EE4-08
EE9-SO14

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

MSc and EEE/EIE PART IV: MEng and ACGI

DIGITAL IMAGE PROCESSING

Corrected copy

Friday, 12 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer THREE questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

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1. (a) Consider an $M \times M$ -pixel gray level image $f(x, y)$ which is zero outside $0 \le x \le M-1$ and $0 \le y \le M-1$. The image intensity is given by the following relationship

$$f(x,y) = \begin{cases} c, & x = x_1, x = x_2, 0 \le y \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

where c is a constant value between 0 and 255 and x_1, x_2 $x_1 \ne x_2$ are constant values between 0 and $M-1$.

(i) Plot the image intensity. [1]

(ii) Find the $M \times M$ -point Discrete Fourier Transform (DFT) of $f(x, y)$. [5]

(iii) Compare the original image and its Fourier Transform. [2]

Hint: The following result holds: $\sum_{k=0}^{N-1} a^k = \frac{1-a^N}{1-a}, a \ne 1$.

(b) Let $f(x, y)$ denote the following constant 4×4 digital image that is zero outside $0 \le x \le 3$, $0 \le y \le 3$, with r a constant value.

$$\begin{bmatrix} r & r & r & r \\ r & r & r & r \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Give the standard Hadamard Transform of $f(x, y)$. [5]

Hint: Use the recursive relationship of the Hadamard matrix and the separability property of the Hadamard Transform.

(c) Consider the population of vectors $\underline{f}$ of the form

$$\underline{f} = \begin{bmatrix} f_1(x, y) \\ f_2(x, y) \\ f_3(x, y) \end{bmatrix}.$$

Each component $f_i(x, y), i = 1, 2, 3$ represents a real image. The population arises from the formation of the vectors across the entire collection of pixels. Consider now a population of vectors $\underline{g}$ which are the Karhunen-Loeve transforms of the vectors $\underline{f}$.

The covariance matrix of the population $\underline{f}$ calculated as part of the transform is

$$\underline{C}_f = \begin{bmatrix} a & b & c \\ b & a & 0 \\ c & 0 & a \end{bmatrix}$$

(i) Find the covariance matrix of the population $\underline{g}$. Provide conditions for a, b, c so that both covariance matrices are valid. [5]

(ii) Suppose that a credible job could be done of reconstructing approximations to the three original images by using one or two principal component images. What would be the mean square error incurred in each case? [2]

2. (a) (i) Knowing that adding uncorrelated images convolves their histograms, how would you expect the contrast of the sum of two uncorrelated images to compare with the contrast of its component images? Justify your answer. [2]

(ii) Consider an $N \times N$ image $f(x,y)$. From $f(x,y)$ create the following image $g(x,y)$:

$$g(x,y) = -2f(x,y) + f(x,y-1) + f(x,y+1).$$

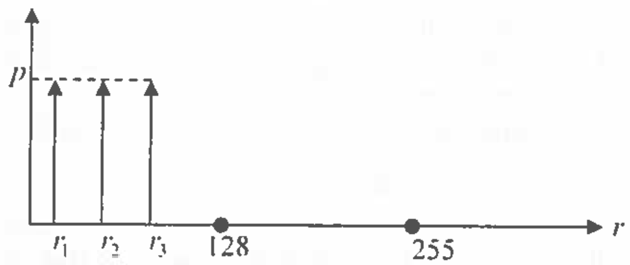
Comment on the histogram of $g(x,y)$ in relation to the histogram of $f(x,y)$. [2]

(b) Propose a method that uses variable-size spatial filters to reduce background noise without blurring the image significantly. [2]

(c) Why are bandpass filters useful in image processing? Justify your answer. Propose a method to obtain a bandpass filtered version of an image using spatial masks. [3]

(d) Propose a method that detects edges in an image along the directions $\pm 45^{\circ}$. [3]

(e) Consider a grey-level image $f(x,y)$ with histogram sketched below.



(i) What can we say about $f(x,y)$? [2]

(ii) Propose an intensity transformation function which will improve the contrast of the image when it is used to modify the intensity of the image. [2]

(iii) Sketch the histogram of the transformed intensity. [2]

(iv) Calculate the mean and the variance of the two images. [2]

3. (a) We are given the noisy version $g(x,y)$ of an image $f(x,y)$. We wish to denoise $g(x,y)$ using a spatially adaptive image denoising mask with coefficients $h(m,n)$. The mask is estimated at each pixel (x,y) based on the local signal variance $\sigma_g^2(x,y)$. The local signal variance is estimated from the degraded image $g(x,y)$ using a local neighborhood around each pixel. The mask coefficients are obtained using the following equation:

$$h_{(x,y)}(m,n) = k_1(x,y)e^{-k_2(x,y)(m^2+n^2)}w(m,n)$$

where $w(m,n)$ is a 5×5-point rectangular window placed symmetrically around the origin, i.e.,

$$w(m,n) = \begin{cases} 1, & -2 \le m \le 2, -2 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}$$

We require that

$$\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} h_{(x,y)}(m,n) = 1$$

(i) Sketch $h_{(x,y)}(m,n)$ for a very small k_2 (close to 0). [3]

(ii) Sketch $h_{(x,y)}(m,n)$ for a very large k_2 (close to ∞). [3]

(iii) Explain why random noise is typically less visible to human viewers in image regions of high detail, such as edge regions, than in image regions of low detail, such as uniform background regions. [3]

(iv) Give one reasonable choice of $k_2(x,y)$ as a function of $\sigma_g^2(x,y)$. Justify your answer. For your choice of $k_2(x,y)$ determine $k_1(x,y)$. [3]

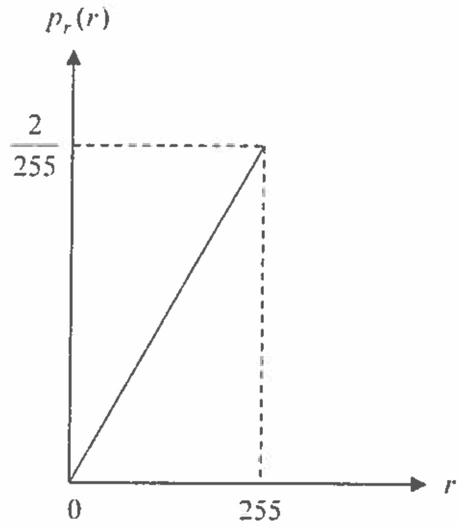
(v) The image restoration system discussed here can exploit the observation stated in (iii). The system, however, cannot exploit the observation that random noise is typically less visible to human viewers in bright areas than in dark areas. How would you modify the above image restoration system so that this additional piece of information can be exploited? [3]

(b) Consider the Constrained Least Squares (CLS) filtering image restoration technique.

(i) Give the general expressions for both the CLS filter estimator and the restored image in both spatial and frequency domains and explain all symbols used. [2]

(ii) Explain the role of regularization parameter in the CLS filtering image restoration technique. [3]

4. (a) Consider an image with intensity $f(x,y)$ that can be modeled as a sample obtained from the probability density function sketched below:

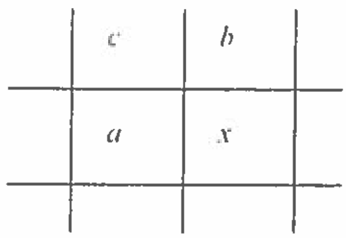


(i) Assume that three reconstruction levels are assigned to quantize the intensity $f(x,y)$. Determine these reconstruction levels using a uniform quantizer. [5]

(ii) Determine the codeword to be assigned to each of the three reconstruction levels using Huffman coding. For your codeword assignment, determine the average number of bits required to represent r. [5]

(iii) Determine the entropy, the redundancy and the coding efficiency of the Huffman code for this example. Comment on the efficiency of Huffman code for this particular set of symbols. [5]

b? (c) In lossless JPEG, one forms a prediction residual using previously encoded pixels in the current line and/or the previous line. Suppose that the prediction residual for pixel with intensity x in the following figure is defined as $r = y - x$ where y is the function $y = a + b - c$.



Consider the case with pixel values $a = 144$, $b = 191$, $c = 190$ and $x = 180$. Find the codeword of the prediction residual y, knowing that the Huffman code for six is 1110.

r

[5]