UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2001

MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

*This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute*
*This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science*

# PAPER C481

# MODELS OF CONCURRENT COMPUTATION

Tuesday 1 May 2001, 14:00
Duration: 120 minutes

*Answer THREE questions*

Paper contains 4 questions
Calculators not required

1a Show using equational reasoning that the following holds in CCS:

$$\tau.a+\tau.(b+\tau.c) = \tau.c+\tau.a+\tau.(b+\tau.c)$$

(The **0**s are omitted.) State any equational laws you use.

b Define *weak bisimulation* and *weak equivalence* ($\approx$).

c Let P,Q,R be CCS processes, and let

$$S =_{df} a.P+\tau.(Q+\tau.R)$$
$$T =_{df} \tau.R+\tau.S$$

i) Show carefully that $S \approx T$.

ii) Is it the case that $S = T$? Explain your answer.

d Let

$$U =_{df} a+\tau.(\tau.b+c)$$
$$V =_{df} a+\tau.(b+c)$$

Show that $U \neq V$.

*The four parts carry, respectively, 20%, 20%, 35%, 25% of the marks.*

2a CCS processes are defined as follows:

$$P =_{df} on.P'$$
$$P' =_{df} \overline{a}.P' + off.P$$
$$Q =_{df} a.Q'$$
$$Q' =_{df} b.Q$$
$$S =_{df} new\ a\ (P|Q)$$

i) Draw a static (flow) diagram to illustrate S.

ii) By using the Expansion Theorem, or otherwise, obtain a process T such that T = S and T is defined without using parallel composition.

b Explain and distinguish the strong and weak sum in Temporal CCS (TCCS).

c A mobile phone (process *Phone*) works as follows. The customer inserts a card into the handset (action *insert*), which gives 100 minutes of talk time. The handset meters the number of minutes taken from when the customer starts a call (action *start*) to when they stop talking (action *stop*). Any number of calls is allowed, but at the end of 100 minutes the talk time is exhausted.

Model the phone in TCCS.

d The phone from above is now modified (process *Phone'*) so that additionally the card expires six months after it is inserted. For convenience, six months is taken to be 250,000 minutes.

Model the modified phone in TCCS.

*The four parts carry, respectively, 35%, 15%, 25%, 25% of the marks.*

3a i) State the laws of structural congruence (≡) for *restriction* in the pi-calculus.

ii) Show that if P is a process and a is not a free name of P then new a P ≡ P.

b i) State the rules of *reaction* in the pi-calculus.

ii) Give the full sequence of reactions of the following process:

$$\text{new b } (\bar{a}\langle b\rangle.\mathbf{0}) \mid f(c).\bar{c}\langle d\rangle.\mathbf{0} \mid a(e).\bar{f}\langle e\rangle.\mathbf{0}$$

c A circular buffer CB consists of n cells connected in a ring. Each cell is capable of storing one item. Items are received on channel *in* and output on channel *out*. The cells input items in cyclic order (i.e. 1,2,...,n,1,...). They also output in cyclic order. Initially the buffer is empty.

Model CB in the pi-calculus. Explain your answer briefly with the aid of a diagram.

*The three parts carry, respectively, 30%, 30%, 40% of the marks.*

4a i) Give the definitions of the prefixing and parallel composition operators in the failures model of CSP, paying attention to alphabets.

ii) Suppose that action a belongs to the alphabets of CSP processes P and Q. Use your definitions in Part (i) to show that

$$a.P \parallel a.Q = a.(P \parallel Q)$$

iii) Suppose that P is constructed only using **0** (the STOP process), prefixing and parallel composition. Is it necessarily the case that P||P = P? Explain your answer.

b Let M be some set of possible messages.

i) Give a failures-style specification in CSP of a *Buffer* carrying messages from M.

ii) Give a failures-style specification of a Buffer of capacity at least 2 carrying messages from M.

iii) A *Bag* is like a Buffer, except that messages can be re-ordered. Give a failures-style specification of a Bag carrying messages from M.

*The two parts carry, respectively, 55%, 45% of the marks.*