

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EEE/EIE PART III/IV: MEng, Beng and ACGI

Corrected Copy

DIGITAL SIGNAL PROCESSING

Wednesday, 15 January 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible

First Marker(s) :	P.A. Naylor
Second Marker(s) :	W. Dai

DIGITAL SIGNAL PROCESSING

1. a) A discrete-time signal $x(n)$ is given by

$$x(n) = -2 + 2\cos\frac{n\pi}{4} + \cos\frac{n\pi}{2} + \frac{1}{2}\cos\frac{3n\pi}{4}.$$

- i) Determine the period in samples of $x(n)$. [3]
- ii) Determine $|X(k)|$, the magnitude spectrum of $x(n)$. [5]
- iii) Draw a labelled sketch of $|X(k)|$. [3]
- iv) Verify Parseval's relation for this case by computing the power in both the time and frequency domains. [3]

- b) Given a discrete-time signal $x(n)$ having Fourier transform

$$F\{x(n)\} = \frac{1}{1 - ae^{-j\omega}}$$

find the Fourier transforms of

- i) $x(n+2)$, [2]
- ii) $x(n) \otimes x(n-2)$, [2]
- iii) $x(n) \otimes x(-n)$, [2]

where \otimes represents circular convolution.

2. The bilinear transform describing a mapping between the s -plane and the z -plane can be written

$$s = \frac{2}{T} \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right).$$

- a) Let z and s be denoted

$$z = re^{j\omega}$$

$$s = \sigma + j\Omega.$$

Explain the result of the bilinear transform on $s = \sigma + j\Omega$ for the cases of $\sigma < 0$, $\sigma = 0$ and $\sigma > 0$. Include illustrative labelled sketches of the s -plane and z -plane.

[5]

- b) Explain what is meant by frequency warping in the context of the bilinear transform and write an expression for the frequency ω in terms of Ω .

[3]

- c) Consider a continuous-time bandpass filter with system function

$$H(s) = \frac{(\Omega_u - \Omega_l)s}{s^2 + (\Omega_u - \Omega_l)s + \Omega_l\Omega_u}$$

where Ω_u and Ω_l are the upper and lower band edge frequencies respectively.

- i) Apply the bilinear transform to convert $H(s)$ to a discrete-time IIR filter $H(z)$ with sampling period T s. (*Hint: Do not consider frequency warping.*) [6]
- ii) Write out the difference equation for the filter's output $y(n)$ given the input signal $x(n)$. [4]
- iii) Draw an illustrative labelled sketch of the magnitude frequency response of $H(z)$. [2]

3. a) Consider a maximally decimated 2-band analysis filter bank directly connected in cascade to a corresponding synthesis filter bank.

- i) Draw a labelled sketch of this analysis-synthesis filter bank employing analysis filters $H_0(z)$ and $H_1(z)$ and synthesis filters $F_0(z)$ and $F_1(z)$. Denote the input signal as $x(n)$ with z-transform $X(z)$, the subband signals as $y_0(n)$ and $y_1(n)$ with z-transforms $Y_0(z)$ and $Y_1(z)$ respectively, and the output of the synthesis filter bank as $\hat{x}(n)$ with z-transform $\hat{X}(z)$. [4]
- ii) Derive expressions for $Y_0(z)$ and $Y_1(z)$ in terms of $X(z)$, $H_0(z)$ and $H_1(z)$. [4]
- iii) Derive an expression for $\hat{X}(z)$ in terms of $X(z)$, $H_0(z)$, $H_1(z)$, $F_0(z)$ and $F_1(z)$. [4]
- iv) Show that the expression for $\hat{X}(z)$ can be written in matrix form including the matrix term [2]

$$\mathbf{F} = \begin{bmatrix} F_0(z) \\ F_1(z) \end{bmatrix}.$$

- b) Consider the system shown in Fig. 3.1 for which the input signal $x(n)$ has the spectrum shown in Fig. 3.2 and $H_B(z)$ is a bandpass filter with magnitude frequency response shown in Fig. 3.3. Draw a labelled sketch of the spectrum of the signal $y(m)$ and explain your answer. [6]

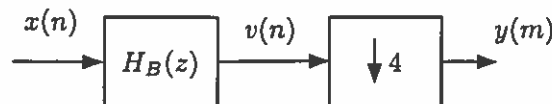


Figure 3.1 Multirate system

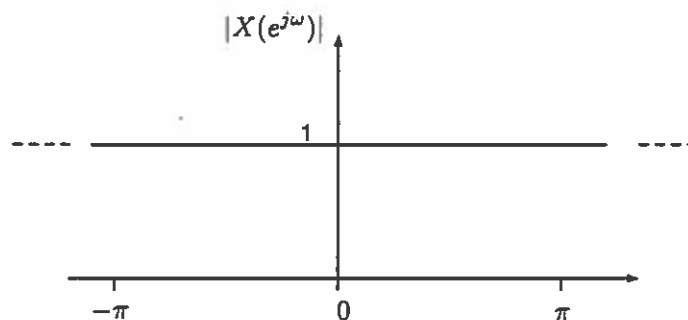


Figure 3.2 Input signal magnitude spectrum

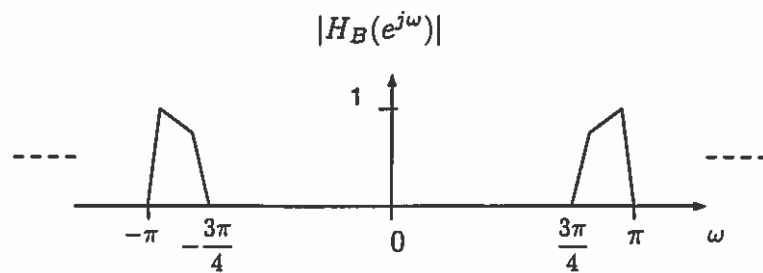


Figure 3.3 Filter magnitude frequency response

4. The Discrete Fourier Transform can be written

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk} \quad k = 0, 1, \dots, N-1.$$

- a) Show that
- i) $W_N^{k+N/2} = -W_N^k$
 - ii) $W_N^{k+N} = W_N^k$ [4]
- b)
- i) Derive the 4-point Radix-2 Decimation-in-Time FFT algorithm and draw the signal flow graph. [7]
 - ii) Write a clear explanation of the terms *Radix-2* and *Decimation-in-Time* in this context. [2]
 - iii) Determine the number of real multiply operations required to compute the 8-point Radix-2 Decimation-in-Time FFT. Ignore multiplications by 0 , $+1$ and -1 . [3]
- c) For a discrete-time signal $x(n)$ of length N samples with DFT $X(k)$, consider a new sequence $y(n)$ of length $2N$ such that

$$y(n) = \begin{cases} x(n/2), & \text{for } n \text{ even} \\ 0, & \text{for } n \text{ odd.} \end{cases}$$

Find an expression for the DFT of $y(n)$ in terms of $X(k)$. [4]