UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
BSc Honours Degree in Mathematics and Computer Science Part II
MSci Honours Degree in Mathematics and Computer Science Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the
Associateship of the Royal College of Science

PAPER C240=MC240

COMPLEXITY AND COMPUTABILITY

Wednesday 12 May 2004, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 Throughout this question you should use the following definitions:

$$\mathbf{S} \equiv \lambda xyz.xz(yz)$$

$$\mathbf{K} \equiv \lambda xy.x$$

$$\mathbf{I} \equiv \lambda x.x$$

a Write down all of the possible reduction sequences for the term **S I I** (there are three of them). Identify which are *standard* reductions.

b Show that the terms $\lambda xz.xz$ and $\lambda x.xz$ are incompatible.

c i) State and prove the Fixed-Point Theorem.

ii) Hence or otherwise define a fixed-point operator **Y**.

iii) Compute the fixed-point of the term **KI**.

d In the *standard* numeral system, 0 is represented by **I** and (n+1) is represented by the pair [**F**,"n"] (where **F** is the term representing false and "n" is the numeral representing n).

i) Write down terms for the successor, predecessor and test for zero functions.

ii) Assuming an encoding for conditionals and a fixed-point operator, write down a term for addition.

iii) Assuming an encoding for conditionals and a fixed-point operator and using your answer from part ii, write down a term for multiplication.

The four parts carry, respectively, 25%, 25%, 25% and 25% of the marks.

2 a i) Define the *Halting Problem*. What does it mean to say that the halting problem is *unsolvable*?

ii) Prove that the Halting Problem is unsolvable.

iii) Explain how the technique of *reduction* can be used to prove a problem unsolvable.

b In this question:

- C denotes the standard typewriter alphabet

- If S is a standard Turing machine and w a word of C, $S[w]$ is a standard Turing machine that overwrites its input with w and then runs S. So $f_{S[w]}(x) = f_S(w)$ for any word x of C

- $EDIT$ is a standard Turing machine such that for any standard Turing machine S and word w of C, $f_{EDIT}(code(S)*w) = code(S[w])$.

- $REMB$ is a standard Turing machine that removes all instances of the character 'b' from its input. (so, for example, $f_{REMB}(abc) = ac$).

- U is a (standard) Universal Turing machine.

Evaluate:

i) $f_U(code(REMB)*\ bibliographic)$

ii) $f_U(f_{EDIT}(code(REMB)*\ books)*papers)$

iii) $f_U(f_{EDIT}(code(U)*code(REMB)*\ papers)*\ books)$

The two parts carry, respectively, 70%, 30% of the marks.

3 In this question you are asked to design a Turing Machine, M, which encodes a string of symbols according to substitution rules which are also given in the input. The Turing Machine alphabet is C, the usual typewriter alphabet.
The input is of the form $w*e$, where

$w = w_1 w_2 .. w_m$ is the input word to be encoded

$e = c_1 d_1 c_2 d_2 c_3 d_3 c_4 d_4 \ldots c_n d_n$ where the c_i, $i = 1, 2, \ldots n$ are symbols of C, and $encode(c_i) = d_i$.

The output is the word $w' = w_1' \, w_2' .. w_m'$, where

$w'_1 = encode(w_1)$, $w'_2 = encode(w_2), .. w'_m = encode(w_m)$.

You may assume that $c_i \neq *$ for $i = 1, 2, \ldots, n$ *and* $w_i \neq *$ for $i = 1, 2, \ldots, m$, and also that every symbol in w appears with its encoding in e.
You should not assume that square 0 is implicitly marked.

a Give

i) the state diagram of your Turing Machine, with an explanation of your notation

ii) a written explanation of how your Turing Machine works.

b i) Define the *time function* of a Turing Machine.

ii) Derive the time function, $time_M(m, n)$ for the Turing Machine M in your answer to part a, and explain your derivation.

The two parts carry, respectively, 70%, 30% of the marks.

4a Let A and B be arbitrary *yes-no* problems. Define what it means to say that A *reduces to* B *in p-time*. (in symbols, $A \leq B$).

b Let $\leq$ be the relation of part a. Prove that if $A \leq B$ and $B \in NP$ then $A \in NP$.

c Let $\leq$ be the relation of part a.

i) Define the class NPC of NP-complete yes-no problems.

ii) Let HCP, PSAT be the Hamiltonian Circuit Problem and the Propositional Satisfaction Problem, respectively.

Let A be a yes-no problem, and suppose that HCP$\leq$ A and $A \leq$ PSAT. Prove directly from your definition in part c i) that A is NP-complete.

[You may assume that HCP and PSAT are NP-complete.]

The three parts carry, respectively, 20%, 50%, 30% of the marks