Model answer to Q 1: bookwork

$(i)\quad \tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'} = 6.9x10^{-3}$

$\therefore Q = \frac{1}{\tan\delta} = 145$

$(ii)\quad \lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_r'}} = 18.6\,[mm]$

$k = \frac{1}{\lambda} = 53.75\,[\lambda / m]$

$\therefore \gamma \approx 1.165\,[Np/m] + j337.7\,[rad/m]$

$(iii)\quad \delta = \frac{1}{\alpha} = 858\,[mm]$

$(iv)\quad l = \lambda/4 = 4.65\,[mm]$

$(v)\quad Power\ Attenuation = e^{-2(\alpha l)}$

$\therefore \alpha l = 0.0217\,[Np/\lambda]$

$\therefore Power\ Attenuation = \alpha l(20\log e) = 8.686\alpha l = 0.188\,[dB/\lambda]$

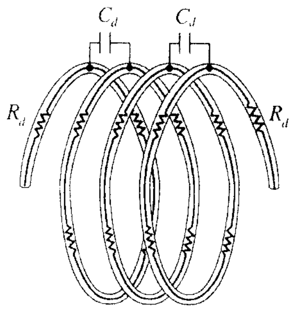
$(vi)\quad \eta = \frac{j\omega\mu}{\gamma} \quad where \quad \omega = 2\pi f \quad and \quad \mu = 4\pi x10^{-7}\,[H/m]$

$\therefore \eta = 233.8 + j0.8\,[\Omega]$

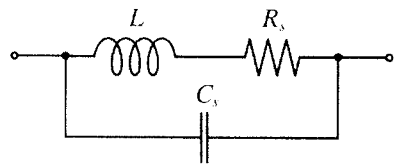
$(vii)\quad P_D = \frac{|E|^2}{\mathrm{Re}\{\eta\}} = 4.3\,[mW/m^2]$

Model answer to Q 2(a): bookwork

RF Inductors



Distributed capacitance and series resistance in the inductor coil.



Equivalent circuit of the high-frequency inductor.

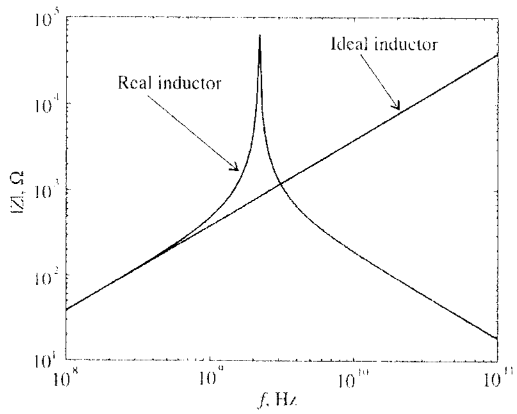
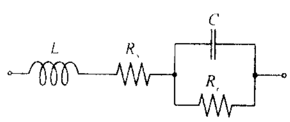
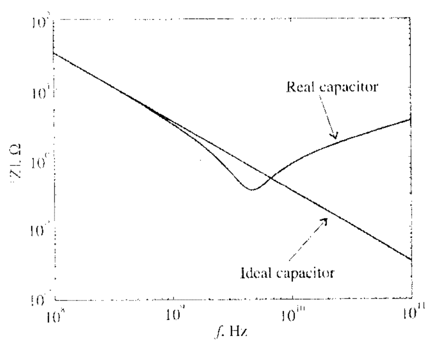


Figure 1-17 Frequency response of the impedance of an RFC.

RF Capacitors

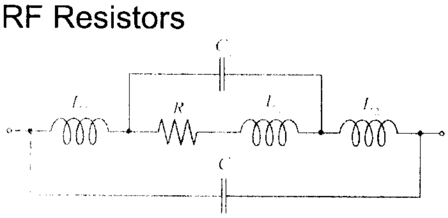


Electric equivalent circuit for a high-frequency capacitor.

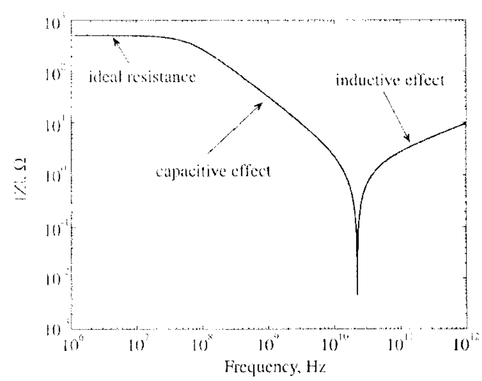


1-12 Absolute value of the capacitor impedance as a function of frequency.

RF Resistors

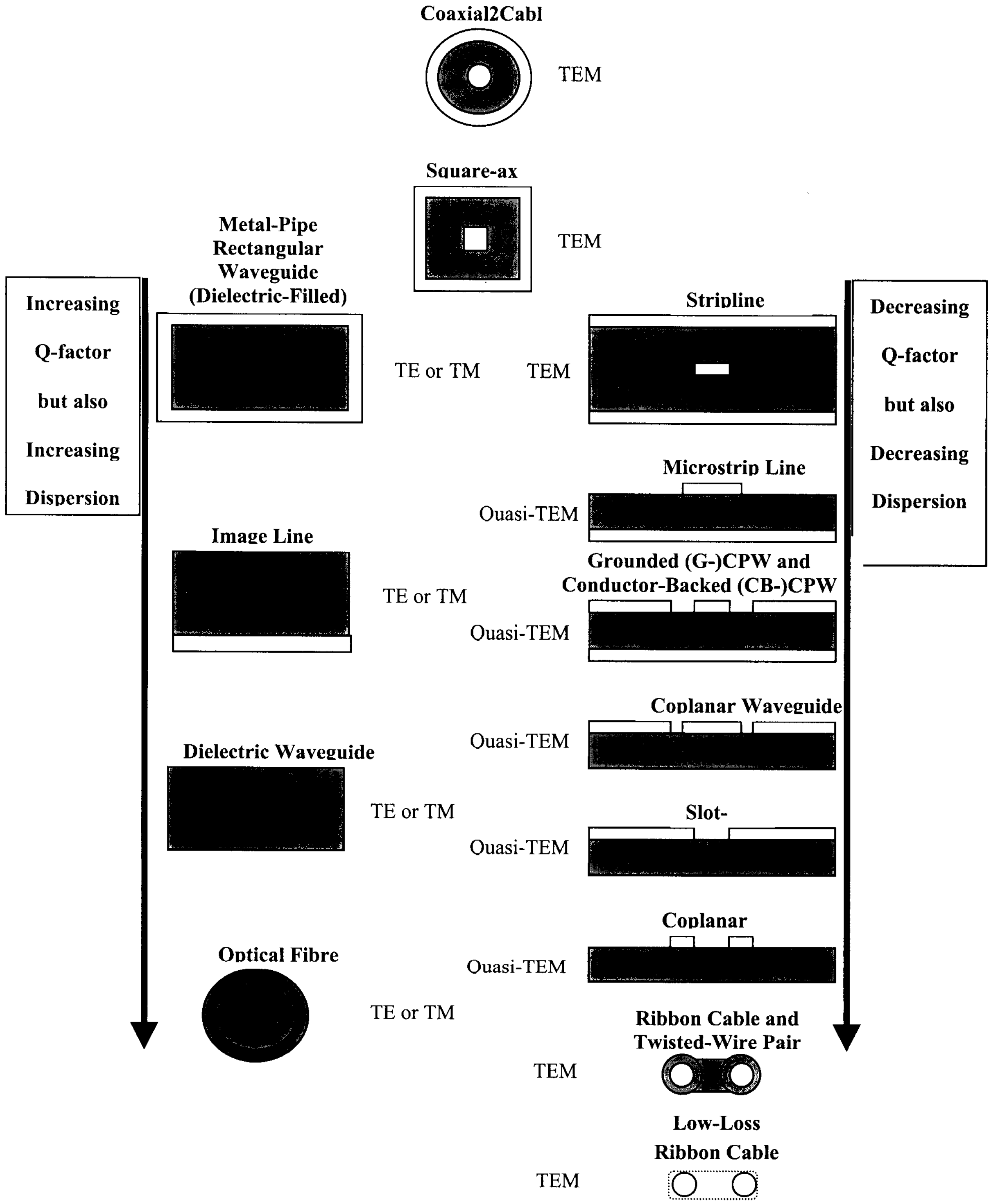


Electric equivalent circuit representation for a high-frequency wire-wound resistor.



1-10 Absolute impedance value of a 500-Ω thin-film resistor as a function of frequency.

Model answer to Q 2(b): bookwork



Model answer to Q 2(c): bookwork

- Propagation can be easily explained by inspection of Maxwell's equations:

$$\nabla x\overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

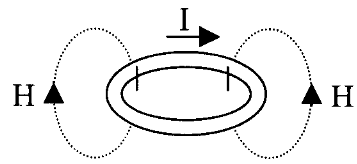
$$\nabla x\overline{H} = \overline{J}_C + \frac{\partial \overline{D}}{\partial t} \qquad \overline{B} = \mu\overline{H}$$

where

$$\nabla \cdot \overline{D} = \rho \qquad \overline{D} = \varepsilon\overline{E}$$

$$\nabla \cdot \overline{B} = 0$$

- Consider a time-varying conduction current (so the free-electron charge is accelerating) flowing around a theoretical loop of lossless wire.



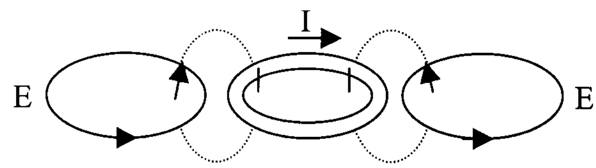
- The conduction current creates a circulating (i.e. curling) H-field, with lines that encircle the current loop. This is represented mathematically by Ampere's Law:

$$\overline{J}_C \rightarrow \nabla x\overline{H}$$

- Since the conduction current is time-varying then so must be the H-field:

$$\overline{J}_C(t) \rightarrow \nabla x\overline{H}(t)$$

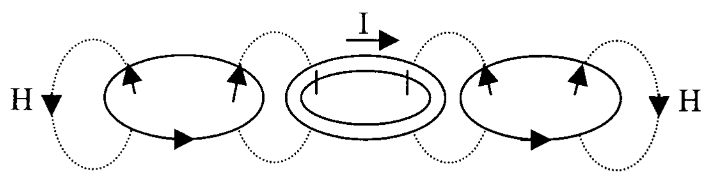
- The time-varying H-field, in-turn, creates a time-varying and circulating (i.e. curling) E-field, with lines that encircle the H-Field loops:



- This is represented mathematically by Faraday's Law:

$$-\mu\frac{\partial \overline{H}(t)}{\partial t} \rightarrow \nabla x\overline{E}(t)$$

- The time-varying E-field, in-turn, creates a time-varying and circulating (i.e. curling) H-field, with lines that encircle the E-Field loops:

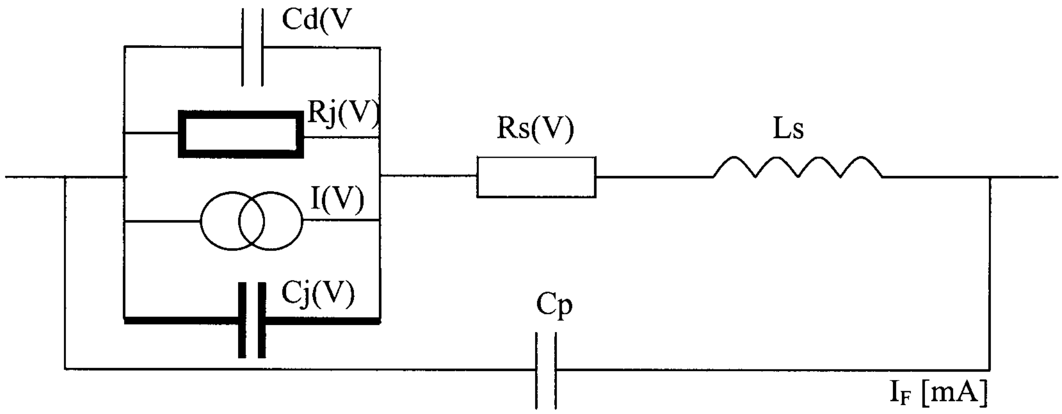


- This is represented mathematically by Maxwell's Law:

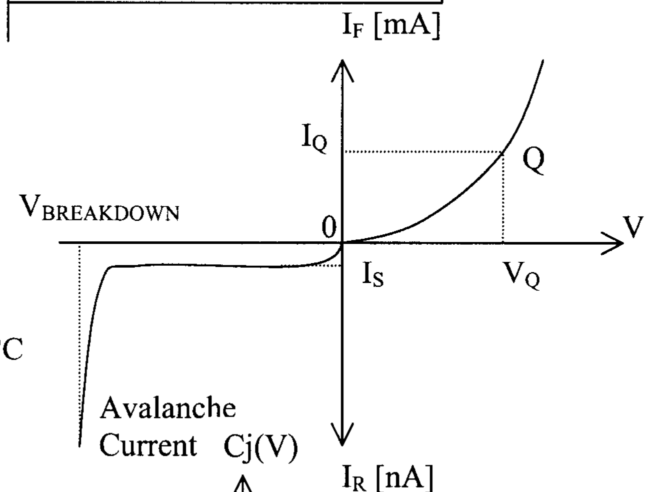
$$\varepsilon\frac{\partial \overline{E}(t)}{\partial t} \rightarrow \nabla x\overline{H}(t)$$

- The time-varying H-field, in-turn, creates a time-varying and circulating E-field, etc.....

Model answer to Q 3(a): bookwork

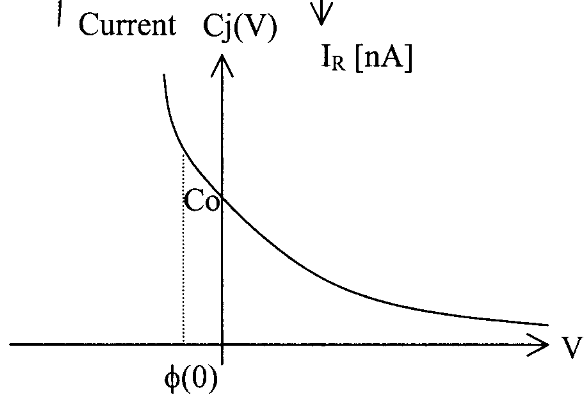


$$I_d = I_s\left(e^{-\frac{V}{nV_{TH}}} - 1\right)$$



where,

V = applied bias potential
n = ideality factor
Thermal voltage, $V_{TH} = \left(\frac{kT}{e}\right)$ = 25.7mV at 24°C



$$C_j(V) = \frac{C_0}{\left(1 + V/\phi(0)\right)^{\gamma}}$$

where,

$C_0 = C_j(V)$ with V=0
V = reverse bias potential
$\phi(0)$ = built-in barrier (or junction or contact) potential
$\phi(0)$ = 0.7-0.8V for Si and 1.2-1.3V for GaAs

γ = (Cj(V) vs. V) slope (or parameter) exponent when plotted on log-log paper
γ = 0.5 diode has an abrupt junction (AJ) doping profile
γ > 0.5 diode has an hyperabrupt junction (HAJ) doping profile

Model answer to Q 3(b): bookwork

The *p-n* junction diode is not used in forward bias mode because the parasitic capacitances (Cj(V) + Cd(V)) effectively shunt RF current around the junction leakage resistance, Rj(V), that is used at low RF frequencies for mixer applications. The main application of a reverse biased *p-n* junction diode at microwave frequencies is to realise a voltage-controlled variable reactor (or varactor) diode. These voltage-controlled capacitors are used widely in the following applications:- analogue phase shifters, voltage-controlled oscillators (VCOs), tunable filters, frequency multipliers, nonlinear delay lines and parametric amplifiers.

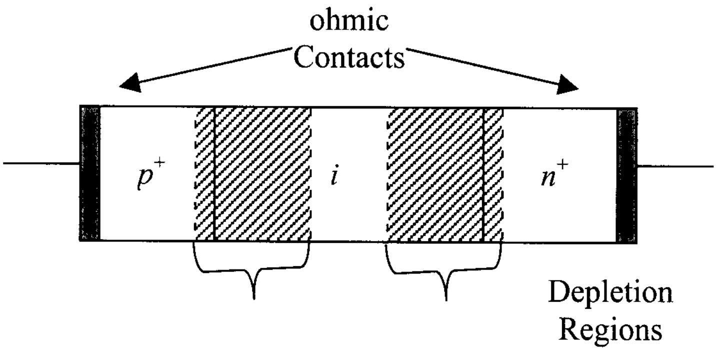
Maximum total capacitance ratio, $TCR = \frac{Cj(V)|_{\max}}{Cj(V)|_{\min}} \Rightarrow \frac{Cj(0)}{Cj(V_{Breakdown})}$

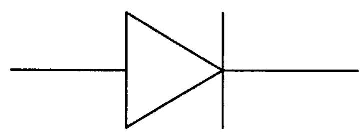
Maximum tuning (or frequency) ratio, $M = \sqrt{\left(1+\frac{V_{\max}}{\phi(0)}\right)^{\gamma}} \Rightarrow \sqrt{\left(1+\frac{V_{Breakdown}}{\phi(0)}\right)^{\gamma}}$

$$M = \sqrt{TCR}$$

Compared to abrupt junctions, hyperabrupt junction diodes provides a larger tuning ratio, because γ is larger with HAJ. It can be seen that $\gamma = 2$ would give a linear tuning characteristic.

Model answer to Q 3(c): bookwork





PIN or p-η-n or p-π-n diode

n-type p-type

light doping in the intrinsic (i)-region

The main applications of PIN diodes are to act as switching and variable resistor (or varistor) diodes. These are used in gain control circuits, for automatic gain control applications and amplitude modulators.

- Forward Bias

The carriers are injected into the intrinsic region, resulting in an increased carrier concentration in the i-region. If a low frequency signal is applied (i.e. $f < f_{\min}$), the diode will behave as a normal p-n junction diode. However, if the frequency is sufficiently high:

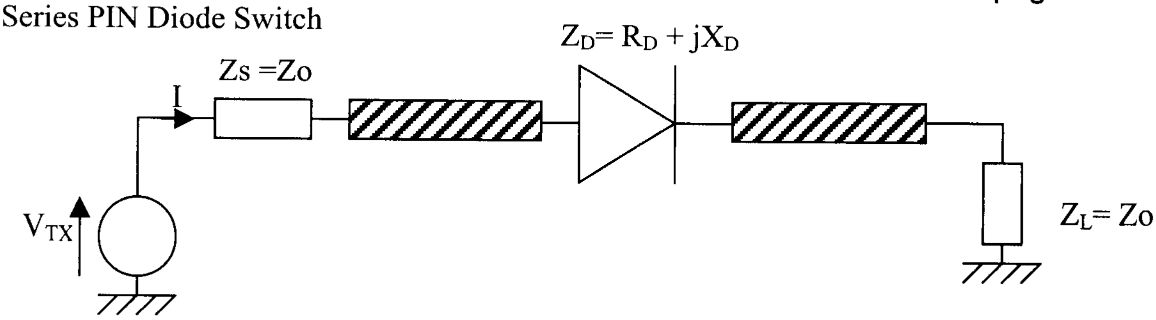
$$\text{i.e. } f \geq f_{\min} = \frac{1}{\textit{carrier recombination lifetime}} \quad \text{e.g.} = 10\text{ MHz}$$

then the carriers in the i-region will tend to oscillate about their mean position and not recombine. As a result, the i-region behaves as a conductor with its conductance dependant on the number of carriers in the i-region – which, in turn, depends on the level of forward bias current. This is known as "conductivity modulation".

- Reverse Bias

The PIN diode acts as a single p-n junction, because the depletion regions extend until the entire i-region is swept free of carriers.

- Series PIN Diode Switch



For large PIN diodes, $Z_D \sim 0$ when forward biased (i.e. when the switch is on) and all the RF power incident on the diode flows straight though it and is absorbed by the load resistance, Z_L. However, $Z_D = R_D + j\, X_D$ when reverse biased (i.e. when the switch is off). Here, the diode has finite isolation and has to hold-off the entire RF voltage, since it is the highest impedance in the loop. If significant reverse bias current is to be prevented from flowing, for all parts of the RF cycle, then:

Model answer to Q 3(d): bookwork

$$V_Q = \frac{V_{BREAKDOWN}}{2}$$

$$\therefore V_{TX}\,|_{peak} = \frac{V_{BREAKDOWN}}{2} \quad and \quad V_{TX}\,|_{RMS} = \frac{V_{BREAKDOWN}}{2\sqrt{2}}$$

$$I = \frac{V_{TX}}{2Zo + Z_D}$$

$$\therefore P_D = \frac{I\,|_{peak}{}^2\, R_D}{2}\,|_{X_D=0} = V_{TX}\,|_{RMS}{}^2\, \frac{R_D}{(2Zo + R_D)^2}$$

$$and,\, P_L = \frac{I\,|_{peak} \cdot I\,|^*_{peak}\, Zo}{2} = V_{TX}\,|_{RMS}{}^2\, \frac{Zo}{(2Zo + R_D)^2 + X_D{}^2}$$

$$when\ Z_D = 0, P_L = P_L\,|_{max}$$

$$\therefore P_L\,|_{max} = V_{TX}\,|_{RMS}{}^2\, \frac{1}{4Zo}$$

$$also, attenuation, \alpha = \frac{P_L(switch\ on) = P_L\,|_{max}}{P_L(switch\ off) = P_L} = \left(1 + \frac{R_D}{2Zo}\right)^2 + \left(\frac{X_D}{2Zo}\right)^2$$

$$Z_D = \frac{Ri}{1 + (\omega CiRi)^2} - j\frac{\omega CiRi^2}{1 + (\omega CiRi)^2} = 10 - j318\Omega$$

$$\therefore \alpha = 11.3 \equiv 10.5dB$$

Model answer to Q 4(a): bookwork

EM simulation can exist in 3 different domains. If l is a dimension of the structure we are simulating, and λ is the wavelength of EM radiation (in the structure, if the structure is dielectric, or in the surrounding space if the structure is conductive), there are 3 domains for solving Maxwell's equations:

- **$\lambda >> l$ Circuit Theory**
- **$\lambda \sim l$ Microwave theory**
- **$\lambda << l$ Geometric Optics**

[3]

Model answer to Q 4(b): bookwork

The differential form Maxwell's equations are discretised on a rectangular grid of size δ. Then the derivatives become differences:

$$\frac{\partial f}{\partial x} = \frac{f(n+1) - f(n-1)}{2\delta}$$

[2]

(note, the side derivative is acceptable as an answer only if clearly stated as a side derivative)

and

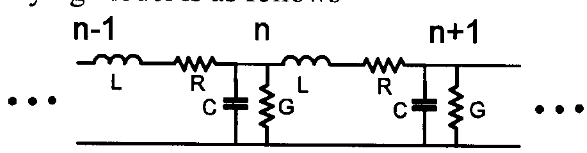
$$\frac{\partial^2 f}{\partial x^2} = \frac{f(n+1) - 2f(n) + f(n-1)}{\delta^2}$$

This way the PDE becomes a sparse linear system, which can be solved with linear algebra techniques.

[2]

Model answer to Q 4(c): bookwork

The transmission line method is useful for electromagnetic simulation at lower frequencies, when $L << \lambda$. The underlying model is as follows



[3]

Model answer to Q 4(d): computed example

the input impedance does not change by adding a segment, therefore:

$$Z = \left(\frac{Z}{j\omega CZ + 1}\right) + j\omega L \Rightarrow j\omega CZ^2 + \omega^2 LCZ - j\omega L = 0$$

with the substitutions:

$z' = \sqrt{\frac{C}{L}} Z$ and $\omega' = \omega\sqrt{LC}$ we find:

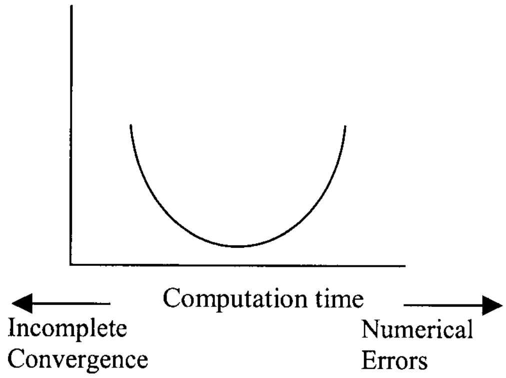
$$z' = \frac{-j\omega'}{2}\left(1 \pm \sqrt{1 - \frac{4}{\omega'^2}}\right)$$

this impedance has a real part if $\omega' < 2 \Rightarrow \omega \quad 2\sqrt{LC}$. It has a reactive (capacitive) component at all frequencies. In the limit of vanishing L,C the cutoff is at infinite frequency and the impedance becomes real, which is the exact result.

[5]

Model answer to Q 4(e): bookwork + application

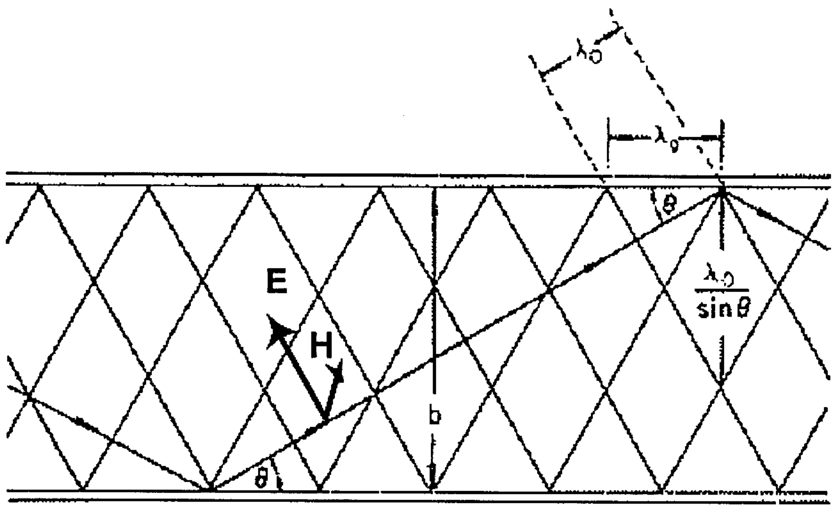
Error versus numerical size.



In the case of TLM method, the incomplete convergence error is due to the difference of the impedance from the exact result. (computation time is a power of the discretisation grid refinement)

[5]

Model answer to Q 5(a): bookwork



A parallel plate waveguide, side view, with wavefronts.

Propagation only as long as $\theta < \pi/2$, i.e as long as the following equation has a solution in ϑ:

$$b = \frac{n\lambda_0}{2\sin(\vartheta)}$$

with λ_0 the free space wavelength. n is the mode index.

The maximum wavelength (cutoff wavelength), for a given n, propagation can occur is:

$$2b = n\lambda_C \Rightarrow \sin\vartheta = \frac{\lambda_0}{n\lambda_C}$$

[3]

Model answer to Q 5(b): bookwork

The guide wavelength is shown in the figure, and given by:

$$\lambda_g = \frac{\lambda_0}{\cos\vartheta}$$

or $\lambda_g = \dfrac{\lambda_0}{\sqrt{1-\left(\dfrac{n\lambda_0}{2b}\right)^2}} = \dfrac{\lambda_0}{\sqrt{1-\left(\dfrac{\lambda_0}{\lambda_c^2}\right)^2}}$

The cutoff wavelength for the nth mode is :

$$\lambda_C = \frac{2b}{n}$$

[3]

Model answer to Q 5(c): bookwork

The phase velocity is $v_p = \dfrac{c_0}{\cos\vartheta} = \dfrac{c_0\lambda_g}{\lambda_0} > c_0$ and the group velocity is the projection of the partial wave phase velocity on the waveguide walls: $v_g = c_0\cos\vartheta$. The product is $v_p v_g = c_0^2$

[2]

Model answer to Q 5(d): bookwork

The equation for the quide wavelength is still valid, except now the cutoff wavelength for the m,n mode of a rectangular waveguide is found in terms of the wavevectors in the x,y directions:

$k^2 = k_x^2 + k_y^2 + k_z^2 = -\dfrac{\omega^2}{c^2}$ with $k_x = \dfrac{m\pi}{a}, k_y = \dfrac{n\pi}{b}$ for the standing waves between the waveguide walls.

$$k_c = \sqrt{k_x^2 + k_y^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \frac{2\pi}{\lambda_c}$$

[6]

Model answer to Q 5(e): computed example

Noticing that a=2b, the cutoff wavelength becomes:

$$\frac{2}{\lambda_c} = \frac{1}{b}\sqrt{\left(\frac{m}{2}\right)^2 + (n)^2}.$$

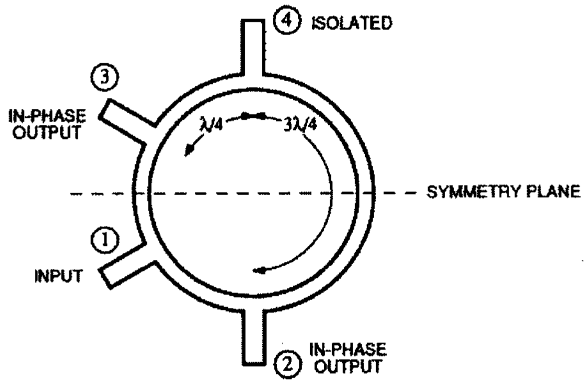
For the (1,0) mode, $f_c = \dfrac{c_0}{\lambda_c} = \dfrac{c_0}{4b} = 7.85GHz$,

The next mode is the (0,1) mode with:

$$f_c = \frac{c_0}{\lambda_c} = \frac{c_0}{2b} = 15.7GHz$$

Model answer to Q 6(a): bookwork

The hybrid ring coupler is an interference device, the interference occurring between CW and CCW propagating waves.



[2]

Model answer to Q 6(b): bookwork

Because of symmetry we only need to consider driving port 1 or port 2.

Driving port 1:

Output at port 2: $L_{CW} = \lambda/4$. $L_{CCW} = 5\lambda/4$ Path difference is λ, ie two partial waves are in phase, and of equal amplitude, so that each ½ input power, ¼ of the cycle delayed relative to Port 1. Same argument for port 3, with CW and CCW paths reversed. Then

$$V_2 = V_3 = \frac{1}{\sqrt{2}} V_1 \exp(-j\pi/4).$$

At port 4 $L_{CW} = \lambda/2$, $L_{CCW} = \lambda$. So there is half wavelength path difference and the port is isolated.

[5]

Driving port 2:

Output at port 1: $L_{CW} = 5\lambda/4$. $L_{CCW} = \lambda/4$ Path difference is λ, ie two partial waves are in phase, and of equal amplitude, so that each ½ input power, ¼ of the cycle delayed relative to Port 2.

$$V_1 = \frac{1}{\sqrt{2}} V_2 \exp(-j\pi/4).$$

At port 4 $L_{CW} = L_{CCW} = 3\lambda/4$, So there is the same path length to both ports, i.e. they are in phase relative to each other, and ¾ of the cycle delayed relative to port 2. This implies ports 1 and 4 are out of phase to each other.

Port 3 is isolated, since CW and CCW paths to it differ by ½ wavelength.

[5]

Model answer to Q 6(c): bookwork

One can arrive at the equivalent half circuit, if any of the following drive vectors (are used:

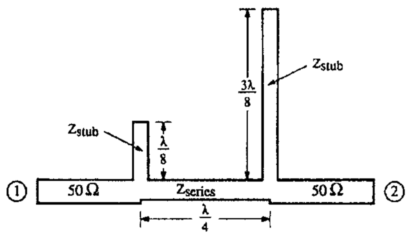
$$a = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \text{(even)},\ b = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \text{(odd)}, c = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \text{(even)},\ d = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \text{(odd)}.$$

[2]

Clearly,

$$\begin{bmatrix} V \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} V(a+b) \text{ and } \begin{bmatrix} 0 \\ V \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} V(c+d)$$

[2]



The stubs are open for the even modes, and shorted for the odd modes.

[2]

Note : student is not expected to use driving vector notation, but reproduce its content.