Master copy - July 08

DIGITAL SIGNAL PROCESSING
SOLUTIONS

1. a) $x_e(n) = \frac{1}{2}(x(n) + x(-n))$

   $x_o(n) = \frac{1}{2}(x(n) - x(-n))$

   b) When $x(n)$ is causal we have:

   $n > 0: \; x_e(n) = x_o(n)$

   $n = 0: \; x_e(0) = x(0)$ and $x_o(0) = 0$

   $n < 0: \; x_e(n) = -x_o(n)$

   c) $Y_R(k) = \sum_{n=0}^{N-1} \left(y_R(n)\cos\frac{2\pi nk}{N} + y_I(n)\sin\frac{2\pi nk}{N}\right)$

   $Y_I(k) = \sum_{n=0}^{N-1} \left(-y_R(n)\sin\frac{2\pi nk}{N} + y_I(n)\cos\frac{2\pi nk}{N}\right)$

   d) If $x(n)$ is real, causal and absolutely summable, then $x_I(n) = 0$ in the above and the Fourier transforms exist, so that:

   $x_e(n) \quad \leftrightarrow \quad X_R(k)$

   $x_o(n) \quad \leftrightarrow \quad X_I(k)$

   By way of the discussion, it should be noted that, from part (b), when a signal is causal, it is determined only by the even part $x_e(n)$. Hence, the DFT is determined only by the real part $X_R(k)$

   e) The first step is to choose the block size and window function for application of the DFT. We need to place a frequency bin exactly at 23.2 and 3.2 kHz. A frequency spacing of 800 Hz will suffice for which the total time of sampling must be $1/800 = 1.3$ ms. At 48 kHz this corresponds to 60 samples. The absolute value of the $Y(4)$ bin would give an indication of signal present at 3.2 kHz which the absolute value of the $Y(29)$ bin would give an indication of the signal present at 23.2 kHz. The music may have strong components also at these frequencies which may give rise to false alarms.

2. a) Bookwork.

b) A cascade of expansion and decimation is used. It is (almost) always essential to expand before decimation so as to maintain the maximum spectral content in tact. Lowpass filtering is applied between the expander and decimator with cutoff frequency corresponding to the lowest Nyquist frequency in the system.

c) A: x0,x3,x6
B: x0,0,x3,0,x6,0
C: y0,0,y1,0,y2,0,y3,0,y5,0,y6,0,
D: y0,0,y3,0,y6

The order of multirate blocks is significant except in the special case of decimation and expansion where the rate change factors are co-prime. This example is just such a special case so that the two systems are in fact equivalent.

d) From the definition of the z-transform

$$\begin{aligned} Y_E(z) &= \sum_{n=-\infty}^{\infty} y_E(n) z^{-n} \\ &= \sum_{k=-\infty}^{\infty} y_E(kL) z^{-kL} \\ &= \sum_{k=-\infty}^{\infty} x(k) z^{-kL} = X(z^L) \end{aligned}$$

For frequency response write $z = e^{j\omega}$ giving

$$Y_E(e^{j\omega}) = X(e^{j\omega L})$$

Y_E is a compressed version of X.

Multiple images of $X(e^{j\omega})$ are created in $Y_E(e^{j\omega})$ between $\omega = 0$ and $\omega = 2\pi$.

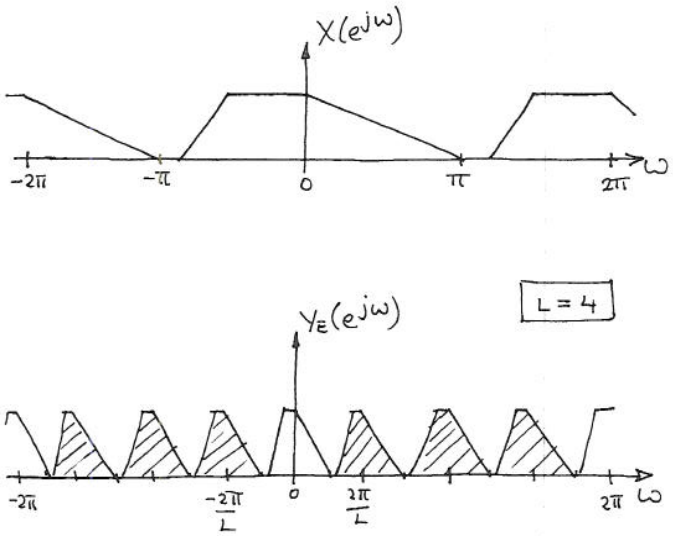

Figure 2.1

3. a) FIR is non-recursive; output depends of weighted sum of current and previous inputs.

$$H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

IIR is recursive, output depends on weighted sum of current input and previous inputs and outputs.

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

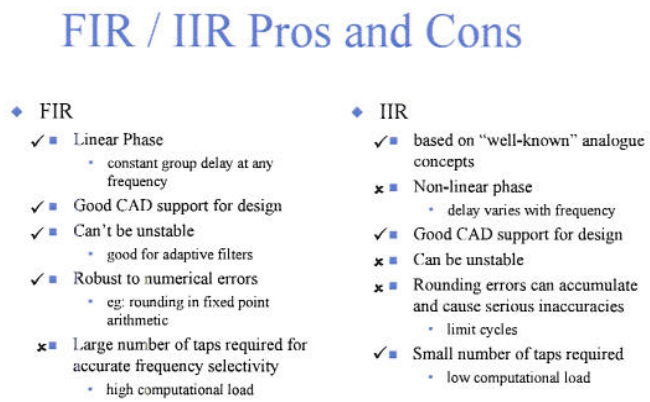

Figure 3.1

The system function follows directly as

$$B(z) = (1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

which has the difference equation

$$y(n) = x(n) + x(n-1) + x(n-2) + x(n-3) + x(n-4).$$

This can be written recursively as

$$y(n) = y(n-1) + x(n) - x(n-5)$$

such that

$$G(z) = \frac{1 - z^{-5}}{1 - z^{-1}}$$

G(z) has nulls when $\cos 5\omega = 1$ so that $\omega = 2\pi/5,\ 4\pi/5, \ldots$.

The magnitude spectral plot is therefore of the following form. Full marks can be obtained from correctly sketch the general shape and noting correctly the frequencies of the nulls.

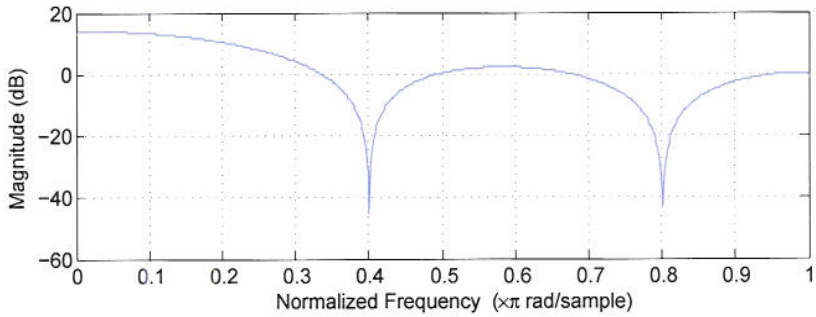
20
0
−20
−40
−60
Magnitude (dB)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Frequency (×π rad/sample)

Figure 3.2

4. a) The signal flow graph is simply of the form of

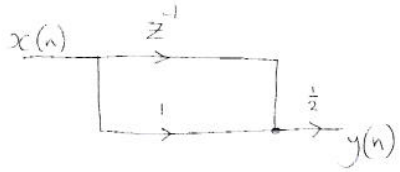

Figure 4.1

b) The frequency response is

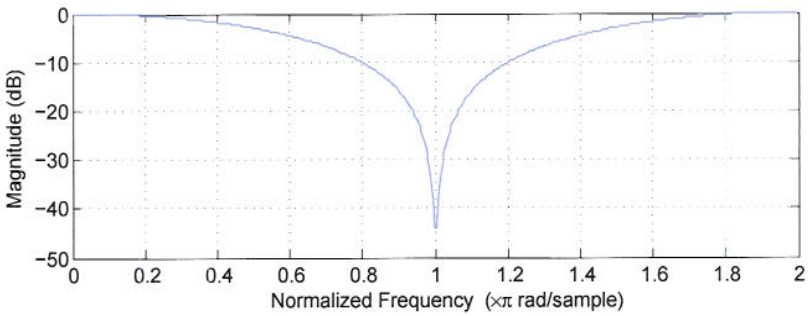

Figure 4.2

c) The filter $H(z)$ has a zero at $z = -1$ and a pole at $z = 0$. To obtain a comb filter we need to modify the frequency axis so that the response is periodic with period $2\pi/M$ instead of being periodic with period 2π. This can be obtained by replacing z with z^M in the system function. Therefore the comb filter $C(z) = H(z^M)$.

d) The signal flow graph is identical to that in Fig. **??** but with z^{-1} replaced by z^{-M}

5. a) $X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$. The region of convergence is the region of the z plane for which the summation in the z-transform converges.

b) i) $X(z) = -1 + z^{+1} - 9z^{-1}, \quad z \neq 0$

ii)

$$\begin{aligned} Y(z) &= \frac{1}{2} Z\{e^{j\omega_0 n} u(n)\} + \frac{1}{2} Z\{e^{-j\omega_0 n} u(n)\} \\ &= \frac{1}{2}\left(\frac{1}{1 - e^{j\omega_0} z^{-1}} + \frac{1}{1 - e^{-j\omega_0} z^{-1}}\right) \\ &= \frac{1 - z^{-1}\cos\omega_0}{1 - 2z^{-1}\cos\omega_0 + z^{-2}} \end{aligned}$$

c) $H(z)$ has poles at $0.8 \pm j0.1$. For a causal system, the ROC must lie outside the pole furthest from the origin. Hence ROC is $|z| > \sqrt{0.8^2 + 0.1^2}$. For stability, the ROC must include the unit circle which it does in this particular case.

d) For input and output samples $p(n)$ and $q(n)$ respectively,

$$q(n) = p(n) + 1.6q(n-1) - 0.65q(n-2).$$

The first three samples are then computed as $[1, 2.6, \ 4.51, \ \ldots]$.

6. a) $\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$

Important properties are that the a.c. function has a maximum at $l = 0$. The periodicity of $\gamma_{xx}(l)$ is equal to the periodicity of $x(n)$.

b) The room can be sketched as follows.

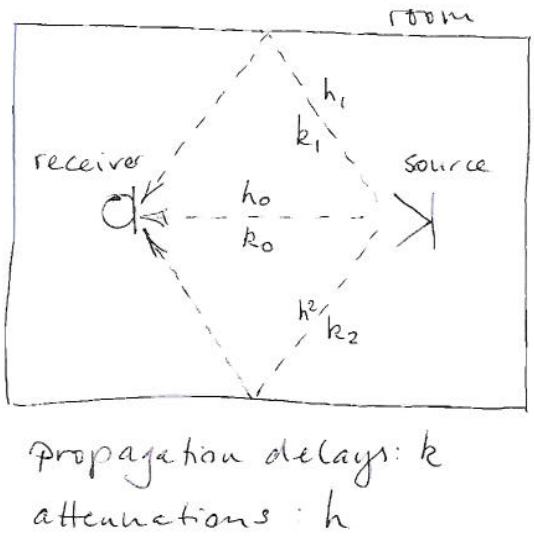

Figure 6.1

The model lacks accuracy because the number of reflections is too few, the delays are not generally integer numbers of samples, h may be dependent on frequency, h and k may be time varying.

$$
\begin{aligned}
\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} & (h_0 s(n-k_0) + h_1 s(n-k_1) + h_2 s(n-k_2)) \cdot \\
& (h_0 s(n-l-k_0) + h_1 s(n-l-k_1) + h_2 s(n-l-k_2)) \\
& = \gamma_{ss}(l)\left(h_0^2 + h_1^2 + h_2^2\right) \\
& + h_0 h_1 \left(\gamma_{ss}(l+k_0-k_1) + \gamma_{ss}(l-k_0+k_1)\right) \\
& + h_0 h_2 \left(\gamma_{ss}(l+k_0-k_2) + \gamma_{ss}(l-k_0+k_2)\right) \\
& + h_1 h_2 \left(\gamma_{ss}(l+k_1-k_2) + \gamma_{ss}(l-k_1+k_2)\right).
\end{aligned}
$$