Imperial College
London

MEng (Engineering) Examination 2017

Year 1

AE1-109 Mechanics

Tuesday 6th June 2017: 14.00 to 16.00
[2 hours]

The paper is divided into Section A and Section B
and contains ***FOUR*** questions.
All questions carry equal marks.
Candidates may obtain full marks for complete answers to ***ALL*** questions.

You must answer each section in a separate answer booklet.

A datasheet is provided

The use of lecture notes is NOT allowed.

Section A

1.

(a) A 4 m long horizontal bar AC of negligible mass supports a mass of 60 kg at one end and is pinned to a wall at the other end as shown in figure 1. The bar is also supported by a string BC. Find the forces applied by the pin and the string on the bar.

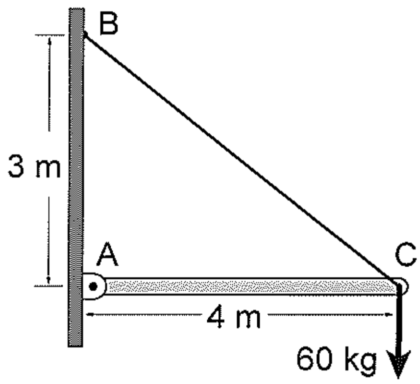

Figure 1

[30%]

(b) Four forces are applied to the machine component ABCD as shown in figure 2. The distance AB is 200 mm (in the x-direction), the distance BC is 160 mm (in the z-direction) and the distance CD is 100 mm (in the y-direction). Replace the four forces with an equivalent force-couple system at A. (Use a vector-based approach).

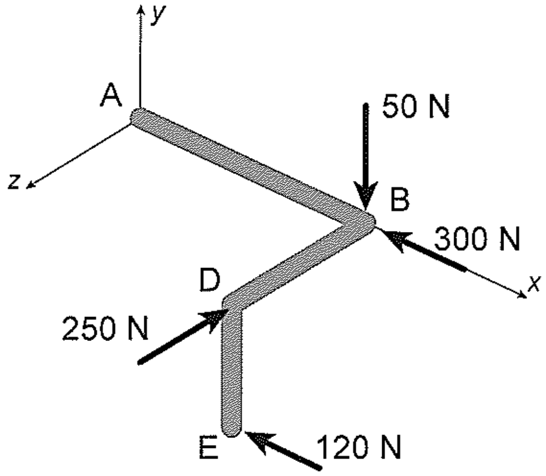

Figure 2

[70%]

2.

a. A beetle walks along the spiral path as shown in figure 3. The path of the beetle can be described by the equation $R = R_0 e^{a\theta}$ where a = 0.182 and R_0 = 5 mm. The beetle's radial distance from the origin of the coordinate system increases at a constant rate of 2 mm/s.

i. Write down the acceleration of the beetle in polar coordinates.

[10%]

ii. Find the tangential and radial components of the acceleration of the bug at $\theta = \pi$.

[40%]

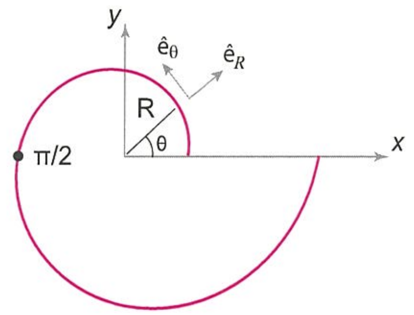

Figure 3

b. The spinning wheel of a stationary exercise bike is brought to rest from 100 rpm by applying a constant braking force over a period of 5 seconds.

i. Find the average angular deceleration of the wheel.

[30%]

ii. Find the number of revolutions before coming to a complete stop.

[20%]

Section B

3. A car of mass m is driven on a long straight road with a constant horizontal velocity v. The undulations in the road may be approximated as sinusoidal with a distance between successive summits of L and a drop of $2h$ between the highest and lowest points as shown in Figure 4. The acceleration due to gravity acts vertically downwards and has value g.

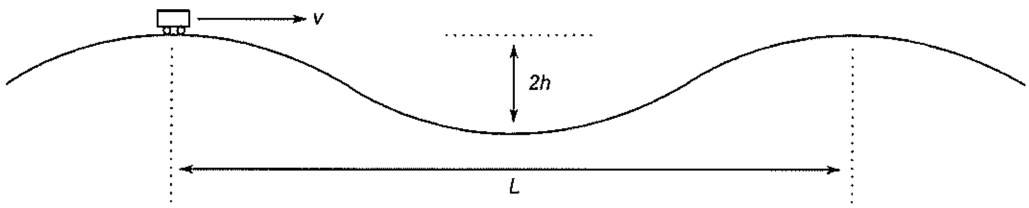

Figure 4

The car's suspension may be approximated as a linear spring of stiffness k in parallel with a velocity-proportional damper with constant of proportionality c. You may assume that the distance between the wheels of the car is significantly less than L.

(a) Define *resonance* for a single-degree-of-freedom damped system. [5%]

(b) By drawing an appropriate free-body diagram determine the equation of motion representing the vertical deflection of the main body of the car with respect to the vertical deflection of the wheels. [15%]

(c) By using a phasor representation determine an expression for the steady-state amplitude of the vertical deflection of the main body of the car. [55%]

(d) As the car is driven along the damper fails, so that it no longer transmits a force. Determine the velocity v that will cause the transmitted displacement to be maximized. State the physical meaning of the corresponding frequency of oscillation. (You may ignore any transient response). [25%]

4. The planet Earth takes 365.256 days (1 sidereal year) to complete a full orbit around the Sun. The planet Mars takes 686.980 days to complete its orbit around the Sun. You may assume as a first approximation that the orbits of both planets are circular and coplanar. The universal gravitational constant $G = 6.674 \times 10^{-11}$ m^3/kg/s^2.

   An exploration mission to Mars is planned which will utilize a Hohmann transfer trajectory.

   (a) Evaluate the radii and tangential velocities of the orbits of Earth and Mars around the Sun. [30%]

   (b) Determine the eccentricity and specific angular momentum of the required transfer trajectory. Also determine the velocity impulses which must be applied to the spacecraft at the beginning and end of the transfer. You may neglect the effects of any local orbital velocities. [30%]

   (c) Determine the transfer time of the spacecraft between Earth and Mars. [30%]

   (d) In reality Mars' orbit has an eccentricity of 0.0934. Without further calculation state how the transfer time can be reduced in practice. [10%]

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1

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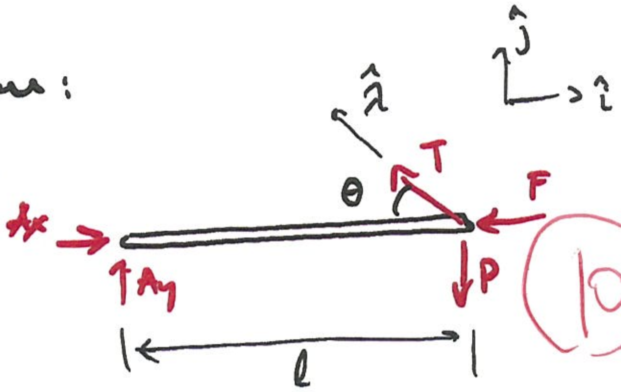
Marks

1a)

Freebody diagram:



10

30

moment balance around point A:

$$\sum \vec{M}_A = 0$$

$$\vec{r}_{C/A} \times T\hat{\lambda} + \vec{r}_{C/A} \times (-P\hat{j}) = \vec{0}$$

$$l\hat{i} \times \underbrace{T(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})}_{= lT\sin\theta\,\hat{k}} + \underbrace{l\hat{i} \times (-P\hat{j})}_{= -lP\hat{k}} = \vec{0}$$

$$(Tl\sin\theta - Pl)\hat{k} = 0$$

$$\Rightarrow \quad \underline{T} = \frac{P}{\sin\theta} = \frac{60\text{kg}}{3/5} = \underline{100\text{kg}}$$

≈ 981.2N

10

force balance $\sum F = 0$ gives

$$(A_x - T\cos\theta)\hat{i} + (A_y + T\sin\theta - P)\hat{j} = \vec{0}$$

$\rightarrow \hat{i}$: $\underline{A_x} = T\cos\theta = 100\text{kg} \cdot 4/5 = \underline{80\text{kg}}$

$\hat{j}$: $\underline{A_y} = P - T\sin\theta = \underline{0}$

785N

785N

10

10

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2

1b)

$R = -420\,N\,\hat{i} \quad -50\,N\,\hat{j} \quad -250\,N\,\hat{k}$

$r_B = 0.2\,m\,\hat{i}$ 10

$r_D = 0.2\,m\,\hat{i} + 0.16\,m\,\hat{k}$ 10

$r_E = 0.2\,m\,\hat{i} - 0.1\,m\,\hat{j} + 0.16\,m\,\hat{k}$ 10

$$M_A^R = \begin{vmatrix} i & j & k \\ 0.2 & 0 & 0 \\ -300 & -50 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0.2 & 0 & 0.16 \\ 0 & 0 & -250 \end{vmatrix}$$ 70

$$+ \begin{vmatrix} i & j & k \\ 0.2 & -0.1 & 0.16 \\ -120 & 0 & 0 \end{vmatrix}$$

$\Rightarrow$ $R = -(420\,N)\hat{i} - (50\,N)\hat{j} - (250\,N)\hat{k}$ 20

$M_A^R = (30.8\,Nm)\hat{j} - (22\cancel{0}\,Nm)\hat{k}$ 20

$|R| = 491$

10 ,f x 1000

1b

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3

2a)

i) i) $\vec{a} = (\ddot{R} - R\dot{\theta}^2)\underline{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\underline{e}_\theta$ 10 10

ii) ii)

we know

$$R = R_0 e^{a\theta}$$

$$\dot{R} = R_0 a e^{a\theta}\dot{\theta}$$

$$\ddot{R} = R_0 \cdot a e^{a\theta}\ddot{\theta} + R_0 a^2 e^{a\theta}\dot{\theta}^2$$

$\dot{R} = 2\,mm/s \quad ; \quad \ddot{R} = 0$

$$R_0\, a e^{a\theta}(\ddot{\theta} + a\dot{\theta}^2) = 0$$

$$\rightarrow \ddot{\theta} = -\frac{\dot{R}^2}{R_0^2\, a e^{2a\theta}} \quad (\sim 10)$$

40

therefore

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\underline{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\underline{e}_\theta$$

$$= \frac{\dot{R}^2}{R_0 a e^{a\theta}}\left[-\frac{1}{a}\underline{e}_R + (2-1)\underline{e}_\theta\right]$$

substitute: $\Rightarrow \vec{a} = \underline{(-13.63\,mm/s^2)}\underline{e}_R + \underline{(2.48\,mm/s^2)}\underline{e}_\theta$

20 20

2a

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4

2b)

i) $\dot{\theta}_0 = 100\,rpm$; $\dot{\theta}_{fin} = 0$; $t = 5s$

α : average constant acceleration

$$\dot{\theta}_{fin} = \dot{\theta}_0 - \alpha t$$

$$\underline{\underline{\alpha}} = \frac{\dot{\theta}_0 - \dot{\theta}_{fin}}{t} = \frac{100\,rpm - 0\,rpm}{5\,sec}$$

(10)

(20)

$$= \frac{60\,rev}{60\,sec} \cdot \frac{1}{5s} = \underline{\underline{0.33\,\frac{rev}{s}}}$$

$= \frac{2}{3}\pi\ rad/sec^2$

$\hat{=} 2.1\ rad/s^2$

30

ii)

$$\underbrace{\theta(t)}_{} = \underbrace{\theta_0 + \dot{\theta}_0 t + \frac{1}{2}(-\alpha)t^2}_{(10?)} = \underbrace{\dot{\theta}_0 t - \frac{1}{2}\alpha t^2}_{(10?)}$$

$$\leadsto \theta = \frac{100\,rev}{60s} \cdot 5s - \frac{1}{2}\, 0.33\,\frac{rev}{s^2} \cdot 25\,s^2$$

20

$\underline{26.175\ rad}$ $= 8.33\,rev - 4.12\,rev$

$4.17 \left(\frac{25}{6}\right)$ $\underline{\underline{= 4.21\,rev}}$ (10)

25

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(1)

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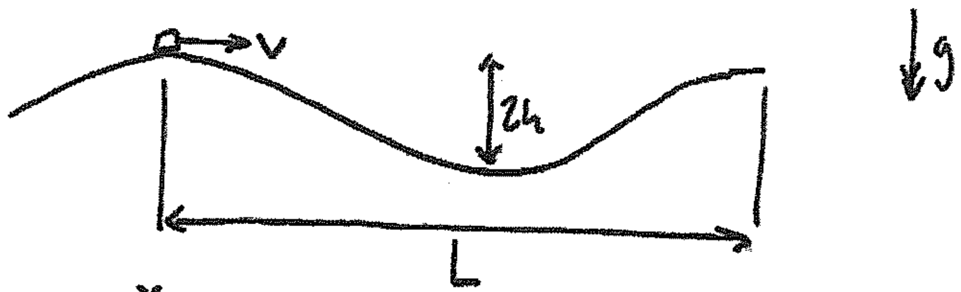
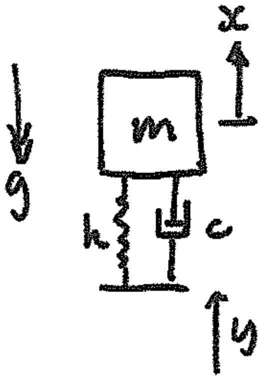
	Marks

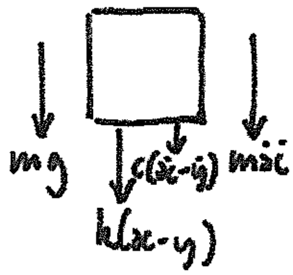
3)





a) Resonance is when the magnitude of steady-state vibration of a forced system is maximized — 5

b)



5

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky - mg$$ — 10

c) $T = \frac{L}{v} = \frac{2\pi}{\omega^*}$ $\quad \omega^* = \frac{2\pi v}{L}$ $\quad y = he^{i\omega^* t}$

$\dot{y} = hi\omega^* e^{i\omega^* t}$ — 5

Steady-state solution:

Try $x = Ae^{i(\omega^* t - \phi)} + C$

$\dot{x} = Ai\omega^* e^{i(\omega^* t - \phi)}$ — 5

$\ddot{x} = -A\omega^{*2} e^{i(\omega^* t - \phi)}$

cont

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(2)

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Marks

3 cont) $C = -\frac{mg}{k}$ (time invariant static deflection due to g) — 5

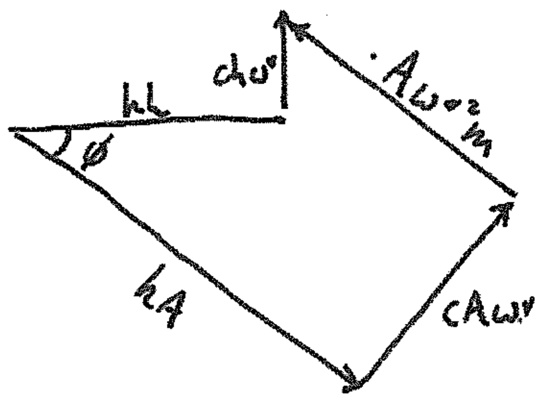
$$-A\omega'^2 m e^{i(\omega' t-\phi)} + cAi\omega' e^{i(\omega' t-\phi)} + kAe^{i(\omega' t-\phi)}$$

$$= chi\omega' e^{i\omega' t} + khe^{i\omega' t}$$ — 10

Phasor representation:

$$-A\omega'^2 m \angle -\phi + cA\omega' \angle \tfrac{\pi}{2}-\phi + kA\angle -\phi$$

$$= ch\omega' \angle \tfrac{\pi}{2} + kh\angle 0$$ — 10



10

$$(kA - A\omega'^2 m)^2 + c^2A^2\omega'^2 = k^2h^2 + c^2h^2\omega'^2$$

$$A^2\left((k-\omega'^2 m)^2 + c^2\omega'^2\right) = h^2(k^2 + c^2\omega'^2)$$

$$\frac{A}{h} = \sqrt{\frac{(k^2 + c^2\omega'^2)}{(k-\omega'^2 m)^2 + c^2\omega'^2}}$$ — 10

cont.

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3 cont.

3

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	Marks
If $c \approx 0$ $\quad \frac{A}{h} = \sqrt{\frac{k^2}{(k - \omega^{*2} m)^2}}$	
transmitted displacement is maximized when	
$(k - \omega^{*2} m) = 0 \qquad \omega^{*} = \sqrt{\frac{k}{m}} = \frac{2\pi V}{L}$	10
$V = \left(\frac{L}{2\pi}\right)\sqrt{\frac{k}{m}}$	10
$\sqrt{\frac{k}{m}}$ is the <u>natural</u> frequency of the system which is identical to the <u>resonant</u> frequency when there is no damping	5

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1

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	Marks

4) a)

$M_s = 1.989 \times 10^{30}$ kg (datasheet)

$G = 6.674 \times 10^{-11}$ m³/kg/s² (given)

$GM = 1.327 \times 10^{20}$ m³/s² — 5

$T_E = 365.256 \times 24 \times 60 \times 60$ s

$\omega_E = 2\pi / T_E = 1.991 \times 10^{-7}$ rad/s

$T_M = 686.980 \times 24 \times 60 \times 60$ s

$\omega_M = 2\pi / T_M = 1.059 \times 10^{-7}$ rad/s

If orbits are circular $e = 0$

$$r = \left(\frac{h^2}{GM}\right)\left(\frac{1}{1 + e\cos\phi}\right)$$

$$r = \frac{h^2}{GM} = \frac{(rv)^2}{GM}$$

$$v = \sqrt{\frac{GM}{r}} = r\omega$$

$$\sqrt{GM} = r^{3/2}\omega$$ — 5

$$r = \sqrt[3]{\frac{GM}{\omega^2}}$$

$r_E = 1.495 \times 10^{11}$ m — 5

$= 150$ million km

$v_E = 29{,}765$ m/s — 5

cont.

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2

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Marks

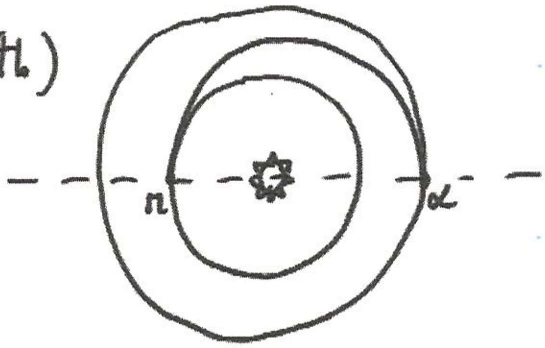
4 cont)

$r_M = 2.278 \times 10^{11}$ m $= 228$ million km — 5

$\left(\omega_M = 1.059 \times 10^{-7} \text{ rad/s}\right)$

$V_M = 24{,}114$ m/s — 5

4b)



$$r_\pi = r_E = \frac{h^2}{GM}\left(\frac{1}{1+e}\right)$$

$$r_\alpha = r_M = \frac{h^2}{GM}\left(\frac{1}{1-e}\right)$$

$$\frac{r_\pi}{r_\alpha} = \frac{1-e}{1+e}$$

Change these.

$$e = \frac{r_\alpha - r_\pi}{r_\alpha + r_\pi} = \frac{r_M - r_E}{r_M + r_E}$$

$e = 0.206$ — 10

$$h_T = \sqrt{r_E GM(1+e)}$$

4.8914×10^{15} $\quad h_T = \cancel{3.969 \times 10^{15}}$ m²/s — 5

Velocity Impulses: $V_\pi = \frac{h_T}{r_\pi} = \cancel{26{,}548}$ 32718 m/s — 5

$V_\alpha = \frac{h_T}{r_\alpha} = \cancel{17{,}423}$ 21,972 m/s

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~~3~~

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Marks

4 cont)

$\Delta V_{\pi} = V_R - V_E = \cancel{3217}$ m/s 2953 — 5

$\Delta V_{\alpha} = V_M - V_{\alpha} = \cancel{+6691}$ m/s 2642. — 5

c) $A_T = \frac{\pi a b}{2}$ $a = \frac{r_\pi + r_\alpha}{2} = 189 \times 10^6$ km — 10

$b = \sqrt{r_\pi r_\alpha} = 185 \times 10^6$ km

$A_T = 5.49 \times 10^{22}$ m^2 — 10

$\frac{dA}{dt} = \frac{A_T}{T} = \frac{h_T}{2}$ $T = \cancel{27.7 \times 10^6}$ s 2.2448 $\times 10^7$ s

$= \cancel{320}$ days transit — 10

259.8 days.

d) As Mars has $e = 0.0934$ its orbit is slightly eliptical. If r_α of the transit is set to r_π of Mars the transit time will be reduced. — 10

(Earth's orbit is also eccentric so similar arguments can be used to reduce the transit time even further.)

Also accept hyperbolic trajectory.

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AE1-109 – Mechanics Data Sheet

Imperial College of Science, Technology & Medicine

Department of Aeronautics

Kepler's Laws of Planetary Motion

Kepler's 1st Law:

$$r = \frac{h^2}{GM(1 + e\cos\phi)}$$

Kepler's 2nd Law:

$$\frac{dA}{dt} = \frac{h}{2}$$

Kepler's 3rd Law:

$$GM = \left(\frac{2\pi}{T}\right)^2 a^3$$

In the above, h is specific angular momentum, G is the universal gravitational constant, M is the mass of the object at the focus, e is trajectory eccentricity, r is radial distance from the focus, and ϕ is the angle from the position at periapsis.

Conservation of Specific Energy

Specific energy is conserved in any given trajectory.

$$\frac{1}{2}v^2 - \frac{GM}{r} = E$$

In the above, v is the speed of the particle, and E is the specific energy. All other variables are as defined above.

Constants

Radius of Earth	$R_E = 6371$ km
Radius of Mars	$R_M = 3396$ km
Radius of Venus	$R_V = 6052$ km
Distance between the Earth and the Sun	$R_{E-S} = 149.6 \times 10^6$ km
Mass of Earth	$M_E = 5.972 \times 10^{24}$ kg
Mass of Sun	$M_S = 1.989 \times 10^{30}$ kg