Q1 a) $N > M$: no congestion will occur. , $i_{max} = M$

$$\pi_i = \left(\frac{\lambda_{i-1}}{\mu_i}\right)\pi_{i-1} = \left(\frac{(M-(i-1))\lambda}{i\mu}\right)\pi_{i-1}$$ 4

Recursive:

$$\pi_i = \binom{M}{i}\alpha^i \pi_0 \quad i = 1, 2, \dots, M \quad \text{and} \quad \alpha = \lambda/\mu$$

$$\pi_0 = \left[\sum_{j=0}^{M}\binom{M}{j}\alpha^j\right]^{-1}$$ 6

let $p = \frac{\alpha}{1+\alpha} \longleftrightarrow \alpha = \frac{p}{1-p}$

multiplying numerator and denominator of π_i by the factor $(1-p)^M$ we get:

$$\pi_i = \frac{\binom{M}{i}p^i(1-p)^{M-i}}{\underbrace{\sum_{j=0}^{M}\binom{M}{j}p^j(1-p)^{M-j}}_{=1}} \quad i = 0, 1, \dots, M$$ 4

(π_i) is binomial (M, p)

Q1 b)

$$\pi_2 = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3!}{2!(3-2)!}\left(\frac{1}{2}\right)^3$$ 6

$= 0.375$

$p = \frac{\lambda}{\mu+\lambda}$

Q2a) M/M/K recursive expression

$$\pi_i = \begin{cases} (A^i/i!)\,\pi_0 & i \le K \\ (A^K/K!)\,\rho^{i-K}\,\pi_0 & i \ge K \end{cases} \qquad A = \lambda/\mu \qquad \rho = A/K$$

Also for $i \ge K$, $i = K+j$ $(j \ge 0)$ and then

$$\pi_{K+j} = \left(\frac{A^K}{K!}\right)\rho^j \pi_0 = \rho^j \pi_K \quad ; \quad \pi_K = \left(\frac{A^K}{K!}\right)\pi_0$$

$$\pi_0 = \frac{1}{(A^K/K!)}\left[\frac{(1-\rho)E_K(A)}{(1-\rho) + \rho E_K(A)}\right] \qquad E_K(A) = \frac{A^K/K!}{\sum_{i=0} A^i/i!}$$ 5

$$P(\text{delay}) = P(W>0) = P(N_t \ge K)$$

$$P(N_t \ge K) = \sum_{i=K}^{\infty} \pi_i = \sum_{j=0}^{\infty} \pi_{K+j} = \sum_{j=0}^{\infty} \pi_K \rho^j$$

$$= \frac{\pi_K}{1-\rho} = \frac{(A^K/K!)\,\pi_0}{1-\rho} = \frac{E_K(A)}{(1-\rho) + \rho E_K(A)} = D_K(A)$$ 5

Q2b) For delay arrivals

$$P(W > \tau \mid Q_t = i) = P(<(i+1) \text{ departures in } (0,\tau))$$
$$= \sum_{j=0}^{i} \frac{(K\mu\tau)^j}{j!} e^{-K\mu\tau}$$ 5

in equilibrium, we know for delay arrivals

$$P(Q_t = i) = (1-\rho)\rho^i$$

Therefore

$$P(W>\tau) = \sum_{i=0}^{\infty} P(Q_t = i)\, P(W>\tau \mid Q_t = i)$$
$$= \sum_{i=0}^{\infty} (1-\rho)\rho^i \sum_{j=0}^{i} \frac{(K\mu\tau)^j}{j!} e^{-K\mu\tau}$$
$$= e^{-K\mu(1-\rho)\tau}$$ 5

(by interchanging summation order and using $\lambda = K\mu\rho$)

Q3a i) Equivalent capacity and admission control
How much traffic can a VP can handle if a prescribed QoS for each traffic class is to be maintained. For example given the capacity of a VP: How many VC can it handled. Or, given a number of active VC, can we admit one more
- Assume ON-OFF traffic sources: peak rate R_p, Average burst length $1/\mu$ and average silence length $1/K$
- single class admission control

i) Large number of sources multiplexed (ignore effect of buffer). A Binomial distribution is approximated quite closely by a normal distribution
ii) If the effect of the access buffer is considered a fluid flow approximation is obtained
Eq capacity = min [sol a, sol b) 5

ii) Leaky Bucket Is one possible approach to control the traffic during the period of a connection. Users found violating their connection agreement may have cells dropped or traffic may be shaped or smoothed to reduce any adverse impact on the network. This is a form of open loop access control.
An interpretation of the leaky bucket algorithm involves the use of a "token pool" buffer. A cell must have a token waiting to be transmitted. Tokens are generated one per D sec, and wait in the buffer, until buffer fills. At this time no further token is generated. Note, that as expected in this case the average throughput drops from the input load (possible cell loss) 5

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Q3(b) The system is M/G/1

mean packet length $= \frac{1}{4}160 + \frac{1}{4}80 + \frac{1}{2}320 = 220 = \frac{11}{4}80$

mean square packet length $= \frac{1}{4}(160)^2 + \frac{1}{4}(80)^2 + \frac{1}{2}(320)^2$

$= 59200 = \left(\frac{4+1+16\cdot 2}{4}\right)(80)^2 = \frac{37}{4}80^2$ 3

hence:

$E(S) = \frac{11}{4}\frac{80}{64} \times 10^{-3} = 3.4375$ (ms)

$E(S^2) = \frac{37}{4}\frac{(80)^2}{(64)^2} \times (10^{-3})^2 = 14.45$ (ms^2)

$\rho = \lambda E(S) = 200 \times 3.4375 \times 10^{-3} = 0.687$ 4

using Pollaczek-Khinchin formula

$E(W) = \left[\frac{\lambda E(S^2)}{2(1-\rho)}\right] = \frac{200 \times 14.45}{2(1-0.687)} \times 10^{-6} = 4.61$ ms 3

$E(T) = E(W) + E(S) = (4.61 + 3.43)$ ms

$= 8$ ms

$E[W] = \frac{E(R)}{1-\rho}$ $\quad E(R) = \frac{1}{2}\lambda E[S^2]$

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Q4a M/G/1 system

$$E[W] = \frac{E[R]}{1-\rho}$$

$$E[R] = \frac{1}{2}\lambda E[S^2]$$

$$E[W] = \frac{\lambda E[S^2]}{2(1-\rho)}$$ 5

Applying Little

$$E[Q_t] = \frac{\lambda^2 E[S^2]}{2(1-\rho)}$$ 5

Q4b For pure chance traffic with $\rho = 13$ and $N = 20$

$B_C = E_{20}(13) = 0.02$ (from chart)

i) mean carried traffic

$$\rho_c = (1-B_c)\rho_o = 12.74 \text{ erlangs}$$ 2

ii) mean channel occupancy is

$$\eta = \frac{\rho_c}{N} = \frac{12.74}{20} = 0.637$$ 2

iii) for $N = \infty$

$$\pi_i = \frac{\rho^i}{i!}e^{-\rho} \qquad i = 0, 1, 2 \ldots$$

Proportion that at most 2 circuits are occupied =

$$\pi_0 + \pi_1 + \pi_2 = e^{-\rho} + \rho e^{-\rho} + \frac{\rho^2}{2}e^{-\rho}$$ 6

$$= 2.26 \times 10^{-4}$$

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Q5a) For a non pre-emptive priority system

$$E[W_1] = \frac{E[R]}{1-\rho_1}, \quad E[W_2] = \frac{E[R]}{(1-\rho_1)(1-\rho_1-\rho_2)}$$

$$E[R] = \frac{1}{2}\lambda E[S^2]$$

$$= \frac{1}{2}\lambda\left[\left(\frac{\lambda_1}{\lambda}\right)E(S_1^2) + \left(\frac{\lambda_2}{\lambda}\right)E(S_2^2) + \left(\frac{\lambda_3}{\lambda}\right)E(S_3^2)\right]$$

$$= \frac{1}{2}\left(\lambda_1 E(S_1^2) + \lambda_2 E(S_2^2) + \lambda_3 E(S_3^2)\right)$$

$$\rho_K = \lambda_K E[S_K] \qquad \rho_1 = .4, \ \rho_2 = .3, \ \rho = .1$$

$$E[R] = \frac{1}{2}\left[2\cdot 3 + 3\cdot 5 + 1\cdot 1\right] = 11$$ 5

$$E[W_1] = \frac{11}{1-0.4} \qquad E[W_2] = \frac{11}{(1-0.4)(1-0.4-0.3)}$$

$$= 18.33 \qquad\qquad = 61.1$$ 5

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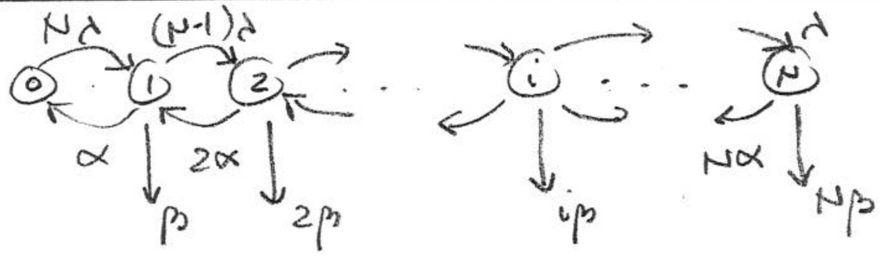
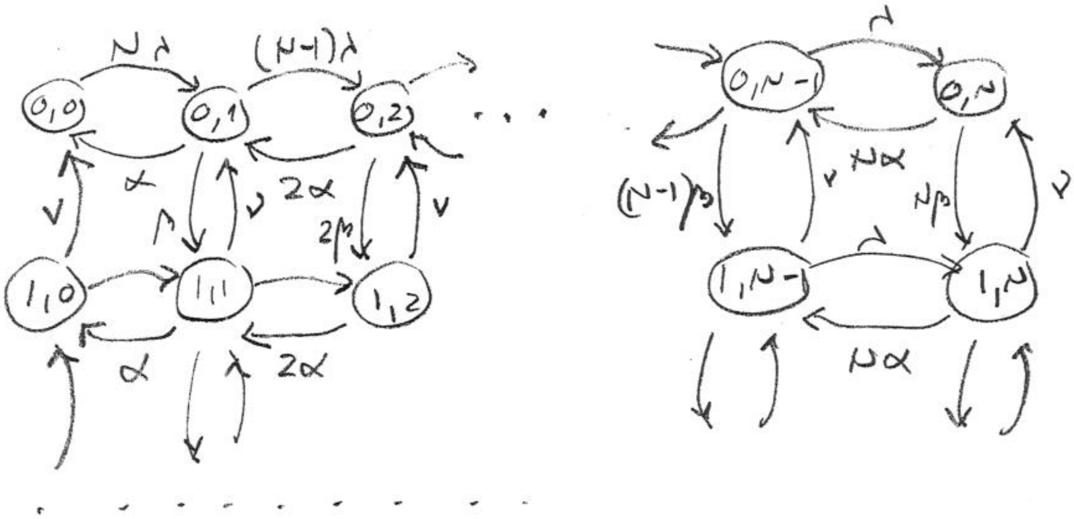
Q5b) i)



2

ii)



3

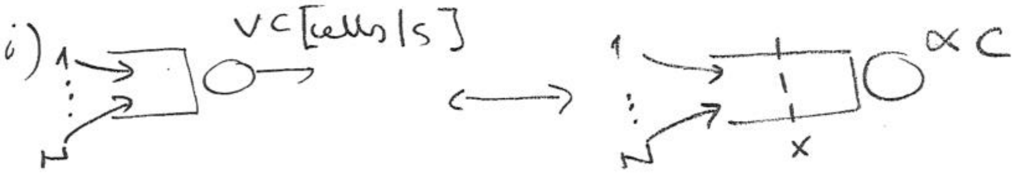
iii) $N\beta\left(\frac{\lambda}{\alpha+\lambda}\right) < \nu$

$N\beta\left(\frac{\lambda}{\alpha+\lambda}\right)$ ~ average cells/s entering the queue

$$\Rightarrow \rho = \frac{N\beta\left(\frac{\lambda}{\alpha+\lambda}\right)}{\nu} < 1$$

5

(6a) i)



- one voice source will generate cells at a rate v [cells/s] during a talk spurt of average length $\frac{1}{\alpha}$ [s].

⇒ x (unit of information) is incremented by v/α cells during a talk spurt

- If the system capacity is vc [cells/s]

⇒ equivalent capacity $\frac{vc}{\frac{v}{\alpha}} = \alpha c$ 5

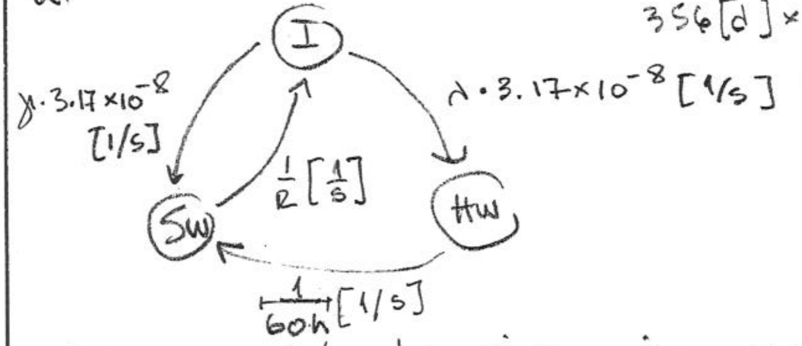
ii) i sources on ⇒ iv [cells/s] ⇒ $\frac{iv}{\frac{v}{\alpha}} = \alpha i$.

$$\frac{\Delta x}{\Delta t} = (i - c)\alpha$$

$i > c$ for the buffer to be filling 5

(6b) i) state space (I) = syst fully operational
(HW) = HW failure
(SW) = SW failure 5

ii)



$356[d] \times 24[h] \times 60[m] \times 60[s]$

Rates need to be given in same scale 5