UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1999

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER 1.9

MATHEMATICAL METHODS AND GRAPHICS

Wednesday, May 12th 1999, 10.00 – 12.00

Answer FOUR questions

For admin. only:
paper contains 6 questions

1. A three dimensional graphics scene made up of polygons is to be drawn in perspective projection viewed from the origin, with the direction of view along the z-axis.

The viewplane has equation z=5, and the viewing window defining the world coordinate system has corners given by the points:

{10,10,10}, {10,-10,10}, {-10,10,10} and {-10,-10,10}.

One of the polygons that makes up the scene is a triangle with corners at the following three dimensional points:

P0={10,40,50}, P1={10,-5,50}, P2={240,40,80}

The scene is to be drawn in a window whose bottom left hand corner is at pixel co-ordinate [128,0] and whose top right hand corner is at pixel co-ordinate [255,127]. The pixel origin is at the bottom left hand side of the screen.

a. What are the x and y co-ordinates of the projections of the points P0, P1, and P2 onto the viewplane in world co-ordinates?

b. Sketch what would be seen in the window on the screen.

c. What is the matrix that calculates the projection, using homogenous co-ordinates?

d. Calculate the values of A,B,C and D in equation pair that carries out the 2D normalisation transformation between the world co-ordinate system defined by the window, and the actual pixel addresses:

Xpix = A x + B
Ypix = C y + D

e. If the user moves the window so that its bottom right hand corner is now at pixel (158, 50), without changing the size, how do the values of A B C and D change?

2. Anti-Aliasing

a. Explain, with a suitable diagram, what is meant by an alias frequency. In what way do alias frequencies manifest themselves in raster images?

b. Explain how the effect of alias frequencies can be reduced by means of a low pass convolution filter. Suggest a suitable filter for this purpose.

c. Explain how super-sampling can be used to reduce alias effects in raster images.

d. What are the advantages and disadvantages of the anti-aliasing methods discussed in parts b and c?

e. Suggest why alias effects can be particularly problematic when mapping texture onto polygons.

turn over

3 a The points $A = (1, 1, 0)$, $B = (1, 0, 1)$ and $C = (0, -1, 0)$ define a plane. The points $D = (2, 1, 0)$ and $E = (1, 4, 3)$ define a line. Find the point at which they intersect.

b For what values of α and β are there unique, no and infinitely many solutions of

$$x + 2y + z = 1$$
$$2x + \alpha y + 2z = \beta$$
$$9x + (2 + 4\alpha) y + 3\alpha z = 2$$

Where there are infinitely many solutions, state their form.

Parts a) and b) carry equal marks.

4 a Find all first and second partial derivatives of

$$f(x, y) = \mathrm{Sin}\sqrt{(x/y)}$$

b Find the stationary point of

$$f(x, y, z) = e^{-(x^2 + y^2 + z^2 - 2x - 4y - 6z)/2}$$

and determine whether it is a maximum, minimum or saddle point.

c A function of three variables is defined by

$$f(x, y, z) = 8x^9 y^{10} z^{11}$$

By considering the total derivative of f (x, y, z) find the approximate percentage change in f if x, y, and z are increased by 1% each.

Parts a), b) and c) carry 25%, 50% and 25% of the marks respectively.

turn over

5 a Find all the roots of the polynomial equation

$$z^6 = 1 + i$$

Illustrate where the solutions lie in an Argand diagram.

b Show that

$$\text{Cos } 5\theta = 16 \text{ Cos}^5\theta - 20 \text{ Cos}^3\theta + 5 \text{ Cos } \theta$$

c Show that

$$\text{Sin } \theta + \text{Sin } 2\theta + \ldots \text{Sin } n\theta$$

$$= \frac{\text{Sin } n\theta}{2} + \frac{\text{Sin } \theta \,(1 - \text{Cos } n\theta)}{2\,(1 - \text{Cos}\theta)}$$

Parts a), b), and c) carry 25%, 25% and 50% of the marks respectively.

6 a A Sequence $\{x_n\}$ is defined by the recurrence relation

$$x_n + 2\,x_{n-1} + x_{n-2} = n\,(-1)^n$$

Find the general solution.

b What are the Maclaurin series for Sin x and Cos x?

Given the result of question 5c, prove that

(i) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

(ii) $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

Parts a) and b) carry equal marks.

End of Paper