

**IMPERIAL COLLEGE LONDON**

**BSc and MSci DEGREES – JANUARY 2015, for Internal Students  
of the Imperial College of Science, Technology and Medicine**

**This paper is also taken for the relevant examination for the  
Associateship**

**ADVANCED CHEMISTRY THEORY IIA**

**Physical Chemistry**

**Thursday 15<sup>th</sup> January 2015, 14:00-15:30**

**PLEASE NOTE THAT IT IS DEPARTMENTAL POLICY THAT  
THESE EXAM QUESTIONS MAY REQUIRE UNDERSTANDING  
OF ANY PRIOR CORE COURSE.**

**USE A SEPARATE ANSWER BOOK FOR EACH  
QUESTION. WRITE YOUR CANDIDATE NUMBER ON  
EACH ANSWER BOOK.**

## 2.P6 – Quantum Chemistry

UNLESS OTHERWISE STATED YOU MAY OMIT NORMALISATION CONSTANTS IN YOUR ANSWER TO THIS QUESTION.

Answer part a) and **EITHER** b) **OR** c) of this question.

a) Answer **ALL** parts of this question.

- i) Explain why the total wave function for indistinguishable particles must be either symmetric or antisymmetric with respect to particle exchange.  
(3 marks)
- ii) Electrons belong to a class of elementary particles known as Fermions – what does this mean in relation to your answer to a) part i)?  
(1 mark)
- iii) For the specific case of two-electron wave functions, explain how spin and space wave functions can be combined to give a complete wave function with the required symmetry properties.  
(2 marks)
- iv) Discuss whether each of the following is an acceptable spin wave function. Explain how the unacceptable spin wave functions may be combined to obtain acceptable spin wave functions :

$$\alpha(1)\alpha(2)$$

$$\alpha(1)\beta(2)$$

$$\beta(1)\alpha(2)$$

$$\beta(1)\beta(2)$$

(2 marks)

- v) In the independent electron approximation, four of the six possible molecular wave functions of  $\text{H}_2^+$  are degenerate. Write down expressions for each of these four degenerate wave functions, taking into account both space and spin.  
(4 marks)
- vi) Explain how electron-electron interactions affect the energies of the four wave functions in a) part v).  
(2 marks)

QUESTION CONTINUED OVERLEAF

b) Answer **ALL** parts of this question.

- i) Write down in  $\hat{T}, \hat{V}$  notation the **full** Hamiltonian for the  $\text{H}_2^+$  molecule. Your expression should include both the electron and the nuclei. (2 marks)
- ii) State the Born-Oppenheimer approximation and use it to obtain an expression for the  $\text{H}_2^+$  electronic Hamiltonian in  $\hat{T}, \hat{V}$  notation. (3 marks)
- iii) Using your expression from bii) write down an expression for the electronic Hamiltonian of the  $\text{H}_2^+$  molecule in terms of the electron mass  $m_e$ , the nuclear separation  $R_{AB}$ , the distance  $r_A$  of the electron from nucleus A and the distance  $r_B$  of the electron from nucleus B. (Your answer should include all relevant fundamental constants). Comment on the signs of each of the four terms. (6 marks)

c) Answer **ALL** parts of this question.

- i) Explain briefly what is meant by the pi electron approximation for conjugated systems. (3 marks)
- ii) State the Hückel approximations, defining clearly in your answer the symbols  $\alpha$  and  $\beta$ . (4 marks)
- iii) Write down the secular determinant for cyclobutadiene ( $\text{C}_4\text{H}_4$ ) in the Hückel approximation in terms of  $\alpha$ ,  $\beta$  and  $E$ , and hence obtain the orbital energies. You may wish to use the result:

$$\begin{vmatrix} a & b & 0 & b \\ b & a & b & 0 \\ 0 & b & a & b \\ b & 0 & b & a \end{vmatrix} = a^2(a^2 - 4b^2)$$

(3 marks)

- iv) What is the delocalisation energy for cyclobutadiene in the Hückel approximation, i.e. how does the average energy per electron in the ground-state of cyclobutadiene compare to the average energy per electron in the ground-state of ethene?

(1 mark)

## 2.P9 – Photochemistry

Answer part a) and **EITHER** part b) **OR** part c) of this question.

a) Answer **ALL** parts of this question.

- i) With the aid of a suitable diagram define the terms internal conversion, intersystem crossing, fluorescence and phosphorescence. Clearly indicate on your diagram singlet and triplet states.

(5 marks)

- ii) Explain what is meant by oscillator strength.

(2 marks)

- iii) A dye molecule has a natural radiative lifetime of 35 ns. In a solution, this dye has a fluorescence lifetime of 900 ps. Calculate the fluorescence quantum yield.

(2 marks)

- iv) Solutions of naphthalene and halogenated naphthalenes were found to have the following photophysical data.

	Fluorescence quantum yield	Phosphorescence quantum yield
Naphthalene	0.55	0.06
1-bromo Naphthalene	$2 \times 10^{-3}$	0.27
1-iodo Naphthalene	$5 \times 10^{-4}$	0.38

Explain the origin of the different fluorescence and phosphorescence yields for these three compounds.

(6 marks)

QUESTION CONTINUED OVERLEAF

b) Answer **BOTH** parts of this question.

- i) Derive the Stern-Volmer equation for bimolecular quenching. (5 marks)
- ii) The emission lifetimes of a dye molecule were measured as function of dissolved oxygen concentration in solution. The data obtained from these studies is given in the table below.

$[\text{O}_2]/(10^{-2} \text{ mol dm}^{-3})$	0	2.2	5.6	8.1	11
$\tau / (10^{-9} \text{ s})$	2.5	1.6	0.9	0.7	0.6

From these data determine the quenching rate constant.

(5 marks)

c) Answer **BOTH** parts of this question.

- i) With the aid of suitable diagrams and equations describe the three main energy transfer mechanisms. (6 marks)
- ii) A donor-bridge-acceptor energy relay has been prepared to test Forster energy transfer theory. The donor moiety has an emission quantum yield of 80% in solution, which is reduced to 30% when covalently bound to the acceptor. Assuming a Forster radius of 4.5 nm for the donor / acceptor pair estimate the length of the bridge. (4 marks)