IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

EEE PART I: MEng, BEng and ACGI

Corrected copy

ENERGY CONVERSION

Friday, 16 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer All questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : O. Sydoruk
Second Marker(s) : B.C. Pal

Formula sheet

Maxwell's equations in integral form

$$\oint_l (\mathbf{E} \cdot \mathrm{d}\mathbf{l}) = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_S (\mathbf{B} \cdot \mathrm{d}\mathbf{S})$$
$$\oint_l (\mathbf{H} \cdot \mathrm{d}\mathbf{l}) = \iint_S (\mathbf{J} \cdot \mathrm{d}\mathbf{S}) + \frac{\mathrm{d}}{\mathrm{d}t} \iint_S (\mathbf{D} \cdot \mathrm{d}\mathbf{S})$$
$$\oint\!\!\!\oint_S (\mathbf{D} \cdot \mathrm{d}\mathbf{S}) = \iiint_V \rho \mathrm{d}V$$
$$\oint\!\!\!\oint_S (\mathbf{B} \cdot \mathrm{d}\mathbf{S}) = 0$$

Gauss's law for electric fields in differential form, Cartesian coordinates

$$\frac{\partial D_x(x,y,z)}{\partial x} + \frac{\partial D_y(x,y,z)}{\partial y} + \frac{\partial D_z(x,y,z)}{\partial z} = \rho(x,y,z)$$

Gauss's law for electric fields in differential form, centrosymmetric distributions, spherical coordinates

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2 D(r)) = \rho(r)$$

Electric flux density and field strength: $\mathbf{D} = \varepsilon_0 \varepsilon_d \mathbf{E}$. Magnetic flux density and field strength: $\mathbf{B} = \mu_0 \mu_r \mathbf{H}$.

Coulomb's law

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0\varepsilon_d r^3}\mathbf{r}$$

The Biot-Savart law

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{[\mathrm{d}\mathbf{l} \times \mathbf{r}]}{r^3}$$

Voltage, potential

$$U_{AB} = \varphi(\mathrm{A}) - \varphi(\mathrm{B}) = \int_{\mathrm{A}}^{\mathrm{B}} (\mathbf{E} \cdot \mathrm{d}\mathbf{l})$$

Electrostatic energy

$$W = \frac{1}{8\pi\varepsilon_0} \sum_{i \neq j} \frac{q_i q_j}{r_{ij}}$$

Capacitance: $C = q/U$. Inductance: $L = \Phi/I$. Force on a charge in electric field: $\mathbf{F} = q\mathbf{E}$; in magnetic field: $\mathbf{F} = q[\mathbf{v} \times \mathbf{B}]$.

Rotating machines. Torque, definition (force perpendicular to arm): $T = Fa$. Torque for a motor with N coils: $T = K\Phi I$, where $K = 2N/\pi$. Back-emf: $e = K\Phi\omega$.

Useful integrals

$$\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{a^2+x^2}) \qquad \int \frac{\mathrm{d}x}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}} \qquad \int \frac{x\mathrm{d}x}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$$

ENERGY CONVERSION

1. a) A linear charge distribution has a length l, and its left end is placed at the origin, as shown in Figure 1.1. The charge density is $\lambda = \lambda_0 \cos(x/l_0)$, where λ_0 and l_0 are constants. Find the total charge of the distribution. [7]

Figure 1.1

b) When a perfect conductor is placed into a static electric field, vector **E** is always directed along the normal to the surface of the conductor. Explain why it follows from here that all points of the conductor surface have the same potential. [8]

c) Figure 1.2 shows major hysteresis loops for two ferromagnetic materials, denoted by labels '1' and '2'. Explain which of the materials will make a stronger permanent magnet? [5]

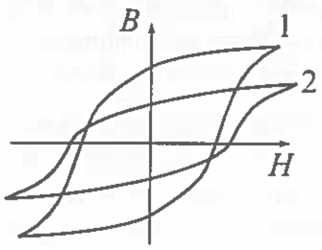

Figure 1.2

d) Explain the reason for laminating ferromagnetic materials used in transformer cores. [7]

e) Figure 1.3 shows two identical current-carrying circular loops in axial and planar configurations. Reasoning qualitatively, explain which of these configurations can provide better power efficiency when used for inductive power transfer. [8]

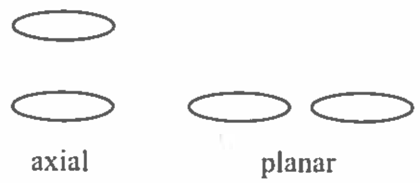

Figure 1.3

f) Starting from the full set of Maxwell's equations show that static electric charges create no magnetic fields. [5]

2. A parallel-plate capacitor is filled with air. It has the area of a plate equal to S, and the distance between the plates equal to d. It is connected to a source with a constant voltage U, as shown in Figure 2.1. Assume that the capacitor plates create the same field as infinitely large planes (i.e. ignore fringing fields). Starting from Gauss's law, find expressions for the following.

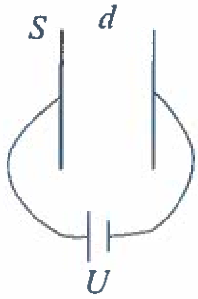

Figure 2.1

a) The electric field strength between the plates. [5]

b) The electric potential in the space between the plates. [5]

c) The capacitance of the capacitor. [5]

d) A uniform dielectric is inserted between the capacitor plates. Assuming that the atoms in the dielectric are polarisable electric dipoles, explain how bound charges appear on the dielectric surfaces. [7]

e) Explain the difference in behaviour of bound charges on the dielectric surfaces and free charges on the capacitor plates. By choosing a surface enclosing both types of charges, write down the expression for Gauss's law. Assume that the surface charge density of the bound charges is proportional to the electric field strength (i.e. $\sigma_{\text{bound}} = \beta E$, where β is a constant) and show how the relative permittivity of the dielectric is introduced. [8]

3. Figure 3.1 shows a schematic of a time-of-flight mass-spectrometer.

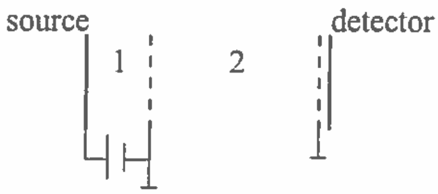

Figure 3.1

a) Explain qualitatively the functions of the regions marked 1 and 2 in Figure 3.1. [10]

b) Write the equations describing the movement of an ion with charge q and mass m inside the two regions. Why is high vacuum required for the operation of a mass-spectrometer? [5]

c) Ion-mobility spectrometers can operate in a gas at the atmospheric pressure. Explain, using qualitative reasoning and calculations, the differences in the motion of ions in an ion-mobility spectrometer and a mass-spectrometer. [10]

d) What is the most important factor limiting the resolution of an ion-mobility spectrometer operating at the atmospheric pressure? [5]