UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1999

MEng Honours Degrees in Computing Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degree in Information Systems Engineering Part IV
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute
Associateship of the Royal College of Science

PAPER 4.77 / I 4.20

COMPUTING FOR OPTIMAL DECISIONS

Thursday, May 6th 1999, 10.00 – 12.00

Answer THREE questions

For admin. only:
paper contains 4 questions

1 a Let $g_i(x)$; $i = 1, \ldots, m$, be a set of convex functions in x and let S be given by

$$S \equiv \left\{ x \;\middle|\; g_i(x) \le b_i \;;\; i = 1, \ldots, m \right\},$$

where b_i are given values. Furthermore, let $x_1, x_2 \in S$ and $0 \le \alpha \le 1$, and determine if

$$\alpha\, x_1 + (1 - \alpha)\, x_2 \in S.$$

b Let f(x) be a convex function in x and S be a non-convex set. Consider the solutions to problem

$$\underset{x}{\text{minimise}} \left\{ f(x) \;\middle|\; x \in S \right\}$$

and demonstrate the possibility, or otherwise, of nonunique solutions using definitions of convexity and nonconvexity.
(All parts carry equal marks)

2 a Solve the equality constrained quadratic programming problem

$$\underset{x}{\text{minimise}} \left\{ \sum_{i=1}^{n} i\, x_i^2 \;\middle|\; \sum_{i=1}^{n} x_i = K \right\},$$

for $K > 0$, by formulating the appropriate Lagrangian and optimality conditions.

b Does this solution minimise the objective function subject to the constraints

$$x_i \ge 0, i = 1, \ldots, n \;;\; \sum_{i=1}^{n} x_i \ge K .$$

c For given $\hat{x}$, A, b, evaluate the solution of

$$\underset{x}{\text{minimise}} \left\{ \tfrac{1}{2}(x - \hat{x})^T(x - \hat{x}) \;\middle|\; A\,x = b \right\},$$

where $\hat{x}$, x, b are vectors and A is a matrix of appropriate dimension and AA^T is nonsingular. Describe how you would assess if this solution also satisfies the problem of optimising the same function subject to the inequalities $A\,x \ge b$. If the solution may be improved, suggest a strategy for further reducing the objective function.
(All parts carry equal marks)

3 a Consider the nonlinear optimisation problem

$$\underset{x}{\text{minimise}} \left\{ f(x) \;\middle|\; g(x) \leq 0, \; A\,x \leq b \right\}$$

where g (x) is a nonlinear constraint, A and b are respectively a given matrix and vector of appropriate dimension. Suggest a hybrid SUMT-Frank-Wolfe algorithm to solve this problem. The algorithm will start from a feasible x_0 for the linear constraints (i.e. $A\,x_0 \leq b$) and will ensure feasibility for the linear constraints for all successive iterates.

b Given the equality constrained problem,

$$\underset{x}{\text{minimise}} \left\{ f(x) \;\middle|\; A\,x = b \right\},$$

suppose, we consider an algorithm by evaluating $-H_k^{-1} \nabla f(x_k)$, for given A, b, H_k, x_k, of appropriate dimension, with H_k positive definite, $A\,x_k = b$, $k = 1, 2, \ldots$ and define the search direction as the solution, d_k, of

$$\underset{d}{\text{minimise}} \left\{ \left(d - \left(-H_k^{-1} \nabla f(x_k)\right)\right)^T H_k \left(d - \left(-H_k^{-1} \nabla f(x_k)\right)\right) \;\middle|\; A\,d = 0 \right\}.$$

Determine d_k, and discuss its descent property for f(x) at x_k. [Hint: The positive definite property of H_k is required to ensure the existence of H_k^{-1} and subsequently to discuss the descent property of d_k.]
(All parts carry equal marks)

4 a A computer manufacturer produces x_i computers during period i, at a cost $f_i(x_i)$. During each period i, there is a demand d_i for the product and a cost $c_i(y_i)$ for holding y_i computers in inventory. The production is restricted to X computers for each period and inventory cannot exceed Y. If in the initial period (i=1), the inventory is given by $y_i = 0$, formulate the nonlinear programming problem for minimising total cost over i = 1, 2, ..., *n* periods in as few variables as possible.

b The manufacturer of three products wants to determine the monthly production volume of each product that maximises profits. The marketing department has specified that the monthly sales volume of product i, denoted by x_i, is related to the selling price (per unit) of product i, denoted by p_i as follows:

$$x_1 = 13 - p_1\,; \quad x_2 = 11 + .2\,p_1 - p_2\,; \quad x_3 = 17 - p_3$$

The cost of production (per unit) for product 1, 2, 3 are 6, 10, 15 respectively. There are a maximum of 1200 hours of machine time and 2000 hours of manpower time available per month. Each product has the following manufacturing requirements:

Product	Machine time required (hrs)	Manpower time required (hrs)
1	.2	.3
2	.4	.5
3	.6	.7

Assuming that all the manufactured products are sold and no inventory is allowed, formulate a decision model to determine the volume for each product to maximise profit. [Do not solve the optimisation problem.]
(All parts carry equal marks)

 Paper 4.77=I4.20
End of paper