

MSc and EEE/EIE PART III/IV: MEng, Beng and ACGI

Thursday, 17 January 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible

First Marker(s) :	J.A. Barria
Second Marker(s) :	T-K. Kim

Special information for students

1. Mean delay for the M/M/1 system may be taken as

$$t_i = \frac{1}{\mu C_i - \lambda_i}$$

where,

$1/\mu$ = Average length of packet [bit/packet]

C_i = Transmission speed link i [bits/s]

μC_i = Service rate (link i) [packet/ s]

λ_i = Arrival rate (link i) [packet/ s]

2. Optimal Routing Problem (ORP)

Min $D(F)$ with respect to $F = \{ F_i \}$

where,
$$D(F) = \sum_{i=1}^L \frac{F_i}{C_i - F_i}$$

and,

C_i = Capacity of link l_i .

F_i = Flow carried by link l_i .

3. Statistical shortest path Routing Problem (SSP)

Aim: minimise the cost function $\mu_p + \Phi(\sigma_p^2)$ for path P . Where $\Phi(\cdot)$ is an arbitrary function, μ_p is the mean of path p and σ_p^2 is the variance of path p .

Note: if edge-weight distributions are assumed mutually independent means and variance are additive. That is, $\mu_p = \sum_{i \in p} \mu_i$ and $\sigma_p^2 = \sum_{i \in p} \sigma_i^2$.

The Questions

1.

- a) Figure 1.1 shows the utilisation of the sliding window flow control as a function of the propagation delay parameter a .

Explicitly derive the utilisation of this flow control mechanism as a function of the parameter a and the window size W . [3]

Calculate and identify in Fig. 1.1 the values of the window size W for the three curves plotted in the figure. [3]

- b) Assume now that you can deploy the Go back N automatic repeat request (ARQ) mechanism.

Explicitly derive the utilisation of this ARQ flow control mechanism and then state clearly all approximations used. [4]

Using Fig. 1.1 as a reference, and for the same window sizes W that you have identified in part a), draw the new curves as a function of the parameter a .

Assume that the probability that a single frame is in error is given by $P = 10^{-3}$. [4]

- c) Introduce and explain Burke's theorem for a system of queues in tandem: describe the system and its assumptions. [3]

Why can't we use Burke's theorem to model switches in tandem within a packet switched network? [3]

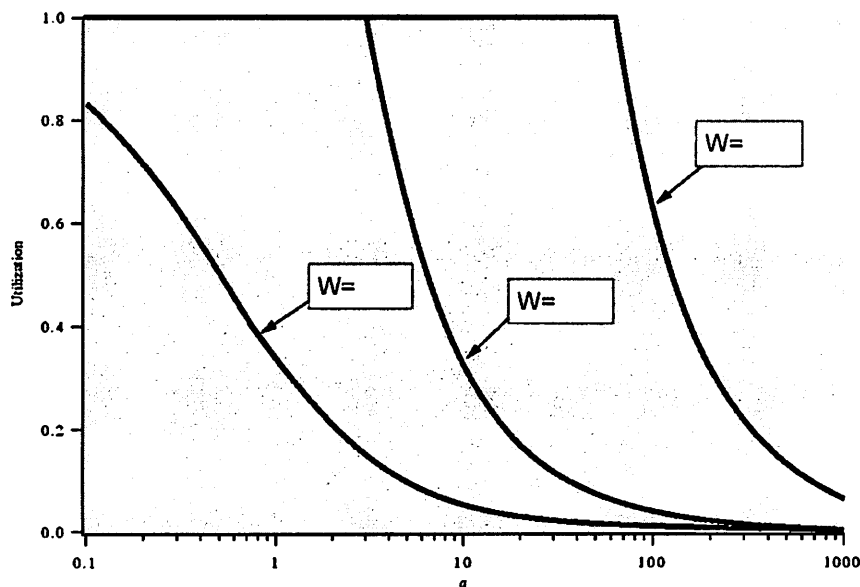


Figure 1.1

2.

a) Assured Forwarding (AF) per hop behaviour, has its roots in the RED (Random Early Detection) queue management scheme.

i) Describe three typical queue management (QM) tasks and explain their functionality. [3]

ii) Explain what a packet scheduling task is and how it differs from QM tasks? [3]

iii) Discuss how the RIO (RED with In/Out) QM scheme mechanism treats conforming and non-conforming packets. [5]

b) For the network shown in Fig. 2.1 a well known objective of network operators is to minimise the mean network delay given by:

$$\text{Min } D(x) = \sum_{i=1}^L \frac{x_i}{C(i) - x_i} \text{ with respect to } x = \{x_i\}$$

Where,

$C(i)$ is the capacity of link l_i , and

x_i is the flow carried by link l_i .

The network requires an upgrade and the two dotted line links: $C(4)$ and $C(5)$ shown in Fig. 2.1 have been selected as possible candidates.

i) If only one link can be upgraded at a time, which link will you choose to deploy first?. Explain and discuss the reasons behind your choice. [5]

ii) Calculate the capacity value of the link chosen in part i) so that the direct traffic – either γ_{21} or γ_{13} – is carried by this link only. [2]

iii) For the topology obtained in part ii) calculate the value of the capacity of the remaining link (i.e. $C(4)$ or $C(5)$) if your objective is for the final upgraded network to carry all direct traffic on the two newly added links. [2]

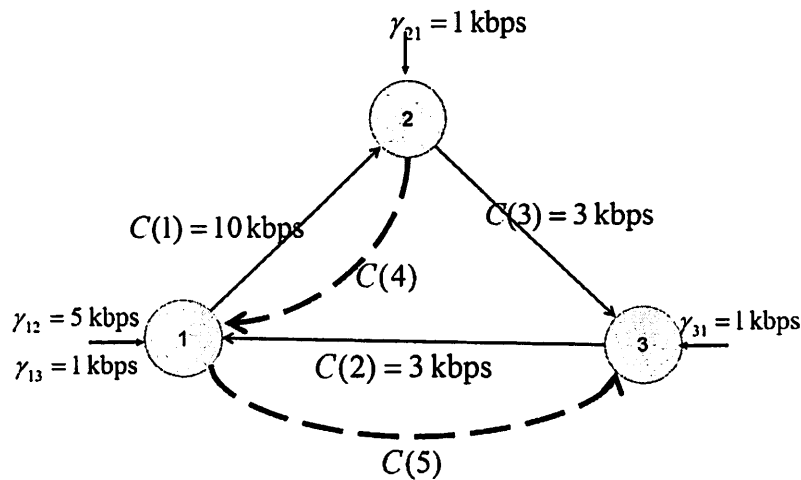


Figure 2.1

3.

a) Adaptive routing algorithms will react to changes in topology and traffic patterns.

i) With the help of the network shown in Fig. 3.1 briefly describe and compare how the Dijkstra and the Bellman-Ford shortest path algorithms work. [4]

ii) Both of the above algorithms are known to converge under static conditions. However, links costs are constantly changing in real world communication networks:

- Give two examples of random events and/or actions by operations personnel that will have an impact on the link cost. [3]

- Give an example as to how link cost changes over time will result in routing instability. Discuss mechanism(s) which could be implemented to minimise the impact of the two identified instabilities. [3]

b) Recently there has been increasing interest in solving shortest path problems using probabilistic edge weights.

i) Explain what it is that we are trying to model when using probabilistic edge weights. [1]

ii) Give two examples of a problem under consideration where it would be appropriate to use this type of probabilistic graph representation. [3]

iii) For the network representation in Fig. 3.1, assume that the edge-weights are Gaussian distributed (with mean μ_i ; variance σ_i^2) and are mutually independent:

- Solve the shortest path problem (from Node 1 to all the rest of the nodes in the network) using the metric $l(i) = \mu_i$.

- Solve the shortest path problem using the metric $l(i) = \mu_i + \sigma_i^2$.

Identify which algorithm you are using and clearly show *all* the iterations in the search for the shortest path solution. [4]

iv) Is it possible to solve the shortest path problem for the metric $l(i) = \mu_i + \sigma_i$ when using any of the algorithms in part 3.a)? Give clear reasons for your answer. [2]

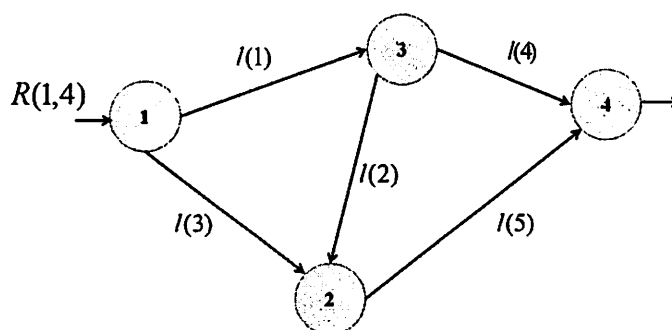


Figure 3.1

$l(i)$	μ_i	σ_i^2
1	1	3
2	2	1
3	4	1
4	4	1
5	1	5

4.

The network utility maximization (NUM) problem and the Nash arbitration solution (NAS) have been extensively studied and applied to networks carrying elastic traffic.

- a) Explain the meaning of the three axiom of fairness required to solve the Nash bargaining problem. [4]
- b) The feasibility set S of possible solutions of a game between two players with utilities functions $u_1(x)$ and $u_2(y)$ is shown in Fig. 4.1.

State clearly the procedure to obtain the NAS, and plot the solution using Fig. 4.1. [3]

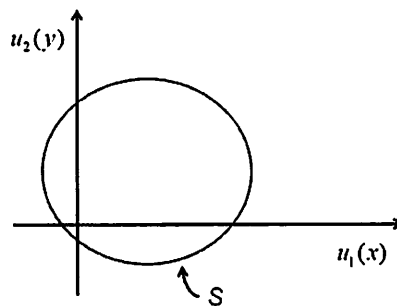


Figure 4.1

- c) Assume that you know the utility function of user i , $u_i(x_i)$, that has been allocated a share, x_i , of the total available resources.

Describe the network utility maximization (NUM) problem and discuss the characteristics of its solution. [4]

- d) For the network shown in Fig. 4.2 solve the flow assignment problem for $x = [x_1, x_2, x_3]$ using the proportional fair allocation principle. [6]

- e) For the allocation of flows in 4.d) show that for any other allocation of feasible flow $y = [y_1, y_2, y_3]$, the following condition holds:

$$\sum_i \frac{y_i - x_i}{x_i} \leq 0$$

[3]

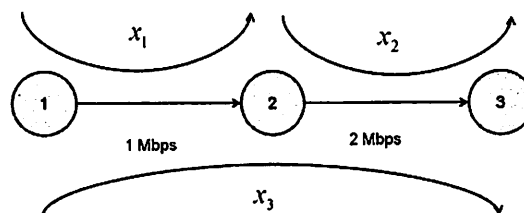


Figure 4.2

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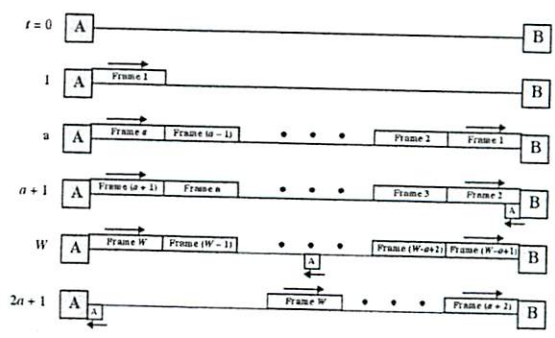
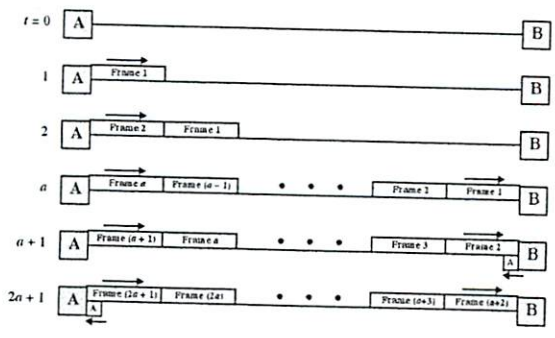
Q1

Error free sliding-window flow control.

a)

NOTE: frame transmission time normalized to 1.
hence a is the propagation delay

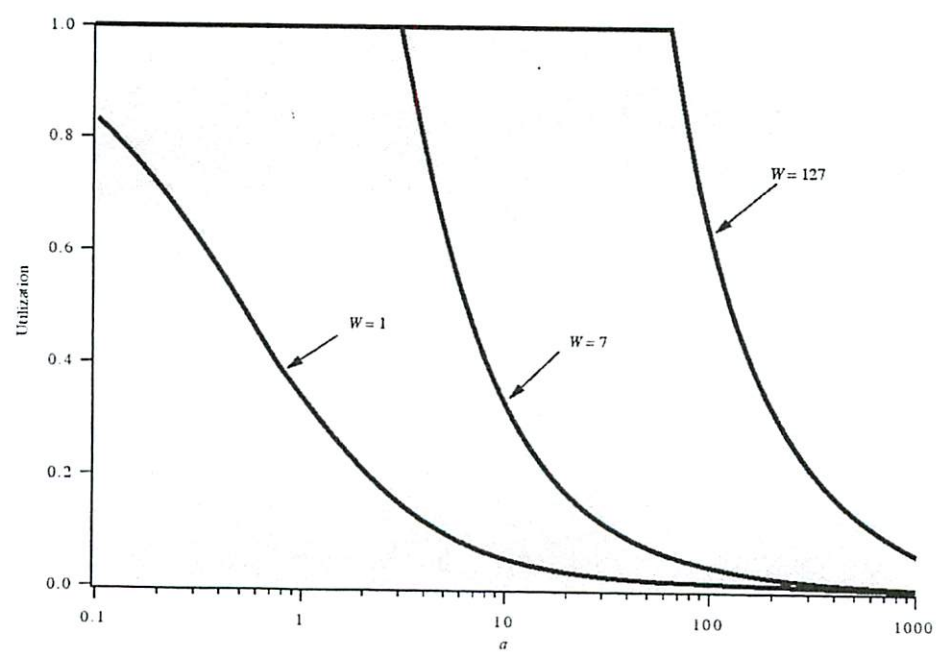
Using the following timing diagram (accept explanation) write



$$W \geq 2a+1$$

$$W < 2a+1$$

$$U = \begin{cases} 1 & W \geq 2a+1 \\ \frac{W}{2a+1} & W < 2a+1 \end{cases}$$



3

3

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Q1
9)

ARQ (Automatic Repeat Request)

We now need to account for the possibility that some frames are repeated because of error

$$U = \frac{T_f}{N_R T_f} \quad \text{where } T_f: \text{time to transmit a single frame}$$

$$T_t: \text{total time the line is engaged}$$

$$N_R: \text{Expected number of transmissions of a frame}$$

Go back to ARQ

$$N_R = \sum_{i=1}^{\infty} f(i) P^{i-1} (1-P) ; P = \text{probability that a single frame is in error}$$

$$f(i) = 1 + (i-1)K = (1-K) + Ki ;$$

(f(i) = the total number of frames transmitted if the original frame must be transmitted i times)

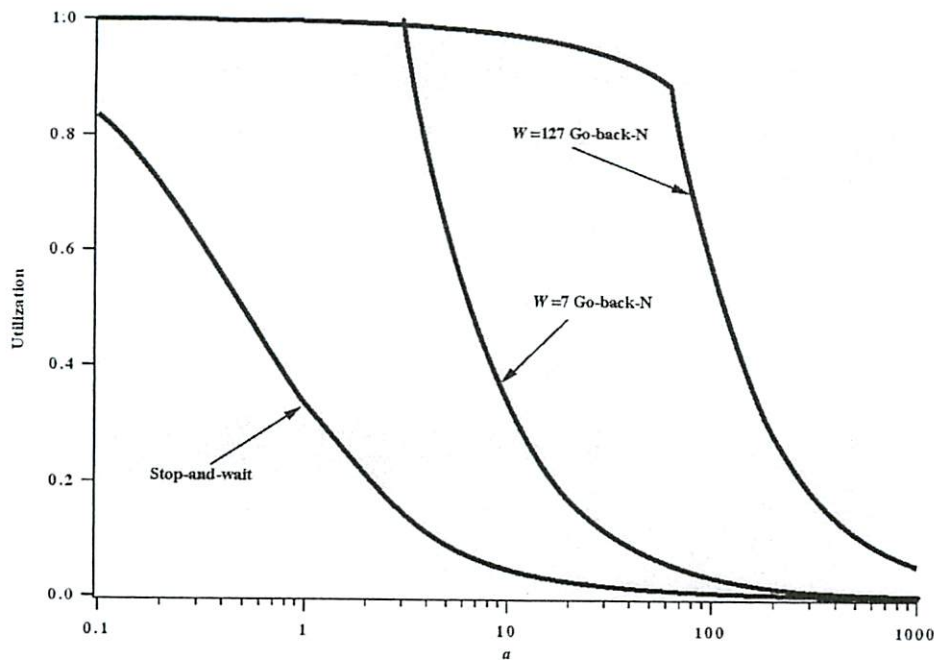
$$N_R = (1-K) \sum_{i=1}^{\infty} P^{i-1} (1-P) + K \sum_{i=1}^{\infty} i P^{i-1} (1-P) = \frac{1-P+KP}{1-P}$$

using timing diagram on previous page:

$$U = \begin{cases} \frac{1-P}{1+2aP} & (w \geq 2a+1) \\ \frac{w(1-P)}{(2a+1)(1-P+wP)} & (w < 2a+1) \end{cases}$$

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Q1 c) Diagram Go back N ARQ with $P=10^{-3}$.

Burke's Theorem



arrivals: Poisson stream : departure

Assume : arrival stream is Poisson (λ)

- service time exponential (μ_1, μ_2)
- service time in Q_1 and Q_2 independent random variable

Then Q_1 and Q_2 behaves like two independent M/M/1 system in series

The assumption that is not valid in a communication network is the third one (service time in Q_1 and Q_2 are independent) since the packet size does not change from one queue to another queue.

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Q2

a)

Queue Management Tasks:

- Move packets to appropriate queue
- Remove packets from a queue on request from packet scheduler
- Drop and remark packets if queue is full or approaching saturation.

Queue management is different from packet scheduling tasks:

- Scheduler decides which packet to send next and per flow bandwidth guarantee, and
- There is no mechanism to control queue size

Random Early Detection (RED) mechanism

- Drop packets from randomly selected flows with some drop probability whenever the queue length exceeds some threshold.
- proactively avoiding queue becoming congested
- Two queue length threshold: If buffer average is below minimum threshold - no dropping of packets; If buffer is above maximum threshold - drop packets with probability 1; If buffer is between drop packets with probability p .

RED with In/Out (DSC) mechanism

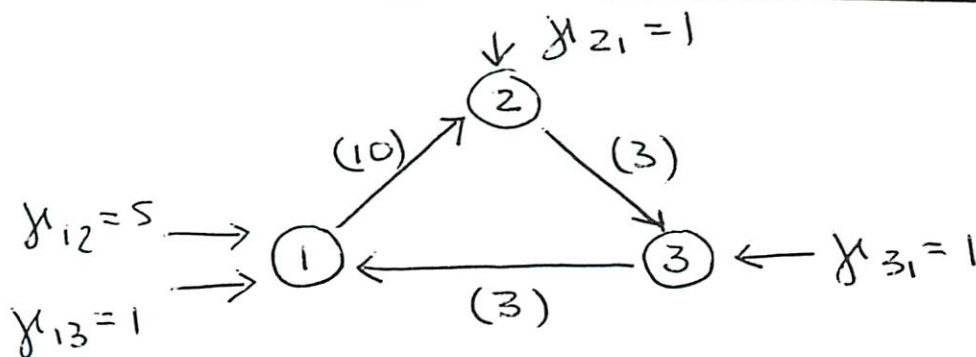
- DSC scheme assumes edge router marking of packets conforming to SLA:
 - Conforming packets, in profile
 - Non-conforming packets, out profile
- Packets out of profile dropped first if a router suffers congestion
- Different parameters used for in/out of profile packets

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Q2
5)

i)

Link alternative $2 \rightarrow 1$

$$\frac{1}{c(5)} = \frac{3}{(3-2)^2} + \frac{3}{(3-2)^2} = 6 \Rightarrow c(5) = \frac{1}{6} = 0.1666$$

Link alternative $1 \rightarrow 3$

$$\frac{1}{c(6)} = \frac{10}{(10-6)^2} + \frac{3}{(3-2)^2} = \frac{10}{16} + 3 \Rightarrow c(6) = 0.2758$$

Link $2 \rightarrow 1$ will start carrying direct traffic at a lower capacity than link $1 \rightarrow 3$

ii)

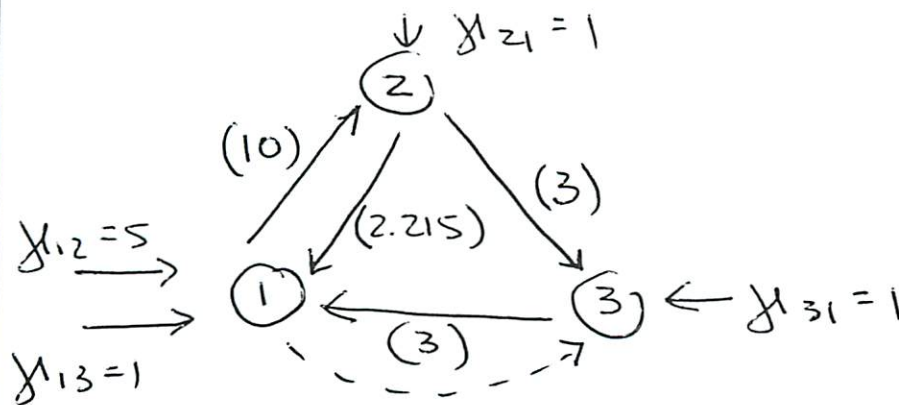
$$\frac{c}{(c-1)^2} = \frac{3}{(3-1)^2} + \frac{3}{(3-1)^2} = 2 + \frac{3}{2^2} = \frac{3}{2}$$

$$\frac{3}{2} (c^2 - 2c + 1^2) - c = 0$$

$$\frac{3}{2} c^2 - 4c + \frac{3}{2} = 0 \Rightarrow c = 2.215$$

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Q2
ii

$$\frac{C}{(C-1)^2} = \frac{10}{(10-5)^2} + \frac{3}{(3-0)^2}$$

$$= \frac{10}{25} + \frac{3}{9} = \frac{11}{15}$$

$$\frac{11}{15}(C^2 - 2C + 1) = C$$

$$\frac{11}{15}C^2 - \frac{37}{15}C + \frac{11}{15} = 0 \quad C = 3.034$$

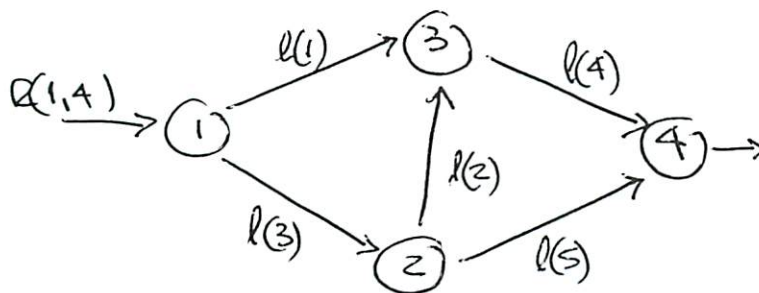
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Q3

a)

i)



Bellman-Ford shortest path algorithm

- find the shortest path to the rest of the nodes in the network using at most one link
- find the shortest path to the rest of the nodes in the network using at most two links
- ...

stop when solution with 'n' links is equal to solution with 'n+1' links

2

Dijkstra shortest path algorithm

- choose always the closest node to the origin node and add it into a set P.
- The idea is to develop the paths in order of increasing path length.
- stop when all nodes are inside set P.

2

ii)

Random events

- change in traffic patterns
- failure of links and/or link degradation

operation personnel

- preventive maintenance
- upgrading network capacity

3

link cost depends on traffic, which in turn depends on the routes chosen, then feedback condition exist, and instability may result

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- Measure average delay over last 10s and transform into link utilization estimate
- Normalize this based on current value and previous results
- Set link cost as a function of average utilization
- Implement mechanism to reduce routing loops

3

Q3

b)

i)

We are trying to model uncertainty due to, for example, variability in model's parameters and/or cost variables

1

ii)

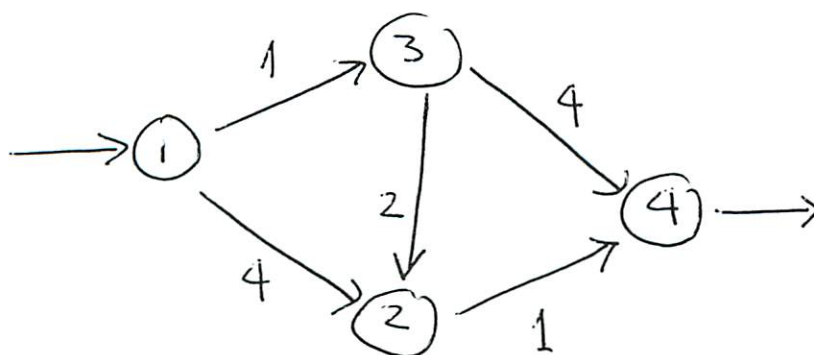
- Maze route in a VLSI CAD considering temperature variation

- Maximising the likelihood of reaching a destination within a specified travel deadline due to uncertainty of vehicular traffic conditions

3

iii)

i) for μp



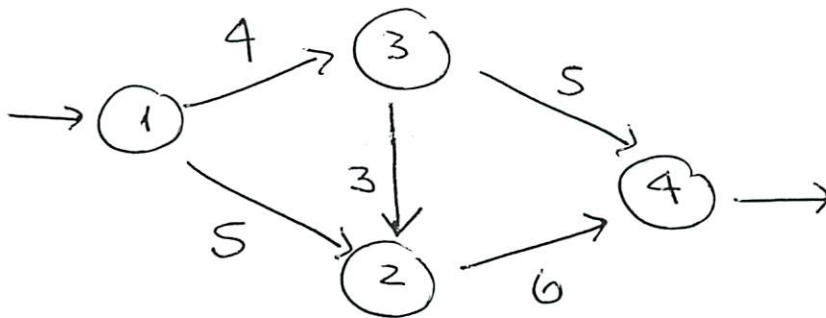
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Q3
v)

2) for $\mu_P + \sigma_P^2$



iv)

It is not possible to use algorithms in part 3a) as it is well known that standard deviation are not additive.

2

2

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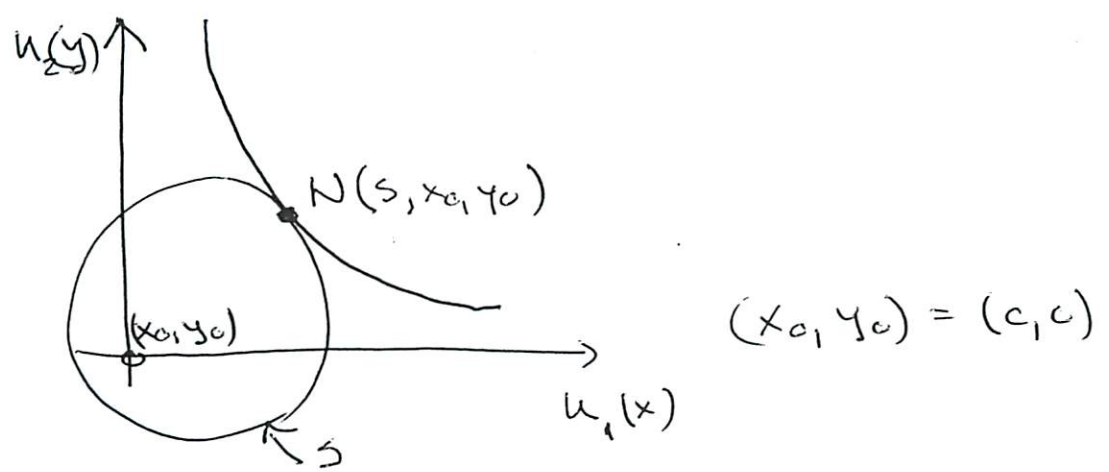
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Q4
a)

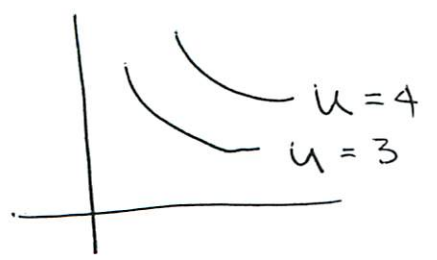
- The linearity property of the solution implies that the bargaining solution is scale invariant i.e. the bargaining solution is unchanged if the performance objectives are affinely (anti) scaled.
- The irrelevant-alternative axiom states that the bargaining point is not affected by enlarging the domain if agreement can be found on a restricted domain
- The symmetry property states that the bargaining point does not depend on the specific labels, i.e. users with the same initial point and objectives will realize the same performance

4

b)



$$U_{\text{sys}} = x y \quad \frac{U_{\text{sys}}}{x} = y$$



Maximize U_{sys} subject to solution in S

$N(s, x_c, y_c) = \text{solution}$

3

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Q4
c)

NUM (Network Utility Maximization)

Assume that associated with user i there is a utility function $u_i(x_i)$ where x_i is, e.g., the flow rate of user i .

A particular welfare function can be obtained by using

$$U = \sum_{i \in F} w_i(x_i) \quad ; \quad F = \text{set of all users}$$

Maximizing U subject to e.g. capacity constraints constitute a NUM problem

Suppose now that the user i can choose an amount of money w_i he/she is willing to spend at price p_i to maximize his/her surplus

$$\max \left[u_i \left(\frac{w_i}{p_i} \right) - w_i \right]; \quad w_i = x_i p_i \text{ or } x_i = \frac{w_i}{p_i}$$

It can be shown that for concave u_i

$$\max_i \sum_i u_i(x_i) \quad \text{is equivalent to} \quad \max_i \sum_i w_i \ln x_i$$

This corresponds to the solution of asymmetric Nash bargaining scheme where bargaining power $a_i = w_i$ and objective $f_i = x_i$

Alternative descriptions and discussions will be accepted

2

2

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Q4
a)

$$\max_x \sum_{i=1}^3 \ln x_i$$

$$\text{subject to: } \begin{aligned} x_1 + x_3 &\leq 1 \\ x_2 + x_3 &\leq 2 \\ x &\geq 0 \end{aligned}$$

$$L(x, \lambda) = \ln x_1 + \ln x_2 + \ln x_3 - \lambda_1 (x_1 + x_3 - 1) - \lambda_2 (x_2 + x_3 - 2)$$

$$\frac{\partial L(x, \lambda)}{\partial x_1} \Rightarrow \frac{1}{x_1} = \lambda_1 \Rightarrow x_1 = \frac{1}{\lambda_1}$$

$$\frac{\partial L(x, \lambda)}{\partial x_2} \Rightarrow \frac{1}{x_2} = \lambda_2 \Rightarrow x_2 = \frac{1}{\lambda_2}$$

$$\frac{\partial L(x, \lambda)}{\partial x_3} \Rightarrow \frac{1}{x_3} = \lambda_1 + \lambda_2 \Rightarrow x_3 = \frac{1}{\lambda_1 + \lambda_2}$$

$$\text{Letting } x_1 + x_3 = 1 \text{ and } x_2 + x_3 = 2$$

$$x_2 - x_1 = 1 \Rightarrow \frac{1}{\lambda_2} - \frac{1}{\lambda_1} = 1 \Rightarrow \frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} = 1$$

$$\lambda_1 - \lambda_2 = \lambda_1 \lambda_2 \Rightarrow \lambda_1 = \lambda_2 (1 + \lambda_1)$$

$$\lambda_2 = \frac{\lambda_1}{\lambda_1 + 1}$$

3

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$$\frac{1}{\lambda_1 + \lambda_2} + \frac{1}{\lambda_1} = 1$$

$$2\lambda_1 + \lambda_2 = \lambda_1(\lambda_1 + \lambda_2) = \lambda_1^2 + \lambda_1\lambda_2$$

$$2\lambda_1 + \frac{\lambda_1}{\lambda_1 + 1} = \lambda_1^2 + \lambda_1\left(\frac{\lambda_1}{\lambda_1 + 1}\right)$$

$$2\lambda_1(\lambda_1 + 1) + \lambda_1 = \lambda_1^2(\lambda_1 + 1) + \lambda_1^2$$

$$1 + 2(1 + \lambda_1) = \lambda_1^2 + \lambda_1 + \lambda_1$$

$$1 + 2 + 2\cancel{\lambda_1} = \lambda_1^2 + 2\cancel{\lambda_1} \Rightarrow \lambda_1 = \sqrt{3}$$

and

$$\lambda_2 = \frac{\sqrt{3}}{\sqrt{3} + 1}$$

$$\lambda_1 = 1.732$$

$$\lambda_2 = 0.634$$

$$x_1 = 1.577$$

$$x_2 = 0.577$$

$$x_3 = 0.422$$

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Q4
e)

given

$$x_1 = 1.577 \quad ; \quad x_2 = 0.577 \quad ; \quad x_3 = 0.422$$

check that for any other allocation y_i

$$\sum_i \frac{y_i - x_i}{x_i} \leq 0$$

3