Use this to estimate the maximum density compression ratio for a strong shock in an ideal monatomic gas, stating any assumptions used.

[6 marks]

(ii) A constant force is applied to the surface of a stationary, low temperature, ideal monatomic gas of density ρ_0 and generates a strong shock moving at velocity U_s into the gas. Show that the pressure of shocked region is

$$P_{shocked} = \frac{2}{\gamma + 1}\rho_0 U_s^{\ 2}$$ stating any assumptions made.

[4 marks]

(iii) If the surface force on this gas is generated by a constant magnetic field of 100 Tesla and ρ_0 = 1kg/m^3 calculate the shock velocity, stating any assumptions made.

[3 marks]

(iv) A strong shock is moving through hydrogen plasma at 10^5 m/s. Assuming the hydrogen plasma is behaving as a perfect gas calculate the post-shock temperature in eV.

[3 marks]

(v) If the initial density of this hydrogen plasma is 1 kg/m^3 calculate whether the kinetic energy flux dissipated by the shock is larger or smaller than the maximum energy flux carried by radiation, stating any assumption made.

[4 marks]

Total [20 marks]

Permeability of free space = $4\pi \times 10^{-7}$ Hm^{-1}
Charge on the electron = 1.6×10^{-19} Coulombs
Stefan Boltzmann constant = 1×10^{9} W m^{-2} eV^{-4}

4.

(i) For a shock propagating into an initially stationary, un-shocked block of material show that the Rankine-Hugoniot jump conditions are:

$$\frac{\rho}{\rho_0} = \frac{U_S}{U_S - u_p} \qquad \text{(conservation of mass)}$$

$$P = \rho_0 U_S u_p \qquad \text{(conservation of momentum)}$$

$$E = \tfrac{1}{2} u_p^2 \qquad \text{(conservation of energy)}$$

Where E is the specific energy (the energy per unit mass), U_S is the shock velocity and u_p is the particle velocity

[5 marks]

(ii) At low pressures, the relationship between U_s and u_p can be written as the Hugoniot;

$$U_S = bu_p + a$$

How does the Hugoniot differ from a thermodynamic process? Using the jump conditions, rewrite the Hugoniot in terms of P and u_p variables and sketch the relationship between P and u_p in a target material.

[4 marks]

(iii) An electromagnetic gun is used to fire a copper flyer at 2000ms^{-1} towards a copper target. When the flyer hits the target, which quantities must be equal on either side of the interface? Sketch the relationship between P and u_p in the flyer, together with the Hugoniot in the target and mark the final state obtained at the interface.

[3 marks]

(iv) Given that copper has an initial density of 8900kgm^{-3} and a and b values of 4000 and 1.5 respectively, estimate the pressure, shock velocity and density in the target when it is hit.

[5 marks]

(v) If the electromagnetic gun is used at higher velocities, reliable equation of state data is difficult to obtain because the impact of the flyer causes the target to melt. As an alternative, the capacitor bank from the electromagnetic gun could be used to drive an isentropic compression experiment. What is meant by an isentropic compression experiment and why might it not melt the target ?

[3 marks]

Total [20 marks]

5.

(i) The Navier Stokes equation at a fixed point in space can be written in the form

$$\rho \frac{\partial \underline{v}}{\partial t} = -\nabla P - \rho g - \nabla \cdot \eta \nabla v - \frac{1}{2}\rho \nabla v^2 - \rho(\nabla \times v) \times v$$

Identify each of the terms and briefly explain their significance.

[5 marks]

(ii) From the Navier Stokes equation show how Bernoulli's principle can be expressed for steady inviscid, irrotational flow, stating any assumptions made.

[3 marks]

(iii) A uniform flow moving in the x direction encounters a flat table top located at y=0 (which is continuous in the Z direction).

If shear stress (above the table) as a function of y is $s = \eta\, \partial v_x/\partial y$

Show that this results in a diffusion equation for velocity.

[4 marks]

(iv) What is meant by Poiseuille flow ? Derive an expression for the flow velocity as a function of radius in a long thin pipe in terms of the viscosity η and the pressure gradient along the pipe.

[5 marks]

(v) If a pressure of 4 atmospheres is applied to one end of a 5cm long, 100 micron diameter pipe filled with a liquid with viscosity of 10^{-3} Pascal seconds, what is the flow rate leaving the pipe in millilitres per second ? Assume the inlet length is very small.

[3 marks]

Total [20 marks]

Atmospheric pressure = $1\text{x}10^5$ Pa

6.

(i) Use diagrams to describe how the Magnus effect can be exploited to cause a football to swerve

[3 marks]

(ii) A Flettner ship or rotorship has a 10m tall rotor of 4m diameter spinning at 120rpm. The effective cross section of the bow is $60m^2$ and the drag coefficient is 0.2. What is maximum speed of the craft through the water if the windspeed is 15m/s ?

[4 marks]

(iii) Sketch the velocity versus radius for a forced vortex, a free vortex and a Rankine vortex. Comment on the vorticity in each case and identity any regions where the behaviour is unphysical.

[4 marks]

(iv) If the maximum velocity in a Rankine vortex in the atmosphere is 20 m/s, estimate the pressure at this point.

[2 marks]

(v) What is a Karman vortex street? Describe how this can drive a damaging resonance.

[3 marks]

(vi) Explain why aerofoils "stall" at low speeds.

[4 marks]

Total [20 marks]

Density of air = 1.2 Kg/m^3
Atmospheric pressure = $1x10^5$ Pa