E1.2 Digital Electronics 1
Solutions 2008

All questions are unseen.

Answer to Question 1 (Compulsory)

a) i)

$$
\begin{aligned}
& ABC+\overline{A}CD+\overline{B}CD \\
&= C(AB+\overline{A}D+\overline{B}D) \\
&= C(AB+D(\overline{A}+\overline{B})) \\
&= C(AB+\overline{AB}D) \\
&= C(AB+D) \\
&= ABC+CD
\end{aligned}
$$

[4]

ii)

$$
\begin{aligned}
& AB+(C+\overline{B})(AB+\overline{C}) \\
&= AB+C(AB+\overline{C})+\overline{B}(AB+\overline{C}) \\
&= AB+ABC+\overline{B}\,\overline{C} \\
&= AB(1+C)+\overline{B}\,\overline{C} \\
&= AB+\overline{B}\,\overline{C}
\end{aligned}
$$

[4]

b)

X \ YZ	00	01	11	10
0	0	1	1	0
1	1	1	0	1

$$
\begin{aligned}
& F(X,Y,Z) \\
&= \overline{X}Z+X\overline{Z}+X\overline{Y} \quad or \\
&= \overline{X}Z+X\overline{Z}+\overline{Y}Z
\end{aligned}
$$

[4]

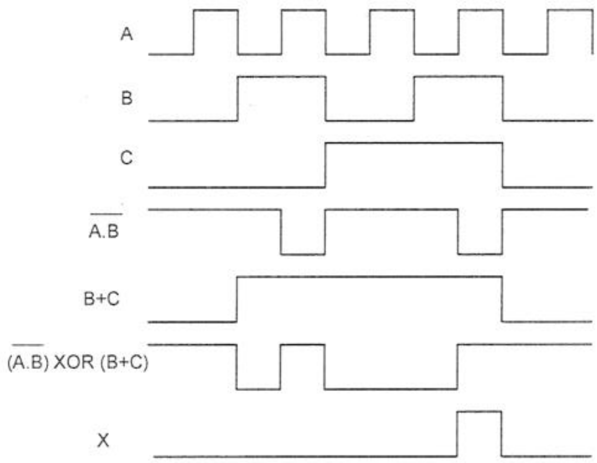
c)

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

$$F=(\overline{A}+\overline{B})(\overline{C}+\overline{D})(\overline{B}+D)$$

[4]

d)



[4]

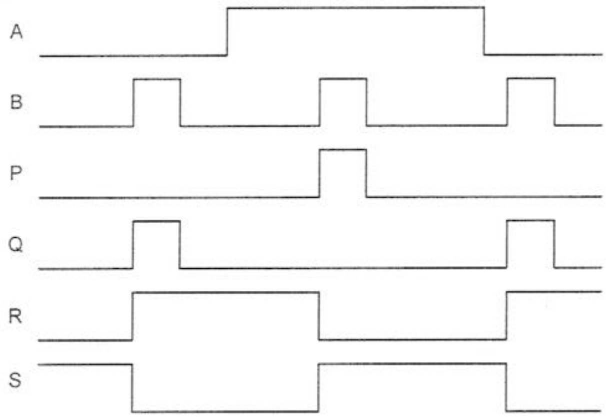
e)

Binary	Hexadecimal	Unsigned Decimal	Signed Decimal	ASCII
0100 0110		70		**‘F’**
	D4	**212**	**-44**	

2 marks each.

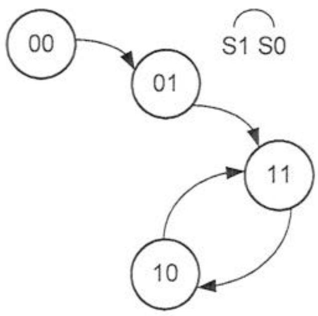
[8]

f) P is reset input (high active), and Q is set input to the flip-flop (R=1 is set state). The two AND gates and the inverter forms a steering circuit, steering the clock pulse on B depending on value of A. When A is high, the clock pulse is steered to reset input and when A is low, the clock pulse is steered to the set input. Hence the waveforms are:



[6]

g)



[6]

Answer to Question 2

a)

B_{in} \ PQ	00	01	11	10
0	0	1	0	1
1	1	0	1	0

D

B_{in} \ PQ	00	01	11	10
0	0	1	0	0
1	1	1	1	0

B_{out}

$$D = \overline{P}\overline{Q}B_{in} + P\overline{Q}\overline{B}_{in} + \overline{P}Q\overline{B}_{in} + PQB_{in} \qquad B_{out} = \overline{P}Q + B_{in}Q + B_{in}\overline{P}$$

[8]

b) Need to expresses both equations as NOR or inverter functions only:

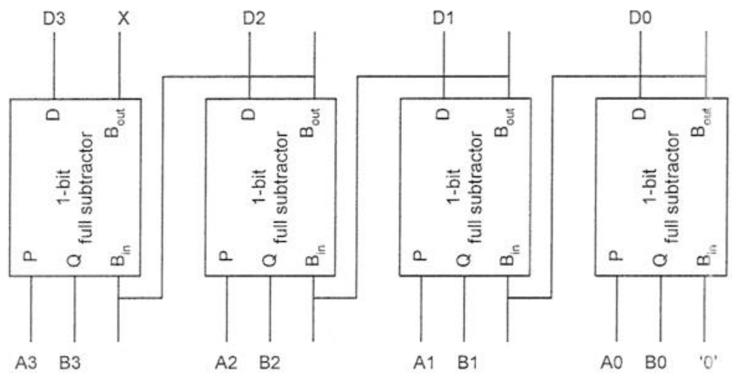
$$\begin{aligned} D &= \overline{P}\overline{Q}B_{in} + P\overline{Q}\overline{B}_{in} + \overline{P}Q\overline{B}_{in} + PQB_{in} \\ &= \overline{\overline{\overline{P}\overline{Q}B_{in} + P\overline{Q}\overline{B}_{in} + \overline{P}Q\overline{B}_{in} + PQB_{in}}} \\ &= \overline{\overline{\overline{\overline{\overline{P}\overline{Q}B_{in}}} + \overline{\overline{P\overline{Q}\overline{B}_{in}}} + \overline{\overline{\overline{P}Q\overline{B}_{in}}} + \overline{\overline{PQB_{in}}}}} \\ &= \overline{\overline{\overline{(P+Q+\overline{B}_{in})} + \overline{(\overline{P}+Q+B_{in})} + \overline{(P+\overline{Q}+B_{in})} + \overline{(\overline{P}+\overline{Q}+\overline{B}_{in})}}} \end{aligned}$$

[4]

$$\begin{aligned} B_{out} &= \overline{P}Q + B_{in}Q + B_{in}\overline{P} \\ &= \overline{\overline{\overline{P}Q + B_{in}Q + B_{in}\overline{P}}} \\ &= \overline{\overline{\overline{\overline{\overline{P}Q}} + \overline{\overline{B_{in}Q}} + \overline{\overline{B_{in}\overline{P}}}}} \\ &= \overline{\overline{\overline{(P+\overline{Q})} + \overline{(\overline{B}_{in}+\overline{Q})} + \overline{(\overline{B}_{in}+P)}}} \end{aligned}$$

[4]

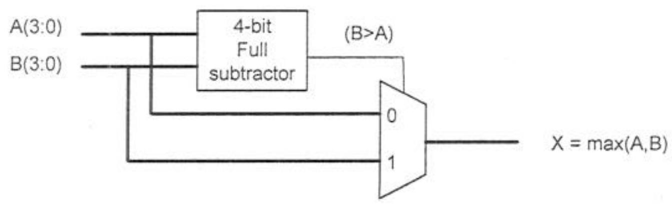
c)



$$X = Borrow\_out \;\; from \;\; MSB = (B > A)$$

[8]

d)



[6]

Answer to Question 3

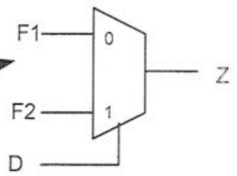
a)

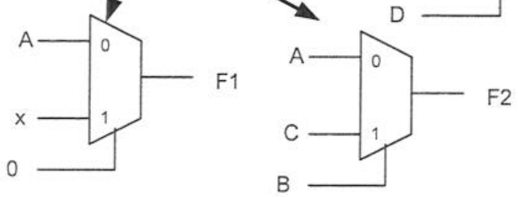
$$
\begin{aligned}
Z &= (D\overline{C} + 1 \bullet C)\overline{B} + (D\overline{A} + 1 \bullet A)B \\
&= (D + C)\overline{B} + (D + A)B \\
&= \overline{B}D + \overline{B}C + BD + AB \\
&= AB + BC + D
\end{aligned}
$$

[10]

b) Need to express the Boolean equation is the form: $Z = F_1\overline{D} + F_2D$

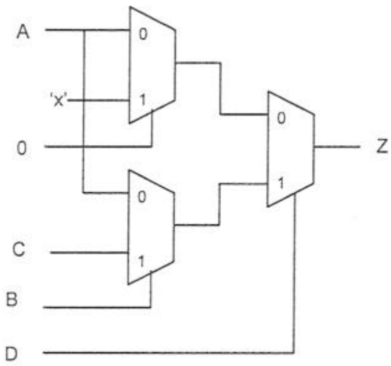
$$
\begin{aligned}
Z &= A\overline{D} + A\overline{B}D + BCD \\
&= A\overline{D} + (A\overline{B} + CB)D \\
&= F_1\overline{D} + F_2D
\end{aligned}
$$





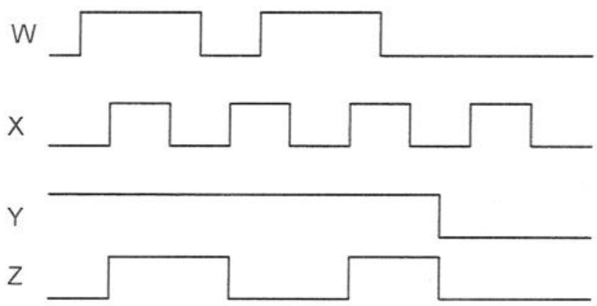
Hence to implement the required equation, we want:

A0 = B0 = A, A1 = ‘x’, SA = ‘0’, B1 = C, SB = B, S =D



[10]

c) This is a positive edge trigger D flip-flop, with asynchronous clear. Data input = W, clock input is X, clear control is Y.



[10]

Answer to Question 4

a)

A	B	X	NextA	NextB	Y
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	0
1	1	1	0	0	0

[6]

b)

NextA	X \ AB	00	01	11	10
	0	0	1	1	1
	1	0	0	0	1

$$NextA = B\overline{X} + A\overline{B}$$

NextB	X \ AB	00	01	11	10
	0	0	0	1	0
	1	1	1	0	1

$$NextB = \overline{A}X + \overline{B}X + AB\overline{X}$$

Y	X \ AB	00	01	11	10
	0	0	0	0	0
	1	1	0	0	1

$$Y = \overline{B}X$$

[14]

c)

This new encoding (known as one-hot encoding) allows very simple implementation of next state equations and outputs. By inspection:

$$NextS1 = S0\,X$$
$$NextS2 = S1\,\overline{X}$$
$$NextS3 = S2\,X$$
$$Next\ S0 = S3\,X$$

$$Y = S0\,X + S2\,X$$

[10]