UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2003

MEng Honours Degree in Information Systems Engineering Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
PhD
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C480=I4.42

AUTOMATED REASONING

Tuesday 29 April 2003, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

NOTE: In all four questions:

- **variables begin with lower case u - z;**
- **names beginning with other lower case letters are functors;**
- **predicates begin with an upper case letter.**

1 a Use the Model Generation Procedure (MG) to derive a model for the following clauses (1) - (7):

(1)	$\neg S \vee T$	(2)	$\neg R \vee Q$
(3)	$\neg P \vee R$	(4)	$\neg Q \vee T$
(5)	$P \vee Q$	(6)	$\neg Q \vee R$
(7)	$\neg P \vee \neg Q$		

Make clear the steps of the method in your answer.

b i) Under which circumstances can the MG procedure immediately return a model M for its arguments (branch B and clauses S)?

State how the true and false atoms are determined in each of these circumstances.

ii) Explain why the assignment(s) given in part (bi) will yield a model for the original sentences.

c i) Draw the initial connection graph G for the clauses (8) - (10):

(8) $P(u, f(v)) \vee Q(v, b)$
(9) $\neg P(b, z) \vee Q(z, b)$
(10) $\neg Q(f(x), x) \vee P(x, x)$

ii) What is the connection graph proof procedure (CGPP) purity rule?

What is an inconsistent link in the CGPP?

iii) Identify an inconsistent link L in G.

Hence, or otherwise, show that the graph G, according to the CGPP, allows the empty graph to be derived.

What does this result signify about the original clauses?

The three parts carry, respectively, 30%, 35%, 35% of the marks.

2 a In the context of a set of rewrite rules R:

i) Describe how the superposition operation derives a new equation from R.

ii) Since rewrite rules are also equations, explain how the superposition operation can be simulated by one or more paramodulation steps using R (as equations).

iii) Illustrate your answer to part (ai) in the context of rewrite rules (11), (12) and (13), by finding a new equation (14).

(11) f(z,z) ==> g(z)
(12) f(s(x), y) ==> s(x)
(13) g(s(w)) ==> w

b Justify the orientation of the rewrite rules (11)-(13) by finding a suitable reduction (termination) ordering.

Apply the ordering to orient (14).

c Explain why rules (11)-(14) are not confluent.

Apply the Knuth Bendix procedure to derive any additional rules needed to form a confluent set.

d Let S be the following set of 3 clauses: {a=b, P(a), ¬P(b)}.

Augment S with an appropriate substitutivity axiom of equality and draw a closed semantic tree for the augmented set of clauses.

Derive a resolution refutation from the tree. *Explain your answer.*

The four parts carry, respectively, 30%, 15%, 30%, 25% of the marks.

3 a Define the hyper-resolution refinement of resolution.
Define θ-subsumption between two clauses C and D.

b In the context of the **Otter** theorem prover:

i) What would be the effect on the "kept resolvents" of each of the (command) settings

set(for_sub) and *set(back_sub)*?

ii) What clauses would be in the sos-list and the usable-list after executing **one** cycle of clause selection and processing of resolvents by Otter for the following input file? *Justify your answer.*

```
clear(order_hyper).
set(for_sub).                list(usable).
set(back_sub).               -P(w, g(w)).
set(hyper_res).              P(d, c).
list(sos).                   -Q(x, y) | -P(x, z) | P(y, x).
Q(u, v) | P(u, v).           end_of_list.
end_of_list.
```

iii) What is the answer to part (bii) if the *order_hyper* flag is set instead of cleared? That is, if hyper-resolution with predicate ordering is used, in which the predicates are ordered alphabetically, such that Q has higher priority than P etc.? *Explain your answer.*

c i) What is a *factor* of a clause?

What is a *safe factor* of a clause?

ii) Give two factors of $Q(x, x) \vee Q(x, y) \vee P(x) \vee P(y) \vee P(a)$, one of which is a *safe factor* and one which is not. State which is which.

iii) Why is a safe factor of C equivalent to C?

Parts a, bi, bii, biii, c carry, respectively, 15%, 10%, 35%, 10%, 30% of the marks.

4 a i) State the soundness property of semantic tableaux (not using free variables).

ii) Explain how the soundness property stated in (ai) can be used to show soundness for free variable tableaux.

b In the context of the Model Elimination (ME) tableaux method and its variations:

i) What is a universal variable in a tableau?

What is the generalised closure rule and how does it (soundly) exploit universal variables? *Briefly justify your answer.*

ii) Illustrate your answer to bi) by comparing the standard ME tableau with a tableau using the generalised closure rule for the clauses (15) - (17), using (15) as top clause.

In particular, your answer should include both a ME tableau **and** a tableau using the generalised closure rule and should also exemplify your justification of the soundness of the generalised closure rule given in part (bi) for the second tableau.

(15) $\neg P(y, x) \vee P(a, x)$
(16) $\neg P(a, b) \vee \neg P(y, y)$
(17) $P(b, x) \vee P(a, x)$

c Let T be a partially developed ME tableau and X be some closed subtree immediately beneath ground literal $\neg L$ which does not use any ancestors of $\neg L$.

i) In what way could L be considered a lemma?

How can the existence of closure X be (soundly) exploited in the rest of the development of T? Justify your answer.

ii) If $\neg L$ is not ground and the closure X does not cause any instantiations of the free variables in $\neg L$, what is the lemma this time?

How can this be (soundly) exploited in the further development of T?

The parts a, bi, bii, c carry, respectively, 15%, 25%, 40%, 20% of the marks.