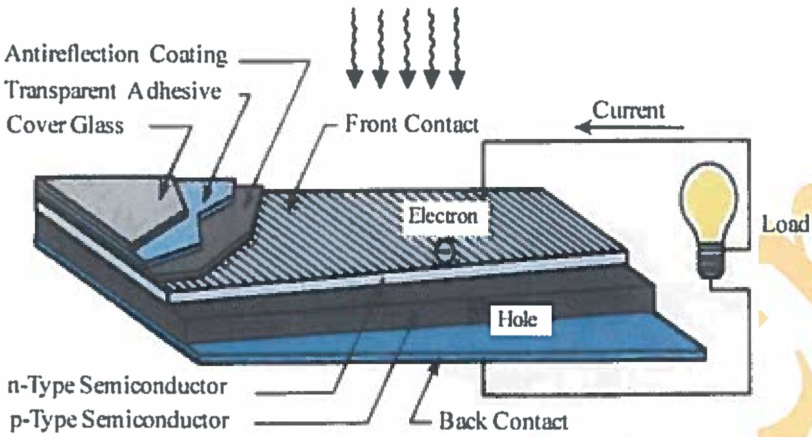
1. This question covers several topics and all parts should be attempted.

a) Consider a typical photovoltaic panel.

i) With the aid of a sketch (showing clearly the intended direction of incident light), describe the construction of a photovoltaic panel. [5]

[Bookwork]
Sketch similar to this required



Key points are:

- *pn* junction
- metalisation in stripes on top surface and solid on bottom
- anti-reflective coating and glass cover
  [3×1 mark plus 2 for quality of sketch]

ii) The choice of width of the metallisation tracks on the top surface of a panel affects the efficiency of the panel. Describe why this is so. [3]

[Bookwork]
Stripes needed to allow light to enter crystal. Thin stripes avoid large fraction of shadow which prevent some light being captured. However, thick stripes needed to reduce resistance of the current collection path.
[1 mark for each reason and 1 for identifying trade-off exists]

iii) List the factors that affect the intensity of light that is incident on a panel. [4]

[Bookwork]

- Length of day given season and location
- Elevation and azimuth of the sun relative to those of the panel
- Air-mass absorbing light energy at certain wavelengths (dependent on path length through atmosphere and therefore position of sun relative to panel).
- Cloud cover causing scattering or shadow
- Buildings, trees, etc. causing shadowing
  [4 marks for four well-described reasons]

b) An impedance connected to a single-phase supply of 230 V draws a current of 12 A that leads the voltage by 15°.

i) Calculate the resistance and reactance of the impedance. [2]

[Standard Calculation]

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$$Z = \frac{V}{I} = \frac{230}{12\angle^{+}15^\circ} = 19.167\angle^{-}15^\circ = 18.51 - j4.96\ \Omega$$

So, the resistance is 19.51 Ω and the reactance -4.96 Ω. [2 marks for accurate answers]

ii) Calculate the real and reactive powers consumed by the impedance [2]

[Standard Calculation]

$P = VI\cos(\phi) = 230 \times 12 \times \cos(-15^\circ) = 2.67$ kW [1 mark for accurate answer and unit]

$Q = VI\sin(\phi) = 230 \times 12 \times \sin(-15^\circ) = -0.71$ kVAr [1 mark for accurate answer and unit]

c) Three impedances, each of 10 + j3 Ω, are connected to a 400 V three-phase supply in delta configuration. Calculate the real and reactive powers consumed. [4]

[Standard Calculation]

For delta connection, phase voltage and line voltage are equivalent. Both are therefore 400 V.

Find phase current $I_P = \frac{V_P}{Z} = \frac{400}{10+j3} = \frac{400}{10.44\angle 16.70^\circ} = 38.31\angle^{-}16.70^\circ$ A

[2 marks for correct current in delta]

Find real power $P = 3V_P I_P cos(\phi) = 3 \times 400 \times 38.31 \times cos(^{-}16.70^\circ) = 44.0$ kW

And reactive power $Q = 3V_P I_P sin(\phi) = 3 \times 400 \times 38.31 \times sin(^{-}16.70^\circ) = 13.2$ kVAr

[2 marks for accurate answers and units]

d) Consider a national-scale AC electricity system.

i) Describe the principal differences between the transmission and distribution networks and include an explanation of why distribution networks use lower voltages than transmission networks. [4]

[Bookwork]

Transmission networks exist to effect bulk transfer of power from geographic regions with more generation than load to regions with more load than generation. They are long distance networks which are built to achieve high availability by having duplicate circuits on all routes and having a high degree of meshing between the nodes. Distribution networks a relatively short distance networks for distributing power within a load centre. They use a variety of voltage levels determined by the volume of load being served. There are double circuit routes at the higher voltages and single circuit routes at the lower voltages but little or no meshing.

[2 marks for mentioning degree of redundancy and availability, difference in function and difference in geographic scale]

The choice of operating voltage is based on achieving the lowest total cost considering operating costs and capital costs. Higher voltages reduce power losses and hence operating costs but have higher capital costs. For long distance routes with relatively few nodes and substations, high voltage achieves a large advantage in power loss for reasonable capital cost. Where distances are short, and connections are many, the lower voltages achieve a better balance between the costs.

[2 marks]

ii) Explain the disadvantages and advantages of using a frequency higher than 60 Hz, such as 400 Hz, and suggest why this option is followed for an aircraft system. [4]

[Bookwork]

Advantages of >60Hz

- Smaller transformers and generators for a given power rating. [1 mark]

Disadvantages of >60Hz

- Large series voltage drops across inductances of lines and larger shunt-currents in capacitance of cables leading to lower power transfers for a given line/cable.
- Need for greater engineering effort in bearings and turbines to accommodate higher rotational speeds. [2 marks]

Answers

Use 400 Hz in aircraft because reduction in weight of transformers and generators is a significant saving in aircraft operations and overcomes the disadvantages (and cable/line runs are short). [1 mark]

e) Consider the characteristics of overhead lines and cables.

i) Explain why the X:R ratio of an overhead line is different for different voltage ratings. [2]

[Bookwork]

High voltage overhead lines have large separation distance between conductors in order to provide sufficient insulation.

Inductance per unit length increases with separation distance, d, according to $L' = \frac{\mu_0}{\pi} \ln\left(\frac{d}{r_C}\right)$ (from formula sheet) [1 mark]

Resistance per unit length is approximately independent of voltage rating because currents are kept in the region of 1 kA and so wire cross-section remains constant. Combining these two factors means that X:R ratio increases with voltage rating.

[1 mark]

ii) Explain why an underground cable has a higher capacitance per unit length than an overhead line. [2]

[Bookwork and interpretation]

Comparing the equations for capacitance of OHL and cables;

$$C' = \frac{\pi \varepsilon_0 \varepsilon_{Rl}}{\ln\left(\frac{d}{r_C}\right)} \quad \text{and} \quad C' = \frac{2\pi \varepsilon_0 \varepsilon_{Rl}}{\ln\left(\frac{r_O}{r_C}\right)}$$

reveals that relative permittivity is important. Cables use plastics of $\varepsilon_R \approx 3$ and OHL use air of $\varepsilon_R \approx 1$ leading to higher capacitance in cables. Also, separation distance in cables (r_O) is smaller than in OHL, d, and both are denominator terms thus also leading to higher capacitance in cables.

[2 marks]

iii) An overhead line is found to have an inductance per unit length of 125 μH/km and is operated at 132 kV and 50 Hz. What is the longest length of line that could be operated with a power transfer of 1,500 MW if the load angle is not to exceed 30°. [4]

[Variation of standard calculation.]

Use standard equation and replace reactance by function of length, l, $P = \frac{V_R V_S}{2\pi f L' l} \sin(\delta)$

$$l = \frac{V_R V_S}{P\, 2\pi f L'} \sin(\delta) = \frac{132\times10^3 \times 132\times10^3}{1{,}500\times10^6 \times 2\pi \times 50 \times 125\times10^{-6}} \times 0.5 = 147.9 \text{ km}$$

[2 marks for correct manipulation of equation; 2 marks for accurate answer]

iv) An overhead line has a series impedance of 0.35 + j 0.40 Ω. Measurements at the sending end are that the voltage is 11.0 kV and real power +8.0 MW flows from sending to receiving end. Estimate the voltage magnitude change across the line and the reactive power that would need (as seen at the sending end) to bring the voltage change to zero. [4]

[Calculation using standard approximation.]

Use $\Delta V \approx \frac{R\, P_S + Z\, Q_S}{|V_S|}$

Answers

With Q=0, $\Delta V \approx \frac{R\,P_S}{|V_S|} = \frac{0.35\times 8\times 10^6}{11\times 10^3} = 254.5$ V [2 marks for accurate answer]

To achieve drop of zero set numerator term to zero by setting $R\,P_S = -Z\,Q_S$

$Q_S = -\frac{Z}{R} P_S = -\frac{0.40}{0.35} \times 8M = -9.14$ MVAr [2 marks for accurate answer]

Answers

2.

A boost switch-mode power supply (SMPS) is to be used to provide a 20 V output from a 4 V input. The inductor has a value of 80 μH and the capacitor has a capacitance of 100 μF and a series resistance of 20 mΩ.

a) Describe the operating principle of the boost SMPS including an explanation of how an output voltage higher than the input voltage is achieved. [5]

[Bookwork]

- Turn-on of the Mosfet imposed the full input voltage across the inductor
- Current increases in the inductor, storing energy as $\frac{1}{2}LI^2$.
- The diode is reverse biased and the capacitor alone supplies energy to the load
- The capacitor has a large stored energy and its voltage changes little during this period.
  [2 marks]
- Turn-off of the Mosfet causes the inductor current to divert to the diode.
- The inductor has a negative voltage imposed across it – the output voltage applied at the RHS and a diode drop (in a negative sense) at the LHS
- The inductor current reduces and stored energy is released to the capacitor and load.
- Charge and energy are added to the capacitor.
  [2 marks]
- With no switching the output voltage settles at the input voltage; each switching cycle adds charge to the output capacitor until the output voltage has risen sufficiently bring the inductor current into steady-state.
  [1 mark]

b)

i) Calculate the value of duty-cycle required assuming continuous conduction. [2]

[Standard calculation. 1 mark for equation; 1 mark for accurate answer.]

$$\frac{V_O}{V_I} = \frac{1}{1-\delta}$$

$$\delta = 1 - \frac{V_I}{V_O} = 1 - \frac{4}{20} = 0.8$$

ii) Calculate the minimum switching frequency for which the SMPS will stay in continuous conduction for an input power of 2 W. [6]

[Variation of standard calculation.]

The input current is $I_I = \frac{P_I}{V_I} = \frac{2}{4} = 0.5$ A [1 mark]

Inductor ripple current must be less than twice this; $\Delta i_L = 1.0\ A$ [2 marks]

The ripple current is defined by $\Delta i_L = \frac{V_I}{L} \times \frac{\delta}{f} = \frac{V_I}{fL} \times \left(1 - \frac{V_I}{V_O}\right)$ [1 mark]

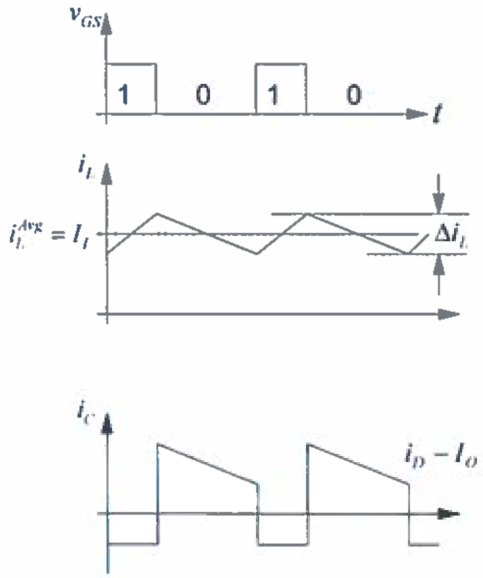
Re-arranging gives expression for frequency, $f = \frac{\delta V_I}{\Delta i_L L} = \frac{0.8 \times 4}{1.0 \times 80 \times 10^{-6}} = 40$ kHz

[1 mark for re-arranged equation, 2 for accuracy]

iii) Sketch the shape of the current through the capacitor during continuous operation and label the sketch with scales for an input power of 12 W and the switching frequency found in (ii). [5]

Answers

[Book work. Sketch similar to the bottom one of these three from the notes.]



[2 marks for sketch with correct slope]

Period is 25 µs; on-time is 20 µs and off-time 5 µs. [1 mark for correct time scale]
$\Delta i_L = 1.0$ A
$I_I = \frac{P_I}{V_I} = \frac{12}{4} = 3.0$ A [2 marks for correct current scale]

iv) Calculate the output voltage ripple under the conditions used in (iii). [5]

[Standard Calculation]
There are two components to the ripple: resistive and capacitive. Peak-to-peak resistive voltage proportion depends on peak-to-peak current through capacitor:
$i_C^{ptp} = i_L^{pk} = I_I + ½\Delta i_L = 3.0 + ½×1 = 3.5\ A$ [1 mark]
$\Delta v_{ESR} = i_C^{ptp} × R_{ESR} = 3.5×0.02 = 70\ mV$ [1 mark]

The charge delivered to the capacitor is most easily found during the transistor on-time (during which the capacitor is discharged by the load current).

$\Delta q = I_O t_{on} = \frac{P_I}{V_O} t_{on} = \frac{12}{20} × 5×10^{-6} = 3$ µC [2 marks]
$\Delta v_C = \frac{\Delta q}{C} = \frac{5×10^{-6}}{100×10^{-6}} = 30$ mV [1 mark]

The output voltage ripple is the sum of these = 30 + 70 = 100 mV

v) The MOSFET used in the SMPS has a channel resistance of $R_{DS(on)}$) = 20 mΩ and turn-on and turn-off energy losses, E_{on} and E_{off} of 5 and 7 µJ respectively, and diode on-state voltage is $V_{AK(on)}$ = 0.6 V. Calculate the total power lost in the semiconductors when the SMPS operates at 15 W. [3]

[Standard calculation]
$P_{loss} = \delta I_{DS}^2 R_{DS(on)} + (1-\delta) I_{AK} V_{AK(on)} + f(E_{on} + E_{off})$
$P_{loss} = 0.8×3^2 ×0.02 + (1 - 0.8)×3×0.6 + 40×10^3×(5+7)×10^{-6} = 0.144 + 0.360 + 0.480$
$= 0.984$
[1 marks for equation and 2 for accuracy]

Answers

vi) One way to reduce the output voltage ripple would be to increase the switching frequency. Without detailed calculation, comment on how effective a doubling of switching frequency would be and the impact on efficiency. [4]

[New interpretation.]
The capacitance term is directly dependent on *f* and so would halve but is only one-third of the total. [1 mark]
The ESR terms is itself formed of a frequency independent term related to the input current and a term dependent on the inductor ripple current. The ripple current term is around a tenth of the total so halving it is not very effective. Overall a reduction of around 20% in voltage ripple is expected. [2 marks]

The switching power loss is approximately half the total power loss so a doubling of frequency adds approximately 50% to power losses. [1 mark]

Answers

3.

a) Explain in outline the operating principle of a three-phase induction machine. [7]

[Book work]

- The stator has three windings displaced (in space) by 120° from each other around the stator.
- The three supply currents are displaced (in time) by 120° from each other.
- Three-phase currents in the three-phase winding create a rotating magnetic field at a speed determined by the supply frequency (and pole-pair number).
  [3 marks]
- The magnetic flux from the stator crosses the air-gap and links with the windings on the rotor.
- The relative motions between rotating flux and rotating rotor winding causes a rate-of-change of flux linkage that voltages in the rotor winding.
  [2 marks]
- The induced voltages drive currents around the closed rotor windings.
- The rotor currents establish a rotor field that interacts with the stator field to develop torque that opposes the relative motion (i.e. causes the rotor to accelerate toward the speed of the stator field).
  [2 marks]

b) A three-phase induction machine with one pole-pair has the following equivalent circuit parameters:

Stator resistance, $R_S = 0.8\ \Omega$.

Stator leakage reactance, $X_S = 2\ \Omega$;

Iron loss resistance, $R_I = 200\ \Omega$;

Magnetising reactance, $X_M = 60\ \Omega$;

Referred rotor leakage reactance, $X_R = 2\ \Omega$;

Referred rotor resistance, $R_R = 0.8\ \Omega$;

The machine is supplied at a phase voltage of 200 V and a frequency of 50 Hz.

i) Initially, the machine runs at 2,930 r.p.m. Calculate the stator current and the real power drawn from the stator supply. [8]

[Standard calculation]

Slip needs to be calculated first from: $s = \frac{\omega_S - \omega_R}{\omega_S}$

Synchronous speed calculated by: $\omega_S = {\omega_E}/{p} = {50 \times 2\pi}/{1} = 314.1\ \text{rad/s}$

Rotor speed is $\omega_R = \frac{2{,}930 \times 2\pi}{60} = 306.8\ \text{rad/s}$

Hence slip is: $s = \frac{314.1 - 306.8}{314.1} = 0.0233$ [2 marks for accurate answer]

Total impedance must be found next:

$$Z_i = R_S + jX_S + \left(jX_M \,//\left(\frac{R_R'}{s} + jX_R \right) \right)$$ [2 marks for correct formulation of impedance]

$Z_T = 23.54 + j14.26 = 27.53\angle 31.2°\ \Omega$

Current is found from the phase voltage $I_S = \frac{V_S}{Z_T} = \frac{230}{27.53\angle 31.2^\circ} = 7.27\angle -31.2^\circ\ \Omega$
[2 marks for accurate answer]

And power calculated for a 3-Ph load $P = 3V_S I_S cos(\phi) = 3\times 230\times 7.27\times \cos(31.2^\circ) = 3.73\ \text{kW}$
[2 marks for accurate answer and units]

ii) For the conditions in (i), calculate the rotor current and power converted to mechanical form. [8]

[Standard calculation]
Rotor current is found from the current divider rule (or otherwise) $I'_R = I_S \frac{jX_m}{jX_m + \frac{R'_R}{s} + jX'_R}$

[2 marks for correct formulation]

$I_R = 5.47\ \angle$-6.21° [2 marks for accurate answer]

Power from standard formula $P_{EM} = 3\ I'^2_R\ R'_R \left(\frac{1-s}{s}\right) = 3\times 5.47^2\times 0.8\times \left(\frac{1-0.0233}{0.0233}\right) = 3.00\ \text{kW}$
[2 mark for correct formula; 2 marks for accurate answer]

iii) Consider now that the mechanical load speeds up to 3,050 r.p.m and recalculate the power exchanged with the supply and comment on this value. [7]

[Variation of standard calculation plus fresh interpretation]
Rotor speed is $\omega_R = \frac{3{,}050\times 2\pi}{60} = 319.4\ \text{rad/s}$

Slip is: $s = \frac{314.1-319.4}{314.1} = -0.0167$ [1 mark for accurate answer]

Total impedance is: $Z_T = -27.56 + j33.48 = 43.37\angle 129.5^\circ\ \Omega$ [1 mark for correctly dealing with negative resistance and 1 mark for accurate overall answer]

Current is found from the phase voltage $I_S = \frac{V_S}{Z_T} = \frac{230}{43.37\angle 129.5^\circ} = 4.61\angle -129.5^\circ\ \Omega$
[1 marks for accurate answer]

Power is $P = 3V_S I_S cos(\phi) = 3\times 230\times 4.61\times \cos(-129.5^\circ) = -1.75\ \text{kW}$
[1 marks for accurate answer]

The machine is running super-synchronous and generating electrical power. The magnitude of the power is smaller than in motoring case because slip is -50 rpm versus +70 rpm and also the losses in the machine subtract from the input mechanical power to yield the stator power (rather than subtracting from the stator power to give the mechanical power).
[2 marks]

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Answers