UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1999

MEng Honours Degrees in Computing Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the Royal College of Science
Associateship of the City and Guilds of London Institute

PAPER 4.80

AUTOMATED REASONING

Thursday, May 6th 1999, 2.00 – 4.00

Answer THREE questions

For admin. only:
paper contains 4 questions

(In all questions variables begin with lower case u - z; other names are constants. Predicates use upper case.)

1 a i) Apply the Model Elimination (ME) tableau procedure to show the unsatisfiability of clauses (1) - (3). Make clear the steps of the procedure in your answer.

$$(1) \quad \neg Q(x,y) \vee \neg Q(y,x)$$
$$(2) \quad Q(g(x), x) \vee Q(b,x)$$
$$(3) \quad Q(x, g(x)) \vee Q(b,x)$$

ii) From the tableau of part (ai), or otherwise, find an unsatisfiable set of 4 ground clauses, each of which is an instance of one of the clauses (1), (2) or (3). (**Hint**: One of the clauses is $\neg$ Q(b,b).)

b Consider the following refinement of the ME tableau procedure for **ground clauses**, which is sound, but not complete:

- Within each clause one literal is chosen to be the ***selected literal.***
- When using an input clause to extend a tableau branch,
  i) the selected literal in the input clause is the leftmost, and
  ii) any literal in the branch can be chosen to unify with the selected literal.

(**e.g.** Suppose a branch contains the literal X and leaf literal Y and $\neg X \vee \neg Y$ is an input clause with selected literal $\neg$X. The branch can be extended by matching with $\neg$X, even though X is not the leaf literal of the branch. The literal $\neg$Y in the input clause cannot be used as it is not the selected literal.)

Using the four clauses given in part (aii), show that the above refinement is not complete; that is, find some literal selection and top clause such that a closed tableau using the four ground instances cannot be found within the refinement. (**Hint:** Use $\neg$ Q(b,b) as top clause.)

c Let G be an atom that does not occur in any given clause, then the following 2 steps, called the add-G operation, make the refinement of part (b) complete:

- For each pair of clauses with complementary selected literals, add the atom G to at least one of the two clauses. G always becomes the selected literal in its clause.
- Add the new clause $\neg$G, which becomes the top clause.

i) Apply the add-G operation to the clauses and selected literals used in part (b). Using the input clauses in the same order as before as far as possible, show that a closed tableau can now be formed within the refinement of part (b).

ii) By considering the relationship of the add-G operation to the locking refinement for clauses, explain why, for a set of ground **unsatisfiable** clauses, it must always be possible to find a pair of clauses to which the add-G operation is applicable.

The three parts carry, respectively, 40%, 20%, 40% of the marks.

2 a State the steps of the hyper-resolution (HR) procedure to show the unsatisfiability of a set of clauses.

b Hyper-resolution can be controlled by the introduction of an atom-ordering, which is a partial order relation $\leq$ defined on atoms such that,

- for ground atoms x and y, $x \leq y$ or $y \leq x$, and
- for arbitrary atoms u and v, $u \leq v$ iff for all ground substitutions θ to variables in u and v, $u\theta \leq v\theta$.

i) Let $\leq$ be an atom-ordering such that , for ground atoms x and y,

- $x \leq y$ iff list(x) is lexicographically before list(y), where list(x) = list formed by flattening the predicate symbol and arguments of x into a list. **e.g.** list(P(a,f(b))) = [P,a,f,b].

Show that $P(a,a) \leq P(y,y)$ is true, but $P(a,f(x)) \leq P(y,f(y))$ is false.

ii) Let the HR procedure be restricted, so that a literal L1 in an *electron* E may be used only if there is no literal L2 in E, different from L1, such that $L2 \leq L1$.

Apply the restricted procedure to the clauses (1) - (6) making clear which literals can be used in each electron.

(1) $\neg P(a)$

(2) $P(x) \vee Q(x, y)$

(3) $\neg R(x, f(y)) \vee P(x)$

(4) $\neg R(u, v) \vee R(u, f(v))$

(5) $Q(c, b) \vee R(a, c)$

(6) $\neg Q(x, y) \vee \neg Q(a, x) \vee \neg Q(c, y)$

c i) Form the initial connection graph for the clauses (1) - (3), (5), (6) of part (bii).

ii) What is the purity rule? Apply the rule to the graph of part (ci).

iii) A link m that connects two literals, such that neither literal is incident to any other link, is called a *unique* link.

Find a unique link in the graph of part (cii) and select it. Give the new graph. What is the benefit of selecting a unique link?

iv) Show that the connection-graph procedure applied to the graph of part (ciii) results in the empty graph. What is the significance of the empty graph?

The three parts carry, respectively, 15%, 40%, 45% of the marks.

Turn over ...

3 a i) State **two** features (other than soundness and completeness) useful for an automated theorem proving method.

ii) ***Very briefly,*** compare resolution and free variable tableau methods with respect to the features in part (ai).

b i) Write down all *non-tautologous* ground instances of the clauses (1)-(3) below, assuming the Herbrand Domain = {a, b}.
Hint: There are 6 in all.

ii) Apply the Davis-Putnam (DP) procedure to show that the set of instances given in part (bi) has no model. Make clear the steps of the procedure in your answer.

$$
\begin{array}{ll}
(1) & \neg P(y, x) \vee P(a,x) \\
(2) & P(a, x) \vee P(b,x) \\
(3) & \neg P(a, b) \vee \neg P(y, y)
\end{array}
$$

c Consider a clausal language with a non-empty Herbrand Domain and no function symbols. The DP procedure is augmented with the following rules in which L(X) is a literal either of the form P(X), or of the form ¬Q(X):

- (S): Select a literal L(X) in a clause C ∨ L(X), where the variables X in the atom do not occur in C. Derive two branches, one containing ∀X. L(X) and the other C. Apply subsumption in both branches if appropriate.
- (U): Apply unit resolution using ∀X. L(X) or a ground literal either to a unit clause to derive the empty clause, or to a non-unit clause D as long as the resolvent subsumes D.
- (I): Select a non-ground clause to which rule (S) cannot be applied and instantiate one variable in the clause in all possible ways. Add to the branch all those so-formed instances of the clause that are not subsumed and remove the selected clause.

i) Apply the modified procedure to show that the clauseset (1)-(3) of part (bii) is unsatisfiable. Assume the Herbrand Domain = {a, b}.

ii) Explain why the following property is true of step (U):

"There is a model of the clauses in a branch B iff
there is a model of the clauses in B after application of step (U)".

The three parts carry, respectively, 15%, 40%, 45% of the marks.

4 a i) State the paramodulation inference rule.

ii) Give the three possible results of paramodulating $f(x) = a$ (not $a = f(x)$) into $P(g(y)) \vee Q(f(y))$.

iii) Choose ***one*** result and give the simulation by resolution of the paramodulation step.

b) Show that the KB procedure, when applied to equations (1) - (3), results in a confluent set of rewrite rules.

(1) $e(1,y) = y$
(2) $n(e(x,y)) = e(n(x), y)$
(3) $n(n(1)) = 1$

Your answer should make clear the steps and possible outcomes of the KB procedure.

c) Suppose R is a canonical set of rewrite rules.

i) What properties does R possess?

ii) How can R be used to decide whether $s = t$ for some ground terms s and t?

iii) ***Briefly***, explain why the method of part (cii) is correct.

The three parts carry, respectively, 30%, 40%, 30% of the marks.

End of paper