

Answer ALL Questions

Question 1 and 2 carries 25 marks each, Question 3 carries 30 marks

1. a) For an AC system with an embedded LCC HVDC link, state the main steps for solving the combined AC-DC power flow problem using a sequential approach. There is no need to write any analytical expression relating the AC and DC side quantities.

[4]

- 1) Solve AC power flow equations to determine the AC voltages at the converter stations (E_{acr} and E_{aci}) using the initial values of converter active and reactive power (P_r, Q_r, P_i, Q_i)
- 2) Use E_{acr} and E_{aci} to solve DC side equations and update P_r, Q_r, P_i, Q_i
- 3) Solution of DC side equations depends on the control mode of rectifier and inverter
- 4) If mismatches in P_r, Q_r, P_i, Q_i are greater than a tolerance, go back to step 1 and solve AC power flow equations until convergence is achieved

[1 mark for each point]

- b) During normal operation of an LCC HVDC link, the rectifier and the inverter operate in current and extinction angle control mode, respectively.

- (i) Under such normal condition, how are the firing angles and transformer tap settings determined at the two ends? There is no need to write any analytical expression.

[4]

- Inverter extinction angle is adjusted to its minimum value γ_{min}
- Rectifier firing angle is adjusted to produce I_{ord}
- Rectifier side tap adjusted to maintain the firing angle within the desired range
- Inverter side tap adjusted to produce the desired (rated) voltage

[1 mark for each point]

- (ii) If the rectifier end is stuck at the minimum permissible firing angle, the inverter end takes up the current control. Under such condition, how are the firing angles and transformer tap settings determined at the two ends? There is no need to write any analytical expression.

[4]

- Rectifier firing angle is fixed at the minimum permissible value α_{\min}
- Inverter extinction angle is adjusted to produce $I_d = I_{ord} - I_m$
- Rectifier side tap is adjusted to maximise direct voltage
- Inverter side tap is adjusted to ensure $\gamma > \gamma_{\min}$ while minimising reactive power consumption

[1 mark for each point]

c) For weak AC systems, a short circuit far from the LCC HVDC terminal could cause the HVDC link to shut down due to a potential runaway problem if the direct current order is held constant.

(i) Explain this runaway problem and its dependence on the AC system strength.

[5]

- For a weak AC system, even a remote short circuit could result in significant reduction in AC voltage at the converter terminal
- Reduction in AC voltage at one end of a LCC HVDC link leads to increased reactive power consumption by the converter station at the opposite end
- If the local capacitor and filter banks are already operating at their rated reactive power capacity, the excess reactive power has to be imported from remote sources (e.g. generators) which incurs large voltage drop in the network, especially for weak AC systems
- Larger voltage drop on the network reduces the AC voltage at the converter terminal
- Reduction in AC voltage at the converter terminal reduces the reactive power generation by the capacitor and filter banks necessitating import of even larger amount of reactive power which in turn reduces AC voltage at the converter further leading to a runaway situation

[1 mark for each point]

(ii) How is the converter control strategy modified to mitigate the above problem?

[3]

- The above runaway situation could be mitigated by adjusting the converter control strategy to include a voltage dependent current order limit (VDCOL)

- In VDCOL, the current order is reduced when the voltage (AC or DC side) falls below a certain threshold
- The revised current order is set according to the low voltage condition often in proportion to the voltage.

[1 mark for each point]

d) Explain the design trade-offs for the smoothing reactors used on the DC side of a LCC HVDC link.

[5]

- Reactance of the smoothing reactor should be high to 1) reduce the ripple current, 2) filter out the harmonics in DC voltage 3) limit fault current at the rectifier end and 4) reduce the possibility of consequent commutation failure
- On the other hand, reactance should be on the lower side to avoid resonance at low non-characteristic frequencies which are difficult to filter out
- Air-gap in the magnetic core of the reactor is necessary to avoid saturation due to DC operation
- Size/volume of the reactor becomes large with increasing air-gap (to prevent saturation)
- Thus, both the reactance value and the design (construction) has to be chosen considering the above conflicting requirements

[1 mark for each point]

2. a) For LCC HVDC links, power flow direction is reversed by changing the polarity of the direct voltage as the direction of direct current is fixed. Explain the implication of this on the following:

(i) Type of sub-sea cables used

[2]

- Polymeric cables are stronger and lighter and particularly suited for sub-sea applications, but they can't withstand slow voltage reversal.
- Hence, such polymeric cables cannot be used for LCC HVDC due to the need for voltage reversal to change the power flow direction. Instead, mass-impregnated cables are commonly used for sub-sea LCC HVDC links.

[1 mark for each point]

(ii) Implementation of a meshed DC grid

[2]

- Power reversal using voltage is not an issue for a point-to-point LCC HVDC link, but it poses a problem for meshed DC grids where voltage polarity reversal affects the power flow in all the DC lines/cables connected to a particular converter station which might not be desirable.
- Instead, power flow direction in individual lines/cables in a meshed grid should be changeable through current flow direction (like in AC systems and VSC HVDC) which is not possible with LCC.

[1 mark for each point]

- b) Neglecting commutation overlap and converter losses, show that the power factor of a typical six-pulse converter used in LCC HVDC links is the ratio of the average direct voltage and the no-load ideal direct voltage. The RMS value of the fundamental component of the AC side current can be considered as $\sqrt{6}I_d/\pi$ where I_d is the constant ripple-free direct current.

[4]

Neglecting commutation overlap, the direct voltage is given by:

$$V_d = V_{d0} \cos \alpha = \left(\frac{3\sqrt{2}}{\pi} E_{LL} \right) \cos \alpha$$

Neglecting converter losses, the active power on AC and DC sides are equal i.e.

$$\sqrt{3}E_{LL}I_{L1} \cos \phi = V_d I_d$$

$$\left(\sqrt{3} E_{LL} \frac{\sqrt{6}}{\pi} I_d \right) \cos \phi = \left(\frac{3\sqrt{2}}{\pi} E_{LL} I_d \right) \cos \alpha$$

Power factor of the converter is

$$\cos \phi = \cos \alpha = \frac{V_d}{V_{d0}}$$

[1 mark for each step]

c) Two separate AC systems with rated 3-phase line voltages of 400 kV and 380 kV are interconnected through a 1000 MW, ± 450 kV bipole LCC HVDC link. At both converter stations, a standard 12-pulse converter arrangement is used for each pole with a commutation resistance of 8Ω for each 6-pulse bridge. The converter station at the 400 kV end acts as the rectifier and controls the direct current at its rated value with the firing angle set at 15° . The inverter end maintains the rated direct voltage with an extinction angle of 16° . The current margin is set at 20%. The minimum limit in firing angle at the rectifier end is 6° . The DC cable resistance is 5Ω for each pole. Neglect the converter losses.

- (i) For the above base case, calculate the reactive power drawn by the converter stations at both ends.

[5]

This problem can be solved in two ways: 1) using per pole calculations OR 2) considering an equivalent monopole representation. The voltage, power and the number of 6-pulse converter bridges would have to be adjusted accordingly. Either approach should fetch full marks. The solution below is using approach (1) but the corresponding answers using approach (2) are also provided in bracket using a different font colour.

Transformer tap-changer action and activation of voltage dependent current order limit (VDCOL) are neglected.

B = no of 6-pulse converter bridges = 2 (4)

R_L = line resistance = 5Ω (10 Ω)

[1 mark]

At inverter end:

Active power and voltage per pole is

$$P_{di} = \frac{1000}{2} = 500 \text{ MW (1000 MW)}, \quad V_{di} = 450 \text{ kV (900 kV)}$$

$$I_d = \frac{P_{di}}{V_{di}} = 1.11 \text{ kA}$$

[1 mark]

$$V_{doi} = \frac{V_{di} + BR_{ci}I_d}{\cos \gamma} = 486.63 \text{ kV}$$

(973.26 kV)

[1 mark]

Total reactive power drawn by the inverter station is:

$$Q_i = 2P_{di} \times \tan \left(\cos^{-1} \frac{V_{di}}{V_{doi}} \right) = 411.61 \text{ MVar}$$

(411.61 MVar)

[1 mark]

At rectifier end:

$$V_{dr} = V_{di} + R_L I_d = 455.56 \text{ kV}$$

(911.11 kV)

$$V_{dor} = \frac{V_{dr} + BR_{cr}I_d}{\cos \alpha} = 490.03 \text{ kV}$$

(980.06 kV)

[1 mark]

Total reactive power drawn by the rectifier station is:

$$Q_r = 2V_{dr}I_d \times \tan \left(\cos^{-1} \frac{V_{dr}}{V_{dor}} \right) = 401.23 \text{ MVar}$$

(401.23 MVar)

[1 mark]

- (ii) If the AC voltage at the rectifier end drops by 10% while that at the inverter end remains at the initial value, compute the change in reactive power drawn by the converter stations at both ends compared to the base case and the new extinction angle (in degrees) at the inverter end.

[6]

This situation is denoted by subscript '1'

At rectifier end:

$$V_{dor1} = 0.9 \times V_{dor} = 441.03 \text{ kV}$$

(882.06 kV)

$$\cos \alpha_1 = \frac{V_{dr} + BR_{cr}I_d}{V_{dor1}} > 1$$

Current control is transferred to the inverter while the rectifier goes into constant firing angle (α_{min}) mode.

[1 mark]

Revised current order is

$$I_{d1} = I_d - I_m = 0.89 \text{ kA}$$

(0.89 kA)

[1 mark]

$$V_{dr1} = V_{dor1} \cos \alpha_{min} - BR_{cr} I_{d1} = 424.39 \text{ kV}$$

(848.78 kV)

[1 mark]

Total reactive power drawn by the rectifier station is:

$$Q_{r1} = 2V_{dr1} I_{d1} \times \tan \left(\cos^{-1} \frac{V_{dr1}}{V_{dor1}} \right) = 213.33 \text{ MVar}$$

(213.33 MVar)

Change in reactive power drawn by the rectifier station is

$$Q_{r1} - Q_r = -187.9 \text{ MVar}$$

(-187.9 MVar)

[1 mark]

At inverter end:

$$V_{di1} = V_{dr1} + R_L I_{d1} = 419.95 \text{ kV}$$

(839.89 kV)

$$Q_{i1} = 2V_{di1} I_{d1} \times \tan \left(\cos^{-1} \frac{V_{di1}}{V_{doi}} \right) = 437.11 \text{ MVar}$$

(437.11 MVar)

Change in reactive power drawn by the inverter station is

$$Q_{i1} - Q_i = 25.5 \text{ MVar}$$

(25.5 MVar)

[1 mark]

$$\gamma_1 = \cos^{-1} \left(\frac{V_{di1} + BR_{ci} I_{d1}}{V_{doi}} \right) = 26.85^\circ$$

(26.85°)

[1 mark]

- (iii) If the AC voltage at the inverter end drops by 15% while that at the rectifier end remains at the initial value, compute firing angle at the rectifier end and the commutation overlap angle (in degrees) at both ends.

[6]

This situation is denoted by subscript '2'

The rectifier station continues to act in constant current control mode while the inverter stays at minimum extinction angle

[1 mark]

At inverter end:

$$V_{doi2} = 0.85 \times V_{doi} = 413.63 \text{ kV}$$

(827.27 kV)

$$V_{di2} = V_{doi2} \cos \gamma - BR_{ci} I_d = 379.83 \text{ kV}$$

(759.67 kV)

[1 mark]

$$V_{di2} = \frac{V_{doi2}}{2} [\cos \gamma + \cos(\gamma + \mu_{i2})]$$

$$\mu_{i2} = \cos^{-1} \left(\frac{2V_{di2}}{V_{doi2}} - \cos \gamma \right) - \gamma = 12.92^\circ$$

(12.92°)

[1 mark]

At rectifier end:

$$V_{dr2} = V_{di2} + R_L I_d = 385.39 \text{ kV}$$

(770.78 kV)

$$\alpha_2 = \cos^{-1} \left(\frac{V_{dr2} + BR_{cr} I_d}{V_{dor}} \right) = 34.64^\circ$$

(34.64°)

[2 marks]

$$V_{dr2} = \frac{V_{dor}}{2} [\cos \alpha_2 + \cos(\alpha_2 + \mu_{r2})]$$

$$\mu_{r2} = \cos^{-1} \left(\frac{2V_{dr2}}{V_{dor}} - \cos \alpha_2 \right) - \alpha_2 = 6.75^\circ$$

(6.75°)

[1 mark]

3. a) Answer the following questions related to the converter shown in Figure 3.1:

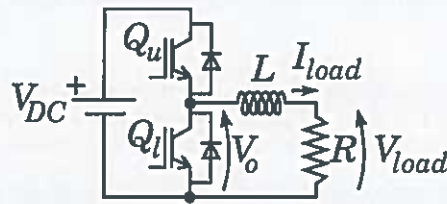


Figure 3.1 – Diagram of a DC-DC converter feeding a resistive load.

- (i) What is the Laplace transfer function between $V_o(s)$ and $I_{load}(s)$?

[3]

The equation of the circuit is: $V_o = L \frac{d}{dt} I_{load}(t) + R I_{load}(t)$

Transforming by Laplace and rearranging the equation we get: $\frac{I_{load}(s)}{V_o(s)} = \frac{1}{Ls + R}$

[3 marks]

- (ii) A closed-loop controller is used to control I_{load} . The closed-loop transfer function between the current reference, I_{load}^* , and the load current, I_{load} , is:

$$\frac{I_{load}(s)}{I_{load}^*(s)} = \frac{1}{\tau_c s + 1}$$

with $\tau_c = 2$ ms.

- What will the steady-state error be for constant I_{load}^* reference values?

[2]

This can be answered by finding out the steady state output for a given reference value and subtracting it from the reference value. The first step can be solved by checking the DC gain from the frequency response by substituting $s := j\omega$ with $\omega := 0$. This gives:

$$\frac{I_{load}(0)}{I_{load}^*(0)} = 1$$

Therefore, for any reference value I , the error will be: $I - I = 0$.

Alternatively, the final value theorem with a step reference could be used instead of the frequency domain analysis, which would lead to the same result. Students can also identify that the closed-loop transfer function corresponds to a first order systems, and identify the DC gain if they remember the standard form.

[2 marks]

- What will the approximate settling time be for step changes of the I_{load}^* reference?

[2]

Students should identify that the closed-loop transfer function corresponds to a first order system with a time constant τ_c . This implies that the system will reach about 63% of the final value in τ_c time. The settling time can have different definitions, a common one is to consider $3\tau_c$, where the system reaches 95% of

the final value. In this question this would give a settling time of 6 ms.
Alternative definitions will be considered correct if they are properly argued.
[2 marks]

- What will the amplitude of I_{load} be if we requested a sinusoidal current of amplitude 1 A and 50 Hz of frequency (specifically: $I_{load}^*(t) = \sin(2\pi 50 t)$)? [4]

The easiest way to solve this question is to check the gain of the closed loop transfer function at the frequency of 50 Hz and then multiply it by the amplitude of the reference signal, which is 1 A. This gives:

$$\begin{aligned} \left\| \frac{I_{load}(j 2 \pi 50 \text{ rad/s})}{I_{load}^*(j 2 \pi 50 \text{ rad/s})} \right\| \times 1A &= \left\| \frac{1}{\tau_c j 2 \pi 50 \text{ rad/s} + 1} \right\| \times 1A \\ &= \frac{1}{\sqrt{1 + (2 \pi 50 \text{ rad/s } \tau_c)^2}} \times 1A = 0.85 A \end{aligned}$$

[4 marks]

- b) Answer the following questions related to the steady-state AC analysis of the three-phase inverter shown in Figure 3.2.

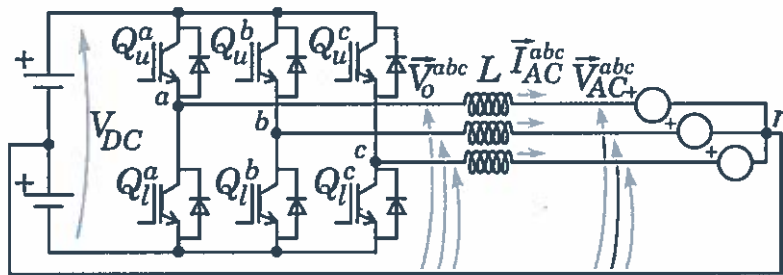


Figure 3.2 – Diagram of an inverter connected to an AC network represented as an AC voltage source.

- (i) Calculate the output current I_{AC} for the inverter to deliver $P=3\text{kW}$ and $Q=300 \text{ VAR}$ to the AC network (the converter generates reactive power) if $V_{AC}=230\text{V}$ (rms phase-to-neutral voltage).

[4]

The three-phase active power at the point of connection of the converter can be expressed as a function of the phase-to-neutral voltage and the phase current as:

$$\bar{S} = P + jQ = 3 V_{AC} \{ \bar{I}_{AC} \}^* = \underbrace{3 V_{AC} \Re \{ \bar{I}_{AC} \}}_P + j \underbrace{3 V_{AC} (-\Im \{ \bar{I}_{AC} \})}_Q$$

where the voltage was used for the reference of angles. Therefore, the current can be found by identifying its real and imaginary terms:

$$\begin{cases} \Re \{ \bar{I}_{AC} \} = \frac{P}{3 V_{AC}} = 4.35 A \\ \Im \{ \bar{I}_{AC} \} = -\frac{Q}{3 V_{AC}} = -0.435 A \end{cases}$$

Therefore: $\bar{I}_{AC} = (4.35 - j0.435)A$ and $I_{AC} = |\bar{I}_{AC}| = 4.37 A$

The sign of the imaginary part can be double-checked by taking into account that if the inverter is generating reactive power, the network must draw reactive power, which implies that the network behaves as an inductor.

[4 marks]

- (ii) What voltage V_o does the converter need to apply in order to operate under the aforementioned conditions if the filter inductance is $X_L = 10 \Omega$?

[2]

The voltage of the inverter can be found by adding the voltage drop of the inductors to the voltage at the point of connection:

$$\bar{V}_o = jX_L \bar{I}_{AC} + V_{AC} = j 10 (4.35 - j0.435) V + 230 V = (234 + j44)V$$

which has a modulus of $V_o = 238V$

[2 marks]

- (iii) What is the minimum V_{DC} required in order for the converter to be able to generate the aforementioned output voltage V_o ? Would this answer to this question change if we removed the connection between the DC bus and the neutral point?

[3]

The maximum peak V_o the converter can generate in the current configuration with the neutral conductor connected to the middle point of the DC bus is $V_{DC}/2$.

Therefore, the converter would need a minimum DC voltage of:

$$V_{DC} \leq 2\sqrt{2}V_{AC} = 2\sqrt{2} \times 238 V = 673 V$$

[2 marks]

If the neutral wasn't connected, the converter would be able to inject 0-sequence triplen-harmonic, which would make the maximum peak voltage be $\frac{1}{\sqrt{3}}V_{DC}$ (about +16% extra voltage). Therefore:

$$V_{DC} \leq \sqrt{3}\sqrt{2}V_{AC} = \sqrt{6} \times 238 V = 583 V$$

[1 marks]

- c) A single-arm converter is used to exchange energy between an AC network and a DC network (see Figure 3.3). Answer the following questions related to this converter:

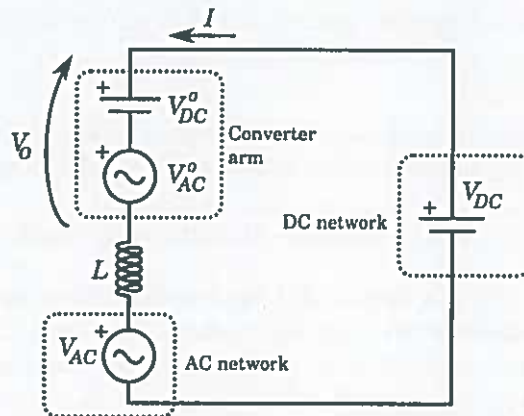


Figure 3.3 – Diagram of a single-arm AC-DC converter exchanging power between an AC network and a DC network.

- (i) Write the AC and DC steady-state equations that relate voltages and currents in the circuit.

[3]

The circuit is linear and therefore, independent steady-state equations can be written for each frequency separately. We express the AC and the DC components of I as \bar{I}_{AC} and I_{DC} . For the AC components, this leads to the following equation:

$$-\bar{V}_{AC}^o - \bar{V}_{AC} = jX\bar{I}_{AC}$$

For the DC components, we get:

$$-V_{DC}^o + V_{DC} = 0$$

[3 marks]

- (ii) Explain briefly what condition has to be met for the converter arm to have zero net power balance.

[2]

Zero net power balance implies that the average power exchanged by the arm with the circuit is equal to zero. Both the voltage and the current across the arm contain AC and DC components, but the crossed terms obtained when multiplying voltage and current give a zero average. This means that the total average net power of the arm is just the summation of the active power due to the AC components and the power due to the DC components alone. These, have to add to zero, which implies:

$$\Re\{\bar{V}_{AC}^o\{\bar{I}_{AC}\}^*\} + V_{DC}I_{DC} = 0$$

[3 marks]

- d) An HVDC link with a rated voltage of $V_{DC}^{RATED} = 320\text{kV}$ and a rated power of 500 MW is protected using a power-electronic HVDC circuit breaker (CB) with a chain of IGBTs in parallel with a chain of varistors. The varistors of the CB are chosen for a clamping voltage of 1.5 times V_{DC}^{RATED} and the IGBTs of the CB have an effective blocking voltage capability of 2 kV each. Answer the following questions:

- i) How many IGBTs will the CB need?

[2]

The CB must have enough IGBTs to block the clamping voltage of the varistors.

This means that the total number of IGBTs will be: $N_{IGBT} = \frac{1.5 \times V_{DC}^{RATED}}{V_{IGBT}} = 240$

[2 marks]

- ii) Calculate the conduction power losses of the converter when operating at rated power considering that the voltage drop of a single IGBT is approximately:

$$V_{IGBT}(I_{IGBT}) = 2V + 0.8 \times 10^{-3} V/A \times I_{IGBT}$$

[3]

The power loss of a single IGBT can be calculated as the product of its current by the voltage drop across the IGBT for that given current. This can be multiplied by the total number of IGBTs in order to obtain the total loss. The rated current of the link can be calculated as: $I_{DC}^{RATED} = \frac{P}{V_{DC}^{RATED}} = 1.56\text{ kA}$. The voltage drop across a single IGBT for that current will be: $V_{IGBT} = 3.25\text{ V}$. Therefore, the power loss of a single IGBT will be: $P_{loss}^1 = 5.1\text{ kW}$, and the total loss will be $P_{loss}^{TOTAL} = 1.22\text{ MW}$.

[3 marks]