EE4-26
EE9-CS5-3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE PART IV: MEng and ACGI

ESTIMATION AND FAULT DETECTION

Thursday, 14 May 10:00 am

Time allowed: 3:00 hours

Corrected Copy

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	T. Parisini
	Second Marker(s) :	D. Angeli

ESTIMATION AND FAULT DETECTION

Information for candidates:

- **One-step ahead Kalman predictor:**

$$\hat{x}(t+1|t) = F\hat{x}(t|t-1) + K(t)[y(t) - H\hat{x}(t|t-1)]$$

- **Kalman predictor gain**

$$K(t) = FP(t)H^\top \left(V_2 + HP(t)H^\top\right)^{-1}, \quad t = 1,2,\ldots$$

- **Riccati equation**

$$P(t+1) = F\left[P(t) - P(t)H^\top \left(V_2 + HP(t)H^\top\right)^{-1} HP(t)\right]F^\top + V_1, \quad t = 1,2,\ldots$$

- **Realization: observer canonical form**

Given

$$Y(s)/U(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots a_1 s + a_0} \quad \text{with} \quad m < n$$

then:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & -a_{n-2} \\ 0 & \cdots & 0 & 1 & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_m \\ 0 \\ \vdots \\ 0 \end{bmatrix} u \\ \\ y = [0 \cdots 0\, 1] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} \end{cases}$$

1. Consider the continuous-time dynamic system depicted in Fig. 1.1.

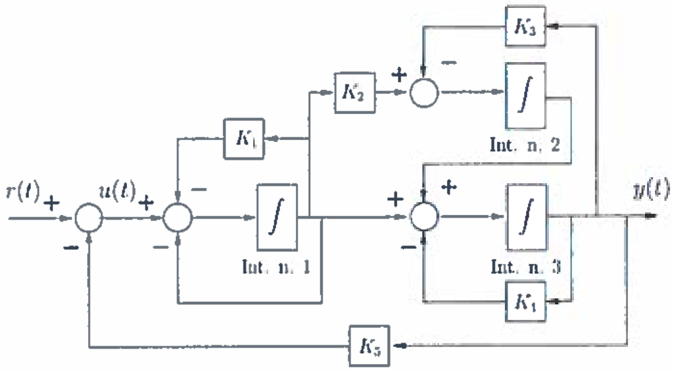

Figure 1.1 Block diagram for Question 1.

made of the interconnection of three integrators denoted by "Int. n. 1", "Int. n. 2", and "Int. n. 3", respectively. The blocks with a symbol K_i written inside represent linear multiplication blocks, that is, the outputs of these blocks are given by the product times K_i of their respective inputs.

a) Determine a state-space description of the whole dynamic system depicted in Fig. 1.1 where $r(t)$ is the input and $y(t)$ is the output.

[3 marks]

b) Set $K_1 = 5$, $K_2 = 6$, $K_3 = 4$, $K_4 = 4$, and $K_5 = 0$. Show that the system is not completely observable by applying the following distinct methods:

i) by analysing the observability matrix. [3 marks]

ii) by inspecting the vector $C(sI - A)^{-1}$, where A is the state matrix and C is the output row vector determined in your answer to Question 1a).
[3 marks]

c) Using the values $K_1 = 5, K_2 = 6, K_3 = 4, K_4 = 4$, and $K_5 = 0$ set in Question 1b), determine a state-space description of the dynamic system depicted in Fig. 1.1 which is equivalent to the one determined in your answer to Question 1a) and that identifies the observable and the non-observable sub-systems. Moreover, determine a basis for the non-observable vector subspace X_{no}.

[8 marks]

d) Suppose that $K_1 = 5$, $K_2 = 6$, $K_3 = 4$, and $K_4 = 4$ (as in Questions 1b) and 1c)) and let now K_5 be arbitrary and $\neq 0$. Does this different setting of K_5 change the observability properties discussed in your answer to Question 1b)? Justify your answer.

[3 marks]

2. Consider the continuous-time dynamic system depicted in Fig. 2.1.

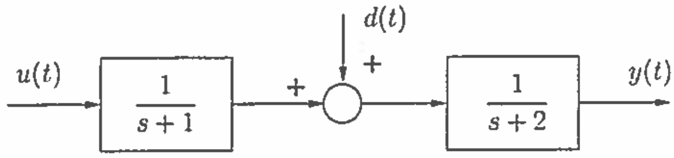

Figure 2.1 Block diagram for Question 2.

a) Suppose that $d(t) = 0, \forall t$. Show that the order of a state-space description of the whole dynamic system depicted in Fig. 2.1 is equal to 2, where $u(t)$ is the input and $y(t)$ is the output. Moreover, determine a state-space description of such a system.

[3 marks]

b) Suppose now that $d(t) = K \cdot 1(t)$ where $K > 0$ is an *unknown* scalar with $1(t)$ denoting the unit-step function.

i) Modify the state equations determined in your answer to Question 2a) and devise an observer-based architecture to estimate the state of the system (according to the state equations determined in your answer to Question 2a)) and to estimate *simultaneously* the scalar K.

[6 marks]

ii) Verify that it is possible to design an observer in the observer-based architecture determined in your answer to Question 2b)-i) to provide the asymptotic estimate of the state vector and of the magnitude K of the input $d(t)$.

[3 marks]

Please do not design the observer in your answers to Questions 2b)-i) and 2b)-ii).

c) Denoting by $\hat{x}(t)$ the estimate of the state $x(t)$ defined in your answer to Question 2a) and by $\widehat{K}(t)$ the estimate of the scalar K, let $e_x(t) = x(t) - \hat{x}(t)$ denote the state estimation error and let $e_K(t) = K - \widehat{K}(t)$ denote the estimation error of the unknown magnitude K of the input $d(t)$.

Defining the vector $e(t) := [e_x(t)^{\top}, e_K(t)]^{\top}$ (the superscript $^{\top}$ stands for "transpose"), suppose that its dynamics obeys

$$\dot{e}(t) = Fe(t).$$

In the context of the observer-based architecture determined in your answer to Question 2b)-i), design a full-order observer such that the eigenvalues of F are:

$$\lambda_1 = -5, \lambda_2 = -5, \lambda_3 = -5$$

[8 marks]

3. Consider the following discrete-time dynamic system affected by state and output disturbances:

$$\begin{cases} x(t+1) = -\frac{1}{3}x(t) + \xi(t) \\ \\ y(t) = 2x(t) + \eta(t) \end{cases} \tag{3.1}$$

where $\xi(\cdot) \sim WGN(0,4)$, $\eta(\cdot) \sim WGN(0,1)$ (Gaussian zero-mean stochastic processes) and the stochastic processes $\xi(\cdot)$ and $\eta(\cdot)$ are supposed to be independent of each other. Moreover, the initial state is a random variable $x(1) \sim WGN(3,9)$ supposed to be independent of $\xi(\cdot)$ and $\eta(\cdot)$.

a) With reference to system (3.1), consider the one-step ahead time-varying optimal Kalman predictor of the state x. Compute 4 values of the Riccati matrix $P(t), t = 1,\ldots,4$ and the corresponding values of the gain $K(t), t = 1,\ldots,4$. Establish whether the sequence of Riccati matrices $P(t)$ eventually converges for increasing values of t to a positive-definite steady-state matrix $\overline{P}$.

[4 Marks]

b) Write the Algebraic Riccati Equation (ARE) and show that the ARE admits an admissible solution $\overline{P}$. Compute the corresponding constant gain $\overline{K}$. Compare the time-behaviors of the sequences $P(t), t = 1,\ldots,4$ and $K(t), t = 1,\ldots,4$ determined in your answer to Question 3a) with the steady-state values $\overline{P}$ and $\overline{K}$. Comment on your findings.

[5 Marks]

c) Write the difference equation yielding the one-step ahead optimal steady-state Kalman prediction $\hat{x}(t+1|t)$ and draw the block-diagram of the predictor.

[3 Marks]

d) Let $e(t) = y(t) - \hat{y}(t|y-1)$ denote the output prediction error.

i) Determine the discrete-time transfer function $G_{\xi e}$ from the noise ξ to the output prediction error e.

[4 Marks]

ii) Determine the discrete-time transfer function $G_{\eta e}$ from the noise η to the output prediction error e.

[4 Marks]

4. Consider the continuous-time control system depicted in Fig. 4.1.

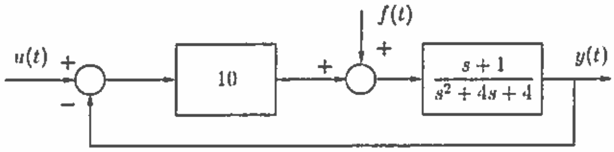

Figure 4.1 Block diagram for Question 4.

where $u(t)$ is a known input and $f(t)$ denotes an actuator fault occurring at time T_0. Suppose that only one actuator fault may occur during the whole time-horizon $t \in (0,\infty)$.

Moreover, suppose that the actuator fault $f(t)$ may only assume the following form:

$$f(t) = Ke^{\alpha t}, \forall t \geq T_0 \tag{4.1}$$

where T_0 denotes the *unknown* time of fault occurrence and where $K > 0$ is an *unknown* scalar whereas $\alpha > 0$ is a positive and *known* scalar.

a) Consider the system depicted in Fig. 4.1 before the possible occurrence of a fault ($t < T_0$).

i) Determine a state-space description of the whole dynamic system depicted in Fig. 4.1, where $u(t)$ is the input and $y(t)$ is the output.

[3 marks]

ii) Denoting by $\hat{x}(t)$ the estimate of the state $x(t)$ referred to the state equations determined in your answer to Question 4a)-i), let $e(t) = x(t) - \hat{x}(t)$ denote the state estimation error and suppose that its dynamics obeys

$$\dot{e}(t) = Fe(t)$$

Design a full-order state observer such that the eigenvalues of F are $\lambda_1 = -4, \lambda_2 = -4$ and determine the time behaviour of the residual

$$\varepsilon(t) = Ce(t)\,,\ \forall t \in (0, T_0)$$

for a given value $\bar{e}$ of the initial estimation error $e(0)$, where C is the output matrix determined in your answer to Question 4a)-i).

[7 marks]

b) Consider now the presence of an actuator fault of the form given in equation (4.1) for $t \geq T_0$.

i) Modify the state equations determined in your answer to Question 4a)-i) and the observer designed in your answer to Question 4a)-ii) so as to be able to construct a fault detection scheme based on the instantaneous estimate $\hat{f}(t)$ of the actuator fault $f(t)$ for $t \geq T_0$. *Please do not attempt to design the fault estimator.*

[7 marks]

ii) Show that the fault estimator of your answer to Question 4b)-i) cannot be designed when $\alpha = 1$ in equation (4.1).

[3 marks]