IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

EEE PART II: MEng, BEng and ACGI

Corrected Copy

DEVICES

Monday, 17 June 2:00 pm

Time allowed: 1:30 hours

There are THREE questions on this paper.

Answer ALL questions. Question One carries 20 marks. Questions Two and Three each carry 15 marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : K. Fobelets
Second Marker(s) : E. Shamonina

Special instructions for invigilators

Special instructions for students

Constants and Formulae

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10}$ cm^{-3} at $T = 300$K
dielectric constant of Si:	$\varepsilon_{Si} = 11$
dielectric constant of SiO_2:	$\varepsilon_{ox} = 4$
thermal voltage:	$V_T = kT/e = 0.026$V at $T = 300$K
charge of an electron:	$e = 1.6 \times 10^{-19}$ C

$$\left.\begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p \frac{dp(x)}{dx} \end{aligned}\right\}$$
Drift-diffusion current equations

$$\left.\begin{aligned} J_n &= \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right) \\ J_p &= \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1\right) \end{aligned}\right\}$$
Diode diffusion currents

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$
Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT}\right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$
Minority carrier injection under bias V

$$\delta c = \Delta c \exp\left(\frac{-x}{L}\right) \text{ with } \begin{cases} \delta c = \delta p_n \text{ or } \delta n_p \\ \Delta c \text{ the excess carrier concentration} \\ \text{at the edge of the depletion region} \end{cases}$$
Excess carrier concentration as a function of distance

$$L = \sqrt{D\tau}$$
Diffusion length

$$D = \frac{kT}{e}\mu$$
Einstein relation

$$C_{diff} = \frac{e}{kT} I\tau$$
Diffusion capacitance

$$i(t) = \frac{Q(t)}{\tau} + \frac{dQ(t)}{dt}$$
Time variation of current and charge.

$$\tau_t = \frac{W_B^2}{2D}$$
Base transit time

$$\delta c_B = C_1 \exp\left(\frac{x}{L_B}\right) + C_2 \exp\left(\frac{-x}{L_B}\right)$$

$$C_1 = \frac{c_B(W_B) - c_{B_0} - \left(c_B(0) - c_{B_0}\right)\exp\left(\frac{-W_B}{L_B}\right)}{2\sinh\left(\frac{W_B}{L_B}\right)}$$

$$C_2 = \frac{\left(c_B(0) - c_{B_0}\right)\exp\left(\frac{-W_B}{L_B}\right) - \left(c_B(W_B) - c_{B_0}\right)}{2\sinh\left(\frac{W_B}{L_B}\right)}$$

Excess minority carrier concentration in base of BJT. EB junction at $x=0$; BC junction at $x=W_B$. c_{B0}: equilibrium concentration. L_B: minority carrier diffusion length in base.

$$w_n = \sqrt{\frac{2\varepsilon}{e}\frac{N_A}{N_A N_D + N_D^2}(V_0 - V)}$$

$$w_p = \sqrt{\frac{2\varepsilon}{e}\frac{N_D}{N_A N_D + N_A^2}(V_0 - V)}$$

Depletion regions in pn diode

$$W_{depl} = \sqrt{\frac{2\varepsilon}{e}\frac{N_A + N_D}{N_A N_D}(V_0 - V)}$$

The Questions

1.

a) The current in pn diodes and BJTs at low forward bias conditions is determined by minority carriers because of (choose one of the following):

i) drift of minority carriers through the neutral device regions.

ii) generation of minority carriers in the neutral device regions.

iii) diffusion of minority carriers through the neutral device regions.

iv) generation of majority carriers in the neutral device regions.

Explain your answer briefly. [5]

b) Short and long material approximations are simplifications of the correct solution of the continuity equation for minority carriers. The three solutions are plotted in Fig. 1.1 for a 300 nm long material. The mobility of the minority carriers is $\mu = 120$ cm^2 V^{-1} s^{-1} and their lifetime is $\tau = 2.88\ 10^{-10}$ s. Which of the approximations gives the smallest error in the current calculation? Explain your answer briefly. [5]

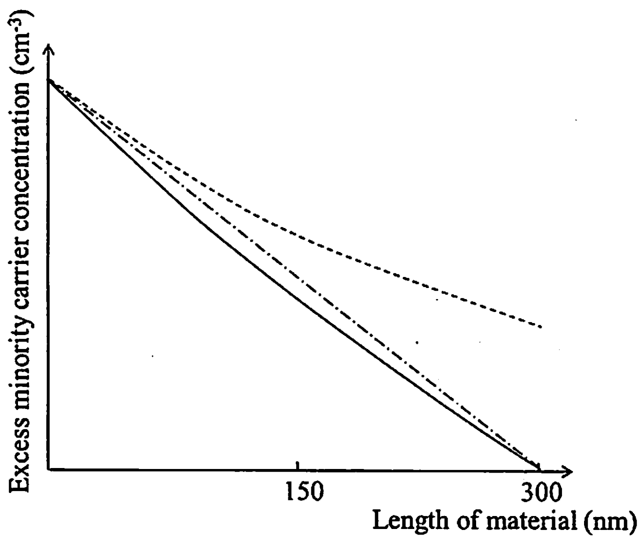


Figure 1.1 Excess minority carrier concentrations. The solid line shows the correct solution; the dashed-dotted line shows the linear approximation and the dashed line shows the exponential approximation.

c) The measurement of the time response of the pn-diode switched from forward to reverse bias is given in Fig. 1.2 (see next page). Sketch, on one figure, the excess minority carrier concentration in one layer of the pn diode as a function of distance, at times $t = t_1$, $t = 0$, $t = t_{sd}$ and $t = t_2$. Assume that the layers are long. [5]

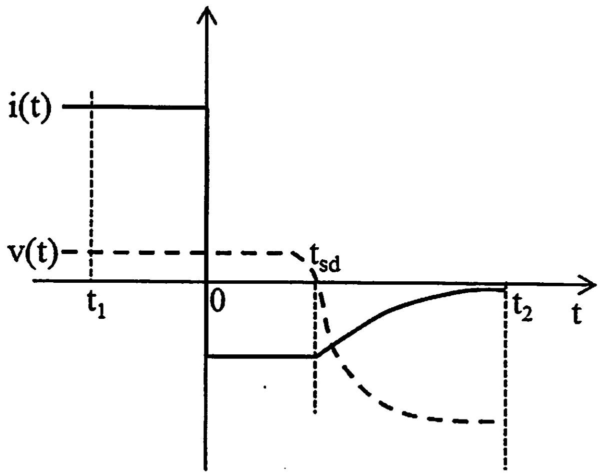


Figure 1.2 The variation of the current *i(t)* and voltage *v(t)* of a switching pn diode.

d) Fig. 1.3 gives a sketch of the minority carrier concentration in the base layer of a pnp BJT.

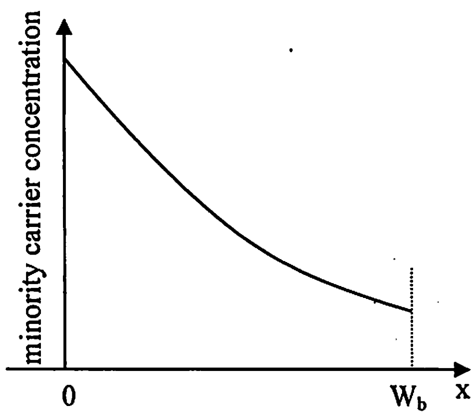


Figure 1.3 Variation of the excess minority carrier concentration in the base.

Copy Fig. 1.3 and add the correct label on the y-axis (δp_n or δn_p). Use your sketch and the definition of diffusion current to explain how the recombination current can be calculated. [5]

2.

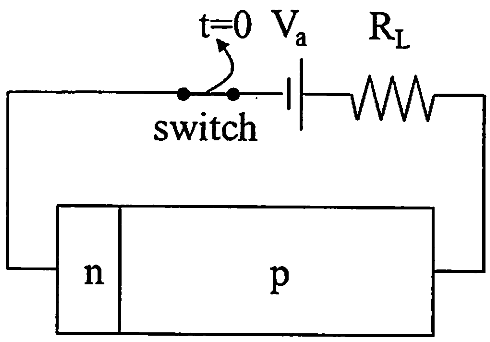


Figure 2.1 Bias circuit for a pn diode with switch.

Table 2.1: Material parameters for the diode in Fig. 2.1.

Material parameters	n-region	p-region
Doping density (cm^{-3})	8×10^{19}	2×10^{17}
Region length (μm)	0.25	500
Minority carrier lifetime (s)	9×10^{-10}	1×10^{-6}
Minority carrier diffusion length (cm)	3×10^{-5}	5×10^{-3}
Cross sectional area (μm^2)	100	100

Temperature is $T = 300$K.

a) Calculate the current flowing through the diode of Fig. 2.1 when a forward bias voltage of $V_d = 0.5$ V appears across the diode. [5]

b) At $t = 0$, the switch in the circuit of Fig. 2.1 is opened. Derive the expression of the time variation of the voltage across the diode, $v_d(t)$. Define all parameters and explain all assumptions you make. [7]

Hint: sketch the excess minority carrier concentration variation. The excess charge concentration variation is : $Q(t) = I\,\tau \exp(-t/\tau)$.

c) Calculate the time needed for the voltage across the diode to settle to 0. [3]

3.

a) Explain briefly the meaning of the term bipolar in the name of the device: bipolar junction transistor (BJT). [3]

b) Assume that an npn BJT has an emitter efficiency of $\gamma = 1$ and that the base is short though recombination is taking place. If the excess minority carrier charge in the base is given by Q_B, give the expression for the base current I_B and the collector current I_C as a function of Q_B. Define all parameters that you use. [5]

c) Calculate the current gain, β in the npn BJT in forward active mode with $\gamma =$ 1. The material parameters are listed in Table 3.1. Take the influence that the reverse biased BC junction has into account. The applied voltages are $V_{EB} =$ 0.5 V and $V_{BC} =$ -1V. The temperature is $T = 300$ K. [7]

Table 3.1 Material parameters of the bipolar junction transistor.

Material parameters	emitter	base	collector
Doping density (cm^{-3})	10^{20}	10^{17}	5×10^{16}
Region length (μm)	0.1	1	500
Diffusion constant ($cm^2\ s^{-1}$)	2	6	20
Minority carrier lifetime (s)	1×10^{-10}	1×10^{-6}	1×10^{-2}
Cross sectional area (μm^2)	100	100	100

The Answers

1.

a)

iii) diffusion of minority carriers through the different device regions

Majority carriers are injected across the junction in forward bias because the **potential barrier decreases** in forward bias. After injection, they become minority carriers. Since the voltage drops occurs mainly across the junction, the movement of these minority carriers in the neutral regions is due to diffusion to the contacts. The contacts have infinite generation and recombination capabilities and thus keep the minority carrier concentration there at the bulk value, creating the **minority carrier gradient**. For all normal pn diode operations the variation in majority carrier concentration can be neglected. [5]

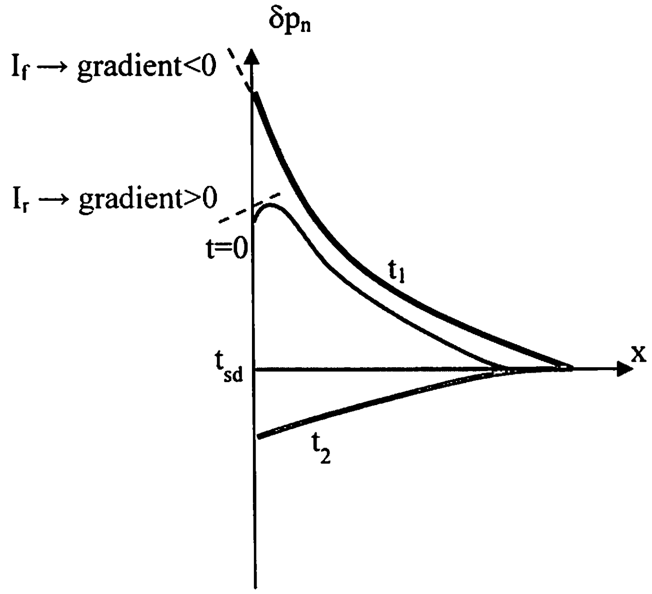
b) First calculate the minority carrier diffusion length. From the formulae list:

$L = \sqrt{D\tau}$ and $D = V_T \mu = 0.026V \times 120 cm^2 V^{-1} s^{-1} = 3.12 cm^2 s^{-1}$

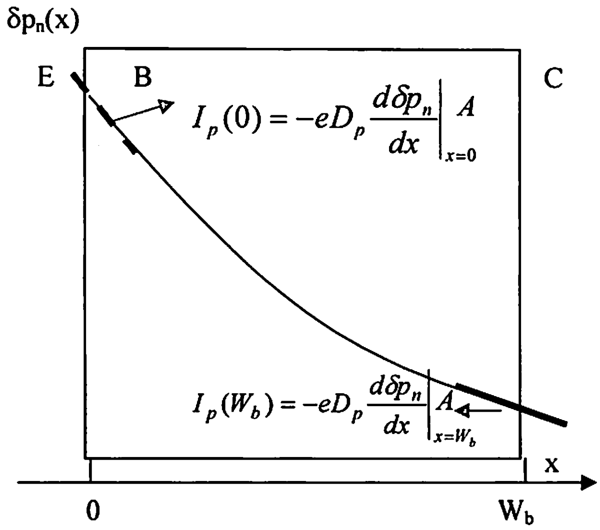
$L = \sqrt{3.12 \times 2.88\, 10^{-10}} cm \approx 3\, 10^{-5} cm = 300\ nm$

The minority carrier diffusion length is equal to the length of the material. In that case the error due to the long material and short material approximation is exactly the same. [5]

c) [5]



d) [5]



The hole current injected across the EB junction at x=0 is proportional to the slope of the excess minority carrier concentration at x=0. The hole current injected across the BC junction is given by the slope of the minority carrier concentration at $x=W_b$. These slopes are different in the case recombination is taken into account. The difference between these two hole currents is the hole current recombined in the base layer.

2.

Table 2.1: all parameters in same unit - cm.

Material parameters	n-region	p-region
$N_{D,A}$ (cm^{-3})	8×10^{19}	2×10^{17}
$X_{n,p}$ (cm)	2.5×10^{-5}	5×10^{-2}
$\tau_{p,n}$ (s)	9×10^{-10}	1×10^{-6}
$L_{p,n}$ (cm)	3×10^{-5}	5×10^{-3}
$D_{p,n}$ ($cm^2\ s^{-1}$) = L^2/τ	1	25
A (cm^2)	1×10^{-6}	1×10^{-6}

a)

n-region is short, p-region is long.

Hole current in n-region:

$$I_p = \frac{e\,D_p\left(p'_n - p_{n_0}\right)A}{X_n} \approx \frac{e\,D_p p'_n\,A}{X_n} = \frac{e\,D_p p_{n_0}\exp\left(\frac{V_d}{V_T}\right)A}{X_n} = \frac{e\,D_p n_i^2\exp\left(\frac{V_d}{V_T}\right)A}{N_D X_n}$$

$$I_p = \frac{1.6\times10^{-19}C\,1\frac{cm^2}{s}\left(1.45\times10^{10}\right)^2 cm^{-6}\exp\left(\frac{0.5}{0.026}\right)10^{-6}cm^2}{8\times10^{19}cm^{-3}\,2.5\times10^{-5}cm} = 3.78\times10^{-12}A$$

Electron current in p-region:

$$I_n \approx \frac{e\,D_n n_{p_0}\exp\left(\frac{V_d}{V_T}\right)A}{L_n} = \frac{e\,D_n n_i^2\exp\left(\frac{V_d}{V_T}\right)A}{N_A L_n}$$

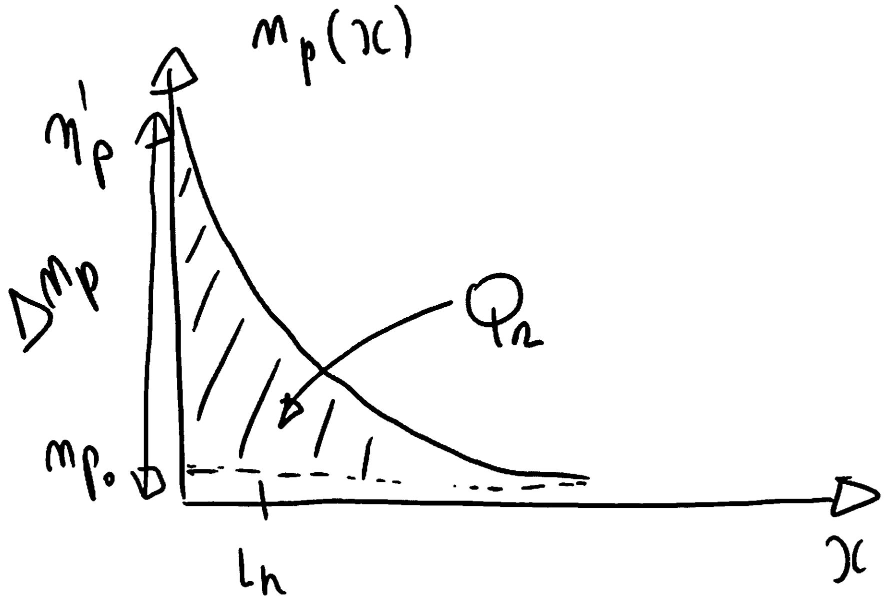
$$I_n \approx \frac{1.6\times10^{-19}C\,25\frac{cm^2}{s}\left(1.45\times10^{10}\right)^2 cm^2\exp\left(\frac{0.5}{0.026}\right)10^{-6}cm^2}{2\times10^{17}cm^{-3}\;5\times10^{-3}cm} = 1.89\times10^{-10}A \qquad [5]$$

$$I_{tot} = I_p + I_n = 1.93\times10^{-10}A \approx I_n = 1.89\times10^{-10}A$$

b)

In the calculation of the forward bias current we see that the electron current in the n region is 100× larger than the hole current. Thus we assume the largest time delay is associated to the largest current component as this holds the largest stored minority carrier charge. The switching experiment is to open circuit, thus charges are removed via recombination. We also see that the recombination time of the electrons in the p-region is much larger than that of holes in the n-region thus the assumption that recombination in the p-region is determining the delay seems appropriate.

Plotting the minority carrier variation in the p-region:



Assume that the long material approximation is acceptable then the excess carrier concentration in the p-layer is given by the formula from the formulae sheet:

$$\delta n_p = \Delta n_p \exp\left(\frac{-x}{L_n}\right)$$

The charge in the p-type region can be calculated by integrating the above equation. However, since we have assumed the long material approximation, we need to put the contact at infinite in the integration.

$$Q_n = -e \times A \times \int_0^\infty \Delta n_p \exp\left(\frac{-x}{L_n}\right) dx = -e \times A \times \Delta n_p \times \int_0^\infty \exp\left(\frac{-x}{L_n}\right) dx$$

$$Q_n = e \times A \times \Delta n_p \times L_n \times \exp\left(\frac{-x}{L_n}\right)\Bigg|_0^\infty = e \times A \times \Delta n_p \times L_n \times (0-1) = -e \times A \times \Delta n_p \times L_n$$

from the formulae list:

$$\Delta n_p = n'_p - n_{p0} = n_{p0}\left(\exp\left(\frac{eV}{kT}\right) - 1\right)$$

If we neglect the change in slope @ x=0 in $n_p(x)$, we can write:

$$Q_n(t) = -e \times A \times L_n \times n_{p0}\left(\exp\left(\frac{e\,v(t)}{kT}\right) - 1\right)$$

On the other hand, the time variation of the charge is given by:

$$Q_n(t) = I_n \tau_n \exp\left(\frac{-t}{\tau_n}\right)$$

$$I_n \tau_n \exp\left(\frac{-t}{\tau_n}\right) = e \times A \times L_n \times n_{p0}\left(\exp\left(\frac{e\,v(t)}{kT}\right) - 1\right)$$

$$\exp\left(\frac{e\,v(t)}{kT}\right) = \frac{I_n \tau_n}{e \times A \times L_n \times n_{p0}} \exp\left(\frac{-t}{\tau_n}\right) + 1$$

$$\frac{e\,v(t)}{kT} = \ln\left(\frac{I_n \tau_n}{e \times A \times L_n \times n_{p0}} \exp\left(\frac{-t}{\tau_n}\right) + 1\right)$$

$$v(t) = V_T \ln\left(\frac{I_n \tau_n}{e \times A \times L_n \times n_{p0}} \exp\left(\frac{-t}{\tau_n}\right) + 1\right)$$

c) Calculate the time needed for the voltage across the diode to become 0. [3]

In principle this could be calculated from:

$$0 = V_T \ln\left(\frac{I_n \tau_n}{e \times A \times L_n \times n_{p0}} \exp\left(\frac{-t_{OFF}}{\tau_n}\right) + 1\right)$$

However ln(y) only becomes zero for y = $-\infty$ this for $t_{OFF} \rightarrow +\infty$

3.

a) The term bipolar means that both holes and electrons (thus both carrier types) play a role in the description of the current of the device. [3]

b) $\gamma = 1$ means that there is no current from base into emitter. The base current is then only composed of recombination re-supply current. In an npn BJT the base current are holes, the collector and emitter current electrons.

$$I_C = I_n = \frac{Q_n}{\tau_t}$$

$$I_B = I_p = \frac{Q_n}{\tau_n}$$

Q_n is the minority carrier excess charge (electrons) in the base.

τ_t is the transit time – the time it takes the injected electron to cross the base from emitter to collector junction.

τ_n is the lifetime of the minority carriers in the base. Is the average time the minority carriers spend in the base before recombining. [5]

c) Once it is established that collector and base current can be expressed in simple expressions as given in b), the current gain $\beta = \frac{I_C}{I_B} = \dfrac{\frac{Q_n}{\tau_t}}{\frac{Q_n}{\tau_n}} = \frac{\tau_n}{\tau_t}$.

The transit time formula is given in the formulae sheet:

$$\tau_t = \frac{W_B^2}{2D_n}$$

The effective base width will be the metallurgic base width minus the depletion width extending into the base region. The depletion region expressions are given in the formulae sheet:

$$w_p = \sqrt{\frac{2\varepsilon}{e}\frac{N_D}{N_A N_D + N_A^2}(V_0 - V)}$$

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.026V \ln\left(\frac{10^{17} \times 5\times10^{16}}{(1.45\times10^{10})^2}\right) = 0.80V$$

$$w_p = \sqrt{\frac{2\varepsilon}{e}\frac{N_D}{N_A N_D + N_A^2}(V_0 - V)} = \sqrt{\frac{2\times11\times8.85\times10^{-14}}{1.6\times10^{-19}}\frac{5\times10^{16}}{5\times10^{16}\times10^{17}+10^{34}}(0.8-(-1))}$$

$$w_p = 8.5\times10^{-6}\,cm$$

The base width becomes: $W_B = 10^{-4} - 8.5\times10^{-6} = 9.15\times10^{-5}\,cm$

The transit time:

$$\tau_t = \frac{W_B^2}{2D_n} = \frac{\left(9.15\times10^{-5}\right)^2}{2\times6} = 6.98\times10^{-10}\,s$$

The current gain:

$$\beta = \frac{\tau_n}{\tau_t} = \frac{10^{-6}}{6.98\times10^{-10}} = 1433$$

Table 3.1 Material parameters of the bipolar junction transistor in cm.

Material parameters	emitter	base	collector
Doping density (cm^{-3})	10^{20}	10^{17}	5×10^{16}
Region length (cm)	0.1×10^{-4}	1×10^{-4}	5×10^{-2}
Diffusion constant ($cm^2\ s^{-1}$)	2	6	20
Minority carrier lifetime (s)	1×10^{-10}	1×10^{-6}	1×10^{-2}
Cross sectional area (cm^2)	1×10^{-6}	1×10^{-6}	1×10^{-6}

Examination Paper Submission document for 2012-2013 academic year.

For this exam, please write the main course code and the course title below.

Code: EE2-10A

Title: Devices

We, the exam setter and the second marker, confirm that the following points have been discussed and agreed between us.

1. There is no full or partial reuse of questions.
2. This examination yields an appropriate range of marks that is well balanced, reflecting the quality of student (with weak students failing, capable students getting at least 40% and bright industrious students obtaining more than 70%)
3. The model answers give a fair indication of the amount of work needed to answer the questions. Each part has a comment indicating to the external examiners the nature of the question; i.e. whether it is bookwork, new theory, a new theoretical application, a calculation for a new example, etc.
4. The exam paper does not contain any grammar and spelling mistakes.
5. The marking schedule is shown in the answers document and the resolution of each allocated mark is better than 3/20 for each question.
6. The examination paper can be completed by the students within time allowed.

Signed (Setter): **Signed (Second Marker):** E. Shamonina

Date: 02/04/2013 **Date: 02/04/2013**

Please submit this form with exam paper and model answers, and associated coursework to the Undergraduate Office on Level 6 by the required submission date.