IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2012

EEE PART IV: MEng and ACGI

**POWER SYSTEM ECONOMICS**

Tuesday, 1 May 10:00 am

Time allowed: 3:00 hours

**There are FOUR questions on this paper.**

**Answer ALL questions.**

*All questions carry equal marks.*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible First Marker(s) : G. Strbac
Second Marker(s) : B.C. Pal

**Instructions to Candidates**

Answer all four questions.

**Useful equations**

None

## Questions

1. Answer the following subquestions.

   (a) Explain briefly the difference between markets with perfect and imperfect competition. [4]

   (b) Consider a market for electrical energy that is supplied by two generating companies whose cost functions are:

   $$C_A = 36 \cdot P_A \quad [£/h]$$

   $$C_B = 31 \cdot P_B \quad [£/h]$$

   where $P_A$ and $P_B$ are production levels of the two generating companies respectively. The inverse demand function for this market is estimated to be:

   $$\pi = 120 - D \quad [£/MWh]$$

   where $\pi$ and $D$ are the electricity price and demand respectively.

   Assuming a Cournot model of competition, form a table to identify the equilibrium point of this market, i.e. market price, demand quantity and profit of each company for different levels of production for each of the companies. Consider a range of production levels for company A of 20MW, 25MW and 30MW and for company B of 25MW, 30MW and 35MW. [8]

   (c) It is observed that one of the cells of the derived Cournot table involves higher profits, for both competing companies, than the cell corresponding to the equilibrium state. Explain why this cell does not express the equilibrium state of the market. [4]

   (d) State the optimality conditions for the evaluation of the exact equilibrium point of this market (there is no need to solve the expression). [4]

2. Answer the following subquestions.

(a) Explain the significance of economic (merit order) dispatch of generation and the system marginal price. [4]

(b) Four generators are available to supply a demand of $D = 472.5\ MW$. The cost of generating power $C_i(P_i)$ for each of the generators $i$ is:

$$C_1(P_1) = 1000 + 15 \cdot P_1 + 0.20 \cdot P_1^2 \quad [£/h]$$

$$C_2(P_2) = 300 + 17 \cdot P_2 + 0.10 \cdot P_2^2 \quad [£/h]$$

$$C_3(P_3) = 150 + 12 \cdot P_3 + 0.15 \cdot P_3^2 \quad [£/h]$$

$$C_4(P_4) = 500 + 2 \cdot P_4 + 0.07 \cdot P_4^2 \quad [£/h]$$

i. Calculate the optimal production of each generator, the system marginal cost, average cost for each of the generators and their respective profits. [6]

ii. Modify the dispatch found in (b)(i) by applying the following constraints on the maximum output of each of the generators:

$$P_1^{max} = 100 \quad [MW]$$

$$P_2^{max} = 120 \quad [MW]$$

$$P_3^{max} = 160 \quad [MW]$$

$$P_4^{max} = 200 \quad [MW]$$

Determine the system marginal cost, the system average cost, the total cost of operating the system and the profits of each generator. [6]

iii. Explain why the profit of generator 1 found in (b)(ii) is negative and describe how this would be avoided in practice. [4]

3. Answer the following subquestions.

(a) Answer the following questions with brief statements: [4]

i. What are the benefits of having a transmission system?

ii. What are the main characteristics of the transmission as a business?

iii. Why is the transmission business regulated?

(b) Consider two-area system, with demand in Area 1 of $D_1 = 670$ $[MW]$ and demand in Area 2 of $D_2 = 80$ $[MW]$. Generator 1 is located in Area 1 and Generator 2 is located in Area 2, with their respective cost functions given by:

$$C_1(P_1) = 143 + 10.66 \cdot P_1 + 0.026 \cdot P_1^2 \qquad [£/h]$$

$$C_2(P_2) = 221 + 12.48 \cdot P_2 + 0.013 \cdot P_2^2 \qquad [£/h]$$

i. Given that the existing capacity of the transmission line connecting the two areas is $F^{max} = 200$ $[MW]$, calculate the constraint costs and locational marginal prices for this system. [4]

ii. The regulators are considering reinforcing the transmission link. Write the expression for network constraint costs as a function of transmission capacity added ($\Delta F$). [4]

iii. The annuitized cost of reinforcing the transmission line is given by

$$C_{inv} = k \cdot L \cdot \Delta F$$

where $k = 175.2$ $[£/MW \cdot km \cdot year]$ (the annuitized cost of reinforcing 1 km of the transmission line for 1 MW) and $L = 300km$ and $\Delta F$ [MW] represents the network capacity added. Plot the cost of network constraints, the cost of network investment and the total system cost as a function of network capacity added ($\Delta F$). Calculate the optimal capacity of the line that should be added between the two areas. [5]

iv. What would be the revenue and profit of investing and operating this transmission link on a merchant basis? [3]

4. Answer the following subquestions.

For the system shown in Figure 4.1, and the accompanied data given in Tables 4.1, 4.2 and 4.3 answer the following questions:

i. Ignoring the impact of transmission constraints, determine the minimum cost dispatch of the generators, the corresponding network flows and show that transmission line 1-2 would be overloaded. [5]

ii. Change the dispatch by increasing generation at bus 3 to eliminate this overload and show that the marginal price at bus 2 would be 11.25 [£/MWh]. [5]

iii. In order to manage the exposure to price fluctuations, the consumer at bus 2 has made a "contract for difference" with the generators at bus 1 at a strike price of 9.5 [£/MWh] and has also purchased a transmission congestion contract that gives the right to transport 60 [MW] from bus 1 to bus 2. Calculate the income from the transmission congestion contract received by the consumer at bus 2 and calculate the unit price that consumers at each bus will pay under this condition. [5]

iv. Because of maintenance on the transmission circuit between busses 2 and 3, the flow through this circuit has to be restricted to 65 [MW]. Show that the price at bus 2 will drop to 5 [£/MWh]. Calculate how much the load at bus 2 will now pay for the electricity consumed, given the new conditions and its contract portfolio. [5]

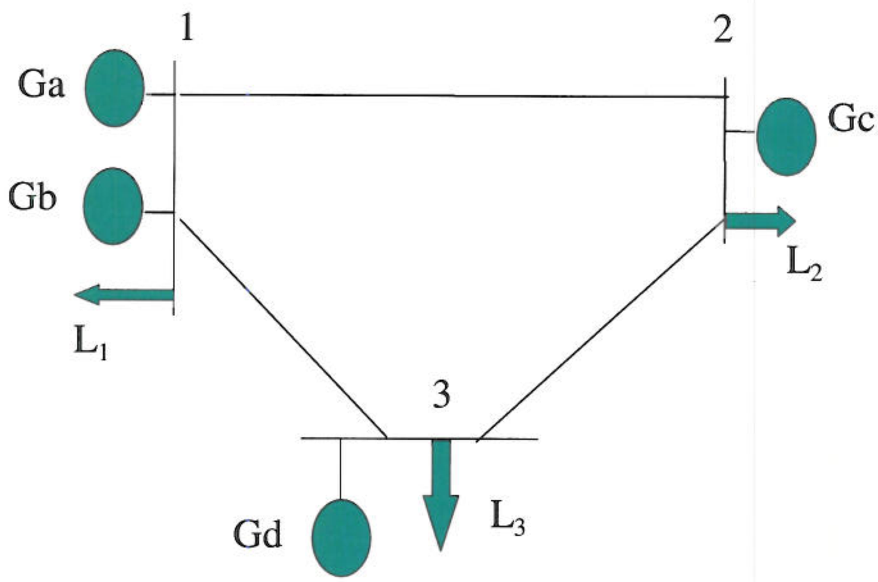



Figure 4.1: The System layout.

| Bus | Load [MW] |
|---|---|
| 1 | 50 |
| 2 | 60 |
| 3 | 300 |

Table 4.1: Load data.

| Generator | Capacity [MW] | Marginal cost [£/MWh] |
|---|---|---|
| Ga | 140 | 7.5 |
| Gb | 285 | 6 |
| Gc | 90 | 14 |
| Gd | 85 | 10 |

Table 4.2: Generator data.

| Line | Per unit reactance | Capacity [MW] |
|---|---|---|
| 1-2 | 0.2 | 126 |
| 1-3 | 0.2 | 250 |
| 2-3 | 0.1 | 100 |

Table 4.3: Line data.

EE4-51

1/12

Power System Economics

Solutions 2012

**Solution** Q1

**(a)** [Answer comes from "Fundamentals of Power Systems Economics", pp. 39-40]

In markets with perfect competition, no market participant has the ability to influence the market price through its individual actions. In other words, market price is a parameter over which participants have no control. This assumption is valid if the number of market participants is large and if none of them controls a large proportion of the production or consumption. Under these circumstances, any supplier who asks more than the market price and any consumer who offers less than the market price will simply be ignored because others can replace their contribution to the market. The price is thus set by the interactions of the buyers and the sellers, taken as groups, and all participants act as price takers.

Under imperfect competition, some producers and/or consumers -called strategic players- control a share of the market that is large enough to enable them to exert market power and manipulate the prices. Prices can be manipulated either by withholding quantity (physical withholding) or by raising (for sellers) / decreasing (buyers) the asking/offered price (economic withholding).

**(b)** In the Cournot model of competition the state of the market is determined by the production decisions made by each firm. We summarize the possible outcomes using a table where all the cells in a column correspond to a given production by company A and the cells in a row correspond to a given production by company B.

Each cell contains four pieces of information arranged in the following format:

| | |
|---|---|
| $D$ | $\Omega_A$ |
| $\Omega_B$ | $\pi$ |

Where:

$\pi$ price $[\frac{£}{MWh}]$

$D$ demand $[MW]$

$\Omega_A$ profit made by company A [£]

$\Omega_B$ profit made by company B [£]

Given the productions $P_A$ and $P_B$ of the two companies, the other quantities are calculated as follows:

$$D = P_A + P_B$$

$$\pi = 120 - D$$

$$\Omega_A = P_A * (\pi - 36)$$

$$\Omega_B = P_B * (\pi - 31)$$

Based on the above, the table expressing the Cournot model of competition for the conditions of the problem is shown below. A Nash equilibrium state implies that given the production level of either of the two companies in this state, the other company cannot achieve a higher profit by following a production level different than the one at the equilibrium. It can thus be observed from the table below that the cell corresponding to $P_A = 25MW$ and $P_B = 30MW$ is an equilibrium point. In other words, given that $P_A = 25MW$, company B achieves its highest profit by producing $P_B = 30MW$ AND given that $P_B = 30MW$, company A achieves its highest profit by producing $P_A = 25MW$.

| $P_B/P_A$ | 20 | | 25 | | 30 | |
|---|---|---|---|---|---|---|
| 25 | 45<br>1100 | 780<br>75 | 50<br>975 | 850<br>70 | 55<br>850 | 870<br>65 |
| 30 | 50<br>1170 | 680<br>70 | 55<br>1020 | 725<br>65 | 60<br>870 | 720<br>60 |
| 35 | 55<br>1190 | 580<br>65 | 60<br>1015 | 600<br>60 | 65<br>840 | 570<br>55 |

**(c)** The cell corresponding to $P_A = 20MW$ and $P_B = 25MW$ involves higher profits than the cell corresponding to $P_A = 25MW$ and $P_B = 30MW$ (identified as the equilibrium state in (b)). However, this cell does not satisfy the properties of Nash equilibrium since: a) given that $P_A = 20MW$, company B can achieve higher profits by producing $P_B = 35MW$ ($\Omega_B = 1190£$) than $P_B = 25MW$ ($\Omega_B = 1100£$) and also b) given that $P_B = 25MW$, company A can achieve higher profits by producing $P_A = 30MW$ ($\Omega_A = 870£$) than $P_A = 20MW$ ($\Omega_A = 780£$).

**(d)** Each generating company is trying to maximize its profit which is equal to the difference between its revenue and its cost:

$$\Omega_A = P_A * \pi(D) - C_A(P_A)$$

$$\Omega_B = P_B * \pi(D) - C_B(P_B)$$

However, we cannot treat these two maximizations as separate optimization problems since they are linked through the demand:

$$P_A + P_B = D$$

$$\pi = 120 - D$$

We can get around this difficulty by expressing the price as a function of the production of the two generators:

$$\pi = 120 - P_A - P_B$$

The profits of the two companies can then be expressed as:

$$\Omega_A = P_A * (120 - P_A - P_B) - 36 * P_A = -P_A^2 + (84 - P_B) * P_A$$

$$\Omega_B = P_B * (120 - P_A - P_B) - 31 * P_B = -P_B^2 + (89 - P_A) * P_B$$

The optimality condition for the maximization problem of company A is:

$$\frac{\partial \Omega_A}{\partial P_A} = -2 * P_A - P_B + 84 = 0$$

While the optimality condition for the maximization problem of company B is:

$$\frac{\partial \Omega_B}{\partial P_B} = -P_A - 2 * P_B + 89 = 0$$

(We only consider the partial derivatives of the profit of a generator with respect to its own output because that is the only variable over which it has control)

Putting these optimality conditions in matrix form we get:

$$\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} P_A \\ P_B \end{bmatrix} = \begin{bmatrix} -84 \\ -89 \end{bmatrix}$$

Solving this linear system we get the following productions for the equilibrium point:

$$P_A = 26.333\ [MW]$$

$$P_B = 31.333\ [MW]$$

From which we can easily find:

$$\pi = 62.333\ [\frac{£}{MWh}]$$

$$D = 57.667\ [MW]$$

$$\Omega_A = 693.44\ [£]$$

$$\Omega_B = 981.78\ [£]$$

**Solution Q2**

**(a)** The merit order dispatch signifies the procedure of dispatching the total load of the system to the available generators on the basis of their bids or marginal costs and in a way that minimizes the total cost of supplying the demand. Once this least-cost dispatch has been determined, the system marginal price signifies the marginal cost of serving an additional unit of demand.

**(b)** In the optimal solution of the problem, the marginal cost of each generator is equal to the system marginal cost $\lambda$ yielding:

$$\frac{\partial C_1(P_1)}{\partial P_1} = 15 + 0.4 * P_1 = \lambda \rightarrow P_1 = \frac{\lambda - 15}{0.4} \quad (1)$$

$$\frac{\partial C_2(P_2)}{\partial P_2} = 17 + 0.2 * P_2 = \lambda \rightarrow P_2 = \frac{\lambda - 17}{0.2} \quad (2)$$

$$\frac{\partial C_3(P_3)}{\partial P_3} = 12 + 0.3 * P_3 = \lambda \rightarrow P_3 = \frac{\lambda - 12}{0.3} \quad (3)$$

$$\frac{\partial C_4(P_4)}{\partial P_4} = 2 + 0.14 * P_4 = \lambda \rightarrow P_4 = \frac{\lambda - 2}{0.14} \quad (4)$$

The satisfaction of the system demand implies that:

$$P_1 + P_2 + P_3 + P_4 = D = 472.5\ [MW] \quad (5)$$

Substituting (1)-(4) in (5) gives for the system marginal cost:

$$\frac{\lambda - 15}{0.4} + \frac{\lambda - 17}{0.2} + \frac{\lambda - 12}{0.3} + \frac{\lambda - 2}{0.14} = 472.5 \rightarrow \lambda = 36.119\ [\frac{£}{MWh}] \quad (6)$$

Substituting (6) in (1)-(4) yields for the optimal production of each generator:

$$P_1 = 52.798\ [MW] \quad (7)$$

$$P_2 = 95.595\ [MW] \quad (8)$$

$$P_3 = 80.397\ [MW] \quad (9)$$

$$P_4 = 243.707\ [MW] \quad (10)$$

Substituting (7)-(10) in the cost functions of the generators yields for the total cost of operating the system:

$$C_{tot} = C_1(P_1) + C_2(P_2) + C_3(P_3) + C_4(P_4) = 12417.68\ [£] \quad (11)$$

The system average cost is:

$$\alpha = \frac{C_{tot}}{P_1 + P_2 + P_3 + P_4} = 26.281\ [\frac{£}{MWh}] \quad (12)$$

The average cost for each of the generators is:

$$\alpha_1 = \frac{C_1(P_1)}{P_1} = 44.499\ [\frac{£}{MWh}] \quad (13)$$

5

$\alpha_2 = \frac{C_2(P_2)}{P_2} = 29.698\ [\frac{£}{MWh}]$ (14)

$\alpha_3 = \frac{C_3(P_3)}{P_3} = 25.925\ [\frac{£}{MWh}]$ (15)

$\alpha_4 = \frac{C_4(P_4)}{P_4} = 21.111\ [\frac{£}{MWh}]$ (16)

The profit for each generator $\Omega_i$ is given by the difference between its respective revenue and cost:

$\Omega_1 = \lambda * P_1 - C_1(P_1) = -442.467\ [£]$ (17)

$\Omega_2 = \lambda * P_2 - C_2(P_2) = 613.841\ [£]$ (18)

$\Omega_3 = \lambda * P_3 - C_3(P_3) = 819.543\ [£]$ (19)

$\Omega_4 = \lambda * P_4 - C_4(P_4) = 3657.522\ [£]$ (20)

**(c)** Given that $P_4 > P_4^{max}$ the solution calculated in **(a)** is not feasible when taking into account the given maximum output limits of the generators. The optimal solution can now be calculated by fixing the output of generator 4 to:

$P_4' = P_4^{max} = 200\ [MW]$ (21)

and satisfying the remaining demand:

$D' = D - P_4' = 272.5\ [MW]$ (22)

through the remaining generators 1,2 and 3. Following the same method as in (a), we get:

$\frac{\lambda'-15}{0.4} + \frac{\lambda'-17}{0.2} + \frac{\lambda'-12}{0.3} = 272.5 \rightarrow \lambda' = 40.154\ [\frac{£}{MWh}]$ (23)

Substituting (23) in (1)-(3) yields for the optimal production of each generator:

$P_1' = 62.885\ [MW]$ (24)

$P_2' = 115.77\ [MW]$ (25)

$P_3' = 93.847\ [MW]$ (26)

Substituting (21) and (24)-(26) in the cost functions of the generators yields for the total cost of operating the system:

$C_{tot}' = C_1(P_1') + C_2(P_2') + C_3(P_3') + C_4(P_4') = 12639.792\ [£]$ (27)

The system average cost is:

$a' = \frac{C_{tot}'}{P_1'+P_2'+P_3'+P_4'} = 26.751\ [\frac{£}{MWh}]$ (28)

The average cost for each of the generators is:

$$a_1' = \frac{C_1(P_1')}{P_1'} = 43.479\ [\frac{£}{MWh}] \quad (29)$$

$$a_2' = \frac{C_2(P_2')}{P_2'} = 31.168\ [\frac{£}{MWh}] \quad (30)$$

$$a_3' = \frac{C_3(P_3')}{P_3'} = 27.675\ [\frac{£}{MWh}] \quad (31)$$

$$a_4' = \frac{C_4(P_4')}{P_4'} = 18.5\ [\frac{£}{MWh}] \quad (32)$$

The profit for each generator is given by the difference between its respective revenue and cost:

$$\Omega_1' = \lambda' * P_1' - C_1(P_1') = -209.096\ [£] \quad (33)$$

$$\Omega_2' = \lambda' * P_2' - C_2(P_2') = 1040.27\ [£] \quad (34)$$

$$\Omega_3' = \lambda' * P_3' - C_3(P_3') = 1171.079\ [£] \quad (35)$$

$$\Omega_4' = \lambda' * P_4' - C_4(P_4') = 4330.8\ [£] \quad (36)$$

**(d)** The negative profit of generator 4 means that the operating cost of the latter is larger than its revenue in the optimal solution and thus it experiences economic losses at the optimal solution (does not recover its operating costs). This is happening since marginal system cost pricing does not guarantee the recovery of the fixed (no-load) costs of the participating generators in combination with the high fixed costs of generator 1. This lack of recovery of some of the generators' fixed costs will mean in the long term that these generators will abandon the business resulting potentially in higher total costs of operating the system or even security problems (inability of the system to satisfy the demand). Therefore, a suitable mechanism compensating the generators' unrecovered fixed costs is required in top of the marginal system cost pricing.

**Solution Q3**

**(a)** A transmission system is required to efficiently and securely transport electric power from generating plants to distribution systems and large consumers so as to: a) minimise fuel costs in the production of electricity by allowing the utilization of those available generating sources having the lowest marginal costs, b) interconnect systems and generating plants to reduce overall generating requirements by taking advantage of the diversity of peak loads (peak loads occur at different times in different systems) and the diversity of generation outages and reserve requirements and c) to enable a competitive trading of electric energy in the marketplace.

Transmission business is characterized by capital intensity, economies of scale, long-lived assets with small re-sale value and long-lead times of construction. The capital intensity means that it is practically unrealistic to construct multiple transmission networks and thus the nature of the transmission business is physically monopolistic. Therefore, the transmission business should be subject to regulation. The regulators fulfil their responsibilities by determining the maximum revenue of the transmission network company and certain quality of supply standards the company has to respect.

**(b)** The cost of network constraints is equal to the difference between the optimal total cost of operating the system with and without taking into account the network constraints. When the network constraints are not taken into account, the optimal solution for the operation of the system is given by the solution of the system of equations (1)-(2):

$$\frac{\partial C_1(P_1^{unc})}{\partial P_1} = \frac{\partial C_2(P_2^{unc})}{\partial P_2} \quad (1)$$

$$P_1^{unc} + P_2^{unc} = D_1 + D_2 \rightarrow P_1^{unc} + P_2^{unc} = 750 \quad (2)$$

which yields:

$$P_1^{unc} = 273.327\ [MW] \quad (3)$$

$$P_2^{unc} = 476.654\ [MW] \quad (4)$$

$$C_{tot}^{unc} = C_1(P_1^{unc}) + C_2(P_2^{unc}) = 14122.294\ [£] \quad (5)$$

When the network constraints are taken into account, the optimal solution in the unconstrained case is not feasible since the flow from busbar 2 to busbar 1 is:

$$F = D_1 - P_1^{unc} = P_2^{unc} - D_2 = 396.673\ [MW] > F^{max} \quad (6)$$

Since the flow cannot be larger than $F^{max}$, the total cost of operating the system in this case is calculated by substituting (7) and (8):

$$P_1^{con} + F^{max} = D_1 \rightarrow P_1^{con} = 470\ [MW] \quad (7)$$

$$P_2^{con} - F^{max} = D_2 \rightarrow P_2^{con} = 280\ [MW] \quad (8)$$

into the given cost functions of the two generators yielding:

$C_{tot}^{con} = C_1(P_1^{con}) + C_2(P_2^{con}) = 15631.2\ [£]$ (9)

Therefore, the cost of network constraints is calculated as (through (5) and (9)):

$C_{net} = C_{tot}^{con} - C_{tot}^{unc} = 1508.906\ [\frac{£}{h}]$ (10)

Since the network line is congested, the marginal prices in the two buses are different. An additional unit of demand in Bus 1 will be satisfied by Generator 1 and thus the locational marginal price at bus 1 is:

$\pi_1 = \frac{\partial C_1(P_1^{con})}{\partial P_1} = 35.1\ [\frac{£}{MWh}]$ (11)

An additional unit of demand in Bus 2 will be satisfied by Generator 2 and thus the locational marginal price at bus 2 is:

$\pi_2 = \frac{\partial C_2(P_2^{con})}{\partial P_2} = 19.76\ [\frac{£}{MWh}]$ (12)

**(c)** The total cost of operating the system when network constraints as a function of $\Delta F$ are taken into account is calculated by substituting (13) and (14):

$P_1^{con} + F^{max} + \Delta F = D_1 \rightarrow P_1^{con} = 470 - \Delta F$ (13)

$P_2^{con} - F^{max} - \Delta F = D_2 \rightarrow P_2^{con} = 280 + \Delta F$ (14)

into the given cost functions of the two generators yielding:

$C_{tot}^{con} = C_1(P_1^{con}) + C_2(P_2^{con}) = 15631.2 - 15.34 * \Delta F + 0.039 * \Delta F^2$ (15)

Therefore, the cost of network constraints is calculated as (through (5) and (15)):

$C_{net} = C_{tot}^{con} - C_{tot}^{unc} = 1508.906 - 15.34 * \Delta F + 0.039 * \Delta F^2$ (16)

**(d)** The optimal level of network capacity that should be built corresponds to the minimum of the total costs composed of cost of network constraints and cost of network investment:

$\min_{\Delta F}(C_{net} + C_{inv})$ (17)

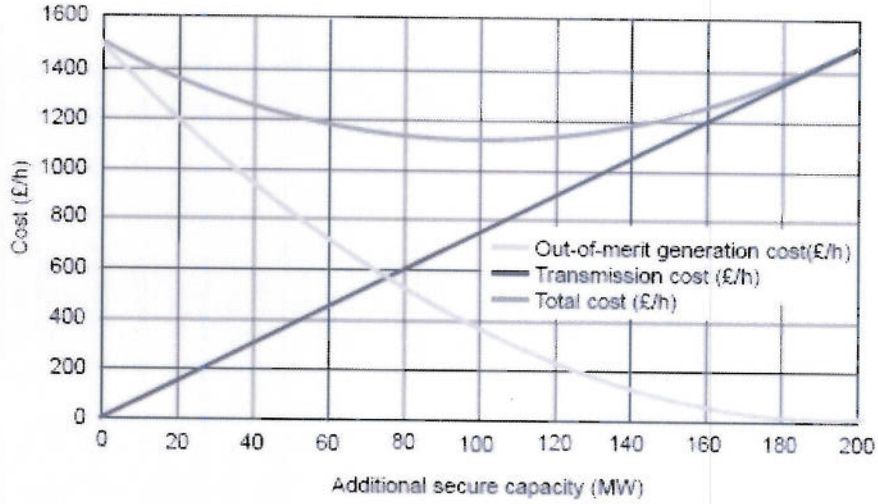
In order to express these two components in the same temporal basis, the cost of network investment -which is given per year- is expressed per hour by dividing it by the number of hours (8760) in a year. Based on this modification, the given data regarding $k$ and $L$ and (16), equation (17) becomes:

$$\min_{\Delta F}(1508.906 - 15.34 * \Delta F + 0.039 * \Delta F^2 + \frac{175.2 * 300}{8760} * \Delta F) \rightarrow$$

$\min_{\Delta F}(1508.906 - 9.34 * \Delta F + 0.039 * \Delta F^2)$ (18)

whose solution is given by:

$\frac{\partial(C_{net}+C_{inv})}{\partial \Delta F} = 0 \rightarrow \Delta F = 119.744\ [MW]$ (19)




**(e)** When the optimal network capacity is built, the new line capacity will be:

$F^{max'} = F^{max} + \Delta F = 319.744\ [MW]$ (20)

The dispatch of the two generators in this case is given by:

$P_1^{con'} + F^{max'} = D_1 \rightarrow P_1^{con'} = 350.256\ [MW]$ (21)

$P_2^{con'} - F^{max'} = D_2 \rightarrow P_2^{con'} = 399.744\ [MW]$ (22)

Thus, the locational marginal prices at the two buses are:

$\pi_1' = \frac{\partial C_1(P_1^{con'})}{\partial P_1} = 28.873\ [\frac{£}{MWh}]$ (23)

$\pi_2' = \frac{\partial C_2(P_2^{con'})}{\partial P_2} = 22.873\ [\frac{£}{MWh}]$ (24)

On a merchant basis, the revenue of operating this transmission link is equal to the congestion surplus, given by:

$CS = (\pi_1' - \pi_2') * F = 1918.464\ [\frac{£}{h}]$ (25)

The investment cost of this link in hourly basis is according to the given data:

$C_{inv} = \frac{k*L}{8760} * F^{max'} = 1918.464\ \left[\frac{£}{h}\right]$ (26)

The profit of investing and operating this transmission link is therefore zero.

**Solution Q4**

a. Since the generators' marginal costs are fixed, the minimum cost dispatch can be calculated by dispatching the generators in an ascending marginal cost order until the total demand of $D_1 + D_2 + D_3 = 410\ [MW]$ is satisfied. We thus get for the minimum cost dispatch:

$P_a = 125\ [MW]$ (1)
$P_b = 285\ [MW]$ (2)
$P_c = 0\ [MW]$ (3)
$P_d = 0\ [MW]$ (4)
$F_{12} = 156\ [MW]$ (5)
$F_{13} = 204\ [MW]$ (6)
$F_{23} = 96\ [MW]$ (7)

Since $F_{12} > F_{12}^{max}$ ($F_{12}^{max} = 126\ [MW]$ according to the data) the branch 1-2 is overloaded.

b. In order to eliminate this overload the output of generator $G_d$ is increased by $x\ [MW]$ and the output of generator $G_a$ (most expensive generator in bus 1) is reduced by $x\ [MW]$. In order to cause a reduction of $F_{12} - F_{12}^{max} = 30\ [MW]$ $x$ should be:

$x = \frac{30}{0.4} = 75\ [MW]$ (8)

Thus, the new generators' dispatch becomes:

$P_a' = 50\ [MW]$ (9)
$P_b' = 285\ [MW]$ (10)
$P_c' = 0\ [MW]$ (11)
$P_d' = 75\ [MW]$ (12)

The new network flows therefore are:
$F_{12}' = 126\ [MW]$ (13)
$F_{13}' = 159\ [MW]$ (14)
$F_{23}' = 66\ [MW]$ (15)

which satisfy the lines' capacities constraints.

We need a total increase of 1 MW in the outputs of $G_a$ and $G_d$ (17) but without increasing the power flow in line 1-2 (18):

$\Delta P_a + \Delta P_d = 1\ [MW]$ (17)

$0.6 * \Delta P_a + 0.2 * \Delta P_d = 0$ (18)

which yields:

$\Delta P_a = -0.5\ [MW]$ (19)

$\Delta P_d = 1.5\ [MW]$ (20)

When these outputs' modifications are carried out the price at bus 2 is:

$$\pi'_2 = -0.5 * MC_A + 1.5 * MC_D = 11.25\ [\frac{£}{MWh}] \quad (21)$$

Which is accepted as the final value of price at bus 2 since it is lower than the value in (16).

c. Although the price at bus 2 is $11.25\ [\frac{£}{MWh}]$ (21), the contract for difference that Load at bus 2 has made at the price of $9.5\ [\frac{£}{MWh}]$ means that it will pay for satisfying its demand:

$$C_2 = 11.25 * D_2 + (9.5 - 11.25) * D_2 = 570\ [\frac{£}{h}] \quad (22)$$

$$I_2 = 60 * (\pi'_2 - \pi_1) \quad (23)$$

Since generator $G_a$ is the cheapest generator whose output has not reached its maximum limit (9)-(12), the price at bus 1 can be easily shown to be:

$$\pi_1 = 7.5\ [\frac{£}{MWh}] \quad (24)$$

Substituting (21) and (24) in (24) gives:

$$I_2 = 225\ [\frac{£}{h}] \quad (25)$$

Therefore, the net cost that load at bus 2 has to pay is (from (22) and (25)):

$$C_2^{net} = C_2 - I_2 = 345\ [\frac{£}{h}] \quad (26)$$

and the unit price that demand at bus 2 will pay is:

$$\pi_2^{net} = \frac{C_2^{net}}{D_2} = 5.75\ [\frac{£}{MWh}] \quad (27)$$

d.

$$y = \frac{1}{0.4} = 2.5\ [MW] \quad (28)$$

Therefore the new dispatch is:

$$P''_a = 47.5\ [MW] \quad (29)$$
$$P''_b = 285\ [MW] \quad (30)$$
$$P''_c = 0\ [MW] \quad (31)$$
$$P''_d = 77.5\ [MW] \quad (32)$$

and the new network flows are:

12/12

$F''_{12} = 125\ [MW]$ (33)
$F''_{13} = 157.5\ [MW]$ (34)
$F''_{23} = 65\ [MW]$ (35)

Price at bus 2 is calculated by following the same methodology as in b. Increase of 1 MW of the output of $G_a$ and $G_d$ can be easily shown to reduce the flow in line 2-3 per 0.4 MW and 0.8 MW respectively as shown on figure in b.

We need a total increase of 1 MW in the outputs of $G_a$ and $G_d$ (17) but without increasing the power flow in line 2-3 (18):

$\Delta P'_a + \Delta P'_d = 1\ [MW]$ (36)

$-0.4\Delta P'_a - 0.8 * \Delta P'_d = 0$ (37)

which yields:

$\Delta P'_a = 2\ [MW]$ (38)
$\Delta P'_d = -1\ [MW]$ (39)

When these outputs' modifications are carried out the flow in line 1-2 becomes (according to the figure in b.):

$F'''_{12} = F''_{12} + \Delta P'_a * 0.6 + \Delta P'_d * 0.2 = 126\ [MW]$ (40)

and thus capacity constraint of line 1-2 is not violated. The price at bus 2 is:

$\pi''_2 = 2 * MC_A - 1 * MC_D = 5\ [\frac{£}{MWh}]$ (41)

Based on its contract for differences, load at bus 2 will now pay for satisfying its demand

$C'_2 = 5 * D_2 + (9.5 - 5) * D_2 = 570\ [\frac{£}{h}]$ (42)

Its income from the transmission congestion contract is now:

$I'_2 = 60 * (\pi''_2 - \pi_1) = -150\ [\frac{£}{h}]$ (43)
Therefore, the net cost that load at bus 2 has to pay is (from (42) and (43)):

$C_2^{net\prime} = C'_2 - I'_2 = 720\ [\frac{£}{h}]$ (44)

and the unit price that demand at bus 2 will pay is:

$\pi_2^{net\prime} = \frac{C_2^{net\prime}}{D_2} = 12\ [\frac{£}{MWh}]$ (45)