EE1-06

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

EEE/EIE PART I: MEng, BEng and ACGI

Corrected copy

INTRODUCTION TO SIGNALS AND COMMUNICATIONS

Wednesday, 30 May 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : K.K. Leung

Second Marker(s) : J.A. Barria

Special Instructions for Invigilator: **None**

Information for Students:

Fourier Transforms

$\cos \omega_o t$ <=> $\pi[\delta(\omega - \omega_o) + \delta(\omega + \omega_o)]$

Some useful trigonometric identities

$$\cos x \cos y = \tfrac{1}{2}\cos(x - y) + \tfrac{1}{2}\cos(x + y)$$
$$\sin(x - y) = \sin x \cos y - \sin y \cos x$$
$$a \cos x + b \sin x = c \cos(x + \theta)$$

where $c = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}(-b / a)$

Complex exponential

$$e^{jx} = \cos x + j \sin x$$

1. This is a general question. (40%)

a. Consider a time-domain signal $f(t) = 1/a$ for $-a/2 \le t \le a/2$ and 0 for $t < -a/2$ or $t > a/2$, and a is a positive constant.

   i. Derive the Fourier transform $F(\omega)$ of $f(t)$. [3]

   ii. Sketch the frequency spectrum of $f(t)$. [2]

   iii. Let $\hat{f}(t) = \lim_{a \to 0} f(t)$. What is the signal $\hat{f}(t)$? Using the definition, what is the Fourier transform $\hat{F}(\omega)$ of $\hat{f}(t)$? [2]

   iv. Derive $\hat{F}(\omega)$ by taking the limit of a to zero in $F(\omega)$ from part i. [2]

b. Let a real, energy signal $x(t)$ consist of three signal components, $a(t)$, $b(t)$ and $c(t)$, as $x(t) = a(t) + b(t) + c(t)$. Further, let E_x, E_a, E_b and E_c be the energy of $x(t)$, $a(t)$, $b(t)$ and $c(t)$, respectively.

   i. Provide an expression for E_x in terms of $a(t)$, $b(t)$ and $c(t)$. [3]

   ii. Identify three sufficient conditions for $E_x = E_a + E_b + E_c$; that is, the energy of $x(t)$ equals the sum of the energies of the signal components. What is the commonly used term for these conditions among $a(t)$, $b(t)$ and $c(t)$? [3]

   iii. Assuming that the conditions identified in part ii are valid, explain whether it is always possible to express an arbitrary energy signal $y(t)$ as $y(t) = \alpha a(t) + \beta b(t) + \gamma c(t)$ for some constants α, β and γ? [2]

   iv. Following part iii, if we can always express any energy signal $y(t)$ as $y(t) = \alpha a(t) + \beta b(t) + \gamma c(t) + \lambda d(t)$ where λ is another constant, what can be said about the relationships between $d(t)$ and the other signal components $a(t)$, $b(t)$ and $c(t)$? [2]

c. Consider the following periodic signal $s(t)$ with period T_0.

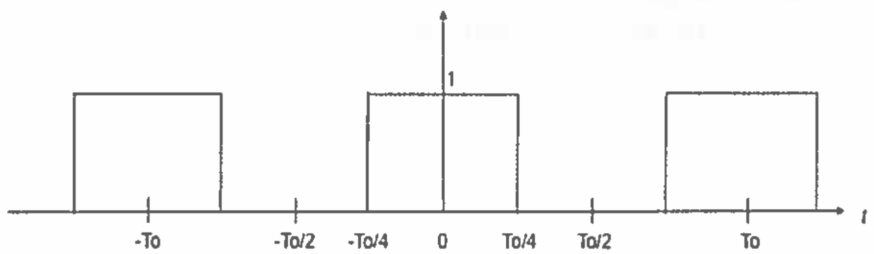

Figure 1. The periodic signal $s(t)$.

Our objective here is to make use of the signal $s(t)$ to generate an amplitude-modulated (AM) signal $\phi(t)$ for a given modulating signal $m(t)$ and a sinusoidal carrier $\cos(\omega_c t)$. Let $M(\omega)$ be the Fourier transform of $m(t)$.

i. Express $s(t)$ as a Fourier series with coefficients a_0, a_n and b_n for integer n from 1 to ∞. [2]

ii. Derive the coefficients a_0, a_n and b_n for integer n from 1 to ∞. [4]

iii. Using results in part ii, provide an expression for $s(t)m(t)$. [2]

iv. Sketch the spectrum of $s(t)m(t)$. [2]

v. From result in part iv, suggest a way to obtain the AM signal $\phi(t)$ and explain why. What is the relationship between T_0 and ω_c in your suggestion? [2]

d. Let $\varphi(t)$ denote a phase-modulation (PM) signal where $m(t)$ is the modulating signal, f_c is the frequency of the sinusoidal carrier, and k_p is the proportionality constant.

i. Give an expression for $\varphi(t)$. [3]

ii. Determine the instantaneous frequency for the PM signal as a function of time. [2]

iii. Assume that $m(t)$ is given by the following diagram.

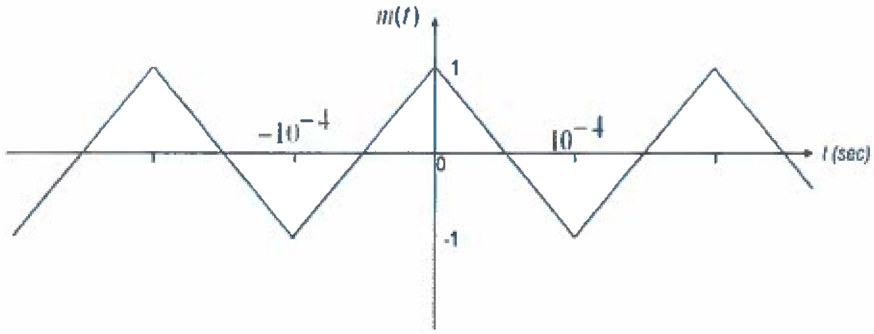

Figure 2. The modulating signal $m(t)$.

Furthermore, let $f_c = 100$ MHz and $k_p = 10\pi$. Determine the maximum and minimum instantaneous frequencies for $\varphi(t)$. [2]

iv. Using results in part iii, sketch the signal $\varphi(t)$. [2]

2. Signals and their transforms. (30%)

   a. Let $\delta(t)$denote the unit impulse at $t=0$. Consider a linear time-invariant (LTI) system for which the unit impulse response function is $h(t)=3/4\delta(t)+1/4\delta(t-T_0/2)$ as shown in Figure 3 below, where T_0 is a fixed positive constant.

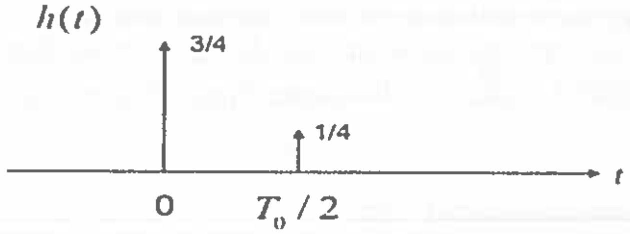

Figure 3. The unit impulse response $h(t)$ for the LTI system.

   i. Input signal $x(t)$ of two pulses in Figure 4 with the assumption of $T_0 = T$ to the system. Determine and sketch the corresponding output signal $z(t)$ of the system. [3]

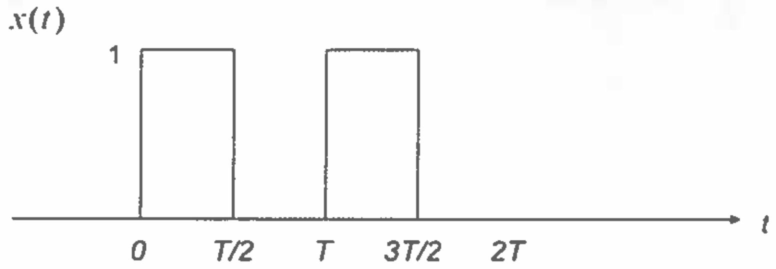

Figure 4. Input signal $x(t)$.

   ii. Repeat part i for the following input signal $y(t)$ in Figure 5. [3]

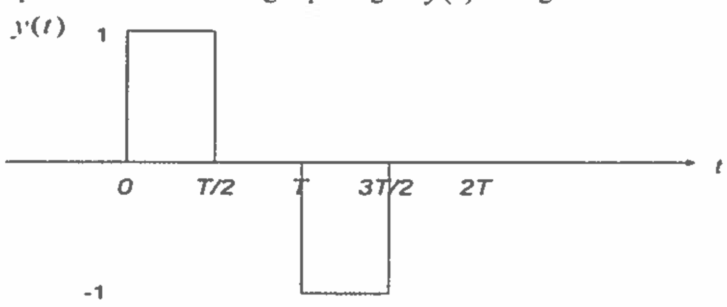

Figure 5. Input signal $y(t)$.

   iii. Suppose that a communication channel is represented by the above linear system. That is, when a unit impulse $\delta(t)$ is transmitted over the channel, the signal at the receiving end of the channel is $h(t)$ as shown in Figure 3. Assume that only periodic pulses with +1 or -1 can be transmitted to represent 1's or 0's, respectively, as in Figure 5, although the pulse period of T can now be varied (and T_0 in $h(t)$ in

Figure 3 remains fixed). Identify the maximum number of pulses per second that can be transmitted over the channel and properly decoded by a simple receiver? Explain your result. [7]

iv. Is it possible for the channel to support a pulse transmission rate higher than the maximum value identified in part iii? If so, explain how to support that. [3]

b. Consider two time-domain signals $f(t)$ and $g(t)$ with their respective Fourier transforms $F(\omega)$ and $G(\omega)$. Let $f(t) * g(t)$ denote the convolution of $f(t)$ and $g(t)$. For convenience, let us use $\Im[v(t)]$ to denote the Fourier transform of a given signal $v(t)$.

i. Give an expression for the convolution $f(t) * g(t)$. [2]

ii. By taking the Fourier transform of result in part i, show that $\Im[f(t) * g(t)] = F(\omega)G(\omega)$. [5]

iii. Give an expression for the convolution $F(\omega) * G(\omega)$. [2]

iv. By taking the inverse Fourier transform of result in part iii, show that

$$\Im[f(t)g(t)] = \frac{1}{2\pi} F(\omega) * G(\omega).$$ [5]

3. Communications techniques. (30%)

a. A continuous-time signal $g(t)$ with bandwidth B Hz is periodically sampled once every T_s seconds and each sample is quantized and encoded into K bits. The information bits are transmitted using the polar non-return-to-zero line coding (i.e., use +1 and -1 to represent 1 and 0 bits, respectively, without returning to zero between two consecutive bits) and the frequency shift keying (FSK) over a communication link. Assume that the link can support a data rate up to R bits per second. Let ω_c be the carrier frequency in radians/second and the amplitude of the modulated signal be denoted by A.

   i. Give an expression for the transmitted FSK signal on the link. [3]
   ii. Provide a block diagram and explain how to demodulate the FSK signal using non-coherent detection. [4]
   iii. To enable correct reception, determine the maximum bandwidth of $g(t)$ in terms of R and K. Use a spectrum diagram to explain your reasoning. [5]

b. Design a wideband frequency modulation (WBFM) system using frequency multipliers as follows. Let $m(t)$ be the modulating signal. A narrow-band FM (NBFM) generator is available to take $m(t)$ as input and generates a narrow-band FM signal with a carrier frequency f_{NB} of 200 kHz and the maximum frequency deviation Δf_{NB} of 30 Hz. As for the WBFM system, the final carrier frequency f_c and the maximum frequency deviation Δf of the WBFM signal are 100 MHz and 61.44 kHz, respectively. Beside the oscillator at 200 kHz for the NBFM generator, a second oscillator of another frequency is available. Furthermore, only multipliers that double the carrier frequency and frequency deviation are available.

   i. By using the NBFM generator, frequency multipliers and a frequency converter as building blocks, draw a block diagram for the WBFM system. Indicate the carrier frequency and the maximum frequency deviation at each step and explain your design. [8]
   ii. What is your selected frequency of the second oscillator? [2]
   iii. What purpose does the second oscillator serve? Provide a mathematical justification for how such use achieves its purpose. [5]
   iv. Is the design of the WBFM system unique? Can the second oscillator be of a different frequency? Explain. [3]