EE4-23
EE9-CS2-1

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

MSc and EEE/EIE PART IV: MEng and ACGI

STABILITY AND CONTROL OF NON-LINEAR SYSTEMS

Friday, 10 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D. Angeli
Second Marker(s) : E.C. Kerrigan

1. Consider the following system of differential equations defined on $\mathbb{R}^2$:

$$\begin{aligned}\dot{x}_1 &= x_1 - x_1x_2 \\ \dot{x}_2 &= x_1x_2 - \frac{2x_2}{1+x_2^2}.\end{aligned}$$

a) Find all equilibria of the system and sketch the nullclines; [3]

b) Show that the coordinate axis are invariant sets; [3]

c) Identify the regions in which the nullclines partition the state-space and sketch the direction of the vector-field in each one of them; [3]

d) Linearize the system around each equilibrium and discuss the local phase-portrait; [5]

e) Sketch the global phase portrait of the system by taking into account the clues collected in items a), b), c) and d) and assuming that solutions in the positive orthant are not bounded (except for the one starting at equilibrium). [6]

2. Consider the following nonlinear system:

$$\dot{x} = (d^2 + d - 6)x$$

with state $x \in \mathbb{R}$ and exogenous disturbance input d.

a) Compute the system's equilibria as a function of d, in particular point out for which values of d there exist multiple equilibria; [2]

b) Let $\mathscr{D} \subset \mathbb{R}$ be a compact interval. Compute $\mathscr{D}$ so that the system is Uniformly Globally Asymptotically Stable; [4]

c) Does there exist a largest possible compact interval $\mathscr{D}$ such that the system is UGAS ? (justify your answer); [3]

d) Consider the following scalar autonomous system:

$$\dot{z} = -z^3 + z$$

where $z \in \mathbb{R}$ is the state variable. Compute its equilibria and explain as a function of initial conditions the asymptotic behaviour of all its solutions; [4]

e) Let us now cascade the two previous subsystems (by letting $d = z$) as follows:

$$\begin{aligned} \dot{z} &= -z^3 + z \\ \dot{x} &= (z^2 + z - 5)x. \end{aligned}$$

Argue that all solutions converge asymptotically to some equilibrium (*Hint: exploit the convergence properties of solutions of the z-subsystem and UGAS of the x-subsystem*); [5]

f) For each initial condition (z_0, x_0) in $\mathbb{R}^2$ compute the corresponding ω-limit set for the cascaded system. [2]

3. Consider the equations of the following nonlinear systems:

$$\begin{aligned}\dot{x}_1 &= -x_1^3 + u \\ \dot{x}_2 &= x_1 - x_2^3 \\ \dot{x}_3 &= x_2^3 - x_3\end{aligned}$$

with state x taking values in $\mathbb{R}^3$, and control input u taking values in $\mathbb{R}$.

a) Compute, for each constant value of u, the corresponding equilibrium. [3]

b) Discuss, for all u, the local asymptotic stability of the equilibria previously computed. [3]

c) For those values of u for which linearization is inconclusive can you argue otherwise ? [2]

d) Let the output equation be defined as

$$y = x_2 + x_3.$$

What is the relative degree with respect to this output selection? Is it globally defined ? Justify your answers. [2]

e) Design an Input-Output linearizing feedback by suitably introducing an auxiliary input v. [3]

f) Write the system of equations in normal form and highlight the internal dynamics. [3]

g) Design, by means of feedback linearization, a controller to steer the output y to zero; what kind of internal stability property can be guaranteed ? (*Hint: are the internal dynamics ISS ?*) [4]

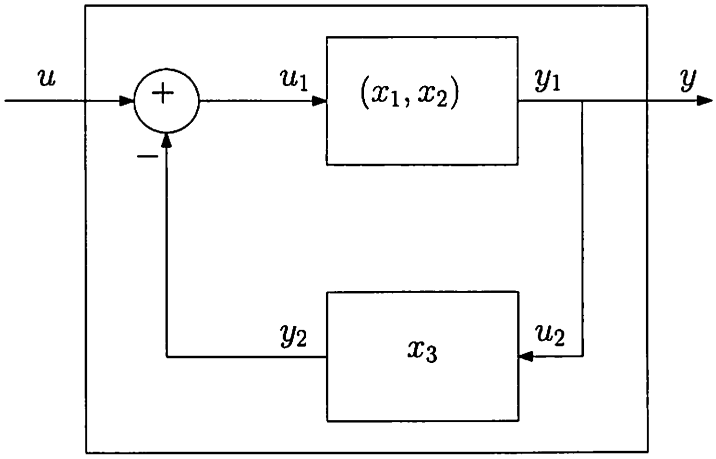

Figure 4.1 Negative feedback interconnection

4. Consider the following nonlinear control system:

$$\begin{aligned}
\dot{x}_1 &= x_2^2 + (u - x_3)x_2 - x_3 + u \\
\dot{x}_2 &= -x_1x_2 + (x_3 - u)x_1 \\
\dot{x}_3 &= -\arctan(x_3) + x_1, \\
\\
y &= x_1
\end{aligned}$$

with state $x = [x_1, x_2, x_3] \in \mathbb{R}^3$, input $u \in \mathbb{R}$ and output $y \in \mathbb{R}$.

a) Regard the (x_1, x_2) equations and x_3 equations as separate subsystems (with suitable inputs and outputs). Argue that the overall system can be recast as a negative feedback interconnection (see Fig. 4.1); [6]

b) Show that the individual subsystems defined in the previous item are passive from their input to the respective output; [6]

c) Show that the overall system is passive, from input u to output y (write explicitly the associated storage function); [3]

d) Show that the origin is an equilibrium of the system for $u = 0$. Is it globally asymptotically stable ? (Justify your answer). [5]

5. Consider the following second-order differential equation:

$$\ddot{y} = -y^3 + y + (y^2/2 - y^4/4 - \dot{y}^2/2 - 1/8)\dot{y}.$$

a) Find a suitable state-space representation of the system; [3]

b) Compute all the equilibrium points and locally linearize the system around each equilibrium; [3]

c) Discuss the local phase portraits previously computed; [3]

d) Consider the function $V(y,\dot{y})$ defined below:

$$V(y,\dot{y}) = \frac{y^4}{4} - \frac{y^2}{2} + \dot{y}^2 2.$$

Compute which of its level sets are invariant.[1] [3]

e) Compute which sub-level sets of V are forward and backwards invariant[2]. [3]

f) Based on the clues collected in items a) to e), sketch a consistent global phase-portrait of the systems solutions. (*Hint: sketch first the qualitative shape of level sets of V for different c values. It is useful to this end to find minimum and saddle points of V*). [5]

[1] A level set of a function V, for any $c \in \mathbb{R}$ is defined as $\mathcal{L}_c = \{(y,\dot{y}) : V(y,\dot{y}) = c\}$.

[2] A sub-level set of a function V, for any $c \in \mathbb{R}$ is defined as $\mathcal{S}_c = \{(y,\dot{y}) : V(y,\dot{y}) \leq c\}$.

6. Consider the following scalar nonlinear system with input:

$$\dot{x} = -\text{sat}(\alpha x + d),$$

with state $x \in \mathbb{R}$, disturbance input $d \in \mathbb{R}$ and parameter $\alpha > 0$. Assume that there exists a class positive definite function ρ such that $\text{sat}(r)r \geq \rho(|r|)$.

a) Show that the system is Input-to-State Stable for any $\alpha > 0$; [4]

b) Estimate the gain from d to x (as a function of α); [4]

c) Consider next the following 2-dimensional system:

$$\begin{aligned} \dot{x}_1 &= -\text{sat}(2x_1 + d) \\ \dot{x}_2 &= -\text{sat}(x_2 + x_1). \end{aligned}$$

Argue that this system is ISS and compute the asymptotic gain from input d to state x (Hint: use $|x|_\infty := \max\{|x_1|, |x_2|\}$ to estimate the ISS gain). [6]

d) Consider the following 3-dimensional system:

$$\begin{aligned} \dot{x}_1 &= -\text{sat}(2x_1 + x_3) \\ \dot{x}_2 &= -\text{sat}(x_2 + x_1) \\ \dot{x}_3 &= -\text{sat}(x_3 + x_2) \end{aligned}$$

Argue that the origin is a globally asymptotically stable equilibrium. (*Recast the system as a feedback interconnection of ISS subsystems*); [6]

SOLUTIONS: STABILITY AND CONTROL OF NONLINEAR SYSTEMS
MASTER IN CONTROL

1. Exercise

a) The system admits 2 equilibria. These are solutions of:

$$\begin{cases} x_1(1-x_2) & = & 0 \\ x_2(x_1 - \frac{2}{1+x_2^2}) & = & 0. \end{cases}$$

Notice that $x_1 = 0$ implies from the second equation $x_2 = 0$. Conversely, $x_2 = 1$, implies from the second equation $x_1 = 1$. The equilibria are therefore in $(0,0)'$ and $(1,1)'$. [3]

b) The coordinate axis have equations $x_1 = 0$ and $x_2 = 0$. Notice that:

$$x_1 = 0 \Rightarrow \dot{x}_1 = x_1(1-x_2) = 0$$

$$x_2 = 0 \Rightarrow \dot{x}_2 = x_2(x_1 - 2/(1+x_2^2)) = 0$$

Hence, $(0, x_2(t))$ and $(x_1(t), 0)$ are solutions of the systems equations provided: $\dot{x}_2(t) = -2x_2(t)/(1+x_2^2(t))$ and $\dot{x}_1(t) = x_1(t)$ respectively. This fact proves invariance of the coordinate axes. [3]

c) The nullclines partition the plane in 9 connected regions, (see Fig. 1.1), denoted by R_1 to R_9 in the picture. By using cardinal point notation for identifying $\dot{x}$ directions, the regions are of type SE,SW,NW, SW,SE, NE, NW, NE and SE respectively. [3]

d) Computing the Jacobian of the vector-field yields:

$$\frac{\partial \dot{x}}{\partial x} = \begin{bmatrix} 1-x_2 & -x_1 \\ x_2 & x_1 - 2(1-x_2^2)/(1+x_2^2)^2 \end{bmatrix}.$$

Evaluating around the equilibrium in $(0,0)$ yields the linearized system:

$$\dot{\delta x} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \delta x.$$

Eigenvalues are in $1, -2$. Hence they are real and have opposite sign. The first equilibrium is therefore a saddle point with unstable manifold along the x_1 axis and stable manifold along the x_2 axis.

Evaluating the Jacobian at the equilibrium in $(1,1)$ yields the linearized system:

$$\dot{\delta x} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} \delta x.$$

The characteristic polynomial is $s^2 - s + 1$. Its roots are in:

$$\frac{1 \pm j\sqrt{3}}{2}.$$

Hence, the second equilibrium is an unstable focus.
(3 marks for saddle and eigenvectors, 2 marks for focus) [5]

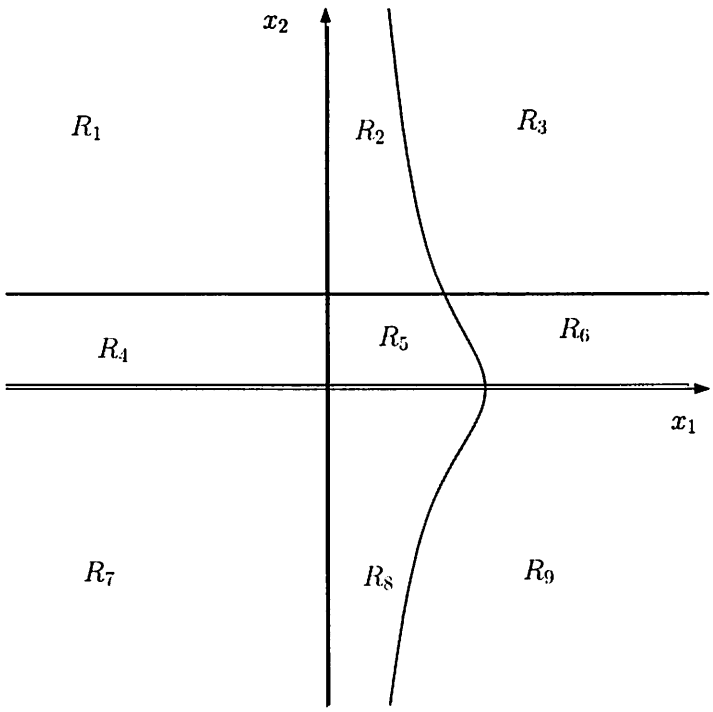

Figure 1.1 Nullclines and regions in which $\mathbb{R}^2$ is partitioned

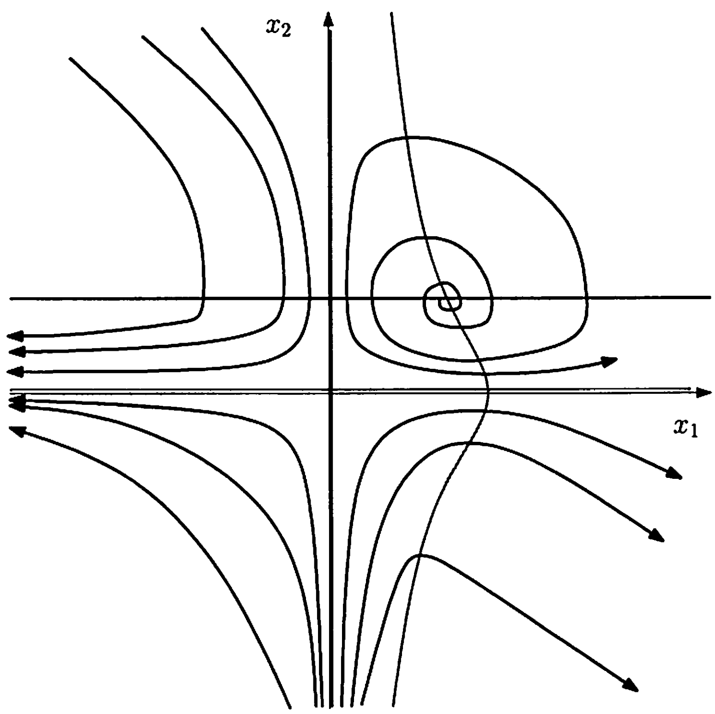

Figure 1.2 Sketch of phase-portrait for Exercise 1

e) A phase portrait consistent with the information collected in items a), b), c) and d) is sketched in Fig. 1.2.

(3 points for local phase-portraits; 3 points for globally consistent phase-portrait) [6]

2. Exercise

a) Equilibria of the system are solutions of $\dot{x} = 0$. Hence, for $(d^2 + d - 6) \neq 0$, we only have one equilibrium point, $x = 0$. If $d \in \{-3, 2\}$ instead, we have infinitely many equilibria, in particular, $\mathbb{R}$ is the set of equilibria. [2]

b) Any interval of the type $[-3 + \varepsilon, 2 - \varepsilon]$ yields UGAS of the system. In fact, taking as a Lyapunov function candidate $V(x) = x^2/2$ we see that:

$$\dot{V} = x^2(d^2 + d - 6) \leq (\varepsilon^2 - 5\varepsilon)x^2.$$

Notice that $\varepsilon^2 - 5\varepsilon < 0$ provided $0 < \varepsilon < 5$ (which is clearly the case). [4]

c) There does not exists a largest compact set such that UGAS is fulfilled. Every interval of the previous type is in fact suitable for UGAS, but the 'limit' interval $[-3, 2]$ is not, since equations read as $\dot{x} = 0$ when $d(\cdot)$ is constant and equal to either 2 or -3. (As a result solutions do not converge to the origin in that case). [3]

d) The system $\dot{z} = -z^3 + z$ has 3 equilibria, in $\{-1, 0, 1\}$. All its solutions converge asymptotically, in particular:

$$z_0 < 0 \Rightarrow \omega(z_0) = \{-1\}$$

$$z_0 = 0 \Rightarrow \omega(z_0) = \{0\}$$

$$z_0 > 0 \Rightarrow \omega(z_0) = \{1\}.$$

[4]

e) For the cascaded system, first of all notice that $z(t)$ is uniformly bounded, regardless of initial conditions (since $z(t)$ is convergent). Hence solutions of $\dot{x} = (z^2 + z - 6)x$ are defined globally in time (a linear system with bounded time-varying coefficients can at most have solutions which diverge exponentially). Regardless of z_0 there exists a finite time T_{z_0} such that $z(t) \in [-3/2, 3/2]$ for all $t \geq T_{z_0}$. Because of what we remarked previously $x(T_{z_0})$ is well defined. Moreover, we may regard $x(t)$, for $t \geq T_{z_0}$ as the solution of system $\dot{x} = (d^2 + d - 6)x$ forced by an input d with values in $[-3/2, 3/2]$. Thanks to the UGAS property we conclude that $x(t)$ approaches 0 asymptotically.
(1 mark for proving forward completeness, 2 marks for realizing that after some times input of downstream system are suitably bounded; 2 additional marks for complete proof) [5]

f) Hence we have the following ω limit sets:

$$z_0 < 0 \Rightarrow \omega([z_0, x_0]) = \{[-1, 0]\}$$

$$z_0 = 0 \Rightarrow \omega([z_0, x_0]) = \{[0, 0]\}$$

$$z_0 > 0 \Rightarrow \omega([z_0, x_0]) = \{[1, 0]\}.$$

[2]

3. Exercise

a) Equilibria are found by solving, for each u the systems of equations $\dot{x} = 0$. Hence we see that:

$$\begin{cases} -x_1^3 + u = 0 \\ x_1 - x_2^3 = 0 \\ x_2^3 - x_3 = 0 \end{cases}$$

Proceeding by substitution we see that $x_3 = x_1 = \sqrt[3]{u}$, and $x_2 = \sqrt[9]{u}$. Hence, for each u there exists a unique equilibrium $[\sqrt[3]{u}, \sqrt[9]{u}, \sqrt[3]{u}]'$. [3]

b) The Jacobian of $f(x,u)$ is given by:

$$\frac{\partial f}{\partial x} = \begin{bmatrix} -3x_1^2 & 0 & 0 \\ 1 & -3x_2^2 & 0 \\ 0 & 3x_2^2 & -1 \end{bmatrix}.$$

In correspondence of equilibria we see that:

$$\left.\frac{\partial f}{\partial x}\right|_{x=[\sqrt[3]{u}, \sqrt[9]{u}, \sqrt[3]{u}]'} = \begin{bmatrix} -3|u|^{2/3} & 0 & 0 \\ 1 & -3|u|^{2/9} & 0 \\ 0 & 3|u|^{2/9} & -1 \end{bmatrix}.$$

For all $u \neq 0$ the spectrum of this matrix is included in the open left half plane, therefore local asymptotic stability follows by the linearization method. For $u = 0$ we have 2 eigenvalues in 0 and therefore this constitutes a critical case. [3]

c) Given the cascaded structure of the system and local asymptotic stability of the x_1, x_2 and x_3 subsystems when considered individually for 0 inputs (viz. $x_1 \equiv 0$ in the $\dot{x}_2$ equation, and $x_2 \equiv 0$ in the $\dot{x}_3$ equations), we can conclude local asymptotic stability also for $u = 0$; [2]

d) In order to decide the relative degree we need to compute Lie derivatives of the output along solutions of the system.

$$\dot{y} = \dot{x}_2 + \dot{x}_3 = x_1 - x_3$$

Since u does not appear in the previous equation, we need to take an extra derivative:

$$\ddot{y} = -x_1^3 + u - x_2^3 + x_3$$

. Since the coefficient of u in $\ddot{y}$ is 1 the relative degree is 2 and is globally defined. [2]

e) An Input-Output linearizing feedback is given by:

$$u = x_1^3 - x_3 + x_2^3 + v$$

[3]

f) The system in normal form reads as follows:

$$\begin{aligned} \ddot{y} &= v \\ \dot{x}_3 &= x_2^3 - x_3 = (y - x_3)^3 - x_3 \end{aligned}$$

The internal dynamics are:

$$\dot{x}_3 = (y - x_3)^3 - x_3 = -x_3 - x_3^3 + 3x_3^2 y - 3x_3 y^2 + y^3.$$

[3]

g) We claim that internal dynamics are ISS, therefore the feedback $v = -\dot{y} - y$ globally asymptotically stabilizes the origin. To see this take as a Lyapunov function $V(x_3) = x_3^2/2$.

$$\dot{V} = -x_3^2 - x_3^4 + 3x_3^3 y - 3x_3^2 y^2 + x_3 y^3 \leq -x_3^2 - x_3^4 + 3|x_3|^3|y| + 3|x_3|^2|y|^2 + |x_3||y|^3$$
$$\leq -x_3^2 - x_3^4 + [3|x_3|^3 + 3|x_3|^2 + 3|x_3|][|y| + |y|^2 + |y^3|].$$

Notice that:

$$\frac{|x_3| + |x_3|^3}{2(3|x_3|^2 + 3|x_3| + 1)} \leq |y| + |y|^2 + |y|^3 \Rightarrow \dot{V} \leq -\frac{x_3^2 + x_3^4}{2}.$$

Since $\frac{r+r^3}{2(3r^2+3r+1)}$ and $r + r^2 + r^3$ are both $\mathcal{K}_\infty$ functions of r, it follows that internal dynamics are ISS as claimed.
(1 mark for the feedback law; 3 marks for the analysis based on ISS) [4]

4. Exercise

a) Notice that a negative feedback interconnection is characterized by the following equations: $y = y_1 = u_2$ and $u_1 = -y_2 + u$. We let $y_2 = x_3$ and rewrite equations as follows:

$$\begin{aligned} \dot{x}_1 &= x_2^2 + u_1 x_2 + u_1 \\ \dot{x}_2 &= -x_1 x_2 - u_1 x_1 \\ y_1 &= x_1 \end{aligned} \qquad \begin{aligned} \dot{x}_3 &= -\mathrm{atn}(x_3) + u_2 \\ y_2 &= x_3 \end{aligned}.$$

(2 marks for definition of negative feedback interconnection, 4 marks for correct identification of subsystems) [6]

b) We want to prove passivity of the individual subsystems. Notice that letting $V_1(x_1, x_2) = x_1^2/2 + x_2^2/2$ yields:

$$\dot{V}_1 = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1 u_1 = y_1 u_1,$$

showing that the (x_1, x_2) subsystem is in fact passive and lossless. For the x_3 subsystem we may let $V_2(x_3) = x_3^2/2$. We see that:

$$\dot{V}_2 = x_3\dot{x}_3 = -\mathrm{atn}(x_3)x_3 + x_3 u_2 \leq y_2 u_2$$

showing that the x_3 subsystem is passive.
(3 marks for each subsystem) [6]

c) The negative feedback system is passive from u to y with storage function $V_1(x_1, x_2) + V_2(x_3)$. [3]

d) We may use the Lyapunov function $V(x_1, x_2, x_3) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$ to prove global asymptotic stability of the origin. In fact V is clearly positive definite and radially unbounded. Moreover, for $u = 0$ we have:

$$\dot{V} = -\mathrm{atn}(x_3)x_3 \leq 0,$$

and therefore its derivative is negative semi-definite. We see that:

$$\{x : \dot{V}(x) = 0\} = \{x : x_3 = 0\}.$$

We would like to show that if $x_3(t)$ is identically equal to 0, then necessarily $x(t) = 0$. In fact:

$$\dot{x}_3(t) \equiv 0 \& x_3(t) \equiv 0 \Rightarrow -\mathrm{atn}(x_3(t)) + x_1(t) \equiv 0 \& x_3(t) \equiv 0 \Rightarrow x_1(t) = 0$$

$$x_1(t) \equiv 0 \Rightarrow \dot{x}_1(t) \equiv 0 \Rightarrow x_2^2(t) \equiv 0.$$

Therefore the origin is the largest invariant set contained in the kernel of $\dot{V}$ and by the Lasalle's criterion the origin is globally asymptotically stable. (2 marks for identifying correct Lyapunov function; 3 marks for Lasalle's argument) [5]

5. Exercise

a) Since the equation is of second order we may pick as state variable $x = [y, \dot{y}]' = [x_1, x_2]'$. Accordingly the systems equations read:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1^3 + x_1 + (x_1^2/2 - x_1^4/4 - x_2^2/2 - 1/8)x_2. \end{aligned}$$

[3]

b) At equilibrium we have $x_2 = 0$ from the first equation and: $x_1 - x_1^3 = 0$ from the second one. Hence, 3 equilibria exist: $[1,0]', [0,0]', [-1,0]'$. Linearizing the system we see that:

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ -3x_1^2 + 1 + (x_1 - x_1^3)x_2 & x_1^2/2 - x_1^4/4 - 1/8 - 3x_2^2/2 \end{bmatrix}.$$

Substituting the equilibrium values in the Jacobian we find the following linearized systems:

$$\dot{\delta x} = \begin{bmatrix} 0 & 1 \\ -2 & 1/8 \end{bmatrix} \delta x,$$

for $x = [\pm 1, 0]'$ or:

$$\dot{\delta x} = \begin{bmatrix} 0 & 1 \\ 1 & -1/8 \end{bmatrix} \delta x,$$

for $x = [0,0]'$. [3]

c) The characteristic polynomial of the first matrix is $s^2 - s/8 + 2$, which has complex conjugate roots with positive real part; hence the local phase-portrait is that of an unstable focus. For the system linearized around $[0,0]'$ instead the characteristic polynomial is: $s^2 + s/8 - 1$ which corresponds to real roots with opposite sign. Hence the local phase-portrait is that of a saddle point. [3]

d) To answer the question we compute the derivative of V along the systems solutions. In particular $V(x_1, x_2) = \frac{x_1^4}{4} - \frac{x_1^2}{2} + \frac{x_2^2}{2}$. Hence:

$$\dot{V}(x_1, x_2) = (x_1^3 - x_1)x_2 + x_2(-x_1^3 + x_1 + (x_1^2/2 - x_1^4/4 - x_2^2/2 - 1/8)x_2$$

$$= x_2^2(x_1^2/2 - x_1^4/4 - x_2^2/2 - 1/8).$$

Notice that for $V = -\frac{1}{8}$ we have $\dot{V} = 0$, which proves invariance of the level set $\mathscr{L}_{-1/8}$. [3]

e) From the previous calculations we also see that:

$$V(x_1, x_2) \geq -1/8 \Rightarrow \dot{V} \leq 0$$

and

$$V(x_1, x_2) \leq -1/8 \Rightarrow \dot{V} \geq 0.$$

Hence, the sublevel set $\mathscr{S}_c$ are forward invariant for all $c \geq -\frac{1}{8}$, whereas they are backwards invariant for all $c \leq -1/8$ (for $c < -1/4$ they are actually empty sets). [3]

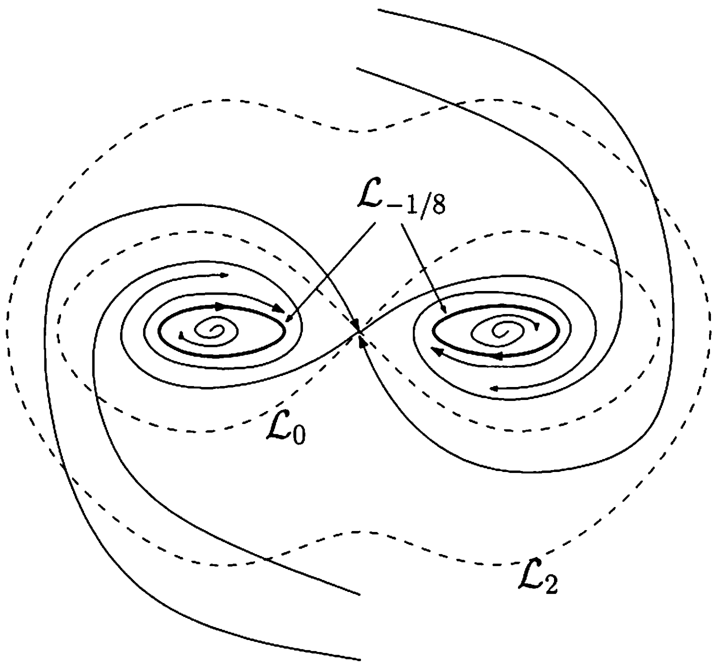

Figure 5.1 Sketch of phase-portrait

f) Since $\mathscr{L}_{-1/8}$ is invariant, the system exhibits two periodic solutions (one for each connected component of $\mathscr{L}_{-1/8}$. A global phase-portrait consistent with the collected clues is shown in Fig. 5.1. [5]

6. Exercise

a) Consider the candidate Lyapunov function $V(x) = x^2/2$; taking its derivative along solutions of the system we see that:

$$\dot{V}(x,d) = x\dot{x} = -x\text{sat}(\alpha x + d).$$

Let $\varepsilon > 0$ be arbitrarily small. We see that :

$$|x| \geq \frac{(1+\varepsilon)}{\alpha}|d| \Rightarrow \dot{V} \leq -\text{sat}(\varepsilon|x|/\alpha)|x|.$$

Hence the system is ISS. [4]

b) The gain from d to x can be estimated in $(1+\varepsilon)/\alpha$. [4]

c) The system is a cascade of ISS systems. The asymptotic gain from d to x_1 is any positive number $> 1/2$; the asymptotic gain from x_1 to x_2 is any positive number > 1. Hence the asymptotic gain from d to x_2 can be estimated to be any positive number bigger than $1/2$. Using the infinity norm to estimate the gain from d to (x_1, x_2) we see that this gain is any positive number $> 1/2$. [6]

d) The x_3 subsystem is ISS with unit gain (any positive number > 1. Overall, the last subsystem can be seen as an interconnection of ISS systems with loop gain $= 1/2 \cdot 1 < 1$. Therefore, by the small gain theorem GAS of the origin holds. [6]

EE4-57 Discrete event systems

1. Paper title, copy right Imperial College London and page number is missing at the bottom of each page.
2. Q1 e, spelling mistake "spuuort".
3. Q2 line 2-3, grammar mistake " these corresponds"
4. Q4 part B, "Can you interpret their meaning?" is not a clear instruction.
5. Mark allocations are not shown in the answers document.

EE 4-23 Stability and control of non-linear systems

1. Paper title, copy right Imperial College London and page number is missing at the bottom of each page.
2. Marks are not shown in the answers document.
3. In the answers document Q3, part f and g are not clearly separated.

EE 4-48 Power System control, measurement and protection

1. Question 3, Figure number should be "Figure 3.1" and the text should be amended accordingly.

EE4-10 Probability of stochastic process

1. Q1 C.iii , grammar problem with "conclude that the probability that some bin empty is smaller than"
2. Q1 d. grammar problem with"we now back to the general setting"
3. Q1 d ii, grammar problem with "assume that we are in room containing m individuals" and also with "two individuals are born the same day of the year".
4. Q2 a, line 2 spelling mistake with "diagramme".
5. Q2 c) iii, answer is not provided in the answers document.
6. Q2 d. Ii. grammar problem with" find average fraction of time that the stock goes up"
7. Q3 b.ii grammar problem with " how long before either of them leave the parking?".
8. Q3 C line 2, grammar problem with " the number of ....constitute ......".
9. Q3C i. Sentence needs clarifying "derive the long run fraction of arriving cars".
10. Q3 c.ii, question instruction is not clear.