UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

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MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
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PAPER C481

MODELS OF CONCURRENT COMPUTATION

Monday 28 April 2003, 14:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 The ISO standard CCR service ensures coordination between two (or more) servers in the sense that all servers carry out a request or none do. The service involves communication between a controller and servers as described below:

- the controller sends requests to all the servers;
- each server sends a reply to the controller saying whether or not it can carry out the request;
- if a server is able and the controller wishes to proceed, then the controller tells the server to prepare to carry out the request; if not, the controller tells the server to ignore the request;
- if the controller tells the server to act, the server carries out the request and then sends an acknowledgement to the controller.

Below is a description of a server S_i:

$$S_i \stackrel{\text{def}}{=} \mathtt{req}_i.(\overline{\mathtt{yes}}_i.Y_i + \overline{\mathtt{no}}_i.S_i)$$
$$Y_i \stackrel{\text{def}}{=} \mathtt{prepare}_i.\overline{\mathtt{act}}_i.\overline{\mathtt{ok}}_i.S_i + \mathtt{ignore}_i.S_i$$

where

- the actions $\mathtt{req}_i$ are requests;
- the actions $\mathtt{yes}_i$ and $\mathtt{no}_i$ indicate whether the server S_i is able to carry out the request;
- the actions $\mathtt{prepare}_i$ and $\mathtt{ignore}_i$ indicate whether the server S_i should prepare for action;
- the action $\mathtt{act}_i$ carries out the request; and
- the action $\mathtt{ok}_i$ is the acknowledgement.

a Give a CCS description of the controller, denoted $\mathtt{Controller}$, in the case when there are two servers.

b Define the process

$$\mathtt{Service} \stackrel{\text{def}}{=} (\mathtt{Controller} \,|\, S_1 \,|\, S_2)\backslash L$$

where L is the set of all actions except for $\mathtt{act}_1$ and $\mathtt{act}_2$.

i) Draw the transition graph of $\mathtt{Service}$. You may simplify the transition graph, by drawing several consecutive τ steps as one step and only including key states.

ii) If you have described the $\mathtt{Controller}$ correctly, you will see from the transition graph that it is weakly bisimilar to

$$D \stackrel{\text{def}}{=} \overline{\mathtt{act}}_1.\overline{\mathtt{act}}_2.D + \overline{\mathtt{act}}_2.\overline{\mathtt{act}}_1.D.$$

Consider the following temporal properties

- eventually $\overline{\mathtt{act}}_1$ is possible;
- always, whenever $\overline{\mathtt{act}}_2$ happens, eventually $\overline{\mathtt{act}}_1$ will be possible.

Express these properties as formulae of CTL$^-$. Which of these properties is satisfied by D, and which by $\mathtt{Service}$? Explain your answers.

The two parts carry, respectively, 40% and 60% of the marks.

2 Consider the process B which specifies a two-place buffer where we abstract from the values stored:

$$\begin{aligned} B &\stackrel{\text{def}}{=} \texttt{in}.B' \\ B' &\stackrel{\text{def}}{=} \texttt{in}.B'' + \overline{\texttt{out}}.B \\ B'' &\stackrel{\text{def}}{=} \overline{\texttt{out}}.B' \end{aligned}$$

It is possible to build this buffer out of cells. Consider the processes (cells) C and D defined by

$$\begin{aligned} C &\stackrel{\text{def}}{=} \texttt{in}.C' \\ C' &\stackrel{\text{def}}{=} \overline{\texttt{write}}.C + \tau.C' \end{aligned}$$

$$\begin{aligned} D &\stackrel{\text{def}}{=} \texttt{write}.D' \\ D' &\stackrel{\text{def}}{=} \overline{\texttt{out}}.D + \tau.D' \end{aligned}$$

a Draw the transition graphs of B and $(C \mid D)\backslash\{\texttt{write}\}$.

b Recall that one definition of weak bisimulation is the following: a binary relation S between processes is a *weak bisimulation relation* if and only if $(E, F) \in S$ implies

if $E \xrightarrow{a} E'$ then $F \stackrel{\hat{a}}{\Longrightarrow} F'$ with $(E', F') \in S$, and

if $F \xrightarrow{a} F'$ then $E \stackrel{\hat{a}}{\Longrightarrow} E'$ with $(E', F') \in S$,

where $\stackrel{\hat{a}}{\Longrightarrow}$ denotes $\stackrel{\tau}{\twoheadrightarrow}\stackrel{a}{\longrightarrow}\stackrel{\tau}{\twoheadrightarrow}$ if $a \neq \tau$ and $\stackrel{\tau}{\twoheadrightarrow}$ if $a = \tau$. Two processes are *weakly bisimilar*, denoted $\approx$, if and only if they are related by a weak bisimulation relation.

i) Give a weak bisimulation relation S which contains B and $(C \mid D)\backslash\{\texttt{write}\}$.

ii) Give a formal proof that $B \approx (C \mid D)\backslash\{\texttt{write}\}$.

The two parts carry, respectively, 20% and 80% of the marks.

3 Recall that the syntax of the Hennessy-Milner logic (HML) is given by the following grammar:

$$\Phi ::= \mathtt{tt} \mid \mathtt{ff} \mid \Phi_1 \vee \Phi_2 \mid \Phi_1 \wedge \Phi_2 \mid \langle K \rangle \Phi_1 \mid [K]\Phi_1$$

where K denotes a set of actions.

a Define the satisfaction relation $E \models \Phi$, where E denotes a CCS process.

b Consider the following two CCS processes which correspond to idealised vending machines:

$$\begin{aligned} \mathtt{Ven} &= \mathtt{1p.tea.Ven} + \mathtt{1p.1p.coffee.Ven} \\ \mathtt{Ven}' &= \mathtt{1p.(tea.Ven}' + \mathtt{1p.coffee.Ven}') \end{aligned}$$

Give simple formulae Φ_1, Φ_2 of HML such that $\mathtt{Ven} \models \Phi_1$, $\mathtt{Ven}' \not\models \Phi_1$, $\mathtt{Ven}' \models \Phi_2$ and $\mathtt{Ven} \not\models \Phi_2$.

c With the processes Ven and Ven′, it is not possible to input two 1ps and return two teas. We therefore adapt Ven′ as follows:

$$\begin{aligned} V_0 &= \mathtt{1p.}(V_1 + \mathtt{1p.}V_2) \\ V_1 &= \mathtt{tea.}V_0 \\ V_2 &= \mathtt{tea.}V_1 + \mathtt{coffee.}V_0 \end{aligned}$$

Prove that $V_0 \models [\mathtt{1p}][\mathtt{1p}](\langle\mathtt{tea}\rangle\langle\mathtt{tea}\rangle\mathtt{tt} \wedge \langle\mathtt{coffee}\rangle\mathtt{tt})$.

d Adapt V_0 to specify a vending machine $\mathtt{AV}_0$, which allows an arbitrary number of 1ps to be entered at any time and which returns any combination of coffees and teas. Draw the (infinite) transition graph of $\mathtt{AV}_0$.

The four parts carry respectively, 30%, 20%, 20% and 30% of the marks.

4a Recall the example of the interaction at a doctors' surgery between a receptionist, one patient, and two doctors. This interaction can be specified in the pi-calculus using the following processes:

$$\begin{aligned}
\mathtt{R} &\stackrel{\text{def}}{=} \mathtt{checkin(n, sym).next(a).\bar{a}\langle n, sym\rangle.R} \\
\mathtt{P(sym)} &\stackrel{\text{def}}{=} \mathtt{(new\ n)}\ \overline{\mathtt{checkin}}\mathtt{\langle n, sym\rangle.n(x)}.P' \\
\mathtt{D} &\stackrel{\text{def}}{=} \mathtt{(new\ a)}\ \overline{\mathtt{next}}\mathtt{\langle a\rangle.a(n, sym).\bar{n}\langle reply(sym)\rangle.D}
\end{aligned}$$

i) Give a precise description, in words, of the interaction between the receptionist, the patient and the doctors.

ii) Show how $\mathtt{D} \mid \mathtt{D} \mid \mathtt{R} \mid \mathtt{P\langle sym\rangle}$ evolves as the patient checks in and is seen by a doctor, illustrating the structural congruence and reduction steps.

b Consider the following extension of the surgery example:

- the doctor records the notes of the meeting between himself and the patient n with identifier a and symptom sym on a central store;
- as well as the doctor sending reply(sym) to a patient, the patient also receives his reference identifier a, and directions to a nurse, hospital or chemist;
- the central store files information using the identifier a;
- a nurse in the surgery interacts with patients via channel nurse, pulls information from the store using identifier a, and returns the result to the patient.

Adapt the pi-processes D and P(sym) to meet this specification, and also give pi-processes corresponding to the nurse and the store.

The two parts carry respectively, 50% and 50% of the marks.

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