Imperial College London

MEng (Engineering) Examination 2017

Year 1

AE1-111 Thermodynamics

Tuesday 30th May 2017: 14.00 to 16.30
[2½ hours]

The paper is divided into Section A and Section B.
Both sections carry the same weight.
Candidates may obtain full marks for complete answers to ***ALL*** questions.

A Data Sheet is attached.

The use of lecture notes is NOT allowed.

Section A

Each question in Section A carries 10% of the total marks for the paper.
Attempt ALL question in this section

1. (a) Hydrogen gas is contained in a cylinder at a temperature of 300 K and a pressure of 400 bar by a moveable piston. The volume occupied by the gas is 0.02 m^3. The surroundings are at a temperature of 300 K and a pressure of 1 bar. Take the gas constant for hydrogen to be 4.12 kJ kg^{-1} K^{-1} and the specific heat capacity at constant volume to be 10.2 kJ kg^{-1} K^{-1}

   What is the mass of hydrogen in the cylinder? [20%]

   (b) The hydrogen undergoes a gradual, isothermal compression to a pressure of 750 bar. Calculate:

   (i) the amount of work required to compress the hydrogen; [20%]

   (ii) the heat transfer, if any. [20%]

   (c) After the compression has occurred,

   (i) explain how the exergy of the hydrogen has altered, if at all; [20%]

   (ii) the maximum quantity of work, if any, that could be obtained in returning the hydrogen to its initial state. [20%]

2. (a) A heat engine working in a cycle absorbs 10 MJ of heat from a reservoir at 750 K, and rejects 7 MJ of heat to a reservoir at 350 K.

   (i) What is the thermal efficiency of the engine? [10%]

   (ii) For the given reservoirs and heat supplied, what power output would correspond to a second law efficiency of 90%? [20%]

(b) Heat engines A and B are each reversible and each also operates between thermal reservoirs at the same hot, T_H, and cold, T_C, temperatures respectively. Show that the thermal efficiency of the two engines must be identical.

[50%]

(c) Given the definition of thermodynamic probability as the number of microstates Ω_k leading to a given macrostate, show that Boltzmann's equation,

$$S = k_B \ln\Omega$$

is consistent with the requirement that the combined entropy of the two systems is equal to the sum of the entropy of each system. [20%]

3. (a) A device operating in the steady state draws in surrounding air at 300 K at a rate of 4.0 $kg.s^{-1}$. The air is returned to the surroundings through two outlets. At one outlet, half the air mass flow exits at 500 K, whilst at the second outlet, the remaining half of the mass flow exits at 350 K. The device produces a power output of 50 kW. Assume the pressure at inlet and at both outlets is 1 bar and neglect all velocities.

(i) Write down the energy equation in a form suitable for analysis of the device, explaining the meaning of all symbols used.

[20%]

(ii) Calculate the heat transfer rate to the device.

[30%]

(b) Sketch a T-s diagram for a low-pressure ratio gas turbine cycle appropriate for ground based power, and give an expression for the thermal efficiency in terms of the relevant process temperatures.

On a separate sketch show how the T-s diagram would be altered if a single intercooling stage were incorporated.

[50%]

4. Consider isentropic flow of a perfect gas through a converging-diverging nozzle.

(a) Show that the exit velocity (v_2) and static pressure P_2 are related to total pressure, P_0 , and total temperature T_0 according to:

$$v_2{}^2 = 2\left(\frac{\gamma R}{\gamma - 1}\right)T_0\left[1 - \left(\frac{P_2}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

and hence find an expression for the maximum theoretical jet exit velocity.

Note: the relation $C_P = \frac{\gamma R}{\gamma-1}$ may be assumed.

[70%]

(b) If the nozzle is choked, calculate the ratio of total to static pressure at the throat for a flow of air. [30%]

5.

A satellite for a proposed mission to Jupiter is designed to utilise two power sources: a photovoltaic solar panel array and a radioisotope thermoelectric generator (RTG). After a journey lasting 6 years, the satellite will enter the planned orbit about Jupiter, after which it will spend a further 12 years transmitting data.
Each source is to provide a minimum power of 50 W throughout the mission.

The following parameters are given:

Panel

Solar conversion efficiency: 20%.

Solar constant (earth) 1300 Wm^{-2}.

Orbital characteristics of Jupiter:

aphelion, 5.45 AU; perihelion 4.95 AU; orbital period 11.9 years.

RTG

Conversion efficiency 8%.

Source: 5.5 MeV alpha emitter;

half-life: 88 years;

activity at launch: 600 GBq/gram (600×10^9 decays s^{-1} g^{-1}).

(a) Calculate the minimum solar panel area required. [50%]

(b) Calculate the minimum mass of the radioisotope required. [50%]

Section B

1.

(a) A ramjet is designed to propel an aircraft at Mach 3 at an altitude where the ambient pressure and temperature are respectively 26.4 kPa and 223 K.
Flow is heated in the combustion chamber to 2000 K. The intake total pressure recovery factor is 90%. The nozzle flow is fully expanded. For the analysis, neglect the added fuel mass and take the ramjet flow as air with constant properties.

Calculate the specific thrust. [25%]

(b) Show that for an ideal ramjet, the specific thrust is given by:

$$\frac{F}{\dot{m}_a a_0} = M_0\left[\sqrt{\tau_b} - 1\right];$$

where $\dot{m}_a$ represents the air mass flow rate, $\sqrt{\tau_b}$ represents the ratio of exhaust to inlet total temperature, and subscript 0 represents inlet conditions, with all other symbols having their usual meaning. State any assumptions made.

Compare also the ideal thrust with that calculated in part (a).

[30%]

(c) Write down an energy balance for the air and fuel flow through the engine and use this to show that the fuel/air ratio may be approximated as:

$$f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{C_P T_{T0}}{\Delta H}\left[\tau_b - 1\right],$$

where T_{T0} is the inlet total temperature and ΔH is the energy released by combustion.
Hence derive an expression for the (weight) specific impulse, $F/_{g\dot{m}_f}$.

[25%]

(d) Using a sketch, compare the ideal ramjet cycle with that of a turbojet with reheat, assuming the maximum reheat temperature is the same as the ramjet maximum temperature. [20%]

2.

(a) A shock wave with a pressure ratio of 20 is propagating in a shock tube filled with air. In the undisturbed air ahead of the shock, the pressure is 2 bar and the temperature is 300 K.

Calculate the static and total temperature of the air flow induced by the shock. [30%]

Calculate the velocity of the flow induced by the shock. [20%]

(b) It is desired to estimate the mean heat transfer coefficient for a sphere in supersonic flow using a thermocouple placed at the centre of a small solid spherical model suspended in a shock tube. The model is of known thermal properties with a high thermal conductivity so that variations in internal temperature are negligible.

Write down an expression relating the rate of temperature increase of the model to the overall heat transfer. [30%]

Hence explain how the mean heat transfer coefficient could be estimated and any assumptions made.

. [10%]

Explain briefly how radiation between the model and the tunnel walls could affect the measurement and how this might be mitigated.

[10%]

AE1-111 Thermodynamics   Data sheet

1: Unless otherwise stated, **air** may be treated as a perfect gas for which

$\gamma = 1.4$, $C_p = 1.005$ kJ/kg K and R = 0.287 kJ/kg K.

2: The **Stefan Boltzmann** constant $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4.

3: Definitions: **Nusselt number**, $Nu = \dfrac{hL}{k}$; **Prandtl number**, $P_r = \dfrac{C_p \mu}{k}$

where L is a characteristic length and all symbols have their usual meanings.

4: The **exergy** of a system at state "1" in surroundings which are at state "*a*" is:

$X = M\phi = M[\,(u_1 - u_a) + P_a(v_1 - v_a) - T_a(s_1 - s_a)\,]$ + K.E. + P.E. etc.,

where all symbols have their usual meaning.

5: Radioactivity. 1 Gigabecquerel (GBq) = 10^9 decays/sec.
1 Megaelectronvolt (MeV) = 1.6×10^{-13} Joules

6: For a perfect gas flowing through a stationary **normal shock wave**

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1} \quad ; \quad \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

$$M_2^2 = \left[(\gamma-1)M_1^2 + 2\right] \Big/ \left[2\gamma\, M_1^2 - (\gamma-1)\right]$$

and for **adiabatic flow**

$$\frac{T_o}{T} = 1 + \frac{\gamma-1}{2} M^2$$

7: Properties of water:

T °C	ρ(kg/m³)	μ(Pa.s)	ν(m²/s)	C_p(kJ/[kgK])	k (W/[mK])
10	999.8	1.308×10^{-3}	$1.308\ 10^{-6}$	4.193	0.582
50	988.0	5.471×10^{-4}	$5.537\ 10^{-7}$	4.181	0.640
100	958.3	2.822×10^{-4}	$2.945\ 10^{-7}$	4.216	0.677

8: Incompressible fully developed **laminar flow in a circular tube**:

(a) constant heat flux at wall: Nu ≈ 4.36
(b) constant wall temperature: Nu ≈ 3.66

9: Incompressible fully developed **turbulent flow in a circular tube**:

$Nu \approx 0.022\, Pr^{0.5} Re^{0.8}$ ($Pr > 0.5$, $Re < 10^6$)

10: The axisymmetric **conduction equation** in cylindrical coordinates can be written:

$$\rho C \frac{\partial T}{\partial t} = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right] + S_V$$

where S_V represents heat generated/unit volume and there is no axial variation.

Course Code and Title **AE1-111 Thermodynamics**: 1

Setter **D J Doorly** With notes from the June exam -

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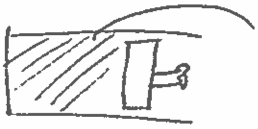
Marks

Q1 (a)



$T = 300$, $P = 4\times10^7$, $v = 0.02$

$PV = MRT$

$$\Rightarrow M = \frac{PV}{RT} = \frac{4\times10^7 \times 0.02}{4120\times300}$$

$M = 0.647$ kg. Ans

Exam notes: Generally good.
Common * errors: Wrong R (air); Forgetting m in calculating W; Assuming isothermal means Q=0; Mistakes in calculating $X_2 - X_1$ → not seeing shortcut.

(b)(i) $P_1 = 400 \rightarrow P_2 = 750$ @ T_{const}.

$$\frac{W}{m} = \int P\,dv \text{ (reversible)} ; \quad Pv = RT \Rightarrow P = \frac{RT}{v}$$

$$\therefore \frac{W}{m} = \int_1^2 \frac{RT}{v}\,dv = RT\ln(v_2/v_1) = RT\ln(P_1/P_2)$$

$$\therefore \frac{W}{m} = 4120\times300\times\ln\left(\frac{400}{750}\right) = -7.7696\times10^5 \text{ J/kg.}$$

$$\therefore W = -0.647(7.77\times10^5) = -5.03\times10^5 \text{ J.}$$

Ans.

(ii) $Q - W = \Delta E$; $\Delta E = 0$ since $\Delta T = 0$

$\therefore Q = W = -5.03\times10^5$ J (heat is from system).

Ans

(c) $\Delta X = X_1 - X_2 = M(U_1 - U_2) + P_a(v_1 - v_2) - T_a(S_1 - S_2)$

$P_a = 1$ bar $T_a = 300$

2

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Marks

Q 1(c) cont'd

$$\Delta X = M\{P_a(v_1 - v_2) - T_a(s_1 - s_2)\}$$

$$\Delta X = P_a(V_1 - V_2) - \underbrace{MRT_a \ln(V_1/V_2)}_{\text{Part b: } -7.7696\times10^5}$$

$$10^5(0.02)\left(1 - \frac{400}{750}\right) = 9.33\times10^2$$

$$\Delta X = -7.7603\times10^5 = X_1 - X_2 .$$

So ΔX is negative $\Rightarrow X_2 > X_1$ so exergy is increased. Ans

(ii) Max work out $= X_2 - X_1 = 7.760\times10^5$ Ans

2 Same as $W_{in} - W_{useless}$ $\int P_a\,dV$.

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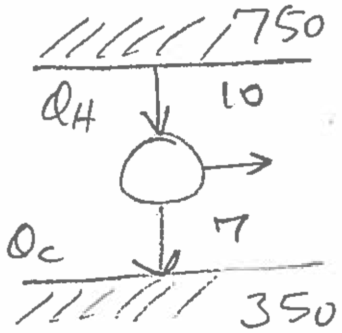
Marks

Q2 (a)

Mostly good



$\eta = \frac{W}{Q_{in}}$; By 1^{st} Law ;

$W = Q_H - Q_c = 3.$

$\therefore \eta = \frac{3}{10} = 30\%$. Ans

$\eta_{II} = \eta / \eta_{carnot}$. $\eta_c = 1 - \frac{T_c}{T_H} = 1 - \frac{350}{750}$

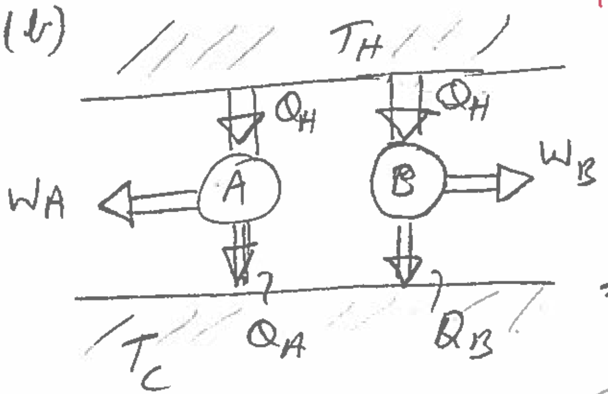
$\therefore \eta_c = 53.3\%$.

If $\eta_{II} = 90\% \Rightarrow \eta = 0.9 \times 53.3 = 48\%$.

$\Rightarrow W = 4.8$ MJ. Ans

Errors: Not understanding η_{II}.

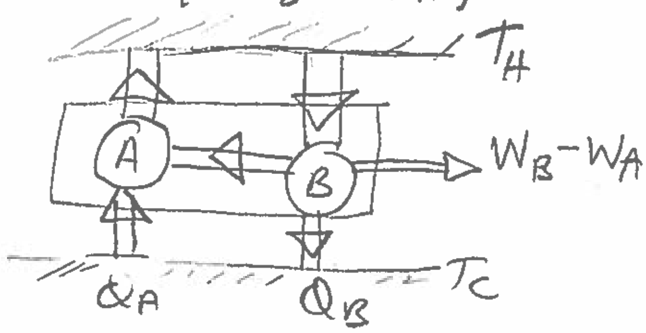
(b)



Say $\eta_B > \eta_A$.

$\Rightarrow \left\{ \begin{array}{l} W_B > W_A \\ Q_B < Q_A \end{array} \right\}$ for same Q_H.

Reverse (A): for compound device, reservoir A is redundant



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Marks

Q 2 (b). Since $W_B > W_A$ by hypothesis there is net Work. and net Q_{in}

$Q_{in} = Q_A - Q_B$ from T_c.

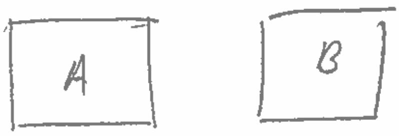
$\therefore$ equivalent to

which violates

Kelvin Planck statement. $\therefore$ Impossible.

* Few good solutions. Many just appealed to $\eta = \eta_c$, forgetting this is a corollary.

Q 2 (c)



For systems A & B, require $S_A + S_B = S_{Total}$.

But $\Omega_{Total} = \Omega_A \times \Omega_B$ by simple combinatorics

Now $\ln(\Omega_A \times \Omega_B) = \ln \Omega_A + \ln \Omega_B$ which is consistent with

$S_{Total} \propto \ln(\Omega_{total})$.

Mostly well answered

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5

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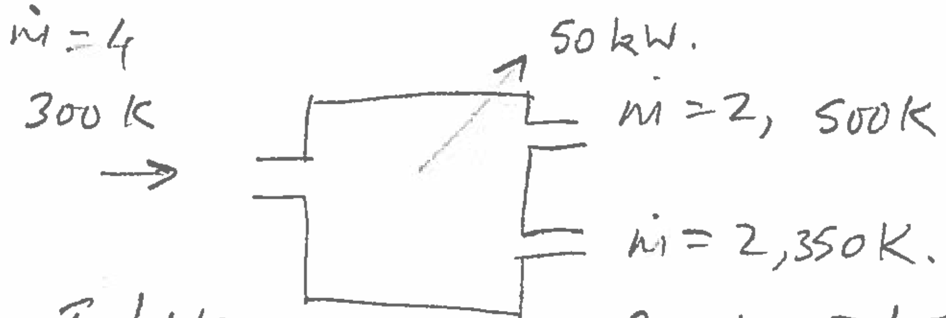
Marks

Q3 (i) Mostly v. good.

$$Q - W_x = \Delta E_{cv} + \sum M_{out} h_{0,out} - \sum M_{in} h_{0,in}$$

$$E_{cv} = \text{C.V. energy change} = M_{final} C_v T_{final} - M_{init} C_v T_{init} + \text{other } \Delta E\text{'s}.$$

$$h_0 = h + \tfrac{1}{2}|V|^2; \quad h = u + Pv = C_p T.$$



In kW:

$C_p = 1.005$ kJ/kg K

$$Q - 50 = 2\left(C_p(500) + C_p\,350\right) - 4 C_p\,300$$

$$Q = 50 + 2\,(502.5 + 351.8) - 4\,(301.5)$$

$$Q = 552.6 \text{ kW}. \quad \underline{\text{Ans}}$$

Common errors: Sign wrong in W_x term.
Using C_v and not C_p in enthalpy term

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Setter D J Doorly Q 3 cont'd

6

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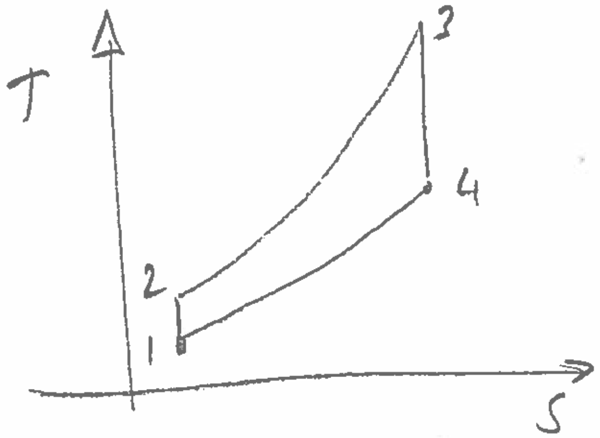
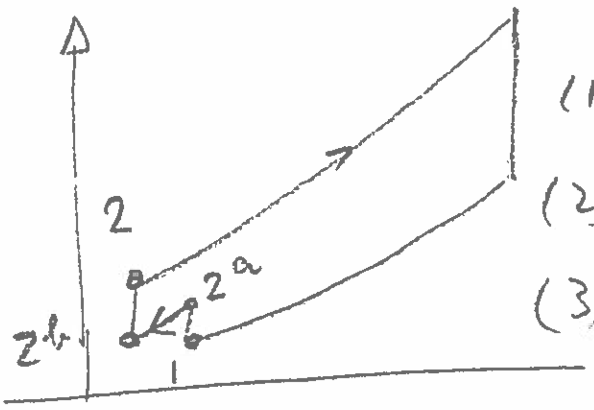
Marks



low $r_p \Rightarrow T_4 > T_2$

$$\eta = \frac{SW}{Q_{in}}$$

$$\Rightarrow \eta = \frac{\cancel{c_p}\left[(T_3 - T_4) - (T_2 - T_1)\right]}{\cancel{c_p}(T_3 - T_2)}$$ Ans.



(1) Part compress to 2^a

(2) Cool to 2^b

(3) Finish compression to 2.

* Common errors:

(1) Drawing T-S diagram for an aero engine, not for ground-based propulsion (including nozzle + inlet).

(2) Only < 50% could sketch intercooling cycle.

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Setter D J Doorly Q4. P7

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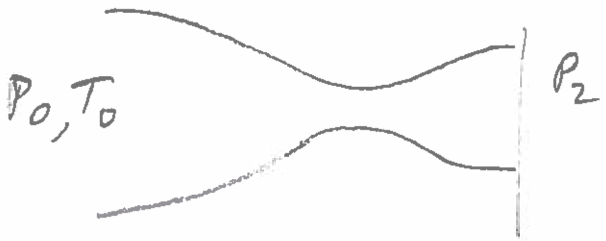
	Marks

Q4. Mixed Results. ~50% fairly good to v. good.

(i) Steady flow energy equation: $h_0 = \text{const.}$



$$C_p T_2 + \tfrac{1}{2} V_2^2 = C_p T_0 .$$

$$\therefore V_2^2 = 2 C_p (T_0 - T_2) = 2 C_p T_0 (1 - T_2/T_0)$$

$$\text{Isentropic} \Rightarrow \frac{T_2}{T_0} = \left(\frac{P_2}{P_0}\right)^{\gamma-1/\gamma} ;$$

$$C_p - C_v = R,\ C_p/C_v = \gamma \Rightarrow C_p = \gamma C_v$$

$$\Rightarrow C_v(\gamma - 1) = R \Rightarrow C_v = \frac{R}{\gamma - 1}$$

$$\Rightarrow C_p = \frac{\gamma R}{\gamma - 1}$$

$$\therefore V_2^2 = 2 \frac{\gamma R}{\gamma - 1} T_0 \left[1 - (P_2/P_0)^{\gamma - 1/\gamma}\right]. \quad \text{Ans}$$ 5

(ii) $V_{2\,max} = \sqrt{2 \frac{\gamma R}{\gamma - 1} T_0}$, when $P_2 \to 0$. 2

* Common errors: Not reading and missing part B. (or not knowing how to attempt?)

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P8

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Marks

Q4 (ii)

Isentropic relation

$$\frac{p_0}{p} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\gamma/\gamma - 1}$$

Choked $\Rightarrow M = 1$ at throat $\Rightarrow \frac{p_0}{p} = \left(1 + \frac{1}{5}\right)^{3.5}$

for air $= \underline{1.893}$.

* This part very easy but many forgot.

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p 9

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Marks

Q5 Reasonable attempts though many did not succeed with part (ii).

(i) For solar panel, require 6

Consider worst case ⇒ furthest distance

— aphelion = 5.45 AU.

$$\dot{Q}_s = \text{Solar flux}\Big|_{r=R_J} = \text{Solar flux}\Big|_{r=R_e} \cdot \left(\frac{R_e}{R_J}\right)^2$$

$$\Rightarrow \dot{Q}_s = \frac{1}{(5.45)^2} \cdot \dot{Q}_e = \frac{1300}{29.7} = 43.8 \text{ W/m}^2.$$

$$\dot{W}_{electrical} = 0.2\,\dot{Q}_s = 8.75 \text{ W/m}^2.$$

$$\Rightarrow \text{area} = \frac{50}{8.75} = 5.71 \text{ m}^2 \quad \text{or approx } (4 \times 0.5 \times 3)$$

* Errors: not taking worst case 5.45 AU.

(ii) For RTG,

Energy per decay $5.5 \text{ MeV} = 5.5 \times 10^6 \times 1.6 \times 10^{-19}$

$= 8.8 \times 10^{-13}$ J.

Activity – $600 \text{ GBq} = 600 \times 10^{19}$ decays/s/gm

Energy/gm $= 600 \times 10^{19} \times 8.8 \times 10^{-13} = 0.528$ W/g

$\dot{Q}_{RTG} = 528$ W/kg.

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Marks

$W_e = 0.08$

$\Rightarrow W_e$ at launch $= 0.08 \times 528$

$= 42.2$ W/kg.

But W_e decays in proportion to radioisotope activity. Mission end $= t_0 + 18$ years.

$N(t) = N_0 e^{-\lambda t}$

When $N = N_0/2 \Rightarrow e^{-\lambda t} = \frac{1}{2} \Rightarrow \lambda t_{\frac{1}{2}} = \ln 2$

$\Rightarrow \lambda = \dfrac{\ln 2}{t^{1/2}}$.

Here $\lambda = \ln 2/88 = 7.877 \times 10^{-3}$

Hence $N(18) = N_0 e^{-7.877(18)} = 0.868 N_0$

$\Rightarrow W_{e(18)} = 36.6$ W/kg.

So Mass required $= \dfrac{50}{36.6} = 1.365$ kg.

(Pu_{238})

* < 25% succeeded in this part.

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Setter D J Doorly Section B . Q1 . 11

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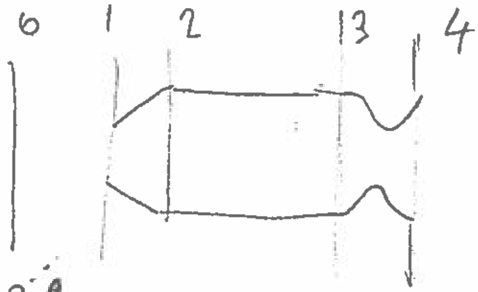
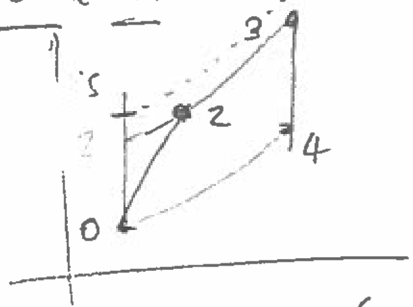
* Mistakes: Confusing P_0, ambient with P_{T0} $= P_{T1}$.

Average 6
Below average performance .

Question B1





$P_0 = 26.4 \times 10^3$

$T_0 = 223\,K \Rightarrow a_0 = \sqrt{\gamma R T_0} = 299.3$

$P_{T,0} = P_0 \left(1 + \frac{\gamma - 1}{2}(3)^2\right)^{3.5} = 36.7\,P_0$.

$P_{T,2} = 0.9\,P_{T,0} = 33.06\,P_0 = 8.728$ bar.

$T_{T,2} = T_{T,0}\left(1 + \frac{M^2}{5}\right) = 624.4\,K$.

$T_{T3} = 2000\,K = T_3$

$C_p T_4 + \frac{1}{2} V_4^2 = C_p T_3$; | $T_4 = T_3 \cdot \left(\frac{P_4}{P_3}\right)^{\gamma - 1/\gamma}$

$T_4 = 2000\left(\frac{1}{33.06}\right)^{1/3.5}$

$T_4 = 736\,K$.

$0.368(2000)$

$\Rightarrow V_4^2 = 2(1005)(2000 - 736)$

$\Rightarrow V_4 = 1594$ m/s.

$\therefore \frac{F}{\dot{m}} = V_4 - V_a = 1594 - 898 = 696$ N/kg/s

* Reasonably attempted

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Setter: D J Doorly

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Marks

Question B1 cont'd.

(b) Ideal ramjet $P_{T4} = P_{T0}$.

Also for fully expanded nozzle, $P_4 = P_0$

$\Rightarrow M_4 = M_0 = M_a$

$$\therefore \frac{F}{\dot m} = V_4 - V_a = M_4 a_4 - M_a a_0$$

$$\frac{F}{\dot m a_0} = M_a\left(\frac{a_4}{a_0} - 1\right)$$

& since $a_4 = \sqrt{\gamma R T_4}$

$$a_4 = \sqrt{\frac{\gamma R T_{T4}}{\left(1+\frac{\gamma-1}{2}M_4^2\right)}}$$

common

$$\Rightarrow \frac{a_4}{a_0} = \sqrt{\frac{T_{T4}}{T_{T0}}} = \sqrt{\tau_b}$$

So $\frac{F}{\dot m a_0} = M_0\left(\sqrt{\tau_b} - 1\right)$. Ans

In this case $\frac{F}{\dot m} = 299.3 \times 3 \times \left\{\sqrt{\frac{2000}{624.4}} - 1\right\}$

$$\frac{F}{\dot m} = 709 \text{ N/kg/s}$$

(c) $\dot m_a C_p T_{T0} + \dot m_f \Delta H \cong \dot m_a C_p T_{T4}$

* Not so well done. Simple energy balance.

$$\Rightarrow \frac{\dot m_f}{\dot m_a} \simeq \frac{C_p(T_{T4} - T_{T0})}{\Delta H} = \frac{C_p T_{T0}}{\Delta H}\left[\tau_b - 1\right]$$

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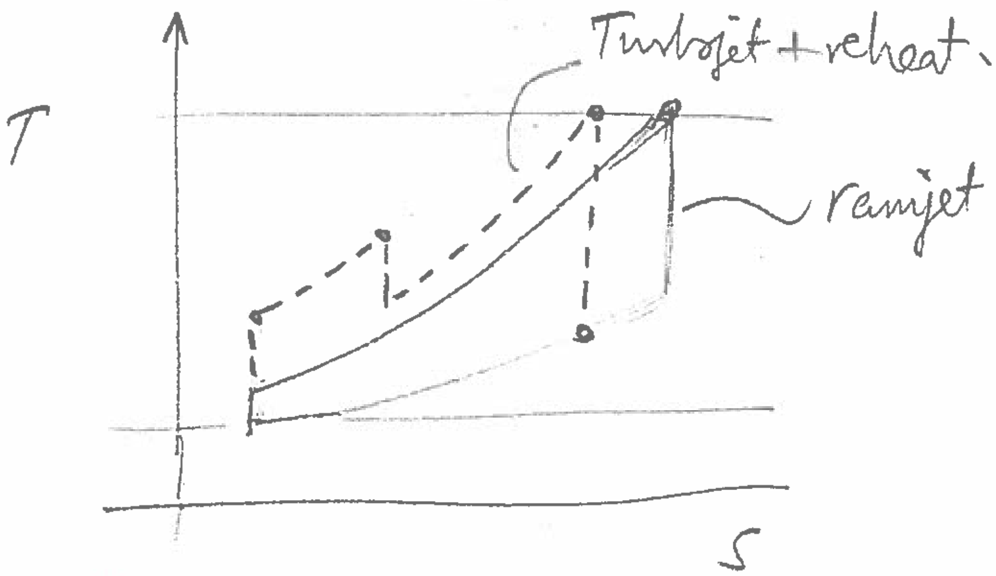
Question B1 cont'd

Impulse

$$\frac{F}{g\dot{m}_f} = \frac{\cancel{\dot{m}_a} a_0 M_0 \sqrt{\tau_\ell - 1}}{g \cdot \cancel{\dot{m}_a} C_p T_{T_0} [\tau_\ell - 1] / \Delta H}$$

$$\Rightarrow \frac{F}{g\dot{m}_f} = \frac{a_0 M_0 \sqrt{\tau_\ell - 1} \cdot \Delta H}{C_p T_{T_0} [\tau_\ell - 1]}$$

(d)



last part well attempted

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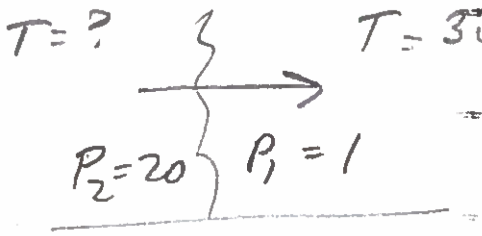
Write on this side only (in ink) between the margins, not more than one solution per sheet please. Solutions must be signed and dated by both exam setter and referee.

Marks

Part B Question 2 cont'd.



Temperature ratio across the shock:

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = \left[\frac{2\gamma M^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}\right] \cdot \left[\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right]^{-1}$$

from datasheet.

Since $P_2/p_1 = 20 \Rightarrow M_{shock} = 4.16$

$\Rightarrow \rho_2/\rho_1 = 4.65 \Rightarrow T_2/T_1 = 4.3 \Rightarrow T_2 = 1290\,K$

$$V_s = M_s a_1 = 4.16\sqrt{\gamma R(300)} = 1{,}444 \text{ m/s}.$$

$$T_{02_{abs}} = T_2 + \frac{\frac{1}{2}V_{2,abs}^2}{C_p}.$$

* Notes: < 20% correctly calculated $T_{02,abs}$.

$V_{2,abs} = V_s - V_{2,rel}$. $\quad V_{2,rel} = V_s/4.65 = 310.5$

$\Rightarrow V_{2,abs} = 1444 - 310.5 = 1133.5$ m/s

$$\Rightarrow T_{02,abs} = 1290 + \frac{(1133.5)^2}{2\times 1005} = \underline{1929\,K}. \quad \underline{\text{Ans.}}$$

[Compare T_{02} in a frame moving with shock; Here $T_{02} = T_2 + V_{rel}^2/2C_p = 1338\,K$.]

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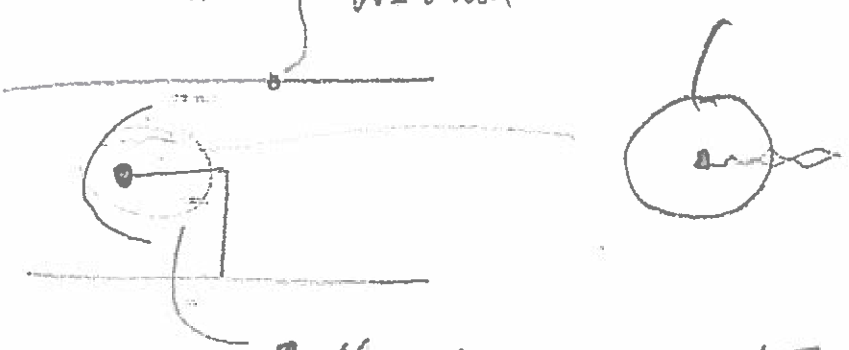
Question B2.



Basic energy balance

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE}{dt}$$ — ~50% managed this

$$\int h\,(T_F - T_S)\,dA - \int \dot{q}_{radiation}\,dA = \rho C V \frac{dT}{dt}$$

Lumped model

$$hA(T_F - T_S) - \varepsilon\sigma(T_S^4 - T_W^4) = \rho C V \frac{dT_S}{dt}.$$

many forgot to include 'A'

For short times, wall temperature ≃ sphere temperature ~ initial temperature

⇒ neglect radiation term

$$\Rightarrow h = \frac{\rho c V \frac{dT_s}{dt}}{A(T_F - T_s)}.$$

measure T_s and compute $\frac{dT_s}{dt}$ ⇒ h. Ans.

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Question B 2

When model heats appreciably, $\dot{q}_r$ is non-negligible. Best to try to reduce $\dot{q}_r$ using low ε coating on sphere and try to match. Only a problem for very small models since $\dfrac{dT}{dt} \propto \dfrac{h}{\rho c R}(T_f - T)$.

* Most talked about 'shielding thermocouple' and missed the point completely.