UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2002

MEng Honours Degree in Information Systems Engineering Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C480=I4.42

AUTOMATED REASONING

Wednesday 8 May 2002, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

NOTE: In all four questions:
- **variables begin with lower case u - z;**
- **other names beginning with lower case letters are functors;**
- **predicates begin with an upper case letter.**

1 a Let R be a set of *confluent* rewrite rules.

i) Define the term "confluent".

ii) Explain why, if R is also terminating, then R rewrites any term t to a unique normal form.

b For the set of equations {(1), (2)}

(1) j(x,x) = x (2) j(y,x) = s(j(x,y))

i) Show they can be oriented as rewrite rules for termination.

ii) Give a counterexample to show the resulting rewrite rules are not confluent.

c Apply the Knuth Bendix procedure to the equations (1), (2) of part b). What is the outcome?

Make clear the steps of the procedure in your answer.

d i) What are the possible outcomes obtainable from the Knuth-Bendix procedure (other than the outcome obtained in part c))?

ii) The Knuth Bendix procedure, which tries to derive a finite set of confluent and terminating rewrite rules, is to be used for the purpose of showing that two different *ground terms* t1 and t2 rewrite into a common term. *Briefly*, describe how, for the two outcomes of the Knuth Bendix procedure that do *not* result in a finite set of confluent and terminating rewrite rules, the results can be partially useful for the above purpose.

The four parts carry, respectively, 20%, 25%, 30%, 25% of the marks.

2 a In the connection graph proof procedure links may be selected according to various criteria, some of which are listed in (i) - (iv) below.

Rank the criteria (i) - (iv), in your opinion, from the *most* useful to the *least* useful, justifying your answer. Use the graph derived from (satisfiable) clauses (3) - (7) below to illustrate your answer.

(i) Select an inconsistent link.
(ii) Select a link L incident to exactly two literals M, N, in different clauses, such that L is the only link incident to M and the only link incident to N.
(iii) Select a link between a unit clause U and a clause C such that the resolvent subsumes C.
(iv) Select a link between two unit clauses.

(3) $P(a) \vee Q(x)$
(4) $\neg R(b)$
(5) $\neg P(x) \vee R(x)$
(6) $\neg Q(a)$
(7) $\neg R(x) \vee Q(x) \vee P(f(x))$

b i) Apply the KE tableau method to the (unsatisfiable) set of sentences (8) - (10):

(8) $((A \to B) \vee C) \to (\neg A \wedge D)$
(9) $A \to (B \vee C)$
(10) $D \to A$

ii) Justify the following statement:

"The KE tableau method, when applied to ground clauses, can simulate the Davis Putnam procedure."

Parts a, bi, bii carry, respectively, 45%, 20%, 35% of the marks.

3 a i) State the result of paramodulating, from left to right, $h(h(x))=x$ into the first argument position of $K(h(z),z)$.

ii) Show how this paramodulation step can be simulated by resolution using the substitutivity axiom

(I) $\neg u=w \vee \neg K(u,v) \vee K(w,v)$

iii) If $h(y)=y$ were to be paramodulated, from left to right, into $K(z,h(z))$ at the *second* occurrence of z, what is the result? Which two substitutivity axioms are needed to simulate this paramodulation step? (There is no need to carry out the simulation.)

b Paramodulation could be approximated in Model Elimination (ME) Tableaux by explicitly using substitutivity axioms such as (I) or those given in part aiii).

Use this approach and the results of part a) to derive a closed ME tableau for the clauses (11) - (14) below together with the appropriate substitutivity axioms. Take clause (14) as the top clause. Make clear the basic steps of the ME method in your answer.

(11) $\neg K(a,a)$
(12) $K(x,y) \wedge K(y,x) \rightarrow y=x$
(13) $K(h(z),z)$
(14) $h(h(x)) = x$

(**Hint**: Use the results of part a).)

c Suppose now that the explicit use of substitutivity axioms is to be incorporated implicitly in a new step called ME-eq.

Suggest possible ways of doing this and comment on your answer. Illustrate your suggestions using the clauses below (or others if you wish)

$P(a)$, $\neg P(b)$, $a=b$

The three parts carry, respectively, 25%, 40%, 35% of the marks.

4 a i) In the context of *hyper-resolution*, what are nucleii and electrons and what is their significance?

ii) Use hyper-resolution to derive the empty clause from

(15) $D(v,v)$
(16) $\neg D(g(g(x)),y) \vee \neg D(y,x)$
(17) $\neg P(y) \vee \neg D(y,a)$
(18) $P(y) \vee D(g(y),y)$

b In the context of the *Otter theorem prover*:

i) How are the two lists *sos* and *usable* employed?

ii) Explain why, if the theorem prover is started in a state with data

```
set(hyper_res).
list(usable).
G(c).
B(a).
G(x) | - G(y).
end_of_list.
list(sos).
- B(x) | - G(x).
end_of_list.
```

it halts with "sos empty".

c Suppose S is a set of clauses with no Herbrand models.

i) Use the *semantic tree method* to show that the set of clauses

$$\{G(c),\ B(a),\ G(x) \vee \neg G(y),\ \neg B(x) \vee \neg G(x)\}$$

taken from the two lists in part bii) has no Herbrand models:

Anotate your answer so that the method is clear.

ii) *Briefly*, explain why consideration of a semantic tree for any unsatisfiable set of clauses S justifies the following statement:

"If S is a set of clauses with no Herbrand model,
then a finite subset of ground (Herbrand) instances of S is unsatisfiable."

Parts a, b, ci, cii carry, respectively, 25%, 25%, 30%, 20% of the marks.