

MSc and EEE/EIE PART IV: MEng and ACGI

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Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge : $e = 1.6 \times 10^{-19} \text{ C}$

permittivity of free space : $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

relative permittivity of silicon : $\epsilon_r = 12$

Planck's constant : $h = 6.63 \times 10^{-34} \text{ J s}$

Boltzmann's constant : $k = 1.38 \times 10^{-23} \text{ J/K}$

speed of light : $c = 3.0 \times 10^8 \text{ m/s}$

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x} \tan(k_{1x}d/2) \text{ and } \kappa = -k_{1x} \cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, state any assumptions or approximations made, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]
- a) An electromagnetic plane wave in a certain medium has a frequency $f = 2.5$ GHz and a phase velocity of 2.1×10^8 m/s. Calculate the magnitude of the spatial frequency k .
 - b) A certain laser diode emits 2.0×10^{16} photons per second, giving a power of 3.0 mW. Calculate the photon wavelength.
 - c) An optical wave passes at normal incidence through a plane boundary between two uniform media of refractive index 1.30 and 1.52 respectively. Calculate the power transmitted into the second medium if the incident power is 5.0 mW.
 - d) Describe the cross-sectional shape of the mode in a step-index single mode optical fibre. Include a sketch of the shape.
 - e) After propagating in a fibre for 25 km, the power of an optical signal has decreased from 0.80 mW to 65 μ W. Calculate the attenuation coefficient of the fibre in dB/km. How does this value compare to the best obtainable values in high performance fibre?
 - f) A photodiode having a quantum efficiency of 0.83 detects an optical signal of 2.5 μ W at a wavelength of 980 nm. Calculate the resulting photocurrent.
 - g) At a certain wavelength, an optical fibre has $d^2n'/d\lambda^2 = 0$, where n' is the effective index. Why does a signal at this wavelength still experience dispersion? What property of the fibre is the dispersion proportional to at this wavelength?
 - h) A silicon p-n photodiode has p and n doping levels of $N_A = 2 \times 10^{20} \text{ m}^{-3}$ and $N_D = 1 \times 10^{20} \text{ m}^{-3}$ respectively, and for a certain bias voltage has a total depletion layer width of 3 μ m. Calculate the magnitude of the electric field at the p-n boundary.
 - i) Briefly describe each of the three energy level transitions made by an electron in a three energy level optical amplifying system, and the role each plays in providing gain.
 - j) A certain step-index, multi-mode silica fibre has a numerical aperture of 0.15. The end of the fibre is cut flat at right angles to the fibre axis, and light propagating in the fibre is transmitted into the air through this surface. Calculate the maximum angle of the output light with respect to the fibre axis. State any assumptions made.

2.

- a) A slab waveguide is formed by a sheet of glass of thickness 3 mm, and index $n = 1.50$, surrounded by air. Calculate the number of TE modes this waveguide can support for a free space wavelength $\lambda_0 = 500 \text{ nm}$. [5]
- b) A symmetric slab waveguide as shown in Fig. 2.1 has a core thickness $d = 6.0 \text{ }\mu\text{m}$, and refractive indices $n_1 = 1.490$ and $n_2 = 1.485$. For a free space wavelength $\lambda_0 = 1.33 \text{ }\mu\text{m}$, how many TE modes will this guide support? [5]
- c) Calculate the effective index n' for each of the TE modes of the guide described in (b). [10]

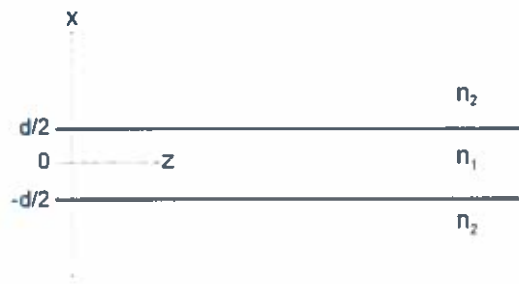


Figure 2.1

3. A laser of peak output power 4 dBm, with nominal wavelength $\lambda_0 = 1330 \text{ nm}$ and spectral width $\Delta\lambda = 1.5 \text{ nm}$, emits into a fibre with an attenuation of 0.35 dB/km and dispersion coefficient 11 ps/nm·km. It is modulated with an on-off keying format at a bit rate B. The receiver has quantum efficiency $\eta = 0.85$ and input resistance $R = 20 \text{ k}\Omega$.
 - a) Assume that the SNR is dominated by thermal noise in the receiver. Hence, derive an expression for the maximum bit rate as a function of fibre length, for an optical SNR of 12. Neglect dispersion, and state any other approximations or assumptions made. [5]
 - b) Assume that the SNR is dominated by shot noise in the receiver. Hence, derive an expression for the maximum bit rate as a function of fibre length, for an optical SNR of 12. Neglect dispersion, and state any other approximations or assumptions made. [5]
 - c) Derive an expression for the bit rate at which the dispersion time is equal to one quarter of the bit time $1/B$, as a function of fibre length. [5]
 - d) On a single graph of bit rate vs. ^{length}~~time~~, sketch the three relations derived in (a), (b) and (c) above, using appropriate scales and ranges for your two axes. Briefly discuss the significance of this graph for the operation of this optical link. [5]

4. a) Describe the structure and operating principles of erbium doped fibre amplifiers. Use diagrams where appropriate. Indicate the main performance criteria, and give typical values for these. Include a definition of amplified spontaneous emission (ASE) and discuss its significance. [8]

- b) The ASE noise spectral density of a fibre amplifier can be approximated as:

$$(I_A^*)^2 = 4 \frac{e^2 \lambda}{hc} G(G-1) \Phi_0$$

where G and Φ_0 are the amplifier gain and input optical power respectively. Show that if a high gain amplifier is placed before the receiver in an optical link, the resulting optical SNR is worse by a factor of $\sqrt{2}$ than without the amplifier, if the latter case is dominated by shot noise. State any assumptions or approximations used. [8]

- c) Describe the general conditions under which an optical amplifier will improve the overall SNR of an optical link. [4]

5. A silicon p-i-n photodiode has intrinsic layer thickness $w_i = 10 \mu\text{m}$, and p and i layer doping levels respectively of $N_A^+ = 2 \times 10^{21} \text{ m}^{-3}$ and $N_D^- = 2 \times 10^{20} \text{ m}^{-3}$. The n layer doping level is labelled as N_D^+ .

- a) Calculate the applied bias voltage V_I at which the intrinsic layer is just fully depleted. [4]

- b) For a bias voltage $V < V_I$, derive an expression for the rate of increase of the total depleted width w with respect to V , i.e. dw/dV , as a function of w . Hence, find a value for the fractional sensitivity of w to V , i.e. $(dw/w)/(dV/V)$. [6]

- c) For a bias voltage V just higher than V_I , derive an approximate expression for the rate of increase of the total depleted width w with respect to V . You may assume $w_i \gg \delta$, where δ is the depleted width in the n layer. State any other assumptions or approximations made. Hence, find the n layer doping level N_D^+ for which the fractional rate of growth of w is only 10% of the fractional rate of increase of V , i.e. $dw/w = 0.1 dV/V$. [6]

- d) Briefly explain why a low sensitivity of w to the applied voltage might be beneficial. [4]

