E4.06
AO9
SO9
ISE4.36

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2006

MSc and EEE/ISE PART IV: MEng and ACGI

OPTICAL COMMUNICATION

Corrected Copy

Wednesday, 17 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 6

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : E.M. Yeatman
Second Marker(s) : A.S. Holmes

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3.0 \times 10^{8}$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x}\tan(k_{1x}d/2) \quad \text{and} \quad \kappa = -k_{1x}\cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]

a) A certain optical receiver has a noise equivalent power of 10 pW/√Hz. Assuming receiver noise dominates, what optical received power will be needed to achieve an optical SNR of 12 if the bit rate is 1 Gbit/s?

b) A certain symmetric slab waveguide supports a single TE mode. If the cladding index of the guide is decreased and all other parameters remain unchanged, will the effective index of this mode increase, decrease or remain the same?

c) A certain symmetric slab waveguide supports three TE modes. If the field amplitude profile is plotted for the highest order mode, how many zero-crossings will it have?

d) A laser diode operating at a nominal wavelength of 1.3 μm has a quantum efficiency $\eta = 1$. What is its slope efficiency?

e) A certain unamplified optical link using a standard p-i-n photodiode detector has a signal-to-noise ratio (SNR) dominated by shot noise. Indicate whether each of the following could improve the SNR: adding an optical amplifier to the link; substituting an avalanche photodiode for the p-i-n diode.

f) Briefly explain, for a fibre supporting several guided modes, why it is not practical to use the modes as different channels carrying separate information.

g) A certain laser transmits pulses of temporal and spectral width $\sigma_o = 0.5\ ns$ and $\sigma_\lambda = 2.0\ nm$ respectively. If these propagate in a fibre of dispersion coefficient $D = 15\ ps/nm{\cdot}km$ at the pulse wavelength, find the fibre length for which dispersion causes the pulse widths to double.

h) Visible light propagates through a sheet of uncoated window glass at approximately normal incidence. If the glass has a refractive index 1.55 in the visible range, estimate the fraction of optical power transmitted.

i) A 5 ns square pulse propagates in standard silica fibre. Approximately how long is it spatially?

j) Name two undesirable effects that can be caused to an optical signal by bending of the fibre in which it is propagating.

2. A symmetric slab waveguide as shown in Fig. 2.1 has a core thickness d = 6 μm, and core and cladding indices of n_1 = 1.48 and n_2 = 1.47 respectively, and supports propagation for a free-space wavelength of 1.50 μm.

a) Find the number of transverse electric (TE) modes supported by the guide. [4]

b) Calculate the effective index for each of the supported TE modes. You may find the plot of Fig 2.2 helpful. [10]

c) For each of the supported modes, determine the distance from the core-cladding interface at which the field amplitude in the cladding has fallen to half its value at the interface. [6]

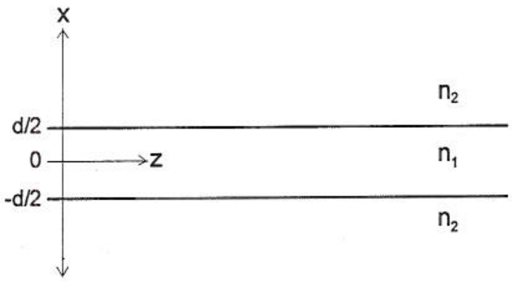

Figure 2.1 Slab waveguide

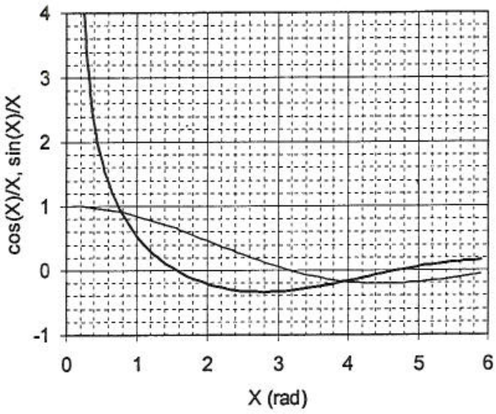

Figure 2.2 cos(X)/X (dark line) and sin(X)/X

3. An optical transmitter couples 10 mW of power, at a nominal wavelength of 1.50 μm, into a fibre having 0.2 dB/km attenuation. The source spectral width is $\sigma_\lambda = 3.0\ nm$ and the fibre dispersion coefficient is $D = 10\ ps/nm{\cdot}km$. The signal is detected by a p-i-n photodiode receiver having a quantum efficiency of 1, and a noise equivalent power of 10 pW/√Hz. An optical signal-to-noise ratio of 12 is required.

a) Calculate the attenuation coefficient α in km^{-1}. [2]

b) Assuming that the achievable bit-rate B is limited only by receiver noise, derive an expression for log(B) in terms of fibre length L, and plot this for a reasonable range of L. [6]

c) Assume instead that B is limited only by the need to keep the dispersion time σ_D below 0.2 bits, derive an expression for log(B) in terms of the fibre length L, and plot this relation on your graph from (b). Hence, determine the length ranges where each effect dominates. [6]

d) Assuming instead that B is limited only by shot noise, derive an expression for log(B) in terms of the fibre length L, and thus show that shot noise is never the limiting factor in this case. [6]

4. A plane wave of TE polarisation is incident on a planar boundary as shown in Fig. 4.1. The indices are $n_1 = 1.50$ and $n_2 = 1.0$. The reflection coefficient in terms of the electric field amplitudes can be given as:

$$\frac{E_r}{E_i} = \frac{k_{ix} - k_{tx}}{k_{ix} + k_{tx}}$$

a) State the boundary condition from which Snell's law can be derived, and show the derivation. [4]

b) Find the critical angle θ_c. [4]

c) Calculate the magnitude of the reflection coefficient, $|E_r/E_i|$, from $\theta_i = 0$ to 90° at 15° intervals and at θ_c. Using this data, sketch $|E_r/E_i|$ vs. θ_i. [6]

d) Calculate the phase change on reflection, ϕ, from $\theta_i = 0$ to 90° at 15° intervals and at θ_c. Using this data, sketch ϕ vs. θ_i. [6]

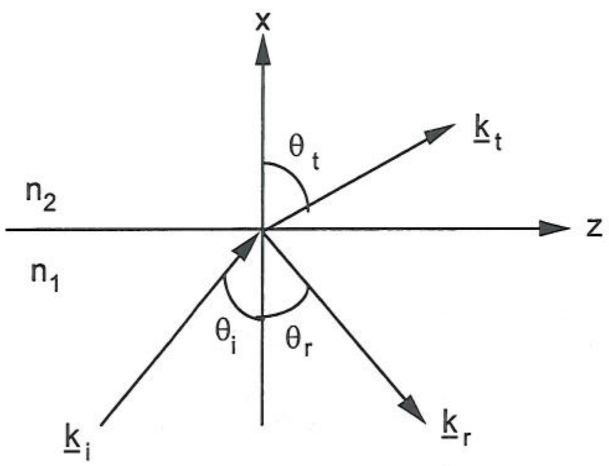

Figure 4.1 Slab waveguide

5. a) Describe the operating principles and structure of an erbium doped fibre amplifier. You should include a schematic diagram of the amplifier indicating its main components, and a energy level diagram for erbium showing the key levels and transitions involved in the amplifier operation. [10]

b) An optical link has 2.5 mW transmitted power, fibre loss of 0.3 dB/km, receiver responsivity of 1 A/W, and the receiver noise is dominated by thermal noise in a 10 kΩ resistor. A certain fibre amplifier has a gain of 25 dB, and a noise figure of 3 dB. Estimate the region of fibre length for which adding the amplifier to the link will improve the signal-to-noise ratio. [10]

6. A silicon p-i-n photodiode has p and n doping levels respectively of $N_A = 2 \times 10^{21}\ m^{-3}$ and $N_D^+ = 10^{21}\ m^{-3}$. The p-layer thickness is 0.5 μm. The attenuation coefficient in Si at the wavelength of interest is given by $\alpha = 0.2 \times 10^6\ m^{-1}$.

a) Find the intrinsic layer thickness w_i such that 80% of photons are absorbed in the intrinsic layer (neglecting Fresnel reflection at the surface). [5]

b) Using w_i as calculated above, find the intrinsic layer doping level N_D^- such that the intrinsic region can be fully depleted by an applied voltage of 4.0 V. [5]

c) Using N_D^- and w_i as calculated above, find the applied bias voltage V such that the electric field amplitude in the intrinsic region varies by 20% (i.e. maximum value is 120% of minimum value). [5]

d) Discuss the main factors to be considered in optimising the electric field magnitude in a p-i-n photodiode. [5]

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Solutions

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A09

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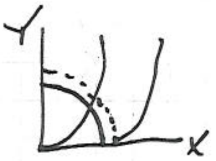
1) a) $SNR = \frac{\Phi_R}{NEP\sqrt{\Delta f}}$, $\Delta f \cong \frac{B}{2} = \frac{1}{2}\times 10^9$

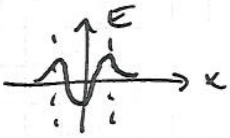
$$\Phi_R = 12\,(10^{-11})\sqrt{.5\times10^9} = \underline{\underline{2.6\ \mu W}}$$

b)



radius of arc increases with NA, thus with Δn. This increases X, thus k_{1x}, which decreases β
∴ 'n' <u>decreases</u>

c) Highest of 3 is $m=2$



$\underline{\underline{2}}$ zero crossings

d) $$S = \eta\frac{hc}{e\lambda} = \frac{1(6.63\times10^{-34})(3\times10^{8})}{1.6\times10^{-19}\times 1.3\times10^{-6}} = 0.956\ \frac{W}{A}$$

e) Neither can improve SNR in this case.

f) - hard to launch them and to separate them at receiver
- modes tend to mix during propagation (e.g. from bending)

g) $\sigma_D = DL\sigma_\lambda$ $\quad \sigma = \sqrt{\sigma_0^2+\sigma_D^2}$

Need $\sigma_D = \sqrt{3}\,\sigma_0$ $\quad L = \frac{\sqrt{3}\,(\cancel{0.5\times10^{-9}}\ 500\,ps)}{15\times 2\,nm} = 28.9\ km$

h) $\frac{P_r}{P_i} = \left(\frac{n_2-n_1}{n_2+n_1}\right)^2 = .046$, so lose 4.6% at each of 2 reflections (front + back).
Neglecting secondary reflections, $\frac{P_T}{P_i} = 90.7\%$

i) $\Delta L = \frac{c}{n}\Delta t \cong \frac{3\times10^8}{1.5}(5\times10^{-9}) = \underline{\underline{1\ m}}$

j) Bending loss, and polarisation mode dispersion.

$\frac{2}{7}$

② a) Cut-off condition for mode m:

$$d = m\lambda_o / 2NA$$

$$m = 2\frac{NA \cdot d}{\lambda_o} = \frac{2\sqrt{1.48^2 - 1.47^2} \cdot 6\times10^{-6}}{1.5\times10^{-6}}$$

$$= 1.37, \quad m_{max} = 1$$

$\underline{\underline{2}}$ modes supported, $m=0$, $m=1$

b) take $X = \frac{k_{ix} d}{2}$ $\quad Y = \frac{Kd}{2}$

Then $X \tan X = Y$, $X \cot X = -Y$

$X^2 + Y^2 = R^2$ with $R = NA \cdot k_o d/2$

Gives $\frac{\cos X}{X} = \pm R^{-1}$ (even modes)

$\frac{\sin X}{X} = \pm R^{-1}$ (odd modes)

here $R = \sqrt{1.48^2 - 1.47^2}\,\pi\,(6/1.5) = 2.152$

$R^{-1} = 0.4647$

From Fig 2.2, $\frac{\cos X}{X} \simeq 0.4647$ at $X \simeq 1.05$

$\sin X / X \simeq 0.4647$ at $X \simeq 2.0$

Then $\underline{\text{refine}}$ these values with calculator to get:

$m=0$: $X = 1.058 = \frac{k_{ix} d}{2}$

$$\frac{k_{ix}}{k_o} = \frac{2\times1.058}{6\times10^{-6}} \times \frac{1.5\times10^{-6}}{2\pi} = 0.08419$$

$$n' = \sqrt{n_1^2 - (k_{ix}/k_o)^2} = \underline{\underline{1.4776}}$$

$m=1$: $X = 1.98 \quad \therefore \frac{k_{ix}}{k_o} = 0.1576$

$$\underline{\underline{n' = 1.4716}}$$

c) Cladding field falls as e^{-Kx}.

$e^{-K\Delta x} = \frac{1}{2}$ for $\Delta x = \ln 2 / K$

$m=0$: $K/k_o = \sqrt{n'^2 - n_2^2} = 0.1497$

$$\Delta x = \frac{(\ln 2)\lambda_o}{2\pi(0.1497)} = \underline{\underline{1.105\ \mu m}}$$

$m=1$ $\quad K/k_o = .0686$, $\Delta x = \underline{\underline{2.42\ \mu m}}$

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(3) a) attenuation in $\frac{dB}{km} = 10 \log e^{-\alpha}$

$$\therefore \alpha = \frac{0.2}{10 \log e} = 0.046 \text{ km}^{-1}$$

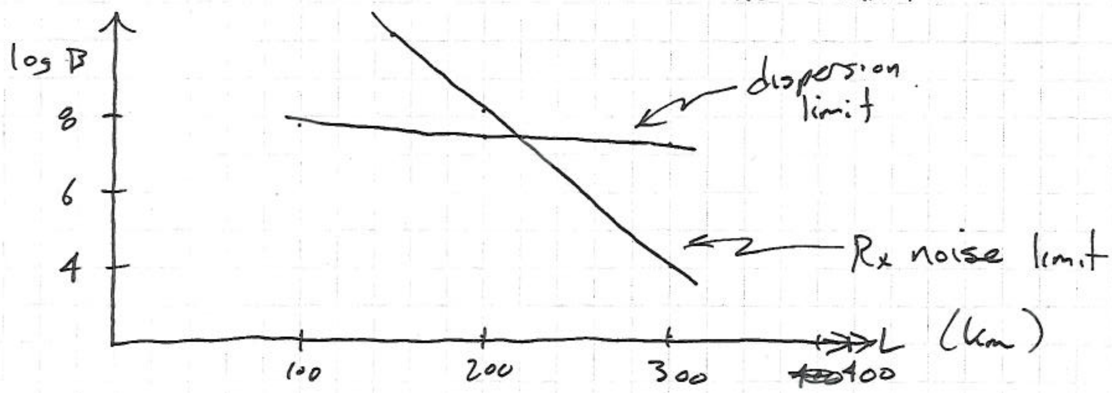
b) $SNR = \frac{\Phi_R}{NEP\sqrt{\Delta f}}$, $\Delta f = \frac{B}{2}$, $\Phi_R = \Phi_T e^{-\alpha L}$

$$\therefore \frac{B}{2} = \left(\frac{\Phi_T e^{-\alpha L}}{NEP \cdot SNR}\right)^2$$

$$\log B = 2 \log\left[\frac{\sqrt{2}\,\Phi_T}{NEP \cdot SNR}\right] - 2\alpha L \log e$$

$$= 2 \log\left[\frac{\sqrt{2} \cdot 10^{-2}}{10^{-11} \cdot 12}\right] - 2 \log e (.046) L$$

$$= 16.1 - 0.04 L$$ } note values above ~10 not realistic



c) $\sigma_D = \sigma_\lambda \cdot D \cdot L = 0.2\left(\frac{1}{B}\right)$

$$B = \frac{0.2}{10^{-11} \cdot 3 \cdot L} = 6.7 \times 10^9 L^{-1}$$

$$\log B = 9.8 - \log L$$

L	log B
100	7.8
200	7.5
300	7.32

Dispersion dominates for $L < 215$ km, receiver noise above.

d) Shot noise: $SNR = \frac{I_{ph}}{\sqrt{2 e I_{ph} \cdot \Delta f}} = \sqrt{\frac{I_{ph}}{eB}}$ $\quad I_{ph} = \frac{e\lambda}{hc}\Phi_T e^{-\alpha L}$

$$\therefore B = \frac{\lambda \Phi_T e^{-\alpha L}}{hc (SNR)^2} = 5.24 \times 10^{14} e^{-\alpha L}$$

$$\log B = 14.7 - \alpha L \log e = 14.7 - .02 L$$

This is above the receiver limit or dispersion limit for any L.

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④ a) $\theta_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\left(\frac{1}{1.5}\right) = \underline{\underline{41.8^\circ}}$

b) k_z must match on boundary, $k_{1z} = k_{2z}$

$k_z = \sqrt{(n_1 k_0)^2 - k_x^2}$

$\therefore\ n_1^2 k_0^2 - k_{1x}^2 = n_2^2 k_0^2 - k_{2x}^2$

$k_x = n k_0 \cos\theta \quad \therefore\ n_1^2 - n_1^2\cos^2\theta_i = n_2^2 - n_2^2\cos^2\theta_t$

$n_1^2 \sin^2\theta_i = n_2^2 \sin^2\theta_t \qquad \underline{\underline{n_1 \sin\theta_i = n_2 \sin\theta_t}}$

c) $\frac{k_{ix}}{k_0} = n_1\cos\theta_i \qquad \frac{k_{tx}}{k_0} = \sqrt{n_2^2 - n_1^2\sin^2\theta_i}$

$E_r/E_i = \frac{k_{ix}/k_0 - k_{tx}/k_0}{\text{''} + \text{''}}, \quad \varphi = -2\tan\left|\frac{k_{tx}}{k_{ix}}\right|$ for $\theta > \theta_c$

We can note that below the critical angle all terms are real so $\varphi = 0$ or π.
Above the critical angle k_{tx} is imaginary, so $\left|\frac{E_r}{E_i}\right| = 1$

θ_i	k_{ix}/k_0	k_{tx}/k_0	$\lvert E_r/E_i \rvert$	φ (deg.)
0	1.5	1	0.2	0
5	1.494	0.991	0.202	0
10	1.477	0.965	0.209	0
15	1.449	0.921	0.222	0
20	1.410	0.858	0.243	0
25	1.359	0.773	0.274	0
30	1.299	0.661	0.325	0
35	1.229	0.510	0.414	0
40	1.149	0.265	0.625	0
45	1.061	j 0.353	1	−37
50	0.964	j 0.516	1	−61
55	0.860	j 0.714	1	−79
60	0.750	j 0.829	1	−96
65	0.634	j 0.921	1	−111
70	0.513	j 0.993	1	−125
75	0.388	j 1.048	1	−139
80	0.261	j 1.087	1	−153
85	0.131	j 1.110	1	−166
90	0	j 1.118	1	−180

d) see above chart, plots overleaf.

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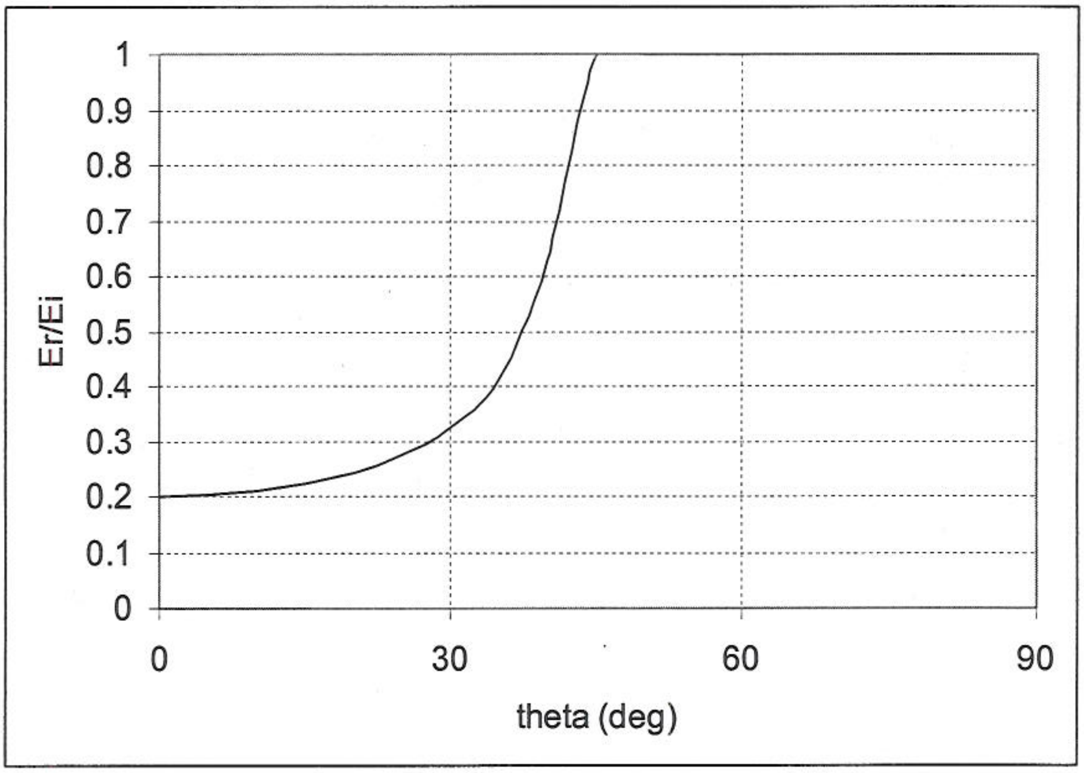
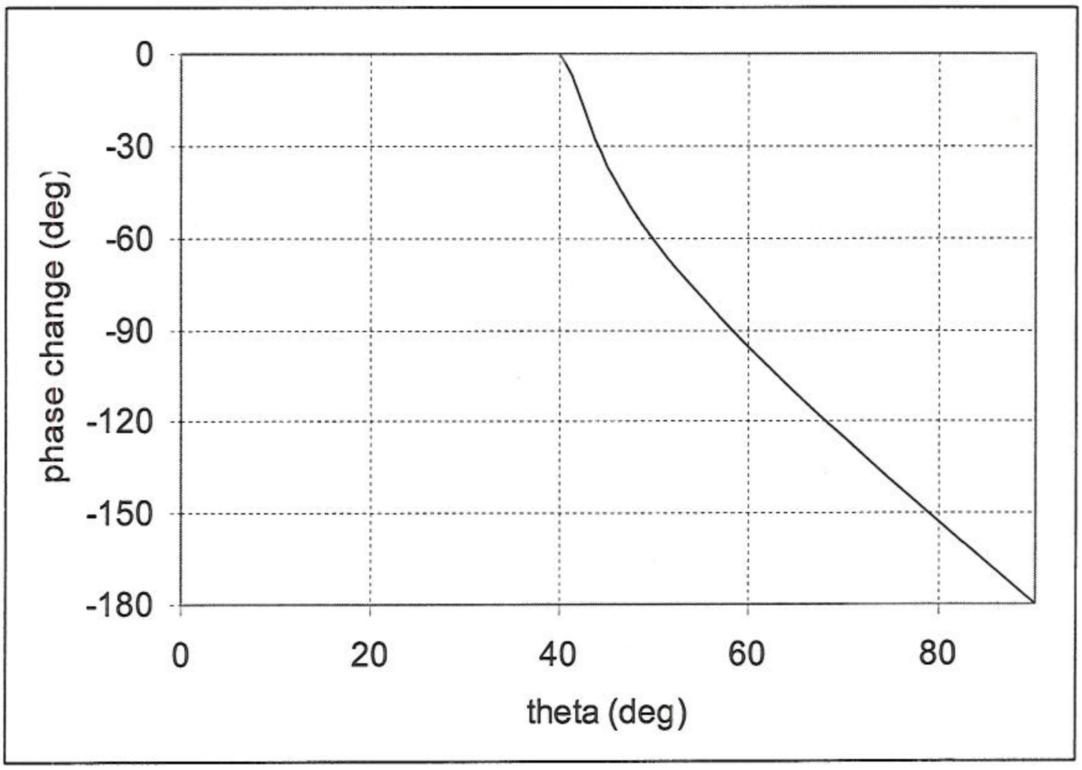


Er/Ei
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
30
60
90
theta (deg)



phase change (deg)
0
-30
-60
-90
-120
-150
-180
0
20
40
60
80
theta (deg)

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Solutions

Q5 a) This is book work, see notes section D, pp 46-47.

b) The optical SNR without the EDFA is given by

$$SNR\cdot\sqrt{\Delta f} = I_{ph} / [2eI_{ph} + 4kT/R]^{1/2}$$

With the amplifier it's approximately:

$$SNR\cdot\sqrt{\Delta f} = GI_{ph} / [2eG^2FI_{ph} + 4kT/R]^{1/2}$$

In this case I have used I_{ph} to indicate the photocurrent if the amplifier was not present, so that it takes the same value as in the previous eqn.

SNR is only improved by the amp if receiver noise dominates over shot noise, i.e. for the longer fibre lengths (lower received power). The minimum length to start to see an advantage will be when the two SNR above are equal. Dividing the latter by the former, squaring, and setting the result equal to 1 we get:

$$G^2[2eI_{ph} + 4kT/R] = [2eG^2FI_{ph} + 4kT/R]$$

$$2eI_{ph}\, G^2(F-1) = (G^2-1)4kT/R$$

Since 25 dB corresponds to $G = 316$, $(G^2-1) \approx G^2$, and 3 dB noise figure gives $F = 2$.

$I_{ph} = kT/eR = 25\text{ mV}/ 10\text{ k}\Omega = 2.5\text{ uA}$. Since responsivity is 1, we need 2.5 uW received power, which is 30 dB less than the transmitted power, so we can get an advantage for fibre lengths above $30/0.3$ = <u>100 km</u>.

6. a) Fraction of captured photons = $\exp(-\alpha x_1) - \exp(\alpha x_2) = 0.8$

Here x1= wp, and x2 = wp + wi.

So $0.8 = \exp(-0.2\times0.5)[1 - \exp(-0.2\times wi)]$ giving wi = 10.8 μm.

b) When the intrinsic layer is just depleted, $E_{max} = -e\, N_D^- w_i/\varepsilon$

And $V = -\frac{1}{2} E_{max}(wi+x)$ where x is the depleted thickness in the p layer, which is given by
$x = N_D^- wi/N_A$

Combining these gives:
$2V\varepsilon = e\, N_D^- w_i^2(1+ N_D^-/N_A)$
$\varepsilon = \varepsilon_r\, \varepsilon_o$
$2V\varepsilon / e\, N_D^- w_i^2 = 4.5\times10^{19}$

$(1/N_A)N_D^{-2} + N_D^- - 4.5\times10^{19} = 0$

Solve by quadratic eqn gives $N_D^- = 4.4\times10^{19}\, m^{-3}$

c) We label the field at top and bottom of the intrinsic layer as E1 and E2. The difference $\Delta E = e\, N_D^- w_i/\varepsilon$, and for a 20% variation E1 = 6 ΔE and E2 = 5 ΔE.

Let us call the depleted lengths in the p and n regions x and y respectively. We can use $N_A ex/\varepsilon = E1$ and $N_D^+ ex/\varepsilon = E2$ to give
$x = 6w_i\, N_D^-/N_A$ and $y = 5w_i\, N_D^-/ N_D^+$.
and $V = \frac{1}{2}\, E1x + \frac{1}{2}\,(E1+E2)\, w_i + \frac{1}{2}E2y = \frac{1}{2}\, E1(x + w_i) + \frac{1}{2}\, E2(y + w_i)$

filling in the expression for E1 and E2 gives

$V = (\frac{1}{2}\, e\, N_D^- w_i^2/\varepsilon)\,(6 + 36\, N_D^-/N_A + 5 + 25\, N_D^-/ N_D^+) = 49.8$ V

d) Bookwork.