EE4-51 Power System Economics

Solution to Question 1

(a) The students are expected to discuss the following:

Economists suggested that unbundling and liberalisation would lead to lower prices and that the economy as a whole would benefit if the supply of electricity became the object of market discipline rather than monopoly regulation or government policy. If companies were allowed to compete freely for the provision of electricity, the efficiency gains arising from this competition would ultimately benefit the consumers. In addition, competing companies would probably choose different technologies. It would therefore be less likely that the consumers would be saddled with the consequences of unwise investments.

(b) The students are expected to discuss the following points:

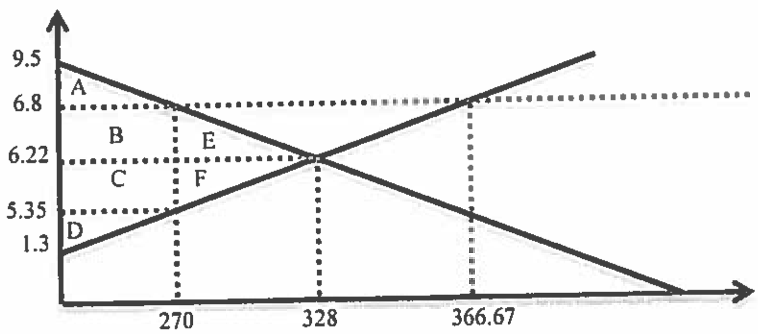
The sources of risk include technical risk (fail to produce or deliver because of technical problem), external risk (fail to produce or deliver because of cataclysmic event) and price risk (having to but at a price much higher than expected or having to sell at a price much lower than expected).

Contracts help to reallocate, spread and share risks as the parties share the risk that the price differs from their expectation or risk is passed to one party which is more willing or able to accept it.

A forward contract is a formal agreement on quantity, quality, price and date of delivery of a commodity, which is paid at the date of delivery and involves unconditional delivery.

As forward contracts are limited to parties who can take physical delivery, a standardised contract is required to reduce the cost of trading for other parties (traders or speculators). Future contracts play this role.

(c)



(i) Inverse demand function: $\pi_D = -0.01Q_D + 9.5$, Q_D *in* kWh, π_D *in* p/kWh

Marginal generation cost = **inverse supply function:** $\pi_S = 0.015Q_S + 1.3,\ \pi_S\ in\ p/kWh$

Equilibrium is located at the point $\pi_S = \pi_D \rightarrow Q^* = 328kWh,\ \pi^* = 6.22\frac{p}{kWh}$, with

-consumer surplus equal to the area A+B+E i.e. $0.5 \cdot 328(9.5 - 6.22) = 537.92\,p$

-producer surplus equal to the area C+D+F i.e. $0.5(6.22 - 1.3)328 = 806.88p$

-total social welfare equal to $806.88 + 537.92 = 1344.8\,p$

In this case, the market does not operate at equilibrium because $\pi_D = 6.8\frac{p}{kWh}$. For this price, the demand is 270kWh and the supply -theoretically- is 366.67kWh. However because electricity supply must equal demand, the supply is 270kWh as well. If it was a different commodity (e.g. apples), then the extra capacity could be bought by the government and stored for future potential situations of shortage. But this cannot be done with electricity.

So the **revenue of the suppliers** is equal to the **demand charges** i.e. $6.8\frac{p}{kWh} \cdot 270kWh = 1{,}836\,p$.
The **consumer surplus** is the area A i.e. $0.5 \cdot (9.5 - 6.8) \cdot 270 = 364.5p$
The **producer surplus** is the area B+C+D i.e. $(6.8 - 5.35)270 + 0.5(5.35 - 1.3)270 = 938.25p$
The **social welfare** is their sum i.e. $1302.75p$
This is a case when for example the state intervenes and sets a price floor in order to help the suppliers by allowing them to sell at a higher –than the equilibrium- price. This policy however results in a reduction of the social welfare by 1344.8-1302.75=42.05p

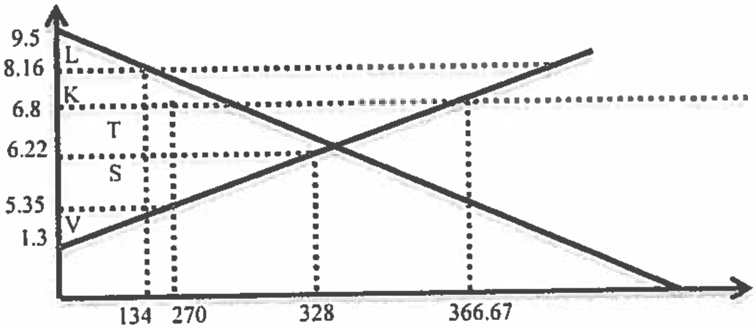
(ii) A 20% increase in price yields a new price of $6.8 \cdot 1.2 = 8.16\frac{p}{kWh}$, where the demand becomes 134kWh.

In this case the **consumer surplus** is $(9.5 - 8.16)134 = 179.56$ (area L) while the **producer surplus** is equal to the area K+T+S+V i.e. $(8.16 - 5.35)134 + 0.5(5.35 - 1.3)134 = 647.89$p

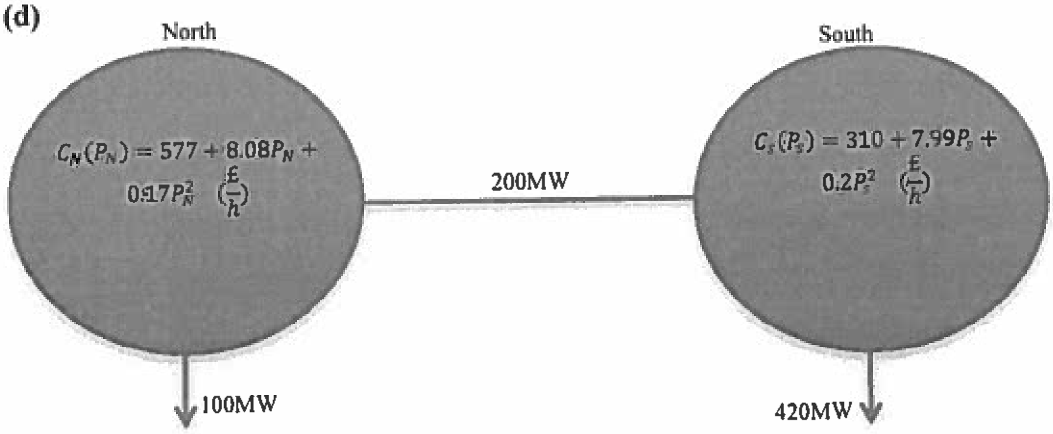
It can be observed that **although the price increased by 20%, the producer surplus decreased by** $\frac{938.25-647.89}{938.25} = 30.95\%$ and the consumer surplus reduced as anticipated.

Compared to the equilibrium state (price $6.22\frac{p}{kWh}$), although the price increased by $\frac{8.16-6.22}{6.22} = 31.19\%$, the producer surplus reduced by $\frac{806.88-647.89}{806.88} = 19.7\%$, and the consumer surplus fell as anticipated.

Hence, it would not be worth for the producer to increase the price by20%.



(d)



First we solve the unconstrained problem:

$$minimize\ C_N(P_N) + C_s(P_s)\ s.t.\ P_N + P_s = 520MW$$
$$L = C_N(P_N) + C_s(P_s) - \lambda(P_N + P_s - 520)$$
$$\frac{\partial L}{\partial P_N} = 0 <=> 8.08 + 0.34P_N = \lambda \rightarrow P_N = \frac{\lambda - 8.08}{0.34}$$
$$\frac{\partial L}{\partial P_S} = 0 <=> 7.99 + 0.4P_s = \lambda \rightarrow P_S = \frac{\lambda - 7.99}{0.4}$$
$$\frac{\lambda - 8.08}{0.34} + \frac{\lambda - 7.99}{0.4} = 520 \rightarrow P_N = 280.95MW, P_S = 239.05MW, \lambda = 103.6\frac{£}{MWh}$$

The **generator payments** are: $R_N = 280.95 \cdot 103.6 = 29{,}106.42£$ and $R_S = 239.05 \cdot 103.6 = 24{,}765.58£$.

The **demand charges** are: $D_N = 100 \cdot 103.6 = 10{,}360$, $D_S = 420 \cdot 103.6 = 43{,}512\ £$ i.e. in the unconstrained version, the flow from northern to southern is 280.95-100=180.95MW<200MW, thus the limit of 200MW is not binding.

In other words, the southern load is satisfied by the flow of 180.95MW that comes from the northern area, plus the southern generation of 239.05MW.
Thus, the marginal prices are equal at both sides, which means that the **cost of constraints** is zero, the **short-run marginal value of transmission** (i.e. the transmission

demand function i.e. the price users are willing to pay) is zero ($\pi_N - \pi_S = 0$), and the **congestion surplus** is also zero.

(e) The students are expected to discuss the following points:

Electricity prices may vary per location as generators of varying costs are located in different locations and the transmission network capacity is limited, implying that it is not always possible to use the cheapest available energy.

Financial Transmission Rights deal with shortfalls in contracts for differences in the presence of network congestion. They are defined between any two nodes in the network and entitle their holders to a revenue equal to the product of the amount of transmission rights bought and the price differential between the two nodes. This amount is exactly what is needed to ensure that a contract for difference can be settled. In conclusion, FTRs completely isolate their holders from the risk associated with congestion in the transmission network. They provide a perfect hedge against variations in nodal prices.

The transmission network business should be regulated because its nature is physically monopolistic (there is only one transmission network-it is not economically viable to construct multiple transmission networks). The regulators fulfil their responsibilities by determining the maximum revenue of the transmission network company and certain quality of supply standards the company has to respect.

Solution to Question 2

(a) Minimize $C_A(P_A) + C_B(P_B) + C_C(P_c)$

s.t.:

$P_A+P_B+P_C=350$

→Lagrange function $L= C_A(P_A) + C_B(P_B) + C_C(P_c) - \lambda(P_A+P_B+P_C-350)$

$\theta L/\theta P_A=0 \Leftrightarrow 1.4+0.08P_A - \lambda=0$

$\theta L/\theta P_B=0 \Leftrightarrow 1.6+0.1P_B - \lambda=0$

$\theta L/\theta P_C=0 \Leftrightarrow 1.8+0.004P_C - \lambda=0$

$P_A+P_B+P_C=350 \Leftrightarrow \lambda=9.021$ \$/h, **P_A= 95.26 MW, P_B=74.21MW, P_C= 180.53 MW**

So the total hourly cost is $C_A(P_A) + C_B(P_B) + C_C(P_c) = 1{,}927.2$ \$/h

(b) Borduria produces energy and the marginal cost of production for each unit is:

$MC_A= 1.4+0.08P_A$

$MC_B= 1.6+0.1P_B$

$MC_C= 1.8+0.04P_C$

It would produce up to the point where the marginal cost of each unit is 8.20 \$/MWh. The rest will be bought from the market at 8.20\$/MWh.

$1.4+0.08P_A = 8.2$ → **P_A= 85MW**

$1.6+0.1P_B = 8.2$ → **P_B= 66 MW**

$1.8+0.04P_C = 8.2$ → **P_C=160 MW**

So the rest 350 – 85-66-160 = 39 MW will be bought from the spot market.

The total minimum cost is $C_1(P_A) + C_2(P_B) + C_3(P_c) = 1911.2$ \$/h

Another way to solve this would be to do the following:

Minimize $C_A(P_A) + C_B(P_B) + C_C(P_c) + B \cdot x$ where B is the amount bought and x the price.

s.t.:

$P_A+P_B+P_C + B =350$

→Lagrange function $L= C_A(P_A) + C_B(P_B) + C_C(P_c) + B \cdot x \ - \lambda(P_A+P_B+P_C+B-350)$

$\theta L/\theta P_A=0$

$\theta L/\theta P_B=0$

$\theta L/\theta P_C=0$

$\theta L/B=0$

$P_A+P_B+P_C + B =350$

The above equations give the same result.

(c) It will produce up to the point where the marginal cost of each unit is 10.2 \$/MWh. The rest will not be produced.

$1.4+0.08P_A = 10.2$ → **P_A= 110 MW**

$1.6+0.1P_B = 10.2$ → **P_B= 86 MW**

$1.8+0.04P_C = 8.2 \rightarrow$ **P_C=210 MW**

So the rest 210+110+86-350 = 56 MW will be sold to the spot market.
The logic is that, if it produces 350MW,the MC was found 9.021$/MWh previously. But given the fact that it can sell it at 10.2£/MWh, this means that it can continue on producing (the MC increases with production) and stop at that point.

The load is supplied at a price 10.2$/MWh so the profits will be:
$350\cdot 10.2 + 56\cdot 10.2 -(15 + 1.4\cdot 110+0.04\cdot 110^2) - (25 + 1.6\cdot 86 + 0.05\cdot 86^2) - (20 + 1.8\cdot 210 + 0.02\cdot 210^2)$ =**1,675.4 $/h.**

To only supply the load, we found from (a) that P_A= 95.26 MW, P_B=74.21MW, P_C= 180.53 MW. So to sell to the market, each unit produces more as follows:
$a=\Delta P_A$= 110 - 95.26, $b=\Delta P_B$=86-74.21 , $c=\Delta P_C$=210-180.53.
So the profit only from selling to the market becomes:
$56\cdot 10.2 -(15 + 1.4\cdot a+0.04\cdot a^2) - (25 + 1.6\cdot b + 0.05\cdot b^2) - (20 + 1.8\cdot c + 0.02\cdot c^2)$= **385.6435 $/h**

This problem could have been formulated as follows:
Minimize $C_A(P_A) + C_B(P_B) +C_C(P_c) - B\cdot 10.2$ where B is the amount sold.
s.t.:
$P_A+P_B+P_C =350 + B$
$\rightarrow$Lagrange function $L= C_A(P_A) + C_B(P_B) +C_C(P_c) + B*10.2 -\lambda(P_A+P_B+P_C+B-350)$
$\theta L/\theta P_A=0$
$\theta L/\theta P_B=0$
$\theta L/\theta P_C=0$
$\theta L/B=0$
$P_A+P_B+P_C =350 + B$
The above equations give the same result.

Cost of producing now the 406 MW
$15 + 1.4\cdot 110+0.04\cdot 110^2 + (25 + 1.6\cdot 86 + 0.05\cdot 86^2) + (20 + 1.8\cdot 210 + 0.02\cdot 210^2)$ =2465.4$/h
//since 110+86+210= 406= 350+56.

In (a) , where there was no cost of supplying to the market, the total hourly cost is $C_A(P_A) + C_B(P_B) +C_C(P_c)$ = 1,927.2 $/h
The difference is : 2465.4 - 1927.22 = 538.19$/h.
The profit should be 571.2(56*10.2) - 538.19 = 33.01$/h.

(d) In this case we want to minimize the production from unit C. So we maximize the production from the other two units to the highest level.

P_A= 100 MW
P_B= 80 MW
P_C=210 MW.

It will not produce more as the MC will grow higher than 10.2 \$/MWh.
So the rest 210+100+80-350 = 40 MW will be sold to the spot market.

So the profit only from selling to the market becomes:
$(a=\Delta P_A= 100 - 95.26,\ b=\Delta P_B=80- 74.21\ ,\ c=\Delta P_C=210-180.53)$
$40\cdot 10.2 - (15 + 1.4\cdot a+0.04\cdot a^2) - (25 + 1.6\cdot b + 0.05\cdot b^2) - (20 + 1.8\cdot c + 0.02\cdot c^2)=$ **259.10 \$/h**

The total profit is: $350\cdot 10.2 + 40\cdot 10.2 -(15 + 1.4\cdot 100+0.04\cdot 100^2) - (25 + 1.6\cdot 80 + 0.05\cdot 80^2) - (20 + 1.8\cdot 210 + 0.02\cdot 210^2) =$ **1670 \$/h**

Solution to Question 3

(a) Under perfect competition, the price is equal to the marginal cost of each generator. Therefore, the following two equations hold:

$$\frac{\partial C_A(P_A)}{\partial P_A} = \pi \tag{1}$$

$$\frac{\partial C_B(P_B)}{\partial P_B} = \pi \tag{2}$$

Since $\pi = 210 - 1.3 \cdot D$ and $D = P_A + P_B$ (demand-supply balance condition), equations (1) and (2) yield respectively:

$$25 + 0.8 \cdot P_A = 210 - 1.3 \cdot (P_A + P_B) \rightarrow P_A = 88.1 - 0.62 \cdot P_B \tag{3}$$

$$27 + 0.6 \cdot P_B = 210 - 1.3 \cdot (P_A + P_B) \rightarrow P_B = 96.32 - 0.68 \cdot P_A \tag{4}$$

The combination of (3) and (4) yields:

$$P_A = 49.15MW \tag{5}$$

$$P_B = 62.82MW \tag{6}$$

The total demand is given by:

$$D = P_A + P_B \rightarrow D = 111.97MW \tag{7}$$

The electricity price is given by:

$$\pi = 210 - 1.3 \cdot D \rightarrow \pi = 64.44\frac{£}{MWh} \tag{8}$$

The profits made by the two generators are given by:

$$\Omega_A = P_A \cdot \pi - 25 \cdot P_A - 0.4 \cdot P_A^2 \rightarrow \Omega_A = 972.19\frac{£}{h} \tag{9}$$

$$\Omega_B = P_B \cdot \pi - 27 \cdot P_B - 0.3 \cdot P_B^2 \rightarrow \Omega_B = 1168.07\frac{£}{h} \tag{10}$$

(b) Models used for the analysis of markets with imperfect competition include:

- **Bertrand model**, where the decision variable of each of the competing firms is the price at which it offers the produced commodity
- **Cournot model**, where the decision variable of each of the competing firms is the quantity of the commodity they produce
- **Supply functions equilibria model**, where the decision variables of each of the competing firms are the parameters of its supply function

- **Agent-based simulation models**, representing more complex interactions between the competing firms

(c) The profits of the two generators are expressed as:

$$\Omega_A = P_A \cdot \pi - C_A(P_A) \tag{11}$$

$$\Omega_B = P_B \cdot \pi - C_B(P_B) \tag{12}$$

Since $\pi = 210 - 1.3 \cdot D$ and $D = P_A + P_B$ (demand-supply balance condition), equations (11) and (12) yield respectively:

$$\Omega_A = 185 \cdot P_A - 1.7 \cdot P_A^2 - 1.3 \cdot P_A \cdot P_B \tag{13}$$

$$\Omega_B = 183 \cdot P_B - 1.6 \cdot P_B^2 - 1.3 \cdot P_A \cdot P_B \tag{14}$$

The optimality conditions for the evaluation of the exact equilibrium point are:

$$\frac{\partial \Omega_A}{\partial P_A} = 0 \rightarrow P_A = 54.41 - 0.38 \cdot P_B \tag{15}$$

$$\frac{\partial \Omega_B}{\partial P_B} = 0 \rightarrow P_B = 57.19 - 0.41 \cdot P_A \tag{16}$$

The combination of (15) and (16) yields:

$$P_A = 38.67MW \tag{17}$$

$$P_B = 41.43MW \tag{18}$$

The total demand is given by:

$$D = P_A + P_B \rightarrow D = 80.1MW \tag{19}$$

The electricity price is given by:

$$\pi = 210 - 1.3 \cdot D \rightarrow \pi = 105.87 \frac{£}{MWh} \tag{20}$$

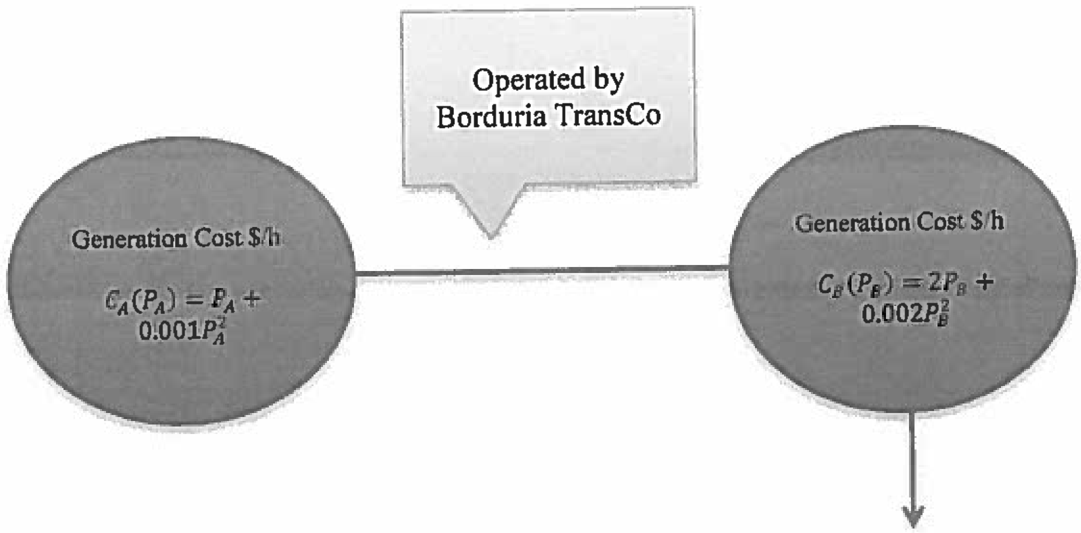
The profits made by the two generators are given by:

$$\Omega_A = P_A \cdot \pi - 25 \cdot P_A - 0.4 \cdot P_A^2 \rightarrow \Omega_A = 2529.09 \frac{£}{h} \tag{21}$$

$$\Omega_B = P_B \cdot \pi - 27 \cdot P_B - 0.3 \cdot P_B^2 \rightarrow \Omega_B = 2752.65 \frac{£}{h} \tag{22}$$

We can observe that under imperfect competition the individual generation levels and the total demand are significantly lower and the price and generation profits are significantly higher with respect to the perfect competition case.

Solution to Question 4



(a) We neglect the transmission line constraint.

__Winter__

To find the optimal levels of generation we need to solve the problem:

Minimize $C_A(P_A) + C_B(P_B)$ *subject to* $P_A + P_B = 3500$

$L = C_A(P_A) + C_B(P_B) - \lambda(P_A + P_B - 3500)$

$\frac{\partial L}{\partial P_A} = 0 \rightarrow P_A = \frac{\lambda-1}{0.002}$

$\frac{\partial L}{\partial P_B} = 0 \rightarrow P_B = \frac{\lambda-2}{0.004}$

$P_A + P_B = 3500 \rightarrow \lambda = 6\frac{\$}{MWh} = MC_A = MC_B, P_A = 2500MW, P_B = 1000MW$

__Summer__

To find the optimal levels of generation we need to solve the problem:

Minimize $C_A(P_A) + C_B(P_B)$ *subject to* $P_A + P_B = 2000$

$L = C_A(P_A) + C_B(P_B) - \lambda(P_A + P_B - 2000)$

$\frac{\partial L}{\partial P_A} = 0 \rightarrow P_A = \frac{\lambda-1}{0.002}$

$\frac{\partial L}{\partial P_B} = 0 \rightarrow P_B = \frac{\lambda-2}{0.004}$

$P_A + P_B = 2000 \rightarrow \lambda = 4\frac{\$}{MWh} = MC_A = MC_B, P_A = 1500MW, P_B = 500MW$

(b) We take into account the transmission line constraint of 1200MW.

Winter

The optimal dispatch becomes: $P_A = 1200MW,\ P_B = 2300MW$ and the prices are $MC_A = 3.4\frac{\$}{MWh} = PriceA,\ MC_B = 11.2\frac{\$}{MWh} = PriceB$

$C_A = 2{,}640\frac{\$}{h},\ C_B = 15{,}180\frac{\$}{h}$

Summer

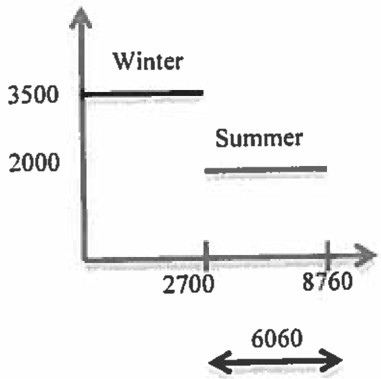
The optimal dispatch becomes : $P_A = 1200MW,\ P_B = 800MW$ and the prices are $MC_A = 3.4\frac{\$}{MWh} = PriceA,\ MC_B = 5.2\frac{\$}{MWh} = PriceB$

$C_A = 2{,}640\frac{\$}{h},\ C_B = 2{,}880\frac{\$}{h}$

(c)

(i)

We take into account the transmission line constraint of 1200MW.



The annual generation costs are:

$C_B^{winter} = 15{,}180\frac{\$}{h}\cdot 2700h = 40{,}986{,}000\ \$$

$C_B^{summer} = 2{,}880\frac{\$}{h}\cdot 6060h = 17{,}452{,}800\ \$$

$C_A^{winter} = 2{,}640\frac{\$}{h}\cdot 2700h = 7{,}128{,}000\ \$$

$C_A^{summer} = 2{,}640\frac{\$}{h}\cdot 6060h = 15{,}998{,}400\ \$$

$C_{A+B}^{annual} = 81{,}565{,}200\ \$$

(ii) The electricity price in winter is 11.2 \$/MWh and in summer is 5.2 \$/MWh.

The annual demand charges are:

$$3500MW\cdot 2700h\cdot 11.2\frac{\$}{MWh} + 2000MW\cdot 6060h\cdot 5.2\frac{\$}{MWh} = 168{,}864{,}000\ \$$$

The annual generation revenues are:

$R_B^{winter} = 2{,}300MW\cdot 11.2\frac{\$}{MWh}\cdot 2700h = 69{,}552{,}000\ \$$

$$R_B^{summer} = 800MW \cdot 5.2\frac{\$}{MWh} \cdot 6060h = 25{,}209{,}600\ \$$$
$$R_A^{winter} = 1200MW \cdot 3.4\frac{\$}{MWh} \cdot 2700h = 11{,}016{,}000\ \$$$
$$R_A^{summer} = 1200MW \cdot 3.4\frac{\$}{MWh} \cdot 6060h = 24{,}724{,}800\ \$$$
$$R_{A+B}^{annual} = 130{,}502{,}400\ \$$$

(iii) The TransCo's total annual revenue is

$$R_{Tra} = 1200MW(11.2 - 3.4) \cdot 2700h + 1200 \cdot (5.2 - 3.4)6060 = 38{,}361{,}600\$$$

We can see that it is equal to the difference between annual demand charges and annual generation revenues.

(d) After the reinforcement, Borduria TransCo's total annual revenue will be:

$$R_{Tra} = 2100MW \cdot (7.6 - 5.2) \cdot 2700h = 13{,}608{,}000\ \$$$
The reinforcement will cost 5,000,000 \$.

Also, total annual demand charges:
$3500 \cdot 2700 \cdot 7.6 + 2000 \cdot 6060 \cdot 4 = 120{,}300{,}000\ \$$ i.e. a reduction of $48,564,000.

In addition, the total annual generation costs will be
$3750 \cdot 6060 + 1500 \cdot 6060 + 6510 \cdot 2700 + 6720 \cdot 2700 = 67{,}536{,}000\$$ i.e. a reduction of 14,029,200$

This investment will benefit all parties in the system: The total generation cost will reduce (i.e. constraints cost will reduce), and total annual demand charges will also reduce.

However, still the total annual revenue of TransCo is higher than its total annuitized investment cost, signifying a degree of underinvestment, Therefore Borduria Regulator will prefer to reject this investment and instead suggest a greater investment such that neither underinvestment nor overinvestment occurs in the system.