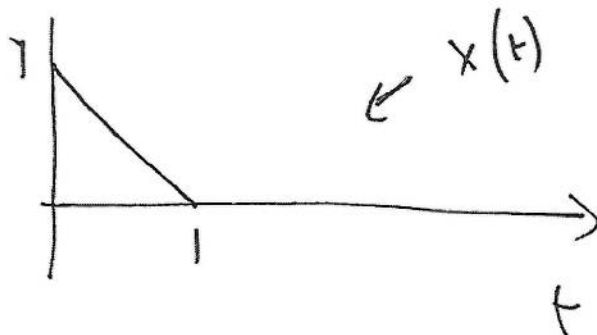
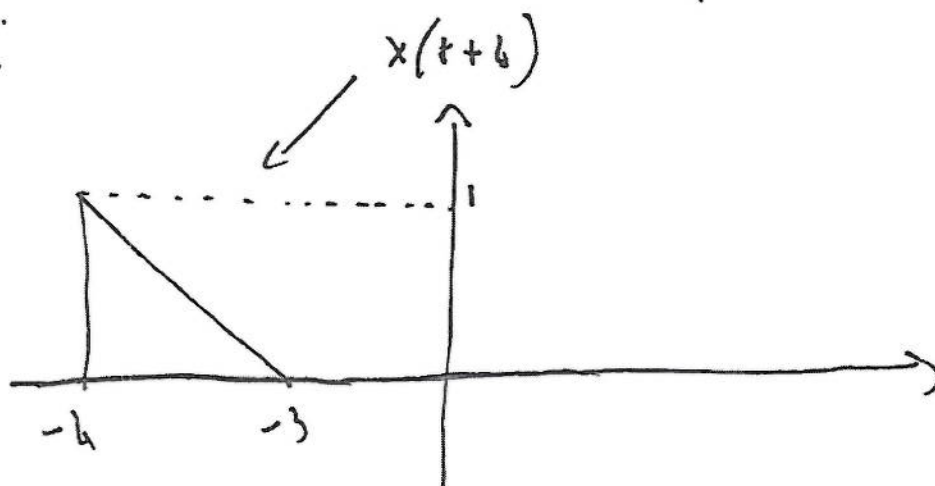


# SOLUTIONS

1.

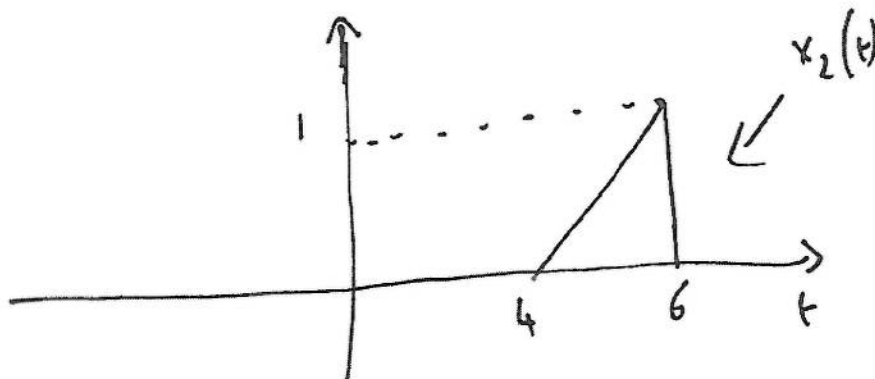


i.



ii.  $x_2(t) = x\left(-\frac{t}{2} + 3\right) = x\left(-\frac{1}{2}(t - 6)\right)$

THEREFORE



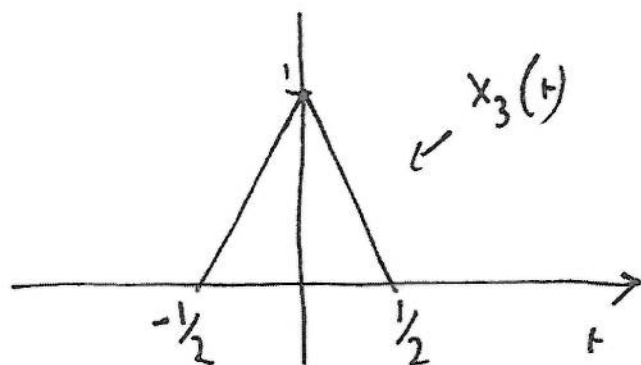
← MANY STUDENTS GOT IT WRONG BECAUSE THEY DIDN'T DO THE OPERATION IN THE RIGHT ORDER.

REMEMBER

$$x_2(t) = x\left(-\frac{t}{2} + 3\right) = x\left(-\frac{1}{2}(t - 6)\right) = 1$$

OR FLIP THE SIGNAL FIRST, RESCALE BY 2, SHIFT BY 6.

iii



(b)

(i) PERIODIC WITH PERIOD  $T=15$

(ii) A-PERIODIC SINCE  $x_3(t) = e^{-t} e^{-jt}$   
AND  $e^{-t}$  IS A-PERIODIC

REMEMBER: EXPONENTIALS  
WITH REAL VALUED  
EXPONENTS ARE A-PERIODIC

(c)

(i) CHARACTERISTIC POLYNOMIAL

$$x^2 + 5x + 4$$

CHARACTERISTIC ROOTS

$$x = \frac{-5 \pm \sqrt{25-16}}{2} = \begin{cases} -4 \\ -1 \end{cases}$$

(ii)  $y(t) = C_1 e^{-t} + C_2 e^{-4t}$

$$y(0) = C_1 + C_2 = 4$$

$$y'(t) \Big|_{t=0} = -C_1 e^{-t} - 4C_2 e^{-4t} \Big|_{t=0}$$

$$= -C_1 - 4C_2 = 1$$

$$\begin{cases} C_1 + C_2 = 4 \\ -C_1 - 4C_2 = 1 \end{cases} \Rightarrow \begin{aligned} C_1 &= \frac{17}{3} \\ C_2 &= -\frac{5}{3} \end{aligned}$$

THUS

$$y(t) = \frac{17}{3} e^{-t} - \frac{5}{3} e^{-4t}$$

(d)

WE USE PARTIAL FRACTION

(i)

$$\frac{s+2}{s^2+6s+5} = \frac{A}{s+5} + \frac{B}{s+1} = \frac{3}{4} \frac{1}{s+5} + \frac{1}{4(s+1)}$$

$$\left( \frac{3}{4} e^{-5t} + \frac{1}{4} e^{-t} \right) u(t)$$

(ii)

IN THIS CASE WE HAVE A ROOT  
WITH MULTIPLICITY 2, THEREFORE

$$\frac{S}{S^2+4S+4} = \frac{A}{S+2} + \frac{B}{(S+2)^2}$$

$$B = -2$$

$$A = 1$$

THEREFORE

$$\frac{S}{S^2+4S+4} = \frac{1}{S+2} - \frac{2}{(S+2)^2} \Leftrightarrow \begin{pmatrix} e^{-2t} & -2te^{-2t} \end{pmatrix} u(t)$$

(2)

$$(i) \quad (s+2)Y(s) = X(s);$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

$$ii \quad X(s) = \frac{1}{s}$$

THEREFORE

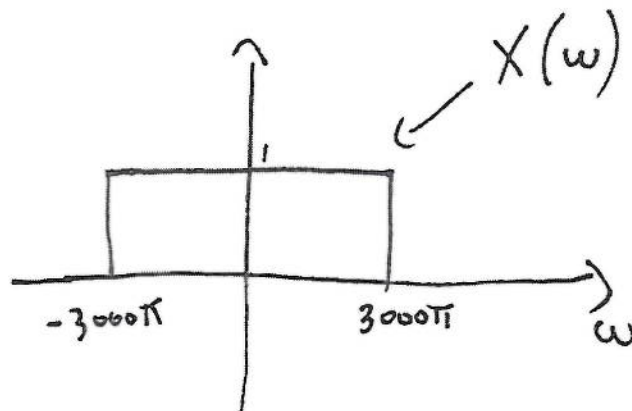
$$Y(s) = \frac{1}{(s+2)s}$$

USING THE FINAL VALUE THEOREM  
WE HAVE THAT

$$y(\infty) = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} \frac{1}{s+2} = \frac{1}{2}$$

(b) USING FOURIER TABLES

$$(i.) 3000 \sin c(3000\pi t) \Leftrightarrow \text{RECT}\left(\frac{\omega}{6000\pi}\right)$$



(ii)

NYQUIST RATE FOR  $x(t)$  :  $f_s = 2 \cdot \frac{3000\pi}{2\pi} = 3\text{KHz}$

NYQUIST RATE FOR  $x^2(t)$  IS  $2f_s = 6\text{KHz}$

THEREFORE NYQUIST RATE

FOR  $x(t) + x^2(t)$  IS  $2f_s = 6\text{KHz}$

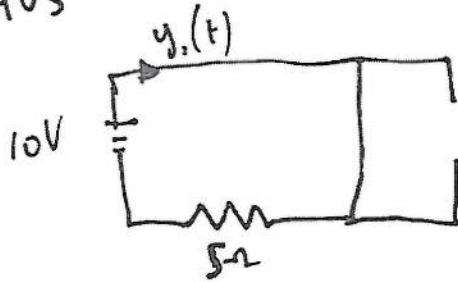
↑ SINCE THE TWO BANDWIDTHS OVERLAP  
THEN NYQUIST RATE IS  
DICTATED BY  $x^2(t)$ .

## QUESTION 2

(a)

IN STEADY STATE (i.e.,  $t < 0$ ) INDUCTORS BEHAVE LIKE SHORT CIRCUITS AND CAPACITORS AS OPEN CIRCUITS

THUS



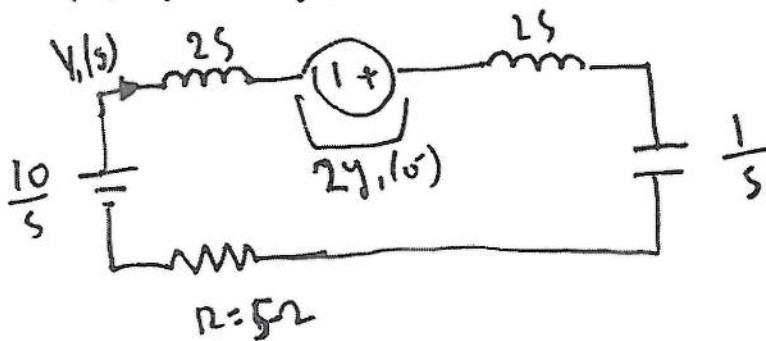
CONSEQUENTLY

$$y_2(0^-) = 0 \quad V_C(0^-) = 0$$

AND

$$y_1(0^-) = \frac{10V}{\Omega} = 10A$$

WHEN  $t \geq 0$  WE HAVE (IN LAPLACE DOMAIN)



THEREFORE

$$2s y_1(s) + 2s y_1(s) + \frac{1}{s} y_1(s) + 5y_1(s) = \frac{10}{s} + 4$$

$$(b) \quad y_1(s) \cdot (4s^2 + 5s + 1) = 10 + 4s$$

NOT  
ANSWERED  
CORRECTLY  
TO THIS  
QUESTION

$$Y_1(s) = \frac{10 + 4s}{4s^2 + 5s + 1}$$

(C) USING PARTIAL FRACTION EXPANSION

$$\begin{aligned} Y_1(s) &= \frac{1}{2} \left( \frac{s + 2s}{s^2 + \frac{5}{4}s + \frac{1}{4}} \right) = \frac{1}{2} \left( \frac{A}{s+1} + \frac{B}{s+\frac{1}{4}} \right) \\ &= \frac{1}{2} \left( -\frac{4}{s+1} + \frac{6}{s+\frac{1}{4}} \right) \\ &= \frac{3}{s+\frac{1}{4}} - \frac{2}{s+1} \end{aligned}$$

$$y_1(t) = \left( 3e^{-t/4} - 2e^{-t} \right) u(t)$$

↑  
SOME STUDENTS GOT POSITIVE ROOTS  
AND THUS POSITIVE EXPONENT.  
REMEMBER A PASSIVE CIRCUIT IS  
STABLE SO  $y(t)$  CANNOT GROW  
ARBITRARY WHEN THE INPUT IS  
BOUNDED.

# QUESTION 3

8

(a)

$$S_1(s) = \frac{1}{s+2}$$

$$S_2(s) = \frac{2}{s+4}$$

(b)

THE TRANSFER FUNCTION OF THE COMPLETE SYSTEM IS:

$$H(s) = \frac{s+2}{s^2+8s+7} \left( \frac{1}{s+2} + \frac{2}{s+4} \right)$$

$$= \frac{\cancel{s+2}}{(s+1)(s+7)} \left( \frac{3s+8}{(\cancel{s+2})(s+4)} \right)$$

$$= \frac{3s+8}{(s+1)(s+4)(s+7)}$$

(c)

$$u(t) \Leftrightarrow \frac{1}{s}$$

THEREFORE

$$Y(s) = \frac{3s+8}{s(s+1)(s+4)(s+7)} = \frac{2}{4s} - \frac{5}{18} \frac{1}{(s+1)} - \frac{1}{9} \frac{1}{(s+4)} + \frac{13}{126} \frac{1}{(s+7)}$$



$$y(t) = \left( \frac{2}{7} - \frac{5}{18} e^{-t} - \frac{1}{9} e^{-4t} + \frac{13}{126} e^{-7t} \right) u(t)$$

(d) THE TRANSFER FUNCTION OF THE FEEDBACK SYSTEM IS :  $H_2(s) = \frac{F(s)}{Y(s)}$  . BUT

$$\cancel{F(s)} = F(s) = K Y(s) - K P(s) F(s)$$

CONSEQUENTLY

$$H_2(s) = \frac{K}{1 + K P(s)} .$$

WE WANT  $H_2(s) \cong H_1^{-1}(s)$  (1)

WHEN  $K P(s) \gg 1$  ~~FOR~~ THEN

$$H_2(s) \cong \frac{1}{P(s)} \quad \text{DPO CONDITION (1) IS}$$

SATISFIED BY CHOOSING  $P(s) = H_1(s)$  .

ONLY A SMALL NUMBER OF STUDENTS ANSWERED THIS CORRECTLY. MANY DID NOT REALIZE THAT WE ARE NOT AFTER A PRECISE VALUE OF  $K$  BUT JUST A VALID RANGE OF VALUES.