Paper Number(s): **E1.3**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

EEE PART I: M.Eng., B.Eng. and ACGI

# DEVICES AND FIELDS

Friday, 15 June 10:00 am

There are FIVE questions on this paper.

There are two sections. Answer THREE questions including at least ONE question from each section.

Use a separate answer book for each section.

Time allowed: 2:00 hours

CORRECTED COPY

Examiners: Wright,S.W., Cozens,J.R., Leaver,K.D. and Green,T.C.

**Formulae and Constants**

For Silicon at 300K:

$N_C$, $N_V$=2x10$^{25}$ m$^{-3}$ $\quad$ $n_i$=1.45x10$^{16}$ m$^{-3}$

$\mu_e$=1300 cm$^2$ V$^{-1}$ sec$^{-1}$ $\quad$ $\mu_h$=500 cm$^2$ V$^{-1}$ sec$^{-1}$

For Silicon Dioxide: $\varepsilon_{0x}=4\mathrm{x}\,\varepsilon_0$

$\varepsilon_0$=8.85x10$^{-12}$ F m$^{-1}$ $\quad$ e=1.6x10$^{-19}$ C $\quad$ k=1.38x10$^{-23}$ JK$^{-1}$

$$f(E)=\frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

$$I_D=\frac{W C_{OX}\mu}{L}\left(V_{GS}-V_T-\frac{V_{DS}}{2}\right)V_{DS} \qquad (V_{GS}-V_T)>V_{DS}>0$$

$$C_{OX}=\frac{\varepsilon_0\varepsilon_{OX}}{t_{OX}} \qquad n=N_C f(E) \qquad n\times p=n_i^2 \qquad \frac{D}{\mu}=\frac{kT}{e}$$

## SECTION A

**Use a separate answer book for each section**

1. A silicon p-n junction diode has an acceptor doping density of $10^{24}$ m$^{-3}$ and a donor doping density of $10^{22}$ m$^{-3}$.

   i Calculate the concentrations of electrons and holes at 300K in both the p and n type regions, far away from the junction. [4]

   ii Sketch an energy level diagram for this diode when unbiased, labelling the Fermi level and the conduction and valence bands. [3]

   iii Give a labelled sketch of the distributions of minority carriers in the diode when it is (a) forward biased and (b) reverse biased, assuming that it behaves as a short diode. [4]

   iv What is the significance of the word 'short' in the description of this diode? [2]

   v From your diagram, deduce an expression for the reverse saturation current density of this diode, and evaluate it given that the n- and p- regions each have a length of $120\mu$m. [5]

   vi What would be the value of the reverse saturation current density if the acceptor doping density is reduced by a factor of $10^3$? [2]

2. Figure 1 shows the dependence of drain current on gate-source voltage for a particular silicon MOSFET measured using a source-drain voltage $V_{DS}$ of 0.1V

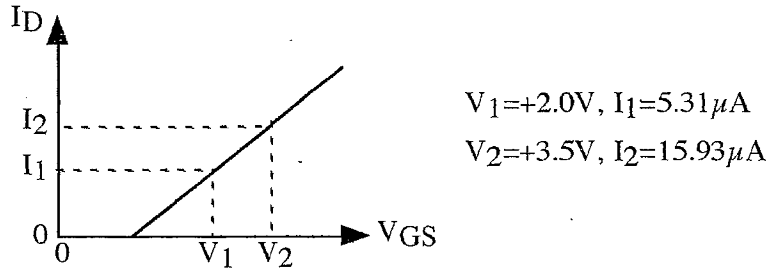


Figure 1 Drain current vs Gate-source voltage characteristic of a MOSFET

i What type of MOSFET is this the characteristic of? [2]

ii What is meant by the threshold voltage? [2]

iii Calculate the values of the threshold voltage and transconductance for this MOSFET. [6]

iv If the device dimensions have the following values, calculate the mobility of the current carriers in the MOSFET - channel length=1$\mu$m, channel width=2$\mu$m, insulator thickness=50nm. [6]

v Why is the value of mobility lower than that which would be measured in a sample of bulk silicon? [2]

vi What would be the dependence of $I_D$ on $V_{GS}$ in the saturation region of operation? [2]

3. i Sketch the form of the Fermi-Dirac distribution function for T=0K and T>0K, showing the position of the Fermi energy, and the value of the function at that energy. [4]

ii State briefly the physical meaning of the Fermi-Dirac distribution function [2]

iii Calculate the differences in energy at 300K between the conduction band and Fermi level for n-type and p-type silicon, doped in each case with $10^{23}$ $m^{-3}$ doping atoms. [6]

iv Sketch the small-signal equivalent circuit of a bipolar transistor, labelling each component in the equivalent circuit. [4]

v Explain briefly how the design of a bipolar transistor can be optimised to increase the current gain [4]

## SECTION B

*Use a separate answer book for each section*

4. State Gauss' Law in electrostatics. [4]

   a) A parallel plate capacitor with plates of area $A$ and separation $d$, has a block of dielectric, of relative permittivity $\varepsilon$, of cross-sectional area $A$, and thickness $t$ ($t<d$), inserted between the plates.

   Find the values of the electric field strength $E$, and the electric flux density $D$, in both the air and the dielectric.

   [4]

   Show that the capacitance of the capacitor is given by

   $$C = A\varepsilon\varepsilon_0 / [\varepsilon\, d - (\varepsilon - 1)t]\ . \qquad [6]$$

   b) The values of the vertical potential gradient at heights of 100 and 1000 metres above the earth's surface are 110 and 25 V/metre respectively. What is the mean electrostatic charge per cubic metre of the atmosphere between these heights? [6]
   ($\varepsilon_0 = 8.85\text{x } 10^{-12}$ F/m )

5. State Ampere's Law in magnetostatics. [4]

An electromagnet is held close to a piece of iron, of mass m, which is lying freely on a wooden surface, as shown in figure 2(a).

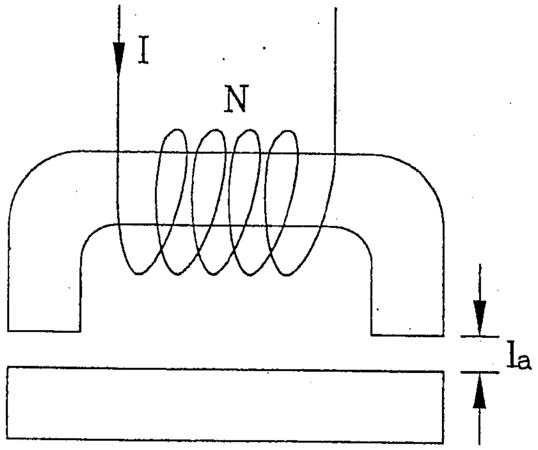


Figure 2(a)

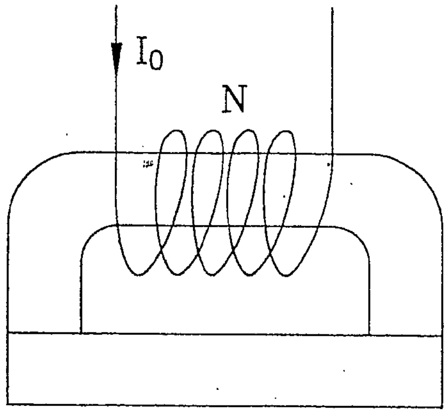


Figure 2(b)

Deduce an expression for the magnetic flux in the iron when a current of I amps flows through the coil, and also for the inductance of the coil. [4,2]

By considering the energy stored in an inductor, or otherwise, deduce an expression for the force pulling the iron piece towards the electromagnet. [4]

Given that the relevant parameters are as follows;

| | | |
|---|---|---|
| Total path length in iron | = | 1m |
| Each air gap | = | 5 mm. |
| Cross Sectional area | = | 25 $cm^2$ |
| Number of turns | = | 100 |
| Mass of iron piece | = | 2 kg |
| Relative permeability of iron | = | $10^4$ |
| $\mu_0$ | = | $4\pi 10^{-7}$ F/m |
| g | = | 9.81 $m/s^2$ |

calculate a value for the minimum current, $I_0$, required to lift the iron from the surface. [4]

If the iron is brought into contact with the electromagnet, as shown in figure 2(b), and the current $I_0$ is maintained, what is the minimum force required to separate the iron from the magnet? [2]

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Section A

Q1

(i) n side $n_n = N_D = 10^{22}\,m^{-3}$

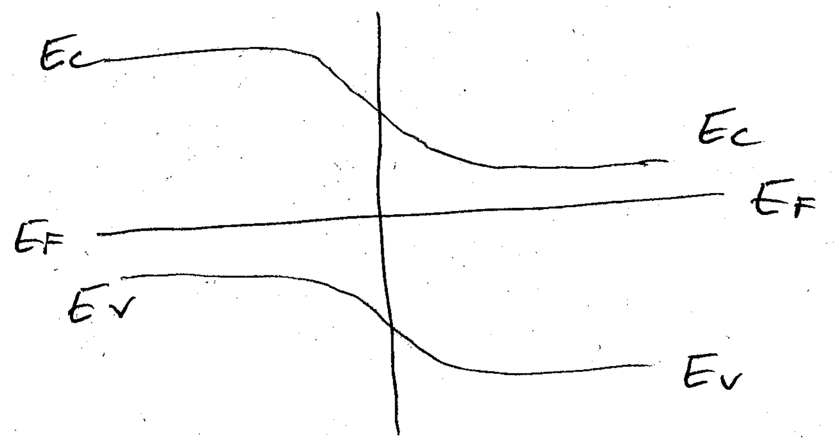
$$p_n = \frac{n_i^2}{n_n} = \frac{(1.45\times10^{16})^2}{10^{22}} = 2.025\times10^{10}\,m^{-3}$$

[4]

p side $p_p = N_A = 10^{24}\,m^{-3}$

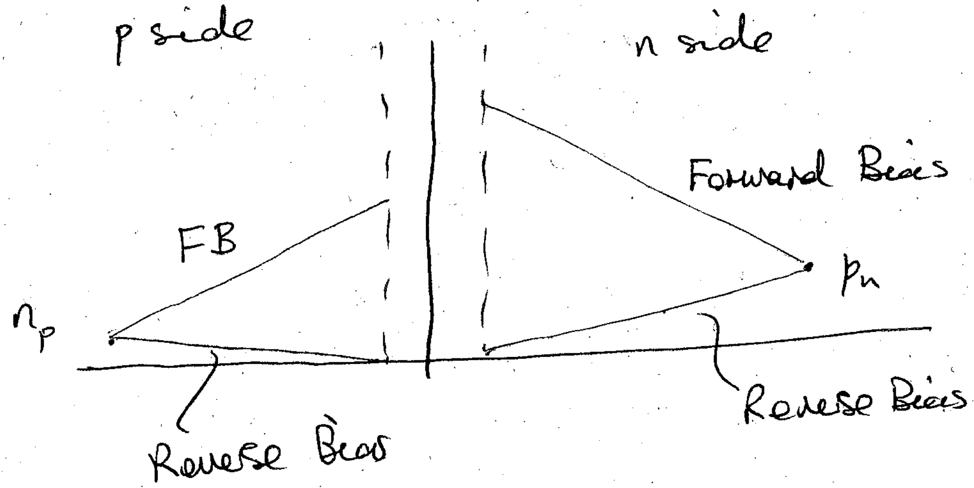
$$n_p = \frac{n_i^2}{p_p} = \frac{(1.45\times10^{16})^2}{10^{24}} = 2.025\times10^{8}\,m^{-3}$$

(ii)




[3]

(iii)




[4]

(iv) A "short" diode - ~~no recombin~~ recombination of minority carriers can be neglected [2]

Minority carrier distributions in (iii) above are then linear

(v) $$J_s = e\left(\frac{D_e n_p}{L_p} + \frac{D_h p_n}{L_n}\right)$$ [2]

$$\frac{D}{\mu} = \frac{kT}{e} \quad \text{so} \quad J_s = kT\left(\frac{\mu_e n_p}{L_p} + \frac{\mu_h p_n}{L_n}\right)$$

$$= 1.38\times10^{-23} \times 300\left(\frac{0.13\times2.025\times10^{8}}{120\times10^{-6}} + \frac{0.05\times2.025\times10^{10}}{120\times10^{-6}}\right)$$

$$= 4.14\times10^{-21}\left(2.19\times10^{11} + 8.44\times10^{12}\right)$$

$$= \underline{3.58\times10^{-8}\ Am^{-2}}$$ [3]

(vi) If $N_A$ and hence $p_p$ reduced by $10^3$

$n_p$ becomes $2.025\times10^{11}\ m^{-3}$

~~so~~ thus first term in above increased by $\times10^3$ [2]

so $$J_{sc} = 4.14\times10^{-21}\left(2.19\times10^{14} + 8.44\times10^{12}\right)$$

$$= \underline{9.42\times10^{-7}\ Am^{-2}}$$

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Q2 (i) This is an n-channel, enhancement mode MOSFET [2]

(ii) Threshold voltage: the gate-source voltage which is needed to cause formation of a conducting channel between source & drain [2]

OR The gate-source voltage which inverts the surface of the silicon

(iii) $I_D = K(V_{GS} - V_T)$ if $V_{DS} \ll (V_{GS} - V_T)$

so $\dfrac{I_{D2}}{I_{D1}} = \dfrac{V_{GS2} - V_T}{V_{GS1} - V_T}$ so $\dfrac{15.93}{5.31} = 3 = \dfrac{3.5 - V_T}{2 - V_T}$ [3]

so $3(2 - V_T) = (3.5 - V_T)$

$V_T = 1.25\text{v}$

$g_m = \dfrac{dI}{dV_{GS}} = \dfrac{I_2 - I_1}{V_2 - V_1} = \dfrac{10.62}{1.5} = 7.08\mu A/\text{Volts}$

[3]

Q2 cont'd

(iv) $I_D = \frac{W}{L}\mu_e C_{ox}(V_{GS}-V_T)V_{DS}$ since $V_{DS} \ll (V_{GS}-V_T)$

where $C_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{t_{ox}} = \frac{4 \times 8.85\times10^{-12}}{50\times10^{-9}} = 7.08\times10^{-4}\ Fm^{-2}$

$g_m = \frac{dI}{dV_{GS}} = \frac{W}{L} C_{ox}\mu V_{DS}$ [6]

so $\mu = \frac{g_m}{\frac{W}{L} C_{ox} V_{DS}} = \frac{7.08\times10^{-6}}{2\times7.08\times10^{-4}\times0.1}$

$= 0.05\ m^2/V\text{sec}$

(v) This is lower than bulk mobility value because of surface scattering of electrons [2]

(vi) In the saturation region [2]

$I_D \propto (V_{GS}-V_T)^2$

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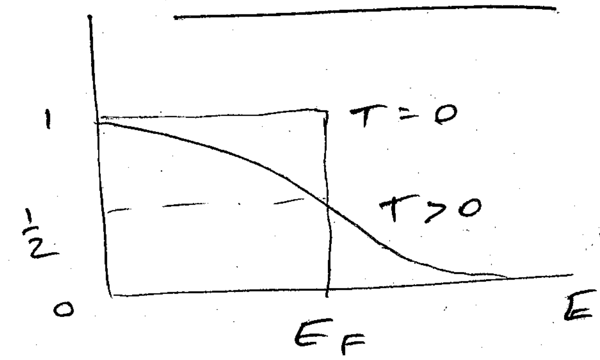
Q3 (i)




[4]

(ii) F-D function – the value at energy E is the probability of a state at energy E being occupied by an election. [2]

(iii) n-type $n = N_c f(E_c) \simeq N_c e^{-\left(\frac{E_c - E_F}{kT}\right)}$

so $E_c - E_F = \frac{kT}{e} \ln\left(\frac{N_c}{n}\right)$

and $n = N_D = 10^{23}\ m^{-3}$ [2]

so $E_c - E_F = \frac{kT}{e} \ln\left(\frac{2\times10^{25}}{10^{23}}\right) = 0.138\,eV$

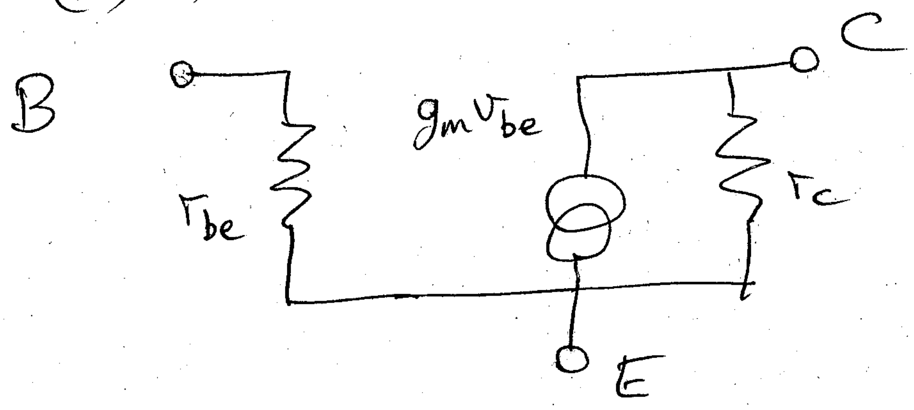
p-type $n = N_c \exp -\left(\frac{E_c - E_F}{kT}\right)$

and $n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A} = \frac{(1.45\times10^{16})^2}{10^{23}}$

$= 2.025\times10^{9}\ m^{-3}$ [2]

so $E_c - E_F = \frac{kT}{e} \ln\left[\frac{2\times10^{25}}{(2.035\times10^{9})}\right] = 0.957\,eV$ [2]

Q3 (iv)




[4]

(v) To improve gain of bipolar transistor

(a) Make base narrow - reduces $I_B$ by reducing minority carrier recombination

increases $I_c$ because steeper concentration gradient in base [4]

(b) Heavily dope emitter - reduce carrier injection from base to emitter.

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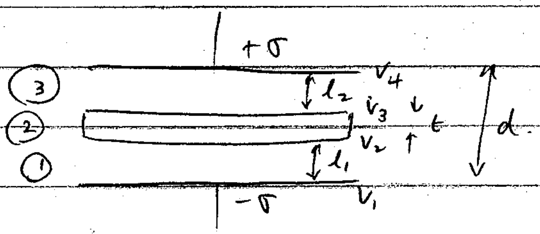
## Section B.

### Answers.

4. $$\oint \underline{D} \cdot d\underline{s} = q$$ [4]

(closed surface) (net charge within closed surface)

a)




From symmetry, lines of electric flux density are normal to the plates; and continuous between them.

$\therefore$ In all regions, $D = \sigma$ (charge/unit area).

In regions 1 & 3, $E = \dfrac{D}{\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$

& in region 2, $E = \dfrac{D}{\varepsilon_0 \varepsilon} = \dfrac{\sigma}{\varepsilon_0 \varepsilon}$ [4]

In all regions, $E = -\dfrac{dV}{dx}$

$\therefore$ in ① $V_2 - V_1 = \left(\dfrac{\sigma}{\varepsilon_0}\right) l_1$

in ② $V_3 - V_2 = \left(\dfrac{\sigma}{\varepsilon_0 \varepsilon}\right) t$

in ③ $V_4 - V_3 = \left(\dfrac{\sigma}{\varepsilon_0}\right) l_2$

Volt drop across whole capacitor $= V_4 - V_1$

$$= \frac{\sigma}{\varepsilon_0}\left[l_1 + l_3 + \frac{t}{\varepsilon}\right]$$

Charge on plates $= \sigma A = Q$,

(PTO).

$$\therefore V = Q \cdot \frac{1}{\varepsilon_0 A} \left\{ (l_1 + l_2) + \frac{t}{\varepsilon} \right\} = \frac{Q}{\varepsilon_0 A} \left\{ (d - t) + \frac{t}{\varepsilon} \right\}.$$

$$\therefore C = \frac{\varepsilon_0 A}{\left[ (d-t) + t/\varepsilon \right]} = \frac{\varepsilon_0 \varepsilon A}{(\varepsilon d - (\varepsilon - 1) t)}$$ [6].

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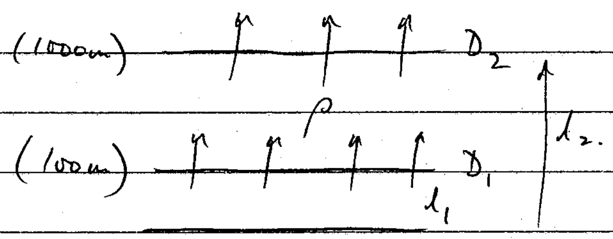
b) Since heights $\ll$ earth's radius, earth is flat!



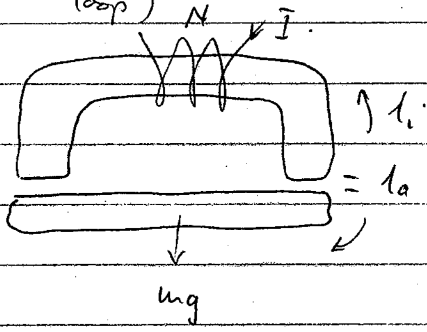
Net flux density / unit area $= D_1 - D_2 = \rho (l_2 - l_1)$.

$$\therefore \rho = \frac{\varepsilon_0 (E_1 - E_2)}{(l_2 - l_1)} = \frac{8.85 \cdot 10^{-12} (110 - 25)}{(1000 - 100)}$$

$$\rho = \underline{0.836 \ pCm^{-3}}.$$ [6].

5. $\oint H.dl = NI$ [4]

(closed loop) (within loop).




$\mu_r = 10^4$.
$A = 25.10^{-4}\ m^2$.
$m = 2kg$.
$N = 10^2$.

From Ampere's Law,

$$H_i l_i + 2H_a l_a = NI$$

$$H = \frac{B}{\mu} = \frac{\phi}{\mu A}.$$

$$\therefore \frac{\phi}{\mu_0 A}\left[\frac{l_i}{\mu_r} + 2l_a\right] = NI.$$

$$\phi = \frac{\mu_0 ANI}{\left[\frac{l_i}{\mu_r} + 2l_a\right]}.$$

$3.11 \times 10^{-5}$

$$= \frac{4\pi.10^{-7}.25.10^{-4}.10^2 I}{\left[10^{-4} + 10^{-2}\right]} = 3.14.10^{-5} I\ Wb.$$ [4]

$$L = \frac{N\phi}{I} = 3.14\ mH.$$ $3.11\ mH$ [2]

Energy in inductor $= \frac{1}{2}LI^2$.

and $F = -\frac{dE}{dl_a}$

$$\therefore F = -\left\{\frac{I_0^2}{2}\mu_0 AN^2\right\}\left\{\frac{-2}{\left(\frac{l_i}{\mu_r} + 2l_a\right)^2}\right\} = mg.$$ [4]

$$\therefore I_0^2 = \frac{mg\left(\frac{l_i}{\mu_r} + 2l_a\right)^2}{\mu_0 AN^2}. \quad \therefore \quad I_0 = 7.9\ A.$$ [4]

7.98

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When air gap is zero,

$$F = \frac{I_0^2 \mu_0 A N^2}{\left(\frac{l_i}{\mu_r}\right)^2} = \frac{7.9^2 \cdot 4\pi.10^{-7} \cdot 25.10^{-4} \cdot 10^4}{10^{-8}}$$

$$F = 1.96 . 10^5 N$$ [2].

$2.00 \times 10^5 N$