UNIVERSITY OF LONDON **[MSc-SC2, E4.01, ISE4.11]**
IMPERIAL COLLEGE OF SCIENCE TECHNOLOGY AND MEDICINE

DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2003

M.Sc and EEE/ISE PART IV: M.Eng. and ACGI

ADVANCED COMMUNICATION THEORY

— There are FOUR questions (Q1 to Q4)
— Answer Question ONE plus TWO other questions.

— Distribution of marks
Question-1: 40 marks
Question-2: 30 marks
Question-3: 30 marks
Question-4: 30 marks

Comments for Question Q1:
— Question Q1 has 20 multiple choice questions numbered 1 to 20.
— Circle the answers you think are correct on the answer sheet provided.
— There is only one correct answer per question.

The following are provided:
- ***A table of Fourier Transforms***
- ***A "Gaussian Tail Function" graph***

Examiners responsible: Dr. A. Manikas

Information for candidates:

The following are provided on pages 2 and 3:

- a table of Fourier Transforms;
- a graph of the 'Gaussian Tail Function'.

Question 1 is in a separate coloured booklet which should be handed in at the end of the examination.

You should answer Question 1 on the separate sheet provided. At the end of the exam, please tie this sheet securely into your main answer book(s).

Special instructions for invigilators:

Please ensure that the three items mentioned below are available on each desk.

- the main examination paper;
- the coloured booklet containing Question 1;
- the separate answer sheet for Question 1;

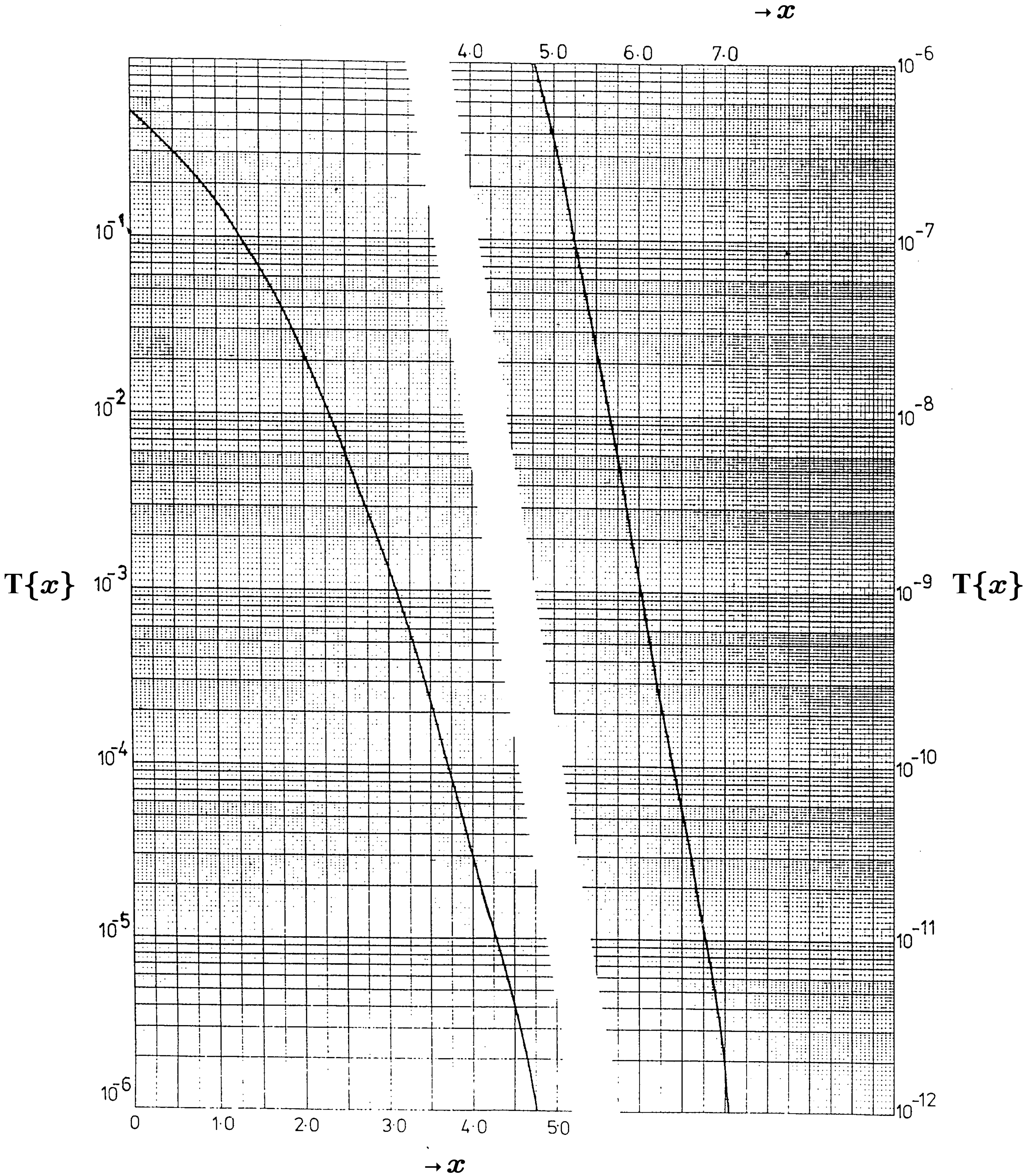
Please remind candidates at the end of the exam that they should tie their Answer Sheet for Question 1 securely into their main answer book, together with supplementary answer books etc.

Please tell candidates they must **NOT** remove the coloured booklet containing Question 1. Collect this booklet in at the end of the exam, along with the standard answer books.

Tail Function Graph

The graph below shows the Tail function $\mathbf{T}\{x\}$ which represents the area from x to ∞ of the Gaussian probability density function N(0,1), i.e.

$$\mathbf{T}\{x\} = \int_x^{\infty} \frac{1}{\sqrt{2\pi}}.\exp\left(-\frac{y^2}{2}\right)dy$$



Note that if $x > 6.5$ then $\mathbf{T}\{x\}$ may be approximated by $\mathbf{T}\{x\} \approx \frac{1}{\sqrt{2\pi}\,.\,x}.\exp\left\{-\frac{x^2}{2}\right\}$

FOURIER TRANSFORMS - TABLES

	DESCRIPTION	FUNCTION	TRANSFORM
1	Definition	$g(t)$	$G(f) = \int_{-\infty}^{\infty} g(t).e^{-j2\pi ft}dt$
2	Scaling	$g(\frac{t}{T})$	$\|T\| \,.\, G(fT)$
3	Time shift	$g(t-T)$	$G(f) \,.\, e^{-j2\pi fT}$
4	Frequency shift	$g(t) \,.\, e^{j2\pi Ft}$	$G(f-F)$
5	Complex conjugate	$g^*(t)$	$G^*(-f)$
6	Temporal derivative	$\frac{d^n}{dt^n} \,.\, g(t)$	$(j2\pi f)^n.\, G(f)$
7	Spectral derivative	$(-j2\pi t)^n.g(t)$	$\frac{d^n}{df^n} \,.\, G(f)$
8	Reciprocity	$G(t)$	$g(-f)$
9	Linearity	$A \,.\, g(t) + B \,.\, h(t)$	$A \,.\, G(f) + B \,.\, H(f)$
10	Multiplication	$g(t) \,.\, h(t)$	$G(f) * H(f)$
11	Convolution	$g(t) * h(t)$	$G(f) \,.\, H(f)$
12	Delta function	$\delta(t)$	1
13	Constant	1	$\delta(f)$

	DESCRIPTION	FUNCTION	TRANSFORM
14	Rectangular function	$\mathbf{rect}\{t\} \equiv \begin{cases} 1 & \text{if } \|t\| < \frac{1}{2} \\ 0 & otherwise \end{cases}$	$\mathbf{sinc}(f) = \frac{sin\, \pi f}{\pi f}$
15	Sinc function	$\mathbf{sinc}(t)$	$\mathbf{rect}(f)$
16	Unit step function	$u(t) = \begin{cases} +1, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$
17	Signum function	$\mathbf{sgn}(t) = \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$	$-\frac{j}{\pi f}$
18	Decaying exponential (two-sided)	$e^{-\|t\|}$	$\frac{2}{1+(2\pi f)^2}$
19	Decaying exponential (one-sided)	$e^{-\|t\|}.u(t)$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
20	Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
21	Lambda function	$\Lambda\{t\} \equiv \begin{cases} 1-t & \text{if} \quad 0 \le t \le 1 \\ 1+t & \text{if} \quad -1 \le t \le 0 \end{cases}$	$\mathbf{sinc}^2(f)$
22	Repeated function	$\mathbf{rep}_T\{g(t)\} = g(t) * \mathbf{rep}_T\{\delta(t)\}$	$\|\frac{1}{T}\|.\mathbf{comb}_{\frac{1}{T}}\{G(f)\}$
23	Sampled function	$\mathbf{comb}_T\{g(t)\} = g(t).\mathbf{rep}_T\{\delta(t)\}$	$\|\frac{1}{T}\|.\mathbf{rep}_{\frac{1}{T}}\{G(f)\}$

The Questions

1. ***This question is bound separately and has 20 multiple choice questions numbered 1 to 20, all carrying equal marks .***

You should answer Question 1 on the separate sheet provided.

Circle the answers you think are correct .

There is only one correct answer per question.

There are no negative marks.

2. For a binary communication channel, design a *minimax* detector with the following costs

$$C_{00} = C_{11} = 0;\ C_{10} = 3;\ C_{01} = 1$$

given that the likelihood functions are

$$p_0(r) = \frac{1}{3}\,\mathrm{rect}\left\{\frac{r}{3}\right\}$$

and

$$p_1(r) = \Lambda\left\{r - 2\right\},$$

and where r is the observed signal at the output of the channel. **[20]**

Find the forward transition matrix $\mathbb{F}$ of this binary channel. **[10]**

3. A speech signal having a maximum frequency of 4kHz is sampled at twice the Nyquist rate and then fed through a 256-level quantizer where each level is encoded using 8-bit codewords. The binary sequence is then fed through a binary PSK direct sequence spread spectrum system which operates in the presence of a jammer of power 1.6 Watts and in the presence of additive white Gaussian noise with double-sided power spectral density 0.5×10^{-12} Watts/Hz. The amplitude of the BPSK signal is 0.693V.

For this system, in which the correlation time is exactly one message bit (i.e. T_{cs}), the jammer power is uniformly spread over 10% of the spread spectrum bandwidth.

a) If the system is fully synchronised the bit error probability is 3×10^{-6}. What is

i) the PN-code rate of the system, **[10]**

ii) the processing gain (PG) of the system **[5]**

b) If the system is not synchronised and the synchronization error is 30% of the correlation time what is the power of the *code noise* at the output of the correlator. **[10]**

Comment on the bit error probability in this case. **[5]**

4. Consider a CDMA system of 256 users where each user has a protection probabilty equal to 10^{-2} and an Anti-jam Margin of 30 dB. Each user employs a feedback shift register of 21 stages, whose feedback connections are described by a primitive polynomial. The system is perfectly power controlled and the received power from each user is equal to $P = 0.1915$W operating in the presence of additive white Gaussian noise of double sided power spectral density 0.5×10^{-6} Watts/Hz.

a) Find the average energy per bit E_b and **[15]**

b) Find the PN-code rate. **[9]**

c) Comment on two major techniques which can be used with the above CDMA system to improve its performance. **[6]**