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### UNIVERSITY OF LONDON

[E2.11 2004]

### **B.ENG. AND M.ENG. EXAMINATIONS 2004**

For Internal Students of Imperial College London

This paper is also taken for the relevant examination for the Associateship of the City & Guilds of London Institute

### INFORMATION SYSTEMS ENGINEERING E2.11

### **MATHEMATICS**

Date Thursday 3rd June 2004 2.00 - 4.00 pm

Answer FOUR questions, to include at least one from Section B

Answers to Section A questions must be written in a different answer book from answers to Section B questions.

[Before starting, please make sure that the paper is complete. There should be SIX pages, with a total of SIX questions. Ask the invigilator for a replacement if this copy is faulty.]

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### Section A

1. If the Fourier transform of  $f(t), -\infty < t < \infty$ , is given by

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt ,$$

show that the Fourier transform of f(-t) is  $\hat{f}(-\omega)$  and that of tf(t) is  $i\frac{d}{d\omega}\hat{f}(\omega)$ .

If for a positive constant a

$$g(t) = \begin{cases} e^{-at} & , & t \ge 0 \\ 0 & , & t < 0 \end{cases}$$

$$h(t) = e^{-a|t|}, -\infty < t < \infty,$$

and

$$k(t) = |t|e^{-a|t|}, -\infty < t < \infty,$$

find  $\hat{g}(\omega)$ .

Hence, or otherwise, show that

$$\hat{h}(\omega) = \frac{2a}{a^2 + \omega^2}$$
 and  $\hat{k}(\omega) = \frac{2(a^2 - \omega^2)}{(a^2 + \omega^2)^2}$ .

Find the Fourier Transform of the function  $\frac{2a}{a^2+t^2}$ .

2. (i) The Laplace transform  $\overline{y}(p)$  of a function y(t) is

$$\overline{y}(p) = \int_0^\infty e^{-pt} y(t) dt$$
.

Show that, assuming y(t) behaves suitably at infinity, the Laplace transforms of

$$y'(t) \equiv \frac{dy}{dt}$$
 and  $y''(t) \equiv \frac{d^2y}{dt^2}$ 

are given respectively by

$$\mathcal{L}\left\{y'\right\} = p\overline{y}(p) - y(0)$$

and

$$\mathcal{L}\{y''\} = p^2 \overline{y}(p) - y'(0) - p y(0).$$

Hence solve the differential equation

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = g(t)$$

for an arbitrary function g(t), for t > 0, given y(0) = y'(0) = 0.

(ii) Evaluate  $\int_C (P dx + Q dy)$  anti-clockwise around the boundary of the region R defined by

$$x^2 + y^2 \le a^2, \quad x \ge 0, \quad y \ge 0,$$

taking

$$P = \frac{x^2}{x+y} \quad \text{and} \quad Q = -\frac{y^2}{x+y} .$$

3. (i) Sketch the region of the xy-plane over which the integral

$$\int_0^1 dx \, \int_{x^2}^1 4 \, x \, e^{y^2} dy$$

is taken. Change the order of integration and hence evaluate the integral.

(ii) Sketch the region of the xy-plane over which the integral

$$\int_0^1 dx \int_x^{\sqrt{2x-x^2}} \frac{y}{x^2 + y^2} dy$$

is taken.

Use polar co-ordinates to show that the value of the integral is  $\frac{1}{2}$ .

4. Find all the poles of  $\frac{1}{z^6+1}$  and the residue at each pole.

For R > 1 let  $C_2$  be the upper semi-circular arc of |z| = R,

directed from +R to -R and let  $C_1$  be the diameter from -R to R.

If C is the semi-circular path  $C_1 + C_2$ , calculate

$$\int_C \frac{dz}{z^6+1} \ .$$

By a careful discussion of the limit as  $R \to -\infty$  show that

$$\int_{-\infty}^{\infty} \frac{dx}{x^6 + 1} = \frac{2\pi}{3} .$$

PLEASE TURN OVER

Section B [E2.11 2004]

5. A batch of IC chips contains 0.5% defectives. Each IC chip is subjected to a test, which gives a positive result if it identifies a chip as defective. The test correctly identifies a chip as defective with probability 0.99. The test misidentifies as defective 2 in every 100 chips. Let D denote the event that an IC chip is defective and T the event that the test identifies a chip as defective.

- (i) What is the probability that the test is positive when applied to a randomly-selected chip?
- (ii) Given that a randomly-chosen IC chip is declared defective by the tester, compute the probability that it is actually defective.

Lifetimes (in million of hours) of IC chips are approximately normally distributed with mean  $\mu$  and variance  $\sigma^2$ . A mainframe manufacturer requires that an IC chip should have a lifetime greater than 4 million hours. He takes a sample of size n=100, with lifetimes denoted by

$$(x_1, \ldots, x_{100})$$
, and observes  $\sum_{i=1}^{100} x_i = 500$ ,  $\sum_{i=1}^{100} x_i^2 = 2508.9$ .

- (iii) Write down expressions for the unbiased estimators for  $\mu$  and  $\sigma^2$ , and compute the corresponding estimates given the data.
- (iv) Using a large-sample approximation, obtain a 95% confidence interval for  $\mu$ .
- (v) Using the unbiased estimates of  $\mu$  and  $\sigma^2$ , evaluate the probability that the lifetime of a randomly selected IC chip is greater than 4 million hours.

(Approximate your results to 2 decimal points).

6. The random variable X has an Poisson distribution with probability mass function

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, ...$$

where  $\lambda > 0$ .

(i) Show that  $E(X) = \lambda$ .

In a manufacturing plant the number of accidents that occurs in a sixmonth period is a Poisson random variable with mean 2. Assuming that the rate of accidents in any six-month period is constant,

- (ii) what is the expected number of accidents per year?
- (iii) what is the probability that there will be no accident in a given month?

Suppose we have a random sample,  $X_1, \ldots, X_N$ , of size N from a Poisson distribution with mean  $\mu > 0$ .

(iv) Show (and verify) that the maximum likelihood estimator for  $\mu$  is

$$\widehat{\mu} = \frac{\sum_{i=1}^{N} X_i}{N} = \overline{X}.$$

(v) Suppose that the following data are observed

Find an estimate of  $\mu$ .

(vi) Express the expectation of the mean of the random sample

$$\overline{X} = \frac{\sum_{i=1}^{N} X_i}{N}$$

in terms of  $\mu$ , and write down the formula for the variance of  $\overline{X}$ .

You may assume that the variance of a Poisson random variable is equal to its mean  $\mu$ .

### END OF PAPER

## MATHEMATICS DEPARTMENT

## MATHEMATICAL FORMULAE

## 1. VECTOR ALGEBRA

$$\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k} = (a_1, a_2, a_3)$$

Scalar (dot) product: a.

 $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ 

Vector (cross) product:

$$\langle \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Scalar triple product:

$$[\mathbf{a}, \mathbf{b}, \mathbf{c}] = \mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \mathbf{b} \cdot \mathbf{c} \times \mathbf{a} = \mathbf{c} \cdot \mathbf{a} \times \mathbf{b} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Vector triple product:  $\mathbf{a} \times (\mathbf{b} \times$ 

 $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$ 

### 2. SERIES

$$(1+x)^{\alpha} = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots$$
 ( $\alpha$  arbitrary,  $|x| < 1$ )

$$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots,$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots,$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots ,$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^n \frac{x^{n+1}}{(n+1)} + \dots (-1 < x \le 1)$$

# 3. TRIGONOMETRIC IDENTITIES AND HYPERBOLIC FUNCTIONS

$$\sin(a+b) = \sin a \cos b + \cos a \sin b;$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b.$$

$$\cos iz = \cosh z$$
;  $\cosh iz = \cos z$ ;  $\sin iz = i \sinh z$ ;  $\sinh iz = i \sin z$ .

## 4. DIFFERENTIAL CALCULUS

(a) Leibniz's formula:

$$D^{n}(fg) = f D^{n}g + \binom{n}{i} Df D^{n-1}g + \ldots + \binom{n}{i} D^{r}f D^{n-r}g + \ldots + D^{n}f g.$$

(b) Taylor's expansion of f(x) about x = a:

$$f(a+h) = f(a) + hf'(a) + h^2f''(a)/2! + \ldots + h^nf^{(n)}(a)/n! + \epsilon_n(h),$$

where  $\epsilon_n(h) = h^{n+1} f^{(n+1)} (a + \theta h)/(n+1)!$ ,  $0 < \theta < 1$ .

(c) Taylor's expansion of f(x, y) about (a, b):

$$f(a+h, b+k) = f(a, b) + [hf_x + kf_y]_{a,b} + 1/2! \left[ h^2 f_{xx} + 2hkf_{xy} + k^2 f_{yy} \right]_{a,b} + \dots$$

(d) Partial differentiation of f(x, y):

i. If 
$$y = y(x)$$
, then  $f = F(x)$ , and  $\frac{dF}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx}$ .

ii. If 
$$x = x(t)$$
,  $y = y(t)$ , then  $f = F(t)$ , and  $\frac{dF}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$ 

iii. If x = x(u, v), y = y(u, v), then f = F(u, v), and

$$\frac{\partial F}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial u}, \quad \frac{\partial F}{\partial v} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial v}$$

(e) Stationary points of f(x, y) occur where  $f_x = 0$ ,  $f_y = 0$  simultaneously. Let (a, b) be a stationary point: examine  $D = [f_{xx}f_{yy} - (f_{xy})^2]_{a.b}$ . If D > 0 and  $f_{xx}(a, b) < 0$ , then (a, b) is a maximum; If D > 0 and  $f_{xx}(a, b) > 0$ , then (a, b) is a minimum; If D < 0 then (a, b) is a saddle-point.

(f) Differential equations:

i. The first order linear equation dy/dx + P(x)y = Q(x) has an integrating factor  $I(x) = \exp[\int P(x)(dx]$ , so that  $\frac{d}{dx}(Iy) = IQ$ .

ii. P(x, y)dx + Q(x, y)dy = 0 is exact if  $\partial Q/\partial x = \partial P/\partial y$ .

## 5. INTEGRAL CALCULUS

- (a) An important substitution:  $\tan(\theta/2)=t$ :  $\sin\theta=2\,t/(1+t^2),\quad\cos\theta=(1-t^2)/(1+t^2),\quad d\theta=2\,dt/(1+t^2).$
- (b) Some indefinite integrals:

$$\int (a^2 - x^2)^{-1/2} dx = \sin^{-1} \left(\frac{x}{a}\right), \quad |x| < a.$$

$$\int (a^2 + x^2)^{-1/2} dx = \sinh^{-1} \left(\frac{x}{a}\right) = \ln \left\{\frac{x}{a} + \left(1 + \frac{x^2}{a^2}\right)^{1/2}\right\}.$$

$$\int (a^{2} + x^{2})^{-1/2} dx = \sinh \left(\frac{x}{a}\right) - \dots \left(\frac{x}{a} + \left(\frac{x^{2}}{a^{2}} - 1\right)^{1/2}\right)$$

$$\int (x^{2} - a^{2})^{-1/2} dx = \cosh^{-1}\left(\frac{x}{a}\right) = \ln\left|\frac{x}{a} + \left(\frac{x^{2}}{a^{2}} - 1\right)^{1/2}\right|.$$

$$\int (a^2 + x^2)^{-1} dx = \left(\frac{1}{a}\right) \tan^{-1} \left(\frac{x}{a}\right).$$

## 6. NUMERICAL METHODS

- (a) Approximate solution of an algebraic equation:
- If a root of f(x) = 0 occurs near x = a, take  $x_0 = a$  and  $x_{n+1} = x_n [f(x_n)/f'(x_n)], n = 0, 1, 2...$

(Newton Raphson method).

- (b) Formulae for numerical integration: Write  $x_n = x_0 + nh$ ,  $y_n = y\left(x_n\right)$ .
- i. Trapezium rule (1-strip):  $\int_{x_0}^{x_1} y(x) dx \approx (h/2) [y_0 + y_1]$ .
- ii. Simpson's rule (2-strip):  $\int_{x_0}^{x_2} y(x) dx \approx (h/3) [y_0 + 4y_1 + y_2]$ .
- (c) Richardson's extrapolation method: Let  $I=\int_a^b f(x)dx$  and let  $I_1,\ I_2$  be two
- estimates of I obtained by using Simpson's rule with intervals h and h/2.
- Then, provided h is small enough,

$$I_2 + (I_2 - I_1)/15$$
,

is a better estimate of  ${\cal I}$  .

## 7. LAPLACE TRANSFORMS

orm	,
Transform	•
tion	

$$F(s) = \int_0^\infty e^{-st} f(t) dt$$

sF(s) - f(0)

df/dt

$$(t)dt$$
 a

$$af(t) + bg(t)$$

$$aF(s) + bG(s)$$

Transform

Function

$$f(t) + cg(t)$$
  
 $f^2 f / dt^2$ 

 $s^2F(s) - sf(0) - f'(0)$ 

-dF(s)/ds

$$d^2f/dt^2$$

F(s-a)

 $e^{at}f(t)$ 

$$\int_0^t f(t)dt$$

$$\int_0^t f(t)dt$$

$$\int_0^t f(t)dt$$

F(s)/s

$$\int_0^t f(t)dt$$

$$\int_0^t f(t)dt$$

 $(\partial/\partial\alpha)F(s,\alpha)$ 

 $(\partial/\partial\alpha)f(t,\,\alpha)$ 

F(s)G(s)

 $\int_0^t f(u)g(t-u)du$ 

$$t^n(n=1,2...$$

$$(n=1, 2\ldots)$$

$$t^n(n=1,\,2\ldots)$$

$$=1, 2...$$

$$n!/s^{n+1}$$
,  $(s > 0)$   
 $\omega/(s^2 + \omega^2)$ ,  $(s > 0)$ 

$$\omega/(s^2+\omega$$
  $t < T$   $e^{-sT/s}$ .

 $1/(s-a), \ (s>a)$ 

 $\cos \omega t$ 

$$\omega/(s^2 + \omega^2), \ (s > 0)$$

$$< T \qquad e^{-sT}/s , (s, T)$$

$$t < T$$
  $e^{-sT/s}$ ,  $(s, T)$   $t > T$ 

$$s/(s^2+\omega^2), \ (s>0)$$
  $H(t-T)=\left\{ egin{array}{ll} 0, & t< T \ 1, & t> T \end{array} 
ight.$ 

### 8. FOURIER SERIES

If f(x) is periodic of period 2L, then f(x+2L) = f(x), and

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L} , \text{ where}$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} dx$$
,  $n = 0, 1, 2, ...$ , and

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L} dx, \quad n = 1, 2, 3, \dots$$

Parseval's theorem

$$\frac{1}{L} \int_{-L}^{L} [f(x)]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} \left( a_n^2 + b_n^2 \right) .$$

1. For the sample space,  $\Omega$ , the impossible event  $\emptyset$ , and events A, B, C:

$$P(\Omega) = 1, \qquad P(\overline{O}) = 0, \qquad P(\overline{A}) = 1 - P(A).$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$P(A \cup B \cup C)$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

Conditional probability: 
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$
 provided that  $P(B) > 0$ .

The odds in favour of A is the ratio  $P(A)/P(\overline{A})$ 

Multiplication rule:

$$P(A \cap B) = P(A \mid B) P(B).$$

Chain rule:

$$P(A \cap B \cap C) = P(A) P(B \mid A) P(C \mid A \cap B).$$

Bayes' rule:

$$P(A \mid B) = \frac{P(A) P(B \mid A)}{P(A) P(B \mid A) + P(\overline{A}) P(B \mid \overline{A})}$$

Independence: Events A and B are independent if  $P(B \mid A) = P(B)$ .

Events A, B, C are independent if  $P(A \cap B \cap C) = P(A)P(B)P(C)$ ,

$$P(A \cap B) = P(A)P(B), \ P(B \cap C) = P(B)P(C), \ P(C \cap A) = P(C)P(A).$$

2. A discrete random variable X has the probability mass function  $\{p_x\} = \{P(X = x)\}$ 

The expectation:  $E(X) = \mu = \sum_x x p_x$ .

From random sample  $x_1, \ldots, x_n$ , the sample mean  $\overline{x} = (1/n) \sum_k x_k$  estimates E(X).

The variance:  $\operatorname{var}(X) = \sigma^2 = E\{(X - \mu)^2\} = E(X^2) - \{E(X)\}^2$ , where  $E(X^2) = \sum_x x^2 p_x$ .

The sample variance:  $s^2 = \frac{1}{n-1} \left\{ \sum_k x_k^2 - \frac{1}{n} \left( \sum_j x_j \right)^2 \right\}$  estimates var (X).

The standard deviation:  $\operatorname{sd}(X) = \sigma = \sqrt{\operatorname{var}(X)}$ .

For grouped data: if the value y is observed with frequency  $n_y$ , then

$$n = \sum_{y} n_{y}, \quad \sum_{k} x_{k} = \sum_{y} y n_{y}, \quad \sum_{k} x_{k}^{2} = \sum_{y} y^{2} n_{y}.$$

Estimated skewness is  $\frac{1}{n-1} \sum_{k} \left( \frac{x_k - \overline{x}}{s} \right)^3$ , estimated kurtosis is  $\frac{1}{n-1} \sum_{k} \left( \frac{x_k - \overline{x}}{s} \right)^4$ 

3. Binomial distribution: X is  $Binomial(n, \theta)$ .

$$p_x = \binom{n}{x} \theta^x (1-\theta)^{n-x} \quad (x=0,1,2,\ldots,n); \qquad \mu = n\theta, \quad \sigma^2 = n\theta(1-\theta).$$

Poisson distribution: X is  $Poisson(\lambda)$ .

$$p_x = \frac{\lambda^x e^{-\lambda}}{x!}$$
  $(x = 0, 1, 2, ...)$  (with  $\lambda > 0$ );  $\mu = \lambda$ ,  $\sigma^2 = \lambda$ .

Geometric distribution: X is  $Geometric(\theta)$ .

$$p_x = (1 - \theta)^{x-1}\theta$$
  $(x = 1, 2, 3, ...);$   $\mu = \frac{1}{\theta}, \ \sigma^2 = \frac{1 - \theta}{\theta^2}.$ 

4. For continuous random variables, the cumulative distribution function (cdf)

$$F(x) = P(X \le x) = \int_{x_0 = -\infty}^{x} f(x_0) dx_0$$

The probability density function (pdf)  $f(x) = \frac{dF(x)}{dx}$ 

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx, \quad E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx, \quad \text{var}(X) = E(X^2) - \{E(X)\}^2.$$

5. Uniform distribution: X is  $Uniform(\alpha, \beta)$ .

$$f(x) = \begin{cases} 1/(\beta - \alpha) & (\alpha < x < \beta), & \mu = (\alpha + \beta)/2, \\ 0 & \text{(otherwise)}. & \sigma^2 = (\beta - \alpha)^2/12. \end{cases}$$

Exponential distribution: X is  $Exponential(\lambda)$ .

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & (0 < x < \infty), & \mu = 1/\lambda, \\ 0 & (-\infty < x \le 0). & \sigma^2 = 1/\lambda^2. \end{cases}$$

Gamma distribution: X is  $Gamma(\nu, \lambda)$ .

$$f(x) = \begin{cases} 1/\Gamma(\nu) \lambda^{\nu} x^{\nu-1} e^{-\lambda x} & (0 < x < \infty), & \mu = \nu/\lambda, \\ 0 & (-\infty < x \le 0). & \sigma^2 = \nu/\lambda^2. \end{cases}$$

Normal distribution: X is  $N(\mu, \sigma^2)$ .

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2\right\} \quad (-\infty < x < \infty); \quad E(X) = \mu, \quad \text{var}(X) = \sigma^2$$

Standard normal distribution: Y is N(0,1).

If X is  $N(\mu, \sigma^2)$ , then  $Y = \frac{X - \mu}{\sigma}$  is N(0, 1).

For Y we write  $\phi(y)$  for the pdf f(y) and  $\Phi(y)$  for the cdf F(y)

6. The lifetime T of a device in continuous operation with pdf f(t) (t > 0):

The reliability at time t: R(t) = P(T > t).

The failure rate or hazard rate: h(t) = f(t)/R(t).

The hazard function:  $H(t) = \int_0^t h(t_0) dt_0 = -\ln\{R(t)\}$ 

The Weibull distribution  $Weibull(\alpha, \beta)$  has  $H(t) = \beta t^{\alpha}$ .

For a system of k devices, which operate independently:

The system reliability, R, is the probability of a path of operating devices.

Let  $R_i = P(D_i) = P(\text{"device } i \text{ operates"}).$ 

A system of devices in series fails if any device fails.

$$R = P(D_1 \cap D_2 \cap \cdots \cap D_k) = R_1 R_2 \cdots R_k.$$

A system of devices in parallel operates if any device operates.

$$R = P(D_1 \cup D_2 \cup \cdots \cup D_k) = 1 - (1 - R_1)(1 - R_2) \cdots (1 - R_k).$$

7. The covariance of X and Y:

$$cov(X,Y) = E\{(X - E(X))(Y - E(Y))\} = E(XY) - E(X)E(Y)$$

The estimate of cov (X, Y) from n pairs of observations  $(x_1, y_1), \ldots, (x_n, y_n)$  is

$$s_{xy} = \frac{1}{n-1} S_{xy}$$
 where  $S_{xy} = \sum_{k} x_k y_k - \frac{1}{n} (\sum_{i} x_i) (\sum_{j} y_j)$ 

The correlation coefficient:  $\rho = \operatorname{corr}(X, Y) = \frac{\operatorname{cov}(X, Y)}{\operatorname{sd}(X) \cdot \operatorname{sd}(Y)}$ 

The sample correlation coefficient:  $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$  estimates  $\rho$ ,

where  $S_{xx} = (n-1)s_{xx}$ ,  $S_{yy} = (n-1)s_{yy}$ , and  $s_{xx}$  and  $s_{yy}$  are  $s^2$  calculated from the xs and ys respectively.

If X and Y have the joint pdf f(x, y):

the marginal pdf for X is 
$$f_X(x) = \int_{-\infty}^{\infty} f(x, y_0) dy_0$$

the conditional pdf for X given Y = y is  $f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)}$  provided  $f_Y(y) > 0$ 

The pdf for 
$$Z=X+Y$$
 is  $f_Z(z)=\int_{x=-\infty}^{\infty}f_X(x)\,f_{Y\mid X}(z-x\mid x)\mathrm{d}x$ 

$$E(X + Y) = E(X) + E(Y), \quad var(X + Y) = var(X) + var(Y) + 2 cov(X, Y)$$

If X is  $N(\mu_1, \sigma_1^2)$ , Y is  $N(\mu_2, \sigma_2^2)$ , and cov (X, Y) = c, then X + Y is  $N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2 + 2c)$ 

8. Chi-squared distribution  $\chi_k^2$ : E(Z) = k, var(Z) = 2k

If 
$$Y_1, \ldots, Y_k$$
 are independent  $N(0,1)$  then  $Z = Y_1^2 + \cdots + Y_k^2$  is  $\chi_k^2$ .

For a random sample from  $N(\mu, \sigma^2)$ ,  $(n-1)s^2/\sigma^2$  is from  $\chi^2_{n-1}$ , and

 $\sqrt{n}(\overline{x}-\mu)/s$  is from  $t_{n-1}$ , the Student t distribution on n-1 degrees of freedom

9. If t estimates  $\theta$ , the standard error of t, se(t), is sd(T), the standard deviation of the sampling distribution of t, and bias(t) =  $E(T - \theta)$ .

The mean square error:  $E\{(T-\theta)^2\} = \operatorname{var}(T) + \{\operatorname{bias}(t)\}^2$ .

If 
$$\overline{x}$$
 estimates  $\mu$ , then bias( $\overline{x}$ ) = 0, se( $\overline{x}$ ) =  $\sigma/\sqrt{n}$ , se( $\overline{x}$ ) =  $s/\sqrt{n}$ , and MSE =  $\sigma^2/n$ .

The likelihood is the joint probability as a function of the unknown parameter  $\theta$ .

$$\ell(\theta; x_1, x_2, \dots, x_n) = P(X_1 = x_1) \cdots P(X_n = x_n)$$
 (discrete distribution)  
 $\ell(\theta; x_1, x_2, \dots, x_n) = f(x_1) f(x_2) \cdots f(x_n)$  (continuous distribution)

The maximum likelihood estimator (MLE) is  $\hat{\theta}$  for which the likelihood is a maximum.

10. If t estimates  $\theta$ , a 95% confidence interval for  $\theta$  is an estimated interval that contains 95% of the sampling distribution of  $\theta$ .

If  $x_1, x_2, \ldots, x_n$  are a random sample from  $N(\mu, \sigma^2)$  and  $\sigma^2$  is known, then the 95% CI for  $\mu$  is  $(\overline{x} - 1.96\sigma/\sqrt{n}, \overline{x} + 1.96\sigma/\sqrt{n})$ .

If  $\sigma^2$  is estimated, then from the table of  $t_{n-1}$  we find  $t_0 = t_{n-1,0.05}$ . Then the 95% CI for  $\mu$  is  $(\overline{x} - t_0 s / \sqrt{n}, \overline{x} + t_0 s / \sqrt{n})$ .

A significance test of  $H_0$  rejects  $H_0$  if, assuming that  $H_0$  is true, a test statistic is in a rejection region of its sampling distribution.

The chi-squared goodness-of-fit test checks how well a fitted distribution fits the data: The frequencies  $n_y$  are grouped so that the fitted frequency  $\hat{n}_y$  for every group exceeds about 5.

 $X^2 = \sum (n_y - \hat{n}_y)^2 / \hat{n}_y$  is referred to the table of  $\chi_k^2$  with significance point p, where k is the number of terms summed, less one for each constraint, eg matching total frequency, and matching  $\bar{x}$  with  $\mu$ .

11. To fit the linear regression model  $y = \alpha + \beta x$  by  $\hat{y} = \hat{\alpha} + \hat{\beta} x$  from observations  $(x_1, y_1), \dots, (x_n, y_n)$ , the least squares fit is  $\hat{\alpha} = \overline{y} - \overline{x}\hat{\beta}$ ,  $\hat{\beta} = S_{xy}/S_{xx}$ .

The residual sum of squares, RSS =  $S_{yy} - \frac{S_{xy}^2}{S_{xx}}$ 

$$\widehat{\sigma^2} = \frac{\text{RSS}}{n-2} \text{ estimates } \sigma^2; \qquad \frac{n-2}{\sigma^2} \, \widehat{\sigma^2} \text{ is } \chi_{n-2}^2$$

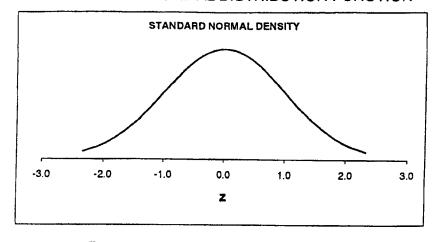
The predictor  $\widehat{y}_x = \widehat{\alpha} + \widehat{\beta}x$  of y when X = x is  $\widehat{\text{var}(\widehat{y}_x)} = \left\{\frac{1}{n} + \frac{(x - \overline{x})^2}{S_{xx}}\right\} \widehat{\sigma^2}$ 

$$\frac{\widehat{\alpha} - \alpha}{\widehat{\operatorname{se}}(\widehat{\alpha})}$$
,  $\frac{\widehat{\beta} - \beta}{\widehat{\operatorname{se}}(\widehat{\beta})}$ ,  $\frac{\widehat{y}_x - \alpha - \beta x}{\widehat{\operatorname{se}}(\widehat{y}_x)}$  are each  $t_{n-2}$ .

A future single observation y' at X = x has prediction error  $\hat{y}_x - y'$  which has variance var  $(\hat{y}_x) + \sigma^2$ 

The 95% prediction interval for y' at X = x is  $\hat{y}_x \pm t_{n-2,0.05} \sqrt{\{\widehat{\text{var}(\hat{y}_x)} + \widehat{\sigma^2}\}}$ 

### THE STANDARD NORMAL DISTRIBUTION FUNCTION



Entries in table are probabilities p such that  $\Phi(z)=p$ 

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7137	0.7190	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.7625	0.7632
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

TABLE OF THE STANDARD NORMAL CDF

Entries in table are ordinates x such that F(x)=p where F(.) is the Student cdf

D. of F.   O.8					Р			
1,3764   3,0777   6,3167   12,7062   31,8210   63,6559   318,2888   2,9200   4,3027   6,9445   9,2250   22,3285   3,09785   1,6377   2,3534   3,1824   4,5407   5,8408   10,2143   4,09410   1,5332   2,1318   2,7765   3,7469   4,6041   7,1729   5,09195   1,4759   2,0150   2,5706   3,3649   4,0321   5,8935   6,09305   1,4149   1,8946   2,3464   2,9979   3,4959   4,7853   8,08889   1,3968   1,8595   2,3060   2,8965   3,3554   4,5008   9,08834   1,3830   1,8331   2,5622   2,8214   3,2498   4,9499   3,8495   4,7853   1,008791   1,3722   1,8125   2,2281   2,7683   3,1693   4,1437   11   0,8755   1,3634   1,7959   2,2010   2,7181   3,1059   4,0248   1,08702   1,3562   1,7823   2,1788   2,6810   3,0545   3,9296   1,08725   1,3562   1,7823   2,1788   2,6810   3,0545   3,9296   1,0861   1,3450   1,7613   2,1448   2,5245   2,9768   3,7874   1,08681   1,3450   1,7613   2,1448   2,5245   2,9768   3,7874   1,08631   1,3341   1,7396   2,1099   2,5524   2,9784   3,6105   1,0862   1,3406   1,7531   2,1315   2,6025   2,9467   3,7329   1,08603   1,3341   1,7396   2,1099   2,5524   2,8784   3,6105   1,08600   1,3277   1,7291   2,0930   2,5395   2,9802   3,6561   1,08601   1,3277   1,7291   2,0930   2,5395   2,8699   3,5793   2,08600   1,3235   1,7207   2,0739   2,5083   2,8188   3,5050   2,08600   1,3235   1,7207   2,0739   2,5083   2,8188   3,5050   2,08561   3,1318   1,7109   2,0687   2,4899   2,6073   3,4850   2,6851   3,1315   1,7139   2,0687   2,4899   2,2073   3,4650   2,6851   1,3115   1,7035   2,0687   2,4899   2,2073   3,4668   2,5861   1,3155   1,7139   2,0687   2,4899   2,2073   3,4668   2,5861   1,3155   1,7139   2,0687   2,4899   2,2073   3,4650   2,6850   1,3168   1,5993   2,0395   2,4852   2,7760   3,3652   2,6893   3,3063   3,6851   3,3070   1,6899   2,0329   2,4874   2,7893   3,4850   2,6851   3,3164   1,6893   2,0211   2,4434   2,4671   2,7683   3,3460   2,6851   3,3063   1,6899   2,0395   2,4486   2,7797   3,4560   3,6861   3,3063   1,6899   2,0395   2,4486   2,7796   3,3450   3,0853   3,0850   1,3065   1,6899	D. of F.	0.8	0.9	0.95		0.99	0.995	0.999
1,0607						31.8210	63.6559	318.2888
1	2	1.0607	1.8856	2.9200	4.3027	6.9645	9.9250	22.3285
5         0.9195         1.4759         2.0150         2.5706         3.3649         4.0321         5.8935           6         0.9057         1.4398         1.9432         2.4469         3.1427         3.7074         5.2075           7         0.8960         1.4149         1.8946         2.3646         2.9979         3.4995         4.7853           8         0.8834         1.3960         1.8331         2.2522         2.8214         3.2498         4.2969           10         0.8791         1.3722         1.8125         2.2281         2.7688         3.1693         4.1437           11         0.8755         1.3602         1.7709         2.1604         2.6603         3.0123         3.8520           13         0.8702         1.3502         1.7709         2.1604         2.6503         3.0123         3.8520           14         0.8613         1.3406         1.7531         2.1185         2.6025         2.9467         3.7329           15         0.8620         1.3406         1.7531         2.1199         2.5835         2.9669         3.8481           16         0.8617         1.3304         1.7396         2.1199         2.6529         2.8812			1.6377	2.3534	3.1824	4.5407	5.8408	10.2143
6         0.9057         1.4398         1.9432         2.4469         3.1427         3.7074         5.2075           7         0.8960         1.4149         1.8946         2.3664         2.9979         3.4995         4.7853           8         0.8889         1.3986         1.8595         2.3060         2.8925         3.3554         4.5008           9         0.8341         1.3830         1.8311         2.6822         2.2814         3.2498         4.2989           10         0.8755         1.3850         1.7759         2.2010         2.7181         3.1058         4.0248           12         0.8726         1.3550         1.7630         2.1788         2.6810         3.0545         3.9296           14         0.8681         1.3450         1.7613         2.1484         2.6245         2.9768         3.7874           15         0.8682         1.3366         1.7531         2.1199         2.6835         2.9208         3.6861           16         0.8630         1.3334         1.7341         2.1099         2.5832         2.8784         3.6160           19         0.8610         1.3277         1.7291         2.0830         2.5395         2.8609			1.5332	2.1318	2.7765	3.7469	4.6041	7.1729
7         0.8850         1.4149         1.8946         2.3646         2.9979         3.4995         4.7503           8         0.8889         1.3830         1.8595         2.3060         2.8965         3.3554         4.5008           9         0.8751         1.3820         1.8125         2.2281         2.7638         3.1053         4.0248           11         0.8755         1.3562         1.7823         2.1788         2.8810         3.0545         3.9296           13         0.8702         1.3562         1.7709         2.1604         2.6503         3.0123         3.8520           14         0.8681         1.3406         1.7531         2.1315         2.6245         2.9768         3.7874           15         0.8662         1.3406         1.7531         2.1315         2.6025         2.9768         3.7829           16         0.8674         1.3334         1.7396         2.1098         2.5699         3.6458           18         0.8620         1.3304         1.7341         2.1099         2.5524         2.8744         3.518           19         0.8501         1.3252         1.7207         2.0796         2.5176         2.8314         3.5271	5	0.9195	1.4759	2.0150	2.5706	3.3649	4.0321	5.8935
7         0.8850         1.4149         1.8946         2.3646         2.9979         3.4995         4.7503           8         0.8889         1.3830         1.8595         2.3060         2.8965         3.3554         4.5008           9         0.8751         1.3820         1.8125         2.2281         2.7638         3.1053         4.0248           11         0.8755         1.3562         1.7823         2.1788         2.8810         3.0545         3.9296           13         0.8702         1.3562         1.7709         2.1604         2.6503         3.0123         3.8520           14         0.8681         1.3406         1.7531         2.1315         2.6245         2.9768         3.7874           15         0.8662         1.3406         1.7531         2.1315         2.6025         2.9768         3.7829           16         0.8674         1.3334         1.7396         2.1098         2.5699         3.6458           18         0.8620         1.3304         1.7341         2.1099         2.5524         2.8744         3.518           19         0.8501         1.3252         1.7207         2.0796         2.5176         2.8314         3.5271	6	0.9057	1.4398	1.9432	2.4469	3.1427	3.7074	5.2075
8         0.8884         1.3968         1.8595         2.3060         2.8955         3.3554         4.5008           9         0.8834         1.3830         1.8331         2.26221         2.28214         3.2498         4.2969           10         0.8795         1.3623         1.7959         2.2010         2.7181         3.1058         4.0248           12         0.8726         1.3502         1.7709         2.1604         2.6503         3.0123         3.8520           13         0.8702         1.3502         1.7709         2.1604         2.6503         3.0123         3.8520           14         0.8681         1.3450         1.7613         2.1484         2.6225         2.9467         7.322           16         0.8647         1.3368         1.7459         2.1199         2.5835         2.9208         3.6861           18         0.8620         1.3304         1.7341         2.1098         2.56524         2.8784         3.6105           19         0.8610         1.3277         1.7291         2.0860         2.5324         2.8603         3.5518           21         0.8553         1.3125         1.7207         2.0865         2.8176         2.8174	7	0.8960	1.4149	1.8946	2.3646	2.9979	3.4995	4.7853
9 0.8834 1.3830 1.8331 2.2522 2.8214 3.2498 4.2959 10 0.8791 1.3722 1.8125 2.2281 2.7638 3.1693 4.1437 11 0.8755 1.3634 1.7959 2.2010 2.7181 3.1058 4.0248 12 0.8726 1.3552 1.7709 2.1604 2.6503 3.0123 3.8520 13 0.8601 1.3450 1.7613 2.1448 2.5245 2.9768 3.7874 15 0.8662 1.3406 1.7531 2.1315 2.6025 2.9467 3.7329 16 0.8647 1.3368 1.7459 2.1199 2.5035 2.9208 3.6861 17 0.8633 1.3334 1.7396 2.1098 2.5669 2.8982 3.6458 18 0.8620 1.3304 1.7341 2.1009 2.5524 2.8784 3.6105 19 0.8610 1.3277 1.7291 2.0930 2.5594 2.8784 3.6105 19 0.8600 1.3253 1.7247 2.0860 2.5280 2.8463 3.5518 20 0.8600 1.3253 1.7247 2.0860 2.5280 2.8453 3.5518 21 0.8575 1.3195 1.7139 2.0867 2.4999 2.8073 3.4550 23 0.8575 1.3195 1.7139 2.0637 2.4999 2.8073 3.4550 24 0.8569 1.3178 1.7091 2.0639 2.4992 2.8073 3.4550 25 0.8562 1.3163 1.7081 2.0595 2.4851 2.7787 3.4502 26 0.8554 1.3150 1.7056 2.0555 2.4786 2.7787 3.4502 27 0.8551 3.1350 1.7061 2.0595 2.4851 2.7787 3.4502 28 0.8546 1.3125 1.7011 2.0484 2.4671 2.7633 3.4082 29 0.8546 1.3125 1.7011 2.0484 2.4671 2.7633 3.4082 29 0.8542 1.3114 1.6991 2.0452 2.4620 2.7564 3.3963 30 0.8538 3.104 1.6973 2.0423 2.4457 2.7385 3.3553 31 0.8534 1.3095 1.6995 2.0395 2.4487 2.7385 3.3553 33 0.8526 1.3006 1.6999 2.0322 2.4411 2.7284 3.3450 34 0.8523 1.3070 1.6999 2.0322 2.4411 2.7284 3.3450 36 0.8517 1.3065 1.6883 2.0281 2.4437 2.7385 3.3553 39 0.8526 1.3006 1.6999 2.0322 2.4411 2.7284 3.3490 30 0.8538 1.3006 1.6999 2.0322 2.4411 2.7284 3.3490 30 0.8509 1.3036 1.6899 2.0322 2.4411 2.7284 3.3490 30 0.8501 1.3065 1.6896 2.0301 2.4377 2.7238 3.3400 30 0.8509 1.3036 1.6899 2.0322 2.4411 2.7284 3.3490 30 0.8501 1.3062 1.6896 2.0301 2.4377 2.7238 3.3400 30 0.8508 1.3002 1.6896 2.0301 2.4377 2.7238 3.3400 30 0.8509 1.3036 1.6899 2.0322 2.4411 2.7284 3.3556 30 0.8507 1.3051 1.6880 2.0211 2.4335 2.7195 3.3256 31 0.8501 1.3062 1.6896 2.0301 2.4377 2.7238 3.3400 30 0.8508 1.3002 1.6896 2.0301 2.4377 2.7238 3.3400 30 0.8508 1.3002 1.6896 2.0301 2.4377 2.7238 3.3400 30 0.8509 1.3036 1.6899 2.0322 2.4481 2.7195 3.3256 30 0.8495 1.3002 1.6			1.3968	1.8595	2.3060	2.8965	3.3554	4.5008
11 0.8755 1.3634 1.7959 2.2010 2.7181 3.1058 4.0248 12 0.8726 1.3552 1.7623 2.1788 2.6810 3.0545 3.9296 13 0.8702 1.3562 1.7709 2.1604 2.6503 3.0123 3.8520 1.7709 2.1604 2.6503 3.0123 3.8520 1.7613 2.1448 2.6245 2.9768 3.7874 15 0.8662 1.3406 1.7531 2.1315 2.6025 2.9467 3.7329 16 0.8647 1.3306 1.7613 2.1448 2.6245 2.9768 3.7874 17 0.8633 1.3334 1.7396 2.1098 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.56025 2.9467 3.7329 2.5033 3.6851 2.50362 2.9208 3.6458 2.5026 2.8452 3.65458 2.9208 3.6458 2.5026 2.8452 2.9764 3.6105 2.0000 2.5524 2.8764 3.6105 2.0000 2.5524 2.8764 3.6105 2.0000 2.5524 2.8764 3.5618 2.0000 2.5220 2.8453 3.5518 2.0000 2.8523 1.3222 1.7207 2.0796 2.5176 2.8314 3.52512 2.0000 2.8523 1.3212 1.7171 2.0739 2.5083 2.8188 3.5050 2.0000 2.8525 2.1365 3.7081 2.0687 2.4929 2.8073 3.4502 2.0000 2.8526 2.1365 3.7081 2.0687 2.4929 2.8073 3.4502 2.0000 2.8526 2.1365 3.7081 2.0595 2.4851 2.7874 3.4502 2.0000 2.8526 2.1863 3.0000 2.0000			1.3830	1.8331	2.2622	2.8214	3.2498	4.2969
12         0.8726         1.3562         1.7823         2.1788         2.6810         3.0545         3.9296           13         0.8702         1.3502         1.7709         2.1604         2.6603         3.0123         3.8520           14         0.8681         1.3450         1.7613         2.1448         2.6245         2.9768         3.7824           15         0.8662         1.3406         1.7531         2.1316         2.6025         2.9467         3.7329           16         0.8647         1.3368         1.7439         2.1199         2.5635         2.9208         3.6861           17         0.8631         3.3304         1.7341         2.1009         2.5524         2.8784         3.6105           19         0.8610         1.3277         1.7247         2.0860         2.5280         2.8453         3.5271           21         0.8591         1.3232         1.7277         2.0796         2.5176         2.8314         3.5271           22         0.8583         1.3187         1.7199         2.0687         2.4999         2.8073         3.4568           24         0.8595         1.3187         1.7199         2.0687         2.4921         2.7770	10	0.8791	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437
13         0.8702         1.3502         1.7709         2.1604         2.6603         3.0123         3.8520           14         0.8661         1.3460         1.7631         2.1418         2.6245         2.9768         3.7874           15         0.8662         1.3466         1.7531         2.1315         2.6025         2.9467         3.7329           16         0.8647         1.3368         1.7459         2.1199         2.5580         2.8982         3.6458           18         0.8620         1.3304         1.7341         2.1009         2.5528         2.8609         3.5793           20         0.8600         1.3253         1.7247         2.0860         2.5376         2.8463         3.5518           21         0.8591         1.3232         1.7207         2.0796         2.5176         2.8314         3.5271           22         0.8583         1.3212         1.7171         2.0739         2.5083         2.8188         3.5050           24         0.8556         1.3153         1.7081         2.0595         2.4851         2.7874         3.4550           24         0.8562         1.3150         1.7081         2.0595         2.4781         2.7787	11	0.8755	1.3634	1.7959	2.2010	2.7181	3.1058	
14         0.8681         1.3450         1.7613         2.1448         2.6245         2.9467         3.7829           15         0.8662         1.3406         1.7531         2.1315         2.6025         2.9467         3.7329           16         0.8633         1.3334         1.7396         2.1098         2.5669         2.8982         3.6458           18         0.8620         1.3304         1.7341         2.1099         2.5524         2.8784         3.6105           19         0.8610         1.3227         1.7247         2.0860         2.5395         2.8609         3.5793           20         0.8501         1.3222         1.7207         2.0796         2.5176         2.8314         3.5271           21         0.8591         1.3222         1.7207         2.0796         2.5176         2.8314         3.5271           22         0.8551         1.3175         1.7199         2.0637         2.4999         2.8073         3.4850           23         0.8556         1.3178         1.7109         2.0639         2.4922         2.7970         3.4668           25         0.8556         1.3178         1.7081         2.06539         2.4222         2.7707	12	0.8726	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296
15         0.8662         1.3406         1.7531         2.1315         2.6025         2.9467         3.7329           16         0.8647         1.3368         1.7459         2.1998         2.5699         2.8982         3.6861           17         0.8630         1.3304         1.7341         2.1098         2.5669         2.8982         3.6458           18         0.8600         1.3277         1.7291         2.0930         2.5395         2.8609         3.5793           20         0.8601         1.3277         1.7207         2.0796         2.5176         2.8314         3.5516           21         0.8591         1.3232         1.7207         2.0799         2.5083         2.8188         3.5050           23         0.8575         1.3195         1.7193         2.0687         2.4992         2.8073         3.4552           24         0.8569         1.3163         1.7091         2.0639         2.4922         2.7797         3.4668           25         0.8562         1.3163         1.7066         2.0555         2.4786         2.7787         3.4550           27         0.8516         1.3125         1.7011         2.0484         2.4671         2.7633	13	0.8702	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520
16         0.8647         1.3368         1.7459         2.1199         2.5835         2.9208         3.6458           17         0.8633         1.3334         1.7936         2.1098         2.5659         2.8982         3.6458           18         0.8620         1.3277         1.7291         2.0930         2.5395         2.8609         3.5773           20         0.8501         1.3252         1.7247         2.0860         2.5280         2.8453         3.5518           21         0.8583         1.3212         1.7171         2.0739         2.5083         2.8188         3.5050           23         0.8575         1.3195         1.7139         2.0687         2.4992         2.8073         3.4550           24         0.8569         1.3178         1.7109         2.0639         2.4922         2.7970         3.4658           25         0.8566         1.3153         1.7081         2.0595         2.4851         2.7874         3.4500           26         0.8557         1.3151         1.7033         2.0518         2.4762         2.7764         3.4502           27         0.8551         1.3174         1.6931         2.0452         2.4620         2.7564	14	0.8681	1.3450	1.7613	2.1448	2.6245	2.9768	
17         0.8633         1.3334         1.7396         2.1098         2.5689         2.8982         3.6458           18         0.8620         1.3304         1.7341         2.1099         2.5524         2.8784         3.6105           19         0.8610         1.3277         1.7291         2.0930         2.5395         2.8609         3.5793           20         0.8501         1.3232         1.7207         2.0796         2.5176         2.8314         3.5271           21         0.8583         1.3212         1.7171         2.0739         2.5083         2.8188         3.5050           23         0.8565         1.3163         1.7091         2.0639         2.4922         2.7970         3.4662           25         0.8566         1.3163         1.7061         2.0555         2.4786         2.7787         3.4502           26         0.8557         1.3150         1.7066         2.0555         2.4786         2.7787         3.4502           27         0.8551         1.3137         1.7033         2.0518         2.4727         2.7707         3.4210           28         0.8546         1.3125         1.7011         2.0484         2.4673         2.7564	15	0.8662	1.3406	1.7531	2.1315	2.6025	2.9467	3.7329
18         0.8620         1.3304         1.7341         2.1009         2.5524         2.8784         3.6105           19         0.8610         1.3277         1.7291         2.0930         2.5395         2.8609         3.5793           20         0.8690         1.3223         1.7207         2.0796         2.5176         2.8314         3.5271           21         0.8591         1.3232         1.7171         2.0739         2.5083         2.8188         3.5050           23         0.8575         1.3195         1.7139         2.0687         2.4999         2.8073         3.4562           24         0.8569         1.3178         1.7109         2.0639         2.4922         2.7770         3.4562           25         0.8562         1.3163         1.7061         2.0555         2.4861         2.7787         3.4510           26         0.8557         1.3150         1.7056         2.0555         2.4762         2.7707         3.4210           27         0.8551         1.3137         1.7033         2.0452         2.4762         2.7564         3.3963           30         0.8524         1.3114         1.6991         2.0452         2.4620         2.7564	16	0.8647	1.3368	1.7459	2.1199	2.5835	2.9208	3,6861
19         0.8610         1.3277         1.7291         2.0930         2.5395         2.8609         3.5578           20         0.8600         1.3253         1.7247         2.0860         2.5280         2.8453         3.5571           21         0.8581         1.3212         1.7171         2.0739         2.5083         2.8188         3.5050           23         0.8575         1.3195         1.7139         2.0687         2.4999         2.8073         3.4650           24         0.8569         1.3178         1.7109         2.0639         2.4922         2.7970         3.4662           25         0.8556         1.3163         1.7066         2.0555         2.4786         2.7787         3.4502           26         0.8551         1.3137         1.7033         2.0518         2.4727         2.7763         3.4202           29         0.8542         1.3114         1.6991         2.0452         2.4670         2.7564         3.3963           30         0.8538         1.3014         1.6991         2.0423         2.4573         2.7503         3.3653           31         0.8540         1.3077         1.6924         2.0345         2.4448         2.7335	17	0.8633	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458
19         0.8610         1.3277         1.7291         2.0930         2.5395         2.8609         3.5578           20         0.8600         1.3253         1.7247         2.0860         2.5280         2.8453         3.5571           21         0.8581         1.3212         1.7171         2.0739         2.5083         2.8188         3.5050           23         0.8575         1.3195         1.7139         2.0687         2.4999         2.8073         3.4650           24         0.8569         1.3178         1.7109         2.0639         2.4922         2.7970         3.4662           25         0.8556         1.3163         1.7066         2.0555         2.4786         2.7787         3.4502           26         0.8551         1.3137         1.7033         2.0518         2.4727         2.7763         3.4202           29         0.8542         1.3114         1.6991         2.0452         2.4670         2.7564         3.3963           30         0.8538         1.3014         1.6991         2.0423         2.4573         2.7503         3.3653           31         0.8540         1.3077         1.6924         2.0345         2.4448         2.7335	18	0.8620	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105
21         0.8591         1.3232         1.7207         2.0796         2.5176         2.8314         3.5271           22         0.8583         1.3212         1.7171         2.0799         2.5083         2.8188         3.5050           23         0.8575         1.3195         1.7139         2.0687         2.4999         2.8073         3.4850           24         0.8569         1.3178         1.7109         2.0639         2.4922         2.7970         3.4660           25         0.8562         1.3163         1.7081         2.0555         2.4786         2.7787         3.4550           26         0.8546         1.3125         1.7011         2.0484         2.4671         2.7633         3.4082           29         0.8546         1.3125         1.7011         2.0484         2.4620         2.7564         3.3963           30         0.8538         1.3014         1.6991         2.0423         2.4528         2.7440         3.3749           32         0.8530         1.3086         1.6939         2.0345         2.4488         2.7333         3.3653           33         0.8526         1.3077         1.6924         2.0345         2.4448         2.7333	19	0.8610		1.7291	2.0930	2.5395	2.8609	3.5793
22         0.8583         1.3212         1.7171         2.0739         2.5083         2.8188         3.5050           23         0.8575         1.3195         1.7139         2.0637         2.4999         2.8073         3.4850           24         0.8569         1.3178         1.7109         2.0639         2.4922         2.7970         3.4650           25         0.8552         1.3163         1.7056         2.0555         2.4851         2.7787         3.4502           26         0.8551         1.3150         1.7056         2.0555         2.4786         2.7787         3.4350           27         0.8546         1.3125         1.7011         2.0482         2.4621         2.7564         3.3963           30         0.8538         1.3104         1.6991         2.0452         2.4520         2.7564         3.3963           31         0.8534         1.3095         1.6955         2.0395         2.4487         2.7385         3.3653           32         0.8530         1.3086         1.69399         2.0322         2.4411         2.7284         3.3460           35         0.8521         1.3077         1.6994         2.0311         2.4377         2.7233	20	0.8600	1.3253	1.7247	2.0860	2.5280	2.8453	
23	21	0.8591	1.3232	1.7207	2.0796	2.5176	2.8314	3.5271
24         0.8569         1.3178         1.7109         2.0639         2.4922         2.7970         3.4668           25         0.8562         1.3163         1.7081         2.0595         2.4851         2.7874         3.4502           26         0.8557         1.3150         1.7055         2.0555         2.4786         2.7787         3.4350           27         0.8551         1.3137         1.7033         2.0518         2.4727         2.7707         3.4210           28         0.8546         1.3125         1.7011         2.0484         2.4671         2.7633         3.4082           29         0.8542         1.3114         1.6991         2.0423         2.4573         2.7500         3.3852           31         0.8534         1.3041         1.6973         2.0423         2.4528         2.7440         3.3749           32         0.8530         1.3086         1.6939         2.0369         2.4487         2.7385         3.3563           33         0.8523         1.3077         1.6924         2.0345         2.4448         2.7333         3.3563           34         0.8523         1.3076         1.6989         2.0312         2.4374         2.7143	22	0.8583	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050
25         0.8562         1.3163         1.7081         2.0595         2.4851         2.7874         3.4502           26         0.8557         1.3150         1.7095         2.05555         2.4786         2.7787         3.4520           27         0.8551         1.3137         1.7033         2.0518         2.4727         2.7707         3.4210           28         0.8546         1.3125         1.7011         2.0484         2.4671         2.7563         3.34982           29         0.8542         1.3114         1.6991         2.0452         2.4620         2.7564         3.3963           30         0.8538         1.3104         1.6973         2.0423         2.4573         2.7500         3.3852           31         0.8534         1.3095         1.6995         2.0369         2.4487         2.7385         3.3563           32         0.8501         1.3077         1.6924         2.0345         2.4448         2.7333         3.3563           34         0.8523         1.3070         1.6909         2.0322         2.4411         2.7284         3.3480           35         0.8520         1.3055         1.68896         2.0381         2.4345         2.7159	23	0.8575	1.3195	1.7139	2.0687	2.4999		
26         0.8557         1.3150         1.7056         2.0555         2.4786         2.7787         3.4350           27         0.8551         1.3137         1.7033         2.0518         2.4727         2.7707         3.4210           28         0.8546         1.3125         1.7011         2.0484         2.4671         2.7663         3.4082           29         0.8542         1.3114         1.6991         2.0452         2.4620         2.7564         3.3963           30         0.8538         1.3104         1.6973         2.0423         2.4573         2.7500         3.3852           31         0.8534         1.3095         1.6995         2.0395         2.4528         2.7440         3.3749           32         0.8530         1.3086         1.6999         2.0369         2.4441         2.7385         3.3553           33         0.8526         1.3077         1.6999         2.0322         2.4411         2.7284         3.3480           35         0.8520         1.3062         1.6896         2.0301         2.4377         2.7238         3.3400           36         0.8517         1.3055         1.6883         2.0281         2.4345         2.7195	24	0.8569	1.3178	1.7109	2.0639	2.4922	2.7970	3.4668
27 0.8551 1.3137 1.7033 2.0518 2.4727 2.7707 3.4210 28 0.8546 1.3125 1.7011 2.0484 2.4671 2.7633 3.4082 29 0.8542 1.3114 1.6991 2.0452 2.4620 2.7564 3.3963 30 0.8538 1.3104 1.6973 2.0423 2.4573 2.7500 3.3852 31 0.8534 1.3095 1.6955 2.0395 2.4528 2.7440 3.3749 32 0.8530 1.3086 1.6939 2.0369 2.4487 2.7385 3.3653 33 0.8526 1.3077 1.6924 2.0345 2.4448 2.7333 3.3563 34 0.8523 1.3070 1.6909 2.0322 2.4411 2.7284 3.3480 35 0.8520 1.3062 1.6896 2.0301 2.4377 2.7238 3.3400 36 0.8517 1.3065 1.6883 2.0281 2.4345 2.7195 3.3326 37 0.8514 1.3049 1.6871 2.0262 2.4314 2.7154 3.3256 38 0.8512 1.3042 1.6860 2.0244 2.4286 2.7116 3.3190 39 0.8509 1.3006 1.6849 2.0227 2.4258 2.7079 3.3127 40 0.8507 1.3013 1.6839 2.0211 2.4233 2.7045 3.3069 41 0.8505 1.3025 1.6829 2.0195 2.4208 2.7012 3.3012 42 0.8503 1.3020 1.6820 2.0181 2.4185 2.6981 3.2959 43 0.8501 1.3016 1.6811 2.0167 2.4163 2.6951 3.2909 44 0.8499 1.3011 1.6802 2.0181 2.4185 2.6981 3.2959 43 0.8501 1.3002 1.6787 2.0129 2.4102 2.6870 3.2771 47 0.8493 1.2998 1.6779 2.0117 2.4083 2.6846 3.2772 49 0.8497 1.3007 1.6794 2.0141 2.4121 2.6896 3.2815 40 0.8497 1.3007 1.6794 2.0141 2.4121 2.6896 3.2815 50 0.8489 1.2991 1.6766 2.0096 2.4098 2.6800 3.2651 50 0.8489 1.2991 1.6750 2.0060 2.4066 2.6822 3.2698 49 0.8490 1.2991 1.6765 2.0060 2.4066 2.6822 3.2698 50 0.8489 1.2991 1.6750 2.0060 2.4066 2.6822 3.2698 50 0.8489 1.2991 1.6750 2.0060 2.4066 2.6822 3.2698 50 0.8489 1.2991 1.6750 2.0060 2.4073 2.6778 3.2614 55 0.8472 1.2994 1.6759 2.0066 2.4033 2.6778 3.2614 55 0.8464 1.2929 1.6654 1.9921 2.3771 2.6430 3.2024 80 0.8461 1.2925 1.6661 1.9967 2.3858 2.6779 3.2108 75 0.8464 1.2929 1.6654 1.9921 2.3771 2.6430 3.2024 80 0.8451 1.2910 1.6602 1.9867 2.3662 2.6268 3.1783 100 0.8452 1.2901 1.6602 1.9867 2.3662 2.6268 3.1783 100 0.8452 1.2901 1.6603 1.9863 2.3710 2.6349 3.1889 90 0.8456 1.2910 1.6603 1.9863 2.3710 2.6349 3.1889 90 0.8456 1.2910 1.6603 1.9867 2.3662 2.6268 3.1783 100 0.8451 1.2958 1.6755 1.9719 2.3551 2.6000 3.1315 200 0.8431 1.2849 1.6510 1.9695 2.3414 2.5956 3.1231	25	0.8562	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502
28	26	0.8557	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350
29         0.8542         1.3114         1.6991         2.0452         2.4620         2.7564         3.3963           30         0.8538         1.3104         1.6973         2.0423         2.4573         2.7500         3.3852           31         0.8534         1.3095         1.6939         2.0369         2.4487         2.7385         3.3563           32         0.8530         1.3086         1.6939         2.0369         2.4487         2.7385         3.3563           34         0.8523         1.3070         1.6909         2.0322         2.4411         2.7284         3.3480           35         0.8520         1.3062         1.6896         2.0301         2.4377         2.7238         3.3400           36         0.8517         1.3055         1.6883         2.0281         2.4345         2.7195         3.3226           37         0.8514         1.3049         1.6871         2.0262         2.4314         2.7154         3.3256           38         0.8512         1.3042         1.6800         2.0227         2.4258         2.7079         3.3127           40         0.8507         1.3031         1.6839         2.0211         2.4233         2.7045	27	0.8551	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210
30         0.8538         1.3104         1.6973         2.0423         2.4573         2.7500         3.3852           31         0.8534         1.3095         1.6955         2.0395         2.4528         2.7440         3.3749           32         0.8530         1.3086         1.6939         2.0369         2.4448         2.7335         3.3653           33         0.8523         1.3070         1.6909         2.0322         2.4411         2.7238         3.3400           35         0.8520         1.3062         1.6896         2.0301         2.4377         2.7238         3.3400           36         0.8517         1.3055         1.6883         2.0281         2.4345         2.7195         3.3326           37         0.8514         1.3049         1.6871         2.0262         2.4314         2.7154         3.3256           38         0.8512         1.3042         1.6860         2.0244         2.4286         2.7116         3.3190           39         0.8509         1.3031         1.6839         2.0211         2.4233         2.7045         3.3069           41         0.8505         1.3025         1.6829         2.0195         2.4208         2.7012	28	0.8546	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082
31 0.8534 1.3095 1.6955 2.0395 2.4528 2.7440 3.3749 32 0.8530 1.3086 1.6939 2.0369 2.4487 2.7385 3.3653 33 0.8526 1.3077 1.6924 2.0345 2.4448 2.7333 3.3563 34 0.8523 1.3070 1.6909 2.0322 2.4411 2.7284 3.3480 35 0.8520 1.3062 1.6896 2.0301 2.4437 2.7283 3.3400 36 0.8517 1.3055 1.6883 2.0281 2.4345 2.7195 3.3326 37 0.8514 1.3049 1.6871 2.0262 2.4314 2.7154 3.3256 38 0.8512 1.3042 1.6860 2.0244 2.4286 2.7116 3.3190 39 0.8509 1.3036 1.6849 2.0227 2.4258 2.7079 3.3127 40 0.8507 1.3031 1.6839 2.0211 2.4233 2.7045 3.3069 41 0.8505 1.3025 1.6829 2.0195 2.4208 2.7012 3.3012 42 0.8503 1.3020 1.6820 2.0181 2.4185 2.6981 3.2959 43 0.8501 1.3016 1.6811 2.0167 2.4163 2.6951 3.2909 44 0.8499 1.3011 1.6802 2.0154 2.4141 2.6923 3.2861 45 0.8497 1.3007 1.6794 2.0141 2.4121 2.6896 3.2815 46 0.8493 1.2998 1.6779 2.0117 2.4083 2.6686 3.2771 47 0.8493 1.2998 1.6779 2.0117 2.4083 2.6686 3.2771 48 0.8493 1.2998 1.6779 2.0117 2.4083 2.6686 3.2771 49 0.8493 1.2991 1.6766 2.0096 2.4049 2.6800 3.2651 50 0.8482 1.2991 1.6766 2.0096 2.4049 2.6800 3.2651 50 0.8482 1.2991 1.6766 2.0096 2.4049 2.6800 3.2651 50 0.8482 1.2991 1.6766 2.0096 2.4049 2.6803 3.2317 65 0.8482 1.2971 1.6730 2.0040 2.3961 2.6682 3.2451 50 0.8484 1.2929 1.6654 1.9971 2.3851 2.6536 3.2204 70 0.8468 1.2938 1.6669 1.9944 2.3808 2.6479 3.2108 75 0.8461 1.2922 1.6654 1.9921 2.3771 2.6430 3.2018 75 0.8461 1.2922 1.6654 1.9921 2.3771 2.6430 3.2018 90 0.8456 1.2910 1.6620 1.9867 2.3539 2.6623 3.2317 65 0.8459 1.2916 1.6630 1.9883 2.3710 2.65349 3.1889 90 0.8456 1.2910 1.6620 1.9867 2.3685 2.6316 3.1832 95 0.8454 1.2905 1.6611 1.9852 2.3662 2.6226 3.1783 100 0.8454 1.2872 1.6551 1.9759 2.3515 2.6090 3.14555 200 0.8431 1.2849 1.5510 1.9695 2.3414 2.5956 3.1231	29	0.8542	1.3114	1.6991	2.0452	2.4620	2.7564	3.3963
32 0.8530 1.3086 1.6939 2.0369 2.4487 2.7385 3.3653 33 0.8526 1.3077 1.6924 2.0345 2.4448 2.7333 3.3563 34 0.8523 1.3070 1.6909 2.0322 2.4411 2.7284 3.3480 35 0.8520 1.3062 1.6896 2.0301 2.4377 2.7238 3.3400 36 0.8517 1.3055 1.6883 2.0281 2.4345 2.7195 3.3326 37 0.8514 1.3049 1.6871 2.0262 2.4314 2.7154 3.3256 38 0.8512 1.3042 1.6860 2.0244 2.4286 2.7116 3.3190 39 0.8509 1.3036 1.6849 2.0227 2.4258 2.7079 3.3127 40 0.8507 1.3031 1.6839 2.0211 2.4233 2.7045 3.3069 41 0.8505 1.3025 1.6829 2.0195 2.4208 2.7012 3.3012 42 0.8503 1.3020 1.6820 2.0181 2.4185 2.6981 3.2959 43 0.8501 1.3016 1.6811 2.0167 2.4163 2.6951 3.2909 44 0.8499 1.3011 1.6802 2.0154 2.4141 2.6923 3.2861 45 0.8497 1.3007 1.6794 2.0141 2.4121 2.6896 3.2815 46 0.8495 1.3002 1.6787 2.0129 2.4102 2.6870 3.2771 47 0.8493 1.2998 1.6779 2.0117 2.4083 2.6846 3.2729 48 0.8490 1.2991 1.6766 2.0096 2.4049 2.6800 3.2651 50 0.8489 1.2991 1.6759 2.0086 2.4033 2.6778 3.26614 50 0.8477 1.2958 1.6759 2.0086 2.4033 2.6778 3.2614 50 0.8477 1.2958 1.6759 2.0086 2.4033 2.6778 3.2614 50 0.8477 1.2958 1.6766 2.0096 2.4033 2.6778 3.2614 50 0.8464 1.2929 1.6654 1.9921 2.3771 2.6430 3.2317 65 0.8482 1.2971 1.6730 2.0040 2.3961 2.6603 3.2317 65 0.8482 1.2971 1.6730 2.0040 2.3961 2.6603 3.2317 65 0.8482 1.2971 1.6760 2.0003 2.3901 2.6603 3.2317 65 0.8482 1.2971 1.6760 2.0003 2.3901 2.6603 3.2317 65 0.8482 1.2971 1.6760 2.0003 2.3901 2.6603 3.2317 65 0.8482 1.2971 1.6760 2.0003 2.3901 2.6603 3.2317 65 0.8464 1.2929 1.6654 1.9921 2.3771 2.6430 3.2024 80 0.8461 1.2922 1.6841 1.9901 2.3739 2.6387 3.1952 85 0.8459 1.2916 1.6630 1.9883 2.3710 2.6349 3.1889 90 0.8456 1.2910 1.6602 1.9867 2.3685 2.6316 3.1832 95 0.8450 1.2910 1.6602 1.9867 2.3662 2.6266 3.1783 100 0.8452 1.2901 1.6602 1.9867 2.3451 2.6008 3.1315 100 0.8452 1.2901 1.6602 1.9860 2.3414 2.5956 3.1231	30	0.8538	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852
33	31	0.8534	1.3095	1.6955	2.0395	2.4528	2.7440	3.3749
34         0.8523         1.3070         1.6909         2.0322         2.4411         2.7284         3.3480           35         0.8520         1.3062         1.6896         2.0301         2.4377         2.7238         3.3400           36         0.8517         1.3055         1.6883         2.0281         2.4345         2.7195         3.3326           37         0.8514         1.3049         1.6871         2.0262         2.4314         2.7154         3.3256           38         0.8512         1.3042         1.6869         2.0227         2.4258         2.7079         3.3127           40         0.8509         1.3031         1.6839         2.0211         2.4233         2.7045         3.3069           41         0.8505         1.3025         1.6829         2.0195         2.4208         2.7012         3.3012           42         0.8503         1.3020         1.6820         2.0181         2.4185         2.6981         3.2959           43         0.8501         1.3016         1.6811         2.0167         2.4163         2.6951         3.2909           44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6923	32	0.8530	1.3086	1.6939	2.0369	2.4487	2.7385	3.3653
35	33	0.8526		1.6924	2.0345	2.4448	2.7333	3.3563
36         0.8517         1.3055         1.6883         2.0281         2.4345         2.7195         3.3326           37         0.8514         1.3049         1.6871         2.0262         2.4314         2.7154         3.3256           38         0.8512         1.3042         1.6860         2.0244         2.4286         2.7116         3.3190           39         0.8509         1.3036         1.6849         2.0227         2.4258         2.7079         3.3127           40         0.8505         1.3021         1.6839         2.0211         2.4233         2.7045         3.3069           41         0.8505         1.3025         1.6829         2.0195         2.4208         2.7012         3.3012           42         0.8503         1.3020         1.6820         2.0181         2.4185         2.6981         3.2959           43         0.8501         1.3016         1.6811         2.0167         2.4163         2.6951         3.2909           44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6826         3.2815           45         0.8497         1.3002         1.6787         2.0129         2.4102         2.6870	34	0.8523	1,3070	1.6909	2.0322	2.4411	2.7284	3.3480
37         0.8514         1.3049         1.6871         2.0262         2.4314         2.7154         3.3256           38         0.8512         1.3042         1.6860         2.0244         2.4286         2.7116         3.3190           39         0.8509         1.3036         1.6849         2.0227         2.4258         2.7079         3.3127           40         0.8507         1.3031         1.6839         2.0211         2.4233         2.7045         3.3069           41         0.8505         1.3025         1.6829         2.0195         2.4208         2.7012         3.3012           42         0.8503         1.3020         1.6820         2.0181         2.4185         2.6981         3.2959           43         0.8501         1.3016         1.6811         2.0167         2.4163         2.6951         3.2909           44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6923         3.2861           45         0.8497         1.3007         1.6787         2.0129         2.4102         2.6870         3.2771           47         0.8493         1.2998         1.6779         2.0117         2.4083         2.6846	35	0.8520	1.3062	1.6896	2.0301	2.4377	2.7238	
38         0.8512         1.3042         1.6860         2.0244         2.4286         2.7116         3.3190           39         0.8509         1.3036         1.6849         2.0227         2.4258         2.7079         3.3127           40         0.8507         1.3031         1.6839         2.0211         2.4233         2.7045         3.3069           41         0.8505         1.3025         1.6829         2.0195         2.4208         2.7012         3.3012           42         0.8503         1.3020         1.6820         2.0181         2.4185         2.6981         3.2959           43         0.8501         1.3016         1.6811         2.0167         2.4163         2.6951         3.2909           44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6923         3.2861           45         0.8497         1.3007         1.6787         2.0141         2.4121         2.6896         3.2815           46         0.8495         1.3002         1.6787         2.0117         2.4083         2.6846         3.2729           48         0.8495         1.2994         1.6772         2.0106         2.4066         2.6822	36	0.8517	1.3055	1.6883	2.0281	2.4345	2.7195	3.3326
39         0.8509         1.3036         1.6849         2.0227         2.4258         2.7079         3.3127           40         0.8507         1.3031         1.6839         2.0211         2.4233         2.7045         3.3069           41         0.8505         1.3025         1.6829         2.0195         2.4208         2.7012         3.3012           42         0.8503         1.3020         1.6820         2.0181         2.4185         2.6981         3.2959           43         0.8501         1.3016         1.6811         2.0167         2.4163         2.6951         3.2909           44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6923         3.2861           45         0.8497         1.3007         1.6794         2.0141         2.4121         2.6896         3.2871           46         0.8495         1.3002         1.6787         2.0129         2.4102         2.6870         3.2771           47         0.8493         1.2998         1.6772         2.01106         2.4066         2.5822         3.2689           49         0.8490         1.2991         1.6766         2.0096         2.4049         2.6800	37	0.8514	1.3049	1.6871	2.0262	2.4314	2.7154	3.3256
40         0.8507         1.3031         1.6839         2.0211         2.4233         2.7045         3.3069           41         0.8505         1.3025         1.6829         2.0195         2.4208         2.7012         3.3012           42         0.8503         1.3020         1.6820         2.0181         2.4185         2.6981         3.2959           43         0.8501         1.3016         1.6811         2.0167         2.4163         2.6951         3.2909           44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6923         3.2861           45         0.8497         1.3007         1.6794         2.0141         2.4121         2.6896         3.2815           46         0.8495         1.3002         1.6787         2.0129         2.4102         2.6870         3.2771           47         0.8493         1.2998         1.6772         2.0106         2.4066         2.6822         3.2689           49         0.8490         1.2991         1.6766         2.0096         2.4049         2.6800         3.2651           50         0.8482         1.2971         1.6730         2.0040         2.3961         2.6683	38	0.8512	1.3042	1.6860	2.0244	2.4286	2.7116	
41         0.8505         1.3025         1.6829         2.0195         2.4208         2.7012         3.3012           42         0.8503         1.3020         1.6820         2.0181         2.4185         2.6981         3.2959           43         0.8501         1.3016         1.6811         2.0167         2.4163         2.6951         3.2909           44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6923         3.2861           45         0.8497         1.3007         1.6784         2.0141         2.4121         2.6896         3.2815           46         0.8495         1.3002         1.6787         2.0129         2.4102         2.6870         3.2771           47         0.8493         1.2998         1.6779         2.0117         2.4083         2.6846         3.2729           48         0.8492         1.2994         1.6772         2.0106         2.4066         2.5822         3.2689           49         0.8490         1.2987         1.6769         2.0096         2.4043         2.6778         3.2614           55         0.8482         1.2971         1.6730         2.0040         2.3961         2.6682	39	0.8509		1.6849	2.0227	2.4258	2.7079	3.3127
42         0.8503         1.3020         1.6820         2.0181         2.4185         2.6981         3.2959           43         0.8501         1.3016         1.6811         2.0167         2.4163         2.6951         3.2909           44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6923         3.2861           45         0.8497         1.3007         1.6794         2.0141         2.4121         2.6896         3.2815           46         0.8495         1.3002         1.6787         2.0129         2.4102         2.6870         3.2771           47         0.8493         1.2998         1.6779         2.0117         2.4083         2.6846         3.2729           48         0.8492         1.2994         1.6766         2.0096         2.4049         2.6800         3.2651           50         0.8489         1.2987         1.6766         2.0096         2.4049         2.6800         3.2651           55         0.8482         1.2971         1.6730         2.0040         2.3961         2.6682         3.2451           60         0.8477         1.2958         1.6706         2.0003         2.3901         2.6603	40	0.8507	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069
43         0.8501         1.3016         1.6811         2.0167         2.4163         2.6951         3.2909           44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6923         3.2861           45         0.8497         1.3007         1.6794         2.0141         2.4121         2.6896         3.2815           46         0.8495         1.3002         1.6787         2.0129         2.4102         2.6870         3.2771           47         0.8493         1.2998         1.6779         2.0117         2.4083         2.6846         3.2729           48         0.8492         1.2994         1.6772         2.0106         2.4066         2.5822         3.2689           49         0.8490         1.2991         1.6766         2.0096         2.4049         2.6800         3.2651           50         0.8489         1.2987         1.6759         2.0086         2.4033         2.6778         3.2614           55         0.8482         1.2971         1.6730         2.0040         2.3961         2.6682         3.2451           60         0.8477         1.2958         1.6706         2.0003         2.3901         2.6603	1	1						
44         0.8499         1.3011         1.6802         2.0154         2.4141         2.6923         3.2861           45         0.8497         1.3007         1.6794         2.0141         2.4121         2.6896         3.2815           46         0.8495         1.3002         1.6787         2.0129         2.4102         2.6870         3.2771           47         0.8493         1.2998         1.6779         2.0117         2.4083         2.6846         3.2729           48         0.8492         1.2994         1.6772         2.0106         2.4066         2.5822         3.2689           49         0.8490         1.2991         1.6766         2.0096         2.4049         2.6800         3.2651           50         0.8489         1.2987         1.6759         2.0086         2.4033         2.6778         3.2614           55         0.8482         1.2971         1.6730         2.0040         2.3961         2.6682         3.2451           60         0.8477         1.2958         1.6706         2.0003         2.3901         2.6603         3.2317           65         0.8472         1.2947         1.6686         1.9971         2.3851         2.6536	ľ	•		1.6820				
45	l .	i e						,
46         0.8495         1.3002         1.6787         2.0129         2.4102         2.6870         3.2771           47         0.8493         1.2998         1.6779         2.0117         2.4083         2.6846         3.2729           48         0.8492         1.2994         1.6772         2.0106         2.4066         2.5822         3.2689           49         0.8490         1.2991         1.6766         2.0096         2.4049         2.6800         3.2651           50         0.8489         1.2987         1.6759         2.0086         2.4033         2.6778         3.2614           55         0.8482         1.2971         1.6730         2.0040         2.3961         2.6682         3.2451           60         0.8477         1.2958         1.6706         2.0003         2.3901         2.6603         3.2317           65         0.8472         1.2947         1.6686         1.9971         2.3851         2.6536         3.2204           70         0.8468         1.2938         1.6669         1.9944         2.3808         2.6479         3.2024           80         0.8461         1.2922         1.6641         1.9901         2.37339         2.6387		1						
47         0.8493         1.2998         1.6779         2.0117         2.4083         2.6846         3.2729           48         0.8492         1.2994         1.6772         2.0106         2.4066         2.6822         3.2689           49         0.8490         1.2991         1.6766         2.0096         2.4049         2.6800         3.2651           50         0.8489         1.2987         1.6759         2.0086         2.4033         2.6778         3.2614           55         0.8482         1.2971         1.6730         2.0040         2.3961         2.6682         3.2451           60         0.8477         1.2958         1.6706         2.0003         2.3901         2.6603         3.2317           65         0.8472         1.2947         1.6686         1.9971         2.3851         2.6536         3.2204           70         0.8468         1.2938         1.6669         1.9944         2.3808         2.6479         3.2108           75         0.8464         1.2929         1.6654         1.9921         2.3771         2.6387         3.1952           85         0.8459         1.2916         1.6630         1.9883         2.3710         2.6349								
48								
49         0.8490         1.2991         1.6766         2.0096         2.4049         2.6800         3.2651           50         0.8489         1.2987         1.6759         2.0086         2.4033         2.6778         3.2614           55         0.8482         1.2971         1.6730         2.0040         2.3961         2.6682         3.2451           60         0.8477         1.2958         1.6706         2.0003         2.3901         2.6603         3.2317           65         0.8472         1.2947         1.6686         1.9971         2.3851         2.6536         3.2204           70         0.8468         1.2938         1.6669         1.9944         2.3808         2.6479         3.2108           75         0.8464         1.2929         1.6654         1.9921         2.3771         2.6430         3.2024           80         0.8461         1.2922         1.6641         1.9901         2.3739         2.6387         3.1889           90         0.8456         1.2916         1.6630         1.9883         2.3710         2.6349         3.1889           90         0.8456         1.2910         1.6620         1.9867         2.3682         2.6286	•							
50         0.8489         1.2987         1.6759         2.0086         2.4033         2.6778         3.2614           55         0.8482         1.2971         1.6730         2.0040         2.3961         2.6682         3.2451           60         0.8477         1.2958         1.6706         2.0003         2.3901         2.6603         3.2317           65         0.8472         1.2947         1.6686         1.9971         2.3851         2.6536         3.2204           70         0.8468         1.2938         1.6659         1.9944         2.3808         2.6479         3.2108           75         0.8464         1.2929         1.6654         1.9921         2.3771         2.6430         3.2024           80         0.8461         1.2922         1.6641         1.9901         2.3739         2.6387         3.1952           85         0.8459         1.2916         1.6630         1.9883         2.3710         2.6349         3.1889           90         0.8456         1.2910         1.6620         1.9867         2.3682         2.6316         3.1832           95         0.8451         1.2905         1.6611         1.9852         2.3662         2.6286		l .						
55         0.8482         1.2971         1.6730         2.0040         2.3961         2.6682         3.2451           60         0.8477         1.2958         1.6706         2.0003         2.3901         2.6603         3.2317           65         0.8472         1.2947         1.6686         1.9971         2.3851         2.6536         3.2204           70         0.8468         1.2938         1.6669         1.9944         2.3808         2.6479         3.2108           75         0.8464         1.2929         1.6654         1.9921         2.3771         2.6430         3.2024           80         0.8461         1.2922         1.6641         1.9901         2.3739         2.6387         3.1952           85         0.8459         1.2916         1.6630         1.9883         2.3710         2.6349         3.1889           90         0.8456         1.2910         1.6620         1.9867         2.3685         2.6316         3.1832           95         0.8454         1.2905         1.6611         1.9852         2.3662         2.6286         3.1783           100         0.8452         1.2901         1.6602         1.9840         2.3642         2.6259		1						
60         0.8477         1.2958         1.6706         2.0003         2.3901         2.6603         3.2317           65         0.8472         1.2947         1.6686         1.9971         2.3851         2.6536         3.2204           70         0.8468         1.2938         1.6669         1.9944         2.3808         2.6479         3.2108           75         0.8464         1.2929         1.6654         1.9921         2.3771         2.6430         3.2024           80         0.8461         1.2922         1.6641         1.9901         2.3739         2.6387         3.1952           85         0.8459         1.2916         1.6630         1.9883         2.3710         2.6349         3.1889           90         0.8456         1.2910         1.6620         1.9867         2.3685         2.6316         3.1832           95         0.8454         1.2905         1.6611         1.9852         2.3662         2.6286         3.1783           100         0.8452         1.2901         1.6602         1.9840         2.3642         2.6259         3.1738           150         0.8440         1.2872         1.6551         1.9759         2.3515         2.6090								
65         0.8472         1.2947         1.6686         1.9971         2.3851         2.6536         3.2204           70         0.8468         1.2938         1.6669         1.9944         2.3808         2.6479         3.2108           75         0.8464         1.2929         1.6654         1.9921         2.3771         2.6430         3.2024           80         0.8461         1.2922         1.6641         1.9901         2.3739         2.6387         3.1952           85         0.8459         1.2916         1.6630         1.9883         2.3710         2.6349         3.1889           90         0.8456         1.2910         1.6620         1.9867         2.3685         2.6316         3.1832           95         0.8454         1.2905         1.6611         1.9852         2.3662         2.6286         3.1783           100         0.8452         1.2901         1.6602         1.9840         2.3642         2.6259         3.1738           150         0.8440         1.2872         1.6551         1.9759         2.3515         2.6090         3.1455           200         0.8434         1.2849         1.6510         1.9695         2.3414         2.5956								
70         0.8468         1.2938         1.6669         1.9944         2.3808         2.6479         3.2108           75         0.8464         1.2929         1.6654         1.9921         2.3771         2.6430         3.2024           80         0.8461         1.2922         1.6641         1.9901         2.3739         2.6387         3.1952           85         0.8459         1.2916         1.6630         1.9883         2.3710         2.6349         3.1889           90         0.8456         1.2910         1.6620         1.9867         2.3685         2.6316         3.1832           95         0.8454         1.2905         1.6611         1.9852         2.3662         2.6286         3.1783           100         0.8452         1.2901         1.6602         1.9840         2.3642         2.6259         3.1738           150         0.8440         1.2872         1.6551         1.9759         2.3515         2.6090         3.1455           200         0.8434         1.2849         1.6510         1.9695         2.3414         2.5956         3.1231								
75								
80         0.8461         1.2922         1.6641         1.9901         2.3739         2.6387         3.1952           85         0.8459         1.2916         1.6630         1.9883         2.3710         2.6349         3.1889           90         0.8456         1.2910         1.6620         1.9867         2.3685         2.6316         3.1832           95         0.8454         1.2905         1.6611         1.9852         2.3662         2.6286         3.1783           100         0.8452         1.2901         1.6602         1.9840         2.3642         2.6259         3.1738           150         0.8440         1.2872         1.6551         1.9759         2.3515         2.6090         3.1455           200         0.8434         1.2858         1.6525         1.9719         2.3451         2.6006         3.1315           250         0.8431         1.2849         1.6510         1.9695         2.3414         2.5956         3.1231								
85         0.8459         1.2916         1.6630         1.9883         2.3710         2.6349         3.1889           90         0.8456         1.2910         1.6620         1.9867         2.3685         2.6316         3.1832           95         0.8454         1.2905         1.6611         1.9852         2.3662         2.6286         3.1783           100         0.8452         1.2901         1.6602         1.9840         2.3642         2.6259         3.1738           150         0.8440         1.2872         1.6551         1.9759         2.3515         2.6090         3.1455           200         0.8434         1.2858         1.6525         1.9719         2.3451         2.6006         3.1315           250         0.8431         1.2849         1.6510         1.9695         2.3414         2.5956         3.1231	l .							- 1
90 0.8456 1.2910 1.6620 1.9867 2.3685 2.6316 3.1832 95 0.8454 1.2905 1.6611 1.9852 2.3662 2.6286 3.1783 100 0.8452 1.2901 1.6602 1.9840 2.3642 2.6259 3.1738 150 0.8440 1.2872 1.6551 1.9759 2.3515 2.6090 3.1455 200 0.8434 1.2858 1.6525 1.9719 2.3451 2.6006 3.1315 250 0.8431 1.2849 1.6510 1.9695 2.3414 2.5956 3.1231	I .	1						
95     0.8454     1.2905     1.6611     1.9852     2.3662     2.6286     3.1783       100     0.8452     1.2901     1.6602     1.9840     2.3642     2.6259     3.1738       150     0.8440     1.2872     1.6551     1.9759     2.3515     2.6090     3.1455       200     0.8434     1.2858     1.6525     1.9719     2.3451     2.6006     3.1315       250     0.8431     1.2849     1.6510     1.9695     2.3414     2.5956     3.1231								
100     0.8452     1.2901     1.6602     1.9840     2.3642     2.6259     3.1738       150     0.8440     1.2872     1.6551     1.9759     2.3515     2.6090     3.1455       200     0.8434     1.2858     1.6525     1.9719     2.3451     2.6006     3.1315       250     0.8431     1.2849     1.6510     1.9695     2.3414     2.5956     3.1231								
150 0.8440 1.2872 1.6551 1.9759 2.3515 2.6090 3.1455 200 0.8434 1.2858 1.6525 1.9719 2.3451 2.6006 3.1315 250 0.8431 1.2849 1.6510 1.9695 2.3414 2.5956 3.1231								4
200 0.8434 1.2858 1.6525 1.9719 2.3451 2.6006 3.1315 250 0.8431 1.2849 1.6510 1.9695 2.3414 2.5956 3.1231								
250 0.8431 1.2849 1.6510 1.9695 2.3414 2.5956 3.1231								
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**PAPER** MATHEMATICS FOR ENGINEERING STUDENTS EXAMINATION QUESTION / SOLUTION ISE 201 nathe QUESTION Please write on this side only, legibly and neatly, between the margins SOLUTION Guer 5 flere wildt = f(w) Dhen F. T & Flation [ f(-t) e-widt == ] = [ (s) e ws (-ds) = \( \xi\) \( \ell\_{10}^{\ightarrow} ds = \frac{7}{6} \( (-\w) \) (2) Defferentiate (i) with w Joseph 2 (e-101) 1+ = d f = J - 1 + 1 + e 101 1+ : FT. of Effe) on idf(w) (5)  $\frac{1}{9(\omega)} = \int_{0}^{\infty} e^{-at} e^{-(\omega)} dt = \frac{e^{-(\alpha+(\omega))t}}{e^{-(\alpha+(\omega))}} = \frac{1}{a_{+(\omega)}}$  Since u > 0h(t) = 9(t) + 9(-t) so 1 (a)=3(a)-3(-a)= = 1 - 1 = 2a

1 (a)=3(a)-3(-a)= = 1 = 2a

1 (a)=3(a)-3(a)= = 1 = 2a to(t) has F. T 1 d ( 1 = -12 = 1 k(t)=tg(t)+(-tg(-t)) hus F.T.  $\frac{1}{(\alpha+i\omega)^2} = \frac{1}{(\alpha-i\omega)^2} = \frac{2(\alpha^2-\omega^2)}{(\alpha^2+\omega^2)^2}$ (3) There was symmetry formule. \$ (e) his F.T. 27 F(-w) So FT. SI 20 00 27 h (-w)= 27 e a/w)

Setter: TR CASH

Setter's signature: 90 Cal

Checker: (JRIDLER Rome

### EXAMINATION QUESTION / SOLUTION

2003 - 2004

ISE 26

**PAPER** 

QUESTION

(2)

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2

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SOLUTION

= - y(0) - p y (p)

Similarly F.T. SI dry or for e- pt dry de = [e-bt dy] + [pe-bt dy of

Consider d = 3 dy +24 = 9(t) and take L.T.

$$50 \quad \overline{y} = \frac{9}{p^2 - 3p+2} = \frac{9}{(p-1)(p-2)} = \left(\frac{1}{p-2} - \frac{1}{p-1}\right) \frac{9}{9}$$

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Checker: CIRIDLER-ROWE

Setter's signature:

### EXAMINATION QUESTION / SOLUTION

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**PAPER** 

ISE 2.6

QUESTION

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SOLUTION 10+0

(2)

$$T = \begin{cases} P \partial x - Q \partial y \\ \nabla x = t, y = 0 \end{cases} \qquad T = \begin{cases} P \partial x + Q \partial y = \int_{t=0}^{\infty} \left( P \frac{\partial x}{\partial r} + Q \frac{\partial y}{\partial t} \right) dt \\ \frac{\partial x}{\partial r} = \frac{\partial x}{\partial r} + \frac{\partial y}{\partial t} = \frac{\partial x}{\partial r} + \frac{\partial x}{\partial t} + \frac{\partial y}{\partial t} = \frac{\partial x}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial x}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial x}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial x}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial x}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial x}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial x}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial x}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r} = \frac{\partial y}{\partial r} + \frac{\partial y}{\partial r$$

$$= -\int_{a}^{b} \left( b \frac{\partial f}{\partial x} - b \frac{\partial f}{\partial y} \right) g + = -\int_{a}^{b} -f \cdot i g + \frac{\partial g}{\partial x} | x$$

$$\overline{L}_{2} = \int_{\theta=0}^{\theta=\pi/L} \left( P \frac{dx}{dx} + Q \frac{dy}{ds} \right) d3$$

$$= \int_{0}^{\pi/2} \frac{a^{2} \cos^{2} a (-a \sin a) - a^{2} \sin^{2} a (a \cos a)}{a \cos a + a \sin a} da$$

$$= \int_{0}^{\pi h} -a^{2} \sin 0 \cos 0 d9 = \left[ -\frac{a^{2}}{5} \sin^{2} 0 \right]_{0}^{\pi h} = \frac{a^{2}}{5}$$

$$= I_{1} - I_{2} - I_{3} = a^{2} h_{1}$$

Setter: JR. CASH

Setter's signature :  $\mathcal{IRC}$ 

Checker: CTRIDLER-ROWE

### EXAMINATION QUESTION / SOLUTION

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PAPER

1SE 26

QUESTION

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SOLUTION

3

2

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(J)

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7

3

1

(2)

(1) 1 yex2, x=

$$= \begin{cases} \left[ 2x^{2} e^{y^{2}} \right]^{\frac{1}{3}} dy$$

$$= \left[ \frac{2y e^{y^{2}} dy}{2y e^{y^{2}} dy} \right]^{\frac{1}{3}} = e^{-1}.$$

$$y: \sqrt{2x-3^{2}}$$

$$x^{2}-2x-4^{2}=0$$

$$x^{2}-2x-4^{2}=0$$

$$x^{2}-2x-4^{2}=0$$

$$\frac{1}{2} = \iint_{\Omega} \frac{r \sin \theta}{12} r dr d\theta$$

$$= \int_{\Omega} \frac{r \cos \theta}{12} r dr d\theta$$

Setter: JB CASH

Setter's signature : In Card

Checker's signature:

Checker:

### EXAMINATION QUESTION / SOLUTION

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ISE 26

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QUESTION

Poles where Z6 =-1 = et = Z= e ( (7/6 + NOT/3) = p is

SOLUTION 4

(10)

When  $0 = \pm \frac{\pi}{6}$ ,  $\pm \frac{\pi}{2}$ ,  $\pm \frac{5\pi}{6}$ .

If we take 2 as one 3 itex values (so  $7^6 = -1$ ) then the

residue of  $\frac{1}{2^{i}}$  at  $\eta$  is  $\frac{1}{\sqrt[3]{7}(7^{i})} = \frac{1}{67^{5}} = -\frac{2}{6}$ 

e in 16 - 1 ( 13 - 1/2) o in h

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46 (- (3 +1/2)

Social = 271 Root e Till - Root e 286/3

= 201 [ - 13 - 1 - 1 - 1 - 1 - 1]/6 = 217

For residue thm  $\int \frac{dz}{z^{6}+1} = \int \frac{dx}{z^{6}+1} \quad \text{on } c_{2} \quad |z^{6}+1| \quad |z| \quad |z^{6}-1| \quad |z| \quad |z|$ 

 $\left| \int \frac{d\tau}{\tau'} \right| \leq \frac{\pi R}{\pi R} \to 0 \text{ as } R \to \infty$ 

 $\frac{1}{5} \frac{c_2}{c_2} \frac{(1+1)}{(1+1)} \frac{R^{-1}}{3} = \int_{C_1}^{\infty} \int_{C_2}^{\infty} \frac{dx}{c_3} \frac{c_3}{x^6 + 1} \frac{dx}{3} = \frac{7i7}{x^6 + 1}$ 

(2)

Setter: J. n. CASH

Setter's signature:

Checker: CTRIDLER- BOWE

EXAMINATION QUESTION / SOLUTION

ISE 2

Ű,

PAPER

2003 -- 2004

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QUESTION

SOLUTION

15

4

$$P(D) = \frac{C.5}{100} \times 0.005$$

Apply Low of ToTal Peobabilities

$$P(T) = P(T|D)P(D) + P(T|D)P(D)$$
  
=  $(0.99 \times 0.005) + (0.02 \times 0.995)$ 

Setter: MARIA DE JORG

Checker: MICROWDER

Setter's signature: U Color Color Color

Checker's signature: MJ Crowdler

### MATHEMATICS FOR ENGINEERING STUDENTS EXAMINATION QUESTION / SOLUTION

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QUESTION

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SOLUTION

+5

$$= \frac{0.005 \times 0.99}{0.00495} = \frac{0.00495}{0.02485} =$$

(c) 
$$\hat{y} = \sum_{i=1}^{\infty} x_i = \bar{x}$$

$$\hat{\mu} = \frac{500}{100} = 5$$

Setter: H. DE IORO

Checker: MITCHOWNER

Checker's signature: MJ Cronoler

Setter's signature: Marie de la Como

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45% confidence interior

$$X \pm 2/2 = 5 \pm 1.96 (0.03)$$

Setter: 12 DE JURO

Checker: MICRONDER Setter's signature: Walled Land Walled

Checker's signature:

MT Crowder

### EXAMINATION QUESTION / SOLUTION

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QUESTION

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SOLUTION

**∤** 5

C. I. 13

(4.9412, 5.0588)

(x)

X~N (5, C.C9)

 $P(X>4) = 1 - P(X \leq 4)$ 

 $P\left(\frac{x-5}{0.3} \le \frac{4-5}{0.3}\right)$ 

 $= 1 - P\left(2 \le \frac{-1}{0.3}\right)$ 

 $21-\int_{1}^{1}-P(2\leq \frac{1}{C3})$ 

= P (2 < 1) = P(75 3,333)

~ 0.9996

2~N(011)

Setter: M. DE ICRC

MICRONDER Checker:

Setter's signature: MCLULCCCOCCCC

Checker's signature:

MJ Crowoler

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QUESTION

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SOLUTION

PROBLEM 2

 $E(x) = \sum_{x=0}^{\infty} se^{-x}$ 

 $=\frac{2}{2}\frac{\lambda^{2}e^{-\lambda}}{(2e^{-1})!}=$ 

 $= e^{-\lambda} \partial_{\alpha} \partial$ 

(b)

> = mean is a 5 months perced

HE mean in a year percol

= 2 ×2 =4

Setter's signature: Muchalung aluma

Checker: MIT CROWDER

Setter: HOR TUPIC

Checker's signature: MJ Complex

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ISE 2

QUESTION

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solution 26

\_\_\_\_

M= mean in a month pertion

 $=\frac{2}{6}=\frac{1}{3}$ 

P(x=0)= 10 = 10 = 165

(cl)

(e)

Like Edrocal

 $L(\mu) = \frac{\pi}{11} \frac{e^{-\mu} e^{-\mu}}{12}$ 

= e-Am =xi N =xi N =xi N =xi

log (L(M)) = - Mm + (Zsei) log M - log (Trseil)

Setter: PE LURUO

Setter's signature: /// Opprovious and signature:

MJ Gandler

Checker: NIJ CRUNDER

### EXAMINATION QUESTION / SOLUTION

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ISE 2

QUESTION

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solution 2.6

$$\frac{\int \log(L(\mu))}{d\mu} = -m + \frac{\sum x_i}{\mu} = 0$$

$$\hat{\mu} = \frac{2xi}{n} = x$$

$$\frac{d^2 \log(L(\mu))}{d\mu^2} = \frac{\sum xi}{\mu^2} co$$

$$\mu = \frac{23}{10} = 2.3$$

$$E(X) = \frac{2}{2}M = M$$

$$V(X) = \frac{1}{2}M$$

Setter: H DE INCLU

Checker: MT CROWNER

Setter's signature: William Color Co

Checker's signature: M.T. Choweler

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