UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2000

MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C480

AUTOMATED REASONING

Friday 19 May 2000, 14:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions

(In all questions variables begin with lower case u - z; other names are constants or functions. Predicates use upper case.)

1 a i) Define the terms *electron*, *nucleus* and *hyper-resolvent* in the context of the hyper-resolution (HR) refinement.

ii) Apply hyper-resolution to derive the empty clause from clauses (1) - (3).

(1) $Q(x, y) \vee Q(y, x)$ (2) $\neg Q(g(z), z)$

(3) $\neg Q(w, g(w)) \vee \neg Q(b,w)$

iii) What is the locking refinement?
Describe a suitable method for choosing indices that allows locking to simulate HR. Justify your answer.

b Negative hyper-resolution is similar to HR, except that the roles of positive and negative literals in electrons and nucleii are reversed.

i) Suppose every literal in a clause C is replaced by its complement to give clause c(C). This will be called the c-transformation. If a set of clauses S is unsatisfiable, why is the set c(S), in which each clause C in S is replaced by c(C), also unsatisfiable?

ii) Explain the meaning of the sentence "Hyper resolution is complete".

iii) Briefly, explain how the following:

the result proved in part bi),
the assumption that HR is complete and
the application of the c-transformation to each clause in S

can be used to justify the completeness of negative hyper-resolution.

iv) Illustrate your answer to part ciii) using clauses (1) to (3).

The two parts carry, respectively, 50%, 50% of the marks.

2 a i) Use the Davis Putnam (DP) procedure to find a model of the clauses (4) - (7). Your answer should make the steps of the procedure clear.

(4)	$A \vee \neg B \vee D$	(5)	$\neg A \vee \neg D$
(6)	$B \vee C$	(7)	$\neg C \vee A$

ii) Let S be a set of ground clauses. Show that, if the splitting operation in the DP procedure yields the two sets of clauses S1 and S2, then
S is satisfiable if and only if S1 is satisfiable or S2 is satisfiable.

b i) Give the initial connection graph for clauses (8) - (13).

(8)	$\neg A(d,e)$	(9)	$\neg B(f)$
(10)	$C(x) \vee B(x)$	(11)	$\neg A(e,f)$
(12)	$A(x,y) \vee \neg B(x) \vee B(y)$	(13)	$\neg C(d)$

ii) Apply the connection graph procedure to the initial graph of part ci); select links by the following criteria, with criterion (A) taking precedence over (B), (B) taking precedence over (C), etc.

(A) Select a link between two facts.
(B) Select a link that leads to a pure resolvent.
(C) Select a link between literals L and M such that it is the only link incident to L and M.
(D) Select a link incident to at least one fact.
(E) Select any link.

iii) Briefly justify the ordering given for the criteria in part cii).

The two parts carry, respectively, 50% , 50% of the marks.

3 a i) Define paramodulation.

ii) Using only paramodulation, and resolution with the clause x=x, show that the clauses (14) - (17) lead to the empty clause.

(14)	$n(n(x)) = x$	(15)	$m(x,x) = n(x)$
(16)	$n(m(x,y)) = m(y,x)$	(17)	$m(x,x) \neq x$

iii) What properties of equality are implicit in the definition of paramodulation?

b Apply the Knuth Bendix procedure to equations (14) - (16). Your answer should make clear the steps of the procedure.

c i) Explain how a confluent set of terminating rewrite rules can be used to decide whether s=t or not, for ground terms s and t. Justify your answer.

ii) In the light of your answer to part b), suggest a modification to your answer to part ci) that will allow you to show

$$m(m(n(a),a), n(n(b))) = m(b,n(m(a, n(a)))).$$

The three parts carry, respectively, 30%, 40%, 30% of the marks.

4 a Apply the Model Elimination (ME) tableau method to show that clauses (18) - (20) are unsatisfiable. Use clause (18) as top clause. Your answer should make clear the steps of the procedure.

(18)	$P(y,x) \vee \neg P(x,b)$	(19)	$\neg P(x,a) \vee \neg P(b,x)$
(20)	$P(a,y) \vee P(y, y)$		

b What is the merging operation of the ME method? Why is it sound for ground ME tableaux?

c For this part, suppose S is a set of Horn clauses (no clause in S has more than one positive literal). Suppose also that a branch is pursued forever until either it closes or fails.

i) If S is unsatisfiable, why must there be at least one clause in S with no positive literal?

NOTE: For parts cii), ciii) and civ) assume the top clause is of the form described in part ci).

ii) Briefly describe the structure of any ME tableau (with top clause as in part ci). In particular, explain why all branches close using extension by a clause from S and none close by ancestor matching.

iii) Suppose closure below a leaf literal L in a branch B fails and that a literal L', such that L subsumes L', occurs in some branch later in the proof search. Briefly, explain why L' must fail in that branch also.

iv) Suppose each negative literal is linked (as in a connection graph) to all positive literals with which it unifies. Briefly, suggest how you might use this information, together with branch failure as in part ciii), to perform some look-ahead pruning of the search for a closed tableau.

The three parts carry, respectively, 35%, 15%, 50% of the marks.