IMPERIAL COLLEGE LONDON

E4.08
SO14
ISE4.33

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

MSc and EEE/ISE PART IV: MEng and ACGI

Corrected Copy

DIGITAL IMAGE PROCESSING

Monday, 19 May 2:30 pm

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : M. Petrou

Second Marker(s) : P.L. Dragotti

1. a) i) What does the term Singular Value Decomposition (SVD) of an image mean?

[5]

ii) If we truncate the singular value expansion of an image, what is the approximation error?

[5]

iii) If A is a matrix, show that the eigenvalues of AA^T are always non-negative numbers.

[20]

iv) If $\mathbf{u}$ is an eigenvector of matrix AA^T with eigenvalue λ, work out the eigenvector of A^TA and the corresponding eigenvalue.

[20]

b) i) Perform the SVD of the following image, showing all your workings:

$$g = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \tag{1.1}$$

[50]

2. a) i) What does "additive, white, Gaussian noise" mean? Explain your answer by considering how a noise value affects the true value of a particular pixel.

[10]

ii) What is the difference between "uncorrelated" and "white" noise? Justify your answer.

[10]

iii) What type of interference is variable illumination? Justify your answer.

[10]

iv) How can we reduce the effect of variable illumination in an image? Include in your answer a drawing of the transfer function of the filter you should use.

[10]

v) For what type of noise will you use a median filter? Justify your answer with a simple example.

[10]

b) You are given the following image:

$$g = \begin{bmatrix} 1 & 0 & 2 & 2 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 & 2 \\ 2 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \qquad (2.1)$$

In the two processings you will have to perform next, do not process the border pixels. Present your two answers as two 5×5 images with the border pixels left blank.

i) Process it with a median filter of size 3×3.

[20]

ii) Process it with a low pass filter with weights:

$$g = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad (2.2)$$

[30]

3. a) The Haar functions are defined as follows:

$$H_0(t) = 1 \text{ for } 0 \le t < 1$$

$$H_1(t) = \begin{cases} 1 & \text{for } 0 \le t < \frac{1}{2} \\ -1 & \text{for } \frac{1}{2} \le t < 1 \end{cases}$$

$$H_{2^p+n}(t)) = \begin{cases} \sqrt{2}^p & \text{for } \frac{n}{2^p} \le t < \frac{n+0.5}{2^p} \\ -\sqrt{2}^p & \text{for } \frac{n+0.5}{2^p} \le t < \frac{n+1}{2^p} \\ 0 & \text{elsewhere} \end{cases} \tag{3.1}$$

for $p = 1, 2, 3, \ldots$ and $n = 0, 1, \ldots, 2^p - 1$.

Work out the 4×4 matrix with which you can obtain the Haar transform of a 4×4 image.

[50]

b) Work out the basis images in terms of which the Haar transform expands a 4×4 image.

[50]

4. a) You have an image that depicts a bright object on a dark background. You know that the object occupies a fraction θ of the image pixels. You want to separate the object from the background by thresholding the image using a threshold t. Work out the equation that you have to solve in order to identify a value of threshold t that minimises the total number of misclassified pixels.

[40]

b) You are told that the grey values of the pixels that make up the object are drawn from probability density function

$$p_o(x) = \frac{1}{2\sigma_o} \exp\left(-\frac{|x-\mu_o|}{\sigma_o}\right) \tag{4.1}$$

while the grey values of the pixels that make up the background are drawn from probability density function:

$$p_b(x) = \frac{1}{2\sigma_b} \exp\left(-\frac{|x-\mu_b|}{\sigma_b}\right) \tag{4.2}$$

If $\mu_o = 60$, $\mu_b = 40$, $\sigma_o = 10$ and $\sigma_b = 5$, find the threshold that minimises the fraction of misclassified pixels, when we know that the object occupies two thirds of the full image.

[60]