Paper Number(s): **E1.5**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

EEE PART I: M.Eng., B.Eng. and ACGI

ENGINEERING MATERIALS

Monday, 18 June 10:00 am

There are FIVE questions on this paper.

Answer THREE questions.

CORRECTED COPY (Q5)

Time allowed: 2:00 hours

Examiners: Syms,R.R.A. and Tate,T.J.

Required Data

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ Wb/Am

$h = 6.62 \times 10^{-34}$ Js

$k = 1.38 \times 10^{-23}$ J/K

$e = 1.6 \times 10^{-19}$ C

$m = 9.1 \times 10^{-31}$ kg

1. a) Discuss the advantages of Si-based accelerometers in car safety systems. What physical properties make silicon suitable for the fabrication of micromechanical devices? **[8]**

   b) A tensile test is performed on a silicon microbeam with dimensions 7.5 mm x 500 μm x 500 μm. Table 1 below shows the results. Plot the experimental stress-strain characteristic, and obtain a value for Young's modulus for Si. **[4]; [2]**

   c) Describe the most common yield mechanism for single-crystal silicon. **[6]**

Load *(N)*	*Extension* *(μm)*
0.5	0.15
1.3	0.35
6.7	2.00
13	4.00
27	7.50

Table 1.

2. a) Define the terms *electronic polarization* and *relative dielectric constant*, explaining how they relate to each other. **[4]**

b) Figure 1 shows a model for the dynamics of a bound electron in a time-varying electric field. Complete the diagram, labelling any important components. Why is the local field E_{Loc} experienced by the electron different from the externally applied field E? **[6]**

c) The equation of motion of the electron is:

$$m\, d^2x/dt^2 + r\, dx/dt + kx = eE_{Loc} \quad \text{where} \quad E_{Loc} = E + P/3\varepsilon_0.$$

Assuming that the electric field can be written in the form $E = E_0 \exp(j\omega t)$, show that the complex relative dielectric constant can found as:

$$\varepsilon_r = 1 + \omega_p^2 / \{(\omega_m^2 - \omega^2) + j\omega\gamma\}$$

and find ω_p, ω_m and γ in terms of the parameters in the equation of motion. **[10]**

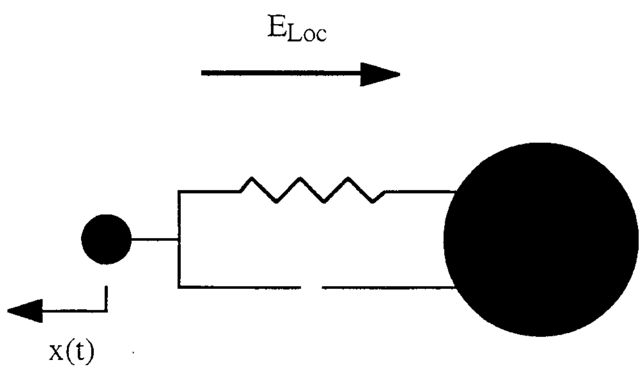

Figure 1.

3. a) Explain what is meant by the following terms: *orientational polarization*, *refractive index*, *birefringence*, *dichroism*, *liquid crystal.* **[5]**

b) Figure 2 below shows an incomplete diagram of a twisted nematic liquid crystal cell. Copy and complete it for the cases i) when the optical transmission is HIGH, and ii) when the transmission is LOW. Explain how a pixellated display is constructed. **[10]**

c) Sketch the variation of transmission through the cell with drive voltage. What changes have been made to this type of display to improve performance? **[5]**

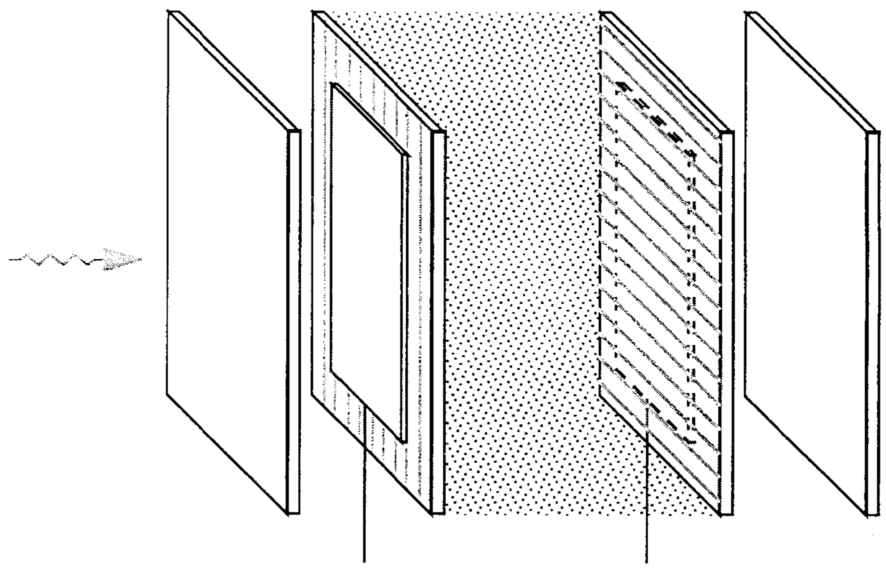

Figure 2.

4. a) The magnetisation of a paramagnet is described by $M = n <\mu_m>$, where n is the dipole density and $<\mu_m>$ is the average component of the dipole moment parallel to the applied field. According to Langevin, $<\mu_m> = \mu_m L\{\mu_m \mu_0 H/kT\}$, where $L(x) = \coth(x) - 1/x$.

Sketch the variation of L(x) with x. Explain the physical situations that may give rise to i) weak and ii) strong dipole alignment. [4]; [4]

b) A particular material has a dipole moment of 10^{-23} Am^2. Estimate the magnetic field required to reach saturation of the Langevin function at i) T = 300 K, and ii) T = 1K. [6]

c) Explain why the formation of magnetic domains in a ferromagnetic material is energetically favourable. What limits the size of domains? [3]; [3]

5. a) Using the Bohr model of an orbiting electron, show that the magnetic moment of a single electron can be written as:

$$\mu_m = neh/4\pi M$$

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where n is an integer. Obtain a numerical value for the Bohr magneton, μ_B. [6]; [2]

b) Iron saturates at a flux density of 2.18 W/m^2. If there are ≈ 8.5×10^{28} Fe atoms/m^3, how many electrons per atom appear to contribute to the magnetisation? [6]

c) Explain briefly the electronic configuration required for strong magnetic effects. From your knowledge of the electronic configuration of Fe, how many electrons might be expected to contribute to its magnetisation? [4]; [2]

E1.5

July 01

Engineering **Materials Paper 2001 - Answers** (11 pages in total)

1\. a) <u>Advantages</u> of Si-based car safety accelerometers include:

Low cost, based on the use of mass-fabrication methods
Repeatability, based on the use of accurate lithographic definition of features
Accuracy, based on the ability to co-integrate low-noise signal conditioning electronics
Reliability, based on the use of very pure single crystal material
Safety, based on the ability to co-integrate self-test features **[4]**

<u>Physical properties</u> that make Si suitable for micromechanical devices include:

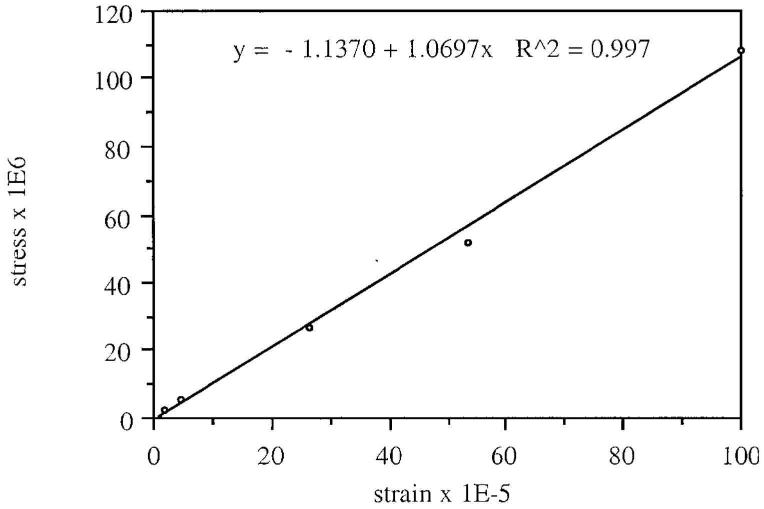
Crystallinity, in any anisotropic wet chemical etching processes used for fabrication
Elasticity, in the spring suspension system
Piezoresistivity, in a strain-based readout system
Semiconductivity, in any integrated electronics **[4]**

b) Stress σ = force ÷ cross-sectional area
Strain ε = change in length ÷ original length
Young's modulus $E = \sigma/\varepsilon$

Cross-sectional area $A = 500 \times 500 \times 10^{-12} = 2.5 \times 10^{-7}$ m^2

Original length $L = 7.5 \times 10^{-3}$ m

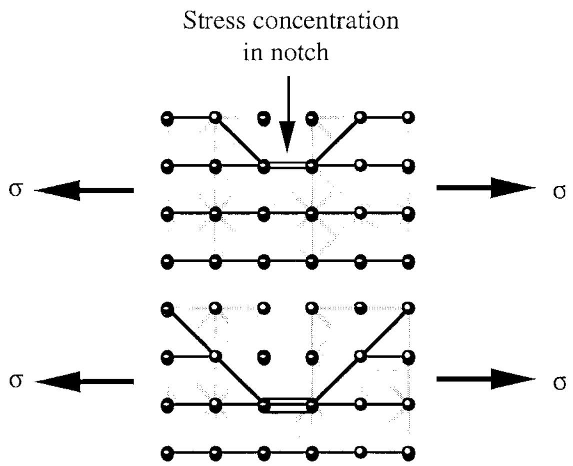
Load (N)	Extension (μm)	Stress ($\times 10^6$ N/m^2)	Strain ($\times 10^{-5}$)
0.5	0.15	2	2
1.3	0.35	5.2	4.67
6.7	2.00	26.8	26.67
13	4.00	52	53.33
27	7.50	108	100



[4]

The slope of the graph gives $E = 1.07 \times 10^{11}$ N/m^2. **[2]**

c) Single-crystal silicon is a brittle material that normally fails by cleavage down crystal planes. The cleave is initiated by the stress concentration of a small crack or notch, as the load that would normally be carried by a bond in the vicinity of the crack is transferred to its neighbours, which then fail. Only a single bond is broken at once, but eventually all bonds fail as the crack propagates through the material at the speed of sound. **[4]**

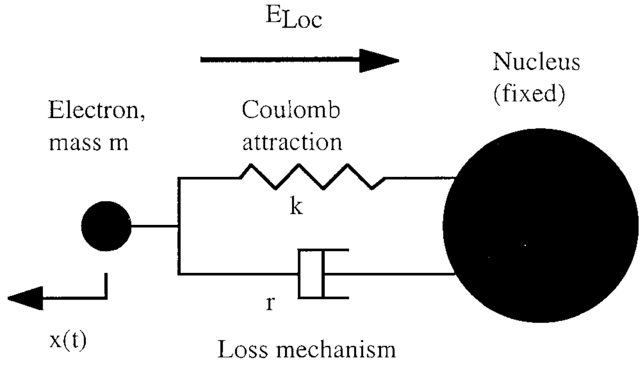


[2]

2. a) *Electronic polarization* is the separation of charge pairs or dipoles under the action of an electric field. If the charges separate by a distance x, and the dipole density is n, the polarization P is P = nex. [2]

Relative dielectric constant is the factor by which the dielectric constant is increased over the value obtained in free space, due to the presence of dipoles (i.e. the presence of matter). $\varepsilon_r = 1 + P/\varepsilon_0 E$ [2]

b)



[4]

The electric field experienced by each separate electron is different from the externally-applied field, because the external field separates the charge pairs, and the separated charges then modify the local field. [2]

c) The equation of motion of the electron is:

$$m\, d^2x/dt^2 + r\, dx/dt + kx = e(E + P/3\varepsilon_0)$$

Multiplying both sides by ne and substituting for P we get:

$$m\, d^2P/dt^2 + r\, dP/dt + kP = ne^2\, (E + P/3\varepsilon_0)$$

Rearranging, we can then obtain:

$$d^2P/dt^2 + \gamma\, dP/dt + \omega_m^2\, P = \varepsilon_0 \omega_p^2\, E$$

where $\gamma = r/m$, $\omega_m^2 = \omega_0^2 - \omega_p^2/3$, $\omega_0^2 = k/m$, and $\omega_p^2 = ne^2/m\varepsilon_0$ is the plasma frequency.

We now assume that the electric field and polarization vary as $E = E_0 \exp(j\omega t)$ and $P = P_0 \exp(j\omega t)$. Substituting and dividing by $\exp(j\omega t)$, we get:

$$-\omega^2\, P_0 + j\omega\, \gamma\, P_0 + \omega_m^2\, P_0 = \varepsilon_0 \omega_p^2\, E_0$$

The amplitude of the polarization can then be found as:

$$P_0 = E_0 \, \varepsilon_0 \omega_p^2 / \{(\omega_m^2 - \omega^2) + j\omega \gamma\}$$

Since $\varepsilon_r = 1 + P/\varepsilon_0 E$, we can now find the relative dielectric constant as:

$$\varepsilon_r = 1 + \omega_p^2 / \{(\omega_m^2 - \omega^2) + j\omega \gamma\}$$

[10]

3a) Electronic polarization is the separation of charge pairs into dipoles by an electric field. *Orientational polarization* is a mechanism that achieves the same effect in materials containing mobile, rod-like dipolar molecules, which can be rotated in the field to align their axes with the field direction. [1]

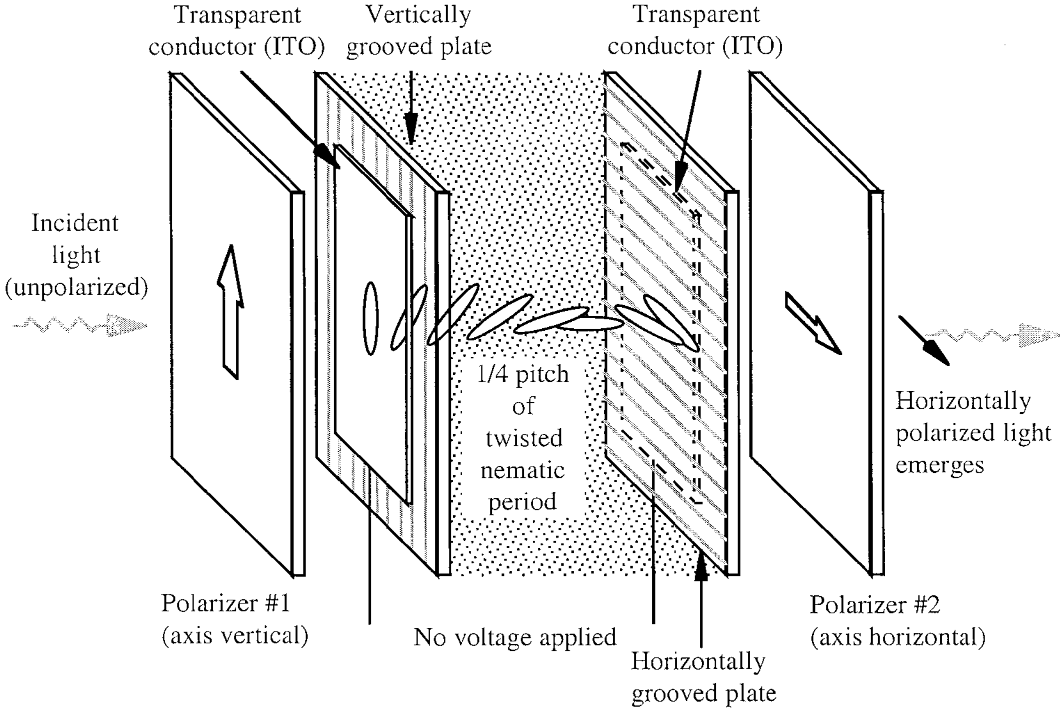
Refractive index is a quantity that describes the effect of matter on the propagation of light and other electromagnetic waves. Electromagnetic waves travel slower in matter by a factor $n = \sqrt{\varepsilon_r}$ (where ε_r is the relative dielectric constant and n is the refractive index). [1]

Birefringence occurs in crystals if there is asymmetry in the atomic arrangement. This asymmetry can make it easier for electrons to be driven in one direction than another, so that the relative dielectric constant varies as the polarization of the optical wave (i.e., the direction of the electric field) varies w.r.t. the crystal axes. It causes twin images when an object is viewed through the crystal. [1]

Birefringent materials can exhibit polarization-dependence of the absorption of an electromagnetic wave, known as *dichroism*, if the losses caused by damping are different for electron motion in different directions. Such material can polarize light, by preferential absorption of the unwanted polarization. [1]

Liquid crystals are materials that combine the properties of crystals (regular arrangement) with the properties of liquids (the possibility of molecular rearrangement). They are fluids containing rod-like, polar organic molecules, which can form a variety of structures with degrees of local order. Their properties can be controlled by composition, and their behaviour modified by an electric field. [1]

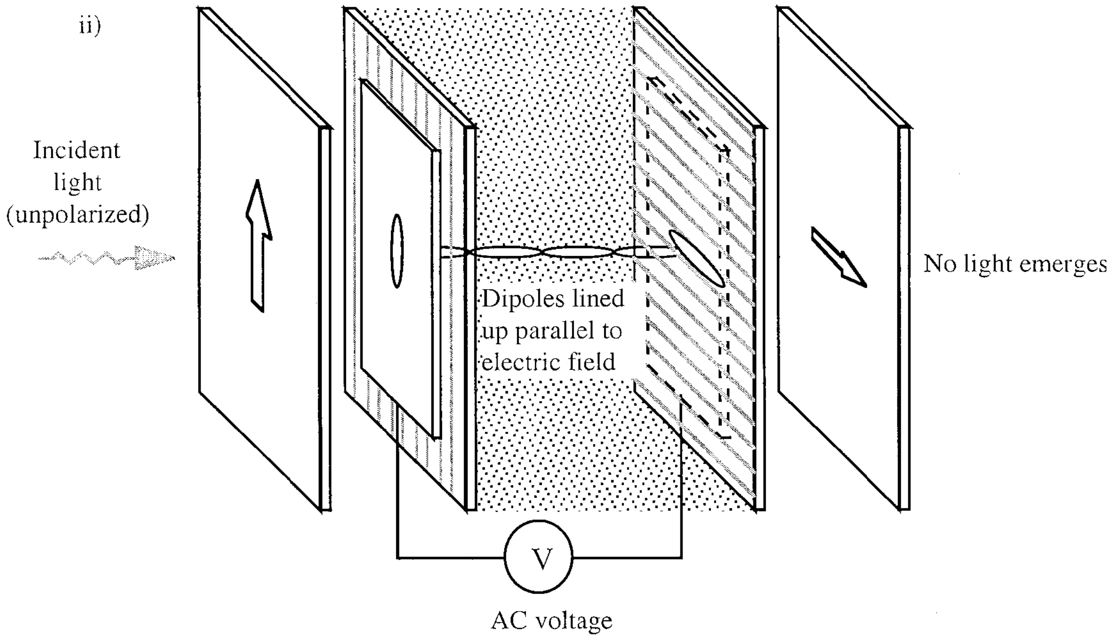
b) Display transmission HIGH:



[4]

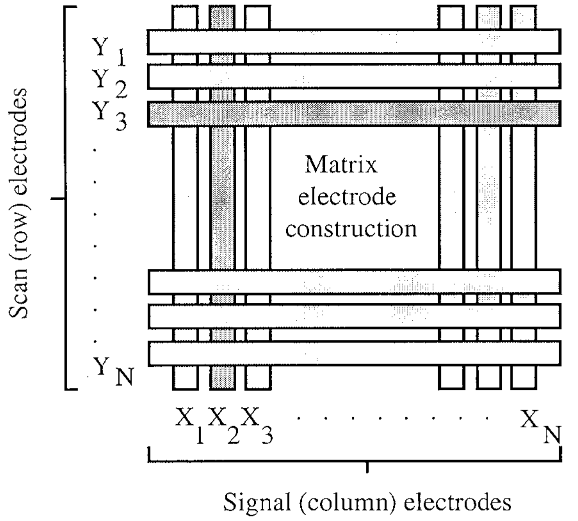
Display transmission LOW:

ii)



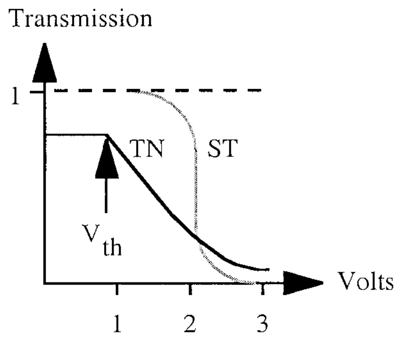
[4]

To form a two-dimensional display, the cell is pixellated. In the figure below, the front and back electrodes are divided into horizontal and vertical strips, respectively. Individual pixels can be addressed by applying the drive to the relevant combination of electrodes; here pixel (2, 3) is being addressed.



[2]

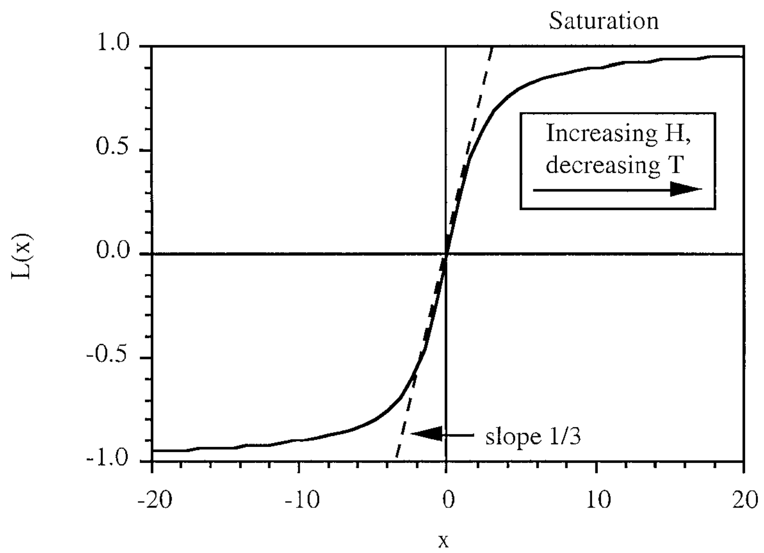
c) The figure below shows the variation of transmission with voltage. The brightness begins to change at a threshold voltage $V_{th} \approx 1$ V, and extinction requires $\approx$ 3V. Total extinction is never achieved. **[2]**



[2]

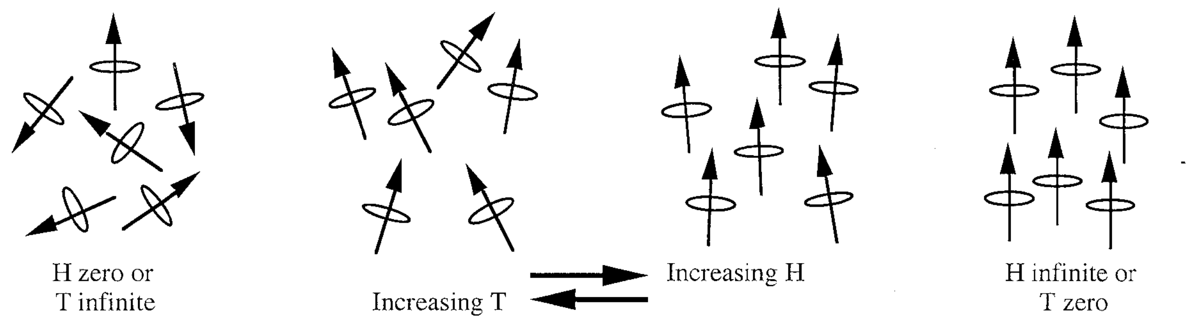
An alternative configuration known as Supertwist (ST), in which the molecules are twisted through 270° rather than 90°, offers superior contrast. **[1]**

4. a) The figure below shows the variation of L(x). For small x, L(x) varies linearly with x, with a slope 1/3. For large x, on the other hand, $L(x) \rightarrow 1$.



[4]

Weak dipole alignment is obtained when x is small, i.e. when H is very small or T is very large. Strong dipole alignment is obtained when x is large, i.e. when H is very large or T is very small. **[2]**



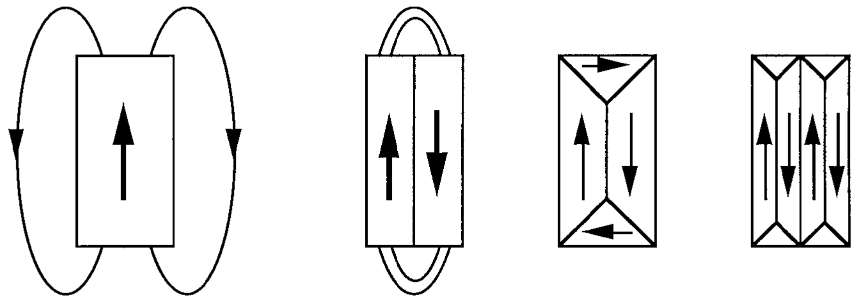
[2]

b) Saturation of the Langevin function L(x) is reached when (say) $x \geq 0$. If $\mu_m\mu_0 H/kT \geq 10$, then the necessary magnetic field is $H \geq 10\ kT / \mu_m\mu_0$. **[2]**

At 300 K, we get $H \geq 10 \times 1.38 \times 10^{-23} \times 300 / \{10^{-23} \times 4\pi \times 10^{-7}\} = 3.29 \times 10^{9}$ A/m. **[2]**

At 1 K, $H = 3.29 \times 10^{9} / 300 = 1.09 \times 10^{7}$ A/m. **[2]**

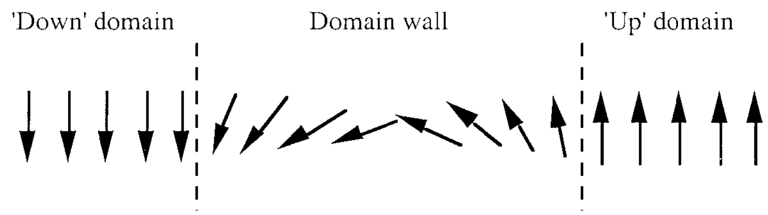
c) Figure (a) below shows a short bar magnet, which is uniformly magnetised. Because the field lines form, closed loops passing outside the magnet, energy must be stored in the external field. Figure (b) shows a similar situation, but now the crystal is divided into two domains, magnetised to saturation in opposite directions. Because the external field can now link the domains, its extent (and hence the energy it stores) is reduced. In c), the crystal is divided into four domains, which confine the field entirely within the crystal; d) shows a further subdivision. In each case, the introduction of domains allows the total energy of the system to be lowered.



a) b) c) d)

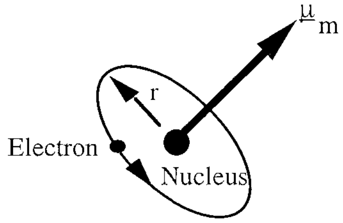
[3]

The argument above suggests that subdivision into domains should continue indefinitely, so that there is no dipole alignment anywhere. Experimentally, this does not occur - domains in Fe are typically $\approx$ 1 mm in size. This is because minimising the energy in the field is not the only consideration. The figure below shows the boundary between two domains. The dipole orientation changes gradually over the domain wall thickness ($\approx$ 1000 Å in Fe). Energy must therefore be stored in this region, because work is required to overcome the tendency of the dipoles to align parallel. A limit to domain size is reached when any reduction in the energy stored in the field matches the increase in the energy stored in the walls.



[3]

5.a) To estimate the dipole moment of an electron orbiting the nucleus of an atom, consider the effect of a single electron orbiting at a radius r at an angular frequency ω as shown below.



This electron orbits a loop of area $A = \pi r^2$, carrying a current $I = e\omega/2\pi$. Its magnetic moment is therefore:

$$\mu_m = IA = e\omega r^2/2$$

Now, the angular momentum of the electron is $J = mr^2\omega$. Consequently, μ_m can be written as:

$$\mu_m = eJ/2m$$

In the Bohr model, the angular momentum of an electron is quantised, so that it can only adopt the values:

$$J = nh/2\pi \quad \text{where} \quad n = \pm 1, \pm 2 \ldots$$

Substituting this result into the expression for μ_m, we then obtain:

$$\mu_m = neh/4\pi m \quad \text{where} \quad n = \pm 1, \pm 2 \ldots$$ **[6]**

The value of μ_m for n = 1 is the Bohr magneton, given by:

$$\mu_B = eh/4\pi m = 1.6 \times 10^{-19} \times 6.62 \times 10^{-34} / \{4\pi \times 9.1 \times 10^{-31}\} = 9.26 \times 10^{-24}\ \mathrm{Am}^2$$ **[2]**

b) At saturation, all the dipoles are aligned. The magnetisation is therefore $M_s \approx n\mu_m$, where μ_m is the dipole moment per atom and n is the atomic density. The magnetic flux is $B_s = \mu_o(H + M_s) \approx \mu_o M_s$.

The dipole moment is therefore $\mu_m = B_s/n\mu_o = 2.18 / \{8.5 \times 10^{28} \times 4\pi \times 10^{-7}\} = 2.04 \times 10^{-23}\mathrm{Am}^2$. **[4]**

Since $\mu_B = 9.26 \times 10^{-24}\ \mathrm{Am}^2$, the number of Bohr magnetons per atom is $n_B = \mu_m/\mu_B = 2.2$. Approximately 2.2 electrons per atom contribute. **[2]**

c) Strong magnetic effects arise in many-electron atoms from having a number of states with unpaired spins. The spin alignment in an isolated atom is usually destroyed when bonds are formed to make a solid. Since the outermost electrons are involved in bonding, strong magnetic effects can still be obtained if the unpaired spins are in an inner shell that is protected by a filled outer shell. **[4]**

An isolated Fe atom has 4 unpaired spins in its (inner) 3d orbital; 4 atoms might therefore be expected to contribute. **[2]**