EE4-57
EE9-CS2-2

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

MSc and EEE/EIE PART IV: MEng and ACGI

DISCRETE-EVENT SYSTEMS

Wednesday, 8 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	D. Angeli
	Second Marker(s) :	E.C. Kerrigan

1. A store has a maximum capacity of 6 boxes; boxes are dispatched to the store in packets of 2 or 3 (events p_2 and p_3, respectively), whereas they are delivered from the store one at a time (event d).

   a) Build a finite deterministic automaton G that models the store and its arrivals and departures of boxes (hint: use as a state space X the integer interval[1] $[0,6]$); [4]

   b) Assume next that events p_2 and p_3 are only partially observable, namely that available sensors are only able to detect arrival of packets (event p) but cannot count how many boxes are coming in. Build a finite non-deterministic automaton G_N to model the store under partial observations; [2]

   c) Let f_N denote the transition map of G_N. Prove that for any integer interval $[i,j]$ it is true that $f_N([i,j],p)$ and $f_N([i,j],d)$ are integer intervals. Write their expressions (as a function of i and j); [4]

   d) Build an observer automaton for G_N in order to estimate the number of boxes in the store after any given admissible sequence of p and d events; [8]

   e) Give an argument to support the statement that the maximum uncertainty at any given time is ± 2 boxes. [2]

[1] An integer interval $[i, i+k]$ is a set of the following type: $\{i, i+1, i+2, \ldots, i+k\}$.

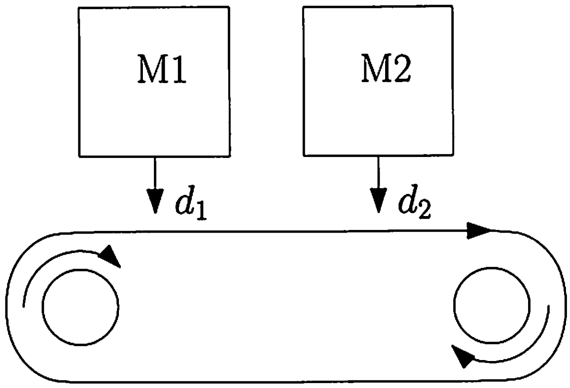

Figure 2.1 A schematic diagram of the belt

2. A factory operates with two machines M_1 and M_2. The machines drop finished pieces on a belt at two different locations, say locations 1 and 2, respectively, and these correspond to events d_1 and d_2. Location 1 is upstream with respect to location 2 by 1 unit of space, where the belt is assumed to move by one unit of space at each 'tick' of a clock (event t). See Fig. 2.1.

a) Build a finite deterministic automaton G_B that models the occupancy of locations 1 and 2 as events d_1, d_2 and t occur. Include an overflow state for the situation in which a piece is dropped at some location where another piece is already present; [5]

b) Assume next that machines M_1 and M_2 take at least 1 unit of time in order to make a new piece. Model each machine M_i by a finite deterministic automaton G_i on alphabet $E_i = \{t, d_i\}$, which represents such a timing constraint; [3]

c) Build a model of the factory by concurrent composition $G_1||G_2||G_B$; [3]

d) Consider next the following specification: "the overflow state is forbidden"; is this specification controllable when the set of uncontrollable events is $E_{uc} = \{d_2, t\}$? Why ? [2]

e) Design an admissible supervisor for the previous specification in the case of $E_{uc} = \{d_1, t\}$; [3]

f) Design a supervisor to achieve the maximal supremal controllable sublanguage in case of $E_{uc} = \{d_2, t\}$; [4]

3. Two transmitters are sharing a channel according to the following protocol:

- Each transmitter has 2 states, an idle state (in which it waits to occupy the channel) and a busy state (in which it starts transmitting and occupying the channel).
- When in the idle state, it waits a random exponentially distributed amount of time with rate λ, which is to be used later as a design parameter;
- When in the busy state it transmits at a some fixed bit-rate (equal for the two transmitters) and the length of packets is exponentially distributed so that transmission times are independent random variables with exponential distribution of rate 1.
- When transmission is completed each transmitter instantaneously enters the idle state. Collisions are also instantaneously detected and lead to a fault in transmission, as well as instantaneous transition of both transmitters to the idle state.

a) Model the protocol described above with a continuous-time homogeneous Markov Chain M (hint: notice that the state in which both transmitters are busy is not needed as transition to the idle state is instantaneous in such a case); [7]

b) Is M an ergodic Markov Chain ? Why ? [2]

c) Compute the steady-state probability distribution as a function of λ; [4]

d) Compute the fraction of time the channel is busy; [1]

e) Compute F, the fraction of time the channel is busy during a transmission that is not interrupted by collisions; [4]

f) Show that there is an optimal value of λ in order to maximize F and compute its value; [2]

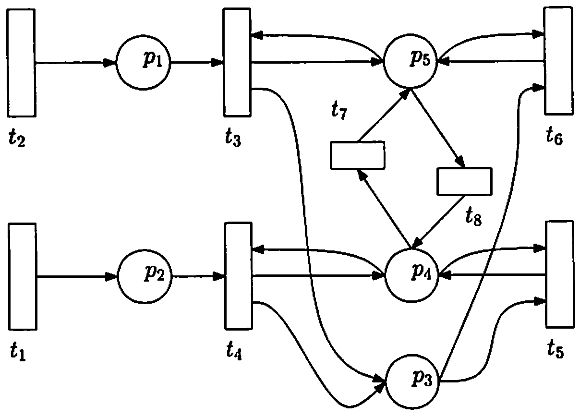

Figure 4.1 Petri Net modeling the elevator

4. The Petri Net in Fig. 4.1 models an elevator operating in a building with 2 floors. In particular, it keeps track of (i) people waiting at their floor for the elevator to come, (ii) people entering and exiting from the elevator and (iii) the floor at which the elevator is currently located.

   a) Explain the physical meaning of each place and transition in the Petri Net; [3]

   b) Write the incidence matrix C associated with the Net; [4]

   c) Find the P-semiflows associated with the Petri Net; explain, in the context of this example, why P-semiflows are of the form just computed; [3]

   d) How could you modify the net if you wanted to add a constraint on the maximum number of people allowed in the elevator ? [3]

   e) Sketch the coverability graph associated with the Petri Net assuming the following initial marking: $M_0 = [0,0,0,0,1]$; [5]

   f) Which places are behaviourally bounded, given the initial marking M_0 ? Are they structurally bounded ? Why ? [2]

SOLUTIONS: DISCRETE EVENT SYSTEMS
MASTER IN CONTROL

1. Exercise

a) The automaton G has an event set $E = \{p_2, p_3, d\}$ and a state-space $X = \{0,1,2,3,4,5,6\}$. Its transition diagram is shown in Fig. 1.1; [4]

b) In order to model G_N it is enough to replace p_2 and p_3 events by p events, as sketched in Fig. 1.2; [2]

c) When an event d occurs the number of boxes decreases by 1; hence, for all $x \geq 1$, $f_N(x,d) = x-1$. As a consequence:

$$f_N([i,j],d) = [\max\{0,i-1\}, \max\{0,j-1\}].$$

When an event p occurs, either 2 or 3 boxes gets added to x. Notice that $x+2$ and $x+3$ are both contiguous integers, so if an interval $[i,j]$ is considered, the union $\bigcup_{x\in[i,j]} f_N(x,p)$ will still be an integer interval. In particular it holds:

$$f_N([i,j],p) = [\min\{i+2,6\}, \min\{j+3,6\}].$$

[4]

d) The transition diagram of the observer automaton is shown in Fig. 1.3; (2 points for correct identification of state-space, 3 points for sound application of algorithm, 3 points for complete observer automaton) [8]

e) It can be seen from Fig. 1.3 that the integer intervals of maximum amplitude are of the type $[i-2, i+2]$ for $i \in \{2,3,4\}$. Hence ± 2 is the maximum uncertainty in the number of boxes currently stored. [2]

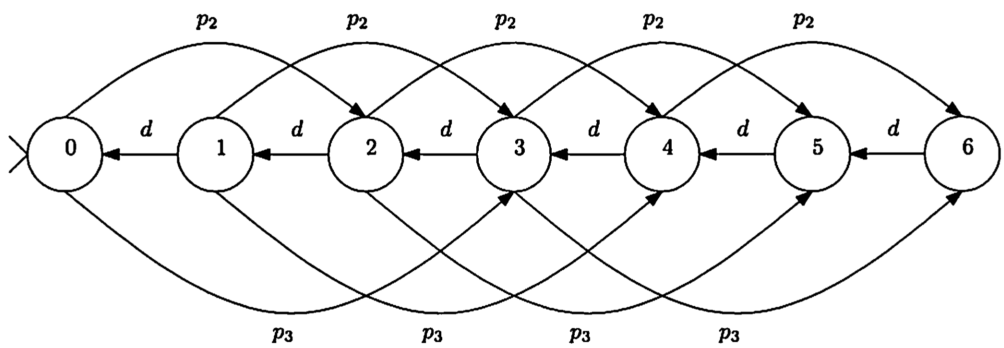

Figure 1.1 Transition diagram of automaton G

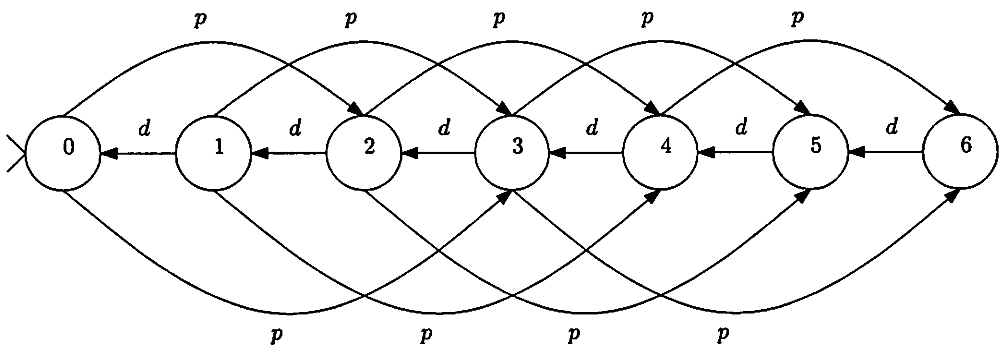

Figure 1.2 Transition diagram of automaton G_N

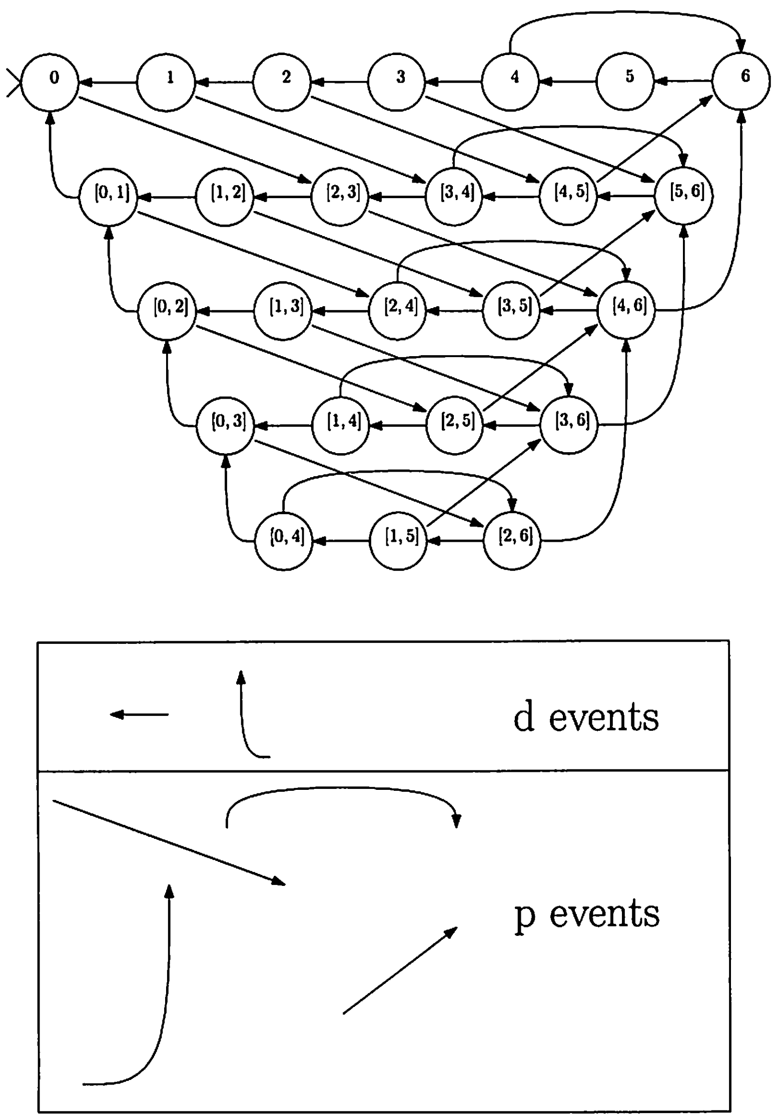

Figure 1.3 The observer automaton $Obs(G_N)$

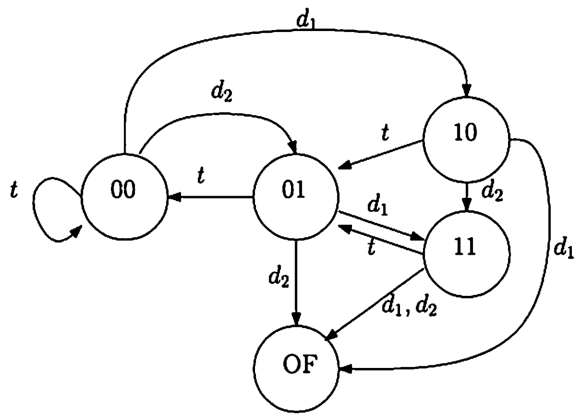

Figure 2.1 The automaton G_B

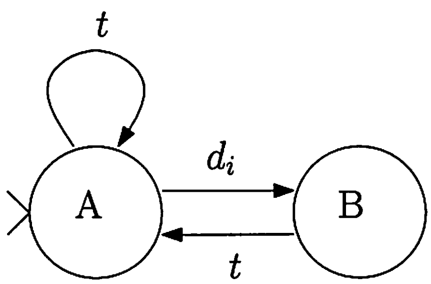

Figure 2.2 The automaton G_i, $i = 1,2$

2. Exercise

a) We include 4 states in the automaton, $\{00, 01, 10, 11\}$ which denote the occupancy (with a 1) or emptyness (with a 0) of locations 1 and 2 respectively. Then, events t simply implements a shift towards the right of the sequence of 2 bits, with a 0 coming in from the left. Events of type d_1 trigger the commutation of the first 0 to 1, or entering an overflow state if such a bit is already a 1. Similarly, events of type d_2 trigger the commutation of the second bit from 0 to 1, or entering the overflow state if the bit is already a 1. The corresponding finite deterministic automaton G_B is sketched in Fig. 2.1.
(2 points for correct state-space, 3 points for correct transition diagram) [5]

b) The automaton representing machine M_i, $i = 1,2$ is sketched in Fig. 2.2.

[3]

c) The model of the factory is obtained by parallel composition $G_1||G_2||G_B$ (see Fig. 2.3). [3]

d) The specification of forbidden overflows is represented by the marked language of the automaton obtained by removing from $G_1||G_2||G_B$ all states beginning with OF (see Fig. 2.4); this specification is not controllable as event d_2 needs to be disabled in states $01AA$, $11BA$ but d_2 belongs to E_{uc}. [2]

e) The specification automaton in Fig. 2.4 also represents an admissible supervisor when $E_{uc} = \{t, d_1\}$ (as it only disables d_2 events). [3]

f) The admissible supervisor S such that $\mathscr{L}(S/G)$ is the supremal controllable sublanguage with respect to the forbidden state specification in the case of

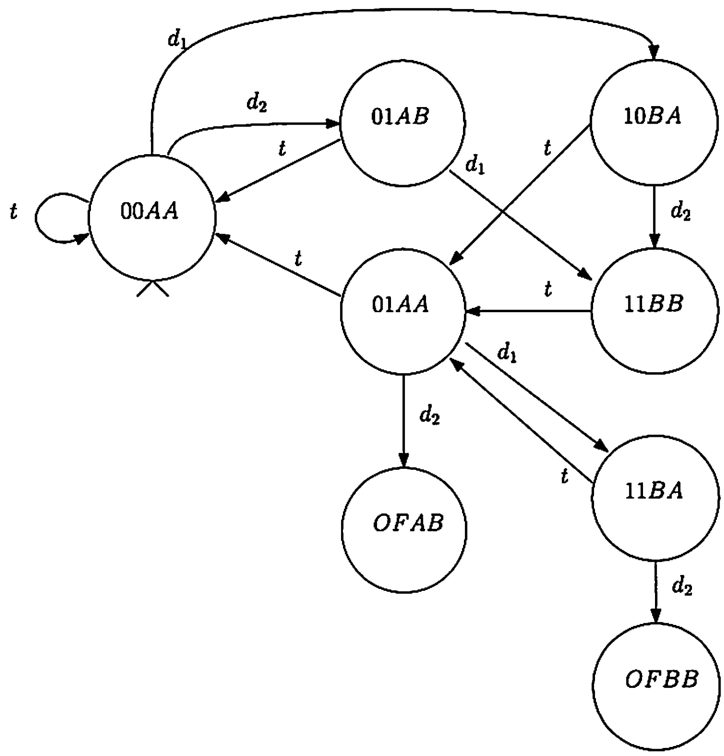

Figure 2.3 The automaton $G_1||G_2||G_B$

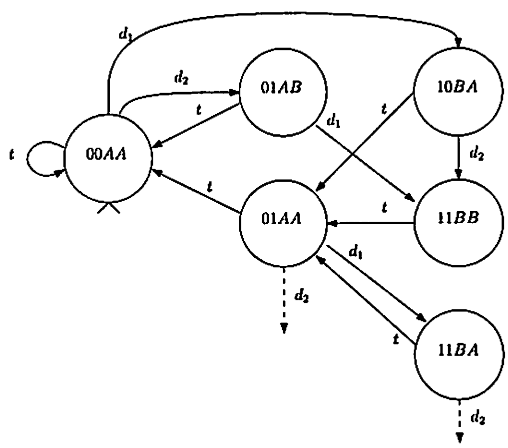

Figure 2.4 The specification automaton

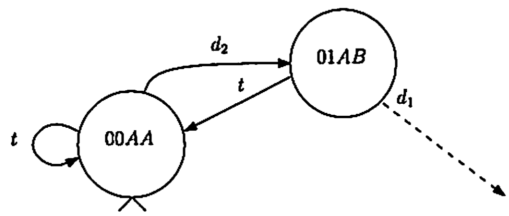

Figure 2.5 The supervisor for the supremal controllable sublanguage

$E_{uc} = \{d_2, t\}$ is shown in Fig. 2.5; notice that this supervisor only disables d_1 events. [4]

3. Exercise

a) The transition diagram of the Markov Chain is shown in Fig. 3.1. Notice that multiple edges link the states BI and IB to II; this is so in order to model the fact that transition to the idle state may occur due to a collision (λ arc) or due to a successfull transmission (1 arc). (3 points for correct state-space, 4 points for correct transition rates) [7]

b) The Markov chain is ergodic since its transition diagram is strongly connected. [2]

c) Letting $\pi = [\pi_{II}, \pi_{BI}, \pi_{IB}]$ we have $\dot{\pi} = \pi Q$ with:

$$Q = \begin{bmatrix} -2\lambda & \lambda & \lambda \\ 1+\lambda & -1-\lambda & 0 \\ 1+\lambda & 0 & -1-\lambda \end{bmatrix}.$$

The associated steady-state probability distribution fulfills the equations:

$$\pi_\infty Q = 0 \qquad \pi_\infty \mathbf{1} = 1.$$

After some computations we see that:

$$\pi_\infty = \left[\frac{2}{9\lambda+3} + \frac{1}{3}, \frac{1}{3} - \frac{1}{9\lambda+3}, \frac{1}{3} - \frac{1}{9\lambda+3}\right].$$

[4]

d) The fraction of time the channel is busy is simply given by:

$$\frac{2}{3} - \frac{2}{9\lambda+3},$$

viz. the time spent in states BI and IB altogether. [1]

e) Notice that whenever the system is in state BI, the probability of it completing successfully a transmission is given by $1/(1+\lambda)$; a similar argument applies to the state IB. Hence, the fraction of busy time spent in transmission which lead to a successfull termination is given by:

$$F(\lambda) = \left(\frac{2}{3} - \frac{2}{9\lambda+3}\right)\frac{1}{1+\lambda}.$$

[4]

f) Notice that $F(0) = 0$ and $\lim_{\lambda\to+\infty} F(\lambda) = 0$; taking derivatives with respect to λ and considering only $\lambda \geq 0$ we see that:

$$F'(\lambda) = 0 \text{ iff } \lambda = \frac{1}{\sqrt{3}}.$$

This is a global maximum. [2]

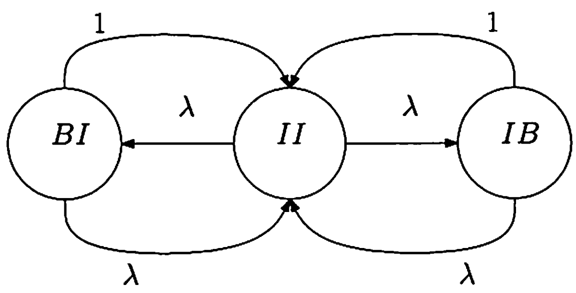

Figure 3.1 Transition diagram of M

4. Exercise

a) The places have the following meaning: p_1 "number of people waiting for the elevator at the first floor", p_2 "number of people waiting at the second floor" (or viceversa); p_5 "is the elevator at the first floor ?" (1 yes, 0 no) p_4 "is the elevator at the second floor ?", p_3 "number of people in the elevator". The transitions have the following meaning: t_2 arrivals at the first floor, t_1 arrivals at the second floor, t_3 people entering the elevator at the first floor, t_4 people entering the elevator at the second floor, t_5 people leaving the elevator at the second floor, t_6 people leaving the elevator at the first floor, t_7 elevator traveling from second to first floor, t_8 elevator traveling from first to second floor. [3]

b) The incidence matrix is as follows:

$$C = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}.$$

[4]

c) There is only one P-semiflow: $[0,0,0,1,1]$. Places p_1 and p_2 cannot be involved in P-semiflows because of transitions t_1 and t_2. The number of people in the elevator cannot be part of any P-semiflows as it receives tokens from p_1 and p_2 which in turn are not part of P-semiflows. The only conserved quantity is associated to the elevator being present in exactly one floor at any given time; hence $M(p_4) + M(p_5)$ is constant in any reachable marking M. [3]

d) In order to enforce a constraint on the maximum number of people it is enough to add a place p_6 to the net whose marking represents the number of free places in the elevator. See Fig. 4.1. [3]

e) The coverability graph associated to the initial marking $[0,0,0,0,1]$ is shown in Fig. 4.2. [5]

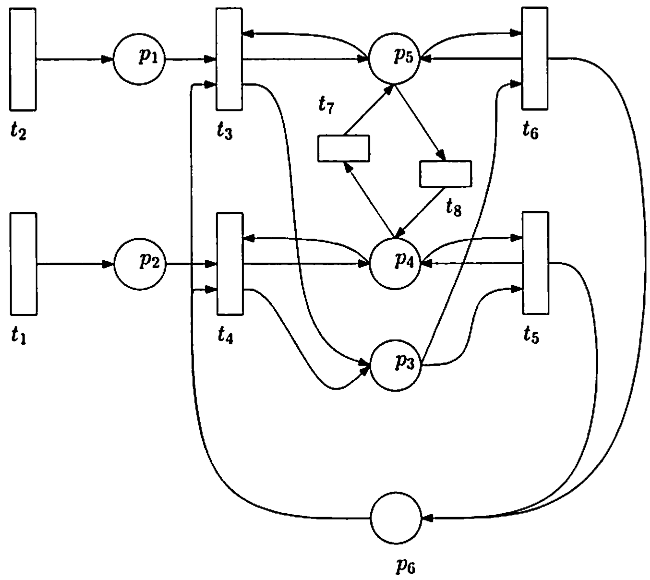

Figure 4.1 Petri Net of elevator with capacity constraint

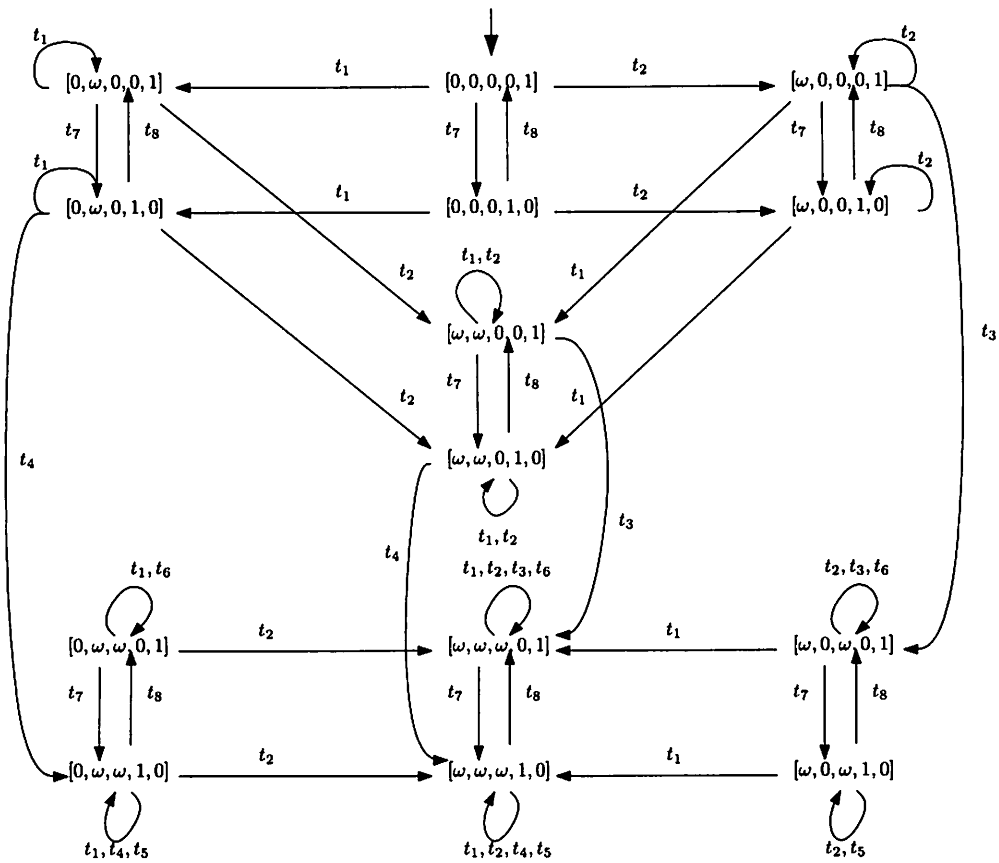

Figure 4.2 Coverability graph

f) The behaviourally bounded places are those where the ω label does not appear, hence p_4 and p_5. They are structurally bounded since $[0,0,0,1,1]$ is a P-semiflow (and hence also a P-decreasing vector) of support $\{p_4, p_5\}$. [2]

EE4-57 Discrete event systems

1. Paper title, copy right Imperial College London and page number is missing at the bottom of each page.
2. Q1 e, spelling mistake "spuuort".
3. Q2 line 2-3, grammar mistake " these corresponds"
4. Q4 part B, "Can you interpret their meaning?" is not a clear instruction.
5. Mark allocations are not shown in the answers document.

EE 4-23 Stability and control of non-linear systems

1. Paper title, copy right Imperial College London and page number is missing at the bottom of each page.
2. Marks are not shown in the answers document.
3. In the answers document Q3, part f and g are not clearly separated.

EE 4-48 Power System control, measurement and protection

1. Question 3, Figure number should be "Figure 3.1" and the text should be amended accordingly.

EE4-10 Probability of stochastic process

1. Q1 C.iii , grammar problem with "conclude that the probability that some bin empty is smaller than"
2. Q1 d. grammar problem with"we now back to the general setting"
3. Q1 d ii, grammar problem with "assume that we are in room containing m individuals" and also with "two individuals are born the same day of the year".
4. Q2 a, line 2 spelling mistake with "diagramme".
5. Q2 c) iii, answer is not provided in the answers document.
6. Q2 d. Ii. grammar problem with" find average fraction of time that the stock goes up"
7. Q3 b.ii grammar problem with " how long before either of them leave the parking?".
8. Q3 C line 2, grammar problem with " the number of ....constitute ......".
9. Q3C i. Sentence needs clarifying "derive the long run fraction of arriving cars".
10. Q3 c.ii, question instruction is not clear.

EE 4-57

Examination Paper Submission document for 2012-2013 academic year.

For this exam, please write the main course code and the course title below.

Code:

Title: DISCRETE EVENT SYSTEMS EE 4-57

We, the exam setter and the second marker, confirm that the following points have been discussed and agreed between us.

1. There is no full or partial reuse of questions.
2. This examination yields an appropriate range of marks that is well balanced, reflecting the quality of student (with weak students failing, capable students getting at least 40% and bright industrious students obtaining more than 70%)
3. The model answers give a fair indication of the amount of work needed to answer the questions. Each part has a comment indicating to the external examiners the nature of the question; i.e. whether it is bookwork, new theory, a new theoretical application, a calculation for a new example, etc.
4. The exam paper does not contain any grammar and spelling mistakes.
5. The marking schedule is shown in the answers document ~~and the resolution of each allocated mark is better than 3/20 for each question.~~
6. The examination paper can be completed by the students within time allowed.

Signed (Setter): David Angeli

Date: **18TH JANUARY 2013**

Signed (Second Marker): E Kerrigan

Date: 21/1/2013

Please submit this form with exam paper and model answers, and associated coursework to the Undergraduate Office on Level 6 by the required submission date.