E4.38
C3.2

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

MODELLING AND CONTROL IN POWER ENGINEERING

Tuesday, 3 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

Please use separate answer books for Sections A and B.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	T.C. Green, B.C. Pal
	Second Marker(s) :	B.C. Pal, T.C. Green

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Section A

1

(a) Describe the steps needed to form a linear state-space model of a switch-mode power supply. [5]

(b) Describe briefly how one obtains the steady-state relationship between output voltage, V_O, and input voltage, V_I, in terms of duty-cycle, Δ, and also the small-signal transfer function between v_O and δ [5]

(c) Figure 1.1 shows the circuit of a Ćuk switch-mode power supply (figure 1.1a) and the current paths that exist when the mosfet is on (figure 1.1b) and when the MOSFET is off (figure 1,1c). Using equation (1.1), as a basis, determine the state-space averaged model of this circuit. You may ignore perturbations in the input voltage, V_I and the resistance of capacitors but should include the series resistance of the inductors. [10]

$$
\begin{aligned}
\dot{\tilde{\mathbf{x}}} &\cong \left(\Delta \mathbf{A}_{\mathrm{On}} + (1-\Delta)\mathbf{A}_{\mathrm{Off}}\right)\tilde{\mathbf{x}} \\
&\quad + \left((\mathbf{A}_{\mathrm{On}} - \mathbf{A}_{\mathrm{Off}})\mathbf{X} + (\mathbf{B}_{\mathrm{On}} - \mathbf{B}_{\mathrm{Off}})\mathbf{U}\right)\tilde{\delta}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\mathbf{y}} &\cong \left(\Delta \mathbf{C}_{\mathrm{On}} + (1-\Delta)\mathbf{C}_{\mathrm{Off}}\right)\tilde{\mathbf{x}} \\
&\quad + \left((\mathbf{C}_{\mathrm{On}} - \mathbf{C}_{\mathrm{Off}})\mathbf{X} + (\mathbf{D}_{\mathrm{On}} - \mathbf{D}_{\mathrm{Off}})\mathbf{U}\right)\tilde{\delta}
\end{aligned}
$$

Equation (1.1)

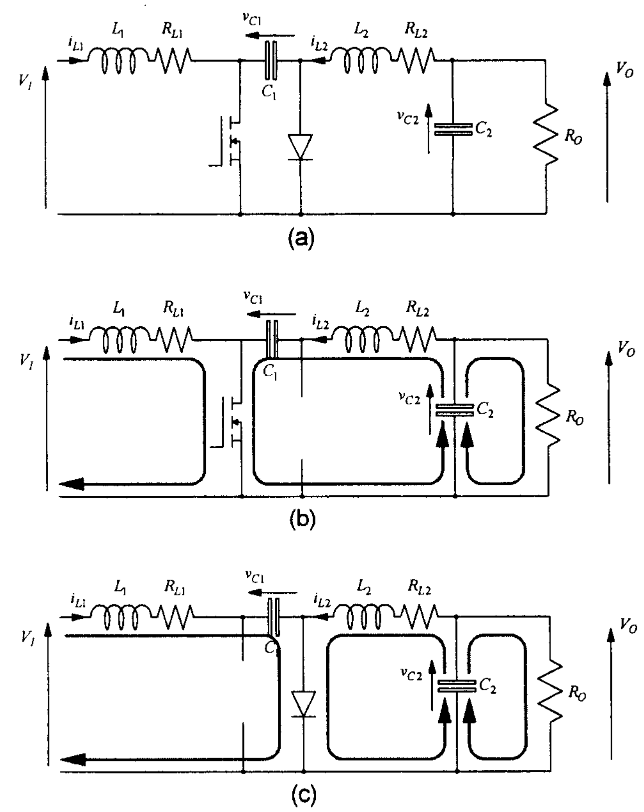


Figure 1.1

2.

The voltage equation for a dq model of an induction machine with the rotor leakage inductance referred to the stator-side and the rotor short-circuit is:

$$[v'_{DQ}] = [R'_{DQ}][i'_{DQ}] + [X'_{DQ}][i'_{DQ}] + [L'_{DQ}]\frac{d}{dt}[i'_{DQ}]$$

where:

$$[v'_{DQ}] = \begin{bmatrix} v_{SD} \\ v_{SQ} \\ 0 \\ 0 \end{bmatrix} \qquad [i_{DQ}] = \begin{bmatrix} i_{SD} \\ i_{SQ} \\ i'_{RD} \\ i'_{RQ} \end{bmatrix}$$

$$[R'_{DQ}] = \begin{bmatrix} R_S & 0 & 0 & 0 \\ 0 & R_S & 0 & 0 \\ 0 & 0 & R_R' & 0 \\ 0 & 0 & 0 & R_R' \end{bmatrix}$$

$$[X'_{DQ}] = \begin{bmatrix} 0 & -\omega L_S & 0 & -\omega M' \\ \omega L_S & 0 & \omega M' & 0 \\ 0 & -P\omega_{slip} M' & 0 & -P\omega_{slip} L'_R \\ P\omega_{slip} M' & 0 & P\omega_{slip} L'_R & 0 \end{bmatrix}$$

$$[L_{DQ}] = \begin{bmatrix} L_S & 0 & M' & 0 \\ 0 & L_S & 0 & M' \\ M' & 0 & L'_R & 0 \\ 0 & M' & 0 & L'_R \end{bmatrix}$$

and:

$$R'_R = R_R\left(\frac{M}{L_R}\right)^2$$

$$L'_R = L_R\left(\frac{M}{L_R}\right)^2$$

$$M' = M\left(\frac{M}{L_R}\right)$$

(i) The rotor leakage inductance is defined as $L'_{LR} = L'_R - M'$. Show that this is zero under the referral given here.

[2]

(ii) Multiply the equation by the transpose of the current vector to obtain an equation for power. Identify the term that expresses the energy converted to mechanical form and from this show that the torque produced is equivalent to that calculated from the un-referred model, that is,

$$T = P\,M'\left(i_{SQ}\,i'_{RD} - i_{SD}\,i'_{RQ}\right)$$

[8]

(iii) Figure 2.1 shows an equivalent circuit of the referred equation using flux linkages such as $\psi'_{RD} = M'\,(i_{SD} + i'_{RD})$. Re-draw the equivalent circuit of figure 2.1 for a case where orientation at $\psi'_{RQ} = 0$ has been achieved and the stator is supplied from a current source not a voltage source. Explain the significance of orienting the model such that $\psi'_{RQ} = 0$ in terms of the torque equation, the equivalent circuit and the dynamics governing the relationships between stator current, flux and torque.

[10]

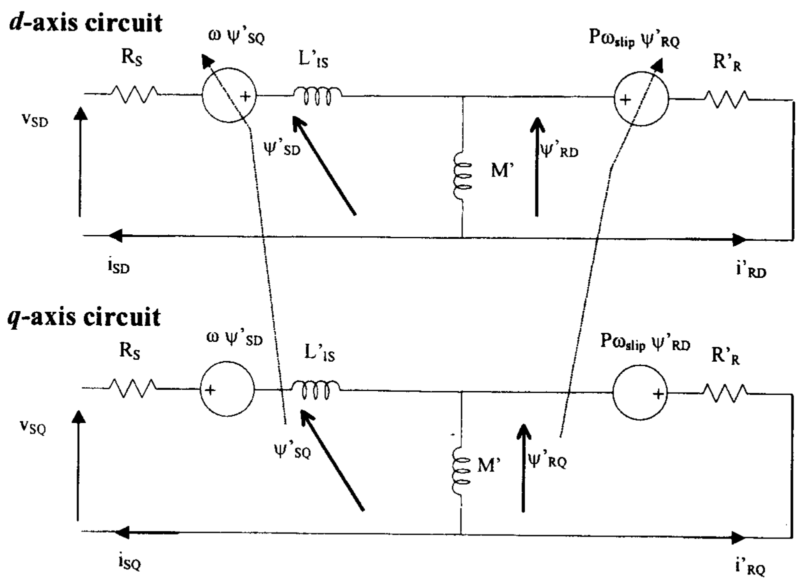


Figure 2.1

3

(a) For each of the signals u_1, u_2 and u_3, state whether they contain positive, negative or zero sequence components. Describe the expected form of transformed signals when the matrices T and T_R are applied. [6]

$$u_1 = \begin{bmatrix} U_1 \cos\left(\omega t + \frac{\pi}{4}\right) \\ U_1 \cos\left(\omega t - \frac{2\pi}{3} + \frac{\pi}{4}\right) \\ U_1 \cos\left(\omega t + \frac{2\pi}{3} + \frac{\pi}{4}\right) \end{bmatrix}$$

$$u_2 = \begin{bmatrix} U_0 + U_2 \cos(\omega t) \\ U_0 + U_2 \cos\left(\omega t - \frac{2\pi}{3}\right) \\ U_0 + U_2 \cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$u_3 = \begin{bmatrix} U_1 \cos(\omega t) \\ U_1 \cos\left(\omega t + \frac{2\pi}{3}\right) \\ U_1 \cos\left(\omega t - \frac{2\pi}{3}\right) \end{bmatrix}$$

$$T = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad T_R = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) The circuit in figure 3.1 is an equivalent circuit of an auto-transformer.

(i) Consider that the input voltages v_{Iabc} and output currents i_{Oabc} are imposed on the circuit and write the circuit equations in matrix form. [4]

(ii) Transform the equations to $dq\gamma$ form. [5]

(iii) Sketch the circuit of the transformed system. [5]

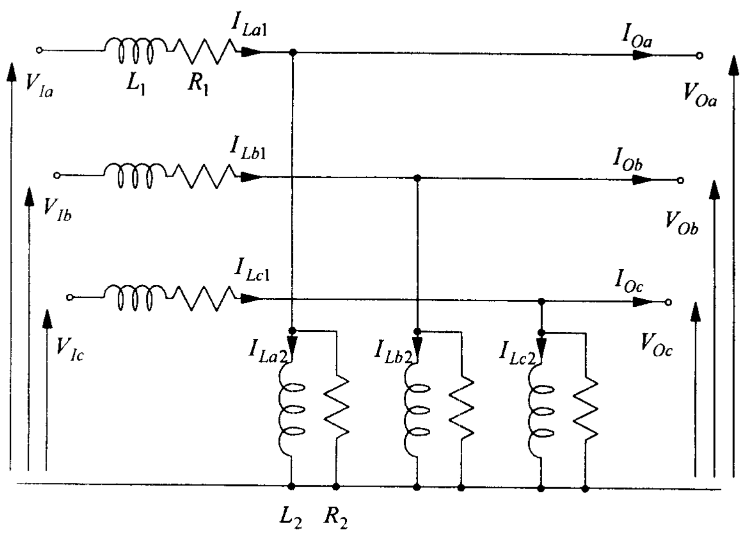


Figure 3.1

Section B

4

a) Discuss the viability of the following statements through brief explanation of the issues involved.

i) It is necessary to have many poles in synchronous machines driven by a hydro turbine. [4]

ii) The rotor of a cylindrical pole machine is made of solid iron whereas the stator is made of laminated steel [4]

iii) A salient pole synchronous machine continues to produce power even when its excitation is lost. [4]

b) A cylindrical rotor synchronous machine is connected to an infinite bus of $V\angle 0$ volts through a resistance of R_s and synchronous reactance X_s ohms. The excitation voltage is $E\angle\delta$. Develop an expression for real and reactive power delivered by the machine. Simplify the expressions when $X_s >> R_s$ and make your comments. [8]

5.

a) Explain your understanding on any four of the following in the context of power system dynamics [4x5 =20]

i) Local mode of oscillation
ii) Small signal stability limit
iii) Thyristor control series capacitor (TCSC)
iv) Synchronising torque
v) Power system stabilizers (PSS)
vi) Impact of load characteristic on stability .

6

a) An on-load tap-changing (OLTC) transformer has N_p and N_s primary and secondary turns respectively. The power frequency leakage impedances are Z_p and Z_s respectively. The nominal values of the impedances at nominal tap position N_{p0}, N_{s0} are Z_{p0} and Z_{s0} respectively. Neglecting the effect of magnetising current, derive the equivalent π model suitable for stability studies and express all the circuit elements of this model in terms of equivalent per unit leakage admittance (Y_e) and off-nominal turns ratio ($c = N_{p0}N_s / N_{s0}N_p$) [10]

b) A round rotor synchronous generator is connected to an infinite bus of 1.0 *pu* voltage through a line of 0.5 *pu* reactance. The machine is delivering 0.9 +j0.29 *pu* power to the system at 60 Hz. Assume that the field flux and mechanical input remain constant. The internal voltage behind a transient reactance of 0.25 *pu* is $0.78\angle\delta$. The H constant of the machine is 8.0 s and the mechanical damping co-efficient (D) is 10.0 *pu* on a common system base. Examine the stability condition of the system through eigen-analysis. Find the frequency and damping of oscillation when the equilibrium is perturbed by sufficiently small amount because of variation in load.

[10]

Master

Solutions –
Modelling & Control in
Power Engineering 2005

[4.38]

1

(a) Describe the steps needed to form a linear state-space model of a switch-mode power supply. [5]

- *Switch-mode circuits are piece-wise linear*
- *State-space models of each state can be formed using the same state vector for each*
- *If the switching between states is at a high frequency compared with the rates of change of the state variable, then the change of state vector over a switching cycle can be approximated by the weighted sum of the changes due to the two states*
- *The resulting model contains products of the duty-cycle and the state vector and/or state matrices*
- *The model is then linearised for perturbations around an operating point.*

(b) Describe briefly how one obtains the steady-state relationship between output voltage V_O and input voltage, V_I in terms of duty-cycle, Δ and also the small-signal transfer function between v_O and δ [5]

If all the perturbation terms are ignored then the remaining terms of the state-space model are simply

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\,\mathbf{U}$$
$$\mathbf{Y} = \mathbf{C}\,\mathbf{X} + \mathbf{D}\,\mathbf{U}$$

where

$$\mathbf{A} = \left(\Delta\mathbf{A}_{\mathbf{On}} + (1-\Delta)\mathbf{A}_{\mathbf{Off}}\right)$$
$$\mathbf{B} = \left(\Delta\mathbf{B}_{\mathbf{On}} + (1-\Delta)\mathbf{B}_{\mathbf{Off}}\right)$$
$$\mathbf{C} = \left(\Delta\,\mathbf{C}_{\mathbf{On}} + (1-\Delta)\mathbf{C}_{\mathbf{Off}}\right)$$
$$\mathbf{D} = \left(\Delta\mathbf{D}_{\mathbf{On}} + (1-\Delta)\mathbf{D}_{\mathbf{Off}}\right)$$

Because steady-state applies, there is no change in the state vector so $\dot{X} = 0$ *and therefore* $\mathbf{X} = -\mathbf{A}^{-1}\,\mathbf{B}\,\mathbf{U}$

Collecting the perturbation terms and ignoring the products of perturbations yields a standard state-space form from which the transfer function can be found in the normal way.

$$\frac{v_O(s)}{\delta(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{E} + \mathbf{F}$$

where $\mathbf{E} = \left(\mathbf{A}_{\mathbf{On}} - \mathbf{A}_{\mathbf{Off}}\right)\mathbf{X} + \left(\mathbf{B}_{\mathbf{On}} - \mathbf{B}_{\mathbf{Off}}\right)\mathbf{U}$ *and* $\mathbf{F} = \left(\mathbf{C}_{\mathbf{On}} - \mathbf{C}_{\mathbf{Off}}\right)\mathbf{X} + \left(\mathbf{D}_{\mathbf{On}} - \mathbf{D}_{\mathbf{Off}}\right)\mathbf{U}$

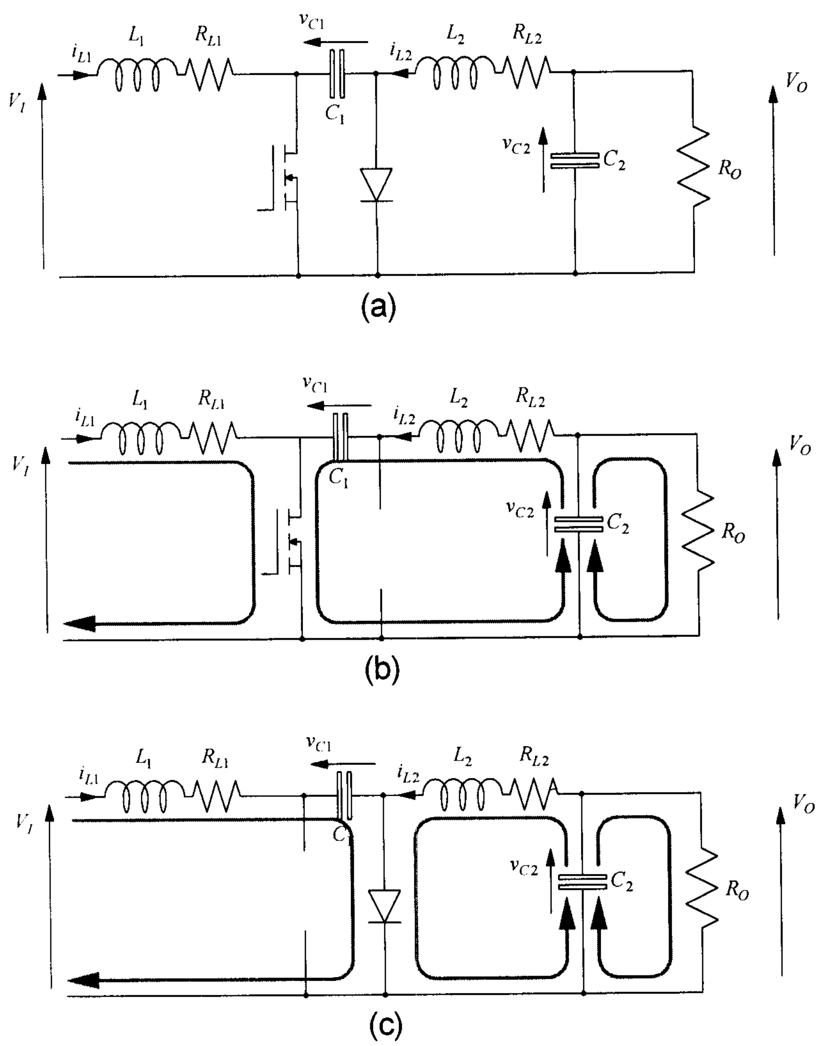
[4.38]

(c) Figure 1.1 shows the circuit of a Ćuk switch-mode power supply and the current paths that exist when the mosfet is on and when the mosfet is off. Using equation (1.1), as a basis, determine the state-space averaged model of this circuit. You may ignore perturbations in the input voltage, V_I and the resistance of capacitors but should include the series resistance of the inductors. [10]

$$\dot{\tilde{\mathbf{x}}} \cong \left(\Delta \mathbf{A}_{\mathrm{On}} + (1-\Delta)\mathbf{A}_{\mathrm{Off}} \right) \tilde{\mathbf{x}} + \left((\mathbf{A}_{\mathrm{On}} - \mathbf{A}_{\mathrm{Off}})\mathbf{X} + (\mathbf{B}_{\mathrm{On}} - \mathbf{B}_{\mathrm{Off}})\mathbf{U} \right)\tilde{\delta}$$

$$\tilde{\mathbf{y}} \cong \left(\Delta \mathbf{C}_{\mathrm{On}} + (1-\Delta)\mathbf{C}_{\mathrm{Off}} \right) \tilde{\mathbf{x}} + \left((\mathbf{C}_{\mathrm{On}} - \mathbf{C}_{\mathrm{Off}})\mathbf{X} + (\mathbf{D}_{\mathrm{On}} - \mathbf{D}_{\mathrm{Off}})\mathbf{U} \right)\tilde{\delta}$$

Equation (1.1)



[4.38]

On-State

$$v_I = i_{L1}R_{L1} + L_1\frac{di_{L1}}{dt}$$

$$v_{C2} + v_{C1} = i_{L2}R_{L2} + L_2\frac{di_{L2}}{dt}$$

$$\frac{dv_{C_1}}{dt} = \frac{-i_{L2}}{C_1}$$

$$\frac{dv_{C_2}}{dt} = \frac{-(i_{L2}+i_O)}{C_2}$$

$$i_O = \frac{v_{C2}}{R_O}$$

$$\mathbf{x} = \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix}$$

$$\mathbf{A}_{\mathbf{On}} = \begin{bmatrix} -\frac{R_{L1}}{L_1} & 0 & 0 & 0 \\ 0 & -\frac{R_{L2}}{L_2} & \frac{1}{L_2} & \frac{1}{L_2} \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & -\frac{1}{C_2} & 0 & -\frac{1}{R_O C_2} \end{bmatrix} \quad \mathbf{B}_{\mathbf{On}} = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{C}_{\mathbf{On}} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{D}_{\mathbf{On}} = [0]$$

Off-state

$$v_I - v_{C1} = i_{L1}R_{L1} + L_1\frac{di_{L1}}{dt}$$

$$v_{C2} = i_{L2}R_{L2} + L_2\frac{di_{L2}}{dt}$$

$$\frac{dv_{C_1}}{dt} = \frac{i_{L1}}{C_1}$$

$$\frac{dv_{C_2}}{dt} = \frac{-(i_{L2}+i_O)}{C_2}$$

$$i_O = \frac{v_{C2}}{R_O}$$

$$\mathbf{A}_{\mathbf{Off}} = \begin{bmatrix} -\frac{R_{L1}}{L_1} & 0 & -\frac{1}{L_1} & 0 \\ 0 & -\frac{R_{L2}}{L_2} & 0 & \frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & -\frac{1}{C_2} & 0 & -\frac{1}{R_O C_2} \end{bmatrix} \quad \mathbf{B}_{\mathbf{Off}} = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{C}_{\mathbf{Off}} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{D}_{\mathbf{Off}} = [0]$$

After averaging

$$\mathbf{A} = \left(\Delta\mathbf{A_{On}} + (1-\Delta)\mathbf{A_{Off}}\right)$$

$$= \Delta\begin{bmatrix} -\frac{R_{L1}}{L_1} & 0 & 0 & 0 \\ 0 & -\frac{R_{L2}}{L_2} & \frac{1}{L_2} & \frac{1}{L_2} \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & -\frac{1}{C_2} & 0 & -\frac{1}{R_O C_2} \end{bmatrix} + (1-\Delta)\begin{bmatrix} -\frac{R_{L1}}{L_1} & 0 & -\frac{1}{L_1} & 0 \\ 0 & -\frac{R_{L2}}{L_2} & 0 & \frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & -\frac{1}{C_2} & 0 & -\frac{1}{R_O C_2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{R_{L1}}{L_1} & 0 & -\frac{(1-\Delta)}{L_1} & 0 \\ 0 & -\frac{R_{L2}}{L_2} & \frac{\Delta}{L_2} & \frac{1}{L_2} \\ \frac{(1-\Delta)}{C_1} & -\frac{\Delta}{C_1} & 0 & 0 \\ 0 & -\frac{1}{C_2} & 0 & -\frac{1}{R_O C_2} \end{bmatrix}$$

$$\mathbf{C} = \left(\Delta\,\mathbf{C_{On}} + (1-\Delta)\mathbf{C_{Off}}\right)$$
$$= \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{E} = \left(\mathbf{A_{On}} - \mathbf{A_{Off}}\right)\mathbf{X} + \left(\mathbf{B_{On}} - \mathbf{B_{Off}}\right)\mathbf{U}$$

$$= \left(\begin{bmatrix} -\frac{R_1}{L_1} & 0 & 0 & 0 \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} & \frac{1}{L_2} \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & -\frac{1}{C_2} & 0 & -\frac{1}{R_O C_2} \end{bmatrix} - \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} & 0 \\ 0 & -\frac{R_2}{L_2} & 0 & \frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & -\frac{1}{C_2} & 0 & -\frac{1}{R_O C_2} \end{bmatrix}\right)\begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix} + \left(\begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}\right)\begin{bmatrix} V_I \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \frac{1}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & 0 \\ -\frac{1}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} I_{L1} \\ I_{L2} \\ V_{C1} \\ V_{C2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{V_{C1}}{L_1} \\ \frac{V_{C1}}{L_2} \\ -\frac{I_{L1} + I_{L2}}{C_1} \\ 0 \end{bmatrix}$$

$$\mathbf{F} = \left(\mathbf{C_{On}} - \mathbf{C_{Off}}\right)\mathbf{X} + \left(\mathbf{D_{On}} - \mathbf{D_{Off}}\right)\mathbf{U} = 0$$

[4.38]

2

The voltage equation for a dq model of an induction machine with the rotor leakage inductance referred to the stator-side and the rotor short-circuit is:

$$[v'_{DQ}] = [R'_{DQ}][i'_{DQ}] + [X'_{DQ}][i'_{DQ}] + [L'_{DQ}]\frac{d}{dt}[i'_{DQ}]$$

where:

$$[v'_{DQ}] = \begin{bmatrix} v_{SD} \\ v_{SQ} \\ 0 \\ 0 \end{bmatrix} \qquad [i_{DQ}] = \begin{bmatrix} i_{SD} \\ i_{SQ} \\ i'_{RD} \\ i'_{RQ} \end{bmatrix}$$

$$[R'_{DQ}] = \begin{bmatrix} R_S & 0 & 0 & 0 \\ 0 & R_S & 0 & 0 \\ 0 & 0 & R_R' & 0 \\ 0 & 0 & 0 & R_R' \end{bmatrix}$$

$$[X'_{DQ}] = \begin{bmatrix} 0 & -\omega L_S & 0 & -\omega M' \\ \omega L_S & 0 & \omega M' & 0 \\ 0 & -P\omega_{slip} M' & 0 & -P\omega_{slip} L'_R \\ P\omega_{slip} M' & 0 & P\omega_{slip} L'_R & 0 \end{bmatrix}$$

$$[L_{DQ}] = \begin{bmatrix} L_S & 0 & M' & 0 \\ 0 & L_S & 0 & M' \\ M' & 0 & L'_R & 0 \\ 0 & M' & 0 & L'_R \end{bmatrix}$$

and:

$$R'_R = R_R\left(\frac{M}{L_R}\right)^2$$

$$L'_R = L_R\left(\frac{M}{L_R}\right)^2$$

$$M' = M\left(\frac{M}{L_R}\right)$$

[4.38]

(i) The rotor leakage inductance is defined as $L'_{LR} = L'_R - M'$. Show that this is zero under the referral given here.

[2]

$$L'_R = \frac{M^2}{L_R} \quad M' = \frac{M^2}{L_R} \quad L'_{lR} = 0$$

(ii) Multiply the equation by the transpose of the current vector to obtain an equation for power. Identify the term that expresses the energy converted to mechanical form and from this show that the torque produced is equivalent to that calculated from the un-referred model, that is,

$$T = P\,M'\left(i_{SQ}\,i'_{RD} - i_{SD}i'_{RQ}\right)$$

[8]

$$\begin{aligned}
p &= i'^{\,T}_{DQ}\; v'_{DQ} \\
&= i'^{\,T}_{DQ}\; R'_{DQ}\, i'_{DQ} \\
&\quad + i'^{\,T}_{DQ}\; X'_{DQ}\, i'_{DQ} \\
&\quad + i'^{\,T}_{DQ}\; L'_{DQ}\,\frac{d\,i'_{DQ}}{dt}
\end{aligned}$$

The power involving X includes both convertion to mechanical form and reactive power. The reactive terms will sum to zero.

$$\begin{aligned}
p_{EM} &= i'^{\,T}_{DQ}\; X'_{DQ}\, i'_{DQ} \\
&= \begin{bmatrix} i_{SD} & i_{SQ} & i'_{RD} & i'_{RQ} \end{bmatrix}
\begin{bmatrix}
-\omega L_S\, i_{SQ} - \omega M' i'_{RQ} \\
+\omega L_S\, i_{SD} + \omega M' i'_{RD} \\
-P\omega_{slip} M' i_{SQ} - P\omega_{slip} L'_R\, i'_{RQ} \\
+P\omega_{slip} M' i_{SD} + P\omega_{slip} L'_R\, i'_{RD}
\end{bmatrix} \\
&= \omega L_S\left(-i_{SD}\, i_{SQ} + i_{SQ}\, i_{SD}\right) \\
&\quad + \omega M'\left(-i_{SD}\, i'_{RQ} + i_{SQ}\, i'_{RD}\right) \\
&\quad + P\,\omega_{slip}\, M'\left(-i'_{RD}\, i_{SQ} + i'_{RQ}\, i'_{SD}\right) \\
&\quad + P\omega_{slip}\, L'_R\left(-i'_{RD}\, i'_{RQ} + i'_{RQ}\, i'_{RD}\right) \\
&= \left(\omega - P\omega_{slip}\right) M'\left(i_{SQ}\, i'_{RD} - i_{SD} i'_{RQ}\right)
\end{aligned}$$

$$\begin{aligned}
T &= \frac{p_{EM}}{\omega_R} = \frac{\left(\omega - P\omega_{slip}\right)}{\left(\dfrac{\omega}{P} - \omega_{slip}\right)} M'\left(i_{SQ}\, i'_{RD} - i_{SD} i'_{RQ}\right) \\
&= P\,M'\left(i_{SQ}\, i'_{RD} - i_{SD} i'_{RQ}\right)
\end{aligned}$$

Applying the referral yields:

$$T = PM\left(\frac{M}{L_R}\right)\left(i_{SQ}\, i_{RD}\left(\frac{L_R}{M}\right) - i_{SD} i_{RQ}\left(\frac{L_R}{M}\right)\right)$$
$$= PM\left(i_{SQ}\, i_{RD} - i_{SD} i_{RQ}\right)$$

(iii) Figure 3 shows an equivalent circuit of the referred equation using flux linkages such as $\psi'_{RD} = M'\,(i_{SD} + i'_{RD})$. Re-draw the equivalent circuit of figure 3 for a case where orientation at $\psi'_{RQ} = 0$ has been achieved and the stator is supplied from a current source not a voltage source. Explain the significance of orienting the model such that $\psi'_{RQ} = 0$ in terms of the torque equation, the equivalent circuit and the dynamics governing the relationships between stator current, flux and torque.

[10]

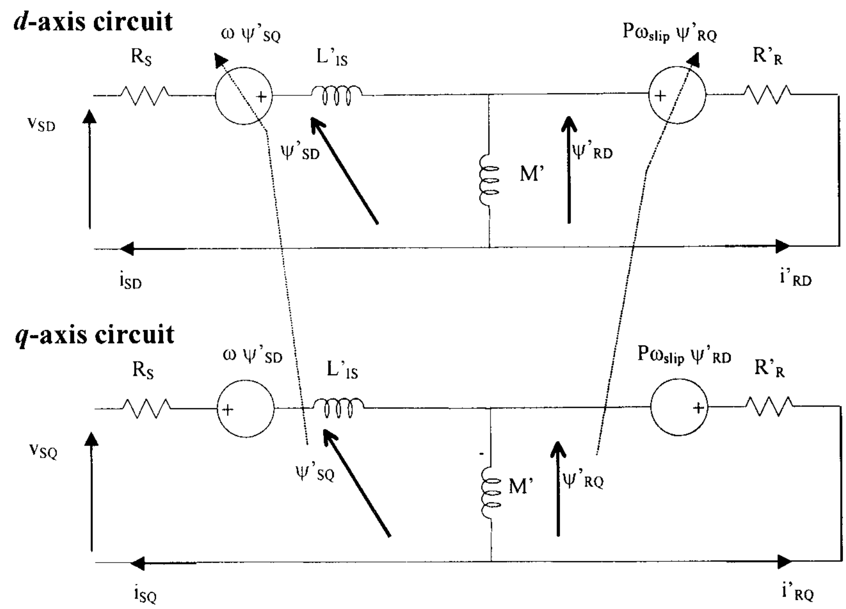
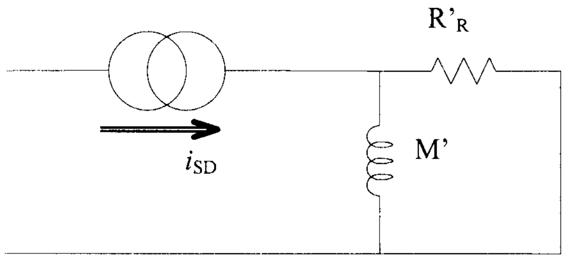


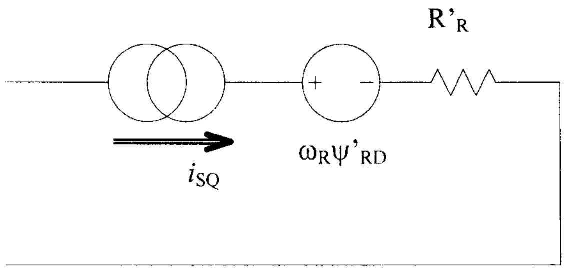
Figure 3

Orienting so that $\psi'_{RQ} = 0$

- *sets to zero the rotational voltage from the rotor of the d-axis circuit*
- *sets to zero the current through the magnetising inductance in the q-axis circuit (i.e. makes it open circuit)*

Supplying the stator from a current source removes the need to model the stator impedance





The d-axis is now a simple R L current divider and it and the d-axis flux has first order dynamics. The time constant of the circuit is long since it involves the magnetising inductance which is large.

$$i_{MD} = i_{SD} \frac{R'_R}{R'_R + s\,M'} = i_{SD} \frac{1}{1 + s\,\tau_R}$$

The q-axis has no dynamics and the rotor current responds instantly to changes in stator current. Thus, if the d-axis current, and hence the flux magnitude, is regulated to be constant, the torque will respond instantly to changes in q-axis stator current.

3

(a) For each of the signals u_1, u_2 and u_3, state whether they contain positive, negative or zero sequence components. Describe the expected form of transformed signals when the matrices T and T_R are applied. [6]

$$u_1 = \begin{bmatrix} U_1 \cos\left(\omega t + \frac{\pi}{4}\right) \\ U_1 \cos\left(\omega t - \frac{2\pi}{3} + \frac{\pi}{4}\right) \\ U_1 \cos\left(\omega t + \frac{2\pi}{3} + \frac{\pi}{4}\right) \end{bmatrix}$$

$$u_2 = \begin{bmatrix} U_0 + U_2 \cos(\omega t) \\ U_0 + U_2 \cos\left(\omega t - \frac{2\pi}{3}\right) \\ U_0 + U_2 \cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$u_3 = \begin{bmatrix} U_1 \cos(\omega t) \\ U_1 \cos\left(\omega t + \frac{2\pi}{3}\right) \\ U_1 \cos\left(\omega t - \frac{2\pi}{3}\right) \end{bmatrix}$$

$$T = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad T_R = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

u1 *is a positive sequence set at an angle of $\pi/4$.*
When transformed by T, two orthogonal components will result with the third term equal to zero.
When further transformed by T_R, the time variation is removed leaving

$$u_{1D} = \sqrt{\tfrac{3}{2}}\, u_1 \cos\left(\tfrac{\pi}{4}\right) \quad u_{1Q} = \sqrt{\tfrac{3}{2}}\, u_1 \sin\left(\tfrac{\pi}{4}\right) \quad u_{1\gamma} = 0$$

u2 *is a sum of a DC zero sequence set and positive sequence set at an angle of 0.*
When transformed by T, two orthogonal components will result with the third term equal to the zero sequence component.
When further transformed by T_R, the time variation is removed and because the positive sequence set is at an angle of zero the quadrature term is equal to zero.

$$u_{2D} = \sqrt{\tfrac{3}{2}}\, u_2 \quad u_{2Q} = 0 \quad u_{1\gamma} = \sqrt{3}\, u_0$$

u3 *is a negative sequence set at an angle of 0.*
When transformed by T, two orthogonal components with negative sequence will result and the third term will be equal to zero.
When further transformed by T_R, the time variation is ***not*** *removed. In fact the rotation of the set is doubled because the transform matrix rotates in the same direction as a negative sequence set.*

[4.38]

(b) The circuit in figure 3.1 is an equivalent circuit of an auto-transformer.

(i) Consider that the input voltages v_{Iabc} and output currents i_{Oabc} are imposed on the circuit and write the circuit equations in matrix form. [4]

(ii) Transform the equations to $dq\gamma$ form. [5]

(iii) Sketch the circuit of the transformed system. [5]

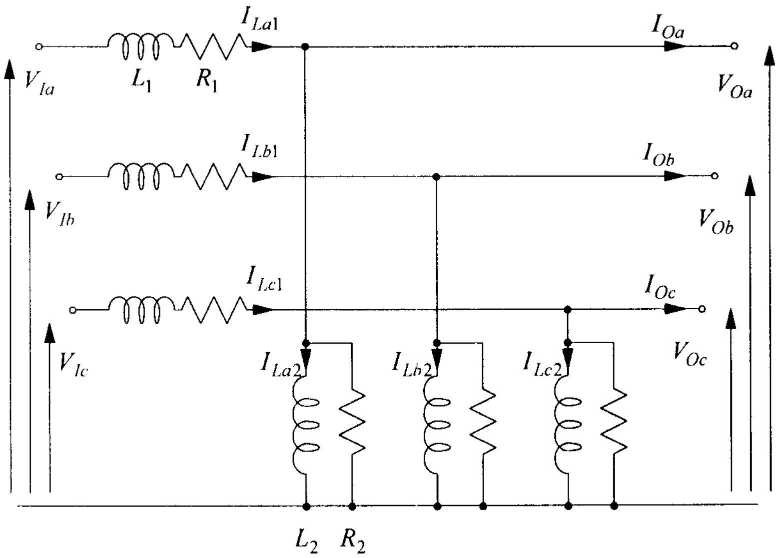


Figure Q3.1

$$v_{Oabc} = R_2\left(i_{L1abc} - i_{L2abc} - i_{Oabc}\right)$$

$$v_{Iabc} = L_1 \frac{di_{L1abc}}{dt} + R_1 i_{L1abc} + R_2\left(i_{L1abc} - i_{L2abc} - i_{Oabc}\right)$$

$$R_2\left(i_{L1abc} - i_{L2abc} - i_{Oabc}\right) = L_2 \frac{di_{L2abc}}{dt}$$

$$v_{Odq\gamma} = R_2\left(i_{L1dq\gamma} - i_{L2dq\gamma} - i_{Odq\gamma}\right)$$

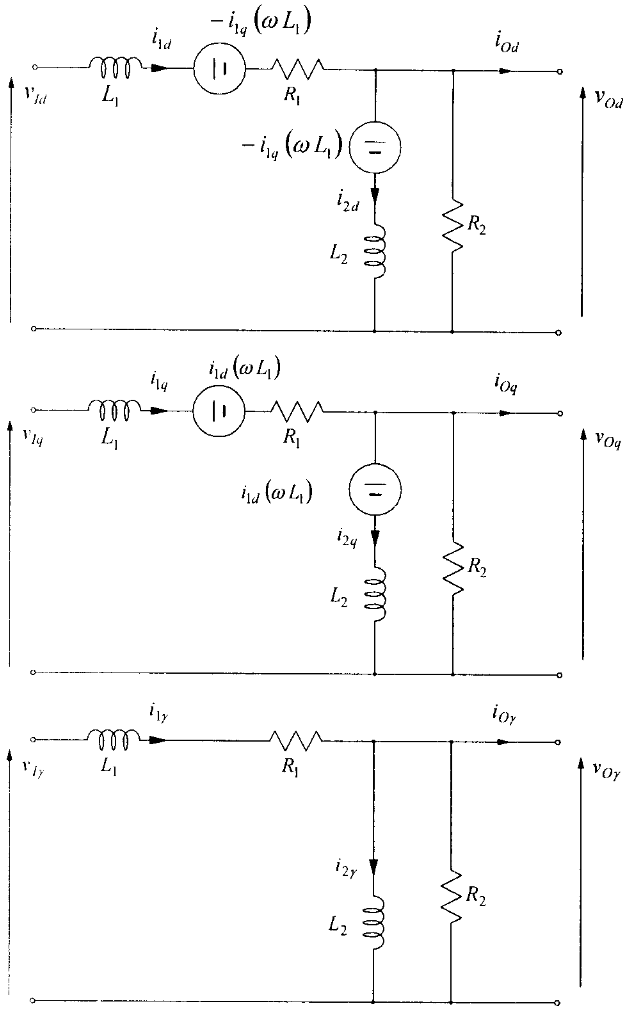
$$v_{Idq\gamma} = L_1 \frac{di_{L1dq\gamma}}{dt} + X_1\, i_{L1dq\gamma} + R_1 i_{L1dq\gamma} + R_2\left(i_{L1dq\gamma} - i_{L2dq\gamma} - i_{Odq\gamma}\right)$$

where $X_1 = \begin{bmatrix} 0 & -\omega L_1 & 0 \\ \omega L_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$$R_2\left(i_{L1dq\gamma} - i_{L2dq\gamma} - i_{Odq\gamma}\right) = L_2 \frac{di_{L2dq\gamma}}{dt} + X_2\, i_{L2dq\gamma}$$

where $X_2 = \begin{bmatrix} 0 & -\omega L_2 & 0 \\ \omega L_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

[4.38]



$-i_{1q}(\omega L_1)$
i_{1d}
i_{Od}
v_{Id}
L_1
R_1
v_{Od}
$-i_{1q}(\omega L_1)$
i_{2d}
L_2
R_2
i_{1q}
$i_{1d}(\omega L_1)$
i_{Oq}
v_{Iq}
L_1
R_1
v_{Oq}
$i_{1d}(\omega L_1)$
i_{2q}
L_2
R_2
$i_{1\gamma}$
$i_{O\gamma}$
$v_{I\gamma}$
L_1
R_1
$v_{O\gamma}$
$i_{2\gamma}$
L_2
R_2

Section B

4

a) Justify the following statements through brief explanation.

i) It is necessary to have many poles in synchronous machines driven by hydro turbine?

Answer: The speed at which the water wheels rotate is very low (of the order of a few hundred rpm). In order for the generator connected to hydro turbine to get connected to the grid of frequency 'f' the following relation has to be satisfied

$$f = \frac{PN}{120}$$

where 'P' is number of poles, N is the speed in rpm. At low speed, a large number of pole is necessary to achieve a grid frequency of 50 or 60 Hz. At low speed the centrifugal force developed is also less requiring simpler arrangement to keep the conductors in place in slots.

[4]

ii) The rotor of a cylindrical pole machine is made of solid iron as opposed to the stator made of laminated steel [4]

Answer: The rotor is excited from a DC source. This does not induce current in rotor body as the DC flux rotates with the rotor body. The EMF generated on the stator winding is AC (usually of 50/60 Hz) and so the stator body is subjected eddy current circulation causing heating of the stator body. The lamination of the stator is to minimise eddy current circulation and hence keep the temperature under control. A solid rotor body also produces damping action when subjected to oscillations.

iii) A salient pole synchronous machine continues to produce power even when its excitation is lost. [4]

Answer: The expression for power developed by a salient pole synchronous machine when stator resistance neglected is:

$$P = \frac{EV_t}{X_d}\sin\delta + \frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

where E is the speed voltage is q-axis , V_t is the terminal voltage , X_d and X_q are direct and quadrature axis reactances; all in the steady state.

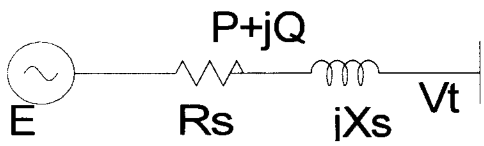
The second term does not depend on the excitation E. Hence, when $X_d \neq X_q$, which is the case with salient pole machines, power is developed even without excitation. This is due to saliency in the magnetic structure producing a reluctance torque (as predicted by the virtual work principle)

b) A cylindrical rotor synchronous machine is connected to an infinite bus of $V\angle 0$ volts through a resistance of R_s and synchronous reactance X_s ohms. The excitation voltage is $E\angle\delta$. Develop an expression for real and reactive power delivered by the machine. [8]

Answer: Let the machine supply current I. The power delivered by the machine is

$S = P + jQ = EI^*$

$I = \dfrac{E\angle\delta - V\angle 0}{R_s + jX_s}$



Induced voltage $\overline{E} = E\angle\delta$,

Terminal voltage (infinite bus voltage): $\overline{V_t} = V_t\angle 0$

Stator impedance: $\overline{Z}_s = R_s + jX_s = Z_s\angle\theta_s$

The complex power delivered by the machine at its terminal is:

$$S = \overline{E}I_s^{\ *}$$

$$I_s^{\ *} = \left(\frac{\overline{E} - V_t}{Z_s}\right)^* = \frac{E}{Z_s}\angle + \theta_s - \delta - \frac{V_t}{Z_s}\angle + \theta_s$$

$$S = -\frac{EV_t}{Z_s}\angle\delta + \theta_s + \frac{E^2}{Z_s}\angle\theta_s$$

$$S = P + jQ;$$

$$P = -\frac{EV_t}{Z_s}\cos(\theta_s + \delta) + \frac{E^2}{Z_s}\cos\theta_s \text{ W/phase}$$

$$Q = -\frac{EV_t}{Z_s}\sin(\theta_s + \delta) + \frac{E^2}{Z_s}\sin\theta_s \text{ VAr/Phase}$$

[6 marks]

When $X_s >> R_s$, $\theta_s \approx 90$ degree and $Z_s \approx X_s$

The expression for P and Q with this assumption is

$$P = \frac{EV_t}{X_s}\sin\delta \ ; \ Q = \frac{E}{X_s}\left(E - V_t\cos\delta\right)$$

Comments: real power varies predominantly with δ and reactive power varies predominantly with E

[2 marks]

5

a) Write short notes on any four of the following in the context of power system dynamics [4x5 =20]

i) Local-mode oscillation

Local-mode oscillation: Local modes are associated with units at a generating station swinging with respect to the rest of the power system. The oscillations are localised at one station. Here the frequency range of the oscillation is 1-2 Hz. This is widely analysed in power system linear stability literature. This treats only the dynamics of the generator under consideration and treats rest of the system as a constant voltage source. This is very useful for machine control design.

ii) Small-signal stability limit

Small signal stability is related to the dynamic behaviour of an interconnected power system when subjected to very small disturbance. Both the angle and voltage stability properties of the system are characterized through a small signal stability limit. Usually the dynamic characteristic of the is assumed linear around an operating equilibrium and eigen-value analysis is performed. The stability margin of the system is assessed by examining the distance of the system poles from the imaginary axis of the eigen-plane. The stability limit is also indicated by the damping ratio, positive for positive margin. The standard analytical tools and techniques in linear system theory are applied.

iii) Thyristor control series capacitor (TCSC)

The basic idea behind series compensation is to reduce line voltage drop by reducing overall line reactance. Consider a SMIB system with line reactance X. The power angle characteristic can be expressed as:

$$P = \frac{EV}{X}\sin\delta .$$

The maximum power transfer limit in the steady state is $\frac{EV}{X}$.

For plant connected to system through a long distance transmission line, X is very high reducing the maximum power transfer limit. If a capacitor X_c (controllable or fixed) is connected in series with the line, the effective transmission impedance is $X_{eff}=X-X_c$. or $X_{eff}=(1-k)X$. where k is the degree of compensation $k= X_c/X$. The power flow equations for this case will be:

$$P = \frac{EV}{(1-k)X}\sin\delta .$$

The maximum power transfer limit is improved as k is kept less than 1.0 always.

For example, a 20% compensation would produce 25% rise maximum transfer capacity. The series compensation level can be controlled with a thyristor control series capacitor and hence power flow can be controlled. The series compensation has been used extensively in the last 50 years throughout the world for compensation of long transmission line.

iv) Synchronising torque

The primary objective for operation of an AC interconnected system is to maintain synchronism. The system does not operate around a single operating condition. The transition from one operating condition to another is made smooth through manual/automatic control. The load angle of an individual generator starts to vary from steady state condition during the transition. This leads to swinging of part of the system with respect to the rest. A variable torque which is counter acting in nature immediately develops to balance the change. This is the inherent torque of the system called thr the synchronising torque that varies linearly with load angle change as long as the change is small. Each generator in the system, depending on its level of loading, will contribute to this synchronising torque. It is the rotor excitation that produces this torque (coupling between rotor and stator magnetic field). The system is held stable during sudden changes in system operating conditions or disturbance with the help of the synchronising torque. All modern day large generators are equipped with very fast acting and high gain excitation voltage control for rapid production of synchronising torque to prevent the system from running away.

v) Power system stabilizers (PSS)

Usually, the high gain fast acting voltage regulation provided to enhance transient stability margin of generator also produces negative damping or oscillatory instability in the low frequency (0.2 to 2.0 Hz) range. Such voltage regulator action was found to introduce negative damping torque at high power output and weak external network conditions represented by long overhead transmission lines, a very common operating situation in power systems in US and Canada. Negative damping gave rise to reduced utilisation of transmission capacity. The conflicting performance of the excitation control loop was resolved by modulating the voltage regulator reference input through an additional signal, which was meant to produce positive damping torque. The control circuitry producing this signal is termed a power system stabilizer (PSS).

The primary objective of a PSS is to introduce a component of electrical torque in the synchronous machine rotor that is proportional to the deviation of the actual speed from the synchronous speed. When the rotor oscillates, this torque acts as a damping torque to counter the oscillation. There is a wide choice available for selecting feedback signals (such as rotor speed, power, bus frequency etc.) and most effective machine for damping oscillation etc. It comprises of a phase compensation block, washout filter and gain with limits on output in both direction. The gain and phase compensation circuit obtained through analysis and design by classical and modern control methods.

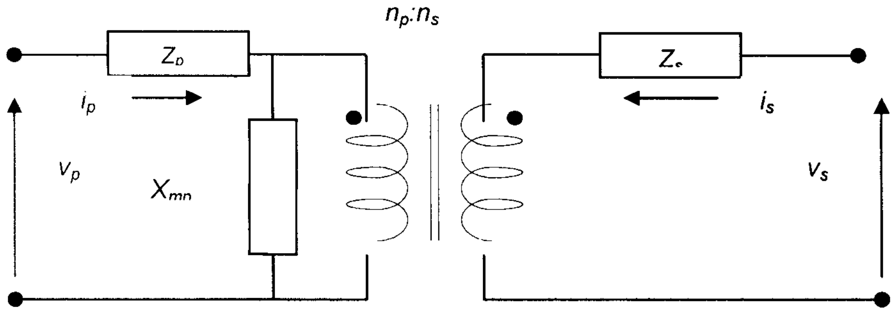
vi) Impact of load characteristic on system stability

The characteristic governing the relationship between current and voltage of a load plays an important part in stability studies. Many instances of service interruptions or even blackouts have been traced to adverse interaction of loads with system controls. The load characteristic changes with seasonal change in demand and weather..Usually, constant power types of load are very detrimental to voltage stability. Majority of the electronically controlled loads tend to draw high currents at constant power and depress voltage. Constant impedance loads are not as detrimental as constant power loads as far as small signal voltage/angle stability is concerned. The loads must be properly modelled for stability studies because controllers designed otherwise will perform unacceptably in the field in most cases. There have been many task forces/study committees in CIGRE/IEEE for load modelling.

6

a) An on-load tap-changing (OLTC) transformer has N_p and N_s primary and secondary turns respectively. The power frequency leakage impedances are Z_p and Z_s respectively. The nominal values of the impedances at nominal tap position N_{p0}, N_{s0} are Z_{p0} and Z_{s0} respectively. Neglecting the effect of magnetising current, derive the equivalent π model suitable for stability studies and express all the circuit elements of this model in terms of equivalent per unit leakage admittance (Y_e) and off-nominal turns ratio ($c = N_{p0}N_s / N_{s0}N_p$) [10]

Answer: The basic equivalent circuit of a two winding transformer with all quantities in physical units are shown in Fig 6.1.



ideal transformer

Fig 6.1: Basic equivalent circuit of a two winding transformer

The subscripts p and s refer respectively to primary and secondary quantities. The magnetizing reactance X_{mp} is very large and is usually neglected in power flow and stability studies. With appropriate choice of base quantities, it is possible to simplify the equivalent circuit by eliminating the ideal transformer. However, the presence of tap-changing features does not allow us to do so and hence it is necessary to consider an off-nominal turns ratio.

From the equivalent circuit in Fig 6.1, the following circuit equations can be written.

$$
\begin{aligned}
v_p &= Z_p i_p + \frac{n_p}{n_s} v_s - \frac{n_p}{n_s} Z_s i_s \\
v_s &= \frac{n_s}{n_p} v_p - \frac{n_s}{n_p} Z_p i_p + Z_s i_s
\end{aligned}
\qquad (6.1)
$$

$Z_p = R_p + jX_p; Z_s = R_s + jX_s$ are primary and secondary impedance respectively; R_p, X_p, R_s, X_s are resistance and reactance in the primary and secondary sides respectively. n_p, n_s are number of primary and secondary winding turns. Let us define nominal values of these parameters, i.e at nominal tap position n_{p0}, n_{s0} as $Z_{p0} = Z_p; Z_{s0} = Z_s$. Assuming that inductances vary as the square of the turns ratio and that resistance is very small, one can assumes without much error, that the impedance also varies in relation to square of the turn ratios.

The equation (6.1) is written as:

$$v_p = \left(\frac{n_p}{n_{p0}}\right)^2 Z_{p0} i_p + \left(\frac{n_p}{n_s}\right) v_s - \left(\frac{n_p}{n_s}\right)\left(\frac{n_s}{n_{s0}}\right)^2 Z_{s0} i_s$$

$$v_s = \frac{n_s}{n_p} v_p - \frac{n_s}{n_p}\left(\frac{n_p}{n_{p0}}\right)^2 Z_{p0} i_p + \left(\frac{n_s}{n_{s0}}\right)^2 Z_{s0} i_s \tag{6.2}$$

The nominal number of turns are related to base voltage as

$$\frac{n_{p0}}{n_{s0}} = \frac{v_{p,base}}{v_{s,base}}; \qquad v_{p,base} = Z_{p,base} i_{p,base}; v_{s,base} = Z_{s,base} i_{s,base}$$

The above equation in *pu* form becomes

$$\bar{v}_p = \bar{n}_p^{\,2} \bar{Z}_{p0} \bar{i}_p + \frac{\bar{n}_p}{\bar{n}_s} \bar{v}_s - \bar{n}_s^{\,2} \frac{\bar{n}_p}{\bar{n}_s} \bar{Z}_{s0} \bar{i}_s$$

$$\bar{v}_s = \frac{\bar{n}_s}{\bar{n}_p} \bar{v}_p - \bar{n}_p^{\,2} \frac{\bar{n}_s}{\bar{n}_p} \bar{Z}_{p0} \bar{i}_p + \bar{n}_s^{\,2} \bar{Z}_{s0} \bar{i}_s \tag{6.3}$$

$$\bar{n}_p = \frac{n_p}{n_{p0}}; \bar{n}_s = \frac{n_s}{n_{s0}}$$

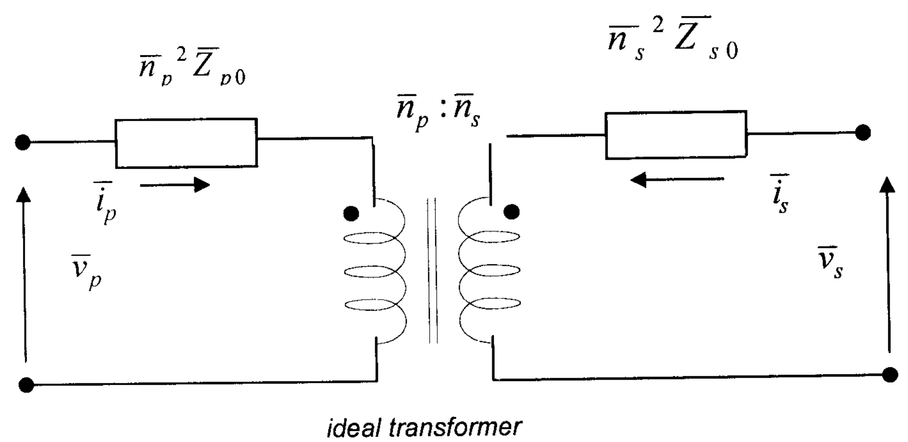


Fig 6.2: per unit equivalent circuit of a two winding transformer

This circuit can be further simplified to Fig 6.3 with following new variables

$$\bar{n} = \frac{\bar{n}_p}{\bar{n}_s} = \frac{n_p n_{s0}}{n_{p0} n_s} \qquad \bar{Z}_e = \bar{n}_s^{\,2}\left(\bar{Z}_{p0} + \bar{Z}_{s0}\right)$$

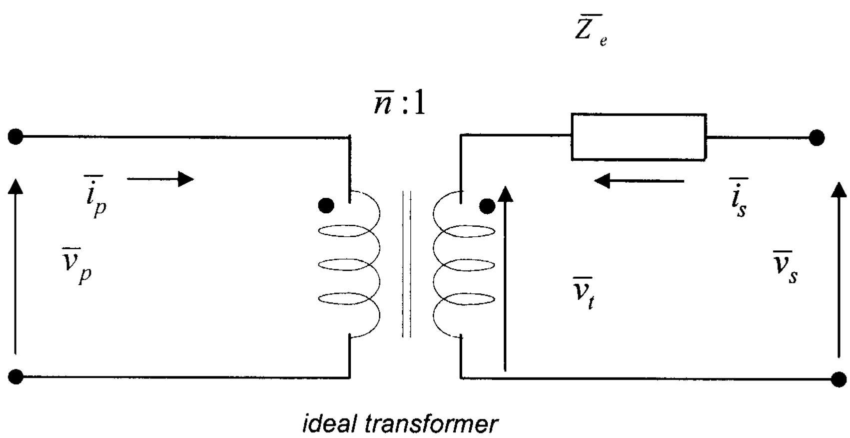


Fig 6.3: standard equivalent circuit of a two winding transformer

We now see that Z_e does not depend on variable $\overline{n}_p$. This perfectly models transformer with fixed tap on one side and under load tap changer (ULTC) on the other side. In power studies this is further reduced as to a π equivalent circuit a follows

$$\begin{aligned} \overline{i}_p &= \left(\overline{v}_p - \overline{n}\overline{v}_s\right)\frac{\overline{Y}_e}{\overline{n}^2} \\ \overline{i}_s &= \left(\overline{n}\overline{v}_s - \overline{v}_p\right)\frac{\overline{Y}_e}{\overline{n}} \\ \overline{Y}_e &= \frac{1}{\overline{Z}_e} \end{aligned} \tag{6.4}$$

The following equivalent circuit can implement the equation set (6.4) which is similar to transmission line equivalent circuit.

This standard equivalent circuit is perfectly valid for three-phase Y/Y and Δ/Δ, where ratios of the line-to-line base voltages are equal the ratios of nominal turns ratios. For Y/Δ or Δ/Y, an additional factor of √3 is introduced because of winding connection and also an angle of 30° is introduced between line-to-line voltages of both side of the transformer. It is usually neglected in power flow studies.

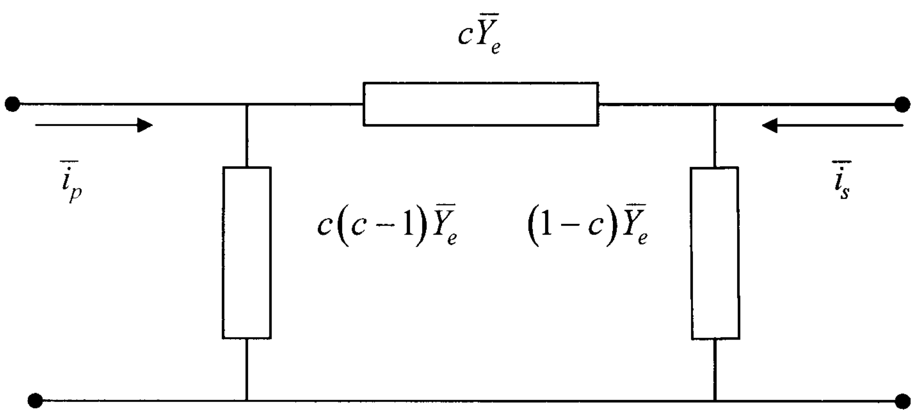


Fig 6.4 Equivalent π circuit

b) A round-rotor synchronous generator is connected to an infinite bus of 1.0 pu voltage through a line of 0.5 pu reactance. The machine is delivering 0.9 +j0.29 pu power to the system at 60 Hz. Assume that the field flux and mechanical power remain constant. The internal voltage behind a transient reactance of 0.25 pu is 0.78$\angle\delta$. The H constant of the machine is 8.0 s and the mechanical damping co-efficient (D) is 10.0 pu on a common system base. Examine the stability condition of the system through eigen-analysis. Find the frequency and damping of oscillation when the equilibrium is perturbed by sufficiently small amount because of variation in load. [10]

Answer: Classical dynamic model of the synchronous machine is applicable here. The power angle equation is

$$P_{elec} = \frac{E_q' V_b}{X_d' + X_l} \sin\delta$$

Given P = 0.9, Vb=1.0, Eq'=0.78, Xd'+Xl= 0.75 p.u, the power angle δ is 60°

The synchronous speed (ω_s) is 2*pi*60 = 377 rad/sec [2 marks]

The swing equations are:

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$M\frac{d\omega}{dt} = P_{mech} - P_{elec} - D(\omega - \omega_s)$$

Where M = $\frac{2H}{\omega_s} = 2*8/377 = 0.04244$ sec^2

Since the equations are expressed in p.u and the speed is in rad/sec, D has to be scaled in sec i.e the value for D for the equation would be 10/377=0.02652 sec [1 mark]

For small signal analysis swing equation needs to be linearised around the given operating condition.

The result of linearization is:

$$\begin{bmatrix} \frac{d\Delta\delta}{dt} \\ \frac{d\Delta\omega}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-E_{q0}' V_{b0}}{M(X_d' + X_l)}\cos\delta_0 & -\frac{D}{M} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix}$$

With the initial values of the parameters the state matrix is

$$A = \begin{bmatrix} 0 & 1 \\ -12.2525 & -0.625 \end{bmatrix}$$ [3 marks]

The eigen-value of this matrix can be found by solving a det(sI-A) = 0 which is $s^2 + 0.625s + 12.2525 = 0$

The roots are $\lambda = -0.3125 \pm i3.4864$

The negative real part of the eigen-values indicates that the system is stable. [3 marks]

The damping and frequency associated with the eigen-value $\lambda = -0.3125 \pm i3.4864$ is
–(-0.3125/abs($\lambda = -0.3125 \pm i3.4864$) = 0.0896 and 3.4864/2*π = 0.554 Hz

[1 mark]