2a i) Choose $f(x) = (1+x)^{1/3}$ or $(1-x)^{\alpha}$ [5]

$f(x) = (1+x)^{1/3}$ $\quad f(0) = 1$

$f'(x) = \frac{1}{3}(1+x)^{-2/3}$ $\quad f'(0) = 1/3$

$f''(x) = -\frac{1}{3}\frac{2}{3}(1+x)^{-5/3}$ $\quad f''(0) = -2/9$ [8]

McL series expansion:

$$f(x) = (1+x)^{1/3} = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + R_2(x)$$

$$= 1 + \frac{x}{3} - \frac{x^2}{9} + R_2(x).$$ [7]

Sub $x = 0.3$

$$\Rightarrow f(0.3) = (1.3)^{1/3} = 1 + \frac{0.3}{3} - \frac{(0.3)^2}{9} + R_2(0.3)$$

$$= 1.09 + R_2(0.3)$$ [5]

ii) $R_2(h) = \frac{h^3}{3!} f'''(0 + \vartheta h)$, $0 \le \vartheta \le 1$ [3]

$R_2(0.3) = \frac{(0.3)^3}{3!} f'''(0.3\vartheta)$ [2]

$f'''(x) = \frac{1}{3}\frac{2}{3}\frac{5}{3}(1+x)^{-8/3}$ [2]

$$R_2(0.3) = \frac{(0.3)^3}{3!}\frac{1}{3}\frac{2}{3}\frac{5}{3}(1 + 0.3\vartheta)^{-8/3}, \quad 0 \le \vartheta \le 1$$

$$\le \frac{(0.3)^3}{3!}\frac{1}{3}\frac{2}{3}\frac{5}{3}$$ since $y = x^{-8/3}$ is $\downarrow$ for the interval and so max for $\vartheta = 0$ [5]

$$= \frac{5}{81}\left(\frac{3}{10}\right)^3 = \frac{1}{600} < 0.0017$$ [3]