UNIVERSITY OF LONDON **[MSc-SC2, E4.01, ISE4.11]**
IMPERIAL COLLEGE OF SCIENCE TECHNOLOGY AND MEDICINE

DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2004

M.Sc and EEE/ISE PART IV: M.Eng. and ACGI

ADVANCED COMMUNICATION THEORY

— There are FOUR questions (Q1 to Q4)
— Answer Question ONE plus TWO other questions.

— Distribution of marks
Question-1: 40 marks
Question-2: 30 marks
Question-3: 30 marks
Question-4: 30 marks

Comments for Question Q1:
— Question Q1 has 20 multiple choice questions numbered 1 to 20 .
— Circle the answers you think are correct on the answer sheet provided.
— There is only one correct answer per question.

The following are provided:
- *A table of Fourier Transforms*
- *A "Gaussian Tail Function" graph*

Examiners responsible: Dr. A. Manikas

Information for candidates:

The following are provided on pages 2 and 3:

- a table of Fourier Transforms;
- a graph of the 'Gaussian Tail Function'.

Question 1 is in a separate coloured booklet which should be handed in at the end of the examination.

You should answer Question 1 on the separate sheet provided. At the end of the exam, please tie this sheet securely into your main answer book(s).

Special instructions for invigilators:

Please ensure that the three items mentioned below are available on each desk.

- the main examination paper;
- the coloured booklet containing Question 1;
- the separate answer sheet for Question 1;

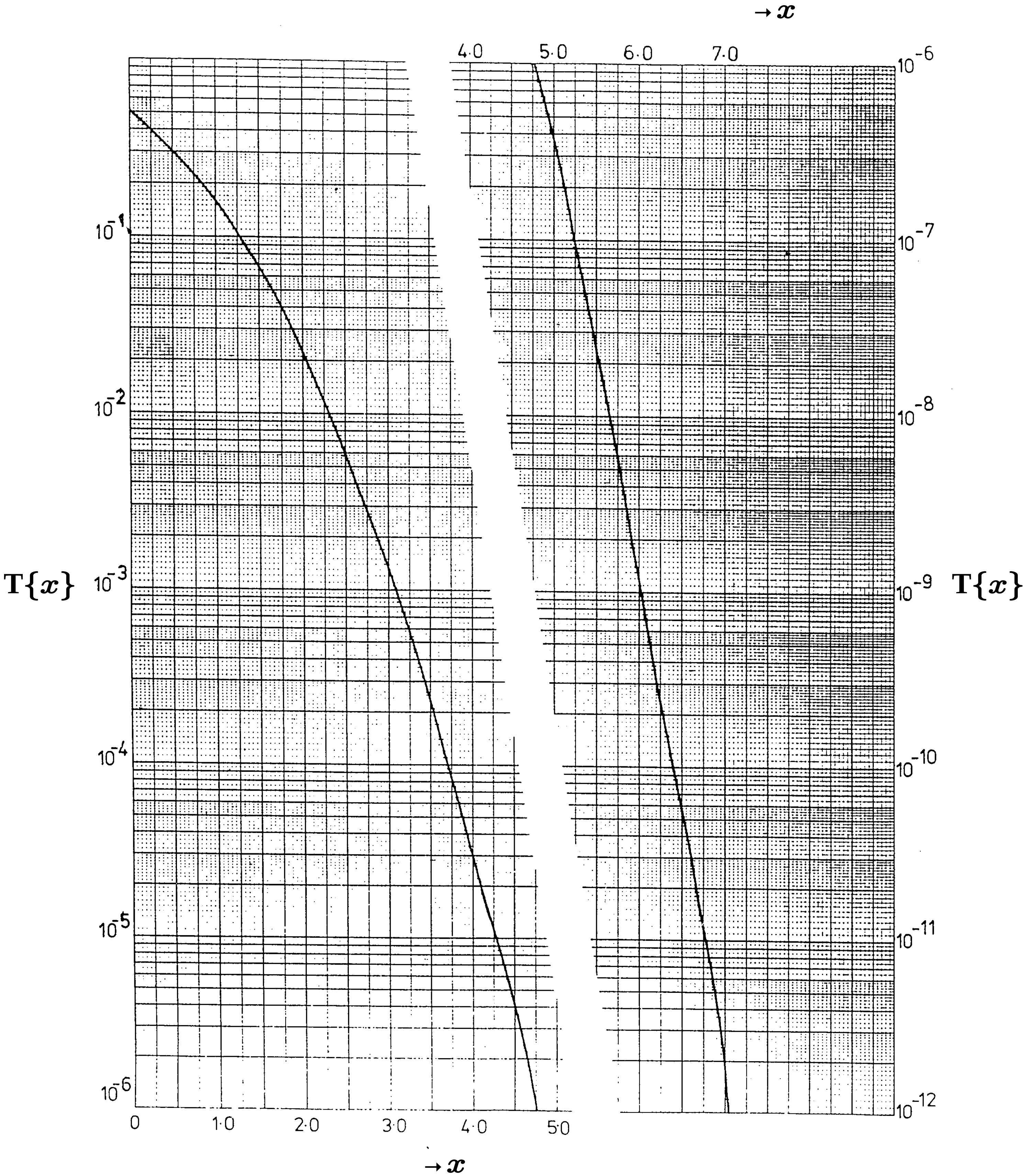
Please remind candidates at the end of the exam that they should tie their Answer Sheet for Question 1 securely into their main answer book, together with supplementary answer books etc.

Please tell candidates they must **NOT** remove the coloured booklet containing Question 1. Collect this booklet in at the end of the exam, along with the standard answer books.

Tail Function Graph

The graph below shows the Tail function $\mathbf{T}\{x\}$ which represents the area from x to ∞ of the Gaussian probability density function N(0,1), i.e.

$$\mathbf{T}\{x\} = \int_x^{\infty} \tfrac{1}{\sqrt{2\pi}}.\exp\left(-\tfrac{y^2}{2}\right)dy$$



Note that if $x > 6.5$ then $\mathbf{T}\{x\}$ may be approximated by $\mathbf{T}\{x\} \approx \frac{1}{\sqrt{2\pi}\,.\,x}.\exp\left\{-\frac{x^2}{2}\right\}$

FOURIER TRANSFORMS - TABLES

	DESCRIPTION	FUNCTION	TRANSFORM
1	Definition	$g(t)$	$G(f) = \int_{-\infty}^{\infty} g(t).e^{-j2\pi ft}dt$
2	Scaling	$g(\frac{t}{T})$	$\|T\| \,.\, G(fT)$
3	Time shift	$g(t-T)$	$G(f) \,.\, e^{-j2\pi fT}$
4	Frequency shift	$g(t) \,.\, e^{j2\pi Ft}$	$G(f-F)$
5	Complex conjugate	$g^*(t)$	$G^*(-f)$
6	Temporal derivative	$\frac{d^n}{dt^n} \,.\, g(t)$	$(j2\pi f)^n.\, G(f)$
7	Spectral derivative	$(-j2\pi t)^n.g(t)$	$\frac{d^n}{df^n} \,.\, G(f)$
8	Reciprocity	$G(t)$	$g(-f)$
9	Linearity	$A \,.\, g(t) + B \,.\, h(t)$	$A \,.\, G(f) + B \,.\, H(f)$
10	Multiplication	$g(t) \,.\, h(t)$	$G(f) * H(f)$
11	Convolution	$g(t) * h(t)$	$G(f) \,.\, H(f)$
12	Delta function	$\delta(t)$	1
13	Constant	1	$\delta(f)$

	DESCRIPTION	FUNCTION	TRANSFORM
14	Rectangular function	$\mathbf{rect}\{t\} \equiv \begin{cases} 1 & \text{if } \|t\| < \frac{1}{2} \\ 0 & otherwise \end{cases}$	$\mathbf{sinc}(f) = \frac{sin\,\pi f}{\pi f}$
15	Sinc function	$\mathbf{sinc}(t)$	$\mathbf{rect}(f)$
16	Unit step function	$u(t) = \begin{cases} +1, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$
17	Signum function	$\mathbf{sgn}(t) = \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$	$-\frac{j}{\pi f}$
18	Decaying exponential (two-sided)	$e^{-\|t\|}$	$\frac{2}{1+(2\pi f)^2}$
19	Decaying exponential (one-sided)	$e^{-\|t\|}.u(t)$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
20	Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
21	Lambda function	$\Lambda\{t\} \equiv \begin{cases} 1-t & \text{if } 0 \le t \le 1 \\ 1+t & \text{if } -1 \le t \le 0 \end{cases}$	$\mathbf{sinc}^2(f)$
22	Repeated function	$\mathbf{rep}_T\{g(t)\} = g(t) * \mathbf{rep}_T\{\delta(t)\}$	$\|\frac{1}{T}\|.\mathbf{comb}_{\frac{1}{T}}\{G(f)\}$
23	Sampled function	$\mathbf{comb}_T\{g(t)\} = g(t).\mathbf{rep}_T\{\delta(t)\}$	$\|\frac{1}{T}\|.\mathbf{rep}_{\frac{1}{T}}\{G(f)\}$

The Questions

1. ***This question is bound separately and has 20 multiple choice questions numbered 1 to 20, all carrying equal marks .***

You should answer Question 1 on the separate sheet provided.

Circle the answers you think are correct .

There is only one correct answer per question.

There are no negative marks.

2. A BPSK direct sequence spread spectrum system (BPSK/DS-SSS) has a PN-code rate of 10 Mchips per second and a binary message rate of 1000 bits per second. The EUE at the receiver's input is 100 and the double-sided power spectral density of the received noise is 0.5×10^{-8} Watts per Hz. For this system, in which the correlation time is exactly one message bit, what would be the receiver's synchronization errors (τ, θ) which would provide code noise power equal to 3.75×10^{-8}W, knowing that if $\tau > T_c$ then the code noise is constant and equal to 1.5×10^{-7}W. **[15]**

Note that the code noise expressions should be proven **[15]**

N.B.: τ represents the PN-code time error and θ denotes the carrier's phase error.

3. Consider a binary digital communication system where the digital modulation scheme being used is described as follows:

> "The input to the digital modulator is a binary sequence of 1's and 0's with the number of 1s being twice the number of zeros. The binary sequence is transmitted as a pulse signal $s(t)$ with a one being sent as $4\Lambda\left(\frac{t}{T_{cs}/2}\right)$ and $zero$ being sent as -2 rect($\frac{t}{T_{cs}}$)."

and the channel noise is assumed to be additive and uniformly distributed between -2 Volts and $+2$ Volts

a) plot the probability density function of $s(t)$ **[2]**

b) plot the probability density function of $r(t) = s(t) + n(t)$
where $n(t)$ represents the noise effects **[5]**

c) identify the likelihood functions $p_0(r)$ and $p_1(r)$ **[3]**

d) design a Bayes Detector (i.e. decision rule) when the following costs apply:
$C_{00} = C_{11} = 0$; $C_{10} = 0.8$; $C_{01} = 1$. **[6]**

e) for the above Bayes detector estimate the

i) the forward transition matrix $\mathbb{F}$ of the system **[6]**

ii) the bit error probability, p_e. **[2]**

iii) the joint-probability matrix $\mathbb{J}$ (i.e. the matrix with elements the probabilities Pr(H_i, D_j) $\forall i,j$ **[6]**

4. Consider a digital cellular DS-BPSK CDMA communication system which employs three directional antennas each having 120° beamwidth, thereby dividing each cell into 3 sectors. The system can support up to 201 users/subscribers and operates with a data bit-rate of 500 kbits/sec in the presence of additive white Gaussian noise of double-sided power spectral density 10^{-9}. With a bit-error-probability for each user of 3×10^{-5}, a power equal to 10 mWatts, and a voice activity factor $\alpha = 0.375$, find the processing gain (PG) of the system. **[30]**