EE4-24
EE9-CS1-3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected copy

DISCRETE-TIME SYSTEMS AND COMPUTER CONTROL

Friday, 20 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	A. Astolfi	[redacted]
	Second Marker(s) :	E.C. Kerrigan	

DTS AND COMPUTER CONTROL

Information for candidates:

- $Z\left(\frac{1}{s}\right) = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$

- $Z\left(\frac{1}{s+a}\right) = \frac{z}{z-e^{-aT}} = \frac{1}{1-z^{-1}e^{-aT}}$

- $Z\left(\frac{1}{s^2}\right) = T\frac{z}{(z-1)^2} = T\frac{z^{-1}}{(1-z^{-1})^2}$

- $Z\left(\frac{1}{s^3}\right) = \frac{T^2}{2}\frac{z(z+1)}{(z-1)^3} = \frac{T^2}{2}\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$

- $Z\left(\frac{b}{(s+a)^2+b^2}\right) = \frac{ze^{-aT}\sin bT}{z^2-2ze^{-aT}\cos bT+e^{-2aT}}$

- Transfer function of the ZOH: $H_0(s) = \frac{1-e^{-sT}}{s}$

- Definition of the w-plane: $z = \frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}$, $w = \frac{2}{T}\frac{z-1}{z+1}$

- Tustin transformation: $s = \frac{2}{T}\frac{z-1}{z+1}$

- Forward Euler: $s = \frac{z-1}{T}$

- Note that, for a given signal r, or $r(t)$, $R(z)$ denotes its Z-transform.

1. Consider the digital control system in Figure 1.

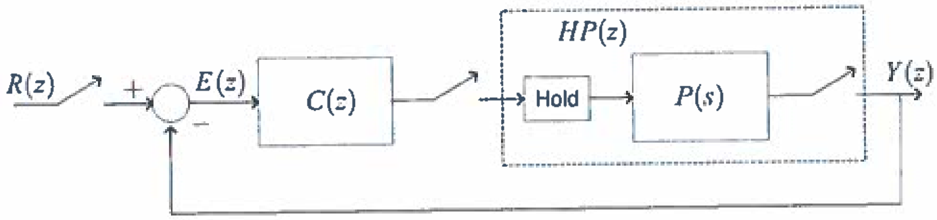

Figure 1: Block diagram for question 1.

Let

$$P(s) = \frac{1}{s\,(s+2)}.$$

Assume the hold is a ZOH.

a) Suppose that the plant $P(s)$ is controlled using the controller

$$C(s) = 20\,\frac{s+2}{s+6},$$

in a unity feedback configuration. Show that the continuous-time closed-loop system is asymptotically stable. Compute the natural angular frequency of the closed-loop system and the angular frequency of the damped oscillations. Determine the period T_d of the damped oscillations.

(Hint: the characteristic polynomial of the closed-loop system is of the form $s^2 + 2\xi\omega_n\, s + \omega_n^2$, in which ω_n is the natural angular frequency and ξ is the damping coefficient. The angular frequency of the damped oscillations is $\omega_d = \omega_n\sqrt{1-\xi^2}$.) [3 marks]

b) Suppose that the controller has to be implemented in digital form. Select a sampling time T to give approximately eight samples per period T_d. (The sampling time should be a multiple of $0.1s$.) [1 mark]

c) Show that the controller $C(s)$ stabilizes the continuous-time closed-loop system in which the plant $P(s)$ has been augmented with a term approximating the effect of the hold. [4 marks]

d) Compute the equivalent discrete-time model $HP(z)$ for the plant interconnected to the hold and the sampler. [4 marks]

e) Discretize the controller $C(s)$ in part a) using the pole-zero correspondence method. Compute explicitly the resulting discrete-time controller. [2 marks]

f) Using the results of parts d) and e) compute the closed-loop transfer function from the input $R(z)$ to the output $Y(z)$ and study its stability properties. [6 marks]

2. Consider the digital control system in Figure 2.

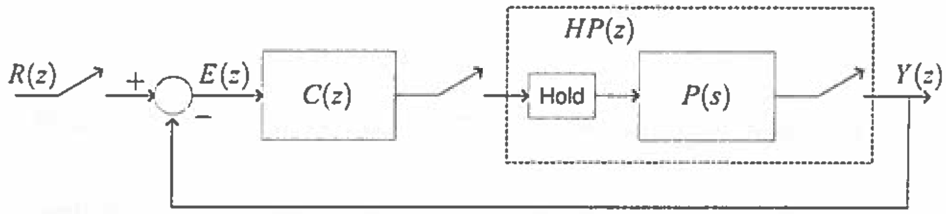

Figure 2: Block diagram for question 2.

Let

$$P(s) = \frac{1}{s\,(10s+1)}.$$

Assume the hold is a ZOH and let the sampling time be $T = 1$. The controller is described by the equation

$$u(kT) = -\frac{1}{2}u((k-1)T) + K\Big(e(kT) - 0.88e((k-1)T)\Big),$$

with $K > 0$ to be determined.

a) Compute the equivalent discrete-time model $HP(z)$ for the plant interconnected to the hold and the sampler. [4 marks]

b) Find the transfer function $C(z)$ of the controller. [4 marks]

c) Determine for which values of $K > 0$ the closed-loop system is asymptotically stable. [6 marks]

d) Determine for which value of K the closed-loop system has a pole at $z = 0$. For this value of K compute all poles of the closed-loop system and identify the slowest mode of the system. [6 marks]

3. Consider a process to be controlled with transfer function

$$P(s) = \frac{1}{s(s+1)}.$$

Assume the system is interconnected to a ZOH and a sampler. Let $T = 0.2$ be the sampling time.

a) Compute the equivalent discrete-time model $HP(z)$ for the plant interconnected to the hold and the sampler. [4 marks]

b) Using the definition of the w-plane determine the transfer function $HP(w)$. [4 marks]

c) Let

$$C(w) = k \frac{1+\frac{w}{a}}{1+\frac{w}{3}},$$

with $k > 0$ and $a > 0$ parameters to be determined. Consider the closed-loop system resulting from the unity feedback interconnection of the controller $C(w)$ with the transfer function $HP(w)$.

i) Determine the characteristic polynomial of the closed-loop system and select numerical values for k and a such that the system has velocity constant $K_v = 2$ and the closed-loop system is asymptotically stable. (Hint: recall that, in the w plane, the velocity constant is given by $K_v = \lim_{w \to 0} w\, C(w)\, HP(w)$.) [6 marks]

ii) Using the numerical values determined in part c.i) compute the discrete-time controller $C(z)$ and study the stability properties of the resulting discrete-time closed-loop system. [6 marks]

4. Consider the digital control system in Figure 4.

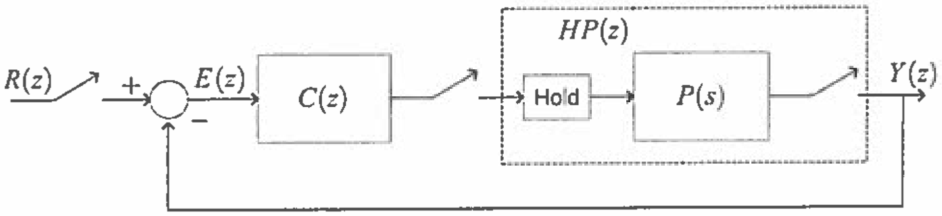

Figure 4: Block diagram for question 4.

Let

$$P(s) = \frac{1}{s-1}.$$

Assume the hold is a ZOH and the sampling time is $T = 1$.

a) Compute the equivalent discrete-time model $HP(z)$ for the plant interconnected to the hold and the sampler. [4 marks]

b) Design a discrete-time controller $C(z)$ such that the closed-loop transfer function from the input $R(z)$ to the output $Y(z)$ has all poles at $z = 0$.
(Hint: design a *minimum order* controller.) [4 marks]

c) Design a discrete-time controller $C(z)$ such that the closed-loop transfer function from the input $R(z)$ to the output $Y(z)$ has all poles at $z = 0$ and the system is of Type 1.
(Hint: design a *minimum order* controller.) [6 marks]

d) Consider the controller $C(z)$ determined in part c) and suppose it has to be implemented with integer numbers, that is define an approximation $C_a(z)$ of the controller in part c) in which the coefficients are selected as the integers part of the coefficients of $C(z)$. (To give some examples, the integer part of 1.765 is 1, the integer part of -2.1345 is -2, the integer part of 12.001 is 12.) Study the stability properties of the resulting closed-loop system and briefly discuss the effect of numerical approximations in discrete-time design. [6 marks]