UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1996

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER 2.6

STATISTICS

Friday, May 17th 1996, 4.00 - 5.30

Answer THREE questions

For admin. only: paper contains
4 questions
4 pages (excluding cover page)

1. a Each of three manufacturers A, B and C is equally likely to have supplied an electronic component. The probabilities of a defective component being supplied by A, B and C are 0.05, 0.03 and 0.01, respectively.

i) What is the probability that a defective component is supplied ?

ii) If a component functions correctly, what is the probability that it was supplied by manufacturer C ?

b Consider a manufacturer who produces components. Before a component is supplied, a test is carried out. If the component is defective, the test finds it defective with probability 0.8, whilst if it functions correctly, the test will incorrectly specify it to be defective with probability 0.10. A component is defective with probability 0.15.

i) Calculate the probability that a component is rejected as a result of the test.

ii) If a component is not found defective as a result of the first test, a second identical and independent test is carried out. A component is only supplied if it passes both tests. What is the probability that a component will be rejected as a result of the series of tests ?

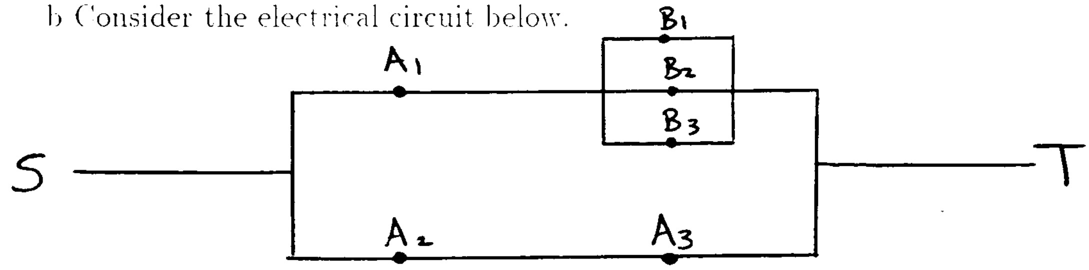
The two parts carry, respectively, 35% and 65% of the marks.

2. a A random variable T is said to have an exponential distribution with parameter λ ($\lambda > 0$) if its probability density function is given by

$$f(t|\lambda) = \begin{cases} \lambda \exp(-\lambda t) & t > 0 \\ 0 & t \leq 0. \end{cases}$$

Determine the distribution function of T.

b Consider the electrical circuit below.



The circuit functions as long as there is a path of functioning components between S and T. The distribution of lifetimes (in hours) of components of type A is exponential with $\lambda = 0.4$ whilst the distribution of lifetimes of components of type B is exponential with $\lambda = 0.5$. The circuit is inspected three hours after it is switched on.

i) Calculate the probabilities that components of type A and type B are still functioning after 3 hours.

ii) Under the assumption that components operate independently, calculate the probability that

1) the upper circuit;
2) the lower circuit; and hence
3) the complete circuit.

functions.

The two parts carry, respectively, 15%, and 85% of the marks.

Turn over ...

3. a The number of corrupted blocks, X, on the hard disc of a type of computer supplied by a particular manufacturer can be well-modelled by a Poisson distribution with mean 0.6. Write down the distribution of X.

b Determine the probability that a particular hard disc contains

i) zero errors;

ii) more than 2 errors.

c A random sample of ten hard discs from the manufacturer are inspected. Denote by Y the number of discs which are free of corrupted blocks. Write down the distribution of Y and hence find the probability that seven of the discs are free of corrupted blocks.

d Another manufacturer supplies hard discs, each of which has a probability of p $(0 < p < 1)$ of being free of corrupted blocks, the numbers on each disc being independent. The manufacturer supplies n discs of which Z are free of corrupt blocks. Write down the ***likelihood function*** for p and determine the maximum likelihood estimator of p.

e A particular shipment of ten discs from the manufacturer in d contains 2 with corrupted blocks, what is the maximum likelihood estimate of p ?

The five parts carry, respectively, 15%, 15%, 30%, 35% and 5% of the marks.

4. a An assembly consists of two rods, A and B, attached lengthways. Each component is manufactured independently. The lengths (in cm) of component A are normally distributed with mean 8cm and standard deviation 0.3cm whilst the lengths (in cm) of component B are normally distributed with mean 11cm and standard deviation 0.5cm. Write down the distribution of the length of the assembly.

b The assembly is acceptable if its length lies between 18.2cm and 19.6cm. Determine the probability that a randomly selected assembly is

(i) too short.
(ii) too long.
(iii) acceptable.

c If, for a particular assembly, the length of component A is known to be 8.1cm, what is the probability that the assembly will be acceptable ?

d The manufacturing process is modified. It is believed that both the mean and the standard deviation of the assembly are altered by the modification. Twelve lengths of the assembly are randomly selected and measured. The following are the lengths (in cm)

18.9, 19.1, 19.0, 18.8, 18.7, 19.0, 19.2, 19.1, 18.7, 18.8, 19.0, 18.8.

Determine a 95% confidence interval for μ, the true mean of the distribution of the lengths of the assembly after the modification. Is there evidence that the mean length of the assembly is significantly reduced by the modification ?

The four parts carry, respectively, 15%, 25%, 25% and 35% of the marks.

End of Paper