UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1997

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute
Associateship of the Royal College of Science

PAPER 3.43 / I3.22

OPERATIONS RESEARCH

Thursday, May 1st 1997, 10.00 - 12.00

Answer THREE questions

For admin. only: paper contains 4 questions

1a A company wishes to blend a new alloy of 40% tin, 35% zinc, and 25% lead from several available alloys having the following properties:

	Alloy				
Property	A1	A2	A3	A4	A5
Percentage tin	60	25	45	20	50
Percentage zinc	10	15	45	50	40
Percentage lead	30	60	10	30	10
Cost (£/lb)	20	18	24	22	26

The objective is to determine the proportions of these alloys that should be blended to produce the new alloy at a minimum cost.

Formulate the linear programming model for the problem. Do not solve it.

b Reformulate the following linear programming problem in standard form and use the simplex algorithm to solve it:

$$\begin{array}{rrrrrrrrr} \max x_0 = & - & x_1 & - & 2x_2 & - & x_3 & & \\ \text{s.t.} & - & x_1 & + & x_2 & + & x_3 & \geq & -1 \\ & & x_1 & + & x_2 & + & 2x_3 & \leq & 4 \\ & & x_1 \geq 0, & & x_2 \text{ free}, & & x_3 \geq 0, & & \end{array}$$

c Having solved the problem numerically, determine whether shadow prices can be easily obtained and, if yes, give them and explain where they belong to and what they mean.

(The three parts carry, respectively, 40%, 40% and 20% of the marks).

2a Let $Ax \leq b,\ x \geq 0$ define the feasible solutions to a linear programming (LP) problem. Assume, $x^{(1)}, x^{(2)}, \ldots, x^{(N)}$ is the list of all basic feasible solutions. Their *convex linear combination* is defined as

$$\bar{x} = \sum_{i=1}^{N} \lambda_i x^{(i)},$$

where $\lambda_i \geq 0,\ i = 1, \ldots, N$ and $\sum_{i=1}^{N} \lambda_i = 1.$

Show that $\bar{x}$ is a feasible solution to the given LP problem.

b Solve the following LP problem using the Two-Phase simplex algorithm:

$$\begin{array}{rrcrcr} \max\ x_0 = & 4x_1 & - & x_2 & & \\ \text{s.t.} & 2x_1 & + & x_2 & \geq & 25 \\ & -\ 3x_1 & + & 2x_2 & \leq & 15 \\ & & x_1,\ x_2 \geq 0. & & & \end{array}$$

(The parts carry, respectively, 50% and 50% of the marks).

3a An investment company is considering six major capital investment opportunities. The investments differ in the estimated long-run profit they are expected to generate as well as in the amount of capital required as shown in the following table in units of millions of £:

	Investment Opportunity					
Estimated long-run profit	19	12	17	21	10	15
Capital required	45	30	36	50	19	34

The total amount available for these investments is £100,000,000. Investment opportunities 3 and 4 are mutually exclusive and so are 5 and 6. Additionally, neither 1 nor 2 can be undertaken unless either 3 or 4 is undertaken. The objective is to select the combination of capital investments that will maximize the total estimated long run profit. Create the $0-1$ integer programming formulation of this problem. Do not solve the problem.

b You are given the following general integer linear programming (LP) problem:

$$\begin{array}{rrcrcl} \max\ x_0 = & x_1 & + & 2x_2 & & \\ \text{s.t.} & 5x_1 & + & 2x_2 & \leq & 16 \\ & 2x_1 & + & 3x_2 & \leq & 11 \\ & \multicolumn{5}{l}{x_1,\ x_2 \text{ integer}} \end{array}$$

Solving the LP relaxation of the problem (integrality constraints ignored), the following optimal tableau is obtained:

	x_1	x_2	x_3	x_4	RHS
x_0	1/3	0	0	2/3	22/3
x_3	11/3	0	1	−2/3	26/3
x_2	2/3	1	0	1/3	11/3

Create a Gomory cut based on the second constraint in the above tableau. Show that this cut excludes the optimal solution of the LP relaxation. Formulate the next LP problem that needs to be solved.

(The parts carry, respectively, 50% and 50% of the marks).

Turn over ...

4a Consider the following payoff matrix for a two-person zero-sum game:

		Player – II					
		1	2	3	4	5	6
	1	1.35	1.2	1.3	1.4	1.5	1.6
	2	1.5	1.35	1.3	1.4	1.5	1.6
Player–I	3	1.4	1.4	1.35	1.4	1.5	1.6
	4	1.3	1.3	1.3	1.35	1.5	1.6
	5	1.2	1.2	1.2	1.2	1.35	1.6
	6	1.1	1.1	1.1	1.1	1.1	1.35

Player–I wishes to maximize his reward, Player–II wishes to minimize his loss. Use the minimax criterion to determine the best strategy for each side. Does the matrix of the game have a saddle point? Characterize the solution you found.

b Given a 2 person zero sum game with a saddle point. Add a constant c to every element of the reward matrix A. Denote the newly obtained matrix by A'. Show that A and A' have the same optimal strategies and that (value of A) = (value of A') + c.

c Formulate the dual of the following linear programming problem:

$$
\begin{array}{rrcrcrcr}
\max\ x_0 = & x_1 & + & 2x_2 & - & 3x_3 & & \\
\text{s.t.} & 3x_1 & + & 2x_2 & - & x_3 & \leq & 4 \\
 & -x_1 & + & 3x_2 & + & x_3 & \geq & 2 \\
 & x_1 & + & x_2 & + & x_3 & = & 14 \\
 & \multicolumn{7}{l}{x_j \geq 0, \quad j = 1, 2, 3.}
\end{array}
$$

(The parts carry, respectively, 40%, 20% and 40% of the marks).

End of Paper