E2.4

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

EEE/ISE PART II: MEng, BEng and ACGI

COMMUNICATIONS 2

Monday, 7 June 2:00 pm

Time allowed: 2:00 hours

There are FIVE questions on this paper.

Question 1 is compulsory.

Answer THREE questions, including Question 1.

All questions carry equal marks

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D.B. Ward

Second Marker(s) : J.A. Barria

Special Information for Invigilators: none

Information for Candidates

Some useful relationships

$$\log_2(x) = 3.32\log_{10}(x)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\cos^2(A) = \frac{1}{2}\left[1 + \cos(2A)\right]$$

$$\sin^2(A) = \frac{1}{2}\left[1 - \cos(2A)\right]$$

$$\cos(A)\sin(A) = \frac{1}{2}\sin(2A)$$

The Questions

QUESTION 1 (Compulsory)

(a) The received signal in an AM communications system is passed through an ideal band-pass filter (BPF) centered on the carrier frequency ω_c and having an appropriate bandwidth. If the additive noise at the input to the BPF has a power-spectral density (PSD) of $N_o/2$ for all frequencies, draw the PSD of the noise component at the output of the BPF. Assume that the bandwidth of the unmodulated message signal is W.

[1 mark]

If this output noise component is represented as $n(t) = \sum_k a_k \cos(\omega_k t + \theta_k)$, show that it can also be written as:

$$n(t) = n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t).$$

[3 marks]

(b) In a baseband analog communications system the received signal $r(t) = s(t) + v(t)$ is passed through an ideal low-pass filter having bandwidth W and unity gain. The signal component $s(t)$ has a power spectral density (PSD) given by:

$$\mathbb{S}(f) = \begin{cases} P_o\left(1 - \frac{|f|}{B}\right), & |f| < B \\ 0, & \text{otherwise,} \end{cases}$$

and the noise component $v(t)$ has a PSD of $N_o/2$ for all frequencies. Derive an expression for the signal-to-noise ratio (SNR) at the output of the low-pass filter.

[4 marks]

(c) Consider a baseband digital system that has 3 equally-probable symbols $\{\alpha, \beta, \gamma\}$. The symbol α is represented by a voltage level of -2 volts, β by 0 volts, and γ by 2 volts. The channel noise is additive white Gaussian noise (i.e., it has a probability density function that is zero-mean and normally distributed). The receiver decision thresholds are set at -1 volt and 1 volt.

Draw and clearly label the probability density function of the received signal for each of the transmitted symbols, indicating the probability of error for each symbol. Comment on the relative probability of error for the three symbols.

[4 marks]

(d) Consider the 4-symbol alphabet $\{A, B, C, D\}$, where the probability of occurrence of each symbol is $p_A = 0.2, p_B = 0.45, p_C = 0.1, p_D = 0.25$. Using the Huffman coding scheme, determine a variable-length source code for this alphabet.

[4 marks]

(e) Briefly describe and contrast circuit switching and packet switching.

[4 marks]

QUESTION 2

(a) A bandpass noise process has the power spectral density shown in Figure 2.1 below. If this bandpass noise is represented by $n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$, sketch and annotate accordingly the power spectral density of the $n_c(t)$ component in each of the following cases:

(i) $f_c = 100$ kHz

[2 marks]

(ii) $f_c = 105$ kHz

[2 marks]

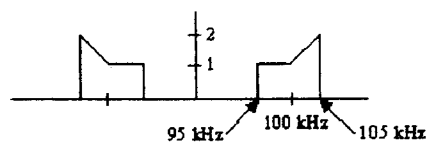

Figure 2.1

(b) Comment on the power spectral densities of typical message and noise components at the output of a receiver in an FM system, and describe how the process of pre- and de-emphasis improves the output signal-to-noise ratio (SNR).

[6 marks]

(c) In the transmission of telephone signals over microwave links, a combination of frequency-division multiplexed (FDM) DSB-SC and FM is to be used. A block diagram of such a system is shown in Figure 2.2 below. Each of the input signals $m_i(t), 1 \le i \le K$ is bandlimited to W, and these signals are DSB-SC modulated on carriers $c_i(t) = A_i \cos(2\pi f_i t)$, where $f_i = 2W(i-1)$, for $1 \le i \le K$. The output of the multiplexer $m(t)$ is the sum of all the DSB-SC modulated signals. The multiplexed signal is then FM modulated on a carrier with amplitude A_c and frequency f_c with a modulation index β.

(i) For $K = 3$, plot a typical spectrum of the multiplexed signal $m(t)$.

[3 marks]

(ii) For $K = 3$, determine the bandwidth of $m(t)$.

[1 mark]

(iii) At the receiver side, the received signal $r(t) = u(t) + n(t)$ is first FM demodulated and then passed through a bank of DSB-SC demodulators. If the power spectral density of the noise at the output of the FM demodulator is $\mathrm{N}(f) = \frac{N_0}{A_c^2} f^2$, derive an expression for the pre-detection noise power at the ith DSB-SC demodulator.

[6 marks]

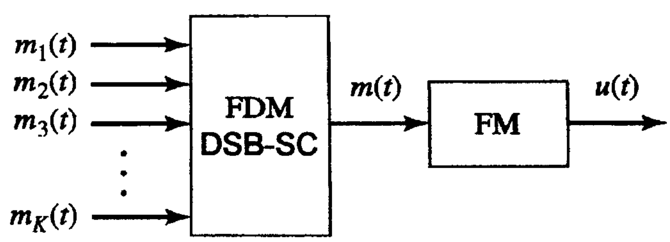

Figure 2.2

QUESTION 3

(a) Explain the process of pulse code modulation, and comment on the issues involved in choosing the number of bits in the quantizer.

[4 marks]

(b) A binary channel with a data rate of 36,000 bits/second is available for pulse code modulated voice transmission. Assume the voice channel has a bandwidth of 3200 Hz and a uniform quantizer is used. Find appropriate values for the number of levels in the quantizer and the sampling rate; explain why you have chosen these particular values.

[3 marks]

(c) The binary bit stream $\{1, 0, 0, 1\}$ is to be transmitted using digital bandpass transmission. Draw and clearly label the transmitted waveform for each of the following schemes:

(i) amplitude shift keying (ASK)

(ii) phase shift keying (PSK)

(iii) frequency shift keying (FSK)

[3 marks]

(d) Having missed out on a banking job in the City, you have taken employment in an engineering consulting firm. Your first task is to set up a central station for simultaneous monitoring of electrocardiograms (ECGs) of 10 hospital patients. The data from the rooms of the 10 patients are brought to a central processing centre over wires and are sampled, quantized, binary encoded, and synchronous time-division multiplexed. The multiplexed data are then transmitted to the monitoring station. A block diagram of the system is shown in Figure 3.1 below. The bandwidth of each ECG signal is 100 Hz. The maximum acceptable error in sample amplitudes is 1% of the peak signal amplitude, and the range of the quantizer matches the peak signal range. The sampling rate is to be at least 50% above the Nyquist rate.

(i) Determine the minimum cable bandwidth needed to transmit the multiplexed data to the monitoring station. Assume that the additive noise in this cable is negligible.

[6 marks]

(ii) It is found that the signal-to-quantization-noise (SQNR) of the received ECG signals in part (i) is not high enough. How could you increase the SQNR by 12 dB without increasing the cable bandwidth?

[4 marks]

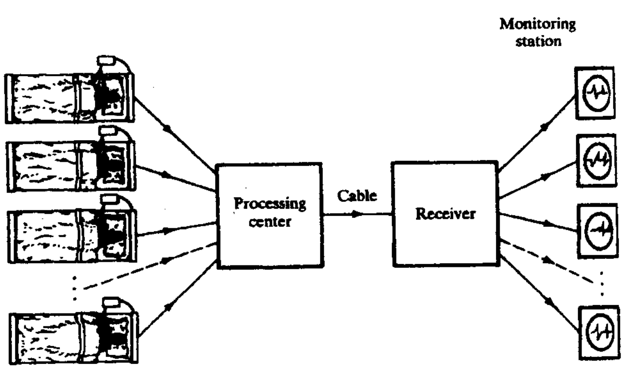

Figure 3.1

QUESTION 4

(a) Write down the Hartley-Shannon expression for channel capacity, explaining and giving units for all terms used. Discuss the practical relevance of this expression.

[4 marks]

(b) An information source can be modelled as a bandlimited process with a bandwidth of 6000 Hz. This process is sampled at the Nyquist rate. It is observed that the resulting samples take values in the set $\mathcal{A} = \{-3, -1, 2, 4\}$ with corresponding probabilities $\{0.3, 0.15, 0.05, 0.5\}$.

What is the entropy of the discrete-time source in bits/sample? What is the entropy in bits/second?

[2 marks]

(c) A discrete memoryless source has the alphabet $\mathcal{A} = \{-3, -1, 0, 1, 3\}$ with corresponding probabilities $\{0.15, 0.1, 0.15, 0.05, 0.55\}$.

Assume that the source is quantized according to the quantization rule:

$$\begin{aligned} q(-3) = q(-1) &= -2, \\ q(0) &= 0, \\ q(1) = q(3) &= 2. \end{aligned}$$

Design a source code for the quantized source, and comment on the average codeword length relative to the entropy.

[4 marks]

(d) A television picture frame is composed of 300,000 basic picture elements (600 picture elements in a horizontal line, and 500 picture elements in a vertical line). Each of these picture elements can assume one of 8 distinguishable brightness levels (such as black and shades of gray) with equal probability.

(i) Determine the information content of a single television picture frame.

[2 marks]

(ii) A television transmission requires 30 such frames to be transmitted per second. If the SNR at the receiver is required to be at least 50 dB, calculate the theoretical bandwidth required of the transmission channel (which is assumed to have additive white Gaussian noise).

[4 marks]

(e) Consider a channel with a bandwidth of B, and additive Gaussian noise having a flat power spectral density of $N_o/2$. Derive an expression for the Hartley-Shannon channel capacity as $B \to \infty$. You may find the following expression useful:

$$\lim_{x \to \infty} x \log_2 \left(1 + \frac{1}{x}\right) = \log_2 e = 1.44$$

[4 marks]

QUESTION 5

(a) Describe and compare statistical time division multiplexing and synchronous time division multiplexing.

Under what conditions would you expect statistical TDM to be more efficient than synchronous TDM.

[6 marks]

(b) Contrast the operation of a LAN using a bus topology with one using a ring topology.

[4 marks]

(c) Consider the signal $x(t) = Ae^{j(\omega t+\theta)}$, where A and ω are constants, and θ is a random variable having a probability density function that is uniformly distributed in the range 0 to $\pi/2$.

(i) Draw the probability density function of θ.

[1 mark]

(ii) Evaluate the mean value of $x(t)$.

[2 marks]

(iii) Evaluate the mean square value of $x(t)$.

[3 marks]

(d) Determine the autocorrelation of the modulated DSB-SC signal $s(t) = m(t)\cos(\omega_c t + \theta)$, where $m(t)$ is a stationary random process and θ is a random variable uniformly distributed over $[0, 2\pi]$. Assume that θ and $m(t)$ are independent.

[4 marks]

Comm'ns II

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MODEL ANSWER and MARKING SCHEME

First Examiner D. Ward.

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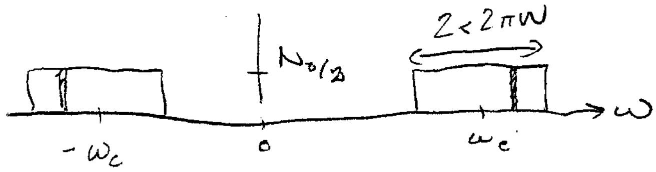
Q1.

(a) PSD is



1.

- let component in narrow frequency band shown be

$n_k(t) = a_k \cos(\omega_k t + \theta_k)$

- let $\omega_k = \omega_k - \omega_c + \omega_c$

$\therefore n_k(t) = a_k \cos[(\omega_k - \omega_c)t + \theta_k] \cos \omega_c t$
$\quad - a_k \sin[(\omega_k - \omega_c)t + \theta_k] \sin \omega_c t$

$\therefore n(t) = \sum_k n_k(t)$
$\quad = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t$

where $n_c(t) = \sum_k a_k \cos[(\omega_k - \omega_c)t + \theta_k]$

$n_s(t) = \sum_k a_k \sin[(\omega_k - \omega_c)t + \theta_k]$ 3.

Q1.

(b)

Avg power of noise: $P_N = \int_{-W}^{W} \frac{N_0}{2}\, df = N_0 W$

Avg power of signal: $P_S = \int_{-W}^{W} P_0\left(1 - \frac{|f|}{B}\right) df$

$$= 2P_0 \int_0^W 1 - \frac{f}{B}\, df$$

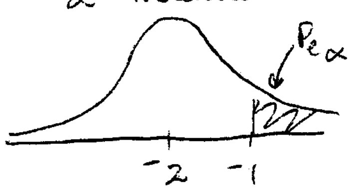
$$= 2P_0 \left[f - \frac{f^2}{2B}\right]_0^W$$

$$= 2P_0\left(W - \frac{W^2}{2B}\right)$$

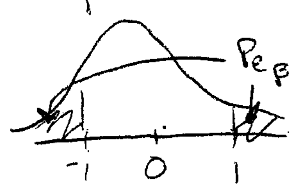
$\therefore SNR = \dfrac{2P_0 W\left(1 - \frac{W}{2B}\right)}{N_0 W} = \dfrac{2P_0\left(1 - \frac{W}{2B}\right)}{N_0}$ 4.

(c)

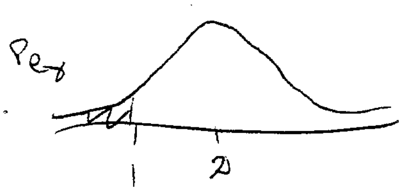
α transmitted



β transmitted



γ transmitted



- β will have twice probability of error of each of other two 4.

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MODEL ANSWER and MARKING SCHEME

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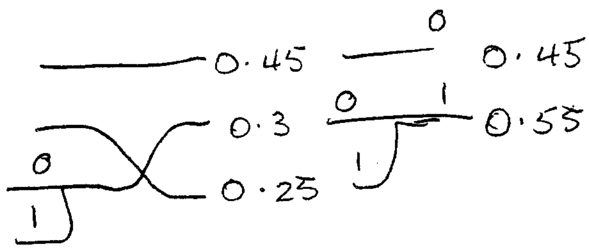
Q1 (d)

0	B	0.45
11	D	0.25
100	A	0.2
101	C	0.1



4.

(e) Circuit switching

- dedicated path exists between stations
- 3 phase: - circuit establishment
  - data transfer
  - circuit disconnect

2.

Packet switching

- station address appended to data packets
- each packet is received by intermediate nodes, stored, transmitted to next node
- no dedicated path
- packets may travel along different routes & arrive in different order

2.

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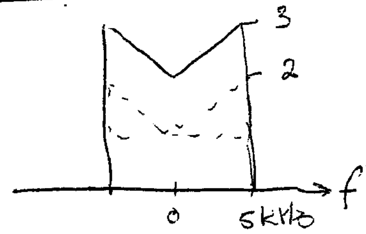
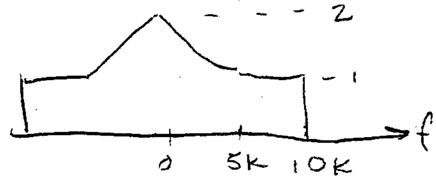
Q2 (a)

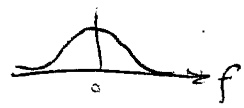
(i) $f_c = 100\text{kHz}$ 2.



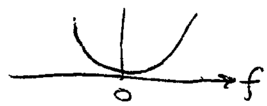
(ii) $f_c = 105\text{kHz}$ 2.



(b) The message component typically has a low-pass type PSD, eg.



whereas at the FM receiver output, the effect of the receiver is to shape the noise PSD as



This means SNR is poor, especially at low frequencies. Pre-emphasis boosts the HF components of the message prior to transmission, thereby increasing SNR at receiver output. De-emphasis then restores the original message spectrum by de-emphasizing HF components so message is undistorted. 6.

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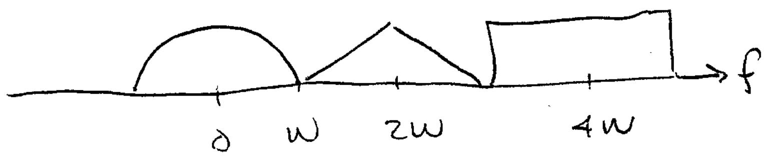
Q2 (c)

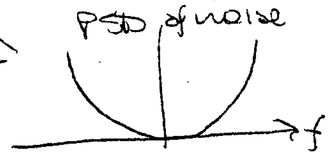
(i)



3.

(ii) $BW = 5W$, or in general $BW = W + 2W(k-1)$ 1.

(iii) PSD of noise



- predetection filter on i^{th} detector has passband of:

$2W(i-1) - W$ to $2W(i-1) + W$

∴ avg noise power after pre-detection filter is:

$$P_N = \int_{L_i}^{U_i} \frac{N_0}{A_c^2} f^2 \, df$$

where $L_i = 2W(i-1) - W$

$U_i = 2W(i-1) + W$

$$= \frac{N_0}{3A_c^2} \left[f^3 \right]_{L_i}^{U_i}$$

6.

Q3 (a)

PCM is a scheme used to represent analog signals digitally. The analog signal is sampled, the amplitude rounded to the nearest one of a finite set of values (quantized), & each quantized sample is encoded into a binary codeword.

The number of bits in the quantizer determines the accuracy with which the amplitude is recorded. The more bits, the less quantization noise, but the higher the required channel bandwidth. 4.

(b)

Choose min sampling rate of 6400 Hz.

This allows $\frac{36,000}{6,400} = 5.625$ bits/sample.

So, choose 5 bits per sample, giving a maximum sampling rate of $\frac{36,000}{5} = 7,200$ Hz.

The higher sampling rate is preferred since it reduces constraint on reconstruction filter. 3.

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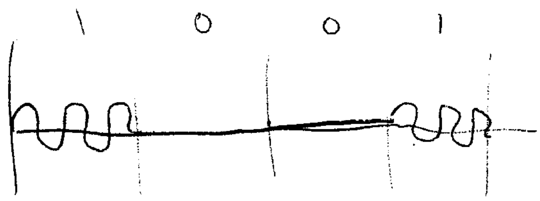
Q3

(c)

(i) ASK 

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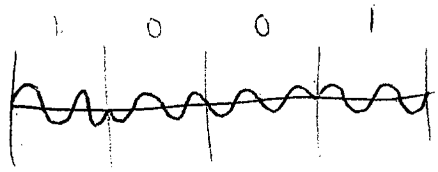
(ii) PSK 

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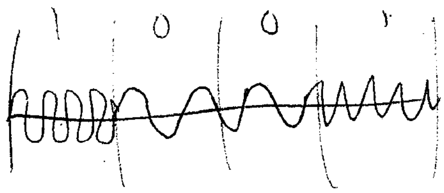
(iii) FSK 

1.

Q3 (a) (i) Nyquist sampling freq is 200 Hz , $\therefore f_s = 300$ Hz

Max quant error is $\Delta/2$ where Δ is quantizer step size

and $\Delta = \frac{2V_p}{L}$ where V_p is peak input amplitude, L is no. of levels.

$\therefore \frac{V_p}{L} = \frac{V_p}{100}$ $\therefore L = 100$ $\therefore$ use $L = 128$,

and $n = \log_2 L = 7$ bits

$\therefore$ Each ECG signal requires : $7 \times 300 = 2{,}100$ bits/sec

$\therefore$ 10 ECG signals require : 21,000 bits/sec.

But, we require 2 bits/sec/Hz

$\therefore$ Cable requires bandwidth of 10·5 kHz. 6.

(ii) Increasing SQNR by 12 dB requires an extra 2 bits in the quantizer, i.e. $n = 9$.

But since each signal can only use 2100 bits/sec we would need to drop sampling frequency to

$f_s = \frac{2100}{9} = 233{\cdot}3$ Hz.

This means there is less guard band between the multiplexed signals, requiring sharper filters at the receiver. 4.

Q4 (a) $C = B\log_2(1+SNR)$

where C is channel capacity : bits/sec
B is bandwidth : Hz
SNR is signal to noise ratio : no units $\left(\frac{W}{W}\right)$

This tells us maximum rate of error-free transmission over a noisy channel, but does not tell us how to encode to achieve this capacity. 4.

(b) $f_s = 12$ kHz

$H = -\sum p_k \log_2 p_k$
$= 1.646$ bits/sample 2.

At 12,000 samples/sec, H is 19,752 bits/sec.

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Q4 (c) For the quantized source, new symbols are $\{-2, 0, 2\}$ with probabilities $\{0.25, 0.15, 0.6\}$

Entropy is: $H = -\sum p_k \log_2 p_k$

$= 1.35$ bits/symbol

Huffman code:

0 (2) 0.6

10 (-2) 0.25

11 (0) 0.15

Avg codeword length: $\bar{L} = \sum p_k l_k$

$= 0.6 \times 1 + 0.25 \times 2 + 0.15 \times 2$

$= 1.4$

Avg codeword length > entropy, as expected from source coding theory. 4.

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Q4 (d)

(i) Each pixel can be represented by $\log_2 8 = 3$ bits, giving total of $3 \times 300{,}000 = 900{,}000$ bits/frame 2.

(ii) For 30 frames/sec, requires:

$$C = 30 \times 900{,}000 = 27 \times 10^6 \text{ bits/sec}$$

But $C = B \log_2(1+SNR)$ where $SNR = 50\,dB = 1 \times 10^5$

$$\therefore B = \frac{C}{\log_2(1+SNR)} = \frac{27 \times 10^6}{\log_2(1 \times 10^5)} = 1.63 \times 10^6 \text{ Hz}$$ 4.

(e) $C = B \log_2(1 + S/N)$ where $N = N_0 B$

$$= B \log_2\left(1 + \frac{S}{N_0 B}\right)$$

$$= \frac{S}{N_0}\left[\frac{N_0 B}{S} \log_2\left(1 + \frac{S}{N_0 B}\right)\right]$$

$$= 1.44 \frac{S}{N_0}.$$ 4.

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Q5 (a)

Synchronous TDM use pre-assigned & fixed time slots, whether or not source has data to send. Statistical TDM only uses slot when source has data to send, but needs to add header information.

Statistical will be more efficient if average source is silent for a period greater than ratio of header info to data length. 6.

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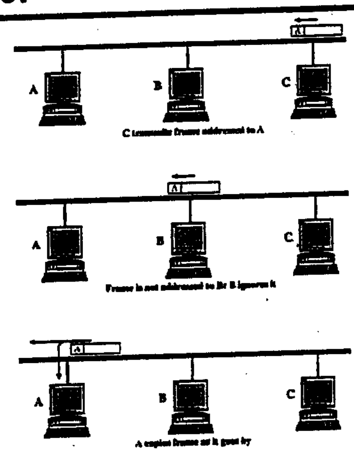
Q5 (b)

Bus topology



- All stations attach directly to linear transmission medium
- Transmission from any station propagates in both directions

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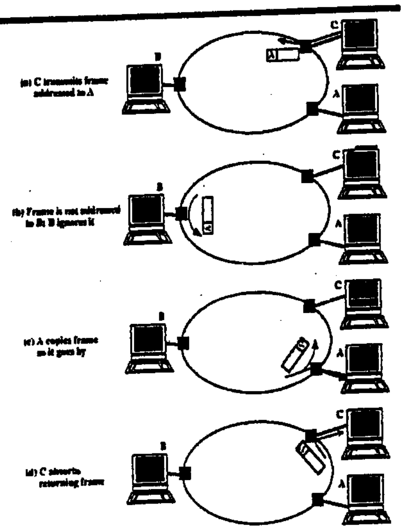
Part II

Ring topology

- Set of repeaters joined in closed loop
- Repeater receives data on one link, and retransmits on other – links are unidirectional
- Each station attaches to network at a repeater



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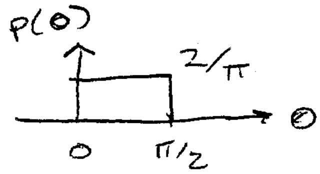
Q5 (c)

(i)



1.

(ii) $E\{x(t)\} = \int_{-\infty}^{\infty} A e^{j(\omega t+\theta)} p(\theta)\, d\theta$

$= \frac{2}{\pi} A e^{j\omega t} \int_0^{\pi/2} e^{j\theta}\, d\theta$

$= \frac{2}{\pi} A e^{j\omega t} \frac{1}{j} [j-1]$ 2.

(iii) $E\{x^2(t)\} = \int_{-\infty}^{\infty} A^2 e^{2j(\omega t+\theta)} p(\theta)\, d\theta$

$= \frac{2}{\pi} A^2 e^{j2\omega t} \int_0^{\pi/2} e^{j2\theta}\, d\theta$

$= \frac{2}{\pi} A^2 e^{j2\omega t} \frac{1}{2j} [-1-1]$

$= \frac{2}{\pi} A^2 e^{j2\omega t}\, j$ 3.

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MODEL ANSWER and MARKING SCHEME

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Q3 (d)

$$R_s(\tau) = E\{s(t)\, s(t+\tau)\}$$

$$= E\{m(t)\cos(\omega_c t+\theta)\, m(t+\tau)\cos[\omega_c(t+\tau)+\theta]\}$$

$$= E\{m(t)m(t+\tau)\}\, E\{\tfrac{1}{2}\cos\omega_c\tau + \tfrac{1}{2}\cos[2\omega_c(t+\tau)+2\theta]\}$$

$$= R_m(\tau)\left[\tfrac{1}{2}\cos\omega_c\tau + \tfrac{1}{2}\int_0^{2\pi}\tfrac{1}{2\pi}\cos(2\omega_c(t+\tau)+2\theta)\,d\theta\right]$$

$$= R_m(\tau)\ \tfrac{1}{2}\cos\omega_c\tau$$

where $R_m(\tau) = E\{m(t)\, m(t+\tau)\}$ 4.