

## Traffic Theory + Queuing Systems SOLUTION

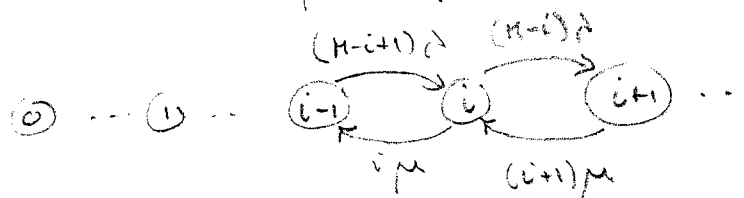
Question Number etc. in left margin

Mark allocation in right margin

Q1 (a) Erlang Model Assumptions (handwritten)

- (a) - Poisson arrival stream: good approx when large number of independent traffic sources  
 - Exponential holding times: good approx for telephone calls  
 - Full-availability access: usually valid  
 - No re-submissions: not strictly valid but model can be modified to allow for this

(b)



Global

$$[(H-i)\lambda + i\mu]\pi_i = (H-i+1)\lambda\pi_{i-1} + (i+1)\mu\pi_{i+1}$$

Local

$$(H-i+1)\lambda\pi_{i-1} = i\mu\pi_i$$

 $H$  = no. sources $\pi_i$  = Eq. state prob. in state  $i$  $\lambda$  = calling rate / free source $\mu$  = release rate / channel

(c)

$$\rho = 8, H = 12$$

$$\rho_c = E_c(8) = 0.05 \text{ (from chart)}$$

(i) mean carried traffic

$$\rho_c = (1 - \rho_c)\rho = 7.3 \text{ Erlangs}$$

(iii) mean channel occupancy

$$\eta = \frac{\rho_c}{H} = 0.6083$$

the prob. idle time =  $\pi_0$ 

$$= 0.00036 \text{ (approx } e^{-\rho_c})$$

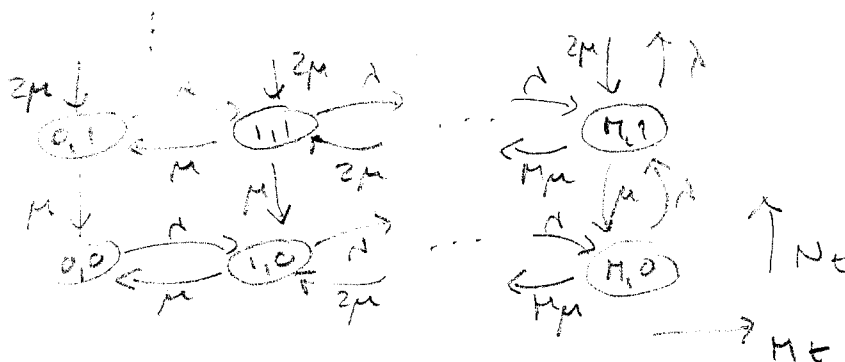
Question Number etc. in left margin

Mark allocation in right margin

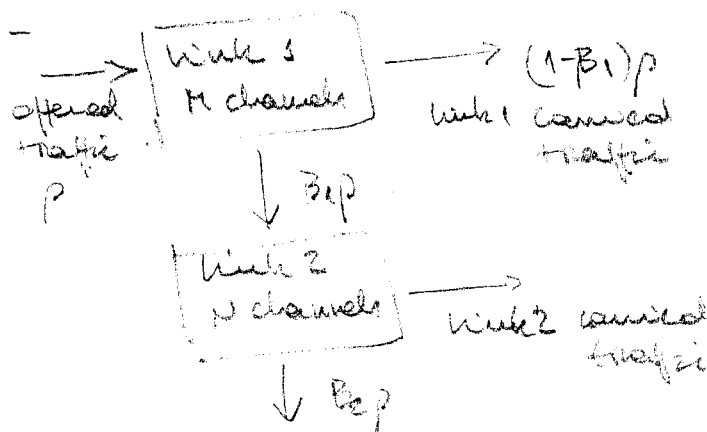
Q2 (a) overflow node (beck work)

(a) let  $\begin{cases} M_1 = \text{no of busy channels on link 1} \\ M_2 = \text{no of busy channels on link 2} \end{cases}$

Assume  $\begin{cases} \text{(i) Poisson arrival stream to link 1, rate } \lambda \\ \text{(ii) Exponential holding times, rate } \mu \end{cases}$



- local balance equation do not hold  
The problem should be solved using global balance equations



link 1 call completion =  $B_1$

link 2 call completion = Arrived rate to link 2 for blocked arrivals

Total arrival rate to link 2

$$= \frac{dB_2}{dB_1} = \frac{B_2}{B_1} = \frac{E_{M+1}(p)}{E_N(p)}$$

3

3

4

Question Number etc. in left margin

Mark allocation in right margin

Q2

(9)

$$K=10, \quad \frac{1}{\mu} = 3 \text{ sec} \quad \rho = \frac{\lambda}{K\mu} = 0.8 \text{ Packets/channel}$$

$$P[\text{Delay}] = \frac{E_K(K\rho)}{1 - \rho[1 - E_K(K\rho)]} = 0.41$$

5

If buffer size is  $Q$  then

$$P[\text{loss}] = \frac{(1-\rho)\rho^Q E_K(K\rho)}{(1-\rho) + \rho(1-\rho^Q) E_K(K\rho)}$$

$$\rho = 0.8 \text{ Packets}$$

$$E_K(K\rho) = 0.122 \text{ (from chart)}$$

$$P[\text{loss}] = 0.030 \text{ for } Q=5$$

5

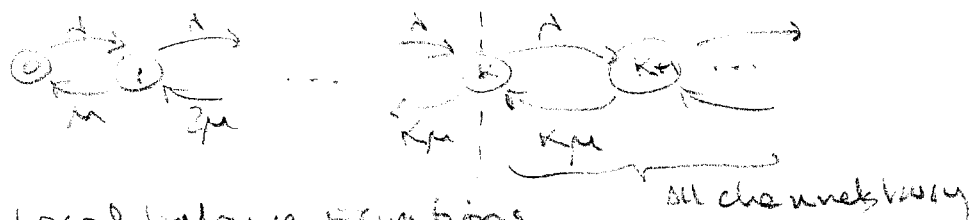
Question Number etc. in left margin

Mark allocation in right margin

Q3  
(a)

→  $\Pi \rightarrow \text{K}$  → (backward/reverse theoretical app)

NET is a BID process with state diagram



Local balance Equations

$$\pi_i = \left( \frac{A}{i\mu} \right) \pi_{i-1} = \left( \frac{A}{i} \right) \pi_{i-1} \quad i \leq K$$

$$\pi_i = \left( \frac{A}{K\mu} \right) \pi_{i-1} = \rho \pi_{i-1} \quad i \geq K$$

$$\pi_i = \left( \frac{A^i}{i!} \right) \pi_0 \quad i \leq K$$

$$\left( \frac{A^K}{K!} \right) \rho^{i-K} \pi_0 \quad i \geq K$$

for  $i \geq K$

$$\pi_{K+j} = \left( \frac{A^K}{K!} \right) \rho^j \pi_0 = \rho^j \pi_K$$

$$\pi_0 = \frac{1}{\frac{A^K}{K!}} \left[ \frac{(1-\rho) E_K(A)}{(1-\rho) + \rho E_K(A)} \right]$$

$$P[\text{Delay}] = P[W > 0] = P[W \geq K] = \sum_{i=K}^{\infty} \pi_i$$

$$= \sum_{j=0}^{\infty} \pi_K \rho^j = \frac{\pi_K}{1-\rho} = \frac{\left( \frac{A^K}{K!} \right) \pi_0}{1-\rho}$$

$$P[\text{Delay}] = \frac{E_K(A)}{(1-\rho) + \rho E_K(A)} = D_K(A)$$

Question Number etc. in left margin

Mark allocation in right margin

Q3

(a)

$$(u) \quad P[Q_t = i | \text{all servers busy}]$$

$$P[Q_t = i | U_t \geq K] = \left\{ \frac{P[N_t = K+i]}{\sum_{j=0}^{\infty} P[N_t = K+j]} \right\}$$

$$= \left\{ \frac{\pi_K \rho^i}{\sum_{j=0}^{\infty} \pi_K \rho^j} \right\}$$

$$P[Q_t = i | \text{delay}] = (1-\rho) \rho^i \quad i=0, 1, 2, \dots$$

7

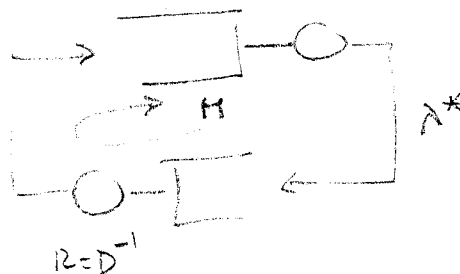
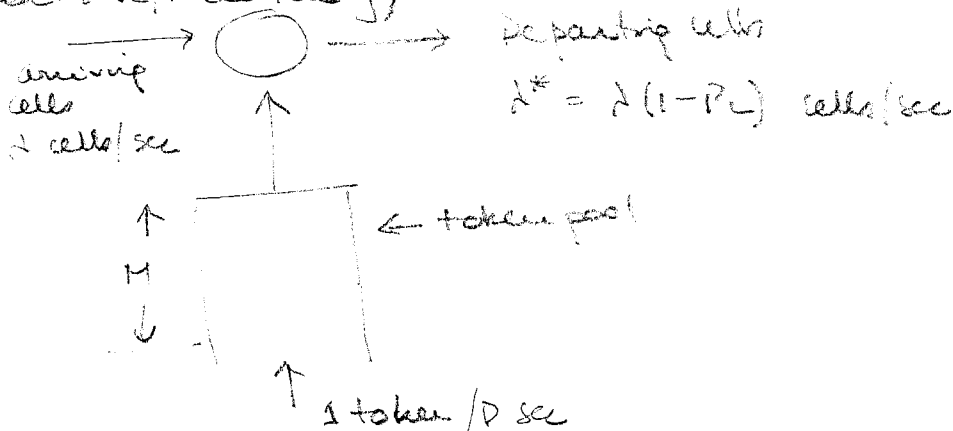
Question Number etc. in left margin

Mark allocation in right margin

Q3

(hook work / new theory)

(b)



closed queueing  
network model,  
leaky bucket M/M/1  
approximation

$$\lambda^* = \lambda (1 - P_L)$$

for maximum occupancy M

$$P_L = \frac{\rho^M (1 - \rho)}{1 - \rho^{M+1}}$$

with  $\rho = \lambda / \mu = \lambda D$

$$\lambda^* = \lambda \left[ \frac{1 - \rho^M}{1 - \rho^{M+1}} \right]$$

Question Number etc. in left margin

Mark allocation in right margin

QA

(a)

Admission control: How much traffic can the network handle if a prescribed QoS for each traffic class is to be maintained while the network utilisation meets some minimum goals. Eg.:

- given a particular VP how many VCs can it handle?
- given a number of active VCs does one admit a new call when a request to set one up arrives?

Discuss meaning and derivation of

$$C_L = \min [C_{CS}, C_{LF}]$$

where

$$C_{CS} = mR_p + \sigma R_p \sqrt{-\ln(2\pi)} - 2\ln \epsilon$$

$$C_{LF} = R_p N \frac{1-\epsilon}{2} + R_p N \sqrt{\left(\frac{1-\epsilon}{2}\right)^2 + k_p}$$

Question Number etc. in left margin

Mark allocation in right margin

Q4

(a)

M/G/1 system

mean message length  $= \left( \frac{5}{4} B \right)$ mean square message length  $= \left( \frac{7}{4} B^2 \right)$ 

$$E(S) = \frac{5}{4} \frac{160}{64} \times 10^{-3} \text{ sec} = 3.125 \text{ msec}$$

$$E(S^2) = \frac{125}{10} \text{ msec}^2 \left( \frac{7}{4} \frac{(160)^2}{(64)^2} \times 10^{-6} \right)$$

$$\rho = \lambda E(S) = 0.9375$$

$$E(R) = \frac{1}{2} \lambda E(S^2) = 1.65 \text{ ms}$$

Priority 1

$$E(W_1) = \left[ \frac{E(R)}{1 - \rho_1} \right]$$

$$\rho_1 = \lambda_1 E(S_1) = 0.5625$$

$$E(S_1) = \frac{160}{64} \times 10^{-3} = 2.5 \times 10^{-3}$$

$$\lambda_1 = \frac{3}{4} \lambda = 0.75 \times 300$$

$$E(W_1) = \left[ \frac{E(R)}{1 - \rho_1} \right] = \frac{1.65}{1 - 0.5625} = 3.97 \text{ ms}$$

$$E(T_1) = E(W_1) + E(S_1) = 6.27 \text{ ms}$$

$$E(W_2) = \left[ \frac{E(W_1)}{1 - \rho_1 - \rho_2} \right] = 60.3 \text{ ms}$$

$$E(T_2) = E(W_2) + E(S_2) = 65.9 \text{ ms}$$

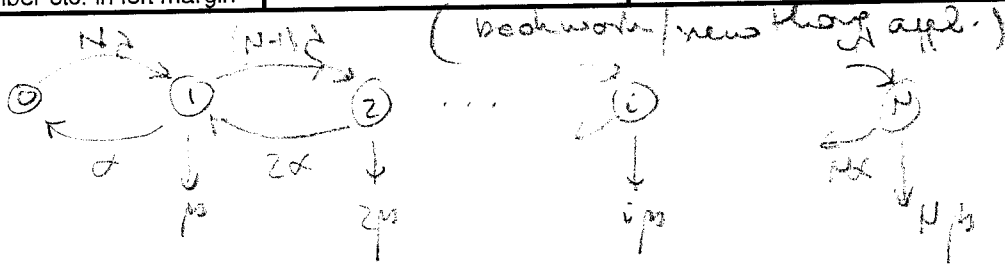
$$E(T) = \frac{2}{3} E(T_1) + \frac{1}{3} E(T_2) = 21.0 \text{ ms}$$



Question Number etc. in left margin

Mark allocation in right margin

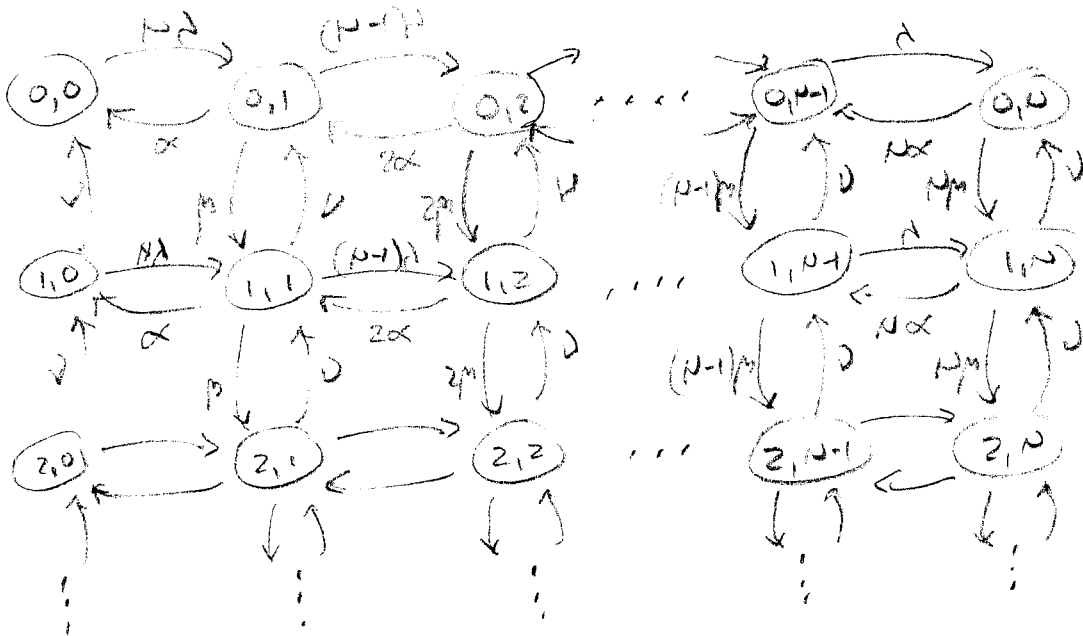
26  
(a)



$$(i) \pi_i = \binom{N}{i} \left( \frac{\lambda}{\mu} \right)^i \left( \frac{\lambda}{\lambda + \mu} \right)^{N-i} = \binom{N}{i} \left( \frac{\lambda}{\mu} \right)^i \left( 1 + \frac{\lambda}{\mu} \right)^{-N}$$

2

(ii)



2

(iii) Average number of cells entering the system (queue).

$$N \rho \left( \frac{\lambda}{\lambda + \mu} \right)$$

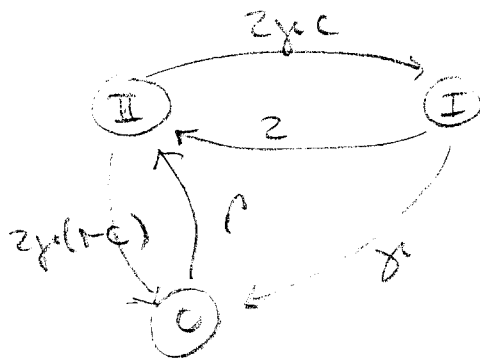
system utilization

$$\frac{N \rho \left( \frac{\lambda}{\lambda + \mu} \right)}{1} = \rho$$

4

Question Number etc. in left margin

Mark allocation in right margin

Q5  
(b)

$$Q = \begin{bmatrix} -0.4 & 0.396 & 0.004 \\ 4.0 & -4.2 & 0.2 \\ 8.0 & 0 & -8.0 \end{bmatrix}$$

solve

$$\begin{bmatrix} -0.4 & 4.0 & 8.0 \\ 0.396 & -4.2 & 0 \\ 0.004 & 0.2 & -8.0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_0 + x_1 + x_2 = 1$$

$$x_0 = 0.085939$$

$$R_0 = 1.6$$

$$x_1 = 0.91482$$

$$R_1 = 1.0$$

$$x_2 = 0.002579$$

$$R_2 = 0.0$$

$$\lim_{t \rightarrow \infty} E(W(t)) = \sum_{i=0}^{\infty} R_i x_i = 1.54431$$

6

6