UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2001

MEng Honours Degree in Information Systems Engineering Part IV
MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C480=I4.42

AUTOMATED REASONING

Thursday 17 May 2001, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

(In all questions variables begin with lower case u - z; other names are constants. Predicates use upper case.)

1 a i) Draw the initial connection graph (call it G) for the clauses (1) - (6)

(1) $P(x) \vee R(y) \vee \neg Q(x,y)$
(2) $S(a)$
(3) $\neg S(z) \vee \neg R(z)$
(4) $\neg P(f(a)) \vee \neg P(f(b))$
(5) $P(u) \vee Q(f(u),u)$
(6) $S(f(a))$

ii) What is the purity reduction rule?

iii) Continually process G, according to the connection graph proof procedure, to eventually derive G', by applying the purity rule and by selecting links that *either* (1) lead to a pure resolvent *or* (2) lead to both parent clauses becoming pure.

iv) What is the significance of the form of the graph G' so obtained?

b Together with either the Model Generation or Davis Putnam procedure for ground clauses the Hyper-linking procedure can be used to detect unsatisfiability of a set of general clauses S by applying these two steps:

- Find a set G of ground instances of S using *Hyper-linking*.
- Apply either the *Model Generation* or *Davis Putnam* procedure to refute G.

i) Find all hyper-link instances of the clauses (1) - (4) of part (a) together with the clauses (7) and (8) below. Explain how you obtain your answer.

(7) $Q(f(u),u) \vee P(f(u))$ (8) $S(b)$

Let the set of ground instances be called SG. (There are 9 in all.)

ii) Apply *either* the Model Generation *or* the Davis Putnam procedure to SG as found in part (bi) to show unsatisfiability of clauses (1) - (4), (7), (8).

iii) Use your answer to part (bii) to find a model of the clauses in SG-{(8)} (i.e. the clauses in SG without clause (8)). Justify your answer.

The two parts carry, respectively, 40%, 60% of the marks.

2 a i) What are a rewrite rule and a rewriting step?

With respect to a given set of rewrite rules

ii) what is a *normal form* of a term?

iii) under what conditions does each term have a unique normal form? Why?

b i) Why is the set of rules {(1), (2)} locally confluent and terminating?

(1) v(f(x)) => f(v(x))
(2) v(a) => a

ii) Show that {(1), (2), (3)} is not locally confluent but is still terminating.

(3) v(v(x)) => f(x)

iii) Apply a method to generate a set of locally confluent and terminating rewrite rules from {(1), (2), (3)}. Your answer should make clear the steps of the method.

iv) Use these rules to decide that $v(f(v(b))) \neq f(b)$, briefly justifying your answer.

c Define paramodulation.
Give all the possible results of paramodulating f(a)=b into P(x,f(x)).

The three parts carry, respectively, 25%, 60%, 15% of the marks.

3 a i) What are the Hyper-resolution and Locking refinements of resolution ?

ii) Explain how indices can be used to achieve a lock-resolution proof that is also a hyper-resolution proof.

How can the indices be used to further restrict the use of electrons?

iii) Use the combination of methods of part (aii) to find a restricted hyper-resolution proof of the empty clause from the clauses (9) - (14)

(9)	$\neg P(u) \vee \neg Q(f(v),v)$
(10)	$S(a)$
(11)	$P(f(a)) \vee P(f(b))$
(12)	$\neg S(z) \vee \neg R(z)$
(13)	$\neg P(x) \vee R(y) \vee Q(x,y)$
(14)	$S(b)$

b Consider the following (additional) clausal deduction rule called (Division) that may be applied to a set of clauses S:

Let $C = D \vee E$, where D and E are clauses of one or more literals, be a clause in S such that the variables in D and E are distinct. Form the two sets of clauses $S' = (S - \{C\}) \cup \{D\}$ and $S'' = (S - \{C\}) \cup \{E\}$

i) Show that if S has a Herbrand model then either S' or S'' has one too.

ii) Use resolution and the Division rule to derive the empty clause from the clauses (9) - (14) of part (aiii).

The two parts carry, respectively, 60% , 40% of the marks.

4 a i) How does the application of the universal instantiation rule differ between its usage in free variable tableaux and standard tableaux?

With respect to *free variable clausal* tableaux:

ii) define the closure step;

iii) how is the extension step restricted in the Model Elimination (ME) procedure?

iv) Find a closed ME tableau for the clauses (15) - (20).

(15) $\neg T(x,y) \vee R(c,y)$
(16) $T(f(a,d),f(a,b))$
(17) $R(x,d)$
(18) $\neg T(x,y) \vee \neg R(c,x)$
(19) $E(x,y) \vee \neg R(x,z) \vee R(x,f(y,z))$
(20) $\neg E(u,v) \vee \neg R(u, f(v,w))$

b i) Describe the clausal tableau refinement called the "Intermediate lemma extension" and use it to derive a closed tableau for the clauses (15) - (20) of part (aiv).

ii) Briefly, say what you regard as the benefits or drawbacks of the refinement.

The two parts carry, respectively, 50%, 50% of the marks.

