[B] Bookwork

[E] New Example

[A] New Application

[T] New Theory

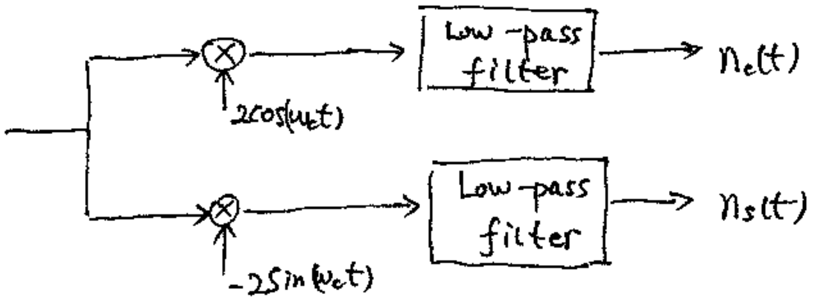
ANSWERS 2013

1. a) i) Uncorrelated: $E[XY] = E[X]\,E[Y]$

independent: $f_{XY}(xy) = f_X(x)\, f_Y(y)$ [4B]

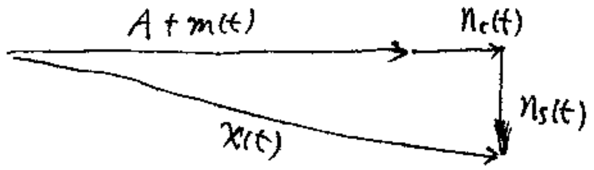
"independent" implies uncorrelated, but the converse is not necessarily true.

ii)



[4B]

b) i)



[3B]

$$y(t) = \text{envelope of } x(t)$$
$$= \sqrt{[A + m(t) + n_c(t)]^2 + n_s^2(t)}$$

When noise is small [3B]

$$y(t) \approx A + m(t) + n_c(t)$$

ii) Digital communication is [3B]

more immune to channel noise;

digital signals can be represented in a uniform format;

easier to process;

flexible and allow for sophisticated functions;

able to provide digital services such as Internet.

Feedback: generally answered well.

Feedback: generally answered ok, but some students mixed up source coding and channel coding.

[4 B]

c) i) Source coding is to compress the data by reducing the number of source bits, e.g., Huffman code.

Channel coding is to introduce redundant bits to enable to detection and correction of errors caused by the channel, e.g., Hamming code.

ii) Hamming codes are a class of linear block codes that can correct a single error. [3 B]

For Hamming codes, $d_{min} = 3$.

iii) $r = n - k = \log_2(n+1) \Rightarrow n = 2^r - 1,$

$k = 2^r - 1 - r.$ [2 B]

First few Hamming codes: [2 B]

$(n, k) = (7, 4),\ (15, 11),\ (31, 26), \cdots$

d) i) [3 B]

$$C_{in} = B \log_2(1 + SNR_{in})$$

ii) [3 B]

$$C_{out} = W \log_2(1 + SNR_{out})$$

iii) For an ideal system, $C_{in} = C_{out}$. [2 B]

$$W \log_2(1 + SNR_{out}) = B \log_2(1 + SNR_{in})$$

Since

$$SNR_{in} = \frac{P}{N_0 B} = \frac{W}{B}\frac{P}{N_0 W} = \frac{W}{B} SNR_{baseband},$$

[2 B]

we have

$$W \log_2(1 + SNR_{out}) = B \log_2\left(1 + \frac{SNR_{baseband}}{B/W}\right)$$

$$SNR_{out} = \left(1 + \frac{SNR_{baseband}}{B/W}\right)^{B/W} - 1.$$ [2 B]

$$\to e^{SNR_{baseband}} \quad \text{as } B/W \to \infty.$$

Some confusions arose here. A common mistake was to miss W in input noise power. Another mistake was the calculation of the limit.

Generally answered well.

2\. i) Stationary process: [5B]

$\mu_X(t) = \mu_X$ doesn't depend on t;

$R_X(t, t+\tau) = R_X(\tau)$ is a function of τ only.

Wiener-khinchine relation: Power spectral density is the Fourier transform of the autocorrelation function.

ii) From the table,

$$R_X(\tau) = N_0 B \operatorname{Sinc}(2B\tau).$$ [2B]

If taken at Nyquist rate, $\tau = \frac{k}{2B}$, $k = 0, \pm 1, \pm 2, \cdots$

Then,

$$R_X(\tau) = N_0 B \operatorname{Sinc}(k) = 0, \quad k = \pm 1, \pm 2, \cdots$$ [3B]

This means the samples are uncorrelated, hence being independent since they are Gaussian.

iii)

Mean: $E[X(t)] = E[a\cos(\omega t + \theta)]$

$$= \frac{1}{2}[a\cos(\omega t) + a\cos(\omega t + \pi)]$$ [2E]

$$= 0$$

Autocorrelation

$$R_X(t, t+\tau) = E[a\cos(\omega t + \theta)\, a\cos(\omega(t+\tau) + \theta)]$$

$$= \frac{1}{2}[a^2\cos(\omega t)\cos(\omega(t+\tau)) + a^2\cos(\omega t + \pi)\cos(\omega(t+\tau) + \pi)]$$ [4E]

$$= a^2\cos(\omega t)\cos(\omega(t+\tau))$$

$$= \frac{a^2}{2}[\cos(\omega(2t+\tau)) + \cos(\omega\tau)]$$

can't get rid of t!

Therefore, it is NOT stationary. [2E]

Generally answered well. A common mistake was to claim it is stationary.

b Since $m(t)$ is Gaussian,

$$P\left(|m(t)| > m_p\right) = 2\,Q\left(\frac{m_p}{\sigma}\right) = 6 \times 10^{-7}$$ [3 A]

where σ is the standard deviation. Thus, $P = \sigma^2$.

From the graph, if $Q(x) = 3 \times 10^{-7}$,

$$x = 5.$$ [3 A]

Therefore, $\frac{m_p}{\sigma} = 5$.

This means $\frac{P}{m_p^2} = \frac{\sigma^2}{m_p^2} = \frac{1}{25}$. [3 A]

Therefore,

$$SNR_{FM} = 3 \times 5^2 \times \frac{1}{25}\, SNR_{baseband}$$ [3 A]

$$= 3\, SNR_{baseband}.$$

Part b was answered less well. A common mistake was to miss factor 2 in the first step. Also, some students couldn't use the graph of Q(x) correctly.

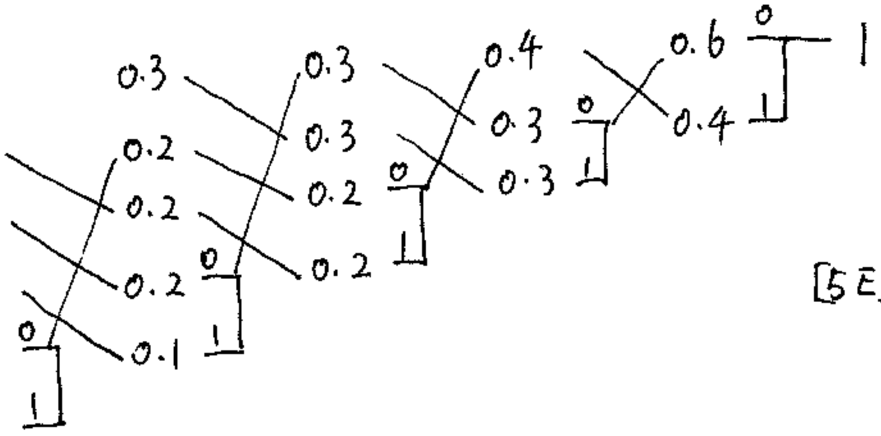
3. a) i) $H(S) = -\sum p_i \log_2 p_i$ [2E]

$$= -\left(0.3\log_2 0.3 + 2\times 0.2\times\log_2 0.2 + 3\times 0.1\times\log_2 0.1\right)$$

$$= 2.45$$

ii)

01	a	0.3
11	i	0.2
000	o	0.2
001	b	0.1
100	e	0.1
101	m	0.1



[5E]

average length

$$L = 2\times 0.5 + 3\times 0.5$$

$$= 2.5$$

Generally answered less well, probably due to the familiar format.

b) i) Systematic, because the information bit appears as is. [2 A]

ii) $G = [1\ 1\ 1\ 1\ 1]$ [2 A]

$$H = \begin{bmatrix} 1 & 1 & & & 0 \\ 1 & & 1 & & \\ 1 & 0 & & 1 & \\ 1 & & & & 1 \end{bmatrix}$$ [2 A]

iii) $d_{min} = 5$ [2 A]

iv) $s = eH^T$

s	e
1 1 1 1	1 0 0 0 0
1 0 0 0	0 1 0 0 0
0 1 0 0	0 0 1 0 0
0 0 1 0	0 0 0 1 0
0 0 0 1	0 0 0 0 1

[5 A]

v)

s	e
0 1 1 1	1 1 0 0 0
1 0 1 1	1 0 1 0 0
1 1 0 1	1 0 0 1 0
1 1 1 0	1 0 0 0 1
1 1 0 0	0 1 1 0 0
1 0 1 0	0 1 0 1 0
1 0 0 1	0 1 0 0 1
0 1 1 0	0 0 1 1 0
0 1 0 1	0 0 1 0 1
0 0 1 1	0 0 0 1 1

[5 A]

Answers were ok, but some students couldn't remember what are the generating and parity-check matrices. Common mistake: don't know what is syndrome and how to calculate it.

vi) For coherent PSK, the raw error probability is

$$P_e = Q\left(\frac{A}{\sigma}\right)$$

$$= Q(4) \approx \quad 3 \times 10^{-5}$$

With majority-rule decoding, ($p =$)

$$P_e = \sum_{i=3}^{5} \binom{5}{i} p^i (1-p)^{5-i}$$ [5 A]

$$= \binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p) + \binom{5}{5} p^5$$

$$= 10\, p^3 (1-p)^2 + 5\, p^4 (1-p) + p^5$$

$$= \quad 2.7 \times 10^{-13}$$

Answers were not good, but a small number of students obtained the correct answer. Common mistakes: that of calculating the raw error probability, and that of identifying the error events of majority-rule decoding.