IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

EEE PART II: MEng, BEng and ACGI

**MATHEMATICS 2B (E-STREAM AND I-STREAM)**

**Corrected copy**

Friday, 26 May 2:00 pm

Time allowed: 1:30 hours

**There are TWO questions on this paper.**

**Answer TWO questions.**

*All questions carry equal marks*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible First Marker(s) : B. Clerckx
Second Marker(s) : D. Nucinkis

## THE QUESTIONS

[25]

1. Consider two continuous random variables $X$ and $Y$ characterized by the following joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} 3(x+y), & 0 < x < 1,\ 0 < y < 1,\ 0 < x+y < 1, \\ 0, & otherwise. \end{cases}$$

a) Compute the probability $P(X+Y \leq 0.5)$.

[4]

b) Compute the marginal probability density function of $X$.

[4]

c) Compute the expectation of $X$, i.e. $E(X)$, and the variance of $X$, i.e. $\mathrm{Var}(X)$.

[4]

d) Compute the marginal probability density function of $Y$.

[2]

e) Compute the expectation of $Y$, i.e. $E(Y)$, and the variance of $Y$, i.e. $\mathrm{Var}(Y)$.

[2]

f) Compute the covariance between $X$ and $Y$, i.e. $\mathrm{Cov}(X,Y)$, and the correlation coefficient between $X$ and $Y$, i.e. $\mathrm{Corr}(X,Y)$.

[2]

g) Are $X$ and $Y$ uncorrelated? Independent? Provide your reasoning.

[2]

h) Compute the conditional probability density function of Y given $X = x$.

[2]

i) Compute the conditional expectation of $Y$ given $X = x$.

[3]

2. a) Consider the independent and identically distributed (i.i.d.) random variables $X_1, X_2, \dots, X_n$ characterized by their probability density function [25]

$$f_X(x) = \begin{cases} 2x, & 0 < x < 1, \\ 0, & otherwise. \end{cases}$$

i) Using the Central Limit Theorem (CLT), compute an approximate value of $P\left(\prod_{i=1}^{n} X_i \leq \exp\left(-\frac{n}{2} + 0.5\sqrt{n}\right)\right)$. Provide your reasoning. [7]

ii) Compute the probability density function of the random variable $U = X_1 X_2$. Provide your reasoning. [6]

iii) Compute the expectation of $U$, i.e. $E(U)$. Provide your reasoning. [2]

b) i) Assume $X \sim N(0,1)$ and take $Y = X^4$. Are $X$ and $Y$ uncorrelated? Independent? Provide your reasoning. [5]

ii) Consider the following statement: If $X$ is a continuous random variable with first moment $m_1$ and second moment $m_2$, then we have $m_1^2 = m_2$. Is the statement correct? If yes, provide a proof. If not, correct the statement and provide a proof. [5]

  