EE3-13 Exam paper with Solutions

Part A – Answer any 2 out of 3 questions in part A

1. Structure of the conventional power system is presented in Figure 1.

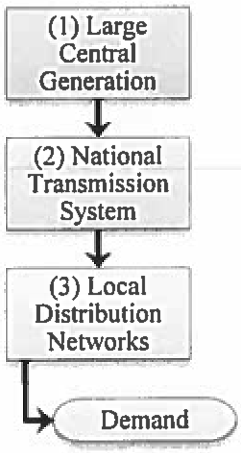


Figure 1

a) List key functions of the three sectors of the electricity system. [3]

- Generation – generate electricity wherever profitable and provides source of system control
- Transmission – responsible for bulk transport of electricity between large power stations and demand centres, minimise fuel costs in the production of electricity by allowing its production all times at the available sources having the lowest marginal costs
- Distribution – take electricity from transmission system and deliver it to the customers at appropriate voltage level - maintain the voltage within statutory limits

[1 mark for descriptions along the each point]

b) Describe briefly how demand-supply balance is maintained in real time. [2.5]

Demand and supply balance must be maintained at all times - any change in demand is met by (almost) instantaneous change in generation. Part-loaded generation is used to maintain the balance of demand and supply in real time.

[2.5 marks for a description along the point]

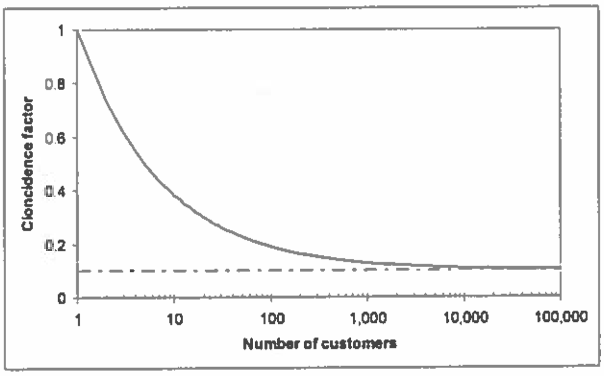
c) Describe briefly how power flows on the transmission network are controlled. [2.5]

Transmission network operates with fixed topology (and fixed impedance). Flows are determined by generation outputs (that matches demand), hence changes in generation outputs at various parts of the network change the flows through the network. Hence the power flow control generally leads to increase in generation costs.

[2.5 marks for a description along the point]

d) Describe the importance of the effect of diversity of demand in planning of power systems. What is the coincidence factor and how it changes with number of consumers? [4]

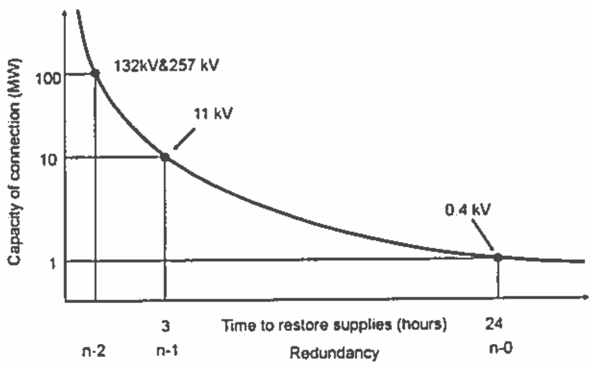
Diversity: peak demand of number individual electricity consumers is lower than the sum of their individual peaks. This is because the peaks of individual consumers do occur at the same time. The larger the number of consumers the more significant the diversity effect is, although there is saturation effect – as shown in the diagram below.



[3 marks for a description along the point + 1 mark for the graphical representation]

e) Describe briefly how security of supply, as one of the key requirements of the electricity system, is delivered. Briefly state the rational behind the power system design standards. [4]

Security of supply is delivered through asset redundancy – generation capacity margin, network design standards (N-2, N-1). Level of redundancy should reflect the balance between the cost of generation / network redundancy and cost of interruptions caused by outages.



[2 marks for a description along the point + 2 marks for the graphical representation]

f) Calculate the probability that the system of 4 generators, each of 100MW with failure rates of 6%, will not meet a demand of 280MW. [4]

Availability of generator is 94%.

Capacity (MW)	Probability	Cumulative probability
400	$94\%^4 = 78.0749\%$	100.0000%
300	$4 \times 94\%^3 \times 6\%^1 = 19.9340\%$	21.9251%
200	$6 \times 94\%^2 \times 6\%^2 = 1.9086\%$	1.9911%
100	$4 \times 94\%^1 \times 6\%^3 = 0.0812\%$	0.0825%
0	$6\%^4 = 0.0013\%$	0.0013%

Probability that demand will not be met is 1.9911%.

[2 marks for calculating individual state probabilities for 200 and 100 MW states + 1 mark for calculating cumulative probability for states 200MW and below + 1 mark for conclusion]

2.

a) Considering a simplified equivalent circuit of a high voltage transmission line presented in Figure 2,

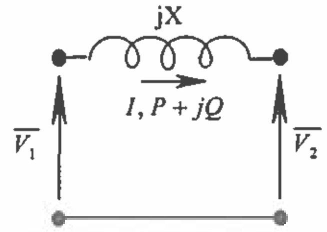


Figure 2

(i) show that the expressions of active and reactive power flow over the transmission line are given by:

$$P = \frac{V_1 V_2}{X}\sin(\delta_1 - \delta_2)$$

$$Q = \frac{V_1^{\,2} - V_1 V_2 \cos(\delta_1 - \delta_2)}{X}$$

[4]

Solution

$$\bar{I} = \frac{\overline{V_1} - \overline{V_2}}{jX}$$

$$S = \overline{V_1}\,\bar{I}^{*}$$

$$= \overline{V_1}\,\frac{\overline{V_1}^{*} - \overline{V_2}^{*}}{-jX}$$

$$= \frac{V_1^{\,2} - V_1 V_2 e^{j(\delta_1 - \delta_2)}}{-jX}$$

$$= \frac{jV_1^{\,2} - V_1 V_2 e^{j(\delta_1 - \delta_2 + 90^\circ)}}{X}$$

$$= \frac{jV_1^{\,2} - V_1 V_2\left[\cos(\delta_1 - \delta_2 + 90^\circ) + j\sin(\delta_1 - \delta_2 + 90^\circ)\right]}{X}$$

$$S = \frac{V_1 V_2}{X}\sin(\delta_1 - \delta_2) + j\frac{V_1^{\,2} - V_1 V_2 \cos(\delta_1 - \delta_2)}{X}$$

$$P = \frac{V_1 V_2}{X}\sin(\delta_1 - \delta_2)$$

$$Q = \frac{V_1^{\,2} - V_1 V_2 \cos(\delta_1 - \delta_2)}{X}$$

[4 marks]

(ii) Discuss the limits of transporting active and reactive power over transmission lines.

[4]

Unlike active power, reactive power cannot be transmitted across long distances: Transmitting reactive power requires a significant voltage drop that would become unacceptable for long distances and/or large amount of reactive power transmitted.

[4 marks]

b) Consider the transformer model in Figure 3. below:

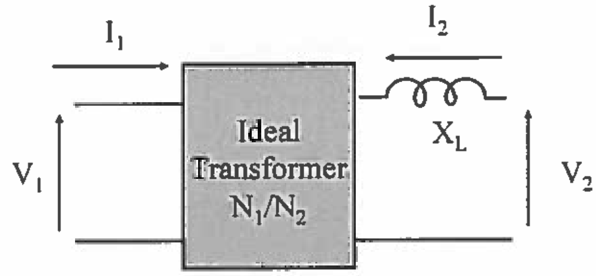


Figure 3

Starting from $Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$ derive the expression for per unit value of the transformer impedance and show that $Z_{1pu} = Z_{2pu}$

[5]

Solution

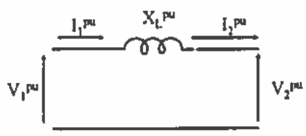
$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

$$Z_1^{pu}\cdot Z_1^{B} = \left(\frac{N_1}{N_2}\right)^2 Z_2^{pu}\cdot Z_2^{B}$$

$$Z_1^{B} = \frac{V_1^{B}}{I_1^{B}} = \frac{V_1^{B\,2}}{S^{B}} \qquad Z_2^{B} = \frac{V_2^{B}}{I_2^{B}} = \frac{V_2^{B\,2}}{S^{B}}$$

$$Z_1^{pu}\cdot\frac{V_{1B}^2}{S_B} = \left(\frac{N_1}{N_2}\right)^2 Z_2^{pu}\cdot\frac{V_2^{B\,2}}{S^{B}}$$

$$\frac{V_1^{B\,2}}{V_2^{B\,2}} = \left(\frac{N_1}{N_2}\right)^2$$



$$\boxed{Z_1^{pu} = Z_2^{pu}}$$

[5 marks]

c) Circuit diagram of a three phase system is shown in Figure 4 below.

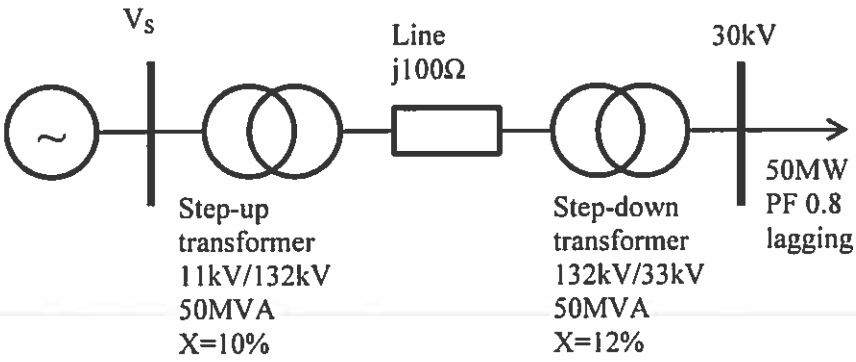


Figure 4

(i) Calculate the corresponding per unit impedances and sketch the per-unit circuit. [3.5]

This is a three-phase system. A load of 50MW at 0.8PF lagging is taken from the 33kV substation which is maintained at 30kV.

- S_{base} = 50MVA for the whole system
- The nominal voltages of the transformers are chosen for the three different zones of the circuit: 11kV, 132kV and 33kV

For the zone at 132kV - base and the per-unit impedance of the transmission line:

$$Z_{base} = \frac{V^2_{base\,LN}}{S_{base}} = \frac{(132kV)^2}{50MVA} = 348.48\ \Omega \qquad Z_{line\,pu} = \frac{Z_{line}}{Z_{base}} = \frac{j100\Omega}{348.48\Omega} = 0.287\ \text{pu}$$

In the zone of the load

$$|S_{load}| = \frac{P}{\cos\varphi} = \frac{50MW}{0.8} = 62.5\ \text{MVA}$$

$$\varphi = \cos^{-1}(0.8) = \pm 36.87^\circ$$

We choose the positive value because we know that the load has a "lagging power factor" which, according to the apparent power convention, refers to a positive Q in the expression S=P+jQ

$$S_{load} = |S_{load}|(\cos\varphi + j\sin\varphi) = 50 + j37.5\ \text{MVA}$$

$$I_{load} = \frac{S^*_{load}}{\sqrt{3}V^*_{load}} = \frac{(50 - j37.5)\ \text{MVA}}{\sqrt{3}\times 30\text{kV}} = 962.25 - j721.69\ \text{A}$$

Base current:

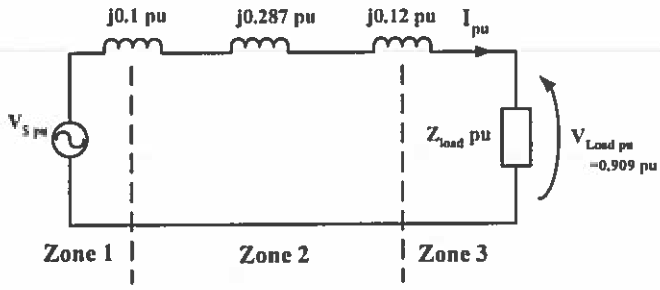
$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}} = \frac{50\ \text{MVA}}{\sqrt{3}\times 33\text{kV}} = 874.77\ \text{A}$$

$$I_{pu} = \frac{I_{load}}{I_{base}} = \frac{(962.25 - j721.69)\text{A}}{874.77\ \text{A}} = 1.1 - j0.825$$

Per-unit voltage:

$$V_{\text{load pu}} = \frac{V_{\text{load}}}{V_{\text{base}}} = \frac{30\text{kV}}{33\text{kV}} = 0.909 \text{ pu}$$

It was not necessary to change the p.u. impedance of the transformers because the chosen Sbase for the whole power network was exactly the same as the transformers' own power base.



[3.5 marks]

(ii) Calculate the magnitude of the sending voltage Vs. [3.5]

The voltage at the source
KVL:

$$Vs_{\text{pu}} = I_{\text{pu}}\left(Z_{\text{T1 pu}} + Z_{\text{line pu}} + Z_{\text{T2 pu}}\right) + V_{\text{load pu}}$$

$$Vs_{\text{pu}} = (1.1 - j0.825)(j0.1 + j0.287 + j0.12) + 0.909$$

$$Vs_{\text{pu}} = 1.327 + j0.558$$

In volts:

$$|Vs| = |Vs_{\text{pu}} V_{\text{base}}| = |(1.327 + j0.558)(11\text{kV})|$$

$$|Vs| = |(14.6 + j6.135)\text{kV}| = 15.84\text{kV}$$

[3.5 marks]

3. A 132 kV network, supplied from a remote generator connected to busbar 1 (Figure 5 below) supplies an industrial site at busbar 3 and a town supplied from busbar 2. Network voltage is controlled by the generator (G) and Static-Var-Compensator (SVC) and is kept at 1 p.u. at busbars 1 and busbar 2. Lengths of the circuits are shown in Figure 3. Individual circuits all have the same unit reactance of x = 0.2 Ω/km, while active power losses can be ignored.

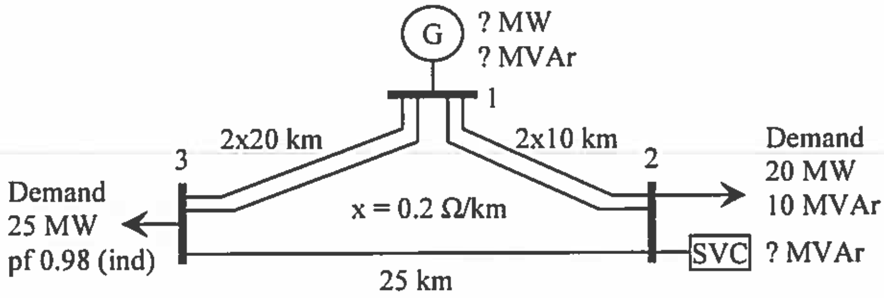


Figure 5

a) Identify the type of busbars for nodes 1, 2 and 3 (Slack, PV or PQ) [3]

Busbar types: 1 – Slack, 2 – PV, and 3 – PQ

[1 mark for each busbar type identified]

b) Assuming a 100MVA base, calculate the per unit values of all system parameters [2]

Per unit calculation:
Sb = 100 MVA
Vb = 132 kV
Zb = 1322 / 100 = 174.24 Ω

Demand at busbar 2 SD2 = 0.2 + j*0.1 p.u.
Demand at busbar 3 SD3 = 0.25 + j*0.25*tan(cos-1(0.98)) = 0.25 + j*0.05 p.u.

Circuits
z12 = j*0.2 * 10 / 2 / 174.24 = j*0.0057 p.u.
z13 = j*0.2 * 20 / 2 / 174.24 = j*0.0115 p.u.
z23 = j*0.2 * 25 / 174.24 = j*0.0287 p.u.

[1 mark for demand per unit values + 1 mark for circuits per unit values]

c) Form the Ybus matrix for this system [2]

Y bus matrix:

y12 = 1/z12 = -j * 174.24 p.u.
y13 = 1/z13 = -j * 87.12 p.u.
y23 = 1/z23 = -j * 34.85 p.u.

$$Y_{bus} = j * \cdot \begin{bmatrix} -261.36 & 174.24 & 87.12 \\ 174.24 & -209.09 & 34.85 \\ 87.12 & 34.85 & -121.97 \end{bmatrix} \text{p.u.}$$

[2 marks]

d) Perform two iterations of the Gauss-Seidel power flow and calculate:
(i) Voltages (p.u.) and angles at all three busbars

[8]

Two iterations of Gauss-Seidel power flow:
- Initialisation
Slack $V_1^0 = V_1^{spec} = 1 + j0$
PV bus $V_2^0 = V_2^{spec} = 1 + j0$; $Q_2^0 = 0$ p.u.; $S_2^0 = (0.2 + j0)$ p.u.
PQ bus $V_3^0 = 1 + j0$; $S_3^0 = -(0.25 + j * 0.05)$ p.u.

- The first iteration

$$V_3^{(1)} = \frac{1}{Y_{33}} \cdot \left(\frac{S_3^*}{V_3^{(0)*}} - Y_{31} \cdot V_1^{(0)} - Y_{32} \cdot V_2^{(0)}\right)$$

$$= \frac{1}{-j121.97} \cdot \left(\frac{-0.25 + j0.05}{1 - j0} - j87.12 \cdot (1 + j0) - j34.85 \cdot (1 + j0)\right)$$

$$= 0.9584 - j0.2323 \quad \text{p.u.}$$

$$\widetilde{V}_2^{(1)} = \frac{1}{Y_{22}} \cdot \left(\frac{S_2^{(0)*}}{V_2^{(0)*}} - Y_{21} \cdot V_1^{(0)} - Y_{23} \cdot V_3^{(1)}\right)$$

$$= \frac{1}{-j209.09} \cdot \left(\frac{-0.2}{1 - j0} - j174.24 \cdot (1 + j0) - j34.85 \cdot (0.9584 - j0.2323)\right) = 1 - j0.0957 \text{p.u}$$

$$V_2^{(1)} = \left|V_2^{spec}\right| \frac{\widetilde{V}_2^{(1)}}{\left|\widetilde{V}_2^{(1)}\right|} = 1 \cdot \frac{1 - j0.0957}{\left|1 - j0.0957\right|} = 0.9955 - j0.0952 \text{ p.u.}$$

$$Q_2^{(1)} = -\operatorname{Im}(V_2^{(1)*} \cdot (Y_{21} \cdot V_1^{(0)} + Y_{22} \cdot V_2^{(1)} + Y_{23} \cdot V_3^{(1)})) = 0.016230 \text{p.u.}$$

$$S_2^{(1)} = \operatorname{Re}(S_2^{(0)}) + jQ_2^{(1)} = -0.2 + j0.01623 \text{ p.u}$$

[5]

- The second iteration

$$V_3^{(2)} = \frac{1}{Y_{33}} \cdot \left(\frac{S_3^*}{V_3^{(1)*}} - Y_{31} \cdot V_1^{(0)} - Y_{32} \cdot V_2^{(1)}\right)$$

$$= \frac{1}{-j121.97} \cdot \left(\frac{-0.25 + j0.05}{0.9584 + j0.2323} - j87.12 \cdot (1 + j0) - j34.85 \cdot (0.9955 - j0.0952)\right)$$

$$= 0.9076 - j0.2305 \quad \text{p.u.}$$

$$\widetilde{V}_2^{(2)} = \frac{1}{Y_{22}} \cdot \left(\frac{S_2^{(1)*}}{V_2^{(1)*}} - Y_{21} \cdot V_1^{(0)} - Y_{23} \cdot V_3^{(2)}\right)$$

$$= \frac{1}{-j209.09} \cdot \left(\frac{-0.2 - 0.016230}{0.9955 + j0.0952} - j174.24 \cdot (1 - j0) - j34.85 \cdot (0.9076 - j0.2305)\right)$$

$$= 0.9917 - \mathrm{j}0.1347 \mathrm{p.u}$$

$$V_2^{(2)} = \left|V_2^{spec}\right| \frac{\widetilde{V}_2^{(2)}}{\left|\widetilde{V}_2^{(2)}\right|} = 1 \cdot \frac{0.9917 - j0.1347}{\left|0.9917 - j0.1347\right|} = 0.9909 \text{ - j}0.1346 \text{ p.u.}$$

$$Q_2^{(2)} = -\mathrm{Im}(V_2^{(2)*} \cdot (Y_{21} \cdot V_1^{(0)} + Y_{22} \cdot V_2^{(2)} + Y_{23} \cdot V_3^{(2)})) = 0.040100 \mathrm{p.u.}$$

$$S_2^{(1)} = \mathrm{Re}(S_2^{(0)}) + jQ_2^{(1)} = -0.2 + \mathrm{j}\,0.040100 \text{ p.u}$$

$$\Delta V_2^{(2)} = \left|V_2^{(2)} - V_2^{(1)}\right| = \left|(0.9909 - j0.1346) - (0.9955 - j0.0952)\right| = 0.0397 \text{ p.u}$$

$$\Delta V_3^{(2)} = \left|V_3^{(2)} - V_3^{(1)}\right| = \left|(0.9076 - j0.2305) - (0.9584 - j0.2323)\right| = 0.0508 \text{ p.u.}$$

[2 marks for initialisation + 1 mark per busbars 2 and 3 voltages per iteration + 1 mark per injected reactive power at busbar 2 per iteration]

(ii) Generator G active and reactive power output

[2]

Generation: 43.5 MW, 9.6 MVAr (sum of flows from slack busbar)

$$S_{12} = V_1^{(0)} \cdot ((V_1^{(0)} - V_2^{(2)}) \cdot y_{12})^* = 0.234473 + j0.015849 \text{ p.u.}$$

$$S_{13} = V_1^{(0)} \cdot ((V_1^{(0)} - V_3^{(2)}) \cdot y_{13})^* = 0.200847 + j0.080464 \text{ p.u}$$

[2 marks]

(iii) Reactive power delivered by the Static Var Compensator (SVC)

[3]

SVC: 4.0 – (-10) = 14 MVAr

[3 marks]

Part B – Answer any 2 out of 3 questions in part B

4.

a) Explain why sub-transient reactance of synchronous generators is considered for fault calculations.

[5]

- During a fault/short-circuit, the current and hence, the magnetic flux in a synchronous machine should increase.
- According to theorem of constant flux linkage the magnetic flux linking a closed winding cannot change instantaneously
- Therefore, excess magnetic flux due to short-circuit current is initially forced through high reluctance paths that do not link the field or damper circuits
- As a result, immediately after the fault the stator inductance (inversely proportional to reluctance) is low leading to high current.
- As the flux moves towards lower reluctance paths, stator inductance increases resulting in lower currents in steady-state.
- Sub-transient reactance is used in fault calculations to capture the above effect immediately after the fault which corresponds to the highest current.

[1 mark for each point]

b) The bus bar arrangement within a 33 kV substation is shown in Figure 4.1. The two sections of the 33 kV bus bar (AB and CD) is separated by a reactor L. Section AB is fed from four identical 10 MVA generators each having 0.2 p.u. sub-transient reactance. Section CD is fed from the power grid through a 50 MVA transformer with a reactance of 0.1 p.u. Each of the circuit breakers M and N has a short circuit capacity of 600 MVA. Consider 50 MVA base for your calculations. For a 3-phase fault at location P,

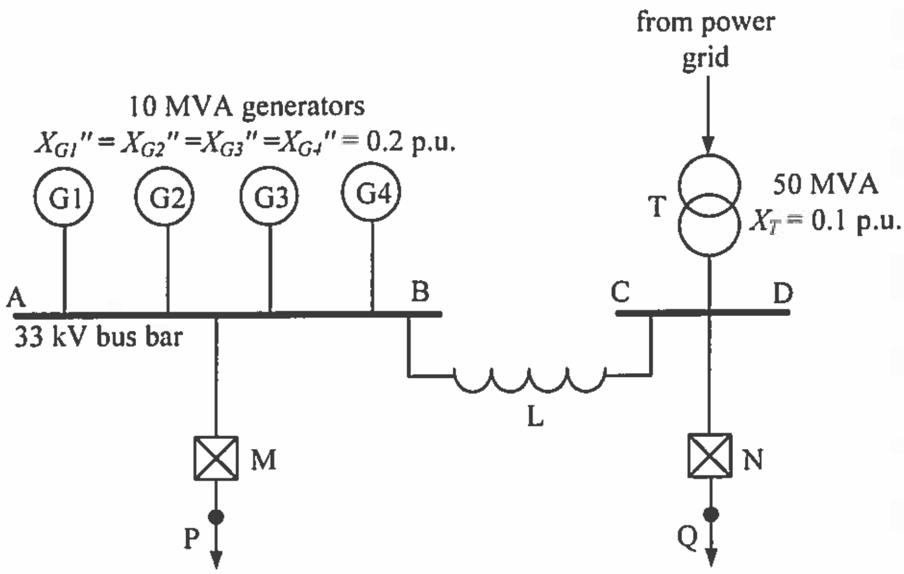


Figure 4.1: Single-line diagram of the 33 kV substation in Question 4(b) and 4(c)

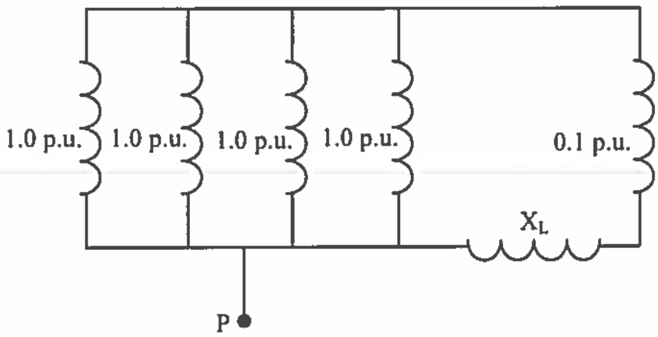
(i) Draw the single line circuit arrangement of the reactances looking into the fault point P. Label the reactance values (except the unknown value of the reactor L) in p.u. with respect to your chosen 50 MVA base.

[3]

Generator sub-transient reactances with respect to the chosen 50 MVA base are

$$X''_{G1} = X''_{G2} = X''_{G3} = X''_{G4} = 0.2 \times \frac{50}{10} = 1.0 \text{ p.u.}$$

The single-line circuit arrangement looking into the fault location P is:



[1 mark for base conversion, 2 marks for circuit diagram]

(ii) Calculate the reactance of the reactor L in p.u. (with respect to 50 MVA base) which would ensure that the circuit breaker M is not overloaded.

[4]

Equivalent reactance looking into the fault point at P is:

$$X_P = \frac{0.25\,(X_L + 0.1)}{0.25 + X_L + 0.1}$$

Short circuit MVA at P is:

$$S_{sc} = \frac{S_{base}}{X_P} = \frac{50\,(X_L + 0.35)}{0.25\,(X_L + 0.1)}$$

If the circuit breaker M is not to be overloaded the above short circuit MVA should not exceed 600 MVA. Hence,

$$\frac{50\,(X_L + 0.35)}{0.25\,(X_L + 0.1)} = 600 \;\Rightarrow\; X_L = 0.05 \text{ p.u.}$$

[2 marks for expression of X_P ; 1 mark each for equation and correct calculation]

(iii) Determine the reactance of the reactor L in ohms.

[3]

Base impedance is:

$$Z_{base} = \frac{(kV_{L\text{-}base})^2}{MVA_{base}} = \frac{33^2}{50} = 21.78\,\Omega$$

The reactance (XL) of the reactor should be:

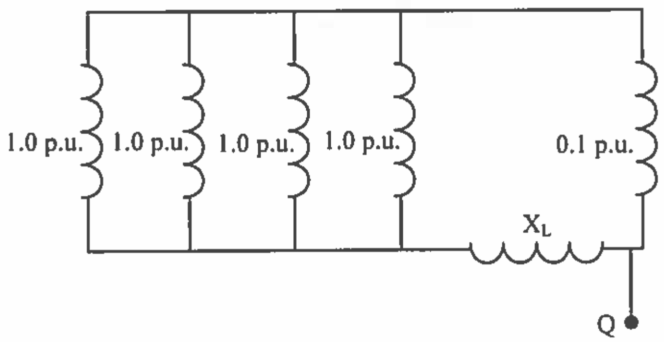
$$X_L\,(\text{in }\Omega) = X_L\,(\text{in p.u.}) \times Z_{base} = 1.089\ \Omega$$

[1 marks each for correct expression for base and absolute reactance; 1 mark for correct calculation]

c) For the same sub-station arrangement as shown in Figure 4.1 and described in part (b) of this question, determine the reactance (in ohms) of the reactor L if the circuit breaker N is not be overloaded due to a 3-phase fault at location Q. Consider 50 MVA base for your calculations.

[5]

The single-line circuit arrangement looking into the fault location Q is:



Equivalent reactance looking into the fault point at Q is:

$$X_P = \frac{0.1\,(X_L + 0.25)}{0.25 + X_L + 0.1}$$

Short circuit MVA at Q is:

$$S_{sc} = \frac{S_{base}}{X_P} = \frac{50\,(X_L + 0.35)}{0.1\,(X_L + 0.25)}$$

If the circuit breaker M is not to be overloaded the above short circuit MVA should not exceed 600 MVA. Hence,

$$\frac{50\,(X_L + 0.35)}{0.1\,(X_L + 0.25)} = 600 \quad \Rightarrow X_L = 0.25\text{ p.u.}$$

The reactance (X_L) of the reactor should be:

$$X_L\,(\text{in }\Omega) = X_L\,(\text{in p.u.}) \times Z_{base} = 5.445\ \Omega$$

[2 marks for expression of X_P; 1 mark each for correct equation and calculation of p.u. and absolute values of X_L]

5.

a) Derive the expression for fault current (in phase domain) due to a line-to-line (LL) fault between phases B and C. The final expression should be in terms of pre-fault voltage and sequence impedances. Neglect fault impedance.

[5]

Fault currents in phase domain:

$I_A = 0, \qquad I_B = -I_C$

Fault condition in sequence domain:

$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_B \\ -I_B \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ (\alpha-\alpha^2)I_B \\ (\alpha^2-\alpha)I_B \end{bmatrix} = \begin{bmatrix} 0 \\ jI_B/\sqrt{3} \\ -jI_B/\sqrt{3} \end{bmatrix}$$

For LL fault, the positive and negative sequence networks are connected in parallel. So,

$$I_{A1} = \frac{V_f}{Z_1 + Z_2}$$

Thus the fault current is given by:

$$I_B = -I_C = \frac{-j\sqrt{3}}{Z_1 + Z_2}V_f$$

[1 mark for phase domain expressions; 2 marks for correct conversion to sequence currents; 1 marks each for expression for positive sequence current and fault current]

b) An unbalanced delta-connected load draws the following line currents: $I_B = 5\angle 30°$ A, $I_C = 3.5\angle 90°$ A. Calculate the magnitude and phase angle of the following:

(i) negative-sequence component of the unbalanced line current drawn by the load.

[4]

For a delta-connected load, the sum of the line currents should be zero. Hence

$$I_A = -(I_B + I_C) = -(5\angle 30^{\circ} + 3.5\angle 90^{\circ}) = 7.4\angle -125.8^{\circ}\ \text{A}$$

Negative sequence component of the current is given by:

$$I_{A2} = \frac{1}{3}(I_A + \alpha^2 I_B + \alpha I_C)$$

$$= \frac{1}{3}(7.4\angle -125.8^{\circ} + 1\angle -120^{\circ} \times 5\angle 30^{\circ} + 1\angle 120^{\circ} \times 3.5\angle 90^{\circ})\text{A}$$

$$= 4.9075\angle -120^{\circ}\ \text{A}$$

$$I_{B2} = \alpha I_{A2} = 4.9075\angle 0^{\circ}\ \text{A}$$

$$I_{C2} = \alpha^2 I_{A2} = 4.9075\angle 120^{\circ}\ \text{A}$$

[1 mark for I_A ; 2 marks for I_{A2} ; 1 mark for I_{B2}, I_{C2}]

(ii) zero-sequence component of the line current drawn by the delta-connected load. [1]

For a delta-connected load, the sum of the line currents should be zero. So zero sequence component of the line current should be zero i.e. there should not be any zero sequence component in the line currents in case of delta-connected loads.

[1 mark]

c) A 50 MVA, 11 kV 3-phase synchronous generator is subjected to three different types of faults at its terminal. Each fault is a solid fault with zero fault impedance. The recorded fault currents for each fault type are as follows:

3-phase fault – 2000 A
LG fault on phase A – 4200 A
LL fault between phases B and C – 2600 A

Assuming 1.0 p.u. pre-fault voltage calculate the positive-, negative- and zero-sequence reatances (in p.u. on 50 MVA base) of the synchronous generator for the following generator neutral grounding arrangements:

(i) Generator neutral is grounded through a reactance of 0.05 ohms [6]

Base (or rated) current is given by:

$$I_{base} = \frac{MVA_{base}}{\sqrt{3} \times kV_{base}} = \frac{50}{\sqrt{3} \times 11} = 2.624 \text{ kA}$$

Recorded fault currents in p.u. for different types of faults are given by:

$$I_{f-3p} = \frac{2}{2.624} = 0.762 \text{ p.u.}$$

$$I_{f-LG} = \frac{4.2}{2.624} = 1.6 \text{ p.u.}$$

$$I_{f-LL} = \frac{2.6}{2.624} = 0.991 \text{ p.u.}$$

For a 3-phase fault, fault current I_f is given by:

$$I_{f-3p} = \frac{V_f}{X_1} \quad \Rightarrow \quad X_1 = \frac{V_f}{I_{f-3p}} = \frac{1}{0.762} = 1.312 \text{ p.u.}$$

For a LL fault, fault current I_f is given by:

$$I_{f-LL} = \frac{\sqrt{3}\, V_f}{X_1 + X_2} \quad \Rightarrow \quad X_1 + X_2 = \frac{\sqrt{3}\, V_f}{I_{f-LL}} = \frac{\sqrt{3}}{0.991} = 1.748 \text{ p.u.} \quad X_2 = 0.436 \text{ p.u.}$$

For a LG fault, fault current I_f is given by:

$$\mathrm{I_{f-LG}} = \frac{3\mathrm{V_f}}{\mathrm{X_1 + X_2 + X_0 + 3X_N}} \quad \Rightarrow \quad \mathrm{X_0} = \frac{3\mathrm{V_f}}{\mathrm{I_{f-LG}}} - (\mathrm{X_1 + X_2 + 3X_N})$$

Reactance of the generator neutral X_N in p.u. is:

$$\mathrm{X_N} = 0.05 \times \frac{50}{11^2} = 0.0207 \text{ p.u.}$$

Zero sequence reactance is:

$$\mathrm{X_0} = \frac{3\mathrm{V_f}}{\mathrm{I_{f-LG}}} - (\mathrm{X_1 + X_2 + 3X_N}) = 0.0649 \text{ p.u.}$$

[1 mark each for correct expression of fault current for each fault type; 1 mark for correct base conversion; 2 marks for calculations]

(ii) Generator neutral is solidly grounded

[4]

Generator neutral impedance is not involved in 3-phase and line-to-line (LL) faults. So the values of the positive and negative sequence reactances would be the same as in part (i).Only the zero sequence impedance would change if the generator neutral is solidly grounded.

Zero sequence reactance in this case would be:

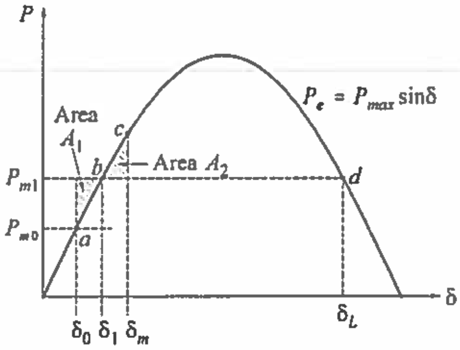
$$\mathrm{X_0} = \frac{3\mathrm{V_f}}{\mathrm{I_{f-LG}}} - (\mathrm{X_1 + X_2}) = 0.127 \text{ p.u.}$$

[1 mark for correct interpretation of X_1 and X_2; 1 mark for correct expression for X_0; 1 mark for calculations]

6.

a) Using the power-angle curve for a round rotor synchronous generator, explain the sequence of events that follows if the mechanical input to the generator is suddenly increased. Label the different positions of mechanical power inputs, rotor angles clearly on the power-angle curve and identify the accelerating and decelerating areas.

[5]



- Rotor accelerates towards 'b' due to excess mech. Power
- At 'b' accelerating power is zero, but rotor speed > synch. speed (ω_0)
- Rotor angle continue to increase, $\delta_1 \rightarrow \delta_m$ but the rotor decelerates as $P_m<P_e$
- At 'c' rotor reaches sync. speed
- As $P_m<P_e$, rotor speed drops below ω_0
- Operating point retraces 'c' → 'b' → 'a' and the same cycle continues in absence of any damping

[1 mark for each point]

b) The rotor of a 50 MVA, 11 kV, 4-pole, 50 Hz 3-phase synchronous generator has a moment of inertia J = 10 × 103 kg-m2.

(i) Calculate the kinetic energy stored (in MJ) in the generator rotor at rated speed.

[2]

Kinetic energy stored in the rotor is:

$$\mathrm{KE} = \frac{1}{2}J\omega_m^2 = \frac{1}{2}J\left[\frac{\omega_s}{(P/2)}\right]^2 = \frac{1}{2}\times 10\times 10^3\times\left[\frac{2\pi\times 50}{(4/2)}\right]^2 = 123.37\ \mathrm{MJ}$$

[1 mark for expression; 1 mark for calculations]

(ii) Determine the inertia constant H (in MJ/MVA) of the generator.

[1]

Inertia constant of the generator is:

$$H = \frac{\mathrm{KE}}{\mathrm{MVA}} = \frac{123.37}{50} = 2.47\ \mathrm{MJ/MVA}$$

[0.5 mark each for expression and calculation]

(iii) If the mechanical input power to the generator is suddenly increased by 25 MW, calculate the acceleration of the rotor (in elect-rad/sec2) using the swing equation. Neglect any instantaneous change in electrical power output of the generator.

[3]

The swing equation can be written as:

$$M\frac{d^2\delta}{dt^2} = \Delta P \quad \text{where } M = \frac{2GH}{\omega_s} = \frac{2 \times 50 \times 2.47}{2\pi \times 50} = 0.785 \text{ MJ-sec/elect-rad}$$

Rotor acceleration is:

$$\frac{d^2\delta}{dt^2} = \frac{\Delta P}{M} = \frac{25}{0.785} = 31.85 \text{ elect-rad/sec}^2$$

[0.5 mark for swing equation, 1 mark each for calculation of *M* and calculation of acceleration; 0.5 mark for correct units]

c) The generator in part (b) of the question is a round-rotor machine which maintains 1.05 p.u. voltage at its terminal while producing 60 MW power output in steady state. The generator is connected to a large power system (infinite bus) through two transmission routes whose equivalent reactance is 0.3 p.u. The voltage of the infinite bus is 0.98 p.u. Neglect the resistance of the generator and the transmission line. Assume a constant the mechanical power input.

(i) Calculate the steady state rotor angle δ0 (in rads).

[2]

In steady state:

$$P_{max1} = \frac{1.05 \times 0.98}{0.3} = 3.43 \text{ p.u.}$$

Actual power output $P_1 = \frac{60}{50} = 1.2$ p.u.

Steady state rotor angle is:

$$\delta_0 = \sin^{-1}\frac{P_1}{P_{max1}} = 0.3574 \text{ rads}$$

[1 mark for correct P_{max1} and P_1; 1 mark for δ_0]

(ii) During a 3-phase short circuit at the sending end of one of the transmission routes the power output of the generator drops to zero. The short circuit is cleared by disconnecting the affected transmission line which increases the equivalent reactance between the generator and the infinite bus to 0.45 p.u. Calculate the maximum allowable rotor angle δ_m (in rads) to preserve stability.

[3]

Maximum allowable rotor angle correspond to the post-fault condition where:

$$P_{max2} = \frac{1.05 \times 0.98}{0.45} = 2.2867 \text{ p.u.}$$

Mechanical power input is constant i.e. $P_l = 1.2$ p.u.

Maximum allowable rotor angle is:

$$\delta_m = \pi - \sin^{-1}\frac{P_1}{P_{max2}} = 2.589 \text{ rads}$$

[1 mark each for interpretation and correct calculation of P_{max2} and δ_m]

(iii) Use equal area criterion to determine the critical clearing angle corresponding to the 3-phase short circuit condition described in part (ii).

[4]

Let's suppose δ_c is the critical clearing angle.

Accelerating area is:

$$A_1 = \int_{\delta_0}^{\delta_c} (P_1 - 0)\, d\delta = 1.2 \times (\delta_c - 0.3574) \text{ p.u-rads}$$

Decelerating area is:

$$A_2 = \int_{\delta_c}^{\delta_m} (P_{max2}\sin\delta - P_1)\, d\delta$$
$$= 2.2867 \times (\cos\delta_c + 0.8512) - 1.2 \times (2.589 - \delta_c) \text{ p.u-rads}$$

Using the equal area criterion:

$$1.2 \times (\delta_c - 0.3574) = 2.2867 \times (\cos\delta_c + 0.8512) - 1.2 \times (2.589 - \delta_c)$$

$$\Rightarrow \delta_c = \cos^{-1} 0.3199 = 1.2452 \text{ rads}$$

[1 mark each for accelerating and decelerating areas; 1 mark for use of EAC and 1 mark for calculations]