## UNIVERSITY OF LONDON IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

## **EXAMINATIONS 1997**

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER 2.9

SIMULATION AND MODELLING Monday, April 21st 1997, 10.00 - 11.30

Answer THREE questions

For admin. only: paper contains 4 questions

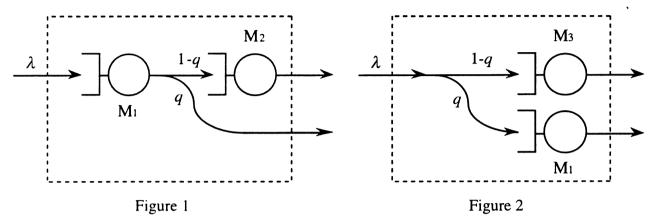
A Surrey-based communications firm wishes to evaluate a new one-way local-area network design which they have called the "Eshernet". Each device "thinks" for a while during which time it prepares its next message for transmission. When it is ready to transmit it first looks to see if the Eshernet is busy. If it is the device stalls and backs off for a randomly generated time after which it retries the transmission request. If not, it claims the Eshernet (i.e. makes it busy) and starts transmitting. At the end of the transmission the Eshernet is released (i.e. marked "not busy") and the device restarts. Each retry operates in the same way as the original request: if the Eshernet is still busy when the retry is attempted the device backs off again for a (different) randomly generated time. All devices are capable of receiving messages from the Eshernet at all times, regardless of whether the device is thinking (i.e. preparing its next message for transmission) or backed-off; any time incurred in processing a received message forms part of the measured think times of the device and so need not be considered separately.

Design a discrete-event simulator for an Eshernet with N attached devices which estimates the utilisation of the Eshernet and the mean number of retries required to complete each transmission. You may assume the existence of an event scheduler such that schedule(e, p, t) arranges for the function call e(p) to be made at simulated time t. You may also assume a server object which includes access functions for claiming and releasing a server and printing its utilisation at the end of the simulation. You may also assume the following support functions:

thinktime() generates a sample device think time transtime() generates a sample transmission time backofftime() generates a sample back-off time

Your solution should identify the (global) state variables which are required to model the system state together with the measurement variables which are required to compute the two performance measures. You should also provide informal pseudocode for each event function and should state how the performance measures can be computed at the end of the simulation. No other pseudocode is required.

A particular processing plant handles two types of jobs: a proportion 100q% of jobs are *simple* jobs which are processed by a machine  $M_1$  each requiring an average service time of  $\mu_1^{-1}$  hours; the remaining 100(1-q)% of jobs are *complex* jobs, which are like simple jobs except they require additional processing by a machine  $M_2$  which takes an average time of  $\mu_2^{-1}$  hours. The arrival rate of jobs to the plant is  $\lambda$  jobs per hour and jobs waiting to be processed are queued up in very large storage areas which are well approximated by infinite capacity queues. The current set up is shown in the form of a queueing network in Figure 1 below.



The owner of the plant wants to reduce the average processing time of jobs and is seeking an alternative set up. She has been offered a new machine  $M_3$  which is capable of processing a complex job in the same mean time as the existing plant (i.e.  $\mu_1^{-1} + \mu_2^{-1}$  hours) and the idea is to trade in machine  $M_2$  for machine  $M_3$  and use  $M_1$  and  $M_3$  in combination as shown in Figure 2. The various parameters have the following values:

$$\lambda = 5$$
  $\mu_1 = 10$   $\mu_2 = 5$   $q = 0.5$ 

- a Assuming that the arrival process is Poisson and that the service times of all three machines are exponentially distributed, compute the following equilibrium quantities for both set-ups:
  - i. The utilisation of each machine
  - ii. The mean number of jobs queueing for each machine (i.e. not including jobs currently being processed)
  - iii. The mean processing time for each job (i.e. the mean time each job spends inside the boxed region in the respective diagram).

On the basis of these calculations what recommendation would you make to the owner of the plant?

b What processing rate for  $M_3$  is required to make the mean processing times the same for both set-ups?

70% of the marks are allocated to part a.

Turn over...

3a The Weibull distribution has a cumulative distribution function (cdf) given by

$$F(x) = 1 - \exp(-(x/\alpha)^{\beta})$$

Describe *one* method by which samples from a Weibull distribution can be computed. As part of your answer you should state any mathematical properties which your method assumes, although you are not required to prove them.

b The time to failure of a number of electronic devices (in thousands of hours) has been measured and the results are summarised in the table below:

Interval (0.0, 0.4] (0.4, 0.8] (0.8, 1.2] (1.2, 1.6] (1.6, 2.0]  $(2.0, 2.4] \ge 2.4$ Frequency 414 1002 889 467 167 48 13

It is suspected that these times have a Weibull distribution and in order to test this the measured data has been used to estimate the parameters  $\alpha$  and  $\beta$  of the distribution; these estimates are  $\alpha = 1.03$ ,  $\beta = 1.97$ . The null hypothesis:

 $H_0$ : The measured data is Weibull-distributed with parameters  $\alpha$  =1.03,  $\beta$  =1.97 is thus proposed.

Perform a  $\chi^2$  test on the data to test this hypothesis at the 10% significance level.

40% of the marks are allocated to part a.

- 4a Explain what is meant by *equilibrium* (or *steady-state*) behaviour and describe one method for estimating when equilibrium has been reached during a discrete-event simulation.
- b Explain how can the method of *batched means* be used to accumulate N *independent* estimates of some unknown quantity during a single simulation run. As part of your answer outline a method for implementing batch means within a discrete-event simulator.
- c Explain how N estimates of some unknown quantity (such as might be produced from a batched means simulation experiment) can be used to compute a *point estimate* and a 90% confidence interval for the unknown quantity. How, statistically, could you halve the width of the 90% confidence interval?

End of paper