2018 Intro Signals and Communication
Exam answers

1. a. i.

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$= \int_{-a/2}^{a/2} \frac{1}{a} e^{-j\omega t} dt$$

$$= \frac{1}{a} \cdot \frac{1}{-j\omega} \cdot e^{-j\omega t} \Big|_{-a/2}^{a/2}$$

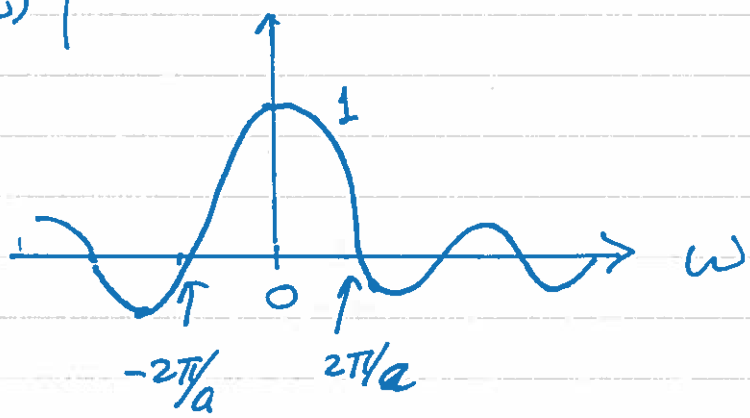
$$= \frac{-1}{ja\omega} \cdot \left[e^{-j\omega a/2} - e^{j\omega a/2} \right]$$

$$= \frac{1}{ja\omega} \cdot \left[\cos(\omega a/2) + j\sin(\omega a/2) - \cos(\omega a/2) + j\sin(\omega a/2) \right]$$

$$\Rightarrow F(\omega) = \frac{2\sin(\omega a/2)}{\omega a}$$

$$F(\omega) = \frac{\sin(\omega a/2)}{\omega a/2}.$$

ii. $|F(\omega)|$



1\. a. iii. $\hat{f}(t) = \lim_{a\to 0} f(t) = \delta(t)$

unit impulse

In that case, $\mathcal{F}[\delta(t)] = 1.$

iv. $$\hat{F}(\omega) = \lim_{a\to 0} F(\omega)$$

$$= \lim_{a\to 0} \frac{\sin(\omega a/2)}{\omega a/2}$$

$$= \left.\frac{\cos(\omega a/2)\cdot \omega/2}{\omega/2}\right|_{a=0}$$

$= 1.$ Same as in part iii.

1\. b. i. $$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-\infty}^{\infty} \left[a(t) + b(t) + c(t)\right]^2 dt$$

ii. $$E_x = \int_{-\infty}^{\infty} a^2(t)\,dt + \int_{-\infty}^{\infty} b^2(t)\,dt$$

$$+ \int_{-\infty}^{\infty} c^2(t)\,dt + 2\int_{-\infty} a(t)b(t)\,dt$$

$$+ 2\int_{-\infty}^{-\infty} b(t)\cancel{d(t)}\,c(t)\,dt + 2\int_{-\infty}^{\infty} c(t)\,a(t)\,dt$$

1. b. ii. $\Rightarrow$ $E_x = E_a + E_b + E_c$

$$+ 2\int_{-\infty}^{\infty} a(t)b(t)dt + 2\int_{-\infty}^{\infty} b(t)c(t)dt$$

$$+ 2\int_{-\infty}^{\infty} c(t)a(t)dt$$

A set of sufficient conditions for $E_x = E_a + E_b + E_c$ is that

$$\int_{-\infty}^{\infty} a(t)\, b(t)\, dt = 0$$

$$\int_{-\infty}^{\infty} b(t)\, c(t)\, dt = 0$$

and

$$\int_{-\infty}^{\infty} c(t)\, a(t)\, dt = 0$$

That is, $a(t)$, $b(t)$ and $c(t)$ are mutually orthogonal to each other.

iii. Given mutually orthogonal among $a(t)$, $b(t)$ and $c(t)$, we ca<u>nnot</u> always express

$$y(t) = \alpha\, a(t) + \beta\, b(t) + \gamma\, c(t)$$

because we are <u>not</u> sure if $a(t)$, $b(t)$ and $c(t)$ are complete i.e., $a(t)$, $b(t)$ and $c(t)$ represent all basis.

1. b. iv. If $y = \alpha a(t) + \beta b(t) + \gamma c(t) + \lambda d(t)$

then $a(t)$, $b(t)$, $c(t)$ and $d(t)$

must be mutually orthogonal to each

other and the signal components are

complete.

1. c. i. $$s(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n w_0 t) + b_n \sin(n w_0 t)$$

where $w_0 = \frac{2\pi}{T_0}$

ii. By inspection, a_0 (dc term):

$$a_0 = 1/2$$

We have $b_n = 0 \quad \forall\, n = 1, 2, \ldots$

because $s(t)$ is an even function in t.

$$a_n = \frac{2}{T_0} \cdot \int_{-T_0/4}^{T_0/4} s(t) \cos(n w_0 t)\, dt$$

$$= \frac{2}{T_0} \cdot \frac{\sin(n w_0 t)}{n w_0} \Big|_{-T_0/4}^{T_0/4}$$

$$= \frac{2}{\cancel{T_0}} \cdot \frac{\cancel{2} \sin(n w_0 \cdot T_0/4)}{n \cdot \frac{\cancel{2}\pi}{\cancel{T_0}}} \cdot$$

1. c. ii. $\Rightarrow \quad a_n = \frac{2}{n\pi} \cdot \sin\left(n \frac{2\pi}{T_0} \cdot \frac{T_0}{4}\right)$

$$a_n = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

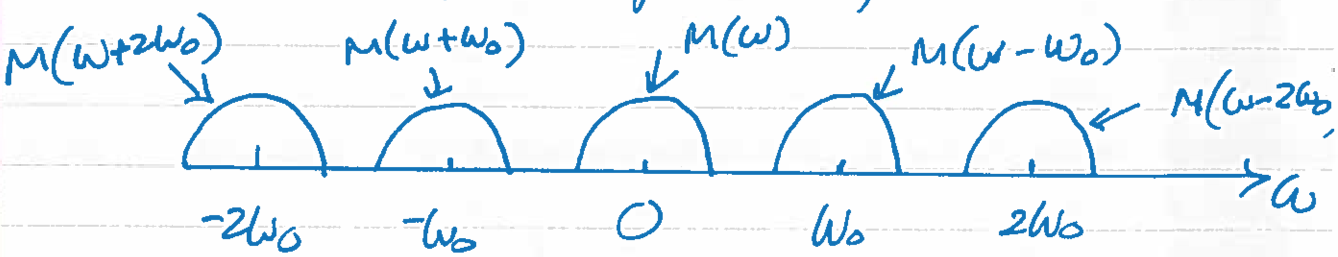
i.e., $\quad s(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos \omega_0 t - \frac{1}{3}\cos(3\omega_0 t) + \frac{1}{5}\cos(5\omega_0 t) + \cdots\right]$

iii. $\quad s(t)\,m(t)$

$$= m(t)\left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t - \frac{1}{3}\cos 3(\omega_0 t) + \frac{1}{5}\cos(5\omega_0 t) + \cdots\right]$$

$$= \left[\frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos(\omega_0 t) - \frac{1}{3}m(t)\cos(3\omega_0 t) + \frac{1}{5}m(t)\cos(5\omega_0 t) + \cdots\right]$$

iv. By inspection of the above expression, each $m(t)\cos(n\omega_0 t)$ represents a shift of $M(\omega)$ spectrum to the frequency $n\omega_0$. So, the spectrum of $s(t)m(t)$ is:



1. c. v. To obtain the AM signal $\phi(t)$,

we can use a bandpass filter on

$s(t)\,m(t)$ with center frequency $\omega_0 = \omega_c$.

That is, $\phi(t) = \frac{2}{\pi} \cdot m(t)\cos(\omega_c t)$

Since $\omega_0 = \omega_c = \frac{2\pi}{T_0}$, we have the relationship

between ω_c & T_0.

1. d. i. The PM signal

$$\phi(t) = A\cos[\omega_c t + k_p m(t)]$$

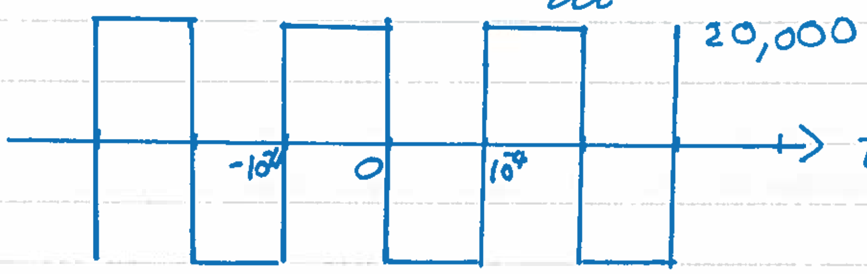
ii. The instantaneous angle is

$$\theta(t) = \omega_c t + k_p m(t)$$

Thus, the instantaneous frequency is

$$\omega_i(t) = \frac{d\theta(t)}{dt} = \omega_c + k_p \frac{dm(t)}{dt}$$

iii. For the given $m(t)$, $\frac{dm(t)}{dt}$ is



1. d. iii.

$$f_i(t) = f_c + \frac{k_p}{2\pi} \cdot \frac{dm(t)}{dt}$$

$$f_i(t)\Big|_{min} = f_c + \frac{k_p}{2\pi} \cdot (-20{,}000)$$

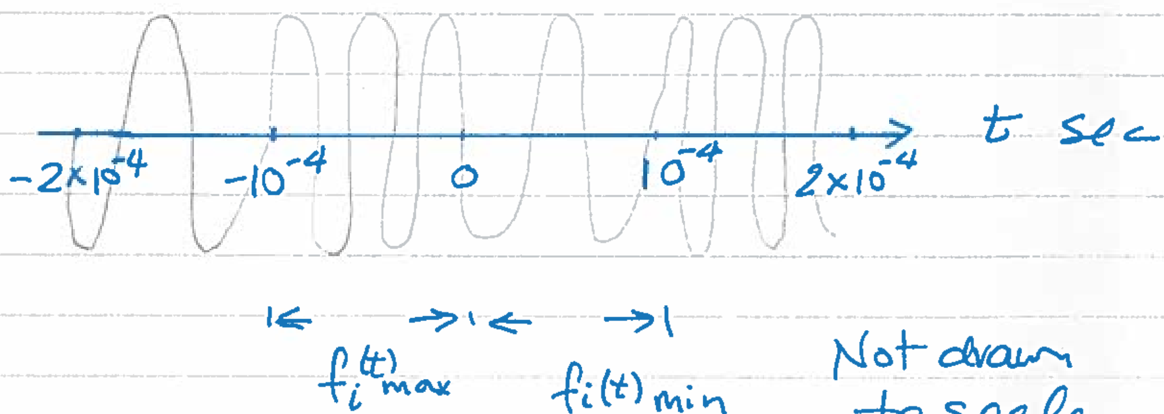
$$= 100\,M + \frac{10\pi}{2\pi}(-2 \times 10^4)$$

$$\Rightarrow \quad f_i(t)\Big|_{min} = 100\,M - 10^5$$

$$= 99.9 \text{ MHz}$$

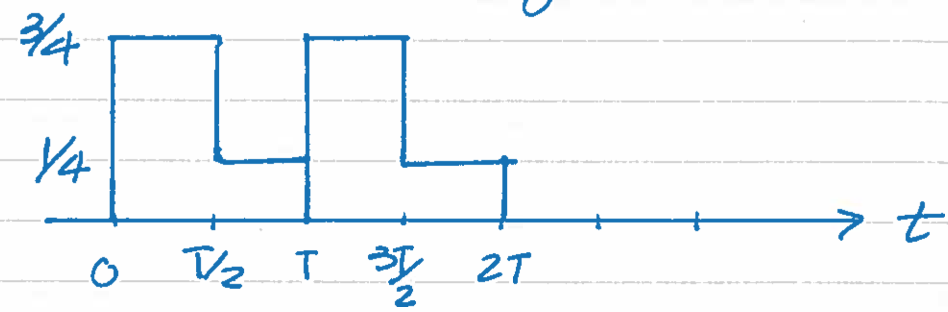
Similarly, $f_i(t)\Big|_{max} = 100.1$ MHz

iv.

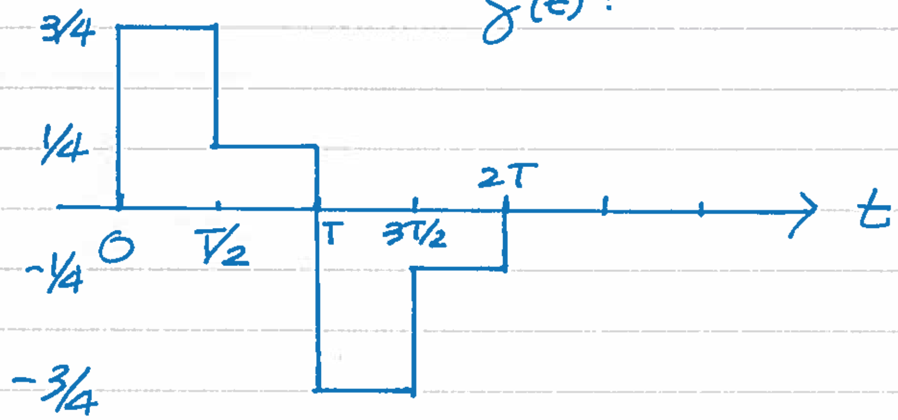


Not drawn to scale

2. a. i. $z(t)$:



ii. $z(t)$:



iii. If the pulse period $T = T_0$, results in parts i and ii reveals that a simple receiver can be used to detect if the received waveform corresponds to a 0 or 1. If $T < T_0$, the received waveform becomes corrupted and simple receivers will not be available to detect the transmitting bits easily.

That is, the maximum number of pulses per sec is $\frac{1}{T_0}$.

2. a. iv. Yes, it is possible to transmit more that $1/T_0$ pulses per second. In that case, sophisticated techniques (e.g., equalization) will be needed to remove the effects due to $\delta(t - T_0/2)$ in $h(t)$ in order to recover the transmitting bits properly.

2. b. i.

$$f(t) * g(t) = \int_{u=-\infty}^{\infty} f(u)\, g(t-u)\, du$$

ii. $\mathcal{F}[f(t) * g(t)]$

$$= \int_{t=-\infty}^{\infty} \int_{u=-\infty}^{\infty} f(u) g(t-u) e^{-j\omega t}\, du\, dt$$

$$= \int_{u=-\infty}^{\infty} \int_{t=-\infty}^{\infty} f(u)\, g(t-u)\, e^{-j\omega t}\, dt\, du$$

$$= \int_{u=-\infty}^{\infty} f(u) \cdot \int_{t=-\infty}^{\infty} g(t-u) e^{-j\omega t}\, dt\, du$$

2.6.ii. $\Rightarrow \mathcal{F}[f(t) * g(t)]$

$$= \int_{u=-\infty}^{\infty} f(u)\, e^{-j\omega u} \underline{\int_{t=-\infty}^{\infty} g(t-u)\, e^{-j\omega(t-u)}\, dt}\, du$$

$$= \int_{u=-\infty}^{\infty} f(u)\, e^{-j\omega u} \cdot G(\omega)\, du$$

$$= G(\omega) \cdot \int_{u=-\infty}^{\infty} f(u)\, e^{-j\omega u}\, du$$

$$\Rightarrow \mathcal{F}[f(t) * g(t)] = F(\omega) \cdot G(\omega).$$

iii. $$F(\omega) * G(\omega) = \int_{u=-\infty}^{\infty} F(u)\, G(\omega - u)\, du$$

iv. $\mathcal{F}^{-1}[F(\omega) * G(\omega)]$

$$= \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} \int_{u=-\infty}^{\infty} F(u)\, G(\omega - u) \cdot du \cdot e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi} \int_{u=-\infty}^{\infty} \int_{\omega=-\infty}^{\infty} F(u)\, G(\omega - u)\, e^{j\omega t}\, du\, d\omega$$

$$= \frac{1}{2\pi} \int_{u=\infty}^{\infty} F(u)\, e^{jut} \int_{\omega=\infty}^{\infty} G(\omega - u)\, e^{j(\omega - u)t} \cdot d\omega \cdot du$$

2.b. iv. $\Rightarrow \mathcal{F}^{-1}\left[F(\omega) * G(\omega)\right]$

$$= \int_{u=-\infty}^{\infty} F(u) \cdot e^{jut} \cdot g(t)\, du$$

$$= g(t) \cdot 2\pi \cdot f(t)$$

$$\Rightarrow \mathcal{F}^{-1}\left[F(\omega) * G(\omega)\right] = 2\pi \cdot f(t) \cdot g(t)$$

Therefore,

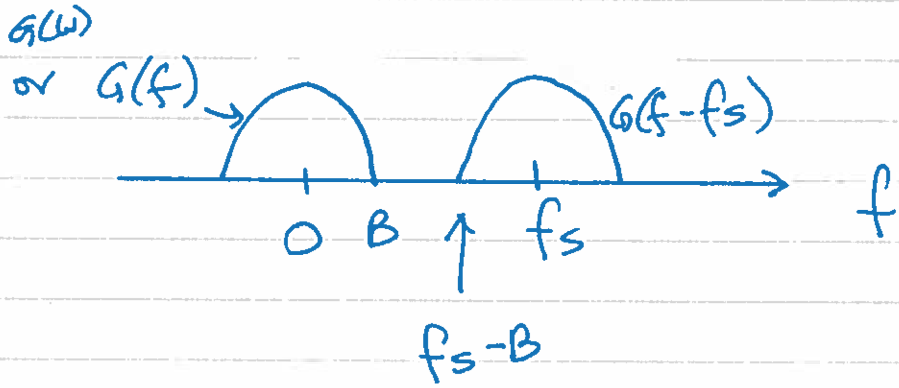
$$\mathcal{F}\left[f(t)\, g(t)\right] = \frac{1}{2\pi} F(\omega) * G(\omega).$$

3. a. iii. Given the maximum data rate for the link R, the maximum sampling rate supported by the link is $f_s = R/K$ samples/sec

By Nyquist sampling requirement,

$f_s \geq 2B$ where B is the bandwidth of $g(t)$

$$\Rightarrow \quad B \leq \frac{R}{2K} \text{ Hz.}$$



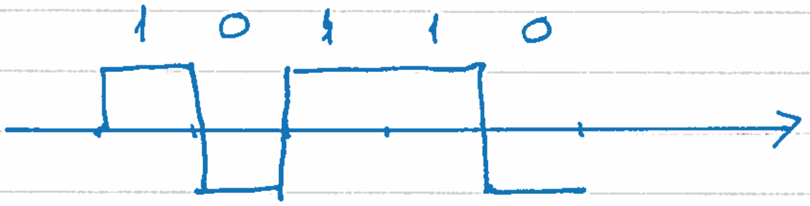
We must have $f_s - B \geq B$

to avoid overlap of the two replica of $G(\omega)$.

$$\Rightarrow \quad \underline{f_s \geq 2B}$$

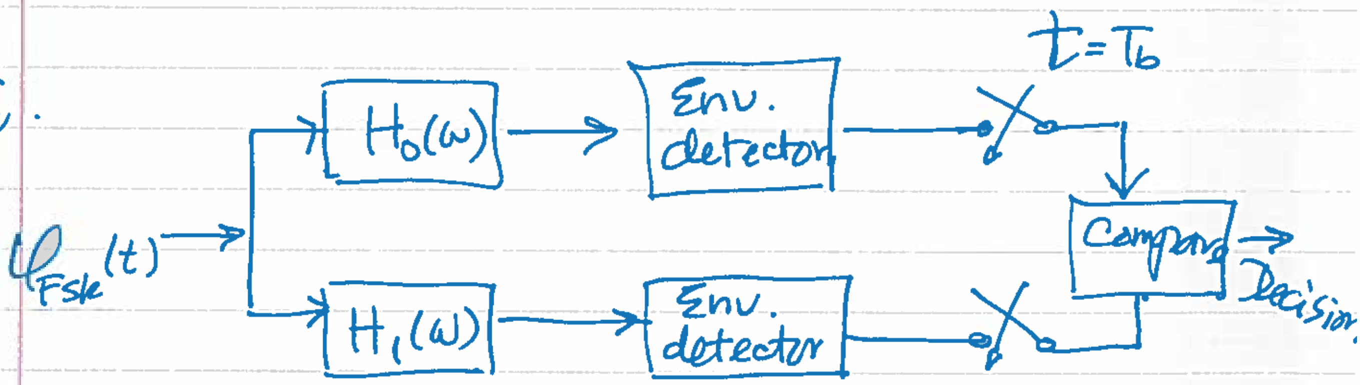
3.a. i. $\varphi_{FSK}(t) = \cos\left[\omega_c t + k_f \int m(t)\,dt\right]$

Since the polar non-return-to-zero line coding has the following waveform,



$\varphi_{FSK}(t)$ has two ~~instansta~~ instantaneous frequencies, which correspond to a 0 or 1 is being transmitted. Specifically, when 1 is sent, the frequency deviation is positive. Otherwise, the deviation is negative.

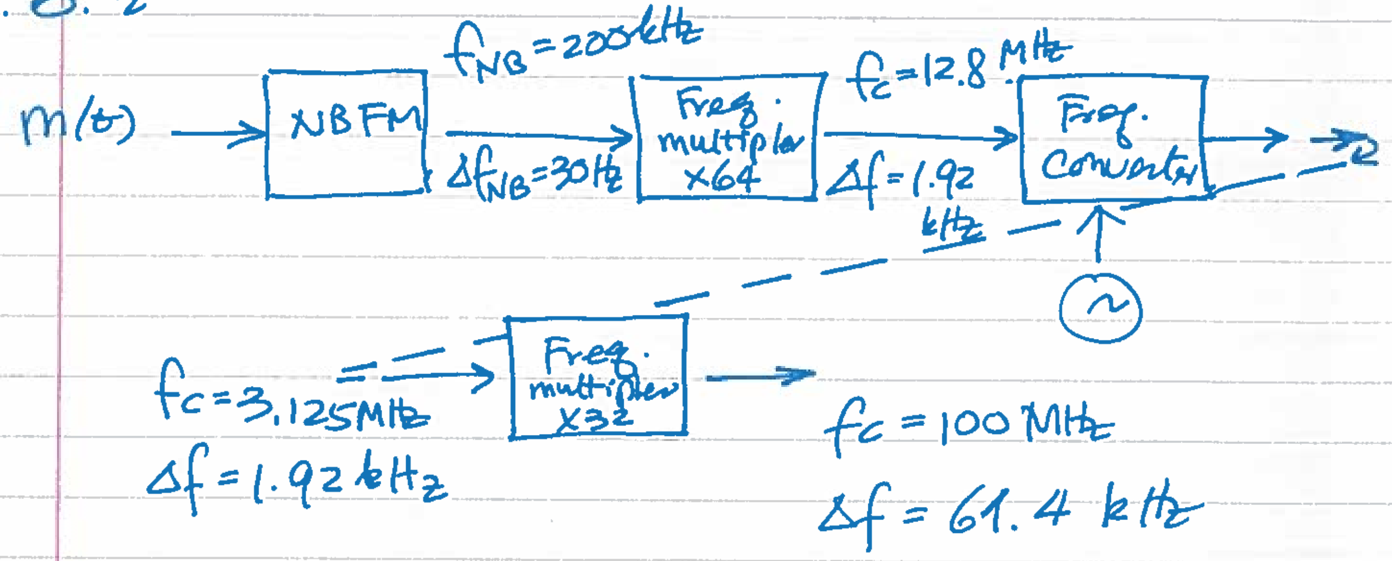
ii.



$H_0(\omega)$ is a filter to band pass the FSK waveform for the ~~[illegible]~~ negative frequency deviation. i.e., 0 is sent.

$H_1(\omega)$ is a band pass filter the waveform for +ve frequency deviation. i.e., 1 is sent.

3.6. i



ii. The second oscillator has a frequency of 9.675 MHz.

iii. The second oscillator (sinusoidal) is used to translate the carrier frequency from 12.8 MHz to 3.125 MHz before the last stage of frequency multipliers.

The frequency converter shifts the input carrier frequency from ω_1 to $\omega_1 - \omega_2$ by using the second oscillator frequency of ω_2.

That is, $\cos(\omega_1 t) \cdot \cos(\omega_2 t)$

$$= \frac{1}{2}\left[\cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t\right]$$

3.b.iii where $\omega_1 = 12.8\ M \times 2\pi$

$$\omega_1 - \omega_2 = 3.125\ M \times 2\pi$$

Therefore, $\omega_2 = 12.8 - 3.125$

$$= 9.675\ MHz\ (\times 2\pi)$$

iv. The design is not unique because for example, we can use a 32x frequency multipliers at the first stage and 64x in the 2nd stage. In that case, the second oscillator frequency should be chosen differently to generate the target $f_c = 100$ MHz at the end.