IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2009

EEE/ISE PART II: MEng, BEng and ACGI

Corrected Copy

CONTROL ENGINEERING

Friday, 29 May 2:00 pm

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : I.M. Jaimoukha, I.M. Jaimoukha
Second Marker(s) : S. Evangelou, S. Evangelou

CONTROL ENGINEERING

1. a) Figure 1.1 illustrates an RLC circuit. The capacitor has capacitance C, the inductor has inductance L and the resistor resistance R. The input is the applied voltage $v_i(t)$ and the output is the voltage across the capacitor $v_o(t)$.

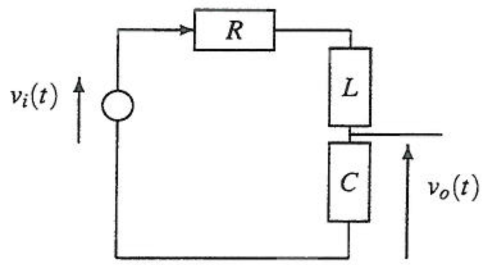


Figure 1.1

i) Determine the transfer function relating v_o to v_i. [4]

ii) Let $v_i(t)$ be a unit step applied at $t = 0$. Use the final value theorem, which should be stated, to find the steady–state value of $v_o(t)$. [5]

iii) Set $L = 0.25\ H$ and $C = 4 \times 10^{-6}\ F$. Derive the value of R so that the response is critically damped. [5]

b) In Figure 1.2 below, $G(s) = \dfrac{s-1}{(s+1)(s+2)}$ and K is a variable gain.

i) Sketch the locus of the closed–loop poles for $0 \leq K < \infty$. [5]

ii) Using the gain criterion, find the value of K for which the closed–loop is marginally stable. [4]

iii) Hence find the range of $K \geq 0$ for which the closed–loop is stable. [4]

c) In Figure 1.2 below, $G(s) = \dfrac{5}{(s+1)^3}$ and K is a variable gain.

i) Draw the Nyquist diagram of $G(s)$ indicating real–axis intercepts. [5]

ii) Take $K = 2$. Use the Nyquist criterion, which should be stated, to determine the number of unstable closed–loop poles. [4]

iii) Take $K = 1$. Determine the gain margin. [4]

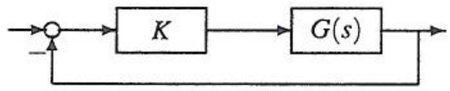


Figure 1.2

2. Consider the feedback system in Figure 2 for voltage regulation. Here, $v_r(t)$ is the reference voltage and $v_o(t)$ is the supplied output voltage. R is a resistance value which is fixed but is unknown.

The op–amp open–loop output voltage E is related to v_e as $E(s) = -G(s)v_e(s)$, where the transfer function $G(s)$:

- is first order,
- has a DC gain A,
- has a time constant of 1 second.

a) Derive an expression for $G(s)$ in terms of A. [6]

b) Derive an expression for $v_e(s)$ in terms of $v_r(s)$ and $v_o(s)$. [6]

c) Derive an expression for $v_o(s)$ in terms of $v_e(s)$. [6]

d) Hence, derive and draw a block diagram representation of the feedback loop. Take the reference to be $-v_r(s)$ and the output to be $v_o(s)$. Indicate the signal $v_e(s)$ on the block diagram. [6]

e) Find the minimum value of the DC gain A such that the closed–loop system has a time constant of 10^{-3} seconds. [6]

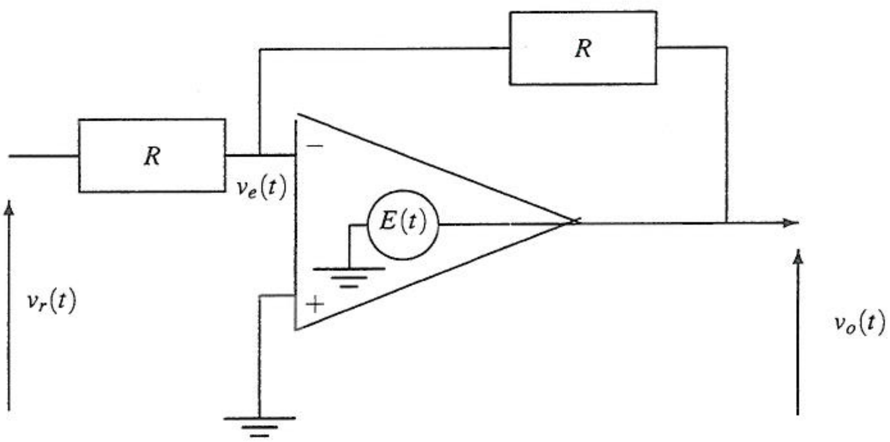


Figure 2

3. Let

$$G(s) = \frac{1}{s(s-1)}$$

and consider the feedback loop shown in Figure 3 below.

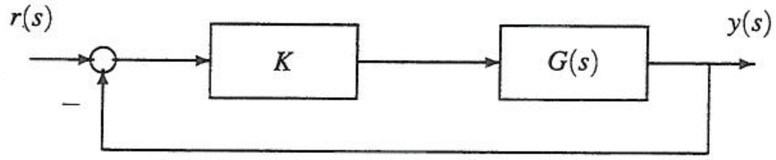


Figure 3

a) Draw the root locus of $G(s)$ accurately for all $K > 0$. [4]

b) Draw the root locus of $G(s)$ accurately for all $K < 0$. [3]

c) Using the answers to Parts (a) and (b), or otherwise, show that there exists no proportional compensator such that the feedback loop is stable. [3]

d) A feedback compensator utilizing rate feedback is required such that the following design specifications are satisfied:

- The closed–loop is stable.
- The closed–loop system has a damping ration $\zeta = 1/\sqrt{2}$.
- The closed–loop response has a settling time of 2 seconds.

  i) Derive the location of the closed–loop poles that satisfy the design specifications. [5]

  ii) Draw a feedback loop incorporating the rate feedback compensator. The compensator should have two design parameters K and K_v which should be clearly shown on the diagram. [5]

  iii) Derive the values of the parameters K_v and K that achieve the design specifications. [5]

  iv) Draw the root locus of the compensated system. [5]

4. Consider the feedback control system in Figure 4 below. Here,

$$G(s) = \frac{2(s+1)}{(s-1)^2}$$

and $K(s)$ is the transfer function of a compensator.

a) Sketch the Nyquist diagram of $G(s)$, clearly indicating the low and high frequency portions, as well as the real–axis intercepts. [6]

b) Let $K(s)$ be a constant compensator $K(s) = K$, where $K > 0$. State the Nyquist stability criterion and use the Nyquist diagram to determine the stability of the close–loop system when:

i) $K > 1$, [4]

ii) $K < 1$, [4]

iii) $K = 1$. [4]

c) Take $K = 1$. Comment on the gain margin for the feedback loop. [6]

d) Without doing any actual design, briefly describe how a phase–lag compensator,

$$K(s) = \frac{1+s/\omega_0}{1+s/\omega_p}, \qquad 0 < \omega_p < \omega_0$$

would affect the stability of the feedback loop. [6]

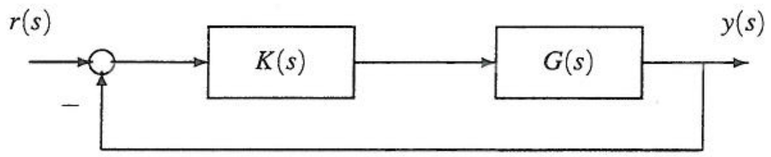


Figure 4

SOLUTIONS Control Engineering 2009

1. a) i) Using the potential divider rule and the impedances we have

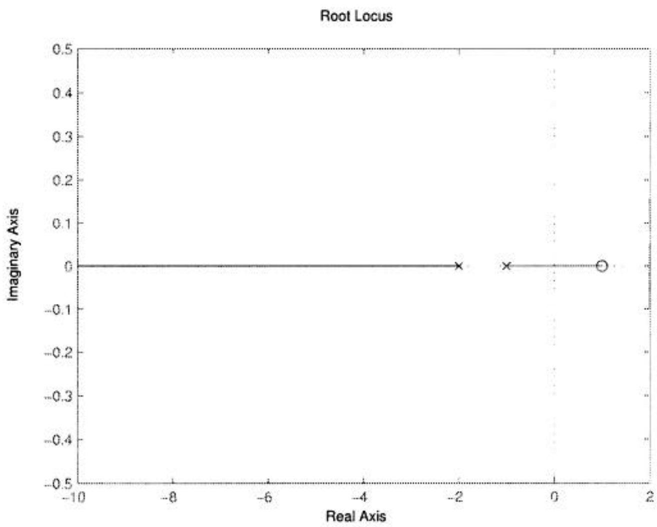
$$G(s) := \frac{v_o(s)}{v_i(s)} = \frac{(LC)^{-1}}{s^2 + sRL^{-1} + (LC)^{-1}}.$$

ii) Using the final value theorem and the fact that $v_i(s) = 1/s$,

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} s v_o(s) = \lim_{s\to 0} sG(s)v_i(s) = \lim_{s\to 0} sG(s)\frac{1}{s} = G(0) = 1.$$

iii) For a critically damped response, the poles are equal and so $R^2 = 4L/C$. Thus $R = 2\sqrt{L/C} = 500\ \Omega$.

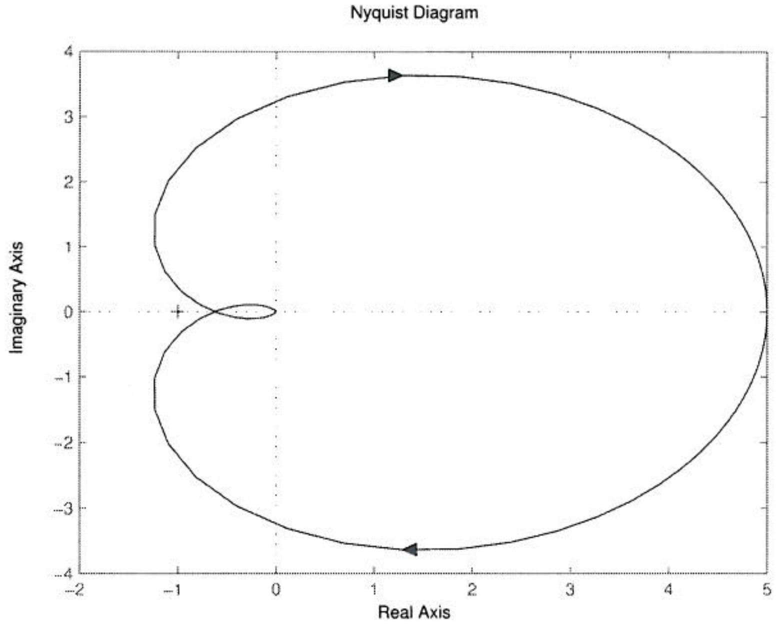
b) i) The root locus is shown below.



ii) The closed-loop is marginally stable when at least one pole is on the imaginary axis and all others are in the left half-plane. It follows from the root locus that the marginal pole is at $s = 0$. Using the gain criterion $K = -1/G(0) = 2$.

iii) It follows from the root locus that the closed-loop is stable for all $0 \le K < 2$.

c) i) The Nyquist diagram is shown below. To find the real-axis intercepts,



we consider the extra gain K necessary for marginal stability for the characteristic equation $1+G(s)K=0$:

$$\text{The Routh array :} \quad \begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 3 & 1+5K \\ s & \frac{8-5K}{3} & \\ 1 & 1+5k & \end{array}$$

Thus $K = 8/5$ and so the intercept is at $-5/8 = -0.625$.

ii) The Nyquist criterion states that $N = Z - P$ where N is the number of clockwise encirclements by $G(s)$ of the point $-1/K$ as s traverses the Nyquist contour, which in this case is equal to 2 ; P is the number of unstable open-loop poles, which in this case is 0; and Z is the number of closed-loop poles. Thus there are $Z = N + P = 2$ unstable closed-loop poles.

iii) When $K = 1$, the Nyquist criterion indicates the closed-loop is stable. The gain margin is the amount of gain that the loop can tolerate before becoming marginally stable and in this case is $1/0.625 = 1.6$.

2. a) A first order transfer function $G(s)$ with DC gain A and time constant 1 second has the form

$$G(s) = \frac{A}{s+1}$$

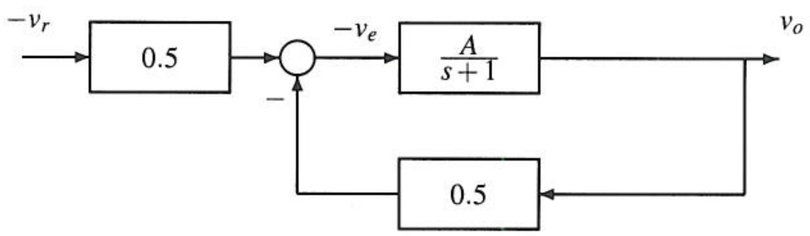
b) Using a potential divider rule at the op–amp input gives

$$\frac{v_e(s) - v_r(s)}{v_o(s) - v_r(s)} = \frac{R}{R+R} = \frac{1}{2} \Rightarrow -v_e(s) = -0.5v_r(s) - 0.5v_o(s).$$

c) At the op–amp output we have

$$E(s) = v_o(s) \Rightarrow v_o(s) = -\frac{A}{s+1}v_e(s).$$

d) Using parts (a) and (b), the block diagram becomes,



e) Using the block diagram in part (d) and a manipulation gives

$$v_o(s) = -\frac{0.5A}{s+1+0.5A}v_r(s).$$

The time constant is now

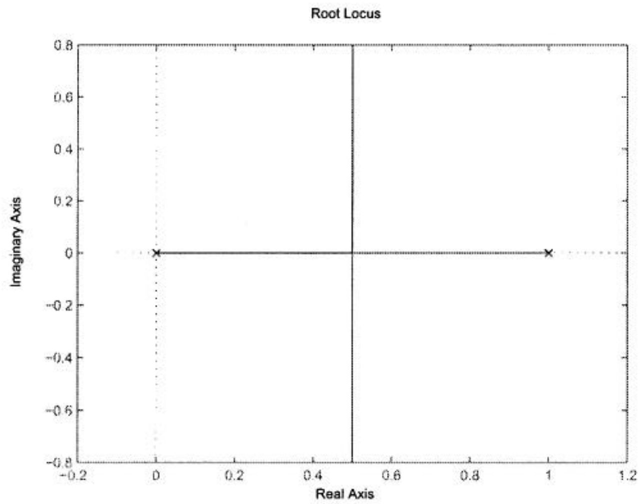
$$T = \frac{1}{1+0.5A}.$$

So, for a time constant $T \leq 10^{-3}$ we need

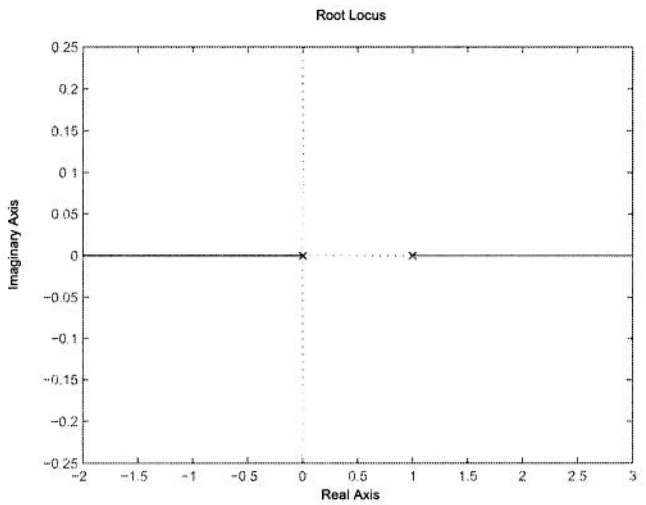
$$\frac{1}{1+0.5A} \leq 10^{-3}$$

or $A \geq 2 \times 10^3$.

3. a) The root locus is shown below.



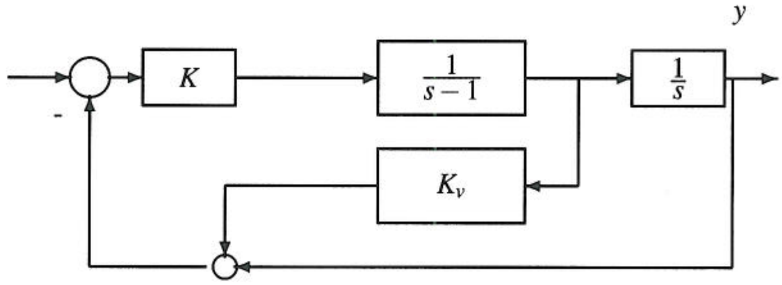
b) The root locus is shown below.



c) It is clear from the root loci above that there is at least one closed-loop pole in the right half plane and so the closed loop is always unstable.

d) i) For $\zeta = 1/\sqrt{2}$ the real and imaginary parts of the pole are equal. For a settling time of 2 seconds, the real part must be equal to -2. Thus the closed-loop poles must be placed at $s_1, \bar{s}_1 = -2 \pm j2$.

ii) The block diagram is shown below.



iii) The characteristic equation is $1 + KK_v \dfrac{s + 1/K_v}{s(s-1)} = 0$. The location of the zero $z = -1/K_v$ can be determined from the angle criterion:

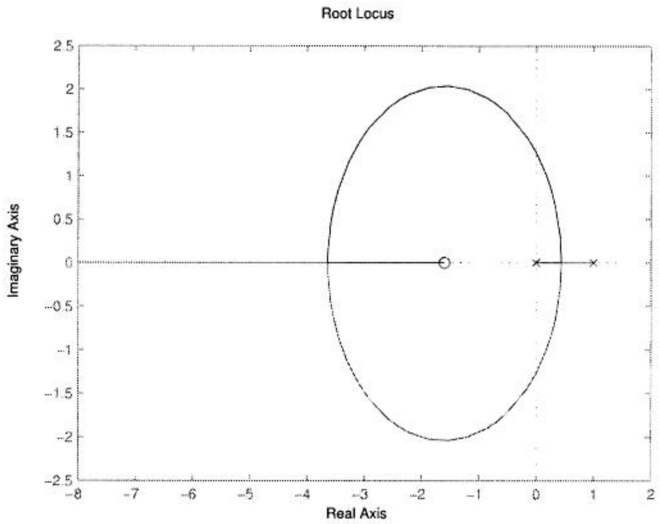
$$\theta = 116^\circ + 146.3^\circ - 180^\circ \sim 101.3^\circ$$

which is satisfied by $z = -1.6$. So, $K_v = 0.625$. Finally, K is obtained from the gain criterion:

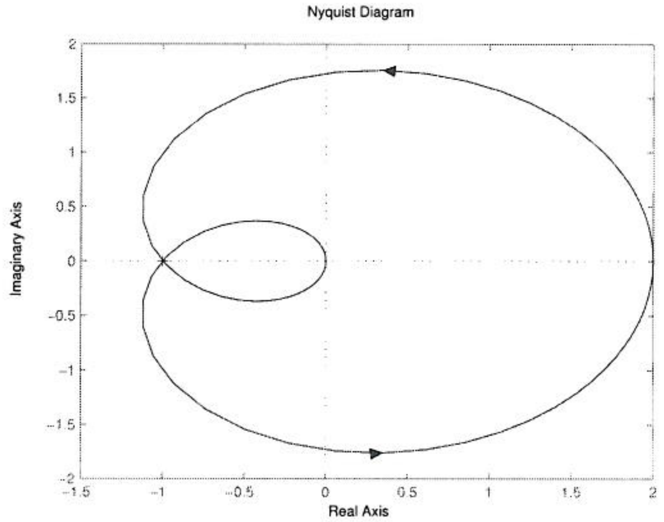
$$KK_v = -s_1(s_1 - 1)/(s_1 + 1.6) = 5 \;\Rightarrow\; K = 8$$

where $s_1 = -2 + j2$.

iv) The root locus of the compensated system is shown below.



4. a) The Nyquist diagram is shown below. The real-axis intercepts can be found by setting the imaginary part of $G(j\omega)$ to zero. This gives $G(j\omega_i) = 2, -1, 0$, respectively.



b) When $K(s) = K$, we have $N = Z - P$ where N is the number of clockwise encirclement by the Nyquist diagram of the point $-K^{-1}$, P is the number of unstable open-loop poles and Z is the number of unstable closed-loop poles. Here, $P = 2$.

i) When $K > 1$, $N = -2$ so $Z = 0$ and the closed-loop is stable.

ii) When $K < 1$, $N = 0$ so $Z = 2$ and the closed-loop is unstable.

iii) It follows that when $K = 1$, the closed-loop is marginally stable.

c) Since the gain can be increased without bound the system has infinite gain margin for increasing gain. Since any decrease in gain will result in an unstable closed-loop, the system has zero gain margin for decreasing gain.

d) The phase-lag compensator has gain close to unity for frequencies below ω_p and gain close to $\frac{\omega_p}{\omega_0} < 1$ for frequencies beyond ω_0. It follows that the compensator will destabilize the feedback loop since the open-loop is marginally stable and any reduction in gain at high frequencies will make the closed-loop system unstable.