

UNIVERSITY OF LONDON

[E1.11 2006]

B.ENG. AND M.ENG. EXAMINATIONS 2006

For Internal Students of Imperial College London

This paper is also taken for the relevant examination for the Associateship of the City & Guilds of London Institute

INFORMATION SYSTEMS ENGINEERING E1.11

MATHEMATICS

Date Tuesday 30th May 2006 10.00 am - 1.00 pm

Answer ANY SEVEN questions

Answers to Section A questions must be written in a different answer book from answers to Section B questions.

[Before starting, please make sure that the paper is complete. There should be SIX pages, with a total of NINE questions. Ask the invigilator for a replacement if this copy is faulty.]

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SECTION A**[E1.11 2006]**

1. (i) Find all possible values of the following complex numbers.

Give your answer in the form $x + iy$ (with x and y real) :

- (a) $(1 + 2i)^2$;
- (b) $\ln(1 + i)$;
- (c) $(-1 + i)^{1/3}$;
- (d) $\operatorname{sech}(1 + i\pi/4)$.

- (ii) Find all the solutions of the equation $\tanh z = 1/2$.

Give your answer in the form $x + iy$ (with x and y real).

2. (i) Differentiate $y = (\tan^{-1} x)^{-1}$.

- (ii) Find the stationary points of $y = x^2 e^{-x^2}$ and classify them as maxima or minima.

Sketch the curve.

PLEASE TURN OVER

3. (i) Using l'Hôpital's Rule or otherwise, evaluate the following limits :

$$(a) \quad \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{2x^2 + 2x - 12} ;$$

$$(b) \quad \lim_{x \rightarrow 1} \frac{\ln x}{\cos\left(\frac{\pi x}{2}\right)} .$$

(ii) Use standard tests to determine whether the series $\sum_{n=1}^{\infty} \frac{e^{n/2}}{\sqrt{n!}}$ converges or diverges.

(iii) Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x^2 + 1)^n}{3^n n^3}$ and investigate the endpoints.

4. Evaluate the following integrals :

$$(i) \quad \int \tan x \, dx ;$$

$$(ii) \quad \int x \sec^2 x \, dx ;$$

$$(iii) \quad \int \cos^2(2x) \, dx ;$$

$$(iv) \quad \int \frac{dx}{x(x^2 + 2x + 1)} .$$

[E1.11 2006]

5. (i) Find the general solution of the 1st order differential equation

$$\frac{dy}{dx} + (\ln x)y = xe^{-x \ln x}.$$

- (ii) Find the solution of the 2nd order differential equation

$$\frac{d^2y}{dx^2} - 8 \frac{dy}{dx} + 16y = e^{4x},$$

with $y = 1, \frac{dy}{dx} = 0$ at $x = 0$.

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SECTION B

6. (i) Let

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 3 & -1 & 4 \\ 1 & -1 & 2 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

Use Gaussian elimination to find all solutions of the system of simultaneous equations

$$Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

- (ii) Find a condition on the entries of a column vector $c = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$

which ensures that the system $Ax = c$ has no solutions, where A is the matrix in (i).

- (iii) Let $B = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$.

Find an invertible 2×2 matrix P such that $P^{-1}BP$ is a diagonal matrix.

7. (i) Let
- $u = x^2 + y^2$
- ,
- $v = xy$
- and
- $f = f(u, v)$
- .

Express $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

- (ii) Let $g(x, y) = x^3y + xy^2 - xy$.

Find the six stationary points of g and determine whether they are maxima, minima or saddle points.

8. Sketch the graph of the function

$$f(x) = |\sin x|.$$

Calculate the Fourier series for $f(x)$, giving the general term.

Deduce that

$$\frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \frac{1}{7 \cdot 9} + \dots = \frac{\pi}{4} - \frac{1}{2}$$

and

$$\frac{1}{3 \cdot 5} - \frac{1}{7 \cdot 9} + \frac{1}{11 \cdot 13} - \frac{1}{15 \cdot 17} + \dots = \frac{\pi}{4\sqrt{2}} - \frac{1}{2}.$$

9. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}(f(t)) = F(s) = \int_0^\infty e^{-st} f(t) dt.$$

- (i) Find $\mathcal{L}(e^{-t} \sin t)$.

(Any rule used must be proved.)

- (ii) Use Laplace transforms to find functions x, y of t satisfying the following simultaneous differential equations :

$$\frac{dx}{dt} + \frac{dy}{dt} + x = 0,$$

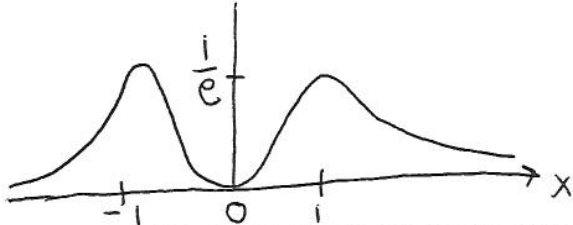
$$\frac{dx}{dt} + 2 \frac{dy}{dt} - x = e^{-t},$$

with $x(0) = \frac{1}{2}$, $y(0) = 0$.

[You may assume that $\mathcal{L}(f'(t)) = -f(0) + s\mathcal{L}(f(t))$.]

END OF PAPER

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1-6
Question 1		Marks & seen/unseen
Parts	(a)(i) $(1+2i)^2 = 1 + 4i + 4i^2 = -3 + 4i$	1
	(ii) $\ln(1+i) = \ln(\sqrt{2}e^{i\frac{\pi}{4} + i2n\pi}) = \ln\sqrt{2} + \frac{i\pi}{4} + i2n\pi$ $= \frac{1}{2}\ln 2 + i(\frac{\pi}{4} + 2n\pi)$	3
	(iii) $(-1+i)^{1/3} = (\sqrt{2}e^{i\frac{3\pi}{4} + i2n\pi})^{1/3}$ $= 2^{1/6}(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}) = 2^{-1/3}(1+i)$ $= 2^{1/6}(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12})$ $= 2^{1/6}(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12})$	4
	(iv) $\operatorname{sech}(1+i\frac{\pi}{4}) = \frac{1}{\cosh(1+i\frac{\pi}{4})} = \frac{2}{e^{1+i\frac{\pi}{4}} + e^{-1-i\frac{\pi}{4}}}$ $= \frac{2}{e^{(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}})} + e^{(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}})}} = \frac{2\sqrt{2}e}{e^2(1+i) + (1-i)} = \frac{2\sqrt{2}e}{(e^2+1) + i(e^2-1)}$ $= \frac{2\sqrt{2}e[(e^2+1) - i(e^2-1)]}{(e^2+1)^2 + (e^2-1)^2} = \frac{2\sqrt{2}e[(e^2+1) - i(e^2-1)]}{2e^4+2} = \frac{\sqrt{2}e(e^2+1)}{e^4+1} - i\frac{\sqrt{2}e(e^2-1)}{e^4+1}$	6
	(b) $\tanh z = \frac{1}{2} \Rightarrow \frac{\sinh z}{\cosh z} = \frac{1}{2} \Rightarrow \frac{e^z - e^{-z}}{e^z + e^{-z}} = \frac{1}{2}$	
	$\Rightarrow 3e^{-z} = e^z \Rightarrow e^{2z} = 3e^{i2n\pi}$	6
	$\Rightarrow 2z = \ln 3 + i2n\pi$	
	$\Rightarrow z = \frac{1}{2}\ln 3 + i n\pi$	
Setter's initials MJB	Checker's initials Mhr	Page number

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1.6
Question 2 ✓		Marks & seen/unseen
Parts	<p>(a) $y = (\tan^{-1} x)^{-1}$ $\Rightarrow \frac{dy}{dx} = -(\tan^{-1} x)^{-2} \frac{d}{dx}(\tan^{-1} x)$</p> <p>Let $v = \tan^{-1} x \Rightarrow \tan v = x$ $\Rightarrow \sec^2 v \frac{dv}{dx} = 1$ $\Rightarrow \frac{dv}{dx} = \frac{1}{\sec^2 v} = \frac{1}{1 + \tan^2 v} = \frac{1}{1 + x^2}$</p> <p>$\Rightarrow \frac{dy}{dx} = -\frac{1}{(\tan^{-1} x)^2} \cdot \frac{1}{1 + x^2}$</p> <p>(b) $y = x^2 e^{-x^2}$ $\Rightarrow \frac{dy}{dx} = 2x e^{-x^2} - 2x^3 e^{-x^2} = 2x(1 - x^2) e^{-x^2}$</p> <p>Stationary points where $\frac{dy}{dx} = 0 \Rightarrow x = 0, \pm 1$</p> <p>$\frac{d^2 y}{dx^2} = 2e^{-x^2} - 4x^2 e^{-x^2} - 6x^2 e^{-x^2} + 4x^4 e^{-x^2} = (2 - 10x^2 + 4x^4) e^{-x^2}$</p> <p>At $x = \pm 1$, $\frac{d^2 y}{dx^2} = (2 - 10 + 4) e^{-1} = -4e^{-1} < 0 \Rightarrow \text{maximum}$</p> <p>At $x = 0$, $\frac{d^2 y}{dx^2} = 2 > 0 \Rightarrow \text{minimum}$</p> 	<p>8</p> <p>9</p> <p>3</p>
Setter's initials MJA	Checker's initials Mhr	Page number

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1.6
Question 3		Marks & seen/unseen
Parts	<p>(a) (i) $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{2x^2 + 2x - 12} = \lim_{x \rightarrow 2} \frac{2x - 1}{4x + 2} = \frac{3}{10}$</p> <p>(ii) $\lim_{x \rightarrow 1} \frac{\ln x}{\cos(\frac{\pi x}{2})} = \lim_{x \rightarrow 1} \frac{1/x}{-\frac{\pi}{2} \sin(\frac{\pi x}{2})} = -\frac{2}{\pi}$</p> <p>(b) $\sum_{n=1}^{\infty} \frac{e^{n/2}}{\sqrt{n!}}$ $a_n = \frac{e^{n/2}}{\sqrt{n!}}$</p> <p>$\rho = \lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = \lim_{n \rightarrow \infty} \left(\frac{e^{\frac{n+1}{2}}}{\sqrt{(n+1)!}} \frac{\sqrt{n!}}{e^{n/2}} \right)$</p> <p>$= \lim_{n \rightarrow \infty} \left(\frac{e^{1/2}}{\sqrt{n+1}} \right) = 0 \Rightarrow \text{convergent.}$</p> <p>(c) $\sum_{n=1}^{\infty} \frac{(x^2 + 1)^n}{3^n n^3}$ $a_n = \frac{(x^2 + 1)^n}{3^n n^3}$</p> <p>$\rho = \lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = \lim_{n \rightarrow \infty} \left(\frac{(x^2 + 1)^{n+1}}{3^{n+1} (n+1)^3} \frac{3^n n^3}{(x^2 + 1)^n} \right)$</p> <p>$= \lim_{n \rightarrow \infty} \left(\frac{x^2 + 1}{3} \frac{1}{(1 + \frac{1}{n})^3} \right) = \frac{x^2 + 1}{3}$</p> <p>Convergent for $\frac{x^2 + 1}{3} < 1 \Rightarrow x^2 < 2 \Rightarrow -\sqrt{2} < x < \sqrt{2}$</p> <p>Endpoints: at $x = \pm\sqrt{2}$, series becomes:</p> <p>$\sum_{n=1}^{\infty} \frac{((\sqrt{2})^2 + 1)^n}{3^n n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3}$ which is convergent.</p>	<p>2</p> <p>3</p> <p>6</p> <p>9</p>
	<p>Setter's initials MJB</p> <p>Checker's initials MJC</p>	Page number

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course Set 6
Question 4		Marks & seen/unseen
Parts	<p>(a) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\ln(\cos x) + c$</p> <p>(b) $\int x \sec^2 x \, dx$ $u = x$ $\frac{dv}{dx} = \sec^2 x$ $\frac{du}{dx} = 1$ $v = \tan x$</p> <p>$= x \tan x - \int \tan x \, dx$</p> <p>$= x \tan x + \ln(\cos x) + c$</p> <p>(c) $\int \cos^2(2x) \, dx$ $= \int \frac{1}{2}(1 + \cos 4x) \, dx$ $= \frac{1}{2}x + \frac{1}{8}\sin 4x + c$</p> <p>(d) $\int \frac{dx}{x(x^2+2x+1)}$</p> <p>$\frac{1}{x(x^2+2x+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+2x+1} \Rightarrow 1 = A(x^2+2x+1) + (Bx+C)x$ $\Rightarrow 1 = x^2(A+B) + x(2A+C) + A$ $\Rightarrow A=1, B=-1, C=-2$</p> <p>$= \int \left\{ \frac{1}{x} - \frac{x+2}{x^2+2x+1} \right\} dx = \int \left\{ \frac{1}{x} - \frac{x+1+1}{(x+1)^2} \right\} dx$ $= \int \left\{ \frac{1}{x} - \frac{1}{x+1} - \frac{1}{(x+1)^2} \right\} dx = \ln x - \ln(x+1) + \frac{1}{x+1} + c$ $= \ln \frac{x}{x+1} + \frac{1}{x+1} + c$</p>	<p>3</p> <p>3</p> <p>4</p> <p>10</p>
	<p>Setter's initials Myt</p> <p>Checker's initials Mhr</p>	Page number

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1.6
Question 5		Marks & seen/unseen
Parts	<p>(a) $\frac{dy}{dx} + (\ln x)y = xe^{-x} \ln x$</p> <p>Integrating factor $e^{\int \ln x dx}$</p> <p>$\int \ln x dx$ $u = \ln x \quad \frac{dv}{dx} = 1 \quad \int \ln x dx = x \ln x - \int dx = x \ln x - x$</p> <p>$\frac{dv}{dx} = \frac{1}{x} \quad v = x$</p> <p>$\Rightarrow \frac{d}{dx} (ye^{x \ln x - x}) = xe^{-x} \ln x e^{x \ln x - x} = xe^{-x}$</p> <p>$\Rightarrow ye^{x \ln x - x} = \int xe^{-x} dx$ $u = x \quad \frac{dv}{dx} = e^{-x}$</p> <p>$\frac{du}{dx} = 1 \quad v = -e^{-x}$</p> <p>$= -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} + C$</p> <p>$\Rightarrow y = e^{-x \ln x} (-1 - x + ce^x)$</p> <p>(b) $\frac{d^2 y}{dx^2} - 8 \frac{dy}{dx} + 16y = e^{4x}$</p> <p>Auxiliary equation: $\lambda^2 - 8\lambda + 16 = 0 \Rightarrow (\lambda - 4)^2 = 0$</p> <p>C.F. $y = (Ax + B)e^{4x}$</p> <p>P.I. $y = Cx^2 e^{4x}$</p> <p>$\Rightarrow 16Cx^2 e^{4x} - 8(4Cx e^{4x} + 2C x^2 e^{4x}) + (2C e^{4x} + 16Cx e^{4x} + 16Cx^2 e^{4x}) = e^{4x}$</p> <p>$\Rightarrow C = \frac{1}{2}$ Full solution: $y = (Ax + B)e^{4x} + \frac{1}{2}x^2 e^{4x}$</p> <p>$y = 1$ at $x = 0 \Rightarrow B = 1$</p> <p>$\frac{dy}{dx} = 0$ at $x = 0 \Rightarrow 0 = x e^{4x} + 2x e^{4x} + A e^{4x} + 4(Ax + 1)e^{4x}$ at $x = 0$</p> <p>$\Rightarrow A = -4$</p>	<p>10</p> <p>10</p>
	<p>Setter's initials H/W</p> <p>Checker's initials mm</p>	Page number

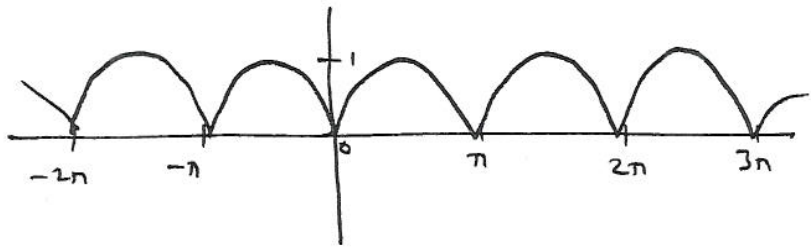
$$\Rightarrow y = (1 - 4x)e^{4x} + \frac{1}{2}x^2 e^{4x}$$

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1.
Question. 6, similar.		Marks & seen/unseen
Parts	<p>(i) Augmented matrix</p> $\left(\begin{array}{ccc c} 2 & -1 & 3 & 0 \\ 3 & -1 & 4 & 0 \\ 1 & -1 & 2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc c} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc c} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ <p>New system: $x_1 - x_2 + 2x_3 = 0$ $x_2 - x_3 = 0$</p> <p>General solution is $x = (-a, a, a)$ (any a).</p> <p>(ii) $\left(\begin{array}{ccc c} 2 & -1 & 3 & c_1 \\ 3 & -1 & 4 & c_2 \\ 1 & -1 & 2 & c_3 \end{array}\right) \rightarrow \left(\begin{array}{ccc c} 1 & -1 & 2 & c_3 \\ 0 & 1 & -1 & c_1 - 2c_3 \\ 0 & 2 & -2 & c_2 - 3c_3 \end{array}\right)$</p> $\rightarrow \left(\begin{array}{ccc c} 1 & -1 & 2 & c_3 \\ 0 & 1 & -1 & c_1 - 2c_3 \\ 0 & 0 & 0 & c_2 - 3c_3 - 2(c_1 - 2c_3) \end{array}\right)$ <p>System has no solutions when constant on RHS of last equation is nonzero, i.e. when</p> $c_2 - 3c_3 - 2(c_1 - 2c_3) \neq 0$ <p>i.e. <u>$2c_1 - c_2 - c_3 \neq 0$</u></p> <p>(iii) Characteristic poly of B is $\begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix}$</p> $= \lambda^2 - 3\lambda - 4 = (\lambda - 4)(\lambda + 1).$ <p>So eigenvalues are 4, -1.</p> <p><u>$\lambda = 4$</u> Eigenvectors $\left(\begin{array}{cc c} -3 & 2 & 0 \\ 3 & -2 & 0 \end{array}\right) \rightarrow a \begin{pmatrix} 2 \\ 3 \end{pmatrix}$</p> <p><u>$\lambda = -1$</u> Eigenvectors $\left(\begin{array}{cc c} 2 & 2 & 0 \\ 3 & 3 & 0 \end{array}\right) \rightarrow b \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$</p>	<p>SIMILAR EGS SEEN</p> <p>6</p> <p>6</p> <p>SIMILAR SEEN</p> <p>2</p> <p>2</p> <p>2</p>
Setter's initials MLW	Checker's initials MJA	Page number 6

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1.
Question. 6, chd.		Marks & seen/unseen
Parts	<p>So $P = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$ will do</p> <hr/>	2
	Setter's initials MLL	Checker's initials mjt
		Page number 7

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1.
Question. 7, where		Marks & seen/unseen
Parts	<p>(i) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot 2x + \frac{\partial f}{\partial v} \cdot y$ ①</p> <p>$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} \cdot 2y + \frac{\partial f}{\partial v} \cdot x$ ②</p> <p>① $\times y$ - ② $\times x$ gives</p> $y f_x - x f_y = f_v (y^2 - x^2)$ <p>$\therefore f_v = \frac{y f_x - x f_y}{y^2 - x^2}$</p> <p>① $\times x$ - ② $\times y$ gives</p> $x f_x - y f_y = f_u (2x^2 - 2y^2)$ <p>$\therefore f_u = \frac{x f_x - y f_y}{2(x^2 - y^2)}$</p> <p>(ii) Here</p> <p>① $g_x = 3x^2y + y^2 - y = y(3x^2 - 1 + y)$</p> <p>② $g_y = x^3 + 2xy - x = x(x^2 - 1 + 2y)$</p> <p>For stationary point, $g_x = g_y = 0$.</p> <p>From ①, $y = 0$ or $3x^2 - 1 + y = 0$</p> <p>From ②, $x = 0$ or $x^2 - 1 + 2y = 0$.</p>	<p><u>UNSEEN</u></p> <p>2</p> <p>2</p> <p>2</p> <p><u>SIMILAR SEEN</u></p> <p>2</p>
	<p>Setter's initials MLL</p> <p>Checker's initials MyD</p>	Page number 8

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE-1																					
Question. 7, cld.		Marks & seen/unseen																					
Parts	<p>If $y = 0$ then $x = 0$ or ± 1.</p> <p>If $x = 0$ then $y = 0$ or 1.</p> <p>If $3x^2 - 1 + y = x^2 - 1 + 2y = 0$, then sub. $y = 1 - 3x^2$, giving $x^2 - 1 + 2(1 - 3x^2) = 0$$\Rightarrow 5x^2 = 1$$\Rightarrow x = \pm \frac{1}{\sqrt{5}}, y = 1 - \frac{3}{5} = \frac{2}{5}.$</p> <p>Hence get 6 stationary points</p> <p>$(0,0), (1,0), (-1,0), (0,1), (\frac{1}{\sqrt{5}}, \frac{2}{5}), (-\frac{1}{\sqrt{5}}, \frac{2}{5})$ 6</p> <p><u>Nature</u> Now</p> <p>$f_{xx} = 6xy, f_{xy} = 3x^2 + 2y - 1, f_{yy} = 2x.$ 2</p> <p>Hence, for $\Delta = f_{xy}^2 - f_{xx} f_{yy}$, get</p> <table><tr><th>pt.</th><th>$(0,0)$</th><th>$(1,0)$</th><th>$(-1,0)$</th><th>$(0,1)$</th><th>$(\frac{1}{\sqrt{5}}, \frac{2}{5})$</th><th>$(-\frac{1}{\sqrt{5}}, \frac{2}{5})$</th></tr><tr><td>$\Delta$</td><td>1</td><td>4</td><td>4</td><td>1</td><td>$-\frac{4}{5}$</td><td>$-\frac{4}{5}$</td></tr><tr><td>f_{xx}</td><td></td><td></td><td></td><td></td><td>> 0</td><td>< 0</td></tr></table> <p>Hence $(0,0), (\pm 1,0), (0,1)$ are <u>saddles</u></p> <p>$(\frac{1}{\sqrt{5}}, \frac{2}{5})$ is a <u>minimum</u></p> <p>$(-\frac{1}{\sqrt{5}}, \frac{2}{5})$ is a <u>maximum</u> 2</p>	pt.	$(0,0)$	$(1,0)$	$(-1,0)$	$(0,1)$	$(\frac{1}{\sqrt{5}}, \frac{2}{5})$	$(-\frac{1}{\sqrt{5}}, \frac{2}{5})$	Δ	1	4	4	1	$-\frac{4}{5}$	$-\frac{4}{5}$	f_{xx}					> 0	< 0	
pt.	$(0,0)$	$(1,0)$	$(-1,0)$	$(0,1)$	$(\frac{1}{\sqrt{5}}, \frac{2}{5})$	$(-\frac{1}{\sqrt{5}}, \frac{2}{5})$																	
Δ	1	4	4	1	$-\frac{4}{5}$	$-\frac{4}{5}$																	
f_{xx}					> 0	< 0																	
	Setter's initials MLL	Checker's initials HYB	Page number 9																				

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1.
Question. 8, solution		Marks & seen/unseen
Parts	<p>Graph:</p>  <p>Fourier series: is a cosine series (even function)</p> $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ <p>where</p> $a_0 = \frac{2}{\pi} \int_0^{\pi} \sin x \, dx = \frac{2}{\pi} [-\cos x]_0^{\pi} = \frac{4}{\pi}$ <p>and for $n \geq 1$,</p> $a_n = \frac{2}{\pi} \int_0^{\pi} \sin x \cos nx \, dx$ $= \frac{1}{\pi} \int_0^{\pi} (\sin(n+1)x - \sin(n-1)x) \, dx$ <p>For $n=1$, this is zero. For $n \neq 1$</p> $= \frac{1}{\pi} \left[\frac{1}{n+1} \cos(n+1)x - \frac{1}{n-1} \cos(n-1)x \right]_0^{\pi}$ $= \begin{cases} 0, & n \text{ odd} \\ -\frac{4}{\pi(n^2-1)}, & n \text{ even.} \end{cases}$ <p>So Fourier series is</p> $\frac{2}{\pi} - \frac{4}{\pi} \left(\frac{1}{2^2-1} \cos 2x + \frac{1}{4^2-1} \cos 4x + \dots \right)$ $= \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2-1} \cos 2nx.$	<p>SIMILAR <u>SEEN</u></p> <p>3</p> <p>9</p>
	<p>Setter's initials MLW</p> <p>Checker's initials MYA</p>	Page number 10

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course IS 1.
Question. 8 w.		Marks & seen/unseen
Parts	<p>Put $x = \frac{\pi}{2}$:</p> $\sin \frac{\pi}{2} = 1 = \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2-1}$ $\therefore \frac{4}{\pi} \left(\frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \dots \right) = 1 - \frac{2}{\pi}$ $\therefore \frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \dots = \frac{\pi}{4} - \frac{1}{2}$ <hr/> <p>Put $x = \frac{\pi}{4}$:</p> $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $= \frac{2}{\pi} - \frac{4}{\pi} \left(\frac{1}{3 \cdot 5} - \frac{1}{7 \cdot 9} + \frac{1}{11 \cdot 13} - \dots \right)$ $\therefore \frac{1}{3 \cdot 5} - \frac{1}{7 \cdot 9} + \frac{1}{11 \cdot 13} - \dots$ $= \frac{\pi}{4} \left(\frac{1}{\sqrt{2}} - \frac{2}{\pi} \right)$ $= \frac{\pi}{4\sqrt{2}} - \frac{1}{2}$ <hr/>	<p>4</p> <p>4</p>
	<p>Setter's initials MLW</p> <p>Checker's initials Mya</p>	Page number 11

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1.
Question.	9, solution.	Marks & seen/unseen
Parts	<p>(i) Let $f(t) = e^{-t} \sin t$. The</p> $\mathcal{L}(f(t)) = F(s) = \int_0^{\infty} e^{-(s+1)t} \sin t \, dt$ $= \left[-\frac{1}{s+1} e^{-(s+1)t} \sin t \right]_0^{\infty} + \int_0^{\infty} \frac{1}{s+1} e^{-(s+1)t} \cos t \, dt$ $= 0 + \frac{1}{s+1} \left[-\frac{1}{s+1} e^{-(s+1)t} \cos t \right]_0^{\infty} - \frac{1}{s+1} \int_0^{\infty} \frac{1}{s+1} e^{-(s+1)t} \sin t \, dt$ $= \frac{1}{(s+1)^2} - \frac{1}{(s+1)^2} F(s)$ $\therefore F(s) \left(1 + \frac{1}{(s+1)^2} \right) = \frac{1}{(s+1)^2}$ $\therefore F(s) = \frac{1}{(s+1)^2 + 1}$ <hr/> <p>(ii) Take Laplace transforms:</p> $\textcircled{1} -\frac{1}{2} + s\mathcal{L}(x) + s\mathcal{L}(y) + \mathcal{L}(x) = 0$ $\textcircled{2} -\frac{1}{2} + s\mathcal{L}(x) + 2s\mathcal{L}(y) - \mathcal{L}(x) = \frac{1}{s+1}$	<p>SIMILAR SEEN</p> <p>6</p> <p>4</p>
	Setter's initials MLL	Checker's initials Page number 12

	EXAMINATION QUESTIONS/SOLUTIONS 2005-06	Course ISE 1
Question. 9, ch.		Marks & seen/unseen
Parts	<p>So</p> <p>① $(s+1) \mathcal{L}(x) + s \mathcal{L}(y) = \frac{1}{2}$</p> <p>② $(s-1) \mathcal{L}(x) + 2s \mathcal{L}(y) = \frac{1}{s+1} + \frac{1}{2} = \frac{s+3}{2(s+1)}$</p> <p>Then $2 \times ① - ②$ gives</p> $(s+3) \mathcal{L}(x) = 1 - \frac{s+3}{2s+2} = \frac{s-1}{2s+2}$ $\therefore \mathcal{L}(x) = \frac{s-1}{2(s+1)(s+3)}$ $= \frac{1}{2} \left(\frac{2}{s+3} - \frac{1}{s+1} \right)$ $\therefore \underline{x = e^{-3t} - \frac{1}{2} e^{-t}}$ <p>From ①, $s \mathcal{L}(y) = \frac{1}{2} - (s+1) \mathcal{L}(x)$</p> $= \frac{1}{2} - \frac{s-1}{2(s+3)} = \frac{2}{s+3}$ <p>So $\mathcal{L}(y) = \frac{2}{s(s+3)} = \frac{2}{3} \left(\frac{1}{s} - \frac{1}{s+3} \right)$</p> $\therefore \underline{y = \frac{2}{3} (1 - e^{-3t})}$	<p>5</p> <p>5</p>
	<p>Setter's initials MLL</p> <p>Checker's initials hjt</p>	Page number 17