Q1 (a) Erlang Model Assumptions   (bookwork)

(a) — Poisson arrival stream: good approx when large number of independent traffic sources

— Exponential holding times: good approx on telephone calls

— Full-availability access: usually valid

— No re-submissions: not strictly valid but model can be modified to allow for this   5

(b)
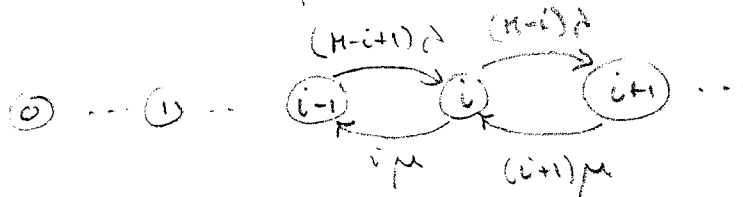
Global

$$[(M-i)\lambda + i\mu]\pi_i = (M-i+1)\lambda\pi_{i-1} + (i+1)\mu\pi_{i+1}$$

Local

$$(M-i+1)\lambda\pi_{i-1} = i\mu\pi_i$$

M = no. sources
π_i = Eq. state prob. in state i
λ = calling rate/free source
μ = release rate/channel   5

(c) $\rho = 8$, $M = 12$

$B_c = E_c(8) = 0.05$ (from chart)

(i) mean carried traffic
$$\rho_c = (1 - B_c)\rho_c = 7.3 \text{ Erlangs}$$

(ii) Mean channel occupancy
$$\eta = \frac{\rho_c}{M} = 3.633$$

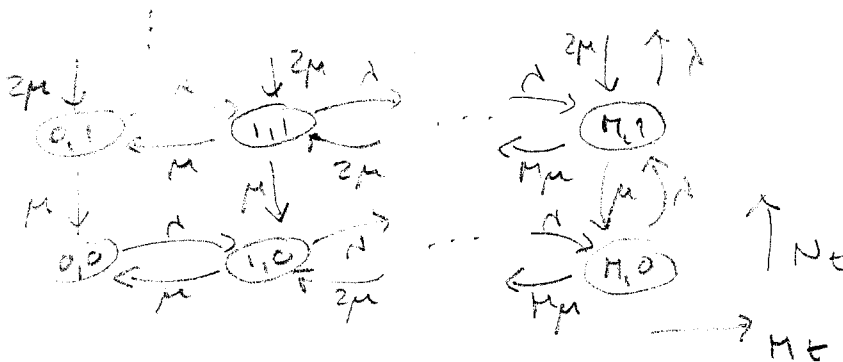(iii) trunk idle time = T_0
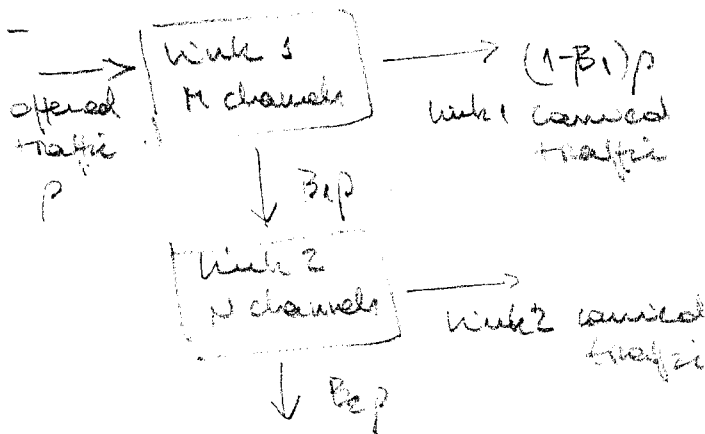= 0.00056 (approx $e^{-\rho_c}$)   10

Q2(a) Overflow model   (Work model)

(a) Let $\begin{cases} M_t = \text{No of busy channels on link 1} \\ N_t = \text{No of busy channels on link 2} \end{cases}$

Assume $\begin{cases} \text{(i) Poisson arrival stream to link 1, rate } \lambda \\ \text{(ii) Exponential holding times, rate } \mu \end{cases}$

— local balance equation do not hold
The problem should be solved using global balance equations

offered traffic ρ → | link 1 M channels | → $(1-\beta_1)\rho$ link 1 carried traffic

↓ $\beta_1 \rho$

| link 2 N channels | → link 2 carried traffic

↓ $\beta_2 \rho$

link 1 call congestion $= \beta_1$

link 2 call congestion $= \dfrac{\text{Arrival rate to link 2 of blocked arrivals}}{\text{Total arrival rate to link 2}}$

$$ = \frac{d\beta_2}{d\beta_1} = \frac{\beta_2}{\beta_1} = \frac{E_{M+N}(\rho)}{E_M(\rho)} $$

3

3

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| Question Number etc. in left margin | Mark allocation in right margin |

Q2

(v)

$K = 10, \quad \dfrac{\lambda}{\mu} = 8 \text{ Erlang} \qquad \rho = \dfrac{\lambda}{K\mu} = 0.8 \text{ Erlangs/channel}$

$P[\text{Delay}] = \dfrac{E_K(K\rho)}{1 - \rho\,[1 - E_K(K\rho)]} = 0.41$

5

If buffer size is Q then

$P[\text{Loss}] = \left[\dfrac{(1-\rho)\,\rho^Q\, E_K(K\rho)}{(1-\rho) + \rho(1-\rho^Q)\, E_K(K\rho)} \right]$
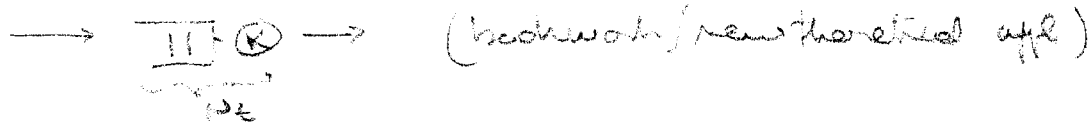
$\rho = 0.8 \text{ Erlangs}$

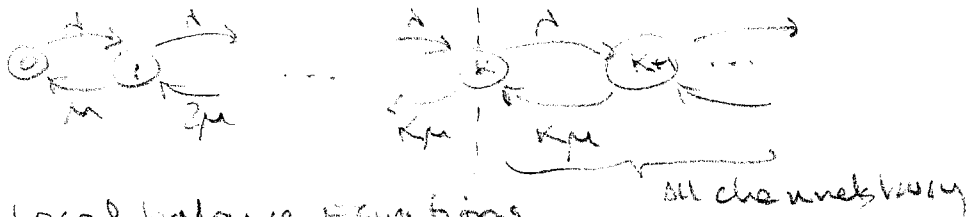$E_K(K\rho) = 0.122 \quad (\text{from chart})$

$P[\text{Loss}] = 0.030 \quad \text{for } Q = 5$

5

Q3
(a)

$$\longrightarrow \boxed{\Pi \cdot K} \longrightarrow \quad (\text{backwork / mean theoretical type})$$

Model in a BD process with state diagram

all channels busy

Local balance Equations

$$\Pi_i = \left(\frac{\lambda}{i\mu}\right)\Pi_{i-1} = \left(\frac{A}{i}\right)\Pi_{i-1} \qquad i \leq K$$

$$\Pi_i = \left(\frac{\lambda}{K\mu}\right)\Pi_{i-1} = \rho \, \Pi_{i-1} \qquad i > K$$

$$\Pi_i = \left(\frac{A^i}{i!}\right)\Pi_0 \qquad i \leq K$$

$$\left(\frac{A^K}{K!}\right)\rho^{i-K}\Pi_0 \qquad i > K$$

for $i > K$

$$\Pi_{K+j} = \left(\frac{A^K}{K!}\right)\rho^j \Pi_0 = \rho^j \Pi_K$$

$$\Pi_0 = \frac{1}{\frac{A^K}{K!}}\left[\frac{(1-\rho)\,E_K(A)}{(1-\rho) + \rho E_K(A)}\right]$$

$$P[\text{Delay}] = P[W > 0] = P[N \geq K] = \sum_{i=K}^{\infty}\Pi_i$$

$$= \sum_{j=0}^{\infty}\Pi_K \rho^j = \frac{\Pi_K}{1-\rho} = \frac{\left(\frac{A^K}{K!}\right)\Pi_0}{1-\rho}$$

$$P[\text{Delay}] = \frac{E_K(A)}{(1-\rho) + \rho E_K(A)} \qquad = D_K(A)$$

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Q3
(a) (i)

$$P[Q_t = i \mid \text{all server busy}]$$

$$P[Q_t = i \mid q_t \geq k] = \left\{ \frac{P[N_t = k+i]}{\sum_{i=0}^{\infty} P[N_t = k+i]} \right\}$$

$$= \left\{ \frac{\pi_k \, p^i}{\sum_{j=0}^{\infty} \pi_k \, p^j} \right\}$$

$$P[Q_t = i \mid \text{Delay}] = (1-p)\, p^i \qquad i = 0, 1, 2, \ldots$$
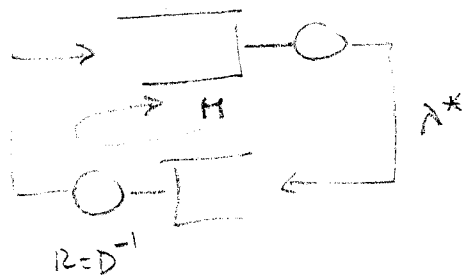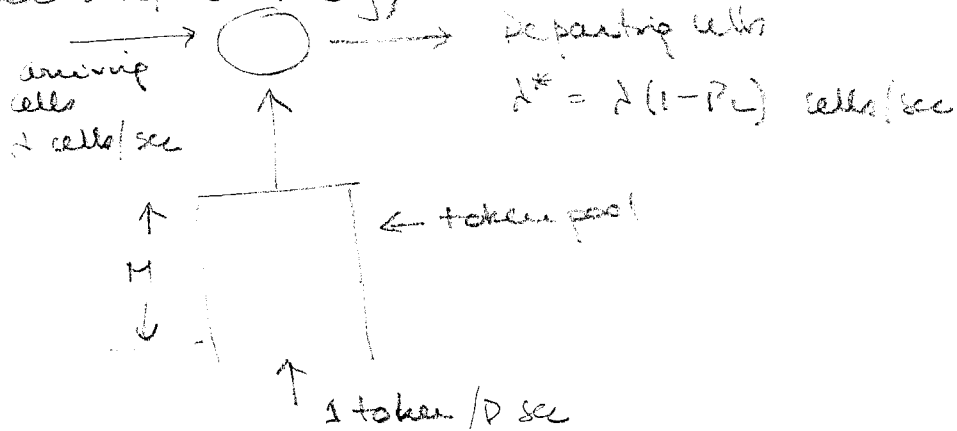
7

Question Number etc. in left margin	Mark allocation in right margin

Q3
(b)

(bookwork / new theory)

arriving
cells
λ cells/sec

departing cells
$\lambda^* = \lambda(1 - P_L)$ cells/sec

← token pool

M

1 token /D sec

M

λ^*

$R = D^{-1}$

Closed queueing
network model,
leaky bucket M/M/1
approximation

$$\lambda^* = \lambda(1 - P_L)$$

for maximum occupancy M

$$P_L = \frac{\rho^M(1-\rho)}{1 - \rho^{M+1}}$$

with $\rho = \lambda/R = \lambda D$

$$\lambda^* = \lambda \left[\frac{1 - \rho^M}{1 - \rho^{M+1}} \right]$$

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Question Number etc. in left margin	Mark allocation in right margin

Q4

(a) Admission control: How much traffic can the network handle if a prescribed QoS for each traffic class is to be maintained while the network utilisation meets some minimum goals. Eg.:

- given a particular VP how many VCs can it handle?
- given a number of active VCs does one admit a new call when a request to set one up arrives?

Discuss meaning and derivation of

$$C_L = \min \left[C_{LG}, C_{LF} \right]$$

where

$$C_{LG} = m R_p + \sigma R_p \sqrt{-\ln(2\pi) - 2\ln \varepsilon}$$

$$C_{LF} = R_p N \frac{1-x}{2} + R_p N \sqrt{\left(\frac{1-x}{2}\right)^2 + \frac{x}{N R_p}}$$

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Paper Code:	Second Examiner:

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Q4
(c)

M/G/1 system

mean message length $= \left(\frac{5}{4} B\right)$

mean square message length $= \left(\frac{7}{4} B^2\right)$

$$E(S) = \frac{5}{4} \frac{160}{64} \times 10^{-3} \text{ sec} = 3.125 \text{ msec}$$

$$E(S^2) = \frac{125}{10} \text{ msec}^2 \quad \left(\frac{7}{4} \frac{(160)^2}{(64)^2} \times 10^{-6}\right)$$

$$\rho = \lambda E(S) = 0.9375$$

$$E(R) = \frac{1}{2} \lambda E(S^2) = 1.65 \text{ ms}$$

Priority 1

$$E(w_1) = \left[\frac{E(R)}{1 - \rho_1}\right]$$

$$\rho_1 = \lambda_1 E(S_1) \qquad = 0.5625$$

$$E(S_1) = \frac{160}{64} \times 10^{-3} = 2.5 \times 10^{-3}$$

$$\lambda_1 = \frac{3}{4} \lambda = 0.75 \times 300$$

$$E(w_1) = \left[\frac{E(R)}{1 - \rho_1}\right] = \frac{1.65}{1 - 0.5625} = 3.77 \text{ ms}$$

$$E(T_1) = E(w_1) + E(S_1) = 6.27 \text{ ms}$$

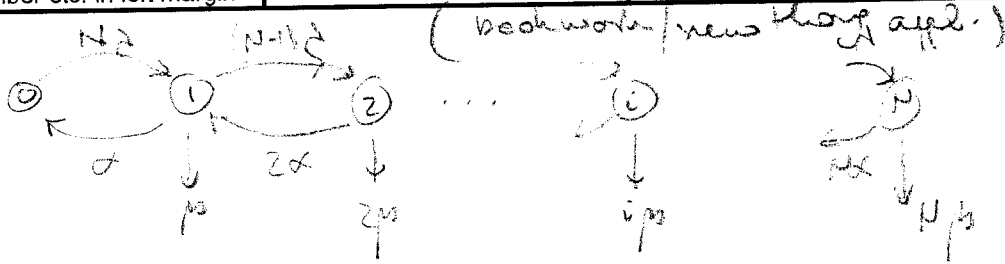$$E(w_2) = \left[\frac{E(w_1)}{1 - \rho_1 - \rho_2}\right] = 60.3 \text{ msec}$$

$$E(T_2) = E(w_2) + E(S_2) = 65.3 \text{ ms}$$

$$E(T) = \frac{3}{4} E(T_1) + \frac{1}{4} E(T_2) = 21.0 \text{ ms}$$

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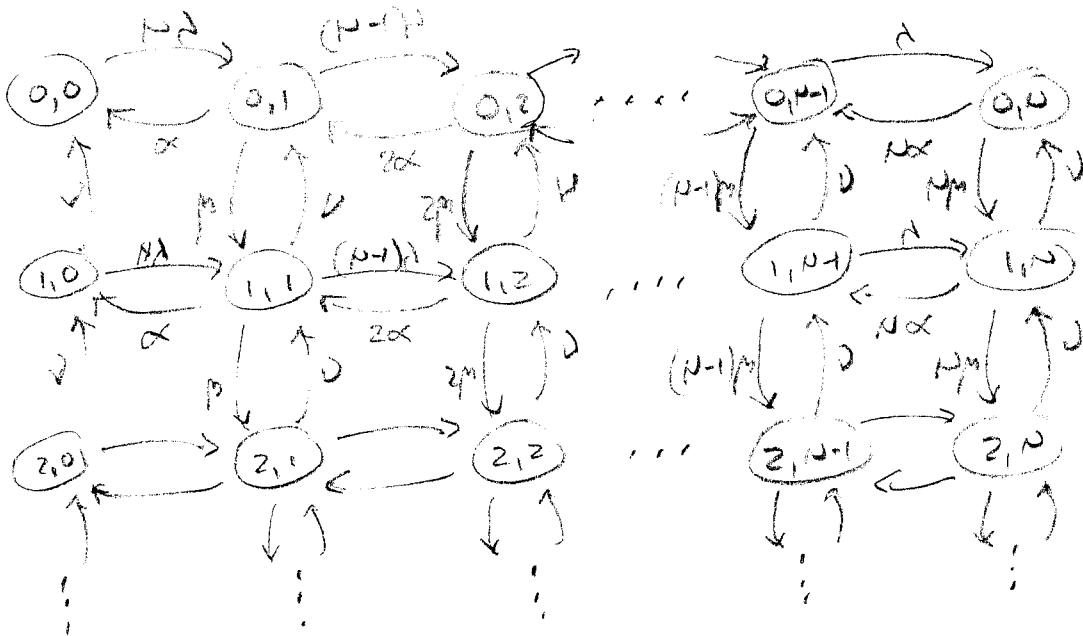| Question Number etc. in left margin | Mark allocation in right margin |

(a)

(bookwork / new theory appl.)

(i) $\pi_c = \binom{N}{i}\left(\frac{\lambda}{\lambda+\alpha}\right)^i\left(\frac{\alpha}{\lambda+\alpha}\right)^{N-i} = \binom{N}{i}\left(\frac{\lambda}{\alpha}\right)^i\left(1+\frac{\lambda}{\alpha}\right)^{-N}$

    2

(ii)

    2

(iii) Average number of cells entering the system (queue)

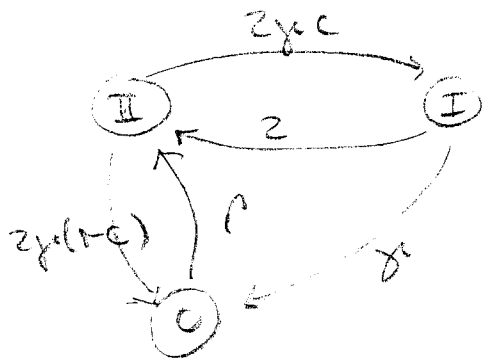$N\mu\left(\frac{\lambda}{\lambda+\alpha}\right)$

system utilisation

$N\rho\left(\frac{\lambda}{\alpha+\rho}\right)$

    4

Q5
(b)

$$Q = \begin{bmatrix} -0.4 & 0.396 & 0.004 \\ 4.0 & -4.2 & 0.2 \\ 8.0 & 0 & -8.0 \end{bmatrix}$$

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solve

$$\begin{bmatrix} -0.4 & 4.0 & 8.0 \\ 0.396 & -4.2 & 0 \\ 0.004 & 0.2 & -8.0 \end{bmatrix} \begin{bmatrix} x_{II} \\ x_1 \\ x_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_{II} + x_1 + x_0 = 1$$

$x_{II} = 0.085580$ $R_{II} = 1.6$

$x_1 = 0.911483$ $R_1 = 1.0$

$x_0 = 0.003579$ $R_0 = 0.0$

$$\lim_{t \to \infty} t(wt) = \sum_{i=0}^{II} R_i x_i = 1.54431$$

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