IMPERIAL COLLEGE LONDON

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DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected Copy

DIGITAL SIGNAL PROCESSING AND DIGITAL FILTERS

Tuesday, 6 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer Question 1 and any TWO other questions

Question 1 is worth 40% of the marks and other questions are worth 30%

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D.M. Brookes

Second Marker(s) : P.T. Stathaki

Digital Signal Processing and Digital Filters

Information for Candidates:

Notation

- All signals and filter coefficients are real-valued unless explicitly noted otherwise.
- Unless otherwise specified, upper and lower case letters are used for sequences and their z-transforms respectively. The signal at a block diagram node V is $v[n]$ and its z-transform is $V(z)$.
- $x[n] = [a, b, c, d, e, f]$ means that $x[0] = a, \ldots x[5] = f$ and that $x[n] = 0$ outside this range.
- $\Re(z)$, $\Im(z)$, z^*, $|z|$ and $\angle z$ denote respectively the real part, imaginary part, complex conjugate, magnitude and argument of a complex number z.

Abbreviations

BIBO	Bounded Input, Bounded Output
CTFT	Continuous-Time Fourier Transform
DCT	Discrete Cosine Transform
DFT	Discrete Fourier Transform
DTFT	Discrete-Time Fourier Transform
LTI	Linear Time-Invariant
MDCT	Modified Discrete Cosine Transform
SNR	Signal-to-Noise Ratio

Standard Sequences

- $\delta[n] = 1$ for $n = 0$ and 0 otherwise.
- $\delta_{\text{condition}}[n] = 1$ whenever "condition" is true and 0 otherwise.
- $u[n] = 1$ for $n \geq 0$ and 0 otherwise.

Geometric Progression

- $\sum_{n=0}^{r} \alpha^n z^{-n} = \frac{1-\alpha^{r+1}z^{-r-1}}{1-\alpha z^{-1}}$ or, more generally, $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1}z^{-r-1}}{1-\alpha z^{-1}}$

Forward and Inverse Transforms

z:	$X(z) = \sum_{-\infty}^{\infty} x[n] z^{-n}$	$x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$
CTFT:	$X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} d\Omega$
DTFT:	$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n] e^{-j\omega n}$	$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$
DFT:	$X[k] = \sum_{0}^{N-1} x[n] e^{-j2\pi \frac{kn}{N}}$	$x[n] = \frac{1}{N} \sum_{0}^{N-1} X[k] e^{j2\pi \frac{kn}{N}}$
DCT:	$X[k] = \sum_{n=0}^{N-1} x[n] \cos \frac{2\pi(2n+1)k}{4N}$	$x[n] = \frac{X[0]}{N} + \frac{2}{N} \sum_{n=1}^{N-1} X[k] \cos \frac{2\pi(2n+1)k}{4N}$
MDCT:	$X[k] = \sum_{n=0}^{2N-1} x[n] \cos \frac{2\pi(2n+1+N)(2k+1)}{8N}$	$y[n] = \frac{1}{N} \sum_{0}^{N-1} X[k] \cos \frac{2\pi(2n+1+N)(2k+1)}{8N}$

Convolution

DTFT: $v[n] = x[n] * y[n] \triangleq \sum_{r=-\infty}^{\infty} x[r] y[n-r] \quad \Leftrightarrow \quad V\left(e^{j\omega}\right) = X\left(e^{j\omega}\right) Y\left(e^{j\omega}\right)$

$v[n] = x[n] y[n] \quad \Leftrightarrow \quad V\left(e^{j\omega}\right) = \frac{1}{2\pi} X\left(e^{j\omega}\right) \circledast Y\left(e^{j\omega}\right) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X\left(e^{j\theta}\right) Y\left(e^{j(\omega-\theta)}\right) d\theta$

DFT: $v[n] = x[n] \circledast_N y[n] \triangleq \sum_{r=0}^{N-1} x[r] y[(n-r)_{\bmod N}] \quad \Leftrightarrow \quad V[k] = X[k] Y[k]$

$v[n] = x[n] y[n] \quad \Leftrightarrow \quad V[k] = \frac{1}{N} X[k] \circledast_N Y[k] \triangleq \frac{1}{N} \sum_{r=0}^{N-1} X[r] Y[(k-r)_{\bmod N}]$

Group Delay

The group delay of a filter, $H(z)$, is $\tau_H(e^{j\omega}) = -\frac{d\angle H(e^{j\omega})}{d\omega} = \Re\left(\frac{-z}{H(z)} \frac{dH(z)}{dz}\right)\Big|_{z=e^{j\omega}} = \Re\left(\frac{\mathscr{F}(nh[n])}{\mathscr{F}(h[n])}\right)$ where $\mathscr{F}()$ denotes the DTFT.

Order Estimation for FIR Filters

Three increasingly sophisticated formulae for estimating the minimum order of an FIR filter with unity gain passbands:

1. $M \approx \frac{a}{3.5\Delta\omega}$
2. $M \approx \frac{a-8}{2.2\Delta\omega}$
3. $M \approx \frac{a-1.2-20\log_{10} b}{4.6\Delta\omega}$

where a =stop band attenuation in dB, b = peak-to-peak passband ripple in dB and $\Delta\omega$ = width of smallest transition band in normalized rad/s.

z-plane Transformations

A lowpass filter, $H(z)$, with cutoff frequency ω_0 may be transformed into the filter $H(\hat{z})$ as follows:

Target $H(\hat{z})$	Substitute	Parameters
Lowpass $\hat{\omega} < \hat{\omega}_1$	$z^{-1} = \frac{\hat{z}^{-1}-\lambda}{1-\lambda\hat{z}^{-1}}$	$\lambda = \frac{\sin\left(\frac{\omega_0-\hat{\omega}_1}{2}\right)}{\sin\left(\frac{\omega_0+\hat{\omega}_1}{2}\right)}$
Highpass $\hat{\omega} > \hat{\omega}_1$	$z^{-1} = -\frac{\hat{z}^{-1}+\lambda}{1+\lambda\hat{z}^{-1}}$	$\lambda = \frac{\cos\left(\frac{\omega_0+\hat{\omega}_1}{2}\right)}{\cos\left(\frac{\omega_0-\hat{\omega}_1}{2}\right)}$
Bandpass $\hat{\omega}_1 < \hat{\omega} < \hat{\omega}_2$	$z^{-1} = -\frac{(\rho-1)-2\lambda\rho\hat{z}^{-1}+(\rho+1)\hat{z}^{-2}}{(\rho+1)-2\lambda\rho\hat{z}^{-1}+(\rho-1)\hat{z}^{-2}}$	$\lambda = \frac{\cos\left(\frac{\hat{\omega}_2+\hat{\omega}_1}{2}\right)}{\cos\left(\frac{\hat{\omega}_2-\hat{\omega}_1}{2}\right)}$, $\rho = \cot\left(\frac{\hat{\omega}_2-\hat{\omega}_1}{2}\right)\tan\left(\frac{\omega_0}{2}\right)$
Bandstop $\hat{\omega}_1 \not< \hat{\omega} \not< \hat{\omega}_2$	$z^{-1} = \frac{(1-\rho)-2\lambda\hat{z}^{-1}+(\rho+1)\hat{z}^{-2}}{(\rho+1)-2\lambda\hat{z}^{-1}+(1-\rho)\hat{z}^{-2}}$	$\lambda = \frac{\cos\left(\frac{\hat{\omega}_2+\hat{\omega}_1}{2}\right)}{\cos\left(\frac{\hat{\omega}_2-\hat{\omega}_1}{2}\right)}$, $\rho = \tan\left(\frac{\hat{\omega}_2-\hat{\omega}_1}{2}\right)\tan\left(\frac{\omega_0}{2}\right)$

1. a) i) Explain what is meant by saying that a linear time-invariant system is "BIBO stable". [1]

ii) The impulse response, $h[n]$, of a linear time-invariant system satisfies $\sum_{n=-\infty}^{\infty} |h[n]| = S$ where $S < \infty$. Prove that the system is BIBO stable and also that $H(z)$ converges for $|z| = 1$. [2]

b) A filter is defined by the difference equation

$$y[n] = \alpha y[n-1] + (1-\alpha)x[n]$$

where $0 < \alpha < 1$ is a real constant.

i) Determine the system function of the filter, $H(z)$, and the impulse response, $h[n]$, for $n = -1, 0, 1, 2$. [2]

ii) State the values of z at which $H(z)$ has a pole or zero. [2]

iii) Determine the frequency at which the filter has a gain of -3 dB. [3]

iv) If the sample frequency is f_s, show that, for $n \geq 0$, the impulse response, $h[n]$, is equal to a sampled version of $h(t) = Ae^{-\frac{t}{\tau}}$ and determine the values of the constants A and τ. [2]

c) Figure 1.1 shows the block diagram of a filter implementation comprising two delays, five multipliers with real-valued coefficients $c_1, \cdots, c_5$ and four adder elements.

i) Show that transfer function $\frac{Y(z)}{X(z)} = \frac{c_3 + c_4 z^{-1} + c_5 z^{-2}}{1 - c_1 z^{-1} - c_2 z^{-2}}$. [3]

ii) Suppose that each multiplier introduces independent additive white noise at its output with power spectral density $S(\omega) = S_0$ and that the noise signals are uncorrelated with $x[n]$. Show that the combined effect of the five noise sources is equivalent to two additive white noise signals at $x[n]$ and $y[n]$ respectively. Hence determine the overall power spectral density, $N(\omega)$, of the noise at $y[n]$. [3]

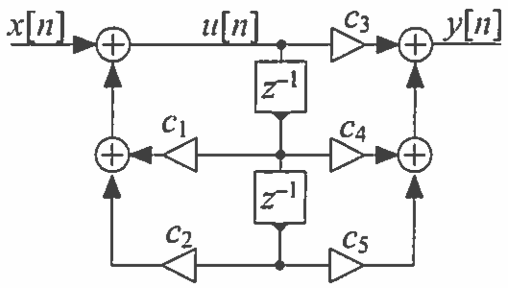

Figure 1.1

d) The impulse response of an antisymmetric FIR filter, $H(z)$, of order M satisfies the relation $h[n[= -h[M-n]$.

i) Show that the magnitude response $|H(e^{j\omega})|$ can be expressed as the absolute value of the sum of N sine waves where $N = \frac{M}{2}$ if M is even and $N = \frac{M+1}{2}$ if M is odd. [3]

ii) Show that $H(e^{j\omega})$ is necessarily zero at $\omega = 0$ but may be non-zero at $\omega = \pi$ if M is odd. Give an example of a filter for which this is the case. [2]

iii) Derive an expression for the phase response, $\angle H(e^{j\omega})$, and determine the group delay, $\tau_H(e^{j\omega}) = -\frac{d\angle H(e^{j\omega})}{d\omega}$. [2]

e) Figure 1.2 shows the analysis and synthesis sections of a subband processing system. The input and output signals are $x[n]$ and $y[n]$ respectively and the intermediate signals are $v_m[n]$, $u_m[r]$ and $w_m[n]$ where $m = 0$ or 1 according to the subband. The corresponding z-transforms are $X(z)$, $Y(z)$ etc.

i) Show that it is possible to express the overall transfer function in the form $Y(z) = \begin{bmatrix} T(z) & A(z) \end{bmatrix} \begin{bmatrix} X(z) \\ X(-z) \end{bmatrix}$ and determine expressions for $T(z)$ and $A(z)$.

You may assume without proof that for $m = 0$ or 1, [3]

$$\begin{aligned} U_m(z) &= \frac{1}{2}\left\{V_m\left(z^{\frac{1}{2}}\right) + V_m\left(-z^{\frac{1}{2}}\right)\right\} \\ W_m(z) &= U_m\left(z^2\right). \end{aligned}$$

ii) Explain why it is normally desirable to have $A(z) \equiv 0$. [2]

iii) Suppose that $H_0(z) = H_1(-z) = G_0(z) = -G_1(-z)$. Show that in this case $A(z) = 0$ and demonstrate how the frequency responses $H_1(e^{j\omega})$, $G_0(e^{j\omega})$ and $G_1(e^{j\omega})$ are related to $H_0(e^{j\omega})$ assuming that $H_0(z)$ is an FIR or IIR filter with real coefficients. [2]

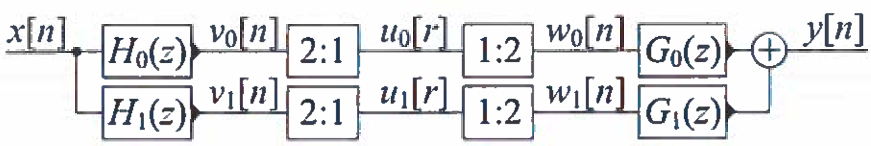

Figure 1.2

f) Figure 1.3 shows an upsampler with real-valued input $x[n]$ and output

$$y[r] = \begin{cases} x\left[\frac{r}{K}\right] & \text{if } K \mid r \\ 0 & \text{otherwise} \end{cases}$$

where $K \mid r$ means K is a factor of r.

i) Show that $Y(z) = X\left(z^K\right)$. [1]

ii) The energy and average power of $x[n]$ are defined respectively as

$$\begin{aligned} E_x &= \sum_{n=-\infty}^{\infty} |x[n]|^2 \\ P_x &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2. \end{aligned}$$

Give expressions for the energy and average power of $y[r]$ in terms of E_x and P_x. [2]

iii) Figure 1.4 shows the power spectral density of $x[n]$ which comprises white noise of unit magnitude together with a bandpass signal component occupying the range $0.5 < \omega < 1$. Sketch the power spectral density of $y[r]$ when $K = 3$ and give the magnitudes of its white noise component and the magnitude and frequency range of all bandpass components. [3]

iv) The diagram of Fig. 1.3 is followed by a lowpass filter to remove spectral images. If $K = 3$ and $x[n]$ is as specified in part iii) above, determine the transition bandwidth and transition band centre frequency of a suitable lowpass filter and explain the reasons for your choices.

[2]

Figure 1.3

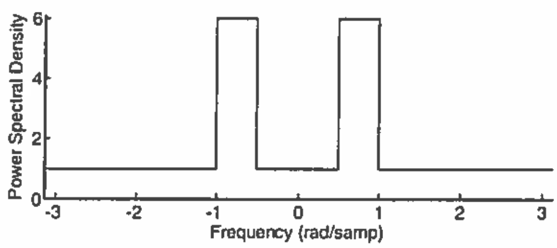

Figure 1.4

2. a) Suppose that $G_1(z) = 1 - pz^{-1}$ and $G_2(z) = 1 - qz^{-1}$ where the constants p and q may be complex. If $q = \frac{1}{p^*}$ show that $|G_1(e^{j\omega})| = \alpha |G_2(e^{j\omega})|$ for all ω and determine an expression for the constant α. [4]

b) Suppose that $H_1(z) = 4 + 14z^{-1} - 8z^{-2}$. Determine the coefficients of $H_2(z)$ such that $|H_1(e^{j\omega})| = |H_2(e^{j\omega})|$ for all ω and that all the zeros of $H_2(z)$ lie inside the unit circle. [4]

c) When designing an IIR filter $H(e^{j\omega}) = \frac{B(e^{j\omega})}{A(e^{j\omega})}$ to approximate a complex target response $D(\omega)$ two error measures that may be used are the weighted solution error, $E_S(\omega)$, and the weighted equation error, $E_E(\omega)$, defined respectively by

$$\begin{aligned} E_S(\omega) &= W_S(\omega)\left(\frac{B(e^{j\omega})}{A(e^{j\omega})} - D(\omega)\right) \\ E_E(\omega) &= W_E(\omega)\left(B(e^{j\omega}) - D(\omega)A(e^{j\omega})\right). \end{aligned}$$

Explain the relative advantages of the two error measures and explain the purpose of the real-valued non-negative weighting functions $W_S(\omega)$ and $W_E(\omega)$. [2]

d) Suppose that $0 \le \omega_1 < \omega_2 < \ldots < \omega_K \le \pi$ is a set of K frequencies and that $A(z) = 1 + [z^{-1}\ z^{-2}\ \cdots\ z^{-N}]\mathbf{a}$ and $B(z) = [1\ z^{-1}\ z^{-2}\ \cdots\ z^{-M}]\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are real-valued coefficient column vectors.

i) Show that it is possible to express the equations $E_E(\omega_k) = 0$ for $1 \le k \le K$ as a set of K simultaneous linear equations in the form $(\mathbf{P}\ \mathbf{Q})\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\end{pmatrix} = \mathbf{d}$.

State the dimensions of the matrices $\mathbf{P}$ and $\mathbf{Q}$ and of the vector $\mathbf{d}$ and derive expressions for the elements of $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{d}$. [4]

ii) Explain how, by separating the real and imaginary parts of $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{d}$, it is possible to obtain a set of simultaneous linear equations for $\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\end{pmatrix}$ in which all coefficients are real-valued. Explain the circumstances under which some of the resultant equations will necessarily have all-zero coefficients. [4]

iii) Explain why it may be desirable to apply the transformation of part b) after obtaining the solution to the equations of part d)ii). [2]

iv) Assuming that $\omega_1 = 0$ and $\omega_K = \pi$, determine the minimum value of K to ensure that the equations of part d) ii) are not underdetermined. [4]

e) Suppose now that $H(z) = \frac{b}{1+az^{-1}}$, that $K = 3$, that $\omega_k = 0.5\,(k-1)\,\pi$, that

$$\begin{aligned} D(\omega) &= \begin{cases} 2 & \text{for } \omega \le 0.25\pi \\ 1 & \text{for } \omega > 0.25\pi \end{cases}. \\ W_E(\omega) &\equiv 1 \end{aligned}$$

Determine the numerical values of the elements of $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{d}$ and hence determine the numerical values of a and b that minimize $\sum_k |E_E(\omega_k)|^2$. [6]

You may assume without proof that the least squares solution to an overdetermined set of real-valued linear equations, $\mathbf{R}\mathbf{x} = \mathbf{q}$, is given by $\mathbf{x} = \left(\mathbf{R}^T\mathbf{R}\right)^{-1}\mathbf{R}^T\mathbf{q}$ assuming that $\mathbf{R}$ has full column rank.

3. a) Figure 3.1 shows the block diagram of a system that multiplies the input sample rate by $\frac{P}{Q}$ where P and Q are coprime with $P < Q$.

i) Explain why the cutoff frequency of the lowpass filter $H(z)$ should be placed at the Nyquist rate of the output signal, $y[m]$ and give the normalized cutoff frequency, ω_0, in rad/sample in terms of P and/or Q.

Using the approximation formula $M \approx \frac{a}{3.5\Delta\omega}$, determine the required filter order M in terms of P and/or Q if the stopband attenuation in dB is $a = 60$ and the normalized transition bandwidth is $\Delta\omega = 0.1\omega_0$. [4]

ii) Using the value of M from part a) i), estimate the average number of multiplications per input sample, $x[n]$, needed to implement the system. [2]

iii) The filter $H(z)$ has a symmetrical impulse response $h[r] = g]r]w[r]$ for $0 \le r \le M$ where $g[r[$ is the impulse response of an ideal lowpass filter with cutoff frequency ω_0 and $w[r]$ is a symmetrical window function.

Derive an expression for the ideal response, $g[r]$, in terms of ω_0, M and r. [4]

b) The filter $H(z)$ is now implemented as a polyphase filter as shown in Fig. 3.2. The filter implementation uses a single set of delays and multipliers with commutated coefficients.

i) State the length of the filter impulse response $h_0[n]$ in terms of M, P and/or Q and give an expression for the coefficients $h_0[n]$ in terms of $h[r]$. [2]

ii) If $x]n] = 0$ for $n < 0$, give expressions for $v[0]$, $v]1]$, $v[2P+1]$ in terms of the input $x[n]$ and the coefficients $h_p[n]$. [2]

iii) Explain how it is possible to eliminate the output decimator by changing both the order and rate at which the coefficient sets, $h_p[n]$ are accessed.

Determine the new coefficient set order for the case $P = 5$ and $Q = 7$. [3]

iv) Determine the number of multiplications per input sample for the system of part b) iii) and the number of distinct coefficients that must be stored. You may assume that $M+1$ is a multiple of P. [2]

c) Suppose now that the sample rate of the input, $x[n]$, is 18 kHz and that the system is implemented as in part b) iii) with the values of a and $\Delta\omega$ as given in part a) i).

Determine the values of P, Q and M when the sample rate of the output, $y[m]$, is (i) 10 kHz and (ii) 10.1 kHz [note that 101 is a prime number].

For each of these cases estimate the number of multiplications per input sample and the number of distinct coefficients that must be stored. [5]

d) In a Farrow filter, the coefficients, $h_p[n]$, are approximated by a low-order polynomial $f_n(t)$ where $t = \frac{p}{P}$ for $0 \le p \le P-1$.

i) Assuming that a rectangular window, $w[r] \equiv 1$, is used in the design of $H(z)$ and that $\omega_0 = \frac{\pi}{P}$, give an expression for the target value of $f_0(t)$ in terms of t, M and P. [3]

ii) If the polynomials, $f_n(v)$, are of order $K = 5$, determine the number of coefficients that must be stored for each of the cases defined in part c).

[3]

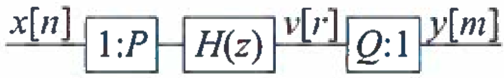

Figure 3.1

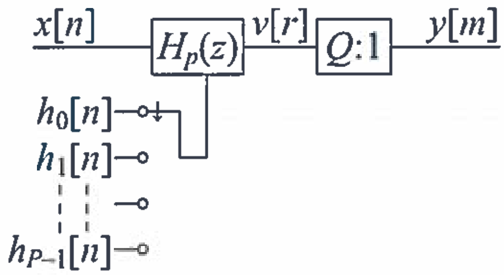

Figure 3.2

4. A complex-valued frequency-modulated signal, $x(t) = a(t)e^{j\phi(t)}$, has a $0\,Hz$ carrier frequency and a peak frequency deviation of $d = 75$kHz. The amplitude, $a(t)$, is approximately constant with $a(t) \approx 1$ and the phase is $\phi(t) = k\int_0^t m(\tau)d\tau$ where k is a constant and $m(t)$ is a baseband audio signal with bandwidth $b = 15$kHz. The signal $x(t)$ is sampled at 400kHz to obtain the discrete-time signal $x[n]$.

a) Carson's rule for the bandwidth of a double-sideband FM signal is $B = 2(d+b)$. Use this to determine the single-sided bandwidth, ω_0, of $x[n]$ in radians/sample. [2]

b) Show that $m(t) = k^{-1}a^{-2}(t)\Im\left(x^*(t)\frac{dx(t)}{dt}\right)$ where $\Im()$ denotes the imaginary part. [4]

c) Figure 4.1 shows a block diagram that implements the equation of part b) in discrete time. Complex-valued signals are shown as bold lines and are represented using their real and imaginary parts. The block labelled "Conj" takes the complex conjugate of its input. The differentiation block, $D(z)$, is designed as an FIR filter using the window method with a target response

$$\overline{D}(e^{j\omega}) = \begin{cases} jc\omega & \text{for } |\omega| \le \omega_1 \\ 0 & \text{otherwise} \end{cases}$$

where c is a scaling constant.

i) Determine the impulse response $\overline{d}[n]$ of $\overline{D}(z)$ in simplified form.[4]

ii) Assuming that $\omega_1 = \frac{\omega_0+\pi}{2}$, draw dimensioned sketches showing the magnitude and phase responses of $\overline{D}(e^{j\omega})$ over the range $-\pi \le \omega \le \pi$.

[3]

iii) Assume that the DTFT of the window function used when designing $D(z)$ has a main lobe width of $\omega = \pm\frac{18}{M+1}$ for a window of length $M+1$. If ω_1 is chosen as $\omega_1 = \frac{\omega_0+\pi}{2}$, determine the smallest value of M that will ensure that the transition in the response of $D(e^{j\omega})$ near $\omega = \omega_1$ lies completely within the range (ω_0, π). [3]

iv) Stating any assumptions, determine the maximum value of c that will ensure $|s[n]| \le 1$ where $s[n]$ is the output of the differentiation block, $D(z)$, as shown in Figure 4.1. [4]

d) An alternative choice for the target response is

$$\widetilde{D}(e^{j\omega}) = \begin{cases} \frac{-jc\omega_1(\pi+\omega)}{\pi-\omega_1} & \text{for } -\pi < \omega \le -\omega_1 \\ jc\omega & \text{for } |\omega| \le \omega_1 \\ \frac{jc\omega_1(\pi-\omega)}{\pi-\omega_1} & \text{for } \omega_1 < \omega \le \pi \end{cases}$$

i) Assuming that $\omega_1 = \frac{\omega_0+\pi}{2}$, draw dimensioned sketches showing the magnitude and phase responses of $\widetilde{D}(e^{j\omega})$ over the range $-\pi \le \omega \le \pi$.

[4]

ii) Outline the relative advantages and disadvantages of using $\widetilde{D}(e^{j\omega})$ rather than $\overline{D}(e^{j\omega})$ as the target response when designing $D(e^{j\omega})$. [2]

e) An alternative structure that avoids any divisions is shown in Fig. 4.2 where the polynomial $f(v)$ is the truncated Taylor series for v^{-1} expanded around $v = 1$. Determine $f(v)$ for the cases when it is (i) a linear expression and (ii) a quadratic expression. In each case determine the gain error (expressed in dB) resulting from the approximation when $a(t) = 1.1$. [4]

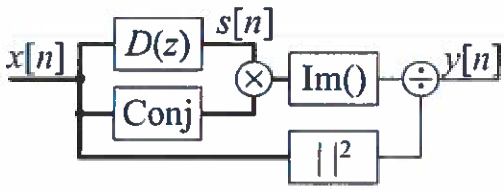

Figure 4.1

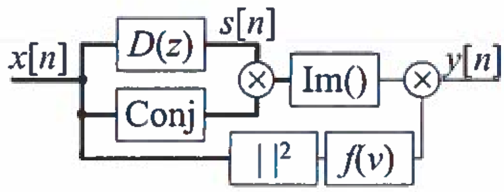

Figure 4.2