Paper Number(s): **E4.18**
AM5

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

MSc and EEE PART IV: M.Eng. and ACGI

RADIO FREQUENCY ELECTRONICS

There are SIX questions on this paper.

Answer FOUR questions.

Time allowed: 3:00 hours

Examiners: Papavassiliou,C. and Payne,A.J.

1. A certain circuit will be used as a filter between a generator of amplitude V_g and impedance Z_g and a load Z_L. This circuit can be represented by an ABCD matrix.

a) State the definition of the ***ABCD*** matrix. From this definition directly compute the voltage transfer function $G=V_l/V_G$ of the circuit of figure 1. in terms of its ***ABCD*** parameters, as well as Z_g and Z_L.

[5 marks]

b) From the definition of the ***ABCD*** matrix directly compute the current transfer function $H=I_l/I_G$ of the circuit in figure 1. in terms of its ***ABCD*** parameters, as well as Z_g and Z_L.

[5 marks]

c) From your answers in questions a) and b) above compute the power gain of this network.

[5 marks]

d) Using component ***ABCD*** matrices compute the voltage and current gain of the tee network in fig. 2. Compute its input impedance if it is terminated in a load Z_L. Show it performs an impedance matching function, but do not compute the component values necessary to implement a particular impedance matching transformer.

[10 marks]

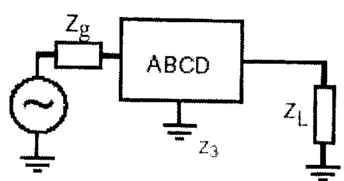


Figure 1: A 2-port network connected as a filter

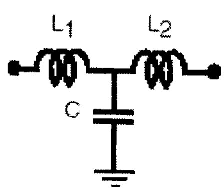


Figure 2: A Tee network filter

2. High speed diodes manufactured on an Integrated Circuit substrate often have one of their terminals, usually the cathode, connected to the substrate. The substrate is, however, also the ground plane for the microstrip lines used for interconnection and impedance matching. We are required to design a monolithic microwave switch using a grounded cathode diode as depicted in figure 3.

a) Describe an impedance transformation approach to make this diode appear in series, so that the switch depicted in figure 4. can be implemented. Implement this impedance transformation approach using microstrip lines. If the phase velocity on a substrate is 1×10^8 m/s what is the required physical length of these lines for a 10 GHz operating frequency?

[5 marks]

b) The diode must be switched on and off to perform the couple/isolate functions of the switch in figure 4. Design appropriate microstrip networks to supply the necessary bias. Note that your circuit should be such that the supply is completely invisible at signal frequencies.

[7 marks]

c) With the aid of a diagram show how we can implement DC isolation between the power supply and the external circuits connected to the two terminals of the switch.

[5 marks]

d) Compute the insertion loss, $\left|\frac{1}{S_{21}}\right|^2$ of your switch when used between 50 Ω lines. Explain how you can minimise the insertion loss of this switch by properly choosing the impedance of your transmission line transformers. What is the minimum insertion loss of this diode in the ON state given that the diode ON resistance is 15 ohms, and the fabrication process permits you to manufacture transmission lines of Z_0 between 20 and 125 ohms?

[8 marks]

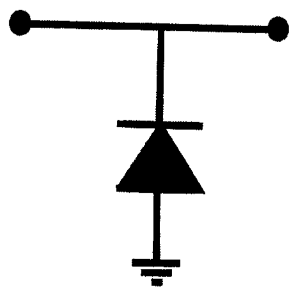


Figure 3: A diode is only available as a shunt connected element

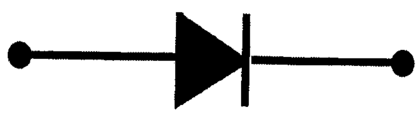


Figure 4: Desired connection for the diode

3. A mobile telephone operator needs you to specify the required transmitter power, and physical size of a base station antenna in order to implement error free communications between the base station and mobile terminals. The following specifications are available:

System	
Operating frequency	1.8GHz
Data bandwidth	56 kBit/s
Analogue Bandwidth	10 kHz
Range	10km
Mobile Terminal	
Antenna effective aperture	16.67 cm^2
Antenna Noise temperature	300 K
Receiver path noise figure	15dB
Transmitter power	10 mW
Base Station	
Antenna temperature	150 K
Receiver path noise figure	5dB
Antenna aperture efficiency	50%
Antenna Electrical efficiency	90%

a) What is the required *S/N* at the end of the receiver path in order to implement error free communications under the system specifications?

[5 marks]

b) Assume the base station is in receive mode, and that a minimum *S/N*=15dB above the theoretical minimum is required for error free reception. What must the base station antenna effective aperture be to ensure error free reception? What is its physical size? What is its directivity? What is its polar extent, given that its azimuthal extent is 50 degrees?

[10 marks]

c) Assume the base station is in transmit mode. What is the minimum transmit power which will result in error-free reception at the mobile terminal? You may assume the same, as in part (b), *S/N* ratio for error free reception.

[10 marks]

($k_B = 1.38 \cdot 10^{-23}$ J/K)

4. An ideal transconductor is defined by the following equations:

$$I_1 = 0$$

$$I_2 = g\, V_1$$

a) Write the $\boldsymbol{Y}$ matrix for an ideal transconductor.

[2 marks]

b) Write the $\boldsymbol{Y}$ matrix for the circuit in fig. 5a, where two transconductors, of transconductance g and –g are connected so that the input of one is the output of the other. This circuit is one of many possible implementations of a gyrator, which we will symbolise as K $(K=1/g)$ (fig 5b). From the $\boldsymbol{Y}$ matrix derive the ABCD matrix for this gyrator. What is its bandwidth?

[4 marks]

c) Derive the ***ABC*D** matrix for a cascade of two gyrators, K_1 and K_2. From your answer draw and clearly label an equivalent circuit for this configuration.

[5 marks]

d) Derive the ***ABCD*** matrices, and draw and clearly label equivalent circuits for each of the networks in figure 6.

[8 marks]

e) Deduce, draw and clearly label equivalent circuits for a series **LC** network substituted for **Z** in fig 6a, and a parallel **LC** network substituted for **Z** in fig 6b, both terminated at Z_T.

[6 marks]

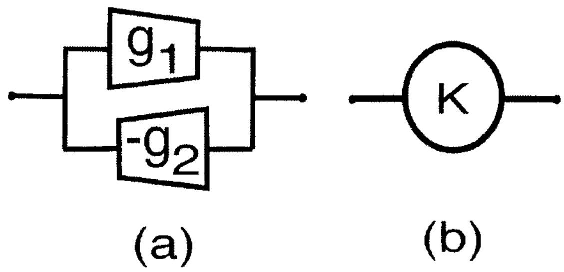


Figure 5 A gyrator (a) implementation, (b) symbol

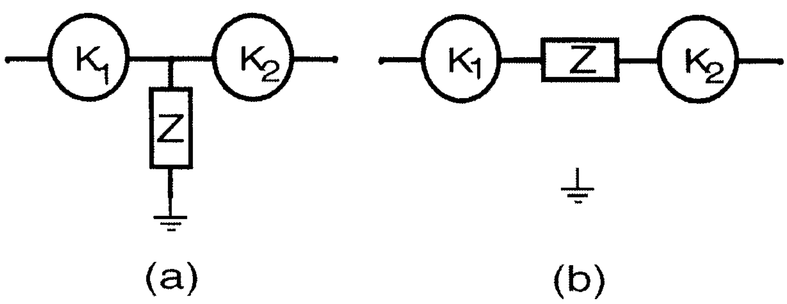


Figure 6 Gyrated circuit elements (a) shunt, (b) series

5) In an RF application we need to isolate our circuit from microwave oven interference, and thus decide to use a 2.45 GHz band reject filter with 100 MHz bandwidth. We will base the design on the 3^{rd} order 1dB Chebyshev normalised lowpass prototype of figure 7.

a) Using this prototype design a 2.45 GHz lumped band reject filter with a stop band of 100 MHz to operate between 50 Ohm source and load.

[10 marks]

b) How can the filter components be implemented using transmission lines? Assume you are working with balanced lines so that both series and shunt connections are allowed. Write equations for these components in terms of the line segment impedances and lengths. Draw a schematic for the filter clearly illustrating the appearance of the filter if made using balanced lines.

[5 marks]

c) Implement the filter using only microstrip resonators and microstrip lines. Write expressions for, but do not compute line lengths and impedances.

[10 marks]

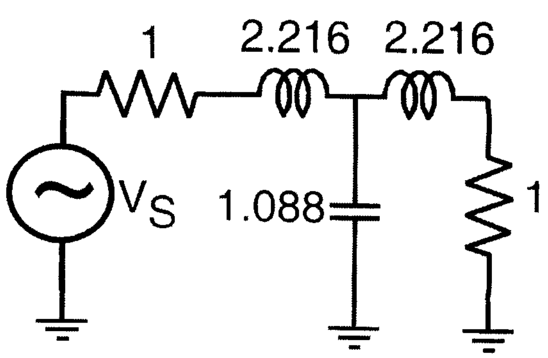


Figure 7: Filter prototype of question 5.

6) The network in figure 8 is often used as an impedance matching transformer between real impedances R and R_{in}.

a) Derive expressions for the components of the impedance matching network in figure 8. You are given the terminating resistances, the frequency of operation, and the desired fractional 3dB bandwidth of the match.

[10 marks]

b) What is the range of ratios between R and R_{in} that you can achieve with this transformer ?

[10 marks]

c) How do you need to modify the component values you calculated in (a) to accommodate complex terminating impedances?

[5 marks]

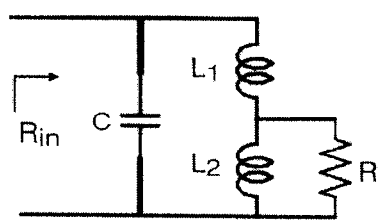


Figure 8: An impedance matching circuit.

Radio Frequency Electronics
2001

RF

Question 1

a) $\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix}$

$V_1 = V_g - Z_s i_1 \Rightarrow i_1 = \frac{V_g - V_1}{Z_S}$ (1)

$V_2 = -Z_L i_2 \Rightarrow i_2 = -\frac{V_2}{Z_L}$ (2)

$V_1 = AV_2 - Bi_2 = AV_2 + B\frac{V_2}{Z_L}$ (3)

$i_1 = CV_2 - DI_2 \Rightarrow \frac{V_g - V_1}{Z_S} = CV_2 + D\frac{V_2}{Z_L} \Rightarrow$

$\frac{V_g}{Z_S} = \frac{AV_2}{Z_S} + \frac{BV_2}{Z_S Z_L} + CV_2 + \frac{DV_2}{Z_L} \Rightarrow$

$V_g = V_2 \cdot \left(A + \frac{B}{Z_L} + CZ_S + D\frac{Z_S}{Z_L}\right) \Rightarrow$

$\frac{V_2}{V_g} = \left(A + \frac{B}{Z_L} + CZ_S + D\frac{Z_S}{Z_L}\right)^{-1} = G$ [5]

(b) The current gain is just from the second eq. in the ABCD definition:

$i_1 = CV_2 - Di_2 = -CZ_L I_2 - DI_2 \Rightarrow$

$I_1 = I_2(-CZ_L - D) \Rightarrow \frac{I_2}{I_1} = -\frac{1}{CZ_L + D} = H$ [5]

(c) $G_P = \frac{P_{out}}{P_{in}} = \frac{-V_2 I_2}{V_1 I_1} = -GH$ (and substitute) [5]

d) $X_{ser\,L} = \begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix}$

$X_{shunt\,C} = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}$

then $X_{filter} = \begin{bmatrix} 1 & j\omega L_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix} \begin{bmatrix} 1 & j\omega L_2 \\ 0 & 1 \end{bmatrix} =$

$$= \begin{bmatrix} 1-\omega^2 L_1 C & j\omega L_1 \\ j\omega C & 1 \end{bmatrix} \begin{bmatrix} 1 & j\omega L_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\omega^2 L_1 C & j\omega(L_1+L_2) - j\omega^3 L_2 L_1 C \\ j\omega C & 1-\omega^2 L_2 C \end{bmatrix}$$

$$G = \left(A + \frac{B}{R_L} + CR_S + D\frac{R_S}{R_L}\right)^{-1} =$$

$$= \left(1-\omega^2 L_1 C + j\omega\frac{(L_1+L_2)}{R_L} - \frac{j\omega^3 L_1 L_2 C}{R_L} + j\omega R_S C + (1-\omega^2 L_2 C)\frac{R_S}{R_L}\right)^{-1}$$

$$H = -(CR_L + D)^{-1} = -(j\omega R_L C + (1-\omega^2 L_2 C))^{-1}$$

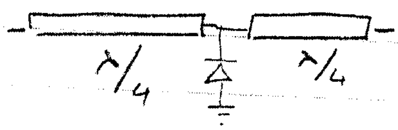
An impedance matching network would present $Z_{in} = Z_S^*$

$$Z_{in} = \frac{V_1}{I_1} \quad \frac{AZ_L + B}{CZ_L + D} = \frac{(1-\omega^2 L_1 C)Z_L + j\omega(L_1+L_2) - j\omega^3 L_2 L_1 C}{j\omega Z_L C + (1-\omega^2 L_2 C)}$$

[10].

Question 2

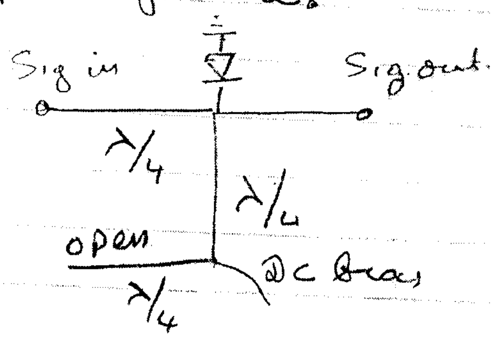
a) Place it between two gyrators. [1]



[2]

$$\left.\begin{array}{l} f = 10^{10}\ Hz \\ c = 10^{10}\ cm/sec \end{array}\right\} \Rightarrow \lambda = 1/cm \Rightarrow l = \lambda/4 = 2.5mm$$ [2]

b). Need to supply DC bias through a λ/4 piece of line:

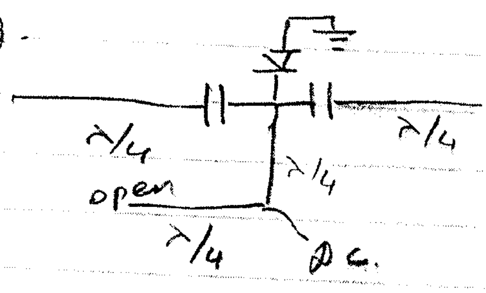


λ/4 for DC bias [4]

λ/4 open to establish DC ground [3]

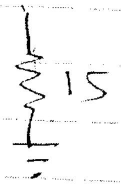
c). Need to introduce two capacitors,

eg.

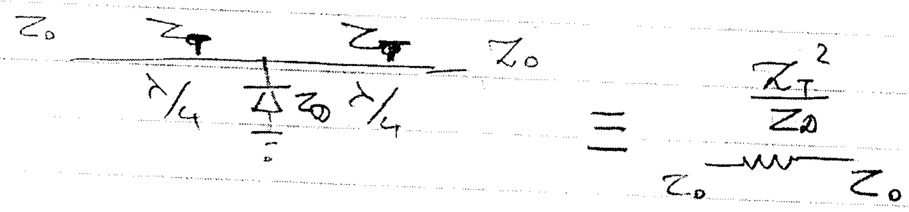


[5]

d) if the diode appears as:



when on, then



or, in reduced units it is a

$$z = \frac{Z_T^2}{Z_D Z_\phi} = \frac{1}{y}.$$

A series impedance has a Y matrix. (in red. units)

$$Y = \begin{pmatrix} y & -y \\ -y & y \end{pmatrix}$$ and the S_{21} element is:

is: $$S_{21} = \frac{-2Y_{21}}{1+Y_{11}+Y_{22}-\Delta Y} = \frac{2y}{1+2y+2y^2} \Rightarrow \left|\frac{1}{S_{21}}\right|^2 = \left(\frac{1+2y+2y^2}{2y}\right)^2$$

since $y = \frac{Z_\phi Z_D}{Z_T^2}$ to minimise the ins loss

to minimise $L = \frac{1}{|S_{21}|}$ $\Rightarrow \frac{dL}{dy} = \frac{y(2+2y)-1-2y-2y^2}{y^2} \times 2L$

$= \frac{-1}{y^2} \times 2L$ i.e. L is monotonically decreasing with y. Then we need to use the minimum possible Z_T for the gyrators, $Z_T = 20\Omega$

then $\quad y = \frac{10 \times 50}{(20)^2} = 1.25$

and $\quad L = \left(\frac{1+2y+2y^2}{2y}\right)^2 = 7.0225 \quad = 8.46\,\text{dB}$

Question 3.

a). From the shannon theorem, if C is the capacity:

$$C = B \log_2(S/N + 1) \Rightarrow S/N = 2^{\frac{C}{B}} - 1$$

it is given: $C = 56 \times 10^3$
$B = 10^4$

$$\Rightarrow S/N = 47.5 = 16.85\,dB$$

[5]

b) By the friis formula,

$$P_{rec} = P_T \cdot \frac{A_{effT}\,A_{effR}}{R^2 \lambda^2}$$

$$\lambda = \frac{c}{1.8GHz} = 0.167\,m$$

the range is given as $R = 10^4 m$.

The mobile terminal power is 1W

and the antenna aperture is $16.67 \times 10^{-4}\,m^2$.

then $P_{rec} = \frac{10^{-2}W \times 16.67 \times 10^{-4} \cdot A_{eff\,B.S.}}{10^8 \times (0.167)^2} =$

$= A_{eff\,BS} \cdot 5.977 \times 10^{-12}\,(W)$

The noise power received is just $P_N = kT_A B =$

$= 1.38 \times 10^{-23} \times 150 \times 10^4 = 2.07 \times 10^{-17}$

therefore, the required minimum signal power is:

$P_{sig} = P_N \cdot S/N \cdot NF \cdot SNR = 2.07\times10^{-17} \times 47.5 \times 3.16 \times 31.6 =$

$= 9.82\times10^{-14}\,W$.

then, the basestation effective aperture is:

$A_{eff} \times 5.977\times10^{-12} = 9.82\times10^{-14} \Rightarrow A_{eff} = 0.164\,m^2$

Since $A_{eff} = A_{phys}\cdot\eta_A \Rightarrow A_{phys} = \frac{0.164\,m^2}{0.5} = 0.328\,m^2$

The directivity is: $G = \eta_{el}\cdot D$. where G is the gain

and $G = \frac{4\pi A_{eff}}{\lambda^2} \Rightarrow D = \frac{1}{\eta_{el}}\cdot\frac{4\pi A_{eff}}{\lambda^2} = \frac{1}{0.9}\,\frac{4\pi\times0.164}{(0.167)^2}$

then $D = 83.6$

In terms of the angular extent the directivity is:

$D \simeq \frac{41000}{\theta\varphi}$. if $\theta = 90$,

$\theta = \frac{41000}{90\times83.6} = 10^\circ$

[10].

c). Once again, the received power is

$$P_{rec} = P_t \cdot \frac{A_{effT} \, A_{effR}}{R^2 \lambda^2}$$

to simplify calculation, we already know P_{rec} for this system, for 10 mW:

$$P_{rec} = \frac{P_{trans}}{10^{-2}} \times 8.82 \times 10^{-14} \; W$$

the required received power is:

$$P_{rec} = k T_{ANT} B \cdot S/N \Big|_{rec} \;\; SNR_{MIN} = \ldots = 20 \times P_{rec_{B.S}}$$

then the transmitter power is $P_{T_{min}} = 200 \, mW$

[5]

Question 4

a). $Y = \begin{pmatrix} 0 & 0 \\ g & 0 \end{pmatrix}$

$$\left(Y_{11} = \frac{\partial I_1}{\partial V_1}\Big|_{V_2=0},\ Y_{12} = \frac{\partial I_1}{\partial V_2}\Big|_{V_1=0},\ Y_{21} = \frac{\partial I_2}{\partial V_1}\Big|_{V_2=0},\ Y_{22} = \frac{\partial I_2}{\partial V_2}\Big|_{V_1=0} \right)$$

[2]

b). $Y = \begin{pmatrix} 0 & -g \\ g & 0 \end{pmatrix}$ (since the two nets are connected opposite to each other, add the Y matrices.

derivation of ABCD from Y:

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ Y_{11} & Y_{12} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -Y_{21} & -Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \frac{1}{Y_{21}} \begin{pmatrix} 1 & 0 \\ Y_{11} & Y_{12} \end{pmatrix} \begin{pmatrix} -Y_{22} & -1 \\ Y_{21} & 0 \end{pmatrix} = \frac{1}{Y_{21}} \begin{pmatrix} -Y_{22} & -1 \\ -\Delta & -Y_{11} \end{pmatrix}$$

for the gyrator: $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \frac{1}{g} \begin{pmatrix} 0 & -1 \\ -g^2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1/g \\ -g & 0 \end{pmatrix}$

the bandwidth is clearly infinite. [4]

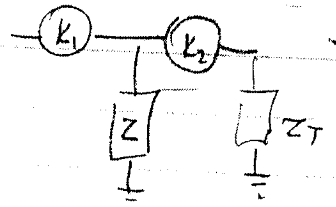
c). $-(k_1)-(k_2)-$ $\quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 0 & -1/g_1 \\ -g_1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1/g_2 \\ -g_2 & 0 \end{pmatrix} = \begin{pmatrix} g_2/g_1 & 0 \\ 0 & g_1/g_2 \end{pmatrix}$

then $V_1 = g_2/g_1\, V_2$

$i_1 = -g_1/g_2\, i_2$.

$Z_{eff} = \frac{V_1}{I_1} = \frac{g_2^2}{g_1^2} Z_T$

So it is a simultaneous voltage and current transformer. [5]

d) 

$$(ABCD) = \begin{pmatrix} 0 & -1/g_1 \\ -g_1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ Y & 1 \end{pmatrix} \begin{pmatrix} 0 & -1/g_2 \\ -g_2 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -Y/g_1 & -1/g_1 \\ -g_1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1/g_2 \\ -g_2 & 0 \end{pmatrix} = \begin{pmatrix} g_2/g_1 & Y/g_2 g_1 \\ 0 & g_1/g_2 \end{pmatrix}$$

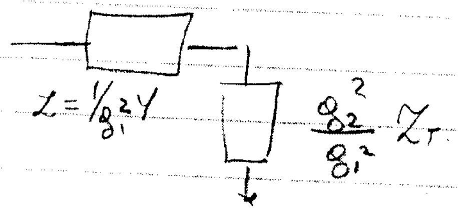
if it is terminated in Z_T then

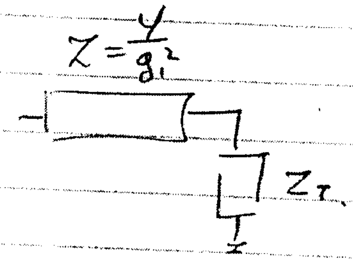
$$-V_2 = -Z_T i_2 \quad \text{and} \quad i_1 = -\frac{g_1}{g_2} i_2$$

$$V_1 = \frac{g_2}{g_1} V_2 - \frac{Y}{g_2 g_1} i_2 = -\frac{g_2}{g_1} Z_T i_2 - \frac{Y}{g_2 g_1} i_2 =$$

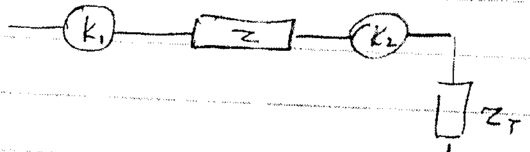
$$= +\frac{g_2^2}{g_1^2} i_1 Z_T + \frac{1}{g_1^2} Y i_1$$

and the equivalent circuit is:



if $g_1 = g_2$ it reduces to 

[4].



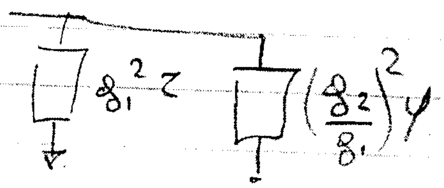
$$(ABCD) = \begin{pmatrix} 0 & -1/g_1 \\ -g_1 & 0 \end{pmatrix} \begin{pmatrix} 1 & Z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1/g_2 \\ -g_2 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & -1/g_1 \\ -g_1 & -g_1 Z \end{pmatrix} \begin{pmatrix} 0 & -1/g_2 \\ -g_2 & 0 \end{pmatrix} = \begin{pmatrix} g_2/g_1 & 0 \\ g_1 g_2 Z & g_1/g_2 \end{pmatrix}.$$

then $V_1 = g_2/g_1 \, V_2$, $V_2 = -Z_T i_2$.

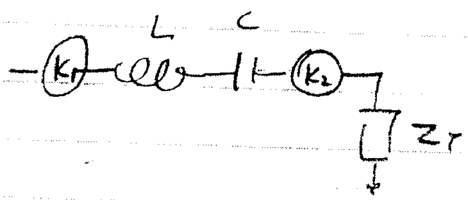
and $i_1 = g_1 g_2 V_2 Z - \frac{g_1}{g_2} i_2 = \left(g_1 g_2 Z + \frac{g_1}{g_2}\frac{1}{Z_T}\right)\frac{g_1}{g_2} V_1$

$$= g_1^2 Z + \left(\frac{g_1^2}{g_2^2}\right)\frac{1}{Z_T}$$



[4]

e).



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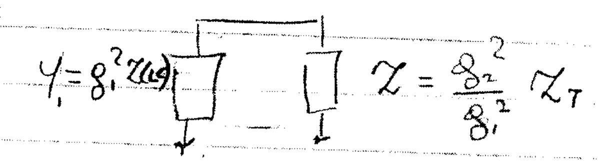
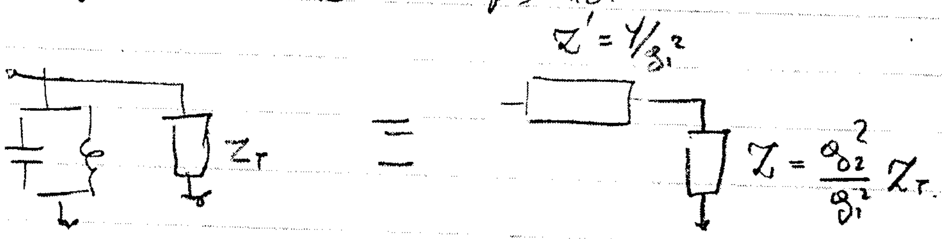


[3]

$Z(LC) = j\omega L + \frac{1}{j\omega C} \Rightarrow Y_1 = g_1^2 j\omega L + \frac{g_1^2}{j\omega C}$.

is a parallel LC, $L' = \frac{C}{g_1^2}$, $C' = g_1^2 L$.

the parallel LC maps to.



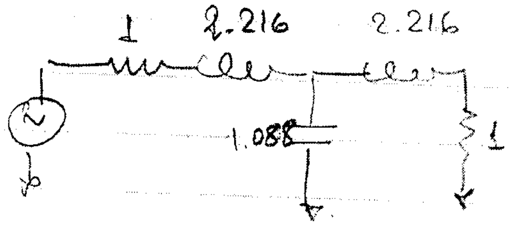
[3]

since $Y(LC) = j\omega C + \frac{1}{j\omega L}$, $Z' = 1/g_1^2 \, j\omega C + 1/_2 \, 1/$.

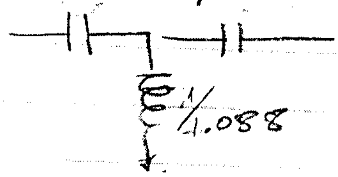
and the equivalent circuit is a series LC with

$L' = \frac{C}{g_1^2}$, $C' = g_1^2 L$.

5)(a)



high pass prototype:



scale for Z and centre frequency.

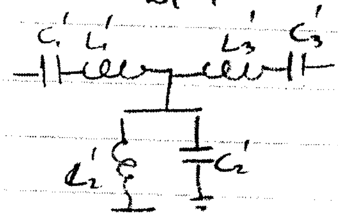
$$C_1' = C_3' = \frac{C_1}{50\,\omega_0} = \frac{1}{2.216}\,\frac{1}{50}\,\frac{1}{2\pi \times 10^8} = 14.3\,pF$$

$$L_2' = \frac{L_1 50}{\omega_0} = \frac{50}{1.088 \times 2\pi \times 10^8} = 289.5\,pH$$

to make it band reject, resonate these with L_1', C_2', L_3' so that $\omega_0^2 = \frac{1}{L_1' C_1'}$

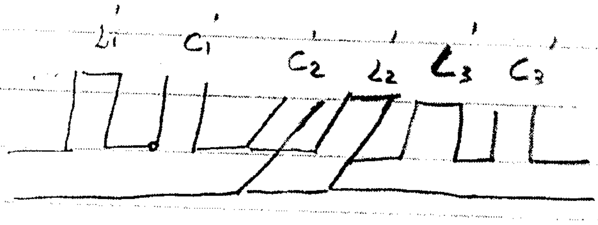
$$\Rightarrow L_1' = \frac{1}{\omega_0^2 C_1'} = 293\,pH = L_3'$$

$$C_2' = \frac{1}{\omega_0^2 L_2'} = 14.4\,pF$$



[5]

b) an inductor can be a shorted stub of length less than $\lambda/4$ and a capacitor an open stub of length less than $\lambda/4$. then the filter becomes:

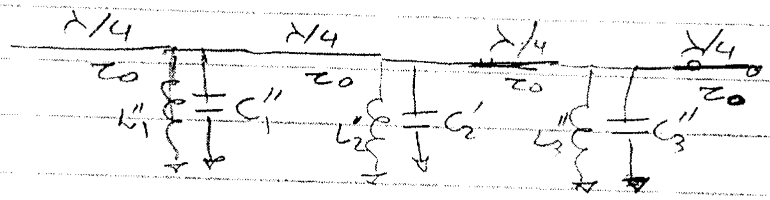


Since $Z_{open} = \dfrac{Z_0}{j\tan kl} \approx \dfrac{Z_0}{j\frac{\omega}{c}l} \Rightarrow C_{eff} = \dfrac{l}{Z_0 c}$

and $Z_{short} = jZ_0 \tan kl \simeq Z_0 j\frac{\omega}{c}l \Rightarrow L_{eff} = \dfrac{Z_0 l}{c}$

[10]

c) We can use $\lambda/4$ transformers as gyrators to turn the filter into:



then, $L_1'' = Z_0^2 C_1' = L_3''$

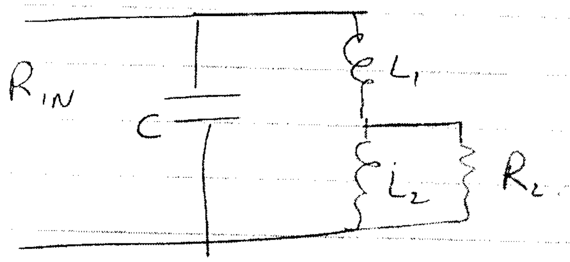
$C_1'' = \dfrac{L_1''}{Z_0^2} = C_3''$

The parallel LC's can be implemented as open $\lambda/4$ stubs with

$L = \dfrac{\pi Z_0}{\ldots}$ and $C = \dfrac{1}{\ldots}$ [10]

Question 6.

a



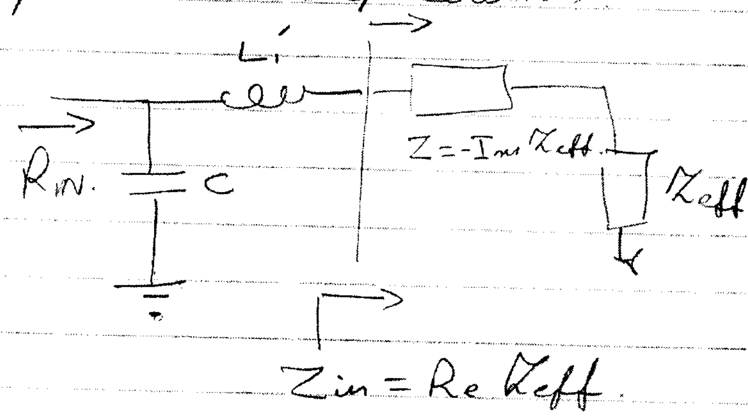
a) Lump L_2, R_L into an $Z_{eff} = \dfrac{j\omega L_2 R_L}{j\omega L_2 + R_L}$

then $Z_{eff} = \dfrac{j\omega L_2}{1 + j\omega \frac{L_2}{R_L}} = R_L \dfrac{j\omega \tau}{1 + j\omega\tau}$ $\quad \tau = \dfrac{L_2}{R_L}$

$$\mathrm{Re}\, Z_{eff} = R_L \frac{\omega^2\tau^2}{1+\omega^2\tau^2} = R_{eff}$$

$$\mathrm{Im}\, Z_{eff} = R_L \frac{\omega\tau}{1+\omega^2\tau^2}$$

this converts to a simple impedance match problem: (step down!)



If Q is specified,

$$Q^2 = \frac{R_{IN}}{R_{eff}} - 1 \Rightarrow Q^2 R_{eff} = R_{IN} - R_{eff} \Rightarrow$$

$$(Q^2+1) R_{eff} = R_{IN}$$

But $R_{eff} = R_L \frac{\xi^2}{1+\xi^2}$ $\left(\xi = \omega\tau = \frac{\omega L_2}{R_L} \right)$

$$\Rightarrow (Q^2+1) \frac{\xi^2}{1+\xi^2} R_L = R_{IN} \Rightarrow (Q^2+1) \frac{\xi^2}{1+\xi^2} \rho = 1$$

$$\Rightarrow \xi^2 \left[(Q^2+1)\rho - 1 \right] = 1 \Rightarrow \frac{\omega^2 L_2^2}{R_L^2} = \frac{1}{[(Q^2+1)\rho - 1]} = \xi^2$$

this defines L_2.

The std. impedance matching with L nets gives:

$$Q R_{eff} = \omega L_1 \Rightarrow Q R_L \frac{\xi^2}{1+\xi^2} = \omega L_1'$$

$$\Rightarrow L_1' = \frac{Q R_L}{\omega} \frac{\xi^2}{1+\xi^2}$$

then

$$\omega L_1 = \omega L_1' - \mathrm{Im}\, Z_{eff} = \omega L_1' - R_L \frac{\xi}{1+\xi^2} =$$

$$= Q R_L \frac{\xi^2}{1+\xi^2} - R_L \frac{\xi}{1+\xi^2} = R_L \frac{Q\xi(\xi-1)}{1+\xi^2}$$

and $\omega C = \frac{Q}{R_{IN}}$

b) By the derivation of L_2 we have:

$$\frac{\omega^2 L_2^2}{R_L^2} = \frac{1}{(Q^2+1)\rho - 1}$$

the constraint on $\rho = \frac{R_{IN}}{R_L}$ is

$$(Q^2+1)\rho \geq 1 \qquad \rho \geq \frac{1}{1+Q^2}.$$

Also, since L_1 needs to be an inductor

it follows $\xi > 1 \Rightarrow \xi^2 > 1 \Rightarrow$

$$\frac{1}{(Q^2+1)\rho - 1} > 1 \Rightarrow 1 > (Q^2+1)\rho - 1 \Rightarrow$$

$$(Q^2+1)\rho < 2 \Rightarrow \rho < \frac{2}{1+Q^2}.$$

$$\Rightarrow \frac{1}{1+Q^2} < \rho < \frac{2}{1+Q^2} \qquad [10]$$

c) By adding the negative of the imag part of the respective impedances.

[5]