Final

E4.04
SC6
ISE4.9

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

MSc and EEE/ISE PART IV: MEng and ACGI

ADVANCED DATA COMMUNICATIONS

Wednesday, 19 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

All questions carry equal marks. The maximum mark for each subquestion is shown in brackets.

Any special instructions for invigilators and information for candidates are on page 1.

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Second Marker(s) : E. Gelenbe

Instructions to Candidates
Useful equations

- When using bthe bit error equations for square QAM as follows
$$P_e = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1} SNR}\right)$$
and cross QAM as follows
$$P_e = 4\left(1 - \frac{1}{\sqrt{2M}}\right) Q\left(\sqrt{\frac{3}{\frac{31}{32}M-1} SNR}\right)$$
we have the following relationships between the bit rate and also the probabilities of errors
$$\bar{b} = 5 \text{ for } P_e = 1.855 \times 10^{-7}$$
and
$$\bar{b} = 4.5 \text{ for } P_e = 1.35 \times 10^{-14}$$

- For Square and cross QAM $P_e = 10^{-6}$ is satisfied when
$$\frac{3}{M-1} SNR = \frac{3}{\frac{31}{32}M-1} SNR = 13.9dB$$

- Fourier transform relationships

$$For\ T = 1$$

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} \frac{1}{1+a \quad \exp\left(j\left(\omega + \frac{2\pi n}{T}\right)\right)} \overset{Fourier\ Transform}{\Longleftrightarrow} \sum_{k=-\infty}^{0} (-a)^k \, \delta\,(t-kT)$$

$$\frac{1}{\sqrt{T}}\,\text{sinc}\left(\frac{t}{T}\right) \overset{Fourier\ Transform}{\Longleftrightarrow} \sqrt{T}\,\text{rect}\,(T\,f)$$

$$\sum_{k=0}^{\infty} (-a)^{2k} = \frac{1}{1-a^2}$$

For $P_e = 10^{-7}$ the gap value $\Gamma = 9.8dB$

For $P_e = 10^{-6}$ the gap value $\Gamma = 8.8dB$

$$\begin{aligned} \text{For } 2 \times \left(1 - \frac{1}{4}\right) Q\left(\sqrt{\frac{3SNR}{15}}\right) &= 5 \times 10^{-7} \\ SNR &= 123.5 \end{aligned}$$

$$2.4 \times Q\left(\sqrt{\frac{10^{1.4}}{1.7}}\right) = 1.45 \times 10^{-4}$$

Questions

1. Answer the following sub-questions

   (a) Show that the following two basis functions are orthonormal [2]

$$\phi_0(t) = \begin{cases} \sqrt{2}\cos(2\pi t) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

$$\phi_1(t) = \begin{cases} \sqrt{2}\sin(2\pi t) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

   (b) Consider the following modulated waveforms

$$x_0(t) = \begin{cases} \sqrt{2}(\cos(2\pi t) + \sin(2\pi t)) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

$$x_1(t) = \begin{cases} \sqrt{2}(\cos(2\pi t) + 3\sin(2\pi t)) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

$$x_2(t) = \begin{cases} \sqrt{2}(3\cos(2\pi t) + \sin(2\pi t)) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

$$x_3(t) = \begin{cases} \sqrt{2}(3\cos(2\pi t) + 3\sin(2\pi t)) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

$$x_4(t) = \begin{cases} \sqrt{2}(\cos(2\pi t) - \sin(2\pi t)) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

$$x_5(t) = \begin{cases} \sqrt{2}(\cos(2\pi t) - 3\sin(2\pi t)) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

$$x_6(t) = \begin{cases} \sqrt{2}(3\cos(2\pi t) - \sin(2\pi t)) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

$$x_7(t) = \begin{cases} \sqrt{2}(3\cos(2\pi t) - 3\sin(2\pi t)) & if\ t \in [0,1] \\ 0 & otherwise \end{cases}$$

$$x_{i+8}(t) = -x_i(t) \quad \text{for } i = 0, \cdots, 7$$

   Draw the constellation points for these waveforms using the basis functions of question 1.a [3]

   (c) For the constellation points given in question 1.b, compute the average energy ε_x and average energy per dimension $\overline{\varepsilon}_x$, where $\overline{\varepsilon}_x = \frac{\varepsilon_x}{N}$ and N is the number of dimensions, for the following cases

i. all signals are equally likely [3]

ii. where [2]

$$p(x_0) = p(x_4) = p(x_8) = p(x_{12}) = \frac{1}{8}$$

and

$$p(x_i) = \frac{1}{24} \text{ for } i = 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15$$

(d) Explain

i. the maximum likelihood (ML) decision rule; [2]

ii. the maximum *a posteriori* (MAP) decision rule. [2]

(e) Outline how the following detectors operate

i. the basis detector; [2]

ii. the signal detector; [2]

iii. the maximum likelihood detector; [2]

iv. the correlation detector; [2]

v. the matched filter demodulator; [2]

vi. the minimum distance decoder. [1]

2. Answer the following sub-questions.

(a) A channel with additive white Gaussian noise has the response shown in Figure 1 with unity gain and no phase distortion up to 50 MHz.

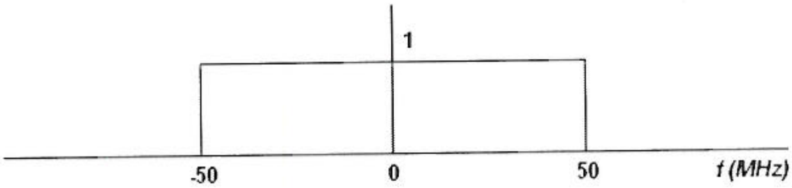


Figure 1. The AWGN channel.

The power spectral density of the noise is -103 dBm/Hz. The transmit power for a QAM modulator is 0 $dBm = \frac{\mathcal{E}_x}{T}$. The initial symbol rate is 1 M $symbols/s$. Answer the following sub-questions.

i. Suggest two ideal basis functions that use the lowest possible frequencies for this channel. [3]

ii. What is the SNR? [3]

iii. What is the data rate R if $P_e \leq 10^{-7}$? [3]

iv. What is the constellation used for your answer in part 2.a.iii? [3]

v. Draw the modulator, and specify input bits, the message m and the mapping into the in-phase component x_I and the quadrature component x_Q. [2]

vi. Draw a simple demodulator. [2]

(b) Either square or cross QAM can be used on an AWGN channel with SNR = 30.2 dB and symbol period T = 10^{-6}. Answer the following sub-questions.

i. Select a QAM constellation and specify a corresponding integer number of bits per symbol, b, for a modem with the highest data rate such that $P_e < 10^{-6}$. [6]

ii. Compute the data rate for part 2.b.i. [3]

3. Answer the following sub-questions.

(a) An $N = 2\bar{N} = 8$ dimensional multi-tone modulation signal is transmitted over a channel with the gain

$$H(f) = 1 + 0.5e^{j2\pi f}.$$

The signal SNR is $\bar{\varepsilon}_x |h|^2 / \sigma^2 = 10$ dB and the average energy $\bar{\varepsilon}_x = 1$. Assuming that target argument of Q-function is 9 dB, calculate the aggregate number of bits $\bar{b}$ per dimension if the total energy is distributed equally among each dimension. [5]

(b) Explain the following resource allocation methods for the High Speed Downlink Packet Access (HSDPA)

i. the equal energy loading algorithm; [4]

ii. the equal signal-to-noise ratio loading algorithm; [4]

iii. the two group resource allocation method. [4]

(c) The Levin-Campello loading algorithm will be used to improve the energy utilization for PAM/QAM signals when transmitting them over the multi-tone modulation channel with $1 + 0.5D^{-1}$. Assume that the system has $N = 8$ dimensions and operates at a bit error rate of $P_e = 10^{-6}$ when the matched filter bound signal-to-noise-ratio $SNR_{MFB} = 10dB$ and the average energy per dimension $\bar{\varepsilon}_x = 1$. Answer the following questions.

i. Create a table of incremental energies $e(n)$ vs. the channel number $n = 0, \cdots, 4$. [2]

ii. Use the EF algorithm to make the average number of bits per dimension $\bar{b} = 1$. [2]

iii. Use the E-Tightening algorithm to find the largest $\bar{b}$. [2]

iv. The total number of bits b obtained in part (3.c.iii) is to be reduced by 2 bits. Use the EF and B-Tightening algorithms to maximize the margin. What is the maximum margin? [2]

4. Answer the following sub-questions.

(a) Show that for any value of the raised cosine spectrum given by equation [9]

$$X_{RC}(f) = \begin{cases} T & 0 \le |f| \le \dfrac{1-\alpha}{2T} \\ \dfrac{T}{2}\left\{1+\cos\left(\dfrac{\pi T}{\alpha}\left(|f| - \dfrac{1-\alpha}{2T}\right)\right)\right\} & \dfrac{1-\alpha}{2T} \le |f| \le \dfrac{1+\alpha}{2T} \\ 0 & \text{otherwise} \end{cases}$$

satisfies

$$\int_{-\infty}^{\infty} X_{RC}(f)\ df = 1$$

(Hint use the fact that $X_{RC}(f)$ satisfies the Nyquist criterion).

(b) A voice-band telephone channel has a passband characteristic in the frequency range $300 < f \le 3000$ Hz.

i. Select a symbol rate and a power efficient constellation size to achieve 9600 bits/sec signal transmission. [4]

ii. If a square-root raised cosine pulse is used for the transmitter pulse select the roll-off factor. Assume that the channel has an ideal frequency response characteristic. [4]

(c) A PAM system transmits time waveforms over a filtered AWGN channel when using the basis function $\varphi(t) = \frac{1}{\sqrt{T}}\operatorname{sinc}\left(\frac{t}{T}\right)$ with $T = 1$ over a channel with a frequency response ($|a| < 1$) :

$$H(\omega) = \begin{cases} \frac{1}{1+a\exp(j\omega)} & |\omega| \le \pi \\ 0 & |\omega| > \pi \end{cases}$$

when the SNR$=\frac{\bar{\mathcal{E}}_x}{\sigma^2} = 15dB$.

i. Find the Fourier Transform of the pulse, $P(\omega)$. [2]

ii. Find the pulse energy $|p|^2$. [1]

iii. Find $Q(D)$, the function characterizing ISI for the channel. [1]

iv. Find the equalizer filter coefficients $W(D)$ for the zero forcing equalizer and MMSE linear equalizer on this channel. [3]

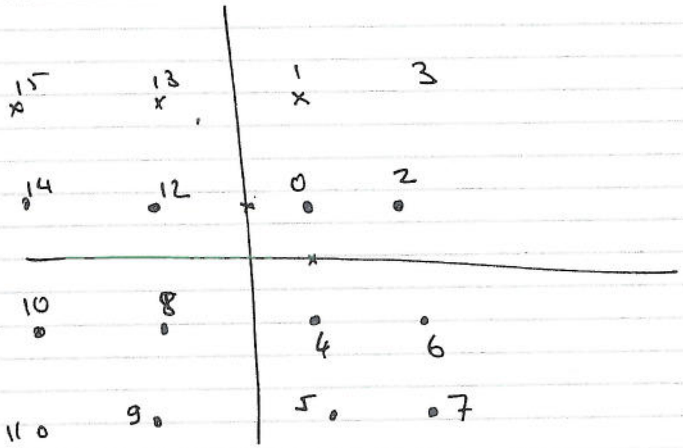
v. If $a = 0$, what data rate is achievable when the time waveforms are transmitted over this channel according to the gap approximation at a probability of error $P_e = 10^{-6}$? [1]

1.a $\int_0^1 \phi_1(t)\,\phi_2(t) = \int_0^1 2\sin(2\pi t)\cos(2\pi t)\,dt = \int_0^1 \sin(4\pi t)\,dt = 0$

$\int_0^1 \phi_1^2(t) = \int_0^1 2\cos^2(2\pi t)\,dt = \int_0^1 [1+\cos(4\pi t)]\,dt = 1$

$\int_0^1 \phi_2^2(t) = \int_0^1 2\sin^2(2\pi t)\,dt = \int_0^1 [1-\cos(4\pi t)]\,dt = 1.$

b.



Signal constellations

c.) i.) $\mathcal{E}_x = \frac{1}{4}(2+18+10+10) = 10.$

$\bar{\mathcal{E}}_x = \frac{10}{2} = 5$

ii.) $\mathcal{E}_x = \frac{4}{8}2 + \frac{4}{24}(10+10+18) = \frac{22}{3}$

$\underline{\mathcal{E}}_x = 11/2$

(d.) (i) Maximum likelihood decision rule.

$$P_{Y/X}(V/X_i) \underset{\hat{m}=m_i}{\geq} P_{Y/X}(Y/X_j)$$

(ii) MAP decision rule.

$$P_X(X_i)\, P_{Y/X}(V/X_i) \underset{\hat{m}=m_i}{\geq} P_{Y/X}(V/X_j)\; P(X_j)$$

(e.) (i.) Basis detector

$$y_i = \int_0^T y(t)\,\phi_i(t)\,dt.$$

$$\vec{Y} = [y_1 \, \ldots \, y_n]^T$$

Basis detector

$$\langle \vec{Y}, \vec{x}_i \rangle + c_i \underset{\hat{m}=m_i}{\geq} \langle \vec{Y}, \vec{x}_j \rangle + c_j$$

$$c_i = \frac{N_0}{2}\log(P_x(x_i)) - \frac{|\bar{x}_i|^2}{2}.$$

ii) use Signal detector

$$\langle \vec{Y}, \vec{x}_i \rangle + c_i \geq \langle \vec{Y}, \vec{x}_j \rangle + c_j$$

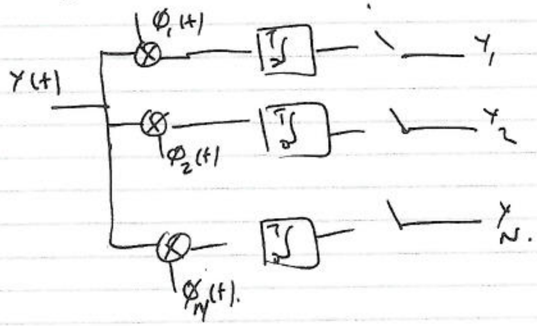
where $\langle \vec{Y}, \vec{x}_j \rangle$ is generated from.

$$\langle \vec{Y}, \vec{x}_j \rangle = Y(t) * x_j(T-t).$$

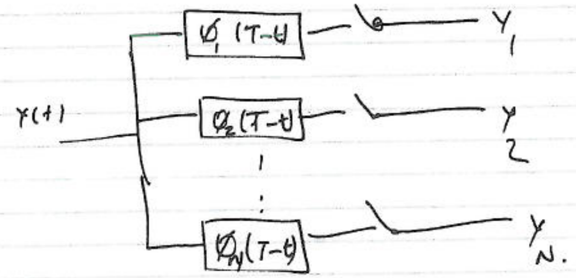
iii) maximum likelihood detector

$$\| \vec{r} - \vec{x}_i \|^2 \underset{\hat{m}=m_i}{\leq} | \bar{V} - \bar{V}_j |^2$$

iv). correlation demodulator.



V.) Matched filter demodulator.



vi) Minimum distance decoder is same as the maximum likelihood detector.

ADVANCED DATA COMMUNICATION.

1) a.) i The symbol rate is 1 M symbol/sec so the symbol period is $T = 10^{-6}$ sec. The bandwidth required is 1 MHz. We are able to choose a carrier frequency in the range [0.5 MHz to 49.5 MHz]. If we select $f = 0.5$ MHz the two basis functions are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \operatorname{sinc}\left(\frac{t}{T}\right) \cos(2\pi f_c t) = \sqrt{2}\, 10^3 \operatorname{sinc}(10^6 t) \cos(10^6 \pi t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \operatorname{sinc}\left(\frac{t}{T}\right) \sin(2\pi f_c t) = \sqrt{2}\, 10^3 \operatorname{sinc}(10^6 t) \sin(10^6 \pi t)$$

a.) ii) $\mathcal{E}_x = 1 \times T = 10^{-6} = -60$ dB

$$\bar{\mathcal{E}}_x = \frac{\mathcal{E}_x}{2} = -60 - 3 = -63 \text{ dB}$$

$$SNR = \frac{\bar{\mathcal{E}}_x}{\sigma^2} = -63 - (-103) = 40 \text{ dB}.$$

iii) We do not know the gap for $P_e = 10^{-7}$. We use the square and cross QAM equations. For square QAM we have

$$P_e = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1} SNR}\right)$$

for cross QAM we have

$$P_e = 4\left(1 - \frac{1}{\sqrt{2M}}\right) Q\left(\sqrt{\frac{3}{\frac{31}{32}M-1}\,SNR}\right)$$

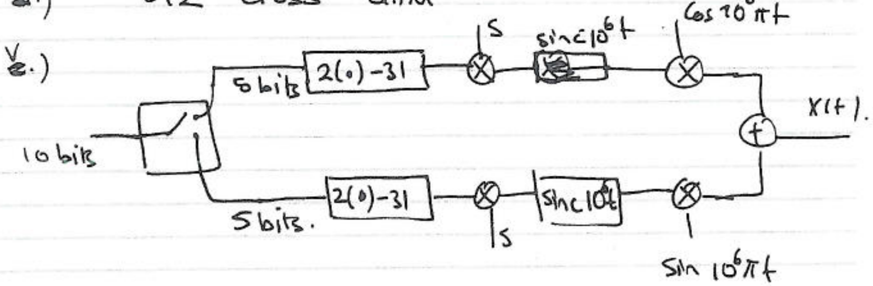
For $\bar{b} = 5$ we have $P_e = 1.855\ 10^{-7}$ and for

$\bar{b} = 4.5$ $P_e = 1.35 \times 10^{-4}$

$$R = \frac{2\bar{b}}{T} = \frac{2 \times 4.5}{10^{-6}} = 9 \text{ Mbps.}$$

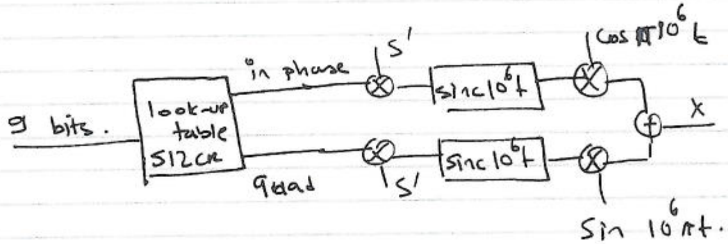
iv d.) 512 cross QAM

v e.)



where $s = \frac{d}{2}\sqrt{\frac{2}{T}} = \frac{d}{\sqrt{2}}10^3$

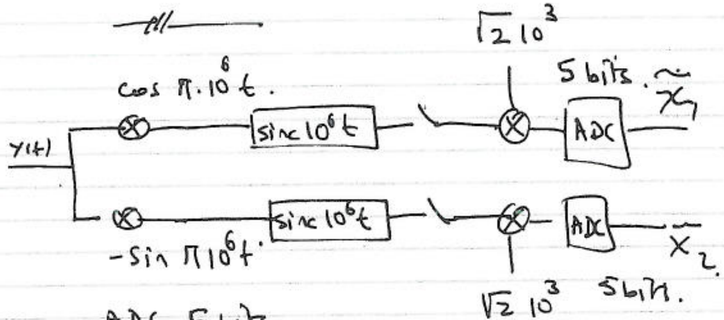
where $d = \sqrt{\frac{12\bar{\mathcal{E}}}{1023}} = \sqrt{\frac{12 \times 10}{1023}} = 2.4$

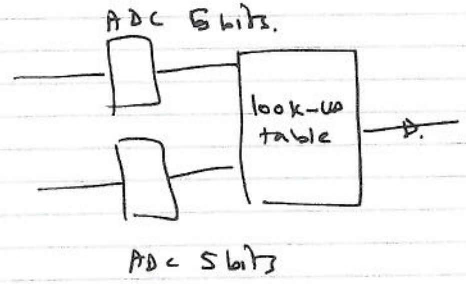


$s' = \frac{d'}{2}\sqrt{\frac{2}{1}} = \frac{d}{\sqrt{2}}10^3$, $d' = \frac{12\bar{\mathcal{E}}_x}{\frac{31}{32}2^{2\bar{b}}-1} = 3.5$

-//-

vi)





2.b The probability of error P_e formulae to use are

$$P_e \le 4\, Q\left(\sqrt{\frac{3\,SNR}{M-1}}\right) \quad \text{for Square QAM}$$

and

$$P_e \le 4\, Q\left(\sqrt{\frac{3\,SNR}{\frac{31}{32}M-1}}\right) \quad \text{for cross QAM.}$$

From the table of $Q(\cdot)$ function we see that to obtain $P_e < 10^{-6}$ we need

$$\frac{3\ SNR}{M-1} = 13.9 \text{ dB} \quad \text{for SQ-QAM.}$$

$$\frac{3\ SNR}{\frac{31}{32}M-1} = 13.9 \text{ dB} \quad \text{for CR-QAM.}$$

Since SQ QAM requires even valued b, hence the highest data rate for SQ-QAM is obtained for b=6. On the other hand CR-QAM requires odd-valued b, and so the highest data rate for CR-QAM is obtained for b=7, which is clearly more than SQ-QAM case.

Thus, to obtain the highest data rate at specified target P_e, we choose $b=7$, ie 128 CR-QAM.

2.b.ii) The data rate is computed as

$$R = \frac{b}{T} = 7\times10^{6} = 7 \text{ Mbps.}$$

2 ~~b.iii) Pro~~ //.

ADC

3.a

$$H(D) = 1 + 0.5D$$

$$h = [1 \quad 0.5]^T$$

$$|h|^2 = \sum_{i=1}^{2} h_i^2 = 1^2 + (0.5)^2 = 1.25$$

$$\bar{\mathcal{E}}_x = 1$$

$SNR_{MFB} = 10$ dB $\qquad SNR_{MFB} = 10 = \dfrac{\bar{\mathcal{E}}_x |h|^2}{\sigma^2}$

$$\sigma^2 = \frac{1 \times 1.25}{10} = 0.125$$

$$SNR_n = \frac{\mathcal{E}_n |H_n|^2}{\sigma^2}$$

for $N = 8$

n	0	1	2	3
$\mathcal{E}_n$	8/7	16/7	16/7	16/7
$\lvert H_n \rvert$	1.5	1.4	1.118	0.737
SNR_n	20.6	17.9	11.43	4.97.
$\bar{b}_n$	1.566	1.479	1.205	0.761

where

$$\bar{b}_n = \frac{1}{2}\log_2\left(1 + \frac{SNR_n}{\Gamma}\right) = \frac{1}{2}\log_2\left(1 + \frac{SNR_N \times 3}{\Gamma}\right)$$

$$\Gamma = 10^{0.9} = 7.94$$

$$\bar{b} = \frac{1 \times 1.566 + 2 \times 1.479 + 2 \times 1.205 + 2 \times 0.761}{8}$$

$$\boxed{\bar{b} = 1.057}$$

3.b

Total SNR at receiver

$SNR = \dfrac{\mathcal{E}_T |h|^2}{2\sigma^2}$ $\qquad$ rate $r = \dfrac{b_T}{T}$ bps.

Channel.

$$H = \begin{bmatrix} h_0 & & 0 \\ \vdots & h_0 & 0 \\ h_{L-1} & & h_0 \\ 0 & h_{L-1} & \\ 0 & & h_{L-1} \end{bmatrix}$$

$$|h|^2 = \sum_{i=0}^{L-1} h_i^2$$

Transmission signature waveform

$$S = [\vec{s}_1 \; \cdots \; \vec{s}_K]$$

Receiver signature waveform

$$Q = [\vec{q}_1 \; - \; - \; \vec{q}_K] = [O^T_{\alpha \times K} \; (HS)^T \; O^T_{(\alpha-L+1)\times K}]^T$$

The covariance matrix of received signal

$$C = Q A^2 Q^H + N_0 I_{N+2K}.$$

$A = \mathrm{diag}(\sqrt{E_1}, \sqrt{E_2} \ldots \sqrt{E_K})$ and $\frac{N_0}{2} = \sigma^2$ is the two sided noise power spectral density.

The SNR at the output k^{th} channel MMSE receiver is.

$$\gamma_k = \frac{E_k \vec{q}_k^H C^{-1} \vec{q}_k}{1 - E_k \vec{q}_k^H C^{-1} \vec{q}_k}$$

Equal energy loading

$$E_k = \frac{E_T}{K}.$$

$$A = \mathrm{diag}(\sqrt{E_1} \ldots \sqrt{E_k}).$$

$$C = Q A^1 Q^H + N_0 I.$$

$$\gamma_k = \frac{E_k \vec{q}_k C^{-1} \vec{q}_k}{1 - E_k \vec{q}_k^H C^{-1} \vec{q}_k}$$

Allocate ~~bit~~ data bit rate per symbol for a given $E_k = \frac{E_T}{K}$.

$$\Gamma(2^{b_p} - 1) \leq \min_k \left(\frac{E_k \vec{q}_k C^{-1} \vec{q}_k}{1 - E_k \vec{q}_k^H C^{-1} \vec{q}_k} \right) < \Gamma(2^{b_p+1} - 1)$$

The HSDPA wastes a total of

$$\text{Wasted SNR} = \sum_{k=1}^{K} \frac{E_k \vec{q}_k^H C^{-1} \vec{q}_k}{1 - E_k \vec{q}_k^H C^{-1} \vec{q}_k} - K\Gamma(2^{b_p} - 1)$$

3.b.ii Equal SNR Loading.

$$E_k = \frac{E_T}{K} \quad \text{for } k = 1 \ldots K.$$

$$A_i = \mathrm{diag}(\sqrt{E_{1(i-1)}} \ldots \sqrt{E_{K(i-1)}})$$

$$C_i = \vec{Q} A_i^2 \vec{Q}^H + N_0 I$$

$$E_k = \frac{\Gamma(2^{b_p} - 1)}{(1 - \Gamma(2^{b_p} - 1)) \vec{q}_k^H C_i^{-1} \vec{q}_k}$$

Each channel is allocated a different E_k to achieve an equal SNR $\Gamma(2^{b_p}-1)$ at the output of the MMSE receiver

we have residual energy

$$E_T - \sum_{k=1}^{K} E_k(b_p)$$

which is not used to transmit any useful information.

3.b.iii) in two group allocation we aim to maximise.

$$r_T = m\, b_{p+1} + (K-m)\, b_p.$$

Determine b_p from

$$0 \leq \left(E_T - \sum_{k=1}^{K} E_k(b_p)\right) < \left(\sum_{k=1}^{K} E_k(b_{p+1}) - \sum_{k=1}^{K} E_k(b_p)\right)$$

and m from

$$0 \leq \left(E_T - \sum_{k=1}^{K} E_k(b_p) - \sum_{J=1}^{m} e_j(b_p)\right) < e_{m+1}(b_p)$$

$$e_j(b_p) = \left(\sum_{k=1}^{j} E_k(b_{p+1}) + \sum_{k=j+1}^{K} E_k(b_p)\right) - \left(\sum_{k=1}^{J-1} E_k(b_{p+1}) - \sum_{k=j}^{K} E_k(b_p)\right)$$

3.c) We will proceed ~~with~~ using the gap approximation

$$E_n(b_n) = \frac{\Gamma}{g_n}\left(2^{\frac{2b_n}{k}}-1\right) \times k \quad \text{where } k=1$$

if PAM and $k=2$ if QAM.

So we first need to find $g_n = \frac{|H_n|^2}{\sigma_n^2}$. From System parameters $\sigma_n^2 = 0.125$. So we have the following table

Subchannel.	0	1	2	3	4
g_n	18	15.6569	10	4.3431	2

Now using the above formula we get.

Subchannel.	0	1	2	3	4
$e_n(1)$	1.2403	0.969	1.5172	3.4932	11.37
$e_n(2)$	5.07	1.938	3.043	6.986	45.523
$e_n(3)$	19.2287	3.876	6.0686	13.97	182.05
$e_n(4)$	81.9150	7.7521	12.1372	27.94	728.234

with the above table it is obvious that the bit allocations are as follows.

ii)

Sub channel	0	1	2	3	4
b_n.	2	3	2	1	0
$\mathcal{E}_n(b_n)$	6.3215	6.783	4.5515	3.4932	0

Bits were chosen in the following order

1, 0, 2, 1, 2, 3, 1, 0

$N * \bar{\mathcal{E}}_x = 8$ so we are way over budget.

Working backwards we get.

iii

Sub channel.	0	1	2	3	4
b_n	1	2	1	0	0
$\mathcal{E}_n(b_n)$	1.2463	2.9	1.5172	0	0

iv) Again we just work backwards.

Subchannel	0	1	2	3	4
b_n	1	1	0	0	0
$\mathcal{E}_n(b_n)$	1.2463	0.969	0	0	0

The margin in this case is $10 \times \log\left(\frac{8}{[illegible]}\right)$

[illegible]

4.9 Substituting the expression $X_{rc}(f)$ in the desired integral we obtain.

$$\int_{-\infty}^{\infty} X_{rc}(f)\,df = \int_{-\frac{1+\alpha}{T}}^{-\frac{1-\alpha}{T}} \frac{T}{2}\left[1 + \cos\frac{\pi T}{\alpha}\left(-f - \frac{1-\alpha}{2T}\right)\right]df$$

$$+ \int_{-\frac{1-\alpha}{T}}^{\frac{1-\alpha}{T}} T\,df$$

$$+ \int_{\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} \frac{T}{2}\left[1 + \cos\frac{\pi T}{\alpha}\left(f - \frac{1-\alpha}{2T}\right)\right]df.$$

$$= \int_{-\frac{1+\alpha}{2T}}^{-\frac{1-\alpha}{2T}} \frac{T}{2}\,df + T\left(\frac{1-\alpha}{T}\right) + \int_{\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} \frac{T}{2}\,df.$$

$$+ \int_{-\frac{1+\alpha}{2T}}^{-\frac{1-\alpha}{2T}} \frac{T}{2}\cos\frac{\pi T}{\alpha}\left(f + \frac{1-\alpha}{2T}\right)df$$

$$+ \int_{\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} \cos\left(\frac{\pi T}{\alpha}\left(f - \frac{1-\alpha}{2T}\right)\right)df.$$

$$= 1 + \int_{-\frac{\alpha}{T}}^{0} \cos\frac{\pi T}{\alpha} x\,dx + \int_{0}^{\alpha/T} \cos\frac{\pi T}{\alpha} x\,dx$$

[illegible]

ADC

$$= 1 + \int_{-\frac{\alpha}{T}}^{\frac{\alpha}{T}} \cos\frac{\pi T}{\alpha} x \, dx = 1 + 0 = 1 .$$

4.b The bandwidth of the channel is

$$W = 3000 - 300 = 2700 \text{ Hz}.$$

Since the minimum transmission bandwidth required for bandpass signalling is R, where R is the rate of transmission we conclude that the maximum value of the symbol rate for the given channel is $R_{max} = 2700$. If an M-ary PAM modulation is used for transmission, then in order to achieve a bit-rate of 9600 bps, with maximum rate of R_{max}, the minimum size of the constellation is $M = 2^k = 16$. In this case the

ADC

symbol rate is

$$R = \frac{9600}{k} = 2400 \text{ symbols/sec.}$$

and the symbol interval is

$$T = \frac{1}{R} = \frac{1}{2400} \text{ sec.}$$

The roll-off factor α of the raised cosine pulse used for transmission is determined by noting that $1200(1+\alpha) = 1350$ and hence $\alpha = 0.125$. Therefore the squared root cosine pulse can have a roll-off of $\alpha = 0.125$.

4.c

i) The pulse response is $p(t) = \phi(t) * h(t)$.

$$p(f) = \phi(f)\, H(f).$$

$$= \left(\sqrt{T}\,\mathrm{rect}(Tf)\right)\left(\frac{1}{1 + a\exp j2\pi f}\,\mathrm{rect}(f)\right)$$

$$= \frac{1}{1 + a\exp(j2\pi f)}\,\mathrm{rect}(f) \quad \text{since } T=1.$$

In terms of ω.

$$P(\omega) = \begin{cases} \dfrac{1}{1+a\exp(j\omega)} & |\omega| \le \pi \\ 0 & |\omega| > \pi. \end{cases}$$

ii) First let us find $P(\exp(-j\omega T))$

$$P(\exp(-j\omega T)) = \frac{1}{T}\sum_{n=-\infty}^{\infty} P\left(\omega + \frac{2\pi n}{T}\right)$$

Since $T=1$ and $P(\omega)=0$ for $|\omega|>\pi$.

$$P(\exp(-j\omega T)) = \frac{1}{1 + a\exp(j\omega)}. \text{ Then.}$$

by inverse Fourier Transform

$$p_k = (-a)^k\, u(-k)$$

Therefore

$$\|P\|^2 = T\sum_{k=-\infty}^{\infty} |p_k|^2$$

$$= \sum_{k=-\infty}^{0} (-a)^{2k} = \frac{1}{1-a^2}$$

iii) By ~~to~~ substituting $\exp(-j\omega T) = D$ into $P(\exp(-j\omega T))$ we get

$$P(D) = \frac{1}{1+aD^{-1}} \quad \text{Therefore}$$

$$Q(D) = \frac{T}{\|P\|^2}P(D)P(D^{-*}) = \frac{1-a^2}{(1+aD)(1+aD^{-1})}$$

iv) ZF equaliser.

$$W_{ZFE}(D) = \frac{1}{\|P\|\, Q(D)} = \frac{(1+aD)(1+aD^{-1})}{\sqrt{1-a^2}}.$$

$$SNR_{MFB} = \frac{\|P\|^2\,\bar{\mathcal{E}}_x}{\sigma^2} = \frac{10^{1.5}}{1-a^2}$$

~~ZF~~ mmse equaliser.

$$W_{MMSE\text{-}LE} = \frac{1}{\|P\|\left(Q(D) + \frac{1}{SNR_{MFB}}\right)}$$

ADC

$$W_{MMSE\text{-}LE} = \frac{\sqrt{1-a^2}}{(1+aD)(1+aD^{-1}) + \frac{1+a^2}{10^{1.5}}}$$

$$= \frac{(1+aD)(1+aD^{-1})}{\sqrt{1-a^2}\left(1+(1+aD)(1+aD^{-1})\,10^{-1.5}\right)}$$

v) when $a=0$

$Q(D)=1$ and $|P|^2=1$ since SNR=15dB

$\Gamma = 8.8\ dB$ at $P_e = 10^{-6}$

$$\bar{b} = \frac{1}{2}\log_2\left(1+\frac{10^{1.5}}{10^{0.88}}\right) = 1.18.$$

Then maximum data rate achievable is

$$R = \frac{b}{T} = \frac{1.18}{1} = 1.18\ \text{bits/s}.$$