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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

MSc and EEE/ISE PART IV: M.Eng. and ACGI

ADVANCED DATA COMMUNICATIONS

Tuesday, 6 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : M.K. Gurcan
Second Marker(s) : A.G. Constantinides

Special Instructions for Invigilators: None

Information for candidates:

Useful equations

Suppose $g(t)$ and $G(f)$ are Fourier transform pairs such that

$$g(t) \Leftrightarrow G(f)$$

where

$$G(f) = \int_{-\infty}^{\infty} g(t)\exp(-j2\pi f t)\,dt$$ and

$$g(t) = \int_{-\infty}^{\infty} G(f)\exp(j2\pi f t)\,df .$$

Then the following Fourier transform relationships might be useful

$$g(t) = rect\left(\frac{t}{T}\right) \quad \Leftrightarrow \quad G(f) = T\,\mathrm{sinc}(f\,T)$$

$$g(t) = \delta(t) \quad \Leftrightarrow \quad G(f) = 1$$

$$x(t) = \mathrm{sinc}\left(\frac{t}{T}\right)\frac{\cos\left(\frac{\pi\,\alpha\,t}{T}\right)}{1-\frac{4\alpha^2\,t^2}{T^2}} \quad \Leftrightarrow \quad X_{RC}(f) = \begin{cases} T, & 0 \le |f| \le \frac{1-\alpha}{2T} \\ \frac{T}{2}\left\{1+\cos\left(\frac{\pi T}{\alpha}\left(|f|-\frac{1-\alpha}{2T}\right)\right)\right\}, & \frac{1-\alpha}{2T} < |f| \le \frac{1+\alpha}{2T} \\ 0, & |f| > \frac{1+\alpha}{2T} \end{cases}.$$

1 a) Consider the signal waveform [6]

$$s(t)=\sum_{i=1}^{n} c_i\, p(t-iT_c)$$

where $p(t)$ is a rectangular pulse of unit amplitude and duration T_c. The $\{c_i\}$ may be viewed as a code vector $\mathbf{C}=[c_1,\ c_2,\ c_3,\ \cdots\ c_n]$ with $c_i=\pm 1$. Show that the filter matched to the waveform $s(t)$ may be realized as a cascade of a filter matched to $p(t)$ followed by a discrete-time filter matched to the vector $\mathbf{C}$. Determine the value of the output of the matched filter at the sampling instant $t=nT_c$.

b) For the QAM signal constellation shown in Figure 1.1,

i) determine the optimum decision boundaries for the detector, assuming that the SNR is sufficiently high so that errors only occur between adjacent points. [2]

ii) Specify a Gray code for the 16 QAM signal constellation shown in Figure 1.1. [2]

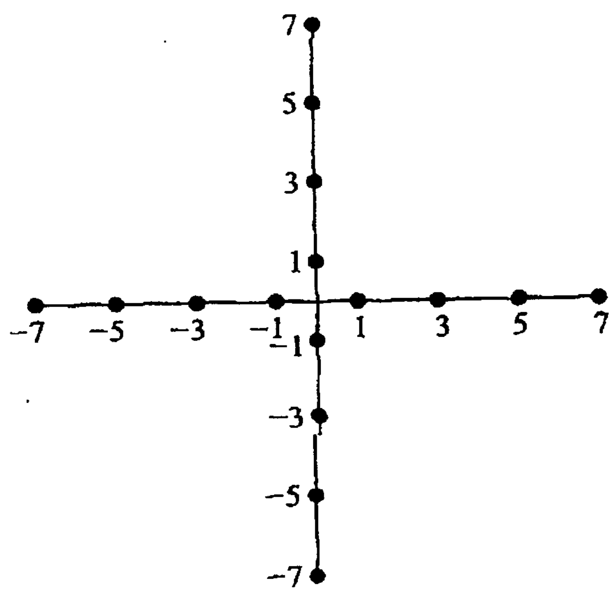

Figure 1.1

Question continued over

c) Consider 8-point QAM constellation shown in Figure 1.2

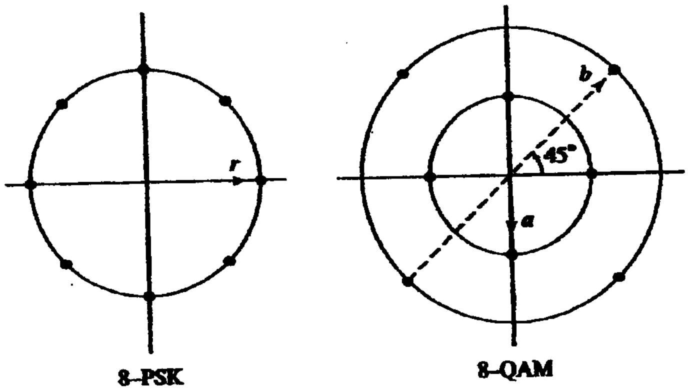

Figure 1.2

i) Assign three data bits to each point of the signal constellation such that nearest (adjacent) points differ in only one bit position. [1]

ii) Determine the symbol rate if the desired bit rate is 90 Mbps. [1]

iii) Compare the SNR required for the 8-point QAM modulation with that of an 8-point PSK modulation having the same error probability. [2]

iv) Which signal constellation, 8-point QAM or 8-point PSK is more immune to phase errors? Explain the reason for your answer. [1]

d) In an MSK signal, the initial state for the phase is either 0 or π radians. Determine the terminal phase state for the following four input pairs of input data [5]

i) 00 ,

ii) 01,

iii) 10,

iv) 11.

2. a) A band-limited signal having bandwidth W can be represented in terms of its samples $\{x_n\}$

$$x(t)=\sum_{n=-\infty}^{\infty} x_n \frac{\sin[2\pi W(t-n/2W)]}{2\pi W(t-n/2W)}.$$

i) Determine the spectrum $X(f)$ and plot $|X(f)|$ for the following case [2]

$x_{-1}=-1,\quad x_0=2,\quad x_1=-1,\quad x_n=0,\quad n\neq -1,0,1.$

ii) Plot $x(t)$ for this case. [2]

iii) If this signal is used as a pulse shaping function for binary transmission, determine the number of received levels possible at the sampling instants $t=nT=n/2W$ and the probabilities of occurrence of the received levels. Assume that the binary digits at the transmitter are equally probable. [2]

b) A 4-KHz band-pass channel is to be used for transmission of data at a rate of 9600 bits/s. If $\frac{1}{2}N_0=10^{-10}$ W/Hz is the spectral density of the additive zero-mean Gaussian noise in the channel, design a QAM modulation scheme and determine the average power that achieves a bit error probability of 10^{-6}. Use a signal pulse with a raised cosine spectrum having a roll-off factor of at least 50 percent. [4]

c) The Nyquist criterion gives the necessary, and sufficient, condition for the spectrum $X(f)$ of the pulse $x(t)$ that yields zero ISI. Prove for any pulse that is band limited to $|f|<1/T$, the zero-ISI condition is satisfied if $\mathrm{Re}[X(f)]$, for $f>0$, consists of a rectangular function, plus an arbitrary odd function, around $f=1/2T$, and $\mathrm{Im}[X(f)]$ is an arbitrary even function around $f=1/2T$. [5]

d) A non-ideal band-limited channel introduces ISI over three successive symbols. The (noise-free) response of the matched filter demodulator sampled at the sampling time kT is [5]

$$\int_{-\infty}^{\infty} s(t)\ \ s(t-kT)\,dt=\begin{cases}\varepsilon_b & k=0\\ 0.9\varepsilon_b & k=\pm1\\ 0.1\varepsilon_b & k=\pm2\\ 0 & \text{otherwise}\end{cases}$$

Determine the tap coefficients of a three-tap linear equalizer that equalizes the channel (received signal) response

$$y_k=\begin{cases}\varepsilon_b & k=0,1\\ 0 & \text{otherwise.}\end{cases}$$

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3. a) The block diagram of a binary convolution coder is shown in Figure 3.1

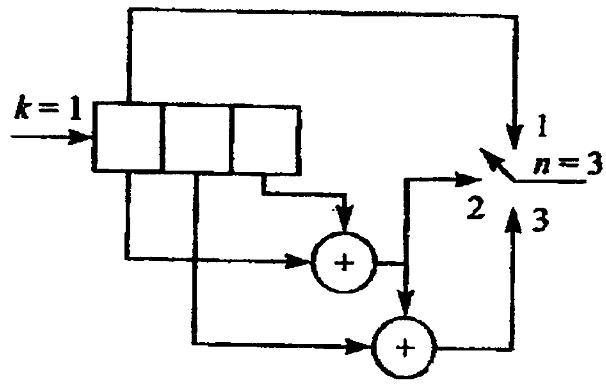

Figure 3.1

i) Draw the state diagram for the code. [3]

ii) Assume that a message has been encoded by this code and transmitted over a binary-symmetric channel. If the received sequence is $\mathbf{r} = (110,\ 110,\ 110,\ 111,\ 010,\ 101,\ 101)$, using the Viterbi algorithm, find the transmitted bit sequence. [3]

b) A trellis-coded signal is formed as shown in Figure 3.2 by encoding one bit by use of a rate ½ convolutional code, while three additional information bits are left uncoded. Perform the set-partitioning of a 32-QAM (cross) constellation which is shown in Figure 3.3 and indicate the subsets in the partition. By how much is the distance between adjacent signal points increased as a result of partitioning? [6]

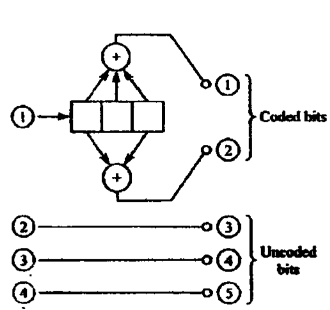

Figure 3.2

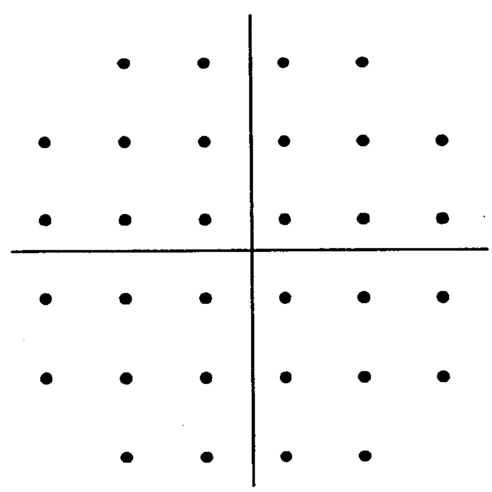

Figure 3.3

Question continued over

c) A trellis coded modulation system uses an 8 PSK signal set with constellation points $S_0 = (0.9239, 0.3827)$, $S_1 = (0.3827, 0.9239)$, $S_2 = (-0.3827, 0.9239)$, $S_3 = (-0.9239, 0.3827)$, $S_4 = (-0.9239, -0.3827)$, $S_5 = (-0.3827, -0.9239)$, $S_6 = (0.3827, -0.9239)$, $S_7 = (0.9239, -0.3827)$. The rate ½ convolution encoder shown in Figure 3.4 is used to partition the signal constellation points in accordance with Ungerberg's set partitioning rule. [8]

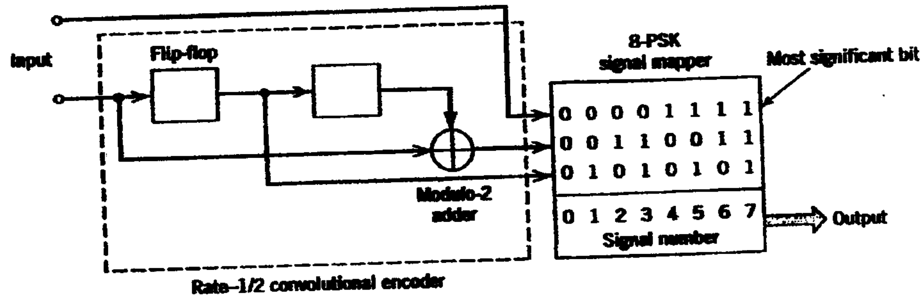

Figure 3.4

The trellis diagram for the encoder is shown in Figure 3.5

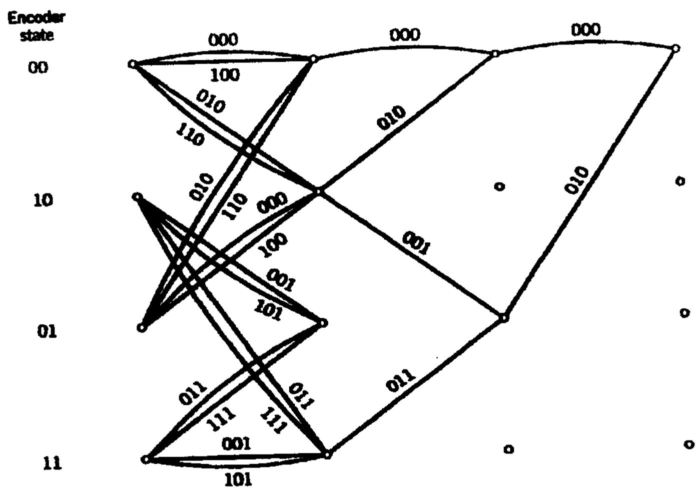

Figure 3.5

Assume that the channel is additive zero-mean white Gaussian noise, and at the output of the matched filters the sequence

$r_1 = (0.910, 0.390)$, $r_2 = (0.390, -0.940)$, $r_3 = (0.340, 0.940)$, $r_4 = (-0.390, 0.950)$, $r_5 = (-0.430, 0.900)$, $r_6 = (0.870, -0.370)$, $r_7 = (-0.370, -0.890)$, $r_8 = (0.880, -0.350)$, $r_9 = (0.930, 0.350)$, $r_{10} = (0.420, 0.910)$, $r_{11} = (-0.400, 0.970)$

is observed over ten symbol periods. What is the most likely transmitted sequence?

4 a) The input SNR as a function of frequency for an OFDM system is [5]

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$$SNR(dB) = 30\exp\left(-\frac{1.5f}{1.1\times10^{-6}}\right)$$ $\leftarrow 30\exp\left(-\frac{1.5f}{1.1\times10^{6}}\right)$

The transmission channel is divided into 256 frequency bins such that the frequency $f_1 = 25$ kHz and $f_{256} = 1124.7$ kHz are the centre frequencies for the first and last bins respectively. Each frequency bin has a bandwidth $\Delta f = 4.3125$ kHz. The symbol period is $T = 250$ μs. Calculate the total bit rate for an average symbol error rate of $P_{e,s} = 0.05$.

b) Assess the cost of the cyclic prefix (used in multi-carrier modulation to avoid ISI) in terms of

i) Extra channel bandwidth. [3]

ii) Extra signal energy. [3]

c) Describe how, for a rate 1/3 turbo encoder, the turbo decoder recursively calculates the probability density functions [9]

$\gamma_k(s',s)$, pdf for moving from state s'_{k-1} to s_k

$\beta_k(s)$, pdf that the data sequence $\tilde{y}_{j>k}$ will be received given that the trellis is in state s_k at time k and

$\alpha_{k-1}(s)$ pdf for being in state s_k and a data sequence $\tilde{y}_{j<k}$ has been received.