Fuzzy Systems Exam Coursework 2000-2001
Deadline April 30th 2001 at 10.00 am *

Download the paper *Interrelationships among fuzzy preference-based choice functions and significance of rationality conditions: a taxonomic and intuitive perspective*, Pankaj Kulshreshtha and B. Shekar, Fuzzy Sets and Systems, Vol 109, 2000, p. 429–445 via the library's electronic journal portal http://www.lib.ic.ac.uk/ejournals/ejnls_titl.htm

This paper is about fuzzy relations expressing the degree to which one alternative is better than another, and the choice set of preferred alternatives that results from such a fuzzy relation. It turns out that there are many possibilities, and the authors criticize the definitions, pointing out counterintuitive consequences. They also discuss how rationality is affected by fuzzy choice. Although this survey article refers to many papers, it is self-contained, and you do not have to look up any other papers, except for the challenge.

Skip the introduction, start reading in section 2 about fuzzy preference relations. Pay attention to the definition of membership function $\mu_R(x, y)$. This is not a fuzzy graph! A crisp preference relation is called transitive if, for alternatives x, y and z, alternative y preferred to x and z preferred to y implies that z is preferred to x. Which property of fuzzy preference relations mentioned in the paper is related to transitivity of crisp alternatives, and how (1/20)?

In the paper, irreflexive means not reflexive. Stick to the definitions in the paper. Consulting references may confuse you.

Now start reading section 3 on choice functions. Equations (3.1)-(3.4) define crisp choice sets. The symbol $\neg$ means "not", the other symbols have been encountered in the course. Fuzzyfy the sets (3.1)-(3.4) using fuzzy truth values (2/20).

Is the strict fuzzy preference relation defined in (3.5) symmetric? Answering a question like this, which carries no marks, will help you understand the paper. Skip line 6 to 26 on page 432, column 1. Do note the definition of

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acyclic preference relation. Read section 3.2 until and including the definition of D-rationality. Skip the rest of section 3 (4 pages).

In section 4, verify, using table 2, the set of non-dominated alternatives with membership function $\mu^{ND}(x)$. How many numbers do you have to compare? How many numbers would you have to compare if there were only three alternatives (1/20)? In the text, "sans x_2" means without x_2. Make sure you understand why, in the second calculation of μ^{ND}, X^{ND} turns out to be x_4, and not x_1. Skip section 4.2.

Read section 5.1. Do you understand that a choice set such as $C(S_2)$ can have several alternatives as elements? Skip sections 5.2, 5.3, 5.4, and 6.

Finally, read section 7 on degree of rationality. Read conditions 1 to 5. A fuzzy preference ordering is a fuzzy set with membership function μ^{ND}, as defined in (3.7) and illustrated in section 4.1. Give your own definition of nearest exact set (1/20). You do not need to be consistent with the references in the paper. Give your own measure of the distance between a fuzzy set and its nearest exact set (1/20). Now read formula (7.1) for the degree of rationality, ignoring the minimum over all relations R that will "N-rationalize $C(S, R)$". It is important that you understand that a fuzzy degree of rationality can be defined based on the distance between the fuzzy preference ordering and its nearest exact or crisp set. Skip section 8.

This is the main part of your exam coursework. Lay down conditions for a fuzzy preference relation that, according to you, represent rationality (2/20). Give examples of such a relation (4/20). Give your own definition of degree of rationality (1/20), and calculate it for the examples you gave (2/20).

In laying down your conditions for rationality, you will choose from among the properties of fuzzy preference relations that you have encountered in the paper, if necessary with minor changes. You earn marks for a judicious and well-justified choice of conditions, not for clever modifications.

To find examples of fuzzy preference relations fulfilling your conditions for rationality, you need to write a program that generates membership values for relations, as in tables 1 to 6 in the paper. To make sure that your examples are different from those of fellow students, one membership value should be derived from the CID number on your Imperial College swipe card in the following way: divide the CID number by the smallest power of 10 so that the result lies between 0 and 1. Use this value for at least one value of μ_R. I advise you to start with relations over three alternatives. If you program runs fast, you can consider relations over four alternatives. You earn marks for examples that are illustrative, and significantly different from each other. It is up to you do decide how many examples you show, and how much explanation you give per example.

If you think the definition of degree of rationality given in the paper is OK, keep it, otherwise make your own changes. Comment on the degree of rationality you obtain for your examples.

The Challenge. If you want to get top marks, you have to do this challenge. However, you will get better marks for a good report without the challenge than for a mediocre report with the challenge solved.

Criticize the work by Amartya K. Sen (Nobel Prize Economics 1998) on Social Choice from the viewpoint of fuzzy choice (5/20). You will need to gain an insight in Sen's work on social choice without reading his works in any detail. Use your literature search skills efficiently for this. Sen has written a lot, and about half is not directly related to social or collective choice and hence not useful for this challenge. This challenge is about comparing ideas. You do not need to do simulations for this, nor do maths.

You could organize your work as follows.

day 1 Read the paper, looking up anything you don't understand in your lecture notes. Plan what you are going to program.

day 2 Do the programming, and debug your program.

day 3 Run the simulations, and collect the results in a form that you can present in your report. Simulations can be in any programming language, on any machine. You can use Matlab or other software packages, but make sure that you have control over the parameters that you want to vary. If you are really desperate, you could use a calculator, or even pen and paper, but this will make this coursework difficult.

day 4 Write the report. It should be maximum six pages (single sided) a4, in a font not smaller than 10 point. You will not get marks for anything exceeding six pages, even if it is appendices. Font size in tables should not be smaller than 10 point either. Describe the problem, and how you have solved it. Describe your simulations, but do not give programme listings. Do not give references to the literature. Make sure you do and answer everything that is asked for in the coursework. If you have problems with formulas in text processing, do them by hand. Do not bind the report, just staple the pages together. Mention your name, and indicate for what degree you are studying.

day 5 Check the consistency and quality of your work. Make last minute changes if necessary. If you feel confident and have the time, tackle the challenge. Resist the temptation to spend more than five 8-hour days of intensive effort on your coursework. You will not be compensated

for it in marks. Just as an exam paper requires a concentrated effort over a few hours, this coursework requires a concentrated effort over a few days.

Do not forget to attend on the "exam" day. Bring a copy of your report with you, and your swipe card. You will be asked one or two questions based on what you have written in your report, to make sure that you have written it yourself. No preparation is necessary.

Good luck.

Dr. P. De Wilde