IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

EEE/EIE PART I: MEng, BEng and ACGI

Corrected Copy

# ANALYSIS OF CIRCUITS

Tuesday, 6 June 10:00 am

Time allowed: 2:00 hours

**There are THREE questions on this paper.**

**Answer ALL questions.**
**Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).**

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible First Marker(s) : D.M. Brookes
Second Marker(s) : P. Georgiou

1. a) Using nodal analysis, calculate the voltages at nodes $X$ and $Y$ of Figure 1.1. [ 4 ]

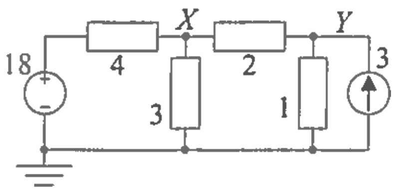


Figure 1.1

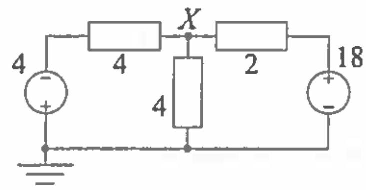


Figure 1.2

b) Use the principle of superposition to find the voltage $X$ in Figure 1.2. [ 4 ]

c) Draw the Thévenin equivalent circuit of the two-terminal network in Figure 1.3 and find the values of its components. [ 4 ]

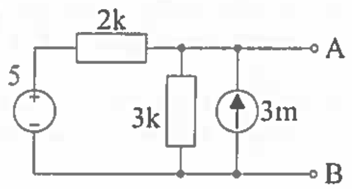


Figure 1.3

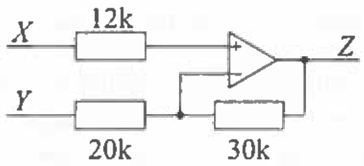


Figure 1.4

d) Assuming the opamp in the circuit of Figure 1.4 is ideal, give an expression for $Z$ in terms of $X$ and $Y$. [ 4 ]

e) The diode in the circuit of Figure 1.5 has a forward voltage of 0.7 V when conducting but is otherwise ideal. Determine the output voltage, $Y$, when
(i) $X = 1$ V,
(ii) $X = 5$ V
(iii) $X = -5$ V. [ 5 ]

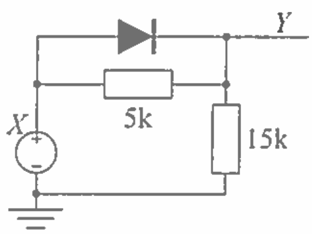


Figure 1.5

# Analysis of Circuits

**Information for Candidates:**

- Numerical answers must be given as fully evaluated decimal values and not as unevaluated arithmetic expressions.

**Notation**

The following notation is used in this paper:

1. The voltage waveform at node $X$ in a circuit is denoted by $x(t)$, the phasor voltage by $X$ and the root-mean-square (or RMS) phasor voltage by $\widetilde{X} = \frac{X}{\sqrt{2}}$. The complex conjugate of $X$ is $X^*$.
2. Component and source values in a circuit are normally given in Ohms, Farads, Henrys, Volts or Amps with the unit symbol omitted. Where an imaginary number is specified, it represents the complex impedance or phasor value.
3. Times are given in seconds unless otherwise stated.
4. Unless otherwise indicated, frequency response graphs should use a linear axis for phase and logarithmic axes for frequency and magnitude.
5. The real and imaginary parts of a complex number, $X$, are written $\Re(X)$ and $\Im(X)$ respectively.

3. Figure 3.1 shows a shows a transmission line of length $L = 10\,\mathrm{m}$ whose characteristic impedance is $Z_0 = 120\,\Omega$ and whose propagation velocity is $u = 2\times 10^8\,\mathrm{m/s}$. Distance along the line is denoted by $x$ and the two points $x = 0$ and $x = L$ are marked in the figure.

At a point $x$ on the line, the line voltage and current are given by $v_x(t) = f_x(t) + g_x(t)$ and $i_x(t) = Z_0^{-1}(f_x(t) - g_x(t))$ where $f_x(t) = f_0(t - u^{-1}x)$ and $g_x(t) = g_0(t + u^{-1}x)$ are the forward and backward waves respectively.

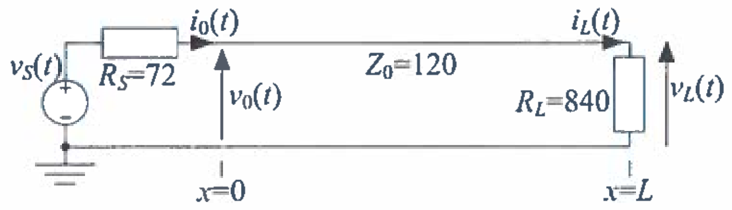


Figure 3.1

a) i) At the position $x = L$, the backward wave is given by $g_L(t) = \rho_L f_L(t)$ where $\rho_L = 0.75$ is the reflection coefficient at $x = L$.

Show that $g_0(t) = \rho_L f_0(t - 2u^{-1}L)$. [ 3 ]

ii) At $x = 0$, show that $v_s(t) = v_0(t) + R_S i_0(t)$. Hence show that $f_0(t)$ can be written in the form $f_0(t) = \tau_0 v_s(t) + \rho_0 g_0(t)$ and determine the numerical values of $\tau_0$ and $\rho_0$. [ 6 ]

iii) By combining the results of parts i) and ii) show that

$$f_0(t) = \tau_0 v_s(t) + \rho_0 \rho_L f_0(t - 2u^{-1}L).$$

Hence prove, by using induction or otherwise, that

$$f_0(t) = \textstyle\sum_{n=0}^{\infty} \tau_0 \rho_0^n \rho_L^n v_s\left(t - 2nu^{-1}L\right).$$ [ 6 ]

b) If the source is a 30 ns pulse given by

$$v_s(t) = \begin{cases} 25.6\,\mathrm{V} & \text{for } 0 \le t \le 30\,\mathrm{ns} \\ 0 & \text{otherwise} \end{cases},$$

draw a dimensioned sketch of the waveform $v_x(t)$ on the line at the point $x = 8\,\mathrm{m}$ for the time interval $0 \le t \le 150\,\mathrm{ns}$. Give the times of all discontinuities and the values of all horizontal portions of the waveform. [ 6 ]

c) Now assume that all voltages and currents are sinusoidal with angular frequency $\omega$. The uppercase letter, $V_x$, denotes the phasor corresponding to $v_x(t)$.

i) The waveform $f_0(t) = A\cos(\omega t + \theta)$ is represented by the phasor $F_0 = Ae^{j\theta}$. Show that $F_x = F_0 e^{-jkx}$ where $k = u^{-1}\omega$. [ 3 ]

ii) By converting the first equation given in part a)iii) into phasor form, determine an expression for $F_0$ in terms of $V_s$. [ 3 ]

iii) Determine an expression for $V_x$ in terms of $V_s$. [ 3 ]

2. The frequency response of a circuit is given by

$$H(j\omega) = \frac{aj\omega}{(j\omega)^2 + 2\zeta\omega_0 j\omega + \omega_0^2}$$

where $a$, $\zeta$ and $\omega_0$ are real numbers.

a) i) By dividing the numerator and denominator of $H(j\omega)$ by $j\omega$ and then multiplying the resultant expression by its complex conjugate, show that $|H(j\omega)|^2 = \dfrac{a^2}{4\zeta^2\omega_0^2 + \left(\omega - \frac{\omega_0^2}{\omega}\right)^2}$. [ 3 ]

ii) Explain why the maximum value of $|H(j\omega)|^2$ occurs when the quantity $\left(\omega - \frac{\omega_0^2}{\omega}\right)$ equals zero. Hence show that the maximum occurs at $\omega = \omega_0$ and determine $|H(j\omega_0)|^2$. [ 2 ]

iii) Find expressions for the two positive values of $\omega$ for which $|H(j\omega)|^2 = \dfrac{a^2}{8\zeta^2\omega_0^2}$ and determine a simplified expression for the difference between them. [ 4 ]

b) Suppose now that $a = 5000\,\mathrm{s}^{-1}$, $\zeta = 0.1$ and $\omega_0 = 5000\,\mathrm{rad/s}$.

i) Determine the low and high frequency asymptotes of $H(j\omega)$. [ 2 ]

ii) Draw a dimensioned sketch showing the high and low frequency asymptotes as well as the true magnitude response, $|H(j\omega)|$. Indicate on your graph in dB the peak value of $|H(j\omega)|$ and the value of the asymptotes at their point of intersection. [ 5 ]

iii) Draw a dimensioned sketch of the straight-line approximation to the phase response, $\angle H(j\omega)$. You may assume without proof that the gradient of the approximation at $\omega_0$ is equal to $-0.5\pi\zeta^{-1}$ radians per decade where "decade" means a factor of 10 in frequency. [ 4 ]

c) i) Show that the frequency response, $\frac{Y(j\omega)}{X(j\omega)}$ of the circuit shown in Figure 2.1 is given by [ 5 ]

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{-j\omega R_2 C}{(j\omega)^2 R_1 R_2 C^2 + 2j\omega R_1 C + 1}.$$

ii) Determine simplified expressions for $a$, $\zeta$ and $\omega_0$ so that the expression given in part c)i) equals that given above for $H(j\omega)$. [ 3 ]

iii) Given that $C = 10\,\mathrm{nF}$, determine the values of $R_1$ and $R_2$ so that $\omega_0 = 5000\,\mathrm{rad/s}$ and $\zeta = 0.1$. [ 2 ]

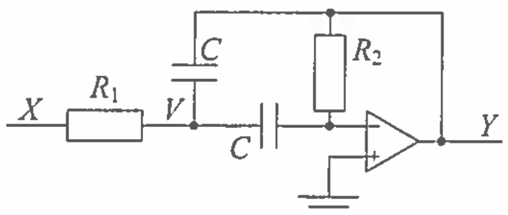


Figure 2.1

f) i) The diagram of Figure 1.6 shows an AC source with r.m.s. voltage $\tilde{V} = 230\,\text{V}$ driving a load with impedance $50 + 25j\,\Omega$ through a line with impedance $2\,\Omega$.

Determine the complex powers, given by $S = \tilde{V} \times \tilde{I}^*$, absorbed both by the load and by the $2\Omega$ resistor. [ 4 ]

ii) A capacitor with impedance $-200j$ is now connected across the load, as indicated in Figure 1.7. Determine the complex powers absorbed both by the load and by the $2\Omega$ resistor. [ 4 ]

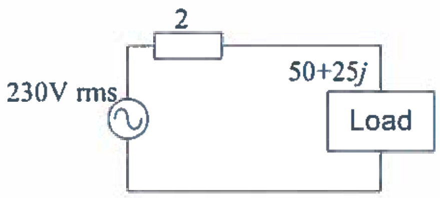


Figure 1.6

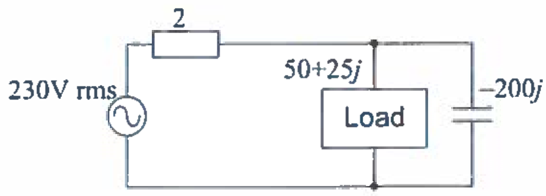


Figure 1.7

g) Determine the gain, $\frac{Y}{X}$, for the block diagram shown in Figure 1.8. The rectangular blocks are drawn with inputs at the left and outputs at the right and have gains of $F$, $G$ and $H$ respectively. The open circle represents an adder/subtractor; its three inputs have the signs indicated on the diagram and its output is $V$. [ 4 ]

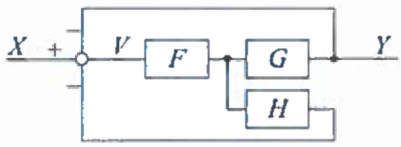


Figure 1.8

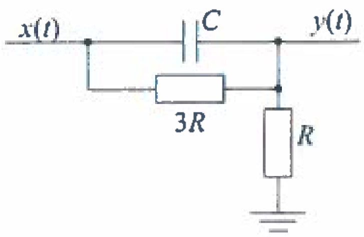


Figure 1.9

h) The input voltage in Figure 1.9 is given by

$$x(t) = \begin{cases} 0 & t < 0 \\ 8\,\text{V} & t \geq 0. \end{cases}$$

i) Determine the time constant of the circuit. [ 2 ]

ii) Determine an expression for $y(t)$ for $t > 0$. [ 5 ]