

1. a) For  $V_C = +5V$  require  $I_E = (10-5)/10k = 0.5mA$

$$I_B = I_E/(1+\beta), \text{ so } I_B = 2.488 \mu A$$

$$\text{But } I_B = (V_C - 0.7)/R_B \text{ assuming } V_{BE} \approx 0.7V$$

$$\Rightarrow R_B = (5-0.7)/2.488 \times 10^{-6} = \underline{\underline{1.73 M\Omega}} \quad [6]$$

b) Circuit is MOSFET current mirror

Both (enh. mode) devices have  $V_D \geq V_G \Rightarrow$  both active

Consider LHS, where  $V_{DS} = V_{GS} = V_D$ :

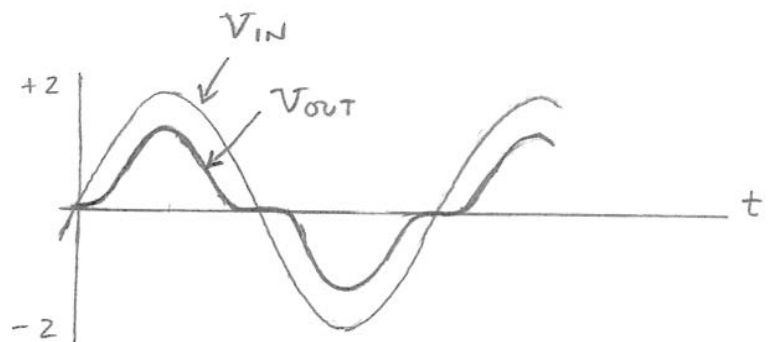
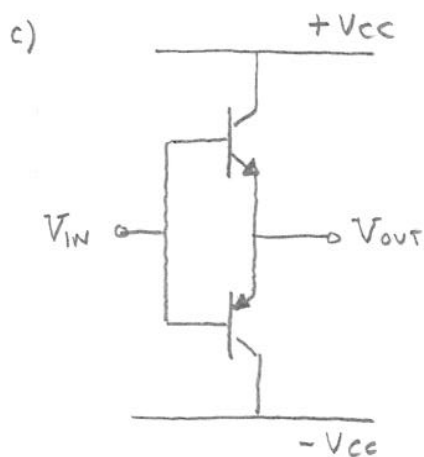
$$I_D = K(V_D - V_t)^2 = \frac{5 - V_D}{15k\Omega} \quad \bar{K} = 0.2mA/V^2, V_t = 1V$$

$$\Rightarrow 3(V_D - 1)^2 = 5 - V_D$$

$$3V_D^2 - 5V_D - 2 = 0$$

$$V_D = \frac{5 \pm \sqrt{25 + 24}}{6} = \frac{5 \pm 7}{6} = -\frac{1}{3} \text{ or } +2V$$

$$\text{MOSFETs matched, so } I = I_D = 0.2m \times (2-1)^2 = \underline{\underline{0.2mA}} \quad [8]$$



Circuit comprises two emitter followers, with NPN acting as follower when  $V_{in} \geq 0.7V$  and PNP acting when  $V_{in} \leq -0.7V$ . Problem is that for  $|V_{in}| \leq 0.7V$  both transistors are off and  $V_{out} \approx 0$

$\Rightarrow$  Cross-over distortion

Description and sketch assume load to ground. [8]

d)  $I_{B2} = I_{E1} = (1+\beta_1)I_{in}$  if Q1 active.

$$I_{out} = I_{C1} + I_{C2} = \beta_1 I_{in} + \beta_2 I_{B2} = [\beta_1 + \beta_2(1+\beta_1)] I_{in} \text{ if both active}$$

$$\text{So } I_{out}/I_{in} = \underline{\underline{\beta_1\beta_2 + \beta_1 + \beta_2}}$$

d) cont'd

Saturation Voltage for Darlington is  $\approx 0.9V$  (since  $V_{E1} = V_{BE2} \approx 0.7$ )

$$\Rightarrow \text{Max. } I_{OUT} \text{ in } 1k\Omega \text{ load is } (12 - 0.9)/1k = \underline{\underline{11.1 \text{ mA}}} \quad [6]$$

e) Depl. mode MOSFET has  $V_{GS} = \phi \Rightarrow$  active if  $V_{DS} \geq |V_t|$   
Assume active initially. Then  $I_D = KV_t^2 = 0.4 \text{ mA}$

But this would imply  $V = 0.4 \text{ mA} \times 10k = 4V$  and hence

$$V_{DS} = 1V < |V_t| \Rightarrow \text{mode must be } \underline{\underline{TRIODE}}$$

For triode mode:

$$I_D = K [2(-V_t)V_{DS} - V_{DS}^2] = (5 - V_{DS})/10k$$

With  $K = 0.1 \text{ mA/V}^2$ ,  $V_t = -2V$  this becomes

$$4V_{DS} - V_{DS}^2 = 5 - V_{DS}$$

$$V_{DS}^2 - 5V_{DS} + 5 = 0$$

$$V_{DS} = \frac{5 \pm \sqrt{25 - 20}}{2} = \frac{5 \pm \sqrt{5}}{2} = \frac{5 \pm 2.24}{2} \Rightarrow 3.62 \text{ or } 1.38V$$

$$V = 5 - V_{DS} = \underline{\underline{3.62V}} \quad [6]$$

f) For steady oscillation we require a solution to the characteristic equation with  $s = j\omega$ , substituting this form of  $s$  gives:

$$-j(1+K)\omega^3 R^3 C^3 - 6\omega^2 R^2 C^2 + j5\omega RC + 1 = 0$$

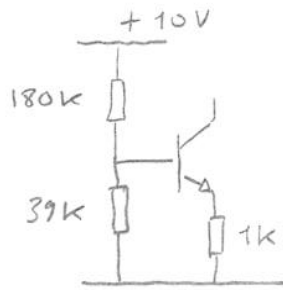
$$\text{Re}\{LHS\} = 0 \Rightarrow \omega = \frac{1}{\sqrt{6}RC} \quad (\text{oscillation frequency})$$

$$\text{Im}\{LHS\} = 0 \Rightarrow (1+K)\omega^2 R^2 C^2 - 5 = 0$$

and with  $\omega^2 R^2 C^2 = 1/6$  this becomes  $1+K - 30 = 0$

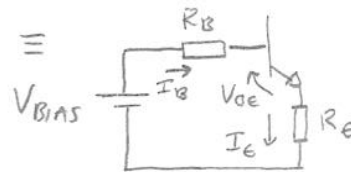
$$\underline{\underline{K = 29}} \quad [6]$$

2. a) Bias cct :



$$V_{BIAS} = \frac{39}{219} \times 10 = 1.78 \text{ V}$$

$$R_B = 39 // 180 = 32.05 \text{ k}$$



$$KVL: I_E R_E + V_{BE} + I_B R_B = V_{BIAS}$$

$$\Rightarrow I_E = \frac{V_{BIAS} - V_{BE}}{R_E + R_B/(1+\beta)}$$

$$= \frac{1.78 - 0.7}{1 + 32.05/201}$$

$$= 0.931 \text{ mA}$$

$$I_C = \alpha I_E = \frac{200}{201} \times 0.931 = \underline{0.926 \text{ mA}}$$

$$V_{out} = 10 - 4.7 \times 0.926 = \underline{5.65 \text{ V}}$$

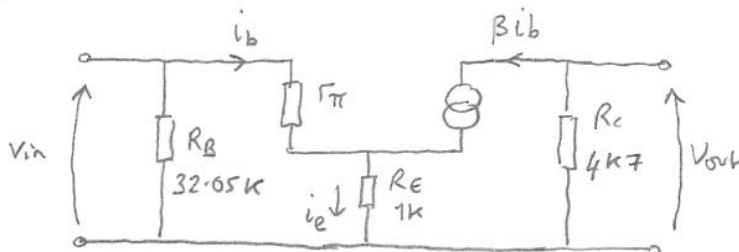
With  $\beta = 150$ ,  $I_E = (1.78 - 0.7) / [1 + 32.05/151] = 0.891 \text{ mA}$

With  $\beta = 250$ ,  $I_E = (1.78 - 0.7) / [1 + 32.05/251] = 0.958 \text{ mA}$

Corresponding  $I_C$  values are 0.885 mA and 0.954 mA

$\beta$  affects  $I_E$  only through  $R_B/(1+\beta)$  term which is small cf  $R_E \Rightarrow$  immunity to  $\beta$  variations [10]

b) SSEC :



$$r_{\pi} = \beta V_T / I_C$$

$$= 5.40 \text{ k}\Omega$$

$$KVL: v_{in} = i_b r_{\pi} + i_e R_E = i_b [r_{\pi} + (1+\beta) R_E] \quad \text{--- (1)}$$

$$\text{Also } v_{out} = -\beta i_b R_C \quad \text{--- (2)}$$

$$\text{(2)/(1)} \Rightarrow v_{out}/v_{in} = \frac{-\beta R_C}{r_{\pi} + (1+\beta) R_E} = \frac{-\alpha R_C}{r_{\pi}/(1+\beta) + R_E} = \frac{-\alpha R_C}{r_e + R_E}$$

Putting  $r_e = 5400/201$ ,  $R_E = 1\text{k}$ ,  $R_C = 4\text{k7}$ ,  $\alpha = \frac{200}{201} \Rightarrow \frac{v_{out}}{v_{in}} = \underline{-4.55}$

$$R_i = R_B // [v_{in}/i_b] = R_B // [r_{\pi} + (1+\beta) R_E] = 32.05\text{k} // 206.4\text{k} = \underline{27.74 \text{ k}\Omega}$$

$$R_o = \underline{4\text{k7}} \text{ by inspection}$$

[14]

c) Gain with  $R_E$  bypassed is obtained by putting  $R_E = 0$  in

given equation,  $A_v \rightarrow -\alpha R_C / r_e = -\frac{\beta R_C}{r_{\pi}} = \frac{-200 \times 4.7}{5.4} = -174$

So ratio =  $\frac{174}{4.55} = \underline{38}$

[6]

3. a) Both MOSFETs carry the same drain current  $I_D$ , and  $V_{G1} = V_{D1} = V_{OUT}$  for lower device

$$\Rightarrow I_D = K_2 (V_G - V_{DD} - V_{t2})^2 = K_1 (V_{OUT} - V_{t1})^2$$

$$\sqrt{\quad} \rightarrow V_{OUT} - V_{t1} = \pm \sqrt{\frac{K_2}{K_1}} (V_G - V_{DD} - V_{t2})$$

Need to take -ve sign for both to be above threshold

$$\Rightarrow V_{OUT} = V_{t1} + \sqrt{K_2/K_1} (V_{DD} + V_{t2} - V_G)$$

as req.

Putting  $V_{t1} = 1V$ ,  $\sqrt{\frac{K_2}{K_1}} = 0.5$ ,  $V_{DD} = 10V$ ,  $V_{t2} = -1V$ ,  $V_G = 4V$

$$\Rightarrow \underline{\underline{V_{OUT} = 3.5V}}$$

Check modes:

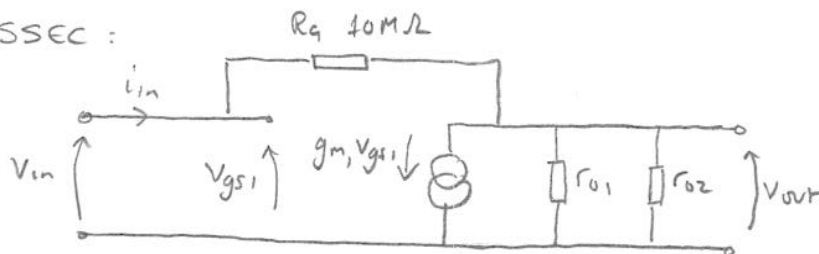
Upper FET req.  $V_{DS} \leq V_{GS} - V_t$   $V_{DS} = 3.5 - 10 = -6.5$   $V_{GS} - V_t = 4 - 10 + 1 = -5$  OK

Lower FET req.  $V_{DS} \geq V_{GS} - V_t$   $V_{DS} = 3.5$   $V_{GS} - V_t = 3.5 - 1 = 2.5$  OK

$$I_D = K_1 (V_{OUT} - V_{t1})^2 = 1.25 \text{ mA}$$

[12]

- b) SSEC:



$$\begin{aligned} r_{01} &= r_{02} = V_A / I_D \\ &= 100V / 1.25 \text{ mA} \\ &= 80 \text{ k}\Omega \end{aligned}$$

$$g_{m1} = 2 \sqrt{K_1 I_D} = 2 \sqrt{0.2 \text{ m} \times 1.25 \text{ m}} = 1 \text{ mA/V}$$

KVL @ o/p  $\Rightarrow$

$$g_{m1} V_{in} + \frac{V_{out}}{r_{01}} + \frac{V_{out}}{r_{02}} + \frac{V_{out} - V_{in}}{R_G} = 0$$

$$\Rightarrow A_v = - \left( g_m - \frac{1}{R_G} \right) \cdot (r_{01} \parallel r_{02} \parallel R_G) = - \underline{\underline{39.8}}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{(v_{in} - v_{out})/R_G} = \frac{R_G}{1 - A_v} = \underline{\underline{245 \text{ k}\Omega}}$$

[14]

- c) For the same bias conditions in lower MOSFET, would require a passive resistor of  $(10 - 3.5)/1.2 \text{ m} = 5.4 \text{ k}\Omega$  as load. Since this is  $\ll r_{02}$ , gain would be smaller.

[4]

- 4 a) When  $V_{in1} = V_{in2} = 0$ , common emitter voltage is  $V_E \approx -0.7V$   
 Tail current in this case is  $I = (10 - 0.7)/20K = \underline{0.465 mA}$   
 Qs are matched and no differential i/p so

$$I_{C1} = I_{C2} = \frac{\alpha I}{2} = \frac{200}{201} \times \frac{0.465}{2} = 0.231 mA = I_C$$

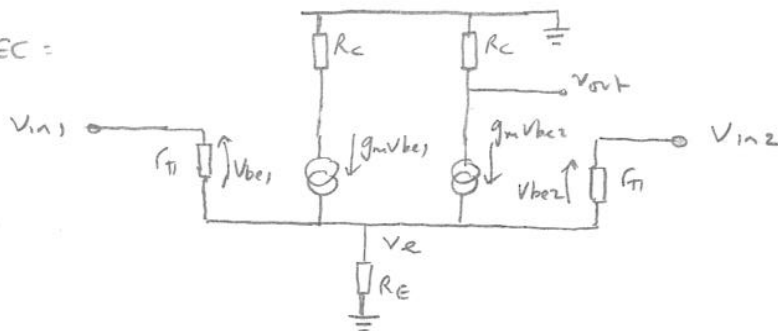
$$V_{out} = 10 - 0.231 \times 20 = \underline{5.37 V}$$

When  $V_{in1} = V_{in2} = -2V$ ,  $V_E \approx -2.7V$

$$\text{In this case } I = (10 - 2.7)/20K = \underline{0.365 mA}$$

$$\Rightarrow I_{C1} = I_{C2} = 0.182 mA \quad \text{and} \quad V_{out} = \underline{6.37 V} \quad [10]$$

b) SSEC =



In case of purely differential i/p, where  $V_{in1} = -V_{in2} = V_d/2$ , we know  $V_E = 0$  from symmetry

$$\Rightarrow V_{be2} = -V_d/2 \quad \text{and} \quad V_{out} = -R_C g_m V_{be2} = R_C g_m \frac{V_d}{2}$$

$$\text{Differential gain is then } A_d = \frac{V_{out}}{V_d} = \frac{R_C g_m}{2} = \frac{R_C I_C}{2 V_T}$$

If common mode voltage  $V_{cm} = 0$ ,  $I_C = 0.231 mA$

$$\Rightarrow \underline{A_d = 92.4}$$

Yes,  $A_d$  will vary with  $V_{cm}$  because it is proportional to  $I_C$  and, as shown in part a),  $I_C$  varies with  $V_{cm}$ . [12]

c)  $A_{cm}$  can be derived by splitting SSEC into two C-E half-ccts

$$\text{This gives } A_{cm} = -\frac{\alpha R_C}{r_e + 2R_E} \approx -\frac{\alpha R_C}{2R_E}$$

In this case we have  $R_C = R_E = 20K$ ,  $\alpha = \frac{200}{201}$  so  $A_{cm} \approx -0.5$   
 (exact formula gives  $-0.496$ )

$A_{cm}$  could be reduced by replacing  $R_E$  with a current mirror; this would all the same bias condition to be achieved while having a much higher small-signal resistance in the tail. [8]