Master copy - Aug. 08

[3.14]

SOLUTIONS - 2008

A07

**EE 3.14 Power Electronics and Machines**

## Information for Students

### Constants:

Boltzmann constant: $1.38\times10^{-23}$ m$^2$ kg s$^{-2}$ K$^{-1}$
Charge on the Electron: $1.6\times10^{-19}$ C

1. A switch-mode power supply (SMPS) is to be designed to provide the high voltage supply of an LCD backlight for a battery operated piece of equipment. The battery voltage is 10 V and the backlight requires 100 V (the polarity of which is not important). Two approaches are to be compared: a buck-boost circuit and a fly-back circuit. Both circuits are to be operated in discontinuous mode.

   a) For the buck-boost circuit an inductor of 20 μH is to be used and the operating frequency will be 20 kHz.

      i) Show that the energy delivered per switch cycle is:

$$E = \frac{(V_I t_{on})^2}{2L}$$ [3]

In discontinuous mode, all of the energy stored in the inductor at the end of the on period is transferred to the output.

$$E = \tfrac{1}{2} L i^2$$

$$\frac{di}{dt} = \frac{V_I}{L}$$

$$i = \frac{V_I}{L} t_{on}$$

$$E = \frac{(V_I t_{on})^2}{2L}$$

      ii) Calculate the duty-cycle required to supply 0.05A to the backlight at the voltage of 100 V. [4]

Power transfer will be the energy transfer per cycle divided by the period.

$$P = V_O I_O = 100 \times 0.05 = 5\,W$$

$$P = \frac{1}{T} \frac{(V_I t_{on})^2}{2L} = \frac{(V_I \delta)^2}{2 f L}$$

$$\delta = \frac{(2 f L P_O)^{\frac{1}{2}}}{V_I}$$

$$= \frac{(2 \times 2 \times 10^4 \times 2 \times 10^{-5} \times 5)^{\frac{1}{2}}}{10} = \frac{4^{\frac{1}{2}}}{10} = 0.2$$

      iii) Confirm whether the circuit will be in discontinuous operation. [3]

Check that diode conduction time is less than off time. Diode conduction time set by ramp down of current.

$$\hat{i} = \frac{V_I}{L}\frac{\delta}{f}$$
$$\left.\frac{di}{dt}\right|_{Off} = \frac{V_O}{L}$$
$$t_{diode} = \frac{\hat{i}L}{V_O} = \frac{V_I}{V_O}\frac{\delta}{f} = \frac{10}{100} \times \frac{0.2}{2\times10^4} = \frac{2}{2\times10^6} = 1\,\mu s$$
$$\hat{t}_{diode} = \frac{(1-\delta)}{f} = 40\,\mu s$$
$$t_{diode} < \hat{t}_{diode}$$

The diode conduction time is well within the limit for discontinuous conduction
Alternatively, consider the balance of current change:

$$0 = \frac{V_I}{L}t_{On} + \frac{V_O}{L}t_{Diode}$$
$$t_{diode} = t_{On}\frac{V_I}{-V_O} = 10\mu\frac{10}{100} = 1\,\mu s$$
$$\hat{t}_{diode} = \frac{(1-\delta)}{f} = 40\,\mu s$$
$$t_{diode} < \hat{t}_{diode}$$

iv) Calculate the voltage imposed across the transistor during the off state. [1]

$$V_T = V_I - V_O = 10 -^- 100 = 110V$$

b) For the flyback circuit a mutually coupled pair of inductors is to be used where the primary inductance is 20 μH and the turns-ratio is 1:6. The operating frequency will again be 20 kHz.

i) Demonstrate that the circuit should be operated with the same duty-cycle as the buck-boost circuit for a load of 0.05 A at 100 V if discontinuous operation is assumed. [2]

Because the frequency is the same, the energy required per switch cycle is the same. Because the primary inductance is the same, the duty-cycle required will be the same.

ii) Confirm whether the circuit will be in discontinuous operation. [3]

Similar calculation to buck-boost case but current ramp down is in secondary. Secondary inductance is found from turns-ratio squared.

$$\hat{i}_1 = \frac{V_I}{L_1}\frac{\delta}{f}$$

$$\hat{i}_1 N_1 = \hat{i}_2 N_2$$

$$\left.\frac{di_2}{dt}\right|_{Off} = \frac{V_O}{L_2}$$

$$t_{diode} = \frac{\hat{i}_2 L_2}{V_O} = \frac{V_I}{V_O}\frac{N_1}{N_2}\frac{L_2}{L_1}\frac{\delta}{f} = \frac{10}{100}\times\frac{6^2}{6}\times\frac{0.2}{2\times 10^4} = \frac{6}{10^6} = 6\,\mu s$$

$$\hat{t}_{diode} = \frac{(1-\delta)}{f} = 40\,\mu s$$

$$t_{diode} < \hat{t}_{diode}$$

Circuit is still just within discontinuous conduction
Alternatively, consider the balance of flux linkage change.

$$0 = \frac{V_I}{N_1}t_{On} + \frac{V_O}{N_2}t_{Diode}$$

$$t_{diode} = t_{On}\frac{V_I}{-V_O}\frac{N_2}{N_1} = 10\mu\frac{10}{100}6 = 6\,\mu s$$

$$\hat{t}_{diode} = \frac{(1-\delta)}{f} = 40\,\mu s$$

$$t_{diode} < \hat{t}_{diode}$$

iii) Calculate the voltage imposed across the transistor in the off state assuming leakage inductance to be negligible. [2]

$$V_T = V_I - \frac{N_1}{N_2}V_O = 10 -^- \frac{100}{6} = 26.7\,V$$

iv) Discuss the relative merits of this circuit with respect to the buck-boost. [2]

Flyback adds isolation which maybe advantageous in general but not called for here. It also lowers the voltage rating required of the transistor at the expense of a second winding on the inductor core. The reduction in voltage rating will not be as favourable as the calculation above indicates because of the voltage overshoot caused by the leakage inductance.

2.

a) Discuss why EMC is an important factor in the design of power electronic equipment. [4]

Many power converters are directly connected to the mains supply and are therefore the main culprit in emitting EM energy directly into the supply system through a conduction path. This could be in terms of LF harmonics or MF components at multiples of a switching frequency. Often the frequencies are ones at which passive filters would be large and costly so actively designing the system to reduce emissions is sensible. Pulsed voltages and currents (or any waveform with high di/dt or dv/dt) have spectrum with considerable energy at high frequencies and because power converters tend to have such waveforms, power electronics that is not properly designed is liable to fail EMC standards and create real problems in practice.

b) An AC/DC power converter based on a boost SMPS is commonly used instead of an uncontrolled diode rectifier in order to meet EMC standards.

i) Explain why a diode rectifier gives a poor current spectrum [3]

A standard diode rectifier with a smoothing capacitor only applies forward bias to the diodes for brief periods near the peaks of the input voltage. These brief periods of conduction give a spectrum rich in harmonics.

ii) Describe how the AC/DC power converter is controlled to operate as a rectifier with good EMC properties and compare the resulting current spectrum to that of the diode rectifier. [6]

A circuit diagram and a control block diagram are required.
Description should include:

- Two conduction loops are available: one that increase inductor current and one that decreases inductor current
- The inductor current and hence input current can be forced to follow a reference wave-shape using either a conventional control loop and a pulse-width modulator or a hysteresis controller.
- The amplitude of the current reference determines the input power drawn. A nested pair of control loops is used. Outer loop sets current amplitude to achieve the desired DC output and the inner current loop forces sinusoidal current.

Spectrum should consist of a line frequency (desired) term plus (undesired) carrier and sideband terms at multiples of the switching frequency. There should be good separation between the desired and undesired elements of the spectrum and some further filtering can be used if required.

iii) Explain why the AC input current of the power converter suffers distortion near the zero-crossing. [2]

When the input voltage is precisely zero the boost circuit is unable to provide the boost from 0V the output voltage. This is observed as an inability to increase the inductor current because the voltage imposed across it is the input voltage (zero). Following the zero-crossing the current will start to increase but at a di/dt less than that required to follow the desired sinewave. Eventually the input voltage rises sufficiently to ramp the current up quickly and regain the desired waveform.

iv) Explain why the DC output of the power converter has a significant voltage ripple at twice the input frequency. [2]

[3.14]

A single phase AC system supplies power as the product of two sinewaves. Thus the input power oscillates at twice line frequency around an average value. The output from the (largely) constant DC voltage will be a constant power at this average value. The excess and deficit of power between input and output is exchanged with the DC side capacitor giving rise to a double frequency voltage ripple.

c) Describe why the EMC properties of a Ćuk SMPS would be expected to be better than those of a Buck-Boost SMPS. [3]

Diagrams of both converter expected.
Buck-boost has input current chopped by transistor and output (capacitor charging) current chopped by a diode. The rectangular waveforms are rich in high order harmonic terms that form conducted emissions.
The Cuk circuit has inductors in both the input and output connections and so the input and output current are essentially smooth. There is a relatively small triangular ripple term that does give rise to emission but a substantially lower amplitudes and with a faster roll-off.

3. This question concerns the operation of power semiconductor devices used in the circuit shown in figure Q3.1.

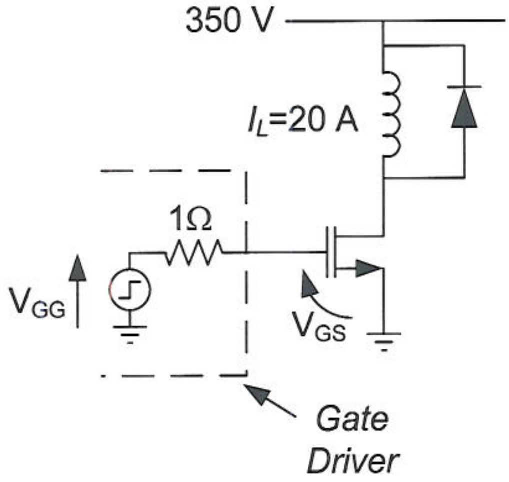


Figure Q3.1 MOSFET switching a diode clamped inductive load

(a) You are required to design a diode for use in the circuit of Figure Q3.1.

(i) Explain why power diodes are generally created with a 3 layer *pin* structure rather than a simple 2 layer *pn* structure. [4]

*[bookwork]*

*Two points should be covered:*

*1. A simple pn diode has a triangular shaped field profile in the depletion layer when in blocking mode. Because the silicon is limited to a maximum field strength, there can only be 1 point on this triangular profile where the maximum field strength is achieved. This means that the silicon is underutilised. A pin diode has a flat field profile in the i region, and thus better use is made of the silicon for voltage blocking.*

*2. The pin diode, when in forward conduction, makes use of conductivity modulation of the i region (holes are injected to the device from one side and electrons from the other). Thus, the on-state conduction losses of a pin diode are less than for a pn diode.*

(ii) Estimate the minimum length required of diode in this application. State any assumptions you make. The maximum field strength before breakdown in silicon is 10 MV/m. [3]

*[simple calculation but requires good understanding to make the right assumptions]*

- *A pin diode will be shorter than a pn diode, so as we are asked for the minimum length that can be use, assume we are designing a pin structure.*
- *Assume that all the voltage is blocked by the i region, and negligible voltage is blocked by the p and n regions and so they have negligible length*

- Then we simply need to block 350 V with a constant field strength of 10 MV/m, which is a length of 35$\mu$m

b) The switching waveforms of the MOSFET shown Figure Q3.1 are shown below in Figure Q3.2.

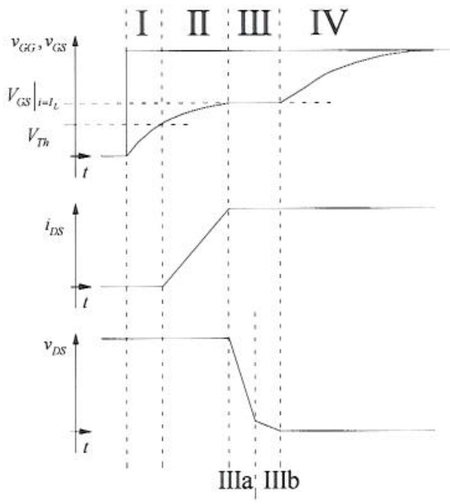


Figure Q3.2 MOSFET switching waveforms

Explain the operation of the circuit in during the turn on process (the periods I to IV in figure Q3.2). [8]

[bookwork]

The students should cover the following main points:

1. The gate capacitance is charged exponentially through the resistor until the threshold voltage is reached.
2. The drain current starts to rise as the gate voltage has now passed the threshold. The gate voltage continues to rise exponentially. The drain voltage is clamped at 350 V until the diode drops out of conduction when the drain current equals 20 A.
1. 3a At this point, the diode drops out of conduction and the drain voltage starts to fall. This fall in drain voltage causes the gate voltage to plateau whilst the Miller capacitance between gate and drain is charged.
2. 3b Once the MOSFET enters the triode region, the gate-drain capacitance increases in value and slows down the rate of fall of drain voltage.
3. The gate voltage continues to rise to the 15 V supplied by the gate driver.

(ii) By calculating the current rise time and voltage fall time, calculate the energy loss in the MOSFET be due to a turn on event. Assume the following: [5]

$$I_D = k(V_{GS} - V_{Th})^2$$

$K$=0.4 A/V$^2$

$V_{Th}$=3 V

$C_{gs}$=7nF and is constant

$C_{gd}$=0.2nF and is constant

$V_{GG}$ transitions instantly from 0 V to 15 V
Drain current rises linearly

*[This question requires the use of familiar equations in an unfamiliar context and requires good understanding of the circuit]*

*The gate voltage rises according to:*

$$V_{GS} = V_{GG}\left[1 - \exp\left(-t / C_{in}R_g\right)\right]$$

*Where $C_{in}=C_{gd}+C_{gs}$ and $R_g=1\Omega$*

*Thus, the time taken for the gate voltage to reach the threshold voltage is:*

$$t = C_{in}R_g \ln\left(1 - \frac{V_{Th}}{V_{GG}}\right)^{-1}$$

*Which is 1.6 ns*

*Phase 2 lasts until the gate voltage is sufficient to maintain 20 A through the MOSFET. This occurs with a gate voltage of:*

$$V_{GS} = \sqrt{\frac{I_D}{k}} + V_{Th}$$

*Which is when $V_{GS}$=10.07 V*

*Thus, the time taken for the gate voltage to rise to this value is again given by:*

$$t = C_{in}R_g \ln\left(1 - \frac{V_{GS}}{V_{GG}}\right)^{-1}$$

*Where $V_{GS}$=10.07 and $C_{in}$ and $R_g$ are as before.*

*Thus, t=8ns*

*Thus, the current rise time is 8-1.6=6.4ns*

*We next need to know the voltage fall time. This is the time taken to charge $C_{gd}$ through a voltage of 350 V.*

*This is done by forcing a constant current, set by (15-10.07)V/1Ω, into $C_{gd}$.*

*The current is 4.93 A.*

*Total charge required is C*V=0.2e-9*350=70 nC*

*This takes 14.2 nS to flow from a 4.93 A source.*

*Thus, the voltage fall time is 14.2 nS*

*Therefore the energy loss in the MOSFET at turn on is 0.5*(14.2e-9+6.4e-9)*350*20=72μW*

4. This question concerns losses in DC-DC power converters:

a) Explain why the switching losses in the circuit of Figure Q4.1 are lower than the switching losses in the circuit of Figure Q4.2, when the supply voltage, on-state steady current and gate drive circuit are the same in each case.

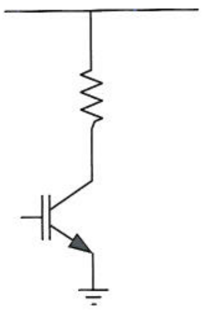

Figure Q4.1 IGBT with resistive load

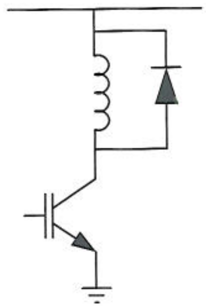

Figure Q4.2 IGBT with inductive load

*[Simple application of knowledge of transistor switching]*

*In the circuit with the resistive load, at turn on the voltage across the IGBT can fall at the same time as the current in the resistive load increases. There is therefore overlap (and must be – as dictated by Ohm's law) of the voltage fall and current rise in the IGBT. At turn off, the voltage across the IGBT rises at the same time as the current through the IGBT falls.*

*In the circuit with the inductive load, the key point is that these rise and fall events happen sequentially, rather than at the same time. At turn on of the IGBT, the voltage across the IGBT cannot fall until the IGBT carries all of the inductor current (as the voltage is clamped by the diode). At turn off, the voltage across the IGBT must rise to the supply voltage until the diode can come into conduction allowing the IGBT current to fall.*

*In short, in the resistive circuit, rise and fall events overlap, but in the inductive circuit, rise and fall events happen in sequence, increasing the length of time for the circuit to switch and thus increasing the losses.*

b) The circuit of Figure Q4.3 shows a buck converter, with 3 additional components added (inside the dotted line)

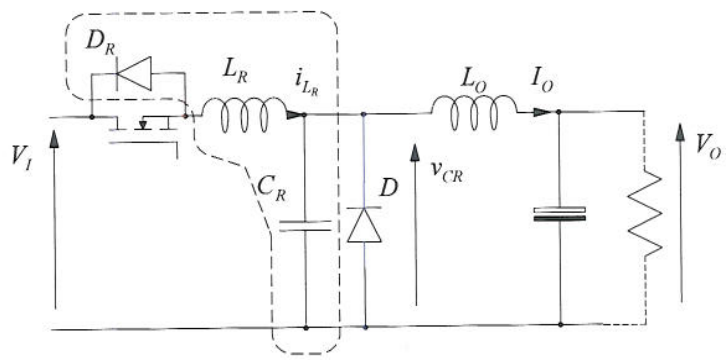


Figure Q4.3 Buck converter with 3 additional components

(i) What type of power converter is this? [1]

*[bookwork]*
*This is a quasi-resonant zero current switched power converter.*

(ii) Explain the purpose of these additional components. How do they influence the turn on and turn off voltage and current waveforms in the MOSFET? [4]

*[bookwork]*

*The additional components turn the original buck converter into a resonant converter with soft switching. This significantly reduces switching losses in the circuit as the MOSFET can be turned off and on at (almost) zero current.*

*At turn on, the MOSFET switches in series with an inductor, $L_R$, which allows the drain source voltage to drop before the current rises, acting as a turn on snubber and reducing turn on losses.*

*The addition of the L and C allow a resonant pulse of current to be created which, assuming correct values are chosen, allows a reversal of current in LR during part of the cycle. Current then flows back through the MOSFET towards the supply and during turn off of the MOSFET, current diverts into the diode, which then clamps the drain source voltage to a low value, reducing turn off losses.*

c) The converter operates with a 50 % duty cycle, and has an output voltage of 5 V with a 10 W load. The freewheeling diode, D, conducts with a duty cycle of 50 %. It is assumed that the diode operates at its maximum allowed junction temperature of 130 $^0$C.

(i) What is the power dissipation in the diode? The diode has a reverse saturation current of $1\times10^{-12}$ A and obeys the Shockley equation.

*[calculation]*

*We need to find the voltage across the diode when conducting. This is given by a rearrangement of the Shockley Equation.*

$$V_d = V_t \ln\left(\frac{I}{I_s} - 1\right)$$

*$V_t$ in this case is given by KT/q=(273+130)*K/q=34.8mV*
*Thus, the voltage across the diode when carrying 0.5 A is 0.94 V*

*Thus, the average power dissipation in the diode is 0.94*0.5*0.5=0.234 W*

(ii) The diode is operated without a heat-sink and as a consequence there is a high thermal resistance between the junction and the air. If this resistance is 60$^0$C/W, what is the maximum room temperature in which the device can operate? [2]

*The thermal resistance needs to allow a flow of 0.234W, which requires a temperature difference of 0.234*60=14$^0$C. Thus, the maximum ambient temperature would be 130-14=116$^0$C*

(iii) The input voltage to the circuit is 24 V. What is the minimum blocking voltage that the diode, D, must be able to withstand without breaking down? Explain your answer. [2]

*The resonant circuit formed by $L_R$ and $C_R$ means that the voltage across the freewheeling diode, D, reaches a value of 2 *Vin. Thus, the diode must be at least rated to 48 V.*

(d) The circuit of Figure Q4.4 shows a buck converter, with 3 additional components added (inside the dotted line) in a different configuration to Figure Q4.3.

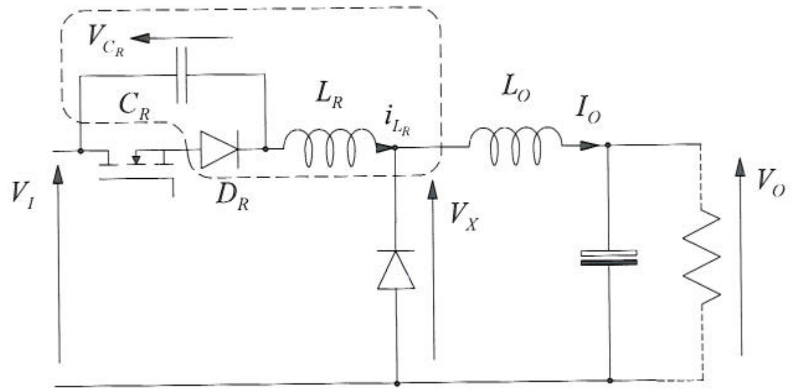


Figure Q4.4 Buck converter with 3 additional components in different configuration

i) What type of converter is this? [1]

*This is a Zero-Voltage Switched Quasi Resonant Converter*

ii) Explain the purpose of these additional components in this circuit. How do they influence the turn on and turn off voltage and current waveforms in the MOSFET? [3]

*In this circuit, turn-off occurs while $v_{CR}$ is zero and is constrained to rise slowly by $C_R$. Thus, turn off losses are minimised. A resonant cycle then follows. Turn-on can occur while $v_{CR}$ is negative and $D_R$ is blocking and thus turn on losses are minimised.*

5. A servo using feedback control of speed is required and a DC motor or an induction motor could be used.

i) Describe the power converters used in each case and the form of modulation to be employed. [5]

Require circuit diagram of 3-phase inverter (6 switch bridge) for IM
Require chopper for DC machine. Full marks requires mention of 1, 2 and 4 quadrant choppers.

ii) Describe the control structures used in each case. (Controlled-slip operation of the induction machine is sufficient.) [7]

Diagrams of control loops with discussion of how speed error is interpreted as a demand for toque either by applying armature current (DC case) or slip (IM case). Note of the inclusion of an explicit current limit in the case of the nested loops of the DC drive and the slip limit (standing in for a current limit) in the IM drive for full marks.

iii) Describe any advantages one choice has over the other. [2]

Primary point is the presence of brushes and commutator in the DC case and the disadvantages that follow from this in terms of maintenance interval, space utilisation and operation in hazardous environments. Further points:

- 6 switches for IM (providing 4Q operation) against 1, 2 or 4 switches for 1, 2, or 4 Q operation of DC.
- Full servo quality control of IM requires vector control with a the higher control computational effort that implies.

iv) State how the supply to each machine is changed to achieve braking torque. [2]

DC machine: applied armature voltage is reduced to be lower in magnitude than the back EMF, current and hence torque reverses.
IM machine: applied stator frequency is dropped to give a synchronous speed below the rotor speed, induced voltages and currents on rotor change phase and hence torque reverses

v) State how the supply to each machine is changed to achieve reverse rotation. [2]

DC machine: applied voltage is reversed in line with reversed back EMF
IM: applied stator voltages rotated in reverse (phase sequence changed or frequency turned negative)

vii) The DC input power required for both drives is to come from a 3-phase mains connection. Describe a circuit that could provide the AC/DC conversion and be able to process power flow in either direction. [2]

Circuit should be a 6-switch inverter circuit with a series inductance as the interface to the mains. The inverter would be controlled to give a sinusoidal set of voltages with a defined phase angle with respect to the mains. This phase angle will control the power flow: lagging will import power; leading will export power.

6.

a) In an open-loop induction machine drive, the stator voltage is varied in proportion to the frequency over the middle range of frequencies. Describe why a different relationship is followed at low and high frequencies. [3]

b) Figure Q3 shows an approximate equivalent circuit for an induction machine (one in which the magnetising inductance has been moved to the stator terminals). It is to be supplied from an inverter with a DC input voltage of 650 V. The designed level of magnetising flux linkage (in RMS form) is $\Psi(\text{design}) = 0.70$ Wb.

The circuit parameters of the machine are:

$L_M = 0.035$ H;
$L_S + L'_R = 0.002$ H;
$R_S = 0.1\ \Omega$;
$R_R = 0.1\ \Omega$;
$P = 1$;

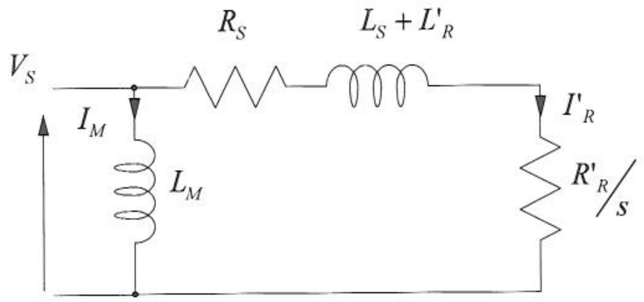


Figure Q3 Approximate equivalent circuit of an induction machine

i) Calculate the required magnetising current and the stator voltage that needs to be applied at 50 Hz to ensure operation at the designed value of flux linkage . [3]

$$I_M = \frac{\Psi}{L_M} = \frac{0.7}{0.035} = 20\ A$$

$$V_S\big|_{50Hz} = 2\pi f \Psi = 2\pi \times 50 \times 0.7 = 220.0 V$$

ii) For 50 Hz operation, the machine will operate at a slip of 50 rpm when producing rated power. Calculate the (referred) rotor current that flows at this condition, its angle with respect to the flux linkage and the torque. [4]

$$s = \frac{\omega_S - \omega_R}{\omega_S} = \frac{n_S - n_R}{n_s} = \frac{50}{3{,}000} = 1/60$$

$$I'_R = \frac{V_S}{R_S + \dfrac{R_R}{s} + j2\pi fL} = \frac{220}{0.1 + 0.1\times 60 + j100\pi \times 0.002} = \frac{220}{6.1 + j0.628} = 35.9\,A \quad \angle -5.88^\circ$$

$$\delta = 90^\circ - \angle I'_R = 84.12^\circ$$

$$T = 3\Psi I'_R \sin\delta$$
$$= 3 \times 0.7 \times 35.9 \times \sin 84.12^\circ = 74.9\ Nm$$

iii) Calculate the stator voltage required at 75 Hz to operate at the designed value of flux linkage. Check the required voltage against the available DC voltage for the inverter and determine if the machine must operate in field weakening. [3]

$$V_S\big|_{25Hz} = 2\pi f\,\Psi = 2\pi \times 75 \times 0.7 = 330.0\,V$$

Assume simple modulation (no third harmonic injection)

$$V_S(\max) = \frac{V_{DC}}{2\sqrt{2}} = \frac{650}{2\sqrt{2}} 2\pi f\,\Psi = 2\pi \times 25 \times 0.7 = 229.8\,V$$

Field weakening imposed at 75Hz by limitation to this maximum frequency.
Calculate weakened flux here or in next part.

$$\Psi\big|_{75Hz} = \frac{V_S(\max)}{2\pi f} = \frac{229.8}{2\pi \times 75} = 0.4877\,Wb$$

iv) By considering Kirchoff's voltage law for the rotor branch, show that $x=1/s$ can be found from the following equation: [2]

$$0 = |V|_S^{\,2} - \left(I'_R 2\pi f\left(L_S + L'_R\right)\right)^2 - I'^{\,2}_R R_S^{\,2} - x\left(2I'^{\,2}_R R'_R R_S\right) - x^2\left(I'^{\,2}_R R'^{\,2}_R\right)$$

Write the voltage equation in vector form and then take the magnitude of the voltage treating the real and imaginary components separately.

$$V_S = I'_R\left(j2\pi f\left(L_S + L'_R\right) + R_S + \frac{R'_R}{s}\right)$$

$$|V_S|^2 = \left(I'_R 2\pi f\left(L_S + L'_R\right)\right)^2 + I'^{\,2}_R\left(R_S^{\,2} + 2\frac{R'_R R_S}{s} + \frac{R'^{\,2}_R}{s^2}\right)$$

$$0 = |V_S|^2 - \left(I'_R 2\pi f\left(L_S + L'_R\right)\right)^2 - I'^{\,2}_R R_S^{\,2} - \frac{2I'^{\,2}_R R'_R R_S}{s} - \frac{I'^{\,2}_R R'^{\,2}_R}{s^2}$$

v) Calculate the slip required when operating at 75 Hz in order to draw the same rotor current as in (b)(ii). [3]

$$0 = V_S^{\,2} - \left(I'_R j2\pi f\left(L_S + L'_R\right)\right)^2 - I'^{\,2}_R R_S^{\,2} - x\left(2I'^{\,2}_R R'_R R_S\right) - x^2\left(I'^{\,2}_R R'^{\,2}_R\right)$$

$$a = -\left(I'^{\,2}_R R'^{\,2}_R\right) = -35.9^2 \times 0.1^2 = -12.888$$

$$b = -\left(2I'^{\,2}_R R'_R R_S\right) = -2 \times 35.9^2 \times 0.1 \times 0.1 = -25.776$$

$$c = V_S^{\,2} - \left(I'_R j2\pi f\left(L_S + L'_R\right)\right)^2 - I'^{\,2}_R R_S^{\,2} = 229.8^2 - 508.8 - 12.888 = 52{,}286$$

$$\frac{1}{s}\bigg|_{75Hz} = \frac{25.776 \pm \sqrt{25.776^2 + 4 \times 12.888 \times 52{,}286}}{2 \times -12.888} = \frac{-25.776 \pm 1641}{25.776}$$

$$s\big|_{75Hz} = 0.01595$$

[3.14]

vi) Calculate the angle of the rotor current and the torque for 75 Hz operation. [2]

$$I'_R = \frac{V_S}{R_S + \frac{R_R}{s} + j2\pi fL} = \frac{229.8}{0.1 + \frac{0.1}{0.01595} + j100\pi \times 0.002} = \frac{229.8}{6.37 + j0.628} = 35.9\,A \quad \angle -5.63^\circ$$

$$\delta = 90^\circ - \angle I'_R = 84.37^\circ$$

$$T = 3\Psi I'_R \sin\delta$$

$$= 3 \times 0.4877 \times 35.9 \times \sin 84.37^\circ = 52.27\ Nm$$

Current computes to same magnitude but slightly different angle, as expected. Torque reduced in proportion to flux linkage.