

MSc and EEE/ISE PART IV: MEng and ACGI

Corrected Copy

Time allowed: 3:00 hours

**Answer THREE questions.**

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      M.K. Gurcan  
Second Marker(s) :      K.K. Leung

**Instructions to Candidates**  
**Useful equations**

$$0.5 \times \operatorname{erfc}(x) = 0.9 \implies x = -1.282$$

The cut-off value  $\gamma_0$  which is a solution for

$$1 = \frac{1}{\gamma_0} \exp\left(-\frac{\gamma_0}{10}\right) - \frac{1}{10} \int_{\frac{\gamma_0}{10}}^{\infty} \frac{\exp(-\gamma)}{\gamma} d\gamma$$

is

$$\gamma_0 = 0.7676$$

The cut-off value  $\gamma_0$  which is a solution for

$$\frac{1}{10} \int_{\gamma_0}^{\infty} \exp\left(-\frac{\gamma}{10}\right) d\gamma = 0.95$$

is

$$\gamma_0 = 0.5129$$

For the cut-off value  $\gamma_0 = 0.5129$  we have

$$\int_{\frac{\gamma_0}{10}}^{\infty} \frac{\exp(-\gamma)}{\gamma} d\gamma = 2.4437$$

1. Answer the following subquestions.

- (a) Under a free-space path loss model, find the transmitter signal power required to obtain a received signal power of 1 dBm for a wireless system with isotropic antennas ( $G_t = 1$ ) and a carrier frequency  $f = 5$  GHz, assuming a distance  $d = 10$  m. [3]
- (b) Consider a mobile radio receiver with noise power -160 dBm within the signal bandwidth of interest. Assume a simplified path-loss model with a reference  $d_0 = 1$  m, constant  $K$  obtained from the free space path-loss formula with omnidirectional antennas and the carrier frequency  $f_c = 1$  GHz, and the path loss exponent  $\gamma = 4$ . For a transmit power of  $P_t = 10$  mW, find the maximum distance between the transmitter and receiver such that the received signal-to-noise power ratio is 20 dB. [6]
- (c) Using the indoor attenuation model, determine the required transmitter signal power for a desired received power of -110 dBm for a signal transmitted over 100 m that goes through 3 floors with attenuation 15 dB, 10 dB, and 6 dB, respectively, as well as 2 double plasterboard walls with the partition loss 3.4 dB. Assume a reference distance  $d_0 = 1$ , path loss exponent  $\gamma = 4$  and constant  $K = 0$  dB. [4]
- (d) Consider a cellular system operating at 900 MHz where propagation follows free-space path-loss with variations from log-normal shadowing with  $\sigma = 6$  dB. Suppose that for an acceptable voice quality a signal-to-noise power ratio of 15 dB is required at the mobile. Assume that the base station transmits at 1 W and its antenna has a 3 dB gain. There is no antenna gain at the mobile and the receiver noise in the bandwidth of interest is -70 dBm. Find the maximum cell radius such that a mobile on the cell boundary will have an acceptable voice quality 90% of the time. [7]

2. Answer the following subquestions.

- (a) Find a formula for the multipath delay spread  $T_m$  for a two-path channel model. Find a simplified formula when the transmitter-receiver separation,  $d$ , is relatively large. Compute  $T_m$  for a transmitter antenna height  $h_t = 10\text{m}$ , and a receiver antenna height  $h_r = 4\text{m}$ , and  $d = 100\text{m}$ . [3]  
[1]  
[1]
- (b) Consider a two-path channel consisting of a direct ray plus a ground-reflected ray, where the transmitter is a fixed base station at height  $h$  and the receiver is mounted on a truck also at height  $h$ . The truck starts next to the base station and moves away at velocity  $v$ . Assume that the signal attenuation on each path follows a free-space path loss model. Find the time-varying channel impulse response at the receiver for the transmitter-receiver separation  $d = vt$  which is sufficiently large for the length of the reflected ray to be approximated by  $r + r' \approx d + 2h^2/d$ . [4]
- (c) Consider a time-invariant indoor wireless channel with a line-of-sight (LOS) component at delay 23 ns, a multipath component at delay 48 ns, and another multipath component at delay 67 ns. Find the delay spread assuming the demodulator synchronizes to
- i. the LOS component, [2]
  - ii. the first multipath component. [1]
- (d) Prove that for  $X$  and  $Y$  independent zero-mean Gaussian random variables with variance  $\sigma^2$ , the distribution of  $Z = \sqrt{X^2 + Y^2}$  is Rayleigh distributed and the distribution of  $Z^2$  is exponentially distributed. [4]
- (e) Answer the following parts
- i. Summarize, in your own words, the main discussion points on flat fading and frequency selective fading. [2]
  - ii. Determine whether individual multipath rays are resolvable for two transmission bandwidths, 1.25 MHz, and 5 MHz when transmitting over a channel with a delay spread of [2]
    - A.  $0.5 \mu\text{s}$ ,
    - B.  $1 \mu\text{s}$  and
    - C.  $6 \mu\text{s}$ .

3. Answer the following subquestions.

- (a) Assume a Rayleigh fading channel with 10MHz transmission bandwidth, where the transmitter and receiver have CSI and the distribution  $p(\gamma)$  of the fading SNR is exponential  $p(\gamma) = \frac{1}{10} \exp\left(-\frac{\gamma}{10}\right)$ .
- i. Find the cut-off value  $\gamma_0$  and the corresponding power adaptation that achieves Shannon capacity on this channel. [2]
  - ii. Compute the Shannon capacity of this channel. [2]
  - iii. Compare your answer in part (a.ii.) with the channel capacity when operating over an additive white Gaussian noise (AWGN) channel with the same average SNR. [2]
  - iv. Compare your answer in part (a.ii.) with the Shannon capacity when only the receiver knows the received signal SNR  $\gamma$ . [2]
  - v. Compare your answer in part (a.ii.) with the zero-outage capacity and outage capacity with outage probability 0.05. [2]
  - vi. Repeat parts (a.ii, iii. and iv.), that is, when using the same average transmission power and the same fading distribution but with mean  $\bar{\gamma} = -5\text{dB}$ ,
    - A. obtain the Shannon capacity for a system with perfect transmitter and receiver side information, [1]
    - B. obtain the Shannon capacity for a system with just receiver side information, and [1]
    - C. describe the circumstances under which a fading channel has a higher capacity than an AWGN channel with the same average SNR and explain why this behavior occurs. [1]
- (b) Show, using Lagrangian techniques, that the optimal power allocation to maximize the capacity of a set of time-invariant block fading channels in parallel is given by the water filling formula as follows [7]

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases}$$

where  $\gamma_0$  is the cut-off value and  $P(\gamma)$  is the transmitted signal power as a function of the signal-to-noise-ratio  $\gamma$  and  $\bar{P}$  is the average transmission power.

4. Answer the following subquestions.

- (a) Consider the Core Network (CN) for the third generation wideband UTRAN/FDD radio system and describe
- i. the functions undertaken by the serving GPRS support node (SGSN) and the gateway GPRS support node (GGSN), [2]
  - ii. the functions for the signalling protocol RANAP when using the control plane for the Iu PS interface which connects the UTRAN to the CN. [2]
- (b) Consider the third generation UTRAN architecture and describe
- i. the logical role of the Radio Network Controller (RNC), [2]
  - ii. the organization of the UMTS signalling plane between the User Equipment (UE) and the Serving Radio Network Controller (SRNC). [2]
- (c) Consider the third generation WCDMA radio interface protocol architecture and describe
- i. the main functions for the Radio Resource Controller (RRC) protocol, [2]
  - ii. how the RRC states operate. [2]
- (d) A direct sequence spread spectrum system (DSSS) with the processing gain  $N = 4$  and the number of parallel channels  $K = 4$  uses a spreading sequence matrix

$$\mathbf{S} = \begin{bmatrix} +0.5 & -0.5 & +0.5 & -0.5 \\ -0.5 & -0.5 & +0.5 & -0.5 \\ +0.5 & -0.5 & -0.5 & +0.5 \\ +0.5 & +0.5 & +0.5 & +0.5 \end{bmatrix}$$

Find

- i. the Gram matrix, [1]
  - ii. the correlation matrix  $\mathbf{S}\mathbf{S}^T$ . [1]
- (e) A DSSS system with the processing gain  $N = 2$  the number of codes  $K = 4$  uses the  $K \times N$  dimensional spreading sequence matrix

$$\mathbf{S}^H = \begin{bmatrix} 0.1968 - 0.9700i & -0.1239 - 0.0715i \\ 0.4752 + 0.1939i & -0.2992 - 0.8044i \\ 0.6720 + 0.1469i & -0.4231 + 0.5898i \\ 0.5329 & 0.8462 \end{bmatrix}$$

Using the matrix  $\mathbf{S}$

- i. Show that the codes satisfy the Welch-Bound-equality conditions. [3]
- ii. Assuming that the received total signal-to-noise ratio is  $\frac{h P_T}{\sigma^2} = 30$ , calculate the signal-to-noise-ratio at the output of each receiver despreading unit when the transmission power is equally distributed. [2]
- iii. Find the sum capacity for the system described in part e.ii. [1]