Answers Question 1 : (new application of theory)

a) We have 3 unknowns, so 3 measurements are needed, at 2 different temperatures.

[2]

b) $X^2 = \sum \left(V_i - \left(aH_i + bT_i + c\right)\right)^2$

[3]

c) Minimum X^2:

$$\frac{\partial X^2}{\partial a} = 0 \Rightarrow \sum H_i \left(V_i - \left(aH_i + bT_i + c\right)\right) = 0$$
$$\frac{\partial X^2}{\partial b} = 0 \Rightarrow \sum T_i \left(V_i - \left(aH_i + bT_i + c\right)\right) = 0$$
$$\frac{\partial X^2}{\partial c} = 0 \Rightarrow \sum \left(V_i - \left(aH_i + bT_i + c\right)\right) = 0$$
$$\Rightarrow$$
$$\begin{bmatrix} \sum H_i^2 & \sum H_i T_i & \sum H_i \\ \sum H_i T_i & \sum T_i^2 & \sum T_i \\ \sum H_i & \sum T_i & N \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum V_i H_i \\ \sum V_i T_i \\ \sum V_i \end{bmatrix}$$

[12]

d) The cross sensitivity is the apparent change of one independent variable due to a change in the other.

Then,

$$S = \frac{\partial H}{\partial T} = \frac{\frac{\partial V}{\partial T}}{\frac{\partial V}{\partial H}} = \frac{b}{a}$$

[3]

Answers question 2: (bookwork)

Any 2 from the following :

Type 1 = Exclusive-OR

If the signal and the VCO are at the same frequency f0 and differing in phase by φ, the output signal will be at a frequency 2f0 and will have a duty cycle $D = \frac{\varphi}{\pi}$ for $0<\varphi<\pi$ and $D = 2 - \frac{\varphi}{\pi}$ for $\pi<\varphi<2\pi$. The output signal has a DC component as shown in figure 4.9. To maximize lock range the type 1 detector requires an equal 50% duty cycle on both its input signals. A PLL with type 1 phase detector will lock at a phase difference of π/2. The XOR gate as a phase detector, despite its simplicity is not very popular, due to its small linear range. However, we will restrict the discussion to this type of detector.

The range of a phase detector can be doubled if it can detect which signal is leading in phase. The following 2 types of phase detector do just this, and have therefore double the range of the type 1 detector. Furthermore, since the input signals are not equivalent, the centre of the linear region of these detectors is when the signals are in phase.

Type 2 : Positive-edge triggered J-K flip-flop.

This detector fires off at leading and trailing edges of the waveforms. It turns out that the duty cycle of the signals is unimportant. This detector has a phase difference range of 2π. When the two signals are in lock the detector output is at a frequency equal to the signal frequency. This ripple is contributes to phase noise in the output of the PLL.

Type 3 : Positive-edge triggered 'tri-state' detector

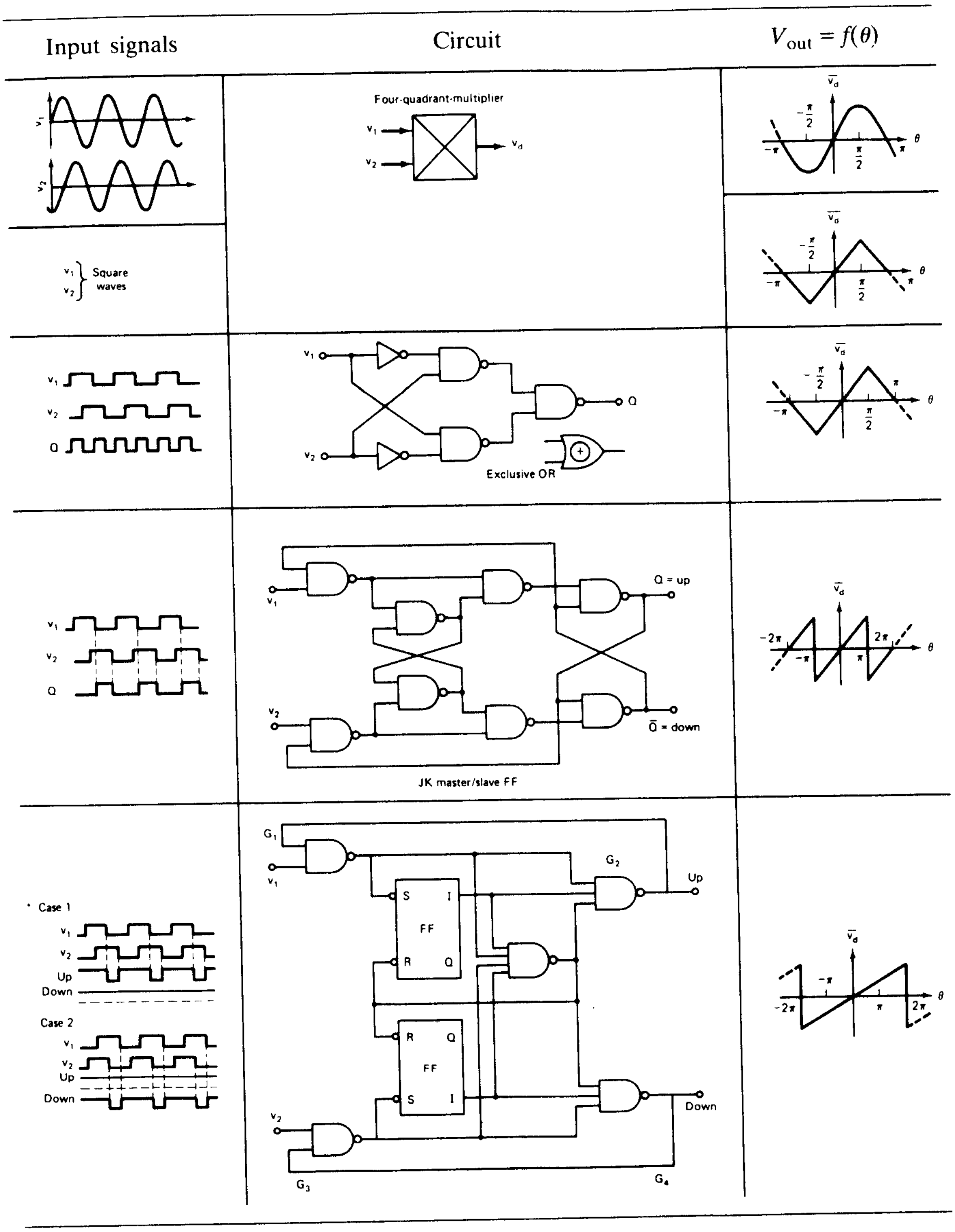
The advantage of this detector is that when the inputs are in lock there is no ripple signal. The range of this detector is between –2π and +2π phase difference. This is probably the most commonly used detector.

The gain of digital detectors is easily computed from their linear range and the logic levels employed.

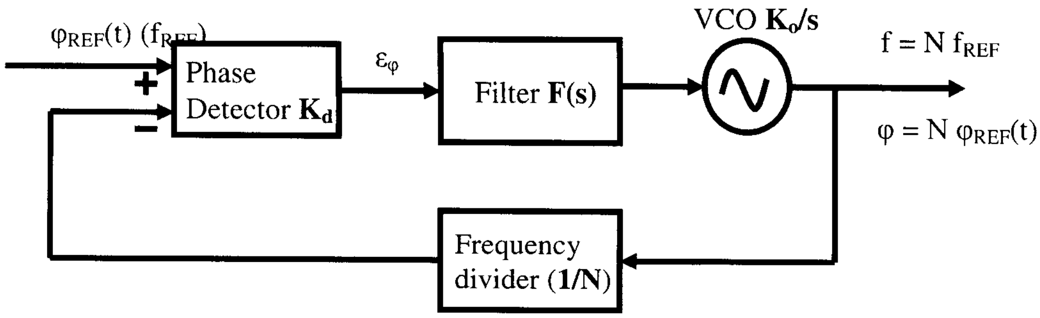
Note that the definitions of type2 and type 3 detectors may be reversed in some references.

[6]

alternatively, the figure from the notes may be used:


Input signals
Circuit
$V_{out} = f(\theta)$
Four-quadrant-multiplier
v_1
v_2
v_d
v_1 } Square waves
v_2
Q
Exclusive OR
Q = up
$\bar{Q}$ = down
JK master/slave FF
G_1
G_2
G_3
G_4
S
R
I
Q
FF
Up
Down
Case 1
Case 2
$\bar{v_d}$
θ
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
$-\pi$
π
-2π
2π

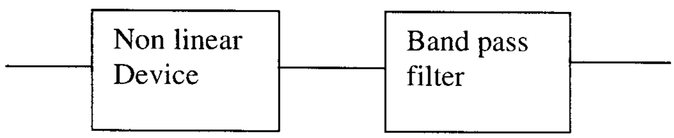
b) PLL frequency multiplier:



Used at lower frequencies and when ease of adjusting the multiplication factor is desired.

[3]

non PLL multiplier:



Typically used at extremely high frequencies as a simpler solution than the PLL. (comb generator)

[3]

c) Simplest multiplier is first order loop and uses type 1(XOR) detector and no filter (uses the implicit low pass filter in the VCO). The transfer function is :

$$B(s) = \frac{\varphi_{out}}{\varphi_{in}} = \frac{NK}{s+K}$$

and the steady state phase error for a $\phi(t) \propto t^n$ disturbance is:

$$\delta\varphi_{SS} = \lim_{s\to 0} s\varepsilon_{\varphi}(s) = \lim_{s\to 0} s\theta(s)\frac{s}{s+K} = \lim_{s\to 0}\frac{as^{1-n}}{s+K}$$

for a step phase disturbance, $\delta\phi_{ss} = 0$, while for a step frequency, $\delta\phi_{ss} \propto \frac{N}{K}$

This loop cannot track continuously changing input frequency (steady state phase error tends to infinity).

[6]

d) a lead-lag filter decouples the loop gain (fast acquisition time) from the Q (phase margin)

[2]

Answers question 3. (bookwork)

a)

Noise factor:

$$F = \frac{S_i}{N_i}\bigg/\frac{S_o}{N_o} = \frac{GN_i + N_a}{N_i}\frac{S_i}{GS_i} = 1 + \frac{N_a}{GN_i} > 1$$

Noise figure :
$N = 10\log F$ ()

A cascade of identical amplifiers with noise factor F, and gain G has an overall noise factor (called the noise measure):

$$M = F + \frac{F-1}{G} + \frac{F-1}{G^2} + \frac{F-1}{G^3} + \cdots = 1 + \sum_{k=01}^{\infty} (F-1)G^{-k} = 1 + \frac{F-1}{1 - \frac{1}{G}}$$

[3]

b) A cascade of non identical amplifiers has a noise factor:

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

[2]

c) **(new computed example)**They must be placed in increasing noise measure order. According to the table below, the correct sequence is 2-3-1.

gain db	nf db	gain(num)	nf(num)	M-1
5	2	3.162278	1.584893	1.855392
10	2.2	10	1.659587	1.732874
15	2.4	31.62278	1.737801	1.761894

[10]

d) The interconnect is effectively an amplifier with G=-.2dB and N=.2dB
It can be lumped after the amplifier, so that the cascade formula applies:

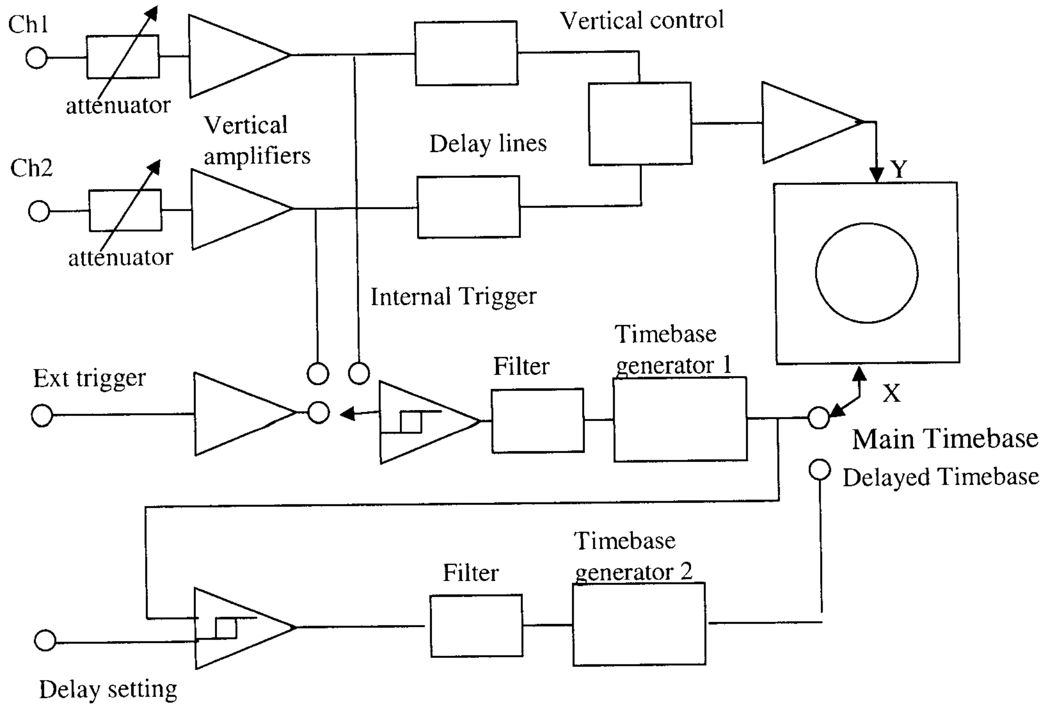
$$F = F_1 + \frac{F_2 - 1}{G_1}$$

with F_2=1.04.. (0.2dB).

All noise measures will increase, so that the first amplifier remains the first. The only question remaining is whether other 2 must exchange order. In the example given this is not the case.

[5]

Answers question 4: (bookwork)

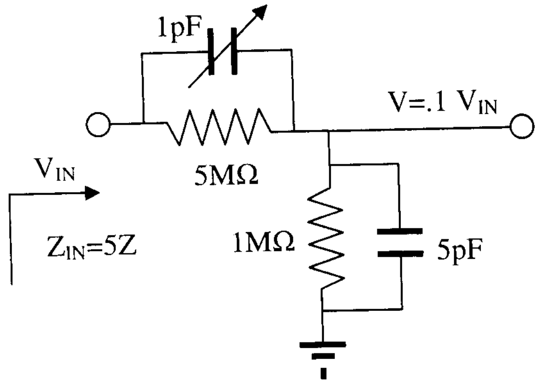


The horizontal sweep system of the oscilloscope consists of the trigger, a one shot sawtooth generator called the **timebase generator** which is of sufficient amplitude for the beam to span horizontally the screen. A horizontal magnification feature, often provided, simply adds a post amplifier to the timebase generator.

The most important part of the oscilloscope is the trigger circuit, which consists of a very fast schmitt trigger, a number of filters, as well as the **delay elements** following the horizontal amplifiers. The delay elements are critical for the correct operation of the oscilloscope. They permit the sweep to start at the trigger event, or even before it.
The trigger circuit effectively operates an enable control for the timebase generator. The quality of the trigger circuit is more often than not the only difference between a cheap and an expensive oscilloscope with apparently similar specifications.
A number of other facilities are usually provided in modern oscilloscopes, including a control to chose between interlacing entire screens or just pieces of a screen between the two vertical channels (**alternate** and **chop** vertical modes) with the obvious bandwidth restrictions arising from the sampling theorem, and the **holdoff** feature, disabling the trigger for some interval after the trigger event, so as to prevent false triggering off artefacts.

[10]

b)



[4]

c)
A practical signal has frequency components in some finite range of frequencies:

$$f_L < f_i < f_H \quad , \quad f_C = \frac{f_L + f_H}{2} \quad , \quad f_C = f_H - f_L \tag{0.1}$$

If the band pass signal is sampled at a frequency $\mathbf{f_s}$ then the reconstruction formula becomes:

$$x(t) = \sum_{n=-\infty}^{\infty} x(n)\, g(t - nT_s) \quad , \quad g(t) = \operatorname{sinc}(2\pi f_s t)\cos(2\pi f_C t) \tag{0.2}$$

It can be shown that a bandpass signal can be uniquely sampled if the Nyquist sampling rate satisfies:

$$2f_B < f_N < 4f_B \tag{0.3}$$

The best case condition (lowest sampling rate) condition occurs when :

$$f_c + f_B/2 = nf_B \tag{0.4}$$

In the limit of small fractional bandwidth we get that the Nyquist rate for bandpass signals satisfies:

$$\lim_{f_c/f_B \to \infty} f_N = 2f_B \tag{0.5}$$

Bandpass sampling is equivalent to mixing plus lowpass sampling, and indeed the relations above can be derived by considering the mixing implied. Bandpass sampling is used in extremely high frequency applications, such as **sampling oscilloscopes** and radar receivers. Bandpass sampling has the drawback that noise power of the sample circuit from aliases of the signal band is aliased into the signal band therefore undermining the signal to noise ratio. The above mean that the input signal needs to be bandpass filtered to a bandwidth less than 1/3 the scope sampling rate. The ultimate limitation will be the vertical amplifier, and trigger jitter.

[6]

Answers Question 5: (computed example)

a) Nonlinearity (absolute or differential), non monotonicity, zero offset, noise, etc.

[3]

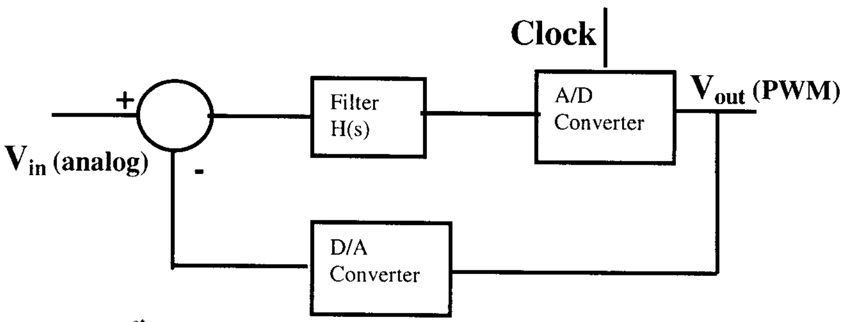
b)
From the signal to quantisation noise ratio, SNQR=1.76+ 6.02N dB, so the 3 bit converter has SNQR=19.82dB .
For a 1mW signal , P_{QN}=10.42μW white over the sampling banwidth, i.e PSD=2.98 pW/Hz. The input signal must be low pass band limited to ½ f_s=1.75MHz

[4]

c) Need to oversample. The necessary minimum sampling frequency for 50 kHz signals is 100kHz. By oversampling the QN PSD drops as the oversampling ratio, hence the amplitude as the QN amplitude as the root of the oversampling ratio. For additional 7 bits of resolution we need 14 octaves oversampling, i.e. f_{min}=18384 . 100kHz = 1.38 GHz (!).

[5]

d) use a 1st order sigmadelta modulator:



With H(s) 1st order LPF at 50 kHz. Then the resolution gain is 1.5bit/bit of oversampling, i.e. only need 5 bits of oversampling (3.2 MHz) to obtain 10 bit resolution.

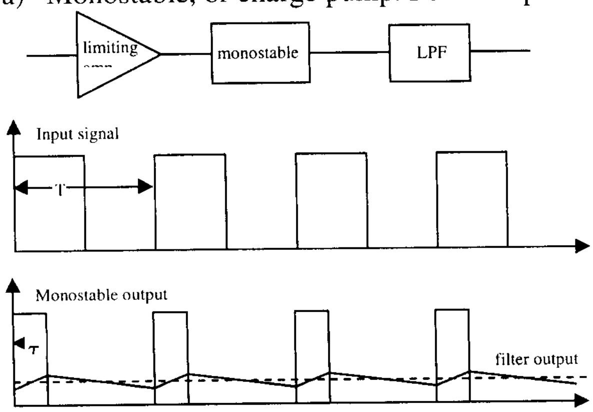
[5]

e) Use a second order filter to get 2.5 bits resolution per bit oversampling. Then, 5 octaves oversampling will give 15.5 bits resolution.

[3]

Answers Question 6. (computed example)

a) Monostable, or charge pump. For example monostable:



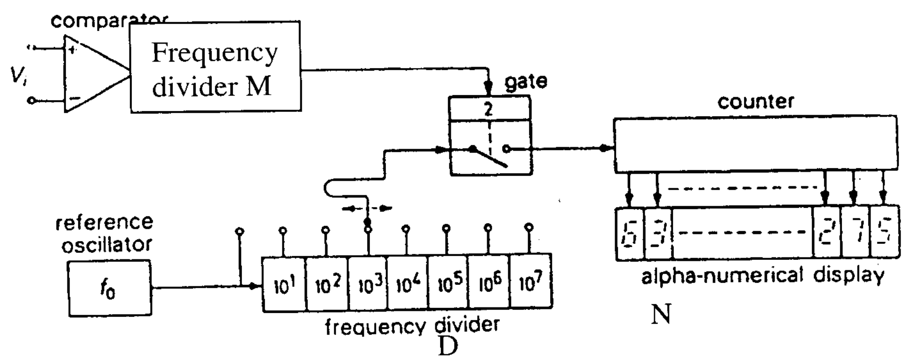
The reading is the DC component of the output, namely:
$V_{out} = V_S D$ (D the duty cycle of the PWM generated), $D = fT_{MS}$ (T_{MS} the monostable pulse width).
Then, the sensitivity is given by:
$K = V_S T_{MS}$
The ripple amplitude is the ripple filtered by the LPF, and clearly depends on the input frequency. Before the filter, assuming all the ripple power is at f,

$$P_{out} = P_{ripple} + V_{out}^2 \Rightarrow V_{ripple}^2 = fV_s^2 T_{MS} - V_s^2 D^2 = fV_s^2 T_{MS} - f^2 V_s^2 T_{MS}^2,$$
$$P_{ripple} = fK\left(V_S - fK\right)$$

I.e. the ripple power has a maximum at 50% duty cycle, i.e. at $f=1/(2T_{MS})$

The worst case accuracy in measurement is:
$$\frac{V_{out}}{V_{ripple}} = \frac{Kf}{\sqrt{Kf\left(V_s - Kf\right)}} = \sqrt{\frac{Kf}{V_S - Kf}} = \sqrt{\frac{1/2V_s}{1/2V_s}} = 1$$
To obtain 1% accuracy we need to LPF the ripple, any solution is acceptable (e.g. a first order LPF at 250 Hz) The pole frequency is the readings rate.

b) Easiest to do with a period counter:



The digital equivalent of the analogue period counting of §1.2.2 is to count, using a fixed reference frequency **F_0/D** , to the duration of one or more periods of the input signal. The count for M periods will be:

$$N = \mathrm{int}\left(MTf_0 / D\right) \Rightarrow f = \frac{Mf_0}{D\left(N \pm 1\right)} \qquad (0.6)$$

By dividing the input frequency by M=50, and D=1 the required accuracy is attained (N>100) . Reading rate is 1000/sec.