IMPERIAL COLLEGE LONDON

Final

E4.06
AO9
SO9
ISE4.36

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

MSc and EEE/ISE PART IV: MEng and ACGI

OPTICAL COMMUNICATION

Friday, 30 April 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 6

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	E.M. Yeatman, E.M. Yeatman
	Second Marker(s) :	A.S. Holmes, A.S. Holmes

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3.0 \times 10^8$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x}\tan(k_{1x}d/2) \quad \text{and} \quad \kappa = -\,k_{1x}\cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, state any assumptions or approximations made, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]

a) A certain slab waveguide supports 4 TE modes, m = 0, 1, 2 and 3. How many TM modes does it support?

b) The square root of the relative permittivity of a material gives what quantity, of interest in optics?

c) An m=2 mode in a slab waveguide has a cross sectional electric field distribution with three peaks. Will the intensity of the central peak be greater than, the same as, or less than that of the two other peaks?

d) A binary signal propagating in a silica optical fibre has bits 0.2 m in length. Estimate the bit rate B.

e) Briefly explain what is meant by an amorphous material, giving an example.

f) How can polarisation maintaining fibre be used to avoid the effects of polarisation dispersion?

g) A p-i-n photodiode with perfect quantum efficiency has an output photocurrent of 0.5 μA. If the incident (free space) wavelength is 1.30 μm, what is the rate of detection of photons, in photons per second?

h) A p-i-n photodiode has an attenuation coefficient at the operating wavelength of 10^5 m^{-1}, and a distance from the surface to the depletion region of 5 μm. What fraction of the absorbed photons are absorbed before reaching the depletion region?

i) Why would a single layer anti-reflection coating not work very well for eyeglasses?

j) For an erbium doped fibre amplifier, estimate the energy level of the metastable state of the erbium ions, relative to the ground state, in eV.

2. A symmetric slab waveguide as shown in Fig. 2.1 has a core thickness d, and a numerical aperture NA = 0.14.

a) Find the value d/λ_o, where λ_o is the free space wavelength, for which the m=0 TE mode of this guide has an electric field distribution E(x) such that $E(d/2) = E(0)/\sqrt{2}$. [10]

b) Does the guide that satisfies the condition in (a) support any additional TE modes, and if so how many? [5]

c) If the guide that satisfies the condition in (a) has a core thickness $d = 4\ \mu m$, at what distance Δx beyond the boundary at $x = d/2$ does the electric field amplitude drop to $\frac{1}{2}E(0)$? [5]

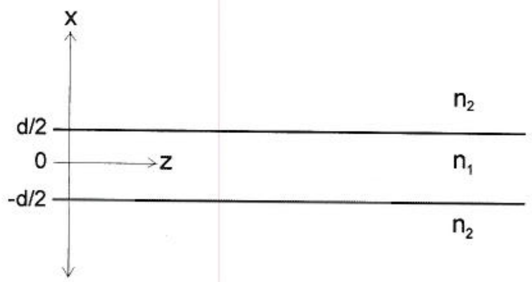

Figure 2.1

3. A certain glass has an index of refraction n given by:

$n(\lambda_o) = n_o + a(\lambda_o - \lambda_c)$

where λ_o is the free space wavelength, λ_c is a constant reference wavelength, n_o is the index of refraction at λ_c, and a is a constant coefficient.

a) Explain why there is no material dispersion in this glass. [4]

b) Show that the group delay τ_g for a propagation length L in this glass is given by:

$$\tau_g = \frac{L}{c}(n_o - a\lambda_c)$$ [8]

c) A certain fibre has a dispersion coefficient D at the operating wavelength. For a transmitted pulse width in time of σ_o, find an expression for the width in time of the received pulse σ_r, in terms of D, σ_o, the transmitted spectral width σ_λ, and the fibre length L. [4]

d) Give an approximate equation indicating the maximum dispersion that can be included in a fibre link analysis as a power budget penalty, in terms of D, L, σ_λ and the bit rate B. Briefly explain why this is valid. [4]

4. a) Give expressions for the optical SNR of a fibre link for each of the two cases: (i) when dominated by shot noise; (ii) when dominated by receiver thermal noise. Define all terms used. [6]

b) A certain optical receiver has a noise equivalent power (NEP) of 10 pW/√Hz, and is used in a link with an operating (free space) wavelength $\lambda_o = 1.50$ μm. Find the received optical power Φ_r for which the noise contributions of shot noise and receiver noise are equal. [6]

c) For the case given in (b), if the required SNR is 12, what will be the maximum bit rate B that can be supported? Indicate any assumptions or approximations made. Is this a realistic bit rate, and if not why not? [8]

5. a) Sketch a typical optical power vs. input current curve for a laser diode, indicating key features. Describe qualitatively the reasons for the existence of a threshold current, and the factors that contribute to it, and briefly discuss its practical implications. [4]

b) A certain Fabry-Perot laser diode has a threshold current of 5.0 mA. Hence, calculate the input current required to reach an output optical power of 8.0 mW, assuming a free space operating wavelength of 1.33 μm and a quantum efficiency = 0.8. [6]

c) Show that the frequency spacing of the longitudinal modes of a laser diode is just the reciprocal of the round-trip time in the cavity. [4]

d) Explain what is meant by internal and external quantum efficiency for an optical source. What are the key factors affecting the quantum efficiencies for (i) light emitting diodes; and (ii) laser diodes. [6]

6. a) A silicon p-n photodiode (Fig. 6.1) has a depletion layer thickness of w, and p and n doping levels respectively of N_A and N_D, with $N_A = 10^{21}$ m^{-3} and $N_D = 4\times10^{20}$ m^{-3}. The quantities w_p and w_n are the depleted widths in the p and n regions respectively. A reverse bias voltage V_b is applied. Find an expression for the full depletion width w as a function of V_b, and the value of V_b for which w = 4 μm. Sketch the electric field distribution E(x), where x is the distance from the photodiode surface. [10]

b) Neglecting Fresnel reflection, find an expression for the quantum efficiency η of the photodiode of (a) if the total p region thickness h = 2.5 μm, and the absorption coefficient $\alpha = 0.8\times10^5$ m^{-1}. Hence find the value of η for V_b = 10.0 V. [5]

c) Show that as the bias voltage V_b is increased from zero, the peak value of electron drift velocity in the depletion region will increase in proportion to the square root of V_b. State any assumptions or approximations made. [5]

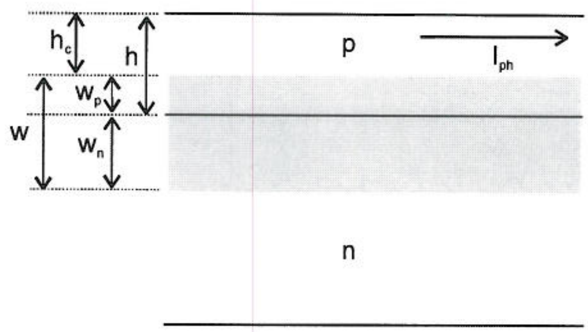

Figure 6.1 p-n photodiode

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SOLUTIONS

1. [20]

a) *A certain slab waveguide supports 4 TE modes, m = 0, 1, 2 and 3. How many TM modes does it support?*

Number of TE modes = number of TM modes, so 4.

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b) *The square root of the relative permittivity of a material gives what quantity, of interest in optics?*

The refractive index, n.

c) *An m=2 mode in a slab waveguide has a cross sectional electric field distribution with three peaks. Will the intensity of the central peak be greater than, the same as, or less than that of the two other peaks?*

The same – all peaks have the same height since mode shape is sinusoidal in core.

d) *A binary signal propagating in a silica optical fibre has bits 0.2 m in length. Estimate the bit rate B.*

$L = tv = (1/B)(c/n)$ so $B = c/nL = 10^9$ bits/s (1 Gbit/s)

e) *Briefly explain what is meant by an amorphous material, giving an example.*

A solid material in which the atoms have no long-range order, including any glass.

f) *How can polarisation maintaining fibre be used to avoid the effects of polarisation dispersion?*

The effective indices of the two principal polarisation modes are sufficiently different that there is negligible mixing between them, so light launched in one will stay in it and the differing velocity of the other will have no spreading effect.

g) *A p-i-n photodiode with perfect quantum efficiency has an output photocurrent of 0.5 μA. If the incident (free space) wavelength is 1.30 μm, what is the rate of detection of photons, in photons per second?*

The rate $r = \Phi/(hc/\lambda)$, $\Phi = I(hc/e\lambda)$ so $r = I/e = 0.5\times10^{-6}/1.6\times10^{-19} = 3.1\times10^{12}$ photons/s. The wavelength doesn't matter.

h) *A p-i-n photodiode has an attenuation coefficient at the operating wavelength of 10^5 m^{-1}, and a distance from the surface to the depletion region of 5 μm. What fraction of the incident photons are absorbed before reaching the depletion region?*

Fraction absorbed = $1 - \exp(-\alpha L) = 1 - \exp(-0.5) = 0.393$ = 39.3%

i) *Why would a single layer anti-reflection coating not work very well for eyeglasses?*

The required wavelength range is too great, so that a single layer which is a quarter wave at the centre wavelength would be far off $\lambda/4$ at the edges.

j) *For an erbium doped fibre amplifier, estimate the energy level of the metastable state of the erbium ions, relative to the ground state, in eV.*

The transition (metastable to ground state energy) has to equal the photon energy, which in eV is $hc/e\lambda$ = 0.829 eV at the EDFA wavelength of about 1.5 μm.

(2) For an even mode such as $m=0$,

a) $E_1(x) = A\cos(k_{1x}x)$

$\therefore \quad E_1(d/2)/E_1(0) = \cos(k_{1x}d/2) = \frac{1}{\sqrt{2}}$

The eigenvalue eq'n for $m=0$ can be written

$$\frac{Kd}{2} = \left(\frac{k_{1x}d}{2}\right)\tan\left(\frac{k_{1x}d}{2}\right)$$

call $Y = Kd/2 \quad X = k_{1x}d/2$

$$Y = X\tan X \qquad (i)$$

and $k_{1z} = k_{2z}$ gives

$$K^2 + k_{1x}^2 = (n_1^2 - n_2^2)k_0^2 = NA^2 k_0^2$$

or $X^2 + Y^2 = NA^2 Z^2 \qquad (ii)$

Combine (i) and (ii)

$$X^2(1+\tan^2 X) = \frac{X^2}{\cos^2 X} = Z^2 NA^2 \quad Z = k_0 d/2 = \pi d/\lambda_0$$

but $\cos X = \frac{1}{\sqrt{2}}$, $\therefore X = \frac{\pi}{4}$ rad

$$\frac{\pi/4}{1/\sqrt{2}} = \frac{\pi d}{\lambda_0} . NA \qquad \frac{d}{\lambda_0} = \frac{\sqrt{2}}{4NA} \cong 2.5$$

b) The cutoff condition for mode m is

$$\frac{d}{\lambda_0} = \frac{m}{2NA} = \frac{1}{2(.14)} = 3.57 \text{ for } m=1$$

So $m=1$ is cut off, <u>no</u> other modes are supported.

c) $E_2(x) = B\exp(-Kx)$

$$\frac{E_2(d/2+\Delta x)}{E_2(d/2)} = \exp(-K\Delta x) = \frac{1/2}{1/\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \therefore \tfrac{1}{2}\ln 2 = K\Delta x$$

$$K = \frac{X\tan X}{d/2} = \frac{2(\pi/4)(1)}{d} = \frac{\pi}{2d} \quad \therefore \Delta x = \frac{\frac{1}{2}\ln 2}{\pi/2d} = \frac{\ln 2}{\pi} . d$$

$$= 0.22d = \underline{0.88\ \mu m}$$

(3)

a.) Dispersion is proportional to $d^2n/d\lambda^2$, which in this case is zero, so there is no dispersion

b) $\tau_g = \frac{L}{v_g}$ $\quad v_g = \frac{d\omega}{dk}$ $\quad k = nk_0$
$k_0 = \omega/c$

$$\therefore \tau_g = L\frac{dk}{d\omega} = L\frac{d}{d\omega}\left(\frac{n\omega}{c}\right)$$

$$= \frac{L}{c}\{n + \omega dn/d\omega\}$$

$$\frac{dn}{d\omega} = \frac{dn}{d\lambda_0}\frac{d\lambda_0}{d\omega} \quad \lambda_0 = \frac{2\pi c}{\omega} \quad \frac{d\lambda_0}{d\omega} = \frac{-2\pi c}{\omega^2} = \frac{-\lambda_0}{\omega}$$

$$\frac{dn}{d\omega} = \frac{dn}{d\lambda_0}\left(\frac{-\lambda_0}{\omega}\right) \quad \therefore \tau_g = \frac{L}{c}\left[n - \lambda_0\frac{dn}{d\lambda_0}\right]$$

$$n = n_0 + a(\lambda_0 - \lambda_c) \quad \frac{dn}{d\lambda_0} = a$$

$$\tau_g = \frac{L}{c}\left\{n_0 + a(\lambda_0 - \lambda_c) - \lambda_0 a\right] = \frac{L}{c}\left[n_0 - a\lambda_c\right]$$

c) $$\sigma_r^2 \cong \sigma_0^2 + \Delta\sigma^2$$

$$\Delta\sigma = D\sigma_\lambda L$$

$$\therefore \sigma_r^2 = \sigma_0^2 + (D\sigma_\lambda L)^2$$

d) We want the pulse spreading, $\Delta\sigma = D\sigma_\lambda L$, to be significantly less than the bit slot $1/B$. If we say $\Delta\sigma < 0.2(1/B)$, then our limit is

$$D\sigma_\lambda L \cong \frac{0.2}{B} \quad D\sigma_\lambda LB \cong 0.2$$

4

a) In general $SNR_{opt} = \frac{I_{ph}}{\sqrt{(I^*)^2 \Delta f}}$

For shot noise $(I^*)^2_{sn} = 2e I_{ph}$

$\therefore$ $SNR = \frac{I_{ph}}{\sqrt{2e I_{ph} \Delta f}} = \sqrt{\frac{I_{ph}}{2e\Delta f}}$ (i)

with I_{ph} = photocurrent, Δf = bandwidth

For thermal noise $(I_{th}^*)^2 = 4kT/R$

$\therefore$ $SNR_{opt} = \frac{I_{ph}}{\sqrt{4kT\Delta f/R}}$ (ii)

where R is the effective load resistance seen by the photodiode.

b) For noise equiv. power, $SNR_{opt} = \frac{\Phi_R}{NEP\sqrt{\Delta f}}$

where Φ_R is the received optical power.

$I_{ph} = \mathcal{R}\Phi_r$ where $\mathcal{R}$ is the receiver responsivity.

Effects will be equal when the two SNR values with only one present are equal, ie

$\frac{\Phi_R}{NEP\sqrt{\Delta f}} = \sqrt{\frac{I_{ph}}{2e\Delta f}} = \sqrt{\frac{\mathcal{R}\Phi_R}{2e\Delta f}}$ $\mathcal{R} = \frac{e\lambda}{hc}$ $(\eta = 1)$

gives: $\frac{\sqrt{\Phi_R}}{NEP} = \sqrt{\frac{\lambda}{2hc}}$ $\Phi_R = \frac{\lambda}{2hc}(NEP)^2$

$= \frac{1.5\times10^{-6}}{2\times6.63\times10^{-34}\times3\times10^{8}}(10^{-11})^2 = 0.4\,mW$

Equivalently, express NEP as $(I^*)^2$: $SNR = \frac{I_{ph}}{\mathcal{R}\cdot NEP\sqrt{\Delta f}}$

$\therefore$ $(I^*)^2 = (\mathcal{R}\cdot NEP)^2 = 2e I_{ph}$

$= 2e\mathcal{R}\Phi_r$

$\therefore$ $\Phi_r = \mathcal{R}(NEP)^2/2e = 0.4\,mW$

4

c) Since there are two equal contributions, the overall SNR will be $\Phi_r / 2NEP\sqrt{\Delta f}$

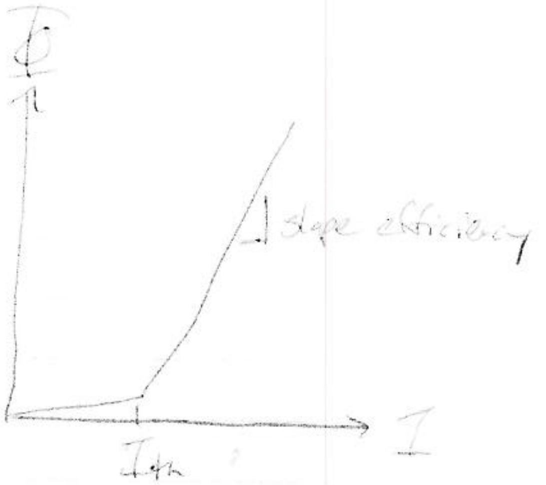
Then if $\Delta f = B/2$ $B \doteq$ bit rate

$$12 = \frac{1\times10^{-4}}{2\cdot10^{-11}\sqrt{B/2}} \qquad B = \frac{2\,(2\times10^{6})^2}{12^2} \simeq 6\times10^{12} \text{ bit/s}$$

6×10^{12}?

This is far beyond the capabilities of modulators & receivers.

⑤ a)



Backmark. Need to mention requirement to reach sufficient photon density in waveguide for stim. emission to overcome spontaneous emission, that I_{th} is temp dependent, and that slope beyond I_{th} approaches $h f / e$

b) $I = I_{th} + \dfrac{\Phi q}{\eta \frac{hc}{e\lambda}} = 5\,\text{mA} + \dfrac{8\,\text{mW}}{0.8} \Big/ \left(\dfrac{1.6\times10^{-19} \times 1.3\times10^{-6}}{6.63\times10^{-34} \times 3\times10^{8}} \right)$

$= 15.45\,\text{mA}$

c) Longitudinal modes have $m\frac{\lambda}{2} = L$ for integers m.

$\therefore \lambda_m = \dfrac{2L}{m} \quad \lambda_{m+1} = \dfrac{2L}{m+1} \quad \Delta\lambda = 2L\left(\dfrac{1}{m} - \dfrac{1}{m+1}\right)$

$\Delta\lambda = \dfrac{-2L}{m(m+1)} \simeq -2L/m^2$

Since $\dfrac{\Delta\lambda}{\lambda}$ is small, $\left|\dfrac{\Delta f}{f}\right| = \left|\dfrac{\Delta\lambda}{\lambda}\right|$, $\Delta f = \dfrac{f}{\lambda}\left(\dfrac{2L}{m^2}\right)$

but $f = c/\lambda_0 = nc/\lambda$, so $\Delta f = \dfrac{nc(2L)}{(\lambda m)^2}$

$\Delta f \simeq nc(2L)/(2L)^2 = \dfrac{nc}{2L} \quad \dfrac{1}{\Delta f} = \dfrac{2L}{nc}$ = round trip time.

d) Should mention:

i) $\eta_i \rightarrow$ radiative vs non-rad recomb,
$\eta_{ext} \rightarrow$ absorption, Fresnel refl. TIR

ii) $\eta_e \rightarrow$ internal absorption vs emission at mirrors per pass
$\eta_i \rightarrow$ spont. vs stim emission, rad vs non-rad.

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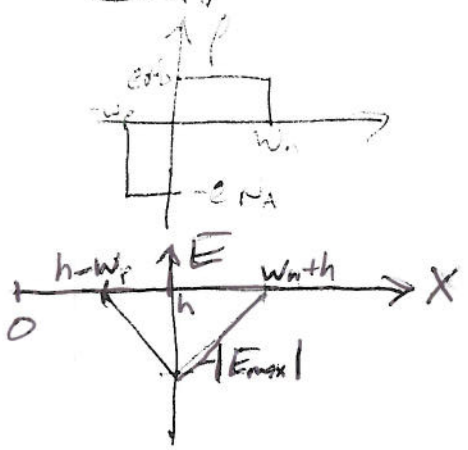
6 a)



cons. of charge

$$w_p N_A = w_n N_D \qquad w = w_p\left(1+\frac{N_A}{N_D}\right)$$

$$|E_{max}| = \frac{e}{\varepsilon} N_A w_p \qquad \left(E = \frac{e}{\varepsilon}\int \rho\, dx\right)$$

$$V_b = -\int E\,dx = \frac{w|E_{max}|}{2}$$

$$= (w_n + w_p)\frac{eN_A}{\varepsilon}w_p = \frac{eN_A}{2\varepsilon}w_p^2\left(1+\frac{N_A}{N_D}\right)$$

$$= \frac{eN_A w^2}{2\varepsilon(1+N_A/N_D)} \therefore w = \sqrt{\frac{2\varepsilon V_b(1+N_A/N_D)}{eN_A}}$$

$$V_b = \frac{1.6\times10^{-19}\cdot 10^{21}\cdot(4\times10^{-6})^2}{2\times12\times8.85\times10^{-12}(1+2.5)} = 3.4\text{V}$$

b) $\eta = e^{-\alpha x_1} - e^{-\alpha x_2} \qquad x_1 = h - w_p,\ x_2 = h + w_n$

$$\eta = e^{-\alpha h}\left[e^{\alpha w_p} - e^{-\alpha w_n}\right]$$

$V_b = 10\text{V} \quad w = \sqrt{\dfrac{2\times12\times8.85\times10^{-12}\times4\times(1+2.5)}{1.6\times10^{-19}\times10^{21}}} = 6.82\ \mu\text{m}$

$w_p = \dfrac{6.82}{3.5} = 1.95\ \mu\text{m} \quad w_n = 2.5w_p = 4.87\ \mu\text{m}$

$$\eta = \exp(-.08\times\cancel{4}^{2.5}_{1})(\exp(.08\times1.95) - \exp(-.08\times4.87))$$
$$= 0.402$$

✓ (40.2%)

c) Assuming drift velocity $v_d \propto E$ (no saturation) then $v_d = \mu_e E \quad \mu_e$ = electron mobility

$V_{dmax} = \mu_e |E_{max}|$

$V_B = \frac{1}{2}|E_{max}|w$

but $w \propto \sqrt{V_b}$

$\therefore |E_{max}| \propto \dfrac{V_b}{\sqrt{V_b}} \propto \sqrt{V_b} \quad \therefore V_{dmax} \propto \sqrt{V_b}$