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Model Answers and Mark Schemes		First Examiner:	Julius Georgiou
Paper Code:	EE 4.17 AO4	Second Examiner:	Esther Rodriguez -Villegas
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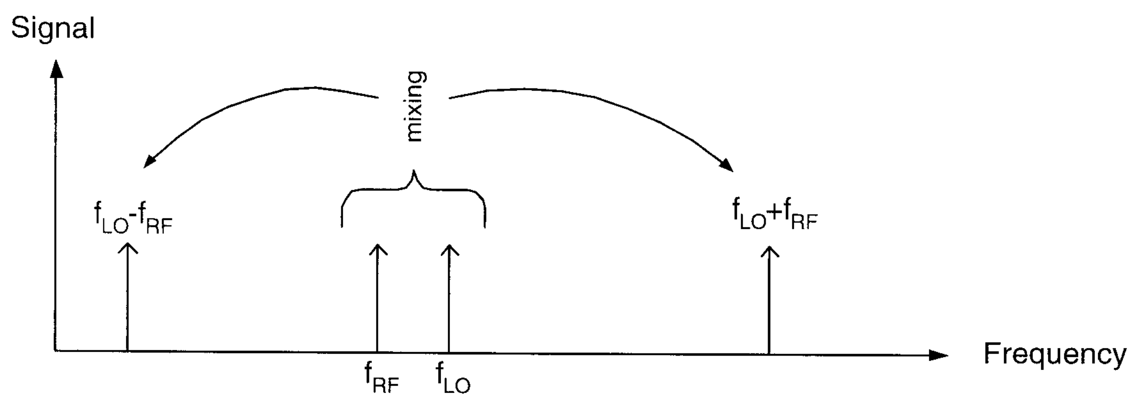
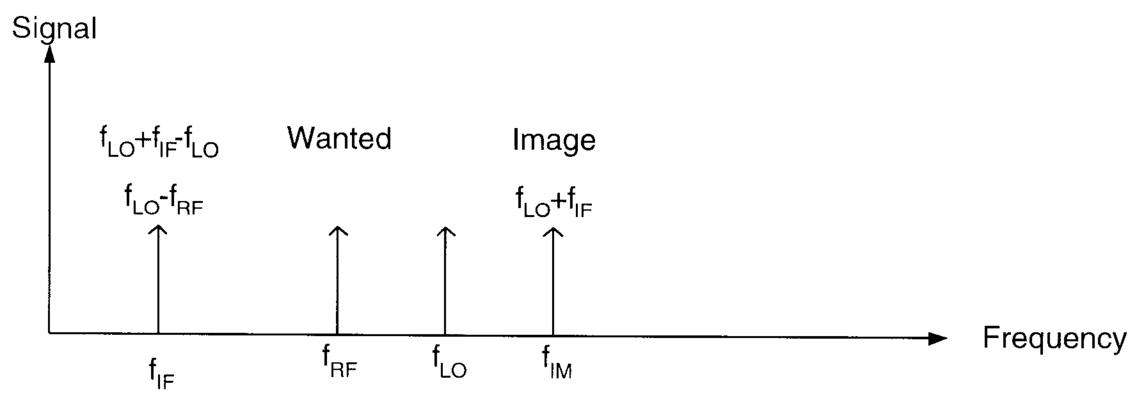
1. (a) (i)The RF signal is converted down to an intermediate frequency (IF) by multiplying it with a local oscillator (LO) reference signal. The multiplication process produces the sum and difference frequencies.



Since the multiplication process does not preserve signal polarity, a signal that is at $f = f_{LO}+f_{IF}$ will also be converted down to the IF by f_{LO} and this will corrupt the required data. This is known as the *image signal.*



(understanding)**[3]**

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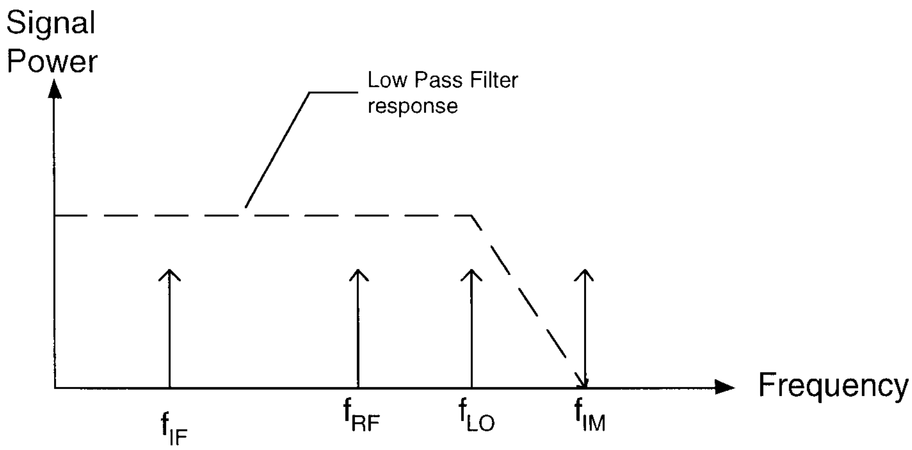
1. (a) (ii) Superheterodyne receiver - A prefilter is used to suppress the image signal before mixing

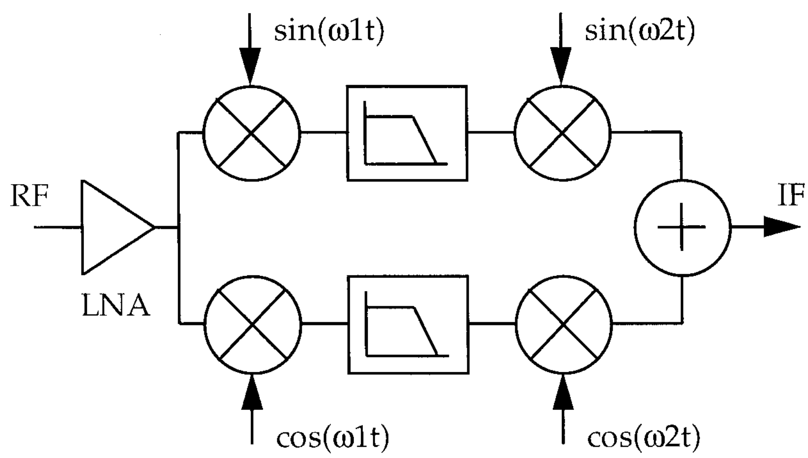


A LPF can be used if $f_{RF}=f_{LO}-f_{IF}$. A HPF must be used if $f_{RF}=f_{LO}+f_{IF}$. The design of this prefilter is eased if the IF is fairly high, as the separation between f_{RF} and $f_{IM} = 2f_{IF}$.

(bookwork) **[4]**

(iii) **Image-reject receiver** – Two parallel downconversion paths are used where the local oscillators are 90° out of phase (quadrature mixing). A further 90° phase shift is then introduced in one of the paths. The net result is that the "wanted" data in each of the paths is in phase, whilst the image data is in antiphase. When the two paths are combined the wanted signals add whilst the image signals cancel.

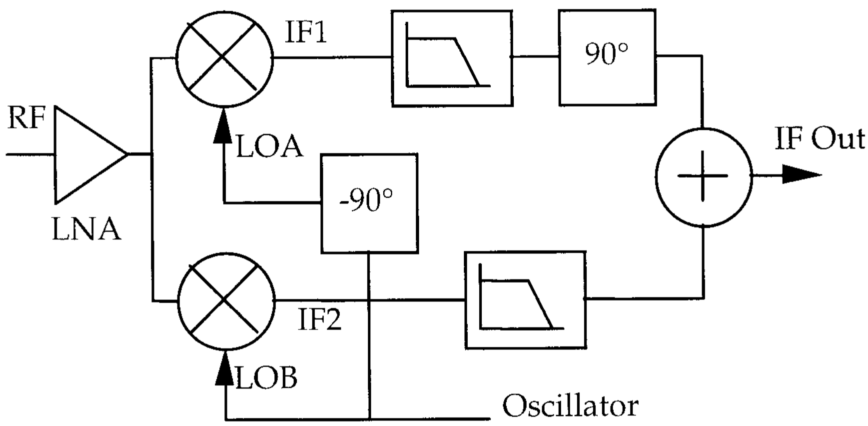
(understanding) **[3]**



Weaver architecture

(bookwork) **[2]**

OR
ALTERNATIVELY



Hartley architecture

(bookwork) **[2]**

1. (b) (i)

Sensitivity =minimum detectable input signal
= Pni(dB)+NF+SNRdet

(bookwork)**[2]**

Pni = available noise power
= kTB
= -174 dBm + 10log B
= -130 dBm for B=25kHz

(new computed example)**[2]**

NF= noise figure
SNRdet=0

-115dBm=-130dBm + NF +SNRdet
=> NF=15dB.

(new computed example) **[1]**

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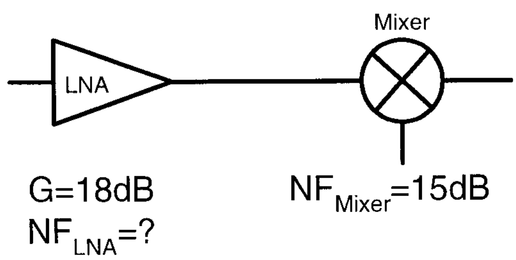
1. (b)(ii)

All 15dB noise is attributed to mixer, if LNA is connected



Required sensitivity = -127 dBm

= $-130+NF_{TOTAL}$

(New application of theory) **[2]**

$NF_{TOTAL} = 3dB \Rightarrow F_{TOTAL}=2$

$F_{TOTAL} = F_{LNA} + (F_{Mixer}-1) / G_{LNA}$

$2 = F_{LNA} + (31.6-1)/63.1$

$\Rightarrow F_{LNA}=1.5$

$\Rightarrow NF_{LNA}=1.8dB$

(New application of theory) **[1]**

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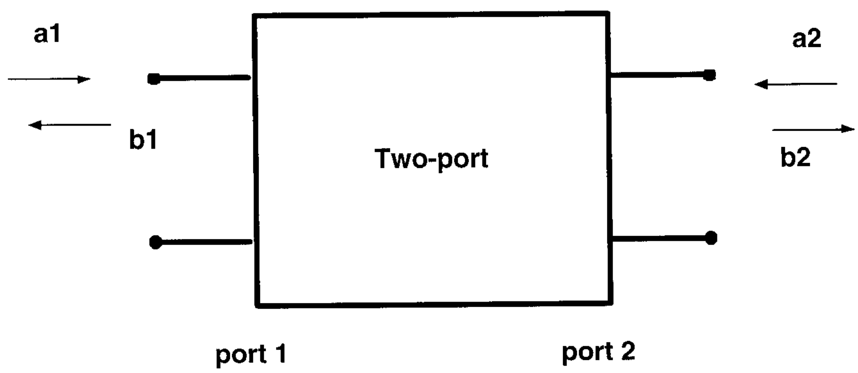
2. (a) z, y and h parameters are particular parameter sets for characterising a 2-port network. At high frequencies, ideal open and short circuits cannot be implemented due to the presence of parasitic capacitances and inductances. S-parameters characterise a two port based on the concept of travelling waves. An incident wave travelling down a transmission line will be reflected to some extent when it reaches a load, and the magnitude of the reflected wave will depend on the relative impedance of the transmission line and load. S-parameters are small-signal quantities which enable impedance matching, input and output reflection coefficients, gain and stability to be determined for a given network.

(understanding) **[3]**



$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad b_2 = S_{21}a_1 + S_{22}a_2$$

a_1 = square root of incident power at port 1
b_1 = square root of reflected power at port 1
a_2 = square root of incident power at port 2
b_2 = square root of reflected power at port 2

The scattering parameters S_{11}, S_{12} etc. depend on the characteristic impedance Z_0. Thus they are defined in relation to a particular external system (usually 50 Ω). To measure s-parameters matching the ports in turn is necessary to set a_1 and a_2 in turn to zero.

eg $S_{11} = (b_1/a_1)_{a2=0}$ $S_{21} = (b_2/a_1)_{a2=0}$ and so on.

(bookwork and understanding) **[4]**

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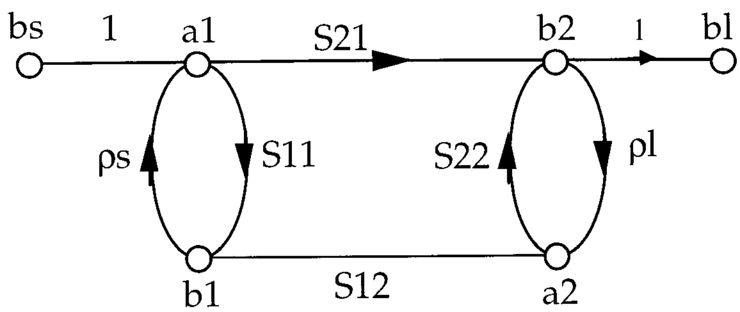
2(b)

(i) Signal flow graph



$b_1 = S_{11}a_1 + S_{12}a_2$

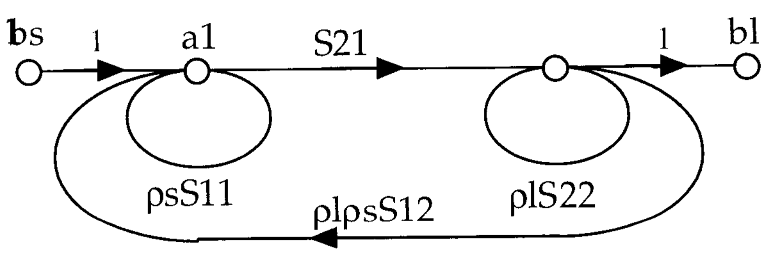
$b_2 = S_{21}a_1 + S_{22}a_2$

$a_1 = bs + \rho s b_1$ $\qquad a_2 = \rho l . b_2$

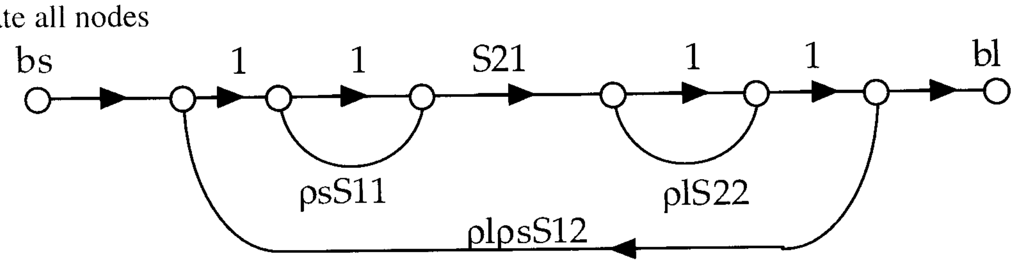
(application of theory) **[2]**

(ii) Simplify

Remove 'don't care' nodes



Separate all nodes



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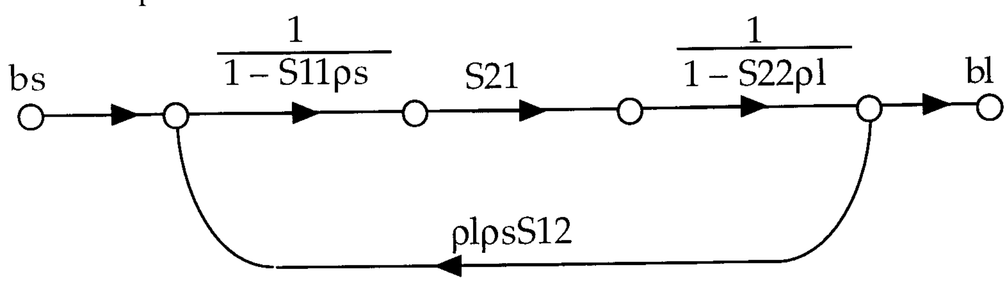
Remove inner loops



Remove outer loop to give transfer function

bs → bl

$$\frac{\frac{1}{1-S11\rho s}\bullet S21\bullet\frac{1}{1-S22\rho l}}{1-\rho lS12\rho s\bullet\frac{1}{1-S11\rho s}\bullet S21\bullet\frac{1}{1-S22\rho l}}$$

(application of theory) **[5]**

2(b)(iii)

S11=0.2+j0.51	S21=1.8+j1.81
S12=0.3+j1.0	S22=0.3+j0.82
ρ_s =0.75+j0.0	ρ_l=0.4+j0.6

$1\text{-}S11\rho_s=1-(0.2+j0.51)\times0.75 = 0.85-j0.38$

$1\text{-}S22\rho_l=1-(0.3+j0.82)(0.4+j0.6) =1.37-j0.51$

$1\text{-}S12\ \rho_l\,\rho_s = 1-[(0.3+j)(0.4+j0.6)\times0.75] = 1.36-j0.44$

Hence numerator of transfer function becomes:

1.8+j1.81 / [(0.85-0.38j)(1.37-0.51j)] = 1.8+j1.81 / 0.97-j0.95

The denominator of transfer function becomes:

1-[(1.36-j0.44)(1.8+j1.81)/(0.97-j0.95)] = 1-[(3.24+j1.67)/(0.97-j0.95)]

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Combining to get transfer function, and multiplying numerator and denominator by (0.97-j0.95) gives:

$(1.8+j1.81)/[(0.97-j0.95)-(3.24+j1.67)]$
$= (1.8+j1.81)/(-2.27-j2.62)$
$=2.55\angle 45^\circ / 3.46\angle 229^\circ$
$=0.73\angle 176^\circ$

(New computed example) **[4]**

(iv) If $\rho_s = \rho_l = 0$ the power transfer bs->bl =S21 =1.8+j1.81=2.55∠45.1°

(New computed example) **[2]**

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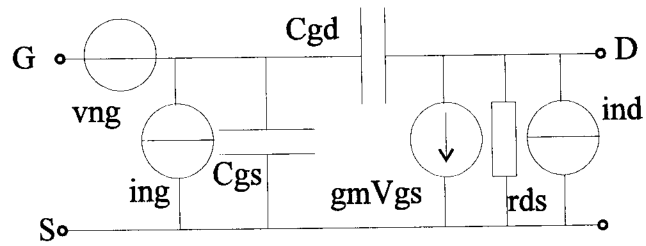
3. (a) (i) Mosfet hybrid-π model



(Bookwork and understanding) **[3]**

(ii)

$$ing^2 = 2qIg\Delta f \ \ A^2$$

is due to shot noise from the gate source leakage current. This was negligible for older processes but with deep submicron processes this is becoming increasingly important.

$$ind^2 = \frac{8kTgm\Delta f}{3} \ A^2$$

is due to thermal noise in the device channel. This can also be represented by an equivalent channel resistance of rd = 3/2gm and using $in^2 = 4kT/R \ \Delta f \ A^2$

$$vng^2 = \frac{k_f \Delta f}{CoxWLf} \ V^2$$

is due to flicker noise generated at the oxide layer under the gate. The expression shown here refers to the noise when referred to the gate of the device.

(Bookwork) **[3]**

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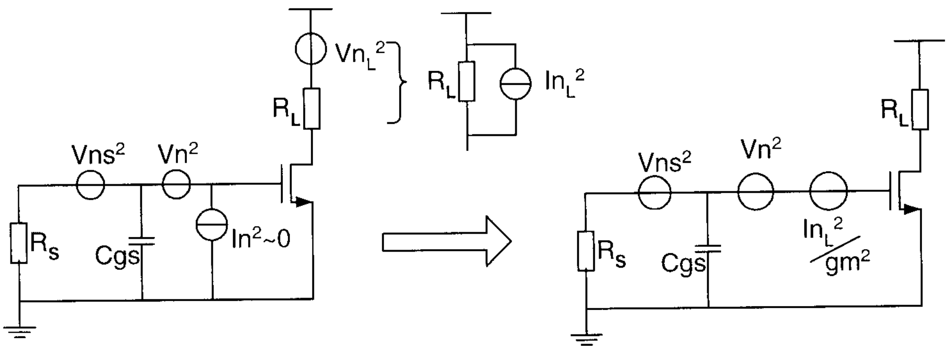
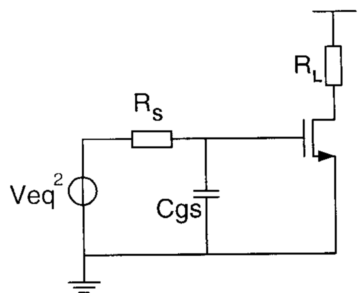
3.(b) (i) Common source configuration



$In^2 \approx 0$
$Vn^2 = Ino^2/gm^2 = 8kT(1+f_l/f)/3gm \ V^2/Hz$
$In_L^2 = 4kT/R_L \ A^2/Hz$
$Vns^2 = 4kTR_S \ V^2/Hz$

(derivation) **[2]**



$$
\begin{aligned}
veq^2 &= vns^2 + vn^2|1+jwCgsRs|^2 + In_L^2|1+jwCgsRs|^2/gm^2 \\
&= vns^2 + vn^2(1+(wCgsRs)^2) + In_L^2(1+(wCgsRs)^2)/gm^2 \\
&= 4kTRs + 8kT(1+f_l/f)\,(1+(wCgsRs)^2)/3gm + (4kT/R_L gm^2)(\,1+(wCgsRs)^2)
\end{aligned}
$$

(derivation and application of theory) **[4]**

Midband equivalent noise:

$veq^2 \quad = 4kTRs+8kT/3gm+(4kT/R_Lgm^2) \;\; V^2/Hz$

3.(b)(ii) $NF=veq^2/vns^2 = 1+2/3gmRs+1/gm^2R_LR_S$

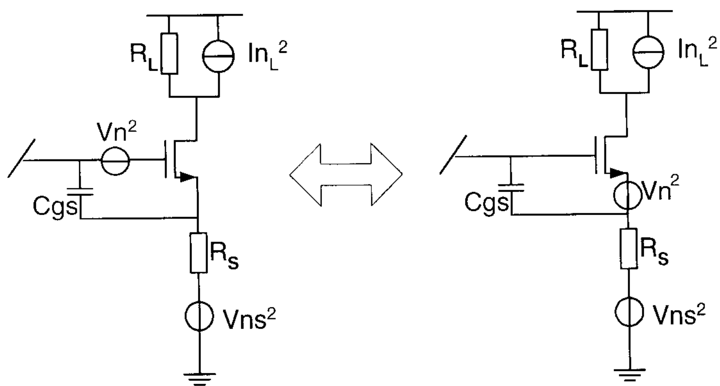
$gm=2\sqrt{(\beta I_D)} \qquad \beta=KW/2L \qquad$ hence $gm=2\sqrt{[50\mu\times25\times1m/(2\times0.5)]}=2.236\times10^{-3}$ S

$NF=1+2/(3\times2.236\times10^{-3}\times150)+1/(5\times10^{-6}\times150\times5\times10^{3}) = \underline{3.254}$

(new computed example) **[3]**

3.(b) (iii)

Common Gate Configuration



(derivation) **[2]**

Midband equivalent input noise (neglecting f_l and C_{gs}):

$veq^2 \;\; = vns^2+vn^2+In_L^2Rs^2$
$\qquad = 4kTRs+8kT/3gm+4kT\, R_S^2\,/R_L \quad V^2/Hz$

$NF=veq^2/vns^2 = 1+2/3gmRs+R_S\,/\,R_L$
$\qquad =1+2/(3\times 2.236\times10^{-3}\times150)+150/5000 = \underline{3.017}$

hence the noise figure goes down.

(new computed example) **[3]**

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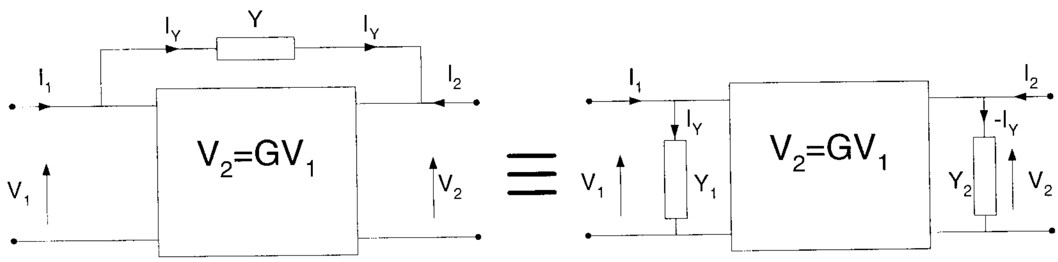
4. (a) (i)The miller transformation an admittance of Y across the input and output of a voltage amplifier of gain G shows that the admittance can be split into two components, one at the input and one at the output.

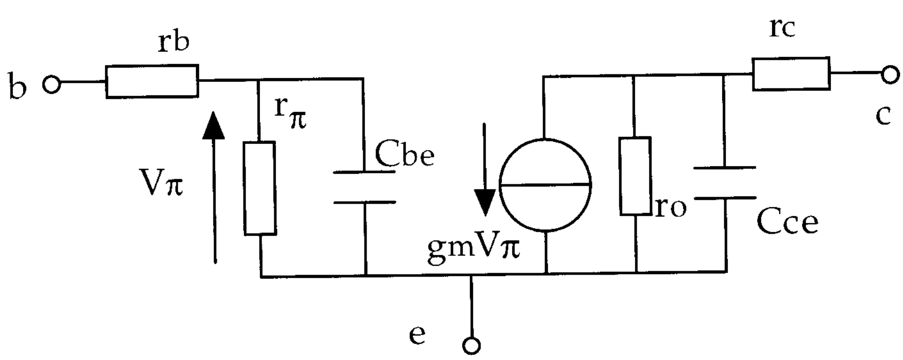


$I_Y=(V_1-V_2)Y$ and $V_2=GV_1$ hence $I_Y= (V_1-GV_1)Y \Rightarrow Y_1=(1-G)Y$

Similarly

$-I_Y=(V_2-V_1)Y$ and $V_1=V_2/G$ hence $-I_Y=(V_2-V_2/G)Y \Rightarrow Y_2=(1-1/G)Y$

The application of this transformation to the the small signal hybrid π model yields



$Cbe = C\pi + C\mu(1 - G)$, $Cce = C\mu(1 - 1/G)$, where G is the voltage gain :
$G = (Vce/Vbe) \cong -g_m R_l$

Thus $Cbe \cong C\pi + g_m R_l C\mu$. and $Cce \cong C\mu$. The output capacitance Cce is often neglected from the small signal model. The approximation $A_V = -g_m R_l$ assumes that $r\pi >> r_b$, and that the load is purely resistive.

(understanding /derivation) **[5]**

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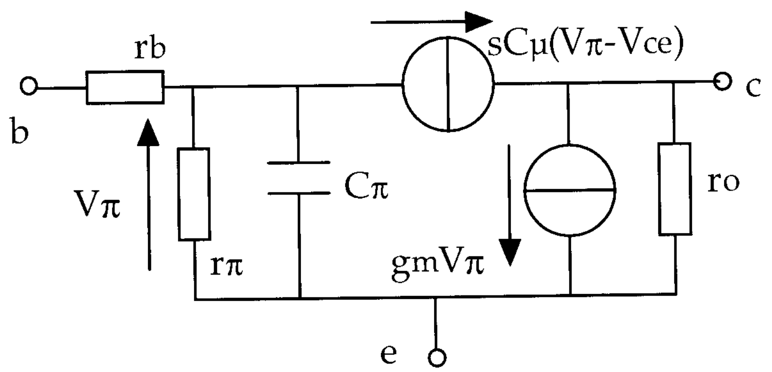
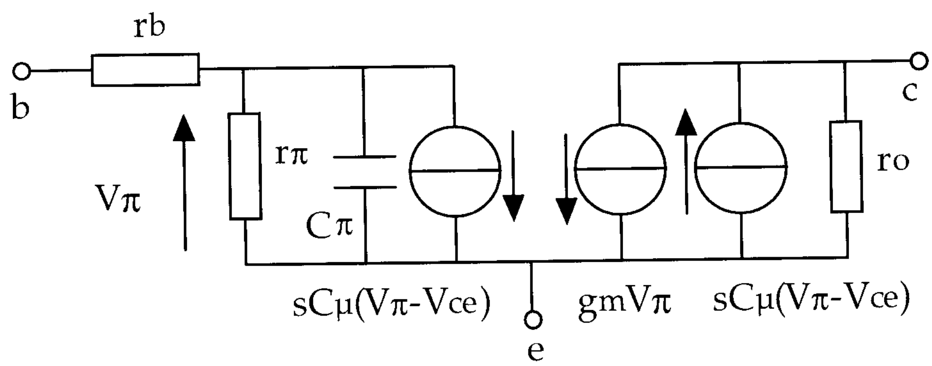
4(a)(ii) At high frequencies however we cannot neglect the gain roll-off due to Cπ and Cμ so the A simplified hybrid-π model which takes the high-frequency gain roll-off into account can be derived:

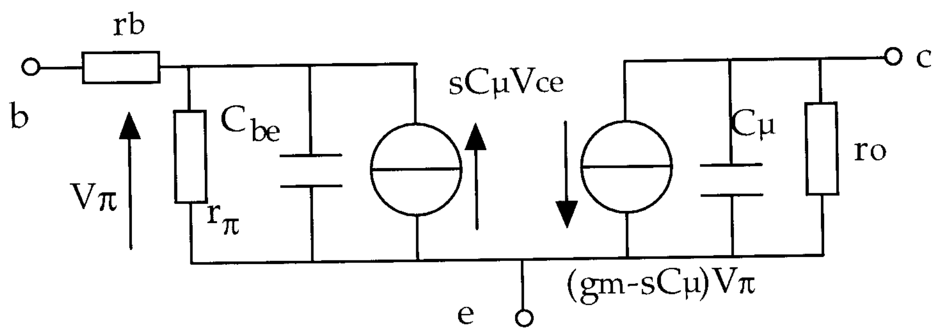
- Cμ is replaced by an equivalent current source sCμ(Vπ - Vce):



- The current source is split between the input and output circuits:



- The input and output component terms are rearranged:



Cbe = Cμ + Cπ

(bookwork/derivation) **[5]**

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4 (b) The input of the amplifier is modelled as

$$Zin=(200\Omega \| 1.5pF)\ \Omega$$
$$=(200 \| 1/j(2\pi\times1.2\times10^{9}\times1.5\times10^{-12}))\ \Omega \quad \text{(at 1.2GHz)}$$
$$=(200 \| 88.42/j)\ \Omega$$

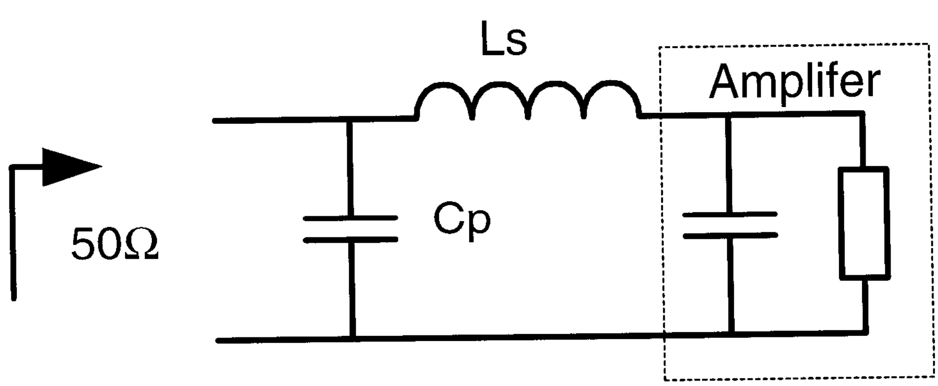
Normalised to 50Ω this gives

$$Zn=(4 \| 1.77/j)\Omega$$

Equivalent admittance

$$Yn=(0.25+j0.56)$$

One way of proceeding is by plotting Yn on the Smith chart as point A. The parallel admittance is converted to a series impedance by reflecting through the centre of the smith chart to give point B, $Z_B=0.7-j1.5\ \Omega$ Adding a normalised series inductance of j1.95 (Ls=12.9nH @ 1.2GHz) takes us to point C, $Z_C=0.7+j0.45$, which is diametrically opposite to point D, $Y_D=1-j0.65$ on the unit circle. By adding a parallel normalised capacitance of 0.65 S $\Rightarrow X_{CPn}=1.538\Omega \Rightarrow X_{CPn}=76.9\Omega$ (Cp=1.7pF @1.2GHz) brings the total conductance to the centre of the chart.



(New computed example) **[10]**

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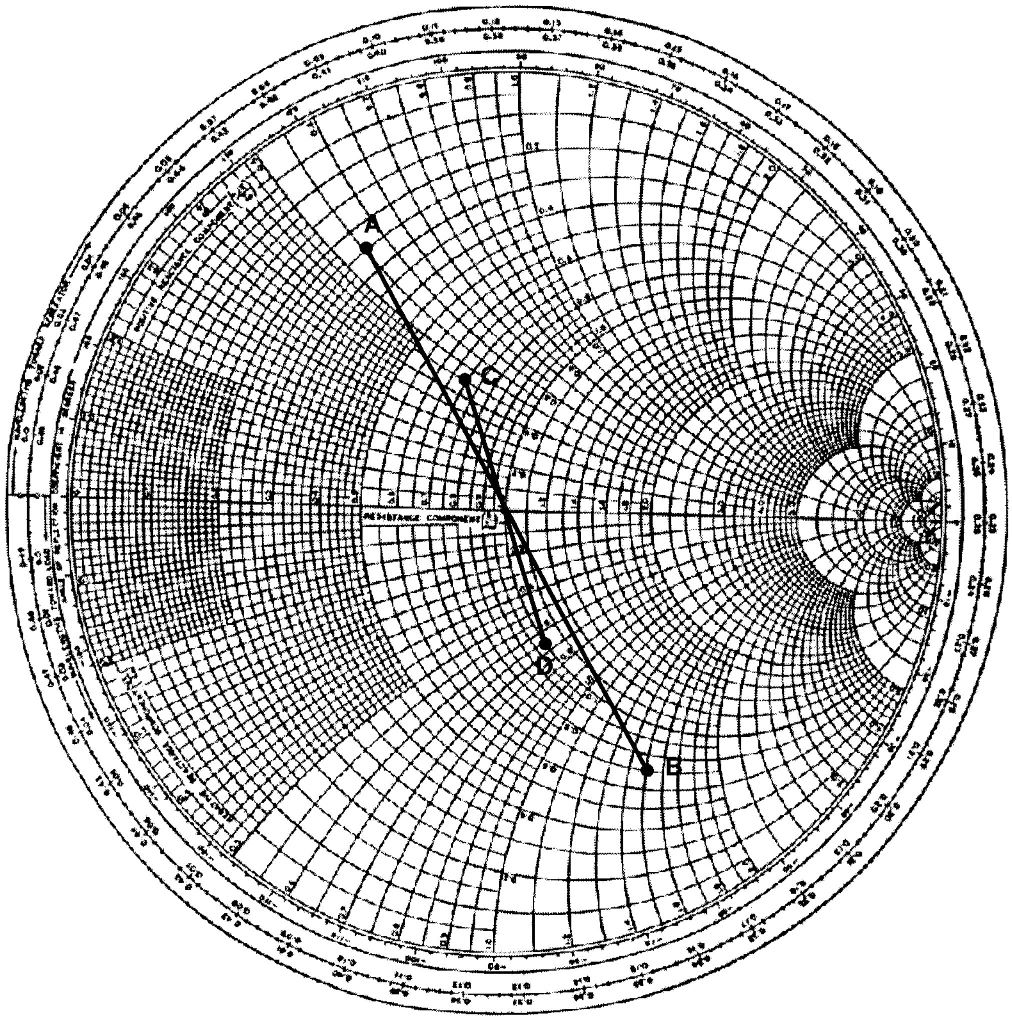

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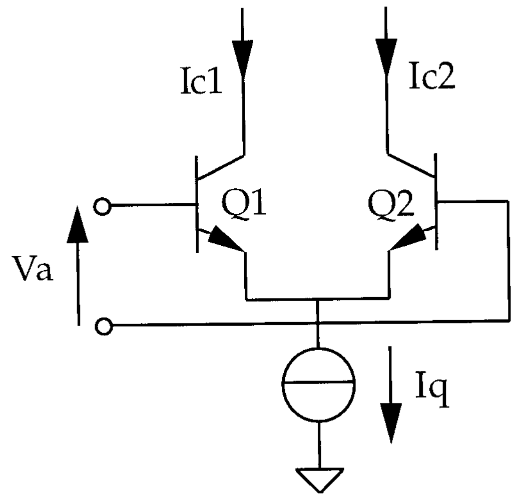
5. (a) (i) For a simple differential pair

$$Iout = Ic1 - Ic2 = Iq \tanh(Va/2Vt)$$



For the double balanced mixer

$$Iout = I_1 - I_2 = (I_{C1} - I_{C2}) + (I_{C3} - I_{C4})$$

$$I_{C1} - I_{C2} = I_{C5} \tanh(V_A/2Vt)$$
$$I_{C4} - I_{C3} = I_{C6} \tanh(V_A/2Vt)$$

Hence

$$Iout = (I_{C5} - I_{C6}) \tanh(V_A/2Vt)$$

But $I_{C5} - I_{C6} = I_Q \tanh(V_B/2Vt)$

Thus

$$Iout = I_Q \tanh(V_B/2Vt) \tanh(V_A/2Vt)$$

For small signals ($V_A, V_B << 2Vt$) ,

$$Iout \cong I_Q (V_B/2Vt)(V_A/2Vt) = I_Q V_B V_A/4Vt^2$$

Conversion gain = $I_Q/4Vt^2 = 1mA/4(25mV)^2 = \underline{0.4}$

(Derivation/bookwork) **[7]**

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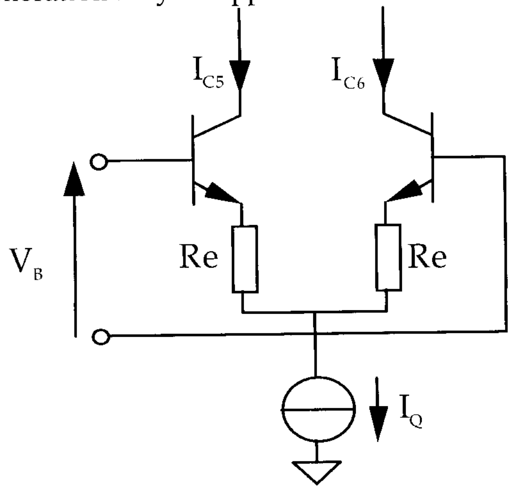
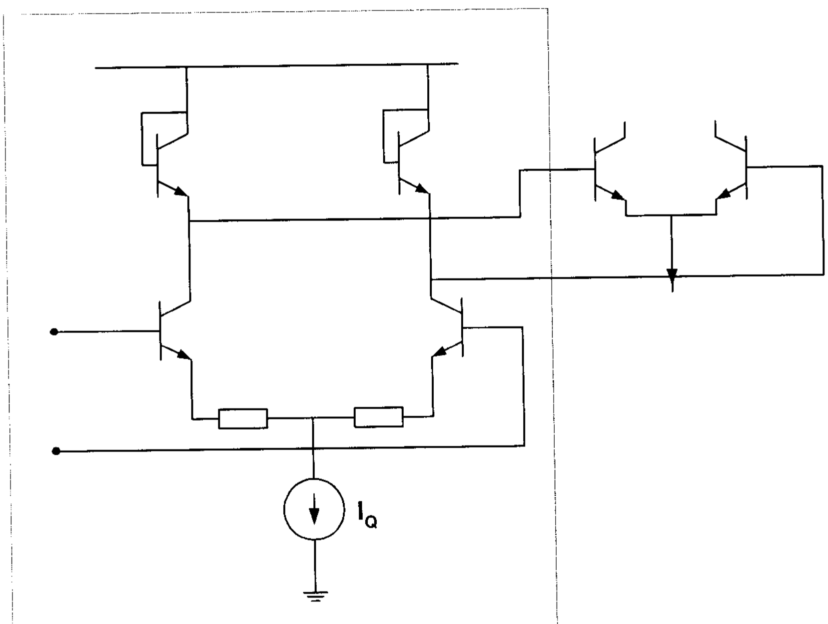
5. (a) (ii) Emitter degeneration may be applied to the bottom transistor to achieve:



I_{C5}-I_{C6}=V_B/Re giving the required linear dependance of Iout on V_B. Applying emitter degeneration to the top pairs will mean that the gain of the top pairs is fixed (Gm=1/Re). We have lost the ability to vary Gm with the tail current, thus we have lost the ability to multiply!



Predistortion circuit

(understanding/bookwork) **[8]**

5(b) Referring to figure 5.2,

$I_{C6}=I_{C7}$

$I_{C7}=I_{C5}+I_B$

⇨ $I_{C5}-I_{C6}=I_{C5}-I_{C7}= -I_B$

Recall for the top pairs of the balanced mixer

$$Iout = (I_{C5} - I_{C6}) \tanh (V_A/2Vt)$$

For signals ($V_{A,} \ll 2Vt$) ,

$$Iout \cong (I_{C5} - I_{C6}).(V_A/2Vt)$$

$$Iout=-I_B V_A/2Vt$$

The output is linearly proportional to the two inputs, with no small signal approximation at I_B. Note that (I_{C5} -I_{C6}) are linearly related to I_B but I_{C5} & I_{C6} are themselves non linear.

(derivation/understanding) **[5]**

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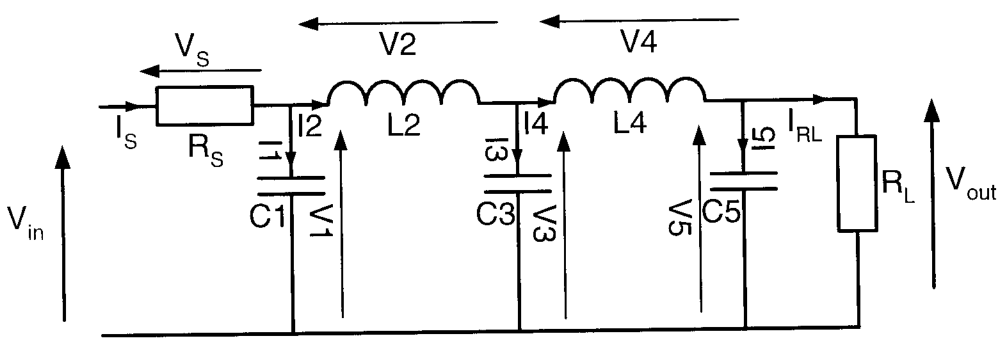
6.(a).
(i)
Advantages of LC ladder approach:
- Are relatively insensitive to component tolerances
- Are insensitive to parasitic capacitances as each node usually has an associated design capacitance that can be reduced so as to compensate for the parasitic
- Component values for an extensive range of filter specifications can be extracted from normalised tables .

Passives are unsuitable for IC implementations since:
- Integrated inductors are not generally available and if they are they have poor performance.
- A passive filter is not easily tuned for component variations.

(bookwork) **[5]**

(ii)



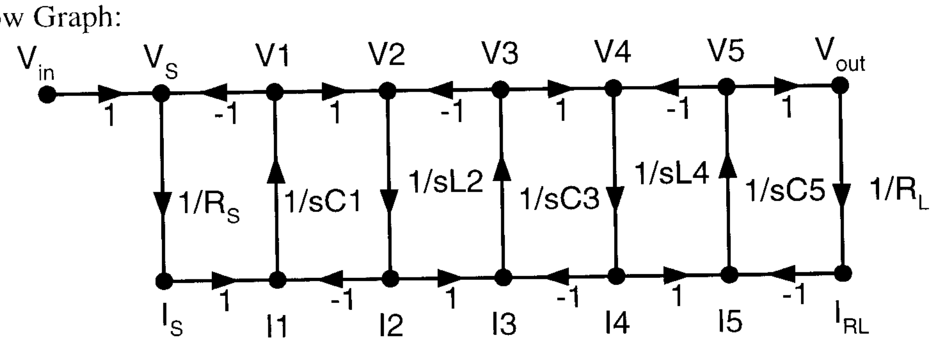
Ladder equations:

$V_S=V_{in}-V1$ $\quad I1=I_s-I2$ $\quad V2=V1-V3$ $\quad I3=I2-I4$ $\quad V4=V3-V5$ $\quad I5=I4-I_{RL}$ $\quad V_{out}=V5$

$I_S=V_s/R_s$ $\quad V1=I1/sC1$ $\quad I2=V2/sL2$ $\quad V3=I3/sC3$ $\quad I4=V4/sL4$ $\quad V5=I5/sC5$ $\quad I_{RL}=V_{out}/R_L$

(Application of methodology) **[3]**

Signal Flow Graph:



(Application of methodology) **[3]**

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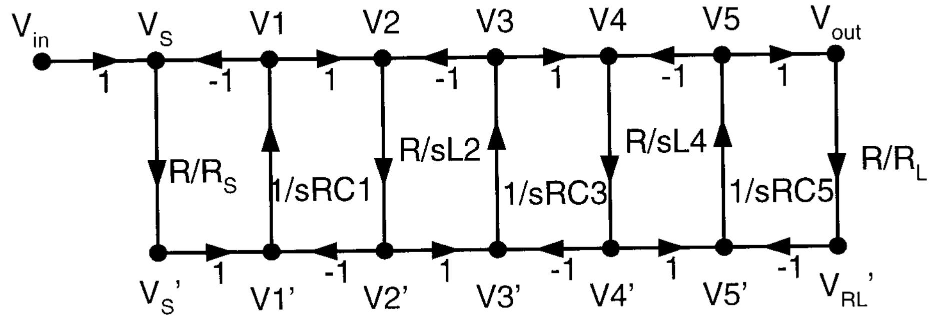
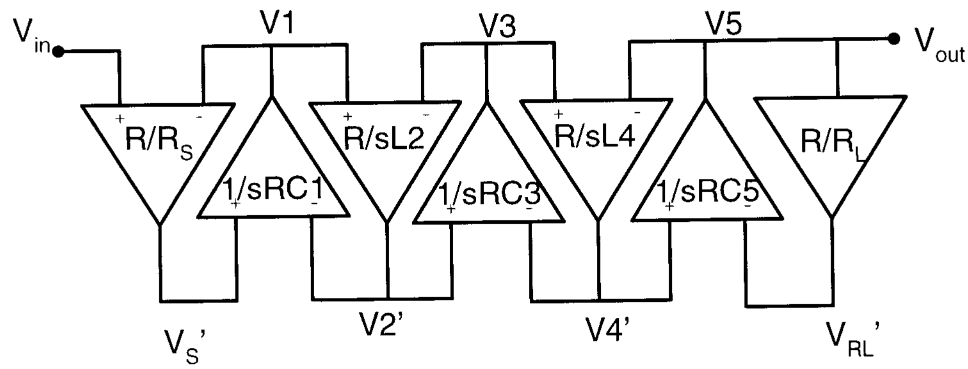
Scaled Signal Flow Graph:



(Application of methodology) **[2]**

Implementation using amplifiers and summing integrators:



(Application of methodology) **[2]**

6.(b)

Considering M1, M2 and M5:

$Id_1 - Id_2 = (Is + Id_5) - (Is - Id_5) = 2Id_5$

Assuming $Vds_5 < Vgs_5 - Vth_5$:

$Id5 = 2\beta(Vgs_5 - Vth_5)Vds_5 - \beta Vds_5^2$

$= GVds_5 - \beta Vds_5^2$

$Vds_5 = Vd_5 - Vs_5 = Vin - Vgs_1 + Vgs_2$

$Vgs_1 = \sqrt{(Id_1/\beta_1)} + Vth_1$ $\qquad$ $Vgs_2 = \sqrt{(Id_2/\beta_2)} + Vth_2$

Assuming M1 and M2 are matched (equal β and Vtho), body effect is negligible (Vth is constant), and $V_Q >> Vin$ and β is large, thus Vgs1, Vgs2 $\approx$ constant,

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Thus $Vds_5=Vin-Vgs_1+Vgs_2 \approx Vin$

$Id_1-Id_2=2Id_5 = 2(G_5Vin-\beta_5Vin^2)$

Similarly
$Id_3-Id_4=2Id_6=2(G_6Vin-\beta_6Vin^2)$

Output current

$Iout=(Id_3-Id_4)-(Id_1-Id_2)$
$=2[G_6Vin-2\beta_6Vin^2]-2[G_5Vin-2\beta_5Vin^2]$
$=2(G_6-G_5)\ Vin$ assuming $\beta_5=\beta_6$

$G5=2\beta_5(Vgs_5-Vth_5)=2\beta_5(V_A-Vs_5-Vth_5)$
$G6=2\beta_6(Vgs_6-Vth_6)=2\beta_6(V_B-Vs_6-Vth_6)$

Assuming $Vth_5=Vth_6$ & $Vs_5=Vs_6$
(ie matched M1->M4), then

$G_6-G_5=2\beta(V_B-V_A)$

Thus

$Iout/Vin=4\beta(V_B-V_A)$

(semi-new derivation) **[5]**