UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2002

BEng Honours Degree in Computing Part III
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C329

COMPUTATIONAL LOGIC

Monday 29 April 2002, 14:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1a i) Convert the propositional sentence (L if N, (M iff R)) into an equivalent pair of clauses each written in conditional ('if') format. Show your reasoning.

ii) From these clauses and the three unit clauses N, ¬M and ¬R, present a refutation of ¬L such that the parents in each step are clearly indicated.

b The following predicates have the intended meanings indicated:

L(X, K)	X is a list having length K
N(K)	$K \in \{0, 1, 2, ...\}$
M(X, U, I)	list X has member U in position I
R(A, I, B)	$A \leq I \leq B$

Write down a single first-order sentence which defines L(X, K) in terms of the N, M and R predicates and which has the same connective structure as shown in part 1ai). Pay careful attention to quantifiers.

c i) Convert your sentence from part 1b into a pair of clauses, using your answer to part 1ai) as a guide. There is no need to show any conversion steps, but pay careful attention to Skolemisation.

ii) Given these clauses, write down the simplest additional unit clauses about N, R and M sufficient to refute ¬L([], 0).

The three parts carry, respectively 25%, 25% and 50% of the marks.

2a Using this program **P** as an example:

```
likes(chris, X) :- likes(X, logic).
likes(bob, logic).
```

define, illustrate and explain the limits $T_P \uparrow \omega$ and $T_P \downarrow \omega$ used to determine whether atoms in **B**(**P**) succeed or fail. Assume that **H** = {chris, bob, logic}.

b For the same program **P**, identify:

i) the smallest symmetric model containing likes(logic, logic);

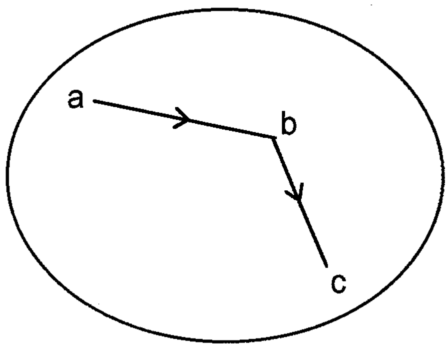
ii) the smallest transitive model.

Note: a symmetric model M is such that, for any X and Y, if likes(X, Y) is in M then so is likes(Y, X)

a transitive model M is such that, for any X, Y, Z, if likes(X, Y) and likes(Y, Z) are in M then so is likes(X, Z).

3a Describe how safe-SLDNF, given a normal query, evaluates the call selected by the given computation rule.

b In a directed graph a terminal node is one having no arcs emerging from it.
A graph is connected when it contains no terminal nodes.
The graph below is not connected because it contains the terminal node c.



This normal program **P1** is intended for testing this graph for connectedness:

connected :- ***fail*** terminal(X).
terminal(X) :- ***fail*** arc(X, Z).
arc(a, b).
arc(b, c).

i) Sketch the evaluation by unsafe Prolog of the query ?- connected using **P1**, and state the answer computed.

ii) Formulate **Comp(P1)** and use it to construct a proof of ¬connected.

iii) Explain the cause of the conflict between the outcomes of i and ii, and explain how safe-SLDNF avoids such a conflict.

c Revise **P1** to obtain a new normal program **P2** which correctly evaluates the query ?- connected under unsafe Prolog.

The three parts carry, respectively 20%, 45% and 35% of the marks.

4 In this Prolog meta-program **D**

```
demo(C, T) :- contains(C, T).
demo(C, T) :-
    contains(C, T1), contains(C, T2),
    resolve(T1, T2, R),
    extend(C, R, Cext),
    demo(Cext, T).
```

the following predicates have the intended meanings indicated:

contains(C, T)	"the clause-set C contains the clause T"
resolve(T1, T2, R)	"R is a resolvent of the clauses T1 and T2"
extend(C, R, Cext)	"extending C with R gives Cext"

Assume that the clauses mentioned in these arguments are always ground. They may be variously definite, indefinite or negative, but contain no ***fail*** calls.

a **D** represents the most general form of the resolution inference system, except that it omits the process of factoring. What is the nature and significance of factoring?

b For each statement below say whether or not it is true in all cases, giving your reasons. If you decide it is not true in some case then give a counter-example.

The symbol □ denotes the empty clause.

i) if ?- demo(C, T) succeeds from **D** then $C \models T$

ii) if $C \models T$ then ?- demo(C, T) succeeds from **D**

iii) if C* is the result of extending C with a unit ground clause $\neg t$ and ?- demo(C*, □) succeeds from **D**, then $C \models t$

iv) if C* is the result of extending C with a ground clause $(\neg t \vee \neg t)$ and $C \models t$, then ?- demo(C*, □) succeeds from **D**

v) if $\neg (C \models T)$, then ?- demo(C, T) fails finitely from **D**

c Given the new predicate

factor(X, F)	"F is the result of factoring the clause X"

add to **D** an extra demo clause which ensures that sufficient factoring occurs.

The three parts carry, respectively, 15%, 70% and 15% of the marks.