E4.55 2014 Answers MEMS & Nanotechnology

Question 1

(a) (i). The electron wave-vectors for region 1 and region 2 are as follows:

$$k_1 = \sqrt{\frac{2m\left(E+V_0\right)}{\hbar^2}}$$

[2]

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

[2]

[Total Marks: 4]

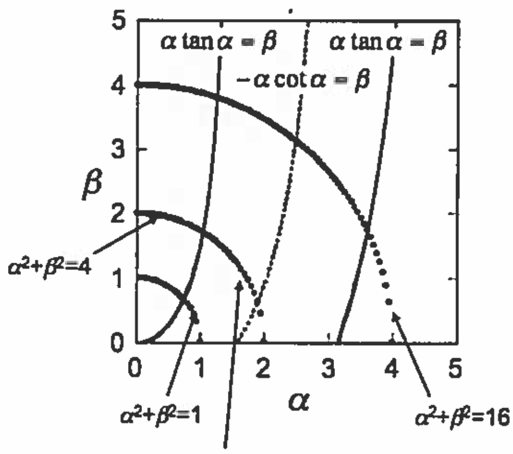
(ii) The equations $\alpha \tan\alpha = \beta$ and $-\alpha \cot\alpha = \beta$ cannot be solved simultaneously, and lead to two separate classes of solutions. A graphical solution is however possible, by noting that:

$$\alpha^2 + \beta^2 = \frac{a^2(k_1^2 + k_2^2)}{4} = \frac{mV_0 a^2}{2\hbar^2}$$

[2]

This is the equation of a circle, and the intersection of this with $\alpha \tan\alpha = \beta$ and $-\alpha \cot\alpha = \beta$ gives us solutions to α and β and hence k_1 and k_2 (see figure below, where different values of $\alpha^2 + \beta^2$ are shown).

[2]



Radius determined by V_0 and a

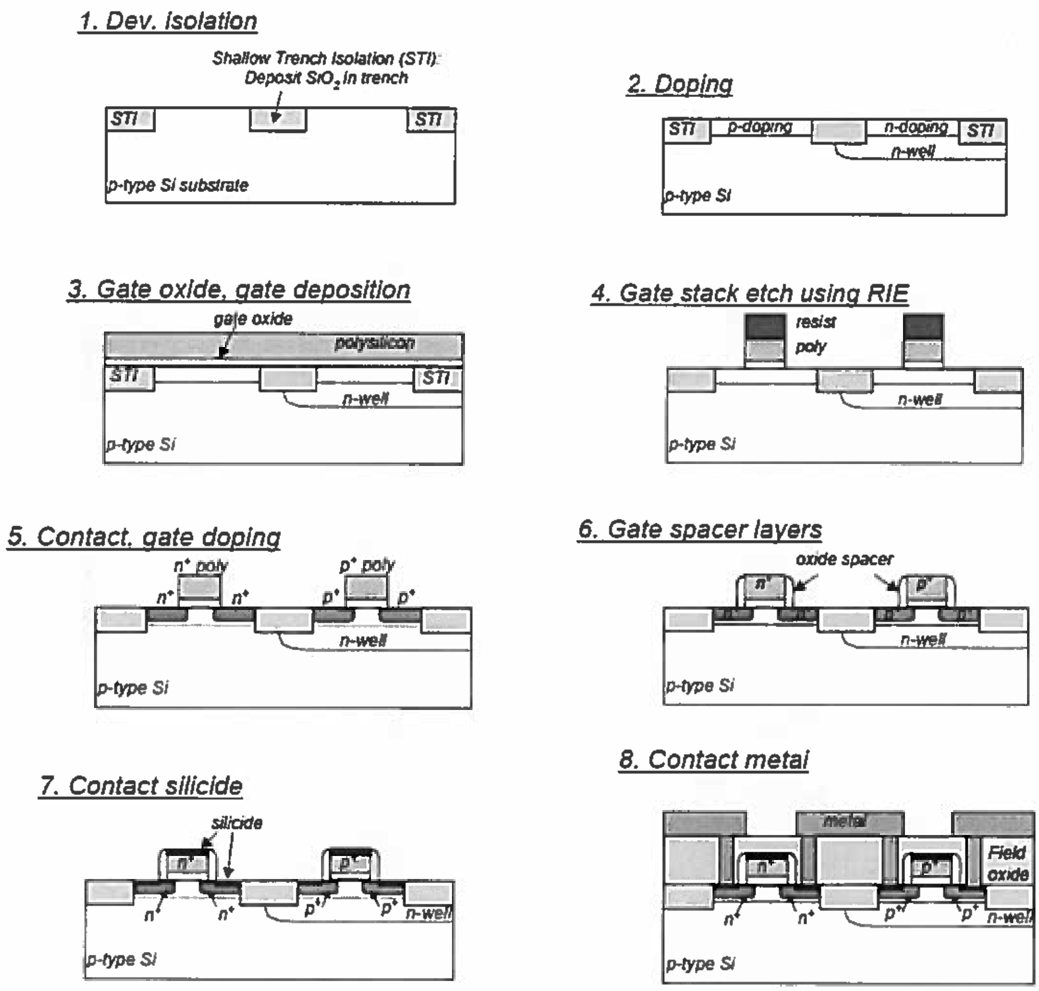
If the intersection of the circle with the x-axis, i.e. for $\beta = 0$, is such that it is less than $-\alpha \cot\alpha = 0$, i.e. $\alpha = \pi/2 = 1.57$, then only one solution exists. If $\alpha < \pi = 3.14$, then 2 solutions can exist and for $\alpha = \pi$, three solutions exist.

Comparison with the condition in the question, $\frac{mV_0a^2}{2\hbar^2} = \pi^2 \Rightarrow \alpha = \pi$ and three solutions exist.

[2]

[Total Marks: 6]

(b) The fabrication process flow for a CMOS inverter is shown diagrammatically below. The required process steps, and the changes to the structure, are shown for each stage.



[1]

[1]

[2]

[1]

[Total Marks: 5]

(c) Charge neutrality over the entire nanowire ⇒

Charge trapped on the surface = charge lost from the interior of the nanowire. If the un-depleted Si core of the nanowire has radius '*r*':

[1]

$\Rightarrow e.N_S.2\pi RL = eN_D.\pi(R^2 - r^2).L$ [1]

$\Rightarrow N_S.2R = N_D (R^2 - r^2)$

$\Rightarrow (N_S/N_D)2R = R^2 - r^2$

$$\Rightarrow r = \sqrt{R^2 - 2R\frac{N_S}{N_D}} = 22.9 \text{ nm} \qquad [2]$$

This give the depletion width $W = R - r = 25 - 22.9 = 2.1$ nm [1]

[Total Marks: 5]

d) The residual stress σ will give rise to an axial load upon release of $P = -\sigma wh$, where w and h are the beam width and height respectively. If this exceeds the critical load then buckling will occur. The critical length of beam beyond which buckling will occur is obtained by setting $P = Pc$, which leads to:

$$-\sigma wh = P_c = \frac{4\pi^2 EI}{L^2} = \frac{\pi^2 Ewh^3}{3L^2} \quad \Rightarrow \quad L = \pi h\sqrt{\frac{-E}{3\sigma}}$$

where we have used the form of I appropriate for a wide beam that will buckle upwards or downwards. With $h = 1.5$ μm, and $E = 160$ GPa, the two stress values given correspond to critical lengths of 243 μm and 109 μm. A reasonable range of lengths would therefore be from 100 to 250 μm.

[5]

e) Advantages of electrostatic and electrothermal actuators: simple construction, down to a single mechanical layer (both); low power consumption (electrostatic); robustness (thermal).

Other types: magnetic (monolithic/hybrid) and piezoelectric (ditto). These might be preferred in applications requiring large force and/or deflection.

[5]

f) The minimum spot size in electron beam lithography is around 10 nm. The factors determining the minimum spot size are diffraction, lens aberrations and the finite source size. Typically it is the interplay between the diffraction limit (which becomes smaller with increasing convergence angle) and lens aberrations (which become worse with increasing convergence angle) that defines the minimum achievable spot. The finite source size may also come into play but is not normally limiting.

The minimum achievable resist feature is larger than the minimum spot size because of scattering in the resist and substrate.

[5]

g) Using the method of sections, the bending moment is given by: $M(x) = p(L-x)\cdot\frac{(L-x)}{2}$, where x is distance along the beam, and L is the beam length. The bending equation is therefore: $\frac{d^2v}{dx^2} = \frac{M}{EI} = p\frac{(L-x)^2}{2EI}$, where E is Youngs modulus and I is the second moment of area. Integrating twice, and applying the boundary conditions $v' = 0$, $v = 0$ at $x = 0$, gives:

$$v = \frac{p}{6EI}\left\{L^3 x + (L-x)^4/4 - L^4/4\right\}$$

Putting $x = L$, the end deflection is obtained as $v_L = \frac{pL^4}{8EI} = \frac{3pL^4}{2Ewh^3}$. The last result assumes the beam is rectangular with cross-section $w \times h$.

This kind of loading arises in a cantilever-based accelerometer.

[5]

Question 2

(a) (i) General 3-D time-dependent form, for $\Psi(r, t)$

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\Psi + V(r,t)\Psi = i\hbar\frac{\partial\Psi}{\partial t}}$$

When V is independent of t, using separation of variables:

Substituting: $\Psi(r,t) = \psi(r)\varphi(t)$ [2]

$$\Rightarrow -\frac{\hbar^2}{2m}\varphi\nabla^2\psi + V(r)\psi\varphi = i\hbar\psi\frac{\partial\varphi}{\partial t}$$

$$\Rightarrow \underbrace{-\frac{\hbar^2}{2m}\frac{1}{\psi}\nabla^2\psi + V(r)}_{\text{Function of } r \text{ alone}} = \underbrace{i\hbar\frac{1}{\varphi}\frac{\partial\varphi}{\partial t}}_{\text{Function of } t \text{ alone}}$$

[4]

[Total marks: 6]

(ii)

Non-trivial solution $\Rightarrow$ both sides equal to non-zero constant E

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{1}{\psi}\nabla^2\psi + V(r) = E = i\hbar\frac{1}{\varphi}\frac{\partial\varphi}{\partial t}$$ [2]

For the <u>time-dependent</u> part:

$$\frac{d\varphi}{dt} + \frac{iE\varphi}{\hbar} = 0 \quad \Rightarrow \quad \varphi = \exp\left(-\frac{iEt}{\hbar}\right)$$ [2]

For the <u>space-dependent</u> part, in <u>1-D</u>:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (V - E)\psi = 0$$

With <u>$E > V$</u>, solutions are propagating waves of the form:

$$\boxed{\psi = A\exp(ikx) + B\exp(-ikx)}$$ [2]

Substituting into the 1-D, space-dependent part:

$$\Rightarrow -\frac{\hbar^2}{2m}(-k^2)\psi + (V-E)\psi = 0$$

$$\boxed{\Rightarrow k = \sqrt{\frac{2m}{\hbar^2}(E-V)}}$$

This gives the complete solution, of the form:

$$\boxed{\Psi = \psi\varphi = \underbrace{A\exp i(kx - \frac{Et}{\hbar})}_{\text{Forward wave}} + \underbrace{B\exp i(-kx - \frac{Et}{\hbar})}_{\text{Backward wave}}}$$ [2]

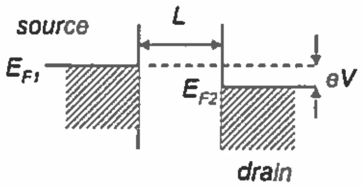
With $\underline{E < V}$, solutions are evanescent waves (exponential decay):

$$\boxed{\psi = C\exp(\alpha x) + D\exp(-\alpha x)} \quad \text{with} \quad \boxed{\alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}}$$

[2]

[Total marks: 10]

(b) An ideal, one-dimension conductor of length L, placed between metal source and drain regions, is shown in the figure below:



In 1-D, in k space, 'volume' occupied by one state in k space = $2\pi/L$

Therefore, the 1-D density of states (i.e. no. of states per unit length L), between k and dk, including a factor of 2 for electron spin is:

$$n(k)dk = 2\left(\frac{1}{L}\right)\left(\frac{L}{2\pi}\right)dk$$ [4]

Here, the factor dk corresponds to the 'volume' increase in k-space, between k and dk, and $1/L$ is the 'volume'.

Furthermore, electron velocity: $v = \frac{p}{m} = \frac{\hbar k}{m}$ [2]

Therefore, current across conductor, assuming $T = 0$ K:

$$I = e\int_0^{\infty} v(k)n(k)f(k)\,dk - e\int_0^{\infty} v(k')n(k')f(k')\,dk'$$ [2]

$$\Rightarrow I = e\int_0^{\infty} \frac{\hbar k}{m}\frac{1}{\pi} f(E)\frac{dk}{dE}dE - e\int_0^{\infty} \frac{\hbar k'}{m}\frac{1}{\pi} f(E')\frac{dk'}{dE'}dE'$$

$$\Rightarrow I = e\int_0^{Ef1} \frac{\hbar k}{m}\frac{1}{\pi}\frac{dk}{dE}dE - e\int_0^{Ef2} \frac{\hbar k'}{m}\frac{1}{\pi}\frac{dk'}{dE'}dE'$$

$$\Rightarrow I = e\int_{Ef2}^{Ef1} \frac{\hbar k}{m}\frac{1}{\pi}\frac{1}{\hbar}\frac{m}{\hbar k}dE$$

where dE/dk was found using $E = \frac{(\hbar k)^2}{2m}$. We then have:

[4]

$$I = \frac{2e}{h}(E_{f1} - E_{f2}) = \frac{2e^2}{h}V$$

$$\Rightarrow G = \frac{I}{V} = \frac{2e^2}{h}$$

[2]
[Total marks: 14]

Question 3

(a) We scale all the device dimensions by a factor $K = 2$. We then have t_{ox}, L, and W changing to t_{ox}/K, L/K, and W/K. This gives us the following new values for the dimensions:

$t_{ox} = 50/2 = 25$ nm, $L = 1000/2 = 500$ nm, and $W = 10000/2 = 5000$ nm.

[2]

In order to keep electric fields F constant, the following changes must be made:

(i) Doping concentration:

In depletion region, Poisson's Eq. $\Rightarrow \dfrac{\partial F_x}{\partial x} + \dfrac{\partial F_y}{\partial y} = \dfrac{-eN_A}{\varepsilon_{Si}\varepsilon_o}$

So, to keep F_x, F_y constant when dimensions are scaled by $1/K$, we must scale N_A and N_D by K:

$\Rightarrow N_A, N_D \rightarrow KN_A, KN_D$

This give us new values of doping concentrations, $N_A = 2 \times 10^{22}$ /m^3, and $N_D = 2 \times 10^{25}$ /m^3.

[4]

(ii) Voltages:

As $F \sim V/L$, reducing L by $L/K \Rightarrow V \rightarrow V/K$. Therefore, the new maximum values of gate and drain voltage are: $V_g = V_d = 2.5/2 = 1.25$ V.

[2]

(iii) Gate capacitance:

As $C = \varepsilon_{ox}\varepsilon_0 A/t_{ox}$, and $A \rightarrow A/K^2$, $t_{ox} \rightarrow t_{ox}/K \Rightarrow C \rightarrow C/K$. Therefore, the new value of the gate capacitance is:
$C = \varepsilon_{ox}\varepsilon_0 A/Kt_{ox} = (4\times 8.854\times 10^{-12}\times 10^{-6}\times 10^{-5})/(2\times 50\times 10^{-9}) = 3.54 \times 10^{-15}$ F.

[4]

(iv) Inversion layer charge density:

$Q_n \sim CV/A$ and $C \rightarrow C/K$, $V \rightarrow V/K$, $A \rightarrow A/K^2$. This $\Rightarrow Q_n$ remains constant. The value for $Q_n \sim CV/A = 7.1\times 10^{-4}$ C/m^2.

[2]

[Total Marks: 14]

(b) For the saturation and sub-threshold currents:

(i) Drift current (linear and saturation region current):

$I_{drift}/W = Q_n v$, where $v = \mu F$ is the carrier velocity. As both Q_n and v are constant, $\Rightarrow I_{drift}/W$ remains constant.

But, as $W \to W/K$, we have $I_{drift} \to I_{drift}/K = I_{drift}/2$

[4]

(ii) Diffusion current (sub-threshold current):

$I_{diff} / W = D_n(dQ_n/dx) = (mkT/e)(dQ_n/dx)$. As Q_n is constant, and $x \to x/K$, $\Rightarrow I_{diff}/W \to K(I_{diff}/W)$, i.e. I_{diff}/W increases by factor K. But, as $W \to W/K$, we find I_{diff} is constant.

(i) and (ii) combined imply that the MOSFET 'on/off' ratio is degraded and it tends not to turn 'off'.

[4]
[Total Marks: 8]

(c) (i) As $F \sim V/L$, reducing L by L/K but keeping V constant $\Rightarrow F \to KF = 2F$. Therefore, electric fields are increased by a factor of 2.

[2]

(ii) For the drift and saturation currents, we need to estimate changes in gate capacitance and inversion layer charge density.

As $C = \varepsilon_{ox}\varepsilon_0 A/t_{ox}$, this scales as before to $C \to C/K$. Therefore, the new value of the gate capacitance is the same as before:

$C = \varepsilon_{ox}\varepsilon_0 A/Kt_{ox} = (4\times 8.854\times 10^{-12}\times 10^{-6}\times 10^{-5})/(2\times 50\times 10^{-9}) = 3.54\times 10^{-15}$ F.

For the inversion layer charge density:
$Q_n \sim CV/A$ and $C \to C/K$, $V \to V$, $A \to A/K^2$. This $\Rightarrow Q_n \to KQ_n$. The value for $Q_n \sim KCV/A = 14.2\times 10^{-4}$ C/m^2.

[2]

Drift current (linear and saturation region current):

$I_{drift}/W = Q_n v$, where $v = \mu F$ is the carrier velocity. As $F \to KF$, and $Q_n \to KQ_n$, we have $v \to Kv$ and this $\Rightarrow I_{drift}/W = K^2 I_{drift}/W = 4I_{drift}/W$.

But, as $W \to W/K$, we have $I_{drift} \to KI_{drift} = 2I_{drift}$

[2]

Diffusion current (sub-threshold current):

$I_{diff} / W = D_n(dQ_n/dx) = (mkT/e)(dQ_n/dx)$. As $Q_n \to KQ_n$, and $x \to x/K$, $\Rightarrow I_{diff}/W \to K^2(I_{diff}/W)$, i.e. I_{diff}/W increases by factor K^2. But, as $W \to W/K$, we find $I_{diff} \to KI_{diff} = 2I_{diff}$.

(i) and (ii) combined imply that the MOSFET does not turn 'off'.

[2]
[Total Marks: 8]

Question 4

a) The main transduction mechanisms are: Electrostatic (or capacitive); Electromagnetic; Piezoresistive; Piezoelectric.

Electrostatic and piezoresistive are the most widely used. The reason is that they can be implemented in a silicon mechanical layer without the need for any additional materials. Magnetic and piezoelectric transduction both require the introduction of materials that are difficult to process. Also, it is difficult to make efficient magnetic circuits using MEMS processes. [6]

b) Using a simple parallel plate approximation, the capacitance in each electrode gap when the moving element is in its rest position is $lh\varepsilon_0/g$.

For the variable gap design, there are $N/3$ electrode fingers on the moving element, and each has one gap contributing to the capacitance C_A between A and COM. When the moving element is displaced by an amount v, every gap changes to $g - v$, and the capacitance becomes $C_{A\_vg} = (N/3)lh\varepsilon_0/(g - v)$. (Have assumed displacement is so as to increase C_A.) [3]

For the variable overlap case, there are $N/2$ electrode fingers on the moving element. Half of these contribute two gaps each to the capacitance C_A, so the total number of gaps contributing to C_A is $N/2$. When the moving element is displaced by an amount v, the overlap changes to $l + v$, and the capacitance becomes $C_{A\_vo} = (N/2)(l + v)h\varepsilon_0/g$. [3]

Differentiating the expressions for C_A with respect to v, and evaluating at $v = 0$, the sensitivities are obtained as $S_{vg} = Nlh\varepsilon_0/(3g^2)$ and $S_{vo} = Nh\varepsilon_0/2g$. The ratio of sensitivities is therefore $S_{vg}/S_{vo} = 2l/(3g)$. [2]

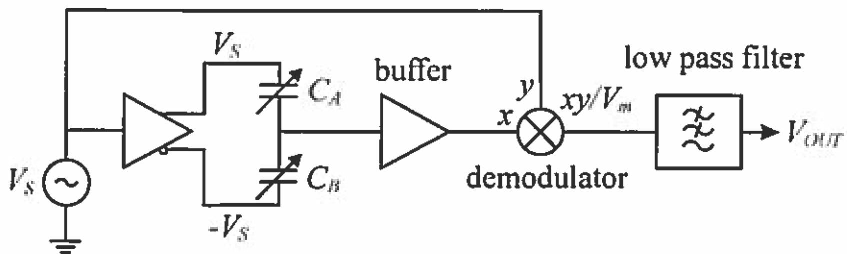
c) The instantaneous electrostatic force in each gap is $lh\varepsilon_0V^2/(2g^2)$ where V is the applied voltage. (This result can be quoted or derived by e.g. virtual work.) [4]

The A and B capacitors will pull the mass in opposite directions, so the net force when the displacement is v will be:

$$F = \frac{N}{3}\frac{lh\varepsilon_0 < V^2 >}{2}\left\{\frac{1}{(g-v)^2} - \frac{1}{(g+v)^2}\right\} \approx \frac{N}{3}\frac{lh\varepsilon_0 < V^2 >}{2}\left\{\frac{4v}{g^3}\right\} = \frac{Nlh\varepsilon_0 V_0^2}{3g^3}v \qquad [4]$$

This force is proportional to v (for small v) and in the direction of v, so the effect is to reduce the suspension stiffness by an amount F/v which is the required result. [2]

d)



[4]

The capacitors C_A and C_B form a potential divider, and the two ends are driven in antiphase so that the voltage at the buffer input is $V_S(C_A - C_B)/(C_A + C_B) = V_S(v/g)$. The rest of the circuit is a standard synchronous detector, so the output is a baseband signal proportional to v. [2]

Question 5

a) A possible fabrication sequence would be:
Thermally oxidize silicon, and pattern oxide by lithography and RIE to define anisotropic etch mask.
Anisotropically etch pyramids, then strip oxide from both sides of wafer by wet etching.
Deposit silicon nitride by e.g. LPCVD, and pattern by lithography and RIE to define cantilevers and land regions.
Deposit gold by e.g. sputtering and pattern by lithography and wet etching to define heaters.
Remove substrate from beneath cantilevers by anisotropic etching from top side. [6]

b) The thermally induced stress in the gold film will generate equal longitudinal and transverse bending moments in the cantilever. These will be given approximately by $M \approx \sigma_f t_f b \times d/2$ where b is the cantilever width (i.e. force × distance from axis passing through centroid). The longitudinal deflection profile can then be obtained from the standard bending equation:

$$\frac{d^2v}{dx^2} = \frac{M}{\tilde{E}I} = \frac{\sigma_f t_f bd/2}{\tilde{E}(bd^3/12)} = \frac{6\sigma_f t_f}{\tilde{E}d^2}$$ [6]

The biaxial modulus is used in this case because the thermally induced stresses are in the plane of the film and isotropic, as are the resulting bending stresses and strains in the cantilever. [2]

c) The steady-state 1D heat flow equation is given as: $\frac{d^2T}{dx^2} + \frac{I^2R}{\kappa A} = 0$ (see front matter).

The second term is constant along the actuator (except near free end), so the equation can be solved by direct integration to give $T(x) = -cx^2/2 + bx + a$, where a and b are constants of integration, and $c = I^2R/\kappa A$. With the boundary conditions $T = 0$ at $x = 0$ (infinite heatsink at root) and $dT/dx = 0$ at $x = L$ (no heat flow out of free end) this gives $T(x) = \frac{c}{2}(2Lx - x^2)$. The maximum temperature rise occurs at $x = L$ and is given by $T_{max} = cL^2/2$. The expression for $T(x)$ can therefore be written as: $T(x) = \frac{T_{\max}}{L^2}(2Lx - x^2)$. [6]

The film stress will be given by $\sigma_f = \tilde{E}_f \Delta\alpha \cdot T$, and combining this with the above results gives:

$$\frac{d^2v}{dx^2} = \frac{6t_f}{\tilde{E}d^2}\tilde{E}_f \Delta\alpha \frac{T_{\max}}{L^2}(2Lx - x^2)$$

Integrating this twice with boundary conditions $v = 0$ and $dv/dx = 0$ at $x = 0$ gives:

$$v(x) = \frac{6t_f}{\tilde{E}d^2}\tilde{E}_f \Delta\alpha \frac{T_{\max}}{L^2}\left(\frac{Lx^3}{3} - \frac{x^4}{12}\right)$$

and putting $x = L$ the end deflection is obtained as:

$$v(L) = \frac{6t_f}{\widetilde{E}d^2}\widetilde{E}_f \Delta\alpha \frac{T_{\max}}{L^2}\frac{L^4}{4} = \frac{3}{2}\frac{L^2 t_f}{d^2}\frac{\widetilde{E}_f}{\widetilde{E}}\Delta\alpha T_{\max}$$

as required. [6]

d) Putting $L = 500$ µm, $d = 1.0$ µm, $t_f = 100$ nm, $\widetilde{E} = 370$ GPa , $\widetilde{E}_f = 214$ GPa, $\Delta\alpha = -10.9 \times 10^{-6}$ and $T_{max} = 50$ °C gives $v(L) = -11.8$ µm. [2]

The probe will deflect downwards (i.e. into the substrate) because the CTE of the gold is higher than that of the silicon nitride. [2]