UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2002

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C343=I3.22

OPERATIONS RESEARCH

Tuesday 23 April 2002, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1a Find the dual of the linear programming problem:

$$\max 4x_1 + x_2$$

subject to

$$\begin{aligned} x_1 + 2x_2 &= 6 \\ x_1 - x_2 &\geq 3 \\ 2x_1 + x_2 &\leq 10 \end{aligned}$$

and

$$x_1 \geq 0, x_2 \geq 0.$$

Do not solve the linear programming problem.

b Give a rule for simply writing down the solution to any linear programming problem of the form:

$$\max \left\{ \sum_{i=1}^{n} c_i x_i \;\middle|\; \sum_{i=1}^{n} x_i = 1;\ x_i \geq 0, i = 1, \ldots, n \right\}.$$

c Formulate a linear programming problem for finding (x_1, x_2) such that:

$$|x_1 + x_2 - 1| + |x_1 + x_2 - 2|$$

is as small as possible.

Do not solve the linear programming problem.

d Formulate a linear programming problem for finding (x_1, x_2) such that the maximum of

$$|x_1 + x_2 - 1| \text{ and } |x_1 + x_2 - 2|$$

is as small as possible.

Do not solve the linear programming problem.

All parts carry equal marks.

2 Consider the following linear programming problem:

$$\max x_0 = 2x_1 + 7x_2 + 4x_3$$

subject to

$$\begin{aligned} x_1 + 2x_2 + x_3 &\leq 10 \\ 3x_1 + 3x_2 + 2x_3 &\leq 10 \end{aligned}$$

and

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

a Construct the dual of this primal problem and use the dual problem — without solving it — to demonstrate that the optimal value of the primal cannot exceed 40.

b Use the simplex algorithm to solve the dual problem.

All parts carry equal marks.

3a Describe the branch and bound algorithm to solve the following integer programming problem:

$$\max x_0 = 2x_1 + 3x_2$$

subject to

$$\begin{aligned} x_1 + 2x_2 &\leq 10 \\ 3x_1 + 4x_2 &\leq 25 \end{aligned}$$

and

$$x_1 \geq 0, x_2 \geq 0; \text{ and } x_1, x_2 \text{ integer.}$$

Hint: When the integer restrictions on x_1 and x_2 are ignored, the solution of the resulting linear program is given by:

$$x_0 = \frac{35}{2},\ x_1 = 5,\ x_2 = \frac{5}{2}.$$

Do not solve the problem but do describe the fathoming rules for Branch and Bound.

b Two secretaries, Eve and Susan, want to divide their main office duties (photocopying, correspondence, typing reports, filing) between them such that each has two different duties, but the total time they spend on office duties is kept to a *minimum*. Their efficiencies in these duties differ. For a given volume of work, the time each would need to perform the corresponding duty is given by the following table:

	Hours per Week Needed			
	Photocopying	Correspondence	Typing Reports	Filing
Eve	9.0	15.6	7.2	5.8
Susan	9.8	14.4	8.6	6.2

Formulate the above as a 0-1 integer programming problem.

Do not solve the problem.

All parts carry equal marks.

4 A two person zero-sum game with an $n \times n$ reward matrix $\mathbf{A}$ is called a *symmetric* game if $\mathbf{A} = -\mathbf{A}^T$.

a Formulate the strategies of the row and column players using linear programming and show that a symmetric game must have a value of zero.

b Show that in a symmetric game if $(x_1^*, x_2^*, \ldots, x_n^*)$ is an optimal strategy for the row player, then $(x_1^*, x_2^*, \ldots, x_n^*)$ is also an optimal strategy for the column player.

All parts carry equal marks.