

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2006

EEE PART III/IV: MEng, BEng and ACGI

**MICROWAVE TECHNOLOGY**

Tuesday, 25 April 10:00 am

Time allowed: 3:00 hours

**There are SIX questions on this paper.**

**Answer FOUR questions.**

*All questions carry equal marks*

Corrected Copy

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible	First Marker(s) :	S. Lucyszyn
	Second Marker(s) :	C. Papavassiliou

## The Questions

1.

- a) Sketch the transverse cross-sections for the guided-wave structures listed below. State two positive attributes, two drawbacks and two common applications for each:

- i) microstrip line [2]
- ii) coplanar waveguide [2]
- iii) slotline [2]
- iv) dielectric-filled metal-pipe rectangular waveguide [2]
- v) image line [2]
- vi) dielectric waveguide [2]

- b) With the aid of diagrams that show the electric field in the dominant modes of propagation, illustrate an efficient transition between the following guided-wave structures:

- i) microstrip to coplanar waveguide [1]
- ii) coplanar waveguide to slotline [1]
- iii) slotline to metal-pipe rectangular waveguide [1]
- iv) dielectric-filled metal-pipe rectangular waveguide to image line [1]
- v) image line to dielectric waveguide [1]

- c) Explain the general mechanisms that limit performance in transitions between dissimilar guided-wave structures. What rules-of-thumb can be adopted to improve efficiency. [3]

2.

- a) Define the following terms for waves travelling on a lossless transmission line and in each case comment on their practical importance:

i) velocity factor [2]

ii) characteristic impedance [2]

iii) return loss [2]

iv) complex reflection coefficient [2]

v) phase delay [2]

- b) A lossless coaxial cable has inductance,  $L$  [H/m] and capacitance  $C$  [F/m]. Give expressions for the characteristic impedance and velocity factor for the line. Calculate the inductance and capacitance of a 1 m length of line if it has an impedance of  $50 \Omega$  and velocity factor or 0.6.

[4]

- c) Calculate the complex reflection coefficient and return loss for reflections at a load having an impedance of  $75 + j 13 \Omega$  connected to a  $50 \Omega$  transmission line. Express the complex reflection coefficient in real and imaginary parts, and also as a modulus and phase angle. How does the complex reflection coefficient change as the reference plane at which it is measured moves towards the generator?

[6]

3.

- a) Quote the vector Helmholtz equation for an E-field in a homogeneous media and then expand this for an orthogonal rectangular coordinate system. Determine the plane wave solution and give the E-field component in the  $y$  direction, as it propagates along the  $z$  direction. [6]
- b) Define what is meant by the term transverse electromagnetic wave, and give a diagram showing the E-field, H-field and direction of propagation, in such a wave. Describe the orientation of a stack of metal plates spaced an arbitrary distance apart that may be introduced into this TEM wave without violating the boundary conditions. [4]
- c) A waveguide is to be designed over the frequency range 110 GHz to 140 GHz, using rectangular metal-pipe filled with a substance having a dielectric constant of 9 in this frequency range. Suggest suitable internal transverse cross-sectional waveguide dimensions, and calculate the guided wavelength at a frequency of 125 GHz. Hint: first suggest a suitable cut-off frequency. Comment on the probable source of attenuation in such a waveguide, and estimate the surface roughness (in microns) of the internal guide walls that it would be possible to tolerate. [6]
- d) With the metal-pipe rectangular waveguide designed in 3c) open-ended, calculate the radiation far-field distance. What is the effect on the radiation pattern with increasing distance in the far-field for this poor antenna? [4]

4.

- a) Draw the simplified circuit of an RF valve amplifier and explain the operation of the thermionic device used. Compare and contrast the main features of this device with the modern day solid-state MESFET. [6]
- b) To increase the frequency of operation, explain how the valve needs to be modified. Explain why the performance of triode valves deteriorates as frequency increases into the microwave spectrum. [4]
- c) If the unwanted parasitics of a triode valve consist of a grid-cathode capacitance of 20 pF and a cathode inductance of 10 nH, derive a general expression for the input resistance and calculate its value at 2 GHz, given a transconductance value of 2 mA/V. Comment on the significance of the input resistance in relation to the voltage gain of the amplifier. For the unwanted parasitic values given, what will the effect be on the voltage gain of the amplifier? [6]
- d) Derive an expression for the Gain-Bandwidth-Product for the valve amplifier in 4a) and calculate this for a 1 mA/V transconductance and a tank having a 10 pF capacitor. How many matched amplifiers would be needed to achieve a Total Power Gain = 40 dB and Bandwidth = 500 MHz? [4]

5.

- a) Give examples of thermionic valves that could still be found within today's home, operating in the KHz, MHz and GHz frequency regions. [3]
- b) With the aid of a diagram, describe in detail the construction and operation of a magnetron and comment on its domestic applications. [10]
- c) Explain the  $\pi$ -mode of operation and how other modes can be suppressed. [5]
- d) Briefly explain a simple method for increasing the output power level of a magnetron valve and give examples of typical power levels for different types of applications. [2]

6.

- a) Define mathematically the voltage and current at any position  $z$  along a uniform length of linear transmission line, and explain how standing waves are formed when this line is terminated by impedance  $Z_T$ . [2]
- b) From first principles, derive the equation for the normalised input impedance of a lossless transmission line having a characteristic impedance  $Z_0$ , physical length  $l$ , and electrical length  $\theta$  and that is terminated by an impedance  $Z_T$ . [6]
- c) From the equation derived in 6b), determine the solution for the input impedance for the following conditions and give an application for each condition if:
- i)  $l = \lambda g/2$  [2]
- ii)  $l = \lambda g/4$  [2]
- iii)  $Z_T = Z_0$  [2]
- iv)  $Z_T = 0$  and sketch the curve of normalized input impedance against distance from the termination impedance. [3]
- v)  $Z_T = \infty$  and sketch the curve of normalized input impedance against distance from the termination impedance. [3]

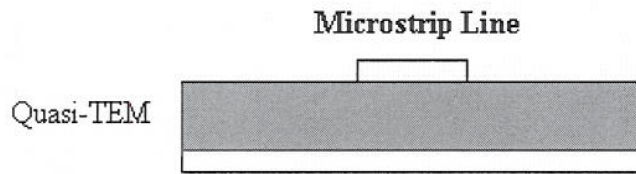


## The Solutions

2006

Model answer to Q 1(a): Bookwork

(i) microstrip line



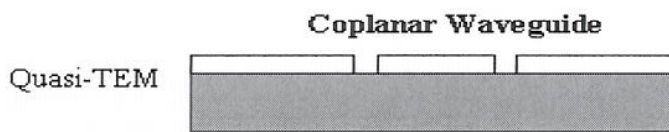
Advantages: easy to manufacture and many models available in CAD software

Disadvantages: frequency dispersive and requires through substrate vias

Applications: general interconnect and impedance matching

[2]

(ii) coplanar waveguide



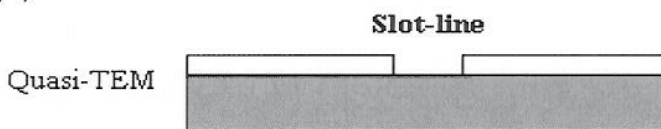
Advantages: easy to manufacture and no through substrate vias are required for grounding

Disadvantages: multi-moding is a problem at discontinuities and models are not so easy to find in CAD software

Applications: highly integrated circuits and millimetre-wave circuits

[2]

(iii) slotline



Advantages: easy to manufacture and easy to implement balanced signal lines

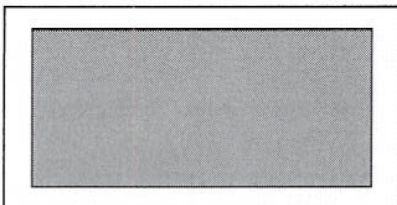
Disadvantages: multi-moding is a problem at discontinuities and models are not so easy to find in CAD software

Applications: feed lines for balanced antennas and balanced mixers

[2]

(iv) dielectric-filled metal-pipe rectangular waveguide

**Metal-Pipe  
Rectangular Waveguide  
(Dielectric-Filled)**



Advantages: very low loss and very high power handling

Disadvantages: frequency dispersive and difficult to integrate active devices

Applications: radio astronomy and radar

Microwave Technology

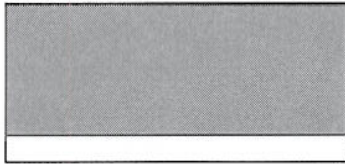
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[2]

(v) image line

**Image Line**



Advantages: very low loss and easy to manufacture

Disadvantages: frequency dispersive and difficult to integrate active devices

Applications: power couplers and low loss millimetre-wave interconnects

[2]

(vi) dielectric waveguide

**Dielectric Waveguide**



Advantages: very low loss and easy to manufacture

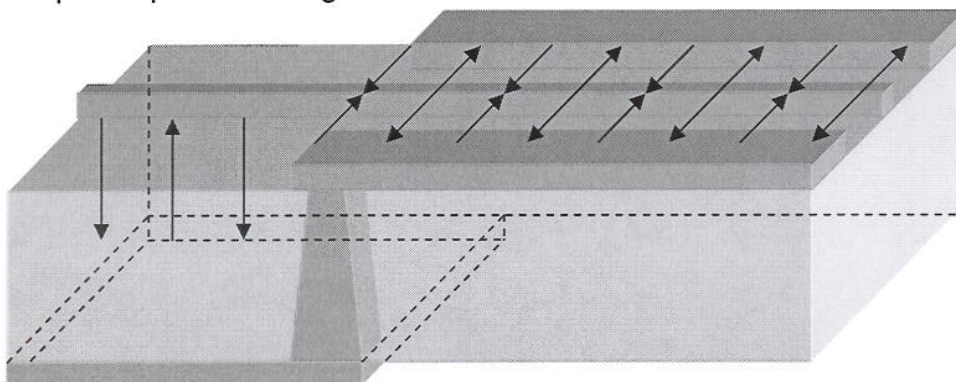
Disadvantages: poor isolation and difficult to integrate active devices

Applications: power couplers and low loss millimetre-wave interconnects

[2]

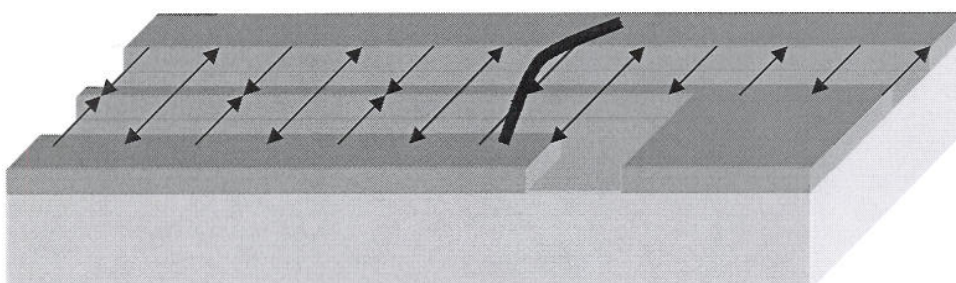
Model answer to Q 1(b): Bookwork

(i) microstrip to coplanar waveguide



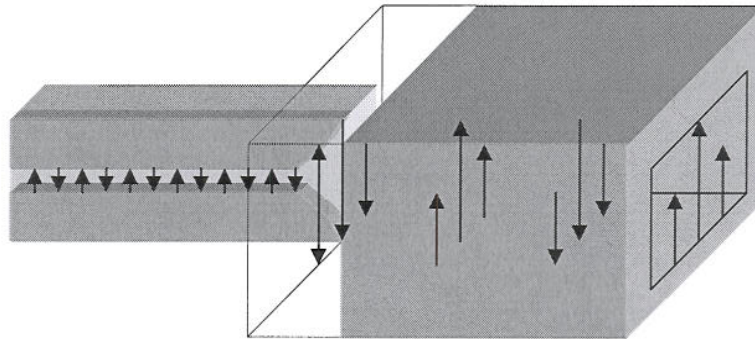
[1]

(ii) coplanar waveguide to slotline



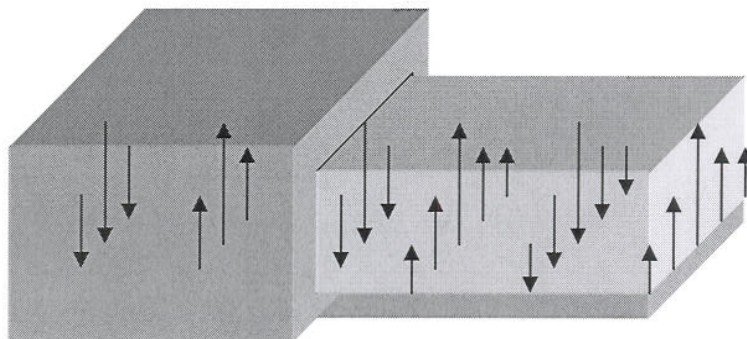
[1]

(iii) slotline to meta-pipe rectangular waveguide



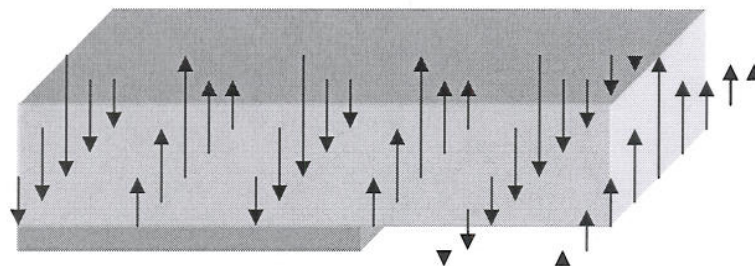
[1]

(iv) dielectric-filled metal-pipe rectangular waveguide to image line



[1]

(v) image line to dielectric waveguide



[1]

### Model answer to Q 1(c): Bookwork

At transitions between different types of guided-wave structures, or indeed at any physical and/ or electrical discontinuities, there may be the possibility of creating unwanted modes of propagation. If these unwanted modes are degenerate in nature then they must be suppressed (e.g. by using bond-wires/underpasses or resonators). If the unwanted modes are evanescent in nature then they will die away within a few guided wavelengths, and so extra lengths of transmission lines are required to allow this to happen so that they do not have enough energy to excite degenerate modes.

[3]



### Model answer to Q 2(a): Bookwork

(i) The velocity factor is the ratio of the speed of waves on the transmission line to the speed of light,  $c$ , in vacuum, where  $c = 3 \times 10^8$  m/s. The velocity factor is a dimensionless number less than unity. It basically represents how much the wave is slowed down inside the transmission line, when compared to propagation in free space.

[2]

(ii) The characteristic impedance is the number of a infinitely long length of transmission line, or of a finite length of line at times before any reflections have arrived back at the measuring source. It is important to match two lengths of different propagating media to the same impedance, in order to avoid reflected waves and associated reflected power. Conversely, two specific impedances are required to generate a specific reflected wave.

[2]

(iii) The return loss is the number of dB by which the return wave power is less than the forward wave power. This is important because it gives a quantitative measure of how much reflected power is generated and thus indicates the level of impedance mismatch.

[2]

(iv) The complex reflection coefficient is the complex ratio of return wave amplitude to forward wave amplitude. This is important, as it allows the exact impedance at a discontinuity to be calculated.

[2]

(v) The phase delay is the amount of phase shift experienced by the wave in travelling along a certain length of transmission line. It is important to be able to calculate this delay so that any reflected or transmitted waves can be combined in-phase or 180° out-of-phase.

[2]

### Model answer to Q 2(b): Computed exercise

$$Z_0 = \sqrt{\frac{L}{C}}; \quad v_p = \frac{1}{\sqrt{LC}} \quad \text{and} \quad VF = \frac{v_p}{c}$$

If  $Z_0 = 50 \Omega$  and  $VF = 0.6$ ,  $L = 87.8 \text{ nH}$  and  $C = 351 \text{ pF}$  for a length of 1 m transmission line.

[4]

### Model answer to Q 2(c): Computed exercise

$$z_L = \frac{Z_L}{Z_0} = 1.5 + j0.26 \Omega$$

$$\rho = \frac{z_L - 1}{z_L + 1} = 0.21 + j0.08 \equiv 0.22 e^{+j21.54}$$

$$\text{Return Loss} = 13 \text{ dB}$$

As the reference plane is moved towards the generator, the modulus of the complex reflection coefficient stays the same (assuming the line is lossless), but the phase angle is reduced by 360° for each  $\lambda/2$  of displacement of the reference plane from the load.

[6]

### Model answer to Q 3(a): Bookwork

The E-field wave (propagation) equation can be obtained from:

$$\nabla^2 \hat{E} + k_m^2 \hat{E} = 0 \quad \text{vector Helmholtz equation for an E-field in a homogeneous media}$$

where, modified wavenumber,  $k_m = \omega \sqrt{\mu\epsilon}$

Therefore, in a rectangular coordinate system:

$$\nabla^2 \hat{E} = \frac{\partial^2 \hat{E}}{\partial x^2} + \frac{\partial^2 \hat{E}}{\partial y^2} + \frac{\partial^2 \hat{E}}{\partial z^2} = -\omega^2 \mu\epsilon \hat{E}$$

In an orthogonal rectangular coordinate system:

$$\nabla^2 E_x = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = -\omega^2 \mu\epsilon E_x$$

$$\nabla^2 E_y = \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = -\omega^2 \mu\epsilon E_y$$

$$\nabla^2 E_z = \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu\epsilon E_z$$

With a plane wave, there is a variation in the field quantities only in one dimension, e.g. the  $z$  direction of propagation:

$$\therefore \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial y^2} = 0 \quad \text{and} \quad \nabla^2 \hat{E} = \frac{\partial^2 \hat{E}}{\partial z^2} = -\omega^2 \mu\epsilon \hat{E}$$

and

$$\nabla^2 E_x = \frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x$$

$$\nabla^2 E_y = \frac{\partial^2 E_y}{\partial z^2} = \gamma^2 E_y$$

$$\nabla^2 E_z = \frac{\partial^2 E_z}{\partial z^2} = \gamma^2 E_z$$

Now, considering only the E-field component in the  $y$  direction, as it propagates along the  $z$  direction:

$$E_y = Ae^{-\gamma z} + Be^{+\gamma z} \quad \text{where} \quad \gamma = jk_m \quad \text{and} \quad k_m = \omega \sqrt{\mu\epsilon}$$

$A$  represents the amplitude of the forward travelling wave, while  $B$  represents the amplitude of the backward travelling wave.

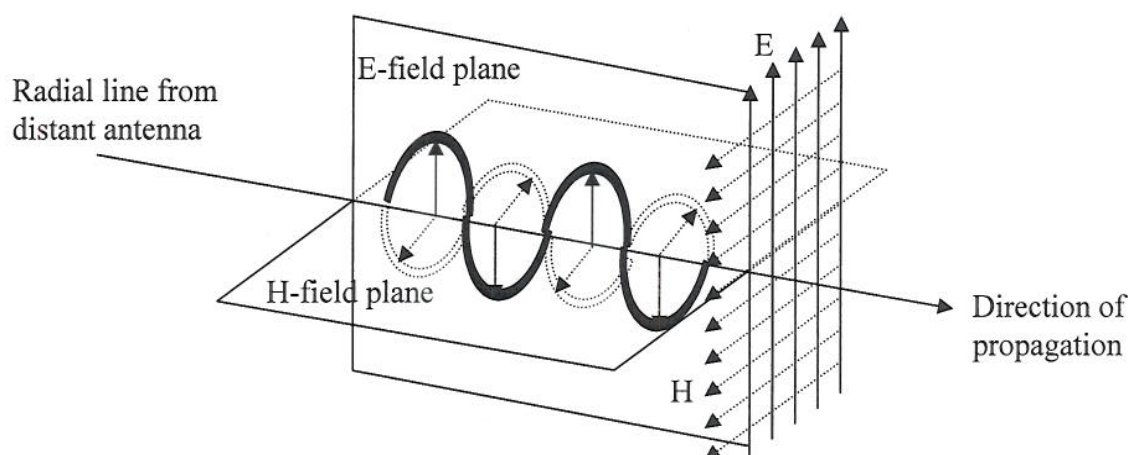
This solution is identical to the voltage waves on a transmission line. In general, further analysis is usually confined to just the forward wave. When time dependency is also considered, the forward wave of the field is represented as:

$$E_y = E_0 e^{(j\omega t - \gamma z)}$$

[6]

Model answer to Q 3(b): Bookwork

In a transverse electromagnetic (TEM) wave, the E-field vector and H-field vector are both at right angles to each other and to the direction of propagation.



The wavefront is the plane that is mutually orthogonal to the E- & H-planes

A stack of metal sheets may be placed so that the E-field is at right angles to the surface of the plates, and the H-field parallel to the surface of the plates, without violating the boundary conditions.

[4]

Model answer to Q 3(c): Computed example

If the dielectric constant is 9, the wave velocity in the material filling the guide is slower than the speed of light in vacuum by a factor  $\sqrt{9} = 3$ . If we assume the cut-off frequency of the required waveguide is 100 GHz, in free space the wavelength would be 3 mm and in the dielectric the wavelength would be 1 mm. The guide cross-sectional dimensions could, therefore, be  $500 \mu\text{m} \times 250 \mu\text{m}$ , as the height of the waveguide is chosen to be half the width. Attenuation would be provided by dielectric loss, and by the surface resistance of the guide walls, since the skin depth would be small. At 125 GHz the free space wavelength in the dielectric would be  $1/1.25 = 0.8 \text{ mm}$ . Using the waveguide formula the guided wavelength is

$$\lambda_g = \frac{\lambda_o}{\sqrt{1 - \left(\frac{\lambda_o}{\lambda_c}\right)^2}} = \frac{1}{\sqrt{\frac{1}{\lambda_o^2} - \frac{1}{\lambda_c^2}}} = \frac{1}{\sqrt{\frac{1}{0.8^2} - 1}} = 1.33 \text{ mm}$$

It would be appropriate to keep the surface roughness below  $\lambda_g/100 = \sim 13 \text{ microns}$ .

[6]

Model answer to Q 3(d): Computed example

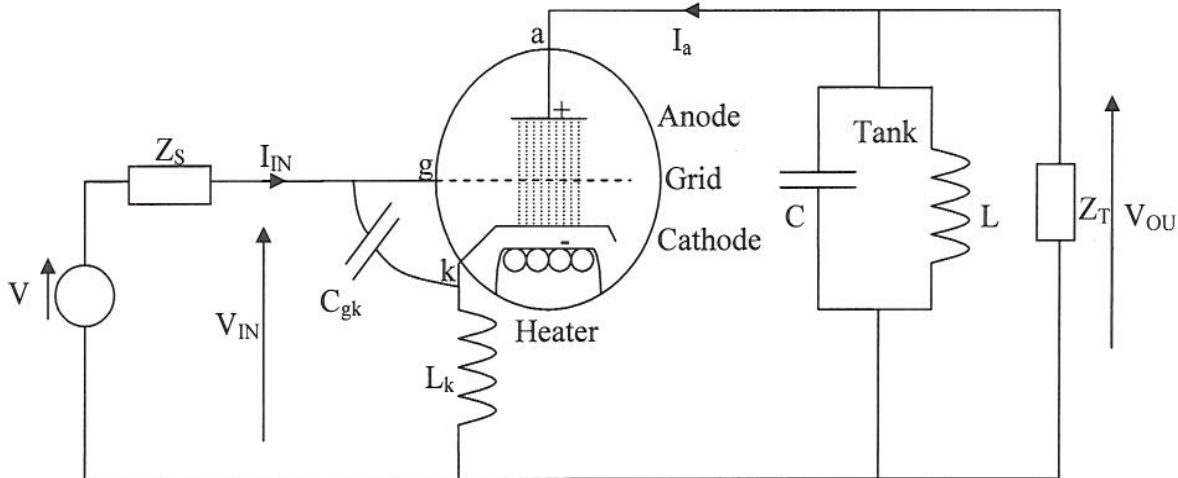
With transverse cross-sectional waveguide dimensions of  $500 \mu\text{m} \times 250 \mu\text{m}$ , the maximum aperture dimension is  $A = \sqrt{(500^2 + 250^2)} = 559 \mu\text{m}$ . Therefore, at 125 GHz, the radiating far field (or Fraunhofer) distance is  $2A^2/\lambda_o = 260 \mu\text{m}$ . Beyond this distance, the radiation pattern of this poor antenna is not a function of distance from the aperture.

[4]



Model answer to Q4(a): Bookwork

**Triode Thermionic Valves (or Gridded Tubes)**



- With the triode valve, a heater is required to heat the cathode electrode (which is negatively biased, with respect to ground). Within the vacuum or inert gas environment, electrons that have absorbed enough thermal energy to overcome the work function of the cathode metal are emitted from the cathode and attracted to the positively biased plate electrode. The resulting flow of electrons can be modulated by controlling the grid-cathode potential. The GaAs MESFET is the modern day analog of the triode valve. With the MESFET, the gate-source potential controls the flow of electrons flowing from the source to drain electrodes. The triode valve consumes a great deal of power for the heaters and required very large positive and negative potentials on the plate and cathode electrodes, respectively. The transconductance is generally low and, therefore, the GAIN-Bandwidth Product (GBP) will be low. In contrast, the MESFET has a higher transconductance and, therefore, higher GBP. Also, this device does not need to be heater or have large bias voltages.

[6]

Model answer to Q4(b): Bookwork

- As frequency increases, the distance between the cathode, grid and plate must decrease. This is because the cathode-plate electron transit time must be kept well below the period of the RF cycle, otherwise:

- The grid and plate signals may not be 180° out-of-phase, which can lead to problems with feedback oscillators.
- The grid can take power from the driving source, since the grid voltage has time to change during the flow of a particular group of electrons from the cathode to the plate.

As a result, at microwave frequencies, the internal dimensions are so small that:

- The large parasitic capacitances could resonate with the lead inductances, within the band of operation.
- Power levels are significantly reduced, since dielectric breakdown of dry air at one atmosphere is only  $\sim 3 \times 10^6$  V/m, and even much less in a vacuum.
- Input impedance and, therefore, voltage gain are reduced.

[4]



### Model answer to Q4(c): Bookwork and Computed Example

If only  $C_{gk}$  and  $L_k$  are considered,

$$I_{OUT} = g_m V_{gk} \text{ and, therefore, } V_{IN} \cong V_{gk} + j\omega L_k I_{OUT} = V_{gk} (1 + j\omega L_k g_m)$$

$$\text{But, } V_{gk} = \frac{I_{IN}}{j\omega C_{gk}}$$

$$\text{Therefore, } Z_{IN} = \frac{V_{IN}}{I_{IN}} = \frac{1 + j\omega L_k g_m}{j\omega C_{gk}}$$

$$\text{Therefore, } Y_{IN} = \frac{1}{Z_{IN}} = \frac{j\omega C_{gk}}{1 + j\omega L_k g_m} \approx \omega^2 L_k C_{gk} g_m + j\omega C_{gk}$$

$$\text{Since, } (1 \pm jA)^{-1} \approx (1 \mp jA) \text{ if } |A| \ll 1$$

$$\therefore R_{IN} \cong \frac{1}{\omega^2 L_k C_{gk} g_m}$$

As the voltage gain of the valve increase, due to an increased transconductance, the input resistance decreases.

$$* \text{ With the tank at resonance, voltage gain } = \frac{V_{OUT}}{V_{IN}} = -g_m R_T$$

$$\text{But, } V_{IN} = \frac{V_S Z_{IN}}{Z_S + Z_{IN}} \text{ and, therefore, } \frac{V_{OUT}}{V_S} = \frac{-g_m R_T}{Z_S / Z_{IN} + 1}$$

When  $f_0 = 2 \text{ GHz}$ ,  $L_k = 10 \text{ nH}$ ,  $C_{gk} = 20 \text{ pF}$  and  $g_m = 2 \text{ mA/V}$

Therefore,  $R_{IN} = 15.8 \Omega$ , i.e. a near short circuit to a driving source impedance of  $Z_S = 50 \Omega$  and, therefore, the overall voltage gain will be low.

[6]

### Model answer to Q4(d): Bookwork and Computed example

$$\text{Output Loaded-Quality factor, } Q_{OUT} = \frac{R_T}{|X_c|} = R_T \omega_0 C$$

$$\text{Therefore, output bandwidth, } \Delta\omega = \frac{\omega_0}{Q_{OUT}} = \frac{1}{R_T C}$$

$$\text{Therefore, for a matched amplifier, gain-bandwidth product, } GBP \cong g_m R_T \frac{\Delta\omega}{2\pi} = \frac{g_m}{2\pi C}$$

For a power gain = 40 dB and bandwidth = 500 MHz it is impossible to meet this specification with the valve amplifier because each stage would have a  $GBP = 16 \times 10^6$  and, therefore, this represents a 30 dB attenuation.

[4]

### Model answer to Q 5(a): Bookwork

In the KHz region, overly-priced Hi-Fi valve amplifiers are still commercially available.

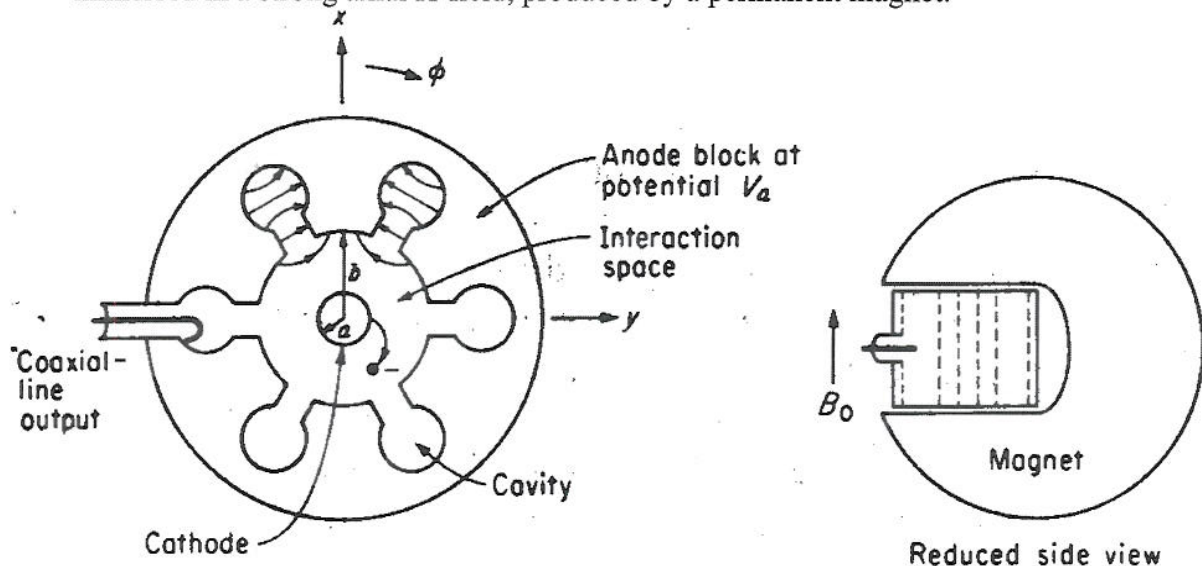
In the MHz region, the cathode-ray tube found in domestic television receivers is still being used in the technologically backward western world.

In the GHz part of the spectrum, the 2.54 GHz magnetron is found in all domestic microwave ovens.

[3]

### Model answer to Q 5(b): Bookwork

- The cylindrical surface of the centre electrode is an electron-emitting cathode. The surrounding block forms the anode, containing N-resonant cavities and these are separated by N-segments. The axial length (i.e. cavity depth) is  $\lambda_0/4$ . The complete structure is immersed in a strong axial H-field, produced by a permanent magnet.



- The magnetron is a self-excited oscillator that converts pulsed DC input power into pulsed RF output power. DC-to-RF conversion takes place in the interaction space between the cathode and the anode. The anode is constructed to form a microwave circuit, consisting of a transmission line loaded with cavities. Initially, a DC voltage is applied between the cathode and the microwave circuit. As the DC voltage is increased, electrons leave the heated cathode and interact with the H-field, causing them to circle the cathode. As the voltage is further increased, the angular velocity of the electrons becomes equal to that of the RF wave travelling along the microwave circuit. When this occurs, there is a spontaneous interaction between the electrons and the microwave circuit – resulting in spokes of space charge that induce microwave power into the slow-wave circuit. Here, electrons move through the RF-field of the resonant cavities. These electrons deliver energy to the resonators, which sustain the oscillations. The RF power is extracted by loose coupling into one of the cavities.

[10]

### Model answer to Q 5(c): Bookwork

- The normal  $\pi$ -mode of operation (as shown above, right) can be regarded as being formed by the superposition of two waves rotating in opposite directions. For an N-cavity magnetron, the angular velocity at which both the electromagnetic wave and the electron spokes rotate is equal to  $2\omega_0/N$ .
- Modes other than the wanted  $\pi$ -mode are suppressed, by making an electrical connection to alternating segments, using two 'strapping rings' at both ends of the block. At higher frequencies, alternate cavities are made with differing depths – forming the 'rising sun' block. Both of these approaches have the effect of increasing the frequency separation between the wanted and the unwanted modes.

[5]

### Model answer to Q 5(d): Bookwork

- Increasing the H-field requires an increased voltage for a given current, and results in a higher RF power output and efficiency.
- Pulsed applications: e.g.  $\sim 200$  KW for airborne radar and  $\sim 20$  MW for ground-based radar. CW applications: e.g. Diathermy and Domestic Microwave Ovens ( $<1$  KW)

[2]

### Model answer to Q 6(a): Bookwork Derivation

The voltage and current on a transmission line can be represented as :

$$V(z) = V_+ e^{-\gamma z} + V_- e^{+\gamma z}$$

$$I(z) = I_+ e^{-\gamma z} + I_- e^{+\gamma z}$$

where,  $V_{\pm}(I_{\pm})$  represents voltage (current) waves at  $z = 0$

and,  $e^{\mp \gamma z}$  represents wave propagation in the  $\pm z$  direction

and the propagation constant,  $\gamma = \alpha + j\beta$

The voltage and current on a transmission line can now be represented as :

$$V(z) = V_+ (e^{-\gamma z} + \rho(0)e^{+\gamma z})$$

$$I(z) = I_+ (e^{-\gamma z} - \rho(0)e^{+\gamma z})$$

The voltage (and current) on the line is composed of a superposition of the incident and reflected waves, which create a "standing wave", due to the mismatched load termination (even if the generator is matched to the line). Here, the incident and reflected wave magnitudes alternately cancel and reinforce one another. This standing wave disappears when the line is said to be "matched", i.e.  $Z_T = Z_0$ , and we are left with just a single wave travelling in the  $+z$  direction.

[2]



### Model answer to Q 6(b): Textbook derivation

Now, the impedance looking into a transmission line that is terminated with a load  $Z_T$  is:

$$Z_{in} = \frac{V(l)}{I(l)} = Z_0 \frac{(e^{+\gamma l} + \rho(0)e^{-\gamma l})}{(e^{+\gamma l} - \rho(0)e^{-\gamma l})} = \frac{(Z_T + Z_0)e^{+\gamma l} + (Z_T - Z_0)e^{-\gamma l}}{(Z_T + Z_0)e^{+\gamma l} - (Z_T - Z_0)e^{-\gamma l}}$$

$$Z_{in} = Z_0 \frac{(Z_T(e^{+\gamma l} + e^{-\gamma l}) + Z_0(e^{+\gamma l} - e^{-\gamma l}))}{(Z_0(e^{+\gamma l} + e^{-\gamma l}) + Z_T(e^{+\gamma l} - e^{-\gamma l}))}$$

$$\text{Therefore, } z_{in} = \frac{Z_{in}}{Z_0} = \frac{z_T + \tanh(\gamma l)}{1 + z_T \tanh(\gamma l)} \Rightarrow \frac{z_T + j \tan \theta}{1 + j z_T \tan \theta} \text{ for a lossless line}$$

[6]

### Model answer to Q 6(c): Textbook derivation

(a) If  $l = \lambda g/2$  then  $Z_{in} = Z_T$ , therefore, no impedance transformation – useful for realising interconnects over a narrow bandwidth.

[2]

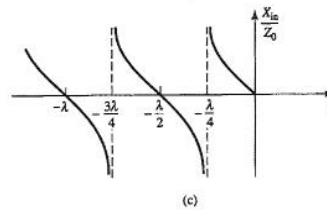
(b) If  $l = \lambda g/4$  then  $Z_{in} = Z_0^2/Z_T$ , therefore, this is a quarter-wavelength impedance transformer – acts as an impedance inverter over a narrow bandwidth.

[2]

(c) If  $Z_T = Z_0$  then  $Z_{in} = Z_0$ , therefore, no impedance transformation – useful for realising interconnects over a very wide bandwidth.

[2]

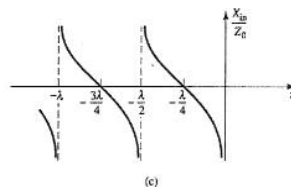
(d) If  $Z_T = 0$  then  $Z_{in} = jZ_0 \tan \theta$  and the impedance is always reactive and periodic along the line, which takes a value from 0 to  $+\infty$  and  $-\infty$  to 0 as  $l$  increases from 0 to  $\lambda g/4$  and  $\lambda g/4$  to  $\lambda g/2$ . This is useful for realising any value of “effective” inductance or capacitance over a narrow bandwidth.



(a) Voltage, (b) current, and (c) impedance ( $R_{in} = 0$  or  $\infty$ ) variation along a short-circuited transmission line.

[3]

(e) If  $Z_T = \infty$  then  $Z_{in} = -jZ_0 \cot \theta$  and the impedance is always reactive and periodic along the line, which takes a value from  $-\infty$  to 0 and 0 to  $+\infty$  as  $l$  increases from 0 to  $\lambda g/4$  and  $\lambda g/4$  to  $\lambda g/2$ . This is useful for realising any value of “effective” capacitance or inductance over a narrow bandwidth.



(a) Voltage, (b) current, and (c) impedance ( $R_{in} = 0$  or  $\infty$ ) variation along an open-circuited transmission line.

[3]