EE2-05

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE/EIE PART II: MEng, BEng and ACGI

SIGNALS AND LINEAR SYSTEMS

Corrected Copy

Friday, 5 June 2:00 pm

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.

Question One carries 40% of the marks. The other 2 questions each carry 30%.

clarification:
Q3 b "at y(t)" added (13:20)

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.L. Dragotti

Second Marker(s) : P.T. Stathaki

Special Information for the Invigilators: none

Information for Candidates

Some Fourier Transforms

$$\mathrm{rect}(\tfrac{t}{\tau}) \iff \tau \mathrm{sinc}(\tfrac{\omega\tau}{2})$$

$$\tfrac{W}{\pi}\mathrm{sinc}(Wt) \iff \mathrm{rect}(\tfrac{\omega}{2W})$$

The unit step function $u(t)$ is defined as:

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

A useful Laplace transform

$$e^{\lambda t}u(t) \iff \tfrac{1}{s-\lambda} \qquad Re\{s\} > \lambda$$

A useful z-transform

$$\gamma^n u[n] \iff \frac{z}{z-\gamma} \qquad |z| > |\gamma|$$

The Questions

1. This question carries 40% of the mark.

   (a) Given the signal

   $$x(t) = \begin{cases} 1-t & \text{for } \ 0 \le t \le 1 \\ 0 & \text{otherwise,} \end{cases}$$

   sketch and dimension each of the following signals:

   i. $x_1(t) = x(-t+2)$ [2]

   ii. $x_2(t) = x(-4t-2)$ [2]

   (b) State with a brief explanation if the causal systems with the following transfer functions are stable or not stable.

   i.
   $$H_1(s) = \frac{1}{s^2+4s+13}$$
   [4]

   ii.
   $$H_2(s) = \frac{1}{s^2+s-2}$$
   [4]

   (c) Consider a linear time invariant (LTI) system satisfying $|h(t)| \le K$, where K is a given number and $h(t)$ is the unit impulse response of the system. Can you claim that the system is BIBO stable? Justify your answer. [4]

Question 1 continues on next page

(d) Given the following two signals

$$x_1(t) = \begin{cases} e^{-t}, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

and

$$x_2(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{otherwise,} \end{cases}$$

compute the convolution $c(t) = x_1(t) * x_2(t)$. [5]

(e) A linear time-invariant system is specified by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 10y(t) = x(t).$$

i. Find the characteristic polynomial, characteristic roots and characteristic modes of this system. [3]

ii. Find the zero-input component of the response $y(t)$ for $t \geq 0$, if the initial conditions are $y(0) = 1$ and $\dot{y}(0) = 0$. [3]

iii. Find the zero-state response assuming $x(t) = e^{-t}u(t)$ where $u(t)$ is the unit step function [Hint: use the Laplace transform]. [3]

iv. Finally find the total response of the system when the input is $x(t) = e^{-t}u(t)$ and the initial conditions are $y(0) = 1$ and $\dot{y}(0) = 0$. [3]

Question 1 continues on next page

(f) Consider a causal LTI system with unit impulse response $h(t)$. The output of $h(t)$ to the input $x(t) = e^{-2t}u(t)$ is $y(t) = e^{-t}u(t) + e^{-2t}u(t)$. Determine $h(t)$. Note that $u(t)$ is the unit step function. [4]

(g) Find the causal inverse z-transform of

$$X[z] = \frac{z}{(z^2 - 5z + 6)}.$$

[3]

2. For the circuit in Fig. 2a, the switch is in a closed position for a long time before $t = 0$, when it is opened instantaneously.

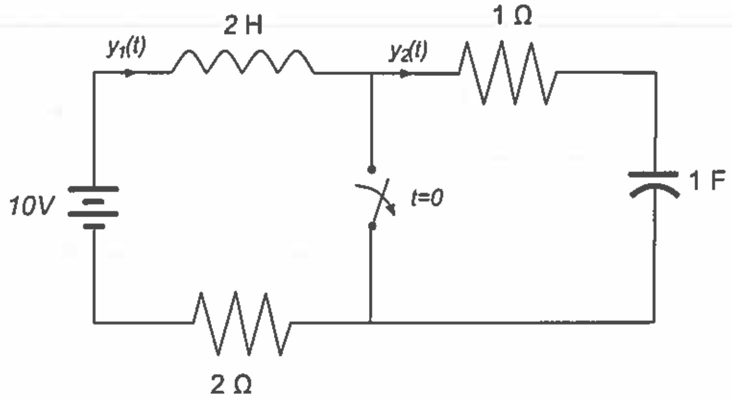

Figure 2a: An electric circuit.

(a) Determine the initial conditions $y_1(0^-)$, $y_2(0^-)$ and $v_C(0^-)$, where $v_C(t)$ is the voltage across the capacitor and $y_1(t)$, $y_2(t)$ are the currents across the two loops in the circuit. [5]

(b) Write the loop equation in the Laplace domain. [5]

(c) Find the exact expression of the current $y_1(t)$ for $t > 0$. [5]

(d) Assume that the circuit has reached a steady-state condition before $t = 10$ (this assumption is only approximately valid), when the switch is closed instantaneously.

   i. Determine the initial conditions $y_1(10^-)$, $y_2(10^-)$ and $v_C(10^-)$. [5]

   ii. Find the expression of $y_1(t)$ for $t > 10$ under the assumption that the system had reached the steady-state before $t = 10$. [5]

(e) Sketch the complete evolution of $y_1(t)$ for $t > 0$. [5]

3. Consider the cascade of two LTI systems shown in Fig. 3a. The first system, A, has unit impulse response $h(t)$. The second system, B, is known to be the inverse of system A and has impulse response $g(t)$.

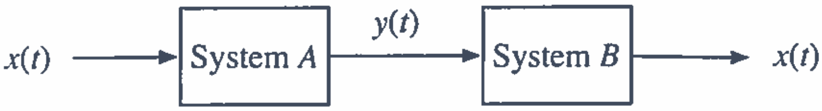

Figure 3a: Block diagram of a cascade of two systems.

(a) Assume that

$$H(s) = \frac{1}{(s^2 + 2s + 1)}.$$

Find the transfer function $G(s)$ of system B. [4]

(b) Consider now the causal system with transfer function $H(s) = (s^2 + 4)$. This system is **not** invertible. Find two everlasting real-valued inputs $x_1(t)$ and $x_2(t)$ that produce the same output. at $y(t)$ [7]

(c) One important use of inverse systems is in situations in which one wishes to remove distortions of some type. A good example of this is the problem of removing echoes from an acoustic signal. Usually the received signal with echoes is modelled as $y(t) = h(t) * x(t)$, where $x(t)$ is the original acoustic signal and $h(t)$ is an LTI system with impulse response:

$$h(t) = \sum_{k=0}^{\infty} h_k \delta(t - kT).$$

Here the echoes occur T seconds apart. The required impulse response $g(t)$ of the inverse system is also a stream of pulses:

$$g(t) = \sum_{k=0}^{\infty} g_k \delta(t - kT).$$

Question 3 continues on next page

i. Determine the equations that the coefficients g_k must satisfy. [6]

ii. Assume that $h_k = 2^{-k}$, solve these equations for g_0, g_1 and g_2. [6]

(d) System A has transfer function $H(s)$. You now want to invert it using the system with feedback depicted in Fig. 3b. The inversion will only be approximate. Determine the responses $H_0(s)$ and $H_1(s)$ that will give you an approximate inversion of system A. Justify your answer. [7]

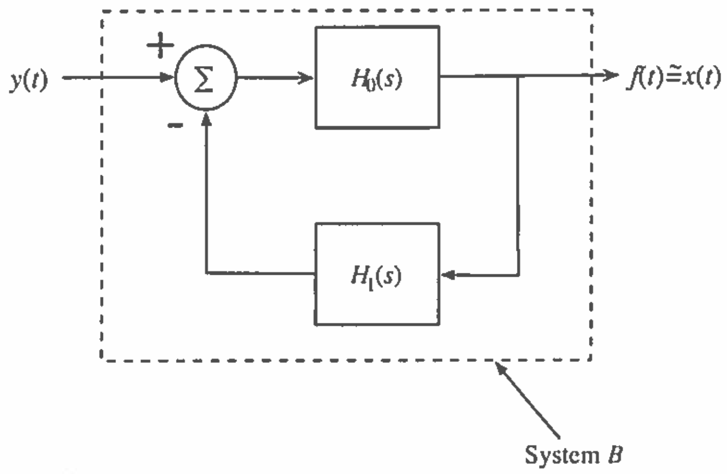

Figure 3b: Block diagram of feedback system.