IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

EEE PART II: MEng, BEng and ACGI

POWER, FIELDS AND DEVICES

Corrected Copy

Tuesday, 31 May 2:00 pm

Time allowed: 3:00 hours

There are NINE questions on this paper.

There are three sections. Answer FIVE questions including at least ONE question from each of sections A, B and C.

Use a separate answer book for each section.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible   First Marker(s) : T.C. Green, K.D. Leaver, R.R.A. Syms

Second Marker(s) : C.A. Hernandez-Aramburo, C.A. Hernandez-Arambu

Information for Candidates

Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho \quad ; \quad \mathbf{D} = \varepsilon \mathbf{E} \qquad\qquad \nabla \cdot \mathbf{B} = 0 \quad ; \quad \mathbf{B} = \mu \mathbf{H}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Physical constants:

$$\varepsilon_0 = 8.854 \text{ x } 10^{-12} \text{ F/m}$$

$$\mu_0 = 4\pi \text{ x } 10^{-7} \text{ H/m}$$

The Questions

Section A

Use a separate answer book for this section

1. A 4-pole induction machine is rated as follows: 415 V line; 50 Hz; 3-phase; star-connected. The equivalent circuit parameters, referred to the stator, are:

Stator resistance	1.4 Ω
Referred rotor resistance	0.7 Ω
Stator leakage reactance	2 Ω
Referred rotor leakage reactance	0.8 Ω
Magnetising reactance	50 Ω
Friction and windage loss	300 W

Iron losses are to be neglected.

a) Draw an equivalent circuit for the machine. [3]

b) Calculate the following for a slip of 0.03:

i) speed; [1]

ii) total input impedance; [4]

iii) stator current; [2]

iv) referred rotor current; [3]

v) electro-magnetic torque; [3]

vi) net mechanical power; [2]

vii) efficiency. [2]

Section A cont'd

2. a) Explain why a neutral conductor is not always necessary for a 3-phase supply and under what circumstances it is necessary. [3]

   b) A 415 V, 3-phase supply is connected to a load composed of three impedances of 10+j25 Ω connected in star. Calculate the line current and the total real power consumed by the load. [3]

   c) Discuss why AC systems are widely used for power distribution and transmission. Explain where and why DC systems are sometimes used. [3]

   d) By considering the power losses in a power transmission line, discuss why a voltage source system is expected to be more efficient than a current source system. [2]

   e) For the following switch-mode power supplies and input (V_I) and output (V_O) voltages, calculate the required duty-cycle for continuous mode operation.

      i) Buck; $V_I = 7$ V, $V_O = 5$ V

      ii) Boost; $V_I = 5$ V, $V_O = 7$ V

      iii) Flyback; $V_I = 7$ V, $V_O = -5$ V [3]

   f) Explain why steel is used as the main material for the stator and rotor of an electrical machine and why the air-gap between them must be kept small. [3]

   g) Explain why an electrical machine has many coils distributed around the circumference of the air-gap and why these are then connected into series groups (or phases). [3]

Section A cont'd

3. A buck switch-mode power supply, SMPS, is to be designed to provide a 5 V output voltage from an input voltage than is nominally 9 V but can vary by ±1.5 V. The output current will be in the range 0.1 A to 2.0 A. It has been decided that the switching frequency of the SMPS should be 250 kHz.

   a) Calculate the range of duty-cycle required. [3]

   b) The proposed inductor is 100 μH. Determine whether the SMPS remains in continuous conduction across the full operating range. [4]

   c) For operation at the nominal input voltage, calculate the maximum value of effective series resistance, ESR, of the capacitor that can be allowed if the output voltage ripple is to remain below 20 mV. [3]

   d) Calculate the capacitance value for the capacitor such that the output voltage ripple attributed to the capacitance is 200 μV. [3]

   e) Consider now that the semiconductor devices used in the Buck SMPS are not perfect. The diode has a forward voltage drop during conduction $V_{AK(on)}$ of 0.7 V. The switched used is a MOSFET with an on-state resistance, $R_{DS(on)}$ of 200 mΩ.

      i) Estimate the power loss in the SMPS at maximum output current and nominal input voltage. [4]

      ii) By noting that the change of inductor current in the on time plus that in the off time sum to zero in steady state, derive an expression for the output voltage as a function of input voltage and duty cycle that accounts for the semiconductor voltage drops. You may assume continuous inductor current conduction. [3]

Section B

Use a separate answer book for this section

4. Four high-voltage wires are supported parallel to one another in air, all in the same x-z plane, as shown in cross-section in Figure 4.1. Wires A & B lie a distance d from the origin, while C & D are a distance c from it. They each carry charges $+q$ or $-q$ per unit length, the charge signs being as shown in Figure 4.1.

   a) Find an expression for the electrostatic force on each of the wires A and B, in terms of the charge q per unit length and the two lengths c and d, and show that the force is exactly zero when

   $$c = d(2+\sqrt{5})$$ [6]

   b) Find a *vector* expression for the electric field strength **E** at a point in space in terms of its coordinates (x,y), and use it to prove that both the y-component and the x-component are zero at the point $(0,a)$, where $a = \sqrt{cd}$. [8]

   c) Sketch a graph of the x-component, $E_x(0,y)$, of the electric field on the y-axis, against y, and use it to help you to draw a sketch showing flux lines between and around the charged wires. [6]

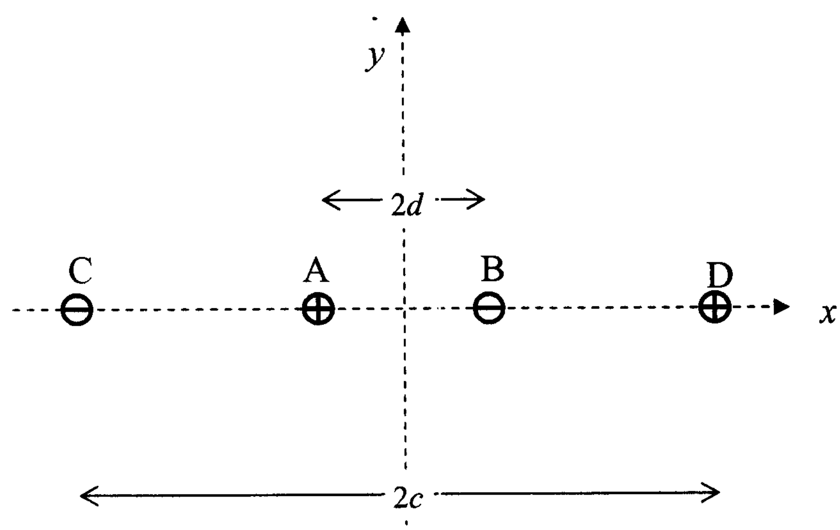


Figure 4.1

Section B cont'd

5. a) Describe what you understand by the term 'skin depth', and use it to explain briefly but carefully, if necessary with the aid of a diagram, why a closed conducting box with thin walls is not a perfect screen of electromagnetic radiation at low frequency, but is highly effective at radio frequencies. You may quote any equations needed to support your explanation. [8]

b) A magnetic field of angular frequency ω inside a metal of conductivity σ is given by the vector equation:

$$\mathbf{H} = (H_{0x}, 0, 0)\exp[j(\omega t - kz)]$$

where H_{0x} and k are constants.

Evaluate curl($\mathbf{H}$), and use a Maxwell equation to show that the corresponding electric field vector is given by

$$\mathbf{E} = \left(0, \frac{-jkH_{0x}}{\sigma}, 0\right)\exp[j(\omega t - kz)]$$ [7]

c) Given that the intrinsic impedance of a metal (its surface impedance) equals $(1+j)p/\sigma$, where p is real, show that the skin depth in the above case equals $(1-j)/k$. [5]

Section B cont'd

6. A strip transmission line has a width b, and a dielectric spacer with a relative permittivity of $\varepsilon_r = 2.0$ and a thickness $d << b$. A sinusoidal voltage wave with an amplitude of 10 V and constant angular frequency ω is travelling along it.

   a) State an equation relating the characteristic impedance of the transmission line to its inductance and capacitance.

   Neglecting fringing fields, deduce an expression for the mean power flowing down the line in terms of the dimensions b and d. [7]

   b) Deduce expressions for the values of the electric and magnetic field strengths in the dielectric spacer as a function of time and distance along the line. You should neglect fringing again. [6]

   c) State an expression for the Poynting vector.

   Evaluate the Poynting vector in the above transmission line and show that it agrees with the result you obtained in part a). [7]

Section C

Use a separate answer book for this section

7. a) Figure 7.1 shows, in plan view, four different types of elastic suspension used in microelectro-mechanical systems (MEMS). Explain briefly how they each operate, and describe any special advantages they might have. [8]

b) A silicon MEMS beam has cross-sectional dimensions of 100 μm x 10 μm. Calculate the value of the second moment of area I for bending of the beam about its two orthogonal axes. A flexure suspension of type i) in Figure 7.1 is fabricated in a 100 μm-thick layer using 10 μm-wide beams, each of length 2 mm. Calculate the lateral stiffness of the suspension. [12]

You may assume the beam bending equation, $d^2y/dx^2 = M/EI$, where x is the distance along the beam, y is the transverse displacement, M is the bending moment, and E is Young's modulus (1.3×10^{11} N/m^2 for Si).

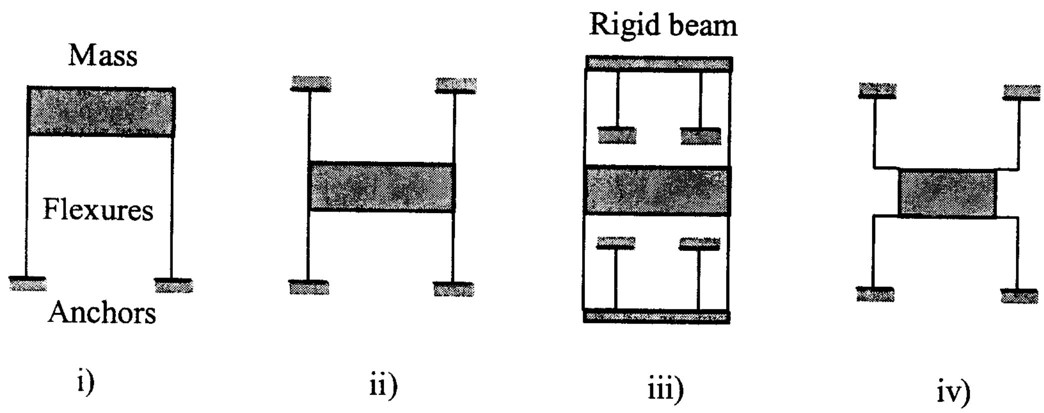


Figure 7.1

Section C cont'd

8. Figure 8.1 shows a silicon micromachined accelerometer with capacitive readout. The comb electrode arrays are not drawn to scale, and in the actual device there are $N = 60$ electrode fingers protruding from the mass. The suspension has an in-plane stiffness of 2.4 N/m. Other key parameters are as follows:

Description	*Symbol*	*Value*
Mechanical layer thickness	h	10 μm
Area of proof mass		0.45 mm^2
Inter-electrode gap	g	5 μm
Electrode overlap	l	300 μm

a) Using a simple parallel-plate model, and ignoring parasitics, estimate the capacitance C_A between terminal A and ground when the mass is in its equilibrium position at zero applied acceleration. Would you expect your estimate to be lower or higher than the actual value? Explain your reasoning. [4]

b) What is the minimum differential capacitance ($C_A - C_B$) that the readout circuit will need to detect if the accelerometer is required to have a resolution of 1 ms^{-2} when operated in open loop? The density of silicon is 2340 kg/m^3. [8]

c) It is decided that the accelerometer should be operated in force-balance mode, with voltages being applied to terminals A and B to hold the mass in a fixed position regardless of applied acceleration. Derive a general expression for the electrostatic holding force, and show that when the control loop is working correctly the force may be expressed as:

$$F = \frac{N\varepsilon_0 lh}{g^2} V_0 V_{BA}$$

where V_0 is the common-mode voltage on the two terminals and V_{BA} is the differential voltage between them. If the common mode voltage is fixed at 5 V, what will be the differential voltage at an applied acceleration of 100 ms^{-2}? [8]

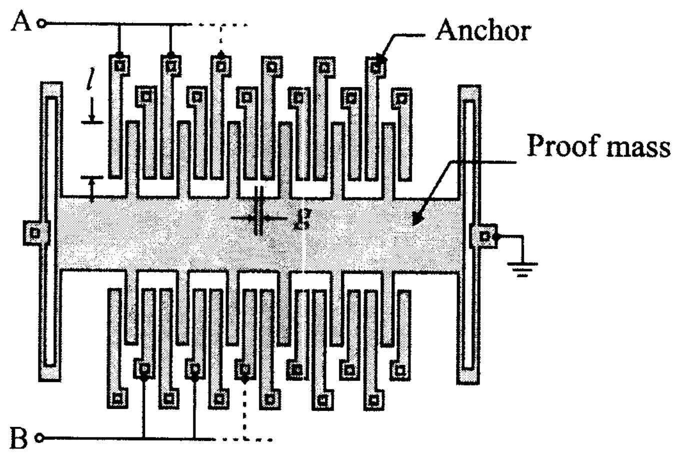


Figure 8.1

Section C cont'd

9\. a) Explain the piezoelectric effect, and sketch a piezoelectric microphone. [6]

b) Describe the construction and operation of a piezoresistive pressure sensor. How is temperature compensation achieved? [8]

c) Explain the Seebeck effect. Table 9.1 below shows the Seebeck coefficients of various materials. Sketch the arrangement of a four-element thermopile, and calculate the voltage generated using platinum and rhodium, when the hot junctions are held at 1000 °C and the cold junctions are at 20 °C. The table suggests that larger voltages would be possible using other material combinations. Explain some potential disadvantages. [6]

Metal	*Seebeck coefficient (μV/K)*	*Metal*	*Seebeck coefficient (μV/K)*
Iron	17.7	Platinum	0.00
Gold	9.20	Palladium	-6.15
Rhodium	8.05	Nickel	-15.50
Aluminium	5.30	Bismuth	-67.85

Table 9.1

POWER FIELDS - DEVICES - SOLUTIONS - 2005
E2.7

1 A 4-pole induction machine is rated as follows: 415 V line; 50 Hz; 3-phase; star-connected. The equivalent circuit parameters, referred to the stator, are:

Stator resistance	1.4 Ω
Referred rotor resistance	0.7 Ω
Stator leakage reactance	2 Ω
Referred rotor leakage reactance	0.8 Ω
Magnetising reactance	50 Ω
Friction and windage loss	300 W

(a) Draw an equivalent circuit for the machine [3]

(b) Calculate the following for a slip of 0.03

(i) speed [1]

(ii) total input impedance [4]

(iii) stator current [2]

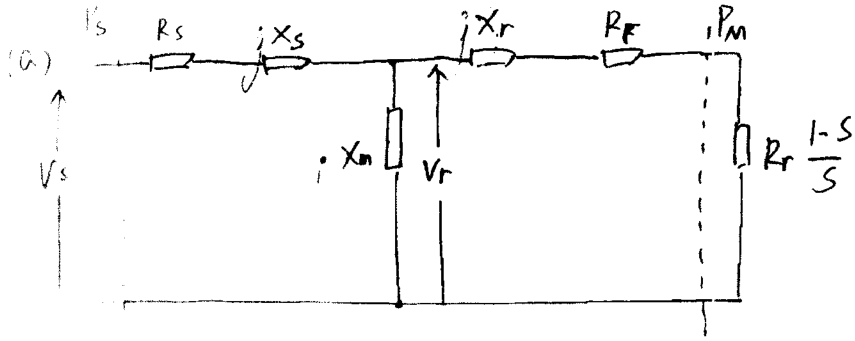
(iv) referred rotor current [3]

(v) electro-magnetic torque [3]

(vi) net mechanical power [2]

(vii) efficiency [2]

(a)



$V_s = \frac{V_{line}}{\sqrt{3}} = \frac{415}{\sqrt{3}}$

$R_S = 1.4\ \Omega$

$X_S = 2\ \Omega$

$R_r = 0.7\ \Omega$

$X_r = 0.8\ \Omega$

$X_m = 50\ \Omega$

$P_{Loss} = 300\,W$

$S = 0.03$

(b)

(i) speed:

Synchronous speed: $\omega_s = \frac{50 \cdot 60}{2} = 1500\ rpm$

Rotor speed: $\omega_R = \omega_s(1-S) = 1500 \cdot (1-0.03) = 1455\ rpm$

(ii) total impedance:

$$Z_{total} = R_s + jX_s + \left[jX_m \,//\, \left(\frac{R_r}{S} + jX_r\right)\right]$$

$$= 20.07 + j11.36 = 23.06 \angle 29.5^\circ\ \Omega$$

(iii) stator current

$$I_s = \frac{V_s}{Z_{total}} = \frac{415/\sqrt{3}}{20.07 + j11.36} = 9.04 - j5.12 = 10.39 \angle -29.5^\circ A$$

(iv) referred rotor current

$$I_r = \frac{V_r}{\frac{R_r}{S} + jX_r} = \frac{V_s - I_s \cdot (R_s + jX_s)}{\frac{R_r}{S} + jX_r} = 9.26 - j0.79 = 9.29 \angle -4.85^\circ A$$

(v) electro-magnetic torque

$$T_e = \frac{P_M}{\omega_R} = \frac{3 \cdot |I_r|^2 \cdot R_r \frac{1-S}{S}}{\omega_R} = \cancel{4.03}\ 38.49\ Nm$$

(vi) net mechanical power

$$P_{NM} = P_M - P_{Loss} = 3 \cdot |I_r|^2 \cdot R_r \frac{1-S}{S} - P_{Loss} = 5564.8\ W$$

(vii) efficiency

$$\eta = \frac{P_{NM}}{P_s} = \frac{P_{NM}}{3 \cdot |V_s| \cdot |I_s| \cdot \cos\varphi} = 85.62\%$$

(a) Explain why a neutral conductor is not always necessary for a 3-phase supply and under what circumstances it is necessary. [3]

- In delta loads the phase voltages are properly defined without a neutral conductor
- In star the phase currents will sum to zero if the phase voltage and phase impedances are balanced and therefore the neutral conductor caries no current and drops no voltage. It can be omitted.
- Only in unbalanced star is a neutral conductor needed to force the phase voltages of the load to match the supply.

(b) A 400 V, 3-phase supply is connected to three impedances of 10+j25 Ω connected in star. Calculate the line current that flows and the real power consumed. [3]

$$V_P = \frac{V_L}{\sqrt{3}} = 230.9V$$

$$I_P = \frac{V_P}{Z_P} = \frac{230.9}{10+j25} = \frac{230.9}{26.93\angle 68.2^\circ} 8.58\angle -68.2^\circ A$$

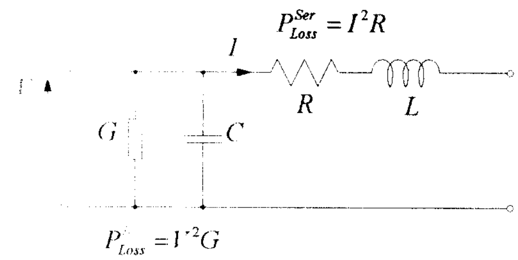
$$P = 3V_P I_P \cos(\phi) = 3\times 230.9\times 8.58\times\cos(-68.2^\circ) = 2.2\,kW$$

(c) Discuss why AC systems are widely used for power distribution and transmission. Explain where and why DC systems are sometimes used. [3]

The traditional way (and still the only cost effective way) of changing voltage levels in a power system is the transformer. Transformers require an AC system because voltage can only be induced by changing flux (E=N.dΦ/dt) and AC is the only way of obtaining continuously changing, but finite, flux. There are exceptions. DC transmission is used to link AC systems of different characteristics and DC is used in some isolated distribution systems such as on large ships

(d) By considering the power losses in a power transmission line, discuss why a voltage source system is expected to be more efficient than a current source system. [2]

A cable or overhead line has a series impedance and a shunt admittance.



The series impedance is the (small) resistance of the copper (or aluminium *etc.*) wire and the inductive reactance of the wire path. The shunt admittance is the (very small) conductivity of the insulators and the capacitive susceptance between wires. Power losses occur in the wire resistance (I^2R) and the insulator conductance (V^2G). In general, insulators are closer to ideal than conductors and the power loss in the insulator is often negligible. Considering only conductor power loss, a voltage source (variable current) system has losses that decrease as the load decreases. A current source system would have constant loss for any load (including no load)

(e) For the following switch-mode power supplies and input (V_I) and output (V_O) voltages, calculate the required duty-cycle for continuous mode operation. [3]

(i) Buck; $V_I = 7$ V, $V_O = 5$ V

(ii) Boost; $V_I = 5$ V, $V_O = 7$ V

(iii) Flyback; $V_I = 7$ V, $V_O = -5$ V

$$\frac{V_O}{V_I} = \delta \quad \delta = \frac{5}{7} = 0.714$$

$$\frac{V_O}{V_I} = \frac{1}{1-\delta} \quad \delta = \frac{7-5}{7} = 0.286$$

$$\frac{V_O}{V_I} = \frac{-\delta}{1-\delta} \quad \delta = \frac{-5}{-5-7} = 0.417$$

(f) Explain why steel is used as the main material for the stator and rotor of an electrical machine and why the air-gap between them must be kept small. [3]

- Desire to produce field flux with as small a winding or magnet as practical
- Need good magnetic path and steel provides this and is a workable material.
- Air-gap is need for clearance between rotating rotor and stationary stator but is a significant reluctance and so is kept short.

(g) Explain why an electrical machine has many coils distributed around the circumference of the air-gap and why these are then connected into series groups (or phases). [3]

- A distributed winding (coils in many slots) makes good use of machine space
- Distributed coils are easier to cool than concentrated windings.
- Individual connections of each coil to a load would be to complex in terms of switches and plugs because number of external connections to loads is reduced.
- Series connection also (by careful design) can cancel harmonic distortion in phase shifted coil voltages to leave phase voltages near sinusoidal.

3 A buck switch-mode power supply, SMPS, is to be designed to provide a 5 V output voltage from an input voltage than is nominally 9 V but can vary by ±1.5 V. The output current will be in the range 0.1 A to 2.0 A. It has been decided that the switching frequency of the SMPS should be 250 kHz.

(a) Calculate range of duty-cycle required [3]

$$\delta = \frac{V_O}{V_I}$$

$$\delta_{Max} = \frac{5}{7.5} = 0.667$$

$$\delta_{Min} = \frac{5}{10.5} = 0.476$$

(b) The proposed inductor is 100 μH. Determine whether the SMPS remains in continuous conduction across the full operating range [4]

Ripple must be less than twice lowest output current to avoid discontinuous operation. Need to find worst case input voltage or check both extremes.

$$\Delta i_L = \frac{V_I - V_O}{L} \cdot \frac{\delta}{f} = \frac{V_I - V_O}{L} \cdot \frac{V_O / V_I}{f} = \frac{V_O}{f\,L}\left(1 - \frac{V_O}{V_I}\right)$$

Largest ripple occurs with largest input voltage. Check the 10.5 V case.

$$\Delta i_L = \frac{V_O}{f\,L}\left(1 - \frac{V_O}{V_I}\right) = \frac{5}{250 \times 10^3 \times 100 \times 10^{-6}}\left(1 - \frac{5}{10.5}\right) = 0.105\,A$$

This less than twice the minimum current and so continuous conduction is ensured in all cases.

(c) For operation at the nominal input voltage, calculate the maximum value of effective series resistance, ESR, of the capacitor that can be allowed if the output voltage ripple is to remain below 20 mV. [3]

$$R_{ESR} = \frac{\Delta v_O}{\Delta i_L} = \Delta v_O \Big/ \frac{V_O}{f\,L}\left(1 - \frac{V_O}{V_I}\right)$$

$$= 20 \times 10^{-3} \Big/ \frac{5}{250 \times 10^3 \times 100 \times 10^{-6}}\left(1 - \frac{5}{9}\right)$$

$$= 225\,m\Omega$$

(d) Calculate the capacitance value for the capacitor such that the output voltage ripple attributed to the capacitance is 200 μV. [3]

$$\Delta v_C = \frac{\Delta i_L}{8fC}$$

$$C = \frac{\Delta i_L}{8f\,\Delta v_C} = \frac{\frac{V_O}{fL}\left(1-\frac{V_O}{V_I}\right)}{8f\,\Delta v_C}$$

$$= \frac{\frac{5}{250\times10^3\times100\times10^{-6}}\left(1-\frac{5}{9}\right)}{8\times250\times10^3\times200\times10^{-6}}$$

$$= 222\ \mu F$$

(e) Consider now that the semiconductor devices used in the Buck SMPS are not perfect. The diode has a forward voltage drop during conduction of $V_{AK(on)}$ 0.7 V. The switched used is a Mosfet with an on-state resistance, $R_{DS(on)}$ of 200 mΩ.

(i) Estimate the power loss in the SMPS at maximum output current and nominal input voltage. [4]

$\delta = 5/9 = 0.556 \quad 1-\delta = 0.444$

Diode loss

$$P_{Diode} = I_{AK}V_{AK(on)}(1-\delta) = 2\times0.7\times0.444 = 0.622\ W$$

Mosfet loss

$$P_{Mosfet} = I_{DS}^{2}R_{DS(on)}\delta = 2^2\times0.2\times0.556 = 0.445\ W$$

(ii) By noting that the change of inductor current in the on time plus that in the off time sum to zero in steady state, derive an expression for the output voltage as a function of input voltage and duty cycle that accounts for the semiconductor voltage drops. You may assume continuous inductor current conduction. [3]

$$\Delta i_L^{On} + \Delta i_L^{Off} = 0$$

$$\Delta i_L^{On} = \delta T\frac{V_I - V_{DS(On)} - V_O}{L}$$

$$\Delta i_L^{Off} = (1-\delta)T\frac{-V_{AK(On)} - V_O}{L}$$

$$V_O = V_I\delta - \left(I_L R_{DS(On)}\delta + V_{AK(On)}(1-\delta)\right)$$

Solution

4 (a) Force per unit length on A in terms of charge q per unit length

$$q\sum_B^D \frac{q'}{2\pi\varepsilon r} = \frac{q^2}{4\pi\varepsilon d} - \frac{q^2}{2\pi\varepsilon(c-d)} - \frac{q^2}{2\pi\varepsilon(c+d)}$$

$$= \frac{q^2}{4\pi\varepsilon d}\left[1 - \frac{4}{\left(\frac{c}{d}-\frac{d}{c}\right)}\right]$$

Force on B is same by symmetry

$= 0$ when $\frac{c}{d} - \frac{d}{c} = 4 \Rightarrow \frac{c}{d} = 2+\sqrt{5}$ [6]

(b) Similarly at (x,y)

$$\underline{E} = \frac{q}{2\pi\varepsilon}\left[\frac{(x+d)\hat{\underline{x}} + y\hat{\underline{y}}}{y^2+(x+d)^2} - \frac{(x-d)\hat{\underline{x}} + y\hat{\underline{y}}}{y^2+(x-d)^2} + \frac{(x-c)\hat{\underline{x}} + y\hat{\underline{y}}}{y^2+(x-c)^2} - \frac{(x+c)\hat{x} + y\hat{\underline{y}}}{y^2+(x+c)^2}\right]$$

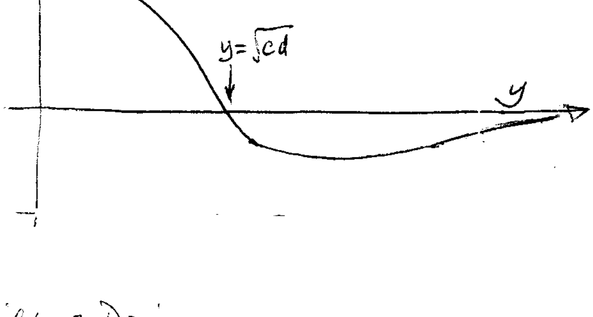
When $x = 0$, $E_y = \frac{q}{2\pi\varepsilon}\left[\frac{y}{y^2+d^2} - \frac{y}{y^2+d^2} + \frac{y}{y^2+c^2} - \frac{y}{y^2+c^2}\right]$

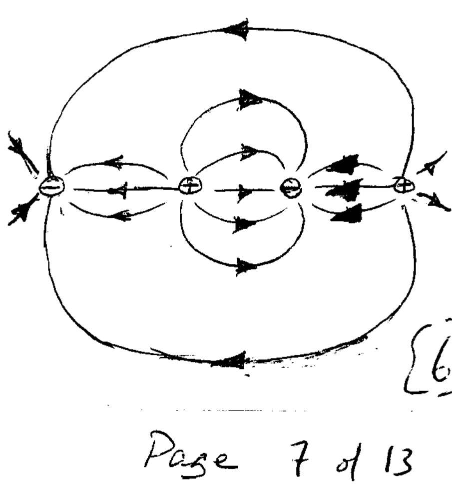
$= 0$

At $(0,a)$: $\frac{2\pi\varepsilon}{q}E_x = \left[\frac{d}{cd+d^2} + \frac{d}{cd+d^2} - \frac{c}{cd+c^2} - \frac{c}{cd+c^2}\right]$

$= \frac{2}{c+d} - \frac{2}{d+c} = 0.$ [8]

(c)





[6]

All problems unseen.

Solution

5. (a) An alternating field at the surface of the conducting wall sets up an e.m. wave in the conductor that decays in amplitude with depth t below the surface as $e^{-t/\delta}$, where δ is the "skin depth". Its magnitude decreases as frequency rises. At low frequency $\delta \gg$ thickness of conductor, so that the e.m. field penetrates to the far surface of the conductor. At high frequencies where $\delta < \frac{1}{2}$ thickness, ~~and then~~ the field at the far surface falls to less than $\frac{1}{10}$th of the amplitude. [8]

Bookwork with unseen variations

(b) $$\underline{\nabla} \times \underline{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\underline{\hat{x}} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\underline{\hat{y}} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\underline{\hat{z}}$$

All terms zero except $\partial H_x/\partial z$, so that

$$\frac{\partial H_x}{\partial z}\underline{\hat{y}} = \sigma \underline{E} \quad \text{(Maxwell's eqn)}$$

Hence $$\underline{E} = \underline{\hat{y}} \cdot \left(-\frac{jk}{\sigma}\right) H_{ox} \exp[j(\omega t - kz)]$$ [7]

As above

(c) Now intrinsic impedance $|E|/|H| = -jk/\sigma = p(1+j)/\sigma \Rightarrow k = p(1+\frac{1}{j})$

$$\Rightarrow \underline{E} = \underline{\hat{y}}\left(-\frac{jk}{\sigma}\right) H_{ox} \exp(-p z)\exp[j(\omega t - p z)].$$

So amplitude of E falls as $\exp(-pz) \Rightarrow \delta = \frac{1}{p} = \frac{k}{(1-j)}$

[5]

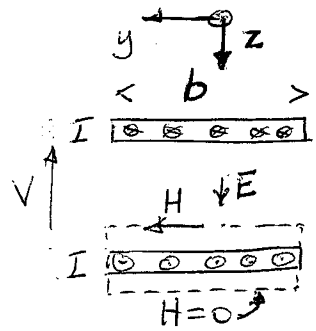
Unseen problem

Solution

6\. (a) $Z_0 = \sqrt{L/C}$ (L, C for same (unit) length)

Assume $V = 10 \exp j(\omega t - kx)$

$$I = \frac{10}{Z_0} \exp j(\omega t - kx)$$



Inductance per unit length is

$L = \dfrac{\mu H d}{I}$ (flux/current)

Amperes law around dotted path $\Rightarrow H = \dfrac{I}{b}$

Capacitance per unit length is

$C = 2\varepsilon_0 b/d$ Hence $\sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{\mu d}{b}\dfrac{d}{2\varepsilon_0 b}} = \dfrac{d}{b}\sqrt{\dfrac{\mu_0}{2\varepsilon_0}}$

Unseen combination of 3 bookwork items

Hence $\overline{VI} = \dfrac{10^2}{2Z_0} = 50\dfrac{b}{d}\sqrt{\dfrac{2\varepsilon_0}{\mu_0}}$ [7]

(b) $|E| = \dfrac{V}{d} = \dfrac{10}{d} \exp j(\omega t - kx)$ — say along z axis

$|H| = \dfrac{I}{b} = \dfrac{10}{\cancel{b}} \cdot \dfrac{\cancel{b}}{d}\sqrt{\dfrac{2\varepsilon_0}{\mu_0}} \exp j(\omega t - kx)$ — along y axis

[6]

Bookwork

(c)

Poynting Vector $= \underline{E} \times \underline{H} = (E_z H_y - E_y H_z)\hat{\underline{x}}$ $\Rightarrow$ power per unit area of cross-section

$$= \frac{10}{d} \cdot \frac{10}{d}\sqrt{\frac{2\varepsilon_0}{\mu_0}} \cos^2(\omega t - kx)$$

$$= \frac{50}{d^2}\sqrt{\frac{2\varepsilon_0}{\mu_0}}\left(1 + \cos 2(\omega t - kx)\right)$$

Mean value of this $\times$ area bd equals value in (a). [7]

Bookwork but in an unseen situation

a) i) Portal frame: Linear suspension, primarily moving in the x-direction. Is not sensitive to intrinsic stress. The use of two parallel beams linked by a stiffer element prevents rotation. [2]

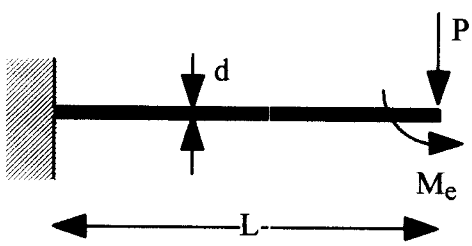
ii) Hammock: Non-linear (stiffening) suspension, primarily moving in the x-direction. Offers increased out-of-plane stiffness. Is sensitive to intrinsic stress. [2]

iii) Folded hammock: Linear suspension, moving primarily in the x-direction. Offers increased out-of-plane stiffness. Is not sensitive to intrinsic stress. [2]

iv) Crab-leg: Linear suspension, moving primarily in the x-direction, but allowing out-of-plane motion by torsion of long beams. Is slightly sensitive to intrinsic stress. [2]

b) The second moment of area of a rectangular section is $I = bd^3/12$, where b and d are the breadth and depth of the beam. For the silicon beam with dimensions 10 μm x 100 μm, the two possible values are:
$I_1 = 100 \times 10^3 \times 10^{-24}/12 = 8.33 \times 10^{-21}\ m^4$
$I_2 = 10 \times 100^3 \times 10^{-24}/12 = 8.33 \times 10^{-19}\ m^4$. [2]

The portal frame problem may be solved by assuming that the horizontal link bar is stiff enough to prevent end-rotation of either of the two flexible beams. For each beam, the beam bending equation must be solved by assuming a transverse end-load P, together with an end-moment M_e sufficient to prevent rotation.



Thus, $M(x) = -P(L - x) + M_e$ and the beam bending equation is:
$d^2y/dx^2 = M/EI = \{-P(L - x) + M_e\}/EI$
Integrating twice we obtain:
$y = \{-P(Lx^2/2 - x^3/6) + M_e x^2/2 + Ax + B\}/EI$

From the boundary conditions at $x = 0$ ($y = 0$, $dy/dx = 0$) we obtain $A = B = 0$
From the boundary conditions at $x = L$ ($dy/dx = 0$) we can find M_e as $M_e = PL/2$
Hence, the complete solution is $y = -P(Lx^2/4 - x^3/6)/EI$
So the end deflection is $y(L) = -PL^3/12EI$ and the stiffness is $k = 12EI/L^3$ [6]
For the two beams together, $k = 24EI/L^3$. The beams will be arranged to bend about the weaker axis.
If $E = 1.3 \times 10^{11}\ N/m^2$ and $I = 8.33 \times 10^{-21}\ m^4$ and $L = 2 \times 10^{-3}$ m
$k = 24 \times 1.3 \times 10^{11} \times 8.33 \times 10^{-21}/8 \times 10^{-9}$ N/m = 3.25 N/m or 3.25 μN/μm. [4]

a) According to parallel plate model, capacitance between adjacent electrode fingers is $\varepsilon_0 lh/g$, so total capacitance between terminal A and ground is $C_A \approx N\varepsilon_0 lh/g$. With $N = 60$, $l = 300$ µm, $h = 10$ µm, $g = 5$ µm this gives $C_A \approx 0.32$ pF.

Fringing will make actual capacitance higher, as will parasitic capacitance through substrate, so estimate is expected to be on low side. [4]

b) Differential capacitance under applied acceleration is:

$$C_A - C_B = N\varepsilon_0 lh\left\{\frac{1}{g+x} - \frac{1}{g-x}\right\} \approx -\frac{2N\varepsilon_0 lh}{g^2}x = -2C_A\frac{x}{g}$$

where x is the displacement of mass relative to substrate (+ve displacement to right). For steady state acceleration a, $x = -ma/k$ where m = proof mass and k = suspension stiffness.

For device in question, m = 2340 x 0.45e-6 x 10e-6 = 1.05e-8 kg, so steady state displacement at acceleration of +1 ms^{-2} is x = -4.375 nm. Corresponding differential capacitance is $C_A - C_B \approx 0.56$ fF. [8]

c) Closure force for single parallel plate actuator (from memory / virtual work / force calc) is $\frac{1}{2}V^2\partial C/\partial g = -A\varepsilon_0 V^2/(2g^2)$. So, general expression for holding force under applied voltages V_A and V_B is:

$$F = -\frac{1}{2}N\varepsilon_0 lh\left\{\frac{V_A^2}{(g+x)^2} - \frac{V_B^2}{(g-x)2}\right\}$$

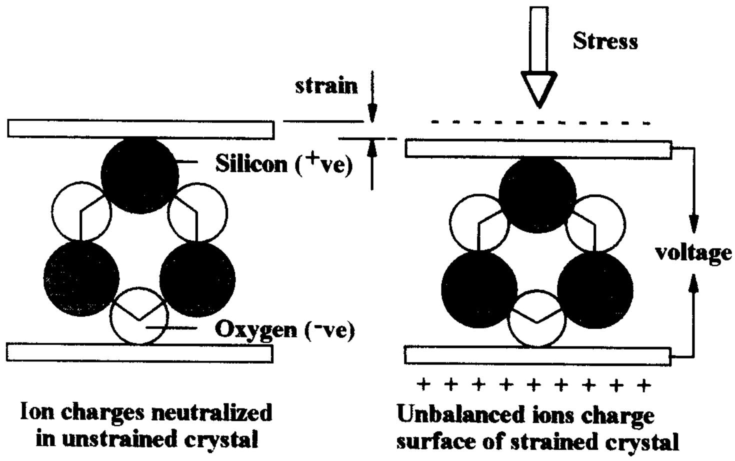
If control loop is working correctly then x = 0 and this reduces to:

$$F = -\frac{N\varepsilon_0 lh}{2g^2}\left(V_A^2 - V_B^2\right) = \frac{N\varepsilon_0 lh}{g^2}V_0 V_{BA} = \frac{C_A V_0}{g}V_{BA}$$

where $V_0 = (V_A + V_B)/2$ is the common-mode voltage and $V_{BA} = V_B - V_A$ is the differential voltage. For an applied acceleration of 100 ms^{-2}, the required force is ma = 1.05 µN, and with V_0 = 5 V the required differential voltage is V_{BA} = 3.3 V. [8]

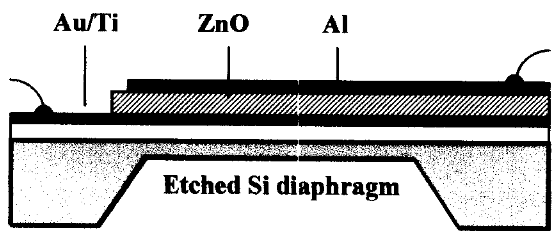
9

a) Piezoelectricity is a tensor effect that occurs only in certain non-centrosymmetric crystalline materials such as quartz (crystalline SiO_2) and ZnO. The effect is reversible, and may be used for sensing and actuation. The application of stress to the crystal creates a surface charge. Because piezoelectricity is a tensor effect, the result depends on the crystal orientation and the direction of the stress. For example, in quartz as shown below the application of stress causes -ve oxygen ions to move closer to the top surface, while +ve silicon ions move closer to the bottom. Similarly, the application of an electric field causes the ions to move so that the crystal expands or contracts, thus creating a strain.



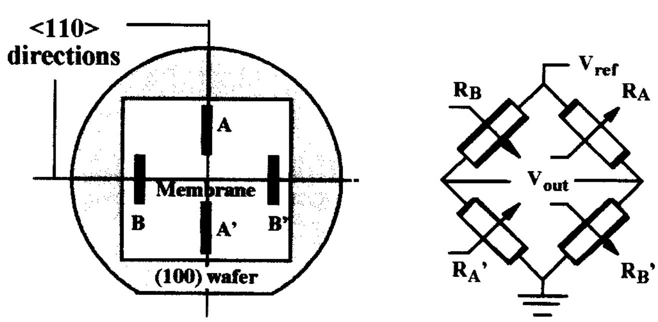
[4]

In a microphone, a piezoelectric layer is deposited on a flexible diaphragm, together with suitable electrodes. The pressure wave accompanying e.g. speech flexes the diaphragm, together with the piezoelectric layer, thus generating a voltage across the electrodes.

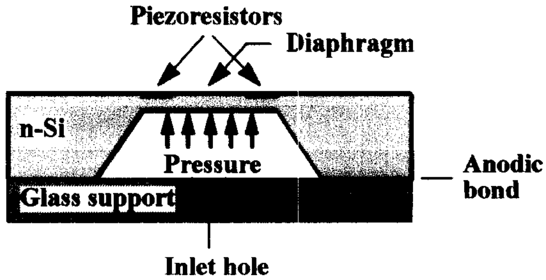


[2]

b) Piezoresistive pressure sensors use diffused channel resistors to read the strain caused by a pressure difference acting across a silicon diaphragm. Usually four resistors are connected in a Wheatstone bridge arrangement.



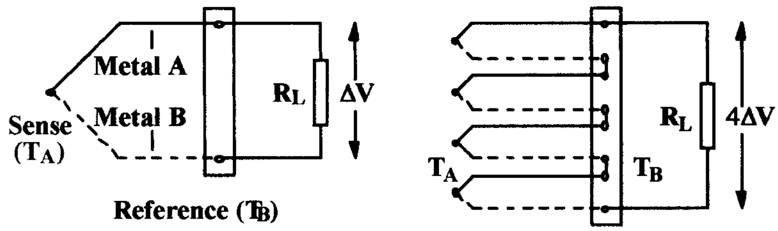
9 cont'd



[6]

Temperature compensation is achieved by using identical resistors (which respond equally to temperature). However, the resistors are placed so that two are affected by transverse strains and two are affected by longitudinal strains, so that deflection of the membrane generates a voltage based on differences in the tensor piezoresistance coefficients. [2]

c) The Seebeck effect refers to the voltage generated in a circuit formed from two dissimilar metals, whose junctions are held at different temperatures. The voltage generated is $\Delta V = (S_A - S_B)\,\Delta T$, where S_A and S_B are the Seebeck coefficients of the two materials and ΔT is the temperature difference between the junctions.



[3]

In a thermopile, larger voltages may be generated from a cascade of junction pairs as shown. For a 4-element platinum-rhodium thermopile, the voltage generated with the temperatures given is $\Delta V = 4 \times (8.05 - 0) \times (1000 - 20) \times 10^{-6} = 0.0316$ V, or 31.6 mV. [2]

Other material combinations may give larger voltages. However, potential issues with high temperature operation are melting point, differences in thermal expansion coefficient (which may cause the joint to shear) and material stability. [1]