EE1-05

1. SOLUTIONS

a) A coaxial cable is used to connect a DC voltage source to a resistive load, with the positive terminal connected to the outer conductor of the cable. Sketch the electric field and magnetic flux in a cross section through the cable. [4]

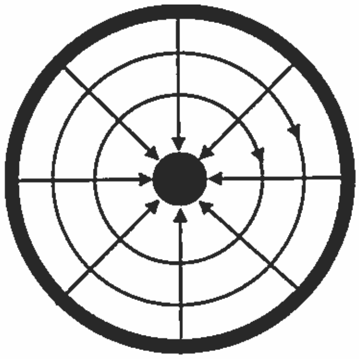


Figure 1 a).

One mark for each for geometry and direction for E and M fields.

b) What is the force between two parallel wires, 50 cm long and 10 cm apart, each carrying a current of 100 mA? [4]

Field from one wire is given by $B = \mu_0 I/2\pi r$ while force on second wire is given by BIL so overall force is given by $F = LI^2/2\pi\mu_0 r$ = 0.5 x 0.1^2 x 4 π x $10^{-7}/(2\pi \times 0.1) = 10^{-8}$ N

c) An iron C-core as shown has a gap length of 0.5 mm, a total path length in the iron of 5 cm, and the coil provides 100 A·turns. If the iron has μ_r = 10,000, calculate the magnetic flux density in the gap, stating any assumptions made. **[4]**

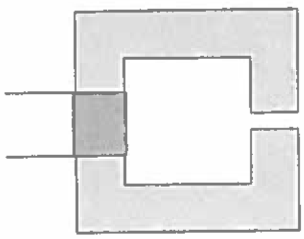
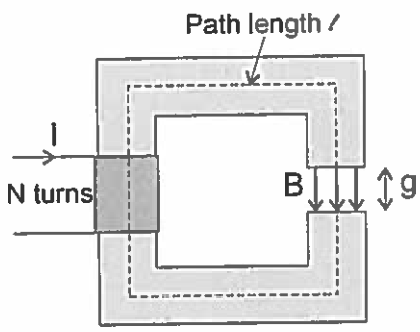


Figure 1 c).

For a closed circuit within the core:



$$\oint H.dl = NI$$

giving:

$$\oint H.dl = H_i l + H_g g = NI$$

for the magnetic fields in the core, H_i, and the gap, H_g. Lines of flux are continuous, so assuming that the gap is small enough to prevent much spreading out of the flux lines in the gap, for the path we can make the approximation $B_g = B_i = B$. In the gap $B_g = \mu_0 H_g$ while assuming a linear relationship between B and H in the core, $B = \mu_0 \mu_r H_i$. Substituting:

$$\frac{Bl}{\mu_0 \mu_r} + \frac{Bg}{\mu_0} = NI$$

$$B = \frac{\mu_0 NI}{l/\mu_r + g}$$

$$\approx \frac{\mu_0 NI}{g}$$

as $l/\mu_r \ll g$. Putting in the values, $B = 0.25$ T

d) A load connected to an 11kV 50Hz AC supply consumes 650kW of real power and draws a current of 60A. Calculate the apparent power, power factor, and magnitude of the load impedance. [4]

$$S = VI = 11 \times 10^3 \times 60 = 0.66MVA$$

$$pf = \frac{P}{S} = \frac{0.65M}{0.66M} = 0.985$$

$$|Z| = \frac{V}{I} = \frac{11 \times 10^3}{60} = 183\,\Omega$$

e) Fig. 1e) shows the IV curve for a photodiode under illumination. Explain the shape of the curve and on your copy of the curve indicate an estimate for the current draw from the photocell that would maximise the power produced.

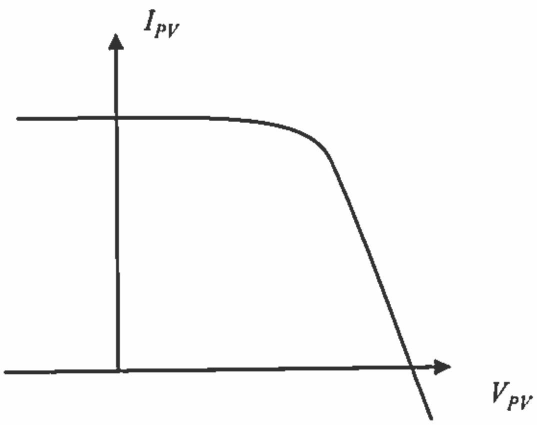


Figure 1 e).

The curve shows that the current is reduced at increasing voltages as the forward current of the photodiode flows in the opposite direction to the photon-induced current. The maximum power will correspond to the largest rectangle from the origin with one vertex on the IV curve:

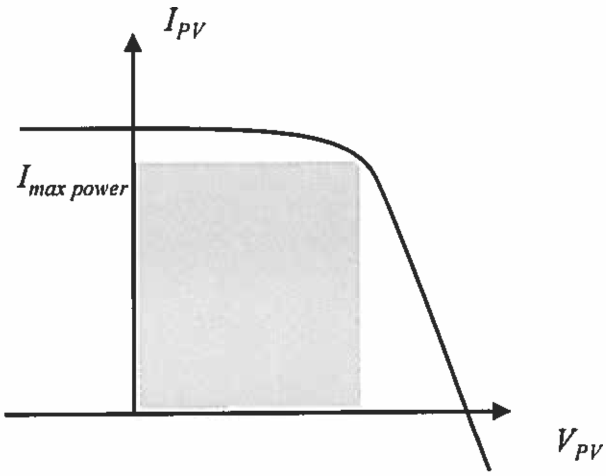


Figure 1 f).

f) An open circuit, 30 turn coil of area 0.05 m^2, initially in the x-y plane, is spun around its x axis at 100 rpm. There is a uniform magnetic field of flux density 0.2 T in the z direction. What will be the peak induced voltage in the coil? [4]

$V = \omega NBA$, $\omega = 2\pi 100/60 = 10.5$ rad/s, $V = 10.5 \times 30 \times 0.2 \times 0.05 = \underline{3.14\text{ V}}$

g) A 10 μH inductor is formed by winding a coil around a toroidal core of relative permeability $\mu_r = 800$. At a coil current of 5 A the core is saturated at a flux density of 1.0 T. By considering the energy density in the core, find its volume in cm^3. [4]

For the inductor the energy is $U = ½LI^2$, while for the core the energy is $U = ½BH \times Vol = (½B^2/\mu_r\mu_o) \times Vol$. Equating the two gives $Vol = \mu_r\mu_o\, LI^2/B^2 = \underline{0.25\text{ cm}^2}$

h) As part of a certain magnetic circuit, magnetic flux passes down the axis of a cylindrical steel rod of length 10 cm, radius 1 cm and relative permeability $\mu_r = 1200$. What is the reluctance of this part of the flux path? [4]

The reluctance $\mathcal{R} = \ell/\mu_r\mu_o A = 0.1/(1200 \times 4\pi \times 10^{-7} \times \pi(0.01)^2) = \underline{211300\text{ F}^{-1}}$

i) A certain simple DC motor in steady state has a back-EMF $e_A = 80$ V, an armature resistance $R_A = 10\ \Omega$, and an armature current $I_A = 2$ A. Calculate its efficiency η. [4]

$P_{mech} = e_A I_A = 160$ W, $P_{loss} = I_A^2 R_A = 40$ W
$\eta = P_{mech} / (P_{mech} + P_{loss}) = 160/200 = \underline{80\%}$

j) For a simple DC motor when the input voltage is suddenly increased, give two factors that may limit the rate at which its speed increases. [4]

One is the moment of inertia of the rotor. The other is the armature inductance, which limits the rate of increase of armature current.

2. SOLUTION

a) A thin, square, conductive plate of side l has a surface normal in the z direction and its two sides in the x and y direction. If the plate is given a charge Q sketch the electric field for a cross section through the centre of the plate in the xz plane and derive an expression for the magnitude of the field near the plate surface but away from the edges. [4]

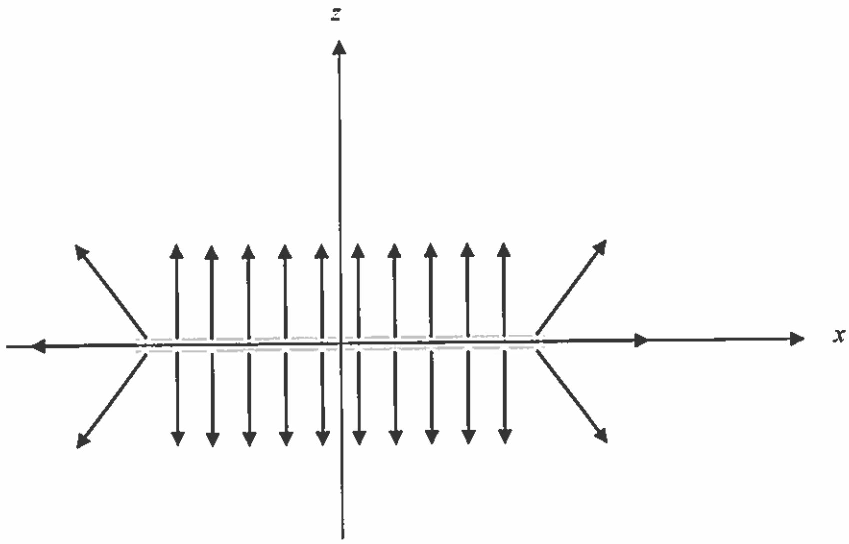


Figure 2 a).

By Gauss' Law, neglecting the flux at the edges of the plate, the electric flux is given by $D \times 2\, l^2 = Q$. Hence $E = D/\varepsilon_0 = Q/(2\varepsilon_0 l^2)$

b) A second identical plate, but with a charge $-Q$, is displaced a distance d, much smaller than l, in the negative z direction. Using superposition, or otherwise, sketch the electric field for the two plates for the same cross section as (a) and derive an expression for the magnitude of the field between the plates away from the edges. Hence derive an expression for the capacitance, C, between two parallel plates. [6]

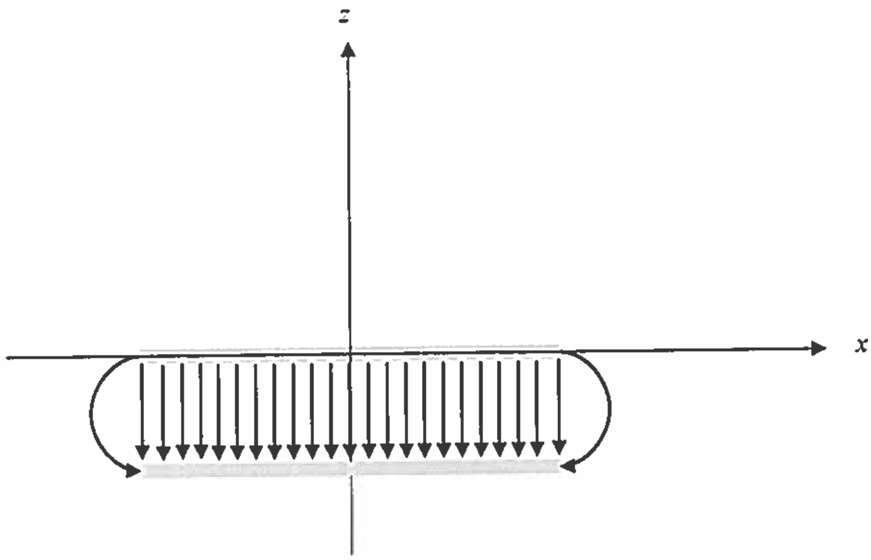


Figure 2 a).

The electric field will now be twice the value in a): $E = Q/(\varepsilon_0 l^2)$
As the field is uniform in the centre of the gap, the voltage between the plates will be $E\ d$. Hence $V = Qd/(\varepsilon_0 l^2)$ and the capacitance is given by $C = Q/V = \varepsilon_0 l^2/d$

c) Derive expressions the capacitance in (a) if the second plate is displaced a distance x (<l/2) horizontally, or z (< d) vertically away from its initial position, and hence derive expressions for dC/dx and dC/dz. [8]

For a horizontal displacement x,
$C = \varepsilon_0 l(l - x)/d$
Hence $dC/dx = -\varepsilon_0 l/d$

For a vertical displacement, z,
$C = \varepsilon_0 l^2/(d - z)$
Hence, $dC/dz = \varepsilon_0 l^2/(d - z)^2$

d) The change in the capacitance in (c) forms the basis for a displacement transducer. What are two advantages in using such a transducer to measure the horizontal rather than vertical displacement? [6]

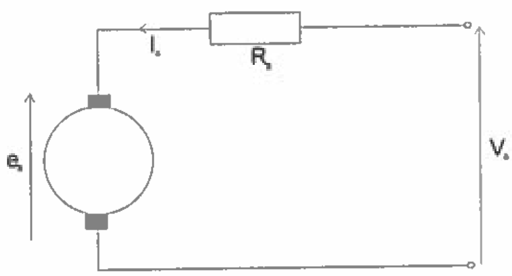
i) The response is linear, with the change in capacitance proportional to the displacement
ii) The transducer has a larger range, with the capacitance changing up to a displacement of $l/2$ for x, while the plates will touch if $z > d$.

e) Suggest a circuit diagram for such a displacement transducer which produces an output that depends on the value of a variable capacitance? [6]

A number of circuits are possible including:
i) a classic LC resonance circuit
ii) a switched circuit that charges the capacitor and then transfers the charge across a resistor
Any circuit that produces the required output, not necessarily optimised, is acceptable.

3. SOLUTION

a) Sketch and fully label an equivalent circuit for a simple DC machine operating in steady state. Below the sketch, for each of the labelled quantities give the full name, e.g. V_a = applied armature voltage. [6]



e_a = back EMF, Ra = armature resistance, Va = applied armature voltage, Ia = armature current

b) A certain DC motor has an armature with $N = 50$ turns, and an applied flux Φ of 100 mWb. The motor torque is measured at both 50 rpm and 100 rpm and found to be 12 Nm and 9 Nm respectively. Calculate the applied armature voltage V_a and the armature resistance R_a. [6]

$K = 2N/\pi = 31.8$, $K\Phi = 3.18$
$T = K\Phi V/R - (K\Phi)^2\omega/R$ (i) divide eqn for T1 by that for T2 giving
$T1/T2 = (V - K\Phi\omega_1)/(V - K\Phi\omega_2)$ (ii)
$\omega_1 = 50*2\pi/60 = 5.23$ rad/s $\omega_2 = 100*2\pi/60 = 10.46$ rad/s
manipulating (ii) gives
$V = (T2\omega_1 - T1\omega_2)K\Phi/(T2 - T1) = \underline{83.2\ V}$
Manipulating (i) gives $R = (K\Phi/T1)(V - K\Phi\,\omega_1) = \underline{17.6\ \Omega}$

c) A certain transformer can be modelled by the equivalent circuit shown in Fig. 3.1. Short circuit and open circuit tests are carried out on it, yielding the results shown in Table 3.1. Calculate the values of the equivalent circuit components R_i, X_m, R_{t1} and X_{t1}. [6]

The open cct test allows calc of the shunt components since I1=I2=0. Then the input power $Pi = V_1^2/Ri$, $Ri = V_1^2/Pi = 3892\ \Omega$
The magnitude of the combined impedance $Z_{oc} = V_1/I_{in} = \underline{2727\ \Omega}$
But this is the parallel combination of a real and an imaginary impedance, so
$Z_{oc} = RiXm/(Ri+Xm)$, giving $Xm = Z_{oc}Ri/(Ri - Z_{oc}) = \underline{3824\ \Omega}$
The open cct test also shows that the turns ratio N1:N2 = 1200:240 (not asked, but needed for (d))
The short cct test allows calculation of the series components, and we can assume the current through the shunt components is negligible (check later), so:
$Pi = I_{in}^2R_{t1}$, $R_{t1} = Pi/I_{in}^2 = \underline{7.07\ \Omega}$
The magnitude of the combined impedance $Z_{sc} = V_1/I_{in} = 8.65\ \Omega$
But this is the series combination of a real and an imaginary impedance, so
$X_{t1}^2 = Z_{sc}^2 - R_{t1}^2 = 25$, $\underline{X_{t1} = 5.0\ \Omega}$
Since both values are more than 100x less than the shunt components, the previous assumption is safe.

d) The transformer of (c) now has a real load of $Z_L = 200\ \Omega$ connected to the secondary terminals. Calculate the efficiency η of the transformer in this case. [6]

First we shift the load to the input side as $Z_L' = (1200/240)^2\, Z_L = 5\ k\Omega$
The modulus of the series load Z_s, i.e. including the series components R_{t1} and X_{t1} but not the shunt components Ri and Xm, will be negligibly affected by X_{t1} since its magnitude is much less than the real part of the load, so $Z_s = 5{,}005\ \Omega$. Then the magnitude of the current through this path, $I_1 = V_1/Z_s$. We can choose an arbitrary V_1 to find the efficiency η. Choosing $V_1 = 100$ V, $I_1 = 20.0$ mA.
If we label the power in Ri, R_{t1}, and Z_L' as Pi, P_{t1} and P_L respectively, then
$\eta = P_L/(P_L + Pi + P_{t1})$. We don't need to shift the load back to the output side to find its power. Then we have:
$Pi = V_1^2/Ri = 2.56$ W
$P_{t1} = I_1^2\,R_{t1} = 2.8$ mW; $P_L = I_1^2\,Z_L' = 2.0$ W; $\eta = 1/4.56 = \underline{21.9\%}$

e) For the transformer of (d), can the power consumed in the load be increased if a capacitor is added in series with the load Z_L? If this is done, will the efficiency η always increase? [6]

No calculation needed – since the capacitor has negative imaginary impedance it can be chosen to cancel the positive imaginary impedance of X_{t1}, which lowers the overall impedance of the combination of the load (with capacitor) and X_{t1} and R_{t1}, thus increasing the current I_1, and this increasing the power in the real load Z_L. The power in R_{t1} will increase in the same proportion, but the power in Ri will not change, so the efficiency must increase.