EE4-57

SOLUTIONS: DISCRETE EVENT SYSTEMS
MASTER IN CONTROL

1. Exercise

   a) The automaton G has an event set $E = \{a_1, a_2, a_3, d_1, d_2, d_3\}$ and a state-space $X = \{000, 001, 010, 011, 100, 101, 110, 111\}$. Its transition diagram is shown in Fig. 1.1;

   b) The automaton including overflows is shown in Fig. 1.2.

   c) The labeling device G_L has event set $E_L = \{o_1\}$, two states, $X_L = \{N, Y\}$, initial state N, and two transitions, $f_L(N, o_1) = Y$ and $f_L(Y, o_1) = Y$.

   d) The parallel composition of G_O and G_L is shown in Fig. 1.3.

   e) The only state of $G_L||G_O$ where o events trigger a transition to more than one state, is $011N$. In fact o may represent either an o_1 or o_2 event, and in the case of o_1 event transition to $111Y$ occurs, while in the case of o_2 event transition to $111N$ occurs.

   f) The diagnoser automaton G_D is represented in Fig. 1.4.

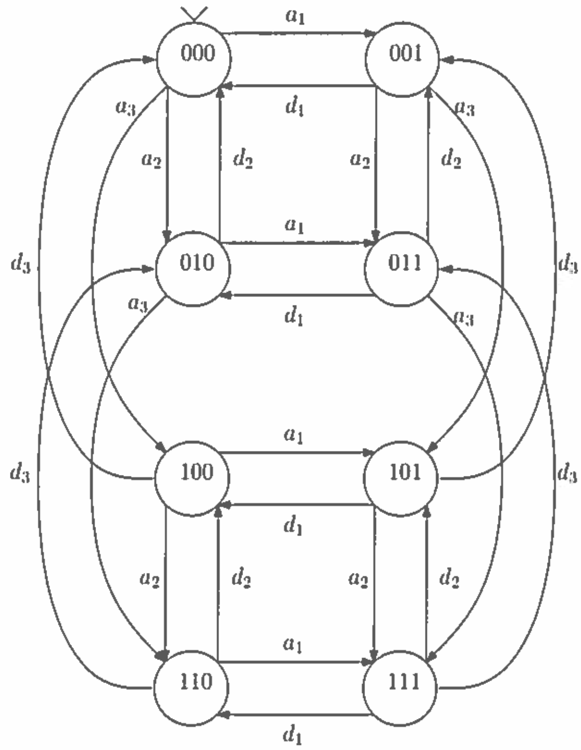

Figure 1.1 Transition diagram of automaton G

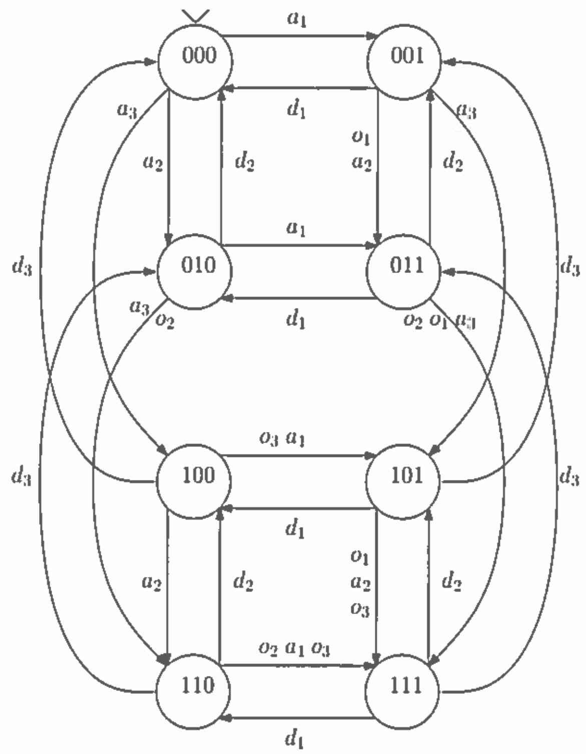

Figure 1.2 Transition diagram of automaton G_O

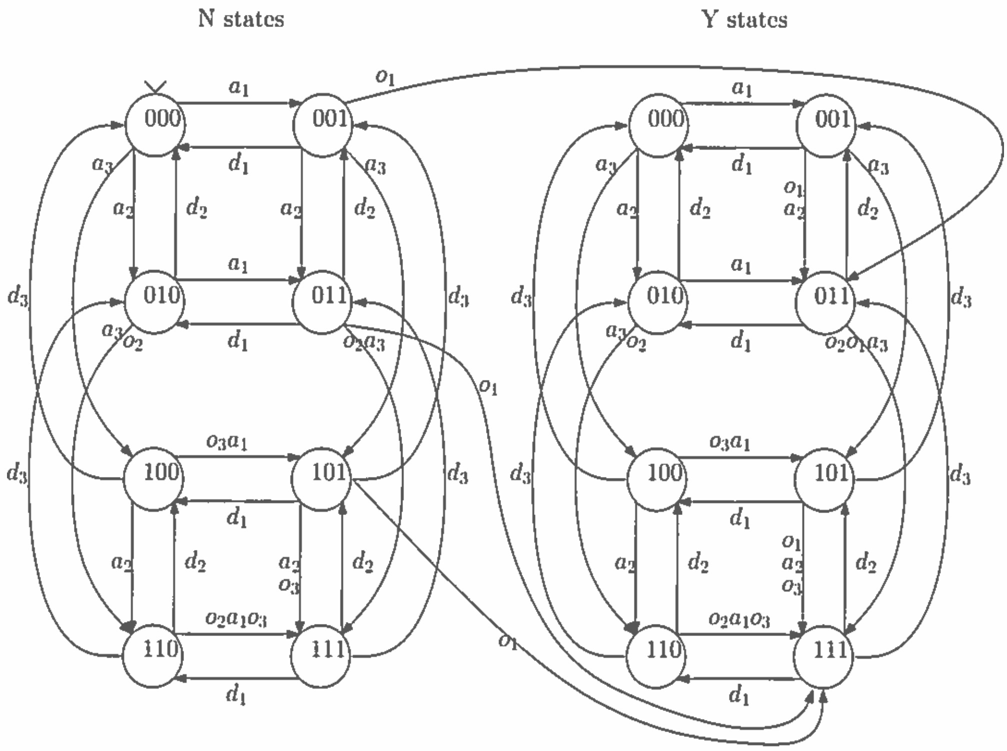

Figure 1.3 Transition diagram of automaton $G_L||G_O$

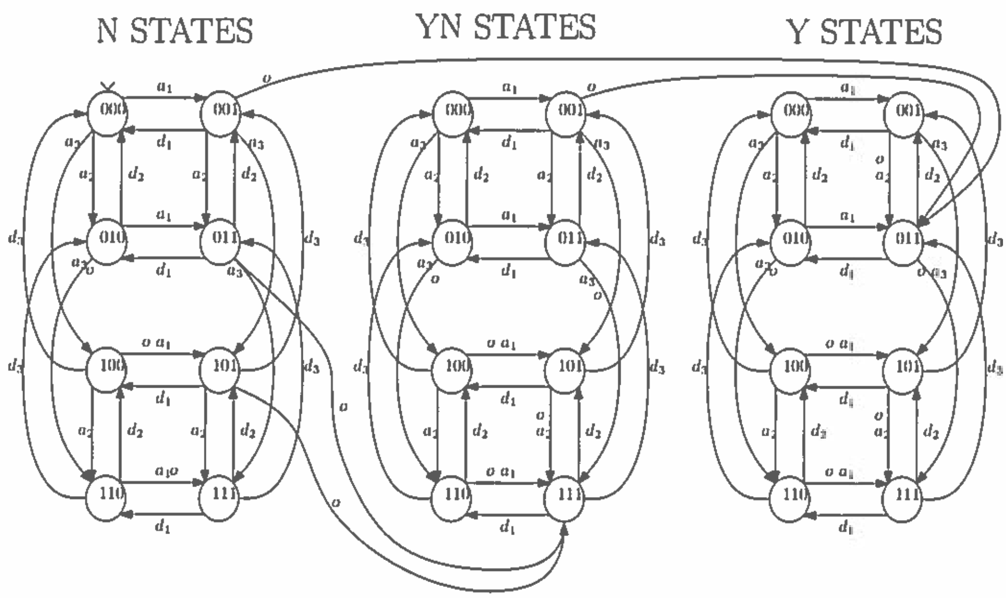

Figure 1.4 Transition diagram of automaton G_D

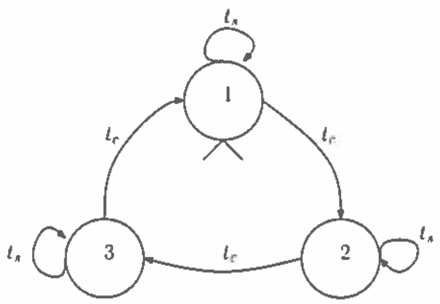

Figure 2.1 Transition diagram of G_A

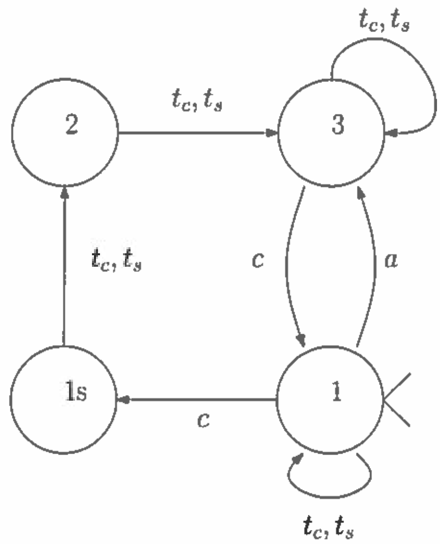

Figure 2.2 Transition diagram of G_B

2. Exercise

a) The automaton G_A has an event set $E_A = \{t_c, t_s\}$ and states $X_A = \{1,2,3\}$. Its transition diagram is shown in Fig. 2.1.

b) The automaton G_B has an event set $E_B = \{a, c, t_c, t_s\}$ and state space $X_B = \{1, 1s, 2, 3\}$. Its transition diagram is shown in Fig. 2.2.

c) The parallel composition $G_A || G_B$ has the transition diagram shown in Fig. 2.3.

d) The automaton H implementing the specification can be obtained simply removing the state 22 from $G_A || G_B$ and associated edges. See Fig. 2.4.

e) Notice that $\mathscr{L}(H)$ is uncontrollable with respect to $\mathscr{L}(G)$ and $E_{uc} = \{t_s, t_c\}$ since in states $1s1$ and $1s2$ events t_c and t_s (respectively) have been disabled.

f) The supremal controllable sublanguage is marked by the automaton $\bar{H}$ shown in Fig. 2.5. Notice that $S(s) = \Gamma_{\bar{H}}(f_{\bar{H}}(11, s))$ is an admissible supervisor since it only disables c events (that are controllable).

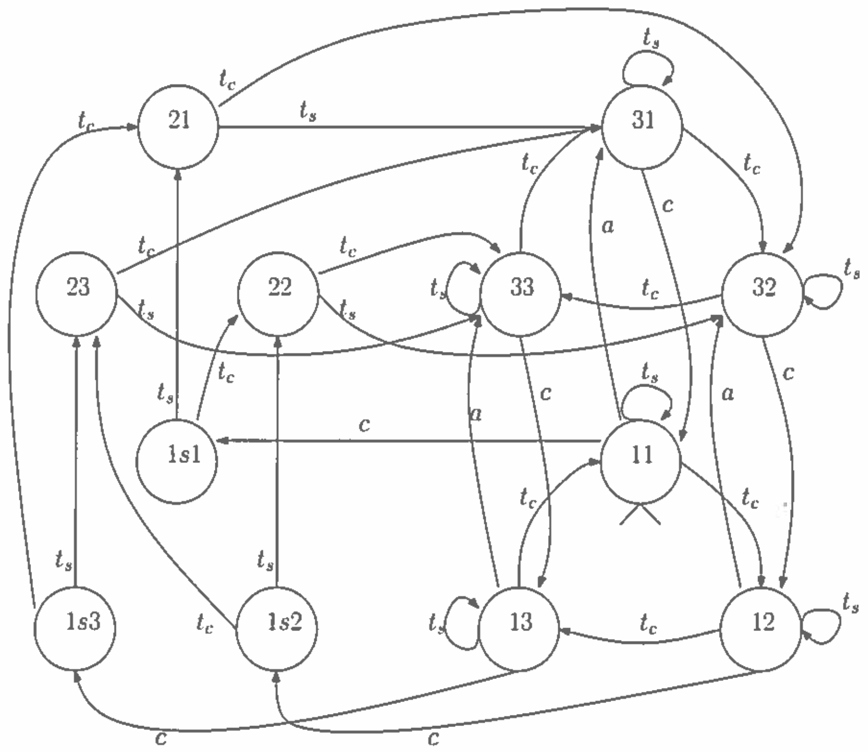

Figure 2.3 Transition diagram of $G_A||G_B$

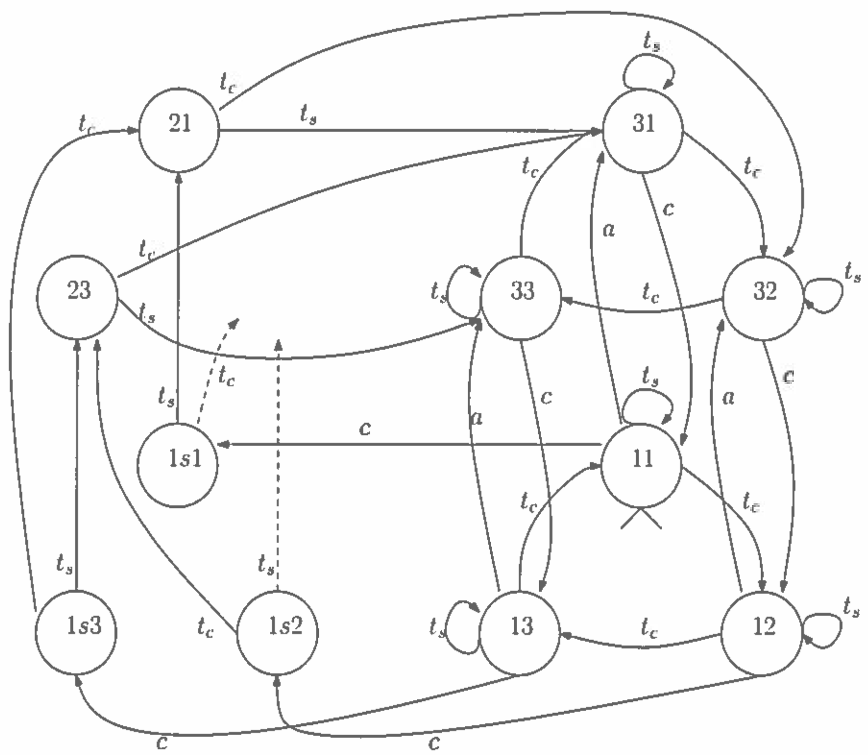

Figure 2.4 Transition diagram of automaton H

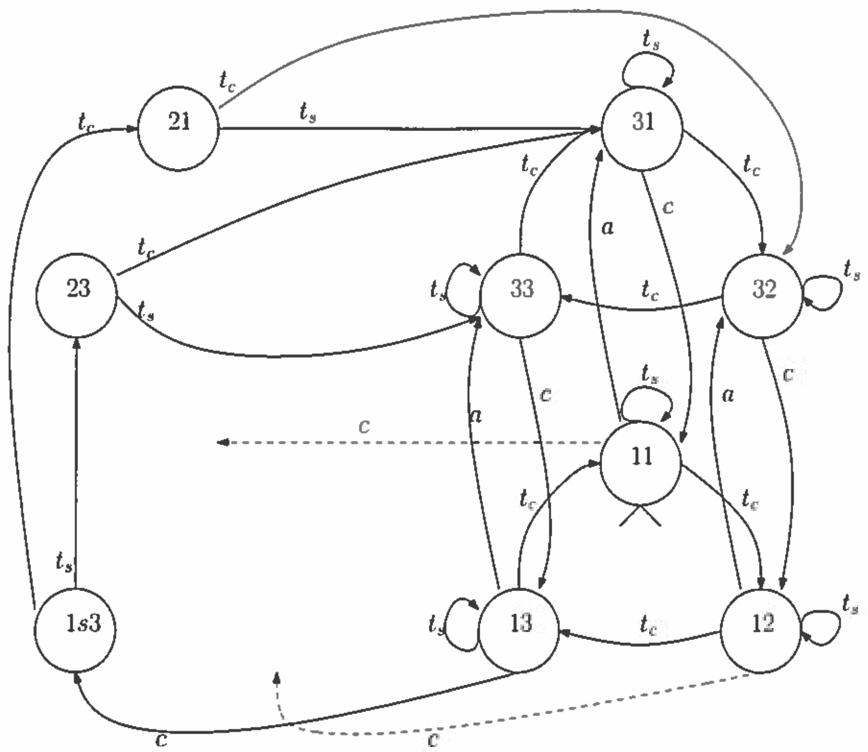

Figure 2.5 Transition diagram of automaton $\tilde{H}$

3. Exercise

a) The Petri Net has 7 transitions, according to events a, p, x, pA, pB, rA, rB and 6 places, representing respectively Tool A available, Tool B available, pieces waiting to be processed, Tool A busy, Tool B busy, processed pieces. The initial marking $M_0 = [1,1,0,0,0,0]'$ as represented in the Figure. See Fig. 3.1

b) When two networks share the same tools and pick up the same unprocessed pieces the model in Fig. 3.2 can be adopted:

c) In case of finite capacity of the storage for processed pieces waiting to be delivered, the Petri Net N can be modified as in Fig. 3.3 (where a capacity of 5 has been represented)

d) The incidence matrix of N is given by:

$$C = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Correspondingly there are two P-invariant vectors of minimal support: $[1,0,0,1,0,0]$ and $[0,1,0,0,1,0]$. These correspond to the tools A and B being conserved resources.

e) The coverability graph associated to $< N, M_0 >$ is shown in Fig. 3.4.

f) The network does not exhibit P-decreasing vectors, except for the 2 P-invariant vectors previously computed. Hence, the structurally bounded places are $\{p_1, p_2, p_4, p_5\}$, viz. those corresponding to the supports of P-invariant vectors. The behaviourally

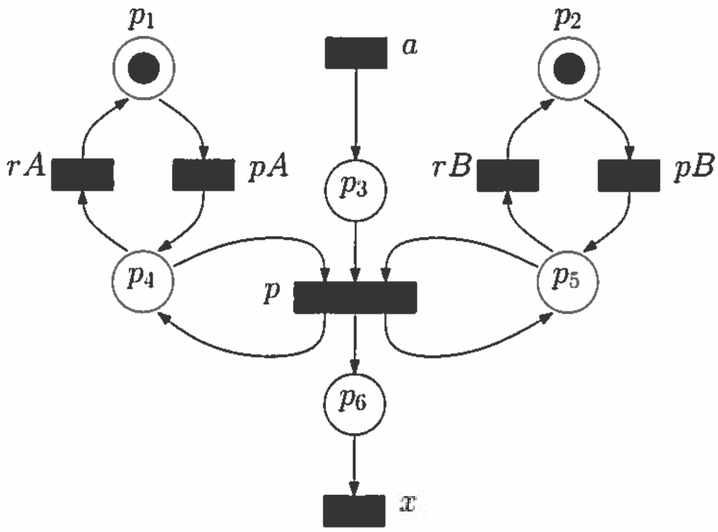

Figure 3.1 The marked Petri Net, $< N, M_0 >$

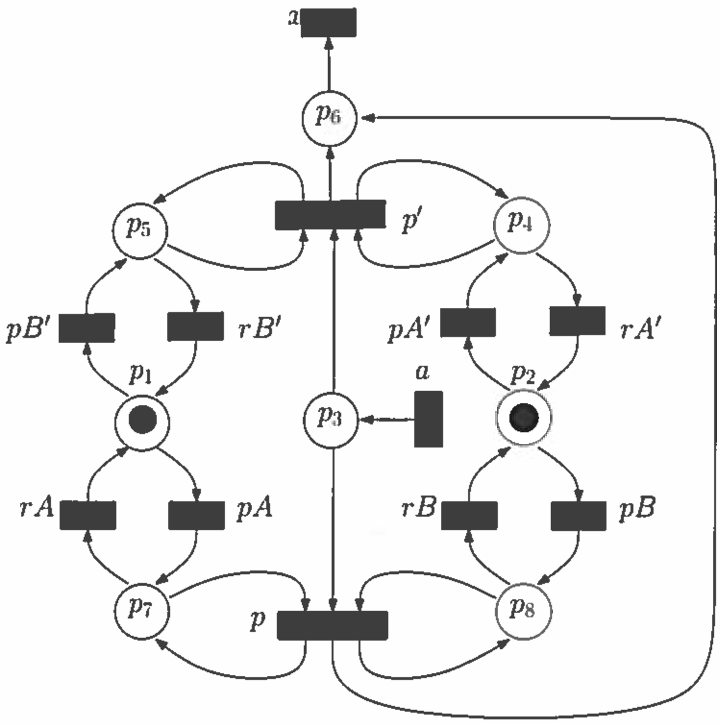

Figure 3.2 Machines sharing tools.

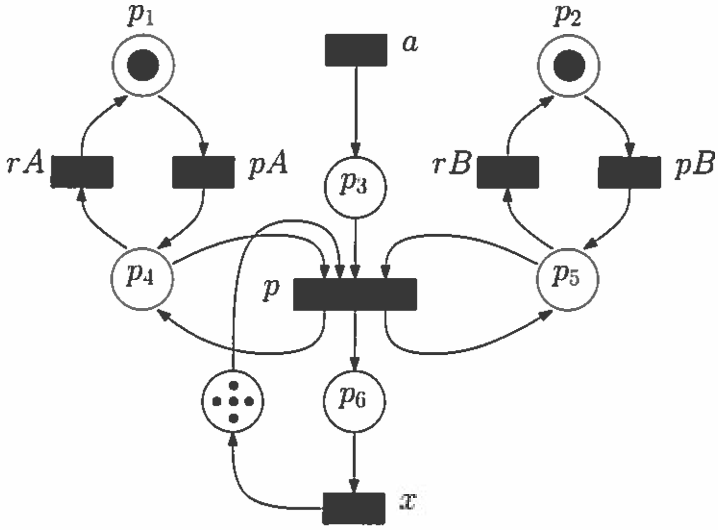

Figure 3.3 A single machine with finite capacity of storage.

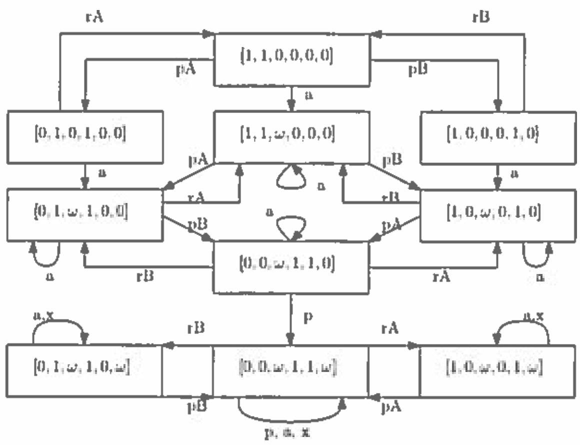

Figure 3.4 The coverability graph

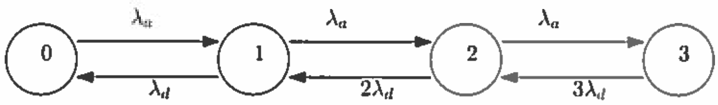

Figure 4.1 Markov chain modeling car park

bounded places are those for which ω does not appear in any nodes of the associated coverability graph. Hence, these are again the places $\{p_1, p_2, p_4, p_5\}$.

4. Exercise

a) Notice that when n cars are parked in the car park, then the distribution of departure times is still normal, with rate $n\lambda_d$. Hence, the Markov chain model of the car park is as in Fig. 4.1; Its equations read:

$$\begin{aligned}
\dot{\pi}_0 &= -\lambda_a\pi_0 + \lambda_d\pi_1 \\
\dot{\pi}_1 &= -(\lambda_d + \lambda_a)\pi_1 + \lambda_a\pi_0 + 2\lambda_d\pi_2 \\
\dot{\pi}_2 &= -(2\lambda_d + \lambda_a)\pi_2 + \lambda_a\pi_1 + 3\lambda_d\pi_3 \\
\dot{\pi}_3 &= -3\lambda_d\pi_3 + \lambda_a\pi_2
\end{aligned}$$

b) The chain is ergodic, hence, there exists a well defined asymptotic probability distribution. This is the unique solution of:

$$\begin{aligned}
0 &= -\lambda_a\pi_0 + \lambda_d\pi_1 \\
0 &= -(\lambda_d + \lambda_a)\pi_1 + \lambda_a\pi_0 + 2\lambda_d\pi_2 \\
0 &= -(2\lambda_d + \lambda_a)\pi_2 + \lambda_a\pi_1 + 3\lambda_d\pi_3 \\
0 &= -3\lambda_d\pi_3 + \lambda_a\pi_2 \\
1 &= \pi_0 + \pi_1 + \pi_2 + \pi_3
\end{aligned}$$

We see that:

$$\pi_1 = \frac{\lambda_a}{\lambda_d}\pi_0 \qquad \pi_2 = \frac{\lambda_a^2}{2\lambda_d^2}\pi_0 \qquad \pi_3 = \frac{\lambda_a^3}{6\lambda_d^3}\pi_0$$

Hence:

$$\pi_0\left(\frac{\lambda_a}{\lambda_d} + \frac{\lambda_a^2}{2\lambda_d^2} + \frac{\lambda_a^3}{6\lambda_d^3}\right) = 1.$$

The average number of cars parked is given by:

$$\pi_1 + 2\pi_2 + 3\pi_3 = \left(\frac{\lambda_a}{\lambda_d} + \frac{\lambda_a^2}{\lambda_d^2} + \frac{\lambda_a^3}{2\lambda_d^3}\right)\pi_0$$

c) We consider next the markov chain with states $\{000, 001, 010, 011, 100, 101, 110, 111\}$ modeling the occupancy of each parking slot. Its transition diagram is shown in Fig. 4.2.

d) Notice that defining $\pi_1 = \pi_{100} + \pi_{110} + \pi_{111} + \pi 101$ and $\pi_0 = \pi_{000} + \pi_{001} + \pi_{010} + \pi_{011}$ yields:

$$\dot{\pi}_0 = -\lambda_a\pi_0 + \lambda_d\pi_1$$

$$\dot{\pi}_1 = -\lambda_d\pi_1 + \lambda_a\pi_0.$$

Hence, these are the equations of a Markov chain with 2 states. The average occupancy of parking slot 1 is therefore $\frac{\lambda_a}{\lambda_a+\lambda_d}$.

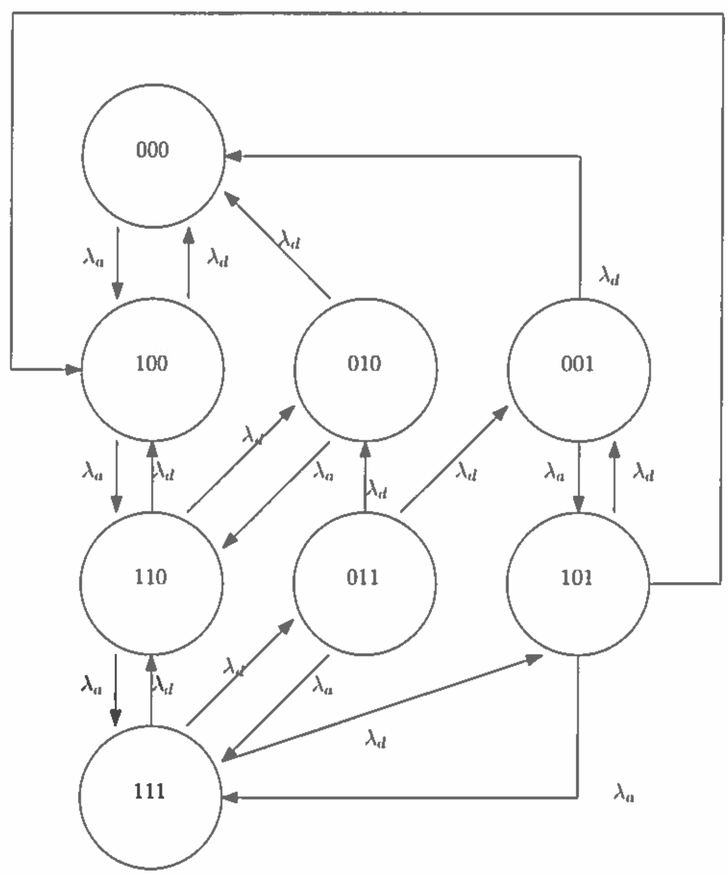

Figure 4.2 Occupancy modeling Markov chain

e) Modeling occupancy of state 2 requires more than 2 states, in fact, when state two is free, the transition probability from a free to a busy state would depend on whether or not slot 1 is already taken.