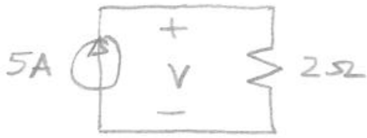
SOLUTIONS

Circuit Analysis
EE 1.1
2008

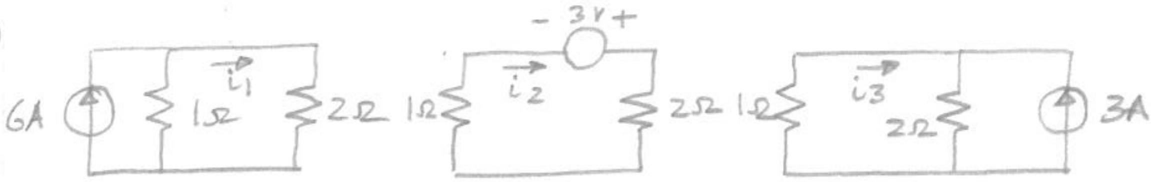
Q1

(a)



$V = 5 \times 2 = 10\ V$

(b)



$i_1 = \frac{1}{3} \times 6 = 2A$ $\quad i_2 = 3/3 = 1A$ $\quad i_3 = (2/3) \times -3 = -2A$

$i = i_1 + i_2 + i_3 = 1A$

(c)

$i_{sc} = 1 + 12/12 + 24/12 = 1 + 1 + 2 = 4A$

$R_{Th} = 3\Omega // 12\Omega // 12\Omega = 3\Omega // 6\Omega = 2\Omega$

$V_{oc} = i_{sc} \times R_{Th} = 4 \times 2 = 8\ V$

(d)

$2V_1 - V_2 = 2 - 1 = 1$ $\qquad 2V_1 - V_2 = 1 \quad (1)$

$(1 + 1/3)V_2 - V_1 = 1 + 1 = 2$ $\qquad -V_1 + \frac{4}{3}V_2 = 2 \quad (2)$

$(1) + 2 \times (2) \rightarrow (8/3 - 1)V_2 = 2 \times 2 + 1 \longrightarrow V_2 = 3V \quad V_1 = 2V$

(e) (i) 0V (ii) 5V iii $\tau = C \times R_2 = 10^{-6} \times 10^{6} = 1\ s$

(iv) $V_c(t) = V_{c0} + (V_{c\infty} - V_{c0})(1 - e^{-t/\tau})$

$= 0 + 5(1 - e^{-t/\tau})$

$= 5 - 5e^{-t/\tau}$

(f) $\bar{V}_s = 4\angle 0^\circ \quad Z_L = j^2/3 \quad Z_C = -j$

$$\bar{V} = \frac{Z_2}{Z_1+Z_2}\bar{V}_s = \frac{1}{1+Z_1Y_2}\bar{V}_s = \frac{4}{1+2\left(-j\frac{3}{2}+j\right)} = \frac{4}{1-j}$$

$$= \frac{4}{\sqrt{2}\angle -45^\circ} = 2\sqrt{2}\,\angle 45^\circ$$

$$v(t) = 2\sqrt{2}\cos(2t + \pi/4)$$

(g) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10^{-3}\cdot 10^{-11})}} = 10^7 \text{ rad/sec}$$

$$Q = \frac{\omega_0 L}{r_s} \;;\; r_s = \frac{\omega_0 L}{Q} = \frac{10^7\, 10^{-3}}{100} = 100\,\Omega$$

$Z_L = -Z_C$, so Gain $= 1/2$

(h) $$i_x = \frac{V_i}{R_i} - \frac{V_s}{R_f}$$

$$V_s = r_m i_x = r_m\left(\frac{V_i}{R_i} - \frac{V_s}{R_f}\right)$$

$$V_s\left(1 + \frac{r_m}{R_f}\right) = \frac{r_m}{R_i}V_i \qquad \frac{V_s}{V_i} = \frac{1}{R_i\left(\frac{1}{r_m}+\frac{1}{R_f}\right)}$$

(i) $$V_1 = (-1)\times 2 = -2V$$

$$V_2 = V_1 = -2V$$

$$V_{out} = V_2 + (V_{out} - V_2)$$

$$= V_2 + (V_2 - 2) = 2V_2 - 2 = -6V$$

(j) $$I_2 = y_{21}V_1 + y_{22}V_2$$

$$I_2 = -V_2/R_L$$

$$-\frac{V_2}{R_L} = y_{21}V_1 + y_{22}V_2$$

$$-V_2\left(\frac{1}{R_L} + y_{22}\right) = y_{21}V_1$$

$$\frac{V_2}{V_1} = -\frac{y_{21}}{\frac{1}{R_L}+y_{22}}$$

2

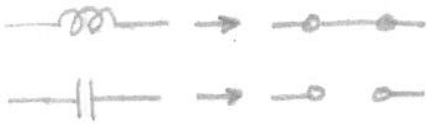
(a)

$$V_L = L\frac{di_L}{dt}$$

$$i_C = C\frac{dV_C}{dt}$$

Inductor is a short circuit

Capacitor is an open circuit



6A current divides equally in resistors. So $i = 3A$, $V = 15\ V$

(b) (i) $\tau = 10^3 \times 10^{-6} = 1\ mS$

(ii) $V_0 = \frac{1}{2} 10\ V = 5\ V$

(iii) $V_\infty = -5V$

(iv)
$$
\begin{aligned}
V(t) &= V_0 + (V_\infty - V_0)(1 - e^{-t/\tau}) \\
&= 5 + (-10)(1 - e^{-t/\tau}) \\
&= -5 + 10e^{-t/\tau}
\end{aligned}
$$

(v)
$$
\begin{aligned}
V(2) &= -5 + 10e^{-2} \\
&=
\end{aligned}
$$

(vi)
$$
\begin{aligned}
\tau_2 &= 10^{-6} \times (10^3 + 2000 // 2000) \\
&= 10^{-6} \times 2000 \\
&= 2mS
\end{aligned}
$$

(c) Pull-up:
$$
\begin{aligned}
\tau_U &= 2500 \times 0.1 \times 10^{-12} \\
&= 250\ pS
\end{aligned}
$$

$$
\begin{aligned}
\text{Minimum charging time} &= 1.75 \times 250 \\
&= 437.5\ pS
\end{aligned}
$$

Pull-down
$$
\begin{aligned}
\tau_D &= 1000 \times 0.1 \times 10^{-12} \\
&= 100\ pS
\end{aligned}
$$

$$
\begin{aligned}
\text{Minimum charging time} &= 1.75 \times 100 \\
&= 175\ pS
\end{aligned}
$$

Minimum clock period governed by pull-up

$$\tau_{min} = 2\tau_u = 875\ pS$$

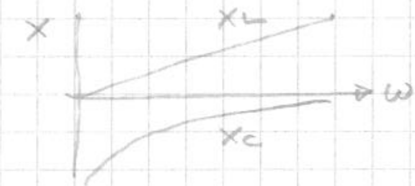
$$\text{Maximum clock rate} = \frac{1}{\tau_{min}} = 1.14\ GHz$$

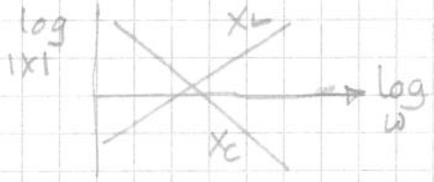
3 (a) Impedance, $Z = \bar{V}/\bar{I}$ where $\bar{V}$ and $\bar{I}$ are phasor forms of element voltage v and current i.

$Z_L = j\omega L \qquad Z_C = 1/(j\omega C)$

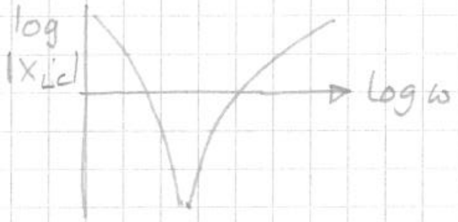
Reactance X is imaginary part of impedance $Z = R + jX$

$X_L = \omega L \qquad X_C = -1/(\omega C)$





$$Z_{LC} = j\omega L + \frac{1}{j\omega C} = j\left(\omega L - \frac{1}{\omega C}\right) \qquad X_{LC} = \omega L - \frac{1}{\omega C}$$



(b)

$$H(j\omega) = \frac{Z_2}{Z_1 + Z_2} = \frac{1}{1 + Z_1 Y_2} = \frac{1}{1 + \left(R + \frac{1}{j\omega C}\right)\left(j\omega C + \frac{1}{R}\right)}$$

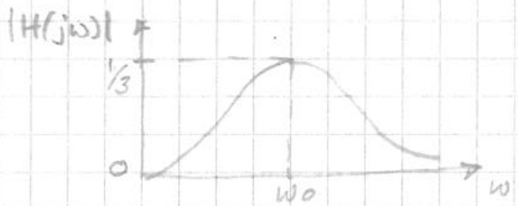
$$= \frac{j\omega CR}{1 + j3\omega CR - (\omega CR)^2}$$

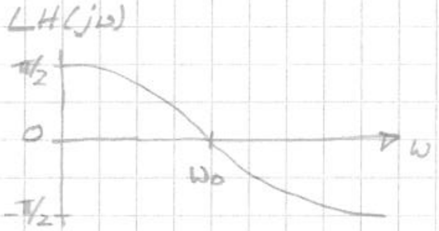
$\angle H(j\omega) = 0$ for $\omega_0 CR = 1$, i.e. $\omega_0 = \frac{1}{CR}$

$$= \frac{1}{10^{-9}\,10^{3}}$$

$$= 10^{\prime}\ \text{Mrad/sec}$$

$|H(j\omega_0)| = 1/3$





(c)

	Z_1	Z_2	Z_3
LP	R	L	C
BP	C	L	R
HP	R	C	L

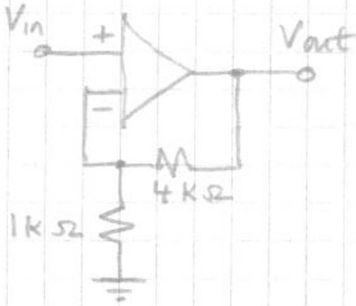
(d)

BPF – Channel-select filter in communication system

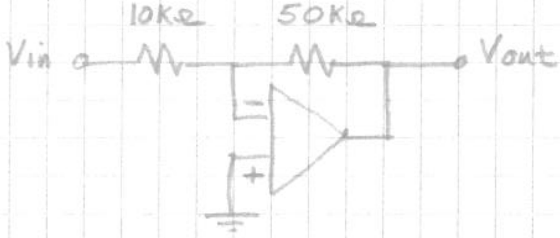
LPF – Anti-alias filter prior to sampling
-or- Reconstruction filter
-or- Removing out-of-band noise

4 (a) Low cost
Short design time

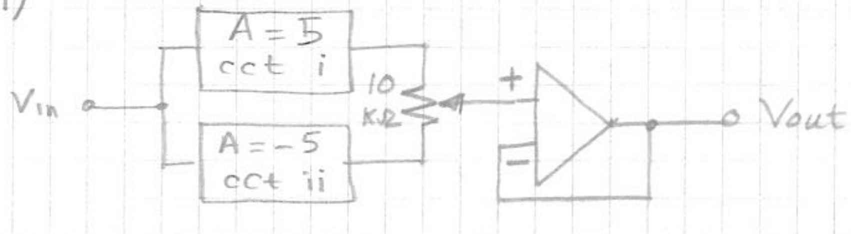
(b) i)



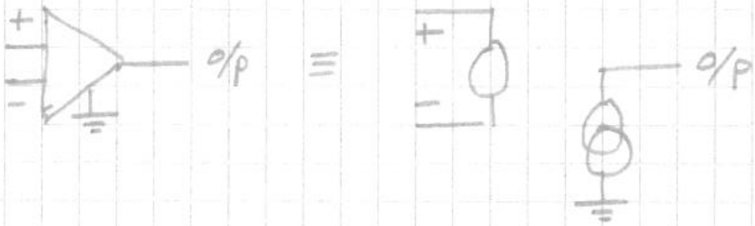
ii)



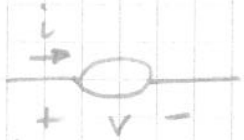
iii)



(c)



Nullator:



$V = 0$
$i = 0$

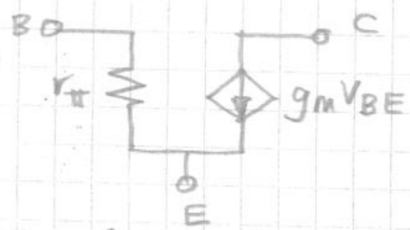
Norator

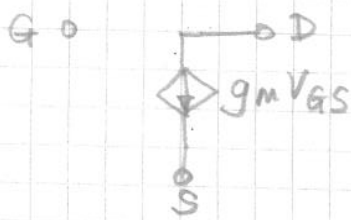
V = arbitrary
i = arbitrary

Non-ideal aspects:
Stability
Finite gain-bandwidth product

(d) (i) For high frequency design
(ii) Designs requiring very low power consumption

(e) BJT (common emitter model) FET





$r_\pi = (1+\beta)V_T/I_{co}$ $g_m = I_{co}/V_T$

$g_m = 2K(V_{GSO} - V_{TH})$

Nodal analysis