

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2012

MSc and EEE/ISE PART IV: MEng and ACGI

Corrected Copy



**MOBILE RADIO COMMUNICATION**

Thursday, 10 May 10:00 am

Time allowed: 3:00 hours

**There are FOUR questions on this paper.**

**Answer THREE questions.**

*All questions carry equal marks. The maximum mark for each subquestion is shown in brackets.*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      M.K. Gurcan  
   Second Marker(s) :      K.K. Leung

## Instructions to Candidates

### Useful equations

Hata model

$$P_{L,urban}(d)dB = 69.55 + 26.16 \log_{10}(f_c) - 13.82 \log_{10}(h_t) \\ - a(h_r) + (44.9 - 6.55 \log_{10}(h_t)) \log_{10}(d).$$

for small to medium sized cities  $f_c > 300$  MHz

$$a(h_r) = 3.2(\log_{10}(11.75h_r))^2 - 4.97dB.$$

Corrections to the urban model are made for suburban and rural propagation,

$$P_{L,suburban}(d) = P_{L,urban}(d) - 2 \left[ \log_{10} \left( \frac{f_c}{28} \right) \right]^2 - 5.4$$

and

$$P_{L,rural}(d) = P_{L,urban}(d) - 4.78 [\log_{10}(f_c)]^2 + 18.33 \log_{10}(f_c) - K.$$

For hexagonal shaped cells

$$SIR = \frac{1}{6} \left( \frac{D}{R} \right)^\gamma.$$

For diamond shaped cells the SIR is related to the number of cells  $N$  in a cluster as

$$SIR = 0.125 (4N)^{\frac{\gamma}{2}}.$$

The critical distance is

$$d_c = \frac{4h_th_r}{\lambda}.$$

The speed of light  $c = 3 \times 10^8$  m/s and  $c = f\lambda$ .

Taylor series approximation is

$$1 + 2a \simeq (1 + a)^2.$$

The free space path loss constant is

$$K = \left( \frac{\lambda}{4\pi d_0} \right)^2.$$

The terms have their usual meaning.

1. Answer the following sub-questions.

- (a) Consider a TDMA cellular system with hexagonally-shaped cells, and path loss exponent  $\gamma = 2$  for all signal propagation in the system. Find the minimum reuse factor  $N$  needed for a target signal-to-interference ratio (SIR) of 10 dB, and the corresponding user capacity assuming a total system bandwidth of 20 MHz and a required signal bandwidth of 100 KHz. [3]
- (b) Consider a cellular system with hexagonal cells of radius  $R = 1$  Km. Suppose that the minimum distance between cell centers using the same frequency must be  $D = 6$  Km to maintain the required SINR.
  - i. Find the required reuse factor  $N$  and the number of cells per cluster. [3]
  - ii. If the total number of channels for the system is 1200, find the number of channels which can be assigned to each cell. [3]
- (c) Consider a TDMA cellular system with diamond shaped cells, path loss exponent  $\gamma = 4$  for all signal propagation in the system, and BPSK modulation. Assume that the received signal exhibits Rayleigh fading. Suppose the users require the average probability of error  $\bar{P}_b = 10^{-3}$  where  $\bar{P} \simeq \frac{0.25}{SNR_0}$  for the average  $SNR_0$ . Assuming the system is interference-limited, find the minimum reuse factor  $N$  needed to meet this performance requirement. Also find the user capacity assuming a total system bandwidth of 20 MHz and a required signal bandwidth of 100 KHz. [3]
- (d) Find the critical distance  $d_c$  under the two-path model for a large macrocell in a suburban area with the base station mounted on a tower or building ( $h_t = 20$ m), the receiver is at height  $h_r = 3$ m, and  $f_c = 2$ GHz. Is this a good size for cell radius in a suburban macrocell? Justify your answer. [3]
- (e) Suppose that instead of a ground reflection, a two-path model consists of a LOS component and a signal reflected off a building to the left (or right) of the LOS path. Where must the building be located relative to the transmitter and receiver for this model to be the same as the two-path model with a LOS component and ground reflection? [4]

2. Answer the following sub-questions.

- (a) For a two-path propagation model with transmitter-receiver separation  $d = 100\text{m}$ ,  $h_t = 10\text{ m}$ , and  $h_r = 2\text{m}$ , find the delay spread between the two signals. [3]
- (b) Consider the two ray model shown in the Figure 2.1, the phase difference  $\Delta\phi$  between the direct line of sight and the reflected path is given by  $\Delta\phi = 2\pi \frac{(x + x' - l)}{\lambda}$  where the transmission wavelength is  $\lambda$ .

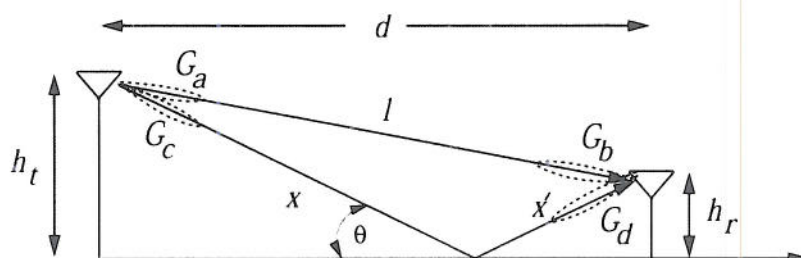


Figure 2.1: Two ray model.

Show how a Taylor series approximation applied to  $\Delta\phi$  results in the approximation [5]

$$\Delta\phi = \frac{4\pi h_t h_r}{\lambda d}.$$

- (c) Consider a receiver with noise power  $-160\text{ dBm}$  within the signal bandwidth of interest. Assume a simplified path loss model with  $d_0 = 1\text{m}$ ,  $K$  obtained from the free space path loss formula with omnidirectional antennas and  $f_c = 1\text{ GHz}$ , and  $\gamma = 4$ . For a transmit power of  $P_t = 10\text{ mW}$ , find the maximum distance between the transmitter and receiver such that the received signal-to-noise power ratio is  $20\text{ dB}$ . [4]
- (d) Consider a linear cellular system using frequency division multiplexing, which may represent one operating along a highway or rural road, as shown in the Figure 2.2.

Each cell is allocated a certain band of frequencies and these frequencies are reused in cells spaced a distance  $d\text{ m}$  away. Assume the system has square cells which are two kilometers in length per side, and that all mobiles transmit at the same power  $P$ . For each of the following propagation



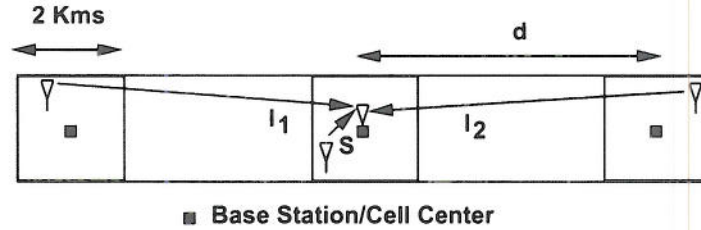


Figure 2.2: Two ray model.

models, determine the minimum distance that the cells operating in the same frequency band must be spaced so that the uplink SNR (the ratio of the minimum received signal-to-interference power (S/I) from mobiles to the base station) is greater than 20 dB. You can ignore all interferers except those from the two nearest cells operating at the same frequency. [5]

- i. Propagation for both signal and interference follow a free-space model.
  - ii. Propagation for both signal and interference follow the simplified path loss model  $P_r = P_t K \left( \frac{d_0}{d} \right)^\gamma$  with  $d_0=100\text{m}$ ,  $K = 1$ , and  $\gamma = 3$ .
  - iii. Propagation for the signal follows the simplified path loss model with  $d_0 = 100\text{m}$ ,  $K = 1$ , and  $\gamma = 2$ , while propagation of the interference follows the same model but with  $\gamma = 4$ .
- (e) Find the median path loss under the Hata model assuming  $f_c = 900 \text{ MHz}$ ,  $h_t = 20\text{m}$ ,  $h_r = 5 \text{ m}$  and  $d = 100\text{m}$  for a large urban city, a small urban city, a suburb, and a rural area. [4]

Explain qualitatively the path loss differences for these 4 environments.

- (f) A car is travelling at a speed of  $v$  and a sinewave is transmitted at 900 MHz from a transmitter to a receiver in the car. The transmitter and receiver are located 4 m above the ground. The scattering function corresponding to a 2-ray model for the channel between the transmitter and receiver is given by

$$S(\tau, \rho) = \begin{cases} \alpha_1 \delta(\tau) & \rho = 35 \text{ Hz} \\ \alpha_2 \delta(\tau - 0.011 \mu\text{s}) & \rho = 33.7 \text{ Hz} \\ 0 & \text{else} \end{cases}$$

where the path gains  $\alpha_1$  and  $\alpha_2$  are determined by the path loss, and multipath fading. Determine how far the receiver is placed from the transmitter and also the speed of the car. [4]

**3.** Answer the following sub-questions.

- (a) Consider a cellular system where the power falloff with distance follows the formula  $P_r(d) = P_t (d_0/d)^\alpha$ , where  $d_0 = 100\text{m}$  and  $\alpha$  is a random variable. The distribution for  $\alpha$  is  $p(\alpha = 2) = .4$ ,  $p(\alpha = 2.5) = .3$ ,  $p(\alpha = 3) = .2$ , and  $p(\alpha = 4) = .1$ . Assume a receiver at a distance  $d = 1000\text{m}$  from the transmitter, with an average transmit power constraint of  $P_t = 100\text{ mW}$  and a receiver noise power of  $.1\text{ mW}$ . Assume both the transmitter and the receiver have Channel Side Information (CSI).
- Compute the distribution of the received SNR. [2]
  - Derive the optimal power control policy for this channel and its corresponding Shannon capacity per unit Hertz (C/B). [2]
  - Determine the zero-outage capacity per unit bandwidth of this channel. [1]
  - Determine the maximum outage capacity per unit bandwidth of this channel. [1]
- (b) Consider the interference channel shown in Figure 3.1.

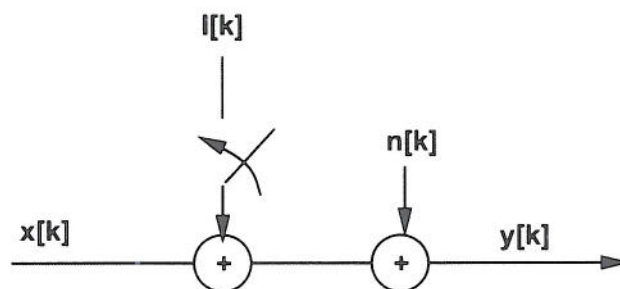


Figure 3.1: Interference channel model.

The channel has a combination of AWGN  $n[k]$  and interference  $I[k]$ . We model  $I[k]$  as AWGN. The interferer is on (i.e. the switch is down) with probability  $.25$  and off (i.e. the switch is up) with probability  $.75$ . The average transmit power is  $10\text{ mW}$ , the noise spectral density is  $10^{-8}\text{W/Hz}$ , the channel bandwidth  $B$  is  $10\text{ KHz}$  (receiver noise power is  $N_0B$ ), and the interference power (when on) is  $9\text{ mW}$ .

- What is the Shannon capacity of the channel if neither the transmitter nor receiver know when the interferer is on? [2]
- What is the capacity of the channel if both transmitter and receiver know when the interferer is on? [2]

- iii. Suppose now that the interferer is a malicious jammer with perfect knowledge of  $\mathbf{x}[k]$  (so the interferer is no longer modeled as AWGN). Assume that neither the transmitter nor receiver have knowledge of the jammer behavior. Assume also that the jammer is always on and has an average transmit power of 10 mW. What strategy should the jammer use to minimize the SNR of the received signal? [2]
- (c) Consider a Multiple-In-Multiple-Out (MIMO) High Speed Downlink Packet Access (HSDPA) system, with a multi-code CDMA transmission operating over a frequency selective multi-path channel. Explain how the mean system value is used to produce the system value upper bound for the capacity as a function of the receiver signal to noise ratio. [3]
- (d) Figure 3.2 shows the successive interference cancellation (SIC) HSDPA MIMO receiver. Describe the SIC HSDPA MIMO receiver operation. [5]

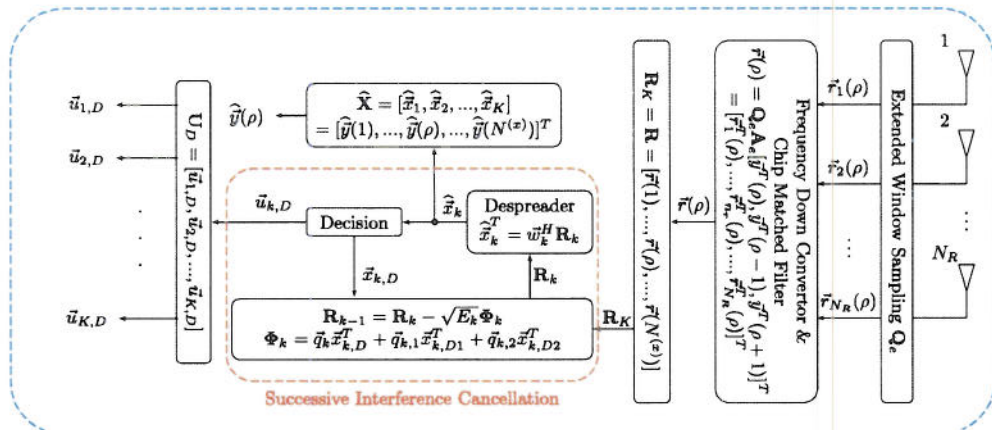


Figure 3.2: HSDPA-MIMO successive interference cancellation receiver.

- (e) For the SIC HSDPA MIMO system, with a multi-code CDMA transmission, operating over a frequency selective multi-path channel explain how the system values are used to iteratively adjust transmission energies. Explain how to calculate the covariance matrix inversions for each channel for a given total received signal to noise ratio. [5]



4. Answer the following sub-questions.

- (a) By considering the dual operation of GSM-UMTS systems, list the GSM network elements which can be reused in the UMTS system. [3]
- (b) The evolution of UMTS towards HSDPA progressed according to planned releases. By considering Release 99 and also Releases 4-10, outline how each release introduced new features to provide improvement upon previous releases. [3]
- (c) Explain how the three step cell search algorithm operates for the frame and slot synchronization in the current HSDPA system. [4]
- (d) The Table 4.1 outlines the receiver categories for the HSDPA Releases R10 and R11.

Release	Category	Number	QAM	MIMO MC	Code	Data MHz
R10	29	15	64	Triple Cell (TC)	.98	63.3
R10	30	15	64	TC MIMO	.98	126.6
R10	31	15	64	Quad Cell (QC)	.98	84.4
R10	32	15	64	QC MIMO	.98	168.8
R11	33	15	64	Hexa Cell (HC)	.98	126.6
R11	34	15	64	HC MIMO	.98	253.2
R11	35	15	64	Octa Cell (OC)	.98	168.8
R11	36	15	64	OC MIMO	.98	337.5

Table 4.1: HSDPA-MIMO specifications for releases R10 and R11.

- Using Table 4.1 explain how the category 29-36 receivers will provide different data rates using various HSDPA-MIMO configurations. [2]
- (e) In the current standards, 4G refers to the IMT-Advanced (International Telecommunications Advanced). Answer the following subquestions
  - i. List the requirements the IMT advanced cellular systems must fulfill. [4]
  - ii. State two basic technologies IMT Advanced is based on. [1]
  - iii. Describe the key features of all suggested 4G technologies. [5]
- (f) Figure 4.1 shows the Long Term Evolution system key parameters. Using these parameters explain how different bandwidth and resource blocks will be used to transmit data at different rates. [3]



## LTE key parameters

Frequency Range	UMTS FDD bands and UMTS TDD bands					
Channel bandwidth, 1 Resource Block=180 kHz	1.4 MHz	3 MHz	5 MHz	10 MHz	15 MHz	20 MHz
	6 Resource Blocks	15 Resource Blocks	25 Resource Blocks	50 Resource Blocks	75 Resource Blocks	100 Resource Blocks
Modulation Schemes	Downlink: QPSK, 16QAM, 64QAM Uplink: QPSK, 16QAM, 64QAM (optional for handset)					
Multiple Access	Downlink: OFDMA (Orthogonal Frequency Division Multiple Access) Uplink: SC-FDMA (Single Carrier Frequency Division Multiple Access)					
MIMO technology	Downlink: Wide choice of MIMO configuration options for transmit diversity, spatial multiplexing, and cyclic delay diversity (max. 4 antennas at base station and handset) Uplink: Multi user collaborative MIMO					
Peak Data Rate	Downlink: 150 Mbps (UE category 4, 2x2 MIMO, 20 MHz) 300 Mbps (UE category 5, 4x4 MIMO, 20 MHz) Uplink: 75 Mbps (20 MHz)					

Figure 4.1: LTE system key parameters.

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1. The answers are

(a) the following equation.

*Solution:* To obtain the reuse factor, we apply ~~with~~ with  $a_1 = .167$  and  $a_2 = 3$  to get

$$N \geq \frac{SIR_0}{a_1 a_2} = \frac{10}{.5} = 20.$$

Now setting  $G = B/B_s = 20 \times 10^6 / 100 \times 10^3 = 200$ , we get  $C_u = G/N = 10$  users per cell that can be accommodated. Typically  $\gamma > 2$ , ~~which is the minimum~~

(b)

$$R = 1 \text{ km}$$

$$D = 6 \text{ km}$$

$$i) N = \frac{A_{cluster}}{A_{cell}} = \frac{\sqrt{3}D^2/2}{3\sqrt{3}R^2/2} = \frac{1}{3}(D/R)^2 = \frac{1}{3}6^2 = 12$$

$$\text{number of cells per cluster} = N = 12$$

where  $G = B/B_s$  is the ratio of the total system bandwidth to the bandwidth required for an individual user.

$$ii) K = \frac{1200}{12} = 100 \text{ channels per cell.}$$

(c)

*Solution:* Treating interference as Gaussian noise, in Rayleigh fading we have  $\bar{P}_b \approx .25/SIR_0$  for  $SIR_0$  the average SIR ratio. The SIR required to meet the  $\bar{P}_b$  target is thus  $SIR_0 = .25/10^{-3} = 250$  (approximately 24 dB). Substituting  $SIR_0 = 250$ ,  $a_1 = .125$ ,  $a_2 = 4$ , and  $\gamma = 4$  into (15.7) yields

$$N \geq \frac{1}{4} \sqrt{\frac{250}{.25}} = 11.18.$$

So a reuse factor of  $N = 12$  meets the performance requirement. For the user capacity we have  $G = B/B_s = 200$ , so  $C_u = G/N = 16$  users per cell can be accommodated. Note that the Gaussian assumption for the interference is just an approximation, which becomes more accurate as the number of interferers grows by the Central Limit Theorem.

(d)

$$h_t = 20 \text{ m}$$

$$h_r = 3 \text{ m}$$

$$f_c = 2 \text{ GHz} \quad \lambda = \frac{c}{f_c} = 0.15$$

$$d_c = \frac{4h_t h_r}{\lambda} = 1600 \text{ m} = 1.6 \text{ Km}$$

This is a good radius for suburban cell radius as user density is low so cells can be kept fairly large. Also, shadowing is less due to fewer obstacles.

(e)

Think of the building as a plane in  $\mathbb{R}^3$

The length of the normal to the building from the top of Tx antenna =  $h_t$

The length of the normal to the building from the top of Rx antenna =  $h_r$

2. (a)

$$d = 100\text{m}$$

$$h_t = 10\text{m}$$

$$h_r = 2\text{m}$$

$$\text{delay spread} = \tau = \frac{x+x'-l}{c} = 1.33 \times$$

(b) The phase difference is  $\Delta\phi = \frac{2\pi(x'+x-l)}{\lambda}$  where

$$x' + x - l = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

which can be simplified to the form

$$x' + x - l = d \left( \sqrt{\left(\frac{h_t + h_r}{d}\right)^2 + 1} - \sqrt{\left(\frac{h_t - h_r}{d}\right)^2 + 1} \right)$$

using Taylor series approximation we have

$$x' + x - l = d \left( \sqrt{\left(\left(\frac{h_t + h_r}{2d}\right)^2 + 1\right)^2} - \sqrt{\left(\left(\frac{h_t - h_r}{2d}\right)^2 + 1\right)^2} \right)$$

simplifications give  $x' + x - l = \frac{2h_t h_r}{d}$  which results in  $\Delta\phi = \frac{2\pi(x'+x-l)}{\lambda} = \frac{2\pi 2h_t h_r}{\lambda d}$ .

(c)

$$P_{\text{noise}} = -160\text{dBm}$$

$$f_c = 1\text{GHz}, d_0 = 1\text{m}, K = (\lambda/4\pi d_0)^2 = 5.7 \times 10^{-4}, \lambda = 0.3, \gamma = 4$$

$$\text{We want } SNR_{\text{recd}} = 20\text{dB} = 100$$

$$\therefore \text{Noise power is } 10^{-19}$$

$$P = P_t K \left(\frac{d_0}{d}\right)^\gamma$$

$$10^{-17} = 10K \left(\frac{0.3}{d}\right)^4$$

$$d \leq 260.7\text{m}$$

(d)

d = distance between cells with reused freq  
p = transmit power of all the mobiles

$$\left(\frac{S}{I}\right)_{\text{uplink}} \geq 20dB$$

(i) Min. S/I will result when main user is at A and Interferers are at B  
 $d_A$  = distance between A and base station #1 =  $\sqrt{2}km$   $d_B$  = distance between B and base station #1 =  $\sqrt{2}km$

$$\left(\frac{S}{I}\right)_{\min} = \frac{P \left[\frac{G\lambda}{4\pi d_A}\right]^2}{2P \left[\frac{G\lambda}{4\pi d_B}\right]^2} = \frac{d_B^2}{2d_A^2} = \frac{(d_{\min} - 1)^2}{4} = 100$$

$\Rightarrow d_{\min} - 1 = 20km \Rightarrow d_{\min} = 21km$  since integer number of cells should be accommodated in distance d  $\Rightarrow d_{\min} = 22km$

(ii)

$$\frac{P_r}{P_u} = k \left[\frac{d_0}{d}\right]^\gamma \Rightarrow \left(\frac{S}{I}\right)_{\min} = \frac{Pk \left[\frac{d_0}{d_A}\right]^\gamma}{2Pk \left[\frac{d_0}{d_B}\right]^\gamma} =$$

$$\frac{1}{2} \left[\frac{d_B}{d_A}\right]^\gamma = \frac{1}{2} \left[\frac{d_{\min} - 1}{\sqrt{2}}\right]^\gamma = \frac{1}{2} \left[\frac{d_{\min} - 1}{\sqrt{2}}\right]^3 = 100$$

$\Rightarrow d_{\min} = 9.27 \Rightarrow$  with the same argument  $\Rightarrow d_{\min} = 10km$

(iii)

$$\left(\frac{S}{I}\right)_{\min} = \frac{k \left[\frac{d_0}{d_A}\right]^\gamma}{2k \left[\frac{d_0}{d_B}\right]^\gamma} = \frac{(d_{\min} - 1)^4}{0.04} = 100$$

$\Rightarrow d_{\min} = 2.41km \Rightarrow$  with the same argument  $d_{\min} = 4km$

(e)

$f_c = 900MHz, h_t = 20m, h_r = 5m, d = 100m$

Large urban city	$PL_{largecity} = 353.52dB$
small urban city	$PL_{smallcity} = 325.99dB$
suburb	$PL_{suburb} = 207.8769dB$
rural area	$PL_{ruralarea/countryside} = 70.9278dB$

As seen, path loss is higher in the presence of multiple reflectors, diffractors and scatterers

(f) The distance travelled by the LOS ray is d and the distance travelled by the first multipath component is

$$2\sqrt{\left(\frac{d}{2}\right)^2 + 4^2} - d = 0.011 \times 10^{-3} \times 3 \times 10^8 = 1.1$$



With this setup we have

$$\begin{aligned} 4 \left( \left( \frac{d}{2} \right)^2 + 4^2 \right) &= (d + 1.1)^2 \\ &= d^2 + 2.2d + 1.1^2 \end{aligned}$$

Hence  $d = 28.541m$ . The Doppler frequency is  $f_D = \frac{v}{\lambda}$  for the LOC component as  $f_D = 35$  the car speed is  $v = \frac{f_D}{3} = 11.66$  m/s. For the reflected signal we have  $\cos \theta = \frac{28.541}{2\sqrt{\left(\frac{28.541}{2}\right)^2 + 4^2}} = \frac{14.27}{14.82} = 0.9629$ . We have

$$f_D = \frac{v \times \cos \theta}{\lambda} = \frac{11.667 \times 0.9629}{\frac{1}{3}} = 33.7 \text{ m/s.}$$

3. (a)

$$SNR_{recvd} = \frac{P_{\gamma}(d)}{P_{noise}} = \begin{cases} 10dB & w.p. 0.4 \\ 5dB & w.p. 0.3 \\ 0dB & w.p. 0.2 \\ -10dB & w.p. 0.1 \end{cases}$$

Assume all channel states are used

$$\frac{1}{\gamma_0} = 1 + \sum_{i=1}^4 \frac{1}{\gamma_i} p_i \Rightarrow \gamma_0 = 0.4283 > 0.1 \quad \therefore \text{not possible}$$

Now assume only the best 3 channel states are used

$$\frac{0.9}{\gamma_0} = 1 + \sum_{i=1}^3 \frac{1}{\gamma_i} p_i \Rightarrow \gamma_0 = 0.6742 < 1 \quad \therefore \text{ok!}$$

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} 1.3832 & \gamma = \gamma_1 = 10 \\ 1.1670 & \gamma = \gamma_2 = 3.1623 \\ 0.4832 & \gamma = \gamma_3 = 1 \\ 0 & \gamma = \gamma_4 = 0.1 \end{cases}$$

$$C/B = 2.3389bps/Hz$$

$$\sigma = 0.7491$$

$$C/B = \log_2(1 + \sigma) = 0.8066bps/Hz$$

$$\left(\frac{C}{B}\right)_{max} = 2.1510bps/Hz \text{ obtained by using the best 2 channel states.}$$

$$\text{With } p_{out} = 0.1 + 0.2 = 0.3$$

(b)

- a) If neither transmitter nor receiver knows when the interferer is on, they must transmit assuming worst case, i.e. as if the interferer was on all the time,

$$C = B \log \left( 1 + \frac{\bar{S}}{N_0 B + \bar{I}} \right) = 10.7Kbps.$$

- b) Suppose we transmit at power  $S_1$  when jammer is off and  $S_2$  when jammer is off,

$$C = B \max \left[ \log \left( 1 + \frac{S_1}{N_0 B} \right) 0.75 + \log \left( 1 + \frac{S_2}{N_0 B + \bar{I}} \right) 0.25 \right]$$

subject to

$$0.75S_1 + 0.25S_2 = \bar{S}.$$

This gives  $S_1 = 12.25mW$ ,  $S_2 = 3.25mW$  and  $C = 53.21Kbps$ .

(c) The jammer should transmit  $-x(t)$  to completely cancel off the signal.

$$\bar{S} = 10\text{mW}$$

$$N_0 = .001 \mu\text{W/Hz}$$

$$B = 10 \text{ MHz}$$

Now we compute the SNR's as:

$$\gamma_j = \frac{|H_j|^2 \bar{S}}{N_0 B}$$

$$\text{This gives: } \gamma_1 = \frac{|1|^2 10^{-3}}{0.001 \times 10^{-6} \times 10 \times 10^6} = 1, \gamma_2 = .25, \gamma_3 = 4, \gamma_4 = 0.0625$$

To compute  $\gamma_0$  we use the power constraint as:

$$\sum_j \left( \frac{1}{\gamma_0} - \frac{1}{\gamma_j} \right)_+ = 1$$

First assume that  $\gamma_0 < 0.0625$ , then we have

$$\begin{aligned} \frac{4}{\gamma_0} &= 1 + \left( \frac{1}{1} + \frac{1}{.25} + \frac{1}{4} + \frac{1}{.0625} \right) \\ \Rightarrow \gamma_0 &= .1798 > 0.0625 \end{aligned}$$

So, our assumption was wrong. Now we assume that  $0.0625 < \gamma_0 < .25$ , then

$$\begin{aligned} \frac{3}{\gamma_0} &= 1 + \left( \frac{1}{1} + \frac{1}{.25} + \frac{1}{4} \right) \\ \Rightarrow \gamma_0 &= .48 > 0.25 \end{aligned}$$

So, our assumption was wrong again. Next we assume that  $0.25 < \gamma_0 < 1$ , then

$$\begin{aligned} \frac{2}{\gamma_0} &= 1 + \left( \frac{1}{1} + \frac{1}{4} \right) \\ \Rightarrow \gamma_0 &= .8889 < 1 \end{aligned}$$

This time our assumption was right. So we get that only two sub-bands each of bandwidth 10 MHz are used for transmission and the remaining two with lesser SNR's are left unused.

Now, we can find capacity as:

$$C = \sum_{j: \gamma_j \geq \gamma_0} B \log_2 \left( \frac{\gamma_j}{\gamma_0} \right)$$

This gives us,  $C = 23.4 \text{ Mbps}$ .

- (c) An HSDPA MIMO scheme constructs a unique covariance matrix  $\mathbf{C}_k$  for  $k = 1, \dots, K^*$  using the iterative covariance matrix relationship to calculate  $\mathbf{C}_k^{-1}$  for use in the detection process. The iterative matrix inversion method is also used to produce the mean square error  $\varepsilon_k$  and the system values  $\lambda_k = 1 - \varepsilon_k$ , for  $k = 1, \dots, K^*$  for use in the maximization of the summation of the discrete transmission rate  $b_T = \sum_{k=1}^{K^*} b_{p_k}$  where  $b_{p_k}$  is the discrete number of bits allocated to each spreading sequence symbol for  $k = 1, \dots, K^*$ . The system values  $\lambda_k$  will also be used for the optimum allocation of energies subject to the energy constraint  $\sum_{k=1}^{K^*} E_k \leq E_T$ . The mean system value will

be  $\lambda_{mean} = \frac{\lambda_T}{K^*} = \frac{\sum_{k=1}^{K^*} \lambda_k}{K^*}$  where the total system value  $\lambda_T$  has its maximum value when  $E_k = \frac{E_T}{K^*}$  for  $k = 1, \dots, K^*$ . For the MMSE receivers, the total system capacity is  $C_T = \sum_{k=1}^{K^*} \log_2 \left( 1 + \frac{\lambda_k}{\Gamma(1-\lambda_k)} \right) \simeq K^* \log_2 \left( 1 + \frac{\lambda_{mean}}{\Gamma(1-\lambda_{mean})} \right)$  where  $\Gamma$  is the gap value. In  $C_T$ , the multiplication of the total channel number with the capacity corresponding to the mean system value  $\lambda_{mean}$  gives a very close approximation to the total capacity.

- (d) An SIC receiver structure is used to reduce the receiver detection complexity. In the SIC receiver implementation, the received signal vectors  $\vec{r}(\rho)$  are collected for  $\rho = 1, \dots, N^{(x)}$  to form the received signal matrix  $\mathbf{R} = [\vec{r}(1), \dots, \vec{r}(N^{(x)})]$ . The receiver is operated by setting  $\mathbf{R}_{K^*} = \mathbf{R}$  to produce a  $N_R(N+L-1) \times N^{(x)}$  dimensional reduced data matrix  $\mathbf{R}_{k-1}$  iteratively using  $\mathbf{R}_{k-1} = \mathbf{R}_k - \sqrt{E_k} \Phi_k$  for  $k = K^*, (K^* - 1), \dots, 1$ . The  $N_R(N+L-1) \times N^{(x)}$  dimensional matrix  $\Phi_k$  is given by  $\Phi_k = \vec{q}_k \vec{x}_{k,D}^T + \vec{q}_{k,1} \vec{x}_{k,D1}^T + \vec{q}_{k,2} \vec{x}_{k,D2}^T$ . The size  $N^{(x)}$  column vector  $\vec{x}_{k,D}$  is the detected data stream and  $\vec{x}_{k,D1} = \mathbf{J}_{N^{(x)}} \vec{x}_{k,D}$  and  $\vec{x}_{k,D2} = \mathbf{J}_{N^{(x)}}^T \vec{x}_{k,D}$  are the row vectors containing ISI symbols received in the previous and the next symbol periods respectively. The contribution of the detected data stream  $\vec{x}_{k,D}$  to the reduced signal matrix  $\mathbf{R}_k$  for channel  $k$  is estimated using  $\sqrt{E_k} \Phi_k$ . The estimated symbol vector  $\vec{x}_{k,D}$  is generated by using each MMSE despreading vector  $\vec{w}_k$  to yield a despread signal vector of  $\hat{\vec{x}}_k^T = \vec{w}_k^H \mathbf{R}_k$  and also an estimate of the corresponding transmitted bit stream  $\hat{u}_k$ . The decoded bit vectors  $\hat{u}_k$  are re-coded at the receiver and re-modulated to regenerate the transmitted symbol vector  $\vec{x}_{k,D}$  at the output of the decision device.
- (e) The SIC based MMSE receiver will have the system values for  $k = 1, \dots, K^*$  as  $\lambda_k = E_k \vec{q}_k^H \mathbf{C}_k^{-1} \vec{q}_k$ . The system value  $\lambda_k$  is reorganized to simplify the signal to noise ratio  $\gamma_k = \frac{\lambda_k}{1-\lambda_k} = E_k \vec{q}_k^H \mathbf{D}_k^{-1} \vec{q}_k$  where  $\mathbf{D}_k = \mathbf{C}_{k-1} + E_k \vec{q}_{k,1} \vec{q}_{k,1}^H + E_k \vec{q}_{k,2} \vec{q}_{k,2}^H = \mathbf{C}_k - E_k \vec{q}_k \vec{q}_k^H$ . This algorithm will dynamically adjust the energies  $E_k$  for  $k = 1, \dots, K^*$  and also  $K^*$  by starting at  $K^* = K$  to return the allocated energies and the numbers of the ordered signature sequences as the elements of the size  $K^*$  vector  $\vec{k}_{order}$ . At the start a set of system values  $\lambda_k$  for  $k = 1, \dots, K^*$  will be produced as the elements of the size  $K^*$  vector  $\vec{\lambda} = [\lambda_1, \dots, \lambda_{K^*}]$  using  $E_k = \frac{E_T}{K^*}$  and  $\mathbf{C}^{-1}$  for a system without an SIC. Signature sequences are ordered for the smallest to largest system values. To find the maximum discrete rate  $b_p$  that maximizes the system throughput  $R_T = K b_p$  in practical systems,  $\lambda_{mean}$  can be used to find the target system value as a function of the discrete bit rate  $\lambda^*(b_p)$ . The bit value which maximizes the system throughput for a practical system is found by comparing  $\lambda^*(b_p)$  with  $\lambda_{mean}$  until the following inequality is satisfied



$\lambda^*(b_p) \leq \lambda_{mean} < \lambda^*(b_{p+1})$ . Thus the achievable total rate  $R_T = Kb_p$  is also found prior to energy allocation. For the SIC systems  $\mathbf{C}_0$  is initialized to only include the noise component  $2\sigma^2$ . Thus  $\mathbf{C}_0^{-1} = \frac{1}{2\sigma^2} \mathbf{I}_{N_R(N+L+1)}$ . The iterative energy allocation in the SIC formulation only requires variable  $E_{k,(i-1)}$  and  $\mathbf{C}_{(k-1)}^{-1}$ . The covariance matrix inverse  $\mathbf{C}_k^{-1}$  is only required after the energy of the  $k^{th}$  channel converges. The energy will be allocated by using the variables  $N, L, N_R, \sigma^2$  and an updated using the vectors  $\vec{q}_k, \vec{q}_{k,1}, \vec{q}_{k,2}$  and  $\vec{k}_{order}$  and also using either the matrix  $\mathbf{C}_k^{-1}$  for the system under consideration with the SIC. The index number of the  $k^{th}$  smallest element of  $\vec{\lambda}$  will be used to re-order the sequences  $\vec{q}_k, \vec{q}_{k,1}$  and  $\vec{q}_{k,2}$ , the allocated energies  $E_k$  and the elements of the vector  $\vec{k}_{order}$ . Here we describe the formulation of the recursive covariance matrix inverse  $\mathbf{C}_k^{-1}$ , and the calculation of  $E_k$  based on  $\mathbf{C}_{k-1}^{-1}$ . The recursive covariance matrix inverse  $\mathbf{C}_k^{-1}$  is express in terms of a linear combination of weighted vectors, covariance matrix inversion of the previous channel  $\mathbf{C}_{k-1}^{-1}$  (or weighted identity matrix inverse  $\mathbf{C}_0^{-1} = \frac{1}{2\sigma^2} \mathbf{I}_{(N+L-1)}$  for the first channel) and the allocated energy for the current channel  $E_k$ . Defining distance vectors of  $\vec{d} = \mathbf{C}_{k-1}^{-1} \vec{q}_k, \vec{d}_1 = \mathbf{C}_{k-1}^{-1} \vec{q}_{k,1}, \vec{d}_2 = \mathbf{C}_{k-1}^{-1} \vec{q}_{k,2}$  and weights  $\xi_1, \xi_2, \xi_3, \xi_4$  and  $\zeta, \zeta_1, \zeta_2$  as follows:  $\xi = \vec{d}^H \vec{q}_k, \xi_1 = \vec{d}_1^H \vec{q}_{k,1}, \xi_2 = \vec{d}_2^H \vec{q}_{k,2}, \xi_3 = \vec{d}^H \vec{q}_{k,1}, \xi_4 = \vec{d}^H \vec{q}_{k,2}$  and  $\zeta_1 = \frac{E_k}{1+E_k\xi_1}, \zeta_2 = \frac{E_k}{1+E_k\xi_2}, \zeta = \frac{E_k}{1+\Gamma(2^b p - 1)}$ . Iterative energy calculation for the  $k^{th}$  channel can be simplified to  $E_{k,i} =$

$$\frac{\gamma_k^*}{\xi - E_{k,(i-1)} \left( \frac{|\xi_3|^2}{1 + E_{k,(i-1)}\xi_1} + \frac{|\xi_4|^2}{1 + E_{k,(i-1)}\xi_2} \right)}$$

the inverse of the recursive covariance matrix  $\mathbf{C}_k^{-1}$  is given by

$$\begin{aligned} \mathbf{C}_k^{-1} = & \mathbf{C}_{k-1}^{-1} - \zeta \vec{d} \vec{d}^H - (\zeta_1 + \zeta \zeta_1^2 |\xi_3|^2) \vec{d}_1 \vec{d}_1^H \\ & - (\zeta_2 + \zeta \zeta_2^2 |\xi_4|^2) \vec{d}_2 \vec{d}_2^H \\ & + \zeta \zeta_1 \left( \xi_3 \vec{d} \vec{d}_1^H + \xi_3^* \left( \vec{d} \vec{d}_1^H \right)^H \right) \\ & + \zeta \zeta_2 \left( \xi_4 \vec{d} \vec{d}_2^H + \xi_4^* \left( \vec{d} \vec{d}_2^H \right)^H \right) \\ & - \zeta \zeta_1 \zeta_2 \left( \xi_3 \xi_4^* \vec{d}_2 \vec{d}_1^H + (\xi_3 \xi_4^*)^* \left( \vec{d}_2 \vec{d}_1^H \right)^H \right) \end{aligned}$$

4. (a) From GPRS network, the following network elements can be reused: Home Location Register (HLR), Visitor Location Register (VLR), Equipment Identity Register (EIR), Mobile Switching Center (MSC) (vendor dependent), Authentication Center (AUC), Serving GPRS Support Node (SGSN) (vendor dependent), Gateway GPRS Support Node (GGSN)
- (b) **Release 99:** 64 kbit/s circuit switch, 384 kbit/s packet switched, Location services, Call services: compatible with Global System for Mobile Communications (GSM), based on Universal Subscriber Identity Module (USIM) Voice quality features - Tandem Free Operation.  
**Release 4:** Edge radio, Multimedia messaging, MExE (Mobile Execution Environment), Improved location services, IP Multimedia Services (IMS).  
**Release 5 :**IP Multimedia Subsystem (IMS), IPv6, IP transport in UTRAN, Improvements in GERAN, MExE, etc. HSDPA.  
**Release 6 :**WLAN integration, Multimedia broadcast and multicast, Improvements in IMS, HSUPA, Fractional DPCH.  
**Release 7 :**Enhanced L2, 64 QAM, MIMO, Voice over HSPA, CPC - continuous packet connectivity, FRLC - Flexible RLC.  
**Release 8 :** DC-HSPA (Dual Cell HSDPA for SISO), HSUPA 16QAM, peak throughput was increased by introducing higher order modulation.  
**Release 9 :**Dual Cell HSDPA introduced for SISO in release 8, was extended to include MIMO.  
**Release 10:** The concept of DC-HSDPA was further extended to include more cells (more carriers). Four carrier HSDPA was introduced.
- (c) **Three step search algorithm:** Primary channel codes are correlated with the incoming signals to identify the starting instants for the time slots. 16 different correlators corresponding to the secondary channel sequences are correlated with the incoming signals to identify which correlator gives the peak value for each slot. Using the sequence of the correlator numbers the group number for the scrambling code sequence is identified for the cell where the UE is. Using the group number 8 sequences corresponding to the sequences in the group are correlated with the incoming signal to identify the scrambling sequence for the cell and also to provide frame synchronization.
- (d) All receivers in categories 29-36 are able to use 64 QAM. Category 29 receiver uses triple cells to achieve up to 64 MHz data rate. Category 30 uses MIMO with TC to achieve 127 MHz data rate. ....
- (e) i. An IMT-Advanced cellular system must fulfill the following requirements:
  - Based on an all-IP packet switched network.

- Peak data rates of up to approximately 100 Mbit/s for high mobility such as mobile access and up to approximately 1 Gbit/s for low mobility such as nomadic/local wireless access.
  - Dynamically share and use the network resources to support more simultaneous users per cell.
  - Scalable channel bandwidth 5–20 MHz, optionally up to 40 MHz.
  - Peak link spectral efficiency of 15 bit/s/Hz in the downlink, and 6.75 bit/s/Hz in the uplink (meaning that 1 Gbit/s in the downlink should be possible over less than 67 MHz bandwidth).
  - System spectral efficiency of up to 3 bit/s/Hz/cell in the downlink and 2.25 bit/s/Hz/cell for indoor usage.
  - Smooth handovers across heterogeneous networks.
  - Ability to offer high quality of service for next generation multimedia support.
- ii. the International Telecommunication Union (ITU) as 4G candidates. Basically all proposals are based on two technologies:
- LTE Advanced standardized by the 3GPP
  - 802.16m standardized by the IEEE (i.e. WiMAX)
- iii. The following key features can be observed in all suggested 4G technologies:
- Physical layer transmission techniques are as follows:
    - \* MIMO: To attain ultra high spectral efficiency by means of spatial processing including multi-antenna and multi-user MIMO
    - \* Frequency-domain-equalization, for example multi-carrier modulation (OFDM) in the downlink or
    - \* single-carrier frequency-domain-equalization (SC-FDE) in the uplink: To exploit the frequency selective channel property without complex equalization
  - Frequency-domain statistical multiplexing, for example (OFDMA) or (single-carrier FDMA) (SC-FDMA, a.k.a. linearly precoded OFDMA, LP-OFDMA) in the uplink: Variable bit rate by assigning different sub-channels to different users based on the channel conditions
  - Turbo principle error-correcting codes: To minimize the required SNR at the reception side
  - Channel-dependent scheduling: To use the time-varying channel



- Link adaptation: Adaptive modulation and error-correcting codes
  - Mobile-IP utilized for mobility
  - IP-based femtocells (home nodes connected to fixed Internet broadband infrastructure)
  - As opposed to earlier generations, 4G systems does not support circuit switched telephony.
  - Most. 4G standards lack soft-handover support, also known as cooperative relaying.
- iv. In 1.4 MHz bandwidth we have 6 resource blocks and guard bands. Each resource block can be used to transmit up to 6 bits per symbols. In 3 MHz bandwidth we have 15 resource blocks. In 5 MHz bandwidth we have 25 resource blocks. In 15 and 25 MHz bandwidths we have 75 and 100 resource blocks respectively.