UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C438=I4.54

COMPLEXITY

Tuesday 11 May 2004, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 a i) What does it mean for a language L to be in NP?

ii) What does it mean for a language L to be *reducible* to a language L' ($L \leq L'$)?

iii) What does it mean for a language L to be *NP-complete*?

iv) The problem HC is defined as follows: Given a graph G, does G have a Hamiltonian circuit (i.e. a Hamiltonian path which returns to its start node)?

Show that HC is NP-complete.

You may assume that the Hamiltonian path problem (HP) is NP-complete.

b i) What does it mean for a language L to be *Cook reducible* to a language L'?

ii) The problem GISOM is the following: Given two graphs G_1 and G_2, is G_1 isomorphic to G_2? The problem GAUTO is the following: Given a graph G, does G have a non-trivial automorphism, i.e. is there an isomorphism from G to itself which is not the identity?

Show that if GISOM $\in$ P then GAUTO $\in$ P.

The two parts carry, respectively, 60%, and 40% of the marks.

2 a i) Show that RCH, the reachability problem for directed graphs, belongs to NL (= NLogspace).

ii) Why is RCH not expected to belong to L (= Logspace)?

b Show that NL $\subseteq$ P.

c Let DOUBLE be the language of doubled words:

$$\text{DOUBLE} = \{ww : w \in \{0,1\}^*\}$$

Show carefully that DOUBLE $\in$ L.

d Given a language $L \subseteq \{0,1\}^*$, let

$$L^* = \{w : w \text{ is a concatenation of zero or more words of } L\}$$

Show that if $L \in$ NL then $L^* \in$ NL.

The four parts carry, respectively, 25%, 25%, 20%, and 30% of the marks.

3 a Give parallel algorithms taking polylogarithmic time for each of the following problems:

i) matrix multiplication

ii) reachability in a directed graph (RCH)

In each case state the time complexity of your algorithm.

b The language EQUAL is defined to be

$$\{x \in \{0,1\}^* : x \text{ has the same number of 0s and 1s}\}$$

Give a parallel algorithm to decide EQUAL. You should minimise the parallel time taken. State the work and parallel time of your algorithm in terms of arithmetical operations.

c i) Define the complexity class NC_j ($j \geq 1$).

ii) Let $L \subseteq \{0,1\}^*$ belong to NC_j. For $x, y \in \{0,1\}^*$ let $y \leq x$ denote that y is an initial substring of x. Define L' by:

$$L' = \{x \in \{0,1\}^* : \text{ for all } y \leq x \text{ we have } y \in L\}$$

Show that $L' \in \mathrm{NC}_j$.

The three parts carry, respectively, 30%, 25%, and 45% of the marks.

4a i) Define the class RP.

ii) Show that if languages L_1 and L_2 belong to RP then so does $L_1 \cap L_2$.

b i) Define SAT and FSAT.

ii) Show that FSAT can be computed in polynomial time given an oracle for SAT.

iii) A *3-colouring* of a graph G is an assignment of colours to the nodes of G in such a way that no two adjacent nodes have the same colour, and no more than three colours are used. 3COL is the problem: given a graph G, does G have a 3-colouring?

State the associated function problem F3COL.

Show how this can be computed in polynomial time given an oracle for 3COL.

The two parts carry, respectively, 40%, and 60% of the marks.