EE4-24
EE9-CS1-3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

MSc and EEE/EIE PART IV: MEng and ACGI

DISCRETE-TIME SYSTEMS AND COMPUTER CONTROL

Friday, 19 May 10:00 am

Time allowed: 3:00 hours

Corrected copy

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

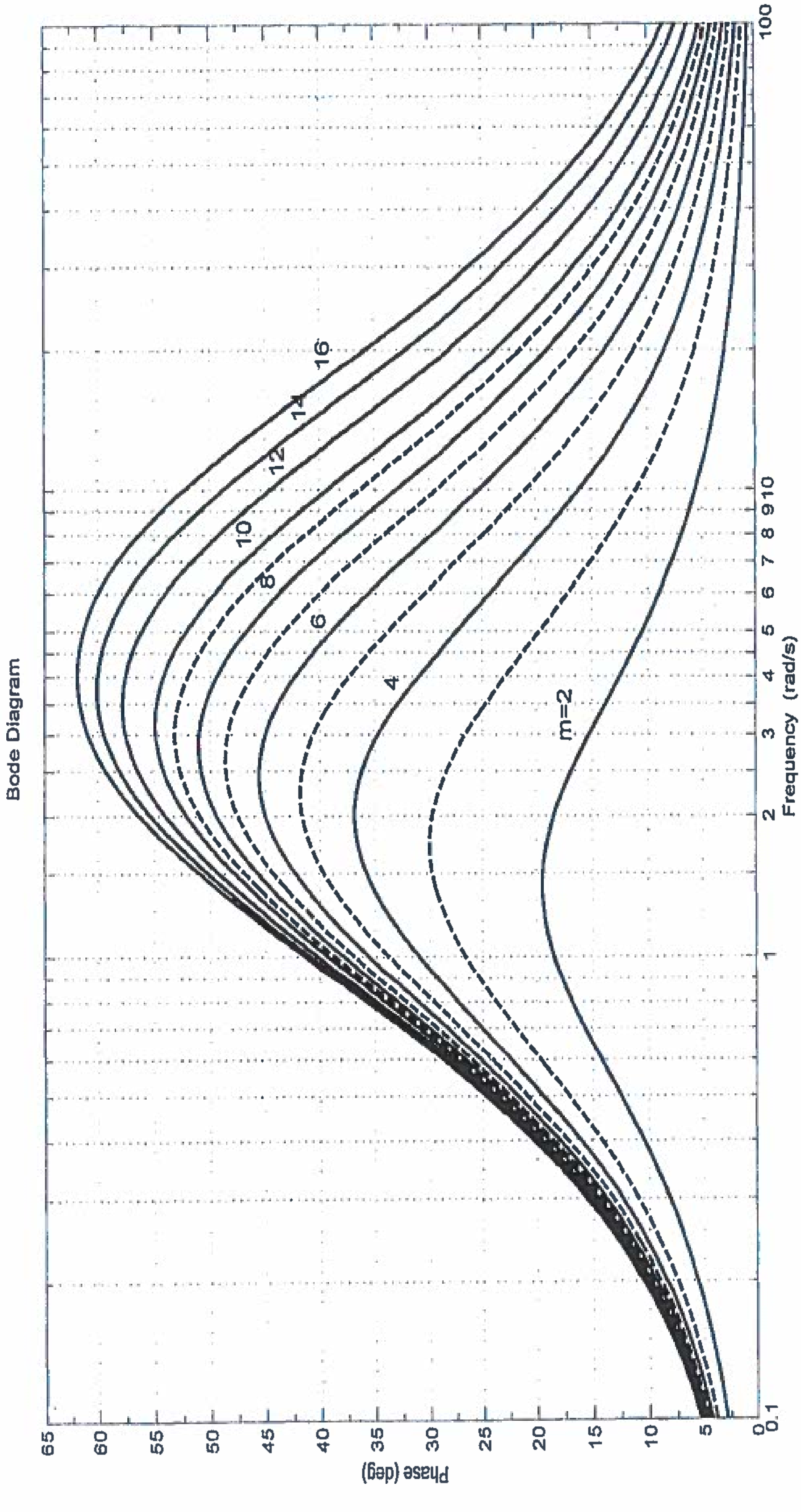
Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : G. Scarciotti
Second Marker(s) : A. Astolfi

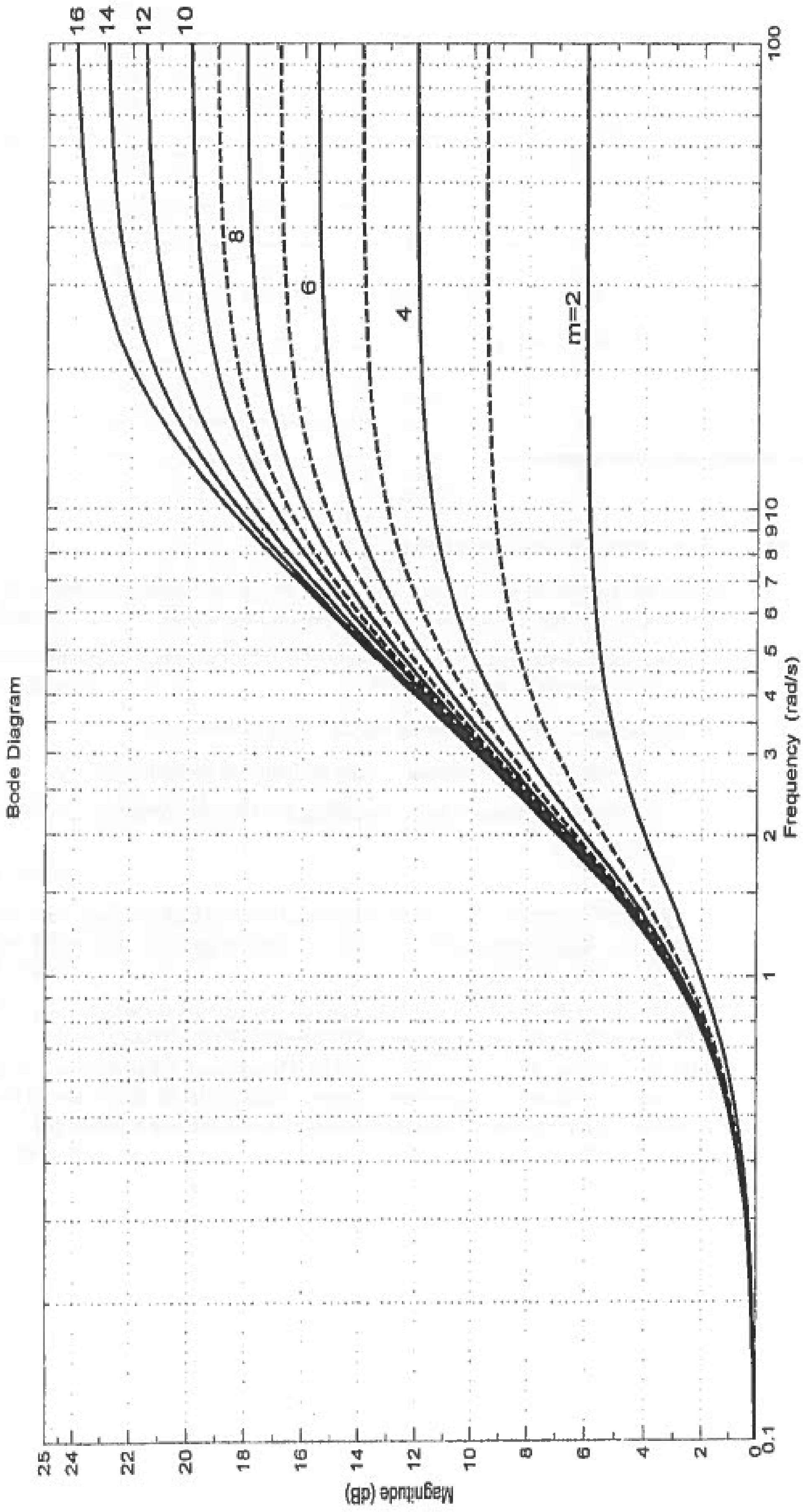
Discrete-time Systems and Computer Control

Information for candidates:

- $Z\left(\frac{1}{s}\right) = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$
- $Z\left(\frac{1}{s+a}\right) = \frac{z}{z-e^{-aT}} = \frac{1}{1-z^{-1}e^{-aT}}$
- $Z\left(\frac{1}{s^2}\right) = T\frac{z}{(z-1)^2} = T\frac{z^{-1}}{(1-z^{-1})^2}$
- $Z\left(\frac{b}{(s+a)^2+b^2}\right) = \frac{ze^{-aT}\sin bT}{z^2-2ze^{-aT}\cos bT+e^{-2aT}}$
- Transfer function of the ZOH: $H_0(s) = \frac{1-e^{-sT}}{s}$
- Definition of the w-plane: $z = \frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}$, $w = \frac{2}{T}\frac{z-1}{z+1}$
- Tustin transformation: $s = \frac{2}{T}\frac{z-1}{z+1}$
- Note that, for a given signal r, or $r(t)$, $R(z)$ denotes its Z-transform, where the underlying sampling time T has been used to sample $r(t)$.
- Consider the characteristic equation $1+K\frac{N(z)}{D(z)} = 0$. $N(z)$ has roots z_i, $i = 1,\dots,m$, and $D(z)$ has roots p_i, $i = 1,\dots,n$. If $n > m$ the asymptotes of the root locus intersect the real axis at $\sigma = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m}$, and radiate out with angles $\theta = \pm q\frac{\pi}{n-m}$, where $q = 1,3,5,\dots$. The break-away or break-in points of the locus are the roots of the polynomial $N(z)\frac{d}{dz}D(z) - D(z)\frac{d}{dz}N(z) = 0$.
- Let $C(s) = \frac{1+\tau s}{1+\frac{\tau}{m}s}$ be a compensator parametrized by a positive integer m and a positive real number τ. The Bode plot of this compensator for $\tau = 1$ and different values of m is shown in the next two pages. The first page shows the phase plot parametrized by m, whereas the second page shows the magnitude plot parametrized by m. Recall that if $\hat{\omega}$ is the frequency at which you want to place the phase peak of the compensator, then $\tau = \frac{\sqrt{m}}{\hat{\omega}}$.



Bode Diagram
Phase (deg)
Frequency (rad/s)
m=2
4
6
8
10
12
14
16
65
60
55
50
45
40
35
30
25
20
15
10
5
0
0.1
1
2
3
4
5
6
7
8
9 10
100


Bode Diagram
Magnitude (dB)
Frequency (rad/s)
m=2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
25
0.1
1
2
3
4
5
6
7
8
9 10
100

1. Consider the digital control system in Figure 1.1.

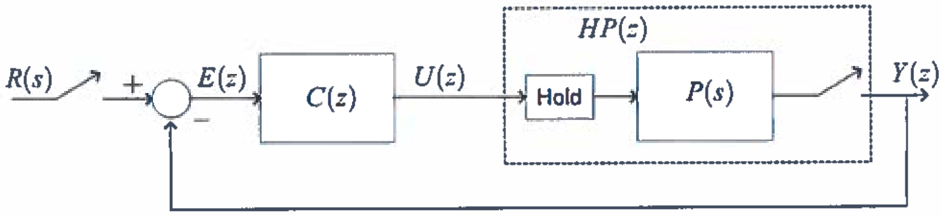


Figure 1.1 Block diagram for question 1.

Consider the continuous-time plant

$$P(s) = \frac{K}{s}$$

controlled using the controller

$$C(s) = \frac{1}{s+a},$$

with $a > 0$, in a unity feedback configuration.

a) Sketch the root locus of $C(s)P(s)$. Determine for which values of K and a the closed-loop continuous-time system is asymptotically stable. [4 marks]

b) Compute the equivalent discrete-time model $HP(z)$ for the plant interconnected to the zero-order hold and the sampler. [3 marks]

c) Discretize the controller $C(s)$ using the following methods.

   i) Impulse response invariance method (call this controller $C_I(z)$).

   ii) Pole-zero correspondence, matching the DC gain (call this controller $C_{PZ}(z)$).

[3 marks]

d) Consider the controller $C_I(z)$ found in part c.i) and sketch the root locus of $C_I(z)HP(z)$. Consider the controller $C_{ZP}(z)$ found in part c.ii) and sketch the root locus of $C_{PZ}(z)HP(z)$. [4 marks]

e) Consider the controller $C_I(z)$ found in part c.i). Determine for which values of K the closed-loop discrete-time system is asymptotically stable. Hence, consider the controller $C_{ZP}(z)$ found in part c.ii). Determine for which values of K the closed-loop discrete-time system is asymptotically stable. Comment on the difference (if any) between these two ranges of K and the one found in part a). [6 marks]

2. a) Consider the shaded s-plane region shown in Figure 2.1. Given the sampling time $T = 3.92699$, explain why the whole shaded region lies in the primary strip. Sketch the z-plane region that corresponds to the shaded s-plane region.

[4 marks]

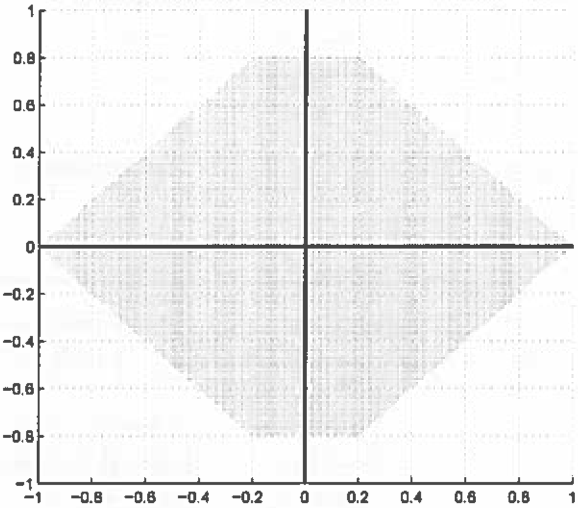


Figure 2.1 Shaded s-plane region for question 2.

b) Consider the digital control system in Figure 2.2. Assume that the samplers are impulsive samplers. Determine the pulse transfer function between the input $R(z)$ and the output $Y(z)$.

[4 marks]

c) Consider the digital control system in Figure 2.3. Assume that the reference signal is $R(z) = 0$. Determine the pulse transfer function between the disturbance $D_1(z)$ and the output $Y(z)$. Explain if the gains of $C_1(z)$ and $C_2(z)$ should be selected large or small to minimize the effect of the disturbance.

[4 marks]

d) Consider the digital control system in Figure 2.4. Assume that the reference signal is $R(z) = 0$. Determine the pulse transfer function between the disturbance $D_2(z)$ and the output $Y(z)$. Explain if the gains of $C_1(z)$ and $C_2(z)$ should be selected large or small to minimize the effect of the disturbance.

[4 marks]

e) Consider the digital control system in Figure 2.5. Assume that the reference signal is $R(z) = 0$. Determine the pulse transfer function between the disturbance $D_3(z)$ and the output $Y(z)$. Explain if the gains of $C_1(z)$ and $C_2(z)$ should be selected large or small to minimize the effect of the disturbance.

[4 marks]

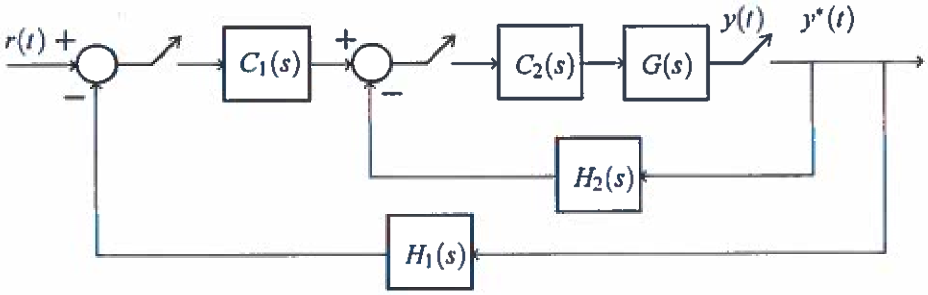


Figure 2.2 Block diagram for part b) of question 2.

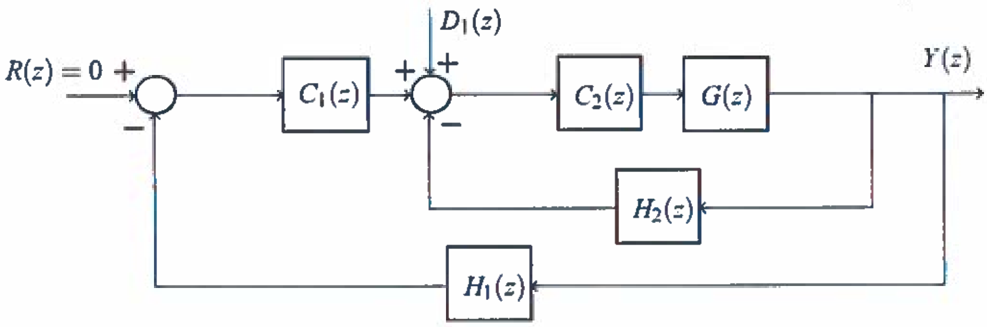


Figure 2.3 Block diagram for part c) of question 2.

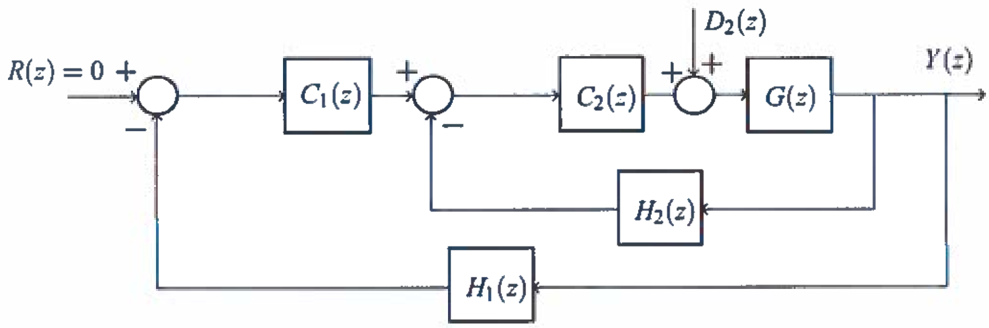


Figure 2.4 Block diagram for part d) of question 2.

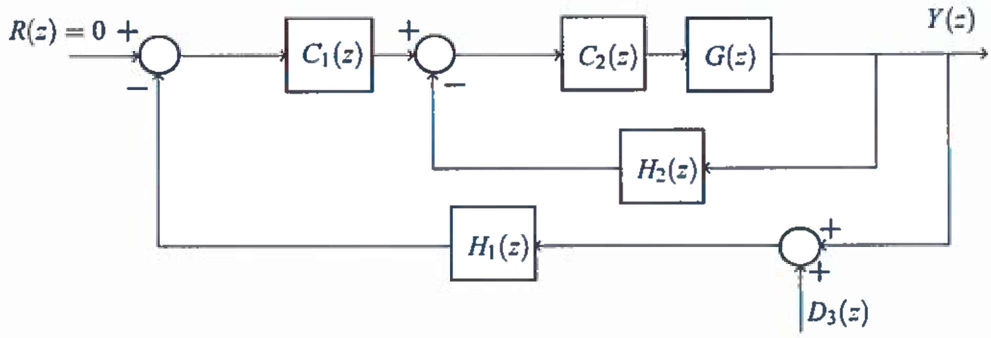


Figure 2.5 Block diagram for part e) of question 2.

3. Consider a process to be controlled with transfer function

$$P(s) = \frac{5}{(s+1)(s+2)}.$$

Assume the system is interconnected to a ZOH and a sampler. Let $T = 0.1$ be the sampling time.

a) Compute the equivalent discrete-time model $HP(z)$ for the plant interconnected to the hold and the sampler. [4 marks]

b) Using the definition of the w-plane show that the transfer function $HP(w)$ is

$$HP(w) = -0.000622\frac{(w+400.33)(w-20)}{(w+1.9934)(w+0.9992)}.$$

[3 marks]

c) Let

$$C_1(w) = K\,\frac{1}{w^r},$$

with r a non-negative integer number. Select numerical values for K and r such that the system has velocity constant $K_v = 1$. Determine the open-loop transfer function obtained from the interconnection of $C_1(w)$ with the transfer function $HP(w)$. (Hint: recall that, in the w plane, the velocity constant is given by $K_v = \lim_{w\to 0} w\, C_1(w)\, HP(w)$.)

[3 marks]

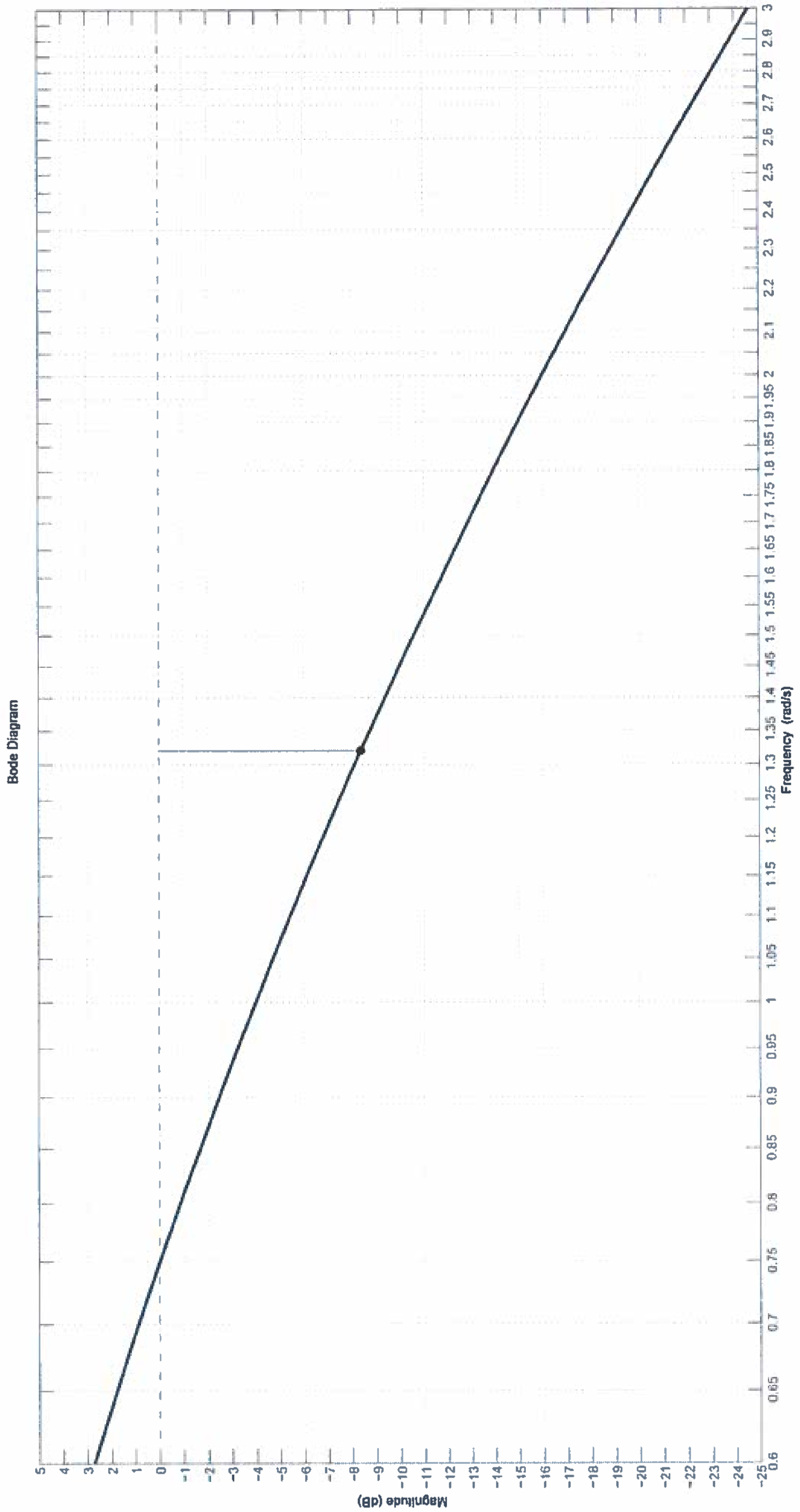
d) Let

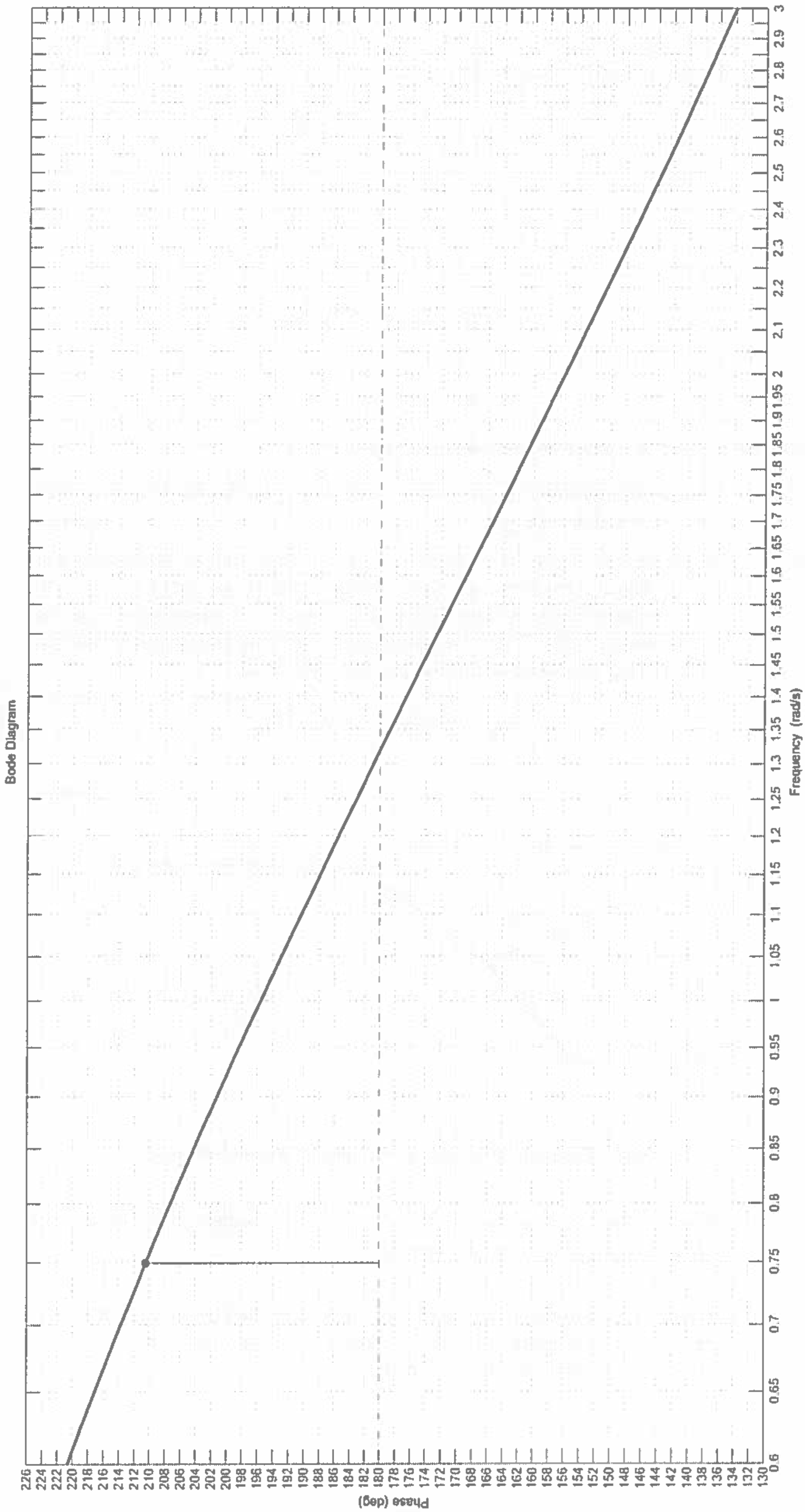
$$C_2(w) = \frac{1+\tau w}{1+\dfrac{\tau}{m}w},$$

with $\tau > 0$ and $m > 0$ to be determined. Consider the closed-loop system resulting from the unity feedback interconnection of the controller $C_2(w)$ with the transfer function $C_1(w)HP(w)$ obtained in part c). Determine the controller $C_2(w)$ such that the phase margin is at least 45°. To help the design, the Bode plot of the correct $C_1(w)HP(w)$ is represented in the next two pages. The Bode plot of the phase-lead compensator is given at the beginning of this exam paper. (Hint: note that more than one design is possible.)

[6 marks]

e) Using the numerical values determined in part c) and d) compute the discrete-time controller $C_1(z)C_2(z)$. [4 marks]


Bode Diagram
Magnitude (dB)
Frequency (rad/s)


Bode Diagram
Phase (deg)
Frequency (rad/s)

4. Consider the digital control system in Figure 4.1.

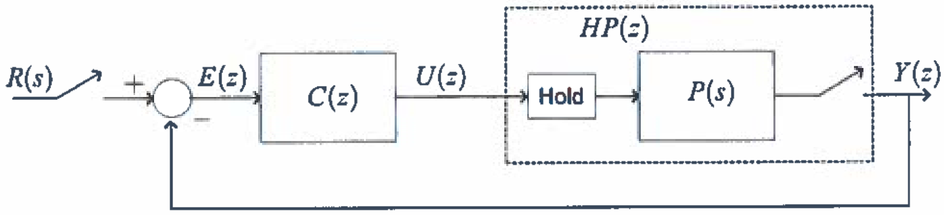


Figure 4.1 Block diagram for question 4.

Let

$$P(s) = \frac{e^{-s}}{2s+1}.$$

Assume the hold is a ZOH and the sampling time is $T = 1$.

a) Compute the equivalent discrete-time model $HP(z)$ for the plant interconnected to the hold and the sampler. [4 marks]

b) Let the desired output $\tilde{y}(t)$ in response to a unit-step input be as shown in Figure 4.2. The curve is zero from 0 sec to 1 sec. It then rises as $\alpha(1-e^{-0.5(t-1)})$, with α to be determined, from 1 sec to 3 sec. Finally it reaches one at 3 sec and it stays constant after 3 sec. Determine the value of α and write the values of $\tilde{y}(kT)$ for $k \geq 0$. Show that the z-transform of $\tilde{y}(kT)$ is

$$\tilde{Y}(z) = \frac{0.6225z^{-2} + 0.3775z^{-3}}{1-z^{-1}}.$$

[6 marks]

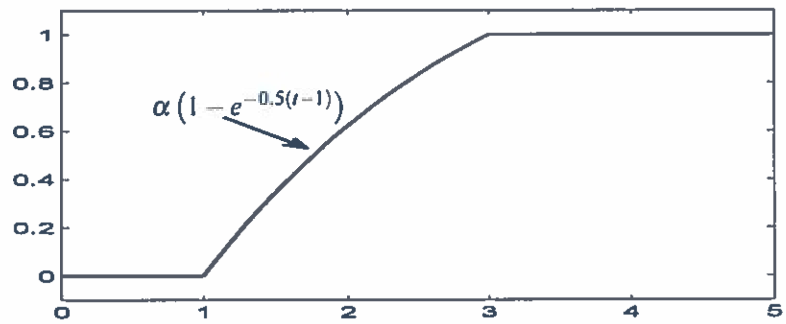


Figure 4.2 Desired output $\tilde{y}(t)$ in response to a unit-step input.

c) Design a discrete-time controller $C(z)$ such that the output of the closed-loop system is $\tilde{Y}(z)$. Show that $C(z)$ has a pole at $z = 1$.

[6 marks]

d) Show that the control signal $u(kT)$ (the signal generated by the controller $C(z)$, see Figure 4.1) is constant for $k \geq 2$. Sketch the signal $u(t)$ for $0 \leq t \leq 5$ ($u(t)$ is the signal obtained by the ZOH action).

[4 marks]