

UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

BEng Honours Degree in Computing Part III
MSc in Computing Science
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

*This paper is also taken for the relevant examinations for the
Associateship of the City and Guilds of London Institute*

*This paper is also taken for the relevant examinations for the
Associateship of the Royal College of Science*

PAPER C381=I3.30

COMPUTATIONAL FINANCE

Thursday 6 May 2004, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

- 1 a i) Consider n risky assets with given expected returns \bar{r}_i ($i = 1, \dots, n$) and covariances of r_i, r_j given by σ_{ij} . Write the mean-variance optimization problem, assuming that there is no shortsale and you can invest at most 40% of your capital on any asset.
- ii) Define the efficient frontier. Explain the risk-averse and the risk-seeking investment strategies.
- b Consider a single factor model where the market portfolio serves as the factor. The portfolio is constructed using these assets whose betas, weights and current prices are presented in the following table. The market portfolio has an expected rate of return of 15% and a standard deviation of 20%. The annual risk free interest rate is 8%.

Stock	Beta	Weights in Portfolio	Current Price
A	1.05	35%	50
B	0.50	45%	70
C	1.85	20%	55

- i) What is the expected rate of return of the portfolio constructed by the three assets?
- ii) Calculate the future expected selling price of the asset A.
Hint: Consider CAPM model.
- c Suppose that the price and payoff values of the securities A, B, C for two states are given in the following table.

Security	Price (£)	Payoff in State 1	Payoff in State 2
A	75	60	110
B	50	40	120
C	80	50	115

- i) Investigate any possibility for an arbitrage opportunity involving securities A, B, C. State your strategy clearly.
- ii) Suppose you have £10,000 capital. What is the outcome of forming the arbitrage portfolio?
Hint: Consider the portfolio constructed by securities A, B to replicate asset C.

The three parts carry, respectively, 30%, 35%, and 35% of the marks.

- 2a Consider two bonds A and B which mature in a year and two years with coupon payments of 12% and 10%, respectively. The face value is £100 and prices of bonds are £102 and £100.
- What are 1-year and 2-year spot rates s_1, s_2 ?
 - Consider the spot rates for 1 and 2 years in the previous part. What is the forward rate $f_{1,2}$ under annual compounding?
- b A company has just paid the dividend of £3.0 per share. For the next year, the dividends are expected as £3.5. After the first year, the company has the expected rate of growth in dividends per year for 2 years as 6% and then 5% per year thereafter. What is the new stock price if the required return remains at 11%?
- c Consider a growing perpetuity with grow rate of g , annuity of a and the annual interest rate of r .
- Show that the present value of cash flow $(a, a(1+g), \dots, a(1+g)^n, \dots)$ at $t = 1, 2, \dots, n, \dots$ gives the present value of a growing perpetuity as

$$PV_{GP} = \frac{a}{r - g}$$

Hint: Use the formula of an infinite geometric progression

$$1 + x + x^2 + x^3 + \dots = \frac{1}{1 - x} \quad 0 < x < 1$$

- Determine the value of a growing perpetuity with annuity of £100 and the grow rate of 5% when the annual interest rate of 10%.

The three parts carry, respectively, 40%, 30%, and 30% of the marks.

- 3 a Consider a binomial lattice model for the stock price process $\{S_n : 0 \leq n \leq 2\}$ and $S_0 = £150$. Let the price rise or fall by 14% at each step. The risk-free interest rate is assumed $r = 7\%$. The strike price of a European Call option is £150.
- i) Compute the risk neutral probabilities and evaluate the stock prices on 2-period binomial lattice.
 - ii) Derive the cash position and stock position for 2 periods on 2-period binomial lattice using “the replicating portfolio method”. Calculate the price of the European call option at $t = 2$.
- b Suppose that a given stock is selling at £550. The risk free interest rate is 5%, the dividend rate is 3%.
- i) What is the price of 6 months forward contract under continuous compounding?
 - ii) Suppose that the price of 6 month forwards contract is £580. Investigate whether there is an arbitrage opportunity in the market. State clearly your positions and cash flows in an arbitrage table.

The two parts carry, respectively, 55%, and 45% of the marks.

- 4 Consider the following quadratic programming problem.

$$\min f(x) = -2x_1 + x_2 + 2x_1^2 - 6x_1x_2 + 5x_2^2$$

subject to

$$-x_1 + x_2 = 5$$

$$x_1, x_2 \geq 0$$

- a Define the Lagrange function of the problem.
- b Write the KKT conditions of the problem.

The two parts carry, respectively, 50% and 50% of the marks