EE1-06 Kin K. Leung SOLUTIONS

2016 Intro. to Signals & Communications Exam

1. a. i. $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

$$= \int_{-b}^{b} a e^{-j\omega t} dt$$

$$= a \int_{-b}^{b} e^{-j\omega t} dt$$

$$= a \cdot \frac{1}{-j\omega} \cdot \frac{e^{-j\omega b} - e^{j\omega b}}{2}$$

$$= a \cdot \frac{1}{-j\omega} \cdot \frac{-2j \sin(\omega b)}{2}$$

$$\Rightarrow F(\omega) = \frac{a \sin(\omega b)}{\omega}.$$

$$F(\omega) = ab \cdot \text{Sinc}(\omega b)$$

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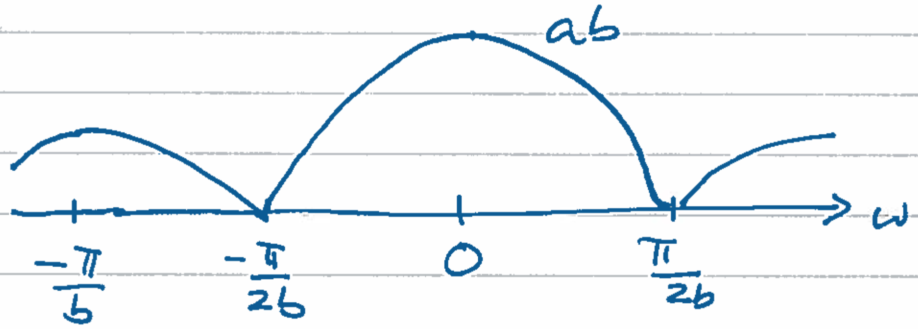
ii)



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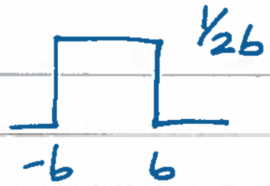
phase $\angle F(\omega) = 0$ because $F(\omega)$ is real

1\. a. iii.

$$f(t) = \lim_{b \to 0}$$ 

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$$\Rightarrow f(t) = \delta(t)$$ $\because$ the area is unit and the width tends to zero

In this special case

$$F(\omega) = \mathcal{F}[\delta(t)] = 1.$$

iv. $$\mathcal{F}[f(t-t_0)]$$

$$= \int_{-\infty}^{\infty} f(t-t_0)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t-t_0)\, e^{-j\omega(t-t_0)} \cdot e^{-j\omega t_0}\, dt$$

$$= e^{-j\omega t_0} \int_{-\infty}^{\infty} f(t-t_0)\, e^{-j\omega(t-t_0)}\, dt$$

$$= e^{-j\omega t_0} \cdot F(\omega)$$

$$\Rightarrow \hat{F}(\omega) = e^{-j\omega t_0} F(\omega)$$

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1. b. i. $\omega_0 = \frac{2\pi}{T_0}$ 1

ii. a_0 is the dc component of the signal $g(t)$. 1

iii. a_n reflects the degree of how $g(t)$ is similar to $\cos(n\omega_0 t)$

b_n : similarity w/ $\sin(n\omega_0 t)$ 2

iv. If $g(t)$ is odd function of t,

$$g(-t) = -g(t)$$

So,

$$g(-t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(-n\omega_0 t)$$

$$= a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) - \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

Since $g(-t) = -g(t)$

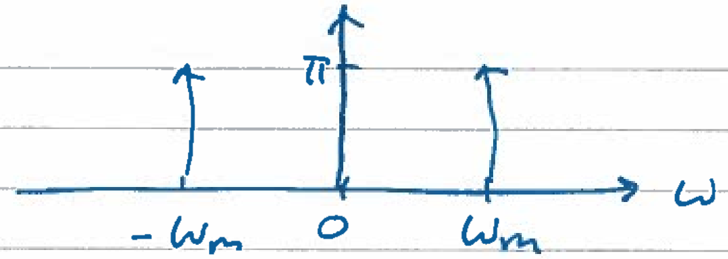
$$\Rightarrow a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) - \cancel{\sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)}$$

$$= -a_0 - \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) - \cancel{\sum_{n=1}^{\infty} b_n \sin(n\omega_0}$$

$$\Rightarrow a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) = 0$$ 3

$$\Rightarrow a_0 = 0, \; a_n = 0 \; \forall \text{ all } n = 1, 2, \dots \infty$$ 3

1. c. i.



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ii. $\phi(t) = m(t)\cos(\omega_c t)$

$= \cos(\omega_m t) \cdot \cos(\omega_c t)$

$= \frac{1}{2}\left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t\right]$

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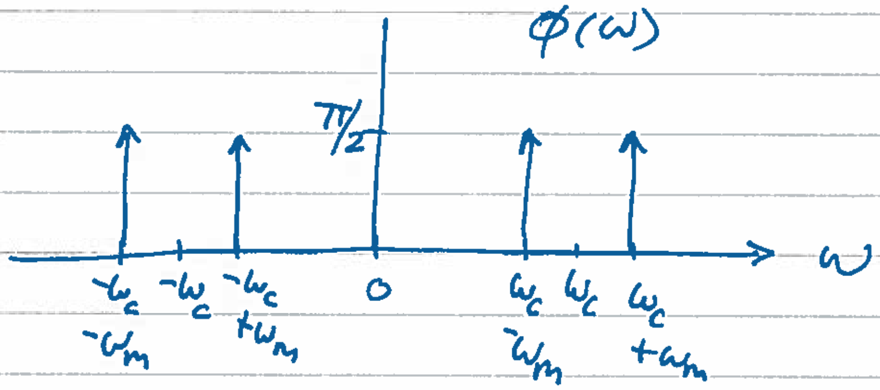
iii.



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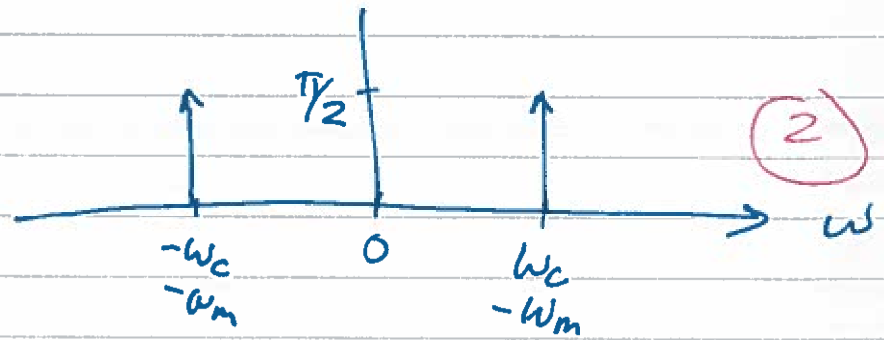
iv. The AM effectively shifts the spectrum of $m(t)$ to the frequency range surrounding the carrier frequency $\pm \omega_c$. 1

v.

$\varphi_{LSB}(t) = \frac{1}{2}\cos(\omega_c - \omega_m)t$ 2

vi.



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1\. d. i. $\phi(t) = A\cos\left(\omega_c t + k_f \int m(\alpha)\,d\alpha\right)$ 2

ii. $\phi'(t) = d\phi(t)/dt$

$= A\left[\omega_c + k_f m(t)\right] \cdot$ 2

$\sin\left[\omega_c t + k_f \int m(\alpha)\,d\alpha\right]$

iii.

$$\phi(t) \longrightarrow \boxed{d/dt} \xrightarrow{\phi'(t)} \boxed{\text{Envelop detector}} \xrightarrow{A(\omega_c + k_f m(t))} \boxed{\text{dc block}} \longrightarrow m(t)$$

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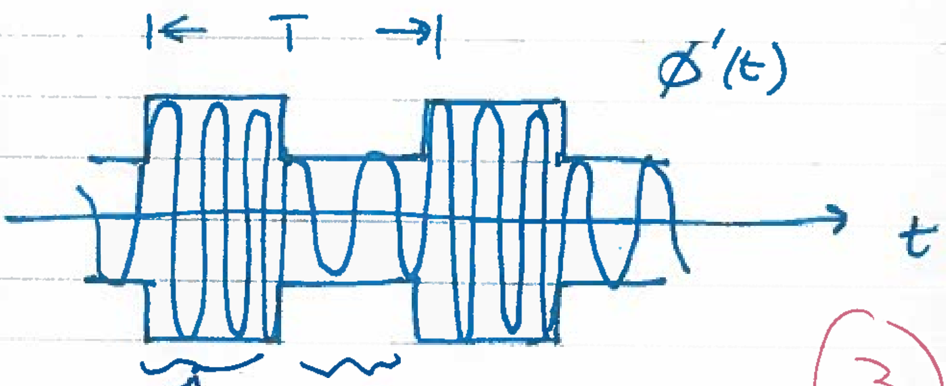
iv.



$W_i = W_c + k_f$ $W_i = W_c - k_f$

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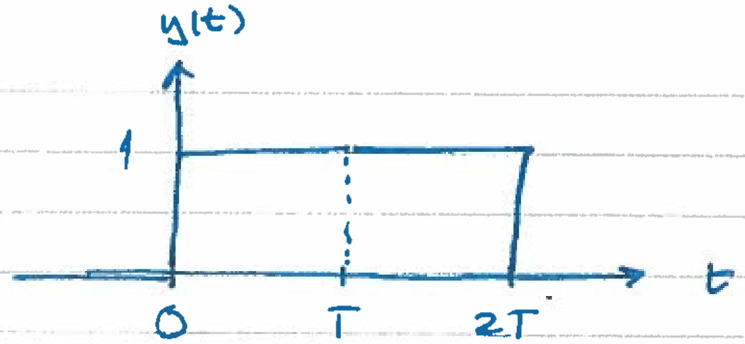
2\. a. i.



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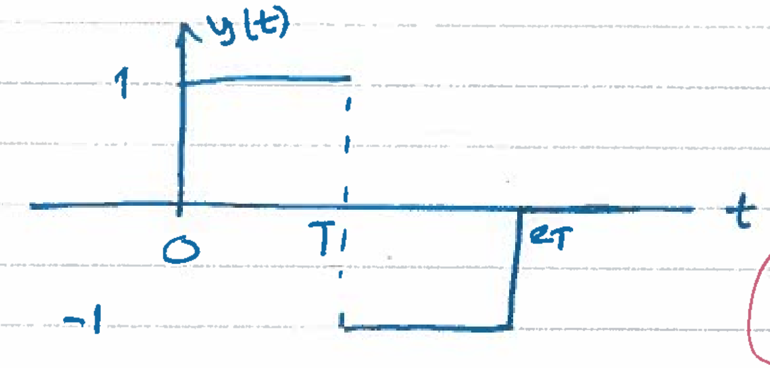
ii.



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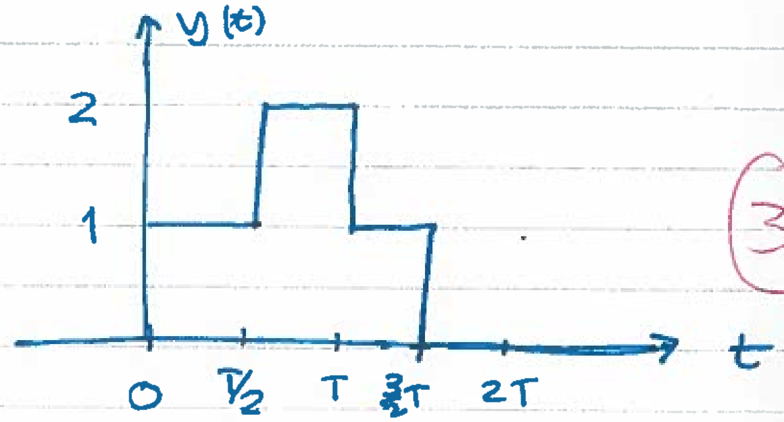
iii.



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iv. maximum signal rate :

$$f = 1/T$$

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It is so because each impulse (signal) has a response time (duration) of T time units (sec). To avoid use of complicated technique for receiving, keep the signal rate of $1/T$ samples per sec will be supported by the channel.

2\. b. i. $f(t) * g(t)$

$$= \int_{u=-\infty}^{\infty} f(u) g(t-u) \, du$$

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ii. $\mathcal{F}[f(t) * g(t)]$

$$= \int_{t=-\infty}^{\infty} \int_{u=-\infty}^{\infty} f(u) g(t-u) e^{-j\omega t} \, du \, dt$$

$$= \int_{u=-\infty}^{\infty} \int_{t=-\infty}^{\infty} f(u) g(t-u) e^{-j\omega t} \, dt \, du$$

$$= \int_{u=-\infty}^{\infty} \int_{t=-\infty}^{\infty} f(u) g(t-u) e^{-j\omega (t-u)} \cdot e^{-j\omega u} \, dt \, du$$

$$= \int_{u=-\infty}^{\infty} f(u) e^{-j\omega u} \cdot \int_{t=-\infty}^{\infty} g(t-u) e^{-j\omega (t-u)} \, dt \, du$$

$$= \int_{u=-\infty}^{\infty} f(u) e^{-j\omega u} \cdot \int_{t'=-\infty}^{\infty} g(t') e^{-j\omega t'} \, dt' \, du$$

where $t' = t - u$

$$= \int_{u=-\infty}^{\infty} f(u) e^{-j\omega u} G(\omega) \, du$$

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$$= G(\omega) \cdot F(\omega).$$

2.b. iii. $F(\omega) * G(\omega)$

$$= \int_{u=-\infty}^{\infty} F(u)\, G(\omega - u)\, du$$

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iv. $\mathcal{F}[f(t)\, g(t)]$

$$= \int_{t=-\infty}^{\infty} f(t)\, g(t)\, e^{-j\omega t}\, dt$$

$$= \int_{t=-\infty}^{\infty} \frac{1}{2\pi} \int_{u=-\infty}^{\infty} F(u)\, e^{jut}\, du\, g(t)\, e^{-j\omega t}\, dt$$

$$= \frac{1}{2\pi} \int_{t=-\infty}^{\infty} \int_{u=-\infty}^{\infty} F(u)\, g(t)\, e^{-j(\omega - u)t}\, du\, dt$$

$$= \frac{1}{2\pi} \int_{u=-\infty}^{\infty} F(u) \int_{t=-\infty}^{\infty} g(t)\, e^{-j(\omega - u)t}\, dt\, du$$

$$= \frac{1}{2\pi} \int_{u=-\infty}^{\infty} F(u) \cdot G(\omega - u)\, du$$

$$= \frac{1}{2\pi} F(\omega) * G(\omega)$$ as result in part iii. shows

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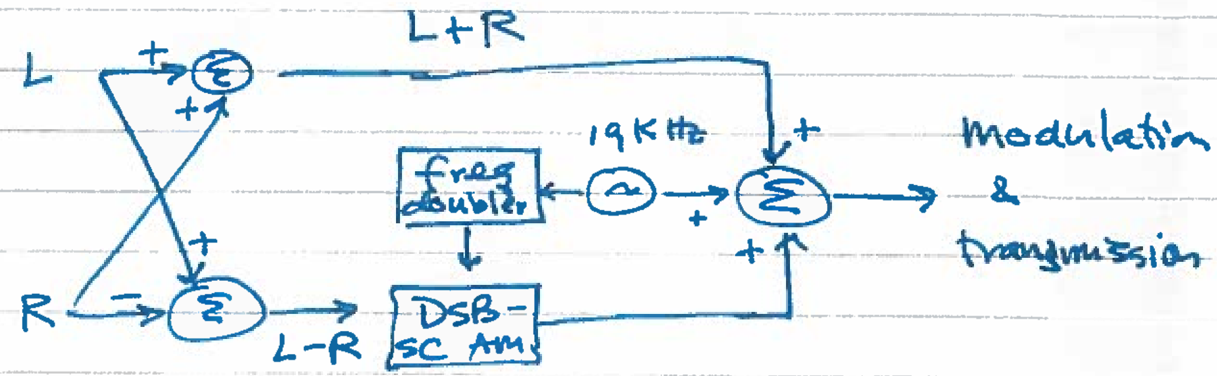
3. a. i.



Form signals : L+R and L−R.

Use & double the pilot signal of 19 KHz.

Form DSB-SC signal for L−R signal using the 38 KHz Carrier.

Transmit the composite baseband signal of

L+R + 19 KHz + DSB-SC of L−R Signal at 38 KHz Carrier

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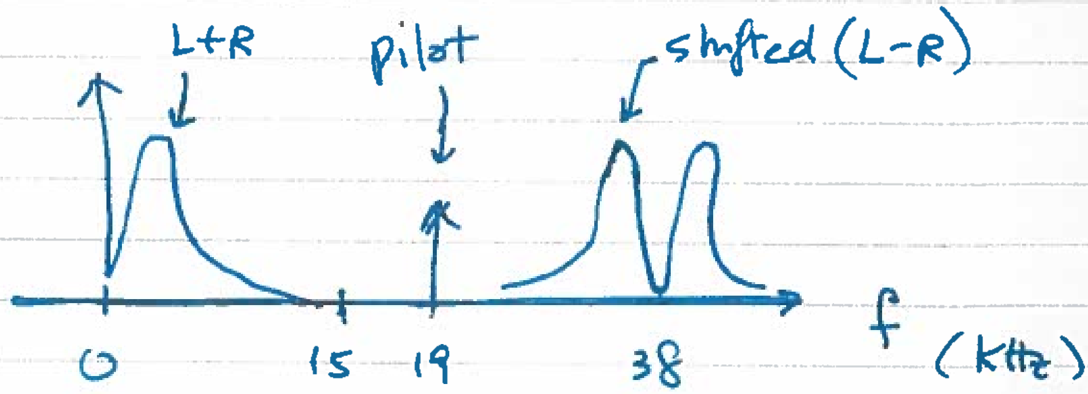
ii.



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3.a. iii. The frequency of the pilot signal of 19 KHz is doubled.

Then, the 38 KHz is multiple with the L-R signal to form the DSB-SC (AM) signal at carrier frequency of 38 KHz.

The frequency multiplication is equivalent to shift the L-R signal to the carrier frequency of 38 KHz.

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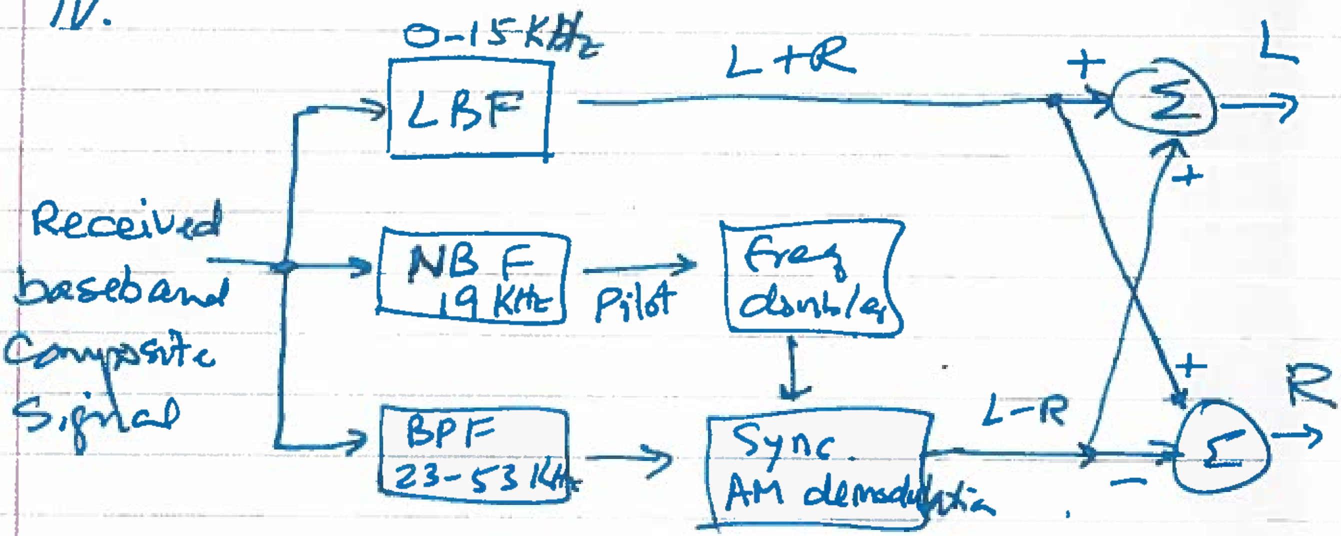
iv.



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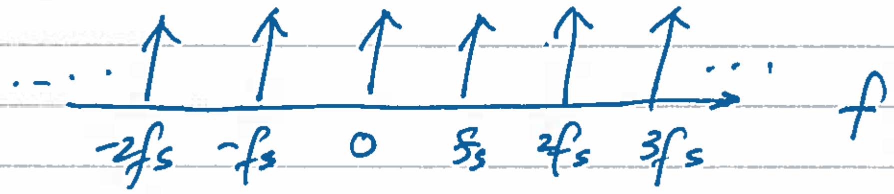
3. b. i. Due to Nyquist Sampling criterion:

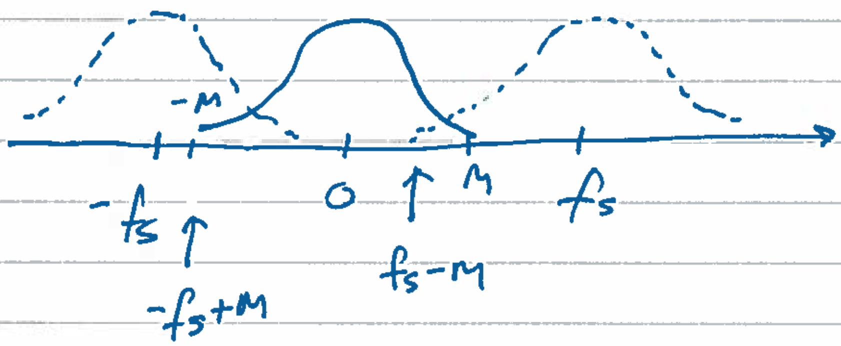
$$f_s \geq 2 \times \text{Bandwidth of } m(t)$$

So, $M = f_s/2$

This is so because the periodic samples ~~is~~ are equal to multiplying a train of impulses with the signal $m(t)$. The train of impulses has a spectrum of



As a result of sampling, the samples of $m(t)$ have the following frequency spectrum:



From the above diagram, in order to recover $m(t)$, we require

$f_s - M \geq M$ to avoid overlap of replica of the original signal spectrum

$$\Rightarrow f_s \geq 2M$$

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3. b. ii. Clearly, $C \geq M$

because the channel bandwidth must be wider than the signal bandwidth in order to avoid loss of information.

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iii. Since $M = f_s/2$,

we have $C \geq f_s/2$

i.e., $f_s \leq 2C$

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iv. $R = 2C \log_2 M$

$= 2C \log_2 4$

$\Rightarrow R = 4C$ bits/sec

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