Model answer to Q 1(a): Bookwork

a) The constitutive terms for a material can be represented as a quad chart:

i) Draw the quad chart, clearly labelling the axes and the regions in terms of the constitutive terms being greater and less than zero.

[2]

ii) In each quadrant, indicate if the refractive index in real or imaginary and positive or negative.

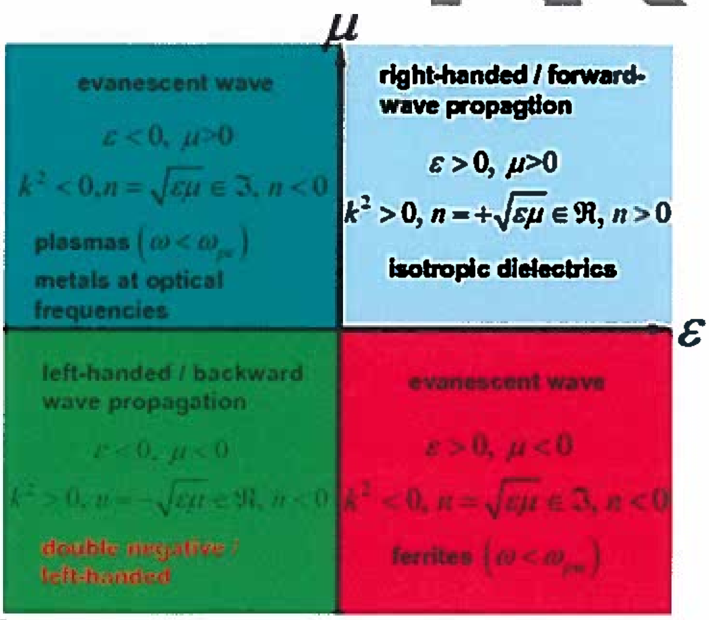
[2]

iii) In each quadrant, indicate if the electromagnetic wave within the material is evanescent or propagating. With the latter, in which direction.

[2]

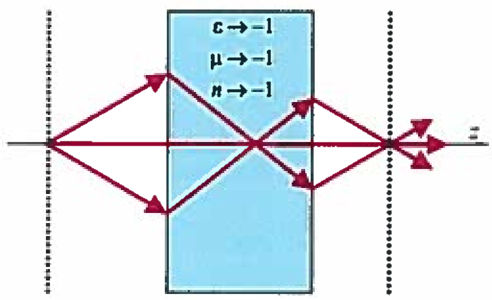
iv) In each quadrant, indicate the type of material that best represents that quadrant.

[2]



Model answer to Q 1(b): Bookwork

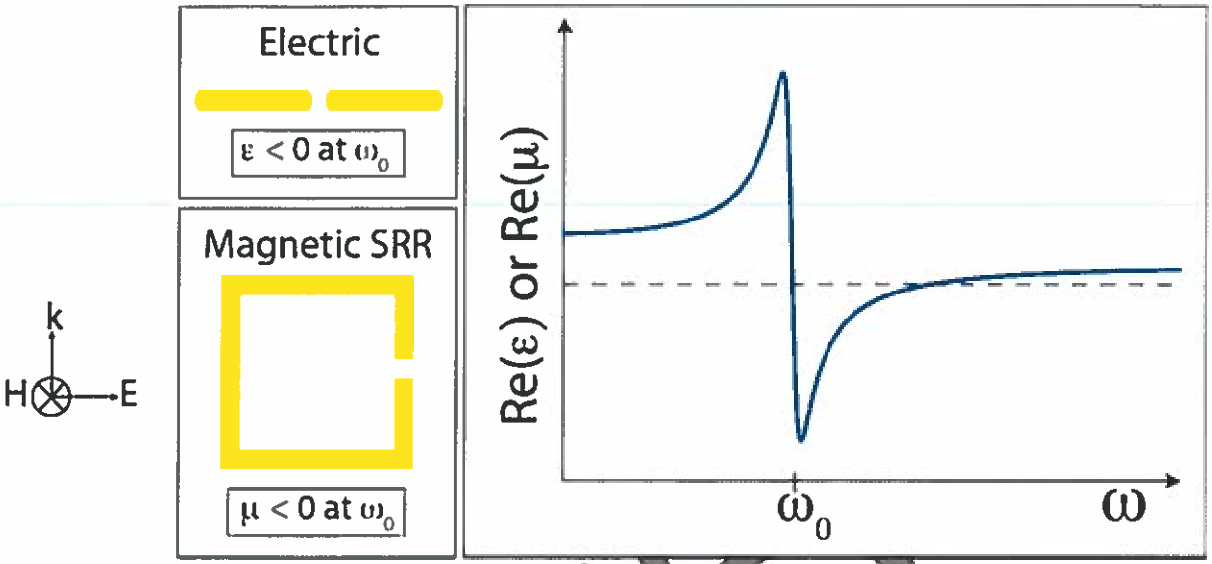
If a material has permittivity and permeability values of minus unity, draw a ray tracing diagram that illustrates how a perfect lens can be made.



[2]

Model answer to Q 1(c): Extended Bookwork

Give one example of how a unit cell can be constructed that result in negative permittivity and permeability values. Sketch the frequency response for each element within the cell and comment on their size.



The resonant electric dipole and magnetic split-ring resonator must be much smaller in size when compared to the wavelength of the electromagnetic wave at resonance.

[4]

Model answer to Q 1(d): New Derivation and Calculated Example

From first principles, and assuming Drude modelling to represent frequency dispersion, derive a simple expression that shows how a non-magnetic metal has a negative permittivity below its plasma frequency. Given a bulk conductivity at DC of 4.1 x 10^7 S/m and phenomenological scattering relaxation time of 27.135 fs, calculate the plasma frequency for gold. From its constitutive terms, at microwave frequencies, can gold be considered as a metamaterial?

EFFECTIVE PERMITTIVITY OF A NORMAL METAL

$$\varepsilon_{eff} = \varepsilon_c - j\frac{\sigma_c}{\omega}$$

INTRINSIC PERMITTIVITY $\varepsilon_c \doteq \varepsilon_0$

INTRINSIC CONDUCTIVITY $\sigma_c = \frac{\sigma_o}{1+j\omega\tau}$ DRUDE MODEL

σ_o = BULK CONDUCTIVITY AT DC

τ = PHENOMENOLOGICAL SCATTERING RELAXATION TIME

$$\therefore\ \varepsilon_{eff} \doteq \varepsilon_0 - j\frac{\sigma_0/\omega}{1+j\omega\tau}$$

$$\therefore\ \varepsilon_{eff} \doteq \left[\varepsilon_0 - \frac{\sigma_0\tau}{1+(\omega\tau)^2}\right] - j\frac{\sigma_0}{\omega[1+(\omega\tau)^2]}$$

$\varepsilon'_{eff} = 0$ AT PLASMA FREQUENCY

$$\therefore\ \varepsilon_0 = \frac{\sigma_0\tau}{1+(\omega_p\tau)^2}$$

$$\therefore\ f_p = \frac{1}{2\pi\tau}\sqrt{\frac{\sigma_0\tau}{\varepsilon_0}-1} = 2{,}079\ \text{THz}$$

The effective permittivity is clearly going to be very negative at microwave frequencies, but since gold is a non-magnetic material with an effective relative permeability very close to unity then it does not have the negative value required to make it a metamaterial. Since gold is one of the elements in the periodic table and has been around since just after the universe was created, it cannot be considered artificial in its bulk form!

[6]

Model answer to Q 2(a): Computed Example

A 10 kΩ.cm high resistivity silicon (HRS) wafer has a measured dielectric constant of 11.64 and loss tangent of 1 x 10^{-4} at 100 GHz.

a) Calculate the bulk effective conductivity and quality factor of the wafer.

$$\varepsilon_r' = 11.64 \quad and \tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'} = 0.0001 \quad \therefore \varepsilon_r'' = 1.164 \times 10^{-3}$$

$$\sigma = \sigma' - j\sigma'' = j\omega\varepsilon_o(\varepsilon_r - 1) \quad and \quad \varepsilon_r = \varepsilon_r' - j\varepsilon_r''$$

$$\therefore \sigma' = \omega\varepsilon_o\varepsilon_r'' \quad and \quad -\sigma'' = \omega\varepsilon_o(\varepsilon_r' - 1)$$

$$\therefore \sigma = 0.0065 - j(-59.19)\ S/m$$

$$Quality\ factor = 1/\tan\delta = 10{,}000$$

[5]

Model answer to Q 2(b): Computed Example

Using the result from (a), how does the real part of conductivity differ from the original value quoted for the high resistivity silicon?

$$\rho_o = 10\ k\Omega \cdot cm = 100\ \Omega \cdot m$$

$$\therefore \sigma_o = \frac{1}{\rho_o} = 0.01\ S/m$$

Therefore, it can be seen that at 100 GHz the conductivity is 65% of the originally quoted value at DC.

[3]

Model answer to Q 2(c): Computed Example

When a plane electromagnetic wave in free space has normal incidence to the HRS, determine a simple expression for the voltage wave reflection coefficient and from this calculate the reflectivity. *Hint: state any simplifying assumption made for the dielectric property of the HRS.*

$$\rho = \frac{\eta - \eta_o}{\eta - \eta_o} \quad where \quad \eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \quad and \quad \eta = \sqrt{\frac{\mu_o\mu_r}{\varepsilon_o\varepsilon_r}} \to \sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r'}}$$

$$\therefore \rho = \frac{1 - \sqrt{\varepsilon_r'}}{1 + \sqrt{\varepsilon_r'}} = -0.547$$

$$\Gamma = |\rho|^2 = 30\%$$

[5]

Model answer to Q 2(d): New Derivation

If the HRS is heavily doped, such that its surface layer behaves like a good conductor, determine an expression for reflectivity, in terms of surface resistance and intrinsic impedance of free space, given that a plane wave in free space has normal incidence. *Hint: the total magnetic field has both forward and backward travelling waves, represented by the following expression:*

$$H(z) = H(0)e^{-\gamma z} + H(0)e^{+\gamma z} \qquad (2.1)$$

All variables have their usual meaning.

$$H(z) = H(0)e^{-\gamma z} + H(0)e^{+\gamma z}$$

$$P_{ABSORBED} = |H(z)|^2_{z=0} R_s = 4|H(0)|^2 R_s$$

$$P_{INCIDENCE} = |H(0)|^2 \eta_o$$

$$\Gamma = \frac{P_{REFLECTED}}{P_{INCIDENCE}} \quad where \quad P_{REFLECTED} = P_{INCIDENCE} - P_{ABSORBED}$$

$$\therefore \Gamma = 1 - \frac{P_{ABSORBED}}{P_{INCIDENCE}} = 1 - 4\frac{R_s}{\eta_o}$$

[5]

Model answer to Q 2(e): Computed Example

Calculate the reflectivity in (d), given a surface impedance of 0.5 Ω.

$$\Gamma = 1 - 4\frac{R_s}{\eta_o} = 1 - 4\frac{0.5}{120\pi} = 98.94\%$$

[2]

Model answer to Q 3(a): Bookwork

a) Very briefly:

i) Explain why electromagnetic cavity resonators are preferred when compared to lumped-element resonators.

Electromagnetic cavity resonators having electrically conducting walls have been exploited for well over seven decades, because of their ability to produce sharp spectral resonances – much sharper than their lumped-element counterparts. Their ability to store electromagnetic energy with very low dissipative losses, from sub-microwave to optical frequencies, has made them an essential component in many systems.

[2]

ii) Explain why spherical cavity resonators are preferred when compared to either rectangular or cylindrical.

Spherical cavity resonators have the highest quality factor, when compared to rectangular or cylindrical cavities when made with the same wall material.

[2]

iii) Give at least three examples of where lossy resonances can be found.

In general, it is highly desirable to have low-loss electromagnetic resonators (e.g., to implement high-performance impedance matching networks and filters). However, there are many examples where lossy resonant structures exist; requiring a more rigorous approach to the modeling of their behavior. Examples include: (1) surface plasmon polaritons, found in nature and engineered; (2) heavy time-domain damping of unwanted resonances (e.g., in DC biasing networks) and even control systems; (3) implementation of ultra wideband frequency-domain networks (e.g. antennas, phase shifters and filters found in ultra-high speed telecommunications and radar systems); (4) unexpected box-mode resonances with low-cost plastic/organic packaging of high frequency devices; and (5) certain metamaterial and graphene structures.

[2]

iv) State the use and limitation of perturbation techniques.

Low loss performance with metal-walled cavities is achieved with the use of "good electrical conductors". Fortunately, in this case, the analysis can be greatly simplified with the use of approximate solutions (e.g., traditional perturbation techniques), but their validity is limited to low losses.

[2]

Model answer to Q 3(b): Bookwork Derivation and Calculated Example

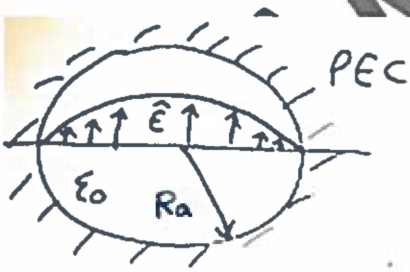
b) Given an air-filled spherical cavity resonator with radius R_a and perfect electrical conducting wall:

i) State the exact dominant mode for this cavity resonator and the associated non-zero field components.

With a dielectric-filled spherical cavity resonator having a PEC wall, the dominant mode is the transvers magnetic TM_{011} (i.e., $m = 0$, $n = p = 1$, where p is associated with the variations along the radial direction, where r is the radial coordinate), where the only non-zero field components are E_r, E_θ and H_φ.

[2]

With the use of a simple sketch that shows the boundary conditions for the electric field distribution, derive a very approximate expression for the ideal resonance frequency of the dominant mode.



$$R_a \sim \lambda_I/4$$

$$\beta_I = \frac{2\pi}{\lambda_I} \sim \frac{\pi}{2R_a}$$

$$\therefore \beta_I R_a \sim \pi/2$$

$$f_I = \frac{\omega_I}{2\pi} = \frac{\beta_I c}{2\pi}$$

[2]

With a 300 µm diameter sphere, calculate the approximate ideal resonance frequency of the dominant mode.

With a 300 µm diameter sphere, the ideal resonance frequency ~500 GHz.

[2]

Model answer to Q 3(c): New Derivation with Calculated Examples

c) If the conducting wall is made using a real conductor that has loss:

i) Define the eigenfrequency in terms of its damped (or undriven) resonance frequency, napier frequency and undamped (or driven) resonance frequencies.

EIGENFREQ. $\hat{f}_0 = f_0' + j f_0''$

f_0': DAMPED RESONANCE FREQUENCY; f_0'': NAPIER FREQUENCY

UNDAMPED RESONANCE FREQUENCY $f_0 = |\hat{f}_0|$

$$\therefore f_0 = \sqrt{(f_0')^2 + (f_0'')^2}$$

[2]

Give expressions for the unloaded quality factor at the damped and undamped resonance frequency. In addition, derive an expression that relates both quality factors to each other.

$$Q_U(f_0) = \frac{\omega_0}{2\omega_0''} \quad ; \quad Q_U(f_0') = \frac{\omega_0'}{2\omega_0''}$$

$$\omega_0^2 = (\omega_0')^2 + (\omega_0'')^2$$

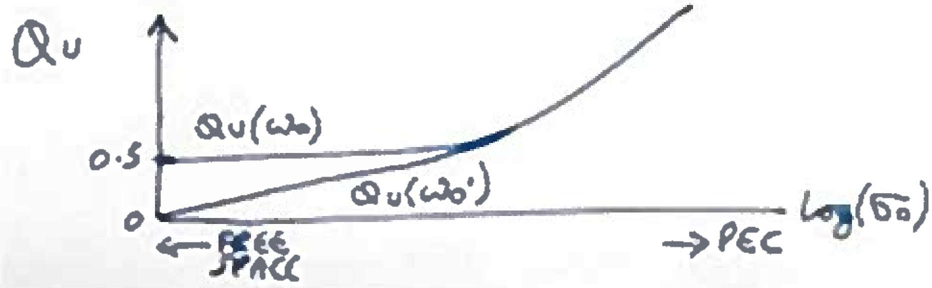
$$\therefore \frac{\omega_0^2}{(2\omega_0'')^2} = \frac{(\omega_0')^2 + (\omega_0'')^2}{(2\omega_0'')^2}$$

$$\therefore Q_U(\omega_0)^2 = Q_U(\omega_0')^2 + \left(\frac{1}{2}\right)^2$$

[2]

Sketch the unloaded quality factors at the damped and undamped resonance frequencies of an infinitely thick cavity resonator, as the conductivity decreases from infinity (giving a perfect electrical conducting wall) to zero (giving free space).

$$\text{IF} \begin{cases} Q_U(\omega_0) \to \infty & ; \quad Q_U(\omega_0') \to \infty \\ Q_U(\omega_0) = \frac{1}{\sqrt{2}} & ; \quad Q_U(\omega_0') = \frac{1}{2} \\ Q_U(\omega_0) = \frac{1}{2} & ; \quad Q_U(\omega_0') = 0 \end{cases}$$



[2]

Model answer to Q 4(a): Bookwork Derivation and Calculated Example

You have been given a quartz crystal sphere, having a radius of $R_a = 150$ µm and dielectric constant of $\varepsilon_r = 3.8$.

a) If the sphere is plated with a perfect electrical conductor (PEC), the wavenumber β_1 is given by the following:

$$\beta_1 R_a = 2.74370$$

Calculate the ideal resonance frequency f_I for the resonator.

$$\beta_I = \frac{\omega_I}{V_p} = 18,291\cdot 3 \text{ rad/s}$$

$$V_p = \frac{c}{\sqrt{\varepsilon_r}} = 153\cdot 9\times 10^6 \text{ m/s}$$

$$f_I = \frac{\omega_I}{2\pi} = 448 \text{ GHz}$$

[3]

Model answer to Q 4(b): Bookwork Derivation and Calculated Example

b) Calculate the geometrical factor Γ for a spherical cavity, given the following:

$$\Gamma = \frac{\mu_0}{\xi}\left(\frac{V}{S}\right); \quad \xi \cong 0.90790$$

where V is the internal cavity volume and S is the inner surface area of the wall.

$$V = 4\pi R_a^3/3$$

$$S = 4\pi R_a^2$$

$$\therefore \left(\frac{V}{S}\right) = \frac{R_a}{3}$$

$$\therefore \Gamma = 69\cdot 2 \text{ pH}$$

[3]

Model answer to Q 4(c): Bookwork Derivation and Calculated Example

If the PEC is replaced by a real metal having loss, the undamped resonance frequency ω_0 can be found using perturbation theory, by solving the following:

$$X_0(\omega_0) - 2(\omega_I - \omega_0)\Gamma = 0$$

Where $X_0(\omega_0)$ is the classical skin-effect model surface reactance and the other variable have their usual meaning. If the infinitely thick lossy metal wall results in 10% detuning, calculate:

i) The undamped resonance frequency and classical skin-effect model surface reactance.

$$10\%\ \text{DETUNING GIVES}\ \omega_0 = 0.9\,\omega_I$$

$$\therefore f_0 = 403.2\ \text{GHz}$$

$$X_0(\omega_0) = 2(\omega_I - \omega_0)\Gamma = 0.2\,\omega_I\Gamma = 39\,\Omega$$

[3]

ii) The conductivity of the wall material. What material could the wall be made from?

$$R_0(\omega_0) = X_0(\omega_0) = \sqrt{\frac{\omega_0\mu_0}{2\sigma_0}}$$

$$\therefore \sigma_0 = \frac{\omega_0\mu_0}{2X_0(\omega_0)^2} = 1{,}047\ \text{S/m}$$

THIS IS COMMENSURATE WITH CARBON

[3]

iii) The unloaded quality factor at the undamped resonance frequency using the extended perturbation theory.

$$Q_u(\omega_0) \sim \frac{\omega_0\Gamma}{R_0(\omega_0)} = \frac{175.3}{39} = 4.5$$

[3]

iv) The complex eigenfrequency.

$$\tilde{f}_0 = f_0\left[\sqrt{1-\left(\frac{1}{2Q_u(\omega_0)}\right)^2} + j\,\frac{1}{2Q_u(\omega_0)}\right]$$

$$\therefore f_0' = 398.2\ \text{GHz}$$

$$f_0'' = 44.8\ \text{GHz}$$

[3]

v) The unloaded quality factor at the damped resonance frequency.

$$Q_u(\omega_0') = \frac{\omega_0'}{2\omega_0''} \sim 4.44$$

[2]

Model answer to Q 5(a): Bookwork

A simple low frequency microwave FET amplifier can be realised by employing conventional microstrip transmission lines for its interconnects, impedance matching and biasing networks.

a) Describe the main problem with conventional microstrip circuits when trying to ground the source connection of a FET. Give two examples of catastrophic circuit failure. Also, state how a conventional microstrip circuit can be modified to reduce this problem.

The main problem with conventional microstrip circuits, when trying to ground the source connection of a FET, is that the through-substrate metal-plated via connection has both parasitic inductance and resistance. This can prevent an oscillator from oscillating or cause amplifiers to oscillate. Both these examples represent catastrophic circuit failure.

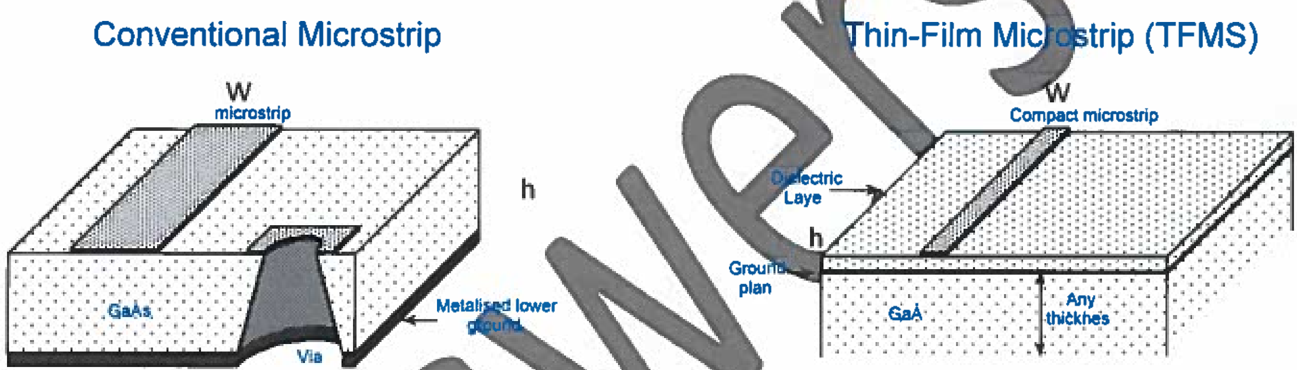
A conventional microstrip circuit can be modified to reduce this problem by turning it into thin-film microstrip.

[3]

Model answer to Q 5(b): Bookwork

Compare and contrast the manufacture of TFMS with conventional microstrip. Use suitable illustrations, showing variables for the main cross-sectional dimensions.

When compared to conventional microstrip, the ground plane is brought up to the top of the substrate. A thin layer of non-conductor defines the dielectric that separates the ground plane from the main signal line. The dielectric layer can be much thinner for TFMS than with conventional microstrip, and easily deposited using a spin-on or lamination bonding process



[4]

Model answer to Q 5(c): Bookwork

Write simple equations, to a first-order approximation, to justify each of the following claims:

The characteristic impedances of a TFMS and conventional microstrip line can be made equal.

i) The characteristic impedances Zo of a TFMS and conventional microstrip line can be the same, since $Zo \propto \left(\frac{h}{W_{effective}}\right)$. Therefore, as long as this ratio stays the same then characteristic impedance can stay the same. This means that W will be much narrower with TFMS than with conventional microstrip.

[3]

Losses in a TFMS are higher than in a corresponding conventional microstrip line.

i) losses in a TFMS is higher than a corresponding conventional microstrip line. The reason is that power dissipated as heat is defined by: $P_{DISSIPATED} = |Js|^2 Rs$, where Js is the surface impedance and Rs is the surface resistance. Since Js is directly proportional to the conduction current density $Jc(0)$ [A/m^2], it follows that as the cross-sectional area of the microstrip's signal track is much smaller with TFMS then $Jc(0)$, Js and $P_{DISSIPATED}$ are all much greater.

[4]

Model answer to Q 5(d): Bookwork

Figure 5.1 shows the photograph of a MMIC amplifier, employing TFMS transmission lines. By inspection, draw the equivalent circuit model of this amplifier and state the role of the two longest TFMS transmission lines. What kind of amplifier topology is this?

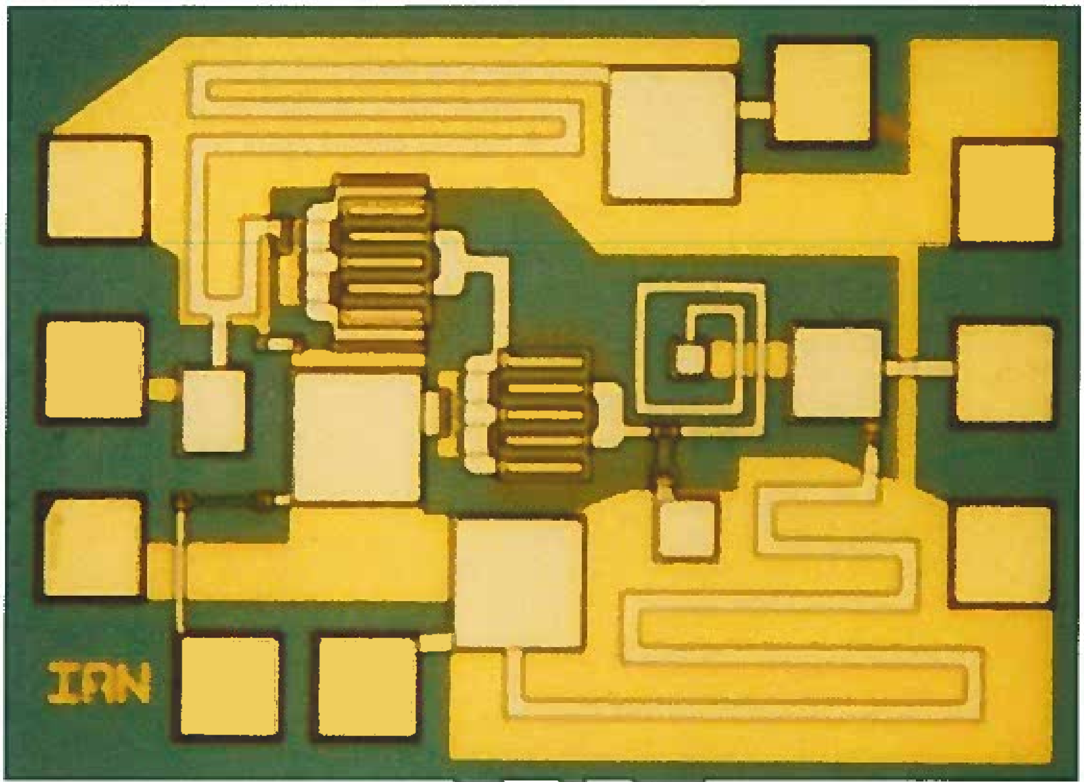
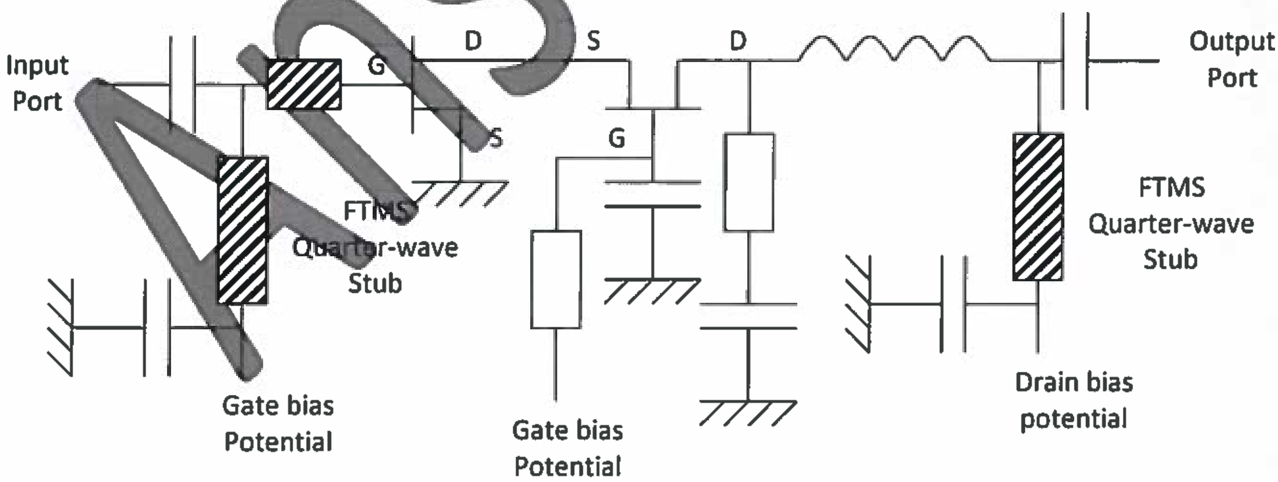


Figure 5.1



The two largest TFMS transmission lines are used as quarter-wave stubs, which act as DC bias chokes. The amplifier has a cascode topology.

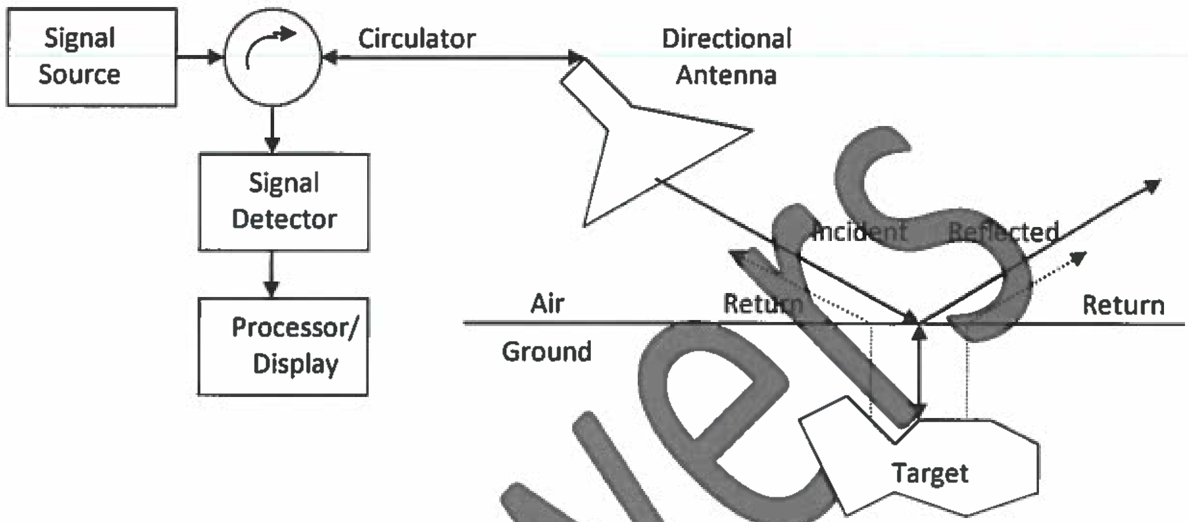
[6]

Model answer to Q 6(a): Bookwork and Computed Example

With conventional ground-penetrating radars (GPRs):

i) With the aid of a diagram, describe the principle of operation.

A pulsed or FMCW (chirp) microwave signal is transmitted. Because the incident signal is at an angle with the ground, the resulting reflected wave is directed away from the antenna. The relatively high permittivity of the ground means that the penetrating wave is refracted towards the normal. A target will then reflect a proportion of the incident signal back to the antenna.



[5]

Briefly discuss the main problems associated with soil composition.

If the water content of the ground is high then the resulting high conductivity (σ) of the ground will limit the depth at which the microwave signal can penetrate. For example, at one skin depth (δ) the amplitude of the wave decreases from its surface value to 37%. Since $\delta = \sqrt{\frac{2}{\mu\omega\sigma'}}$, if the conductivity of the soil is doubled then the operating frequency (ω) must be halved, to achieve the same value of δ. [The student should also comment on the effects of multiple layers, having different electrical properties, as this will create multiple-reflections.]

[5]

If the dynamic range of a GPR is 120 dB and at 10 GHz the attenuation of the sand is 10 dB/m, calculate the maximum depth the radar can detect an ideal target. Comment on the result of this depth if the sand is wet. What assumption has to be made with respect to impedance mismatching?

Depth = Dynamic Range/ (2 Attenuation) = 5 m
If the sand is wet, its conductivity increases. As a result, the radar is not much use at this frequency. The frequency should be reduced by a few orders of magnitude in order to achieve the same depth.

[5]

Briefly discuss the main problems associated with a non-ideal buried target.

The wavelength of the microwave signal needs to be smaller than the size of the target, in order to achieve adequate resolution. Therefore, the size of the target sets a limit on the lowest frequency. Also, the target may not reflect much of the incident signal.

[5]