M3S1/M4S1

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March 27, 2014

BSc and MSci EXAMINATIONS (MATHEMATICS)

May-June 2014

M3S1/M4S1 Statistical Theory I

- 1. (a) (i) What is the sufficiency principle?
 - (ii) What is its importance?
 - (iii) Explain what is meant by a *minimal sufficient statistic* for a family of distributions parameterised by an unknown parameter θ .
 - (b) For cases (i) and (ii) below find, giving your reasoning, a minimal sufficient statistic for positive θ .
 - (i) From a random sample $x = \{x_1, x_2, \dots, x_n\}$ having the probability density function

$$f(x|\theta) = \begin{cases} \theta e^{\theta(\theta-x)} & (x > \theta), \\ 0 & (x \le \theta). \end{cases}$$

(ii) In bio-assay, $P(\text{positive response at dosage }z) = P(X=1\,|\,z,\theta) = \frac{e^{\theta z}}{1+e^{\theta z}}$ Here X_1,X_2,\ldots,X_n are independent Bernoulli random variables with

$$P(X_k = x_k | z_k, \theta) = \frac{e^{\theta z_k x_k}}{1 + e^{\theta z_k x_k}} \quad (x_k \in \{0, 1\})$$

where $z = \{z_1, z_2, \dots, z_n\}$ are known constants.

- 2. (a) What is the monotone likelihood ratio criterion? What is its importance?
 - (b) (i) From the combined independent random samples $\boldsymbol{x} = \{x_1, x_2, \dots, x_n\}$ from $Poisson(\theta)$ and $\boldsymbol{y} = \{y_1, y_2, \dots, y_n\}$ from $Poisson(c\theta)$, where c > 0 is a known constant, show that $t(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=1}^{n} (x_i + y_i)$ is sufficient for θ .
 - (ii) Find the most powerful size α test of $H_0: \theta \leq 1$ against $H_1: \theta > 1$, where α is small. Give your reasoning.
 - (iii) Consider the size α test of $H_0^*: \theta = 1$ against $H_1: \theta > 1$. Obtain a normal approximation for the distribution of T = t(X, Y) for given θ and large n.

If ξ is such that $\alpha = P(T > \xi | \theta = 1)$, find an approximation for ξ . Obtain an approximation to the power function $\beta(\theta)$.

- 3. (a) What is meant by a pivotal quantity?
 - (b) Let $x = \{x_1, x_2, \dots, x_n\}$ be a random sample from a distribution having probability density function

$$f(x|\theta) = \begin{cases} \frac{x}{\theta} \exp\left(-\frac{1}{2} \frac{x^2}{\theta}\right) & (x > 0), \\ 0 & (x \le 0), \end{cases}$$

where θ is an unknown positive parameter.

- (i) Obtain the efficient total score $U_{\bullet}(\theta)$.
- (ii) From the form of $U_{\bullet}(\theta)$ write down
 - the total Fisher information $I_{\bullet}(\theta)$,
 - the maximum likelihood estimator $\widehat{\theta}$ of θ ,
 - $\operatorname{var}(\widehat{\theta})$.
- (iii) Which theorem guarantees that $var(\widehat{\theta})$ minimises the variance over all unbiased estimators of θ ?
- (iv) Show that $Z = \hat{\theta}/\theta$ is a pivotal quantity having a Gamma(n, 1) distribution.
- (v) From (iv) construct a $100(1-\alpha)\%$ confidence interval for θ having equal tail probabilities for small α .
- 4. (a) State the Lehmann-Scheffé Theorem for finding a minimum variance unbiased estimator (MVUE).
 - (b) For a random sample $x = \{x_1, x_2, \dots, x_n\}$ from the Delayed Exponential distribution having probability density function

$$f(x|\theta) = \begin{cases} e^{-(x-\theta)} & (x > \theta), \\ 0 & (x \le \theta), \end{cases}$$

by considering the pivotal quantity $Z=X-\theta_{\rm i}$ or otherwise, find the distribution of $X_{\rm min}$.

Find an unbiased estimator of θ that is a function of X_{\min} alone.

- (c) (i) Find a non-zero function h(t), for which $E\{h(T)\}=0$, to show that the $Uniform(-\theta,\theta)$ family of distributions, where $\theta>0$, is not complete.
 - (ii) Let $x = \{x_1, x_2, \dots, x_n\}$ be a random sample from $Uniform(-\theta, \theta)$ $(\theta > 0)$, and let the prior probability for θ be Pareto with probability density function

$$\pi(\theta \mid \alpha, \beta) = \frac{\beta \alpha^{\beta}}{4\beta+1} H(\theta > \alpha)$$

where α and β are known positive constants, and H(A)=1 if A is true, and 0 if A is false.

Show that the posterior distribution for θ is $Pareto(\alpha^*, \beta^*)$, where α^* and β^* are to be determined.

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Question		Marks & seen/unseen
Parts a) i)	The sufficiency principle is that: if statistic t is sufficient for the family of distributions parameterised by B, then analysis of the data	Borhwork
ùλ	should be only through t. Possible responses regarding its importance are: " the reduction of a set of data to t.	2
ùĭ)	etc eg criteria for most powerful tests of composite hypotheses, makes " Edentifying MVIE A minimal sufficient statistic is a function of every sufficient statistic, so any further reduction	2.
	would not yield a sufficient statistic. It is essentially unique through the equivalence relation $t(x) = t(y)$ if likelihood satural $\ell(\theta;x)/\ell(\theta;y)$ does not depend on θ .	2
Ы) i)	$\ell(\theta; x) = (\theta e^{\theta^2})^n e^{-n\theta x} H(x_{min} > \theta)$ $= g(\theta, x) = g(\theta, x, x_{min})$ so π , x_{min} are jointly sufficient for θ by Neyman frestonisation. $\ell(\theta; x) = \frac{1}{2} (\theta, x) + \frac{1}{2$	Unscen
رين	$\frac{\ell(\theta;z)}{\ell(\theta;y)} = e^{-n\theta(x-y)} \frac{H(x_{min}>\theta)}{H(y_{min}>\theta)}$ This does not depend on θ when $x=y$ & $x_{min}=x_{max}$ $\ell(\theta;x,z) = e^{\theta Z z_{k} x_{k}} / \prod (1+e^{\theta z_{k}})$ So $t(x) = \sum_{k} x_{k}$ is sufficient for θ .	7
	Minimal sufficient because dimension 1. cannot be reduced fruither. At logich $L(\theta; \mathbf{x}, \mathbf{z}) = \theta \mathbf{z}_{\mathbf{z}} \mathbf{x}_{\mathbf{k}} - \mathbf{z} \ln(1 + e^{\theta \mathbf{z}_{\mathbf{k}}})$ $L(\theta; \mathbf{x}, \mathbf{z}) - L(\theta; \mathbf{y}, \mathbf{z}) = \theta \mathbf{z}(\mathbf{x}) - \mathbf{t}(\mathbf{y}) \mathbf{z}$ does not dynadate if too = tiy. Setter's initials Checker's initials	7
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Parts a)	The monotone likelihood ratio initerior holds if the likelihood vatio $\lambda(x)$ is a non-increasing	Bothwork
	for non-devicating) function of $t(x)$, a rafficient statistic for θ .	
	Its importance is that if the criberion is satisfied, the test is UMP.	3
b) i).	$f_{X,Y}(X,y \theta) = \{(\theta; x,y,c) = \prod_{i=1}^{n} \left\{ \frac{e^{x_i} e^{-\theta}}{x_i!} \cdot \frac{(i\theta)^{y_i} e^{-c\theta}}{y_i!} \right\}$ $= \frac{e^{zy_i}}{\prod_{i=1}^{n} y_{i!}} \theta^{z(x_i+y_i)} e^{-(c+1)n\theta}$	Uascen
	Let $E(x,y) = E(x_i + y_i) \otimes w = (c+1)n$ then E is sufficient for θ by Neyman factorisation.	3
(1)	tightenhouse L(B; Z, M) = the U - WO	
	$\frac{\partial L}{\partial \theta} = \frac{t}{\theta} - w$, $\frac{\partial^2 L}{\partial \theta^2} = -\frac{t}{\theta^2} < 0$ so $L \uparrow as \theta \uparrow$ So enterior is satisfied. (Note: mle $\theta = \frac{t}{w}$)	5
	Note: $H_0: \theta = 0 < v. H_1: \theta = 0 > 1$ is MP by Neymann- Holds for all θ . θ .	
ü <i>i</i>)	So the MP test is to reject $H_0: 0 \le 1$ if t is too large, Under $H_0^*: 0 = 1$ v. $H_i: 0 > 1$, $T = Z(X_i + Y_i)$ E(T) = w0, vor $(T) = w0$ (by independence)	- -
	$Z = \frac{T - E(T)}{\sqrt{Var(T)}} = \frac{T - w\theta}{\sqrt{w\theta}} \sim N(0, 1) \text{ (for large n)}$	
	$P(T>\xi \theta) = P(Z>\frac{\xi-w\theta}{Jw\theta}) \sim 1 - \overline{P}(\frac{\xi-w\theta}{Jw\theta})$ Although T is integer valued (as inverted we come	
	Atthough T is integer valued, for large n we can approximate ξ . If $\theta = 1$, for given size α , $\alpha \approx 1 - \sqrt[4]{(\frac{\xi - w}{\sqrt{u}})}$	
	i.e. 5 ≈ w + √√ (1-α)	
	$s_{p} \beta(\theta) \approx 1 - \tilde{Q} \left(\frac{\tilde{s} - w\theta}{J_{we}} \right)$	7
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Question 3		Marks & seen/unseen
Parts a)	A juvital quantity $Z(X,B)$ has a known sampling distribution that does not depend on B	Sten 2
b) i)	$ \ln f = \ln x - \ln \theta - \frac{1}{2} \frac{x^2}{\theta} $ $ \frac{\partial \ln f}{\partial \theta} = -\frac{1}{\theta} + \frac{1}{2} x^2 \left(\frac{1}{-\theta^2}\right) \text{so} \mathcal{U}(\theta) = \frac{1}{\theta^2} \left(\frac{1}{2} \chi^2 - \theta\right) $	unreen
	$U_{s}(\theta) = \frac{n}{\theta^{2}} \left(\frac{1}{2} \overline{X^{2}} - \theta \right)$	4
<i>ii</i>)	$I_{\lambda}(\theta) = \frac{n}{\theta^{2}}$	2
:	MLE 0 = ME = = = = = = = = = = = = = = = = =	2
	$Var(\hat{\theta}) = 1/I, (\theta) = \frac{\theta^2}{n}$	2
iii) iv)	Cramer-Rao Theorem	2
,	Let $y = \frac{x^2}{2\theta}$ $dy = \frac{x}{\theta}dx$ $\int_0^x f(x_c \theta)dx_0 = \int_0^x e^{-\frac{1}{20}x_c^2} \frac{1}{\theta}x_cdx_0$ $= \int_0^y e^{-y_c}dy_c = 1 - e^{-y}$ so $Y = \frac{X^2}{2\theta}$ is Exponential(1)	
	$Z = \sum_{i=1}^{n} Y_{i}^{2} = \frac{1}{20} \sum_{i=1}^{n} X_{i}^{2} = \frac{\hat{\theta}}{\theta} \text{ is } Gamma(n, 1)$	3
ν)	$\frac{1}{2} \alpha = P(\hat{P} < c) = P(\hat{P} > \hat{P})$	
	so the $100(1-\alpha)\%$ CI for θ is $(\frac{\hat{\theta}}{9}, \frac{\hat{\theta}}{6})$	3
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Question A Question A Parts a) If 5 is a complete sufficient statistic, then any function of S is a MVUE of its expectation. b) Pivot $Z=X=0$ is Exponential(1) $P(Z_{min}>z)=P(\text{each }Z;>z)=(e^{-z})^n=e^{-nz}$ (z>0) 50 Z_{min} is Exponential(n) $E(Z_{min})=\frac{1}{n}$ so $E(n(X_{min}-\theta))=1$ i.e. $E(X_{min}-\frac{1}{n})=\theta$ 50 X_{min} is an unbiased estimator of θ and is a function of n atome. c)) $E(X)=0$, $E(X)=0$ for example. 3 $E(X)=0$, $E(X)=0$ for example. $E(X)=0$, $E(X)=0$ for example. $E(X)=0$, $E(X)=0$		M3S1/M4S1 EXAMINATION SOLUTIONS	M3S1
Question A Parts a) If S is a complete sufficient statistic, then any function of S is a MVUE of its expectation. b) Pivot $Z = X - \theta$ is Exponential(1) $P(Z_{min} \times z) = P(\text{each } Z_i \times z) = (e^{-Z})^n = e^{-nZ} (z \times 0)$ 50 Z_{min} is Exponential(n) $E(Z_{min}) = \frac{1}{n}$ so $E(n(X_{min} - \theta)) = 1$ i.e. $E(X_{min} - \frac{1}{n}) = \theta$ so X_{min} if an embrasced estimator of θ and is a function of n atome. c) $E(X) = 0$, $E(X) = 0$ for example. d) $\pi(\theta x, \beta, x) = \frac{px^{\theta}}{\theta^{\theta+1}} H(\theta \times x) \cdot \frac{1}{(2\theta)^n} H(\theta \times x_{min}) H(\theta \times x_{min})$ $C(e^* \alpha^* \theta^*) H(\alpha^* \alpha \otimes x^*)$ where $x^* = \max\{x, x_{min} , x_{max} \}$, $y^* = g + n$. Setter's initials Checker's initials Page number			
Parts a) If S is a complete sufficient statistic, then any function of S is a MVUE of its expectation. b) Pivot $Z = X - \theta$ is Exponential (1) $P(Z_{min} > z) = P(\text{each } Z_1 > z) = (e^{-Z})^n = e^{-nz}$ (2>0) 5. Z_{min} is Exponential (n) $E(Z_{min}) = \frac{1}{n}$ so $E(n(X_{min} - \theta)) = 1$ i.e. $E(X_{min} - \frac{1}{n}) = \theta$ so $X_{min} + \frac{1}{n}$ is an untriasted estimator of θ and is a function of n atome. c) $E(X) = 0$, $E(X) = 0$ for excumple. 3. $\pi(\theta x, \beta, \chi) = \frac{\beta n \theta}{\beta \beta n} H(\theta > \alpha) \cdot \frac{1}{(2\theta)^n} H(\theta > x_{min}) H(\theta > x_{min})$ $C(\theta) = \frac{1}{n} $	Question	Triay Dallo Edit	
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Setter's initials $E(Z_{min}) = \frac{1}{n} \text{ so } E(n(X_{min} - \theta)) = 1$ i.e. $E(X_{min} - \frac{1}{n}) = \theta$ so $X_{min} - \frac{1}{n}$ is an unbriased estimator of θ and is a function of n alone. $E(X) = 0, E(X) = 0 \text{ for exemple.}$ $E(X) = 0, E(X) = 0, E(X) = 0 \text{ for exemple.}$ $E(X) = 0, E(X) = 0, E(X) = 0 \text{ for exemple.}$ $E(X) = 0, E(X) = 0, E(X) = 0 \text{ for exemple.}$ $E(X) = 0, E(X) = 0, E$,	
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Setter's initials $E(X) = 0, E(\overline{X}) = 0 \text{for example.}$ $\pi(\theta \alpha,\beta,\chi) = \frac{\beta \kappa^{\frac{1}{2}}}{\theta \beta^{+1}} H(\theta > \alpha) \cdot \frac{1}{(2\theta)^n} H(-\theta < \epsilon \alpha c h \times \kappa < \theta)$ $\propto \frac{1}{\theta \beta^{+n+1}} H(\theta > \alpha) H(\theta > x_{min}) H(\theta > x_{max})$ $\propto \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x \theta > \alpha^{+})$ $\omega^{+} = \frac{1}{\theta \beta^{+n}} H(\alpha x$		and is a function of n alone.	6
$\frac{1}{\theta \beta + 1} H(\theta > \alpha) \cdot \frac{1}{(2\theta)^n} H(\theta > x_{min}) H(\theta > x_{max})$ $\propto \frac{1}{\theta \beta + 1} H(\theta > \alpha) H(\theta > x_{min}) H(\theta > x_{max})$ $\propto \frac{1}{\theta \beta + 1} H(\theta > \alpha) H(\theta > x_{min}) H(\theta > x_{max})$ where $\alpha^* = \max_{\theta \in \beta} \alpha \cdot x_{min} , x_{max} $, $\beta^* = \beta + n$ Setter's initials Checker's initials Page number	c)i)	E(X) = 0, $E(X) = 0$ for example.	3
Setter's initials $ \frac{G^* x^* x^*}{\theta p^*} + (x^* y^*) + (x^* y^*$	ü)		
Setter's initials Checker's initials Page number		$\propto \frac{1}{\theta \beta^{+n+1}} H(\theta > \alpha) H(\theta > x_{min}) H(\theta > x_{min})$,
Setter's initials Checker's initials Page number		$\propto \frac{B_* \propto 4B_*}{8 B_*} H(\approx 38 \times 4)$	
Setter's initials Checker's initials Page number		Where $\alpha^* = \max\{\alpha, x_{min} , x_{max} \}$	a
		p* = B+n.	, ,
		Setter's initials Chapter's Interior	Page gumber
		GRECKER'S INICIAIS	4 H