Corrected Copy (Q2)

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

EEE/ISE PART I: MEng, BEng and ACGI

ANALYSIS OF CIRCUITS

Friday, 3 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D.M. Brookes
Second Marker(s) : P. Georgiou

Information for Candidates:

The following notation is used in this paper:

1. The voltage waveform at node X in a circuit is denoted by $x(t)$, the phasor voltage by X and the root-mean-square phasor voltage by $\tilde{X} = \frac{X}{\sqrt{2}}$.
2. Component and source values in a circuit are normally given in Ohms, Farads, Henrys, Volts or Amps with the unit symbol omitted. Where an imaginary number is specified, it represents the complex impedance or phasor value.
3. Times are given in seconds unless otherwise stated.

1. (a) Using nodal analysis calculate the voltages at nodes X and Y in *Figure 1.1.* [5]

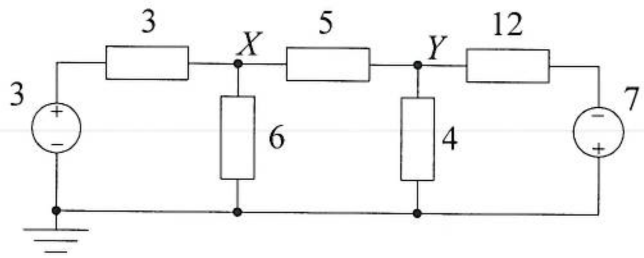


Figure 1.1

(b) Use the principle of superposition to find the voltage V in *Figure 1.2.* [5]

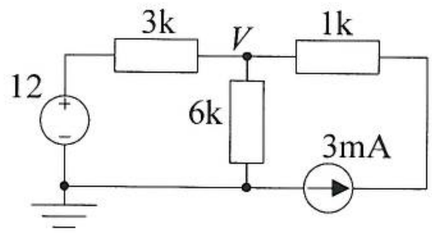


Figure 1.2

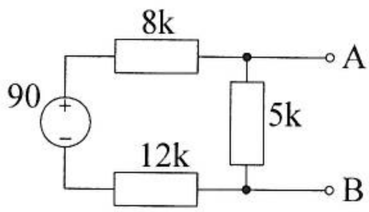


Figure 1.3

(c) Draw the Thévenin equivalent circuit of the network in *Figure 1.3* and find the values of its components. [5]

(d) Assuming the opamp in the circuit of *Figure 1.4* is ideal, give an expression for Z in terms of X. [5]

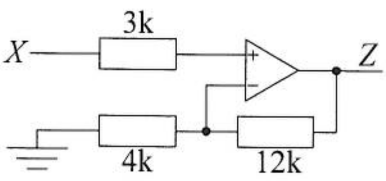


Figure 1.4

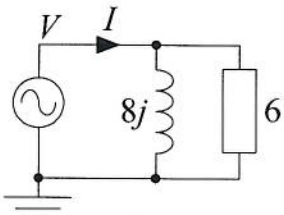


Figure 1.5

(e) (i) The phasor representing the voltage at V in *Figure 1.5* has the value $24j$. Determine the phasor current I in the form $a + jb$. [2]

(ii) Determine the complex impedance of the parallel L-R combination in the form $r\angle\theta$. [2]

(iii) If $\omega = 500$ rad/s, calculate the value of the inductance in Henries. [1]

(f) (i) Show that the frequency response of the circuit shown in Figure 1.6 is [1]

$$\frac{Y}{X} = \frac{1}{j\omega RC + 1}$$

(ii) Give expressions for the low and high frequency asymptotes of the response. [1]

(iii) Draw separate graphs showing straight-line approximations to the magnitude and phase responses of the circuit. Indicate on your graphs the corner frequency values and the values of any horizontal portions of the responses. [3]

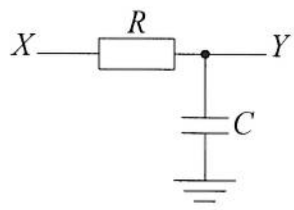


Figure 1.6

(g) (i) Determine the angular frequency, ω_0, at which the impedances of the inductor and capacitor in the circuit of *Figure 1.7* have the same magnitude. [2]

(ii) Determine the value of the phasor X at the frequency ω_0 if the phasor V has the value 10. [3]

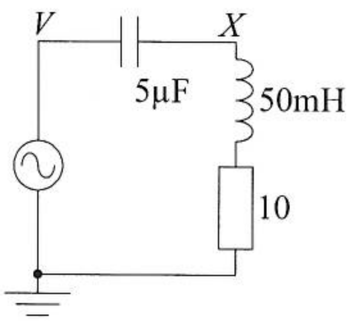


Figure 1.7

(h) In *Figure 1.8*, the voltage at X is $x(t) = 5\sin\omega t$. Sketch a graph showing the waveform at Y. Indicate on your graph the maximum and minimum values taken. Assume that the diode has a forward voltage drop of 0.7 V and is otherwise ideal. [5]

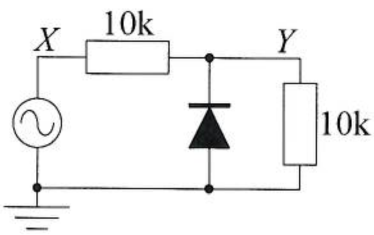


Figure 1.8

2. (a) Assuming that the op-amp in the circuit of *Figure 2.1* is ideal, give an expression for the gain $\frac{Y}{X}$. State clearly any assumptions you make. [4]

(b) Determine the transfer function, $\frac{Y}{X}(j\omega)$ of the circuit shown in *Figure 2.2*. Give expressions for its low and high frequency asymptotes and for its corner frequencies. [10]

(c) Draw a dimensioned sketch of the straight-line approximation to the magnitude response, $\left|\frac{Y}{X}(j\omega)\right|$, when $C = 10$ n, $R_1 = 10$ k and $R_2 = 25$ k. Indicate on your sketch, the values of the corner frequencies in Hz and the gain of any horizontal portions of the response. [6]

" using logarithmic axes "

(d) Explain how the response would be changed if the two resistors were interchanged. [4]

(e) With $C = 10$ n, select values for R_1 and R_2 so that the corner frequencies are at 500 Hz and 5 kHz and the gain of the horizontal portion of the transfer function is unity. [6]

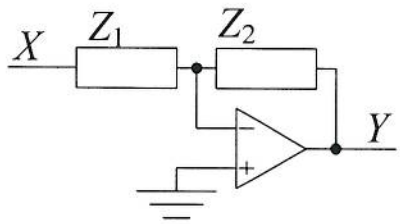


Figure 2.1

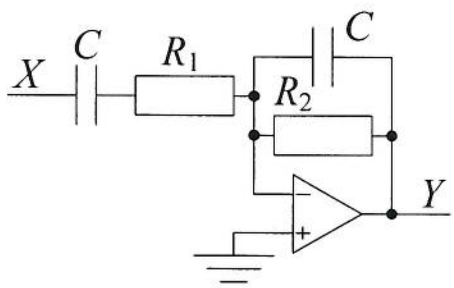


Figure 2.2

3. In the circuit of *Figure 3.1* the complex impedance of the inductor is jX and the phasor voltage of the source is V at an angular frequency $\omega = 500$ rad/s.

(a) Give an expression for the average power dissipation of resistor R in terms of V, R, X and Y. [8]

(b) Prove that the value of R that maximizes its average power dissipation is given by [6]

$$R = \sqrt{X^2 + Y^2}.$$

(c) If $V = -10j$, $X = 500$, $Y = 10$ and $R = 50$ determine the complex power absorbed by each of the components and the phasor voltage W in the form $a + jb$. [8]

(d) Now suppose that the component values are the same as in part (c), but the waveform V is now given by

$$v(t) = \begin{cases} 0 & \text{for } t < 0 \\ 10 \sin \omega t & \text{for } t \geq 0 \end{cases}$$

as shown in *Figure 3.2*.

Determine an expression for the waveform $w(t)$ for $t \geq 0$. Calculate the numerical values of all quantities in the expression. [8]

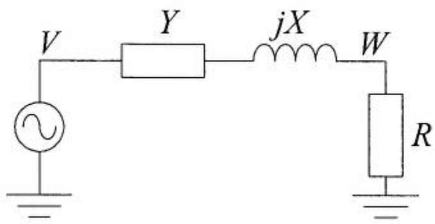


Figure 3.1

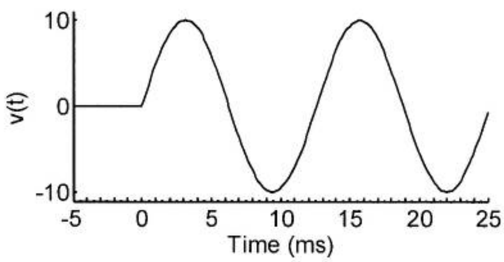


Figure 3.2

2011 E1.1: Analysis of Circuits - Solutions

Key to letters on mark scheme: B=Bookwork, C=New computed example, A=Analysis of new circuit, D=design of new circuit

1. (a) Nodal equation at X gives $\frac{X-3}{3}+\frac{X}{6}+\frac{X-Y}{5}=0$ from which $21X-6Y=30$. [This simplifies to $7X-2Y=10$.] [2A]

Nodal equation at Y gives $\frac{Y-X}{5}+\frac{Y}{4}+\frac{Y+7}{12}=0$ from which $12X-32Y=35$. [2A]

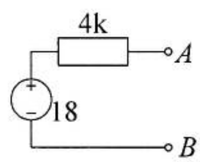
Taking 4 times the first equation minus 7 times the second gives $200Y=-125$ from which $Y=-0.625$. Substituting this into the second equation gives $12X=15$ from which $X=1.25$. [1A]

(b) Setting the current source to zero (open circuit) gives a potential divider with $V=12\times\frac{6}{9}=8$. Setting the voltage source to zero (short circuit) gives 3k and 6k resistors in parallel which are equivalent to 2k. Hence the voltage due to the current source is $2\times 3=6$. Combining these gives $V=8+6=14$. [5A]

(c) The Thévenin resistance (obtained by setting the voltage source to zero) is $5||(8+12)=5||20=4\text{ k}\Omega$. [2A]

The open circuit voltage is just the voltage across the 5k resistor. The circuit is a potential divider so this is $V_{Th}=90\times\frac{5}{8+5+12}=18$ V. [2A]

The Thévenin equivalent is therefore:



[1A]

(d) There is no current through the 3k resistor, so V+ will equal X. The amplifier is a non-inverting amplifier, so $Z=\left(1+\frac{12}{4}\right)X=4X$. [5A]

(e) (i) $I=\frac{24j}{8j}+\frac{24j}{6}=3+4j$. [2A]

(ii) $Z=\frac{6\times 8j}{6+8j}=3.84+2.88j=4.8\angle 0.644=4.8\angle 36.9°$ [2A]

(iii) $j\omega L=8j$ so $L=\frac{8}{\omega}=16$ mH [1A]

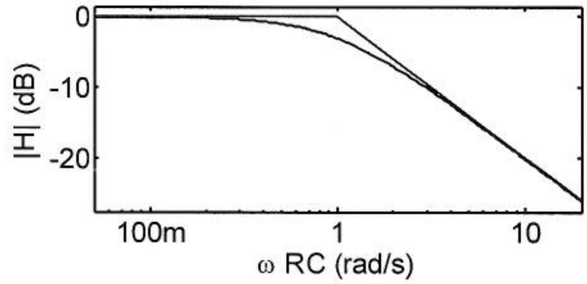
(f) (i) This circuit is a potential divider, so its transfer function is [1A]

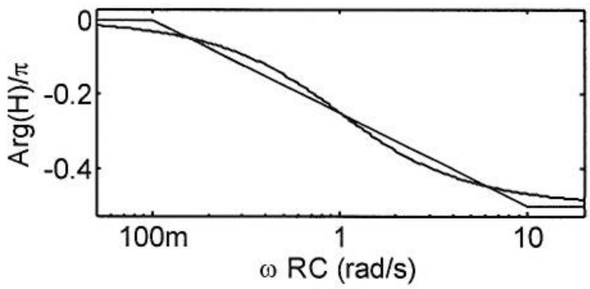
$$\frac{Y}{X}=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}=\frac{1}{j\omega RC+1}$$

(ii) The LF asymptote is 1 and the HF asymptote is $\frac{1}{j\omega RC}$. [1A]

(iii) The magnitude response corner frequency is at $\omega=\frac{1}{RC}$ with the LF asymptote having a gain of 0 dB.

The phase corner frequencies are at $\omega=\frac{0.1}{RC}$ and $\omega=\frac{10}{RC}$. The horizontal phase asymptotes are at 0 and $-\frac{\pi}{2}$ respectively. [3A]

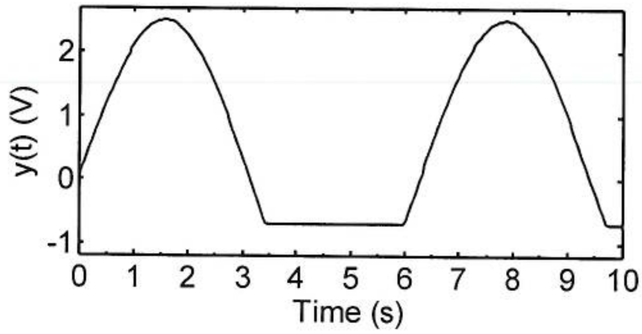




(g) (i) We require $\frac{1}{\omega_0 C}=\omega_0 L$ from which we get $\omega_0=\sqrt{\frac{1}{LC}}=2000$. [2A]

(ii) $\frac{1}{\omega_0 C}=\omega_0 L=100$ so $\frac{X}{V}=\frac{10+100j}{10+100j-100j}=\frac{10+100j}{10}=1+10j$.
Hence $X=10+100j$. [3A]

(h) When the diode is off, $y(t) = 0.5x(t)$, however when the diode is on, $y(t) = -0.7$. Thus $y(t) = \max(0.5x(t), -0.7)$ which gives the graph below. The maximum value of $y(t)$ is 2.5 and the minimum is -0.7. [5A]



2. (a) The gain is $\frac{Y}{X} = -\frac{Z_2}{Z_1}$. We assume that there is no current into the input terminals of the op-amp and that the op-amp gain is infinite: this implies that negative feedback will result in the input terminals having the same voltage. [4A]

(b) Refering to the previous part, $Z_1 = R_1 + \frac{1}{j\omega C} = \frac{j\omega R_1 C+1}{j\omega C}$ and $Z_2 = \frac{R_2 \times \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{j\omega R_2 C+1}$.

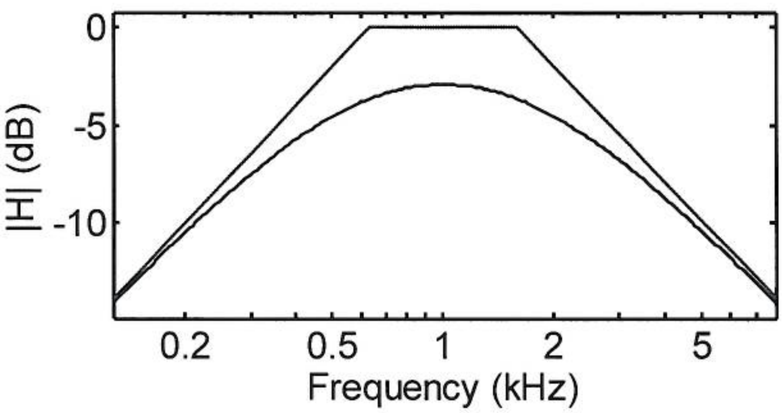
Substituting these expressions into the gain equation from part (a) gives [6A]

$$\frac{Y}{X} = -\frac{j\omega R_2 C}{(j\omega R_1 C + 1)(j\omega R_2 C + 1)}.$$

The LF asymptote is $j\omega R_2 C$ and the high frequency asymptote is $\frac{1}{j\omega R_1 C}$.

The corner frequencies are $\frac{1}{R_1 C}$ and $\frac{1}{R_2 C}$. [4A]

(c) With the values given, the corner frequencies are 4k and 10k rad/s = 637 Hz and 1592 Hz. Between these frequencies, the straight line approximation gives a gain of 0 dB. [6A]



(d) We now have $R_1 = 25\text{k}$ and $R_2 = 10\text{k}$, The corner frequencies remain the same but the lowest-valued corner frequency is now $\frac{1}{R_1 C}$. If we calculate the value of the LF asymptote at this frequency, we find that the mid-band gain has been reduced to $\frac{R_2}{R_1} = 0.4 = -8$ dB. [4A]

(e) We need $[R_1, R_2] = \frac{1}{2\pi f C} = [3.18\text{k}, 31.8\text{k}]$. Note that they must be in this order, or else the mid-band gain is -20 dB. [6D]

3. (a) The current is $= \frac{V}{R+Y+jX}$. The power dissipated in R is $|\tilde{I}|^2 R = \frac{1}{2}|I|^2 R$. Substituting for I gives [8A]

$$P = \frac{|V|^2}{2} \times \frac{R}{(R+Y)^2 + X^2}$$

(b) We want to find the value of R that makes $\frac{dP}{dR} = 0$. We can ignore the constant factor and need only consider the numerator of $\frac{dP}{dR}$. This gives (from the quotient rule):

$$\frac{dP}{dR} \propto \{(R+Y)^2 + X^2\} \times 1 - R \times \{2(R+Y)\} = -R^2 + Y^2 + X^2$$

Setting this to zero gives $R = \sqrt{X^2 + Y^2}$. [6A]

(c) From part (a) we have $I = \frac{V}{R+Y+jX} = (-19.7 - 2.37j)\text{mA} = 19.9\angle - 173°\text{ mA}$.

Hence $W = IR = -0.986 - 0.118j = 0.993\angle - 173°$ [2A]

The complex power absorbed by a component with impedance Z is $|\tilde{I}|^2 Z$. We can calculate $|\tilde{I}|^2 = \frac{1}{2}|I|^2 = 0.394 \times 10^{-3}$. Therefore the complex power absorbed by Y, jX and R is respectively 3.94, $197j$ and 19.7 mW. [6A]

(d) We have already calculated the steady state phasor $W = -0.986 - 0.118j$. This implies that the waveform $w(t) = -0.986 \cos \omega t + 0.118 \sin \omega t + Ae^{-\frac{t}{\tau}}$.

We can see that $\tau = \frac{L}{R+Y} = \frac{X}{\omega(R+Y)} = 16.7\text{ ms}$. 5

We know that $w(t)$ cannot have a discontinuity at $t = 0$ because the current through the inductor cannot change instantly and so must be zero at time $t = 0+$.

It follows that $A = 0.986$. [8A]