IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

MSc and EEE PART IV: MEng and ACGI

TRAFFIC THEORY & QUEUEING SYSTEMS

Thursday, 9 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : J.A. Barria
Second Marker(s) : D.P. Mandic

Special instructions for students

1. Erlang Loss formula recursive evaluation:

$$E_N(\rho) = \frac{\rho E_{N-1}(\rho)}{N + \rho E_{N-1}(\rho)}$$
$$E_0(\rho) = 1$$

2. Engset Loss formula recursive evaluation (for a fixed M and $p = \alpha/1+\alpha$):

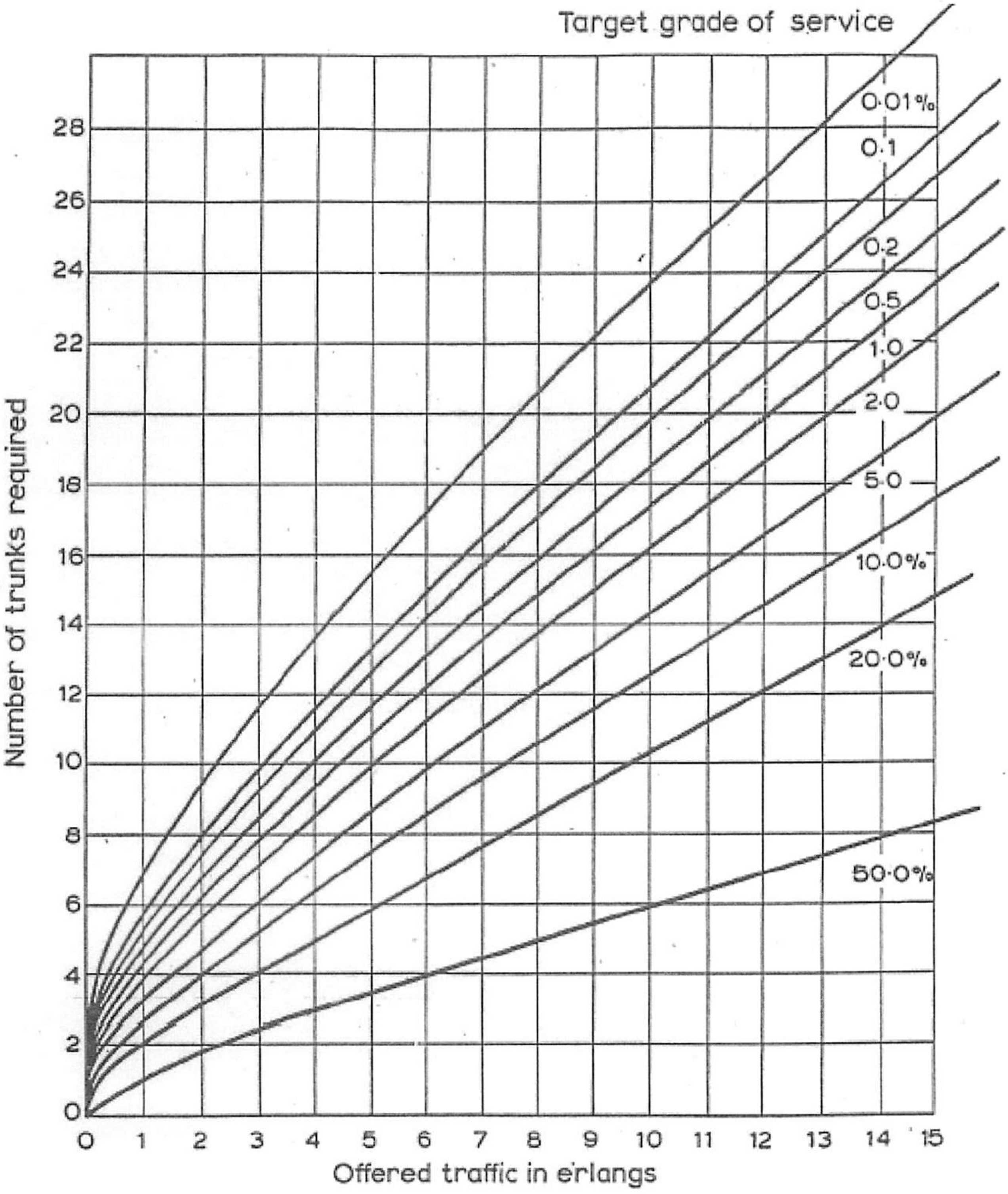
$$e_N = \frac{(M-N+1)\alpha e_{N-1}}{N+(M-N+1)\alpha e_{N-1}}$$
$$e_0 = 1$$
$$\alpha = \lambda/\mu$$

3. Traffic capacity on basis of Erlang B formula (next page).

Note: for large ρ, N is approximately linear: $N \approx 1.33\rho + 5$

4. Expected residual time

$$E[R] = \frac{1}{2}\lambda E[S^2]$$



Traffic capacity on basis of Erlang B. formula.

The Questions

1.

a)

i) Describe in your own words a Poisson traffic source in terms of its probability of generating demand.

[3]

ii) The system shown in Fig. 1.1 is composed of M identical Poisson sources (mean arrival rate λ) which are offering traffic to an N channel link.

Derive the equilibrium traffic distribution if M >> N. Assume full availability. State clearly all the assumptions made.

[5]

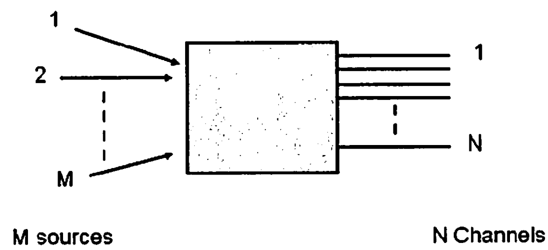


Figure 1.1

b) Two buildings are connected to the same public switching exchange via a shared 4-channel access link.

One building has 200 telephone lines installed which offer a traffic level of 0.2 Erlangs per non-busy telephone.

The other building has only 10 telephone lines installed which offer a traffic level of 0.01 Erlangs per non-busy telephone.

i) Derive the Birth/Death (B/D) model for the system described. Carefully state all the assumptions in your derivations.

[4]

ii) Derive the equilibrium standard equivalent finite-source model. Clearly show all the steps of your derivations.

[5]

iii) How many equivalent sources $\overline{M}$ have the standard equivalent finite-source model ?

[3]

2.

a) Pure chance traffic is offered to an M-channel communications link, denoted by Link 1. When Link 1 is saturated the overflow traffic is fed to an N-channel link, denoted by Link 2; as shown in Figure 2.1.

i) Derive the state transition diagram of the 2-dimensional Markov chain representing the system described above.

Note: Assume that the offered traffic is pure chance traffic with parameters (λ, μ).

[4]

ii) If you know the following parameters of the system: $\lambda = 180$ (calls/hour); $1/\mu = 4$ (m); $M = 12$, and $N = 8$.

Determine:

- the mean traffic carried on Link 1. [2]
- the mean traffic carried on Link 2. [2]
- the call congestion for Link 2 (i.e. the fraction of calls offered to Link 2 which get blocked) [2]

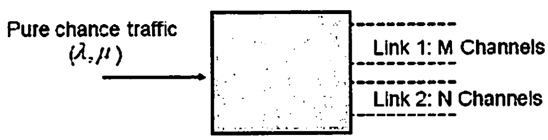


Figure 2.1

b) Consider a single-channel packet transmission link with a FIFO input buffer. The transmission rate of the link is 64 (kbits/s).

The arrival stream of packets is Poisson with mean arrival rate of 80 (packets/s) and the length of the packet is geometrically distributed with mean packet length of 700 bits.

i) Determine the probability that the packet will not have to wait for transmission.

[5]

ii) Determine the probability that a packet will have to wait for more than 20 (ms) before its transmission starts.

[5]

3.

a) For the non-preemtive priority queue shown in Fig. 3.1 derive the expected waiting time of a class j arrival. [5]

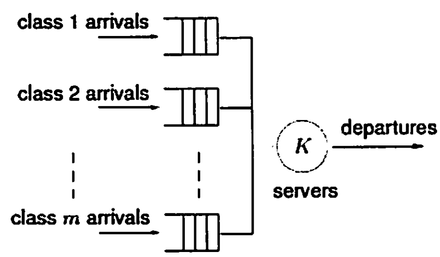
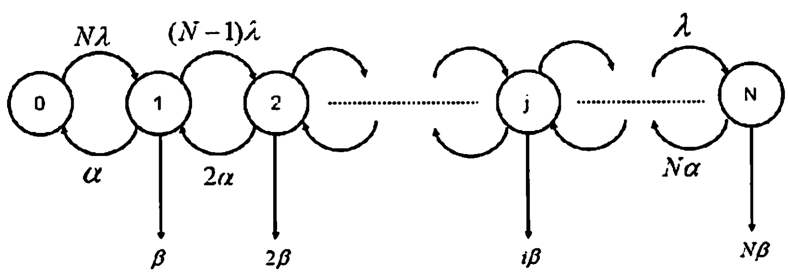


Figure 3.1.

b) For the N Independent ON-OFF voice multiplexer model shown in Fig. 3.2:

i) Identify the underlying ON-OFF single voice model. [3]

ii) Using the identified ON-OFF single voice model derive the probability that j out of N sources are active. [4]



Note: β (packets/s) is the rate at which voice packets are being offered to a multiplexer link.

Figure 3.2.

c) In the context of a Broadband network answer the following questions.

i) Define the equivalent capacity of a set of *connections* multiplexed on a link. [3]

ii) Describe two equivalent capacity approximations known to you if the measure of link load is loss of packet or buffer overflow probability. Discuss what characteristics of the *connections* are being captured by each one of the identified approximations. [5]

4.

a) A 24-channel link is being offered on average 420 (calls/hour). The calls are on average 2 (m) long.

i) Determine the call blocking probability B_c of the link described. [4]

ii) Calculate the mean carried traffic. [3]

b) Consider a multiprocessor system consisting of ***n*** processors whose base model is shown in Fig 4.1. Each processor has a constant rate of failure = λ (failures/hour) and a coverage factor c.

i) Explain what the state space {0,1, ...3} is representing. [2]

ii) Is this system repairable or non-repairable ? Clearly argue your answer. [2]

iii) Define a suitable reward structure to analyse the system availability. [3]

iv) Modify the base model of Fig. 4.1. to consider the following two maintenance policies:

- If the system is in any state *j* in which one or more processors are in faulty conditions, reconfigure the systems to state *j-1*. Continue to reconfigure the system until all processors of the system are fully operational. [3]

- If the system fails (i.e. no processor is operational) restart the system to its full capacity. [3]

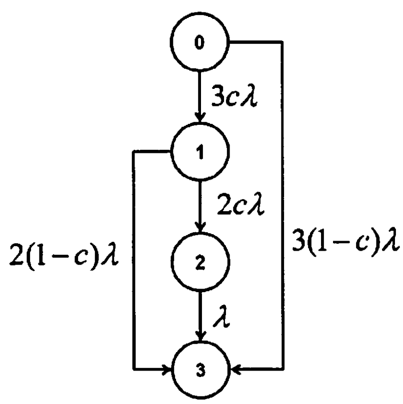


Figure 4.1

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Model Answers and Mark Schemes — First Examiner: J. Barria

Paper Code: EE4.05 / EE9-S07 — Second Examiner:

Question Number etc. in left margin — Mark allocation in right margin

a) i) Poisson source : a source which, when idle, generates a demand in $(t, t+\Delta t)$ with probability $\lambda \Delta t$. 3

ii) Traffic distribution for M sources, N channels

$$(i\mu)\pi_i = (M-i+1)\lambda \pi_{i-1}$$

$$\pi_i = \binom{M}{i}\alpha^i \pi_0 \qquad \alpha = \lambda/\mu$$

$$\pi_0 = \sum_{j=0}^{i_{max}} \binom{M}{j}\alpha^j \qquad \alpha = \frac{p}{1-p}$$

for $M \le N$ $\quad \pi_i = \binom{M}{i} p^i (1-p)^{M-i} \qquad 0 \le i \le M$

for $M > N$ $\quad \pi_i = K\binom{M}{i} p^i (1-p)^{M-i} \qquad 0 \le i \le N$

and K = normalising constant

For $M \gg N$

$$\binom{M}{i} p^i (1-p)^{M-i} \sim \frac{Mp}{i!} e^{-Mp}$$

for $i \ll M$

Therefore the distribution is truncated Poisson 5

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Q1 b)

α = offered traffic per non busy source

Assumptions

- Exponential holding times
- identical holding time distribution
- full-availability access

B/D model $0 < i \leq 4$

$$(i\mu)\pi_i = [(M_1 - i + 1)\lambda_1 + (M_2 - i + 1)\lambda_2]\pi_{i-1}$$ 4

where $M_1 = 200 \quad (\lambda_1/\mu) = 0.01$

$M_2 = 10 \quad (\lambda_2/\mu) = 0.2$

$$(i\mu)\pi_i = [(M_1\lambda_1 + M_2\lambda_2) - (i-1)(\lambda_1 + \lambda_2)]\pi_{i-1}$$

$$= [(M' - i + 1)\lambda']\pi_{i-1} \quad 0 < i \leq 4$$ 5

where $\lambda' = \lambda_1 + \lambda_2$

$$M' = M_1\left(\frac{\lambda_1}{\lambda_1 + \lambda_2}\right) + M_2\left(\frac{\lambda_2}{\lambda_1 + \lambda_2}\right)$$

$$M' = \frac{400}{21} \sim 20$$ 3

31

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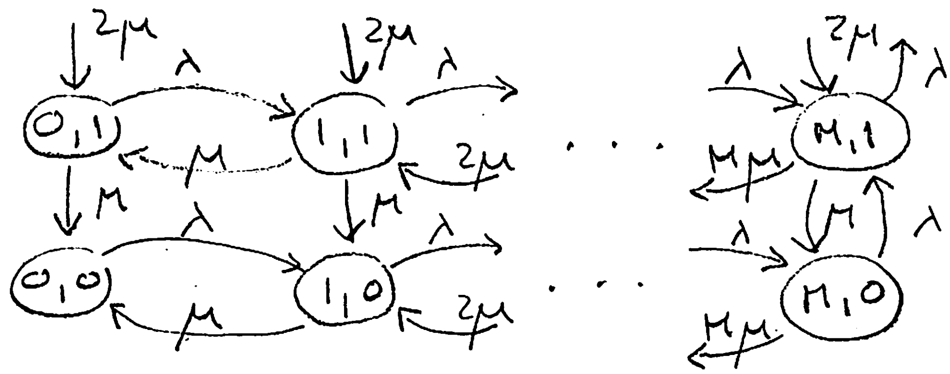
Q2 a) i)



4

ii)

offered Traffic to link 1

$\rho = \frac{180}{60} \times 4 = 12$ Erlangs

call congestion on link 1

$B_1 = E_{12}(12) = 0.2$

carried traffic on link 1

$\rho_1 = \rho(1 - B_1) = 9.6$ Erlangs 2

call congestion for links 1 and 2

$B_2 = E_{20}(12) = 0.01$ 2

carried traffic on link 2

[Traffic on 1 + 2] − [Traffic on 1]

$= \rho(B_1 - B_2) = 2.3$ Erlangs 2

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Q2 b)

Geometric message length distribution can be approximated by an exponential distribution

mean service time $= \frac{700}{64000}$ [s]

service rate $\mu = 91.4$ [s^{-1}]

Arrival rate $\lambda = 80$ [s^{-1}]

Offered traffic $\rho = \frac{\lambda}{\mu} = 0.875$ Erlang

i) $P[W=0] = P[\text{channel idle}] = 1-\rho = 0.125$ 5

ii) $P[W>0.02] = P[W>0]\, P[W>0.02 \mid W>0]$

$$= \rho\, e^{-\mu(1-\rho)0.02}$$

$$= 0.875\, e^{-0.23}$$

$$= 0.695$$

5

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Q3 a

λ_K = arrival rate

S_K = service time

W_K = waiting time

Q_K = queue-length seen on arrival

offered traffic in class k : $\rho_K = \lambda_K E[S_K]$

Total offered traffic : $\rho = \sum_{k=1}^{m} \rho_K$

stability condition $\sum_{k=1}^{m} \rho_K < 1$

Using mean value analysis, let

R = residual service time seen by an arbitrary arrival

For class 1 arrivals

$$E[W_1] = E[R] + E[Q_1]E[S_1]$$
$$= E[R] + \lambda_1 E[W_1] E[S_1]$$

$$E[W_1] = \frac{E[R]}{1-\rho_1}$$

For class 2 arrivals

$$E[W_2] = E[R] + (E[Q_1]E[S_1] + E[Q_2]E[S_2]) + (\lambda_1 E[W_2])E[S_1]$$

$$E[W_2] = \frac{E[R]}{(1-\rho_1)(1-\rho_1-\rho_2)}$$

. . .

$$E[W_K] = \frac{E[R]}{(1-\sigma_{K-1})(1-\sigma_K)} \quad ; \quad \sigma_{K-1} = \sum_{i=1}^{K-1} \rho_i$$ 5

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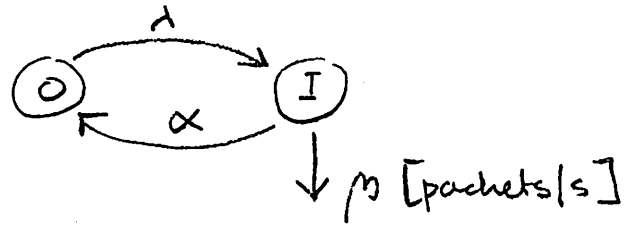
Q3 b)



3

For an N independent ON-OFF voice multiplexer the probability of j out of N source active:

$$\pi_j = \binom{N}{j}\left(\frac{\lambda}{\lambda+\alpha}\right)^j\left(\frac{\alpha}{\lambda+\alpha}\right)^{N-j}$$

4

c) i) The equivalent capacity of a number of connections multiplexed on a link is defined as the amount of bandwidth required to achieve a desired grade of service, for example, buffer overflow probability. 3

ii) - The fluid flow approximation estimates the equivalent capacity when the impact of individual connections characteristic is critical. 2

- The stationary approximation estimates the bandwidth requirements when the effect of statistical multiplexing is of significance.

3

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Q4 a) i) calling rate 420 [calls/h] = 7 [calls/m]

mean call duration = 2 [m]

offered traffic = 7 × 2 = 14 Erlangs

$B_c = E_{24}(14) = 0.005$ (from chart) 4

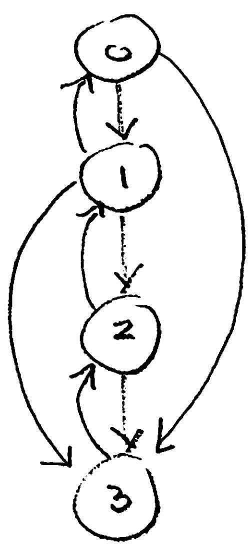
ii) carried traffic = $14(1 - B_c) = 13.93$ Erlangs 3

Q4 b) i) The state space represents the number of faulty processors 2

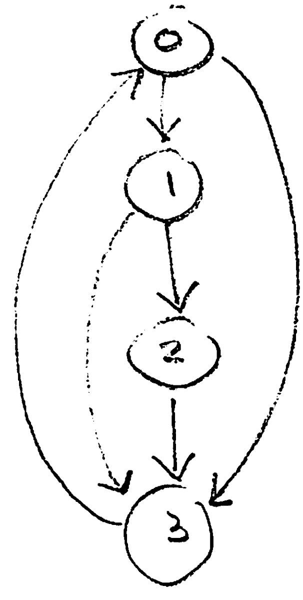
ii) The system is non-repairable as there is no rate of transition between a state N and N−i ($i \le N$). 2

iii) A suitable reward structure when analysing system availability can be to assign a reward 1 to all operational states and reward ϕ to the system faulty state (state = 3) 3

iv)



(3)



(3)