EE4-55

Solution 1:

a) Quantum confinement in the spherical nanocrystal will raise the bottom of the conduction band and lower the top of the valance band by the corresponding confinement energy $E_k = [\pi^2\hbar^2]/[2m_{eff}m_0(d/2)^2]$, where $\hbar = h/2\pi$ is reduced Planck's constant, d is the diameter, m_0 is the electron rest mass, and m_{eff} is the relevant effective mass. This increases the band gap E_g. Confinement of electron within a small volume, from Heisenberg's Uncertainty Principle, will remove the requirement for momentum conservation in the e-h recombination process.

The energy E_p of the emitted photon is then given by:

$$E_p = \hbar\omega_p = E_g + [\pi^2\hbar^2]/[2m_e m_0(d/2)^2] + [\pi^2\hbar^2]/[2m_h m_0(d/2)^2]$$
$$= 1.1 + 0.06 + 0.11 = 1.27 \text{ eV} = 2.03 \times 10^{-19} \text{ J}$$

[Marks: 3]

The wavelength of the emitted photon is then given by:

$\lambda = c/f = 2\pi c/\omega_p \qquad 2\pi c\hbar/E_p = 980$ nm.

[Marks: 2]

If the answer uses the confinement energy $E = [\pi^2\hbar^2]/[2m_{eff}m_0 d^2]$ in 1-D, then 1 mark may be deducted.

b)

i) The wave vectors in regions I, II, and III are as follows:

$$k_I = \sqrt{\frac{2m}{\hbar^2}(E-V_2)}\ ,\ k_{II} = \sqrt{\frac{2m}{\hbar^2}E}\ \ ,\ \ k_{III} = \sqrt{\frac{2m}{\hbar^2}(E-V_1)}$$

[Marks: 1+1+1]

ii) Figure A1.2a shows the ground state $\psi_{1,\text{sngl}}$ in a single quantum well of width L, where a central 'cos(kx)' section matches to exponential tails within the barriers on either side. In a double quantum well (Fig. A1.2b and Fig. A1.2c), if the ground states in the two wells overlap, then two possibilities exist, both of which satisfy the wave function matching conditions $\psi_1 = \psi_2$ and $\psi_1' = \psi_2'$ at a potential boundary. These give the symmetric wave function ψ_1 (Fig. A1.2b), and the anti-symmetric wave function ψ_2 (Fig. A1.2c). If the central potential barrier width D tends to zero, then ψ_1 tends to the ground state of a single potential well of width $2L$ (the 'peaks' of ψ_1 move together) and ψ_2 tends to the first excited state of a single potential well of width $2L$ (the 'zero' remains at the centre of the new potential well). Therefore, ψ_1 is the ground state of the double potential well, with energy E_1, and ψ_2 is associated with the first excited state of the double potential well, with energy E_2.

Fig. A1.2a

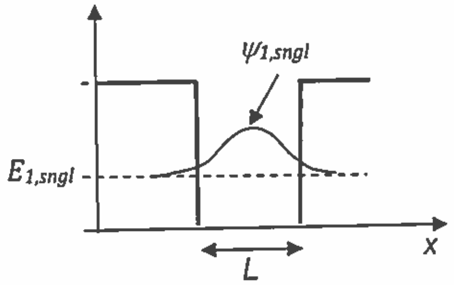


Fig. A1.2b

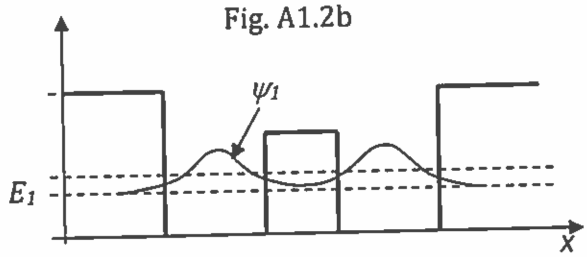
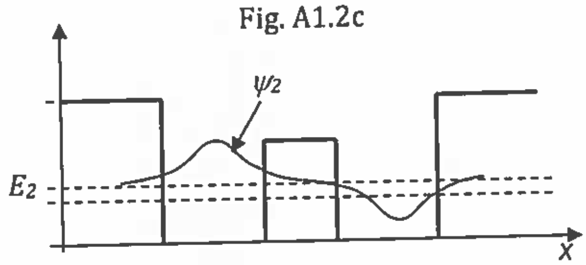


Fig. A1.2c



[Marks: 3]

c) Assume the wave-function Ψ is represented by a travelling wave:

$$\Psi = A\exp i(kx - \omega t) \quad , \quad \text{where } k = 2\pi/\lambda$$

For the free particle in a region with potential energy V:

Total energy = kinetic energy + potential energy

$$\Rightarrow \qquad E = \frac{p^2}{2m} + V$$

where p is the momentum. Furthermore, $p = \hbar k$.

[Marks: 2]

Using the wave-function Ψ, we see that:

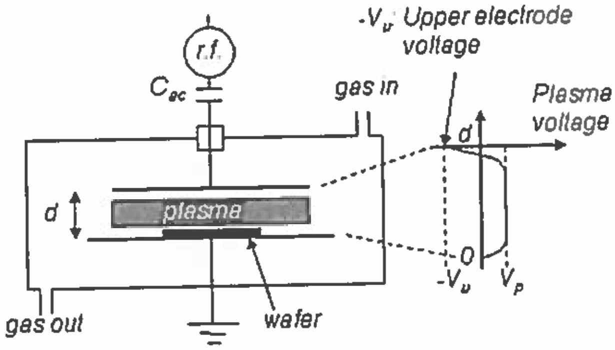
$$\frac{\partial \Psi}{\partial t} = -i\omega\Psi \quad \text{and} \quad \frac{\partial^2 \Psi}{\partial x^2} = -k^2\Psi$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

where $\Psi\Psi^*$ represents the probability density.

[Marks: 2]

d)



The system uses a glow discharge, i.e. weakly-ionised plasma (>90% neutral particles). An rf. electric field E_{rf} is used to generate the plasma, as follows. Free electrons are accelerated by E_{rf}. These collide with gas atoms, generating more free electrons and ions and increasing the plasma density. However, reduction in the number of electrons and ions due to collision with the electrodes lowers the plasma density. The two processes then lead to an equilibrium plasma density. The plasma gas is chosen to generate reactive species, e.g. F or Cl, and these species etch the wafer. Both mechanical (ion sputtering) and chemical etching occurs in the etching process. The ion sputtering is caused by a reduced potential at either plate compared to the plasma potential, associated with charge build-up on the plates. Positive ions then bombard the wafer surface, leading to mechanical, anisotropic etching.

[Marks: 5]

Solution 1:

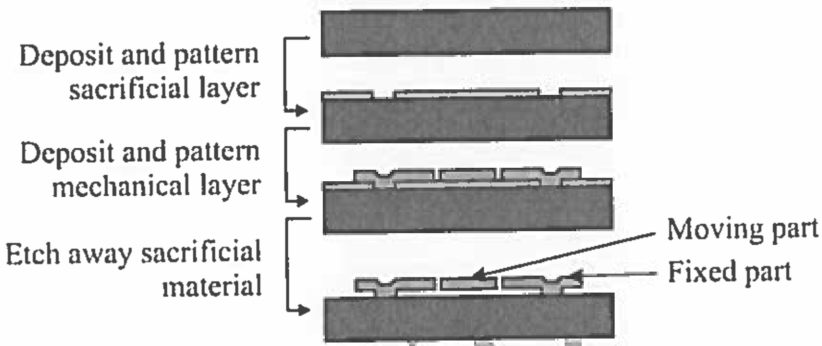
e) The main materials are:

- a mechanical material – usually polysilicon
- a sacrificial material – usually (doped) silicon dioxide

The main processes are:

- LPCVD (low pressure chemical vapour deposition) for depositing the mechanical and sacrificial layers
- Lithography and RIE (reactive ion etching) for patterning the deposited layers
- Sacrificial etch – usually either HF wet etch or HF vapour etch – to remove sacrificial material. [3]

Process flow:



[2]

f) The mass of the moving element is $m = \rho A h$, with $\rho = 2330$ kg/m^3, $A = 1\times10^{-6}$ m^2, $h = 1\times10^{-5}$ m. This gives $m = 2.33\times10^{-8}$ kg. The resonant frequency is then $f_0 = (1/2\pi)\sqrt{(k/m)} = 2.33$ kHz.

The damping constant associated with Couette flow beneath the mass is $c = \mu A/d$ where d is the gap between mass and substrate. The resonator Q is then given by $Q = m\omega_0/c = m\omega_0 d/\mu A$. With $\omega_0 = 14.65\times10^3$ rad/s, $d = 2$ µm, $\mu = 1.8\times10^{-5}$ Ns/m^2, and other values as above, this gives $Q = 37.9$. [3]

Since m scales as L^3, and k scales as L^1, we know that f_0 will scales as L^{-1}. It follows that Q will scale as L^1. The frequency will therefore double, and the Q will be halved, when the device is scaled down by a factor of 2. [2]

g) These equations are statements of Newton's second law ("f = ma") as applied to the gyroscope proof mass. The gyroscope is based on a two-axis resonator, and the equations are resolved along the principal axes which are uncoupled in the absence of any input rotation. The variables v_x and v_y are the proof displacements along x and y.

Considering either one of the equations: the first term is the inertial (i.e. "ma") term; the second term represents the damping force; the third term represents the restoring force due to the spring suspension; the fourth term represents cross-coupling between the axes which arises from the Coriolis force when there is an input rotation about the z-axis. It is this term that is key to the gyroscope action. The final term on the right is the actuator driving force. [3]

Assuming the x-axis is driven at the resonant frequency and at constant amplitude A, the cross-coupling will cause excitation at the resonant frequency on the y-axis. In steady state the inertial and spring force terms in the y-axis equation will cancel out (resonance), and since $F_y = 0$ the y-axis equation will reduce to $c_y \dot{v}_y + 2m\Omega\dot{v}_x = 0$. From this it follows that the amplitude on the sensed axis is $B = -2m\Omega A / c$, and the ratio of amplitudes is $B / A = -2m\Omega / c$. [2]

h) The main actuation mechanisms are: electrostatic (ES), electrothermal (ET), piezoelectric (PE) and electromagnetic (EM).

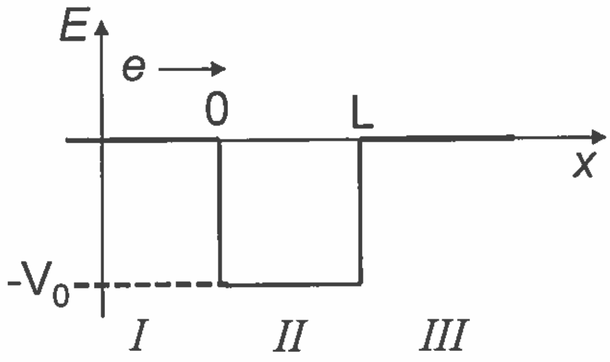
ES and ET actuators are easiest to implement because they can be fabricated in a single mechanical layer. EM actuators are most difficult because they require magnetic materials and tend to be geometrically more complex if they include an efficient magnetic circuit.

ES, PE and EM actuators are all fast. ET actuators are slower because they are limited by the thermal time constant of the structure.

ES and EM are limited to relatively low force levels by breakdown and Joule heating respectively. PE and ET actuators can generally develop larger forces. [5]

Solution 2:

From a sketch of the given potential well, electrons can only approach the well for $E > 0$.



a) Wave vectors in regions I, II and II are as follows:

Region I: $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$ Region II: $k_1 = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$

Region III: $k_3 = k_1 = \sqrt{\frac{2mE}{\hbar^2}}$

[Marks: 2+2+2]

b) Wave functions in regions I, II and II are as follows:

Region I: $\psi_1 = e^{ik_1x} + re^{-ik_1x}$ Region II: $\psi_2 = Ae^{ik_2x} + Be^{-ik_2x}$

Region III: $\psi_3 = \tilde{t}e^{ik_1x}$

Here, the incident wave is assumed to have an amplitude of 1 to simplify the calculation, and in region III, there is no reflected wave, as electrons are only incident from the left.

[Marks: 2+2+2]

c) We now apply boundary conditions:

At $x = 0$, $\psi_1\big|_{x=0} = \psi_2\big|_{x=0}$ and $\psi_1'\big|_{x=0} = \psi_2'\big|_{x=0}$

$\Rightarrow 1 + r = A + B$ (Eq. 1)

$k_1 - k_1 r = k_2(A - B)$ (Eq. 2)

And, at $x = L$, $\psi_2\big|_{x=L} = \psi_3\big|_{x=L}$ and $\psi_2'\big|_{x=L} = \psi_3'\big|_{x=L}$

$\Rightarrow Ae^{ik_2L} + Be^{-ik_2L} = t = \tilde{t}e^{ik_1L}$ (Eq. 3)

$$k_2\left(Ae^{ik_2L} - Be^{-ik_2L}\right) = k_1 t \qquad \text{(Eq. 4)}$$

[Marks: 2]

Equations (1) – (4) may be solved simultaneously:

$$(1) + (2)/k_1 \Rightarrow 2 = A\left(1 + \frac{k_2}{k_1}\right) + B\left(1 - \frac{k_2}{k_1}\right) \qquad \text{(Eq. 5)}$$

[Marks: 2]

and

$$(3) + (4)/k_2 \Rightarrow 2Ae^{ik_2L} = t\left(1 + \frac{k_1}{k_2}\right)$$

$$\Rightarrow A = \left(1 + \frac{k_1}{k_2}\right)\frac{t}{2}e^{-ik_2L} \qquad \text{(Eq. 6)}$$

[Marks: 2]

Substituting A in Eq. (3) gives:

$$B = \left(1 + \frac{k_1}{k_2}\right)\frac{t}{2}e^{+ik_2L} \qquad \text{(Eq. 7)}$$

[Marks: 2]

Substituting A and B in Eq. (5) then gives the amplitude transmission coefficient t:

$$t = \frac{4k_1k_2}{\left(k_1 + k_2\right)^2 e^{-ik_2L} - \left(k_1 - k_2\right)^2 e^{ik_2L}} \qquad \text{(Eq. 8)}$$

[Marks: 2]

d) The reflection coefficient $R = 0$ when the transmission coefficient $T = |t|^2 = 1$, as $R = 1 - T$. For this condition, using Eq. 8:

$$\Rightarrow \left(\left(k_1 + k_2\right)^2 e^{-ik_2L} - \left(k_1 - k_2\right)^2 e^{ik_2L}\right)^2 = \left(4k_1k_2\right)^2$$

[Marks: 2]

$$\Rightarrow \left(k_1^2 + k_2^2\right)\left(e^{-ik_2L} - e^{ik_2L}\right) + 2k_1k_2\left(e^{-ik_2L} + e^{ik_2L}\right) = \pm 4k_1k_2$$

$$\Rightarrow -2i\left(k_1^2 + k_2^2\right)\sin k_2L + 4k_1k_2\cos k_2L = \pm 4k_1k_2$$

[Marks: 4]

Inspecting this equation, the solution $k_2L = n\pi$ gives the condition for $R = 0$.

[Marks: 2]

Solution 3:

a) The energy band diagrams along the line XX' are shown in Fig. A3.1 below. The 'flat-band' condition is also shown for completeness (this is not asked in the question):

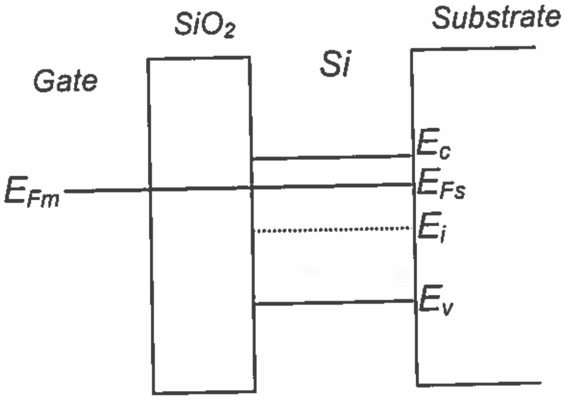


Fig. A3.1a. 'Flat-band'

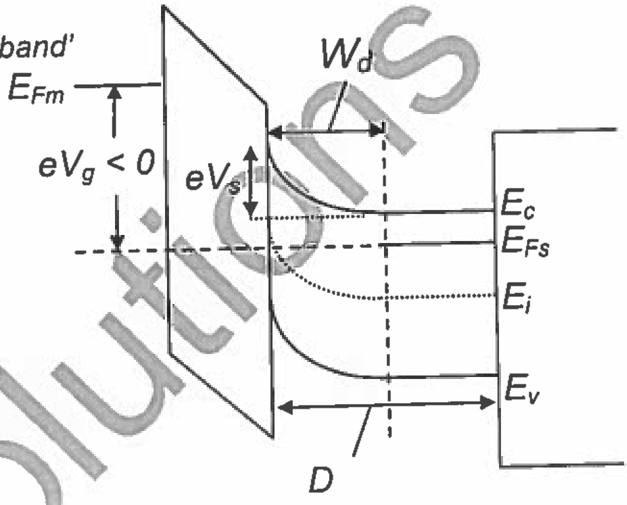


Fig. A3.1b. Depletion

Here, give 1 mark for showing the different band-gaps in SiO_2 and Si, 1 mark for showing the effect of V_g on the Fermi energy, 2 marks for the correct band bending and 1 mark for marking the depletion region.

[Marks: 5]

b) The charge density $\rho(x)$ vs. x, along the line XX', is shown in Fig. A3.2 below:

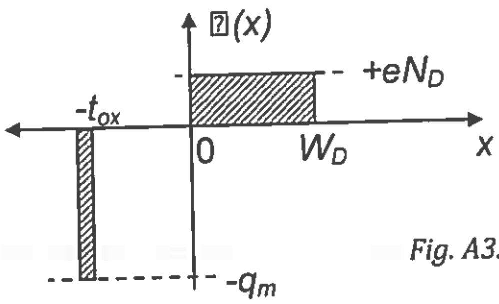


Fig. A3.2

Here, give 2 marks each for the correct charge profiles in the gate and Si regions, and 1 mark for locating these correctly.

[Marks: 5]

c) Poisson's equation, in the Si, is as follows:

$$-\frac{\partial^2 V}{\partial x^2} = \frac{\partial F}{\partial x} = \frac{\rho}{\varepsilon_{Si}\varepsilon_0} = \frac{eN_D}{\varepsilon_{Si}\varepsilon_0}$$

$\therefore$ Solving for electric field F:

$$F = \int \frac{eN_D}{\varepsilon_{Si}\varepsilon_0} dx = \frac{eN_D x}{\varepsilon_{Si}\varepsilon_0} + c$$

[Marks: 2]

Boundary condition $F = 0$ at $x = W_D \Rightarrow$

$$0 = \frac{eN_D W_D}{\varepsilon_{Si}\varepsilon_0} + c \Rightarrow c = -\frac{eN_D W_D}{\varepsilon_{Si}\varepsilon_0}$$

$$\Rightarrow F = -\frac{eN_D W_D}{\varepsilon_{Si}\varepsilon_0}\left(1 - \frac{x}{W_D}\right)$$

[Marks: 4]

Integrating again, for potential $V(x)$:

$$V = -\int F dx = +\int \frac{eN_D W_D}{\varepsilon_{Si}\varepsilon_0}\left(1 - \frac{x}{W_D}\right) dx = \frac{eN_D W_D}{\varepsilon_{Si}\varepsilon_0}\left(x - \frac{x^2}{2W_d}\right) + c$$

[Marks: 2]

Boundary condition $V = 0$ at $x = W_D \Rightarrow$

$$c = -\frac{eN_D W_D^2}{2\varepsilon_{Si}\varepsilon_0}$$

$$\Rightarrow V = -\frac{eN_D W_D^2}{2\varepsilon_{Si}\varepsilon_0}\left(-\frac{2x}{W_D} + \left(\frac{x}{W_D}\right)^2 + 1\right)$$

$$\Rightarrow V = -\frac{eN_D W_D^2}{2\varepsilon_{Si}\varepsilon_0}\left(1 - \frac{x}{W_D}\right)^2$$

[Marks: 4]

$\therefore$ At $x = 0$, the surface potential V_s is:

$$V_s = -\frac{eN_D W_D^2}{2\varepsilon_{Si}\varepsilon_0}$$

[Marks: 2]

d) The threshold voltage V_{th} corresponds to the voltage where the Si layer is completely depleted, i.e. $W_D = D$. We then have:

$$V_{th} = V_{ox} + V_s = -\frac{eN_D W_D}{C_{ox}} + V_s$$

Give 4 marks for seeing this, and writing the correct expression.

[Marks: 4]

Substituting for V_s from part (c), and using $W_D = D$ gives V_{th}:

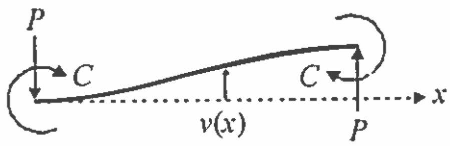
$$V_{th} = -\frac{eN_D D}{C_{ox}} - \frac{eN_D D^2}{2\varepsilon_{Si}\varepsilon_0}$$

Here, gate capacitance $C_{ox} = \varepsilon_0 \varepsilon_{ox}/t_{ox}$.

[Marks: 2]

Solution 4:

a) We need to solve the bending equation for this problem:



The bending moment a distance x along the beam is $M = P(L-x) - C$, and we know that the couple C must satisfy $2C = PL$ for equilibrium, so the bending equation is:

$$M = P(L/2 - x) = EIv''$$

Integrating twice, and applying the boundary conditions $v(0) = v'(0) = 0$, the deflection profile is obtained as:

$$\mathrm{v(x) = P(Lx^2/4 - x^3/6)/EI}$$

The end deflection is therefore $v(L) = PL^3/12EI$, and the transverse stiffness is $k_T = \frac{P}{v(L)} = \frac{12EI}{L^3} = \frac{w^3hE}{L^3}$, where we have used $I = \frac{w^3h}{12}$. [6]

From Hooke's law, an axial stress of F/wh will lead to an axial strain $\delta L/L = F/(whE)$. The axial stiffness is therefore $k_A = \frac{F}{\delta L} = \frac{whE}{L}$. [3]

From our expressions for k_T and k_A we can see that $\frac{k_T}{k_A} = \frac{w^3hE}{L^3}\frac{L}{whE} = \frac{w^2}{L^2}$ as required. [1]

b) The axial and transverse displacements u_A and u_T arise from the components of the load P that act parallel to and normal to the beam respectively. We can therefore write:

$$k_A u_A = P\cos\theta \quad ; \quad k_T u_T = P\sin\theta \qquad \ldots.(1)$$ [2]

From the sketch on the right of Figure 4.1, we can see that the geometrical relationships between Δx, Δy, u_A and u_T are:

$$\Delta x/2 = u_T\sin\theta + u_A\cos\theta \quad ; \quad \Delta y = u_T\cos\theta - u_A\sin\theta \quad \ldots.(2)$$ [3]

Taking the ratio of equations (2), we obtain:

$$m = \frac{\Delta y}{\Delta x} = \frac{u_T\cos\theta - u_A\sin\theta}{2(u_T\sin\theta + u_A\cos\theta)} = \frac{1-(u_A/u_T)\tan\theta}{2[\tan\theta + (u_A/u_T)]}$$

and from equations (1) we can see that $u_A/u_T = \alpha\cot\theta$. Using this relation, and the given relation $u = \tan\theta$, the expression for m becomes:

$$m = \frac{1-\alpha}{2(u+\alpha/u)}$$ as required. [5]

c) i) The v-beam has $l = 1.5$ mm, $w = 20$ µm and $\theta = 1.5°$, so we have $u = 0.0262$ and $\alpha = 1/5625 = 0.000178$. Substituting these values gives $m = 15.16$.

The displacement Δx of the mass under a static acceleration a is given by $\Delta x = a / \omega_0^2$ where ω_0 is the (radian) resonant frequency. For the accelerometer in question, $\omega_0 = 2\pi \times 12000 = 75398$ rad/s. At the minimum detectable acceleration, $\Delta x = 1/15.16 = 0.066$ nm. Substituting for Δx and ω_0 we obtain $a = 0.375$ m/s^2 = 0.038 g. [6]

c) ii) The sensitivity could be increased without using mechanical amplification by lowering the resonant frequency, and hence the bandwidth of the device. This could be done either by increasing the mass or reducing the suspension stiffness. The advantage of using mechanical amplification is that it allows the sensitivity to be increased without loss of bandwidth. [4]

Solution 5:

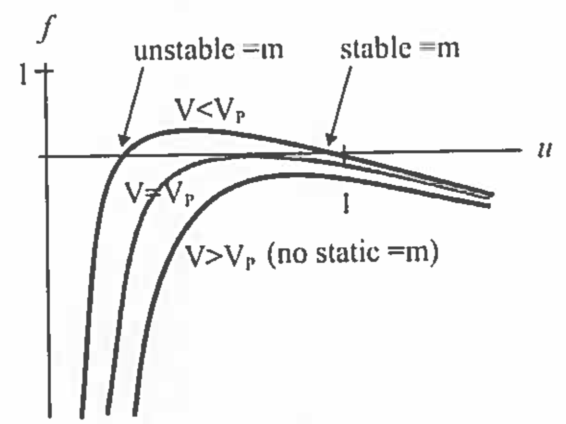
a) i) The electrostatic force on the moveable plate is $F_e = -A\varepsilon_0 E^2/2$ where $E = V/g$ is the electric field in the gap (this can be quoted). The only other force acting on the moveable plate is the spring force $F_k = k(g_0 - g)$, so the total force is:

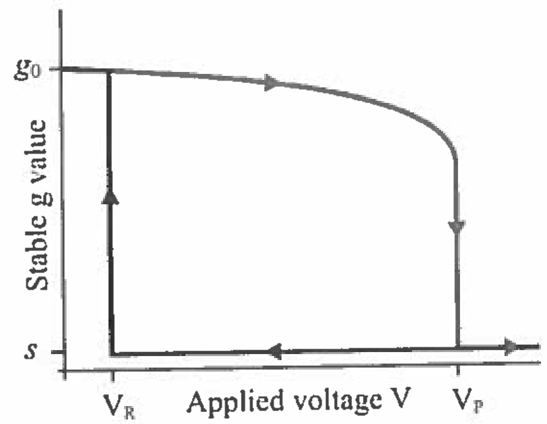
$$F = F_e + F_k = k(g_0 - g) - \frac{A\varepsilon_0 V^2}{2g^2}$$

Dividing through by kg_0, and using the substitutions $a = A\varepsilon_0 V^2/2kg_0^3$ and $u = g/g_0$, the above becomes:

$$\frac{F}{kg_0} = f = 1 - u - \frac{a}{u^2} \quad \text{as required.} \qquad (1) \qquad [6]$$

a) ii)





[4]

The LH graph shows the variation of force with gap for different values of V (and hence a). When $V < V_P$ there are two equilibrium points (i.e. points where $f = 0$). One is stable, and the other is unstable, but the moveable plate naturally settles at the stable one (and never reaches the unstable one) if the applied voltage is increased from zero. For $V > V_P$ there is no equilibrium point, and the total force is always negative so the plate snaps down. Once this has occurred, the voltage must be reduced to the point where the electrostatic force at $g = s$ falls below the spring force. This results in hysteretic behaviour as shown in RH graph. [4]

b) The conditions at the point of snap-down are $f = 0$ and $\partial f/\partial u = 0$. Differentiating the force equation gives:

$$\frac{\partial f}{\partial u} = -1 + \frac{2a}{u^3}$$

So, when $\partial f/\partial u = 0$ we have $a = u^3/2$. Substituting this value of a into equation (1), and imposing the condition $f = 0$, we find that $u = 2/3$ at the point of snap-down. It follows that $a = 4/27$ at the point of snap-down, and so the snap-down voltage is given by:

$$\frac{4}{27} = \frac{A\varepsilon_0 V_P^2}{2kg_0^3} \quad \Rightarrow \quad V_P = \sqrt{\frac{8kg_0^3}{27A\varepsilon_0}}$$ [4]

c) When the device is placed in series with the capacitor C, only a fraction of the drive voltage appears across the actuator, as determined by the potential divider rule. The electrostatic force becomes:

$$F_e = -\frac{A\varepsilon_0}{2g^2}\frac{V^2C^2}{(C + C_0/u)^2} = -\frac{A\varepsilon_0 V^2}{2(u+b)^2 g_0^2}$$

where we have used the fact that the actuator capacitance at an arbitrary position is C_0/u.

The total force is obtained as $F = F_e + F_k$, with F_k as in part a), and dividing through by kg_0 gives:

$$f = \frac{F}{kg_0} = 1 - u - \frac{a}{(u+b)^2} \quad \text{as required.}$$ [4]

The 'b' in the denominator of the last term reduces the electrostatic force. This raises curve $f(u)$ and shifts the stationary point to the left.

We look for a value of b that will place the snap-down point at $g = s$, i.e. when the moveable plate is at its lowest possible position. If such a value of b exists, and is applied, then snap-down will never actually occur (since for lower applied voltages the force curve will always be higher and will have a stable equilibrium point, and by the time the snap-down voltage is reached the moveable plate will already be touching the spacers).

Applying the conditions $f = 0$ and $\partial f/\partial u = 0$ gives:

$$1 - u - \frac{a}{(u+b)^2} = 0 \quad \text{and} \quad -1 + \frac{2a}{(u+b)^3} = 0 \qquad (2)$$

Combining these relations we obtain the condition:

$$1 - 3u/2 - b/2 = 0$$

For this to apply at $g = s$, we require that $b = (2 - 3s/g_0)$. Larger values of b will also give stable behaviour (since increasing b always raises the force curve). The condition for stable behaviour is therefore:

$$b = \frac{C_0}{C} \geq (2 - 3s/g_0) \quad \text{or} \quad C \leq \frac{C_0}{(2 - 3s/g_0)}$$ [4]

d) To find the pull-down voltage in the limiting case, we set $b = (2 - 3s/g_0)$ and $u = s/g_0$ in either of the equations (2) to find the value of a. This gives $a = 4(1 - s/g_0)^3$. The corresponding voltage is:

$$V = \sqrt{\frac{8k(g_0 - s)^3}{A\varepsilon_0}} \approx 3\sqrt{3}V_P \quad \text{if } s \ll g_0$$ [4]

