IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2006

EEE PART I: MEng, BEng and ACGI

ENGINEERING MATERIALS

Friday, 2 June 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	W.T. Pike, W.T. Pike
	Second Marker(s) :	T.J. Tate, T.J. Tate

Special instructions for students

Fundamental constants

Permittivity of free space, ε_0 = 8.85 x 10^{-12} F/m
Permeability of free space, μ_0 = 4π x 10^{-7} H/m
Planck's constant, h = 6.6 x 10^{-34} Js
Boltzmann's constant, k = 1.38 x 10^{-23} J/K
Electron charge, e = 1.6 x 10^{-19} C
Electron mass, m = 9.1 x 10^{-31} kg
Speed of light, c = 3.0 x 10^{8} ms^{-1}

Schrödinger's equation

General form:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

In one dimension:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

In spherical coordinates:

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$$

Free-electron theory

Density of states (3D):

$$g(E) = \frac{1}{\pi^2\hbar^3}(m)^{3/2}\sqrt{2E}$$

Fermi energy

$$E_f = \frac{\hbar^2\pi^2}{2m}\left(\frac{3n}{\pi}\right)^{2/3}$$

Fermi distribution

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$$

Electrons in semiconductors

Effective mass:

$$m_e^* = \frac{\hbar^2}{d^2E(k)/dk^2}$$

Concentration of electrons in a semiconductor of bandgap E_g:

$$n = \frac{1}{\sqrt{2}\hbar^3}\left(\frac{m_e^* kT}{\pi}\right)^{3/2} e^{-\frac{(E_g - E_f)}{kT}}$$

$$= N_c e^{-\frac{(E_g - E_f)}{kT}}$$

Concentration of holes

$$p = \frac{1}{\sqrt{2}\hbar^3}\left(\frac{m_h^* kT}{\pi}\right)^{3/2} e^{-\frac{E_f}{kT}}$$

$$= N_v e^{-\frac{E_f}{kT}}$$

Polarization

Lorentz correction for local field:

$$\mathbf{E}_{loc} = \mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}$$

Electronic polarization:

$$P_0 = \frac{\varepsilon_0 \omega_p^2 E_0}{\omega_m^2 - \omega^2 + j\omega\gamma}$$

where

$$\gamma = \frac{r}{m},$$

$$\omega_m^2 = \omega_0^2 - \frac{\omega_p^2}{3},$$

$$\omega_0^2 = k/m,$$

$$\omega_p^2 = \frac{ne^2}{m\varepsilon_0}.$$

Orientational Polarization:

Static:

$\mathrm{P} = n\mu\, \mathrm{L}(\mu E/kT)$ where $\mathrm{L}(x) = \coth(x) - 1/x$

Dynamic:

$$P_0 = \frac{P_s}{1 + \mathrm{j}\omega\tau},$$

Magnetism

Magnet dipole due to electron angular momentum:

$$\boldsymbol{\mu}_m = -\frac{e\mathbf{L}}{2m}$$

Magnet dipole due to electron spin:

$$\boldsymbol{\mu}_m = -\frac{e\mathbf{S}}{m}$$

Paramagnetism:

$$M = n\mu_m \mathrm{L}\left(\frac{\mu_m \mu_0 H}{kT}\right)$$

The Questions

1. This question refers to an investigation of the properties of the imaginary semiconductor, Imperium:

a) Show that an electron gun with an accelerating voltage V accelerates electrons to a velocity v given by $v = \sqrt{\frac{2eV}{m}}$, where e is the charge and m the mass of an electron. [4]

b) From the result in part (a) show that a 100kV electron gun produces electrons with a wavelength of 3.9×10^{-12} m. [4]

c) What is the angle of the first diffraction peak of the 100kV electrons of (b) passing through a thin semiconductor crystal of Imperium which has an atomic spacing 0.14 nm? [4]

d) Imperium has a band gap of 1.1 eV. What is the longest wavelength of light is able to excite an electron across its band gap? [4]

e) What is the occupancy of the valence- and conduction-band electron states of the undoped Imperium at very low temperatures? How does this change as the semiconductor is heated up? [4]

f) What is the difference between the atoms of the two types of dopants than can make Imperium n- or p-type? How do dopants change the occupancy of states in the semiconductor? [4]

g) The balance of forces that determines the atomic spacing of Imperium can be illustrated by plotting the interatomic forces involved against the atomic separation (see fig. 1). What do the labels (1) to (8) represent? [8]

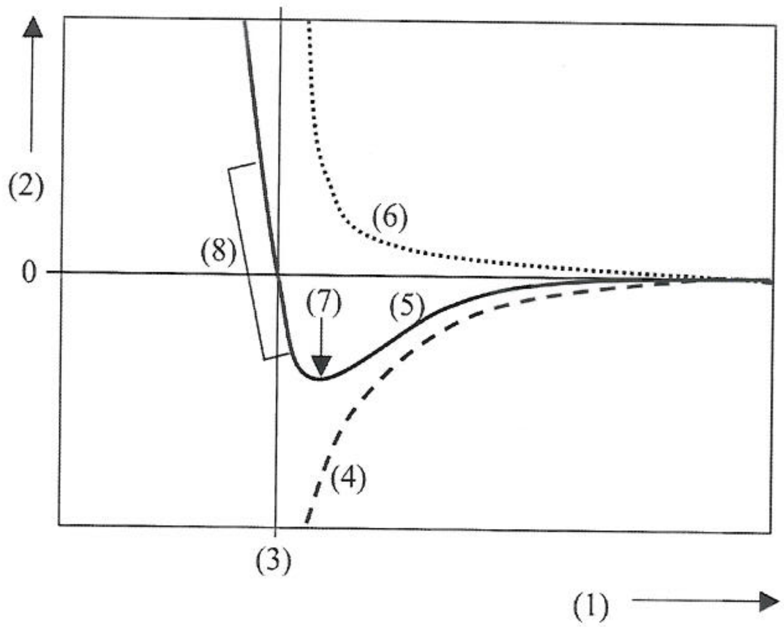


Figure 1

h) Imperium shows both electronic and molecular polarisation in response to an electromagnetic wave. Describe these two types of polarisation and the relative frequencies at which they would be expected to contribute to Imperium's refractive index. [4]

i) Is Imperium likely to be diamagnetic, paramagnetic or ferromagnetic? Explain your reasoning. [4]

2.

a) The one-dimensional Schroedinger equation for a particle is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x).$$

Identify the meaning of all the variables and constants in the equation, and associate each of the three terms in the equation with the total, potential, and kinetic energy of the particle. [8]

b) Show that the wavefunction

$$\psi(x)=\psi_0\sin kx,\quad 0\le x\le L,$$
$$\psi(x)=0,\quad \text{otherwise.}$$

is a solution for the one-dimensional Schroedinger equation for a particle of mass m in an infinitely deep quantum well, whose potential is given by

$$V(x)=0,\quad 0\le x\le L,$$
$$V(x)=\infty,\quad \text{otherwise.}$$

with the the wavenumbers of the particle having the values of $k=n\pi/L,\quad n=1,2,3...$ and the energy of the particle take values

$$E=\frac{n^2\hbar^2\pi^2}{2mL^2},\quad n=1,2,3...$$ [12]

c) For an electron in an infinitely deep quantum well of width 1 nm, calculate the two longest wavelengths of light that can excite a transition of the electron between the quantum states of the well. [10]

3.

a) Explain the difference between a semiconductor and a metal on the basis of the energy distribution of electron states. [6]

b) Fig. 3 shows a plot of the energy states of a model semiconductor at a temperature of $T = 0\text{K}$ (absolute zero). Occupancy of a state by an electron is represented by a filled circle. Electron spin is neglected. The states have a spacing of ΔE and the energy gap is $2\Delta E$. Show all the possible four arrangements of the electrons in the available states if the temperature of the semiconductor is raised to a temperature T, where $kT = 5\Delta E$. Hence determine and plot the average occupancy of each state at this temperature and identify the energy of the Fermi level for this semiconductor. [16]

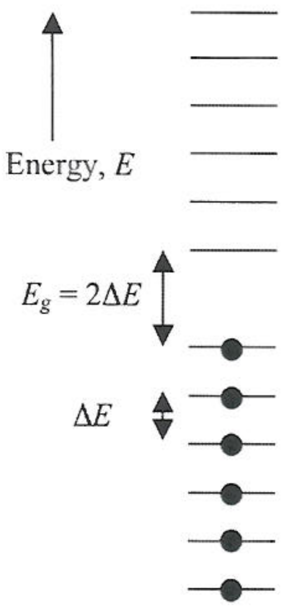


Figure 3

c) In a real semiconductor only electron states near to the band gap (either just below the top of the valence band or just above the bottom of the conduction band) change their occupancy with temperature or doping. Explain why this is so. [8]

4.

a) Fig. 4 shows schematically the construction of a magnetic read/write head. The head gap has a size g and the velocity of the magnetic medium under the head is v. Reproduce and label the components of fig. 4 and explain how it can operate both to write and read data to and from the magnetic medium. [14]

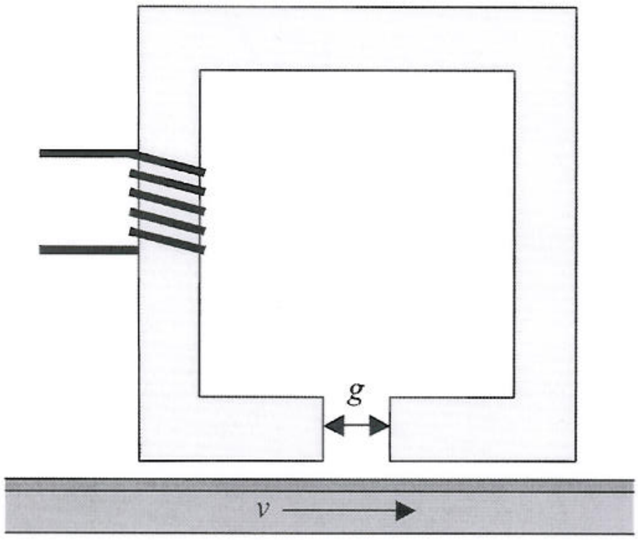


Figure 4

b) Show that when such a head is used to write an audio signal to a magnetic tape, the maximum audio frequency that can be recorded will be limited to $f_{max} = v/2g$. Typically, audio magnetic tape uses a velocity of 20 cm/s and a head gap of 10 μm. What is the maximum audio frequency that can be recorded for these values? [8]

c) How have other approaches to magnetic recording increased the data write rate compared to the audio magnetic tape discussed in (b)? [8]

EI.5 ENGINEERING MATERIALS 2006 Master Copy. June 06

The Answers

1.

a) Initial potential energy of electron is given by eV. Final kinetic energy of electron is given by $1/2\ mv^2$. Equating the two gives: $v = \sqrt{\dfrac{2eV}{m}}$. [4]

b) From $p = mv = h/\lambda$,

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= \frac{h}{\sqrt{2meV}} \\ &= \frac{6.6\times10^{-34}}{\sqrt{2\times1.6\times10^{-19}\times9.1\times10^{-31}\times10^{5}}}\text{m} \\ &= 3.9\times10^{-12}\text{m}\end{aligned}$$

[4]

c) The crystal acts like a 2-D diffraction grating. Diffraction peak when $m\lambda = d\sin\theta$. Diffraction angle is given by:

$$\begin{aligned}\theta \approx \sin\theta &= \lambda/d \\ &= \frac{3.9\times10^{-12}}{0.14\times10^{-9}} \\ &= 0.028 \text{ radians} \\ &= 28 \text{ mrad} \\ &= 1.6^{0}\end{aligned}$$

d)

$$\begin{aligned}E &= hf = hc/\lambda; \\ \lambda &= hc/E \\ &= \frac{6.6\times10^{-34}\times3\times10^{8}}{1.1\times1.6\times10^{-19}}\text{m} \\ &= 1.1\times10^{-6}\text{m} \\ &= 1.1\mu\text{m}\end{aligned}$$

[4]

e) At low temperatures the valence states are completely occupied and the conduction states are completely empty. As the semiconductor is heated up some electrons are thermally excited from the valence band to the conduction band, leaving states empty in the valence band and occupied in the conduction band. [4]

f) n-type dopants have one extra electron compared to the semiconductor. This electron is weakly bound when the dopant substitutes for a semiconductor atom and can be easily thermally excited to produce an electron in the conduction band. P-type dopants have one less electron. When they substitute for a semiconductor atom, valence electrons can be easily excited into the dopant state leaving an unoccupied state (hole) in the valence band.

g) 1) interatomic spacing

2) interatomic force

3) equilibrium spacing,

4) attractive force

5) total force

6) repulsive force

7) point of instability

8) linear elastic region – Hooke's law applies. [8]

h) Electronic polarisation involves the motion of the electrons about the nucleus induced by the electric field of the EM wave. Molecular polarisation is the motion of the atoms in response to the wave. As the later involves much heavier particles (nuclei compared to electrons) it occurs at much lower frequencies, and increasing the refractive index below these frequencies. [4]

i) Imperium has a full outer shell in the solid state and no d-shell electrons in its crystalline form. This is likely to make it a diamgnet.

2. (a) S.E.:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

h-bar: Planck's constant divided by 2 Pi

m: the mass of the particle

ψ: the wavefunction of the particle

x: the spatial dimension

$V(x)$: the potential energy, varying as a function of x

E: the total energy of the particle

The first term is associated with the kinetic energy, the second term with the potential energy and the third term is the total energy. [8]

b) First solve S.E. for the two regions, and then apply the boundary conditions:

The electron is trapped in the quantum well, and therefore $\psi(x)$ must be zero outside the well; alternative explanation - to avoid $V(x)$ $\psi(x)$ blowing up in S.E., $\psi(x)$ must be zero when $V(x)$ is infinite.

Inside the quantum well, $V(x)$ is zero, so S.E. becomes:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}=E\psi(x)$$

Substituting for $\psi=\psi_0\sin k_n x$ gives

$$\frac{\hbar^2k^2}{2m}\psi_o\sin kx=E\psi_o\sin kx$$

$$\Rightarrow E=\frac{\hbar^2k^2}{2m}$$

The boundary conditions automatically match at $x = 0$ through the choice of a sine for the wavefunction. At $x = L$, we have

$$\sin kL=0$$

$$\Rightarrow kL=n\pi$$

for integer n. $n = 0$ is not a solution as this would imply ψ is zero everywhere, and hence there is no electron in the state. Hence we obtain:

$$k=\frac{n\pi}{L},\quad n=1,2,3...$$

and $$E=\frac{n^2\hbar^2\pi^2}{2mL^2},\quad n=1,2,3...$$

c) The energy differences between two states, n_1 and n_2 will be:

$$\Delta E=\frac{\hbar^2\pi^2}{2mL^2}(n_1^2-n_2^2)$$

Hence the two lowest energy transitions will be

$$n = 1 \text{to} 2 : \Delta E = 3\frac{\hbar^2\pi^2}{2mL^2}$$

$$n = 2 \text{to} 3 : \Delta E = 5\frac{\hbar^2\pi^2}{2mL^2}$$

And the corresponding wavelengths will be

$$hf = \frac{hc}{\lambda} = \Delta E,$$

$$\lambda = \left(\frac{1}{3},\frac{1}{5}\right) \times \frac{4mL^2c}{h}$$

$$= \left(\frac{1}{3},\frac{1}{5}\right) \times \frac{8 \times 9.1 \times 10^{-31} \times \left(10^{-9}\right)^2 \times 3 \times 10^8}{6.62 \times 10^{-34}} \text{m}$$

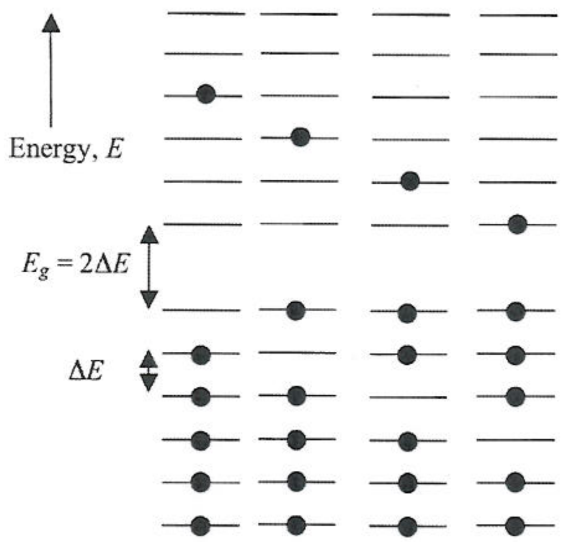
$$= \left(\frac{1}{3},\frac{1}{5}\right) \times 3.3\mu\text{m}$$

$$= 1.1 \text{ and } 0.66\,\mu\text{m}$$

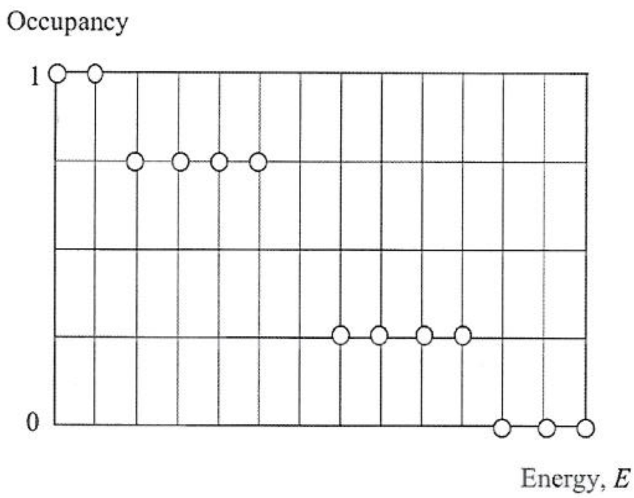
3.

a) A couple of band diagrams would probably be the best way to illustrate this. The major difference is that the in a metal there are available empty states just above the occupied states, while in a semiconductor, there are none as there is an energy gap between the occupied valence-band states and the unoccupied conduction band states. [6]

b) The states can be occupied as:



The occupancy plot will be:



The Fermi level, where the occupancy is 0.5, would be found in the middle of the band gap. [16]

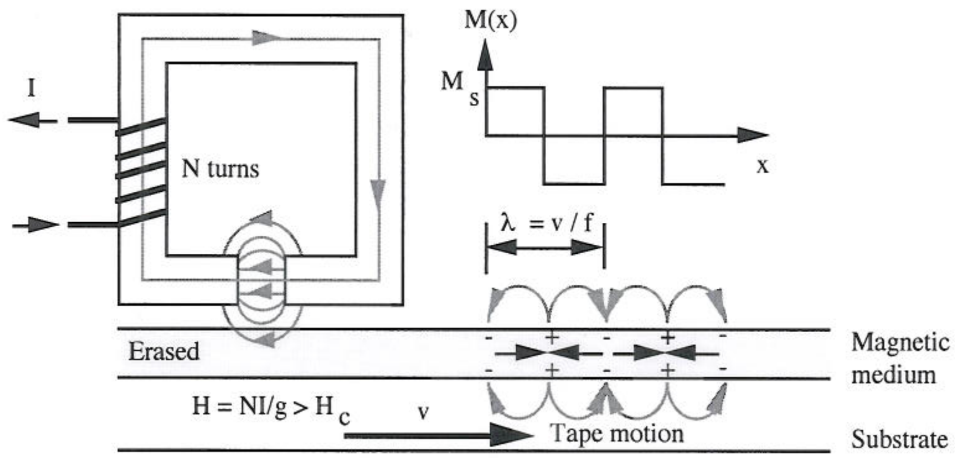
c) Thermal excitation only affects theses levels near the conduction band edge as the tail of the Fermi distribution, indicating occupancy in the conduction band, is falling off very fast with temperature, at room temperature and for typical band gaps. In fact a full analysis shows the peak in occupancy is just 1/2kT above the conduction band edge, or 1/80 eV. The argument for hole in the valence band is very similar, now with electrons only close to the band gap being thermally excited out of the valence band, leaving a hole behind.

Doping introduces states which are just inside the band gap. Again thermal excitation is sufficient to just excite donor electrons into the conduction band, or valence electrons into the acceptor levels. [8]

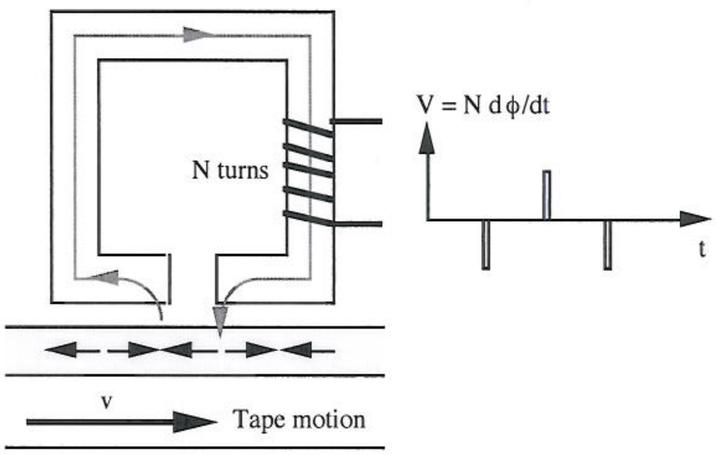
4.

a)

The figure below shows the operation of a magnetic recording head. A plastic tape coated with small magnetic particles is moved at constant speed past a magnetic core, which has a short air gap between two pole pieces. The current I flowing through the N-turn coil creates a magnetic field in the core, which in turn creates a field in the gap. The tape is magnetised by the fringing field near the gap. Assuming that this field is greater than the coercive field of the tape, the tape is magnetized to saturation with a dipole direction that depends on the sign of the write current.



Magnetic tape is read out by a head of similar construction. The magnetic flux of the pattern stored on the tape links into the magnetic circuit of the core. Time variations of the linked flux ϕ then result in an induced EMF of $Nd\phi/dt$ in the read coil. The read voltage is then proportional to the *derivative* of the write current.



[14]

b) To produce one cycle of a vibration requires magnetistation of portions of the tape both along and against the velocity vector. Hence the minimum tape length for one audio wavelength is $2g$, as g is the minimum length of magnetisation. If the tape is moving at a velocity v, the time taken to write one cycle will be $2g/v$, and the corresponding audio frequency will be $v/2g$. For the figures given this is 10 kHz. [8]

c) Increasing the magnetic media velocity is possible in a rigid hard drive compared to a flexible tape player.

Decreasing the gap will also speed up the write rate. However, this must be accompanied with a reduction of the head-medium distance to ensure sufficient stray field is present to write above the coercive field of the media. It will also be necessary to ensure that the domain size of the magnetic material is smaller than the head gap. Finally, alignment will become critical if close tolerances are to be maintained.

[8]