Imperial College London

BSc/MSci EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

LASER TECHNOLOGY

For 4th-Year Physics Students

Friday, 18th May 2012: 10:00 to 12:00

Answer ALL parts of Section A, ONE question from Section B and ONE question from Section C.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the 4 answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in 4 answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

SECTION A

1. (i) The steady-state heat diffusion equation for a solid-state laser rod, assuming uniform heating and radial heat flow, is given by:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{Q}{\kappa}$$

where $T = T(r)$ is the temperature distribution as a function of radial coordinate r relative to the centre of the rod ($r = 0$); Q is the heating power per unit volume and κ is the thermal conductivity of the rod.

(a) Solve this equation to show that the temperature distribution in the rod is parabolic. [2 marks]

(b) Describe briefly the difficulties that this temperature distribution can cause to the operation of the solid-state laser. [2 marks]

(ii) A semiconductor diode laser has band-gap energy 1.9 eV, a series resistance of 0.4 Ω and produces an output of 0.4 W for a drive current of 0.5 A

(a) Calculate the electrical-to-optical efficiency of the diode laser. [2 marks]

(b) Estimate the wavelength of the diode laser and hence suggest what semiconductor material is being used. [2 marks]

(iii) Draw a sketch of a ”double-clad” fibre laser amplifier, labelling the key features of the design, and explain how the structure guides high-power multimode diode pump radiation whilst supporting single mode laser output. [2 marks]

[Total 10 marks]

2. (i) Explain briefly the origin of the nonlinear optical response, using diagrams where necessary. Write down an expression for the polarisation of the medium, P, as a function of the electric field E that is valid in the nonlinear regime, defining all symbols used and giving their units. Estimate the electric field strength required for the second order polarisation to be 0.1% of the linear polarisation in the nonlinear crystal ADP ($NH_4H_2PO_4$). Take $n = 1.5$ and $\chi^{(2)} = 1.5 \times 10^{-12}$ m/V. [4 marks]

(ii) The coupled wave equations for second harmonic generation are

$$\begin{aligned} \frac{d\widetilde{E}_\omega}{dz} &= -c_1\chi^{(2)}\widetilde{E}^*_\omega(z)\widetilde{E}_{2\omega}(z)\exp(-i\Delta kz) \\ \frac{d\widetilde{E}_{2\omega}}{dz} &= -c_2\chi^{(2)}\widetilde{E}^2_\omega(z)\exp(i\Delta kz) \end{aligned}$$

where $\widetilde{E}_\omega(z)$ and $\widetilde{E}_{2\omega}(z)$ are the complex amplitudes for the pump and second harmonic fields, c_1 and c_2 are constants and the other symbols have their usual meanings. Obtain an expression for $\widetilde{E}_{2\omega}(z)$ in the limit of very low pump depletion for a nonlinear medium of length L with $\chi^{(2)}(z) = \chi_0 \exp(-i\Delta kz)$. Comment on your result. [3 marks]

(iii) In a noncollinear sum frequency generation (SFG) scheme, the k-vectors of the input waves are given by $\mathbf{k_1} = k_1(\cos\theta_1\hat{\mathbf{x}} + \sin\theta_1\hat{\mathbf{y}})$, $\mathbf{k_2} = k_2(\cos\theta_2\hat{\mathbf{x}} - \sin\theta_2\hat{\mathbf{y}})$ and the output wave k-vector is $\mathbf{k_3} = k_3\hat{\mathbf{x}}$. Sketch the k-vectors for the phase-matched interaction, clearly labelling the angles involved. If $\theta_1 = 30°$ and $k_1 = \sqrt{3}k_2$, find θ_2 and k_3 . [3 marks]

[Total 10 marks]

SECTION B

3.

A Nd:YAG rod laser with wavelength $\lambda = 1064$ nm is pumped by diode laser radiation at wavelength 808 nm with up to 100 W of diode pump power. The Nd:YAG laser has an output power of 14.4 W at 50 W diode pumping and an output power of 30.4 W at maximum diode pump power.

(i) Assuming a linear power characteristic, what is the threshold pump power and slope efficiency of the laser with respect to diode pump power? [2 marks]

The Nd:YAG laser cavity has plano-concave mirrors, with back mirror with 100% reflectivity and radius of curvature $R = 2$ m and the output coupler is a partially reflecting plane mirror with transmission $T = 6\%$ and the intra-cavity round-trip loss is $L_0 = 2\%$. The cavity length is $L = 1$ m (you can neglect effects of refractive index of laser crystal in your answer). The radius of the laser rod is $r_0 = 0.8\dot{m}$m and is located near the minimum waist location (and rod length can be considered to be negligibly short).

(ii) *Derive* an expression for the size of the minimum beam waist radial size (w_0) in terms of cavity parameters R and L. State the location where the minimum waist occurs and calculate the size of w_0 for this cavity. [4 marks]

(iii) Calculate a value for the quantum efficiency (η_Q) of the diode pumped Nd:YAG laser? [1 mark]

(iv) Estimate the beam overlap efficiency (η_B) of the TEM_{00} mode in the laser rod. [2 marks]

(v) Calculate the "output coupling" efficiency (η_C) of the laser relating to the chosen output coupling transmission T and the intra-cavity loss of the laser L_0. [2 marks]

(vi) Deduce if the slope efficiency of the laser is consistent with the combination of these efficiency factors. [1 mark]

(vii) Calculate the increase in beam overlap efficiency (η_B) of the TEM_{00} mode by moving the laser rod to the location in the cavity where the mode size is largest. [3 marks]

[Total 15 marks]

4. (i) What are the advantages of using a laser for cutting compared to a mechanical tool? [2 marks]

A CO_2 laser used for sheet metal cutting operates at wavelength 10.6 microns with $M^2 = 4$ spatial beam quality parameter and output power 10 kW. It has output beam diameter of 10 mm and is focussed by a lens with focal length 100 mm onto a metal sheet with thickness 2 mm.

(ii) What is the diameter of the focal spot? [2 marks]

(iii) Estimate a value for the depth of focus with which the focal spot needs to be located at the work-piece. [2 marks]

(iv) If an energy per unit volume of 10^{10} J/m^3 is required to vaporise the metal sheet material, at what speed can the sheet be cut by this laser if the metal surface has 40% absorption efficiency? State any assumptions made in your calculation. [4 marks]

(v) What is the danger to the lens used in this cutting system and how can this be reduced? [1 mark]

An excimer laser is used as a cutting tool to perform eye surgery to correct short and long sightedness.

(vi) Explain what this procedure entails. [2 marks]

(vii) State what excimer gas is commonly used and its wavelength. Explain why this laser is chosen for this task. [2 marks]

[Total 15 marks]

SECTION C

5.

Consider a Type I second harmonic generation (SHG) phase-matching scheme given by $e_\omega + e_\omega \rightarrow o_{2\omega}$ where o and e refer to ordinary and extraordinary waves and ω and 2ω are the pump and second harmonic frequencies respectively.

(i) Make a clear sketch of the geometry of the interaction for a practical scheme in a uniaxial crystal, taking care to show the relative polarisation of all fields as well as any angles involved. [3 marks]

(ii) Obtain the phase-matching condition in terms of the refractive indices involved. By making carefully labelled sketches of these indices as a function of frequency for a normally dispersive uniaxial crystal, show that critical phase-matching can be achieved for a positive uniaxial crystal but not for a negative uniaxial crystal. [4 marks]

$LiGaTe_2$ (LGT) is a positive uniaxial crystal. Its Sellmeier equations, valid in the $0.8 - 10\,\mu$m range, are given by

$$\begin{aligned} n_o^2(\lambda) &= 6.24921 + \frac{0.42592}{\lambda^2 - 0.0531} - 0.00149\lambda^2 \\ \bar{n}_e^2(\lambda) &= 6.70825 + \frac{0.5667}{\lambda^2 - 0.01964} - 0.001\lambda^2 \end{aligned}$$

where λ is in μm, n_o is the ordinary refractive index, and $\bar{n}_e$ is the principal value of the extraordinary refractive index.

(iii) Calculate the Type I SHG ($ee - o$) phase-matching angle in LGT for a pump laser wavelength of 4.5μm. You may use that

$$n_e(\omega, \theta) = \left[\frac{\cos^2\theta}{n_o^2(\omega)} + \frac{\sin^2\theta}{\bar{n}_e^2(\omega)}\right]^{-\frac{1}{2}}.$$

[4 marks]

(iv) Briefly explain what is meant by the phase-matching acceptance angle, $\Delta\theta_{acc}$ for SHG. Calculate a numerical value for $\Delta\theta_{acc}$ for the situation described in iii) for a LTG crystal of length $L = 2\,$mm. You may use

$$\Delta\theta_{acc} = \frac{5.568}{L\left|\frac{\partial \Delta k}{\partial \theta}|_{\theta_{PM}}\right|}$$

where θ_{PM} is the phase-matching angle. [4 marks]

[Total 15 marks]

6. (i) Show that even-order nonlinear optical processes are not possible in a centro-symmetric medium. Briefly explain why the intensity-dependent refractive index (IDRI) can occur in any optical medium. [2 marks]

(ii) A laser field $E(t) = E_1 \cos \omega_1 t + E_2 \cos \omega_2 t$ propagates through a medium of nonlinear refractive index n_2. By considering a polarisation response $P = \epsilon_0 \chi^{(1)} E + \epsilon_0 \chi^{(3)} E^3$ show that the field at frequency ω_1 travels with an effective refractive index $n_L + n_2(I_1 + 2I_2)$, where n_L is the low-intensity index and I_j is the intensity of the field at ω_j. You may use $\cos^3 \theta = \frac{3}{4} \cos \theta + \frac{1}{4} \cos 3\theta$, $cos^2\theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$ and

$$n_2 I = \frac{3\chi^{(3)} |E|^2}{8n_L}$$

[4 marks]

(iii) Now consider the phase-modulation arising from the result obtained in (ii). Show that the frequency shift of the field initially at ω_1 after propagating a distance L in the medium us given by

$$\Delta\omega_1 = -\frac{n_2 \omega_1 L}{c} \left[\dot{I}_1(t) + 2\dot{I}_2(t) \right]$$

If I_1 and I_2 are Gaussian pulses given by $I_j(t) = I_{0j} \exp\left(-t^2/\tau_j^2\right)$, make a careful sketch of $\Delta\omega_1$ as a function of time for $I_{01} \approx 4I_{02}$ and $\tau_1 \approx 4\tau_2$. [3 marks]

(iv) Show that for $|t| \ll \tau_j$ the chirp parameter $\beta = d\omega/dt$ for the field initially at ω_1 is given by

$$\beta_1 \approx \frac{2\omega_1 L n_2}{c} \left(\frac{I_1}{\tau_1^2} + \frac{2I_2}{\tau_2^2} \right)$$

where c is the speed of light. [3 marks]

(v) Calculate β_1 for the case of the two pulses coupled into a 1-m long capillary filled with neon gas ($n_2 = 7.4 \times 10^{-21}$ cm^2/W) with $\lambda_1 = 800$ nm, $I_{01} = 10^{14}$ W/cm^2, $I_{02} = 2.5 \times 10^{13}$ W/cm^2, $\tau_1 = 100$ fs and $\tau_2 = 25$ fs. Hence make an estimate of the spectral bandwidth $\Delta\lambda$ generated due to phase-modulation. [3 marks]

[Total 15 marks]