

[Overall comments:

Candidates seemed to have sufficient time to complete the paper and the overall results were good. Average for the paper was 62.2% and for each question 23.9/40; 19.4/30 and 18.9/30. Question 2 was something that divided the class with many finding it easy and scoring highly and some not being well prepared at all.]

1. This question covers several topics and all parts should be attempted.

- a) Describe the different roles undertaken by transmission networks and distribution networks. Describe some of the technical and configurational differences between these networks that follow from these different roles.

[6]

[Bookwork]

2 marks for roles; 2 marks technical differences; full marks requires reasons for differences.

Key difference in the roles is that transmission networks are for bulk transmission of power between regions of a country or continent to move power between regions rich in generation to regions rich in load. This is essentially locational arbitrage whereas distribution networks are principally for distributing power from a bulk supply point to the various load customers in the locality. They also have a role in accepting power from small-scale generation customers.

Because transmission operates over long distances and with relatively few connection points, it is cost effective to use EHV (circa 400 kV) in order to reduce losses. The short distances and dense connection points means distribution occurs at a succession of lower voltages. Transmission uses a highly meshed set of double-circuit routes and an N-2 or N-1 redundancy rule to provide a high degree of availability because of the economic impact on large numbers of customers supply outages occur. Distribution uses radial lines double-circuit for large load groups and single for smaller ones which a lower availability but small numbers of customers affected by particular outages. (Could also mention differences in voltage control strategy).

- b) Maintaining the proper rotational speed of generators is important in AC electricity systems.

- i) Describe how it is that, once connected, a generator stays synchronised with the rest of the system.

[4]

[This question referred to synchronous generators in power plants, which require synchronism to operate. A number of candidates explained the operation of an induction machine instead; some wrongly labelled this machine as synchronous. The original intended marking scheme was applied to those describing a synchronous machine. Generously, 1 mark was given to those who successfully described the operation of an induction machine and identified it correctly.]

[Bookwork]

[3 marks for relevant points; 1 mark for constructing logical argument].

A generator is a spinning mass that will accelerate if there is a net torque or, alternatively, a power imbalance. For equilibrium, the power transferred out of the windings into the AC system must match the mechanical power applied by the steam turbine (or other prime mover) and then the rotational speed will hold constant at value equivalent to the AC network frequency. The exported power depends on the sine of the angle difference between the AC network voltage and the generated EMF. If for some reason the machine is perturbed and runs faster than the grid frequency, the angle difference will start increasing; the power exported will start increasing and there will be a net power deficit on the generator which will decelerate the machine back toward equilibrium.

- ii) Describe how the frequency of an AC network is maintained at its target value.

[4]

[This question was skipped by many candidates and some repeated the same discussion as in (b.i), only those giving a basic but argued description of the role of a governor (automatic generation control) gained full marks.]

[Bookwork]

[3 marks for relevant points; 1 mark for constructing logical argument].

A group of generators will stay synchronised as described in (b.i) but their speed can fall or rise as a group if the total prime mover power differs from the consumed electrical power. This speed increase or fall can be readily detected and used to adjust the steam inlet (or similar) to one or more of the generators. Such a device is known as a governor. Typically, the majority of plant are contracted to supply power only and their governors are inactive (by use of a dead-band). A small number of generators are contracted to run at less than full output so as to be able to regulate up or down and have their governors active. In emergencies, all governors will move out of their dead-bands and activate.

- c) A set of three impedances of  $10+j2\ \Omega$  form a delta connected load. Calculate the line current magnitude and real and reactive powers that are drawn by this load from a three-phase supply of 400 V.

[8]

[Standard calculation]

Note that phase voltage = line voltage = 400 V for delta connection [1 mark]

Calculate phase current and line current. [3 marks]

$$Z = 10 + j2 = 10.2 \angle 11.31^\circ$$

$$I_p = \frac{V_p}{Z} = \frac{400}{10.2 \angle 11.31^\circ} = 39.2 \angle -11.31^\circ \text{ A}$$

$$|I_L| = \sqrt{3}|I_p| = 67.9 \text{ A}$$

Calculate power [1 mark for method; 1 mark for each answer plus 1 for correct units]

$$P = \sqrt{3} V_L I_L \cos(\phi) = \sqrt{3} \times 400 \times 67.9 \times \cos(11.31^\circ) = 46.1 \text{ kW}$$

$$Q = \sqrt{3} V_L I_L \sin(\phi) = \sqrt{3} \times 400 \times 67.9 \times \sin(11.31^\circ) = 9.2 \text{ kVar}$$

- d) Consider a Buck switch-mode power supply, SMPS.

- i) Describe the operation of the SMPS.

[5]

[Bookwork]

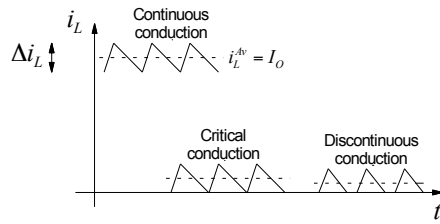
Answer should include description of two different current paths depending on the state of the switch, the role of the diode in providing an alternative path of the transistor; the store and release of energy in the inductor and charging of the capacitor. Can also be described in terms of a chopper giving the correct average voltage followed by a filter.

- ii) Describe the difference between continuous and discontinuous current operation of an SMPS.

[3]

[Bookwork]

In continuous conduction, inductor current ripples up and down around an average value in such a way that the current does not reach zero at its lower point. There is always current in the inductor and the diode must conduct for all of the time the switch is off. In discontinuous mode, the current decreases during the diode conduction time and reaches zero before the switch turns back on. Thus there is a period when the inductor current is zero and neither diode nor switch is conducting. (Diagram not essential but may help)



- iii) Derive an equation for the output voltage of a Buck SMPS in continuous operation, stating the basis for each step.

[5]

[Bookwork]

[2 marks for equations; 3 for assumptions and logic]

Starting assumption is that inductor current is in cyclic steady-state such that there is not net change over a period.

$$\Delta I(on) + \Delta I(diode) = 0$$

Rate of change of current can be found from the voltage appearing across the inductor. A reasonable assumption in most cases is that the voltage drops across the semiconductor are small compared to other voltages present and that the resistance of the inductor is negligible. We also assume that the output capacitor is large enough to allow the output voltage to be treated as constant.

$$\Delta I(on) = t_{on} \times \left. \frac{di}{dt} \right|_{on} = t_{on} \frac{V_i - V_{DS} - V_o}{L} \approx t_{on} \frac{V_i - V_o}{L}$$

$$\Delta I(diode) = t_{diode} \times \left. \frac{di}{dt} \right|_{diode} = t_{diode} \frac{-V_{AK} - V_o}{L} \approx t_{diode} \frac{-V_o}{L}$$

Equating terms and using the fact of continuous conduction to equate diode conduction time to off time allows the following rearrangement

$$\frac{V_o}{V_i} = \frac{t_{on}}{t_{on} + t_{diode}}$$

$$V_o = V_i \frac{t_{on}}{T} = V_i \delta$$

- e) What are the factors that lead to a PV panel not converting all of the incident sunlight energy into electrical energy for use?

[5]

[The purpose of the question was to test candidates' understanding of the physics behind PV generation and the trade-offs of the design of a PV cell. Some candidates failed to identify the band-gap characteristics as a source of loss of generated energy and described other losses of energy in the process of distributing the electrical energy, which was not asked.]

[Bookwork]

There are inefficiencies arising from using a band-gap to absorb energy from photons. Photons with less energy than the band-gap will not be captured (and their energy not converted). Photons with more energy will be captured but only the band-gap fraction is converted to electrical energy; the rest is lost as heat. Some of the converted energy is lost in the resistance of the bulk material and the

metalisation. Some incident sunlight is reflected off the top protective layer of the panel or at other interfaces before the junction is reached. Some of the active region of the panel is obscured by the top-side metalisation strips and photons falling here are not absorbed.

2.

- a) A distribution network operator wishes to establish a distribution connection with a 25 km route and operating at a phase voltage of 20 kV. Two options are being considered, a cable and an overhead line. The parameters of the cable are a resistance per unit length of  $R'_{CAB} = 30 \text{ m}\Omega/\text{km}$  and an inductive reactance of  $X'_{CAB} = 90 \text{ m}\Omega/\text{km}$ . For the overhead line the parameters are  $R'_{OHL} = 50 \text{ m}\Omega/\text{km}$  and an inductive reactance of  $X'_{OHL} = 30 \text{ m}\Omega/\text{km}$ . Estimate the voltage drop expected of each route option when the power consumed at the receiving end is 40 MW (per phase) at a power factor of 0.85 lagging. Neglect the shunt conductance effect in both cases.

[10]

[Intention was for the word “estimate” to clear the way for using the approximate voltage drop equation and candidates who did that found the answer easily.]

Those who attempted to use the full equations found it very difficult. One has to solve the P and Q equations simultaneously (because  $V_R$  and  $\delta$  are both unknown) – this is very difficult since the equations are non-linear. Some people assumed  $\delta$  was the power factor angle which is simply incorrect.

Some candidates estimated the line current from the apparent power and multiplied this by the line impedance to get a voltage drop. This does give the change in voltage magnitude as was wanted but the question wording was perhaps not clear enough here. So, if this approach was applied correctly, it gained full marks.]

[Application of standard equations]

Approximate voltage drop equation is sufficient.

First calculate the parameters for the full route length [2 marks]

$$R_{CAB} = R'_{CAB} \times l = 0.03 \times 25 = 0.75 \Omega$$

$$X_{CAB} = X'_{CAB} \times l = 0.09 \times 25 = 2.25 \Omega$$

$$R_{OHL} = R'_{OHL} \times l = 0.05 \times 25 = 1.25 \Omega$$

$$X_{OHL} = X'_{OHL} \times l = 0.03 \times 25 = 0.75 \Omega$$

Next find the reactive power [4 marks]

[An occasional mistake here was to assume that the 40 MW was the value for S and then use  $\cos()$  and  $\sin()$  to find P and Q. The units of megawatt make it plain that the value is the real power P. An other occasional mistake was to take the negative angle. Lagging power factor means lagging current. With current behind the voltage the power factor angle (and impedance angle) is positive.]

$$Q = S \sin \phi$$

$$S = \frac{P}{pf} = \frac{40 \text{ M}}{0.85} = 47.06 \text{ MVA}$$

$$\phi = \cos^{-1}(0.85) = 31.79^\circ$$

$$Q = 47.06 \text{ M} \times \sin(31.79) = 24.79 \text{ MVar}$$

Then apply voltage drop equation [4 marks]

$$\Delta V \approx \frac{RP_s + XQ_s}{|V_s|}$$

$$\Delta V_{CAB} \approx \frac{R_{CAB}P_s + X_{CAB}Q_s}{|V_s|} = \frac{0.75 \times 40M + 2.25 \times 24.79M}{20k} = 4.28kV$$

$$\Delta V_{OHL} \approx \frac{R_{OHL}P_s + X_{OHL}Q_s}{|V_s|} = \frac{1.25 \times 40M + 0.75 \times 24.79M}{20k} = 3.42kV$$

- b) A 3-phase, 2 pole-pair, induction machine is star-connected and provided with a supply with a phase voltage of 500 V, 50 Hz. The winding parameters of the equivalent circuit of the machine, referred to the stator, are:

stator resistance	1.0 $\Omega$ ,
stator leakage reactance	3.0 $\Omega$ ,
referred rotor resistance	0.75 $\Omega$ ,
referred rotor leakage reactance	2.0 $\Omega$ .

The magnetising reactance and resistance will be neglected.

When driving a particular mechanical load, the machine is observed to spin at 1,440 rpm.

- i) Calculate the slip.

[4]

[Standard calculation]

$$\omega_s = \frac{2\pi f}{p} = 50\pi \text{ rad/s}$$

$$n_s = \frac{f \times 60}{p} = 1,500 \text{ rpm}$$

$$s = \frac{1500 - 1440}{1500} = 0.04$$

- ii) Calculate the stator current in complex form.

[6]

[Standard calculation but with major simplification of ignoring magnetising components. 3 marks for method; 3 marks for accurate numerical answer]

[A very few candidates misunderstood reactance and multiplied the values by  $2\pi f$  as if the value had been an inductance (in henries) not a reactance (in ohms)]

$$Z_T = R_s + jX_s + \frac{R'_r}{s} + jX'_r$$

$$Z_T = 1.0 + j3.0 + \frac{0.75}{0.04} + j2.0 = 19.75 + j5.0 \Omega$$

$$I_s = \frac{V_s}{Z_T} = \frac{500}{19.75 + j5.0} = \frac{500}{20.37 \angle -14.21^\circ} = 24.55 \angle -14.21^\circ \text{ A}$$

Equivalent answer in Cartesian form is 23.79 + j6.02 A

iii) Calculate the electro-magnetic torque.

[4]

[Variation of standard calculation]

[Units are  $Nm$  and not  $Nm^{-1}$  which would be very different. One must use the square of the current magnitude. This is not the same as the magnitude of the square. Using the square of the current and obtaining a complex torque is certainly incorrect.]

Note that rotor current equals stator current if magnetising current is being neglected.

$$\begin{aligned} T &= 3 |I'_R|^2 R'_R \frac{(1/s - 1)}{\omega_R} \\ &= 3 \times 24.55^2 \times 0.75 \times \frac{(1/0.04 - 1)}{1440 \times 2\pi/60} \\ &= 215.8 \text{ Nm} \end{aligned}$$

iv) Calculate the efficiency (ignoring magnetising and friction losses).

[6]

[Standard calculation possible via several routes]

Input power [2 mark]

$$\begin{aligned} P_{In} &= 3V_s I_s \cos(\phi) = 3 \times 500 \times 24.55 \times \cos(14.21^\circ) \\ &= 35.70 \text{ kW} \end{aligned}$$

Output power [2 mark]

$$\begin{aligned} P_{Mech} &= 3 |I'_R|^2 R'_R (1/s - 1) \\ &= 3 \times 24.55^2 \times 0.75 \times (1/0.04 - 1) \\ &= 32.55 \text{ kW} \end{aligned}$$

Efficiency [1 mark plus 1 mark for good accuracy]

$$\eta = \frac{P_{Mech}}{P_{In}} = \frac{32.55k}{35.70k} = 91.2\%$$

3.

A boost SMPS has the following design properties.

Input voltage	$V_I = 6.0 \text{ V}$
Output voltage	$V_O = 24.0 \text{ V}$
Maximum output current	$I_O^{\max} = 1.0 \text{ A}$
Switching frequency	$f = 50 \text{ kHz}$
Inductor	$L = 1.5 \text{ mH}$
Capacitor	to be determined

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- a) Consider the boost SMPS operating in continuous mode.
- i) Calculate the duty-cycle at which the circuit should be operated to achieve an output voltage of  $V_O = 24.0 \text{ V}$ .

[3]

[Manipulation of standard equation]

$$\begin{aligned}\frac{V_O}{V_I} &= \frac{1}{1-\delta} \\ V_O - V_O\delta &= V_I \\ \delta &= \frac{V_O - V_I}{V_O} = \frac{24-6}{24} \\ &= \frac{3}{4} = 0.75\end{aligned}$$

- ii) Calculate the ripple component of the inductor current.

[3]

[Standard Equation]

[Occasional mistake was to use the output voltage in this equation]

$$\begin{aligned}\Delta I_L &= \frac{V_I}{L} \frac{\delta}{f} = \frac{6 \times \frac{3}{4}}{1.5\text{m} \times 50\text{k}} \\ &= 0.06 \text{ A}\end{aligned}$$

- iii) Find the average value of the current through the diode and the average current through the inductor when the load draws the maximum output current.

[4]

[Application of basic principle]

Average diode current must equal average output current [2 marks]

$$I_D = I_O = 1.0 \text{ A}$$

Average inductor current must equal the input current which can be found from conservation of power if SMPS is assumed lossless. [2 marks]

$$I_L = I_I = I_O \frac{V_O}{V_I} = 1.0 \times \frac{24}{6} = 4.0 \text{ A}$$

- iv) By assuming that the voltage ripple across the effective series resistance (ESR) of the capacitor is much larger than that across the capacitance, specify an ESR to achieve a voltage ripple at the output of less than 50 mV.

[5]

[Application of standard method: 3 marks for method, 2 for accurate answer]

Resistive term is peak-to-peak capacitor current (equal to ptp inductor current) multiplied by ESR



$$R_{ESR} < \frac{\Delta V_R}{I_L + \frac{1}{2}\Delta I_L} = \frac{50m}{4.0 + \frac{1}{2} \times 0.06} \\ < 12.4 \text{ m}\Omega$$

- b) The SMPS is constructed with a MOSFET which has an on-state resistance of  $R_{DS(on)}$  of  $0.2 \Omega$  and turn-on and turn-on and turn-off energy losses of  $5 \mu\text{J}$  and  $7 \mu\text{J}$ , respectively. The diode has a forward conduction voltage of  $V_{AK(on)}$  of  $0.8 \text{ V}$ .

- i) Calculate the power loss in the MOSFET when the SMPS operates at maximum output power.

[4]

[Standard calculation; 2 mark for method; 2 for accuracy]

[Most common problem was to neglect the switching loss term. Some candidates did not use the correct current for the conduction power loss.]

Power loss in a MOSFET is sum of conduction and switching loss. This MOSFET carries the inductor current of  $4.0 \text{ A}$  when conducting but does so only for  $0.75$  of the period.

$$P_{Loss-M} = \delta I_{DS}^2 R_{DS} + (E_{On} + E_{Off}) f = 0.75 \times 4^2 \times 0.2 + 12 \times 10^{-6} \times 50 \times 10^3 = 2.4 + 0.6 = 3.0 \text{ W}$$

- ii) Calculate the power loss in the diode when the SMPS operates at maximum output power.

[3]

[Standard calculation; 2 mark for method; 1 for accuracy]

Power loss in a diode is sum of conduction loss only. This diode carries the inductor current of  $4.0 \text{ A}$  when conducting but does so only for  $0.25$  of the period.

[Since this loss is a linear function of current (unlike the  $I^2 R$  term for a Mosfet), it is possible to multiply the average current ( $1 \text{ A}$ ) by the voltage drop ( $0.8 \text{ V}$ ) to get the right answer. Using average current and still including the  $(1-\delta)$  term applies the averaging over a cycle twice and so is incorrect.]

$$P_{Loss-D} = (1-\delta) I_{AK} V_{AK} = 0.25 \times 4 \times 0.8 = 0.8 \text{ W}$$

- iii) Calculate the maximum thermal impedance that can be allowed between the MOSFET and ambient air (at  $30^\circ\text{C}$ ) if the MOSFET junction temperature is not to exceed  $100^\circ\text{C}$ .

[3]

[Standard calculation; 2 marks for method; 1 for accuracy, including units]

[Question asked for Mosfet only, assuming the diode was on a separate heat-sink, so it was incorrect to include the diode losses.]

Temperature rise proportional to heat power and thermal impedance.

$$R_{Th-JA} = \frac{T_J - T_A}{P_{Loss-M}} = \frac{100 - 30}{3} = 23.3 \text{ K/W}$$

- c) A suggestion is made that the components of this SMPS could be made smaller if the switching frequency were increased. Comment on what you would expect to be achieved in the size of inductor, capacitor and heat sink for the redesigned power supply.

[5]

[Interpretation of new problem, 1 mark for making each of 3 basic points and 2 mark for quality of those points.]

[Most common mistake was to say that the capacitor could decrease in size with frequency when in this design it was limited by the ESR.]

If the objective is to achieve a similar electrical performance in terms of ripple in components then the inductor value (an approximate size) could be decreased in proportion to the frequency increase. As equation in (a)(ii) shows [1 mark].

The equation in (a)(iv) shows that frequency does not affect the capacitor choice if the capacitor choice is based on ESR (not even indirectly if ripple current is unchanged, and ripple current is a minor factor anyway) so no saving is made here. [1 mark]

The power losses increase significantly with frequency so the heatsink would get larger [1 mark]