Q1. The fractional transition BW increases the rate of fall off of the frequency response, and for linear phase FIR filters this is clearly related to the "frequency" of the maximum cosine term in the amplitude response. Thus the higher this frequency, the narrower the transition width and hence the relationship. 4

For a single stage decimation we have:–

$$32\,\text{kHz} \longrightarrow \boxed{D:1\downarrow} \longrightarrow 800\,\text{kHz}$$

the FIR filter has an order $N \cong \frac{40}{20} \cdot \frac{32{,}000}{50}$

or $N = 1{,}280$

The computational complexity is $1{,}280 \times 32 \times 10^3$ or $\approx 41 \times 10^6$ mults/second. 3

As a two-stage operation we have:–

The first stage can have wider transition BW by 10× then $N_1 = \frac{40}{20} \cdot \frac{32{,}000}{500} = 128$

and the associated computational complexity is 4×10^6

The second stage has the required transition BW but at a reduced sampling rate ie 3.2 kHz

Hence $N_2 = \frac{40}{20} \cdot \frac{3{,}200}{50} = 128$ 8

Thus the total computational complexity is 8×10^6 – a gain of 5×

The computational complexity can be reduced further by a multistage decimation with the early stages arranged to have higher decimation rates. 5

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AS Alu

Q2. <u>Computational complexity</u> in DFT is taken to be the total number of complex multiplications required for its computation. (Sometimes implicit symmetries can be used for its reduction).

<u>Twiddle factors</u> are phasing factors in the form $\exp(-j\frac{2\pi}{N_i} k_i n_j)$ between stages that modify partial computations in a multi-stage DFT evaluation. 3

For N-point DFT there are N complex multiplications per point producing a total of $O(N^2)$ multiplications.

For
$$\left.\begin{aligned} n &= \langle An_1 + Bn_2 \rangle_N \\ k &= \langle Ck_1 + Dk_2 \rangle_N \end{aligned}\right\} \quad N = N_1 N_2$$
we have,

$$X(k) = X(\langle Ck_1 + Dk_2 \rangle_N) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(\langle An_1 + Bn_2 \rangle_N) W_N^P$$

where $P = (An_1 + Bn_2)(Ck_1 + Dk_2)$ & $W_N = e^{-j\frac{2\pi}{N}}$

Thus $W_N^P = W_N^{ACn_1k_1} \cdot W_N^{ADn_1k_2} \cdot W_N^{BDn_2k_2} \cdot W_N^{BCn_2k_1}$

For the complete removal of twiddle factors we need

$$\langle AD \rangle_N = 0 \quad \langle BC \rangle_N = 0 \quad \langle AC \rangle_N = N_2$$ 10
$$\text{and} \quad \langle BD \rangle_N = N_1$$

Let $\langle N_1^{-1} \rangle_{N_2} = \alpha$ or $\langle \alpha N_1 \rangle_{N_2} = 1$ or $\alpha N_1 = \beta N_2 + 1$

$\langle N_2^{-1} \rangle_{N_1} = \gamma \qquad \gamma N_2 = \delta N_1 + 1$

Then
$$\langle AC \rangle_N = \langle N_2(\delta N_1 + 1) \rangle_N = N_2$$
$$\langle BD \rangle_N = \langle N_1(\beta N_2 + 1) \rangle_N = N_1$$

$\langle AD \rangle_N = \langle N_1 N_2 \langle N_1^{-1} \rangle_{N_2} \rangle_N = 0$ i.e. multiple of $N_1 N_2$ and similarly with $\langle BC \rangle_N$ 3

The algorithm maps a 1-D array $\{x(n)\}$ to a 2-D array. Then a line-by-line followed by a column-by-column N_1 & N_2 point (1D) DFTs (or v.v.) are carried out to return frequency samples to $\{X(\langle Ck_1 + Dk_2 \rangle_N)\}$ 4

All 20 PTS

Q3. Let the output at the intermediate adder be U.
Then in the z-transform domain we can write

$$U(z) = X(z) + \alpha z^{-1} U(z) \quad \text{or} \quad U(z) = X(z) / (1 - \alpha z^{-1})$$

At the output adder we have

$$Y(z) = -\alpha X(z) + (1-\alpha^2).z^{-1} U(z)$$

$$= -\alpha X(z) + (1-\alpha^2).z^{-1} \frac{X(z)}{1-\alpha z^{-1}}$$

or $$\frac{Y(z)}{X(z)} = -\alpha + \frac{(1-\alpha^2) z^{-1}}{1-\alpha z^{-1}} = \frac{-\alpha + \alpha^2 z^{-1} + z^{-1} - \alpha^2 z^{-1}}{1-\alpha z^{-1}}$$

Thus

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}} \qquad \text{———} (1)$$

To show that H(z) is allpass we can write the above as

$$H(z)\Big|_C = z^{-1} \frac{(1-\alpha z)}{1-\alpha z^{-1}} = z^{-1} \frac{(1-\alpha z^{-1})^*}{(1-\alpha z^{-1})}\Bigg|_{C:|z|=1}$$

Thus $$|H(z)|_{C:|z|=1} = 1.$$ 3

From equ(1) we can write

$$Y(z)(1-\alpha z^{-1}) = X(z)(z^{-1} - \alpha)$$

and hence

$$Y(z) = \alpha z^{-1} Y(z) + X(z)(z^{-1} - \alpha) \qquad \text{———} (2)$$

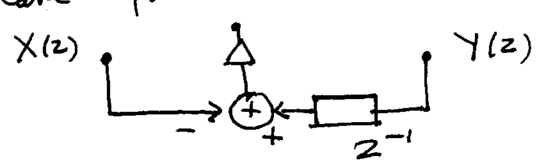
For single multiplier realisation write equ(2) as

$$Y(z) = z^{-1} X(z) + \alpha [z^{-1} Y(z) - X(z)] \qquad \text{—} (3)$$

At this stage we can have many SFGs depending on any additional constraints to be taken into consideration.

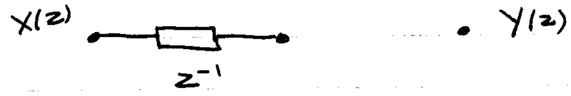
A direct non-canonic realisation is to generate separately the components on the RHS of equ(3).

With the input and output nodes defined we have for the second term

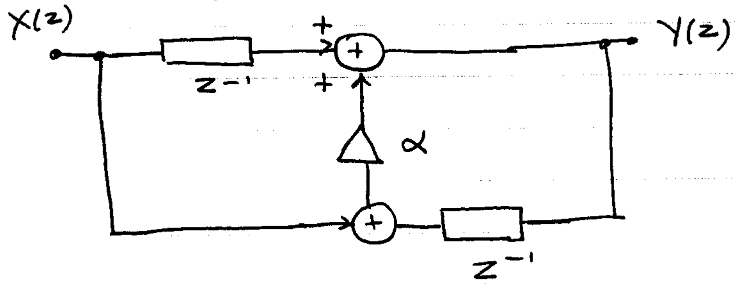


Allpass

while the first term of equ (3) is directly implementable as



On combining the two SFGs we obtain 4



Impulse response $h(n)$

$$h(n) = \mathcal{Z}^{-1}\left[\frac{z^{-1}-\alpha}{1-\alpha z^{-1}}\right]$$

$$= \mathcal{Z}^{-1}\left[\frac{-\alpha}{1-\alpha z^{-1}} + \frac{z^{-1}}{1-\alpha z^{-1}}\right]$$

$$= \mathcal{Z}^{-1}\left[\frac{-\alpha}{1-\alpha z^{-1}}\right] + {}^{-1}\left[\frac{z^{-1}}{1-\alpha z^{-1}}\right]$$

$$= -\alpha\,(\alpha)^{n} + \alpha^{n-1} \qquad n \geq 1$$

$$= -\alpha \qquad n = 0$$

$$= 0 \qquad n < 0$$

When $T_{rev} = 800$ msecs we have $10 \times 800 = 8{,}000$ samples i.e.

$$\alpha^{8{,}000}(1-\alpha^2) = \frac{1}{100}(1-\alpha^2)$$ 5

or $\alpha = 0.9994$

Problems of precision are likely to be encountered with such a value of α in a fixed point realisation. 4

A richer impulse response can be obtained when the allpass is of higher order. 4

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[initials] PTS

Q4 For the given system we have

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 1 & H_{12}(z) \\ H_{21}(z) & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

The new signals U_1 and U_2 are given by

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 1 & -G_{12}(z) \\ -G_{21}(z) & 1 \end{bmatrix}\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

By combining the above we have

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 1 & H_{12}(z) \\ H_{21}(z) & 1 \end{bmatrix}\begin{bmatrix} 1 & -G_{12}(z) \\ -G_{21}(z) & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - H_{12}(z)\,G_{21}(z) & H_{12}(z) - G_{12}(z) \\ H_{21}(z) - G_{21}(z) & 1 - H_{21}(z)\,G_{12}(z) \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Thus there are two possible outcomes

(A) the main diagonal is zero

ie. $H_{12}(z) - G_{12}(z) = 0$

$H_{21}(z) - G_{21}(z) = 0$

giving

$$U_1 = (1 - H_{12}(z)\cdot G_{21}(z))\,X_1$$

$$U_2 = (1 - H_{21}(z)\cdot G_{12}(z))\,X_2$$

(B) the minor diagonal is zero

ie.

$1 - H_{12}(z).\,G_{21}(z) = 0$

$1 - H_{21}(z)\cdot G_{12}(z) = 0$

Ae PTS

giving

$$U_1 = [H_{12}(z) - G_{12}(z)] X_2$$
$$U_2 = [H_{21}(z) - G_{21}(z)] X_1$$ 12

The second case requires

$$G_{12}(z) = \frac{1}{H_{21}(z)}$$

$$G_{21}(z) = \frac{1}{H_{12}(z)}$$

If the mixing matrix (ie channel transfer functions) have zeros on or near the circumference of the unit circle then the dynamic range requirements for $G_{12}(z)$ and $G_{21}(z)$ would be large and may not be practically attainable.

Moreover if $H_{12}(z)$ and $H_{21}(z)$ are non-minimum phase then $G_{12}(z)$ and $G_{21}(z)$ would be unstable and hence the entire scheme unrealisable. 8

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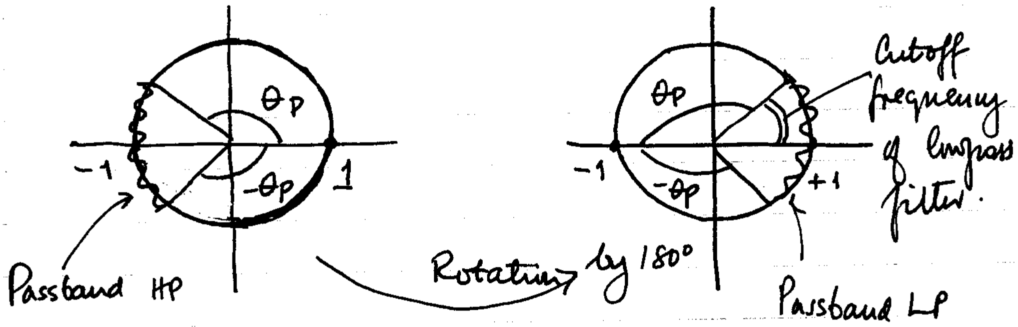
AW PTS

Q5 The z-plane sketch for the given $H_{HP}(z)$ is



Rotation by 180° is equivalent to replacing z by -z.
From the figures it follows that the cutoff frequency of the lowpass filter θ_c will be such that

$$\theta_c + \theta_p = \pi$$

Let

$$H_{HP}(z) = \sum_{n=-\infty}^{+\infty} h_{HP}(n)\, z^{-n}$$

so that $$H_{LP}(z) = H_{HP}(-z) = \sum_{n=-\infty}^{+\infty} h_{HP}(n) \cdot (-1)^n\, z^{-n}$$

i.e. $$h_{LP}(n) = (-1)^n \cdot h_{HP}(n)$$ 5

For bandsplitting into two equal bands $\theta_p = \theta_c = \frac{\pi}{2}$
Since alternate coefficients of the impulse are of opposite sign to their corresponding counterparts in the complementary filter, and in absolute values they are all equal we can group even indexed terms together and odd indexed terms together. Their sum would produce one filter while their difference would produce the complementary filter.

$$G(z) = \sum_{n=-\infty}^{+\infty} h_{LP}(n)\,(e^{-j\theta_0} z)^{-n} + \sum_{n=-\infty}^{+\infty} h_{LP}(n)\,(e^{+j\theta_0} z)^{-n}$$ 5

$$= \sum_{n=-\infty}^{+\infty} h_{LP}(n)\, [e^{-jn\theta_0} + e^{+jn\theta_0}] \cdot z^{-n}$$

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ie. the impulse response is given by

$$g(n) = 2h_{LP}(n) \cdot \cos n\theta_0$$

The bandwidth is then equal to

$$BW = (\theta_0 + \theta_c) - (\theta_0 - \theta_c) = 2\theta_c$$

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In realisable systems the amplitude response does not necessarily fall off rapidly to zero outside the passband.

Hence the addition of shifted versions of the LP response will produce interactions between the passband and/or stopband of one with the corresponding part of the other. These interactions will combine destructively due to phase relationships.

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