Imperial College London

MEng (Engineering) Examination 2016

Year 1

AE1-102 Aircraft Performance

Friday 3rd June 2016: 14.00 to 15.00
[1 hour]

There are *TWO* questions.
Candidates may obtain full marks for complete answers to ***BOTH*** questions.

A table of the standard atmosphere and a data sheet are provided.

The use of lecture notes is NOT allowed.

1. (a) Show that for propeller driven aircraft, the power to weight ratio (P/W) necessary to maintain steady level flight is given by:

$$\frac{P}{W} = \frac{V_{imD}}{2\sqrt{\sigma}\eta_p(L/D)_{\max}}\left(\overline{V}^3 + \frac{1}{\overline{V}}\right)$$

[30%]

(b) Using the relation above show that the power required is minimum when

$$C_L = \sqrt{\frac{3C_{D_o}\pi A\!\!R}{k}}.$$

[30%]

(c) A light aircraft, powered by a non-supercharged piston engine is under development. The aircraft has the following design characteristics:

- aircraft mass $m = 970$ kg,
- wing loading $W/S = 880$ N/m^2,
- zero-lift drag coefficient $C_{D_0} = 0.05$,
- wing aspect-ratio $A\!\!R = 7.5$,
- loading efficiency $k = 1.4$ and
- propeller efficiency $\eta_p = 0.8$.

The design specifications require an absolute ceiling of 4 km. If the variation of maximum engine power output with altitude in the troposphere is given by

$$P/P_0 = \sigma^{1.1}$$

what is the minimum engine power required at sea level? [40%]

2. (a) What are the different types of drag typically acting on an aircraft? Plot the typical variation of each type of drag with respect to Mach number. Comment on what typically happens to drag around $M = 0.8$ for airliners. [30%]

(b) Figure 1 shows the specific excess power (P_S) plot obtained when flight testing a jet trainer at a wing loading $W/S = 4937$ N/m^2. The aircraft has the following characteristics:

- wing reference area $S = 16.7$ m^2,
- wing aspect ratio $\text{Æ} = 7$,
- loading efficiency $k = 1.25$,
- specific fuel consumption $c = 0.45$ 1/hour
- maximum sea-level thrust to weight ratio $F_{N_0}/W = 0.352$.

The aircraft's thrust may be assumed to be independent of Mach number and dependent on altitude only. In the troposphere you may use $F_N/F_{N_0} = \sigma^{0.7}$ and $F_N/F_{N_1} = \sigma$ in the stratosphere.

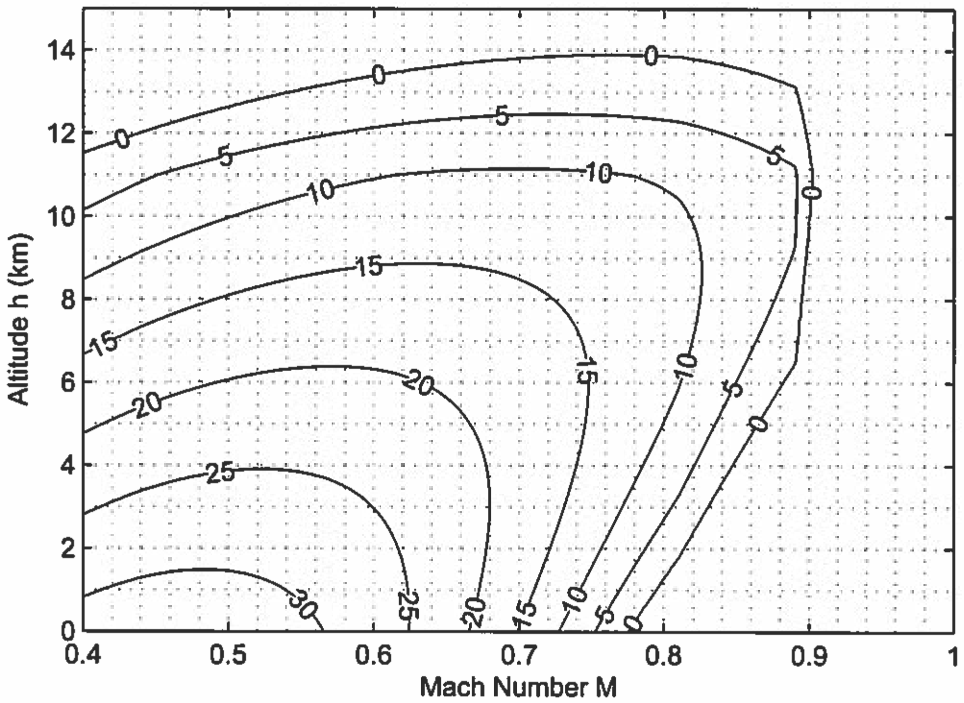

Figure 1: Variation of specific excess power (in m/s) with altitude and Mach number

Based on figure 1:

i. approximately calculate the aircraft's minimum drag Equivalent Airspeed V_{imD}; [20%]

ii. approximately calculate the aircraft zero lift drag coefficient C_{D_0}; [30%]

iii. what is the maximum endurance of this aircraft if 1400 kg of fuel is available? State any assumptions made. [20%]

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Setter: Errikos Levis

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Write on this side only (in ink) between the margins, not more than one solution per sheet please. Solutions must be signed and dated by both exam setter and referee.

Marks

1a) Find an expression for P/w in steady level flight. (n=1)

Here Lift = Weight & Thrust = Drag. (1)

& for props

$Power_{req} = Vel \times Thrust / \eta_P$, where η_P = prop efficiency.

Hence

$$\frac{P}{W} = \frac{V\,T}{W\eta_P} \quad \text{and from (1)} \quad \frac{P}{W} = \frac{V \cdot D}{L\,\eta_P} = \frac{V}{L/D\,\eta_P}$$

To express in non-dimensional terms

$$\frac{L}{D} = \frac{W}{D_{min}}\frac{D_{min}}{D} = \left(\frac{L}{D}\right)_{max}\frac{1}{\bar{D}}, \quad \bar{D} = \frac{1}{2}\left(\bar{V}^2 + \frac{1}{\bar{V}^2}\right)$$

$$V = \frac{1}{\sqrt{\sigma}}V_i = \frac{V_{imD}\bar{V}}{\sqrt{\sigma}}$$

Combining

$$\frac{P}{W} = \frac{V_{imD}\bar{V}\bar{D}}{\eta_P\sqrt{\sigma}(L/D)_{max}} = \frac{V_{imD}}{2\eta_P(L/D)_{max}}\left(\bar{V}^3 + \frac{1}{\bar{V}}\right)$$

30

b) To find minimum P/w, differentiate by $\bar{V}$ and set to zero

$$\frac{d(P/w)}{d\bar{V}} = \frac{V_{imD}}{2\eta_P\sqrt{\sigma}(L/D)_{max}}\left(3\bar{V}^2 - \bar{V}^{-2}\right) = 0$$

$$\Rightarrow 3\bar{V}^2 - \bar{V}^{-2} = 0 \Rightarrow \bar{V}^4 = 1/3 \Rightarrow \bar{V}_{minP} = \sqrt[4]{1/3}$$

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Remembering that $L = W$

$$C_L = \frac{W}{\frac{1}{2}\rho_0 V_i^2 S} = \frac{W}{\frac{1}{2}\rho_0 V_{imD}^2 \bar{V}^2 S}\ , \quad \text{where } V_{imD} = \sqrt{\frac{2W}{\rho_0 S}}\sqrt[4]{\frac{k}{\pi AR\, C_{D_0}}}$$

$$V_{imD}^2 = \frac{2W}{\rho_0 S}\sqrt{\frac{k}{\pi AR\, C_{D_0}}}$$

$$\bar{V}^2 = \sqrt{1/3}$$

$$C_L = \frac{W}{\frac{\rho_0 S}{2}\frac{2W}{\rho_0 S}\sqrt{\frac{k}{\pi AR\, C_{D_0}}}\sqrt{\frac{1}{3}}} \Rightarrow$$

Hence $C_L @ \bar{V} = \sqrt[4]{1/3} = \dfrac{1}{\sqrt{\dfrac{k}{3\pi AR\, C_{D_0}}}} = \sqrt{3\pi AR\, C_{D_0}/k}$ 30

c) Absolute ceiling will be reached when Power available = Min power required.

$$P = P_0\,\sigma^{1.1} = W\left(P/W\right)_{min}$$

$$\Rightarrow P_0 = \frac{W}{\sigma^{1.1}}\left(P/W\right)_{min} \qquad W = 970 \times 9.81 = 9515.7\,N$$

$$\sigma_{4k} = 0.6688$$

$$\left(\frac{P}{W}\right)_{min} = \frac{V_{imD}}{2\sqrt{\sigma}\,\eta_p\,(L/D)_{max}}\underbrace{\left[\left(\frac{1}{3}\right)^{3/4} + 3^{1/4}\right]}_{\approx 1.7548} = 5.7914\ m/s\ (W/N)$$

$$V_{imD} = \sqrt{\frac{2W}{\rho_0 S}}\sqrt[4]{\frac{k}{\pi AR\, C_{D_0}}} = 39.6\ m/s$$

$$(L/D)_{max} = \frac{1}{2}\sqrt{\frac{\pi AR}{k\, C_{D_0}}} = 9.17$$

$$\Rightarrow P_0 = \frac{9515.7 \times 5.7914}{0.6688^{1.1}} = 85.7\ kWatt$$

40

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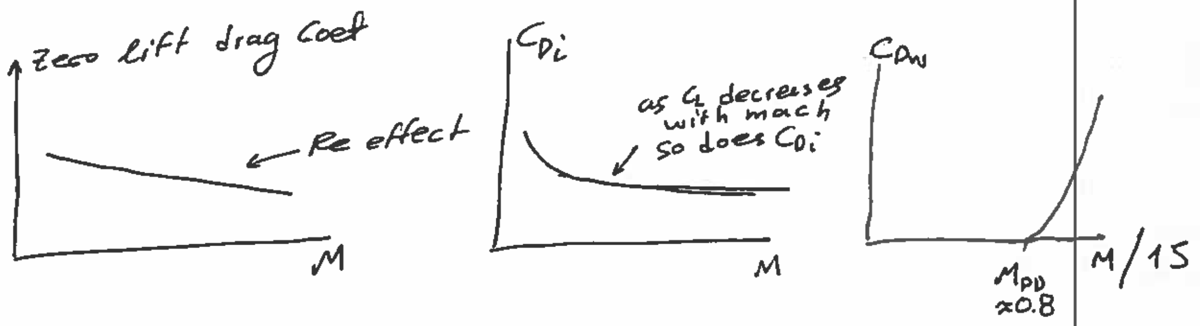
③

2a) There are three major drag components.

1) Zero lift drag: caused by viscous effects & typicaly includes pressure drag effects /5

2) Lift dependent/induced drag: caused by the finite nature of wings, is proportional to C_L^2. /5

3) Wave drag: Appears at high subsonic/transonic Mach numbers & is due to the formation of shocks due to local supersonic flow regions. /5



M /15

2bi) Use the P_s plot to approximate V_{imD}.

When aircraft is at its abs. ceiling it is operating @ min Drag = $M_{minD} \approx 0.77$ @ 14km

$a_{14km} = 295.5$ & $\sigma_{14km} = 0.1852$

$\Rightarrow V_{imD} = \sqrt{\sigma}\, V_{mD} = \sqrt{\sigma}\, a M_{mD} = \underline{\underline{97.9 \text{ m/s}}}$

20

(ii) At the ceiling $F_N = D_{min} = \frac{W}{(L/D)_{max}}$ as $(L/D)_{max} = \frac{W}{D_{min}}$

As in stratosphere (14km)

$F_N = \frac{F_N}{F_{N_1}} \frac{F_{N_1}}{F_{N_0}} F_{N_0} = \frac{\rho}{\rho_1}\left(\frac{\rho_1}{\rho_0}\right)^{0.7} F_{N_0} = \frac{\rho_0}{\rho_0}\frac{\rho_0}{\rho_1}\left(\frac{\rho_1}{\rho_0}\right)^{0.7} F_{N_0} = 1.439\, F_{N_0}\, \sigma$

$\Rightarrow$ ~~F~~

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So since

$$\left(\frac{L}{D}\right)_{max} = \frac{W}{F_N} = \frac{W}{1.4390\,\sigma F_{N_0}} \quad , \quad \frac{F_{N_0}}{W} = 0.352$$

$$\Rightarrow \left(\frac{L}{D}\right)_{max} = \underline{\underline{10.66}}$$

since $\left(\frac{L}{D}\right)_{max} = \frac{1}{2}\sqrt{\frac{\pi A R}{k C_{D_0}}}$, $C_{D_0} = \frac{\pi AR}{4k\left(\frac{L}{D}\right)^2_{max}} =$

$$= \underline{\underline{0.0387}}$$

30

(ii) Assuming constant C_L ~~& velo~~ and thus L/D

$$E = \frac{-L/D}{c} \ln\left(\frac{W_{fin}}{W_{init}}\right)$$

$$\frac{1}{SE} = \dot{m}_{fuel} = cF_N = \frac{cW}{L/D}$$

$$\Rightarrow E = \int SE\, dW = \int \frac{L/D}{cW}\, dW$$

max endurance @ $L/D = (L/D)_{max}$

$$E = \frac{-(L/D)_{max}}{c} \ln\left(\frac{W_{fin}}{W_{init}}\right)$$

$$W_{int} = 4937 \times 16.7 = 82447.9$$

$$\Rightarrow E = -\frac{10.66}{0.45} \ln(0.833)$$

$$W_{fin} = W_{int} - 1400 \times 9.81 = 68713.9$$

$$= \cancel{4.338\,hrs}$$

$$\Rightarrow \frac{W_{fin}}{W_{int}} = 0.833$$

$$= \underline{4.316\ hrs}$$

20

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY & MEDICINE

M Eng Part I Mechanics of Flight – Aircraft Performance

Data and Formulae

The total drag coefficient is given by

$$C_D = C_{D_0} + \frac{k}{\pi \mathrm{AR}} C_L^2.$$

The total drag curve is given by

$$D = \left(A \cdot V_i^2 + B \cdot \frac{n^2}{V_i^2} \right),$$

where n is the load factor ($n=1$ for steady level flight) and

$$\mathrm{A} = C_{D_0} \tfrac{1}{2} \rho_0 S$$

$$\mathrm{B} = \frac{\mathrm{kW}^2}{\left(\frac{1}{2}\rho_0 S\right)(\pi \mathrm{AR})}.$$

The minimum drag and EAS for minimum drag are

$$D_{\min} = 2n\sqrt{AB} \quad \text{and} \quad V_{imD} = (B/A)^{1/4} n^{1/2}.$$

V_{imD} and D_{min} are used to define the relative speed and relative drag respectively:

$$\overline{V} = \frac{V_i}{V_{imD}\big|_{n=1}} \quad \text{and} \quad \overline{D} = \frac{D}{D_{\min}\big|_{n=1}}.$$

where $\overline{D}$ satisfies

$$\overline{D} = \frac{1}{2}\left(\overline{V}^2 + \frac{n^2}{\overline{V}^2} \right)$$

The maximum lift-to-drag ratio, which is constant for a given aircraft at a given configuration and weight, is

$$(L/D)_{\max} = \frac{L}{D_{\min}} = \frac{1}{2}\sqrt{\frac{\pi \mathrm{AR}}{kC_{D_0}}} = \frac{1}{2}\frac{W}{\sqrt{AB}}$$

For steady level flight, thrust equals drag and the drag equation becomes

$$V_i^4 - \frac{F_N}{A} V_i^2 + \frac{B}{A} = 0,$$

from which two EAS solutions can be obtained:

$$V_{i1}^2 = \frac{F_N}{2A} + \sqrt{\left(\frac{F_N}{2A}\right)^2 - \frac{B}{A}} \quad \text{and} \quad V_{i1}^2 = \frac{F_N}{2A} - \sqrt{\left(\frac{F_N}{2A}\right)^2 - \frac{B}{A}}.$$

In steady level flight in the stratosphere, ceiling density ratio is given by

$$\sigma_{\max alt} = \frac{2\sqrt{AB}}{1.439 F_{N0}}.$$

The range of an aircraft flying at constant velocity and constant lift coefficient is

$$R_1 = \left(\frac{V_{\mathrm{mD},1}(L/D)_{\max}}{c}\right)\left[\frac{2}{\overline{V}+\overline{V}^{-3}}\right]\ln\left(\frac{W_{init}}{W_{fin}}\right).$$

Range of an aircraft flying at constant density and constant lift coefficient:

$$R_2 = \left(\frac{V_{\mathrm{mD},1}(L/D)_{\max}}{c}\right)\left(\frac{4\overline{V}}{\overline{V}^2+\overline{V}^{-2}}\right)\left(1-\sqrt{\frac{W_{fin}}{W_{init}}}\right)$$

In both cases, maximum range is obtained by flying at $\overline{V} = \sqrt[4]{3}$.

The gradient of climb is

$$(L/D)_{\max} \sin(\Gamma) = \tau - \frac{1}{2}\left(\overline{V}^2 + \frac{1}{\overline{V}^2}\right).$$

The rate of climb is

$$(L/D)_{\max} \overline{v}_c = \tau \cdot \overline{V} - \frac{1}{2}\left(\overline{V}^3 + \frac{1}{\overline{V}}\right)$$

where $\overline{v}_c = \dfrac{v_c}{V_{mD}}$.

List of Symbols:

AR	= wing aspect ratio (= b^2/S)
b	= wing span
c	= specific fuel consumption (Weight of fuel consumption per unit thrust per sec.)
C_D	= drag coefficient
C_{D_0}	= lift-independent drag coefficient
C_L	= lift coefficient
D	= drag
D_{min}	= drag at the minimum drag speed
F_N	= thrust
F_{N0}	= thrust at sea-level
k	= loading efficiency
L	= lift
$(L/D)_{max}$	= maximum lift-to-drag ratio
R	= range, for steady level flight
S	= wing surface area
σ	= air density ratio
τ	= thrust-to-drag ratio (= $F_N/D_{\mathrm{min}}\|_{n=1}$)
V	= true airspeed (TAS)
V_i	= equivalent airspeed (EAS)
V_{imD}	= EAS for minimum drag
$V_{\mathrm{mD,I}}$	= TAS for minimum drag at beginning of cruise (when weight is W_{init})
W_{init}	= aircraft weight at the beginning of the cruise
W_{fin}	= aircraft weight at the end of the cruise