E4.51

1/17

Answers to Power Systems Economics 2008

Part 1

1. Answer the following questions.

 a) Explain why the spot prices of electricity are often volatile. Aid your explanation with typical supply and demand curves for electricity.

[Text book exercise]
For a full marks answer, the student needs to briefly explain that small reductions in supply cause large changes in peak prices, and also that small increases in peak demand cause large changes in peak prices.

[4]

 b) Explain why a managed spot market is needed in a competitive electricity market.

[Text book exercise]
The economic consequences of a system collapse are enormous; consequently the system balance must be maintained at almost any cost. This balance cannot be left to the market forces alone because it requires that the gaps between load and generation are filled quickly and in an organised manner. The managed spot market (run by the system operator) is the mechanism by which this delicate balance is achieved.

[2]

 c) What are the "Distribution Network Use of Service (DNUoS)" and "Transmission Network Use of Service (TNUoS)" charges in the electricity supply industry? Who sets them in the UK?

[Text book exercise]
DNUoS and TNUoS refer to the charges that the Distribution and Transmission Network Operators (respectively) are entitled to make for transporting the electrical energy across their networks. These charges are associated to maintaining and managing the network. In the UK these charges are set by Ofgem, the market Regulator.

[2]

d) Show, from first principles and using a Cournot model of competition, that the marginal cost of a certain producer can be expressed as a function of the market price, the producer's market share and the price elasticity of the demand.

[Text book exercise]

$$\max_{y_i} \{ y_i \cdot \pi(Y) - c(y_i) \}$$

Production of generator i

$$Y = y_1 + \cdots + y_n$$

is the total industry output

$$\frac{d}{dy_i} \{ y_i \cdot \pi(Y) - c(y_i) \} = 0$$

$$\pi(Y) + y_i \frac{d\pi(Y)}{dy_i} = \frac{dc(y_i)}{dy_i}$$

$$\pi(Y) \left\{ 1 + \frac{y_i}{Y} \frac{Y}{dy_i} \frac{d\pi(Y)}{\pi(Y)} \right\} = \frac{dc(y_i)}{dy_i}$$

$$\pi(Y) \left\{ 1 - \frac{s_i}{|\varepsilon(Y)|} \right\} = \frac{dc(y_i)}{dy_i}$$

Where:

$$\varepsilon = -\frac{\frac{dy}{y}}{\frac{d\pi}{\pi}} = -\frac{\pi}{y} \cdot \frac{dy}{d\pi}$$ and $$s_i = \frac{y_i}{Y}$$

[4]

e) The bids made by generators in a unit commitment-based pool market normally have several components in addition to price and quantity. List at least three of these additional components.

[Text book exercise]

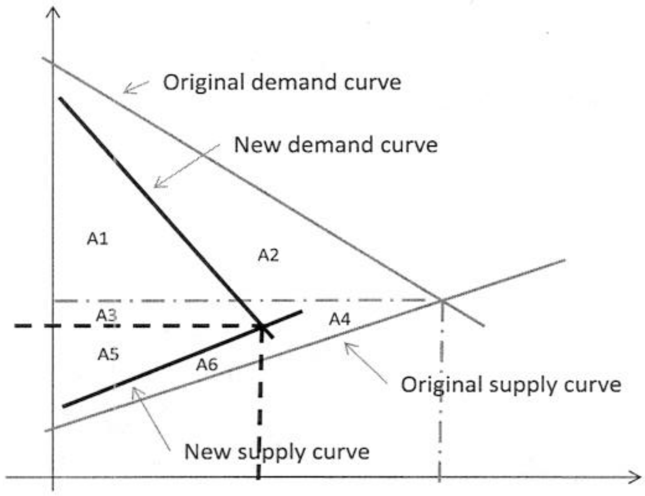
In a unit commitment-based pool, the generators submit bids with several components, some of them are as follows: piecewise linear marginal price curve, start-up price, minimum and maximum power, minimum up time, min down time.

[2]

f) First sketch typical supply-demand curves for a generic perfect competitive market. Consider next that the suppliers suffer a small increase in their production costs (which they wish to pass on to the consumers) and, at the same time, the consumers collectively lose a significant amount of interest in buying the product.

i) draw on top of your original plot the new demand-supply curves for the new market conditions.

[Direct extension of the theory seen in lectures]



[2]

ii) with the aid of the original and new supply-demand curves, explain how the original consumers' surplus is re-distributed.

[Direct extension of the theory seen in lectures]

From the figure:
Original customers' surplus: A1+A2
New customers' surplus: A1+A3

Therefore, the consumer's surplus re-distribution is as follows:
A1 remains as consumers' surplus.
A2 is original consumers' surplus lost due to the new market conditions.
A3 is new consumers' surplus gained from the suppliers' profits.

[2]

iii) with the aid of the original and new supply-demand curves, explain how the original suppliers' profits are re-distributed.

[Direct extension of the theory seen in lectures]

From the figure:
Original suppliers' profits: A3+A4+A5+A6
New suppliers' profits: A5

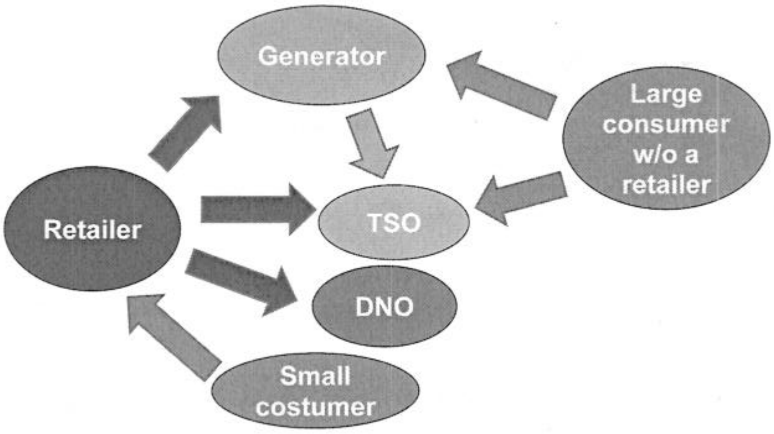
Therefore the suppliers' surplus re-distribution is as follows:
A3 is original suppliers' profits lost to new customers' surplus.
A4 is original suppliers' profits lost due to the new market conditions .
A5 remains as suppliers' profits.
A6 is original suppliers' profits lost to the new market conditions.

[2]

2. Answer the following questions.

a) Sketch a diagram showing the "cash flow" between market participants in the UK electricity market, and explain it briefly. Differentiate between small and large consumers. Neglect any deviations from contractual positions.

[Text book exercise]



In brief:
Small consumers pay their retailers. Retailers pay generators the energy they bought from them; and pay DNOs and the TSO a certain fee for using the distribution and transmission network, respectively. Generators pay the TSO for using the transmission network. Large consumers without a retailer pay their corresponding generator and the TSO.

[4]

b) Briefly explain why electricity markets may lend themselves to be imperfect competitive markets.

[Text book exercise]
For full-marks the students should cover the following points:

- **There tend to be fairly large participants able to influence the energy price by their actions.**
- **The low elasticity of the demand facilitates the abuse of market power.**
- **Locational effects could prevent the free flow of the commodity.**
- **Electricity networks are natural monopolies.**

[3]

c) Explain why producers taking part in a pool market do not have any incentives to bid at a price different to their marginal cost of production.

[Text book exercise]

If a generator bids higher than its marginal cost, it could become an extra-marginal producer and miss an opportunity to sell at a profit. If the generator bids lower than its marginal cost it could have to produce at a loss. Any of these two options bring in a certain amount of risk to the producer that would rather be avoided. On the other hand, if the producer bids at its marginal cost, it gets paid the market price if it is a marginal or infra-marginal producer. This is the preferred option for a producer.

[2]

d) Briefly explain the obstacles to increase the elasticity of the electricity demand.

[Text book exercise]
For full marks the student should explain the following issues:

- **The presence of tariffs**
- **The need for faster communication across the several participants**
- **The need for storage**
- **The fact that it may not be in the best interests of the generating companies because increasing the elasticity reduces the average price of electricity.**

[3]

e) Assume that there are only two firms (A and B) supplying a certain electricity market, and that they compete against each other. Their respective production costs are as follows:

$C_A=30*P_A$ [£/h]

$C_B=40*P_B$ [£/h]

P_A and P_B represent the energy production (in MWh) of firms A and B, respectively. The inverse demand curve for this market is given by:

$\pi = 120 - D$ [£/MWh]

Where π is the market price and D is the demand quantity.

Using a Cournot model of competition, calculate the following:

i) The values for P_A and P_B at the Nash equilibrium point.

[Numerical example of the theory seen in lectures]

The profits for firm A can be calculated as follows:
$\Omega_A(P_A, P_B) = \pi(D) \times P_A - C_A$

For firm B:
$\Omega_B(P_A, P_B) = \pi(D) \times P_B - C_B$

Because A and B are the only two firms supplying the market:
$D = P_A + P_B$

The optimality condition for A is:

$\partial \Omega_A/\partial P_A = 0$
$\partial\{(120-P_A-P_B) \times P_A - 30P_A\}/\partial P_A = 0$
$120 - 2P_A - P_B - 30 = 0$
$90 - 2P_A - P_B = 0$ …….. eq. 1

The optimality condition for B is:
$\partial \Omega_B/\partial P_B = 0$
$\partial\{(120-P_A-P_B) \times P_B - 40P_B\}/\partial P_B = 0$
$120 - P_A - 2P_B - 40 = 0$
$80 - P_A - 2P_B = 0$ …….. eq. 2

Solving eqs. 1 and 2 simultaneously we obtain:
$90 - 2P_A - P_B = 90 - 2P_A - (80-P_A)/2 = 0$ (eq. 2 into eq.1)
$50 - 1.5P_A = 0$

Therefore $P_A = 50/1.5 = 33.33$ MWh

From eq. 1 we have $P_B = 90-2(P_A) = 90-2(33.33) = 23.34$ MWh

[6]

ii) The amount of demand supplied and the market price at this equilibrium point.

$D = P_A + P_B = 56.67$ MWh
$\pi = 120 - D = 120 - 56.67 = 63.33$ £/MWh

[2]

3. Answer the following questions.

a) In the short term, the demand of electricity does not change considerably with respect to price, why does electricity have such a low elasticity?

[Direct extension of theory seen in lectures]
The two components of elasticity, cross-elasticity and self-elasticity, are rather low for the case of electricity as a commodity. Electricity has very few substitutes for most of its applications; therefore, its cross-elasticity is rather low. In terms of self-elasticity, there are several issues to consider:

- **The costs of electricity may be a small part of the cost of the total benefit that it is obtained from it (e.g. banking operations) and therefore there is no incentive to reduce demand upon an increase in price.**
- **If a customer pays a flat rate for the electricity used, this customer becomes unaware of any price fluctuations in the short term, and has no incentive to change his demand.**
- **Reducing or increasing demand usually requires of a fairly complex control system to be linked with the price of electricity, or have a person especially appointed to perform this task. Few consumers can afford any of these solutions.**

[4]

b) When is it said that a firm is exercising its market power?

[Text book exercise]
A firm exercises its market power when it reduces its output (physical withholding), or when it raises its offer price (economic withholding) in order to change the market price.

[1]

c) For the current British market structure, briefly explain what the system prices are, and what the market index price is.

[Text book exercise]
The system prices (system sell price and system buy price) reflect the expenses incurred by the system operator in maintaining the system into balance. The system sell price represents the cost of selling any energy excess; the system buy price represents the cost of buying any shortage of power. The market index price reflects the wholesale price of electricity in the short term markets.

[3]

d) Consider that the following conditions for a particular trading session in the British market:

- The Market Index Price (MIP) is 55.30 [£/MWh]
- The System Sell Price (SSP) is 42.27[£/MWh]
- The System Buy Price (SBP) is 55.30 [£/MWh]

i) Note that the MIP has the same value as the SBP, what can you infer from this coincidence in respect to the balancing operations?

[Numerical example of the theory seen in lectures]
This coincidence suggests that the market was "long"; i.e., the total volume of Balancing Services Adjustment Data (BSAD) sales and bids was larger than the total volume of BSAD purchases and offers. In these conditions the SBP takes the value of the MIP; otherwise the SBP would have been calculated as a weighted average of the net BSAD purchases and offers, which (very likely) would have produced a different value.

[4]

ii) Calculate the amount paid (or charged) to a generator that delivered 20MWh less than the amount specified in its contractual position.

[Numerical example of the theory seen in lectures]
The generator is said to be "short" and it pays the volume it didn't deliver at the SBP: (20MWh) x (55.30 £/MWh)=£1,106

[2]

iii) Calculate the amount paid (or charged) to a consumer who consumed 30MWh less than the amount specified in its contractual position.

[Numerical example of the theory seen in lectures]
The consumer is paid the energy it didn't consume at the SSP: (30MWh) x (42.27 £/MWh)= £1, 268.10

[2]

e) Explain briefly two major challenges for the integration of the Pan-European electricity market.

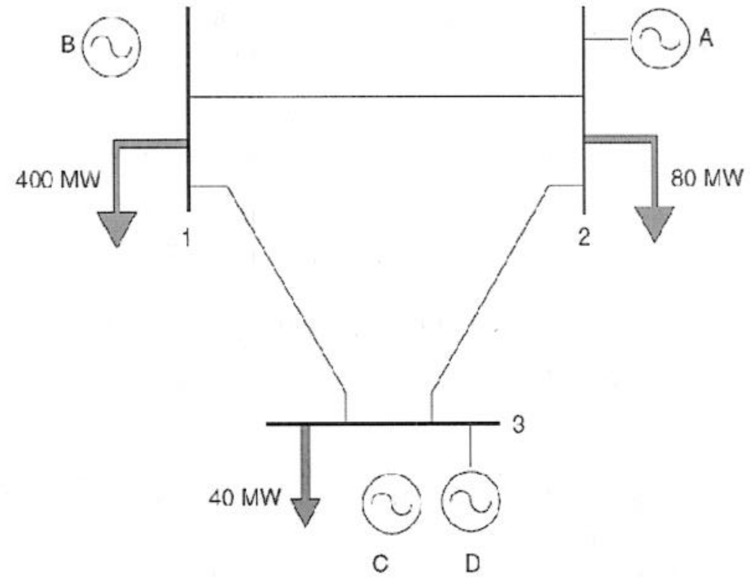
[This question summarises some discussions held during lectures]
For full marks the student should explain briefly any two of the following issues:

- **Increase the powers of the market regulators**
- **Differences in competition level and market structures across the several participating countries**
- **Increase transmission capacity**
- **Reduce circulating power flows**

[4]

Part 2.

4. The three-bus power system is shown in Figure 4, along with the parameters of the transmission system.



Generator	Capacity (MW)	Marginal Cost ($/MWh)
A	150	12
B	200	15
C	150	10
D	400	8

Branch	Reactance (p.u.)	Capacity (MW)
1-2	0.2	250
1-3	0.3	250
2-3	0.3	250

Figure 4 A three-bus system and its parameters

a) Total demand equals 520 MW, and the generators are dispatched in the order of increasing marginal cost (D, C, A, B), which results in following generator outputs for the unconstrained case:

$$P_A = 0 \text{ MW}$$
$$P_B = 0 \text{ MW}$$
$$P_C = 120 \text{ MW}$$
$$P_D = 400 \text{ MW}$$

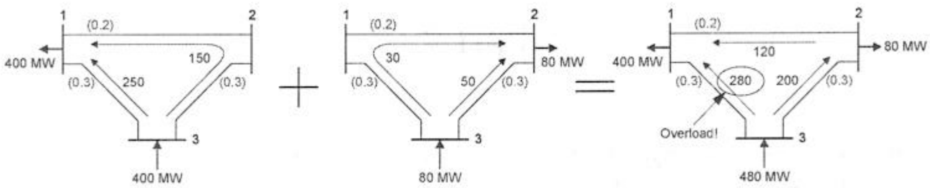
The marginal price of electricity is the same for all nodes, and is equal to the marginal generation cost of generator C, which is $\pi = 10\ \$/\text{MWh}$.

The total cost of unconstrained dispatch is therefore:

$$C_U = 120 \cdot 10 + 400 \cdot 8 = 4{,}400\ \$$$

[2]

b) The line flows can be found using the principle of superposition, as shown in the diagram below.



The net injection of 480 MW at bus 3 is decomposed into the injection of 400 MW absorbed at bus 1, and injection of 80 MW absorbed at bus 2. The line flows then follow from the network topology:

$$F_{21} = 400 \cdot \frac{0.3}{0.8} - 80 \cdot \frac{0.3}{0.8} = 120 \text{ MW}$$

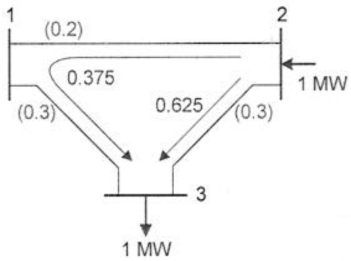
$$F_{32} = 400 \cdot \frac{0.3}{0.8} + 80 \cdot \frac{0.5}{0.8} = 200 \text{ MW}$$

$$F_{31} = 400 \cdot \frac{0.5}{0.8} + 80 \cdot \frac{0.3}{0.8} = 280 \text{ MW}$$

Obviously, the line 1-3 is overloaded by 30 MW.

[3]

c) Redispatch 1: Increase output from generator A and reduce output from generator C so that the flow in line 1-3 is reduced to 250 MW. The amount to reduce can be found by observing the impact of 1 MW injected at bus 2 and absorbed at bus 3:



The amount needed to relieve the capacity violation is 30 MW, which makes the necessary output of generator A:

$$P_A' = \frac{30}{0.375} = 80 \text{ MW}$$

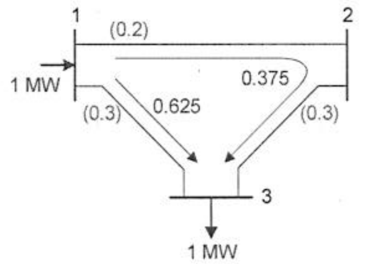
The output of generator C is reduced by the same amount, i.e. $P_C' = 40$ MW. The total cost of dispatch is now:

$$C_C' = 400 \cdot 8 + 40 \cdot 10 + 80 \cdot 12 = 4{,}560 \,\$$$

This makes the cost of constraint for this case equal to: $\Delta C' = 120$ \$.

The line flows are now: $F_{21} = 150$ MW, $F_{32} = 150$ MW, $F_{31} = 250$ MW.

Redispatch 2: Increase output from generator B and reduce the output of generator C to avoid overloading the line 1-3. The impact of this redispatch solution is depicted below:



In order to relieve the net loading of line 1-3 by 30 MW, the necessary output of generator B is:

$$P_B'' = \frac{30}{0.625} = 48 \text{ MW}$$

The output of generator C is reduced by 48 MW, i.e. $P_C'' = 72$ MW. Total generation cost in this case is:

$$C_C'' = 400 \cdot 8 + 72 \cdot 10 + 48 \cdot 15 = 4{,}640 \,\$$$

The cost of constraint is now: $\Delta C'' = 200\ \$$.

The line flows in this case are: $F_{21} = 102$ MW, $F_{32} = 182$ MW, $F_{31} = 250$ MW.

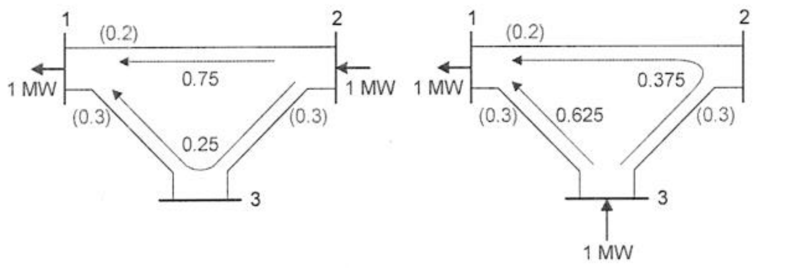
Obviously, the first redispatch (using generator A to relieve overloading) is preferable to second one, since it results in lower additional cost as compared to the unconstrained dispatch.

[7]

d) As concluded in (iii), the optimal redispatch is obtained by increasing the output of generator A, and reducing accordingly the output of generator C.

Nodal price at bus 3 is simply the marginal cost of generator C ($\pi_3 = 10$ \$/MWh), which is the marginal generator at that bus. Similarly, at bus 2 the generator A is marginal (generator C cannot increase output without violating the constraints), so its marginal cost determines the nodal price: $\pi_2 = 12$ \$/MWh.

The nodal price at bus 1 (being the cost of supplying 1 MW of additional electricity at that bus) is obtained by simultaneously increasing the output of generator A and decreasing the output of generator C by a smaller amount. The rationale behind this is that the reduction of generator C output by 1 MW offloads the line 1-3 by more than the additional flow coming from injecting 1 MW from generator A, as shown by the Figure below.



Net of the redispatch has to result in 1 MW being supplied to bus 1:

$$\Delta P_A + \Delta P_C = 1 \text{ MW}$$

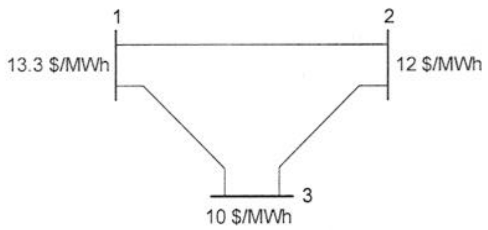
On the other hand, the loading of line 1-3 has to remain the same:

$$0.25\Delta P_A + 0.625\Delta P_C = 0$$

Solving the last two equations yields: $\Delta P_A = 1.667$ MW, $\Delta P_C = -0.667$ MW. This finally makes the marginal price at bus 1 equal to:

$$\pi_1 = 1.667 \cdot 12 - 0.667 \cdot 10 = 13.33\ \$/\text{MWh}$$

In summary, all nodal prices are shown in the following figure:



[5]

e) The congestion surplus is obtained as the difference between the amount the demand pays and the amount received by the generators, assuming all electricity is priced according to nodal marginal prices:

$$S' = (D_1\pi_1 + D_2\pi_2 + D_3\pi_3) - (P_A'\pi_2 + P_C'\pi_3 + P_D\pi_3)$$
$$= 1333.33\ \$$$

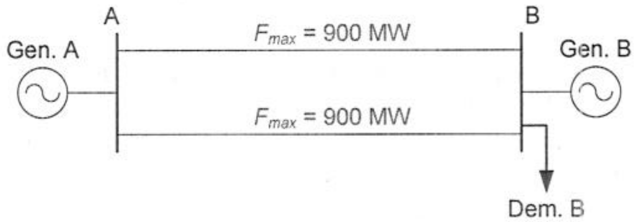
On the other hand, the sum of surpluses associated with individual lines is:

$$S'' = F_{21}(\pi_1 - \pi_2) + F_{32}(\pi_2 - \pi_3) + F_{31}(\pi_1 - \pi_3)$$
$$= 1333.33\ \$$$

Unsurprisingly, the two sums are exactly the same.

[3]

5. The transmission system operated by the Borduria Transco is shown in the figure below.



a) Marginal generation costs are as follows:

$$\pi_A = \frac{\mathrm{d}C_A(P_A)}{\mathrm{d}P_A} = 1 + 0.002P_A$$

$$\pi_B = \frac{\mathrm{d}C_B(P_B)}{\mathrm{d}P_B} = 2 + 0.004P_B$$

The optimal dispatch for the unconstrained case can be obtained by solving the following system of equations:

$$\pi_A = \pi_B, \text{i.e. } 1 + 0.002P_A = 2 + 0.004P_B$$

$$P_A + P_B = D_B$$

The flow of electricity is in direction from A to B, in the amount of P_A (since there is no load at bus A). Depending on the season, this results in following generator outputs and system marginal prices:

Winter: $P_A^W = 2{,}500\text{ MW},\ P_B^W = 1{,}000\text{ MW},\ \pi^W = \pi_A^W = \pi_B^W = 6\text{ £/MWh}$.

Summer: $P_A^S = 1{,}500\text{ MW},\ P_B^S = 500\text{ MW},\quad \pi^S = \pi_A^S = \pi_B^S = 4\text{ £/MWh}$.

[3]

b) We can assume that the transmission system is operated according to the (n–1) security criterion, i.e. the flow on the lines must not exceed the ratings even if one of the lines goes out of operation unexpectedly. This means that the total flow from bus A to bus B cannot exceed the capacity of one line, i.e. 900 MW.

In our system, this implies that the output of generator A cannot be higher than 900 MW. The remainder of the load is then covered by generators in area B ($P_B = D_B - P_A$). Due to the existence of constraints, the marginal prices at two nodes are now different, and are calculated according to generator's marginal costs:

Winter: $\mathrm{P_A^W} = 900\text{ MW},\quad \mathrm{P_B^W} = 2{,}600\text{ MW},\ \pi_\mathrm{A}^\mathrm{W} = 2.8\text{ £/MWh}$, $\pi_\mathrm{B}^\mathrm{W} = 12.4\text{ £/MWh}$.

Summer: $\mathrm{P_A^S} = 900\text{ MW},\quad \mathrm{P_B^S} = 1{,}100\text{ MW},\ \pi_\mathrm{A}^\mathrm{S} = 2.8\text{ £/MWh}$, $\pi_\mathrm{B}^\mathrm{S} = 6.4\text{ £/MWh}$.

[3]

c) Let us denote winter and summer duration with τ_w and τ_s, respectively.

i) The total annual generation costs can be obtained by multiplying seasonal hourly costs with the respective season duration:

$$\begin{aligned} C_G &= \tau_w[C_A(P_A^W) + C_B(P_B^W)] + \tau_s[C_A(P_A^S) + C_B(P_B^S)] \\ &= 2500\left(P_A^W + 0.001P_A^{W2} + 2P_B^W + 0.002P_B^{W2}\right) \\ &\qquad + 6260\left(P_A^S + 0.001P_A^{S2} + 2P_B^S + 0.002P_B^{S2}\right) \\ &= 90{,}700{,}800\text{ £/yr} \end{aligned}$$

[2]

ii) Total annual revenue received by the generators is:

$$\begin{aligned} R_G &= \tau_w(P_A^W\pi_A^W + P_B^W\pi_B^W) + \tau_s(P_A^S\pi_A^S + P_B^S\pi_B^S) \\ &= 146{,}745{,}600\text{ £/yr} \end{aligned}$$

Total payments by customers are:

$$\begin{aligned} C_D &= \tau_w D_B^W\pi_B^W + \tau_s D_B^S\pi_B^S \\ &= 188{,}628{,}000\text{ £/yr} \end{aligned}$$

[3]

iii) Total annual transmission revenue received by the Borduria Transco is:

$$\begin{aligned} R_T &= \tau_w P_A^W(\pi_B^W - \pi_A^W) + \tau_s P_A^S(\pi_B^S - \pi_A^S) \\ &= 41{,}882{,}400\text{ £/yr} \end{aligned}$$

This amount is (expectedly) equal to the difference between generator revenues and demand payments ($R_T = C_D - R_G$).

[2]

d) To see if the construction of the third line is economically justified, we need to compare the relevant construction cost with the benefits arising from having a third line available. This benefit is reflected in the reduced cost of transmission constraints. The constraint cost is calculated as the difference between the actual (redispatched) generation cost and the generation cost for the unconstrained case.

From the previous results it is easy to see that the unconstrained generation cost is equal to $C_G^0 = 64{,}740{,}000$ £/yr. This makes the constraint cost in the system with two lines equal to $\Delta_{900} = C_G(900) - C_G^0 = 25{,}960{,}800$ £/yr.

With the third line in operation, the available transmission capacity increases to 1,800 MW. This has the following effect on seasonal generator outputs and marginal costs:

<u>Winter</u>: $P_A^W = 1{,}800$ MW, $P_B^W = 1{,}700$ MW, $\pi_A^W = 4.6$ £/MWh, $\pi_B^W = 8.8$ £/MWh.

<u>Summer</u>: $P_A^S = 1{,}500$ MW, $P_B^S = 500$ MW, $\pi^S = \pi_A^S = \pi_B^S = 4$ £/MWh.

Using the same expression as above, the annual generation cost in this case is $C_G(1800) = 68{,}415{,}000$ £/yr, which means that the constraint cost is now reduced to $\Delta_{1800} = C_G(1800) - C_G^0 = 3{,}675{,}000$ £/yr. The reduction in constraint cost due to the construction of the third line is $\Delta_{900} - \Delta_{1800} = 22{,}285{,}800$ £/yr. Since this amount is significantly higher than the cost of constructing a new line, we conclude that the addition of the third line is justified.

[7]

6. The system is defined by the following figure and generation cost functions:

$$C_X(P_X) = 10P_X + 0.01P_X^2 \text{ [\$/h]}$$

$$C_Y(P_Y) = 20P_Y + 0.015P_Y^2 \text{ [\$/h]}$$

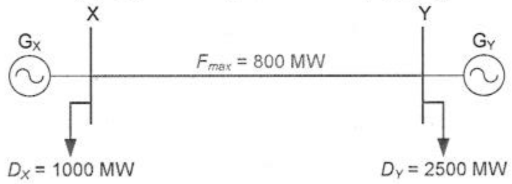


Figure 6 A simple two-bus power system

a) We first need to find the marginal generation costs at both nodes:

$$\pi_X = \frac{\mathrm{d}C_X(P_X)}{\mathrm{d}P_X} = 10 + 0.02P_X \text{ [\$/MWh]}$$

$$\pi_Y = \frac{\mathrm{d}C_Y(P_Y)}{\mathrm{d}P_Y} = 20 + 0.03P_Y \text{ [\$/MWh]}$$

Generator outputs can be found by making their marginal costs equal, while keeping the total output equal to total demand:

$$10 + 0.02P_X = 20 + 0.03P_Y$$

$$P_X + P_Y = D_X + D_Y$$

Solving this system yields: $P_X = 2{,}300$ MW, $P_Y = 1{,}200$ MW, $\pi_X = \pi_Y = 56$ \$/MWh.

[2]

b) In the constrained case we have:

$$P_X = D_X + F_{max} = 1{,}800 \text{ MW}$$
$$P_Y = D_Y - F_{max} = 1{,}700 \text{ MW}$$
$$\pi_X = 46 \text{ \$/MWh}$$
$$\pi_Y = 71 \text{ \$/MWh}$$

Generator profits can be found as the difference between the revenue from the market and the generation cost:

$$\Pi_X = P_X \cdot \pi_X - C_X(P_X) = 82{,}800 - 50{,}400 = 32{,}400 \text{ \$/h}$$
$$\Pi_Y = P_Y \cdot \pi_Y - C_Y(P_Y) = 120{,}700 - 77{,}350 = 43{,}350 \text{ \$/h}$$

[3]

c) The revenue of generator X from selling the CfD volume of $V_{CfD} = 650$ MW to the market is equal to:

$$R_{GX} = V_{CfD} \cdot \pi_X = 29{,}900 \text{ \$/h}$$

Since the generator would normally expect to effectively receive $650 \cdot 60 = 39{,}000$ \$/h from the CfD, it expects to receive 9,100 \$/h in addition to the market revenues.

The amount that the customer at Y is paying for the same amount at its local marginal price is:

$$C_{DX} = V_{CfD} \cdot \pi_Y = 46{,}150 \text{ \$/h}$$

Since the customer expects to effectively purchase electricity at 39,000 \$/h using the CfD, it would additionally require to receive 7,150 \$/h.

In summary, both the generator and the customer would expect to receive cash based on the CfD, with the total shortfall being equal to 16,250 \$/h.

[4]

d) In order to balance out the cash flows between itself and its customer, generator X purchases Financial Transmission Rights (FTRs) in the volume of 680 MW. This entitles the generator to receive additional revenue in the amount of:

$$R_{FTR} = V_{FTR} \cdot (\pi_Y - \pi_X) = 17{,}000 \text{ \$/h}$$

Out of that amount, the generator would pay the customer the expected 7,150 \$/h (a CfD obligation), making the customer's net payment equal to 39,000 \$/h, as expected from the CfD. On the other hand, the remaining 9,850 \$/h are added to the generator's market revenue. Consequently, by using FTRs the generator is able to fulfil its CfD obligation towards its customer, as well as lock in revenues, even receiving a larger amount than customer payments, due to the larger volume of FTR purchase.

The total profit of generator X is the sum of its market profit from selling to the pool (Π_X), the net cash flow from the CfD, and the revenue received based on holding the FTRs:

$$E_X = \Pi_X + V_{CfD} \cdot (\pi_{CfD} - \pi_Y) + V_{FTR} \cdot (\pi_Y - \pi_X) = 42{,}250\ \$/\mathrm{h}$$

It has to be noted that the above sum does not include the price paid by the generator for purchasing the FTRs.

[5]

e) The generator outputs and the corresponding nodal marginal prices for the case when transmission capacity is reduced to 500 MW can be found as follows:

$$P_X = D_X + F_{red} = 1{,}500\ \mathrm{MW}$$
$$P_Y = D_Y - F_{red} = 2{,}000\ \mathrm{MW}$$
$$\pi_X = 40\ \$/\mathrm{MWh}$$
$$\pi_Y = 80\ \$/\mathrm{MWh}$$

Generator profits from the market activities alone are calculated as the difference between their market revenues and generation costs:

$$\Pi_X = P_X \cdot \pi_X - C_X(P_X) = 60{,}000 - 37{,}500 = 22{,}500\ \$/\mathrm{h}$$
$$\Pi_Y = P_Y \cdot \pi_Y - C_Y(P_Y) = 160{,}000 - 100{,}000 = 60{,}000\ \$/\mathrm{h}$$

Obviously, the market profit of the more expensive generator (Y) increases, while the market profit of the cheaper generator (X) becomes lower as the available transmission capacity reduces.

The cash flows resulting from the CfD between generator X and a customer at the node Y are as follows. The amount the customer is paying for the CfD volume at its local marginal price is now equal to:

$$C_{DX} = V_{CfD} \cdot \pi_Y = 52{,}000\ \$/\mathrm{h}$$

Since the effective cost for the customer according to the CfD should be 39,000 \$/h, this means that the customer now expects to receive 13,000 \$/h from the generator X.

The revenue of generator X from selling the CfD volume at its local market is equal to:

$$R_{GX} = V_{CfD} \cdot \pi_X = 26{,}000\ \$/\mathrm{h}$$

The generator's revenue is also 13,000 \$/h short of its CfD contractual sum. This is compensated by the revenue from the FTR, which amounts to:

$$R_{FTR} = V_{FTR} \cdot (\pi_Y - \pi_X) = 27{,}200\ \$/\mathrm{h}$$

This revenue can then be used to cover both customer's and generator's shortfalls (13,000 \$/h each), and bring their net revenue/payment into agreement with the provisions of the CfD. The rest of the sum (1,200 \$/h) is an extra profit of generator X.

Total profit of generator X (including all market and financial activities) is:

$$E_X = \Pi_X + V_{CfD} \cdot (\pi_{CfD} - \pi_Y) + V_{FTR} \cdot (\pi_Y - \pi_X) = 36{,}700\ \$/\mathrm{h}$$

Although the profit of generator X is smaller than in the case where transmission capacity was 800 MW (and that is because of the fraction of its output not covered by CfD and FTRs), it is still significantly higher than without using the CfD and FTRs to hedge against the unfavourable movements in prices and unpredictable events in the transmission network.

It is also worth noting that the revenue collected by the TSO due to congestion is $500 \cdot 40 = 20{,}000\ \$/\mathrm{h}$, while its obligation towards generator X (being a holder of 680 MW of FTRs) amount to 27,200 $/h, meaning that TSO is incurring a loss of 7,200 $/h because it sold more FTRs (680 MW) than its available capacity (500 MW).

[6]