EE2-05

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

EEE/EIE PART II: MEng, BEng and ACGI

Corrected Copy

SIGNALS AND LINEAR SYSTEMS

Tuesday, 31 May 2:00 pm

Time allowed: 2:00 hours

Q.3 (c)(ii) TWICE THE BANDWIDTH → TWICE THE ESSENTIAL BANDWIDTH

There are THREE questions on this paper.

Answer ALL questions.

Question One carries 40% of the marks. The other 2 questions each carry 30%.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.L. Dragotti
Second Marker(s) : P.T. Stathaki

Special Information for the Invigilators: none

Information for Candidates

Some Fourier transforms

$$\mathrm{rect}(\tfrac{t}{\tau}) \iff \tau \mathrm{sinc}(\tfrac{\omega\tau}{2})$$

$$\tfrac{W}{\pi}\mathrm{sinc}(Wt) \iff \mathrm{rect}(\tfrac{\omega}{2W})$$

The unit step function $u(t)$ is defined as:

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Some useful Laplace transforms

$$e^{\lambda t}u(t) \iff \tfrac{1}{s-\lambda} \qquad Re\{s\} > \lambda$$

$$t^n e^{\lambda t}u(t) \iff \tfrac{n!}{(s-\lambda)^{n+1}} \qquad Re\{s\} > \lambda$$

A useful z-transform

$$\gamma^n u[n] \iff \frac{z}{z-\gamma} \qquad |z| > |\gamma|$$

The Questions

1. This question carries 40% of the mark.

   (a) Given the signal $x(t)$ shown in Fig. 1a, sketch and dimension each of the

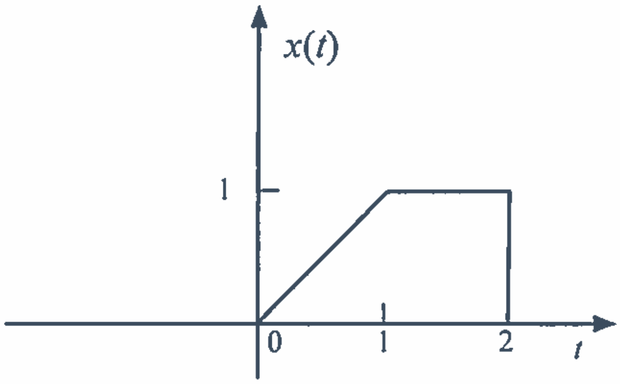

Figure 1a: A continuous-time signal

following signals:

   i. $x_1(t) = x(t)[u(t) - u(t-1)]$, where $u(t)$ is the unit step function. [2]

   ii. $x_2(t) = x(-4t+2)$ [2]

   iii. $x_3(t) = x(-t/2)$ [2]

   (b) Find the even and odd components of $2e^{-t}u(t)$. [2]

Question 1 continues on next page

(c) State with a brief explanation if the causal systems with the following transfer functions are stable or not stable.

i.

$$H_1(s) = \frac{1}{s^2 + 2s + 1}$$

[4]

ii.

$$H_2(s) = \frac{1}{s^2 + 2s + 2}$$

[4]

(d) Consider the two unit impulse responses, $h_1(t), h_2(t)$, sketched in Fig. 1b. State with a brief explanation if

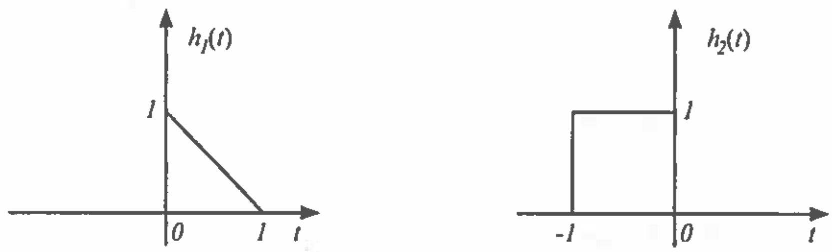

Figure 1b: Two impulse responses

i. the linear time invariant system $h_1(t)$ is memoryless or with memory, causal or non-causal, [2]

ii. the linear time invariant system $h_2(t)$ is causal or non-causal. [2]

Question 1 continues on next page

(e) Find the unit impulse response $h(t)$ of a system with input-output relation given by:

$$y(t) = \int_t^{\infty} e^{-2(t-\tau)} x(\tau) d\tau.$$

[2]

(f) A linear time-invariant system is specified by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 5y(t) = \frac{dx(t)}{dt}.$$

i. Find the characteristic polynomial, characteristic roots and characteristic modes of this system. [3]

ii. Find the zero-input component of the response $y(t)$ for $t \geq 0$, if the initial conditions are $y(0) = 1$ and $\dot{y}(0) = 1$. [3]

iii. Find the zero-state response assuming $x(t) = e^{-5t}u(t)$ where $u(t)$ is the unit step function. [3]

iv. Finally find the total response of the system when the initial conditions are $y(0) = 1$ and $\dot{y}(0) = 1$ and the input is $x(t) = e^{-5t}u(t)$. [3]

(g) Find the inverse Laplace transform of

$$X(s) = \frac{1}{(s^2 + 3s + 2)(s + 3)}$$

[3]

(h) Find the z-transform of the following sequence and state the region of convergence

$$x[n] = \left(\frac{1}{5}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[n].$$

[3]

2. Consider the linear time-invariant (LTI) system shown in Fig. 2a. The rectangular boxes compute the first order derivative of the incoming signal, the circular components are adders or subtractors, the triangular components provide linear gains (i.e., a triangle with a gain a_0 has an output $y(t) = a_0 x(t)$ where $x(t)$ is the input).

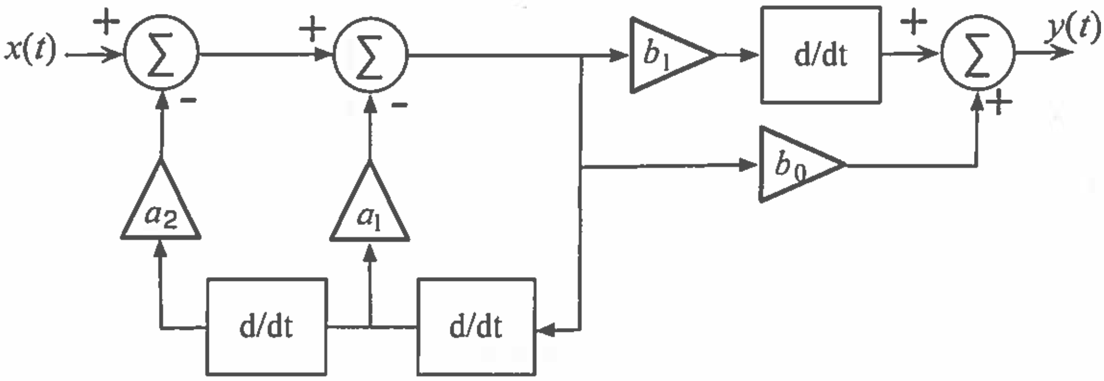

Figure 2a: A linear time invariant system.

(a) Derive the transfer function $H(s)$ of the system. [10]

(b) Find the linear differential equation relating the output $y(t)$ to the input $x(t)$. [6]

(c) Assume that $b_1 = a_1 = 0$, determine the gains b_0 and a_2 that give the output

$$y(t) = 2e^{jt} + e^{j2t}$$

when the input is

$$x(t) = e^{jt} + e^{j2t}.$$

[7]

(d) Assume now that $b_1 = b_0 = 1$, $a_2 = 2$ and $a_1 = 3$, find the exact expression of $y(t)$ when $x(t) = e^{-t}u(t)$. [7]

3. Consider the feedback system shown in Fig. 3a and assume that $H_1(s) = K$, where K is a constant. (Constant K is a REAL number)

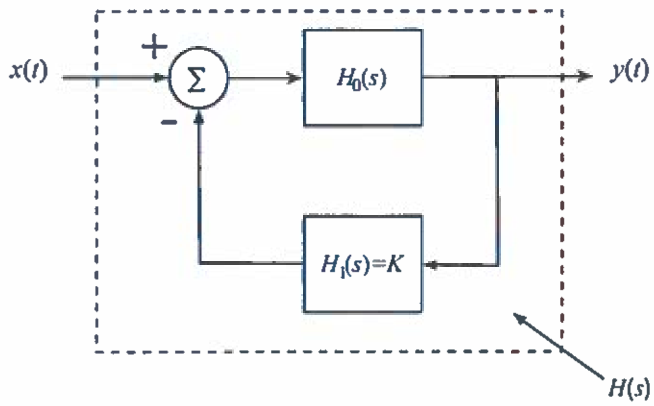

Figure 3a: A feedback system

(a) Determine the transfer function $H(s) = Y(s)/X(s)$ of the feedback system [6]

(b) Assume that

$$H_0(s) = \frac{2}{s-3}.$$

Specify the range of values of K for which the feedback system $H(s)$ is stable. [8]

Question 3 continues on next page

(c) We now want to use the feedback to increase the bandwidth of $H_0(s)$. Assume that

$$H_0(s) = \frac{2}{s+2}.$$

i. We define the essential bandwidth of this system as the frequency at which its magnitude frequency response, $|H_0(\omega)|$, is $1/\sqrt{2}$ times its magnitude at $\omega = 0$. What is the essential bandwidth of $H_0(\omega)$? [8]

ii. Find the value of K such that the system with feedback $H(s)$ has essential bandwidth that is exactly twice the bandwidth of $H_0(s)$. [8]