IMPERIAL COLLEGE LONDON

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DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

MSc and EEE/ISE PART IV: M.Eng. and ACGI

DIGITAL IMAGE PROCESSING

Friday, 16 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T. Stathaki
Second Marker(s) : T.J.W. Clarke

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Special instructions for invigilators: None

Information for candidates: None

1. (a) (i) Explain why it is common to work only with unitary transforms. **[1]**

(ii) In a specific experiment it is observed that the amplitude of the Fourier transform of an image exhibits a circular symmetry around the origin. State the implications of this observation as far as the original image is concerned. **[3]**

(b) Let $f(x,y)$ denote an $M \times N$-point two-dimensional (2-D) sequence that is zero outside $0 \le x \le M-1$, $0 \le y \le N-1$. In implementing the 2-D Discrete Hadamard Transform (DHT) of $f(x,y)$, we relate $f(x,y)$ to a new $M \times N$-point sequence $H(u,v)$.

(i) Define the sequence $H(u,v)$ in terms of $f(x,y)$. **[3]**

(ii) Find the Hadamard Transform of the following image

$$\begin{bmatrix} 5 & 6 & 8 & 10 \\ 6 & 6 & 5 & 7 \\ 4 & 5 & 3 & 6 \\ 8 & 7 & 5 & 5 \end{bmatrix}.$$

[Hint: You may use the recursive relation property of the Hadamard matrix $H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}$ knowing that the 2×2 Hadamard matrix is $H_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$].

[6]

(c) Consider the population of vectors $\underline{f}$ of the form

$$\underline{f} = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \\ f_3(x,y) \end{bmatrix}.$$

Each component $f_i(x,y), i = 1,2,3$ represents an image. The population arises from the formation of the vectors across the entire collection of pixels.

Consider now a population of vectors $\underline{g}$ of the form

$$\underline{g} = \begin{bmatrix} g_1(x,y) \\ g_2(x,y) \\ g_3(x,y) \end{bmatrix}$$

where the vectors $\underline{g}$ are the Karhunen-Loeve transforms of the vectors $\underline{f}$.

The covariance matrix of the population $\underline{f}$ calculated as part of the transform is

$$\underline{C}_{\underline{f}} = \frac{1}{25}\begin{bmatrix} 5 & 2 & -1 \\ 2 & 5 & -2 \\ -1 & -2 & 5 \end{bmatrix}$$

with eigenvectors

$$\underline{e}_1 = \begin{bmatrix} 0.4544 \\ -0.7662 \\ -0.4544 \end{bmatrix} \quad \underline{e}_2 = \begin{bmatrix} -0.5418 \\ -0.6426 \\ 0.5418 \end{bmatrix} \quad \underline{e}_3 = \begin{bmatrix} -0.7071 \\ 0 \\ -0.7071 \end{bmatrix}$$

(i) Find the covariance matrix of the population $\underline{g}$. **[4]**

(ii) Suppose that a credible job could be done of reconstructing approximations to the 3 original images by using one or two principal component images. What would be the mean square error incurred in doing so in both cases? Express your answer as a percentage of the maximum possible error. **[3]**

2. (a) An image has the gray level probability density function $p_r(r) = 4r^3$, $0 \le r \le 1$. It is desired to transform the gray levels of this image so that they will have the specified probability density function $p_z(z) = 2z$, $0 \le z \le 1$. Assume continuous quantities and find the transformation that will accomplish this. **[4]**

(b) Let $f(x,y)$ denote an $M \times N$ image that is zero outside $0 \le x \le M-1$, $0 \le y \le N-1$. Suppose that the pixel intensities are represented by 8 bits. The mean and the variance of all the pixels in $f(x,y)$ can be calculated as follows:

$$\mu = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$$

$$\sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (f(x,y) - \mu)^2$$

Let $h(n)$ be the histogram of the 8-bit image, $f(x,y)$, so that the index n ranges from 0 to 255, and $h(n_1)$ is the number of pixels in $f(x,y)$ with value n_1. Derive expressions for calculating the mean and variance of the image using only the histogram $h(n)$. **[7]**

(c) For each of the image disturbances listed below propose an appropriate enhancement technique. Justify your choice, stating clearly any assumptions you made and drawbacks of the proposed techniques.

(i) Salt and pepper noise. **[3]**

(ii) Uncorrelated zero mean additive noise. **[3]**

(iii) Low contrast where edge detail should be enhanced but region information should be preserved. **[3]**

3. We are given the blurred and noisy version $g(x,y)$ of an image $f(x,y)$ such that in lexicographic ordering

$$g = Hf + n$$

where H is the degradation matrix which is assumed to be block-circulant, and n is the noise term which is assumed to be zero mean, independent from the original image and white. Moreover, f and g are the lexicographically ordered original and degraded image respectively.

(a) (i) Consider the Wiener filtering image restoration technique. Write down without proof the expressions for both the Wiener filter estimator and the restored image both in the spatial domain and the frequency domain. Explain all symbols used. **[5]**

(ii) Discuss the relation between Wiener filtering and both inverse and pseudo-inverse filtering. **[5]**

(b) Let $g(x,y)$ be a degraded only by noise image that can be expressed as

$$g(x,y) = f(x,y) + n(x,y)$$

where $f(x,y)$ is the original image and $n(x,y)$ is a background noise. In Wiener filtering we assume that the Fourier transform $S_{ff}(u,v)$ of the autocorrelation function of the original image is available.

One method of estimating $S_{ff}(u,v)$ is to model the autocorrelation function as follows

$$R_{ff}(k,l) = \underset{(x,y)}{E}\{f(x,y)f(x+k,y+l)\} = \rho_1^{|k|}\rho_2^{|l|}$$

with ρ_1, ρ_2 unknown parameters with $|\rho_1| < 1$, $|\rho_2| < 1$. In that case it can be shown that

$$S_{ff}(u,v) = \frac{1-\rho_1^2}{1+\rho_1^2 - 2\rho_1\cos(u)} \cdot \frac{1-\rho_2^2}{1+\rho_2^2 - 2\rho_2\cos(v)}$$

(i) Assuming $n(x,y)$ is a zero mean, white noise with unknown variance σ_n^2 and independent of $f(x,y)$, write down without proof the expressions for the Wiener filter estimator and the restored image in the frequency domain as functions of ρ_1, ρ_2 and σ_n^2. **[5]**

(ii) Develop a method of estimating ρ_1 and ρ_2 using the autocorrelation function samples of $g(x,y)$. **[5]**

4. (a) Consider an image with intensity $f(x,y)$ that can take three possible values A, B, C with the following probabilities shown in Figure 4.1 below.

   (i) Determine the codeword to be assigned to the above set of symbols using differential coding followed by Huffman coding. Specify what the reconstruction level is for each codeword. For your codeword assignment, determine the average number of bits required to represent r. **[6]**

   (ii) Determine the entropy and the redundancy of the Huffman code in (i). **[4]**

(b) In lossless JPEG, one forms a prediction residual using previously encoded pixels in the current line and/or the previous line. Suppose that the prediction residual for pixel with intensity x in the following Figure 4.2 is defined as $r = y - x$ where y is the function $y = \dfrac{a+b}{2}$.

   (i) In which part of the lossless JPEG is Huffman coding used and why? **[6]**

   (ii) Consider the case with pixel values $a = 100$, $b = 191$ and $x = 180$. Find the codeword of the prediction residual y, knowing that the Huffman code for six is 1110. **[4]**

Intensity	**Probability**
A	$\frac{1}{6}$
B	$\frac{2}{6}$
C	$\frac{3}{6}$

Figure 4.1

c	b	
a	x	

Figure 4.2