EE4-55
EE9-AO11

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

MSc and EEE PART IV: MEng and ACGI

MEMS AND NANOTECHNOLOGY

Thursday, 19 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer Question 1.
Answer Question 2 OR Question 3.
Answer Question 4 OR Question 5.

Question 1 carries 40% of the marks. Remaining questions carry 30% each.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : Z. Durrani, A.S. Holmes,
Second Marker(s) : A.S. Holmes, Z. Durrani,

Information for Candidates

There is no supplementary information with this paper.

This question is compulsory

1. a) Calculate the minimum wavelength, λ, for a photon emitted from a spherical Si nanocrystal of 5 nm diameter, in the presence of electron-hole recombination. You may use effective masses for electrons and holes in Si of 1 and 0.55, respectively. The rest mass of an electron $m_0 = 0.91\times10^{-30}$ kg, the Planck constant $h = 6.625\times10^{-34}$ Js, the speed of light $c = 3\times10^8$ m/s, and the band-gap in bulk silicon $E_g = 1.1$ eV. [5]

   b) Figure 1.1 shows a double potential well, where two potential wells of width L are separated by a potential barrier of width D and height V_1. Here, the energy within the wells is zero and D is narrow enough such that the wave functions overlap.

      i) Write down the wave vectors k_I, k_{II}, and k_{III}, for the wave functions in regions I, II and III, respectively. [3]

      ii) Sketch, without solving Schrödinger's equation, the wave functions corresponding to the first two energy levels E_1 and E_2. Justify, briefly, your argument for the shapes of the wave functions. [3]

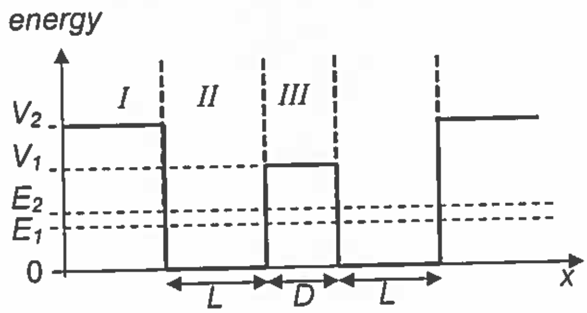

Figure 1.1

   c) Assuming the wave function of an electron of mass m travelling in a potential V is given by $\Psi = A.\exp[i(kx - \omega t)]$, derive the 1-D time-dependent form of Schrödinger's equation. [4]

   d) Using a suitable diagram, explain the operation of a parallel plate reactive-ion etching system. [5]

Question 1 continues on the next page.

Question 1 continued.

e) Outline the main materials and process steps involved in a silicon surface micromachining process. Illustrate your answer by sketching a process flow for a surface micromachined device having one mechanical layer. [5]

f) A silicon resonator is fabricated on a bonded silicon-on-insulator wafer. The thickness of the mechanical layer is 10 μm, and the buried oxide layer is 2 μm thick. The proof mass has an area of 1 mm × 1 mm, and the suspension is designed to have a stiffness of 5 N/m.

Calculate the resonant frequency of the device. Also estimate the Q of the resonator assuming this is dominated by viscous drag in the air gap between the mass and the substrate. How would these parameters change if the linear dimensions of the device were scaled down by a factor of 2? The viscosity of air is 1.8×10^{-5} Ns/m^2, and the density of silicon is 2330 kg/m^3. [5]

g) The equations of motion for a linear vibratory gyroscope are given by:

$$m\ddot{v}_x + c_x\dot{v}_x + k_x v_x - 2m\Omega\dot{v}_y = F_x$$
$$m\ddot{v}_y + c_y\dot{v}_y + k_y v_y + 2m\Omega\dot{v}_x = F_y$$

Explain the physical significance of the various terms in these equations. Also, assuming that the gyroscope is operated in open loop, and that the resonant frequencies are matched, derive an expression for the ratio of sensed to driven amplitudes when the device is subject to rotation at a constant rate. [5]

h) List the main actuation mechanisms used in MEMS devices, and briefly compare them in terms of speed, force and ease of implementation. [5]

End of Question 1.

2. A beam of electrons, travelling along the x-axis in the positive x direction, is incident on the potential well shown in Figure 2.1 below.

The potential energy is given by:

$$E = 0, \text{ for } x < 0 \text{ and } x > L$$
$$E = -V_0, \text{ for } 0 < x < L$$

a) For the electron wave functions, write down the wave vectors k_1 in region I, k_2 in region II, and k_3 in region III. [6]

b) Write down the general forms of the electron wave functions ψ_1 in region I, ψ_2 in region II, and ψ_3 in region III. [6]

c) Hence, by applying the boundary conditions for solutions to Schrödinger's equation, find the amplitude transmission coefficient t for this potential. [10]

d) By inspecting the amplitude transmission coefficient t, find the condition that must be satisfied for there to be no reflected electron wave. [8]

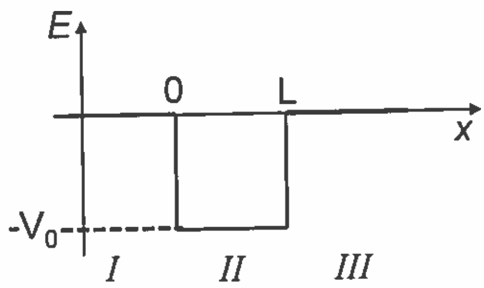

Figure 2.1

3. Figure 3.1 shows an n-channel, depletion mode silicon-on-insulator MOSFET, where the channel is formed by a Si layer of thickness D, lying on a SiO_2 substrate. The Si channel doping concentration is N_D. A top SiO_2 layer of thickness t_{ox} forms the gate oxide.

a) Assuming that 'flat-band' conditions exist at gate voltage $V_g = 0$ V and drain-source voltage $V_{ds} = 0$ V, sketch the energy bands along the line XX' for carrier depletion in the n-channel. Your diagram should show the conduction and valance band edges E_c and E_v, and the intrinsic level E_i, in the Si. The diagram should also show the Fermi energy E_F, and the width of the depletion region W_D. [5]

b) Sketch the charge density per unit volume $\rho(x)$ along the line XX'. [5]

c) Hence, by solving Poisson's equation, $-\frac{\partial^2 V}{\partial x^2} = \frac{\partial F}{\partial x} = \frac{\rho(x)}{\varepsilon_0 \varepsilon_{Si}}$, in the channel region, find the potential V_s at the Si – gate oxide interface, at $x = 0$. Here, ε_0 is the permittivity of free space, ε_{Si} is the relative permittivity of Si, F is the electric field and V is the potential. [14]

d) Using the result of Part c), write down an expression for the threshold voltage V_{th}, in terms of the Si layer thickness D. [6]

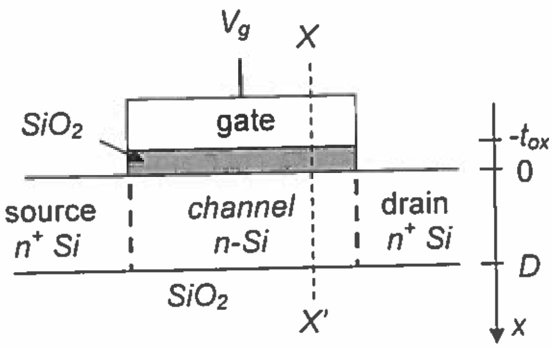

Figure 3.1

4. a) Starting from the elementary bending equation, derive an expression for the transverse stiffness k_T of a surface micromachined flexure. Also obtain an expression for the axial stiffness k_A, and hence show that the ratio of transverse to axial stiffness is:

$$\frac{k_T}{k_A} = \alpha = \left(\frac{w}{l}\right)^2$$

where w is the width of the beam and l is its length. [10]

b) Figure 4.1 shows a v-beam structure consisting of two flexures connected in series with an angle between them. Such a structure can be used as a mechanical amplifier since, depending on the angle θ, a small x-displacement at the free end can lead to a relatively large y-displacement of the central block.

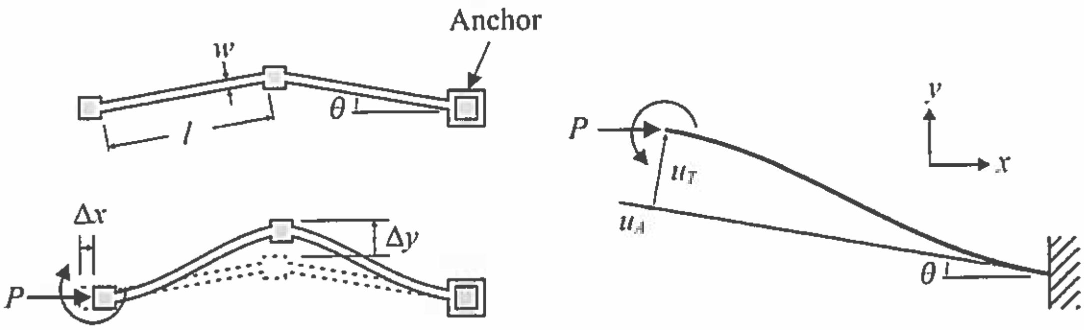

Figure 4.1

Considering the deflected structure, and focusing on the right-hand flexure as detailed on the right of Figure 4.1, derive expressions for the axial and transverse end displacements, u_A and u_T, in terms of the load P and the beam stiffnesses. Also write down the geometrical relationships between u_A, u_T, Δx and Δy, and hence show that the mechanical amplification of the structure may be expressed as:

$$m = \frac{\Delta y}{\Delta x} = \frac{(1-\alpha)}{2(u+\alpha/u)}$$

where $u = \tan(\theta)$ and α is defined as in Part a). [10]

c) i) A v-beam amplifier is used to enhance the sensitivity of an MEMS accelerometer with optical readout. The free end of the v-beam is connected to the proof mass, and the lateral deflection of the central block is read by an optical transducer. The v-beam structure has l = 1.5 mm, w = 20 μm, and θ = 1.5°. If the mechanical resonance frequency of the accelerometer is 12 kHz, and the transducer has a resolution of 1 nm, what will be the minimum detectable static acceleration? [6]

ii) What other approaches could have been employed at the design stage to enhance the sensitivity, assuming the same transducer? Does mechanical amplification offer any advantages over the approaches you suggest? Explain your answer. [4]

5. Figure 5.1 shows the model for an out-of-plane electrostatic actuator comprising a moveable plate on an elastic suspension and a fixed lower plate. The electrode gap at zero applied voltage is g_0, and spacers either side of the lower plate set the minimum gap, s.

a) i) Derive an expression for the total upward force F on the moveable plate, and show that it can be expressed in normalised form as:

$$f = 1 - u - \frac{a}{u^2}$$

where $f = F/(kg_0)$ is the normalised force, $u = g/g_0$ is the normalised gap, and $a = A\varepsilon_0 V^2/(2kg_0^3)$. [6]

ii) Sketch the variation of f with u for several values of a, and hence explain the origin of snap-down instability. Also sketch the variation of gap with applied voltage. [8]

b) State the conditions that apply at the point of snap-down, and hence derive an expression for the critical voltage, V_P, at which snap-down occurs. [4]

c) It is suggested that snap down can be avoided by placing a fixed capacitor C in series with the actuator rather than driving it directly from a voltage source. Show that in this case the normalised force becomes:

$$f = 1 - u - \frac{a}{(u+b)^2}$$

where $b = C_0/C$, with C_0 being the rest capacitance of the actuator at zero applied voltage. By considering the snap-down condition in this case, show that the actuator will not exhibit snap-down provided the series capacitance satisfies:

$$C \leq \frac{C_0}{(2 - 3s/g_0)}$$ [8]

d) If C has the maximum allowed value derived in Part c), and $s \ll g_0$, show that the voltage required to bring the moveable plate to the bottom of its travel is approximately $V = 3\sqrt{3}V_P$. [4]

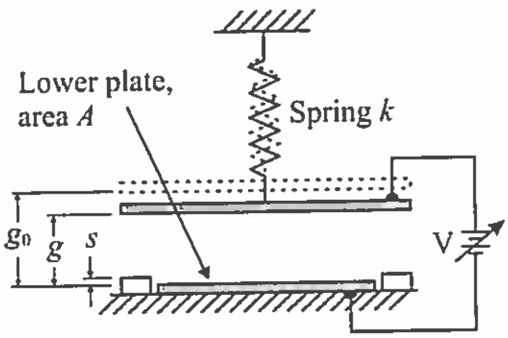

Figure 5.1

