Q1 a i) Total calls per hour 280 calls/h → 4.66 calls/m

Average duration of call 3 m.

Offered traffic = 4.66 × 3 = 13.98 Erlangs 3

ii) N = 21 from traffic capacity Erlang chart 4

iii) Loss probability = B_c = 0.002

Carried traffic = 13.98 [1 − 0.002] = 13.95 Erlangs 3

Q1 b i) $\lambda = 173$ calls / h $1/\mu = 20$ s $= E(S)$

$\lambda = 2.88333$ calls / m $1/\mu = 0.333$ m

$\rho = 0.96111$

$$E[Q_t] = \frac{\rho^2}{1-\rho} =$$ 3

ii) $$E[W] = \frac{\rho}{1-\rho} E(S) =$$ 3

iii) $$E[Q_t^2] = \sum i \rho (1-\rho)\rho^i = \rho \sum i^2 (1-\rho)\rho^i$$

$$= \rho\left(\rho\left(\frac{2}{(1-\rho)^2} - \frac{1}{1-\rho}\right)\right) = \rho^2\left(\frac{2}{(1-\rho)^2} - \frac{1}{1-\rho}\right)$$

$$\mathrm{Var}[Q_t] = \rho^2\left(\frac{2}{(1-\rho)^2} - \frac{1}{1-\rho}\right) - \left(\frac{\rho^2}{1-\rho}\right)^2$$

$$= \frac{\rho^2}{(1-\rho)^2}\left(1 + \rho - \rho^2\right) =$$ 4

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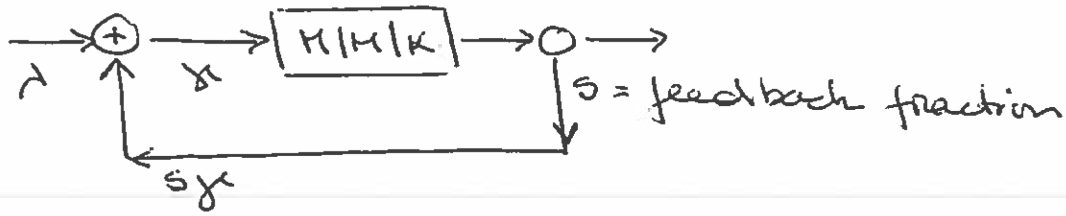
Q2 a) i)



Total arrival rate $\gamma = \lambda + s\gamma \Rightarrow \gamma = \dfrac{\lambda}{1-s}$

If μ is the service rate per channel :

offered load per channel $= \dfrac{\gamma}{K\mu} = \dfrac{\lambda}{K\mu(1-s)}$

for a stable system: $\dfrac{\lambda}{K\mu(1-s)} < 1 \Rightarrow \dfrac{\lambda}{K\mu} < 1-s$

$\rho = \dfrac{\lambda}{K\mu}$ (offered traffic entering the system) $\Rightarrow \rho < 1-s \Rightarrow s < 1-\rho$ 4

ii) $K = 1$ a) without feedback $E(N_t) = \dfrac{\rho}{1-\rho}$ $\rho = \dfrac{\lambda}{\mu}$

a) with feedback $E(N_t) = \dfrac{\rho'}{1-\rho'}$ $\rho' = \dfrac{\gamma}{\mu} = \dfrac{\lambda}{\mu(1-s)}$

$= \dfrac{\rho}{(1-s-\rho)}$ 6

Q2 b) Link speed 64000 bits/s

message service rate $\mu = \dfrac{64000}{700}\ s^{-1}$ $\rho_M = \dfrac{\lambda}{\mu} = 0.875 = \rho_2$

message arrival rate $\lambda = 80\ s^{-1}$ 2

Acknowledgement service time $= \dfrac{2\times 32}{64000}\ s = 1\ ms$

$\rho_A = 80 \times 0.001$
$= 0.08 = \rho_1$

for a non-pre-emptive priority system: 2

mean message waiting time $= \dfrac{R}{(1-\rho_1)(1-\rho_1-\rho_2)} =$

$R = \frac{1}{2}\left[\lambda_1 E[S_1^2] + \lambda_2 E(S_2^2)\right]$

$\frac{1}{2}\left[\dfrac{\lambda_1}{\mu_1^2} + \dfrac{\lambda_2}{\mu_2^2}\right] = 4.83\ ms$ $= \dfrac{4.83}{0.92 \times 0.045}$ 6

$= 0.1166\ s$

Q3 a) Mean value analysis of M|G|1 system

define R_i = residual service time = time until first departure seen by ith arrival

$$E[W_i] = E[R_i] + E[Q_i\, E[S]]$$
$$= E[R_i] + E[Q_i]\, E[S]$$

since Poisson arrivals see an unbiassed sample of queue behaviour

$$E[W] = E[R] + E[Q]\, E[S] \qquad \text{but also } E[Q] = \lambda E[W]$$

$$E[W] = \frac{E[R]}{1-\rho} \qquad \rho = \lambda E[S]$$

$$E[R_t] = \lim_{T\to\infty} \frac{1}{T}\int_0^T R_t\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \sum_{i=1}^{M_T} \left(\frac{1}{2} s_i^2\right) = \lim_{T\to\infty} \frac{1}{2}\left(\frac{M_T}{T}\right)\left[\frac{1}{M_T}\sum_{i=1}^{M_T} s_i^2\right]$$

(M_T is the number of completed services in $[0,T]$)

$$E[R_t] = \frac{1}{2}\lambda E[S^2]$$

$$E[W] = \frac{\lambda E[S^2]}{2(1-\rho)}$$ 4

for an M|D|1 system

S = constant $= h \Rightarrow E[S^2] = h^2$ $(\mathrm{Var}(S) = 0)$

$$E[W] = \frac{\rho}{2(1-\rho)} h = \frac{\rho}{2(1-\rho)} E[S]$$ 4

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Q3 b)

$$F_i(t+\Delta t, x) = [N-(i-1)]\lambda\Delta t\, F_{i-1}(t,x) + (i+1)\alpha\Delta t\, F_{i+1}(t,x) + \{1-[(N-i)\lambda + i\alpha]\Delta t\} F_i[t, x - \underbrace{(i-c)\alpha\Delta t}_{\Delta x}] + o(\Delta t)$$

$$-F_i(t,x) = -F_i(t,x)$$

$$F_i(t+\Delta t, x) - F_i(t,x) = h_1\Delta t\, F_{i-1}(t,x) + h_2\Delta t\, F_{i+1}(t,x) + h_3\Delta t\, F_i(t, x-\Delta x) + F_i(t,x-\Delta x) - F_i(t,x)$$

$$\frac{F_i(t+\Delta t,x) - F_i(t,x)}{\Delta t} = h_1 F_{i-1}(t,x) + h_2 F_{i+1}(t,x) + h_3 F_i(t,x-\Delta x) + \frac{F_i(t,x-\Delta x) - F_i(t,x)}{\Delta x}\,\frac{\Delta x}{\Delta t}$$

$\Delta t, \Delta x \to 0$

$$\frac{\partial F_i(t,x)}{\partial t} = h_1 F_{i-1}(t,x) + h_2 F_{i+1}(t,x) + h_3 F_i(t,x-\Delta x) - (i-c)\alpha\frac{\partial F_i(t,x)}{\partial x}$$ 6

$$h_1 = [N-(i-1)]\lambda, \quad h_2 = (i+1)\alpha, \quad h_3 = -[(N-i)\lambda + i\alpha]$$

ii) In statistical equilibrium: $\frac{\partial F_i(x,t)}{\partial t} = 0$ and $F_i(t,x) \to F_i(x)$

$$(i-c)\alpha\frac{dF_i(x)}{dx} = [N-(i-1)]\lambda F_{i-1}(x) - [(N-i)\lambda + i\alpha]F_i(x) + (i+1)\alpha F_{i+1}(x)$$

in matrix form:

$$\frac{dF(x)}{dx}D = F(x)M \Rightarrow \frac{dF(x)}{dx} = F(x)M'$$ 4

iii) This expression represents a set of first-order linear differential equations for which the solution is well-known to be a sum of exponentials 2

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Q4 a

i) ON-OFF source traffic source:

R_p = peak Rate
$1/\beta$ = average burst length
$1/\alpha$ = average silence length

probability source is ON $p = \frac{\alpha}{\alpha+\beta}$

Equivalent capacity $= C_L = mR_p + k\sigma R_p$

$m R_p$ = mean $\Rightarrow m = Np$ for N ON-OFF sources

σR_p = standard deviation $\Rightarrow \sigma^2 = m(1-p)$ for N ON-OFF sources

$k = k(\text{QoS})$

$$C = \frac{C_L}{R_p} = m + k\sigma$$
$$= Np + k\sqrt{Np(1-p)} \quad \text{and } k \sim P_L \text{ or } E$$

$$P_L = \sum_{i=J_0}^{N} \frac{(i-C)\pi_i}{m} \qquad E = \sum_{j=j}^{N} \pi_i$$

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ii) For a large number of sources multiplexed, for $N \gg 1$, $p \ll 1$:

$$\pi_i = \binom{N}{i} p^i (1-p)^{N-i} \quad \text{(binomial)}$$

can be approximated quite closely by the normal distribution ($m = Np$, $\sigma^2 = Np(1-p)$)

$$E = \int_{J_0}^{\infty} \frac{e^{-\frac{(x-m)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}\, dx$$

if $(C-m) > 3\sqrt{2}\,\sigma$

$$E = \frac{\sigma e^{-\frac{(C-m)^2}{2\sigma^2}}}{\sqrt{2\pi}\,(C-m)}$$

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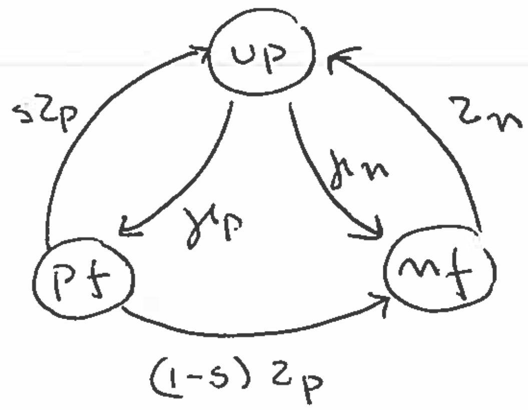
Q4 v) i) $E = \{\text{up, process failure, node failure}\}$ 3

ii)



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iii)

$$
\begin{array}{c|ccc}
 & \text{up} & \text{pf} & \text{nf} \\
\hline
\text{up} & -(\gamma_p+\gamma_n) & \gamma_p & \gamma_n \\
\text{pf} & s z_p & -z_p & (1-s) z_p \\
\text{nf} & z_n & \phi & -z_n
\end{array}
= Q
$$

3