Q1

i) M|M|K system

$$P(W>z \mid Q_t = i) = \sum_{j=0}^{i} \frac{(K\mu z)^j}{j!} e^{-K\mu z}$$

$$P(Q_t = i) = (1-\rho)\rho^i$$

$$P(W>z) = \sum_{i=0}^{\infty} P(Q_t=i)\,P(W>z \mid Q_t=i)$$

$$= \sum_{i=0}^{\infty} (1-\rho)\rho^i \sum_{j=0}^{i} \frac{(K\mu z)^j}{j!} e^{-K\mu z}$$

$$= e^{-K\mu(1-\rho)z}$$ 6

ii) $K = 20$

$\lambda = 10$ calls/minute

$1/\mu = 1$ minute

$$A = \frac{\lambda}{\mu} \quad \rho = \frac{A}{K} = \frac{\lambda}{K\mu} = \frac{10}{20} = 0.5$$

$$P(W > 0.5\,m) = e^{-20(1-0.5)0.5}$$

$$= e^{-5} = 0.00674$$ 4

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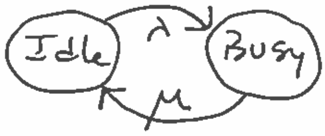
Question Number etc. in left margin — Mark allocation in right margin

Q1b i) ON-OFF source model, description and discussions



In equilibrium $P(\text{source is busy}) = \dfrac{\lambda}{\lambda+\mu}$

$$= \frac{\alpha}{1+\alpha} = p$$ 2

ii If there are M non-interacting sources, each with $P(\text{busy}) = p$

Then the number of busy sources is binomial with parametre (M, p) $\Rightarrow$

The number of busy channel is binomial (M, p)

$N \geq M$

$$\pi_i = \frac{\binom{M}{i} p^i (1-p)^{M-i}}{\sum_{j=0}^{M} \binom{M}{j} p^j (1-p)^{M-j}} \quad i = 0, \dots M$$ 5

iii Engset model

M sources acting independently

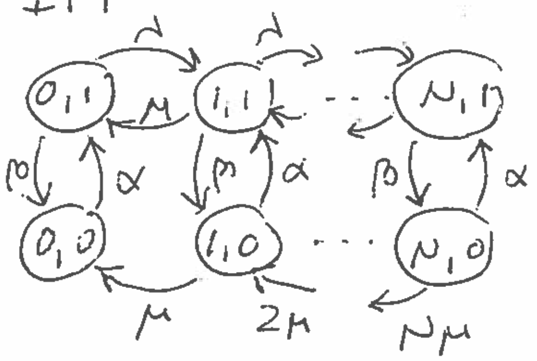
N channels / circuits

M and N of similar magnitude

j channels busy $\Leftrightarrow$ j sources busy

$\Rightarrow$ total arrival rate to the link will fall as N_t increases. And the total offered traffic will be less than that predicted by the Erlang model 3

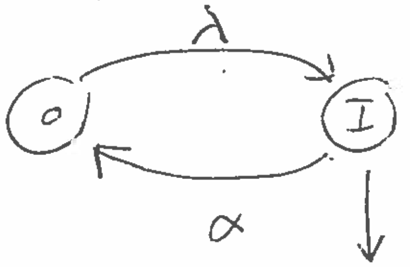
Q2a) i) IPP



- Describe and Interpret the model
- Discuss the model

4

ii Two state model



v packets/second

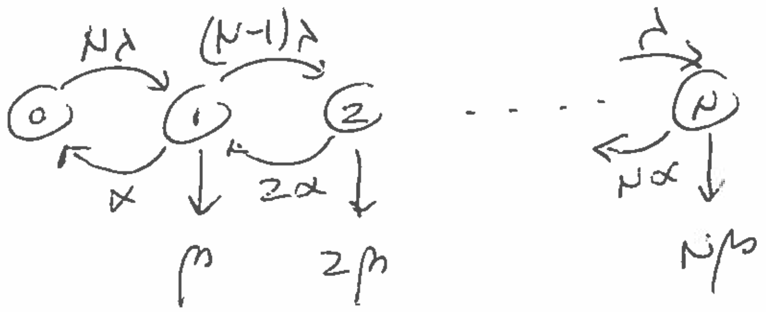
0 = silent (inactive)

I = talk spurt (active)

- Describe and interpret the model
- Discuss the model

3

iii MMPP : N multiplexed voice sources



(Poisson rates)

- Describe and Interpret the model
- Discuss the model

3

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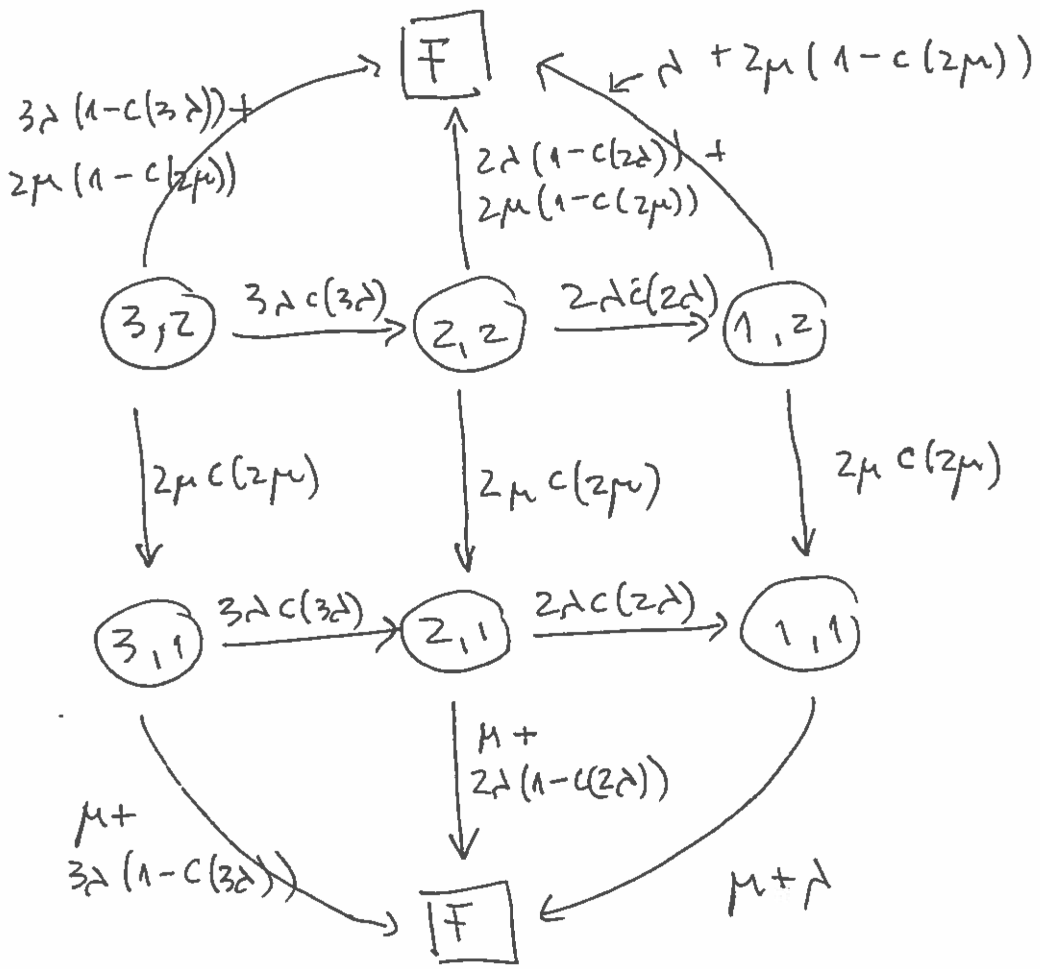
Q2b

i) (# processors up, # buses up)

3,2	3,1	3,0
2,2	2,1	2,0
1,2	1,1	1,0
0,2	0,1	0,0

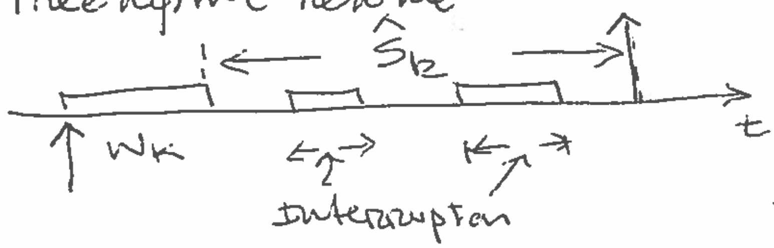
F 4



3

3

Q3a) i) Preemptive resume



$T_K = W_K + \hat{S}_K$

W_K = waiting time

$\hat{S}_K$ = effective service time

T_K = transit time 3

ii Define $E[R_K] = \frac{1}{2} \sum_{i=1}^{k} \lambda_i E[S_i^2]$

$$E[W_K] = \frac{E[R_K]}{(1-\sigma_{K-1})(1-\sigma_K)}$$

V_K = work brought into the system, during $\hat{S}_K$ by higher-priority arrivals

$$E[V_K] = \sum_{i=1}^{k-1} \left(\lambda_i E[\hat{S}_K]\right) E[S_i] \quad , \quad \rho_i = \lambda_i E[S_i]$$

$$= \left(\sum_{i=1}^{k-1} \rho_i\right) E[\hat{S}_K] = \sigma_{K-1} E[\hat{S}_K]$$

$$E[\hat{S}_K] = E[S_k] + E[V_K] = E[S_k] + \sigma_{K-1} E[\hat{S}_K]$$

$$E[\hat{S}_k] = \frac{E[S_k]}{1-\sigma_{K-1}} \quad \text{and} \quad E[T_K] = E[W_K] + E[\hat{S}_K]$$

$$E[T_K] = \frac{E[R_K]}{(1-\sigma_{k-1})(1-\sigma_k)} + \frac{E[S_k]}{1-\sigma_{k-1}}$$ 4

iii Discussion on the preemptive restart case 3

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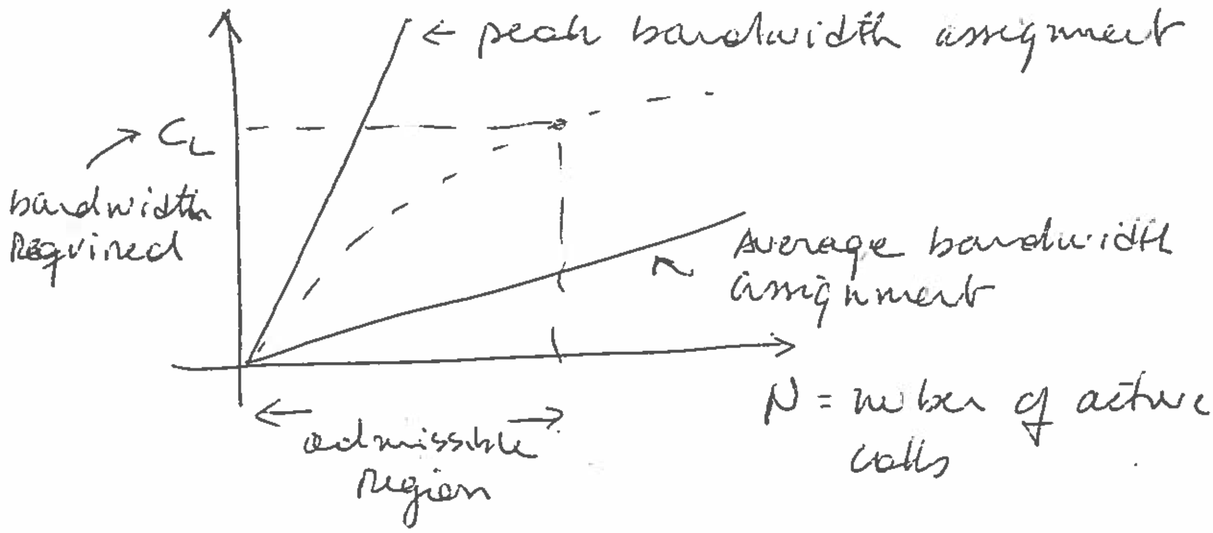
Q3b

i) In ATM terms, a particular VP that has already been set up is designed to provide a given QoS. This VP is assigned the capacity C_L. The question is to determine how many calls or virtual connections can be handled. 3

ii



Average bandwidth assignment: best multiplexing strategy. But it may be unacceptable in terms of cell loss

Peak rate assignment: guarantees no cell loss. But there might be periods in which the VP is under used. It provide a lower bound on the number of cells that can be accepted 2

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Q2b iii

$C_L > m R_p$ $\quad m$ = mean $\quad R_p$ = peak rate

$C_L = (m + k\sigma) R_p$ $\quad \sigma$ = standard deviation

$k = k(QoS)$

$m = Np$

p for an ON-OFF source is $\frac{\alpha}{\alpha+\beta}$

$\frac{\alpha}{\alpha+\beta} = p$ = probability source is ON

$$\sigma^2 = Np(1-p) = m(1-p)$$

$$C = \frac{C_L}{R_p} \rightarrow C = m + k\sigma$$

$$C = Np + k\sqrt{Np(1-p)}$$

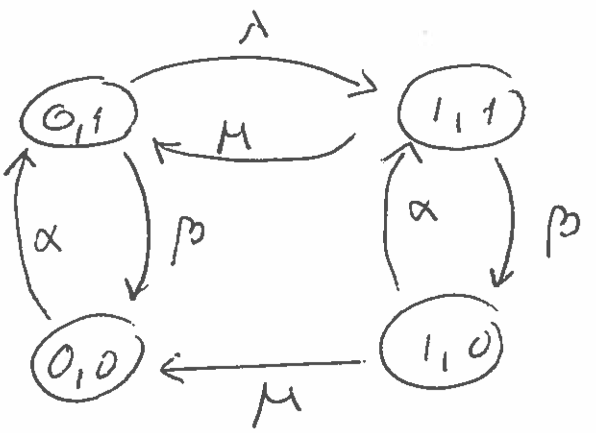
$$K(QoS) \sim P_L$$

$$P_L = \sum_{i=j_0}^{N} \frac{(i-c)\pi_i}{m}$$

5

Q4 a)

i)



$\lambda = 3 \qquad \mu = \frac{1}{3}$

$\alpha = 0.145 \qquad \beta = 7.09$

state space (N_t, Y_t)

N_t = number of busy channels on the overflow link

Y_t = 0, arrival stream is OFF
1, arrival stream is ON — 5

ii) global balance equations

$$\alpha \pi_0 = \beta \sigma_0 + \mu \pi_1$$

$$(\mu + \lambda)\sigma_0 = \alpha \pi_0 + \mu \sigma_1$$

$$(\alpha + \mu)\pi_1 = \beta \sigma_1$$

$$(\beta + \mu)\sigma_1 = \alpha \pi_1 + \lambda \sigma_0$$

where

$\pi_0 = P(N_t = 0, Y_t = 0)$

$\sigma_0 = P(N_t = 0, Y_t = 1)$

$\pi_1 = P(N_t = 1, Y_t = 0)$

$\sigma_1 = P(N_t = 1, Y_t = 1)$ — 5

4b)

i) $\rho = \frac{\lambda}{K\mu} = 0.8$ Erlangs / channel (operator)

$$P[\text{Delay}] = \frac{E_K(K\rho)}{1-\rho[1-E_K(K\rho)]} = 0.41$$

from Graph Traffic capacity on basis of Erlang B formula:

$E_{10}(8) \sim 0.12$ 6

ii $E[W \mid \text{Delay}] = \frac{1}{\lambda} E[Q_t \mid \text{Delay}]$

$$= \frac{1}{\lambda}\frac{\rho}{1-\rho} = \frac{1}{K\mu}\frac{1}{1-\rho}$$

$$= \frac{1}{\mu}\frac{1}{K(1-\rho)} =$$ 4