IMPERIAL COLLEGE LONDON

EE4-05
EE9-CS7-22
EE9-SO7

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE PART IV: MEng and ACGI

TRAFFIC THEORY & QUEUEING SYSTEMS

Corrected Copy

Tuesday, 5 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : J.A. Barria
Second Marker(s) : D.P. Mandic

Special instructions for students

1. Erlang Loss formula recursive evaluation:

$$E_N(\rho) = \frac{\rho E_{N-1}(\rho)}{N + \rho E_{N-1}(\rho)}$$

$$E_0(\rho) = 1$$

2. Engset Loss formula recursive evaluation (for a fixed M and $p = \alpha/1+\alpha$):

$$e_N = \frac{(M-N+1)\alpha e_{N-1}}{N+(M-N+1)\alpha e_{N-1}}$$

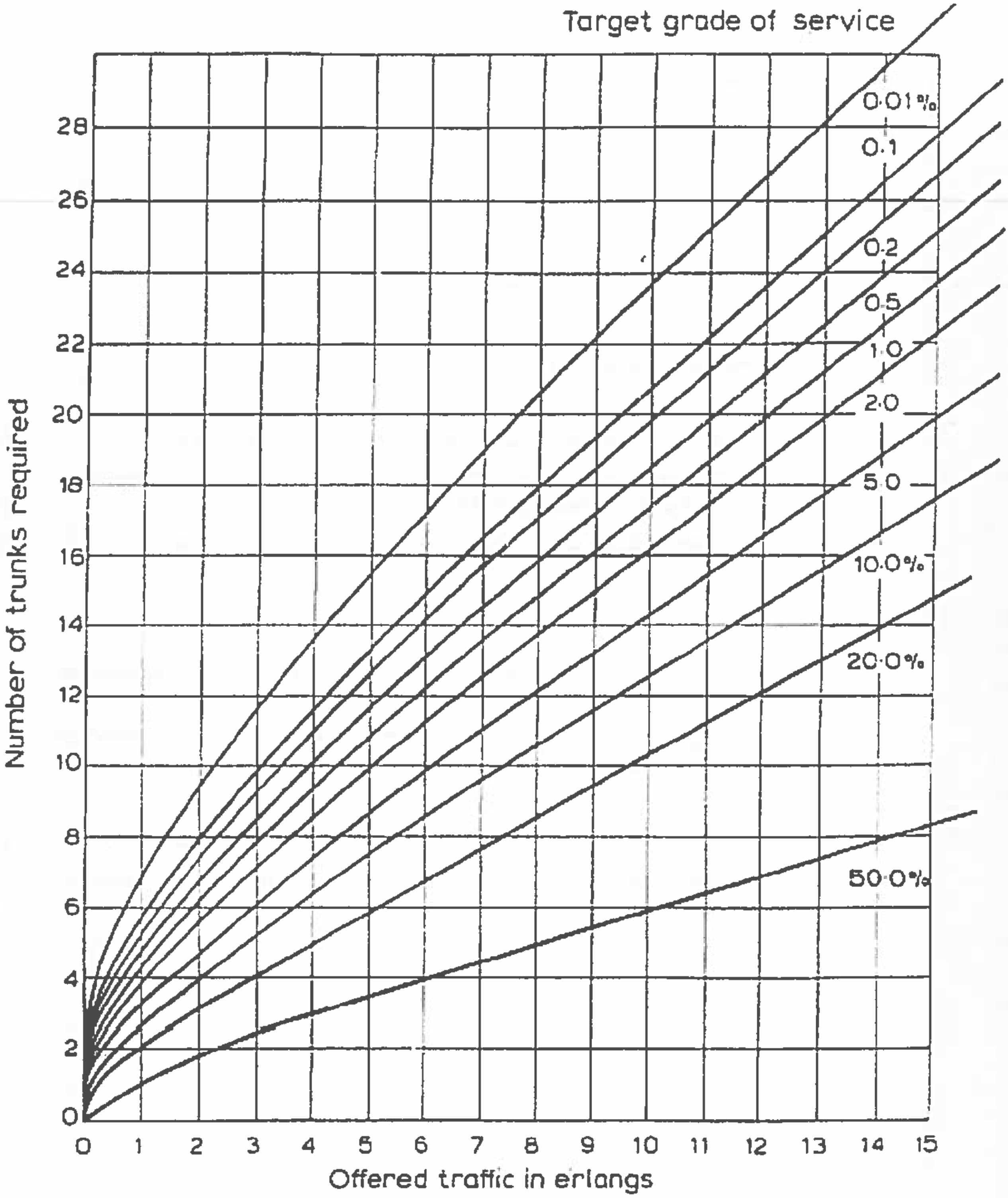
$$e_0 = 1$$

$$\alpha = \lambda/\mu$$

3. Traffic capacity on basis of Erlang B formula (next page).

4. Expected residual time

$$E[R] = \frac{1}{2}\lambda E[S^2] = \frac{1}{2}\sum_{k=1}^{m}\lambda_k E[S_k^2]$$



Traffic capacity on basis of Erlang B. formula.

The Questions

1.

a) A switching exchange serves three communities with the following traffic activity characteristics:

Community	Calls per hour
A	60
B	140
C	80

The average duration of a call is 3 minutes.

i) Determine the total offered traffic to the switching exchange. [3]

ii) Estimate the numbers of channels of the exchange link if it is operating with a loss probability of 0.002. [4]

ii) Determine the total traffic carried by the switching exchange. [3]

b) Poisson traffic is offered to a single communication channel via an infinite FIFO buffer.

Assume that the average rate of arrival of the Poisson stream is 170 calls per hour and that the average duration of the calls is 25 seconds.

i) Determine the value of the buffer mean queue length. [3]

ii) Determine the value of the mean waiting time in the system. [3]

iii) Derive the expression for the variance of the queue length and determine its value. [4]

2.

a) For an $M/M/K$ queueing system, a fraction s of the departing items from such a system are fed back to the input with zero delay.

i) Show that the closed-loop system will only be stable if $s \le (1-\rho)$, where ρ is the offered traffic per channel entering the system. [4]

ii) For $K = 1$, show that the above feedback increases the equilibrium mean number of items in the systems by a factor of $(1-\rho)/(1-\rho-s)$. [6]

b) A Poisson stream of messages for which the lengths are approximately exponentially distributed is offered to a 64 Kbit/s single-channel transmission link which is buffered by a FIFO infinite buffer.

The total traffic to be transmitted consists of:

(1) Messages that have mean arrival rate of 4800 messages per minute, and the mean message length is 700 bits.

(2) A stream of acknowledgments with mean arrival rate of 80 acknowledgements per second; each acknowledgement is 64 bits long.

Determine the mean waiting time of a message assuming that the acknowledgements have non-pre-emptive priority over the messages. [10]

3.

a) Derive the expression for the expected waiting time of an $M/D/1$ queueing system (D stand for deterministic service time) using mean value analysis.

State clearly all intermediate steps and assumptions made. [8]

b) Figure 3.1 represents a fluid flow model of N multiplexed independent ON-OFF sources.

Denote by $F_i(t,x)$ the probability distribution function at time t, with the buffer occupancy of the system in state x and i sources in a talk spurt.

i) Derive an analytical expression for $\partial F_i(t,x)/\partial t$. [6]

ii) Assuming that the stationarity conditions hold, derive the statistical equilibrium equations for $\partial F_i(x)/\partial x$. [4]

iii) Comment on the characteristic of the solution to the equations derived in ii). [2]

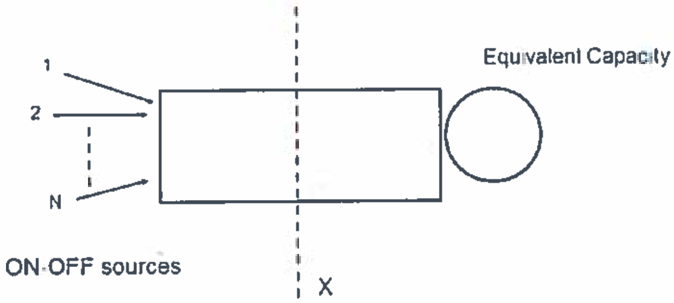


Figure 3.1.

4.

a) Using the ON-OFF source model shown in Fig. 4.1:

i) Define and derive a simple expression of equivalent capacity in terms of the mean and standard deviation of N ON-OFF multiplexed sources. [5]

ii) For an N ON-OFF source multiplexer, derive an approximation to the probability of being in an overflow state:

$$E = \sum_{i=J_0}^{N} \pi_i$$

where π_i is the probability that the system is in state i, and J_0 is the overload state. [5]

Hint: Assume a large number $(N >> 1)$ of ON-OFF multiplexed sources, each one of these sources having a small probability of being in the ON state $(p << 0)$.

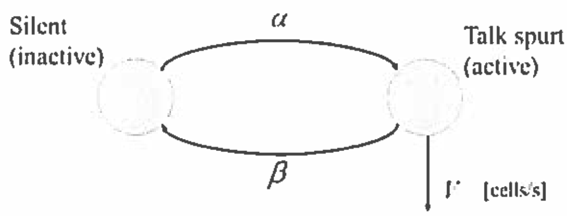


Figure 4.1

b) An on-line Internet service can fail due to two possible events: i) the service underlying process crashes at a rate γ_p, or the node hosting the process fails at a rate γ_n.

If the process crashes, an attempt is made to restart the process at a rate τ_p and succeeds with probability s. If the restart is unsuccessful (with probability $(1-s)$), an attempt to repair the node is made at a rate τ_n.

If a node fails, the node's average repair time is $1/\tau_n$.

i) Define the state space of the system. [3]

ii) Derive the Markov chain representation of the system. [4]

iii) Derive the generator matrix of the Markov chain of the system. [3]