IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE/EIE PART III/IV: MEng, BEng and ACGI

Corrected copy

DIGITAL SIGNAL PROCESSING

Monday, 14 December 9:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.A. Naylor
Second Marker(s) : W. Dai

DIGITAL SIGNAL PROCESSING

1. a) Given a discrete-time signal $x(n)$, write down the formula for the z-transform $X(z)$ and hence find the z-transform of

$$x(n) = [-10,\ \underset{\uparrow}{10},\ 5,\ 2].$$

. [2]

b) Consider a linear system with system function $H(z)$. Explain what is meant by the term *Region of Convergence* in the context of the z-transform and state the relationship between the *Region of Convergence* and the *stability* of $H(z)$. [2]

c) Next consider

$$P(z) = \frac{1}{1 - pz^{-1}}$$

and the unit step function $u(n)$.

i) If the inverse z-transform of $P(z)$ corresponds to a causal signal, write an expression in the discrete-time domain for this causal signal and state the *Region of Convergence.* [1]

ii) If the inverse z-transform of $P(z)$ corresponds to an anticausal signal, write an expression in the discrete-time domain for this anticausal signal and state the *Region of Convergence.* [1]

d) For the causal system

$$Q(z) = \frac{5z + 15.4}{z^2 + 5.2z + 1}$$

find the inverse z-transform of $Q(z)$ and explain whether or not $Q(z)$ is stable. [5]

e) The block diagram of Figure 1.1 shows a discrete-time system with system function $H(z)$. The z-transforms of the input signal $x(n)$ and the output signal $y(n)$ are $X(z)$ and $Y(z)$ respectively. Evaluate the system function $H(z)$ in terms of the constant scalar coefficients d_1, d_2 and state the key property of $H(z)$. [9]

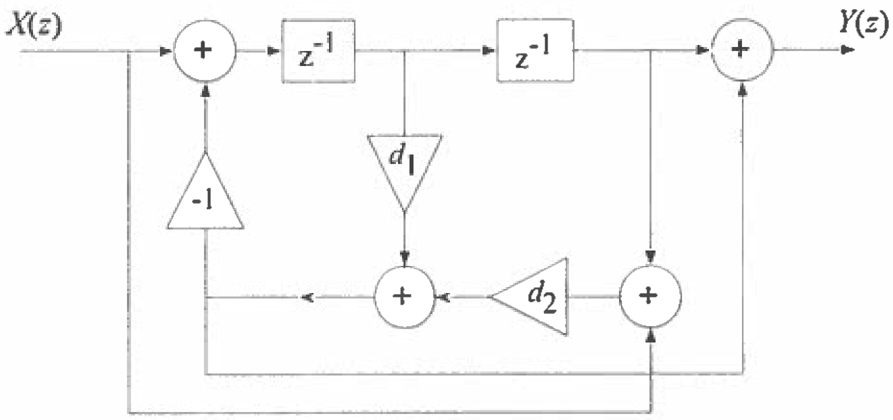

Figure 1.1 Signal flow graph

2. Consider a discrete-time filter with finite impulse response for which the input is denoted $x(n)$, the output is denoted $y(n)$, n is the discrete-time index and

$$y(n) = -0.5x(n) - 0.45x(n-2).$$

a) Draw a labelled sketch plot of the z-plane and indicate on the plot the positions of the poles and zeros of this filter. [4]

b) Write an expression for the transfer function of the filter. [2]

c) Write an expression for the magnitude of the frequency response of this filter. [3]

d) Draw a labelled sketch of the magnitude of the frequency response of this filter and mark on the sketch the values of the magnitude in dB at frequencies of 0, $\pi/2$, and π. [6]

e) Define group delay for discrete-time filters and estimate the group delay of this filter in seconds at a frequency of 2 kHz given that the sampling frequency is 16 kHz. [5]

3. a) Consider the discrete-time signal $x(n) = 2a^n u(n)$ where $u(n)$ is the unit step function and $|a| < 1$.

i) Write down an expression for the spectrum of this signal. [2]

ii) In an example of a multirate signal processing system, the signal $x(n)$ is decimated by a factor of 2. Explain, with an illustrative sketch, the effect that such decimation has in the frequency domain and hence determine the spectrum of the signal after downsampling. [3]

b) i) State and explain the Noble Identities. [3]

ii) Figure 3.1 shows two multirate signal processing systems.

Denoting the input samples as $x(n) = \{x0,\ x1,\ x2,\ \ldots\}$ and $y(n) = \{y0,\ y1,\ y2,\ \ldots\}$, find the corresponding samples at the points in the Figure marked A, C and, hence, the first 6 samples at each of the points marked B and D. Comment on the importance of the order of operation of multirate processing blocks such as those in Figure 3.1. [6]

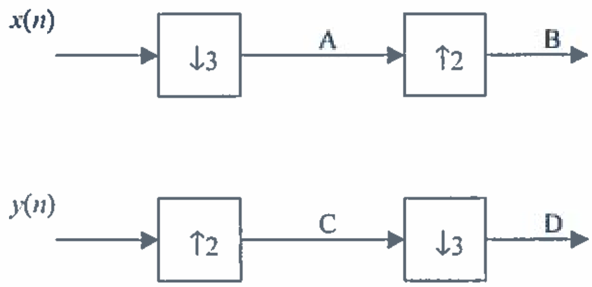

Figure 3.1 Multirate signal processing systems.

c) Consider a DSP system operating on an input signal $x(n)$ sampled at a rate of 1000 samples per second. Using multirate signal processing techniques, design a system to delay $x(n)$ by 300 μs. Show your design in terms of a labelled block diagram together with a detailed explanation of its operation. [6]

4. Consider a discrete-time signal $x(n)$ of length N samples and having N-point DFT $X(k)$.

a) If $X(k)$ is a real sequence, what conditions must be satisfied by $x(n)$, assuming $x(n)$ is real? Give an example for $x(n)$ of length 6 samples for which $X(k)$ is real. [4]

b) Let $x(n) = [1,\ -1,\ 0,\ 2]$. Calculate $X(k)$. [6]

c) If $x(n)$ satisfies the condition $x(n) = x(N-1-n)$ and N is an even number, show that $X(N/2) = 0$. [3]

d) If $x(n) = -x(N-1-n)$, show that $X(0) = 0$. [3]

e) Now consider the magnitude and phase of $X(k)$. If $x(n)$ is real, what conditions must be satisfied by the magnitude and phase of $X(k)$. Draw a labelled sketch of the magnitude and phase spectra of $X(k)$ for an illustrative example of a case for which $x(n)$ is real. [4]