EE4-05
EE9-CS7-22
EE9-SO7

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

MSc and EEE PART IV: MEng and ACGI

Corrected copy

TRAFFIC THEORY & QUEUEING SYSTEMS

Tuesday, 23 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible   First Marker(s) : J.A. Barria
Second Marker(s) : D.P. Mandic

Special instructions for students

1. Recursive evaluation of Erlang Loss formula:

$$E_N(\rho) = \frac{\rho E_{N-1}(\rho)}{N + \rho E_{N-1}(\rho)}$$
$$E_0(\rho) = 1$$

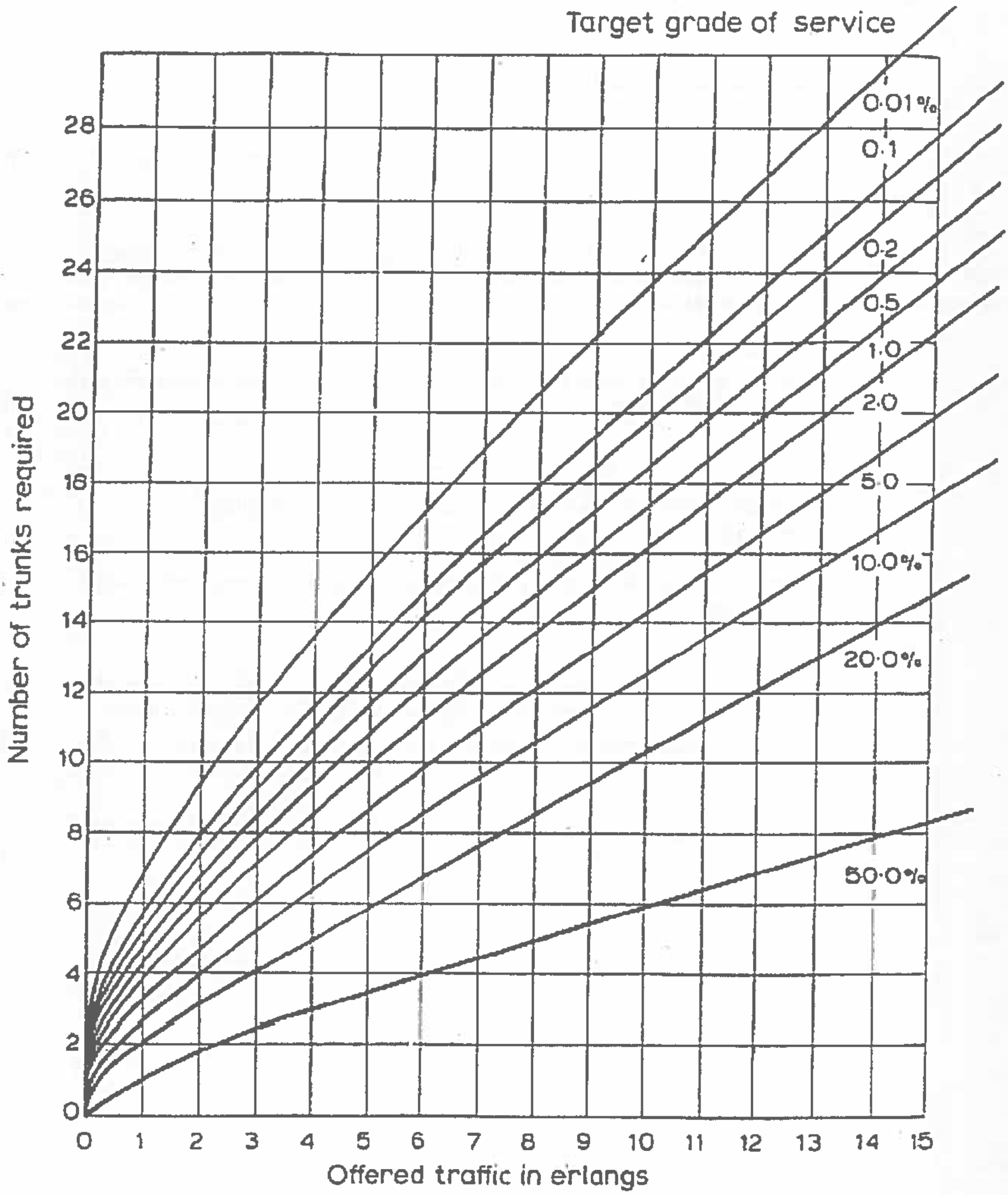
2. Recursive evaluation of Engset Loss formula (for a given M and $p = \alpha/1+\alpha$):

$$e_N = \frac{(M-N+1)\alpha e_{N-1}}{N + (M-N+1)\alpha e_{N-1}}$$
$$e_0 = 1$$
$$\alpha = \lambda/\mu$$

3. Traffic capacity on the basis of Erlang B formula (next page).

4. Expected residual time

$$E[R] = \frac{1}{2}\lambda E[S^2] = \frac{1}{2}\sum_{k=1}^{m}\lambda_k E[S_k^2]$$



Traffic capacity on basis of Erlang B. formula.

The Questions

1.

a) In the context of modelling a Markov process:

i) Give two examples of Birth/Death (B/D) processes known to you. [6]

ii) Choose one of the B/D introduced in i) and derive the corresponding Global Balance Equations (GBE) and the Local Balance Equations (LBE). [6]

iii) Starting with, for example, the derived LBE in ii) show that the LBE and GBE are equivalent for the same process. [2]

b) A multichannel link is required to be designed with a loss probability of $B_c = 0.02$.

The link average calling rate is 1800 calls per hour and the average call duration is 3 minutes.

i) Using the provided graph of the system capacity based on the Erlang B formula, and for large values of the offered traffic ρ, derive a linear approximation of the number of trunks required (N) as a function of ρ. [2]

ii) Using the approximation derived in i) estimate: the size (N) of the link and the total carried traffic. [4]

2.

a) For a $M/G/1$ queue system:

i) Define, for the $i-th$ arrival, the residual waiting time variable and the mean waiting time for a $M/G/1$ queue system. [2]

ii) Show that the expected waiting time can be expressed as a function of the expected residual time. [4]

iii) Show that the expected waiting time is a function of the mean squared service time. [3]

b) For a $M/M/K$ queue system:

i) Derive the queue length distribution for delayed arrivals. [3]

ii) Derive the expected queue length for delayed arrivals. [3]

iii) Derive the variance of the queue length for delayed arrivals. [3]

iv) Derive the unconditional queue length distribution for the $M/M/K$ queue system. [2]

Note: state clearly all the steps in your derivations.

3.

a) Consider the following problem:

A Poisson stream of messages with mean rate of $\lambda = 250$ messages per second arrives to a queue with infinite buffer capacity. Sixty percent (60%) of the messages are composed of a single size 160 bits long packet. The reminder 40% of messages consists of two packets of size 160 bits long each.

The channel transmission rate is 64 Kbits/s.

For a non-pre-emptive priority treatment of messages of a single size 160 bits long packets:

i) Derive the mean transit time for the single packet messages. [6]

ii) Derive the mean transit time for two packets messages. [6]

iii) Derive the overall mean message transit time. [2]

b) Using Fig 3.1 as a reference:

Assume that the numbers of channels of Link 1 is $M = 12$ and that the number of channels of Link 2 is $N = 8$.

Assume also that the offered traffic to the system is $\rho = 10$ Erlangs.

i) Calculate the Link 1 blocking probability B_1 and, the Link 2 blocking probability B_2. [3]

ii) Derive the carried traffic on Link 1 and the carried traffic on Link 2. [3]

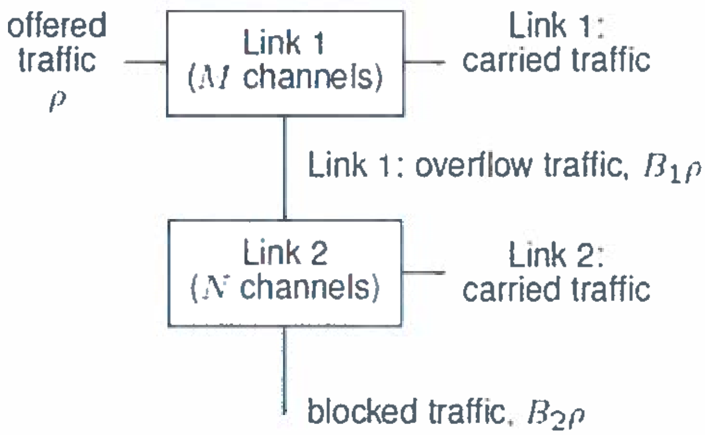


Figure 3.1.

4.

a) For an $M/M/K/N$ queuing system, the buffer has a limited capacity measured in buffer stage units. Also consider that the following characteristics of the system are known to you:

- the failure rate of each server is λ,
- the failure rate of any of the buffer stage is γ,
- the repair rate of the servers is μ,
- the repair rate of the buffer is τ.

The system is in a faulty condition if any of the buffer stage is in failure and/or all the servers units fail.

i) Define the state space of the system. [4]

ii) Construct the base model of the associated Performability model. Clearly show all the transition rates. [6]

iii) Clearly identify the faulty states. [2]

b) In the context of a fluid flow model (FFM) framework:

i) State clearly and discuss the assumption made in a FFM framework with respect to the packets being generated during an active period of an ON-OFF packet source. [2]

ii) For a statistical multiplexer's access buffer: derive step by step an equation that you can use to obtain the stationary probability that the buffer occupancy is greater than x, given that there are i ON-OFF sources active. [6]