

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2016

EEE PART I: MEng, BEng and ACGI

**SEMICONDUCTOR DEVICES**

Corrected Copy

Tuesday, 31 May 10:00 am

Time allowed: 2:00 hours

**There are THREE questions on this paper.**

**Answer ALL questions.**

*Question One carries 50% of the marks. Questions Two and Three each carry 25%.*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      K. Fobelets

Second Marker(s) :      S. Lucyszyn



**Special instructions for invigilators**

**Special instructions for students**

*Do not use red nor green ink.*

## Constants

permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$
permeability of free space:	$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10} \text{ cm}^{-3} \text{ at } T = 300 \text{ K}$
dielectric constant of Si:	$\epsilon_{Si} = 11$
dielectric constant of SiO <sub>2</sub> :	$\epsilon_{ox} = 4$
thermal voltage:	$V_T = kT/e = 0.026 \text{ V at } T = 300 \text{ K}$
charge of an electron:	$e = 1.6 \times 10^{-19} \text{ C}$
Planck's constant:	$h = 6.63 \times 10^{-34} \text{ Js}$
Bandgap of Si:	$E_G = 1.12 \text{ eV at } T = 300 \text{ K}$
Effective density of states of Si:	$N_C = 3.2 \times 10^{19} \text{ cm}^{-3} \text{ at } T = 300 \text{ K}$ $N_V = 1.8 \times 10^{19} \text{ cm}^{-3} \text{ at } T = 300 \text{ K}$

## Formulae

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

Schrödinger's equation  
in one dimension

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Fermi distribution

$$n_i = \sqrt{N_V N_C} \exp\left(\frac{-E_G}{2kT}\right)$$

Intrinsic carrier concentration

$$n = N_C e^{\frac{(E_C - E_F)}{kT}}$$

Concentration of electrons

$$p = N_V e^{\frac{(E_V - E_F)}{kT}}$$

Concentration of holes

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon}$$

Poisson equation in 1  
dimension

$$\left. \begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p \frac{dp(x)}{dx} \end{aligned} \right\}$$

Drift and diffusion current  
densities in a semiconductor

$$I_{DS} = \frac{\mu C_{ox} W}{L} \left( (V_{GS} - V_{th}) V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Current in a MOSFET

$$J_n = \frac{eD_n n_{p0}}{L_n} \left( e^{\frac{eV}{kT}} - 1 \right)$$

$$J_p = \frac{eD_p p_{n0}}{L_p} \left( e^{\frac{eV}{kT}} - 1 \right)$$

Current densities for a pn-  
junction with lengths  $L_n$  &  $L_p$

$$V_{bi} = \frac{kT}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT}\right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$

Minority carrier injection  
under bias  $V$

$$D = \frac{kT}{e} \mu$$

Einstein relation

$$w_n(V) = \left[ \frac{2\epsilon(V_{bi} - V)N_A}{e(N_A + N_D)N_D} \right]^{1/2} \quad \& \quad w_p(V) = \left[ \frac{2\epsilon(V_{bi} - V)N_D}{e(N_A + N_D)N_A} \right]^{1/2}$$

Depletion widths under bias  $V$

$$W_{depl}^{\max} = 2 \left[ \frac{\epsilon kT \ln\left(\frac{N_{\text{substrate}}}{n_i}\right)}{e^2 N_{\text{substrate}}} \right]^{1/2}$$

Maximum depletion width

1.

a) Energy band diagrams

i) Sketch the energy band diagram, including  $E_C$ ,  $E_V$ ,  $E_F$ ,  $E_i$  and  $E_G$  of n-type Si. [4]

ii) Give the values of  $E_C$ ,  $E_F$ , and  $E_i$  with respect to  $E_V$  for the Si in i) when the doping concentration is  $2 \times 10^{17} \text{ cm}^{-3}$ . [6]

iii) Draw the energy band diagram,  $E_C$ ,  $E_V$ ,  $E_i$  of the Si in ii) when a voltage of 0.5 V is applied as shown in Fig. 1.1. Give the values of  $E_C$ , and  $E_i$  at  $x = 0$  and  $1 \mu\text{m}$  with respect to  $E_V$ . [6]

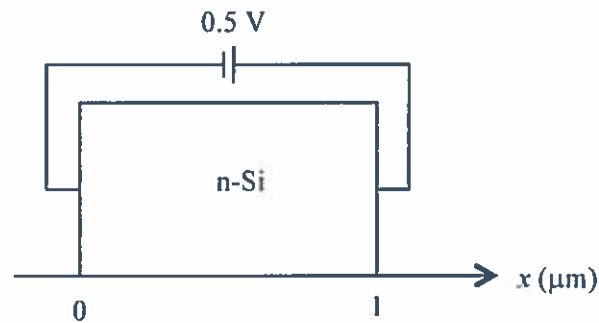


Figure 1.1: n-Si with a length of  $1 \mu\text{m}$  and an applied voltage of 0.5 V. No voltage drops across the ideal ohmic contacts.

iv) Add the electric field vector to the drawing in iii) and give its amplitude. [4]

b) pn-diode

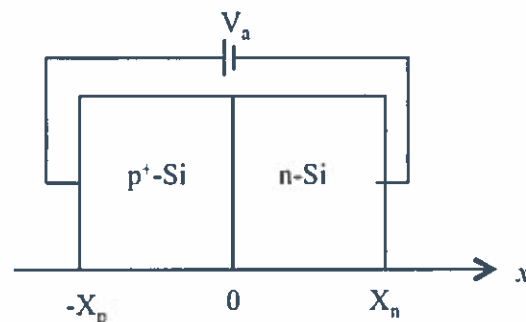


Figure 1.2: p-n diode under bias  $V_a$ . No voltage drops across the ideal ohmic contacts.

i) Sketch the energy band diagram, including  $E_C$ ,  $E_V$ , and  $E_F$  of the pn-diode given in Fig. 1.2 when  $N_A = 2 \times N_D$ . Ensure that your sketch is consistent with the relative doping densities in the layers. [6]

ii) Fill in the blank spaces with one of the following words: forward, reverse, diffuse, drift, left-to-right, right-to-left, 1, 2, 4, larger or smaller.

ii1) The pn diode in Fig. 1.2 is.....biased.

ii2) Electrons.....from.....across the depletion region.

ii3) Holes.....from.....across the n-type region.

ii4) The depletion width in the p-type region is... $\times$ .....than that in the p-type region.

n-type

[4]

c) BJT

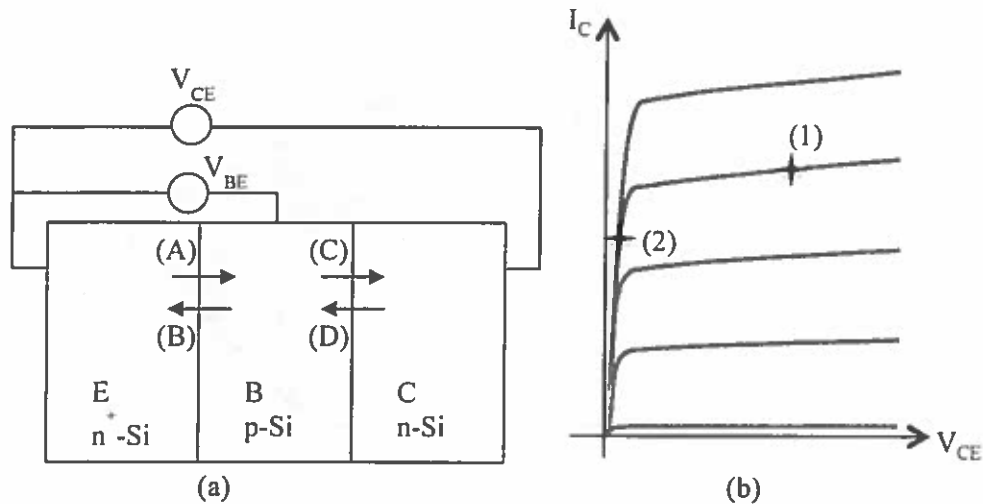


Figure 1.3: (a) Material cross section of npn BJT with bias circuit (common-emitter configuration) (b) Output characteristic of the npn BJT in (a). Two bias points are indicated with a  $\overset{(i)}{+}$  ( $i = 1, 2$ ).

- i) Give the bias condition (forward or reverse) across the EB and the BC junction for bias point (1) and (2) on the output characteristics in Fig. 1.3(b). [4]
- ii) Which type of carriers (electrons, holes or both) needs to be injected into the base by the base current at bias point (1)? [2]
- iii) Give the expression of the current gain  $\beta$  in terms of (A), (B), (C), (D) in Fig. 1.3(a) for the BJT biased in bias point (1). (A), ..., (D) indicate the currents associated with the carrier injection indicated by the arrows. [4]

d) MOSFET

- i) Sketch the energy band diagram,  $E_C$ ,  $E_V$  and  $E_F$  of an n-channel enhancement mode MOSFET from the gate through the oxide into the substrate in the middle of the channel and at  $V_{GS} = V_{th}$  and  $V_{DS} = 0$  V. Indicate  $V_{GS}$  on the sketch. [6]
- ii) Derive the expression of the transconductance of an n-channel enhancement mode MOSFET for  $V_{GS} > V_{th}$  and  $V_{DS} > V_{GS} - V_{th}$ . [4]

## 2. BJT

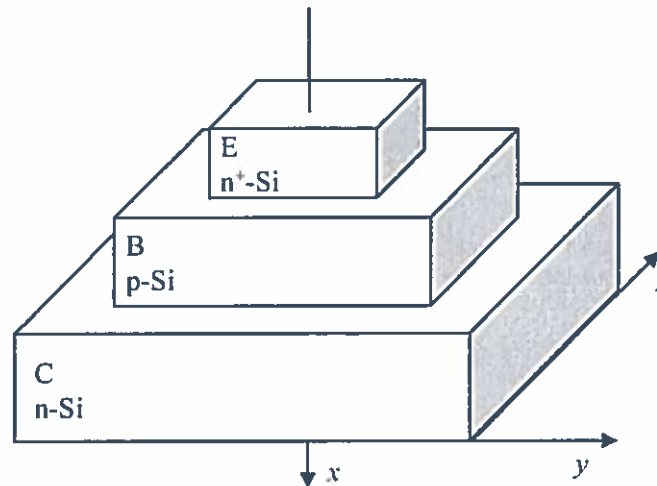


Figure 2.1: Material cross section of a BJT. Transport of carriers is in the  $x$ -direction. The carrier concentrations and currents are homogeneous in the  $y$ - $z$  plane in each layer. The layers of the BJT are shorter than the minority carrier diffusion length.

Table 2.1: Device parameters

	Layer length ( $\mu\text{m}$ )	Layer area ( $\mu\text{m}^2$ )	Doping density ( $\text{cm}^{-3}$ )	Diffusion constant minority carriers ( $\text{cm}^2 \text{s}^{-1}$ )
emitter	0.4	$40 \times 40$	$5 \times 10^{18}$	4
base	0.8	$50 \times 50$	$5 \times 10^{16}$	20
collector	1	$60 \times 60$	$1 \times 10^{16}$	10

- What is the meaning of the expression: “diffusion length of the minority carriers”? Use a maximum of 30 words to answer. [4]
- Calculate the value of the minority carrier concentration at the contacts and the edges of the depletion regions for a forward bias across the EB junction of 0.26 V and a reverse bias of amplitude 1 V across the BC junction. The contacts are ideal. [6]
- Plot the minority carrier concentration variation along the  $x$ -axis (in Fig. 2.1) in each layer of the BJT. Take the depletion region at the BC junction into account. Label the axes. [6]
- Calculate the emitter, base and collector current for the BJT with bias given in b) and taking the depletion region at the BC junction into account. [9]



### 3. MOSFET

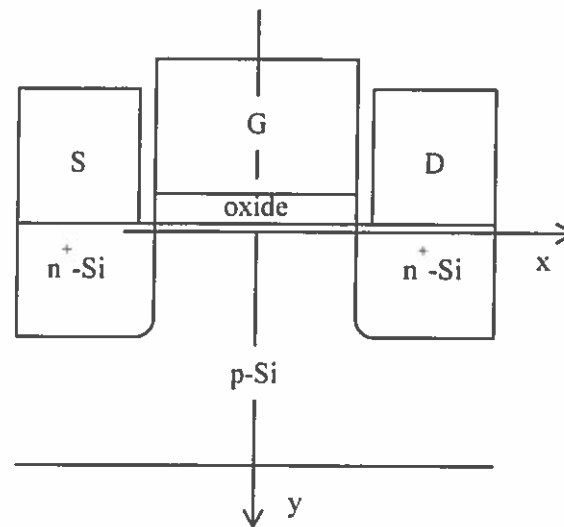


Figure 3.1: Material cross section of a MOSFET.  $\phi_p < \phi_G$

Table 3.1: Device parameters:

Substrate doping ( $\text{cm}^{-3}$ )	$5 \cdot 10^{16}$
Ohmic contact doping ( $\text{cm}^{-3}$ )	$10^{20}$
Gate length ( $\mu\text{m}$ )	2
Gate width ( $\mu\text{m}$ )	5
Oxide thickness (nm)	10
Electron mobility ( $\text{cm}^2 \text{V}^{-1} \text{s}^{-1}$ )	400
Hole mobility ( $\text{cm}^2 \text{V}^{-1} \text{s}^{-1}$ )	200
Threshold voltage (V)	2

- Why is the electron mobility higher than the hole mobility in Si? Use a maximum of 30 words to answer. [4]
- Sketch the energy band diagram, including  $E_C$ ,  $E_V$  and  $E_F$  along the y-direction in Fig. 3.1. The relationship between the workfunction of the substrate and the gate is:  $\phi_G - \phi_p = 1 \text{ V}$  and the bias is zero on all contacts. [6]
- Plot the gate capacitance  $C_{GS}(V_{GS})$  for  $V_{GS} < 0$  to  $V_{GS} > V_{th}$ , of the MOSFET in Fig. 3.1 and  $V_{DS}$  in the triode region.  $V_{th}$  is the threshold voltage. Comment on each voltage  $V_{GS}$  where  $C_{GS}$  deviates from a constant value. [9]
- Calculate the maximum and minimum value of  $C_{GS}$ . Use the device parameters given in Table 3.1. [6]

