IMPERIAL COLLEGE
LONDON

[E303/ISE3.3]

DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2004

EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI

SOLUTIONS 2004
COMMUNICATION SYSTEMS

ANSWER to Q1

1)	**A**	**B**	**C**	**D**	**E**
2)	**A**	**B**	**C**	**D**	**E**
3)	**A**	**B**	**C**	**D**	**E**
4)	**A**	**B**	**C**	**D**	**E**
5)	**A**	**B**	**C**	**D**	**E**
6)	**A**	**B**	**C**	**D**	**E**
7)	**A**	**B**	**C**	**D**	**E**
8)	**A**	**B**	**C**	**D**	**E**
9)	**A**	**B**	**C**	**D**	**E**
10)	**A**	**B**	**C**	**D**	**E**
11)	**A**	**B**	**C**	**D**	**E**
12)	**A**	**B**	**C**	**D**	**E**
13)	**A**	**B**	**C**	**D**	**E**
14)	**A**	**B**	**C**	**D**	**E**
15)	**A**	**B**	**C**	**D**	**E**
16)	**A**	**B**	**C**	**D**	**E**
17)	**A**	**B**	**C**	**D**	**E**
18)	**A**	**B**	**C**	**D**	**E**
19)	**A**	**B**	**C**	**D**	**E**
20)	**A**	**B**	**C**	**D**	**E**

ANSWER to Q2

a)

$$\left.\begin{array}{l} b_0 = -4V \\ b_4 = +4V \end{array}\right\} \Rightarrow \Delta = \frac{b_4 - b_0}{Q} = 2V$$

$Q = 4$

$\therefore$ end points $b_0 = -4V$ & o/p levels $m_1 = -3V$

$b_1 = -2V$ $m_2 = -1V$

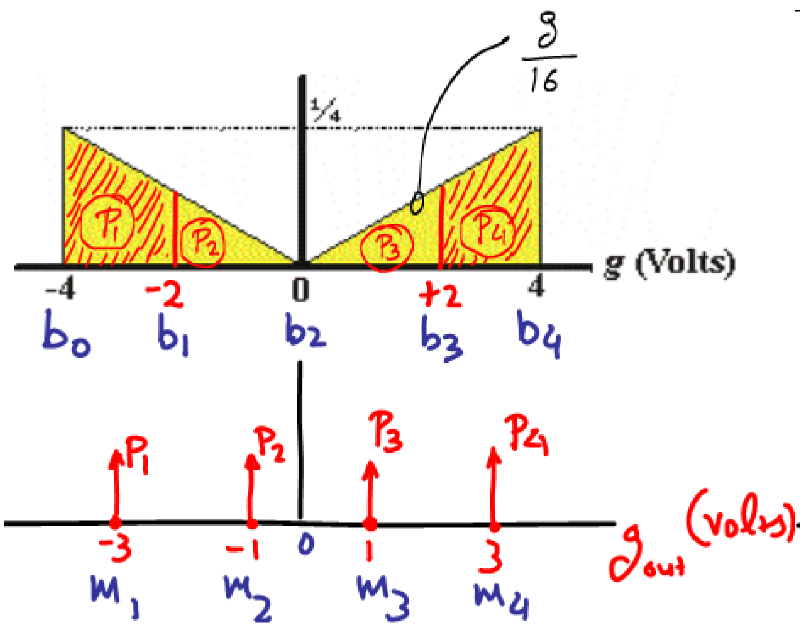
$b_2 = 0V$ $m_3 = 1V$

$b_3 = 2V$ $m_4 = 3V$

$b_4 = 4V$

[5]

b)



$P_1 = P_4$

$P_2 = P_3$

$$P_3 = \Pr(g_{out} = 1V) = \Pr(0 < g < 2V)$$
$$= \int_0^{2V} \text{pdf}_g(g)\, dg$$
$$= \int_0^{2V} \frac{g}{16}\, dg$$
$$= \frac{1}{16} \left. \frac{g^2}{2} \right]_0^2 = \frac{1}{8} = P_2$$

$$P_4 = \Pr(g_{out} = 3V) = \Pr(2 < g < 4V)$$
$$= \int_2^{4V} \text{pdf}_g(g)\, dg$$
$$= \int_2^{4V} \frac{g}{16}\, dg$$
$$= \frac{1}{16} \left. \frac{g^2}{2} \right]_2^4 = \frac{3}{8} = P_1$$

$$\therefore P_{g_{out}} = (-3)^2 \cdot P_1 + (-1)^2 \cdot P_2 + 1^2 \cdot P_3 + 3^2 \cdot P_4 = \frac{56}{8} = 7.$$

$$P_{n_q} = \frac{\Delta^2}{12} = \frac{4}{12} = \frac{1}{3} = 0.333V$$

$$SNR_q = \frac{7}{1/3} = 21$$

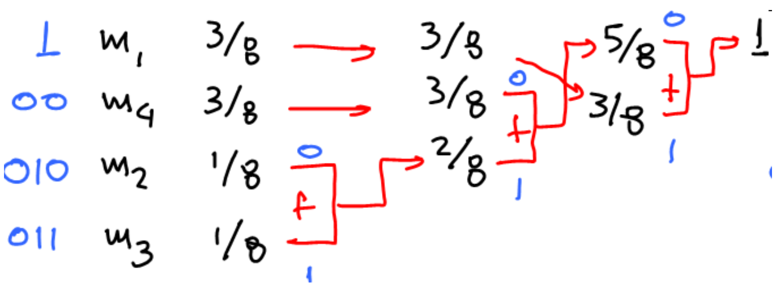
[6]

c)

$$H_M = -2\left(\frac{1}{8}\log_2\frac{1}{8} + \frac{3}{8}\log_2\frac{3}{8}\right)$$
$$= 1.8113 \ \frac{\text{bits}}{\text{symbol}}$$

[3]

d)



[7]

e)

$$\bar{l} = 1\times\frac{3}{8} + 2\times\frac{3}{8} + 3\times\frac{1}{8} + 3\times\frac{1}{8}$$
$$= \frac{15}{8} = 1.8750\ \frac{\text{bits}}{\text{symbol}}$$

[3]

f)

$$r_{inf} = \underbrace{r_M}_{2\times 8K} \times \underbrace{H_M}_{1.8113} = 28.9804\ \frac{\text{bits}}{\text{sec}}$$

[6]

$$r_{data} = r_M \times \underbrace{\bar{l}}_{1.8750} = 30\ \frac{\text{bits}}{\text{sec}}$$

ANSWER to Q3

a)

$$P_e = \underbrace{\Pr(r_2|m_1)\cdot\Pr(m_1)}_{\Pr(r_2,m_1)} + \underbrace{\Pr(r_1|m_2)\cdot\Pr(m_2)}_{\Pr(r_1,m_2)} = \underbrace{0.1\times 0.25}_{0.025} + \underbrace{0.2\times 0.75}_{0.15} = 0.175$$

[6]

b)

$$\underline{q} = \begin{bmatrix} \Pr(r_1) \\ \Pr(r_2) \end{bmatrix} = \underline{\underline{\mathbb{F}}}\cdot\underline{p} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}\begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$$

$$\underline{\underline{\mathbb{J}}} = \underline{\underline{\mathbb{F}}}.\mathrm{diag}(\underline{p}) = \begin{bmatrix} 0.9\times 0.25 & 0.2\times 0.75 \\ 0.1\times 0.25 & 0.8\times 0.75 \end{bmatrix} = \begin{bmatrix} 0.225 & 0.15 \\ 0.025 & 0.6 \end{bmatrix}$$

[12]

$$H_{R|M} = -\left\| \underline{\underline{\mathbb{J}}} \odot \log_2 \underline{\underline{\mathbb{F}}} \right\|_{1+} = 0.658695$$

c)

$$H_{Mut} = H_R - H_{R|M}$$

$$H_R = \underline{q}^T.\log_2(\underline{q}) = 0.9544$$

$$\therefore H_{Mut} = 0.9544 - 0.658695 = 0.295705$$

[12]

ANSWER to Q4

a)

code rate : 1/3

constraint length 3

[5]

b)

generator polynomials

1) $1+D^2$

2) D^2

3) $1+D+D^2$

[6]

c)

$$G_c = \begin{bmatrix} 101 & 001 & 111 & 000 & 000 & \cdots \\ 000 & 101 & 001 & 111 & 000 & \cdots \\ 000 & 000 & 101 & 001 & 111 & \cdots \\ \vdots & \text{etc} & & \cdots & & \end{bmatrix}$$

[9]

d)

if $\underline{x} = [\overset{\text{first}}{1}\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ \overset{\text{last}}{0}]$

then $\underline{x} \cdot \underline{\underline{G}} = [101\ 001\ 010 \text{ etc}]$.

[10]