UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2003

BEng Honours Degree in Computing Part III
MSc in Computing Science
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the
Associateship of the Royal College of Science

PAPER C381=I3.30

COMPUTATIONAL FINANCE

Thursday 8 May 2003, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1a Consider a portfolio of a risk free asset A and two risky stocks B, C. Suppose that there exist three states with probabilities of 0.20, 0.45, and 0.35. The asset returns for each state s_i for $i = 1, 2, 3$ are given in the following table.

| States | Probability | Rates of Return | | |
		Risk Free Asset A	Stock B	Stock C
s_1	0.20	0.05	0.08	0.03
s_2	0.45	0.05	0.05	0.09
s_3	0.35	0.05	0.07	0.15

  (i) Let w_A, w_B and w_C be the weights of assets. Compute the expected rate of return and expected risk of the portfolio constructed by assets A, B and C.

  (ii) Write the mean-variance Markowitz portfolio optimization model, assuming that there is no shortsale and you can invest at most 30% of your capital on asset A, 25% on asset B and 45% on asset C.

b Suppose that the price and payoff values of the securities A, B, C for two states are given in the following table.

| Security | Price (£) | Payoff Values (£) | |
		State 1	State 2
A	50	50	100
B	55	30	120
C	125	77.5	79.5

  (i) How can you produce an arbitrage opportunity involving securities A, B, C. State your strategy clearly.

  (ii) Suppose you have £100k capital. What is the outcome of forming the arbitrage portfolio?

  Hint: *Replicate the portfolio constructed by securities A, B to asset C.*

  (Each part carries 50% [=30% + 20%] of the marks).

2a Consider a bond which has maturity of 2 years, pays coupon payments of £8. The face value at the end of the second year is £100.

    (i) Compute the Macaulay duration and convexity if the yield to maturity is 5%?

    (ii) Derive the new price of the bond using the modified duration and convexity if the yield to maturity decreases 1%. Compare your result with the actual price of the bond obtained with the new yield.

b A company has just paid the dividend of £5 per share. The company has expected the rate of growth in dividends per year for 2 years as 8% and then 5% per year thereafter. What is the new stock price if the required return remains at 9%?

c Suppose that you want to buy a £170,000 property. You get a loan with the terms of £20,000 deposit and repayment of the rest in monthly instalments in 25 years. The fixed interest rate is 8% p.a. for the life of the loan and compounds monthly. What is the monthly payment?

Hint: *Use the following formulae as necessary.*

The Macaulay Duration:

$$D = \frac{\sum_{t=1}^{T} t \times PV(C_t)}{P}$$

Convexity:

$$C = \frac{1}{P(1+\lambda)^2} \sum_{k=1}^{n} k(k+1)PV(a_k)$$

The present value of Constant Growth Dividend:

$$P_t = \frac{D_{t+1}}{r_e - g}$$

The present value of n period annuity:

$$PV_A = \frac{a.d(1-d^n)}{1-d}$$

(The three parts carry, respectively 50% [= 25% + 25%], 30% and 20% of the marks).

3a   Consider a binomial lattice model for the stock price process $\{S_n : 0 \leq n \leq 2\}$ and $S_0 = \pounds 125$. Let the price rise or fall by 12% at each step. The risk–free interest rate is assumed $r = 6\%$. The contract we wish to price is a European call option with strike price £110 at time 2.

   (i)   Compute the risk neutral probabilities.

   (ii)   Evaluate the stock prices on 2-period binomial lattice.

   (iii)   Derive the cash position and stock position for 2 periods on 2-period binomial lattice using "the replicating portfolio method". Calculate the option price.

   b   Suppose that the stock is selling at £250. The risk free interest rate is 12% and the dividend rate is 6% and compounded continuously.

   (i)   What is the price of 1 year forward contract?

   (ii)   Suppose that the price of forward contract is £280. Investigate whether there is an arbitrage opportunity in the market. State clearly your positions and cash flows in an arbitrage table.

   (The two parts carry, respectively, 60%[= 15% + 15% + 30%], and 40%[= 15% + 25%] of the marks).

4a You are given the following quadratic programming (QP) problem:

$$\min f(x_1, x_2) = \begin{array}{ccccccc} -2x_1 & + & x_2 & + & x_1^2 & - & 2x_1x_2 & + & 6x_2^2 \end{array}$$
$$\begin{array}{rcrcrcr} \text{s.t.} & -2x_1 & - & 3x_2 & \leq & -4 \\ & -x_1 & + & 4x_2 & \geq & -3 \\ & & & x_1, & x_2 & \geq & 0. \end{array}$$

(i) Determine the Lagrange function of the given QP problem in matrix/vector form.

(ii) Write the KKT optimality conditions of the above QP problem in equation form. Explain your work.

b (i) Let $f(x)$ be defined in the following way:

$$f(x) = \left\{ \begin{array}{ll} x^3 & \text{if } x < 0, \\ x^2 & \text{if } x \geq 0. \end{array} \right.$$

Is $f(x)$ continuous in the $[-1, 1]$ interval?
Is $f(x)$ differentiable in the $[-1, 1]$ interval?
Justify your answers.

(ii) The existence of the AB matrix product does not necessarily mean that BA also exists. Even if both are square and of the same dimension it is generally not true that $AB = BA$. Equality holds in special cases only. Consider the following two matrices

$$A = \begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix}, \quad B = \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix}$$

where a, b, c and d are scalars. Compute $AB - BA$. What conditions must the scalars satisfy in order to make $AB = BA$? Give a constructive answer.

(The two parts carry, respectively, 60% [= 30% + 30%] and 40% [= 20% + 20%] of the marks).