EE3-12 Optoelectronics Solutions 2015

1. a) Polarization is represented by the electric field vector of an electromagnetic wave.

[1]

Unpolarized sources (such as the sun, incandescent lamps, and LEDs) emit light whose E-field points in random directions. Polarized sources (such as lasers) emit light whose E-field points in a single direction.

[2]

'TE' stands for 'transverse electric, and 'TE incidence' implies that the electric field vector is perpendicular to a plane in which an electromagnetic problem is drawn.

[2]

b) Maxwell's equations (given) are:

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

Similarly, the material equations are

$\underline{\mathbf{J}} = \sigma\,\underline{\mathbf{E}}$, $\underline{\mathbf{D}} = \varepsilon\,\underline{\mathbf{E}}$, $\underline{\mathbf{B}} = \mu\,\underline{\mathbf{H}}$.

Assuming that the medium is an optical one, we can take $\sigma = 0$, $\mu = \mu_0$ and $\varepsilon = \varepsilon_0\varepsilon_r$.

Hence:

$\mathrm{div}(\underline{\mathbf{D}}) = 0$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\mu_0\,\partial\underline{\mathbf{H}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \varepsilon_0\varepsilon_r\,\partial\underline{\mathbf{E}}/\partial t$

[2]

Taking the curl of the 3rd equation:

$\mathrm{curl}\,\{\mathrm{curl}(\underline{\mathbf{E}})\} = -\mu_0\,\mathrm{curl}\,\{\partial\underline{\mathbf{H}}/\partial t\} = -\mu_0\,\partial\{\mathrm{curl}\,\underline{\mathbf{H}}\}/\partial t = -\mu_0\varepsilon_0\varepsilon_r\,\partial^2\underline{\mathbf{E}}/\partial t^2$

[2]

Using the identity $\mathrm{curl}\,\{\mathrm{curl}(\underline{\mathbf{F}})\} = \mathrm{grad}\,\{\mathrm{div}(\underline{\mathbf{F}})\} - \nabla^2\underline{\mathbf{F}}$, we then get:

$\mathrm{grad}\,\{\mathrm{div}(\underline{\mathbf{E}})\} - \nabla^2\underline{\mathbf{E}} = -\mu_0\varepsilon_0\varepsilon_r\,\partial^2\underline{\mathbf{E}}/\partial t^2$

The 1st equation implies that $\mathrm{div}(\varepsilon\underline{\mathbf{E}}) = 0$. Hence, $\mathrm{div}(\underline{\mathbf{E}})$ and $\mathrm{grad}\,\{\mathrm{div}(\underline{\mathbf{E}})\}$ are zero, so:

$\nabla^2\underline{\mathbf{E}} = \mu_0\varepsilon_0\varepsilon_r\,\partial^2\underline{\mathbf{E}}/\partial t^2$

[2]

Assuming now that the field is oscillating at angular frequency ω, we can write:

$\underline{\mathbf{E}} = \underline{\mathbf{E}}\,\exp(j\omega t)$

$\partial\underline{\mathbf{E}}/\partial t = j\omega\,\underline{\mathbf{E}}\,\exp(j\omega t)$

$\partial^2\underline{\mathbf{E}}/\partial t^2 = -\omega^2\,\underline{\mathbf{E}}\,\exp(j\omega t)$

$\nabla^2\underline{\mathbf{E}} = -\omega^2\mu_0\varepsilon_0\varepsilon_r\,\underline{\mathbf{E}}$

[3]

Assuming now that the field is polarized in the y-direction, we can write $\underline{\mathbf{E}} = E_y\,\underline{\mathbf{j}}$, and:

$\nabla^2 E_y = -\omega^2\mu_0\varepsilon_0\varepsilon_r\,E_y$

[1]

c) Expanding, the wave equation can be written as:

$\partial^2E_y/\partial x^2 + \partial^2E_y/\partial y^2 + \partial^2E_y/\partial z^2 = -\omega^2\mu_0\varepsilon_0\varepsilon_r\,E_y$

However, if E_y is a function only of x and z, the equation simplifies to:

$\partial^2E_y/\partial x^2 + \partial^2E_y/\partial z^2 = -\omega^2\mu_0\varepsilon_0\varepsilon_r\,E_y$

[1]

One possible solution might be a plane wave, $E_y = E_0\exp(-jkz)$. In this case:

$\partial E_y/\partial x = 0$

$\partial E_y/\partial z = -jk\,E_y$

$\partial^2E_y/\partial z^2 = -k^2\,E_y$

Consequently the solution is valid provided $k^2 = \omega^2\mu_0\varepsilon_0\varepsilon_r$

[2]

Another solution might be an inhomogeneous wave, $E_y = E_0\exp(\gamma x)\exp(-j\beta z)$. In this case:

$\partial E_y/\partial x = \gamma\,E_y$ $\qquad\partial^2E_y/\partial x^2 = \gamma^2\,E_y$

$\partial E_y/\partial z = -j\beta\,E_y$ $\qquad\partial^2E_y/\partial z^2 = -\beta^2\,E_y$

Consequently the solution is valid provided $\gamma^2 - \beta^2 = -\omega^2\mu_0\varepsilon_0\varepsilon_r$ or $\gamma^2 - \beta^2 + k^2 = 0$

[2]

2. a) The sensor is based on a Mach-Zehnder interferometer.

[1]

The two components labelled C_1 and C_2 are both directional couplers.

[1]

Their function is to split the input beam equally between the reference and sensing arms, and to recombine the two beams after they have passed through the two arms.

[2]

b) For unity input amplitude, the amplitudes at X and Y after passing through C_1 are:

$A_1 = \cos(\kappa L)$, $A_2 = -j \sin(\kappa L)$

where L is the length of the coupling region and κ is the coupling coefficient.

Assuming the coupler provides an equal power split, $\kappa L = \pi/4$ and we can write:

$A_1 = 1/\sqrt{2}$, $A_2 = -j/\sqrt{2}$

[2]

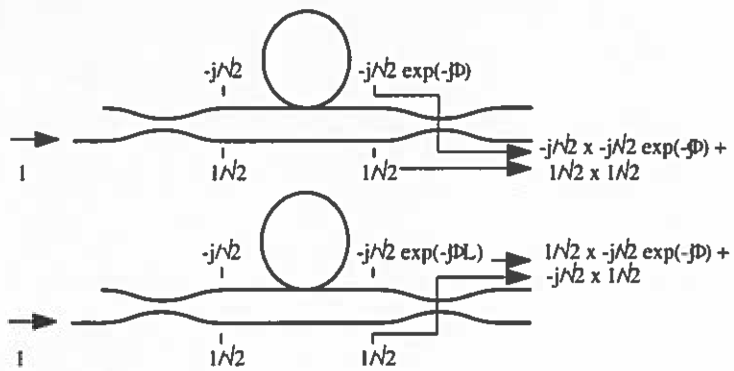
After passing through the reference and sensing arms, the two amplitudes (ignoring any common phase delay) are:

$A_1' = 1/\sqrt{2}$, $A_2' = (-j/\sqrt{2}) \exp(-j\Phi)$

After passing through the coupler C_2 (assumed to be the same as C_1) the amplitudes are:

$A_1'' = (1/\sqrt{2})(1/\sqrt{2}) + (-j/\sqrt{2})(-j/\sqrt{2}) \exp(-j\Phi) = \{1 - \exp(-j\Phi)\}/2$

$A_2'' = (1/\sqrt{2})(-j/\sqrt{2}) + (1/\sqrt{2})(-j/\sqrt{2}) \exp(-j\Phi) = -j\{1 + \exp(-j\Phi)\}/2$



[4]

The output powers are then:

$P_1 = A_1A_1^* = \{1 - \exp(-j\Phi)\}\{1 - \exp(+j\Phi)\}/4$

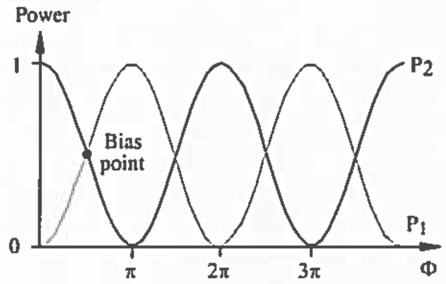
$P_1 = \{2 - \exp(-j\Phi) - \exp(-j\Phi)\}/4 = \{1 - \cos(\Phi)\}/2 = \sin^2(\Phi/2)$

$P_2 = A_2A_2^* = \{1 + \exp(-j\Phi)\}\{1 + \exp(+j\Phi)\}/4$

$P_1 = \{2 + \exp(-j\Phi) + \exp(-j\Phi)\}/4 = \{1 + \cos(\Phi)\}/2 = \cos^2(\Phi/2)$

[2]

c) The variation in the two output powers as a function of Φ can then be drawn as:



[2]

Since $\Phi = \beta \Delta L$, where ΔL is the path imbalance, the output powers are therefore sensitive to changes in ΔL. If these can be cause by temperature, the device can act as a thermometer; alternatively, if these can be caused by strain the device can act as a strain sensor.

[2]

The sensitivity $dP_{1,2}/d\Phi$ is clearly not constant, and is zero whenever $\Phi = \nu\pi$.

[2]

The sensitivity is maximised when $\Phi = \pi/2, 3\pi/2, 5\pi/2 \ldots$ and consequently one of these conditions should be chosen as the bias point.

[2]

3. a) A Y-junction consists of a fork in a single mode waveguide (a). Its operation can be understood in terms of the characteristic modes that it supports (b). At the left-hand end, it supports a single, symmetric mode. At the right-hand end, it supports two modes. The symmetric mode now has two peaks, and a further double-peaked anti-symmetric mode starts to exist part way through the fork.

[2]

An input to the single guide at the left-hand end (c) corresponds to excitation of the symmetric mode. As this evolves, the power is distributed equally, so that the device provides a 50 : 50 power split. Hence the two output amplitudes are $1/\sqrt{2}$ and $1/\sqrt{2}$.

[2]

Equal excitation of both guides at the right-hand (d) again corresponds to excitation of the symmetric mode. This arrangement involves a time reversal of (c) but with the excitation increased by a factor of $\sqrt{2}$, so the output amplitude is increased by the same factor.

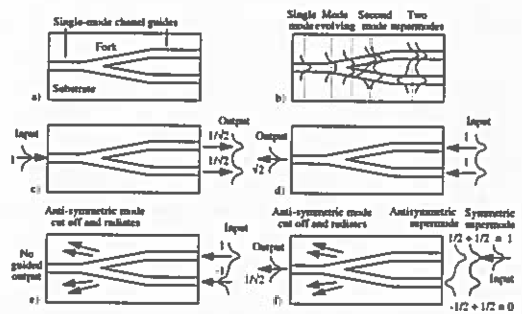
[2]

Equal excitation of both guides at the right-hand end, but in anti-phase (e), now corresponds to excitation of the antisymmetric mode. This will become cut off part way through the fork, so that all the power is radiated. Hence, there will be no guided output.

[2]

Excitation of only one of the guides at the right hand end (f) corresponds to excitation of both the symmetric and antisymmetric modes together, with equal amplitude. Since the antisymmetric mode must be radiated at the point where it cuts off, only the symmetric mode emerges. Since this carries half the power, the output amplitude is $1/\sqrt{2}$.

[2]



b) The amplitudes at each stage can be found by expanding the excitation pattern into symmetric and antisymmetric components, and then assuming that the antisymmetric mode is lost by radiation. For i) we get:

Input	Intermediate	Symm mode	Asymm mode	Output
$1/\sqrt{2}$	$1/\sqrt{2}$	0	$1/\sqrt{2}$	
				$0^{(3)}$
	$-1/\sqrt{2}^{(1)}$	0	$-1/\sqrt{2}$	
-1				
	$-1/\sqrt{2}$			$-1/\sqrt{2}^{(2)}$

[4]

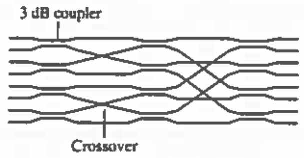
For ii) we get:

Output	Symm mode	Asymm mode	Intermediate	Symm mode	Asymm Mode	Input
				1/2	1/2	1
	$-1/\sqrt{2}$	$+2/\sqrt{2}$	$1/\sqrt{2}^{(2)}$			
				1/2	-1/2	0
$-1^{(3)}$						
				-3/2	+3/2	0
	$-1/\sqrt{2}$	$-2/\sqrt{2}$	$-3/\sqrt{2}^{(1)}$			
				-3/2	-3/2	-3

[6]

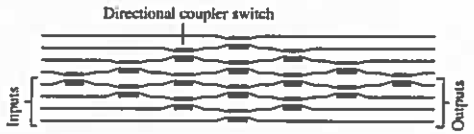
5. a) Component layouts are as follows:

Star coupler



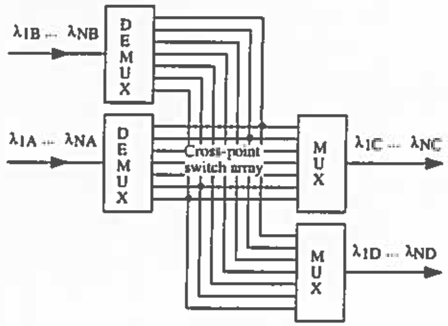
[2]

Non-blocking switch array



[2]

Wavelength routing switch



[2]

b) The coupled mode equations are:

$dA_1/dz + j\kappa A_2 = 0$

$dA_2/dz + j\kappa A_1 = 0$

Differentiating the upper equation:

$d^2A_1/dz^2 + j\kappa\, dA_2/dz = 0$

Substituting using the upper equation:

$d^2A_1/dz^2 + \kappa^2 A_1 = 0$

Hence, $A_1 = A\cos(\kappa z) + B\sin(\kappa z)$ and $dA_1/dz = -\kappa A\sin(\kappa z) + \kappa B\cos(\kappa z)$

[3]

The boundary conditions are $A_1(0) = 1$, $A_2(0) = 0$.

Using the upper equation, these can be rewritten as $A_1(0) = 1$, $dA_1(0)/dz = 0$.

From the first condition, $A = 1$; from the second condition, $B = 0$

Hence $A_1 = \cos(\kappa z)$ and $dA_1/dz = -\kappa\sin(\kappa z)$

Using the upper equation, $A_2 = (1/j\kappa)\, dA_1/dz = j\sin(\kappa z)$

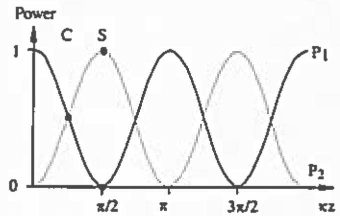
[3]

The two output powers are then:

$P_1 = A_1A_1^* = \cos^2(\kappa z)$ $P_2 = A_2A_2^* = \sin^2(\kappa z)$

And the variation with of the two powers with κz can then be sketched as shown below.

[2]



[2]

For a 3dB coupler, an equal power split is needed, for example at point C when $\kappa z = \pi/4$.

For a directional coupler switch we require a 100% cross-coupling efficiency (which can be switched off electro-optically) as at point S when $\kappa z = \pi/2$.

[2]

The star coupler uses 3 dB splitters, and consequently requires $\kappa z = \pi/4$. If the coupling coefficient is $\kappa = 0.5\ \text{mm}^{-1}$, the length needed is $z = \pi/(4\kappa) = \pi/2$ mm = 1.57 mm.

[1]

The switch array and wavelength switch both use switch elements, and consequently require $\kappa z = \pi/2$. If the coupling coefficient is $\kappa = 0.5\ \text{mm}^{-1}$, the length needed is $z = \pi/(2\kappa) = \pi$ mm = 3.14 mm.

[1]

5. a) The relationship between the energy gained or lost in a band-to-band transition and the wavelength of the absorbed or emitted photon is $eE_g = hc/\lambda_g$, where:

$e = 1.6 \times 10^{-19}$ is the electron charge

$h = 6.62 \times 10^{-34}$ is Planck's constant

$c = 3 \times 10^{8}$ m/s is the speed of light

Hence, $E_g = hc/e\lambda_g$ (eV) and $\lambda_g = hc/eE_g$ (m) and the numerical entries can be completed as below. The band structure follows from knowledge that InP is the most-common substrate material for near-infrared emitters, while Si is not as an optoelectronic material.

Material	E_g (eV) at 300 K	λ_g (μm)	Band structure
Ge	0.66	**1.88**	Indirect gap
InP	1.35	0.92	**Direct gap**
GaP	2.2	**0.56**	Indirect gap
Si	**1.12**	1.11	**Indirect gap**
GaAs	1.42	0.87	Direct gap

[5]

To absorb light, we require the photon energy to be greater than that needed for a band-to-band transition, i.e. $hc/\lambda > hc/\lambda_g$. Consequently, we must have $\lambda < \lambda_g$. If $\lambda = 1.55$ μm, the only suitable material is Ge.

[2]

b) Radiative recombination involves electron-hole recombination events that generate quanta of light, or photons. Non-radiative recombination involves recombination events that generate quanta of heat, or phonons. Only the former is useful.

[2]

Total recombination is described by the electron rate equation:

$dn/dt = r_e n = r_{rr} n + r_{nr} n$ where

r_e is the total recombination coefficient

r_{rr} is the radiative recombination coefficient

r_{nr} is the non-radiative recombination coefficient

This equation can be written alternatively as:

$dn/dt = n/\tau_e = n/\tau_{rr} + n/\tau_{nr}$ where

τ_e is the total recombination lifetime

τ_{rr} is the radiative recombination lifetime

τ_{nr} is the non-radiative recombination lifetime

[2]

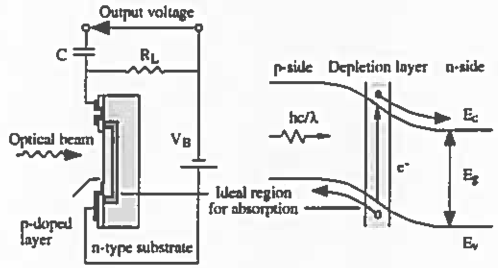
The relation between the lifetimes is clearly $1/\tau_e = 1/\tau_{rr} + 1/\tau_{nr}$

Clearly, only radiative transitions are useful. The quantum efficiency of the light-generation process is therefore:

$\eta = r_{rr}/r_e = \tau_e/\tau_{rr}$

[2]

c) Construction and band diagram of a surface entry photodiode are typically as below.



[4]

The ideal region for photon absorption is the depletion layer, since the high electric field in each case will separate the photo-generated electron-hole pairs and force them towards the contacts without recombination.

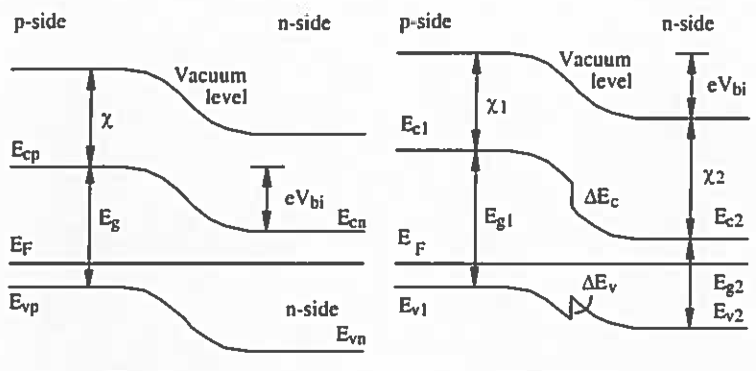
[1]

Photons with short wavelength and high energy will be absorbed before reaching the depletion region. Photons with long wavelength and low energy will be absorbed after passing through the depletion region. In each case the net effect will be a reduction in quantum efficiency.

[2]

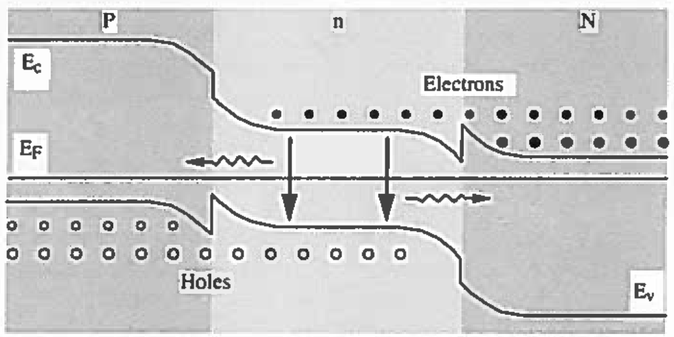
6. a) A homojunction is a junction between two materials with the same energy gap E_g. The band diagrams for the valence and conduction bands follow similar smooth, S-shaped curves, with identical energy barriers for electrons and holes in the conduction and valence band, respectively. In contrast, a heterojunction is a junction between two materials with different energy gaps E_{g1} and E_{g2}. This arrangement allows different energy barriers to be provided for electrons and holes.

[2]



[3]

The figure below shows the band diagram of a PnN heterojuction in equilibrium.



[2]

The importance of the design in optoelectronics is that:

If the refractive index of the central layer is higher than that of the two outer layers, the arrangement forms three-layer slab waveguide, a confining structure for photons. The arrangement also allows large, confining energy barriers at different positions for injected electrons and holes, consequently providing a region within which the population can be inverted that is spatially aligned with the region within which photons are confined. These conditions are ideal for promoting stimulated emission.

[3]

b) Consider the idealised model of a laser shown below. Assuming the field at position 1 is $E_1 = E_0$, the fields at subsequent points are:

$E_2 = E_0 \exp(gL) \exp(-j\beta L)$

$E_3 = E_0 R_1 \exp(gL) \exp(-j\beta L)$

$E_4 = E_0 R_1 \exp(2gL) \exp(-j2\beta L)$

$E_5 = E_0 R_1R_2 \exp(2gL) \exp(-j2\beta L)$

For oscillation, we require $E_1 = E_5$, so $R_1R_2 \exp(2gL) \exp(-j2\beta L) = 1$

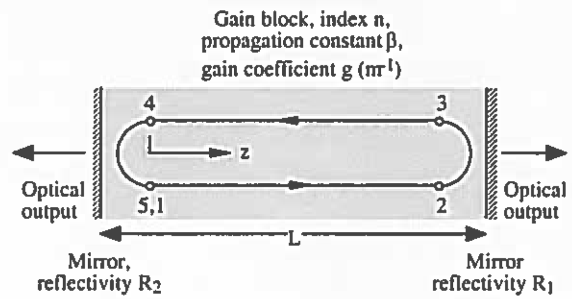
[1]

Since the RHS is real, this leads to separate gain and phase conditions:

$R_1R_2 \exp(2gL) = 1$

$2\beta L = 2\nu\pi$

[2]



[2]

d) If n = 3.5, the reflectivity of the cleaved end mirrors must be:

$R = (n - 1)/(n + 1) = 2.5/4.5 = 0.555$

[1]

Re-arranging the gain condition, the gain coefficient must be:

$g = (1/2L)\ \log_e(1/R^2) = \log_e(1/0.555^2)/(2 \times 500 \times 10^{-6}) = 1177\ m^{-1}$.

[2]

Re-arranging the second condition, we get:

$(2\pi n/\lambda)L = \nu\pi$, where n is an integer

Hence, the λ_ν corresponding to a particular value of ν is $\lambda_\nu = 2nL/\nu$

The spectral separation between to adjacent modes is therefore:

$\Delta\lambda = \lambda_\nu - \lambda_{\nu+1} = 2nL\{1/\nu - 1/(\nu+1)\} = 2nL/\{\nu(\nu+1)\}$

We can approximate this as:

$\Delta\lambda = 2nL/\nu^2 = \lambda_\nu^2/2nL$

For $\lambda_\nu = 1.5\ \mu m$, $n = 3.5$ and $L = 500\ \mu m$ we then obtain:

$\Delta\lambda = (1.5 \times 10^{-6})^2/(2 \times 3.5 \times 500 \times 10^{-6}) = 6.43 \times 10^{-10}\ m = 0.643\ nm$

[2]