

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2011

EEE/ISE PART II: MEng, BEng and ACGI

## **COMMUNICATIONS 2**

Wednesday, 15 June 2:00 pm

Time allowed: 2:00 hours

**There are THREE questions on this paper.**

**Answer ALL questions.**

**Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).**

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible	First Marker(s) :	C. Ling
	Second Marker(s) :	J.A. Barria

## EXAM QUESTIONS

1. a) i) Explain the terms “noise”, “external noise”, and “internal noise”. [ 3 ]
- ii) Explain the terms “white noise”, “Gaussian noise”, and “additive white Gaussian noise”. [ 3 ]
- iii) Consider bandpass noise  $n(t) = n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$  which has the power spectral density shown in Fig. 1.1. Draw the power spectral density (PSD) of baseband noise  $n_c(t)$  or  $n_s(t)$  if the center frequency is chosen as: [ 4 ]
  - $f_c = 7 \text{ Hz}$
  - $f_c = 10 \text{ Hz}$

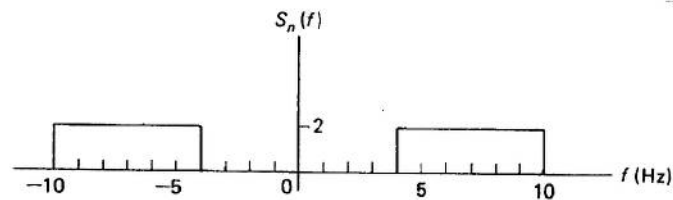


Figure 1.1 PSD of bandpass noise.

- b) i) Explain the advantages and disadvantages of synchronous detection and envelope detection respectively for standard amplitude modulation (AM). [ 5 ]
- ii) With help of a diagram, explain the operation of coherent detection for frequency shift keying (FSK). [ 5 ]
- c) i) Name two primary resources in communications. [ 2 ]
- ii) Write down the expression of the capacity of a Gaussian channel with bandwidth  $W$ . [ 2 ]
- iii) What does the channel coding theorem say about the relation between transmission rate  $R$  and channel capacity  $C$ ? [ 3 ]
- iv) Consider a telephone line channel. If the signal to noise ratio (SNR) is 20 dB and the bandwidth available is 4 kHz, calculate the corresponding channel capacity. [ 3 ]

- d) Consider an information source generating the random variable  $X$  with probability distribution

$x_k$	$x_1$	$x_2$	$x_3$	$x_4$
$P(X = x_k)$	0.4	0.2	0.25	0.15

- i) Calculate the entropy of this source. [ 3 ]
- ii) Construct a binary Huffman code. [ 5 ]
- iii) Compute the average codeword length. [ 2 ]

2. a) Figure 2.1 shows the diagram of the FM receiver. The bandpass filter has bandwidth  $B_T$ , while the baseband low-pass filter has bandwidth  $W$ . Let the FM signal be  $s(t) = A \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau]$  and assume the bandpass noise  $w(t) = n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$  has single-sided power spectral density  $N_0$ .

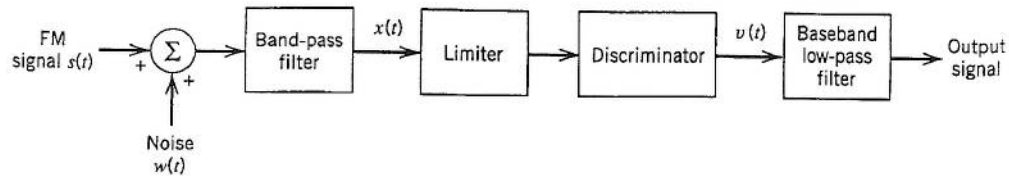


Figure 2.1 Block diagram of the FM receiver.

- i) Explain the function of each of the blocks in Figure 2.1. [ 5 ]
  - ii) Given the noise at the output of the discriminator
 
$$f_i(t) = \frac{1}{2\pi A} \frac{dn_s(t)}{dt},$$
 derive an expression for its power spectral density. [ 5 ]
  - iii) Sketch power spectral densities of  $n_s(t)$ ,  $f_i(t)$ , and the noise at the output of the lowpass filter. [ 5 ]
- b) Consider pre-emphasis and de-emphasis.
- i) Show that in order to achieve flat noise power spectral density at the output of the FM receiver, the ideal de-emphasis filter has a transfer function  $H_{de}(f) = 1/f$  within the message bandwidth. [ 5 ]
  - ii) Discuss why an FM transmitter with a corresponding pre-emphasis filter  $H_{pre}(f) = f$  is essentially phase modulation. [ 5 ]
- c) The signal-to-noise ratio (SNR) improvement factor is defined as

$$I = \frac{\text{Noise power without pre/de-emphasis}}{\text{Noise power with pre/de-emphasis}}.$$

Derive an expression of the improvement factor  $I$  for the scaled ideal de-emphasis filter  $H_{de}(f) = f_0/f$ , then compute the gain in dB for the parameter  $W = 15$  kHz and  $f_0 = 3$  kHz. [ 5 ]

3. a) A uniform quantizer for PCM has  $2^n$  levels. The input signal is

$$m(t) = [A + A \cos(\omega_1 t)] \cos(\omega_2 t)$$

where  $\omega_1 \neq \pm \omega_2$ . Assume the dynamic range of the quantizer matches that of the input signal.

- i) Work out the signal power. [ 3 ]
- ii) Write down the probability density function of the quantization noise and the quantization noise power. [ 3 ]
- iii) Work out the SNR in dB at the output of the quantizer. [ 3 ]
- iv) Determine the minimum value of  $n$  such that the output SNR is no less than 60 dB. [ 3 ]
- v) What can be done to increase the output SNR? [ 3 ]

- b) An  $(n, k)$  linear block code has the following parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad (3.1)$$

- i) What are the values of  $n$  and  $k$ ? [ 2 ]
- ii) Give a systematic generator matrix  $G$  of this code. [ 3 ]
- iii) Compute the syndrome table for a single error. [ 5 ]
- iv) The vector  $y = [1000001]$  is received. Find the syndrome and hence the most likely data bits. [ 5 ]

## ANSWERS

B — Bookwork

E — New examples

A — New applications

## EE2-4 COMMUNICATIONS 2

1. a) i) Noise refers to unwanted waves that disturb communications.

[3B]

External noise: interference from nearby channels, human-made noise, natural noise for external.

Internal noise: noise from within electronic devices, such as thermal noise.

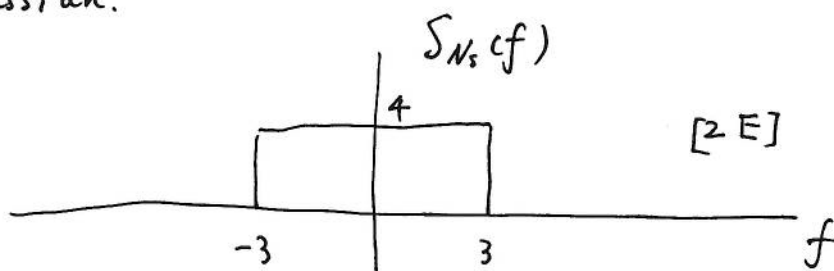
- ii). White noise: the PSD is constant

Gaussian noise: the PDF is Gaussian.

[3B]

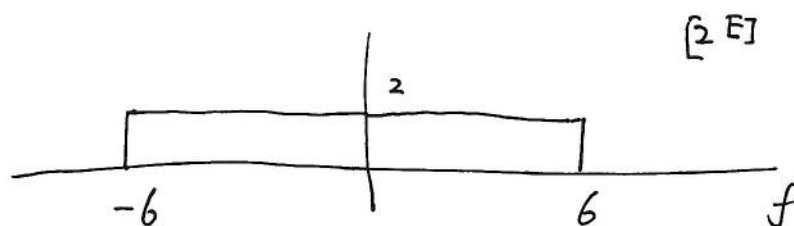
additive white Gaussian noise: noise is additive, white, and Gaussian.

- iii).  $f_c = 7 \text{ Hz}$



[2E]

$$f_c = 10 \text{ Hz}$$



[2E]

- b) i) Synchronous detection: good noise performance, complex sync circuit.

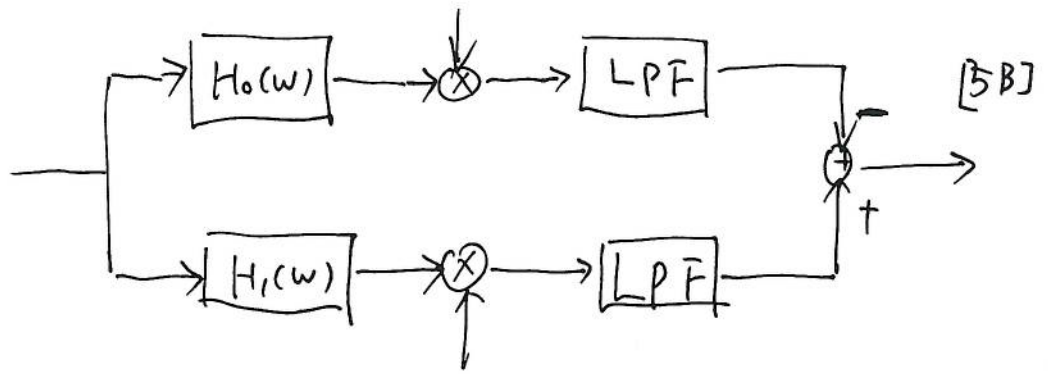
[2B]

Envelope detection: simple circuit, good performance at high SNR, but suffers from threshold effect.

[3B]



ii)



The BPFs  $H_0(w)$  and  $H_1(w)$  have central frequency  $f_0$  and  $f_1$ , respectively.

c) i) power, bandwidth [2B]

ii)  $C = W \log_2(1 + \text{SNR})$  [2B]

iii) Channel coding theorem; As long as  $R \leq C$ , we can achieve reliable communication over a noisy channel (i.e., with arbitrarily small probability of error); conversely, it is impossible to transmit messages without error if  $R > C$ . [3B]

iv)  $20 \text{ dB} \Rightarrow \text{SNR} = 100$  [3E]

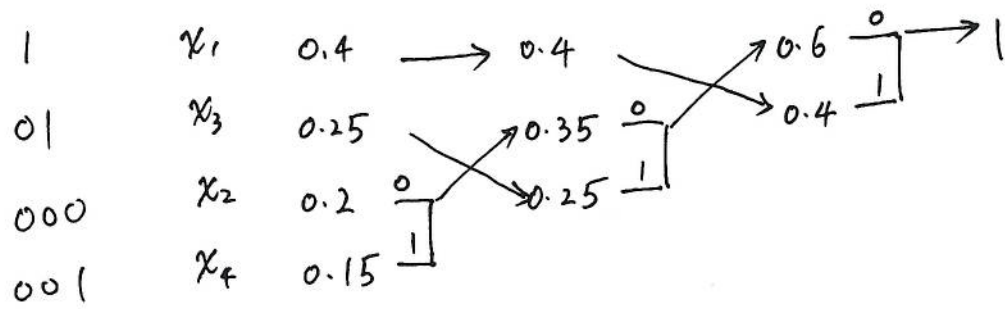
$$C = 4 \text{ K} \times \log_2(1 + 100) = 26.6 \text{ kbps}$$

d) i)  $H(X) = - \sum p(x_k) \log p(x_k)$  [3E]

$$= -0.4 \times \log 0.4 - 0.2 \times \log 0.2 - 0.25 \times \log 0.25 - 0.15 \times \log 0.15$$

$$= 1.9$$

ii)



[5 E]

iii)

$$L = 1 \times 0.4 + 2 \times 0.25 + 3 \times 0.35$$

$$= 1.95$$

[2 E]



2. a) i) The bandpass filter removes out-of-band noise. [5B]

The Limiter results in a constant envelope.

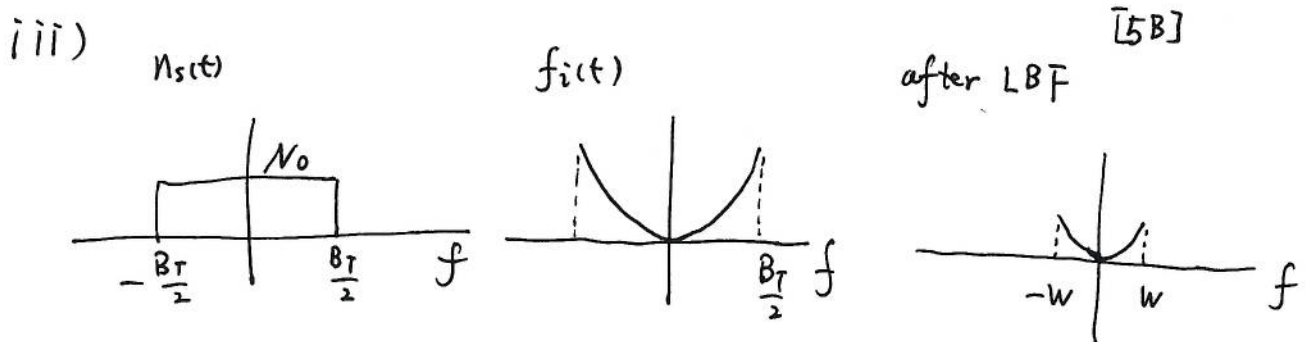
The discriminator outputs the deviation in the instantaneous frequency, i.e., it recovers the message signal.

Low-pass filter: has a bandwidth  $W$ . It passes the message and removes out-of-band noise.

ii) This can be viewed as a linear system with transfer function  $\frac{1}{2\pi A} j2\pi f = j\frac{f}{A}$ . [3B]

Therefore, PSD for  $f_i(t)$  is

$$\frac{f^2}{A^2} N_0, \quad |f| \leq \frac{B_T}{2}$$



b) i) It's clear that to equalize the PSD  $\frac{f^2}{A^2} N_0$ , we need a de-emphasis filter  $H_{de}(f) = \frac{1}{f}$ . [5E]

ii)  $H_{pre}(f) = f$  is a differentiation circuit. Thus the signal becomes  $s(t) = A \cos[2\pi f_c t + k_f m(t)]$ , which is PM. [5A]

C) Without deemphasis, the noise power is

$$P_N = \int_{-W}^W \frac{f^2}{A^2} N_0 df = \frac{2}{3} \frac{W^3 N_0}{A^2} \quad [1B]$$

With deemphasis, the noise power is

$$\begin{aligned} P_N &= \int_{-W}^W \frac{f^2}{A^2} N_0 \cdot \frac{f_0^2}{f^2} df \\ &= \int_{-W}^W \frac{f_0^2 N_0}{A^2} df \\ &= \frac{2 W f_0^2 N_0}{A^2} \end{aligned} \quad [2A]$$

Then,

$$\bar{I} = \frac{\frac{2 W^3 N_0}{3 A^2}}{\frac{2 W f_0^2 N_0}{A^2}} = \frac{W^2}{2 f_0^2} \quad [1A]$$

When  $W = 15 \text{ kHz}$ ,  $f_0 = 2.1 \text{ kHz}$

$$\bar{I} = \frac{15^2}{2 \cdot 2.1^2} = 25.5 \quad [1A]$$

In dB,

$$I(\text{dB}) = 14 \text{ dB}.$$

3. a) i)  $P_S = \frac{A^2}{2} + \frac{A^2}{4} = \frac{3}{4} A^2$  [3 E]

ii) The quantization noise has a uniform PDF

$$f(x) = \frac{1}{\Delta}, \quad |x| < \frac{\Delta}{2}$$

Power:  $P_N = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} x^2 \cdot \frac{1}{\Delta} dx$  [3 E]

$$= \frac{2}{\Delta} \cdot \frac{1}{3} \left(\frac{\Delta}{2}\right)^3$$

$$= \frac{\Delta^2}{12}$$

The signal range is  $[-A, A]$ .

$$\Delta = \frac{2A}{2^n} = \frac{A}{2^{n-1}}$$

iii)  $SNR = \frac{P_S}{P_N} = \frac{\frac{3}{4} A^2}{\frac{(A/2^{n-1})^2}{12}} = \frac{9}{4} \cdot 2^{2n}$  [3 E]

$$SNR(\text{dB}) = 6n + 3.5 \text{ dB}$$

iv)  $n = 10$  so that  $SNR \geq 60 \text{ dB}$  [3 E]

v) increase  $n$ . [3 B]

b) i)  $n=7$   $k=4$  [2 E]

ii)  $G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$  [3 E]

iii)  $S = e \cdot H^T$  [5 A]

Syndrome table

s	e
1 1 1	1 0 0 0 0 0 0
1 1 0	0 1 0 0 0 0 0
1 0 1	0 0 1 0 0 0 0
0 1 1	0 0 0 1 0 0 0
1 0 0	0 0 0 0 1 0 0
0 1 0	0 0 0 0 0 1 0
0 0 1	0 0 0 0 0 0 1

iv)  $S = y \cdot H^T$  [5 A]

$= (1 1 0)$

$\Rightarrow e = (0 1 0 0 0 0 0)$

$\Rightarrow$  the second bit is wrong

$\Rightarrow$  sent codeword  $x = [1 1 0 0 0 0 1]$

$\Rightarrow$  data bits  $= [1 1 0 0]$