UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2000

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BEng Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C337=I3.18

SIMULATION AND MODELLING

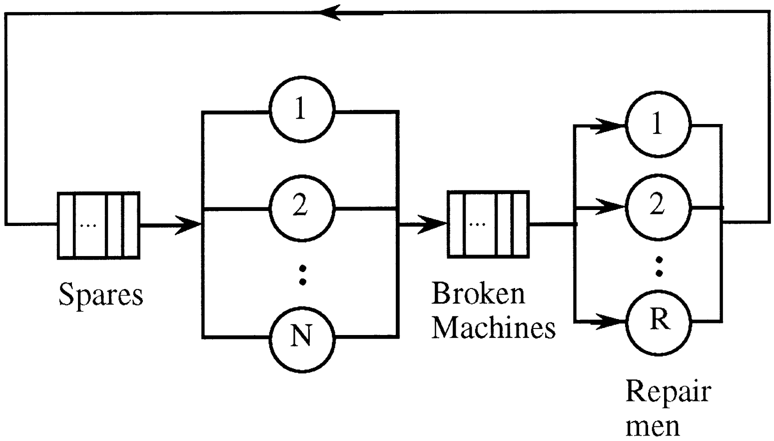
Wednesday 10 May 2000, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions

1 A canning factory has the capacity to run N labelling machines concurrently, each of which sticks labels to cans. From time to time these machines break down. Fortunately they are easily replaced so the factory owner keeps a set of S spare machines which can be substituted whilst the broken machines are repaired. A total of R repair men are available to repair broken machines which are then returned to the pool of spares for later use. Each repair man works exclusively on one machine until it is repaired, before moving on to the next broken machine (if there is one). The repair times are well approximated by a known mathematical distribution which can be sampled using the function `RepSample()`. Similarly the time between failures for each machine has a well known mathematical distribution which can be sampled using the function `UpTimeSample()`. The factory runs 24 hours a day with the repair men operating in shifts so that there are always exactly R of them on duty.

The system can be described by the following (closed) queueing network.



Design a simulation program for this system which will estimate the average number of machines in operation (i.e. actually labelling cans) at any time. You may assume that the time to replace a broken machine with a working one is small in comparison to the repair time and so can be assumed to be zero.

You may use any notation you wish so long as your solution is clear and readable. You need not define any library procedures you use, e.g. those for scheduling an event, provided their meaning is clear, e.g. via suitably obvious identifiers, or annotations. Avoid unnecessary detail.

3a A new generator has been developed for producing identically-distributed random samples with mean 1/4 and variance 1/16. Describe how the central limit theorem leads to a method for generating standard normal (N(0,1)) samples by combining samples from this generator. The efficiency of your method will be taken into account when awarding credit.

3b Explain briefly the *rejection method* for generating samples from a distribution with density function $f(x)$. As part of your answer, explain the principal factor(s) which determine the efficiency of the rejection method.

3c A generator has been developed to sample a distribution with cumulative distribution function

$$F(x) = 1 - \exp(-x^2) \qquad x \geq 0$$

and density function

$$f(x) = 2x \exp(-x^2) \qquad x \geq 0$$

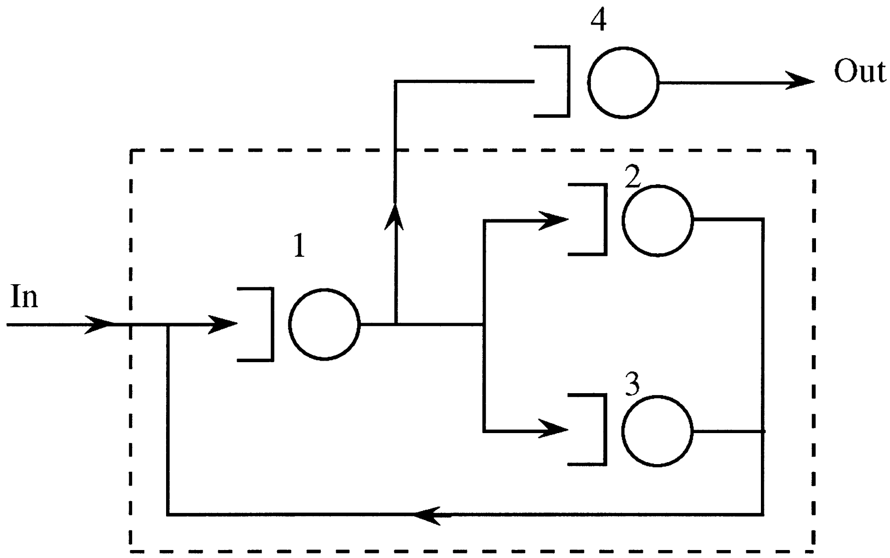
After 4000 samples it has produced the following histogram:

interval:	[0.0,0.3)	[0.3,0.6)	[0.6,0.9)	[0.9,1.2)	[1.2,1.5)	[1.5,1.8)	[1.8,2.1)
observed frequency:	380	816	1062	888	474	301	79

In order to test whether the generator is performing correctly, perform a χ^2 test on the data at the 5% significance level to test the null hypothesis H0: the observed data has the above distribution.

Note: The three parts carry 30%, 30% and 40% of the marks respectively.

2 Consider the following queueing system:



Jobs enter the system according to a Poisson process at the average rate of 1 job every 10 seconds. Each job first circulates around the subsystem (shown by the dotted box) before being post-processed by service centre 4. The subsystem comprises a general-purpose processor (centre 1) and two special-purpose processors (service centres 2 and 3). Within the subsystem each job visits the first special-purpose processor (centre 2) 100 times on average and the second special-purpose processor (centre 3) 124 times on average. A job spends on average 40ms at each visit to the general-purpose processor, an average of 50ms and 40ms respectively on each visit to the two special-purpose processors (centres 2 and 3) and an average of 8 seconds at the post-processor (centre 4). Every job entering the subsystem eventually leaves after a finite expected time. All queues are assumed to be very large and well approximated by infinite capacity FIFO queues. The service times throughout may be assumed to be exponentially distributed.

a How many times on average does each job visit the general-purpose processor (centre 1)? Justify your answer.

b At what rate do jobs arrive at the post-processor (centre 4)? Justify your answer.

c What is the average time spent by each job in the (dotted) subsystem? Show your working.

d What is the mean response time of (i.e. the mean time spent in) the whole system?

e How fast would the general-purpose processor (centre 1) have to be in order to halve the current mean response time?

Note: The five parts carry 10%, 10%, 40%, 20% and 20% of the marks respectively.

4a A discrete-event simulation is executed n times with identical parameters but different random number seeds in order to produce estimates $X_1, \ldots, X_n$, of some unknown quantity A.

i What, mathematically, is meant by a *p% confidence interval* (or *p% interval estimate*) for A?

ii In computing a confidence interval for the population mean of the X_i $(1 \leq i \leq n)$, why is important that the X_i are independent?

ii Is it always the case that each of the X_i $(1 \leq i \leq n)$ must lie within the bounds of the confidence interval? Explain your answer.

4b A discrete-event simulation has been written to estimate the mean waiting time for a customer in a queue. Nine independent estimates were obtained as follows:

5.14 5.29 4.78 5.01 4.67 4.91 5.54 5.30 4.49

i Compute the 95% confidence interval for the mean waiting time

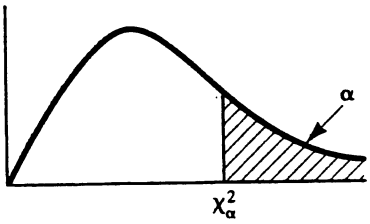
ii On the basis of the above data, and using the tables provided, what is the probability that the mean waiting time is less than or equal to 5.27?

iii If, within a single execution of the simulation, you were to measure the waiting times of *successive* customers as they left the queue, why would you expect the waiting times *not* to be independent?

Note: The two parts carry 40% and 60% of the marks respectively.

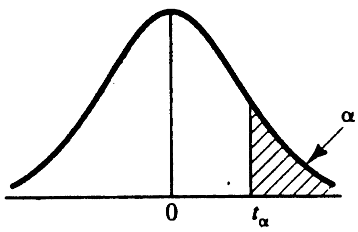
End of paper.

PERCENTAGE POINTS OF THE CHI-SQUARE DISTRIBUTION
WITH V DEGREES OF FREEDOM



ν	$\chi^2_{0.005}$	$\chi^2_{0.01}$	$\chi^2_{0.025}$	$\chi^2_{0.05}$	$t^2_{0.10}$
1	7.88	6.63	5.02	3.84	2.71
2	10.60	9.21	7.38	5.99	4.61
3	12.84	11.34	9.35	7.81	6.25
4	14.96	13.28	11.14	9.49	7.78
5	16.7	15.1	12.8	11.1	9.2
6	18.5	16.8	14.4	12.6	10.6
7	20.3	18.5	16.0	14.1	12.0
8	22.0	20.1	17.5	15.5	13.4
9	23.6	21.7	19.0	16.9	14.7
10	25.2	23.2	20.5	18.3	16.0
11	26.8	24.7	21.9	19.7	17.3
12	28.3	26.2	23.3	21.0	18.5
13	29.8	27.7	24.7	22.4	19.8
14	31.3	29.1	26.1	23.7	21.1
15	32.8	30.6	27.5	25.0	22.3
16	34.3	32.0	28.8	26.3	23.5
17	35.7	33.4	30.2	27.6	24.8
18	37.2	34.8	31.5	28.9	26.0
19	38.6	36.2	32.9	30.1	27.2
20	40.0	37.6	34.2	31.4	28.4
21	41.4	38.9	35.5	32.7	29.6
22	42.8	40.3	36.8	33.9	30.8
23	44.2	41.6	38.1	35.2	32.0
24	45.6	43.0	39.4	36.4	33.2
25	49.6	44.3	40.6	37.7	34.4
26	48.3	45.6	41.9	38.9	35.6
27	49.6	47.0	43.2	40.1	36.7
28	51.0	48.3	44.5	41.3	37.9
29	52.3	49.6	45.7	42.6	39.1
30	53.7	50.9	47.0	43.8	40.3
40	66.8	63.7	59.3	55.8	51.8
50	79.5	76.2	71.4	67.5	63.2
60	92.0	88.4	83.3	79.1	74.4
70	104.2	100.4	95.0	90.5	85.5
80	116.3	112.3	106.6	101.9	96.6
90	128.3	124.1	118.1	113.1	107.6
100	140.2	135.8	129.6	124.3	118.5

PERCENTAGE POINTS OF THE STUDENTS t
DISTRIBUTION WITH ν DEGREES OF FREEDOM



ν	$t_{0.005}$	$t_{0.01}$	$t_{0.025}$	$t_{0.05}$	$t_{0.10}$
1	63.66	31.82	12.71	6.31	3.08
2	9.92	6.92	4.30	2.92	1.89
3	5.84	4.54	3.18	2.35	1.64
4	4.60	3.75	2.78	2.13	1.53
5	4.03	3.36	2.57	2.02	1.48
6	3.71	3.14	2.45	1.94	1.44
7	3.50	3.00	2.36	1.90	1.42
8	3.36	2.90	2.31	1.86	1.40
9	3.25	2.82	2.26	1.83	1.38
10	3.17	2.76	2.23	1.81	1.37
11	3.11	2.72	2.20	1.80	1.36
12	3.06	2.68	2.18	1.78	1.36
13	3.01	2.65	2.16	1.77	1.35
14	2.98	2.62	2.14	1.76	1.34
15	2.95	2.60	2.13	1.75	1.34
16	2.92	2.58	2.12	1.75	1.34
17	2.90	2.57	2.11	1.74	1.33
18	2.88	2.55	2.10	1.73	1.33
19	2.86	2.54	2.09	1.73	1.33
20	2.84	2.53	2.09	1.72	1.32
21	2.83	2.52	2.08	1.72	1.32
22	2.82	2.51	2.07	1.72	1.32
23	2.81	2.50	2.07	1.71	1.32
24	2.80	2.49	2.06	1.71	1.32
25	2.79	2.48	2.06	1.71	1.32
26	2.78	2.48	2.06	1.71	1.32
27	2.77	2.47	2.05	1.70	1.31
28	2.76	2.47	2.05	1.70	1.31
29	2.76	2.46	2.04	1.70	1.31
30	2.75	2.46	2.04	1.70	1.31
40	2.70	2.42	2.02	1.68	1.30
60	2.66	2.39	2.00	1.67	1.30
120	2.62	2.36	1.98	1.66	1.29
∞	2.58	2.33	1.96	1.645	1.28