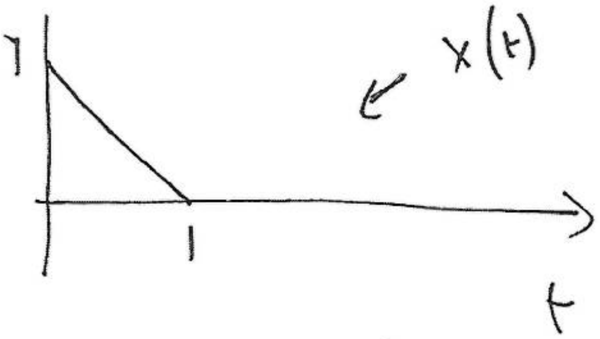
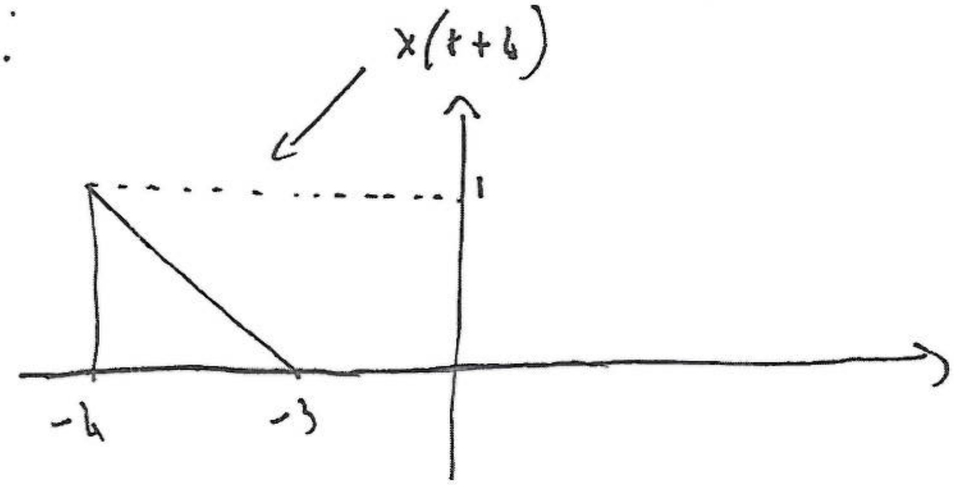
# SOLUTIONS

1.

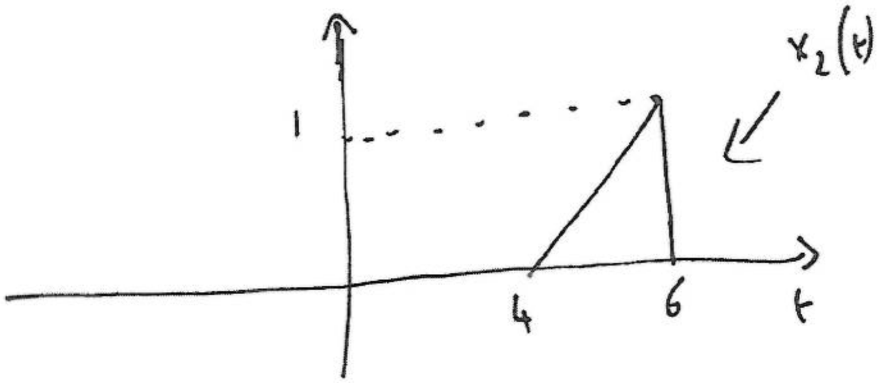



i.




ii. $x_2(t) = x\left(-\frac{t}{2}+3\right) = x\left(-\frac{1}{2}(t-6)\right)$

THEREFORE



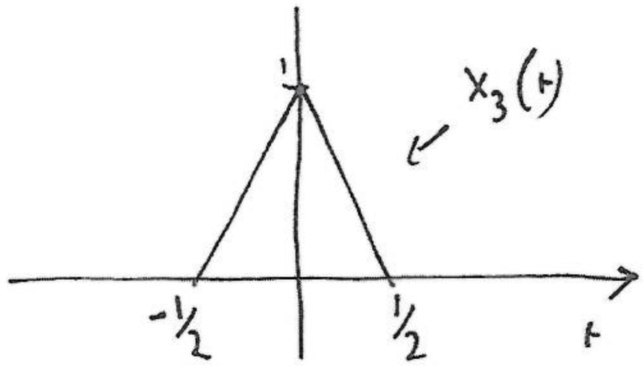

MANY STUDENTS GOT IT WRONG BECAUSE THEY DIDN'T DO THE OPERATION IN THE RIGHT ORDER.

REMEMBER

$x_2(t) = x\left(-\frac{t}{2}+3\right) = x\left(-\frac{1}{2}(t-6)\right) \Rightarrow$

FLIP THE SIGNAL FIRST, RESCALE BY 2, SHIFT BY 6.

iii




(b)

(i) PERIODIC WITH PERIOD $T=15$

(ii) A-PERIODIC SINCE $x_3(t) = e^{-t} e^{-jt}$ AND $e^{-t}$ IS A-PERIODIC

REMEMBER : EXPONENTIALS WITH REAL VALUED EXPONENTS ARE A-PERIODIC

(c) (i) CHARACTERISTIC POLYNOMIAL

$x^2 + 5x + 4$

CHARACTERISTIC ROOTS

$$x = \frac{-5 \pm \sqrt{25-16}}{2} = \begin{cases} -4 \\ -1 \end{cases}$$

(ii) $y(t) = c_1 e^{-t} + c_2 e^{-4t}$

$y(0) = c_1 + c_2 = 4$

$$y'(t)\Big|_{t=0} = -C_1 e^{-t} - 4C_2 e^{-4t}\Big|_{t=0}$$

$$= -C_1 - 4C_2 = 1$$

$$\begin{cases} C_1 + C_2 = 4 \\ -C_1 - 4C_2 = 1 \end{cases} \Rightarrow \quad C_1 = \frac{17}{3} \quad C_2 = -\frac{5}{3}$$

THUS

$$y(t) = \frac{17}{3} e^{-t} - \frac{5}{3} e^{-4t}$$

(d)

WE USE PARTIAL FRACTION

(i)

$$\frac{s+2}{s^2+6s+5} = \frac{A}{s+5} + \frac{B}{s+1} = \frac{3}{4}\frac{1}{s+5} + \frac{1}{4(s+1)}$$

$$\Updownarrow$$

$$\left(\frac{3}{4} e^{-5t} + \frac{1}{4} e^{-t}\right) u(t)$$

(ii)

IN THIS CASE WE HAVE A ROOT WITH MULTIPLICITY 2, THEREFORE

$$\frac{s}{s^2+4s+4} = \frac{A}{s+2} + \frac{B}{(s+2)^2}$$

$$B = -2$$

$$A = 1$$

THEREFORE

$$\frac{s}{s^2+4s+4} = \frac{1}{s+2} - \frac{2}{(s+2)^2} \Longleftrightarrow \left(e^{-2t} - 2t\,e^{-2t}\right)u(t)$$

(e)

(i) $(s+2)\,Y(s) = X(s)$;

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

ii $X(s) = \frac{1}{s}$

THEREFORE

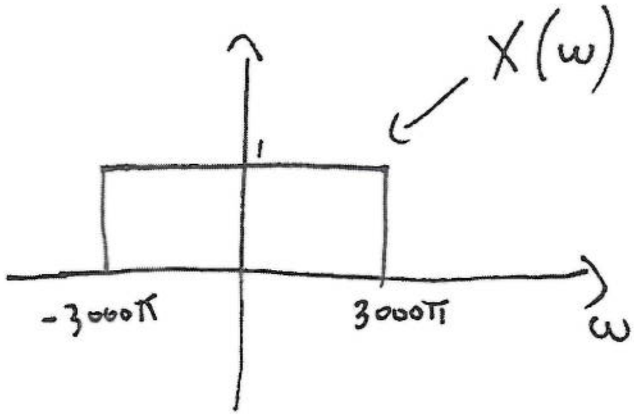
$$Y(s) = \frac{1}{(s+2)s}$$

USING THE FINAL VALUE THEOREM WE HAVE THAT

$$y(\infty) = \lim_{s \to 0} sY(s) = \lim_{s\to 0} \frac{1}{s+2} = \frac{1}{2}$$

(b) USING FOURIER TABLES

(i.) $3000 \,\mathrm{SINC}(3000\pi t) \Leftrightarrow \mathrm{RECT}\left(\frac{\omega}{6000\pi}\right)$




(ii)

NYQUIST RATE FOR $x(t)$ : $f_s = 2 \cdot \frac{3000\pi}{2\pi} = 3\,\mathrm{KHz}$

NYQUIST RATE FOR $x^2(t)$ IS $2f_s = 6\,\mathrm{KHz}$

THEREFORE NYQUIST RATE FOR $x(t) + x^2(t)$ IS $2f_s = 6\,\mathrm{KHz}$

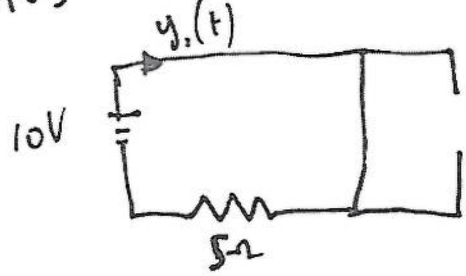
SINCE THE TWO BANDWIDTH OVERLAP THEN NYQUIST RATE IS DICTATED BY $x^2(t)$.

# QUESTION 2

(a)

IN STEADY STATE (i.e., $t<0$) INDUCTORS BEHAVE LIKE SHORT CIRCUITS AND CAPACITORS AS OPEN CIRCUITS

THUS




MOST ANSWERED CORRECTLY TO THIS QUESTION

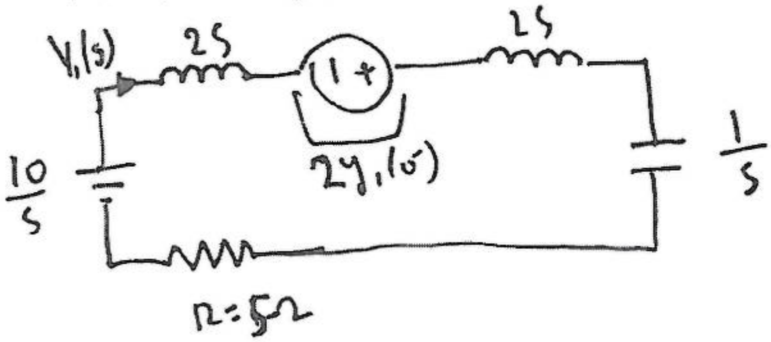
CONSEQUENTLY

$$y_2(0^-) = 0 \qquad V_c(0^-) = 0$$

AND

$$y_1(0^-) = \frac{10V}{R} = 2A$$

WHEN $t \geq 0$ WE HAVE (IN LAPLACE DOMAIN)




THEREFORE

$$2s\,Y_1(s) + 2s\,Y_1(s) + \frac{1}{s}Y_1(s) + 5Y_1(s) = \frac{10}{s} + 4$$

(b)

$$Y_1(s)\left(4s^2 + 5s + 1\right) = 10 + 4s$$

$$Y_1(s) = \frac{10+4s}{4s^2+5s+1}$$

(c) USING PARTIAL FRACTION EXPANSION

$$Y_1(s) = \frac{1}{2}\left(\frac{5+2s}{s^2+\frac{5}{4}s+\frac{1}{4}}\right) = \frac{1}{2}\left(\frac{A}{s+1} + \frac{B}{s+1/4}\right)$$

$$= \frac{1}{2}\left(-\frac{4}{s+1} + \frac{6}{s+1/4}\right)$$

$$= \frac{3}{s+1/4} - \frac{2}{s+1}$$

$$y_1(t) = \left(3e^{-t/4} - 2e^{-t}\right)u(t)$$

SOME STUDENTS GOT POSITIVE ROOTS AND THUS POSITIVE EXPONENT. REMEMBER A PASSIVE CIRCUIT IS STABLE SO $y(t)$ CANNOT GROW ARBITRARY WHEN THE INPUT IS BOUNDED.

# QUESTION 3

(a)

$$S_1(s) = \frac{1}{s+2},$$

$$S_2(s) = \frac{2}{s+4}.$$

(b) THE TRANSFER FUNCTION OF THE COMPLETE SYSTEM IS:

$$H_1(s) = \frac{s+2}{s^2+8s+7}\left(\frac{1}{s+2} + \frac{2}{s+4}\right)$$

$$= \frac{\cancel{s+2}}{(s+1)(s+7)}\left(\frac{3s+8}{\cancel{(s+2)}(s+4)}\right)$$

$$= \frac{3s+8}{(s+1)(s+4)(s+7)}$$

(c)

$$u(t) \Leftrightarrow \frac{1}{s}$$

THEREFORE

$$Y(s) = \frac{3s+8}{s(s+1)(s+4)(s+7)} = \frac{2}{7s} - \frac{5}{18}\frac{1}{(s+1)} - \frac{1}{9}\frac{1}{(s+4)} + \frac{13}{126}\frac{1}{(s+7)}$$

$$y(t) = \left(\frac{2}{7} - \frac{5}{18} e^{-t} - \frac{1}{9} e^{-4t} + \frac{13}{126} e^{-7t}\right) u(t)$$

(d) THE TRASFER FUNCTION OF THE FEEDBACK SYSTEM IS: $H_2(s) = \frac{F(s)}{Y(s)}$. BUT

$$F(s) = K\,Y(s) - K\,P(s)\,F(s)$$

CONSEQUENTLY

$$H_2(s) = \frac{K}{1 + K P(s)}.$$

WE WANT $H_2(s) \cong H_1^{-1}(s)$ (1)

WHEN $KP(s) >> 1$ ~~THAT~~ THEN

$H_2(s) \simeq \frac{1}{P(s)}$ AND CONDITION (1) IS

SATISFIED BY CHOOSING $P(s) = H_1(s)$.

ONLY A SMALL NUMBER OF STUDENTS ANSWERED THIS CORRECTLY. MANY DID NOT REALIZE THAT WE ARE NOT AFTER A PRECISE VALUE OF K BUT JUST A VALID RANGE OF VALUES.