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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

MSc and EEE PART III/IV: MEng, BEng.and ACGI

POWER ELECTRONICS AND MACHINES

Thursday, 12 May 10:00 am

Time allowed: 3:00 hours

Corrected Copy

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T.C. Green

Second Marker(s) : B.C. Pal

1

(a) Draw the circuit diagram of a zero-current switched quasi-resonant converter (ZCS-QRC) based on a buck circuit. Identify the components that are used in addition to the standard buck circuit. [3]

(b) Explain how the ZCS-QRC achieves low power loss during switching. Explain why low switching power loss is an advantage when designing a power converter to occupy a small physical volume. [5]

(c) Sketch the waveform of the current through the resonant inductor and voltage across the resonant capacitor. Identify the time at which the switch is turned on and turned off. Describe what governs the on and off times. [3]

(d) A ZCS-QRC is built with resonant components L_R = 500 nH and C_R = 100 nF. It is used to convert a battery voltage in the range 5.2 to 6.8 V to an output voltage 3.3 V. Calculate the following:

(i) the resonant period [2]

(ii) the approximate off-time of the circuit [3]

(iii) the maximum output power consistent with ZCS operation over the range of battery voltage. [4]

[3.14]

2

(a) For a 3-phase inverter (DC/AC converter):

(i) Sketch a circuit diagram of the principal components required [3]

(ii) Describe either the PWM or space-voltage vector methods of generating the signals to control the on/off state of the switches and include a description of how the magnitude and frequency of the AC voltage is set [5]

(iii) Describe the form of frequency spectrum expected of the output voltage and clearly distinguish between the desired and undesired parts of the spectrum. [4]

(b) An inverter can be used to give bidirectional power flow between an AC and a DC system.

(i) Describe how the power flow direction and magnitude can be set. [4]

(ii) Describe the advantages this circuit and mode of operation have over a simple diode rectifier. [4]

3. In the context of semiconductor devices designed for use in power electronics, answer the following questions.

(a) Why is a large cross sectional area required? [2]

(b) Why is the die required to be deep? [2]

(c) For a junction required to block high voltage, why is one side of the junction more heavily doped than the other? [3]

(d) What is high level injection? [3]

(e) Why is the forward voltage drop of a p^+n^- power diode significantly greater than the 0.7 V expected of a signal diode? [3]

(f) Why is a vertical double-diffused structure used for a MOSFET? [4]

(g) Why does the addition of an extra junction to the MOSFET to form the IGBT lower the on-state voltage drop? [3]

[3.14]

4

(a) Describe the relationships between power, torque, speed and size of an electric motor. [3]

(b) Describe, with examples, how the shape and layout of a motor might be varied to achieve high acceleration. [4]

(c) Describe how commutation is achieved in a brushless DC machine [3]

(d) A standard chopper is used to supply a brushed DC machine which has a separate circuit supplying field current. The chopper has a DC supply of 200 V. The machine has a maximum armature current of 30 A and maximum field current of 1 A. When the armature is rotated at 1,000 rpm with a field current of 1 A the induced voltage measure across the open circuit armature was 130 V. The armature has a resistance of 0.5 Ω

(i) Calculate the maximum torque available from the machine [2]

(ii) Calculate the chopper duty-cycle required to achieve maximum torque at 500 rpm. [2]

(iii) Estimate the maximum speed that can be achieved without field weakening and the output power this achieves. [3]

(iv) Calculate the degree of field weakening required to operate at 2,500 rpm and maximum armature current. Also calculate the torque and output power achieved. [3]

5

(a) Derive an expression for the output voltage to input voltage ratio of a Ćuk switch-mode power supply (SMPS), shown in figure 5.1, and state any assumptions made. [5]

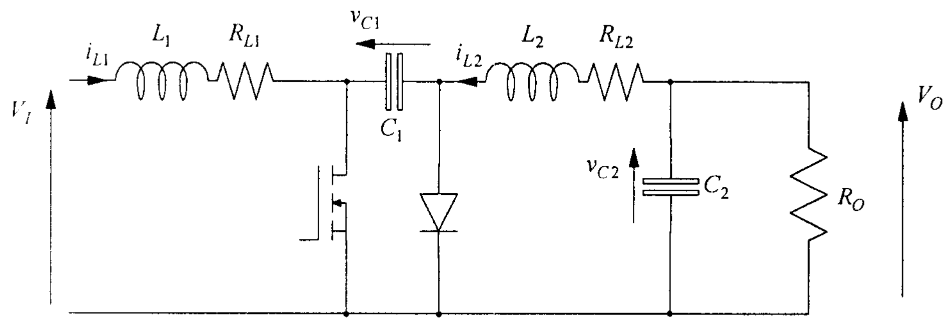


Figure 5.1

(b) A Ćuk SMPS is to be used to provide a -5 V output from a 9 V input. The maximum output current will be 1.2 A. For a switching frequency of 100 kHz, choose inductors to maintain the input and output current ripples at 10 mA and 20 mA respectively. [6]

(c) It is found that for the inductor materials used, the size of the inductor is related to maximum stored energy it can support. The energy density achieved is 180 J/m^3. Calculate the volume of inductor required for the design in (b) and calculate the inductor value that could be used in a buck-boost SMPS to achieve the same voltage ratio as (b) and use the same volume of material. [5]

(d) Compare the Ćuk SMPS in (b) with the buck-boost SMPS in (c) in terms of the maximum voltage and currents required of the transistor and the maximum ripple in the input current and output currents. [4]

6

Figure 6.1 shows a transistor switching an inductive load. The load current can be considered constant. The supply voltage, V_{DC} is 700 V and the load current, I_L is 50 A.

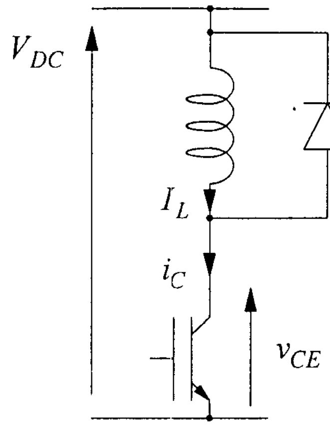


Figure 6.1

(i) Sketch the shape of the collector-emitter voltage and collector current waveforms at turn-on. [2]

(ii) Calculate the turn-on energy loss if the current rises at a rate of 400×10^6 A/s and the voltage falls at 20×10^9 V/s. [3]

(iii) Sketch the circuit of a turn-on snubber and the associated voltage and current waveforms assuming that the current does not complete its rise before the voltage has fallen. [3]

(iv) Choose snubber components such that the rate of rise of current is never greater than 200×10^6 A/s and snubber reset takes 2 µs. [4]

(v) Calculate the turn-on loss of the transistor with the snubber present and ~~the power~~ loss in the snubber resistor. [5]

12 40 ENERGY

(vi) The diode is a snap recovery diode. At turn-on of the switch the current continues rising above the load current at the same rate of change until a charge of 0.9 µC has been removed from the diode. At this point the junction recovers and no more current flows. Calculate the diode recovery time and peak recovery current and redraw the turn-~~off~~ on waveforms for the case without the snubber. [3]

12:05

9.C.

Power Electronics

Solutions 2005

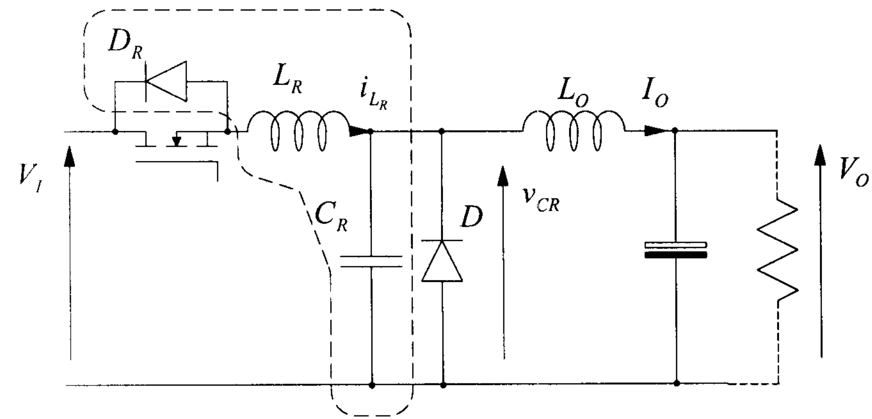
[3.14]

A07

1

(a) Draw the circuit diagram of a zero-current switched quasi-resonant converter (ZCS-QRC) based on a buck circuit. Identify the components that are used in addition to the standard buck circuit. [3]

A ZCS QRC. The QRC includes an additional L and C for each switch and an additional diode.

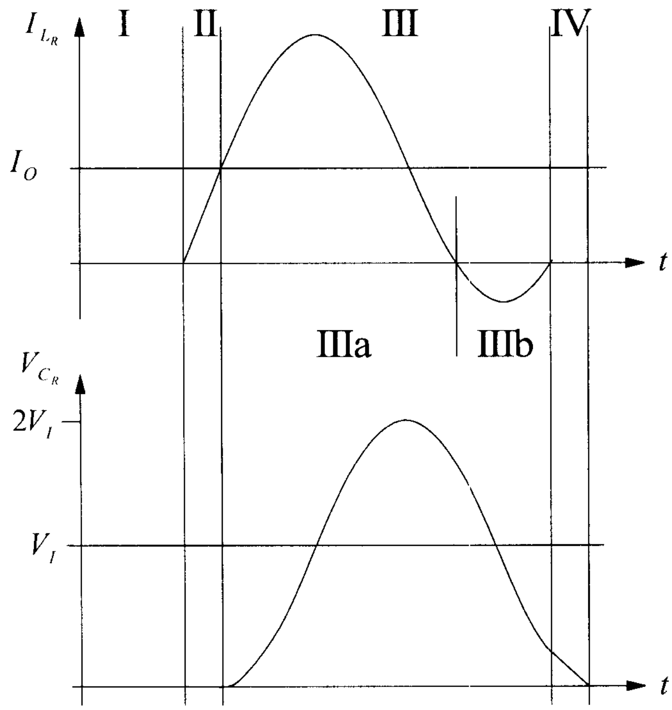


(b) Explain how the ZCS-QRC achieves low power loss during switching. Explain why low switching power loss is an advantage when designing a power converter to occupy a small physical volume. [5]

With the inductor placed in series with the switch, the resonant circuit is excited by turning on the switch and at a later point in the cycle the current will reverse. At this point the additional diode conducts in place of the switch and the switch can be turned off when carrying no current and with a small reverse voltage. At turn on, the current will be initially zero and constrained to rise slowly by the series inductor. The voltage will be large but will fall while the current is still small. Overall, the voltage-current product during switching is small. By achieving a small switching power loss the penalty against raising the switching frequency is reduced and by operating at high frequency smaller valued inductors and capacitors can be used in the main circuit while maintaining the same ripple voltage specification. Smaller valued L and C are also physically smaller (for the same V&I).

(c) Sketch the waveform of the current through the resonant inductor and voltage across the resonant capacitor. Identify the time at which the switch is turned on and turned off. Describe what governs the on and off times. [3]

Turn-on can occur as a free event and appears in the diagram as the point at which the inductor current begins to rise. Turn-off can occur at any point while i_L is negative. The conduction period of the inductor is set by the resonant period and this is considered the on-time of the circuit. The on-time of the gate of the switch is approximately ¾ of the resonant period. The off time is chosen to give an average voltage across C_R equal to the desired output voltage.



(d) A ZCS-QRC is built with resonant components L_R = 500 nH and C_R = 100 nF. It is used to convert a battery voltage in the range 5.2 to 6.8 V to an output voltage 3.3 V. Calculate the following:

(i) the resonant period [2]

$$T_R = 2\pi\sqrt{L_R C_R} = 2\pi\sqrt{500\times10^{-9}\times100\times10^{-9}}$$
$$= 1.405\ \mu s$$

(ii) the approximate off-time of the circuit [3]

$$\frac{V_O}{V_I} \approx \frac{T_R}{T} = \frac{T_R}{T_R + t_{Off}}$$

$$t_{Off} \approx T_R \frac{V_I - V_O}{V_O}$$

$$V_I = 5.2\ V$$

$$t_{Off} = 1.405\mu\times\frac{5.2-3.3}{3.3} = 0.81\ \mu s$$

$$V_I = 6.8\ V$$

$$t_{Off} = 1.405\mu\times\frac{6.8-3.3}{3.3} = 1.49\ \mu s$$

(iii) the maximum output power consistent with ZCS operation over the range of battery voltage. [4]

The current in the resonant inductor during resonant action is:

$$i_{L_R} = I_O + \frac{V_I}{\omega_R L_R}\sin(\omega_R t)$$

This is composed of a load current term and an oscillatory term. Turn-off under ZCS conditions is only achieved if this current turns negative at some point and therefore the load current must be smaller than the amplitude of the resonant term.

$$V = 5.2V$$

$$I_O^{Max} = \frac{V_I}{\omega_R L_R} = \frac{V_I}{\sqrt{L_R / C_R}} = \frac{5.2}{\sqrt{500n / 100n}} = 2.33\ A$$

$$P_O^{Max} = 7.7\ W$$

$$V = 6.8\ V$$

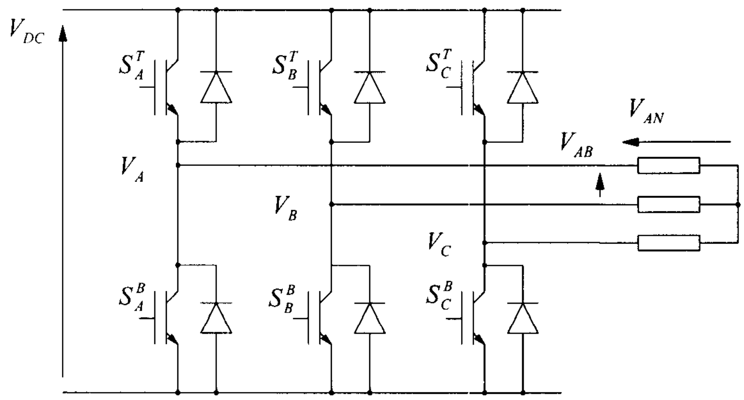
$$I_O^{Max} = \frac{6.8}{\sqrt{500n / 100n}} = 3.04\ A$$

$$P_O^{Max} = 10.0\ W$$

2.

(a) For a 3-phase inverter (DC/AC converter):

(i) Sketch a circuit diagram of the principal components required [3]



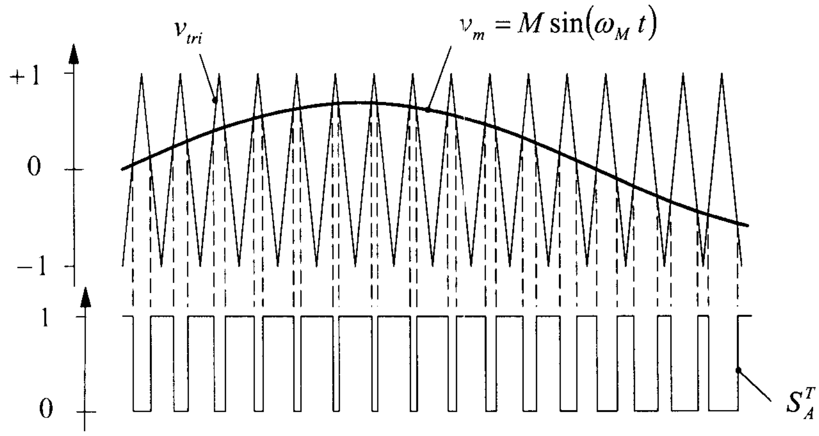
(ii) Describe either the PWM or space-voltage vector methods of generating the signals to control the on/off state of the switches and include a description of how the magnitude and frequency of the AC voltage is set [5]

A sinusoidal reference signal of unit magnitude is established for each phase voltage. The signals are compared with a common triangle wave carrier to generate the commutation signals.

$$v_m = M\sin(\omega_M t) \quad \text{where } 0 \le M \le 1$$

$$\delta = \tfrac{1}{2} + \tfrac{1}{2}m \qquad \text{(Eqn 2.1)}$$

where M is known as the depth of modulation and sets the magnitude of the AC voltage being synthesised and v_m is the instantaneous modulating signal.



The resulting phase voltages are

$$v_A = \tfrac{1}{2}V_{DC} + \tfrac{1}{2}V_{DC}M\sin(\omega t) + \text{carier and sideband terms}$$

$$v_B = \tfrac{1}{2}V_{DC} + \tfrac{1}{2}V_{DC}M\sin\left(\omega t - \tfrac{2\pi}{3}\right) + \text{carier and sideband terms}$$

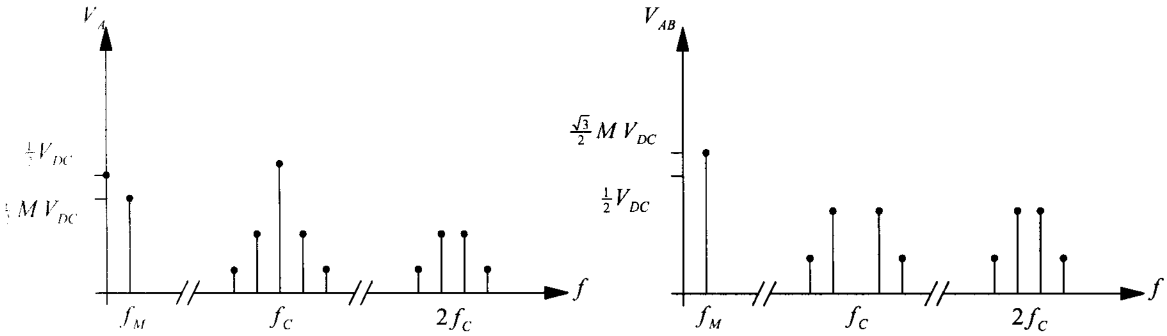
$$v_C = \tfrac{1}{2}V_{DC} + \tfrac{1}{2}V_{DC}M\sin\left(\omega t + \tfrac{2\pi}{3}\right) + \text{carier and sideband terms}$$

The DC terms are common mode and can be ignored.

The reference signals are often generated by a look up table. The frequency with which the look-up table is scanned dictates the AC supply frequency. The amplitude can be attenuated by multiplying the samples by a scaling factor.

(ii) Describe the form of frequency spectrum expected of the output voltage and clearly distinguish between the desired and undesired parts of the spectrum. [4]

The spectrum of the phase voltages consists of the desired base-band term plus a DC offset, a carrier and sidebands at the switching frequency and further carrier and sideband terms at multiples of the switching frequency. The undesirable DC term is not present in the line voltages because it is common-mode. The higher frequency carrier and sideband terms are naturally attenuated by inductive loads or can be attenuated by filters.

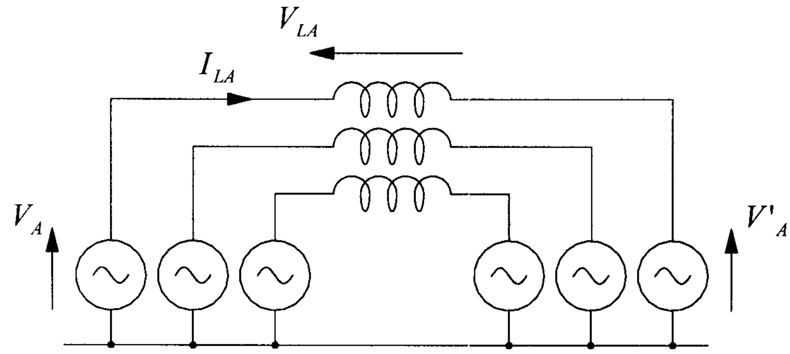


Frequency spectra of the phase and line voltages of a three-phase inverter

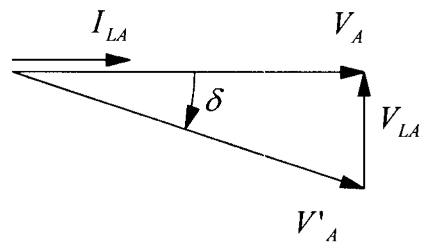
(b) An inverter can be used to give bidirectional power flow between an AC and a DC system.

(i) Describe how the power flow direction and magnitude can be set. [4]

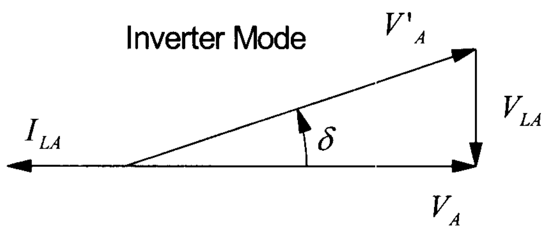
Power flow through the inverter can be controlled in the same manner as a standard synchronous machine. The phase difference (or load angle), δ across an interface inductor can be set by the PWM system and used to control the real power flow. The voltage magnitude could also be used to control the reactive power flow. The inverter is able to operate in all four quadrants of the $S=P+jQ$ plane. This is useful for regenerative systems since power can be returned to the AC system. It is common to implement current control in this class of converter and separately control the in-phase (real power) and in-quadrature (reactive power) components of current. This might well be performed in the *dq*-domain following Clark and Park transforms of the variables.



Rectifier Mode



Inverter Mode



(ii) Describe the advantages this circuit and mode of operation have over a simple diode rectifier. [4]

A simple diode rectifier has two major problems: (i) the output voltage can not be controlled and (ii) the input (AC) current is not sinusoidal and is rich in harmonics. The inverter operated as a rectifier draws sinusoidal current by imposing a sinusoidal voltage drop across an inductor and thereby operates within harmonic current standards and at a high power factor. The control of real power flow can be used to control the charging and discharging of the DC-side capacitor and thereby control or regulate its voltage.

3. In the context of semiconductor devices designed for use in power electronics, answer the following questions.

(a) Why is a large cross sectional area required? [2]

A large cross sectional area is required to keep the current density reasonably low for a given current to avoid excessive forward voltage drop and power dissipation.

(b) Why is the semiconductor deep? [2]

A deep device is required to achieve a high reverse breakdown voltage. If the doping level is high, the electric field strength is high and avalanche breakdown occurs. If the doping is lowered then avalanche breakdown can be avoided but the depletion layer will be deeper for a given voltage and the semiconductor must be deep enough to support this without punch-through.

(c) For a junction required to block high voltage, why is one side of the junction more heavily doped than the other? [3]

It is sufficient to lightly dope one side of the junction to keep the peak electric field strength low (to avoid avalanche). By lightly doping only one side, only that side suffers from a high ohmic voltage drop in forward conduction. By keeping the other side highly doped it is possible to achieve high-level injection in the lightly doped side in forward conduction to avoid excessive forward voltage drop.

(d) What is high level injection? [3]

In forward conduction, carriers are injected across the junction and the minority carrier concentrations on both sides rise. On the lightly doped side, the natural minority carrier concentration is only slightly below intrinsic levels and not much below majority carrier concentration. The rise in minority carriers requires a rise in majority carriers to maintain charge neutrality outside the deletion region. For a lightly doped region with a large minority carrier injection, both the minority and majority carrier concentrations rise significantly and both are above intrinsic levels. This is high level injection.

(e) Why is the forward voltage drop of a $p^{+}n^{-}$ power diode significantly greater than the 0.7 V expected of a signal diode? [3]

The light doping required of a high voltage diode combined with the large injection of high forward currents results in power diodes operating under high-level injection. Under HLI, the drift voltage required across the neutral region is equal to the voltage drop across the junction and the forward voltage drop can be twice that expected of a junction alone. This is also an ohmic voltage drop across the neutral regions but this is kept relatively small by the high carrier concentrations expect of HLI.

(f) Why is a vertical double-diffused structure used for a MOSFET? [4]

The standard lateral mosfet supports off-state voltage drop across a depletion region formed between a lightly doped body and a heavily doped drain diffusion. The depletion layer grows mostly in the body in the space also used for the on-state channel. The design of the channel is compromised. The doubled diffused structure uses a large lightly doped drain region in which the off-state depletion layer can grow and allowing the channel to form in a short body region as required for good (low) on-state resistance.

(g) Why does the addition of an extra junction to the MOSFET to form the IGBT lower the on-state voltage drop? [3]

The junction is added to the drain side of the device and is forward biased by positive drain-source voltages. The drain is lightly doped and of low conductivity. The extra diffusion to form the junction is heavily doped. In forward conduction, the junction injects minority carriers into the large lightly doped drain region to the extent of high level injection. The conductivity of the drain is modulated (reduced) and the large ohmic voltage drop in the drain of a high-voltage power mosfet is avoided in the IGBT.

4.

(a) Describe the relationships between power, torque, speed and size of an electric motor. [3]

The torque developed by a motor is proportional to the volume of the rotor where the constants of proportionality are the magnetic flux density in the pole pieces and the (one-dimensional) current density around the circumference of the stator. These two constants are fixed by material properties. The power developed is the torque multiplied by the speed.

(b) Describe, with examples, how the shape and layout of a motor might be varied to achieve high acceleration. [4]

To achieve high acceleration, a high torque to moment-of-inertia ratio is required. Where the load inertia dominates, this mean simply a high torque and therefore little constraint is place on the shape. Where the load inertia is low and the motor inertia is significant it is important to use a low radius, long length machine. Additional measures aim to reduce the mass of the rotor. One approach is to use a hollow (annular) rotor or an ironless rotor in which only the winding itself rotates and all magnetic material is stationary. This could use a radial flux design with an armature suspended only at one end or an axial flux (disc) design.

(c) Describe how commutation is achieved in a brushless DC machine [3]

The armature conductors are stationary and the magnets rotate. There is no mechanical commutator. As the magnets rotate their position is sensed (perhaps with a Hall-effect sensor) and feed to a commutation logic circuit. The position is used to establish current references for the armature phases. Power mosfets used under PWM control are used to force currents to flow in the appropriate directions in each phase and to change direction as the magnets move.

(d) A standard chopper is used to supply a brushed DC machine which has a separate circuit supplying field current. The chopper has a DC supply of 200 V. The machine has a maximum armature current of 30 A and maximum field current of 1 A. When the armature is rotated at 1,000 rpm with a field current of 1 A the induced voltage measure across the open circuit armature was 130 V. The armature has a resistance of 0.5 Ω

(i) Calculate the maximum torque available from the machine [2]

$$k_A\phi\big|_{I_F=1A} = \frac{E}{\omega} = \frac{130}{1000\times\frac{2\pi}{60}} = 1.24\ Vs/rad$$

$$T = k_A\phi I_A = 1.24\times 30 = 37.2\ Nm$$

(ii) Calculate the chopper duty-cycle required to achieve maximum torque at 500 rpm. [2]

$$V_A = E + IR = k_A\phi\,\omega + \frac{T}{k_A\phi}R_A = 130\times\frac{500}{1000} + 30\times 0.5 = 80.0\ \text{V}$$

$$\delta = \frac{V_A}{V_{DC}} = \frac{80}{200} = 0.4$$

(iii) Estimate the maximum speed that can be achieved without field weakening and the output power this achieves. [3]

[3.14]

$$V_{\times}^{Max} = 200\,V$$

$$V_A^{Max} = k_A \phi \, \omega^{Max} + I_A^{Max} R_A$$

$$\omega^{Max} = \frac{V_A^{Max} - I_A^{Max} R_A}{k_A \phi} = \frac{200 - 30 \times 0.5}{1.24} = 149.2\,rad/s$$

$$n^{Max} = 1{,}424.7\,rpm$$

$$P_{Out} = E_A I_A^{Max} = 185 \times 30 = 5.5\,kW$$

(iv) Calculate the degree of field weakening required to operate at 2,500 rpm and maximum armature current. Also calculate the torque and output power achieved. [3]

$$V_A^{Max} = k_A \phi \, \omega^{Max} + I_A^{Max} R_A$$

$$k_A \phi' = \frac{V_A^{Max} - I_A^{Max} R_A}{\omega^{Max}} = \frac{200 - 30 \times 0.5}{2{,}500 \times \frac{2\pi}{60}} = 0.707$$

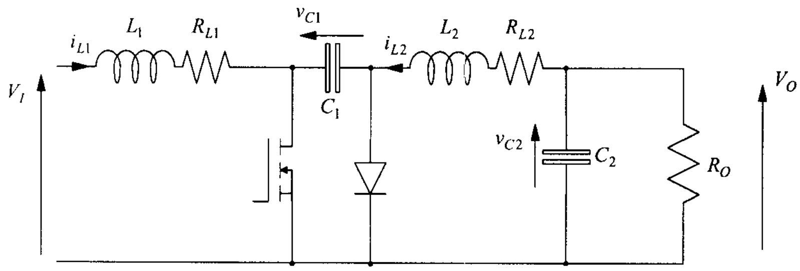
$$\frac{\phi'}{\phi} = \frac{0.707}{1.24} = 57\%$$

$$T^{Max\,'} = T^{Max} \times 0.57 = 21.2\,Nm$$

$$P_{Out} = E_A I_A^{Max} = 185 \times 30 = 5.5\,kW$$

5.

(a) Derive an expression for the output voltage to input voltage ratio of a Ćuk switch-mode power supply (SMPS) and state any assumptions made. [5]



Assume

- C_1 and C_2 are large, V_{C1} and V_{C2} do not change significantly during switching period T
- Both L_1 and L_2 are in continuous conduction and the currents in them are in steady-state

$$\Delta i_{L1}{}^{On} + \Delta i_{L1}{}^{Off} = 0$$

$$\frac{V_1}{L_1}\delta T + \frac{V_1 - V_{C1}}{L_1}(1-\delta)T = 0$$

$$\frac{V_{C1}}{V_1} = \frac{1}{1-\delta}$$

$$\Delta i_{L2}{}^{On} + \Delta i_{L2}{}^{Off} = 0$$

$$\frac{V_{C2} + V_{C1}}{L_2}\delta T + \frac{V_{C2}}{L_2}(1-\delta)T = 0$$

$$\frac{V_{C2}}{V_{C1}} = -\delta$$

$$\frac{V_{C2}}{V_1} = \frac{V_{C2}}{V_{C1}} \times \frac{V_{C1}}{V_1} = \frac{-\delta}{1-\delta}$$

(b) A Ćuk SMPS is to be used to provide a -5 V output from a 9 V input. The maximum output current will be 1.2 A. For a switching frequency of 100 kHz, choose inductors to maintain the input and output current ripples at 10 mA and 20 mA respectively. [6]

$$V_{C2} - \delta V_{C2} = -\delta V_1$$

$$\delta = \frac{V_{C2}}{V_{C2} - V_1} = \frac{-5}{-5-9} = \frac{5}{14} = 0.357$$

$$\Delta i_{L1} = \frac{V_1}{L_1} \times \frac{\delta}{f_{sw}}$$

$$L_1 = \frac{V_1}{\Delta i_{L1}} \times \frac{\delta}{f_{sw}} = \frac{9}{0.01} \times \frac{0.357}{100 \times 10^3} = 3.2\ mH$$

$$\Delta i_{L2} = \frac{V_2}{L_2} \times \frac{1-\delta}{f_{sw}}$$

$$L_2 = \left| \frac{V_2}{\Delta i_{L2}} \times \frac{1-\delta}{f_{sw}} \right| = \frac{5}{0.02} \times \frac{1-0.357}{100 \times 10^3} = 1.6\ mH$$

(c) It is found that for the inductor materials used, the size of the inductor is related to maximum stored energy it can support. The energy density achieved is 180 J/m^3. Calculate the volume of inductor required for the design in (b) and calculate the inductor value that could be used in a buck-boost SMPS to achieve the same voltage ratio as (b) and use the same volume of material. [5]

Output Inductor (ignoring ripple contribution to maximum current)

$$E_2 = \tfrac{1}{2} L_2\, I_2^{\,2} = 0.5 \times 1.6 \times 10^{-3} \times 1.2^2 = 1.16\ mJ$$

$$Vol_2 = \frac{1.16m}{180} = 6.43 \times 10^{-6}\ m^3 = 6.43\ cm^3$$

Input Inductor (ignoring ripple contribution to maximum current)

$$I_1 \approx I_2 \frac{V_2}{V_1} = 1.2 \times \frac{5}{9} = 0.667$$

$$E_1 = \tfrac{1}{2} L_1\, I_1^{\,2} = 0.5 \times 3.0 \times 10^{-3} \times 0.667^2 = 0.667\ mJ$$

$$Vol_1 = \frac{0.667m}{180} = 3.70 \times 10^{-6}\ m^3 = 3.70\ cm^3$$

Total Volume 10.13 cm^3

Buck-Boost will use the same duty-cycle and have same input and output current. The inductor current is given by:

$$\delta I_L = I_1$$

$$I_L = \frac{0.667}{0.357} = 1.87\ A$$

Allowed inductor value is:

$$L = \frac{Vol \times 180}{\tfrac{1}{2} I_L^{\,2}} = \frac{10.13 \times 10^{-6} \times 180}{0.5 \times 1.87^2} = 1.04\ mH$$

(d) Compare the Ćuk SMPS in (b) with the buck-boost SMPS in (c) in terms of the maximum voltage and currents required of the transistor and the maximum ripple in the input current and output currents. [4]

The input and out ripples in the Ćuk are as specified and are 10 mA and 20mA. For the buck-boost the input and output currents are chopped and have an amplitude equal to the inductor current which is 1.87A. The ripple and consequent interference [produced by the buck-boost is much larger than the Ćuk.

The transistor in the Ćuk carries $I_{l1}+I_{L2}$ = 0.667+ 1.2 = 1.87 A when on. In the buck boost the current is also 1.87 A. In the off state the transistor must support $V_{C1}=V_1-V_{C2}$ = 9--5=14V for the Ćuk and the same voltage. There is no penalty in semiconductor rating for using the Ćuk over the buck-boost.

6.

Figure 6.1 shows a transistor switching an inductive load. The load current can be considered constant. The supply voltage, V_{DC} is 700 V and the load current, I_L is 50 A.

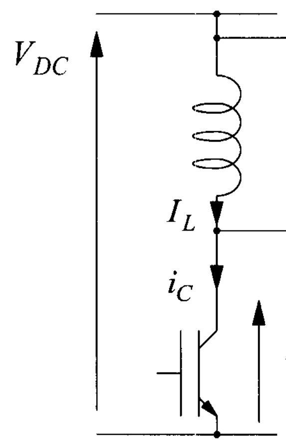
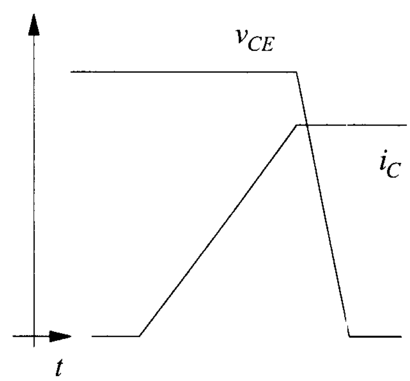


Figure 6.1

(i) Sketch the shape of the collector-emitter voltage and collector current waveforms at turn-on. [2]



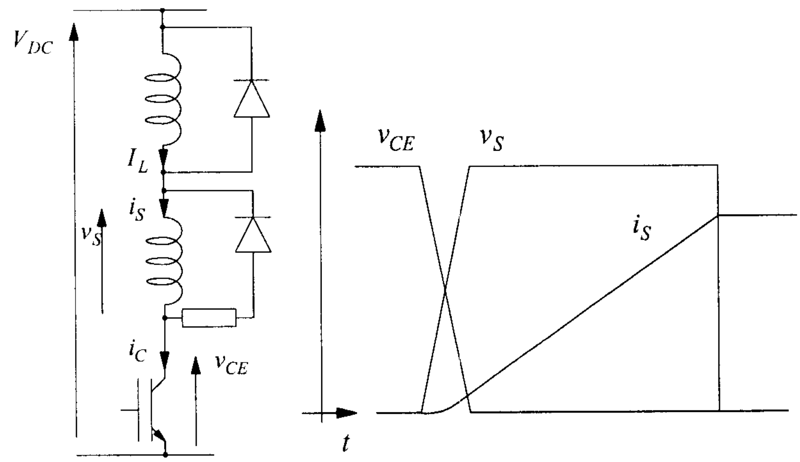
(ii) Calculate the turn-on energy loss if the current rises at a rate of $400\text{x}10^6$ A/s and the voltage falls at $20\text{x}10^9$ V/s. [3]

$$t_{ri} = \frac{I_L}{{di}/{dt}} = \frac{50}{400 \times 10^6} = 125\ ns$$

$$t_{fv} = \frac{V_{DC}}{{dv}/{dt}} = \frac{700}{20 \times 10^9} = 35\ ns$$

$$E_{Sw-On} = \tfrac{1}{2} V_{DC} I_L \left(t_{ri} + t_{fv}\right) = \tfrac{1}{2} \times 700 \times 50 \times \left(35n + 125n\right) = 2.8\ mJ$$

(iii) Sketch the circuit of a turn-on snubber and the associated voltage and current waveforms assuming that the current does not complete its rise before the voltage has fallen. [3]



(iv) Choose snubber components such that the rate of rise of current is never greater than 200 x10^6 A/s and snubber reset takes 2 μs. [4]

Maximum rate of rise occurs with full supply voltage across L_S.

$$L_S = \frac{V_{DC}}{di/dt} = \frac{700}{200\times10^6} = 3.5\ \mu H$$

Snubber reset can be considered to take 5 time constants

$$5\tau = \frac{5L_S}{R_S} = 2\times10^{-6}$$

$$R_S = \frac{5\times3.5\times10^{-6}}{2\times10^{-6}} = 8.75\ \Omega$$

Choose a 10 Ω resistor.

(v) Calculate the turn-on loss of the transistor with the snubber present and the power loss in the snubber resistor. [5]

$$E_{Sw-On} = \int^{t_{fv}} v_{CE}\cdot i_C\cdot dt$$

$$v_{CE} = V_{DC}\left(1 - t/t_{fv}\right)$$

$$for\ 0 < t < t_{fv}$$

$$i_C = i_S = \int \frac{v_S}{L_S}\cdot dt = \int \frac{V_{DC} - v_{CE}}{L_S}\cdot dt$$

$$= \int \frac{V_{DC}\frac{t}{t_{fv}}}{L_S}\cdot dt$$

$$= \frac{V_{DC}}{L_S}\cdot\frac{t^2}{2t_{fv}}$$

[3.14]

$$E_{Sw-On} = \int_0^{t_{fv}} V_{DC}\left(1-\frac{t}{t_{fv}}\right)\cdot\frac{V_{DC}}{L_S}\cdot\frac{t^2}{2t_{fv}}\cdot dt$$

$$= \frac{V_{DC}}{L_S}\left[\frac{t^3}{6t_{fv}} - \frac{t^4}{8t_{fv}^{\ 2}}\right]_0^{t_{fv}} = \frac{V_{DC}}{L_S}\left[\frac{t_{fv}^{\ 2}}{6} - \frac{t_{fv}^{\ 2}}{8}\right]$$

$$= \frac{V_{DC}}{L_S}\cdot\frac{t_{fv}^{\ 2}}{24} = \frac{700\times(35n)^2}{24\times 3.5\mu} = 10.2\ nJ$$

$$E_{R_S} = \tfrac{1}{2} L_S I_L^{\ 2}$$

$$= \tfrac{1}{2}\times 3.5\times10^{-6}\times 50^2 = 4.4\ mJ$$

(vi) The diode is a snap recovery diode. At turn-on of the switch the current continues rising above the load current at the same rate of change until a charge of 0.9 μC has been removed from the diode. At this point the junction recovers and no more current flows. Calculate the diode recovery time and peak recovery current and redraw the turn-off waveforms for the case without the snubber. [3]

$$I_{rr} = \frac{di}{dt} t_{rr}$$

$$Q_{rr} = \tfrac{1}{2} I_{rr} t_{rr} = \tfrac{1}{2}\frac{di}{dt} t_{rr}^{\ 2}$$

$$t_{rr}^{\ 2} = \frac{2Q_{rr}}{\frac{di}{dt}} = \frac{2\times 0.9\times10^{-6}}{400\times10^{6}}$$

$$t_{rr} = 67\ ns$$

$$I_{rr} = 26.8\ A$$