Imperial College London

MEng (Engineering) Examination 2017

Year 1

AE1-101 Introduction to Aerodynamics

Monday 5th June 2017: 14.00 to 16.00
[2 hours]

The paper is divided into Section A and Section B
There are ***FOUR*** questions. All questions carry the same weight
Candidates may obtain full marks for complete answers to ***ALL*** questions.

You must answer each section in a separate answer booklet

The equations of motion for steady, two-dimensional, viscous flow are as follows:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

The use of lecture notes is NOT allowed.

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Section A

1. (a) Determine the dimensions of the following quantities in the form $M^{\alpha}L^{\beta}T^{\gamma}$:

   i. total pressure,
   ii. streamfunction,
   iii. wall shear stress,
   iv. dynamic viscosity,
   v. mass flux and
   vi. drag coefficient. [30%]

   (b) Two immiscible, incompressible, viscous fluids having the same densities, ρ, but different viscosities, μ_1 and μ_2 are contained between two infinite horizontal, parallel plates as shown in figure 1. The bottom plate is fixed and the upper plate moves with a constant velocity U. The motion of the fluid is caused entirely by the movement of the upper plate; assume there is no pressure gradient in the x-direction. The fluid velocity and shear stress are continuous across the interface between the two fluids. Assume laminar flow.

   i. Starting from the $u-$component equations provided on the front page of the exam sheet and the divergence condition, determine the velocity profiles within the two fluids. Justify any assumptions. Express your answer in terms of U, μ_1 and μ_2. [55%]
   ii. Determine the vorticity distribution between the plates. [15%]

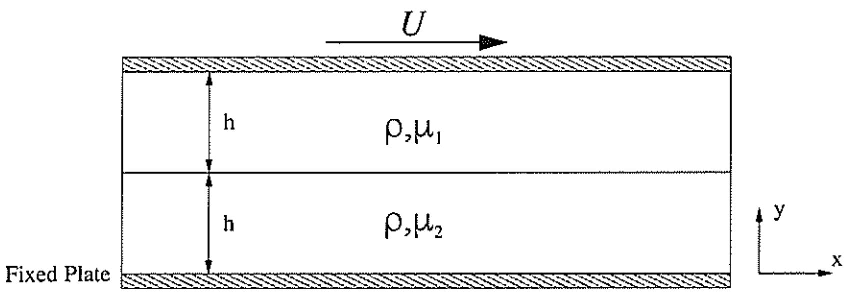

Figure 1

2. (a) State Bernoulli's equation and define under what conditions this equation is valid? [20%]

   (b) In a contraction of a wind tunnel with a circular cross section the pressure difference between two static pressure tappings is found to give a difference in a manometer height reading of h. The diameter of the contraction at the location of the first tapping is 0.15m and the diameter of the contraction at the second pressure tapping is D.

   Determine the mass flow rate Q through the contraction as a function of D, the acceleration due to gravity g, the height h, the manometer fluid density ρ_m and the air density ρ_a. [40%]

   (c) The velocity in a water channel is sometimes determined by the use of a device called a Venturi flume. As shown in figure 2 this device consists of a bump on the bottom of the channel. If the water surface dips a distance of 0.06 m over a bump of height 0.15 m under the conditions shown in figure 2, what is the value of velocity V_2 in m/s? Assume that the velocity is uniform and viscous effects are negligible. [40%]

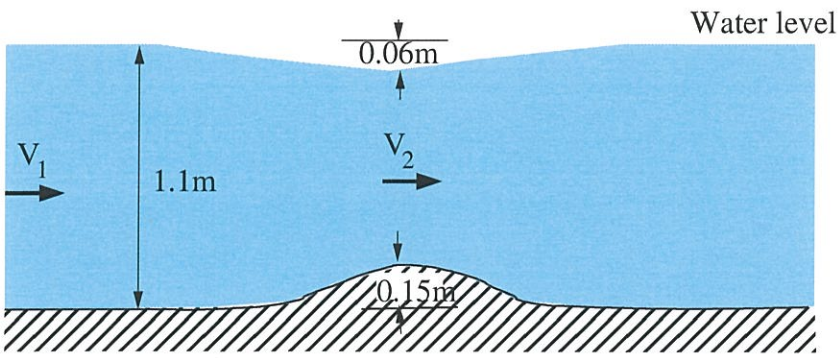

Figure 2

Section B

3. (a) Define total, dynamic, static and hydrostatic pressure. Explain with the aid of a labelled diagram, the use of a Pitot-static tube and manometer for the measurement of air velocity in incompressible flow. [35%]

(b) Consider the control volume shown in figure 3 centred around a curved streamline of radius of curvature, R. Applying the principle of conservation of momentum within the control volume normal to the streamline, show that the pressure drop, $\partial p/\partial r$, normal to the streamline is

$$\frac{\partial p}{\partial r} = \frac{\rho U_\theta^2}{R},$$

where ρ is the fluid density and U_θ is the velocity tangential to the streamline. Assume steady flow conditions. [45%]

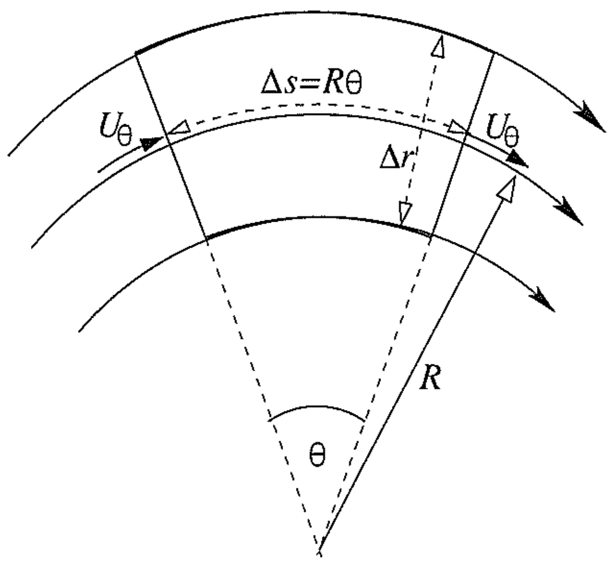

Figure 3

(c) How can this expression be used to explain the observation that there is a lower pressure on the upper surface of an aerofoil with a positive angle of attack? [20%]

4. (a) With the aid of sketches, explain why an aerofoil in inviscid flow with no circulation produces no lift. In your sketch, identify the stagnation points. [25%]

(b) Give the Kutta condition. Illustrate its effect with the aid of sketches and explain how this represents the real viscous flow. [25%]

(c) Steady flow of a viscous fluid over a flat plate results in the boundary layer development as shown in figure 4. At the leading edge of the plate, the velocity profile may be considered to be of uniform magnitude U. Along and above the outer edge of the boundary layer the x-component of fluid velocity is also assumed to have a magnitude equal to U. The x-direction velocity profile at position A is determined to be of the form

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}.$$

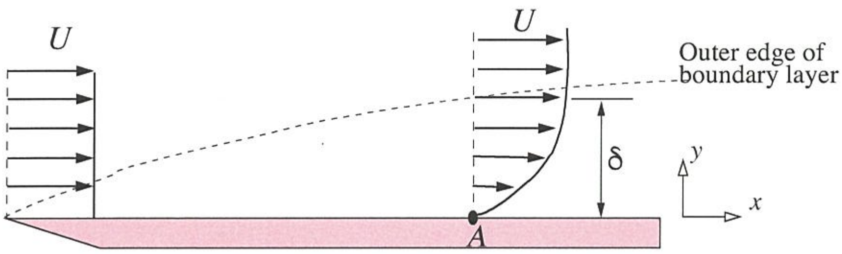

Figure 4

i) If the flow is incompressible with density ρ, determine the mass flow rate per unit span up to a height δ in the boundary layer at point A. [20%]

ii) Comparing the shape of the two velocity profiles in figure 4, it is evident that the one at A has a reduced mass flow rate compared to that of the inviscid flow profile at A. Write down this difference. What does the difference between the mass flow rate calculated in part (i) and the leading edge mass flow rate up to a height δ represent? [30%]

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Marks

1 (a) Total pressure has same units as pressure $\frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$. 5

(b) Streamfunction $u = \frac{\partial \psi}{\partial y} \Rightarrow \frac{L}{T} = \frac{\psi}{L}$. 5

So $\psi \sim \frac{L^2}{T}$

(c) Wall shear stress = $\frac{\text{Force}}{\text{Area}} = ML^{-1}T^{-2}$ 5

(d) Dynamic Viscosity $\tau_w = \mu \frac{dv}{dy}$ 5

$\mu = \frac{ML^{-1}T^{-2}}{1/T} = ML^{-1}T^{-1}$

(e) Mass flux $\rho u A = ML^{-3}LT^{-1}L^2$ 5

$= MT^{-1}$.

(f) Drag Coefficient $\Rightarrow$ Dimensionless $= M^0L^0T^0$ 5

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From steady x-component momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \nu\frac{\partial^2 u}{\partial x^2} \quad (1)$$

For fully developed flow $\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2} = 0$ — 5

From divergence condition $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

we know $\frac{\partial v}{\partial y} = 0 \Rightarrow v(x) = \text{const.}$ — 5

but at walls we know $v = 0 \Rightarrow v = 0$ everywhere

So eq^n (1) becomes

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$ — 5

told that $\frac{dp}{dx} = 0$ and so

$$\frac{\partial^2 u}{\partial y^2} = 0 \Rightarrow u_1 = a_1 y + b_1$$
$$u_2 = a_2 y + b_2$$ — 5

Imposing boundary condition

In fluid (1) $u_1(2h) = U \Rightarrow U = a_1 2h + b_1 \quad (1)$ — 5

In fluid (2) $u_2(0) = 0 \Rightarrow b_2 = 0 \quad (2)$

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At the interface

$$u_1(h) = u_2(h) \Rightarrow a_1 h + b_1 = a_2 h \quad (3)$$ 5

also

$$\tau_1(h) = \tau_2(h)$$ 5

$$\mu_1 \frac{\partial u_1}{\partial y}(h) = \mu_2 \frac{\partial u_2}{\partial y}(h)$$

where $\frac{\partial u_1}{\partial y} = a_1 \quad \frac{\partial u_2}{\partial y} = a_2$ 5

So $\mu_1 a_1 = \mu_2 a_2 \quad (4)$

(3) $-$ (1) $\quad a_2 h - U = a_1 h - 2a_1 h$

$$a_2 h - U = -a_1 h$$

using (4) $\quad a_2 h - U = -\frac{\mu_2 a_2 h}{\mu_1}$

$$a_2 h \left(1 + \frac{\mu_2}{\mu_1}\right) = U$$

$$a_2 = \frac{U}{h} \frac{1}{1 + \frac{\mu_2}{\mu_1}} = \frac{U}{h} \frac{\mu_1}{\mu_1 + \mu_2}$$

$$a_1 = \frac{\mu_2}{\mu_1} a_2 = \frac{U}{h} \frac{\mu_2}{\mu_1 + \mu_2}$$ 5

$$b_1 = (a_2 - a_1) h = U \left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)$$ 5

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So finally

$$u_1 = \frac{U}{h}\frac{\mu_2}{\mu_1+\mu_2}\, y + U\frac{(\mu_1-\mu_2)}{(\mu_1+\mu_2)}.$$ 5.

$$u_2 = \frac{U}{h}\frac{\mu_1}{\mu_1+\mu_2}\, y$$

(b) Vorticity $\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ 5

and so

$$\omega_1 = -\frac{U}{h}\frac{\mu_2}{\mu_1+\mu_2}$$ 5

$$\omega_2 = -\frac{U}{h}\frac{\mu_1}{\mu_1+\mu_2}.$$ 5

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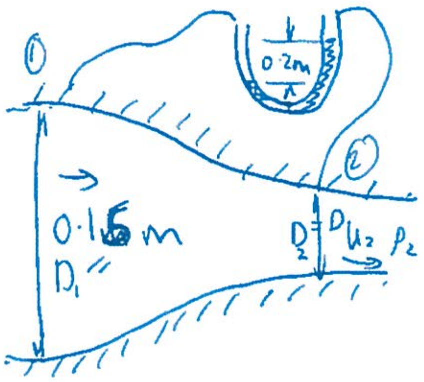
(a) Bernoulli's equation

$$p + \frac{1}{2}\rho U^2 = P_0$$ 20

Valid in steady, incompressible and irrotational flow.

(b)



5

From Bernoulli's

$$P_1 + \frac{1}{2}\rho U_1^2 = P_2 + \frac{1}{2}\rho U_2^2 \Rightarrow P_1 - P_2 = \frac{1}{2}\rho_a (U_2^2 - U_1^2)$$ 5

From hydrostatics

$$P_1 + \rho_m g h_1 = P_2 + \rho_m g h_2$$ 5.

$$P_1 - P_2 = \rho_m g (h_2 - h_1)$$

Equating: $\frac{1}{2}\rho_a(U_2^2 - U_1^2) = \rho_m g (h_2 - h_1) = \rho_m g h$ ① 5.

From mass conservation $U_1 A_1 = U_2 A_2$

$$U_1 \frac{D_1^2 \pi}{4} = U_2 \frac{D_2^2 \pi}{4} \Rightarrow U_1 = U_2 \frac{D_2^2}{D_1^2}$$ ② 10

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(1),(2)

$$U_2^2\left(1 - \frac{D_2^4}{D_1^4}\right) = \frac{2\rho_m g h}{\rho_a}.$$

$$U_2 = \sqrt{\frac{2\rho_m g h}{\rho_a}\frac{D_1^4}{D_1^4 - D_2^4}}$$

$$Q = \rho_a U_2 A_2 = \rho U_2 \frac{D_2^2 \pi}{4}$$

$$= \frac{\rho_a \pi}{4} \cdot D_2^2 \sqrt{\frac{2\rho_m g h}{\rho_a}\frac{D_1^4}{(D_1^4 - D_2^4)}} = \frac{\rho_a \pi}{4}\sqrt{\frac{2\rho_m}{\rho_a}\frac{g h D_1^4 D_2^4}{(D_1^4 - D_2^4)}}$$

$$= \frac{\cancel{\rho} \pi}{4}\sqrt{\frac{2\rho_m \rho_a g h\, 0.15^4 D^4}{(0.15^4 - D^4)}}$$

10

(c) Since the flow is inviscid we can apply Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$ (strictly speaking P is static plus hydrostatic pressure)

$$P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2)$$

10

This change in pressure causes change in height of the free surface which gives rise to the pressure (hydrostatic) difference, i.e.

$$P_1 - P_2 = \rho g h.$$

10

and so $\frac{1}{2}\cancel{\rho}(V_2^2 - V_1^2) = \rho g h = 0.06 \cancel{\rho} g$

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From mass Conservation

$$V_1 \times 1.1 = V_2 (1.1 - 0.15 - 0.06)$$

$$= V_2 \, 0.89$$

$$V_1 = V_2 \frac{0.89}{1.1}$$ 10

So $$V_2^2 \left(1 - \left(\frac{0.89}{1.1}\right)^2\right) = 0.12 g$$

$$V_2 = \sqrt{\frac{0.12 \times 9.81}{1 - \left(\frac{0.89}{1.1}\right)^2}} = \sqrt{\frac{1.1772}{0.34537}}$$ 10

$$= 1.846 \text{ m/s}$$

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Question ~~3~~ 8

~~1~~ (1)

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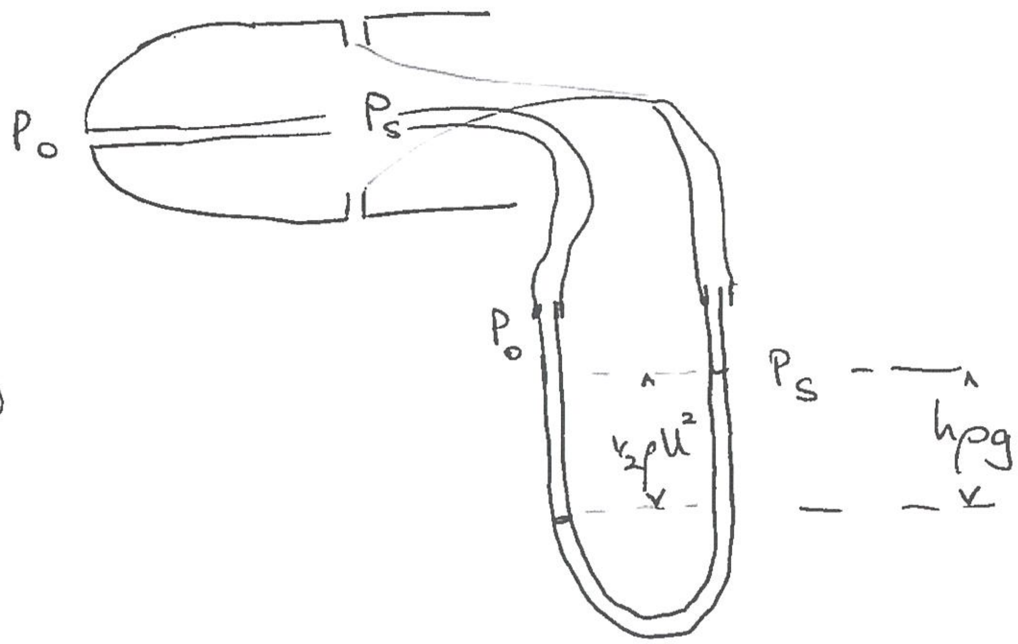
5 • Total $P_o = p_s + \frac{1}{2}\rho U^2$ $\xrightarrow{U}$

5 • dynamic pressure $= \frac{1}{2}\rho U^2$

10 • static pressure : the normal force per unit area on a surface due to the time rate of change of momentum of the gas molecules impacting on or crossing the surface. Pressure is a scalar, P_s

5 • hydrostatic pressure $= h\rho g$

10



$\frac{1}{2}\rho U^2 = h\rho g$

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Question 3

~~1~~ (2) 9

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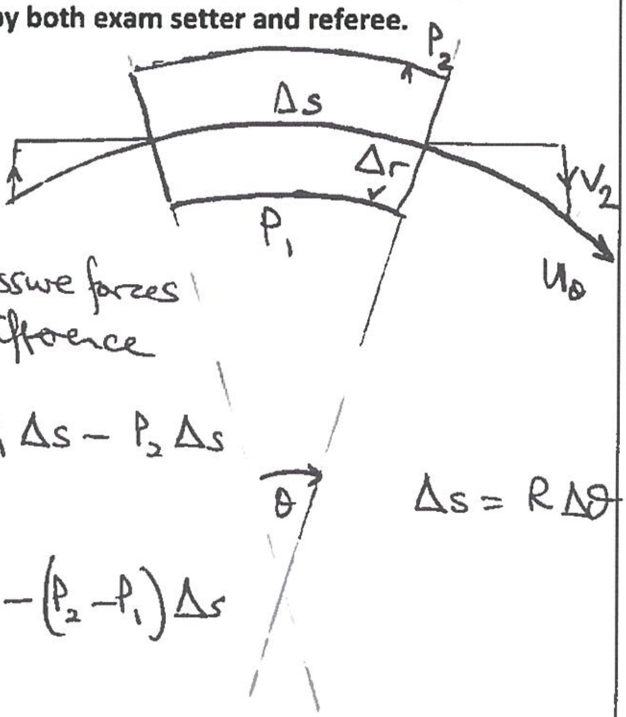
(b) $V_1 = U_\theta \sin\theta/_2 \simeq U_\theta \frac{\theta}{2}$

$V_2 = -U_\theta \sin\theta/_2 \simeq -U_\theta \theta/_2$

Vertical



Net momentum flux = pressure forces difference

$$\rho U_\theta V_2 \Delta r - \rho U_\theta V_1 \Delta r = P_1 \Delta s - P_2 \Delta s$$

$\Delta s = R\Delta\theta$

$$\rho U_\theta \Delta r \left(-U_\theta \frac{\theta}{2} - U_\theta \frac{\theta}{2}\right) = -(P_2 - P_1)\Delta s$$

$$-\rho \frac{U_\theta^2}{R} = -\frac{(P_2 - P_1)}{\Delta r}$$

$$\frac{\partial p}{\partial r} = \rho \frac{U_\theta^2}{R} \quad \text{— (1)}$$

pressure force = apparent centrifugal force

45

(c) With either positive α, or positive camber, convex curvature (as the flow sees it) reduces pressure on suction surface but opposite happens on pressure surface.

$P_1 < P_2$ according to (1) — P_1 could be on the surface.

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Question 4

~~2~~ (1) 10

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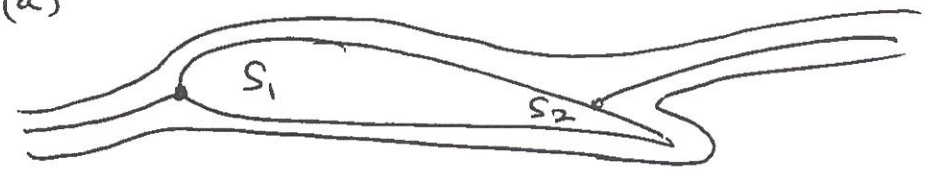
(a)



inviscid flow, stagnation points S_1, S_2. Despite there being curved streamlines, no lift is produced, no drag either. At trailing edge curvature of streamlines is such that ∞ suction is generated there. This balances forces caused by streamline curvature elsewhere such that net vertical & horizonatal forces $\longrightarrow 0$. 25

(b) Kutta condition: flow must leave trailing edge "smoothly". Consequently rear stagnation point appears <u>at</u> t.e

There must be circulation about the aerofoil for this to happen. Kutta-Joukowski $L = \rho U \Gamma$ hence lift is generated, in a real viscous flow which $\rightarrow$ a <u>boundary layer</u>.

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Question 4

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~~2~~ (2)

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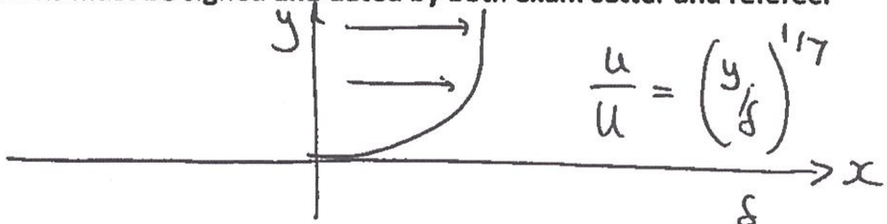
(c) (i)



$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

mass flow rate per unit width (z) $= \int_0^\delta \rho u \, dy$

$\eta = y/\delta$

$$= \rho \delta U \int_0^1 \eta^{1/7} \, d\eta = \rho \delta U \frac{7}{8} \left[\eta^{8/7} \right]_0^1$$

$$= \underline{\frac{7}{8} \rho \delta U}$$

20

(ii) Boundary layer constitutes a mass flux deficit per unit width

$$= \rho \delta U \int_0^1 \left(1 - \frac{u}{U}\right) d\eta = \rho \delta U \int_0^1 1 - \eta^{1/7} \, d\eta$$

integral is just the same

So provided they realise that inviscid mass flux is $\rho U \delta$ they can just write down

$$\underline{\tfrac{1}{8} \rho \delta U.}$$

This is caused by the no-slip condition & subsequent viscous stresses in the boundary layer

30

100