UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

PAPER C382

TYPE SYSTEMS FOR PROGRAMMING LANGUAGES

Thursday 29 April 2004, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 a Give the definition of

i) Lambda Terms.

ii) Curry types.

iii) Curry type assignment for the Lambda Calculus.

b Give, in functional programming language notation, the algorithm that calculates the principal Curry pair for lambda terms.

c Assuming the existence of booleans, add the *conditional* language construct to expressions, and extend, if needed, the notion of reduction to deal with this addition. Give the natural extension to Curry's type assignment system for the expressions that deals with this new construct.

d Extend the principal type algorithm with a case for the new construct of the previous part.

The four parts carry, respectively, 20%, 25%, 25%, and 30% of the marks.

2a Give the syntax definition for ML-terms, and the essential rules in the notion of reduction of the ML-language.

b Give the syntax definition for ML-types, and give the type assignment rules for the Milner type assignment system.

c Take the algebraic data types *Int* and *List*:

$$\begin{array}{lcl} n & ::= & 0 \mid (\mathit{Succ}\, n) \\ l & ::= & \mathit{Nil} \mid (\mathit{Cons}\, n\, l) \end{array}$$

Assuming the following extensions to ML:

- a prefix addition '+' with type $Int \rightarrow Int \rightarrow Int$.
- a conditional language construct '*cond*' with type $Bool \rightarrow \sigma \rightarrow \sigma \rightarrow \sigma$,
- a test for zero '=0' with type $Int \rightarrow Bool$, and
- a function '-1' with type $Int \rightarrow Int$,

express the multiplication function as an ML-*expression*, and give a derivation that types this term; you can assume that all added functions are treated as constants.

d Abbreviating your answer to the previous part by D :: $\vdash \times : Int \rightarrow Int \rightarrow Int$, and using the information for the previous part, write an ML-*expression* that represents the (infinite) list of all square numbers. Show that this term is typeable.

The four parts carry, respectively, 20%, 30%, 30%, and 20% of the marks.

3 a Give, for Term Rewriting Systems, the definition of

i) Terms.

ii) Rewrite rules and reduction.

iii) Curry type assignment for term rewriting ($\vdash_{\mathcal{E}}$).

b Given the term rewriting system

$$\begin{array}{lcl} \mathsf{B}\,x\,y\,z & \rightarrow & x\,(y\,z) \\ \mathsf{C}\,x\,y\,z & \rightarrow & x\,z\,y \\ \mathsf{K}\,x\,y & \rightarrow & x \\ \mathsf{S}\,x\,y\,z & \rightarrow & x\,z\,(y\,z) \end{array}$$

Give an environment that makes these rules typeable. You do not need to give derivations, just the types.

c Add the following rules to the system above.

$$\begin{array}{lcl} \mathsf{S}\,(\mathsf{K}\,x)\,(\mathsf{K}\,y) & \rightarrow & \mathsf{K}\,(x\,y) \\ \mathsf{S}\,(\mathsf{K}\,x)\,y & \rightarrow & \mathsf{B}\,x\,y \\ \mathsf{S}\,x\,(\mathsf{K}\,y) & \rightarrow & \mathsf{C}\,x\,y \end{array}$$

Show, perhaps using an abbreviated notation, that the system is still typeable using the same environment.

d Add now also the rule

$$\mathsf{S}\,(\mathsf{K}\,x)\,\mathsf{I} \rightarrow x$$

Show that the system is no longer typeable using the same environment. What would you have to change to make the system typeable?

The four parts carry, respectively, 30%, 20%, 30%, and 20% of the marks.

4a Give the definition of

i) Intersection types.

ii) Intersection type assignment for the Lambda Calculus.

b Show

i) $\emptyset \vdash_\cap \lambda xy.xy : (\sigma{\to}\tau){\to}(\rho{\cap}\sigma){\to}\tau$.

ii) $\emptyset \vdash_\cap \lambda xyz.xz(yz) : (\alpha{\to}\omega{\to}\gamma){\to}\omega{\to}\alpha{\to}\gamma$

iii) $\emptyset \vdash_\cap (\lambda xyz.xz(yz))(\lambda ab.a) : \omega{\to}\sigma{\to}\sigma$ (use the previous result without repeating the whole structure).

c Compare the sets of types assignable to the terms $(\lambda xyz.xz(yz))(\lambda xy.x)(\lambda x.x)$ and $\lambda x.x$ in the Curry system and in the intersection system. Motivate your answer.

The three parts carry, respectively, 35%, 40%, and 25% of the marks.