UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2002

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C240=MC240

ALGORITHMS, COMPLEXITY AND COMPUTABILITY

Friday 26 April 2002, 14:00
Duration: 90 minutes
(Reading time 5 minutes)

Answer THREE questions

Paper contains 4 questions
Calculators not required

1a Design a 2-tape Turing machine M, with the standard typewriter alphabet C as input, which interleaves data from two strings as follows:

given $w = w_1w_2w_3\ldots w_n$ and $z = z_1z_2z_3\ldots z_m$ where each of the w_i $1 \leq i \leq n$ and each of the z_j $1 \leq j \leq m$ is a symbol in C which is not $*$,

$$f_M(w*z) = w_1z_1w_2z_2w_3z_3\ldots w_nz_nz_{n+1}\ldots z_m \text{ where } m \geq n,$$

$$f_M(w*z) = w_1z_1w_2z_2w_3z_3\ldots w_mz_mw_{m+1}w_{m+2}\ldots w_n \text{ where } m < n.$$

i) draw a diagram of your Turing machine and explain the notation you have used.

ii) explain the operation of the Turing machine in your answer to part a i).

b Derive an expression for the time function $time_M(n,m)$ of the Turing machine M in your answer to part a, explaining each component of your expression clearly.

The two parts carry, respectively, 70%, 30% of the marks.

2a Explain the *Halting Problem*. What does it mean to say that the Halting Problem is unsolvable?

b Explain the technique of reduction and how it can be used to show that a problem is unsolvable.

c Let C be the standard typewriter alphabet, and C* the set of words over C. The symbol $*$ is used as a delimiter. For any standard Turing machine S and word w of C, let the partial function $f : C^* \longrightarrow \{0,1\}$ be given by

$$f(code(S)*w) = \begin{cases} 1, \text{ if S halts on input w} \\ 0, \text{ otherwise.} \end{cases}$$

Prove, either directly or by reduction of the Halting Problem, that there is no Turing machine M such that $f_M = f$.

The three parts carry, respectively, 30%, 30%, 40% of the marks.

3a Explain what is meant by a *minimal spanning tree* (MST) of a connected weighted graph G. Define the *separation property* of a spanning tree for a connected weighted graph.

b Let T be any spanning tree of a connected weighted graph G. Prove that if T has the separation property, then T is a MST of G.
(*Hint*: given a spanning tree T which has the separation property; suppose that T is not minimal and from this derive a contradiction.)

c Explain how the result in part b is used in Prim's algorithm.

d Use Prim's algorithm to find a MST of the graph in fig. 1. List the edges of your MST.

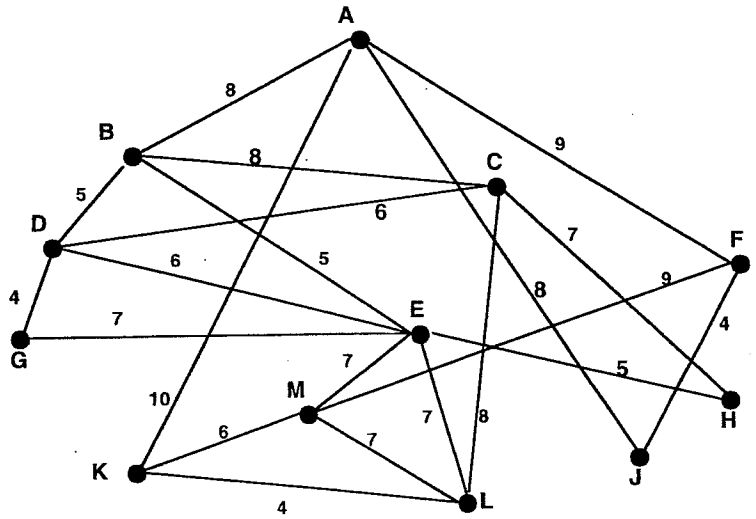

fig. 1

The four parts carry, respectively, 20%, 50%, 10%, 20% of the marks.

4a Define each of the following sets of yes/no problems:

i) P
ii) NP
iii) NPC

and give the relationships between them.

b Define each of the following problems:

i) the Hamiltonian Circuit Problem
ii) PSAT - the Propositional Satisfaction Problem
iii) the problem of finding the Shortest Path between two nodes in a connected weighted graph.

c For each of the problems listed in part b, state, justifying your answer, to which of the sets P, NP, NPC each of them belongs

The three parts carry, respectively, 40%, 15%, 45% of the marks.