EE2-10A

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE PART II: MEng, BEng and ACGI

**DEVICES**

Corrected Copy

Monday, 15 June 2:00 pm

Time allowed: 1:30 hours

**There are TWO questions on this paper.**

**Answer ALL questions. Question One carries 20 marks and Question Two carries 30 marks.**

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible First Marker(s) : K. Fobelets
Second Marker(s) : S. Lucyszyn

## Special instructions for invigilators

## Special instructions for students

*Do not use red nor green ink.*

**Constants and Formulae**

| | |
|---|---|
| permittivity of free space: | $\varepsilon_o = 8.85 \times 10^{-12}$ F/m |
| permeability of free space: | $\mu_o = 4\pi \times 10^{-7}$ H/m |
| intrinsic carrier concentration in Si: | $n_i = 1.45 \times 10^{10}$ cm$^{-3}$ at $T = 300$K |
| dielectric constant of Si: | $\varepsilon_{Si} = 11$ |
| dielectric constant of $SiO_2$: | $\varepsilon_{ox} = 4$ |
| thermal voltage: | $V_T = kT/e = 0.026$V at $T = 300$K |
| charge of an electron: | $e = 1.6 \times 10^{-19}$ C |

$$\left.\begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n\frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p\frac{dp(x)}{dx} \end{aligned}\right\}$$

Drift-diffusion current equations

$$\left.\begin{aligned} \frac{\partial\delta n}{\partial t} &= D_n\frac{\partial^2\delta n}{\partial x^2} - \frac{\delta n}{\tau_n} \\ \frac{\partial\delta p}{\partial t} &= D_p\frac{\partial^2\delta p}{\partial x^2} - \frac{\delta p}{\tau_p} \end{aligned}\right\}$$

Continuity equations of minority carriers

$$\left.\begin{aligned} J_n &= \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right) \\ J_p &= \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1\right) \end{aligned}\right\}$$

Text-book diode diffusion currents

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT}\right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$

Minority carrier injection under bias $V$

$$\delta c = \Delta c \exp\left(\frac{-x}{L}\right) \text{ with } \begin{cases} \delta c = \delta p_n \text{ or } \delta n_p \\ \Delta c \text{ the excess carrier concentration} \\ \text{at the edge of the depletion region} \end{cases}$$

Excess carrier concentration as a function of distance when recombination occurs – long layer approximation.

$$L = \sqrt{D\tau}$$

Diffusion length

$$D = \frac{kT}{e}\mu$$

Einstein relation

$$C_{diff} = \frac{e}{kT}I\tau$$

Diffusion capacitance

$$i(t) = \frac{Q(t)}{\tau} + \frac{dQ(t)}{dt}$$

Time variation of current and charge

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon}$$

Poisson equation in 1 dimension

$$\delta c_B = C_1 \exp\left(\frac{x}{L_B}\right) + C_2 \exp\left(\frac{-x}{L_B}\right)$$

$$C_1 = \frac{c_B(W_B) - c_{B_0} - \left(c_B(0) - c_{B_0}\right)\exp\left(\frac{-W_B}{L_B}\right)}{2\sinh\left(\frac{W_B}{L_B}\right)}$$

$$C_2 = \frac{\left(c_B(0) - c_{B_0}\right)\exp\left(\frac{-W_B}{L_B}\right) - \left(c_B(W_B) - c_{B_0}\right)}{2\sinh\left(\frac{W_B}{L_B}\right)}$$

Excess minority carrier concentration in the base of a BJT

$C_1$, $C_2$: integration constants
EB junction is at $x = 0$
BC junction is at $x = W_B$
$c_{B0}$: equilibrium concentration
$L_B$: minority carrier diffusion length in base

$$w_n = \sqrt{\frac{2\varepsilon}{e}\frac{N_A}{N_A N_D + N_D^2}(V_0 - V)}$$

$$w_p = \sqrt{\frac{2\varepsilon}{e}\frac{N_D}{N_A N_D + N_A^2}(V_0 - V)}$$

$$W_{depl} = \sqrt{\frac{2\varepsilon}{e}\frac{N_A + N_D}{N_A N_D}(V_0 - V)}$$

Depletion regions in pn diode

1.

a) Why is the minority carrier concentration maintained at a semiconductor-metal interface? Support your explanation with appropriate sketches. [5]

b) The net rate of variation of minority carriers, $U$, is given by the simple model:

$$U_n = \frac{n_p - n_{p_0}}{\tau_n}$$

$$U_p = \frac{p_n - p_{n_0}}{\tau_p}$$

i) In which region of the pn diode does the relationship $U_n$ apply? [2]

ii) What are the signs of $U_n$ and $U_p$ in a forward biased pn diode? Provide a brief explanation for your answer. [3]

c) A Si pn diode with cross sectional area $A = 10^4$ μm$^2$ has the following material parameters:

$N_A = 2.8\times10^{17}$ cm$^{-3}$ (acceptor doping)

$N_D = 4.5\times10^{18}$ cm$^{-3}$ (donor doping)

$\tau_p = 1.4\times10^{-7}$ s (lifetime of minority carrier holes)

$\tau_n = 10^{-6}$ s (lifetime of minority carrier electrons)

$L_p = 5\times10^{-4}$ cm (diffusion length of minority carrier holes)

$L_n = 4.6\times10^{-3}$ cm (diffusion length of minority carrier electrons)

$X_p = 50\times10^{-3}$ cm (length of p-type region)

$X_n = 5\times10^{-3}$ cm (length of n-type region)

i) For the diode under a 0.05 V forward bias, sketch the minority carrier concentration variation in the p-type region as a function of distance using both the short and long layer approximation. Label the axes with minimum and maximum values. You may ignore the depletion region. [4]

ii) Calculate the difference in stored minority carrier charge in the p-type region for the diode in c) i) between the short and long layer approximation. You may ignore the depletion region and include the equilibrium charge. [6]

2.

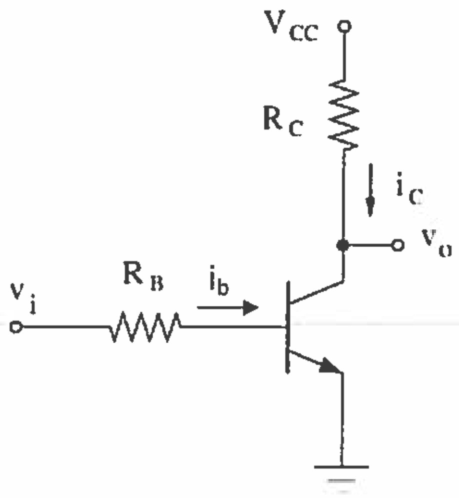


*Figure 2.1: an npn BJT with bias circuit.*

a) DC characteristics of the circuit in Fig. 2.1.

i) Draw four output characteristics $I_C$ against $V_{CE}$ for the BJT for base currents: $I_{B1} < I_{B2} < I_{B3} < I_{B4}$. Associate each base current to the correct characteristic. [5]

ii) Give the equation for the load line and add it to the plot in a) i). The load line should drive the BJT from forward active mode into saturation. Indicate the value of the current and voltage at $V_{CE} = 0$ V and $I_C = 0$ A, respectively. Label the current and voltage where the BJT goes into saturation with $I_{sat}$ and $V_{sat}$, respectively. [5]

b) Consider the circuit in Fig. 2.1 operating as a switch.

i) Plot the transfer function $v_o$ as a function of $v_i$ for $0 \leq v_i \leq v_{end}$, such that $0 \leq i_B \leq i_{Bend}$ with $\beta \times i_{Bend} > V_{CC}/R_C$.
Label the low and high voltage on the $v_o$ axis consistent with a) ii). [5]

ii) Similarly, plot the variation of $i_C$ as a function of $v_i$ for the same range as in b) i). Add the current $\beta \times i_B$ to the plot.
Label the low and high current on the $i_C$ axis consistent with a) ii). [5]

c) Transients of the circuit in Fig. 2.1.

i) Consider the case where the BJT in Fig. 2.1 switches from OFF ($i_B = 0$ A) to ON ($i_B > V_{CC}/(\beta \times R_C)$) at $t = 0$ s. Derive the expression for the variation of the collector current $i_C$ as a function of time $t$. Assume $\gamma = 1$. [5]

ii) How would the switch-ON delay change if the BJT in Fig. 2.1 is switched from a reverse biased emitter-base junction rather than from $i_B = 0$ A? Use suitable equations and relevant drawings to explain your answer. [5]