ENERGY CONVERSION: ANSWERS

Question 1. [a) = 7], [b) = 8], [c) = 5], [d) = 7], [e) = 8], [f) = 5]

a) (calculated problem) Since the density is varying with distance, we can find the total change by integrating [1]

$$q = \int_0^l \lambda \mathrm{d}x \qquad [1]$$

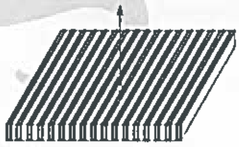
Integrating yields

$$q = \lambda_0 l_0 \sin\frac{l}{l_0} \qquad [5]$$

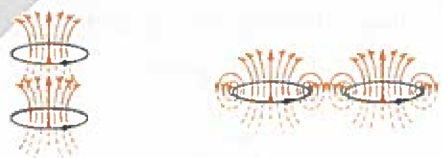
b) (bookwork) Let us choose two points (denoted by A and B) on the surface of the conductor. The voltage between them can be found as $U_{AB} = \int_A^B (\mathbf{E}\cdot \mathrm{d}\mathbf{l})$ [2]. The integration path between the two points can be arbitrary [1]. Along any path, the field is normal to the surface and hence will be perpendicular to dl. Therefore $(\mathbf{E}\cdot \mathrm{d}\mathbf{l}) = 0$ anywhere on any path between points A and B [3]. Therefore, the voltage between any two points is zero, from which follows that the potential of all point is the same [2].

c) (bookwork) Permanent magnets are magnetised in the absence of the external magnetic field [2]. A stronger permanent magnet will therefore have larger retentivity: the value of B for $H = 0$. Therefore, material 1 will be a stronger permanent magnet [3].

d) (bookwork) Laminating means inserting thin dielectric layers into ferromagnetic materials [1]. Such widely used ferromagnetic materials as iron and nickel are conductors, and a time varying magnetic field will excite currents in them ('eddy currents') [3], which would lead to energy loss [1]. The dielectric materials prevent eddy currents and reduce loss [2].



e) (reasoning/assimilation of bookwork)



The power efficiency of inductive power transfer will be higher for a higher coupling coefficient between the loops [2]. The coupling coefficient is defined as $\kappa = M/L$, where L is the inductance of the loops and M is the mutual inductance between them [1]. Therefore, the configuration capable of higher M will potentially provide higher power efficiencies [1]. Of the two the axial configuration can provide better coupling due to two reasons: first, the loops can be placed closer to each other than in the planar configuration (where the minimum distance is at least the diameter of the loop) [1]; second, the magnetic field lines are directed vertically on the surface of a loop [1]. Both reasons lead to a potentially higher flux produced by one loop crossing the surface of the other one, and therefore, to a higher mutual inductance [2].

f) (bookwork) Since we deal with a static situation, $d/dt = 0$, and Maxwell's equations separate into two independent sets, for electric and magnetic fields [2]. The equations for electric charges are of the form

$$\oint(\mathbf{E}\cdot d\mathbf{l}) = 0 \quad \oint(\mathbf{D}\cdot d\mathbf{S}) = q \qquad [1]$$

The equations for the magnetic fields do not contain electric charges

$$\oint(\mathbf{B}\cdot d\mathbf{S}) = 0 \quad \oint(\mathbf{H}\cdot d\mathbf{l}) = I \qquad [1]$$

Therefore, static charges create electric fields but do not create magnetic fields [1].

Question 2. [a) = 5], [b) = 5], [c) = 5], [d) = 7], [e) = 8]

a) (calculated problem) Since we assume that the field between the plates in the same as that between two infinitely large planes, we can state that the field lines are horizontal and the field strength is the same on a plane parallel to the plates [1]. The total field in the the sum of the fields created by each plate [1]. Applying Gauss's law to a single plate, we find that the field is constant [1] and equal to $q/(2\varepsilon_0 S)$, where q is the charge of a plate. The total field between the plates is twice stronger [1]. This implies that the potential is varying linearly between the plates, so that the field strength is equal to $E = U/d$ [1].

b) By assigning zero potential to the negatively charged plate the plate we find [2]

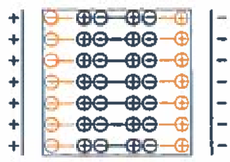
$$\varphi = \int_0^x E dx = \frac{Ux}{d} \qquad [3]$$

where x is calculated from the plate with the zero potential.

c) By definition $C = q/U$ [1]. On the other hand, from a) we have $q = \varepsilon_0 SU/d$ [1], so that

$$C = \frac{\varepsilon_0 S}{d} \qquad [3]$$

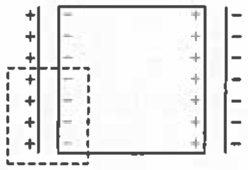
d) (bookwork) When a uniform dielectric is inserted between the capacitor plates, its atoms will polarise as shown in the figure below [3]. Inside the dielectric the positive and negative charges due to neigh-



bouring dipoles compensate each other [2]. However, on the dielectric surfaces, there are uncompensated positive a negative charges [1]. These will have a constant charge density σ_{bound} [1].

e) (bookwork) One difference between the free charges on the capacitor plates and the bound charges on the dielectric surface is that the bound charges appear due to the existence of the free ones, but not the other way round [1]. In other words, removing the dielectric from the capacitor plates will not remove the free charges, whereas removing the capacitor will make bound charges disappear [1]. Also, bound charges cannot move away inside the dielectric and will not lead to a conduction current flowing it. On the other hand, free charges can move inside the conductor [1].

Let us choose a cylindrical surface that encloses both a capacitor plate and a surface of the dielectric, as shown in the Figure below.



The surface encloses both free and bound charges. The electric field lines are horizontal, and so Gauss's law $\oint(\mathbf{D}\cdot\mathrm{d}\mathbf{S}) = q$ takes the form

$$\varepsilon_0 E = \sigma_{\text{free}} - \sigma_{\text{bound}} \quad [2]$$

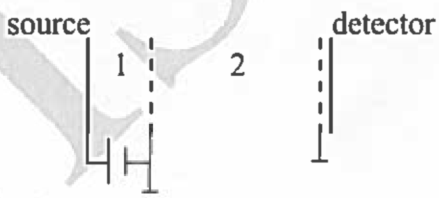
However, $\sigma_{\text{bound}} = \beta E$ leading to

$$\varepsilon_0\left(1+\frac{\beta}{\varepsilon_0}\right)E = \sigma_{\text{free}}$$

which allows us to introduce the relative permittivity of the dielectric as

$$\varepsilon_{\text{d}} = 1 + \frac{\beta}{\varepsilon_0} \quad [3]$$

Question 3. [a) = 10], [b) = 5], [c) = 10], [d)=5]



a) (bookwork) Ions first enter region 1. A uniform electric field exists in this region, created by the dc voltage source [1]. So, the function of this region is to accelerate ions [2], and we call it the acceleration region [1]. No field exists in region 2 [1]. Ions exiting the acceleration region will then drift in region 2 with a constant velocity [2], and we can call it the drift region [1]. The drift time in this region will be proportional to the velocity, which itself will be proportional to the charge-to-mass ratio of the ions. Measuring the time-of-flight allows determining the charge-to-mass ratio [2].

b) (bookwork) For region 1 (acceleration), where a uniform field exists, we have

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = qE \quad [3]$$

For region 2, where no field exists, we have

$$m\frac{d^2x}{dt^2} = 0 \quad [2]$$

c) (bookwork) The two equations from the previous answer describe ion movement in vacuum, without collisions with neutral molecules [1]. When such collisions are present, the equation of motion can be modified as

$$\frac{dv}{dt} + \frac{v}{\tau} = \frac{q}{m}E \quad [3]$$

where v is the ion velocity and τ is the collision time. The second term in the left-hand side represents the collisions, and at atmospheric pressure in an ion-mobility spectrometer, these will dominate [2], so that the first term on the left-hand side can be neglected [2]. As a result, the equation for ion motion in an ion mobility spectrometer can be written as

$$\frac{v}{\tau} = \frac{q}{m}E \quad \text{or} \quad v = KE \quad [2]$$

where K is the ion mobility. One consequence is that the ion drift time is proportional to the field strength in an ion mobility spectrometer, whereas it is not in a mass-spectrometer.

d) (reasoning/assimilation of bookwork) Collisions of ions with neutrals is a random process and will lead to ion diffusion [2]. As a result, an initially narrow ion pulse entering an ion-mobility spectrometer will spread as the ions drift inside the spectrometer. Diffusion will make it difficult to separate ions with different values of mobility, limiting the resolution [3].