UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1997

MEng Honours Degrees in Computing Part IV
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute

PAPER 4.80

AUTOMATED THEOREM PROVING

Monday, April 28th 1997, 2.00 - 4.00

Answer THREE questions

For admin. only: paper contains 4 questions

1 ai) The splitting rule of the Davis Putnam Procedure (DPP) for propositional sets of clauses, splitting on atom A, transforms a branch containing clauses S into two branches containing clauses S1 and S2.

What are the two sets S1 and S2 in terms of A and the clauses in S?

ii) Use **Fact I** below, together with the fact that every reduction step of the DPP maintains unsatisfiability, to explain why, when the DPP is applied to an unsatisfiable set of Horn clauses (clauses with a maximum of one positive literal) a reduction step is always possible until the empty clause is derived.

Fact I: Any inconsistent set of clauses always has at least one clause that contains only positive literals.

b If S is a set of clauses (not necessarily Horn clauses) the following is an algorithm for applying the DPP to S in an attempt to show unsatisfiability:

1. apply non-splitting reduction rules if possible;
2. select an atom A that occurs positively in as many (non-unit) clauses as possible and apply the splitting rule to A;
3. repeat steps 1. and 2.

i) Apply the algorithm to show unsatisfiability of the following set of clauses:

$$\{ \neg B \vee \neg A,\ B \vee \neg C \vee \neg A,\ C \vee \neg D \vee \neg A,\ D \vee B,$$
$$A \vee B \vee \neg E,\ A \vee E \vee \neg B,\ A \vee E \vee \neg D,\ \neg E \vee \neg B \}$$

ii) Explain why the method must eventually result in all non-terminated branches containing only Horn clauses.

c How could the semantic tree method be used to show the unsatisfiability of a set of clauses? What is the drawback of this method?

The three parts carry, respectively, 30%, 45%, 25% of the marks.

2 ai) Define paramodulation.

ii) How are critical pairs formed in the Knuth-Bendix Procedure (KBP)?

iii) Explain how the two operations of rewriting and superposition can be seen as instances of paramodulation.

iv) What properties of equality are used in a simulation of a paramodulation step by resolution? Illustrate your answer by considering the two possibilities for paramodulation of

f(x) = g(x) into P(f(f(b)), c).

bi) Find a partial ordering of terms, based on the number of symbols in a term, such that the rewrite rules 1 and 2 below are reducing. Justify your answer.

1. g(f(x,y)) => f(y,g(x))
2. f(x,x) => a

ii) Use the KBP to derive the following rewrite rules starting from rules 1. and 2.

3. f(x,g(x)) => g(a)
4. g(g(a)) => a
5. f(g(a),a) => g(a)

Make sure you show that the new rules are reducing by the ordering of part bi).

c Explain *briefly* how each of the possible outcomes of the KBP can be applied to the problem of checking t1=t2 for ground terms t1 and t2.

Note: x, y are variables and a, b, c are constants.

The three parts carry, respectively, 45%, 35% , 20% of the marks.

3 ai) State the restrictions on resolution that are imposed in standard Hyper-resolution (HR). What are electrons and nucleii?

ii) For any set of clauses which Herbrand interpretation makes each electron false and each nucleus true?

iii) List the "electrons" - false clauses and "nucleii" - true clauses from the set of clauses below for the Herbrand interpretation that makes R-atoms and S-atoms true and T-atoms false.

$$\{S(x) \vee T(x), \neg S(x) \vee \neg T(x), S(a), T(c), R(a,b), R(b,c), \neg R(x,y) \vee T(x) \vee \neg T(y)\}$$

Apply HR using these electrons and nucleii to derive the empty clause.

b Describe the book-keeping of the Connection Graph Procedure (CGP), illustrating your answer using the set of clauses given in part aiii).

In particular, explain the "lookahead" and "lookbehind" features of the CGP.

Note: x, y are variables and a, b, c are constants.

The two parts carry, respectively, 55%, 45% of the marks.

Turn over ...

4 a Explain how model elimination (or SL-resolution) can be seen as a *linear tableau* development method.

Use the model elimination method to show the unsatisfiablility of the set of clauses

$$\{\neg Q(x,y) \vee \neg Q(y,x),\ \ Q(g(x), x) \vee Q(b,x),\ \ Q(x, g(x)) \vee Q(b,x)\}$$

b A particular disjunction of literals, called a Qclause, is defined as follows:

A Qliteral is a literal or a universally quantified literal.
A Qclause is a (possibly universally quantified) disjunction of Qliterals.

The model elimination method is to be adapted to Qclauses. Suggest what modifications might be made and *briefly* justify their soundness.

c The concept of Qclauses of part b) is further refined as follows:

A Qconjunction is a conjunction of one or more Qliterals.
A Qclause is a (possibly universally quantified) disjunction of Qconjunctions.

Suggest additional modifications to the model elimination method to accommodate Qclauses.

Illustrate your answer by showing the unsatisfiability of the two Qclauses

$$\{\forall x\ [\forall y.\ P(y, x) \vee (P(x,c) \wedge P(a,c))]\ ,\ \neg P(c,c) \vee \neg P(a, c) \vee \forall\ y.\neg P(y, y)\}$$

Note: x, y are variables and a, b, c are constants.

The three parts carry, respectively, 40%, 30%, 30% of the marks.

End of paper