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UNIVERSITY OF LONDON [II(3)E 2004]

B.ENG. AND M.ENG. EXAMINATIONS 2004

For Internal Students of Imperial College London

This paper is also taken for the relevant examination for the Associateship of the City & Guilds of London Institute

PART II : MATHEMATICS 3 (ELECTRICAL ENGINEERING)

Wednesday 2nd June 2004 2.00 - 5.00 pm

Answer EIGHT questions.
Answers to Section A *questions must be written in a different answer book from answers to* Section B *questions.*

Corrected Copy

[Before starting, please make sure that the paper is complete; there should be 7 pages, with a total of 12 questions. Ask the invigilator for a replacement if your copy is faulty.]

1. Consider the complex mapping

$$w = \frac{z}{z-1}$$

from the $z-$plane where $z = x + iy$ to the w-plane where $w = u(x,y) + iv(x,y)$.

(i) Show that circles $(x-1)^2 + y^2 = a^2$ in the z-plane map to circles

$$(u - 1)^2 + v^2 = a^{-2}$$

in the w-plane. For some value of $a \neq 1$, make separate sketches of each circle. Show that for any value of $a \neq 1$, if both fixed points of the mapping (that is, points that satisfy $w = z$) lie inside one of the circles then they must lie outside the other and vice-versa.

(ii) Show also that the y-axis in the z-plane maps to the circle centred at $(\frac{1}{2}, 0)$ and radius $\frac{1}{2}$ in the w-plane.

(iii) Given the straight lines in the z-plane of the form $y = m(x-1)$, show that for arbitrary finite values of m, these map to the lines

$$mu + v = m$$

in the w-plane.

2. By choosing a suitable closed contour C in the upper half of the complex plane for the complex integral

$$\oint_C \frac{e^{2iz}\,dz}{(z^2+4)(z^2+9)},$$

use the Residue Theorem to show that

$$\int_{-\infty}^{\infty} \frac{\cos 2x\,dx}{(x^2+4)(x^2+9)} = \frac{\pi}{5}\left(\frac{e^{-4}}{2} - \frac{e^{-6}}{3}\right).$$

PLEASE TURN OVER

3. The contour integral

$$\oint_C \frac{e^{iz}}{z}\, dz$$

is taken around a closed contour C that contains no poles. C is comprised of

(i) A semi-circle in the upper half-plane of radius R;

(ii) A small semi-circular indentation around the pole at $z = 0$ that has radius r and which lies in the upper half-plane ;

(iii) Those two parts of the x-axis, namely $(-R,\, 0) \to (-r,\, 0)$ and $(r,\, 0) \to (R,\, 0)$, that connect the two semi-circles.

By considering the integral I in the limits $r \to 0$ and $R \to \infty$, show that

$$\int_{-\infty}^{\infty} \frac{\sin x}{x}\, dx \;=\; \pi\,.$$

4. The square-wave $\Pi(t)$, the tent function $\Lambda(t)$ and the sinc-function $\operatorname{sinc} t$, are defined respectively by

$$\Pi(t) \;=\; \begin{cases} 1, & -1/2 \le t \le 1/2\,, \\ 0, & \text{otherwise}\,. \end{cases}$$

$$\Lambda(t) \;=\; \begin{cases} 1+t, & -1 \le t \le 0\,, \\ 1-t, & 0 \le t \le 1\,, \\ 0, & \text{otherwise}\,, \end{cases}$$

$$\operatorname{sinc} t \;=\; \frac{\sin(t/2)}{(t/2)}\,.$$

(i) Show that the Fourier transform of $\Pi(t)$ is given by

$$\overline{\Pi}(\omega) \;=\; \operatorname{sinc}\omega\,.$$

(ii) Show that the Fourier transform of $\Lambda(t)$ is given by

$$\overline{\Lambda}(\omega) \;=\; \operatorname{sinc}^2\omega\,.$$

(iii) Given that

$$\int_{-\infty}^{\infty} \frac{e^{ipt}}{t}\, dt \;=\; \begin{cases} +i\pi & p > 0 \\ -i\pi & p < 0 \end{cases}$$

where p is an arbitrary real number, show directly that the Fourier transform of $\operatorname{sinc} t$ is $2\pi\Pi(\omega)$.

5. Show that the Dirac delta-function has an integral representation of the form

$$\int_{-\infty}^{\infty} e^{\pm i\tau\omega}\, d\omega \;=\; 2\pi\delta(\tau)$$

or with τ and ω reversed.

Hence, prove Plancherel's integral relation between the two functions $f(t)$ and $g(t)$ and their Fourier transforms $\overline{f}(\omega)$ and $\overline{g}(\omega)$

$$\int_{-\infty}^{\infty} f(t)g^*(t)\, dt \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{f}(\omega)\overline{g}^*(\omega)\, d\omega\,,$$

where $*$ represents the complex conjugate.

If $f(t) = e^{-|t|}$ and $g(t) = \cos\Omega t$, where Ω is a constant frequency, show that

$$\int_{-\infty}^{\infty} e^{-|t|}\cos\Omega t\, dt \;=\; \frac{2}{1+\Omega^2}\,.$$

6. A function $y(t)$ satisfies the differential equation

$$\ddot{y} \,+\, 5\dot{y} \,+\, 6y \;=\; f(t),$$

subject to the initial conditions $y(0) = y_0$; $\dot{y}(0) = -2y_0$, where y_0 is a constant. $f(t)$ is a given function of t. Using a Laplace transform and the Laplace Convolution Theorem, obtain the solution of this differential equation in the form

$$y(t) \;=\; y_0\, e^{-2t} \;+\; \int_0^t \left\{e^{-2(t-u)} \;-\; e^{-3(t-u)}\right\} f(u)\, du\,.$$

PLEASE TURN OVER

7. P and Q are continuous functions of x and y with continuous first partial derivatives in a simply connected region R with a piecewise smooth boundary C. Use Green's Theorem in a plane to find a two-dimensional vector $\boldsymbol{u}$, defined in terms of P and Q, to show that Green's theorem can be re-expressed as the two-dimensional version of the Divergence Theorem

$$\oint_C \boldsymbol{u}\,.\,\boldsymbol{n}\,ds \;=\; \int\int_R \mathrm{div}\boldsymbol{u}\,dxdy$$

where $\boldsymbol{n}$ is the unit normal to the curve C.

If $\boldsymbol{u} = \boldsymbol{i}\,x^2 + \boldsymbol{j}\,y^2$ and R is the region between the pair of parabolae $2y = x^2$ and $2x = y^2$ in the first quadrant, evaluate the double integral directly to show that

$$\int\int_R \mathrm{div}\boldsymbol{u}\,dxdy \;=\; 24/5$$

Green's Theorem in a plane says that

$$\oint_C (P\,dx \;+\; Q\,dy) \;=\; \int\int_R \left(\frac{\partial Q}{\partial x} \;-\; \frac{\partial P}{\partial y}\right)\; dxdy.$$

8. The double integral I is given by

$$I \;=\; \int\!\!\int_R (x+y)^n\, f(x^2-y^2)\,dx\,dy\;,$$

where $n > 0$ is an integer and f is an arbitrary function. The domain of integration R is the finite region in the $x-y$ plane enclosed by the lines $x = 0,\; y = 0$ and $y = 1-x$.

(i) Show that, after the variable transformation,

$$u \;= x^2 \;-\; y^2\,,\quad v \;=\; x \;+\; y\;,$$

the integral can be written as

$$I \;=\; \frac{1}{2}\int_0^1 \; v^{n-1}\left(\int_{-v^2}^{v^2}\; f(u)\,du\right)dv\;.$$

(ii) Evaluate the integral for the special case $n = 2,\;\; f(u) \equiv e^u$.

(iii) Evaluate the integral for the special case $n = 0,\;$ and $\;f(u) \equiv 1$.

Give a simple interpretation of your result.

9. A vector field $\boldsymbol{F}$ is defined as

$$\boldsymbol{F}(x, y, z) = 2x \sin z \, \boldsymbol{i} + ze^y \, \boldsymbol{j} + (x^2 \cos z + ae^y) \, \boldsymbol{k} ,$$

where a is a constant.

(i) Find div $\boldsymbol{F}$ and curl $\boldsymbol{F}$.

(ii) Find the value of a for which there exists a scalar function $\phi(x, y, z)$ such that $\boldsymbol{F} = \nabla\phi$ and find $\phi(x, y, z)$.

(iii) Find $(\boldsymbol{F} \cdot \boldsymbol{\nabla})\phi$ and $\nabla^2\phi$ for the ϕ obtained in (ii).

10. (i) The two-dimensional vector field $\boldsymbol{F}$ is defined by $\boldsymbol{F} = (y \cos x, \; 3y + \sin x, \; 0)$.

Show from Green's theorem in the plane that the line integral

$$I = \int_C (F_1 \, dx + F_2 \, dy)$$

depends only on the end points A and B of the path C and is otherwise independent of C.

Find a potential function $\phi(x, y)$ such that $\boldsymbol{F} = \boldsymbol{\nabla}\phi$ and hence evaluate the integral I when A is the point $(0, 0)$ and B is the point $(\pi/2, 1)$.

(ii) Let R be the region in the first quadrant of the xy-plane bounded by the ellipse $(x/2)^2 + y^2 = 1$ and the lines $x = 0, \; y = 0$. Let C be the boundary of R taken in the counter-clockwise direction.

Using Green's theorem in the plane, evaluate the line integral

$$\oint_C (y^2 \, dx - x^2 \, dy) .$$

Green's theorem in the plane states that

$$\oint_C (f \, dx + g \, dy) = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dx \, dy ,$$

where C is the counter-clockwise boundary of the region R.

PLEASE TURN OVER

SECTION B

11. In a certain electrical subsystem three components are arranged as shown below: c1 and c2 are in series, and c3 is in parallel with them, and the components function independently. The probability of failure of component c1 is p_1, that of c2 is p_2, and that of c3 is p_3. Calculate the probability of failure of the system.

```
-----+--c1----c2--+-----
     |            |
     +-----c3-----+
```

The cost of the system with just c1 and c2 is k, the additional cost of installing c3 is l, and the cost of a system failure is m. Show that installation of c3 is justified in terms of expected cost if l/m is less than a certain function of p_1, p_2 and p_3.

12. The table below shows the bivariate probability distribution of two random variables, X_1 and X_2.

	$X_1 = 1$	2	3
$X_2 = 1$	0.12	0.06	0.22
2	0.05	0.02	0.13
3	0.13	0.02	0.25

(i) Calculate the marginal distributions of X_1 and X_2.

(ii) Calculate the conditional distribution of X_1 given $X_2 = 3$.

(iii) Compute $\mathrm{E}(X_1)$, $\mathrm{E}(X_2)$, $\mathrm{var}(X_1)$, $\mathrm{var}(X_2)$, $\mathrm{E}(X_1X_2)$, and $\mathrm{cov}(X_1, X_2)$.

(iv) Are X_1 and X_2 correlated? Are they independent? Give your reasoning.

END OF PAPER

M A T H E M A T I C S   D E P A R T M E N T

MATHEMATICAL FORMULAE

1. VECTOR ALGEBRA

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} = (a_1,\, a_2,\, a_3)$$

Scalar (dot) product: $\mathbf{a}\,.\,\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$

Vector (cross) product:

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Scalar triple product:

$$[\mathbf{a},\, \mathbf{b},\, \mathbf{c}] = \mathbf{a}\,.\,\mathbf{b}\times\mathbf{c} = \mathbf{b}\,.\,\mathbf{c}\times\mathbf{a} = \mathbf{c}\,.\,\mathbf{a}\times\mathbf{b} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Vector triple product: $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{c}\,.\,\mathbf{a})\mathbf{b} - (\mathbf{b}\,.\,\mathbf{a})\mathbf{c}$

2. SERIES

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \ldots \quad (\alpha \text{ arbitrary, } |x| < 1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots\,,$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^n\frac{x^{2n}}{(2n)!} + \ldots\,,$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + \ldots\,,$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^n\frac{x^{n+1}}{(n+1)} + \ldots (-1 < x \le 1)$$

3. TRIGONOMETRIC IDENTITIES AND HYPERBOLIC FUNCTIONS

$$\sin(a+b) = \sin a\,\cos b + \cos a\,\sin b\,;$$

$$\cos(a+b) = \cos a\,\cos b - \sin a\,\sin b\,.$$

$$\cos iz = \cosh z\,; \quad \cosh iz = \cos z\,; \quad \sin iz = i\sinh z\,; \quad \sinh iz = i\sin z\,.$$

4. DIFFERENTIAL CALCULUS

(a) Leibniz's formula:

$$D^n(fg) = f\,D^n g + \tbinom{n}{1} Df\,D^{n-1}g + \ldots + \tbinom{n}{r} D^r f\,D^{n-r}g + \ldots + D^n f\;g\,.$$

(b) Taylor's expansion of $f(x)$ about $x = a$:

$$f(a+h) = f(a) + hf'(a) + h^2 f''(a)/2! + \ldots + h^n f^{(n)}(a)/n! + \epsilon_n(h)\,,$$

where $\epsilon_n(h) = h^{n+1}f^{(n+1)}(a+\theta h)/(n+1)!$, $0 < \theta < 1$.

(c) Taylor's expansion of $f(x,\, y)$ about $(a,\, b)$:

$$f(a+h,\, b+k) = f(a,\, b) + [hf_x + kf_y]_{a.b} + 1/2!\,\left[h^2 f_{xx} + 2hkf_{xy} + k^2 f_{yy}\right]_{a.b} + \cdots$$

(d) Partial differentiation of $f(x,\, y)$:

i. If $y = y(x)$, then $f = F(x)$, and $\dfrac{dF}{dx} = \dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dx}$.

ii. If $x = x(t)$, $y = y(t)$, then $f = F(t)$, and $\dfrac{dF}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$.

iii. If $x = x(u,\, v)$, $y = y(u,\, v)$, then $f = F(u,\, v)$, and

$$\frac{\partial F}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}\,, \qquad \frac{\partial F}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}\,.$$

(e) Stationary points of $f(x,\, y)$ occur where $f_x = 0$, $f_y = 0$ simultaneously.
Let $(a,\, b)$ be a stationary point: examine $D = [f_{xx}f_{yy} - (f_{xy})^2]_{a.b}$.
If $D > 0$ and $f_{xx}(a,\, b) < 0$, then $(a,\, b)$ is a maximum;
If $D > 0$ and $f_{xx}(a,\, b) > 0$, then $(a,\, b)$ is a minimum;
If $D < 0$ then $(a,\, b)$ is a saddle-point.

(f) Differential equations:

i. The first order linear equation $dy/dx + P(x)y = Q(x)$ has an integrating factor $I(x) = \exp[\int P(x)(dx]$, so that $\dfrac{d}{dx}(Iy) = IQ$.

ii. $P(x,\, y)dx + Q(x,\, y)dy = 0$ is exact if $\partial Q/\partial x = \partial P/\partial y$.

5. INTEGRAL CALCULUS

(a) An important substitution: $\tan(\theta/2) = t$:
$\sin\theta = 2t/(1+t^2)$, $\cos\theta = (1-t^2)/(1+t^2)$, $d\theta = 2\,dt/(1+t^2)$.

(b) Some indefinite integrals:

$$\int (a^2 - x^2)^{-1/2} dx = \sin^{-1}\left(\frac{x}{a}\right), \quad |x| < a.$$

$$\int (a^2 + x^2)^{-1/2} dx = \sinh^{-1}\left(\frac{x}{a}\right) = \ln\left\{\frac{x}{a} + \left(1 + \frac{x^2}{a^2}\right)^{1/2}\right\}.$$

$$\int (x^2 - a^2)^{-1/2} dx = \cosh^{-1}\left(\frac{x}{a}\right) = \ln\left|\frac{x}{a} + \left(\frac{x^2}{a^2} - 1\right)^{1/2}\right|.$$

$$\int (a^2 + x^2)^{-1} dx = \left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right).$$

6. NUMERICAL METHODS

(a) Approximate solution of an algebraic equation:

If a root of $f(x) = 0$ occurs near $x = a$, take $x_0 = a$ and
$x_{n+1} = x_n - [f(x_n)/f'(x_n)]$, $n = 0, 1, 2 \ldots$

(Newton Raphson method).

(b) Formulae for numerical integration: Write $x_n = x_0 + nh$, $y_n = y(x_n)$.

i. Trapezium rule (1-strip): $\int_{x_0}^{x_1} y(x)dx \approx (h/2)[y_0 + y_1]$.

ii. Simpson's rule (2-strip): $\int_{x_0}^{x_2} y(x)dx \approx (h/3)[y_0 + 4y_1 + y_2]$.

(c) Richardson's extrapolation method: Let $I = \int_a^b f(x)dx$ and let I_1, I_2 be two estimates of I obtained by using Simpson's rule with intervals h and $h/2$.

Then, provided h is small enough,

$$I_2 + (I_2 - I_1)/15,$$

is a better estimate of I.

7. LAPLACE TRANSFORMS

Function	Transform	Function	Transform
$f(t)$	$F(s) = \int_0^\infty e^{-st} f(t)dt$	$af(t) + bg(t)$	$aF(s) + bG(s)$
df/dt	$sF(s) - f(0)$	d^2f/dt^2	$s^2F(s) - sf(0) - f'(0)$
$e^{at}f(t)$	$F(s-a)$	$tf(t)$	$-dF(s)/ds$
$(\partial/\partial\alpha)f(t,\alpha)$	$(\partial/\partial\alpha)F(s,\alpha)$	$\int_0^t f(t)dt$	$F(s)/s$
$\int_0^t f(u)g(t-u)du$	$F(s)G(s)$		
1	$1/s$	$t^n (n = 1, 2 \ldots)$	$n!/s^{n+1}$, $(s > 0)$
e^{at}	$1/(s-a)$, $(s > a)$	$\sin\omega t$	$\omega/(s^2+\omega^2)$, $(s > 0)$
$\cos\omega t$	$s/(s^2+\omega^2)$, $(s > 0)$	$H(t-T) = \begin{cases} 0, & t < T \\ 1, & t > T \end{cases}$	e^{-sT}/s, $(s, T > 0)$

8. FOURIER SERIES

If $f(x)$ is periodic of period $2L$, then $f(x + 2L) = f(x)$, and

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}, \text{ where}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx, \quad n = 0, 1, 2, \ldots, \text{ and}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx, \quad n = 1, 2, 3, \ldots$$

Parseval's theorem

$$\frac{1}{L}\int_{-L}^{L} [f(x)]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right).$$

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MATHEMATICS FOR ENGINEERING STUDENTS

EXAMINATION QUESTION / SOLUTION

2003 – 2004

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PAPER II (3)

QUESTION

SOLUTION /E1

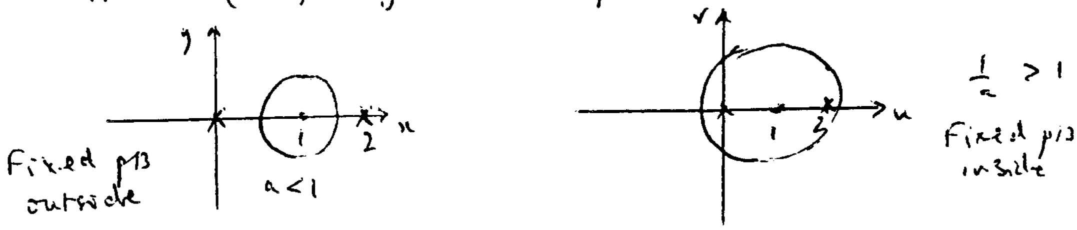
$$w = u + iv = \frac{z}{z-1} \quad \text{or} \quad w - 1 = \frac{1}{z-1}$$

$$\therefore \; u - 1 + iv = \frac{1}{(x-1) + iy} = \frac{x - 1 - iy}{(x-1)^2 + y^2}$$

$$\therefore \; (u-1) = \frac{x-1}{(x-1)^2+y^2} \qquad v = -\frac{y}{(x-1)^2+y^2} \quad — \; (*)$$ 2

1) $$(u-1)^2 + v^2 = \frac{1}{(x-1)^2+y^2},$$

Circles $(x-1)^2 + y^2 = a^2$ maps to $(u-1)^2 + v^2 = \frac{1}{a^2}$. 3



1

Fixed pts $w = z$ are $w = z = 0, 2$. When $a > 1$ the pictures are reversed. 1

2) $x = 0$ is the y-axis : in $(*)$

$$u - 1 = -\frac{1}{1+y^2} \qquad v = -\frac{y}{1+y^2}$$

$$v^2 + (u-1)^2 = \frac{1}{1+y^2} = -(u-1)$$ 4

$\therefore \; (u - 1 + \tfrac{1}{2})^2 + v^2 = \frac{1}{4}$ Completing the square.

or $(u - \tfrac{1}{2})^2 + v^2 = (\tfrac{1}{2})^2$ Circle centred at $(\tfrac{1}{2}, 0)$ radius $\tfrac{1}{2}$.

3) $y = m(x-1)$: in $(*)$

$$u - 1 = \frac{1}{m^2+1} \cdot \frac{1}{x-1}, \qquad v = -\frac{m}{m^2+1} \cdot \frac{1}{x-1}$$ 4

$$\therefore \; m(u-1) + v = 0$$

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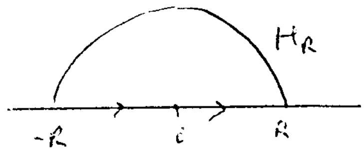
PAPER 3

QUESTION

SOLUTION E2



H_R : circle $z = Re^{i\theta}$

$\theta : 0 \to \pi$ 1

$$f(z) = \frac{e^{2iz}}{(z^2+4)(z^2+9)}$$ has 4 poles at $z = \pm 2i$, $z = \pm 3i$ 1

Choose only those in upper ½-plane.

Residue of $f(z)$ at $z = 2i$ is $\dfrac{e^{-4}}{5 \times 4i}$ 2

" " " " $z = 3i$ is $\dfrac{e^{-6}}{-5 \times 6i}$ 2

RESIDUE THM:

$$\oint_C f(z)\,dz = 2\pi i \left\{ \frac{e^{-4}}{20i} - \frac{e^{-6}}{30i} \right) = \frac{\pi}{5}\left(\frac{e^{-4}}{2} - \frac{e^{-6}}{3} \right)$$ 1

Now $$\oint_C f(z)\,dz = \int_{H_R} f(z)\,dz + \int_{-R}^{R} \frac{e^{2ix}\,dx}{(x^2+4)(x^2+9)}$$ 2

$\lim_{R\to\infty} \int_{H_R} f(z)\,dz = 0$ by Jordan's Lemma

i) Only poles in upper ½-plane

ii) $m = 2 > 0$ 3

iii) $\dfrac{1}{(z^2+4)(z^2+9)} \underset{R\to\infty}{\longrightarrow} 0$

∴ In the limit $R \to \infty$,

$$\frac{\pi}{5}\left(\frac{e^{-4}}{2} - \frac{e^{-6}}{3} \right) = \int_{-\infty}^{\infty} \frac{\cos 2x + i \sin 2x}{(x^2+4)(x^2+9)}\,dx$$ 2

Sine-part of integral zero as it is odd.

$$\therefore \int_{-\infty}^{\infty} \frac{\cos 2x\,dx}{(x^2+4)(x^2+9)} = \frac{\pi}{5}\left(\frac{e^{-4}}{2} - \frac{e^{-6}}{3} \right)$$ 1

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Checker : X. WU Checker's signature : Xuesong

MATHEMATICS FOR ENGINEERING STUDENTS

EXAMINATION QUESTION / SOLUTION

2003 – 2004

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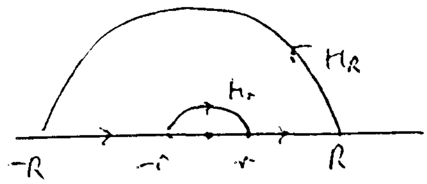
PAPER 3

QUESTION

SOLUTION E3



C: $H_R : \theta : 0 \to \pi$
$H_r : \theta : \pi \to 0$
x-axis: $(-R \to -r)$
$(r \to R)$ 1

$f(z) = \dfrac{e^{iz}}{z}$ has one simple pole at $z = 0$.

The contour has been specifically chosen to exclude pole at $z=0$

Since C excludes the pole, $f(z)$ analytic everywhere in C.

$$\oint_C \frac{e^{iz}}{z}\,dz = 0 \qquad \text{Cauchy's Thm} \quad 2$$

$$\therefore\ 0 = \oint_C \frac{e^{iz}}{z}\,dz = \int_{H_R} \frac{e^{iz}}{z}\,dz + \int_{H_r} \frac{e^{iz}}{z}\,dz + \left(\int_{-R}^{-r} + \int_{r}^{R}\right)\frac{e^{ix}}{x}\,dx \quad 2$$

H_r: circle $z = re^{i\theta}$ $\theta : \pi \to 0$ $\displaystyle\int_{H_r} = \int_\pi^0 \frac{e^{ire^{i\theta}}}{re^{i\theta}}\, ire^{i\theta}\,d\theta$ 3

$$\therefore \lim_{r\to 0} \int_{H_r} = -i\int_0^\pi d\theta = -i\pi$$

H_R: circle $z = Re^{i\theta}$ $\theta : 0 \to \pi$. Jordan's Lemma says that

$\displaystyle\lim_{R\to\infty} \int_{H_R} \frac{e^{iz}}{z}\,dz = 0$ provided i) Only sings of $f(z)$ are poles ✓ 3

ii) $m\,(=1) > 0$ ✓

iii) $\dfrac{1}{|z|} \to 0$ as $R \to \infty$ ✓.

$$\therefore \lim_{\substack{r\to 0\\ R\to\infty}} \left(\int_{-R}^{-r} + \int_{r}^{R}\right)\frac{e^{ix}}{x}\,dx = i\pi \quad 2$$

or $\displaystyle\int_{-\infty}^{\infty} \frac{e^{ix}}{x}\,dx = i\pi$ $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x}{x}\,dx = 0$ as $\dfrac{\cos x}{x}$ odd. 2

$$\therefore \int_{-\infty}^{\infty} \frac{\sin x}{x}\,dx = \pi$$

Setter : J.D. GIBBON Setter's signature : J D. Gibbon

Checker : X. WU Checker's signature : Xuesong

MATHEMATICS FOR ENGINEERING STUDENTS

EXAMINATION QUESTION / SOLUTION

2003 – 2004

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PAPER 3

QUESTION

SOLUTION E4

1) $$\bar{\Pi}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t}\,\Pi(t)\,dt$$

$$= \int_{-1/2}^{1/2} e^{-i\omega t}\,dt = -\frac{1}{i\omega}\left[e^{-i\omega/2} - e^{i\omega/2}\right]$$

$$= \frac{-i}{\omega}\left(e^{i\omega/2} - e^{-i\omega/2}\right) = \sin\omega/2 \,/\, \omega/2 = \operatorname{sinc}\omega$$ 3

2) $$\bar{\Lambda}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t}\Lambda(t)\,dt = \int_{-1}^{0}(1+t)e^{-i\omega t}dt + \int_{0}^{1}(1-t)e^{-i\omega t}dt$$

$$= \left[\left(\frac{1}{\omega^2} + \frac{i}{\omega}\right)\left(1 - e^{i\omega}\right) + \frac{i}{\omega}e^{i\omega}\right] - \frac{e^{-i\omega}}{\omega^2} - \left(\frac{i}{\omega} - \frac{1}{\omega^2}\right)$$

$$= \frac{2}{\omega^2} - \frac{1}{\omega^2}\left(e^{i\omega} + e^{-i\omega}\right) = \frac{2}{\omega^2}(1 - \cos\omega)$$ 6

$$= \frac{4}{\omega^2}\sin^2\tfrac{1}{2}\omega = \operatorname{sinc}^2\omega$$

[Comes from $\int t e^{-i\omega t}dt = \left[\frac{1}{\omega^2} + \frac{it}{\omega}\right]e^{-i\omega t}$]

3) $$\int_{-\infty}^{\infty} \frac{e^{ipx}}{x}dx = \int_{-\infty p}^{\infty p}\frac{e^{iq}}{q}dq = \begin{cases} i\pi & p > 0 \\ -i\pi & p < 0\end{cases}$$

$$\therefore \int_{-\infty}^{\infty} \frac{e^{-i\omega t}\sin t/2}{t/2}dt = 2\int_{-\infty}^{\infty}\frac{\sin\theta}{\theta}e^{-2i\omega\theta}d\theta \qquad t/2 = \theta$$ 3

$$= \frac{1}{i}\int_{-\infty}^{\infty}\frac{e^{i\theta p_1}}{\theta}d\theta - \frac{1}{i}\int_{-\infty}^{\infty}\frac{e^{i\theta p_2}}{\theta}d\theta \qquad \begin{matrix} p_1 = 1 - 2\omega \\ p_2 = -1 - 2\omega\end{matrix}$$

$$= \begin{cases} 0 & p_1 \,\&\, p_2 \text{ same sign}; \quad \omega < -\tfrac12 \ (p_1, p_2 > 0), \quad \omega > \tfrac12 \ (p_1 < 0, p_2 < 0) \\ \frac{2\pi i}{i} = 2\pi & p_1 > 0, p_2 < 0 \text{ (opposite signs)} \quad -\tfrac12 < \omega < \tfrac12 \end{cases}$$

(The case $p_1 < 0$, $p_2 > 0$ is not possible) 3

$$= 2\pi\begin{Bmatrix} 1 & -\tfrac12 < \omega < \tfrac12 \\ 0 & \text{otherwise}\end{Bmatrix} = 2\pi\,\Pi(\omega)$$

Setter : J.D. GIBBON

Setter's signature : J.D. Gibbon

Checker : X. WU

Checker's signature : Xuesong

PAPER 3
QUESTION
SOLUTION E5

Consider $\bar{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$

$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}(\omega) e^{i\omega t} d\omega$

$\therefore f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(t') e^{-i\omega t'} dt'\right) e^{i\omega t} d\omega$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} e^{i\omega(t-t')} d\omega\right) f(t') dt'$

Now the δ-function has the property:

$f(t) = \int_{-\infty}^{\infty} \delta(t-t') f(t') dt'$ — Convention $e^{+it(\omega-\omega')}$ if $\bar{f}(\omega)$ & $f(t)$ reversed. 5

so (*) $2\pi\delta(t-t') = \int_{-\infty}^{\infty} e^{\pm i\omega(t-t')} d\omega$
Signs irrelevant

Bookwork $\therefore \int_{-\infty}^{\infty} f(t) g^*(t) dt = \int_{-\infty}^{\infty} dt \left(\int_{-\infty}^{\infty} \bar{f}(\omega) e^{i\omega t} d\omega\right)\left(\int_{-\infty}^{\infty} \bar{g}(\omega') e^{i\omega' t} d\omega'\right)^* \frac{1}{4\pi^2}$

$= \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} e^{i(\omega-\omega')t} dt\right) \bar{f}(\omega) \bar{g}^*(\omega') d\omega d\omega'$

$= \frac{1}{(2\pi)^2} \int\int_{-\infty}^{\infty} 2\pi\, \delta(\omega-\omega') \bar{f}(\omega) \bar{g}^*(\omega') d\omega d\omega'$ Using (*) above 5

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}(\omega) \bar{g}^*(\omega) d\omega$

If $f(t) = e^{-|t|} = \begin{cases} e^{-t} & t>0 \\ e^{t} & t<0 \end{cases} \Rightarrow \bar{f}(\omega) = \int_0^{\infty} e^{-t-i\omega t} dt + \int_{-\infty}^{0} e^{t-i\omega t} dt$

$\therefore \bar{f}(\omega) = \frac{1}{1+i\omega} + \frac{1}{1-i\omega} = \frac{2}{1+\omega^2}$ 5

Unseen $\bar{g}(\omega) = \frac{2\pi}{2}[\delta(\omega-\Omega) + \delta(\omega+\Omega)]$

$\therefore \int_{-\infty}^{\infty} e^{-|t|} \cos\Omega t\, dt = \frac{2\pi}{2\pi}\int_{-\infty}^{\infty} \frac{1}{2}\cdot\frac{2}{1+\omega^2}[\delta(\omega-\Omega) + \delta(\omega+\Omega)] d\omega$

$= \frac{2}{1+\Omega^2}$ or directly from the t-integral.

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MATHEMATICS FOR ENGINEERING STUDENTS

EXAMINATION QUESTION / SOLUTION

2003 – 2004

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PAPER 3

QUESTION

SOLUTION E6

$\ddot{y} + 5\dot{y} + 6y = f(t)$ $\quad\mathcal{L}\dot{y} = s\bar{y}(s) - y(0)$ Tables

$\mathcal{L}\ddot{y} = s^2\bar{y}(s) - sy(0) - \dot{y}(0)$

$\therefore s^2\bar{y}(s) - sy(0) - \dot{y}(0) + 5s\bar{y}(s) - 5y(0) + 6\bar{y}(s) = \bar{f}(s)$

$$(s^2+5s+6)\bar{y}(s) = (s+5)\bar{y}(0) + \dot{y}(0) + \bar{f}(s)$$
$$= (s+3)\bar{y}(0) + \bar{f}(s) \quad \text{if } \dot{y}(0) = -2y(0)$$

6

$$\therefore \bar{y}(s) = \frac{\bar{f}(s)}{(s+3)(s+2)} + \frac{\bar{y}(0)}{s+2}$$

Now inverse transform: shift theorem $\mathcal{L}^{-1}\left(\frac{1}{s+2}\right) = e^{-2t}$

$$\frac{\bar{f}(s)}{(s+3)(s+2)} = \frac{\bar{f}(s)}{s+2} - \frac{\bar{f}(s)}{s+3}$$

4

$$\therefore y(t) = y(0)e^{-2t} + \mathcal{L}^{-1}\left(\frac{\bar{f}(s)}{s+2}\right) - \mathcal{L}^{-1}\left(\frac{\bar{f}(s)}{s+3}\right)$$

Now the convolution theorem says that

$$\mathcal{L}^{-1}\left(\bar{f}(s)\bar{g}(s)\right) = f(t) * g(t) \qquad g_1(s) = \frac{1}{s+2} \quad g_2(s) = \frac{1}{s+3}$$

$g_1(t) = e^{-2t} \quad g_2(t) = e^{-3t}$

3

$$\therefore f(t) * g_1(t) = \int_0^t e^{-2(t-u)} f(u)\,du$$
$$f(t) * g_2(t) = \int_0^t e^{-3(t-u)} f(u)\,du$$

Altogether; $\quad \bar{y}(s) = y(0)e^{-2t} + \int_0^t \left[e^{-2(t-u)} - e^{-3(t-u)}\right] f(u)\,du$ 2

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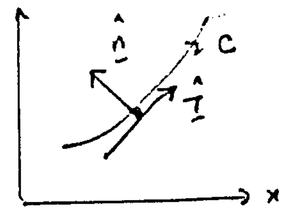
Consider a curve C with unit tangent vector $\hat{\underline{T}}$ & normal $\hat{\underline{n}}$



$$\hat{\underline{T}} = d\underline{r}/ds = \hat{\underline{i}}\left(\frac{dx}{ds}\right) + \hat{\underline{j}}\left(\frac{dy}{ds}\right)$$

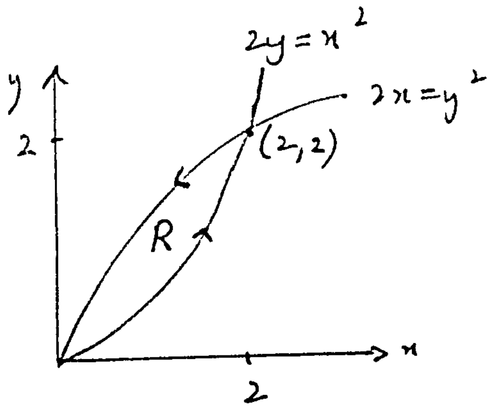
but, because $\hat{\underline{n}}\cdot\hat{\underline{T}} = 0 \Rightarrow \hat{\underline{n}} = \pm\left(\hat{\underline{i}}\frac{dy}{ds} - \hat{\underline{j}}\frac{dx}{ds}\right)$

Choose a vector $\underline{u}$ such that $u = (Q\hat{\underline{i}} - P\hat{\underline{j}})$

$$\therefore\quad \underline{u}\cdot\hat{\underline{n}} = \left(P\frac{dx}{ds} + Q\frac{dy}{ds}\right) \Rightarrow P\,dx + Q\,dy = \underline{u}\cdot\hat{\underline{n}}\,ds$$

and $\text{div}\,\underline{u} = Q_x - P_y$. Hence, Green's Theorem gives

$$\oint_C \underline{u}\cdot\hat{\underline{n}}\,ds = \iint_R \text{div}\,\underline{u}\;dx\,dy \quad \text{if } C \text{ is closed.}$$



$$\underline{u} = x^2 + y^2 \Rightarrow \text{div}\,\underline{u} = 2(x+y)$$

$$\therefore \iint_R \text{div}\,\underline{u}\;dx\,dy = 2\iint_R (x+y)\,dx\,dy$$

$$= 2\int_0^2 \left\{ \int_{\frac{1}{2}x^2}^{\sqrt{2x}} (x+y)\,dy \right\} dx$$

$$= 2\int_0^2 \left(xy + \tfrac{1}{2}y^2 \right)_{\frac{1}{2}x^2}^{\sqrt{2x}} dx$$

$$= 2\int_0^2 \left(\sqrt{2}\,x^{3/2} + x - \tfrac{1}{2}x^3 - \tfrac{1}{8}x^4 \right) dx$$

$$= 2\left[\sqrt{2}\cdot\tfrac{2}{5}\cdot x^{5/2} + \tfrac{1}{2}x^2 - \tfrac{1}{8}x^4 - \tfrac{1}{40}x^5 \right]_0^2$$

$$= 2\left[2^4/5 + 2 - 2 - 32/40 \right]$$

$$= 8/5\,[4 - 1] = 24/5$$

Setter : J. D. GIBBON

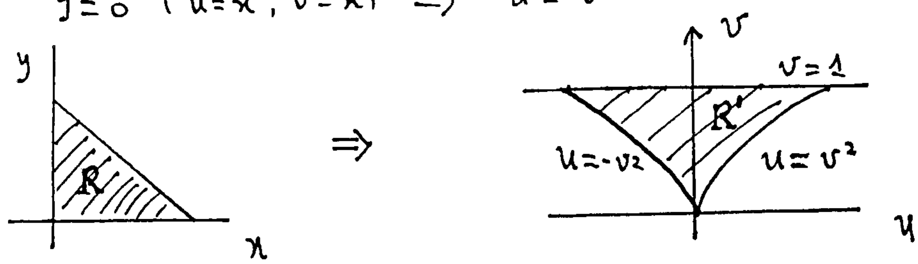
Setter's signature : J. D. Gibbon

Checker : [illegible]

Checker's signature : [illegible]

(i) Transformation of boundaries:

- $x+y=1 \Rightarrow v=1$
- $x=0 \; (u=-y^2,\ v=y) \Rightarrow u=-v^2$
- $y=0 \; (u=x^2,\ v=x) \Rightarrow u=v^2$ 3



2

Jacobian: $\dfrac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2x & -2y \\ 1 & 1 \end{vmatrix} = 2(x+y) = 2v, \quad \dfrac{\partial(x,y)}{\partial(u,v)} = \dfrac{1}{2v}$

$$I = \iint_{R'} v^n f(u) \frac{1}{2v}\, du\, dv = \frac{1}{2}\int_0^1 v^{n-1}\left\{\int_{-v^2}^{v^2} f(u)\,du\right\} dv$$ 3

(ii) For $n=2$, $f(u) \equiv e^u$

$$I = \frac{1}{2}\int_0^1 v\left\{\int_{-v^2}^{v^2} e^u\,du\right\}dv = \frac{1}{2}\int_0^1 v(e^{v^2} - e^{-v^2})\,dv$$

$$= \frac{1}{4}(e^{v^2} + e^{-v^2})\Big|_0^1 = \underline{\frac{1}{4}(e + e^{-1} - 2)}$$ 4

(iii) For $n=0$, $f \equiv 1$

$$I = \frac{1}{2}\int_0^1 v^{-1}\left\{\int_{-v^2}^{v^2} du\right\}dv = \int_0^1 v\,dv = \frac{1}{2}$$ 2

$\therefore \; I = \frac{1}{2}$

In this case $I = \iint_R dx\,dy = \underline{\text{Area of } R = \tfrac{1}{2}\times 1 \times 1 = \tfrac{1}{2}}$ 1

15

Setter: X. WU — Setter's signature: Xuesong Wu

Checker: R L JACOBS — Checker's signature: R.L. Jacobs

(i) $\operatorname{div}\underline{F} = \underline{\nabla}\cdot\underline{F} = 2\sin z + ze^y - x^2\sin z = (2-x^2)\sin z + ze^y$ 2

$$\operatorname{curl}\underline{F} = \underline{\nabla}\times\underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x\sin z & ze^y & x^2\cos z + ae^y \end{vmatrix}$$

$$= (a-1)e^y\,\underline{i} - (2x\cos z - 2x\cos z)\,\underline{j} + 0\,\underline{k}$$

$\underline{\operatorname{Curl}\underline{F} = (a-1)e^y\,\underline{i}}$ 3

(ii) $a = 1.$ $\quad \frac{\partial\phi}{\partial x} = 2x\sin z \implies \phi = x^2\sin z + f(y,z)$

$\frac{\partial\phi}{\partial y} = ze^y \implies \frac{\partial f}{\partial y} = ze^y,\ f = ze^y + g(z)$

$\therefore\ \phi = x^2\sin z + ze^y + g(z)$

$\frac{\partial\phi}{\partial z} = x^2\cos z + e^y \implies x^2\cos z + e^y + g'(z) = x^2\cos z + e^y$

i.e. $g'(z) = 0$, $g = \text{Const.}$

$\therefore\ \underline{\phi = x^2\sin z + ze^y + C}$ 5

(iii) $(\underline{F}\cdot\underline{\nabla})\phi = 2x\sin z\,\frac{\partial\phi}{\partial x} + ze^y\,\frac{\partial\phi}{\partial y} + (x^2\cos z + e^y)\,\frac{\partial\phi}{\partial z}$

$= \underline{4x^2\sin^2 z + z^2e^{2y} + (x^2\cos z + e^y)^2}$ 3

$$\nabla^2\phi = \frac{\partial^2}{\partial x^2}\phi + \frac{\partial^2}{\partial y^2}\phi + \frac{\partial^2}{\partial z^2}\phi = 2\sin z + ze^y - x^2\sin z$$

$\therefore\ \nabla^2\phi = \underline{(2-x^2)\sin z + ze^y}\quad (= \underline{\nabla}\cdot\underline{F})$ 2

15

Setter : X. WU Setter's signature : Xuesong

Checker : R L JACOBS Checker's signature : R L Jacobs

MATHEMATICS FOR ENGINEERING STUDENTS

EXAMINATION QUESTION / SOLUTION

2003 – 2004

E 10

PAPER 3

QUESTION

SOLUTION C5

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(i) Consider two paths C_1 and C_2 with the same endpoints A and B. Consider composite path

$C = C_1 \bar{-} C_2$ taken counter clockwise

Then $\oint_C (F_1\,dx + F_2\,dy)$

$$= \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$

In this case $F_1 = y\cos x$

$F_2 = \sin x + 3y$

$$\therefore \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 0$$

$$\therefore \oint_C = 0 \Rightarrow \int_{C_1} (F_1\,dx + F_2\,dy) = \int_{C_2} (F_1\,dx + F_2\,dy)$$

5

Hence integral is independent of path.

Now $\frac{\partial \varphi}{\partial x} = y\cos x$ — (1), $\frac{\partial \varphi}{\partial y} = \sin x + 3y$ — (2)

(1) $\Rightarrow \varphi = y\sin x + h(y)$. Subst. into (2) to get

$\sin x + \frac{dh}{dy} = \sin x + 3y \Rightarrow h = \frac{3}{2}y^2 + c$

4

$\therefore \varphi(x,y) = y\sin x + \frac{3}{2}y^2 + c$.

$\therefore \int_A^B = \varphi(\pi/2, 1) - \varphi(0,0) = 1 + \frac{3}{2} + c - c$

$= 5/2$.

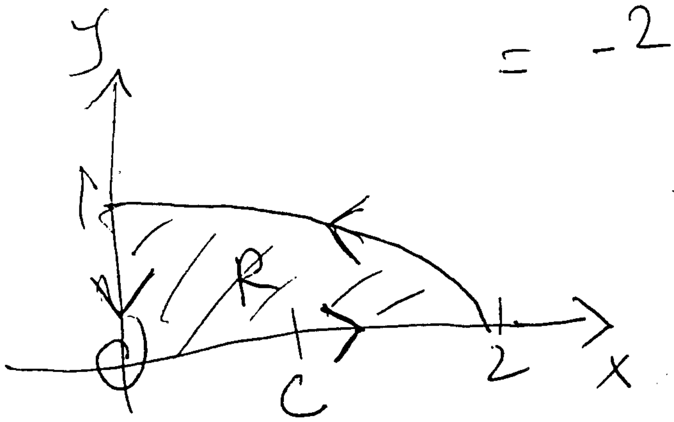
1

Setter : R.L. Jacobs Setter's signature : R.L. Jacobs

Checker: J.D. Gibbon Checker's signature :

(ii) $$\oint_C (y^2\,dx - x^2\,dy) = \iint_R (-2x-2y)\,dx\,dy$$

$$= -2\int_{y=0}^{1}\left[\int_{x=0}^{+2(1-y^2)^{1/2}} (x+y)\,dx\right]dy$$



$$= -2\int_{y=0}^{1}\left[2(1-y^2) + y\,2(1-y^2)^{1/2}\right]dy$$

$$= -2\left[2y - \frac{2y^3}{3} - \frac{2}{3}(1-y^2)^{3/2}\right]_0^1$$

$$= -2\left[2 - \frac{2}{3} - 0 - 0 + 0 + \frac{2}{3}\right]$$

$$= -4.$$

5

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MATHEMATICS FOR ENGINEERING STUDENTS

EXAMINATION QUESTION / SOLUTION

2003-2004

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PAPER 3 EE2

QUESTION 11

SOLUTION 11

1.

$$P(system\ \ failure) = P\{(c1\ fails \cup c2\ fails) \cap (c3\ fails)\}$$

$$= \{1 - P(c1\ does\ not\ fail \cap c2\ does\ not\ fail)\}\ P(c3\ fails)$$

$$= \{1 - (1 - p_1)(1 - p_2)\}p_3.$$

5

$$E(cost\ without\ c3) = k + \{1 - (1 - p_1)(1 - p_2)\}m$$

$$E(cost\ with\ c3) = k + l + \{1 - (1 - p_1)(1 - p_2)\}p_3 m$$

5

$$\text{difference} = \{1 - (1 - p_1)(1 - p_2)\}(1 - p_3)m - l$$

$$> 0 \text{ if } l/m < \{1 - (1 - p_1)(1 - p_2)\}(1 - p_3)$$

5

15

Setter: MJ CROWDER

Setter's signature: MJ Crowder

Checker: R COLEMAN

Checker's signature: R Coleman

PAPER 3 EE2

QUESTION 12

SOLUTION 12

2.

(i) marginal X_1: $p_1(1) = 0.3,\ p_1(2) = 0.1,\ p_1(3) = 0.6$

marginal X_2: $p_2(1) = 0.4,\ p_2(2) = 0.2,\ p_2(3) = 0.4$ 4

(ii) conditional $X_1 \mid X_2 = 3$:

$p_{1|2}(1) = 0.13/0.4 = 0.325,$

$p_{1|2}(2) = 0.02/0.4 = 0.05,$

$p_{1|2}(3) = 0.25/0.4 = 0.625,$ 4

(iii) $\mathrm{E}(X_1) = \sum_{x_1} x_1 p_1(x_1) = 2.3$, $\mathrm{E}(X_2) = 2.0$

$\mathrm{var}(X_1) = \sum x_1^2 p_1(x_1) - \mu_1^2 = 6.1 - 2.3^2 = 0.81$, $\mathrm{var}(X_2) = 4.8 - 2.0^2 = 0.8$

$\mathrm{E}(X_1X_2) = \sum x_1 x_2 p(x_1, x_2) = 4.62$

$\mathrm{cov}(X_1, X_2) = 4.62 - (2.3 \times 2.0) = 0.02$ 5

(iv) Yes, because covariance not zero.

No, because covariance not zero, or e.g. $p(1,2) \neq p_1(1)p_2(2)$ 2

15

Setter: MJ CROWDER Setter's signature: MJ Crowder

Checker: R COLEMAN Checker's signature: R Coleman