EE4-54
EE9-CS4-1

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

MSc and EEE/EIE PART IV: MEng and ACGI

PREDICTIVE CONTROL

Corrected Copy

Wednesday, 29 April 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : E.C. Kerrigan
Second Marker(s) : S.A. Evangelou

1. Suppose we are given a continuous-time system of the form

$$\frac{dq(t)}{dt} = Fq(t) + Gv(t), \quad y(t) = Hq(t),$$

where the state $q(t) \in \mathbb{R}^n$, input $v(t) \in \mathbb{R}^m$ and output $y(t) \in \mathbb{R}^p$. The matrices F, G and H have compatible dimensions.

The system is controlled with a computer at a sample rate of h seconds with a *first-order hold* on the input, i.e. the trajectory $v(\cdot)$ is piecewise linear and continuous with the rate of the input

$$\frac{dv(t)}{dt} = u_k, \forall t \in [kh, kh+h), k \in \mathbb{Z},$$

where k denotes the sample instant and u_k is a discrete-time input signal given by the computer.

The above sampled-data system can be converted into an equivalent discrete-time model of the form

$$x_{k+1} = Ax_k + Bu_k, \quad y_k = Cx_k,$$

where the subscript denotes the signal at time $t = kh$.

Suppose $q(0)$ and $v(0)$ are given. You are required to compute a sequence $\bar{u} := [u_0^T \cdots u_{N-1}^T]^T$ that would steer the state and input of the continuous-time system to the origin, i.e.

$$q(Nh) = 0, \quad v(Nh) = 0.$$

while ensuring that the following constraints are satisfied:

$$\|v(t)\|_\infty \leq 1, \quad \left\|\frac{dv(t)}{dt}\right\|_\infty \leq 1, \quad \forall t \in [0, Nh).$$

a) Give a suitable definition for the state vector x_k. [2]

b) Give a suitable definition for C. [2]

c) Give the definition of the exponential of a matrix X. [2]

d) Use the matrix exponential to show how one would be able to compute A and B, given F and G. State the dimension of all your matrices. [4]

e) Show that the equality constraints can be converted to the form

$$Pq(0) + Qv(0) + R\bar{u} = 0.$$

State the dimension of all your matrices. [4]

f) Show that the inequality constraints above can be converted to the form

$$L\bar{u} \leq Mv(0).$$

State the dimension of all your matrices. [6]

2. In the following, $x \in \mathbb{R}^n$, with A, b, C and d having compatible dimensions. For a vector $v \in \mathbb{R}^m$, the i^{th} component of the vector v^+ is defined as

$$v_i^+ := \max\{0, v_i\},\ i = 1, \ldots, m.$$

The optimal value f^* of a function $f : \mathbb{R}^n \to \mathbb{R}$ is denoted by

$$f^* := \min_x f(x).$$

a) Show that if $f(x) := \|Ax - b\|_2^2$, then one can compute f^* by formulating and solving a system of linear equations. [2]

b) Show that if $f(x) := \|Ax - b\|_1^2$, then one can compute f^* by formulating and solving a quadratic program. [6]

c) Show that if $f(x) := \|Ax - b\|_\infty^2$, then one can compute f^* by formulating and solving a quadratic program. [2]

d) Show that if $f(x) := \|Ax - b\|_1 + \|(Cx - d)^+\|_\infty$, then one can compute f^* by formulating and solving a linear program. [6]

e) Show that if $f(x) := \|Ax - b\|_2^2 + \|(Cx - d)^+\|_2^2$, then one can compute f^* by formulating and solving a quadratic program. [4]

3. We are interested in solving the following optimal control problem:

$$\min_{u_0,u_1,\ldots,u_{N-1}} \|Px_N\|_\infty + \sum_{k=0}^{N-1}\left[\|Qx_k\|_2^2 + \|Ru_k\|_1\right]$$

subject to

$$x_0 = \hat{x},\ x_{k+1} = Ax_k + Bu_k, \quad k = 0,1,\ldots,N-1,$$

where the state $x_k \in \mathbb{R}^n$, input $u_k \in \mathbb{R}^m$ and weights P, Q and R are square matrices. Assume an estimate of the current state $\hat{x}$ is given.

You are required to convert the above problem into an equivalent QP of the form

$$\min_\theta \frac{1}{2}\theta' H\theta + c'\theta$$

subject to

$$M\theta \leq f.$$

a) Why can the solution to the above optimal control problem not be found by solving a single set of linear equations, even if there are no inequality constraints on the state or input? [1]

b) Give a suitable definition for θ. [3]

c) Give an expression for M in terms of the problem data. [6]

d) How many rows and columns does M have? [2]

e) Give an expression for f in terms of the problem data. [2]

f) Give an expression for H in terms of the problem data. [3]

g) Give an expression for c in terms of the problem data. [3]

4. In the following, please keep your answers clear and concise, using no more than 20 words per point, e.g. if the answer is worth 3 points, as in part a) below, then you should use no more than 60 words, etc. State the number of words you used for each answer. Marks will be deducted if you have gone over the limit. Equations are not allowed.

a) Explain when predictive control can be considered to be an optimal control method. [2]

b) Why are numerical methods, such as QP solvers, used to compute the solution to predictive control problems on-line, rather than computing off-line an explicit control law? [2]

c) Explain briefly what the difference is between an interior point QP solver and an active set QP solver. [4]

d) What is the main difference between 'receding horizon control' and 'decreasing horizon control'? [2]

e) Give an example of an application where it is preferable to implement a decreasing horizon controller, rather than a receding horizon controller. Give a reason for your answer. [2]

f) Explain how you would design a predictive controller for an autonomous car that would steer the car back into its lane if it had to go outside the lane to avoid an accident? [2]

g) Why is nonlinear control theory, such as Lyapunov's direct method, used to prove stability of receding horizon controllers, even if the system to be controlled is linear? [2]

h) Explain briefly how you would design a predictive controller to ensure that a constant input disturbance is rejected. [4]