Paper Number(s): **E4.06**
AS3
SO9
ISE4.36

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

MSc and EEE/ISE PART IV: M.Eng. and ACGI

OPTICAL COMMUNICATION

Wednesday, 1 May 10:00 am

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions Two to Six.

All questions carry equal marks.

Corrected Copy

Time allowed: 3:00 hours

Examiners responsible:

First Marker(s): Yeatman,E.M.

Second Marker(s): Leaver,K.D.

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3 \times 10^{8}$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x}\tan(k_{1x}d/2) \quad \text{and} \quad \kappa = -\, k_{1x}\cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, and give a brief (one or two lines) explanation where appropriate. All parts have equal value.

a) A certain dielectric symmetric slab waveguide supports two TE modes, having mode numbers m=0 and m=1. Which of these modes has a larger fraction of its field distribution in the core of the guide?

b) A step index silica fibre has a core diameter of 6 μm and an index difference of 0.01. What is the cutoff wavelength in nm for single mode operation?

c) Certain types of glass have lower intrinsic minimum attenuation than silica. What property makes this possible? Is this lower minimum found at longer or shorter wavelength than that of silica?

d) The spread in pulse length due to material dispersion in a fibre is generally proportional to the second derivative of index with respect to wavelength, i.e. $d^2n/d\lambda^2$. Why is the material dispersion not zero at wavelengths where this second derivative is zero, and in what way does the dispersion depend on $n(\lambda)$ in this case?

e) What would a suitable thickness and index be for an anti-reflection coating on the end of a silica fibre emitting into air, if the free space wavelength is 1330 nm?

f) A LED emits light in the wavelength range 780 – 790 nm. What is the bandgap energy E_g, in electron volts, in the active region?

g) A cooled LED emits radiation primarily in the wavelength range 1530 – 1570 nm. Estimate its operating temperature.

h) A Fabry-Perot laser diode of cavity length 350 μm operating at a free space wavelength of 1550 nm has longitudinal modes spaced at 1 nm. Calculate the effective index n for the guided modes in the laser.

i) A p-i-n photodiode with quantum efficiency η= 0.8 has a responsivity of 0.85 A/W at a particular operating wavelength. What is this wavelength?

j) A certain erbium doped fibre amplifier has significant gain over a bandwidth of 40 nm. Estimate the width ΔE in eV of the metastable energy level (assuming the ground state has the same width).

2. a) A symmetric slab waveguide supports three TE modes. Show the relative shapes of these modes by sketching the electric field amplitudes as a function of x. The core-cladding boundaries are at $x = d/2$ and $x = -d/2$. [6]

b) Show that regardless of the number of TE modes supported in a symmetric slab waveguide, the ratio $|E(0)| / |E(d/2)|$ decreases monotonically with increasing m for even modes m = 0, 2, 4.., where m is the mode number and E(0) and E(d/2) are the field amplitudes at the centre of the core and at the core-cladding boundary respectively. [6]

c) Define α to be the ratio of the waveguide transverse wavevector amplitude in the cladding to that of the core, i.e.

$\alpha = K/k_{1x}$

for a particular even mode. Define γ to be the ratio between the waveguide width and the free space wavelength, i.e.

$\gamma = d/\lambda_o$.

Find an expression for the numerical aperture (NA) of a symmetric slab waveguide, in terms of α and γ. [8]

3. a) An optical source transmits +3 dBm into a 80 km fibre with attenuation 0.5 dB/km at the operating wavelength of 1550 nm. This is received in a p-i-n photodiode with quantum efficiency $\eta = 0.8$. The receiver noise can be approximated by a load resistance of 10 kΩ. Find the relative magnitude of shot and thermal noise square spectral density in this case. Neglect connector, reflection and other excess losses. [4]

b) Show that when shot noise is the dominant noise source in an optical receiver, the signal-to-noise ratio (SNR) in terms of electrical power is equal to the number of photons per bit. Assume on-off keying modulation and a quantum efficiency $\eta = 1$. [6]

c) An erbium doped fibre amplifier (EDFA) whose noise figure is 6 dB at the operating wavelength is now added in front of the receiver to the link described in (a). Find the maximum improvement in SNR that can be given by this amplifier. [6]

d) What quantity primarily influences the noise figure of an optical amplifier? For what value of this quantity is the noise figure minimised, and what is the minimum noise figure? [4]

4. a) Show that the spread in pulse length due to material dispersion in a fibre is proportional to the second derivative of index with respect to wavelength, i.e. $d^2n/d\lambda^2$. [6]

b) A certain receiver has a noise equivalent power (NEP) of 5 pW/√Hz. For a transmitted power of 1 mW and a fibre attenuation of 0.6 dB/km, find an expression for the maximum bit rate B as a function of fibre length (assuming NEP is the limiting factor), and sketch this relation over a practical range of fibre lengths. Assume on-off keying, a signal bandwidth at the Nyquist limit, and a minimum optical SNR of 12 dB. Why is this approximation invalid for shorter fiber lengths? [8]

c) Using the relationship found in (b), find the minimum length, and corresponding bit rate, for which the dispersion penalty is below 1 dB, for a dispersion coefficient of 10 ps/nm·km. You may use Fig. 4.1 below. The source spectral width σ_λ is 0.15 nm (and is not affected by the signal bandwidth). [6]

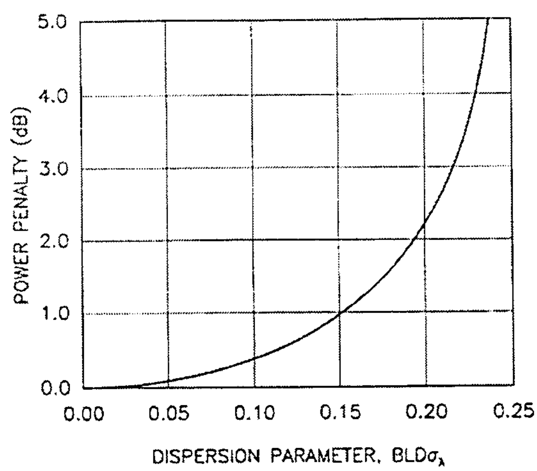

Figure 4.1 Dispersion penalty vs. dispersion parameter

5. a) Briefly describe the four factors which reduce the external quantum efficiency in a light emitting diode (LED), and ways in which their effects can be reduced. [6]

b) For a homostructure LED emitting into air, find the fraction of emitted photons lost through each of the four mechanisms of part (a), and hence calculate the external quantum efficiency η_{ext}, if none of the special measures to improve it have been used. Assume an attenuation coefficient of 10^3 cm^{-1}, and that the active region emits photons equally in all directions. The distance from the active region to the surface is 5 μm, and the refractive index of the semiconductor is 3.6. State any other assumptions or approximations made. [6]

c) Calculate the quantum efficiency for this same LED emitting into guided modes of a multi-mode fibre with a numerical aperture of 0.1. [4]

d) Explain why silicon is not well suited for making LEDs. [4]

6. a) A silicon p-i-n photodiode has intrinsic layer thickness w_i = 10 μm, and p, i and n doping levels respectively of $N_A^+ = 10^{21}$ m^{-3}, N_D^-, and $N_D^+ = 10^{21}$ m^{-3}. The electron and hole velocities can be approximated as linearly proportional to applied field, with mobilities of 0.15 m^2/Vs and 0.06 m^2/Vs respectively, up to a saturation velocity of 10^5 m/s (for both electrons and holes). Find the applied electric field amplitude at which the electrons reach their saturation velocity. Hence, find the value of N_D^- such that the electron velocity reaches the saturation value at the p-i junction, and drops 10% below this value across the intrinsic region. Calculate also the corresponding applied voltage needed to reach this condition. [10]

b) For the structure and conditions of part (a), find the maximum time for a carrier pair to be swept out of the depletion region for a photon absorbed in the intrinsic region (you may neglect the propagation in the depleted parts of the n^+ and p^+ regions). You may find the following integral useful:

$$\int \frac{dx}{C+ax} = \frac{1}{a}\ln(C+ax)$$

[5]

c) Explain the structure and functioning of an avalanche photodiode. Use sketches of the charge and field distribution to illustrate your explanation. [5]

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1 a) $m=0$ has more field in core (farther from cut-off).

b) For single mode $V = 2.405 = r_0 k_0 NA$, $k_0 = 2\pi/\lambda$

$NA = \sqrt{2n\Delta n}$ $2n \simeq 3$, $2.405 = \dfrac{3\times10^{-6}\times 2\pi \times \sqrt{.03}}{\lambda}$

$\lambda = 1357$ nm

c) These glasses have lower molecular vibration frequencies, or phonon energies, so IR absorption is at longer λ. Minimum attenuation will be at longer λ than for SiO_2.

d) $D \propto d^3n/d\lambda^3$, Dispersion is not zero since $\Delta\lambda$ is finite, so $\frac{d^2n}{d\lambda^2}$ not zero over all of $\Delta\lambda$

e) $n = \sqrt{n_1 n_2} = \sqrt{1.5\times1} = 1.225$.

$t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{1330}{4\times1.225} = 271$ nm

f) $E_g = \dfrac{hc}{\lambda_g}$, corresponding to lowest energy or longest λ

$\therefore E_g = \dfrac{1.24}{.79} = 1.57$ eV

g) $\Delta E = \dfrac{1.24}{1.53} - \dfrac{1.24}{1.57} = 0.021$ eV $\simeq 2kT$

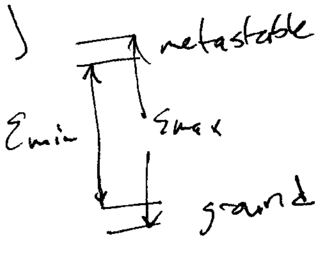
$T = \dfrac{.021\times1.6\times10^{-19}}{2\times1.38\times10^{-23}} = 122$ K $= -151$°C

h) $\dfrac{\lambda_0}{2n'} m = L$ $\dfrac{1.55}{2n'} m = 350$ $\dfrac{1.551(m+1)}{2n'} = 350$ $\dfrac{m}{n'} = \dfrac{700}{1.55} = 451.613$

$\dfrac{m-1}{n'} = \dfrac{700}{1.551} = 451.322$

$\dfrac{1}{n'} = .291$ $n' = 3.4$

i) $R = \dfrac{\eta e\lambda}{hc}$ $\lambda = \dfrac{hcR}{\eta e} = \dfrac{6.6\times10^{-34}\times3\times10^{8}\times0.85}{0.8\times1.6\times10^{-19}} = 1.31\ \mu$m

j) 

typical range would be 1520 - 1560 nm (has to around 1.55 μm band)

$\Delta E = \dfrac{1.24}{1.52} - \dfrac{1.24}{1.56} = .021$ eV,

split this between the two states, metastable width is 10 meV

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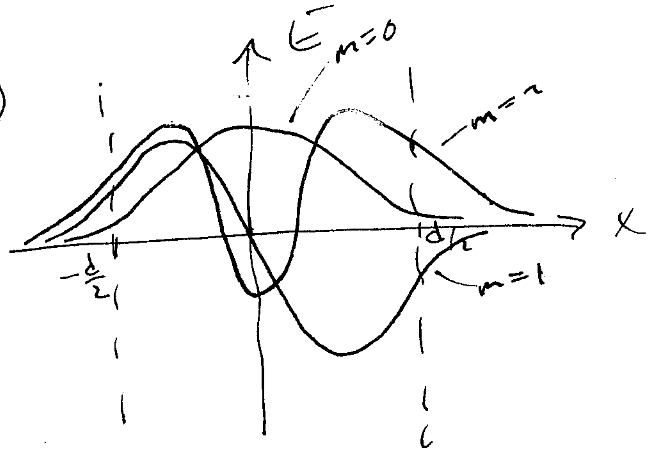
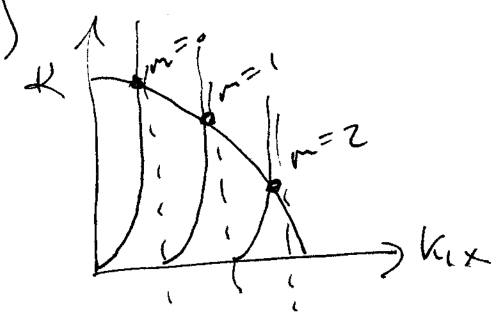
2 a)



b)



From the K-k_{1x} diagram we can see that as m rises, K drops and k_{1x} rises.

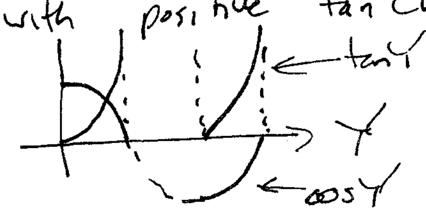
For even modes, within core $E(x) = E_0 \cos k_{1x}x$ and $E(0) = E_0$

so $E(d/2) = E_0 \cos(k_{1x}d/2)$

so $\dfrac{|E(0)|}{|E(d/2)|} = \dfrac{1}{|\cos(k_{1x}d/2)|}$

But for even modes $K = k_{1x}\tan(k_{1x}d/2)$.

Since K is dropping and k_{1x} rising, K/k_{1x} must be dropping as m increases and $\tan(k_{1x}d/2)$ must be falling. Also, we are only concerned with positive $\tan(k_{1x}d/2)$. Taking $Y = k_{1x}d/2$,



In either quadrant as tanY rises, |cosY| drops monotonically. Since $\tan(k_{1x}d/2)$ falls with increasing m, $\dfrac{1}{|\cos(k_{1x}d/2)|}$ must also fall!

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2 c) $\alpha = K/k_{1x}$ $\quad \gamma = d/\lambda_0$

We know $K = k_{1x}\tan(k_{1x}d/2)$.

We can use $k_{1z} = k_{2z}$ to derive $K^2 + k_{1x}^2 = (n_1^2 - n_2^2)k_0^2$

but $NA = \sqrt{n_1^2 - n_2^2}$

so $K^2 + k_{1x}^2 = NA^2(2\pi/\lambda_0)^2$

$$\alpha^2 + 1 = NA^2\left(\frac{2\pi}{\lambda_0 k_{1x}}\right)^2$$

$$\alpha = \tan(k_{1x}d/2) \quad \therefore \quad k_{1x} = \frac{2}{d}\tan^{-1}\alpha$$

$$\alpha^2 + 1 = NA^2\left(\frac{\pi d}{\lambda_0 \tan^{-1}\alpha}\right)^2$$

$$NA = \sqrt{\alpha^2+1}\ \frac{\tan^{-1}\alpha}{\pi\gamma}$$

3 a) $80\text{ km} \times 0.5\text{ dB/km} = 40\text{ dB}$

$3 - 40 = -37\text{ dBm} = .2\,\mu W$

$$I_{ph} = \frac{\eta e\lambda}{hc}\Phi = \frac{0.8 \times 1.6\times10^{-19} \times 1.55 \times .2\times10^{-12}}{6.6\times10^{-34}\times3\times10^{8}} = .2\,\mu A$$

$$I_{sh}^{*2} = 2eI_{ph} = 2\times1.6\times10^{-19}\times.2\times10^{-6} = 6.4\times10^{-26}\ A^2\cdot s$$

$$I_{th}^{*2} = \frac{4kT}{R} = \frac{4\times1.38\times10^{-23}\times300}{10^{4}} = 1.66\times10^{-24}\ A^2\cdot s$$

$$\frac{I_{sh}^{*2}}{I_{th}^{*2}} = .039$$

b) In shot noise limit, $SNR_{opt} = \dfrac{I_{ph}}{\sqrt{2eI_{ph}}\sqrt{\Delta f}}$

$$SNR_{elec} = SNR_{opt}^2 = \frac{I_{ph}^2}{2eI_{ph}\Delta f}$$

$$= \frac{I_{ph}}{2e\Delta f} = \frac{I_{ph}}{eB}.$$ Taking $I_{ph} = \frac{e}{h\nu}\Phi$, $\Phi/h\nu = \dot{N}$ (photons/sec)

then $SNR_{elec.} = \dfrac{\dot{N}}{B} = \dfrac{\text{photons/sec}}{\text{bits/sec}} = \dfrac{\text{photons}}{\text{bit}}$

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3 c) The optical amplifier degrades the electrical SNR by NF compared to the shot noise limit. Here NF = 6 dB = 4

$$SNR_{elec} = \frac{I_{ph}^2}{I^{*2}\Delta f} = \frac{I_{ph}'^2}{(8eGI_{ph}' + \frac{4kT}{R})\Delta f} = \frac{G^2 I_{ph}^2}{(8eG^2 I_{ph} + 4kT/R)\Delta f}$$

where G is the gain and I_{ph}' is the amplified signal.

so $SNR_{max} = I_{ph}/8e\Delta f$

Without the amplifier $SNR_{elec} = \frac{I_{ph}^2}{(2eI_{ph} + \frac{4kT}{R})\Delta f}$

Compared to this the amplified SNR is better by:

$$\frac{I_{ph}/8e\Delta f}{I_{ph}^2/((2eI_{ph} + \frac{4kT}{R})\Delta f)} = \frac{2eI_{ph} + 4kT/R}{8eI_{ph}}$$

$$= \frac{1}{4}\left(1 + \frac{4kT/R}{2eI_{ph}}\right)$$

From (a), $\frac{2eI_{ph}}{4kT/R} = .039$

Max. improvement $= \frac{1}{4}\left(1 + \frac{1}{.039}\right) \simeq 6.7$

3 d) The NF ~~is~~ mainly determined by the inversion ratio, ie the relative population of the excited state. If this state is full & the ground state is ~~optimum~~ empty, the inversion ratio = 1, and the NF is at its lowest possible value 3 dB.

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4 a) Group delay $\tau_g = \frac{L}{V_g}$, $V_g = \frac{d\omega}{dk}$

$\therefore \Delta\tau_g = \Delta(L\, dk/d\omega)$

$\frac{dk}{d\omega} = \frac{dk}{d\lambda}\frac{d\lambda}{d\omega}$ (taking λ as free space wavelength)

$k = \frac{2\pi}{\lambda}n \quad \frac{dk}{d\lambda} = \frac{-2\pi n}{\lambda^2} + k\frac{dn}{d\lambda}$

$\omega = \frac{2\pi c}{\lambda} \quad \therefore \frac{d\lambda}{d\omega} = \frac{-2\pi c}{\omega^2} = \frac{-\lambda}{\omega}$

$$\frac{dk}{d\omega} = \frac{2\pi n}{\lambda\omega} - \frac{\lambda k}{\omega}\frac{dn}{d\lambda} = \frac{n}{c} - \frac{\lambda}{c}\frac{dn}{d\lambda}$$

$$\Delta\tau_g = \frac{\partial\tau_g}{\partial\lambda}\Delta\lambda = \frac{L\cdot\Delta\lambda}{c}\cdot\frac{\partial}{\partial\lambda}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

$$= \frac{L\Delta\lambda}{c}\left(\cancel{\frac{dn}{d\lambda}} - \cancel{\frac{dn}{d\lambda}} - \lambda\frac{d^2n}{d\lambda^2}\right) = \frac{L\cdot\Delta\lambda\cdot\lambda}{c}\cdot\frac{d^2n}{d\lambda^2}$$

b) Nyquist $\Delta f = B/2$

Optical SNR $= \frac{\Phi_R}{NEP\sqrt{\Delta f}} = 16$ (= 12dB)

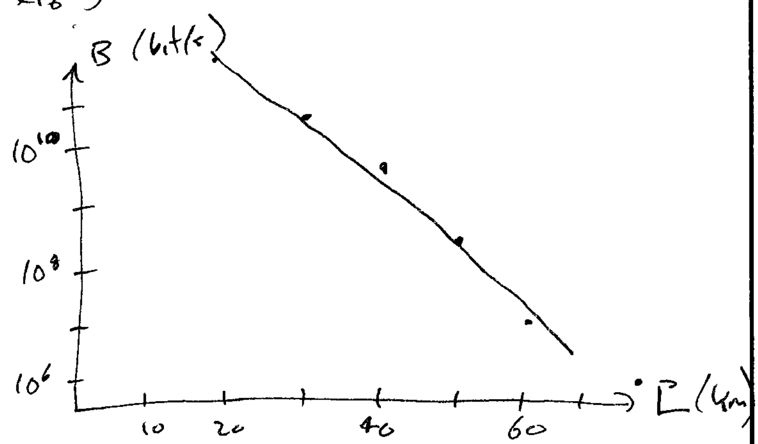
$\Phi_R = \Phi_0 e^{-\alpha L}$ loss per km in dB $= 10\log_{10} e^{-\alpha} = -0.6$

$\alpha = 0.6/10\log_{10}(e) = 0.138\ \text{km}^{-1}$

$\Phi_R = 10^{-3}e^{-0.138L} = 5\times10^{-12}\times16\times\sqrt{B/2}$

$$B = 2\left(\frac{10^{-3}e^{-0.14L}}{5\times10^{-12}\times16}\right)^2 = 3.1\times10^{14}e^{-.28L}$$

L (km)	B (bit/s)
10	19×10^{12}
20	1.1×10^{12}
30	70×10^{9}
40	4.2×10^{9}
50	260×10^{6}
60	16×10^{6}
80	58×10^{3}



Below about 25 km, shot noise dominates over NEP so the approximation above is completely inaccurate. Dispersion will also dominate (see c)

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$BLD \equiv \frac{km\ s\ nm}{s\ nm\,km}$

4 c) $BLD_x \lessapprox 0.15$

$$BL \times 10^{-11} \frac{s}{nm \cdot km} \times 0.15\ nm = 0.15$$

$$BL = 10^{11}\ km/s = 3.1 \times 10^{14}\, L e^{-0.28L}$$

$$\therefore\ L = \frac{e^{0.28L}}{3.1 \times 10^{3}}$$

we can solve this by successive approximation to get $L \doteq 42$ km.

5 a) i) half the light goes down towards the back of the LED. If a heterostructure is used to reduce absorption, then a mirror could be placed at the base to reflect this light back to the emitting surface

ii) Light is absorbed as it travels to the surface. This can be reduced by a heterostructure - the region above the active region being given a wider band-gap.

iii) Fresnel reflection at surface - an AR coating reduces this.

iv) Total internal reflection - photons beyond the critical angle, e.g. $\sin^{-1}\left(\frac{1}{n_s}\right)$ for semi-conductor to air, are reflected. Can be improved by a hemispherical cap.

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5 b) i) 50% lost downwards.

iv) It is sensible to determine the TIR next. We have $\theta_c = \sin^{-1}\left(\frac{1}{3.6}\right) = 16^\circ$

and $f = \dfrac{\int_0^{\theta_c} d\Omega}{\int_0^{\pi/2} d\Omega}$ where $d\Omega = 2\pi \sin\theta \, d\theta$

then $f = \dfrac{\int_0^{\theta_c} \sin\theta \, d\theta}{\int_0^{\pi/2} \sin\theta \, d\theta} = \dfrac{\cos\big|_{\theta_c}^{0}}{\cos\big|_{\pi/2}^{0}} = \dfrac{1-\cos\theta_c}{1-0}$

$= .039$

So of the 50% going upward, 96.1% of these photons are lost.

ii) Since we no longer care about photons outside θ_c, the path length to the surface only varies by $\cos\theta_c$, ie a max increase of $\simeq 4\%$, so we can approximate re-absorption by $e^{-\alpha x} = e^{-10^3 \times 5\times10^{-4}} = 0.61$

so another 39% of the remaining photons are lost.

iii) Again, we will use the limited angular range and take a normal incidence approximation for Fresnel reflection, ie

$R = \left(\dfrac{3.6-1}{3.6+1}\right)^2 = 0.32$, so 32% more are lost.

Overall, $\eta_{ext} = \frac{1}{2} \times f \times e^{-\alpha x} \times (1-R)$

$= \frac{1}{2} \times .039 \times 0.61 \times 0.68 = 0.81\%$

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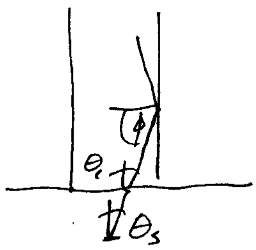
5 c) $NA = 0.1 \simeq \sqrt{2n\Delta n}$

taking $n \simeq 1.5$, $\Delta n = \frac{0.1^2}{3} = .0033$



Critical angle in fibre $= \sin^{-1}\left(\frac{n_o}{n_c}\right)$

$= \sin^{-1}\left(\frac{n}{n+\Delta n}\right) \simeq \sin^{-1}\left(\frac{1.5}{1.5+\Delta n}\right) = 78°$

So $\theta_c = 90 - 78 = 12°$

By Snell's law, max angle from axis in LED given by $\sin\theta_s = \frac{n_c}{n_s}\sin\theta_c = \frac{1.5}{3.6}\sin(12)$

$\theta_s = 5°$

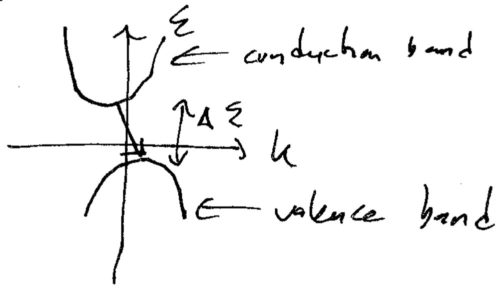
So $f = 1 - \cos 5° = .0038$

and $R = \left(\frac{3.6-1.5}{3.6+1.5}\right)^2 = .17$

$e^{-\alpha x}$ is unchanged

$\eta = \frac{1}{2} \times .0038 \times .61 \times .83 = 0.096\ \%$

d) Silicon is an indirect band-gap semiconductor. The lowest ~~—~~ ΔE transitions involve a change in momentum which cannot be provided by a photon, so a combined photon-phonon emission is needed. This is statistically very unlikely so internal quantum efficiency for radiative recombination is very low



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Model Answers and Mark Schemes First Examiner: EMY $\frac{9}{10}$

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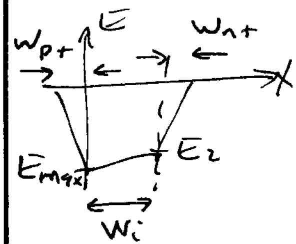
6 a) $V_{sat} = 10^5\ m/s$, $v = E\mu$

to reach V_{sat} for electrons, $E = \frac{10^5}{0.15} = 6.6\times10^5\ \frac{V}{m}$



$E_{max} = -6.6\times10^5\ V/m$

depleted widths in p+ and n+ regions we call W_{n+} and W_{p+}.

$$E = \int \frac{\rho\,dx}{\varepsilon} \quad \text{so} \quad E_{max} = -\frac{N_A^+ e W_{p+}}{\varepsilon_r\varepsilon_0}$$

$$\therefore\ W_{p+} = \frac{6.6\times10^5\times12\times8.85\times10^{-12}}{10^{21}\times1.6\times10^{-19}} = 0.44\ \mu m$$

We allow field to drop 10% in W_i, so

$$E_2 = -0.9(6.6\times10^5) = -5.94\times10^5\ V/m$$

and $\Delta E = \frac{|E_{max}|}{10} = \frac{N_D^- e W_i}{\varepsilon_r\varepsilon_0}$ $\therefore$ $N_D^- = \frac{\varepsilon_r\varepsilon_0|E_{max}|/10}{eW_i}$

$$N_D^- = \frac{12\times8.85\times10^{-12}\times6.6\times10^4}{1.6\times10^{-19}\times10^{-5}} = \boxed{4.4\times10^{18}\ m^{-3}}$$

Voltage applied is the area under $E(x)$:

$$\Delta V = \frac{|E_{max}|W_{p+}}{2} + 0.95|E_{max}|W_i + \frac{0.9|E_{max}|W_{n+}}{2}$$

$$W_{n+} = \frac{\varepsilon_r\varepsilon_0\times0.9|E_{max}|}{N_D^+ e} = 0.9W_{p+} = 0.40\ \mu m$$

$$\Delta V = 6.6\times10^5\left(\frac{0.44}{2} + 0.95\times10 + \frac{0.9\times0.4}{2}\right)\times10^{-6} = \boxed{6.53\ V}$$

b) Maximum time is taken for absorption at N^+ end of depletion region, for hole to return to p^+ end.

$$\tau = \int\frac{dx}{v} = \frac{1}{\mu}\int\frac{dx}{E} \qquad E(x) = E_{max}\left(1 - \frac{0.1x}{10^{-5}}\right) = E_{max}(1-10^4x)$$

$$\tau = \frac{1}{\mu}\int_0^{10^{-5}}\frac{dx}{E_{max}(1-10^4x)}$$

$$= \frac{1}{.06\times6.6\times10^5}\left.\frac{\ln(1-10^4x)}{-10^4}\right|_0^{10^{-5}} = \frac{1}{4\times10^4\times10^4}\ln(1.1) \simeq 260\ ps$$

If we include the n^+ and p^+ parts, the integral doesn't converge.

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Model Answers and Mark Schemes First Examiner: EMY

Paper Code: E4.06 Second Examiner: KDL

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Question Number etc. in left margin | Mark allocation in right margin

6 c) An avalanche photodiode has a doping distribution so as to create a short high field region near the base of the depletion region. In this section, electrons travelling towards the base are accelerated sufficiently that their collisions cause ionisation, and the number of carriers produced is multiplied by M. This is a kind of controlled dielectric breakdown. It is useful to overcome thermal noise but causes a reduction in SNR compared to shot noise due to statistical nature of avalanche process