IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

EEE PART IV: MEng and ACGI

POWER SYSTEM ECONOMICS

Wednesday, 8 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

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Power System Economics - The Questions

Question 1

(a) Describe the significance of the system marginal price and explain why it varies with time. **[4]**

(b) Three generators are available to supply a demand of $D = 400\ [MW]$. The cost of generating power $C_i(P_i)$ corresponding to each of the generators i is:

$$C_1(P_1) = 577 + 8.08 * P_1 + 0.17 * P_1^2\ [\frac{£}{h}]$$

$$C_2(P_2) = 310 + 7.99 * P_2 + 0.20 * P_2^2\ [\frac{£}{h}]$$

$$C_3(P_3) = 156 + 8.11 * P_3 + 0.47 * P_3^2\ [\frac{£}{h}]$$

Calculate the optimal production of each generator, the system marginal cost, average cost for each of the generators and their respective profits. **[8]**

(c) Modify the dispatch found in **(b)** that respects the constraints on the minimum and maximum outputs of each of the generators:

$P_1^{min} = 150\ [MW]$ and $P_1^{max} = 250\ [MW]$

$P_2^{min} = 120\ [MW]$ and $P_2^{max} = 200\ [MW]$

$P_3^{min} = 120\ [MW]$ and $P_3^{max} = 180\ [MW]$

Determine the system marginal cost, the system average cost, the total cost of operating the system and the profits of each generator. **[4]**

(d) The profit of one of the generators in **(c)** is negative. Explain why this is the case. If the generators are free to participate in the market based on their profitability, discuss how that generator can avoid the economic losses **[4]**

Question 2

(a) Explain the difference between markets with perfect and imperfect competition. List the models that can be used for the analysis of markets with imperfect competition. **[4]**

(b) Derive the short-run profit maximization conditions for a generation participant under perfect competition and imperfect competition modelled through a Cournot model. **[4]**

(c) Consider a market for electrical energy that is supplied by two generating companies with the two corresponding cost functions:

$$C_A = P_A^2 \ [\frac{£}{h}]$$

$$C_B = P_B^2 + 15 * P_B \ [\frac{£}{h}]$$

The inverse demand curve for this market is estimated to be:

$$\pi = 60 - D \ [\frac{£}{GWh}]$$

Assuming a Cournot model of competition, form a table to calculate the Nash equilibrium point of this market, i.e. market price, demand quantity and profit of each company for different levels of productions for each of the companies. Consider range of productions of company A at 7GW, 10GW and 13GW and of company B at 6GW, 8GW and 10GW.

[12]

Question 3

(a) Describe the significance of

(i) contracts for differences and **[3]**

(ii) financial transmission rights. **[3]**

(b) Consider two areas, Borduria and Syldavia, which are linked via a transmission line that can carry $F_{max} = 400\ [MW]$. There are two Generating companies in Borduria and two generating companies in Syldavia, with their respective cost functions and maximum output limits given by:

Borduria Power (BP): $C_{BP}(P_{BP}) = 15 * P_{BP}\ [\frac{£}{h}]$, $P_{BP}^{max} = 800\ [MW]$

Borduria Gen (BG): $C_{BG}(P_{BG}) = 19 * P_{BG}\ [\frac{£}{h}]$, $P_{BG}^{max} = 800\ [MW]$

Syldavia Energy (SE): $C_{SE}(P_{SE}) = 35 * P_{SE}\ [\frac{£}{h}]$, $P_{SE}^{max} = 1200\ [MW]$

Syldavia Supply (SS): $C_{SS}(P_{SS}) = 43 * P_{SS}\ [\frac{£}{h}]$, $P_{SS}^{max} = 1200\ [MW]$

Both Borduria and Syldavia also have retail companies (Borduria Retail and Syldavia Retail) that sell energy to end consumers in the corresponding areas. Demand in Borduria is $D_B = 500MW$ while demand in Syldavia is $D_S = 1500MW$.

Independent System Operator schedules generation by minimising the overall generation costs, while satisfying network constraints. Calculate the optimal generation dispatch, power flow between Bordura and Syldavia, the locational marginal prices in the two areas and congestion surplus. **[7]**

(c) Borduria Power (seller) and Syldavia Retail (buyer) have a Contract for Difference for 300MWh (for this particular hour), with a Strike Price of 33£/MWh and Syldavia as the delivery point. Given that Borduria Power bought 300MW of Financial Transmission Rights in the capacity auction, calculate the revenue and the profit of Borduria Power and the charges of Syldavia Retail, for this particular setting. **[7]**

Question 4

(a) Consider a two area system, with demand in Area A $D_A = 2000\ [MW]$ while demand in Area B is $D_B = 1000\ [MW]$. Generator A is in Area A and generator B is located in Area B, with their respective cost functions given by:

$$C_A(P_A) = 20 * P_A + 0.015 * P_A^2\ [\frac{£}{h}]$$

$$C_B(P_B) = 15 * P_B + 0.01 * P_B^2\ [\frac{£}{h}]$$

Determine the optimal generation dispatch, locational marginal prices, total cost of operating the system, cost of constraints and congestion surplus for:

i) The case when the capacity of the transmission link between the two areas is not binding **[3]**

ii) The case when no transmission link exists between the two areas **[3]**

(b) Given that the annuitized cost of building the transmission line is given by $C_{inv} = k * L * F$, where F is the capacity of the line, $L = 300km$ (length of the line) and $k = 175.2\ [£/(MW.km.year)]$, calculate and plot:

i) The cost of constraints as a function of F **[3]**

ii) The transmission demand function **[3]**

iii) The transmission supply function **[2]**

(c) Determine the optimal capacity that should be built, when the transmission is:

i) regulated activity and **[3]**

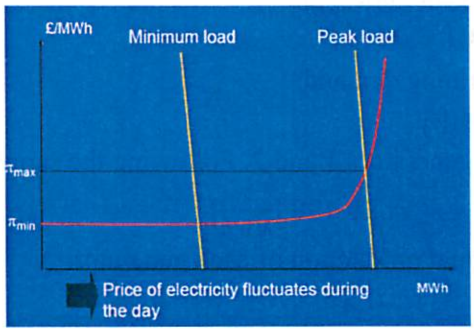
ii) operated as a merchant company. **[3]**

Solutions

Question 1

(a) The system marginal price signifies the marginal cost of serving an additional unit of demand at the least-cost dispatch of the total load to the available generators. **[2]**

In contrast with other commodities, electrical energy cannot be economically stored in large quantities and thus it must be produced exactly when it is consumed by the inflexible demand. Moreover, the electrical demand changes and the generation side is characterized by a production cost which varies with the size of the load. In combination with the fluctuation of the demand from a minimum to a peak value, temporal variations in the system marginal price emerge. **[2]**



(b) In the optimal solution of the problem, the marginal cost of each generator is equal to the system marginal cost λ yielding:

$\frac{\partial C_1(P_1)}{\partial P_1} = 8.08 + 0.34 * P_1 = \lambda \rightarrow P_1 = \frac{\lambda - 8.08}{0.34}$

$\frac{\partial C_2(P_2)}{\partial P_2} = 7.99 + 0.4 * P_2 = \lambda \rightarrow P_2 = \frac{\lambda - 7.99}{0.4}$

$\frac{\partial C_3(P_3)}{\partial P_3} = 8.11 + 0.94 * P_3 = \lambda \rightarrow P_3 = \frac{\lambda - 8.11}{0.94}$

The satisfaction of the system demand implies that:

$P_1 + P_2 + P_3 = D = 400\ [MW]$ **[3]**

This gives for the system marginal cost:

$\frac{\lambda - 8.08}{0.34} + \frac{\lambda - 7.99}{0.4} + \frac{\lambda - 8.11}{0.94} = 400 \rightarrow \lambda = 69.541\ [\frac{£}{MWh}]$ **[1]**

and for the optimal production of each generator:

$P_1 = 180.768\ [MW]$

$P_2 = 153.878\ [MW]$

$P_3 = 65.352\ [MW]$ **[1]**

This yields for the total cost of operating the system:

$C_{tot} = C_1(P_1) + C_2(P_2) + C_3(P_3) = 16561.2\ [£]$ **[1]**

The system average cost is:

$\alpha = \frac{C_{tot}}{P_1 + P_2 + P_3} = 41.403\ [\frac{£}{MWh}]$

The average cost for each of the generators is:

$\alpha_1 = \frac{C_1(P_1)}{P_1} = 42.003\ [\frac{£}{MWh}]$

$\alpha_2 = \frac{C_2(P_2)}{P_2} = 40.78\ [\frac{£}{MWh}]$

$\alpha_3 = \frac{C_3(P_3)}{P_3} = 41.213\ [\frac{£}{MWh}]$ **[1]**

The profit for each generator Ω_i is given by the difference between its respective revenue and cost:

$\Omega_1 = \lambda * P_1 - C_1(P_1) = 4978.08\ [£]$

$\Omega_2 = \lambda * P_2 - C_2(P_2) = 4425.657\ [£]$

$\Omega_3 = \lambda * P_3 - C_3(P_3) = 1851.323\ [£]$ **[1]**

(c) Given that $P_3 < P_3^{min}$ the solution calculated in **(a)** is not feasible when taking into account the given output limits of the generators. The optimal solution can now be calculated by fixing the output of generator 3 to:

$P_3' = P_3^{min} = 120\ [MW]$

and satisfying the remaining demand:

$D' = D - P_3' = 280\ [MW]$ **[2]**

through the remaining generators 1 and 2. Following the same method as in **(a)**, we get:

$\frac{\lambda - 8.08}{0.34} + \frac{\lambda - 7.99}{0.4} = 280 \rightarrow \lambda' = 59.498\ [\frac{£}{MWh}]$

This yields for the optimal production of each generator:

$P_1' = 151.229\ [MW]$

$P_2' = 128.77\ [MW]$ **[1]**

This yields for the total cost of operating the system:

$C_{tot}' = C_1(P_1') + C_2(P_2') + C_3(P_3') = 18239.28\ [£]$

The system average cost is:

$a' = \frac{C_{tot}'}{P_1' + P_2' + P_3'} = 45.598\ [\frac{£}{MWh}]$

The average cost for each of the generators is:

$a_1' = \frac{C_1(P_1')}{P_1'} = 37.604\ [\frac{£}{MWh}]$

$a_2' = \frac{C_2(P_2')}{P_2'} = 36.151\ [\frac{£}{MWh}]$

$a_3' = \frac{C_3(P_3')}{P_3'} = 65.81\ [\frac{£}{MWh}]$

The profit for each generator is given by the difference between its respective revenue and cost:

$\Omega_1' = \lambda' * P_1' - C_1(P_1') = 3310.957\ [£]$

$\Omega_2' = \lambda' * P_2' - C_2(P_2') = 3006.343\ [£]$

$\Omega_3' = \lambda' * P_3' - C_3(P_3') = -757.44\ [£]$ **[1]**

(d) The negative profit of generator 3 means that the operating cost of the latter is larger than its revenue in the optimal solution and thus it experiences economic losses at the optimal solution (does not recover its operating costs). This is because the combination of the high variable costs of generator 3 and its high minimum output limit with respect to the initial unconstrained dispatch calculated in **(b)**. **[2]**

Since the generators are free to participate in the market based on their profitability, generator 3 can avoid the negative profits (economic losses) by decommitting at the time period examined in the problem. [2]

Question 2

(a) In markets with perfect competition, no market participant has the ability to influence the market price through its individual actions. In other words, market price is a parameter over which participants have no control. Under imperfect competition, some producers and/or consumers -called strategic players- can exert market power and manipulate the prices. Prices can be manipulated either by withholding quantity (physical withholding) or by raising (for sellers) / decreasing (buyers) the asking/offered price (economic withholding).
A market operating under perfect competition is characterised by a large number of market participants, small shares of the total production or consumption controlled by each of them and significant price elasticity of demand. A market operating under imperfect competition is characterised by a small number of market participants, large shares of the total production or consumption controlled by some of them and low price elasticity of demand [2]
Models used for the analysis of markets with imperfect competition include:

- **Bertrand model**, where the decision variable of each of the competing firms is the price at which it offers the produced commodity
- **Cournot model**, where the decision variable of each of the competing firms is the quantity of the commodity they produce
- **Supply functions equilibria model**, where the decision variables of each of the competing firms are the parameters of its supply function

[2]

(b) The short-run profit of a generation participant i is given by the difference between its revenue and its cost $c_i(y_i)$. The former is given by the product of the market price π and the generation participant's quantity produced y_i. Under perfect competition, generation participants do not have the ability to influence the market price through their individual actions and thus the market price is independent of their produced quantity. Therefore, the short-run profit maximization condition for participant i is given by:

$\max_{y_i}(\pi * y_i - c_i(y_i))$

which yields:

$\frac{d(\pi * y_i - c_i(y_i))}{dy_i} = 0$

and thus:

$\pi = \frac{dc_i(y_i)}{dy_i}$ [2]

Under imperfect competition, generation participants can influence the prices through their individual actions and thus the market price is a function of the quantity produced by the participants. If $Y = y_1 + .. + y_n$ denotes the total quantity produced by all n generation participants in the market, the short-run profit maximization condition for participant i is given by:

$\max_{y_i}(\pi(Y) * y_i - c_i(y_i))$

Which yields:

$\frac{d(\pi(Y)*y_i-c_i(y_i))}{dy_i}=0$

$$\pi(Y)+y_i*\frac{d\pi(Y)}{dy_i}=\frac{dc_i(y_i)}{dy_i}$$

$$\pi(Y)*\left\{1+\frac{y_i}{Y}\frac{Y}{dy_i}\frac{d\pi(Y)}{\pi(Y)}\right\}=\frac{dc_i(y_i)}{dy_i}$$

$$\pi(Y)*\left\{1-\frac{s_i}{|\varepsilon(Y)|}\right\}=\frac{dc_i(y_i)}{dy_i}$$

Where:

$s_i=\frac{y_i}{Y}$ is the market share of generation participant i and

$\varepsilon=-\frac{\frac{dy}{y}}{\frac{d\pi}{\pi}}=-\frac{\pi}{y}*\frac{dy}{d\pi}$ is the price elasticity of the commodity demand **[2]**

(c) In the Cournot model of competition the state of the market is determined by the production decisions made by each firm. We summarize the possible outcomes using a table where all the cells in a column correspond to a given production by company A and the cells in a row correspond to a given production by company B.

Each cell contains four pieces of information arranged in the following format:

D	Ω_A
Ω_B	π

Where:

π price $[\frac{£}{GWh}]$

D demand $[GW]$

Ω_A profit made by company A [£]

Ω_B profit made by company B [£] **[3]**

Given the productions P_A and P_B of the two companies, the other quantities are calculated as follows:

$D=P_A+P_B$

$\pi=60-D$

$\Omega_A=P_A*\pi-P_A^2$

$\Omega_B=P_B*\pi-P_B^2-15*P_B$

Based on the above, the table expressing the Cournot model of competition for the conditions of the problem is shown below. **[3]**

A Nash equilibrium state implies that given the production level of either of the two companies in this state, the other company cannot achieve a higher profit by following a production level different than the one at the equilibrium. It can thus be observed from the table below that the cell corresponding to $P_A=13GW$ and $P_B=8GW$ is an equilibrium point. In other words, given that $P_A=13GW$, company B achieves its highest profit by producing $P_B=8GW$ AND given that $P_B=8GW$, company A achieves its highest profit by producing $P_A=13GW$. **[6]**

P_B/P_A	7		10		13	
6	13	280	16	340	19	364

	156	47	138	44	120	41
8	15	266	18	320	21	338
	176	45	152	42	128	39
10	17	252	20	300	23	312
	180	43	150	40	120	37

Question 3

(a) ***Contracts for differences***: In many markets, producers and consumers of electricity are obliged to trade solely through a centralized market. In such situations, parties often resort to *contracts for differences* that operate in parallel with the centralized market (and are equivalent to bilateral agreements with forward and future contracts aimed at reducing their exposure to price risks). In a contract for difference, the parties agree on a *strike price* and an amount of the commodity. They then take part in the centralized market like all other participants. Once trading on the centralized market is complete, the contract for difference is settled as follows:

· If the strike price agreed in the contract is higher than the centralized market price, the buyer pays the seller the difference between these two prices times the amount agreed in the contract.

· If the strike price is lower than the market price, the seller pays the buyer the difference between these two prices times the agreed amount.

A contract for difference thus insulates the parties from the price on the centralized market while allowing them to take part in this market.

[3]

Financial Transmission Rights: Financial Transmission Rights deal with shortfalls in contracts for differences in the presence of network congestion. They are defined between any two nodes in the network and entitle their holders to a revenue equal to the product of the amount of transmission rights bought and the price differential between the two nodes. This amount is exactly what is needed to ensure that a contract for difference can be settled. In conclusion, FTRs completely isolate their holders from the risk associated with congestion in the transmission network. They provide a perfect hedge against variations in nodal prices. **[3]**

(b) In the case where network constraints are not taken into account, the optimal generation dispatch is calculated through a merit order dispatch due to the linearity of the generators' cost functions:

$P_{BP} = 800\ [MW]$

$P_{BG} = 800\ [MW]$

$P_{SE} = 400\ [MW]$

$P_{SS} = 0\ [MW]$ **[1]**

In this case, the flow on the transmission line is:

$F = P_{BP} + P_{BG} - D_B = D_S - P_{SE} - P_{SS} = 1100\ [MW]$

Since $F > F_{max}$, the line is congested, which means that the final flow is:

$F' = F_{max} = 400\ [MW]$ **[1]**

and the generation dispatch should be modified, according to:

$P'_{BP} + P'_{BG} = D_B + F_{max} \rightarrow P'_{BP} + P'_{BG} = 900\ [MW]$
$P'_{SE} + P'_{SS} = D_S - F_{max} \rightarrow P'_{SE} + P'_{SS} = 1100\ [MW]$
Based on this and according to the merit order dispatch, the optimal generation dispatch in the constrained case is:
$P'_{BP} = 800\ [MW]$
$P'_{BG} = 100\ [MW]$
$P'_{SE} = 1100\ [MW]$
$P'_{SS} = 0\ [MW]$ **[2]**
The marginal generator in Borduria is Borduria Gen; therefore the locational marginal price in Borduria is:
$\pi'_B = 19\ [\frac{£}{MWh}]$
The marginal generator in Syldavia is Syldavia Energy; therefore, the locational marginal price in Syldavia is:
$\pi'_S = 35\ [\frac{£}{MWh}]$ **[2]**
Finally, the congestion surplus is:
$CS = (\pi'_S - \pi'_B) * F' = 6400\ [\frac{£}{h}]$ **[1]**

(c) Based on the results of (b), when the Contract for Differences and the contracted Financial Transmission Rights are not taken into account, the cost, revenue and profit of Borduria Power are:
$C_{BP} = 15 * P'_{BP} = 12{,}000\ \left[\frac{£}{h}\right]$
$R_{BP} = \pi'_B * P'_{BP} = 15{,}200\ \left[\frac{£}{h}\right]$
$\Pi_{BP} = R_{BP} - C_{BP} = 3{,}200\ \left[\frac{£}{h}\right]$ **[1]**
And the charges of Syldavia Retail are:
$E_{SR} = \pi'_S * D_S = 52{,}500\ [\frac{£}{h}]$ **[1]**
The revenue of Borduria Power (seller) from the Contract for Differences is:
$R_{BP}^{CFD} = (33 - \pi'_S) * 300 = -600\ [\frac{£}{h}]$ **[1]**
And accordingly the expense of Syldavia Retail (buyer) from the Contract for Differences is:
$E_{SR}^{CFD} = (33 - \pi'_S) * 300 = -600\ [\frac{£}{h}]$ **[1]**
The revenue of Borduria Power from the contracted Financial Transmission Rights is:
$R_{BP}^{FTR} = (\pi'_S - \pi'_B) * 300 = 4800\ [\frac{£}{h}]$ **[1]**
The net revenue and profit of Borduria Power are:
$R_{BP}^{NET} = R_{BP} + R_{BP}^{CFD} + R_{BP}^{FTR} = 19400\ [\frac{£}{h}]$
$\Pi_{BP}^{NET} = R_{BP}^{NET} - C_{BP} = 7400\ [\frac{£}{h}]$ **[1]**
The net charges of Syldavia Retail are:
$E_{SR}^{NET} = E_{SR} + E_{SR}^{CFD} = 51900\ [\frac{£}{h}]$ **[1]**

Question 4

(a) i) When the capacity of the transmission link is not binding, the optimal generation dispatch is calculated as if both generators and both demand are connected to the same bus and is given by the solution of the system of equations:

$$\frac{\partial C_A(P_A^{unc})}{\partial P_A} = \frac{\partial C_B(P_B^{unc})}{\partial P_B} \rightarrow 20 + 0.03 * P_A^{unc} = 15 + 0.02 * P_B^{unc}$$

$$P_A^{unc} + P_B^{unc} = D_A + D_B \rightarrow P_A^{unc} + P_B^{unc} = 3000$$

which yields:

$$P_A^{unc} = 1100\ [MW]$$

$$P_B^{unc} = 1900\ [MW]$$

Since $P_A^{unc} < D_A$ and $P_B^{unc} > D_B$, the flow on the transmission link is from area B to area A and is equal to:

$$F^{unc} = D_A - P_A^{unc} = P_B^{unc} - D_B = 900\ [MW]$$

The locational marginal price is the same for the two areas and is given by:

$$\pi = 20 + 0.03 * P_A^{unc} = 15 + 0.02 * P_B^{unc} = 53\ [\frac{£}{MWh}]$$

The total cost of operating the system is calculated by setting the values determined above in the cost functions of the two generators:

$$C_{tot}^{unc} = C_A(P_A^{unc}) + C_B(P_B^{unc}) = 104{,}750\ [£]$$

Since the capacity of the transmission link is not binding, the cost of constraints is zero. The congestion surplus is also zero, since the locational marginal price is the same for the two areas. **[3]**

ii) When no transmission link exists between the two areas, generator A will satisfy demand in area A and generator B will satisfy demand in area B, meaning:

$$P_A^{no} = D_A = 2000\ [MW]$$

$$P_B^{no} = D_B = 1000\ [MW]$$

The locational marginal prices in the two areas are different. An additional unit of demand in Area A will be satisfied by Generator A and thus the locational marginal price at area A is:

$$\pi_A = \frac{\partial C_A(P_A^{no})}{\partial P_A} = 80\ [\frac{£}{MWh}]$$

An additional unit of demand in Area B will be satisfied by Generator B and thus the locational marginal price at area B is:

$$\pi_B = \frac{\partial C_B(P_B^{no})}{\partial P_B} = 35\ [\frac{£}{MWh}]$$

The total cost of operating the system is calculated by setting the values determined by above in the cost functions of the two generators:

$$C_{tot}^{no} = C_A(P_A^{no}) + C_B(P_B^{no}) = 125{,}000\ [£]$$

The cost of constraints is equal to the difference between the total cost of operating the system in this case and the respective cost in the case where the capacity of the transmission link is not binding:

$$CC^{no} = C_{tot}^{no} - C_{tot}^{unc} = 20250\ [£]$$

The congestion surplus is zero since the capacity of the transmission link connecting the two areas is also zero. **[3]**

(b) i) The total cost of operating the system when a line of capacity F connects the two areas is calculated by substituting:

$P_A^F = D_A - F$

$P_B^F = D_B + F$

into the cost functions of the two generators, yielding a total cost of operating the system:

$C_{tot}^F = C_A(P_A^F) + C_B(P_B^F) = 125{,}000 - 45 * F + 0.025 * F^2$

The cost of constraints is equal to the difference between this cost and the respective cost in the case where the capacity of the transmission link is not binding:

$CC^F = C_{tot}^F - C_{tot}^{unc} = 20{,}250 - 45 * F + 0.025 * F^2$

For $F = 0$: $CC^F = 20{,}250$ [£] (the value calculated in (a) ii) – no transmission)

For $F = 900$ $[MW]$ (the value calculated in (a) i)): $CC^F = 0$ (unconstrained transmission)

The plot $CC^F = f(F)$ is a quadratically, strictly-decreasing curve starting from the point $(F = 0, CC^F = 20{,}250)$ and ending to point $(F = 900, CC^F = 0)$ **[3]**

ii) The demand function for transmission is calculated as the price differential between the two areas as a function of the line capacity F, which yields:

$\Pi_D(F) = \pi_A(F) - \pi_B(F) = (20 + 0.03 * P_A^F) - (15 + 0.02 * P_B^F) = 45 - 0.05 * F$

For $F = 0$: $\Pi_D = 45$ $[\frac{£}{MWh}]$ (no transmission)

For $F = 900$ $[MW]$: $\Pi_D = 0$ (unconstrained transmission)

The plot $\Pi_D = f(F)$ is a linear decreasing curve starting from the point $(F = 0, \Pi_D = 45)$ and ending to the point $(F = 900, \Pi_D = 0)$ **[3]**

iii) The supply function for transmission is given by the marginal cost of building the transmission line, expressed in an hourly basis:

$\Pi_S(F) = \frac{k*L}{8760h} = 6 \left[\frac{£}{MWh}\right]$

The plot $\Pi_D = f(F)$ is a constant curve equal to 6 irrespectively of the value of F **[2]**

(c) i) A regulated transmission company seeks to maximise the social welfare, by building a line that will exactly finance itself, meaning that the annual revenues from the congestion of the line are equal to the annual investment required for building the line. Therefore, the optimal capacity that it would build satisfies the equality between the demand and the supply for transmission:

$\Pi_D(F) = \Pi_S(F) \rightarrow 45 - 0.05 * F = 6 \rightarrow F = 780$ $[MW]$ **[3]**

ii) A merchant transmission company seeks to maximize its profits. Since its revenues will be given by the congestion surplus -which is equal to the product of the volume of energy transported and the price differential between the two areas- it has an incentive to under-invest in order to keep the price differential in a high level. Therefore, the optimal capacity that it would build maximizes its profits:

$\max_F(REVENUE(F) - COST(F))$

Its revenue is equal to the congestion surplus which is given by:

$REVENUE(F) = CS(F) = \Pi_D(F) * F = 45 * F - 0.05 * F^2$

Its cost is given by the cost of reinforcing the transmission line, expressed in an hourly basis, which is:

$COST(F) = 6 * F$

The above yield:

$\max_F(45 * F - 0.05 * F^2 - 6 * F) \rightarrow F = 390\ [MW]$ **[3]**