IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2017

EEE/EIE PART II: MEng, BEng and ACGI

SIGNALS AND LINEAR SYSTEMS

Corrected copy

Tuesday, 30 May 10:00 am

Time allowed: 2:00 hours

Correction Q3 p.8

There are THREE questions on this paper.

Answer ALL questions.

Question One carries 40% of the marks. The other 2 questions each carry 30%.

Any special instructions for invigilators and information for candidates are on page 1.

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Second Marker(s) : P.T. Stathaki

Special Information for the Invigilators: none

Information for Candidates

Some Fourier transforms

$$\mathrm{rect}(\tfrac{t}{\tau}) \iff \tau\,\mathrm{sinc}(\tfrac{\omega\tau}{2})$$

$$\tfrac{W}{\pi}\mathrm{sinc}(Wt) \iff \mathrm{rect}(\tfrac{\omega}{2W})$$

The unit step function $u(t)$ is defined as:

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Some useful Laplace transforms

$$e^{\lambda t}u(t) \iff \frac{1}{s-\lambda} \qquad Re\{s\} > \lambda$$

$$t^n e^{\lambda t}u(t) \iff \frac{n!}{(s-\lambda)^{n+1}} \qquad Re\{s\} > \lambda$$

Time-shifting property of the Laplace transform

$$x(t-t_d) \iff X(s)e^{-st_d}$$

Frequency-shifting property of the Laplace transform

$$x(t)e^{s_0 t} \iff X(s-s_0)$$

Initial Value Theorem:

$$\lim_{t\to 0} x(t) = x(0^+) = \lim_{s\to\infty} sX(s)$$

Final Value Theorem:

$$\lim_{t\to\infty} x(t) = x(\infty) = \lim_{s\to 0} sX(s)$$

The Questions

1. This question carries 40% of the mark.

   (a) Given the signal $x(t)$ shown in Fig. 1a, sketch and dimension each of the following signals:

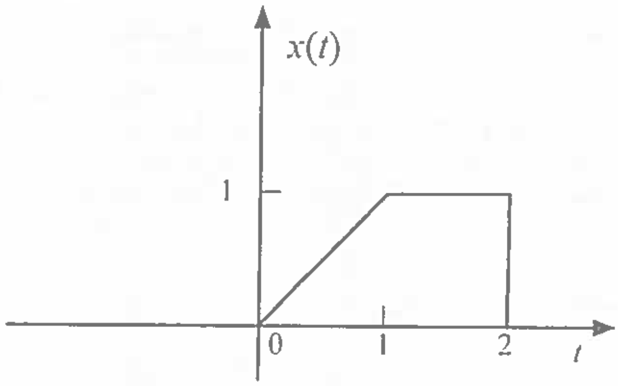

Figure 1a: A continuous-time signal

   i. $x_1(t) = x(-3t)$ [2]

   ii. $x_2(t) = x(-t/2 + 3)$ [2]

   (b) Find the even and odd components of $x(t) = e^{-t/2}\cos(t)u(t)$. [2]

   (c) State whether the causal linear-time invariant (LTI) system with the following unit impulse response is stable or not. Explain your answer:

$$h(t) = te^{-t}u(t)$$

[2]

Question 1 continues on next page

(d) State whether the causal linear-time invariant (LTI) system with the following transfer function is stable or not. Explain your answer:

$$H(s) = \frac{1}{s^2 + 2s + 5}$$

[2]

(e) In many applications, it is often undesirable for the step response of a system to overshoot its final value (remember that for step response we mean the response of a system when the input is the step function). Show that if $h(t)$, the impulse response of a LTI system, is always greater than or equal to zero, the step response of the filter is a monotonically nondecreasing function and therefore will not have overshoot.

[5]

(f) Given the following two signals

$$x_1(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

and

$$x_2(t) = \begin{cases} e^{2t}, & 0 \le t \le 1 \\ 0, & \text{otherwise,} \end{cases}$$

compute the convolution $c(t) = x_1(t) * x_2(t)$.

[5]

(g) Compute the Laplace transform of $x(t) = e^{-t}u(t-2)$

[2]

Question 1 continues on next page

(h) Find the inverse Laplace transform of

$$X(s) = \frac{1}{(s^2 + 4s + 3)(s^2 + 2s + 1)}$$

[4]

(i) A linear time-invariant system is specified by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 12y(t) = x(t).$$

i. Find the characteristic polynomial and characteristic roots of this system. [2]

ii. Find the zero-input component of the response $y(t)$ for $t \geq 0$, if the initial conditions are $y(0) = 1$ and $\dot{y}(0) = 0$. [2]

iii. Find the zero-state response assuming $x(t) = e^{-t}u(t)$ where $u(t)$ is the unit step function. [2]

iv. Finally find the total response of the system when the initial conditions are $y(0) = 1$ and $\dot{y}(0) = 0$ and the input is $x(t) = e^{-t}u(t)$. [2]

(j) A signal $x(t)$ is sampled with sampling period $T = 0.01$ sec leading to the samples $x_n = x(nT)$.

i. What is the highest frequency that $x(t)$ can contain if aliasing is to be avoided in the conversion process? [2]

ii. Assume $x(t) = 84\text{sinc}(84\pi t)$.

A. Sketch and dimension the Fourier transform of $x(t)$. [2]

B. Is the sampling period $T = 0.01$ sec small enough for aliasing to be avoided? Justify your answer. [2]

2. Consider the system connected in parallel as depicted in Fig. 2a. Both S_1 and S_2 are causal systems.

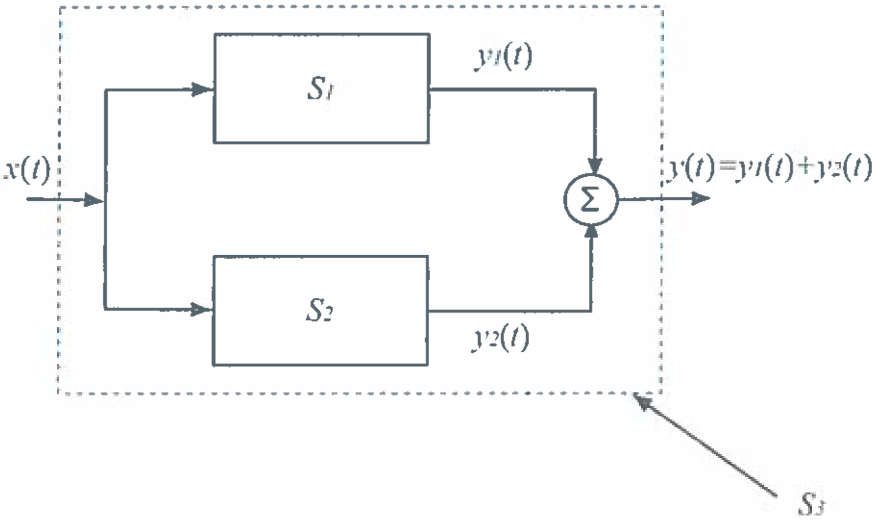

Figure 2a: A parallel system.

The linear system S_1 has the following input/output relationship:

$$\frac{d^2 y_1(t)}{dt^2} + a_1 \frac{dy_1(t)}{dt} + a_2 y_1(t) = x(t)$$

and the LTI system S_2 has the following transfer function

$$H_2(s) = \frac{1}{s + b_1}.$$

Question 2 continues on next page

(a) Find the transfer function $H_1(s)$ of system S_1. [5]

(b) Find the transfer function $H_3(s)$ of the parallel connected system S_3. [5]

(c) Your aim now is to determine the real-valued coefficients a_1, a_2 and b_1 using the following information: The poles of $H_1(s)$ are all real and the region of convergence (ROC) of $H_1(s)$ is $Re\{s\} > -1$. Moreover, $\lim_{t\to\infty} y(t) = 1$ and $\lim_{t\to\infty} y_1(t) = 1/2$ when the input $x(t) = u(t)$. [8]

(d) Consider now a linear time-invariant (LTI) system whose input response $g(t)$ is real and whose transfer function is $G(s)$. Assume that the input is $x(t) = e^{-3t}\cos t$ and that the corresponding output $y(t)$ is well defined for this input.

i. If you were allowed to determined $G(s)$ for only a single value of s, which value would you pick in order to obtain an explicit expression for the output $y(t)$ corresponding to the above input $x(t)$? [8]

ii. Suppose it is known that $y(0) = 0$ and that the first order derivative of $y(t)$ at zero is $\dot{y}(0) = 1$, determine the exact time-domain expression of $y(t)$.
[Hint: use the identity $cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$]. [4]

3. You have been given two systems S_1 and S_2 with transfer functions

$$H_1(s) = \frac{1}{s+a}$$

and

$$H_2(s) = \frac{1}{s+b},$$

respectively. Here a and b are real-valued constants. You have been asked to combine them in order to realize a new system S_3 with transfer function

$$H_3(s) = \frac{s+2}{s^2+6s+9}.$$

You decide to use the feedback configuration shown in Fig. 3a.

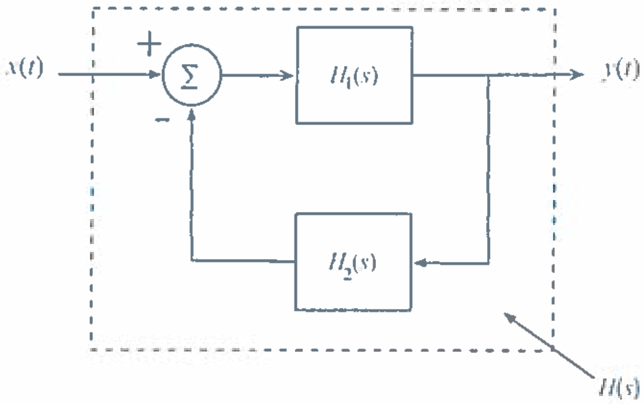

Figure 3a: A feedback system

(a) Determine the transfer function $H(s) = Y(s)/X(s)$ of the feedback system. [5]

(b) Determine a and b so that $H(s) = H_3(s)$. [5]

Question 3 continues on next page

$-3t$

(c) The system in part (b) outputs $y(t) = [-\frac{1}{4}e^{-3t} + \frac{1}{2}te^{-t} + \frac{1}{4}e^{-t}]u(t)$ when the input is $x(t) = e^{-t}u(t)$. Determine the exact expression of the output $y(t)$ assuming the input is $x(t) = e^{-t}[u(t) - u(t-5)]$. [5]

(d) Assume now that $x(t)$ is the combination of a signal $f(t)$ and its delayed version: $x(t) = f(t) + f(t-T)$. You want your feedback system to remove the delayed version. Therefore, design $H_1(s)$ and $H_2(s)$ so that $y(t) = f(t)$. Please note that $H_1(s)$ and $H_2(s)$ are no longer limited to the form given at the beginning of Question 3. [7]

(e) You are now given three systems with the following transfer functions:

$$H_1(s) = \frac{1}{s+a},$$

$$H_2(s) = \frac{1}{s+b}$$

and

$$H_3(s) = K,$$

where a, b, and K are all real-valued constants. You have been asked to combine them in order to realise a system with the following transfer function:

$$H(s) = \frac{2}{s^2 + 5s + 6}.$$

Find a configuration that gives you the desired transfer function and determine the constants a, b and K. You want a and b such that $H_1(s)$ and $H_2(s)$ are stable. Also note that the solution might not be unique. [Hint: consider using a configuration with feedback.] [8]