EE2-10B

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EEE PART II: MEng, BEng and ACGI

Corrected Copy

FIELDS

Monday, 16 June 2:00 pm

Time allowed: 1:30 hours

There are THREE questions on this paper.

There are THREE questions. Question One carries 40 marks. Question Two and Question Three carries 30 marks each.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : R.R.A. Syms
Second Marker(s) : S. Lucyszyn

Vector Calculus and Electromagnetic Fields 2014 – Formula Sheet

- Vectors (Cartesian co-ordinates):

$\underline{a} \cdot \underline{b} = \mathbf{a_x\, b_x + a_y\, b_y + a_z\, b_z}$

$\underline{a} \times \underline{b} = \{\mathbf{a_y b_z - a_z b_y}\}\, \underline{i} + \{\mathbf{a_z b_x - a_x b_z}\}\, \underline{j} + \{\mathbf{a_x b_y - a_y b_x}\}\, \underline{k}$

- Differential operators (Cartesian co-ordinates)

$\nabla = \partial/\partial x\, \underline{i} + \partial/\partial y\, \underline{j} + \partial/\partial z\, \underline{k}$

$\nabla\phi = \partial\phi/\partial x\, \underline{i} + \partial\phi/\partial y\, \underline{j} + \partial\phi/\partial z\, \underline{k}$

$\nabla \cdot \underline{F} = \partial F_x/\partial x + \partial F_y/\partial y + \partial F_z/\partial z$

$\nabla \times \underline{F} = \{\partial F_z/\partial y - \partial F_y/\partial z\}\, \underline{i} + \{\partial F_x/\partial z - \partial F_z/\partial x\}\, \underline{j} + \{\partial F_y/\partial x - \partial F_x/\partial y\}\, \underline{k}$

$\nabla^2\phi = \partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 + \partial^2\phi/\partial z^2$

- Identities:

$\nabla \cdot (\phi\underline{F}) = \underline{F} \cdot \nabla\phi + \phi\, \nabla \cdot \underline{F}$

$\nabla \cdot (\phi\nabla\psi - .\psi\nabla\phi) = \phi\, \nabla^2\psi - \psi\, \nabla^2\phi$

$\nabla \times \nabla \times \underline{F} = \nabla(\nabla \cdot \underline{F}) - \nabla^2\underline{F}$

- Integral theorems:

$\int\int_A \underline{F} \cdot d\underline{a} = \int\int\int_V \nabla \cdot \underline{F}\, dv$

$\int_L \underline{F} \cdot d\underline{L} = \int\int_A (\nabla \times \underline{F}) \cdot d\underline{a}$

- Maxwell's equations – integral form

$\int\int_A \underline{\mathbf{D}} \cdot d\underline{a} = \int\int\int_V \rho\, dv$

$\int\int_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = -\int\int_A \partial\underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \int\int_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

- Maxwell's equations – differential form

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

- Material equations

$\underline{\mathbf{J}} = \sigma\, \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon\, \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu\, \underline{\mathbf{H}}$

- Electromagnetic waves (pure dielectric media)

Time dependent vector wave equation $\nabla^2\underline{E} = \mu_0\varepsilon\, \partial^2\underline{E}/\partial t^2$

Time independent scalar wave equation $\nabla^2\underline{E} = -\omega^2\mu_0\varepsilon_0\varepsilon_r\, \underline{E}$

For z-going, x-polarized waves $d^2E_x/dz^2 + \omega^2\mu_0\varepsilon_0\varepsilon_r\, E_x = 0$

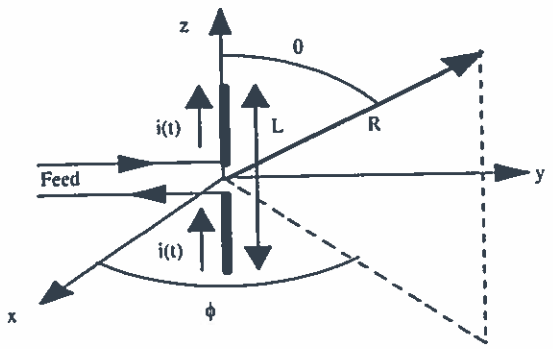
Where $\underline{E}$ is a time-independent vector field

- Antenna formulae

Far-field pattern of half-wave dipole

$E_\theta = j\, 60 I_0 \{\cos[(\pi/2)\cos(\theta)] / \sin(\theta)\} \exp(-jkR)/R$; $H_\phi = E_\theta/Z_0$

Here I_0 is peak current, R is range and $k = 2\pi/\lambda$



Power density $\underline{S} = 1/2\ \mathrm{Re}\ (\underline{E} \times \underline{H}^*) = S(R, \theta)$

Normalised radiation pattern $F(\theta, \phi) = S(R, \theta, \phi) / S_{max}$

Directivity $D = 1/ \{1/4\pi \int\int_{4\pi} F(\theta, \phi) \sin(\theta)\, d\theta\, d\phi\}$

Gain $G = \eta D$ where η is antenna efficiency

Effective area $A_e = \lambda^2 D/4\pi$

Friis transmission formula $P_r = P_t\, (\eta_t\eta_r A_t\, A_r/R^2\lambda^2)$

1. a) Briefly explain the following terms, and their particular application in electrostatics:

i) Potential function

ii) Equipotential

ii) The gradient of a potential function

[5]

b) If ϕ is a general two-dimensional potential function, show that $\nabla\phi$ is always perpendicular to the equipotentials of ϕ. What are the implications for electrostatics?

[5]

c) A two-dimensional function is defined in Cartesian co-ordinates as $\phi(x, y) = \exp\{-(x^2 + y^2)\}$ Sketch the equipotentials. Evaluate and sketch the spatial variation of $\nabla\phi$. Show that $\nabla\phi$ is indeed perpendicular to the equipotential lines in this case.

[5]

d) Is the function $\phi(x, y) = \exp\{-(x^2 + y^2)/a^2\}$, where 'a' is a constant, a valid solution of Laplace's equation?

[5]

e) Evaluate the integral $_0\int^1 {}_0\int^{2x} f(x, y)\, dy\, dx$, with $f(x, y) = x^2y$.
Change the order of integration of the integral, and show that the same result is obtained.

[5]

f) Evaluate $\oint_L \underline{\mathbf{F}} \cdot d\underline{\mathbf{L}}$ for the vector field $\underline{\mathbf{F}} = 3\,\underline{i} - 3\,\underline{j}$, assuming that L is the closed path shown in Figure 1a, starting and ending at (0, 0).

[5]

g) By evaluating the integral of a general vector field $\underline{\mathbf{F}}(x, y, z)$ over the surface of the cube shown in Figure 1b, prove Gauss' Theorem.

[10]

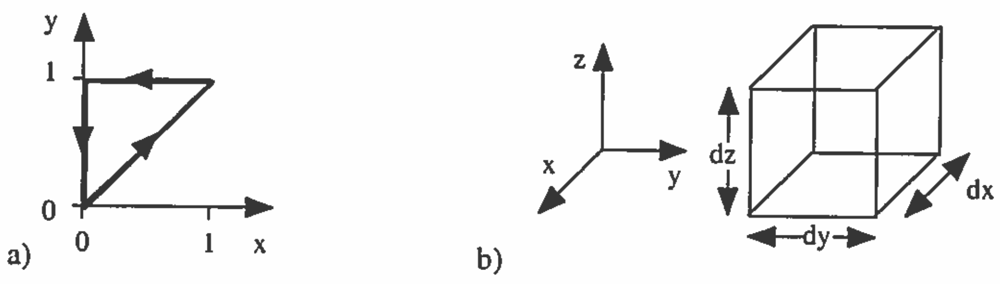


Figure 1.

2. Using diagrams and formulae where appropriate, explain briefly each of the following:

a) The ionosphere **[6]**

b) Dispersion diagram **[6]**

c) Matched splitters **[6]**

d) The skin effect **[6]**

e) Total internal reflection **[6]**

3. a) Describe three factors that limit the effectiveness of free-space communication, noting their implications for systems operating in different regions of the electromagnetic spectrum.

[9]

b) Omitting any possible angular dependence, the time-independent scalar wave equation for electric fields can be written in spherical co-ordinates as $d^2E/dr^2 + (2/r)\, dE/dr + \omega^2\mu_0\varepsilon_0 E = 0$. Which (if any) of the trial fields below is a possible wave solution?

i) $E(r) = E_0 \exp(-jk_0r)$

ii) $E(r) = (E_0/r) \exp(-jk_0r)$

iii) $E(r) = (E_0/r^2) \exp(-jk_0r)$

[6]

c) Explain briefly the significance of the time-averaged power density or irradiance. How does the irradiance vary with distance for a spherical wave?

[5]

d) A free-space radio link operating at 100 MHz frequency uses a lossless receiving antenna based on an array of half-wave dipoles. What is the length of each dipole? Assuming the directivity of the array is 100, what is its effective area? The received power in the radio link above is 1 mW at a distance of 1 km. Assuming that the transmitting antenna is isotropic, estimate the transmitter power.

[6]

e) How is the received power increased if the transmitting antenna is replaced by a similar dipole array? Estimate the maximum link length that may now be achieved if the minimum signal power needed for successful reception is 10 μW.

[4]