

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

EEE/ISE PART I: MEng, BEng and ACGI

Corrected Copy

ANALOGUE ELECTRONICS 1

Friday, 3 June 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.

Answer Q1 and any two of questions 2-4.

Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	A.S. Holmes, A.S. Holmes
	Second Marker(s) :	S. Lucyszyn, S. Lucyszyn

1. **This question is compulsory.** You should attempt all six parts. State clearly any assumptions made in your calculations.

- a) For the BJT in Figure 1.1, choose the values of R_B and R_C to give a collector current of 1 mA and a collector voltage of +5 V.

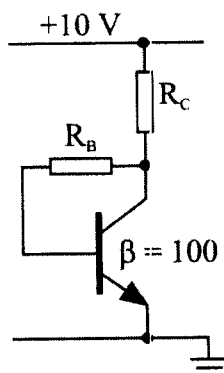


Figure 1.1

[6]

- b) Determine the drain current and drain voltage of the MOSFET in Figure 1.2. What is the minimum supply voltage for which the MOSFET will remain active?

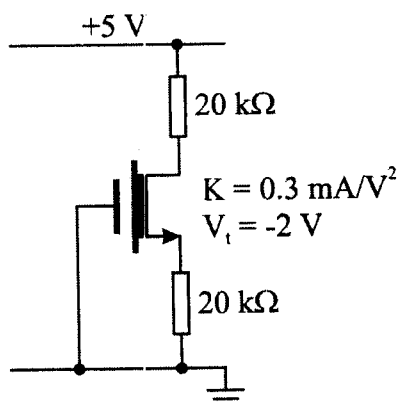


Figure 1.2

[8]

- c) Show that the output current I of a simple BJT current mirror comprising two matched transistors is related to the input current I_{REF} by the following equation:

$$I = \frac{I_{REF}}{1 + 2/\beta} \quad [4]$$

- d) Sketch circuits showing a Class B push-pull output stage and one possible configuration for a Class AB push-pull output stage. Explain briefly the advantages of the Class AB configuration. [10]

Question 1 continues on the next page...

Question 1 continued:

- e) Figure 1.3 shows an n-channel, depletion mode MOSFET connected as an active load. Sketch the I-V characteristic of this device for $V \geq 0$, and annotate your graph to identify clearly the triode and active regions.

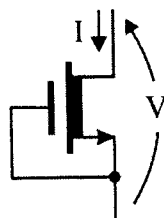


Figure 1.3

[5]

- f) Using the resistance reflection rule, or otherwise, determine the small-signal output resistance of the circuit in Figure 1.4.

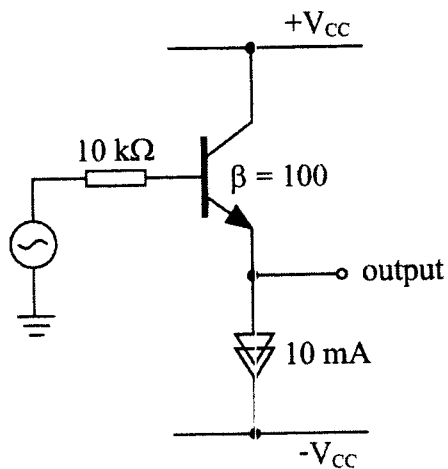


Figure 1.4

[7]

2. The signal v_s from a high-impedance source is to be amplified using a simple amplifier based on an n-channel enhancement mode MOSFET, as shown in Figure 2.1.

- Determine the quiescent values of the drain current and drain voltage, and show that the MOSFET is saturated under quiescent conditions. State clearly any assumptions made in your calculations. [10]
- Draw a small-signal equivalent circuit of the amplifier, and calculate its small-signal voltage gain. Also calculate the small-signal input resistance of the amplifier, taking into account the bias resistors, and hence determine the overall in-circuit voltage gain v_{out}/v_s in the mid-band i.e. at frequencies for which the coupling capacitor C is effectively short-circuit. [15]
- By what ratio would the gain v_{out}/v_s be increased if the $10\text{ k}\Omega$ resistor were replaced by a depletion type active load with an Early voltage of 80 V , assuming the same quiescent current? [5]

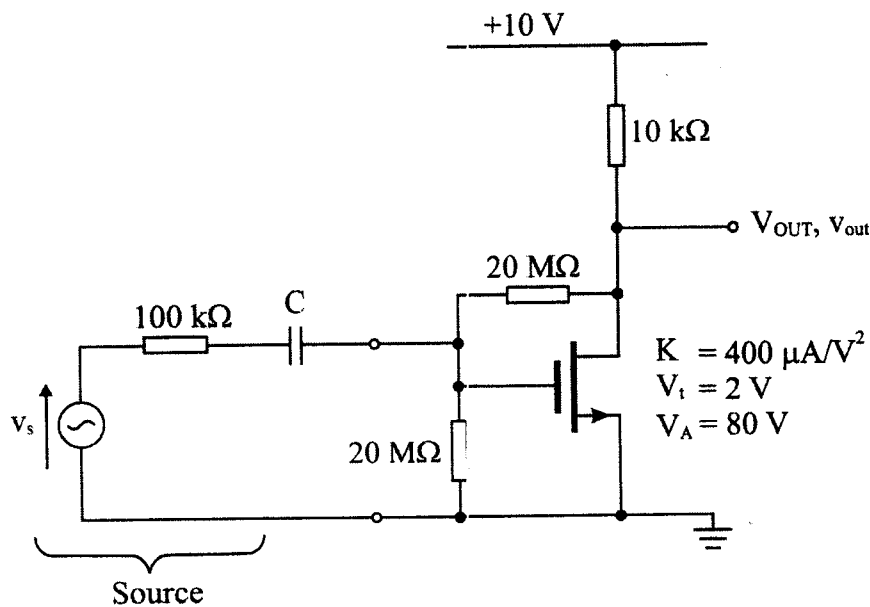


Figure 2.1

3. This question relates to the Wilson current mirror shown in Figure 3.1. You may assume that the three transistors are matched.

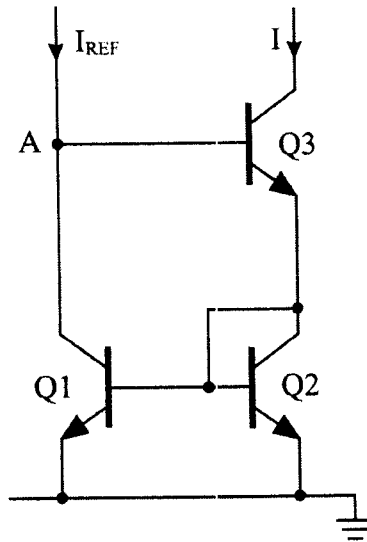
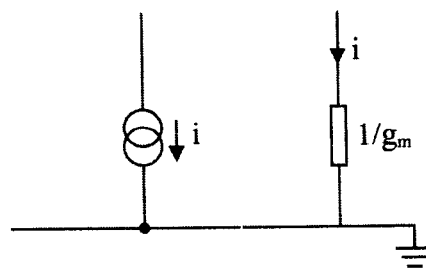


Figure 3.1

- a) By applying Kirchhoff's current law at node A, and making use of the fact that Q1 and Q2 form a simple BJT current mirror, show that the currents I and I_{REF} are related as follows:

$$\frac{I_{REF}}{I} = \left(1 + \frac{2}{\beta(\beta + 2)} \right) \approx 1 + 2/\beta^2 \quad [10]$$

- b) Draw a small-signal equivalent circuit (SSEC) of the simple BJT current mirror formed by Q1 and Q2, and show that, by assuming $r_{be} \gg 1/g_m$ and neglecting output resistances, it can be reduced to the following approximate form:



[10]

- c) Draw a small-signal equivalent circuit of the complete Wilson current mirror, including the output resistance of Q3, but using the SSEC given in part b) to represent Q1 and Q2. By applying a test signal to the output terminal, or otherwise, show that the small-signal output resistance R_o of the circuit you have drawn is:

$$R_o = \left[1 + \frac{1}{2} \left(\beta + \frac{1}{g_m r_o} \right) \right] r_o \approx \frac{1}{2} \beta r_o \quad [10]$$

4. Figure 4.1 shows a differential amplifier in which all four transistors are matched. You may neglect base currents in any large-signal calculations, and in parts a), b) and c) you may assume that the transistors have infinite output resistance.

- a) Choose the value of R to give a tail current of $I = 0.5$ mA. Assuming this value of R , what is the quiescent output voltage of the circuit when $V_{IN1} = V_{IN2}$ and all transistors are active? [6]
- b) Show that, provided none of the transistors enters saturation, the output voltage V_{OUT} , for arbitrary V_{IN1} and V_{IN2} , may be expressed as:

$$V_{OUT} = \frac{10}{1 + \exp(-40V_D)} \quad [10]$$

where $V_D = V_{IN1} - V_{IN2}$ is the differential input voltage.

- c) By evaluating $d(V_{OUT})/dV_D$ at $V_D = 0$, or otherwise, determine the small-signal differential gain of the amplifier. Also calculate the differential input resistance for small signals if $\beta = 200$. [10]
- d) What will be the common-mode rejection ratio of the amplifier if the transistors have an Early voltage of 100 V? You may neglect the output resistances of $Q1$ and $Q2$ when evaluating the common-mode gain. [4]

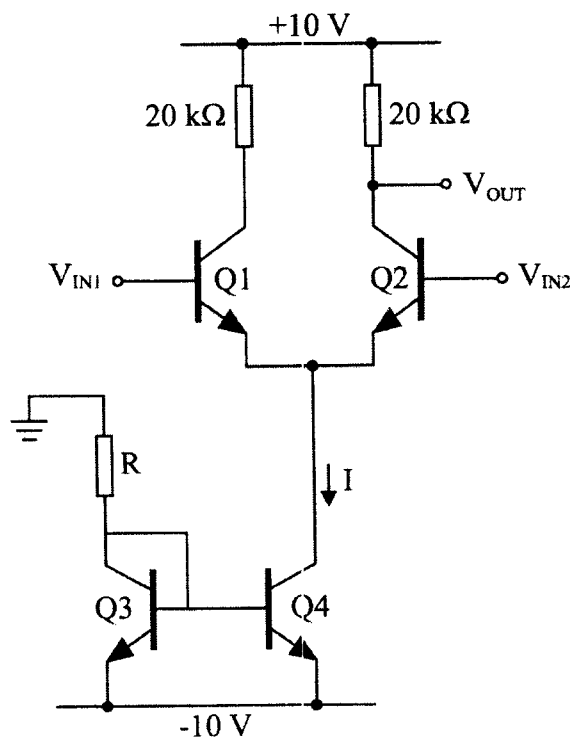


Figure 4.1

1 (a) For $I_c = 1\text{mA}$ require $I_B = 1\text{mA}/100 = 10\text{ }\mu\text{A}$

$$I_B = \frac{V_C - V_B}{R_B} \Rightarrow R_B = \frac{5 - 0.7}{10^{-5}} = \underline{\underline{430\text{ k}\Omega}}$$

$$R_c \text{ carries emitter current} \Rightarrow R_c = \frac{10 - 5}{1\text{mA}} \times \frac{100}{101} = \underline{\underline{4.95\text{ k}\Omega}}$$

Assumed $V_{CE} = 0.7\text{ V}$

[6]

(b) $V_S = I_D R_S \dots \textcircled{1}$ and $V_D = 5 - I_D R_D \dots \textcircled{2}$

and, assuming FET is active, $I_D = K(V_{GS} - V_t)^2 = K(2 - V_S)^2 \dots \textcircled{3}$

Eliminating I_D from $\textcircled{1}$ & $\textcircled{3} \Rightarrow V_S = R_S K (2 - V_S)^2$

$$R_S K = 6\text{ V} \Rightarrow 6V_S^2 - 25V_S + 24 = 0$$

$$\Rightarrow V_S = \frac{25 \pm 7}{12} = \cancel{2.67} \text{ or } 1.5$$

(sub threshold)

From $\textcircled{1}, \textcircled{2}$ $I_D = V_S / R_S = \underline{\underline{75\text{ }\mu\text{A}}}$ and $V_D = 5 - 1.5 = \underline{\underline{3.5\text{ V}}}$

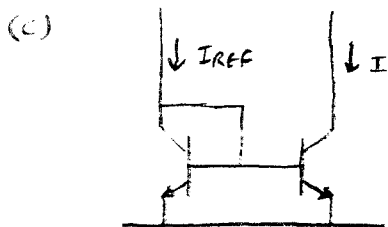
Check Mode: $V_{DS} = 3.5 - 1.5 = 2$, $V_{GS} - V_t = -1.5 - -2 = +0.5$

\Rightarrow ACTIVE assumption was correct.

Min supply voltage is when $V_{DS} = 0.5\text{ V}$, in which case

$$V_{\text{supply}} = V_S + V_{DS} + I_D R_D = 1.5 + 0.5 + 1.5 = \underline{\underline{3.5\text{ V}}}$$

[8]



$$I_{REF} = I_c + 2I_B \quad (\text{KCL at LHS})$$

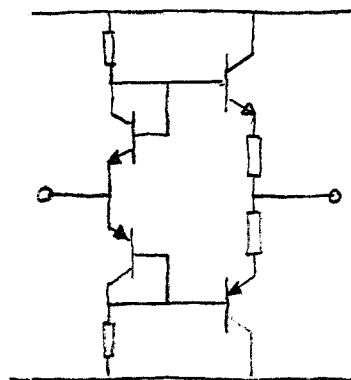
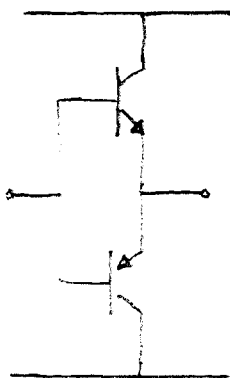
$$= I_c (1 + 2/\beta)$$

$$I = I_c = I_{REF} / (1 + 2/\beta)$$

[4]

(d) Class B:

Class AB: (one possible soln)

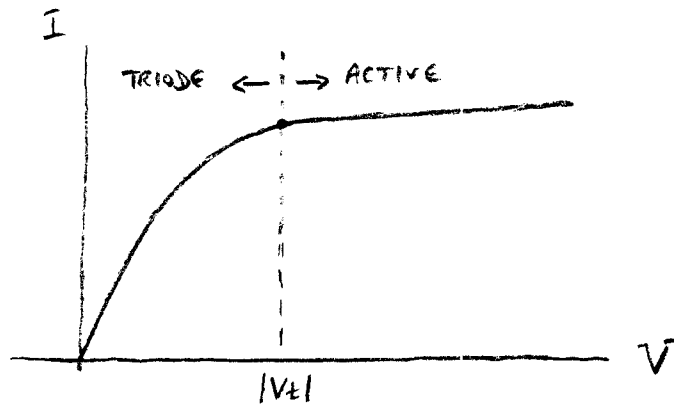


Cont'd...

1(d) cont'd Main advantage of class AB over class B is that cross-over distortion is much lower because at least one o/p transistor is conducting at all times. Advantage over class A is lower power consumption

[10]

(e)



[5]

(f) using resistance reflection rule :

$$R_o = \frac{R_s + r_{\pi}}{(1+\beta)} = \frac{R_s}{(1+\beta)} + r_e$$

where R_s = source resistance = $10 \text{ k}\Omega$

$$r_e = \text{emitter resistance} = V_T / I_E = \frac{25 \text{ mV}}{10 \text{ mA}} = 2.5 \Omega$$

$$\Rightarrow R_o = \frac{10 \text{ k}}{101} + 2.5 = \underline{\underline{101.5 \Omega}}$$

[7]

2 (a) $I_G = 0 \Rightarrow$ Bias network imposes constraint $V_G = V_D/2$
 Know that $V_G > V_t$ (since, if FET were not conducting, V_G would be +5V which would not be self-consistent)
 and $V_D = 2V_G > V_G - V_t \Rightarrow$ FET is ACTIVE

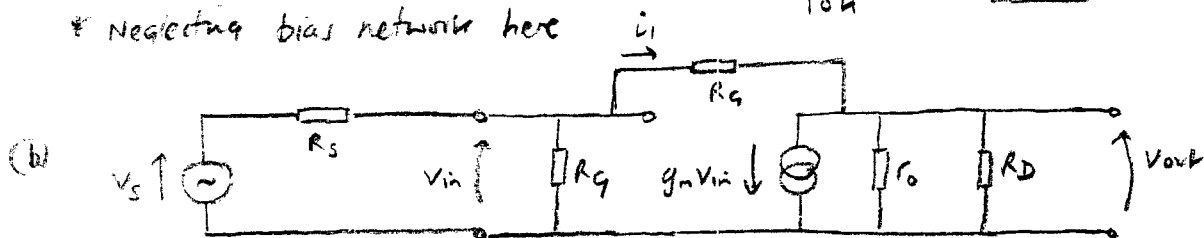
* I_D satisfies: $I_D = \frac{+10 - V_D}{R_D} = K \left(\frac{V_D}{2} - V_t \right)^2$

$K R_D = 4V \Rightarrow V_D^2 - 7V_D + 6 = 0$, $V_D = 1$ or $V_D = 6$

$I_D = \frac{10 - 6}{10k} = \underline{0.4 \text{ mA}}$

[10]

* Neglecting bias network here



KCL @ o/p $\Rightarrow g_m V_{in} + \frac{V_{out}}{r_o} + \frac{V_{out}}{R_D} + \frac{V_{out} - V_{in}}{R_G} = 0$

$\Rightarrow A_v = \frac{V_{out}}{V_{in}} = -(g_m - 1/R_G) \cdot (r_o \parallel R_D \parallel R_G)$

$g_m = 2\sqrt{K I_D} = 0.8 \text{ mA/V}$, $r_o = V_A/I_D = 200 \text{ k}\Omega$

$R_D = 10 \text{ k}\Omega$, $R_G = 20 \text{ M}\Omega \Rightarrow A_v = \underline{-7.61}$

$i_i = \frac{V_{in} - V_{out}}{R_G} = \frac{V_{in}(1 - A_v)}{R_G} \Rightarrow V_{in}/i_i = R_G/(1 - A_v)$
 and $R_{in} = R_G \parallel [R_G/(1 - A_v)] = \underline{2.08 \text{ M}\Omega}$

Overall gain: $\frac{V_{out}}{V_s} = A_v \times \frac{R_{in}}{R_{in} + R_s} = \underline{-7.26}$

[15]

(c) With active load $R_D \rightarrow V_A/I_D = 200 \text{ k}\Omega$

$\Rightarrow A_v \rightarrow -(g_m - 1/R_G)(r_o/2 \parallel R_G) = -79.6$

$R_{in} \rightarrow 248 \text{ k}\Omega$

Overall gain $\frac{V_{out}}{V_s} \rightarrow A_v \times \frac{R_{in}}{R_{in} + R_s} = -56.7$

[5]

Ratio: $\frac{56.7}{7.26} = 7.8$

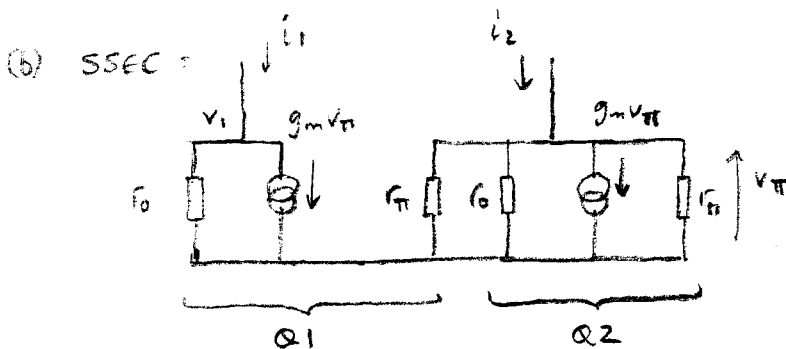
3 (a) KCL at A: $I_{REF} = I_{B3} + I_{C1} = I/\beta + I_{C1} \dots \textcircled{1}$

$Q1, Q2$ form CM $\Rightarrow I_{C1} = \frac{I_{E3}}{1 + 2/\beta} = \frac{I}{\alpha} \cdot \frac{1}{(1 + 2/\beta)} \dots \textcircled{2}$

$\textcircled{1}$ in $\textcircled{2} \Rightarrow \frac{I_{REF}}{I} = \frac{1}{\beta} + \frac{(\beta+1)}{\beta} \cdot \frac{1}{1 + 2/\beta}$

$$= \frac{(2+\beta) + \beta(\beta+1)}{\beta(2+\beta)} = \frac{\beta^2 + 2\beta + 2}{\beta(2+\beta)}$$

$$= 1 + \frac{2}{\beta(\beta+2)} \approx 1 + \frac{2}{\beta^2} \quad \text{since } \beta \gg 1 \quad [10]$$



KCL @ RHS:

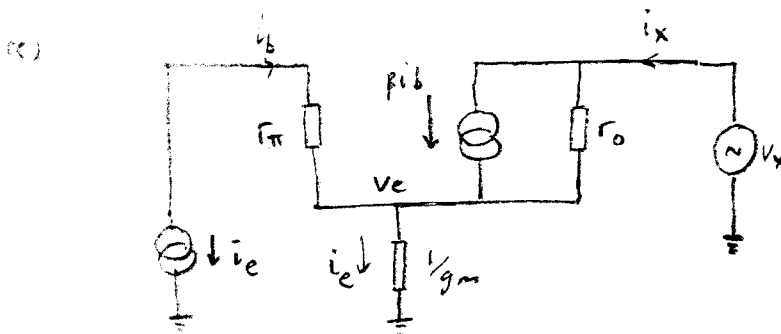
$$i_2 = 2\frac{v_{\pi}}{r_{\pi}} + \frac{v_{\pi}}{r_o} + g_m v_{\pi}$$

$$\frac{v_{\pi}}{i_2} = \left(\frac{r_{\pi}}{2} \parallel r_o \parallel \frac{1}{g_m} \right)$$

If $\frac{1}{g_m} \ll r_{\pi}$, and r_o neglected then $v_{\pi} = \left(\frac{1}{g_m} \right) i_2$
and RHS can be approximated as resistor $1/g_m$

KCL @ LHS: $i_1 = g_m v_{\pi} + \frac{v_1}{r_o} \approx g_m v_{\pi}$

$$= g_m v_{\pi} \approx g_m \cdot \left(\frac{1}{g_m} \right) i_2 = i_2 \quad [10]$$



KCL for Q3:

$$i_b + i_x = i_e$$

$$\text{and } i_b = -i_e$$

$$\Rightarrow i_x = 2i_e \dots \textcircled{1}$$

KCL @ o/p: $i_x = \frac{v_x - v_e}{r_o} + \beta i_b = \frac{v_x}{r_o} - \frac{i_e}{g_m r_o} - \beta i_e \dots \textcircled{2}$

$\textcircled{1} \rightarrow \textcircled{2} \Rightarrow i_x \left[1 + \frac{1}{2} \left[\beta + \frac{1}{g_m r_o} \right] \right] = \frac{v_x}{r_o}$

$$= \frac{v_x}{i_x} = R_o = r_o \left[1 + \frac{\beta}{2} + \frac{1}{2g_m r_o} \right] \approx \frac{r_o \beta}{2} \quad [10]$$

since $\beta \gg 1$ and $g_m r_o \gg 1$

4 (a) Assuming ideal current mirror $I = (0 - V_{B3})/R$

$V_{B3} \approx -9.3V$, so for $I = 0.5mA$ require $R = \underline{18.6k\Omega}$
 when $V_{in1} = V_{in2}$, $I_{C1} = I_{C2} = I/2 = 0.25mA$

$$V_{out} = +10 - I_{C2} \times 20k = \underline{+5V} \quad [6]$$

(b) Large signal eqns for $Q1$ & $Q2$ ($L \rightarrow R$) are:

$$I_{C1} = I_S \exp\left(\frac{V_{in1} - V_E}{V_T}\right) \dots (1) \quad I_{C2} = I_S \exp\left(\frac{V_{in2} - V_E}{V_T}\right) \dots (2)$$

where V_E = (common) emitter voltage of $Q1/Q2$

$$(1)/(2) \Rightarrow I_{C1}/I_{C2} = \exp(V_D/V_T) \dots (3)$$

$$KCL \text{ at common emitter: } I_{C1} + I_{C2} = I \dots (4)$$

$$(3) + (4) \Rightarrow I_{C2} [1 + \exp(V_D/V_T)] = I$$

$$V_{out} = V_{CC} - I_{C2} R_C = V_{CC} - \frac{I R_C}{1 + \exp(V_D/V_T)}$$

$$V_{CC} = 10V, I R_C = 10V$$

$$\Rightarrow V_{out} = 10 \left[1 - \frac{1}{1 + \exp(V_D/V_T)} \right] = \frac{10}{1 + \exp(-40V_D)}$$

$$\text{since } V_T = \frac{1}{40} V \quad [10]$$

(c) From given equation $\frac{dV_{out}}{dV_{in}} = \frac{-10}{[1 + \exp(-40V_D)]^2} \cdot -40 \exp(-40V_D)$

$$\text{Evaluating at } V_D = 0 \Rightarrow A_v = \underline{+100}$$

$$\text{Differential i/p resistance is } R_i = 2r_{be} = \frac{2\beta}{g_m} = \frac{2\beta V_T}{I_C}$$

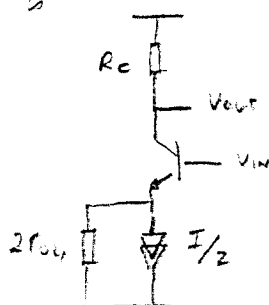
$$\text{where } I_C = 0.25mA$$

$$\Rightarrow R_i = \underline{40k\Omega} \quad [10]$$

(d) Common mode gain depends on current mirror v/p resistance for half ckt is

for this ckt (C.E. amp w/ R_E)

$$A_{cm} \approx -\frac{R_C}{2r_{o4}} = -\frac{R_C I}{2V_A} = -\frac{R_C I_C}{V_A} = -0.05$$



$$\Rightarrow CMRR = \frac{100}{0.05} = 2000 \quad [4]$$

$$[= 66dB]$$