EE4-65
EE9-SO27

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

WIRELESS COMMUNICATIONS

Monday, 12 May 10:00 am

Time allowed: 3:00 hours

There are THREE questions on this paper.

Answer THREE questions.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : B. Clerckx

Second Marker(s) : K.K. Leung

Important information for students

Notations:

(a) A $n_r \times n_t$ MIMO channel consists in n_r receive antennas and n_t transmit antennas.

(b) a, $\mathbf{a}$, $\mathbf{A}$ denote a scalar, vector and matrix respectively.

(c) $\mathbf{A}^H$ denotes conjugate transpose (Hermitian).

(d) $\mathbf{A}^*$ denotes conjugate.

(e) $\mathbf{A}^T$ denotes transpose.

(f) $|a|$ denotes the absolute value of scalar a.

(g) $\|\mathbf{a}\|$ denotes the (Euclidean) norm of vector $\mathbf{a}$.

(h) "i.i.d." means "independent and identically distributed".

(i) "CSI" means "Channel State Information".

(j) "CSIT" means "Channel State Information at the Transmitter".

(k) "CDIT" means "Channel Distribution Information at the Transmitter".

(l) $\mathcal{E}\{.\}$ denotes Expectation.

(m) $\mathrm{Tr}\{.\}$ denotes the Trace of a matrix.

Assumptions:

(a) The CSI is assumed to be always perfectly known to the receiver.

(b) The receiver noise is a $n_r \times 1$ vector with i.i.d. entries modeled as zero mean complex additive white Gaussian noise with variance σ_n^2.

Some useful relationships:

(a) $\|\mathbf{A}\|_F^2 = \mathrm{Tr}\{\mathbf{A}\mathbf{A}^H\} = \mathrm{Tr}\{\mathbf{A}^H\mathbf{A}\}$

(b) $\mathrm{Tr}\{\mathbf{AB}\} = \mathrm{Tr}\{\mathbf{BA}\}$

(c) $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$

THE QUESTIONS

1. [40]

a) Consider the Multiuser Multiple-Input Single-Output (MU-MISO) transmission of two independent streams to two independent users. The transmitter is equipped with 4 antennas while each user is equipped with a single receive antenna. The Channel State Information (CSI) is perfectly known to the transmitter. Denoting the vector of transmitted symbols as $\mathbf{c} = [c_1, c_2]^T$ with c_1 and c_2 intended for user 1 and 2 respectively, the received signals are written as $y_1 = \mathbf{h}_1\mathbf{P}\mathbf{c} + n_1$ and $y_2 = \mathbf{h}_2\mathbf{P}\mathbf{c} + n_2$ at user 1 and 2, respectively. The precoder $\mathbf{P}$ is made of two columns $\mathbf{p}_1$ and $\mathbf{p}_2$, each column subject to a power constraint $\|\mathbf{p}_i\|^2 = 1$, $i = 1,2$. The channel vector of user 1 is given by

$$\mathbf{h}_1 = \begin{bmatrix} 1 & 2 & -1 & 1 \end{bmatrix}$$

while that of user 2 is given by

$$\mathbf{h}_2 = \begin{bmatrix} 3 & 1 & 4 & -1 \end{bmatrix}.$$

At the transmitter we would like to apply a precoder P with the lowest possible complexity such that the multi-user interference is completely nulled out.

i) Derive the expression of the precoder $\mathbf{P}$. Provide your reasoning. [3]

ii) What kind of precoder is this? Explain your result. [3]

b) Figure 1.1 displays the average Error Probability of one scheme (i.e., one transmission and reception strategy) vs. SNR for point-to-point channels with i.i.d. Rayleigh slow fading and four different antenna configurations (a) to (d). The CSI is unknown to the transmitter.

i) What is the diversity gain (at high SNR) achieved by that scheme in each antenna configuration? Provide your reasoning. [4]

ii) For each scenario (a) to (d), identify an antenna configuration (i.e., n_t and n_r) and the corresponding transmission/reception strategy that can achieve such diversity gain. Provide your reasoning. [4]

c) Figure 1.2 displays the ergodic capacity of point-to-point i.i.d. Rayleigh fast fading channels with Channel Distribution Information at the Transmitter (CDIT) for five antenna ($n_r \times n_t$) configurations (denoted as (a) to (e)) with $n_t + n_r = 8$.

i) What is the achievable (spatial) multiplexing gain (at high SNR) for cases (a), (b), (c), (d) and (e)? Provide your reasoning. [5]

ii) For (a), (b), (c), (d) and (e), identify an antenna configuration, i.e. n_t and n_r, satisfying $n_t + n_r = 8$ that achieves such multiplexing gain.

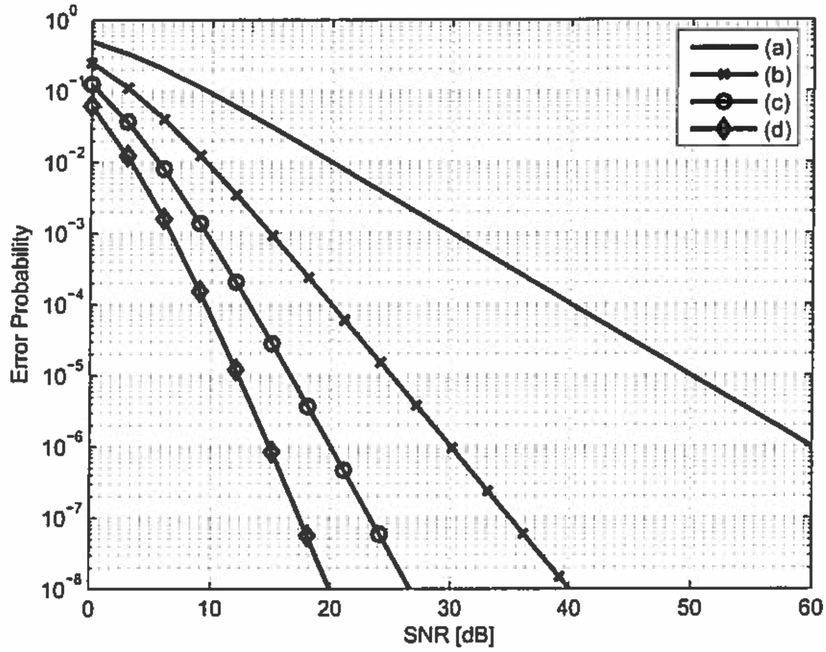

Figure 1.1 Average Error Probability vs. SNR.

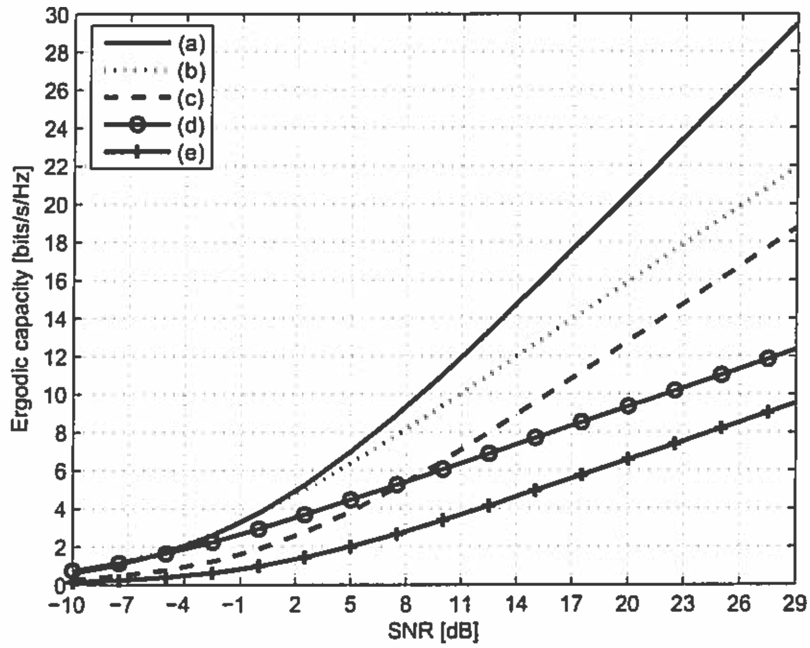

Figure 1.2 Ergodic capacity vs. SNR.

Provide your reasoning.

[5]

d) Consider a MISO channel $\mathrm{h} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}$ with two transmit antennas and one receive antenna, constant over two consecutive symbol durations. Symbols c_1 and c_2 are transmitted over the channel during two symbol durations using an Alamouti space-time code.

i) Write the received signals at the receiver. [2]

ii) Provide a simple receiver strategy to detect c_1 and c_2. [3]

iii) What happens if the same transmission strategy is applied to a channel that is not constant over two consecutive symbol durations? Provide your reasoning. [3]

e) Consider the very strong interference regime of a two-user SISO interference channel.

i) Explain why a per-user multiplexing gain of 1 is achievable and which receiver strategy allows to achieve such multiplexing gain? [4]

ii) Is there any condition on the Signal to Noise Ratio (SNR) and Interference to Noise Ratio (INR) of each link for such strategy to be valid?

[4]

2. [30]

Discuss the validity of the following statements. Detail your argument.

a) In a two-user SISO Broadcast Channel, for any ordering of the channels, the capacity region can be achieved with superposition coding with Successive Interference Cancellation (SIC).

[6]

b) Consider a point-to-point i.i.d. MISO Rayleigh slow fading channel with 4 transmit antennas and 1 receive antenna and a transmission strategy based on partial transmit channel knowledge consisting of transmitting a single stream using quantized precoding. The codebook of precoders is given by

$$\mathcal{W} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

This transmission strategy achieves a diversity gain of 4.

[6]

c) In point-to-point MISO and in multi-user MISO, the multiplexing gain (at high SNR) is not affected by the quality of Channel State Information at the Transmitter (CSIT).

[6]

d) In a cellular system where users are uniformly located in a cell and have the same Quality of Service (QoS), the proportional fair (PF) scheduler is a fair scheduler that benefits from multi-user diversity in the limit of very small scheduling time scale.

[6]

e) A narrowband transmission using a transmission strategy characterized by the following set of codewords

$$\mathbf{a} = \begin{bmatrix} a & a & b & c \end{bmatrix},$$
$$\mathbf{b} = \begin{bmatrix} a & d & a & b \end{bmatrix},$$
$$\mathbf{c} = \begin{bmatrix} b & a & a & b \end{bmatrix},$$

with $a = \frac{1}{\sqrt{2}}(1+j)$, $b = \frac{1}{\sqrt{2}}(-1+j)$, $c = \frac{1}{\sqrt{2}}(-1-j)$ and $d = \frac{1}{\sqrt{2}}(1-j)$ being the four constellation symbols taken from a unit average energy QPSK constellation, and a Maximum Likelihood (ML) receiver, achieves a diversity gain of 2 in i.i.d. fast Rayleigh fading channels with a single receive antenna and a single transmit antenna.

[6]

3. [30]

Assume a downlink narrowband transmission in a cellular network consisting of 2 cells, each equipped with one transmitter, denoted as transmitter 1 in cell 1 and transmitter 2 in cell 2. In cell 1, there are two terminals (denoted as terminal 1 and 2) and in cell 2, there is a single terminal (denoted as terminal 3). The two transmitters are equipped with 2 antennas and the terminals equipped with 1 receive antenna. In cell 1, each receiver perfectly estimates and reports its channel state information to transmitter 1 and transmitter 1 schedules the two terminals at a time and sends using MU-MISO linear precoding a total of 2 streams, one stream per terminal. In cell 2, transmitter 2 sends a single stream to terminal 3. Each transmitter is subject to a total transmit power P.

a) Write an expression for the received signal of terminal 1 in cell 1 in terms of channel parameters, precoders and transmit symbol vectors. Clearly define each variable and identify the terms responsible for the intra-cell interference (also called multi-user interference) and inter-cell interference in your expression.
[4]

b) Derive the expression of the rate achievable by terminal 1 in cell 1.
[4]

c) Assume that terminal 1 in cell 1 has the ability to recommend one precoder to transmitter 2 so as to maximize its rate. In order to decrease the feedback overhead, the precoder recommended by terminal 1 has to be selected in a codebook (with codewords normalized to have unit norm). Two codebooks, each containing four precoders, are available

$$\mathscr{W}_1 = \left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix} \right\}$$

and

$$\mathscr{W}_2 = \left\{ \begin{bmatrix} -0.1612 - 0.7348i \\ -0.5135 - 0.4128i \end{bmatrix}, \begin{bmatrix} -0.0787 - 0.3192i \\ -0.2506 + 0.9106i \end{bmatrix}, \begin{bmatrix} -0.2399 + 0.5985i \\ -0.7641 - 0.0212i \end{bmatrix}, \begin{bmatrix} -0.9541 \\ 0.2996 \end{bmatrix} \right\}.$$

i) Assuming that the channel vector between transmitter 2 and terminal 1 is given by $\begin{bmatrix} 1 & e^{\pi/3} \end{bmatrix}$ and that codebook $\mathscr{W}_1$ is used, which precoder will terminal 1 recommend to transmitter 2? Provide your reasoning.
[3]

ii) Assuming that the channel vector between transmitter 2 and terminal 1 is random and has entries modeled as i.i.d. circularly symmetric complex Gaussian variables, which codebook (among $\mathscr{W}_1$ and $\mathscr{W}_2$) would you recommend to use? [4]

d) Assume the interference from transmitter has been completely mitigated (i.e. there is no interference from transmitter 2 to terminal 1 and 2) and focus exclusively on cell 1, i.e. transmitter 1 and its two terminals.

i) Derive the expression of the transmit precoder such that the two terminals do not experience any multi-user interference. Provide your reasoning.
[5]

ii) Derive the total multiplexing gain that is achieved at high SNR with such a transmit precoder. Provide your reasoning.

[5]

iii) Let us improve the design of part i) such that the sum-rate is maximized under the constraint that the two terminals do not experience any multi-user interference. In particular, derive the expressions of the transmit precoding and power allocation strategy (assuming a fixed total power constraint) for transmitter 1 such that the sum-rate is maximized under the constraint that the two terminals do not experience any multi-user interference. Provide your reasoning. [5]