UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1998

MEng Honours Degrees in Computing Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the Royal College of Science
Associateship of the City and Guilds of London Institute

PAPER 4.90

FUNCTIONAL PROGRAMMING - FOUNDATIONS
Thursday, May 14th 1998, 10.00 - 12.00

Answer THREE questions

For admin. only: paper contains 4 questions

1a i) Give the (formal) definition of $M[x := N]$.

ii) Show that for all A, B, $(\lambda xyz.zxy)AB(\lambda xy.x) = A$.

iii) Show that $\lambda xy.x \# \lambda xy.y$

b i) Given a term M what is the fixed point of M? (write down the lambda term given by the fixed point theorem)

ii) Call a term *massive* if its β−reduction graph is infinite (i.e. has an infinite number of nodes). Show that for all lambda terms M the fixed point of M is massive.

c i) What is a strongly normalising term?

ii) Is a massive term strongly normalising ?(justify your answer)

2a i) Define the set of de Bruijn terms.

ii) Find two lambda terms M, N such that: M and N are not identical (as lambda terms) but they are translated into the same de Bruijn term.

b i) Translate $((\lambda xyz.xyzyx)(\lambda xy.x))(\lambda xy.x)$ into the De Bruijn notation.

ii) Which are the terminal configurations of the Krivine machine?

c Call a term M *slim* if no internal redex is met in any reduction sequence starting from M.

i) Show that any normal form is slim.

ii) Is a head normal form slim?

iii) Show that if M is slim there is no difference in reducing M by leftmost-outermost and leftmost-innermost.

3a i) Define the types of PCF.

ii) Define the rank of a type.

b i) Define the transition rules for the PCF constant `cond`.

ii) Define a PCF term of type `int` $\rightarrow$ `int` which implements the following function f:

$$f(0) = 1, f(n+1) = 2 + f(n)$$

c i) Call a PCF term M *insensitive* if for all ground contexts $C[\ ]$, the evaluation of $C[M]$ terminates. Is there any insensitive PCF term? (justify your answer).

ii) Call a PCF term M *sensitive* if there exists a ground context $C[\ ]$, such that the evaluation of $C[M]$ terminate. Is there any sensitive PCF term? (justify your answer).

4a i) Give the definition of a domain.

b Let $(D, \leq)$ be a domain and $f : D \rightarrow D$ a continuous function. Suppose that exists $d \in D$ such that $d \leq f(d)$.

i) Prove that the sequence $d, f(d), f^2(d), \ldots, f^n(d), \ldots$ is a chain.

ii) Prove (using (i)) that $\sqcup_{n\geq 0} f^n(d)$ is a fixed point for f.

c i) Let $(D, \leq)$ be a domain and $f : D \rightarrow D$ a continuous function which is not decreasing (i.e. for all d, $f(d) \not< d$). Suppose that D has a greatest element. Show then that f has a greatest fixed point.

End of paper