EE2-08B

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE PART II: MEng, BEng and ACGI

MATHEMATICS 2B (E-STREAM AND I-STREAM)

Friday, 29 May 2:00 pm

Time allowed: 1:30 hours

Corrected Copy

There are TWO questions on this paper.

Answer TWO questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible | First Marker(s) : B. Clerckx
Second Marker(s) : D. Nucinkis

THE QUESTIONS

[30]

1. a) Consider the continuous random variable X characterized by the following probability density function

$$f_X(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & otherwise. \end{cases}$$

i) Compute the cumulative distribution function of X, i.e. $F_X(x)$.

[2]

ii) Compute the expectation of X, i.e. $E(X)$, and the variance of X, i.e. $\text{Var}(X)$.

[4]

iii) Compute the moment generating function of X, i.e. $m_x(t)$. Explain how to make use this function to find the expectation and variance of a random variable. Apply this principle to X.

[4]

iv) By making use of Chebyshev's Inequality, determine a bound on

$$P\left(\left|X - \frac{1}{3}\right| \geq \frac{1}{4}\right).$$

Compute then the exact value of this probability.

[4]

b) Consider the continuous random variable X characterized by the following probability density function

$$f_X(x) = \begin{cases} \frac{2x}{\theta^2}, & 0 \leq x \leq \theta, \\ 0, & otherwise. \end{cases}$$

We observe the random sample $X_1, \ldots, X_n$ (of size n).

i) Determine the method of moment estimator of θ (denoted as $\tilde{\theta}$).

[4]

ii) Compute the expectation and the variance of this estimator.

[4]

iii) Is this estimator biased or unbiased? Provide your reasoning.

[4]

iv) Assume n large. Compute an estimate of $P\left(\tilde{\theta} \geq \theta\right)$. Provide your reasoning. [4]

[20]

2. Consider two continuous random variables X and Y characterized by the following joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-(x+y)}, & 0 < x < y < +\infty, \\ 0, & otherwise. \end{cases}$$

a) Compute the expectation of X, i.e. $E(X)$.

[4]

b) Make the change of variables $U = Y - X$, $V = X$ and compute the joint probability density function $f_{U,V}(u,v)$.

[4]

c) Compute the marginal probability density function of U and V, i.e. $f_U(u)$ and $f_V(v)$.

[2]

d) Are U and V independent? Provide your reasoning.

[2]

e) Compute the conditional probability density function of U given V, i.e. $f_{U|V}(u|v)$.

[2]

f) Compute the conditional expectation of U given V, i.e. $E(U|V)$.

[2]

g) By making use of f), compute the conditional expectation of Y given X, i.e. $E(Y|X)$.

[2]

h) By making use of g), compute the expectation of Y, i.e. $E(Y)$.

[2]