

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2014

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

**MEMS AND NANOTECHNOLOGY**

Wednesday, 14 May 10:00 am

Time allowed: 3:00 hours

**There are FIVE questions on this paper.**

**Answer Question 1.**

**Answer Question 2 OR Question 3.**

**Answer Question 4 OR Question 5.**

*Question 1 carries 40% of the marks. Remaining questions carry 30% each.*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible

First Marker(s) : Z. Durrani, A.S. Holmes, Z. Durrani

Second Marker(s) : A.S. Holmes, Z. Durrani, A.S. Holmes



### Information for Candidates

The steady state, 1-dimensional heat flow equation for a beam with distributed electrical heating is:

$$\kappa A \frac{d^2 T}{dx^2} + I^2 R = 0$$

where  $T(x)$  is the temperature variation along the beam,  $\kappa$  is the thermal conductivity of the beam material,  $A$  is the beam cross-sectional area,  $I$  is the applied current and  $R$  is the resistance per unit length of the heater.

For an elastic beam that is built in at both ends, the critical axial load  $P_c$  at which buckling will occur is given by:

$$P_c = \frac{4\pi^2 EI}{L^2}$$

where  $E$  is Young's modulus,  $I$  is the second moment of area, and  $L$  is the beam length.

**This question is compulsory**

1. a) A one-dimensional rectangular potential well of width  $a$  and depth  $-V_0$  is shown in Figure 1.1.

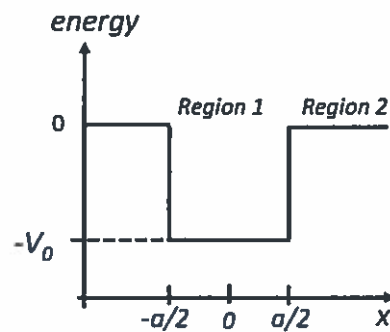


Figure 1.1

- (i) Write down the wave-vectors  $k_1$  in region 1, and  $k_2$  in region 2, for electron wave-functions for this potential well. [4]
- (ii) The solution to Schrödinger's equation for the potential well must satisfy the following:

$$\alpha \tan \alpha = \beta$$

Or  $-\alpha \cot \alpha = \beta$

Where  $\alpha = a \cdot k_1/2$  and  $\beta = a \cdot ik_2/2$ . Using these equations, find the number of bound states that can exist within the potential well for the condition:

$$\frac{mV_0a^2}{2\hbar^2} = \pi^2 \quad [6]$$

- b) Starting from a bulk Si wafer, show using suitable diagrams the fabrication process flow for a CMOS inverter. Your diagrams should show the cross-section through the inverter and state the process steps involved. [5]

**Question 1 continues on the next page**

### Question 1 continued

- c) A Si nanowire, shown in Figure 1.2(a), has radius  $R = 25 \text{ nm}$  and length  $L = 1 \text{ }\mu\text{m}$ . The nanowire is doped  $n$ -type at a density  $N_D = 5 \times 10^{24} \text{ m}^{-3}$ . Defect states exist on the nanowire surface, with density  $N_S = 10^{16} \text{ m}^{-2}$ , and can trap electrons from within the nanowire. The defect states lie at an energy  $E_t$  corresponding to a position slightly below the middle of the Si band gap  $E_g$  (Figure 1.2(b)). What is the width of the surface depletion region (shown as the shaded region in Figure 1.2(a))?

[5]

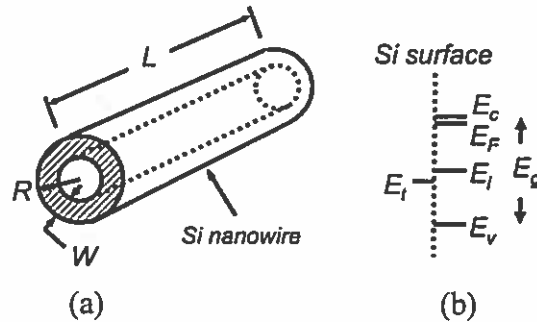


Figure 1.2

- d) A series of straight, built-in beams is included in a mask design to allow monitoring of the residual film stress of an LPCVD polysilicon process. If the polysilicon layer thickness is  $1.5 \text{ }\mu\text{m}$ , and it is known that the residual stress is somewhere in the range  $-20$  to  $-100 \text{ MPa}$ , what range of beam lengths should be used?

[5]

You may assume a value of  $160 \text{ GPa}$  for the Young's modulus of polysilicon.

- e) What advantages do electrostatic and electrothermal actuators have over other types of microactuator? What other types are available, and in what kinds of applications might these alternatives be preferred?
- f) Roughly what is the minimum spot size that can be achieved in electron beam lithography, and what factors determine this limit? Why is the minimum feature size that can be defined in a resist layer larger than this?
- g) Write down the bending equation for a cantilever subject to a uniformly distributed transverse load  $p \text{ (N/m)}$  acting along its entire length. Solve the bending equation to obtain an expression for the end deflection in terms of the beam dimensions and Young's modulus. In what kind of MEMS device might this kind of loading arise?

[5]

[5]

[5]

End of Question 1.

2. a) Given the three-dimensional, time-dependent Schrödinger equation:

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + V(r, t) \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

where  $V(r, t)$  and  $\Psi(r, t)$  are the potential and the wave-function in three dimensions, respectively:

- (i) Show that if  $V$  is independent of time, the Schrödinger equation may be separated into time-dependent and space-dependent equations. [6]
  - (ii) Hence, solve these equations to find the general 1-dimensional form of the solution for  $\Psi(r, t)$  in the case  $V = \text{constant}$ . [10]
- b) An ideal, one-dimension conductor of length  $L$  is placed between metal source and drain regions. The Fermi energies of the source and drain regions are  $E_{F1}$  and  $E_{F2}$  respectively. If the one-dimensional 'volume' occupied by one state in  $k$ -space is  $2\pi/L$ , show that the conductance of an electron channel along the conductor is given by  $G = 2e^2/h$ . [14]

3. An  $n$ -channel Si MOSFET is shown in Figure 3.1. The device has gate length  $L = 1 \mu\text{m}$ , gate width  $W = 10 \mu\text{m}$ , and gate oxide thickness  $t_{ox} = 50 \text{ nm}$ . The doping concentration in the source and drain regions is  $N_D = 10^{25} \text{ m}^{-3}$ , and the bulk doping concentration  $N_A = 10^{22} \text{ m}^{-3}$ . The maximum value of the drain voltage  $V_d$ , and gate voltage,  $V_g$ , is 2.5 V.

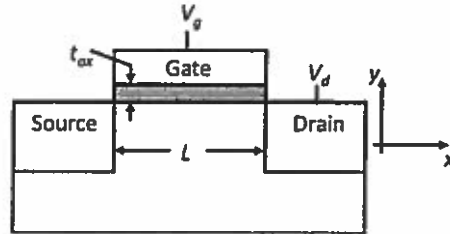


Figure 3.1

- The MOSFET dimensions are to be scaled down by a factor of 2, under constant electric field scaling conditions. For the reduced device size, estimate values for doping concentrations, maximum applied voltages, inversion layer charge density, and gate capacitance. [14]
- By what factor does the current in the saturation and sub-threshold regions change? [8]
- If the maximum values of the drain and gate voltages remain fixed at 2.5 V, but other parameters are scaled, what effect does this have on the electric fields, saturation current, and sub-threshold current in the device? [8]

You may use  $\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$ . The dielectric constant for  $\text{SiO}_2$  is  $\epsilon_{ox} = 4$ .

4. a) What are the main transduction mechanisms used in MEMS sensors? Which two approaches are most widely used, and why? What factors limit the use of the other mechanisms? [6]
- b) Figure 4.1 shows two configurations for in-plane capacitive position sensing, based on variable gap and variable overlap respectively. In each case, the mechanical layer has height  $h$ , and the total number of electrode fingers (including all fixed and moving electrodes) is  $N$ . The overlap  $l$  and the gap  $g$  are the same for both configurations.

For each configuration, derive approximate expressions for the capacitance between the A and COM terminals when the moving element is displaced from its equilibrium position by a distance  $v$ . Hence show that the sensitivities of the two configurations are in the ratio  $2l/3g$ . [8]

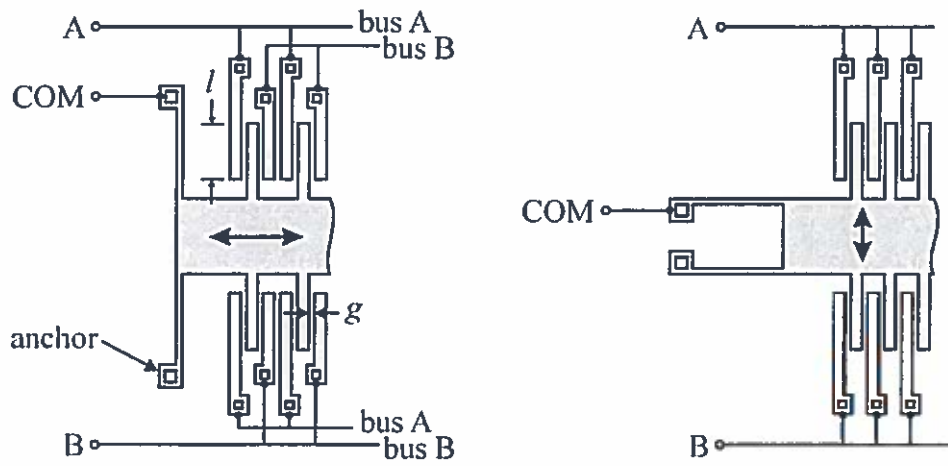


Figure 4.1

- c) It is proposed to use the variable gap configuration in an accelerometer. The nature of the readout circuit is such that the COM terminal is held at ground potential, while sinusoidal voltages of amplitude  $V_0$  are applied to the A and B terminals. Show that, under these conditions, the transducer will exert a force on the proof mass that increases with displacement and effectively reduces the suspension stiffness by an amount  $\delta k$  where:

$$\delta k \approx \frac{N\epsilon_0 l h}{3g^3} V_0^2 \quad [10]$$

- d) Sketch the block diagram of a possible readout circuit for the transducers in Figure 4.1, and briefly explain its operation. [6]



5. Figure 5.1 shows a thermally actuated probe for a dip-pen nanolithography system. The actuator is a material bimorph comprising a silicon-nitride cantilever with a thin layer of gold on its upper surface. The probe tip is integrated at the end of the cantilever.

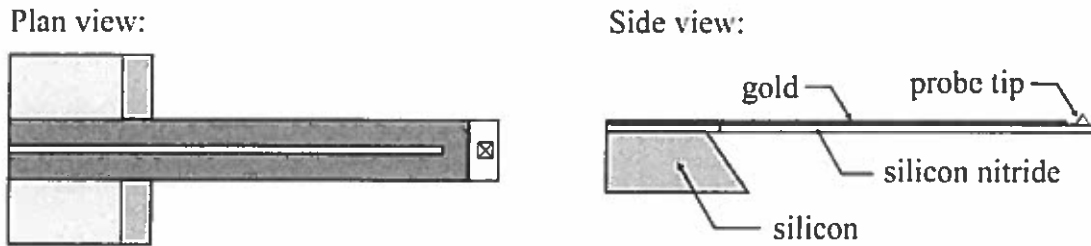


Figure 5.1

- a) Assuming that the probe tip is to be formed by depositing silicon nitride over an anisotropically etched pyramid on a (100) silicon wafer, suggest a possible fabrication process for the entire probe (starting from unprocessed silicon). You should list the process steps and materials but you do not need to describe the steps in detail. [6]
- b) By considering the bending moment due to the stress  $\sigma_f$  in the gold film, show that the deflection profile  $v(x)$  of the cantilever when heated is expected to satisfy:

$$\frac{d^2 v}{dx^2} = \frac{6\sigma_f t_f}{\tilde{E}d^2}$$

where  $\tilde{E}$  is the biaxial modulus of the silicon nitride, and  $t_f$  and  $d$  are the gold and silicon nitride thicknesses respectively. Explain why the biaxial modulus (as opposed to Young's modulus) appears in this equation. [8]

- c) Assuming that heat flows only by conduction in the cantilever, and that the substrate is a perfect heat sink, determine the temperature profile along the cantilever when the gold layer is carrying a current. Using this result, and your earlier result in part b), show that the end deflection of the probe is expected to be:

$$v(L) = \frac{3}{2} \frac{L^2 t_f}{d^2} \frac{\tilde{E}_f}{\tilde{E}} \Delta\alpha T_{\max}$$

where  $L$  is the cantilever length,  $\tilde{E}_f$  is the biaxial modulus of gold,  $\Delta\alpha$  is the difference between the thermal expansion coefficients, and  $T_{\max}$  is the maximum temperature rise (above ambient). [12]

- d) If the probe is  $500 \mu\text{m}$  long, with  $d = 1.0 \mu\text{m}$  and  $t_f = 100 \text{ nm}$ , what will be the maximum achievable deflection assuming the temperature rise is limited to  $50^\circ\text{C}$ ? In which direction will the probe deflect? [4]

You may assume  $\tilde{E} = 370 \text{ GPa}$ ,  $\tilde{E}_f = 214 \text{ GPa}$ , and that the thermal expansion coefficients are  $3.3 \times 10^{-6} \text{ K}^{-1}$  for silicon nitride and  $14.2 \times 10^{-6} \text{ K}^{-1}$  for gold.

