EE1-06

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE/EIE PART I: MEng, BEng and ACGI

INTRODUCTION TO SIGNALS AND COMMUNICATIONS

Corrected Copy

Tuesday, 26 May 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : K.K. Leung
Second Marker(s) : C. Papavassiliou

Special Instructions for Invigilator: **None**

Information for Students:

Some Fourier Transforms

$\cos \omega_o t$ <=> $\pi[\delta(\omega - \omega_o) + \delta(\omega + \omega_o)]$

Some useful trigonometric identities

$$\cos x \cos y = \tfrac{1}{2}\cos(x-y) + \tfrac{1}{2}\cos(x+y)$$
$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$
$$a\cos x + b\sin x = c\cos(x+\theta)$$

where $c = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}(-b/a)$

Complex exponential

$$e^{jx} = \cos x + j\sin x$$

1. This is a general question. (40%)

a. Consider a rectangular pulse defined by $f(t)=1$ for $-a<t<a$ and $f(t)=0$ otherwise where a is a positive constant.

i. Derive the Fourier transform $F(\omega)$ of $f(t)$. [2]

ii. Sketch the frequency spectrum of $f(t)$. [2]

iii. If $f(t)$ is the unit impulse response of a linear time-invariant system, can the system be realizable in practice and why? [2]

b. Consider the exponential Fourier series for a periodic signal $g(t)$ with period T_0:

$$g(t)=\sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}.$$

i. Express ω_0 in terms of T_0. [1]

ii. Prove that different signal components, $e^{jm\omega_0 t}$ and $e^{jn\omega_0 t}$ for $m \neq n$, are orthogonal to each other. [3]

iii. What is the integral value of the product of $e^{jn\omega_0 t}$ and $e^{-jn\omega_0 t}$ for all $n=-\infty$ to ∞ over the period T_0? [1]

iv. Using results in parts ii and iii, derive an expression for D_n as an integral of $g(t)$ and $e^{-jn\omega_0 t}$ by the following steps: Multiply both sides of the Fourier series for $g(t)$ given above with $e^{-jm\omega_0 t}$ and integrate both sides over the period T_0. [3]

v. In the frequency domain, what does each signal component $e^{jn\omega_0 t}$ represent? [2]

vi. Give a physical interpretation of the Fourier series coefficients D_n's. [2]

c. Consider the full amplitude modulation (AM) signal with its waveform denoted by $\phi_{AM}(t)$. Let $m(t)$ be the modulating signal at the baseband, ω_c be the carrier angular frequency in radians/second, and A be the amplitude of the carrier.

i. Give an expression for $\phi_{AM}(t)$. [2]

ii. What does the AM do on the modulating signal $m(t)$ from the frequency-domain perspective and why? [2]

iii. If $m(t)$ has a bandwidth of B Hz, what is the bandwidth for the AM signal? [2]

iv. What is the power P_c of the carrier component of the AM signal? [2]

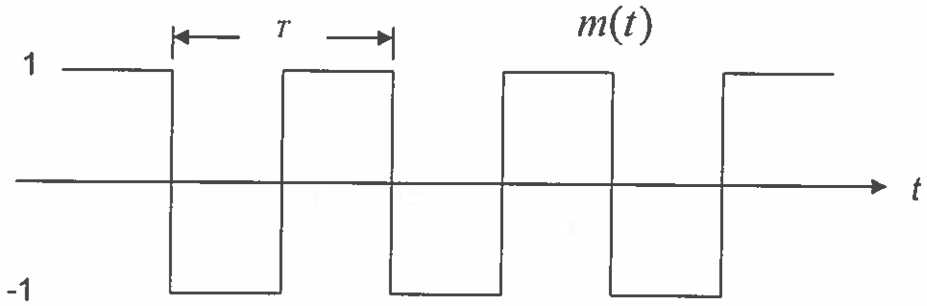
v. Express the power P_s of the sideband signal in terms of

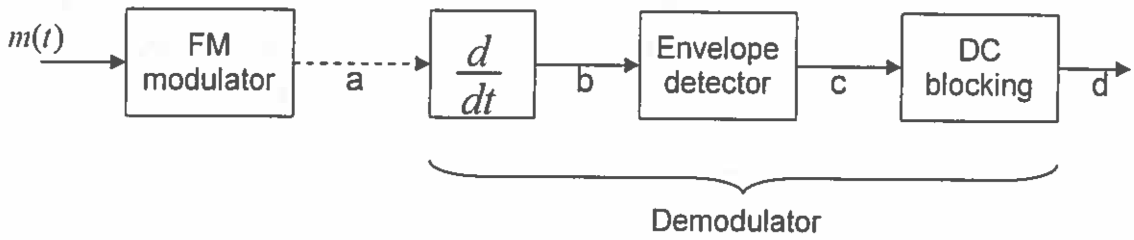
$$E[m^2(t)] \equiv \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} m^2(t)dt.$$ [2]

vi. Obtain the power efficiency of the AM signal. [2]

1. This is a general question. (Continued)

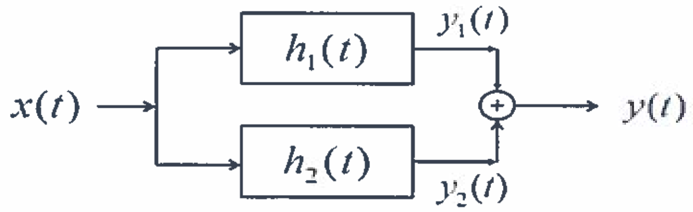
   d. The following diagrams present a periodic square wave $m(t)$ and a block diagram for the frequency modulation (FM) and demodulation process, respectively. The periodic square wave $m(t)$ modulates the frequency of a sinusoidal carrier with the center frequency of $f_c = 200$ kHz, producing a resultant frequency deviation of $\Delta f = 20$ kHz. The carrier amplitude is assumed to be A. Sketch the waveforms at points a, b, c and d and indicate the corresponding signal frequency where appropriate. [10]





2. Signals and their transforms. (30%)

   a. Consider a linear time-invariant (LTI) system, which consists of two LTI sub-systems 1 and 2, as shown in the following diagram. The same input signal, $x(t) = e^{-t}$ for $t \ge 0$ and $x(t) = 0$ for $t < 0$, is applied to both sub-systems. The unit impulse response function and the output of the sub-system i are denoted by $h_i(t)$ and $y_i(t)$, respectively, for $i = 1$ and 2. The output of the entire system $y(t)$ is equal to $y_1(t) + y_2(t)$. Furthermore, let $h_i(t) = \delta(t - T_i)$ for sub-system $i = 1$ and 2 where T_1 and T_2 are positive constants with $T_2 > T_1$.



      i. Express $y_1(t)$ as a convolution integral of $x(t)$ and $h_1(t)$. [3]
      ii. Use result in part i to obtain a closed-form expression for $y_1(t)$ and sketch $y_1(t)$. [4]
      iii. Give a physical interpretation of the effects of $h_1(t)$ and $h_2(t)$ on the input $x(t)$. [2]
      iv. Let the unit impulse response function for the entire system be denoted by $h(t)$. Express $h(t)$ in terms of $h_1(t)$ and $h_2(t)$. [2]
      v. Let $X(\omega)$, $Y(\omega)$, $H_1(\omega)$ and $H_2(\omega)$ be the Fourier transform of $x(t)$, $y(t)$, $h_1(t)$ and $h_2(t)$, respectively. Obtain an expression of $Y(\omega)$ in terms of $X(\omega)$, $H_1(\omega)$ and $H_2(\omega)$. [4]

   b. Consider a signal $f(t)$ and its Fourier transform $F(\omega)$. For convenience, let us use $\Im[g(t)]$ to denote the Fourier transform of any given function $g(t)$.

      i. From the definition of Fourier transform, show that $\Im[f(t-T)] = F(\omega)e^{-j\omega T}$ where T is a positive constant. [4]
      ii. From the result in part i, does the magnitude of the signal spectrum change when the signal is shifted in time? What is changed in the transform due to the time shift? [2]
      iii. For a positive constant a, show that $\Im[f(at)] = \frac{1}{a}F(\frac{\omega}{a})$. [4]
      iv. If $a > 1$, what is the physical interpretation of the transform result in part iii? What is the corresponding interpretation for $0 < a < 1$? [3]
      v. Based on results in parts iii and iv, what can be said about the Fourier transform of a signal that changes rapidly in time, when compared with that of another signal that varies very slowly in time? [2]

3. Communications techniques. (30%)

a. Design a wide-band frequency modulation (WBFM) system with the final carrier frequency $f_c = 100$ MHz and the maximum frequency deviation $\Delta f = 61.44$ kHz as follows. Assume that a narrow-band FM (NBFM) signal is available and given by

$$\phi(t) = A\cos[2\pi f_{NB} t + k_f \int m(u)du]$$

where A is the carrier amplitude, $f_{NB} = 200$ kHz is the narrow-band carrier frequency, k_f is a proportionality constant and $m(t)$ is the modulating signal. Let the maximum frequency deviation of this NBFM signal be $\Delta f_{NB} = 30$ Hz, which is the maximum of $k_f \int m(u)du$ over all time t. Assume that a non-linear device is available to take an input signal of $x(t)$ and produce an output of $y(t)$ according to the relationship of $y(t) = a\,x(t) + b\,x^2(t)$ where a and b are constants. A frequency converter is also available, which can move a signal in a frequency band centered at f_c to another frequency at f_c' in the frequency domain.

i. Show that when $\phi(t)$ is input to the non-linear device, another FM signal that doubles the carrier frequency and the frequency deviation of $\phi(t)$ can be obtained from the output $y(t)$. (As a result, the device is referred to as the *frequency multiplier* below.) [4]

ii. Using such frequency multipliers and only one frequency converter as building blocks, devise and draw a block diagram for the specified WBFM system. Explain how the frequency multipliers and converter can achieve the design specifications. [8]

iii. What is the oscillator frequency required by the frequency converter? Provide a mathematical justification for the selected oscillator frequency. [6]

b. Consider a noiseless communication channel with a bandwidth of B Hz, which is used to transmit periodic signal samples at a rate of f_S samples per second.

i. What is the maximum value of f_S? [2]

ii. Draw frequency spectrum diagram(s) to explain your result in part i. [7]

iii. If each sample is represented by one of M signal levels (where M is a positive integer), what is the maximum bit rate the channel can support? [3]