IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

EXAMINATIONS 2017

Mathematics for Signals and Systems

There are THREE questions in this paper. Answer ALL questions. All questions carry equal marks.

Time allowed 3 hours.

Special Information for the Invigilators: none

Information for Candidates: none

The Questions

1. (a) Consider the following subspace of $\mathbb{R}^4$:

$$S = \text{span}\left\{ \begin{bmatrix} 0 \\ -1 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ -5 \\ -5 \end{bmatrix} \right\}.$$

i. Find a basis for S. [3]

ii. Use the Gram-Schmidt method to orthogonalize your basis [3]

(b) Find a basis for the null space of

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & -5/3 \\ 1/2 & 1 & 1/2 & -1/6 \\ -1 & -1 & 0 & 1 \end{bmatrix}.$$

[5]

(c) Consider the matrix

$$\mathbf{A} = \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

Compute its inverse. [5]

(d) Consider the matrix

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix},$$

and assume that $\det(\mathbf{A}) = 10$. Compute $\det(5\mathbf{A})$, $\det(3\mathbf{A}^{-1})$, $\det(3\mathbf{A}^3)$ [4]

2. (a) Given the matrix

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -0.5 \end{bmatrix},$$

state whether B is the pseudo-inverse of

$$\mathbf{A} = \begin{bmatrix} 0.5 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

Justify your answer. [5]

(b) Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & a & b \\ 1 & 0 & 1 \end{bmatrix}.$$

For what values of a, b does $\mathbf{A}$ have an inverse? Justify your answer. [5]

(c) Consider the system of equations

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}.$$

Find the minimum norm solution. [5]

(d) Given a Hermitian symmetric matrix $\mathbf{A} \in \mathbb{C}^{n \times n}$ prove that its eigenvalues are real. [5]

3. (a) Let

$$\mathbf{p}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \qquad \mathbf{p}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

and

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}.$$

Determine the nearest vector $\hat{\mathbf{x}}$ in $S = \text{span}\{\mathbf{p}_1, \mathbf{p}_2\}$, where nearest is in the least-squares sense. [5]

(b) Consider the finite difference equations

$$x_{n+1} = -7x_n + 10y_n$$

and

$$y_{n+1} = -5x_n + 8y_n.$$

Given that $x_0 = 1$ and $y_0 = 0.5$, find x_4 and y_4. [5]

(c) Calculate the singular values of the following matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix}.$$

[4]

(d) Find the singular value decomposition of

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix}.$$

[6]