EE4-25
EE9-CS1-2

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE/EIE PART IV: MEng and ACGI

Correction
Q1, 10,20

DESIGN OF LINEAR MULTIVARIABLE CONTROL SYSTEMS

Monday, 18 May 10:00 am

Time allowed: 3:00 hours

Corrected Copy

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	I.M. Jaimoukha
	Second Marker(s) :	E.C. Kerrigan

© Imperial College London

1. Let the n-th order transfer matrix $G(s)$ have a state space realisation

$$G(s) \stackrel{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$$

and let

$$A'Q + QA + C'C = 0.$$

and

$$AP + PA' + BB' = 0$$

for some $Q = Q'$ and $P = P'$.

Suppose that

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & 0 \end{bmatrix},$$

and

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix}$$

where $\mathscr{R}^{n_1 \times n_1} \ni P_1 \succ 0$ and $\mathscr{R}^{n_1 \times n_1} \ni Q_2 \succ 0$ and where $n_1 < n$. Assume that A has no eigenvalues on the imaginary axis.

a) Derive a decomposition of the system into its controllable/uncontrollable and observable/unobservable subsystems. The derivation should include proofs that the subsystems have the relevant properties. [8]

b) Prove that the controllable and observable subsystem is stable. [5]

c) State the conditions that guarantee that the system is stabilisable and detectable. [2]

d) Draw a diagram illustrating the subsystems of $G(s)$. [5]

Copyright Imperial College London

2. Consider the $\mathscr{H}_\infty$ filter for estimating x shown in Figure 2.

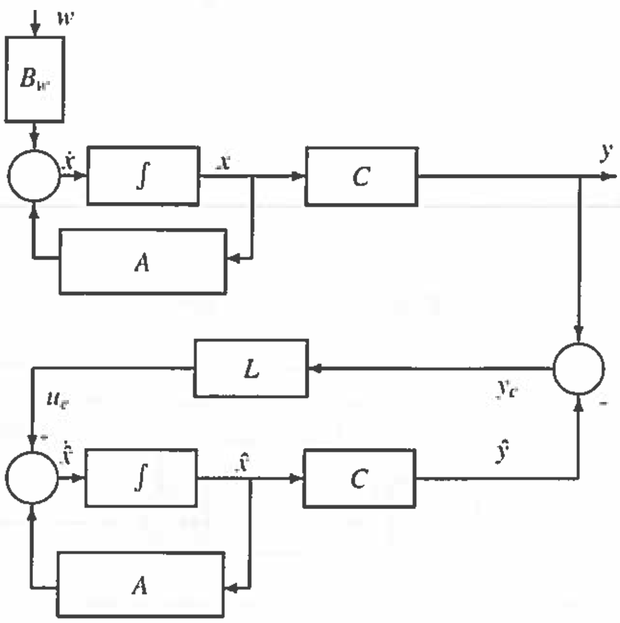

Figure 2

Let $z = x - \hat{x}$ and let $T_{zw}(s)$ denote the transfer matrix from w to z. A stabilizing filter gain matrix L is to be designed such that, for $\gamma > 0$, $\|T_{zw}\|_\infty < \gamma$.

a) Derive a state-space realisation for the transfer matrix $T_{zw}(s)$. [5]

b) Use the Bounded Real Lemma, stated in Question 5 below, to derive necessary and sufficient conditions for $\|T_{zw}\|_\infty < \gamma$. These conditions should be in the form of matrix inequality constraints. [10]

c) Suggest a transformation of variables that turns the matrix inequality derived in Part (b) above into a linear matrix inequality and write down this inequality. [5]

 Copyright Imperial College London

3. Consider the feedback configuration in Figure 3. Here, $G(s)$ is a nominal plant model and $K(s)$ is a compensator. The stable transfer matrices $\Delta_a(s)$ and $\Delta_m(s)$ represent additive and multiplicative uncertainties on the nominal model.

The design specification are to synthesize a compensator $K(s)$ such that the feedback loop is internally stable when:

(i) $\Delta_a = 0$ and $\|\Delta_m(j\omega)\| \leq |w_m(j\omega)|, \forall\omega$, and,

(ii) $\Delta_m = 0$ and $\|\Delta_a(j\omega)\| \leq |w_a(j\omega)|, \forall\omega$.

where $w_a(s)$ and $w_m(s)$ are appropriate weighting functions.

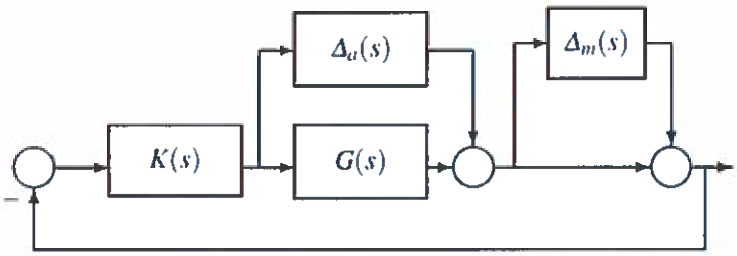

Figure 3

a) Derive $\mathcal{H}_\infty$–norm bounds, in terms of $G(s)$, $K(s)$, $w_a(s)$ and $w_m(s)$ that are sufficient to achieve the robust design specifications. [6]

b) Define suitable cost signals $z_1(s)$ and $z_2(s)$, external signal $w(s)$, measured signal $y(s)$ and control signal $u(s)$ and draw a block diagram, showing all these signals, as well as suitable weighting functions. [6]

c) Hence derive a generalised regulator formulation of the design problem that captures the sufficient conditions. [8]

Copyright Imperial College London

4. Consider the regulator in Figure 4 for which $x(0) = x_0$. A stabilizing state-feedback gain matrix F is to be designed such that the cost function

$$J = \left\| \begin{bmatrix} u \\ z \end{bmatrix} \right\|_2^2$$

is minimized.

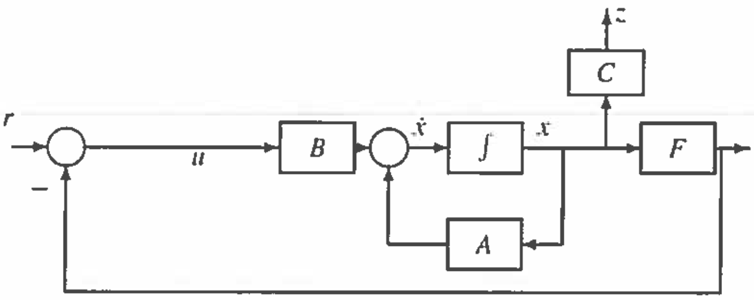

Figure 4

a) State the assumptions needed on the matrices A, B and C to obtain a solution and define a suitable Lyapunov function $V(t)$. [2]

b) Obtain an expression for $\int_0^\infty \dot{V}(t)dt$. State clearly any assumptions made. [2]

c) Find an expression for F that minimizes J. Give also the algebraic Riccati equation that needs to be satisfied. What is the minimum value of J? [6]

d) Prove now that the closed loop system in Figure 4 is stable. State the assumptions required to guarantee stability. [4]

e) By evaluating an expression for $T_{zr}(s)$ in Figure 4 and using the Bounded Real Lemma (given in Question 5 below), prove that $\|T_{zr}\|_\infty < 1$. [6]

Copyright Imperial College London

5. a) Consider a state–variable model described by the dynamics

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t) + Du(t),$$

and let $H(s) = D + C(sI - A)^{-1}B$ denote the corresponding transfer matrix. Suppose there exists a $P = P' \succ 0$ such that

$$\begin{bmatrix} A'P + PA + C'C & PB + C'D \\ B'P + D'C & D'D - \gamma^2 I \end{bmatrix} \prec 0.$$

i) Prove that A is stable. [4]

ii) By defining suitable Lyapunov and cost functions and completing a square, prove that

$$\|H\|_\infty < \gamma.$$

[6]

b) Consider the output injection problem shown in Figure 5. Note that the signal w enters the system at two points. Let $T_{yw}(s)$ denote the transfer matrix from w to y. An internally stabilizing output injection gain matrix L is to be designed such that, for a given $\gamma > 0$, $\|T_{yw}\|_\infty < \gamma$.

i) Derive a state space realization for $T_{yw}(s)$. [4]

ii) By using the answer to Part (a) above, or otherwise, derive sufficient conditions for the existence of a feasible L. Your conditions should be in the form of the existence of certain solutions to linear matrix inequalities. [6]

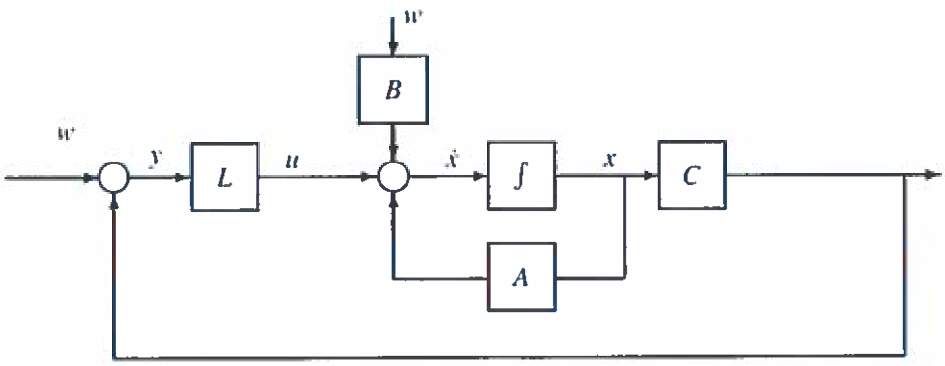

Figure 5

Copyright Imperial College London

6. Consider the regulator shown in Figure 6. Assume that

(i) The pair (A, B) is controllable

(ii) $x(0) = 0$

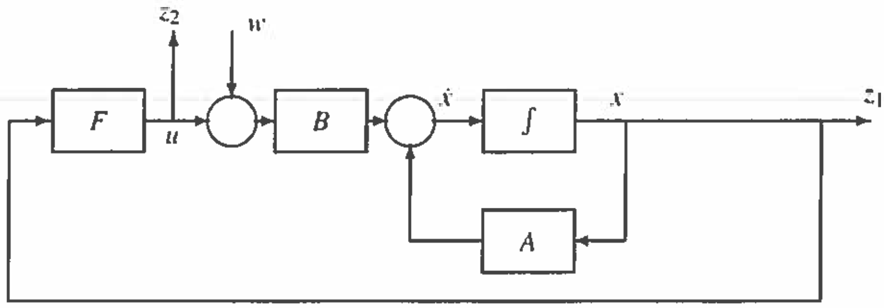

Figure 6

Let

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

and let $H(s)$ denote the transfer matrix from w to z.

A stabilizing static output feedback gain matrix F is to be designed such that, for $\gamma > 0$, $\|H\|_\infty < \gamma$.

a) Write down the generalized regulator system for this design problem. [6]

b) By using the Lyapunov function $V(t) = x(t)'Xx(t)$, where X is to be determined, derive sufficient conditions for the solution of the design problem.

Your conditions should be in the form of the existence of a certain solution to an algebraic Riccati equation.

It should also include an expression for F and an expression for the worst-case disturbance w. [10]

c) Suppose that A is stable. Show that $\gamma = 1$ is feasible. [4]

Copyright Imperial College London