E4.03
AS5
SO10
ISE4.3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

MSc and EEE/ISE PART IV: MEng and ACGI

Corrected Copy

**MOBILE RADIO COMMUNICATION**

Tuesday, 1 May 10:00 am

Time allowed: 3:00 hours

**There are FOUR questions on this paper.**

**Answer THREE questions.**

*All questions carry equal marks. The maximum mark for each subquestion is shown in brackets.*

**Any special instructions for invigilators and information for candidates are on page 1.**

| | | |
|---|---|---|
| Examiners responsible | First Marker(s) : | M.K. Gurcan |
| | Second Marker(s) : | K.K. Leung |

## The Questions

1. Answer the following subquestions.

   (a) For the two-ray path-loss model, derive an approximate expression for the distance values below the critical distance, $d_c$, at which signal nulls occur. [7]

   (b) The following table lists a set of empirical path-loss measurements.

   Take $\lambda = 0.4248$ m

| Distance from Transmitter | $P_r/P_t$ |
|---|---|
| 5 m | -60 dB |
| 25 m | -80 dB |
| 65 m | -105 dB |
| 110 m | -115 dB |
| 400 m | -135 dB |
| 1000 m | -150 dB |

   where $P_t$ and $P_r$ are transmitted and received signal powers.

   i. Find the parameters of a simplified path-loss model, plus log normal shadowing that best fit this data. [3]

   ii. Find the path loss at 2000 m based on this model. [2]

   iii. Find the outage probability at a distance $d$ assuming the received power at $d$ due to path-loss alone is 10 dB above the required power for non-outage. [2]

   (c) Consider a two-path channel with impulse response

$$h(t) = \alpha_1 \delta(\tau) + \alpha_2 \delta(\tau - 22 \times 10^{-9} s).$$

   Find the transmission delay $\tau$, as well as $\alpha_1$ and $\alpha_2$, assuming a free space path loss on each path, and a reflection coefficient of -1. Assume the transmitter and receiver are located 8 meters above the ground and the carrier frequency is 900 MHz. [6]

**2.** Answer the following subquestions.

(a) For a Rayleigh fading channel with average power $\bar{P}_r$= 30dB, and Doppler frequency $f_D = 10$ Hz, compute the average fade duration for target fade values $P_0$= 0 dB, $P_0$= 15 dB, and $P_0$= 30dB. [5]

(b) Consider the following two-ray channel scattering function obtained when applying a 900 MHz sinusoidal input to the channel:

$$S(\tau,\rho) = \begin{cases} \alpha_1 \delta(\tau) & \rho = 70 \\ \alpha_2 \delta(\tau - 22 \times 10^{-9}\text{s}) & \rho = 49.5\text{Hz} \\ 0 & \text{else,} \end{cases}$$

where $\alpha_1$ and $\alpha_2$ are determined by path loss, shadowing, and multipath fading. Assume the transmitter and receiver used to send and receive the sinusoidal signal are located 8 meters above the ground.

i. Find the distance and velocity between the transmitter and receiver. [3]

ii. For the distance computed in part i., is the path loss as a function of distance proportional to $d^{-2}$ or $d^{-4}$? [3]

iii. Does a 30 KHz voice signal transmitted over this channel experience flat fading or frequency-selective fading? [2]

(c) Let a scattering function $S(\tau, \rho)$ be nonzero over $0 \leq \tau \leq 0.1$ ms and -0.1 $\leq \rho \leq$ 0.1 Hz. Assume that the power of the scattering function is approximately uniform over the range where it is nonzero.

i. What is the multipath spread and the Doppler frequency spread of the channel? [1]

ii. If the input to this channel is two identical sinusoidal signals separated in time by $\Delta t$, what is the minimum value of $\Delta f$ for which the channel response to the first sinusoidal signal is approximately independent of the channel response to the second sinusoidal signal? [2]

iii. For two sinusoidal inputs to the channel $u_1(t)$= $\sin 2\pi ft$ and $u_2(t) = sin 2\pi f(t + \Delta t)$, what is the minimum value of $\Delta t$ for which the channel response to $u_1(t)$ is approximately independent of the channel response to $u_2(t)$? [2]

iv. Will this channel exhibit flat fading or frequency-selective fading for a typical voice channel with a 3 KHz bandwidth? Examine the nature of fading for a cellular channel having a bandwidth of 30 KHz. [2]

**3.** Answer the following subquestions.

(a) Consider an additive white Gaussian noise (AWGN) channel with bandwidth 50 MHz, received power 10 mW, and a noise power spectral density (PSD) $N_0 = 2 \times 10^{-9}$ W/Hz. If the received signal power is doubled, by how much is the capacity increased? If the channel bandwidth is doubled, by how much is the capacity increased? [3] [3]

(b) Consider a flat-fading channel where, for a fixed transmit power $\bar{P}$, the received signal-to-noise-ratio (SNR) is one of four values: $\gamma_1 = 30$ dB, $\gamma_2 = 20$ dB, $\gamma_3 = 10$ dB, and $\gamma_4 = 0$ dB. The probability associated with each state is $p_1 = 0.2$, $p_2 = 0.3$, $p_3 = 0.3$, and $p_4 = 0.2$ respectively. Assume both transmitter and receiver have Channel Side Information.

i. Find the optimal power control policy $P(\gamma_i)/\bar{P}$ , $i = 1, \cdots, 4$, for this channel and its corresponding Shannon capacity per unit of bandwidth. [3]

ii. Find the channel inversion power control policy for this channel and the associated zero-outage capacity per unit of bandwidth. [3]

iii. Find the truncated channel inversion power control policy for this channel and the associated outage capacity per unit bandwidth and also the associated cutoff $\gamma_0$ for the following three different outage probabilities: $p_{out} = 0.1$, $p_{out} = 0.01$, and $p_{out}$ corresponding to the value that achieves the maximum capacity with outage. [4]

(c) Consider two users simultaneously transmitting to a single receiver in an AWGN channel. Assume the users have equal received power of 10 mW and total noise at the receiver in the bandwidth of interest of 0.1 mW. The channel bandwidth for each user is 20 MHz.

i. Suppose that the receiver decodes one of the users' signal (signal 1) and the second user's signal acts as AWGN. What is the capacity of the channel associated with the decoded signal 1? [2]

ii. Suppose that after decoding signal 1, the decoder re-encodes it and subtracts it from the received signal, what is the Shannon capacity of the second user's channel? [2]

**4.** Answer the following subquestions.

(a) Consider the third generation wideband UTRA/FDD radio system and describe how the User Equipment, when it is first turned on, identifies the random access physical channel parameters using the following channels:

i. the Primary Synchronization Channel, [1]

ii. the Secondary Synchronization Channel, [1]

iii. the Pilot Channel, [1]

iv. the Primary Common Control Physical Channel (PCCPCH), [1]

v. the Secondary Common Control Physical Channel (SCCPCH). [1]

(b) In connection with the Global Systems Mobile ( GSM ) radio system, explain

i. the framing structure for the control channel, [1]

ii. how the logical control channels are multiplexed over the physical channel, [2]

iii. how the network switching subsytem is modified to handle packet transmission for use in the General Packet Radio Service (GPRS). [1]

(c) Consider a wideband direct sequence spread spectrum (WCDMA) system, where a total of $K$spreading signature waveforms are used to spread the information data bits over the downlink. Assume that both transmitter and receiver have knowledge of both the channel gain, $h_k$, and the channel-SNR, $\frac{h_k}{\sigma^2}$, for each code where the term $\sigma^2$ is the noise variance. Given that the transmitter adjusts the transmission power $P_k$ for each code $k$ while maintaining a minimum SNIR requirement $\gamma_k^* \geq 0$ at the output of the receiver detector in accordance with the Perron-Frobenius theorem, produce an expression for the power $P_k$ as a function of the inverse-channel-SNR. [4]

(d) Describe how the sum-capacity for random CDMA codes is upper bounded by the Welch-Bound-Equality limit. [3]

Explain how the iterative water filling and the discrete bit loading algorithms can be used to maximize the sum-capacity for random parallel codes in WCDMA systems. [4]

1.a SIGNAL NULLS OCCUR WHEN $\Delta\phi = (2n+1)\pi$

$$\frac{2\pi(x'+x-l)}{\lambda} = (2n+1)\pi$$

$$\frac{2\pi}{\lambda}\left[\sqrt{(h_t+h_r)^2+d^2} - \sqrt{(h_t-h_r)^2+d^2}\right] = \pi(2n+1)$$

$$\sqrt{(h_t+h_r)^2+d^2} - \sqrt{(h_t-h_r)^2+d^2} = \frac{\lambda}{2}(2n+1)$$

Let $m = (2n+1)$

$$\sqrt{(h_t+h_r)^2+d^2} = m\frac{\lambda}{2} + \sqrt{(h_t-h_r)^2+d^2}$$

square both sides

$$(h_t+h_r)^2+d^2 = m^2\frac{\lambda^2}{4} + (h_t-h_r)^2 + d^2 + m\lambda\sqrt{(h_t-h_r)^2+d^2}$$

$x = (h_t+h_r)^2$, $y = (h_t-h_r)^2$, $x-y = 4h_t h_r$

$$x = m^2\frac{\lambda^2}{4} + y + m\lambda\sqrt{y+d^2}$$

$$\Rightarrow d = \sqrt{\left[\frac{1}{m\lambda}\left(x - m^2\frac{\lambda^2}{4} - y\right)\right]^2 - y}$$

$$d = \sqrt{\left(\frac{4h_t h_r}{(2n+1)\lambda} - \frac{(2n+1)\lambda}{4}\right)^2 - (h_t-h_r)^2}, \quad n \in \mathbb{Z}$$

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MODEL ANSWER and MARKING SCHEME

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1.6

i

$$\frac{P_r}{P_t}\,dB = 10\log_{10} k - 10\gamma\log_{10}\frac{d}{d_0}$$

using least squares we get

$$10\log_{10} k = -29.42\ dB$$

$$\gamma = 4$$

ii

$$PL(2000) = 10\log_{10} k - 10\gamma\log_{10} d$$

$$= -161.76\ dB$$

iii

Receiver power can be assumed to be Gaussian with variance $\sigma^2_{\psi_{dB}}$

$$X \sim N(0, \sigma^2_{\psi_{dB}})$$

$$\text{Prob}(X < -10) = \text{Prob}\left(\frac{X}{\sigma_{\psi_{dB}}} < \frac{-10}{\sigma_{\psi_{dB}}}\right)$$

$$= 6.512 \times 10^{-4}$$

1.C $h(t) = \alpha_1 \delta(t-\tau) + \alpha_2 \delta(t-(\tau+0.22\mu s))$

$G_r = G_t = 1$

$h_t = h_r = 8m$

$f_c = 900 \text{ MHz}, \quad \lambda = c/f_c = 1/3, \quad R = -1$

$$\text{DELAY SPREAD} = \frac{x+x'-l}{c} = 0.022 \times 10^{-6} s$$

$$\Rightarrow \frac{2\sqrt{8^2+\left(\frac{d}{2}\right)^2} - d}{c} = 0.022 \times 10^{-6} s$$

$$\Rightarrow d = 16.1 \text{ m}$$

$$\therefore \tau = \frac{d}{c} = 53.67 \text{ ns}$$

$$\alpha_1 = \left(\frac{\lambda}{4\pi} \frac{\sqrt{G_L}}{L}\right)^2 = 2.71 \times 10^{-6}$$

$$\alpha_2 = \left(\frac{\lambda}{4\pi} \frac{\sqrt{R.G_r}}{x+x'}\right) = 1.37 \times 10^{-6}$$

2.a $P_r = 30$ dBm

$f_D = 10$ Hz

$$\rho = \sqrt{10^{\frac{(P_0 - P_r)}{10}}} \Rightarrow \begin{array}{l} \rho = \sqrt{10^{-3}} = 0.03165 \text{ for } P_0 = 0 \text{ dBm} \\ \rho = \sqrt{10^{-1.3}} = 0.1778 \text{ for } P_0 = 15 \text{ dBm} \\ \rho = \sqrt{10^{-0.5}} = 0.5623 \quad P_0 = 25 \text{ dBm} \end{array}$$

$$P_0 = 0 \text{ dBm}, \quad \bar{t}_z = \frac{e^{\rho^2} - 1}{\rho f_D \sqrt{2\pi}} = 0.0013 s$$

$$P_0 = 15 \text{ dBm}, \quad \bar{t}_z = 0.0072 s$$

$$P_0 = 25 \text{ dBm}, \quad \bar{t}_z = 0.0264 s$$

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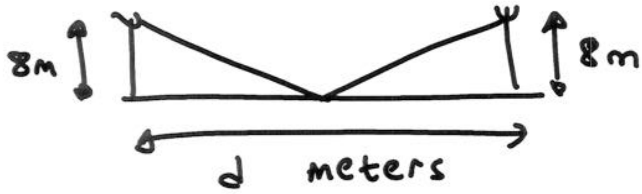
2.b

$$S(\tau,\rho) = \begin{cases} \alpha_1 \delta(\tau) & \rho = 70\,\text{Hz} \\ \alpha_2 \delta(\tau - 0.022\mu s) & \rho = 49.5\,\text{Hz} \\ 0 & \text{else} \end{cases}$$

The antenna set up is




From the figure, the distance travalled by the LOS ray is d and the distance travalled by the first multipath component is

$$2\sqrt{\left(\frac{d}{2}\right)^2 + 64}$$

Given this set up, we can plot the arrival of the LOS ray and the multipath ray that bounces of the ground on a time axis as shown in the above diagram. So we have

$$2\sqrt{\left(\frac{d}{2}\right)^2 + 8^2} - d = 0.022 \times 10^{3} \times 3 \times 10^{8}$$

$$\Rightarrow 4\left(\frac{d^2}{4} + 8^2\right) = 6.6^2 + d^2 + 2d(6.6)$$

$$\Rightarrow d = 16.1\,\text{m}.$$

2-b continued

$$f_D = v\cos(\theta)/\lambda \qquad v = f_D\lambda/\cos(\theta).$$

For the LOS ray, $\theta = 0$ and for the multipath component, $\theta = 45°$. We can use either of these rays and the corresponding $f_D$ value to get $v = 23.33$ m/s.

—///—

ii

$$d_c = \frac{4h_t h_r}{\lambda}$$

$d_c = 768$ m. Since $d \ll d_c$, power fall-off is proportional to $d^{-2}$.

—///

iii

$T_m = 0.022\ \mu s$, $B^{-1} = 0.33\ \mu s$.

Since $T_m \ll B^{-1}$, we have flat fading.

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2.c i

$T_m \simeq 0.1$ msec $= 100\,\mu$sec

$B_d \simeq 0.1$ Hz

Answers based on $\mu_{\tau_m}$ or $\sigma_{\tau_m}$ are fine too.

Notice, that based on the choice of either $T_m$, $\mu_{\tau_m}$ or $\sigma_{\tau_m}$, the remaining answers will be different too.

ii

$B_c \simeq \frac{1}{T_m} = 10$ kHz.

$\Delta f > 10$ kHz for $u_1 \perp u_2$

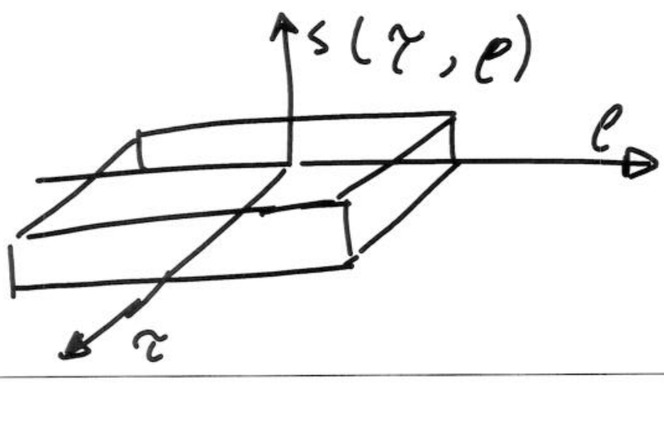
iii

$(\Delta t)_c = 10$ s

iv

3 kHz $< B_c \Rightarrow$ flat

30 kHz $> B_c \Rightarrow$ frequency selective.



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3.a $\text{Noise} = 50\times10^{-6}\times2\times10^{-9} = 0.1\ \text{w}$

$\text{Signal} = 0.01\ \text{w}$

$\text{SNR} = 0.01/0.1 = 0.1$

$C = 50\times\log_2(1+0.1) = 6.8752\ \text{Mbps}$

Doubling power $C = 50\times\log_2(1+0.2)$

$P_{new} = 20\ \text{mW},\ C = 13.1517$

$(\text{for } x \ll 1,\ \log(1+x) \simeq x)$

$\text{capacity increase} = 13.1517 - 6.8752$

$= 6.2765\ \text{Mbps}$

B = 100 MHz, Notice that both the bandwidth and noise power will increase. So C = 7 Mbps

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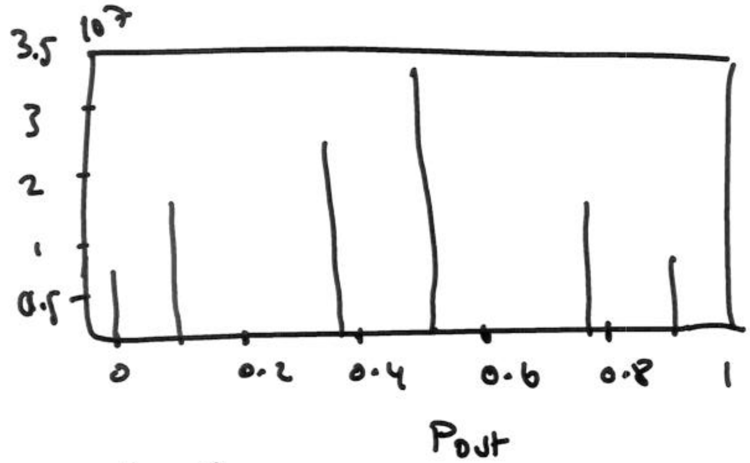
3.b i We suppose that all channel states are used

$$\frac{1}{\gamma_0} = 1 + \sum_{i=1}^{4} \frac{1}{\gamma_i} p_i \Rightarrow \gamma_0 = 0.8109$$

$$\frac{1}{\gamma_0} - \frac{1}{\gamma_4} > 0 \quad \therefore \text{ true}$$

$$\frac{P(\gamma_i)}{\bar{P}} = \frac{1}{\gamma_0} - \frac{1}{\gamma_i}$$




$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} 1.2322 & \gamma = \gamma_1 \\ 1.2232 & \gamma = \gamma_2 \\ 1.1332 & \gamma = \gamma_3 \\ 0.2332 & \gamma = \gamma_4 \end{cases}$$

$$\frac{C}{B} = \sum_{i=1}^{4} \log\left(\frac{\gamma_i}{\gamma_0}\right) p(\gamma_i)$$

$$= 5.2853 \text{ bps/Hz.}$$

3.b ii

$$\sigma = \frac{1}{E\left[\frac{1}{\gamma}\right]} = 4.2882$$

$$\frac{P(\gamma_i)}{\bar{P}} = \frac{\sigma}{\gamma_i}$$

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} 0.0043 & \gamma = \gamma_1 \\ 0.0029 & \gamma = \gamma_2 \\ 0.4288 & \gamma = \gamma_3 \\ 4.2882 & \gamma = \gamma_4 \end{cases}$$

$$\frac{C}{B} = \log(1+\sigma) = 2.4028 \text{ bps/Hz}$$

3.b iii

To have $P_{out} = 0.1$ or $0.01$ we will have to use all the sub-channels as leaving any of these will result in a $P_{out}$ of at least 0.2 ∴ truncated channel power control policy and associated spectral efficiency are the same as zero-outage case in part b.

To have $P_{out}$ that maximizes C with truncated channel inversion, we get

$$\max \frac{C}{B} = 4.1462 \text{ bps/Hz}, \quad P_{out} = 0.5$$

$$\frac{C_T}{B} = (0.3+0.2) \times \log_2\left(1 + \frac{1}{\frac{0.2}{10^3} + \frac{0.3}{10^2}}\right)$$

$$= 0.5 \log_2\left(1 + \frac{1}{0.0032}\right)$$

$$= 0.5 \times 8.923 = 4.1462$$

$\gamma_1 = 10^3$ $P_1 = 0.2$

$\gamma_2 = 10^2$ $P_2 = 0.3$

$\gamma_3 = 10$ $P_3 = 0.3$

$\gamma_4 = 1$ $P_4 = 0.4$

4.a i.) book work.
- primary synchronization channel is used to get slot timing information

ii) secondary synchronization channel is used to get frame timing information.

iii) Pilot channel is used to get primary scrambling code information.

iv.) Primary common broadcast channel is used to get the channelazation codes for the random access channel.

v) Secondary control chanel is used to transfer information over forward access and random access channels.

—//—

4.b i.) each frame consists of 8 slots.
51 frames is a multi frame
26 multi frames is a super frame.

—///—

ii) Each multi frame is organized as follows

10 channels:
- 1 frequency correction channel
- 1 Synchronization channel
- 4 Broadcast channels
- 4 paging channels

above arrangement is repeated five times.

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4.b iii SGSN (Serving GPRS Support Node)
GGSN ( Gateway GPRS Support Node)
are added to handle packet switching
and to link other packet switched
networks.

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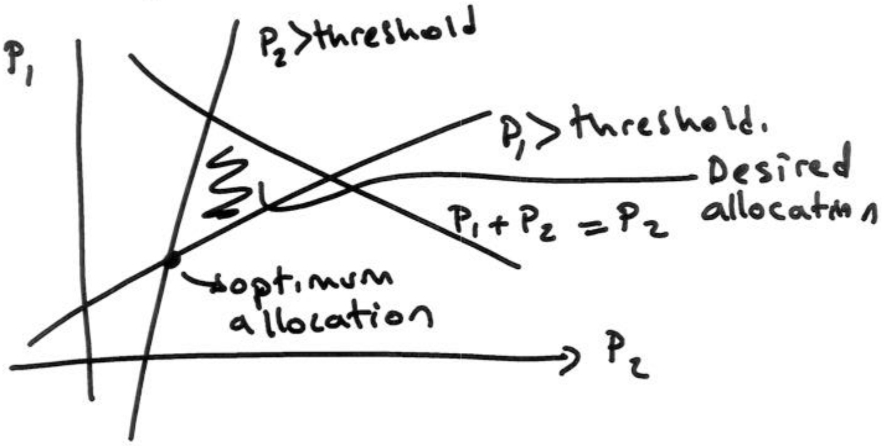
4.c

$$P_j \geq \gamma_j^* \sum_{k=j} P_k (c_j s_k)^2 + \frac{\gamma_j^* \sigma^2}{h} c_j^T c_j$$

where $\gamma_j^*$ is the desired SNR.



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4.d WBE

$$\lim_{K\to\infty} \frac{1}{2}\sum_{k=1}^{K} \log_2(1+\gamma_k) = \frac{1}{2}\log_2\left(1+\frac{\gamma}{N}\right)^N$$

**IWF**

$$\max_{p}\ b = \frac{1}{2}\sum_{k=1}^{K} \log_2\left(1+\frac{\gamma_k}{G}\right)$$

$$\text{ST} \quad P_T = \sum_{k=1}^{K} p_k$$

let

$$\gamma_k = p_k \frac{h_k (w_k s_k)^2}{\sum p_i (w_i s_k)^2 + \sigma^2 (w_k^T w_k)} = p_k g_k$$

$k^0 = k$

for $i = 1$ to $L$
  $k^i = k^{i-1}$
  for $k = 1$ to $k^i$.
    calculate $w_k$ using $p_k^{i-1}$
    calculate $g_k^i$ with $w_k$ and $p_k^{i-1}$
  end.
  Sort channels $g_1^i > g_2^i > \dots > g_{k^i}^i$
  find lagrange constant $K_L = \frac{1}{k^i}\left[P_T + \sum \frac{G}{g_k^i}\right]$
  for $k = 1$ to $k^i$ do
    $p_k^i = \max\left(0, K_L - \frac{G_i}{g_k^i}\right)$
  end
end

4.d Discrete bit loading.

SNRs on each sub-channel

$$Q_k = [Q_{K\times 1}]_k, \quad \text{where} \quad Q_k = \frac{1}{2}\log_2\left(1+\frac{\gamma_k}{G}\right)$$

$$\gamma_k = [\gamma_{K\times 1}]_k, \quad \text{where} \quad \gamma_k = G\left(2^{2Q_k}-1\right)$$

$$\gamma_k = \frac{h_k p_k (w_k^T s_k)^2}{\sum h_k p_j (w_k s_j)^2 + \sigma^2 (w_k^T w_k)}$$

equating two SNR equations lead to

$$G\left(2^{2Q_k}-1\right) = \frac{h_k p_k (w_k^T s_k)^2}{\sum h_k p_j (w_k^T s_j)^2 + \sigma^2 (w_k^T w_k)}$$

Re organise equation.

$$(I-F)p = u$$

$$f_{k,j} = [F_{K\times K}]_{k,j} = \begin{cases} 0 & \text{if } i=j \\ G\left(2^{2Q_k}-1\right)\dfrac{(w_k^T s_j)^2}{(w_k s_k)^2} & i\neq j \end{cases}$$

$$u_k = \frac{\sigma^2 G\left(2^{2Q_k}-1\right)}{h_k}\,\frac{w_k^T w_k}{(w_k^T s_k)^2}$$

4d $P = (I - F)^{-1} u$

energy distribution

$$\mathcal{E}(Q) = \tau P = \tau (I - F)^{-1} u$$

incremental energy

$$\mathcal{E}(Q) = \mathcal{E}(Q + \beta_{r_{k+}} + v_k) - \mathcal{E}(Q)$$

$$S = \sum_{k=1}^{K} \mathcal{E}_k$$

Algorithm

$S \Leftarrow 0$

while loop does not break do

$m \Leftarrow \arg\min_i \left[ \dfrac{e_i(Q)}{\beta_{r_{k+1}} - \beta_{r_i}} \right]$

$S \Leftarrow S + e_m(Q)$

if $S < E_T$ then.

$Q < Q + (\beta_{r_{m+1}} - \beta_{r_m}) v_m$

else

break.

end if.

end while

$Q_e \Leftarrow Q.$

return $Q_e$.