E4.06, ISE 4.36, A09, S09

OPTICAL COMMUNICATION SOLUTIONS 2003

(1) a) Need $V < 2.405$ CE

$V = r_0 k_0 \sqrt{n_c^2 - n_s^2} \cong \frac{r_0 2\pi}{\lambda_0} \sqrt{2n\,\Delta n}$

$2.405 = (3\mu m\,(2\pi)/1.502\mu m)\sqrt{2\times 3\times \Delta n}$

gives $\Delta n \cong \underline{0.013}$

b) Since the V number is proportional to $\sqrt{n_1^2 - n_2^2}$, raising the Δn will increase the number of supported modes BW

c) 1600 - 2000 nm is at the long wavelength side of the attenuation minimum, in this region interatomic vibrations dominate the loss, esp Si-O stretching vibration BW

d) For an LED we estimate ΔE to be ~2kT. At room temp this is ~ 50 meV. The minimum energy will be $E_g \cong 1.24\,eV\cdot\mu m / 1.31\mu m = 0.947 eV$ CE

Then $\lambda_{min} \cong 1.24\,eV\cdot\mu m / (0.947 + .05) = 1.244\,\mu m$

$\Delta\lambda \cong 1310 - 1244 = \underline{66\ nm}$

e) Long haul systems work at ~1310 or 1550 nm wavelength. Si, with a bandgap of 1.1 eV, is transparent at those wavelengths BW

CE = computed example
BW = bookwork
NT = new theory
TA = theor. application

1

f) 40 Gbit/s (purely memory this one!) BW
Limited by electronics & dispersion.

g) DFB lasers use gratings as the reflective elements to form the resonant cavity, which are λ selective, and so can have a much narrower spectrum than F-P lasers. BW

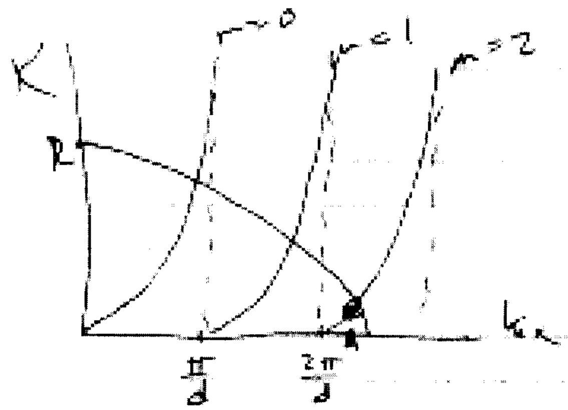
h) The maximum slope efficiency ($\eta = 1$) for a laser is $\frac{hc}{e\lambda}$. In this case CE

$$S_{max} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 0.78 \times 10^{-6}} = 1.59 \text{ W/A}$$

i) The pump laser λ must be $< \lambda_{signal}$, must correspond to a strong absorption band for Er that results in an efficient transition to the metastable state, and must be manufacturable as a high power laser diode. BW

j) Conservation of power applies. Input -10 dBm $= 0.1$ mW. First output port, -20 dBm $= 0.01$ mW, leaving 0.09 mW on the other output, $= \underline{\underline{-10.46 \text{ dBm}}}$. CE

2) a)



TA

$\frac{\pi}{d} = 0.5236 \times 10^6\ m^{-1}$

$k_{1x} = 1.19 \times 10^6\ m^{-1}$ is slightly > $2\pi/d$

$\therefore\ m = 2$

$\frac{k_{1x}d}{2} = 3.57 \qquad K = k_{1x}\tan\frac{k_{1x}d}{2} = 0.543 \times 10^6\ m^{-1}$

$\sqrt{k_{1x}^2 + K^2} = 1.308 \times 10^6 = R < 3\pi/d$

therefore $m = 2$ is the highest mode

There are 3 modes, $m = 0, 1, 2$.

b) $n' = \frac{c}{v_p} = \frac{3}{2.027} = 1.480$

TA

$k_{1x}^2 + \beta^2 = n_c^2 k_0^2$

$(k_{1x}/k_0)^2 + n'^2 = n_c^2$

$n_c = \sqrt{1.480^2 + \left(\frac{1.19 \times 10^{?}}{2\pi}\right)^2} = 1.507$

c) $\frac{k_{1x}d}{2} = X \qquad \frac{Kd}{2} = Y$

TA

$X^2 + Y^2 = (Rd/2)^2$

$Y^2 = X^2\tan^2 X$

$X^2(1+\tan^2 X) = \frac{X^2}{\cos^2 X} = (Rd/2)^2$

$X = \frac{Rd}{2}\cos X = \left(\frac{1.308 \times 6}{2}\right)\cos X = 3.924\cos X$

From diagram we estimate that the $m=0$ solution has $\frac{k_{1x}d}{2} \cong \frac{\pi}{2}$. With this starting point,

trial and error gives $X = 1.247$

$k_{1x} = 2X/d = 0.4156 \times 10^6\ m^{-1}$

$n' = \sqrt{n_1^2 - \left(\frac{k_{1x}}{k_0}\right)^2} = 1.504$

③

a) $\tau_g = \frac{L}{c}\left(n - \lambda \, dn/d\lambda\right)$ NT

We can estimate $\Delta\tau_g$ in this case using 2 terms of the series expansion:

$$\Delta\tau_g \simeq \frac{d\tau_g}{d\lambda}\Delta\lambda + \frac{d^2\tau_g}{d\lambda^2}\frac{\Delta\lambda^2}{2}$$

$$\frac{d\tau_g}{d\lambda} = \frac{L}{c}\left(\frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda\frac{d^2n}{d\lambda^2}\right) = -\frac{L}{c}\left(\lambda\frac{d^2n}{d\lambda^2}\right)$$

$$\frac{d^2\tau_g}{d\lambda^2} = \frac{-L}{c}\left(\frac{d^2n}{d\lambda^2} + \lambda\frac{d^3n}{d\lambda^3}\right)$$

Taking $d^2n/d\lambda^2 = 0$:

$$|\Delta\tau_g| = (L/c)(\lambda d^3n/d\lambda^3)\Delta\lambda^2/2$$

b) $\sigma_R^2 = \sigma_0^2 + \sigma_D^2$, $\sigma_D = DL\cdot\sigma_\lambda$ TA

here $\sigma_\lambda \simeq \sigma_{\lambda s}$

$$\sigma_R^2 = \sigma_0^2 + (DL\sigma_{\lambda s})^2$$

$$(\sigma_R/\sigma_0)^2 = 1 + \left(\frac{DL\sigma_{\lambda s}}{\sigma_0}\right)^2 = 1.25^2 = 1.56$$

$(DL\sigma_{\lambda s}/\sigma_0) = 0.75$, need $\sigma_0 > 1.33\, D\cdot L\cdot\sigma_{\lambda s}$

c) For transform limited pulses $\sigma_\omega\sigma_0 = \frac{1}{2}$ NT

But $\sigma_\lambda/\lambda \simeq \sigma_\omega/\omega$, $\sigma_\lambda \simeq \frac{\lambda^2}{2\pi c}\sigma_\omega$

This gives $\sigma_\lambda = \frac{1}{2\sigma_0}\left(\frac{\lambda^2}{2\pi c}\right)$

$\sigma_R^2 = \sigma_0^2 + A^2/\sigma_0^2$ where $A = DL\lambda^2/4\pi c$

$$\frac{\partial\sigma_R^2}{\partial\sigma_0} = 2\sigma_0 - \frac{2A^2}{\sigma_0^3} = 0 \text{ for } \sigma_0 = \sqrt{A}$$

So $\sigma_0 = \sqrt{\frac{DL\lambda^2}{4\pi c}}$, giving $\frac{\sigma_R}{\sigma_0} = \sqrt{2}$.

(4) a) $\eta = \dfrac{1}{1+\tau_{rr}/\tau_{nr}} = \dfrac{1}{1+1/5} = 0.833$ CE

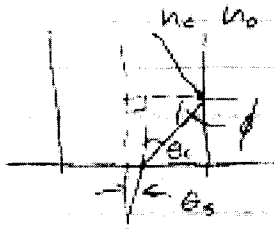
b) The 4 mechanisms & solutions are: BW

i) light goes downward
- add a mirror at base, but need either a heterostructure or very short path length

ii) absorption before leaving semiconductor
- heterostructure used so bandgap is greater outside active region

iii) surface (Fresnel) reflection
- antireflection coating, $\lambda/4$ thick, $n = \sqrt{n_1 n_2}$

iv) Total internal reflection
- add a dome of glass/plastic, or shape semiconductor surface to reduce angle.

c) 

need $\sin\phi > n_0/n_c$ TA

$\therefore \cos\phi < \sqrt{1-(n_0/n_c)^2}$

$\cos\phi = \sin\theta_c$

$\sin\theta_c < \sqrt{n_c^2-n_0^2}/n_c$

$n_s \sin\theta_s = n_c \sin\theta_c$

$\sin\theta_s < \sqrt{n_c^2-n_0^2}/n_s$

Need $\theta_s < \sin^{-1}(NA/n_s)$ where $NA = \sqrt{n_c^2-n_0^2}$
$= 0.1$
$n_s \approx 3.5$
$\theta_{max} \approx 1.64°$

$f = \int_0^{\theta_{max}} 2\pi \sin\theta \, d\theta \Big/ \int_0^{\pi} 2\pi \sin\theta \, d\theta$

$= [\cos\theta]_0^{\theta_{max}} / [\cos\theta]_0^{\pi} = (1-\cos\theta)/2 \approx \theta^2/4$

$f \approx \sin^2\theta/4 = \dfrac{NA^2}{4n_s^2} = \dfrac{.1^2}{4(3.5)^2} \approx .00020$

d) $\tau = L/v_g \quad \Delta\tau_g \approx \Delta n L/c$ TA
$= n' L/c \quad$ Need $\Delta\tau_g < (\tfrac{1}{4})\tfrac{1}{B}$

gives $B < c/4L\Delta n$

5

a) Shot noise is given in terms of equivalent photocurrent per $\sqrt{\Delta f}$ as

$I_s^* = \sqrt{2eI_{ph}}$ TA

We need to convert this to be expressed w.r.t. received optical power Φ_R. Assuming $\eta = 1$, $I_{ph} = e\Phi_R/h\nu = e\lambda\Phi_R/hc$

then $\Phi_s^* = \frac{hc}{e\lambda}\sqrt{2e^2\lambda\Phi_R/hc} = \sqrt{2hc\Phi_R/\lambda}$

$\Phi_s^* = 5\,pW/\sqrt{Hz}$ for

$$(5\times10^{-12})^2 = \frac{2(1.6\times10^{-19})^2\,1.5\times10^{-6}\,\Phi_R}{6.65\times10^{-34}\times3\times10^8}$$

$\Phi_R = 94\,\mu W$

b) $SNR = \dfrac{\Phi_R}{[NEP + \Phi_s^*](\Delta f)^{1/2}}$ TA

$SNR = 10\,dB = 10$, $\Phi_s^* = \sqrt{\dfrac{2e^2\lambda\Phi_R}{hc}}$

$\Delta f = B/2$

Where receiver noise dominates:

$10 \simeq \dfrac{\Phi_T e^{-\alpha L}}{NEP\sqrt{B/2}}$, $\dfrac{10^2 B}{2} = \left(\dfrac{10^{-2}e^{-\alpha L}}{5\times10^{-12}}\right)^2$

$B = 8\times10^{16} e^{-2\alpha L}$

$\alpha = \dfrac{dB/km}{4.3} = 0.1\,km^{-1}$

$\log B = 16.9 - 0.2L\log e = 16.9 - .087L$

Where shot noise dominates:

$SNR \simeq \dfrac{\Phi_R}{\sqrt{\dfrac{2hc\Phi_R}{\lambda}}\sqrt{\Delta f}}$ $\qquad 10\sqrt{\dfrac{B}{2}} = \sqrt{\dfrac{\lambda\Phi_R}{2hc}}$

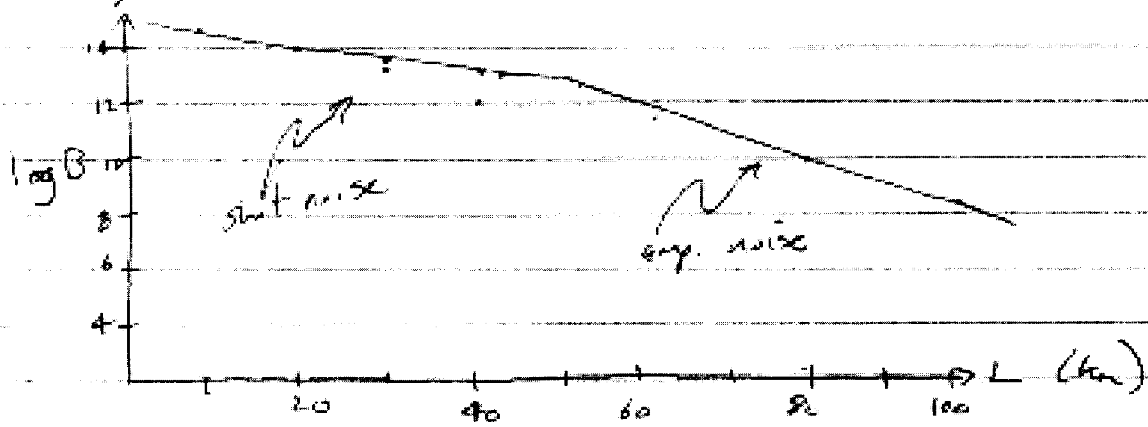
$B = \dfrac{\lambda\Phi_T e^{-\alpha L}}{100hc}$ $\qquad \log B = \log\left(\dfrac{1.5\times10^{-6}\times10^{-2}}{100\times6.6\times10^{-34}\times3\times10^{8}}\right)$

$- 0.1L\log e$

5 b) (continued)

For shot noise case

$\log B = 14.9 - .043L$



c) With the amplifier, Φ_R increases by 20 dB, NEP is unaffected where NEP dominates: 7A

$$\frac{10^2 B}{2} = \left(\frac{e^{-\alpha L}}{5\times10^{-12}}\right)^2 \quad B = e^{-.2L} \times 8\times10^{20}$$

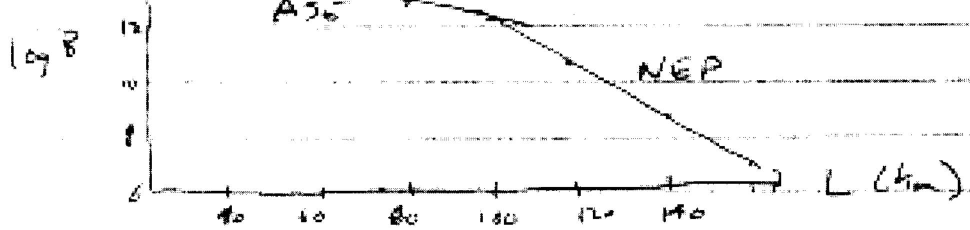
$\log B = 21 - 0.087L$

For the shot/ASE case:

4 dB is a ratio of 2.5 in electrical SNR, 1.58 in optical SNR.

$$\frac{10^2 B}{2} = \frac{1}{2.5}\left(\frac{1.5\times10^{-6}\, e^{-\alpha L}}{2\times 6.6\times10^{-34}\times 3\times10^{8}}\right)$$

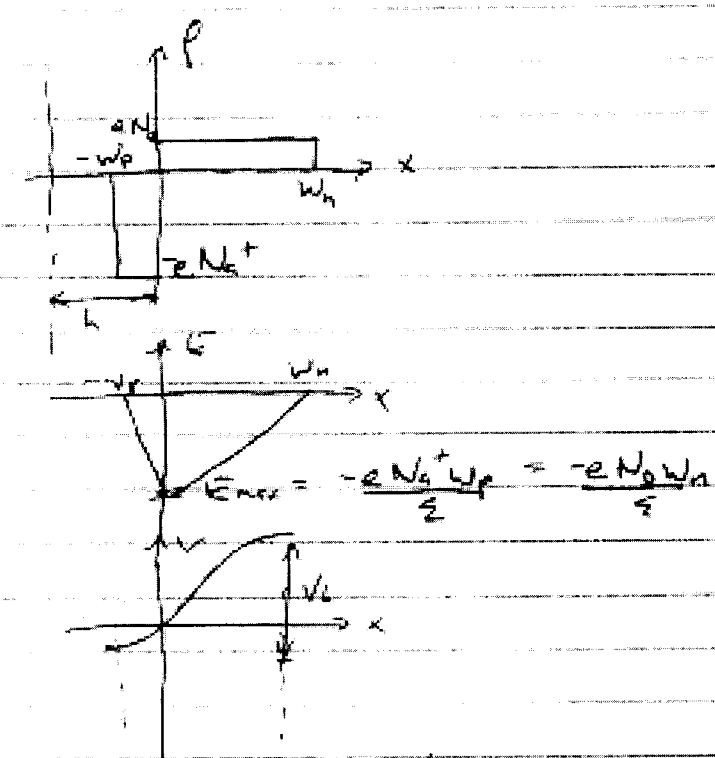
$B = 3\times10^{16} e^{-.1L} \qquad \log B = 16.5 - .043L$



6

a)

7A



$$V_b = |\mathcal{E}_{max}|\,(w_n + w_p)/2$$

$$w_n = \frac{N_A^+ w_p}{N_D} \qquad V_b = \left(\frac{eN_A^+ w_p}{\varepsilon}\right)\left(\frac{N_A^+}{N_D}+1\right) w_p/2$$

$$V_b = \frac{eN_A^+}{2\varepsilon}\left(\frac{N_A^+}{N_D}+1\right) w_p^2 = \frac{1.6\times10^{-19}\times10^{20}}{2\times12\times8.85\times10^{-12}}\,(6)\,w_p^2$$

$w_p = 1.49\sqrt{V_b}$ (in μm)

$w_p = 10\ \mu m$ for $V_b = 45.2\ V$

$W = (5+1)w_p = 8.9\sqrt{V_b} = 10\ \mu m$ for $V = 1.25\ V$

b) $\eta = e^{-\alpha x_1} - e^{-\alpha x_2}$

7A

$x_1 = h - w_p \qquad x_2 = h + w_n$

$$\eta = e^{-\alpha(h-w_p)} - e^{-\alpha(h+5w_p)}$$
$$= e^{-\alpha h}\left(e^{\alpha w_p} - e^{-\alpha 5 w_p}\right) \qquad \alpha h = 10^5\times10^{-5} = 1$$

$$\eta = e^{-1}\left(e^{1.49\times10^5\sqrt{V_b}} - e^{-7.45\times10^5\sqrt{V_b}}\right)$$

To get $\eta = 0.9$. $\quad 0.9e = e^{\alpha w_p} - e^{-5(\alpha w_p)} = 2.446$

gives $\alpha w_p = 0.9 \quad w_p = 0.9\times10^{-5}\ m = 9\ \mu m = 1.49\sqrt{V_b}$

$V_b = 36.5\ V$

(6)

c) If $R \propto (\text{residual } p \text{ thickness})^{-1}$ then NT

$$R = \frac{A}{h - w_p}$$

and we know $C \propto \frac{1}{w_n + w_p}$ i.e. $C = \frac{B}{w_n + w_p}$

But $w_n + w_p = 6 w_p$

$$RC = \frac{AB/6}{(h - w_p) w_p}$$

$\frac{\partial RC}{\partial w_p} = 0$ at $\frac{\partial \left(\frac{1}{RC}\right)}{\partial w_p} = 0 = \frac{6}{AB}(h - 2w_p)$

$w_p = \frac{h}{2} = 5\,\mu m$