IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

EEE/EIE PART II: MEng, Beng and ACGI

COMMUNICATION SYSTEMS

Friday, 7 June 2:00 pm

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : C. Ling
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EXAM QUESTIONS

Information for Students

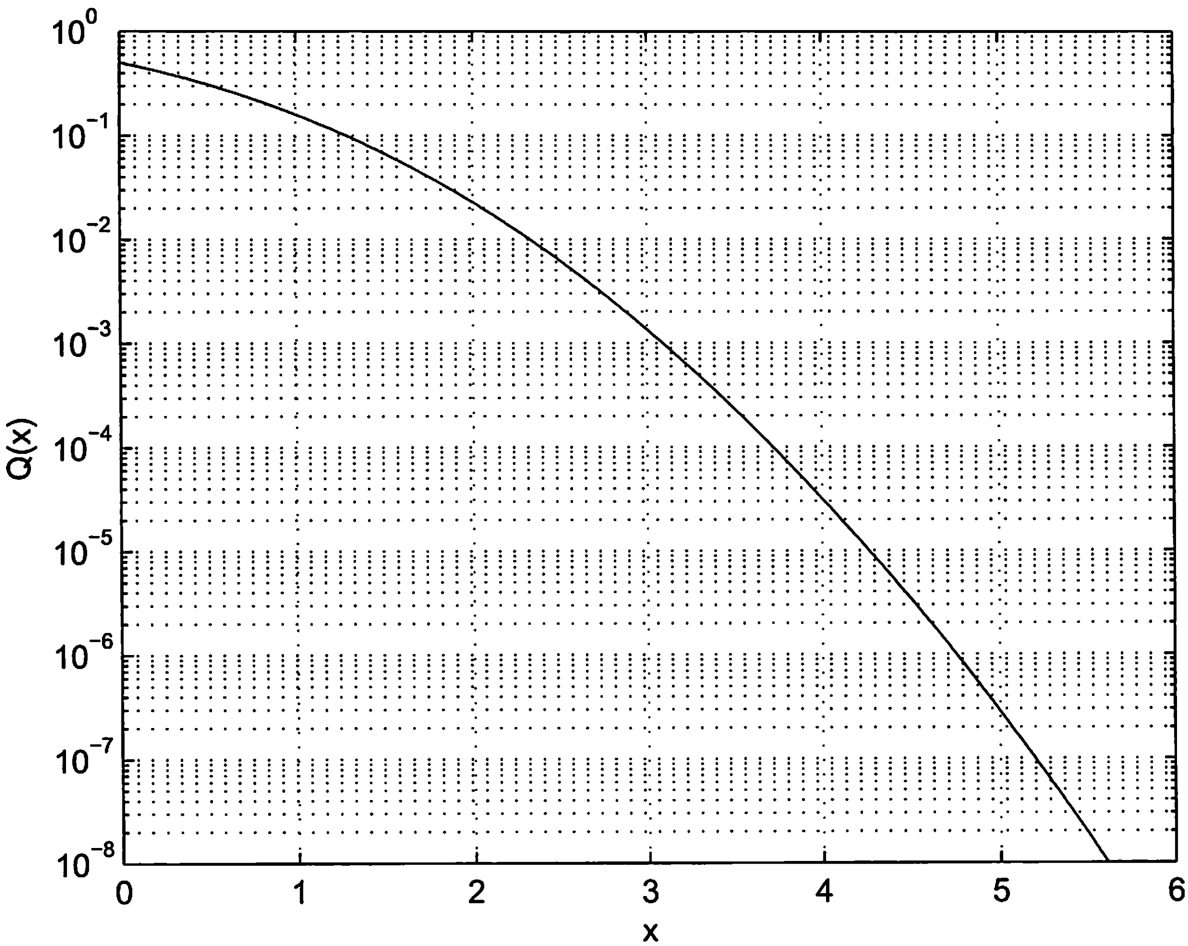


Figure 0.1 The graph of the Q-function.

TABLE 4-2
Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$	Comments on Derivation
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} \tau f$	Direct evaluation
2.	$2W \operatorname{sinc} 2Wt$	$\Pi\left(\frac{f}{2W}\right)$	Duality with pair 1, Example 4-7
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 \tau f$	Convolution using pair 1
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	Direct evaluation
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$	Differentiation of pair 4 with respect to α
6.	$\exp(-\alpha\lvert t\rvert),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$	Direct evaluation
7.	$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f\tau)^2}$	Direct evaluation
8.	$\delta(t)$	1	Example 4-9
9.	1	$\delta(f)$	Duality with pair 7
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	Shift and pair 7
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$	Duality with pair 9
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$	Exponential representation of cos and sin and pair 10
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$	
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$	Integration and pair 7
15.	$\operatorname{sgn} t$	$(j\pi f)^{-1}$	Pair 8 and pair 13 with superposition
16.	$\frac{1}{\pi t}$	$-j \operatorname{sgn}(f)$	Duality with pair 14
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda}\, d\lambda$	$-j \operatorname{sgn}(f) X(f)$	Convolution and pair 15
18.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s),$ $f_s = T_s^{-1}$	Example 4-10

1. a) i) Consider two random variables X and Y. Explain the notions "uncorrelated" and "independent". Discuss the relation between "uncorrelated" and "independent". [4]

ii) Draw a diagram showing how to obtain in-phase component $n_c(t)$ and quadrature component $n_s(t)$ from the bandpass noise $n(t)$. [4]

b) i) With the help of a phasor diagram, write down an equation for the envelope of the output of the AM envelope detector in the presence of noise $n(t)$, and give an approximation when the noise is small. [6]

ii) Name three advantages of digital communications, when compared to analog communications. [3]

c) i) Explain and compare source coding and channel coding. Give examples. [4]

ii) What are Hamming codes? What is its minimum Hamming distance? [3]

iii) Given the parameters (n,k) where n is the codeword length and k is the number of information bits, what is the relation between n and k for Hamming codes?. [2]

iv) Give the pairs (n,k) for the fist three Hamming codes. [2]

d) Consider a modulated communication system. At the receiver side, the predetection filter bandwidth is B, while the postdetection filter bandwidth is W. Suppose the predetection signal-to-noise ratio (SNR) is SNR_{in}, while the postdetection SNR is SNR_{out}.

i) Write down the formula for the maximum rate at which information may arrive at the receiver. [3]

ii) Write down the formula for the maximum rate at which information may leave the receiver. [3]

iii) Derive SNR_{out} of an ideal communication system as a function of $SNR_{baseband}$. Discuss the limit as the ratio $B/W \to \infty$. [6]

2. a) i) What is a stationary random process? What is the Wiener-Khinchine relation between the power spectral density and the autocorrelation function? [5]

ii) Given the power spectral density of the ideal low-pass white Gaussian noise

$$S_N(f) = \begin{cases} \frac{N_0}{2}, & |f| < B; \\ 0, & \text{otherwise.} \end{cases}$$

Write down the autocorrelation function using the Wiener-Khinchine relation.
Calculate the autocorrelation between samples taken at the Nyquist rate.
Discuss the meaning of your finding. [5]

iii) Consider the random process

$$X(t) = a\cos(\omega t + \Theta)$$

where a and ω are constants and Θ is a binary random variable taking values of 0 or π equiprobably, i.e., $P(\Theta = 0) = 1/2$ and $P(\Theta = \pi) = 1/2$. Determine whether this is a stationary or nonstationary process, by computing the mean and autocorrelation function. [8]

b) The output SNR of the FM receiver is given by

$$\text{SNR}_{\text{FM}} = 3\beta^2 \frac{P}{m_p^2} \text{SNR}_{\text{baseband}} \tag{2.1}$$

where P and m_p are the power and peak amplitude of the message, respectively. Assume the deviation ratio $\beta = 5$ and the message $m(t)$ is a zero-mean Gaussian random process.
Compute SNR_{FM} as a function of $\text{SNR}_{\text{baseband}}$, when the overload probability is 6×10^{-7} (i.e., the probability $P(|m(t)| > m_p) = 6 \times 10^{-7}$).
(Hint: use the graph of the Q-function.) [12]

3. a) Consider a sequence of the English alphabet with their probabilities of occurrence given by

Letter	a	b	i	l	m	o
Probability	0.3	0.1	0.2	0.1	0.1	0.2

i) Calculate the entropy of this source. [2]

ii) Construct a Huffman code and find the average codeword length. [5]

b) Repetition codes represent the simplest type of linear block codes. In particular, a single message bit is encoded into a block of n identical bits, producing an $(n,1)$ code. Let $n=5$ in this question.

i) Do you consider a repetition code to be systematic or non-systematic? Explain your answer. [2]

ii) Write down the generator matrix and parity-check matrix of a $(5,1)$ repetition code. [4]

iii) Determine the minimum Hamming distance. [2]

iv) Compute the syndrome table for all possible single-error patterns.

[5]

v) Compute the syndrome table for all possible 10 double-error patterns. [5]

vi) Let us apply this $(5,1)$ code to a PSK-modulated system with signal-to-noise ratio $\frac{A}{\sigma}=4$.
Compute the raw error probability of PSK and the decoding error probability after majority-rule decoding.
(Hint: Majority-rule decoding works as follows: if in a block of n received bits, the number of 0s exceeds the number of 1s, the decoder decides in favor of a 0; otherwise, it decides in favor of a 1.) [5]

[B] Bookwork

[E] New Example

[A] New Application

[T] New Theory

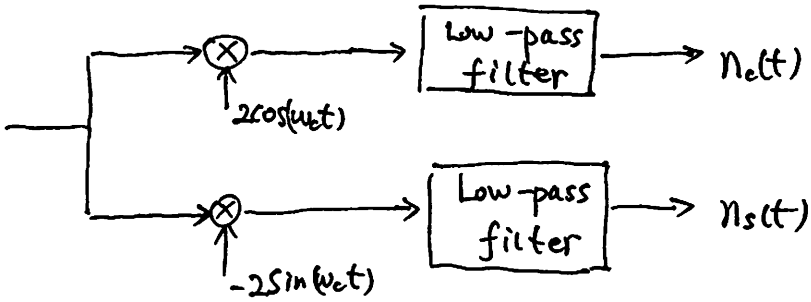
ANSWERS

1. a) i) Uncorreated: $E[XY] = E[X]\,E[Y]$

independent: $f_{XY}(xy) = f_X(x)\, f_Y(y)$ [4B]

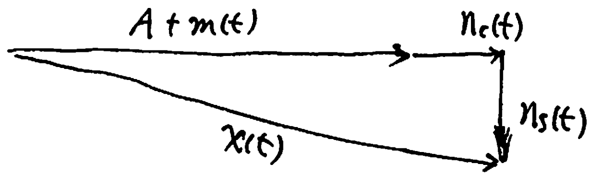
"independent" implies uncorreleted, but the converse is not necessarily true.

ii)



[4B]

b) i)



[3B]

$y(t)$ = envelope of $x(t)$

$$= \sqrt{[A + m(t) + n_c(t)]^2 + n_s^2(t)}$$

When noise is small [3B]

$$y(t) \approx A + m(t) + n_c(t)$$

ii) Digital communication is [3B]

more immune to channel noise;

digital signals can be represented in a uniform format;

easier to process;

flexible and allow for sophisticated functions;

able to provide digital services such as Internet.

c) i) Source coding is to compress the data by reducing the number of source bits, e.g., Huffman code. [4 B]

Channel coding is to introduce redundant bits to enable to detection and correction of errors caused by the channel, e.g., Hamming code.

ii) Hamming codes are a class of linear block codes that can correct a single error.

[3 B]

For Hamming codes, $d_{min} = 3$.

iii) $r = n - k = \log_2(n+1) \Rightarrow n = 2^r - 1,$
$k = 2^r - 1 - r.$ [2 B]

First few Hamming codes:

[2 B]

$(n, k) = (7, 4), (15, 11), (31, 26), \cdots$

d) i) $C_{in} = B \log_2(1 + SNR_{in})$ [3 B]

ii) $C_{out} = W \log_2(1 + SNR_{out})$ [3 B]

iii) For an ideal system, $C_{in} = C_{out}$.

[2 B]

$$W \log_2(1 + SNR_{out}) = B \log_2(1 + SNR_{in})$$

Since

$$SNR_{in} = \frac{P}{N_0 B} = \frac{W}{B} \frac{P}{N_0 W} = \frac{W}{B} SNR_{baseband},$$

we have [2 B]

$$W \log_2(1 + SNR_{out}) = B \log_2\left(1 + \frac{SNR_{baseband}}{B/W}\right)$$

$$SNR_{out} = \left(1 + \frac{SNR_{baseband}}{B/W}\right)^{B/W} - 1.$$ [2 B]

$$\rightarrow e^{SNR_{baseband}} \quad \text{as} \quad B/W \rightarrow \infty.$$

2. i) Stationary process: [5B]

$$\mu_X(t) = \mu_X \quad \text{doesn't depend on } t;$$

$$R_X(t, t+\tau) = R_X(\tau) \quad \text{is a function of } \tau \text{ only.}$$

Wiener-khinchine relation: Power spectral density is the Fourier transform of the autocorrelation function.

ii) From the table,

$$R_X(\tau) = N_0 B \operatorname{Sinc}(2B\tau).$$ [2B]

If taken at Nyquist rate, $\tau = \frac{k}{2B}$, $k = 0, \pm 1, \pm 2, \dots$

Then,

$$R_X(\tau) = N_0 B \operatorname{Sinc}(k) = 0, \quad k = \pm 1, \pm 2, \dots$$ [3B]

This means the samples are uncorrelated, hence being independent since they are Gaussian.

iii)
Mean: $E[X(t)] = E[a\cos(\omega t + \theta)]$

$$= \frac{1}{2}[a\cos(\omega t) + a\cos(\omega t + \pi)]$$ [2E]

$$= 0$$

Autocorrelation

$$R_X(t, t+\tau) = E[a\cos(\omega t + \theta)\, a\cos(\omega(t+\tau) + \theta)]$$

$$= \frac{1}{2}[a^2\cos(\omega t)\cos(\omega(t+\tau)) + a^2\cos(\omega t + \pi)\cos(\omega(t+\tau) + \pi)]$$ [4E]

$$= a^2\cos(\omega t)\cos(\omega(t+\tau))$$

$$= \frac{a^2}{2}[\cos(\omega(2t+\tau)) + \cos(\omega\tau)]$$

can't get rid of t!

Therefore, it is NOT stationary. [2E]

b Since $m(t)$ is Gaussian,

$$P\left(|m(t)| > m_p\right) = 2\,Q\left(\frac{m_p}{\sigma}\right) = 6 \times 10^{-7}$$ [3A]

where σ is the standard deviation. Thus, $P = \sigma^2$.

From the graph, if $Q(x) = 3 \times 10^{-7}$,

$$x = 5.$$ [3A]

Therefore, $\frac{m_p}{\sigma} = 5$.

This means $\frac{P}{m_p^2} = \frac{\sigma^2}{m_p^2} = \frac{1}{25}$. [3A]

Therefore,

$$SNR_{FM} = 3 \times 5^2 \times \frac{1}{25} SNR_{baseband}$$ [3A]

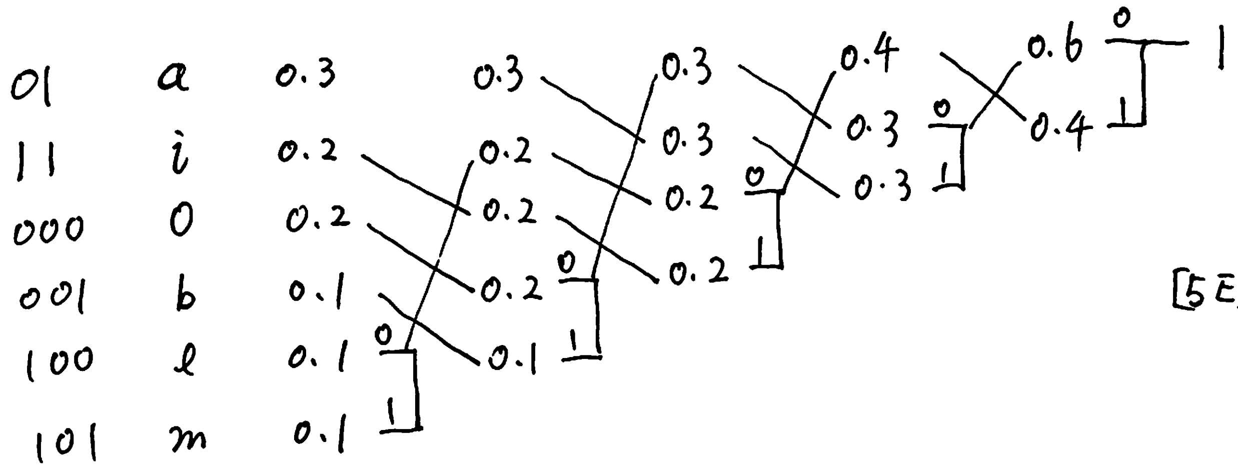
$$= 3\, SNR_{baseband}.$$

3. a) i) $H(S) = -\sum p_i \log_2 p_i$ [2E]

$$= -(0.3\log_2 0.3 + 2\times 0.2\times \log_2 0.2 + 3\times 0.1\times \log_2 0.1)$$

$$= 2.45$$

ii)



[5E]

average length

$$L = 2\times 0.5 + 3\times 0.5$$

$$= 2.5$$

b) i) Systematic, because the information bit appears as is. [2 A]

ii) $G = [1\ 1\ 1\ 1\ 1]$ [2 A]

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$ [2 A]

iii) $d_{min} = 5$ [2 A]

iv) $s = eH^T$

s	e
1111	10000
1000	01000
0100	00100
0010	00010
0001	00001

[5 A]

v)

s	e
0111	11000
1011	10100
1101	10010
1110	10001
1100	01100
1010	01010
1001	01001
0110	00110
0101	00101
0011	00011

[5 A]

vi) For coherent PSK, the raw error probability is

$$P_e = Q\left(\frac{A}{2\sigma}\right)$$

$$= Q(2) \approx 2.2 \times 10^{-2}$$

With majority-rule decoding, $(p = 2.2 \times 10^{-2})$

$$P_e = \sum_{i=3}^{5} \binom{5}{i} p^i (1-p)^{5-i}$$ [5A]

$$= \binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p) + \binom{5}{5} p^5$$

$$= 10\, p^3 (1-p)^2 + 5\, p^4 (1-p) + p^5$$

$$= 1 \times 10^{-4}$$