IMPERIAL COLLEGE LONDON

E4.05
C5.22
SO7

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

MSc and EEE PART IV: MEng and ACGI

TRAFFIC THEORY & QUEUEING SYSTEMS

Corrected Copy

Thursday, 15 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : J.A. Barria

Second Marker(s) : M.M. Draief

Special instructions for students

1. Erlang Loss formula recursive evaluation:

$$E_N(\rho) = \frac{\rho E_{N-1}(\rho)}{N + \rho E_{N-1}(\rho)}$$
$$E_0(\rho) = 1.$$

2. Engset Loss formula recursive evaluation (for a fixed M and $p = \alpha/1+\alpha$):

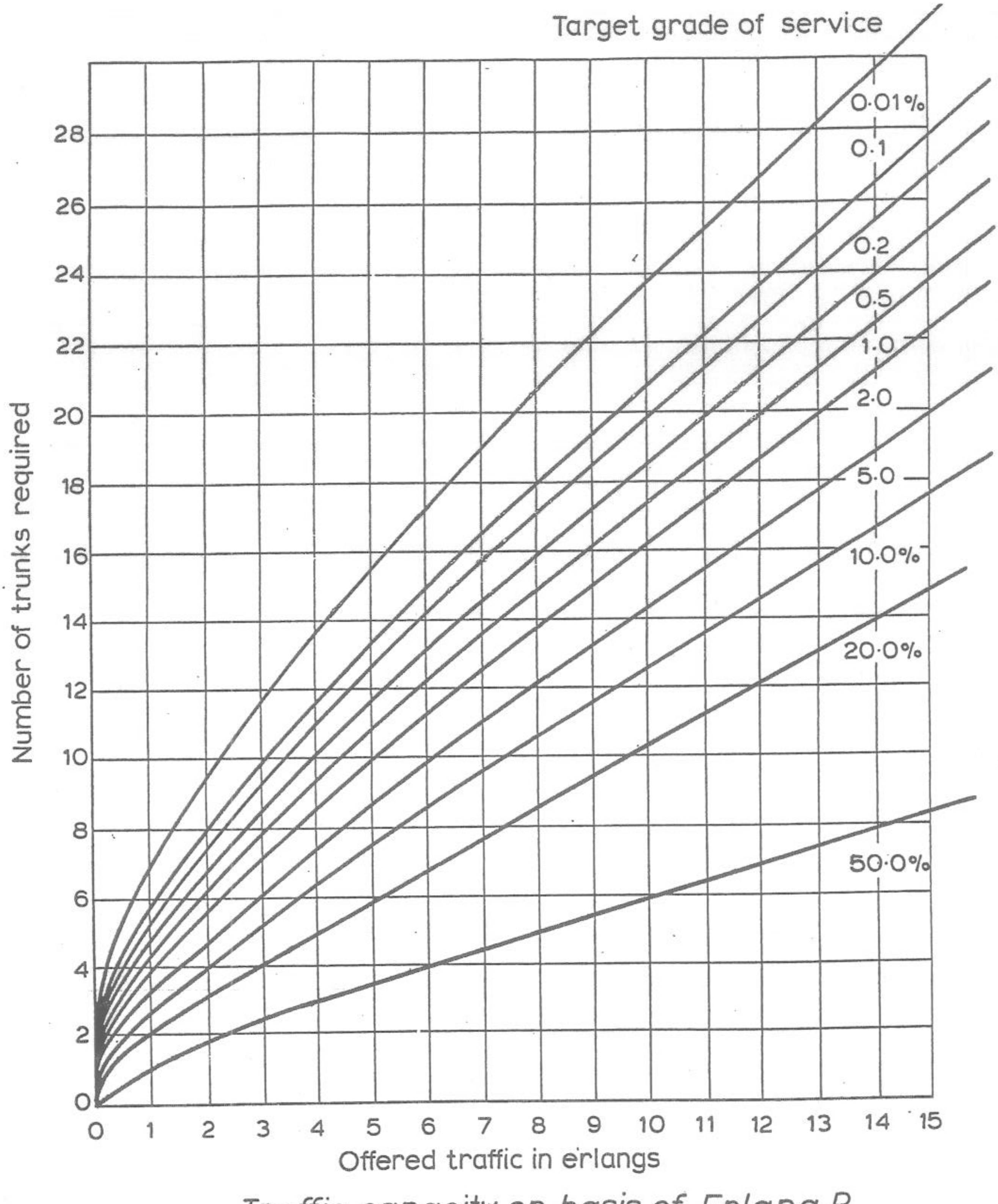
$$e_N = \frac{(M-N+1)\alpha e_{N-1}}{N + (M-N+1)\alpha e_{N-1}}$$
$$e_0 = 1.$$
$$\alpha = \lambda/\mu.$$

3. Traffic capacity on basis of Erlang B formula (next page).

Note: for large ρ, N is approximately linear: $N \approx 1.33\rho + 5$

4. Expected residual time

$$E[R] = \frac{1}{2}\sum_{k=1}^{m} \lambda_k E[S_k^2]$$



Traffic capacity on basis of Erlang B. formula.

1.

a) For the system in Figure 1.1 assume:
- The M sources act independently.
- The call arrival rate from a free source is Poisson with parameter λ.
- The link is composed of N channels.
- The channel holding time is a negative exponential with parameter $1/\mu$.
- Assume M ~ N and that there is full availability access.

i) Show that for $N > M$ the equilibrium state probability distribution π_i is binomial (M, p).

[8]

ii) Using the source model in Fig. 1.2 define and derive p.

[6]

b) Consider the system of part a) and assume the following parameters:
- N = 10 (channels link).
- M = 3 (sources of traffic).
- The call rate of a free source = 0.5 [calls/minute].
- The mean holding time = 2 [minutes].

i) Derive the steady state probability of two (2) circuits occupied π_2.

[6]

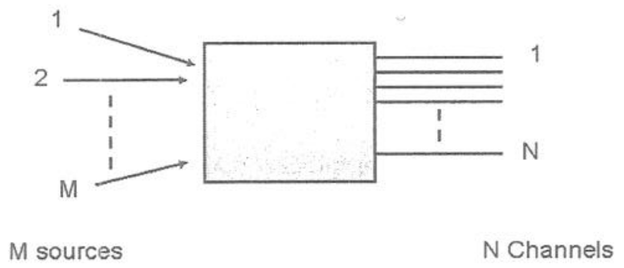


Figure 1.1

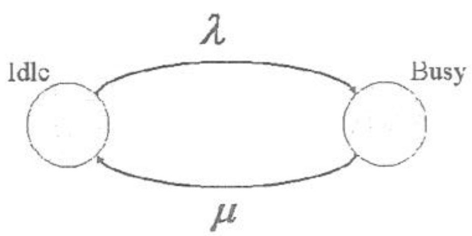


Figure 1.2

2.

a) For an M/M/K system:

i) Derive the probability of delay $P(W > 0)$.

[10]

State clearly all assumptions made.
Explain clearly all steps of your derivations

b) For an M/M/K system:

i) Derive the waiting time distribution $P(W > \tau)$ for delay arrivals.

[10]

State clearly all assumptions made.
Explain clearly all steps of your derivations

3.

a)

i) Define and discuss the usefulness of the Equivalent Capacity function in ATM networks. Highlight the approximations used to derive it.

[5]

ii) Describe and discuss the operations of a Leaky Bucket algorithm in the context of an ATM network.

[5]

b) A Poisson stream of packets arrives to a single-channel communication link at a rate of $\lambda = 200$[packets/s] and are allowed to join a large buffer size with FIFO queue discipline with transmission rate 65[Kbits/s].

The arrivals consist of a random mixture of the following packet sizes:

Packet size [bits]	Probability of Arrival
160	25 %
80	25 %
320	50 %

i) Determine the packets mean waiting time

[7]

ii) Determine the expected total packet transit time. [3]

4.

a) For an M/G/1 system:

- Derive the expected queue length $E[Q_t]$.

[10]

State clearly all assumptions made.
Explain clearly all steps of your derivations

b) A Poisson arrival stream of calls which have exponential holding times is offered to a 20-channel communications link.

Given that the measured link traffic is 13 Erlangs:

i) Obtain the mean of the carried traffic.

[2]

ii) Obtain the mean channel occupancy.

[2]

iii) If the number of channel is set very high ($N = \infty$) derive the proportion of time that at most two (2) circuits are occupied.

[6]

5.

a) For a non pre-emptive priority system.

$$\begin{aligned}
&\text{Class 1 arrivals}: \lambda_1 = 2[1/s] \quad E[S_1] = 0.2[s] \quad E[S_1^2] = 3[s^2] \\
&\text{Class 2 arrivals}: \lambda_2 = 3[1/s] \quad E[S_2] = 0.1[s] \quad E[S_2^2] = 5[s^2] \\
&\text{Class 3 arrivals}: \lambda_3 = 1[1/s] \quad E[S_3] = 0.1[s] \quad E[S_3^2] = 1[s^2]
\end{aligned}$$

-Derive the expected waiting time of class arrival 1, $E[W_1]$, and class arrival 2, $E[W_2]$.

[10]

b) A multiplexor is being offered the traffic of N independent ON-OFF traffic sources (see Fig. 5.1).

i) Derive and depict its MMPP model representing the N multiplexed sources.

[2]

ii) Derive and depict the state space representation of the multiplexer with service rate ν [cells/s].

[3]

iii) Assume that the service rate is ν [cells/s] and, when one source is active, the arrival is Poisson with rate β [cells/s].
- Derive the maximum number of multiplexed sources allowed into the system.

[5]

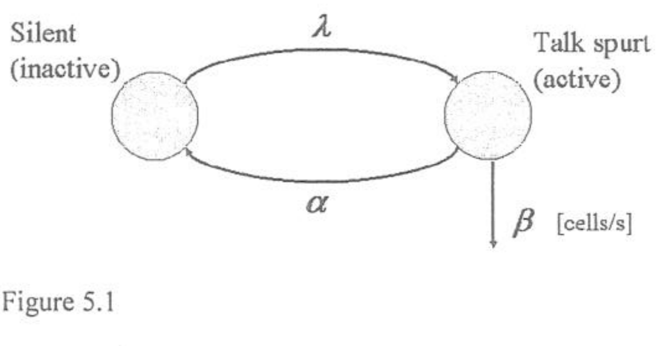


Figure 5.1

6.

a) An N-independent ON-OFF sources multiplexer can be represented by the fluid flow model in Fig 6.1.

i) For the model in Fig 6.1: define the variable x and derive the value of Equivalent Capacity.

[5]

ii) For the model in Fig. 6.1: give the condition for the buffer to be filling.

[5]

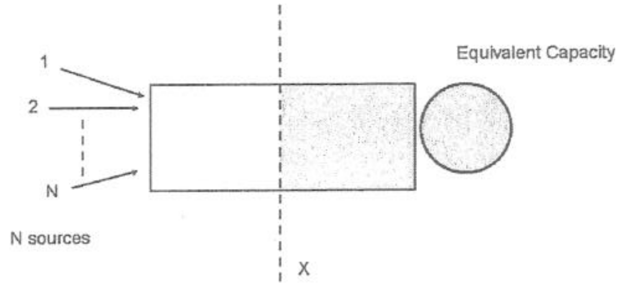


Figure 6.1

b) Assume the following characteristics of a degradable system which consists of two processors working in micro-synchronism:

Hardware failure rate = λ [failures/year]

Software failure rate = γ [failures/year]

Mean time to partial re-boot of the system = r [seconds]

Mean time to solve a hardware problem = h [hours]

- If the system is affected by a software failure, the system needs to perform a partial re-boot to go back to fully operational condition.
- If the system is affected by a hardware failure, after the hardware is resolved the system need to perform a partial re-boot to go back to fully operational condition.

i) Define the state space of the system, and [5]

ii) Derive the associated Markov chain.

[5]