UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2003

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
BSc Honours Degree in Mathematics and Computer Science Part II
MSci Honours Degree in Mathematics and Computer Science Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C240=MC240

ALGORITHMS, COMPLEXITY AND COMPUTABILITY

Friday 2 May 2003, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1a Design a 1-tape Turing Machine, $M = (Q, \Sigma, I, q_0, \delta, F)$, where:

The input alphabet, I = C, the typewriter alphabet without the blank symbol,'^'. The whole alphabet $\Sigma = I \cup \{\wedge\}$.
M takes input of the form w*v, where w and v are each words of I which do not include the symbol '*'. M halts and succeeds if the sequence of symbols in v is the sequence in w in reverse order, and halts and fails otherwise.

Give the state diagram for your Turing Machine and describe its operation.

b Explain what the *time function* of a Turing Machine is.
Derive a time function for the Turing Machine given as your answer to part a, and explain your derivation.

c Design a 2-tape Turing machine which takes a binary number, with the least significant digit in square 0 of the tape, and outputs the same value as a number to base 4.
Note: your TM should handle marking of square 0 explicitly.

The three parts carry, respectively, 40%, 20%, 40% of the marks.

2a What is the *Church-Turing Thesis*? Explain why it cannot be proved but could possibly be disproved. What kinds of evidence for the thesis are there?

b Explain what the *Halting Problem* is. What does it mean to say that the Halting Problem is *unsolvable*?

c Let $g: C^* \rightarrow C^*$ be a partial function that tells us whether the output of a standard Turing machine on a given input is the word 'imperial' or not; that is, for any standard Turing machine S and word w of C,

$$g(\text{code}(S)*w) = \begin{cases} y & \text{if } f_S(w) = \text{'imperial'} \\ n & \text{otherwise} \end{cases}$$

Show that there is no Turing machine M such that $f_M = g$

The three parts carry, respectively, 40%, 30%, 30% of the marks.

3 Given a connected weighted graph G = (E, V, w), where V is the set of nodes, E is the set of edges, and w is the weight relation, Kruskal's Algorithm which finds a Minimum Spanning Tree, T, of G can be expressed as follows:

- put the edges in E into a heap, H, ordered with a lowest weight edge at the root.
- represent the spanning tree T as the set of edges it contains; initialise T to { }.
- while T does not span G (i.e. while there is any node in G which is not in T)
  remove the shortest edge e from the heap H and rebuild H.
  if $T \cup \{e\}$ has no cycle, $T = T \cup \{e\}$.
  end while.

a Prove that Kruskal's Algorithm generates a Minimal Spanning Tree (MST) of the graph G.

b Derive an expression for the time function of this algorithm.

c Use Kruskal's Algorithm to find a MST of the graph in fig.1. Show your detailed working.

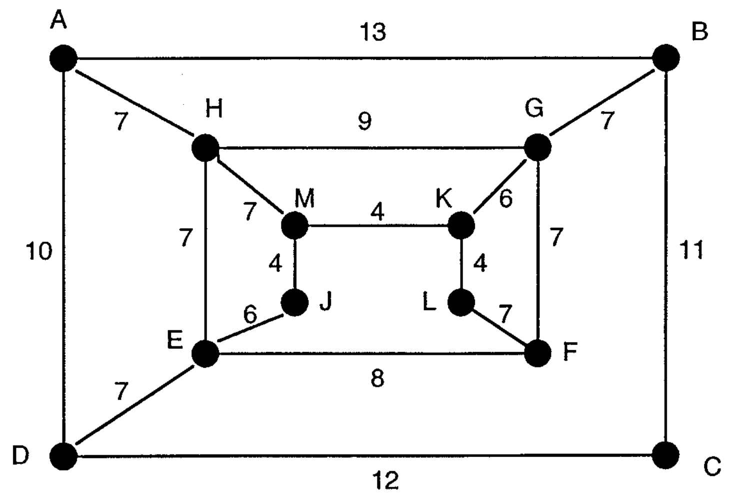

fig 1.

The three parts carry, respectively, 50%, 25%, 25% of the marks.

4a Let A, B, C be yes/no problems. What is meant by

i) $A \in P$

ii) $B \in NP$

iii) $C \in NPC$

b Define a non-deterministic Turing machine (NDTM) and describe briefly how a NDTM operates.

What is meant by "the NDTM N solves the yes/no problem A"?

c Outline the proof of Cook's Theorem, which states that PSAT $\in$ NPC. Explain the importance of this result.

The three parts carry, respectively, 20%, 35%, 45% of the marks.