EE4-51 Power System Economics

Solution to Question 1

Discussion along the following line is needed for full mark.

(i) Unbundling refers to the process of separating the activities of generation, transmission, distribution and retail of electricity, previously performed by a single vertically integrated utility, into different entities i.e. companies. This then allows for the competition to be introduced in the generation and retail business, while transmission and distribution are organised as regulated monopolies, ensuring fair and open network access to all market participants.
The reasons for liberalising electricity markets are the increased efficiency in the supply of electricity and the lower cost of electricity to consumers. This is achieved through introducing competition both at the wholesale level (generators compete to sell electrical energy) and the retail level (consumers choose from whom they buy electricity). [3.5]

(ii) The *supply curve* indicates the value that the market price should take to make it worthwhile for the aggregated producers to supply a certain quantity of the commodity to the market. Similarly, the *demand curve* describes the quantity of products that the aggregated consumers are willing to purchase for a particular price in a given market. Demand curve is normally downward sloping, while the supply curve is normally upward sloping. *Market clearing price* or the *equilibrium price* is the price for which the quantity that the suppliers are willing to provide is equal to the quantity that the consumers wish to obtain. *Social welfare* is the sum of the net consumers' surplus and of the producers' profit, and it quantifies the overall benefit that arises from trading. [3.5]

(iii) In situations where transmission network causes the generation dispatch to be constrained (i.e. the generators cannot be used in the most cost-efficient way), they segment the market and the price of electricity is not the same at each bus of the system. This is because the marginal generator is not the same in all parts of the network, but rather varies with location, as do the locational prices.
The *cost of constraints* is the cost of making the network secure and it is calculated as the difference between the cost of the constrained dispatch (dispatch when network constraints are taken into account) and the cost of the economic dispatch (dispatch when network constraints are not taken into account). The *congestion cost (surplus)* is the difference between demand charges and generation payments, emerging from the congestion in the network. It is calculated as the sum of products of differences between the nodal prices at any two buses and the flow on the transmission line between those two buses:

$$S_{cong} = \sum_{i,j} (\pi_j - \pi_i) F_{i-j}$$

[3.5]

(iv) The transmission operator's revenue is given by: $R = \pi_T \cdot F = 7.2F - 0.015F^2$. The capacity resulting in maximum revenue is obtained by differentiating this expression and making it equal to zero: (3)

$$7.2 - 0.03F = 0 \Rightarrow F = 240 \text{ MW}$$

(v) The optimal capacity from the perspective of global welfare satisfies the fact that:
Marginal value of transmission = Marginal investment cost of transmission
Value of transmission is given by: $\pi_T = 7.2 - 0.015F$ (3)
Marginal investment cost of transmission (hourly) = 13,140 / 8,760 = 1.5 £/MWh
Hence, $7.2 - 0.015F = 1.5 \Rightarrow F = 380$ MW.

(vi) FTRs are market instruments defined between any two nodes in the network, and they entitle their holders to a revenue equal to the product of the amount of transmission rights bought and the price differential between the two nodes: $R_{FTR} = F(\pi_1 - \pi_2)$. This enables a contract for difference between a producer at location 1 and a consumer at location 2 to be settled even in the presence of congestion in the network. (3.)

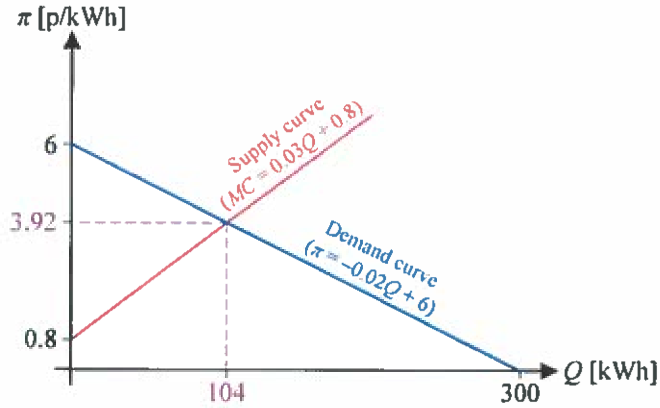
Solution to Question 2

(i) Marginal production cost: $MC = \dfrac{dC}{dQ} = 0.03Q + 0.8\,[\text{p/kWh}]$

The equilibrium price and demand:

$$\pi = MC \Rightarrow -0.02Q + 6 = 0.03Q + 0.8$$
$$\Rightarrow \pi_{eq} = 3.92 \text{ p/kWh},\ Q_{eq} = 104 \text{ kWh}$$

Sketch of the supply and demand functions:



(4)

(ii) Total production cost $= C(104) = 0.015\cdot 105^2 + 0.8\cdot 105 = 245.44$ p

Average production cost = Total production cost / 104 kWh = 2.36 p/kWh

Supplier's revenue $= 104\cdot 3.92 = 407.68$ p

Supplier's profit = Supplier's revenue – Production cost = $407.68 - 245.44 = 162.24$ p

Demand charges = supplier's revenue = 407.68 p

Gross demand benefit $= 0.5\cdot(6 + 3.92)\cdot 104 = 515.84$ p

Consumer's surplus $= 0.5\cdot(6 - 3.92)\cdot 104 = 108.16$ p

[4]

(iii) Consumption: $Q = \dfrac{6-\pi}{0.02} = \dfrac{6-4.5}{0.02} = 75 \text{ kWh}$

Demand charges and suppliers' revenues are: $Q\cdot\pi = 75\cdot 4.5 = 337.5\text{p}$

Consumer's surplus $= \dfrac{1}{2}\cdot 75\cdot(6-4.5) = 56.25\,\text{p}$

The percentage change in quantity in reaction to a 1% increase in price is quantified as the *price elasticity*, and is calculated as:

$$\varepsilon = \frac{\pi}{Q}\cdot\frac{dQ}{d\pi} = \frac{4.5}{75}\cdot\frac{d}{d\pi}\left(\frac{6-\pi}{0.02}\right) = \frac{4.5}{75}\cdot(-50) = -3$$

Thus, the quantity bought would drop by 3%.

[3]

(iv) Producer's surplus = Producer's revenue – production cost

$$= 337.5 - C(75) = 337.5 - 144.375 = 193.125\text{p}$$

Social welfare = Producer's surplus + Consumer's surplus
= 193.125 + 56.25 = 249.375 (2)

(v) The quantity that would be purchased by the consumer is:

$$Q_D = \frac{6-3.6}{0.02} = 120 \text{ kWh}$$

However, the producer will only produce up to the level where this price equals its marginal cost: $MC = 0.03Q + 0.8 = 3.6 \Rightarrow Q_S = 93.33\text{kWh}$

Producer's revenue = 93.33 kWh · 3.6 p/kWh = 336 p

Producer's surplus = 336 – C(93.33) = 336 – 205.33 = 130.67 p

Price that the consumer would be willing to pay for the quantity of 93.33 kWh is:

$\pi = -0.02 \cdot 93.33 + 6 = 4.13$ p

Consumer's surplus is given by the area of the trapezoid defined by the following (π,Q) pairs: (3.6, 0), (3.6, 93.33), (4.13, 93.33) and (6, 0). Its area is equal to:

$$\frac{(6-3.6)+(4.13-3.6)}{2} \cdot 93.33 = 136.89\text{p}$$

Finally, social welfare is the sum of producer's and consumer's surpluses, and is equal to 267.56 p, which is 18.18 p higher than in (ii). (4)

(vi) At the equilibrium point we have:

Demand payments = Producer's revenue = 3.92 p/kWh · 104 kWh = 407.68 p

Producer's surplus = 407.68 – C(104) = 407.68 – 245.44 = 162.24 p

$$\text{Consumer's surplus} = \frac{104 \cdot (6-3.92)}{2} = 108.16 \text{ p}$$

Social welfare = Consumer's surplus + Producer's surplus = 270.40 p

Social welfare at the equilibrium point is larger than welfares obtained in cases (iii) and (v). The welfare loss in (ii) is 21.03 p, and in (iv) it is 2.84 p. This illustrates that the social welfare is maximised when the price is determined by the intersection of supply and demand curves, and that any administrative price fixing tends to reduce the global welfare. (1)

Solution to Question 3

a. In the unconstrained case marginal costs of all active generators should be the same. Unit C has a constant marginal cost which does not depend on its output level. If units A and B reached the output corresponding to 45 £/MWh, we would have:

$$12 + 0.1P_A = 45 \rightarrow P_A^* = 330 \text{ MW}$$
$$18 + 0.2P_B = 45 \rightarrow P_B^* = 135 \text{ MW}$$

This suggests that the total output would need to be at least 465 MW in order for the marginal cost to reach 45 £/MWh, and the total load in our three-bus system is only 300 MW. Hence, the marginal cost will be lower than 45 £/MWh and unit C will not be used. Dispatch of units A and B is found from the following system of equations:

$$MC_A = MC_B \rightarrow 12 + 0.1P_A = 18 + 0.2P_B$$
$$P_A + P_B = 240 + 60 = 300$$

This yields: $P_A = 220$ MW, $P_B = 80$ MW, $P_C = 0$ MW. All nodal prices are equal to: $\pi_1 = \pi_2 = \pi_3 = 12 + 0.1 \cdot 220 = 34$ £/MWh.
The cost functions of the two units are found by integrating the marginal cost functions (assuming the constant to be zero):

$$C_A(P_A) = 12P_A + 0.05P_A^2,\ C_B(P_B) = 18P_B + 0.1P_B^2$$

The hourly costs are therefore: C_A = 5060 £/h, C_B = 2080 £/h, and the total hourly cost is 7140 £/h. (3)

b. The line flows are obtained from the following system of linear equations:

$$F_{12} + F_{13} = P_A = 220$$
$$F_{23} - F_{12} = P_B - D_B = 20$$
$$F_{12} \cdot X_{12} + F_{23} \cdot X_{23} - F_{13} \cdot X_{13} = 0$$

The solution to this system is the following: F_{12} = 50 MW, F_{23} = 70 MW, F_{13} = 170 MW. (4)

c. With the 160 MW limit on line 1-3, we have a 10 MW overload in the unconstrained case. In order to relieve the overloading, it is necessary to decrease output from unit A and increase from unit B by the same amount. The change in outputs needs to result in a decrease of flow in line 1-3 by 10 MW. It will depend on the network topology, as specified in the following system of equations:

$$\Delta P_A + \Delta P_B = 0$$

$$\Delta P_A \cdot \frac{X_{12} + X_{23}}{X_{12} + X_{23} + X_{13}} + \Delta P_B \cdot \frac{X_{23}}{X_{12} + X_{23} + X_{13}} = 0.75\Delta P_A + 0.25\Delta P_B = -10$$

The solution to this system is: $\Delta P_A = -20$ MW, $\Delta P_B = 20$ MW. This means that the new dispatch will be: $P_A = 200$ MW, $P_B = 100$ MW, $P_C = 0$ MW.
Line loadings for this case will be: $F_{12} = 40$ MW, $F_{23} = 80$ MW, $F_{13} = 160$ MW. (5)

d. The hourly cost of this dispatch, using the same approach as in a), is: C_A = 4400 £/h, C_B = 2800 £/h, and the total hourly cost is 7200 £/h.
The hourly cost of security is the difference between generation costs in constrained and unconstrained case: $C_{sec} = 7200 - 7140 = 60$ £/h. (3)

e. Marginal cost of electricity at every node is the cost of supplying an additional MWh at that node. Marginal generator at bus 1 is unit A, and its marginal cost is:

$$\pi_1 = 12 + 0.1 \cdot 200 = 32 \text{ £/MWh}$$

Marginal generator at bus 2 is unit B, as unit A cannot increase its output due to the flow limit on line 1-3. Therefore, nodal price at bus 2 is:

$$\pi_2 = 18 + 0.2 \cdot 100 = 38 \text{ £/MWh}$$

For the bus 3, we cannot simply increase outputs of units A or B by 1 MW as this would overload the line 1-3. Due to network topology, an increase of output of unit A by 1 MW causes an overloading of line 1-3 by 0.75 MW, while the same increase by unit B causes an overload of 0.25 MW. Therefore, in order to supply 1 MW of electricity to bus 3 without increasing the loading of line 1-3, we need to solve the following equations:

$$\delta P_A + \delta P_B = 1$$
$$0.75 \cdot \delta P_A + 0.25 \cdot \delta P_B = 0$$

Solving this yields: $\delta P_A = -0.5$ MW, $\delta P_B = 1.5$ MW, which means that unit A needs to reduce its output by 0.5 MW, and unit B needs to increase output by 1.5 MW in order to supply additional 1 MW to bus 3. The nodal price for bus 3 is therefore:

$$\pi_3 = 1.5MC_B - 0.5MC_A = 41 \text{ £/MWh}$$

An alternative approach would be to supply the additional 1 MW by unit C, but its marginal cost is higher than the above price, so it will not be used.
The sum of congestion surpluses across all three lines is obtained as follows:

$$\text{Surplus} = (\pi_2 - \pi_1)F_{12} + (\pi_3 - \pi_2)F_{23} + (\pi_3 - \pi_1)F_{13} = 1920 \text{ £/h}$$

(5)

Solution to Question 4

a. Since there is no interconnection, generator outputs are equal to local demand levels:

$$P_A^W = D_A^W = 100 \text{ MW},\ P_A^S = D_A^S = 70 \text{ MW}$$
$$P_B^W = D_B^W = 400 \text{ MW},\ P_B^S = D_B^S = 250 \text{ MW}$$

Marginal costs are obtained from generators' costs:

$$\pi_A = MC_A \rightarrow \pi_A^W = 10 + 0.02\cdot 100 = 12 \text{ £/MWh},\ \pi_A^S = 10 + 0.02\cdot 70 = 11.4 \text{ £/MWh}$$
$$\pi_B = MC_B \rightarrow \pi_B^W = 11 + 0.04\cdot 400 = 27 \text{ £/MWh},\ \pi_B^S = 11 + 0.04\cdot 250 = 21 \text{ £/MWh}$$

Generator revenues are equal to demand charges:

$$R_A = \pi_A\cdot D_A \rightarrow R_A^W = 12\cdot 100 = 1200 \text{ £/h},\ R_A^S = 11.4\cdot 70 = 798 \text{ £/h}$$
$$R_B = \pi_B\cdot D_B \rightarrow R_B^W = 27\cdot 400 = 10{,}800 \text{ £/h},\ R_B^S = 21\cdot 250 = 5250 \text{ £/h}$$

Marginal value of transmission is found as the difference between the two nodal prices:

$$\pi_T^W = \pi_B^W - \pi_A^W = 15 \text{ £/MWh},\ \pi_T^S = \pi_B^S - \pi_A^S = 9.6 \text{ £/MWh}$$

[4]

b. The optimal dispatch for the unconstrained case is obtained from the following pair of equations:

$$MC_A = MC_B$$
$$P_A + P_B = D_A + D_B$$

This system yields the following solutions:

$$P_A = \frac{1 + 0.04(D_A + D_B)}{0.06},\quad P_B = \frac{0.02(D_A + D_B) - 1}{0.06}$$

The optimal unconstrained dispatch is:

Winter: $P_A = 350$ MW, $P_B = 150$ MW
Summer: $P_A = 230$ MW, $P_B = 90$ MW

Marginal costs: $\pi_A^W = \pi_B^W = 17$ £/MWh
$\pi_A^S = \pi_B^S = 14.6$ £/MWh

Generator revenues: $R_A^W = 5950$ £/h, $R_B^W = 2550$ £/h
$R_A^S = 3358$ £/h, $R_B^S = 1314$ £/h

Marginal value of transmission equals zero in both seasons (as the capacity is unconstrained).

[4]

c.

(i) $F = 80$ MW

Marginal prices are found using: $\pi_A = 10 + 0.02(P_A + F)$, $\pi_B = 11 + 0.04(P_B - F)$. This yields:

$$\pi_A^w = 10 + 0.02 \cdot 180 = 13.6 \text{ £/MWh}, \ \pi_A^s = 10 + 0.02 \cdot 150 = 13 \text{ £/MWh}$$
$$\pi_B^w = 11 + 0.04 \cdot 320 = 23.8 \text{ £/MWh}, \ \pi_B^s = 11 + 0.04 \cdot 170 = 17.8 \text{ £/MWh}$$

Hourly generator payments / demand charges:

$$R_A = \pi_A \cdot D_A \rightarrow R_A^w = 13.6 \cdot 180 = 2448 \text{ £/h}, \ R_A^s = 13 \cdot 150 = 1950 \text{ £/h}$$
$$R_B = \pi_B \cdot D_B \rightarrow R_B^w = 23.8 \cdot 320 = 7616 \text{ £/h}, \ R_B^s = 17.8 \cdot 170 = 3026 \text{ £/h}$$

Hourly congestion surpluses:

$$S_{cong} = (\pi_A - \pi_B)F \rightarrow S_{cong}^w = (23.8 - 13.6) \cdot 80 = 816 \text{ £/h},$$
$$S_{cong}^s = (17.8 - 13) \cdot 80 = 384 \text{ £/h}$$

Annual investment cost: $I_{ann} = k\,\lambda\,F = 37 \cdot 300 \cdot 80 = 888{,}000$ £/yr.

Annual revenue is calculated from hourly congestion surpluses and period durations d_w and d_s:

$$R_{ann} = S_{cong}^w d_w + S_{cong}^s d_s = 816 \cdot 2500 + 384 \cdot 6260 = 4{,}443{,}840 \text{ £/yr}$$

Finally, the annual profit is found as the difference between the congestion revenue and investment cost:

$$\Omega_{ann} = R_{ann} - I_{ann} = 3{,}555{,}840 \text{ £/yr}$$ (3.5)

(ii) $F = 160$ MW

Marginal prices:

$$\pi_A^w = 10 + 0.02 \cdot 260 = 15.2 \text{ £/MWh}, \ \pi_A^s = 10 + 0.02 \cdot 230 = 14.6 \text{ £/MWh}$$
$$\pi_B^w = 11 + 0.04 \cdot 240 = 20.6 \text{ £/MWh}, \ \pi_B^s = 11 + 0.04 \cdot 90 = 14.6 \text{ £/MWh}$$

Hourly generator payments / demand charges:

$$R_A = \pi_A \cdot D_A \rightarrow R_A^w = 15.2 \cdot 260 = 3952 \text{ £/h}, \ R_A^s = 14.6 \cdot 230 = 3358 \text{ £/h}$$
$$R_B = \pi_B \cdot D_B \rightarrow R_B^w = 20.6 \cdot 240 = 4944 \text{ £/h}, \ R_B^s = 14.6 \cdot 90 = 1314 \text{ £/h}$$

Hourly congestion surpluses:

$$S_{cong} = (\pi_A - \pi_B)F \rightarrow S_{cong}^w = (20.6 - 15.2) \cdot 160 = 864 \text{ £/h},$$
$$S_{cong}^s = (14.6 - 14.6) \cdot 160 = 0 \text{ £/h}$$

Annual investment cost: $I_{ann} = k\,\lambda\,F = 37 \cdot 300 \cdot 160 = 1{,}776{,}000$ £/yr.

Annual revenue:

$$R_{ann} = S_{cong}{}^{w} d_w + S_{cong}{}^{s} d_s = 864 \cdot 2500 + 0 \cdot 6260 = 2{,}160{,}000 \text{ £/yr}$$

Annual profit:

$$\Omega_{ann} = R_{ann} - I_{ann} = 384{,}000 \text{ £/yr}$$

[3.5]

d.

(i) A merchant transmission company (making money from buying electricity in one region and selling it in another) seeks to maximise its annual profit, and would therefore prefer the first option ($F = 80$ MW), where the profit is larger by an order of magnitude.

(ii) A regulated transmission company normally wants to maximise the system-level benefits of the transmission network. This means that the congestion surplus collected by the network operator needs to be offset by the investment cost of building the interconnecting line. Ideally, since allowable profits are already calculated into the investment cost, the goal would be to build the capacity at the level which yields zero profits. Out of the two cases, clearly the second one ($F = 160$ MW) is closer to the ideal situation, since the over-recovery of cost is much smaller than for the 80 MW link.

[5]