Part 1

# Biomedical Engineering
BE1-HMECH1
Mechanics 1, Main Exam

30/05/2017, 14.00-15.30
Duration: 90 minutes

The paper has THREE COMPULSORY questions
Answer ALL THREE question(s).

Each question is worth 100 marks

Marks for questions and parts of questions are shown next to the question. The marks for questions (and parts thereof) are indicative, and they may be slightly moderated at the discretion of the examiner.

**Question 1**

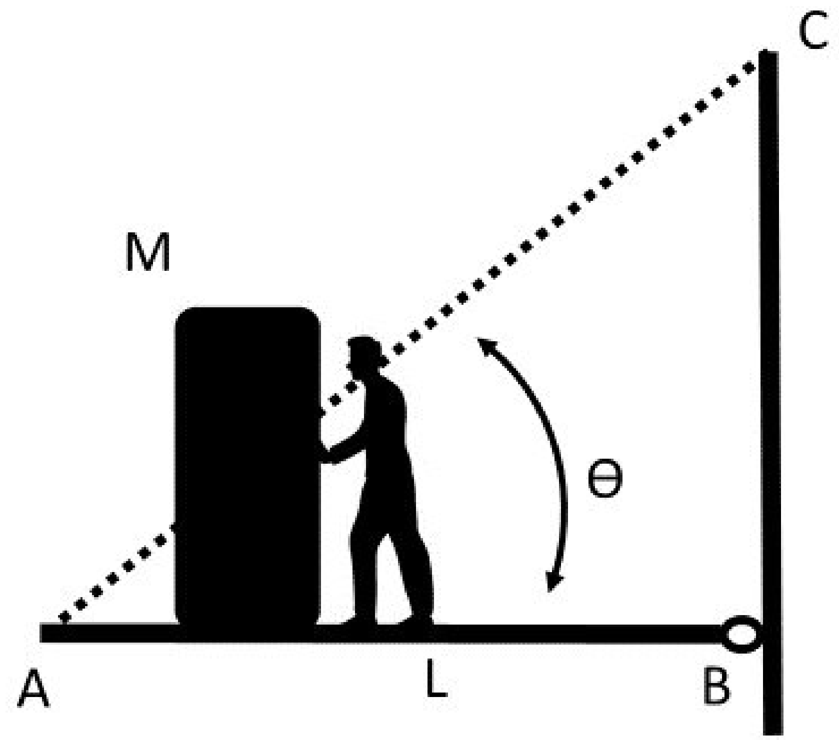



**Figure 1**

A Swing Bridge consists of a horizontal deck AB of length L and mass m, a tower BC, and supporting cable AC. The bridge deck is hinged at B. A container of mass M on a sled is being pushed across the bridge by a student of negligible mass, as shown.

a) If the distance of the trolley M from the end of the bridge A is distance x, draw a free body diagram for the bridge span AB. **(20 marks)**

b) Derive an expression for the tension in the cable AC in terms of M, m, x, L and all other relevant parameters. **(10 marks)**

c) If M is 160 kg, L is 25 m, m is 15 kg and Θ is 45° , what is the maximum tension in the cable AC? **(25 marks)**

d) If the student weighs 80 kg, and pushes from a position 1m behind the Centre of Mass of the container,

   i. Draw a new Free Body Diagram **(20 marks)**

   ii. calculate the reaction forces at the hinge B. **(25 marks)**

**Question total: 100 marks**

**Question 2**

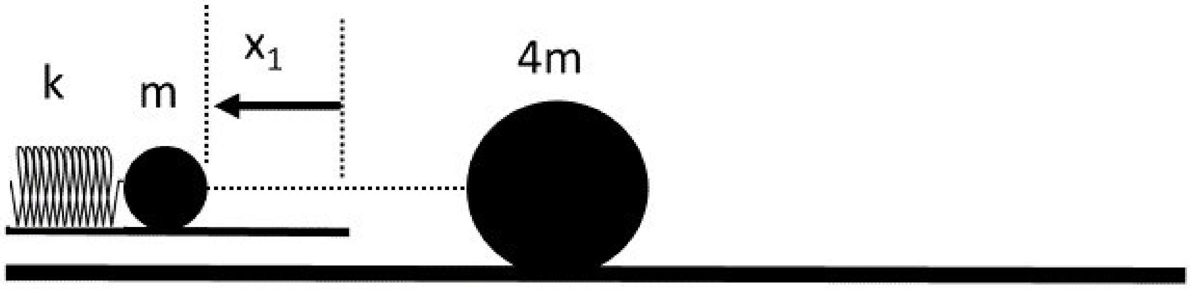



**Figure 2**

The apparatus shown in Figure 2 fires a steel ball at a larger steel ball sat stationary on a frictionless surface as shown.

a) Ball 1(of mass m) is compressed against the spring of constant k a distance of $x_1$, and then released such that it strikes Ball 2 (of mass 4m) in line with its Centre of Mass. If the balls strike with perfect elastic collisions,
   i. Derive an expression for the velocity of Ball 1 after the spring has fired it . **(15 marks)**
   ii. derive expressions for the final velocity of Ball 1 and Ball 2 in terms of m, k, $x_1$ and any other parameters you may need. **(35 marks)**

b) If the surface is not frictionless, but the coefficient of friction between the ball and the surface is 0.1;
   i. Describe in words what will happen **(15 marks)**
   ii. derive mathematical expressions for the final motion of Ball 1 and ball 2 **(35 marks)**

**Question total: 100 marks**

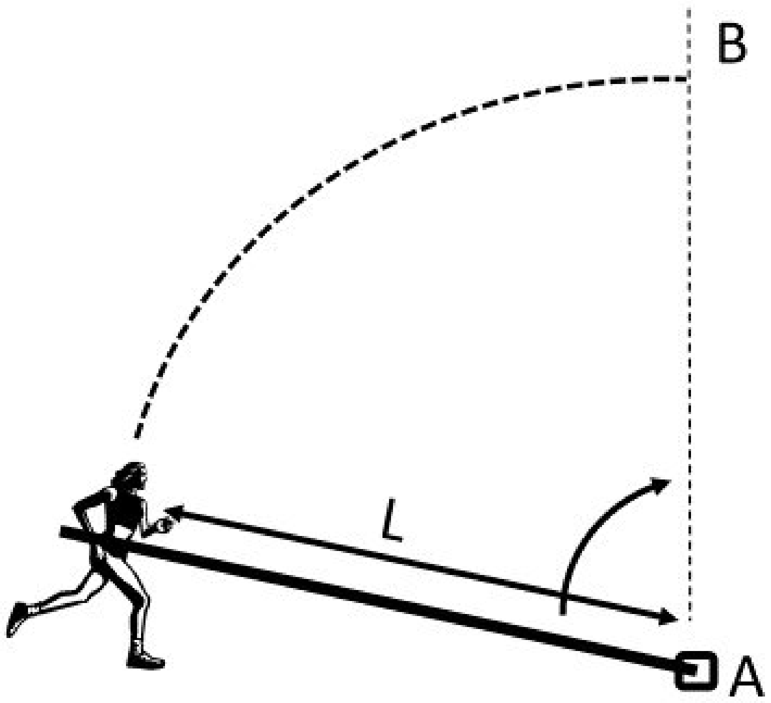



**Figure 3**

**Question 3.**

Student A is a keen canal -vaulter, whose legs are able to generate a maximum continuous force of Q Newtons in the horizontal direction.

a) If the maximum speed at which the student can run is S $ms^{-1}$ , working from first principles, and showing each stage of your derivation, derive an expression for the distance required to reach that speed. **(25 marks)**

b) If they are running at their max speed of S $ms^{-1}$ and then plant their pole in a ground socket and convert their linear motion into circular motion about the point of the pole which is L m from their Centre of Mass (assume they act as a simple pendulum with radius L and mass m), derive an expression for their tangential velocity at the top of the arc? **(25 marks)**

c) If that maximum speed S is 9.2 $ms^{-1}$ , their body mass 65 kg, and the point of the pole is 2.4 m from their Centre of Mass (assume they act as a simple pendulum with radius 2.4 m and mass 65 kg), what will be their tangential velocity at the top of the arc? (assume their Centre of Mass starts 1m above ground level). **(15 marks)**

**d)** If they let go of the pole at this point (the highest point on their swing), how far will they travel before hitting the ground (assume their Centre of Mass is just 10 cm above their point of impact when they hit). **(35 marks)**

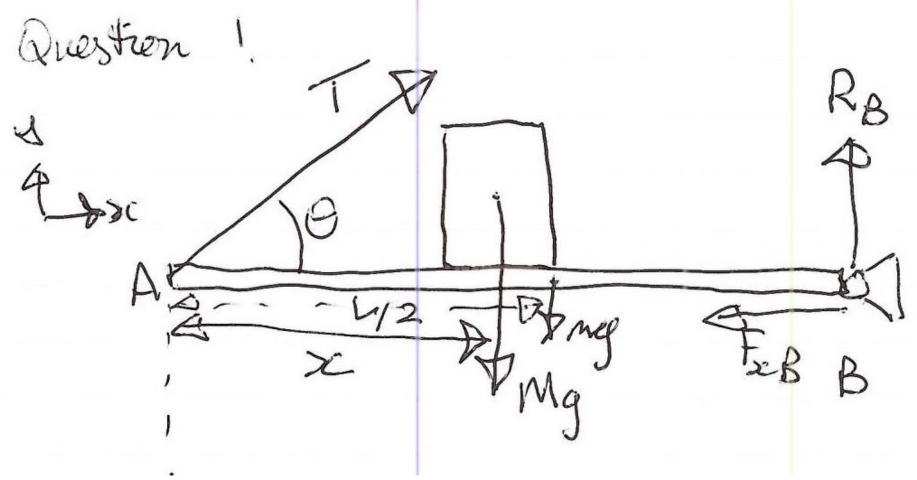
**Question total: 100 marks**

| Department of Bioengineering Examinations – 2016-17 Session Confidential |
|---|
| MODEL ANSWERS and MARKING SCHEME |
| First Examiner: [redacted] |
| Paper: **BE1-HMECH1 – Mechanics 1, Main Exam** Question: **1**<br>Page **1** of **3** |

**Question 1** A Swing Bridge consists of a horizontal deck AB of length L and mass m, a tower BC, and supporting cable AC. The bridge deck is hinged at B. A container of mass M on a sled is being pushed across the bridge by a student of negligible mass, as shown.

**a)** If the distance of the trolley M from the end of the bridge A is distance x, draw a free body diagram for the bridge span AB. .

**20 marks**




**20**

**b)** Derive an expression for the tension in the cable AC in terms of M, m, x, L and all other relevant parameters..

**10 marks**

Take moments about B

$$\Sigma M_B \quad -T.L.\sin\theta + Mg(L-x) + \frac{mgL}{2} = 0$$

$$\therefore \quad TL\sin\theta = Mg(L-x) + \frac{mgL}{2}$$

**10**

**c)** If M is 160 kg, L is 25 m, m is 15 kg and Θ is 45° , what is the maximum tension in the cable AC?

**30 marks**

Maximum tension occurs when $x = 0$, when the mass is at A.

$$\Rightarrow T L \sin\theta = MgL + \frac{mgL}{2}$$

$$T = \frac{gL\left(M + \frac{m}{2}\right)}{L \sin\theta}$$

$$= \frac{9.81\,(160 + 7.5)}{\cancel{25} \sin 45}$$

$$= \cancel{92.858}\ N.$$

(10 min

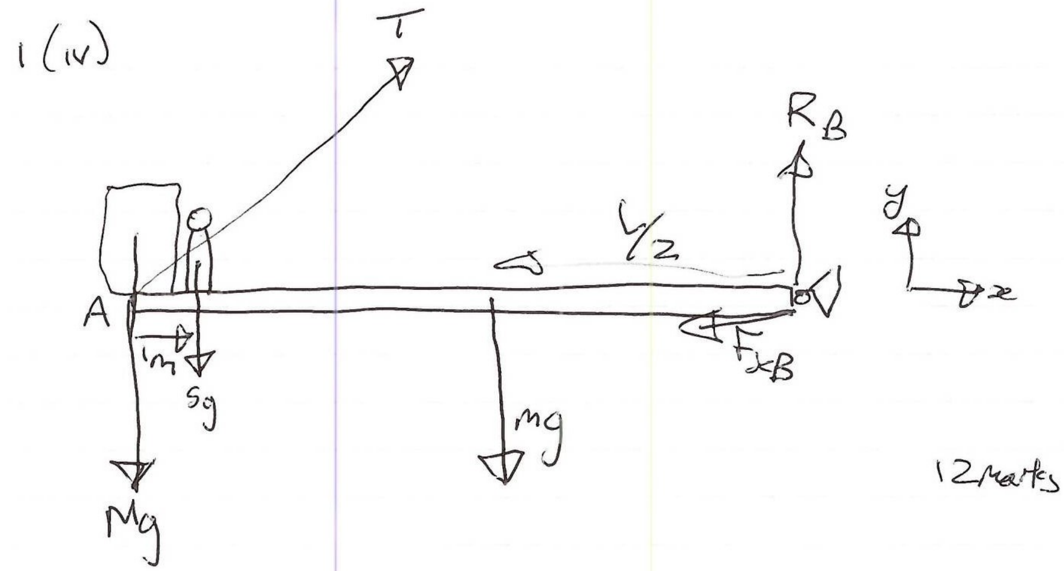
$$= 2323.523\ N$$

30

**d)** If the student weighs 80 kg, and pushes from a position 1m behind the Centre of Mass of the container,

**i)** Draw a new Free Body Diagram

**20 marks**




20

**ii)** re-calculate the maximum tension in the cable.

**20 marks**

(b) Again, moments about B

$$\Sigma M_B \quad + MgL - TL\sin\theta + Sg(L-1) + mg\frac{L}{2} = 0$$

$$\therefore \quad TL\sin\theta = MgL + Sg(L-1) + mg\frac{L}{2}$$

$$\therefore \quad T = \frac{160\times g\times 25 + 80\times g\times 24 + 15g\times 7.5}{25\times 0.7071}$$

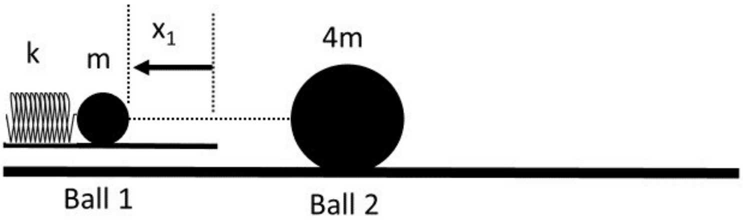
$$= \frac{39240 + 18835.2 + 1103.625}{25\times 0.7071}$$

$$= \frac{59178.825}{25\times 0.707}$$

$$= 3347.66 \text{ N}$$

20

| **Department of Bioengineering Examinations – 2016-17 Session Confidential** |
|---|
| MODEL ANSWERS and MARKING SCHEME |
| First Examiner: [redacted] |
| Paper: **BE1-HMECH1 – Mechanics 1, Main Exam** Question: **2**<br> |




**Question 2** The apparatus shown in Figure 2 fires a steel ball at a larger steel ball sat stationary on a frictionless surface as shown.

**a)** Ball 1(of mass m) is compressed against the spring of constant k a distance of $x_1$, and then released such that it strikes Ball 2 (of mass 4m) in line with its Centre of Mass. If the balls strike with perfect elastic collisions,.

**i)** Derive an expression for the velocity of Ball after the spring has fired it in terms of m, k, $x_1$ and any other parameters you may need

**15 marks**

Ball one spring $F = -kx_1$

energy $E = \frac{1}{2}kx_1^2 = \frac{1}{2}mV^2$

$\therefore kx_1^2 = mV^2$

$\therefore V^2 = \frac{kx_1^2}{m}$ ...... (1)

**15**

**ii)** derive expressions for the final velocity of Ball 1 and Ball 2 in terms of m, k, $x_1$ and any other parameters you may need..

**35 marks**

At collision, momentum is conserved. $\therefore L_1 = L_2$

$$\therefore mV_{1i} + 4m(0) = mV_{1f} + 4mV_{2f}$$

$$\therefore mV_{1f} = mV_{1i} - 4mV_{2f}$$

$$\therefore V_{1f} = V_{1i} - 4V_{2f} \quad \ldots (2)$$

Energy is also conserved $\quad E_1 = E_2$

$$\therefore \tfrac{1}{2}mV_{1i}^2 + \tfrac{4m}{2}(0)^2 = \tfrac{1}{2}mV_{1f}^2 + \tfrac{4m}{2}V_{2f}^2$$

$$\therefore V_{1f}^2 = V_{1i}^2 - 4V_{2f}^2 \quad \ldots (3)$$

Substitute from (2) above

$$\Rightarrow (V_{1i} - 4V_{2f})^2 = V_{1i}^2 - 4V_{2f}^2$$

$$\therefore (V_{1i}^2 - 8V_{1i}V_{2f} + 16V_{2f}^2 = V_{1i}^2 - 4V_{2f}^2$$

$$\therefore 16V_{2f}^2 + 4V_{2f}^2 = 8V_{1i}V_{2f}$$

$$\therefore 20V_{2f}^2 = 8V_{1i}V_{2f}$$

$$\therefore 20V_{2f} = 8V_{1i}$$

$$V_{2f} = \frac{8}{20}V_{1i} = \frac{V_{1i}}{2.5} \quad \ldots (4)$$

from (2) above $\quad V_{1f} = V_{1i} - 4V_{2f}$

$$= V_{1i} - 4\left(\frac{V_{1i}}{2.5}\right) = V_{1i} - \left(\frac{10V_{1i}}{2.5}\right)$$

$$= \frac{7.5V_{1i}}{2.5}$$

$$= \cancel{V_{1i}}\ 3V_{1i} \quad (4)$$

35

**b)** If the surface is not frictionless, but the coefficient of friction between the ball and the surface is 0.1.

**i)** Describe in words what will happen

**15 marks**

Ball 1 will strike ball 2, which will start to move sideways in the positive x direction, but will also start to rotate/roll. The linear velocity will be defined by the collision equations as derived above.
The angular velocity (rolling speed) will be determined by the frictional force and the Moment of Inertia of ball 2.

**15**

iii. **iii)** derive mathematical expressions for the final motion of Ball 1 and ball 2

**35 marks**

2(d)

from (1) $V_{1i}^2 = \frac{k x_i^2}{m}$ $\quad x_i = 0.2$

$m = 0.25$

$k = 16$

$\Rightarrow V_{1i} = \frac{16 \times 0.04}{0.25} = 2.56 \text{ ms}^{-1}$

$\Rightarrow V_{2f} = \frac{2.56}{2.5} = 1.025 \text{ ms}^{-1}$

fire another ball at it $\quad V_{1i}$ again 2.56

$V_{2i}$ now 1.025

Momentum is conserved so

$m2.56 + 4m \times 1.025 = mV_{1f} + 4mV_{2f} \quad \ldots (6)$

energy is also conserved

$\frac{1}{2}m(2.56)^2 + 2m(1.025)^2 = \frac{1}{2}mV_{1f}^2 + 2mU_{2f}^2 \quad \ldots (7)$

$\therefore (2.56)^2 + 4(1.025)^2 = V_{1f}^2 + 4V_{2f}^2$

from (6) above $\quad 2.56\cancel{m} + 4.1\cancel{m} = \cancel{m}V_{1f} + 4\cancel{m}V_{2f}$

$\therefore V_{1f} = 6.66 - 4V_{2f}$

from (7)

$6.5536 + 4.2025 = V_{1f}^2 + 4V_{2f}^2$

$\therefore 10.8561 = (6.66 - 4V_{2f})^2 + 4V_{2f}^2$

$\therefore 4V_{2f}^2 + 16V_{2f}^2 - 26.64V_{2f} - 10.8561 = 0$

$\therefore 12V_{2f}^2 - 26.64V_{2f} - 10.8561 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad \therefore V_{2f} = \frac{26.64 \pm \sqrt{(26.64)^2 - 4(12 \times 10.8561)}}{24}$

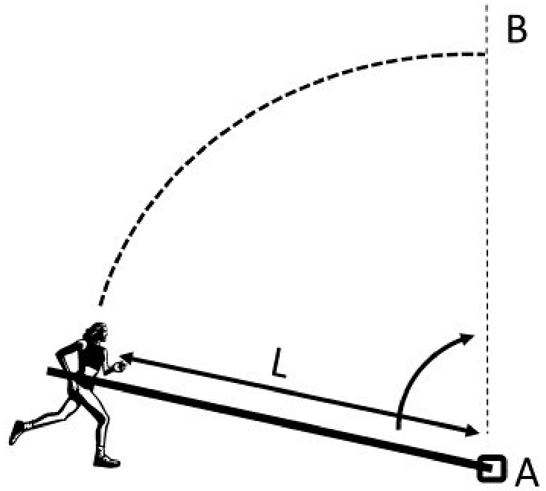
$= \frac{26.64 \pm \sqrt{(709.6896 - 521.088}}{24}$

$= \frac{26.64 \pm 13.733}{24}$

$= 1.683 \quad \text{ms}^{-1}$

**Marks:**

| |
|---|
| **Department of Bioengineering Examinations – 2016-17 Session Confidential** |
| MODEL ANSWERS and MARKING SCHEME |
| First Examiner: [redacted] |
| Paper: **BE1-HMECH1 – Mechanics 1, Main Exam** Question: **3**<br>Page**3** of **3** |




**Question 3** Student A is a keen canal -vaulter, whose legs are able to generate a maximum continuous force of Q Newtons in the horizontal direction.

**a)** If the maximum speed at which the student can run is S $ms^{-1}$ , working from first principles, and showing each stage of your derivation, derive an expression for the distance required to reach that speed..

**25 marks**

3.

$a = K \, ms^{-2}$

$0 \to K$ $\quad \sum F_x \quad K = ma$

$\therefore a = \frac{K}{m}$

integrate $\therefore \quad V_x = \frac{kt}{m} + V_0 \quad (=0)$

integrate $\quad = s$

$x = \frac{kt^2}{2m} + c \quad (=0)$

at time $t_2 \quad V_2 = s = \frac{kt_2}{m}$

$\therefore t_2 = \frac{sm}{K}$

at time $t_2 \quad x = \frac{kt_2^2}{2m} = \frac{k}{2m}\left(\frac{sm}{k}\right)^2$

$= \frac{sm}{2k}$

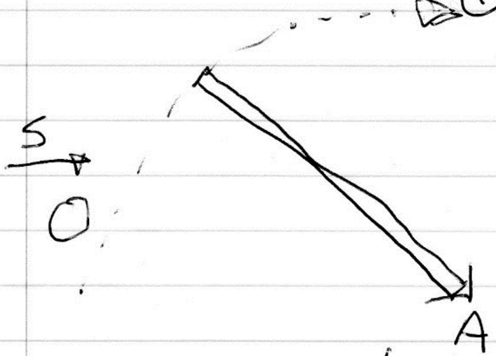
**b)** If they are running at their max speed of S $ms^{-1}$ and then plant their pole in a ground socket (A) and convert their linear motion into pendulum motion about the point of the pole which is L m from their Centre of Mass (assume they act as a simple pendulum with radius L and mass m), derive an expression for their tangential velocity at the top of the arc (B)?.

**20 marks**

| |
|---|
| The answer to the subpart.<br>**Marks:** |

Mechanics Exam

3(b)




(1) At start of pendulum motion all their energy is kinetic,

$$E_1 = K_1 = \frac{1}{2}mV^2 = \frac{1}{2}mS^2$$

energy is conserved.

so, at the top of the pendulum swing if their CoM is $h_1$ above the ground at the start

$$\cancel{E_2 = U_2 + K_2 = mg}$$

then their vertical height gained $\Delta h = L - h$

$$E_2 = U_2 + K_2 = mg\Delta h + \frac{1}{2}mV_2^2$$

$$= mg(L-h) + \frac{1}{2}mV_2^2 = \frac{1}{2}mS^2$$

$$\therefore \quad V_2^2 = S^2 - 2g(L-h)$$

$$\therefore \quad V_2 = \sqrt{S^2 - 2g(L-h)}$$

**c)** If that maximum speed is 9.2 $ms^{-1}$ , their body mass 65 kg, and the point of the pole is 2.4 m from their Centre of Mass (assume they act as a simple pendulum with radius 2.4 m and mass 65 kg), what will be their tangential velocity at the top of the arc? (assume their Centre of Mass starts 1m above ground level)..

**20 marks**

T

(b) at pole plant $V = 9.2$ m/s

so $E = \frac{1}{2}mV^2 = \frac{65}{2}(9.2)^2$

at top of pendulum swing $E = \frac{1}{2}mv^2 + mgh$

$\therefore \frac{\cancel{65}}{2}(9.2)^2 = \cancel{65} \times g \times 1.4 + \frac{\cancel{65}}{2} V_2^2$

$\therefore \frac{V_2^2}{2} = \frac{(9.2)^2}{2} - 1.4g$

$\therefore V_2^2 = (9.2)^2 - 2.8g =$

$= 84.64 - 27.468$

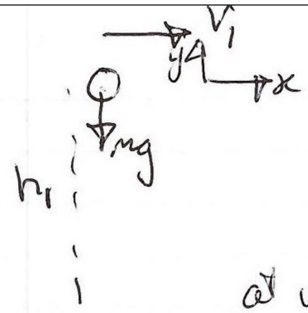
$= 57.172$

$\therefore V_2 = 7.56 \; ms^{-1}$

**20**

**d)** If they let go of the pole at this point (the highest point on their swing), how far will they travel before hitting the ground (assume their Centre of Mass is just 10 cm above their point of impact when they hit).

**35 marks**




when they release, $V_1 = 7.56\ ms^{-1}$
and
altitude = 2.4 m.

at impact, " = 0.1 m.

$\Sigma F_y \quad -mg = ma \quad \therefore a = -g$

integrate

$\therefore V_y = -gt + \cancel{y_0}$

(set $y = 0$ at release) so $y_0 = 0$ $\quad V_0 = 0$

$V_y = -gt$

integrate

$\Rightarrow y = -\dfrac{gt^2}{2} + y_0^{(=0)}$

$\therefore 2.3 = \dfrac{gt^2}{2}$

$\therefore t^2 = \dfrac{4.6}{9} = \cancel{0.715}s.$

$\Sigma F_x \quad 0 = ma \qquad t = 0.715\ s.$

$\therefore a = 0$

integrate $\therefore V_x = V_0 = 7.56\ ms^{-1}$

integrate

$x = V_0 t = 7.56 \times 0.715\,s$

$= 5.405\ m.$