IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

EXAMINATIONS 2013

Signals and Linear Systems

There are THREE questions in this paper. Answer to ALL questions. The FIRST question carries 40% of the mark and other TWO questions carry 30% of the mark.

Time allowed 2 hours.

Special Information for the Invigilators: none

Information for Candidates

Some Fourier Transforms

$$\mathrm{rect}(\tfrac{t}{\tau}) \quad \Longleftrightarrow \quad \tau\mathrm{sinc}(\tfrac{\omega\tau}{2})$$

$$\tfrac{W}{\pi}\mathrm{sinc}(Wt) \quad \Longleftrightarrow \quad \mathrm{rect}(\tfrac{\omega}{2W})$$

Time-shifting property of the Laplace transform

$$x(t-t_d) \Longleftrightarrow X(s)e^{-st_d}$$

Two useful Laplace transforms

$$e^{\lambda t}u(t) \Longleftrightarrow \frac{1}{s-\lambda}$$

$$t^n e^{\lambda t}u(t) \Longleftrightarrow \frac{n!}{(s-\lambda)^{n+1}}$$

The Questions

1. This question carries 40% of the mark.

   (a) Given the signal:

   $$x(t) = \begin{cases} 1-t & \text{for} \quad 0 \le t \le 1 \\ 0 & \text{otherwise,} \end{cases}$$

   sketch and dimension each of the following signals:

   i. $x_1(t) = x(t+4)$

   [2]

   ii. $x_2(t) = x(-t/2+3)$

   [2]

   iii. $x_3(t) = x(2t) + x(-2t)$

   [2]

   (b) Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its period.

   i. $x_2(t) = \cos(\frac{2\pi}{5}t) + \cos(\frac{2\pi}{3}t)$

   [4]

   ii. $x_3(t) = e^{-(1+j)t}$

   [4]

   (c) A linear time-invariant system is specified by the following differential equation:

   $$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt} + x(t).$$

   i. Find the characteristic polynomial and the characteristic roots of this system.

   [4]

   ii. Find the zero-input component of the response $y(t)$ for $t \ge 0$, if the initial condition is $y(0) = 4$ and $\dot{y}(0) = 1$.

   [4]

Question 1 continues on next page

(d) Find the Inverse Laplace transforms of the following functions:

i.

$$\frac{s+2}{s^2+6s+5}$$

[4]

ii.

$$\frac{s}{s^2+4s+4}$$

[4]

(e) Consider the linear time-invariant system whose input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t).$$

i. Find the transfer function of the system.

[3]

ii. Assuming that $x(t) = u(t)$ is the unit step function, find the final value of the output $y(t)$. Specifically, find $y(\infty) = \lim_{t\to\infty} y(t)$.

[3]

(f) Consider the signal $x(t) = 3000\text{sinc}(3000\pi t)$

i. Sketch and dimension the Fourier transform of $x(t)$

[2]

ii. Determine the Nyquist sampling rate for $x(t) + x^2(t)$

[2]

2. For the circuit in Fig. 2a, the switch is in a closed position for a long time before $t = 0$, when it is opened instantaneously.

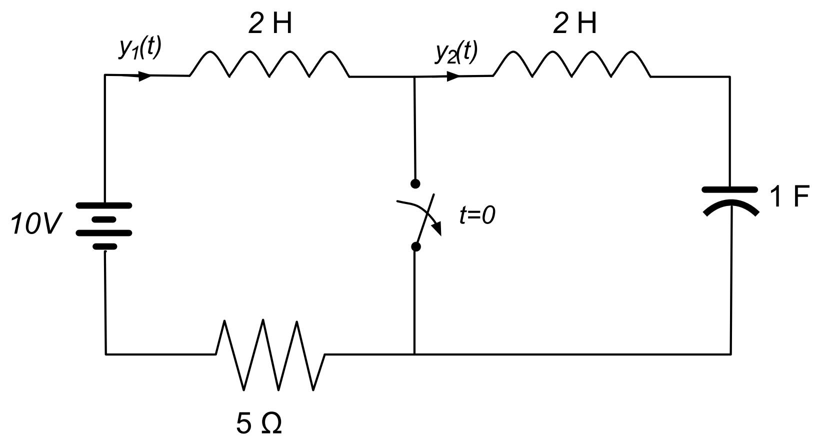

Figure 2a: An electric circuit.

(a) Determine the initial conditions $y_1(0^-)$, $y_2(0^-)$ and $v_C(0^-)$, where $v_C(t)$ is the voltage across the capacitor.

[10]

(b) Write the loop equation in the Laplace domain.

[10]

(c) Find $y_1(t)$ for $t \geq 0$.

[10]

3. Consider the system depicted in Fig. 3a. Here the linear system S_1 has the

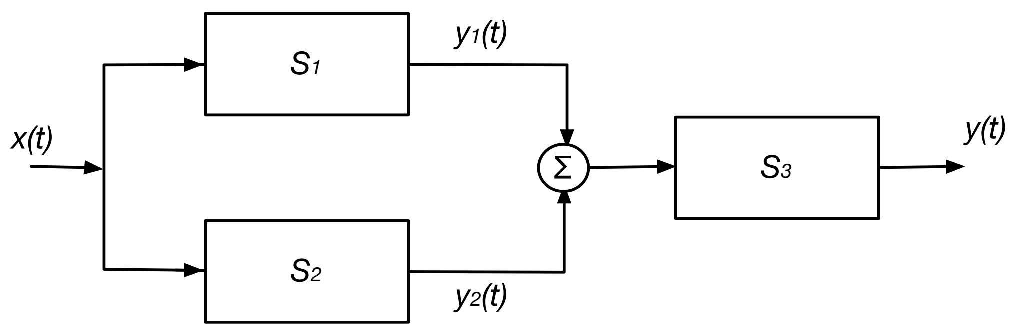

Figure 3a: Block diagram of a linear system.

following input/output relationship:

$$\frac{dy_1}{dt} + 2y_1(t) = x(t),$$

the system S_2 has the following input/output relationship:

$$\frac{dy_2}{dt} + 4y_2(t) = 2x(t),$$

finally, the system S_3 has the following transfer function:

$$S_3(s) = \frac{s+2}{s^2+8s+7}.$$

(a) Find the transfer function of S_1 and S_2.

[5]

(b) Find the transfer function of the complete system.

[5]

Question 3 continues on next page

(c) Assume the system was at rest when it was excited by the input $x(t) = u(t)$, determine the exact expression of the output $y(t)$ for $t \geq 0$.

[10]

(d) The output $y(t)$ of the system of Fig. 3a is then fed to the system of Fig. 3b. The aim of this second system is to invert the effect on $x(t)$ of the system of Fig. 3a. Therefore, determine the gain K and the transfer function $P(s)$ that ensure that $f(t)$ is approximately equal to $x(t)$.

[10]

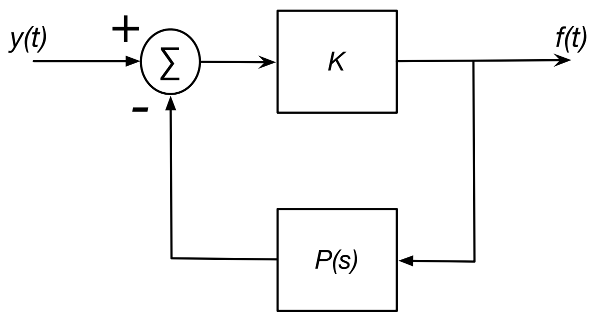

Figure 3b: A feedback system used to invert the system of Fig. 3a.