E2.3B

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

EEE PART II: MEng, BEng and ACGI

Corrected Copy

POWER

Wednesday, 6 June 2:00 pm

Time allowed: 1:30 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T.C. Green, T.C. Green

Second Marker(s) : C.A. Hernandez-Aramburo, C.A. Hernandez-Aramburo

1.

(a) Explain why transformers are a key element of an electricity supply system [4]

(b) Explain why the core of a transformer is made of steel and why the steel is laminated. [4]

(c) Explain why a switch-mode power supply is more efficient than a linear regulator [4]

(d) Derive the voltage transfer ratio of a boost switch-mode power supply stating any assumptions made [4]

(e) Explain how currents are caused to flow in the rotor of an induction machine and how this leads to development of a torque on the shaft of the machine [4]

(f) Briefly describe the different physical processes by which power is lost as heat in an induction machine. [4]

(g) Discuss how frequency is controlled in an AC electricity supply system [4]

(h) Discuss the difference between real power and reactive power [4]

(i) A voltage difference between two nodes in an electricity network causes a current and a voltage to flow. Explain why magnitude difference is associated with reactive power flow and angle difference with real power flow. [4]

(j) Discuss the mix of primary energy sources used for electricity generation in the UK and note any which are renewable or low-carbon. [4]

2.

(a) For both the buck and boost switch-mode power supplies, SMPS:

(i) draw the circuit diagram; [6]

(ii) state the voltage transfer ratio for the continuous conduction mode; [3]

(iii) state why the boost SMPS would be expected to produce higher output voltage ripple than the buck for a similar output current. [3]

(b) A buck SMPS is required to convert 36 V to 9 V and provide up to 10 A of output current. The design must ensure an output voltage ripple of 100 mV or less. It may be assumed that the circuit will work in the continuous conduction mode. Certain elements of the design have already been chosen as follows:

Inductor	$L = 150\ \mu H$
Capacitor	$C = 10{,}000\ \mu F$
Capacitor series resistance	$R_{ESR} = 120\ m\Omega$
MOSFET on resistance	$R_{DS(on)} = 0.1\ \Omega$
MOSFET turn-on loss	$E_{on} = 14\ \mu J$ when switched on at 10 A
MOSFET turn-off loss	$E_{off} = 20\ \mu J$ when switched off at 10 A
MOSFET thermal resistance	$R_{ThJS} = 6.0$ K/W (junction to heatsink)
MOSFET maximum temperature	$\theta_{Jmax} = 120^\circ C$ (at junction)

Calculate the following:

(i) required duty-cycle; [3]

(ii) maximum inductor ripple current if output voltage ripple is not to exceed 100 mV (assume that the capacitor voltage ripple is dominated by R_{ESR} and the capacitance can be ignored); [3]

(iii) required switching frequency of the MOSFET; [5]

(iv) total power loss in the MOSFET; [4]

(v) required thermal resistance between heat-sink and air assuming an ambient air temperature of $\theta_A = 40^\circ C$. [3]

3. A 1-pole-pair, 3-phase induction machine is connected in star to a 400 V (line), 50 Hz supply. The machine is rated to run with a slip of 2.5%. The equivalent circuit parameters, referred to the stator, are:

Stator resistance	0.5 Ω
Referred rotor resistance	0.8 Ω
Stator leakage reactance	0.35 Ω
Referred rotor leakage reactance	0.8 Ω
Magnetising reactance	23 Ω
Magnetising Resistance	100 Ω

For operation at rated slip calculate the following

(i) shaft speed [3]

(ii) total input impedance [7]

(iii) stator current [5]

(iv) referred rotor current [5]

(v) mechanical power [5]

(vi) efficiency [5]

4. A factory has a real power requirement of 5.5 MW and draws an inductive reactive power of 1.0 MVAr. It is to be supplied with a three-phase supply at 11.0 kV and is located 20 km from a substation. The line to be used to connect between substation and factory has a resistance per unit length of 0.05 Ω/km and an inductive reactance per unit length of 0.07 Ω/km.

(i) Calculate the apparent power, power factor and angle between the phase voltage and the phase current. [5]

(ii) Calculate the phase current and the impedance per phase of the load assuming the load to be star connected. [6]

(iii) Calculate the voltage magnitude required at the substation to ensure 11.0 kV is present at the factory. [6]

(iv) Calculate the power lost in the line in transmitting the power to the factory. [5]

(v) Calculate the power lost in transmission if the line were operated a 33.0 kV with a 33 kV to 11 kV transformer present at the factory. The transformer has a total winding resistance of 0.2 Ω referred to the 11 kV side. [5]

(vi) Determine the distance the between factory and substation for which it is better (in terms of power loss) to operate the line at 11 kV rather than 33 kV. [3]

Master

E2.3B - SOLUTIONS 2007

Question 1 is compulsory.
Answer question 1 and any two of questions 2-4
Question 1 caries 40% and the other questions carry equal weighting of 30%.

1.

(a) Explain why transformers are a key element of an electricity supply system [4]

- High voltage required for efficiency in bulk transmission
- Variety of voltages needed to balance capital cost and running cost at different power levels
- Ability to change voltage between sections of network is key
- Transformers remain the only cost-effective means to do this

(b) Explain why the core of a transformer is made of steel and why the steel is laminated. [4]

- Steel has a high permittivity and relatively high saturation flux density
- Steel therefore gives a low magnetising current and low conduction losses associated with the magnetising current
- However, hysteresis and eddy current losses are high in steel
- Laminating with lamination plane set to allow flux path but break up eddy current path helps reduce eddy currents and their associated power loss

(c) Explain why a switch-mode power supply is more efficient than a linear regulator [4]

- Linear regulator use transistors to "drop" voltage in a potential divider with the load. The product of the current supplied and the dropped voltage is the power lost and this is inevitably high if the voltage step-down ratio is large
- Switch-mode circuits avoid this by always using the transistor in full-on or fully-off conditions in which it dissipates little power. The voltage transformation takes place in passive magnetic components.

(d) Derive the voltage transfer ratio of a boost switch-mode power supply stating any assumptions made [4]

Any increase in inductor current during the time that the transistor is on must be balanced by a decrease during the time that the diode conducts. The applied voltages determine the increases and decreases.

$$\Delta I_{(on)} + \Delta I_{(diode)} = 0$$

$$\frac{V_I}{L}\cdot t_{on} + \frac{V_I - V_O}{L}\cdot t_{diode} = 0$$

Rearranging and then assuming that continuous inductor current (diode conducts for all of the off time) yields:

$$\frac{V_o}{V_i} = \frac{t_{on} + t_{diode}}{t_{diode}} = \frac{T}{t_{diode}} = \frac{1}{1-\delta}$$

Assumptions:

- The output-side capacitor is considered to be very large such that the voltage across it does not vary significantly during one cycle of transistor switching,
- The voltage drops across the semiconductors are negligible,
- The resistance of the inductor is negligible.

(e) Explain how currents are caused to flow in the rotor of an induction machine and how this leads to development of a torque on the shaft of the machine [4]

- A three-phase stator winding supplied with a three-phase current produces a rotating magnetic field
- If the rotor rotates at any other speed than the speed of the field, the conductors of the rotor are subjected to a changing flux proportional to the speed difference
- Voltages induced in the rotor by the rate of change of flux linkage cause currents to flow in the closed paths of the rotor.

(f) Briefly describe the different physical processes by which power is lost as heat in an induction machine. [4]

- Ohmic power loss in the stator conductors
- Ohmic power loss in the rotor conductors
- Hysteresis power loss in the steel core
- Eddy current power loss in the steel core

(g) Discuss how frequency is controlled in an AC electricity supply system [4]

- System frequency is set by the rotational speed of the synchronous generators in the system
- Any difference between the mechanical power put into a generator and the electrical power taken out will cause the speed to change by adding to or subtracting from the kinetic energy of the rotating mass
- The generators are all synchronised and so rise and fall in frequency together
- Frequency control is exerted through governors which measure the shaft speed of one or more of the generators and adjust the power input (via a steam valve for instance) in proportion to the frequency error as a form of proportional control of frequency

(h) Discuss the difference between real power and reactive power [4]

- Both are average powers concerned with how instantaneous power evolves over a steady-state cycle.
- Real power is a permanent transfer of energy from one point to another normally as a result of permanent conversion of energy between electrical form and another form. Real power results from the component of current that is in phase with the voltage (or vice versa)
- Reactive power is a shuffling of energy into and out of a reactive component such as an inductor or capacitor. Electrical energy is stored in a magnetic or electric field as that field intensity is increased and released again later in the cycle as the intensity decreases. There is no permanent change of energy from one form to another. Reactive power results from the component of current that is in quadrature to the voltage (or vice versa)

(i) A voltage difference between two nodes in an electricity network causes a current and a voltage to flow. Explain why magnitude difference is associated with reactive power flow and angle difference with real power flow. [4]

- The voltage difference across the branch impedance sets the current flow and the power flow.

- $$\overline{S}_S = \overline{V}_S \overline{I}_{SR}^* = \overline{V}_S \frac{\overline{V}_S^* - \overline{V}_R^*}{\overline{Z}_{SR}^*}$$
- $$P_S = \frac{V_S^2}{Z_{SR}}\cos(\theta) - \frac{V_S V_R}{Z_{SR}}\cos(\theta+\delta)$$
  $$Q_S = \frac{V_S^2}{Z_{SR}}\sin(\theta) - \frac{V_S V_R}{Z_{SR}}\sin(\theta+\delta)$$
- If the impedance is dominated by reactance , as it is for HV overhead lines then
- $$P_S \approx \frac{V_S V_R}{X_{SR}}\sin(\delta)$$
  $$Q_S \approx \frac{V_S^2}{X_{SR}} - \frac{V_S V_R}{X_{SR}}\cos(\delta)$$
- Q is affected by the difference between V_S and V_R but P isn't
- Q is hardly affected by delta but P is.

(j) Discuss the mix of primary energy sources used for electricity generation in the UK and note any which are renewable or low-carbon. [4]

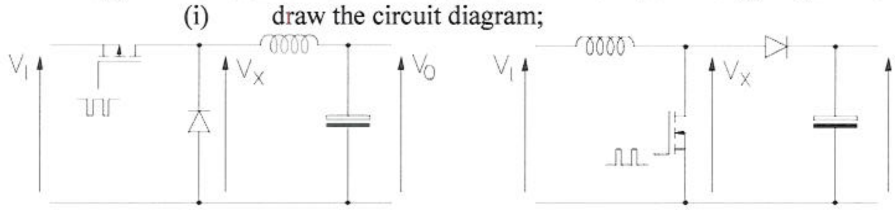
- The UK is dominated by coal, gas and nuclear with small amounts of hyrdo, wind and imports from France
- Wind is small but rising fast; hydro is not growing
- Wind and hydro are low carbon and renewable
- Nuclear fission is low carbon but not renewable
- Other points can be raised

Section B

2.

(a) For both the buck and boost switch-mode power supplies, SMPS:

(i) draw the circuit diagram; [6]



(ii) state the voltage transfer ratio for the continuous conduction mode; [3]

$$buck\ \frac{V_O}{V_I} = \delta \qquad boost\ \frac{V_O}{V_I} = \frac{1}{1-\delta}$$

(iii) state why the boost SMPS would be expected to produce higher output voltage ripple than the buck for a similar output current. [3]

Charging current in boost case is the diode current and is therefore a trapezoidal pulse train with an amplitude equal to the peak inductor current whereas the charging current in the buck cases is the ripple component of the inductor current which, in continuous mode, is necessarily less than the peak inductor current. For similar output currents, the boost passes a higher current through the ESR of the capacitor.

(b) A buck SMPS is required to convert 36 V to 9 V and provide up to 10 A of output current. The design must ensure an output voltage ripple of 100 mV or less. It may be assumed that the circuit will work in the continuous conduction mode. Certain elements of the design have already been chosen as follows:

Inductor	$L = 150\ \mu H$
Capacitor	$C = 10{,}000\ \mu F$
Capacitor series resistance	$R_{ESR} = 120\ m\Omega$
MOSFET on resistance	$R_{DS(on)} = 0.1\ \Omega$
MOSFET turn-on loss	$E_{on} = 14\ \mu J$ when switched on at 10 A
MOSFET turn-off loss	$E_{off} = 20\ \mu J$ when switched off at 10 A
MOSFET thermal resistance	$R_{ThJS} = 6.0$ K/W (junction to heatsink)
MOSFET maximum temperature	$\theta_{Jmax} = 120^\circ C$ (at junction)

Calculate the following:

(i) required duty-cycle; [3]

$$\delta = \frac{V_O}{V_I} = \frac{9}{36} = 0.25$$

(ii) maximum inductor ripple current if output voltage ripple is not to exceed 100 mV (assume that the capacitor voltage ripple is dominated by R_{ESR} and the capacitance can be ignored); [3]

$$\Delta i_L = \Delta i_C = \frac{\Delta v_{ESR}}{R_{ESR}} = \frac{0.1}{0.12} = 0.8333$$

(iii) required switching frequency of the MOSFET; [5]

$$\Delta i_L = \Delta i_C$$

$$\Delta i_L = \frac{V_I - V_O}{L} \cdot \frac{\delta}{f}$$

$$f = \frac{V_I - V_O}{L} \cdot \frac{\delta}{\Delta i_C} = \frac{(36-9) \times 0.25}{150 \times 10^{-6} \times 0.8333} = 54.0\, kHz$$

(iv) total power loss in the MOSFET; [4]

$$P_{Loss} = \delta I_L^{\ 2} R_{DS(on)} + (E_{On} + E_{Off})f = 0.25 \times 10^2 \times 0.2 + (14 \times 10^{-6} + 20 \times 10^{-6}) \times 54 \times 10^3 = 4.336\, W$$

(v) required thermal resistance between heat-sink and air assuming an ambient air temperature of θ_A = 40°C. [3]

$$\theta_J - \theta_A = P_{Loss}(R_{ThJS} + R_{ThSA})$$

$$R_{ThSA} = \frac{\theta_J - \theta_A}{P_{Loss}} - R_{ThJS} = \frac{120-40}{4.336} - 6.0 = 12.45\, K/W$$

3. A 1-pole-pair, 3-phase induction machine is connected in star to a 400 V (line), 50 Hz supply. The machine is rated to run with a slip of 2.5%. The equivalent circuit parameters, referred to the stator, are:

Stator resistance	0.5 Ω
Referred rotor resistance	0.8 Ω
Stator leakage reactance	0.35 Ω
Referred rotor leakage reactance	0.8 Ω
Magnetising reactance	23 Ω
Magnetising Resistance	100 Ω

For operation at rated slip calculate the following

(i) shaft speed [3]

$$\omega_S = 2\pi P\ f_E = 100\pi \quad or \quad 3{,}000\, r.p.m.$$

$$\omega_R = \omega_S(1-s) = 97.5\pi \quad or \quad 2{,}925\ r.p.m$$

(ii) total input impedance [7]

An incorrect value was used for R_R' and so numerical answers are in error.

$$Z_R = \frac{R'_R}{s} + jX'_R = \frac{0.35}{0.025} + j0.8 = 14 + j0.8\ \Omega = 14.023\angle 3.27°\ \Omega$$

$$Z_M = X_M \mathbin{//} R_M = \frac{j23 \times 100}{100 + j23} = \frac{2300\angle 90°}{102.611\angle 12.95°} = 22.415\angle 77.05°\ \Omega = 5.023 + j21.845\ \Omega$$

$$Z_{In} = Z_S + Z_M \mathbin{//} Z_R = 0.5 + j0.8 + \frac{22.415\angle 77.05° \times 14.023\angle 3.27°}{14 + j0.8 + 5.023 + j21.845} = 0.5 + j0.8 + \frac{314.326\angle 77.32°}{19.023 + j26.868}$$

$$= 0.5 + j0.8 + \frac{314.326\angle 77.32°}{32.921\angle 54.70°} = 0.5 + j0.8 + 9.548\angle 22.62° = 0.5 + j0.8 + 8.813 + j3.672$$

$$= 9.313 + j4.472\ \Omega$$

(iii) stator current [5]

$$I_S = \frac{V_S}{Z_{In}} = \frac{400}{\sqrt{3} \times 10.331\angle 25.67°} = 22.35\angle -25.67°$$

(iv) referred rotor current [5]

$$I_R = I_S \frac{Z_M}{Z_M + Z_R} = 22.35\angle 25.67° \times \frac{22.415\angle 77.05°}{32.921\angle 54.70°} = 15.22\angle -3.32°$$

(v) mechanical power [5]

$$P_{Mech} = 3I'^{\,2}_R\ R'_R\left(\frac{1}{s} - 1\right) = 3 \times 15.22^2 \times 0.35\left(\frac{1}{0.025} - 1\right) = 9.486\ kW$$

(vi) efficiency [5]

$$\eta = \frac{P_{Mech}}{P_{In}} = \frac{P_{Mech}}{\sqrt{3}V_L I_S \cos(\phi)} = \frac{9486}{\sqrt{3} \times 400 \times 22.35 \times \cos(-25.67°)}$$

$$= \frac{9486}{13956} = 68.0\%$$

4. A factory has a real power requirement of 5.5 MW and draws an inductive reactive power of 1.0 MVAr. It is to be supplied with a three-phase supply at 11.0 kV and is located 20 km from a substation. The line to be used to connect between substation and factory has a resistance per unit length of 0.05 Ω/km and an inductive reactance per unit length of 0.07 Ω/km.

(i) Calculate the apparent power, power factor and angle between the phase voltage and the phase current. [5]

$$S = \sqrt{P^2 + Q^2} = \sqrt{5.5^2 + 1} = 5.590\ MVA$$

$$p.f. = \frac{P}{S} = \frac{5.5}{5.59} = 0.984$$

$$\angle V - \angle I = \phi =^{+} \cos^{-1} 0.984 = 10.30°$$

(ii) Calculate the phase current and the impedance per phase of the load assuming the load to be star connected. [6]

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{11}{\sqrt{3}} = 6.351\ kV$$

$$I_P = \frac{S}{3V_P} = \frac{5.59 \times 10^6}{3 \times 6.351 \times 10^3} 293.4\ A$$

$$\overline{Z}_P = \frac{\overline{V}_P}{\overline{I}_P} = \frac{6.351 \times 10^3}{293.4\angle -10.30°} = 21.64\angle 10.30°\ \Omega$$

(iii) Calculate the voltage magnitude required at the substation to ensure 11.0 kV is present at the factory. [6]

$$\overline{V}_S = \overline{V}_R + \overline{I}_L \overline{Z}_L = 6351\angle 0° + 293.4\angle -10.30° \times 20 \times 0.0860\angle 54.46° = 6351 + 504.65\angle 44.16°$$

$$= 6351 + 362 + j352 = 6722\angle 3.0°$$

$$V_S = 6.722\ kV$$

or

$$V_{LS} = 11.642\, kV$$

(iv) Calculate the power lost in the line in transmitting the power to the factory. [5]

$$P_{Loss} = 3I_L{}^2 R_L = 3 \times 293.4^2 \times 20 \times 0.05 = 258\ kW$$

(v) Calculate the power lost in transmission if the line were operated a 33.0 kV with a transformer 33 kV to 11 kV transformer present at the factory. The transformer has a total winding resistance of 0.2 Ω referred to the 11 kV side. [5]

$$I_{L-33kV} = I_{L-11kV} \frac{11kV}{33kV} = \frac{293.4}{3} = 97.8\ A$$

$$P_{Loss-33kV} = P_{Loss-11kV}\left(\frac{11kV}{33kV}\right)^2 + 3I_{L-11kV}{}^2 R_T = \frac{258 \times 10^3}{9} + 3 \times 293.4^2 \times 0.2 = 80\ kW$$

(vi) Determine the distance the between factory and substation for which it is better (in terms of power loss) to operate the line at 11 kV rather than 33 kV. [3]

Using d to represent the distance at which the power loss of the two approaches is equal and using prime to indicate a quantity per unit length

$$d\,P'_{L-33kV} + P_{Tran} = dP'_{L-11kV}$$

$$d = \frac{P_{Tran}}{P'_{L-11kV} - P'_{L-33kV}} = \frac{I_{L-11kV}{}^{2} R_{T}}{I_{L-11kV}{}^{2} R'_{L}\left(1-\frac{1}{9}\right)} = \frac{9}{8} \times \frac{0.2}{0.05} = 4.5\ km$$

2

So, for distances less than 4.5 km operation at 11 kV produces less loss.