

**Part A – Answer any 2 out of 3 questions in Part A**

1. a) Consider a single phase AC circuit shown in Figure 1.1. Expressions for generator voltage and current are given by:

$$v(t) = \sqrt{2}V \sin \omega t$$

$$i(t) = \sqrt{2}I \sin(\omega t - \phi)$$

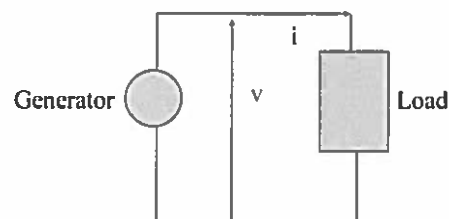


Figure 1.1: Single phase AC circuit

- (i) Show that the expression for the instantaneous power can be written in the following form:

$$p(t) = \underbrace{P(1 - \cos 2\omega t)}_{A(t)} - \underbrace{Q \sin 2\omega t}_{R(t)}$$

and write the expressions for P and Q

[4]

$$p(t) = \sqrt{2} \cdot V \cdot \sin(\omega t) \cdot \sqrt{2} \cdot I \cdot \sin(\omega t - \phi) = V \cdot I \cdot \cos(\phi) - V \cdot I \cdot \cos(2\omega t - \phi)$$

$$p(t) = V \cdot I \cdot \cos(\phi) - V \cdot I \cdot [\cos(2\omega t) \cdot \cos(\phi) + \sin(2\omega t) \cdot \sin(\phi)]$$

$$p(t) = \underbrace{V \cdot I \cdot \cos(\phi) \cdot [1 - \cos(2\omega t)]}_{A(t)} - \underbrace{V \cdot I \cdot \sin(\phi) \cdot \sin(2\omega t)}_{R(t)}$$

$$p(t) = \underbrace{P \cdot [1 - \cos(2\omega t)]}_{A(t)} - \underbrace{Q \cdot \sin(2\omega t)}_{R(t)}$$

$$P = V \cdot I \cdot \cos(\phi)$$

$$Q = V \cdot I \cdot \sin(\phi)$$

$$p(t) = v(t) \cdot i(t) = VI \cos \phi - VI \cos(2\omega t - \phi)$$

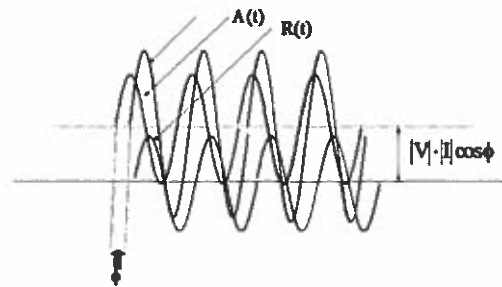
$$p(t) = \underbrace{VI \cos \phi (1 - \cos 2\omega t)}_{A(t)} - \underbrace{VI \sin \phi \sin 2\omega t}_{R(t)}$$

$$P = VI \cos \phi$$

$$Q = VI \sin \phi$$

- (ii) Sketch functions  $p(t)$  and its components  $A(t)$  and  $R(t)$  on the same diagram.

[3]



- (iii) Write the expression for  $A(t)$  and  $R(t)$  for a three phase system (no need for a formal derivation). Explain  $R(t)$ .

[3]

Three phase system:

$$p_3(t) = v_a(t) \cdot i_a(t) + v_b(t) \cdot i_b(t) + v_c(t) \cdot i_c(t)$$

$$p_3(t) = 3VI \cos \phi = 3P$$

Instantaneous power in each phase:

$$p(t) = \underbrace{P(1 - \cos 2\omega t)}_{A(t)} - \underbrace{Q \sin 2\omega t}_{R(t)}$$

$$p_3(t) = 3VI \cos \phi = 3P \quad P_{3\phi} = 3P_{1\phi}$$

Although three phase currents add up to zero, but they are still very much in evidence in each phase, hence:

$$Q_{3\phi} = 3Q$$

- b) Find the peak load of an 11/0.4 kV substation supplying 370 households not using electricity for heating purposes (Type A), and 80 households with electric heating (Type B). Peak demands of individual households are 10 kW and 20 kW, respectively. Coincidence coefficient for Type A households is  $j_{A\infty} = 0.2$ , and for Type B  $j_{B\infty} = 0.5$ . Assume that peaks of both groups of consumers coincide.

[4]

Substation peak load is estimated at:

$$P_S = N_A \cdot \left( j_{A\infty} + \frac{1 - j_{A\infty}}{\sqrt{N_A}} \right) \cdot P_A + N_B \cdot \left( j_{B\infty} + \frac{1 - j_{B\infty}}{\sqrt{N_B}} \right) \cdot P_B$$

$$P_S = 370 \cdot \left( 0.2 + \frac{1 - 0.2}{\sqrt{370}} \right) \cdot 10 + 80 \cdot \left( 0.5 + \frac{1 - 0.5}{\sqrt{80}} \right) \cdot 20 = 1297 \text{ kW}$$

- c) Consider a system supplied with three generators with given capacities and availabilities as in Figure 1.2.

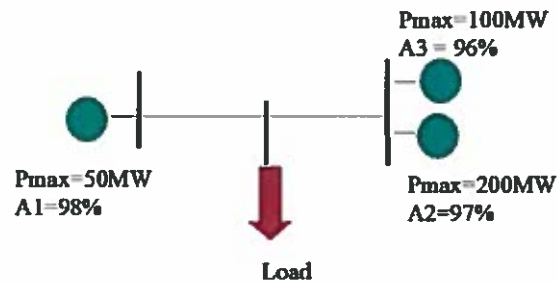


Figure 1.2: A system with 3 generators

The states in which this generation system can find itself are given in Table 1.1.

Table 1.1: System state probabilities

| STATE  | State Probability | Probability that Generation is equal to or greater than State |
|--------|-------------------|---|
| 350 MW |                   |   |
| 300 MW |                   |   |
| 250 MW |                   |   |
| 200 MW |                   |   |
| 150 MW |                   |   |
| 100 MW |                   |   |
| 50 MW  |                   |   |
| 0 MW   |                   |   |

- (i) Calculate state probabilities for this system and the probability that the generation will be greater than the given state.

[3]

| STATE  | State Probability                       | Probability that Generation is equal to or greater than State |
|--------|---|---|
| 350 MW | $98\% \cdot 97\% \cdot 96\% = 91.258\%$ | 91.258%   |
| 300 MW | $2\% \cdot 97\% \cdot 96\% = 1.862\%$   | $91.258\% + 1.862\% = 93.120\%$                               |
| 250 MW | $98\% \cdot 97\% \cdot 4\% = 3.802\%$   | $93.120\% + 3.802\% = 96.922\%$                               |
| 200 MW | $2\% \cdot 97\% \cdot 4\% = 0.078\%$    | $96.922\% + 0.078\% = 97.000\%$                               |
| 150 MW | $98\% \cdot 3\% \cdot 96\% = 2.822\%$   | $97.000\% + 2.822\% = 99.822\%$                               |
| 100 MW | $2\% \cdot 3\% \cdot 96\% = 0.058\%$    | $99.822\% + 0.058\% = 99.880\%$                               |

|       |                                      |                                  |
|-------|--------------------------------------|----------------------------------|
| 50 MW | $98\% \cdot 3\% \cdot 4\% = 0.118\%$ | $99.880\% + 0.118\% = 99.998\%$  |
| 0 MW  | $2\% \cdot 3\% \cdot 4\% = 0.002\%$  | $99.998\% + 0.002\% = 100.000\%$ |

- (ii) If the system peak load is 260 MW, find the probability that generation will not be able to meet it.

[3]

Probability that generation will meet 260 MW is 93.120%. Therefore, probability that generation will not meet 260 MW is  $100\% - 93.120\% = 6.880\%$ .

2. a) A 33/11kV 15MVA transformer has a leakage reactance of  $4\Omega$  as seen from the HV side  
 (i) Calculate the leakage reactance as seen from the LV side [2]

$$X_{lv} = X_{hv} \cdot \frac{U_{lv}^2}{U_{hv}^2} = 0.4444\Omega$$

- (ii) Calculate the p.u. impedance at the HV side and show that this is the same as the pu impedance as seen from the LV side [3]

$$Z_{base\ hv} = \frac{U_{hv}^2}{S_b} = \frac{33^2 \cdot 10^6}{15 \cdot 10^6} = 72.6\Omega$$

in per unit

$$x = \frac{X_{hv}}{Z_{base\ hv}} = \frac{4}{72.6} = 0.0551\text{ p.u.}$$

$$X_{lv} = X_{hv} \cdot \frac{U_{lv}^2}{U_{hv}^2} = 0.4444\Omega, Z_{base\ lv} = \frac{U_{lv}^2}{S_b} = \frac{11^2 \cdot 10^6}{15 \cdot 10^6} = 8.06\Omega$$

in per unit

$$x = \frac{0.4444}{8.06} = 0.0551\text{ p.u.}$$

- (iii) What is the significance of per-unit system in the analysis of power systems? [2]

Per unit system provides the basis for the application of conventional circuit theory and methods to multi-voltage levels power system problems. The key benefit is transforming multi-voltage level system into a single voltage level circuit.

- b) A turbo-generator feeds into a very strong network that maintains the terminal voltage  $V_t = 1\text{ p.u.}$  (Figure 2.1).

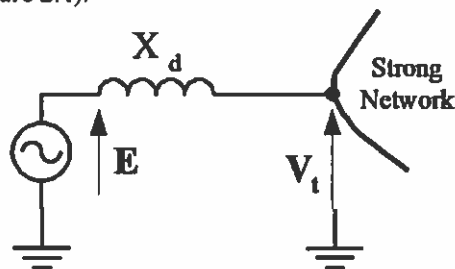


Figure 2.1: A generator connected to a very strong system

The synchronous reactance of the generator is equal to 1 p.u. Initially, the generator runs overexcited with  $E = 1.5\text{ p.u.}$  with real power output of 0.25 p.u. Calculate:

- (i) The power angle and reactive power output for this initial operating condition.

[4]

$$S = \frac{V_1 V_2}{X} \sin(\delta_1 - \delta_2) + j \frac{V_1^2 - V_1 V_2 \cos(\delta_1 - \delta_2)}{X}$$

$$P = \frac{V_1 V_2}{X} \sin(\delta_1 - \delta_2)$$

$$Q = \frac{V_1^2 - V_1 V_2 \cos(\delta_1 - \delta_2)}{X}$$

$$0.25 = \frac{1.5 \times 1.0}{1.0} \sin \delta \Rightarrow \sin \delta = 0.1667 \Rightarrow \delta = 9.6^\circ$$

$$Q = \frac{1.5 \times 1.0}{1.0} \cos \delta - \frac{(1.0)^2}{1.0} = 0.48 \text{ pu}$$

- (ii) The active and reactive power delivered to the system when the turbine torque doubles.

[3]

$$0.5 = \frac{1.5 \times 1.0}{1.0} \sin \delta \Rightarrow \delta = 19.5^\circ$$

$$Q = \frac{1.5 \times 1.0}{1.0} \cos \delta - \frac{(1.0)^2}{1.0} = 0.41 \text{ pu (slight drop in reactive output)}$$

- (iii) The active and reactive power delivered to the system for an increase in the internal voltage E by 20% (from the initial condition).

[3]

$$0.25 = \frac{1.8 \times 1.0}{1.0} \sin \delta \Rightarrow \delta = 8^\circ$$

$$Q = \frac{1.8 \times 1.0}{1.0} \cos \delta - \frac{(1.0)^2}{1.0} = 0.78 \text{ pu (63% increase from the original value of 0.48 p.u.)}$$

- (iv) Explain how the active and reactive power outputs of a synchronous generator are controlled.

[3]

Active power output is controlled through adjusting generator governor valves, while reactive power output / consumption is controlled through adjusting excitation (rotor DC current control)

3. a) List 3 objectives of power flow calculations.

[3]

Purpose of load flow is to:

- Analyse the loading of transmission circuits and voltage profile under different loading conditions and generation dispatches
- Analyse effects of rearranging circuits and incorporating new circuits on system loading and voltage profile
- Analyse effects of temporary loss of generation and transmission circuits on system loading
- Analyse effects of injecting in-phase and quadrature voltages on system loading and voltage profile
- Initialise stability calculations

- b) Explain briefly why an iterative method is required to determine nodal voltages in power networks.

[4]

Traditional methods (nodal analysis, loop analysis) yield systems of complex, linear equations. In power systems, however, sources and loads are defined in terms of power, not voltage, current or impedance and hence the equations are non-linear. System of non-linear equations are solved iteratively.

- c) Explain briefly why reactive power cannot be transported over long distances across transmission networks.

[4]

$$Q = \frac{V_1^2 - V_1 V_2 \cos(\delta_1 - \delta_2)}{X}$$

Unlike active power, reactive power cannot be transmitted across long distances as transmitting Q requires a voltage drop that would become unacceptable for long distances. Furthermore, since  $X \gg R$ , the reactive losses are much larger than the active losses and the transmission of Q would be inefficient

- d) In Figure 3.1, the reactances of transmission circuits and bus bar loads are given in per unit using a common base

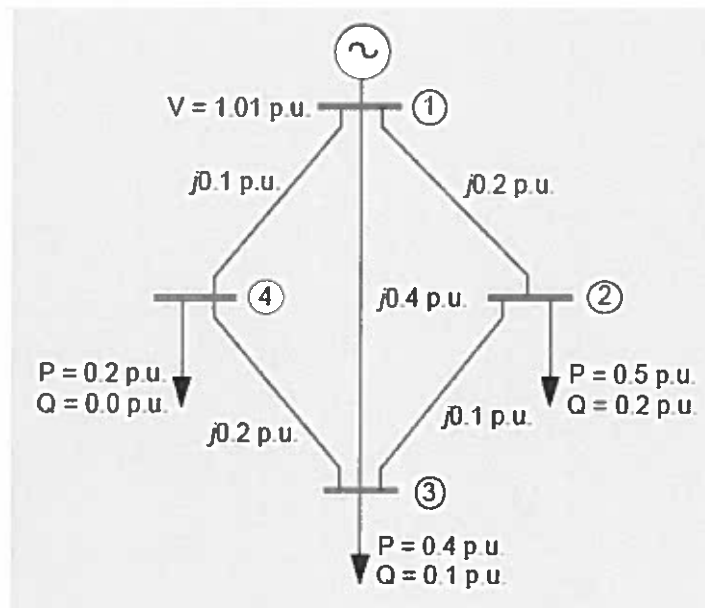


Figure 3.1: Network diagram and data

- (i) Form the bus admittance matrix  $Y_{BUS}$  for this network.

[4]

$$Y_{bus} = \begin{bmatrix} -j17.5 & j5 & j2.5 & j10 \\ j5 & -j15 & j10 & 0 \\ j2.5 & j10 & -j17.5 & j5 \\ j10 & 0 & j5 & -j15 \end{bmatrix}$$

- (ii) Using bus bar 1 as the slack (reference) bus bar, carry out the first iteration of a Gauss-Seidel load-flow algorithm to determine the voltage at all bus bars. Assume the initial voltages of all bus bars to be 1.01 p.u.

[5]

Assume  $V_2^{(0)} = 1.01$  p.u.,  $V_3^{(0)} = 1.01$  p.u. and  $V_4^{(0)} = 1.01$  p.u.

$$\begin{aligned} V_1^{(1)} &= \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{V_2^{(0)*}} - Y_{21} V_1 - Y_{23} V_3^{(0)} \right] \\ &= \frac{1}{-j15} \left[ \frac{-0.5 + j0.2}{1.01} - j5 \times 1.01 - j10 \times 1.01 \right] \\ &= 0.9968 - j0.033 \end{aligned}$$



$$\begin{aligned}
 \mathbf{V}_3^{(1)} &= \frac{1}{\mathbf{Y}_{33}} \left[ \frac{P_3 - jQ_3}{\mathbf{V}_3^{(0)*}} - \mathbf{Y}_{31} \mathbf{V}_1 - \mathbf{Y}_{32} \mathbf{V}_2^{(1)} - \mathbf{Y}_{34} \mathbf{V}_4^{(0)} \right] \\
 &= \frac{1}{-j17.5} \left[ \frac{-0.4 + j0.1}{1.01} - j2.5 \times 1.01 - j10 \times (0.9968 - j0.033) - j5 \times 1.01 \right] \\
 &= 0.9968 - j0.0415
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{V}_4^{(1)} &= \frac{1}{\mathbf{Y}_{44}} \left[ \frac{P_4 - jQ_4}{\mathbf{V}_4^{(0)*}} - \mathbf{Y}_{41} \mathbf{V}_1 - \mathbf{Y}_{43} \mathbf{V}_3^{(1)} \right] \\
 &= \frac{1}{-j15} \left[ \frac{-0.2}{1.01} - j10 \times 1.01 - j5 \times (0.9968 - j0.0415) \right] \\
 &= 1.0056 - j0.027
 \end{aligned}$$

**Part B – Answer any 2 out of 3 questions in part B**

4. a) For short-circuit analysis, why is it reasonable to assume 1.0 p.u. pre-fault voltages and zero pre-fault currents?

[5]

Assumption 1: All pre-fault voltage magnitudes are 1.0 pu

In practice under normal operation voltages at all the bus bars are nearly 1.0 pu. This is achieved by voltage control devices like excitation system of generators, transformer tap changer action

Assumption 2: All pre-fault currents are zero

- Change in current due to fault is quite large, typically 10-20 pu
- Sub-transient current is mostly reactive while pre-fault current is predominantly resistive. Hence total current (sub-transient + pre-fault) magnitude can be assumed to be the larger of the two

4. b) Explain with the help of constant flux linkage theorem why there is a decaying AC component in the stator current which flows as a result of a three-phase short circuit on the terminal of a synchronous generator.

[5]

Due to a three-phase fault on the generator terminal, the current increases. The magnetic flux also increases as a result. According to the constant flux linkage theorem, flux linking a closed winding cannot change instantaneously. Therefore, the magnetic flux due to short-circuit stator current is forced through high reluctance paths that do not link the field (rotor) or damper circuits. Thus, the stator inductance (inversely proportional to reluctance) is initially low leading to high current. As the flux moves towards lower reluctance paths, stator inductance increases resulting in relatively lower currents in steady-state compared to the transient or sub-transient conditions. This explains the decaying AC component in the stator current.

4. c) Four identical generators are connected to two bus bars, A and B as shown in Figure 4.1. The rating of each generator is 13 kV, 60 MVA and their sub-transient and transient reactance is 0.15 pu and 0.3 pu, respectively. A feeder is supplied from bus bar A through a step-up transformer rated at 30 MVA with 10% leakage reactance. Bus bars A and B are connected through a reactor with reactance  $X$ . Neglect the pre-fault current and fault impedance and assume the pre-fault voltage to be 1.0 pu. Choose 60 MVA as the system base. Consider fault current contribution from the generators only.
- i) Calculate the value of the reactance  $X$  in pu if the three-phase short-circuit level at the feeder side of the transformer (marked by point C in Figure 4.1) is to be limited to 240 MVA
- ii) Using the value of  $X$  from part (i) calculate the voltage at bus bar A during a three-phase fault at the feeder side of the transformer (marked by point C in Figure 4.1)

[6]

[4]

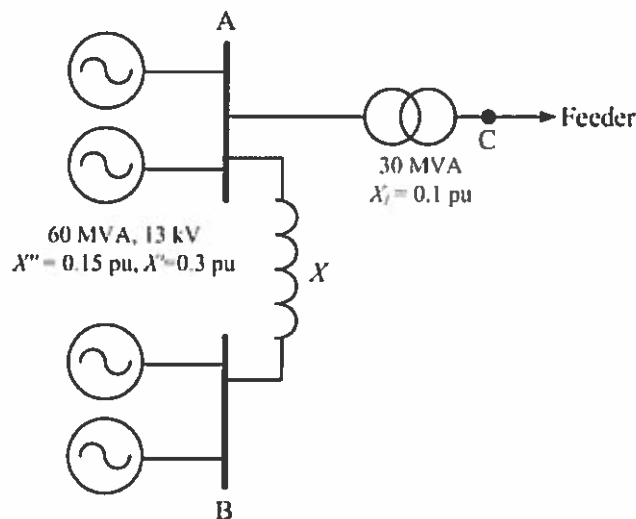


Figure 4.1: Single line diagram for the system in Problem 4(b)

Let's consider  $S_{base} = 60 \text{ MVA}$

Fault level at C should be limited to  $240 / 60 = 4 \text{ p.u.}$

Therefore  $Z_{eq}$  should be  $= 1/4 = 0.25 \text{ p.u.}$

Equivalent impedance looking into the fault location at C is:

$$\begin{aligned}
 Z_{eq} &= \left\{ \left[ \frac{0.15}{2} \right] // \left[ \frac{0.15}{2} + X \right] \right\} + 0.2 \\
 &= \frac{0.075(0.075 + X)}{0.075 + 0.075 + X} + 0.2 \\
 &= \frac{0.0056 + 0.075X}{0.15 + X} + 0.2
 \end{aligned}$$

By equating  $Z_{eq}$  to 0.25,

$$Z_{eq} = \frac{0.0056 + 0.075X}{0.15 + X} + 0.2 = 0.25$$

$$\therefore X = 0.076 \text{ p.u.}$$

$$\begin{aligned}
 \text{Voltage at bus bar A} &= 4 \times 0.2 = 0.8 \text{ p.u.} \\
 &= 13 \times 0.8 = 10.4 \text{ kV}
 \end{aligned}$$

5. a) Starting from the voltages and currents in the phase domain during a fault condition show how the positive, negative and zero sequence networks would be connected for a line-to-line (LL) fault between phases B and C.

[5]

For a LL fault between phase B and C, the fault currents and voltages are:

$$I_a = 0$$

$$I_c = -I_b$$

$$V_b - V_c = Z^f I_b$$

Voltages and currents in sequence domain are:

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 \\ (\alpha - \alpha^2)I_b \\ -(\alpha - \alpha^2)I_b \end{bmatrix}$$

$$I_{a1} = -I_{a2}, I_{a0} = 0$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b - Z^f I_b \end{bmatrix}$$

$$3(V_{a1} - V_{a2}) = (\alpha - \alpha^2)Z^f I_b = 3Z^f I_{a1}$$

$$V_{a1} - V_{a2} = Z^f I_{a1}$$

As the positive and zero sequence currents are equal and opposite and the difference between the positive and negative sequence voltages is the drop across the fault impedance, For LL faults, the positive and negative sequence networks are connected in parallel across the fault impedance.

The zero sequence network has no role in LL faults.

5. b) A 20 kV, 500 MVA three-phase generator has a star connected stator winding with the neutral point grounded through a  $1\Omega$  resistor. The positive, negative and zero sequence sub-transient reactance for the generator are 0.2 pu, 0.16 pu and 0.14 pu, respectively based on the rating of the generator. The generator supplies a delta-star connected 20 kV/275 kV, 550 MVA step-up transformer with its neutral point solidly grounded as shown in Figure 5.1. The leakage reactance of the transformer is 0.15 pu. Neglect the pre-fault current and fault impedance and assume the pre-fault voltage to be 1.0 pu. Choose 60 MVA as the system base. Consider fault current contribution from the generator only and assume zero contribution from the system on the 275 kV side of the transformer.

- i) Calculate the fault current (in kA) due to a three-phase fault on the 275 kV side of the transformer.

[4]

- ii) Calculate the fault current (in kA) due to a line-to-ground (LG) fault on the 275 kV side of the transformer. [6]
- iii) Compare the fault currents for a three-phase and line-to-ground (LG) fault on the 275 kV side of the transformer if the transformer connection is changed to star-star with the neutral points solidly connected on both sides. [5]

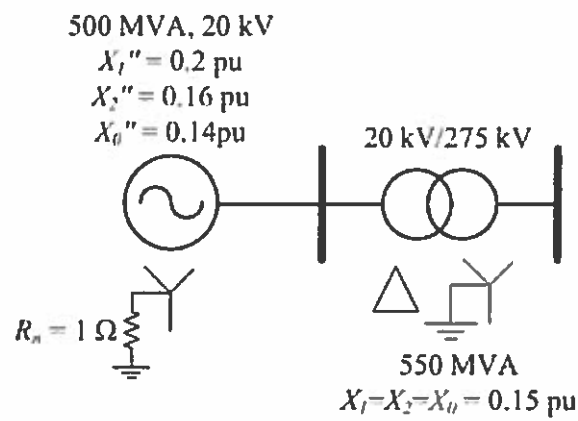


Figure 5.1: Single line diagram for the system in Problem 5(b)

For 500 MVA, 275 kV base

$$I_{base} = \frac{500 \times 10^6}{\sqrt{3} \times 275 \times 10^3} = 1049.7 \text{ A}$$

For 500 MVA, 20 kV base

$$I_{base} = \frac{500 \times 10^6}{\sqrt{3} \times 20 \times 10^3} = 14.43 \text{ kA}$$

$$Z_{base} = \frac{20^2}{500} = 0.8 \Omega$$

On 500 MVA generator base:  $X_1 = 0.2$ ,  $X_2 = 0.16$ ,  $X_0 = 0.14$

For transformer:  $X_1 = X_2 = X_0 = 0.15 \times 500/550 = 0.136 \text{ p.u.}$

Grounding resistor =  $1/0.8 = 1.25 \text{ p.u.}$

(i) For a three-phase fault on 275 kV bus bar

$$I_f = \frac{1}{0.2 + 0.136} = 2.98 \text{ p.u.}$$

$$= 2.98 \times 1049.7 = 3.124 \text{ kA}$$

(ii) For line-to-ground (LG) fault on 275 kV bus bar

Positive sequence reactance  $Z_1 = 0.2 + 0.136 = 0.336 \text{ p.u.}$

Negative sequence reactance  $Z_2 = 0.16 + 0.136 = 0.296 \text{ p.u.}$

For delta-star connection with neutral grounded, the zero sequence current would flow to ground through the star side but cannot flow on the line (generator) side of the delta winding. Hence, zero sequence reactance  $Z_0 = 0.136 \text{ p.u.}$

The fault current  $I_f$

$$I_f = \frac{3}{0.336 + 0.296 + 0.136} = 3.906 \text{ p.u.} = 4.1 \text{ kA}$$

(iii) Once the transformer connection is changed to star-star with the neutral points solidly connected on both sides, the zero sequence current would flow through the transformer and the generator and its neutral grounding resistor.

Thus the zero sequence impedance in this case would be:

$$Z_0 = \sqrt{(0.136 + 0.14)^2 + (3 \times 1.25)^2} = 3.76 \text{ p.u.}$$

The fault current  $I_f$

$$I_f = \frac{3}{0.336 + 0.296 + 3.76} = 0.683 \text{ p.u.} = 0.717 \text{ kA}$$

Changing the transformer connection to star-star with the neutral points solidly connected on both sides reduces the LG fault current to 0.717 kA (compared to 4.1 kA with delta-star with neutral grounded connection). The three-phase fault current would remain the same.

6. a) What is transient stability in the context of power systems?

[2]

Transient stability is the ability of power system to maintain synchronism when subjected to a severe disturbance like faults on transmission lines, transformers, buses; loss of generation and/or loads.

6. b) Mention two main factors that influence the transient stability of power systems along with an example of each of the two factors in the context of a short-circuit in a simple system comprising a single generator connected to an infinite bus bar.

[3]

Transient stability depends on (i) initial operating condition and (ii) severity of the disturbance.

An example of initial operating condition is the initial power angle ( $\delta_0$ ) or power transfer level and that of severity of the disturbance is magnitude and/or duration of the fault.

6. c) A round-rotor synchronous generator is connected to an infinite bus bar via a generator transformer and two parallel overhead lines. The transformer has a leakage reactance of 0.15 p.u. and each transmission line has a reactance of 0.4 p.u. Under normal operating condition the generator is supplying 0.8 p.u. active power at a terminal voltage of 1.0 p.u. The generator has a transient reactance of 0.2 p.u. All impedance values are based on the generator rating and the voltage of the infinite bus bar is 1 p.u. Calculate the critical clearing angle (in degrees) if a three-phase solid fault occurs on the sending (generator) end of one of the transmission line circuits and is cleared by disconnecting the faulted line. Assume the electrical power output of the generator to be zero during the fault. Neglect resistances.

[8]

Pre-fault

$$P_0 = 0.8 \text{ p.u.}$$

$$X = 0.2 + 0.15 + 0.4/2 = 0.55 \text{ p.u.}$$

$$P_2 \approx 1/0.55 = 1.82 \text{ p.u.}$$

$$\delta_0 = \sin^{-1}(0.8/1.82) = 27.02^\circ$$

During the fault  $P = 0$

After the fault

$$X = 0.2 + 0.15 + 0.4 = 0.75 \text{ p.u.}$$



$$P_2 \approx 1/0.75 = 1.32 \text{ p.u.}$$

$$\delta_2 = 180^\circ - \sin^{-1}(0.8/1.22) = 142.7^\circ$$

From equal area criteria:

$$\int_{\delta_0}^{\delta_1} P_0 d\delta + \int_{\delta_1}^{\delta_2} (P_0 - P_2 \sin \delta) d\delta = 0$$

$$P_0(\delta_2 - \delta_0) + P_2 \cos \delta_2 - P_2 \cos \delta_1 = 0$$

$$0.8 \times (142.7^\circ - 27.02^\circ) \times \pi / 180^\circ + 1.32 \cos 142.7^\circ - 1.32 \cos \delta_1 = 0$$

$$\cos \delta_1 = \frac{1.615 - 1.05}{1.32}$$

$$\delta_1 = 64.65^\circ$$

6. d) A round-rotor generator connected to an infinite bus bar through two parallel 132 kV lines in parallel, each having a reactance of 70  $\Omega$ /phase. The rating of the generator is 60 MW at power factor 0.9 lagging and has a transient reactance of 0.3 p.u. and an inertia constant 3 kWs/kVA. The generator is delivering 1.0 pu active power to the infinite bus bar. A three-phase symmetrical fault occurs halfway along one line and is cleared by disconnecting the faulted line. The generator internal voltage is 1.05 p.u. and the infinite bus bar voltage is 1.0 p.u. From steady state stability considerations, determine maximum allowable power transfer during the (i) pre-fault, and (ii) post-fault conditions. Neglect resistances. Assume a system base of 100 MVA.

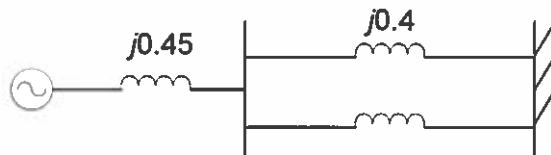
[4+3]

Let's assume a system base 100 MVA

$$\text{On 100 MVA base, } Z_{\text{base}} = 132^2/100 = 174.24$$

$$\text{Line reactance} = 70/174.24 = 0.4 \text{ p.u.}$$

$$\text{Generator transient reactance on 100 MVA base} = 0.3 \times 100 / (60/0.9) = 0.45 \text{ p.u.}$$



(i) Pre-fault reactance =  $j0.45 + j0.4/2 = j0.65 \text{ p.u.}$

Maximum power transfer corresponds to the peak of the sinusoidal power-angle curve under pre-fault condition:

$$\text{Maximum power transfer} = 1.05 \times 1.0 / 0.65 = 1.62 \text{ p.u.}$$

(ii)  $\text{Post-fault reactance} = j0.45 + j0.4 = j0.85 \text{ p.u.}$

Maximum power transfer corresponds to the peak of the sinusoidal power-angle curve under post-fault condition:

$$\text{Maximum power transfer} = 1.05 \times 1.0 / 0.85 = 1.24 \text{ p.u.}$$