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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

EEE/ISE PART II: MEng, BEng and ACGI

**CONTROL ENGINEERING**

Thursday, 29 May 2:00 pm

Time allowed: 2:00 hours

**There are FOUR questions on this paper.**

**Q1 is compulsory.**
**Answer Q1 and any two of questions 2-4.**
**Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).**

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible First Marker(s) : I.M. Jaimoukha, I.M. Jaimoukha

Second Marker(s) : S. Evangelou, S. Evangelou

1. (a) Figure 1.1 shows a mass-spring system where $K$, $D$ and $M$ have the standard interpretation. The signal $u(t)$ represents an applied force and $y(t)$ the displacement from the rest position.

   (i) Derive the differential equation relating $u(t)$ to $y(t)$. **[3]**

   (ii) Evaluate the transfer function relating $u(s)$ to $y(s)$. **[3]**

   (iii) Take $K = M = 1$ in appropriate units. Determine the value of $D$ for which the response to a step input is critically damped. **[3]**

   (iv) Take $K = 2$, $M = 1$ and $D = 3$ in appropriate units. Determine the response to a unit step input. **[3]**

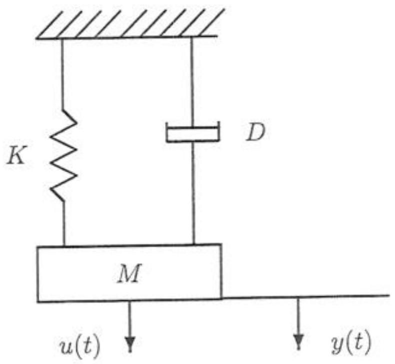


*Figure 1.1*

(b) In Figure 1.2, $G(s) = \dfrac{s-1}{s^2+1}$ and $K$ is a variable gain.

   (i) Sketch the locus for $0 \leq K < \infty$. You should evaluate the breakaway point and the angle of departure from the poles. **[5]**

   (ii) Use the Routh array to derive the range of values of $K$ for which the closed-loop is stable. **[5]**

   (iii) Derive the value of $K$ for which the closed–loop response is critically damped. **[4]**

(c) Consider the feedback loop in Figure 1.2 with $G(s) = \dfrac{4}{s-1}$.

   (i) Sketch the Nyquist diagram of $G(s)$. **[6]**

   (ii) Take $K = 1$. Use the Nyquist diagram to determine the number of unstable closed-loop poles. **[4]**

   (iii) Take $K = 1$. Evaluate the gain margin. **[4]**

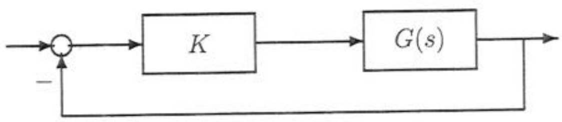


*Figure 1.2*

2. Consider the feedback arrangement shown in Figure 2 which is used to simulate a control system. The operational amplifiers $O1$, $O2$ and $O3$ are in the negative feedback mode, so that the positive input terminal is earthed and the inputs are connected to the negative terminals. Assume that the operational amplifiers are ideal, so that we can make the virtual earth assumption.

Take $R = 1$ and $C = 1$ in appropriate scaled units, while $C1$ is a design parameter.

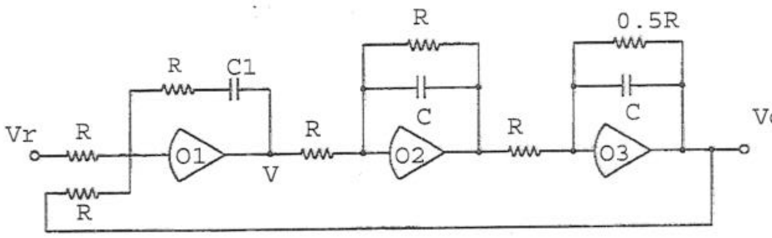


*Figure 2*

(a) Derive the transfer functions between $Vr(s)$ and $V(s)$ and between $Vo(s)$ and $V(s)$. **[6]**

(b) Derive the transfer function between $V(s)$ and $Vo(s)$. **[6]**

(c) Hence, derive and clearly draw a block diagram representation of the feedback loop. The feedback loop should contain two blocks: one block, denoted by $G(s)$, represents the plant and the second block, denoted by $K(s)$ represents the compensator. Take the reference signal to be $-Vr(s)$ and the output signal to be $Vo(s)$. Indicate clearly the signal $V(s)$ and the error signal $E(s) = (-Vr(s)) - Vo(s)$. What type of compensator is $K(s)$? **[6]**

(d) Suppose that $Vr(t)$ is a unit step applied at $t = 0$. Find the steady state value of the error signal $E_{ss}^{step} := \lim_{t\to\infty} E(t)$. **[6]**

(e) Suppose that $Vr(t)$ is a unit ramp applied at $t = 0$ and suppose that $\epsilon > 0$ is given. Find the maximum value of $C1$, in terms of $\epsilon$, such that the steady state value of the error signal $E_{ss}^{ramp} := \lim_{t\to\infty} E(t)$ satisfies $|E_{ss}^{ramp}| \le \epsilon$ . **[6]**

3. Consider the feedback control system shown in Figure 3 below. Here,

$$G(s) = \frac{s+1}{(s+2)^4}$$

and $K(s)$ is the transfer function of the compensator.

(a) For $K(s) = k$, a constant compensator, draw the root locus accurately as $k$ varies in the range $0 \leq k \leq \infty$. **[6]**

(b) Take $K(s) = k$ where $k > 0$. Find the range of values of $k$ for which the closed loop is stable. **[6]**

(c) Take $K(s) = k$ where $k > 0$. Use the answer to Part (b) to find the value of $k$ for which the closed loop is marginally stable. For this value of $k$, what is the corresponding frequency of oscillation? **[6]**

(d) Design a proportional-plus-derivative compensator such that the following design specifications are simultaneously satisfied:

i. The closed loop is stable.

ii. The closed–loop system should have two real poles and a pair of complex conjugate poles.

iii. The settling time for the complex poles is at most $4s$.

iv. The damping ratio of the complex poles is $\frac{1}{\sqrt{2}}$.

Draw the root locus of the compensated system. **[12]**

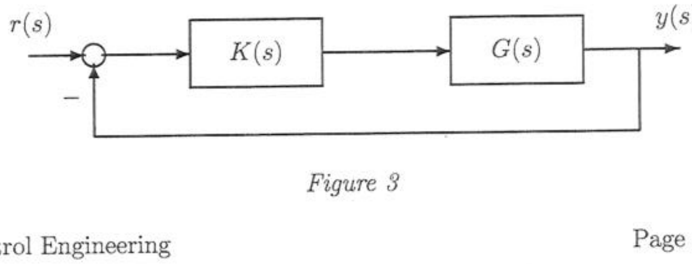


*Figure 3*

4. Consider the feedback control system in Figure 4 below. Here,

$$G(s) = \frac{4}{(s+1)^3}$$

and $K(s)$ is the transfer function of a compensator.

(a) Sketch the Nyquist diagram of $G(s)$, indicating the low and high frequency portions. Also, calculate the real–axis intercepts. **[8]**

(b) Take $K(s) = 1$ in Figure 4.

(i) Use the Nyquist diagram to determine the number of unstable closed-loop poles. **[4]**

(ii) Determine the phase and gain margins. **[7]**

(c) (i) Explain what is meant by a Proportional-plus-Integral (PI) compensator. Your answer should include an expression for such a compensator, and a description of its frequency response. **[5]**

(ii) Without doing any actual design, describe how a PI compensator would effect the stability margins and the steady–state tracking properties of the loop.

Where should the zero be placed to reduce the destabilizing effect of the PI compensator? **[6]**

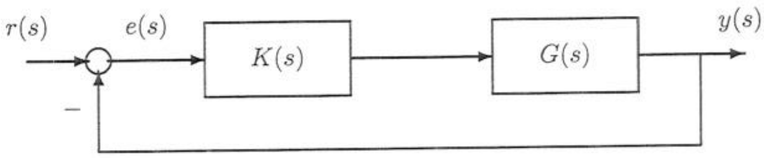


*Figure 4*