EE4-57
EE9-CS2-2

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected Copy

DISCRETE-EVENT SYSTEMS

Friday, 22 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D. Angeli
Second Marker(s) : E.C. Kerrigan

1. A robot moves on a circular platform with 5 positions, one labeled 0, corresponding to its center, and 4 labeled, respectively, A, B, C and D, corresponding to the 4 cardinal points (North, South, East, West). Movements normally occur along the North-South direction, events n and s, respectively, or along the East-West direction, events e and w, respectively. Occasionally the robot moves diagonally in a clockwise direction, event d.

a) Build a finite deterministic automaton G that models the movements of the robot, assuming it is in position 0 at initial time. [4]

b) Design next a labeling automaton G_L whose state is meant to keep track of whether events of type d have occurred or not. [3]

c) Compute the concurrent composition $G_L||G$. [3]

d) Assume that events of type d are unobservable. Build a *diagnoser* automaton, which is able to classify strings in $\{n,s,e,w\}^*$ by returning whether or not an event of type d has occurred (yes/maybe/not). [6]

e) Assume that the diagnoser automaton marks all strings in $\{n,s,e,w\}^*$ for which a d event has definitely occured in the original automaton G. Denote this language by L_D. Find the minimal automaton G_M such that $\mathscr{L}_m(G_M) = L_D$. [4]

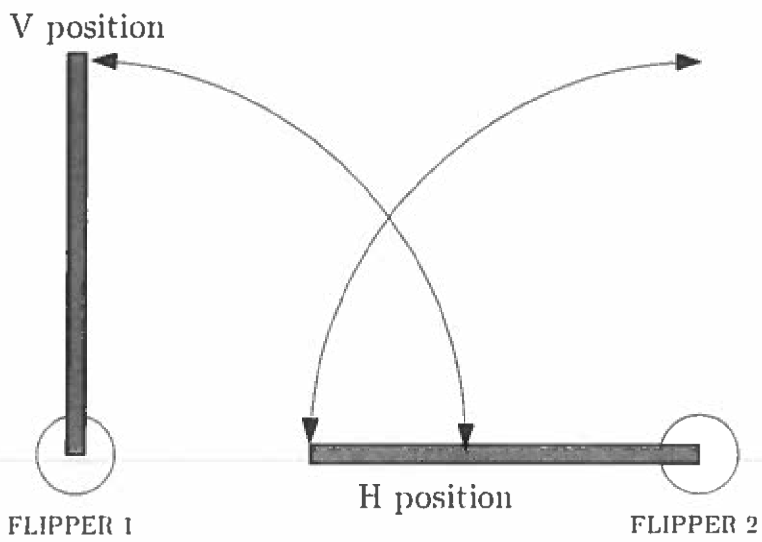

Figure 2.1 Flippers layout

2. A "flipper" is a device that can move instantaneously between two positions, vertical and horizontal, as shown in Fig. 2.1, by rotating 90 degrees. Assume that two flippers are available, F_1 and F_2, and that their rotation (from vertical to horizontal and vice versa) is associated with events f_1 and f_2, respectively.

a) Build finite deterministic automatons F_i, $i = 1,2$ that keep track of the positions of the individual flippers, as triggered by events f_i. Assume that both automata start out in the vertical position. [3]

b) Build a finite deterministic automaton F describing the simultaneous evolutions of both flippers. How can you express this automaton in terms of the original automatons F_1, F_2? [3]

c) Assume that the flippers are arranged as in the Figure 2.1; in particular, their tips are in contact when both flippers are in the horizontal position. Build an automaton G_{sp} that generates the language corresponding to the following specification: flippers are allowed to be both horizontal, but the last flipper to reach the horizontal position has to be the first one to rotate to vertical, so as to avoid collisions during the rotation. [4]

d) Assume that event f_2 is uncontrollable (that is, flipper 2 moves independently of our control). Is $\mathscr{L}(G_{sp})$ a controllable specification? (Justify your answer). [4]

e) Find the maximal controllable sublanguage $\mathscr{L}(G_{sp})^{\uparrow C}$ and represent it as the generated language of a suitable automaton. [6]

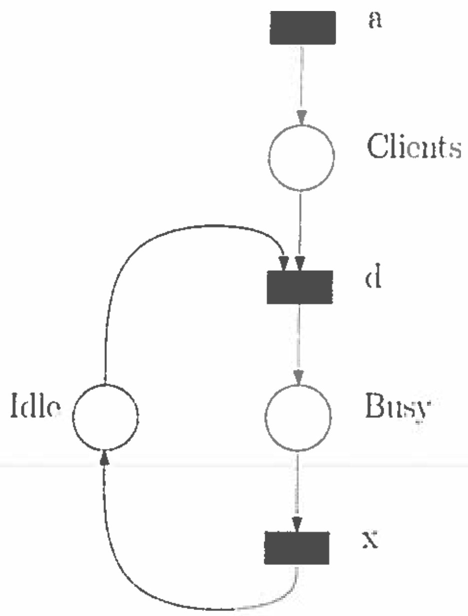

Figure 3.1 Single queue Petri Net

3. The Petri Net in Figure 3.1 represents a simple model of a queue feeding to a single server, where transition a models arrivals, transition d models departures from the queue and transition x clients who have been served.

a) Inspired by the layout of a single queue, build a model of two queues in parallel, with each one feeding to his own server. [2]

b) Modify the previous network and design a marked Petri Net so that a capacity constraint is enforced that only allows for n clients in total to be waiting in the two queues. Which is the appropriate initial marking? [4]

c) Is the net designed in part b) structurally bounded? Justify your answer by computing the P-semiflows of the net. [4]

d) Modify the model in part b) by allowing clients from each queue to be served from the other server provided the other server is not busy, the other queue is empty and the total office capacity has been occupied. [4]

e) Is the original Petri Net in Figure 3.1 structurally bounded? Does it have structurally bounded places? Justify your answer. [2]

f) Build the coverability graph of the Petri Net in Figure 3.1 for initial marking $[0, 1, 0]^T$, (where places are in the following order: Clients, Idle, Busy). [4]

4. A car rental company (with only 1 car for the time being) receives requests for cars to be hired at time instants randomly distributed according to an exponential probability distribution of rate λ_a. If the car is not available the request is declined, whereas if the car is available, it is instantaneously rented out. The same car, usually, returns to the base after some time of utilization distributed according to an exponential probability distribution of rate λ_b. This happens unless a fault of the car occurs, in which case the car is diverted to a mechanics workshop, fixed, and returned to the customer. Assume that faults and fixing times also occur in agreement with an exponential probability distribution of rate λ_f and λ_w, respectively.

a) Model the stochastic automaton that results by means of a continuous time Markov chain that keeps track of the time spent by the car at the base, with customers or in the workshop. [3]

b) Write the Kolmogorov equations that govern the evolution of the probability of being in each of the states of the Markov chain described in part a). [4]

c) Is the chain ergodic? (Justify your answer). Compute the fraction of time the car is being used by a customer. [4]

d) How would you modify the Markov chain if the company had two cars taking into account that the rate at which cars are requested won't be affected? [4]

e) In the case of a single car, how much time does it take in average for a hired car to be returned to the base (counting from the moment the car is hired and taking into account the possibility of faults)? [5]