# IMPERIAL COLLEGE LONDON

**B.Eng, M.Eng, and ACGI Examinations  2017 - 2018**
# Part 1

# BE1-HMATH1    Mathematics I

**31 May 2018 14:00 - 15:30**
**(duration: 90 minutes)**

**All three questions are compulsory.**

**Please answer each question in separate answer book.**

**A list of formulae is provided separately.**

**Each question is worth 100 marks.**
**Marks for questions and parts of questions are shown next to the question. The marks for questions (and parts thereof) are indicative, and they may be slightly moderated at the discretion of the Examiner.**

**Question 1**   This question has two parts.

**a)**   Find the complex Fourier series expansion of $f(x) = \cos x$ from $-\pi$ to $\pi$, to show that $\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$.

**50 marks**

**b)**   **i)**   Find all the solutions to the complex equation

$$z^2 + \bar{z}^2 = 0 \, .$$

**20 marks**

**ii)**   Find all the solutions to the complex equation

$$z^6 + 7z^3 - 8 = 0 \, .$$

**20 marks**

**iii)**   Draw the solutions from i) and ii) on an Argand diagram.

**10 marks**

*The two parts carry equal marks.*

**Question 2**    This question has two parts.

    **a)**    Find the general solution $y(x)$ of the following differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3x^2 + \sin x = 0.$$

**40 marks**

    **b)**    **i)**    Solve the following differential equation in the domain $D \equiv (-\infty, \infty)$:

$$\frac{2xy}{x^2 + 1} - x - \left[1 - \ln\left(x^2 + 1\right)\right] y' = 0$$

with the condition $y(0) = 1$.

**30 marks**

        **ii)**    Sketch the function
$$y(x) = \log(x^2) + x$$

Identify clearly the domain of the function, and whether the function intercepts with $x$-axis and the $y$-axis (no need to provide the exact expressions of the intercepts). Furthermore, identify, if exist, the local maxima/minima, intervals in which $y(x)$ is increasing and decreasing, intervals in which $y(x)$ is concave up and down, and asymptotes.

**30 marks**

*The two parts carry, respectively, 40%, and 60% of the marks.*

**Question 3 a)** Consider the three points, $O(0, 0, 0)$, $A(2, 1, 1)$ and $B(1, 3, 2)$.

    **i)** Find the area of the parallelogram which is spanned by the vectors <u>OA</u> and <u>OB</u>.

**20 marks**

    **ii)** Find an equation of the plane $\Pi$ containing the points $A$, $B$ and $O$.

**10 marks**

**b)** Determine whether the set of three vectors $\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$ is linearly independent or dependent.

**20 marks**

**c)** Find the derivative of $x^x$.

**25 marks**

**d)** Consider a graph described by $xy + x^2 y^2 = 6$. Obtain the two points on this graph whose x-coordinate is 1. For each of these two points, find an equation of the tangent line (a straight line that touches the graph at the point in question).

**25 marks**

*The four parts carry, respectively, 30%, 20%, 25%, and 25% of the marks.*