EE4-06
EE9-AO9
EE9-SO9

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

MSc and EEE/EIE PART IV: MEng and ACGI

OPTICAL COMMUNICATION

Wednesday, 8 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 6

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	E.M. Yeatman, E.M. Yeatman
	Second Marker(s) :	A.S. Holmes, A.S. Holmes

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3.0 \times 10^{8}$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$\kappa = k_{1x}\tan(k_{1x}d/2)$ and $\kappa = -\,k_{1x}\cot(k_{1x}d/2)$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, state any assumptions or approximations made, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]

a) A certain optical receiver detects 10^{11} photons/s, at a nominal wavelength of 1.46 μm. What is the equivalent received optical power in dBm?

b) A certain symmetric slab waveguide supports a single TE mode. How many TM modes will this guide support?

c) Briefly explain the physical significance of the imaginary part of the refractive index of a material.

d) A silica optical fibre has a numerical aperture of 0.16. Estimate the index difference Δn.

e) An optical fibre link is running at 1.2 Gbit/s. Estimate the number of bits propagating in a 3 km long fibre at any one moment.

f) Calculate the optimal thickness and refractive index of an anti-reflection coating acting between a glass of index 1.52 and air, for a free space wavelength of 1.15 μm.

g) Why can photodiodes for use in high performance fibre optic communication systems not be based on silicon?

h) A laser diode with quantum efficiency 0.88 has a slope efficiency of 1.16 W/A. Calculate the output (free space) wavelength.

i) An optical point-to-point link has a fibre length of 30 km and an attenuation coefficient of 0.35 dB/km. Which would you expect to have a worse effect on the signal-to-noise ratio: doubling the cable length, or doubling the bit-rate? Assume thermal noise dominates in all cases, and give a brief derivation or reasoning.

j) What is the principal attenuation mechanism in high purity silica optical fibre at nominal wavelengths less than 1 μm?

2. On pg. 1 the eigenvalue equations are given for TE modes in a symmetric slab waveguide as shown in Fig. 2.1

a) What are the boundary conditions leading to these equations? What fundamental physical principle or principles give rise to these? [4]

b) Derive the eigenvalue equations using a field profile approach. [10]

c) Derive the additional equation from which, along with the eigenvalue equations, the mode indices of a waveguide of this type can be calculated, and describe a process by which this calculation could be done. [6]

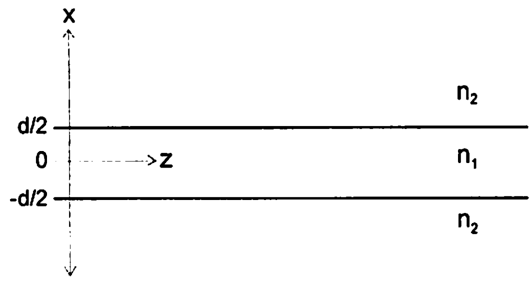

Figure 2.1 Slab waveguide

3. a) How are group and phase velocity each defined for a propagating electromagnetic wave? What is the main physical significance of group velocity? [6]

b) In the wavelength range of interest, the refractive index of a sample of silica glass can be approximated by

$$n(\lambda) \cong D_o + D_l\,\lambda_o^{-2} - D_2\,\lambda_o^{2} \qquad (5.1)$$

where D_o = 1.45, D_l = 0.003 μm^2 and D_2 = 0.0032 μm^{-2}. Find expressions for the phase and group velocities v_p and v_g in this material, as functions of D_o, D_l, D_2, and λ_o and c. Hence show that v_g is always < v_p in this case. [8]

c) For the glass described above, find the wavelength of zero material dispersion. [6]

4. a) Briefly describe the four factors which reduce the external quantum efficiency in a light emitting diode (LED), and ways in which their effects can be reduced. [6]

b) For an LED emitting into air, find the fraction of emitted photons lost through each of the four mechanisms of part (a), and hence calculate the external quantum efficiency η_{ext}, if none of the special measures to improve it have been used. Assume an attenuation coefficient of 0.5×10^{2} cm^{-1}, and that the active region emits photons equally in all directions. The distance from the active region to the surface is 10 μm, and the refractive index of the semiconductor is 3.7. State any other assumptions or approximations made. [6]

c) Calculate the quantum efficiency for this same LED emitting into guided modes of a multi-mode fibre with a numerical aperture of 0.15. [4]

d) Describe the advantages of laser diodes over light emitting diodes for optical communication applications. [4]

5. a) Describe and discuss the important attenuation mechanisms in optical fibres, and the influence these have in choice of operating wavelengths. Use diagrams and equations where appropriate. [6]

b) An optical receiver can be implemented as a photodiode in series with a resistor, followed by a voltage amplifier. State the principal noise sources in this case, and draw a noise equivalent circuit for the receiver. [4]

c) A receiver with an effective input resistance of 10 kΩ is at the end of 25 km of step index, multi-mode fibre with index difference 0.003 and attenuation of 0.7 dB/km at the operating wavelength of 1510 nm. The transmitted power is 2 μW. Making an appropriate assumption for the required bandwidth, find the maximum bit-rate B for this link in the two cases

i) if the link is limited by the SNR requirement of 12, assuming perfect quantum efficiency in the receiver, and that thermal noise is the dominant noise source;
ii) if the link is limited by dispersion.

Which is the more restricting factor for the parameters given? [10]

6. A silicon avalanche photodiode (Fig. 6.1) has intrinsic and avalanche layer thicknesses of w_i and w_A respectively, acceptor doping levels N_{A+}, N_{Ai} and N_{AA} in the p^+, intrinsic and avalanche layers respectively, and donor doping level N_D in the n layer.

a) Assuming that a sufficient bias voltage has been applied to fully deplete both the intrinsic and avalanche regions, derive expressions for the difference in electric field strength ΔE_i across the intrinsic region, and the difference in electric field strength ΔE_A across the avalanche region, in terms of the doping levels and layer thicknesses. Hence, derive an expression for the minimum bias voltage, V_{min}, required to fully deplete these two layers. [8]

b) Calculate V_{min} as derived in (a) for the following parameter values:

$w_i = 8$ μm, $w_A = 3$ μm

$N_{A+} = 10^{21}$ m^{-3}, $N_{AA} = 4\times10^{21}$ m^{-3} , $N_{Ai} = 10^{20}$ m^{-3} and $N_D = 6\times10^{21}$ m^{-3}

Plot and dimension the electric field profile E(x) for this case where the bias voltage equals V_{min}. On the same graph, sketch additional field distributions for bias voltages higher than and lower than this value. [8]

c) Discuss the necessary and the desirable properties of the electric field distribution for such a device, and some factors limiting the optimisation of the field distribution. [4]

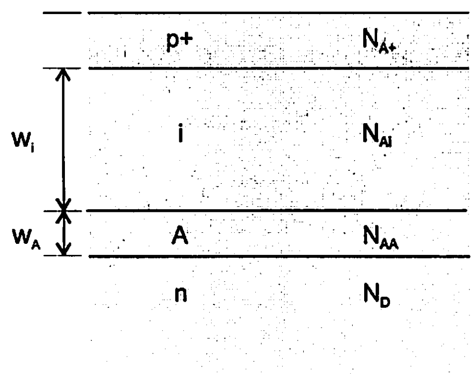

Figure 6.1

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Optical Communication 2013
Solutions

1

a) $P_R = \frac{hc}{\lambda} \times \dot{N} = \frac{6.63\times10^{-34}\times3\times10^{8}\times10^{4}}{1.46\times10^{-6}} = 13.6\,nW = \underline{-48.7\ dBm}$

b) Cutoff conditions are the same so one TM mode.

c) Imaginary part of n is associated with loss and attenuation. Gives an exponential decay factor in waveform.

d) $NA \cong \sqrt{2n\cdot\Delta n}$ $\therefore$ $\Delta n \cong \frac{NA^2}{2n} = \frac{.16^2}{2\times1.46} = \underline{8.8\times10^{-3}}$

e) $\Delta z = \Delta t \cdot \frac{c}{n} \cong \frac{3\times10^{8}}{1.5}\times\frac{1}{1.2\times10^{9}} = \frac{1}{6}\,m$

$N = \frac{L}{\Delta z} = \frac{3000}{1/6} = \underline{18{,}000 \text{ bits}}$

f) $n_{NA} = \sqrt{n_1 n_2} = \sqrt{1.52\times1} = \underline{1.233}$

$t = \frac{\lambda_0}{4n} = \frac{1.15}{4\times1.233} = \underline{0.233\ \mu m}$

g) Because of its bandgap just above 1eV, Si is transparent at the main wavelengths of 1.3 & 1.55 μm.

h) $S = \eta\frac{h c}{e\lambda}$ $\therefore$ $\lambda = \frac{\eta h c}{e S} = \frac{0.88\times6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times1.16}$

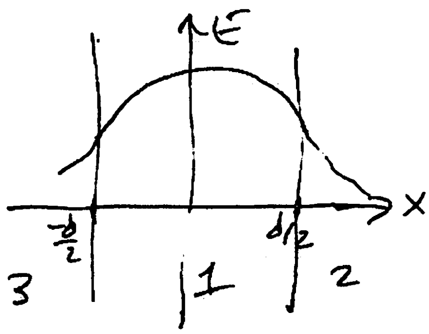
$= \underline{0.94\ \mu m}$

i) Doubling cable adds 30 km × .35 = 10.5 dB of loss so SNR_{opt} is reduced by 10.5 dB. Doubling bit rate (and thus δf) reduces SNR_{opt} by a factor $\sqrt{2}$, a reduction of 1.5 dB. Doubling length is worse.

j) Below 1μm Rayleigh scattering dominates.

(2) a) E is continuous at boundaries
$\frac{dE}{dx}$ is continuous at boundaries

b)



$E_1(x) = A' \exp(jk_{1x}x) + A'' \exp(-jk_{1x}x)$

$E_2(x) = B \exp(-Kx)$

where $k_{2x} = jK$ is imaginary because $\theta > \theta_c$

Guide is symmetric so solutions are symmetric (S) or anti-symmetric (AS).

$S: A' = A'' \therefore E_1(x) = A\cos(k_{1x}x)$
$E_1(x = d/2) = E_2(x = d/2) \therefore A\cos\left(\frac{k_{1x}d}{2}\right) = B\exp\left(-\frac{Kd}{2}\right)$ (i)

$dE_1/dx = -k_{1x}A\sin(k_{1x}x)$ $\quad \frac{dE_2}{dx} = -KB\exp(-Kx)$

$\therefore \quad -k_{1x}A\sin(k_{1x}d/2) = -KB\exp(-Kd/2)$ (ii)

Divide (ii) by (i) gives $k_{1x}\tan\left(\frac{k_{1x}d}{2}\right) = K$

follow same procedure for AS case to get 2nd eigenvalue eq'n.

c) use phase matching at boundary, ie $k_{z1} = k_{z2}$

Since $k_z^2 + k_x^2 = (n k_0)^2$ then this gives $(n_1 k_0)^2 - k_{1x}^2 = (n_2 k_0)^2 + K^2$

this gives $K^2 + k_{1x}^2 = (n_1^2 - n_2^2)k_0^2$.
Defining $X = \frac{k_{1x}d}{2}$ and $Y = \frac{Kd}{2}$ this gives $X^2 + Y^2 = R^2$

(ie a circular arc) with $R = \sqrt{n_1^2 - n_2^2}\, k_0 d/2$
and eigenvalue equation (S) becomes $X\tan X = Y$
Combining these gives:

$$X^2 + X^2\tan^2 X = R^2 = X^2(1 + \tan^2 X) = \frac{X^2}{\cos^2 X}$$

ie $X = \pm R\cos X$. This can be solved by successive approximation. From X get k_{1x}, from that can calculate β and thus n'.

3

a) $V_p = \omega/k \qquad V_g = \frac{d\omega}{dk}$

V_g describes the speed that pulses propagate

b) $k = nk_o = n\omega/c$

$$V_g = \frac{1}{dk/d\omega} \qquad \frac{1}{V_g} = \frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}$$

$$\frac{dn}{d\omega} = \frac{dn}{d\lambda_o}\frac{d\lambda_o}{d\omega} \qquad \omega = k_o c = \frac{2\pi c}{\lambda_o}$$

$$\frac{d\lambda_o}{d\omega} = \frac{-2\pi c}{\omega^2} = -\frac{\lambda_o}{\omega}$$

$$\frac{dn}{d\omega} = -\frac{\lambda_o}{\omega}\frac{dn}{d\lambda_o}$$

$$\frac{1}{V_g} = \frac{1}{c}\left(n - \lambda_o dn/d\lambda_o\right)$$

$$n = D_o + D_1\lambda_o^{-2} - D_2\lambda_o^2$$

$$\frac{dn}{d\lambda_o} = -2D_1\lambda_o^{-3} - 2D_2\lambda_o$$

$$\frac{1}{V_g} = \frac{1}{c}\left(n + \lambda_o(2D_1\lambda_o^{-3} + 2D_2\lambda_o)\right)$$

$$= \frac{1}{c}\left(D_o + D_1\lambda_o^{-2} - D_2\lambda_o^{-2} + 2D_1\lambda_o^{-2} + 2D_2\lambda_o^2\right)$$

$$V_g = \frac{c}{D_o + 3D_1\lambda_o^{-2} + D_2\lambda_o^2}$$

$$V_p = \frac{c}{n} = \frac{c}{D_o + D_1\lambda_o^{-2} - D_2\lambda_o^2}$$

since $V_g = \dfrac{c}{n + 2D_1\lambda_o^{-2} + 2D_2\lambda_o^2}$

and the denominator is always $> n$, $V_g < V_p$

c) $dn/d\lambda = -2D_1\lambda_o^{-3} - 2D_2\lambda_o$

$$\frac{d^2n}{d\lambda^2} = 6D_1\lambda_o^{-4} - 2D_2 = 0 \quad \text{at zero dispersion}$$

$$3D_1/D_2 = \lambda_o^4 \qquad \lambda_o = \sqrt[4]{3 \times .003/.0032}$$

$$= 1.295\ \mu m$$

(4)

a) i) half the light goes downwards. This can be recovered by a heterostructure to reduce absorption outside ~~depletion~~ active region, and mirror at bottom surface.

ii) Light is absorbed between active region and surface. Again, reduce by heterostructure (higher E_g in upper part).

iii) Fresnel reflection at upper surface from index difference – reduce by AR coating.

iv) Total Int. Reflection – photons not within $\theta_c = \sin^{-1}\left(\frac{1}{n_s}\right)$ of normal will not escape. Can be reduced by hemispherical cap.

b) i) 50% lost downwards.

ii) Calculate TIR next. $\theta_c = \sin^{-1}\left(\frac{1}{3.7}\right) = 15.7°$

fraction emitted $f = \int_0^{\theta_c} d\Omega \Big/ \int_0^{\pi/2} d\Omega \qquad d\Omega = 2\pi \sin\theta \, d\theta$

$\therefore f = 1 - \cos\theta_c = 0.0372$

96.3% of upwards light is lost.

iii) Within θ_c, path length to surface varies little, so fraction re-absorbed $\cong 1 - \exp(-\alpha L)$ with $L = 10 \times 10^{-6}$ m, $\alpha = 0.5 \times 10^4$ m^{-1}, $\exp(-\alpha L) = 0.951$

4.9% lost.

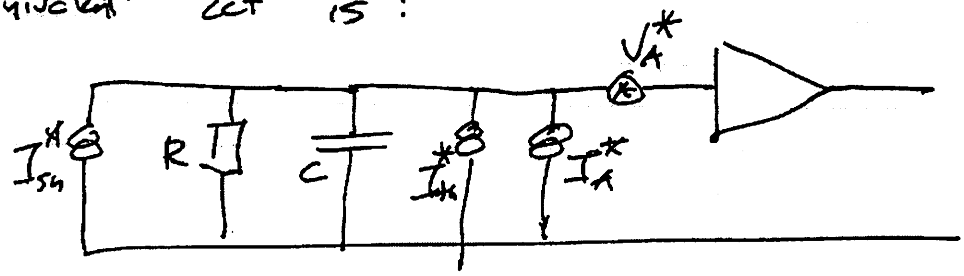
iv) We will use normal incidence to approximate Fresnel reflection: $R = \left(\frac{3.7-1}{3.7+1}\right)^2 = .33$

Overall $\eta_{ext} = \frac{1}{2} \times f \times e^{-\alpha L} \times (1-R) = \underline{1.18\%}$

(5)

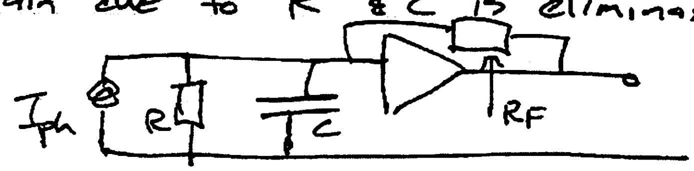
a) Discussion should include UV and IR absorption, Rayleigh scattering, and the underlying mechanisms of each of these, impurity absorption, particularly water. A plot of these should be sketched over a sensible wavelength range. Bending loss should be mentioned. The λ dependence of Rayleigh scattering ($\propto \lambda^{-4}$) should be indicated.

b) Noise sources are shot noise, thermal noise, and amplifier voltage and current noise. Noise equivalent cct is:



They should explain that R & C are combined components from :- load and amplifier input resistance
- diode and amplifier input capacitance

An alternative configuration is the transimpedance amplifier. In this case the photodiode is effectively grounded, so the frequency dependence of gain due to R & C is eliminated.



This also means the frequency dependence of SNR is largely eliminated.

c)

(5) c) For thermal noise, $SNR_{opt} = \dfrac{I_{ph}}{\sqrt{\dfrac{4kT}{R}\cdot\dfrac{B}{2}}}$

(assumes bandwidth requirement $\Delta f = B/2$)

Attenuation is $25 \times .7 = 17.5$ dB

$\therefore \ \Phi_R = 2\mu W \times 10^{-1.75} = 35.6 \text{ nW}$

For $\eta = 1$, $I_{ph} = \Phi_R \times \dfrac{e\lambda}{hc} = \dfrac{1.6\times10^{-19} \times 1.51\times10^{-6}}{6.63\times10^{-34}\times3\times10^{8}} \times 35.6\times10^{-9}$

$= 43.2 \text{ nA}$

$B_{max} = \left(\dfrac{I_{ph}}{SNR}\right)^2 \times \dfrac{2R}{4kT} = \left(\dfrac{43.2\times10^{-9}}{12}\right)^2 \dfrac{\times 2\times10^4}{4\times1.38\times10^{-23}\times300}$

$\cong 16$ Mbit/s.

For Dispersion case, in step index multimode, modes are spread evenly with $n_2 < n' < n_1$

so $\dfrac{\Delta\tau_g}{\tau_g} \cong \dfrac{\Delta n}{n}$, and $\tau_g \cong \dfrac{n'L}{c}$ $\quad n' \cong 1.46$

So $\Delta\tau_g \cong \dfrac{.003}{1.46} \times \dfrac{1.46\times25\times10^3}{3\times10^8} \cong 250$ nsec

need $\Delta\tau_g < \frac{1}{4}\left(\frac{1}{B}\right)$ $\quad B_{max} \cong \dfrac{1}{4}\dfrac{1}{\Delta\tau_g}$

$B_{max} = \dfrac{1}{4\times250\times10^{-9}} = 10^6 = 1$ Mbit/s

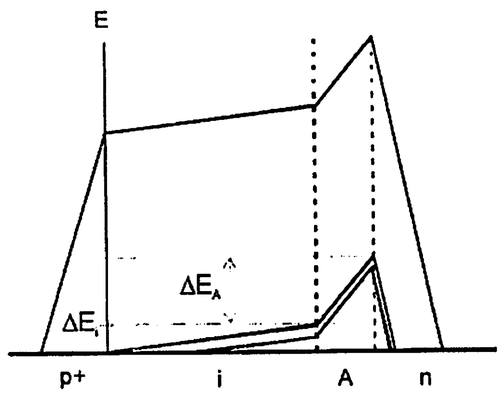
So dispersion is the stronger limiting factor.

6.

a) In one dimension $E = (e/\varepsilon)\int\rho dx$, so for the fully depleted intrinsic layer $\Delta E_i = (e/\varepsilon)w_iN_{Ai}$. Similarly $\Delta E_A = (e/\varepsilon)w_AN_{AA}$. At V_{min}, the p+ layer is undepleted, so E=0 at the p-i boundary, so at the A-n boundary $E = \Delta E_i + \Delta E_A$. Charge balance gives $(w_iN_{Ai} + w_AN_{AA}) = w_nN_D$ with w_n the depleted thickness in the n layer. The voltage is now the area under the middle curve in the plot below, i.e. $V_{min} = ½ w_i\Delta E_i + w_A\Delta E_i + ½w_A\Delta E_A + ½w_n(\Delta E_A + \Delta E_i)$. Combining with the expressions above gives:

$$V_{\min} = \frac{e}{\varepsilon}\left[w_iN_{Ai}\left(w_A + \frac{w_i}{2}\right) + \frac{w_AN_{AA}}{2}\left(w_A + \frac{w_iN_{Ai} + w_AN_{AA}}{N_D}\right)\right]$$

b) Inserting the values gives $\Delta E_i = 1.2 \times 10^6$ V/m, $\Delta E_A = 18 \times 10^6$ V/m, $V_{min} = 54.6$ V.



c)

Ideally the field should be at the level to give the saturation drift velocity throughout the depleted region, but not high enough to cause breakdown, except in the avalanche region, where it should be just above the breakdown level. In practice the field cannot be at the saturation level through most of the p+ level since it has to rise from zero, so we keep this layer thin by using a high doping level. Similarly the field has to rise linearly through the avalanche layer so cannot be uniform, and the n layer needs to be highly doped for the same reason as the p+.