EE3-12

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

EEE PART III/IV: MEng, BEng and ACGI

Corrected copy

OPTOELECTRONICS

Thursday, 14 December 9:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : R.R.A. Syms
Second Marker(s) : O. Sydoruk

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Fundamental constants

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ m kg/C^2

$c = 3 \times 10^8$ m/s

$h = 6.62 \times 10^{-34}$ Js

Maxwell's equations – integral form

$\iint_A \underline{\mathbf{D}} \cdot d\underline{a} = \iiint_V \rho \, dv$

$\iint_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = - \iint_A \partial\underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \iint_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

Maxwell's equations – differential form

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

Material or constitutive equations

$\underline{\mathbf{J}} = \sigma \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu \underline{\mathbf{H}}$

Vector calculus (Cartesian co-ordinates)

$\mathrm{grad}(\phi) = \partial\phi/\partial x \, \underline{i} + \partial\phi/\partial y \, \underline{j} + \partial\phi/\partial z \, \underline{k}$

$\mathrm{div}(\underline{\mathbf{F}}) = \partial \mathbf{F}_x/\partial x + \partial \mathbf{F}_y/\partial y + \partial \mathbf{F}_z/\partial z$

$\mathrm{curl}(\underline{\mathbf{F}}) = \underline{i} \{\partial \mathbf{F}_z/\partial y - \partial \mathbf{F}_y/\partial z\} + \underline{j} \{\partial \mathbf{F}_x/\partial z - \partial \mathbf{F}_z/\partial x\} + \underline{k} \{\partial \mathbf{F}_y/\partial x - \partial \mathbf{F}_x/\partial y\}$

$\mathrm{curl}\,\{\mathrm{curl}(\underline{\mathbf{F}})\} = \mathrm{grad}\,\{\mathrm{div}(\underline{\mathbf{F}})\} - \nabla^2\underline{\mathbf{F}}$

$\iint_A \underline{\mathbf{F}} \cdot d\underline{a} = \iiint_V \mathrm{div}(\underline{\mathbf{F}}) \, dv$

$\int_L \underline{\mathbf{F}} \cdot d\underline{L} = \iint_A \mathrm{curl}(\underline{\mathbf{F}}) \cdot d\underline{a}$

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1. a) The time-independent scalar wave equation for electric fields can be written in the form:

$$\partial^2 E_y/\partial x^2 + \partial^2 E_y/\partial z^2 + n^2 k_0^2 E_y = 0$$

What type of waves can this equation describe? What are n and k_0?

[4]

b) Prove that the field $E_{y1} = E_0 \exp\{-jnk_0(z \cos(\theta) + x \sin(\theta))\}$ is a solution. What does E_{y1} represent?

[5]

c) Relying on the result of a) but without using a detailed mathematical proof, explain why the field:

$$E_{y2} = E_0 [\exp\{-jnk_0(z \cos(\theta) + x \sin(\theta))\} + \exp\{-jnk_0(z \cos(\theta) - x \sin(\theta))\}]$$

must also be a possible solution. Rewrite this solution in modal form, as a new field E_{y2}'. What do the two fields E_{y2} and E_{y2}' represent?

[7]

d) Write down an expression for the electric field of an inhomogeneous wave, and show that it is also a solution to the scalar wave equation.

[4]

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2. a) Explaining the geometry, write down Snell's Law, and show how the critical angle is calculated. Assuming the refractive index of glass is 1.5, what is the critical angle for a glass-air interface?

[7]

b) The reflection coefficient Γ_E for TE incidence is:

$$\Gamma_E = \{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)\} / \{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)\}$$

Show that all the incident power is reflected once the angle of incidence θ_1 is greater than the critical angle. Sketch the variation of power reflectivity $|\Gamma_E|^2$ with θ_1 for incidence from either side of a glass-air interface.

[7]

c) The eigenvalue equation for a symmetric slab waveguide formed from a core of thickness h and refractive index n_1 surrounded by a cladding of refractive index n_2 is:

$$\tan(\kappa h/2) = \gamma/\kappa \qquad \text{(symmetric modes)}$$
$$\tan(\kappa h/2) = -\kappa/\gamma \qquad \text{(antisymmetric modes)}$$

where:

$\kappa = \sqrt{(n_1^2k_0^2 - \beta^2)}$, $\gamma = \sqrt{(\beta^2 - n_2^2k_0^2)}$, β is the propagation constant and $k_0 = 2\pi/\lambda$.

What phenomena occur at cutoff? To what values do γ, β and κ tend at this point? Derive the cutoff condition for all modes.

[6]

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3. a) Briefly explain the operation of a directional coupler. What is meant by the term 'synchronous coupler'?

[5]

b) In a synchronous coupler, the equations governing the variation of the amplitudes A_1 and A_2 of the modes in the two guides with distance z are:

$$dA_1/dz = -j\kappa A_2$$
$$dA_2/dz = -j\kappa A_1$$

Here κ is the coupling coefficient. Without solving the equations, show that they conserve power.

[4]

c) Find the outputs at a distance L, assuming an input to Guide 1. What value of L is required to obtain a 3 dB coupler, assuming that $\kappa = 0.25\ mm^{-1}$?

[4]

d) Figure 3.1 shows a device based on a fibre 3 dB coupler. The two right-hand fibres have been cut to unequal lengths, and their ends have been metal-coated to form perfect mirrors M. Assuming unity input power, find the output power as a function of the length difference ΔL and propagation constant β. What might the device be used for?

[7]

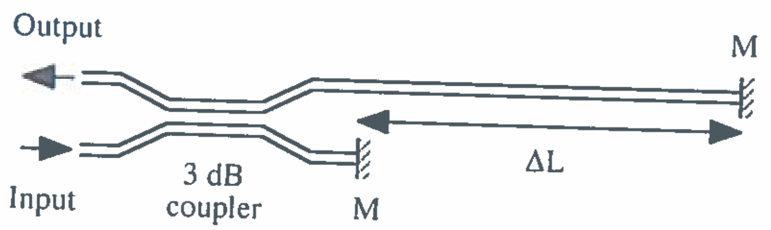

Figure 3.1.

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4. a) Which of the following materials could be electro-optic: Si, GaAs and InP? Why is InP used in fibre telecommunications?

[5]

b) Briefly explain the process of epitaxy.

[3]

c) Sketch the layer structure, refractive index profile and transverse field profile of a planar double heterostructure waveguide in the GaAs/GaAlAs materials system.

[4]

d) Figure 4.1 shows the cross-section of an InP homostructure channel-guide phase modulator. Explain the function of the layers. How is phase modulation achieved, i) in forward bias, and ii) in reverse bias? Which method allows the fastest modulation?

[8]

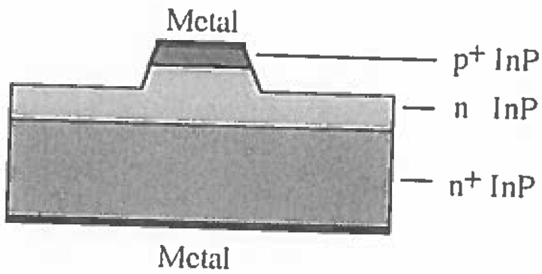

Figure 4.1

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5. a) Describe the three main physical processes that can occur between electrons, holes and photons in optoelectronics, in each case indicating whether the process is dominant in a laser, photodiode or light emitting diode.

[8]

b) Explain the difference between indirect- and direct-gap semiconductors. What is the consequence of silicon being an indirect-gap material?

[6]

c) Explain the difference between semiconductor homojunctions and heterojunctions, and sketch their energy band diagrams in equilibrium. What is the main advantage of the heterojunction in optoelectronics?

[6]

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6. a) The rate equations for a light emitting diode can be written in the form:

$$dn/dt = I/ev - n/\tau_e$$

$$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$$

Explain the processes modelled by the terms on the right-hand sides of the equations. How should the equations be modified for a laser diode?

[7]

b) What is meant by the active volume v of a laser diode? Explain qualitatively the difference between the electron lifetime and the photon lifetime. Show that the photon lifetime in a laser is $\tau_p = L/\{v_g \log_e(1/R_1R_2)\}$, where L is the cavity length, v_g is the group velocity and R_1 and R_2 are the reflectivities of the cavity end-mirrors.

[6]

c) Above threshold, and in the steady state, show that the electron density is clamped to $n = n_0 + 1/G\tau_p$, where n_0 is the electron density at transparency and G is the gain constant. Show that the threshold current is $I_t = nev/\tau_e$, and derive an expression for the variation of the optical power output with current during lasing. What is the slope efficiency dP/dI at 1.5 μm wavelength?

[7]