IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2012

EEE PART III/IV: MEng, BEng and ACGI

ELECTRICAL ENERGY SYSTEMS

Tuesday, 22 May 2:30 pm

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer 2 questions from Section A and 2 questions from Section B. Use a separate answer book for each section.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : G. Strbac, B. Chaudhuri
Second Marker(s) : B. Chaudhuri, G. Strbac

Part A – Answer any 2 out of 3 questions in Part A

1. a) Why is the per-unit system used in analysis of power systems?

[4]

b) An SI representation of a transformer shown in Figure 1.1 (a) is composed of an ideal transformer and leakage reactance. Derive an equivalent per-unit representation shown in Figure 1.1(b).

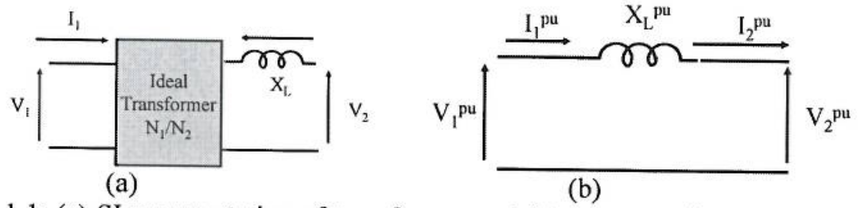


Figure 1.1: (a) SI representation of transformer and (b) corresponding per unit representation of transformer

[12]

c) A 33kV/11kV 15MVA transformer has a leakage reactance of 4.5 Ω as seen from the HV side. Calculate the pu impedance and show that this is the same as the pu impedance as seen from the LV side.

[4]

2. In Figure 2.1 a simplified equivalent circuit of a transmission line is presented.

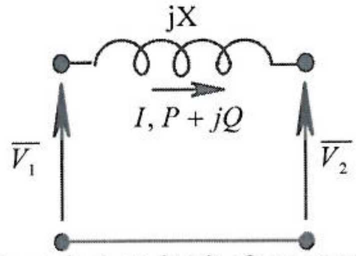


Figure 2.1: equivalent circuit of a transmission line

a) Show that expressions for active and reactive power transmitted across the line are given by:

$$P = \frac{V_1 V_2}{X} \sin(\alpha_1 - \alpha_2)$$

$$Q = \frac{V_1^2 - V_1 V_2 \cos(\alpha_1 - \alpha_2)}{X}$$

[10]

b) Estimate the maximum power that can be transmitted across a 500km 275kV transmission line with reactance of 0.42 Ohm/km.

[4]

c) Using the expression for reactive power, explain why reactive power is not transmitted over long distances. For the case when 120MW of active power is transmitted over the 275 kV circuit in (b), calculate the maximum amount of reactive power that can be transmitted if the maximum voltage drop that can be tolerated is 4%.

[4]

d) What other problems do you expect with 500km long 275kV transmission line?

[2]

3. Consider the simple power system in Figure 3.1. As shown in the figure, the generator maintains the voltage at 1pu at its terminals (node 1), while the compensator connected to node 2 maintains the voltage at 1.03 pu. Active and reactive power demand connected to node 2 are 0.5pu and 0.3pu respectively. Resistance and reactance of the transmission line are 0.02pu and 0.12 pu respectively.

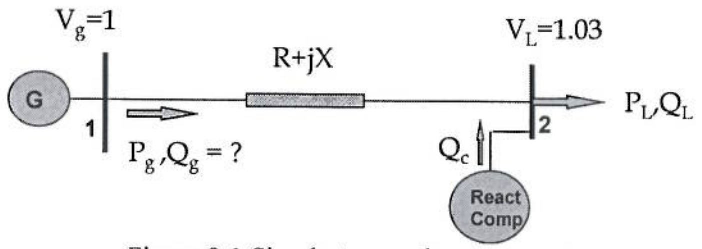


Figure 3.1 Simple two-node power system

a) Form the Ybus matrix for the system. [3]

b) Perform 3 iterations of the Gauss Seidel load flow. [10]

c) Calculate the output of the reactive power compensator. [3]

d) Calculate losses in the line and active and reactive power generated by the generator. [4]

Part B – Answer any 2 out of 3 questions in part B

4. a) State the four steps for calculating the fault current for balanced faults without making any assumption about the currents and voltages under pre-fault condition.

[5]

b) Explain using the theorem of constant flux linkage why the sub-transient reactance of a synchronous generator is less than its transient reactance and why the former is normally used for short circuit analysis.

[5]

c) A three-phase fault occurs at point F at the far end of the line in the system shown in Figure 4.1. Considering 30 MVA as the system base calculate the fault current in Amperes. Assume the pre-fault voltages to be 1.0 pu and neglect pre-fault currents.

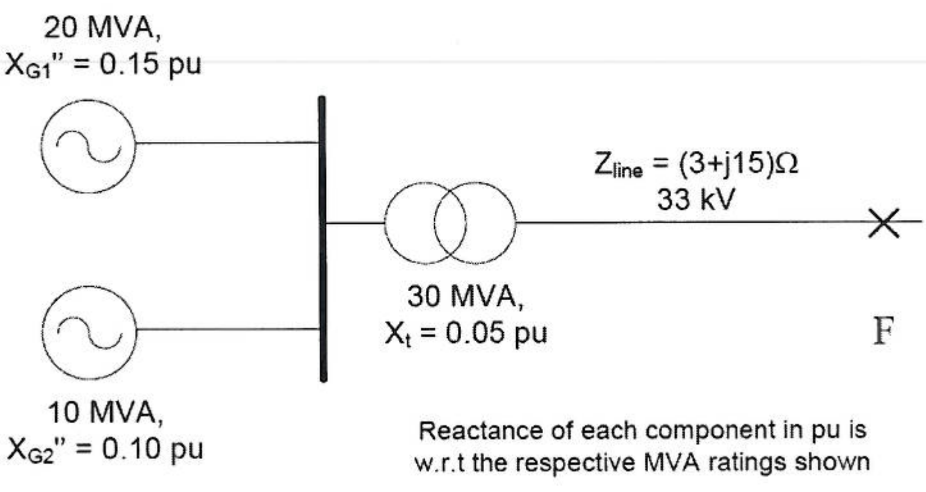


Figure 4.1: Two synchronous generators connected to a transmission line through a transformer

[10]

5. a) Use the method of symmetrical components to prove that the positive and negative sequence currents cannot exist in the neutral wire of a star connection.

[4]

b) Show that the three sequence networks are connected in parallel for line to line to ground (LLG) fault affecting phases B and C with zero fault impedance.

[6]

c) The positive-, negative-, and zero-sequence reactance of a 20 MVA, 13.2 kV three-phase synchronous generator are 0.3 pu, 0.2 pu and 0.1 pu, respectively. The generator is solidly grounded and is operating at no-load. Calculate the fault current (in Amperes) to ground for a line to line to ground (LLG) fault affecting phases B and C at the generator terminal. Neglect resistance of the generator and fault impedance.

[10]

6. a) Explain why operation in the region of positive slope on the standard power angle characteristics implies steady-state stability while that on the negative slope region implies instability. Explain without using the swing equation.

[5]

b) Derive an expression for critical clearing angle (δ_{cr}) in terms of the initial load angle (δ_0) assuming zero electrical power output during fault and identical standard power-angle characteristics during pre-fault and post-fault.

[5]

c) A synchronous generator with a maximum power output of 500 MW is operating at a power angle of $\delta_0 = 8^\circ$. The mechanical power input to the generator is increased suddenly.

i) Show that the power angle δ_m corresponding to the maximum allowable mechanical power input without loss of stability is given by the following equation:

$$(3 - \delta_m)\sin\delta_m - \cos\delta_m = 0.99$$

where δ_m is in radians.

[7]

ii) If the solution to the above equation in part (i) is $\delta_m = 50^\circ$, by how much can the mechanical power output increase suddenly without loss of stability?

[3]

SOLUTIONS 2012 EE3-13

Part A – Answer any 2 out of 3 questions in part A

1.

a) Why is the per-unit system used in analysis of power systems? [4]

For the full mark a short discussion around the following reasons should be listed:

Multiple voltage levels: 400kV, 275 kV, 132 kV

Makes circuit analysis rather confusing

Impedance of transformers depends on side

Normalise all quantities to help understanding

Avoid confusion due to transformers

b) An SI representation of a transformer shown in Figure 1.1 (a) is composed of an ideal transformer and leakage reactance. Derive an equivalent per-unit representation shown in Figure 1.1 (b). [12]

$$\frac{V_1}{V_2}=\frac{N_1}{N_2}$$

Choose:

$$\frac{V_{1B}}{V_{2B}}=\frac{V_{1\,Nom}}{V_{2\,Nom}}=\frac{N_1}{N_2}$$

$$\frac{V_1^{\,pu}}{V_2^{\,pu}}=\frac{\dfrac{V_1}{V_{1B}}}{\dfrac{V_2}{V_{2B}}}=\frac{V_1}{V_2}\times\frac{V_{2B}}{V_{1B}}=\frac{N_1}{N_2}\times\frac{N_2}{N_1}=1$$

$$V_1^{\,pu}=V_2^{\,pu}$$

$$\frac{I_1^{pu}}{I_2^{pu}}=\frac{\dfrac{I_1}{I_1^B}}{\dfrac{I_2}{I_2^B}}=\frac{I_1}{I_2}\cdot\frac{I_2^B}{I_1^B}$$

$$I_1^B=\frac{S^B}{V_1^B},\qquad I_2^B=\frac{S^B}{V_2^B}\qquad\Rightarrow\qquad \frac{I_2^B}{I_1^B}=\frac{V_1^B}{V_2^B}=\frac{N_1}{N_2}$$

$$\frac{I_1}{I_2}=\frac{N_2}{N_1}$$

$$\frac{I_1^{pu}}{I_2^{pu}}=\frac{N_2}{N_1}\cdot\frac{N_1}{N_2}=1\quad\Rightarrow\quad\boxed{I_1^{pu}=I_2^{pu}}$$

$$Z_1=\left(\frac{N_1}{N_2}\right)^2 Z_2$$

$$Z_1^{pu}\cdot Z_1^B=\left(\frac{N_1}{N_2}\right)^2 Z_2^{pu}\cdot Z_2^B$$

$$Z_1^B=\frac{V_1^B}{I_1^B}=\frac{V_1^{B\,2}}{S^B}\qquad Z_2^B=\frac{V_2^B}{I_2^B}=\frac{V_2^{B\,2}}{S^B}$$

$$Z_1^{pu}\cdot\frac{V_{1B}^2}{S_B}=\left(\frac{N_1}{N_2}\right)^2 Z_2^{pu}\cdot\frac{V_2^{B\,2}}{S^B}$$

$$\frac{V_1^{B\,2}}{V_2^{B\,2}}=\left(\frac{N_1}{N_2}\right)^2$$

$$\boxed{Z_1^{pu}=Z_2^{pu}}$$

c) A 33kV/11kV 15MVA transformer has a leakage reactance of 4.5 Ω as seen from the HV side. Calculate the pu impedance and show that this is the same as the pu impedance as seen from the LV side. [4]

Per unit reactance has the same numerical value from both sides of the transformer. First we calculated the impedance values from both sides and then the impedance per unit values from each one.

$$Z_1 = 4.5\Omega$$

$$Z_2 = \left(\frac{N_2}{N_1}\right)^2 Z_1 = \left(\frac{1}{3}\right)^2 Z_1 = 0.5\Omega$$

$$Z_{B1} = \frac{(33KV)^2}{(15\ MVA)} = 72.60\Omega$$

$$Z_{B2} = \frac{(11KV)^2}{(15\ MVA)} = 8.066\Omega$$

$$Z_{1pu} = \frac{4.5\Omega}{72.60\Omega} = 0.06198$$

$$Z_{2pu} = \frac{0.5\Omega}{8.066\Omega} = 0.06198$$

2

a) Show that expressions for active and reactive power transmitted across the line are given by:

$$P = \frac{V_1 V_2}{X} \sin(\alpha_1 - \alpha_2)$$

$$Q = \frac{V_1^2 - V_1 V_2 \cos(\alpha_1 - \alpha_2)}{X}$$

[12]

$$\mathbf{I} = \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathrm{j}X}$$

$$\mathbf{S} = \mathbf{V}_1 \mathbf{I}^*$$

$$\mathbf{S} = \mathbf{V}_1 \left(\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathrm{j}X} \right)^*$$

$$\mathbf{S} = \frac{V_1^2 - \left(V_1 \mathrm{e}^{j\alpha_1}\right)\left(V_2 \mathrm{e}^{-j\alpha_2}\right)}{-\mathrm{j}X} = \frac{V_1^2 - \left(V_1 V_2 \mathrm{e}^{\mathrm{j}(\alpha_1 - \alpha_2)}\right)}{-\mathrm{j}X}$$

$$\mathbf{S} = \frac{\mathrm{j}V_1^2 - \mathrm{j}\left(V_1 V_2 \mathrm{e}^{-\mathrm{j}(\alpha_1 - \alpha_2)}\right)}{X} = \frac{\mathrm{j}V_1^2 - \left(V_1 V_2 \mathrm{e}^{-\mathrm{j}(\alpha_1 - \alpha_2 + 90^\circ)}\right)}{X}$$

$$\mathbf{S} = \frac{\mathrm{j}V_1^2 - V_1 V_2 \left[\cos(\alpha_1 - \alpha_2 + 90^\circ) + \mathrm{j}\sin(\alpha_1 - \alpha_2 + 90^\circ)\right]}{X}$$

$$\mathbf{S} = \frac{-V_1 V_2 \cos(\alpha_1 - \alpha_2 + 90^\circ) + \mathrm{j}\left[V_1^2 + V_1 V_2 \sin(\alpha_1 - \alpha_2 + 90^\circ)\right]}{X}$$

b) Estimate the maximum power that can be transmitted across a 275kV transmission line with reactance of 0.42 Ohm/km. [4]

If the transmission line resistance is R=0 then, theoretically the maximum transferred active power value will be when

$(\alpha_1 - \alpha_2) = 90^\circ$

$\sin(90^\circ) = 1$

Then

$$P = \frac{V_1 V_2}{X} = \frac{(275\text{kV})(275\text{kV})}{\left(0.42\ \Omega/_{\text{km}}\right)(\#\ \text{km})}$$

i.e. 50Km

$$\text{P} = \frac{(275\text{kV})(275\text{kV})}{\left(0.42\ \Omega/_{\text{km}}\right)(50\text{km})} = 3{,}601.19\text{MW}$$

P(500km) = 360MW

c) Using the expression for reactive power, explain why reactive power is not transmitted over long distances. For the case when 120MW of active power is transmitted over the 275 kV circuit in (b), calculate the maximum amount of reactive power that can be transmitted if the maximum voltage drop that can be tolerated is 4%. [4]

If the equation is rearranged

$$Q = \frac{V_1^2 - V_1 V_2 \cos(\alpha_1 - \alpha_2)}{X} = \frac{V_1(V_1 - V_2 \cos(\alpha_1 - \alpha_2)}{X}$$

Then is clear that the Q value is dependent of the difference

$V_1(V_1 - V_2 \cos(\alpha_1 - \alpha_2)$

Considering an small angle

$(\alpha_1 - \alpha_2) \approx 0$ then

$\cos(\alpha_1 - \alpha_2) \approx 1$ and Q value will be dependent of the voltage drop

$(V_1 - V_2)$ in

$$Q = \frac{V_1(V_1 - V_2)}{X}$$

Then for a 4% drop

$$Q = \frac{275\text{ kV}\,(275\text{ kV} - 264\text{ kV})}{\left(0.42\ \Omega/_{\text{km}}\right)(l(\text{km}))} = \frac{3025\ (\text{kV})^2}{\left(0.42\ \Omega/_{\text{km}}\right)(l(\text{km}))}$$

i.e. 50 Km

Q(50Km)=144 MVar

Q(500km) = 14.4 MVAr

For the case where 120 MW are transferred

50Km

$$\sin(\alpha_1 - \alpha_2) = \frac{PX}{V_1V_2} = \frac{(120\ \text{MW})\left(0.42\ \Omega/_{\text{km}}\right)(50\ \text{km})}{(275\ \text{kV})(264\ \text{kV})} = 3.4710 \times 10^{-2}$$

$$(\alpha_1 - \alpha_2) = \text{arsin}\{\sin(\alpha_1 - \alpha_2)\} = 1.9891^\circ$$

$$\cos(1.9891) = 0.9993$$

$$Q(50\ \text{km}) = \frac{275\ \text{kV}\,(275\ \text{kV} - (264\ \text{kV})(0.9993))}{\left(0.42\ \Omega/_{\text{km}}\right)(50\ \text{km})} = 146.13\ \text{MVAr}$$

500Km

$$\sin(\alpha_1 - \alpha_2) = \frac{PX}{V_1V_2} = \frac{(120\ \text{MW})\left(0.42\ \Omega/_{\text{km}}\right)(500\ \text{km})}{(275\ \text{kV})(264\ \text{kV})} = 0.34710$$

$$(\alpha_1 - \alpha_2) = \text{arsin}\{\sin(\alpha_1 - \alpha_2)\} = 20.31^\circ$$

$$\cos(1.9891) = 0.9378$$

$$Q(500\ \text{km}) = \frac{275\ \text{kV}\,(275\ \text{kV} - (264\ \text{kV})(0.9378))}{\left(0.42\ \Omega/_{\text{km}}\right)(500\ \text{km})} = 35.89\ \text{MVAr}$$

d) What other problems do you expect with 500km long 275kV transmission line?

[2]

500km line will generate significant reactive power under low load condition and would hence require very significant reactive power / voltage management.

3.

a) The admittance bus matrix Form the Ybus matrix for the system. [3]

$$z_{12} = z_{21} = 0.02 + j0.12$$

$$y_{12} = y_{21} = \frac{1}{0.02 + j0.12} = 1.351 - j8.108108$$

$$\mathbf{I}_1 = y_{12}\left(\mathbf{V}_1 - \mathbf{V}_2\right)$$

$$\mathbf{I}_2 = y_{12}\left(\mathbf{V}_2 - \mathbf{V}_1\right) = y_{12}\left(-\mathbf{V}_1 + \mathbf{V}_2\right)$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} y_{12} & -y_{12} \\ -y_{12} & y_{12} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1.351 - j8.108108 & -1.351 + j8.108108 \\ -1.351 + j8.108108 & 1.351 - j8.108108 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 1.351 - j8.108108 & -1.351 + j8.108108 \\ -1.351 + j8.108108 & 1.351 - j8.108108 \end{bmatrix}$$

b) Perform 3 iterations of the Gauss Seidel load flow [10]

We can transform the PQ Bus 2 and its compensator by a PV bus if we make

$Q_c = Q_2 + Q_L$

Q_c = Reactive power of Capacitor

Q_2 = Reactive Power for the PV bus

Q_L = Reactive Power of the Load

Then we can define the buses

Bus1 is the slag then

$V_1 = 1$

$\delta_1 = 0$

$P_1 = ?$

$Q_1 = ?$

Bus 2 is the PV bus

$V_2 = 1.03$

$\delta_2 = ?$

$P_2 = -0.5$

$Q_2 = ?$

Iteration 1

The next will be the iterative equations

$$Q_2^k = -\mathrm{imag}\left\{V_2^*\left[Y_{21}V_1 + Y_{22}V_2\right]\right\}$$

$$V_2^k = \frac{1}{Y_{22}}\left(\frac{\left(P_2 + Q_2^k\right)^*}{\left(V_2^{k-1}\right)^*} - Y_{21}V_1\right)$$

We can assume the initial voltage of Bus 2

$$V_2^0 = 1.03 + j0.0$$

$$Q_2^0 = -\mathrm{imag}\left\{(1.03 + j0.0)^*\left[(-1.351 + j8.108108)(1 + j0.0_1) + (1.351 - j8.108108)(1.03 + j0.0)\right]\right\}$$

$$Q_2^0 = 0.2505$$

$$V_2^1 = \frac{1}{(1.351 - j8.108108)}\left(\frac{(-0.5 + j0.2505)^*}{((1.03 + j0.0))^*} - (-1.351 + j8.108108)(1 + j0.0)\right)$$

$$V_2^1 = 1.0195 - j0.0631$$

$$V_{2corr}^1 = 1.03\frac{(1.0195 - j0.0631)}{|1.0195 - j0.0631|} = 1.0280 - j0.0636$$

$$Q_2^1 = -\mathrm{imag}\left\{(1.0280 - j0.0636)^*\left[(-1.351 + j8.108108)(1 + j0.0_1) + (1.351 - j8.108108)(1.0280 - j0.0636)\right]\right\}$$

$$Q_2^1 = 0.3525$$

Iteration 2

$$V_2^2 = \frac{1}{(1.351 - j8.108108)}\left(\frac{(-0.5 + j0.3525)^*}{(1.0280 - j0.0636)^*} - (-1.351 + j8.108108)(1 + j0.0)\right)$$

$$V_2^2 = 1.0273 - j0.0669$$

$$V_{2corr}^2 = 1.03\frac{(1.0273 - j0.0669)}{|1.0273 - j0.0669|} = 1.0278 - j0.0669$$

$$Q_2^2 = -\mathrm{imag}\left\{(1.0278 - j0.0669)^*\left[(-1.351 + j8.108108)(1 + j0.0_1) + (1.351 - j8.108108)(1.0278 - j0.0669)\right]\right\}$$

$$Q_2^2 = 0.3587$$

$$V_2^3 = \frac{1}{(1.351 - j8.108108)}\left(\frac{(-0.5 + j0.3587)^*}{(1.0278 - j0.0669)^*} - (-1.351 + j8.108108)(1 + j0.0)\right)$$

$$V_2^3 = 1.0278 - j0.0672$$

$$V_{2corr}^3 = 1.03\frac{(1.0278 - 0.0672i)}{|1.0278 - 0.0672i|} = 1.0278 - j0.0672$$

$$Q_2^3 = -\mathrm{imag}\left\{(1.0278 - j0.0672)^*\left[(-1.351 + j8.108108)(1 + j0.0_1) + (1.351 - j8.108108)(1.0278 - j0.0672)\right]\right\}$$

$$Q_2^3 = 0.3591$$

The next is a table of unknown variables

Iteration	δ_2	Q_2	$\Delta\delta$	ΔQ_2
0	0	0.2505	–	–
1	-3.5427	0.3525	3.5427	0.1020
2	-3.7266	0.3587	0.1839	0.0062
3	-3.7389	0.3591	0.0123	0.0004

c) Calculate the output of the reactive power compensator. [3]

The output of the reactive compensator

$$Q_c = Q_2 + Q_L$$

$$Q_c = 0.3591 + 0.3 = 0.6591\text{pu}$$

d) Calculate losses in the line and active and reactive power generated by the generator. [4]

First method

$$S_1 = V_1 I_1^* = (V_1)\left[Y_{12}V_2 + Y_{11}V_1\right]^*$$

$$S_1 = V_1 I_1^* = (1 + j0.0)\left[(-1.351 + j8.108108)(1.0278 - j0.0672) + (1.351 - j8.108108)(1 + j0.0)\right]^*$$

$$S_1 = 0.5070 - j0.3162$$

$$P_1 = 0.5070$$

$$Q_1 = -0.3162$$

$$S_{loss} = S_1 + S_2 = (0.5070 - j0.3162) + (-0.4999 + j0.3591)$$

$$S_{loss} = 0.0071 + j0.0428$$

$$P_R = 0.0071$$

$$Q_X = 0.0428$$

Second method

$$I_{12} = (V_1 - V_2^3)y_{12} = 0.5070 + j0.3159$$

$$|I_{12}| = |(V_1 - V_2^3)y_{12}| = 0.5974$$

$$P_R = (|I_{12}|)^2 R = 0.0071 \text{ pu}$$

$$Q_X = (|I_{12}|)^2 X = 0.0428 \text{ pu}$$

Part B – Answer any 2 out of 3 questions in part B

4. a) State the four steps for calculating the fault current for balanced faults without making any assumption about the currents and voltages under pre-fault condition.

[5]

1. Obtain the pre-fault steady state solution V_o and I_o (from power flow analysis)
2. Replace the reactance of synchronous machines (generator and motor) by their sub-transient values
3. Excite the passive Thevenin equivalent (emf sources shorted) at the fault point by NEGATIVE of the pre-fault voltage in series with fault impedance. Compute sub-transient voltage and current at all points of interest
4. Fault current is obtained by adding the results of steps 1 (pre-fault) and 3 (sub-transient): Superposition

b) Explain using the theorem of constant flux linkage why the sub-transient reactance of a synchronous generator is less than its transient reactance and why the former is normally used for short circuit analysis.

[5]

According to the theorem of constant flux linkage, in a closed R-L circuit with a finite source of emf, the flux linkage cannot change instantaneously. Immediately after the occurrence of a fault, the excess flux due to the fault current is not allowed to link the rotor magnetic circuit as a result of which the excess flux is forced through the peripheral parts of the rotor. As the reluctance involved is high, the reactance is relatively low initially in the sub-transient phase. Depending on the time-constant of the rotor circuit the excess flux due to fault current gradually links the rotor getting through the transient on to the steady state. Thus with less reluctance path for the flux during transient condition than sub-transient, the sub-transient reactance of a synchronous generator is lower than its transient value.

Faults are usually cleared within 4-5 cycles i.e. 100 ms which is typically the duration up to which sub-transient condition persists. Hence, sub-transient reactance is considered for short circuit analysis.

c) A three-phase fault occurs at point F at the far end of the line in the system shown in Figure 4.1. Considering 30 MVA as the system base calculate the fault current in Amperes. Assume the pre-fault voltages to be 1.0 pu and neglect pre-fault currents.

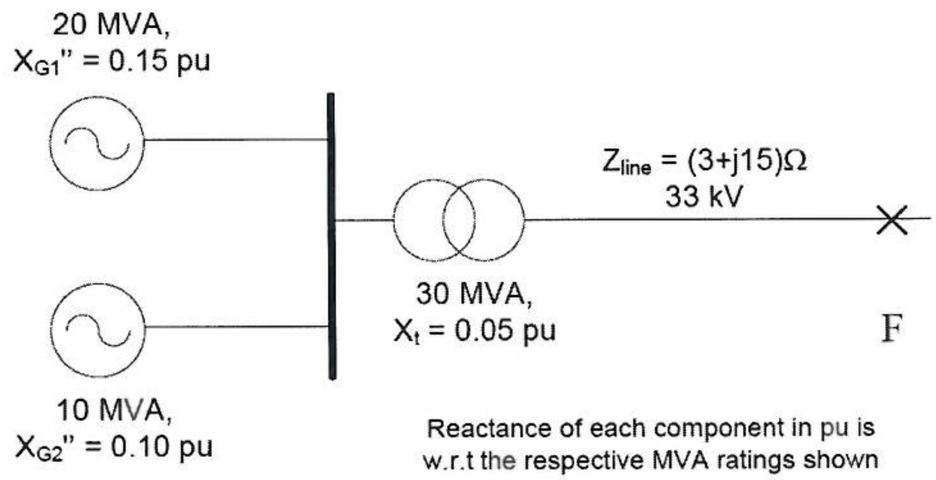


Figure 4.1: Two synchronous generators connected to a transmission line through a transformer

[10]

The pu values of the reactance and impedance with respect to 30 MVA system base are:

$$X_{G1}'' = 0.15\frac{30}{20} = 0.225\ pu,\ X_{G2}'' = 0.1\frac{30}{10} = 0.3\ pu .X_t = 0.05\ pu$$

$$Z_{line} = (3 + j5)\frac{30}{33^2} = (0.0826 + j0.4132)\ pu$$

Equivalent impedance from the generator neutral to the fault is

$$Z_{total} = (X_{G1}'' \parallel X_{G2}'') + jX_t + Z_{line} = 0.0826 + j0.5918 = 0.5975\angle 82^\circ\ pu$$

Fault current in pu is

$$I_f = \frac{1}{Z_{total}} = 1.6736\angle -82^\circ\ pu$$

Base current is

$$I_B = \frac{30\times10^6}{\sqrt{3}\times33\times10^3} = 524.86\ A$$

Fault current in Amperes

$$I_f = I_f(pu)\times I_B = 878.5\,A$$

5. a) Use the method of symmetrical components to prove that the positive and negative sequence currents cannot exist in the neutral wire of a star connection.

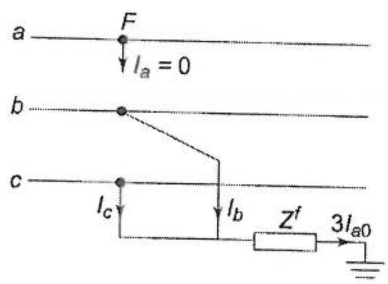
[4]

$$
\begin{aligned}
I_n &= I_a + I_b + I_c \\
&= 3I_0 + I_1(1+\alpha^2+\alpha) + I_2(1+\alpha+\alpha^2)
\end{aligned}
$$

Thus the neutral current comprises only of zero sequence component without any positive or negative sequence component.

b) Show that the three sequence networks are connected in parallel for line to line to ground (LLG) fault affecting phases B and C with zero fault impedance.

[6]



Fault condition in phase domain

$I_a = 0$

$V_b = V_c = Z^f\,(I_b + I_c)$

Fault condition in sequence domain

$I_{a0} + I_{a1} + I_{a2} = 0$

$$
\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}
$$

$V_{a1} = V_{a2}$

$$
V_{a0} - V_{a1} = \frac{1}{3}\left(2-\alpha-\alpha^2\right)V_b = V_b = 3Z^f I_{a0}
$$

For zero fault impedance, Zf, the voltage across the three sequence networks are the same and the sum of the current through them is zero. This implies that they are connected in parallel.

c) The positive-, negative-, and zero-sequence reactance of a 20 MVA, 13.2 kV three-phase synchronous generator are 0.3 pu, 0.2 pu and 0.1 pu, respectively. The generator is solidly grounded and is operating at no-load. Calculate the fault current (in Amperes) to ground for a line to line to ground (LLG) fault affecting phases B and C at the generator terminal. Neglect resistance of the generator and fault impedance.

[10]

From the parallel connection of the sequence networks for LLG fault with zero fault impedance the positive sequence current is:

$$I_{a1} = \frac{1\angle 0^\circ}{j0.3 + \dfrac{j0.2 \times j0.1}{j0.2 + j0.1}} = -j2.73\ pu$$

Voltage across the sequence networks are:

$$V_{a1} = E_{a1} - I_{a1}Z_1 = 1\angle 0 - (-j2.73)\times(j0.3) = 0.181\ pu$$
$$V_{a2} = V_{a0} = V_{a1} = 0.181\ pu$$

Currents through the negative and zero sequence networks are:

$$I_{a2} = -\frac{V_{a2}}{Z_2} = j0.905\ pu,\ I_{a0} = -\frac{V_{a0}}{Z_0} = j1.81\ pu$$

Fault current in pu

$$I_f = I_b + I_c = 3I_{a0} = j5.43\ pu$$

Fault current in Amperes

$$I_f = I_f(pu) \times \frac{20 \times 10^3}{\sqrt{3} \times 13.2} = 4750\ A$$

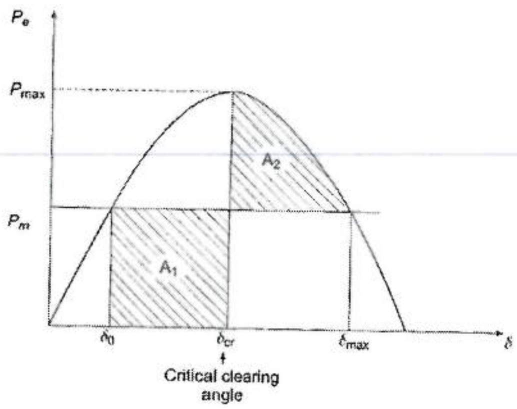
6. a) Explain why operation in the region of positive slope on the standard power angle characteristics implies steady-state stability while that on the negative slope region implies instability. Explain without using the swing equation.

[5]

If the operating point is in the positive slope region, any increase in mechanical input leads to acceleration of the rotor resulting in increase in load angle and hence electrical power output. Thus increase/decrease in mechanical power input implies corresponding increase/decrease of electrical power output which helps maintain stability. On the other hand, if the operating point is in the negative slope region, any increase in mechanical input results in increase in load angle as before, but on this occasion increase in load angle leads to reduction in electrical power output because of the negative slope. This is a runway situation with increase in mechanical input causing reduction in electrical power output and hence, the situation is unstable.

b) Derive an expression for critical clearing angle (δ_{cr}) in terms of the initial load angle (δ_0) assuming zero electrical power output during fault and identical standard power-angle characteristics during pre-fault and post-fault.

[5]



$$\delta_{\max} = \pi - \delta_0$$

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)d\delta = P_m(\delta_{cr} - \delta_0)$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{\max}} (P_{\max}\sin\delta - P_m)d\delta$$

$$= P_{\max}(\cos\delta_{cr} - \cos\delta_{\max}) - P_m(\delta_{\max} - \delta_{cr})$$

For stability

$$A_2 = A_1$$

$$\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0\right]$$

c) A synchronous generator with a maximum power output of 500 MW is operating at a power angle of $\delta_0 = 8^\circ$. The mechanical power input to the generator is increased suddenly.

i) Show that the power angle δ_m corresponding to the maximum allowable mechanical power input without loss of stability is given by the following equation:

$$(3 - \delta_m)\sin\delta_m - \cos\delta_m = 0.99$$

where δ_m is in radians.

[7]

ii) If the solution to the above equation in part (i) is $\delta_m = 50^\circ$, by how much can the mechanical power output increase suddenly without loss of stability?

[3]

i) The power-angle characteristics of the synchronous generator is given by $P = P_{\max}\sin\delta$ where $P_{\max} = 500\ MW$ and the initial operation is at power angle $\delta_0 = 8^\circ$

If P_m is the maximum mechanical power input possible at which point the power angle is δ_m then for stability accelerating area should be equal to the decelerating area.

$$\int_{\delta_0}^{\delta_m}(P_m - P_{\max}\sin\delta)d\delta = \int_{\delta_m}^{\pi-\delta_m}(P_{\max}\sin\delta - P_m)d\delta$$

$$\Rightarrow (\pi - \delta_m - \delta_0)\sin\delta_m - \cos\delta_m - \cos\delta_0 = 0$$

Putting $\delta_0 = 8^\circ = 0.13885\ rad$ in the above equation we get

$$(3 - \delta_m)\sin\delta_m - \cos\delta_m = 0.99$$

ii) Maximum mechanical power input P_m corresponding to $\delta_m = 50^\circ$ is

$P_m = 500 \times \sin 50^\circ = 383.02\ MW$

Initial mechanical power input is $P_0 = 500 \times \sin 8^\circ = 69.6\ MW$

Without loss of stability the synchronous generator can accommodate $P_m - P_0 = 313.42\ MW$ increase in mechanical power input.