UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2002

MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

*This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute*
*This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science*

# PAPER C438

# COMPLEXITY

Friday 26 April 2002, 14:30
Duration: 120 minutes

*Answer THREE questions*

Paper contains 4 questions
Calculators required

1a i) Let $\Sigma$ be a finite alphabet, and let $L, L' \subseteq \Sigma^*$ be languages. What does it mean for L to be reducible to L′ ($L \leq L'$) in polynomial time?

ii) The problems UHC (respectively DHC) are defined as follows: Given an undirected (respectively directed) graph G, does G have a Hamiltonian circuit?
Show UHC $\leq$ DHC.

b The word x is a *merge* of words y and z if it is got by combining x and y into a single word while preserving symbol order within x and within y. For instance, the word adbec is a merge of abc and de. Given languages $L_1, L_2 \subseteq \Sigma^*$, let

$$\text{merge}(L_1,L_2) = \{x \in \Sigma^*: x \text{ is a merge of a word of } L_1 \text{ with a word of } L_2\}$$

Show that if $L_1, L_2$ are in NP then $L = \text{merge}(L_1,L_2)$ is also in NP.

c A weakly negative Horn (WNH) clause is a clause with at most one negative literal (i.e. negated propositional variable). The problem NHORNSAT is: given a set of WNH clauses, is it satisfiable?

i) Is the following set of WNH clauses satisfiable?

$$\neg x,\ x \vee y,\ x \vee \neg y \vee z,\ y \vee \neg z$$

Give a brief justification.

ii) Show that NHORNSAT is in P. [Hint: consider clauses with a single literal.]

2a i) Let $\Sigma$ be a finite alphabet. What does it mean for a function $f(x): \Sigma^* \to \Sigma^*$ to be *logspace computable*?

ii) Define the class NLOGSPACE ('NL' for short).

iii) What does it mean for a language L to be *NL-complete*?

b A form of the Szelepscényi-Immerman Theorem states the following: Let G be a directed graph and x a node of G. Let N(x) be the set of nodes reachable from x. Then $|N(x)|$ is computable in NL.

Use this result (and its proof) to show that co-NL = NL.

c i) An *independent set* I is a subset of the nodes of a graph G such that no two nodes in I are adjacent in G.
Let k be a natural number. The problem k-IND is: Given a graph G, does G have an independent set of size k?
Show that k-IND is in LOGSPACE.

ii) A graph is *acyclic* if it has no cycles. The problem ACYCLIC is: Given a directed graph G, is it acyclic?
Explain why ACYCLIC is in NL.

d A directed graph is *connected* if there is a path from each node to every other node. The problem DCONN is: Given a directed graph G, is it connected?
Show that DCONN is NL-complete.

You may assume that the following problem (RCH) is NL-complete: Given a directed graph G, and nodes x, y, is y reachable from x?

 

3a i) Describe Rabin's randomized algorithm for primality testing, explaining the terms you use.

ii) Explain why false positive but no false negative results can arise when using the algorithm.

b Estimate how many iterations of Rabin's algorithm would be required when testing a number n around $2^{100}$, to ensure that if the algorithm declares n prime, the probability of n being composite is less than one in a million. [It may help to know that $\ln 2 \approx 0.69$.]

c Define the class RP, explaining the terms you use. Give an example of a problem (or language) in RP.

d Show carefully that if $L_1, L_2 \in$ RP then $L_1 \cap L_2 \in$ RP.

4a What is the Hamiltonian Path problem (HAM)? What is the corresponding function problem FHAM?

b Show that if HAM $\in$ P then FHAM can be solved in polynomial time.

c Define the classes FNP, FP of function problems, explaining the terms you use.

d Show that P = NP if and only if FP = FNP. [Binary search may be useful.]

*The four parts carry, respectively, 20%, 25%, 20%, 35% of the marks.*