IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

EEE/EIE PART III/IV: MEng, BEng and ACGI

DIGITAL SIGNAL PROCESSING

Corrected copy

Wednesday, 13 December 9:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.A. Naylor

Second Marker(s) : W. Dai

DIGITAL SIGNAL PROCESSING

1. a) Figure 1.1 shows a digital signal processing system containing 4 subsystems, A, B, C and D with impulse responses as shown in Table 1.

Subsystem	Impulse response
A	$h_a(n)$
B	$h_b(n)$
C	$h_c(n)$
D	$h_d(n)$

Table 1

Write an expression for $h(n)$, where $h(n)$ is the impulse response of the complete LTI system with input $x(n)$ and output $y(n)$. [2]

Determine $h(n)$ for the following definitions of $h_a(n)$, $h_b(n)$, $h_c(n)$, $h_d(n)$. [3]

$$h_a(n) = \{0.5, 0.3, 0.5\}$$
$$h_b(n) = (n+1)u(n)$$
$$h_c(n) = h_b(n)$$
$$h_d(n) = \delta(n-1).$$

Determine the output of the LTI system, $y(n)$, when the input is

$$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3).$$

[4]

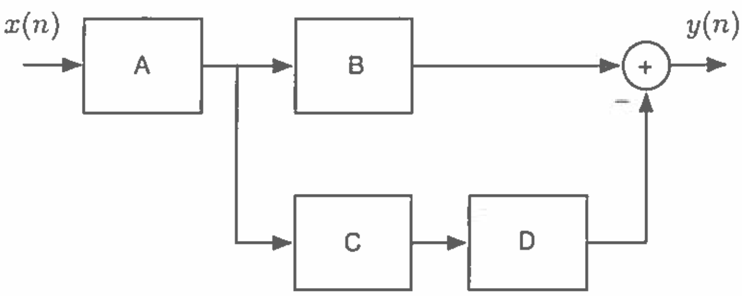

Figure 1.1

b) An LTI system, with input signal $x(n)$ and output signal $y(n)$ is characterized by the difference equation

$$y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2).$$

Determine the impulse response and the unit step response of this system. [6]

c) Consider two classes of discrete-time systems: recursive and nonrecursive. State the important differences between these two classes of systems. [1]

Derive the difference equation of a recursive system for which the output $y(n)$ is the cumulative average of a signal $x(n)$. Draw the signal flow diagram of this system. [4]

2. a) i) Given an M-fold decimator with input $x(n)$ and output $y_D(n)$, write down the expression for $y_D(n)$ in terms of $x(n)$. [2]

ii) Given an L-fold expander with input $x(n)$ and output $y_E(n)$, write down the expression for $y_E(n)$ in terms of $x(n)$. [2]

b) Next consider the multirate signal processing system shown in Fig. 2.1.

i) Write down $Y(z)$ in terms of $X(z)$ for this system. [4]

ii) In Fig. 2.2, the magnitude spectrum of $X(z)$ is shown for the frequency range $-\pi \leq \omega \leq \pi$. Now consider the case when the LTI system $H(z)$ is an ideal lowpass filter with normalized cut-off frequency $\frac{\pi}{2}$.

Draw a labelled sketch of the magnitude spectrum of $Y(z)$ covering the range of frequencies $-2\pi \leq \omega \leq 4\pi$. Explain the main features of this magnitude spectrum sketch.

Does aliasing occur? Explain your answer. [4]

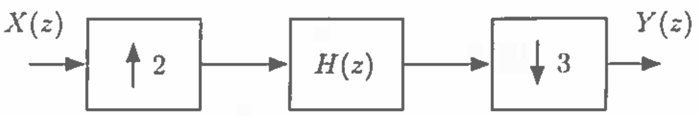

Figure 2.1

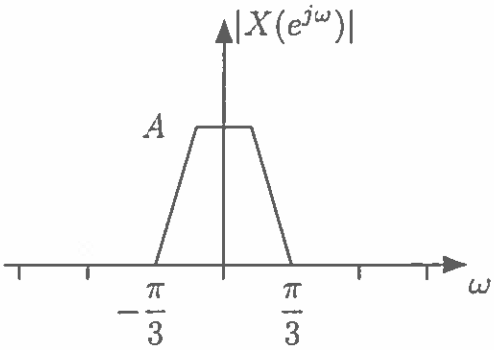

Figure 2.2

c) State the Noble identities relating to multirate signal processing. Hence, redraw the system in Fig. 2.1 such that the computation of $H(z)$ is performed at the lowest possible sampling frequency. [4]

d) i) Write down an expression for the 3-phase Type 1 polyphase decomposition of a filter $H(z)$.

ii) Redraw Fig. 2.1 using the 3-phase Type 1 polyphase decomposition.

iii) Redraw your solution to part c) using 3-phase Type 1 polyphase filtering.

iv) State the relative merits of the structures resulting from (ii) and (iii).

[4]

3. a) Using a few sentences and one or more relevant diagrams, describe the similarities and differences between the continuous-time Fourier transform, the DTFT and the DFT? [6]

b) i) With discrete-time index n, consider a signal $x(n)$ for $n = 0, 1, \ldots, 5$. State and explain a sufficient condition on $x(n)$ such that its Fourier transform $X(e^{j\omega})$ is real valued. [2]

ii) Find $x(n)$ for $X(e^{j\omega}) = \{6, -5, 3, 4, 3, -5\}$. [5]

c) A continuous-time analogue signal containing an audio recording of bird song has a bandwidth of 4 kHz. It is required to compute the spectrum of this signal using an N-point DFT with $N = 2^m$, for integer m, and a frequency resolution $\Delta f \leq 50$ Hz.

i) What is the minimum sampling frequency required?

ii) What is the minimum number of samples required?

iii) What is the minimum duration of the analogue signal required?

[3]

d) By exploiting the DFT, prove that

$$\sum_{l=-\infty}^{\infty} \delta(n + lN) = \frac{1}{N} \sum_{k=0}^{N-1} e^{j2\pi kn/N}.$$

[4]

4. a) Consider a general discrete-time system with system function

$$H(z) = \frac{Y(z)}{X(z)}.$$

With reference to the polynomials $X(z)$ and $Y(z)$, explain the meaning of, and define criteria for, the underlined terms in the following statements.

i) The system $H(z)$ is an <u>unstable</u> system.

ii) The system $H(z)$ is a <u>non-causal</u> system.

iii) The system $H(z)$ is a <u>maximum phase</u> system.

[3]

b) Consider a linear time-invariant system

$$H(z) = \frac{(z-\frac{1}{3})(z+2)(z^2+\frac{1}{9})}{(z^2+2z+5)(z^2-4z+13)}.$$

i) Find the roots of $H(z)$ and draw a labelled sketch showing the roots on the z-plane. [6]

ii) Determine any regions of convergence of $H(z)$ and state whether the inverse z-transform associated with each region of convergence is left-sided, right-sided or two-sided. Comment on the stability of $H(z)$. [5]

c) Given the signal representation in the z-domain

$$X(z) = z^3 + 2z^2 - 2 + \frac{2}{z^2+5z+4}, \quad |z| > 4$$

find the corresponding signal representation in the time domain using the inverse z-transform. [6]