UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1999

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER 2.6

STATISTICS

Tuesday, April 27th 1999, 4.00 – 5.30

Answer THREE questions

For admin. only:
paper contains 4 questions

1 a A computer returned under guarantee is given two initial tests, H and W. From past returns, the probability that it passes H is 0.7, and the probability that it passes W is 0.6. If it has passed W, it will pass H with probability 0.9.

Find, showing your reasoning, the probability that the computer passes

i both tests,

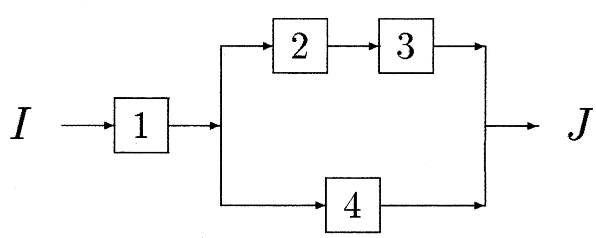
ii exactly one test,

iii neither test.

If two computers are chosen independently at random, find

iv the probability that one passes H but not W, and the other passes W but not H.

b For the system of four devices shown, each independently may or may not operate. The probabilities of the devices being operational are 0.95, 0.90, 0.95 and 0.80, respectively, for the devices labelled 1, 2, 3 and 4. The *system reliability* is the probability that there is a path of operating devices from I to J. Find the system reliability, showing your reasoning.



2 The lifetime, T, in hours of a certain type of electronic component is a random variable having the Exponential probability density function

$$f(t) = \begin{cases} A\,e^{-\frac{1}{1200}t} & (t \geq 0), \\ 0 & (t < 0), \end{cases}$$

where A is a constant.

a Find the value of A.

b Show that the mean and the standard deviation of T are both 1200 hours.

c Find an expression for the proportion of such components that will survive beyond t hours.

d Find the median lifetime of a component.

e Find and sketch the hazard function for the lifetime distribution.

f Explain briefly when the Exponential distribution might be appropriate for modelling the lifetimes of electronic components.

The six parts carry respectively 10%, 20%, 20%, 20%, 20% and 10% of the marks.

Turn over ...

3 a If the random variable X is normally distributed with mean μ and standard deviation σ, and $P(X < 1) = 0.05$, and $P(X > 4) = 0.20$, find μ and σ.

b A low-noise transistor is being developed to reduce the mean noise level to below the 2.5 dB level of those currently in use. It is known that the noise level of this type of transistor follows a Normal distribution.

i The noise levels of a random sample of 9 of the new transistors have a mean of 1.8 dB and a standard deviation of 0.8 dB. Find the P-value for a test of whether this reduction is achieved.

ii Based on the P-value obtained in (i), what conclusion can be reached regarding whether a noise reduction has been achieved? Give your reasons.

The new transistors will not be commercially viable if they do not reduce the mean noise level to below 2 dB.

iii Determine approximately how many of the new transistors must be measured to distinguish between a mean of 2.5 dB and a mean of 2.0 dB, at a 5% significance level with a power of 80%.

The two parts carry respectively 40% and 60% of the marks.

4 A computer algorithm has been modified to reduce the mean completion time of a statistical application. We wish to test, using the evidence given below, whether or not the hypothesis of equal mean completion times can be rejected. The completion times, in milliseconds, under the standard and under the modified algorithms are known to be independently normally distributed with the same variance.

Algorithm	Standard	Modified
Sample size	6	8
Sample mean	13.6	10.7
Sample standard deviation	2.6	3.1

a State the null and alternative hypotheses for this problem.

b By considering both 5% and 1% significance levels, test whether the data offer support that the modified algorithm reduces the mean completion time. Show your reasoning.

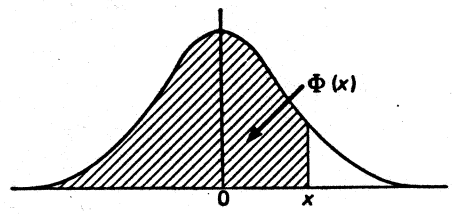
c If the variance were known to be 2.9, would this alter your findings? Give your reasons.

The three parts carry respectively 20%, 60% and 20% of the marks.

End of Paper

THE NORMAL DISTRIBUTION FUNCTION

he function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is he probability that a random variable, normally distributed vith zero mean and unit variance, will be less than or equal o x. When $x < 0$ use $\Phi(x) = 1-\Phi(-x)$, as the normal istribution with zero mean and unit variance is symmetric bout zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

			0·9994		0·99990		0·99995
3·075		3·263		3·731		3·916	
	0·9990		0·9995		0·99991		0·99996
3·105		3·320		3·759		3·976	
	0·9991		0·9996		0·99992		0·99997
3·138		3·389		3·791		4·055	
	0·9992		0·9997		0·99993		0·99998
3·174		3·480		3·826		4·173	
	0·9993		0·9998		0·99994		0·99999
3·215		3·615		3·867		4·417	
	0·9994		0·9999		0·99995		1·00000

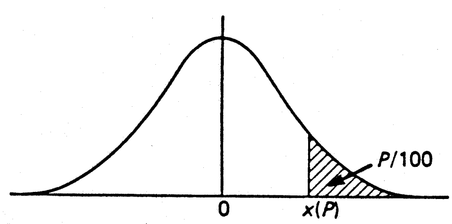
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

table gives percentage points $x(P)$ defined by the ion

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

is a variable, normally distributed with zero mean and ariance, $P/100$ is the probability that $X \geqslant x(P)$. The P per cent points are given by symmetry as $-x(P)$, ne probability that $|X| \geqslant x(P)$ is $2P/100$.



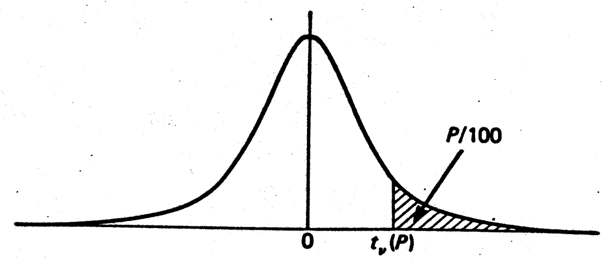
P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	·2887	·6172	0·8165	1·0607	·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	·2767	·5844	0·7649	0·9785	·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	·2707	·5686	0·7407	0·9410	·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	·2648	·5534	·7176	·9057	·134	·440	1·943	·447	3·143	3·707	5·208	5·959
7	·2632	·5491	·7111	·8960	·119	·415	1·895	·365	2·998	3·499	4·785	5·408
8	·2619	·5459	·7064	·8889	·108	·397	1·860	·306	2·896	3·355	4·501	5·041
9	·2610	·5435	·7027	·8834	·100	·383	1·833	·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	·2596	·5399	·6974	·8755	·088	·363	·796	·201	·718	3·106	4·025	·437
12	·2590	·5386	·6955	·8726	·083	·356	·782	·179	·681	3·055	3·930	·318
13	·2586	·5375	·6938	·8702	·079	·350	·771	·160	·650	3·012	3·852	·221
14	·2582	·5366	·6924	·8681	·076	·345	·761	·145	·624	2·977	3·787	·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	·2576	·5350	·6901	·8647	·071	·337	·746	·120	·583	·921	·686	4·015
17	·2573	·5344	·6892	·8633	·069	·333	·740	·110	·567	·898	·646	3·965
18	·2571	·5338	·6884	·8620	·067	·330	·734	·101	·552	·878	·610	3·922
19	·2569	·5333	·6876	·8610	·066	·328	·729	·093	·539	·861	·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	·2566	·5325	·6864	·8591	·063	·323	·721	·080	·518	·831	·527	·819
22	·2564	·5321	·6858	·8583	·061	·321	·717	·074	·508	·819	·505	·792
23	·2563	·5317	·6853	·8575	·060	·319	·714	·069	·500	·807	·485	·768
24	·2562	·5314	·6848	·8569	·059	·318	·711	·064	·492	·797	·467	·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	·2560	·5309	·6840	·8557	·058	·315	·706	·056	·479	·779	·435	·707
27	·2559	·5306	·6837	·8551	·057	·314	·703	·052	·473	·771	·421	·690
28	·2558	·5304	·6834	·8546	·056	·313	·701	·048	·467	·763	·408	·674
29	·2557	·5302	·6830	·8542	·055	·311	·699	·045	·462	·756	·396	·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	·2555	·5297	·6822	·8530	·054	·309	·694	·037	·449	·738	·365	·622
34	·2553	·5294	·6818	·8523	·052	·307	·691	·032	·441	·728	·348	·601
36	·2552	·5291	·6814	·8517	·052	·306	·688	·028	·434	·719	·333	·582
38	·2551	·5288	·6810	·8512	·051	·304	·686	·024	·429	·712	·319	·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	·2547	·5278	·6794	·8489	·047	·299	·676	2·009	·403	·678	·261	·496
60	·2545	·5272	·6786	·8477	·045	·296	·671	2·000	·390	·660	·232	·460
120	·2539	·5258	·6765	·8446	·041	·289	·658	1·980	·358	·617	·160	·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291