

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2008

EEE PART II: MEng, BEng and ACGI

Corrected Copy

**DEVICES AND FIELDS (A)**

Monday, 16 June 2:00 pm

Time allowed: 2:00 hours

**There are SIX questions on this paper.**

**Question ONE and Question FOUR are compulsory. Answer Question One, Question Four, plus one additional question from Section A and one additional question from Section B.**

*Questions One and Four are each weighted at 20%. Remaining questions are each weighted at 30%.*

*Use a separate answer book for each section.*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      K. Fobelets, R.R.A. Symms  
   Second Marker(s) :      W.T. Pike, C.A. Hernandez-Aramburo

### **Special instructions for invigilators**

This exam consists of **2 sections**. **Section A: Devices** and **Section B: Fields**. Each section has to be solved in their respective answer books. Check that 2 different answer books are available for the students.

Questions 1 and 4 are obligatory. Q1 needs to be solved in answer book A and Q4 in answer book B.

### **Special instructions for students**

Use different answers books for each section:

**Devices:** answer book A

**Fields:** answer book B

Questions 1 and 4 are obligatory. Q1 needs to be solved in answer book A and Q4 in answer book B.

### Constants and Formulae for section A: Devices

permittivity of free space:  $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

permeability of free space:  $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

intrinsic carrier concentration in Si:  $n_i = 1.45 \times 10^{10} \text{ cm}^{-3} \text{ at } T = 300\text{K}$

dielectric constant of Si:  $\epsilon_{Si} = 11$

dielectric constant of SiO<sub>2</sub>:  $\epsilon_{ox} = 4$

thermal voltage:  $kT/e = 0.026\text{V at } T = 300\text{K}$

charge of an electron:  $e = 1.6 \times 10^{-19} \text{ C}$

$$\left. \begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p \frac{dp(x)}{dx} \end{aligned} \right\} \text{ Drift-diffusion current equations}$$

$$I_{DS} = \frac{\mu C_{ox} W}{L} \left( (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right) \text{ MOSFET current}$$

$$\left. \begin{aligned} J_n &= \frac{eD_n n_p}{L_n} \left( e^{\frac{eV}{kT}} - 1 \right) \\ J_p &= \frac{eD_p p_n}{L_p} \left( e^{\frac{eV}{kT}} - 1 \right) \end{aligned} \right\} \text{ Diode diffusion currents}$$

$$V_0 = \frac{kT}{e} \ln \left( \frac{N_A N_D}{n_i^2} \right) \text{ Built-in voltage}$$

$$c = c_0 \exp \left( \frac{eV}{kT} \right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases} \text{ Minority carrier injection under bias } V$$

$$\delta c = \Delta c \exp \left( \frac{-x}{L} \right) \text{ with } \begin{cases} \delta c = \delta p_n \text{ or } \delta n_p \\ \Delta c \text{ the excess carrier concentration at the edge of the depletion region} \end{cases} \text{ Excess carrier concentration as a function of distance}$$

$$L = \sqrt{D\tau} \text{ Diffusion length}$$

$$D = \frac{kT}{e} \mu \text{ Einstein relation}$$

$$W_{depl} = \left[ \frac{2\epsilon V_0}{e} \frac{N_A + N_D}{N_A N_D} \right]^{1/2} \text{ Depletion width in pn diode}$$

$$C_{diff} = \frac{e}{kT} I \tau \text{ Diffusion capacitance}$$

## Constants and Formulae for section B: Fields

### Vector calculus (Cartesian co-ordinates)

$$\nabla = \underline{i} \partial/\partial x + \underline{j} \partial/\partial y + \underline{k} \partial/\partial z$$

$$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$$

$$\text{grad}(\phi) = \nabla \phi = \underline{i} \partial\phi/\partial x + \underline{j} \partial\phi/\partial y + \underline{k} \partial\phi/\partial z$$

$$\text{div}(\underline{F}) = \nabla \cdot \underline{F} = \partial F_x/\partial x + \partial F_y/\partial y + \partial F_z/\partial z$$

$$\text{curl}(\underline{F}) = \nabla \times \underline{F} = \underline{i} \{ \partial F_z/\partial y - \partial F_y/\partial z \} + \underline{j} \{ \partial F_x/\partial z - \partial F_z/\partial x \} + \underline{k} \{ \partial F_y/\partial x - \partial F_x/\partial y \}$$

Where  $\phi$  is a scalar field and  $\underline{F}$  is a vector field

### Maxwell's equations – integral form

$$\iint_A \underline{D} \cdot d\underline{a} = \iiint_V \rho \, dv$$

$$\iint_A \underline{B} \cdot d\underline{a} = 0$$

$$\int_L \underline{E} \cdot d\underline{L} = - \iint_A \partial \underline{B} / \partial t \cdot d\underline{a}$$

$$\int_L \underline{H} \cdot d\underline{L} = \iint_A [\underline{J} + \partial \underline{D} / \partial t] \cdot d\underline{a}$$

Where  $\underline{D}$ ,  $\underline{B}$ ,  $\underline{E}$ ,  $\underline{H}$ ,  $\underline{J}$  are time-varying vector fields

### Maxwell's equations – differential form

$$\text{div}(\underline{D}) = \rho$$

$$\text{div}(\underline{B}) = 0$$

$$\text{curl}(\underline{E}) = -\partial \underline{B} / \partial t$$

$$\text{curl}(\underline{H}) = \underline{J} + \partial \underline{D} / \partial t$$

### Material equations

$$\underline{J} = \sigma \underline{E}$$

$$\underline{D} = \epsilon \underline{E}$$

$$\underline{B} = \mu \underline{H}$$

### Theorems

$$\iint_A \underline{F} \cdot d\underline{a} = \iiint_V \text{div}(\underline{F}) \, dv - \text{Gauss' theorem}$$

$$\int_L \underline{F} \cdot d\underline{L} = \iint_A \text{curl}(\underline{F}) \cdot d\underline{a} - \text{Stokes' theorem}$$

$$\text{curl} \{ \text{curl}(\underline{F}) \} = \text{grad} \{ \text{div}(\underline{F}) \} - \nabla^2 \underline{F}$$

## The Questions

### SECTION A: SEMICONDUCTOR DEVICES

1. This question is **compulsory**

- What are the minority carriers in an n-type semiconductor device? [2]
- Calculate the minority and majority carrier concentration of n-type Si doped with a donor concentration of  $N_D = 5 \times 10^{17} \text{ cm}^{-3}$  at room temperature. [4]
- There exist two types of currents, caused by different physical processes. Give the physical process that causes each current. [4]
- In fig. 1 electrons are injected at a constant rate (steady-state) into intrinsic Si at  $x=0$ . Sketch the *excess* electron concentration from  $x=0$  to  $x=L$  when recombination of carriers occurs.  $L_n$  is the diffusion length of electrons and  $L \gg L_n$ . [6]

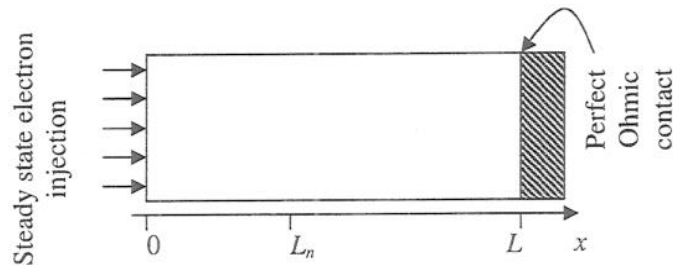


Figure 1: Steady-state electron injection into intrinsic Si at  $x=0$ . The ohmic contact  $x=L$  is ideal.

- Explain briefly the cause for the switch-off delay in a bipolar junction transistor (BJT) with respect to charges in the different layers. [4]

## 2. Bipolar Junction Transistors (BJTs)

Given: a Si  $n^+pn$  BJT with doping concentration:  $N_{D-E} = 10^{19} \text{ cm}^{-3}$  in the emitter,  $N_{A-B} = 10^{16} \text{ cm}^{-3}$  in the base and  $N_{D-C} = 10^{17} \text{ cm}^{-3}$  in the collector. The width of the layers are  $x_e = 200 \text{ nm}$  (emitter),  $W_b = 200 \text{ nm}$  (base) and  $x_c = 1 \text{ }\mu\text{m}$  (collector). Emitter and base can be assumed “short”, the collector is “long”. The Ohmic contacts are ideal.

- Sketch the energy band diagram including  $E_c$ ,  $E_v$ ,  $E_G$  and  $E_F$  across the BJT from emitter through base to collector when no external bias is applied.  
Ensure that the relative distances between the energy levels are consistent with the doping density. Label all parameters in your graph. [5]
- Sketch, on one graph, the *excess* minority carrier concentration in all layers of the BJT when it is biased in forward active mode ( $V_{EB} > 0$  &  $V_{BC} < 0$ ). Include depletion widths.  
Ensure that the relative distances between the excess carrier levels at the junctions are consistent with the doping density. Label all parameters in your graph. Make the graph sufficiently large to make all features clear. [8]
- Give the expression of both the collector current  $I_C$  and the base current  $I_B$  as a function of material parameters: doping densities, layer widths and diffusion constants. Ensure that the material parameters are labelled per layer. [8]
- Calculate the current gain  $\beta$  for this transistor given the diffusion constants of the minority carriers in each layer:  $D_E = 12 \text{ cm}^2/\text{s}$ ,  $D_B = 20 \text{ cm}^2/\text{s}$  and  $D_C = 14 \text{ cm}^2/\text{s}$  (respectively emitter, base and collector minority carrier diffusion constants). [5]
- Explain why SPICE uses the Ebers-Moll equations to describe BJT large signal behaviour. [4]

### 3. pn diode

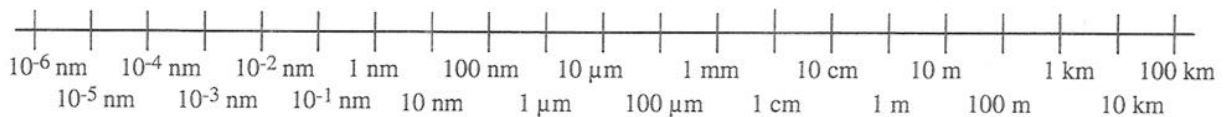
Given: a Si n<sup>+</sup>p diode. The doping concentrations are  $N_D = 10^{19} \text{ cm}^{-3}$  and  $N_A = 10^{16} \text{ cm}^{-3}$ . The width of the layers are  $x_n = 200 \text{ nm}$  (n<sup>+</sup> layer) and  $x_p = 1 \text{ } \mu\text{m}$  (p layer). Thus the n-layer is “short”, the p-layer “long”. The diffusion length of the minority carrier holes is  $L_p = 400 \text{ nm}$  and minority carrier electrons is  $L_n = 500 \text{ nm}$ . The Ohmic contacts are ideal.

- Sketch the energy band diagram including  $E_c$ ,  $E_v$ ,  $E_G$  and  $E_F$  and the built-in voltage  $V_0$  across the n<sup>+</sup>p diode when no external bias is applied.  
Ensure that the relative distances between the energy levels are consistent with the doping density. Label all parameters in your graph. [5]
- Sketch the *excess* minority carrier concentration in all layers of the diode in forward bias. Included the depletion region.  
Ensure that the relative distances between the excess carrier levels at the junctions are consistent with the doping density. Label all parameters in your graph. [8]
- Based on the equations given in the formulae sheet, give the expression of the stored excess charge  $Q_n$  and  $Q_p$  in each layer of the forward biased diode as a function of the material parameters: doping densities, layer widths and diffusion constants. [8]
- Give a reasonable approximation for the value of the current density flowing through the diode when you know that the excess carrier concentration at the edge of the depletion width in the p region is  $2 \times 10^{12} \text{ cm}^{-3}$ . The lifetime of the minority carriers in the p region is  $\tau = 0.1 \text{ } \mu\text{s}$ .  
State all approximations you make. [8]
- When the given diode is switched from on to open circuit (off) at time  $t_{\text{off}}$ . What is the sign (positive, negative or zero) of the:
  - the current through the diode immediately after  $t_{\text{off}}$ ?
  - the voltage across the diode immediately after  $t_{\text{off}}$ ?
 Give a *brief* reason for each case. [4]

## SECTION B: ELECTROMAGNETIC FIELDS

4. This question is **compulsory**

- a. State the key contributions to electromagnetism and its applications made by the following: [7]
- Wilhelm Roentgen and Heinrich Hertz
  - Alexander Graham Bell, Guglielmo Marconi and John Logie Baird
  - John Tyndall, and Charles Kao and George Hockham
- b. Figure 2 shows the entire electromagnetic spectrum. On a copy, indicate the ranges of: [7]
- Gamma rays
  - X-rays
  - Ultraviolet light
  - Visible light
  - Infrared waves
  - Microwaves
  - Radio waves



*Figure 2: The electromagnetic spectrum.*

- c. Free-space terrestrial communications can operate using radio waves, microwaves or light waves. Point out any difficulties that each might suffer, and how these might be overcome. [6]



5.

- a. Explain the difference between phase velocity and group velocity. Find the group velocity of a signal consisting of a pair of sinusoidal travelling waves of equal amplitude, with frequencies  $\omega + \Delta\omega$  and  $\omega - \Delta\omega$  [6]

- b. The relationship between the angular frequency  $\omega$  and the propagation constant  $k$  for electromagnetic waves in the ionosphere may be shown to have the form:

$$\omega = \sqrt{\omega_p^2 + c^2 k^2}$$

Here  $c$  is the velocity of light and  $\omega_p$  is the plasma frequency.

- i. Derive expressions for the variation of the phase velocity and the group velocity with  $\omega$  [6]
- ii. What are the corresponding results for the atmosphere (which has  $\omega_p = 0$ )? [6]
- iii. Sketch the dispersion diagram for the ionosphere and find the value of the velocity of the envelope ( $v_g$ ) when  $\omega$  tends to  $\omega_p$ . What is the significance of this result for information propagation? [6]
- iv. Find a solution for  $k$  when  $\omega < \omega_p$ . What form of wave does this solution describe? Explain the significance of this result for waves incident on the ionosphere from the atmosphere. [6]

6. Figure 3 shows the cross section of a transmission line constructed from two parallel wires of radius  $a$  separated by a distance  $d$  (with  $a \ll d$ ) and surrounded by air
- Assuming that the wires carry charges  $\pm q$ :
    - Sketch the electric field lines and the equipotential lines. [9]
    - Estimate the electric flux density at the point P. [3]
    - Hence, find the voltage between the wires, and the capacitance per unit length of the line. [6]
  - Assuming now that the wires carry currents  $\pm I$ :
    - Estimate the magnetic flux density at the point P. [3]
    - Hence, find the flux between the wires and the inductance per unit length of the line. [6]
    - Find the phase velocity of the line. [3]

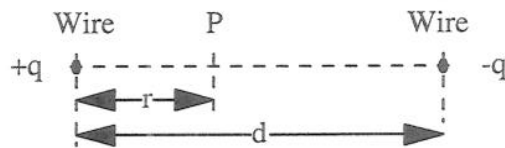


Figure 3: the cross section of a transmission line constructed from two parallel wires.