UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2001

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
BSc Honours Degree in Mathematics and Computer Science Part II
MSci Honours Degree in Mathematics and Computer Science Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C240=MC240

ALGORITHMS, COMPLEXITY AND COMPUTABILITY

Tuesday 1 May 2001, 16:00
Duration: 90 minutes
(Reading time 5 minutes)

Answer THREE questions

Paper contains 4 questions
Calculators not required

Section A *(Use a separate answer book for this Section)*

1 In this question you are asked to design a *Turing Machine*, (TM) to sort sequences of values.

a Design a 2-tape TM, S, with input alphabet I = {a, b, c, d, e, f, *}, which takes input of the form w*z, where $* \notin$ w and $* \notin$ z, and w is a permutation of {a, b, c, d, e, f}, giving the sort order of these symbols;
z is the sequence of symbols to be sorted, such that the input/output function of S is: f_S(w*z) = y, where y contains the symbols of z ordered according to w.
For example

f_S (bcdeaf*beadfeed) = bddeeeaf.

Give

i) a state diagram of your TM, S, with an explanation of your notation

ii) a brief written description of how your TM works

b Explain what is meant by the *time function* of a TM.

c Derive an expression for the time function for S, the TM given in your answer to part a.

The three parts carry, respectively, 65%, 10%, 25% of the marks.

2a i) Describe the *Universal Turing Machine*, U

ii) Give the input-output function of U

iii) Explain the operation of U

b Assume S is any standard Turing machine and w is any word of C (the typewriter alphabet).

Given the standard Turing machines DUP, EDIT, with input-output functions $f_{DUP}(w) = w*w$, $f_{EDIT}(code(S)*w) = code(S[w])$ respectively, the standard Turing machine ODD which selects the odd-indexed symbols in its input, (so that, for example, f_{ODD} (imperial)=ipra), and the Universal Turing Machine, U:

Evaluate

i) $f_U(f_U(code(EDIT)*code(ODD)*abracadabra)*sesame)$

ii) $f_U(f_U(f_U(f_U(code(DUP)*code(EDIT))*code(ODD)*london)*paris)*berlin)$.

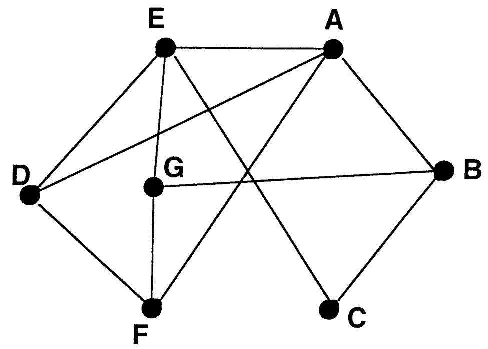
c i) Define the *Halting Problem*.What does it mean to say that the halting problem is not computable? Describe briefly how this result may be proved.

ii) Explain why the result mentioned in part c ii) is important in the solution of other problems.

The three parts carry, respectively, 50%, 20%, 30% of the marks.

Section B *(Use a separate answer book for this Section)*

3a Starting at node A and using breadth-first or depth-first search, find a spanning tree of the following graph. State which search strategy you use, and give a diagram showing the edges of the tree.



b For n>1, how many edges do the following have?

i) A Hamiltonian circuit of a graph with n nodes

ii) A minimal spanning tree of a connected weighted graph with n nodes

iii) A complete graph with n nodes

Briefly justify your answer in each case.

c Let n be a positive integer. Give a formula for the maximum number of edges that a graph with n nodes and two connected components can have. Show all your working.

The three parts carry, respectively, 30%, 40%, 30% of the marks.

4 In this question, A and B denote arbitrary yes-no problems. The *complementary problem* of A (in symbols, co-A), is the yes-no problem whose yes instances are the no-instances of A, and whose no-instances are the yes-instances of A.

a i) Define the classes P and NP of yes-no problems

ii) Explain what is meant by the statement "A reduces to B in polynomial time" a (in symbols, A≤B)

b Show that if A is in P then so is co-A

c Explain why the method of part b does not prove that if A is in NP then so is co-A

d Show that if A≤B then co-A≤co-B

The four parts carry, respectively, 35%, 25%, 25%, 15% of the marks.