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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2012

MSc and EEE/ISE PART IV: MEng and ACGI

OPTICAL COMMUNICATION

Wednesday, 16 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 6

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : E.M. Yeatman, E.M. Yeatman

Second Marker(s) : A.S. Holmes, A.S. Holmes

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3.0 \times 10^8$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x}\tan(k_{1x}d/2) \quad \text{and} \quad \kappa = -\,k_{1x}\cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, state any assumptions or approximations made, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]

a) Light with a nominal wavelength of 1.20 μm forms a standing wave in a glass having refractive index 1.50. How far apart will the nulls in the standing wave be?

b) A slab waveguide supports 3 TE modes, m = 0, 1 and 2, for a nominal wavelength λ_o of 1.30 μm. What is the minimum number of TE modes that will be supported for λ_o = 0.65 μm?

c) A silica optical fibre has a numerical aperture of 0.25. Estimate the refractive index difference Δn.

d) A certain optical receiver detects a signal at 2.5 Gbit/s with a received power of -40 dBm. What is the equivalent received energy per bit in Joules?

e) Briefly describe the main advantage of graded index over step index optical fibre.

f) The square root of the ratio given by the permeability of free space divided by the permittivity of free space gives what quantity, of interest in optics?

g) Why can spreading loss in free space communications systems not be given in units of dB/km?

h) What important type of noise in optical communications cannot be overcome by using an optical amplifier? Briefly explain why this is so.

i) The mobility of a charge carrier is defined as the ratio of which two quantities?

j) Estimate the spectral width in nm of an LED operating at a nominal wavelength of 560 nm.

2. A symmetric slab waveguide as shown in Fig. 2.1 has a core thickness d, and core and cladding indices of $n_1 = 1.47$ and $n_2 = 1.46$ respectively, and supports propagation for a free-space wavelength of $\lambda_o = 1.30$ μm.

a) Find the range of values of d for which exactly 3 transverse electric (TE) modes are supported by the guide (i.e. m = 0,1,2). [4]

b) Sketch the typical cross sectional shapes of m = 0, 1 and 2 modes in terms of the electric field amplitude E(x). [4]

c) Calculate the effective index for the m=2 mode for each of the two values of d at the ends of the range calculated in (a). You may find the plot of Fig 2.2 helpful. [6]

d) Show that when the m=2 mode is just at the limit of being cut off, the modal angle θ_i with respect to the x direction is given by $\cos(\theta_i) = \lambda_o/n_1 d$. [6]

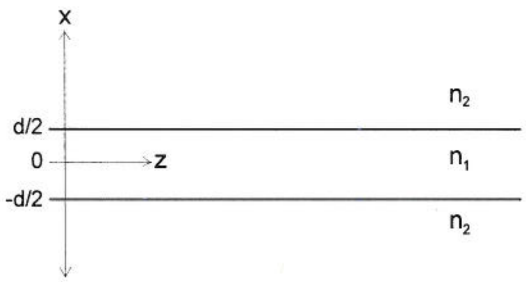

Figure 2.1 Slab waveguide

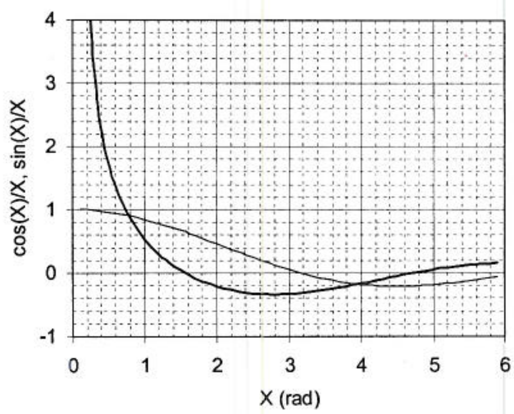

Figure 2.2 cos(X)/X (dark line) and sin(X)/X

3\. a) Give the boundary conditions for the electric fields at the core-cladding interfaces, for TE modes propagating in symmetric slab waveguides. Hence, and using equations for the field distributions of such modes, derive the relevant eigenvalue equations. [5]

b) Give the additional equation that must be satisfied along with the eigenvalue equations and which therefore allows the modes to be identified. Show that this equation relates to a particular boundary condition. Is this condition also valid for TM modes? [5]

c) The condition for a slab or cylindrical step index waveguide to be single mode involves a combination of core thickness (or diameter) and index difference, so that, for example, a larger thickness is possible if a smaller index difference is used. What is the principal benefit of a larger core size in such a case, and what is a possible disadvantage of the reduced index difference? [5]

d) Show that for a symmetric slab waveguide, if the m=1 TE mode is just supported (i.e. nearly cut off), its mode angle θ_i is equal to the critical angle for total internal reflection at the core-cladding boundary. [5]

4\. a) A certain Fabry-Perot laser diode has a cavity length of 650 μm and an effective index for the cavity modes of 3.65. The nominal wavelength is 1350 nm. Find the spacing between the nominal wavelengths of the longitudinal modes in nm. [4]

b) For the laser in (a), estimate the number of longitudinal modes in the spectrum. [4]

c) The laser is then modified by adding distributed Bragg reflectors. Calculate a suitable period Λ for these. What is the main advantage of such structures? [4]

d) Using a quantum efficiency of 0.8, calculate the slope efficiency S of this laser. Hence, if a small sinusoidal current of amplitude 10 μA is added to a suitable bias current to bring the laser above threshold, what will be the amplitude of the sinusoidal component of the output optical power? [4]

e) Explain why the maximum modulation rate of laser diodes can be much higher than that of an LED. [4]

5. a) Describe and discuss the important attenuation mechanisms in optical fibres, and the influence these have in choice of operating wavelengths. Use diagrams and equations where appropriate. [5]

b) A certain optical receiver uses a voltage amplifier configuration. A signal is received for which the photocurrent generates a voltage in the load resistor of average amplitude 200 mV. In this case, which noise source is more significant, shot noise or thermal noise? Explain your reasoning, and state any approximations or assumptions made. [5]

c) A receiver with an effective input resistance of 10 kΩ, and responsivity 0.8 A/W, is at the end of a single-mode fibre with attenuation of 0.2 dB/km at the operating wavelength of 1510 nm. The transmitted power is 5 mW. Assuming an optical SNR of 12 is required and that thermal noise dominates, find an expression for the maximum bit-rate B as a function of fibre length L. [5]

d) The same fibre as in (c) has a dispersion coefficient of 10 ps/nm·km. The transmitted spectral width is 0.5 nm. Find the maximum bit-rate B, as a function of fibre length L, for which the pulse spreading is not more than 0.25 bit slots. For what range of lengths L is this more restrictive than thermal noise as calculated in (c)? [5]

6. A silicon p-i-n photodiode has *p*, *i* and *n* doping levels respectively of N_A, N_D^- and N_D^+, with $N_D^- = 10^{19}\ m^{-3}$ The distances from the diode surface to the top and bottom of the intrinsic layer are $X_1 = 1.5$ μm and $X_2 = 6.5$ μm respectively.

a) Find the optical attenuation coefficient which maximises the fraction of incident photons which are absorbed in the intrinsic layer of this device. [4]

b) Find the value of N_D^+ which allows the peak electric field magnitude to be only 10% greater than the magnitude at the bottom of the intrinsic layer, while only depleting 0.5 μm of the *n* layer. Then, find the value of N_A which allows these conditions to be achieved while only depleting 0.5 μm of the *p* layer. [8]

c) Find the applied bias voltage V such that the conditions in (b) are achieved and the peak electric field magnitude is 4×10^5 V/m. [4]

d) Discuss why it may be beneficial to minimise the distances to which the depletion region extends into the *n* and *p* layers. [4]

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1. a) $\lambda = \lambda_0 / n = 1.200/1.5 = 0.8\ \mu m$
The nulls form every half wavelength so their separation is $\underline{0.4\mu m}$

b) If the $m=2$ mode is supported for $1.30\mu m$ then the $m=4$ will be for half the wavelength. Then we have at least $m=0$ to 4 = $\underline{5 \text{ modes}}$. ($m=5$ may also be supported).

c) $N.A. = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1+n_2)\Delta n}$
Estimating $n_1+n_2 = 2n = 3.0$, $\Delta n = \frac{N.A.^2}{3.0} = \frac{.25^2}{3} = \underline{\underline{0.21}}$

d) $-40\,dBm = 0.1\,\mu W = 10^{-7}\ J/s$
energy/bit $= \frac{10^{-7}}{2.5\times10^{9}} = \underline{4\times10^{-17}\ J}$

e) For multi-mode fibre of a given Δn, for graded index the mode indices are concentrated together, and so the pulse spreading (inter-modal dispersion) is less.

f) $\sqrt{\mu_0/\varepsilon_0} = Z_0$, the impedance of free space.
$Z_0 \cong 375\,\Omega$

g) Only exponential loss mechanisms (fractional signal loss per unit distance is constant) have a fixed value of dB/km. Spreading loss is geometric - power falls as $1/L^2$ - so does not give a constant value in dB/km

Optical Comm. 2012 Solutions.

1. (continued)

h) Shot noise cannot be overcome by optical amplification since it is inherent in the propagating signal – the amplifier can only overcome noise added beyond the amplifier (typically receiver noise).

i) Mobility μ is the ratio of drift velocity to electric field amplitude $\mu = v_d/E$

j) $|\Delta E/E| \cong |\Delta\lambda/\lambda|$
The photon energy E in eV $= \dfrac{hc}{e\lambda} = \dfrac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times560\times10^{-9}}$

$E = 2.21$ eV

We can estimate ΔE as $\sim 2kT \cong 50$ meV

$\Delta\lambda \cong \dfrac{50}{2210}\times 560 = \underline{\underline{12.6 \text{ nm}}}$

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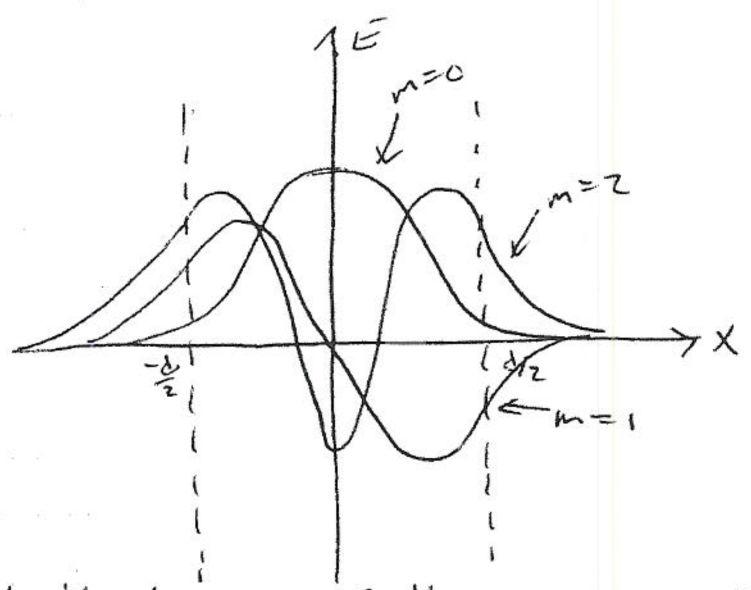
2. a) The cut-off condition for mode m is :

$$d = \frac{m(\lambda_o/2)}{\sqrt{n_1^2 - n_2^2}}$$

The range of d for which 3 modes are supported is between the cut-off for $m=2$ and for $m=3$.

$$\frac{2(0.65)}{0.171} < d < \frac{3(0.65)}{0.171} \qquad \underline{7.60\mu m < d < 11.4\mu m}$$

b)



c) At the beginning of the range, $m=2$ is just cut off, so its effective index n' is simply the cladding index $\underline{n' = n_2 = 1.46}$

For the other end, for the R-k_{1x} diagram, $R = 3\pi/2$ (since $m=3$ is just cut off). Then $\frac{\cos X}{X} = \frac{2}{\pm 3\pi} = \pm 0.212$

Fig 2.2 gives us $X \cong 1.2$, then successive approximation gives $X = 1.245$. $k_{1x} = \frac{2X}{d} = .227 \mu m^{-1}$

2.(c) (continued)

We know for $m=2$, $X > \pi$, so the first value for which $\cos X / X \cong 0.2$ is $X \cong 4$. Successive approximation then gives a more precise $X = 3.78$

Then $k_{1x} = \frac{2X}{d}$ $\quad n' = \sqrt{n_1^2 - \left(\frac{2X}{d} \cdot \frac{\cancel{\lambda_0}}{2\pi}\right)^2}$

$$n' = \sqrt{1.47^2 - \left(\frac{3.78 \times 1.3}{\pi \times 11.4}\right)^2} = \underline{1.424}$$

d) When $m=2$ is at the verge of being cut off,
$X = \pi$, $k_{1x} = 2\pi/d$
But $k_{1x} = n_1 k_0 \cos\theta_i$ $\therefore \cos\theta_i = \frac{(2\pi/d)}{n_1 k_0}$

$$\therefore \underline{\cos\theta_i = \frac{\lambda_0}{n_1 d}}$$

3. a) (from lecture notes part A)

b) The condition is phase matching along the boundaries, giving $K_{1z} = K_{2z}$, from which the $k_{ix}^2 + K^2 = R^2$ equation is derived (see notes). This condition is valid also for TM.

c) The main advantage of a larger core is to ease alignment and improve coupling efficiency when joining a fibre to another fibre or a laser or amplifier. The disadvantage of a reduced Δn is that the mode is weakly guided so bending loss in particular will be worse.

d) When $m = 1$ is just cut-off, $k_{1x} = \pi/d$ (see notes Fig. 3.3), and from the circular arc:

$$R = NA\,k_0 = \pi/d$$

So $k_{1x} = NA\,k_0$ (this is true of any mode at cut-off). But $k_{1x} = n_1 k_0 \cos\theta_i$, giving

$$\cos\theta_i = \frac{N.A.}{n_1} = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} = \sqrt{1 - (n_2/n_1)^2}$$

$$\therefore \sin\theta_i = \sqrt{1 - \cos^2\theta} = n_2/n_1.$$

But $\sin\theta_c = n_2/n_1$ so $\underline{\theta_i = \theta_c}$

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4. a) $L = m\lambda/2 = m\lambda_0/2n'$

$\lambda_0 = 2n'L/m$

$\therefore \frac{\Delta\lambda_0}{\Delta m} = -\frac{2n'L}{m^2}$, $\left|\frac{\Delta\lambda_0}{\Delta m}\right| = \frac{\lambda_0^2}{2n'L}$

$= \frac{(1.35)^2}{2(3.65)(650)} = 3.8\times10^{-4}\,\mu m = \underline{0.38\ nm}$

b) as in Q1 j), and assuming the overall spectrum is of the same width as for an LED, we take

$\Delta E \cong 2kT = 50\,meV$

The photon energy in eV in this case is $\frac{2.21(560)}{1350} = 0.917\,eV$

Then $\Delta\lambda \cong \frac{50}{917}\times 1350 \cong 74\ nm$

Using the spacing from (a) $74/.38 \cong \underline{195}$ modes.

c) We simply need $\Lambda = \lambda_0/2n' = \frac{1350}{2(3.65)} = \underline{185\ nm}$

This gives wavelength selective reflection and so allows only a single long. mode to be supported, thus narrowing the overall $\Delta\lambda$.

d) $S = \frac{hc}{e\lambda}\eta = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times1.35\times10^{-6}}\times0.8 = 0.737\ W/A$

So a $10\,\mu A$ signal gives a $\underline{7.37\,\mu W}$ variation.

e) The LED speed is limited by the turn-off time which is limited by the spontaneous recombination of remaining carriers. With a laser, light still in the cavity causes rapid depletion of carriers by stimulated recombination.

5. a) (see notes part 6)

b) The noise square spectral densities are:

$$(I_{th}^*)^2 = 4kT/R \qquad (I_{sh}^*)^2 = 2eI_{ph}$$

Then the ratio is: $\dfrac{(I_{th}^*)^2}{(I_{sh}^*)^2} = \dfrac{2kT/e}{I_{ph}R}$

kT/e is the thermal voltage = 25 mV at room temp.
$I_{ph}R$ is the voltage across the load resistor.

Then $\dfrac{(I_{th}^*)^2}{(I_{sh}^*)^2} = \dfrac{0.05\,V}{0.2\,V} = 0.25$

so the thermal noise is smaller and shot noise dominates.

c) Taking $\Delta f = B/2$

$$SNR = \frac{I_{ph}}{(4kT/R)^{1/2}(B/2)^{1/2}} \qquad I_{ph} = \Phi_T \mathcal{R} e^{-\alpha L}$$

$$B_{max} = \frac{2R}{4kT(SNR)^2} \times \mathcal{R}^2 \Phi_T^2 e^{-2\alpha L}$$

$$= \frac{2\times 10^4}{(0.1\times 1.6\times 10^{-19})(12)^2} \times 0.8^2 \times (5\times 10^{-3})^2 e^{-2\alpha L} = 0.14\times 10^{18} e^{-2\alpha L}$$

$$\log B_{max} = 17.1 - 2(0.2)L\log e = \underline{17.1 - 0.17L}$$

d) $D \cdot \Delta\lambda \cdot L = 0.25/B_{max}$

$$B_{max} = \frac{0.25}{10^{-11}\times 0.5\times L} = 5\times 10^{10}/L$$

$$\log B_{max} = 10.7 - \log L$$

The two restrictions give the same B_{max} for $L = 47.5$ km. For shorter L dispersion is the stronger restriction.

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6. a) Neglecting Fresnel reflection (which will be independent of α), the fraction of photons absorbed in the intrinsic region is $\eta = e^{-\alpha x_1} - e^{-\alpha x_2}$

To find the optimum:

$$\frac{\partial \eta}{\partial \alpha} = -x_1 e^{-\alpha x_1} + x_2 e^{-\alpha x_2} = 0$$

$$\frac{x_2}{x_1} = e^{\alpha(x_2 - x_1)}$$

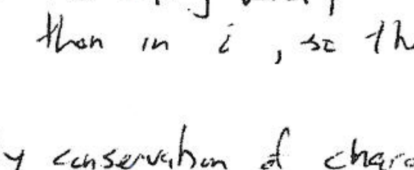
$$\alpha = \frac{\ln(6.5/1.5)}{5\,\mu m} = \underline{0.29\ \mu m^{-1}}$$

b) The magnitude of E rises by 10% from x_2 to x_1 and drops by 100% from x_2 to the bottom of the depleted region, ie over $5\,\mu m$ and $0.5\,\mu m$ respectively. Since the slope dE/dx is proportional to the doping level, this must be $100\times$ higher in n than in i, so that $\underline{N_D^+ = 10^{21}\ m^{-3}}$

By conservation of charge, $N_A(0.5\,\mu m) = N_D^-(5\,\mu m) + N_D^+(0.5\,\mu m)$

This gives $N_A = 10 N_D^- + N_D^+ = \underline{1.1\times 10^{21}\ m^{-3}}$

c) The peak electric field $|E_{max}|$ will be at x_1. We can find the applied voltage by integrating under the $E(x)$ distribution:



$$\Delta V = (4\times10^5 \times 0.5\times10^{-6})/2 + (3.82\times10^5 \times 5\times10^{-6}) + (3.64\times10^5 \times 0.5\times10^{-6})/2 = \underline{2.10\ V}$$

d) Minimising the depletion of p and n makes the depletion thickness (& capacitance) less dependent on V, keeps average $|E|$ (and thus v_d) high, and keeps a conductive path in p.