EE4-06

2016 Optical Comm : Solutions

(1)

a) The refractive index n is the $\sqrt{\ }$ of the relative permittivity ε_r, ie $n = \sqrt{\varepsilon/\varepsilon_0}$

b) Each bit is $10^3/5\times10^3 = 0.2\,m$, velocity in fibre $\cong 2\times10^8\,m/s$, time per bit $0.2/2\times10^8 = 1\,ns$

$B = 10^9$ bit/s

c) Higher order modes are more weakly confined, ie have more light in cladding, so $\underline{m=0}$ has higher proportion in core

d) 6 dBm = 4 mW. Energy/photon $= hc/\lambda_0 = 1.5\times10^{-19}$ J

$\dot{N} = 4\times10^{-3}/1.5\times10^{-19} = 2.67\times10^{16}$

e) $S = \eta \dfrac{hc}{e\lambda} = \dfrac{0.85\times6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times1510\times10^{-9}} = \underline{0.70\ W/A}$

f) Using $I^{*2} = 4kT/R$, the noise power density is $I^{*2}R = \underline{4kT}$

g) L is reduced, I_{ph} will increase, I^*_{sh} will increase, I^*_{th} is unchanged so shot noise is now greater

h) Silicon is an indirect bandgap semiconductor. Thus only a small fraction of e-h recombinations are radiative, ie produce a photon.

i) $NA = \sqrt{n_1^2 - n_2^2} \cong \sqrt{2n\cdot\Delta n}$ $\quad n \cong 1.5$

$NA \cong \sqrt{3\times\Delta n}$ $\quad \Delta n \cong \dfrac{0.28^2}{3} = \underline{0.026}$

j) $E = hc/\lambda$

$\Delta E \cong (hc/\lambda^2)\Delta\lambda$ Taking $\Delta E \cong 2kT$

$\Delta\lambda = 2kT\cdot\lambda^2/hc \cong \underline{10.0\ nm}$

(2) a) For an even mode such as $m=0$.

$E_1(x) = A\cos(k_{1x}x)$

$\therefore E_1(d/2)/E_1(0) = \cos(k_{1x}d/2) = 1/\sqrt{2}$

The eigenvalue eq'n for $m=0$:

$$\frac{Kd}{2} = \left(\frac{k_{1x}d}{2}\right)\tan\left(\frac{k_{1x}d}{2}\right) \rightarrow Y = X\tan X \ (i)$$

and $k_z = k_{2z}$ gives:

$$K^2 + k_{1x}^2 = NA^2 k_0^2$$

or $X^2 + Y^2 = NA^2 Z^2$ with $Z = k_0 d/2$ (ii)

Combine (i) and (ii):

$$X^2(1+\tan^2 X) = \frac{X^2}{\cos^2 X} = Z^2 NA^2$$, and $\cos X = \frac{1}{\sqrt{2}}$ so

$$X = \pi/4 \text{ rad.} \qquad \frac{\pi/4}{1/\sqrt{2}} = \frac{\pi d}{\lambda_0} \cdot NA$$

$$d/\lambda_0 = \frac{\sqrt{2}\,\pi/4}{\pi \cdot NA} = \frac{\sqrt{2}}{4 \times 0.19} = 1.86$$

b) The cutoff condition for mode m is:

$d/\lambda_0 = m/2 \cdot NA = 1/(2 \times .19) = 2.63$

So $m=1$ is cutoff so there are no other modes

c) $E_2(x) = B\exp(-Kx)$

$$\frac{E_2(d/2 + \Delta x)}{E_2(d/2)} = \exp(-K\Delta x) = \frac{1/2}{1/\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\therefore \frac{1}{2}\ln 2 = K \cdot \Delta x$

$$K = \frac{X\tan X}{d/2} = \frac{2(\pi/4)(1)}{d} = \frac{\pi}{2d}$$

$$\Delta x = \frac{\frac{1}{2}\ln 2}{\pi/2d} = \frac{\ln 2}{\pi} \cdot d = \underline{0.99\,\mu m}$$

3 a)

In general, $SNR_{opt} = \dfrac{I_{ph}}{\sqrt{(I^*)^2 \cdot \Delta f}}$

For shot noise $(I^*)^2 = 2e\, I_{ph}$

$\therefore SNR = \dfrac{I_{ph}}{\sqrt{2e\, I_{ph} \cdot \Delta f}} = \sqrt{\dfrac{I_{ph}}{2e \cdot \Delta f}}$

Where I_{ph} is the photocurrent, Δf the bandwidth

For thermal noise $(I^*)^2 = 4kT/R$.

$\therefore SNR = \dfrac{I_{ph}}{\sqrt{4kT \cdot \Delta f/R}}$; where R is the load resistance.

b) For noise equivalent power, $SNR_{opt} = \dfrac{\Phi_R}{NEP\sqrt{\Delta f}}$

where Φ_R is the received optical power.

$I_{ph} = \mathcal{R}\,\Phi_R$ with $\mathcal{R}$ the responsivity

The effects will be equal if the SNR values taking account of each noise source separately are equal

$\dfrac{\Phi_R}{NEP \cdot \Delta f} = \sqrt{\dfrac{I_{ph}}{2e \cdot \Delta f}} = \sqrt{\dfrac{\mathcal{R}\Phi_R}{2e \cdot \Delta f}}$ $\quad \mathcal{R} = \dfrac{e\lambda}{hc}$ for $\eta = 1$

This gives : $\Phi_R = \dfrac{\lambda}{2hc} \cdot (NEP)^2 = \dfrac{1.55 \times 10^{-6} \times (8 \times 10^{-11})^2}{2 \times 6.63 \times 10^{-34} \times 3 \times 10^8}$

$\Phi_R = 0.25$ mW

Equivalently, express NEP as $(I_x)^2 \rightarrow SNR = \dfrac{I_{ph}}{\mathcal{R} \cdot NEP\sqrt{\Delta f}}$

$(I^*)^2 = (\mathcal{R} \cdot NEP)^2 = 2e\, I_{ph}$

$= 2e\mathcal{R}\Phi_R$

$\Phi_R = \mathcal{R}(NEP)^2/2e = 0.25$ mW

c) Since we have to add the two noise sources,

$SNR_{opt} = \dfrac{\Phi_R}{\sqrt{2}\, NEP\sqrt{\Delta f}}$ (noise sources add incoherently)

. And $B = \Delta f/2$

$B = 2\Phi_R^2/2 \cdot NEP^2 \cdot SNR^2 = (0.25 \times 10^{-3})^2/(8 \times 10^{-12})^2 (12)^2 =$

$= 7 \times 10^{12}$ bit/s

This is for above transmitter & detector capabilities

(4)

a) $v_p = \omega/k \qquad v_g = \frac{d\omega}{dk}$

v_g describes the speed that pulses propagate.

b) $k = nk_o = n\omega/c$

$$v_g = \frac{1}{dk/d\omega} \qquad \frac{1}{v_g} = \frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}$$

$$\frac{dn}{d\omega} = \frac{dn}{d\lambda_o}\frac{d\lambda_o}{d\omega} \qquad \omega = k_o c = \frac{2\pi c}{\lambda_o}$$

$$\frac{d\lambda_o}{d\omega} = \frac{-2\pi c}{\omega^2} = -\frac{\lambda_o}{\omega}$$

$$\frac{dn}{d\omega} = -\frac{\lambda_o}{\omega}\frac{dn}{d\lambda_o}$$

$$\frac{1}{v_g} = \frac{1}{c}\left(n - \lambda_o dn/d\lambda_o\right)$$

$$n = D_o + D_1\lambda_o^{-2} - D_2\lambda_o^2$$

$$\frac{dn}{d\lambda_o} = -2D_1\lambda_o^{-3} - 2D_2\lambda_o$$

$$\frac{1}{v_g} = \frac{1}{c}\left(n + \lambda_o\left(2D_1\lambda_o^{-3} + 2D_2\lambda_o\right)\right)$$

$$= \frac{1}{c}\left(D_o + D_1\lambda_o^{-2} - D_2\lambda_o^{-2} + 2D_1\lambda_o^{-2} + 2D_2\lambda_o^2\right)$$

$$v_g = \frac{c}{D_o + 3D_1\lambda_o^{-2} + D_2\lambda_o^2}$$

$$v_p = \frac{c}{n} = \frac{c}{D_o + D_1\lambda_o^{-2} - D_2\lambda_o^2}$$

since $v_g = \frac{c}{n + 2D_1\lambda_o^{-2} + 2D_2\lambda_o^2}$

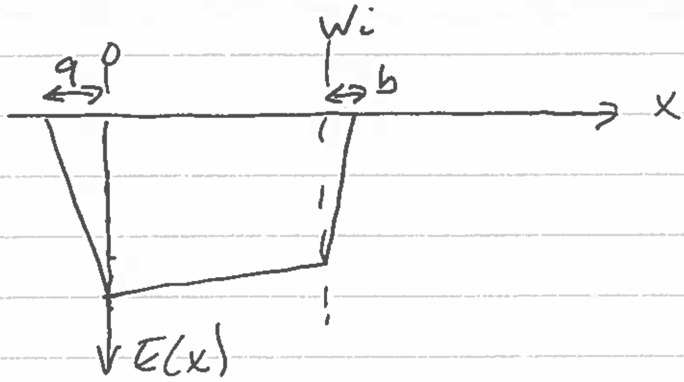
and the denominator is always $> n$, $v_g < v_p$

c) $dn/d\lambda = -2D_1\lambda_o^{-3} - 2D_2\lambda_o$

$$\frac{d^2n}{d\lambda^2} = 6D_1\lambda_o^{-4} - 2D_2 = 0 \text{ at zero dispersion.}$$

$$3D_1/D_2 = \lambda_o^4 \qquad \lambda_o = \sqrt[4]{3 \times .0028/.0026}$$
$$= 1.34\ \mu m$$

5 a)



$$E_{sat} = \frac{v_{dsat}}{\mu} = \frac{10^5}{0.125} = 8\times10^5\ \text{V/m} \quad \text{(electrons)}$$

$$= \frac{10^5}{.05} = 2\times10^6\ \text{V/m} \quad \text{(holes)}$$

We want

$$E(x=a) = 8\times10^5\ \text{V/m}$$

$$E(x=w_i) = 7.2\times10^5\ \text{V/m}$$

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon} \quad \therefore\ \rho = \varepsilon_0 \varepsilon_r \Delta E / W_i = \frac{12\times8.85\times10^{-12}\times8\times10^4}{8\times10^{-6}}$$

(in intrinsic region) $= 1.06\ \text{C/m}^3$

But $\rho \cong e N_D^- \quad \therefore\ N_D^- = \dfrac{1.06}{1.6\times10^{-19}} = \underline{6.6\times10^{18}\ \text{m}^{-3}}$

To get voltage need ~~a~~ a and b.

$$\frac{E_{max}}{a} = \frac{eN_A^+}{\varepsilon_r\varepsilon_0} \quad \therefore\ a = \frac{8\times10^5}{2\times10^{21}\times1.6\times10^{-19} / 12\times8.85\times10^{-12}} = 0.27\ \mu\text{m}$$

$$b = \frac{\varepsilon_r\varepsilon_0\times0.9\,E_{max}}{eN_D^+} = \frac{12\times8.85\times10^{-12}\times0.9\times8\times10^5}{1.6\times10^{-19}\times10^{21}} = 0.48\ \mu\text{m}$$

$$V = -\int E\,dx = 0.95\,E_{max}W_i + \tfrac{1}{2}E_{max}\,a + \tfrac{1}{2}\times0.9\,E_{max}\,b$$

$$= 8\times10^5\left(0.95(8) + 0.5(.27) + .5\times0.9\times.48\right)\times10^{-6}$$

$$= \underline{6.36\ \text{V}}$$

b) Largest time will be for holes to return from bottom of intrinsic region.

$$E(x) = E_{max}(1 - x/80) \qquad (x \text{ in } \mu m)$$

$$V_d = \mu_h E(x) = 0.05(8\times10^5)(1 - x/8)$$

$$= 4\times10^4(1 - x/80) = 5\times10^2(80 - x) \ \mu m/s$$

$$= 5\times10^8(80-x) \ m/s$$

$$dt = dx/v(x) \quad \therefore \quad \Delta t = +\int_8^0 \frac{dx}{V(x)}$$

$$\Delta t = \frac{-1}{5\times10^8}\int_0^8 \frac{dx}{80-x} = -\frac{1}{5\times10^8}\Big[\ln(80-x)\Big]_0^8$$

$$= \frac{\ln(80/72)}{5\times10^8} = 0.21 \ ms$$

c) In a photoconductor response speed can be traded off against responsivity in the design, using geometry, but the main disadvantage for communications is the large dark current