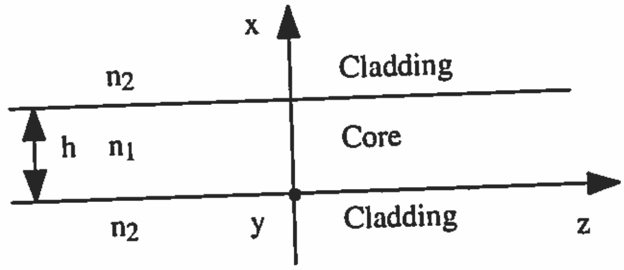
EE3-12

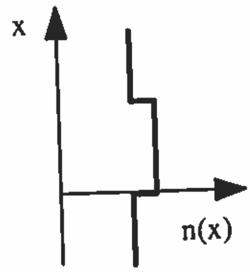
Optoelectronics Solutions 2014

1a) The guide is symmetric, so the refractive indices of the upper and lower cladding layers must match.



[2]

The guide is formed from uniform slabs, so the index variation is in the shape of a top-hat.



[2]

The refractive indices must satisfy $n_1 > n_2$ if total internal reflection is to occur.

[1]

b) The scalar wave equation is:

$$\partial^2 E_{yi}/\partial x^2 + \partial^2 E_{yi}/\partial z^2 + n_i^2 k_0^2 E_{yi} = 0$$

Where E_{yi} is the electric field and n_i is the refractive index in the i^{th} layer, $k_0 = 2\pi/\lambda$ is the free-space propagation constant and λ is the wavelength .

[1]

Modal solutions have the form:

$$E_{yi}(x, z) = E_i(x) \exp(-j\beta z)$$

Where $E_i(x)$ is the transverse field in the i^{th} layer and β is the modal propagation constant.

Substituting into the wave equation yields the waveguide equation

$d^2E_i/dx^2 + (n_i^2k_0^2 - \beta^2)E_i = 0$

[2]

If $n_i^2k_0^2 - \beta^2 > 0$, solutions have the form $E_i(x) = E\cos(\kappa x - \phi)$, where $\kappa = \sqrt{(n_i^2k_0^2 - \beta^2)}$

If $n_i^2k_0^2 - \beta^2 < 0$, solutions have the form $E_i(x) = E\exp(\pm\gamma x)$, where $\gamma = \sqrt{(\beta^2 - n_i^2k_0^2)}$

[2]

c) The boundary conditions are continuity of the tangential electric and magnetic field at the interfaces. The electric field is wholly tangential, so $E_1 = E_2$ on $x = 0$ and $E_1 = E_3$ on $x = d$.

[1]

The magnetic field can be found from the electric field. For time-independent fields, $\nabla \times \underline{E} = -j\omega\mu_0 \underline{H}$ where ω is the angular frequency and μ_0 is the permeability of free-space.

Consequently, $\underline{H} = (j/\omega\mu_0) \nabla \times \underline{E}$. In general:

$\nabla \times \underline{E} = \{\partial E_z/\partial y - \partial E_y/\partial z\}\ \underline{i} + \{\partial E_x/\partial z - \partial E_z/\partial x\}\ \underline{j} + \{\partial E_y/\partial x - \partial E_x/\partial y\}\ \underline{k}$

Since E only has a y-component here:

$\nabla \times \underline{E} = \{0 - \partial E_y/\partial z\}\ \underline{i} + \{0 - 0\}\ \underline{j} + \{\partial E_y/\partial x - 0\}\ \underline{k}$

Thus, $\underline{H} = (j/\omega\mu_0)\ \{-\partial E_y/\partial z\ \underline{i} + \partial E_y/\partial x\ \underline{k}\}$

[2]

The only tangential component is $H_z = (j/\omega\mu_0)\ \partial E_y/\partial x$

If we write this as $H_{zi} = H_i \exp(-j\beta z)$, then $H_i = (j/\omega\mu_0)\ dE_i/dx$

Matching implies that we require $dE_1/dx = dE_2/dx$ on $x = 0$ and $dE_1/dx = dE_3/dx$ on $x = d$

[1]

d) Assuming the solutions:

$E_1(x) = E\cos(\kappa x - \phi)$	$E_2(x) = E'\exp(\gamma x)$	$E_3(x) = E''\exp\{-\gamma(x - h)\})$
$dE_1/dx = -\kappa E\sin(\kappa x - \phi)$	$dE_2/dx = \gamma E'\exp(\gamma x)$	$dE_3/dx = -\gamma E''\exp\{-\gamma(x - h)\})$

[2]

Boundary matching gives:

$E\cos(\phi) = E'$ and $-\kappa E\sin(-\phi) = \gamma E'$

$E\cos(\kappa h - \phi) = E''$ and $-\kappa E\sin(\kappa h - \phi) = -\gamma E''$

Dividing the equations together in pairs then gives:

$\tan(\phi) = \gamma/\kappa$ and $\tan(\kappa h - \phi) = \gamma/\kappa$

[2]

Now $\tan(\kappa h - \phi) = \{\tan(\kappa h) - \tan(\phi)\} / \{1 + \tan(\kappa h)\tan(\phi)\}$ so

$\{\tan(\kappa h) - \gamma/\kappa\} / \{1 + \tan(\kappa h)\,\gamma/\kappa\} = \gamma/\kappa$

$\tan(\kappa h) - \gamma/\kappa = \gamma/\kappa\,\{1 + \tan(\kappa h)\,\gamma/\kappa\}$

$\tan(\kappa h) = (2\gamma/\kappa) / \{1 - (\gamma/\kappa)^2\}$

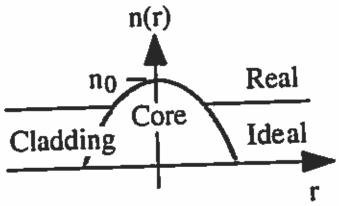
This is the eigenvalue equation, since it is only a function of the propagation constant β.

[2]

2. a) Silica fibre is fabricated using a two-step process. A preform is fabricated first, typically from SiO_2 containing a radial variation of GeO_2 dopant. The preform is much shorter and fatter than the final fibre, but contains the same refractive index profile. Several methods can be used to form the preform, including deposition onto a silica rod (outside vapour phase oxidation, or OVPO), deposition inside a silica tube (IVPO), and deposition on the end of a rod (vapour axial deposition, or VAD). Typically the glass is formed from a gas-phase reaction and the feed rates of the SiO_2 and GeO_2 precursors are varied to achieve the desired core profile. The preform is then fed into a furnace so that its tip is melted, and a thread of fibre is drawn from the tip. As the fibre leaves the furnace, it cools and solidifies. It can then be coated in a plastic jacket and wound onto a spool.

[4]

b) Real and ideal variations in refractive index:

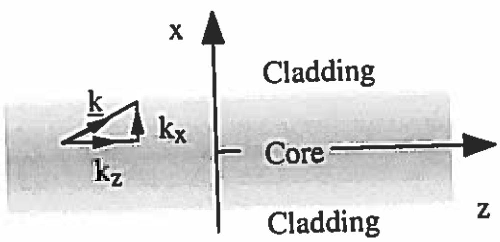


[3]

Although a refractive index value less than unity is not physical, there is no difficulty in allowing it to exist in the model; the guided mode will be sufficiently localised that the light is mainly confined near the axis, where the refractive index is always greater than unity.

[1]

c) The trajectory equation is constructed by assuming that the local slope of the ray can be found from the x- and z- components of the propagation vector shown below.



[2]

Thus, we have $dx/dz = k_x/k_z$ where $k_z^2 + k_x^2 = k^2$

Here $k_z = \beta$ is the modal propagation constant, $k = k_0 n(x)$ is the local propagation constant, $k_0 = 2\pi/\lambda$ and $n(x) = n_0\sqrt{\{1 - (x/x_0)^2\}}$ is the local refractive index.

Hence $\beta\, dx/dz = \sqrt{(k^2 - \beta^2)}$ and $\beta^2(dx/dz)^2 = k_0^2 n_0^2\{1 - (x/x_0)^2\} - \beta^2$

[2]

Now assume the sinusoidal trajectory $x = A \sin(Bz + \psi)$ so $dx/dz = AB \cos(Bz + \psi)$

Hence $\beta^2 A^2 B^2 \cos^2(Bz + \psi) = k_0^2 n_0^2\{1 - (A^2/x_0^2) \sin^2(Bz + \psi)\} - \beta^2$

Re-arranging: $\beta^2 A^2 B^2 \cos^2(Bz + \psi) + (k_0^2 n_0^2 A^2/x_0^2) \sin^2(Bz + \psi)\} = k_0^2 n_0^2 - \beta^2$

[2]

The only way the equation can be satisfied is if the coefficients of the $\cos^2$ and $\sin^2$ terms are the same, so that these add together to give a value that no longer depends on z.

Hence we require $\beta^2 A^2 B^2 = (k_0^2 n_0^2 A^2/x_0^2)$ so $B = (k_0 n_0/\beta x_0)$

The main equation then reduces to $k_0^2 n_0^2 A^2/x_0^2 = k_0^2 n_0^2 - \beta^2$ so $A = x_0\sqrt{(1 - \beta^2/k_0^2 n_0^2)}$

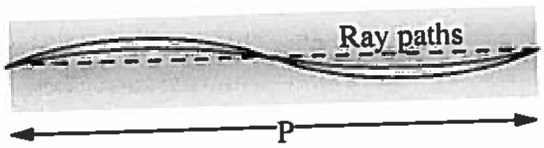
The complete solution is then: $x = x_0\sqrt{(1 - \beta^2/k_0^2 n_0^2)} \sin(k_0 n_0 z/\beta x_0 + \psi)$

[2]

d) For weakly varying guides, $\beta \approx k_0 n_0$.

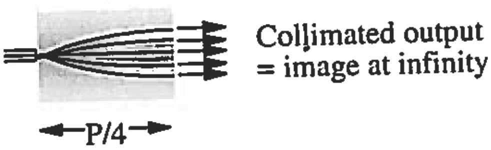
The solution then approximates to $x = A \sin(z/x_0 + \psi)$

All trajectories then have the same period $P = 2\pi x_0$.



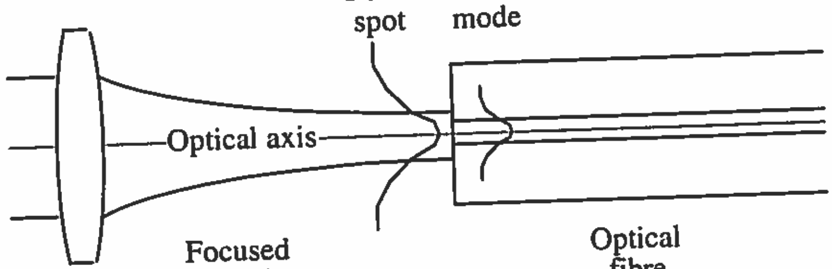
[2]

A quarter-pitch GRIN-rod lens can then form a collimated output from a point source.



[2]

3. a) End-fire coupling involves the use of a lens to focus a free-space Gaussian beam down onto the cleaved end face of an optical fibre, thus:



[2]

High coupling efficiency is obtained if the beam is aligned to the fibre core, and the radius of the focal spot matches the mode field radius.

[2]

b) The overlap integral between two transverse fields $E_1(x, y)$ and $E_2(x, y)$ is:

$<E_1, E_2> = {}_{-\infty}\int^{\infty} {}_{-\infty}\int^{\infty} E_1(x, y)\, E_2(x, y)\, dx\, dy$

Two guided modes $E_\mu(x, y)$ and $E_\nu(x, y)$ are orthogonal if their overlap is zero, so:

$<E_\mu, E_\nu> = {}_{-\infty}\int^{\infty} {}_{-\infty}\int^{\infty} E_\mu(x, y)\, E_\nu(x, y)\, dx\, dy = 0$

[2]

In excitation problems, an incident field E_{inc} is used to excite a waveguide. Assuming the incident field can be represented as a summation of the possible modes E_μ weighted by amplitude coefficients A_μ, we may write:

$E_{inc}(x, y) = {}_\mu\Sigma\, A_\mu\, E_\mu(x, y)$

Multiplying both sides by a further modal field $E_\nu(x, y)$ and integrating, we get:

$<E_{inc}, E_\nu> {}_{-\infty}\int^{\infty} {}_{-\infty}\int^{\infty} E_{inc}(x, y)\, E_\nu(x, y)\, dx\, dy = {}_{-\infty}\int^{\infty} {}_{-\infty}\int^{\infty} {}_\mu\Sigma\, A_\mu\, E_\mu(x, y)\, E_\nu(x, y)\, dx\, dy$

Modal orthogonality implies that the product terms on the RHS will integrate to zero unless $\nu = \mu$. Hence:

$<E_{inc}, E_\mu> = {}_{-\infty}\int^{\infty} {}_{-\infty}\int^{\infty} E_{inc}(x, y)\, E_\mu(x, y)\, dx\, dy = A_\mu\, {}_{-\infty}\int^{\infty} {}_{-\infty}\int^{\infty} E_\mu^2(x, y)\, dx\, dy = A_\mu <E_\mu, E_\mu>$

Consequently, the amplitude A_μ is

$A_\mu = <E_{inc}, E_\mu> / <E_\mu, E_\mu>$

[4]

The excitation efficiency η is:

η = Power in desired mode / Incident power

The power in the μ^{th} mode is $P_\mu = A_\mu^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_\mu^2(x, y)\, dx\, dy = A_\mu^2 \langle E_\mu, E_\mu \rangle$

The power in the incident field is $P_{inc} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_{inc}^2(x, y)\, dx\, dy = \langle E_{inc}, E_{inc} \rangle$

Hence the coupling efficiency is:

$\eta = \{\langle E_{inc}, E_\mu \rangle / \langle E_\mu, E_\mu \rangle\}^2 . \langle E_\mu, E_\mu \rangle / \langle E_{inc}, E_{inc} \rangle$

Or:

$\eta = \{\langle E_{inc}, E_\mu \rangle\}^2 / \{\langle E_\mu, E_\mu \rangle \langle E_{inc}, E_{inc} \rangle\}$

[3]

c) For a modal field $E_l(r) = E_0 \exp(-r^2/a^2)$:

$\langle E_l, E_l \rangle = \int_0^\infty E_0^2 \exp(-2r^2/a^2)\, 2\pi r\, dr$, or

$\langle E_l, E_l \rangle = -(a^2/4)\, 2\pi E_0^2 \int_0^\infty \exp(-2r^2/a^2)\, (-4r/a^2)\, dr$

$\langle E_l, E_l \rangle = -(\pi a^2/2)\, E_0^2\, {}_0[\exp(-2r^2/a^2)]^\infty = (\pi a^2/2)\, E_0^2$

For an excitation field $E_{inc}(r) = E'_0 \exp(-r^2/b^2)$ we clearly have:

$\langle E_{inc}, E_{inc} \rangle = (\pi b^2/2)\, E'^2_0$

[3]

For the two fields together, we have:

$\langle E_{inc}, E_l \rangle = \int_0^\infty E_0 \exp(-r^2/a^2)\, E'_0 \exp(-r^2/b^2)\, 2\pi r\, dr$, or

$\langle E_{inc}, E_l \rangle = -\{a^2b^2/2(a^2+b^2)\}\, 2\pi\, E_0E'_0 \int_0^\infty \exp\{-r^2(a^2 + b^2)/a^2b^2\}\, \{-2r(a^2 + b^2)/a^2b^2\}\, dr$,

or

$\langle E_{inc}, E_l \rangle = -\pi\{a^2b^2/(a^2+b^2)\}\, E_0E'_0\, {}_0[\exp\{-r^2(a^2 + b^2)/a^2b^2\}]^\infty$, or

$\langle E_{inc}, E_l \rangle = \pi\{a^2b^2/(a^2+b^2)\}\, E_0E'_0$

[2]

Hence the coupling efficiency is:

$\eta = \{\langle E_{inc}, E_\mu \rangle\}^2 / \{\langle E_l, E_l \rangle \langle E_{inc}, E_{inc} \rangle\}$ or

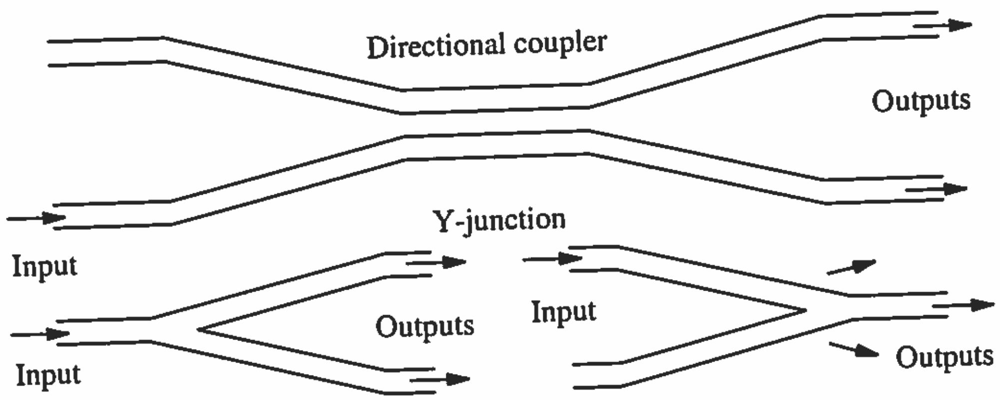
$\eta = \{\pi[a^2b^2/(a^2+b^2)]\, E_0E'_0\}^2 / \{(\pi a^2/2)\, E_0^2\, (\pi b^2/2)\, E'^2_0\}$ or:

$\eta = 4a^2b^2/(a^2+b^2)^2$

[2]

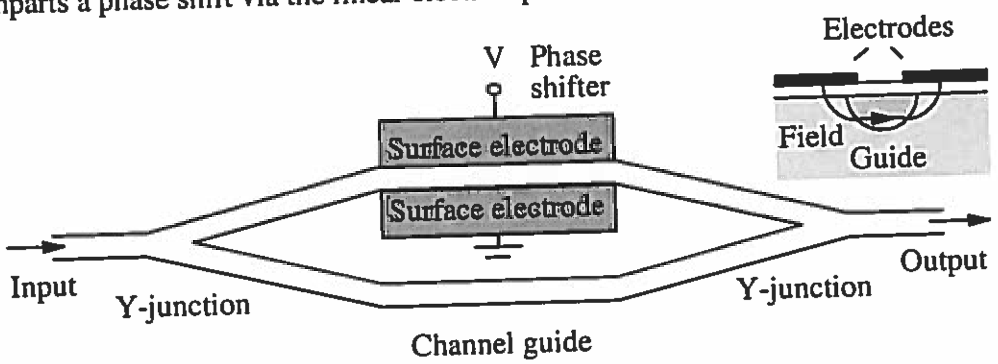
4. a) Directional couplers and Y-junctions are both channel guide devices that may be used as passive splitters. Couplers have four guided ports, while Y-junctions have three guided ports and one port connected to radiation. Consequently, couplers are (in principle) lossless, whichever port is used, while Y-junctions can radiate during asymmetric excitation.

[2]



[2]

b) The figure below shows a Y-junction based electro-optic MZI. Devices of this type are typically constructed in $LiNbO_3$, using channel guides formed by in-diffusion of Ti. Surface electrodes are used to create an electric field passing through the guide that then imparts a phase shift via the linear electro-optic effect.



[4]

The device operates as follows.

- The first Y-junction splits the input power equally. Assuming input modal amplitude of A_0, the amplitudes in the upper and lower guides are both $A_0/\sqrt{2}$.
- The two guided modes propagate to the right. The upper guide passes through an electro-optic phase shifter, which imparts a phase shift of ϕ.

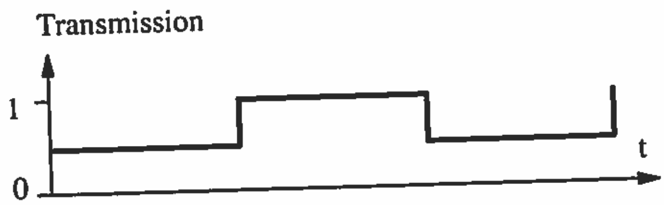
- Ignoring the common mode phase shift, the amplitudes at the output Y-junction are then $(A_0/\sqrt{2}) \exp(-j\phi)$ and $A_0/\sqrt{2}$.
- These amplitudes can be written as $(A_0/\sqrt{2}) \exp(-j\phi/2) \{\cos(\phi/2) - j \sin(\phi/2)\}$ and $(A_0/\sqrt{2}) \exp(-j\phi/2) \{\cos(\phi/2) + j \sin(\phi/2)\}$, respectively.
- The symmetric component $A_0 \exp(-j\phi/2) \cos(\phi/2)$ emerges from the output guide, while the antisymmetric component $j A_0 \exp(-j\phi/2) \sin(\phi/2)$ is radiated when the higher order mode in the Y-junction reaches cutoff.
- The transmitted power therefore varies as $AA^* = \cos^2(\phi/2)$ while the radiated power varies as $\sin^2(\phi/2)$.

[4]

The interferometer is used to convert phase modulation into amplitude modulation. An external modulator is generally still preferred to direct modulation of a laser, because it can operate at much higher frequency (up to 100 GHz, compared with a few GHz).

[2]

c) i) Assuming the voltage-length product is $VL_\pi = 50$ V mm, the application of 5 V to a 5 mm long device will generate a phase shift of $\pi \times 5 \times 5 / 50 = \pi/2$. At this point, the transmission will be $\cos^2(\pi/4)$, or 0.5. When the drive voltage is zero, the transmission will be $\cos^2(0) = 1$. Thus, the transmission will alternate between 1 and 0.5 as shown below.



[2]

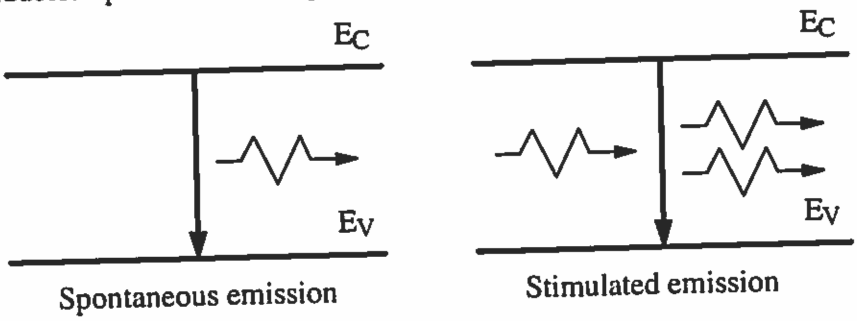
ii) When the drive voltage is 10 V, the phase shift will be π. At this point, the transmission will be zero. The same will be true for -10 V, so the transmission will be zero throughout.

[2]

ii) When the drive voltage is 15 V, the phase shift will be $3\pi/2$. The transmission will therefore follow a $\cos^2$ curve for 3/4 of a period, and then abruptly reset as shown below.

[2]

5. a) The energy band diagrams below show the two possible methods of generating light in semiconductor optoelectronics, spontaneous and stimulated emission.

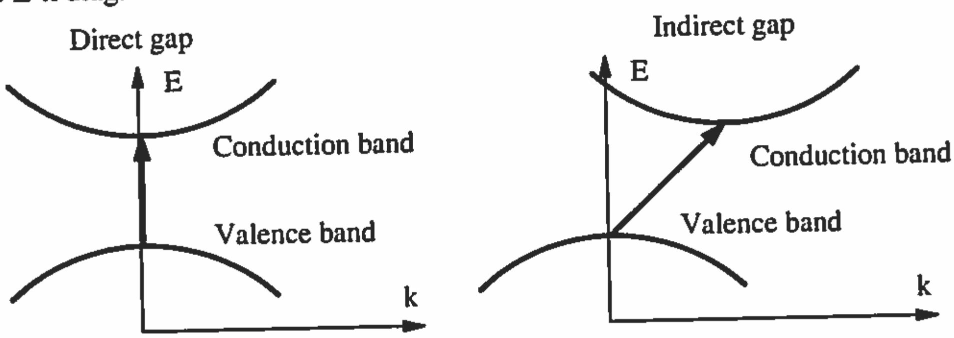


[2]

In each case, downward transition of an electron from the conduction band to the valence band leads to the generation of a photon of energy $hc/\lambda = E_C - E_V$. However, in spontaneous emission, the photon is emitted with a range of possible wavelengths, and random phase, direction and polarization. In stimulated emission, generation of a photon is triggered by a stimulating photon, to which the emitted photon is identical. Thus, by arranging for travelling-wave amplification stimulated emission in a laser structure, photons with identical wavelength, phase, direction and polarization are created. These properties allow the generation of high-power single-frequency carrier waves for optical communication systems that may be easily coupled into low-numerical aperture optical fibres.

[4]

b) The E-k diagrams of direct and indirect gap semiconductors are shown below.



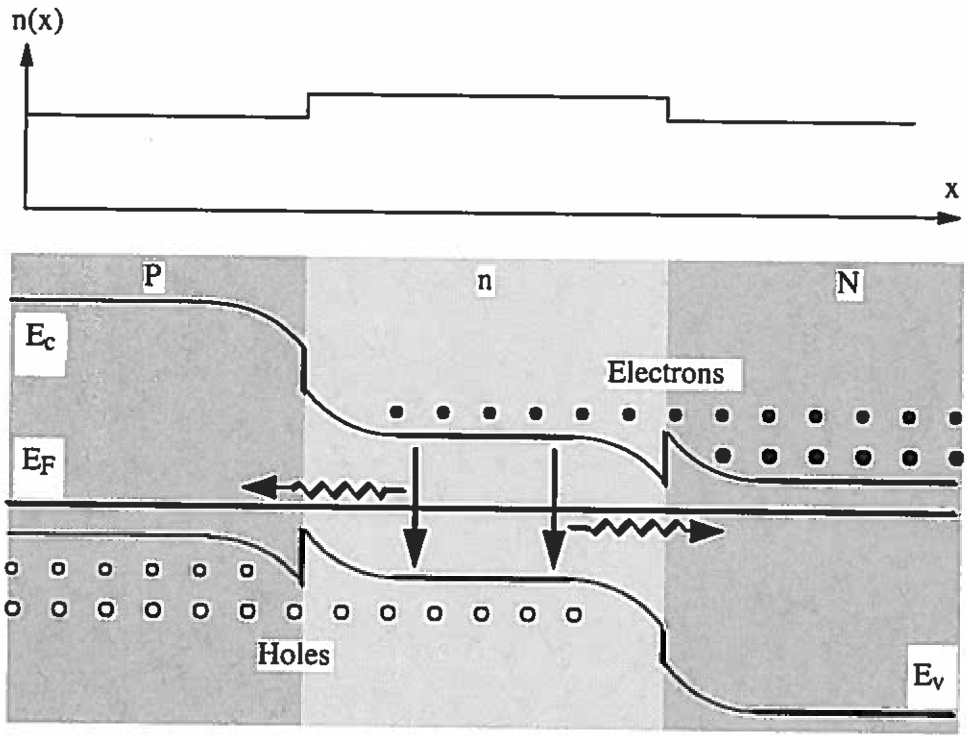
[2]

In the former case, the minimum of the conduction band is directly above the maximum of the valence band. Consequently a band-to-band transition can be induced almost entirely with energy. The necessary energy may be obtained from a photon (which carries significant energy, but little momentum).

In the latter case, the conduction band minimum is offset, so that a source of momentum is also needed in a band-to-band transition. The additional momentum may be obtained from a phonon (which carries significant momentum, but little energy). The need for both photons and phonons to interact with electrons greatly reduces the likelihood of band-to-band transitions, making emission rates very low in indirect-gap materials.

[4]

c) The figure below shows a P-n-N double heterostructure.



[4]

The structure consists of three layers of semiconductor: a layer of p-type material with wide bandgap and low refractive index, a layer of n-type material with small bandgap and high index and finally a layer of n-type material with wide bandgap and low index.

In equilibrium, the band diagram is constructed as follows. The position of the conduction band in each layer with respect to the Fermi level is first found from the doping level. The Fermi levels in each layer are then aligned. The spatial variation of the vacuum level, which follows the standard p-n junction variation at (say) the LH interface, is then found. The conduction band is then drawn in, by measuring down from the vacuum level by the value of the electron affinity χ for each layer. The valence band is finally drawn in, by measuring down from the conduction level by the value of the energy gap $E_C - E_V$ for each layer.

Differences in energy gap between the layers then result in abrupt steps in the conduction band and valence band at the interfaces. In the diagram shown, there is a large step for electrons in the conduction band at the P-n interface, but only a small step at the n-N interface. Conversely, there is a large step for holes in the valence band at the n-N interface, but only a small step at the P-n interface.

As a result, barriers for electron and hole motion are set up at different points on either side of the central active layer, so that electrons and holes injected under forward bias are prevented from diffusing away. At the same time, the transverse variation in refractive index provides an optical waveguide. Electrons, holes and photons are therefore confined to the active region so the rate of stimulated emission in a laser structure can be maximised.

[4]

6. The rate equations for a light emitting diode are:

$$dn/dt = I/ev - n/\tau_e$$

$$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$$

a) The quantities involved in the equations are n (electron density) and ϕ (photon density)

[1]

The terms on the right hand side of each equation are:

I/ev – rate of injection of electrons (per unit volume)

n/τ_e - rate of recombination of electrons (per unit volume)

n/τ_{rr} - rate of radiative recombination due to spontaneous emission (per unit volume)

ϕ/τ_p – rate of loss of photons as they escape (per unit volume)

[4]

The time constants τ_e and τ_{rr} are the total electron lifetime, and the radiative recombination lifetimes. They are related by:

$1/\tau_e = 1/\tau_{rr} + 1/\tau_{nr}$, where τ_{nr} is the non-radiative recombination lifetime.

[2]

τ_p is a function of the device geometry and is not related to τ_e and τ_{rr} at all.

[1]

b) To model a semiconductor laser, terms describing the generation of light by stimulated emission/absorption should be added:

$$dn/dt = I/ev - n/\tau_e - \mathbf{G\phi(n - n_0)}$$

$$d\phi/dt = n/\tau_{rr} + \mathbf{G\phi(n - n_0)} - \phi/\tau_p$$

Here G is the gain coefficient and n_0 is the electron density needed for transparency.

[2]

In the steady state, the equations reduce to:

$$I/ev - n/\tau_e - G\phi(n - n_0) = 0$$

$$n/\tau_{rr} + G\phi(n - n_0) - \phi/\tau_p = 0$$

Above threshold, we may neglect the spontaneous emission term n/τ_{rr}. The lower equation approximates to:

$$G\phi(n - n_0) - \phi/\tau_p = 0 \text{ or}$$

$$G(n - n_0) - 1/\tau_p = 0$$

So the electron density during lasing is:

$$n = n_0 + 1/G\tau_p$$

This value is independent of the drive current I.

[2]

The rate of escape of photons (per unit volume) is:

$$\phi/\tau_p = G\phi(n - n_0) = I/ev - n/\tau_e$$

Or:

$$\phi/\tau_p = (I - nev/\tau_e)/ev$$

Or:

$$\phi/\tau_p = (I - I_{th})/ev$$

Where $I_{th} = nev/\tau_e$ is the threshold current.

[2]

The total rate of escape of photons is:

$$\Phi = \phi v/\tau_p = (I - I_{th})/e$$

Assuming each photon carries an energy hc/λ, the power output is $P = \Phi hc/\lambda$, or:

$$P = (I - I_{th})\,(hc/\lambda e)$$

This value clearly varies linearly with current.

[2]

c) Re-arranging the expression for I_{th}, the electron density during lasing is $n = I_{th}\tau_e/ev$

The active volume is $v = 1\ \mu m \times 0.1\ \mu m \times 250\ \mu m = 25 \times 10^{-18}\ m^3$.

If $I_{th} = 20 \times 10^{-3}$ A and $\tau_e = 10^{-9}$ sec, the electron density is:

$n = 20 \times 10^{-3} \times 10^{-9} / (1.6 \times 10^{-19} \times 25 \times 10^{-18}) = 5 \times 10^{24}\ m^{-3}$.

[2]

The output power is $P = (I - I_{th})\,(hc/\lambda e)$

Assuming $\lambda = 1.55\ \mu m$ and $I = 30$ mA, the optical output power is:

$P = (30 - 20) \times 10^{-3} \times 6.62 \times 10^{-34} \times 3 \times 10^{8} / (1.55 \times 10^{-6} \times 1.6 \times 10^{-19})$ W = 8 mW

[2]

