IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

EEE PART IV: MEng and ACGI

POWER SYSTEM ECONOMICS

Tuesday, 17 May 2:30 pm

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : G. Strbac
Second Marker(s) : B.C. Pal

The Questions

Question 1

a) Explain how contracts help to reallocate, share and spread risk.

[3]

b) How can option contracts be used for risk management? Explain the meaning of exercise price and option fee.

[3]

c) What factors favour the exercise of market power in electricity markets?

[3]

d) Derive and discuss the analytical expression showing how much power a generating unit should produce as a function of the price of electricity in a perfectly competitive market. Using the derived expression, calculate the power generation of a unit which generation cost is given by $C(P_g) = 2P_g^2 - 3P_g\ [£/h]$ for a market price of $\pi_m = 15\ [£/MWh]$.

[3]

e) Explain why electricity prices may vary with location. What is the difference between constraint costs and congestion costs (surplus)?

[4]

f) Discuss and contrast shortly the approach taken by:

i. Merchant transmission
ii. Consortium
iii. Regulated company

when building a transmission interconnector.

[3]

g) The demand function of a transmission interconnector is given in the form of: $\pi_T = 7.2 - 0.015F\ [£/MWh]$ (where F is the capacity of the line in [MW]).

i. Determine the capacity that would maximise the revenue to the transmission operator.
ii. If the annuitized investment cost of building the interconnector can be expressed as a linear function of its capacity: $C = 13{,}140F\ [£]$, estimate the capacity that should be built to maximise the benefit for the entire system.

[3]

h) Define Financial Transmission Rights and explain how these are used.

[3]

Question 2

a) A manufacturer estimates that its variable cost for manufacturing a given product is given by the following expression:

$$C(q) = 2.5q^2 + 200q \text{ [\$]}$$

where C is the total cost and q is the quantity produced.

i. Derive an expression for the marginal cost of production.
ii. Derive expressions for the revenue and the profit when the widgets are sold at marginal cost.

[2]

b) The inverse demand function of a group of consumers for a given type of widgets is given by the following expression:

$$\pi = -10q + 2{,}000 \text{ [\$/}unit\text{]}$$

where q is the demand and π is the unit price for this product.

i. Determine the maximum consumers' surplus. Explain why the consumers will not be able to realize this surplus.
ii. For a price π of 1,000 [$\$/unit$], calculate the consumption, the consumers' gross surplus, the revenue collected by the producers and the consumers' net surplus.
iii. If the price π increases by 20%, calculate the change in consumption and the change in the revenue collected by the producers.

[5]

c) Economists estimate that the supply function for the widget market is given by the following expression:

$$q = 0.2\pi - 40 \text{ [}units\text{]}$$

i. Calculate the demand and price at the market equilibrium if the demand is as defined in Part b).
ii. For this equilibrium, calculate the consumers' gross surplus, the consumers' net surplus, the producers' revenue, the producers' profit and the global welfare.

[6]

d) Calculate the effect on the market equilibrium of Part c) of the following interventions:

i. A minimum price of \$900 per widget.

ii. A maximum price of \$600 per widget.
iii. A sales tax of \$450 per widget.

In each case, calculate the market price, the quantity transacted, the consumers' net surplus, the producers' profit and the global welfare. Illustrate your calculations using diagrams.

[12]

Question 3

a) What are the conditions for perfect competition? How can a company in an imperfect competitive environment influence the market price? What is the relationship between the marginal cost of production and the market price in perfectly and imperfectly competitive markets?

[6]

b) Derive the set of equations required to find the equilibrium in a market where the competition is represented by the Cournot model. Assume that: there are two companies participating in the market (A and B), which generation cost functions are $C_A(P_A)$ and $C_B(P_B)$ respectively, the demand level for electrical energy is D, and that the inverse demand function for this market is $\pi(D)$.

[7]

c) Consider a market for electrical energy that is supplied by two generating companies whose generation costs are:

$$C_A = 40P_A \ [£/h]$$
$$C_B = 60P_B \ [£/h]$$

The inverse demand function of the market is given by:

$$\pi = 200 - 2D \ [£/MWh]$$

where π is the price in $[£/MWh]$ and D is the demand in $[MW]$.

i. If we assume a Bertrand model for the competition in this market, calculate the production of each of the companies, the resulting market price and profits made by both companies.

ii. If we now assume a Cournot model for the competition in this market, calculate the production of each of the companies, the resulting market price and profits made by both companies using the expressions derived in Part b).

[8]

d) What is the principal difference between Bertrand and Cournot competition models?

[4]

Question 4

a) Consider the Three-bus power system shown in Fig 4.1. Table 4.1 shows the data about the generators connected to this system. Calculate the unconstrained economic dispatch for the loading conditions shown in Fig 4.1.

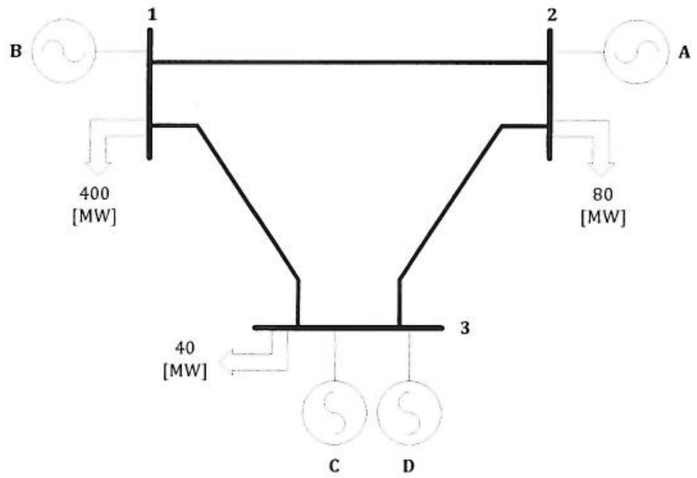

Fig 4.1: Three-bus power system

Table 4.1: Generators characteristics

Generator	Capacity [MW]	Marginal Cost [$/MWh]
A	150	12
B	200	15
C	150	10
D	400	8

[1]

b) Table 4.2 gives the branch data for the Three-bus power system described in Part a). Calculate the flow that would result if the generating units were dispatched as calculated in Part a). Demonstrate that branch 1-3 is overloaded.

Table 4.2: Transmission branches characteristics

Branch	Reactance [p.u.]	Capacity [MW]
1-2	0.2	250
1-3	0.3	250
2-3	0.3	250

[4]

c) Show how this overloaded can be eliminated by:

i. Increasing the output of generator B.

ii. Increasing the output of generator A.

Calculate the cost of corresponding redispatch i and ii described above. Which redispatch is preferable and why?

[14]

d) Calculate the nodal prices for the three-bus power system when the generating units have been optimally redispatched to relieve the constraint violations identified in Part b).

[6]

Question 5

Consider the Two-bus power system shown in Fig 5.1. Assume that the demand is constant and insensitive to price and that energy is sold at its marginal cost of production and that there are no limits on the output of the generators. The marginal cost of production of the generators connected to buses A and B are given respectively by the following expressions:

$$MC_A = 20 + 0.03P_A \ [\$/MWh]$$
$$MC_B = 15 + 0.02P_B \ [\$/MWh]$$

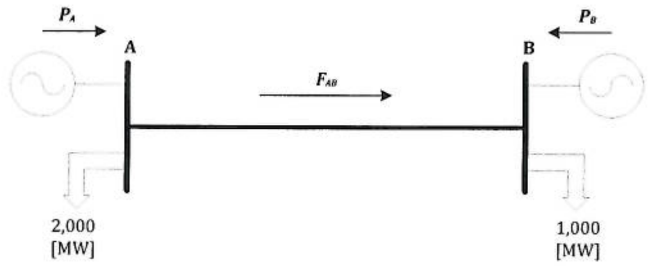

Fig 5.1: Two-bus power system

a) Plot the marginal value of transmission as a function of the capacity of the transmission line connecting buses A and B.

[2]

b) Determine the transmission demand function for the system of Part a).

[6]

c) Calculate the hourly long-range marginal cost of the transmission line of Part a) assuming that the line is 500 $[km]$ long and that the amortized variable cost of building the line is 210 $\left[{}^{\$}/_{MW \cdot km \cdot year}\right]$.

[3]

d) Determine the optimal capacity of the transmission line for the proposed power system.

[4]

e) Calculate the nodal prices and merchandising surplus for the optimal transmission capacity built.

[10]

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The Answers 2011

Question 1

Part a)

Relocation of risk: contracts allow risk to be passed to someone who is either more willing to bear the risk or has more control over the source of risk.

Risk Sharing: Selling future production to a buyer at a fixed price protects the buyer from falling prices. Buyer, by committing to purchase the output, is now also exposed to a risk of falling prices in the future. By having the contract, the buyer is sharing the risk with the seller.

Risk Spreading: Risk is generally "diversifiable", in that fluctuations in one commodity price are generally uncorrelated with prices of the other commodities that do not belong to the same category. Speculators will have a large of number of contracts in a variety of different commodities and hence the risk will be spread.

Part b)

An ***option contract*** is an agreement between two parties, giving one party the right but not the obligation to buy goods from or sell goods to the other party at a certain price and at a certain time in the future. A call option gives its holder the right to buy a given amount of a commodity at a price called the ***exercise price***. A put option gives its holder the right to sell a given amount of a commodity at the exercise price. Whether the holder of an option decides to exercise its rights under the contract depends on the spot price for the commodity. When an option contract is agreed, the seller of the option receives a non-refundable ***option fee*** from the holder of the option. The buyer of the option on the other hand gets a guarantee that he/she will be able to buy/sell a commodity for at least the option exercise price, which can be used for risk management.

Part c)

Factors:

i. Small number of competitors.
ii. Inelasticity of the demand for electricity in the short term.
iii. Transmission congestion that limits the number of effective participants.

Part d)

A generating unit tries always to maximize its profit, i.e. revenue minus costs. Then:

$$\underset{P_g\geq 0}{\text{Max}}\{\pi_m P_g - C(P_g)\}$$

where:

- P_g : Output of the generation unit.
- π_m : Market price.
- $C(P_g)$: Production cost function of the generation unit.

The optimal value of P_g is then obtained differentiating and equating to zero:

$$\frac{d}{dP_g}\{\pi_m P_g - C(P_g)\} = 0$$

which leads to:

$$\pi_m = \frac{dC(P_g)}{dP_g}$$

This means that the generation unit should increase its production up to the point where the marginal cost of production is equal to the market price.

If the generation cost of a unit is $C(P_g) = 2P_g^2 - 3P_g$ $[\pounds\ /\ h]$ and the market price $\pi_m = 15$ $[\pounds/MWh]$, then the optimum production using the expression derived before is:

$$\pi_m = \frac{dC(P_g)}{dP_g} \quad \Rightarrow \quad 15 = 4P_g - 3 \quad \Rightarrow \quad P_g = 3\ [MW]$$

Part e)

In situations where transmission network causes the generation dispatch to be constrained (i.e. the generators cannot be used in the most cost-efficient way), they segment the market and the price of electricity is not the same at each bus (node) of the system. This is because the marginal generator is not the same in all parts of the network, but rather varies with location, as do the locational (nodal) prices.

The ***cost of constraints*** is the cost of making the network secure and it is calculated as the difference between the cost of the constrained dispatch (dispatch when network constraints are taken into account) and the cost of the economic dispatch (dispatch when network constraints are not taken into account).

The ***congestion cost (surplus)*** is the difference between demand charges and generation payments, emerging from the congestion in the network. It is calculated as the sum of products of differences between the nodal prices at any two buses and the flow on the transmission line between those two buses. Then:

$$S_{cong} = \sum_{i,j} (\pi_j - \pi_i) \cdot F_{ij}$$

Part f)

i. A merchant transmission company has a tendency to under-invest and then build less than the optimum capacity.

ii. A consortium has a tendency to over-invest and then build more than the optimum capacity.

iii. A regulated company will invest the right amount of capacity in order to maximize the social welfare.

Part g)

i. The transmission operator's revenue is given by:

$$R = \pi_T F = 7.2F - 0.015F^2 \ [£/h]$$

Then, the capacity resulting in maximum revenue is obtained by differentiating this expression and making it equal to zero:

$$\frac{dR}{dF} = 7.2 - 0.03F = 0 \quad \Rightarrow \quad F = 240 \ [MW]$$

ii. The optimal capacity from the perspective of global welfare satisfies the fact that:

$$Marginal\ value\ of\ transmission = Marginal\ investment\ cost\ of\ transmission$$

The value of transmission is given by:

$$\pi_T = 7.2 - 0.015F \ [£/MWh]$$

The marginal investment cost of transmission (hourly) is given by:

$$Marginal\ investment\ cost\ of\ transmission = {}^{13,140}/_{8760} = 1.5 \ [£/MWh]$$

Hence,

$$7.2 - 0.015F = 1.5 \quad \Rightarrow \quad F = 380 \ [MW]$$

Part h)

FTRs are market instruments defined between any two nodes in the transmission network, and they entitle their holders to a revenue equal to the product of the amount of transmission rights bought and the price differential between the two nodes: $R_{FTR} = F(\pi_1 - \pi_2)$. This enables a contract for difference between a producer at location 1 and a consumer at location 2 to be settled even in the presence of congestion in the network.

Question 2

Part a)

i. Expression for the marginal cost of production:

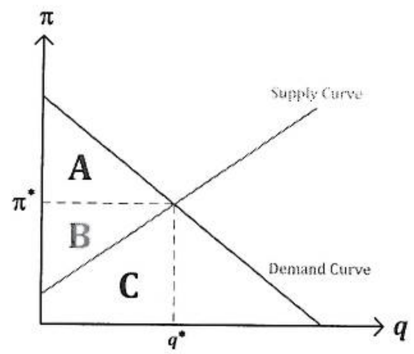
$$MC(q) = \frac{dC(q)}{dq} = 5q + 200 \; [\$/unit]$$

ii. Expressions for the revenue and the profit when the widgets are sold at marginal cost:

$$Revenue = \pi \cdot q = MC(q) \cdot q = 5q^2 + 200q \; [\$]$$

$$Profit = Revenue - Costs = 5q^2 + 200q - 2.5q^2 - 200q = 2.5q^2 \; [\$]$$

Part b)



i. Maximum consumers' surplus:

$$Max\ consumers'surplus \quad \Leftrightarrow \quad \pi = 0 \quad \Rightarrow \quad CS_{Max} = 200{,}000 \; [units]$$

It cannot be realized because it is unlikely that producers will sell for nothing.

ii. If price π is 1,000 $[\$/unit]$:

$$\pi = 1{,}000 \; [\$/unit] \quad \Rightarrow \quad 1{,}000 = -10q + 2{,}000 \quad \Rightarrow \quad q = 100 \; [units]$$

$$Consumers'Gross\ Surplus = \boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} = \frac{2{,}000 + 1{,}000}{2} \cdot 100 = 150{,}000 \; [\$]$$

$$Producers'Revenue = \boldsymbol{B} + \boldsymbol{C} = 1{,}000 \cdot 100 = 100{,}000 \; [\$]$$

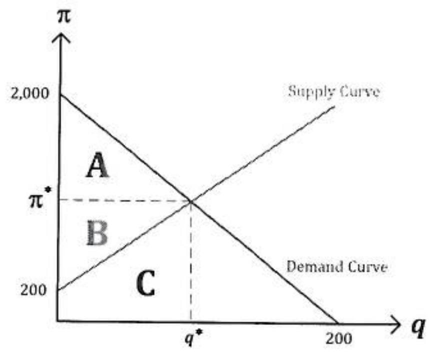
$$Consumers'Net\ Surplus = \boldsymbol{A} = \frac{(2{,}000 - 1{,}000) \cdot 100}{2} = 50{,}000 \; [\$]$$

iii. If the price π increases by 20%:

$$\pi = 1{,}200\ [\$/unit] \quad \Rightarrow \quad 1{,}200 = -10q + 2{,}000 \quad \Rightarrow \quad q = 80\ [units] \quad \Rightarrow \quad \Delta q = 20\%$$

$$Producers'Revenue = \boldsymbol{B} + \boldsymbol{C} = 1{,}200 \cdot 80 = 96{,}000\ [\$] \quad \Rightarrow \quad \Delta PR = 4\%$$

Part c)



i. The demand and price at the market equilibrium if the demand is as defined in Part b) are:

$$Market\ Equilibrium \quad \Rightarrow \quad Demand = Supply \quad \Rightarrow \quad q_{Demand}(\pi) = q_{Supply}(\pi)$$

$$200 - \frac{\pi}{10} = 0.2\pi - 40 \quad \Rightarrow \quad \pi^* = 800\ [\$/unit]$$

$$\pi^* = 800\ [\$/unit] \quad \Rightarrow \quad q^* = 120\ [units]$$

ii. The consumers' gross surplus, the consumers' net surplus, the producers' revenue, the producers' profit and the global welfare.

$$Consumers'Gross\ Surplus = \boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} = \frac{(2{,}000 + \pi^*)}{2} \cdot q^* = 168{,}000\ [\$]$$

$$Consumers'Net\ Surplus = \boldsymbol{A} = \frac{(2{,}000 - \pi^*) \cdot q^*}{2} = 72{,}000\ [\$]$$

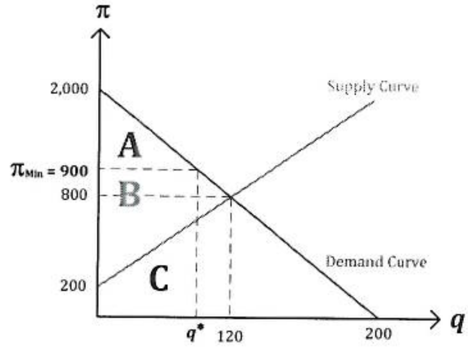
$$Producers'Revenue = \boldsymbol{B} + \boldsymbol{C} = \pi^* \cdot q^* = 96{,}000\ [\$]$$

$$Producers'Profit = Revenue - Costs = \boldsymbol{B} = \frac{(\pi^* - 200) \cdot q^*}{2} = 36{,}000\ [\$]$$

$$Global\ Welfare = Consumers'Net\ Surplus\ + Producers'Profit = \boldsymbol{A} + \boldsymbol{B} = 108{,}000\ [\$]$$

Part d)

i. A minimum price of \$900 per widget.



$$\pi_{Min} = 900\ [\$/unit] \quad \Rightarrow \quad q_{Demand}(\pi_{Min}) = 200 - \frac{\pi_{Min}}{10} \quad \Rightarrow \quad q^* = 110\ [units]$$

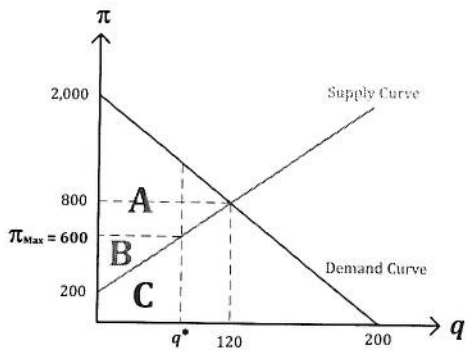
$$\pi_{Min} = 900\ [\$/unit] \quad \Rightarrow \quad \pi^* = 900\ [\$/unit]$$

$$Consumers'Net\ Surplus = A = \frac{(2{,}000 - \pi_{Min}) \cdot q^*}{2} = 60{,}500\ [\$]$$

$$Producers'Profit = Revenue - Costs = B = \pi_{Min} \cdot q^* - \frac{\left(200 + \pi_{Supply}(q^*)\right)}{2} \cdot q^*$$
$$= 46{,}750\ [\$]$$

$$Global\ Welfare = Consumers'Net\ Surplus\ + Producers'Profit = A + B$$
$$= 107{,}250\ [\$]$$

ii. A maximum price of \$600 per widget.



$$\pi_{Max} = 600\ [\$/unit] \quad \Rightarrow \quad q_{Supply}(\pi_{Max}) = 0.2\pi_{Max} - 40 \quad \Rightarrow \quad q^* = 80\ [units]$$

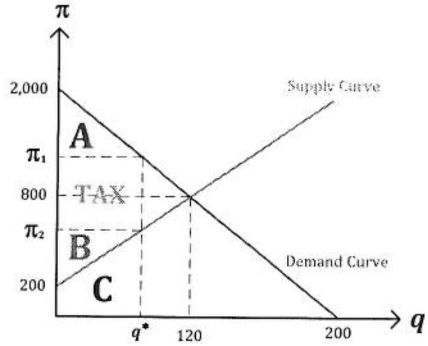
$$\pi_{Max} = 600\ [\$/unit] \quad \Rightarrow \quad \pi^* = 600\ [\$/unit]$$

$$Consumers' Net\ Surplus = A = \frac{(2{,}000 - 600 + \pi_{Demand}(q^*) - 600)}{2} \cdot q^* = 80{,}000\ [\$]$$

$$Producers' Profit = Revenue - Costs = B = \frac{(\pi_{Max} - 200) \cdot q^*}{2} = 16{,}000\ [\$]$$

$$Global\ Welfare = Consumers' Net\ Surplus\ + Producers' Profit = A + B$$
$$= 96{,}000\ [\$]$$

iii. A sales tax of \$450 per widget.



$$TAX \quad \Rightarrow \quad \pi_1 - \pi_2 = 450\ [\$/unit] \quad \Rightarrow \quad \pi_{Demand}(q^*) - \pi_{Supply}(q^*) = 450$$
$$\Rightarrow \quad q^* = 90\ [units]$$

$$q^* = 90\ [units] \quad \Rightarrow \quad \pi^* = \pi_{Demand}(q^*) = 1{,}100\ [\$/unit]$$

$$Consumers' Net\ Surplus = A = \frac{(2{,}000 - \pi_{Demand}(q^*)) \cdot q^*}{2} = 40{,}500\ [\$]$$

$$Producers' Profit = Revenue - Costs = B = \frac{(\pi_{Supply}(q^*) - 200) \cdot q^*}{2} = 20{,}250\ [\$]$$

$$Global\ Welfare = Consumers' Net\ Surplus\ + Producers' Profit\ + TAX$$
$$= A + B + TAX = 101{,}250\ [\$]$$

Question 3

Part a)

In a perfectly competitive market no participant has the ability to influence the market price through its individual actions (i.e. all participants act as price takers). This assumption is valid if the number of market participants is large and if none of them controls a large proportion of the production or consumption. Perfect competition is a highly desirable goal because it encourages efficient market behaviour.

Strategic market players can manipulate the prices either by withholding quantity (physical withholding) or by raising the asking price (economic withholding), compared to the perfectly competitive case.

In the presence of imperfect competition market prices tend to be higher than producers' marginal costs (which determine the price in perfectly competitive markets).

Part b)

We have two companies (A and B) participating in the market for electrical energy, in which, the competition is assumed to be represented by the Cournot Model.

If we assume that the cost functions of companies A and B are $C_A(P_A)$ and $C_B(P_B)$ respectively, then the profit function for each of those companies will be defined as follows:

$$\Omega_A(P_A, P_B) = \pi(D) \cdot P_A - C_A(P_A)$$

$$\Omega_B(P_A, P_B) = \pi(D) \cdot P_B - C_B(P_B)$$

Each company want to maximize their profit. Then the optimality conditions are given by:

$$\frac{\partial}{\partial P_A}\{\Omega_A(P_A, P_B)\} = 0 \quad \Rightarrow \quad \frac{\partial \Omega_A(P_A, P_B)}{\partial P_A} = \pi(D) + P_A \frac{\partial \pi(D)}{\partial P_A} - \frac{dC_A(P_A)}{dP_A} = 0$$

$$\frac{\partial}{\partial P_B}\{\Omega_B(P_A, P_B)\} = 0 \quad \Rightarrow \quad \frac{\partial \Omega_B(P_A, P_B)}{\partial P_B} = \pi(D) + P_B \frac{\partial \pi(D)}{\partial P_B} - \frac{dC_B(P_B)}{dP_B} = 0$$

And we know that:

$$D = P_A + P_B \quad \Rightarrow \quad \pi(P_A, P_B) \quad \Rightarrow \quad \frac{\partial \pi(D)}{\partial P_A} = \frac{d\pi(D)}{dD} \cdot \frac{dD}{dP_A} \quad \& \quad \frac{\partial \pi(D)}{\partial P_B} = \frac{d\pi(D)}{dD} \cdot \frac{dD}{dP_B}$$

Finally, the set of equations required to solve the problem and find the equilibrium are:

$$(1)\quad \pi(D) + P_A \frac{d\pi(D)}{dD} \cdot \frac{dD}{dP_A} - \frac{dC_A(P_A)}{dP_A} = 0$$

$$(2)\quad \pi(D) + P_B \frac{d\pi(D)}{dD} \cdot \frac{dD}{dP_B} - \frac{dC_B(P_B)}{dP_B} = 0$$

$$(3)\quad D = P_A + P_B$$

Part c)

i. In the Bertrand competition model, Company A will set its price marginally below the generation cost of Company B in order to squeeze it out of the market. The market price will therefore be:

$$\pi = 60\ [£/MWh]$$

The demand level for this price will be:

$$D = 100 - \frac{\pi}{2} = 70\ [MW]$$

Then, all the demand will be supplied by Company A.

$$P_A = 70\ [MW]$$
$$P_B = 0\ [MW]$$

Finally, the profit made by companies A and B will be:

$$Profit_A = (\pi - 40) \cdot P_A = 1{,}400\ [£/h]$$

$$Profit_B = (\pi - 60) \cdot P_B = 0\ [£/h]$$

ii. Using the expressions derived in Part b):

(1) $160 - 2D - 2P_A = 0$
(2) $140 - 2D - 2P_B = 0$
(3) $D = P_A + P_B$

Solving these equations:

$$P_A = 30\ [MW]$$
$$P_B = 20\ [MW]$$

$$D = 50\ [MW]$$

The market price will therefore be:

$$\pi = 200 - 2D \quad \Longrightarrow \quad \pi = 100\ [£/MWh]$$

Finally, the profit made by companies *A* and *B* will be:

$$Profit_A = (\pi - 40) \cdot P_A = 1{,}800\ [£/h]$$

$$Profit_B = (\pi - 60) \cdot P_B = 800\ [£/h]$$

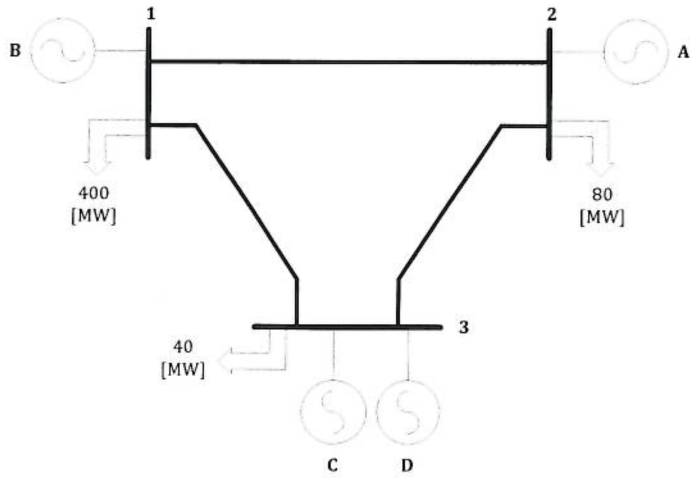
Part d)

The key difference between Bertrand and Cournot competition models is that the first models imperfect competition by introducing competition on prices, and the latter assumes competition takes place through companies choosing which quantity to produce in order to maximise profits.

Question 4

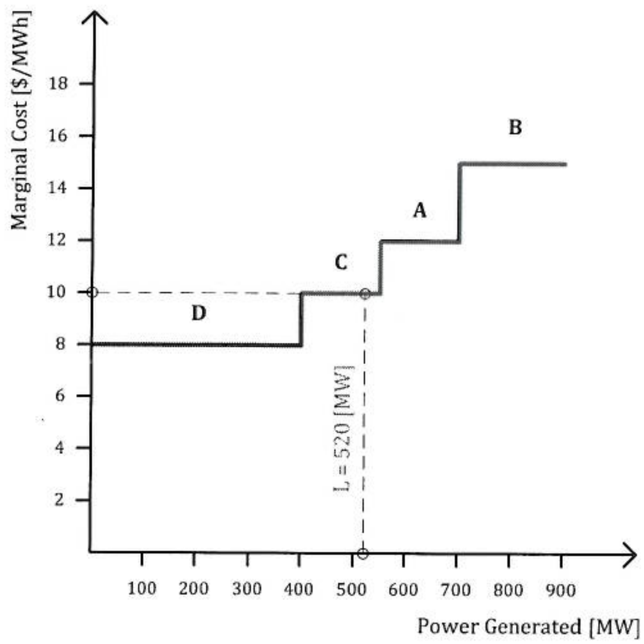
Part a)

We have the following Three-bus power system:



Generator	Capacity [MW]	Marginal Cost [$/MWh]
A	150	12
B	200	15
C	150	10
D	400	8

Since there are no transmission constrains, the output of all the generators can be stacked in order of marginal cost as shown in the following figure:



Using the figure, we can see that for a system load of $L = 400 + 40 + 80 = 520\ [MW]$ the marginal cost (and hence the price) is $10\ [\$/MWh]$. Furthermore, the units are dispatched as follows:

$$P_D = 400\ [MW]$$
$$P_C = 120\ [MW]$$
$$P_A = P_B = 0\ [MW]$$

Part b)

This problem can be solved using the superposition principle and also directly. To this effect, we write the power balance equation at two buses and KVL around the loop.

Bus 1	: $P_B - 400 = F_{12} + F_{13}$
Bus 2	: $P_A - 80 = -F_{12} + F_{23}$
Bus 3	: $P_C + P_D - 40 = -F_{13} - F_{23}$
Loop equation :	$0.2F_{12} + 0.3F_{23} - 0.3F_{13} = 0$

Putting these equations in matrix form gives:

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} F_{12} \\ F_{13} \\ F_{23} \end{bmatrix} = \begin{bmatrix} P_A - 80 \\ P_C + P_D - 40 \\ 0 \end{bmatrix}$$

Substituting $P_A = 0\ [MW]$, $P_B = 0\ [MW]$, $P_C = 120\ [MW]$ and $P_D = 400\ [MW]$ in these equations, we get:

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} F_{12} \\ F_{13} \\ F_{23} \end{bmatrix} = \begin{bmatrix} -80 \\ -480 \\ 0 \end{bmatrix}$$

Solving these equations, we get:

$$F_{12} = -120\ [MW]$$
$$F_{13} = -280\ [MW]$$
$$F_{23} = -200\ [MW]$$

The flow on line 1-3 thus exceeds its maximum capacity by $30\ [MW]$.

Part c)

i. The first method consists in increasing the output of generator B and decreasing by the same amount the output of generator C. Decreasing the output of generator D is not desirable as it is cheaper than generator C. To calculate how big this increase should be to remove the violation of the flow limit on line 3-1, consider an injection of $+1\ [MW]$ at bus 1 and an injection of $-1\ [MW]$ at bus 3. This pair of injection causes a flow in the network that divides itself as follows:

$\frac{0.3}{(0.2+0.3)+0.3} \cdot 1 = 0.375\ [MW]$ along the path 1-2-3

$\frac{(0.2+0.3)}{(0.2+0.3)+0.3} \cdot 1 = 0.625\ [MW]$ along the path 1-3

Since we use a linear (dc) model, we can say that to remove 30 $[MW]$ overload on line 3-1, we therefore need to increase the output of generator B by:

$$\frac{30}{0.625} = 48\ [MW]$$

The constrained dispatch is then:

$$P_A = 0\ [MW]$$
$$P_B = 48\ [MW]$$
$$P_C = 72\ [MW]$$
$$P_D = 400\ [MW]$$

Using the nodal and loop equations, the flows are calculated solving the following linear system:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} F_{12} \\ F_{13} \\ F_{23} \end{bmatrix} = \begin{bmatrix} P_B - 400 \\ P_C + P_D - 40 \\ 0 \end{bmatrix}$$

$$F_{12} = -102\ [MW]$$
$$F_{13} = -250\ [MW]$$
$$F_{23} = -182\ [MW]$$

Then, we can see that this dispatch does not cause a violation of the line flow constrains on any other line.

The cost of this dispatch is:

$$C_{Total} = 48 \cdot 15 + 72 \cdot 10 + 400 \cdot 8 = 4{,}640\ [\$]$$

which represents an increase of \$240 compared to the case where the network constrains are not considered.

ii. The other method to remove the constrain violation consists in increasing the output of generator A and decreasing the output of generator C by the same amount. To calculate how big this increase should be to remove the violation of the flow limit on line 3-1, consider an injection of $+1\ [MW]$ at bus 2 and an injection of $-1\ [MW]$ at bus 3. This pair of injection causes a flow in the network that divides itself as follows:

$\frac{0.3}{(0.2+0.3)+0.3} \cdot 1 = 0.375\ [MW]$ along the path 2-1-3

$\frac{(0.2+0.3)}{(0.2+0.3)+0.3} \cdot 1 = 0.625\ [MW]$ along the path 2-3

Since we use a linear (dc) model, we can say that to remove 30 $[MW]$ overload on line 3-1, we therefore need to increase the output of generator B by:

$$\frac{30}{0.375} = 80\ [MW]$$

The constrained dispatch is then:

$$P_A = 80\ [MW]$$
$$P_B = 0\ [MW]$$
$$P_C = 40\ [MW]$$
$$P_D = 400\ [MW]$$

Using the nodal and loop equations, the flows are calculated solving the following linear system:

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} F_{12} \\ F_{13} \\ F_{23} \end{bmatrix} = \begin{bmatrix} P_A - 80 \\ P_C + P_D - 40 \\ 0 \end{bmatrix}$$

$$F_{12} = -150\ [MW]$$
$$F_{13} = -250\ [MW]$$
$$F_{23} = -150\ [MW]$$

Once again, this redispatch does not cause a violation of the line flow constrains on any other line. The cost of this constrained dispatch is:

$$C_{Total} = 80 \cdot 12 + 40 \cdot 10 + 400 \cdot 8 = 4{,}560\ [\$]$$

which represents an increase of $\$160$ compared to the case where the network constrains are not considered.

Even though it re-dispatches a larger amount of MW, the second constrained dispatch is preferable to the first one because its cost is smaller. It is thus the optimal constrained dispatch.

Part d)

The nodal price at each bus is given by the cost of one additional MW of load at each node. Therefore, the price at bus 3 is 10 [$\$/MWh$] because the next MW of load would be generated locally by generator C because it is the cheapest generator not operating at its upper limit. An additional MW of load at node 2 would have to be produced by generator A. Producing it with generator C would cause a violation of the line flow constrain on line 3-1. Producing it with generator B would be more expensive than with generator A. The price at node 2 is therefore 12 [$\$/MWh$]. An additional MW of loaf at bus 1 requires a redispatch of A and C to minimize the cost increase while maintaining the flow on line 3-1 within limits. Extracting an additional an additional 1 [MW] at bus 1 and generating it at bus 3 causes the following change in the flow on line 1-3:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} \Delta F_{12} \\ \Delta F_{13} \\ \Delta F_{23} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \Delta F_{31} = 0.625 \ [MW]$$

Similarly, extracting an additional 1 [MW] at bus 1 and generating it at bus 2 causes the following change in the flow on line 1-3:

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0.2 & -0.3 & 0.3 \end{bmatrix} \begin{bmatrix} \Delta F_{12} \\ \Delta F_{13} \\ \Delta F_{23} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \Delta F_{31} = 0.250 \ [MW]$$

Therefore, if we want that the flow on line 3-1 remains unchanged (because it is already at its limit), we must change the productions by generators C and A in such a way that:

$$\Delta F_{31} = 0 = 0.625 \cdot \Delta P_C + 0.250 \cdot \Delta P_A$$

At the same time, since we are increasing the load by 1 [MW], we must also have:

$$\Delta P_A + \Delta P_C = 1$$

Solving the system consisting of the previous two equations we get:

$$\Delta P_C = -0.667 \ [MW]$$
$$\Delta P_A = 1.667 \ [MW]$$

To supply an additional MW of load at bus 1 without violating the network constrains, we must therefore increase the output of generator A and decrease the output of generator C.

The nodal price at bus 1 is thus given by a linear combination of the marginal cost of production of these two generators.

$$\pi_1 = -0.667 \cdot 10 + 1.667 \cdot 12 = 13.33\ [\$/MWh]$$

Finally:

$$\pi_1 = 13.33\ [\$/MWh]$$

$$\pi_2 = 12\ [\$/MWh]$$

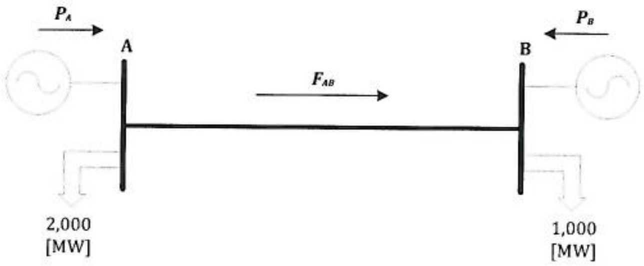
$$\pi_3 = 10\ [\$/MWh]$$

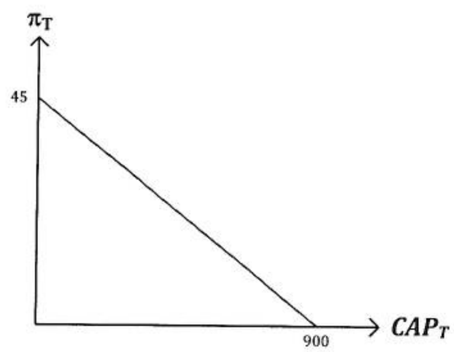
Question 5

Part a)

We assume that the demand is constant and insensitive to price and that energy is sold at its marginal cost of production and that there are no limits on the output of the generators. The marginal cost of production of the generators connected to buses A and B are given respectively by the following expressions:

$$MC_A = 20 + 0.03P_A\ [\$/MWh]$$
$$MC_B = 15 + 0.02P_B\ [\$/MWh]$$





where:

- π_T : Marginal Value of Transmission, measured in $[\$/MWh]$.
- CAP_T : Capacity of the transmission line connecting buses A and B, measured in $[MW]$.

Part b)

We can write the following two power flow equations:

$$F_{AB} = P_A - D_A$$

$$F_{BA} = -F_{AB} = P_B - D_B$$

Therefore:

$$P_A = F_{AB} + D_A$$

$$P_B = D_B - F_{AB}$$

Since the price that users of the transmission system are willing to pay is given by the difference between the prices for electrical energy at the two buses, we have:

$$\pi_T = 20 + 0.03P_A - 15 - 0.02P_B$$

$$\pi_T = 20 + 0.03F_{AB} + 60 - 15 + 0.02F_{AB} - 20$$

$$\pi_T = 45 + 0.05F_{AB}$$

Since the marginal cost at B is smaller than at the marginal cost at A the flow is positive from B to A, we should rewrite the equation in terms of F_{BA} as follows:

$$\pi_T = 45 - 0.05F_{BA}\ [\$/MWh]$$

This is the transmission demand function.

Part c)

The hourly long-range marginal cost of the transmission line is:

$$c_T(T) = \frac{kl}{\tau_0} = \frac{210 \cdot 500}{8{,}760} = 11.98 \approx 12.00\ [\$/MWh]$$

Part d)

The optimal transmission capacity is such that the supply and demand for transmission are in equilibrium. Therefore:

$$\pi_T = c_T(T)$$

Combining the transmission demand function obtained in Part b) and the transmission supply function determined in Part c), we get that:

$$45 - 0.05F_{BA} = 12$$

Then, the optimal capacity for these conditions is 660 $[MW]$.

Part e)

Since the marginal cost at B is smaller than at the marginal cost at A the flow is positive from B to A, and then:

$$P_A = D_A - F_{BA} \quad \Rightarrow \quad P_A = 1{,}340\ [MW]$$

$$P_B = D_B + F_{BA} \quad \Rightarrow \quad P_B = 1{,}660\ [MW]$$

The nodal prices at buses A and B therefore will be:

$$\pi_A = MC_A = 20 + 0.03P_A \quad \Rightarrow \quad \pi_A = 60.2\ [\$/MWh]$$

$$\pi_B = MC_B = 15 + 0.02P_B \quad \Rightarrow \quad \pi_B = 48.2\ [\$/MWh]$$

Finally, the merchandising surplus is:

$$Merchandising\ Surpluss = (\pi_A - \pi_B)F_{BA} = \pi_T F_{BA}\ [\$/h]$$

$$Merchandising\ Surpluss = 7{,}920\ [\$/h]$$

