E4.16
AO5

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2006

MSc and EEE PART IV: MEng and ACGI

CURRENT-MODE ANALOGUE SIGNAL PROCESSING

Wednesday, 26 April 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : E. Drakakis
Second Marker(s) : E. Rodriguez-Villegas

Special instructions for students

$$\sinh(x) = \frac{\exp(x) - \exp(-x)}{2}$$

$$\cosh(x) = \frac{\exp(x) + \exp(-x)}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\text{sech}(x) = \frac{1}{\cosh(x)}$$

$$\text{sech}^2(x) + \tanh^2(x) = 1$$

Question 1

The transfer function of an Infinite Impulse Response (IIR) filter of order N is given by:

$$y(m)=\sum_{n=1}^{N}a(n)y(m-n)+\sum_{n=0}^{N}b(n)x(m-n)$$

In this question the design of an IIR filter of order 2 will be outlined using switched current cells. This is a low-pass filter with a break frequency $f_B = 0.35 f_S$ (f_S is the switching frequency). The filter nominal pass-band gain is unity. The filter coefficients are:

j	a(j)	b(j)
0	-----	0.44
1	0.76	0.88
2	0.45	0.44

(a) Draw a block diagram of such a filter, in terms of unity delay elements, gain elements and summing junctions. [3]

(b) Design a switched current "branching element" cell which for a given input generates two outputs each equal to half the input. Discuss the effect of transistor mismatch. [5]

(c)

i) Use the divide-by-two branching element from part (b) to design one of the filter coefficients of part (a) to a coefficient absolute tolerance of $5 \cdot 10^{-3}$. Draw your circuit diagram in terms of branching cells, NOT in terms of transistors. (hint: you will need to construct a binary representation of the filter coefficient by multiplying it by a suitable power of 2, and use several branching stages).

ii) Estimate the number of control clock phases required for each coefficient.

iii) Estimate the number of transistors, including switches and bias current sources required to implement the filter using this technique.

iv) Estimate the number of phases needed to run this filter. If the transistors have a transit frequency of 10 GHz, what is the maximum signal frequency the filter can handle? Explain your answer.

[12]

Question 2

In this question "x" denotes the current input terminal of a current conveyor, and "y" denotes the other input terminal.

(a) Describe the function of an ideal 3rd generation current conveyor, and write an equation relating the voltages and currents at its terminals. What is the application of the CCIII? [5]

(b) i) What is the terminal impedance of an ideal CCIII on the "x" terminal if the "y" terminal is grounded?

ii) What is the terminal impedance of a CCIII on the "y" terminal if the "y" terminal is grounded?

iii) Without drawing a circuit diagram describethe kind of feedback connections and the magnitude of the loop gain of each required to approximately obtain the terminal characteristics of an ideal CCIII using real devices. Discuss the consequences of such a circuit topology. [5]

(c) Draw the circuit diagram of a current difference amplifier of gain +10 using only CCII- current conveyors. [5]

(d) Use your result from Question 2(c) above, together with any other current conveyors needed to design a differential-to-single-ended current mode 2nd order active high pass filter following the Sallen-Key methodology. [5]

Question 3

In this question CFOA means Current feedback op-amp

(a)

i) Write a matrix equation describing the output of an ideal CFOA in terms of its inputs.

ii) Write an equation for the transfer function of a CFOA with a finite transimpedance, but otherwise ideal.

iii) Draw a circuit diagram for a non-inverting voltage amplifier constructed around an finite gain but otherwise ideal CFOA.

iv) Write an expression for the gain of this amplifier in terms of the CFOA gain and any resistors used. What is the limit of this expression as the transimpedance goes to infinity? [5]

(b)

i) Write an equation describing the frequency dependent transfer function of a realistic CFOA in terms of its inputs. Include the effect of a finite, frequency dependent transimpedance only.

ii) What is the low frequency gain of this model? What is the bandwidth? What is the gain-bandwidth product? [7]

(c)

i) Extend the model of part (b) to include a finite input impedance at the inverting input. Assign a single pole model, at a time constant τ_1, to this input impedance.

ii) Show that the resulting non-inverting amplifier is a band-pass filter. Calculate the centre frequency and quality factor of this filter. Assume the low frequency transimpedance is much larger than any other impedance in the problem.

iii) Comment on your result. How does the filter Q scale with the ratio of the input pole frequency (typically GHz) to the transimpedance pole frequency (typically kHz)?

[8]

Question 4

(a) Derive the bipolar translinear principle with reference to a loop containing 2m base-emitter junctions. State all the assumptions that you make and list the conditions which must be satisfied in order for this principle to be valid. [3]

(b) Compare the translinear principle relations for the MOS and the Bipolar cases [2]

(c) Figure 4.1 illustrates a translinear circuit whose differential current output realises a trigonometric approximation.

(i) Express the currents I_3 and I_4 in terms of I and I_x. [3]

(ii) Next, express the differential output $I_2 - I_1$ in terms of I and I_x and show that when $I_x = yI$ holds: $I_2 - I_1 = \dfrac{2}{3}\dfrac{\left(2y - y^3\right)}{1 + y^2} I$. You may assume that the transistors' beta value is large. [4]

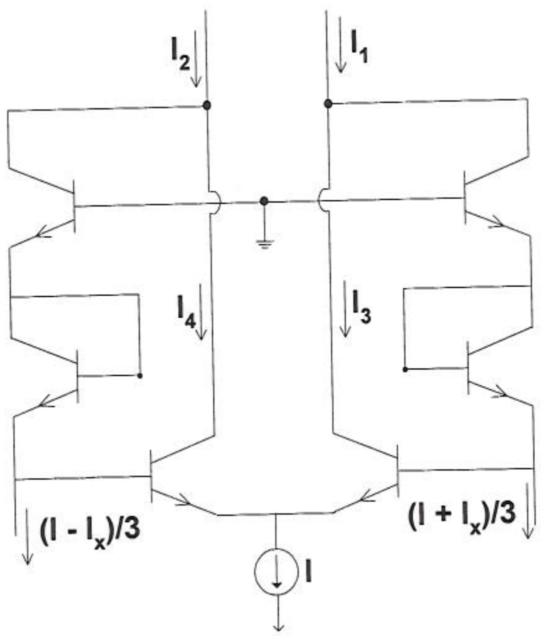


Figure 4.1

(d) Figure 4.2 illustrates a translinear circuit whose output current I_z can implement a variety of trigonometric approximations.

(i) Express the current I_2 in terms of I, I_x and the output current I_z.

[2]

(ii) Next, express the output current I_z in terms of I, I_x and the emitter area A. [3]

(iii) Determine the emitter area A so that

$$I_z = 1.05I + 3.45I_X + 0.7\left[\frac{I_X^{\ 2}}{I} - \frac{I_X^{\ 3}}{I^2}\right].$$ [3]

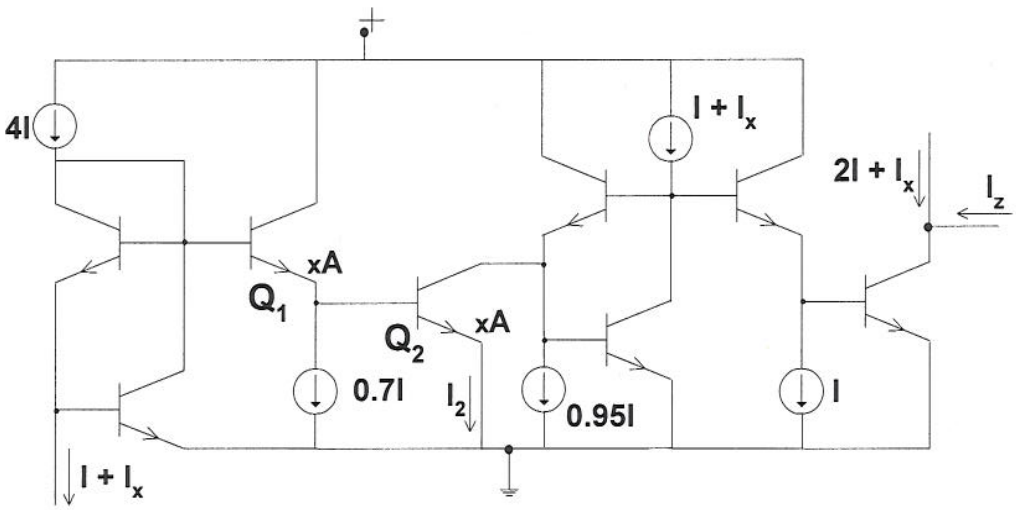


Figure 4.2

Question 5

(a) Figure 5.1 illustrates a general companding circuit.

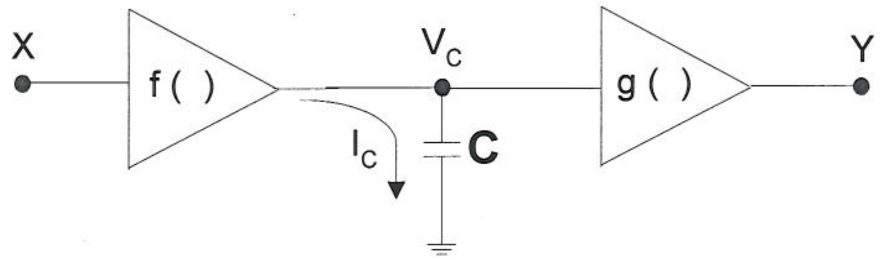


Figure 5.1

(i) Derive the condition under which the circuit operates as an input-output linear integrator. [2]

(ii) If the input signal X is a current of value I_{in} and the output signal Y is a current $g(V_C) = I_T \tanh(\alpha V_C)$ with $\alpha = V_T^{-1}$ and I_T a constant with dimensions of current, determine the function $f(\ )$ which ensures that the circuit operates as an input-output linear companding integrator. [2]

(b) You are given the following state-space:

$$\dot{x}_1(t) = -\omega_0 x_1(t) + \omega_0 x_2(t)$$
$$\dot{x}_2(t) = -2\omega_0 x_1(t) + \omega_0 x_2(t) + \omega_0 U(t)$$
$$y(t) = x_1(t)$$

where $U(t)$ denotes the input, $y(t)$ denotes the output, $x_1(t)$ and $x_2(t)$ are state variables and a dot above a variable denotes time-differentiation. The input U is tuned appropriately to initially start oscillation.

(i) Show that the transfer function of the above oscillator is provided by the relation: $H(s) = \dfrac{\omega_0^2}{s^2 + \omega_0^2}$ [3]

(ii) Using the exponential mappings $x_j = I_0 \, exp\left(\frac{V_j}{V_T}\right) (j = 1,2)$ and

$U = I_S \, exp\left(\frac{V_U}{V_T}\right)$ show that the above linear state-space equations can be transformed into non-linear log-domain design equations (I_S denotes the reverse saturation current of a bipolar junction transistor). [6]

(iii) Sketch the transistor level implementation of the log-domain oscillator which realises these design relations and choose DC bias current values to give an oscillation frequency $\omega_0 = 2\pi(5 \times 10^6)$ rad/s. You may assume that all capacitors to be used are of value 7.5 pF. [7]

Question 6

(a) Derive and sketch the adjoint network of a resistor, a nullor and a voltage amplifier [6]

(b)

(i) Figure 6.1 illustrates a typical current amplifier. Show that

$$\frac{i_{out}}{i_{in}} = \frac{A_i\left(1+\frac{R_2}{R_1}\right)}{A_i+\left(1+\frac{R_2}{R_1}\right)}$$ with A_i denoting the gain of the amplifier.

[3]

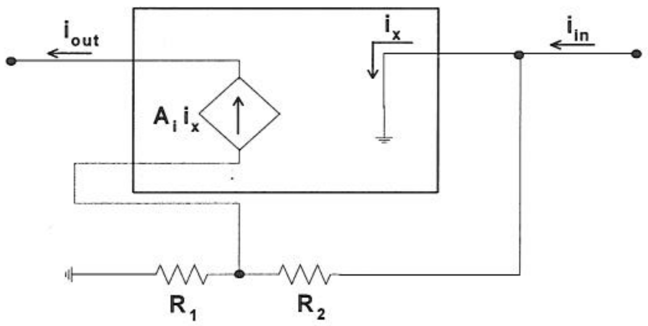


Figure 6.1

(ii) Figure 6.2 illustrates a current-mode Sallen-Key filter. Based on the results you derived in (i) show that its transfer function is provided by the relation $\frac{I_{out}(s)}{I_{in}(s)} = \frac{G}{(RC)^2 s^2 + (3-G)(RC)s + 1}$ where $G = 1 + \frac{R_2}{R_1}$ and $A_i >> 1$.

[5]

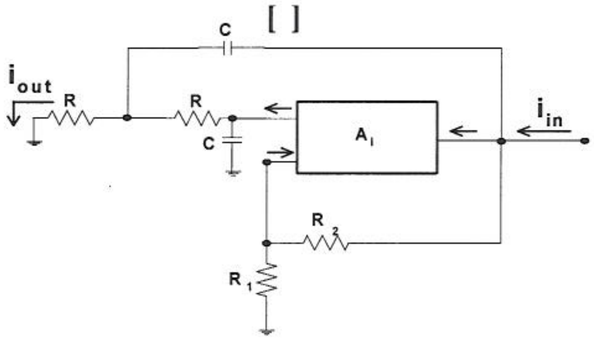


Figure 6.2

(c) Figure 6.3 shows the architecture of a simple current-follower, where the Symbols CM represent current-mirrors with an arrow marking their input side.

(i) Derive expressions for d.c. input offset voltage and small-signal input resistance at node X. [2]

Explain with the aid of a diagram in each case, how the circuit can be modified to:

(ii) Reduce the d.c. offset without increasing the small-signal input resistance [2]

(iii) Reduce the small signal input resistance without increasing the value of I_0. [2]

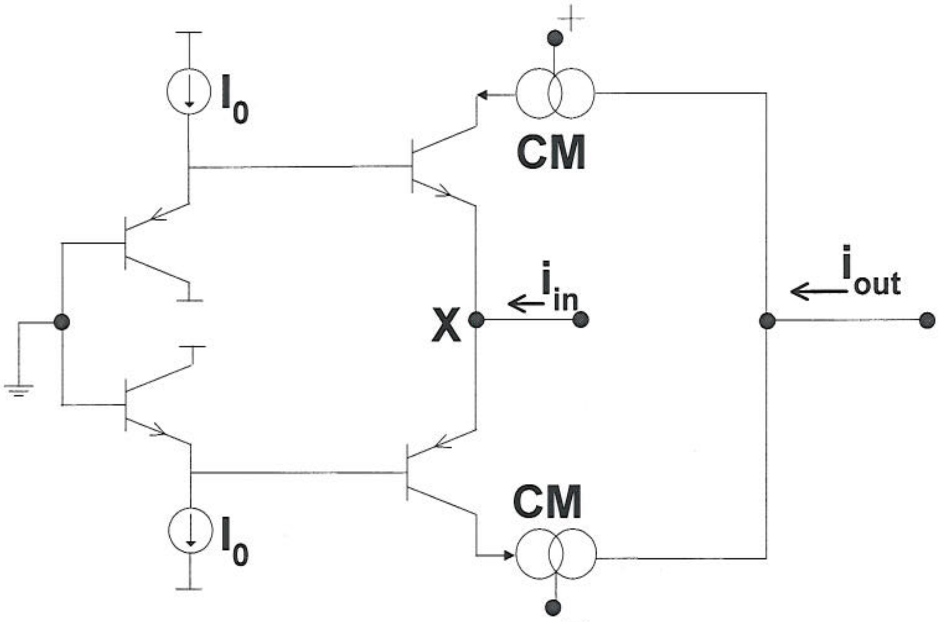
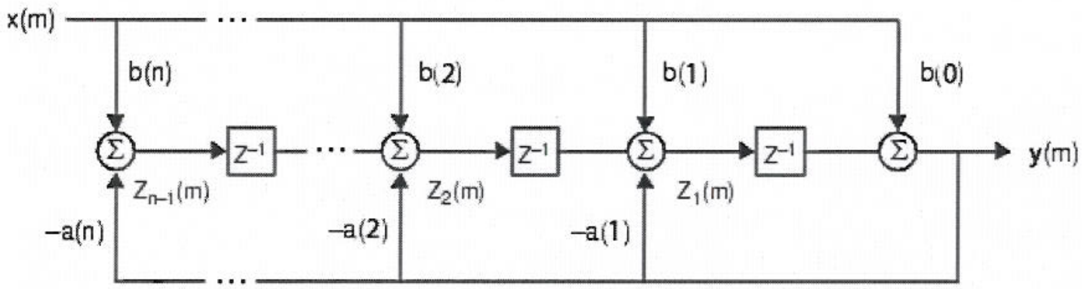


Figure 6.3

Current-Mode Analogue Signal Processing E4.16
A05
Answers - Q1 - Q3 - 6 pages. 2006

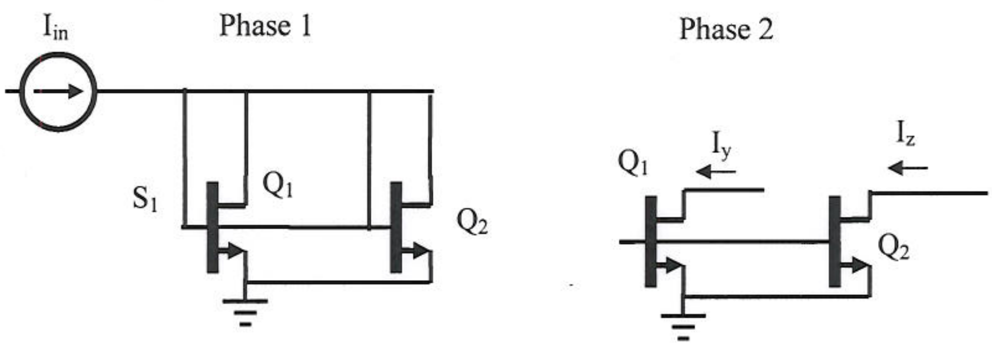
ANSWER Q1:

a) **[bookwork]**



[3]

b) **[computed example]**



[3]

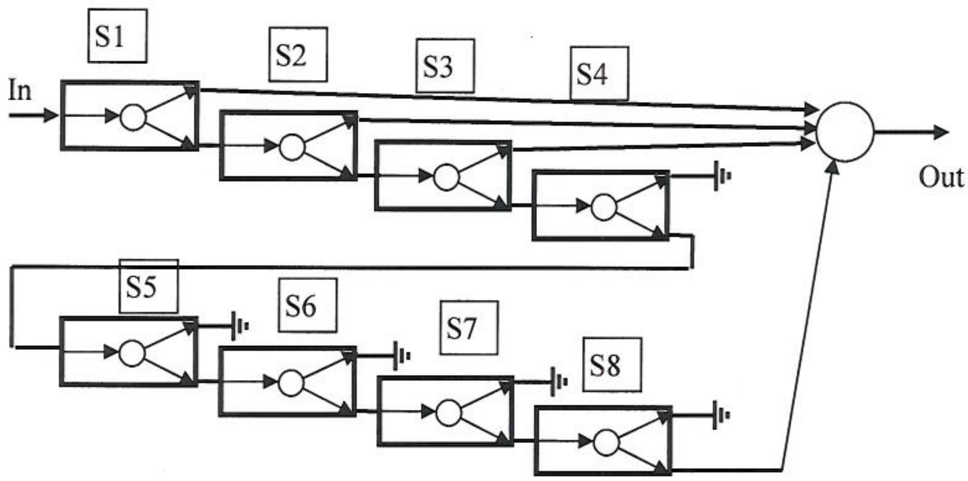
Transistor mismatch has the effect that $I_y = \frac{I}{2}(1+\varepsilon)$, $I_z = \frac{I}{2}(1-\varepsilon)$

[2]

c) **[computed example]**
A binary representation of a coefficient can be implemented by the branching elements. Each stage needs a separate phase, so an N-bit representation needs N phases: For example, 0.88 = 0.11100001 in the required 8 bit accuracy.

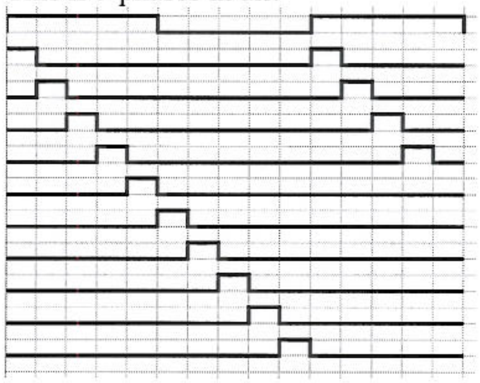
[2]

A block diagram of a circuit which can do this is:



[4]

Similar circuits are needed for each of the other 4 coefficients.
The coefficient itself needs 8 phases (1 for each bit, plus an extra one for the signal duplication so that 1 copy is fed to the coefficient and 1 to the delay element.
This is 9 phases in all:



[2]

To estimate the number of transistors, each duplicator is 2 memory + 6 switches+2 current bias sources = 10 FET x8 =80
Each delay-duplicator has to be a CCI so it is another 3 memories +6 switches+ 3 current sources=12 FET x 6 = 72
So the total circuit has about 152 transistors and 9 phases.

[2]

The transistors need to settle to 0.005 so each subphase (9+1=10 in a period) needs to be such that

$e^{-2\pi f_T T} \geq 0.995 \Rightarrow T \geq \dfrac{1}{400\pi f_T}$. For $f_T = 10GHz \Rightarrow T \geq 126ns$. The sampling frequency

needs to be 10 times lower than the switch phase frequency, i.e. $f_s \leq \dfrac{1}{1.26\mu s} = 796kHz$

and the Nyquist frequency is again half this so $f_N = 388kHz$

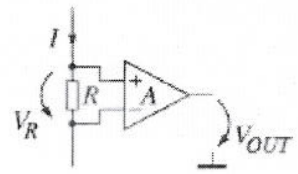
[2]

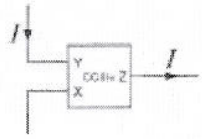
Note: Partial of full credit will be awarded for any reasonable answer to these estimates.

ANSWERS Q2:

a) **[bookwork]**

$$\begin{bmatrix} i_y \\ v_x \\ i_z \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_y \\ i_x \\ v_z \end{bmatrix}$$





This is an ideal current meter. The two inputs together behave like an ideal node

[5]

b) **[interpretation of theory]**

From the behavioural equation, if y is grounded, $v_x = 0 \Rightarrow Z_x = 0$.

Similarly, if x is grounded, $v_y = 0 \Rightarrow Z_y = 0$.

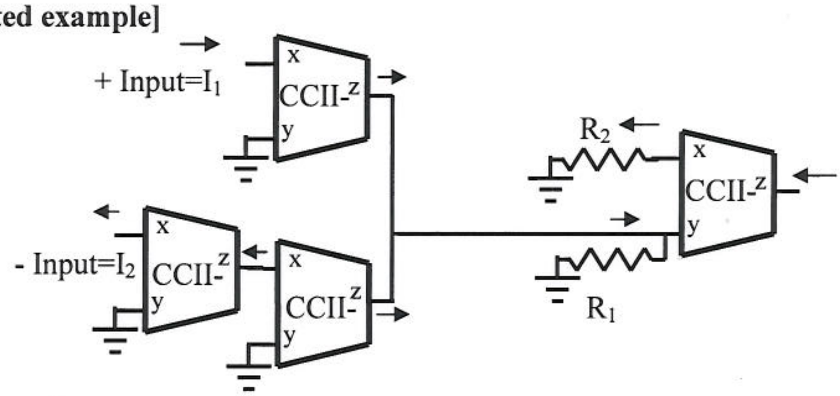
To obtain zero terminal impedance on the terminals we need unity gain positive feedback connected in series with each of the inputs. Since the output is required to be a current source the connection at the output side is a shunt connection. So a series-shunt positive feedback connection at each of the inputs is required. The opposite current polarity requires an extra inversion in one of the feedback connections.

Such a circuit is likely to be unstable in a real implementation. Alternatively, a very large loop gain shunt-series connection is required, severely restricting the bandwidth of the device.

(Note: Either answer will be accepted as correct)

[5]

c) **[computed example]**

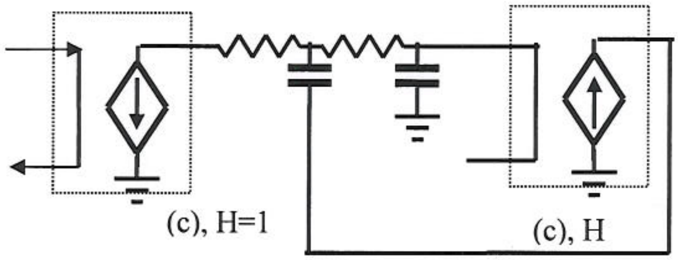


$$\left.\begin{aligned} &V_x = V_y \\ &V_x = -i_x R_2 \\ &V_y = (I_1 - I_2) R_1 \end{aligned}\right\} \Rightarrow i_x R_2 = -(I_1 - I_2) R_1 \Rightarrow$$

$$i_z = -i_x = \frac{R_1}{R_2}(I_1 - I_2) \Rightarrow R_1 = 10 R_2$$

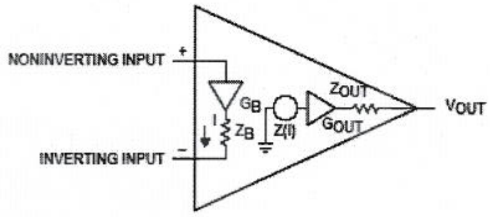
[5]

d) **[computed example]**

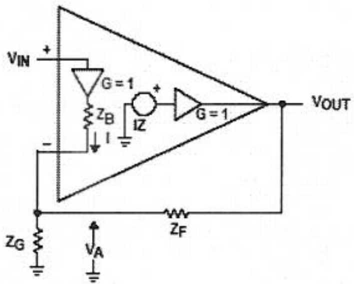


[5]

ANSWER Q3.
a) **[bookwork+extensions]**



$v_{out} = Z_T I_{IN}$, for an ideal CFOA $Z_T = \infty$.
The non-inverting amplifier diagram is the same as with a voltage op-amp:



$V_{out} = Z_T I$ and the gain goes to infinity as $Z_T \to \infty$, with the input current out of the inverting input. The inverting input voltage follows that of the non-inverting input.

Since:

$$\left.\begin{aligned} & V_- = V_+ \\ & i_- + \frac{V_-}{R_G} + \frac{(V_- - V_{out})}{R_F} = 0 \\ & V_{out} = -Z_T i_- \end{aligned}\right\} \Rightarrow G = \frac{V_{out}}{V_+} = Z_T\left(\frac{1}{R_G} + \frac{1}{R_F}\right) \Big/ \left(1 + \frac{Z_T}{R_F}\right)$$

$$\lim_{Z \to \infty} G = 1 + \frac{R_F}{R_G} \equiv G_0$$

[5]

b) Let $Z_T = \dfrac{Z_0}{1 + s\tau}$. Then,

$$G = \frac{V_{out}}{V_+} = \frac{Z_0}{1 + s\tau}\left(\frac{1}{R_G} + \frac{1}{R_F}\right) \Big/ \left(1 + \frac{Z_0}{R_F(1 + s\tau)}\right) = \frac{Z_0}{R_F + Z_0} G_0 \left(\frac{1}{1 + \dfrac{R_F}{R_F + Z_0} s\tau}\right)$$

The low frequency gain is clearly:

$$G(0) = \frac{G_0}{R_F / Z_0 + 1}$$

the bandwidth: $\omega_0 = \dfrac{1}{\tau}\left(1 + \dfrac{Z_0}{R_F}\right) \simeq \dfrac{Z_0}{\tau R_F}$ depends only on R_F, an the gain bandwidth product is:

$$\omega_0 G(0) = \frac{G_0}{R_F / Z_0 + 1} \simeq \frac{Z_0}{\tau R_F}\left(1 + \frac{R_F}{R_G}\right)$$, which is clearly not a constant.

[7]

c) we allow for a finite $R_{in} = R_0 / (1 + s\tau_1)$:

$$\left.\begin{array}{l} V_- = V_+ + i_- R_{in} \\ i_- + \dfrac{V_-}{R_G} + \dfrac{(V_- - V_{out})}{R_F} = 0 \\ V_{out} = -Z i_- \end{array}\right\} \Rightarrow G = \frac{V_{out}}{V_+} = Z_T\left(\frac{1}{R_G} + \frac{1}{R_F}\right) \Big/ \left(1 + \frac{R_{in}}{R_G} + \frac{R_{in}}{R_F} + \frac{Z_T}{R_F}\right)$$

$$\Rightarrow G = \frac{Z_T G_0}{R_F} \Big/ \left(1 + \frac{R_{in} G_0}{R_F} + \frac{Z_T}{R_F}\right) = \frac{Z_T G_0}{R_F + G_0 R_{in} + Z_T} = \frac{Z_0}{(1 + s\tau)} \frac{G_0}{R_F + \dfrac{R_0 G_0}{1 + s\tau_1} + \dfrac{Z_0}{1 + s\tau}} =$$

$$= \frac{Z_0 G_0 (1 + s\tau_1)}{R_F (1 + s\tau_1)(1 + s\tau) + R_0 G_0 (1 + s\tau) + Z_0 (1 + s\tau_1)} =$$

$$= \frac{Z_0 G_0}{R_F + Z_0 + G_0 R_0} \frac{1 + s\tau_1}{1 + \left(\dfrac{R_F(\tau + \tau_1) + G_0 R_0 \tau + Z_0 \tau_1}{R_F + Z_0 + G_0 R_0}\right) s + s^2 \dfrac{R_F \tau \tau_1}{R_F + Z_0 + G_0 R_0}} =$$

The $LPF + BPF$, both 2nd order.

with $G_0 = 1 + \dfrac{R_F}{R_G}$

two filter transfer functions share natural frequency and Q:

$$\omega_0 = \sqrt{\frac{1 + Z_0/R_F + G_0 R_0/R_F}{\tau \tau_1}}, \quad Q = \frac{R_F + Z_0 + G_0 R_0}{R_F(\tau + \tau_1) + G_0 R_0 \tau + Z_0 \tau_1} \sqrt{\frac{\tau \tau_1}{1 + Z_0/R_F + G_0 R_0/R_F}}$$

if Z_0 is by far the largest term, as is usually the case,

$$\omega_0 = \sqrt{\frac{1 + Z_0/R_F + G_0 R_0/R_F}{\tau \tau_1}} \simeq \sqrt{\frac{Z_0}{R_F \tau \tau_1}}, \quad Q = \sqrt{\frac{\tau}{\tau_1}} \sqrt{\frac{R_F}{Z_0}}$$

An attempt to increase the bandwidth clearly increases the Q, as does an attempt to overcompensate the amplifier. The break frequency now varies with the square root of R_F, and the gain-bandwidth product is:

$$GBP = \frac{Z_0 G_0}{R_F + Z_0 + G_0 R_0} \sqrt{\frac{Z_0}{R_F \tau \tau_1}} \simeq \sqrt{\frac{Z_0 G_0}{R_F \tau \tau_1}}$$ much closer to a constant.

Answers - Q4 - Q6 - 15 pages
Current-mode Analogue SP

Question 4

a) Assumptions & Conditions [Bookwork]

equal number of CW & ACW npn junctions

-"- -"- -"- CW & ACW pnp -"-

Same V_T (i.e. temperature) for all devices

All npn (pnp) devices have same current density J_{SN} (J_{SP})

When the above hold:

$$CW \sum_{j=1}^{m} V_{be_j} = ACW \sum_{j=1}^{m} V_{be_j} \Rightarrow$$

$$\Rightarrow CW \sum_{j=1}^{m} V_T \ln\left[\frac{I_{C_j}}{J_{s_j} A_j}\right] = ACW \sum_{j=1}^{m} V_T \ln\left[\frac{I_{C_j}}{J_{s_j} A_j}\right] \Rightarrow$$

$$\Rightarrow CW \prod_{j=1}^{m}\left[\frac{I_{C_j}}{J_{s_j} A_j}\right] = ACW \prod_{j=1}^{m}\left[\frac{I_{C_j}}{J_{s_j} A_j}\right] \Rightarrow$$

$$\Rightarrow CW \prod_{j=1}^{m}\left[\frac{I_{C_j}}{A_j}\right] = ACW \prod_{j=1}^{m}\left[\frac{I_{C_j}}{A_j}\right]$$ 3 marks

b) MOS TLP: $$CW \sum_{j=1}^{m} \sqrt{\frac{I_{D_j}}{K_j}} = ACW \sum_{j=1}^{m} \sqrt{\frac{I_{D_j}}{K_j}}$$

[Bookwork]

$$K_j = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)$$

Disadvantages:
- "Sum of roots" is not as widely useful as bipolar "products"
- MOS square law holds over a smaller current range than the bipolar exponential law
- process parameters do not cancel "automatically" for "mixed" (NMOS & PMOS) TL loops

Advantage : MOS TL circuits do not suffer from beta errors. (2 marks)

c) [Exercise based on taught theory]

i) $I_3 + I_4 = I$

$$\left[\frac{I-I_x}{3}\right]^2 I_4 = I_3 \left[\frac{I+I_x}{3}\right]^2 \Rightarrow \frac{I_4}{I_3} = \left[\frac{I+I_x}{I-I_x}\right]^2 \Rightarrow$$

$$\frac{I_4 + I_3}{I_3} = \frac{I}{I_3} = \frac{[I+I_x]^2 + [I-I_x]^2}{[I-I_x]^2} \Rightarrow$$

$$I_3 = \frac{[I-I_x]^2 I}{[I+I_x]^2 + [I-I_x]^2}$$

$$I_4 = \frac{[I+I_x]^2 I}{[I+I_x]^2 + [I-I_x]^2}$$ (3 marks)

ii)

$$I_2 = I_4 + \frac{I-I_x}{3} = \frac{I-I_x}{3} + \frac{[I+I_x]^2 I}{[I+I_x]^2 + [I-I_x]^2}$$

$$I_1 = I_3 + \frac{I+I_x}{3} = \frac{I+I_x}{3} + \frac{[I-I_x]^2 I}{[I+I_x]^2 + [I-I_x]^2} \Rightarrow$$

$$I_2 - I_1 = -\frac{2I_x}{3} + \frac{[\cancel{I^2} + \cancel{I_x^2} + 2II_x - \cancel{I^2} - \cancel{I_x^2} + 2II_x]\, I}{I^2 + I_x^2 + \cancel{2II_x} + I^2 + I_x^2 - \cancel{2II_x}} \Rightarrow$$ (4 marks)

$$I_2 - I_1 = -\frac{2I_x}{3} + \frac{4II_x}{2(I^2+I_x^2)} \overset{(I_x = yI)}{=} -\frac{2yI}{3} + \frac{4yI^{\cancel{3}\,2}}{2(\cancel{I^2}+y^2\cancel{I^2})} \Rightarrow$$

$$I_2 - I_1 = \left[\frac{2y}{1+y^2} - \frac{2y}{3}\right] I = \frac{6y - 2y(1+y^2)}{3(1+y^2)} I = 2\,\frac{(2y - y^3)}{3(1+y^2)}\, I$$

d) i) Applying the TLP & KCL yields:

$$(2I+I_x+I_z)\times I = (0.95I+I_z)(I+I_x) \Rightarrow$$

$$\Rightarrow \quad I_z = \frac{(2I+I_x+I_z)I}{I+I_x} - 0.95I$$

2 marks

ii)

TLP $\Rightarrow$

$$\frac{I_z}{A}\,\frac{0.7I}{A} = \underbrace{(I+I_x)}_{}\,[4I-\overbrace{(I+I_x)}]$$

$$\left[\frac{(2I+I_x+I_z)I}{I+I_x} - 0.95I\right](0.7I) = A^2\,\overbrace{(I+I_x)}\,(3I-I_x) \Rightarrow$$

$$\Rightarrow \quad \frac{(2I+I_x+I_z)I}{(I+I_x)} = \frac{A^2(I+I_x)(3I-I_x)}{0.7I} + 0.95I \Rightarrow$$

$$\Rightarrow \quad 2I+I_x+I_z = \frac{I+I_x}{I}\,\frac{A^2(I+I_x)(3I-I_x)+0.665I^2}{0.7I}$$

$$\Rightarrow \quad \boxed{I_z = \frac{A^2(I+I_x)^2(3I-I_x)+0.665I^2(I+I_x)}{0.7I^2} - 2I - I_x}$$

3 marks

iii) $$I_z = \frac{A^2(I^2+I_x^2+2I I_x)(3I-I_x)+0.665 I^2(I+I_x)-1.4 I^3-0.7 I^2 I_x}{0.7 I^2} \Rightarrow$$

$$\Rightarrow I_z = \left(\frac{3A^2-0.735}{0.7}\right)I + \left(\frac{5A^2-0.035}{0.7}\right)I_x + \frac{A^2}{0.7}\left[\frac{I_x^2}{I} - \frac{I_x^3}{I^2}\right]$$

for $\underline{A = 0.7}$ $\Rightarrow$

$$I_z = 1.05\, I + 3.45\, I_x + 0.7\left[\frac{I_x^2}{I} - \frac{I_x^3}{I^2}\right]$$

3 marks

Question 5 [Exercise based on taught material] PP.5

a) i) $y = g(V_c)$ for linear operation

$f(x) = I_c = C\,\dot{V}_c \qquad y = K\int x\,dt \Rightarrow$

$$\Rightarrow \frac{dy}{dt} = KX \Rightarrow \frac{dy}{dV_c}\dot{V}_c = KX \Rightarrow$$

$$\Rightarrow \frac{dg(V_c)}{dV_c}\frac{I_c}{C} = KX \Rightarrow$$

$I_c \to f(x)$

condition for linear integration

$$\Rightarrow \boxed{\frac{dg(V_c)}{dV_c}\frac{f(x)}{C} = KX} \Rightarrow$$

$$\Rightarrow f(x) = \frac{KCX}{\frac{dg(V_c)}{dV_c}}$$

2 marks

ii) $g(V_c) = I_T \tanh(aV_c)$

$g(V_c) \to Y$

$$\frac{dg(V_c)}{dV_c} = \frac{I_T}{V_T}\operatorname{sech}^2(aV_c) = \frac{I_T}{V_T}\frac{1}{\cosh^2(aV_c)} \Rightarrow$$

2 marks

$$\frac{dg(V_c)}{dV_c} = \frac{I_T}{V_T}\left[1 - \tanh^2(aV_c)\right] = \frac{\cancel{I_T}}{V_T}\frac{\left[I_T^2 - Y^2\right]}{I_T^{\cancel{2}}} \Rightarrow$$

$\tanh^2(aV_c) \to (Y/I_T)^2$

$$f(x) = \frac{KCX}{\frac{I_T^2 - Y^2}{V_T I_T}} = \boxed{(KCV_T)\frac{I_T X}{I_T^2 - Y^2}}$$

b) [New Exercise based on taught material] pp. 6

$$\dot{x}_1 + \omega_0 x_1 - \omega_0 x_2 = 0$$

$$2\omega_0 x_1 + \dot{x}_2 - \omega_0 x_2 = \omega_0 U(t)$$

$$y = x_1$$

i) Applying Laplace transform:

$$(s+\omega_0)\, X_1(s) - \omega_0\, X_2(s) = 0$$

$$2\omega_0\, X_1(s) + (s-\omega_0) X_2(s) = \omega_0\, U(s)$$

$$D = \begin{vmatrix} s+\omega_0 & -\omega_0 \\ 2\omega_0 & s-\omega_0 \end{vmatrix} = s^2 - \omega_0^2 + 2\omega_0^2 = s^2 + \omega_0^2$$

$$D_{X_1} = \begin{vmatrix} 0 & -\omega_0 \\ \omega_0 U(s) & (s-\omega_0) \end{vmatrix} = \omega_0^2\, U(s)$$

$$\Rightarrow \quad X_1(s) = \frac{\omega_0^2}{s^2+\omega_0^2} U(s) \Rightarrow \frac{X_1(s)\,[\equiv Y(s)]}{U(s)} = \frac{\omega_0^2}{s^2+\omega_0^2}$$

3 marks

ii) $X_1 = I_1 e^{V_1/V_T} \Rightarrow \dot{x}_1 = x_1 \dfrac{\dot{V}_1}{V_T}$

PP. 7

$X_2 = I_2 e^{V_2/V_T} \Rightarrow \dot{x}_2 = x_2 \dfrac{\dot{V}_2}{V_T}$

$\mho = I_4 e^{V_4/V_T} \Rightarrow$

$$\left.\begin{aligned} &\cancel{x_1} \frac{\dot{V}_1}{V_T} + w_0 \cancel{x_1} - w_0 \frac{x_2}{\cancel{x_1}} = 0 \\ &2w_0 \frac{x_1}{x_2} + \cancel{x_2} \frac{\dot{V}_2}{V_T} - w_0 \cancel{x_2} = w_0 \frac{\mho(t)}{x_2} \end{aligned}\right\} \Rightarrow$$

$$\left[\begin{aligned} &C\dot{V}_1 + (CV_T w_0) = (CV_T w_0) \frac{I_2}{I_1} e^{\frac{V_2 - V_1}{V_T}} \\ &C\dot{V}_2 + 2(CV_T w_0) \frac{I_1}{I_2} e^{\frac{V_1 - V_2}{V_T}} = (CV_T w_0) + (CV_T w_0) \frac{I_4}{I_2} e^{\frac{V_4 - V_2}{V_T}} \end{aligned}\right]$$

Setting $I_1 = I_2 = I_0$ & $I_4 = I_S \Rightarrow$

$$\Rightarrow \boxed{\begin{aligned} &C\dot{V}_1 + I_0 = I_0 e^{\frac{V_2 - V_1}{V_T}} \\ &C\dot{V}_2 + 2 I_0 e^{\frac{V_1 - V_2}{V_T}} - I_0 = I_S e^{\frac{V_4 - V_2}{V_T}} \end{aligned}}$$

$Y = X_1 = I_0 e^{V_1/V_T}$ $\qquad \underline{\underline{CV_T w_0 = I_0}}$

6 marks

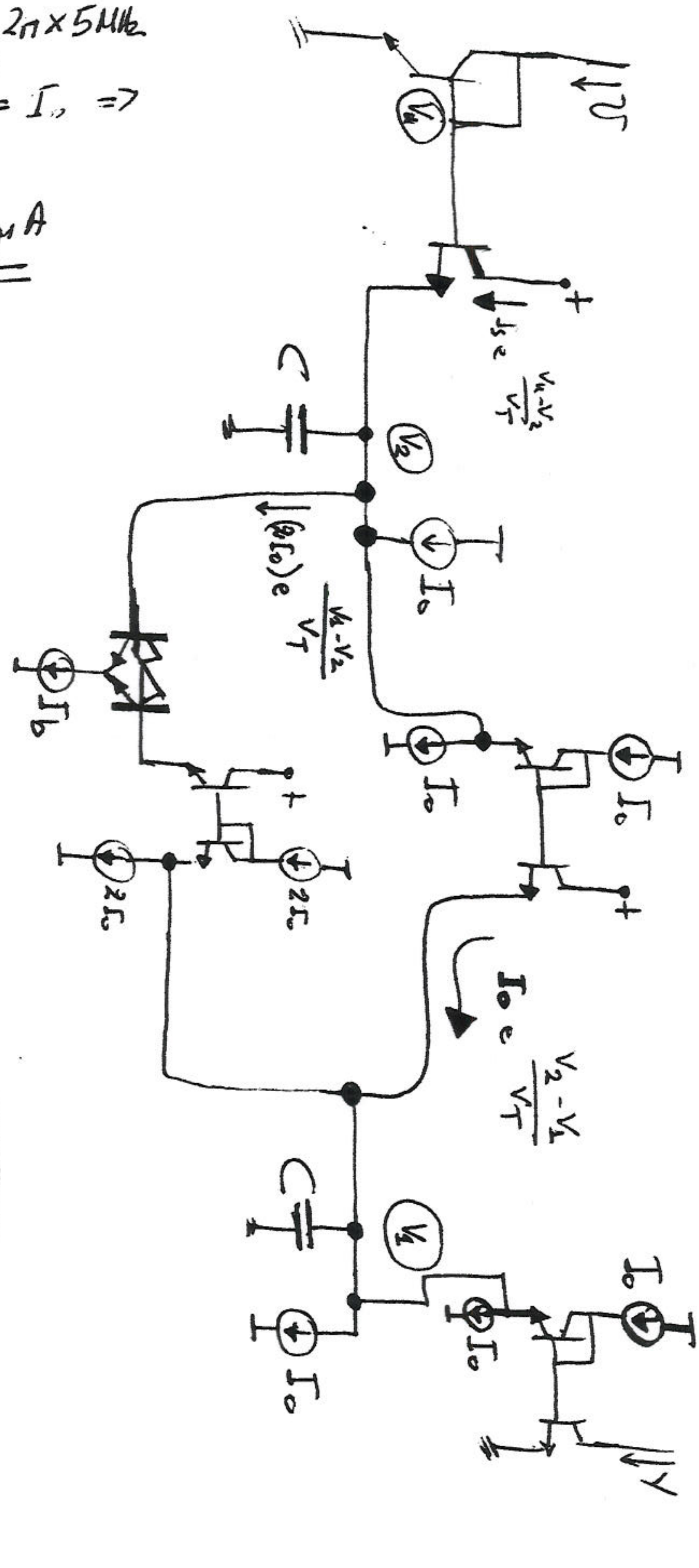
PP. 8

iii)

7.5 p   2π × 5 MHz

$C V_T \omega_0 = I_0 \Rightarrow$

$I_0 = 6.13\,\mu A$



7 marks

Question 6

[Bookwork]

a) Two N-port networks A & B must satisfy the following relation in order to be adjoint:

$$\sum_{n=1}^{N} \left(V_{A_n} I_{B_n} - V_{B_n} I_{A_n} \right) = 0$$

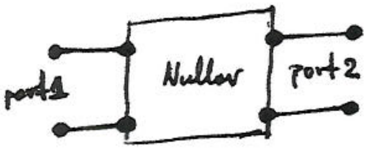
with V_{A_n} denoting the voltage of the n-th port of network A, etc.

Resistor $\quad V_A = R_A I_A$, one port

$$\left.\begin{array}{l} V_A I_B - V_B I_A = 0 \Rightarrow \dfrac{V_B}{I_B} = \dfrac{V_A}{I_A} \\ \text{But } \dfrac{V_A}{I_A} = R_A \end{array}\right\} \Rightarrow V_B = I_B \underset{R_B!}{\underset{\downarrow}{R_A}} \Rightarrow$$

$\Rightarrow$ the adjoint of a resistor is a resistor of the same value $\quad R_A \Longleftrightarrow R_B (= R_A)$

Nullor



$V_A = [V_{A_1} \; V_{A_2}] = [0 \; x]$

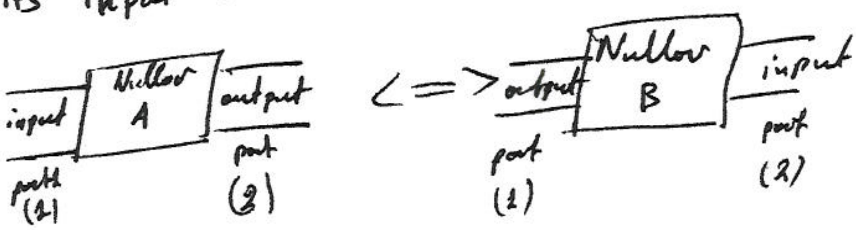
$I_A = [I_{A_1} \; I_{A_2}] = [0 \; x]$

x denotes "any value"

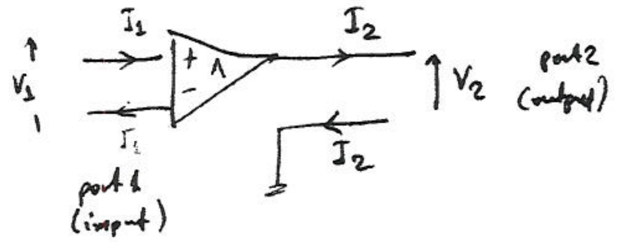
Here I want: $\underset{0}{\underset{\downarrow}{V_{A_1}}} I_{B_1} - V_{B_1} \underset{0}{\underset{\downarrow}{I_{A_1}}} + \underset{x}{\underset{\downarrow}{V_{A_2}}} I_{B_2} - V_{B_2} \underset{x}{\underset{\downarrow}{I_{A_2}}} = 0$

Hence when $V_B = [V_{B_1} \; V_{B_2}] = [x \; 0]$ and $I_B = [I_{B_1} \; I_{B_2}] = [x, 0]$ the relation is satisfied

Thus, the adjoint of a nullor is another nullor with its input & output ports interchanged



Voltage amplifier

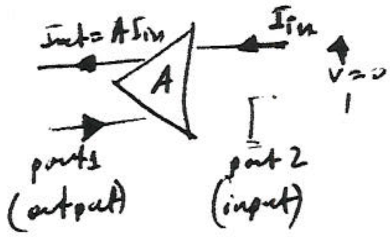


$$V_A = [V_{A_1} \; V_{A_2}] = [V_{in} \; AV_{in}]$$
$$I_A = [I_{A_1} \; I_{A_2}] = [0 \; \times]$$

Hence, $$\underbrace{V_{A_1}'}_{V_{in}} I_{B_1} - V_{B_1} \underbrace{I_{A_1}'}_{0} + \underbrace{V_{A_2}'}_{AV_{in}} I_{B_2} - V_{B_2} \underbrace{I_{A_2}'}_{\times} = 0$$

and when
$$V_B = [V_{B_1} \; V_{B_2}] = [\times \;\; 0]$$
$$I_B = [I_{B_1} \; I_{B_2}] = [-AI_{in} \; I_{in}]$$
then the relation is satisfied

Thus, the adjoint of an ideal voltage amplifier is a current amplifier with input & output ports interchanged:



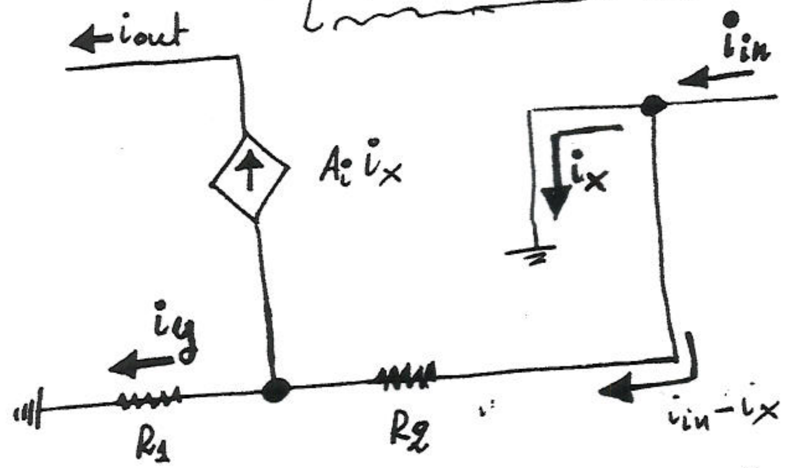
6 marks

[Exercise based on taught material]

PP. 11

b)

i)



$$\left.\begin{array}{ll} (1) & (i_{in} - i_x) R_2 + i_y R_1 = 0 \\ (2) & (i_{in} - i_x) = i_y + A_i i_x \end{array}\right\} \Rightarrow \left.\begin{array}{l} (i_{in} - i_x) R_2 + \left[i_{in} - (A_i + 1) i_x\right] R_1 = 0 \\ i_y = i_{in} - (A_i + 1) i_x \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[R_2 \; i_x + R_1 (A_i + 1) i_x\right] = (R_2 + R_1) i_{in} \Rightarrow$$

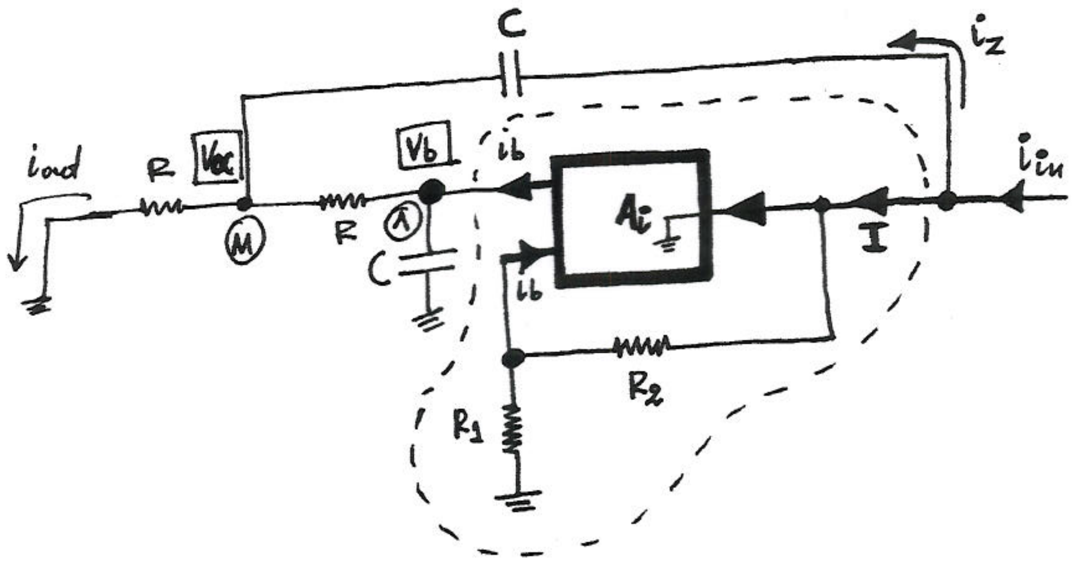
$$\Rightarrow \quad i_x = \frac{R_2 + R_1}{R_2 + R_1(A_i + 1)} = \frac{R_2 + R_1}{R_1\left[A_i + \frac{R_2 + R_1}{R_1}\right]} = \frac{1 + \frac{R_2}{R_1}}{A_i + \left(1 + \frac{R_2}{R_1}\right)} \Rightarrow$$

$$\Rightarrow \quad \frac{i_{out}}{i_{in}} = \frac{A_i\left(1 + \frac{R_2}{R_1}\right)}{A_i + \left(1 + \frac{R_2}{R_1}\right)}$$

When $A_i \gg 1$, then $\frac{i_{out}}{i_{in}} = 1 + \frac{R_2}{R_1}$

3 marks

ii)



When $A_i \gg 1 \Rightarrow i_b = GI = G(i_{in} - i_z) \Rightarrow$

$\Rightarrow i_b = G(i_{in} + C\dot{V}_a) \Rightarrow \boxed{i_b(s) = G[i_{in}(s) + CSV_a(s)]}$ (1)

KCL at (M): $i_z = i_{out} + \frac{V_a - V_b}{R} \Rightarrow$

$\Rightarrow -C\dot{V}_a = \frac{V_a}{R} + \frac{V_a - V_b}{R} \Rightarrow$

$\Rightarrow \frac{V_b}{R} = \frac{2}{R} V_a + C\dot{V}_a \Rightarrow \frac{V_b(s)}{R} = \left[\frac{2}{R} + CS\right] V_a(s) \Rightarrow$

$\Rightarrow \boxed{V_b(s) = [2 + RCS]\, V_a(s)}$ (2)

KCL at (1): $\frac{V_a - V_b}{R} + i_b = C\dot{V}_b \Rightarrow$

$\frac{V_a}{R} + i_b = \frac{V_b}{R} + C\dot{V}_b \Rightarrow \frac{V_a(s)}{R} + i_b(s) = \left[\frac{1}{R} + CS\right] V_b(s) \Rightarrow$

$\Rightarrow \boxed{V_a(s) + R\, i_b(s) = [1 + RCS]\, V_b(s)}$ (3)

Substituting (1) & (2) into (3):

$V_a(s) + RG[i_{in}(s) + CS\,V_a(s)] = [1 + RCS][2 + RCS]\, V_a(s) \Rightarrow$

$G(RC)S\,V_a(s)$

$\Rightarrow [2 + RCS + 2RCS + (RC)^2 s^2 - G(RC)s]\, \overset{R\, i_{out}(s)}{\bcancel{V_a(s)}} = \cancel{R}G\, i_{in}(s) \Rightarrow$

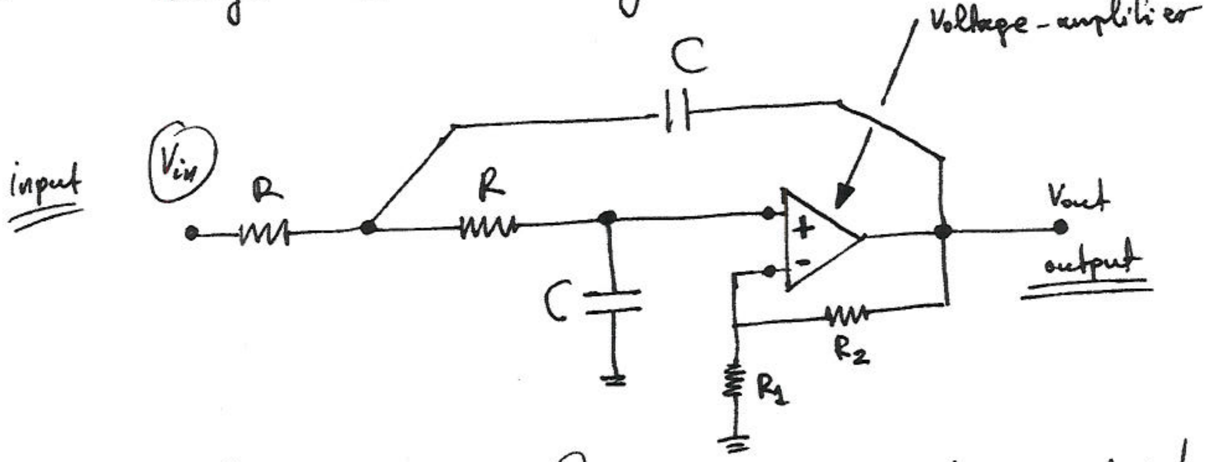
-1

$V_a(s) = R\, i_{out}(s)$

$\Rightarrow \boxed{\frac{i_{out}(s)}{i_{in}(s)} = \frac{G}{(RC)^2 s^2 + (3-G)CS + 1}}$

4 marks

iii) Voltage-mode Sallen-Key filter

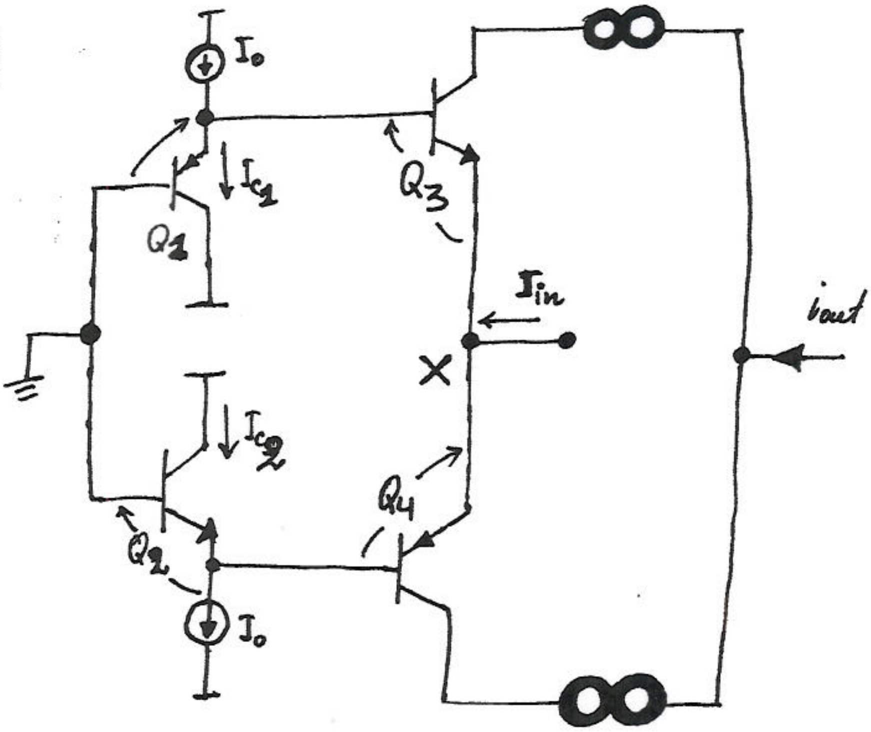


$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{G}{(RC)^2 s^2 + (3-G) C s + 1}$$

because adjoint networks have the same transfer function!

1 marks

c)



[Bookwork]

pp. 14

i)

When $I_{in} = 0$, $I_{c_3} = I_{c_4}$ (assuming infinite beta & equal areas)

By TLP: $I_{c_1} I_{c_2} = I_o^2 = I_{c_3} I_{c_4}$ } =>

=> $I_{c_3} = I_{c_4} = I_o$

DC offset voltage $\quad V_{offset} (\equiv V_X) + V_{be_3} - V_{be_4} = 0$ =>

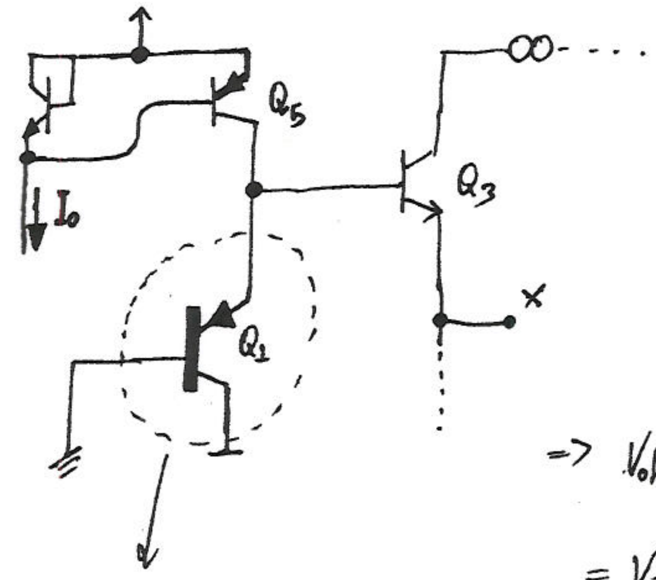
=> $V_{offset} = V_{be_4} - V_{be_3} = V_T \ln\left(\frac{I_o}{I_{sp}} \frac{I_{sn}}{I_o}\right)$ =>

=> $V_{offset} = V_T \ln\left(\frac{I_{sn}}{I_{sp}}\right)$

Small-signal R_X $\quad R_X = r_{e3} // r_{e4} \simeq \frac{V_T}{2 I_o}$

2 marks

ii) Use of scaled current sources:



$$V_{be_6} = V_T \ln\left(\frac{I_o}{I_{sn}}\right)$$

$$I_{C_5} = I_{sp}\, e^{\frac{V_T \ln\left(\frac{I_o}{I_{sn}}\right)}{V_T}} \Rightarrow$$

$$I_{C_5} = \left(\frac{I_{sp}}{I_{sn}}\right) I_o = I_{C_1} \Rightarrow$$

$$\Rightarrow V_{offset} = V_{be_1} - V_{be_3} =$$

$$= V_T \ln\left(\frac{I_{C_1}}{I_{sp}}\right) - V_T \ln\left(\frac{I_o}{I_{sn}}\right) =$$

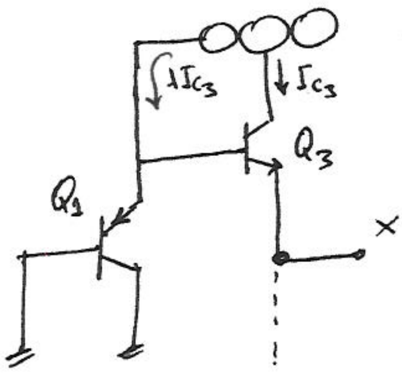
$$= V_T \ln\left[\frac{\cancel{I_{sp}}}{I_{sn}} \frac{I_o}{\cancel{I_{sp}}}\right] - V_T \ln\left(\frac{I_o}{I_{sn}}\right) \Rightarrow$$

"In other words Q_1 "looks like" an npn despite being pnp!

$\Rightarrow V_{offset} \rightarrow 0$

2 marks

iii) Use of local current feedback



$$R_x = \frac{V_x}{I_x} = \frac{V_{be_1} - V_{be_3}}{I_x} \Rightarrow$$

$$R_x = \frac{V_T \ln\left[\frac{\lambda \cancel{I_{C_3}}}{I_{sp}} \frac{I_{sn}}{\cancel{I_{C_3}}}\right]}{I_x} \Rightarrow$$

$$R_x = V_T \ln\left\{\frac{\lambda I_{sn}}{I_{sp}}\right\}$$

$\lambda \rightarrow 1$

$V_{be_1} - V_{be_3} \simeq$ constant

$\Rightarrow R_x \downarrow$

2 marks