

Digital Electronics 1 EE1-02

Solutions

Question 1

1. a) (i)

$$\begin{aligned}
 & \overline{(A + \bar{B})} \overline{(AC\bar{D})} \\
 &= \overline{(A + \bar{B})} + AC\bar{D} \\
 &= \overline{(A + \bar{B})} + A(\bar{C} + D) \\
 &= \bar{A} + B + A\bar{C} + AD \\
 &= \bar{A} + \bar{C} + B + AD \\
 &= \bar{A} + \bar{C} + B + D
 \end{aligned}$$

[3]

(ii)

$$\begin{aligned}
 & \overline{(A \oplus BC)} \overline{(A + B + C)} \\
 &= \overline{(\bar{A}BC + ABC)} (\bar{A}\bar{B}\bar{C}) \\
 &= \overline{(\bar{A}(\bar{B} + \bar{C}) + ABC)} (\bar{A}\bar{B}\bar{C}) \\
 &= \bar{A}\bar{B}\bar{C}
 \end{aligned}$$

[3]

b)

$$f(A, B, C, D) = \Sigma(0, 2, 4, 5, 6, 7, 8, 10, 15)$$

		CD			
		00	01	11	10
AB	00	1	0	0	1
	01	1	1	1	1
	11	0	0	1	0
	10	1	0	0	1

$$\Rightarrow f = \bar{B}\bar{D} + \bar{A}\bar{D} + \bar{A}B + BCD$$

Here, 1 mark for drawing the Karnaugh map, 1 for filling it out correctly, 1 for the correct grouping, and 1 for the final expression.

[4]

c)

$$f = A\bar{B}\bar{C} + B\bar{C}\bar{D} + \bar{A}\bar{B}C + \bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D}$$

		CD			
		00	01	11	10
AB	00	0	1	1	1
	01	1	0	0	0
	11	1	0	0	0
	10	1	1	0	1

$$\Rightarrow f = (A+B+C+D)(\bar{B}+\bar{D})(\bar{B}+\bar{C})(\bar{A}+\bar{C}+\bar{D})$$

Here, 1 mark for drawing the Karnaugh map, 1 for filling it out correctly, 1 for the correct grouping, and 1 for the final expression.

[4]

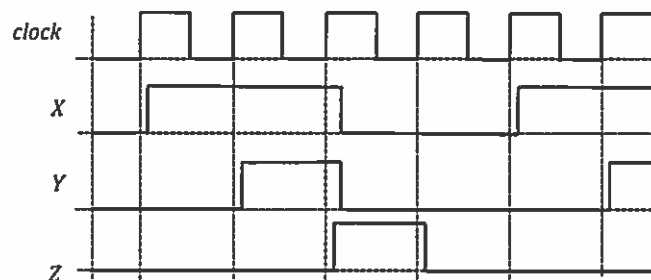
d)

Hexadecimal	Octal	Signed binary	Signed Decimal
97		1001 0111	-105
BD	275		-67

Give 2 marks per answer.

[8]

e)



Give 2 marks per waveform X, Y and Z.

[6]

f) (i) The truth table for the full-adder is as follows:

A	B	C _{in}	Σ	C _{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Give 1 mark for correctly showing the possible values of A, B and C_{in}, and 1 mark each for Σ and C_{out}.

[3]

(ii) Using a Karnaugh map, simplified Boolean expressions for Σ and C_{out} are as follows:

Σ

		BC _{in}	00	01	11	10
A	0	0	0	1	0	1
	1	0	1	0	1	0

C_{out}

		BC _{in}	00	01	11	10
A	0	0	0	0	1	0
	1	0	1	1	1	1

$$\begin{aligned}
 \Sigma &= \bar{A}.\bar{B}.C_i + \bar{A}.B.\bar{C}_i + A.\bar{B}.\bar{C}_i + A.B.C_i \\
 &= A \oplus B \oplus C_i \\
 C_{out} &= AB + AC_i + BC_i
 \end{aligned}$$

Give 1 mark for using Karnaugh maps or Boolean algebra for simplification, and 1 mark each for the correct final expressions for Σ and C_{out}.

[3]

- g) (i) Initially, write the truth table for f and g as follows:

X	Y	Z	f	g
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	1

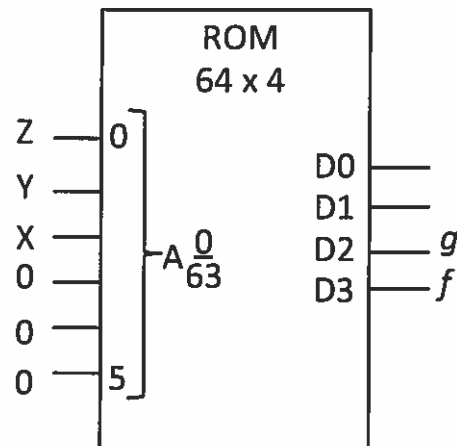
Comparison with the hexadecimal data given in Fig. 1.3(b) shows that if $D3 = f$, $D2 = g$, and $D1 = D0 = 0$ for the first eight data locations, then the correct sequence is obtained in Fig. 1.3(b). Furthermore, if address line $A2 = X$, $A1 = Y$ and $A0 = Z$, then these data locations are accessed correctly. This gives the following complete table:

A[5:0]	D[3:0]
00	0
01	C
02	8
03	0
04	C
05	4
06	0
07	C

Give 1 mark for each correct entry.

[4]

(ii) The connections to address and data lines are shown below. Note that $A_5 = A_4 = A_3 = 0$, to allow the first eight data locations to be addressed.

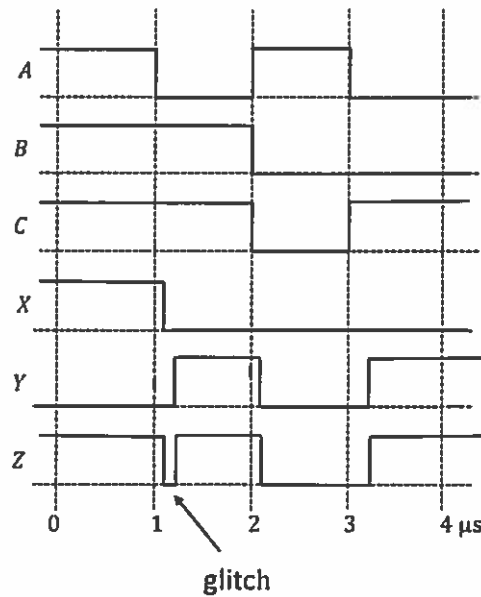
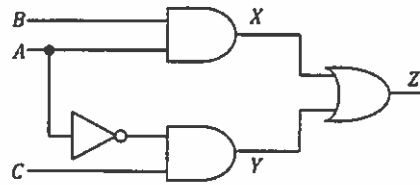


Give 1 mark for correct address line connections and 1 mark for correct data line connections.

[2]

Question 2

2. a) (i)



2 marks each for the waveforms for X, Y and Z. These should show the switching delay, and the glitch in Z.

[6]

(ii) To prevent a static 1 hazard, i.e. a glitch which transitions from 1 – 0 – 1, we require covering '1's in the Karnaugh map which are adjacent but in different groups. For the function in the circuit:

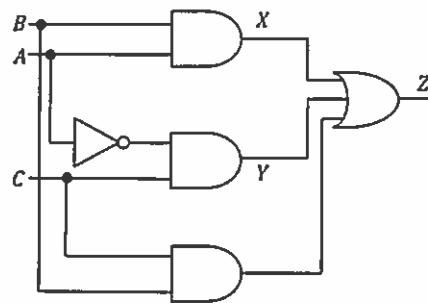
$$Z = AB + \bar{A}C$$

The Karnaugh map is as follows:

		BC			
		00	01	11	10
A	0	0	1	1	0
	1	0	0	1	1

Here the grouping BC will eliminate the hazard, giving the Boolean expression and circuit as follows:

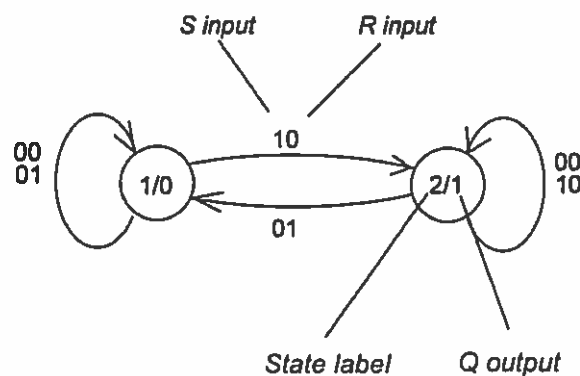
$$Z = AB + \bar{A}C + BC$$



2 marks for Karnaugh map, 1 mark for the introduction of the correct extra term or Boolean expression, and 1 for the redesigned circuit diagram.

[4]

(b) (i) The state diagram for the SR flip-flop is as follows:



1 mark for states, 1 mark for interconnections, and 2 for labelling.

[4]

(ii) To find the characteristic Boolean equations for the flip-flop, we first draw the assigned state table from the state diagram:

Present Output	Next output inputs: SR			
	00	01	11	10
0	0	0	X	1
1	1	0	X	1

Re-drawing this as a Karnaugh map for next state output Q^+ , with inputs Q (the present state), S and R :

$Q \backslash SR$	00	01	11	10
0	0	0	X	1
1	1	0	X	1

The Karnaugh map gives the following Boolean equations:

$$Q^+ = Q\bar{R} + S$$

$$\bar{Q}^+ = \bar{Q}\bar{S} + R$$

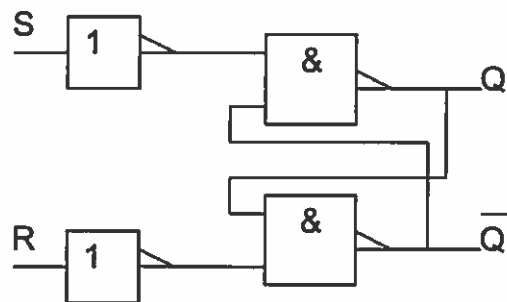
1 mark each for the assigned state table and Karnaugh map, and 1 mark each for the Boolean equation.

[4]

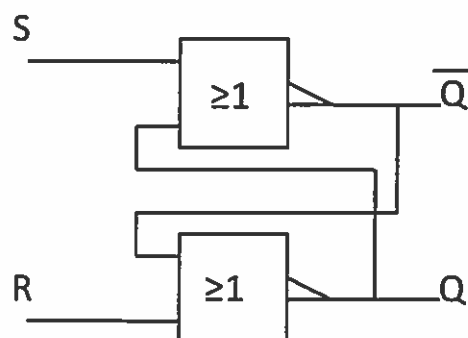
(iii) Rewrite the Boolean equations using NAND gates only, and draw the circuit diagram:

$$Q^+ = \overline{\overline{(Q\bar{R})} \cdot \bar{S}}$$

$$\bar{Q}^+ = \overline{\overline{(Q\bar{S})} R}$$



By replacing the NAND gates by their alternative symbols (NOR with inverted inputs), and cancelling inversion bubbles where required, we obtain the NOR version of the circuit:



1 mark for NAND implementation, 1 mark for the NAND circuit, and 2 for the NOR circuit.

[4]

(c) (i) The MUXs may be used to directly write give the following Boolean equations. This requires understanding the MUX IEEE symbol and operation.

$$X = A\bar{B}\bar{C} + A\bar{B}C = A\bar{B}$$

$$Y = \bar{A}B + A\bar{B}$$

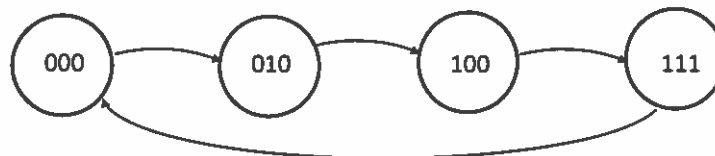
2 marks each for the Boolean equation.

[4]

(ii) Initially, draw the state transition table for the FSM:

Present state	Next state
Q2Q1Q0 A B C	Q2 ⁺ Q1 ⁺ Q0 ⁺
0 0 0	0 1 0
0 0 1	0 1 0
0 1 0	1 0 0
0 1 1	1 0 0
1 0 0	1 1 1
1 0 1	1 1 1
1 1 0	0 0 0
1 1 1	0 0 0

If the initial state for the FSM is 000, then the Moore diagram is the following. Note that not all states in the table above are used here:

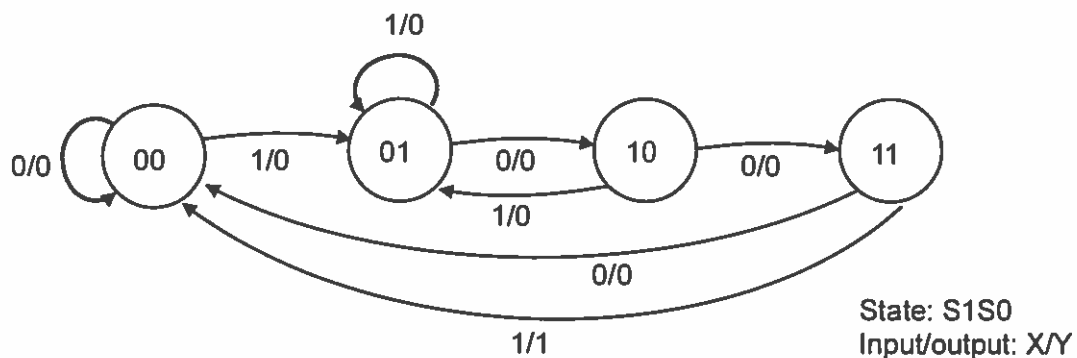


2 marks for the state transition table and 2 marks for the Moore diagram.

[4]

Question 3

(a) Mealy diagram for the FSM:



Give 2 marks for correct number of states, 4 marks for correct interconnections, and 2 marks correct labelling of the interconnections.

[8]

b)

State transition table for the FSM:

Current state		Input	Next state		Output
S1	S0	X	S1+	S0+	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	0	0	0
1	1	1	0	0	1

Give 1 mark for each correct row of the table.

[8]

c)

The transition table for the J-K flip-flop is the following:

Transition	J	K
0 → 0	0	X
0 → 1	1	X
1 → 0	X	1
1 → 1	X	0

Give 1 mark for the table.

Using this table, it is possible to draw Karnaugh maps for $J1$, $K1$ and $J0$, $K0$, as a function of $S1$, $S0$ and X , to give the next states $S1+$ and $S0+$. Y may be obtained directly from $S1$, $S0$ and X .

For $S1+$:

K-map for $J1$:

X	$S1S0$			
	00	01	11	10
0	0	1	X	X
1	0	0	X	X

$$J1 = \bar{X}S0$$

$$K1 = X + S0$$

K-map for $K1$:

X	$S1S0$			
	00	01	11	10
0	X	X	1	0
1	X	X	1	1

For $S0+$:

K-map for $J0$:

X	$S1S0$			
	00	01	11	10
0	0	X	X	1
1	1	X	X	1

$$J0 = X + S1$$

$$K0 = \bar{X} + S0$$

K-map for $K0$:

X	$S1S0$			
	00	01	11	10
0	X	1	1	X
1	X	0	1	X

Give 2 marks for each K-map with its associated equation.

For Y :

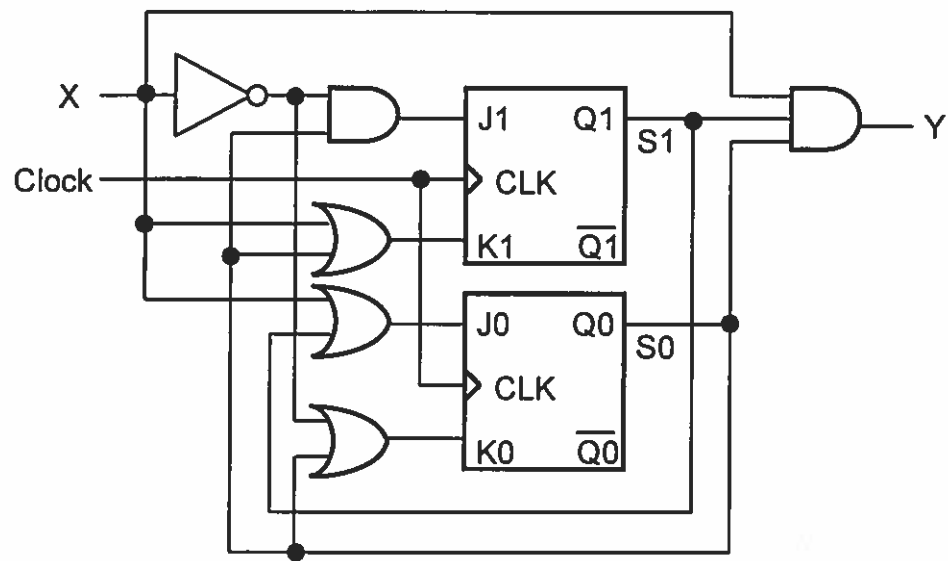
Y can be found directly from the state transition table:

$$Y = S1S0X$$

Give 1 mark for this.

[10]

d)



Give 1 mark for correct number of flip-flops, 2 marks for correct gates and 1 for correct interconnections.

[4]