

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2014

EEE PART II: MEng, BEng and ACGI

Corrected Copy

**CONTROL ENGINEERING**

Wednesday, 4 June 2:00 pm

Time allowed: 2:00 hours

There are **THREE** questions on this paper.

**Answer ALL questions.**

**Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).**

**Any special instructions for invigilators and information for candidates are on page 1.**

|                       |                    |                |
|-----------------------|--------------------|----------------|
| Examiners responsible | First Marker(s) :  | I.M. Jaimoukha |
|                       | Second Marker(s) : | S. Evangelou   |



1. a) Consider the mechanical system illustrated in Figure 1.1 where all the symbols have the standard interpretation. The input is the applied force  $f(t)$  and the output is the displacement  $y(t)$ . Take  $M=K_2=D=1$  in appropriate units.

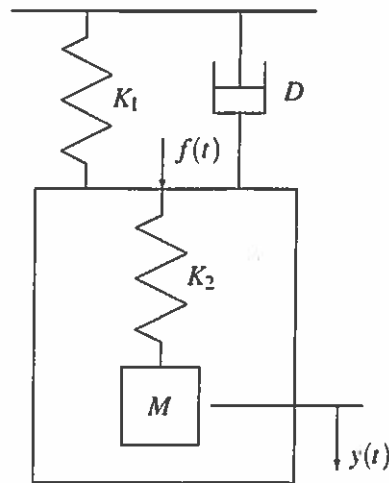


Figure 1.1

- i) Determine the transfer function  $G(s)$  relating  $y$  to  $f$ . [ 5 ]
  - ii) Use the Routh array to find the range of values of  $K_1$  for stability. [ 5 ]
  - iii) Find the value of  $K_1$  for which  $G(s)$  is marginally stable. For this value of  $K_1$ , what are the poles of  $G(s)$ ? [ 5 ]
  - iv) Let  $f(t)$  be a unit step applied at  $t=0$ . Use the final value theorem, which should be stated, to find the steady-state value  $y_{ss}$  of  $y(t)$  in terms of  $K_1$ . What is the value of  $K_1$  for which  $y_{ss} = 2$ ? [ 5 ]
- b) In Figure 1.2 below,  $G(s) = 2/(s^3 - 1)$  and  $K(s)$  is a compensator.
- i) Draw the Nyquist diagram of  $G(s)$ . [ 5 ]
  - ii) Let  $K(s) = k$  be a constant compensator. Use the Nyquist criterion, which should be stated, to determine how many unstable or marginally stable poles the closed-loop has for all  $k$ . [ 5 ]
  - iii) Use the Routh-Hurwitz stability criterion to determine if the closed-loop can be stabilised using a PD compensator. [ 5 ]
  - iv) Show that the closed-loop can be stabilised using the compensator

$$K(s) = k \frac{s^2 + s + 1}{s^2 + 2s + 3}$$

for some  $k > 0$ .

[ 5 ]



Figure 1.2

2. Consider the feedback control system in Figure 2.1 below. Here,

$$G(s) = \frac{1}{s^3 + as^2 + bs + c}$$

represents an uncertain model where it is only known that

$$a > 0, \quad b > 0, \quad c > 1, \quad ab - c \geq 2. \quad (2.1)$$

$K(s)$  is the transfer function of a compensator.

- a) Sketch a typical Nyquist diagram of  $G(s)$ , indicating the low and high frequency portions. Use the Routh array to find the real-axis intercepts. [ 8 ]
- b) Let  $K(s) = K$  be a nondynamic compensator. State the Nyquist stability criterion and use the Nyquist diagram to determine the number of unstable or marginally stable closed-loop poles for all values of  $K$ . [ 8 ]
- c) What is the value of the gain margin for  $G(s)$ ? [ 3 ]
- d) Derive the minimum value of the gain margin for all  $a, b, c$  satisfying the relations in equation (2.1). [ 3 ]
- e) Suppose that it is known that  $G(s)$  has an adequate phase margin and that you have the option of either using a phase-lead or a phase-lag compensator. Which would you choose? Justify your choice. [ 8 ]

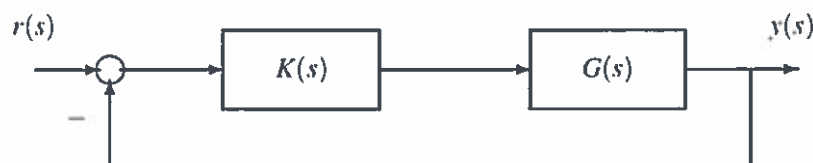


Figure 2.1

3. Consider the feedback loop shown in Figure 3.1 below. Here

$$G(s) = \frac{-2s}{s^2 + k(s+z) + 1}$$

where  $k > 0$  and  $z \geq 0$  are design parameters. It is required to find  $k$  and  $z$  such that the following specifications in response to a step reference signal are satisfied

- The settling time is at most 4 seconds.
- The response is oscillatory with a maximum overshoot of 5%.

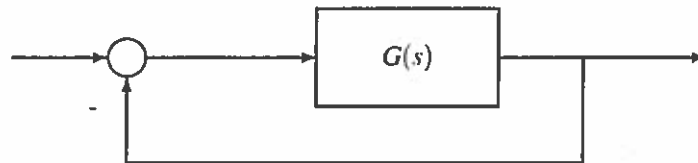


Figure 3.1

- Find the location of the closed-loop poles that achieves the design specifications above. [ 5 ]
- For this part, take  $z = 0$ .
  - Sketch the locus of the closed-loop poles as  $k$  varies from 0 to  $\infty$ . [ 5 ]
  - Hence show that a setting time of  $4s$  is achievable but the resulting response is not oscillatory. Which closed-loop pole achieves this modified specification? What is the corresponding value of  $k$ ? [ 5 ]
- In this part, you are required to find the values of  $k$  and  $z$  that achieve the design specification.
  - Use the angle criterion to choose the value of  $z$  so that the locus of the closed-loop poles as  $k$  varies from 0 to  $\infty$  includes the closed-loop pole evaluated in Part (a) above. [ 5 ]
  - For this value of  $z$ , sketch the locus of the closed-loop poles as  $k$  varies from 0 to  $\infty$ . [ 5 ]
  - Finally, use the gain criterion to find the value of  $k$  that achieves the design specifications. [ 5 ]

