IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2009

EEE PART IV: MEng and ACGI

Corrected Copy

MEMS AND NANOTECHNOLOGY

Monday, 18 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer Question 1.
Answer Question 2 OR Question 3.
Answer Question 4 OR Question 5.

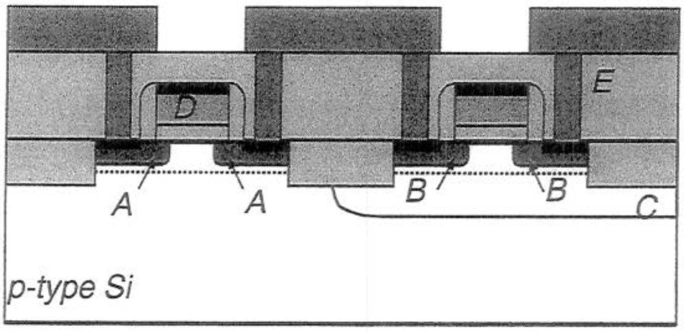
Question 1 carries 40% of the marks. Remaining questions carry 30% each.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	Z. Durrani, A.S. Holmes, Z. Durrani
	Second Marker(s) :	A.S. Holmes, Z. Durrani, A.S. Holmes

This question is compulsory.

1. a) The diagram below shows a cross-section through a CMOS inverter. What are the regions, A, B, C, D, and E?

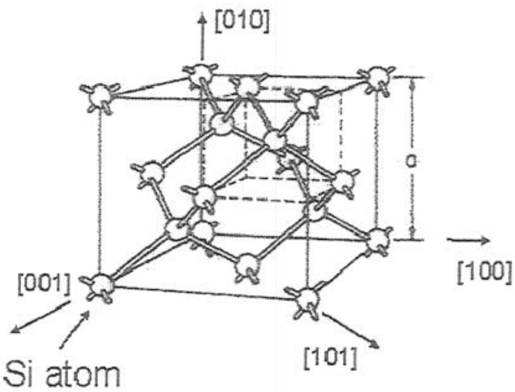


[5]

b) Using a suitable diagram, explain the operation of a parallel plate reactive-ion etching system. [5]

c) Using suitable diagrams, explain the vapour-liquid-solid growth mechanism for a silicon nanowire. [5]

d) The diagram below shows the unit cell in a Si crystal:



Consider the {100} and {110} planes in the unit cell. For planes that include the origin, sketch the arrangement of atoms in the planes. For the intersections of these planes with the unit cell of side a, what are the lengths of the sides of the planes? [5]

Question 1 continues on the next page.

Question 1 continued.

e) Derive an expression for the total force on the moving plate in a pull-down electrostatic actuator with a spring suspension. Explain, with the aid of a diagram, why such an actuator exhibits *snap-down* instability. [6]

f) A square nitride membrane is to be fabricated by anisotropically etching from the back side of a (100) silicon wafer which is coated both sides with silicon nitride. If the membrane is to have an area of 1 mm^2, the silicon wafer is 500 μm thick, and the etchant has an anisotropy of 40, what size of opening should be made in the nitride layer on the back side? [4]

g) Sketch the structure of a typical electrothermal shape bimorph actuator, and explain its operation. Also derive an approximate expression for the tip deflection in terms of appropriate dimensions and the average temperature rises in the different sections. [5]

h) Sketch the layout of a typical silicon membrane pressure sensor with piezo-resistive readout, and briefly explain its operation. [5]

2. Consider a one-dimensional potential well where the potential energy $V(x)$ is given by:

$$V(x) = 0, \quad \text{for } 0 \le x \le L$$
$$V(x) = \infty, \quad \text{for } x < 0 \text{ and } x > L$$

a) Solve the one-dimensional, time independent Schrödinger Equation, $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} - E\psi(x) = 0$, for a particle in the potential well, and derive expressions for the particle energy levels E_n in the well. [10]

b) Derive expressions for the corresponding normalised wave functions ψ_n, and the probability density functions corresponding to ψ_n. [8]

c) Sketch the wave functions, and the corresponding probability density functions, for $n = 1$, 2 and 3. [6]

d) For a potential well of finite depth, where $V(x) = -V_0$, for $0 \le x \le L$, and $V(x) = 0$ elsewhere, sketch only the wave functions, and the corresponding probability density functions, for $n = 1$ and 2. How does the peak value of the probability density for $n = 1$ in this case compare to the peak value in a well with sides of infinite energy? [6]

3. A FET is fabricated on a Si nanowire, as shown in Fig. 3(a). The nanowire is a cylinder of radius R = 25 nm, doped n-type at a density $N_D = 10^{24}$ m^{-3}. The nanowire is surrounded by an oxide shell of thickness t_{ox} = 10 nm. A metal gate (not shown) surrounds the oxide shell completely. Defect states exist on the nanowire surface, with density $N_S = 10^{16}$ m^{-2}, at an energy E_t lying within the Si band gap E_g at the surface (Fig. 3(b)). Electrons from the nanowire are trapped in the surface defect states, creating a surface depletion region (shaded region in Fig. 3(a)) and leaving a conducting core of radius 'r'.

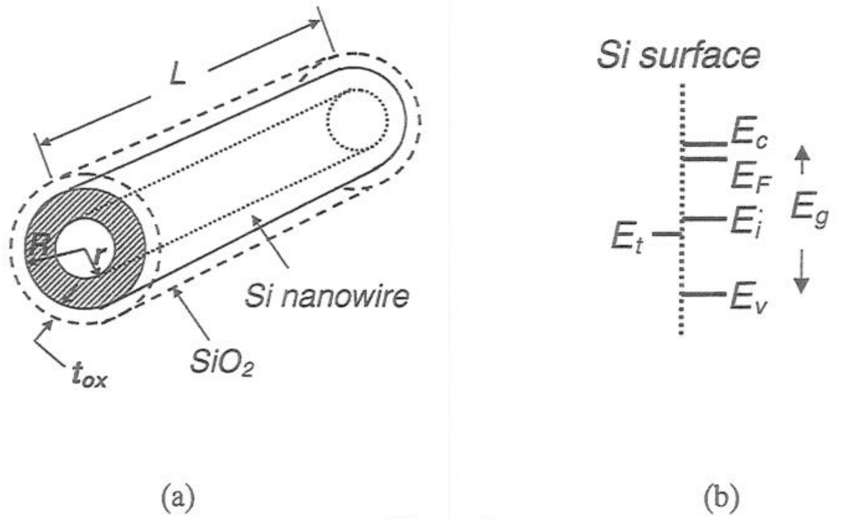


Figure 3

a) Use charge neutrality to calculate the radius 'r'. [10]

b) Calculate the surface potential V_s relative to the axis of the nanowire. You may use the expression $V(x) = \dfrac{eNx^2}{2\varepsilon_r\varepsilon_0}$ for the potential across a one-dimensional, uniformly depleted region, where N is the doping concentration, and ε_r is the dielectric constant of the material. Sketch the band diagram along the nanowire diameter, assuming equal work functions in the metal and nanowire. [6]

c) Estimate the 'flat-band' voltage, V_{FB}. [6]

d) Hence calculate the threshold voltage in the device, V_{th}. [8]

You may use $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m and $e = 1.6 \times 10^{-19}$ C. The dielectric constants for Si and SiO_2 are ε_{si} = 11.9 and ε_{ox} = 3.9 respectively.

4. a) By considering the forces on an elementary section, show that the lateral deflection $v(x, t)$ of an undamped vibrating beam satisfies the wave equation:

$$EI\frac{\partial^4 v}{\partial x^4}+m\frac{\partial^2 v}{\partial t^2}=0$$

where m is the mass per unit length of the beam, and E and I are respectively its Young's modulus and second moment of area. [8]

b) Verify that the following is a general solution of the wave equation in part a):

$$v(x,t)=[A\cos kx+B\sin kx+C\cosh kx+D\sinh kx]\cdot\exp(j\omega t)$$

Also derive the mathematical relationship between ω and k. [6]

c) Explain why the boundary conditions at the free end of a vibrating cantilever are:

$$\frac{\partial^2 v}{\partial x^2}=0\ ;\ \frac{\partial^3 v}{\partial x^3}=0$$

By applying these conditions, along with the boundary conditions at the built-in end, show that k must satisfy the eigenvalue equation:

$$\cos(kL)\cosh(kL)=-1$$

where L is the length of the cantilever. [8]

d) A chemical sensor is fabricated from a silicon cantilever with a selectively absorbing polymer coating of thickness 1.0 μm covering its upper surface. The polymer has a density of 1500 kg/m^3 and its Young's modulus is negligible in comparison to that of silicon. If the cantilever is 10 μm wide, 150 μm long, and 5 μm deep, calculate the frequency of the sensor's lowest order vibrational mode. Also determine the minimum detectable mass of analyte if the electronics can detect a frequency shift of 10 Hz. [8]

Assume values of 113 GPa and 2330 kg/m^3 for the Young's modulus and density of silicon. The smallest positive root of the equation $\cos(x)\cosh(x)=-1$ is $x = 1.875$.

5. Figure 5 shows a silicon micromachined accelerometer with capacitive readout. The comb electrode arrays are not drawn to scale, and in the actual device there are $N = 40$ electrode fingers protruding from the mass. The mechanical layer thickness is $h = 5$ μm, the nominal inter-electrode gap is $g = 1$ μm, and the electrode overlap length is $l = 100$ μm. The proof mass has an area of 0.12 mm^2. The suspension consists of four folded springs in which the flexures all have length 200 μm and width 4 μm.

a) Calculate the suspension stiffness, assuming E = 113 GPa for silicon. [8]

b) Using a simple parallel-plate model, and ignoring parasitics, estimate the capacitance C_A between terminal A and ground when the mass is in its equilibrium position at zero applied acceleration. Would you expect your estimate to be lower or higher than the actual value? Explain your reasoning. [8]

c) Derive a general expression for differential capacitance $C_{diff} = (C_A - C_B)$ as a function of proof mass displacement. Hence determine the magnitude of C_{diff} when the accelerometer is operated in open loop and subject to an acceleration of 5 ms^{-2}. The density of silicon is 2330 kg/m^3. [6]

d) It is decided that the accelerometer should be operated in force-balance mode, with actuation voltages being applied to terminals A and B. Derive a general expression for the electrostatic holding force, and show that when the control loop is working correctly the force may be expressed as:

$$F = \frac{N\varepsilon_0 lh}{g^2} V_0 V_{BA}$$

where V_0 is the common-mode voltage on the two terminals and V_{BA} is the differential voltage between them. If the common mode voltage is fixed at 5 V, what will be the differential voltage at an applied acceleration of 100 ms^{-2}? [8]

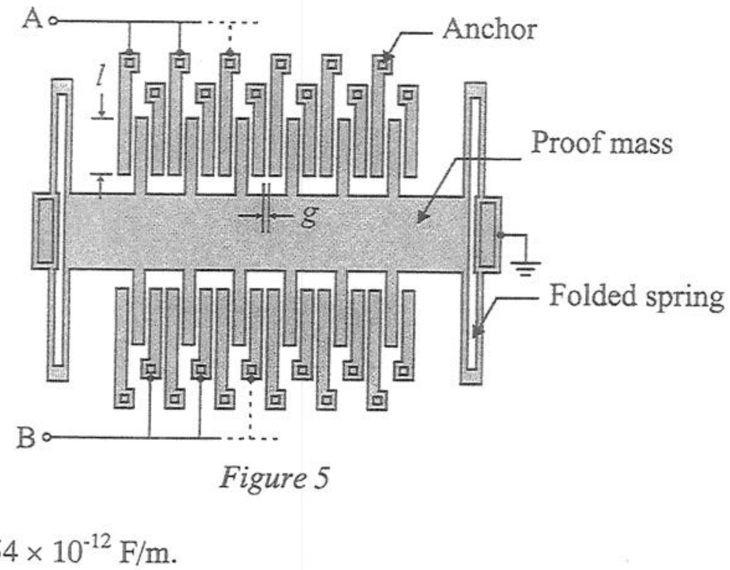
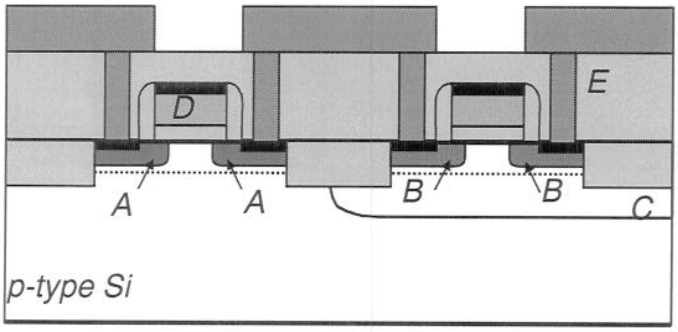


Figure 5

You may use $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

Question 1:

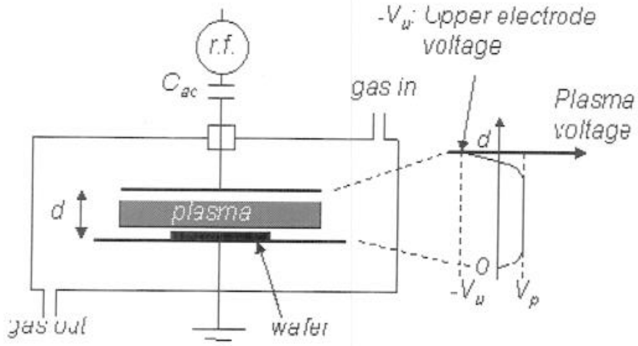
a) The diagram below shows a cross-section through a CMOS inverter. What are the regions, A, B, C, D, and E?



Answer:
A: *n*+ source and drain regions of the *n*-channel MOSFET.
B: *p*+ source and drain regions of the *p*-channel MOSFET.
C: *n*-well, for the fabrication of the *p*-channel MOSFET.
D: Gate, fabricated using metal or heavily-doped polysilicon
E: Field oxide, for contact metallisation support and device isolation. [5]

b) Using a suitable diagram, explain the operation of a parallel plate reactive-ion etching system.

Answer:

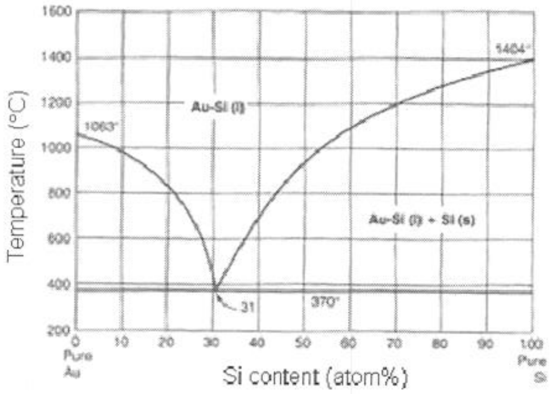


The system uses a glow discharge, i.e. weakly-ionised plasma (>90% neutral particles). An rf. electric field E_{rf} is used to generate the plasma, as follows. Free electrons are accelerated by E_{rf}. These collide with gas atoms, generating more free electrons and ions and increasing the plasma density. However reduction in the number of electrons and ions due to collision with the electrodes lowers the plasma density. The two processes then lead to an equilibrium plasma density. The plasma gas is chosen to generate reactive species, e.g. F or Cl, and these species etch the wafer. Both mechanical (ion sputtering) and chemical etching occurs in the etching process. The ion sputtering is caused by a reduced potential at either plate compared to the plasma potential, associated with charge build-up on the plates. Positive ions then bombard the wafer surface, leading to mechanical, anisotropic etching. [5]

c) Using suitable diagrams, explain the vapour-liquid-solid growth mechanism for a silicon nanowire.

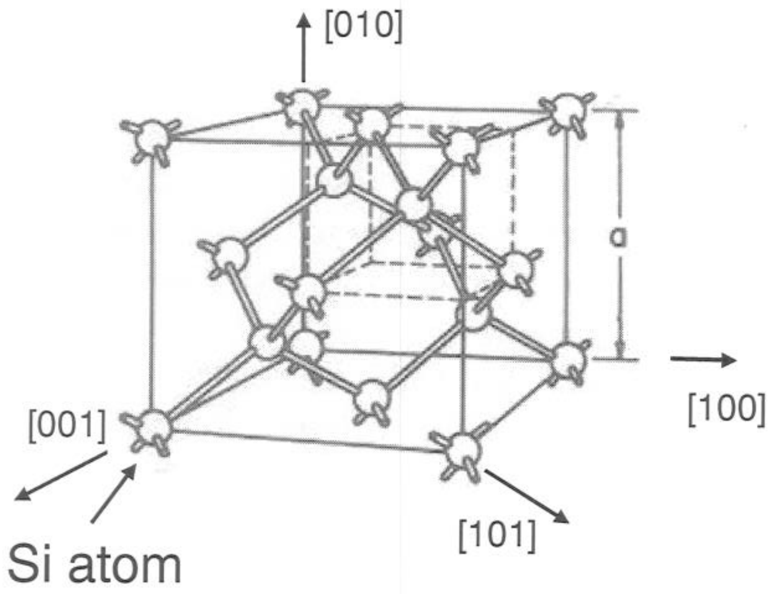
Answer:

The vapour-liquid-solid (VLS) growth mechanism uses metal nanoparticles (typically Au or Fe) as a catalyst to grow Si nanowires. The wires may be grown by CVD, or PECVD, from SiH_4 gas, in the presence of Au nanoparticles on a substrate. The Au-Si phase diagram is shown below (this may be shown schematically):



The phase diagram predicts that Au nanoparticles would form a liquid Au-Si alloy at temperature $T > 363°C$ (Eutectic point). For this temperature range, during the CVD/PECVD process, super-saturation of Si in the nanoparticle alloy leads to precipitation of solid Si and the formation of a solid Si/liquid Si/Au-Si alloy interface. Subsequent growth of a 1-D Si nanowire occurs from this interface. [5]

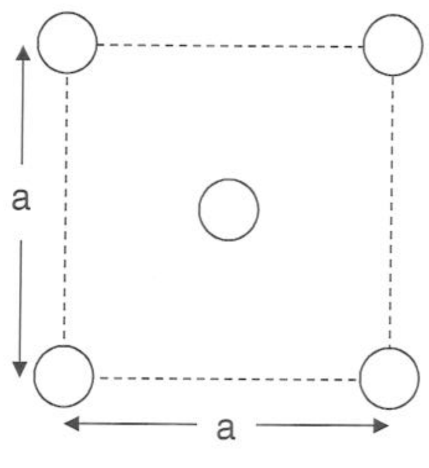
d) The diagram below shows the unit cell in a Si crystal:



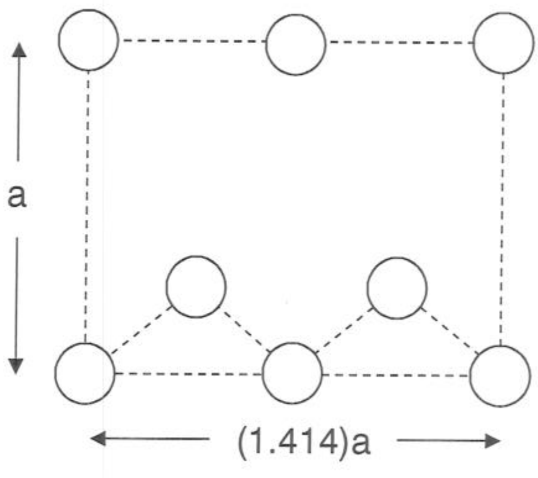
Consider the {100} and {110} planes in the unit cell. For planes that include the origin, sketch the arrangement of atoms in the planes. For the intersections of these planes with the unit cell of side a, what are the lengths of the sides of the planes?

Answer:

{100} plane:



{110} plane:



[5]

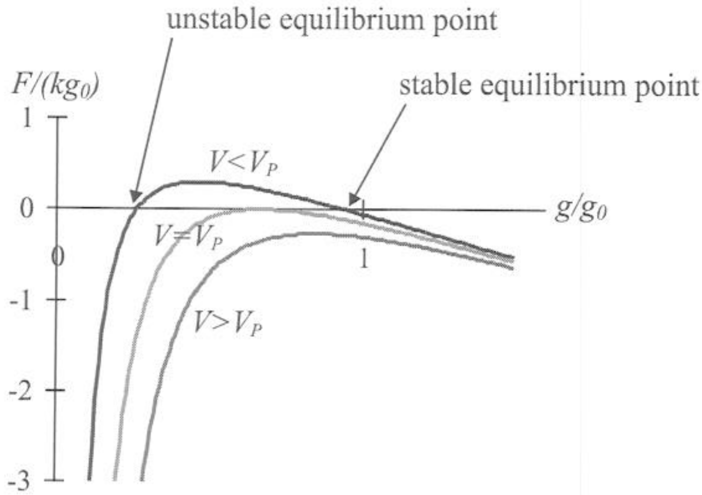
E4.53

e) Derive an expression for the total force on the moving plate in a pull-down electrostatic actuator with a spring suspension. Explain, with the aid of a diagram, why such an actuator exhibits *snap-down* instability.

Answer:

Using virtual work, electrostatic force is $F_e = \partial U / \partial g$, where U is the electrostatic stored in the capacitor, and g is the gap between the plates. U is given by: $U = CV^2/2 = \varepsilon_0 AV^2/(2g)$, so $\partial U/\partial g = F_e = -\varepsilon_0 AV^2/(2g^2)$. The only other force acting on the moveable plate is the spring force $F_k = k(g_0 - g)$, so the total force is:

$$F = F_k + F_e = k(g_0 - g) - \frac{\varepsilon_0 AV^2}{2g^2}$$



When $V < V_P$ there are two equilibrium points (i.e. points where $F = 0$). One is stable, and the other is unstable, but the moveable plate naturally settles at the stable one (and never reaches the unstable one) if the applied voltage is increased from zero. For $V > V_P$ there is no equilibrium point, and the total force is always negative so the plate snaps down.

[6]

f) A square nitride membrane is to be fabricated by anisotropically etching from the back side of a (100) silicon wafer which is coated both sides with silicon nitride. If the membrane is to have an area of 1 mm^2, the silicon wafer is 500 μm thick, and the etchant has an anisotropy of 40, what size of opening should be made in the nitride layer on the back side?

Answer:

The cavity etched from the back side will have {111} sidewalls which will be inclined at an angle of $\tan^{-1}(\sqrt{2})$ to the wafer surface. The opening at the back side will therefore be larger than the membrane, the difference in side length being $\sqrt{2}$D where D is the wafer thickness. Also, the mask on the back side will be undercut by an amount $\sqrt{6}$D/S where S is the anisotropy. The mask opening should therefore be a square with side given by:

$$1 \text{ mm} + \sqrt{2}D - \sqrt{6}D/S = 1.677 \text{ mm}$$

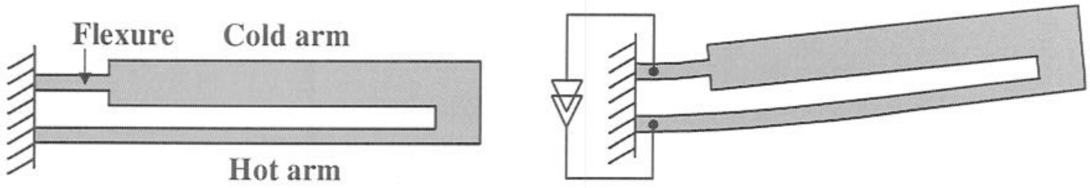
(This assumes that the etch time is just sufficient to go right through the wafer. A smaller aperture could be used with a longer etch time.)

[4]

g) Sketch the structure of a typical electrothermal shape bimorph actuator, and explain its operation. Also derive an approximate expression for the tip deflection in terms of appropriate dimensions and the average temperature rises in the different sections.

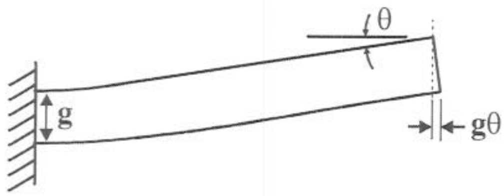
Answer:

Typical structure:



Current passed through structure causes differential heating because cold arm has lower resistance (and hence less heat generated) per unit length, and better heat conduction. The structure deflects to allow differential expansion of the hot and cold arms.

The difference in length between the hot and cold sides is approximately $\alpha(LhTh - LcTc - LfTf)$ where Lh, Lc and Lf are the lengths of the hot arm, cold arm and flexure respectively, and Th, Tc and Tf are the corresponding average temp rises. This must be equal to $g\theta$, where θ is the (small) angular deflection and g is the gap between the beams:



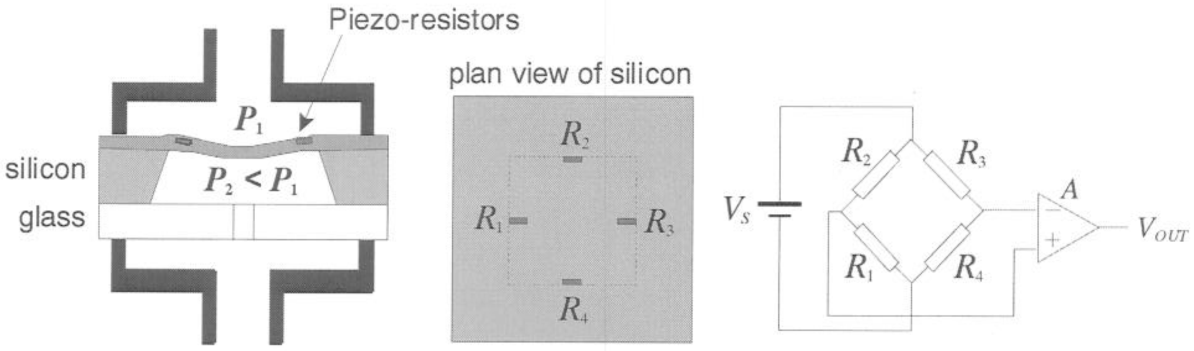
Assuming the flexure bends as a circular arc, and the cold arm remains straight, the tip deflection is then:

$$v = (Lc + Lf/2).\theta \approx \alpha(Lc + Lf/2).(LhTh - LcTc - LfTf)/g$$ [5]

h) Sketch the layout of a typical silicon membrane pressure sensor with piezo-resistive readout, and briefly explain is operation.

Answer:

Typical layout (with readout circuit):

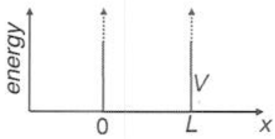


Pressure difference leads to deflection of the membrane which is accompanied by bending stress. Typically piezoresistors are placed at the positions of maximum stress, and connected in a bridge as shown. Stress in R1 and R3 is primarily longitudinal, while stress in R2 and R4 is primarily transverse. This allows for differential measurement. [5]

Answer, question 2:

a) Potential energy given by:

$$\boxed{\begin{array}{l} V(x) = 0,\ \ 0 \le x \le L \\ V(x) = \infty,\ \ x < 0,\ x > L \end{array}}$$



Particle cannot exist ($\psi = 0$) where $V(x) = \infty$, $\Rightarrow$ particle trapped in the well

Within the well, space-dependent Schrödinger equation $\Rightarrow$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} - E\psi(x) = 0$$

General solution is of the form:

$$\psi = A\sin kx + B\cos kx \qquad \text{where} \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

Boundary conditions:

Substituting $\psi = 0$ at $x = 0$ in the general solution $\Rightarrow$

$$B = 0 \quad \Rightarrow \quad \psi = A\sin kx$$

Substituting $\psi = 0$ at $x = L$ in this $\Rightarrow$

$$\psi = A\sin kL = 0 \quad \Rightarrow \quad kL = n\pi \qquad \text{where } n = 0, 1, 2\ldots.$$

This gives the particle energy levels:

$$\boxed{E_n = \frac{\hbar^2 k^2}{2m} = \frac{n^2\pi^2\hbar^2}{2mL^2}} \qquad \text{The energies are quantised}$$

[10]

b) To find the full solution for ψ, we normalise the wave-function:

$$\int_{-\infty}^{+\infty}\psi^2(x)dx = 1 \quad \Rightarrow \quad \int_0^l A^2\sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

$$\Rightarrow A^2\int_0^l \frac{1}{2}\left(1 - \cos\frac{2n\pi x}{L}\right)dx = 1 \quad \Rightarrow A = \sqrt{\frac{2}{L}}$$

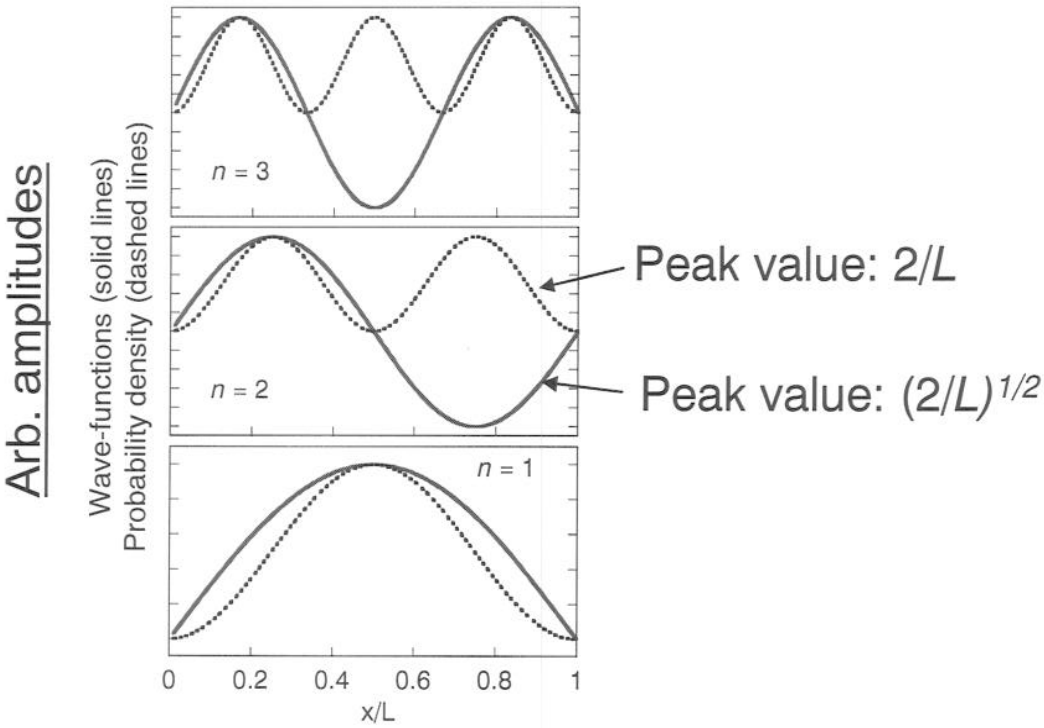
$$\boxed{\Rightarrow \psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}}$$

Probability density functions $\psi_n\psi_n{}^*$ are then given by:

$$\boxed{\psi_n\psi_n{}^* = \frac{2}{L}\sin^2\frac{n\pi x}{L}}$$

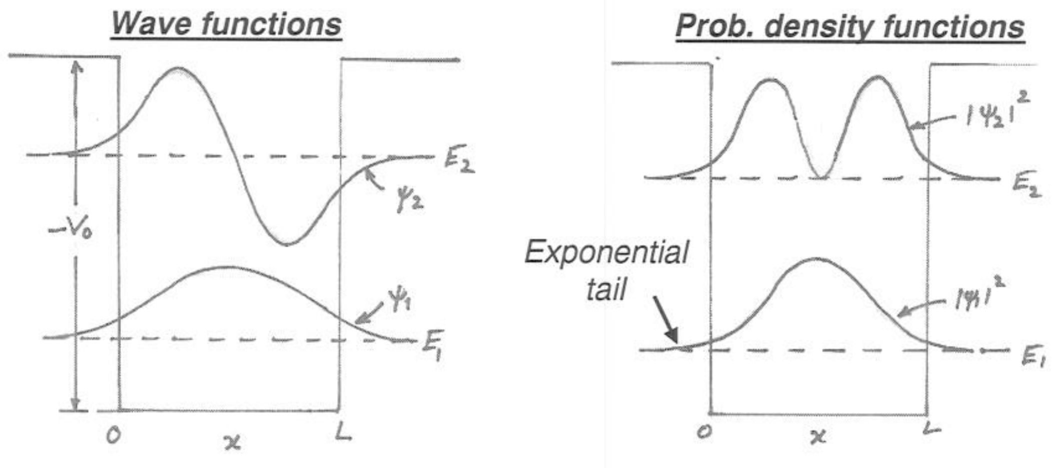
[8]

c) Sketches of the wave functions, and the probability density functions, in the infinite potential well for $n = 1$, 2, and 3 are as follows:



[6]

d) Sketches of the wave functions, and the probability density functions, in the finite potential well for $n = 1$ and 2 are as follows:



The peak value of the probability density function for $n = 1$ in the finite potential well is less than that in the infinite potential well. This is because there is a non-zero, decaying probability that the particle exists outside the well.

[6]

Answer, question 3:

a) Charge neutrality over the entire nanowire $\Rightarrow$

Charge trapped on the surface = charge lost from the interior of the nanowire.

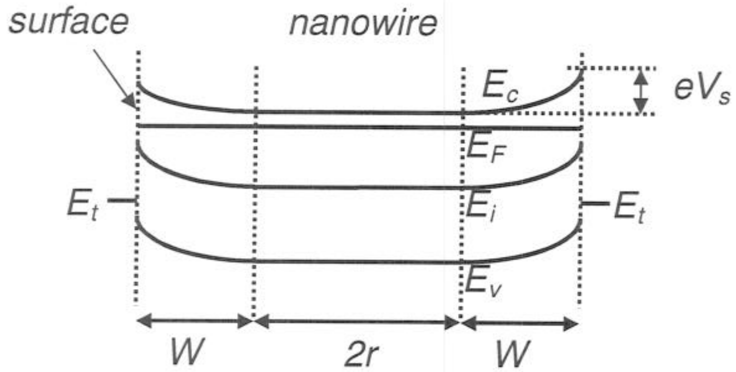
$$\Rightarrow e.N_S.2\pi RL = eN_D.\pi(R^2 - r^2).L$$
$$\Rightarrow N_S.2R = N_D\,(R^2 - r^2)$$
$$\Rightarrow (N_S/N_D)2R = R^2 - r^2$$
$$\Rightarrow r = \sqrt{R^2 - 2R\frac{N_S}{N_D}} = 11.2 \text{ nm}$$

[10]

b) Width of depleted region = $W = R - r$ = 25 nm - 11.2 nm = 13.8 nm

$\therefore$ surface potential $V_s = \dfrac{eN_D W^2}{2\varepsilon_r \varepsilon_0}$ = 0.145 V

Band diagram along the nanowire diameter:



[6]

c) For 'Flat-band' voltage, we need the gate oxide capacitance per unit area, C_{ox}:

$$C_{ox} = \varepsilon_{ox}\varepsilon_0 / t_{ox} = 3.45 \text{ mF/m}^2$$

For a surface charge density Q_s, 'Flat-band' voltage = $V_{FB} = -Q_s/C_{ox} = +eN_S/C_{ox}$

$$\Rightarrow V_{FB} = +0.464 \text{ V}$$

[6]

d) This is a <u>depletion mode</u> device. The threshold voltage V_{th} corresponds to the depletion across the entire nanowire, i.e. for depletion width $W = R$. It is negative in value, as this is necessary to deplete an n-type nanowire. The threshold voltage is then given by:

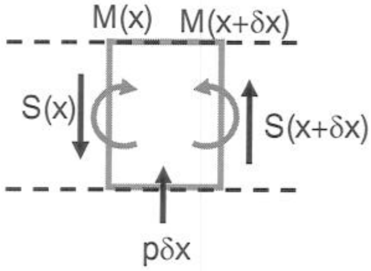
$V_{th} = -|$voltage needed to fully deplete the nanowire – flat band voltage$|$

$= -(eN_D R/C_{ox} - V_{FB}) = -0.695$ V

[8]

Answer, question 4:

a)



Resolving forces: $S(x+\delta x) - S(x) + p\delta x = 0$ which, in the limit $\delta x \rightarrow 0$, leads to $p = -\partial S/\partial x$
Similarly, taking moments gives $S = -\partial M/\partial x$
Combining these with the bending equation $M = EI\partial^2 v/\partial x^2$ we obtain $EI\partial^4 v/\partial x^4 = p$.
If the load p is purely inertial then $p = -m\partial^2 v/\partial t^2$ where m is mass per unit length.
$\Rightarrow$ required result. [8]

b) By direct evaluation, we find that the given form of v satisfies $\partial^4 v/\partial x^4 = k^4 v$ and $\partial^2 v/\partial t^2 = -\omega^2 v$, so it is a solution of the wave equation provided:

$$EIk^4 = m\omega^2$$ [6]

c) $\partial^2 v/\partial x^2$ is proportional to the bending moment, and $\partial^3 v/\partial x^3$ is proportional to the shear force, and since both of these are zero at the free end of a cantilever, the given boundary conditions apply.

The boundary conditions at the built-in end are $v = 0$ and $\partial v/\partial x = 0$. Applying these to the general solution:

$$v = 0 \text{ at } x = 0 \Rightarrow A + C = 0$$
$$v' = 0 \text{ at } x = 0 \Rightarrow B + D = 0$$

And for the free end:

$$v'' = 0 \text{ at } x = L \Rightarrow A(\cos kL + \cosh kL) + B(\sin kL + \sinh kL) = 0$$
$$v''' = 0 \text{ at } x = L \Rightarrow A(\sin kL - \sinh kL) - B(\cos kL + \cosh kL) = 0$$

Eliminating A and B from the last two equations gives the required result. [8]

d) From part b), and the given root of the eigenvalue equation, we know that the frequency of the lowest mode is:

$$f_0 = (1/2\pi)(1.875/L)^2\sqrt{(EI/m)}$$

$I = bd^3/12 = 10 \times 5^3 \times 10^{-24}/12 = 1.042 \times 10^{-22}\ m^4$
$m = 10 \times 10^{-6} \times (5 \times 10^{-6} \times 2330 + 1 \times 10^{-6} \times 1500) = 1.315 \times 10^{-7}$ kg/m
With E = 113 GPa and L = 150 μm this gives f_0 = 235 kHz.

Since $f_0 \propto 1/\sqrt{m}$ we know that $\delta f_0/f_0 = -\frac{1}{2}\,\delta m/m$, and hence that $\delta m = -2m\delta f_0/f_0$
Putting δf_0 = 10 Hz, the minimum detectable mass is $2mL\delta f_0/f_0$ = 1.7 picograms. [8]

Answer question 5:

a) The suspension stiffness is twice that of a single flexure i.e. $k = 24EI/L^3 = 2Ew^3h/L^3$. With $E = 113$ GPa, $w = 4$ μm, $d = 5$ μm and $L = 200$ μm, this gives $k = 9$ N/m. [8]

b) According to parallel plate model, capacitance between adjacent electrode fingers is $\varepsilon_0 lh/g$, so total capacitance between terminal A and ground is $C_A \approx N\varepsilon_0 lh/g$. With $N = 40$, $l = 100$ μm, $h = 5$ μm, $g = 1$ μm this gives $C_A \approx 0.18$ pF.

Fringing will make actual capacitance higher, as will parasitic capacitance through substrate, so estimate is expected to be on low side. [8]

c) Differential capacitance under applied acceleration is:

$$C_A - C_B = N\varepsilon_0 lh\left\{\frac{1}{g+x} - \frac{1}{g-x}\right\} \approx -\frac{2N\varepsilon_0 lh}{g^2}x = -2C_A\frac{x}{g}$$

where x is the displacement of mass relative to substrate (+ve displacement to right). For steady state acceleration a, $x = -ma/k$ where m = proof mass and k = suspension stiffness.

For device in question, m = 2330 x 0.12e-6 x 5e-6 = 1.40e-9 kg, so steady state displacement at acceleration of +5 ms^{-2} is x = -0.78 nm. Corresponding differential capacitance is $C_A - C_B \approx 0.28$ fF. [8]

d) Closure force for single parallel plate actuator (from memory / virtual work / force calc) is $\frac{1}{2}V^2\partial C/\partial g = -A\varepsilon_0 V^2/(2g^2)$. So, general expression for holding force under applied voltages V_A and V_B is:

$$F = -\frac{1}{2}N\varepsilon_0 lh\left\{\frac{V_A^2}{(g+x)^2} - \frac{V_B^2}{(g-x)2}\right\}$$

If control loop is working correctly then x = 0 and this reduces to:

$$F = -\frac{N\varepsilon_0 lh}{2g^2}\left(V_A^2 - V_B^2\right) = \frac{N\varepsilon_0 lh}{g^2}V_0 V_{BA} = \frac{C_A V_0}{g}V_{BA}$$

where $V_0 = (V_A + V_B)/2$ is the common-mode voltage and $V_{BA} = V_B - V_A$ is the differential voltage. For an applied acceleration of 100 ms^{-2}, the required force is $ma = 0.14$ μN, and with $V_0 = 5$ V the required differential voltage is $V_{BA} = 156$ mV. [8]