Imperial College London

BSc/MSci EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

MECHANICS AND RELATIVITY

For 1st-Year Physics Students

Wednesday, 9th May 2012: 10:00 to 12:00

Answer ALL questions from section A and TWO questions from section B.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the FOUR answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in FOUR answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

Go to the next page for questions

SECTION A

1. Show all working throughout

(i) A particle of mass 130 g is projected at initial velocity 3.0 ms^{-1} into a medium providing a retarding force of $0.45e^{0.25v}$ N where v is the particle's velocity in the medium in ms^{-1}. No other forces act on the particle. Find the particle's initial acceleration and the time taken to slow to half its original velocity. [6 marks]

(ii) A uniform ladder of length 10.0 m and weight 195 N resting on a rough floor leans against a smooth wall at an angle of 30.0° to the vertical. Sketch a labelled free body diagram for the ladder and find the magnitude of all the forces on it. [6 marks]

(iii) Steel pellets moving at 12 ms^{-1} strike a plate set at 45° to their line of motion. The pellets are thereby deflected by 90° without a change in speed. If the flow rate is 0.5 kgs^{-1} find the magnitude of the average force on the plate. [5 marks]

(iv) An object of mass 30.0 g slides all the way down an inclined plane of length 50.0 cm set at 60.0° to horizontal. The coefficient of dynamic friction between the object and the plane is zero at the top and increases linearly with distance to 0.250 at the bottom. Find the energy dissipated by friction during the slide. [5 marks]

(v) A person of mass 40.0 kg is on the edge of a uniform roundabout of radius 2.0 m and moment of inertia 160 kg.m^2 initially rotating at 1.0 rad.s^{-1}. The person walks to the centre of the roundabout. Treating the person as a point mass, find the kinetic energy increase in the system and explain where the increase in kinetic energy has come from. [5 marks]

[Total 27 marks]

2. The Lorentz transformations are

$$\begin{aligned} ct' &= \gamma(ct - \beta x), \\ x' &= \gamma(x - \beta ct), \\ y' &= y, \\ z' &= z, \end{aligned}$$

where all the symbols have their usual meaning.

(i) Given that $y' = y$, why are the y-velocities u_y and u'_y different in the primed and unprimed frames? [2 marks]

(ii) Procyon is a binary star system 11 light-years distant as measured in the inertial frame of the Earth. Some future astronauts visit Procyon from Earth by travelling at a constant velocity of 0.6c until they reach the system. (Procyon is at rest in the Earth's frame.)

(a) Why is the spaceship considered an inertial frame? [1 mark]

(b) Calculate the initial distance to the star as measured by the astronauts as they pass the Earth. [1 mark]

(c) How long would the journey from the Earth to Procyon take in the astronauts' frame? In the Earth's frame? [2 marks]

(d) Disaster! Procyon explodes as a supernova during the trip. Light from the supernova reaches the spaceship 10 years into its journey, according to its clock. Draw a space-time diagram from the ship's inertial frame, showing the world lines of the spaceship, Earth, Procyon and the light from the supernova explosion. [3 marks]

[Total 9 marks]

SECTION B

3. This question is about circular motion at constant speed.
Marks will be awarded for showing working and words of explanation.

(i) Define (a) velocity and (b) acceleration. Explain how it is possible for an object to accelerate whilst travelling at constant speed. [5 marks]

(ii) Using diagrams where appropriate, show that if an object travels along the arc of a circle of radius r at speed v then the magnitude of its acceleration towards the centre of the circle is given by $\frac{v^2}{r} = \omega^2 r$ where ω is the object's angular velocity. [10 marks]

(iii) In a conical pendulum a mass M is attached to a light string of length l which hangs from a fixed point and rotates at constant angular velocity ω such that the pendulum traces out a cone of constant radius r with the string at angle α to the vertical.
Sketch a free body diagram for the mass and use Newton's Second Law to show that a solution for the angle of rotation is $\cos\alpha = \frac{g}{l\omega^2}$. [10 marks]

(iv) Explain why the solution derived in (iii) can only be valid for $\omega \geq \sqrt{\frac{g}{l}}$ and discuss how the erroneous solution for the $\omega < \sqrt{\frac{g}{l}}$ has appeared. Hence sketch a physically realistic graph of α against ω and explain what happens to the linear velocity of the mass as the angular velocity reaches the threshold value. [7 marks]

[Total 32 marks]

4. This question is about planetary orbits in the Solar System
Marks will be awarded for showing working and words of explanation.

(i) State Newton's Law of Universal Gravitation [2 marks]

(ii) What is meant by a point mass? Explain *qualitatively* why a sphere of uniform density distribution acts as a point mass outside its radius. [5 marks]

(iii) A planet of mass m has a circular orbit of radius R about the Sun (mass M). Use Newton's Laws of Motion and the law stated in (i) to find expressions for the planet's (a) speed, (b) period of orbit and (c) kinetic energy. Assume $m \ll M$. [9 marks]

(iv) If a planet is jolted to a slightly larger radius, $R + \Delta R$, where $\frac{\Delta R}{R} \ll 1$, show that the new period is $T + \Delta T$ and the new velocity is $v - \Delta v$ where $\Delta T \approx \frac{3\pi\Delta R}{v}$ and $\Delta v = \frac{\pi\Delta R}{T}$. Estimate these changes if the Earth were jolted 100 km further than its current radius of 1.50×10^{11} m. The mass of the Sun is 1.99×10^{30} kg. [10 marks]

(v) The Sun is losing mass at a constant rate of 4×10^{9} kg.s^{-1}. If the Earth's orbital radius remains constant, estimate the change in length of the Earth's year over the span of accurate astronomical observation. State any assumptions you make. Describe qualitatively how realistic the assumption the radius remaining constant is, and discuss how a more appreciable mass loss may affect planetary orbits in general. [6 marks]

[Total 32 marks]

5. This question is about toppling and rolling objects.
Marks will be awarded for showing working and words of explanation.

(i) Define angular momentum, $\mathbf{L}$, and torque, $\mathbf{\Gamma}$. Use these definitions to show that the rate of change of angular momentum of a particle is proportional to the resultant external torque on it. [7 marks]

(ii) A stationary bicycle wheel is placed on its rim on rough ground. It topples over.

(a) Sketch a free body diagram for the wheel when it is at an arbitrary angle to the vertical and label the forces. Explain qualitatively what happens to the magnitude and direction of each of the forces during the toppling process. [7 marks]

(b) If the wheel has mass m, radius r and moment of inertia about the contact point with the ground I write down a time dependent differential equation for the angle the wheel makes with the vertical. Comment on under what conditions an exact, analytical, solution to this differential equation is possible. [4 marks]

(c) Describe, using vector diagrams if appropriate, what happens to the angular momentum of the bicycle wheel about the contact point with the ground during the toppling process. [3 marks]

(iii) The bicycle wheel is now rolled over the ground at an angle to the vertical. Sketch a free body diagram for the wheel and explain using vector diagrams why the wheel now resists toppling. Further explain why as the wheel slows it starts to lean progressively more until toppling does occur. [7 marks]

(iv) Outline with the aid of sketches and force diagrams why a wheel with a hard rim on a hard surface will roll for a longer time than a wheel with a soft rim on a soft surface. Why do cyclists sometimes prefer to ride with soft tyres? [4 marks]

[Total 32 marks]

6. A lamp emits very (infintesimally) short flashes of light separated by time T in its rest frame, which we'll call S_L. It moves with velocity v towards a point P where an observer records the arrival times of the light from each flash. Call the receiver's frame S_R.

(i) What is the velocity of the light as observed from frame S_L and from S_R? [1 mark]

(ii) What is the time separation T_R between flashes in the receiver's frame? [2 marks]

(iii) Sketch a diagram in frame S_R showing the position of the lamp at $t = 0$ when it flashes first, its position at time T_R when it flashes again, and the the light pulses from these two flashes. Calculate the distance L between successive pulses as observed in S_R. [5 marks]

(iv) Calculate the frequency f_R at which pulses arrive at P, and from this derive the Doppler shift formula,

$$f_R = f\sqrt{\frac{1+v/c}{1-v/c}},$$

where $f = 1/T$. [6 marks]

(v) Using the relation between frequency and wavelength $c = \lambda f$ rewrite the Doppler shift formula in terms of λ. How would the wavelength change if the light was moving away from P? [3 marks]

(vi) In a spiral galaxy individual stars rotate in a flat plane about the galactic centre. A spiral galaxy with a diameter of 100,000 light-years is observed edge-on from Earth. At either edge of the galaxy, light from a hydrogen emission line is recorded at a wavelength of 535.6 nm and 803.4 nm. This unshifted wavelength is measured to be 656 nm in a lab on Earth. What is the rotational period of this galaxy? [10 marks]

(vii) Hydrogen in a different spiral galaxy, also observed edge on, has measured wavelengths of 726.0 nm and 558 nm at each edge of the galaxy. Calculate the velocities these Doppler shifts imply. Without doing any detailed calculations, comment on the possible motion of this galaxy. [5 marks]

[Total 32 marks]