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Department of mathematics
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Assignment 2

The project should be submitted as one zip- or tar-file to i.shevchenko@imperial.ac.uk by the due date. The file should contain all codes used to generate your results and a pdf-file of the report. The assignment must include a pledge that this is all your own work, your name and CID. Any marks received for the assignment are only indicative and may be subject to moderation and scaling.

Exercise 1 (Taylor series methods for scalar ODEs) % of CW mark: 2.5

Solve the initial value problem

$$x' = -8x - 40(e^{-t/8} - 1),\ x(0) = 100,\ t \in [0, 20] \tag{1}$$

with the TS(2) and TS(3) methods.

a) Compute the numerical solution with the time steps $h = \{0.1, 0.05, 0.025\}$.

b) Study how the global error e_n of each method depends on h.

c) Explain your findings.

Exercise 2 (Taylor series methods for systems of ODEs) % of CW mark: 2.5

Solve the initial value problem

$$x'' + 3x' + 2x = t^2,\ x(0) = 1,\ x'(0) = 0,\ t \in [0, 2] \tag{2}$$

with the Trapezoidal rule and TS(2) method.

a) Compute the numerical solution with the time steps $h = \{0.1, 0.05, 0.025\}$.

b) Study how the global error e_n of each method depends on h.

c) Explain the results.

Exercise 3 (Linear multistep methods for systems of ODEs) % of CW mark: 3.0 Mastery Component

Solve the initial value problem (the Duffing oscillator)

$$\begin{aligned} &x'' + \delta x' + \beta x + \alpha x^3 = \gamma \cos(\omega t),\ x(0) = 0,\ x'(0) = 0,\ t \in [0, 100], \\ &\alpha = \omega = 1.0,\ \beta = 0.0,\ \delta = 0.05,\ \gamma = 0.3 \end{aligned} \tag{3}$$

with the two-step Adams-Bashforth method (AB(2)).

a) Compute the numerical solution with the time steps $h = \{0.1, 0.05, 0.025\}$.

b) Study how the global error e_n depends on h.

c) Explain how you start the AB(2) and why you do it this way.

d) Explain the results.