Imperial College London

MEng (Engineering) Examination 2017

Year 1

AE1-110 Introduction to Structural Analysis

Thursday 1st June 2017: 14.00 to 16.00
[2 hours]

The paper is divided into Section A and Section B and contains ***FOUR*** questions.

In each section, the FIRST question has HALF the weight of the SECOND question.

Candidates may obtain full marks for complete answers to ***ALL*** questions.

You must answer each section in a separate answer booklet.

A data sheet is provided.

The use of lecture notes is NOT allowed.

Section A

Note that question 1 is worth half the marks of question 2.

1. (a) For each of the three pin-jointed frameworks shown in Figure 1a determine whether it is statically determinate, statically indeterminate, a mechanism, or a combination of these. If the framework is a mechanism sketch a possible deformed strain-free configuration, and state what assumptions must be made for this deformed configuration to occur. [50%]

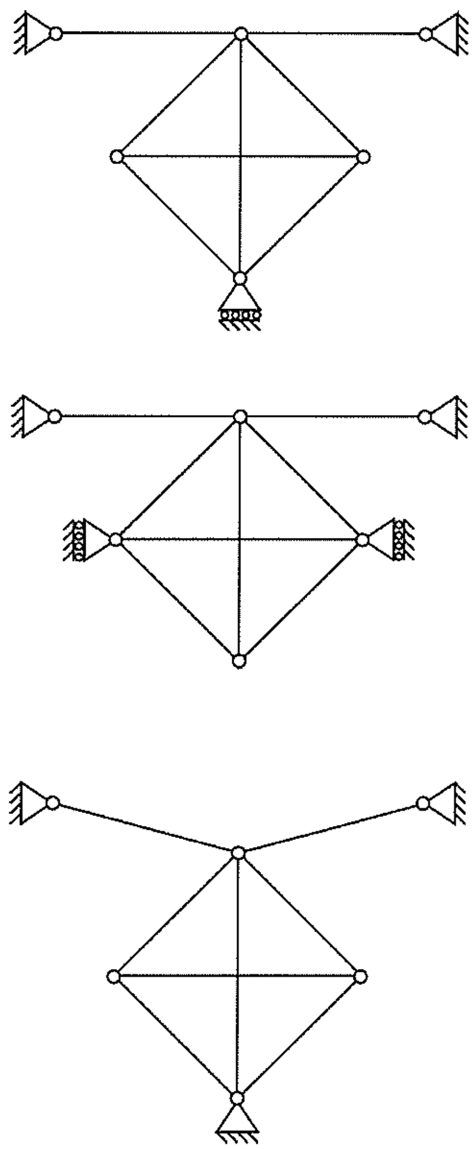

Figure 1a

[CONTINUED OVERLEAF]

(b) In the pin-jointed framework shown in Figure 1b all members have constant cross section area A and Young's modulus E. All horizontal and vertical members have initial length L, and both diagonal members are oriented at $45°$ to the horizontal axis. A vertical downward load P is applied as shown to node 4. The bar forces resulting from this applied load are as shown in Table 1

Table 1

Bar	$T(\times P)$
12	$-\frac{1}{2}$
14	$\frac{1}{\sqrt{2}}$
23	$-\frac{1}{2}$
24	0
34	$\frac{1}{\sqrt{2}}$

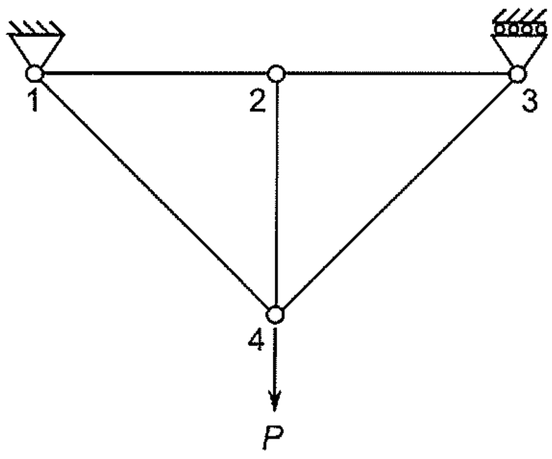

Figure 1b

i. Evaluate the corresponding bar extensions. [15%]

ii. Using the virtual work method evaluate the vertical deflection of node 4. [25%]

iii. Without further calculation write down the horizontal displacement of node 4 and state your reasoning. [10%]

2. In the pin-jointed framework shown in Figure 2 all members have constant cross section area A, Young's modulus E and coefficient of thermal expansion α. All horizontal and vertical members have initial length L, and both diagonal members are oriented at $45°$ to the horizontal axis.

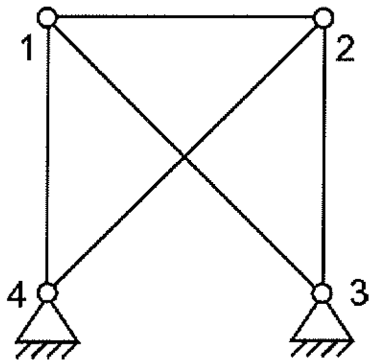

Figure 2

(a) Confirm that the structure shown in Figure 2 is statically-indeterminate with one redundancy. [5%]

(b) Bars 13 and 24 are both heated to raise their temperature by ΔT. Determine the resulting nodal deflections. [60%]

(c) Bar 24 is disconnected from the structure. Bar 13 continues to be heated as before. Determine the new nodal deflections of node 1. [35%]

Section B

Note that question 3 is worth half the marks of question 4.

3.

(a) Figure 3a shows a cantilever beam which has a constant flexural stiffness of EI for $0 \leq z < L/3$ and $2EI$ for $L/3 \leq z \leq L$ and is subjected to a point load, P, at its free end.

Determine the rotation (slope) of the beam at $z = 0$. [60%]

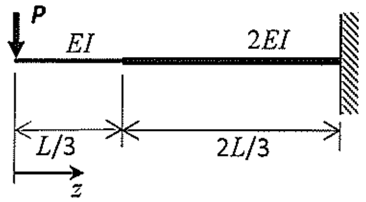

Figure 3a

(b) Figure 3b shows a uniform solid circular section bar of radius 40 mm. The bar is rigidly supported at one end and is subjected to two torques as shown.

i. Determine the reaction torque at $z = 0$. [10%]

ii. Determine and sketch the distribution of internal torque in the bar. [20%]

iii. Determine the maximum shear stress in the bar. [10%]

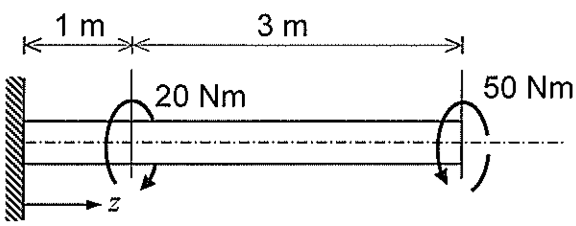

Figure 3b

4.

(a) Derive equations for the shear force and bending moment distributions for the beam shown in Figure 4a. Sketch the shear force and bending moment diagrams indicating maximum and minimum values and clearly marking the direction of shear distortion and bending curvature. [30%]

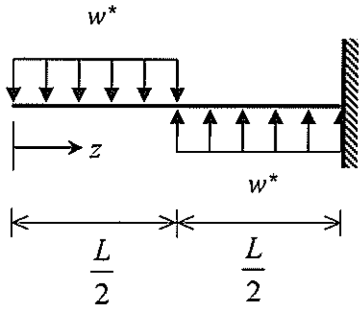

Figure 4a

(b) Figure 4b shows a simply supported beam subjected to a downward point load at $z = L/3$. The flexural stiffness of the beam is EI for $0 \leq z < L/3$ and $2EI$ for $L/3 \leq z < L$. Determine the vertical deflection of the beam at $z = L/3$. [35%]

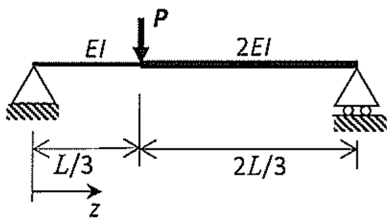

Figure 4b

(c) Figure 4c shows a uniform simply supported beam subjected to a uniformly distributed load. The bending moment distribution for the beam and the vertical deflection at $z = 0$ are also given in the figure.

Figure 4d shows the same beam subjected to a unit point load at z = 0. The bending moment distribution and the vertical deflection at $z = 0$ are given.

[CONTINUED OVERLEAF]

Figure 4e shows the same beam loaded as in Figure 4c but with an additional simple support at $z = 0$.

For the beam shown in Figure 4e:

i. Calculate the reaction force at $z = 0$. [10%]

ii. Determine and sketch the bending moment distribution in the beam for $0 \leq z < L/2$. Indicate the values and locations of any maxima and minima and clearly mark the direction of curvature. [20%]

iii. Without any further calculation sketch the deflected shape of the beam for $0 \leq z < L/2$ indicating positions of zero slope, zero curvature, maximum curvature and minimum curvature. [5%]

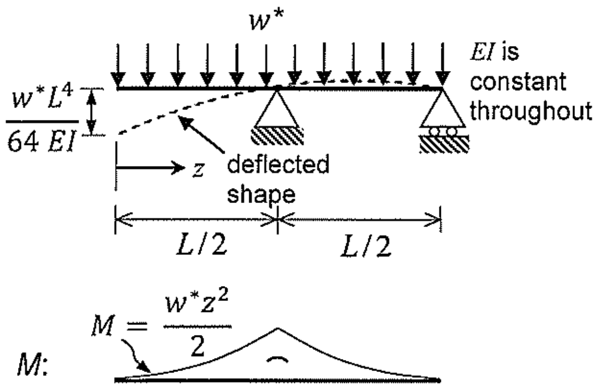

Figure 4c

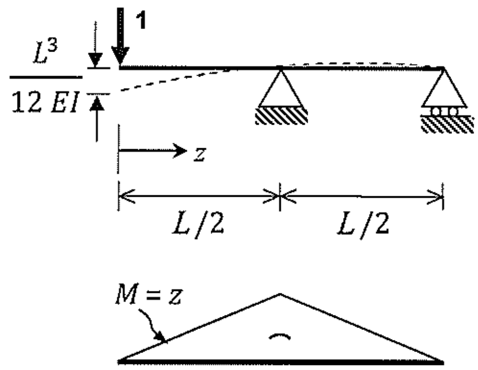

Figure 4d

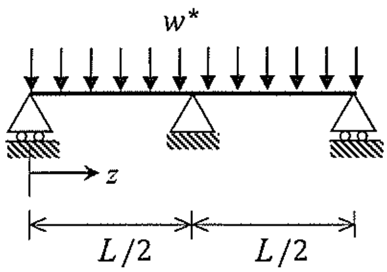

Figure 4e

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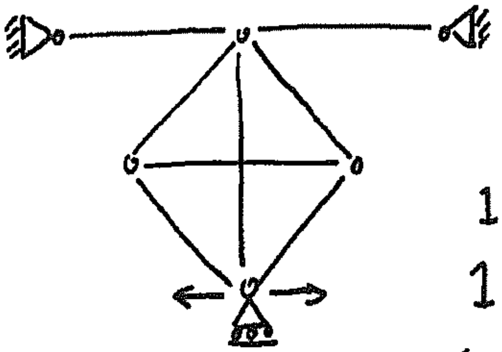
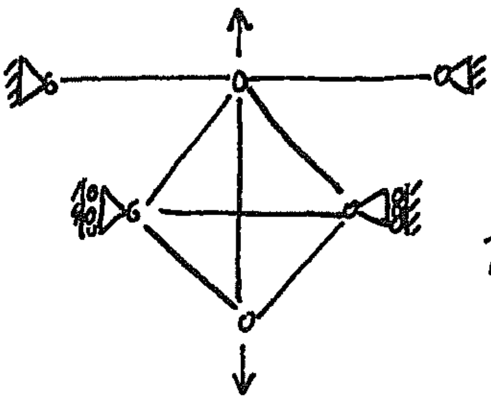
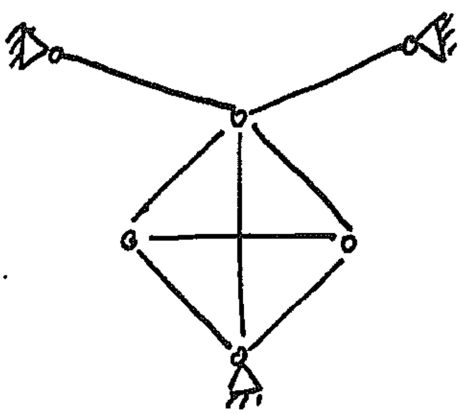
	Marks
1a)	
	
$b + r - 2j$	
$8 + 5 - 12 = 1$	5
1 redundancy	5
1 inf. mech. by inspection assuming rollers are horizontal	5
sketch	5
	
$b + r - 2j$	
$8 + 6 - 12 = 2$	5
2 redundancies	5
1 inf. mech by inspection assuming rollers are vertical	5
sketch	5
	
$b + r - 2j$	
$8 + 6 - 12 = 2$	5
statically indeterminate with 2 redundancies	5

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(2)

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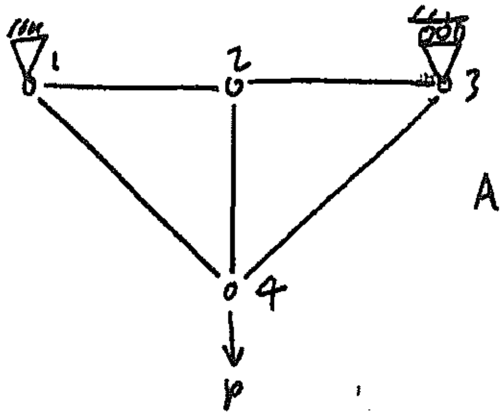
Marks

1b)



A, E

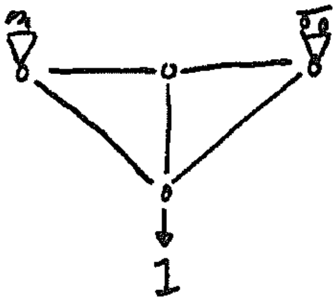
(i)

BAR	Length ($\times L$)	T ($\times P$)	$e\left(\times \frac{PL}{AE}\right)$	T^*_{v4}
12	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
14	$\sqrt{2}$	$1/\sqrt{2}$	1	$1/\sqrt{2}$
23	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
24	1	0	0	0
34	$\sqrt{2}$	$1/\sqrt{2}$	1	$1/\sqrt{2}$

10

for $\frac{PL}{AE}$ 5

ii)



$$1.v_4 = \left(\frac{1}{4} + \frac{1}{\sqrt{2}} + \frac{1}{4} + \frac{1}{\sqrt{2}}\right)\frac{PL}{AE}$$ 5

$$v_4 = \left(\frac{1}{2} + \sqrt{2}\right)\frac{PL}{AE} \text{ downwards}$$ 20

iii) $u_4 = 0$ by symmetry 10

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Marks

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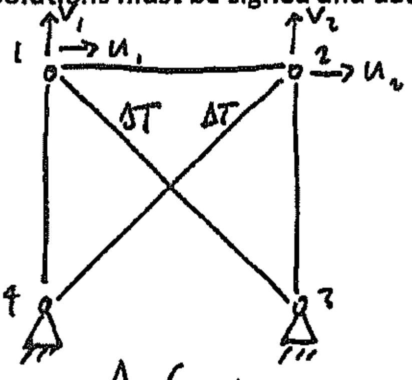
2)



A, E, α

a) $b + r - 2j$

$5 + 4 - 8 = 1$

& by inspection statically indeterminate with 1 redundancy — 5

b)

BAR	Length ($\times L$)	e	
12	1	$e_{12} = u_2 - u_1$	$\varepsilon_{12} = \frac{1}{L}(u_2 - u_1)$
13	$\sqrt{2}$	$e_{13} = \frac{1}{\sqrt{2}}(v_1 - u_1)$	$\varepsilon_{13} = \frac{1}{2L}(v_1 - u_1)$
14	1	$e_{14} = v_1$	$\varepsilon_{14} = \frac{1}{L} v_1$
23	1	$e_{23} = v_2$	$\varepsilon_{23} = \frac{1}{L} v_2$
24	$\sqrt{2}$	$e_{24} = \frac{1}{\sqrt{2}}(u_2 + v_2)$	$\varepsilon_{24} = \frac{1}{2L}(u_2 + v_2)$

10

$$\frac{1}{L}(u_2 - u_1) = \frac{T_{12}}{AE}$$

$$\frac{1}{2L}(v_1 - u_1) = \frac{T_{13}}{AE} + \alpha \Delta T$$

$$\frac{1}{L} v_1 = \frac{T_{14}}{AE}$$

$$\frac{1}{L} v_2 = \frac{T_{23}}{AE}$$

$$\frac{1}{2L}(u_2 + v_2) = \frac{T_{24}}{AE} + \alpha \Delta T$$

10

cont.

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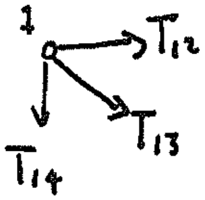
Marks

2 cont)



$$T_{12} + \frac{T_{13}}{\sqrt{2}} = 0$$

$$T_{14} + \frac{T_{13}}{\sqrt{2}} = 0$$

10

$$\begin{cases} \dfrac{AE}{L}(u_2 - u_1) + \dfrac{AE}{2\sqrt{2}L}(v_1 - u_1) - \dfrac{1}{\sqrt{2}}AE\alpha\Delta T = 0 \\ \dfrac{AE}{L}v_1 + \dfrac{AE}{2\sqrt{2}L}(v_1 - u_1) - \dfrac{1}{\sqrt{2}}AE\alpha\Delta T = 0 \end{cases}$$

10

By symmetry $u_2 = -u_1$, $v_1 = v_2$

$$\begin{cases} -2u_1 + \dfrac{1}{2\sqrt{2}}(v_1 - u_1) - \dfrac{1}{\sqrt{2}}L\alpha\Delta T = 0 \\ v_1 + \dfrac{1}{2\sqrt{2}}(v_1 - u_1) - \dfrac{1}{\sqrt{2}}L\alpha\Delta T = 0 \end{cases}$$

$$v_1 = -2u_1 \quad , \quad u_1 = -\frac{v_1}{2}$$

$$v_1 + \frac{1}{2\sqrt{2}}v_1 + \frac{1}{4\sqrt{2}}v_1 = \frac{1}{\sqrt{2}}L\alpha\Delta T$$

$$v_1\left(\frac{4\sqrt{2} + 2 + 1}{4\sqrt{2}}\right) = \frac{1}{\sqrt{2}}L\alpha\Delta T$$

(0.462)

$$v_1 = \left(\frac{4}{3 + 4\sqrt{2}}\right)L\alpha\Delta T = v_2$$

5 5

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cont.

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5 (3)

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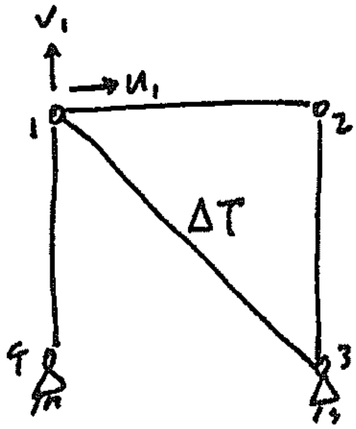
Marks

2 cont) $u_1 = -\left(\frac{2}{3+4\sqrt{2}}\right)L\alpha\Delta T = -u_2$ 5 5

(0.231)

c)



Structure is now statically determinate 5

$\varepsilon_{13} = \alpha\Delta T$

$e_{13} = \sqrt{2}L\alpha\Delta T$ 10

all other extensions are 0

Same compatibility equations as before.

$e_{14} = 0 \quad \therefore \quad v_1 = 0$ 10

$e_{13} = -\frac{u_1}{\sqrt{2}}$

$u_1 = -2L\alpha\Delta T$ 10

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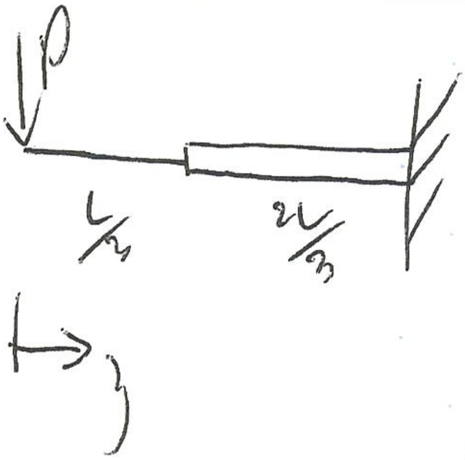
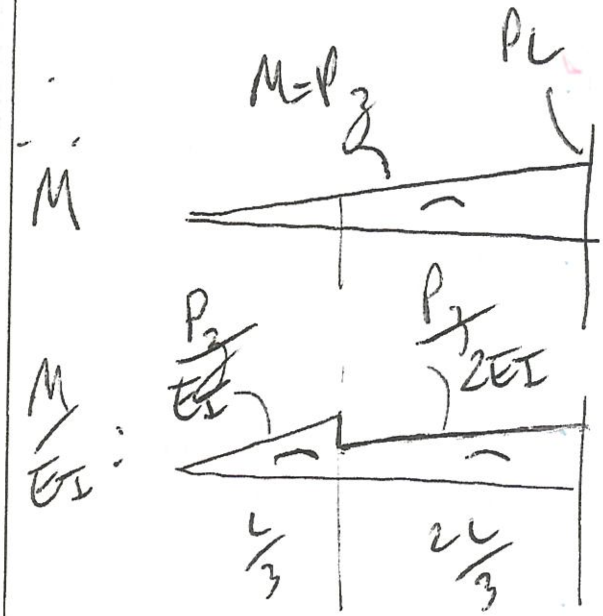
3a



Moments equilibrium about cut

$\Rightarrow M - Pz = 0$

$\therefore M = Pz$



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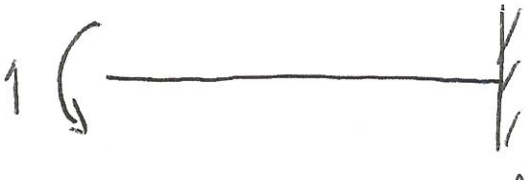
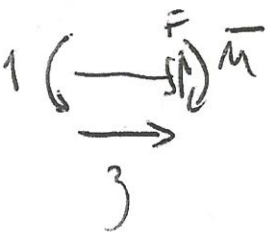
To determine rotation at $z=0$, apply a unit virtual moment:





Moments eq. about cut $\Rightarrow$ $\bar{M} - 1 = 0$ $\therefore \bar{M} = 1$

5

3a Integrating from curvature M 10

Approach (1)

$0 < z < L/3$ $\quad \dfrac{d^2v}{dz^2} = -\dfrac{Pz}{EI}$ (i) 5

$L/3 < z < L$ $\quad \dfrac{d^2v}{dz^2} = -\dfrac{Pz}{2EI}$ (ii) 5

boundary conditions $\dfrac{dv}{dz} = 0,\ v = 0$ at $z = L$

we want $\dfrac{dv}{dz}$ at $z = 0$

use (ii) & integrate $\Rightarrow \dfrac{dv}{dz} = \dfrac{-Pz^2}{4EI} + C_1$ 5

at $z = L$, $\dfrac{dv}{dz} = 0 \therefore 0 = \dfrac{-PL^2}{4EI} + C_1$ 5 5

$\therefore \dfrac{dv}{dz} = -\dfrac{Pz^2}{4EI} + \dfrac{PL^2}{4EI}$

$\therefore \dfrac{dv}{dz}\Big|_{z=L/3} = \dfrac{P}{4EI}\left(L^2 - \dfrac{L^2}{9}\right)$

$= \dfrac{8PL^2}{36EI}$ 5

use (i) & integrate $\Rightarrow \dfrac{dv}{dz} = -\dfrac{Pz^2}{2EI} + C_2$ 5

at $z = L/3$, $\dfrac{dv}{dz} = \dfrac{8P}{36EI}L^2 = \dfrac{-PL^2}{18EI} + C_2 \therefore C_2 = \dfrac{10PL^2}{36EI}$ 5 5

$\therefore \dfrac{dv}{dz}\Big|_{z=0} = \dfrac{10PL^2}{36EI} = \dfrac{5}{18}\dfrac{PL^2}{EI}$ 5

Approach (2)

as approach (1)

We note at $z = L$, $\dfrac{dv}{dz} = 0$

the change in slope between $z = 0$ and $z = L$ =

$\left[\dfrac{dv}{dz}\right]_0^L = \cancel{\left[\int \dfrac{d^2v}{dz^2}\, dz\right]^L}$

$= \left[\dfrac{dv}{dz}\Big|_{z=L} - \dfrac{dv}{dz}\Big|_{z=0}\right]$ 5

$= \int_0^L \dfrac{d^2v}{dz^2}\, dz = \int_0^{L/3} -\dfrac{Pz}{EI}\, dz + \int_{L/3}^{L} \dfrac{-Pz}{2EI}\, dz$ 5 5

$= \left[\dfrac{-Pz^2}{2EI}\right]_0^{L/3} + \left[\dfrac{-Pz^2}{4EI}\right]_{L/3}^{L} = \dfrac{-PL^2}{EI}\left(\dfrac{1}{18} + \dfrac{4}{18}\right)$ 5 5

$= -\dfrac{5}{18}\dfrac{PL^2}{EI}$. Using $\dfrac{dv}{dz}\Big|_{z=L} = 0$ 5

then $\dfrac{dv}{dz}\Big|_{z=0} = \dfrac{5}{18}\dfrac{PL^2}{EI}$ 5

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By principle of virtual work

$$1.\theta = \int_{\substack{\text{beam}\\ \text{length}}} \frac{M}{EI}\cdot\bar{M}\,dz$$

$$= \int_0^{L/3} \frac{Pz}{EI}\cdot 1\,dz + \int^{L} \frac{Pz}{2EI}\cdot 1\cdot dz$$ 5, 5

$$= \left[\frac{Pz^2}{2EI}\right]_0^{L/3} + \left[\frac{Pz^2}{4EI}\right]_{L/3}^{L}$$ 5, 5

$$= \frac{PL^2}{EI}\left(\frac{1}{18} + \frac{1}{4} - \frac{1}{36}\right)$$ 5, 5

$$\underline{\theta = \frac{10}{36}\frac{PL^2}{EI}}$$ 5

consider its & cl with earlier error

16

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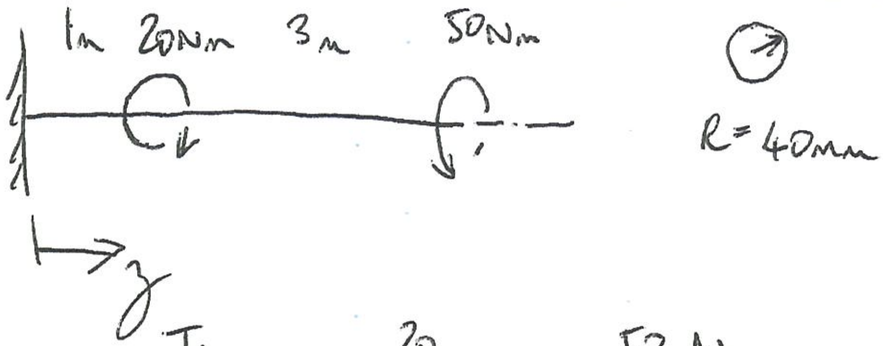
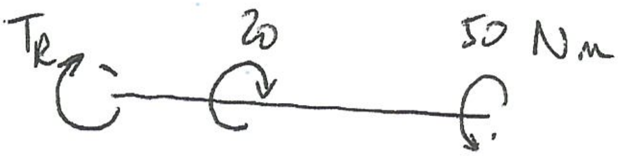
9

Marks

3b.



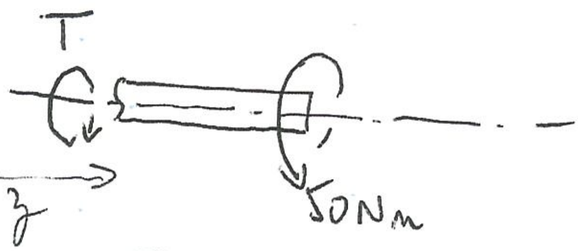
i) FBD :



Torsional equilibrium : $T_R + 20 - 50 = 0$

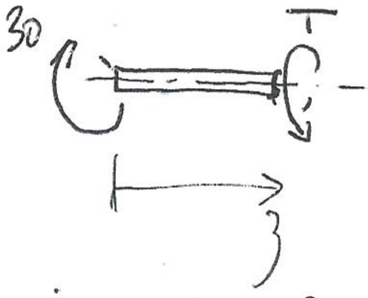
$\therefore \underline{T_R = 30\ \text{Nm}}$ 10

ii) $1 < z < 3$



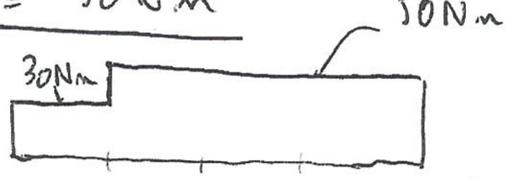
$T - 50 = 0 \quad \therefore \underline{T = 50\ \text{Nm}}$ 7

$0 < z < 1$



$T - 30 = 0 \quad \therefore \underline{T = 30\ \text{Nm}}$ 7

$\therefore$ Internal torque distribution :



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20

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10

(ii) | Marks

Max int. torque = 50 Nm

Max shear stress $= \dfrac{T_{max} R}{J}$

(data sheet)

$J = \dfrac{\pi R^4}{2} = \dfrac{\pi \times 40^4}{2}$ mm^4 — 5

$\therefore$ Max shear stress $= \dfrac{50 \times 10^3 \times 2 \times 40}{\pi \times 40^4}$

$= 0.5$ N/mm^2 — 5

40

4a – using differential relationships

$0 < z < \frac{L}{2}$

$\frac{dF}{dz} = -w = w^*$

$F = w^* z + C_1$

at $z = 0$ $F = 0$, $\therefore C_1 = 0$, $\therefore \underline{F = w^* z}$

$\frac{dM}{dz} = F \therefore M = \frac{w^* z^2}{2} + C_2$

at $z = 0$, $M = 0$, $\therefore C_2 = 0$, $\therefore \underline{M = \frac{w^* z^2}{2}}$

from overall equilibrium

$F|_{z=L} = 0$,

$M|_{z=L} = \frac{w^* L^2}{4}$

(11)

$\frac{L}{2} < z < L$

$\frac{dF}{dz} = -w = -w^*$

$\therefore F = -w^* z + C_3$

at $z = L$, $F = 0$ $\therefore 0 = -w^* L + C_3$ $\therefore C_3 = w^* L$

$\therefore \underline{F = w^*(L - z)}$

$\frac{dM}{dz} = F = w^*(L - z)$ $\therefore M = w^*\left(Lz - \frac{z^2}{2}\right) + C_4$

at $z = L$, $M = \frac{w^* L^2}{4}$ $\therefore \frac{w^* L^2}{4} = w^*\left(L^2 - \frac{L^2}{2}\right) + C_4$

$C_4 = -\frac{w^* L^2}{4}$

$\therefore \underline{M = w^*\left(Lz - \frac{z^2}{2} - \frac{L^2}{4}\right)}$

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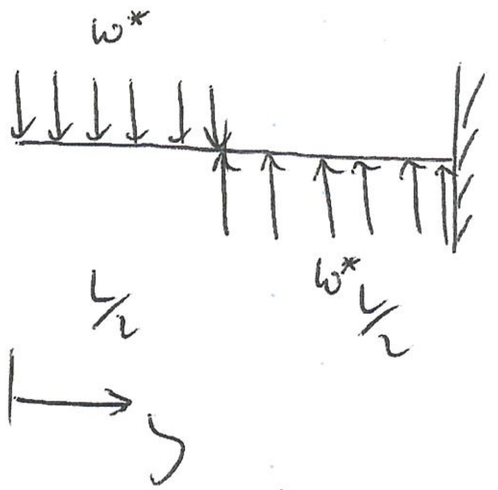
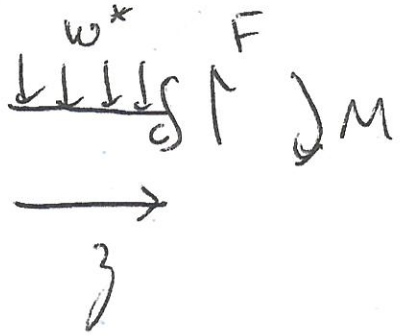
4 a) i)



$0 < z < \frac{L}{2}$



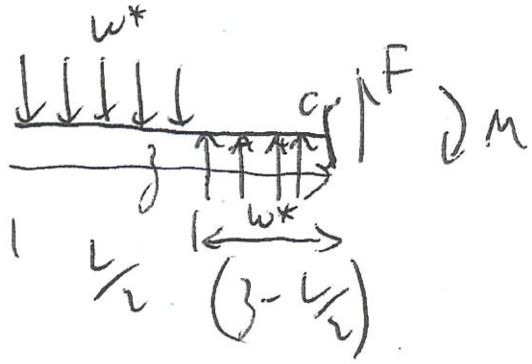
3

Vert eq: $w^* z - F = 0 \quad \therefore \underline{F = w^* z}$ 3

Moments eq C: $M - w^* \frac{z^2}{2} = 0 \quad \therefore \underline{M = w^* \frac{z^2}{2}}$ 3

9

$\frac{L}{2} < z < L$



3

Vert eq: $w^* \frac{L}{2} - w^*\left(z - \frac{L}{2}\right) - F = 0$ 3

$\therefore F = w^*(L - z)$

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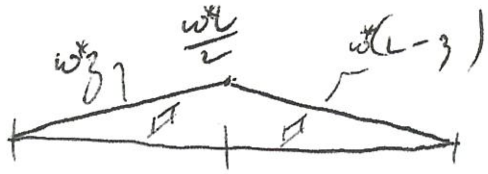
Moment eq ↻: $M - w^* \frac{L}{2}\left(z - \frac{L}{2} + \frac{L}{4}\right) + w^*\left(z - \frac{L}{2}\right)^2 \frac{1}{2} = 0$

$$\therefore M = \frac{w^* L}{2}\left(z - \frac{L}{4}\right) - \frac{w^*}{2}\left(z - \frac{L}{2}\right)^2$$

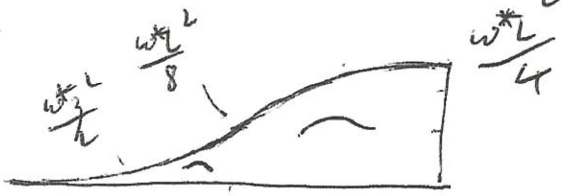
$$= w^*\left(\frac{Lz}{2} - \frac{L^2}{8} - \frac{z^2}{2} + \frac{Lz}{2} - \frac{L^2}{8}\right)$$

$$= w^*\left(-\frac{z^2}{2} + Lz - \frac{L^2}{4}\right)$$ 6 12

Shear force diagram 4

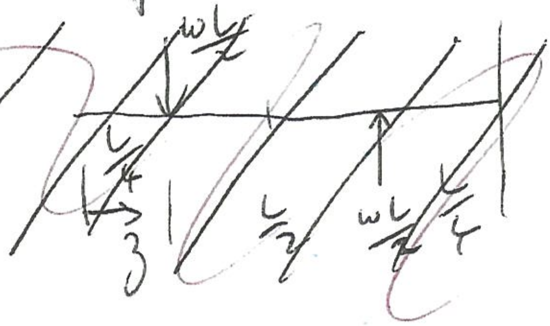


Bending moment diagram 5



30

~~4a i.~~



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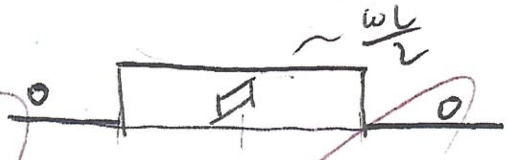
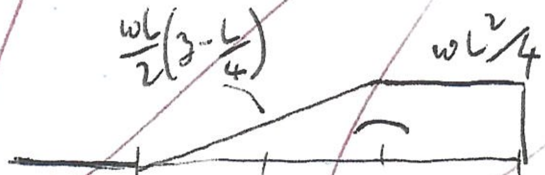
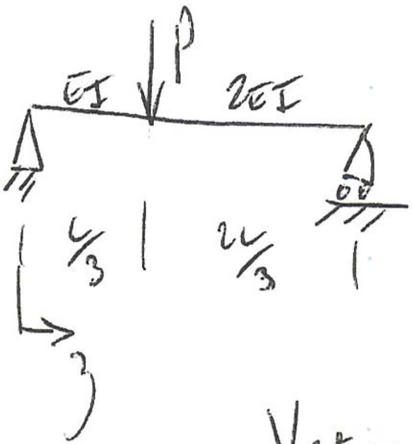
Shear force diagram:

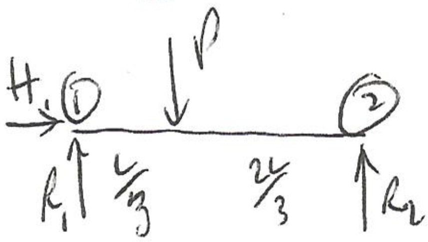


Bending moment diag:



4b





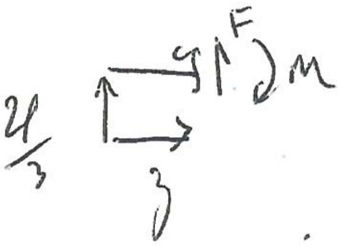
Horiz eq: $\underline{H_1 = 0}$

Vert eq: $R_1 + R_2 = P$

Moments eq ①: $\frac{PL}{3} - R_2 L = 0 \quad \therefore \underline{R_2 = \frac{P}{3}}$ 3

$\therefore$ from vert eq $\underline{R_1 = \frac{2P}{3}}$ 3

$0 \le z \le L/3$



Moments eq:

$$\frac{2P}{3}z + M = 0$$

$$\therefore \underline{M = -\frac{2P}{3}z} \checkmark$$ 3

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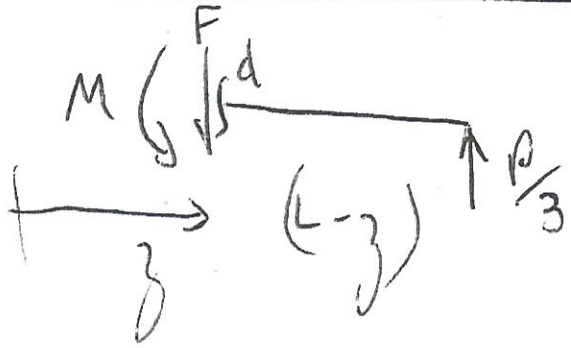
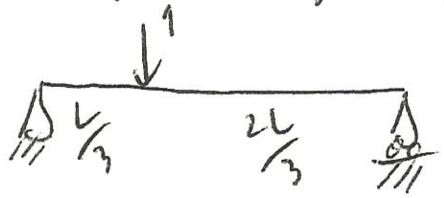
$\frac{L}{3} \leq z \leq L$



Moments eq. d: $\quad -M - \frac{P}{3}\cdot(L-z) = 0$

$$\therefore \underline{M = \frac{P}{3}(z-L)}$$ 3

For deflection at $z = L/3$, apply a unit virtual load :



$\therefore \bar{M} = -\frac{2}{3}z \quad 0 \leq z \leq L/3$ 3

$\& \; \bar{M} = \frac{1}{3}(z-L) \quad \frac{L}{3} \leq z \leq L$?

$$\therefore \delta = \int_0^{L/3} -\frac{2P}{3EI} z \left(-\frac{2z}{3}\right) dz + \int_{L/3}^{L} \frac{P(z-L)}{3 \times 2EI} \cdot \frac{(z-L)}{3} \cdot dz$$ 3, 3

$$= \frac{4P}{9EI} \left[\frac{z^3}{3}\right]_0^{L/3} + \frac{P}{18EI}\left[\frac{(z-L)^3}{3}\right]_{L/3}^{L}$$ 3, 3

integration error but units 0 $\frac{1}{3}$ $\frac{1}{3}$

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$$\therefore \delta = \frac{4P}{9EI}\left(\frac{L^3}{81}\right) + \frac{P}{18EI}\left(\frac{8L^3}{81}\right)$$

$$= \frac{P}{18}\frac{L^3}{EI}\left(\frac{8}{81} + \frac{8}{81}\right)$$

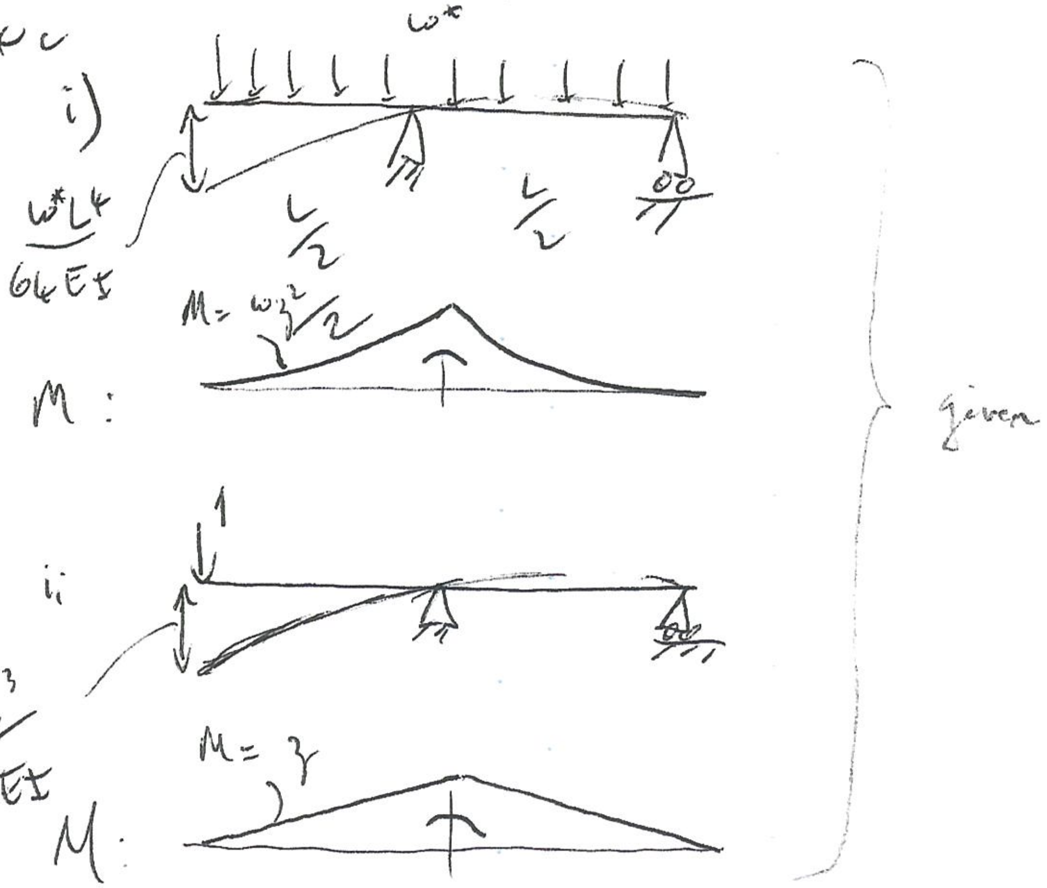
$$= \frac{8}{9}\frac{PL^3}{EI}\cdot\frac{1}{81} = \underline{\frac{8}{729}\frac{PL^3}{EI}}$$

consistent with earlier errors but wrong 2/5

5

35

4c



(17)

$$\frac{d^2v}{dz^2} = \frac{-w^*}{EI}\left(\frac{z^2}{2} - \frac{3Lz}{16}\right)$$

$$\frac{dv}{dz} = \frac{-w^*}{EI}\left(\frac{z^3}{6} - \frac{3Lz^2}{32}\right) + C$$

at $z = \frac{L}{2}$, $\frac{dv}{dz} = 0$ $\therefore$ $0 = \frac{-w^*}{EI}\underbrace{\left(\frac{L^3}{48} - \frac{3L^3}{\cancel{32}\,128}\right)}_{-\frac{L^3}{384}} + C$

$$= \frac{wL^3}{384EI} + C$$

$$\therefore C = -\frac{wL^3}{384EI}.$$

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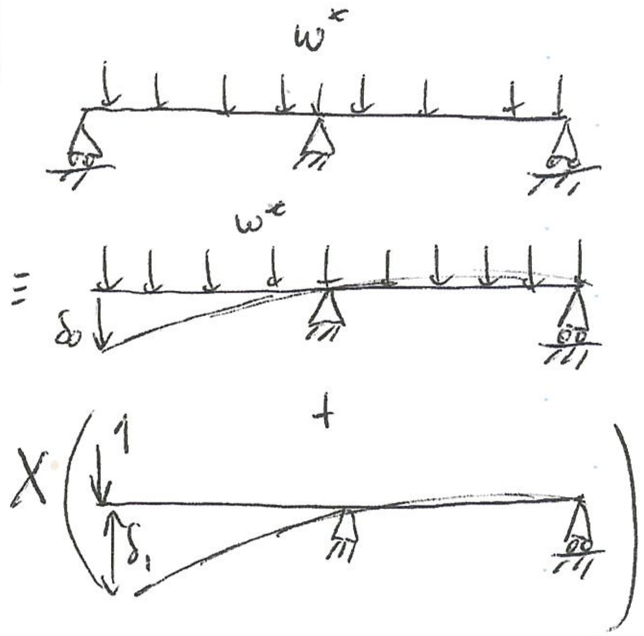

5

We require $\delta_0 + X\delta_1 = 0$

$$\therefore \frac{w^* L^4}{64EI} + X \frac{L^3}{12EI} = 0$$

$$\therefore X = -\frac{3}{16} w^* L$$ 5

ie reaction at $z = 0$ is $\frac{3}{16} w^* L$ upwards 10

From superposition for $0 \le z \le L/2$

$$M = \frac{w^* z^2}{2} - \frac{3}{16} w^* L . z = w^*\left(\frac{z^2}{2} - \frac{3L}{16} z\right)$$ 5

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at $z=0$, $M=0$; at $z=\frac{L}{2}$, $M=\frac{w^*L^2}{32}$

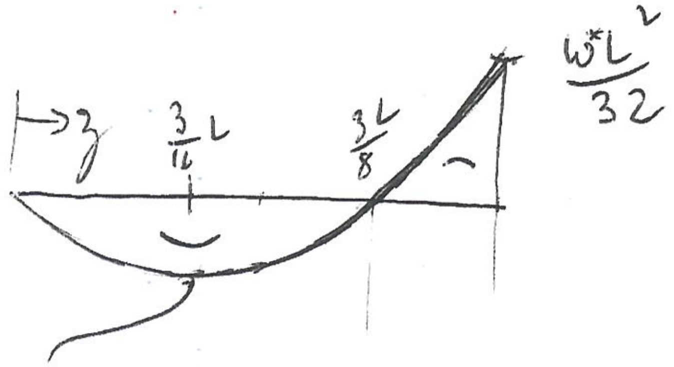
also $M=0$ at $\frac{z}{2}-\frac{3L}{16}=0 \Rightarrow z=\frac{3}{8}L$

$\frac{dM}{dz}=w^*\left(z-\frac{3L}{16}\right)$ $\therefore$ at $z=\frac{3L}{16}$, $\frac{dM}{dz}=0$ 10

$\frac{d^2M}{dz^2}=w^*$ $\therefore$ $M_{MIN}=w^*\left(\frac{9L^2}{2\times256}-\frac{9}{256}L^2\right)$

$= -\frac{9}{512}w^*L^2$

M diagram



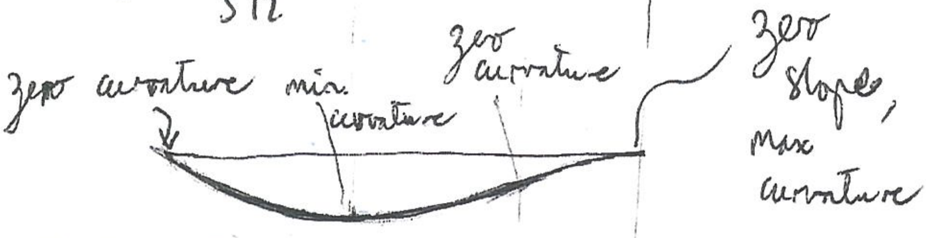
5

missing curvature 4/5

20

$M_{MIN}=-\frac{9}{512}w^*L^2$

Deflected shape :



5 5

35

4b By integration from curvature (20)

first part to determine M $\quad 0 < z < L/3 \quad M = -\frac{2P}{3} z$

same as virtual work sol $\quad ; \quad L/3 < z < L \quad M = \frac{P}{3}(z - L)$

3, 3 reaction, 3, 3 Ms

<u>LHS</u> $(0 < z < L/3)$

$$\frac{d^2 v}{dz^2} = \frac{2P}{3EI} z$$ 1

$$\therefore \frac{dv}{dz} = \frac{\cancel{2}P}{3EI} z^2 + C_1 \quad \& \quad v = \frac{P z^3}{9EI} + C_1 z + C_2$$ 2 2

at $z = 0$, $v = 0 \Rightarrow \underline{C_2 = 0}$ 1

<u>RHS</u> $(L/3 < z < L)$

$$\frac{d^2 v}{dz^2} = -\frac{P}{6EI}(z - L)$$ 1

$$\therefore \frac{dv}{dz} = -\frac{P}{6EI}\left(\frac{z^2}{2} - Lz\right) + C_3 \quad \& \quad v = \frac{-P}{6EI}\left(\frac{z^3}{6} - \frac{Lz^2}{2}\right) + C_3 z + C_4$$ 2 2

at $z = L$, $v = 0 \Rightarrow 0 = \frac{-P}{6EI}\left(-\frac{L^3}{3}\right) + C_3 L + C_4$

$$\therefore \underline{C_4 = \frac{-PL^3}{18EI} - C_3 L}$$ 2

at $z = L/3 \quad \left.\frac{dv}{dz}\right|_{LHS} = \left.\frac{dv}{dz}\right|_{RHS}$

LAST PAGE (21)

$$\therefore \frac{P}{27EI}L^2 + C_1 = -\frac{PL^2}{6EI}\left(\frac{-5}{18}\right) + C_3$$ 2

$$\therefore C_1 = \frac{PL^2}{108} + C_3$$ 2

also at $z = L/3$ $\quad v|_{LHS} = v|_{RHS}$

$$\therefore \frac{PL^3}{243EI} + \frac{PL^3}{324} + \frac{C_3 L}{3} = \frac{2PL^3}{243} + C_3\frac{L}{3} - \frac{PL^3}{18EI} - C_3 L$$ 2

$$\frac{7}{972}\frac{PL^3}{EI} + \frac{C_3 L}{3} = -\frac{23}{486}PL^3 - \frac{2C_3 L}{3}$$

$$\therefore \frac{53}{972}\frac{PL^3}{EI} = -C_3 L$$

$$\therefore C_3 = -\frac{53}{972}\frac{PL^2}{EI}$$ 2

$$\& \quad C_1 = \cancel{-\frac{455}{486}PL^2} \; -\frac{11}{243}PL^2$$

$$\therefore \text{at } z = L/3, \quad v = \frac{P}{9EI}\left(\frac{L}{3}\right)^3 + -\frac{11}{243}L^3 \cdot \frac{L}{3}$$

$$= -\frac{8}{729}\frac{PL^3}{EI}$$ 2

35

$-8/729$ $\quad -8/729$

Introduction to Structural Analysis Data Sheet

Constitutive Stress/strain Law (Hooke's Law):

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \upsilon\sigma_{yy} - \upsilon\sigma_{zz}\right) \text{ etc.}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \text{ etc.}$$

Compatibility:

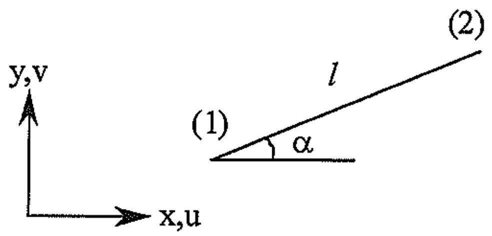
$$\varepsilon_{xx} = \frac{du}{dx} \text{ etc.}$$

Elastic constants:

$$\text{Shear modulus, } G = \frac{E}{2(1+\upsilon)}$$

$$\text{Bulk modulus, } K = \frac{E}{3(1-2\upsilon)}$$

Stretch of a pin-jointed bar in terms of end displacements:



$$\Delta l = (u_2 - u_1)\cos\alpha + (v_2 - v_1)\sin\alpha$$

Virtual Work (unit load) theorem for pin-jointed frameworks:

$$\bar{1}.\delta = \sum \bar{T}_{ij} . e_{ij}$$

Virtual Work (unit load) theorem for beams:

$$\bar{1}.\delta = \int \frac{\bar{M} M}{EI} dz$$

Stress–moment–curvature relationships for beams:

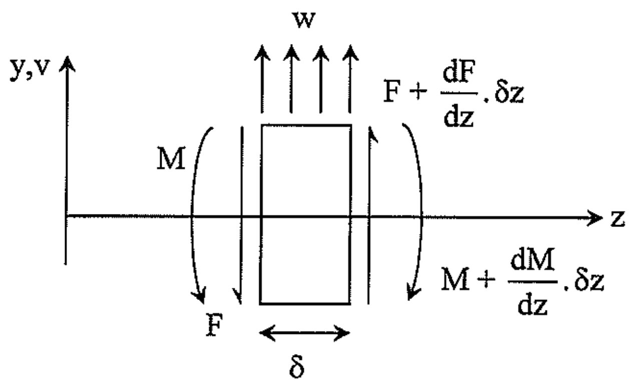
$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} \quad \text{where } \frac{1}{R} = -\frac{d^2v}{dz^2}$$

Stress-torque-twist relationship for circular section tubes:

$$\frac{\tau}{r} = \frac{T}{J} = G.\frac{d\theta}{dz}$$

Load–shear–moment relationship:

$$-w = \frac{dF}{dz}; \; F = \frac{dM}{dz}$$



The torsion constant for a thick circular tube of outer and inner radii R_0 and R_I is

$$J = \frac{\pi}{2}\left(R_0^{\,4} - R_I^{\,4}\right)$$

and for a thin-walled tube of mid-line radius R and wall thickness t, $J = 2\pi R^3 t$.

Unit load method for singly redundant beam:

$$X.\delta_1 + \delta_0 = 0$$

$$\text{where } \delta_0 = \int \frac{M_0 . \overline{M}}{EI} dz; \; \delta_1 = \int \frac{\overline{M}^2 \, dz}{EI}$$