UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C491

KNOWLEDGE REPRESENTATION

Thursday 6 May 2004, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 $\mathrm{Th}(S)$ denotes the classical truth functional consequences of a set of formulas S.

a What does it mean to say that Th is a *classical consequence operator*?

Let W be a set of formulas. Explain why there are three possible answers to a query ?- α (α any closed formula) when the content of a database with base W is taken to be $\mathrm{Th}(W)$.

b Reiter's 'Closed World Assumption' can be formalised as follows. Let W be a set of formulas, and $\mathcal{P}$ a set of atoms.

$$cwa^{\mathcal{P}}(W) =_{\text{def}} \mathrm{Th}(\, W \cup \{\neg p \mid p \in \mathcal{P} \text{ and } p \notin \mathrm{Th}(W)\}\,)$$

Let $\mathcal{P} = \{p(a), p(b), p(c), q(a), q(b), q(c)\}$.

i) Let $W = \{p(a),\ \neg q(c),\ p(c) \vee q(c)\}$. What answers are correct for the following queries on database $cwa^{\mathcal{P}}(W)$?

?- $p(a)$?- $p(b)$?- $p(c)$?- $q(c)$

ii) Let $W' = W \cup \{\forall x\, (p(x) \vee q(x))\}$. What answers are correct for the above queries on database $cwa^{\mathcal{P}}(W')$?

c The property of 'cautious monotony' for $cwa^{\mathcal{P}}$ may be stated as follows:

If $\alpha \in cwa^{\mathcal{P}}(W)$ then $cwa^{\mathcal{P}}(W) \subseteq cwa^{\mathcal{P}}(W \cup \{\alpha\})$, for any formula α.

Use $W = \{p \vee q\}$ and a suitable choice of $\mathcal{P}$ and α to show that cautious monotony fails for $cwa^{\mathcal{P}}$.

d Show that $cwa^{\mathcal{P}}$ satisfies cumulative transitivity ('cut') by showing each of the following in turn, for any set of formulas W and X. Item (iii) is the statement of cumulative transitivity.

Let $neg^{\mathcal{P}}(W) =_{\text{def}} \{\neg p \mid p \in \mathcal{P} \text{ and } p \notin \mathrm{Th}(W)\}$.

i) $neg^{\mathcal{P}}(W \cup X) \subseteq neg^{\mathcal{P}}(W)$.

ii) If $X \subseteq cwa^{\mathcal{P}}(W)$ then $W \cup X \cup neg^{\mathcal{P}}(W \cup X) \subseteq cwa^{\mathcal{P}}(W)$.

iii) If $X \subseteq cwa^{\mathcal{P}}(W)$ then $cwa^{\mathcal{P}}(W \cup X) \subseteq cwa^{\mathcal{P}}(W)$.

The four parts carry, respectively, 25%, 20%, 25%, and 30% of the marks.

2 a Define the *answer set semantics* of an *extended* logic program.

Explain how an extended logic program can be translated to an equivalent *normal* logic program.

In what sense *precisely* are the two programs equivalent?

b Consider the following extended logic program P_{birds}:

$$\begin{aligned} \mathit{can\_fly}(X) &\leftarrow \mathit{bird}(X),\ \text{not}\ \mathit{ab\_bird}(X) \\ \neg\mathit{can\_fly}(X) &\leftarrow \mathit{bird}(X),\ \mathit{ab\_bird}(X) \\ \mathit{bird}(X) &\leftarrow \mathit{ostrich}(X) \\ \mathit{ab\_bird}(X) &\leftarrow \mathit{ostrich}(X) \end{aligned}$$

Compute the answer set(s) of the program $P_{\text{birds}} \cup \{\mathit{bird}(\mathit{jim}),\ \neg\mathit{can\_fly}(\mathit{jim})\}$.

Can *jim* fly according to this program? That is, what answer is correct to the query ?- $\mathit{can\_fly}(\mathit{jim})$?

c Use splitting sets to determine the answer sets (stable models) of the following program:

$$\begin{array}{llll} p \leftarrow & \text{not}\ q & \quad r \leftarrow & q \\ q \leftarrow & \text{not}\ p & \quad s \leftarrow & \text{not}\ r \\ r \leftarrow & p & \quad t \leftarrow & \text{not}\ s \end{array}$$

d Determine the answer sets (stable models) of the following program:

$$\begin{array}{llll} a \leftarrow & \text{not}\ \hat{a} & \quad f \leftarrow & \hat{a}, \hat{b},\ \text{not}\ f \\ \hat{a} \leftarrow & \text{not}\ a & \quad f \leftarrow & a, b,\ \text{not}\ f \\ b \leftarrow & \text{not}\ \hat{b} & & \\ \hat{b} \leftarrow & \text{not}\ b & & \end{array}$$

The four parts carry, respectively, 30%, 25%, 25%, and 20% of the marks.

3 $\mathrm{Th}(S)$ denotes the classical truth functional consequences of a set of formulas S.

a Define the closure $\mathrm{Cn}_R(W)$ of a set of formulas W under classical consequence Th and rules R.

State (without proof) the characterisation of $\mathrm{Cn}_R(W)$ in terms of the 'base operator' $\mathrm{B}_R(S) =_{\text{def}} S \cup \{\gamma \mid \frac{\alpha}{\gamma} \in R \text{ and } \alpha \in \mathrm{Th}(S)\}$.

b Define the *extensions* of a default theory (D, W), stating the definition in terms of the closure of a 'reduct' of the default rules D.

Use the corresponding base operator to construct an inductive characterisation of an extension E of (D, W).

c Consider the following default theory:

$$\begin{aligned} D &= \{\frac{\neg q : sp}{sk}, \frac{p : ph, \neg sk}{ph}, \frac{ph : \neg sp}{\neg sp}\} \\ W &= \{p \vee q, \neg q\} \end{aligned}$$

Determine which of the following two sets of formulas are extensions of (D, W).

$$\mathrm{Th}(\{p, \neg q, ph, \neg sp\})$$
$$\mathrm{Th}(\{p, \neg q, sk, \neg sp\})$$

d Write down the standard translation of the default theory (D, W) of part (c) into an equivalent *extended logic program.*

In what sense *precisely* are the two equivalent?

e How do the following two formulas of autoepistemic logic translate to rules of a default theory in the standard translation?

i) $\mathrm{L}\,p \wedge \neg \mathrm{L}\,q \wedge \neg \mathrm{L}\,\neg r \rightarrow s$

ii) $p \wedge \neg \mathrm{L}\,q \wedge \neg \mathrm{L}\,\neg r \rightarrow s$

The five parts carry, respectively, 20%, 20%, 30%, 20%, and 10% of the marks.

4 a How is default persistence of fluents typically formulated as a frame axiom in situation calculus using an 'abnormality predicate' and circumscription?

Outline *briefly* how the circumscription works.

What is *chronological minimisation*?

b How is default persistence of fluents obtained in the event calculus?

c In the action language $\mathcal{C}+$ how are the following expressed as static and/or fluent dynamic laws?

i) The default value of fluent f is v.

ii) The fluent constant g is inertial. (g is not necessarily Boolean.)

iii) There is no state in which the fluent $status$ has value on and value off.

iv) The (Boolean) action $switch$ changes the value of fluent $status$ from on to off and from off to on.

v) The (Boolean) action $open$ is not executable when $status = off$.

Write out the static and/or fluent dynamic laws in full, and not just as $\mathcal{C}+$ abbreviations.

d For each of the laws in part (c) write out its translation as rules of the corresonding causal theory. Assume that the maximum length of paths is n.

Do not forget to include the required 'exogeneity' rules.

e Define a *model* of a causal theory Γ.

What is the relationship between a $\mathcal{C}+$ action description D and models of the causal theory Γ_n^D to which it is translated?

The five parts carry, respectively, 25%, 10%, 25%, 25%, and 15% of the marks.