EE4-65
EE9-SO27

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

WIRELESS COMMUNICATIONS

Monday, 18 May 10:00 am

Time allowed: 3:00 hours

There are THREE questions on this paper.

Answer THREE questions.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : B. Clerckx
Second Marker(s) : K.K. Leung

Important information for students

Notations:

(a) A $n_r \times n_t$ MIMO channel consists in n_r receive antennas and n_t transmit antennas.

(b) a, $\mathbf{a}$, $\mathbf{A}$ denote a scalar, vector and matrix respectively.

(c) $\mathbf{A}^H$ denotes conjugate transpose (Hermitian).

(d) $\mathbf{A}^*$ denotes conjugate.

(e) $\mathbf{A}^T$ denotes transpose.

(f) $|a|$ denotes the absolute value of scalar a.

(g) $\|\mathbf{a}\|$ denotes the (Euclidean) norm of vector $\mathbf{a}$.

(h) "i.i.d." means "independent and identically distributed".

(i) "CSI" means "Channel State Information".

(j) "CSIT" means "Channel State Information at the Transmitter".

(k) "CDIT" means "Channel Distribution Information at the Transmitter".

(l) $\mathscr{E}\{.\}$ denotes Expectation.

(m) $\mathrm{Tr}\{.\}$ denotes the Trace of a matrix.

Assumptions:

(a) The CSI is assumed to be always perfectly known to the receiver.

(b) The receiver noise is a $n_r \times 1$ vector with i.i.d. entries modeled as zero mean complex additive white Gaussian noise with variance σ_n^2.

Some useful relationships:

(a) $\|\mathbf{A}\|_F^2 = \mathrm{Tr}\{\mathbf{A}\mathbf{A}^H\} = \mathrm{Tr}\{\mathbf{A}^H\mathbf{A}\}$

(b) $\mathrm{Tr}\{\mathbf{AB}\} = \mathrm{Tr}\{\mathbf{BA}\}$

(c) $\det(\mathbf{I}+\mathbf{AB}) = \det(\mathbf{I}+\mathbf{BA})$

(d) $\mathrm{Tr}\{\mathbf{ABB}^H\mathbf{A}^H\} = \mathrm{vec}(\mathbf{A}^H)^H(\mathbf{I}\otimes\mathbf{BB}^H)\,\mathrm{vec}(\mathbf{A}^H)$

(e) Gaussian Q-function

$$Q(x) \triangleq P(y \geq x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp\left(-\frac{y^2}{2}\right) dy$$

(f) Craig's formula

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2(\beta)}\right) d\beta$$

(g) Chernoff bound

$$Q(x) \leq \exp\left(-\frac{x^2}{2}\right)$$

(h) The moment generating function of a Hermitian quadratic form in complex Gaussian random variable $y = \mathbf{z}\mathbf{F}\mathbf{z}^H$, where $\mathbf{z}$ is a circularly symmetric complex Gaussian vector with mean $\overline{\mathbf{z}}$ and a covariance matrix $\mathbf{R}_z$ and $\mathbf{F}$ a Hermitian matrix, is given by

$$M_y(s) \triangleq \int_0^\infty \exp(sy)\, p_y(y)\, dy = \frac{\exp\left(s\overline{\mathbf{z}}\mathbf{F}\left(\mathbf{I} - s\mathbf{R}_z\mathbf{F}\right)^{-1}\overline{\mathbf{z}}^H\right)}{\det\left(\mathbf{I} - s\mathbf{R}_z\mathbf{F}\right)}$$

THE QUESTIONS

1. [40]

a) Consider the Downlink Multiuser Multiple-Input Single-Output (MU-MISO) transmission of two independent streams to two independent users. The transmitter is equipped with 4 antennas while each user is equipped with a single receive antenna. The Channel State Information (CSI) is perfectly known to the transmitter. Denoting the vector of transmitted symbols as $\mathbf{c} = [c_1, c_2]^T$ with c_1 and c_2 intended for user 1 and 2 respectively, the received signals are written as $y_1 = \mathbf{h}_1\mathbf{P}\mathbf{c} + n_1$ and $y_2 = \mathbf{h}_2\mathbf{P}\mathbf{c} + n_2$ at user 1 and 2, respectively. The precoder $\mathbf{P}$ is made of two columns $\mathbf{p}_1$ and $\mathbf{p}_2$, each column subject to a power constraint $\|\mathbf{p}_i\|^2 = 1$, $i = 1,2$. The channel vector of user 1 is given by

$$\mathbf{h}_1 = \begin{bmatrix} 1 & 1+j & 1 & 1-j \end{bmatrix}$$

while that of user 2 is given by

$$\mathbf{h}_2 = \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix}.$$

At the transmitter we would like to design the precoder $\mathbf{p}_1$ so as to maximize the SNR at user 1 and guarantee that the multi-user interference is completely nulled out.

i) Derive the expression of the precoder $\mathbf{p}_1$. Provide your reasoning.

[3]

ii) What kind of precoder is this? Explain your result.

[3]

b) Figure 1.1 displays the average Symbol Error Probability of three schemes vs. SNR for point-to-point channels with i.i.d. Rayleigh slow fading. The three schemes are (1) Alamouti scheme without CSIT in a two-transmit one-receive MISO channel with BPSK, (2) matched beamforming (also called transmit MRC) with perfect CSIT in a two-transmit one-receive MISO channel with BPSK, (3) uncoded SISO transmission with BPSK.

i) Map schemes (1), (2) and (3) to curves (a), (b) and (c) in Figure 1.1 and provide your reasoning.

[3]

ii) From the analytical expressions of the error probability of (b) and (c), explain the gap between those two curves.

[5]

c) Figure 1.2 displays the ergodic capacity of point-to-point i.i.d. Rayleigh fast fading channels with Channel Distribution Information at the Transmitter (CDIT) for five antenna ($n_r \times n_t$) configurations (denoted as (a) to (e)) with $n_t + n_r = 7$.

i) What is the achievable (spatial) multiplexing gain (at high SNR) for each of cases (a) to (e)? Provide your reasoning.

[5]

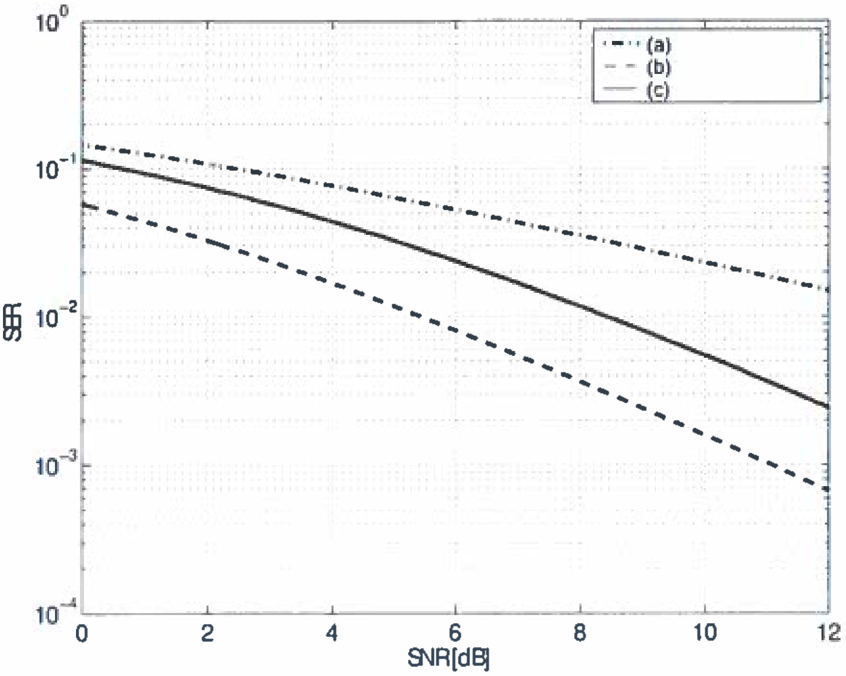

Figure 1.1 Average Symbol Error Probability vs. SNR.

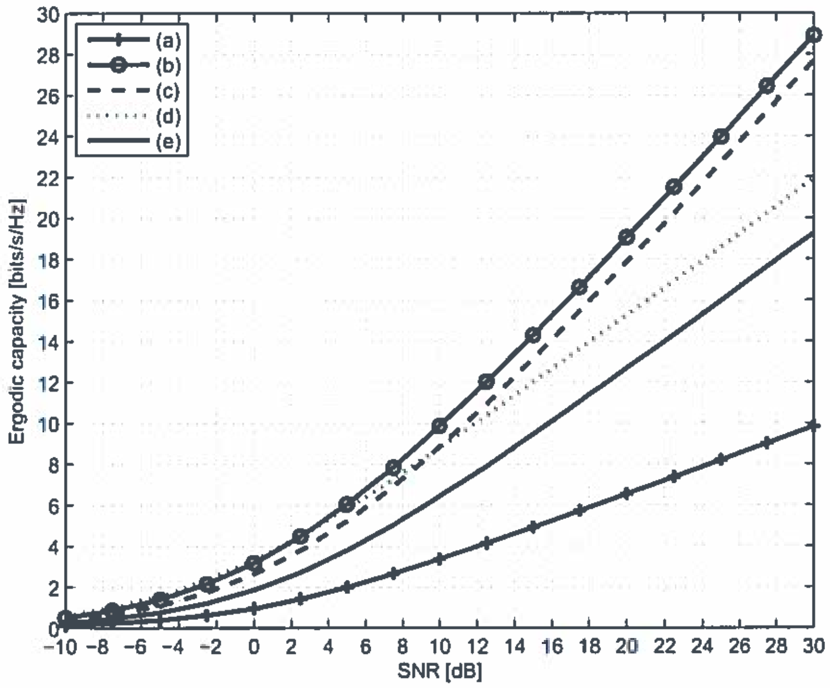

Figure 1.2 Ergodic capacity vs. SNR.

ii) For each of cases (a) to (e), identify the antenna configuration, i.e. n_t and n_r, satisfying $n_t + n_r = 7$ that achieves such multiplexing gain. Provide your reasoning.

[5]

d) Consider two point-to-point 1×2 MISO deployment scenarios (with two transmit antennas) where the channel can be modeled as Rayleigh fading with a spatial correlation matrix $\mathbf{R}_{t,1}$ and $\mathbf{R}_{t,2}$, respectively. In the first deployment, the angle spread and antenna spacing are 4° and 0.5λ (with λ being the wavelength), respectively. In the second deployment, they are 20° and 4λ, respectively.

i) Which deployment would lead to a higher spatial correlation? Provide your reasoning. [2]

ii) Assuming no CSIT and no CDIT and a transmit diversity scheme based on O-STBC using QAM, which of the deployment scenarios would lead to a lower average error probability at high SNR? Provide your reasoning. [3]

iii) Assuming perfect CSIT and a matched beamforming transmission strategy using QAM, which scenario would lead to a larger array gain? Provide your reasoning. [3]

e) Consider Spatial Multiplexing with QAM in an i.i.d. slow Rayleigh fading MIMO channel with n_t transmit antennas and n_r receive antenna. CSI is assumed perfectly known to the receiver but is unknown to the transmitter. ML decoding is assumed at the receiver.

i) Starting from the conditional pairwise error probability and by making use of the Chernoff Bound, derive an upper bound on the average pairwise error probability. [6]

ii) Infer from (i) the diversity gain achieved by that scheme at high SNR. [2]

2. [30]

a) Show that the sum-rate capacity of a two-user SISO Broadcast Channel (BC) is achieved by allocating the transmit power to the strongest user.

[6]

b) Consider the transmission $\mathbf{y} = \mathbf{H}\mathbf{c}' + \mathbf{n}$ with perfect CSIT over a deterministic point to point MIMO channel with two transmit and two receive antennas whose matrix is given by

$$\mathbf{H} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

where a and b are complex scalars with $|a| \geq |b|$. The receiver is subject to AWGN noise such that the noise variances on receive antenna 1 and 2 are given by $\sigma_{n,1}^2$ and $\sigma_{n,2}^2$, respectively. Assume $\sigma_{n,1}^2 \leq \sigma_{n,2}^2$. The input covariance matrix is given by $\mathbf{Q} = \mathcal{E}\{\mathbf{c}'\mathbf{c}'^H\}$ and is subject to the transmit power constraint $\mathrm{Tr}\{\mathbf{Q}\} \leq P$. Compute the capacity with perfect CSIT of that deterministic channel. Explain your reasoning.

[6]

c) Consider a narrowband transmission using a transmission strategy characterized by the following set of codewords

$$\mathbf{a} = \begin{bmatrix} a & b & c & d \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} b & c & d & a \end{bmatrix},$$

$$\mathbf{c} = \begin{bmatrix} c & d & a & b \end{bmatrix},$$

$$\mathbf{d} = \begin{bmatrix} d & a & b & c \end{bmatrix},$$

with $a = \frac{1}{\sqrt{2}}(1+j)$, $b = \frac{1}{\sqrt{2}}(-1+j)$, $c = \frac{1}{\sqrt{2}}(-1-j)$ and $d = \frac{1}{\sqrt{2}}(1-j)$ being the four constellation symbols taken from a unit average energy QPSK constellation. What is the diversity gain that can be achieved with a Maximum Likelihood (ML) receiver in i.i.d. fast Rayleigh fading channels with a single receive antenna and a single transmit antenna? Provide your reasoning.

[6]

d) Discuss the validity of the following statements. Detail your argument.

i) In a two-user Gaussian SIMO Multiple Access Channel (one antenna at the transmitters and multiple antennas at the receiver) over the deterministic channels $\mathbf{h}_1$ and $\mathbf{h}_2$, an increase of user 1 rate leads to a decrease of user 2 rate.

[6]

ii)

$$\mathcal{E}\left\{\log_2\left(1+\rho|h|^2\right)\right\} \geq \log_2\left(1+\rho\mathcal{E}\left\{|h|^2\right\}\right) = \log_2(1+\rho)$$

where h is a circularly symmetric complex Gaussian random variable with $\mathcal{E}\left\{|h|^2\right\} = 1$ and ρ is the average SNR. Detail your argument.

[6]

3. [30]

Assume an uplink narrowband transmission in a cellular network consisting of 2 cells. Each cell is equipped with one receiver, denoted as receiver 1 in cell 1 and receiver 2 in cell 2. In cell 1, there are two terminals (denoted as terminal 1 and 2). Similarly, in cell 2, there are also two terminals (denoted as terminal 3 and 4). The two receivers are each equipped with n_r receive antennas and the terminals equipped with 1 transmit antenna. The channel state information is assumed perfectly known to both receivers. Terminals are scheduled in the same time/frequency resource and each terminal transmits one stream with a transmit power P. Denote as stream 1 and 2 the stream transmitted by terminal 1 and 2, respectively.

a) Write an expression of the received signal at receiver 1 in cell 1 in terms of channel parameters and transmit symbol vectors. Clearly define each variable and identify the terms responsible for the intra-cell interference (also called multi-user interference) and inter-cell interference in your expression from the perspective of stream 1.

[5]

b) We want to apply a linear combiner to detect stream 1 at receiver 1 in cell 1 such that the intra-cell interference is completely nulled out.

i) What is the smallest number of receive antennas required? Let us denote it by $n_{r,min}$. Explain your rationale.

[3]

ii) Assuming $n_r = n_{r,min}$, derive the expression of such a combiner and the achievable rate of stream 1. Is the choice unique? If not, what is the most suitable choice of combiner?

[4]

iii) Assuming $n_r > n_{r,min}$, derive the expression of such a combiner and the achievable rate of stream 1. Is the choice unique? If not, what is the most suitable choice of combiner?

[4]

iv) In the high SNR regime (e.g. P large), what is the multiplexing gain achieved by stream 1? Explain your rationale.

[4]

c) We want to improve the rate of stream 1 by applying a combiner at receiver 1 in cell 1 such that the intra-cell and inter-cell interference is completely nulled out.

i) What is the smallest number of receive antennas required? Let us denote it by $n'_{r,min}$. Explain your rationale.

[3]

ii) Assuming $n_r = n'_{r,min}$, derive the expression of such a combiner and the achievable rate of stream 1.

[4]

iii) In the high SNR regime (e.g. P large), what is the multiplexing gain achieved by stream 1? Explain your rationale and compare with b).

[3]