UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2003

BEng Honours Degree in Computing Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C382

TYPE SYSTEMS FOR PROGRAMMING LANGUAGES

Monday 12 May 2003, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1) a. Give the definition of

A. Lambda Terms.

B. Curry types.

C. Curry type assignment for the Lambda Calculus.

b. Give, in functional programming language notation, the algorithm that calculates the principal Curry pair for lambda terms.

c. Give a definition of substitution on types, and show that substitution is sound with respect to type assignment in Curry's system.

d. What is *Polymorphism*? Give an extension of the syntax definition for Lambda Calculus, as done for ML, that allows you to introduce polymorphism. Give the natural extension to Curry's type assignment system for the Lambda Calculus that deals with this new construct.

The four parts carry, respectively, 20%, 25%, 35%, 15%, and 15% of the marks.

2) Take the algebraic data type *Int*

$$n ::= 0 \mid (Succ\ n).$$

and prefix multiplication '*Times*'.

Likewise, take the algebraic data type *IntList*

$$l ::= Nil \mid (Cons\ n\ l)$$

a. Express, using the information above, the factorial function as a *term rewriting system*, and give an *environment* that makes your definition typeable, and show the necessary *derivation.*

b. Using your answer to the previous part, extend the rewrite system with a *rule*, and a *single term* (so write a program) that calculates the list of all factorial numbers. Show that the new rule and the term are typeable.

c. Assuming the existence of a conditional language construct '*if b then* E_1 *else* E_2', a test for zero '*IsZero*', and a function '*MinusOne*', using the information above, express the factorial function as an ML-*expression.*

d. Write an ML-*expression* that represents the list of all factorial numbers; you can abbreviate your answer to the previous part (i.e. write *Fac* rather than the full ML-term), .

The four parts carry 25%, 35%, 20%, and 20% of the marks.

3) a. Give, for Term Rewriting Systems, the definition of

A) Terms.

B) Rewrite rules and reduction.

C) Curry type assignment ($\vdash_{\mathcal{E}}$).

b. Given the term rewriting system

$$\begin{aligned}
\mathsf{B}\ x\ y\ z &\to x\ (y\ z)\\
\mathsf{C}\ x\ y\ z &\to x\ z\ y\\
\mathsf{K}\ x\ y &\to x\\
\mathsf{S}\ x\ y\ z &\to x\ z\ (y\ z)
\end{aligned}$$

Give an environment that makes these rules typeable. You do not need to give derivations, just the types.

c. Add the following rules to the system above.

$$\begin{aligned}
\mathsf{S}\ (\mathsf{K}\ x)\ (\mathsf{K}\ y) &\to \mathsf{K}\ (x\ y)\\
\mathsf{S}\ (\mathsf{K}\ x)\ y &\to \mathsf{B}\ x\ y\\
\mathsf{S}\ x\ (\mathsf{K}\ y) &\to \mathsf{C}\ x\ y
\end{aligned}$$

Show, perhaps using an abbreviated notation, that the system is still typeable using the same environment.

d. Add now also the rule

$$\mathsf{S}\ (\mathsf{K}\ x)\ \mathsf{I} \to x$$

Show that the system is no longer typeable using the same environment. What would you have to change to make the system typeable?

The four parts carry, respectively, 30%, 20%, 30%, and 20% of the marks.

4) a. Give the definition of

A) Intersection types.

B) Intersection type assignment for the Lambda Calculus.

b. Show

A) $\emptyset \vdash_\cap \lambda xy.xy : (\sigma \to \tau) \to (\rho \cap \sigma) \to \tau$.

B) $\emptyset \vdash_\cap \lambda xyz.xz(yz) : (\alpha \to \omega \to \gamma) \to \omega \to \alpha \to \gamma$

C) $\emptyset \vdash_\cap (\lambda xyz.xz(yz))(\lambda ab.a) : \omega \to \sigma \to \sigma$ (use the previous result without repeating the whole structure).

c. Compare the sets of types assignable to the terms $(\lambda xyz.xz(yz))(\lambda xy.x)(\lambda x.x)$ and $\lambda x.x$ in the Curry system and in the intersection system. Motivate your answer.

The three parts carry, respectively, 35%, 40%, and 25% of the marks.

End of Paper