Power Engineering

Information for Candidates

Switch-Mode Power Supplies

Voltage ratio equations for buck SMPS $\frac{V_o}{V_i} = \delta$ or $\frac{V_O}{V_I} = \frac{1}{1 + \frac{2fLI_O}{V_I\delta^2}}$

Voltage ratio equations for boost SMPS $\frac{V_o}{V_i} = \frac{1}{1-\delta}$ or $\frac{V_o}{V_i} = \frac{1}{1 - \frac{V_i\delta^2}{2fLI_i}}$

Three-Phase Systems

Line Voltages and Current

Star $V_L = \sqrt{3}V_P \quad I_L = I_P$ Delta $V_L = V_P \quad I_L = \sqrt{3}I_P$

Power
$$P_{3\Phi} = 3V_P I_P \cos(\phi) = \sqrt{3}V_L I_L \cos(\phi)$$

Induction Machines

$$\omega_S = \frac{\omega_E}{P} \quad s = \frac{\omega_S - \omega_R}{\omega_S} \quad T_{em} = \frac{3 I_R'^2 R_R'}{\omega_R}\left(\frac{1-s}{s}\right)$$

Photovoltaic Systems

$$I_{PV} = I_{Ph} - I_{AK} - I_{Sh} \quad I_{AK} = I_0\left[e^{\frac{V_{PV}+I_{PV}R_S}{K_I v_T}} - 1\right]$$

Power Flow in Lines and Cables

Cable Parameters

$$R'_{LF} = \frac{1}{\sigma_C \pi r_C^2} + \frac{1}{\sigma_C 2\pi r_O t_O} \quad L' = \frac{\mu_0}{2\pi}\ln\left(\frac{r_O}{r_C}\right) \quad C' = \frac{2\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(\frac{r_O}{r_C}\right)} \quad G' = \frac{2\pi\sigma_I}{\ln\left(\frac{r_O}{r_C}\right)}$$

OHL Parameters (approximate form)

$$R'_{LF} = \frac{2}{\sigma_C \pi r_C^2} \quad L' = \frac{\mu_0}{\pi}\ln\left(\frac{d}{r_C}\right) \quad C' = \frac{\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(\frac{d}{r_C}\right)} \quad G' = \frac{\pi\sigma_I}{\ln\left(\frac{d}{r_C}\right)}$$

Power Flow (full form)

$$P_S = \frac{V_S^2}{Z_{SR}}\cos(\theta) - \frac{V_S V_R}{Z_{SR}}\cos(\theta + \delta)$$

$$Q_S = \frac{V_S^2}{Z_{SR}}\sin(\theta) - \frac{V_S V_R}{Z_{SR}}\sin(\theta + \delta)$$

Voltage Drop (approximate form)

$$\Delta V = |V_S| - |V_R| \approx \frac{RP_S + XQ_S}{|V_S|}$$

Answers

1. This question covers several topics and all parts should be attempted.

a) Consider a national scale electricity system based on AC.

i) Describe the principal difference between the transmission and distribution networks and explain why transmission networks use higher voltages than distribution networks. [6]

Transmission networks exist to effect bulk transfer of power from geographic regions with more generation than load to regions with more load than generation. They are long distance networks which are built to achieve high availability by having duplicate circuits on all routes and having a high degree of meshing between the nodes. Distribution networks a relatively short distance networks for distributing power within a load centre. They use a variety of voltage levels determined by the volume of load being served. There are double circuit routes at the higher voltages and single circuit routes at the lower voltages but little or no meshing.
[4 marks for mentioning degree of redundancy and availability, difference in function and difference in geographic scale]
The choice of operating voltage is based on achieving the lowest total cost considering operating costs and capital costs. Higher voltages reduce power losses and hence operating costs but have higher capital costs. For long distance routes with relatively few nodes and substations, high voltage achieves a large advantage in power loss for reasonable capital cost. Where distances are short, and connections are many, the lower voltages achieve a better balance between the costs.
[2 marks]

ii) Most countries of the world operate their electricity networks at a nominal frequency of 50 or 60 Hz, however, in practice the frequency is not strictly constant. Explain why the frequency varies a little around its nominal value. [4]

The frequency is determined by the rotational speed of the generators of the system. These generators are locked-in to each other and run in synchronism but their combined speed will still increase if there is a net torque applied to their combine inertia. Thus if there is a mismatch between the power (torque) applied to the generator shafts by the prime movers and the power extracted (reaction torque) by the electrical loads then the speed will change. If there is an excess of electrical load then the "missing" power is extracted from the kinetic energy of the system until such time as extra power is applied from the prime movers.

b) Consider power transfer in AC networks.

i) A single-phase cable operated at 50 Hz and 120 kV has a capacitance per unit length of 0.25 μF/km. Calculate the capacitive charging current for a 75 km length of cable. [2]

Use standard formula and assume that approximately the same voltage is found along the length of the cable.

$$I_C = \frac{V}{X_C} = V \times 2\pi \times f \times d \times C' = 120k \times 2\pi \times 50 \times 75 \times 0.25\mu = 706.9 \quad A$$

ii) A single-phase overhead line has a series impedance which is purely reactive with a value of 18 Ω and is operated at a voltage of 400 kV. Calculate the phase angle that would exist between sending and receiving end voltages when transferring 1,000 MW. [4]

Use standard formula for power transfer across inductive impedance.

$$P = \frac{V^2}{X}\sin(\delta)$$

$$\delta = \sin^{-1}\left(\frac{XP}{V^2}\right) = \sin^{-1}\left(\frac{18\times10^9}{\left(400\times10^3\right)^2}\right) = 6.46^\circ$$

iii) A three-phase load is composed of impedances of 10 + j2 Ω connected in star. It is connected to a supply with a line voltage of 400 V. Calculate the real and reactive power drawn. [4]

[2 marks for method, 2 for accurate answers]
First find phase voltage and phase current.

$$V_P = \frac{V_L}{\sqrt{3}} = 230.9 \quad V$$

$$I_P = \frac{V_P}{Z} = \frac{230.9}{10+j2} = 22.65\angle{-11.3^\circ}$$

$$P = 3V_P I_P\cos(\theta) = 3\times230.9\times22.65\times\cos(-11.3^\circ) = 15.39 \quad kW$$

$$Q = 3V_P I_P\sin(\theta) = 3\times230.9\times22.65\times\sin(-11.3^\circ) = 3.08 \quad kVAr$$

c) Compare buck and boost switch mode power supplies (SMPSs) through the following steps.

i) For a fixed input voltage, sketch the shape of the output voltage as a function of the duty-cycle of the switch (assuming continuous conduction) for each SMPS. [3]

Answer in form graph expected but important aspects of shape are as follows. For Buck case the graph is linear starting at the origin and reaching V_I at δ=1. For Boost case, graph is non-linear, starts at V_I pass through 2 V_I at δ=0.5 and tends to infinity.

ii) Sketch the shape of the currents flowing in the inductor, diode and capacitor for each type of SMPS. [5]

Answer in form graph expected but important aspects of shape are as follows. Inductor current and diode current are similar for Buck and Boost, inductor is DC with triangular ripple, diode is trapezoid with downward slope matching part of inductor current [2 marks]. Capacitor current is different in the two cases, for Buck it is the triangular ripple portion of the inductor current [1 mark], for boost it is the diode current shift down by the output current such that it has an average value of zero [2 marks].

iii) For similar magnitudes of output current and the same choice of capacitor, compare the amplitudes of output voltage ripple. [2]

The trapezoidal pulse of capacitor current in boost is about the amplitude of the input current (which is large than the output current for step up operation). The triangular ripple of the Buck is smaller than the output current. Thus the voltage drop across the ESR and the C is larger in the Boost case.

d) Consider a typical photovoltaic panel.

i) Describe the operating principle of a photovoltaic cell and the role of the band-gap energy in determining the efficiency of the cell. [5]

The cell uses the photoelectric effect in which a photon of sufficient energy can ionise an atom in the lattice of a semiconductor and generate a hole-electron pair. That pair of charge carries are swept out of the depletion region and appear as external current. [2 marks]
To generate the hole-electron pair, the photon must have at least the energy of the semiconductor band-gap. Photons with less energy are not absorbed and this is an aspect of the inefficiency of the cell. Photons with more energy than the band-gap create a pair but only of the energy of gap is gained as useful energy, the remainder is lost by the electron to the lattice as heat which is also an inefficiency. [3 marks]

ii) A PV cell was tested at 25°C. It produced a short-circuit current of 0.8 A under an irradiance of 250 W/m^2. The reverse saturation current of the cell was measured as 1×10^{-10} A. You may assume that the cell has a very small series resistance, a very large shunt resistance and an ideality factor of 1. Boltzmann's constant is $k = 1.380\times10^{-23}$ and the electronic charge is $q = 1.602\times10^{-19}$ C. Calculate the open circuit voltage at irradiances of 250 and 1,000 W/m^2. [5]

In the open-circuit case all of the photoelectric current flows through the diode-like junction governed by the diode equation:

$$I_{AK} = I_0\left[e^{\frac{q V_{AK}}{K_I kT}} - 1\right]$$

Setting K=1 and re-arranging give the equation for the voltage in terms of the current.

$$V_{AK} = \frac{kT}{q}\ln\left[\frac{I_{AK}}{I_0} + 1\right]$$

At 250 W/m^2, the current is 0.8 A and so:

$$V_{AK} = \frac{1.380\times10^{-23}}{1.602\times10^{-19}} ln\left[\frac{0.8}{10^{-10}} + 1\right] = 0.5854$$

[3 marks]

At 1,000 W/m^2, the current is four times greater, i.e. 3.2 A and so:

$$V_{AK} = \frac{1.380\times10^{-23}}{1.602\times10^{-19}} ln\left[\frac{3.2}{10^{-10}} + 1\right] = 0.6209$$

[2 marks]

2.
A **buck** switch-mode power supply, SMPS, is to be designed to provide a 5 V output voltage from a 24 V input. The output current will be in the range 0.1 A to 2.0 A and it is intended that the SMPS operates in continuous conduction mode across this range. It has been decided to design for a switching frequency of 150 kHz.

A suitable MOSFET has been identified for the SMPS with the following properties:

On resistance $R_{DS(on)} = 0.25\ \Omega$
Turn-on loss $E_{on} = 13\ \mu J$ when switched on at 2.0 A
Turn-off loss $E_{off} = 7\ \mu J$ when switched off at 2.0 A

The diode to be used has an on-state voltage of 1.0 V.

i) Calculate the duty-cycle at which the switch will operate. [2]

Use standard formula

$$\delta = \frac{V_O}{V_I} = \frac{5}{24} = 0.2083$$

ii) Calculate the power losses in the semiconductors and the efficiency (ignoring other losses) when operating at maximum output power. [8]

Mosfet conduction loss [2 marks]

$$P_{Cond} = I_O^2 R_{DS} \delta = 2.0^2 \times 0.25 \times 0.2083 = 0.2083 \quad W$$

Mosfet switching loss [2 marks]

$$P_{Sw} = \left(E_{on} + E_{Off}\right) f = \left(13\mu + 7\mu\right) \times 150k = 3.0 \quad W$$

Diode conduction loss [2 marks]

$$P_{Diode} = I_O V_{AK}\left(1-\delta\right) = 2.0 \times 1.0 \times \left(1-0.2083\right) = 1.58 \quad W$$

Total loss [2 marks]

$$P_{Loss} = P_{Cond} + P_{Sw} + P_{Diode} = 0.2083 + 3.0 + 1.58 = 4.79 \quad W$$

iii) Choose an inductor to ensure continuous conduction. [4]

Critical conduction condition is when output current is at half the inductor ripple current. Choose inductor to set the ripple current accordingly. This will be the minimum inductor value required. [2 marks for method; 2 for accurate answer].

$$\Delta I_L = 2 I_O^{Critical} = 2 I_O^{Min} = 2 \times 0.1 = 0.2 \quad A$$

$$\Delta I_L = \frac{V_I - V_O}{L} \times \frac{\delta}{f}$$

$$L = \frac{V_I - V_O}{\Delta I_L} \times \frac{\delta}{f} = \frac{\left(24-5\right) \times 0.2083}{0.2 \times 150 \times 10^3} = 132 \quad \mu H$$

iv) Choose a capacitor to ensure that the output voltage ripple is less than 40 mV. You may work on the basis that 80% of the ripple will arise from the effective series resistance of the capacitor (ESR) and 20% from the capacitance. [6]

The inductor current ripple goes on to flow through the capacitor and cause the voltage ripple. Standard formulae apply.
[2 marks for method; 2 for each accurate answer].

$$\Delta v_{ESR} = 0.8 \times \Delta v_O = 0.032$$

$$R_{ESR} = \frac{\Delta v_{ESR}}{\Delta I_L} = \frac{0.032}{0.2} = 0.16 \quad \Omega$$

$$\Delta v_C = 0.2 \times \Delta v_O = 0.008$$

$$C = \frac{\Delta I}{8 f \Delta v_C} = \frac{0.2}{8 \times 150 \times 10^3 \times 0.008} = 20.8 \quad \mu F$$

v) Calculate the switching frequency that would be needed to improve the efficiency to 80%. [6]

Reducing the switching frequency will create a proportionate decrease in power loss due to switching. Begin by calculating the allowed total power loss for the efficiency given and then fund allowed switching loss.
[4 marks for method; 2 for accurate answer].

$$\eta = \frac{P_O}{P_O + P_{Loss}}$$

$$P_{Loss} = \left(\frac{1}{\eta} - 1\right) P_O = \left(\frac{1}{0.8} - 1\right) \times 2 \times 5 = 2.5 \quad W$$

$$P_{Sw} = P_{Loss} - P_{Cond} - P_{Diode} = 2.5 - 0.2083 - 1.58 = 0.7083 \quad W$$

$$f = \frac{P_{Sw}}{E_{On} + E_{Off}} = \frac{0.7083}{(13\mu + 7\mu)} = 35.4 \quad kHz$$

vi) What design changes would be needed (trends rather than numerical values are sufficient) as a consequence of the change of frequency determined in part (v)? [4]

The lower overall power loss from the lower switching loss will allow a smaller heat-sink to be used. The lower frequency would cause a larger ripple current and so a larger range of output current causing discontinuous operation. If the constraint of part (iii) is retained then a large inductor (in inverse proportion to frequency reduction) will be required. If this is done then the ESR specification for the capacitor is unchanged but a large capacitance value is required. This may not change the capacitor choice if it was the ESR that was the dominant factor.

3.

A 2 pole-pair, 3-phase induction machine is star-connected and supplied at 50 Hz. The equivalent circuit of the machine is shown in Figure Q3 for which the parameters, referred to the stator, are:

stator resistance	0.04 Ω,
stator leakage reactance	0.4 Ω,
referred rotor resistance	0.055 Ω,
referred rotor leakage reactance	0.4 Ω,
magnetising resistance	20 Ω,
magnetising reactance	8 Ω.

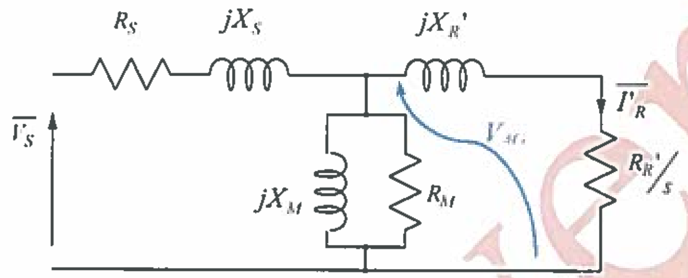

Figure Q3

The machine is rated to develop a torque of 500 Nm at a slip of 0.03. Calculate the following for operation at rated torque:

i) the rotor speed in rad/s [2]

First find synchronous speed and then apply slip equation

$$\omega_s = \frac{\omega_e}{p} = \frac{2\pi 50}{2} = 157.08 \quad rad/s$$

$$\omega_R = \omega_S(1-s) = 157.08 \times (1-0.03) = 152.36 \quad rad/s$$

ii) the mechanical output power [3]

$$P_{Mech} = T\omega_R = 500 \times 152.36 = 76.2 \quad kW$$

iii) the referred rotor current required to develop this power [5]

Take standard equation for mechanical power as function of rotor current and re-arrange. [3 marks for method; 2 for accurate answer]

$$P_{Mech} = \frac{3\, I_R'^2\, R_R'}{\omega_R}\left(\frac{1-s}{s}\right)$$

$$I_R' = \sqrt{\frac{P_{Mech}}{3\, R_R'}\left(\frac{s}{1-s}\right)}$$

$$= \sqrt{\frac{76{,}200}{3 \times 0.055}\left(\frac{0.03}{1-0.03}\right)} = 119.5 \quad A$$

iv) the voltage appearing across the rotor branch (denoted V_{AG} in the equivalent circuit) in magnitude and angle form [5]

With the rotor current established in (iii), multiplication by the rotor branch impedance will yield the voltage. [3 marks for method; 2 for accurate answer]. Angle given with respect to rotor current.

$$Z'_R = \frac{R'_R}{s} + jX'_R = \frac{0.055}{0.03} + j0.4 \quad \Omega$$

$$V_{AG} = I'_R Z'_R = 119.5 \times \left(\frac{0.055}{0.03} + j0.4\right) = 224.3\angle 12.3^\circ \quad V$$

v) the magnetising current and stator current (in any convenient form) [5]

The rotor branch voltage established in (iv) also appears across the magnetising components and so currents through them can be found as two separate terms and summed to give magnetising current. Adding on the rotor current yields the stator current. [2 marks for method; 3 for accurate answers]. Angle given with respect to rotor current.

$$V_{AG} = 224.3\angle 12.3^\circ \quad V$$

$$I_{RM} = \frac{V_{AG}}{R_M} = \frac{224.3\angle 12.3^\circ}{20} = 11.21\angle 12.3^\circ = 10.95 + j2.39 \quad A$$

$$I_{XM} = \frac{V_{AG}}{jX_M} = \frac{224.3\angle 12.3^\circ}{j8} = 28.03\angle -77.7^\circ = 5.97 - j27.39 \quad A$$

$$I_M = I_{RM} + I_{XM} = 16.93 - j25.00 \quad A$$

$$I_S = I_M + I'_R = 136.43 - j25.00 \quad A$$

vi) the magnitude of the required stator voltage [5]

Adding the stator voltage drop to the rotor branch voltage yields the required stator voltage (alternatively one could multiply the stator current by the total impedance of the machine). [2 marks for method; 3 for accurate answer]

$$Z_S = R_S + jX_S = 0.04 + j0.4 \quad \Omega$$

$$V_S = V_{AG} + I_S Z_S = 224.3\angle 12.3^\circ + (136.43 - j25.00)(0.04 + j0.4)$$
$$= 234.5 + j101.4 \quad V$$

Which is a magnitude of 255.5 V

vii) the efficiency at the rated torque [5]

Input power is found from stator voltage and current and output power was found in (ii). [2 marks for method and 3 for accurate answer]

$$P_I = 3\mathrm{Re}\{V_S I_S^*\} = 3\mathrm{Re}\{(234.5 + j101.4) \times (136.43 + j25.00)\}$$
$$= 88.39 \quad kW$$

$$\eta = \frac{P_{Mech}}{P_I} = \frac{76.2}{88.39} = 86.2\%$$