IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

EEE PART I: MEng, BEng and ACGI

Corrected Copy

DEVICES AND FIELDS

Tuesday, 1 June 10:00 am

10.19am Q4 d) ε_r should be μ_r

Time allowed: 2:00 hours

There are SIX questions on this paper.

Question ONE and Question FOUR are compulsory. Answer Question One and Question Four, plus one additional question from Section A and one additional question from Section B.

Questions One and Four each carry 20% of the marks; remaining questions each carry 30% of the marks.

Use a separate answer book for each section.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	K. Fobelets, E.M. Yeatman
	Second Marker(s) :	E.M. Yeatman, B.C. Pal

Special instructions for invigilators

This exam consists of ***2 sections****. Section A:* ***Devices*** *and section B:* ***Fields****. Each section has to be solved in its respective answer book. Check that 2 different answer books are available for the students and remind them of this instruction, and of the rubric (see front cover).*

Special instructions for students

Use different answers books for each section:

Devices*: answer book* ***A***

Fields*: answer book* ***B***

Constants and Formulae

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10}$ cm^{-3} at $T = 300$K
dielectric constant of Si:	$\varepsilon_{Si} = 11$
dielectric constant of SiO_2:	$\varepsilon_{ox} = 4$
thermal voltage:	$kT/e = 0.026$V at $T = 300$K
charge of an electron:	$e = 1.6 \times 10^{-19}$ C

$$\left.\begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n\frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p\frac{dp(x)}{dx} \end{aligned}\right\}$$

Drift and diffusion current densities in a semiconductor

$$I_{DS} = \frac{\mu C_{ox}W}{L}\left((V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Current in a MOSFET

$$\left.\begin{aligned} J_n &= \frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}} - 1\right) \\ J_p &= \frac{eD_p p_{n_0}}{L_p}\left(e^{\frac{eV}{kT}} - 1\right) \end{aligned}\right\}$$

Diffusion current densities in a pn-junction

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT}\right) \text{with} \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$

Minority carrier injection under bias V

$$D = \frac{kT}{e}\mu$$

Einstein relation

$$w_n = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}\frac{N_A}{N_A N_D + N_D^2}(V_0 - V)}$$

$$w_p = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}\frac{N_D}{N_A N_D + N_A^2}(V_0 - V)}$$

Depletion width in the n and p-type region

SECTION A: SEMICONDUCTOR DEVICES

1. This question is **obligatory**.

a) Give the name and the sign of the charge of the two types of mobile carriers that can exist in a semiconductor. [4]

b) Sketch the energy band diagram (E_c, E_v, E_F, E_G) of both n-type and p-type Si (clearly indicate which band diagram belongs to which semiconductor). [4]

c) Currents occurring in semiconductors can be a consequence of two types of physical processes, define both and give a brief description for each one. [4]

d) Give the reason for the small current that flows in a reverse biased pn-diode. [4]

e) Sketch the basic material cross section of an n-channel enhancement mode MOSFET. Define all layers and contacts. [4]

2. A n^+p diode has the following parameters:

Donor doping density: $N_D = 10^{19}$ cm^{-3}.
Acceptor doping density: $N_A = 10^{15}$ cm^{-3}.
Diffusion length of the minority carriers: $L_p = 10^{-2}$ cm (holes).
$L_n = 10^{-3}$ cm (electrons).
Diffusion constant of the minority carriers: $D_p = 10$ cm^2/s (holes).
$D_n = 20$ cm^2/s (electrons).
Area of the diode: $A = 10^{-2}$ cm^2.
The physical length of the n and p type region is exactly equal to the minority carrier diffusion length.
The temperature is: $T = 300$K.

a) Draw the energy band diagram (including E_c, E_v, E_F and E_G) for this diode:

i) when no bias is applied. [6]

ii) under forward bias. Indicate the positive and negative terminals and the direction of both the net hole and net electron flux (h-flux, n-flux) on the diagram. [6]

Ensure that the relative distances between the energies are correct for both cases and between cases. Define all layers.

b) Give and prove the relative magnitude of the electron current density, J_n, and the hole current density, J_p, when the diode is *forward* biased. [4]

c) Give and prove the relative magnitude of the depletion width extending into the n-type region, w_n, and the p-type region, w_p, when the diode is *reverse* biased. [4]

d) Which carrier type, electrons or holes, contributes most to the reverse bias current of the diode? Prove your answer. [4]

e) Re-design the diode such that for the same bias the current *density* is increased by a factor of 10. You can only change one parameter and you are not allowed to change the doping densities. Ensure your answer is realistic. Define all parameters in your final answer. [6]

3. A certain p-type enhancement mode MOSFET (pMOS) has the following parameters:

Donor doping density: $N_D = 10^{15}$ cm^{-3}.
Acceptor doping density: $N_A = 10^{19}$ cm^{-3}.
Diffusion constant of the majority carriers: $D_p = 10$ cm^2/s (holes) &
$D_n = 20$ cm^2/s (electrons).
Gate length and width are respectively: $L_g = 2$ μm and W = 10 μm.
Oxide thickness: $t_{ox} = 100$ nm.
The temperature is: $T = 300$K

a) Draw the energy band diagram (including E_c, E_v, E_F and E_G) for this MOSFET from gate to substrate (gate and semiconductor have the same work function).

i) when no bias voltages are applied. [6]

ii) when the drain-source voltage V_{DS} = 0V and the gate-source voltage $V_{GS} = V_{th_p}$. V_{th_p} is the threshold voltage of the pMOS. The source is connected to ground. [6]

Ensure that the relative distances between the energies are correct for both cases and between cases. Define all layers.

b) Referring to the MOSFET described above:

i) What is the sign of V_{th_p} ? Explain your answer briefly. [4]

ii) What is the relative magnitude of V_{GS} and V_{th_p} for the MOSFET in inversion? Explain your answer briefly. [4]

ii) What is the sign of the drain voltage at saturation for this pMOS? Prove your answer. [4]

c) Fig. 3.1 gives a CMOS circuit based on the pMOS given above and an nMOS. The threshold voltage of the nMOS is V_{th_n} with $|V_{th_p}| = |V_{th_n}|$. Find the values for the gate geometry L_g and W of the nMOS such that the ON current through it has the same magnitude as the ON current through the pMOS. [6]

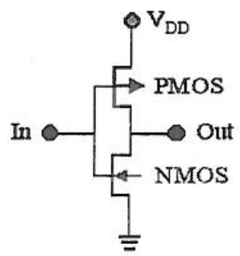


Figure 3.1: CMOS circuit with pMOS and nMOS (n-channel enhancement mode MOSFET) connected in series and the gates at the same bias.

SECTION B: FIELDS

4. This question is **obligatory**. Answer all parts. Each part is worth 4 marks.

a) Two point charges, of charge +0.01 nC and –0.01 nC, lie on the x axis at $x = -1$ cm and +1 cm respectively. Find the magnitude and the direction of the electric field E at the origin ($x = 0$).

b) The electric field in a certain region is uniform and points in the $+x$ direction. It is found that 6 mJ of work is required to move a charge of 3 μC from $x = +10$ cm to $x = 0$. Find the magnitude of the electric field.

c) A region of free space has equal energy densities for the electric and magnetic fields in that region. Find a value for the ratio E/H. What is this value commonly called?

μ_r

d) At some position in a linear ferromagnetic material, having a relative permeability $\varepsilon_r = 1200$, the magnetic field strength H = 700A/m. Calculate magnetic flux density B at the same position. If H is in the +x direction, what will be the direction of B?

e) A DC motor in steady state is producing 0.1 Nm of torque at a speed of 1200 rad/s, with an armature current of 2.0 A. Calculate the back EMF in this condition.

5. A coaxial cable cross-section is shown in Figure 5.1(a) below. The radii of the inner and outer conductors are $a = 4\ mm$ and $b = 10\ mm$ respectively, and the relative permittivity of the dielectric between them $\varepsilon_r = 4$.

a) Using Gauss' Law, derive an expression for the electric field in the dielectric, $E(r)$, where r is the radial distance from the cable axis, for a certain charge per unit length $+\rho$ on the inner conductor. [8]

b) Using the expression derived in (a), derive an expression for the capacitance per unit length of the cable, and calculate the value of this for the parameter values given above. [8]

c) Find the voltage that must be applied between the conductors to reach a maximum field strength within the dielectric of 5 MV/m. [6]

d) The uniform dielectric material is now replaced by a double layer of dielectric as shown in Figure 5.1(b), with an inner layer of relative permittivity ε_{r1} and outer radius c, and an outer layer of relative permittivity ε_{r2} and outer radius b. If $\varepsilon_{r1} = 4$, $c = 6\ mm$, and a and b are as given above, find the value of ε_{r2} such that the maximum field strength within the outer dielectric is the same as that within the inner dielectric. [8]

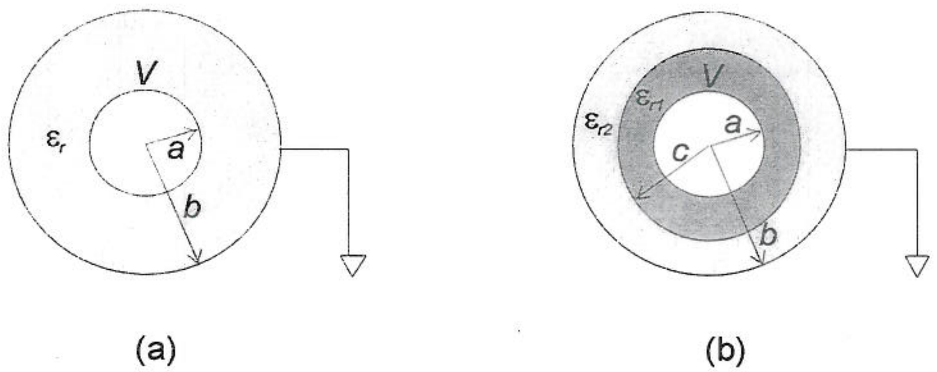


Figure 5.1

6. A long straight wire is carrying a current I, as shown in Figure 6.1. Near it is placed a rectangular, open circuit coil of N turns and dimensions $a \times b$, at a distance c from the straight wire as shown. Both the wire and the coil lie in the x-y plane and are in air.

a) Derive an expression for the magnetic flux density B at a point in the centre of the coil. If the direction of the current is as indicated in Figure 6.1, what will be the direction of B at this point? [6]

b) Derive an expression for the total magnetic flux Φ passing through the coil, as a function of the current I and the dimensions a, b and c. Do not use the approximation $c >> a$. [6]

c) If the current I is sinusoidal according to $I(t) = I_o sin(\omega t)$, derive an expression for the voltage induced in the coil. Hence, calculate the rms coil voltage if $a = 4\ cm$, $b = 6\ cm$ and $c = 2\ cm$, $N = 20$, $I_o = 100\ mA$ and $\omega = 100\pi\ rad/s$. [6]

d) Calculate the mutual inductance between the coil and the straight wire, for the parameter values given in part (c). [6]

e) If a resistor is connected between the terminals of the coil, so that a current can flow in the coil, and the current in the wire is as in (c), will the flux passing through the coil be increased, decreased or unchanged compared to the open circuit case? Give a brief explanation of your answer. [6]

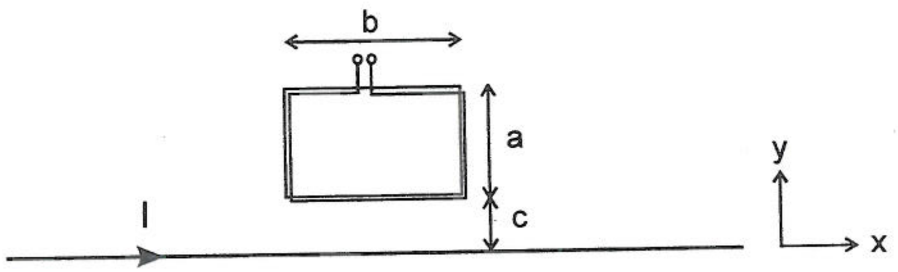


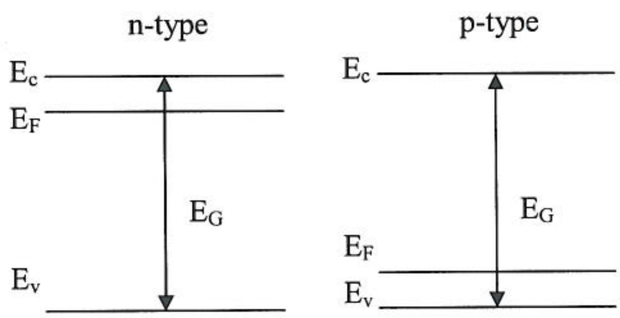
Figure 6.1

SECTION A: SEMICONDUCTOR DEVICES - **ANSWERS** 2010

1. This question is **obligatory**.

a) electrons, negative charge q=-e, e>0
holes, positive charge q=e. [4]

b)

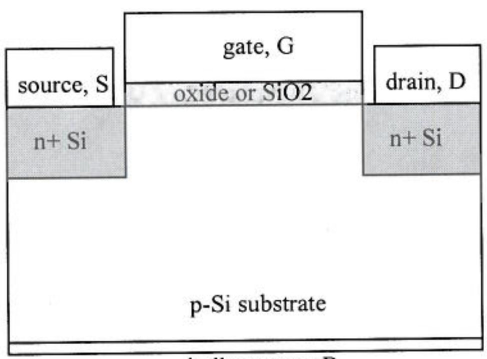


Important:
1. E_F is closer to E_c for n-type and E_F is closer to E_v for p-type
2. E_G is the same for both [4]

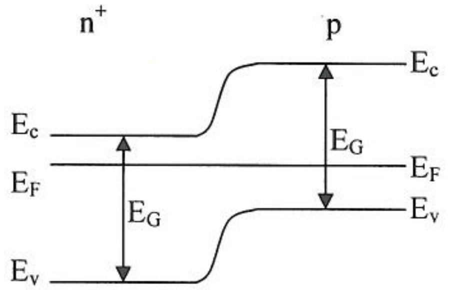
c) Drift current is a consequence of an electric field (both internal as well as externally applied)
Diffusion current is a consequence of a gradient in carrier (electron or hole) concentration. [4]

d) In reverse bias, the carrier type injected across the junction is the minority carrier, thus holes from n to p and electrons from p to n. Since the density of minority carriers available is small, the current is limited by this density and is thus small. [4]

e) [4]



2. a) i)

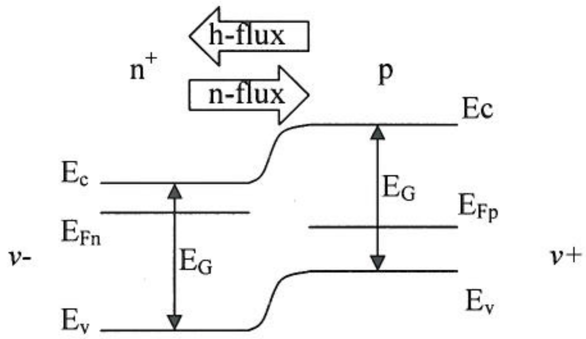


[6]

Important rules:

1. E_F needs to be constant.
2. for n-type E_c-E_F < E_F-E_v ; for p-type E_c-E_F > E_F-E_v
3. $(E_c\text{-}E_F)_n < (E_F\text{-}E_v)_p$ (the distance between Fermi level and conduction band in the n-layer needs to be smaller than the distance between Fermi level and valence band in the p-type layer because n is more heavily doped than p.
4. the bandgap in n and p type layers are the same.

ii)



[6]

Important rules:

5. E_F not constant. E_{Fn} should be higher than E_{Fp}.
6. rule 2 should still be applied (no change with previous diagram)
7. rule 3: $(E_c\text{-}E_F)_n < (E_F\text{-}E_v)_p$ should still be applied (no change with previous diagram)
8. the bandgap in n and p type layers are the same and the same as in previous diagram.
9. the potential barrier between the n and p layers should be smaller than in the previous diagram. The difference should be approximately the difference in E_{Fn} and E_{Fp}.

b) From the formulae list copy. [4]

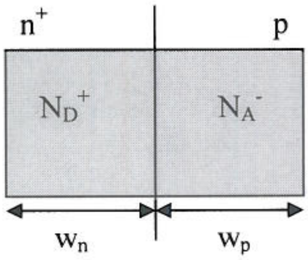
The electron current density of the pn diode: $J_n = \frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)$ and hole current density of the pn diode: $J_p = \frac{eD_p p_{n_0}}{L_p}\left(e^{\frac{eV}{kT}} - 1\right)$

Rewrite the expression as a function of doping density:

$$J_n = \frac{eD_n n_i^2}{L_n N_A}\left(e^{\frac{eV}{kT}} - 1\right) \& J_p = \frac{eD_p n_i^2}{L_p N_D}\left(e^{\frac{eV}{kT}} - 1\right)$$

Since $\frac{D_n}{L_n} > \frac{D_p}{L_p}$ and $N_A << N_D$, $J_n > J_p$. Thus electron current density is larger than hole current density.

c) The influence of the voltage is not important because it is the same for w_n and w_p. Thus the relationship is still the same as for zero bias. Then we can apply charge neutrality across the depletion region (see figure) $|Q_n|=|Q_p|$: negative charge in the p-side depletion region is equal to the positive charge in the n-side depletion region or: e N_A w_p A = e N_D w_n A, A cross sectional area. Thus when N_D>N_A $\mathbf{w_p > w_n}$ for the equality to be true. [4]



d) The reverse bias current can be extracted from the formulae in the list by removing the exponential term (see calculations in b):

$-I_{off} = \frac{eD_n n_i^2}{L_n N_A} + \frac{eD_p n_i^2}{L_p N_D} \approx \frac{eD_n n_i^2}{L_n N_A}$ since $\frac{D_n}{L_n} > \frac{D_p}{L_p}$ and $N_A << N_D$ thus the reverse bias current is determined by electrons coming from the p-type region.

The other way to explain this is by stating that the off current is governed by minority carrier injection across the junction thus e- from p to n and h+ from n to p. There are more e^- in the p-type region than h^+ in the n-type: $n_{p_0} > p_{n_0}$ because $\frac{n_i^2}{N_A} > \frac{n_i^2}{N_D}$ since $N_A << N_D$. [4]

e) If doping density and voltage cannot be changed then the only parameter remaining is the length of the n or p region.

$$J_{new} = I_{0_{new}}\left(e^{\frac{eV}{kT}} - 1\right) = 10 \times J_{old} = 10 \times \left(\frac{eD_n n_i^2}{L_n N_A} + \frac{eD_p n_i^2}{L_p N_D}\right)\left(e^{\frac{eV}{kT}} - 1\right)$$

$$I_{0_{new}} = 10 \times \left(\frac{eD_n n_i^2}{L_n N_A} + \frac{eD_p n_i^2}{L_p N_D}\right)$$

$$\left(\frac{eD_n n_i^2}{X_p N_A} + \frac{eD_p n_i^2}{X_n N_D}\right) = 10 \times \left(\frac{eD_n n_i^2}{L_n N_A} + \frac{eD_p n_i^2}{L_p N_D}\right)$$

Definition of parameters: X_p is the length of the p-type layer, X_n is the length of the n-type layer. Remember holes diffuse in the n-type layer and electrons diffuse in the p-type layer.

If we keep the length of the n-layer unchanged thus X_n remains L_p (X_n=L_p) and change only the length of the p-layer X_p then:

$$\left(\frac{eD_n n_i^2}{X_p N_A}+\frac{eD_p n_i^2}{L_p N_D}\right)=10\times\left(\frac{eD_n n_i^2}{L_n N_A}+\frac{eD_p n_i^2}{L_p N_D}\right)$$

$$\left(\frac{eD_n n_i^2}{X_p N_A}\right)=10\times\left(\frac{eD_n n_i^2}{L_n N_A}\right)$$

$$X_p=\frac{L_n}{10}=10^{-4}\,cm=1\mu m$$

Thus the ten times increase in current can be obtained by a ten times decrease in the length of the p-layer or more accurately by making the length of the p-layer 10 time smaller than the electron diffusion length. This is a realistic solution.

If we would have chosen to keep the length of the p-layer unchanged thus X_p remains L_n ($X_p=L_n$) and change only the length of the n-layer X_n then:

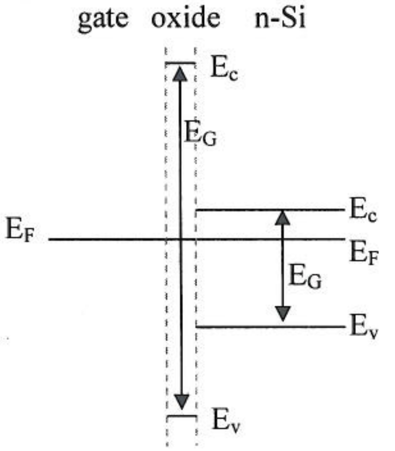
$$\left(\frac{eD_n n_i^2}{L_n N_A}+\frac{eD_p n_i^2}{X_n N_D}\right)=10\times\left(\frac{eD_n n_i^2}{L_n N_A}+\frac{eD_p n_i^2}{L_p N_D}\right)$$

$$\left(\frac{eD_p n_i^2}{X_n N_D}\right)=10\times\left(\frac{eD_n n_i^2}{L_n N_A}+\frac{eD_p n_i^2}{L_p N_D}\right)\approx 10\times\left(\frac{eD_n n_i^2}{L_n N_A}\right)\quad [N_D>>N_A]$$

$$X_n=\frac{D_p N_A L_n}{10\times D_n N_D}=\frac{10\times 10^{15}L_n}{10\times 20\times 10^{19}}=5\times10^{-6}L_n=5\times10^{-9}\,cm=5\times10^{-11}\,m$$

This would mean that the width of the n-type layer should have to become smaller than an atomic distance, thus this is not a realistic solution. [6]

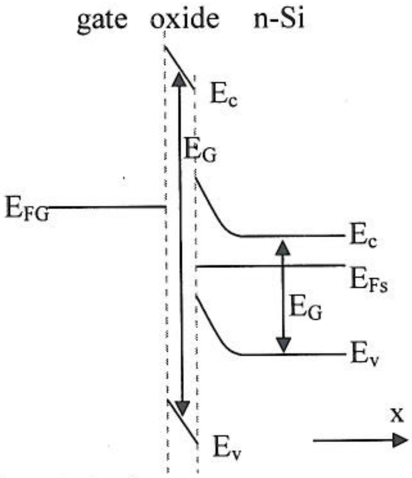
3. a) i) [6]



Important rules:

1. E_F needs to be constant.
2. Si needs to be n-type of an enhancement mode pMOS thus E_c-E_F < E_F-E_v
3. the bandgap of Si needs to be smaller than that of oxide
4. the bands are flat because there is no workfunction difference and no voltage applied.

ii)



Important rules:

5. E_F is not constant.
6. $E_{FG} > E_{Fs}$ the Fermi level in the gate should be higher than the Fermi level in the semiconductor because a negative voltage is applied to the gate for creating inversion.
7. Far from the junction Si E_c-E_F < E_F-E_v and E_c-E_F still the same as without bias.
8. At the junction $(E_F\text{-}E_v)_{@junction} = (E_c\text{-}E_F)_{away\ from\ junction}$ (definition of threshold voltage)
9. the bandgaps should not have changed
10. the bands must tilt with a negative gradient in the +x direction, both in oxide as well as in semiconductor.

[6]

b) i) $V_{th_p} < 0V$ because in order to create a channel holes have to be attracted. Holes are positively charged and are thus attracted by a negative potential. [4]

ii) $V_{GS} < V_{th_p}$, V_{GS} needs to be more negative than the threshold voltage to first repel the electrons and then attract sufficient holes. [4]

ii) a pMOS needs a negative drain voltage with respect to the source.

$$V_{DS}^{sat} = V_{GS} - V_{th_p} = -|V_{GS}| - \left(-|V_{th_p}|\right) = -|V_{GS}| + |V_{th_p}| \text{ since } |V_{GS}| > |V_{th_p}| \rightarrow V_{DS}^{sat} < 0V$$ [4]

c) From the formulae list copy the expression for drain current [6]

$$I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right)$$

CMOS switching is done with the FETs in saturation, thus the currents become saturation currents:

$$pMOS \quad I_{DS}^{sat} = \frac{\mu_p C_{ox} W_p}{L_{Gp}}\left(V_{GS} - V_{th_p}\right)^2$$

$$nMOS \quad I_{DS}^{sat} = \frac{\mu_n C_{ox} W_n}{L_{Gn}}\left(V_{GS} - V_{th_n}\right)^2$$

The oxide does not change because both FETs are defined on the same wafer for CMOS. The threshold voltages are the same (given). Thus for the currents to be the same:

$$\frac{\mu_p C_{ox} W_p}{L_{Gp}} = \frac{\mu_n C_{ox} W_n}{L_{Gn}}$$

$$\frac{W_n}{L_{Gn}} = \frac{\mu_p W_p}{\mu_n L_{Gp}}$$

Using Einstein's equation (see formulae list): $D = \frac{kT}{e}\mu$ the above equation becomes:

$$\frac{W_n}{L_{Gn}} = \frac{D_p W_p}{D_n L_{Gp}} = \frac{10}{20} \times \frac{W_p}{L_{Gp}} = \frac{W_p}{2 \times L_{Gp}}$$

Thus the ratio of the width of the gate to the length of the gate of the nMOS needs to be half that of the pMOS. There is a whole range of possibilities. However in industry the gate lengths are kept constant for all FETs thus the most appropriate and realistic solution is to change the gate width.

Thus the gate geometry of the nMOS should be:

$$L_{Gn} = L_{Gp} = 2\mu m$$

$$W_n = \frac{W_p}{2} = \frac{10\mu m}{2} = 5\mu m$$

SECTION B: FIELDS

4. This question is mandatory. Answer all parts. Each part is worth 4 marks.

a) Two point charges, of charge +0.01 nC and –0.01 nC, lie on the x axis at $x = -1$ cm and +1 cm respectively. Find the magnitude and the direction of the electric field E at the origin ($x = 0$).
For one charge, $E = Q/(4\pi\varepsilon_o r^2) = 10^{-11}/(4\pi\times8.85\times10^{-12}\times10^{-4}) = 900$ V/m. Total field is double this (two charges contribute equally): 1800 V/m.

b) The electric field in a certain region is uniform and points in the $+x$ direction. It is found that 6 mJ of work is required to move a charge of 3 μC from $x = +10$ cm to $x = 0$. Find the magnitude of the electric field.
$W = QV$, $E = \Delta V/\Delta x$, so $E = W/(Q\Delta x) = 6\times10^{-3}/(3\times8.85\times10^{-6}\times0.1) = 20$ kV/m.

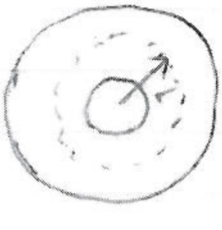
c) A region of free space has equal energy densities for the electric and magnetic fields in that region. Find a value for the ratio E/H. What is this value commonly called?
In free space, $U_E' = \frac{1}{2}\varepsilon_o E^2$, $U_M' = \frac{1}{2}\mu_o H^2$, setting them equal gives $E/H = (\mu_o/\varepsilon_o)^{1/2} = 377\ \Omega$. This is known as the impedance of free space.

d) At some position in a linear ferromagnetic material, having a relative permeability $\varepsilon_r = 1200$, the magnetic field strength H = 700A/m. Calculate magnetic flux density B at the same position. If H is in the +x direction, what will be the direction of B?
$B = \mu H = \mu_r\mu_o H$, so $B = 4\pi\times10^{-7}\times 1200 \times 700 = 1.06$ T. Since B and H have the same direction, the direction of B is $+x$.

e) A DC motor in steady state is producing 0.1 Nm of torque at a speed of 1200 rad/s, with an armature current of 2.0 A. Calculate the back EMF in this condition.
The output mechanical power, $P = T\omega$, equals the useful electrical power, $P = e_a I_a$, so $e_a = T\omega/I_a = 0.1 \times 1200 / 2 = 60$ V.

5.

a)



define a Gauss surface as a cylinder of radius r, $a<r<b$, of length L

No flux flows through the "end cap" surfaces (by symmetry). On the curved surface, D is uniform and $\perp$ surface, giving

$$\oint_S \underline{D}\cdot d\underline{S} = D(2\pi r L) = Q_{enc} = \rho L$$

$$\therefore \quad D = \rho/2\pi r \quad \therefore \quad E = \rho/\varepsilon_r \varepsilon_0 2\pi r$$

b) $Q = CV$ so $\frac{C}{L} \equiv C' = \frac{\rho}{V}$

$$V = -\int \underline{E}\cdot d\underline{l} = \int_a^b \frac{\rho}{\varepsilon_r \varepsilon_0 2\pi r}\,dr = \frac{\rho}{\varepsilon_r \varepsilon_0 2\pi} \ln\left(\frac{b}{a}\right)$$

$$C' = \frac{\varepsilon_r \varepsilon_0 2\pi}{\ln(b/a)} \quad \text{in this case} = \frac{4 \times 8.85 \times 10^{-12}(2\pi)}{\ln(2.5)}$$

$$= 2.43 \times 10^{-10} \text{ F/m}$$

c) $E(r) = \frac{\rho}{2\pi\varepsilon r}$ $\quad E_{max} = \frac{\rho}{2\pi\varepsilon_r\varepsilon_0 a} = \frac{C'V}{2\pi\varepsilon_r\varepsilon_0 a}$

$$= \frac{V}{\ln(b/a)\, a} \qquad V_{max} = E_{max} \times a \ln\left(\frac{b}{a}\right)$$

$$= 5\times 10^6 \times 4 \times 10^{-3} \times \ln(2.5)$$

$$= 18.3 \text{ kV}$$

d) $D(r) = \rho/2\pi r \quad a<r<b$ (in both materials)

$E_1(r) = \rho/2\pi\varepsilon_0\varepsilon_{r1} r \quad E_2(r) = \rho/2\pi\varepsilon_0\varepsilon_{r2} r$

$$E_{1max} = \frac{\rho}{2\pi\varepsilon_0\varepsilon_{r1} a} = E_{2max} = \frac{\rho}{2\pi\varepsilon_0\varepsilon_{r2} c} \quad \therefore \varepsilon_{r1} a = \varepsilon_{r2} c$$

$$\varepsilon_{r2} = 4\left(\frac{4\text{mm}}{6\text{mm}}\right) = 2.67$$

(6)

a) $B = \dfrac{\mu_0 I}{2\pi h}$ at a distance h from a long straight wire carrying I

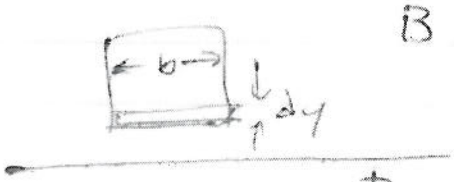
(can be derived from Ampere's Law)

– assumes $\mu_r = 1$ (air)

here the distance $h = c + a/2$ (center of coil)

$\therefore \quad B = \dfrac{\mu_0 I}{2\pi(c + a/2)}$. By right hand rule, B points out of the page (+z)

b) $\Phi = \int_S B \cdot dA$ with S the cross-section of the coil.



B is uniform in horizontal strips of width dy, and $B = \mu_0 I / 2\pi y$

$$\therefore \quad \Phi = \int_c^{c+a} \frac{\mu_0 I b\, dy}{2\pi y} = \frac{\mu_0 I b}{2\pi} \int_c^{c+a} \frac{dy}{y}$$

$$\Phi = \frac{\mu_0 b I}{2\pi} \ln\left(\frac{c+a}{c}\right)$$

c) $V_c(t) = -N \dfrac{d\Phi}{dt} = \dfrac{\mu_0 N b}{2\pi} \ln\left(\dfrac{c+a}{c}\right) \dfrac{dI}{dt}$

$= \dfrac{\mu_0 N b}{2\pi} \ln\left(\dfrac{c+a}{c}\right) \omega I_0 \cos\omega t$

$$V_{rms} = \frac{1}{\sqrt{2}} \times \frac{\mu_0 N b \ln\left(\frac{c+a}{c}\right)}{2\pi} \omega I_0$$

$$= \frac{1}{\sqrt{2}} \times \frac{4\pi \times 10^{-7}}{2\pi} \times 20 \times 6 \times 10^{-2} \times \ln\left(\frac{6}{2}\right) 100\pi \times 0.1$$

$$= 5.86\ \mu V$$

d) $M_{21} = N_2 \Phi_2 / I_1$ taking 2 as coil, 1 as wire and taking Φ_2 from (b):

$$M_{21} = \frac{\mu_0 b N_2 \ln\left(\frac{c+a}{c}\right)}{2\pi} = 264\ nH$$

e) Flux is decreased, as the coil current creates additional flux and this acts to reduce the rate of change in Φ – since $\Phi(t)$ is still sinusoidal the magnitude must also be reduced.