UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2000

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C233

COMPUTATIONAL TECHNIQUES

Tuesday 9 May 2000, 14:00
Duration: 90 minutes
(Reading time 5 minutes)

Answer THREE questions

Paper contains 4 questions

1a Assume 6-digit decimal arithmetic. Below, there are two numbers, a and b, and their approximations.

Accurate value	Approximation
$a = 1000.00$	1001.99
$b = 99.9000$	100.200

Determine the error of the approximations using all measures of error you know. Is the approximation of a or b more accurate? Why?

b Which of the following two matrices is positive definite? How can you determine that? Explain your work.

$$\mathbf{A} = \begin{bmatrix} 9 & -6 & 0 \\ -6 & 5 & 2 \\ 0 & 2 & 20 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 9 & -6 & 0 \\ -6 & 4 & 2 \\ 0 & 2 & 16 \end{bmatrix}$$

c Assume that matrices **A**, **B** and **C** have appropriate dimensions for the operations below. Prove that

(i) $(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$.

(ii) $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$.

(The three parts carry, respectively, 20%, 50% and 30% of the marks).

2a Which of the following two sets of vectors is linearly independent (if any)?

$$\begin{array}{lcl} \mathbf{a}_1 = [1, -3/2, 1]^T & & \mathbf{b}_1 = [1, -3, 1]^T \\ \mathbf{a}_2 = [-9, 8, -3]^T & \text{or} & \mathbf{b}_2 = [-9, 21, -3]^T \\ \mathbf{a}_3 = [7, -5, 1]^T & & \mathbf{b}_3 = [7, -5, 1]^T \end{array}$$

Explain the method you use.

b Given matrix $\mathbf{A}$, determine its ℓ_1, ℓ_2 and ℓ_∞ norm:

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 5 \\ 0 & 3 & 6 \end{bmatrix}.$$

c Matrix $\mathbf{A}$ is called *skew symmetric* if $\mathbf{A}^T = -\mathbf{A}$. What is the shape of $\mathbf{A}$? What are the diagonal elements of $\mathbf{A}$? Show that if $\mathbf{A}$ is skew symmetric then $\mathbf{A}^T\mathbf{A} = \mathbf{A}\mathbf{A}^T$!

(The three parts carry, respectively, 30%, 50% and 20% of the marks).

3a Let $\mathbf{A}$ be an $m \times m$ positive definite matrix. Prove that all diagonal elements (a_{ii}, for $i = 1, \ldots, m$) are strictly positive. Explain your work.

b Show that for any matrix $\mathbf{C} \in \mathbb{R}^{m \times n}$ both $\mathbf{C}^T\mathbf{C}$ and $\mathbf{C}\mathbf{C}^T$ are symmetric. Determine the dimensions of the symmetric matrices.

c Under what conditions are the following matrix equalities true?

(i) $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + 2\mathbf{A}\mathbf{B} + \mathbf{B}^2$.

(ii) $(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B}) = \mathbf{A}^2 - \mathbf{B}^2$.

d Let I be the half-open interval $(-1, 0]$ and $d(x, y) = |x - y|$ be the distance for any $x, y \in I$.

(i) Show that d is a metric on I.

(ii) Show that $x_n = \dfrac{1-n}{n}$, for $n = 1, 2, \ldots$ is a Cauchy sequence in the metric d.

(iii) Is (I, d) complete? Justify your answer.

(The four parts carry, respectively, 15%, 20%, 20% and 45% of the marks).

4a You are given matrix $\mathbf{A} \in \mathbb{R}^{3\times 3}$ in the following form:

$$\mathbf{A} = \begin{bmatrix} 1 & a_{12} & 0 \\ 0 & a_{22} & 0 \\ 0 & a_{32} & 1 \end{bmatrix}$$

and an arbitrary vector $\mathbf{b} \in \mathbb{R}^3$.

Show that $\mathbf{Ab} = \mathbf{b}$ if $b_2 = 0$. Discuss the usefulness of this simple fact. Generalise the observation for $m \times m$ matrices that differ from $\mathbf{I}_m$ only in one column. What can you conclude for the eigensystem of $\mathbf{A}$?

b Let matrix $\mathbf{P}$ be given in the following partitioned form:

$$\mathbf{P} = \begin{bmatrix} \mathbf{c}^T & 1 \\ \mathbf{A} & \mathbf{0} \end{bmatrix},$$

where $\mathbf{A}$ is an $m \times m$ nonsingular matrix with inverse $\mathbf{A}^{-1}$, $\mathbf{c}$ is an m-vector and $\mathbf{0}$ is the m dimensional null vector.

Determine the inverse of $\mathbf{P}$ symbolically in a partitioned form. Verify your answer. What is the dimension of $\mathbf{P}$? What are the dimensions of the submatrices in $\mathbf{P}^{-1}$?

c Find a local minimum or maximum for

$$f(x, y) = x^2 + y - \frac{1}{3}y^3.$$

Explain your work.

(The three parts carry, respectively, 20%, 40% and 40% of the marks).