E3.13

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

EEE PART III/IV: MEng, BEng and ACGI

Corrected Copy

ELECTRICAL ENERGY SYSTEMS

Wednesday, 4 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

Please use separate answer books for Sections A and B.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	C.A. Hernandez-Aramburo, B.C. Pal
	Second Marker(s) :	B.C. Pal, C.A. Hernandez-Aramburo

Section A

1. The one-line diagram of a three-phase system is shown below. Impedances are marked in per-unit in a 100 MVA, 400 kV base. The load at bus 2 is $S_2 = 15.93\,\text{MW} - \text{j}33.4\,\text{Mvar}$, and the load at bus 3 is $S_3 = 77\,\text{MW} + \text{j}14\,\text{Mvar}$. The voltage at bus 3 is held at $400 \angle 0^\circ\,\text{kV}$. Working in per-unit, determine the voltage at buses 2 and 1.

[20]

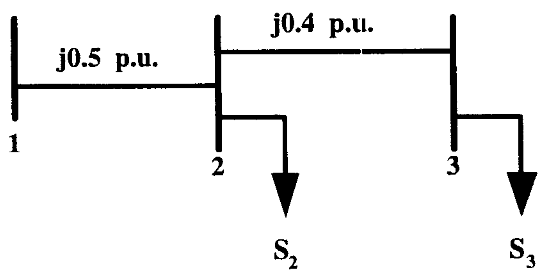

Figure 1.1: 3-bus system

2 A three-phase 200 km transmission line has the following characteristics:

- Rated voltage: 345 kV
- Series impedance: $z = 0.025 + j0.30 \quad \Omega/\mathrm{km}$
- Shunt admittance: $y = j4.0 \times 10^{-6} \quad \mathrm{S/km}$
- At full-load condition, this transmission line delivers 700 MW at 0.98 leading power factor at the receiving-end. Under this load condition, the receiving-end voltage is 0.95 of the rated voltage.

Using a medium-length model for the transmission line, determine the following:

(a) The characteristic impedance of the transmission line [4]

(b) The ABCD model of the transmission line [10]

(c) The sending-end voltage at full-load conditions [6]

3. The figure below shows the one-line diagram of a three-bus power system. Bus 1 is the slack bus, bus 2 is a generator bus and bus 3 is a load bus. The line impedances are marked in per-unit in a 100 MVA base. Answer the following:

(a) Determine the bus admittance matrix of the system

[3]

(b) For a load-flow analysis, state what the known and unknown variables are for each bus.

[6]

(c) If the Newton-Raphson method is to be used to solve the system:

(i) write the equation for the real power at bus 2, and write also the equations for the real and reactive power at bus 3.

[6]

(ii) write the elements of the following matrices:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

Do not write the full expression for all the elements of the Jacobian matrix. Do, however, write the full expression for the first element of that matrix (J_{11}).

[5]

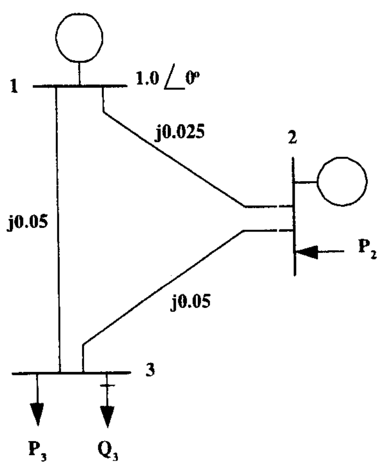

Figure 3.1: 3-bus power system network

Section B

4. (a) Why is it so important to optimally allocate generation levels to various units in a power system? [4]

(b) What is meant by *incremental cost* in power generation? [2]

(c) A power system has 'n' generators with unit operating cost function $F_1(P_1), F_2(P_2), ...F_n(P_n)$. Prove that the cost of operating the system would reach minimum when incremental cost of generation of each generator is the same. Assume no loss in the system. [6]

(d) An area of an interconnected power system has two fossil-fuel units operating on economic dispatch. The variable operating costs of these units are given by

$$C_1 = 8P_1 + 0.005P_1^2 \quad \$/hr \qquad 300 \leq P_1 \leq 1200 \qquad (4.1)$$

$$C_2 = 7P_2 + 0.009P_2^2 \quad \$/hr \qquad 400 \leq P_2 \leq 1000 \qquad (4.2)$$

where P_1 and P_2 are in MW. Determine the power output of each unit and the incremental operating cost(s), if the system is to operate at minimum cost to meet a load demand of 1200 MW. Transmission losses are neglected. [8]

5. (a) Why should not a power system be allowed to operate beyond permissible range of frequency variation? [5]

(b) An 11 kV supply busbar is connected to an 11/132 kV, 100 MVA, 8 percent reactance transformer. The transformer feeds a 132 kV transmission link consisting of an overhead line of impedance (0.014+j0.035) p.u and a cable of impedance (0.03+j0.01) p.u. in parallel. If the receiving end is to be maintained at 132 kV when delivering 80 MW, 0.95 p.f. lagging, calculate the real power and reactive power carried by the line and the cable. All p.u. values relate to 100 MVA and 132 kV base. [15]

6 (a) Why is the method of symmetrical component analysis so useful in fault analysis? [4]

(b) A Y-connected unbalanced load draws currents in abc sequence as $I_a = 10\angle 10^0$, $I_b = 15\angle 100^0$ and $I_c = 20\angle 150^0$. Evaluate the positive, negative and zero sequence component of these currents [7]

(c) Consider the one-line diagram of a simple power system shown in Figure 6.1. System data in per-unit (p.u.) on appropriate MVA base are given as follows:

Synchronous generators:

G1:	500 MVA	16 kV	$X_1 = X_2 = 0.15$	$X_0 = 0.05$
G2:	500 MVA	16 kV	$X_1 = X_2 = 0.15$	$X_0 = 0.05$

Transformers:

T1:	500 MVA	16/400 kV	$X_1 = X_2 = X_0 = 0.08$
T2:	500 MVA	16/400 kV	$X_1 = X_2 = X_0 = 0.08$

Transmission lines:

TL12:	100 MVA	400 kV	$X_1 = X_2 = 0.05$	$X_0 = 0.15$
TL13:	100 MVA	400 kV	$X_1 = X_2 = 0.025$	$X_0 = 0.075$
TL23:	100 MVA	400 kV	$X_1 = X_2 = 0.025$	$X_0 = 0.075$

The neutral of each generator is grounded through a current limiting reactor of 0.03 p.u. on 100 MVA base. All transformer neutrals are solidly grounded. The generators are operating at no-load with a voltage of 1.05 p.u. Draw the positive, negative and zero sequence networks after expressing all the parameters in 100 MVA base. [9]

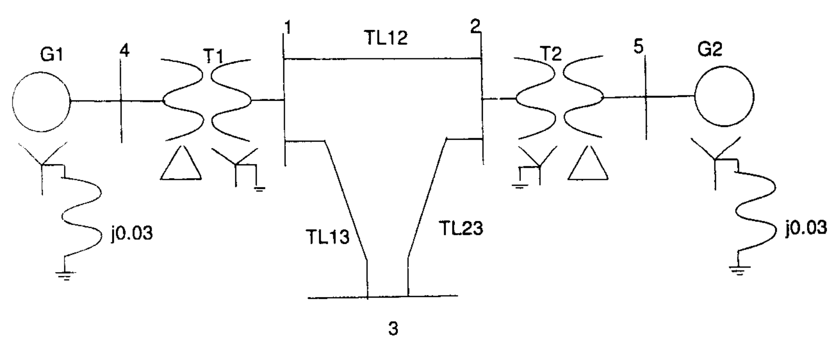

Figure 6.1: A simple power system

Master

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

Department of Electrical and Electronic Engineering

Examinations 2004 / 05

MODEL ANSWERS

E 3.13 Electric Energy Systems

Authors: Dr C. A. Hernandez and Dr B. Pal

Problem 1. calculation of new example

$$I_{23} = \frac{S_3^*}{V_3^*} = \frac{(0.77 + j0.14)^*}{1} = 0.77 - j0.14$$

$$\begin{aligned}
V_2 &= I_{23}Z_{23} + V_3 \\
&= (0.77 - j0.14)(j0.4) + 1 \\
&= 1.0560 + j0.308 \\
&= 1.1\angle 16.26^\circ \text{p.u.} \\
&= 1.1\angle 16.26^\circ \times 400\text{kV} = 440\angle 16.26^\circ \quad \text{kV}
\end{aligned}$$

$$\begin{aligned}
I_{S2} &= \frac{S_2^*}{V_2^*} \\
&= \left[\frac{(0.1593 - j0.334)^*}{(1.0560 + j0.308)^*}\right] \\
&= 0.0540 + j0.332
\end{aligned}$$

$$\begin{aligned}
I_{12} &= I_{23} + I_{S2} \\
&= 0.77 - j0.14 + 0.0540 + j0.332 \\
&= 0.8240 + j0.1920
\end{aligned}$$

$$\begin{aligned}
V_1 &= I_{12}Z_{12} + V_2 \\
&= (0.8240 + j0.1920)(j0.5) + 1.0560 + j0.308 \\
&= 0.9600 + j0.7200 \\
&= 1.2\angle 36.87^\circ \text{p.u.} \\
&= 1.2\angle 36.87^\circ \times 400\text{kV} = 480\angle 36.87^\circ \quad \text{kV}
\end{aligned}$$

Problem 2. calculation of new example

a. Characteristic impedance

$$\begin{aligned} Zc &= \sqrt{\frac{z}{y}} = \sqrt{\frac{0.025 + j0.30}{j4.0 \times 10^{-6}}} \\ &= 274.10 - j11.40\Omega = 274.3355\angle(-2.3818°)\Omega \end{aligned}$$

b. ABCD model

$$\begin{aligned} Z &= zl = (0.025 + j0.30) \times 200 = 5 + j60 \quad [\Omega] \\ Y &= yl = j4.0 \times 10^{-6} \times 200 = j8.0 \times 10^{-4} \quad [\mathrm{S}] \end{aligned}$$

$$\begin{aligned} A &= 1 + \frac{YZ}{2} = 1 + \frac{(j8.0 \times 10^{-4})(5 + j60)}{2} = 0.9760 + j0.0020 \\ B &= Z = 5 + j60 \\ C &= Y\left[1 + \frac{YZ}{4}\right] = -8.00 \times 10^{-7} + j7.9040 \times 10^{-4} \\ D &= A = 0.9760 + j0.0020 \end{aligned}$$

c. Sending-end voltage at full-load

$$\begin{aligned} S_R &= |S_R|\angle\cos^{-1}(PF) \\ &= \frac{700}{0.98}\angle\cos^{-1}(PF) \\ &= 714.2857\angle(-11.4783°) \quad [\mathrm{MVA}] \end{aligned}$$

The minus sign is because it is a leading power factor.

$$\begin{aligned} I_R^* &= \frac{S_R}{\sqrt{3}V_R} \\ &= \frac{714.2857\angle(-11.4783°)\,[\mathrm{MVA}]}{\sqrt{3}(0.95 \times 345 \quad \mathrm{kV})} \\ &= 1.2583\angle(-11.4783°)[\mathrm{kA}] \\ I_R &= 1.2583\angle(11.4783°)[\mathrm{kA}] = (1.2331 + \mathrm{j}0.2504)[\mathrm{kA}] \end{aligned}$$

$$\begin{aligned} V_S &= AV_R + BI_R \\ &= (0.9760 + j0.0020) \times (0.95 \times 345\,\mathrm{kV}) + (5 + \mathrm{j}60) \times (1.2583\angle(11.4783°)) \\ &= 320.15\angle 13.718°[\mathrm{kV}] = (311.03 + \mathrm{j}75.894)[\mathrm{kV}] \end{aligned}$$

Problem 3. calculation of new example

(a) Admittance matrix

$$Y = \begin{bmatrix} -j60 & j40 & j20 \\ j40 & -j60 & j20 \\ j20 & j20 & -j40 \end{bmatrix} =: \begin{bmatrix} 60\angle-\frac{\pi}{2} & 40\angle\frac{\pi}{2} & 20\angle\frac{\pi}{2} \\ 40\angle\frac{\pi}{2} & 60\angle-\frac{\pi}{2} & 20\angle\frac{\pi}{2} \\ 20\angle\frac{\pi}{2} & 20\angle\frac{\pi}{2} & 40\angle-\frac{\pi}{2} \end{bmatrix}$$

(b) Known and unknown variables

- Bus 1 (slack bus). **Known:** $|V_1|$, δ_1. **Unknown:** P_1, Q_1
- Bus 2 (generator bus). **Known:** P_2, $|V_2|$. **Unknown:** Q_2, δ_2
- Bus 3 (Load bus). **Known:** P_3, Q_3. **Unknown:** $|V_3|$, δ_3

(c) Expressions for active and reactive power
From the lecture notes regarding the application of the Newton-Raphson method to the power flow problem:

$$\begin{aligned} P_i &= \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \\ Q_i &= -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \end{aligned}$$

Therefore, the pertinent equations for buses 2 and 3 are as follows:

$$\begin{aligned} P_2 &= 40|V_2||V_1|\cos\left(\frac{\pi}{2} - \delta_2 + \delta_1\right) + 60|V_2||V_2|\cos\left(-\frac{\pi}{2}\right) + \\ &\quad 20|V_2||V_3|\cos\left(\frac{\pi}{2} - \delta_2 + \delta_3\right) \\ P_3 &= 20|V_3||V_1|\cos\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) + 20|V_3||V_2|\cos\left(\frac{\pi}{2} - \delta_3 + \delta_2\right) + \\ &\quad 40|V_3||V_3|\cos\left(-\frac{\pi}{2}\right) \\ Q_3 &= -20|V_3||V_1|\sin\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) - 20|V_3||V_2|\sin\left(\frac{\pi}{2} - \delta_3 + \delta_2\right) + \\ &\quad -40|V_3||V_3|\sin\left(-\frac{\pi}{2}\right) \end{aligned}$$

(d) Jacobian matrix

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_3 \end{bmatrix}$$

The first element of the Jacobian matrix is:

$$\frac{\partial P_2}{\partial \delta_2} = 40|V_2||V_1|\sin\left(\frac{\pi}{2} - \delta_2 + \delta_1\right) + 20|V_2||V_3|\sin\left(\frac{\pi}{2} - \delta_2 + \delta_3\right)$$

Problem 4.

(i) (**Bookwork**) In thermal power plants, fuels (oil, gas and coal) used are very costly. The price fluctuates with international market conditions couple with demand and supply. The amount of fuel used in generating power in the range of GW is astronomical. A slight variation in price causes billions of dollars of hike in fuel bills to the power companies. The cost of electric energy production rises accordingly. This has the knock on effect on the cost of finished products from industries where electricity is one of the key inputs. This suggests that various units in a system must share their outputs in an efficient manner. This is why it is so important to have optimum allocations of generation. **[4marks]**

(ii) (**Bookwork** Usually two costs are associated with power generation: fixed cost and variable cost. The fixed cost does not (or even if it is very slightly) change with power output. The variable cost obviously changes with power output. The main component of this cost is cost of the fuel (coal, gas, oil etc.) Generator/turbine of specific design and make will have specific characteritic per unit output. This value also does not remain fixed over the whole output range. At any point the derivative of variable cost with respect to power output (usually in MW)is known as *incremental cost.* **[2marks]**

(iii) (**Bookwork**) Let us assume that the generators are producing $P_1, P_2, P_3 ... P_n$ to supply a load of P_L. Under the assumption of no loss,

$$P_L = \sum_{i=1}^{n} P_i \tag{0.1}$$

The total cost of generation is

$$F_T = \sum_{i=1}^{n} F_i P_i \tag{0.2}$$

For optimal cost of operation, Lagrange multiplier method can be used. The Lagrange function (L) is the combination of cost and operational constraint described in (0.1). Based on this argument the following can be written

$$L = \sum_{i=1}^{n} F_i P_i + \lambda \left(P_L - \sum_{i=1}^{n} P_i \right) \tag{0.3}$$

where λ is lagrange multiplier. The derivative of L with respect to P_i will produce

$$\frac{\partial L}{\partial P_i} = \frac{dF_i(P_i)}{dP_i} - \lambda \tag{0.4}$$

for $i = 1ton$. The derivative of L with respect to λ is

$$\frac{\partial L}{\partial \lambda} = P_L - \sum_{i=1}^{n} P_i \tag{0.5}$$

At optimal point all these derivatives must be zero. This means the incremental cost of all the generator must be same at the optimal operating condition and the load and generation constraint is satisfied(0.5) **[6marks]**

(iv) When both units are in operation, the condition of optimal operating cost is reached when incremental fuel costs for both of them are equal. In the absence of transmission loss, the total load equals total generation. i.e. at optimal point the following has to satisfy.

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \lambda \ (in \quad R\$/\text{MWhr}) \tag{0.6}$$

$$P_{load} = P_1 + P_2 + P_3 \tag{0.7}$$

Now,

$$\frac{dC_1}{dP_1} = 8.0 + 0.010P_1 \tag{0.8}$$

$$\frac{dC_2}{dP_2} = 7.0 + 0.018P_2 \tag{0.9}$$

Upon substitution of (0.8) to (0.9) into (0.6) and carrying out necessary manipulations including (0.7) the following final form is obtained

$$0.010P_1 - \lambda = -8.0 \tag{0.10}$$

$$0.018P_2 - \lambda = -7.0 \tag{0.11}$$

$$P_1 + P_2 = 1200 \tag{0.12}$$

[4marks]

The solution is

$$P_1 = 735.714 \quad \text{MW} \tag{0.13}$$

$$P_2 = 464.286 \quad \text{MW} \quad \text{MW} \tag{0.14}$$

$$\lambda = 15.35 \quad \$/\text{MWhr} \tag{0.15}$$

[4marks]

Problem 5.

(i) (**Bookwork**) A significant component of industrial loads comprise of induction motors. The speeds at which these motor run depends on supply frequency. At times it is desired to have constancy in drive speed in some process industries. The supply frequency in these cases should be as constant as possible. In a power plant, reduction in frequency by 1 Hz, significantly affects the performance of the system. The auxiliary loads comprise of large feed pumps to maintain desired pressure of the steam circulating in boiler and turbines. The drop in supply frequency to boiler feed pump motor will cause drop in steam pressure and hence the power output of the generator. Low frequency would also result in high magnetisation or over fluxing in transformers and motors. The consequence of this is larger magnetic losses leading to increased temperature rise and shortening of life of these costly components. Constancy in frequency is very important for time keeping devices like clocks and timers. **[5marks]**

(ii) (**New computed example**) Fig 3.1 describes the problem. Since

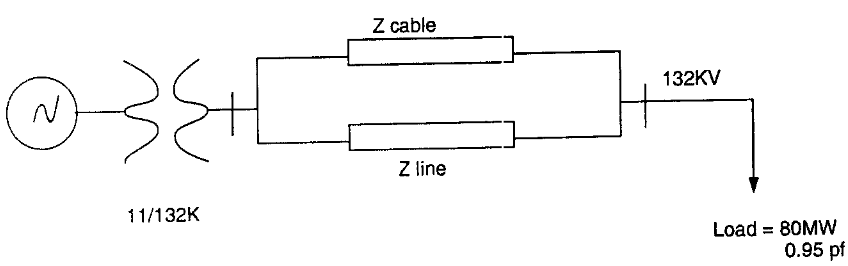

Figure 5.1: The circuit description of Problem 5

100 MVA and 132 kV line voltage is given as base and parameters are all expressed in p.u. on this base, it will be easy to work in p.u. here. The transformer data would also not be necessary as we are to find flow in the line and cable. $Z_{cable} = 0.03 + \jmath 0.01$ and $Z_{line} = 0.014 + \jmath 0.035$ The combined impedance of the cable and line is $0.0143 + \jmath 0.0124$ **[3marks]**
The magnitude of the current at the receiving end is $I = 80.0/100/0.95 = 0.8421$. The complex value of current from the power factor of 0.95 is found to be $0.80 - \jmath 0.2629$ The current through cable is $I_{cable} = 0.0529 + \jmath 0.0383$ and that through line is $I_{line} = 0.2971 - \jmath 0.3013$ **[3 marks]**

Sending end voltage is

$$V_s = 1.0 + (0.80 - j0.2629) \times 0.0143 + j0.0124 \quad (0.16)$$

[3 marks]

The amount of power carried by the line is

$$V_s \times I_{line} \times baseMVA = 29.96(MW) + j30.76(MVAR) \quad (0.17)$$

[3 marks]

The amount of power carried by the cable is

$$V_s \times I_{line} \times baseMVA = 51.06(MW) - j3.58(MVAR) \quad (0.18)$$

Note the cable is carrying leading reactive power. **[3 marks]**

Problem 6.

(i) (**Bookwork**) The method of symmetrical components is very useful in analyzing unbalanced circuits. Most of the faults encountered in electricity supply system leads to unbalanced operating conditions. Usually the network parameters are balanced. The advantage with symmetrical components is that three independent balanced components are obtained from an unbalanced system. These three balanced components are suitably connected at fault point depending on the nature of fault to evaluate fault currents. **[4marks]**

(ii) (**New computed example**) The transformation matrix T relating phase variables with sequence component variables is given by

$$T = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \qquad (0.19)$$

where $a = 1\angle 120^0$. In view of this one can write the following

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad (0.20)$$

On substituting the values of $I_a, I_b and I_c$ and necessary computation the following is obtained: $I_0 = 9.4531\angle{-110.81^0}, I_1 = 11.1583\angle{+44.33^0}$ and $I_2 = 5.2724\angle{-7.61^0}$

(iii) (**New computed example**) The first task is to express all the reactances in common MVA base of 100 MVA. Use the base conversion formula

$$Z_{new}\,(p.u.) = Z_{old}\,(p.u.)\left(\frac{base - kV_{old}}{base - kV_{new}}\right)^2\left(\frac{MVA_{new}}{MVA_{old}}\right) \qquad (0.21)$$

This conversion produces the following values in p.u.: (note positive sequence parameters which will also be negative sequence as they are same in the problem)

G1 $X_{1g1} = X_{2g1} = 0.03, X_{0g1} = 0.01,\ X_{0g1n} = 3 \times 0.03 = 0.09$
G2 $X_{1g2} = X_{2g2} = 0.03, X_{0g2} = 0.01,\ X_{0g2n} = 3 \times 0.03 = 0.09$
T1 $X_{1T1} = X_{2T1} = X_{0T1} = 0.016$
T2 $X_{1T1} = X_{2T1} = X_{0T1} = 0.016$
TL12 $X_{1L12} = X_{2L12} = 0.05, X_{0L12} = 0.15$
TL13 $X_{1L12} = X_{2L12} = 0.025, X_{0L12} = 0.075$
TL23 $X_{1L12} = X_{2L12} = 0.025, X_{0L12} = 0.075$ **[2marks]**

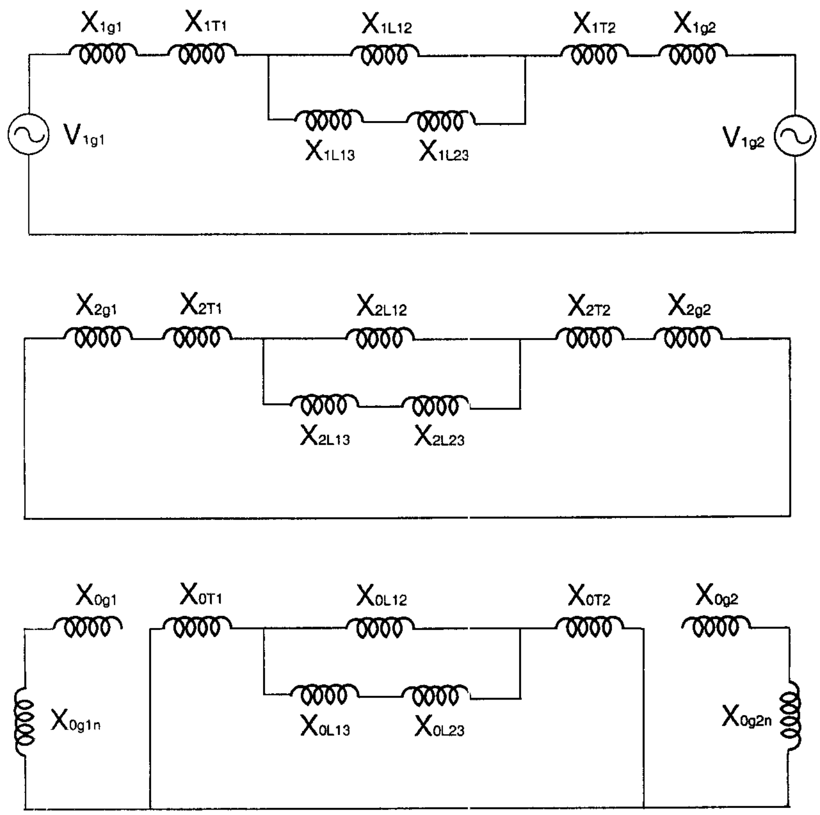

Figure 6.1: The sequence networks representation of the system

The second task is to break this system into three sequence network. This is shown in Fig6.1 where the detail representation for each network is shown.

The poitive sequence network contains voltage sources of 1.05 p.u. voltage. . **[2marks]**

The negative sequence network is easy to obtain once the positive sequence network is obtained. This network would not have any voltage source as negative sequence voltage is not generated. Since all the negative sequence reactance of generator, line and transformer are equal to respective positive sequence reactances as seen in the problem, **[2marks]**

It is interesting to see that the path for zero sequence current is open between bus #5 and #2 because of delta-star transformer T2. Hence the zero sequence network would be different. There would be no zero sequence component current coming from generator G2 because of this broken path. **[3marks]**