EE4-08
EE9-SO14

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected copy

DIGITAL IMAGE PROCESSING

Friday, 11 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.T. Stathaki
Second Marker(s) : T-K. Kim

1. (a) Consider a $(2M+1) \times (2M+1)$ - pixel gray level real image $f(x,y)$ which is zero outside $-M \le x \le M$ and $-M \le y \le M$. Show that:

(i) $F(-u,-v) = F^*(u,v)$ where $F(u,v)$ is the two-dimensional Discrete Fourier Transform (DFT) of $f(x,y)$. [2]

(ii) In order for the image to have the imaginary part of its two-dimensional Discrete Fourier Transform equal to zero, the image must be symmetric around the origin (even). [2]

(iii) In order for the image to have the real part of its two-dimensional Discrete Fourier Transform equal to zero, the image must be antisymmetric around the origin (odd). [2]

(b) Consider the eight images shown in the figures below (A to H). Using knowledge of symmetry properties of the two-dimensional Discrete Fourier Transform and not exact calculation of it, list which image(s) will have a two-dimensional Discrete Fourier Transform $F(u,v)$ with the following properties:

(i) The imaginary part of $F(u,v)$ is zero for all u,v. [2]

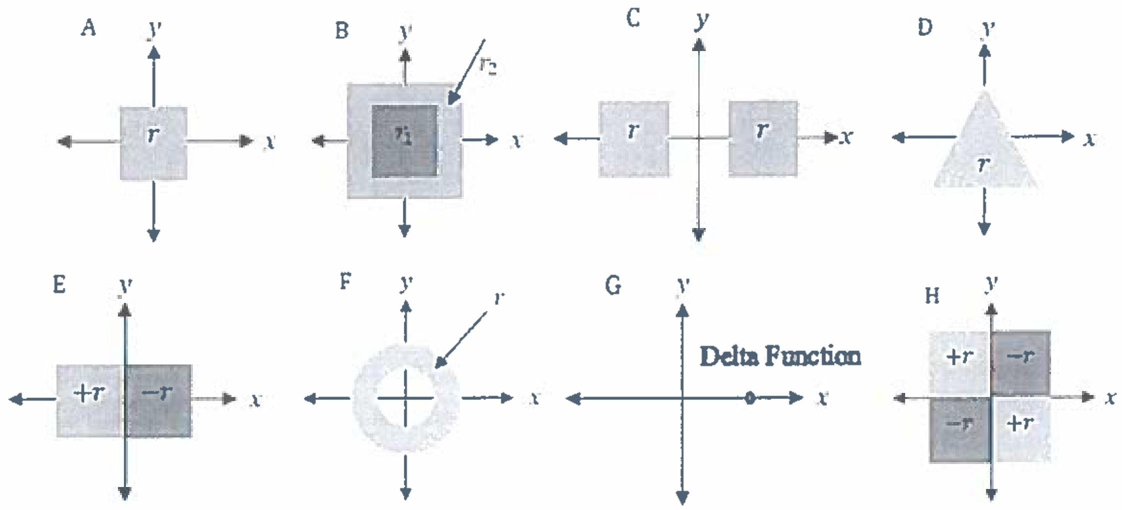
(ii) $F(0,0) = 0$ [2]

(iii) $F(u,v)$ has circular symmetry. [2]

(iv) The real part of $F(u,v)$ is zero for all u,v. [2]

In the figures below r, r_1, r_2 are the positive constant intensities of the corresponding gray shaded areas. White areas correspond to pixel intensities which are zero.

[Hint: You can use the statements of (a)-(ii) and (a)-(iii) above].



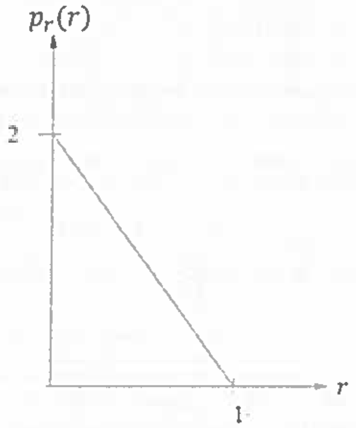
(c) Consider the images $f_1(x,y)$ and $f_2(x,y)$ defined below with M even.

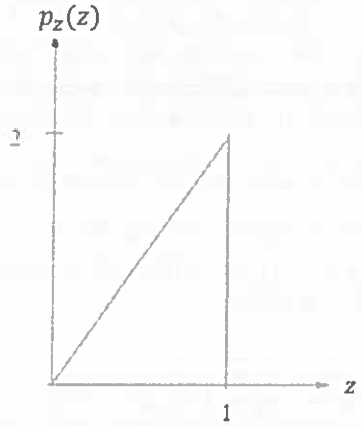
$$f_1(x,y) = \begin{cases} r_1 & 1 \le x \le \frac{M}{2}, 1 \le y \le M \\ r_2 & \frac{M}{2} < x \le M, 1 \le y \le M \end{cases} \quad \text{and } f_2(x,y) = \begin{cases} r_1 & 1 \le x \le M, 1 \le y \le \frac{M}{2} \\ r_2 & 1 \le x \le M, \frac{M}{2} < y \le M \end{cases}$$

(i) Transform the images $f_1(x,y)$ and $f_2(x,y)$ into a new set of two images using the Karhunen-Loeve Transform (KLT) in order to achieve data reduction and demonstrate whether data reduction can be achieved in that set of images using the KLT. [4]

(ii) Comment on whether you could obtain the result of (c)-(i) above using intuition rather than by explicit calculation. [2]

2. (a) An image has the gray level probability density function (or histogram normalized by the number of pixels) $p_r(r)$ shown in the left figure below.





(i) Find the pixel transformation $s = T(r)$ such that after transformation the image has a flat PDF, i.e., the transformation which accomplishes histogram equalisation. Assume continuous variables r, s. [2]

(ii) Find the pixel transformation function $z = f(r)$ such that the transformed image will have the PDF denoted by $p_z(z)$ shown in the right figure above. Assume continuous variables r, z. [2]

(b) Two images have the same histogram. Which of the following properties must they have in common? Justify your answer.

(i) Same total power (the power is the sum of squares of pixel values). [2]

(ii) Same entropy. Recall that the entropy of an image is $H = -\sum_{k=0}^{K-1} p_k \log_2(p_k)$ where K is the number of gray levels and p_k is the probability associated with gray level k. [2]

(iii) Same degree of pixel-to-pixel correlation. [2]

(c) Consider the image I below and the filters F and L.

$$
\mathrm{I} = \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 1 \end{array}
\qquad
\mathrm{F} = \begin{array}{ccc} & 1/8 & \\ 1/8 & 1/2 & 1/8 \\ & 1/8 & \end{array}
\qquad
\mathrm{L} = \begin{array}{ccc} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{array}
$$

(i) Explain the type of filters F and L and their main function. Justify your answer. [2]

(ii) Convolve the image I with the filter F above and compute the output image (assume ones outside the input image and round down the output pixel values to the nearest integer). [2]

(iii) Apply a 3 by 3 median filter to the same image I to produce a 3 by 3 output image. Assume ones outside the image. [2]

(iv) From the result in part (c)-(iii) above demonstrate one general advantage of the median filter. [2]

(v) Show that convolving the image I with the filter L is equivalent to locally subtracting an amplified original value of the image from a five-point local mean. How would you explain the later without doing any calculations? [2]

3. (a) A sampled image $f(x,y)$ is distorted by convolution with either the space invariant point spread function h_1 or the function h_2 where:

$$h_1(x,y) = \delta(x,y) + \delta(x,y-1) + \delta(x,y+1) + \delta(x-1,y) + \delta(x+1,y)$$
$$h_2(x,y) = 5\delta(x,y) + \delta(x,y-1) + \delta(x,y+1) + \delta(x-1,y) + \delta(x+1,y)$$

Assuming that the distorted images also contain random additive noise, then in one image the distortion can be effectively removed using an Inverse Fourier filter, while the other requires the Pseudo Inverse filter. Which image must use the Pseudo Inverse filter and why?

[For the two-dimensional Discrete Fourier Transform of the non-causal functions h_1 and h_2 you can use the relationship $H_i(u,v) = \frac{1}{(2M+1)^2}\sum_{-M}^{M}\sum_{-M}^{M} h_i(x,y)\, e^{-j\frac{2\pi}{(2M+1)}(ux+vy)}$, $i = 1,2$ with $(2M+1)$ the extended size we use for all images by zero padding, in order to apply circular convolution.]

[5]

(b) Consider an image $f(x,y)$ moving at a speed s past a camera whose exposure time is Δt. The distortion that motion causes can be modelled in the two-dimensional frequency domain as the following function:

$$H(u,v) = \frac{\sin(\pi s \Delta t u)}{\pi s \Delta t u}$$

(i) Explain why the Inverse filtering technique is unsuitable for restoring this image. [2]

(ii) Calculate the appropriate Wiener filter to apply to the above restoration problem. Assume that the additive noise in the Fourier domain is white, with power spectral density $S_{nn}(u,v) = s\Delta t$, while the original undistorted version of the image has power spectral density $S_{ff}(u,v) = 1/u$ where u, v are the spatial frequencies. What is the Wiener filter value at $u = u_0 = 1/(s\Delta t)$? [6]

(c) We wish to restore an image convolved with an unknown two-dimensional filter (mask). The image intensity is mainly zero, but part of the image contains a single intensity rectangular object which in the distorted image appears as given below. Determine this mask assuming only that we know that the mask is a 3x3 pixel function and that the sum of the 9 values is 1. [Hint: consider the symmetries of the mask.] [7]

Convolved Image

0	0	0	0	0	0	0	0	0	0	0
0	0	0	2	2	2	2	2	2	0	0
0	0	2	6	8	8	8	8	6	2	0
0	0	2	8	10	10	10	10	8	2	0
0	0	2	8	10	10	10	10	8	2	0
0	0	2	8	10	10	10	10	8	2	0
0	0	2	8	10	10	10	10	8	2	0
0	0	2	8	10	10	10	10	8	2	0
0	0	2	6	8	8	8	8	6	2	0
0	0	0	2	2	2	2	2	2	0	0
0	0	0	0	0	0	0	0	0	0	0

4. (a) A binary image is to be Huffman coded in blocks of M pixels. The successive pixels are independent from each other and 5% of the pixels are 1. Find the Huffman code, its redundancy and its coding efficiency for:

(i) $M = 1$ [3]

(ii) $M = 2$ [5]

(iii) $M = 3$ [7]

(b) In lossless JPEG, one forms a prediction residual using previously encoded pixels in the current line and/or the previous line. Suppose that the prediction residual for pixel with intensity x in the following figure is defined as $r = y - x$ where y is the function $y = a + b - c$.

	c	b	
	a	x	

(i) Describe briefly the procedure of coding the prediction residual in the Lossless JPEG Standard.

[3]

(ii) Consider the case with pixel values $a = 101$, $b = 191$, $c = 101$ and $x = 191$. Find the codeword of the prediction residual ~~y~~, knowing that the Huffman code for 0 is 00.

r not y

[2]

announced at 11-35