EE4-55
EE9-AO11

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

MSc and EEE PART IV: MEng and ACGI

MEMS AND NANOTECHNOLOGY

Corrected copy

Monday, 22 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer Question 1.
Answer Question 2 OR Question 3.
Answer Question 4 OR Question 5.

Question 1 carries 40% of the marks. Remaining questions carry 30% each.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : Z. Durrani, A.S. Holmes, Z. Durrani
Second Marker(s) : A.S. Holmes, Z. Durrani, A.S. Holmes

Information for Candidates

The following physical constants may be used:

electron charge:	$e = 1.6 \times 10^{-19}$ C
electron mass:	$m_e = 9.1 \times 10^{-31}$ kg
Planck's constant:	$h = 6.63 \times 10^{-34}$ Js
Permittivity of free space:	$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

The series expansions for $\cosh(x)$ and $\sinh(x)$ up to and including 6$^{\text{th}}$ order terms are:

$$\cosh(x) = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^6}{720} + \ldots.$$

$$\sinh(x) = x + \frac{x^3}{6} + \frac{x^5}{120} + \ldots.$$

This question is compulsory

1. a) Figure 1.1 below shows a cross-section through a CMOS inverter. What are the regions A, B, C, and D?

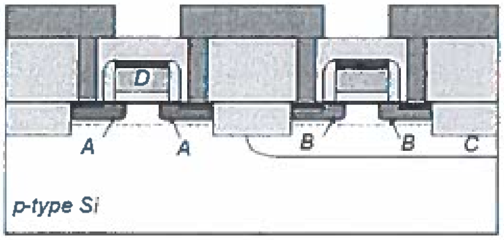

Figure 1.1

[4]

b) By approximating a Si atom within the crystal as a hydrogenic atom, construct the hybridised wave functions $|2p_x\rangle$, $|2p_y\rangle$ and $|2p_z\rangle$. Sketch these wave functions, and the $|2s\rangle$ wave function.

You may use the $|2s\rangle$ and $|2p\rangle$ states of the hydrogenic atom, in spherical coordinates (r, θ, ϕ), given below:

$$|2s\rangle = \psi_{200} \sim \left(1 - \frac{Zr}{2a_0}\right) e^{\frac{-Zr}{2a_0}}$$

$$|2p_0\rangle = \psi_{210} \sim e^{\frac{-Zr}{2a_0}} r\cos\theta$$

$$|2p_{\pm 1}\rangle = \psi_{21,\pm 1} \sim e^{\frac{-Zr}{2a_0}} e^{\pm i\phi} r\sin\theta$$ [8]

c) Using suitable diagrams, explain how the unit cell in a Si crystal arises from two face centred cubic cells. [4]

d) Using suitable diagrams, explain briefly the operation of a double barrier heterostructure resonant tunnelling diode. [4]

Question 1 continues on the next page.

Question 1 continued.

e) Write down the bending equation for a cantilever subject to a uniformly distributed transverse load p (N/m) acting along its entire length. Solve the bending equation to obtain an equation for the deflection profile of the beam. Also derive an expression for the maximum tensile stress in the beam. [5]

f) Describe briefly the main features of bulk silicon micromachining technology, focusing on the key process step and the kinds of structures that can be produced. [5]

g) An electron beam lithography system has an acceleration voltage of 50 kV and a beam convergence angle of 1 milliradian. Estimate the resolution that would be achieved by this system if it were limited only by diffraction. In practice what other factors will limit the minimum achievable feature size in a resist layer? [5]

h) Explain briefly with the aid of diagrams the operational principles and typical geometries for material bimorph and shape bimorph electrothermal actuators. Highlight the differences in functionality and any advantages one type may offer over the other. [5]

End of Question 1.

2. A Si nanowire FET is shown in Figure 2.1(a). The nanowire is a cylinder of radius $R = 25$ nm, doped n-type at a density $N_D = 2 \times 10^{24}$ m^{-3}. An oxide shell of thickness $t_{ox} = 15$ nm surrounds the nanowire, and a metal gate (not shown) surrounds the oxide shell. The defect state density on the nanowire surface is $N_S = 10^{16}$ m^{-2}, at an energy E_t lying within the Si band gap E_g (Fig. 2.1(b)). Electrons from the nanowire are trapped in the surface defect states, creating a surface depletion region (shaded region in Fig. 2.1(a)) and leaving a conducting core of radius 'r'.

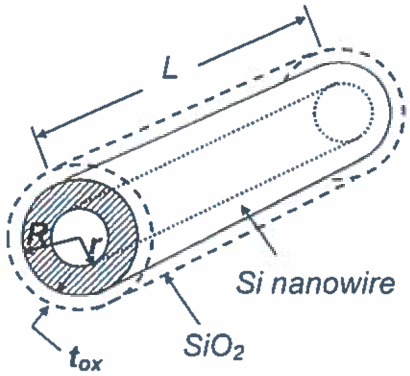

Figure 2.1(a)

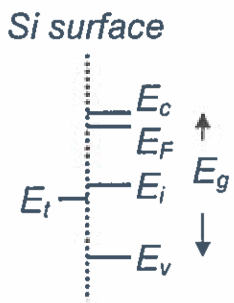

Figure 2.1(b)

a) Use charge neutrality to calculate the radius 'r'. [10]

b) Calculate the surface potential V_s relative to the axis of the nanowire. You may use the expression $V(r) = \dfrac{eNr^2}{2\varepsilon_r \varepsilon_0}$ for the potential along r, in a uniformly depleted region, where N is the doping concentration, and ε_r is the dielectric constant of the material. Sketch the band diagram along the nanowire diameter, assuming equal work functions in the metal and nanowire. [8]

c) Estimate the 'flat-band' voltage, V_{FB}. [6]

d) Hence calculate the threshold voltage in the device, V_{th}. [6]

The dielectric constants for Si and SiO_2 are $\varepsilon_{Si} = 11.9$ and $\varepsilon_{ox} = 3.9$ respectively.

3. Electron waves, with amplitudes A, B, C, D, α and β, travelling parallel to the x-axis, are incident and reflected from, and within, a potential barrier as shown in Figure 3.1.

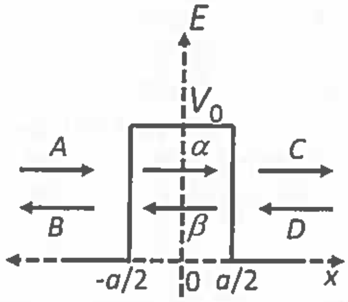

Figure 3.1

The potential energy and wave vectors for the electron wave functions are given by:

$E = 0$, for $x < -a/2$ and $x > a/2$, wave vector for travelling waves k_1.
$E = V_0$, for $-a/2 < x < a/2$, wave vector for exponential decays k_2.

The transmission matrices at the boundaries $-a/2$ and $a/2$ are given by:

$$\Gamma_1 = \frac{1}{2\,\mathrm{Re}[k]}\begin{pmatrix} k\exp\left(k^*\dfrac{a}{2}\right) & k^*\exp\left(k\dfrac{a}{2}\right) \\ k^*\exp\left(-k\dfrac{a}{2}\right) & k\exp\left(-k^*\dfrac{a}{2}\right)\end{pmatrix}, \text{ at } x = -a/2 \qquad \text{Eq. 3.1}$$

$$\Gamma_2 = \frac{i}{2\,\mathrm{Im}[k]}\begin{pmatrix} -k\exp\left(k^*\dfrac{a}{2}\right) & k^*\exp\left(-k\dfrac{a}{2}\right) \\ k^*\exp\left(k\dfrac{a}{2}\right) & -k\exp\left(-k^*\dfrac{a}{2}\right)\end{pmatrix}, \text{ at } x = a/2 \qquad \text{Eq. 3.2}$$

where $k = k_2 + ik_1$ and k^* is the complex conjugate.

a) By using the boundary transmission matrices Γ_1 and Γ_2, find the transmission matrix Γ_{single} for the barrier. [12]

b) Hence, show that the determinant of $\Gamma_{single} = 1$. Here, you may use the expression $(c)^4 + (c^*)^4 - 2|c|^4 = -16a^2b^2$, for a complex number $c = a + ib$. [10]

c) Using the results of parts a) and b), write down an expression for the transmission coefficient for intensity T, for a wave incident only from the left hand side of the barrier. [8]

4. Figure 4.1 shows a surface micromachined accelerometer with capacitive readout and an electrostatic actuator which is used only for self-testing. The sensing and actuation electrode arrays are similar in design, comprising a number of cells each with a moving electrode positioned between two fixed electrodes. The drawing is not to scale, and in the actual device there are 42 cells in the sensing section and 12 cells in the actuator. The mechanical layer thickness is 2 μm, the nominal gap between electrode fingers is $g = 1.3$ μm, and the electrode overlap length is $l = 100$ μm. The suspension consists of four folded springs in which the flexures are 120 μm long and 2.5 μm wide. The area of the proof mass is 0.05 mm^2.

   a) Why do you think the designer of the accelerometer chose to use a variable gap electrode configuration as opposed to a variable overlap one? For what kind of application might variable overlap be preferred? [4]

   b) Calculate the stiffness of the suspension and the mechanical resonance frequency of the device. [10]

   c) Estimate the capacitance C_A between the A and COM terminals when the proof mass is in its rest position with zero applied acceleration. Also derive an expression for the differential capacitance $C_{diff} = (C_A - C_B)$ as a function of proof mass displacement. Hence determine the magnitude of C_{diff} when the accelerometer is subject to an acceleration of 10 ms^{-2} along the sensing direction. [10]

   d) During self-test a DC voltage is applied between either terminal C or terminal D and the COM terminal to emulate an applied acceleration. What voltage should be applied to emulate an applied acceleration of 10 ms^{-2}? [6]

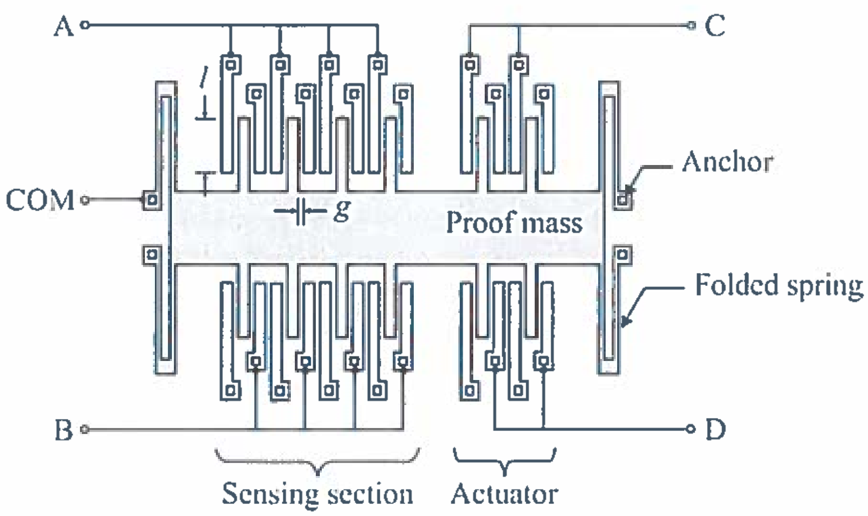

Figure 4.1

You should assume values of $E = 160$ GPa and $\rho = 2330$ kg/m^3 respectively for the Young's modulus and density of polysilicon.

5. a) Figure 5.1 shows a flexure of length L, subject to both a transverse end load P and an axial load F. Derive the bending equation for this situation, and verify that it is satisfied by the following solution:

$$v(x) = A\cosh(\kappa x) + B\sinh(\kappa x) + \frac{v_L}{2} + \frac{P}{2F}(2x - L)$$

where $v_L = v(L)$ is the end deflection, $\kappa = \sqrt{F/(EI)}$, with E and I having their usual meanings, and A and B are constants to be determined from the boundary conditions. [10]

b) By considering the boundary conditions at $x = 0$, obtain expressions for A and B, and hence show that the transverse stiffness $k = P/v_L$ of the flexure can be expressed in the form:

$$k = \frac{F}{L} \cdot \frac{\kappa L[1+\cosh(\kappa L)]}{\kappa L[1+\cosh(\kappa L)] - 2\sinh(\kappa L)}$$

Also show that, when the axial load is small, the stiffness may be written approximately as:

$$k \approx \frac{12EI}{L^3}\left[1 + \frac{FL^2}{10EI}\right]$$

[10]

c) Figure 5.2 shows the suggested layout for a micromechanical resonator which is to be fabricated in nickel on a silicon substrate. The suspension is to consist of four flexures, each flexure being 200 μm long and 10 μm wide.

It is proposed to use the stress induced by differential thermal expansion to compensate for the temperature variation of Young's modulus, so as to maintain a constant resonant frequency independent of temperature. Show that this should indeed be possible with the proposed configuration, and calculate the required gap g between the roots of the flexures. The temperature coefficient of the Young's modulus of nickel is −0.0286 %/K, and the thermal expansion coefficients of silicon and nickel are $2.5 \times 10^{-6}\ \mathrm{K}^{-1}$ and $13.3 \times 10^{-6}\ \mathrm{K}^{-1}$ respectively. [10]

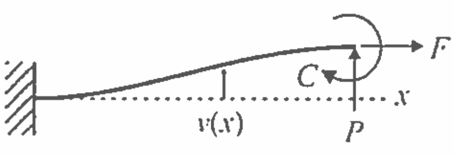

Figure 5.1

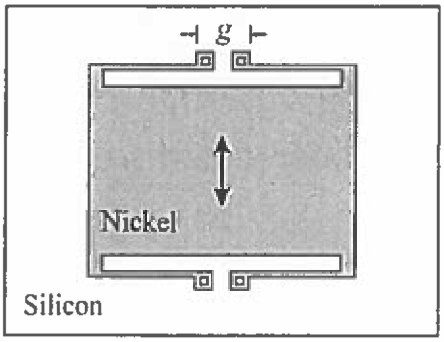

Figure 5.2

