Microwave technology

The Solutions for E3.18 and AO12, 2016

<u>Model answer to Q 1(a): Bookwork</u>

i) Faraday Law of Electromagnetic Induction

$$\nabla x\hat{E} = -\frac{\partial \hat{B}}{\partial t} \Rightarrow -j\omega\mu\hat{H} \quad [V/m]$$

$$\textit{where constitutive relationship,} \quad \hat{B}\,[T \text{ or } Wb/m^2] = \mu\,[H/m]\,\hat{H}\,[A/m]$$

i.e. electromotive force induced in a closed circuit is proportional to the rate of change in the magnetic flux density threading the circuit.

[3]

ii) Ampere's Law of Magnetomotive Force

$$\nabla x\hat{H} = \hat{J}c + \hat{J}_D \Rightarrow \sigma_o\hat{E} + \hat{J}_D \quad [A/m^2]$$

$$\textit{where} \quad \hat{J}c\,[A/m^2] = \sigma_o\,[S/m]\,\hat{E}\,[V/m] \Rightarrow 0 \quad \text{with in a perfect insulator}$$

i.e. conduction current creates a closed loop of magnetic field. Note that conduction current is simply a surface current density, where the surface is transverse to the direction of current flow (i.e. *width x depth*). In addition, Maxwell discovered that a magnetic field can also be created by a displacement current. Note that displacement (or electric flux density) is simply a surface charge density.

$$\hat{J}_D = \frac{\partial \hat{D}}{\partial t} = \varepsilon\frac{\partial \hat{E}}{\partial t} \Rightarrow j\omega\varepsilon\hat{E} \quad [A/m^2]$$

$$\textit{where constitutive relationship,} \quad \hat{D}\,[C/m^2] = \varepsilon\,[F/m]\,\hat{E}\,[V/m]$$

For example, when a time-varying conduction current flows through the leads of a parallel-plate capacitor, then an equal displacement or "wireless" current must also flow between its plates, thus creating a closed loop of magnetic field between the plates.

[3]

iii) Gauss's Law: Electric form

$$\nabla\cdot\hat{D} = \rho \Rightarrow 0 \quad \text{with no stored charges} \quad [C/m^3]$$

i.e. an E-field is created by a stored electric charge. Note that a volume charge density is used here.

[2]

iv) Gauss's Law: Magnetic form

$$\nabla\cdot\hat{B} = 0 \quad [Wb/m^3]$$

i.e. a H-field is not created by a stored magnetic charge. Therefore, it must exist in a closed loop.

[2]

Model answer to Q 1(b): Bookwork

The electromagnetic wave is periodic in both time (t) and space (z), i.e. $e^{(j\omega t-\gamma z)}$ where γ = jq and q = complex wavenumber.

Therefore:

$$\nabla x\hat{E} = -\frac{\partial \hat{B}}{\partial t} \Rightarrow -jqE = -j\omega\mu H$$

$$\therefore \eta = \frac{E}{H} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{jq}$$

$$but, q = \frac{2\pi}{\lambda} = \frac{\omega\sqrt{\varepsilon_r}}{c} = \omega\sqrt{\mu\varepsilon_o\varepsilon_r}$$

$$\therefore \eta = \sqrt{\frac{\mu_o\mu_r}{\varepsilon_o\varepsilon_r}}$$

[3]

Model answer to Q 1(c): Bookwork

$$\gamma = \frac{j\omega\mu}{\eta} = \alpha + j\beta$$

$$skin\ depth = \frac{1}{\alpha}$$

$$wavelength = \frac{2\pi}{\beta}$$

[2]

Model answer to Q 1([illegible]ookwo[illegible]

Given ε_r [illegible]53 and σ_o = [illegible] at 15 GH[illegible]

It is assum[illegible] that [illegible] distille[illegible] water is treated as a homogeneous and isotropic material. Also, distille[illegible] water [illegible] non-ma[illegible]etic material and the following first order approximation is assumed[illegible]

$$\varepsilon_r'' = \frac{\sigma_o}{\omega\varepsilon_o}.$$

$$\varepsilon_o = \frac{1}{c^2\mu_o} \quad and\ c = 3\times10^8\ m/s \quad and\ \mu_o = 4\pi\times10^{-7}\ H/m$$

$$\therefore \varepsilon_o = 8.842\ pF/m$$

$$\therefore \varepsilon_r'' = 37.2$$

$$\therefore \eta = \frac{377}{\sqrt{53 - j37.2}} = 44.7 + j14.1\,\Omega$$

$$\therefore \gamma = \frac{j\omega\mu}{\eta} = 762 + j2411 \quad \therefore \delta = 1.31\ mm \quad and \quad \lambda = 2.61\ mm$$

[5]

Model answer to Q 2(a): Bookwork Derivation

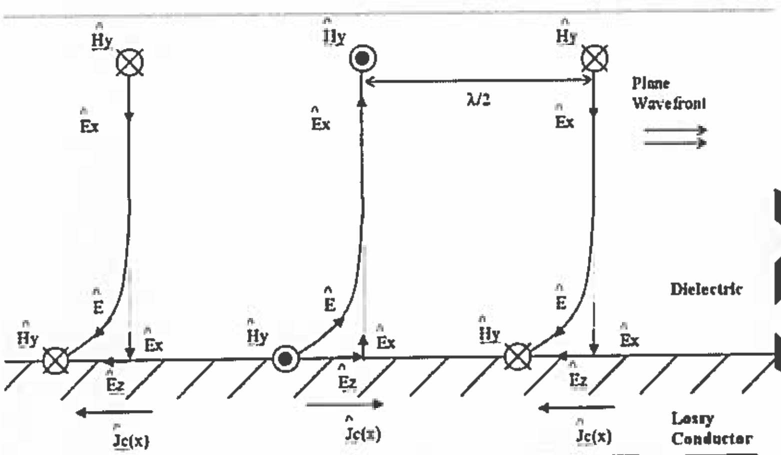
Ez(x=0) = Zs Js, where **Js = n x Hy** and **n** = normal unit vector and Zs = surface impedance

Therefore, when σo is made finite, a tangential electric field exists, since the surface impedance is no longer zero.

[2]

Model answer to Q 2(b): Bookwork

Since Ez(0) exists, the resultant E-field in the normal metal leans forward, just above the surface of the conductor, i.e. **E = xEx + zEz**. Therefore, just above the surface, the wave is not pure TEM, because the E-field, H-field and direction of propagation are not mutually orthogonal.



[5]

Model answer to Q 2(c): Bookwor[illegible]erivation

Well above the surface th[illegible]inting v[illegible] Pz [illegible]-x x Hy.
Now, since Ez and Hy exi[illegible] ins[illegible] the conduct[illegible] a wave can propagate inside this material, i.e. with Poynting vector **Px(x)** [illegible](x) x **Hy**[illegible] whe[illegible]x) = Ez(0)e-γx and Hy = Ez/Zs. If a wave propagates inside th[illegible]al, the asso[illegible]ted E-fiel[illegible]ill induce a conduction current,
Jc(x) = σo [illegible](x)[illegible](0)e-γx.

[5]

Model answe[illegible] Q 2(d[illegible]kwor[illegible]erivation

$$Js = \int_0^{\infty} Jc(x).dx$$

$$Js = \frac{Jc(0)}{-\gamma}\left[e^{-\gamma x}\right]_0^{\infty}$$

$$\therefore Jc(0) = \gamma Js$$

$$\gamma = \sigma_o Zs$$

[4]

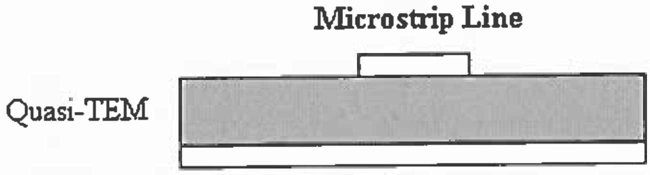
Model answer to Q 2(e): Bookwork Derivation

At the surface of the conductor, the conduction current leads the surface current by 45°, since **Jc(0)** = σo Zs **Js** = σo (√2 $e^{+j\pi/4}$ Rs) **Js**

[4]

Model answer to Q 3(a): Bookwork

(i) microstrip line



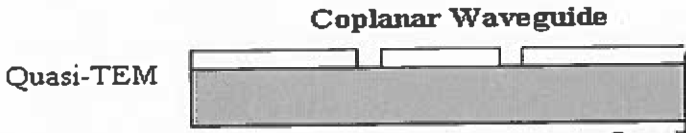
Advantages: easy to manufacture and many models available in CAD software

Disadvantages: frequency dispersive and requires through substrate vias

Applications: general interconnect and impedance matching

[2]

(ii) coplanar waveguide



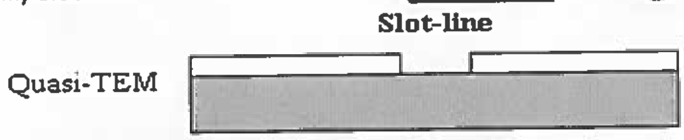
Advantages: easy to manufacture and no through substrate vias are required for grounding

Disadvantages: multi-moding is a problem at discontinuities and models are not so easy to find in CAD software

Applications: highly integrated circuits and m... wave circuits

[2]

(iii) slotline



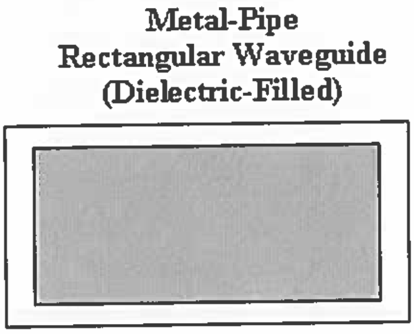
Advantages: easy to manufacture and easy to implement balanced signal lines

Disadvantages: multi-moding is a problem at discontinuities and models are not so easy to find in CAD software

Applications: feed lines for balanced antennas and balanced mixers

[2]

(iv) dielectric-filled metal-pipe rectangular waveguide



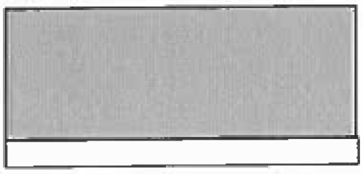
Advantages: very low loss and very high power handling

Disadvantages: frequency dispersive and difficult to integrate active devices

Applications: radio astronomy and radar [2]

(v) image line

Image Line



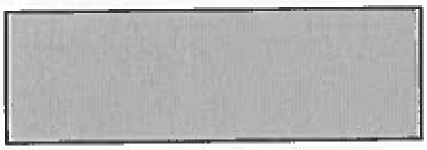
Advantages: very low loss and easy to manufacture

Disadvantages: frequency dispersive and difficult to integrate active devices

Applications: power couplers and low loss millimetre-wave interconnects [2]

(vi) dielectric waveguide

Dielectric Waveguide



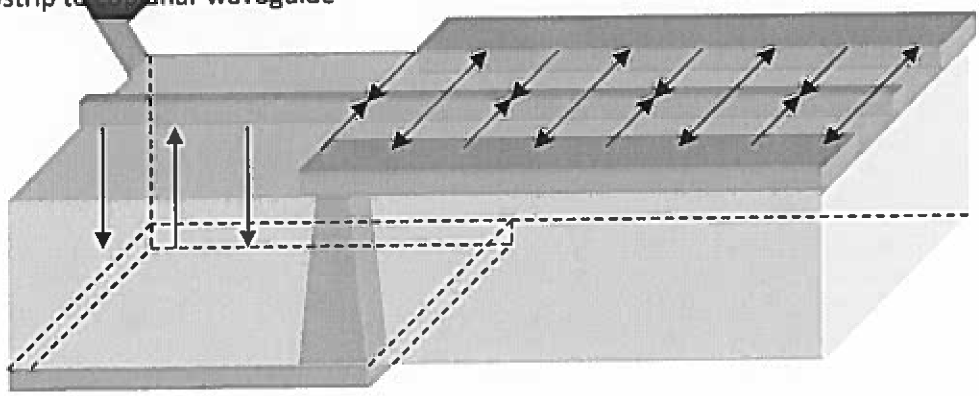
Advantages: very low loss and easy to manufacture

Disadvantages: poor isolation and difficult to integrate active devices

Applications: power couplers and low loss millimetre-wave interconnects [2]

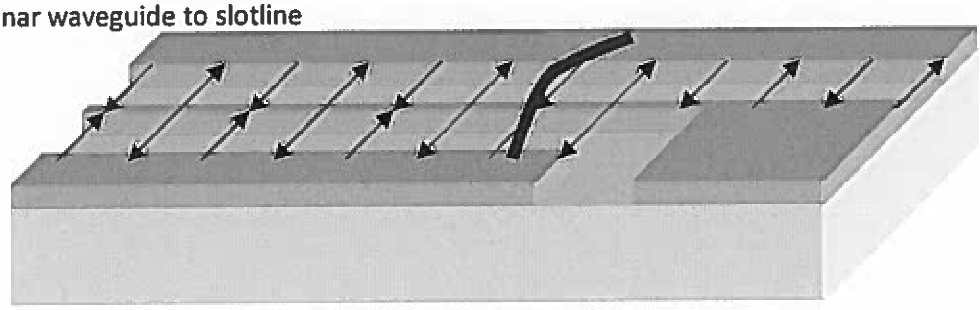
<u>Model answer to Q 3(b): Bookwork</u>

(i) microstrip to coplanar waveguide



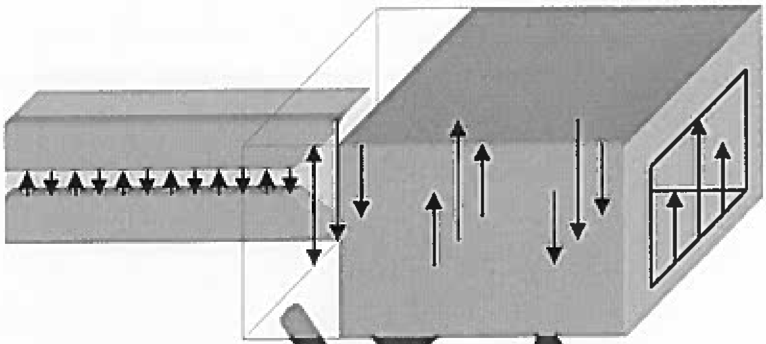
[1]

(ii) coplanar waveguide to slotline



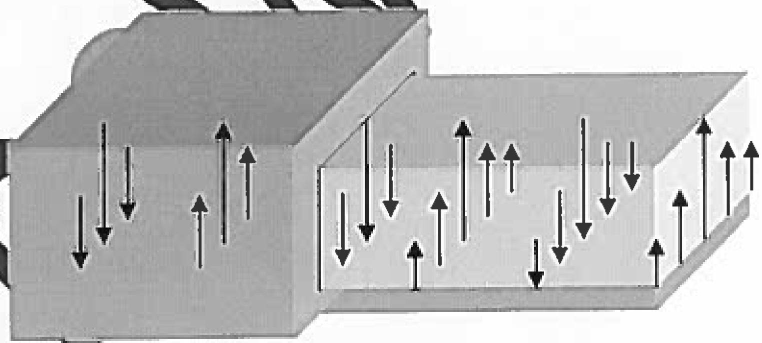
[1]

(iii) slotline to meta-pipe rectangular waveguide



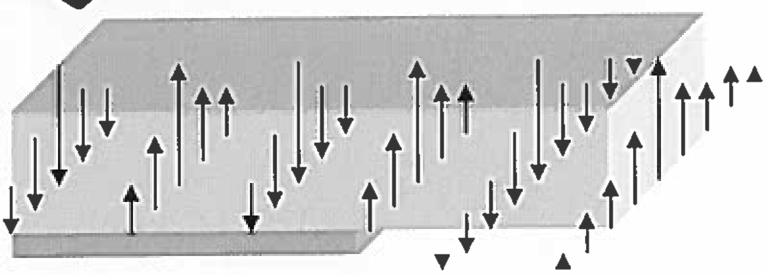
[1]

(iv) dielectric-filled metal-pipe rectangular waveguide to image line



[1]

(v) image line to dielectric waveguide



[1]

Model answer to Q 1(c): Bookwork

At transitions between different types of guided-wave structures, or indeed at any physical and/ or electrical discontinuities, there may be the possibility of creating unwanted modes of propagation. If these unwanted modes are degenerate in nature then they must be suppressed (e.g. by using bondwires/underpasses or resonators). If the unwanted modes are evanescent in nature then they will die away within a few guided wavelengths, and so extra lengths of transmission lines are required to allow this to happen so that they do not have enough energy to excite degenerate modes.

[3]

Model answer to Q 4(a): Bookwork and Derivation Exercise

The voltage and current on the line can be represented as:

$$V(z) = V_+\left(e^{-\gamma z} + \rho(0)e^{+\gamma z}\right)$$
$$I(z) = I_+\left(e^{-\gamma z} - \rho(0)e^{+\gamma z}\right)$$

$$\text{It can be found that}: V_+ = 0.5\left(V(0) + ZoI(0)\right) \quad \text{and} \quad V_- = 0.5\left(V(0) - ZoI(0)\right)$$

$$\therefore \text{incident wave power}, P_+ = \frac{|V_+|^2}{Zo} \quad \text{and} \quad \text{reflected wave power}, P_- = \frac{|V_-|^2}{Zo}$$

If Zo is taken to be purely real, the time-average power flow down the line is:

$$\mathrm{P(z)} = \mathrm{Re}\{V(z)I(z)^*\} = \mathrm{Re}\left\{V_+\left(e^{-\gamma z} + \rho(0)e^{+\gamma z}\right)I_+\left(e^{-\gamma z} - \rho(0)e^{+\gamma z}\right)^*\right\}$$

$$\text{where}, \rho(z) = \rho(0)e^{+2\gamma z} \equiv \rho(0)e^{+j2\beta z} \quad \text{for a lossless line}$$

$$\mathrm{P(z)} = \mathrm{Re}\left\{\frac{|V_+|^2}{Zo}(1+\rho(z))(1-\rho(z))^*\right\} = \mathrm{Re}\left\{\frac{|V_+|^2}{Zo}(1+\rho(z))(1-\rho(z)^*)\right\} = \frac{|V_+|^2}{Zo}\left(1-|\rho(z)|^2\right)$$

$$\text{but}, |\rho(z)| = |\rho(0)| \quad \text{for a lossless transmission line}$$

$$\therefore \mathrm{P(z)} = \frac{|V_+|^2}{Zo}\left(1-|\rho(0)|^2\right) = P_+\left(1-\frac{P_-}{P_+}\right) \equiv \frac{|V_+|^2}{Zo}\left(1-|\rho(0)|^2\right) = P_+\left(1-\frac{P_-}{P_+}\right) = (P_+ - P_-)$$

This shows that, for a lossless transmission line, time-average power flow is independent of the line length and equal to the incident wave power minus the reflected wave power.

[5]

Model answer to Q 4(b): Bookwork and Derivation Exercise

The guided wavelength, λg, is defined as the distance between two successive points of equal phase on the wave at a fixed instance in time. The phase velocity of a wave is defined as the speed at which a constant phase point travels down the line. Frequency dispersion is said to occur when $\beta \neq \omega$.constant. Dispersion can occur when $vp = f(\omega)$, i.e. when $Dk = f(\omega)$. It can be shown that zero dispersion in a lossy line can also occur, but only when $RC = GL$:

$$\gamma^2 = (R + j\omega L)(G + j\omega C) \quad \text{and} \quad RC = GL$$

$$\therefore \alpha(\omega) = \alpha(0) = \sqrt{RG} \neq f(\omega) \quad \text{and} \quad \beta = \omega\sqrt{LC}$$

$$\text{also, Group Velocity}, Vg = \frac{\partial\omega}{\partial\beta} = \frac{1}{\sqrt{LC}} \equiv vp \neq f(\omega)$$

[5]

Model answer to Q 4(c): Bookwork

$$Zin = j\omega L + \frac{Zo\frac{1}{j\omega C}}{Zo + \frac{1}{j\omega C}} \equiv Zo$$

$$\therefore Zo = \frac{j\omega L}{2}\left(1 \mp \sqrt{1 - \frac{4}{\omega^2 LC}}\right) = \frac{j\omega L}{2}\left(1 \mp \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}\right)$$

$\therefore$ *Cut-off frequency*, $fc = \frac{1}{\pi\sqrt{LC}}$ representing the bandwidth, i.e. when $\frac{4}{\omega^2 LC} \equiv 1$

$$Zo = \begin{cases} \sqrt{\frac{L}{C}} & when \quad \omega << \omega_c \quad i.e.\ purely\ real \\ Complex & when \quad 0 << \omega < \omega_c \\ j\sqrt{\frac{L}{C}} & when \quad \omega = \omega_c \quad i.e.\ purely\ imaginary \\ j\omega L & when \quad \omega >> \omega_c \quad i.e.\ purely\ imaginary \end{cases}$$

Note that this cut-off frequency is also referred to as the Bragg frequency, which is twice the cut-off frequency of a traditional lossless LC filter.

[5]

Model answer to Q 4(d): Computed Example

$$Zin(\omega_c) = jZo(\omega << \omega_c) \quad and \quad |\rho(\omega_c)|^2 = 1 \Rightarrow 0\,dB \quad and \quad |\tau(\omega_c)|^2 = 1 - |\rho(\omega_c)|^2 = 0 \Rightarrow -\infty\,dB$$

[5]

Model answer to Q 5(a): Bookwork

The structure has the ideal (i.e. lossless) dominant-mode guided wavelength given by the following textbook expression:

$$\lambda_{g\_ideal} = \frac{\lambda_o}{\sqrt{1 - \left(\frac{\lambda_o}{\lambda_c}\right)^2}} = \frac{\lambda_o}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

where, λ_o is the free space wavelength; $\lambda_c = 2a$ is the ideal cut-off wavelength; a is the internal width dimension of the RRWG; $f_c = c/2a$ is the ideal cut-off frequency for the dominant TE_{10} mode; and c is the speed of light in free space.

[3]

Model answer to Q 5(b): Calculated Example

For a = 200 μm: $f_c = c/(2a)$ =750 GHz and $\lambda_{g\_ideal} = 566.7\,\mu m$

[2]

Model answer to Q 5(c): Bookwork

The corresponding textbook expression for the resonant frequencies for the TE_{mnl} modes in an ideal (i.e. lossless) cavity is given by:

$$\gamma = \hat{x}\gamma_x + \hat{y}\gamma_x + \hat{z}\gamma_z = jk$$

$$k = |\gamma| = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$k_x = \frac{m2\pi}{\lambda_x} \quad \lambda_x = 2a; \quad k_y = \frac{n2\pi}{\lambda_y} \quad \lambda_x = 2b; \quad k_z = \frac{l2\pi}{\lambda_z} \quad \lambda_z = 2a;$$

$$k_{mnl\_ideal} = \frac{2\pi f_{mnl\_ideal}}{c} \quad \therefore f_{mnl\_ideal} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

[4]

<u>Model answer to Q 5(d): New Derivation</u>

For the dominant TE_{101} mode this becomes:

$$f_{101\_ideal} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} = \sqrt{\frac{3}{8}}\left(\frac{c}{a}\right) \text{ when } d = \sqrt{2}a$$

$$f_{101\_ideal} = \sqrt{1.5}f_c \text{ when } d = \sqrt{2}a$$

[2]

<u>Model answer to Q 5(e): Computed Example</u>

For a = 200 μm: b = 100 μm, d = [illegible] μm, f_{101_ideal} = 918 GHz

[2]

<u>Model answer to Q 5(f): Bookwork</u>

$$Q_U = 2\pi\frac{\text{Time-average energy stored at a resonant frequency}}{\text{Energy dissipated in one period of this frequency}} \text{ (Dimensionless)}$$

$$Q_{Uo} = \omega_{oo}\frac{U}{P_{DISS}} \text{ (Dimensionless)}$$

[1]

<u>Model answer to Q 5(g): New Derivation</u>

$$\lambda_{101\_ideal} = \frac{2\pi c}{\omega_{101\_ideal}} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2}} = 2\sqrt{\frac{2}{3}}a \quad for \quad d = \sqrt{2}a$$

[2]

<u>Model answer to Q 5(h): Calculated Example</u>

$$\delta = \sqrt{\frac{2}{\omega_{101\_ideal}\mu_o\sigma_o}} = 78nm \quad at\ 918\,GHz$$

[2]

<u>Model answer to Q 5(i): Calculated Example</u>

$$Q_U = \frac{\lambda_{101\_ideal}}{4\delta}\left\{\frac{2b\left(a^2+d^2\right)^{\frac{3}{2}}}{\left[2b\left(a^3+d^3\right)+ad\left(a^2+d^2\right)\right]}\right\} = \frac{2}{\delta_{So}}\left(\frac{3\sqrt{2}a}{4(5\sqrt{2}+1)}\right) \quad for \quad d = \sqrt{2}a$$

$$\therefore Q \approx 674 \quad for\ a = 200\mu m\ and\ d = \sqrt{2}a$$

[2]

<u>Model answer to Q 6(a): Bookwork</u>

In the kHz region, overly-priced Hi-Fi valve amplifiers are still commercially available.

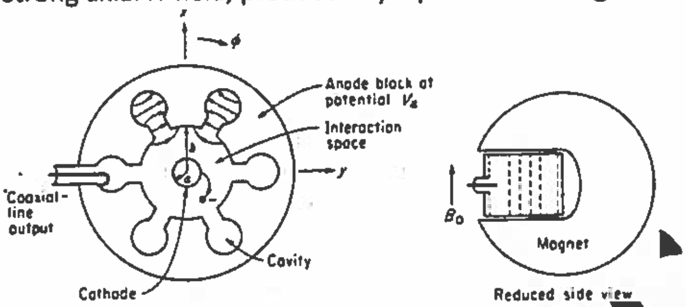
In the MHz region, the cathode-ray tube found in domestic television receivers is still being used in the technologically backward western world.

In the GHz part of the spectrum, the 2.54 GHz magnetron is found in all domestic microwave ovens.

[3]

Model answer to Q 6(b): Bookwork

- The cylindrical surface of the centre electrode is an electron-emitting cathode. The surrounding block forms the anode, containing N-resonant cavities and these are separated by N-segments. The axial length (i.e. cavity depth) is $\lambda o/4$. The complete structure is immersed in a strong axial H-field, produced by a permanent magnet.



- The magnetron is a self-excited oscillator that converts pulsed DC input power into pulsed RF output power. DC-to-RF conversion takes place in the interaction space between the cathode and the anode. The anode is constructed to form a microwave circuit, consisting of a transmission line loaded with cavities. Initially, a DC voltage is applied between the cathode and the microwave circuit. As the DC voltage is increased, electrons leave the heated cathode and interact with the H-field, causing them to circle the cathode. As the voltage is further increased, the angular velocity of the electrons becomes equal to that of the RF wave travelling along the microwave circuit. When this occurs, there is a spontaneous interaction between the electrons and the microwave circuit – resulting in spokes of space charge that induce microwave power into the slow-wave circuit. Here, electrons move through the RF-field of the resonant cavities. These electrons deliver energy to the resonators, which sustain the oscillations. The RF power is extracted by loose coupling into one of the cavities.

[10]

Model answer to Q 6(c): Bookwork

- The normal π-mode of operation (as shown above, right) can be regarded as being formed by the superposition of two waves rotating in opposite directions. For an *N*-cavity magnetron, the angular velocity at which both the electromagnetic wave and the electron spokes rotate is equal to $2\omega_o/N$.

- Modes other than the wanted π-mode are suppressed, by making an electrical connection to alternating segments, using two 'strapping rings' at both ends of the block. At higher frequencies, alternate cavities are made with differing depths – forming the 'rising sun' block. Both of these approaches have the effect of increasing the frequency separation between the wanted and the unwanted modes.

[5]

Model answer to Q 6(d): Bookwork

- Increasing the H-field requires an increased voltage for a given current, and results in a higher RF power output and efficiency.

- Pulsed applications: e.g. ~ 200 KW for airborne radar and ~ 20 MW for ground-based radar. CW applications: e.g. Diathermy and Domestic Microwave Ovens (<1 KW)

[2]