UNIVERSITY OF LONDON [C145 2001]

B.ENG. AND M.ENG. EXAMINATIONS 2001

For Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examination for the Associateship of the City and Guilds of London Institute.

COMPUTING C145

MATHEMATICAL METHODS AND GRAPHICS

Date 2001 10.00 - 12.00

Answer FOUR questions

[*Before starting, please make sure that the paper is complete. There should be a total of SIX questions. Ask the invigilator for a replacement if this copy is faulty.*]

1. Transformations and projections:

   (i) The homogeneous coordinate of a point is [2, 5, 8, 2]. What is its Cartesian coordinate?

   (ii) What is meant by the term affine transformation?

   (iii) Give one example of a non-affine transformation.

   (iv) Assuming a left hand axis system, give the transformation matrices which will:

      (a) produce a rotation through θ degrees about the y-axis,

      (b) produce a rotation through ϕ degrees about the x-axis.

   (v) A wireframe scene is to be drawn with the viewpoint at the origin. The direction of view is defined by the vector [0, −1, 5]. It is to be drawn using perspective projection on a plane at a distance of 2 units from the viewpoint. Calculate the required transformation matrix, by combining a rotation matrix about the y axis, a rotation about the x axis and a projection matrix.

(The five parts carry 10%, 10%, 10%, 20% and 50% of the marks respectively.)

2. A plane can be completely defined by specifying the vector from the origin to the closest point on the plane (providing the plane does not go through the origin). This vector is of course normal to the plane.

(i) For a plane defined by the vector $\mathbf{S} = [S_x,\ S_y,\ S_z]$, show that its vector equation can be written in the form $\mathbf{S.P} = |\mathbf{S}|^2$, where $P = [x,\ y,\ z]$ is a point on the plane. Expand the vector equation to obtain the Cartesian equation of the plane.

(ii) Write a procedure in your favourite pseudocode (or programming language) that will determine whether a given point lies on the same side of a plane as the origin. The plane is defined by a position vector $\mathbf{S}$ as above.

(iii) A graphics scene is to be viewed in perspective projection. The viewing window is on the $x - y$ plane, bounded by the lines $x = 2$, $x = -2$, $y = 2$ and $y = -2$. The centre of projection is at the point $[0,\ 0,\ -2]$, and the viewing direction is along the z-axis. Explain how your pseudocode of part *(ii)* could be used to determine whether a line of the scene is completely visible or not.

(iv) A flight simulator is set up as in part (iii), but has in addition a front clipping plane defined by the vector $[0,\ 0,\ 1]$. Briefly describe the purpose of the front clipping plane.

(The four parts carry 30%, 25%, 25% and 20% of the marks respectively.)

3. (i) Using row operations (Gauss-Jordan process), find the inverse of

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix} .$$

Hence, or otherwise, solve the simultaneous equations

$$\begin{aligned} x + 2y + z &= 4 , \\ 2x + y + z &= 4 , \\ 3x + y + z &= 5 . \end{aligned}$$

(ii) Determine the form of the solutions to the singular simultaneous equations

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 , \\ 2x_1 - x_2 + x_3 - x_4 &= 0 , \\ 3x_1 + x_2 + 2x_3 + x_4 &= 0 , \\ 4x_1 + 2x_2 + 3x_3 + 2x_4 &= 0 . \end{aligned}$$

(Parts (i) and (ii) carry equal marks.)

4. (i) For the function $u = x^2 \ln \left(\frac{x}{y}\right)$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u .$$

(ii) Find all four stationary points of the function

$$u(x, y) = x^3 + xy^2 + x^2 y - 6x + y^3 - 6y .$$

(iii) A function of four variables is given by

$$f(x_1, x_2, x_3, x_4) = 6x_1^2 x_2^3 x_3^4 x_4^5 .$$

By considering the total derivative of f, find the approximate percentage change in f if x_1 and x_2 are each increased by 2% whilst x_3 and x_4 are each decreased by 1%.

(Parts (i), (ii) and (iii) carry 15%, 45% and 40% of the marks respectively.)

5. (i) Find all roots of the polynomial equation

$$z^4 = i$$

and illustrate where the solutions lie on the Argand diagram.

(ii) Show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1\,.$$

Hence show that

$$\cos 22.5^o = \frac{\sqrt{2+\sqrt{2}}}{2}\,.$$

(iii) Determine the equation of the curve satisfying the relation

$$|z-1| = |z-3|\,,$$

where $z = x + iy$ and sketch the curve on an Argand diagram.

(Parts (i), (ii) and (iii) carry 40%, 40% and 20% of the marks respectively.)

6. (i) Find the general solution to the recurrence relation

$$x_n - 8x_{n-1} + 16x_{n-2} = (n+1)\,4^n\,.$$

(ii) Determine the unique solution for which

$$x_1 = 4, \quad x_2 = 16\,.$$

(Parts (i) and (ii) carry 75% and 25% of the marks respectively.)

END OF PAPER