IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

COMMUNICATIONS 2

Corrected Copy

Wednesday, 28 May 2:00 pm

Time allowed: 2:00 hours

There are FIVE questions on this paper.

Answer THREE questions.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	D.B. Ward
	Second Marker(s) :	J.A. Barria

1. (a) Consider the system shown in Figure 1.1 below,

$$x(t) \longrightarrow \boxed{H(f)} \longrightarrow y(t)$$

Figure 1.1

where $x(t)$ is the input signal, $y(t)$ is the output signal, and $H(f)$ is the transfer function of the system.

(i) If $S_x(f)$ is the power spectral density (PSD) of the input signal, write down an expression for the PSD of the output signal. [1]

(ii) If the transfer function of the system is: $H(f) = \dfrac{1}{1 + j\left(\dfrac{f}{f_o}\right)}$

and the input has a uniform PSD, $S_x(f) = N_o / 2$, write down an expression for the PSD of the output. Also, draw a labeled sketch of the PSD of the output. [4]

(iii) Write down an expression for the autocorrelation of the output signal in terms of the PSD of the output signal. [1]

(iv) Hence, write down an expression for the average power of the output signal in terms of the autocorrelation of the output signal. [1]

(b) Bandpass noise can be represented by the expression:

$$n(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

(i) Explain why this representation is used, with reference to evaluating the SNR performance of modulated analog systems. [4]

(ii) If the PSD of $n(t)$ is shown in Figure 1.2 below, draw a labeled diagram of the PSD of each of $n_c(t)$ and $n_s(t)$. [3]

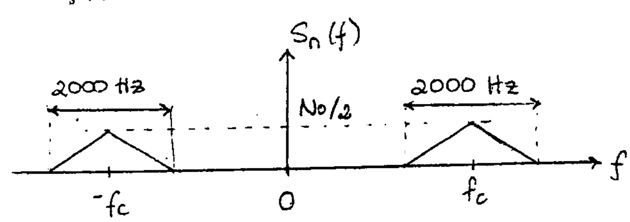

Figure 1.2

(c) Consider the random signal $x(t) = a\cos(\omega_c t)$, where ω_c is a constant, and a is a random variable that is uniformly distributed in the range $[-A, A]$. Evaluate both the mean and mean-square values of the signal $x(t)$. [6]

2. (a) Consider a communications channel with a maximum error-free transmission rate of 1500 bits/second. Assume the source produces symbols at a rate of 1000 symbols/second. The source alphabet consists of the symbols $\{A, B, C, D, E\}$, which have probabilities of occurrence of $p_A = 0.07$, $p_B = 0.7$, $p_C = 0.04$, $p_D 0.14$, $p_E = 0.05$.

Assuming that we want to achieve reliable communication, answer each of the following questions providing a justification for your answer.

(i) Can a binary code with equal codeword length be used over this channel? [1]

(ii) Is it theoretically possible to use a variable-length code over this channel? [3]

(iii) Can a first-order Huffman code be used over this channel? [7]

(b) Write down the Hartley-Shannon channel capacity formula, explaining and giving units for each symbol used. What is the significance of this formula? [4]

(c) Using the Hartley-Shannon channel capacity formula, show that for the ideal analog communications system shown in Figure 2.1 below, there is an exponential relationship between channel bandwidth and the output SNR. [5]

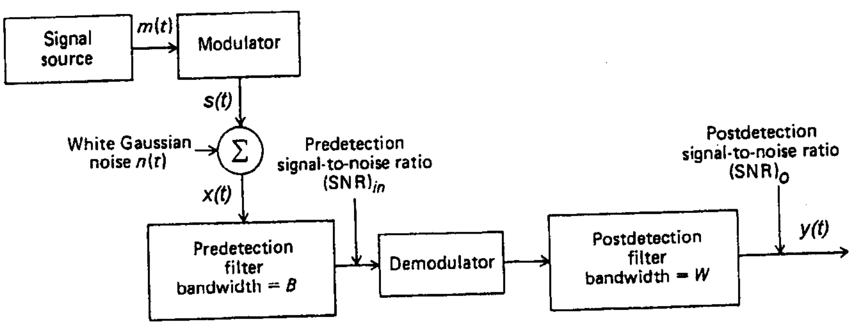

Figure 2.1

NOTE: You may find the following formula useful: $\log_2 x = 3.32 \log_{10} x$

3. (a) A sinusoidal message signal whose frequency is less than 1000 Hz, modulates a carrier $c(t) = 10^{-3}\cos(2\pi f_c t)$. The modulation scheme is conventional AM and the modulation index is $\mu = 0.5$. The channel noise is additive white with power spectral density (PSD) of $N_o/2 = 10^{-12}$ W/Hz. At the receiver, the signal is processed as shown in Figure 3.1 below. The frequency response of the bandpass noise-limiting filter is shown in Figure 3.2 below.

(i) Find both the signal power and the noise power at the output of the bandpass noise-limiting filter. [4]

(ii) Find the output SNR. [4]

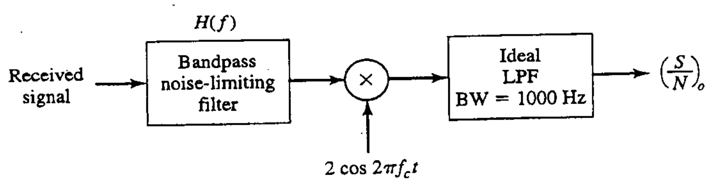

Figure 3.1

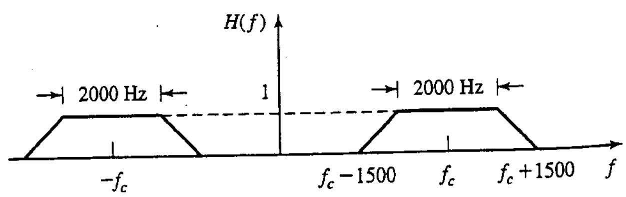

Figure 3.2

(b) Consider an FM modulated waveform:

$$s(t) = A\cos\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt\right)$$

where $m(t)$ is the message signal, and f_c is the carrier frequency.

(i) Derive an expression for the pre-detection SNR at the receiver (i.e., just after the bandpass noise-limiting filter in the receiver). Assume that the transmission bandwidth is given by Carson's rule as: $B_T = 2(\beta + 1)W$
where β is the deviation ratio, and W is the message bandwidth. Assume the channel noise has a flat PSD of $N_o/2$. [4]

(ii) If the pre-detection SNR satisfies the threshold requirement: $SNR_{\text{pre}} > 10$

then the output SNR is given by: $SNR_{\text{out}} = 3\beta^2 \frac{P}{m_p^2} SNR_{\text{base}}$,

where P is the average message power, m_p is the peak message amplitude, SNR_{base} is the SNR of an equivalent baseband system having the same transmitted power.

Using your results from part (i), find an expression for the output SNR at threshold. [8]

4. (a) Consider a baseband digital system that has three symbols. The symbol "1" is represented by A volts, the symbol "0" is represented by 0 volts, and the symbol "-1" is represented by $-A$ volts. The channel noise is a low-pass stationary process having the probability density function shown in Figure 4.1 below. The decision device at the output of the receiver operates as follows:

$$\text{Symbol} = \begin{cases} "1", & \text{if } y > T \\ "0", & \text{if } -T < y < T \\ "-1", & \text{if } y < -T \end{cases}$$

where y is the sampled signal at the receiver.

(i) Find an expression for the probability of error for each symbol. Hence find the probability of error for each symbol in the special case where $T = A/2$.

[7]

(ii) Write down an expression for the threshold value that should be used such that the probability of error for each symbol is the same?

[3]

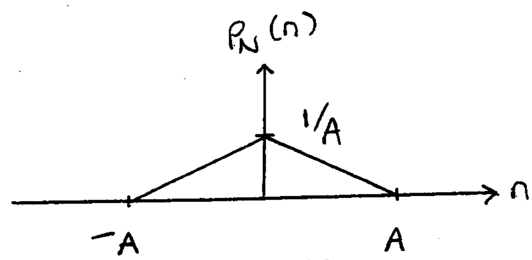

Figure 4.1

(b) A signal can be modeled as a low-pass stationary process $X(t)$ whose probability density function at any time t_0 is given in Figure 4.2 below.

The bandwidth of this process is 5 kHz, and it is desired to transmit it using a pulse-code modulation (PCM) system.

(i) If a uniform 4-bit quantizer is used, what is the resulting signal-to-quantization-noise ratio (SQNR)? Assume that the quantizer range is [-1,1] volts.

[6]

(ii) If the available bandwidth of the channel is 40 kHz and sampling is done at the Nyquist rate, what is the highest achievable SQNR?

[4]

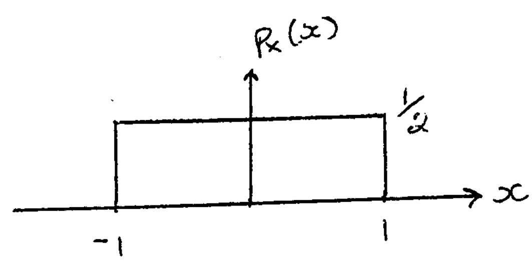

Figure 4.2

5. Answer any two of the following subsections (a), (b), and (c).

(a) Describe the mechanism for communication between two peers at layer N in a network, with reference to protocols and protocol architectures.

[10]

(b) Describe the functions performed by each of the seven layers of the OSI model.

[10]

(c) Describe the operation of a broadcast network, with reference to bus and ring topologies, and medium access control.

[10]

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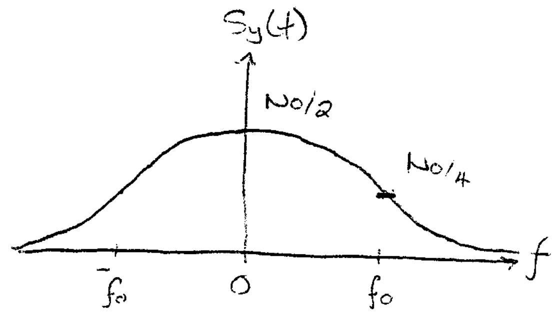
Q1 (a)

(i) $S_Y(f) = |H(f)|^2 \, S_X(f)$

(ii) $|H(f)|^2 = \dfrac{1}{1+(f/f_0)^2}$

$\therefore S_Y(f) = \dfrac{N_0/2}{1+(f/f_0)^2}$



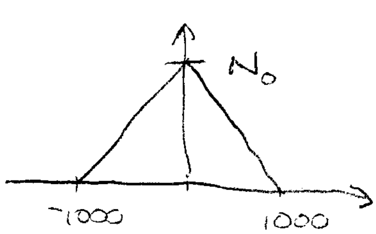
(iii) $R_Y(\tau) = \int_{-\infty}^{\infty} S_Y(f)\, e^{j2\pi f\tau}\, df$

(iv) $P_Y = R_Y(0) = \int_{-\infty}^{\infty} S_Y(f)\, df.$

(b) (i) For modulated signals, the receiver always includes a pre-detection BPF to limit out-of-band noise. This means that noise entering the receiver is bandpass.

Also, the demodulator operates by shifting frequencies close to the carrier to baseband. Because the band-limited noise representation consists of noise terms modulated by a carrier, the given representation ensures that the effect of demodulation on the noise can be easily evaluated analytically. Hence, one can derive a analytical expression for the SNR at the output.

(ii)



Both PSD's are given by this diagram

(c) $x(t) = a \cos \omega t$

$$p(a) = \begin{cases} \frac{1}{2A}, & |a| < A \\ 0, & \text{otherwise} \end{cases}$$

Mean: $E\{x\} = \int_{-\infty}^{\infty} x(a) p(a) \, da$

$$= \frac{1}{2A} \int_{-A}^{A} a \cos \omega t \, da$$

$$= \frac{\cos \omega t}{2A} \left[\frac{1}{2} a^2 \right]_{-A}^{A}$$

$$= 0$$

Mean square: $E\{x^2\} = \int_{-\infty}^{\infty} x^2(a) \, p(a) \, da$

$$= \frac{1}{2A} \int_{-A}^{A} a^2 \cos^2 \omega t \, da$$

$$= \frac{\cos^2 \omega t}{2A} \left[\frac{1}{3} a^3 \right]_{-A}^{A}$$

$$= \frac{\cos^2 \omega t}{2A} \frac{1}{3} 2A^3$$

$$= \frac{1}{3} A^2 \cos^2 \omega t$$

Q2. (a)

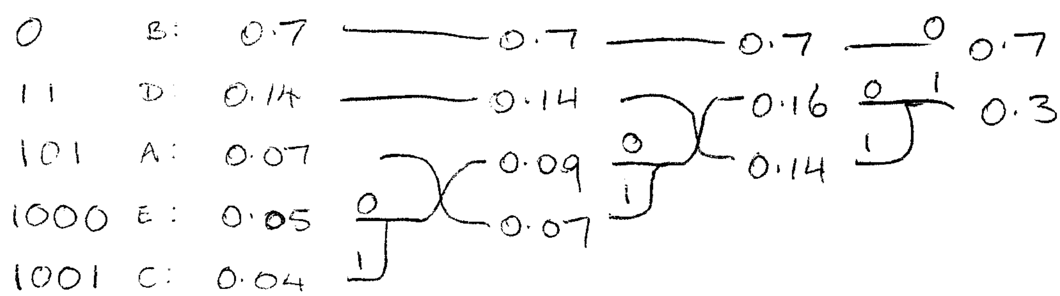
(i) For 5 symbols, would require a 3-bit word. This would require a transmission of $3 \times 1000 = 3\text{kb/s}$. The channel cannot handle this rate, so NO.

(ii) The entropy of the alphabet gives the minimum variable length code.

$$H(S) = -\sum p_k \log_2 p_k$$
$$= 1.427$$

With an average code length of 1.427 bits/sym, this would have a rate of 1427 bits/sec, which is below 1500 bps. So YES

(iii)



$$\bar{L} = \sum p_k l_k$$
$$= 0.7 \times 1 + 0.14 \times 2 + 0.07 \times 3 + 0.05 \times 4 + 0.04 \times 4$$
$$= 1.55$$

This requires a rate of 1550 bits/sec which is above the transmission rate, so NO.

(b)

$$C = B \log_2 \left(1 + P_S/P_N\right)$$

C : channel capacity, bits per second
B : channel bandwidth, Hz
P_S: average signal power at receiver, W
P_N: average noise power at receiver, W

This formula gives the maximum rate of reliable information transfer. Above this rate it is not possible to find a coding scheme that provides reliable transfer.

(c) Let B be bandwidth of channel, & W be bandwidth of message. Let SNR_{in} be pre-detection SNR, & let SNR_o be output SNR.

Maximum rate of info into receiver:

$$C_{in} = B \log_2 (1 + SNR_{in})$$

Maximum rate of info out:

$$C_{out} = W \log_2 (1 + SNR_o)$$

But these must be equal, so

$$B \log (1 + SNR_{in}) = W \log (1 + SNR_o)$$

$$\therefore \quad (1 + SNR_{in})^{B/W} = 1 + SNR_o$$

$$\therefore \quad SNR_o = (1 + SNR_{in})^{B/W} - 1$$

Hence, there is an exponential relationship between SNR_{out} & bandwidth

Q3. (a) For AM we have:

$$s(t) = [A + m(t)] \cos \omega_c t$$

Carrier is: $c(t) = A \cos \omega_c t \quad \therefore A = 10^{-3}$

Mod'n index:

$$\mu = \frac{m_p}{A} = 0.5 \quad \therefore m_p = 0.5 \times 10^{-3} \quad \therefore m_p = A/2$$

$\therefore$ Message signal:

$$m(t) = 0.5 \times 10^{-3} \cos \omega_m t \quad , \omega_m < 2\pi \times 1000$$

(i) Predetection avg signal power:

$$P_S = \langle [(A + A/2 \cos \omega_m t) \cos \omega_c t]^2 \rangle$$

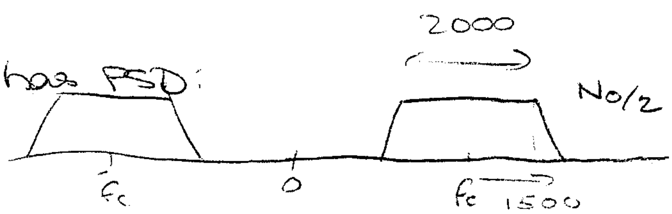
$$= \langle (A + A/2 \cos \omega_m t)^2 \cos^2 \omega_c t \rangle$$

$$= \langle A^2 (1 + \tfrac{1}{2} \cos \omega_m t)^2 \cos^2 \omega_c t \rangle$$

$$= \frac{A^2}{2}\left(1 + \frac{1}{4} \times \frac{1}{2}\right)$$

$$= \frac{10^{-6}}{2}\left(1 + \frac{1}{4} \times \frac{1}{2}\right) = 5.625 \times 10^{-7} \text{ W}$$

P_N has PSD:



$$\therefore P_N = \cancel{\tfrac{N_0}{2} \times 2000} \; 2 \times \left(\frac{N_0}{2} \times 2000 + \frac{N_0}{2} \times 500\right)$$

$$= N_0 \times 2500$$

$$= 10^{-12} \times 5000 = 50 \times 10^{-10} \text{ W}$$

(iii) The receiver output is found as follows.
After the multiplier, we have:

$$x(t) = \Big[(A + m(t)) \cos \omega_c t + n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t \Big] \times 2 \cos \omega_c t$$

$$= 2\left[A + m(t) + n_c(t)\right] \cos^2 \omega_c t - 2 n_s(t) \sin \omega_c t \cos \omega_c t$$

And after LPF this becomes:

$$y(t) = \cancel{A} + m(t) + n_c(t)$$

ignore due to DC block.

$\therefore$ Signal power, $P_S = E\{m^2\}$

$$= \frac{m_p^2}{2} = \frac{1}{2} \frac{A^2}{4} = \frac{A^2}{8} = \frac{10^{-6}}{8} = 1.25 \times 10^{-7}\ W$$

Noise power, $P_N = 50 \times 10^{-10}\ W$

$\therefore$ SNR $= 25$ or $14\ dB$

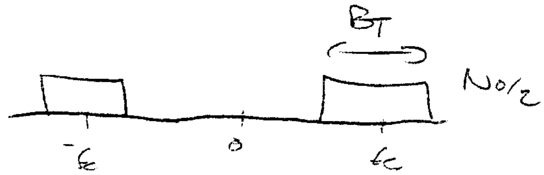
(b)

(i) Predetection signal is:

$$x(t) = A\cos[\omega_c t + 2\pi k_f \phi(t)] + n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

Signal power: $P_S = A^2/2$

Noise has PSD as:

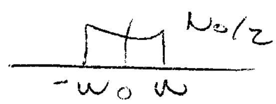


$$\therefore P_N = 2 \times \frac{N_0}{2} \times B_T$$

$$= N_0 B_T = N_0\, 2(\beta+1)W$$

$$\therefore SNR_{pre} = \frac{A^2}{4N_0(\beta+1)W}$$

(ii) $SNR_{base} = \dfrac{P_T}{N_0 W}$



$$= \frac{A^2}{2N_0 W}$$

Above threshold we have: $SNR_0 = 3\beta^2 \dfrac{P}{m_p^2} \times \dfrac{A^2}{2N_0 W}$

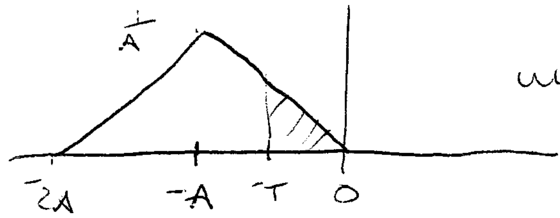
But at threshold: $\dfrac{A^2}{4N_0(\beta+1)W} = 10 \quad \therefore \dfrac{A^2}{2N_0 W} = 20(\beta+1)$

$$\therefore SNR_0 = 3\beta^2 \frac{P}{m_p^2} \times 20(\beta+1)$$

$$= 60\,\beta^2(\beta+1)\frac{P}{m_p^2}$$

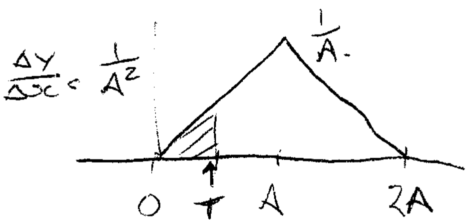
Q4 1(a)

(i) Consider the symbol "-1". The pdf at the output is:



where the shaded region is $P_e(1)$

Similarly, for symbol "1":

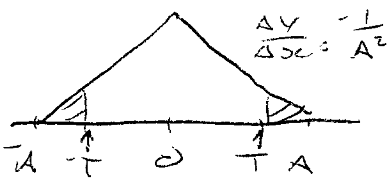


by symmetry we have $P_{e(-1)} = P_{e1}$

$$\therefore P_{e1} = \frac{1}{2} \times T \times \frac{1}{A^2} \times T$$

$$= \frac{T^2}{2A^2}$$

For symbol "0" we have:



$$\therefore y = \frac{-1}{A^2} T + \frac{1}{A}.$$

$$\therefore P_{e0} = 2 \times \frac{1}{2} \times (A-T) \times \left(\frac{-1}{A^2} T + \frac{1}{A} \right)$$

$$= (A-T) \frac{1}{A} \left(1 - \frac{T}{A}\right)$$

$$= A\left(1 - \frac{T}{A}\right) \frac{1}{A} \left(1 - \frac{T}{A}\right)$$

$$= \left(1 - \frac{T}{A}\right)^2$$

(ii) For equal probabilities, we require:

$$\frac{T^2}{2A^2} = \left(1 - \frac{T}{A}\right)^2 = 1 - \frac{2T}{A} + \frac{T^2}{A^2}$$

$$\therefore T^2/2 = A^2 - 2TA + T^2$$

$$\therefore \frac{1}{2} T^2 - 2AT + A^2 = 0$$ Solving for T gives threshold.

(b)

(i) $P_S = E\{x^2\} = \int_{-\infty}^{\infty} x^2 \, p(x) \, dx$

$$= \frac{1}{2} \int_{-1}^{1} x^2 \, dx$$

$$= \frac{1}{2} \times \frac{1}{3}\left(x^3\right)_{-1}^{1} = \frac{1}{3}$$

$P_N = \dfrac{\Delta^2}{12}$ but $\Delta = \dfrac{2}{2^n}$ $\therefore \Delta^2 = \dfrac{4}{2^{2n}}$

$$\therefore SQNR = \frac{1}{3} \times 12 \times \frac{2^{2n}}{4}$$

$$= 2^8 = 256 = 24 \text{ dB}$$

(ii) PCM requires a BW of $B_T = nW$ for Nyquist sampling

$\therefore$ If $B_T = 40\text{kHz}$, $W = 5\text{kHz}$ $\therefore n_{max} = 8$.

$\therefore SQNR_{max} = 2^{16} = 65536 = 48 \text{ dB}$

Q5 (a)

PROTOCOLS & ARCHITECTURES

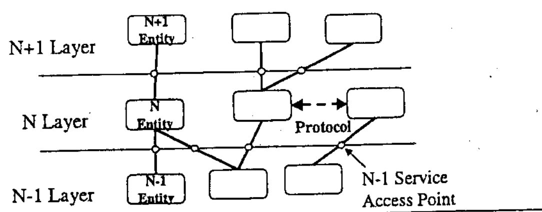
- High level of cooperation between communicating stations
- Example – file transfer:
  - source must activate communication path to destination
  - source must ascertain that destination is prepared to receive data
  - file transfer application on source must ascertain that file management program on destination is prepared
  - may require format translation if file formats incompatible
- Network architecture: protocols and protocol architectures

Protocol

- Used for communication between similar entities on different systems
- Entity: anything capable of sending/receiving data (eg. file transfer packages, email)
- System: physically distinct object that contains one or more entities (eg. computers)
- Key elements of protocol:
  - syntax (eg. data format, signal levels)
  - semantics (eg. control info, error handling)
  - timing (eg. speed matching, sequencing)

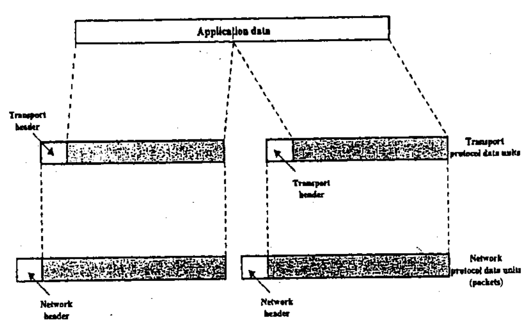
Protocol architecture

- Break cooperation logic into sub tasks, each implemented separately – layers
- Each layer offers services to higher layers, hiding implementation details
- Layer N on one system talks to layer N on other – layer N protocol
- Between adjacent layers is interface that defines operations/services lower layer offers to upper layer



Protocol data unit

- PDU – combination of data from next higher layer and control info



Q5 (b)

Application

Provides access to the OSI environment for users and also provides distributed information services.

Presentation

Provides independence to the application processes from differences in data representation (syntax).

Session

Provides the control structure for communication between applications; establishes, manages, and terminates connections (sessions) between cooperating applications.

Transport

Provides reliable, transparent transfer of data between end points; provides end-to-end error recovery and flow control.

Network

Provides upper layers with independence from the data transmission and switching technologies used to connect systems; responsible for establishing, maintaining, and terminating connections.

Data Link

Provides for the reliable transfer of information across the physical link; sends blocks (frames) with the necessary synchronization, error control, and flow control.

Physical

Concerned with transmission of unstructured bit stream over physical medium; deals with the mechanical, electrical, functional, and procedural characteristics to access the physical medium.

- Application layer (Layer 7)
  - contains general purpose applications such as file transfer, email

- Presentation layer (Layer 6)
  - defines syntax used between application entities, and offers a set of data transformation services to applications
  - example services include data compression and encryption

- Session layer (Layer 5)
  - example services include allowing a user to login to a remote system, transfer files between two systems

- Transport layer (Layer 4)
  - basic function is to accept data from Session Layer, split it into smaller units, pass these to the Network Layer, and ensure that pieces all arrive correctly at the other end

- Network layer (Layer 3)
  - provides for transfer of info between end systems across network
  - system engages in dialogue with network to specify destination address and to request network facilities
  - key issues is determining how packets are routed from source to destination

- Data link layer (Layer 2)
  - higher layers can assume error-free transmission over link
  - attempts to make Physical Layer reliable

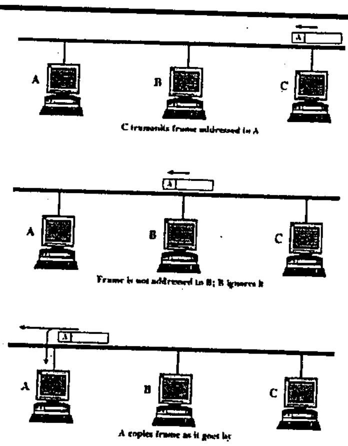
- Physical layer (Layer 1)
  - provides for transmission of bits over a physical medium, including procedures to activate and deactivate a physical circuit
  - only layer at which a direct physical connection may exist between source and destination
  - above Physical Layer, each protocol entity sends data down to next layer to get access to peer entity

Q5 (c)

BROADCAST COMMUNICATION NETWORKS

- Have following characteristics:
  - no switching devices in basic architecture
  - data transmitted by one station received by many, often all, other stations – requires address to indicate for whom transmission is intended
  - stations share common transmission medium – requires Medium Access Control (MAC) technique

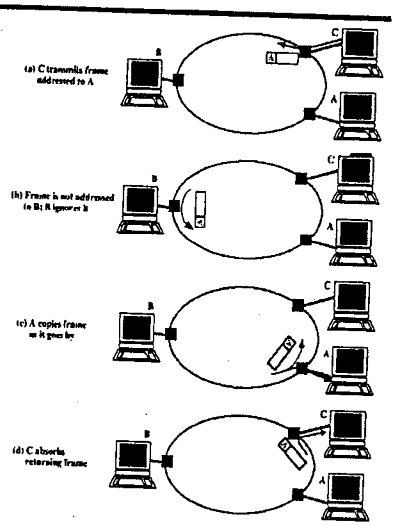
Bus topology



- All stations attach directly to linear transmission medium
- Transmission from any station propagates in both directions

Ring topology

- Set of repeaters joined in closed loop
- Repeater receives data on one link, and retransmits on other – links are unidirectional
- Each station attaches to network at a repeater



Medium access control (MAC)

- Many devices share network's transmission capacity, but only one can successfully transmit at a time
- Where should control be exercised?
  - (a) centralized
    - designated controller has authority to grant access to network
    - station wishing to transmit must wait to receive permission
  - (b) distributed
    - stations collectively perform MAC function to dynamically determine order in which stations transmit

Advantages of centralized:

- greater control
- simple logic at each station
- avoids coordination problem

Disadvantages of centralized:

- single point of failure
- may act as bottleneck

MAC (cont.)

- How should control be exercised?
  - (a) synchronous – specific capacity dedicated to each connection
  - (b) asynchronous – dynamic allocation of capacity
    - (i) round robin
      - each station in turn is given opportunity to transmit, station may decline, or may transmit data subject to upper bound
    - (ii) reservation
      - time on medium is divided into slots, and station wishing to transmit reserves future slots for extended period
    - (iii) contention
      - no control is exercised, all stations content for transmission time
      - simple to implement
      - performance tends to collapse under heavy load