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UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
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MSc and EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI

VHDL AND LOGIC SYNTHESIS

Friday, 26 April 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

Time allowed: 3:00 hours

Examiners responsible:

First Marker(s): Clarke, T.J.W.

Second Marker(s): Cheung, P.Y.K.

Corrected copy (3A)

Q6 10.15

diag

Q5 wording 10.20

Q26 wording 10.25

Special information for invigilators: Students may bring any written or printed aids into this examination.

Information for candidates: None.

1. *Figure 1.1* shows the entity of a hardware sine function generator *sinegen* with input *x*, output *y*, and generic parameters *n*, *m*. The two's complement signed number *x* has *n* bits to the left, and *m* bits to the right, of its fixed point. It can thus represent a real number in the range $(2^{n-1} - 2^{-m})$ to -2^{n-1} . The *sinegen* entity implements the sine function over this range by using one instance of the entity *sine_lookup*, also in *Figure 1.1*. *Sine_lookup* is a combinational block you are given that implements the sine function over the limited range $0-2\pi$. The *sinegen* entity operates as in *Figure 1.2*. The input *x* is first changed to its absolute value, *xabs*, and then, by subtracting the appropriate multiple of 2π , to a number *xbase* in the range $0-2\pi$, which is passed to the input of *sine_lookup*. The output *y* is driven from the output of *sine_lookup*, or its negation, as shown in *Figure 1.2*. Fixed point arithmetic, with *m* bits to the left of the fixed point, is used throughout to represent real numbers, and the implementation of *sinegen* is purely combinational.

- a) Assuming that the output of *sine_lookup* is the sine of its input, explain how *sinegen* implements $y = \sin(x)$, and why the lengths of *xabs*, *xbase*, *y* are as specified in *Figure 1.2*. [5]

- b) Let *i* be any integer > 0 . If u_i is a number in the range 0 to $2^{i-1} \cdot 2\pi$, explain why the output of a comparator/subtractor/multiplexor:

$$u_{i-1} = \text{if } u_i < 2^{i-1} \cdot 2\pi \text{ then } u_i \text{ else } u_i - 2^{i-1} \cdot 2\pi$$

lies in the range 0 to $2^{i-1} \cdot 2\pi$. Hence draw a diagram indicating how such units, cascaded, can be used to convert *xabs* into *xbase*. How many units are required? [5]

- c) Write a synthesisable architecture for *sinegen*. You may assume that an implementation for entity *sine_lookup* is given, and that *m*, *n* are chosen such that all numeric vectors in *sinegen* may be represented by VHDL integers without overflow. Implement the numbers u_i as an array of vectors, each of length $n+m$. Your solution may use functions from the package *utils*, described in the lectures. [10]

```

ENTITY sinegen IS
  GENERIC( n : INTEGER := 8;
           m : INTEGER := 8
         );
  PORT( x : IN  STD_LOGIC_VECTOR(m+n-1 DOWNTO 0);
        y : OUT STD_LOGIC_VECTOR(m+1 DOWNTO 0));
END sinegen;

ENTITY sine_lookup IS
  GENERIC( m: integer);
  PORT( x: std_logic_vector( m+2 DOWNTO 0);
        y: OUT std_logic_vector( m+1 DOWNTO 0)
      );
END sine_lookup;
```

Figure 1.1

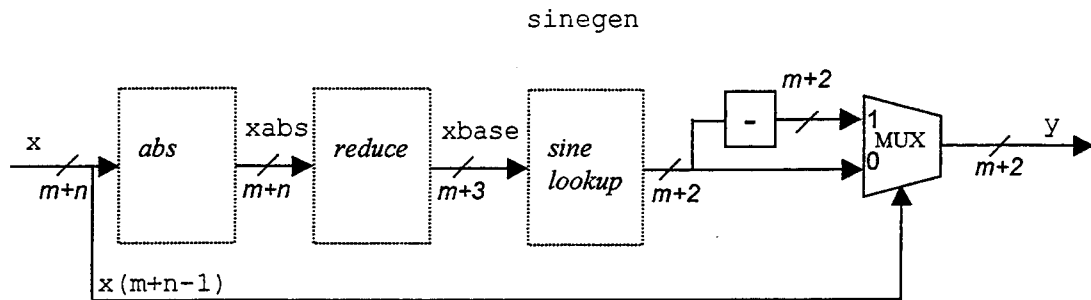


Figure 1.2

2. This question requires you to write a testbench for the entity `sinegen` defined in question 1.

You are given a VHDL package `maths` in library `mathlib` containing functions:

```

IMPURE FUNCTION random( l: integer; h: integer) RETURN integer;
FUNCTION sin( x : real) RETURN real;

```

The function `random`, when called, returns a new pseudo-random integer in the range 1 to `h` inclusive ($1 < h$). The function `sin` returns the sine of its parameter `x`, which specifies an angle in radians.

- a) Why is the function `random` declared IMPURE?

[2]

- b) Write a testbench entity for `sinegen` that has generic `testnum`, and performs `testnum` tests with pseudo-random input stimulus. The parameters `n`, `m` should also be generic parameters of your testbench, with default values of 20 and 8 respectively. Each test should check that the output is within 2^{-m} of the value computed by the `sin` function.

[12]

- c) The entity `sine_lookup` in Figure 1.1 is implemented as a lookup table. Discuss the relative merits of pseudo-random and exhaustive testing of `sinegen`, assuming that each separate test takes 1ms to execute.

[6]

3.

- a) *Figure 3.1* shows a critical path from X to Z in a circuit. Each of the blocks F is defined by: $B = P.Q + P.A + Q.A$. By applying controllability factoring at point Y , derive an equivalent circuit with reduced critical path length. What is your control function C ?

[10]

- b) The VHDL fragment in *Figure 3.2* defines y as a Boolean function of $x(i)$, where x has type `std_logic_vector(2 downto 0)`. Write a truth table for y , and compute two ROBDDs for y using variable orders: $x(0), x(1), x(2)$, and $x(2), x(1), x(0)$ respectively.

[10]

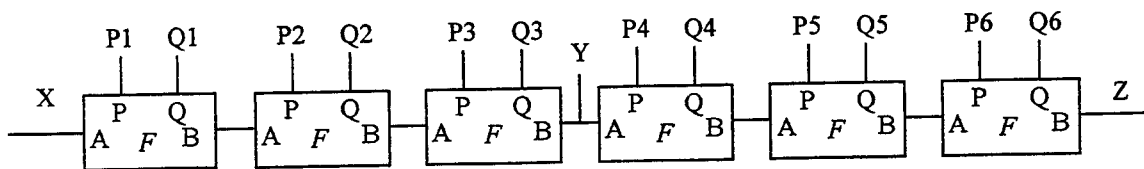


Figure 3.1

```

PROCESS(x)
BEGIN
  IF UNSIGNED(x) > 2 THEN
    y <= '1';
  ELSE
    y <= '0';
  END IF;
END PROCESS;

```

Figure 3.2

4. *Figure 4.1* shows the entity `mult16_16` of a custom VLSI 16*16 bit unsigned multiplier. In order to use this in a synchronous circuit it is proposed to implement a positive edge triggered clocked entity `mult32_32`, as in *Figure 4.1*. The I/O timing for `mult32_32` is shown in *Figure 4.2*. The inputs `a,b` are stable shortly after the rising edge of `clk`. The output `y` will be valid either 2, or 3 cycles after the corresponding inputs `a,b`. The output `ready` will be '1' during any clock cycle in which new inputs may be presented. The output `y` will be valid 2 or 3 cycles after `a, b` according to whether `ready` is '1' or '0' during the cycle after that in which `a, b` is presented. The inputs `a, b` are don't care for the 2nd cycle of a 3 cycle operation. The `reset` input provides a synchronous reset.

`Mult32_32` uses three `mult16_16` units to implement 32*32 bit multiplication with 32 bit result. If `al,ah` and `bl,bh` are the unsigned low and high 16 bit words of inputs `a & b` respectively, output `y` is calculated as:

$$y = \text{floor}(2^{-16}(al*bh+bl*ah)+ ah*bh)$$

The multiplication delay of `mult32_32` is determined by that of each component `mult16_16` unit, which has a propagation delay dependent only on the value of one of its inputs: `m`. If $m < 2^8$ the `mult16_16` delay is under 1 clock cycle, otherwise it is between 1 and 2 clock cycles. You may assume that the flip-flop setup times, and all other combinational delays, are negligible, so that the `mult_16_16` delay determines the required number of clock cycles for each multiplication. Data inputs and outputs of `mult32_32` are registered, so providing the minimum 2 cycle delay from input to output. There are no other clocked registers in the datapath of `mult32_32`.

- a) *Figure 4.3* shows the state diagram of an FSM that will generate the required timing, from an input `c`. The signal `c` is a function of `a,b` and equal to '1' when the corresponding operation must take 3 cycles. Rewrite the state diagram including `ready` as an output.

[5]

- b) Sketch an implementation of the datapath of `mult32_32`, implemented so as to minimise delay for `a, b < 224`. What is `c` as a function of `a,b`?

[5]

- c) Assuming that `mult16_16` is synthesisable, implement in VHDL a synthesisable architecture for `mult32_32`.

[10]

```

ENTITY mult16_16 IS
  PORT(
    m : IN  STD_LOGIC_VECTOR(15 DOWNTO 0);
    n : IN  STD_LOGIC_VECTOR(15 DOWNTO 0);
    p : OUT STD_LOGIC_VECTOR(31 DOWNTO 0)
  );
END mult16_16;

ENTITY mult32_32 IS
  PORT( clk   : IN  STD_LOGIC;
        reset : IN  STD_LOGIC;
        a     : IN  STD_LOGIC_VECTOR(31 DOWNTO 0);
        b     : IN  STD_LOGIC_VECTOR(31 DOWNTO 0);
        y     : OUT STD_LOGIC_VECTOR(31 DOWNTO 0);
        ready : OUT STD_LOGIC;
  );
END mult32_32;

```

Figure 4.1

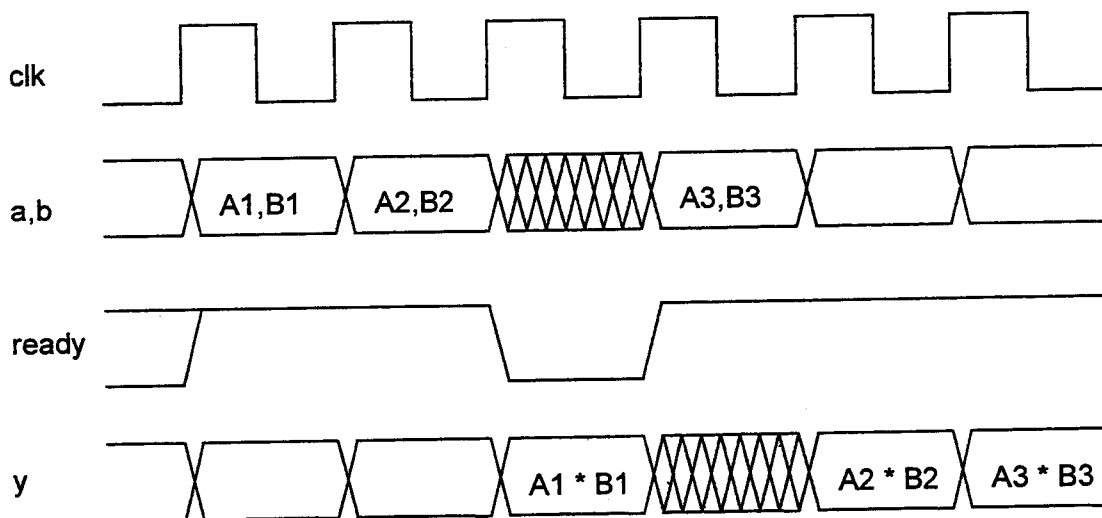


Figure 4.2

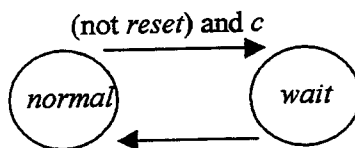


Figure 4.3

5. *Figure 5.1* on page 8 contains a procedure `read_cycle` that communicates with process `mem_driver_proc` via shared variables `mem_request_cycle`, `mem_data`, `mem_address`, and signal `mem_ack`.
- a) In VHDL both signals and shared variables can be used for inter-process communication. Why in the code of *Figure 5.1* on page 8 is `mem_ack` required to be a signal, whereas `mem_request_cycle` must be a shared variable? Discuss whether `mem_data`, `mem_address` could be signals in the cases:
- (i) `read_cycle` is called in a single process.
 - (ii) `read_cycle` is called in multiple processes.

[10]

- b) Write a VHDL procedure:

```
delay_by_clocks_and_deltas( SIGNAL clk: IN std_logic;
                           m: in INTEGER; n: IN INTEGER);
```

that when called will wait until m 0→1 transitions of `clk` have occurred, then wait a further n simulation deltas, then return. Your procedure should work correctly for all non-negative values of m and n .

[10]

6. *Figure 5.1* on page 8 gives VHDL source for an entity `test_mem_driver` with a behavioural architecture, and a package `comms` containing procedure `read_cycle`. The `test_mem_driver` entity has a positive edge active clock `clk`, and interfaces to a RAM through address and read data busses, as illustrated in *Figure 6.1*.
- a) Initially `mem_request_cycle` is false. Draw the waveforms of all signals and shared variables used in `test_mem_driver`, until the final (indefinite) wait statement in process `p1` is executed. You must indicate precise timing of all signal and shared variable transitions, including simulation deltas where relevant.

[15]

- b) It is intended that a call to `read_cycle` will initiate a 1 `clk` cycle long read of the RAM, at the address specified by the value of `addr`, after which the procedure will return. During what time window after a clock edge must `read_cycle` be called for this behaviour to result?

[5]

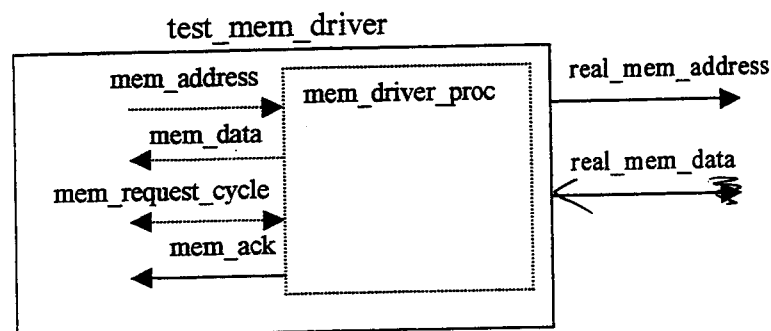


Figure 6.1


```

ENTITY test_mem_driver IS
    PORT (real_mem_address : OUT INTEGER;
          real_mem_data    : IN  STD_LOGIC_VECTOR( 7 DOWNTO 0);
          );
END test_mem_driver;

ARCHITECTURE behav OF test_mem_driver IS
    SIGNAL clk      : STD_LOGIC;
    SIGNAL mem_ack   : BOOLEAN;
BEGIN

    clkgen : PROCESS
    BEGIN
        clk <= '0';
        WAIT FOR 50 ns;
        clk <= '1';
        WAIT FOR 50 ns;
    END PROCESS clkgen;

    mem_driver_proc : PROCESS
    BEGIN
        FOR i IN 1 TO 10 LOOP
            WAIT FOR 0 ns;
        END LOOP;
        IF mem_request_cycle THEN
            real_mem_address <= mem_address;
            WAIT UNTIL clk'EVENT AND clk = '1';
            mem_data        := real_mem_data;
            mem_ack          <= true;
            mem_request_cycle := false;
            WAIT FOR 0 ns;
            mem_ack          <= false;
        ELSE
            real_mem_address <= 0;
            WAIT UNTIL clk'EVENT AND clk = '1';
            WAIT FOR 0 ns;
            mem_data        := (OTHERS => 'X');
        END IF;
    END PROCESS mem_driver_proc;

    p1 : PROCESS
        VARIABLE a, b : STD_LOGIC_VECTOR( 7 DOWNTO 0);
    BEGIN
        WAIT UNTIL clk'EVENT AND clk = '1';
        WAIT FOR 0 ns;
        WAIT FOR 0 ns;
        read_cycle( 1, a, mem_ack, clk);
        read_cycle( 2, b, mem_ack, clk);
        WAIT;
    END PROCESS p1;

END behav;

```

Figure 5.1 (continued on next page)

```
PACKAGE comms IS
```

```
    SHARED VARIABLE mem_request_cycle : BOOLEAN := false;
    SHARED VARIABLE mem_address       : INTEGER;
    SHARED VARIABLE mem_data          : STD_LOGIC_VECTOR( 7 DOWNT0 0);
```

```
    PROCEDURE read_cycle(
        addr      : IN  INTEGER;
        VARIABLE data : OUT STD_LOGIC_VECTOR(7 DOWNT0 0);
        SIGNAL    ack  : IN  BOOLEAN;
        SIGNAL    clk  : IN  STD_LOGIC);
```

```
END PACKAGE comms;
```

```
PACKAGE BODY comms IS
```

```
    PROCEDURE read_cycle(
        addr      : IN  INTEGER;
        VARIABLE data : OUT STD_LOGIC_VECTOR(7 DOWNT0 0);
        SIGNAL    ack  : IN  BOOLEAN;
        SIGNAL    clk  : IN  STD_LOGIC) IS
    BEGIN
        WAIT FOR 0 ns;
        WAIT FOR 0 ns;
        WHILE mem_request_cycle = true LOOP
            WAIT UNTIL clk'EVENT AND clk = '1';
            WAIT FOR 0 ns;
            WAIT FOR 0 ns;
        END LOOP;
        mem_request_cycle := true;
        mem_address       := addr;
        WAIT UNTIL ack;
        data               := mem_data;
    END read_cycle;
```

```
END PACKAGE BODY comms;
```

Figure 5.1 (continued from previous page)

ANSWERS - VHDL & Logic Synthesis

Question 1.

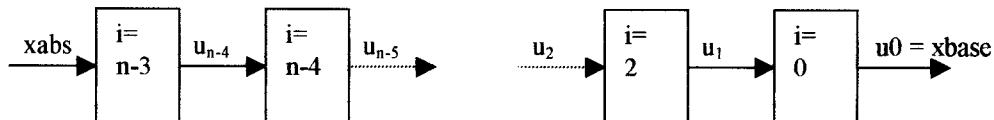
a)

If the input is negative, it is negated and the output negated to compensate. The *reduce* stage subtracts a multiple of 2π and hence does not alter answer. Finally the output of *reduce* is in the correct range for *sine_lookup* to return the correct answer.

$xabs$ is always positive, its max value (1.000) uses what was the sign bit in x , hence length $xabs$ also $n+m$. $xbase$ ranges unsigned up to $2\pi \Rightarrow 3$ bits to left of point, $m+3$ bits overall. Finally y must range from -1 to 1, hence $m+2$ bits.

[5]

- b) If the *then* part is taken, u_{i-1} is $< 2^{i-1} \cdot 2\pi$. Otherwise, the subtraction forces u_{i-1} to be in the required range. $xabs$ has max value 2^{n-1} . Hence $\text{ceiling}(n-1-\log_2(2\pi)) = n-3$ stages are needed to reduce value to max 2π .



[5]

- c) Using the definitions given in the architecture declaration section of Figure 1.2, write a synthesisable architecture for *sinegen*.

P. 65.
J. S. Chong

```

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
USE ieee.std_logic_arith.ALL;
USE work.utils.ALL;
USE work.sine_lookup;

ARCHITECTURE rtl OF sinegen IS
    CONSTANT cnum : INTEGER := n-3;
    CONSTANT pi: real := 3.1415926
    TYPE x2_arr IS ARRAY (0 TO cnum-1) OF STD_LOGIC_VECTOR(x'RANGE);
    SIGNAL x2_pre : x2_arr;
    SIGNAL xabs : STD_LOGIC_VECTOR(x'RANGE);
    SIGNAL x2 : STD_LOGIC_VECTOR(m+2 DOWNT0 0);
    SIGNAL x4 : STD_LOGIC_VECTOR(m+1 DOWNT0 0);
BEGIN

    stage1 : PROCESS(x)
    BEGIN
        IF SIGNED(x) < 0 THEN
            xabs <= -SIGNED(x);
        ELSE
            xabs <= x;
        END IF;
    END PROCESS stage1;

    stage2 : PROCESS(x2_pre, xabs)
    VARIABLE i : INTEGER;
    VARIABLE tmp : STD_LOGIC_VECTOR(x'RANGE);
    BEGIN
        FOR i IN 0 TO cnum-1 LOOP
            tmp := xabs;
            IF i /= cnum-1 THEN
                tmp := x2_pre(i+1);
            END IF;
            IF UNSIGNED( tmp) > INTEGER(REAL(2**(i+m))*2.0*pi) THEN
                x2_pre(i+1) <= UNSIGNED(tmp) -
                    INTEGER(REAL(2**(i+m))*2.0*pi);
            ELSE
                x2_pre(i+1) <= tmp;
            END IF;
        END LOOP;

        xbase <= x2_pre(0) (m+2 DOWNT0 0);
    END PROCESS stage2;

```

```
stage4: ENTITY sine_lookup GENERIC MAP(m=>m)
        PORT map(x=>xbase,y=>x4);
```

```
p_out: PROCESS(x,x4)
begin
    IF x(x'left) = '1' THEN
        y <= 0-unsigned(x4);
    else
        y <= x4;
    END IF;
```

```
END process p_out;
```

```
END rtl;
```

[10]

Question 2.

This question requires you to write a testbench for the entity *sinegen* defined in question 1.

- a) Why is the function *random* declared IMPURE?

It must return a different value each time it is called, using a shared variable to store state. Therefore it must be declared impure.

[2]

- b) Write a testbench entity for *sinegen*.

```
LIBRARY ieee;
LIBRARY mathlib
    USE ieee.std_logic_1164;
    USE ieee.std_logic_arith;
    USE mathlib.maths;

ENTITY sinetest IS
    GENERIC( testnum : INTEGER;
            m       : INTEGER := 20;
            n       : INTEGER := 8
            );
END sinetest;

ARCHITECTURE behav OF sinetest IS
    SIGNAL x_i: std_logic_vector(m+n-1 downto 0);
    SIGNAL y_i: std_logic_vector(m+1 downto 0);
BEGIN

    dut: ENTITY sinegen(x_i, y_i);

    dotest: PROCESS
    BEGIN
        FOR i = 1 TO testnum LOOP
            x_i <= random(0, 2**(n+m)-1);
            WAIT FOR 100 ns;
            y_real := REAL(conv_integer(SIGNED(y_i)))/REAL(2**m);
            x_real := REAL(conv_integer(SIGNED(x_i)))/REAL(2**m);
            ASSERT ABS(y_real - sin(x_real)) < (1.0 / REAL(2**m))
                REPORT "Bad output : x = " & REAL'IMAGE(x_real) & ", y = " & REAL'IMAGE(y_real);
            SEVERITY warning;
        END LOOP;
        REPORT "Test finished";
        WAIT;
    END PROCESS dotest;

end;
```

[12]

- c) Discuss the relative merits of pseudo-random and exhaustive testing of *sinegen*, assuming that each separate test takes 1ms to execute.

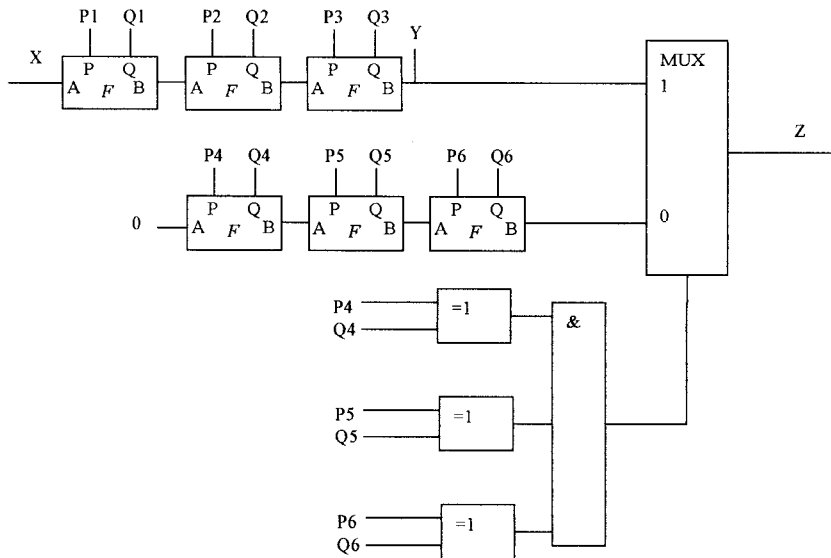
For exhaustive testing execution time is 2^{n+m} ms. This is feasible for say $n+m < 25$. Above this size the test time becomes unpleasantly large. Pseudo-random testing will provide a faster test, of reasonably good quality. If *sine_lookup* is based on a ROM lookup every location should be tested if possible => exhaustive test of all values $< \pi/2$. Corner cases should be added: $x = \text{max negative input}$, $\text{max positive input}$, $x = \pi/2$ (sin is max) $x = -\pi/2$ (sin is min).

[6]

Question 3

- a) By applying controllability factoring at point Y, derive an equivalent circuit with reduced critical path length. What is your control function C?

$$C = (P4 \text{ xor } Q4).(P5 \text{ xor } Q5).(P6 \text{ xor } Q6).$$

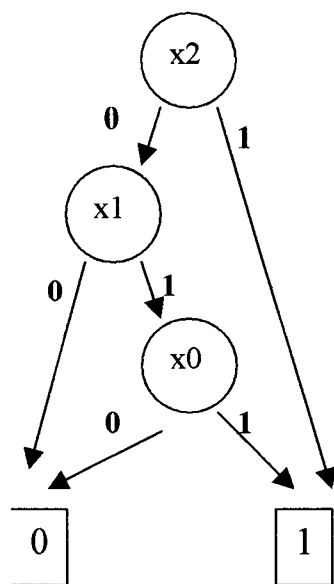


[10]

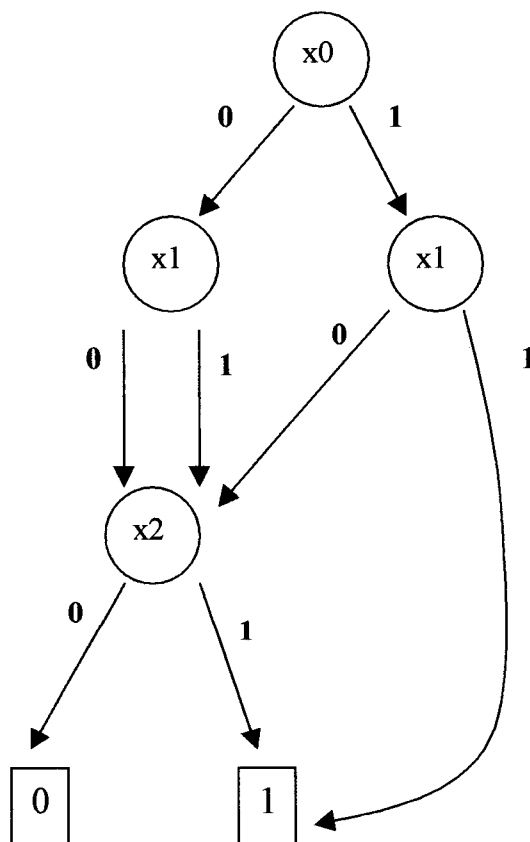
- b) Write a truth table for y , and compute two ROBDDs for y using variable orders: $x(0), x(1), x(2)$, and $x(2), x(1), x(0)$ respectively.

X2	X1	X0	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Order: $x(2), x(1), x(0)$



Order $x(0), x(1), x(2)$

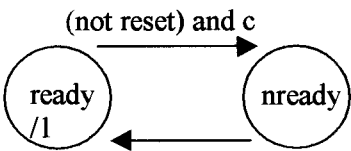


[10]

Question 4

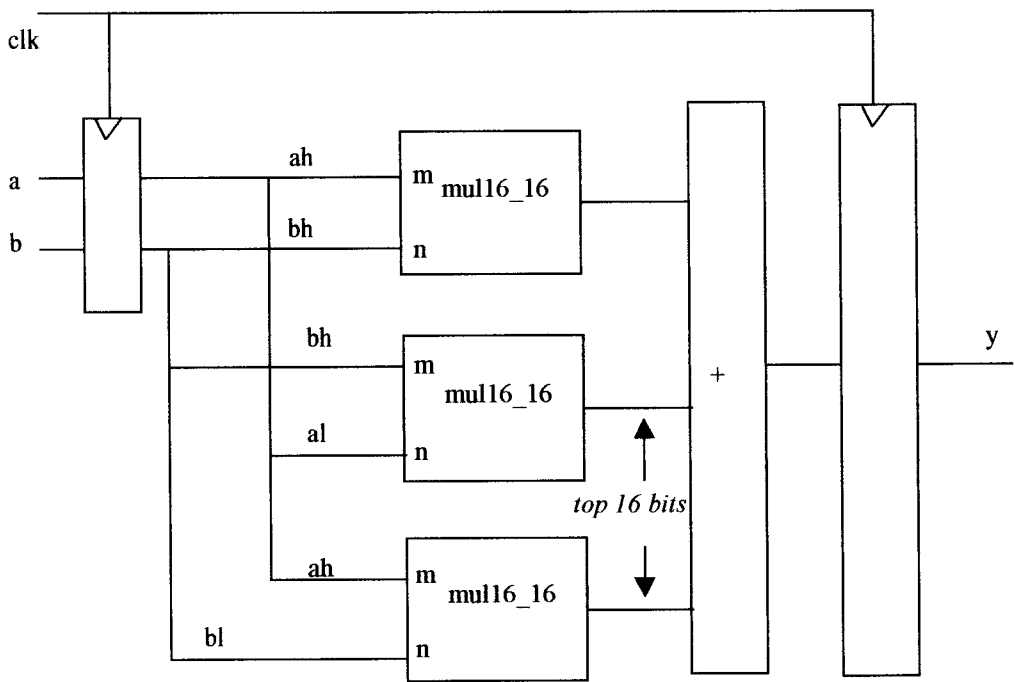
a)

Output: ready (default 0).



[5]

b)



c will be 0 iff $a < 2^{24}$ and $b < 2^{24}$.

[5]

c) Implement in VHDL the architecture of *mult32_32*.

[10]

```
ARCHITECTURE rtl OF mult32_32 IS
    SIGNAL p1, p2, p3          : STD_LOGIC_VECTOR(31 DOWNTO 0);
    SIGNAL a_big, b_big, ready_int : STD_LOGIC;
    SIGNAL al, ah, bl, bh      : STD_LOGIC_VECTOR(15 DOWNTO 0);
    SIGNAL fsmwait             : STD_LOGIC;  --FSM state bit
    SIGNAL c                   : STD_LOGIC;  --wait condition
BEGIN

    in_reg : PROCESS
    BEGIN
        WAIT UNTIL clk'EVENT AND clk = '1';
        IF fsm_wait = '0' THEN
            al <= a(15 DOWNTO 0);
            ah <= a(31 DOWNTO 16);
            bl <= b(15 DOWNTO 0);
            bh <= b(31 DOWNTO 16);
        END IF;
    END PROCESS in_reg;

    c <= a_big or b_big;

    m1 : ENTITY mul16_16(ah, bl, p1);
    m2 : ENTITY mul16_16(bh, al, p2);
    m3 : ENTITY mul16_16(ah, bh, p3);

    out_reg : PROCESS
    BEGIN
        y <= p3 + (p1(31 DOWNTO 16)+p2(31 DOWNTO 16));
    END PROCESS out_reg;

    compare : PROCESS(a, b)
    BEGIN
        a_big <= UNSIGNED(a(31 DOWNTO 24)) /= 0;
        b_big <= UNSIGNED(b(31 DOWNTO 24)) /= 0;
    END PROCESS compare;

    fsm : PROCESS
    BEGIN
        WAIT UNTIL clk'EVENT AND clk = '1';
        IF reset = '1' THEN
            fsmwait <= '0';
        ELSE
            fsmwait <= c AND NOT fsmwait;
        END IF;
    END PROCESS fsm;

    ready <= NOT fsmwait;

END ARCHITECTURE rtl;
```

Question 5.

- a) mem_ack is waited on by read_cycle, hence must be signal. mem_cycle_request is driven in both read_cycle and mem_driver_proc, hence must be shared variable. mem_data could be a signal, the extra 1 delta delay would not matter since mem_ack has similar delay. mem_address could be a signal, although to be safe at all times a 1 delta delay would need to be added to mem_driver_proc, between checking mem_cycle_request and reading mem_address. However if read_cycle were called from multiple processes this would not be possible.

[10]

b)

```
PROCEDURE delay_by_clocks_and_deltas( SIGNAL clk : IN STD_LOGIC;
                                     m       : IN INTEGER;
                                     n       : IN INTEGER) IS
BEGIN
    FOR i IN 1 TO n LOOP
        WAIT UNTIL clk'EVENT AND clk = '1' AND clk'LAST_VALUE = '0';
    END LOOP;

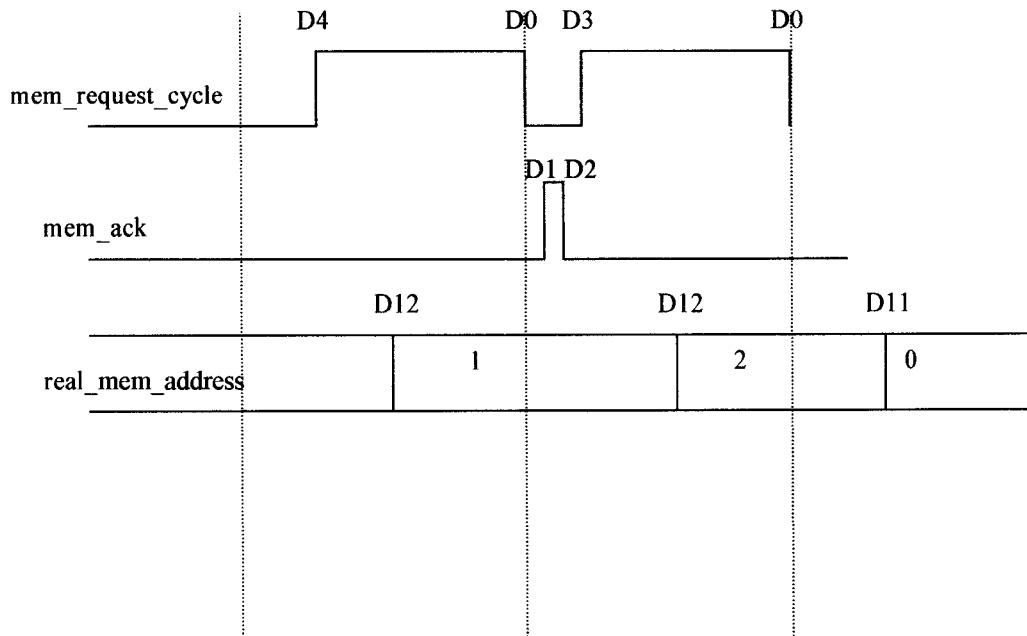
    FOR i IN 1 TO m LOOP
        WAIT FOR 0 ns;
    END LOOP;
END PROCEDURE delay_by_clocks_and_deltas;
```

[10]

Question 6.

- a) Draw the waveforms of all signals and shared variables used in `test_mem_driver`, until the final (indefinite) wait statement in process `p1` is executed. You must indicate precise timing of all signal and shared variable transitions, including delta delays where relevant.

D = number of deltas from clock edge. All times are referenced from clock rising edge, at 50, 150, 250. NB clock edge is actually at delta 1 in this architecture, so add 1 for true deltas. `mem_data` changes on falling edge of `mem_request_cycle`. `mem_addr` changes 1 delta after rising edge of `mem_request_cycle`.



[15]

- b) During what time window after a clock edge will `read_cycle` have this behaviour?

`mem_request_cycle` is tested 11delta after the clock edge by `mem_driver` proc. For it to be certainly read as set, it must be set 10 delta after clock => `read_cycle` executed 8 delta after clock edge. `mem_request_cycle` is tested by `read_cycle` 2 delta after the call, and reset by `mem_driver_proc` on the clock edge. So window is clock edge to clock edge + 8 delta.

[5]