

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EIE PART II: MEng, Beng and ACGI

FEEDBACK SYSTEMS

Wednesday, 4 June 2:00 pm

Time allowed: 1:30 hours

There are THREE questions on this paper.

Answer ALL questions. Question 1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	I.M. Jaimoukha
	Second Marker(s) :	S. Evangelou

1. a) Consider the mechanical system illustrated in Figure 1.1 where all the symbols have the standard interpretation. The input is the applied force $f(t)$ and the output is the displacement $y(t)$. Take $M = K_2 = D = 1$ in appropriate units.

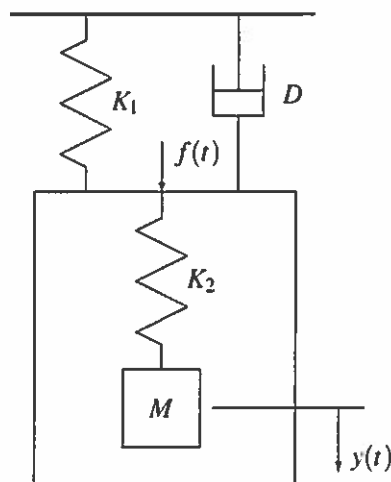


Figure 1.1

- i) Determine the transfer function $G(s)$ relating y to f . [5]
 - ii) Use the Routh array to find the range of values of K_1 for stability. [5]
 - iii) Find the value of K_1 for which $G(s)$ is marginally stable. For this value of K_1 , what are the poles of $G(s)$? [5]
 - iv) Let $f(t)$ be a unit step applied at $t = 0$. Use the final value theorem, which should be stated, to find the steady-state value y_{ss} of $y(t)$ in terms of K_1 . What is the value of K_1 for which $y_{ss} = 2$? [5]
- b) In Figure 1.2 below, $G(s) = 2/(s^3 - 1)$ and $K(s)$ is a compensator.
- i) Draw the Nyquist diagram of $G(s)$. [5]
 - ii) Let $K(s) = k$ be a constant compensator. Use the Nyquist criterion, which should be stated, to determine how many unstable or marginally stable poles the closed-loop has for all k . [5]
 - iii) Use the Routh-Hurwitz stability criterion to determine if the closed-loop can be stabilised using a PD compensator. [5]
 - iv) Show that the closed-loop can be stabilised using the compensator

$$K(s) = k \frac{s^2 + s + 1}{s^2 + 2s + 3}$$

for some $k > 0$.

[5]

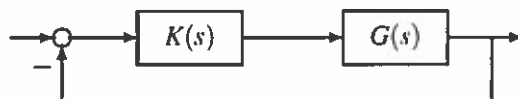


Figure 1.2

2. Consider the feedback control system in Figure 2.1 below. Here,

$$G(s) = \frac{1}{s^3 + as^2 + bs + c}$$

represents an uncertain model where it is only known that

$$a > 0, \quad b > 0, \quad c > 1, \quad ab - c \geq 2. \quad (2.1)$$

$K(s)$ is the transfer function of a compensator.

- Sketch a typical Nyquist diagram of $G(s)$, indicating the low and high frequency portions. Use the Routh array to find the real-axis intercepts. [8]
- Let $K(s) = K$ be a nondynamic compensator. State the Nyquist stability criterion and use the Nyquist diagram to determine the number of unstable or marginally stable closed-loop poles for all values of K . [8]
- What is the value of the gain margin for $G(s)$? [3]
- Derive the minimum value of the gain margin for all a, b, c satisfying the relations in equation (2.1). [3]
- Suppose that it is known that $G(s)$ has an adequate phase margin and that you have the option of either using a phase-lead or a phase-lag compensator. Which would you choose? Justify your choice. [8]

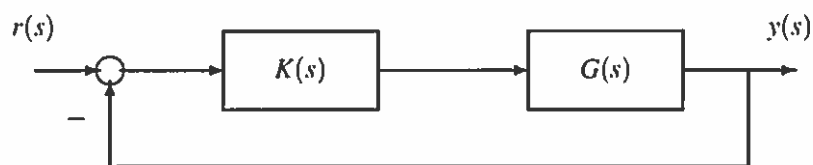


Figure 2.1

3. Consider the feedback loop shown in Figure 3.1 below.

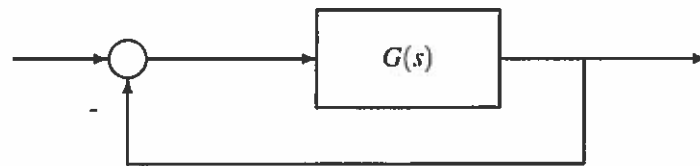


Figure 3.1

- a) Suppose that

$$G(s) = \frac{-2s}{s^2 + ks + 1}$$

where $k > 0$ is a design parameter. It is required to find k such that closed-loop response to a step reference signal is critically damped with a settling time of $4s$.

- i) Find the location of the closed-loop poles that achieves the design specification. [5]
- ii) Derive the closed-loop characteristic equation. [5]
- iii) Find the value of k that achieves the design specification. [5]

- b) Suppose that

$$G(s) = \frac{-2s}{s^2 + k(s+z) + 1}$$

where $k > 0$ and $z \geq 0$ are design parameters. It is required to find k and z such that closed-loop response to a step reference signal achieves the following design specifications:

- The settling time is at most 4 seconds.
 - The response is oscillatory with a maximum overshoot of 5%.
- i) Find the location of the closed-loop poles that achieves the design specification. [5]
 - ii) Derive the closed-loop characteristic equation. [5]
 - iii) Find the values of k and z that achieve the design specifications. [5]