UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1999

MEng Honours Degrees in Computing Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the Royal College of Science
Associateship of the City and Guilds of London Institute

PAPER 4.99

MODAL AND TEMPORAL LOGIC
Thursday, April 29th 1999, 10.00 – 12.00

Answer THREE questions

For admin. only:
paper contains 4 questions

1a i) Define a Kripke frame, and a Kripke model.

ii) Explain what it means for a modal formula to be *valid* in a Kripke model.

iii) Explain what it means for a modal formula to be *valid* in a Kripke frame.

b For each frame property listed below, write down a modal formula that is valid in a Kripke frame if and only if the frame has the property.

i) reflexivity, ii) transitivity, iii) symmetry.

c Use Sahlqvist's algorithm to show that the formula

$$\Diamond p \wedge \Diamond(q \wedge \neg p) \rightarrow \Box(p \vee q)$$

is valid in a Kripke frame if and only if for every world *t* of the frame, there are *at most two* worlds accessible from *t*.

d Using p-morphisms, or otherwise, show that there is no modal formula that is valid in a Kripke frame if and only if for every world *t* of the frame, there are *at least two* worlds accessible from *t*. You may quote standard results on p-morphisms.

The four parts carry, respectively, 20%, 15%, 40%, 25% of the marks.

2a Express the following in temporal logic. Use suitable atoms, and *U, S, F, P, G, H, T, Y* as you wish.

i) While the dollar goes up, the pound will go down.

ii) The pound will not go up until Britain joins the common European currency. [Do not assume Britain will ever join.]

iii) As soon as the pound starts to go down, the French franc will go up.

b Write down the tableau rules dealing with the temporal connectives *F, G, P,* and *H,* and with transitivity of flows of time. State, without proof, a soundness–completeness theorem for temporal tableaux.

c Using a temporal tableau, show that $\vdash GF p \wedge G(p \rightarrow Hq) \rightarrow GF(p \wedge q)$.

d Define temporal connectives *W* ('weak until') and *B* ('before'), as follows. For a temporal model $M = (T,<,h)$, and a time $t \in T$, let:

$M \vDash (C\ W\ D)(t)$ iff either $M \vDash C(u)$ for all $u>t$,
or $M \vDash D(u)$ for some $u>t$ such that $M \vDash C(v)$ for all v with $t<v<u$.

$M \vDash (C\ B\ D)(t)$ iff for all $u>t$, if $M \vDash D(u)$ then $M \vDash C(v)$ for some v with $t<v<u$.

Show that each of *W, B,* and *U* (until) can express the other two — e.g., by writing a formula using only *U* that is equivalent to C *W* D, etc.

3 In Halpern and Shoham's interval temporal logic the relations underlying the six basic possibility-style operators <A>, <$\underline{A}$>, <B>, <$\underline{B}$>, <E>, <$\underline{E}$>, are illustrated in the figure below, relative to the current interval, where as is usual, [A]P ↔ ¬ <A>¬ P, etc. (The temporal scale need not be continuous).

current interval

A (Adjoins)
B (Begins with)
E (Ends with)

$\underline{A}$
$\underline{B}$
$\underline{E}$

Some derived operators:
D (During)
L (Later)
O (Overlaps)
[EndPoint]

a Give formal definitions for the <A>P, <B>P, and <E>P to be satisfied in a model **M**, at a current interval [s,t], relative to the condition that P be satisfied in **M** at some interval.

b Explain why the necessary-style modalities [A], [B] and [E] can be viewed as normal modalities of a Kripke model. Indicate why the rule of necessitation (universal generalisation), and the K axiom hold for these modalities.

c Provide definitions for the conditions <D>P, <L>P, <O>P, and [Endpoint] P in terms of the basic operators.
Hint. For [Endpoint] P, recall that a world may have no worlds accessible from it.

d Let each interval [s,t] be represented as a point in the Cartesian plane:

i) indicate which points in the plane are possible worlds.

ii) show the region relative to [s,t] which is accessible under the operator <A>

iii) define <A>P in terms of (reflexive) modal operators <n>, <s>, <e>, and <w>, representing respectively, a point due North, South, East, and West of the current point.

The four parts of this question carry, respectively, 20%, 20%, 25%, and 35% of the marks.

[Turn over

4 a i) Briefly describe and justify the global axioms which are accepted for a normal modal logic of knowledge, and indicate the distinction made between knowledge and belief.

ii) As a formalisation of an agent's belief or knowledge, a normal modal logic forces omniscience. What does this mean? How does omniscience arise with these logics?

iii) Let $K_i\, p$ express the fact that agent i knows that p holds. Define a modality K^* for the common knowledge of two agents.

iv) Explain why common knowledge is difficult to establish in general.

b i) Explain what is meant by a non-rigid designator in modal logic.
Give an every day example from computer programming.

ii) Describe an interpretation in a Kripke model which makes the formula $[\lambda x.\Diamond P(x)](a)$ true at a world w.

iii) It has been observed that equality cannot in general be substituted into modal contexts. Although we know the morning star m and the evening star e are equal (in the sense that each designates the planet Venus), the ancient Greeks did not know this.

Use predicate abstraction to explain this paradox by contrasting the two different modal formalisations of *know* $e{=}m$.

Parts a and b carry, respectively, 60% and 40% of the marks.

[End of Paper

