EE4-05
EE9-SO7
EE9-FPN2-01

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

MSc and EEE PART IV: MEng and ACGI

TRAFFIC THEORY & QUEUEING SYSTEMS

Corrected copy

Monday, 21 May 10:00 am

Time allowed: 3:00 hours

Page 3: Fig 1.1
and
Page 7: Fig 4.1

Arrows are very tiny
can hardly ~~see~~ see them
on diagrams

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : J.A. Barria
Second Marker(s) : D.P. Mandic

Special information for students

1. Erlang Loss formula recursive evaluation:

$$E_N(\rho) = \frac{\rho E_{N-1}(\rho)}{N + \rho E_{N-1}(\rho)}$$

$$E_0(\rho) = 1$$

2. Engset Loss formula recursive evaluation (for a fixed M and $p = \alpha/1+\alpha$):

$$e_N = \frac{(M-N}{N+(M-}$$

$$e_0 = 1$$

$$\alpha = \lambda/\mu$$

Hand it out. Point 5 is missing from exam paper

3. Traffic capacity on basis of Erlang B formula (r

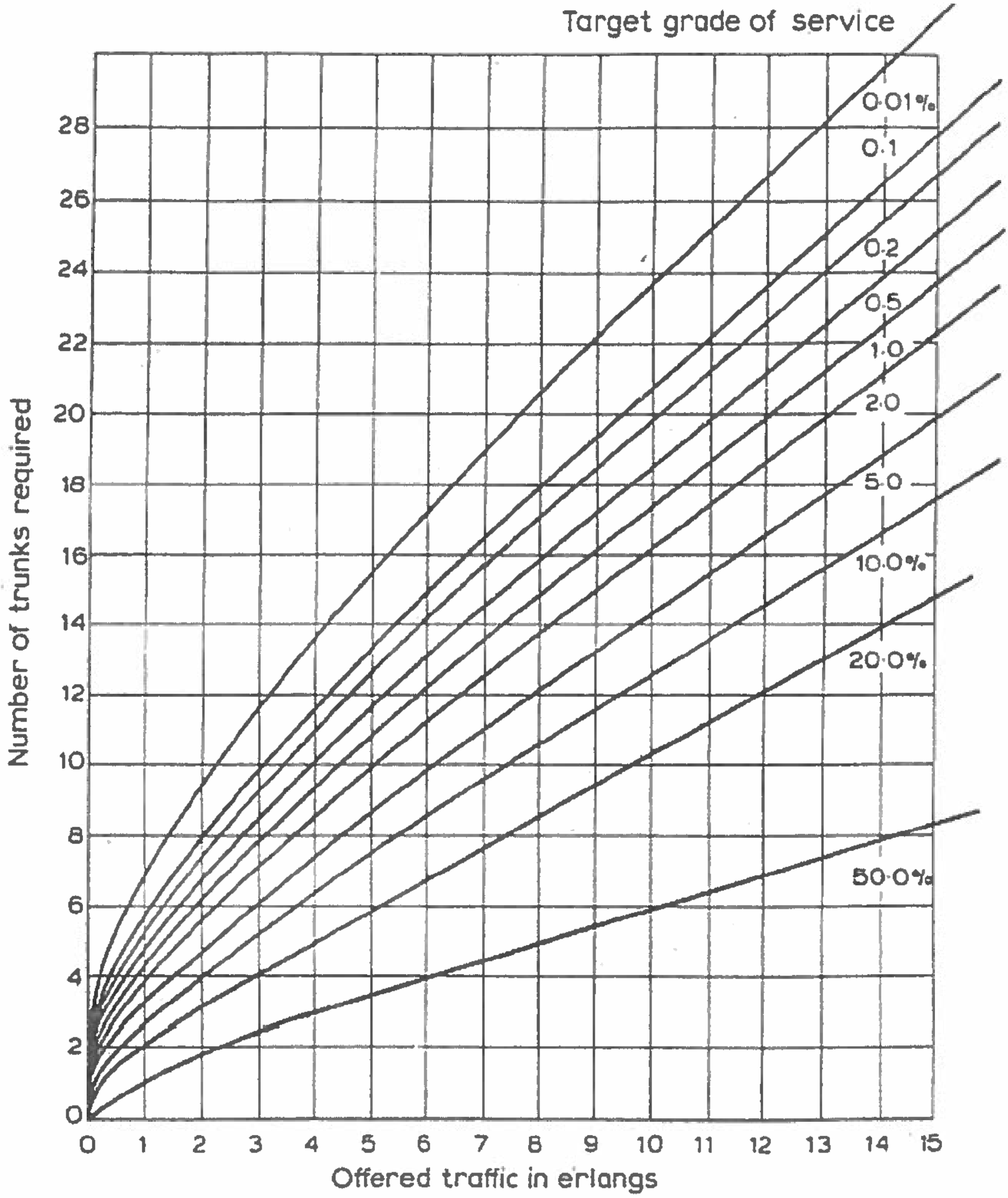
4. Expected residual time

$$E[R] = \frac{1}{2}\lambda E[S^2] = \frac{1}{2}\sum_{k=1}^{m}\lambda_k E[S_k^2]$$

5. Uniform distribution, $unif(a,b)$, where $b = X_{max}$:

$$E[X] = \frac{a+b}{2}$$

$$Variance[X] = \frac{1}{12}(b-a)^2$$



Traffic capacity on basis of Erlang B formula.

Special information for students

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$$e_N = \frac{(M-N+1)\alpha e_{N-1}}{N+(M-N+1)\alpha e_{N-1}}$$
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3. Traffic capacity on basis of Erlang B formula (next page).

4. Expected residual time

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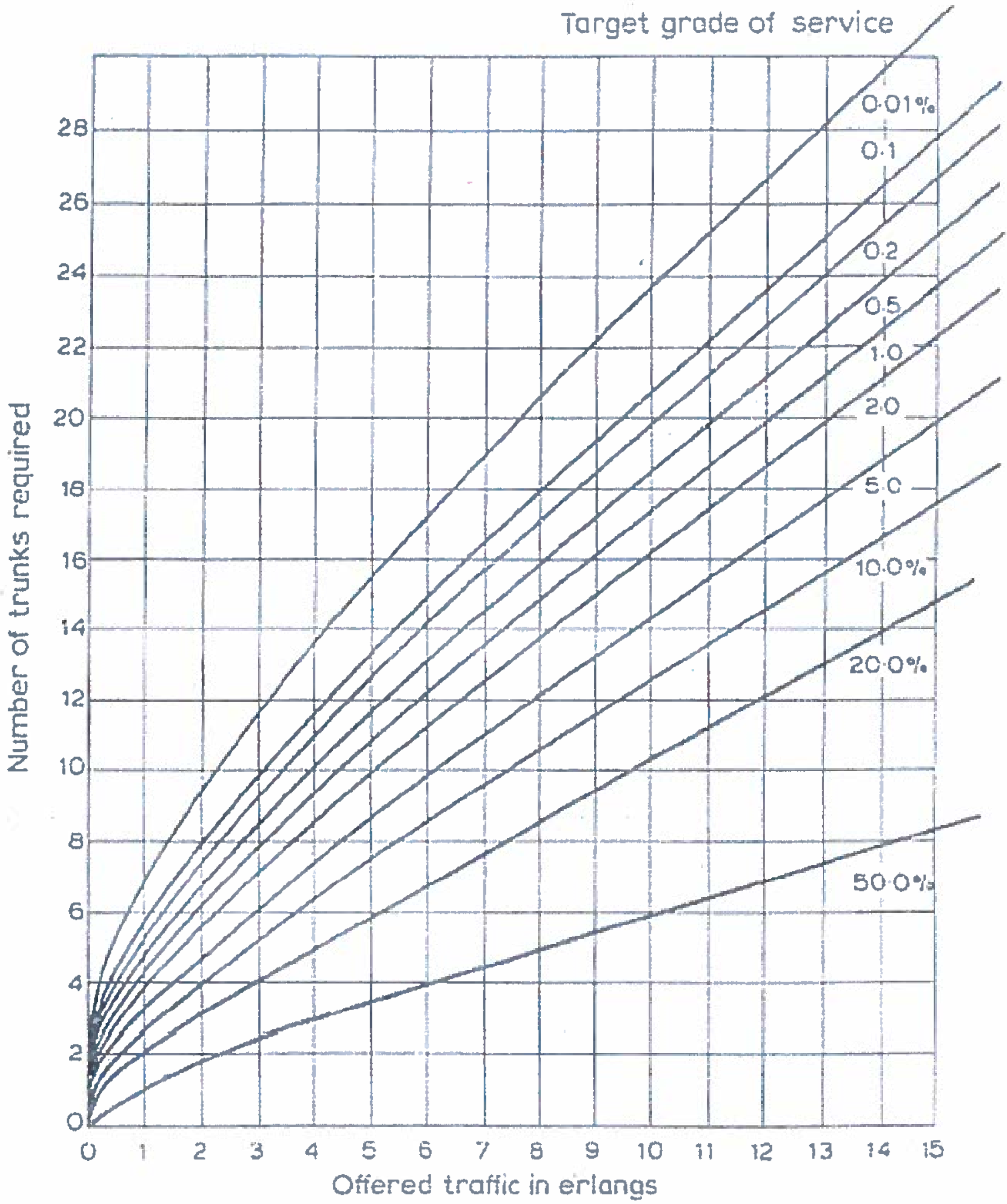

Traffic capacity on basis of Erlang B formula.

The Questions

1.

a) Traffic from M independent sources, is offered to an N channel communication link.

Each source behaves as represented by the Markov chain represented in ***Fig. 1.1.***

Assuming that $N \geq M$:

i) Show that the probability distribution of the number of busy channels is given by a binomial probability with parameters M and p. [6]

ii) Derive explicitly p as a function of the system parameters λ, μ, M and N. [4]

b) Two type of calls are being offered to a link with N channels. Both type of calls can be represented as pure chance traffic with parameters (λ_i, μ_i) for $i = 1,2$.

Note: When there are three type 2 calls using the link no type 1 call can be accepted. Also note that when there are six type 1 calls using the link no type 2 calls can be accepted.

i) Define the state space of the system and set up the B/D model for the traffic on the link. [4]

ii) Show how to derive the blocking probability of type 1 and type 2 calls using the B/D model derived in i). [6]

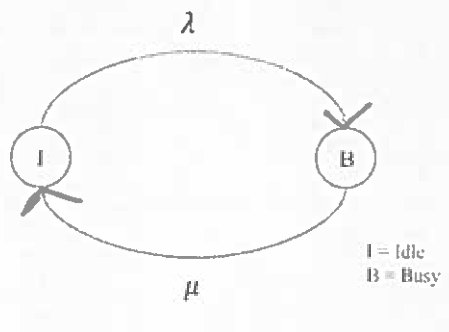

Figure 1.1.

2.

a) A Poisson stream of messages is offered to a communication link with a large buffer at a rate of 27000 messages per minute. The message lengths are uniformly distributed from 1 packet to 100 packets and the link can transmit at a maximum rate of 2.048 Mbits/s.

i) If the system operates with a first in first out ($FIFO$) queue discipline, and all the packets are of length 80 bits, determine the mean message waiting time of the system. [7]

ii) Assume now that the buffer discipline is changed to a shortest job first. Discuss how the mean waiting time for a 1-packet message (top priority) and the mean waiting time for a 100-packet message (lowest priority) will be affected. [3]

b) For an $M/M/K$ queuing system operating with a $FIFO$ queue discipline, the arrivals could be served immediately if any one server is idle. If all servers are busy, arrivals could join a waiting buffer.

i) Derive the queue length distribution seen by arrivals which find all K servers busy.

Note: Starting from the balance equations show explicitly all the steps of your derivation. [4]

ii) For the arrivals that find all K servers busy derive the waiting time distribution. [6]

3.

a) For the $M/M/K/N$ queuing system ($N = K + B$) the set of local balance equations is given by:

$$\pi_i = \left(\frac{A^i}{i!}\right)\pi_0 \text{ for } 0 \le i \le K$$
$$= \left(\frac{A^K}{K!}\right)\rho^{i-K}\pi_0 \text{ for } K \le i \le K + B$$
$$\pi_0 = \frac{1}{(A^K/K!)}\left[\frac{(1-\rho)E_K(A)}{(1-\rho) + \rho(1-\rho^B)E_K(A)}\right]$$

For a system with $K = 2$ and $N = 4$, the messages arrive at a rate of 40 messages/s and each server can process messages at a rate of 40 messages per second.

Assume also that you know that the probability of the messages being blocked is 0.0435.

i) Derive the mean queue length for arrivals that find K servers busy. [4]

ii) Derive the mean waiting time for messages that have to wait in the buffer (before transmission). [5]

3.

b) A system is composed of two processors which are working in micro-synchronism. When both processors are up and running, the normalised processing capability is 16.0 jobs per second. When only one processor is up and running only essential real-time jobs are performed and hence the system normalised processing capability is 10.0 jobs per second.

Assuming that:

- The processors work independently and each one has a failure rate of 1 failures in five hours.

- If one processor is out-of-service it can be repaired at a rate of 4 repairs per hour.

- If the system is down (both processors are out-of-service), the complete system can be replaced at a rate of 9 replacements per hour.

The system is required to operate under the following specifications/constraints:

- System availability $>$ 0.999

- Long term average number of jobs per second $\geq$ 15.0.

i) Define the state space of the system and its underlying Markov chain. [3]

ii) Does the system comply with the specification of its steady state availability? [4]

iii) Will the system be able to complete on average 15.0 jobs per second? [4]

4.

a) Using the single voice source model shown in *Fig. 4.1.*:

i) Construct a Markov modulated Poisson process model for an N-multiplexer voice source model. [4]

ii) Derive the steady state arrival rate. [2]

iii) If the N-multiplexer voice source model is fed into a server with capacity $1/\nu$ cells/s, state the condition for the system to be stable. [4]

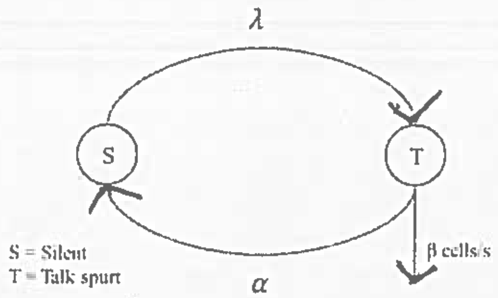

Figure 4.1

b) In the context of admission control, we are interested in finding the maximum number of sources N that could be multiplexed on a link with capacity C_L. Considering the one source model behaviour shown in Fig. 4.2, where $1/\alpha$ is the duration of the mean silence period and $1/\beta$ is the average duration of the period in which the sources are offering cells to the multiplexer. Assuming exponential holding times in both (ON and OFF) states:

i) For N-multiplexed sources derive the average rate of active sources. For the same N-multiplexed sources derive the variance σ^2 of the number of sources. [6]

ii) Derive an expression for C_L as a function of σ and R_p (the peak rate in cells per second) and the number of multiplexed sources N. [4]

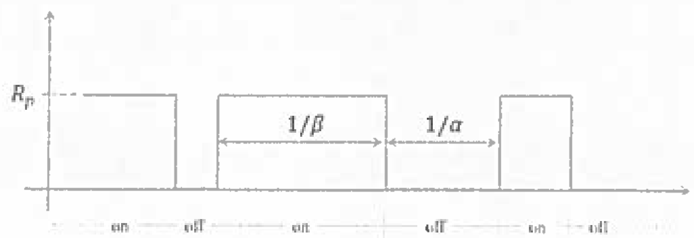

Figure 4.2