UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1998

MEng Honours Degrees in Computing Part IV
MEng Honours Degree in Information Systems Engineering Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute
Associateship of the Royal College of Science

PAPER 4.77 / I4.20

COMPUTING FOR OPTIMAL DECISIONS
Tuesday, April 28th 1998, 2.00 - 4.00

Answer THREE questions

For admin. only: paper contains 4 questions

1 a A computer company has divided its market into geographical areas A and B. If £x_1 is spent on advertisements in area A then $60\sqrt{\left(\frac{x_1}{100}\right)}$ computers will be sold in A. If £x_2 is spent on advertisements in B then $40\sqrt{\left(\frac{x_2}{100}\right)}$ computers will be sold in B. In A, the sale price is £1000/computer, with production and distribution costs £500/computer. In B, the sale price is £900/computer, with production and distribution costs £400/computer. The advertisement budget is limited to £1 million. Determine the advertising expenditure to optimise total profit for the company.

b Establish the convexity of the problem

$$\min \left\{ x_1 + x_2 \;\middle|\; x_1^2 + x_2^2 \leq 0;\, x_1, x_2 \geq 0 \right\}$$

(i.e. establish the convexity of the objective, each constraint and the intersection of the constraints). Use this property to show that the solution is a global optimum.

2 a Consider a parent company P which owns factories A, B, C. P is considering reinvesting its profits to improve the productivity of A, B and C. The total amount available to reinvest is £M million. It has been estimated that investment of £ω_i in factory i (i = A, B, C) will yield a return in terms of increased profits of r_i *per £ invested*, and

$$r_i = \alpha_i \left(\frac{\omega_i}{100000}\right)^{\gamma_i} + \beta_i\, \epsilon_i\,; \quad i = A, B, C,$$

where α_i, γ_i, β_i are given nonzero constants, ϵ_i is a zero mean, normally distributed random variable with given constant covariance $\mathcal{E}[\epsilon_i \epsilon_j] = q_{ij}$, i, j = A, B, C. Let Q denote the covariance matrix. Assume the production of A, B, C do not interfere with each other except through Q. The investment in A should be at least double the amount of the investment in B. It is desired to determine an optimal investment policy which maximises expected profit and minimises expected return for P. Formulate a robust mean-variance optimisation investment problem for P, to determine ω_i, i = A, B, C.

b Assuming that you only have access to a linear programming software, discuss an algorithm you might implement to solve the above problem.

3 a Consider

$$\mathbb{S} \equiv \left\{ v \in \Re^n \;\middle|\; N^T v = b \right\}$$

where $\Re^n$ is the n-dimensional vector space, N is a given $n \times m$ real matrix and b is a given m-dimensional vector and $b \neq 0$. An $n \times n$ symmetric matrix Q is said to be negative definite on $\mathbb{S}$ if

$$x^T Q x < 0 \; ; \; \forall \; x \in \mathbb{S}.$$

Formulate a mathematical programming problem to check if a given Q is negative definite on $\mathbb{S}$.

b Assume that the columns of N are linearly independent, Q may be positive or negative definite on $\mathbb{S}$ but it might be singular on $\Re^n$. Suggest a procedure for solving the above problem.

4 a Given $\hat{x}$, solve the problem of finding x minimising the objective $a^T x$, subject to the restriction $(x - \hat{x})^T Q (x - \hat{x}) = 1$ for a given symmetric positive definite matrix Q.

b Let g (x) an m-dimensional vector, each element of which is a nonlinear function of $x \in \Re^n$. Derive the feasible steepest descent direction at some given point x_0 for the problem

$$\min \left\{ g^T(x)\, g(x) \;\middle|\; H^T x = b \right\}$$

in terms of the $g(x_0)$, its gradient, H and b. H is an $n \times m$ matrix whose columns are linearly independent. [You do not need to discuss further the steepest descent algorithm.]

c Formulate an algorithm to solve the problem

$$\min \left\{ g^T(x)\, g(x) \;\middle|\; x \geq 0 \right\}$$

by considering the second order Taylor series expansion of $g^T(x)\, g(x)$.

End of paper