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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI

DIGITAL SIGNAL PROCESSING

Friday, 2 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.A. Naylor

Second Marker(s) : D.P. Mandic

Special Instructions for Invigilators: None

Information for Candidates:

Sequence	z-transform
$\delta(n)$	1
$u(n)$	$\dfrac{1}{1-z^{-1}}$
$a^n u(n)$	$\dfrac{1}{1-az^{-1}}$
$\left(r^n \cos\omega_0 n\right)u(n)$	$\dfrac{1-(r\cos\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}$

Table 1 : z-transform pairs

$\delta(n)$ is defined to be the unit impulse function.
$u(n)$ is defined to be the unit step function.

Numbers in square brackets against the right margin of the following pages are a guide to the marking scheme.

[4]

1. (a) Write down the expression for $x(n)$ as the inverse discrete Fourier transform of $X(k)$. Briefly explain the meaning of each term in the expression and give an illustrative example of a practical application of the inverse discrete Fourier transform.

(b) Explain the meaning of the terms *minimum phase, maximum phase* and *mixed phase* in the context of causal stable FIR filters. [4]

(c) Given $X(k) = [1,2,1,2]$, determine $x(n)$. [6]

Consider that $x(n)$ is the impulse response of an FIR filter. Plot the roots of the filter's transfer function on the *z*-plane. Sketch graphs showing the main features of the filter's magnitude response, in dB, and phase response. Comment on whether this filter is minimum phase, maximum phase or mixed phase. [6]

2. (a) Define the autocorrelation function, $\gamma_{xx}(l)$, of a real-valued signal $x(n)$ and give a short description of autocorrelation including its method of computation and the significance of l. [4]

(b) Write down the important properties of $\gamma_{xx}(l)$. [3]

(c) The short-term autocorrelation function of a signal $x(n)$ can be defined as [13]

$$\gamma_{ST}(k,n) = \sum_{m=-\infty}^{\infty} x(m)w(n-m)x(m+k)w(n-m-k)$$

for which $w(n)$ is a window function.

Show that $\gamma_{ST}(k,n)$ can be computed using a structure of the form of Figure 2.1 and determine an expression for the digital filter $h_k(n)$.

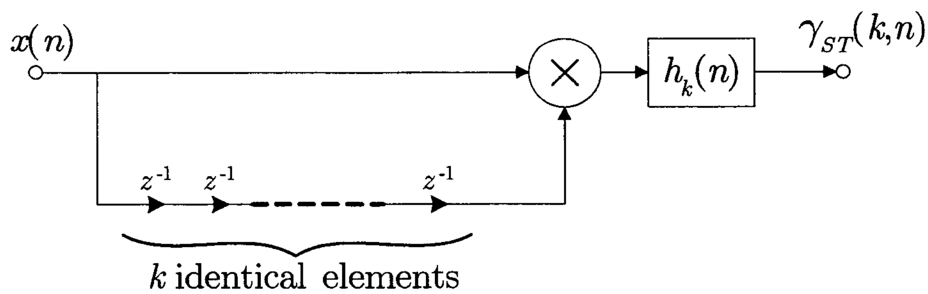

Figure 2.1

3. (a) A filter $A(z)$ is required to be a stable allpass filter. Write down the necessary conditions [10]

(i) in the frequency domain and

(ii) in the z-domain.

Briefly describe a practical application of allpass filters.

Give the z domain system function for a second order allpass filter using the real filter coefficients λ_i for integer i.

(b) Derive and sketch the signal flow graph of a first-order allpass filter that employs only one non-trivial multiplication operation per output sample. [10]

4. Consider a multirate system consisting of an analysis filter bank followed by a synthesis filter bank. The filter banks have two bands of equal bandwidth. The input signal to the system is $x(n)$ and the output is $\hat{x}(n)$.

(a) Draw a signal flow diagram of the system and label all signals in the system. Give an expression for each signal in terms of $X(z)$, the z-transform of $x(n)$. [5]

(b) Consider a half-band lowpass prototype filter $H_0(z)$. Write down expressions for the filters in your multirate system in terms of $H_0(z)$. Give reasons for your choices. [3]

(c) Show how each of the filters in the multirate system can be represented in a 2-phase, Type 1 polyphase form and, hence, draw and label the signal flow diagram of the analysis and synthesis filter banks using filters in Type 1 polyphase filters. [6]

(d) State the <u>Noble Identities</u> and briefly describe how they can be used to improve the efficiency of multirate systems. [2]

(e) Using the Nobel Identities, redraw the two-band multirate system in an <u>efficient</u> Type 1 polyphase form. Calculate the ratio of the number of filtering computations required in your efficient implementation compared to the number required in the direct implementation of the system described in part (a). [4]

5. (a) Consider the discrete-time signal $x(n)$ and the continuous-time signal $x_a(t)$, with $x_a(nT) = x(n)$ for sampling period *T*.

(i) Express $x(n)$ in terms of the superposition of complex exponentials. [2]

(ii) State an expression for the amplitudes of the exponentials. [2]

(iii) Hence or otherwise, show that the Fourier Transform of $x(n)$ is periodic with period 2π. [3]

(iv) Show how $x_a(t)$ can be reconstructed from $x(n)$ stating any necessary conditions. [4]

(b) A particular LTI system has input $x(n)$ and output $y(n)$. An experiment was conducted which employed an input signal

$$x(n) = \begin{cases} 1, & n = 0,1 \\ 0, & \text{otherwise} \end{cases}$$

and the corresponding output was found to be

$$y(n) = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}.$$

(i) Determine the frequency response of the system using the DTFT. [5]

(ii) Sketch the magnitude response for $N = 3$. [4]

6. (a) Consider the two-sided function $x(n) = \alpha^n$. [6]

(i) Write down the definition of the z-transform of $x(n)$.

(ii) By considering $x(n)$ as the sum of two one-sided functions, show that $x(n) = \alpha^n$ does not have a z-transform.

(b) Using long division, find the inverse z-transform of [6]

$$H(z) = \frac{1 + 2z^{-1}}{1 + 0.4z^{-1} - 0.12z^{-2}}.$$

Hence determine the first 5 samples of the impulse response of $H(z)$

(c) Find a causal, stable, IIR equalizer $G(z)$ such that $\left|H(e^{j\omega})G(e^{j\omega})\right| = 1$. [8]