Imperial College London

MEng (Engineering) Examination 2017

Year 1

AE1-106 Properties of Materials

Thursday 8th June 2017: 14.00 to 17.00
[3 hours]

The paper is divided into Section A and Section B
and contains ***THREE*** questions.
All questions carry equal marks.
Candidates may obtain full marks for complete answers to ***ALL*** questions.

You must answer each section in a separate answer booklet.

A data sheet is attached.

The use of lecture notes is NOT allowed.

Section A

1.

(a) A cubic specimen of initial side length $L_0 = 10$ mm is subject to the simultaneous action of a uniform increase in temperature $\Delta T = +150\,\mathrm{K}$ and a hydrostatic compressive stress field $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -10\,\mathrm{MPa}$. The material is an isotropic elastic metal with thermal expansion coefficient $\alpha = 10^{-5}\,\mathrm{K}^{-1}$ and bulk modulus $K = 80\,\mathrm{GPa}$. Calculate the final side length of the cube L.

[10%]

(b) A component is made from an isotropic metal with mechanical properties $E = 100$ GPa; $\nu = 0.3$; $\sigma_Y = 500\,\mathrm{MPa}$. The stress state at a point in this component is given, in a certain Cartesian reference system, by

$$[\sigma] = \begin{pmatrix} 100 & 100 & 0 \\ 100 & 200 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathrm{MPa}.$$

The material obeys the von Mises yield criterion.

i. Check that the given state of stress is not sufficient to initiate plasticity at this point.

[10%]

ii. Calculate the corresponding elastic strain tensor at this point.

[10%]

(c) A metallic circular cylinder of initial cross-section $A_0 = 100\,\mathrm{mm}^2$ and height $L_0 = 20\,\mathrm{mm}$ is made from a perfectly plastic metal with $E = 100$ GPa; $\sigma_Y = 300\,\mathrm{MPa}$. The cylinder is axially compressed by $\Delta L = -5\,\mathrm{mm}$ and subsequently unloaded. Calculate (i) the final height of the cylinder, (ii) the true plastic strain and (iii) the applied force at the onset of plastic deformation.

[20%]

[CONTINUED OVERLEAF]

(d) A ductile metal sample of hardness $H = 1\,\mathrm{GPa}$ is tested with a Vickers indenter to a maximum force of $20\,\mathrm{N}$. Calculate the diagonal length of the square indentation left after the experiment.

[10%]

(e) A metallic rope is made from 20 straight filaments, each of diameter $D = 0.1$ mm; 12 of these filaments are made from a Titanium alloy and 8 are made from a Copper alloy. The two alloys have identical elastic properties ($E = 100\,\mathrm{GPa}$, $\nu = 0.3$) but dissimilar plastic response: the Titanium alloy can be taken as perfectly plastic $\left(\sigma_Y^{\mathrm{Ti}} = 500\,\mathrm{MPa}\right)$ while the Copper alloy is linearly hardening with yield stress $\sigma_Y^{\mathrm{Cu}} = 200\,\mathrm{MPa}$ and nominal hardening modulus $H^{\mathrm{Cu}} = 40\,\mathrm{GPa}$. A length of the rope $L_0 = 1\,\mathrm{m}$ is progressively elongated by application of a tensile force; it can be assumed that the 20 filaments are in a state of uniaxial stress and are subject to equal axial strains. Determine (i) which of the two materials will first yield, and the corresponding total force carried by the rope; (ii) the total force carried by the rope when both materials yield.

[20%]

(f) Briefly define the degree of polymerisation and the glass transition temperature T_g of a polymer. Explain the fundamental difference between a thermoset polymer and an elastomer.

[10%]

(g) i. Sketch the uniaxial compressive stress versus strain response of a foam, naming and briefly describing its salient phases.

ii. Define *porosity* and *relative density* of a foam; write a mathematical formula relating these properties.

iii. Calculate the axial elastic modulus of a honeycomb made from Aluminium ($E = 70\,\mathrm{GPa}$) if the relative density is 0.05.

[10%]

2.

(a) A rod is manufactured from a unidirectional carbon fibre - epoxy composite of negligible porosity and fibre volume fraction of $\varphi = 0.65$. The properties of the fibres (F) and matrix (M) are

$$E^F = 220 \text{ GPa}, \ \rho^F = 1600 \text{ kg m}^{-3}; \ \ E^M = 2.5 \text{ GPa}, \ \rho^M = 1350 \text{ kg m}^{-3}.$$

The fibres are aligned in the axial direction of the rod. Calculate the density of the rod as well as the axial and radial elastic moduli.

[15%]

(b) A horizontal cantilever beam of length $L = 3$ m and square cross-section of area $t \times t$ needs to be designed. The beam, of density ρ, is required to carry only its own self-weight without yielding and with a tip deflection of exactly $\delta = 0.15$ m. The maximum deflection and the maximum stress for a cantilever subject to its own self-weight are given by

$$\delta_{\max} = \frac{3\rho g L^4}{2Et^2}; \quad \sigma_{\max} = \frac{3\rho g L^2}{t},$$

where $g = 9.81 \text{ m/s}^2$ is the acceleration due to gravity.

i. Derive the material merit index which should be maximised to obtain a design of minimum cost (assume that cost C is proportional to the mass m, $C = mc$ where c is the cost per unit mass).

[15%]

ii. Select which of the following two materials is most appropriate for this design: steel ($E = 210 \text{GPa}, \ \sigma_Y = 350 \text{MPa}, \ \rho = 8000 \text{kg m}^{-3}, \ c = 1.5$ £/kg) or an Aluminium alloy ($E = 70$ GPa, $\sigma_Y = 200 \text{MPa}, \ \rho = 2700 \text{kg m}^{-3}$, $c = 5$ £/kg). Calculate the optimal value of t.

[10%]

[CONTINUED OVERLEAF]

(c) Define the Mode I strain energy release rate G_I. Define the critical strain energy release rate G_{IC}; how is this quantity related to surface tension, for both ductile and brittle materials?

[10%]

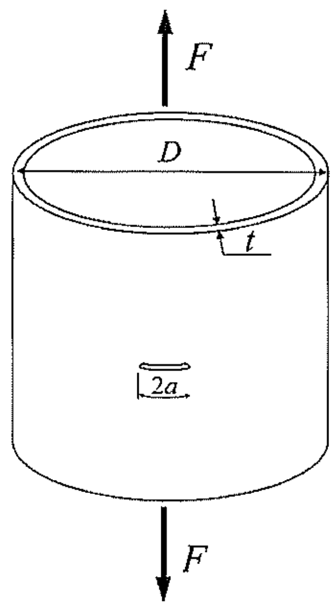

Figure 1. Sketch of the cracked glass tube described in question 2 (d).

(d) A thin-walled glass tube of diameter $D = 200\ \mathrm{mm}$ and wall thickness $t = 2\ \mathrm{mm}$ is loaded in axial tension by a force F, as shown in Figure 1. Glass has fracture toughness $K_{Ic} = 0.5\ \mathrm{MPa}\sqrt{\mathrm{m}}$. Due to an accidental impact, the tube contains a small through-thickness crack of length $2a = 4\ \mathrm{mm}$ oriented in the circumferential direction. Calculate the force at which the tube will fracture.

[10%]

(e) Describe briefly, with the aid of suitable graphs, the phenomena of (i) strain rate sensitivity, (ii) stress relaxation and (iii) creep in the uniaxial response of viscous materials.

[15%]

[CONTINUED OVERLEAF]

(f) State the main reason for the brittle response of ceramics. Discuss briefly, with the aid of a sketch, the effect of confinement on the compressive response of ceramics.

[10%]

(g) A foam cylinder of diameter $D = 10\ \mathrm{mm}$ and length L is supported at one end by a rigid surface and impacted at the opposite end by a rigid cylindrical projectile of the same diameter D, mass $M = 0.05\ \mathrm{kg}$ and velocity $V = 100\ \mathrm{m/s}$. The foam has plateau strength $\sigma_Y = 350\ \mathrm{MPa}$ and nominal densification strain of 0.7. Calculate the deceleration of the projectile and the minimum length L of the foam cylinder to ensure that this will not undergo full densification.

[15%]

Section B

3.

(a)

Material	Structure	Melting Temperature °C
Material A	FCC	1085
Material B	Liquid at room temperature	0
Material C	Rock Salt	801
Material D	Diamond lattice	3550
Material E	Zinc Blende	1412
Material F	Gas at room temperature	- 189

Materials A-F in the table above are (in random order): Sodium chloride (NaCl), Argon (Ar), Copper (Cu), Diamond, Water (H_2O) and Silicon carbide (SiC).

i. Identify materials A-F. Indicate the type of atomic bond present in each at room temperature. Briefly describe typical characteristics of each bond.

[12%]

ii. State which of the six materials you would expect to have: the highest modulus of elasticity; the highest hardness; the highest ductility; the highest electrical conductivity. Briefly explaining your reasoning.

[4%]

iii. Material A has an FCC crystal structure and an atomic radius of 0.1278 nm. Assume the atoms are hard spheres that touch each other along the face diagonals of the FCC unit cell as shown in Figure 2.

How many Material A atoms are in one unit cell?

[2%]

Calculate a theoretical value for the density of Material A. The atomic mass of Material A is 63.45 g/mol and Avogadro's number R is 6.02×10^{23} atoms/mol.

[8%]

[CONTINUED OVERLEAF]

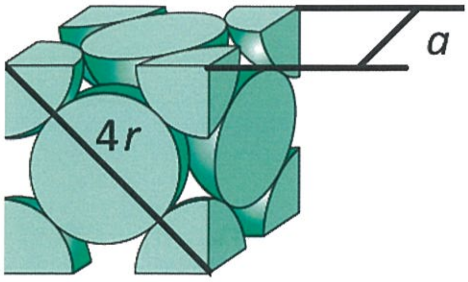

Figure 2. FCC unit cell in which a is the lattice parameter and r is the atomic radius.

(b) With reference to Figure 3, the potential energy U of two atoms separated by a distance r is given by:

$$U = -\frac{A}{r^m} + \frac{B}{r^n} \qquad (1)$$

r_0 is the equilibrium interatomic spacing. Derive an expression for the equilibrium bond energy U_0 in terms of parameters A, B, n and m.

Sketch a potential energy versus separation distance curve for the atom pair, and sketch beneath this curve the related force versus separation distance curve. Mark U_0, r_0, the regions of attraction force and repulsion force on the curves.

[16%]

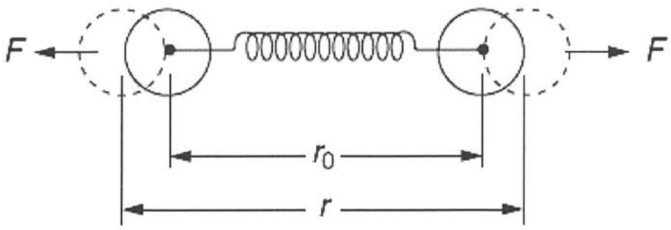

Figure 3. Idealisation of two atoms in a molecule.

[CONTINUED OVERLEAF]

(c) Sketch three-dimensional views of a cubic unit cell, showing

i. a (100) plane, a (110) plane, a (111) plane and a (221) plane.

[4%]

ii. a [100] direction, a [111] direction and a [210] direction.

[3%]

iii. One slip plane for the face centred cubic (FCC) crystal structure in three-dimensional view and label the slip directions in the plane. What is the slip system for this structure?

[5%]

(d) Calculate the resolved shear stress on the (111) $[0\bar{1}1]$ slip system of a unit cell in an FCC Nickel single crystal if a stress of 13.7 MPa is applied in the $[001]$ direction of a unit cell. Use a schematic drawing to explain your calculation.

[14%]

(e) i. The yield strength of an undeformed 70%Cu-30%Zn brass alloy is 150 MPa when the grain diameter d is 10 μm and 100 MPa when the grain diameter d is 28 μm. Predict the yield strength of this alloy when the average grain diameter is 1×10^{-3} mm. Assume the Hall-Petch equation below applies.

$$\sigma_{yield} = \sigma_0 + k_y d^{-0.5}$$

[6%]

ii. A sheet of the same Cu-Zn alloy is to be cold-rolled from 0.07 to 0.04 cm. Calculate the percent cold work, assuming the width change of the sheet can be neglected. Using the graph in Figure 4, estimate the tensile strength (in ksi), yield strength (in ksi) and elongation.

[CONTINUED OVERLEAF]

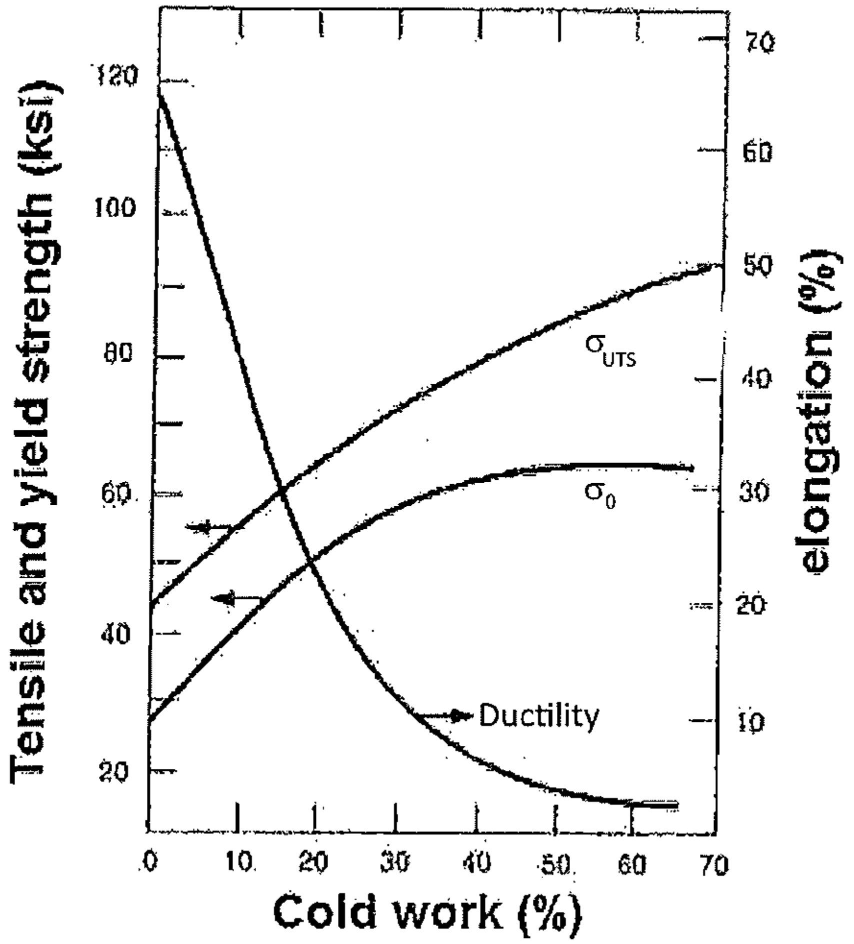

Figure 4. Percent cold work vs. tensile strength, yield strength and elongation for 70 wt% Cu – 30 wt% Zn alloy.

[10%]

iii. Name the two reinforcing mechanisms present in (i) and (ii). List the other two strengthening mechanisms that are also commonly used in metallic materials. Briefly explain these four mechanisms, discussing the role of dislocations in each of them.

[16%]

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Setter VL Tagarielli:

Marks

$\underline{1(a)}$

Use principle of superposition (shear strains are absent due to symmetry of loading):

$$\varepsilon_V^{\,thermal} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\alpha\Delta T;\ \varepsilon_V^{\,stress} = \frac{\sigma_H}{K} = \frac{p}{K};\ \varepsilon_V^{\,total} = 3\alpha\Delta T + \frac{\sigma_H}{K} = 3\,10^{-5}\,150 - \frac{10}{80000} =$$

$$= 4.5\ 10^{-3} - 0.125\ 10^{-3} = 4.375\ 10^{-3}.$$

$$V = V_0 + \Delta V = V_0\left(1+\varepsilon_V^{\,total}\right) = L^3 \Rightarrow L = L_0\sqrt[3]{\left(1+\varepsilon_V^{\,total}\right)} = 10.0145\ \text{mm}.$$

10

$\underline{1(b)}$

(i)

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[\left(100-200\right)^2 + \left(100-0\right)^2 + \left(200-0\right)^2 + 6\left(100^2+0+0\right)\right]} = 244.9 < \sigma_Y$$

10

(ii)

$$G = E/\left[2\left(1+\nu\right)\right] = 38.46\ \text{GPa}$$

$$\varepsilon_x = \frac{\sigma_1}{E} - \frac{\nu}{E}\left(\sigma_2\right) = 4\ 10^{-4}$$

$$\varepsilon_y = \frac{\sigma_2}{E} - \frac{\nu}{E}\left(\sigma_1\right) = 17\ 10^{-4}$$

$$\varepsilon_z = -\frac{\nu}{E}\left(\sigma_1+\sigma_2\right) = -9\ 10^{-4}$$

$$\gamma_{xy} = \tau_{xy}/G = 26\ 10^{-4}$$

$$\gamma_{xz} = \tau_{xz}/G = 0$$

$$\gamma_{yz} = \tau_{yz}/G = 0$$

10

$\underline{1(c)}$

$$\Delta L = -5\ \text{mm} \Rightarrow \varepsilon_n = \Delta L/L_0 = -0.25\ \ (>\varepsilon_y = \sigma_Y/E = 0.003);\ F_Y = \sigma_Y A_0 = 30\,\text{kN};$$

$$\varepsilon_n^{pl} = \varepsilon_n - \varepsilon^{el} = \varepsilon_n - \sigma/E = -0.25 - (-300/100000) = -0.25 + 0.003 = -0.247;$$

$$\Delta L^{pl} = L_0\varepsilon_n^{pl} = -4.94\,\text{mm}.$$

$$L = L_0 + \Delta L = 20 - 4.94 = 15.06\ \text{mm}.$$

$$\varepsilon^{pl} = \ln\left(1+\varepsilon_n^{pl}\right) = \ln\left(1-0.247\right) = -0.2837.$$

20

$\underline{1(d)}$

$$H = F/A = \frac{2F}{d^2} \Rightarrow d = \sqrt{\frac{2F}{H}} = 200\ \mu\text{m}.$$

10

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1(e)

$\varepsilon_Y^{Ti} = \sigma_Y^{Ti} / E = 0.005$; $\varepsilon_Y^{Cu} = \sigma_Y^{Cu} / E = 0.002$; equal strains, Cu yields first at $\varepsilon = \varepsilon_Y^{Cu}$.

$A_0 = \pi D^2 / 4 = 7.854\ 10^{-3}\ \text{mm}^2$;

$F_Y^{Cu} = 12 A_0 \sigma^{Cu} + 8 A_0 \sigma^{Ti} = A_0 \left(12 E \varepsilon_Y^{Cu} + 8 E \varepsilon_Y^{Cu}\right) = 20 A_0 E \varepsilon_Y^{Cu} = 31.416\ \text{N}.$

Ti filaments will yield at a strain $\varepsilon = \varepsilon_Y^{Ti} = 0.005$ and stress $\sigma^{Ti} = \sigma_Y^{Ti}$.

At the (common) strain ε_Y^{Ti} the Cu filaments will be at a stress:

$\sigma^{Cu} = \sigma_Y^{Cu} + H^{Cu}\left(\varepsilon - \varepsilon_Y^{Cu}\right) = \sigma_Y^{Cu} + H^{Cu}\left(\varepsilon^{Ti} - \varepsilon_Y^{Cu}\right) = 320\ \text{MPa}.$

$F_Y^{Cu+Ti} = 12 A_0 \sigma^{Cu} + 8 A_0 \sigma_Y^{Ti} = 30.159 + 31.416 = 61.575\ \text{N}.$ 20

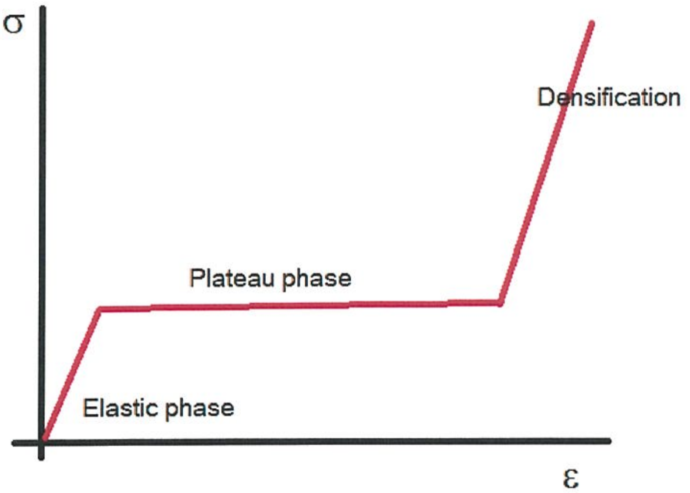
1(f)

DP: average number of repetitions of a monomer in a polymeric chain, quantifying the length of individual molecules;

Tg: Temperature at which secondary bonds melt, eliminating mechanical side-ways interactions between polymer chains.

Elastomer: cross-linked polymer above Tg; Thermoset: cross-linked polymer below Tg. 10

1(g)



Elastic: parent material deforms elastically; Plateau: progressive crushing/buckling/plastic bending of cell walls at approximately constant stress; Densification: compaction of the foam and stiffening due to self-contact of cell walls.

$$f = \frac{V_{pores}}{V_{total}};\ \bar{\rho} = \frac{V_{solid}}{V_{total}} = 1 - f.$$

$E_{honeycomb} = \bar{\rho} E_{solid} = 0.05\ \ 70 = 3.5\ \text{GPa}.$ 10

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2(a)

$\rho = \varphi\rho^F + (1-\varphi)\rho^M = 1512.5 \text{ kg m}^{-3}$;

$E_{axial} = \varphi E^F + (1-\varphi)E^M = 143.875 \text{ GPa}$;

$E_{radial} = \left(\frac{\varphi}{E^F} + \frac{(1-\varphi)}{E^M}\right)^{-1} = 6.995 \text{ GPa}$. 15

2(b)

$\sigma_{\max} = \frac{3L^2\rho g}{t} \le \sigma_Y \Rightarrow t \ge \frac{3L^2\rho g}{\sigma_Y}$, take $t = \frac{3L^2\rho g}{\sigma_Y}$.

$C = mc = t^2 L\rho c = 9L^5 g^2 \frac{\rho^3 c}{\sigma_Y{}^2} = 210468 \frac{\rho^3 c}{\sigma_Y{}^2} \Rightarrow M_{I_{\min C}} = \frac{\sigma_Y{}^2}{\rho^3 c}$. 15

$M_{I_{\min C}}{}^{Steel} = 159505$; $M_{I_{\min C}}{}^{Al\ alloy} = 406442 > M_{I_{\min C}}{}^{Steel}$; choose Al alloy (S.I. units).

$\delta_{\max} = \frac{3\rho g L^4}{2Et^2} = 0.15 \Rightarrow t^2 = 7946.1 \frac{\rho^{Al\ alloy}}{E^{Al\ alloy}} \Rightarrow t = 17.5 \text{ mm}$. 10

2(c)

$G_I = -\frac{dU}{dA}$ quantifies the decrease of elastic strain energy (-dU) in a cracked body when the cracked area increases (by dA).

At fracture (unstable crack propagation) in a ductile material it is

$-dU > 2\gamma dA + (\text{plastic dissipation at crack tip}) = G_{Ic} dA$.

Plastic dissipation is absent in a brittle materials, therefore $G_{Ic} = 2\gamma$. 10

2(d)

Fracture occurs when

$\sigma\sqrt{\pi a} = \frac{F}{\pi D t}\sqrt{\pi a} \ge K_{Ic} \Rightarrow F \ge \frac{\pi D t K_{Ic}}{\sqrt{\pi a}} = 7.926 \text{ kN}$.

Using net cross-section or formula for area of thick tubes also acceptable - but unnecessary. 10

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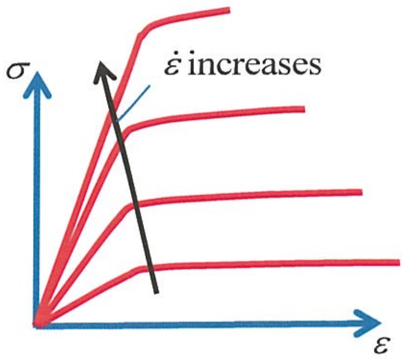
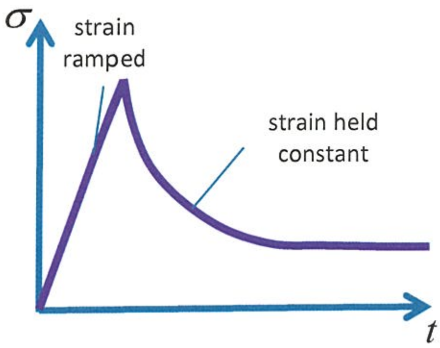
2(e)

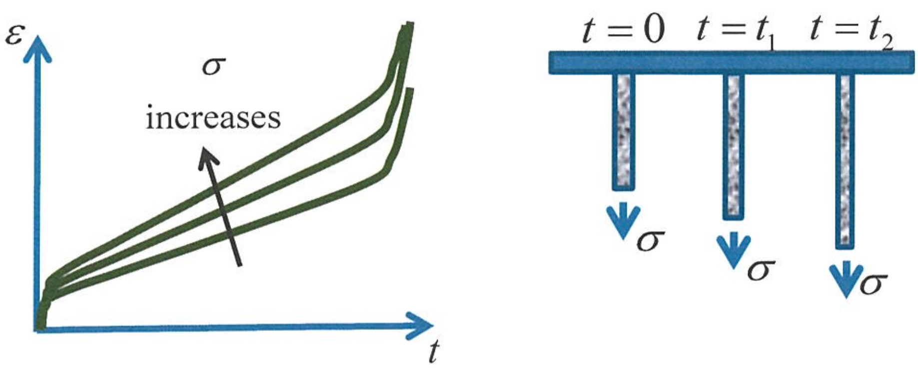
(i) Strain rate sensitivity: dependence of mechanical response upon imposed strain rate.



(ii) Stress relaxation: decrease in stress as a function of time while strain is held constant.



(iii) Creep: increase in strain as a function of time while stress is held constant.



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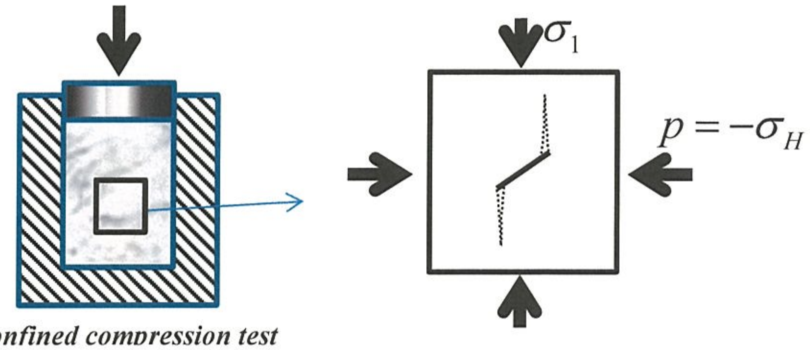
$2(f)$

Brittle response due to presence of defects such as inclusions, grain boundaries, voids and micro-cracks originating from processing. Confinement induces a state of high triaxiality (relatively high hydrostatic stress), due to impeded Poisson's lateral contraction. This increases frictional contact between crack faces and opposes dilation consequent to crack opening. The macroscopic effect is an increased strength and the possible occurrence of dislocation plasticity, due to the high load required to cause crack propagation.



Confined compression test

10

$2(g)$

$$a = \frac{\sigma_Y \pi D^2}{4M} = 5.497\ 10^5\ \text{m/s}^2; \quad \sigma_{pl}^{\max} \varepsilon_D A L > \frac{MV^2}{2} \Rightarrow L > \frac{2MV^2}{\sigma_{pl}^{\max} \pi D^2 \varepsilon_D} = 129.9\,\text{mm}.$$

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3a (i)

A: Cu. Metallic bond. Strong bond, non-directional. Stiff and strong, high melting point, conductive material. 2

B. Water. Hydrogen bond. Weak; directional bond. low melting point. 2

C. NaCl. Ionic bond. Non-directional bond. high melting point. Good insulators. 2

D. Diamond, Covalent bond. Very strong bond; highly directional. Stiff, anisotropic material. No conductivity. High Tmelt 2

E. ~~Silicon~~ SiC Silicon Carbide, Covalent. Very stiff and strong. High Tmelt. Good insulators. 2

F. Argon (Ar). Van der Waals. Weak bonds. Non-directional. Low melting point. 2

(ii) Highest Modulus. Diamond. Strongest covalent bonding. Stiffest "springs" between atoms. 1

Highest Hardness: Diamond. Covalent bonding, highest melting point. 1

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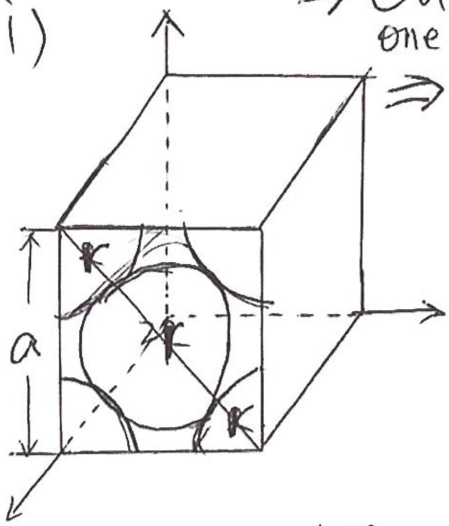
Highest ductility: Cu. Metallic bonding (non directional), fcc structure, large number of available slip systems. 1

Highest electrical conductivity: Cu. Metallic bonding (nondirectional). Free electrons are very mobile, allowing good conducting. 1

(iii)

$\Rightarrow$ Cu has $\frac{6}{2}+\frac{8}{8}=3+1=4$ atoms in one unit cell. 2



$\Rightarrow$ For FCC unit,

$$a^2+a^2=(r+2r+r)^2$$

$$2a^2=(4r)^2$$

$$\therefore 2a^2=16r^2$$

$$a=\frac{4r}{\sqrt{2}}$$ 1

$$a=\frac{4r}{\sqrt{2}}=\frac{4\times 0.1278\,nm}{\sqrt{2}}=0.361\,nm$$ 1

each copper atom has a mass:

$63.45\,(g/mol)\div 6.02\times10^{23}\,(atom/mol)$

the mass of Cu atoms in FCC unit is:

$$m=\frac{4\text{ atoms}\times 63.45\text{ g/mol}}{6.02\times10^{23}\text{ atoms/mol}}=4.22\times10^{-22}\,g$$ 2

the volume V of Cu unit is: $V=a^3=\left(0.361\,nm\times\frac{10^{-9}m}{nm}\right)^3=4.7\times10^{-29}m^3$ 2

$\therefore$ density of Cu $\rho=\frac{m}{V}=\frac{4.22\times10^{-22}}{4.7\times10^{-29}}=8.98\times10^{-6}\,g/m^3=8.98\,g/cm^3$ 2

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3b. $U = -\frac{A}{r^m} + \frac{B}{r^n}$

$$\frac{du}{dr} = m\frac{A}{r^{m+1}} - n\frac{B}{r^{n+1}}$$ 2

at $r = r_0$

$$\frac{du}{dr} = 0 \quad \therefore \quad m\frac{A}{r_0^{m+1}} - n\frac{B}{r_0^{n+1}} = 0$$ 3

$$m\frac{A}{r_0^{m+1}} = n\frac{B}{r_0^{n+1}}$$

$$r_0^{m-n} = \frac{mA}{nB}$$

$$r_0 = \sqrt[m-n]{\left(\frac{mA}{nB}\right)}$$ 2

$$U_0 = -\frac{A}{r_0^m} + \frac{B}{r_0^n} = -\frac{A}{\left(\frac{mA}{nB}\right)^{m \cdot \frac{1}{m-n}}} + \frac{B}{\frac{mA}{nB}^{\frac{n}{m-n}}}$$

$$= \frac{-A \cdot (nB)^{\frac{m-n}{m}}}{(mA)^{\frac{m}{m-n}}} + \frac{B \cdot (nB)^{\frac{m-n}{n}}}{(mA)^{\frac{n}{m-n}}}$$ 2

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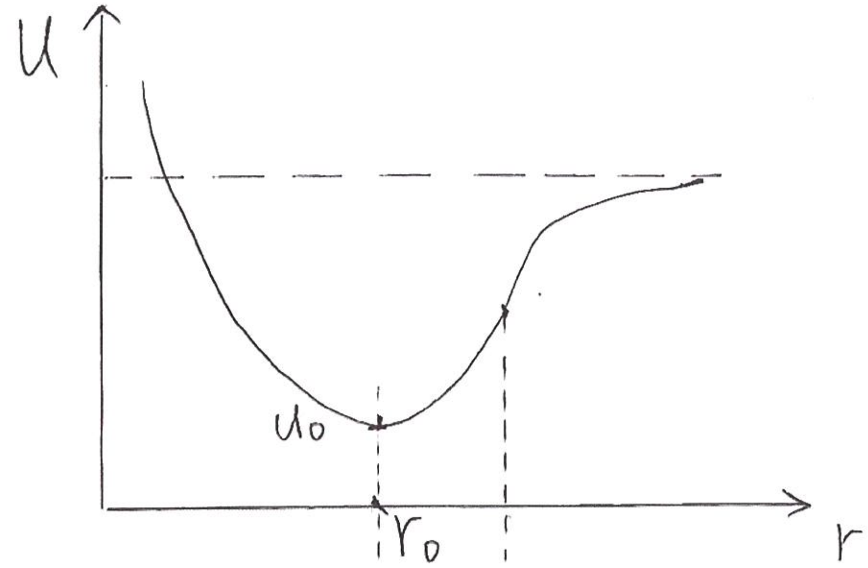

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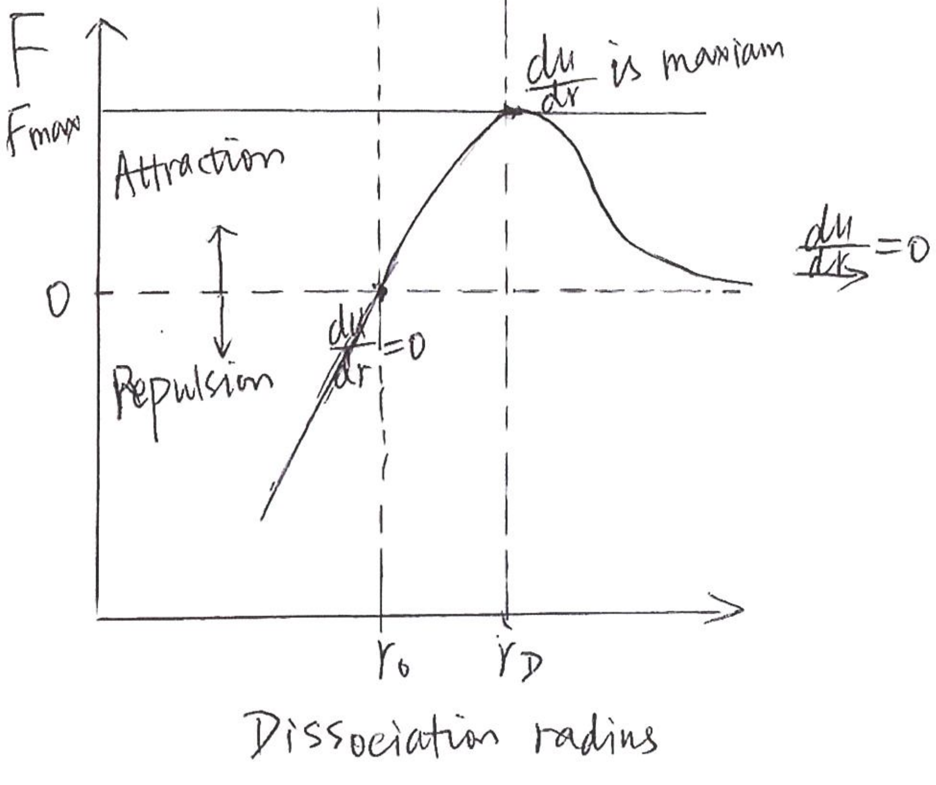

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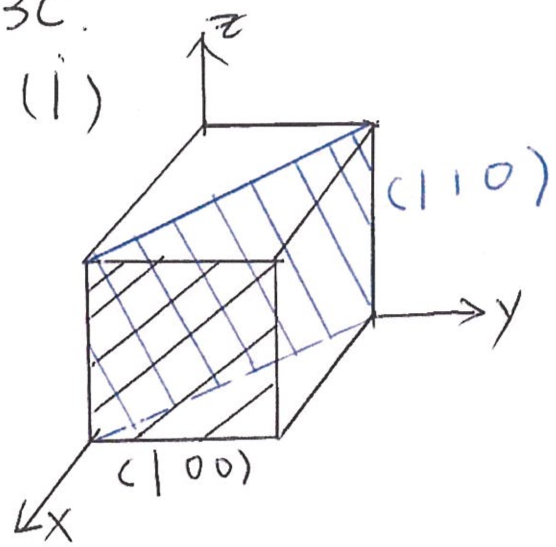
3C.

(i)



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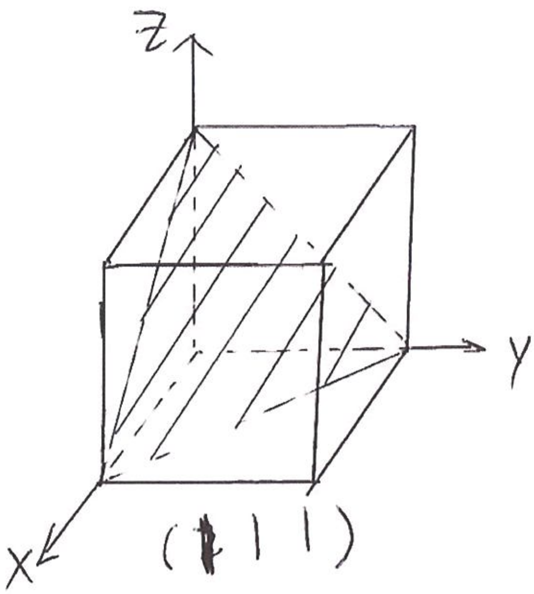

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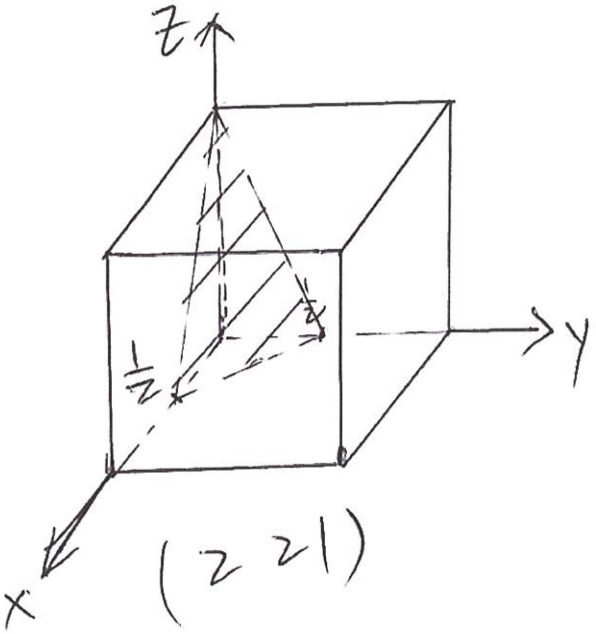

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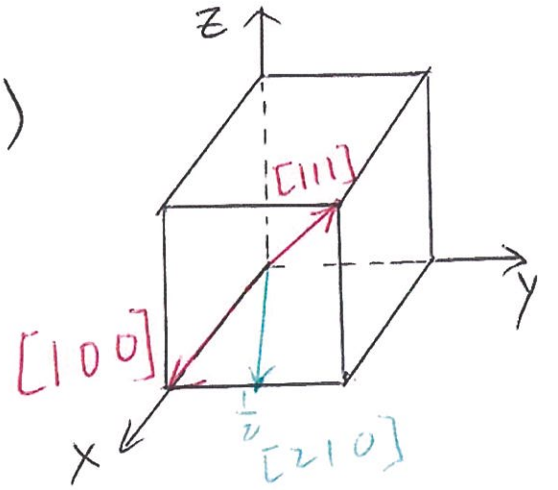
(ii)



3

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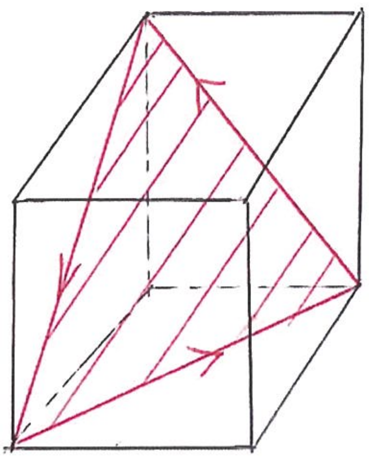
(iii) Slip planes {111}, directions <110> 4



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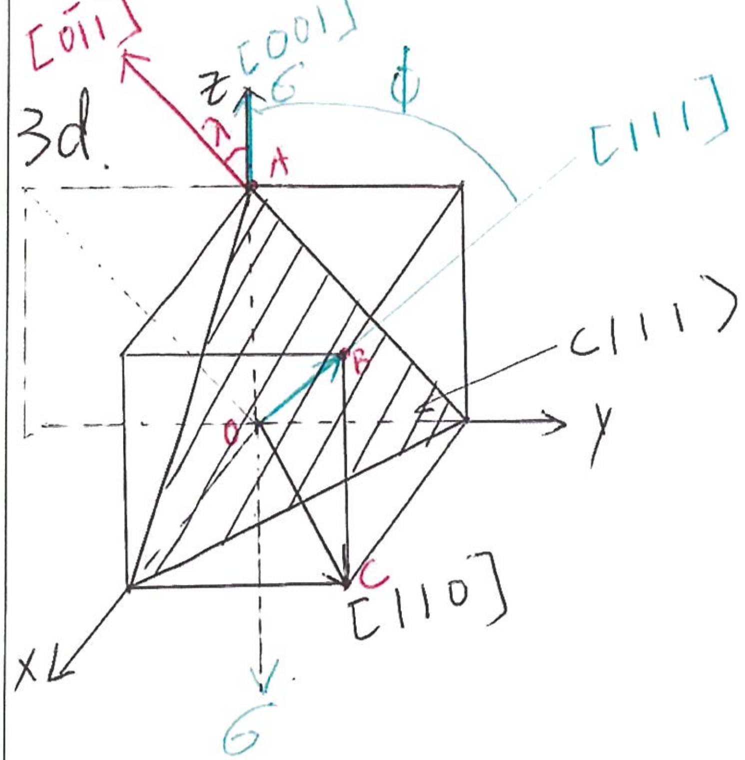
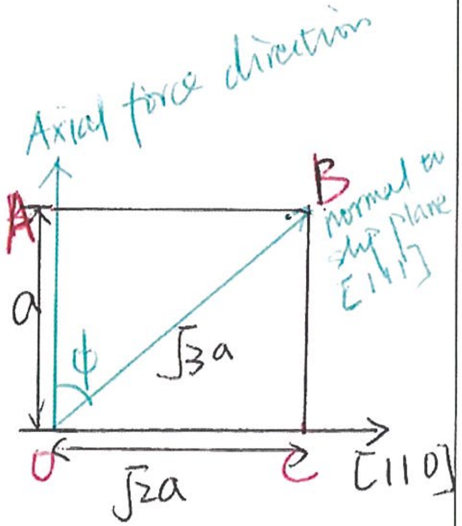
3d.





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By geometry angle λ between the applied stress and the slip direction is 45°. In the cubic system the direction indices of the normal to a crystal plane are the same as the Miller indices of the crystal plane. Therefore the normal to 2 the (1 1 1) plane that is the slip plane is the [1 1 1] direction.

$\cos\phi = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$, $\phi = 54.74°$ 2

resolved sheer stress 2

$\tau_r = \sigma \cos\lambda \cos\phi$

$= 13.7 MPa \times \cos 45° \times \cos 54.74°$ 2

$= 5.6 MPa$

3e.(i) $150 = \sigma_0 + k_y (10)^{-0.5}$

$100 = \sigma_0 + k_y (28)^{-0.5}$

$k_y = 392.94\ MPa\ (\mu m)^{-0.5}$ 2

$\sigma_0 = 25.74\ MPa$ 2

$\sigma_{yield} = \sigma_0 + k_y (1)^{-0.5}$

$= 418.68\ MPa$ 2

(ii) (1) $\%CW = \frac{0.7 - 0.4}{0.7} = 42.9\%$ 4

(2) From Figure 3, read out
tensile strength $\doteq$ 84 ksi, yield strength $\doteq$ 64 ksi 6
elongation $\doteq$ 5%

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Write on this side only (in ink) between the margins, not more than one solution per sheet please. Solutions must be signed and dated by both exam setter and referee.

Marks

(iii)

(i) Grain refinement: Grain boundaries act as obstacles to dislocation motion (Dislocation pile ups at grain boundaries) and the increased obstacle density leads to a strengthening effect. 2 3

(ii) Work hardening: Plastic deformation leads to the multiplication of dislocations (or lengthening of dislocations) increasing dislocation density in the material. As dislocations interact with each other and can act as obstacles to each others motion this leads to strengthening of the material. Such as forging, rolling, drawing and extrusion, and cold work. 2 3

Other two strengthening mechanisms are:

Precipitation hardening

By choice of suitable alloying system and thermal treatment it is possible to form closely spaced precipitates within the crystal host lattice. These precipitates act as obstacles to dislocation motion and have to be overcome either by cutting through them or bowing around them, which leads to a strengthening effect. 3

Solid solution strengthening

By solving atoms of a different species in the host lattice dislocation motion can be hindered. The atomic mismatch between the atoms leads to stress/strain fields that interact with the dislocations and lead to reduced mobility and, hence, strengthening. 3

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Stress – strain definitions and equations of elasticity.

$$\sigma_n = \frac{F}{A_0}; \quad \sigma_t = \frac{F}{A} = \sigma_n\left(1+\varepsilon_n\right) \quad \left(\text{for plastically incompressible solids}\right);$$

$$\varepsilon_n = \frac{\Delta L}{L_0}; \; \varepsilon_t = \ln\left(1+\varepsilon_n\right);$$

$$\tau = G\gamma; \quad G = \frac{E}{2\left(1+\nu\right)};$$

$$\varepsilon_{V_n} = \frac{\Delta V}{V_0}; \quad \sigma_H = -p = \frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3};$$

$$\sigma_H = K\varepsilon_V; \quad K = \frac{E}{3\left(1-2\nu\right)}$$

$$\varepsilon^{th} = \alpha\,\Delta T$$

$$\begin{cases} \varepsilon_1 = \dfrac{\sigma_1}{E} - \dfrac{\nu}{E}\left(\sigma_2+\sigma_3\right) + \alpha\,\Delta T \\ \varepsilon_2 = \dfrac{\sigma_2}{E} - \dfrac{\nu}{E}\left(\sigma_1+\sigma_3\right) + \alpha\,\Delta T \\ \varepsilon_3 = \dfrac{\sigma_3}{E} - \dfrac{\nu}{E}\left(\sigma_1+\sigma_2\right) + \alpha\,\Delta T \\ \gamma_{12} = \tau_{12}/G \\ \gamma_{13} = \tau_{13}/G \\ \gamma_{23} = \tau_{23}/G \end{cases}$$

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{dl}{ldt} = V/l$$

Von Mises stress in an arbitrary Cartesian reference system x, y, z:

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{xx} - \sigma_{zz}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + 6\left(\tau_{xy}{}^2 + \tau_{xz}{}^2 + \tau_{yz}{}^2\right)\right]}.$$

Atomic bond

$$U(r) = -\frac{A}{r^m} + \frac{B}{r^n}, \quad n > m; \quad F(r) = \frac{dU}{dr};$$

$$S(r) = \frac{dF}{dr} = \frac{d^2U}{dr^2}; \quad S_0 = S(r_0)$$

Fracture mechanics

$$K_I = Y\sigma\sqrt{\pi a} \geq K_{IC} \quad \text{at unstable crack propagation (fracture)}$$

$$G_{IC} = \frac{K_{IC}{}^2}{E}$$

Two – phase composites

$$\rho = \varphi_\mathrm{f}\rho_\mathrm{f} + \left(1 - \varphi_\mathrm{f}\right)\rho_\mathrm{m};$$

$$E_1 = \varphi_f E_f + \left(1 - \varphi_f\right)E_m;$$

$$\frac{1}{E_2} = \frac{\varphi_f}{E_f} + \frac{\left(1 - \varphi_f\right)}{E_m}; \quad \frac{1}{G_2} = \frac{\varphi_f}{G_f} + \frac{\left(1 - \varphi_f\right)}{G_m}$$

Cellular solids (foams)

$$\bar{\rho} = \frac{\rho_{foam}}{\rho_{parent\ material}}; \quad f = \frac{V_{pores}}{V_{total}} = 1 - \bar{\rho}.$$