

General comments on EE2-21 Feedback Systems paper 2012

1. The students have done relatively well on this question, scoring approximately 78%.
 - (a) This is a mechanical modeling question and is a somewhat typical study group question. Figure 1.1 has come up in questions before.
 - i. Typical study group question.
 - ii. Typical study group question.
 - iii. Typical study group question.
 - iv. A bit tricky, since it requires the steady-state value of the derivative of a variable (rather than the variable itself).
 - v. A bit tricky since it uses all the results above. It also asks for a physical interpretation.
 - (b) This is a Nyquist diagram/Routh-Hurwitz question and is mostly typical of study group questions. Figure 1.2 has come up in questions before.
 - i. Typical study group question.
 - ii. Typical study group question.
 - iii. Typical study group question, however, can be done much more quickly if the student uses the answer to Part (1.b.ii) above.
 - iv. Typical study group question.
 - v. Typical study group question, however, it can be done more quickly if the students use Parts (1.b.iii) and (1.b.iv) above.
 - vi. Typical study group question.
2. This question combines knowledge about Nyquist analysis and the Routh-Hurwitz criterion in a slightly non-standard way for compensator design. The students did less well on this question, scoring an average mark of 61%.
 - (a) Standard study group question.
 - (b) The Nyquist diagram can be more easily drawn if the students make use of Part (2.a) above.
 - (c) This uses the extended Nyquist stability criterion in that it requires the determination of closed-loop stability for all possible gains. The students tend to make elementary mistakes in signs, inversions and inequalities.
 - (d) This part is quite tricky since there are two ways of achieving the specifications of a compensated gain margin of 2. In the first, $K = 0.25$, which results in an infinite phase-margin (since the resulting Nyquist diagram is within the unit circle). For the second, we can take $K = -0.5$ (typically the students discount compensators with negative gain), which results in a phase-margin of 180° . Many students expect phase-margins between 0° and 90° .
3. This is a relatively straightforward design question that uses basic concepts of stability from a system's characteristic equation. It turned out to be less tricky than I expected, and the students did relatively well scoring approximately 69%.
 - (a) Most students got the closed-loop poles right, but many did not give the correct comment on the closed-loop stability.
 - (b) Most students got the closed-loop poles right, but many did not give the correct comment on the closed-loop stability.
 - (c) Most students seem to have understood the concept of PD compensator design.
 - i. This is standard study group question, and most students did well.
 - ii. This was from the notes, and most students got it right.
 - iii. This is also mostly from the notes, and most students got it right.
 - iv. A standard question, although a few students couldn't evaluate the limit.

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2012

ISE PART II: MEng, BEng and ACGI

Corrected Copy

FEEDBACK SYSTEMS

Friday, 1 June 2:00 pm

Time allowed: 1:30 hours

There are **THREE** questions on this paper.

Answer **ALL** questions. Question 1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	I.M. Jaimoukha
	Second Marker(s) :	S. Evangelou

1. a) Figure 1.1 shows a mass-spring system where K , D and M have the standard interpretation. The signal $u(t)$ represents an applied force and $y(t)$ the displacement from the rest position.

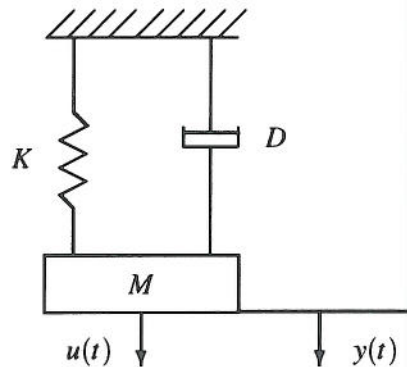


Figure 1.1

- i) Derive the differential equation relating $u(t)$ to $y(t)$. [3]
 - ii) Evaluate the transfer function relating $u(s)$ to $y(s)$. [3]
 - iii) Let $u(t)$ be a unit impulse applied at $t=0$. For this part of the question, take $M=1$, $D=3$ and $K=2$ in appropriate units. Evaluate $y(t)$. [3]
 - iv) Take $K=0$ and let $u(t)$ be a unit step applied at $t=0$. Find the terminal velocity $v_{ss} = \lim_{t \rightarrow \infty} v(t)$ where $v(t) = \dot{y}(t)$. [3]
 - v) Take $K=0$, $M=75\text{kg}$ and let $u(t) = 75 \times g$ where $g = 10\text{ms}^{-2}$. Find the value of D for which the terminal velocity as defined above is 2ms^{-1} . Comment on your answer. [4]
- b) In Figure 1.2 below, $G(s) = 4/(s+1)^3$ and K is a gain.
- i) Determine the steady-state error for a unit step reference signal assuming the closed-loop is stable. [4]
 - ii) Use the Routh Hurwitz criterion to determine the range of values of K for closed-loop stability. [4]
 - iii) Determine the value of $K > 0$ for which the closed-loop is marginally stable. What is the frequency of the resulting oscillations? [4]
 - iv) Sketch the Nyquist diagram of $G(s)$, indicating the low and high frequency portions. [4]
 - v) Let $K=1$. Use the Nyquist criterion, which should be stated, to show that the closed-loop is stable. Find the gain margin. [4]
 - vi) Let $K=10$. Use the Nyquist criterion to show that the closed-loop is unstable. How many unstable poles does the closed-loop have? [4]

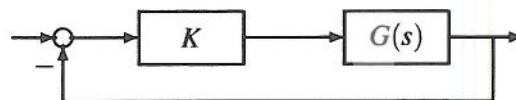


Figure 1.2

2. Consider the feedback control system in Figure 2.1 below. Here,

$$G(s) = \frac{2(s-1)}{(s+1)^2}$$

and $K(s)$ is the transfer function of a compensator.

- a) Let $K(s)$ be a constant compensator $K(s) = K$. Construct a Routh array to find the values of K , call them K_1 and K_2 , such that the closed-loop is marginally stable with $K_1 < K_2$. [6]
- b) Sketch the Nyquist diagram of $G(s)$, clearly indicating the low and high frequency portions. Use the Routh array above to find the real-axis intercepts. [8]
- c) Let $K(s)$ be a constant compensator $K(s) = K$. State the Nyquist stability criterion and use the Nyquist diagram to determine the number of unstable close-loop poles when:
- i) $-\infty < K < K_1$, [2]
- ii) $K_1 < K < 0$, [2]
- iii) $0 < K < K_2$. [2]
- iv) $K_2 < K < \infty$. [2]
- d) Design a constant compensator $K(s) = K$ so that the closed-loop is stable and the gain margin of the compensated system is equal to 2. Comment on the phase-margin of the compensated system. [8]

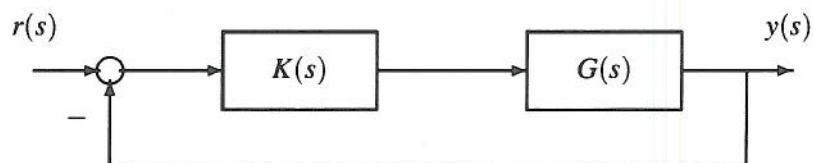


Figure 2.1

3. Let

$$G(s) = \frac{1}{s^2}$$

and consider the feedback loop shown in Figure 3.1 below.

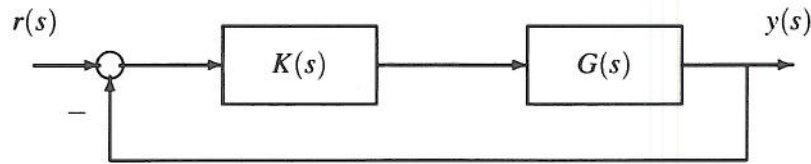


Figure 3.1

- a) Let $K(s) = K$ be a constant compensator with $K > 0$. Find the closed-loop characteristic equation in terms of K . Comment on the closed-loop stability as K varies from 0 to ∞ . [5]
- b) Let $K(s) = -K$ be a constant compensator with $K > 0$. Find the closed-loop characteristic equation in terms of K . Comment on the closed-loop stability as K varies from 0 to ∞ . [5]
- c) A PD compensator $K(s) = K_P + sK_D$, with $K_P > 0$ and $K_D > 0$ is required such that the following design specifications are satisfied:
- The closed-loop is stable.
 - The closed-loop system has a damping ratio $\zeta = 1/\sqrt{2}$.
 - The closed-loop step response has a settling time of 4 seconds.
- i) Derive the location of the closed-loop poles that satisfy the design specifications. [5]
- ii) Write down the closed-loop characteristic equation in terms of K_P and K_D . [5]
- iii) Derive the values of the parameters K_P and K_D that achieve the design specifications. [5]
- iv) For the compensated system, use the final value theorem to evaluate the steady-state error when $r(t) = t$. [5]

SOLUTIONS: Feedback Systems 2012

1. a) i) Applying Newton's laws on the mass,

$$u(t) = M\ddot{y}(t) + D\dot{y}(t) + Ky(t).$$

- ii) Taking Laplace transforms,

$$\frac{y(s)}{u(s)} = \frac{1}{Ms^2 + Ds + K}.$$

- iii) Since $u(t)$ is a unit impulse, $u(s) = 1$. Putting in the numbers,

$$y(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

and so

$$y(t) = e^{-t} - e^{-2t}.$$

- iv) Taking $v(t) = \dot{y}(t)$, the differential equation satisfied by $v(t)$ is

$$u(t) = M\dot{v}(t) + Dv(t)$$

and the transfer function is

$$\frac{v(s)}{u(s)} = \frac{1}{Ms + D}.$$

Since $u(s) = 1/s$, using the final value theorem,

$$v_{ss} = \lim_{s \rightarrow 0} sv(s) = 1/D.$$

- v) Putting in the numbers, $u(s) = 750/s$ and so $v_{ss} = 750/D$. Therefore $D = 375$. This answer could represent the evaluation of a damping value for safe landing for, e.g. a parachutist.

- b) i) The error signal is given by $e(s) = \frac{r(s)}{1+KG(s)}$ and so, using the final value theorem, $e_{ss} = \lim_{s \rightarrow 0} \frac{1}{1+KG(s)} = \frac{1}{1+4K}$.
- ii) The characteristic equation for the closed-loop is

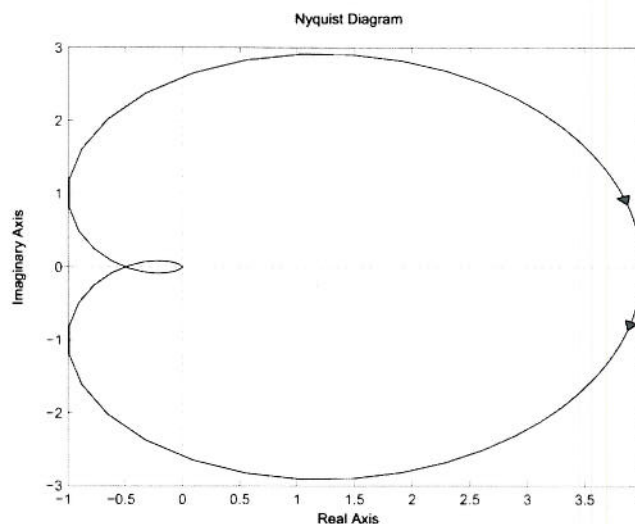
$$1 + KG(s) = 1 + \frac{4K}{(s+1)^3} = 0 \Rightarrow s^3 + 3s^2 + 3s + 1 + 4K = 0$$

The Routh array is:

s^3	1	3
s^2	3	$1+4K$
s	$0.75(2-K)$	
1	$1+4K$	

For stability we need the first column to be positive, so $-0.25 < K < 2$.

- iii) When $K = 2$ the third row is zero and so the closed-loop is marginally stable. The auxiliary equation is given by $3(s^2 + 3) = 0$ and so the resulting frequency of oscillations is $\sqrt{3}$ rad/s.
- iv) The Nyquist diagram is shown below.



- v) When $K = 1$, we need the real-axis intercept. This can be obtained from Part (iii) above as -0.5 . The Nyquist criterion states that $N = Z - P$, where N is the number of clockwise encirclements by the Nyquist diagram of the point $-K^{-1} = -1$, P is the number of unstable open-loop poles and Z is the number of unstable closed-loop poles. Since $G(s)$ is stable, $P = 0$. From the diagram, $N = 0$ and so $Z = 0$ and the closed-loop is stable.
- vi) When $K = 10$, $N = 2$ and so $Z = 2$. Therefore there are two unstable closed-loop poles.

2. a) The characteristic equation for the closed-loop is

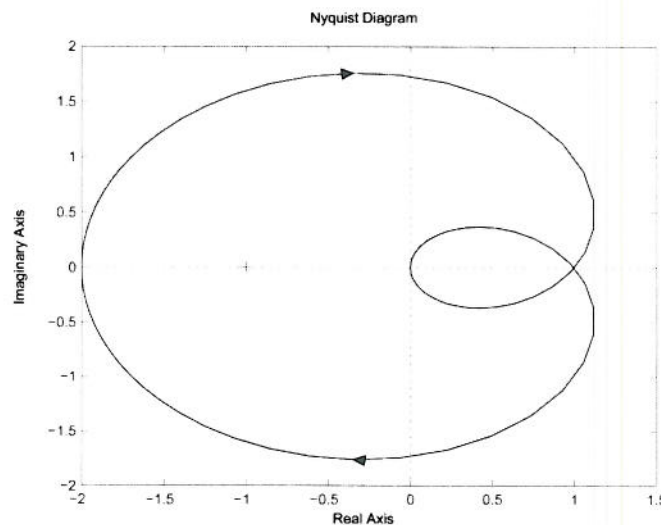
$$1 + KG(s) = 1 + \frac{2K(s-1)}{(s+1)^2} = 0 \Rightarrow s^2 + 2(1+K)s + (1-2K) = 0$$

The Routh array is:

$$\begin{array}{c|cc} s^2 & 1 & 1-2K \\ s & 2(1+K) & \\ 1 & 1-2K & \end{array}$$

Therefore $K_1 = -1$ and $K_2 = 0.5$.

- b) The Nyquist diagram is shown below. The real-axis intercepts can be found as $-1/K_1$, $-1/K_2$, or 1, -2 as well as 0.



- c) When $K(s) = K$, we have $N = Z - P$, where N is the number of clockwise encirclements by the Nyquist diagram of the point $-K^{-1}$, P is the number of unstable open-loop poles and Z is the number of unstable closed-loop poles. Here, $P = 0$.
- i) When $-\infty < K < -1$, $N = 2$ so $Z = 2$.
 - ii) When $-1 < K < 0$, $N = 0$ so $Z = 0$.
 - iii) When $0 < K < 0.5$, $N = 0$ so $Z = 0$.
 - iv) When $0.5 < K < \infty$, $N = 1$ so $Z = 1$.
- d) For closed-loop stability we need $-1 < K < 0.5$. An inspection of the Nyquist diagram shows that for a gain margin of 2, the compensated system must have a real-axis intercept at -0.5 . This implies that $K = 0.25$. Since the Nyquist diagram of the compensated system $KG(s)$ lies within the unit circle centred at the origin, the phase margin is infinite.

3. a) The closed-loop characteristic equation is

$$1 + KG(s) = 0$$

or

$$s^2 + K = 0.$$

It follows that the closed-loop poles are given as

$$s = \pm j\sqrt{K}$$

and so the closed-loop is marginally stable for all K .

- b) The closed-loop characteristic equation is

$$1 - KG(s) = 0$$

or

$$s^2 - K = 0.$$

It follows that the closed-loop poles are given as

$$s = \pm \sqrt{K}$$

and so the closed-loop is unstable for all K .

- c) i) For $\zeta = 1/\sqrt{2}$, the real and imaginary parts of the pole are equal. For a settling time of 4 seconds, the real part must be equal to -1 . Thus the closed-loop poles must be equal to

$$s_1, \bar{s}_1 = -1 \pm j.$$

- ii) The characteristic equation is

$$1 + \frac{K_P + sK_D}{s^2} = 0$$

or

$$s^2 + sK_D + K_P = 0.$$

- iii) Since the closed-loop poles must be equal to $s_1, \bar{s}_1 = -1 \pm j$, it follows that $K_D = 2$ and $K_P = 2$.

- iv) The error signal for the compensated system is

$$e(s) = \frac{r(s)}{1 + K(s)G(s)}$$

with

$$G(s) = 1/s^2, K(s) = 2(s+1), r(s) = 1/s^2.$$

Using the final value theorem

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} se(s) = \lim_{s \rightarrow 0} \frac{s}{s^2 + 2(s+1)} = 0.$$