UNIVERSITY OF LONDON

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

# EXAMINATIONS 2000

MEng Honours Degrees in Computing Part IV

MSc in Advanced Computing

for Internal Students of the Imperial College of Science, Technology and Medicine

*This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute*

# PAPER C486

# DEDUCTIVE DATABASES

Monday 15 May 2000, 10:00

Duration: 120 minutes

*Answer THREE questions*

Paper contains 4 questions

1a State the three defining properties of a *classical consequence operator*.

Show that every such operator is *transitive*.

b A consequence operator Cn (not necessarily classical) is said to be *cumulative* iff it satisfies *inclusion* and has the following property

$$\text{if } A \subseteq B \subseteq \mathrm{Cn}(A) \text{ then } \mathrm{Cn}(A) = \mathrm{Cn}(B)$$

i) Show that, for any cumulative operator Cn, $\mathrm{Cn}(\mathrm{Cn}(A)) = \mathrm{Cn}(A)$.

ii) Suppose $T$ is any set of sentences. Show that the consequence operator $\mathrm{Cn}_T$ defined as

$$\mathrm{Cn}_T(A) =_{\mathrm{def}} \mathrm{Cn}(T \cup A)$$

is cumulative if Cn is cumulative.

c A consequence operator Cn is *supraclassical* when $\mathrm{Th}(A) \subseteq \mathrm{Cn}(A)$ for all sets of sentences $A$, where Th denotes classical truth-functional consequence.

Show that the following hold for any supraclassical cumulative operator Cn:

i) $\mathrm{Th}(\mathrm{Cn}(A)) = \mathrm{Cn}(A)$

ii) if $B \subseteq \mathrm{Cn}(A)$ and $C \subseteq \mathrm{Th}(B)$ then $C \subseteq \mathrm{Cn}(A)$

iii) if $\{\ \mathit{canary}, \neg(\mathit{yellow} \wedge \mathit{blue})\ \} \mathrel{|\!\sim} \mathit{yellow}$ then
$\{\ \mathit{canary}, \neg(\mathit{yellow} \wedge \mathit{blue})\ \} \mathrel{|\!\sim} \neg \mathit{blue}$ (where $A \mathrel{|\!\sim} \alpha$ iff $\alpha \in \mathrm{Cn}(A)$ ).

*The three parts carry, respectively, 25%, 35%, 40% of the marks.*

2a Explain what is meant by the '*model theoretic*' and '*proof theoretic*' approaches to databases. How do the two approaches differ as regards the database itself, queries, and answers to queries?

b Explain what is meant by the *epistemic* or *metalevel* reading of integrity constraints on proof theoretic databases, and how such integrity constraints can be formulated by extending the language of the database with an additional modal operator.

Illustrate your answer by showing one integrity constraint that is satisfied and one that is not satisfied by the following (classical) first-order logic database

$$\{\ p(a),\ p(b),\ q(a),\ p(c) \vee q(c)\ \}$$

c What is the definition of integrity constraint satisfaction for model theoretic databases?

Explain carefully why the epistemic/metalevel reading of integrity constraint satisfaction is not distinguishable from the standard definition in the case of model theoretic databases.

d Consider $D = \{\ p(x) \leftarrow q(x),\ \ q(a),\ \ q(b)\ \}$ ($x$ is a variable).

Write down the contents of comp($D$) and give one example of a closed query on comp($D$) for which the correct answer is neither 'yes' nor 'no'.

*The four parts carry equal marks.*

3a Let IDB be a set of definite clauses and EDB a set of ground unit clauses. Define the closure of EDB under IDB, $Cl_{\mathrm{IDB}}(\mathrm{EDB})$, and state (without proof) its characterisation in terms of Herbrand models and least fixpoints. (It is not necessary to define the immediate consequence operator.)

b Present (in any style) the algorithm for *semi-naive* bottom-up evaluation of the 'derived tuples' $s = Cl_{\mathrm{IDB}}(\mathrm{EDB}) - \mathrm{EDB}$.

Show the semi-naïve evaluation of the following example:

IDB:
$$h(x) \leftarrow g(x)$$
$$h(x) \leftarrow q(x, y), h(y)$$

EDB:
$q(a, b)$ $g(a)$
$q(b, c)$ $g(c)$
$q(c, d)$
$q(d, a)$

Indicate clearly the feature of semi-naïve evaluation that distinguishes it from naïve evaluation.

c Write down the *magic set* transformation of the example in part (b) for the query $h(c)$ ?.

What is the magic set transformation for the query $h(x)$ ? ($x$ a variable)?

d Suppose IDB is replaced by the following clauses:

IDB':
$$h(x) \leftarrow g(x)$$
$$h(x) \leftarrow \forall y (\, q(x, y) \rightarrow g(y)\,)$$

Show the ABW iterated fixpoint construction for IDB' on the EDB of part (b). You will need to transform the second clause.

What does this construction compute?

*The four parts carry, respectively, 20%, 30%, 20%, 30% of the marks.*

4a In Reiter's default logic, what are *normal* and *non-normal* default rules?

Define the *extension* of a default theory $\langle D, W \rangle$.

b Exams are typically unpleasant, except that exams at Imperial College are typically not unpleasant.

Suppose that the above is represented as Reiter default rules as follows:

$$\frac{exam(x) : unpleasant(x)}{unpleasant(x)} \qquad \frac{ICexam(x) : \neg unpleasant(x)}{\neg unpleasant(x)}$$

Explain carefully why this pair of defaults (together with the non-default rule that an Imperial College exam is also an exam) does not represent the exception structure adequately.

Suggest one modification to the default rules that does capture the exception structure correctly.

c In autoepistemic logic, what is a *stable expansion*?

Show how the example in part (b) may be formulated in autoepistemic logic. Your answer should cover both the original representation suggested in the question and your modified form.

d Show how the example of part (b) may be formulated using

i circumscription

ii maxiconsistent explanations ('Poole systems').

It is sufficient to present a formulation that represents the exception structure correctly. It is not necessary to include definitions of the terms circumscription and maxiconsistent explanation themselves.

*The four parts carry equal marks.*