EE1 - 3

General comments on EE1 Devices paper 2012

Q1.

a. Typical study group question
b. Trick question because of the misinterpretation of suggestion 2. This is actually an ionised atom in the lattice, cannot move, thus is not a carrier.
c. This question is similar to the one asked last year on which the students did very badly. However this is pure bookwork. This is fundamental knowledge to solve the depletion width of pn junctions. It is really surprising that many students do not understand what $\rho(x)$ means within the different regions of a pn junction.
d. Similar question to one of the study group questions and a standard question in an exam.
e. This question is partially bookwork. Many students forgot to draw the depletion regions underneath the ohmic contact doped areas into the substrate. These are pn-junction and should be surrounded by depletion regions.
f. Simple bookwork question. Basic principle of MOSFET.
g. This issue has been discussed multiple times in the lectures and can be easily derived from equations in the formulae list.

Q2.

a. Question not asked before, but information on this topic is available in the course notes and doping of semiconductors is explained by similar drawings. Many students did not realise exactly what had to be drawn as the question is unfamiliar.
b. This question is not straightforward bookwork and requires some reasoning or drawings of energy band diagrams. Students did in general very well on this question with very good reasoning.
c. i) is a standard band diagram question

   ii) this is a challenging question, not just bookwork. Many students solved this question by just drawing a square potential well as seen in the introductory lectures. However, the inversion layer of a MOSFET is not a square quantum well but a triangular one. Thus the equation for V(x) is linear between oxide-semiconductor junction and there where the field in the semiconductor becomes zero. This can be easily seen in the band diagram drawn in i) but 80% of the students did not make that connection. Some students realised that the quantum well is triangular but seemed not to be able to write an equation for it. This might indicate that the students still do not understand what V(x) is in the Schrodinger equation.

Q3.

a. The challenge in the question is summarising the key aspects. Keyword in the answer are potential barrier reduces, diffusion current, minority carrier gradient, contact conditions and reference to zero bias condition. Many students have difficulty in connecting these words/sentences to the drawings of energy band diagram and minority carrier concentration.
b. Bookwork: understanding the difference between fluxes and currents applied to BJTs. These are normally explained in simple pn diodes but the principles remain the same in short BJTs.

c. i) bookwork, most students had no problem with this question some students still do not understand that the electron current in calculated in the p-region and hole current in the n-region, therefore there were errors in IC and IB.
ii) not bookwork, quite challenging, introduction of a new semiconductor with a different bandgap. Students need to realise that a different bandgap has an influence on different material parameters. Two obvious parameters are the mobility and the change in intrinsic carrier concentration. Mobility value was given in formulae list and the Einstein equation that links the mobility to the diffusion constant too – this is the easy part. The intrinsic carrier concentration normally divides away in β in a homojunction, but not in a heterojunction! This aspect is really difficult and you only realise it when you work out the equations without missing steps out.

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2012

EEE PART I: MEng, BEng and ACGI

SEMICONDUCTOR DEVICES

Friday, 15 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.

Question One carries 40% of the marks. Questions Two and Three each carry 30%.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	K. Fobelets
	Second Marker(s) :	S. Lucyszyn

Constants

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10}$ cm^{-3} at T = 300K
dielectric constant of Si:	$\varepsilon_{Si} = 11$
dielectric constant of SiO_2:	$\varepsilon_{ox} = 4$
thermal voltage:	$V_T = kT/e = 0.026$V at T = 300K
charge of an electron:	$e = 1.6 \times 10^{-19}$ C
Planck's constant:	$h = 6.63 \times 10^{-34}$ Js
Bandgap Si:	$E_G = 1.12$ eV at T = 300K
Bandgap Ge:	$E_G = 0.66$ eV at T = 300K
Effective density of states of Si:	$N_C = 3.2 \times 10^{19}$ cm^{-3} at T = 300K $N_V = 1.8 \times 10^{19}$ cm^{-3} at T = 300K
Effective density of states of Ge:	$N_C = 1.0 \times 10^{19}$ cm^{-3} at T = 300K $N_V = 5.0 \times 10^{18}$ cm^{-3} at T = 300K

Formulae

$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$	Schrödinger's equation in one dimension
$f(E)=\frac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$	Fermi distribution
$n_i=\sqrt{N_V N_C}\exp\left(\frac{-E_G}{2kT}\right)$	Intrinsic carrier concentration
$n=N_c\exp\left(-\frac{(E_c-E_F)}{kT}\right)$	Concentration of electrons
$p=N_v\exp\left(\frac{(E_v-E_F)}{kT}\right)$	Concentration of holes
$\frac{dE}{dx}=\frac{\rho(x)}{\varepsilon}$	Poisson equation in 1 dimension
$\left.\begin{aligned}J_n(x)&=e\mu_n n(x)E(x)+eD_n\frac{dn(x)}{dx}\\J_p(x)&=e\mu_p p(x)E(x)-eD_p\frac{dp(x)}{dx}\end{aligned}\right\}$	Drift and diffusion current densities in a semiconductor
$I_{DS}=\frac{\mu C_{ox}W}{L}\left((V_{GS}-V_{th})V_{DS}-\frac{V_{DS}^2}{2}\right)$	Current in a MOSFET
$\left.\begin{aligned}J_n&=\frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}}-1\right)\\J_p&=\frac{eD_p p_{n_0}}{L_p}\left(e^{\frac{eV}{kT}}-1\right)\end{aligned}\right\}$	Current densities for a pn-junction with lengths L_n & L_p
$V_0=\frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$	Built-in voltage
$c=c_0\exp\left(\frac{eV}{kT}\right)\text{ with}\begin{cases}c=p_n\text{ or }n_p\\c_0\text{ bulk minoritycarrierconcentraton}\end{cases}$	Minority carrier injection under bias V
$D=\frac{kT}{e}\mu$	Einstein relation
$W_{depl}(V)=\left[\frac{2\varepsilon(V_{bi}-V)}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\right]^{1/2}$	Total depletion width under bias V

1.

a)

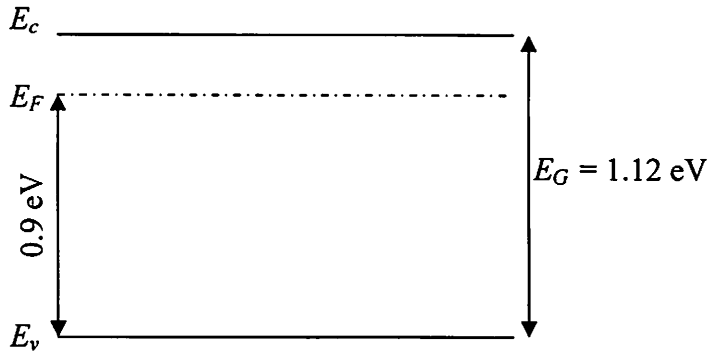


Figure 1.1: The energy band diagram for doped Si.

i) Give the doping type for the semiconductor in Fig. 1.1. Explain why. [4]

ii) Calculate the doping density at room temperature for the semiconductor in Fig. 1.1. You can assume complete ionisation. [6]

b) Choose, from the following list, the correct description for a hole.

1. a hole is a void in a semiconductor.
2. a hole is a positively charged Si atom that has lost its electron.
3. a hole is a positively charged carrier.
4. a hole is the lack of a charged carrier in the conduction band. [2]

c)

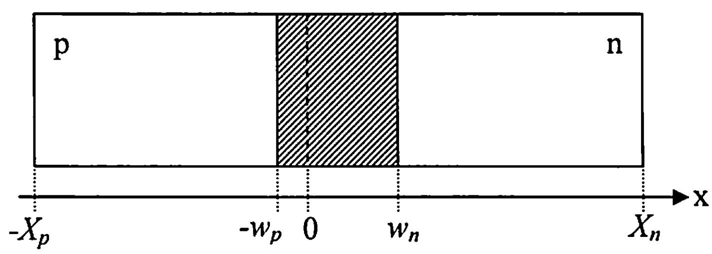


Figure 1.2: Material cross-section of a pn diode. X_p, X_n is the length of the p respectively n region. w_p and w_n is the length of the depletion region in the p respectively n region.

Sketch the variation of the charge density $\rho(x)$ as a function of x from $-X_p < x < X_n$ for the pn diode given in Fig. 1.2. Ensure the relative magnitudes are correct. Give the relevant parameters on the axes. [8]

Question continues on next page.

d) Sketch the energy band diagram (E_c, E_v, E_F, E_G) from source to drain through the channel for an n-channel enhancement mode MOSFET in inversion and for small V_{DS} in the triode region. Ensure all energy and distance scales are consistent relative to each other. [6]

e) Plot the material cross-section of an n-channel enhancement mode MOSFET at pinch-off. Indicate the inversion and depletion regions and the doping type for each region. [6]

f)

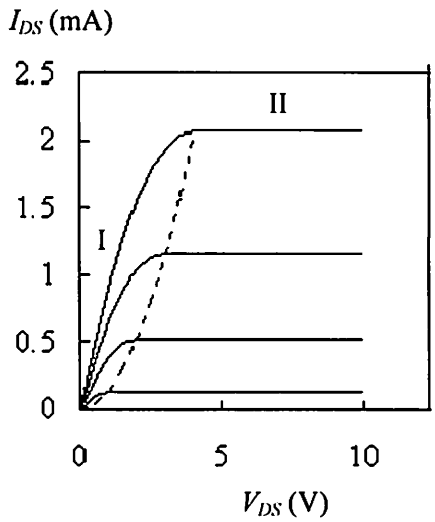


Figure 1.3: Output characteristic of an enhancement mode n-channel MOSFET. Two regions I and II are indicated on the plot for respectively small and large drain voltage.

Give the simplified expression for the drain-source current I_{DS} for both regions I and II in Fig. 1.3. The general expression for the MOSFET current is given in the formulae list on p. 2. [4]

g) Explain why the doping in the emitter of a bipolar junction transistor (BJT) is higher than that in the base. [4]

2.

a) Take boron-doped silicon with a doping concentration of 5×10^{16} cm^{-3}. Boron is a group 3a element and silicon a group 4a element.

i) Sketch a covalent bonding scheme with 14 silicon atoms and 2 boron atoms. Indicate the presence of the valence electrons in this scheme. [5]

ii) The mobility of the majority carriers in this semiconductor is 400 cm^2/Vs. Will the mobility change when the doping concentration is increased? Explain your answer briefly. [5]

b) The boron-doped silicon material of (a) is used as the bulk for an enhancement mode MOSFET. The gate oxide is ideal and has a thickness of t_{ox} = 2 nm.

i) Give the type of inversion carriers for this MOSFET. [2]

ii) Explain what will happen to the threshold voltage when we increase the oxide thickness t_{ox}? [4]

iii) Explain what will happen to the threshold voltage when a homogeneous distribution of negative fixed charges has been introduced into the gate oxide during the fabrication process? [4]

c) Take the MOSFET of (b) with a substrate doping of 5×10^{16} cm^{-3} boron atoms and an ideal oxide of thickness of t_{ox} = 2 nm.

i) Sketch the energy band diagram (E_c, E_v, E_F, E_G), from the gate into the bulk (x-direction), for this MOSFET. The gate voltage should cause an inversion carrier concentration at the Si/SiO$_2$ interface, n_s to be larger than the majority carrier concentration in the bulk, c_b. [5]

ii) Under strong inversion, the inversion layer in the MOSFET of (c) i) forms a triangular quantum well. In order to calculate the position of the energy levels in this quantum well, the Schrödinger equation, $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$, needs to be solved. Give an approximate expression for the potential variation of the quantum well region, in terms of the potential drop across the Si/SiO$_2$ surface V_s. [5]

3.

a) The assumptions made to extract the expression for the currents in a pn junction are that the applied electric field occurs only across the depletion region and the depletion region contains no free carriers.

Under these assumptions, explain the occurrence of an electron and hole current in a forward biased pn junction [4]. Use sketches of energy band diagrams [3] and carrier concentrations [3] to illustrate your explanation. [10]

b) Figure 3.1 gives a cross section of both an npn (i) and a pnp (ii) BJT. The BJTs are both biased in forward active mode in common base configuration.

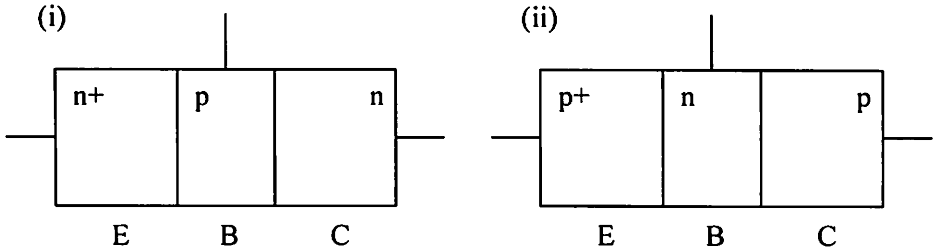


Figure 3.1: i) a cross-section of an npn BJT, ii) a cross-section of a pnp BJT.

i) Redraw Fig. 3.1(i) giving the bias voltages between the contacts and indicating clearly the direction of the emitter, base and collector current, I_E, I_B and I_C. [5]

ii) Redraw Fig. 3.1(ii) giving the bias voltages between the contacts and indicating clearly the direction of the main electron and hole fluxes in each material layer. [5]

c) The expression for the base and collector current in a short pnp BJT is given by:

$$I_B = \frac{e\,D_i\,c_i}{X}\exp\left(\frac{V_i}{V_T}\right)$$

$$I_C = \frac{e\,D_j\,c_j}{Y}\exp\left(\frac{V_j}{V_T}\right)$$

with subscript i, j the doping type of the relevant layer (n or p) ; c_i, c_j the relevant carrier concentration in the material under zero bias ; D_i, D_j the diffusion constants, X, Y the width of the relevant layers and V_i, V_j the voltage drop across the relevant junction.

i) Define the parameters: c_i, c_j, D_i, D_j, X, Y, V_i, V_j. Choose from: n_n, n_p, p_p, p_n, respectively the electron majority, electron minority, hole majority and hole minority carrier concentration. D_n, D_p, and X_E, X_B, X_C, respectively emitter, base and collector width. V_{EB}, V_{BC}, respectively the emitter-base and the base-collector voltage. [5]

ii) The temperature is $T = 300$ K. How does the current gain change when Si is replaced with Ge in the base of the BJT, while keeping the same geometry and doping density. Assume that the mobility of both the electrons and holes in Ge is double that of Si. [5]

ANSWERS 2 of 2 E1-3

1. a)

i) n-type doping because the Fermi level E_F lies closer to the conduction band minimum E_C than the valence band maximum E_v.

[4]

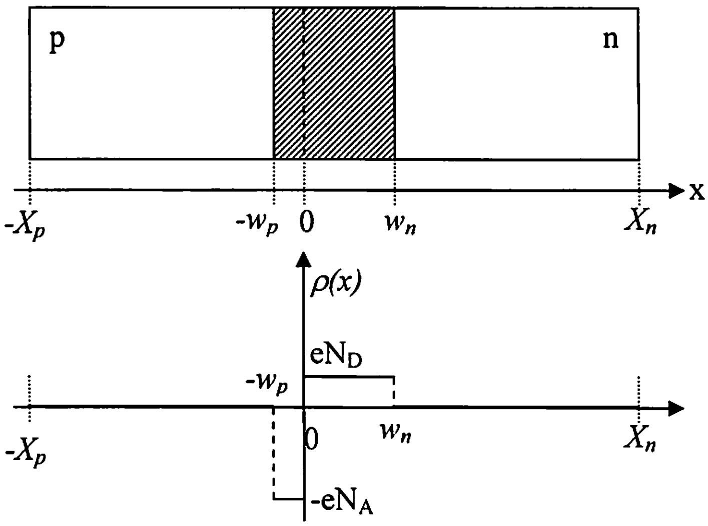
ii) Take formula from list on p.2: $n = N_c e^{-\frac{(E_g - E_F)}{kT}}$. If complete ionisation is assumed then the doping density is equal to the majority carrier concentration. $N_D = n$. $E_c - E_F = 1.12\text{ eV} - 0.9\text{ eV} = 0.22\text{ eV}$.

$$N_D = n = 3.2 \times 10^{19} e^{-\frac{0.22}{0.26}} = 1.37 \times 10^{19}\ cm^{-3}$$ [6]

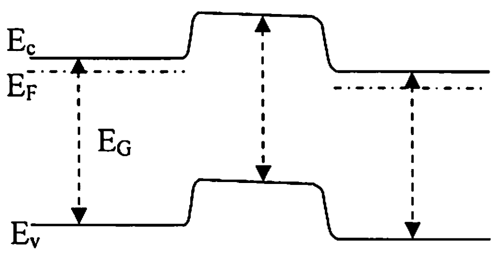
b) Choose from the following list the correct description for a hole.

3. a hole is a positively charged carrier. [2]

c) Note that the area in the p-depletion region must be the same to that in the n-depletion region due to charge neutrality. [8]

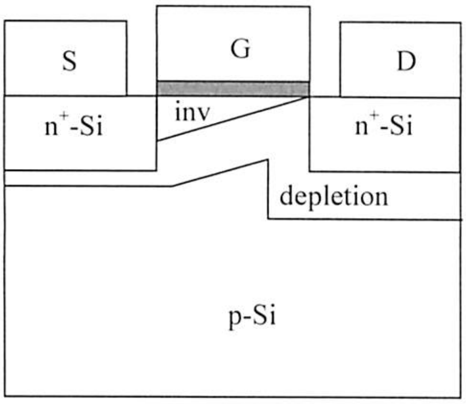


d) [6]



Channel region must be n-type but less than the ohmic contact regions. There should be a small tilt in the Ec and Ev curves in the channel region (middle). The depletion region and the barrier should be slightly larger at the drain side (right)

e) Note that the depletion region should be larger at the drain side. [6]



f) Region I : triode region $I_{DS} = \frac{\mu C_{ox} W}{L}(V_{GS} - V_{th})V_{DS}$

Region II : saturation region $I_{DS} = \frac{\mu C_{ox} W}{2L}(V_{GS} - V_{th})^2$ [4]

g) The emitter doping must be higher than the base doping for high current gain.

$\beta = \frac{I_C}{I_B}$. The currents are proportional to the minority carrier concentrations: [4]

$$J_n = \frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)$$

$$J_p = \frac{eD_p p_{n_0}}{L_p}\left(e^{\frac{eV}{kT}} - 1\right)$$

(formula list)

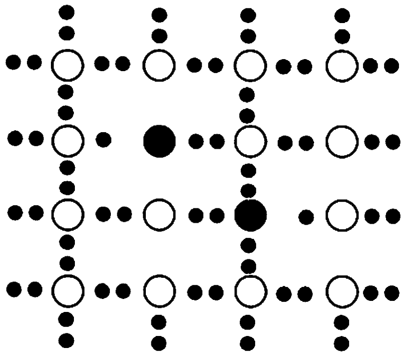
Thus, if the minority carrier concentration in the emitter is lower than in the base, then the minority carrier current in the emitter (= base current) will be higher than in the base (collector current). Minority carrier concentrations are inversely proportional to the doping density:

$$n_{p0} = \frac{n_i^2}{N_A}$$

$$p_{n0} = \frac{n_i^2}{N_D}$$

2.

a) i) [5]



Open circles are the Si atoms, black circles the B atoms and the small circles the electrons in the bond.

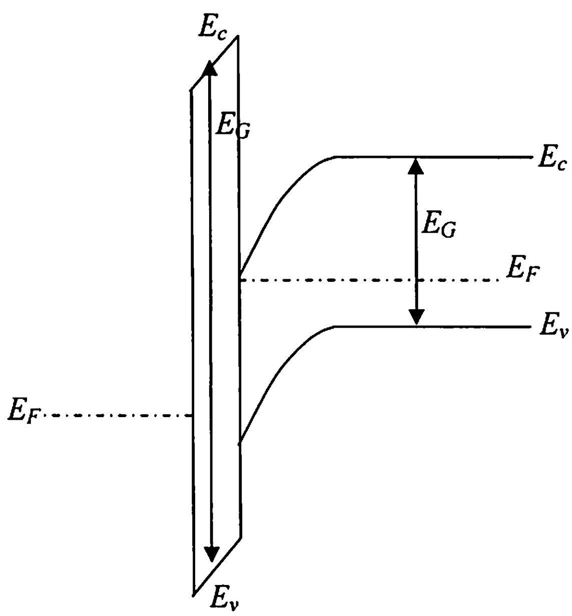
ii) Yes, the mobility will lower due to impurity scattering. Mobility is directly proportional to the average time between scattering event. When more doping atoms are present the probability to scatter increases and the average time between scattering events also decreases. [5]

b) i) electrons. [2]

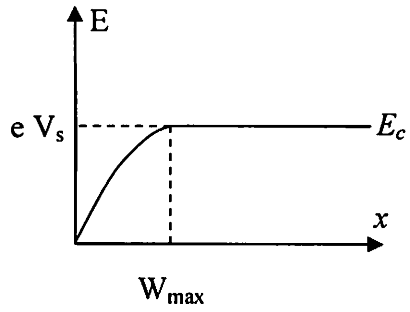
ii) When the oxide thickness increases a larger part of the applied voltage will be dropped across the oxide, thus more gate voltage will need to be applied to attract the same density of carriers in the inversion layer. Same energy band bending needs to be maintained in the semiconductor at the Si/SiO_2 interface. [4]

iii) The introduction of negative fixed charges in the oxide will attract majority carrier holes to the Si/SiO_2 interface due to the requirement for charge neutrality. Thus we will need to apply a higher gate voltage to obtain the threshold condition in the Si channel. This is because we first have to apply a gate voltage to repel the additional holes attracted by the fixed oxide charge. This voltage is in addition to what needs to be applied when no fixed negative charges are present. [4]

c) i) [5]



ii) The variation of the potential energy of the electrons is plotted in the figure below. [5]



Assumptions are that the depletion region is the maximum depletion width and that the width of the inversion layer is negligible compared to W_{max}. The voltage drop across the Si at the interface is denoted with V_s. We take the bottom of the conduction band at x=0 to be E=0.

To first approximation we can assume that the variation of the potential in the depletion region is linear. Equation for V(x) = A x + B

@ x = 0, V(x) = 0 → B = 0 ; @ x = W_{max}, V(x) = V_s/e → A = V_s/(W_{max} e)

Thus: V(x) = V_s x/(W_{max} e) with $W_{max} = \left[\frac{2\varepsilon V_s}{e}\left(\frac{1}{N_A}\right)\right]^{1/2}$

For more accurate answer, the Poisson equation needs to be solved:

This approach is not required is the linear approximation is taken.

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon}$$

$$\frac{dE}{dx} = \frac{-eN_A}{\varepsilon}$$

$$E = \frac{-eN_A}{\varepsilon}x + C$$

$$@x = w_p, E = 0$$

$$E = \frac{-eN_A}{\varepsilon}\left(x - w_p\right)$$

$$E = -\frac{dV}{dx}$$

$$\frac{dV}{dx} = \frac{eN_A}{\varepsilon}\left(x - w_p\right)$$

$$V = \frac{eN_A}{\varepsilon}\left(\frac{x}{2} - w_p\right)x + C$$

$$@x = w_p, V = V_s$$

$$V_s = \frac{-eN_A}{\varepsilon}\frac{w_p^2}{2} + C$$

$$C = V_s + \frac{eN_A}{\varepsilon}\frac{w_p^2}{2}$$

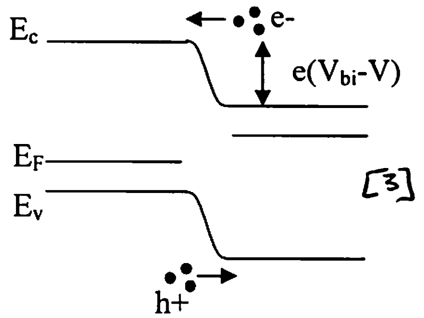
$$V = \frac{eN_A}{\varepsilon}\left(\frac{x}{2} - w_p\right)x + V_s + \frac{eN_A}{\varepsilon}\frac{w_p^2}{2}$$

with

$$w_p = \left[\frac{2\varepsilon V_s}{e\,N_A}\right]^{1/2}$$

3.

a) The currents that are flowing are minority carrier diffusion currents. The explanation is based on the gradients of the minority carriers in the neutral regions. [10]



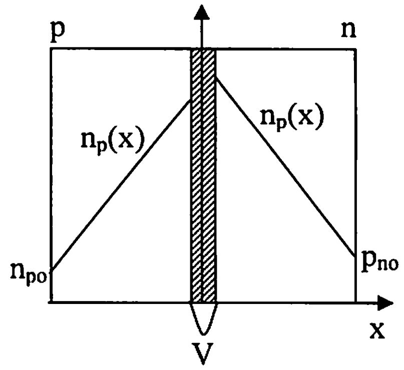
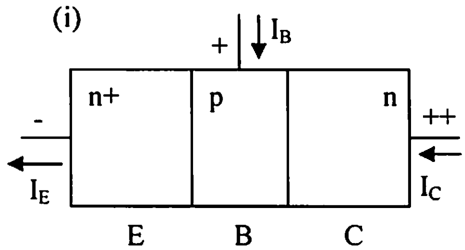


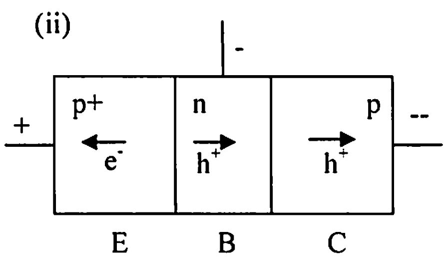
Figure 3.1: left: energy band diagram of pn junction under forward bias V. Right: the variation of the minority carriers under forward bias.

When the diode is under zero bias, drift and diffusion currents across the junction are equal and opposite. This is due to a build-in voltage V_{bi} that causes a potential barrier between p and n. When a forward bias ,V is applied, the potential barrier is lowered, see fig. 3.1 left. This lowering of the potential barrier allows carriers at a higher energy to cross the junction. Once the carriers have crossed, they become minority carriers and a build-up of minority carriers occurs at the edge of the depletion width. Because the ohmic contacts impose the bulk minority carrier concentration to be maintained at the contacts, a minority carrier gradient is built up, see fig. 3.1 right. Since there now exists a carrier concentration variation, these carriers will diffuse to the contacts and cause the forward bias diode current. [4]

b) i) EB forward biased, BC reverse biased (thus potential on C more positive than on B). [5]



ii) EB forward biased, BC reverse biased (thus potential on C more negative than on B). Note that there is also a small reverse bias fluxes of e^- from C $\rightarrow$ B and h^+, this is not a main contributor. . [5]



c) The expression for the base and collector current in a short pnp BJT is given by:

with subscript i, j the doping type of the relevant layer (n or p) ; c_i, c_j the relevant carrier concentration in the material under zero bias ; X, Y the width of the relevant layers and V_i, V_j the voltage drop across the relevant junction.

i) $c_i = n_p, D_i = D_n, X = X_E, V_i = V_{EB}$.
$c_j = p_n, D_j = D_p, X = X_B, V_i = V_{EB}$. [5]

ii) Calculate the ratio of the current gain of a Si BJT and a Ge BJT with the same geometry and doping density in the different device layers. $T = 300\text{K}$. [5]

$$\beta = \frac{I_C}{I_B} = \frac{D_p\, p_n X_E}{D_n\, n_p X_B}$$

Using the Einstein equation $D = \frac{kT}{e}\mu$ (formulae list) and the expression of the minority carriers as a function of doping density

$p_n = \frac{n_i^2}{N_D}$ & $n_p = \frac{n_i^2}{N_A}$ (by heart).

The current gain for the BJT with the Si base

$$\beta_{Si} = \frac{I_C}{I_B} = \frac{D_p\, p_n X_E}{D_n\, n_p X_B} = \frac{\mu_{p\_Si}}{\mu_{n-Si}} \frac{N_A}{N_D} \frac{X_E}{X_B}$$

The current gain for the BJT with the Ge base

$$\beta_{Ge} = \frac{I_C}{I_B} = \frac{D_p\, p_n X_E}{D_n\, n_p X_B} = \frac{\mu_{p\_Ge}}{\mu_{n-Si}} \frac{n_{i\_Ge}^2}{n_{i\_Si}^2} \frac{N_A}{N_D} \frac{X_E}{X_B}$$

$$\frac{\beta_{Ge}}{\beta_{Si}} = \frac{\mu_{p\_Ge}}{\mu_{p\_Si}} \frac{n_{i\_Ge}^2}{n_{i\_Si}^2}$$

Using the expression for the intrinsic carrier concentration as a function of the bandgap (formulae list).

$$n_i = \sqrt{N_V N_C}\exp\left(\frac{-E_G}{2kT}\right)$$

$$\frac{\beta_{Ge}}{\beta_{Si}} = \frac{\mu_{p\_Ge}}{\mu_{p\_Si}} \frac{N_{V\_Ge} N_{C\_Ge}}{N_{V\_Si} N_{C\_Si}} \exp\left(\frac{E_{G\_Si} - E_{G\_Ge}}{kT}\right)$$

$$\frac{\beta_{Ge}}{\beta_{Si}} = 2 \times \frac{1 \times 0.5}{3.2 \times 1.8} \exp\left(\frac{0.46}{0.026}\right) = 8.4\ 10^6$$