Imperial College London

BSc/MSci EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

CONCEPTS IN DEVICE PHYSICS

For 4th-Year Physics Students

Monday, 21st May 2012: 10:00 to 12:00

Answer THREE questions; ONE from Section A and TWO from Section B
All questions carry equal marks.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the 3 answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in 3 answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

SECTION A

1. (i) Consider a $Ga_{1-x}Al_xAs/GaAs/Ga_{1-x}Al_xAs$ quantum well of width a assuming a 50:50 band offset. Draw the physical structure of the quantum well and indicate the device growth direction, z. Draw a separate electron energy diagram showing the valence and conduction bands of the resulting structure and label the growth direction. [2 marks]

The Schrödinger equation solutions in the QW are not Bloch states but are instead given by

$$\psi_{QW} = u(\mathbf{r}) \times f(z) \times e^{i\mathbf{k}_{\parallel}.\mathbf{r}} \qquad (1)$$

Explain the difference between this expression and the form of a Bloch state stating any assumptions that lead to Equation 1. Which is the confinement direction? [2 marks]

(ii) Using the infinite potential approximation find expressions for the quantised energies of electrons in the conduction band and from your answer find expressions for the difference between the energies of these states and the energies of hole states in the valence band. You may assume that the energy gap in the QW is E_{gap}, and that the conduction and valence bands have effective masses m_e^* and m_h^*, respectively. [3 marks]

(iii) The matrix element that determines the transition rate for photon-induced transitions is

$$M =< \psi'|e\mathbf{r_{dipole}}.\mathbf{E_0}|\psi > , \qquad (2)$$

where ψ' is the final state and ψ is the initial state of the electron. The electron dipole is given by $e\mathbf{r_{dipole}}$ and the electric field due to the photon is $\mathbf{E_0}$. If the electric field is perpendicular to the growth direction, use the infinite potential well approximation to investigate:

(a) the possibility of inter-band transitions in terms of quantisation of initial and final states

(b) a condition on the allowed intra-band transitions stating any assumptions that you make. [4 marks]

Without explicit calculation, describe how your answers would change if the true, finite well was instead modelled. [1 mark]

(iv) A researcher has fabricated a $Ga_{1-x}Al_xAs/GaAs/Ga_{1-x}Al_xAs$ QW but he did not take good lab notes and cannot remember the thickness of the well or the concentration of Aluminium he used. Luckily, a colleague is able to measure the absorption coefficient as a function of incident photon energy, and in a separate experiment, the absorption spectrum of pure, unconfined GaAs. From her data (Figure 1) and the band gap variation of $Ga_{1-x}Al_xAs$ given in Equation 3, estimate:

(a) the width, a, of the GaAs layer; [2 marks]

(b) an upper bound on the amount of aluminium used (x). What features of the absorption spectrum give you confidence in your answer? [6 marks]

[This question continues on the next page . . .]

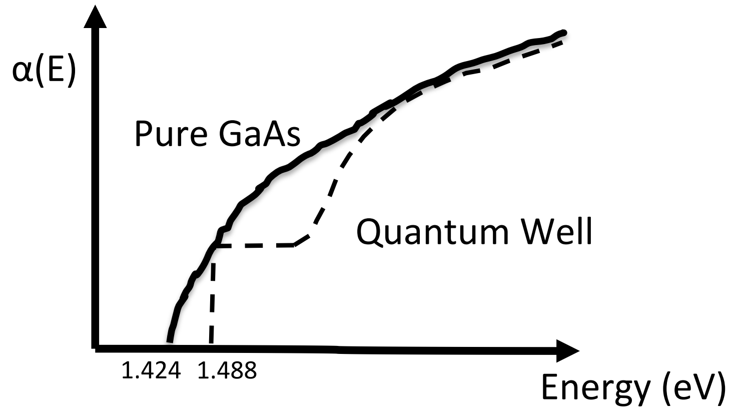

Figure 1: Variation of absorption coefficient with incident photon energy for the QW.

$$E_{gap} = [1.424 + 1.247x]\ eV \tag{3}$$

You may assume a 50:50 band offset, and that the electron and hole effective masses are given by $m_e^* = 0.067m_e$ and $m_h^* = 0.5m_e$, respectively, where m_e is the bare electron mass.

[Total 20 marks]

[You may assume that the following wave function is of Bloch form:

$$\psi_{\mathbf{k}} = u_{\mathbf{k}}(\mathbf{r}) \times F(x, y, z) = u_{\mathbf{k}}(\mathbf{r}) \times e^{i\mathbf{k}.\mathbf{r}}\,, \tag{4}$$

where the subscripts indicate labelling of quantum states in reciprocal space. It is a solution of the one-electron Schrodinger equation with periodic potential $V(\mathbf{r}) = V(\mathbf{r} + \mathbf{R})$ where $\mathbf{R}$ is a lattice vector.]

2. (i) Explain briefly the difference between *localised* and *itinerant* models of magnetism. What is orbital quenching, and in which elements is it typically found? What distinguishes a paramagnet from a ferromagnet? [7 marks]

(ii) Consider a paramagnetic material described by the localised model. If each atomic site is of spin $S = \frac{1}{2}$ and angular momentum is quenched what is the energy of the site moment in a magnetic field, H? [2 marks]

(iii) By using the partition function, or otherwise, show that the maximum entropy of the moment for such spins is $k_B \ln(2)$ per site. Without calculation, sketch the variation in site entropy with temperature, assuming that the material remains paramagnetic throughout the temperature range of your plot. [2 marks]

(iv) A magnetic field is applied to this paramagnet. Add a second curve to your sketch to show the variation of entropy with temperature when a fixed magnetic field is applied. What sign has the temperature change that results from applying the magnetic field adiabatically and why does temperature change without a change in entropy? [2 marks]

(v) Below a certain (Curie) temperature, the material becomes ferromagnetic. On a new plot, without calculation sketch the variation in entropy as a function of temperature in zero magnetic field, focusing on the region of the Curie temperature. Add another curve in finite field, explaining your reasoning and comment on the adiabatic temperature change. [2 marks]

(vi) From the expression for Gibbs Free Energy of a material in a magnetic field $dG = -SdT - MdH$, derive a Maxwell relation and from this show that the effect of a small magnetic field dH, when applied adiabatically, is to change the temperature of the material by an amount given by

$$dT = -\frac{T}{C_p(T,H)}\left(\frac{\partial M}{\partial T}\right)_H dH\,.$$

Does this expression yield the sign of the adiabatic temperature change that you predicted in part (v)? [5 marks]

[Total 20 marks]

[You may assume the following relations:
The partition function is given by $Z = \sum_{states} e^{-E/k_B T}$
Entropy is given by $S = \frac{\partial}{\partial T}(k_B T \ln Z)$.
The g-factor for orbital angular momentum is 1 and for spin is 2.]

SECTION B

3. (i) Sketch the equilibrium energy band diagram for a metal (calcium) and a p-type semiconductor out of contact and in contact, assuming an ideally clean interface (i.e. no interface states), using the following materials parameters:
Metal work function (Φ_M) = - 3 eV, Semiconductor valence band (E_V) = -6 eV
Semiconductor work function (Φ_S) = -5.5 eV, Bandgap energy (E_G) = 3.5 eV

From this diagram calculate the energy barriers for holes crossing from the metal electrode to the p-type semiconductor (M→S) and from the p-type semiconductor to the metal electrode (S→M). Based on your calculations, comment on the type of the contact formed between the metal electrode and the p-type semiconductor. [5 marks]

(ii) Sketch the band diagram of the contact in (i) under a |1| Volt reverse bias. [1 mark]

(iii) In a rectifying contact formed between a metal and an n-type semiconductor under equilibrium the n-type semiconductor is depleted of mobile electrons to a depth (W). This is the so-called depletion region width. Using the simplified depletion approximation derive an expression for W assuming the contact is an ideal rectifying metal-semiconductor junction and that the thickness of the semiconductor is much greater than W. [4 marks]

(iv) A metal-oxide-semiconductor (MOS) capacitor is based on a bilayer gate dielectric comprising a 50 nm thick layer of ZrO_2 (ϵ_r = 10), a 100 nm thick layer of HfO2 (ϵ_r = 20), a thick n-type silicon layer (ϵ_r = 3.9) as the semiconductor and a highly conductive metal as the gate electrode. Assuming an active capacitor area (A) of 1 cm^2 calculate the device capacitance in accumulation. [3 marks]

(v) Sketch the high frequency capacitance-voltage (CV) characteristic for a typical MOS capacitor based on n-type silicon as the semiconductor and briefly describe the main operating regimes. Which are the voltages that demarcate these regimes? [2 marks]

(vi) The maximum operating frequency (f_{MAX}) of an n-channel silicon transistor with channel length L = 500 nm measured at a drain-source voltage (V_{DS}) of +5 V, is 100 GHz. Assuming that the transistor is ideal, calculate the effective carrier mobility of the semiconducting channel. [2 marks]

(vii) Sketch the schematic device structure of an n-p-n bipolar junction transistor (BJT) and identify the necessary biasing conditions so the device can be operated in active mode (i.e. as a current amplifier). Assuming that the collector efficiency (α) of the transistor is 0.991, calculate the current gain (β) of the device in common emitter configuration. [3 marks]

[Total 20 marks]

4. (i) Briefly describe the operation of a p-n junction as an LED. [3 marks]

(ii) State what is meant by the quantities *quantum efficiency* and *power conversion efficiency* of an LED. [2 marks]

(iii) The following inorganic semiconductor LED data was obtained from an electronics catalogue:

	I_F (typical)/mA	V_F (at I_F typical)	Peak wavelength (nm)
Red	10	2	635
Green	10	2	562

where I_F is the forward current and V_F is the forward voltage dropped across the diode at the given forward current. Assume the diode construction is n++p, so that the current is almost entirely due to the flow of electrons. For each type of diode calculate:

(a) An estimate of the band gap energy of the semiconductor (in eV); [1 mark]

(b) The optical power emitted by the LED assuming 30% radiative recombination efficiency, perfect out-coupling of the emitted light and negligible re-absorption; [1 mark]

(c) The input power to the diode. [1 mark]

(d) The power conversion efficiency. [1 mark]

(iv) There is a discrepancy here. If the radiative recombination efficiency were 100% then the green LED would appear to give out more optical power than electrical power supplied. The data in the table must therefore be wrong. Assuming that your estimate of the band gap is accurate to around 50 meV identify the erroneous data and explain why it cannot have the value given in the table. [2 marks]

(v) Efficient LED operation depends in part on efficient extraction of light. For an interface with air, the critical angle for total internal reflection θ_c can be derived from Snells law:

$$\sin\theta_c = \frac{1}{n}$$

where n is the refractive index of the semiconductor material.

(a) By considering the solid angle of the escape cones Ω_c and a perfect back surface mirror (a specular reflector with $R = 1$) show that the fraction of light that can be coupled out of the front surface of a planar LED is given by

$$\frac{\Omega_c}{4\pi} = 1 - \frac{1}{n}\sqrt{n^2 - 1}$$

Here Ω_c defines the total solid angle of the escape cones for light propagating in the forward and backward directions with respect to the front surface. [4 marks]

[This question continues on the next page . . .]

(b) Hence quantify the advantage of an organic LED material with a refractive index n of 1.6 over that of an inorganic LED material with a refractive index n of 3.3 in terms of light extraction. [1 mark]

(c) The results of part (b) illustrate the importance of strategies to improve light extraction. Give two examples of ways to increase the fraction of light that can be coupled out of an LED chip. [2 marks]

(vi) The calculations above show the advantage of organic materials over inorganic ones in terms of light extraction. However, organic materials generally have lower radiative efficiencies. Considering the types of excitons formed in organic materials explain the main reason why this is the case. [2 marks]

[Total 20 marks]

5. Cylindrical nanomagnets are of interest as a data storage medium. Consider a disk-shaped cobalt nanomagnet with radius R=100 nm and thickness t=10 nm. The nanodisks have a single in-plane easy axis of magnetisation. The energy per unit volume associated with magnetising along the easy axis is defined by the anisotropy constant K_u. The energy cost per unit length associated with rotating the magnetization is defined by the exchange stiffness A. The magnetic parameters of cobalt are:
Saturation magnetization, M_S = 1.44 MAm^{-1}
Anisotropy constant K_u = 410 kJm^{-3}
Exchange stiffness A=31 pJm^{-1}.

(i) Calculate the magnetic moment, M, of the nanodisk uniformly magnetised along its easy axis. [2 marks]

(ii) Calculate the energy barrier to reversing the nanodisks magnetic moment by coherent rotation. [2 marks]

(iii) A magnetic field H is applied parallel to the easy axis direction. Assuming that the magnetic flux density B is dominated by the contribution from the applied field, calculate the Zeeman energy when the field and magnetisation are:

(a) parallel;

(b) antiparallel. [2 marks]

(iv) Therefore estimate the coercive field of the nanodisk. [3 marks]

(v) Assume the cobalt is uniformly magnetised and that the anisotropy energy density has the form $K_u \sin^2(\theta)$.

(a) Sketch the total energy in zero applied field as a function of θ, the angle between the magnetisation direction and the easy axis . Label the anisotropy energy barrier. On the same graph sketch energy vs. θ in positive applied fields where:

(b) $H < H_c$;

(c) $H = H_c$;

(d) $H > H_c$.

[6 marks]

(vi) In nanodisks, uniform magnetisation leads to stray fields, which have a demagnetising energy cost. This demagnetising energy can be avoided by the formation of a domain wall. The domain wall has an energy cost from both the exchange (E_{ex}) and the anisotropy (E_A), which both depend on the domain wall width, w.

(a) Describe briefly whether E_{ex} and E_A favour narrow or wide domain walls. Consider the limiting cases $w \to 0$ and $w \to \infty$. [3 marks]

(b) Assuming the equilibrium domain wall width is $\pi\sqrt{AK_u}$ state whether the formation of a domain wall is possible in disks of this size. [2 marks]

[Total 20 marks]

6. Write short notes on FOUR of the following topics using equations and figures where appropriate.

(i) Sketch the circuitry of an active matrix liquid crystal display (AM-LCD) and active matrix organic light-emitting diodes (AM-OLED) sub-pixel. Briefly explain why different sub-pixel architectures are required. [5 marks]

(ii) Organic semiconductors are a promising set of materials for a host of emerging applications in the area of large area opto-/electronics. Assuming the energy band gap of an intrinsic small-molecule organic semiconductor is 2 eV and the Fermi energy level (E_F) is located at -4 eV, calculate the HOMO and LUMO levels of the molecule and briefly explain their physical meaning. [5 marks]

(iii) Discuss electrical and optical confinement and how they are achieved in a typical heterojunction semiconductor laser based on the material AlGaAs/GaAs. [5 marks]

(iv) Discuss the advantages of using quantum wells (QWs) in inorganic semiconductor LED and heterostructure laser design. [5 marks]

(v) Methods of sensing magnetic fields. [5 marks]

(vi) Compare hard disks using in-plane and perpendicular magnetic storage. [5 marks]

[Total 20 marks]