

EE4-57

SOLUTIONS: DISCRETE EVENT SYSTEMS MASTER IN CONTROL

1. Exercise

- a) The automaton G has an event set $E = \{a_1, a_2, a_3, d_1, d_2, d_3\}$ and a state-space $X = \{000, 001, 010, 011, 100, 101, 110, 111\}$. Its transition diagram is shown in Fig. 1.1;
- b) The automaton including overflows is shown in Fig. 1.2.
- c) The labeling device G_L has event set $E_L = \{o_1\}$, two states, $X_L = \{N, Y\}$, initial state N , and two transitions, $f_L(N, o_1) = Y$ and $f_L(Y, o_1) = N$.
- d) The parallel composition of G_O and G_L is shown in Fig. 1.3.
- e) The only state of $G_L || G_O$ where o events trigger a transition to more than one state, is $011N$. In fact o may represent either an o_1 or o_2 event, and in the case of o_1 event transition to $111Y$ occurs, while in the case of o_2 event transition to $111N$ occurs.
- f) The diagnoser automaton G_D is represented in Fig. 1.4.

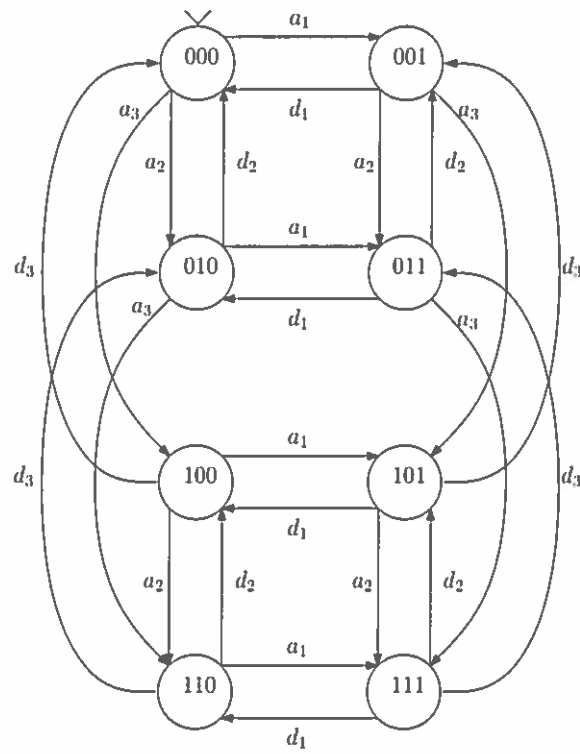


Figure 1.1 Transition diagram of automaton G

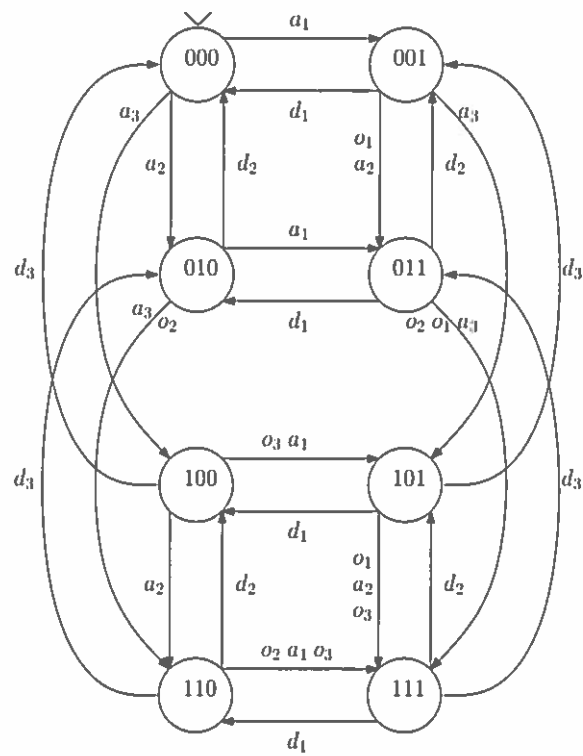


Figure 1.2 Transition diagram of automaton G_O

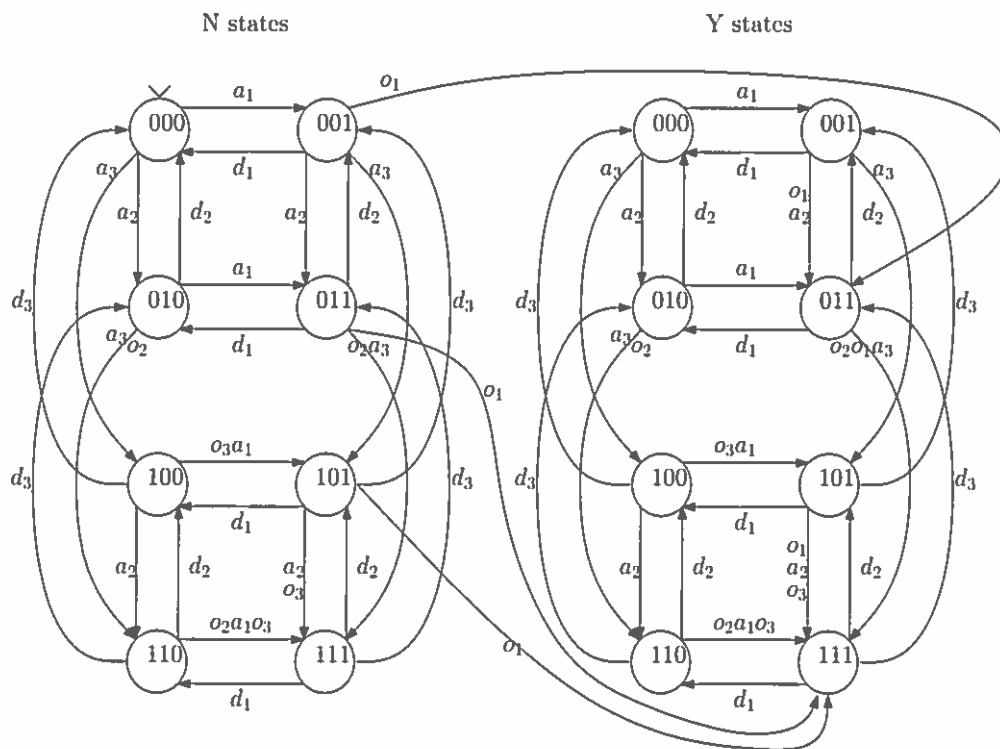


Figure 1.3 Transition diagram of automaton $G_L || G_O$

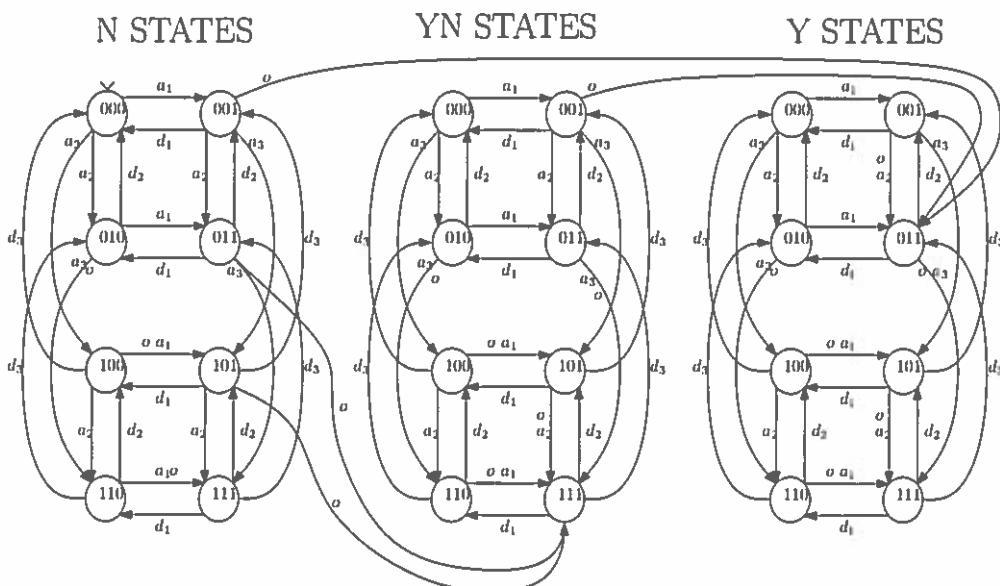


Figure 1.4 Transition diagram of automaton G_D

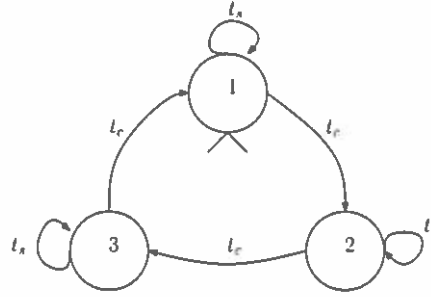


Figure 2.1 Transition diagram of G_A

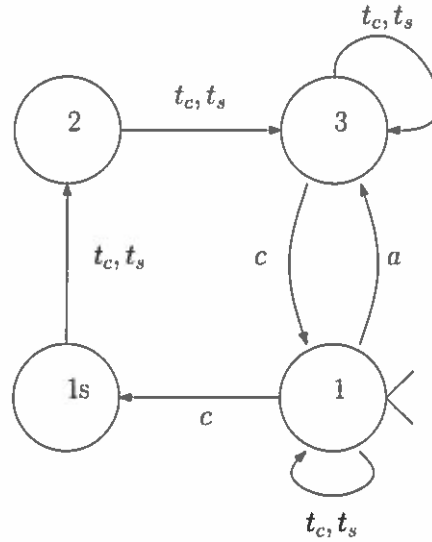


Figure 2.2 Transition diagram of G_B

2. Exercise

- The automaton G_A has an event set $E_A = \{t_c, t_s\}$ and states $X_A = \{1, 2, 3\}$. Its transition diagram is shown in Fig. 2.1.
- The automaton G_B has an event set $E_B = \{a, c, t_c, t_s\}$ and state space $X_B = \{1, 1s, 2, 3\}$. Its transition diagram is shown in Fig. 2.2.
- The parallel composition $G_A || G_B$ has the transition diagram shown in Fig. 2.3.
- The automaton H implementing the specification can be obtained simply removing the state 22 from $G_A || G_B$ and associated edges. See Fig. 2.4.
- Notice that $\mathcal{L}(H)$ is uncontrollable with respect to $\mathcal{L}(G)$ and $E_{uc} = \{t_s, t_c\}$ since in states 1s1 and 1s2 events t_c and t_s (respectively) have been disabled.
- The supremal controllable sublanguage is marked by the automaton \hat{H} shown in Fig. 2.5. Notice that $S(s) = \Gamma_{\hat{H}}(f_{\hat{H}}(11, s))$ is an admissible supervisor since it only disables c events (that are controllable).

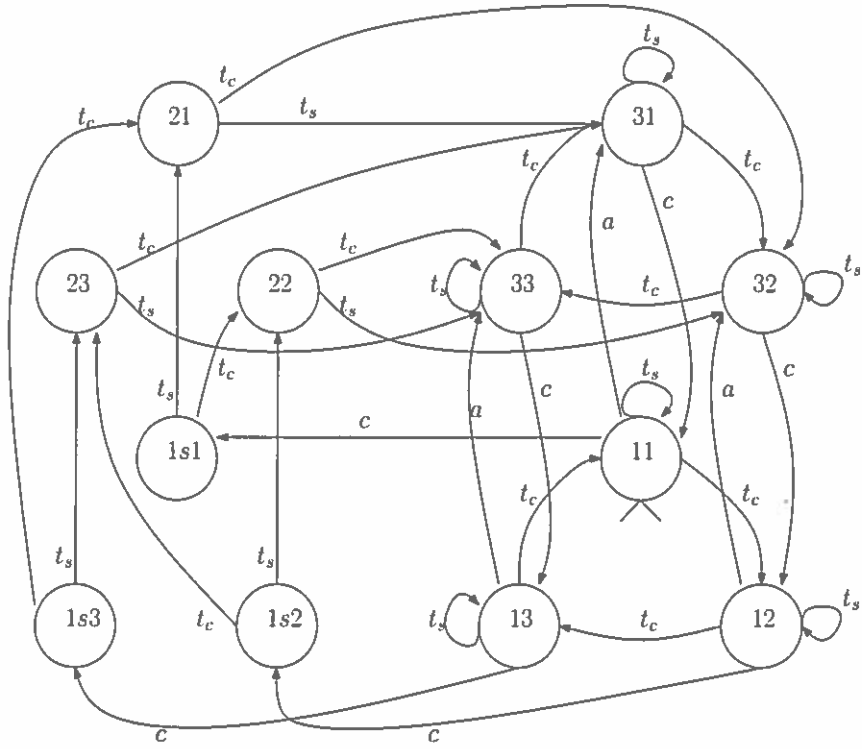


Figure 2.3 Transition diagram of $G_A || G_B$

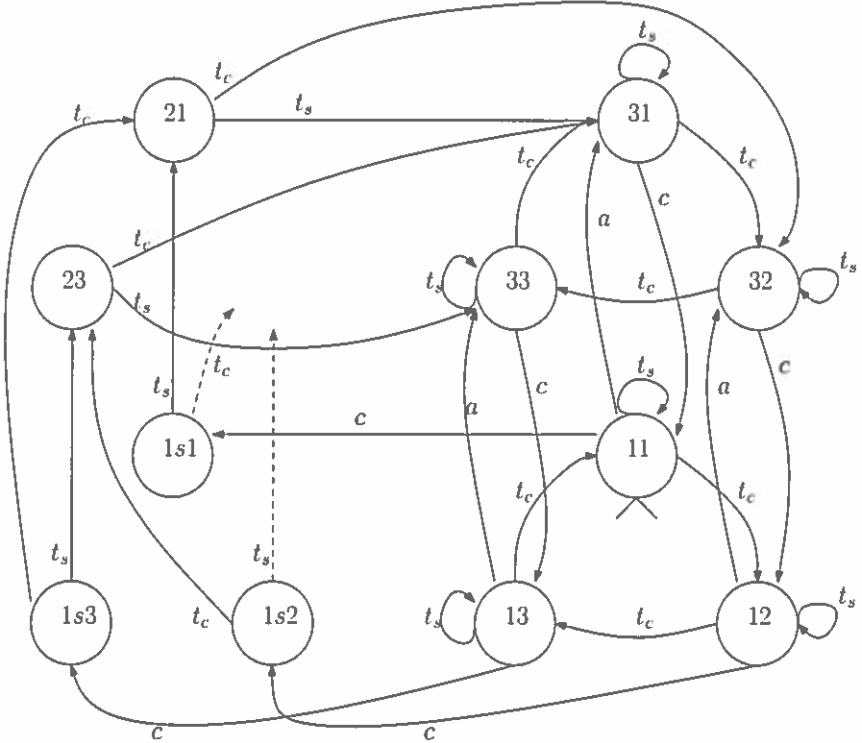


Figure 2.4 Transition diagram of automaton H

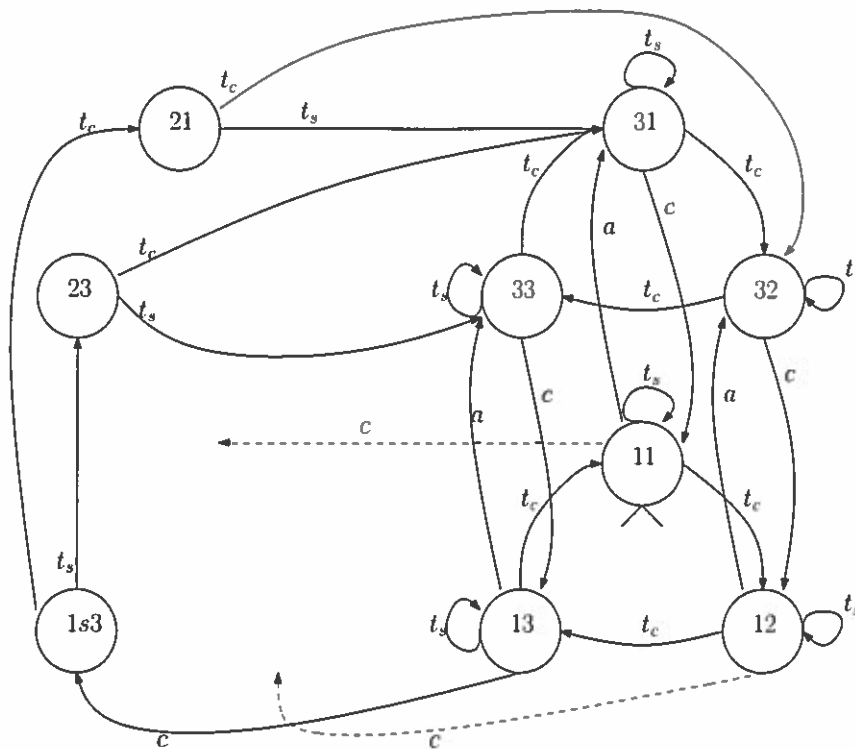


Figure 2.5 Transition diagram of automaton \tilde{H}

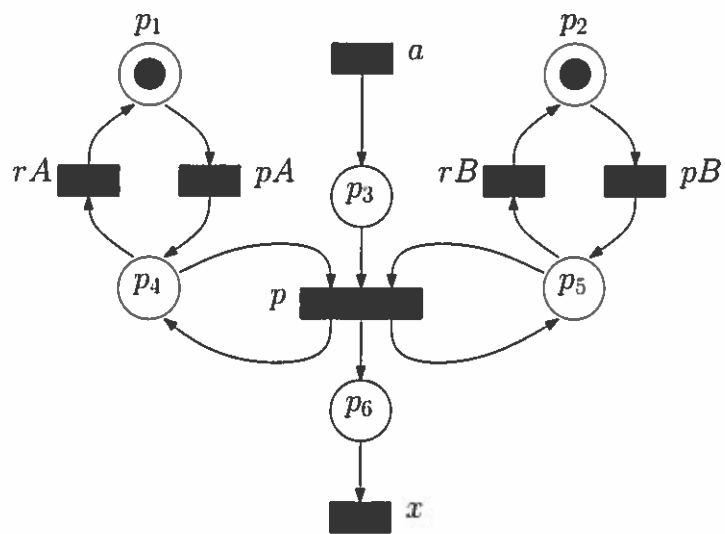


Figure 3.1 The marked Petri Net, $\langle N, M_0 \rangle$

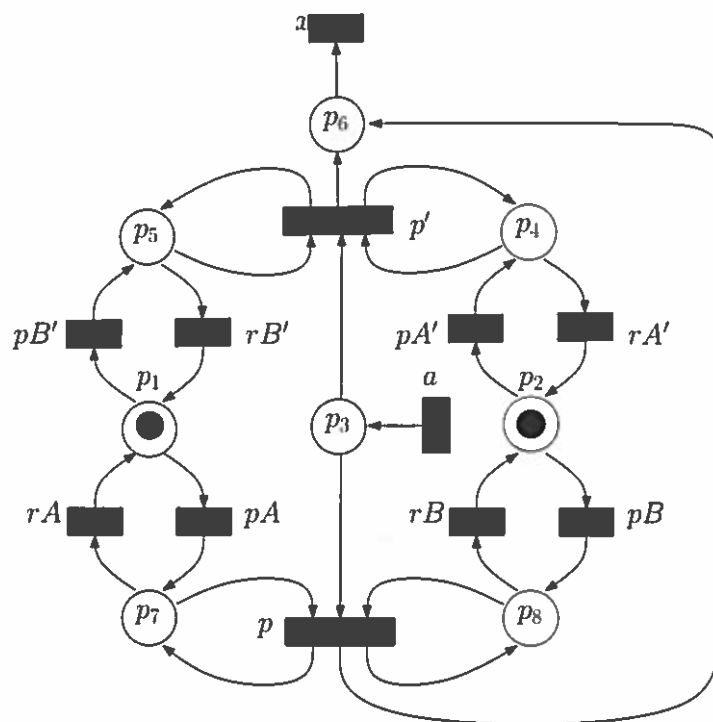


Figure 3.2 Machines sharing tools.

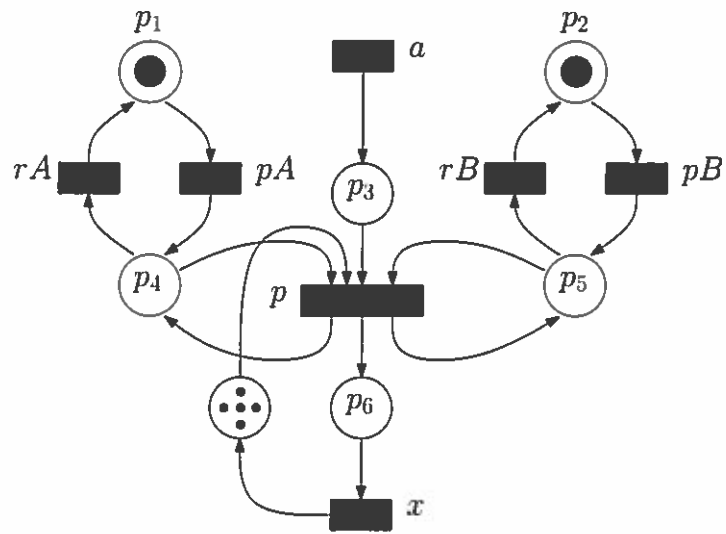


Figure 3.3 A single machine with finite capacity of storage.

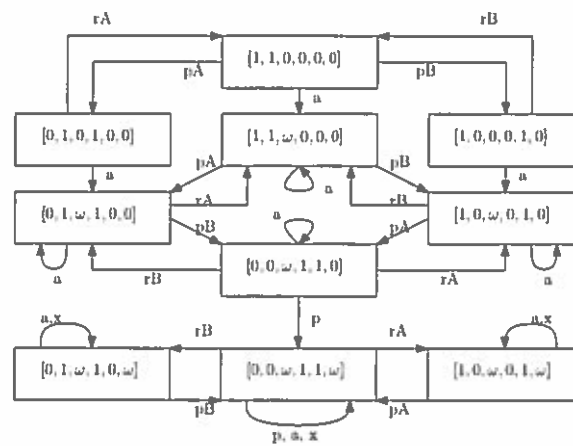


Figure 3.4 The coverability graph

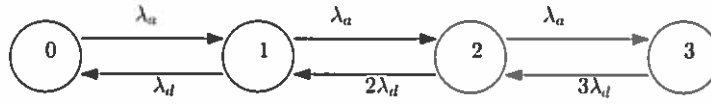


Figure 4.1 Markov chain modeling car park

bounded places are those for which ω does not appear in any nodes of the associated coverability graph. Hence, these are again the places $\{p_1, p_2, p_4, p_5\}$.

4. Exercise

- a) Notice that when n cars are parked in the car park, then the distribution of departure times is still normal, with rate $n\lambda_d$. Hence, the Markov chain model of the car park is as in Fig. 4.1; Its equations read:

$$\begin{aligned}
 \dot{\pi}_0 &= -\lambda_a \pi_0 + \lambda_d \pi_1 \\
 \dot{\pi}_1 &= -(\lambda_d + \lambda_a) \pi_1 + \lambda_a \pi_0 + 2\lambda_d \pi_2 \\
 \dot{\pi}_2 &= -(2\lambda_d + \lambda_a) \pi_2 + \lambda_a \pi_1 + 3\lambda_d \pi_3 \\
 \dot{\pi}_3 &= -3\lambda_d \pi_3 + \lambda_a \pi_2
 \end{aligned}$$

- b) The chain is ergodic, hence, there exists a well defined asymptotic probability distribution. This is the unique solution of:

$$\begin{aligned}
 0 &= -\lambda_a \pi_0 + \lambda_d \pi_1 \\
 0 &= -(\lambda_d + \lambda_a) \pi_1 + \lambda_a \pi_0 + 2\lambda_d \pi_2 \\
 0 &= -(2\lambda_d + \lambda_a) \pi_2 + \lambda_a \pi_1 + 3\lambda_d \pi_3 \\
 0 &= -3\lambda_d \pi_3 + \lambda_a \pi_2 \\
 1 &= \pi_0 + \pi_1 + \pi_2 + \pi_3
 \end{aligned}$$

We see that:

$$\pi_1 = \frac{\lambda_a}{\lambda_d} \pi_0 \quad \pi_2 = \frac{\lambda_a^2}{2\lambda_d^2} \pi_0 \quad \pi_3 = \frac{\lambda_a^3}{6\lambda_d^3} \pi_0$$

Hence:

$$\pi_0 \left(\frac{\lambda_a}{\lambda_d} + \frac{\lambda_a^2}{2\lambda_d^2} + \frac{\lambda_a^3}{6\lambda_d^3} \right) = 1.$$

The average number of cars parked is given by:

$$\pi_1 + 2\pi_2 + 3\pi_3 = \left(\frac{\lambda_a}{\lambda_d} + \frac{\lambda_a^2}{\lambda_d^2} + \frac{\lambda_a^3}{2\lambda_d^3} \right) \pi_0$$

- c) We consider next the markov chain with states $\{000, 001, 010, 011, 100, 101, 110, 111\}$ modeling the occupancy of each parking slot. Its transition diagram is shown in Fig. 4.2.
- d) Notice that defining $\pi_1 = \pi_{100} + \pi_{110} + \pi_{111} + \pi_{101}$ and $\pi_0 = \pi_{000} + \pi_{001} + \pi_{010} + \pi_{011}$ yields:

$$\dot{\pi}_0 = -\lambda_a \pi_0 + \lambda_d \pi_1$$

$$\dot{\pi}_1 = -\lambda_d \pi_1 + \lambda_a \pi_0.$$

Hence, these are the equations of a Markov chain with 2 states. The average occupancy of parking slot 1 is therefore $\frac{\lambda_a}{\lambda_a + \lambda_d}$.

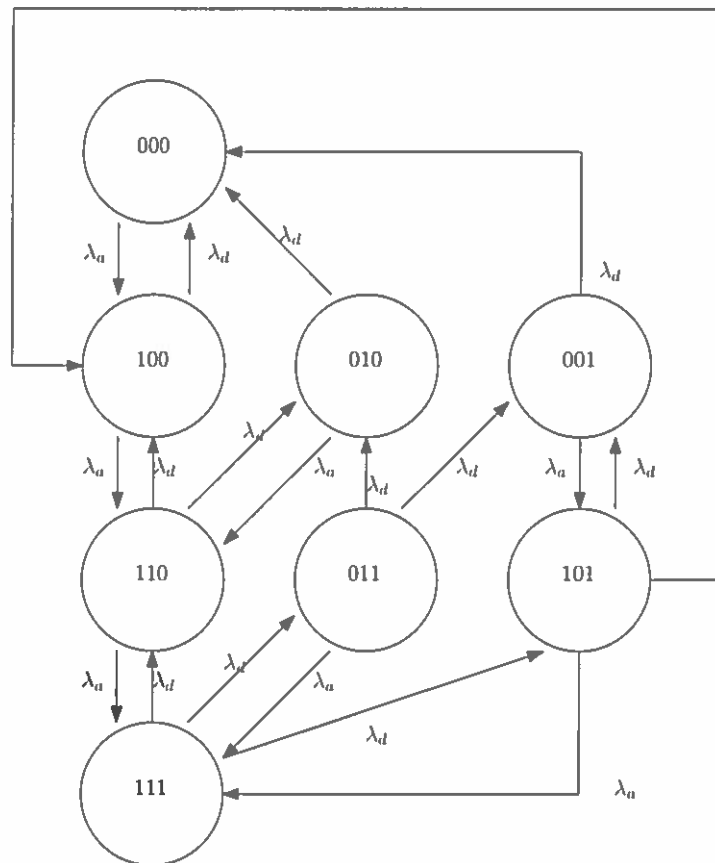


Figure 4.2 Occupancy modeling Markov chain

- e) Modeling occupancy of state 2 requires more than 2 states, in fact, when state two is free, the transition probability from a free to a busy state would depend on whether or not slot 1 is already taken.