

IMPERIAL COLLEGE LONDON

E4.03
AS5
SO10
ISE4.3

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

MSc and EEE/ISE PART IV: MEng and ACGI

Corrected Copy

MOBILE RADIO COMMUNICATION

Wednesday, 4 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	M.K. Gurcan
	Second Marker(s) :	K.K. Leung

Special Instructions for Invigilators: None

Information for candidates :

Q function values

z	Q(z)
0	0.5
0.0251	0.49
0.0502	0.48
0.0753	0.47
0.1004	0.46
0.1257	0.45
0.151	0.44
0.1764	0.43
0.2019	0.42
0.2275	0.41
0.2533	0.4
0.2793	0.39
0.3055	0.38
0.3319	0.37
0.3585	0.36
0.3853	0.35
0.4125	0.34
0.4399	0.33
0.4677	0.5

z	Q(z)
0.4677	0.32
0.4959	0.31
0.5244	0.3
0.5534	0.29
0.5828	0.28
0.6128	0.27
0.6433	0.26
0.6745	0.25
0.7063	0.24
0.7388	0.23
0.7722	0.22
0.8064	0.21
0.8416	0.2
0.8779	0.19
0.9154	0.18
0.9542	0.17
0.9945	0.16
1.0364	0.15
1.0803	0.14

z	Q(z)
0.9945	0.16
1.0364	0.15
1.0803	0.14
1.1264	0.13
1.175	0.12
1.2265	0.11
1.2816	0.1
1.3408	0.09
1.4051	0.08
1.4758	0.07
1.5548	0.06
1.6449	0.05
1.7507	0.04
1.8808	0.03
2.0537	0.02
2.3263	0.01

1)

- a) An in-building radio system operates at the transmission frequency $f_c = 900$ MHz, and has cells of radius 100 m. Assuming free-space path loss model, and non-directional antennas at the transmitters and receivers, answer the following
- i) What is the required transmit power at the base station such that all terminals within the cell receive a minimum power of 10 μ W? [3]
- ii) How will this change if the system frequency is 5 GHz? [2]

- b) Over an indoor radio transmission link, the terms $P_{t,\text{dBm}}$ and $P_{r,\text{dBm}}$ represent the dB power levels for the transmitted and received signals. At distance d_i , the simplified path loss model estimates the received signal power in dB from

$$P_{r,\text{dBm}} = P_{t,\text{dBm}} + K - 10 \cdot \gamma \cdot \log_{10}(d_i)$$

where $K = 20 \log_{10}(\lambda/4\pi)$ is the free space path loss at unit distance, and γ is the path loss exponent.

Consider the set of empirical measurements of $P_{r,\text{dBm}} - P_{t,\text{dBm}}$ given in the Table 1.1 for the system at 900 MHz.

Distance, d_i , from transmitter	$P_{r,\text{dBm}} - P_{t,\text{dBm}}$
10 m	-70 dB
20 m	-75 dB
30 m	-90 dB
100 m	-100 dB
300 m	-125 dB

Table 1.1

The minimum-mean-square-error (mmse) between the empirical dB power measurements and the dB power estimate for the simplified model is

$$F(\gamma) = \sum_{i=1}^5 [M_{\text{measured}}(d_i) - M_{\text{model}}(d_i)]^2$$

Where $M_{\text{measured}}(d_i)$ is the path loss measurements given in Table 1.1 and $M_{\text{model}}(d_i) = K - 10 \cdot \gamma \cdot \log_{10}(d_i)$ at distance d_i .

- i) By differentiating the mmse with respect to γ , find the path loss exponent which minimises the mmse. [8]

Question continued over

ii) With this path loss exponent and a transmit power of 1 mW (0dBm), calculate the received power at 150 m for the simplified path loss model. [2]

c) Consider a cellular system operating at 900 MHz where propagation follows free space path loss (the path loss exponent $\gamma = 2$) with variations from log normal shadowing with $\sigma = 6$ dB. Suppose that for acceptable voice quality a signal-to-noise power ratio of $SNR_{\min} = 15$ dB is required at the mobile. Assume the base station transmits at $P_t = 1$ W (30 dBm) and its antenna has a gain of $G_{t,dB} = 10 \log_{10} G_t = 3$ dB. There is no antenna gain at the mobile and the receiver noise in the bandwidth of interest is -70 dBm.

Under path loss and shadowing, outage probability $P_{out}(P_{\min}, d)$ is defined to be the probability that the received power, $P_r(d)$, at a given distance, d , falls below P_{\min} , i.e., $P_{out}(P_{\min}, d) = p(P_r(d) \leq P_{\min})$. For the combined path loss and shadowing

$$p(P_r(d) \leq P_{\min}) = 1 - Q\left(\frac{P_{\min,dBm} - (P_{t,dBm} + K - 10 \cdot \gamma \cdot \log_{10}(d))}{\sigma}\right)$$

where the Q function (see useful information section on page 1) is defined as

$$Q(z) = \frac{1}{2} \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

The term $K = 20 \log_{10}(\sqrt{G_t} \lambda / 4\pi)$ is the free space path loss at reference distance unity.

Find the maximum cell size so that a mobile on the cell boundary will have acceptable voice quality 90% of the time. [7]

d) Consider a channel with Rayleigh fading and average received power $P_r = 20$ dBm.

Find the probability that the received power is below 10 dBm. [3]

- 2) a) The multipath power delay profile, $A_c(\tau)$, is the autocorrelation function of the multi-path channel impulse response. The average delay spread, μ_{T_m} , and rms delay spread, σ_{T_m} are defined in terms of the power delay profile as

$$\mu_{T_m} = \frac{\int_0^{\infty} \tau A_c(\tau) d\tau}{\int_0^{\infty} A_c(\tau) d\tau} \quad \text{and}$$

$$\sigma_{T_m} = \sqrt{\frac{\int_0^{\infty} (\tau - \mu_{T_m})^2 A_c(\tau) d\tau}{\int_0^{\infty} A_c(\tau) d\tau}}$$

A linearly modulated waveform with symbol period, T_s , experiences significant inter-symbol-interference (ISI) if $T_s \ll \sigma_{T_m}$.

- i) The multipath power delay profile is modelled as a one-sided exponential distribution:

$$A_c(\tau) = \frac{1}{\bar{T}_m} e^{-\frac{\tau}{\bar{T}_m}}, \quad \tau \geq 0$$

Show that the average delay spread is $\mu_{T_m} = \bar{T}_m$. Find the rms delay spread σ_{T_m} . [3]

- ii) Consider a wideband channel with power delay profile

$$A_c(\tau) = \begin{cases} e^{-\frac{\tau}{0.00001}}, & 0 < \tau \leq 20 \mu\text{sec} \\ 0 & \text{else} \end{cases}$$

(01) Find the mean and rms delay spreads of the channel. [3]

(02) Find the maximum symbol rate such that a linearly modulated signal transmitted through this channel does not experience ISI. [2]

- iii) A linearly modulated signal is transmitted through a channel which experiences severe ISI. A maximum likelihood sequence detector (MLSD) is used as the decision device.

Explain how the channel impulse response rotation method can be used to maximise the minimum distance of the MLSD receiver. [11]

Question continued over

b) Consider a deferred-first transmission Aloha system.

- i) Suppose that the transmission rate is $R = 10$ Mbps and packets consist of 1000 bits. [2]

For what packet arrival rate λ will the system achieve the maximum throughput.

What is the effective data rate associated with this throughput?

- ii) Explain how the joint-drift analysis is used to stabilise the system to operate at the maximum throughput. [4]

- 3) a) The SIR for a CDMA uplink with non-orthogonal codes under the standard Gaussian assumption is given as $SIR = 3G/(K - 1)$. Where $G=128$ is the ratio of spread bandwidth to signal bandwidth, and K is the number of users. Assume that the bandwidth of the information signal prior to spreading is $B_s = 9.765$ KHz.
- i) Calculate how many users the CDMA uplink can support if the required SINR on a channel is 10 dB. [3]
 - ii) How many could be supported within the same total bandwidth for an FDMA system? Comment on the result. [2]
- b) Consider the third generation wideband UTRA/FDD radio system and describe how the User Equipment, when it is first turned on, identifies the random access physical channel parameters using the following channels: [10]
- i) the Primary Synchronization Channel,
 - ii) the Secondary Synchronization Channel,
 - iii) the Pilot Channel,
 - iv) the Primary Common Control Physical Channel (PCCPCH), and
 - v) the Secondary Common Control Physical Channel (SCCPCH).
- c) Consider a mobile system downlink channel, with additive white Gaussian noise, used as the downlink broadcast channel with total transmission power limited to 20 mW. The downlink bandwidth is limited to 150 kHz and will be used to broadcast messages to 2 users with each having unity link path gain. The noise power spectral densities for the link 1 and 2 are $7 \cdot 10^{-9}$ W/Hz and $2 \cdot 10^{-8}$ W/Hz respectively. Suppose user 1 requires a data rate of 310 kbps. Using the Corporate broadcast method, find the data rate allocated to user 2 under the following allocation schemes:
- i) Fixed power time division, [4]
 - ii) Equal bandwidth frequency division, and [3]
 - iii) Superposition coding. [3]

4. a) The parameters, used for a service in the third generation FDD wideband UTRA/FDD radio system downlink (DL), are shown in Table 4.1.

	Service 1, DCH#1
Transport block size	164 bits
TTI	40 ms
Bit rate	4.1 kbps
CRC	16 bits
Coding	Convolutional rate 1/3

Table 4.1 parameters for the 4.1 kbps data service.

Describe how a 4.1 kbps data service can be mapped to the downlink dedicated physical channel. [6]

- b) Consider an ad hoc wireless network with three users. Users 1 and 2 require a received SINR of $\gamma_1^* = \gamma_2^* = 7$ dB whereas user 3 requires an SINR of $\gamma_3^* = 10$ dB. Assume all receivers have the same noise power $n_k = 1$, for $k=1,2,3$, and there is no processing gain to reduce interference, i.e. $\rho = 1$. Assume a matrix of gain values, indexed by the transmitter receiver numbers, is given by

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 0.85, & 0.05, & 0.03 \\ 0.08, & 0.95, & 0.116 \\ 0.054, & 0.034, & 0.75 \end{bmatrix}$$

The SINR at the intended receiver on link k is given by

$$\gamma_k = g_{kk} P_k / \left(n_k + \rho \sum_{j \neq k} g_{kj} P_j \right) \text{ where } P_k > 0 \text{ is the transmission power for}$$

user k . Assume that $\mathbf{P} = [P_1 \ P_2 \ P_3]^T$ is the column vector for the

transmission powers, $\mathbf{u} = [\gamma_1^* n_1 / g_{11}, \gamma_2^* n_2 / g_{22}, \gamma_3^* n_3 / g_{33}]^T$ is the column

vector of noise powers scaled by the SIR constraints and channel gains,

and \mathbf{F} is matrix with $F_{kj} = \begin{cases} 0 & k = j \\ \gamma_k^* g_{kj} \rho / g_{kk} & k \neq j \end{cases}$ with $k, j \in \{1, 2, 3\}$.

- i) Confirm that the vector equation $(\mathbf{I} - \mathbf{F})\mathbf{P} \geq \mathbf{u}$ is equivalent to the SINR constraints of each user. [8]

- ii) Show that a feasible power vector exists for this system such that all users achieve their desired SINRs. [4]

Question continued over

- c) In IEEE 802.11a standard for the wireless LANs, $N = 64$ subcarriers are generated over 20MHz bandwidth (equal to the sampling rate $1/T_s$). Only 48 subcarriers are actually used for data transmission, with the outer 12 subcarriers not used to minimize interference, and 4 subcarriers are used as pilot symbols for channel estimation. The cyclic prefix consists of $\mu = 16$ samples, so the total number of samples associated with each OFDM symbol, including both data samples and the cyclic prefix, is 80. The same code and modulation are used for *all* the subcarriers at any given time. The error correction code is a convolutional code with one of three possible coding rates: $r = 1/2$, $2/3$, or $3/4$. The modulation types that can be used on the subchannels are BPSK, QPSK, 16QAM, or 64QAM.
- i) What is the bandwidth for each subchannel? [1]
 - ii) What is the maximum delay spread for which ISI is removed? [1]
 - iii) What is the symbol time per subchannel? [1]
 - iv) What is the minimum data rate corresponding to BPSK? [1]
 - v) What is the maximum data rate that can be transmitted? [1]
 - vi) Find the data rate by assuming 16 QAM modulation and rate $2/3$ coding is used. [2]

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question 1 Page 1 out of 2

Question labels in left margin

Marks allocations in right margin

1-a. i we must find the transmit power such that the terminals at the cell boundary receive the minimum required power. we obtain a formula for the required transmit power

$$P_T = P_r \left(\frac{4\pi d}{\sqrt{G_T} \lambda} \right)^2 \quad \text{where } G_T = 1, \lambda = \frac{c}{f_c} = 0.33 \text{ m}, d = 10 \text{ m}$$

$$\text{and } P_r = 10 \mu\text{W}$$

$$P_T = 1.45 \text{ W} = 1.61 \text{ dBW} \quad \text{and} \quad 10 \log_{10} \left(\frac{P_T}{0.001} \right) \text{ dBm}$$

$$= 10 \log_{10} \left(\frac{1.45}{0.001} \right) = 31.61 \text{ dBm}$$

1-a. ii At 5 GHz $\lambda = 0.06$ so $P_T = 43.9 \text{ kW} \Rightarrow 10 \log_{10} \left(\frac{43.9 \times 10^3}{0.001} \right) = 76.42 \text{ dBm}$

1-b) $K = 20 \log_{10} \left(\frac{0.333}{4\pi} \right) = -31.54$

$$F(x) = (-70 + 31.54 + 10x)^2 + (-75 + 31.54 + 13.01x)^2 + (-90 + 31.54 + 16.79x)^2$$

$$+ (-110 + 31.54 + 20x)^2 + (-125 + 31.54 + 24.77x)^2$$

$$= 21676.3 - 11654.9x + 1571.47x^2$$

Differentiating $F(x)$ relative to x and setting it to zero

$$\frac{\partial F(x)}{\partial x} = -11654.9 + 3142.94x = 0 \Rightarrow x = 3.71$$

1-b) ii with $K = -31.54$, $x = 3.71$ and $P_r = 0 \text{ dBm}$ we have

$$P_{T, \text{dBm}} = P_{r, \text{dBm}} + K - 10 \log_{10}(d) = 0 - 31.54 - 10 \times 3.71 \times \log_{10}(150) = -112.27 \text{ dBm}$$

1-c

$$1 - Q(z) = 0.1 \Rightarrow Q(z) = 0.9$$

$$2Q(z) = \text{erfc} \left(\frac{z}{\sqrt{2}} \right) = 1 - \text{erf} \left(\frac{z}{\sqrt{2}} \right)$$

$$\text{erf} \left(\frac{z}{\sqrt{2}} \right) = 1 - 2Q(z) = 1 - 2 \times 0.9 = 1 - 1.8 = -0.8$$

$$\text{erf} \left(\frac{z}{\sqrt{2}} \right) = -0.8 \Rightarrow \frac{z}{\sqrt{2}} = -0.9062$$

$$\therefore z = \sqrt{2} \times (-0.9062) = -1.2816$$

$$Q(z) = Q(-1.2816) = Q \left(\frac{P_{\min} - P_T - K + 20 \log_{10} d}{6} \right)$$

$$P_{\min} - (-70) = 15 \Rightarrow P_{\min} = -70 + 15 = -55$$

$$K = 20 \log_{10} \left(\frac{\sqrt{2} \times 0.33}{4\pi} \right) = -28.6$$

$$6 \times (-1.2816) = -55 - 30 + 28.6 + 20 \log_{10} d$$

$$d = 10^{\frac{48.71}{20}} = 272 \text{ m.}$$

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question / Page 2 out of 12

Question labels in left margin

Marks allocations in right margin

1d

 $P_r = 20 \text{ dBm} = 100 \text{ mW}$.

we want to find the probability that $z^2 < 10 \text{ dBm} = 10 \text{ mW}$
we have exponential pdf for the Rayleigh signal power

$$P(z^2 < 10) = \int_0^{10} \frac{1}{100} e^{-\frac{x}{100}} dx = 0.095$$

(3)

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question 2 Page 3 out of 14

Question labels in left margin

Marks allocations in right margin

2.a.i

$$\int_0^{\infty} A_c(\tau) d\tau = \int_0^{\infty} \frac{1}{T_m} \exp\left(-\frac{\tau}{T_m}\right) d\tau = 1$$

$$\mu_{T_m} = \frac{1}{T_m} \int_0^{\infty} \tau \exp\left(-\frac{\tau}{T_m}\right) d\tau = T_m$$

$$\sigma_{T_m}^2 = \frac{1}{T_m} \int_0^{\infty} \tau^2 \exp\left(-\frac{\tau}{T_m}\right) d\tau - \mu_{T_m}^2 = 2T_m - T_m^2 = T_m^2$$

2.a.ii

$$\mu_{T_m} = \frac{\int_0^{20 \times 10^{-6}} \tau \exp(-\tau/0.0001) d\tau}{\int_0^{20 \times 10^{-6}} \exp(-\tau/0.0001) d\tau} = 6.87 \mu\text{sec}$$

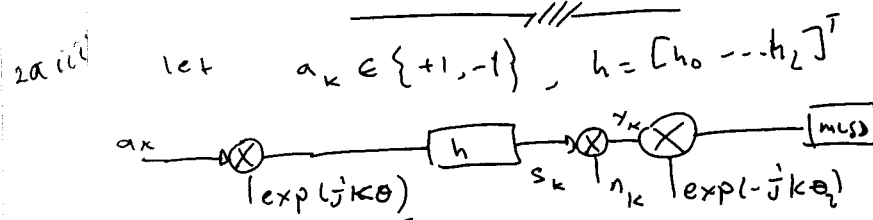
$$\sigma_{T_m} = \sqrt{\frac{\int_0^{20 \times 10^{-6}} (\tau - \mu_{T_m})^2 \exp(-\tau) d\tau}{\int_0^{20 \times 10^{-6}} \exp(-\tau) d\tau}} = 5.25 \mu\text{sec}$$

we see that the mean delay is roughly equal to its rms.

To avoid ISI we require that $T_s > 10\sigma_{T_m}$

which gives

$T_s = 52.5 \mu\text{s}$ or symbol rate $R_s = 1/T_s = 19.04 \text{ kpsps}$.



at the output of the channel

$$s_k = a_k^T h$$

where $a_k = [a_{k,0}, a_{k,1}, a_{k,2}, \dots, a_{k,L-1}]^T$

$$y_k = s_k + n_k \quad \text{with} \quad E(n_k^2) = \frac{N_0}{2}$$

For MLD

$$\hat{a} = \arg \max_a P(y|a)$$

let

$$s_i = [a_{i,0}, a_{i,1}, a_{i,2}, \dots, a_{i,L-1}]^T \quad i = 0, \dots, 2^L - 1$$

output symbol

$$s_{i,j,k} = [a_{i,j,k}, s_i^T] \cdot h$$

$$b_{M_{i,j,k}} = |y_k - s_{i,j,k}|^2 = |y_k - [a_{i,j,k}, s_i^T] h|^2$$

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question 2 Page 4 out of 13

Question labels in left margin

Marks allocations in right margin

The branch metric comprises of three terms
 the received symbol y_k ,
 the data vector $[a_i, s_i^T]$ and
 the detection response h
 let the error vectors be

$$e_1 = [-2]^T$$

$$e_2 = [-2, +2]^T$$

$$e_3 = [-2, +2, -2]^T$$

$$e_4 = [-2, -2]^T$$

$$e_5 = [-2, -2, -2]^T$$

$$e_6 = [-2, 0, +2]^T$$

$$e_7 = [+2, 0, +2]^T$$

$$e_8 = [-2, +2, +2]^T$$

$$e_9 = [-2, -2, +2]^T$$

The error event vector is $d_i = e_i * h$, the squared
 error event distance is $d_i^2 = |d_i|^2$

and the minimum distance is given by

$$d_{\min, h}^2 = \min_i (d_i^2) \quad i = 1 \dots N$$

$$d_{\min, h}^2 = \min_i h^H E_i^T E_i h \quad \text{for } i = 1 \dots N$$

where

$$E_i = \begin{bmatrix} e_{i,1} & 0 & 0 & 0 & 0 & 0 \\ e_{i,2} & e_{i,1} & & & & \\ \vdots & & e_{i,1} & & & \\ e_{i,p+1} & & & & & -e_{i,1} \\ 0 & & & & & e_{i,p+1} \end{bmatrix}$$

where $e_i = [e_{i,1}, \dots, e_{i,p+1}]^T$

The BER for the MCD

$$P_e \approx KQ\left\{\sqrt{\frac{d_{\min, h}^2}{4E_{\min}^2}}\right\}$$

(2)

(2)

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question 2 Page 5 out of 13

Question labels in left margin

Marks allocations in right margin

$$\gamma_{\min} = \frac{d_{\min,h}^2}{4 \|h\|^2}$$

if we multiply a_k with $\exp(jk\theta_2)$ the received signal becomes

$$r_k = \exp(jk\theta_2) a_k^T S_{0,L}^{\theta_2} h$$

where

$$S_{0,L}^{\theta_2} = \begin{bmatrix} \exp(-j\theta_2) & 0 & 0 & 0 & 0 \\ 0 & \exp(j2\theta_2) & 0 & 0 & 0 \\ 0 & & \ddots & & \\ 0 & & & \ddots & \\ 0 & & & & \exp(jL\theta_2) \end{bmatrix}$$

The rotated received signal becomes

$$r_k = r_k \exp(-jk\theta_2) = a_k^T S_{0,L}^{\theta_2} h + n_k \exp(-jk\theta_2)$$

The branch metric is modified to

$$b_{m,i,j,k} = |r_k - [a_{i,j,k} \ S_i^T] S_{0,L}^{\theta_2} h|^2$$

The detection response is modified to

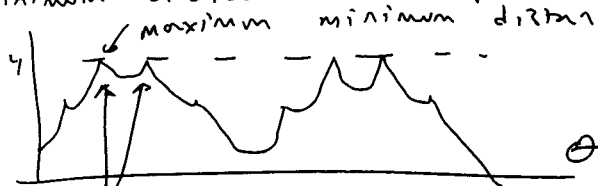
$S_{0,L}^{\theta_2} h$ and the minimum distance as a function θ_2

$$d_{\min}^2(\theta) = \min_i h^T S_{0,L}^{\theta_2} E_i^T E_i S_{0,L}^{\theta_2} h$$

Simplifications lead to

$$d_{\min}^2(\theta_2) = \min_i \left\{ |e_i^T h|^2 + 2 \sum_{n=1}^{P_L+1} \sum_{a=0}^{L-1} \sum_{b=a+1}^L \operatorname{Re} \{ h_a^* h_b e_{i,n-a} e_{i,n-b} \exp(j(b-a)\theta) \} \right\}$$

minimum distance as a function of θ_2



operate system at these angles.

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question ~~1~~ 2 Page 6 out of 12

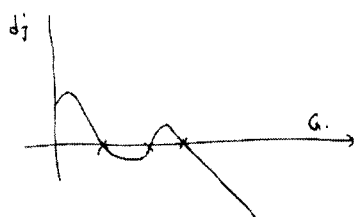
Question labels in left margin

Marks allocations in right margin

2b) The throughput T is maximised for $L = \lambda T = 1$, where λ is the packet arrival rate and τ is the packet duration. With 10 Mbps transmission rate and 1000 bits/packet, $\tau = 1000 / 10^7 = 0.1 \text{ ms}$. Thus $\lambda = L / \tau = 1 / 10^{-4} = 10^4$ packets/second maximises throughput. The throughput for $L=1$ is $T=0.37$ so the effective data rate is $TR = 3.7 \text{ Mbps}$.

Joint drift: dj

$$dj = dn - G \cdot ds$$



$$dj(h=0) = \lambda$$

$$\lim_{G \rightarrow \infty} dj(G) = -\infty$$

$\therefore dj$ has at least one, at most three roots. Roots with positive going slopes are the settling points. In order to avoid bistable operation, we must have one root with positive going slope at the optimum operating point.

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question 3 Page 7 out of 12

Question labels in left margin

Marks allocations in right margin

3a. To determine how many users can be supported we invert the SIR expression to get

$$K \leq \frac{3G}{SIR} + 1 = \frac{256}{20} + 1 = 39.4$$

and since K must be an integer, the system can support 39 users.

In FDMA we have

$$K = \frac{1.25 \times 10^6}{9.765 \times 10^3} = 128$$

So the total bandwidth of 1.25 MHz can support 128 channels of 9.765 kHz. This calculation implies that FDMA is three times more efficient than non-orthogonal CDMA under the standard Gaussian assumption code cross correlation.

3.b

The primary synchronisation channel transmits the synchronisation code at every base station.

The handset primary synchronisation code and finds the strongest peak. This peak identifies the slot time.

In order to find the frame timing the handset cross correlates 64 synchronisation group codes to identify the scrambling codes families used on the pilot and common broadcast channels.

Once the scrambling code family is identified, using each one of 8 scrambling codes belonging to the family and also the data transmitted on the pilot channel the handset cross correlates the received signal against the signature waveform. The scrambling code highest correlation identifies the scrambling code used in the cell.

Using this scrambling code, the handset cross correlates the signature waveform against the received signal to decode the data on the broadcast channel

(3)

(2)

(10)

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question 3 Page 8 out of 12

Question labels in left margin

Marks allocations in right margin

From this data, handset identifies the signature waveforms used on the Forward Access channel, paging channel, random access channel and acquisition indication channels.

The handset randomly selects one of the signature waveforms and transmits over the random access channel. Successfully received random access preambles are acknowledged on the indication channel.

When the handset receives the acknowledgement it transmits a message to inform the base station what it wants to do.

The base station uses forward channel to inform the handset which physical channels are assigned to the mobile to start the two way communication.

3.6
4

user 1 has a rate of $R_1 = \tau B \log_2 \left(1 + \frac{P}{\eta_1 B}\right)$

$$\text{where } P = 20 \times 10^{-3} \text{ W} \\ \eta_1 = 7 \times 10^{-9} \text{ W/Hz} \\ B = 150 \times 10^3 \text{ Hz}$$

$$R_1 = 310 \text{ kbps}$$

$$R_1 = \tau B \log_2 \left(1 + \frac{P}{\eta_1 B}\right) = \tau \cdot 150 \times 10^3 \log_2 \left(1 + \frac{20 \times 10^{-3}}{7 \times 10^{-9} \times 150 \times 10^3}\right) = 310 \times 10^3$$

$$= 6.48807 \times 10^5 \tau = 310 \times 10^3$$

$$\therefore \tau = 0.4778$$

For user 2 the rate is

$$R_2 = (1 - \tau) B \log_2 \left(1 + \frac{P}{\eta_2 B}\right) = 230 \text{ kHz}$$

80

In equal bandwidth frequency division we require

$$R_1 = 0.5 B \log_2 \left(1 + \frac{P_1}{\eta_1 B}\right) = 310 \cdot 10^3 \text{ bps. solving this for } P_1$$

$$P_1 = 0.5 \eta_1 B (2^{(R_1/B)} - 1) \text{ yields } P_1 = 0.0087$$

$$\text{setting } P_2 = P - P_1 = 20 \times 10^{-3} - 8.7 \times 10^{-3} = 11.3 \times 10^{-3} \text{ W}$$

$$\text{we get } R_2 = 0.5 B \log_2 \left(1 + \frac{P_2}{\eta_2 B}\right) = 232 \times 10^3 \text{ bps}$$

(4)

(3)

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question 3 Page 9 out of 12

Question labels in left margin

Marks allocations in right margin

3-C-ii

with superposition coding we have

$$R_1 = B \log_2 \left(1 + \frac{P_1}{n_1 B} \right) = 310 \times 10^5 \text{ bps}$$

gives

$$P_1 = n_1 B (2^{R_1/B} - 1) \text{ yields } P_1 = 3.3 \times 10^{-3} \text{ W}$$

Then

$$R_2 = B \log_2 \left(1 + \frac{P - P_1}{n_2 B + P_1} \right) = 278.6 \times 10^5 \text{ bps.}$$

—//—

(3)

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

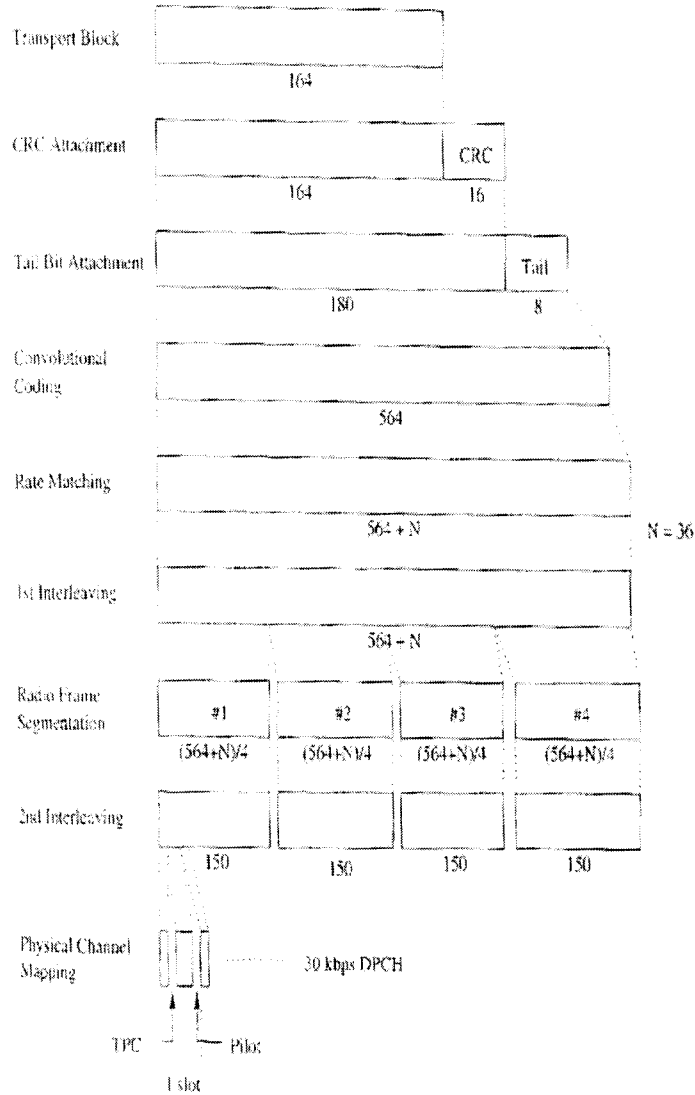
Second Examiner: Leung, K.K.

Question ~~10~~ 4 Page 10 out of 12

Question labels in left margin

Marks allocations in right margin

4-a



6

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question 4 Page 11 out of 12

Question labels in left margin

Marks allocations in right margin

4b

We have SINR at the input of the decision device

$$\gamma_i = \frac{p_i g_{ii}}{e \sum_{j \neq i} p_j g_{ij} + n_i} \quad \text{where } \gamma_i > \gamma_i^* \leftarrow \text{required SNIR}$$

hence

$$\frac{p_i g_{ii}}{e \sum_{j \neq i} p_j g_{ij} + n_i} > \gamma_i^*$$

re organising the above equation

$$p_i - \sum_{j \neq i} p_j \frac{e \gamma_i^* g_{ij}}{g_{ii}} = \frac{n_i \gamma_i^*}{g_{ii}}$$

$$\text{let } u = \left[\frac{\gamma_1^* n_1}{g_{11}}, \frac{\gamma_2^* n_2}{g_{22}}, \dots, \gamma_N^* \right]^T$$

$$F \text{ is a matrix with } F_{ij} = \begin{cases} \frac{e \gamma_i^* g_{ij}}{g_{ii}} & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

in matrix form if

$(I-F)P > u$ and if the maximum eigenvalue of F is less than unity we have power allocations which satisfy the required SNIR values

4bii

$$G = \begin{bmatrix} 0.85 & 0.05 & 0.03 \\ 0.08 & 0.95 & 0.116 \\ 0.054 & 0.034 & 0.75 \end{bmatrix}$$

$$\gamma_{\text{th}}^* = [6, 7, 9]^T$$

$$\gamma_i^* = [3.9811, 5.0119, 7.9433]^T$$

$$F = \begin{bmatrix} 0 & 0.2342 & 0.1405 \\ 0.4221 & 0 & 0.6120 \\ 0.5719 & 0.3601 & 0 \end{bmatrix} \Rightarrow (I-F)P = u$$

gives

$$P = [11.4358, 22.108, 20.4588]^T$$

(7)

(1)

(4)

Confidential

MODEL ANSWERS and MARKING SCHEME

First Examiner: Gurcan, M.K.

Paper Code : E4.03, SO10, ISE4.3

Second Examiner: Leung, K.K.

Question 4 Page 12 out of 12

Question labels in left margin

Marks allocations in right margin

A/c-

Since the bandwidth B is 20 MHz and 64 subcarriers evenly spaced over the bandwidth, the subcarrier bandwidth is

$$B_N = \frac{20 \text{ MHz}}{64} = 312.5 \text{ kHz}$$

Since $\mu = 16$ and $1/T_s = 20 \text{ MHz}$ the spread for which ISI is removed

$$T_m < \mu T_s = \frac{16}{20 \times 10^6} = 4 \mu\text{sec}$$

which corresponds to delay spread in an indoor environment.

Including both the OFDM symbol and cyclic prefix, there are $16 + 64 = 80$ samples per OFDM symbol time so the symbol time

$$T_N = 80 T_s = \frac{80}{20 \times 10^6} = 4 \mu\text{s}$$

The rate per sub channel is $\log M/T_N$. The minimum data rate for this system, corresponding to BPSK (1 bit/symbol) a rate = $1/2$ code, and into account that only 48 subcarriers actually carry usable data, is given by

$$R_{\min} = 48 \text{ Subcarriers} \times \frac{1/2 \text{ bit}}{\text{code bit}} \times \frac{1 \text{ code bit}}{\text{Subcarrier Symbol}} \times \frac{1 \text{ Subcarrier Symbol}}{4 \times 10^{-6} \text{ seconds}} = 6 \text{ Mbps}$$

The maximum data rate that can be transmitted is

$$R_{\max} = 48 \text{ Subcarriers} \times \frac{3/4 \text{ bit}}{\text{code bit}} \times \frac{6 \text{ coded bits}}{\text{Subcarrier Symbol}} \times \frac{1 \text{ Subcarrier Symbol}}{4 \times 10^{-6} \text{ seconds}} = 54 \text{ Mbps.}$$

Naturally a wide range of data rates between these two extremes is possible

(2)