UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1996

MEng Honours Degrees in Computing Part IV
MSc Degree in Foundations of Advanced Information Technology
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute

PAPER 4.99

MODAL AND TEMPORAL LOGIC

Friday, May 17th 1996, 10.00 - 12.00

Answer THREE questions

For admin. only: paper contains
4 questions
2 pages (excluding cover page)

1a Briefly explain the concept of a modal logic, and distinguish :
(i) a possible worlds model,
(ii) a Kripke model, and
(iii) a canonical Kripke model
for a modal logic.

What is a normal modal logic?
What is the connection between possible truth and necessary truth for such a logic?

b Show that a rule of necessitation is valid for normal modal logic (*i.e.* that if α is a theorem, then that α is necessary, is valid in a Kripke model).

Why does the corresponding rule of "possibilitation" not hold?

Show that Kripke models for standard modal logics where "possibilitation" does hold are models for logics which are deontic in the usual axiomatic sense.

2a Define the standard meaning of:
(i) the unary operator *Next* ,
(ii) the normal (Priorian) future operator <*F*>, and
(iii) the (irreflexive) binary operator *Until*,
over the temporal frame $\langle \mathrm{N}, < \rangle$ of natural numbers with the usual order relation.

b Show that a propositional temporal logic, L_U, over $\langle \mathrm{N}, < \rangle$, with *Until* as the only temporal operator, can express the sense of both *Next* and <*F*>.

Show also that a similar temporal logic, L_F, with <*F*> as only as the only temporal operator, cannot express the sense of *Next*.
(*Hint*, consider a model in which a proposition p is *true* and *false* for arbitrarily large time points, and show that for this model a formula in which p is the only propositional variable is equivalent, in L_F, to *true, false*, p, or $\neg p$, but that this is not so in a logic L_N with *Next*).

c Suggest an axiomatisation in the temporal logic of part (a) of the action modality $\boldsymbol{a}$,
where we intend that when the formula $\boldsymbol{a}\,\alpha$ holds at n in the frame $\langle \mathrm{N}, < \rangle$ and the action is initiated at n then it will terminate at some m, such that $n < m$, where α holds.

The so called frame problem for temporal and action logics is the need to make assumptions about which propositions are unaffected by change. Can you suggest a rule which could help?

3a Briefly justify the use of the normal logic KD45 as a logic for the belief of an ideal agent, where, as usual, the code indicates axioms which ensure the accessibility relation on a Kripke frame is serial, transitive, and Euclidean.
(*You should justify each axiom, but you are not asked to prove the correspondence between axiom and class of models*).

Why do we say that this is a logic for an *ideal* agent?
In which way would a corresponding logic of knowledge differ?

b Briefly explain the additional concepts of a first order modal logic of knowledge and belief which enable the propositions expressed below to be consistent in a world where London and Londres are the same, and where Pierre believes something if he knows it.

Pierre knows that London is not pretty.
Pierre believes that Londres is pretty.

Exhibit a consistent model.
Show that *Pierre does not know that London =Londres.*

4a Let the modality E be defined by the schema $Ep \equiv Dp \wedge C\neg p$, where D is a normal reflexive (KT) modality and $\neg C\neg$ a normal deontic (KD) modality.

Show, by tableaux or otherwise, that axiom K holds for E, but that $\neg E\neg\bot$ is a theorem, and thence the rule of necessitation fails, so E is not a normal modality.

b Consider a multi-modal specification of updates to a dynamic knowledge base, with distinct modalities D (for new data), K (know), N (temporal next), and E ("bring about that"). Suggest reasonable interaction axioms which ensure *firstly* that updates will include new data, *secondly* that consistent existing knowledge is preserved.

If this were to be the knowledge system of a crude but conscious agent, what additional concepts would you expect in the symbolic language?