Question 1

(a) Describe the significance of the concept of system marginal price and explain why it varies with time in the case of electricity.

[4]

(b) Four generators are available to supply a demand of $D = 472.5\,[MW]$. The cost of generating power $C_i(P_i)$ corresponding to each of the generators i is:

$$C_1(P_1) = 500 + 3 * P_1 + 0.06 * P_1^2 \; [\frac{£}{h}]$$

$$C_2(P_2) = 300 + 17 * P_2 + 0.10 * P_2^2 \; [\frac{£}{h}]$$

$$C_3(P_3) = 1300 + 12 * P_3 + 0.15 * P_3^2 \; [\frac{£}{h}]$$

$$C_4(P_4) = 150 + 15 * P_4 + 0.22 * P_4^2 \; [\frac{£}{h}]$$

Calculate the optimal production of each generator, the system marginal cost, the system average cost, and the profits of each generator.

[6]

(c) Assume that the maximum output limit of each generator is:

$$P_1^{max} = 230\,[MW]$$

$$P_2^{max} = 120\,[MW]$$

$$P_3^{max} = 160\,[MW]$$

$$P_4^{max} = 80\,[MW]$$

Determine the optimal production of each generator, the system marginal cost, the system average cost and the profits of each generator. Based on the results, briefly explain the impact of considering these maximum output limits on the total cost of operating the system and the profits of the four generators.

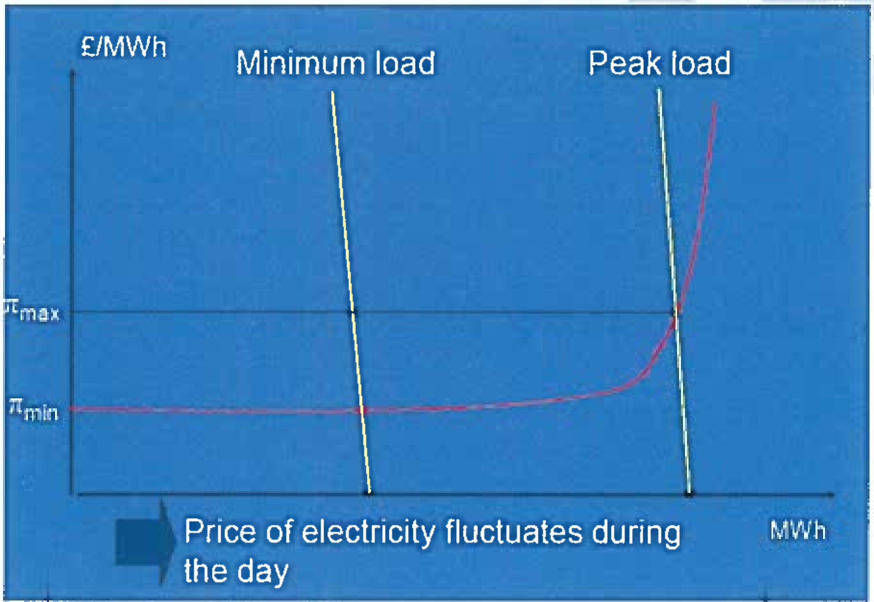
[6]

(d) In the solution of **(b)** and **(c)**, the profit of one of the generators is negative. Explain why this happens, what this means for this generator, as well as why and how it should be avoided in practice.

[4]

Solution to Question 1

(a) The system marginal price signifies the marginal cost of serving an additional unit of demand at the least-cost dispatch of the total load to the available generators. In contrast with other commodities, electrical energy cannot be economically stored in large quantities and thus it must be produced exactly when it is consumed by the inflexible demand. Moreover, the electrical demand is not the same every hour, day, month etc and the generation side is characterized by a production cost which varies with the size of the load. The combination of these effects explains why the system marginal price varies with time. This is demonstrated in the figure below. The supply curve of the generation side exhibits a "hockey-stick" shape, being rather flat at the area of low loads, where cheap base generators are employed for the satisfaction of the demand, and rather steep at the area of high loads, where expensive peaking generators are also employed to cover the demand. In combination with the fluctuation of the demand from a minimum to a peak value, temporal variations in the system marginal price emerge.



(b) In the optimal solution of the problem, the marginal cost of each generator is equal to the system marginal cost λ yielding:

$$\frac{\partial C_1(P_1)}{\partial P_1} = 3 + 0.12 * P_1 = \lambda \rightarrow P_1 = \frac{\lambda - 3}{0.12} \quad (1)$$

$$\frac{\partial C_2(P_2)}{\partial P_2} = 17 + 0.2 * P_2 = \lambda \rightarrow P_2 = \frac{\lambda - 17}{0.2} \quad (2)$$

$$\frac{\partial C_3(P_3)}{\partial P_3} = 12 + 0.3 * P_3 = \lambda \rightarrow P_3 = \frac{\lambda - 12}{0.3} \quad (3)$$

$$\frac{\partial C_4(P_4)}{\partial P_4} = 15 + 0.44 * P_4 = \lambda \rightarrow P_4 = \frac{\lambda - 15}{0.44} \quad (4)$$

The satisfaction of the system demand implies that:

$$P_1 + P_2 + P_3 + P_4 = D = 472.5\ [MW] \quad (5)$$

Substituting (1)-(4) in (5) gives for the system marginal cost:

$$\frac{\lambda-3}{0.12}+\frac{\lambda-17}{0.2}+\frac{\lambda-12}{0.3}+\frac{\lambda-15}{0.44}=472.5 \rightarrow \lambda=34.668\ [\frac{£}{MWh} \tag{6}$$

Substituting (6) in (1)-(4) yields for the optimal production of each generator:

$$P_1 = 263.900\ [MW] \tag{7}$$

$$P_2 = 88.340\ [MW] \tag{8}$$

$$P_3 = 75.560\ [MW] \tag{9}$$

$$P_4 = 44.700[MW] \tag{10}$$

Substituting (7)-(10) in the cost functions of the generators yields for the total cost of operating the system:

$$C_{tot} = C_1(P_1) + C_2(P_2) + C_3(P_3) + C_4(P_4) = 12375.665\ [£] \tag{11}$$

The system average cost is:

$$\alpha = \frac{C_{tot}}{P_1+P_2+P_3+P_4} = 26.192\ [\frac{£}{MWh}] \tag{12}$$

The profit for each generator Ω_i is given by the difference between its respective revenue and cost:

$$\Omega_1 = \lambda * P_1 - C_1(P_1) = 3678.593\ [£] \tag{13}$$

$$\Omega_2 = \lambda * P_2 - C_2(P_2) = 480.396\ [£] \tag{14}$$

$$\Omega_3 = \lambda * P_3 - C_3(P_3) = -443.603\ [£] \tag{15}$$

$$\Omega_4 = \lambda * P_4 - C_4(P_4) = 289.580\ [£] \tag{16}$$

(c) Given that $P_1 > P_1^{max}$ the solution calculated in **(b)** is not feasible when taking into account given maximum output limits of the generators. The optimal solution can now be calculated by fixing the output of generator 1 to:

$$P_1' = P_1^{max} = 230\ [MW] \tag{17}$$

and satisfying the remaining demand:

$$D' = D - P_1' = 242.5\ [MW] \tag{18}$$

through the remaining generators 2, 3 and 4. Following the same method as in (b), we get:

$$\frac{\lambda'-17}{0.2}+\frac{\lambda'-12}{0.3}+\frac{\lambda'-15}{0.44}=242.5 \rightarrow \lambda'=37.864\ [\frac{£}{MWh}] \tag{19}$$

Substituting (19) in (2)-(4) yields for the optimal production of each generator:

$$P_2' = 104.321[MW] \tag{20}$$

$$P_3' = 86.214\ [MW] \tag{21}$$

$$P_4' = 51.964[MW] \quad (22)$$

Substituting (17) and (20)-(22) in the cost functions of the generators yields for the total cost of operating the system:

$$C_{tot}' = C_1(P_1') + C_2(P_2') + C_3(P_3') + C_4(P_4') = 12498.795\ [£] \quad (23)$$

The system average cost is:

$$a' = \frac{C_{tot}'}{P_1'+P_2'+P_3'+P_4'} = 26.452\ [\frac{£}{MWh}] \quad (24)$$

The profit for each generator Ω_i is given by the difference between its respective revenue and cost:

$$\Omega_1' = \lambda * P_1' - C_1(P_1') = 4344.786\ [£] \quad (25)$$

$$\Omega_2' = \lambda * P_2' - C_2(P_2') = 788.296\ [£] \quad (26)$$

$$\Omega_3' = \lambda * P_3' - C_3(P_3') = -185.065\ [£] \quad (27)$$

$$\Omega_4' = \lambda * P_4' - C_4(P_4') = 444.063\ [£] \quad (28)$$

When the maximum output limits are considered, the output of generator 1 needs to be restricted to a lower value and therefore the outputs of the other three generators are increased. Given that generator 1 is characterised by the lowest variable cost, this new dispatch is less costly (i.e. it increases the total cost of operating the system, the system marginal cost and the average system cost). On the other hand, this new dispatch increases the profit of all the generators. For generators 2-4 this is driven by the increase of both their outputs and the system marginal price, while for generator 1 this is driven by the increase of the system marginal price despite the reduction of its output.

(d) The negative profit of generator 3 reflects the fact that the operating cost of this generator is larger than its revenue and thus it experiences economic losses at market clearing price. This is happening since marginal system cost pricing does not guarantee the recovery of the fixed (no-load) costs of the participating generators in combination with the high fixed costs of generator 3. This lack of recovery fixed costs will mean in the long term that these generators would abandon the business. Therefore, a suitable mechanism compensating the generators' unrecovered fixed costs are implemented which provides additional revenue, on top of the marginal system cost pricing.

Question 2

(a) Explain the difference between markets with perfect and imperfect competition. List the models that can be used for the analysis of markets with imperfect competition.

[6]

(b) Consider a market for electrical energy that is supplied by two generating companies with the two corresponding cost functions:

$$C_A(P_A) = 25 \cdot P_A + 0.40 \cdot P_A^2 \, [\frac{£}{h}]$$

$$C_B(P_B) = 27 \cdot P_B + 0.30 \cdot P_B^2 \, [\frac{£}{h}]$$

The inverse demand curve for this market is estimated to be:

$$\pi = 210 - 1.3 \cdot D \, [\frac{£}{MWh}]$$

Assuming perfect competition, calculate the electricity price, level of demand, production levels and the profits made by the generating companies.

[4]

(c) Assuming a Cournot model of competition:

(i) Form a table to calculate the Nash equilibrium point of this market, i.e. market price, demand quantity and profit of each company for different levels of productions for each of the companies. Consider the level of production of company A at 37MW, 39MW and 41MW and company B at 37MW, 39MW and 41MW.

[8]

(ii) Based on the results, briefly explain the impacts of imperfect competition on the market outcome.

[2]

Solution to Question 2

(a) In markets with perfect competition, no market participant has the ability to influence the market price through its individual actions. In other words, market price is a parameter over which participants have no control. A market operating under perfect competition is characterised by a large number of market participants, small shares of the total production or consumption controlled by each of them and significant price elasticity of demand. Under imperfect competition, some producers and/or consumers -called strategic players- can exert market power and manipulate the prices. Prices can be manipulated either by withholding quantity (physical withholding) or by raising (for sellers) / decreasing (buyers) the asking/offered price (economic withholding). A market operating under imperfect competition is characterised by a small number of market participants, large shares of the total production or consumption controlled by some of them and low price elasticity of demand.

Models used for the analysis of markets with imperfect competition include:

- **Bertrand model**, where the decision variable of each of the competing firms is the price at which it offers the produced commodity
- **Cournot model**, where the decision variable of each of the competing firms is the quantity of the commodity they produce
- **Supply functions equilibria model**, where the decision variables of each of the competing firms are the parameters of its supply function
- **Agent-based simulation models**, representing more complex interactions between the competing firms

(b) Under perfect competition, the price is equal to the marginal cost of each generator. Therefore, the following two equations hold:

$$\frac{\partial C_A(P_A)}{\partial P_A} = \pi \qquad (1)$$

$$\frac{\partial C_B(P_B)}{\partial P_B} = \pi \qquad (2)$$

Since $\pi = 210 - 1.3 \cdot D$ and $D = P_A + P_B$ (demand-supply balance condition), equations (1) and (2) yield respectively:

$$25 + 0.8 \cdot P_A = 210 - 1.3 \cdot (P_A + P_B) \rightarrow P_A = 88.1 - 0.62 \cdot P_B \qquad (3)$$

$$27 + 0.6 \cdot P_B = 210 - 1.3 \cdot (P_A + P_B) \rightarrow P_B = 96.32 - 0.68 \cdot P_A \qquad (4)$$

The combination of (3) and (4) yields:

$$P_A = 49.15\ [MW] \qquad (5)$$

$$P_B = 62.82\ [MW] \qquad (6)$$

The total demand is given by:

$$D = P_A + P_B \rightarrow D = 111.97\ [MW] \qquad (7)$$

The electricity price is given by:

$$\pi = 210 - 1.3 \cdot D \rightarrow \pi = 64.44\ [\frac{£}{MWh}] \quad (8)$$

The profits made by the two generators are given by:

$$\Omega_A = P_A \cdot \pi - 25 \cdot P_A - 0.4 \cdot P_A^2 \rightarrow \Omega_A = 972.19\ [\frac{£}{h}] \quad (9)$$

$$\Omega_B = P_B \cdot \pi - 27 \cdot P_B - 0.3 \cdot P_B^2 \rightarrow \Omega_B = 1168.07[\frac{£}{h}] \quad (10)$$

(c) (i) In the Cournot model of competition the state of the market is determined by the production decisions made by each firm. Possible outcomes should be described by a table using cell in the following format:

D	Ω_A
Ω_B	π

where:

π price $[\frac{£}{MWh}]$

D demand $[MW]$

Ω_A profit made by company A $[\frac{£}{h}]$

Ω_B profit made by company B $[\frac{£}{h}]$

Productions P_A and P_B of the two companies, price and profits are calculated as follows:

$$D = P_A + P_B \quad (11)$$

$$\pi = 210 - 1.3 \cdot D \quad (12)$$

$$\Omega_A = P_A * \pi - 25 \cdot P_A - 0.40 \cdot P_A^2 \quad (13)$$

$$\Omega_B = P_B * \pi - 27 \cdot P_B - 0.30 \cdot P_B^2 \quad (14)$$

Based on the above, the table expressing the Cournot model of competition for the conditions of the problem is in table below.

P_B/P_A	37		39		41	
37	74	2738	76	2753.4	78	2755.2
	2800.9	113.8	2704.7	111.2	2608.5	108.6
39	76	2641.8	78	2652	80	2648.6
	2827.5	111.2	2726.1	108.6	2624.7	106
41	78	2545.6	80	2550.6	82	2542
	2841.3	108.6	2734.7	106	2628.1	103.4

Production of $P_A = 39MW$ and $P_B = 41MW$ corresponds to the equilibrium point.

(ii) We can observe that under imperfect competition the individual generation levels and the total demand are significantly lower and the price and generation profits are significantly higher with respect to the perfect competition case. This is because producers exert market power in order to manipulate the prices and increase their profits beyond the perfect competition levels.

Question 3

(a) List and discuss key characteristics of electricity transmission as a stand-alone business.

[5]

(b) Consider a two area system, with demand in Area A $D_A = 1500\ [MW]$ while demand in Area B is $D_B = 400[MW]$. Generator A is in Area A and generator B is located in Area B, with their respective cost functions given by:

$$C_A(P_A) = 1000 + 10 * P_A + 0.01 * P_A^2\ [\frac{£}{h}]$$

$$C_B(P_B) = 500 + 5 * P_B + 0.005 * P_B^2\ [\frac{£}{h}]$$

Determine the optimal generation dispatch, locational marginal prices, total cost of operating the system, cost of constraints and congestion surplus for:

i) The case where the capacity of the transmission link between the two areas is not binding

ii) The case where no transmission link exists between the two areas

[5]

(c) If the annuitized cost of building the transmission line is given by $C_{inv} = k * L * F$, where F is the capacity of the line, $L = 750km$ (length of the line) and $k = 120\ [£/(MW.km.year)]$, calculate:

i) The transmission demand function

ii) The transmission supply function

iii) The congestion surplus as a function of F

[5]

(d) Determine the optimal capacity that should be built if i) the transmission is a regulated activity and ii) the transmission is operated as a merchant company.

[5]

Solution to Question 3

(a) A brief discussion of the following points is expected:

Transmission is a natural monopoly Because of their visual impact on the environment, construction of competing transmission corridors along similar routes is not feasible. Like all monopolies that provide an essential service, electricity transmission must be regulated to ensure that it delivers an economically optimal combination of quality of service and price.

Transmission is a capital-intensive business The cost of investment in transmission is significantly higher that cost of operating the system. Making efficient investment decisions is thus critical aspect of running a transmission company.

Transmission assets have a long life Most transmission equipment is designed for an expected life of more than 40 years. Generation and demand conditions can change over such a long period. A transmission line that is built on the basis of erroneous forecasts may therefore be used at only a fraction of its rating.

Transmission investments are irreversible Once a transmission line has been built, it cannot be redeployed cost effectively to another location - resale value of installed assets is very low. Owners of transmission networks therefore need to live with the consequences of their investment decisions for a very long time. A large investment that is not used as much as was initially expected is called a *stranded investment.*

Transmission investments are lumpy Manufacturers sell transmission equipment in only a small number of standardized voltage and MVA ratings. It is therefore often not possible to build a transmission facility with rating that exactly matches the need.

Economies of scale Because of high fixed cost component of building transmission infrastructure, the average cost of transmitting electricity generally decreases with the amount transported.

(b) i) When the capacity of the transmission link is not binding, the optimal generation dispatch is calculated as if both generators and both demand are connected to the same bus and is given by the solution of the system of equations:

$$\frac{\partial C_A(P_A^{unc})}{\partial P_A} = \frac{\partial C_B(P_B^{unc})}{\partial P_B} \rightarrow 10 + 0.02 * P_A^{unc} = 5 + 0.01 * P_B^{unc} \qquad (1)$$

$$P_A^{unc} + P_B^{unc} = D_A + D_B \rightarrow P_A^{unc} + P_B^{unc} = 1900 \qquad (2)$$

which yields:

$$P_A^{unc} = 466.667\ [MW] \qquad (3)$$

$$P_B^{unc} = 1433.333\ [MW] \qquad (4)$$

Since $P_A^{unc} < D_A$ and $P_B^{unc} > D_B$, the flow on the transmission link is from area B to area A and is equal to:

$$F^{unc} = D_A - P_A^{unc} = P_B^{unc} - D_B = 1033.333\ [MW] \qquad (5)$$

The locational marginal price is the same for the two areas and is given by:

$$\pi = 10 + 0.02 * P_A^{unc} = 5 + 0.01 * P_B^{unc} = 19.333\ [\frac{£}{MWh}] \tag{6}$$

The total cost of operating the system is calculated by setting the values determined above in the cost functions of the two generators:

$$C_{tot}^{unc} = C_A(P_A^{unc}) + C_B(P_B^{unc}) = 25783.333\ [£] \tag{7}$$

Since the capacity of the transmission link is not binding, the cost of constraints is zero. The congestion surplus is also zero, since the locational marginal price is the same for the two areas.

ii) When no transmission link exists between the two areas, generator A will satisfy demand in area A and generator B will satisfy demand in area B, meaning:

$$P_A^{no} = D_A = 1500\ [MW] \tag{8}$$

$$P_B^{no} = D_B = 400\ [MW] \tag{9}$$

The locational marginal prices in the two areas are different. An additional unit of demand in Area A will be satisfied by Generator A and thus the locational marginal price at area A is:

$$\pi_A = \frac{\partial C_A(P_A^{no})}{\partial P_A} = 40\ [\frac{£}{MWh}] \tag{10}$$

An additional unit of demand in Area B will be satisfied by Generator B and thus the locational marginal price at area B is:

$$\pi_B = \frac{\partial C_B(P_B^{no})}{\partial P_B} = 9\ [\frac{£}{MWh}] \tag{11}$$

The total cost of operating the system is calculated by setting the values determined by above in the cost functions of the two generators:

$$C_{tot}^{no} = C_A(P_A^{no}) + C_B(P_B^{no}) = 41800\ [£] \tag{12}$$

The cost of constraints is equal to the difference between the total cost of operating the system in this case and the respective cost in the case where the capacity of the transmission link is not binding:

$$CC^{no} = C_{tot}^{no} - C_{tot}^{unc} = 16016.67\ [£] \tag{13}$$

The congestion surplus is zero since the capacity of the transmission link connecting the two areas is also zero.

(c) i) The demand function for transmission is calculated as the price differential between the two areas as a function of the line capacity F:

$$\Pi_D(F) = \pi_A(F) - \pi_B(F) \tag{14}$$

The price at each area depends on the output of the respective generator:

$$\pi_A(F) = (10 + 0.02 * P_A^F) \tag{15}$$

$$\pi_B(F) = (5 + 0.01 * P_B^F) \quad (16)$$

The generators' outputs depend on F:

$$P_A^F = D_A - F \quad (17)$$

$$P_B^F = D_B + F \quad (18)$$

By substituting (15)-(18) in (14) we get:

$$\Pi_D(F) = 31 - 0.03 * F \quad (19)$$

ii) The supply function for transmission is given by the marginal cost of building the transmission line, expressed in an hourly basis:

$$\Pi_S(F) = \frac{k * L}{8760h} = 10.274 \left[\frac{£}{MWh}\right] \quad (20)$$

iii) The congestion surplus is given by:

$$CS(F) = \left(\pi_A(F) - \pi_B(F)\right) * F \quad (21)$$

By substituting (19) into (21) we get:

$$CS(F) = 31 * F - 0.03 * F^2 \quad (22)$$

(d) i) A regulated transmission company seeks to maximise the social welfare, by building a line that will exactly finance itself, meaning that the annual revenues from the congestion are equal to the annual investment. Therefore, the optimal capacity that should be build would satisfy the equality between the demand and the supply function for transmission:

$$\Pi_D(F) = \Pi_S(F) \to 31 - 0.03 * F = 10.274 \to F = 690.868\ [MW] \quad (23)$$

ii) A merchant transmission company seeks to maximize its profits. Since its revenues will be given by the congestion surplus -which is equal to the product of the volume of energy transported and the price differential between the two areas- it would have an incentive to under-invest in order to keep the price differential in a high level. Therefore, the optimal capacity that it would build maximizes its profits:

$$\max_F(REVENUE(F) - COST(F))$$

Its revenue is equal to the congestion surplus which is given by:

$$REVENUE(F) = CS(F) = 31 * F - 0.03 * F^2$$

Its cost is given by the cost of reinforcing the transmission line, expressed in an hourly basis, which is:

$$COST(F) = 10.274 * F$$

The above yield:

$$\max_F(31 * F - 0.03 * F^2 - 10.274 * F) \to F = 345.434\ [MW] \quad (24)$$

Question 4

(a) Explain the significance of option contracts, the types of option contracts and the meaning of exercise price and option fee.

[6]

(b) For the system shown in Figure Q4, and the data given in Tables Q4-1 and Q4-2, determined the unconstrained economic dispatch (ignoring network capacity constraints), power flows in each line as well as the resulting nodal prices.

[4]

(c) Demonstrate that line 1-3 would be overloaded if the capacity limits of the lines are considered and the dispatch calculated in (b) is implemented. Show how this overload can be eliminated by:

i) increasing the output of generator A.

ii) increasing the output of generator B.

Calculate the hourly cost of options i) and ii) above. Which of the two options is preferable and why?

[5]

(d) Calculate the nodal prices when the more preferable of the two options compared in (c) is implemented.

[5]

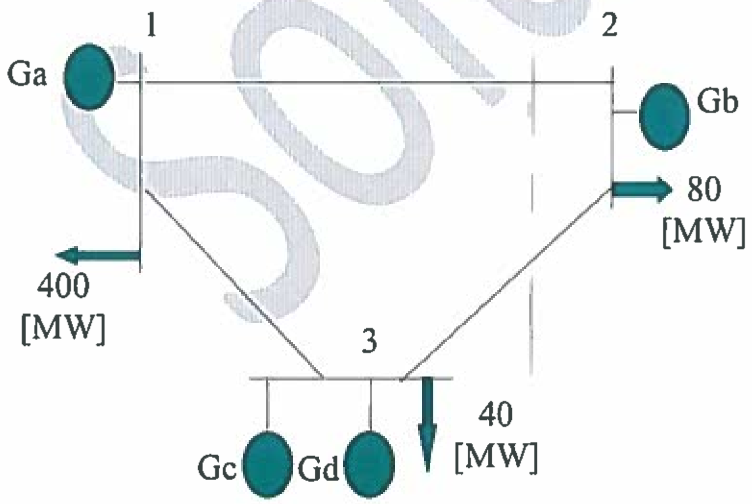


Figure Q4: System layout

Table Q4-1 Generator data

Generator	Capacity (MW)	Marginal cost (£/MWh)
Ga	200	15
Gb	150	12
Gc	150	10
Gd	400	8

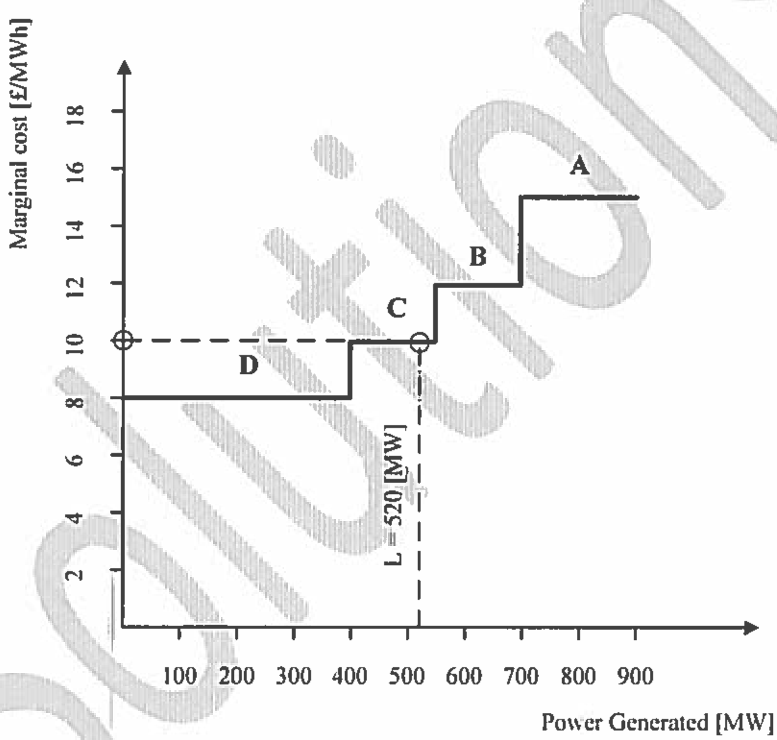
Table Q4-2 Line data

Line	Per unit reactance	Capacity (MW)
1-2	0.2	250
1-3	0.3	250
2-3	0.3	250

Solution to Question 4

(a) In some cases, participants may prefer contracts with a conditional delivery, which means contracts that are exercised only if the holder of the contract decides that it is in its interest to do so. Such contracts are called *options* and come in two varieties: *calls* and *puts*. A call option gives its holder the right to buy a given amount of a commodity at a price called the *exercise price*. A put option gives its holder the right to sell a given amount of a commodity at the exercise price. Whether the holder of an option decides to exercise its rights under the contract depends on the spot price for the commodity. When an option contract is agreed, the seller of the option receives a non-refundable *option fee* from the holder of the option. The buyer of the option on the other hand gets a guarantee that they will be able to buy/sell a commodity for at least the option exercise price, which can be used for risk management.

(b) When transmission constraints are ignored, the output of all the generators can be stacked in order of marginal cost as shown in the following figure:



Using the figure, we can see that for a system load of $L = 400 + 40 + 80 = 520MW$ the marginal cost (and hence the nodal price) is:

$$\pi = 10\ [\frac{£}{MWh}] \quad (1)$$

Furthermore, the units are dispatched as follows:

$$P_D = 400\ [MW] \quad (2)$$

$$P_C = 120\ [MW] \quad (3)$$

$$P_A = P_B = 0[MW] \quad (4)$$

The line flow problem can be solved using the superposition principle and also directly. To this effect, we write the power balance equation at two buses and KVL around the loop.

Bus 1: $P_A - 400 = F_{12} + F_{13}$ (5)

Bus 2: $P_B - 80 = -F_{12} + F_{23}$ (6)

Bus 3: $P_C + P_D - 40 = -F_{13} - F_{23}$ (7)

Loop equation: $0.2F_{12} + 0.3F_{23} - 0.3F_{13} = 0$ (8)

Putting these equations in the matrix form gives:

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} F_{12} \\ F_{13} \\ F_{23} \end{bmatrix} = \begin{bmatrix} P_B - 80 \\ P_C + P_D - 40 \\ 0 \end{bmatrix} \quad (9)$$

Substituting $P_A = 0\ [MW]$, $P_B = 0[MW]$, $P_C = 120[MW]$ and $P_D = 400[MW]$ in (9), we get:

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} F_{12} \\ F_{13} \\ F_{23} \end{bmatrix} = \begin{bmatrix} -80 \\ 480 \\ 0 \end{bmatrix} \quad (10)$$

Solving these equations, we get:

$F_{12} = -120\ [MW]$ (11)

$F_{13} = -280\ [MW]$ (12)

$F_{23} = -200[MW]$ (13)

(c) From part b), the flow on line 1-3 exceeds its maximum capacity by $30[MW]$.

i) The first approach requires increase in the output of generator A and corresponding decrease of the output of generator C. To calculate how big this increase should be to remove the violation of the flow limit on line 3-1, marginal impact ($1[MW]$) is considered - this pair of injection causes changes in flows as follows:

$\frac{0.3}{(0.2+0.3)+0.3} \cdot 1 = 0.375[MW]$ along the path 1-2-3 (14)

$\frac{(0.2+0.3)}{(0.2+0.3)+0.3} \cdot 1 = 0.625[MW]$ along the path 1-3 (15)

Since linear (dc) power flow model is used, in order to reduce $30[MW]$ in line 3-1, output of generator A should be increased by:

$\frac{30}{0.625} = 48\ [MW]$ (16)

The constrained dispatch is then:

$P_A = 48\ [MW]$ (17)

$$P_B = 0\ [MW] \quad (18)$$

$$P_C = 72[MW] \quad (19)$$

$$P_D = 400[MW] \quad (20)$$

Using the nodal and loop equations, the flows are calculated solving the following linear system:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} F_{12} \\ F_{13} \\ F_{23} \end{bmatrix} = \begin{bmatrix} P_A - 400 \\ P_C + P_D - 40 \\ 0 \end{bmatrix} \quad (21)$$

Hence:

$$F_{12} = -102\ [MW] \quad (22)$$

$$F_{13} = -250\ [MW] \quad (23)$$

$$F_{23} = -182[MW] \quad (24)$$

This dispatch does not cause a violation of the line flow constrains in any other line.

The cost of this dispatch is:

$$C_{Total} = 48 \cdot 15 + 72 \cdot 10 + 400 \cdot 8 = 4640[£] \quad (25)$$

ii) The other method to remove the constraint violation consists in increasing the output of generator B and decreasing the output of generator C by the same amount. In this case:

$$\frac{0.3}{(0.2+0.3)+0.3} \cdot 1 = 0.375[MW] \quad \text{along the path 2-1-3} \quad (26)$$

$$\frac{(0.2+0.3)}{(0.2+0.3)+0.3} \cdot 1 = 0.625[MW] \quad \text{along the path 2-3} \quad (27)$$

Since linear (dc) power flow model is used, in order to reduce $30[MW]$ in line 3-1, output of generator B should be increased by:

$$\frac{30}{0.375} = 80\ [MW] \quad (28)$$

The constrained dispatch is then:

$$P_A = 0\ [MW] \quad (29)$$

$$P_B = 80\ [MW] \quad (30)$$

$$P_C = 40[MW] \quad (31)$$

$$P_D = 400[MW] \quad (32)$$

Using the nodal and loop equations, the flows are calculated solving the following linear system:

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} F_{12} \\ F_{13} \\ F_{23} \end{bmatrix} = \begin{bmatrix} P_B - 80 \\ P_C + P_D - 40 \\ 0 \end{bmatrix} \tag{33}$$

Hence:

$$F_{12} = -150\ [MW] \tag{34}$$

$$F_{13} = -250\ [MW] \tag{35}$$

$$F_{23} = -150[MW] \tag{36}$$

This re-dispatch does not cause a violation of the line flow constrains on any other line. The cost of this constrained dispatch is:

$$C_{Total} = 80 \cdot 12 + 40 \cdot 10 + 400 \cdot 8 = 4560[£] \tag{37}$$

Even though it re-dispatches a larger amount of MW, the second constrained dispatch is preferable to the first one because its cost is lower.

(d) The nodal price at each bus is given by the cost of one additional MW of load at each node. Therefore, the price at bus 3 is $10\left[\frac{£}{MWh}\right]$ because the next MW of load would be generated locally by generator C because it is the cheapest generator not operating at its limit. An additional MW of load at node 2 would have to be produced by generator B. Producing it with generator C would cause a violation of the line flow constraint on line 3-1. Producing it with generator A would be more expensive than with generator B. The price at node 2 is therefore $12\left[\frac{£}{MWh}\right]$. An additional MW of load at bus 1 requires a redispatch of generators B and C to minimize the cost increase while maintaining the flow on line 3-1 within limits.

Extracting an additional $1[MW]$ at bus 1 and generating it at bus 3 causes the following change in the flow on line 1-3:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} \Delta F_{12} \\ \Delta F_{13} \\ \Delta F_{23} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \Delta F_{31} = 0.625[MW] \tag{38}$$

Similarly, extracting an additional $1[MW]$ at bus 1 and generating it at bus 2 causes the following change in the flow on line 1-3:

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} \Delta F_{12} \\ \Delta F_{13} \\ \Delta F_{23} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \Delta F_{31} = 0.250[MW] \tag{39}$$

Therefore, as the flow on line 3-1 remains unchanged (because it is already at its limit), productions by generators C and B would be determined by:

$$\Delta F_{31} = 0 = 0.625 \cdot \Delta P_C + 0.250 \cdot \Delta P_B \tag{40}$$

At the same time, increasing the load by $1[MW]$ would require:

$$\Delta P_B + \Delta P_C = 1 \tag{41}$$

Solving the system consisting of the previous two equations:

$$\Delta P_C = -0.667[MW] \tag{42}$$

$$\Delta P_B = 1.667[MW] \tag{43}$$

To supply an additional MW of load at bus 1 without violating the network constraints, will require increase of output of generator B and decrease of output of generator C.

The nodal price at bus 1 is thus given by a linear combination of the marginal cost of production of these two generators.

$$\pi_1 = -0.667 \cdot 10 + 1.667 \cdot 12 = 13.33[\frac{£}{MWh}] \tag{44}$$

Finally:

$$\pi_1 = 13.33[\frac{£}{MWh}] \tag{45}$$

$$\pi_2 = 12[\frac{£}{MWh}] \tag{46}$$

$$\pi_3 = 10[\frac{£}{MWh}] \tag{47}$$