EE4-26
EE9-CS5-3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

ESTIMATION AND FAULT DETECTION

Wednesday, 21 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T. Parisini
Second Marker(s) : D. Angeli

ESTIMATION AND FAULT DETECTION

1. Consider the continuous-time closed-loop control system depicted in Fig. 1.1.

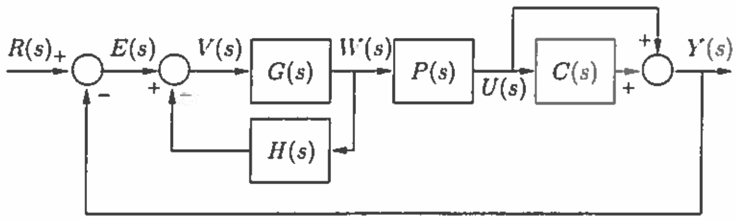

Figure 1.1 Block diagram for Question 1.

where $C(s)$, $G(s)$, $P(s)$, and $H(s)$ are transfer functions of reachable and observable single-input single-output systems (without common factors between numerator and denominator).

a) Determine the transfer function $G_{cl}(s)$ from the input $R(s)$ to the output $Y(s)$, where $G_{cl}(s)$ should be expressed in terms of the generic transfer functions $C(s)$, $G(s)$, $P(s)$, and $H(s)$.

[3 marks]

b) Now, given

$$G(s) = \frac{1}{s+1}; \quad H(s) = 9; \quad P(s) = \frac{s+10}{(s+3)^2}; \quad C(s) = \frac{1}{s+2}$$

determine a state-space description of the whole closed-loop control system depicted in Fig. 1.1.

[3 marks]

c) Verify that the closed-loop control system depicted in Fig. 1.1 is not completely observable.

[7 marks]

d) Determine one of non-observable poles of the closed-loop control system depicted in Fig. 1.1.

[7 marks]

2. Consider the continuous-time dynamic system described by the following state equations:

$$\begin{cases} \dot{x}_1 = -x_1 + 2x_2 + u \\ \dot{x}_2 = -x_1 - x_2 \\ y = x_1 + 3x_2 \end{cases} \tag{2.1}$$

a) Verify that a full-order observer providing the asymptotic estimate of the state vector can be determined.

[3 marks]

b) Denoting by $\hat{x}(t)$ the estimate of the state $x(t)$ of the system described by equations (2.1), let $e(t) = x(t) - \hat{x}(t)$ denote the state estimation error and suppose that its dynamics obeys

$$\dot{e}(t) = Fe(t)$$

where λ_1, λ_2 are the eigenvalues of F. Design a full-order state observer such that

$$\lambda_1 = -2, \lambda_2 = -4$$

[8 marks]

c) Consider the state equations (2.1) in Question 2a). A reduced-order observer can be designed to provide an estimate $\hat{x}_2$ of x_2 and use the output equation to calculate the estimate $\hat{x}_1$ of x_1. Consider the estimation error $\varepsilon_2(t) = x_2(t) - \hat{x}_2(t)$ and suppose that its dynamics obeys

$$\dot{\varepsilon}_2(t) = \tilde{f}\varepsilon_2(t)$$

Design a reduced-order state observer such that

$$\tilde{f} = -2$$

[9 marks]

3. Consider the following discrete-time dynamic system affected by state and output disturbances:

$$\begin{cases} x(t+1) = \frac{1}{2}x(t) + v_1(t) \\ y(t) = -\frac{1}{3}x(t) + v_2(t) \end{cases} \tag{3.1}$$

where $v_1(\cdot) \sim WGN(0,1)$, $v_2(\cdot) \sim WGN(0,1)$ (Gaussian zero-mean stochastic processes) and the stochastic processes $v_1(\cdot)$ and $v_2(\cdot)$ are supposed to be independent to each other.

a) Referring to system (3.1), consider the one-step ahead optimal steady-state Kalman predictor of the state x. Write the Algebraic Riccati Equation (ARE) and show that the ARE admits an admissible solution $\bar{P}$. Compute the corresponding constant gain $\bar{K}$.

[4 Marks]

b) Compute the variance of the estimation error, $\text{var}[x(t) - \hat{x}(t|t-1)]$, and the variance of the process, $\text{var}[x(t)]$. Compare $\text{var}[x(t) - \hat{x}(t|t-1)]$ with $\text{var}[x(t)]$ and comment on your findings.

[5 Marks]

c) Write the difference equation yielding the one-step ahead optimal steady-state Kalman prediction $\hat{x}(t+1|t)$ and draw the block-diagram of the predictor.

[3 Marks]

d) Suppose that an *unknown constant* fault $\bar{u}$ affects the state equation (3.1), that is

$$\begin{cases} x(t+1) = \frac{1}{2}x(t) + v_1(t) + \bar{u} \\ y(t) = -\frac{1}{3}x(t) + v_2(t) \end{cases}$$

Formulate an estimation problem in such a way that it is possible to *simultaneously* predict the state x and the fault $\bar{u}$ by a one-step-ahead optimal steady-state Kalman predictor. (*Please do not attempt to design such Kalman predictor.*)

[8 Marks]

4. Consider the continuous-time dynamic system described by the following state equations:

$$\begin{cases} \dot{x}_1 = -x_1 + 2x_2 + u \\ \dot{x}_2 = -3x_2 + f \\ y = x_1 \end{cases} \tag{4.1}$$

where u is a known input and f denotes a fault affecting the second state equation. Suppose that only one fault may occur during the whole time-horizon $t \in (0, \infty)$.

Moreover, suppose that the fault f may only assume one among the following two forms:

$$f_1(t) = \sin(t), \forall t \geq T_0$$

or

$$f_2(t) = 1, \forall t \geq T_0$$

where T_0 denotes the unknown time of fault occurrence.

a) Consider system (4.1) before the possible occurrence of a fault ($t < T_0$), that is

$$\begin{cases} \dot{x}_1 = -x_1 + 2x_2 + u \\ \dot{x}_2 = -3x_2 \\ y = x_1 \end{cases} \tag{4.2}$$

Denoting by $\hat{x}(t)$ the estimate of the state $x(t)$ of the system described by equations (4.2), let $e(t) = x(t) - \hat{x}(t)$ denote the state estimation error and suppose that its dynamics obeys

$$\dot{e}(t) = Fe(t)$$

where λ_1, λ_2 are the eigenvalues of F. Design a full-order state observer such that

$$\lambda_1 = -1, \lambda_2 = -1$$

and determine the time behaviour of the output residual

$$\varepsilon(t) = Ce(t), \forall t \in (0, T_0)$$

for a given value $\tilde{e}$ of the initial estimation error $e(0)$.

[6 marks]

b) Consider a fault detection scheme based on the observer designed in the answer to Question 4a. A fault is detected at time T_d if the output residual $\varepsilon(T_d)$ satisfies

$$\varepsilon(T_d) > \bar{\varepsilon}(T_d)$$

where $\bar{\varepsilon}(t)$ is a suitable detection threshold.

Supposing that $|x(0)| \leq 10$, determine a detection threshold $\bar{\varepsilon}(t)$ allowing to detect in a finite time T_d a fault $f = f_1$ or $f = f_2$ occurring at some finite time $T_0 < T_d$.

[6 marks]

c) Show that it is possible to design two fault estimators, one to estimate fault $f = f_1$ and one to estimate fault $f = f_2$. (*Please do not attempt to design such estimators.*)

[8 marks]