UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1996

MEng Honours Degrees in Computing Part IV
MSc Degree in Foundations of Advanced Information Technology
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute

PAPER 4.80

AUTOMATED THEOREM PROVING
Wednesday, May 8th 1996, 10.00 - 12.00

Answer THREE questions

For admin. only: paper contains
4 questions
3 pages (excluding cover page)

(In all questions x, y, z are variables and are implicitly universally quantified. Other argument names are constants.)

1a Explain briefly how a set of rewrite rules R might be used to prove s = t for some ground terms s and t.

Why must the rules R be terminating and confluent?

b i) Show that the set of rules {(1), (2)} is locally confluent and terminating.

(1) v(f(x)) => f(v(x))
(2) v(a) => a

ii) Show {(1), (2), (3)} is not locally confluent but is still terminating.

(3) v(v(x)) => f(x)

Describe and apply a method to generate a set of confluent rules from {(1), (2), (3)}.

c What generalisation of the method of part a) is needed to prove equations of the form ∃ [u = v], where the terms u and v may contain variables?

Use this generalisation to prove

∃ z [v(f(v(z) = f(z)]

The three parts carry, respectively, 25%, 50% 25% of the marks.

2a Explain briefly how the method of free variable tableaux can be used as a Horn clause theorem prover where the top-clause has only negative literals.

b i) Explain how the methods of Model Elimination and Intermediate Lemma Extension extend the prover described in part a) to general clauses.

ii) *For the Lemma Extension method only*, illustrate your answer to part bi) using the clause-set

(1) ¬ Qx ∨ ¬ Px
(2) Qa ∨ Sx
(3) ¬ Rx ∨ Px
(4) ¬ Sa ∨ ¬ Sb
(5) Px ∨ Rx

c Call an unbound variable that occurs in only one open branch of an uncompleted Model Elimination tableau a *universal* variable.

Describe, using the clause-set of part bii) for illustration, how universal variables can be exploited in the Model Elimination method.

The three parts carry, respectively, 25%, 50% , 25% of the marks.

3a i) Explain briefly how a resolution proof can be derived from a closed semantic tree for an unsatisfiable set of clauses.

ii) For the clause-set

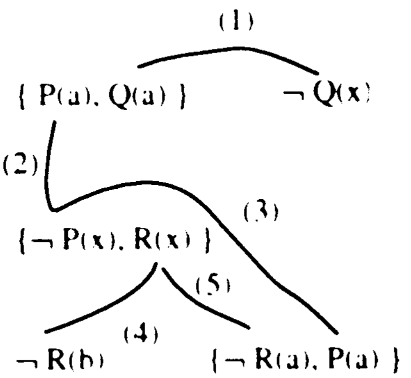
(1) $Qx \vee Px$
(2) $\neg P(x) \vee Rx$
(3) $\neg Rx$
(4) $\neg Qa \vee \neg Qb$

find a closed semantic tree and the corresponding refutation such that the derived resolution proof resolves on literals in a clause with priorities: Q before P before R.

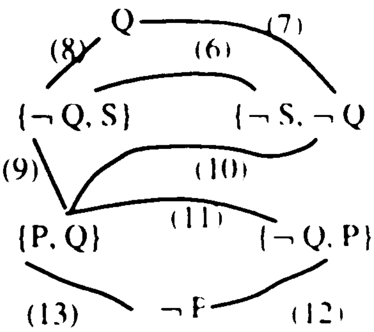
b In the connection graph proof procedure links may be selected according to various criteria, some of which are listed in (i) - (vi).

Rank the criteria, in your opinion, from the most useful to the least useful, justifying your answer. Use the graphs (A) and (B) below to illustrate your answers.

(i) Select an inconsistent link;
(ii) Select a link L incident to exactly two literals M, N, in different clauses, such that L is the only link incident to M and the only link incident to N;
(iii) Select a link L that gives a tautology;
(iv) Select a link L that divides the graph into two disjoint subgraphs;
(v) Select a link between a unit clause U and a clause C such that the resolvent subsumes C;
(vi) Select a link between two unit clauses.



(A)



(B)

The two parts carry, respectively, 50%, 50% of the marks.

Turn over ...

4a What are the equality axioms that are necessary to simulate by resolution the paramodulation of $a = b$ into $P(f(x),g(x))$?

Give the simulation for paramodulating $a=b$ into the first argument of $P(f(x),g(x))$.

b i) Explain how locks can be used to achieve a lock-resolution proof that is a hyper-resolution proof. In particular, mention any differences there might be between the simulation of hyper-resolution by lock-resolution and the standard hyper-resolution method.

How can the locks be used to further restrict the use of electrons?

ii) Use the method of part bi) to find a restricted hyper-resolution proof of the empty clause from the clauses

(1) $Qx \vee Px$

(2) $\neg Px \vee \neg Rx$

(3) $\neg Qa \vee \neg Sx$

(4) $Sa \vee Sb$

(5) $Rx \vee \neg Px$

c The following new rule is proposed for the propositional Davis Putnam procedure:

"If $A \vee B1 \vee \ldots \vee Bn$ and $D \vee B1 \vee \ldots \vee Bn$ are two clauses, $n \geq 0$, such that A and D are complementary, then replace both clauses by $B1 \vee \ldots \vee Bn$."

Justify the soundness of the new rule (i.e. show that P is satisfiable iff P' is satisfiable, where P and P' are respectively the clause-sets before and after application of the rule).

Discuss whether this rule would be useful. (Hint: Consider splitting on A.)

The three parts carry, respectively, 25%, 45%, 30% of the marks.

End of paper