EE1-01

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2015

EEE/EIE PART I: MEng, BEng and ACGI

ANALYSIS OF CIRCUITS

Corrected Copy

Friday, 5 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D.M. Brookes

Second Marker(s) : P. Georgiou

ANALYSIS OF CIRCUITS

Information for Candidates:

The following notation is used in this paper:

1. The voltage waveform at node X in a circuit is denoted by $x(t)$, the phasor voltage by X and the root-mean-square (or RMS) phasor voltage by $\widetilde{X} = \frac{X}{\sqrt{2}}$. The complex conjugate of X is X^*.
2. Component and source values in a circuit are normally given in Ohms, Farads, Henrys, Volts or Amps with the unit symbol omitted. Where an imaginary number is specified, it represents the complex impedance or phasor value.
3. Times are given in seconds unless otherwise stated.
4. Unless otherwise indicated, frequency response graphs should use a linear axis for phase and logarithmic axes for frequency and magnitude.

1. a) Using nodal analysis, calculate the voltages at nodes X and Y of Figure 1.1. [5]

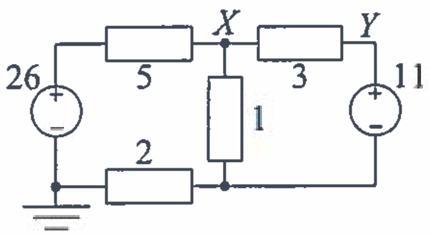

Figure 1.1

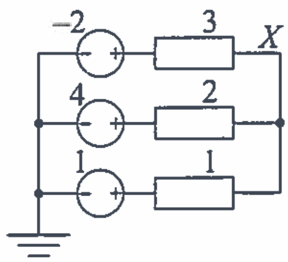

Figure 1.2

b) Use the principle of superposition to find the voltage X in Figure 1.2. [5]

c) Draw the Thévenin equivalent circuit of the network in Figure 1.3 and find the values of its components. [5]

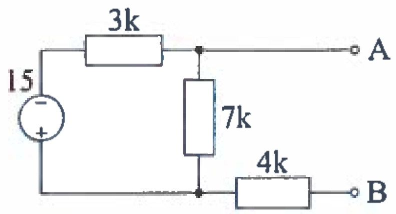

Figure 1.3

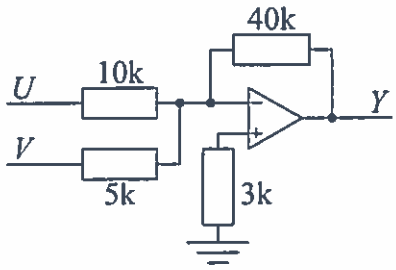

Figure 1.4

d) Assuming the opamp in the circuit of Figure 1.4 is ideal, give an expression for Y in terms of U and V. [5]

e) The waveform, $x(t)$, is a periodic triangle wave of amplitude ± 4 V as shown in Figure 1.5. The waveform is applied to the input, X, of the circuit shown in Figure 1.6. The diode has a forward voltage drop of 0.7 V and is otherwise ideal.

Determine the maximum and minimum values of the waveform $y(t)$ and determine the input voltage, x_0, at which the diode turns on. [5]

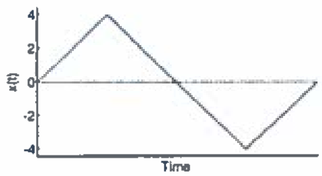

Figure 1.5

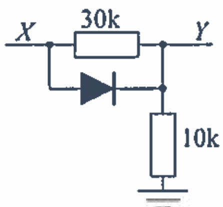

Figure 1.6

f) Determine the gain, $\frac{Y}{X}$, for the block diagram shown in Figure 1.7. The rectangular blocks are drawn with inputs at the left and outputs at the right and have gains of F and G respectively. The open circles represent adder/subtractors; their inputs have the signs indicated on the diagram and their outputs are V and W respectively. [5]

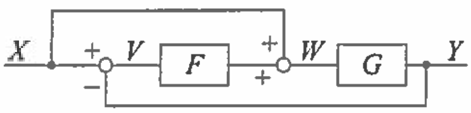

Figure 1.7

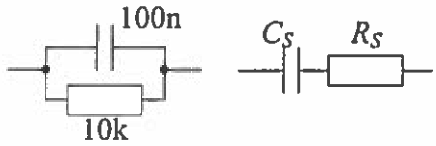

Figure 1.8

g) i) Determine C_S and R_S so that the two networks in Figure 1.8 have the same impedance at $\omega_0 = 2000\,\mathrm{rad/s}$.

ii) Using logarithmic axes for both frequency and impedance sketch a graph showing the impedance of both networks for the frequency range $20 < \omega < 200000$. [5]

h) The waveform, $x(t)$, shown in Figure 1.9 is applied to the input, X, of the circuit shown in Figure 1.10. Determine the time constant of the circuit and the amplitude of the transient.

Hence draw a dimensioned sketch of the waveform at Y . [5]

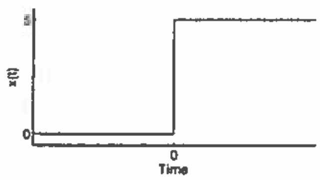

Figure 1.9

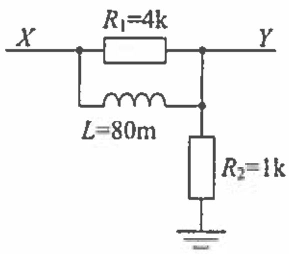

Figure 1.10

2. A second order transfer function is given by

$$H(j\omega) = \frac{-G}{\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\frac{j\omega}{\omega_0} + 1}$$

where G, ω_0 and ζ are positive real numbers.

a) Determine the magnitude and phase of $H(j\omega)$ at [4]

i) $\omega = 0$,

ii) $\omega = \omega_0$,

iii) $\omega \gg \omega_0$.

b) If we define $\phi(\omega) = \angle H(j\omega)$, show that $\phi(\omega) = \tan^{-1}\left(\frac{2\zeta\omega_0\omega}{\omega^2-\omega_0^2}\right)$ and hence show that its derivative at ω_0 equals $\phi'(\omega_0) = \frac{-1}{\zeta\omega_0}$. [6]

c) Suppose that $G = 5$, $\zeta = 0.8$ and $\omega_0 = 10^4$ rad/s.

i) Sketch a dimensioned graph of $|H(j\omega)|$ in decibels using a logarithmic frequency axis. Your graph should include a sketch of the true magnitude response in addition to the high and low frequency asymptotes. [3]

ii) Sketch a dimensioned graph of $\angle H(j\omega)$ using a linear phase axis in radians and a logarithmic frequency axis. [3]

d) Fig. 2.1 shows the circuit diagram of a filter circuit. Use nodal analysis to show that the frequency response of the filter is given by

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{-R_2}{R_1R_2R_3C_1C_2(j\omega)^2 + (R_1R_2 + R_1R_3 + R_2R_3)C_1 j\omega + R_1}.$$

[6]

e) Find expressions for G, ω_0 and ζ in terms of the component values when the frequency response of the filter is expressed in the form given for $H(j\omega)$ above. [4]

f) If $R_2 = 60\,\text{k}\Omega$ and $R_3 = \frac{50}{3}\,\text{k}\Omega$ determine values for R_1, C_1 and C_2 such that $G = 5$, $\zeta = 0.8$ and $\omega_0 = 10^4$ rad/s. [4]

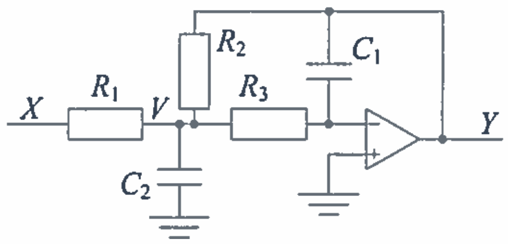

Figure 2.1

3. The circuit of Fig. 3.1 shows a transmission line of length L driven by a voltage source $v_S(t)$ through a resistor, R_S. The characteristic impedance and propagation velocity of the line are Z_0 and u respectively. The phasor corresponding to the waveform $v_S(t)$ is written V_S and similarly for other waveforms.

The voltage and current waveforms at a distance x from the source are given respectively by

$$\begin{aligned} v_x(t) &= f_x(t) + g_x(t) \\ i_x(t) &= Z_0^{-1}\left(f_x(t) - g_x(t)\right) \end{aligned}$$

where $f_x(t) = f_0\left(t - u^{-1}x\right)$ and $g_x(t) = g_0\left(t + u^{-1}x\right)$ are the forward and backward waves at a distance x from the source.

a) Show that if $f_0(t) = A\cos(\omega t + \phi)$ then the phasors F_x and F_0 satisfy

$$F_x = F_0 e^{-j\omega u^{-1}x}.$$

Determine a similar expression relating G_x and G_0. [5]

You may assume without proof that the phasor corresponding to $A\cos(\omega t + \phi)$ is $Ae^{j\phi}$.

b) Use the load equation $V_L = I_L R_L$ to show that G_0 can be written in the form $G_0 = \rho_L e^{j\theta} F_0$ and determine expressions for the real-valued constants ρ_L and θ. [5]

c) By applying Kirchoff's current law at the point marked $v_0(t)$ in Fig. 3.1, show that F_0 may be expressed as $F_0 = \tau_S V_S + \rho_S G_0$ and determine expressions for the real-valued constants τ_S and ρ_S. [5]

d) Eliminate G_0 between the answers to parts b) and c) to obtain an expression for F_0 in terms of V_S. [4]

e) Suppose that $R_S = 25\,\Omega$, $R_L = 400\,\Omega$, $Z_0 = 100\,\Omega$, $L = 10\,\text{m}$, $u = 1.5 \times 10^8\,\text{m/s}$, $V_S = 10j$ and $\omega = 6 \times 10^7\,\text{rad/s}$.

Determine the phasors V_0 and I_0. [6]

f) Calculate the complex power supplied by V_S and the average power absorbed by R_S. Hence deduce the average power absorbed by R_L. [5]

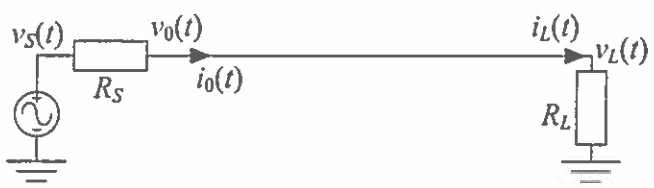

Figure 3.1