IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EEE/EIE PART III/IV: MEng, Beng and ACGI

Corrected Copy

CONTROL ENGINEERING

Wednesday, 22 January 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : A. Astolfi
Second Marker(s) : D. Angeli

CONTROL ENGINEERING

1. Consider a linear, single-input, single-output, continuous-time, system of dimension $n = 3$, that is $x = [x_1, x_2, x_3]'$, with

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

a) Show that the system is controllable and observable. [4 marks]

b) Compute $\dot{y}$ and write $\dot{y}$ as a function of the state x. [2 marks]

c) Consider the feedback law

$$u = -k^2 y - k\dot{y}.$$

Show that there exists a value $k_\star > 0$ such that the closed-loop system is asymptotically stable for all $k > k_\star$.
(Hint: compute the characteristic polynomial of the closed-loop system and then use Routh test.) [6 marks]

d) Compute $\ddot{y}$ and write $\ddot{y}$ as a function of the state x and the input u. [2 marks]

e) Consider now the feedback law

$$u = -k^3 y - k^2 \dot{y} - k\ddot{y}.$$

Discuss for which values of k this feedback law is well-defined and show that there exists a value $k_\circ > 0$ such that the closed-loop system is asymptotically stable for all $k > k_\circ$. [6 marks]

2. Consider a linear, continuous-time, system described by the equations

$$\dot{x}_1 = u_1, \qquad \dot{x}_2 = u_2, \qquad \dot{x}_3 = u_3,$$

with $x_i(t) \in I\!R$ and $u_i(t) \in I\!R$, for $i = 1,2,3$, and the problem of designing the feedback signals u_i, for $i = 1,2,3$, such that

$$\lim_{t\to\infty}(x_1(t) - x_2(t)) = \lim_{t\to\infty}(x_2(t) - x_3(t)) = 0. \qquad (\star)$$

Let

$$u_1 = \alpha_1(x_2 - x_1) \qquad u_2 = \alpha_2(x_1 - x_2) + \alpha_3(x_3 - x_2) \qquad u_3 = \alpha_4(x_2 - x_3),$$

with α_i constant, for $i = 1,2,3,4$.

a) Write the equations describing the closed-loop system in the form $\dot{x} = Ax$, with $x = [x_1, x_2, x_3]'$. Write explicitly the matrix A. [4 marks]

b) Show that the matrix A has always an eigenvalue equal to zero. [2 marks]

c) Select the parameters of the matrix A such that the matrix has one eigenvalue equal to -1 and one eigenvalue equal to -3.
(Hint: the selection is not unique. Any selection is acceptable.) [6 marks]

d) Using the parameters selected in part c) show that the condition $(\star)$ holds. This can be shown as follows.

   i) Let $z_{12} = x_1 - x_2$ and $z_{23} = x_2 - x_3$. Write differential equations for the variables z_{12} and z_{23}. [2 marks]

   ii) Write the differential equations in part d.i) in the form

   $$\begin{bmatrix} \dot{z}_{12} \\ \dot{z}_{23} \end{bmatrix} = F \begin{bmatrix} z_{12} \\ z_{23} \end{bmatrix}.$$

   Write explicitely the matrix F and show that it has two eigenvalues equal to -1 and -3. [4 marks]

   iii) Exploiting the results of part d.ii) show that condition $(\star)$ holds. [2 marks]

3. Consider the discrete-time system described by the equations

$$x(k+1) = \begin{bmatrix} a_1 & 1 \\ a_0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} a_1 \\ a_0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) + u(k),$$

in which a_0 and a_1 are constant.

a) Show that the sequence $y(k)$ is such that

$$y(k+2) - a_1 y(k+1) - a_0 y(k) = u(k+2).$$

[8 marks]

b) Study the reachability and controllability properties of the system as a function of a_0 and a_1. [6 marks]

c) Study the observability properties of the system as a function of a_0 and a_1. [2 marks]

d) Determine values of a_0 and a_1 such that the output sequence is generated by a relation of the form

$$y(k+1) - \alpha y(k) = u(k+1),$$

for some α constant.
(Hint: exploit the results of part b) in your selection.) [4 marks]

4. Consider a nonlinear, discrete-time, system described by the equations

$$x^+ = f(x,u) = \begin{bmatrix} \alpha \sin x_2 \\ -\alpha \sin x_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u,$$

with $\alpha \in (0,1]$ and constant, $x(t) = [x_1(t), x_2(t)]' \in I\!R^2$ and $u(t) \in I\!R$.

Assume $u = 0$.

a) Show that the point $(0,0)$ is the only equilibrium of the system. [4 marks]

b) Compute the linearization of the system around the equilibrium point $(0,0)$ and write explicitly the matrices A and B of the linearized system.

[4 marks]

c) Study the stability properties of the linearized system as a function of α. (Recall that $\alpha \in (0,1]$.)

[4 marks]

d) Consider the linearized system determined in part b). Design a linear state feedback control law which assigns all eigenvalues of the closed-loop system to zero. [4 marks]

e) Consider the linearized system determined in part b) with output $y = Cx$, and $C = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$. Study the observability properties of the linearized system.

Assume $c_1^2 + c_2^2 \neq 0$ and design an observer such that the estimation error system has both eigenvalues at 0. [4 marks]