Paper Number(s): **E3.13**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE PART III/IV: B.Eng., M.Eng. and ACGI

ELECTRICAL ENERGY SYSTEMS

Thursday, 25 April 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

Corrected Copy

Time allowed: 3:00 hours.

Examiners responsible:

First Marker(s): Pal,B. and Popovic,D.
Second Marker(s): Popovic,D. and Pal,B.

Special instructions for invigilators: None

Information for candidates: None

1. (a) Describe briefly the influence of voltage magnitude, voltage angle and reactive power transfer on real power transfer between two active sources connected by an inductive reactance.

[8]

(b) For the system shown in Figure 1, all quantities are per-phase per-unit values. Calculate the reactive power injected by a capacitor at bus 2 so that $|V_2| = 1$pu. In this case, what is the phase angle of the voltage at bus 2?

[12]

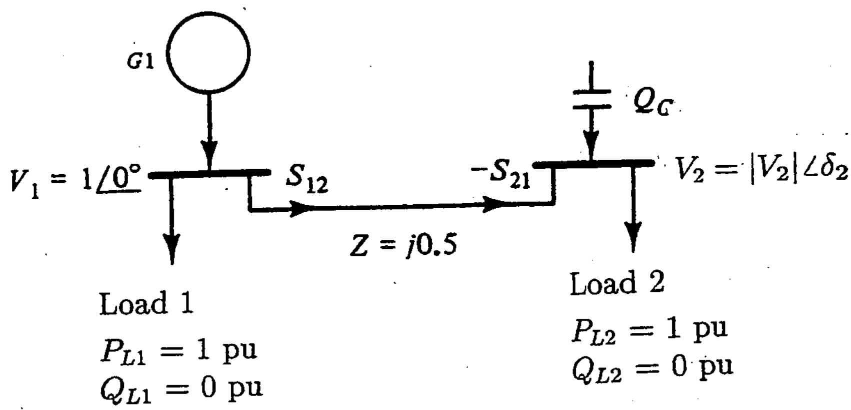

Figure 1.1

2. A 300km, 500kV three-phase transmission line has a series reactance $x = 0.34\Omega/\text{km}$ and a shunt admittance $y = j4.5 \cdot 10^{-6}\text{S/km}$. Line losses are neglected.

   (a) Calculate surge impedance Z_c, propagation constant γl, the ABCD parameters, the wavelength λ of the line, and the surge impedance loading in MW. [8]

   (b) Rated line voltage is applied to the sending end of the line. Calculate the receiving-end voltage when the receiving end is terminated by

   (i) an open circuit [3]

   (ii) the surge impedance of the line [3]

   (iii) one-half of the surge impedance. [3]

   (c) Calculate the theoretical maximum real power that the line can deliver when rated voltage is applied to both ends of the line. [3]

3. For the power system shown in Figure 2, the per-unit admittance matrix Y_{BUS} is given by

$$Y_{BUS} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 \\ -2+j6 & 2.5-j7.5 & -0.5+j1.5 \\ -1+j3 & -0.5+j1.5 & 1.5-j4.5 \end{bmatrix}.$$

The per-unit bus voltages and injections are also given in the Figure.

(a) For each bus k, specify the bus type, and determine which of the variables V_k, δ_k, P_k and Q_k are input data and which are unknowns.

[5]

(b) Assume an initial estimate of $V_2 = 1\angle 0^o$ and $\delta_3 = 0^o$, and calculate the bus real and reactive power mismatches to be used in the first iteration of the Newton-Raphson power flow method.

[10]

(c) Set up the linearized system of equations $J\,\Delta x = \Delta y$ that are solved at each iteration of the Newton-Raphson power flow method. Do not solve the equations.

[5]

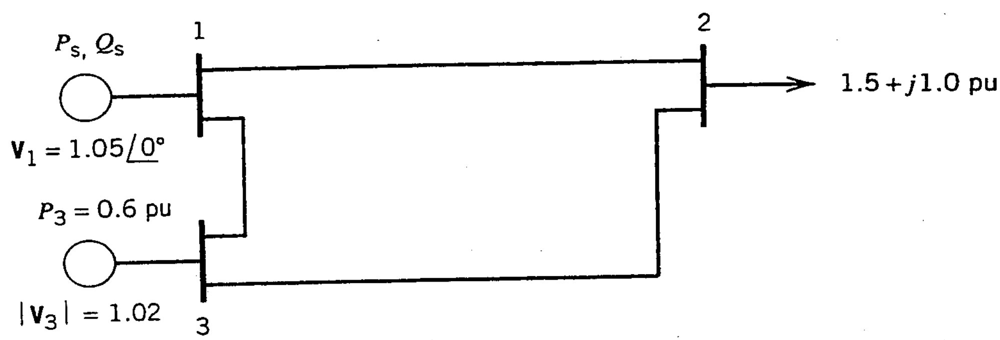

Figure 3.1

4. (a) Why is it so important to optimally allocate power generations to various units in a power system?

[5]

(b) Three units with the following specifications are listed:

Unit 1: Coal fired:
Max output = 1200 MW
Min output = 350 MW
Input-output curve:

$$H_1 = 510 + 7.2P_1 + 0.00142P_1^2 \quad \text{(Mbtu/h)}$$

Unit 2: Oil fired:
Max output = 1000 MW
Min output = 300 MW
Input-output curve:

$$H_2 = 310 + 7.85P_2 + 0.00142P_2^2 \quad \text{(Mbtu/h)}$$

Unit 3: Oil fired:
Max output = 800 MW
Min output = 200 MW
Input-output curve:

$$H_3 = 178 + 7.97P_3 + 0.00482P_3^2 \quad \text{(Mbtu/h)}$$

Fuel costs for the three units are 1.2 R/Mbtu, 1.0 R/Mbtu and 1.1 R/Mbtu respectively. 'R' is any fictitious currency symbol.

Determine the economic operating point for these three units when delivering a total of 2000 MW.

[15]

5. (a) What are the various methods of voltage control?

[6]

(b) Generator A of rating 200 MW and generator B of rating 350 MW have governor droops of 5 per cent and 8 per cent, respectively, from no load to full load. They are the only supply to an isolated system whose nominal frequency is 50 Hz. The corresponding generator speed is 3000 rpm. Initially, generator A is at 0.5 p.u load and generator B is at 0.65 p.u. load, both running at 50 Hz. Find the no-load speed of each generator if it is disconnected from the system. Also determine the output of B when A reaches its rating.

[14]

6. (a) V_a, V_b and V_c are phase voltages and V_0, V_1 and V_2 are sequence components in a three phase system. If S_{ph} and S_s are powers in phase and symmetrical components respectively, show that $S_{ph} = 3S_s$. [6]

(b) Equipment ratings for the five-bus power system shown in Figure 3 are as follows:
Generator G1: 50 MVA, 12 kV, $X'' = 0.2$ per unit
Generator G2: 100 MVA, 15 kV, $X'' = 0.2$ per unit
Transformer T1: 50 MVA, 12 kV Y/138 kV Y, $X = 0.1$ per unit
Transformer T2: 100 MVA, 15 kV Y/138 kV Y, $X = 0.1$ per unit 12 & 42
Each 138-kV line: X_l= 40Ω. Δ
A three-phase short circuit occurs at bus 5, where the pre-fault voltage is 15kV. Pre-fault load current is neglected.

(i) Draw the positive-sequence reactance diagram in per-unit on a 100-MVA, 15kV base. [8]
(ii) Determine the Thévenin equivalent at the fault. [4]
(iii) Determine the subtransient fault current in per-unit and in kA. [2]

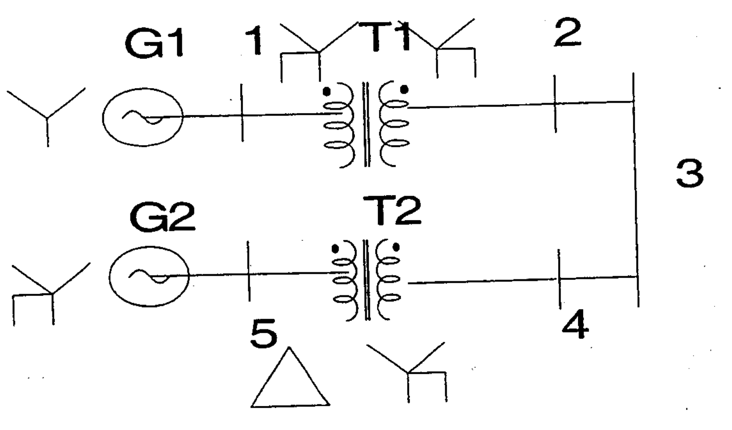

Figure 6.1

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Model Answers and Mark Schemes

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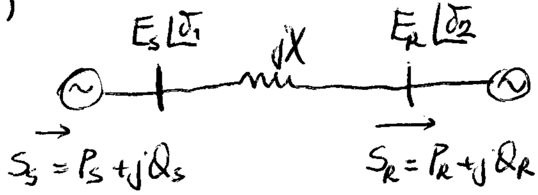
B. PAL (Ques 4, 5, 6)

Paper Code: E3.13
ELECTRICAL ENERGY SYSTEMS

Second Examiner:
B. PAL (Ques 1, 2, 3)
D. POPOVIĆ (Ques 4, 5, 6)

Question Number etc. in left margin | Mark allocation in right margin

1 a)



1) P transfer depends mainly on the angle between source voltages 2

2) Q transfer depends mainly on voltage magnitudes 2

3) Q cannot be transmitted over long distances (large voltage gradient would be required) 2

4) an increase in Q transfer causes an increase in P, Q losses 2

b) $S_S = E_S I^* = V_1 I^* = V_1\left(\frac{V_1 - V_2}{Z}\right)^*$

$S_R = E_R I^* = V_2 I^* = V_2\left(\frac{V_1 - V_2}{Z}\right)^* = -\frac{|V_2|^2}{|Z|}e^{j\angle Z} + \frac{|V_1||V_2|}{|Z|}e^{j\angle Z}\cdot e^{-j\delta_{12}}$

$V_1 = |V_1|\angle\delta_1 \qquad \delta_{12} = \delta_1 - \delta_2 \qquad Z = jX = Xe^{j90^\circ}$

$V_2 = |V_2|\angle\delta_2$

$\Rightarrow P_R = \frac{|V_1|\,|V_2|}{X}\sin\delta_{12}$ 2

$Q_R = \frac{|V_1|\,|V_2|}{X}\cos\delta_{12} - \frac{|V_2|^2}{X}$ 2

} received at bus 2

$P_R = P_{L2} = 1\ pu$ 1

$Q_R = Q_{L2} - Q_C = 0 - Q_C = -Q_C$ 1

$\Rightarrow Q_C = \frac{|V_2|^2}{X} - \frac{|V_1|\,|V_2|}{X}\cos\delta_{12}$

$P_R = \frac{|V_1||V_2|}{X}\sin\delta_{12} = \frac{1\cdot 1}{0.5}\sin\delta_{12} = 2\sin\delta_{12}$ 2

$P_R = 1\,pu$

$\Rightarrow 2\sin\delta_{12} = 1 \Rightarrow \delta_{12} = 30^\circ \Rightarrow \delta_1 - \delta_2 = 30^\circ \Rightarrow \delta_2 = -30^\circ$ 2

$Q_C = \frac{|V_2|^2}{X} - \frac{|V_2|\,|V_1|}{X}\cos\delta_{12} = 2 - 2\cos 30^\circ = 0,268$ 2

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Model Answers and Mark Schemes

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2 a) $Z_c = \sqrt{\frac{z}{y}} = 274.9\ \Omega$ 1

$\gamma l = \sqrt{zy}\ l = j0.3711\ \text{pu}$ 1

$\gamma l = j(\beta l),\quad \beta l = 0.3711$

$A = D = \cos(\beta l) = 0.9319 \angle 0^\circ\ \text{pu}$ 2

$B = \underbrace{jZ_c \sin(\beta l)}_{X'} = j274.9 \sin(0.3711) = j99.69\ \Omega$ 1

$C = j\frac{1}{Z_c} \sin(\beta l) = j1.319 \cdot 10^{-3}\ \Omega$ 1

$\lambda = \frac{2\pi}{\beta} = 5079\ \text{km}$ 1

$SIL = \frac{V_{rated}^2}{Z_c} = \frac{500^2}{274.9} = 909.4\ \text{MW}\ (3\phi)$ 1

b) (i) $V_R = \frac{V_S}{A} = \frac{500}{0.9319} = 536.5\ \text{kV}$ 3

(ii) $V_R = V_S = 500\ \text{kV}$ 3

(iii) $V_S = \cos(\beta l)\ V_R + jZ_c \sin(\beta l)\ \frac{V_R}{\frac{1}{2}Z_c}$

$= [\cos(\beta l) + j2\sin(\beta l)]\ V_R$ 2

$\Rightarrow V_R = \frac{500}{\cos 0.3711 + j2\sin 0.3711} = 423.4\ \text{kV}$ 1

c) Theoretical maximum real power:

$P_{max} = \frac{V_S V_R}{X'} = \frac{500 \cdot 500}{99.69} = 2508\ \text{MW}$ 3

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3 a)

bus	type	input data	unknown
1	swing	$V_1,\ \delta_1$	$P_1,\ Q_1$
2	load	$P_2,\ Q_2$	$V_2,\ \delta_2$
3	generator	$P_3,\ V_3$	$\delta_3,\ Q_3$

5

b) Power flow equations

$$P_K = V_K \sum_{n=1}^{N} V_n \left(G_{Kn} \cos(\delta_K - \delta_n) + B_{Kn} \sin(\delta_K - \delta_n) \right)$$

$$Q_K = V_K \sum_{n=1}^{N} V_n \left(G_{Kn} \sin(\delta_K - \delta_n) - B_{Kn} \cos(\delta_K - \delta_n) \right)$$

$$\delta_{Kn} = \delta_K - \delta_n$$

$$\Delta y = y - f(x)$$

$$y = \begin{bmatrix} P_2 \\ P_3 \\ Q_2 \end{bmatrix} = \begin{bmatrix} P_{G2} - P_{L2} \\ P_{G3} - P_{L3} \\ Q_{G2} - Q_{L2} \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0.6 \\ -1 \end{bmatrix} \text{pu}$$ 1

$K=2$:

$$P_2(0) = V_2 \left[V_1 (G_{21} \cos\delta_{21} + B_{21} \sin\delta_{21}) + V_2 G_{22} + V_3 (G_{23} \cos\delta_{23} + B_{23} \sin\delta_{23}) \right] = -0.11 \text{pu}$$ 2

$k=3$:

$$P_3(0) = V_3 \left[V_1 (G_{31} \cos\delta_{31} + B_{31} \sin\delta_{31}) + V_2 (G_{32} \cos\delta_{32} + B_{32} \sin\delta_{32}) + V_3 G_{33} \right] = -0.02 \text{pu}$$ 2

$$Q_2(0) = V_2 \left[V_1 (G_{21} \sin\delta_{21} - B_{21} \cos\delta_{21}) - V_2 B_{22} + V_3 (G_{23} \sin\delta_{23} - B_{23} \cos\delta_{23}) \right] = -0.33 \text{pu}$$ 2

$$\Delta P_2 = P_2 - P_2(0) = -1.5 + 0.11 = -1.39 \text{pu}$$ 1

$$\Delta P_3 = P_3 - P_3(0) = 0.6 - (-0.02) = 0.62 \text{pu}$$ 1

$$\Delta Q_2 = Q_2 - Q_2(0) = -1 - (-0.33) = -0.67 \text{pu}$$ 1

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3 (cont)

c) $J \cdot \Delta x = \Delta y$ 1

$$\Delta x = \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix}, \qquad \Delta y = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} -1.39 \\ 0.62 \\ -0.67 \end{bmatrix}$$ 2

$$J = \begin{bmatrix} \frac{dP_2}{d\delta_2} & \frac{dP_2}{d\delta_3} & \frac{dP_2}{dV_2} \\ \frac{dP_3}{d\delta_2} & \frac{dP_3}{d\delta_3} & \frac{dP_3}{dV_2} \\ \frac{dQ_2}{d\delta_2} & \frac{dQ_2}{d\delta_3} & \frac{dQ_2}{dV_2} \end{bmatrix}$$ 2

20

MODEL ANSWERS

3:13: Electrical Energy Systems
Dr. Bikash Pal

Q4 (a)

In thermal power plants, fuels (oil, gas and coal) used are very costly. The price fluctuates with international market conditions couple with demand and supply. The amount of fuel used in generating power in the range of GW is astronomical. A slight variation in price causes billions of dollars of hike in fuel bills to the power companies. The cost of electric energy production rises accordingly. This has the knock on effect on the cost of finished products from industries where electricity is one of the key inputs. This suggests that various units in a system must share their outputs in an efficient manner. This is why it is so important to have optimum allocations of generation. [5 marks]

(b) Convert input-output curve into cost curve multiplying through fuel cost.

This would give rise to :

$$C_1 = 5C_1 = 510 + 8.64\,P_1 + 0.001704\,P_1^2$$
$$C_2 = 310 + 7.85\,P_2 + 0.00142\,P_2^2$$
$$C_3 = 178 + 7.97\,P_3 + 0.00482\,P_3^2$$

[2 marks]

The incremental costs (R/MW) is

$$\frac{dC_1}{dP_1} = 8.64 + 0.003408\,P_1 = \lambda$$
$$\frac{dC_2}{dP_2} = 7.85 + 0.00284\,P_2 = \lambda$$
$$\frac{dC_3}{dP_3} = 7.97 + 0.00964\,P_3 = \lambda$$
$$P_1 + P_2 + P_3 = 2000$$

[3 marks]

Solve for P_1, P_2, P_3, λ

Ans: $P_1 = 647.1, P_2 = 1054, P_3 = 298, (MW)\ \lambda = 10.8$ [5]

Note Unit #2 exceeds maximum limit, hence $P_2 = 1000MW$;

Unit #1 and 3 have to provide 1000 MW in optimal fashion

Following can be used to solve for P_1, P_3, λ

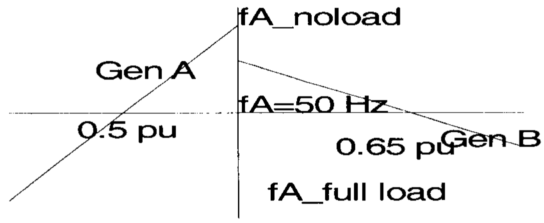
$$\frac{dC_1}{dP_1} = 8.64 + 0.003408\,P_1 = \lambda$$
$$\frac{dC_3}{dP_3} = 7.97 + 0.00964\,P_3 = \lambda$$
$$P_1 + P_3 = 2000$$

ans; $P_1 = 687.46\ MW,\ P_3 = 312.53\ MW;\ \lambda = 10.98$ [5 marks]

Q5 (a) The students are expected to write a couple of sentences on the following:

Injection of reactive power (shunt capacitors/reactors, series capacitors, synchronous compensators), tap changing transformers, booster transformer, phase shifting transformer, Flexible AC transmission system (FACT) devices such as Static Var compensator (SVC), Static compensator (STATCOM), Unified power flow controllers (UPFC) etc. [6 marks]

(b) Generator A and B respectively loaded to 0.5 p.u. and 0.65 p.u of full load. This corresponds to 50 Hz or 3000 rpm.
A typical representative (not complete or to scale) droop characteritic can be drawn to help solve the problem.



Say at this droop setting and power output of Gen A, the no load frequency is $f_A^{'}$ and full load frequency is $f_A^{''}$ and at 0.5 p.u it is as stated in the problem f_A (1.p.u or 50 Hz). For Gen B the no load frequency is $f_B^{'}$ and full load frequency is $f_B^{''}$ and at 0.5 p.u it is as stated in the problem f_B (1.p.u or 50 Hz). With the help of droop relationship, the following expression can be written:

$$\frac{100}{200} = \frac{f_A^{'} - f_A}{f_A^{'} - f_A^{''}} = \frac{f_A^{'} - f_A}{0.05} \qquad \text{[7 marks]}$$
$$f_A^{'} = 1.025 f_A \rightarrow 3075 rpm$$

Similarly for Gen B

$$\frac{227.5}{350} = \frac{f_B^{'} - f_B}{f_B^{'} - f_B^{''}} = \frac{f_B^{'} - f_B}{0.08} \qquad \text{[3 marks]}$$
$$f_B^{'} = 1.052 f_B \rightarrow 3156 rpm$$

$f_A^{''}$ corresponding to 200 MW loading on Gen A can be found from the above expression to be $0.975 f_A$. This has to be the frequency of Gen B as well. Corresponding to this frequency, the output of Gen B can be found from the solution of P_B

$$\frac{227.5}{P_B} = \frac{1.052 f_B - f_B}{1.052 f_B - 0.975 f_B} = 0.675; \qquad \text{[4 marks]}$$
$$P_B = 337 MW$$

Q6

(a) The expression for power in a three-phase circuit is

$$S_{ph} = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = V_{ph}^{t} I_{ph}^{*} \qquad \text{[2 marks]}$$

The phase and sequence components are connected through transformation matrix

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \text{ i.e } V_{ph} = TV_s \quad S_{ph} = (TV_s)^t (TI_s)^*$$

$$= \left(T^t T^*\right) V_s^{\,t} I_s^{\,*} = 3S_s \quad \text{[4 marks]}$$

(b) The first task is to express every parameter in common base of 100 MVA and 15 kV on G2. For transmission line base kV 138. For G1 and T1 it is 12 kV.

The following relationship has to be used to arrive at p.u. value on a common base.

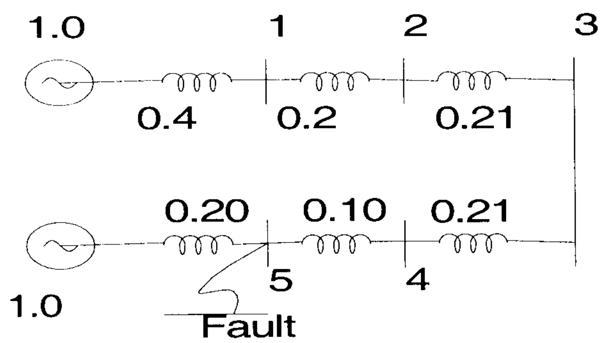
$$Z_{new}(pu) = Z_{old} * \frac{Base - MVA_{new}}{Base - MVA_{old}} \left(\frac{Base - kV_{old}}{Base - kV_{new}} \right)^2 pu$$

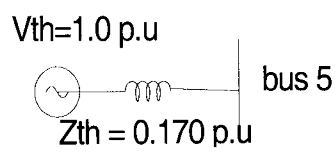
This produces positive sequence reactance in p.u for
Gen #1: 0.40; Gen #2 : 0.2; Transformer #1 : 0.2; Transformer #2: 0.1pu,

(i) Base impedance in ohm in 138kV line side is 138*138/100; Positive sequence reactance of line in each segment (between bus #2 & bus #3 and bus #2 & bus #4 is 0.21 p.u. [8 marks]

(ii) The prefault voltage at bus 5 is 15kV and so it will be 1.0 p.u, since load current is neglected, the thevenin voltage would be 1.0 p.u, The Thevenin's reactance is 0.170 p.u. [4 marks]

(iii) The fault current is 1/.170 = 5.89 pu. The base current at bus 5 is 100/(1.732*15) kA = 3.85 kA; So the fault current in kA = 5.89*3.85 = 22.67 kA. [2 marks]
The positive sequence network and Thevenin equivalent is shown in the diagrams





The credits for drawing positive sequence diagram and Thevenin's diagram are included in the marking of part (i) and (ii)