UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2000

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
BEng Honours Degree in Mathematics and Computer Science Part II
MEng Honours Degree in Mathematics and Computer Science Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C240=MC240

ALGORITHMS, COMPLEXITY AND COMPUTABILITY

Wednesday 10 May 2000, 16:00
Duration: 90 minutes
(Reading time 5 minutes)

Answer THREE questions

Paper contains 4 questions

1 a What is the *Church-Turing Thesis*? Explain why it cannot be proved but could possibly be disproved. What kinds of evidence for the thesis are there?

b Explain what the *halting problem* is. What does it mean to say that the halting problem is *unsolvable*?

c Let $g : C^* \rightarrow C^*$ be a partial function that "tells us whether or not a standard Turing machine halts and succeeds <u>on input w*w</u>". That is, for any standard Turing machine S and word w of C,

$$g(\text{code}(S)*w) = \begin{array}{ll} y & \text{if S halts and succeeds on input } w*w \\ n & \text{otherwise} \end{array}$$

Show that g is not computable.

The three parts carry, respectively, 30%, 30%, 40% of the marks.

2 Design a one- or two-tape Turing Machine, M, with input alphabet {a,b,c,...,z,*} which given an input word of the form w_1*w_2, halts and succeeds if w_1 is a substring of w_2, and halts and fails otherwise. You may assume that w_1 and w_2 do not contain the symbol *, and that square 0 is implicitly marked.

a *Either* write a pseudocode description of your Turing Machine

or draw a diagram of your Turing machine and explain the notation you have used.

b Explain briefly how your Turing Machine works.

c Derive an expression for the time function $time_M(m,n)$ for the number of steps M executes for the input word w_1*w_2, where the length of w_1 is m and the length of w_2 is n .

The three parts carry, respectively, 50%, 20%, 30% of the marks.

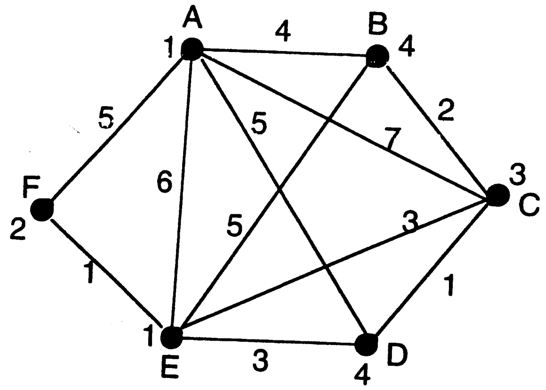
3 a Explain the meaning of the terms

i) *weighted graph,*

ii) *minimal spanning tree (MST) of a connected weighted graph.*

b Briefly describe an algorithm to find a minimal spanning tree of a weighted graph. (Explain its operation, but do not prove its correctness.)

c An electricity company wishes to build a grid of overhead cables to supply towns A-F with electricity from a substation at C:



The annual cost of operating a cable between two towns is the sum of:

(i) overheads and customs & excise duties in each town (the numbers written next to the towns), and

(ii) line maintenance costs (the number written on the edge between the towns).

So for example, operating a cable BC from B to C costs 4+3+2 = £9 million per year. Operating the grid consisting of five cables AB, BC, AD, AE and EF costs (1+4+4) + (4+3+2) + (1+4+5) + (1+1+6) + (1+2+1) = £40M per year.

Briefly describe an algorithm that would find a most economical grid in a situation like this. Use it to find such a grid in the map above. Is there a unique most economical grid in this case?

The three parts carry, respectively, 30%, 30%, 40% of the marks.

4a Let A and B be arbitrary *yes-no* problems. Explain what is meant by saying that A *reduces to* B in *p-time* (in symbols, A ≤B).

b Let ≤ be the relation of part a. Prove that if A≤ B and B∈ NP then A ≤NP.

c Let ≤ be the relation of part a.

i) Define the class NPC of NP-complete *yes-no* problems.

ii) Describe briefly two methods of determining whether a new problem, C, is in NPC.

The three parts carry, respectively, 20%, 50%, 30% of the marks.