

① a) Need  $V < 2.405$  CE

$$V = \frac{r_0 k_0}{d_0} \sqrt{n_c^2 - n_s^2} \approx \frac{r_0 2\pi}{d_0} \sqrt{2n \Delta n}$$

$$2.405 = \left( \frac{3 \mu\text{m} (2\pi)}{1.502 \mu\text{m}} \right) \sqrt{2 \times 3 \times \Delta n}$$

gives  $\Delta n \approx 0.013$

b) Since the V number is proportional to  $\sqrt{n_c^2 - n_s^2}$ , raising the  $\Delta n$  will increase the number of supported modes BW

c) 1600 - 2000 nm is at the long wavelength side of the attenuation minimum, in this region interatomic vibrations dominate the loss, esp Si-O stretching vibration. BW

d) For an LED we estimate  $\Delta E$  to be  $\sim 2kT$ . At room temp this is  $\sim 50 \text{ meV}$ . The minimum energy will be  $E_g \approx 1.24 \text{ eV} \cdot \mu\text{m} / 1.31 \mu\text{m} = 0.947 \text{ eV}$  CE

Then  $\lambda_{\text{min}} \approx 1.24 \text{ eV} \cdot \mu\text{m} / (0.947 \pm 0.05) = 1.244 \mu\text{m}$

$\Delta \lambda \approx 1310 - 1244 = \underline{66 \text{ nm}}$

e) Long haul systems work at  $\sim 1310$  or  $1550 \text{ nm}$  wavelength. Si, with a bandgap of 1.1 eV, is transparent at those wavelengths BW

CE = computed example  
BW = backwork  
NT = new theory  
TA = theor. application.

①

f) 40 Gbit/s (purely memory this one!) BW  
 Limited by electronics & dispersion.

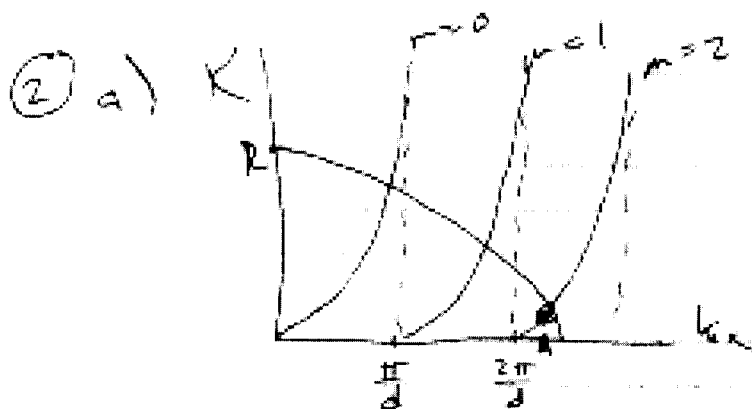
g) DFB lasers use gratings as the reflective elements to form the resonant cavity, which are  $\lambda$  selective, and so can have a much narrower spectrum than F-P lasers. BW

h) The maximum slope efficiency ( $\eta = 1$ ) for a laser is  $\frac{hc}{e\lambda}$ . In this case CE

$$S_{max} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 0.78 \times 10^{-6}} = 1.59 \text{ W/A}$$

i) The pump laser  $\lambda$  must be  $< \lambda_{signal}$  must correspond to a strong absorption band for Er that results in an efficient transition to the metastable state, and must be manufacturable as a high power laser diode. BW

j) Conservation of power applies. Input CE  
 -10 dBm = 0.1 mW. First output port,  
 -20 dBm = 0.01 mW, leaving 0.09 mW  
 on the other output, = -10.46 dBm.



TA

$$\frac{\pi}{d} = 0.5236 \times 10^6 \text{ m}^{-1}$$

$$k_{ix} = 1.19 \times 10^6 \text{ m}^{-1} \text{ is slightly } > 2\pi/d$$

$$\therefore m = 2$$

$$k_{ixd} = \frac{3\pi}{2} \quad K = k_{ix} \text{ backward} = 0.542 \times 10^6 \text{ m}^{-1}$$

$$\sqrt{k_{ix}^2 + K^2} = 1.308 \times 10^6 = R < 3\pi/d$$

Therefore  $m = 2$  is the highest mode

There are 3 modes,  $m = 0, 1, 2$ .

b)  $n' = \frac{c}{v_p} = \frac{3}{2.027} = 1.480$

TA

$$k_{ix}^2 + \beta^2 = n_c^2 k_0^2$$

$$(k_{ix}/k_0)^2 + n'^2 = n_c^2$$

$$n_c = \sqrt{1.480^2 + \left(\frac{1.19 \times 10^6}{2\pi}\right)^2} = 1.507$$

c)  $\frac{k_{ixd}}{2} = X \quad \frac{K_d}{2} = Y$

TA

$$X^2 + Y^2 = (Rd/2)^2$$

$$Y^2 = X^2 \tan^2 X$$

$$X^2 (1 + \tan^2 X) = \frac{X^2}{\cos^2 X} = (Rd/2)^2$$

$$X = \frac{Rd}{2} \cos X = \left(\frac{1.308 \times 6}{2}\right) \cos X = 3.924 \cos X$$

From diagram we estimate that the  $m=0$  solution has  $\frac{k_{ixd}}{2} \approx \frac{\pi}{2}$ . With this starting point,

$$\text{trial and error gives } X = 1.247$$

$$k_{ix} = 2X/d = 0.4156 \times 10^6 \text{ m}^{-1}$$

$$n' = \sqrt{n_c^2 - \left(\frac{k_{ix}}{k_0}\right)^2} = 1.504$$

3)

$$a) Z_g = \frac{L}{c} (n - \lambda \frac{dn}{d\lambda}) \quad NT$$

We can estimate  $\Delta Z_g$  in this case using 2 terms of the series expansion:

$$\Delta Z_g = \frac{dZ_g}{d\lambda} \Delta \lambda + \frac{d^2 Z_g}{d\lambda^2} \frac{\Delta \lambda^2}{2}$$

$$\frac{dZ_g}{d\lambda} = \frac{L}{c} \left( \frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda \frac{d^2 n}{d\lambda^2} \right) = -\frac{L}{c} \left( \lambda \frac{d^2 n}{d\lambda^2} \right)$$

$$\frac{d^2 Z_g}{d\lambda^2} = -\frac{L}{c} \left( \frac{d^2 n}{d\lambda^2} + \lambda \frac{d^3 n}{d\lambda^3} \right)$$

Taking  $d^2 n / d\lambda^2 = 0$ :

$$|\Delta Z_g| = (L/c) (\lambda \frac{d^3 n}{d\lambda^3}) \Delta \lambda^2 / 2$$

$$b) G_R^2 = G_o^2 + G_D^2, \quad G_D = DL \cdot G_{15} \quad TA$$

here  $G_{15} \approx G_{15}$

$$G_R^2 = G_o^2 + (DL G_{15})^2$$

$$(G_R/G_o)^2 = 1 + \left( \frac{DL G_{15}}{G_o} \right)^2 = 1.25^2 = 1.56$$

$$(DL G_{15}/G_o) = 0.75, \text{ need } G_o > 1.33 DL \cdot G_{15}$$

$$c) \text{ Far transform limited pulses, } G_o G_s = \frac{1}{2} \quad NT$$

$$\text{But } G_s/\lambda = G_o/\omega, \quad G_s \approx \frac{\lambda^2}{2\pi c} G_o$$

$$\text{This gives } G_s = \frac{1}{2G_o} \left( \frac{\lambda^2}{2\pi c} \right)$$

$$G_R^2 = G_o^2 + A^2/G_o^2 \quad \text{where } A = DL\lambda^2/4\pi c$$

$$\frac{dG_R^2}{dG_o} = 2G_o - \frac{2A^2}{G_o^3} = 0 \quad \text{for } G_o = \sqrt{A}$$

$$\text{So } G_o = \sqrt{\frac{DL\lambda^2}{4\pi c}}, \text{ giving } \frac{G_R}{G_o} = \sqrt{2}$$

④ a)  $\eta = \frac{1}{1 + Z_{in}/Z_{out}} = \frac{1}{1 + 1/5} = 0.833$  CE

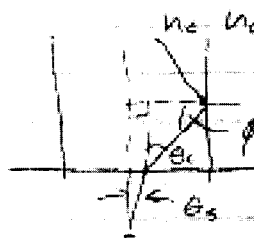
b) The 4 mechanisms & solutions are: BW

i) light goes downward  
- add a mirror at base, but need either a heterostructure or very short path length

ii) absorption before leaving semiconductor  
- heterostructure used so bandgap is greater outside active region

iii) surface (Fresnel) reflection  
- antireflection coating,  $\lambda/4$  thick,  $n = \sqrt{n_1 n_2}$

iv) Total internal reflection  
- add a dome of glass/plastic, or shape semiconductor surface to reduce angle.

c)  need  $\sin \theta_c > n_s/n_c$   
 $\therefore \cos \phi < \sqrt{1 - (n_s/n_c)^2}$  TA  
 $\cos \phi = \sin \theta_c$   
 $\sin \theta_c < \sqrt{n_c^2 - n_s^2} / n_c$   
 $n_s \sin \theta_s = n_c \sin \theta_c$   
 $\sin \theta_s < \sqrt{n_c^2 - n_s^2} / n_s$   
 Need  $\theta_s < \sin^{-1}(NA/n_s)$  where  $NA = \sqrt{n_c^2 - n_s^2}$   
 $f = \frac{\int_0^{\theta_{max}} 2\pi \sin \theta d\theta}{\int_0^{\pi} 2\pi \sin \theta d\theta}$   $n_c = 3.5$   $n_s = 1.5$   $\theta_{max} \approx 1.47^\circ$   
 $= \frac{[1 - \cos \theta]_0^{\theta_{max}}}{[1 - \cos \theta]_0^{\pi}} = \frac{(1 - \cos \theta_{max})/2}{2} \approx \theta^2/4$   
 $f \approx \sin^2 \theta/4 = \frac{NA^2}{4n_s^2} = \frac{.1^2}{4(3.5)^2} = .00020$

d)  $\tau = L/v_g$   $\Delta \tau_g \approx \Delta n L/c$  TA  
 $= n L/c$  Needs  $\Delta \tau_g < (1/4) \tau$   
 gives  $B < c/4L \Delta n$

5)

a) Shot noise is given in terms of equivalent photocurrent per  $\sqrt{\Delta f}$  as

$$I_s^* = \sqrt{2e I_{ph}}$$

TA

We need to convert this to be expressed w.r.t. received optical power  $\Phi_R$ . Assuming

$$\eta = 1, I_{ph} = e\Phi_R/h\nu = e\lambda\Phi_R/hc$$

$$\text{then } I_s^* = \frac{hc}{e\lambda} \sqrt{2e^2 \lambda \Phi_R / hc} = \sqrt{2hc\Phi_R/\lambda}$$

$$I_s^* = 5 \mu W / \sqrt{Hz} \text{ for}$$

$$(5 \times 10^{-6})^2 = \frac{2(16 \times 10^{-19})^2 1.5 \times 10^6 \Phi_R}{6.65 \times 10^{-7} \times 3 \times 10^8}$$

$$\Phi_R = 94 \mu W$$

b)  $SNR = \frac{\Phi_R}{[NEP + \Phi_s^*] (\Delta f)^{1/2}}$

TA

$$SNR = 10 \text{ dB} = 10$$

$$\Phi_s^* = \sqrt{2e^2 \lambda \Phi_R / hc}$$

$$\Delta f = B/2$$

Where receiver noise dominates:

$$10 = \frac{\Phi_R e^{-\alpha L}}{NEP \sqrt{B/2}}, \quad 10^2 B = \left( \frac{10^2 e^{-2\alpha L}}{5 \times 10^{-12}} \right)^2$$

$$B = 8 \times 10^6 e^{-2\alpha L}$$

$$\alpha = \frac{dB/km}{4.3} = 0.1 \text{ km}^{-1}$$

$$\log B = 16.9 - 0.24 \log e = 16.9 - .087 L$$

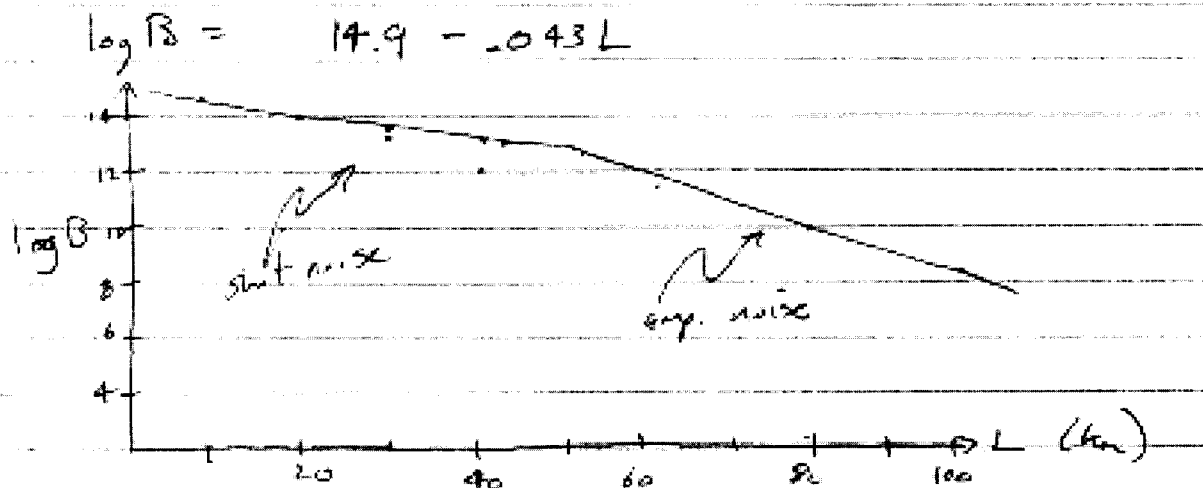
Where shot noise dominates:

$$SNR = \frac{\Phi_R}{\sqrt{2hc\Phi_R/\lambda} \sqrt{\Delta f}}, \quad 10 \sqrt{B} = \sqrt{\frac{\lambda \Phi_R}{2hc}}$$

$$B = \frac{\lambda \Phi_R^2 e^{-2\alpha L}}{100 hc}, \quad \log B = \log \left( \frac{15 \times 10^6 \times 10^2}{100 \times 6.6 \times 10^{-19} \times 3 \times 10^8} \right) - 0.11 L \log e$$

⑤ b) (continued)

for shot noise case



c) With the amplifier,  $\Phi_p$  increases by 20 dB, NEP is unaffected where NEP dominates:

$$\frac{10^3 B}{2} = \left( \frac{e^{-\alpha L}}{5 \times 10^{12}} \right)^2 \quad B = e^{-.2L} \times 8 \times 10^{20}$$

$$\log B = 21 - .087 L$$

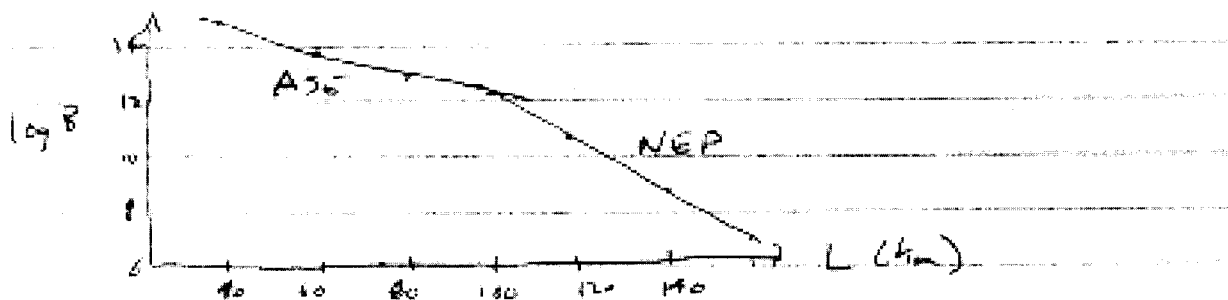
for the shot/ASE case:

4 dB is a ratio of 2.5 in electrical SNR,

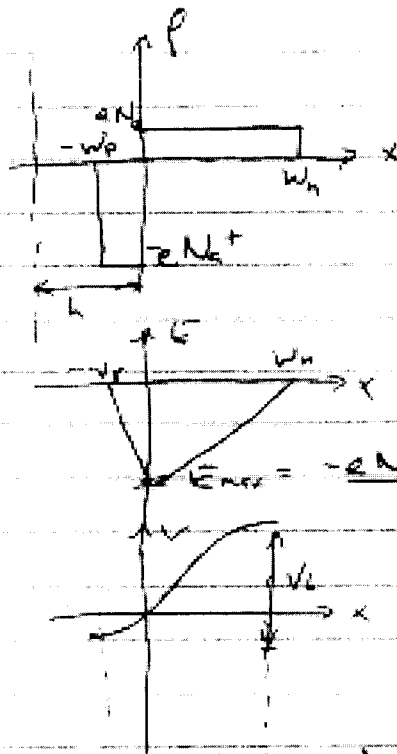
1.5 dB in optical SNR.

$$\frac{10^3 B}{2} = \frac{1}{2.5} \left( \frac{1.5 \times 10^6 e^{-\alpha L}}{2 \times 10^{-14} \times 3 \times 10^5} \right)$$

$$B = 3 \times 10^{16} e^{-.043 L} \quad \log B = 16.5 - .043 L$$



6)



TA

$$V_b = |E_{max}| (w_n + w_p) / 2$$

$$w_n = \frac{N_A^+ w_p}{N_D} \quad V_b = \left( \frac{eN_A^+ w_p}{\epsilon} \right) \left( \frac{N_A^+ + 1}{N_D} \right) w_p / 2$$

$$V_b = \frac{eN_A^+}{2\epsilon} \left( \frac{N_A^+}{N_D} + 1 \right) w_p^2 = \frac{16 \times 10^{-9} \times 10^{20}}{2 \times 17 \times 8.85 \times 10^{-12}} (6) w_p^2$$

$$w_p = 1.49 \sqrt{V_b} \quad (\text{in } \mu\text{m})$$

$$w_p = 10 \mu\text{m} \quad \text{for } V_b = 46.2 \text{ V}$$

$$w = (5+1) w_p = 8.9 \sqrt{V_b} = 10 \mu\text{m} \quad \text{for } V = 1.25 \text{ V}$$

b)  $q = e^{-\alpha x_1} - e^{-\alpha x_2}$

TA

$$x_1 = h - w_p \quad x_2 = h + w_n$$

$$q = e^{-\alpha(h-w_p)} - e^{-\alpha(h+w_n)}$$

$$q = e^{-\alpha h} (e^{\alpha w_p} - e^{-\alpha w_n}) \quad \alpha h = 10^5 \times 10^{-5} = 1$$

$$q = e^{-1} (e^{1.49 \times 10^5 \sqrt{V_b}} - e^{-7.85 \times 10^5 \sqrt{V_b}})$$

$$\text{To get } q = 0.9: \quad 0.9e = e^{\alpha w_p} - e^{-\alpha w_n} = 2.946$$

$$\text{gives } \alpha w_p = 0.9 \quad w_p = 0.9 \times 10^{-5} \text{ m} = 9 \mu\text{m} = 1.49 \sqrt{V_b}$$

$$V_b = 36.5 \text{ V}$$



⑥

c) If  $R \propto (\text{residual p thickness})^{-1}$  then NT

$$R = \frac{A}{h - w_p}$$

and we know  $C \propto \frac{1}{w_p t_{wp}}$  i.e.  $C = \frac{B}{w_p t_{wp}}$

$$\text{But } w_p + t_{wp} = b$$

$$RC = \frac{AB/b}{(h - w_p)w_p}$$

$$\frac{\partial RC}{\partial w_p} = 0 \quad \text{at} \quad \frac{\partial}{\partial w_p} \left( \frac{1}{RC} \right) = 0 = \frac{b}{Ab} (h - 2w_p)$$

$$w_p = \frac{h}{2} = 5 \mu\text{m}$$