EE4-08
EE9-SO14

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

MSc and EEE/EIE PART IV: MEng and ACGI

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DIGITAL IMAGE PROCESSING

Friday, 6 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.T. Stathaki
Second Marker(s) : T-K. Kim

1. a) Let $f(x,y)$ denote a digital image of size 256×256 pixels. In order to compress this image, we take its Discrete Cosine Transform $C(u,v), u,v = 0,\dots,255$ and keep only the Discrete Cosine Transform coefficients for $u = 0,\dots,n$ and $v = 0,\dots,n$ with $0 \le n < 255$. The remaining of the Discrete Cosine Transform coefficients are replaced with zero values. The percentage of total energy of the original image that is preserved in that case is given by the formula $an + b + 85$ with a,b constant parameters. Furthermore, the energy that is preserved if $n = 0$ is 85%. Find the constants a,b. **[4]**

b) Let $f(x,y)$ denote a digital image of size $M \times N$ pixels that is zero outside $0 \le x \le M-1$, $0 \le y \le N-1$, where M and N are integers and powers of 2. In implementing the standard Discrete Hadamard Transform of $f(x,y)$, we relate $f(x,y)$ to a new $M \times N$ point sequence $H(u,v)$.

(i) State the main disadvantage of the Discrete Hadamard Transform. **[2]**

(ii) In the case of $M = N = 2$ and $f(x,y) = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ calculate the Hadamard transform coefficients of $f(x,y)$ **[4]**

c) Consider the population of vectors $\underline{f}$ of the form

$$\underline{f}(x,y) = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \\ f_3(x,y) \end{bmatrix}.$$

Each component $f_i(x,y)$, $i = 1,2,3$ represents an image of size $M \times M$ where M is even. The population arises from the formation of the vectors $\underline{f}$ across the entire collection of pixels (x,y). The three images are defined as follows:

$$f_1(x,y) = \begin{cases} r_1 & 1 \le x \le \dfrac{M}{2}, 1 \le y \le M \\ r_2 & \dfrac{M}{2} < x \le M, 1 \le y \le M \end{cases}$$

$$f_2(x,y) = r_3,\ 1 \le x \le M,\ 1 \le y \le M$$

$$f_3(x,y) = r_4,\ 1 \le x \le M,\ 1 \le y \le M$$

The parameters r_1, r_2, r_3, r_4 are constants.

Consider now a population of random vectors of the form

$$\underline{g}(x,y) = \begin{bmatrix} g_1(x,y) \\ g_2(x,y) \\ g_3(x,y) \end{bmatrix}$$

where the vectors $\underline{g}$ are the Karhunen-Loeve (KL) transforms of the vectors $\underline{f}$.

(i) Find the images $g_1(x,y)$, $g_2(x,y)$ and $g_3(x,y)$ using the Karhunen-Loeve (KL) transform. **[8]**

(ii) Comment on whether you could obtain the result of c)-(i) above using intuition rather than by explicit calculation. **[2]**

2. a) A 3×3 two dimensional filter is separable if its response can be written in the form:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix}\begin{bmatrix} d & e & f \end{bmatrix}$$

where a,b,c,d,e,f are constant coefficients. Consider the 3×3 two dimensional filters with coefficients shown below:

$$\frac{1}{9}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9A-1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

with A a constant real number slightly greater than 1.

For each filter given above, answer the following questions.

(i) Is it a separable filter? If yes, present the vertical and horizontal filters. [3]

(ii) What is the functionality of the filter? Explain your reasoning. [3]

b) Consider a grey level image $f(x,y)$ of size 256×256 pixels, with $1\le x,y\le 256$, which has the following intensities:

$$f(x,y)=\begin{cases} r+1 & 1\le x\le 12 \text{ and } 1\le y\le 12 \\ r & 13\le x\le 16, 1\le y\le 16 \text{ and } 1\le x\le 12, 13\le y\le 16 \\ r+2 & \text{elsewhere} \end{cases}$$

with $0\le r\le 253$.

(i) Sketch the image $f(x,y)$ and comment on its visual appearance. Justify your answer. [3]

(ii) Apply global histogram equalisation on the above image. Comment on the visual appearance of the resulting equalised image. [3]

(iii) Apply local histogram equalisation on the above image using non-overlapping image patches of size 16×16 pixels. Comment on the visual appearance of the resulting locally equalised image. [3]

(iv) Based on the above observations, which of the two types of equalisation processes would you choose for the visual improvement of the particular image? Justify your answer. [2]

c) Propose a method that uses spatial filters of variable sizes to reduce background noise without blurring the image significantly. [3]

3. We are given the degraded version g of an image f such that in lexicographic ordering

$$g = Hf + n$$

where H is the degradation matrix which is assumed to be block-circulant and n is the noise term which is assumed to be zero-mean, white and independent of the image f. All images involved have size $N \times N$ after extension and zero-padding.

a) (i) Consider the Inverse Filtering image restoration technique. Give the general expressions for both the Inverse Filtering estimator and the restored image in both spatial and frequency domains and explain all symbols used. [5]

(ii) In a particular scenario the degradation process can be modelled as a linear filter with the two dimensional impulse response given below:

$$h(x,y) = \begin{cases} 1 & -1 \le x \le 1,\ y = 0 \\ 0 & \text{elsewhere} \end{cases}$$

Estimate the frequency pairs for which Inverse Filtering cannot be applied. [5]

b) (i) Consider the Constrained Least Squares (CLS) filtering image restoration technique. Give the general expressions for both the CLS filter estimator and the restored image in both spatial and frequency domains and explain all symbols used. [5]

(ii) In a particular scenario, the degradation process can be modelled as a linear filter with the transfer function given below:

$$H(u,v) = \sqrt{2\pi}\sigma(u^2 + v^2)e^{-j2\pi^2\sigma^2(u^2+v^2)}$$

In the above formulation σ is a constant parameter. Generate the expression of the CLS filter in frequency domain by assuming that the high pass filter used in CLS is a Laplacian filter.

[5]

4. a) (i) Give the definition of a Discrete Memoryless Source (DMS). [3]

(ii) Consider a set of symbols generated from a DMS. Give the minimum number of bits per symbol that we can achieve if we use Huffman coding for the binary representation of the symbols. Explain what type of probabilities the symbols must possess in order to achieve the minimum number of bits per symbol using Huffman coding. [3]

(iii) Provide a scenario where Huffman coding would not reduce the number of bits per symbol from that achieved using fixed number of bits per symbol. [2]

b) The following Figure 4 shows a list of 7 symbols and their probabilities. It is assumed that these symbols are generated by a Discrete Memoryless Source (DMS).

Symbol	**Probability**
k	0.05
l	0.2
u	0.1
w	0.05
e	0.3
r	0.2
?	0.1

Figure 4

(i) Derive a Huffman code taking into consideration that in a particular transmission system, the probability of a 1 being transmitted as 0 is zero and the probability of a 0 being transmitted as a 1 is 0.05. [4]

(ii) Calculate the compression ratio. [4]

(iii) In the particular transmission system described in b)(i) above, find the probability of a codeword equal to 100 being transmitted wrongly. [4]