UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1999

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER 2.14

COMPUTATIONAL TECHNIQUES
Tuesday, May 11th 1999, 2.00 – 3.30

Answer THREE questions

For admin. only:
paper contains 4 questions

1a Given the following three vectors:

$\mathbf{a}_1 = [1, -3/2, 1]^T$
$\mathbf{a}_2 = [-9, 8, -3]^T$
$\mathbf{a}_3 = [7, -5, 1]^T$.

Determine whether they are linearly dependent or independent.

b Assume that matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ have appropriate dimensions for the operations below. Prove that

(i) $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$.

(ii) $(\mathbf{ABC})^T = \mathbf{C}^T\mathbf{B}^T\mathbf{A}^T$.

c You are given two sets of linear equations:

$$\begin{array}{rcrcrcr} 4x_1 & - & x_2 & + & 4x_3 & = & 10 \\ -x_1 & + & 9x_2 & - & 2x_3 & = & 13 \\ 4x_1 & - & x_2 & + & 4x_3 & = & 10 \end{array} \qquad (1)$$

and

$$\begin{array}{rcrcrcr} 4x_1 & - & 2x_2 & + & 2x_3 & = & 8 \\ -2x_1 & + & 10x_2 & - & x_3 & = & 14 \\ 2x_1 & - & x_2 & + & 2x_3 & = & 8 \end{array} \qquad (2)$$

One of them can be solved by Cholesky factorisation. Identify it, explain your choice and apply Cholesky factorisation to solve it.

(The three parts carry, respectively, 30%, 20% and 50% of the marks).

2a Given matrix $\mathbf{D}$, determine its ℓ_2 norm $||\mathbf{D}||_2$:

$$\mathbf{D} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}.$$

b Assume 6-digit decimal arithmetic. Below, there are two numbers, a and b, and their approximations.

Accurate value	Approximation
$a = 99.9000$	100.100
$b = 10.0000$	10.0998

Determine the error of the approximations using all measures of error you know. Which approximation is better? Why?

c Matrix $\mathbf{A}$ is called *skew symmetric* if $\mathbf{A}^T = -\mathbf{A}$. What is the shape of $\mathbf{A}$? What are the diagonal elements of $\mathbf{A}$? Show that if $\mathbf{A}$ is skew symmetric then $\mathbf{A}^T\mathbf{A} = \mathbf{A}\mathbf{A}^T$!

(The three parts carry, respectively, 50%, 20% and 30% of the marks).

3a Under what conditions are the following matrix equalities true?

(i) $(\mathbf{X}+\mathbf{Y})^2 = \mathbf{X}^2 + 2\mathbf{XY} + \mathbf{Y}^2$.

(ii) $(\mathbf{X}+\mathbf{Y})(\mathbf{X}-\mathbf{Y}) = \mathbf{X}^2 - \mathbf{Y}^2$.

b Let I be the half-open interval $[1, 2)$ and $d(x, y) = |x - y|$ be the distance for any $x, y \in I$.

(i) Show that d is a metric on I (key points: nonnegativity, when $= 0$, symmetry, triangular inequality).

(ii) Show that $x_n = \dfrac{2n-1}{n}$, for $n = 1, 2, \ldots$ is a Cauchy sequence in the metric d.

(iii) Is (I, d) complete? Justify your answer.

c Let $\mathbf{B}$ be an $m \times m$ nonsingular matrix with inverse $\mathbf{B}^{-1}$, $\mathbf{a}$ an m-vector and $\mathbf{0}$ the m dimensional null vector. $\mathbf{A}$ is given in the following partitioned form:

$$\mathbf{A} = \begin{bmatrix} 1 & \mathbf{a}^T \\ \mathbf{0} & \mathbf{B} \end{bmatrix}$$

Determine $\mathbf{A}^{-1}$ symbolically in a partitioned form. What is the dimension of $\mathbf{A}$? What are the dimensions of the submatrices in $\mathbf{A}^{-1}$?

(The three parts carry, respectively, 20%, 40% and 40% of the marks).

Turn over ...

4a Find a local minimum or maximum for

$$f(x,y) = x^2 + y - \frac{1}{3}y^3.$$

Explain your work.

b Let

$$\mathbf{A} = \begin{bmatrix} 1 & a_{12} & 0 \\ 0 & a_{22} & 0 \\ 0 & a_{32} & 1 \end{bmatrix}$$

and $\mathbf{b} \in \mathbb{R}^3$ an arbitrary vector.

Show that $\mathbf{Ab} = \mathbf{b}$ if $b_2 = 0$. Discuss the usefulness of this simple fact. Generalise the observation for $m \times m$ matrices that differ from $\mathbf{I}_m$ only in one column.

c Show that for any matrix $\mathbf{C}$ both $\mathbf{C}^T\mathbf{C}$ and $\mathbf{C}\mathbf{C}^T$ are symmetric.

(The three parts carry, respectively, 45%, 25% and 30% of the marks).

End of Paper