IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

EEE PART I: MEng, BEng and ACGI

ENGINEERING MATERIALS

Wednesday, 8 June 10:00 am

Corrected Copy

TJT

Q4. 10.52

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : W.T. Pike, W.T. Pike

Second Marker(s) : T.J. Tate, T.J. Tate

Special Information for Invigilators: None

Information for Candidates:

Fundamental constants

Permittivity of free space, ε_0 = 8.85 x 10^{-12} F/m
Permeability of free space, μ_0 = 4π x 10^{-7} H/m
Planck's constant, h = 6.62 x 10^{-34} Js
Boltzmann's constant, k = 1.38 x 10^{-23} J/K
Electron charge, e = 1.6 x 10^{-19} C
Electron mass, m = 9.1 x 10^{-31} kg
Speed of light, c = 3.0 x 10^{8} ms^{-1}

Schrödinger's equation

General form:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

In one dimension:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

In spherical coordinates:

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$$

Free-electron theory

Density of states (3D):

$$g(E) = \frac{1}{\pi^2\hbar^3}(m)^{3/2}\sqrt{2E}$$

Fermi energy

$$E_f = \frac{\hbar^2\pi^2}{2m}\left(\frac{3n}{\pi}\right)^{2/3}$$

Fermi distribution

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$$

Electrons in semiconductors

Effective mass:

$$m_e^* = \frac{\hbar^2}{d^2E(k)/dk^2}$$

Concentration of electrons in a semiconductor of bandgap E_g:

$$n = \frac{1}{\sqrt{2}h^3}\left(\frac{m_e^* kT}{\pi}\right)^{3/2} e^{-\frac{(E_g - E_f)}{kT}}$$

$$= N_c e^{-\frac{(E_g - E_f)}{kT}}$$

Concentration of holes

$$p = \frac{1}{\sqrt{2}h^3}\left(\frac{m_h^* kT}{\pi}\right)^{3/2} e^{-\frac{E_f}{kT}}$$

$$= N_v e^{-\frac{E_f}{kT}}$$

Polarization

Lorentz correction for local field:

$$\mathbf{E}_{loc} = \mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}$$

Electronic polarization:

$$P_0 = \frac{\varepsilon_0 \omega_p^2 E_0}{\omega_m^2 - \omega^2 + j\omega\gamma}$$

where

$$\gamma = \frac{r}{m},$$

$$\omega_m^2 = \omega_0^2 - \frac{\omega_p^2}{3},$$

$$\omega_0^2 = k/m,$$

$$\omega_p^2 = \frac{ne^2}{m\varepsilon_0}.$$

Orientational Polarization:

Static:

$$\mathrm{P} = n\mu\,\mathrm{L}(\mu E/kT)$$

where

$$\mathrm{L}(x) = \coth(x) - 1/x$$

Dynamic:

$$P_0 = \frac{P_s}{1 + j\omega\tau},$$

Magnetism

Magnet dipole due to electron angular momentum:

$$\boldsymbol{\mu}_m = -\frac{e\mathbf{L}}{2m}$$

Magnet dipole due to electron spin:

$$\boldsymbol{\mu}_m = -\frac{e\mathbf{S}}{m}$$

Paramagnetism:

$$M = n\mu_m \mathrm{L}\left(\frac{\mu_m \mu_0 H}{kT}\right)$$

The Questions

1. (a) Show that the sinusoid wavefunction $\psi(x) = \psi_0 \sin kx$ is a solution of the 1-D Schrödinger equation for a constant potential, $V(x) = V_0$ and hence find the wavenumber, k, of the wavefunction [4]

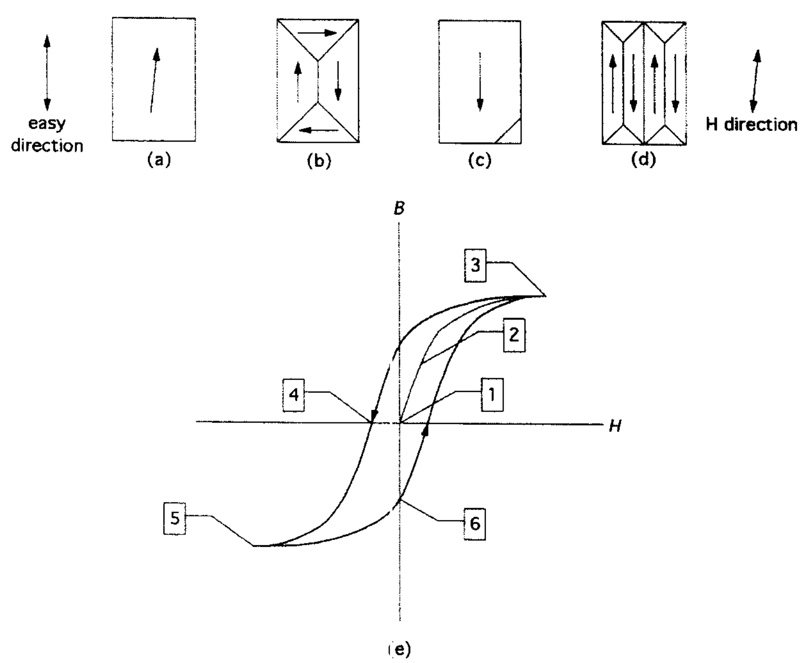
(b) Explain how the electronic polarisation of a block of material placed in a static electric field leads to the creation of surface charge on that block. Determine the relationship between the direction of the electric field and the sign of the charge on a surface of the block. [4]

(c) Explain how the direction of the Hall voltage can be used to determine the charge of the carriers in a material [4]

(d) Name the three major mechanisms which contribute to the polarisation of a material in an electric field and explain briefly how each contribution occurs. [4]

(e) Show diagrammatically how the propagation of an edge dislocation through a block of material can result in an inelastic shear of that material. [4]

(f) Figs. 1 (a – d) show the domain structure of a magnet under various magnetizing conditions. The easy direction of magnetization for the material and the direction of the external magnetic field is as indicated. Match each of the domain structures to one of the locations indicated in the magnetization curve of fig. 1(e).



[4]

Fig. 1: (a – d) domain structure a magnet under various magnetizing conditions with (e) the magnetization curve for this material.

(g) The potential energy $V(r)$ of a bond can be expressed as a power law in the bond length, r,

$$V(r) = \frac{A}{r^p} - \frac{B}{r^q}$$

show that the equilibrium bond length, r_0, is given by

$$r_0 = \left(\frac{pA}{qB}\right)^{\frac{1}{p-q}}.$$

[4]

(h) Explain how the magnetoresistive properties of a material can be used to form the basis of a digital magnetic data reader. [4]

(i) Derive expressions for the effective capacitance and the leakage resistance of a parallel-plate capacitor, plate area A, separation d, with a dielectric $\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$ and excitation at frequency ω. [4]

(j) What is the probability that a state situated at the Fermi level will be occupied by an electron? [4]

2. (a) An overhead power line, consisting of 20 steel cables of 2 mm diameter each, hangs between pylons with a total tension in the line of 10 kN. Determine the fractional increase in the length and decrease in the diameter due to the tension in the line for steel with a Young's modulus of 200 GPa and a Poisson's ratio of 0.3. Hence determine the increase in the resistance compared to the same power line installed in a stress-free environment underground. [22]

(b) What materials properties might be expected to affect the resistance of the overhead power line from summer to winter due to the seasonal variations in temperature, and why? [8]

3. (a) A p-type semiconductor with a doping concentration n_D has an energy gap, E_g, and the impurity levels are E_D above the top of the valence band. Draw a fully labelled energy diagram for this semiconductor. [6]

(b) Show how the electrons distribute themselves within the energy diagram and the location of the carriers at the following temperatures, T:

i. $T = 0$ K

ii. room temperature where $E_D < kT < E_g$

iii. high temperature, $kT > E_g$

Indicate the Fermi level in each case [14]

(c) Sketch the variation of the majority carrier concentration with temperature, identifying the freeze-out, intrinsic and extrinsic regimes for a doped semiconductor. Mark n_D and room temperature (300 K) on the appropriate axes. [10]

4. (a) In the Weiss theory of magnetism, the ferromagnetic solution for magnetisation is obtained by replacing the magnetic field H experienced by a material by $H + \lambda M$. Show how this leads to the equation

$$\alpha x - \beta H = L(x) \qquad (1)$$

where

$$x = \frac{\mu_0 \mu_m (H + \lambda M)}{kT}$$

and determine expressions for α and β. [10]

By considering the graphical solution to equation 1, show how Weiss theory predicts hysteresis in ferromagnetic materials and sketch the shape of the magnetization curve. [12]

How does the shape of this curve predicted from Weiss differ from that in question 1(e), and why? In what circumstances will the Weiss curve be applicable? [8]

1(f)

Notified in both rooms at 10.52.

E1 5 Engineering materials
2005
Solutions

The Answers

1. (a) Substituting $\psi(x) = \psi_0 \sin kx$ and $V(x) = V_0$ into 1-D Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \text{ [1]}$$

gives

$$\frac{\hbar^2}{2m}k^2 + V_0 = E \text{ [2]}$$

and so

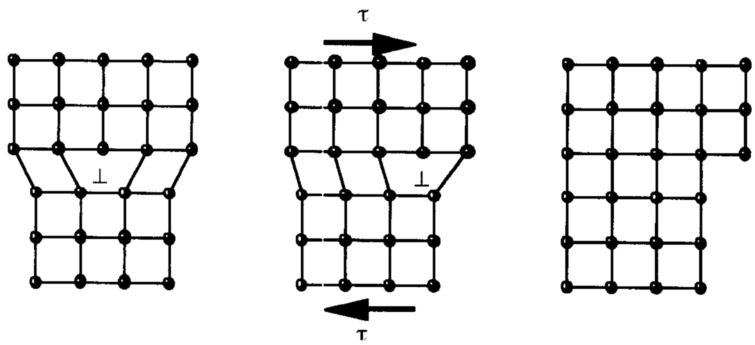
$$k = \frac{\sqrt{2m(E - V_0)}}{\hbar} \text{ [1]}$$

[4]

(b) Electric field causes the charge separation between the nucleus and the surrounding electrons [1], with the lighter electrons displaced against the direction of the electric field [1]. Overall in the bulk of the material the displaced charge densities produce no overall charge accumulation, but at the surface, electrons can be displaced a little out of the material [1], producing an overall negative charge for the surface normal against the field direction, and a corresponding positive charge for surface normal in the field direction [1]. [4]

(c) Pass a current through a block of material with a magnetic field perpendicular to the current flow [1]. The charge carriers will experience a Lorentz force that will deflect them to one side of the material, leaving a a deficit of the carriers charge on the other side of the material [1]. The resulting field (which can be determined through the Hall voltage) will build up until it exactly cancels the Lorentz force on the carries, and the carriers will then travel undeflected [1]. As the Lorentz force depends on the sign of the charge on the carriers, the resulting direction of the Hall voltage will indicate the charge [1]. [4]

(d) Electronic: response of individual electrons to an exciting electric field[1]; molecular: response of polar or ionic atoms, casing a distortion of the bonds[1]; orientational; response of molecular dipoles in aligning with the exciting electrical field [2] [4]



[4]

(e)

(f) a – 3 [1]; b – 4 [1]; c – 6 [1]; d – 1 [1] [4]

(g) At the equilibrium position potential energy is a minimum:

$$\frac{dV(r)}{dr} = -\frac{pA}{r^{p+1}} + \frac{qB}{r^{q+1}} = 0, [2]$$

Hence at $r = r_0$

$$r_0 = \left(\frac{pA}{qB}\right)^{\frac{1}{p-q}} [2]$$

[4]

(h) A small block of magnetoresisitve material is mounted on the read head with a connection to either side of the block[1]. As the head is scanned over the magnetic medium, its resistance will change dependent on the state of magnetization of the medium which can be measured as a change in current flow if a constant voltage is applied across the block [2]. This can form the basis of digital data storage. In this case, the bit coding might be magnetized = 1, unmagnetised = 0 [1].

[4]

(i) First substitute $\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$ into the normal formula for capacitance: $C = \frac{\varepsilon_0(\varepsilon_r' - j\varepsilon_r'')A}{d}$ [1]

The admittance of this can be written:

$$Y = j\omega C = \frac{j\omega\varepsilon_0\varepsilon_r' A}{d} + \frac{\omega\varepsilon_0\varepsilon_r'' A}{d} \quad [1]$$
$$= j\omega C_{eff} + 1/R_{eff}$$

from which

$$C_{eff} = \frac{\varepsilon_0\varepsilon_r' A}{d}$$
$$R_{eff} = \frac{d}{\omega\varepsilon_0\varepsilon_r'' A} \quad [2]$$

[4]

(j) From the Fermi distribution:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_f}{kT}\right)} \text{ [1]}$$

when $E = E_f$, $f(E)$ =0.5 [2], which implies there is a 50% probability of finding an electron in the state [1].

[4]

2. From definition of Young's modulus

$$\varepsilon = \sigma / E$$
$$= \frac{T}{AE} \quad [4]$$
$$= \frac{4T}{n\pi d^2 E}$$

In this case, therefore, where T = 10,000 N, n = 20 and d = 2mm, E = 200 GPa , ε = 0.00080 [3]. From the definition of Poisson's ratio,

$\varepsilon_{radial} = -\nu\varepsilon_{axial}$ [2]

and so the diameter will decrease by 0.3 x 0.00080 = 0.00024 [3].

As the resistance of the wire is given by $R = \dfrac{\rho l}{A}$ [2]

the resistance of the stressed wire will be given by

$$R = \frac{\rho l_o(1+\varepsilon_{axial})}{A_0(1+\varepsilon_{radial})^2}$$
$$\approx R_0(1+\varepsilon_{axial} - 2\varepsilon_{radial}) \; [6]$$
$$\approx R_0(1+\varepsilon_{axial}(1+2\nu))$$

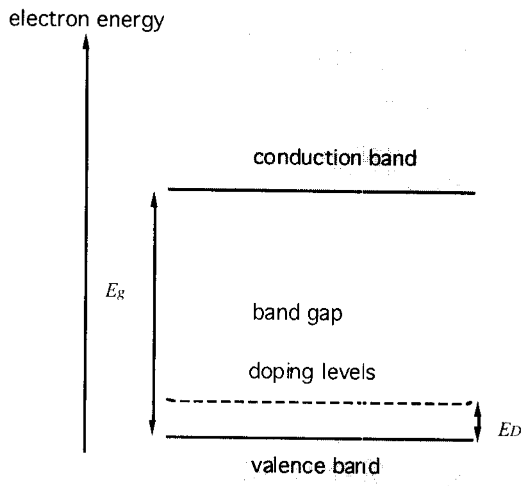
and so the fractional change in the resistance is

$$\frac{R-R_0}{R_0} \approx \varepsilon_{axial}(1+2\nu)$$
$$\approx 0.0008(1+0.6) \; [2]$$
$$\approx 0.0013$$

[22]

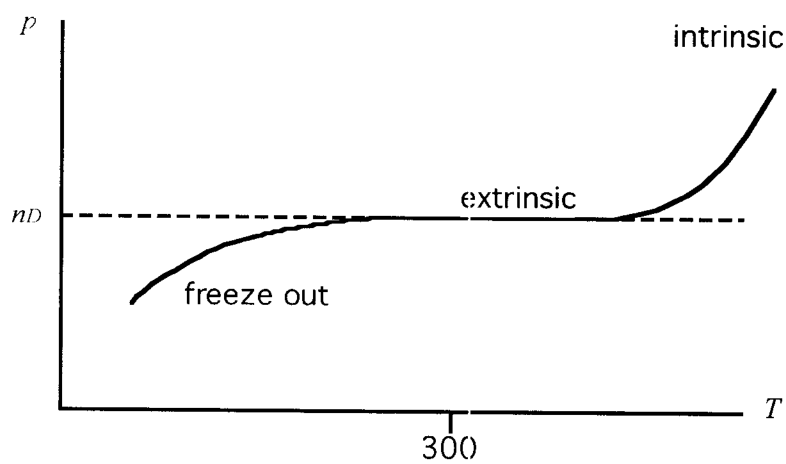
The resistivity of the steel will increase with temperature [1]. The dimensions of the wire will also change as Young's modulus will drop with increased temperature (it will get softer) causing additional elongation and thinning of the wire [2]. All the effects so far will cause the resistance to increase [1]. Finally the wire will tend to expand through thermal expansion in summer [1]. This will increase the resistance due to axial expansion, but reduce the resistance due to the increase in the cross sectional area [1]. The latter will be twice the effect of the former, causing an overall decrease in the resistivity [1]. Overall the resistance change could be in either direction, depending on the relative sizes of the thermal coefficients of resistivity, Young's modulus and expansion [1]. [8]

3.



[6]

i. T = 0 K: no carriers found anywhere - all states are occupied [2]. Fermi level is midway between valence band and doping levels[2].
ii. room temperature where $E_D < kT < E_g$ - carriers (holes) are all at the top of the valence band [2], with a concentration of n_D – the dopants are all thermalised i.e. electrons populating all the dopant levels [2], leaving the holes behind. Fermi level is between the doping level and mid gap [2].
iii. high temperature, $kT > E_g$ - carriers found at the top of the valence band and the bottom of the conduction band [2]. The dopant levels will still all be occupied, but their contribution to the carrier population will be swamped by concentration of electron and hole carriers produced by thermalisation across the band gap [2]. Fermi level is mid gap [2]. [14]



[10]

4. Substituting H by $H + \lambda M$ in $M = n\mu_m \mathrm{L}\left(\frac{\mu_m \mu_0 H}{kT}\right)$

gives

$M = n\mu_m \mathrm{L}\left(\frac{\mu_m \mu_0 (H + \lambda M)}{kT}\right)$[1]

If $x = \dfrac{\mu_0 \mu_m (H + \lambda M)}{kT}$ then

$M = \dfrac{kTx}{\lambda \mu_0 \mu_m} - \dfrac{H}{\lambda}$ [1]

and so

$\dfrac{kTx}{\lambda \mu_0 \mu_m} - \dfrac{H}{\lambda} = n\mu L(x)$ [2]

which can be rewritten as

$\alpha x - \beta H = \mathrm{L}(x)$ (1)

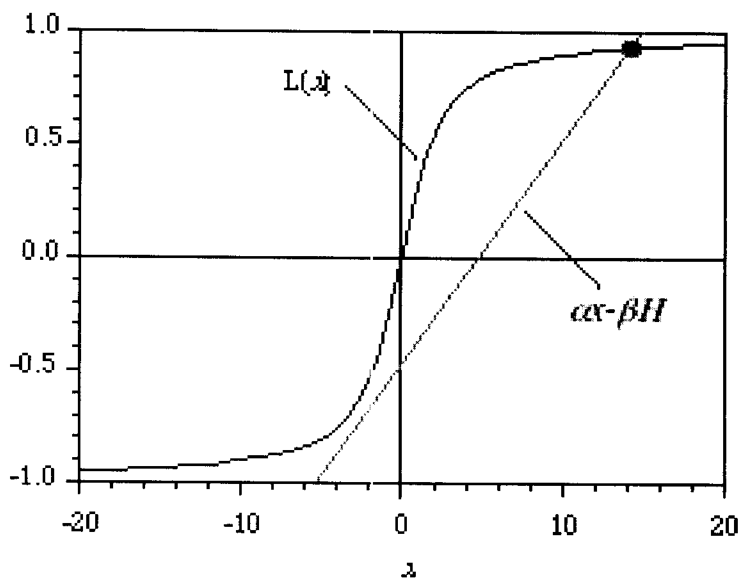
with

$\alpha = \dfrac{kT}{\lambda n \mu_0 \mu_m^2}$ [2]

and

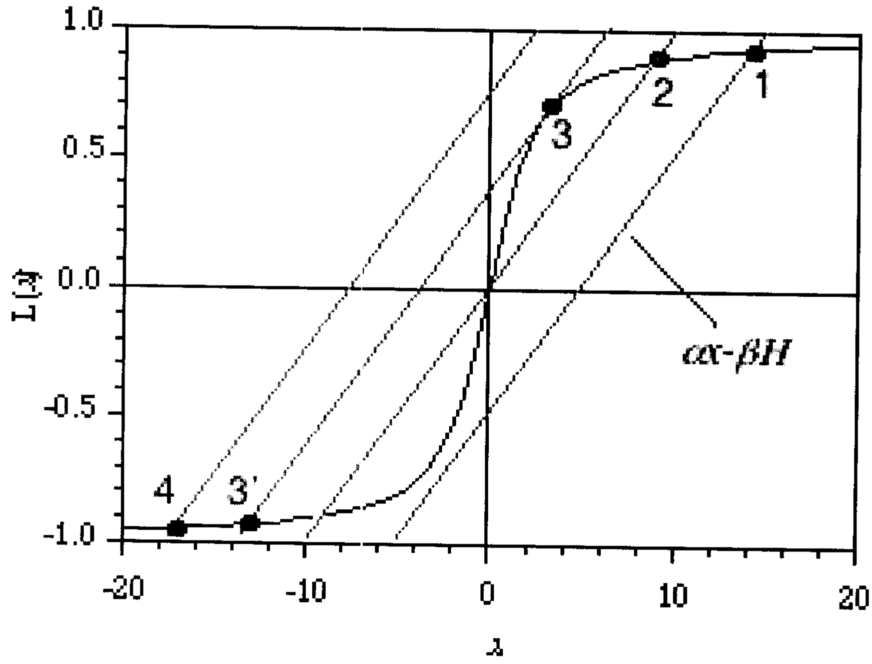
$\beta = \dfrac{1}{\lambda n \mu_m}$ [2] [8]

Graphical solution is given by the intersection of the straight line with the Langevin function in (1): [2]



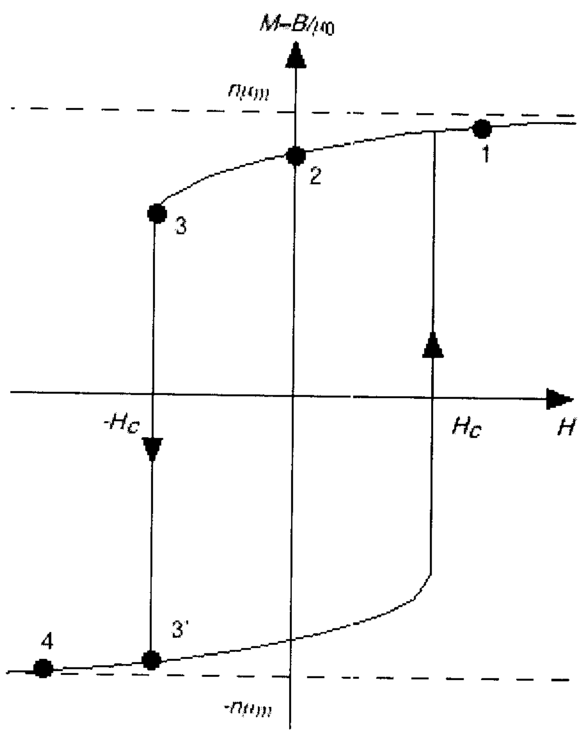
[2]

which gives:



[4]

which in turn gives a magnetization curve



[4] [12]

This curve, in contrast to the hystersis curve of question 1, has vertical segments [2]. This is because Weiss theory does not include the effect of domains [2]. As the domain walls require energy for motion, there is no sudden change in the diraction of magnetization. Additional magnetic energy needs to be used to drive the domain-wall motion, which causeas the actual hysteresis curve to have a sloping characteristic [2].

If there are no domain walls the magnet should behave as Weiss predicted. This is the case for micrometer-sized domains [2]. [8]