E2.3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

Corrected Copy

Q1(c) corrected.
Q5 corrected.

POWER, FIELDS AND DEVICES

Tuesday, 27 May 2:00 pm

Time allowed: 3:00 hours

There are NINE questions on this paper.

There are three sections. Answer FIVE questions including at least ONE question from each of sections A, B and C.

Use a separate answer book for each section.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T.C. Green, K.D. Leaver, R.R.A. Syms
Second Marker(s) : D. Popovic, A.S. Holmes, W.T. Pike

Information for Candidates

Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho \quad ; \quad \mathbf{D} = \varepsilon \mathbf{E} \qquad\qquad \nabla \cdot \mathbf{B} = 0 \quad ; \quad \mathbf{B} = \mu \mathbf{H}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Physical constants and material parameters:

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$e = 1.602 \times 10^{-19}$ C

$kT = 0.025$ eV at 290 K

In SiO_2 $\quad \varepsilon = 4\varepsilon_0$

In silicon $\varepsilon = 11.7\varepsilon_0$

1.

(a) Explain in outline terms the operating principle of an induction machine. [4]

(b) State which components of the equivalent circuit of an induction machine give rise to power losses which affect the efficiency of the machine. Describe the physical processes which cause these power losses. [4]

(c) A three-phase induction machine has the following equivalent circuit parameters:

Number of pole-pairs, $P = 1$;

Magnetising reactance, $X_M = 60\ \Omega$;

Iron loss resistance, $R_I = 200\ \Omega$;

Referred leakage reactance of rotor, $X_R = 2\ \Omega$;

Leakage reactance of stator, $X_S = 2\ \Omega$;

Referred rotor resistance, $R_R = 0.8\ \Omega$;

Stator resistance, $R_S = 0.8\ \Omega$.

On test with a stator phase voltage of 200 V and supply frequency of 50 Hz, the machine drew a stator current of 8.19 A at a phase angle of 31.2° lagging when running at a speed of 2930 r.p.m.

(i) Calculate the input power [3]

(ii) Calculate the power loss in the magnetising components [4]

(iii) Given that the magnitude of the rotor current is 5.63 A under the circumstance described, calculate the efficiency of the machine. [5]

2.

(a) Derive an equation for the ratio of output voltage to input voltage for the step-down switch-mode power supply (SMPS) shown in figure 2 for continuous conduction mode. State any assumptions that are made in the derivation. [4]

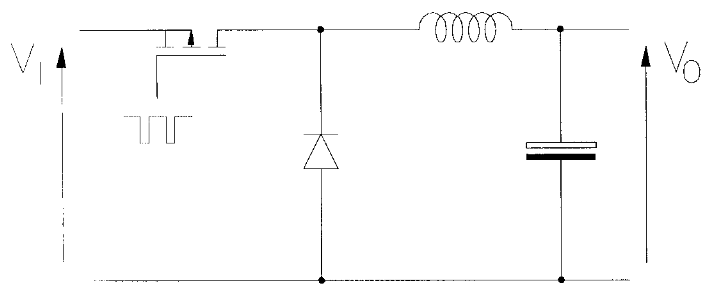


Figure 2

(b) A step-down SMPS is to be used in a laptop computer. During discharge, the battery of the laptop has a voltage that varies from 15.0 V to 13.8 V. The power supply is required to convert this voltage to a constant output voltage of 3.3 V. The output voltage should have a ripple of 10 mV or less.

The switching frequency of the power supply has been set at 100 kHz and it is to be assumed that the SMPS will be in continuous conduction.

An initial assumption of the circuit design is that the ripple of the voltage across the output capacitor arises solely from effective series resistance (ESR) of that capacitor.

(i) Starting with a capacitor with an ESR of 100 mΩ determine the inductor current ripple that can be allowed. [2]

(ii) Choose an inductor value [6]

(iii) The full model of the capacitor is a series combination of a capacitance of 1,000 µF, an ESR of 100 mΩ and an effective series inductance (ESL) of 100 nH. Sketch the shape of the voltage ripple caused by each component of the model and calculate the magnitude of the ripple across each component. [6]

(c) In deriving the voltage transfer ratio of an SMPS it is common to assume that the voltage across the capacitor is constant. In view of the answer to part (b)(iii), is that assumption safe. [2]

3. For each part that follows, decide whether the statement is true, partially true or false. Marks are awarded for the reasons you give for the choice.

(a) High voltages are used in power transmission because they give the best efficiency [2]

(b) Only two wattmeters are required to measure the power consumed by a three-phase load. [2]

(c) For use in a 50 Hz power system, generators must spin at a speed of 3,000 r.p.m. [2]

(d) A high switching frequency is used in a switch-mode power supply so that physically small inductors and capacitors can be used [2]

(e) A high switching frequency is used in a switch-mode power supply so that a physically small heat-sink can be used. [2]

(f) With the same output current and the same capacitor, a Buck and a Boost switch-mode power supply will give the same output voltage ripple. [2]

(g) A neutral line is not required in a three-phase system. [2]

(h) Sinewave voltages are used in a power system because that is the natural waveform of an AC generator. [2]

(i) An electricity supply system for the UK could be built using only wind turbines as a source of energy. [2]

(j) Energy crops (also known as biomass) can be used to generate electricity without adding to greenhouse gasses. [2]

Section B
Use a separate answer book for this section

4. (a) The potential at a radial distance r from a thin straight wire carrying charge q per unit length is

$$V = -\frac{q}{2\pi\varepsilon}\ln\frac{r}{r_0}$$

Show that r_0 is the radius at which the potential is zero. Explain carefully why the two electrostatic problems in Figures 4(a) and 4(b), that are described in the caption to Figure 4, are equivalent.

[7]

(b) Write down an expression for the potential at a point (2a,y) in the problem (b) in Figure 4, in terms of the charge on the wire. Hence find the y-component of the electric field at the point (2a,y) and use the resulting expression to find the charge per unit area on the plate at the point (2a,0).

[8]

(c) Approximately where on the grounded conductor in Figure 4(b) is the charge per unit area at its greatest? Where on it does the charge per unit area become zero? Explain your reasoning in each case.

[5]

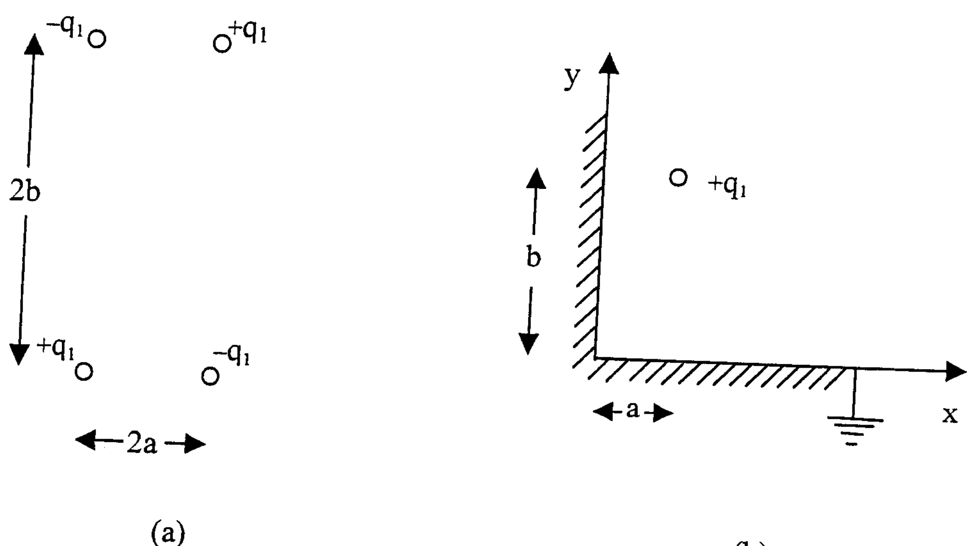


Figure 4 (a) A cross-section through four very long parallel wires that lie normal to the paper and intersect it at the corners of a rectangle. Each wire carries a charge $+q_1$ or $-q_1$ per unit length.

(b) A cross-section through a single wire lying parallel to two grounded conducting flat plates that intersect at right angles.

5. (a) Use Maxwell's equations together with the vector equality

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

to show that the vector magnetic field **H** inside a metal with high conductivity σ and permeability μ obeys the equation

$$\nabla^2 \mathbf{H} = \mu\sigma \frac{\partial \mathbf{H}}{\partial t}$$

(5) [5]

Announced 15:00.

(b) Each conductor of a very long strip transmission line, a part of which is illustrated in Figure 5, carries an alternating current $I = I_0 \exp(j\omega t)$ in the direction shown.

With the aid of Ampères law (or otherwise) and suitable assumptions, find an expression for the magnitude H of the magnetic field strength at the point A, just above the surface of the lower conductor. [5]

(c) Each conductor in Figure 5 has width b, conductivity 10^7 Sm^{-1} and a relative permeability of unity. Using equation (5) above, show that, inside the conductor, H decays exponentially with distance from the surface, and that it reaches 1/e of its value at the point A, at a depth δ, where

$$\delta = 1/\sqrt{(2\pi\omega)} \text{ m.}$$

You may assume that the conductor's thickness is very large compared with δ. [10]

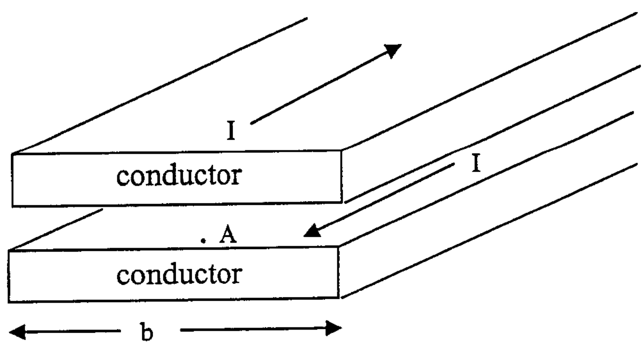


Figure 5

6. (a) A transmission line on which the phase velocity is c has an input voltage of the form $V = V_0 \sin wt$. Write down expressions for the voltage and current in the line, in terms of distance x along the line and time t.

What is meant by the *characteristic impedance* of a transmission line? [6]

(b) The conductors of a coaxial cable have radii b and a (b>a), the outer being grounded. The line carries a travelling electromagnetic wave of fixed frequency. At a particular instant, the voltage at a particular location is V_1 and the current is I_1. Find expressions for the electric and magnetic field strengths at an arbitrary radius r, in terms of V_1 and I_1. [10]

(c) Hence show that the ratio of the fields is given by

$$\frac{E}{H} = \frac{2\pi}{\ln(b/a)} Z_0$$

where Z_0 is the characteristic impedance of the cable. [4]

Section C
Use a separate answer book for this section

7. (a) The bending equation for a cantilever beam, loaded and constrained as shown in Figure 7 below, is of the form:

$$EI\frac{d^2v}{dx^2} = C - Pv \qquad (7.1)$$

What do the terms E, I and C in this equation represent? [4]

(b) By solving Equation 7.1 subject to appropriate boundary conditions, show that the deflection profile v(x) when the beam is buckled is of the form:

$$v(x) = \frac{v_0}{2}\left(1 - \cos(2\pi x / L)\right)$$

where v_0 is the deflection at the centre of the beam and L is the beam length. Also show that the end load P is given by:

$$P = \frac{4\pi^2 EI}{L^2}$$ [10]

(c) A buckling electrothermal actuator is constructed from a silicon beam with built-in supports at both ends. The beam is 1 mm long, 20 μm wide, and 3 μm high. Calculate the average temperature rise required to buckle the beam, assuming a thermal expansion coefficient of $\alpha = 2.5 \times 10^{-6}\ K^{-1}$ for silicon. [6]

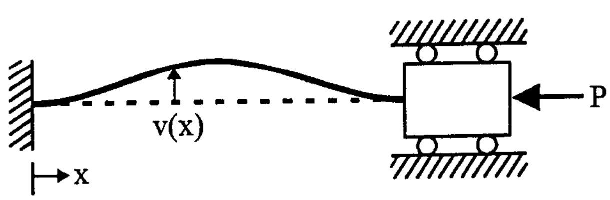


Figure 7

8. (a) What advantages do electrostatic and electrothermal actuators have over other types of micro-actuator? In what kinds of applications might other types of actuator be preferred?

[4]

(b) Using the principle of virtual work, or otherwise, show that the force developed by a parallel plate electrostatic actuator is:

$$F = -\frac{\varepsilon_0 A}{2g^2} V^2$$

where A is the plate area, g is the gap between the plates, and V is the applied voltage. Derive a scaling law for this force, stating clearly any assumptions you make.

[6]

(c) Sketch the structure of a typical electrothermal shape bimorph actuator, and explain the operation of the device. Also derive an approximate expression for the tip deflection in terms of the gap between the beams, and the lengths and average temperature rises of the hot arm, the cold arm and the flexure. [10]

9\. (a) Explain the difference between the *piezoelectric* and *piezoresistive* effects. Is silicon a piezoelectric material? Sketch the layout of a silicon membrane pressure sensor with piezoresistive readout, and explain its operation. [12]

(b) Explain why a photodiode is a more useful device than a photoconductive detector. Define the *responsivity* of a photodiode. Why does the quantum efficiency of a photodiode tend to fall at both short and long wavelengths? [8]

1.

(a) Explain in outline terms the operating principle of an induction machine. [4]

- *Applied AC stator establishes an alternating flux in each winding*
- *Three windings position displaced by 120° supplied by voltages phase displaced by 120° create three fluxes which sum to a constant magnitude and smoothly rotating angular position.*
- *Rotating flux cuts rotor bars and induces voltages along bars (for all speeds of rotation of the rotor except synchronous speed)*
- *Rotor currents are caused to flow in the closed rotor circuit.*
- *Flux produced by rotor rotates synchronously with the stator flux but is displaced by a small angle.*
- *Torque is exerted on the rotor so as to close the angle. This acts to accelerate the machine toward synchronous speed (which includes decelerating the rotor if it is super synchronous)*

(b) State which components of the equivalent circuit of an induction machine give rise to power losses which affect the efficiency of the machine. Describe the physical processes which cause these power losses. [4]

Only the resistive components give rise to power losses, viz., stator resistance. Iron loss resistance and the (referred) rotor resistance. The stator and rotor resistance are normal ohmic components arising from the resistivity of the winding material (copper and aluminium respectively in a standard cage induction machine). The iron loss resistance is an approximate model of losses occurring from hysteresis in the magnetisation characteristic of the steel and from eddy current induced in the steel.

(c) A three-phase induction machine has the following equivalent circuit parameters:

Number of pole-pairs, $P = 1$;
Magnetising reactance, $X_M = 60\ \Omega$;
Iron loss resistance, $R_I = 200\ \Omega$;
Referred leakage reactance of rotor, $X_R = 2\ \Omega$;
Leakage reactance of stator, $X_S = 2\ \Omega$;
Referred rotor resistance, $R_R = 0.8\ \Omega$;
Stator resistance, $R_S = 0.8\ \Omega$.

On test with a stator phase voltage of 200 V the machine drew a stator current of 8.19 A at a phase angle of 31.2° lagging when running at a speed of 2900 r.p.m.

(i) Calculate the input power [3]

$$\begin{aligned} P_{In} &= 3V_S\, I_S \cos(\angle V_S - \angle I_S) \\ &= 3 \times 200 \times 8.19 \times \cos(-2.55°) \\ &= 4.20\ kW \end{aligned}$$

(ii) Calculate the power loss in the magnetising components [4]

$$
\begin{aligned}
V_{AG} &= V_S - I_S\left(R_S + jX_S\right) \\
&= 200 - 8.19\angle -31.2°(0.8 + j2.0) \\
&= 186.2V
\end{aligned}
$$

$$
\begin{aligned}
P_{Iron} &= 3\frac{V_{AG}^{\ 2}}{R_I} \\
&= \frac{3 \times 193.4^2}{200} \\
&= 520\ W
\end{aligned}
$$

(iii) Given that the magnitude of the rotor current is 5.63 A under the circumstance described, calculate the efficiency of the machine. [5]

Several approaches possible.

$$
s = \frac{3{,}000 - 2{,}930}{3{,}000} = 0.023
$$

$$
I_R = \frac{V_{AG}}{R_R/_s + jX_R} = \frac{186.0}{0.8/_{0.023} + j2.0} = 5.42\,A
$$

$$
\begin{aligned}
\eta &= \frac{P_{Mech}}{P_{In}} = \frac{3I_R^{\ 2}R_R\left(1/_s - 1\right)}{P_{In}} \\
&= \frac{3 \times 5.42^2 \times \left(1/_{0.023} - 1\right)}{4.2 \times 10^3} \\
&= 70.2\%
\end{aligned}
$$

2.

(a) Derive an equation for the ratio of output voltage to input voltage for the step-down switch-mode power supply (SMPS) shown in figure 2 for continuous conduction mode. State any assumptions that are made in the derivation. [4]

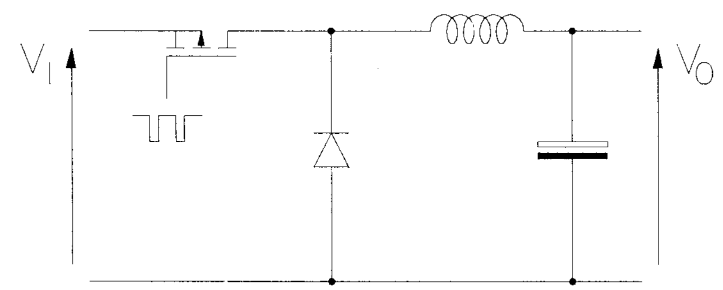


Figure 2

Assume that:

- *the capacitor value is large and therefore the output/capacitor voltage is approximately constant over the switching period of the transistor*
- *the inductor current is in periodic steady-state*
- *the voltage drops across the semiconductors (while they are in conduction) are negligible*
- *the voltage drops across the parasitic resistance of the inductor and capacitor are negligible.*

In periodic steady-state:

$$\Delta i_L(On) + \Delta i_L(Diode) = 0$$

$$\Delta i_L(On) = \frac{di_L}{dt} t_{On} = \frac{V_I - V_O}{L} t_{On}$$

$$\Delta i_L(Diode) = \frac{di_L}{dt} t_{Diode} = \frac{-V_O}{L} t_{Diode}$$

$$\frac{V_O}{V_I} = \frac{t_{On}}{t_{On} + t_{Diode}}$$

For the continuous conduction case, $t_{Diode} = t_{Off}$ *and the output/input ratio is simply the duty-cycle of the switch,* δ.

$$\frac{V_O}{V_I} = \delta$$

(b) A step-down SMPS is to be used in a laptop computer. During discharge, the battery of the laptop has a voltage that varies from 15.0 V to 13.8 V. The power supply is required to convert this voltage to a constant output voltage of 3.3 V. The output voltage should have a ripple of 10 mV or less.

The switching frequency of the power supply has been set at 100 kHz and it is to be assumed that the SMPS will be in continuous conduction.

An initial assumption of the circuit design is that the ripple of the voltage across the output capacitor arises solely from effective series resistance (ESR) of that capacitor.

(i) Starting with a capacitor with an ESR of 100 m determine the inductor current ripple that can be allowed. [2]

For a Buck SMPS in continuous conduction, the current ripple in the inductor is the same as the current ripple in the capacitor.

$$i_L^{ptp} = i_C^{ptp} = \frac{v_O^{ptp}}{R_{ESR}} = \frac{0.01}{0.1} = 0.1\,A$$

(ii) Choose an inductor value [6]

Considering the on-state:

$$\Delta i_L^{On} = \frac{V_I - V_O}{L} \cdot \frac{\delta}{f}$$

$$L = \frac{V_I - V_O}{i_L^{ptp}} \cdot \frac{\delta}{f}$$

However, V_I and therefore also , is not constant and L must be chosen for the worst case condition.

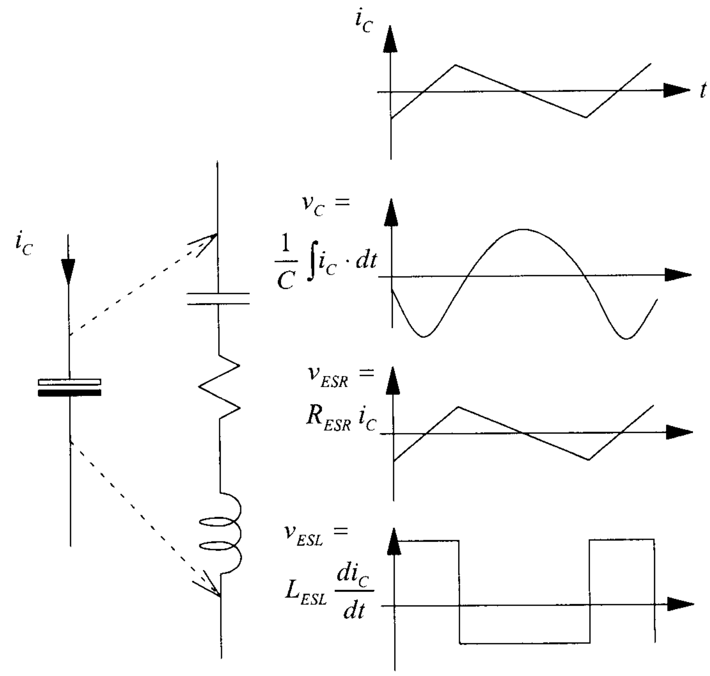
$$\delta = \frac{V_O}{V_I}$$

$$L = \frac{V_O\left(1 - {}^{V_O}\!/_{V_I}\right)}{i_L^{ptp} f}$$

The current ripple is to less than or equal to some limit so the largest of the indicated range of L should be chosen. Thus, the highest input voltage is the limiting case.

$$L = \frac{3.3\left(1 - {}^{3.3}\!/_{15.0}\right)}{0.1 \times 100 \times 10^3} = 257\ \mu H$$

(iii) The full model of the capacitor is a series combination of a capacitance of 1,000 µF, an ESR of 100 mΩ and an effective series inductance (ESL) of 100 nH. Sketch the shape of the voltage ripple caused by each component of the model and calculate the magnitude of the ripple across each component. [6]



$$v_{ESR}{}^{ptp} = i_C{}^{ptp} R_{ESR} = 0.1 \times 0.1 = 10\, mV$$

$$v_C{}^{ptp} = \frac{1}{C}\int^{T/2} i_C . dt = \frac{1}{8 f C} = \frac{1}{8 \times 100 \times 10^3 \times 10^{-3}} = 1.25\ mV$$

$$v_L{}^{ptp} = L_{ESL}\left(\left.\frac{di}{dt}\right|_{On} - \left.\frac{di}{dt}\right|_{Off}\right) = \frac{L_{ESL}}{L}\left(\left(V_I - V_O\right) - \left(-V_O\right)\right) = \frac{100 \times 10^{-9}}{257 \times 10^{-6}} \times 15.0 = 5.8\ mV$$

(c) In deriving the voltage transfer ratio of an SMPS it is common to assume that the voltage across the capacitor is constant. In view of the answer to part (b)(iii), is that assumption safe. [2]

The three ripple voltage terms are all small compared with the DC component of output voltage and the assumption is safe. The capacitive element might be the first choice to include in a more detailed derivation but the ripple across the capacitive element is the smallest term.

3. For each part that follows, decide whether the statement is true, partially true or false. Marks are awarded for the reasons you give for the choice.

(a) High voltages are used in power transmission because they give the best efficiency [2]

For a given power, transmission at a high voltage will occur with a lower current and since the dominant loss mechanism for reasonable voltages is the ohmic loss in the series resistance this will give the best efficiency. True

(b) Only two wattmeters are required to measure the power consumed by a three-phase load. [2]

If the 3-phase load is connected without a neutral (3-wire, 3-phase) then two wattmeters are sufficient since the current in the third wire is implicitly measured since the three currents sum to zero. If a neutral wire is present (4-wire, 3-phase) and the load is unbalanced then this is no longer true and a third wattmeter is required. Partially True

(c) For use in a 50 Hz power system, generators must spin at a speed of 3,000 r.p.m. [2]

This is one of the options. A two pole-pair machine would need to spin at 3,000 r.p.m. (which is 50 revolutions per second). A generator with a higher pole-pair number would need to be spun at a fraction of this speed. Not True

(d) A high switching frequency is used in a switch-mode power supply so that physically small inductors and capacitors can be used [2]

For a given current ripple and voltage ripple specification, a power supply can be designed with smaller valued passive components since a higher rate of change of current or voltage can be allowed over a smaller period. Smaller valued components (for a given voltage or current) have a smaller physical volume). True

(e) A high switching frequency is used in a switch-mode power supply so that a physically small heat-sink can be used. [2]

Heatsink size will depend on the heat power to be removed. The power loss (heat) will increase with frequency and so a larger, not smaller, heatsink is required at high frequency. False

(f) With the same output current and the same capacitor, a Buck and a Boost switch-mode power supply will give the same output voltage ripple. [2]

The shape and amplitude of the current in the capacitor is completely different in the two circuits (even with other things equal). The boost converter has a "chopped" current flowing through the diode and into the capacitor/load combination. The ripple developed across the capacitor will be much larger in the boost circuit. False.

(g) A neutral line is not required in a three-phase system. [2]

For delta 3-phase loads there is no opportunity for a neutral wire. For star three-phase loads a neutral would carry no current anyway if the load is balanced and can be omitted. For an unbalanced star the absence of a neutral conductor will result in a "neutral point shift" which is tolerable if it is small. True (mostly)

(h) Sinewave voltages are used in a power system because that is the natural waveform of an AC generator. [2]

A realistic generator with a small air-gap between the stator steel and the rotor steel will have a radial flux pattern which will give flat-topped (trapezoidal) voltages across each coil. Only careful design involving several position displaced coils connected in series yield an approximately sinusoidal voltage. False.

(i) An electricity supply system for the UK could be built using only wind turbines as a source of energy. [2]

The UK has considerable wind energy resource which in principle could provide the annual electrical energy use. However the generation would be intermittent and seasonal. Both short term and long term energy storage would be needed to match supply and demand, both which are difficult in practice. There are also considerable obstacles and much opposition to deploying the number of turbines required. Not true in practice.

(j) Energy crops (also known as biomass) can be used to generate electricity without adding to greenhouse gasses. [2]

Energy crops (short rotation coppice, sugar beet, rape seed oil) will release CO_2 when burnt but this will have been CO_2 absorbed while growing within the previous year. Taken over a yearly cycle there is negligible net CO_2 production. True

4. (a) When $r = r_0$, $V = \frac{q}{2\pi\varepsilon} \ln 1 = 0$.

[type a)]

In diagram (a), potential is zero by symmetry on both bisectors of the lines joining wires at $\pm V_1$.

(b) At $(2a, y)$; $V = V_1 + V_2 + V_3 + V_4$ where $V_n = \pm \frac{q}{2\pi\varepsilon} \ln \frac{r_n}{r_0}$

[type b)]

$$\Rightarrow V = \frac{q}{2\pi\varepsilon}\left[\ln\sqrt{\frac{a^2+(b-y)^2}{a^2}} - \ln\sqrt{\frac{9a^2+(b-y)^2}{a^2}} - \ln\sqrt{\frac{(b+y)^2+a^2}{b^2}} + \ln\sqrt{\frac{(b+y)^2+9a^2}{b^2}}\right]$$

$$\Rightarrow E_y(y) = -\frac{\partial V}{\partial y} = \frac{q}{4\pi\varepsilon}\left[\frac{-2(b-y)}{(b-y)^2+a^2} - \frac{(-2(b-y))}{(b-y)^2+9a^2} - \frac{2(b+y)}{(b+y)^2+9a^2} + \frac{2(b+y)}{(b+y)^2+9a^2}\right]$$

$$E_y(y=0) = \frac{q}{2\pi\varepsilon_0}\left[\frac{-4b}{b^2+a^2} + \frac{4b}{b^2+9a^2}\right]$$

$D = \varepsilon_0 E_y = \sigma$ (charge per unit area) — proved by using Gauss' law.

(c) Max charge density where $E_x(2a)$ is greatest, near point $(b, 0)$

[type c)]

Zero charge where $E_y = E_x = 0$, i.e. at $(0,0)$.

5. (a) When conductivity σ is high,

$$\nabla \times \underline{H} = \underline{J} + \partial \underline{D}/\partial t \simeq \sigma \underline{E} \quad (1), \text{ neglecting } \frac{\partial D}{\partial t}$$

[type a)]

and $$\nabla \times \underline{E} = -\partial \underline{B}/\partial t. = -\mu \frac{\partial H}{\partial t} \quad \text{---}(2)$$

$$\nabla \times (\nabla \times \underline{H}) = \nabla(\nabla . \underline{H}) - \nabla^2 \underline{H}$$

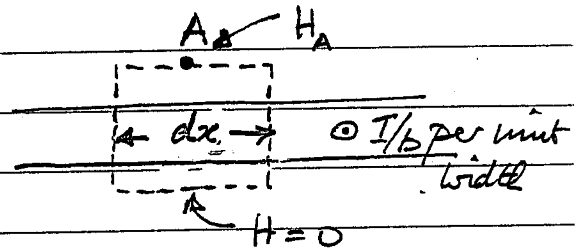
$$= -\nabla^2 \underline{H} \quad \text{since } \nabla . \underline{B} = \mu \nabla . \underline{H} = 0.$$

Hence $$\nabla^2 \underline{H} = -\nabla \times (\sigma \underline{E}) = \sigma \mu \frac{\partial H}{\partial t} \quad \text{using } (2)$$

(b) Assume $b \gg a$ and current is uniformly distributed, so H is purely horizontal at A and zero below the conductor.

[type d)]

Then using Amperes law around a loop as shown,



$$H_A dx = \frac{I}{b} dx$$

$$\Rightarrow H_A = \frac{I}{b}$$

[type d)] (c) Just inside conductor, $H = H_A$ (proved with loop of height dy)

Trial solution of eqn (5) in question when $H_z = H_y = 0$ is

$$H_x = H_0 \exp j(\omega t + Ky)$$

Substituting into (5):

$$(jK)^2 H_0 \exp j(\omega t + Ky) = \mu\sigma j\omega H_0 \exp j(\omega t + Ky)$$

Hence $K^2 = -j\omega\mu\sigma$, i.e. $K = \pm(1-j)\sqrt{\frac{\omega\mu\sigma}{2}}$

Hence $$H_x = H_0 \exp\left(\mp y\sqrt{\frac{\omega\mu\sigma}{2}}\right) \exp j\left[\omega t \pm y\sqrt{\frac{\omega\mu\sigma}{2}}\right]$$

Amplitude falls by $\frac{1}{e}$ where $y = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{4\pi\,\omega}}$

ANSWER Q6

6. (a) $V = V_0 \sin \omega(t - \frac{x}{c})$

[type a)] $I = I_0 \sin \omega(t - \frac{x}{c})$

Characteristic impedance $= \dfrac{V(x,t)}{I(x,t)}$

(b) Between the conductors, using Amperes law around
[type d)] a circular path, radius r:

$2\pi r H(r) = I_1 \Rightarrow H(r) = \dfrac{I_1}{2\pi r}.$

Using Gauss' law on a cylinder of radius r & unit length that is coaxial with the cable,

$2\pi r \varepsilon E(r) = Q$ (charge per unit length on inner)

$\Rightarrow E(r) = Q/2\pi\varepsilon r$

Now $V = -\int E dr = \dfrac{-Q \ln r}{2\pi\varepsilon} + \text{const}$

If $V=0$ at $r=b$, const $= Q \ln b/2\pi\varepsilon$

and $V = V_1$ at $r=a$, so that $\dfrac{Q \ln(b/a)}{2\pi\varepsilon} = V_1$

i.e. $E(r) = \dfrac{V_1}{r \ln b/a}$

(c) Hence $\dfrac{E(r)}{H(r)} = \dfrac{2\pi r}{r \ln \frac{b}{a}} \cdot \dfrac{V_1}{I_1} = \dfrac{2\pi}{\ln \frac{b}{a}} \cdot Z_0.$

[type c)]

7. (a) E = Young's modulus
I = second moment of area of beam
C = couple exerted by roller support

[4]

(b) Equation is of the form $d^2v/dx^2 + k^2v = C/EI$, where $k^2 = P/EI$.

General solution is:

$$v = A\cos kx + B\sin kx + C/P$$

BCs at built-in end: $v = 0$, $v' = 0$ at $x = 0$ $\Rightarrow$ $B = 0$ and $A = -C/P$
BCs at roller support: $v = 0$, $v' = 0$ at $x = L$ $\Rightarrow$ $kL = 2m\pi$

Only lowest order mode ($m = 1$) occurs in practice, so deflection profile is:

$$v = (C/P)[1 - \cos(2\pi x/L)]$$

Max deflection occurs at $x = L/2$, and is given by $v_0 = 2C/P$. Required result follows.

Since $k^2 = P/EI$, and $k = 2\pi/L$, it follows that $P = 4\pi^2EI/L^2$ as required.

[10]

(c) Beam is wider than it is high, so it will buckle out of plane. Relevant form for I is therefore $I = wh^3/12$.

Axial load due to heating is $P = E.wh.\alpha\Delta T$. Beam will buckle when this reaches the critical load given by equation at end of part b), i.e. when:

$$Ewh\alpha\Delta T = 4\pi^2E(wh^3/12)/L^2$$

Rearranging gives: $\Delta T = \pi^2h^2/(3L2\alpha)$

Putting $h = 3\ \mu m$, $L = 1000\ \mu m$, $\alpha = 2.5 \times 10^{-6}$ gives $\Delta T = 12\ °C$.

[6]

8. (a) Advantages: simple construction (both); low power consumption (electrostatic); robustness (thermal).
Other types might be preferred in applications requiring large force and/or deflection.
[4]

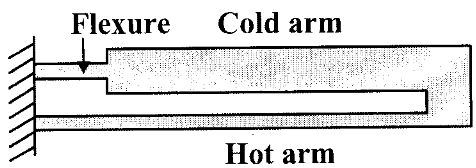
(b) Using virtual work, force is $F = \partial U/\partial g$, where U is the electrostatic stored in the capacitor, and g is the gap between the plates.

U is given by: $U = \frac{1}{2}CV^2 = A\varepsilon_0 V^2/(2g)$, so:

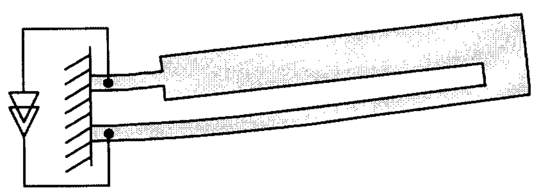
$$\partial U/\partial g = F = -A\varepsilon_0 V^2/(2g^2) \quad \text{as required}$$

Assuming the electric field $E = V/g$ remains constant as the device is scaled down, F scales as the plate area, and so $F \propto L^2$.
[6]

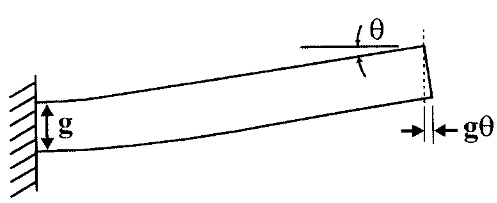
(c) Typical structure:



Current passed through structure causes differential heating because cold arm has lower resistance (and hence less heat generated) per unit length, and better heat conduction. The structure deflects to allow differential expansion of the hot and cold arms:



The difference in length between the hot and cold sides is approximately (LhTh – LcTc – LfTf). This must be equal to $g\theta$, where θ is the angular deflection and g is the gap between the beams:



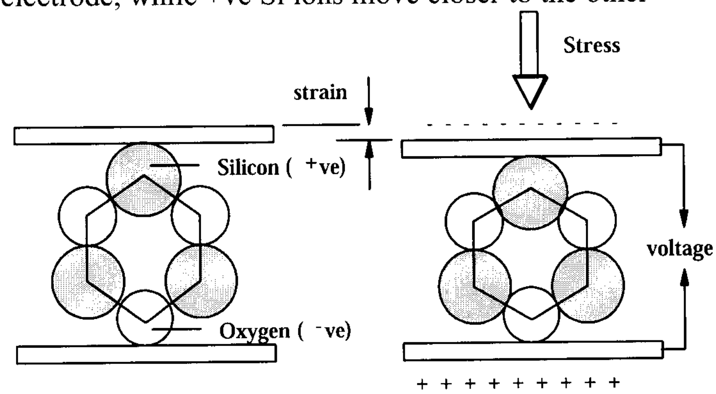
Assuming the flexure bends as a circular arc, and the cold arm remains straight, the tip deflection is then:

$$v = (Lc + Lf/2).\theta \approx (Lc + Lf/2). (LhTh - LcTc - LfTf)/g$$

[10]

9 (a) *Piezoelectricity* - application of a stress creates a surface charge.

- Example: in quartz (crystalline SiO_2), application of stress can causes -ve O ions to move closer to one electrode, while +ve Si ions move closer to the other



Ion charges neutralized in unstrained crystal

Unbalanced ions charge surface of strained crystal

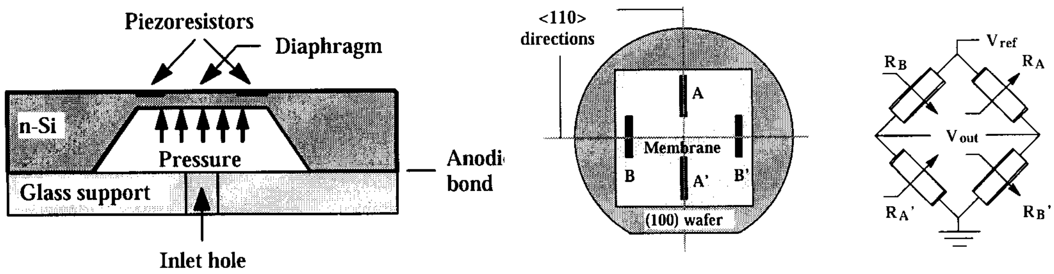
- The effect is dependent on the directions of the *stress* and the *surface normal*
  $P_i = {}_{j=1}\Sigma^6 \, d_{ij}\sigma_j$
  P_i is polarization (charge per unit area)
  σ_j is stress component in j^{th} direction
  d_{ij} is $(i, j)^{th}$ coefficient of 6 x 3 tensor
  Many elements of piezoelectric tensor are zero
- Stress creates charge, and hence electric field $\Leftrightarrow$ Electric field creates strain
  Effect is therefore *reversible* and can be used for actuation
- Effect only occurs in non-centro-symmetric materials. No effect in Si.

[4]

Piezoresistivity - application of stress alters resistivity
- Effect is dependent on the directions of the *stress* and *current flow*
  $\Delta\rho_i/\rho_i = {}_j\Sigma \, \Pi_{ij}\sigma_j$ ρ_i is resistivity in i^{th} direction
  σ_j is stress in j^{th} direction
  Π_{ij} is $(i, j)^{th}$ coefficient of 6 x 3 tensor
- In Si, there are 3 non-zero coefficients: Π_{11}, Π_{12} and Π_{44}. Values depend on doping.
- In p-type resistors, orientation of maximum sensitivity (Π_{44}) coincides with edge of etched diaphragm (<110>). The effect depends on whether the stress is longitudinal or transverse. $\Delta R/R \approx \{\sigma_L - \sigma_T\} \, \Pi_{44}/2$.

[4]

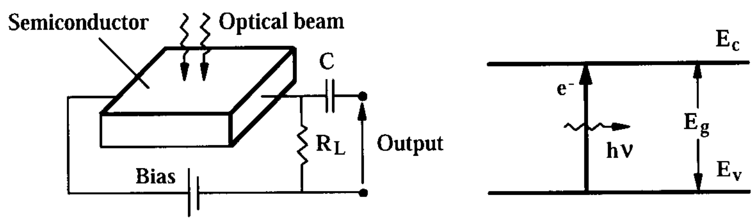
Membrane-based pressure sensor with piezoresistive readout: Four similarly orientated piezoresistors are located on an etched diaphragm, and arranged in a bridge circuit. Since A- and B-type resistors are strained differently, the arrangement is sensitive to deflection of the membrane by a pressure difference. However, use of four similar piezoresistors provides temperature compensation.



[4]

(b) *Photoconductive detectors* - valence electron is promoted to conduction band, by the absorption of a photon of energy $h\nu > E_g$

- Rise in conductivity is then caused by an increase in free carriers
- However, device is either <u>slow</u> (long recombination time) or <u>inefficient</u> (short recombination time); device cannot be both fast and efficient.
-



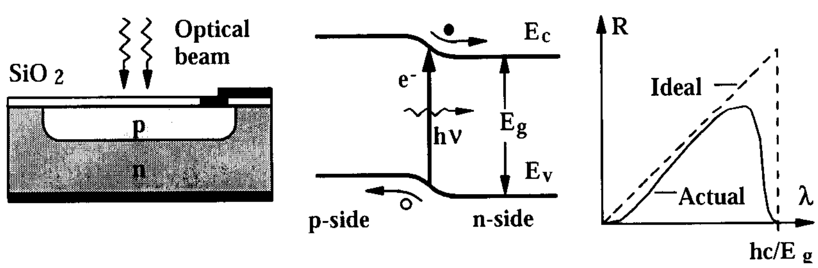
Photoconductive detector principle; band diagram

[3]

- In contrast, *photodiodes* can be made fast <u>and</u> efficient by separating the carrier pairs in the built-in field of a diode, so that they cannot recombine.
- For optical power P, no. of photons/sec is $P/h\nu = P\lambda/hc$
- Photocurrent I_P is then $I_P = \eta eP\lambda/hc$

  Where η is *quantum efficiency* and accounts for non-ideality
- Responsivity $R = I_p/P$ is $R = \eta e\lambda/hc$

  The responsivity is linearly dependent on λ, but zero for $\lambda > hc/E_g$



Photodiode: layout, band diagram and responsvity

[3]

The quantum efficiency falls at short wavelengths because highly energetic photons are absorbed before they reach the deplection layer, and at long wavelengths because photons pass right through the depletion layer without being absorbed.

[2]