

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2008

MSc and EEE PART III/IV: MEng, BEng. and ACGI

Corrected Copy

## **INSTRUMENTATION**

Friday, 16 May 10:00 am

Time allowed: 3:00 hours

**There are SIX questions on this paper.**

**Answer FOUR questions.**

*All questions carry equal marks*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      C. Papavassiliou  
Second Marker(s) :      S. Lucyszyn

## The Questions

1. The symbol in Figure 1(a) represents the digital potentiometer of Figure 1(b). The switch position is determined by the value of the input  $S$ , as shown. The circuit in Figure 1(c) senses the digital input  $S$ .
  - a) Write an expression for the output voltage as a function of the switch setting  $S$ , of the potentiometer of Figure 1(c) if  $R_L = \infty$ . [3]
  - b) Write an expression for gain from the input  $S$  to the output of the circuit in Figure 1(c). Treat  $S$  as a continuous variable. What is the mean gain of the sensor? [3]
  - c) What condition must the load resistor  $R_L$  satisfy for the output of the circuit in Figure 1(c) to be monotonic in  $S$ ? [2]
  - d) Assume that  $R_L \gg R$ . Use a Taylor expansion to write an expression for the output voltage of the sensor up to the leading power term in  $1/R_L$ . [2]
  - e) Use the result from (d) to estimate the maximum absolute nonlinearity of the sensor. [4]
  - f) Use the result from (d) to estimate the maximum differential non-linearity of the sensor. [4]
  - g) What condition must  $R_L$  satisfy so that the sensor output is within 1% of its ideal value  $V_0 = V_R S / 10$ ? [2]

HINT: You can use the ratio  $\rho = R_L / R$  to simplify calculations.

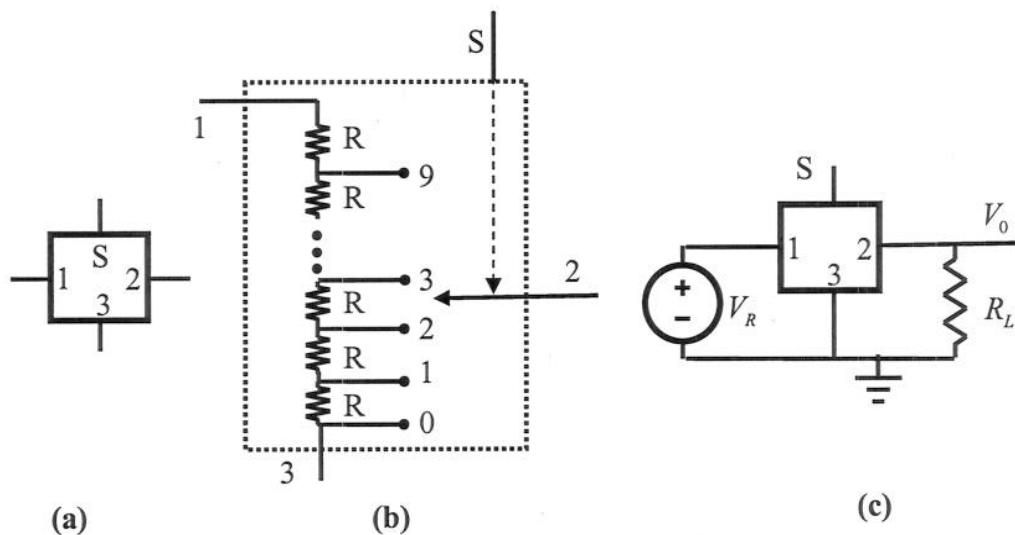


Figure 1: Diagrams for Question 1

2.

- a) Define, very briefly, each of the following quantities for a sensor.
- i. Sensitivity [1]
  - ii. Threshold [1]
  - iii. Zero Offset [1]
  - iv. Absolute non-linearity [1]
  - v. Differential Non-linearity [1]
- b) Explain when a sensor can be considered to be monotonic. Comment on the monotonicity of the following devices:
- a. XOR gate when used as a phase detector
  - b. Analogue multiplier when used as a phase detector.
  - c. Ideal D/A converter
- [6]
- c) Briefly discuss the difference between linear and non-linear memory effects in a sensor. [2]
- d) What is the cross-sensitivity of a sensor? How can it be used to enhance measurements? [3]
- e) Draw a diagram for a chopper amplifier. Explain the operation of the chopper amplifier. Why is the chopper amplifier equivalent to a communications link? What type of modulation can be used in a chopper amplifier? [4]

3. The excess noise of a particular amplifier was modelled by three voltage noise sources  $V_{n1}$ ,  $V_{n2}$ ,  $V_{n3}$  connected in series to its output, as shown in Figure 3. The amplifier is operated at  $T = 293\text{K}$  over a 20 Hz - 20 kHz bandwidth. The source and load impedances are  $R_S = R_L = 600\ \Omega$ .

The voltage noise sources are described as:

- $V_{n1}$  is equal to the open circuit noise voltage developed across a  $1\text{ k}\Omega$  resistor. Furthermore  $V_{n1}$  exhibits pink noise with a 1 kHz corner frequency.
- $V_{n2}$  is equal to the noise voltage developed on a  $2\text{ k}\Omega$  resistor when a noise current  $I_N = g_m V_{n1}$ , with  $g_m = 1\text{ mS}$ , passes through it. You may assume the  $2\text{ k}\Omega$  resistor is otherwise noiseless.
- $V_{n3}$  is equal to the noise voltage developed on a noiseless  $100\text{ k}\Omega$  resistor when driven by a 10 mA DC current source.

- a) Write an expression for the frequency dependence of the power spectral density of  $V_{n1}$ .  
Write an expression for the root-mean-square magnitude of  $V_{n1}$  over the operating frequency band.

[5]

- b) Write an expression for the frequency dependence of the power spectral density of  $V_{n2}$ .  
Write an expression for the root-mean-square magnitude of  $V_{n2}$  over the operating frequency band.

[3]

- c) Write an expression for the frequency dependence of the power spectral density of  $V_{n3}$ .  
Write an expression for the root-mean-square magnitude of  $V_{n3}$  over the operating frequency band.

[3]

- d) Write an expression for the total, input-referred, noise voltage over the operation bandwidth. Calculate the magnitude of the input referred noise over the operation bandwidth.

[5]

- e) Define the noise Figure of an amplifier. Calculate the noise Figure of the amplifier in Figure 3 if its voltage gain is 20 dB.

[4]

Note:  $k_B T = 4 \times 10^{-21}\text{ J}$  at  $T = 293\text{K}$ .

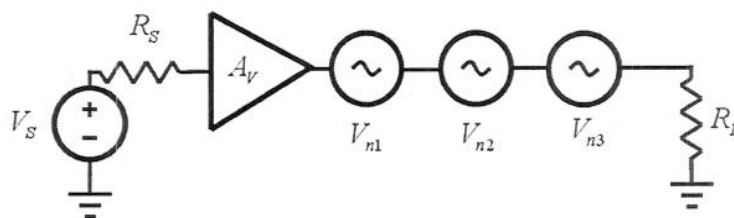


Figure 3: Diagram for Question 3

4. The block diagram for a measurement instrument is shown in Figure 4. You may assume that the frequency multiplier multiplies the frequency of the VCO output by a factor of  $N$ , introducing no delay.
- a) Write the linearised loop equation for the circuit in Figure 4, assuming its output is phase locked to the input. What is its steady state ( $s=0$ ) response? What is the function of this circuit? [4]
  - b) Write an expression for the loop gain of the circuit in Figure 4. Calculate the phase margin of this loop if the filter is a first order low pass filter and the multiplication constant of the frequency multiplier is very large. What happens if the phase detector introduces a small delay? [4]
  - c) What is a comb generator? Explain what happens if a comb generator is put in the place of the frequency multiplier in Figure 4. [4]
  - d) Briefly describe the transfer oscillator, its function and its similarities and its differences to a PLL. Draw a block diagram for a transfer oscillator. [4]
  - e) Describe how frequencies greater than 10 GHz can be measured with 1 Hz accuracy using transfer oscillators and/or other methods. [4]

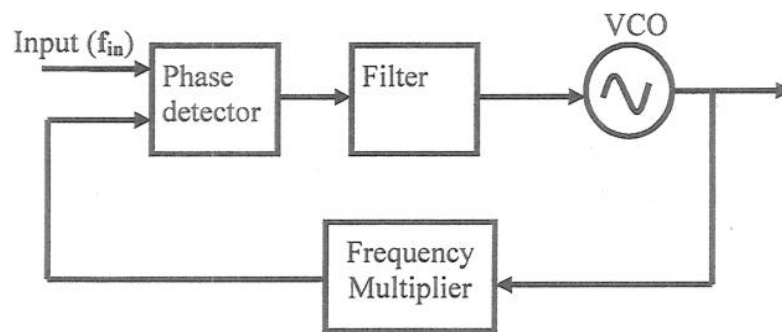


Figure 4: The instrument for question 4.

5.

- a) List and briefly describe four ways in which a real A/D converter deviates from ideality. [4]
- b) Calculate the RMS quantisation noise power of an ideal 1-bit A/D converter operating at a conversion rate of  $5 \times 10^6$  samples/s. The input signal is a 1V amplitude sinusoid and the Thevenin impedance of the signal source is  $1 \text{ k}\Omega$ . What is the frequency dependence of the quantisation noise power density? [4]
- c) Calculate the ratio of quantisation to thermal noise power if the input of the A/D converter in part (b). Assume the signal source has an adequate noise bandwidth to satisfy the sampling criterion. What is the maximum practical number of bits to convert the input, so that the Least Significant Bit is always larger than the thermal noise amplitude? [4]
- d) Describe how the converter in part (b) can be used, without feedback, to give a 12 bit digital output. What is the maximum output reading rate in this case? [4]
- e) Describe how the converter in part (b) can be used, with feedback which includes a first order low pass filter, to give a 12 bit digital output. What is the maximum sample rate in this case? [4]

6.

a)

- i. Define the period, frequency and phase of a signal.
- ii. Can frequency and phase be defined for any signal?
- iii. Explain why measuring the phase of a periodic signal is not a good measurement of time.

[4]

b)

- i. Draw and label a diagram for a feedback amplifier.
- ii. Use the diagram to state the necessary and sufficient conditions for a feedback amplifier to oscillate. (HINT: to determine the sufficient condition think of the start up of the oscillator)
- iii. What is the oscillation frequency of an oscillator satisfying these conditions?
- iv. Provide an example of an oscillator based around a feedback amplifier satisfying the conditions of oscillation.

[4]

c) Consider a ring oscillator that consists of a loop of analogue inverters. Assume each of the inverters subjects the signal to a delay  $T$ .

- i. Specify the magnitude of the gain of each inverter so that the ring can oscillate.
- ii. At what frequency will this ring oscillator oscillate if the number of inverters in the loop is odd?
- iii. At what frequency will this ring oscillator oscillate if the number of inverters in the loop is even?
- iv. Show that if an odd ring oscillator supports oscillation at a frequency  $f$  it also supports oscillation at the odd harmonics of  $f$ .

[6]

d) An unknown signal has a frequency nearly equal to that of a reference oscillator available in the lab. To accurately measure the unknown frequency, it is proposed that both a frequency and period counter measurements are performed using the same divider value. Derive an expression for the uncertainty in this measurement.

[6]