IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

EEE/EIE PART I: MEng, BEng and ACGI

Corrected copy

INTRODUCTION TO SIGNALS AND COMMUNICATIONS

Thursday, 26 May 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : K.K. Leung
Second Marker(s) : C. Papavassiliou

Special Instructions for Invigilator: **None**

Information for Students:

Fourier Transforms

$$\cos \omega_o t \quad \Leftrightarrow \quad \pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$$

Some useful trigonometric identities

$$\cos x \cos y = \tfrac{1}{2}\cos(x-y)+\tfrac{1}{2}\cos(x+y)$$
$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$
$$a\cos x + b\sin x = c\cos(x+\theta)$$

where $c=\sqrt{a^2+b^2}$ and $\theta=\tan^{-1}(-b/a)$

Complex exponential

$$e^{jx} = \cos x + j\sin x$$

1. This is a general question. (40%)

a. Consider a time-domain signal $f(t) = a$ for $-b \le t \le b$ and 0 otherwise, where b is positive.

i. Derive the Fourier transform $F(\omega)$ of $f(t)$. [4]

ii. Sketch the frequency spectrum of $f(t)$. [3]

iii. Consider a special case of $f(t)$ where $a = \frac{1}{2b}$ and b is reduced to zero (i.e., $b \to 0$). What is the Fourier transform of this special case of $f(t)$? [2]

iv. Now obtain $\hat{f}(t)$ by shifting $f(t)$ by t_o amount of time. That is, $\hat{f}(t) = f(t - t_o)$. Let $\hat{F}(\omega)$ denote the Fourier transform of $\hat{f}(t)$. How are $\hat{F}(\omega)$ and $F(\omega)$ related to each other? [3]

b. The trigonometric Fourier series for a periodic signal $g(t)$ with period T_0 is given by

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t).$$

i. Express ω_0 in terms of T_0. [1]

ii. Physically, what does the coefficient a_0 represent? [1]

iii. Give a physical interpretation of the coefficients a_n and b_n for $n = 1, 2, ..., \infty$. [2]

iv. If $g(t)$ is an odd function of t, what can be said about any of the coefficients, a_0, a_n and b_n for $n = 1, 2, ..., \infty$, and why? [3]

c. Consider the modulating signal $m(t) = \cos(\omega_m t)$.

i. Sketch the spectrum of $m(t)$. [2]

ii. Give an expression $\phi(t)$ for the amplitude-modulated signal, Double-Side-Band with Suppressed Carrier (DSB-SC), with the modulating signal $m(t)$ and the carrier angular frequency, ω_c radians per second, where $\omega_c > 2\omega_m$. [2]

iii. Sketch the spectrum of the DSB-SC signal in part ii. [2]

iv. From the frequency-domain perspective, what is the effect of the amplitude modulation on the modulating signal $m(t)$. [1]

v. From the spectrum for $\phi(t)$, filter out the upper-side-band (USB) spectrum to obtain the lower-side-band (LSB) $\varphi_{LSB}(t)$. Write the expression for $\varphi_{LSB}(t)$. [2]

vi. Sketch the spectrum of $\varphi_{LSB}(t)$. [2]

1. This is a general question. (Continued)

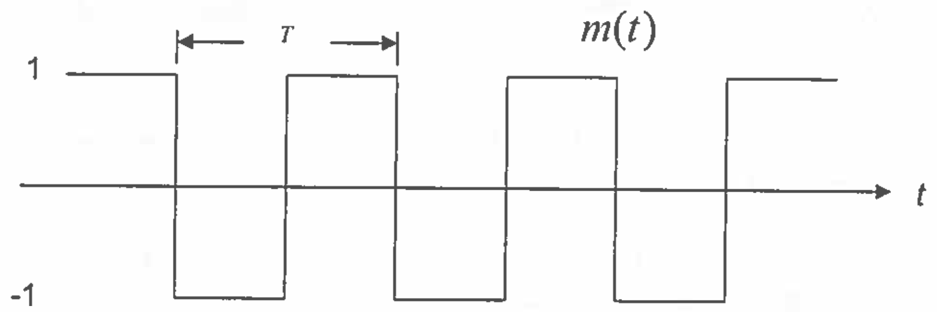
d. Let $\phi(t)$ denote the frequency-modulation (FM) waveform of the modulating signal $m(t)$ with k_f being the proportionality constant for the frequency deviation, and A and ω_c being the amplitude and angular frequency of the carrier (in radians per second), respectively.

i. Give an expression for $\phi(t)$. [2]

ii. Give an expression for the output (denoted by $\phi'(t)$) of an ideal differentiator with $\phi(t)$ as input. [2]

iii. Using the result in part ii, provide a block diagram and explain how the FM signal can be demodulated. [3]

iv. For the modulating signal $m(t)$ given below, sketch the waveform $\phi'(t)$. Give the parameter values associated with the waveform in your diagram. [3]



2. Signals and their transforms. (30%)

a. Consider a linear time-invariant (LTI) system as follows. The unit impulse response function of the system is given by $h(t)=1$ for $0<t<T$ and $h(t)=0$ otherwise, where T is a positive constant. Let $\delta(t)$ denote the unit impulse at $t=0$.

   i. Assume that a signal $x(t)=\delta(t)+\delta(t-T)$ is input to the system. Determine and sketch the output signal $y(t)$ of the system. [3]

   ii. Repeat part i for an input signal of $x(t)=\delta(t)-\delta(t-T)$. [3]

   iii. Repeat part i for an input signal of $x(t)=\delta(t)+\delta(t-T/2)$. [3]

   iv. Now, treat the above linear system as a communication channel. In other words, when a unit impulse $\delta(t)$ is transmitted over the channel, the signal received at the receiving end is $h(t)$. Assume that only periodic unit impulses with positive or negative magnitude can be transmitted to represent 1's or 0's, respectively, as in part ii. Suggest the maximum signal rate in terms of the number of unit impulses per second for transmission over the channel and proper decoding by a simple receiver? Explain your result. [6]

b. Consider two signals, $f(t)$ and $g(t)$, and their respective Fourier transforms $F(\omega)$ and $G(\omega)$. Let $\Im[h(t)]$ denote the Fourier transform of any given function $h(t)$.

   i. Give an expression for the convolution integral of the two signals, $f(t)$ and $g(t)$. Let the convolution integral be denoted by $f(t)*g(t)$. [3]

   ii. By the definition of Fourier transform, show that $\Im[f(t)*g(t)]=F(\omega)G(\omega)$. [5]

   iii. Give an expression for the convolution integral of the two signals, $F(\omega)$ and $G(\omega)$. Let the convolution integral be denoted by $F(\omega)*G(\omega)$. [2]

   iv. Show that $\Im[f(t)g(t)]=(1/2\pi)F(\omega)*G(\omega)$. (Hint: Result in part iii may be useful at the end of the derivation.) [5]

3. Communications techniques. (30%)

a. The purpose of a radio communication system is to transmit and receive audio signals of right and left-hand channels, which are designated as R and L channels, as in the stereo radio broadcast. We focus here on the system design for the baseband signals without considering the modulation scheme for use in the actual transmission over the air. The audio signal of each channel has a bandwidth of 15 kHz and a total bandwidth of 55 kHz is available to carry the composite signals of R and L channels in baseband. A key design requirement is to enable mono-receivers to readily receive the combined signals (i.e., the sum) of R and L channels without the ability to separate them. On the other hand, stereo receivers can receive and separate the signals of the R and L channels. A sinusoidal (pilot) signal of 19 kHz is available for the design.

   i. Draw a block diagram and explain your transmitter design using the aforementioned specifications and parameters. [6]
   ii. Draw a frequency spectrum diagram and explain the composite baseband signals of both the R and L channels in your design. [4]
   iii. Provide a mathematical explanation or physical interpretation of how the pilot signal of 19 kHz is used and serves its purpose in the design. [4]
   iv. Draw a block diagram and explain your stereo receiver design for receiving signals of the R and L channels. [4]

b. Use a noiseless communication channel with a bandwidth of C Hz to transmit periodic samples of an analog signal $m(t)$ at a rate of f_S samples per second.

   i. Ignore the communication channel and just consider the signal sampling at this moment. For a given sampling rate f_S, what is the maximum bandwidth of $m(t)$ so that the signal can be properly recovered from the samples? Let the maximum bandwidth of $m(t)$ be denoted by M. Draw frequency spectrum diagram(s) to explain your result. [6]
   ii. How should C and M be related for successful reception of $m(t)$ over the communication channel? Why? [2]
   iii. What is the maximum value of f_S, which can be supported by the channel? [2]
   iv. If each sample is represented by one of four possible signal levels, what is the maximum bit rate the channel can support? [2]