EE2-08B

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

EEE PART II: MEng, BEng and ACGI

Corrected Copy

MATHEMATICS 2B (E-STREAM AND I-STREAM)

Tuesday, 24 May 2:00 pm

Time allowed: 1:30 hours

There are TWO questions on this paper.

Answer TWO questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : B. Clerckx
Second Marker(s) : D. Nucinkis

THE QUESTIONS

[25]

1. a) Consider two continuous random variable X and Y characterized by the following joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} 2x, & 0 < x < 1, 0 < y < 1 \\ 0, & otherwise. \end{cases}$$

i) Compute the correlation coefficient between X and Y, i.e. $\mathrm{Corr}(X,Y)$.

[4]

ii) Compute the probability that X is smaller than or equal to 0.25 given that Y is larger than or equal to 1/3, i.e. $P(X \leq 0.25 \,|\, Y \geq 1/3)$.

[4]

iii) Compute the variance of $3X - 2Y + 5$, i.e. $\mathrm{Var}(3X - 2Y + 5)$.

[4]

iv) Are X and Y independent? Provide your reasoning.

[4]

b) Consider the continuous random variable X characterized by the following probability density function

$$f_X(x) = \begin{cases} \theta\,(1-x)^{\theta-1}, & 0 < x < 1, \\ 0, & otherwise. \end{cases}$$

with $\theta > 0$.

i) We observe a random sample of size n drawn from the above distribution. Determine the maximum likelihood estimator of θ.

[6]

ii) If the observed random sample is given by the following data, using results from i), compute the estimate of θ.

0.214; 0.108; 0.015; 0.186; 0.054; 0.487; 0.062; 0.095; 0.052; 0.111;
0.519; 0.061; 0.046; 0.180; 0.351; 0.439; 0.088; 0.176; 0.041; 0.288

[3]

2. a) Consider two independent continuous random variables X_1 and X_2, each characterized by the probability density function [25]

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & otherwise. \end{cases}$$

Compute the probability that $X_1 + X_2$ is larger or equal to 1, i.e. $P(X_1 + X_2 \geq 1)$.

[4]

b) Consider two continuous random variables X and Y characterized by the conditional probability density function

$$f_{Y|X}(y|x) = \begin{cases} \frac{1}{x}\exp\left(-\frac{y}{x}\right), & 0 < y < \infty, 0 < x < 1 \\ 0, & otherwise. \end{cases}$$

and the marginal probability density function

$$f_X(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & otherwise. \end{cases}$$

i) Compute the joint probability density function $f_{X,Y}(x,y)$.

[2]

ii) Show that the conditional expectation of Y given X, i.e. $E(Y|X)$, is given by X.

[3]

iii) By making use of ii), compute the expectation of Y, i.e. $E(Y)$.

[4]

iv) By making use of ii), compute the variance of Y, i.e. $\mathrm{Var}(Y)$.

[6]

c) Consider the random variable U, uniformly distributed between 0 and 1. Compute the probability density function of $1 - \sqrt{1-U}$.

[6]

