UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1998

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER 2.14

COMPUTATIONAL TECHNIQUES

Wednesday, May 13th 1998, 2.00 - 3.30

Answer THREE questions

For admin. only: paper contains 4 questions

1a Assume 6-digit decimal arithmetic. Below, there are two numbers, x and y, and their approximations.

Accurate value	Approximation
$x = 1.00000$	1.00199
$y = 9.00000$	8.99694

Determine the error of the approximations using all measures of error you know. Which approximation is better? Why?

b Matrix $\mathbf{A}$ is called *skew symmetric* if $\mathbf{A}^T = -\mathbf{A}$. What is the shape of $\mathbf{A}$? What are the diagonal elements of $\mathbf{A}$? Show that if $\mathbf{A}$ is skew symmetric then $\mathbf{A}^T\mathbf{A} = \mathbf{A}\mathbf{A}^T$!

c Given $\mathbf{A}$, determine its ℓ_2 norm $||\mathbf{A}||_2$:

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}.$$

(The three parts carry, respectively, 20%, 30% and 50% of the marks).

2a Let I be the half-open interval $(0,1]$ and $d(x,y) = |x-y|$ be the usual distance on I.

(i) Show that d is a metric on I (key points: nonnegativity, when $= 0$, symmetry, triangular inequality).

(ii) Show that $x_n = 1/n$ for $n = 1, 2, \ldots$ is a Cauchy sequence in the metric d.

(iii) Is (I, d) complete? Justify your answer.

b You are given two sets of linear equations:

$$\begin{array}{rcrcrcr} 4x_1 & - & x_2 & + & 4x_3 & = & 6 \\ -x_1 & + & 9x_2 & - & 2x_3 & = & 15 \\ 4x_1 & - & x_2 & + & 4x_3 & = & 6 \end{array} \tag{1}$$

and

$$\begin{array}{rcrcrcr} 4x_1 & - & 2x_2 & + & 2x_3 & = & 6 \\ -2x_1 & + & 10x_2 & - & x_3 & = & 15 \\ 2x_1 & - & x_2 & + & 2x_3 & = & 6 \end{array} \tag{2}$$

One of them can be solved by Cholesky factorisation. Identify it, explain your choice and apply Cholesky factorisation to solve it.

c Given the following three vectors:
$\mathbf{a}_1 = [2, -3, 2]^T$
$\mathbf{a}_2 = [-9, 8, -3]^T$
$\mathbf{a}_3 = [7, -5, 1]^T$.
Determine whether they are linearly dependent or independent.

(The three parts carry, respectively, 40%, 40% and 20% of the marks).

Turn over ...

3a Which of the following functions have local extreme points (minimum or maximum), and if so, where? Justify your answer.

(i) $f(x, y) = 1 - xy$

(ii) $f(x, y) = x^2 - y^3$

(iii) $f(x, y) = x^2 + y^2$

b Assume that matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ have appropriate dimensions for the operations below.

(i) Prove that $(\mathbf{ABC})^T = \mathbf{C}^T\mathbf{B}^T\mathbf{A}^T$.

(ii) Prove that $(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$.

c Show that for any matrix $\mathbf{A}$ both $\mathbf{A}^T\mathbf{A}$ and $\mathbf{AA}^T$ are symmetric.

(The three parts carry, respectively, 45%, 20% and 35% of the marks).

4a Let $\mathbf{B}$ an $m \times m$ nonsingular matrix with inverse $\mathbf{B}^{-1}$, $\mathbf{c}$ an m-vector and $\mathbf{0}$ the m dimensional null vector. $\mathbf{A}$ is given in the following partitioned form:

$$\mathbf{A} = \begin{bmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{c}^T & 1 \end{bmatrix}$$

Determine $\mathbf{A}^{-1}$ symbolically in a partitioned form. What is the dimension of $\mathbf{A}$? What are the dimensions of the submatrices in $\mathbf{A}^{-1}$?

b Under what conditions are the following equalities true?

(i) $(\mathbf{A}+\mathbf{B})^2 = \mathbf{A}^2 + 2\mathbf{AB} + \mathbf{B}^2$.

(ii) $(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B}) = \mathbf{A}^2 - \mathbf{B}^2$.

c Given matrices $\mathbf{A}$ and $\mathbf{B}$:

$$\mathbf{A} = \begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix},$$

where a, b, c and d are scalars. Compute $\mathbf{AB} - \mathbf{BA}$. Give conditions for $\mathbf{AB} = \mathbf{BA}$.

(The three parts carry, respectively, 40%, 20% and 40% of the marks).

End of Paper