IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

EEE PART II: MEng, BEng and ACGI

POWER, FIELDS AND DEVICES

Monday, 24 May 2:00 pm

Time allowed: 3:00 hours

Corrected Copy

There are NINE questions on this paper.

There are three sections. Answer FIVE questions including at least ONE question from each of sections A, B and C.

Use a separate answer book for each section.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T.C. Green, K.D. Leaver, R.R.A. Syms
Second Marker(s) : D. Popovic, A.S. Holmes, W.T. Pike

Information for Candidates

Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho \quad ; \quad \mathbf{D} = \varepsilon \mathbf{E} \qquad\qquad \nabla \cdot \mathbf{B} = 0 \quad ; \quad \mathbf{B} = \mu \mathbf{H}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Physical constants and material parameters:

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$e = 1.602 \times 10^{-19}$ C

kT = 0.025 eV at 290 K

In SiO_2 $\varepsilon = 4\varepsilon_0$

In silicon $\varepsilon = 11.7\varepsilon_0$

Section A

[2.03]

1.

A boost switch-mode power supply (SMPS) is to be used to provide a 12 V output from a 5 V input. The inductor has a value of 80 μH and the capacitor has a capacitance of 470 μF and a series resistance of 20 mΩ. *(Question is more meaningful if C=150 μF)*

(a) Sketch the circuit diagram of the boost converter. [2]

(b) Sketch the waveforms of the currents through the inductor and capacitor for continuous inductor current. [3]

(c) Calculate the value of duty-cycle required assuming continuous conduction. [2]

(d) Calculate the minimum switching frequency for which the SMPS will stay in continuous conduction for an input power of 2 W. [5]

(e) Calculate the output voltage ripple when using the switching frequency found in part (d) but with an input power of 15 W. [6]

(f) Calculate the switching frequency required to achieve an output voltage ripple of 100 mV with an input power of 15 W. [2]

2.

(a) Explain the terms real power, reactive power and power factor. [4]

(b) Figure 2 shows the output voltage of a buck switch-mode power supply as a function of output current when operated in open-loop with a set duty-cycle. Explain its shape. [4]

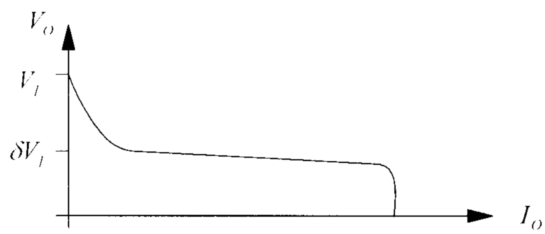

Figure 2

(c) Explain why 3-phase systems are favoured in power systems over, say 1-phase or 5-phase systems. [4]

(d) Discuss the sources from which the UK derives its electrical energy and comment on how this might change in future. [4]

(e) Explain the operating principle of an induction machine. [4]

[2.03]

3.

A three-phase induction machine has the following parameters:

Number of pole-pairs, $P = 2$;
Stator resistance, $R_S = 0.08\ \Omega$;
Stator leakage reactance, $X_S = 0.5\ \Omega$;
Magnetising reactance, $X_M = 10\ \Omega$;
Iron loss resistance, $R_M = 30\ \Omega$;
Referred rotor resistance, $R'_R = 0.07\ \Omega$;
Referred rotor leakage reactance, $X'_R = 0.5\ \Omega$;

The supply to the machine is 50 Hz with a line voltage of 400 V
In operation, the machine runs at a speed of 1,445 r.pm.

Calculate the following:

(a) the slip [2]

(b) the stator current [6]

(c) the electromagnetic torque developed [6]

(d) the efficiency of the machine [6]

Section B

Use a separate answer book for this section

4. Two very long electrodes, one grounded ($V = 0$ V) and one held at +10 V, are separated by an insulator. Their shapes are to be chosen so that the potential in the region between the electrodes varies in Cartesian coordinates according to the equation

$$V = 20xy$$

where V is in Volts, the units of both x and y are mm, and the electrodes do not extend beyond the ranges $x = 0$ to 7 mm or $y = 0$ to 7 mm.

Show that Laplace's equation is satisfied by the above expression, and determine the shapes of the two electrodes. Give a description of those shapes in words.

Determine the coordinates (less than 7 mm) at which the electric field strength at the surface of the high-voltage conductor is the least, and find its value there. [13]

Show that the charge per unit area on the grounded conductor rises linearly with distance from a unique line, and give the location of that line. [7]

Section B cont'd

5. A strip transmission line consists of three long parallel strips of width w, as illustrated in Figure 5. Two strips are grounded, and sandwiched between them are two nonmagnetic insulators of equal thickness $w/5$ and relative permittivity 2.5, one on either side of the live conductor.

   Neglecting all fringing fields, calculate from basic principles the inductance and capacitance per unit length. Hence find the characteristic impedance of this line and the propagation delay over 50 cm. [10]

   One end of this line is then connected to a transmission line consisting of just one pair of conductors of the same width w, separated by a thickness t_2 of the same dielectric material. What should the value of t_2 be in order that no reflections should occur at the junction?

   For which of these two transmission lines do you expect your results to give the better approximation to the true impedance, and why? [10]

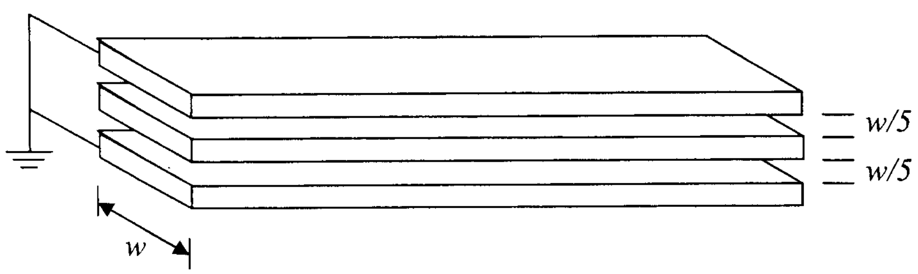

Figure 5

6. By considering sinusoidal travelling waves, prove that the velocities of all electromagnetic waves of whatever shape are the same in a uniform ideal dielectric having a relative permeability of unity and a relative permittivity greater than unity, *only if* the permittivity is independent of frequency. State all assumptions that you make. You may assume without proof the wave equation:

$$\nabla^2 \boldsymbol{E} = \mu\varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

What are the consequences of this result for the propagation of a *modulated* carrier wave through space? [12]

With the aid of one of Maxwell's equations, find a vector expression for the magnetic flux density $\boldsymbol{B}$ that accompanies the electric field $\boldsymbol{E}_x$ of an x-polarised sinusoidal travelling wave of angular frequency ω, and hence show that the ratio of their magnitudes E_x/B equals the velocity of the wave. Do NOT quote an expression for E/H without proof. [8]

Section C

Use a separate answer book for this section

7. (a) A micromechanical oscillator is formed from a silicon mass mounted on an etched silicon cantilever beam. Following a design change, each dimension of the device is to be halved, together with the oscillation amplitude. Explain how the following will alter:

   (i) the stiffness of the cantilever;
   (ii) the static deflection of the cantilever due to the weight of the mass;
   (iii) the natural frequency of the oscillator;
   (iv) the maximum velocity of the mass at resonance;
   (v) the maximum strain energy stored in the cantilever. [10]

   (b) Hammock and folded flexures may both be used as elastic supports for vibrating microsystems. Sketch both arrangements, and explain any differences between them. [4]

   (c) Figure 7 shows a cantilever of length L, depth d and breadth b, formed in a material of Young's modulus E. Derive an expression for the transverse stiffness of the beam against the load P, assuming that its free end is constrained against rotation by the moment M_e. Hence or otherwise, find an expression for the in-plane stiffness of a portal suspension formed from two cantilevers linked by a rigid strut.

   [6]

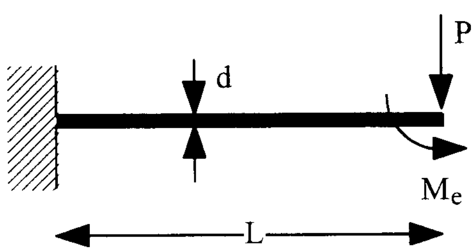

Figure 7

8. (a) Figure 8 shows an electrothermal actuator comprising a polyimide cantilever with a thin metal film heater on its upper surface. Suggest a possible fabrication sequence for this device, assuming the polyimide is deposited as a continuous film and patterned by reactive ion etching using the metal as a mask. [4]

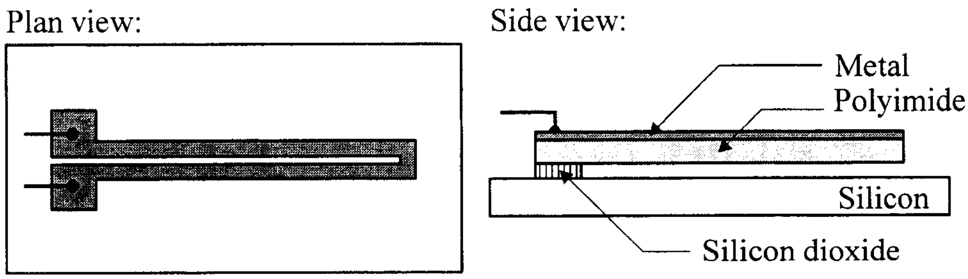

Figure 8

(b) By considering the bending moment due to the stress σ_f in the metal film, show that the deflection profile $v(x)$ of the actuator when heated is expected to satisfy:

$$\frac{d^2v}{dx^2} = \frac{6\sigma_f t_f}{Ed^2}$$

where E is the Young's modulus of the polyimide, t_f and d are the film thickness and cantilever depth respectively, and x is distance along the cantilever, measured from the root. [8]

(c) The temperature profile $T(x)$ along a heated cantilever is measured and found to be of the form:

$$T(x) = \frac{T_{\max}(2Lx - x^2)}{L^2}$$

where T_{max} is the maximum temperature increase, and L is the cantilever length. Using this result, together with the result in part (b), show that the end deflection $v(L)$ is expected to be:

$$v(L) = \frac{3}{2}\frac{L^2 t_f}{d^2}\frac{E_f}{E}\Delta\alpha T_{max}$$

where $\Delta\alpha$ is the difference between the thermal expansion coefficients of the polyimide and the metal, and E_f is the Young's modulus of the metal. [8]

Section C cont'd

9. (a) (i) Figure 9 shows an electrostatically actuated, capacitive RF shunt switch based on a simple metal bridge structure. Explain with the aid of a force-deflection plot, or otherwise, why a device of this type exhibits *snap-down* behaviour. Also sketch the variation of bridge height with applied voltage. [6]

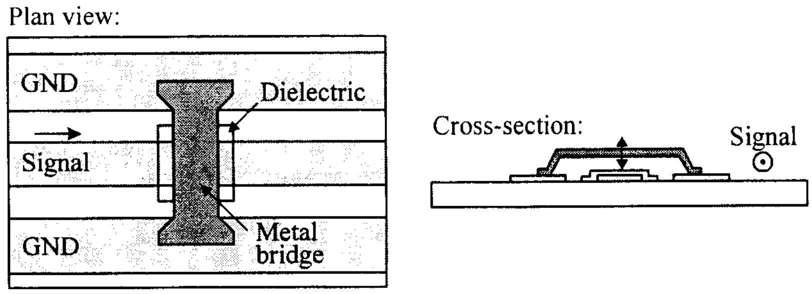

Figure 9

(ii) Show that the open-state insertion loss of the switch in Figure 9 may be written as:

$$|S_{21}|^2 \approx 1 - \frac{\omega^2 C_u^2 Z_o^2}{4}$$

where C_u is the open-state switch capacitance, Z_o is the characteristic impedance of the transmission line, and ω is the angular frequency of the signal. You should assume $\omega C_u Z_o << 1$. [4]

(b) (i) Explain why Gaussian beams are important in miniaturised optical systems. [2]

(ii) For a Gaussian optical beam, the variations of the beam radius w and the radius of phase front curvature R with distance z from the waist are given by:

$$w^2 = w_0^2\{1 + (z/z_0)^2\}$$
$$R = z\ \{1 + (z_0/z)^2\}$$

Here $z_0 = k_0 w_0^2/2$, where w_0 is the waist radius, and $k_0 = 2\pi/\lambda$, where λ is the wavelength. At $\lambda = 1.5\ \mu m$, a particular Gaussian beam has a phase front curvature of 1 m^{-1} and a radius of 1 mm. Estimate the distance from the waist, and the waist radius. [8]

Solutions - Power, Fields & Devices 2004

1) Power

[2.03]

1.

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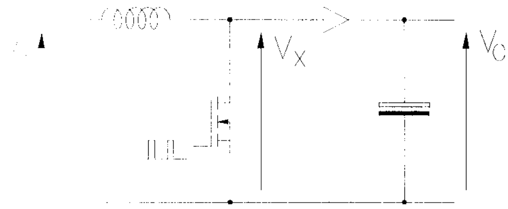
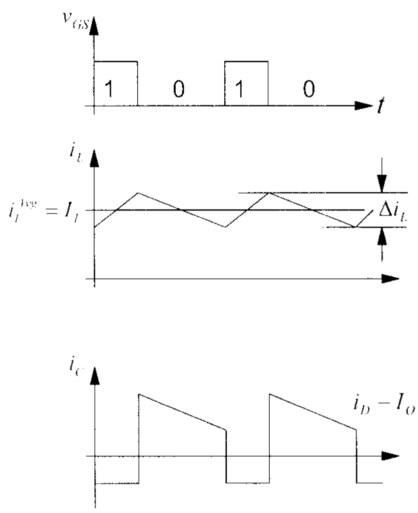
(a) Sketch the circuit diagram of the boost converter. [2]

Book work



(b) Sketch the waveforms of the currents through the inductor and capacitor for continuous inductor current. [3]

Book work (1 mark for inductor current; 2 marks for capacitor current)



(c) Calculate the value of duty-cycle required assuming continuous conduction. [2]

Computed example (1 mark for equation; 1 mark for numerical answer)

$$\frac{V_O}{V_I} = \frac{1}{1-\delta}$$

$$\delta = 1 - \frac{V_I}{V_O} = 1 - \frac{5}{12} = 0.5833$$

(d) Calculate the minimum switching frequency for which the SMPS will stay in continuous conduction for an input power of 2 W. [5]

New computed example

The input current is

$$I_I = \frac{P_I}{V_I} = \frac{2}{5} = 0.4\,A$$ *(1 mark)*

Inductor ripple current must be equal to or less than twice this; $\Delta i_L = 0.8\,A$ *(2 marks)*

The ripple is

$$\Delta i_L = \frac{V_I}{L}\cdot\frac{\delta}{f} = \frac{1}{fL}\cdot V_I\left(1-\frac{V_I}{V_O}\right)$$

(1 mark each)

$$f = \frac{1}{\Delta i_L L}\cdot V_I\left(1-\frac{V_I}{V_O}\right) = \frac{5\times 0.5833}{0.8\times 80\times 10^{-6}} = 45.6\,kHz$$

(e) Calculate the output voltage ripple when using the switching frequency found in part (d) but with an input power of 15 W. [6]

New computed example

There are two components to the ripple: resistive and capacitive

Peak to peak resistive voltage proportion to peak to peak current:

$$i_C^{ptp} = i_L^{pk} = I_I + \tfrac{1}{2}\Delta i_L = \frac{P}{V_I} + \tfrac{1}{2}\Delta i_L = \frac{15}{5} + \tfrac{1}{2}\times 0.8 = 3.4\,A$$

$$\Delta v_{C\ ESR} = i_C^{ptp} R_{ESR} = 3.4\times 0.02 = 0.068\,V$$

(1 mark for current; 1 for voltage equation and 1 for numerical answer))

The charge delivered to the capacitor is most easily found during the transistor on time (during which the capacitor is discharged by the load current).

$$\Delta q = I_O\frac{\delta}{f} = \frac{P}{V_O}\cdot\frac{\delta}{f} = \frac{15}{12}\times\frac{0.5833}{45.6\times 10^3} = 16.0\,\mu C$$

$$\Delta v_C = \frac{\Delta q}{C} = \frac{16.0\mu}{470\mu} = 0.034\ V$$

$$\Delta q = I_O\frac{\delta}{f} = \frac{P}{V_O}\cdot\frac{\delta}{f} = \frac{15}{12}\times\frac{0.5833}{45.6\times 10^3} = 16.0\,\mu C$$

$$\Delta v_C = \frac{\Delta q}{C} = \frac{16.0\mu}{150\mu} = 0.107\ V$$

(2 marks for pair of equations; 1 mark for numerical answer)

The output voltage ripple is the sum of these

$$\Delta v_O = 0.034 + 0.068 = 0.102\ V$$

$$\Delta v_O = 0.107 + 0.068 = 0.175\ V$$

(f) Calculate the switching frequency required to achieve an output voltage ripple of 100 mV with an input power of 15 W. [2]

New computed example

There are two approaches: consider L fixed or consider keeping discontinuous boundary at 2W. The second approach is taken here and it implies that the voltage drop across the ESR is unchanged. The capacitive element of voltage drop must be reduced to 32 mV.

$$f = \frac{P}{V_O}\cdot\frac{(\delta)}{\Delta v_C C} = \frac{15}{12}\times\frac{0.5833}{0.032\times 470\times 10^{-6}} = 48.5\,kHz$$

[2.03]

$$f=\frac{P}{V_O}\cdot\frac{(\delta)}{\Delta v_C\cdot C}=\frac{15}{12}\times\frac{0.5833}{0.032\times150\times10^{-6}}=151.9\,kHz$$

The other approach …

$$\Delta V_O=\frac{1}{C}I_O\frac{\delta}{f}+R_{ESR}\left(I_I+\frac{V_I\delta}{2Lf}\right)$$

$$f=\frac{\frac{I_O\delta}{C}+\frac{V_I\delta R_{ESR}}{2L}}{\Delta V_O-I_I R_{ESR}}$$

2.

(a) Explain the terms real power, reactive power and power factor. [4]

Book work

Real power is the average value of the instantaneous power and represents the power permanently converted into another form such as dissipation as heat. It is associated with the component of current that is in phase with the voltage.

Reactive power is a measure of the portion of instantaneous power that flows as short term storage of energy in reactive components (inductors and capacitors). It is associated with the component of current that is in quadrature to the voltage.

The power factor is the ratio of real to apparent power in which apparent power is the product of the RMS voltage and RMS current. Power factor reflects how effective the current flow is in transmitting real power. In a sinusoidal system power factor is equal to the cosine of the phase shift between voltage and current.

(b) Figure 2 shows the output voltage of a buck switch-mode power supply as a function of output current when operated in open-loop with a set duty-cycle. Explain its shape. [4]

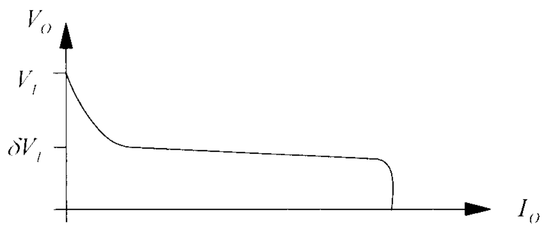


Figure 2

Book work and interpretation

Three regions are evident.

At low currents: the circuit is in discontinuous inductor current mode. The output voltage is an approximately reciprocal function of output current.

At moderate currents: the output voltage is almost constant at δV_I as derived for the ideal circuit. It has a small negative slope which occurs because of voltage drops across the resistive part of the inductor and the mosfet and the voltage drop across the diode.

At high currents, a current limit circuit has intervened and reduced the duty-cycle to reduce the output voltage and limit the current.

(c) Explain why 3-phase systems are favoured in power systems over, say 1-phase or 5-phase systems. [4]

Bookwork

A single-phase system does not make effective use of a generator. The generator is used most effectively by using coils around the complete circumference of the stator. Connecting these in series for a 1-phase machine cause significant voltage cancellation and poor utilisation.

A five phase connection of generator coils involves very little voltage cancellation and so good utilisation. However this arrangement requires 5 conduction paths in all transmission lines, circuit breakers and transformers. This is an expensive arrangement.

A three-phase system is the lowest phase number for which a balanced system with no neutral conductor can be arranged. It also has a good utilisation of the generators with only slightly more voltage cancellation than higher phase number systems

(d) Discuss the sources from which the UK derives its electrical energy and comment on how this might change in future. [4]

Bookwork

The system is quite diverse. Coal, nuclear and natural gas are the three main sources with nuclear, at 20% being the smallest of these. A small portion, around 3-4% is presently produced from renewable sources.

Natural gas is widely used in combined-cycle gas turbines. At present the gas is produced in the UK but the UK will become a net gas importer later this decade.

Coal is the traditional source for the UK but was significantly displaced by gas with the advent of CCGT technology and the availability of cheap long term contracts.

Nuclear power is generated from three different generations of plant. The first generation Magnox are part way through being decommissioned. Second generation AGC plant are projected to stay in service for another 6-12 years. The is only one third generation PWR plant

Hydro power is well established, particularly in northern Scotland but the amount is small and there is little scope for further development.

The targets for renewable energy are 10% by 2010 and 20% by 2020. The 2010 target is being advanced through wind turbines, largely on land to date but off shore in future.

(e) Explain the operating principle of an induction machine. [4]

Book work

There are windings for AC current on the stator and the rotor of the machine. The rotor winding is not normally externally supplied and is simply short-circuit.

The stator is supplied with alternating current. A three-phase current set in a three-phase winding creates a resultant magnetic field that has a constant magnitude but whose angular position rotates.

The rotating flux cuts the conductors of the rotor and the rate-of-change of flux linkage induces voltages along the bar. These voltages drive currents through the short-circuited winding. The currents react with the stator field to create a torque. The torque acts to reduce the rate-of-change of flux and therefore acts to accelerate the rotor towards the same rotational speed as the stator field. Thus torque is produced at all speeds except synchronous speed and the larger the speed difference between the physical rotor and the stator field, the larger the induced voltage. The torque also increases with slip for small values of slip.

[2.03]

3.

A three-phase induction machine has the following parameters:

Number of pole-pairs, P=2;
Stator resistance, RS=0.08;
Stator leakage reactance, XS=0.5;
Magnetising reactance, XM=10;
Iron loss resistance, RM=30;
Referred rotor resistance, RR=0.07;
Referred rotor leakage reactance, XR=0.5;

The supply to the machine is 50 Hz with a line voltage of 400 V
In operation, the machine runs at a speed of 1,445 r.pm.

Calculate the following:

(a) the slip [2]

Computed example

$$n_S = \frac{60f}{P} = 1{,}500\ r.p.m.$$

$$s = \frac{n_S - n_R}{n_S} = \frac{1{,}500 - 1{,}445}{1{,}500} = 0.367$$

(b) the stator current [6]

Computed example (3 marks for equation for impedance; 1 for numerical answer; 1 for correct phase voltage and 1 for numerical answer for current)

$$Z_T = R_S + jX_S + R_M \mathbin{//} jX_M \mathbin{//}\left(\frac{R_R}{s} + jX_R\right)$$

$$\frac{R_R}{s} + jX_R = 1.91 + j0.5\,\Omega$$

$$R_M \mathbin{//} jX_M = 3 + j9\,\Omega$$

$$Z_T = 1.6848 + \mathrm{j}1.2001\,\Omega$$

$$I_S = \frac{V_S}{Z_T} = \frac{\frac{1}{\sqrt{3}}400}{1.6848 + \mathrm{j}1.2001} = 90.9368 - \mathrm{j}64.7751$$
$$= 111.6\angle -35.4^\circ$$

(c) the electromagnetic torque developed [6]

Computed example

[2.03]

$$T = 3\frac{I'^2_R R'_R\left(\frac{1}{s}-1\right)}{\omega_R}$$

$$3\frac{I'^2_R R'_R}{s\,\omega_S}$$ *(3 marks)*

$$I'_R = I_S \frac{jX_M // R_M}{R_M // jX_M // \left(R'_R/s + jX'_R\right)}$$

$$= 88.5895 - j44.3044\ \text{A}$$

$$= 99.05\ A$$

(1 mark for method; 1 for numerical answer)

$$T = 3\frac{99.05^2 \times \frac{0.07}{0.0367}}{2\pi\frac{50}{2}} = 357.4\ Nm$$ *(1 mark)*

(d) the efficiency of the machine [6]

Computed example – several approaches possible

(1 mark for output/input idea; 1 mark for output power; 2 marks for input power; 2 marks for numerical answer)

$$\eta = \frac{P_O}{P_I}\times 100\% = \frac{3I'^2_R R'_R\left(\frac{1}{s}-1\right)}{3V_S I_S \cos(\phi)}\times 100\%$$

$$= 85.9\%$$

<u>Q4 Solution</u> (SECTION B : fields)

Laplace's eqn. is satisfied -- by proving that

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial z^2} = 0.$$

$V = 0$ when $x=0$ or $y=0$. – grounded conductor has right-angled cross section
$V = 10V$ when $xy = 0.5$, i.e. H.V. electrode has parabolic cross-section in the xy plane – a curved strip that approaches the grounded conductor at the closest distance of $0.5/7 = 0.0714$ mm at its two extreme edges.

$$\underline{\nabla} V = 20y\,\hat{\underline{x}} + 20x\,\hat{\underline{y}} \;;\; |\underline{\nabla} V| = 20(x^2+y^2)^{1/2}.$$

On high-V electrode :

$$|\nabla V| = 20\left(x^2 + \frac{1}{4x^2}\right)^{1/2}$$

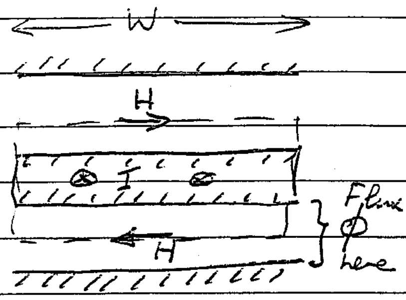
$$\Rightarrow \frac{1}{20}\frac{\partial |E|}{\partial x} = \left[\frac{2x \cdot \frac{1}{2}\frac{16x^3}{(4x^4+1)^{1/2}} - 2(4y^4+1)^{1/2}}{4y^2}\right]$$

$= 0$ when $y^4 = 1/4$, i.e. $y = 1/\sqrt{2}$ & $x = 1/\sqrt{2}$.

– Second differentiation is needed to prove it is a minimum.
Value of E at this point is $|\underline{\nabla} V| = 20$ V/mm

On grounded conductor, $\underline{\nabla} V = 20y$ on y-axis & $20x$ on x-axis
Use of Gauss' law on any suitable surface enclosing an infinitesimal width of this electrode is sufficient to prove that surface charge $\propto \underline{\nabla} V$ & is proportional to x and y i.e. proportional to distance from z-axis.

Q5 Solution E2-3 B (Fields)



Capacitance $= \dfrac{2\varepsilon W}{W/5}$ /metre
(any method)
$= 25\varepsilon_0$ /metre

Inductance : Method A.
Ampères law around eg. path shown $\Rightarrow 2HW = I$
Hence flux $\phi = \mu H \dfrac{W}{5}$ per unit length (this amount of flux encloses current I in centre conductor)

$$\Rightarrow L = \frac{\phi}{I} = \mu\left(\frac{W/5}{2W}\right) = \frac{\mu_0}{10} \text{ per metre.}$$

Alternative method B:
Treat as two identical lines connected in parallel, each with
$C' = \dfrac{\varepsilon W}{W/5}$ and $L' = \mu\dfrac{(W/5)}{W}$

When paralleled, $C = 2C'$ and $L = \frac{1}{2}L'$

Then $Z = \sqrt{L/C} = \sqrt{\dfrac{\mu_0}{250\varepsilon_0}} = 23.8\,\Omega$ and Delay $= \dfrac{1}{2}\sqrt{LC} = 5.3$ns

For no reflections, Z's must be identical in value.
i.e $Z_2 = \sqrt{\dfrac{\mu t_2}{W}\dfrac{t_2}{\varepsilon W}} = \dfrac{t_2}{W}\sqrt{\dfrac{\mu_0}{2.5\varepsilon_0}} = 23.8\,\Omega \Rightarrow \dfrac{t_2}{W} = \dfrac{1}{10}$

Better approximations to correct values when $t \ll W$, and/or $\varepsilon_r \gg 1$.

Solution Q6

6. Consider a wave of a single frequency ω.

Let $\underline{E} = \underline{E}_0 \exp j(\omega t - kz)$ where $\underline{E}_0 \perp Z$

Then $\nabla^2 \underline{E} = \frac{\partial^2 \underline{E}}{\partial z^2} = -k^2 \underline{E}_0 \exp j(\omega t - kz) = -k^2 \underline{E}$

And $\frac{\partial^2 \underline{E}}{\partial t^2} = -\omega^2 \underline{E}$ Hence $k^2 = \mu \varepsilon \omega^2$ to satisfy wave eqn.

Now the phase moves at a velocity at which $(\omega t - kz) = \text{constant}$,

i.e. at velocity $\frac{z}{t} = \frac{\omega}{k} = \sqrt{\frac{1}{\mu\varepsilon}}$ – independent of ω if

ε is independent of ω when $\mu = \mu_0$.

Principle of superposition (assumption) permits any wave shape to be decomposed into a sum over many frequencies, hence any shape travels at this frequency without change.

Consequence is that modulated signals travel without distortion in any such medium – empty space (or air if frequency not too high). Conversely, they are distorted where $\varepsilon_r = f(\omega)$!

Simplest proof:

$\underline{\nabla} \times \underline{E} = -\partial \underline{B}/\partial t \Rightarrow \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{x} = -\frac{\partial B_x}{\partial t}\hat{x} \Rightarrow \frac{\partial B_x}{\partial t} = 0$

Similarly $\frac{\partial B_z}{\partial t} = 0$, but $\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{y} = -\frac{\partial B_y}{\partial t}\hat{y} \Rightarrow -jkE_{0x} \exp j(\omega t - kz) = -\frac{\partial B_y}{\partial t}$

Hence $B_y = \int \frac{\partial B_y}{\partial t} dt = \frac{k}{\omega} E_{0x} \exp j(\omega t - kz)$ and $\frac{B_y}{E_x} = \frac{k}{\omega} = \frac{1}{c}$

[Alternatively use $\underline{\nabla} \times \underline{H} = \underline{J} + \partial \underline{D}/\partial t$]

c) Devices

7\. a) *Bookwork*

(i) The stiffness of the cantilever is $k = 3EI/L^3$, where $I = bd^3/12$ is the second moment of area and E is Young's modulus. Hence, $k = Ebd^3/4L^3$, so the stiffness scales as O[L]. The stiffness will therefore reduce by a factor of 2 if all dimensions are reduced by a factor of 2.

[2]

(ii) The static deflection is $X_s = mg/k$, where m is the mass and g is the acceleration due to gravity. The mass scales as $O[L^3]$, while the stiffness scales as O[L]. Hence, the static deflection scales as $O[L^2]$. The static deflection will therefore reduce by a factor of 4 if all dimensions are reduced by a factor of 2.

[2]

(iii) The natural (angular) frequency of a mass-spring oscillator is $\omega_0 = \sqrt{(k/m)}$. The stiffness scales as O[L], while the mass scales as $O[L^3]$. The resonant frequency therefore scales as $O[L^{-1}]$. The resonant frequency will therefore increase by a factor of 2 if all dimensions are reduced by a factor of 2.

[2]

(iv) If the position of the mass varies as $x(t) = X \sin(\omega t)$, the velocity of the mass varies as $v(t) = X\omega \cos(\omega t)$. The peak velocity then varies as $v_{max} = X\omega$. If all dimensions are reduced by a factor of 2, the resonant frequency will increase by a factor of 2; if the oscillation amplitude X is also reduced by a factor of two, the peak velocity will scale as $O[L^0]$ and hence be unaltered.

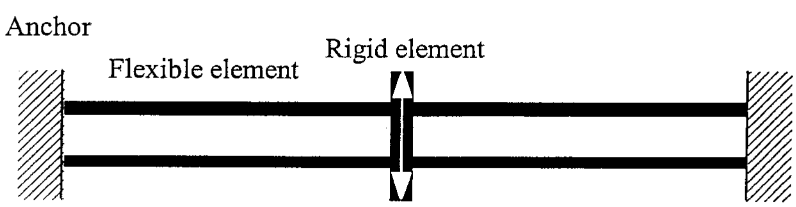
[2]

(v) The peak energy stored in the cantilever is $E = 1/2\ kX^2$, which scales as $O[L^3]$. If all dimensions are reduced by a factor of 2, the stiffness will reduce by a factor of 2. If the oscillation amplitude X is also reduced by a factor of two, the peak energy stored will reduce by a factor of 8.

[2]

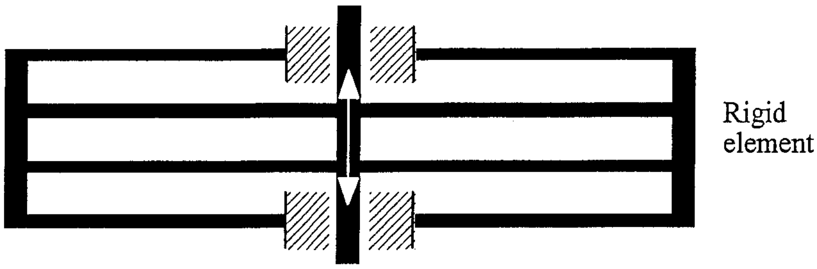
b) *Bookwork*

In the hammock suspension, the moving part is suspended from parallel beams that are built in to an anchor at both ends. The suspension can be made very stiff against out-of-plane displacement by using thin, deep beams. The stiffness is not constant, but increases quadratically with displacement because of tensile axial strain. Non-linear behaviour then arises. The stiffness is also dependent on intrinsic stress in the mechanical layer.



[2]

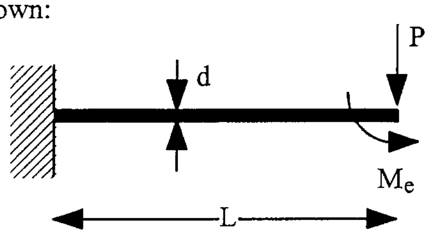
In the folded flexure, the suspension is doubled back on itself as shown. Because the rigid linking beams are not constrained axially, almost perfect compensation for axial strains is achieved. Much greater lateral displacement may then be obtained without non-linearity, and intrinsic stress has little effect.



[2]

c) *Bookwork*

For the cantilever shown:



The beam bending equation is:
$d^2y/dx^2 = M/EI = \{-P(L - x) + M_e\}/EI$
Where the second moment of area is $I = bd^3/12$

Integrating twice we obtain:
$y = \{-P(Lx^2/2 - x^3/6) + M_e x^2/2 + Ax + B\}/EI$

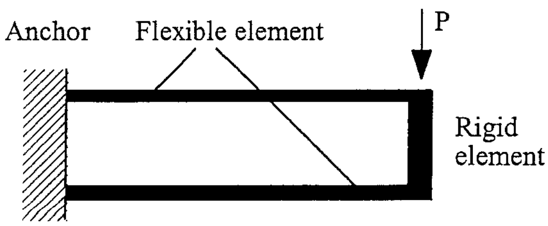
From the boundary conditions at $x = 0$ ($y = 0$, $dy/dx = 0$) we obtain $A = B = 0$
From the boundary conditions at $x = L$ ($dy/dx = 0$) we can find M_e as $M_e = PL/2$

Hence, the complete solution is $y = -P(Lx^2/4 - x^3/6)/EI$

So the end deflection is $y(L) = -PL^3/12EI$ and the stiffness is $k = 12EI/L^3$

[4]

In the portal frame, there are two cantilevers in parallel, and the rigid link bar acts to prevent rotation. The stiffness is therefore twice the result above, namely $k = 24EI/L^3$.



[2]

8. a) *New application of taught principles*
A possible sequence would be:
Grow or deposit sacrificial oxide
Deposit [and cure] polyimide
Vacuum deposit metal film, and pattern by photolithography and wet etching
Reactive ion etch to transfer metal pattern into polyimide
Wet etch sacrificial oxide to release structure

[4]

b) *Bookwork*
The bending moment due to the stress σ_f in the thin film is given by $M = \sigma_f.(t_f b).d/2$ where b is the cantilever width (stress x film cross-sectional area x distance from the neutral axis). [3]

Combining this with the standard bending equation v''=M/EI gives:

$$\frac{d^2v}{dx^2} = \frac{\sigma_f t_f bd/2}{E \cdot (bd^3/12)} = \frac{6\sigma_f t_f}{Ed^2}$$

[5]

c) *New application of theory*
The film stress is arising from frustrated thermal expansion, so we have:

$$\sigma_f = E_f \Delta\alpha T$$ [2]

Combining this with the results given, we obtain:

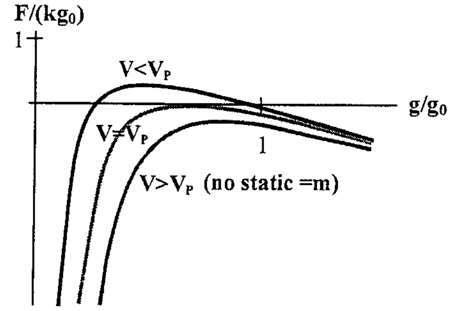
$$\frac{d^2v}{dx^2} = \frac{6t_f}{d^2}\frac{E_f}{E}\Delta\alpha T_{max}\frac{[2Lx - x^2]}{L^2}$$

Integrating twice, with boundary conditions v'(0) = 0 and v(0) = 0, and then putting x = L, the required result is obtained. [6]

13 ½ 15

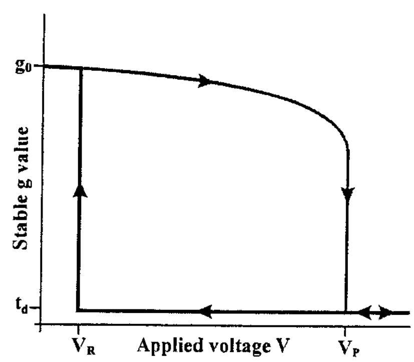
9. a) *Bookwork*

The combination of a parallel plate actuator and a linear spring gives a non-linear force-displacement curve with a single maximum:



Below the critical voltage, Vp, there is a stable equilibrium where F = 0, dF/dg < 0. For V > Vp, there is no equilibrium point and the bridge snaps down. Once snap-down has occurred, V must be reduced until F $\rightarrow$ 0 at minimum gap before bridge will be released.

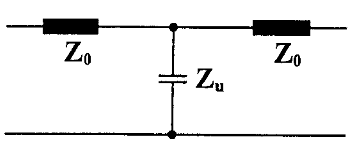
The resulting variation of bridge height g with applied voltage exhibits hysteresis:



[6]

b) *Bookwork*

We need to calculate $|S_{21}|^2$ for a transmission line with capacitive shunt:



Using the standard transmission line formula:

$$S_{21} = 2(Z_u//Z_o)/[(Z_u//Z_o) + Z_o]$$
$$= 1/[1 + Z_o/(2Z_u)]$$

where $Z_u = 1/(j\omega C_u)$ is the impedance of the shunt capacitor. From this it follows that:

$$|S_{21}|^2 = 1/[1 + Z_o^2/(4|Z_u|^2)]$$
$$\approx 1 - Z_o^2/(4|Z_u|^2) \quad \text{when } |Z_u| >> Zo$$

[4]

c) *Bookwork*
Gaussian beams are important in miniaturised optical systems because they represent bounded optical beams that may propagate in a known and controllable manner. The beam size may be kept within limits by periodically refocusing using lenses.

[2]

d) *New application of theory and computed example*
For a Gaussian beam, the variations of the beam radius and the radius of phase front curvature with distance are $w^2 = w_0^2\{1 + (z/z_0)^2\}$ and $R = z\ \{1 + (z_0/z)^2\}$. Here $z_0 = k_0w_0^2/2$, where w_0 is the waist radius and $k_0 = 2\pi/\lambda$, where λ is the wavelength. Hence, $w^2 = 2z_0/k_0\{1 + (z/z_0)^2\}$.

Defining $\alpha = z/z_0$, we can write:
$w^2 = (2z/\alpha k_0)(1 + \alpha^2)$ and $R = (z/\alpha^2)(\ 1 + \alpha^2)$
Hence, $w^2/R = 2\alpha/k_0$, so $\alpha = k_0w^2/2R = (\pi/\lambda)\ w^2/R$
For the parameters given, $\alpha = (\pi/1.5 \times 10^{-6})\ (10^{-3})^2/1 = 2.09439$

[2]

Re-arranging, we get: $z = \alpha\pi w^2/\{\lambda(1 + \alpha^2)\}$
For the parameters given:
$z = 2.09439\ \pi \times (10^{-3})^2/\{1.5 \times 10^{-6} \times (1 + 2.09439^2)\} = 0.81435$ m.

[2]

Hence, $z_0 = 0.81435 / 2.09439 = 0.3888$ m.

[2]

Re-arranging the expression for z_0, we get $w_0 = \sqrt{(2z_0/k_0)} = \sqrt{(\lambda z_0/\pi)}$. For the parameters given, $w_0 = \sqrt{(1.5 \times 10^{-6} \times 0.3888/\pi)} = 4.309 \times 10^{-4}$ m, or 0.43 mm.

[2]

.5 5 15