IMPERIAL COLLEGE LONDON

BEng, MEng and ACGI Examinations 2015-16

Part 1

Biomedical Engineering
BE1-MECH 1
Mechanics 1, Main Exam

14/06/2016, 11.00 – 13.00
Duration: 120 minutes

The paper has 4 (FOUR) questions.
Answer all 4 questions.
Each question is worth 100 marks.

Marks for questions and parts of questions are shown next to the question. The marks for questions (and parts thereof) are indicative, and they may be slightly moderated at the discretion of the examiner.

Question 1.

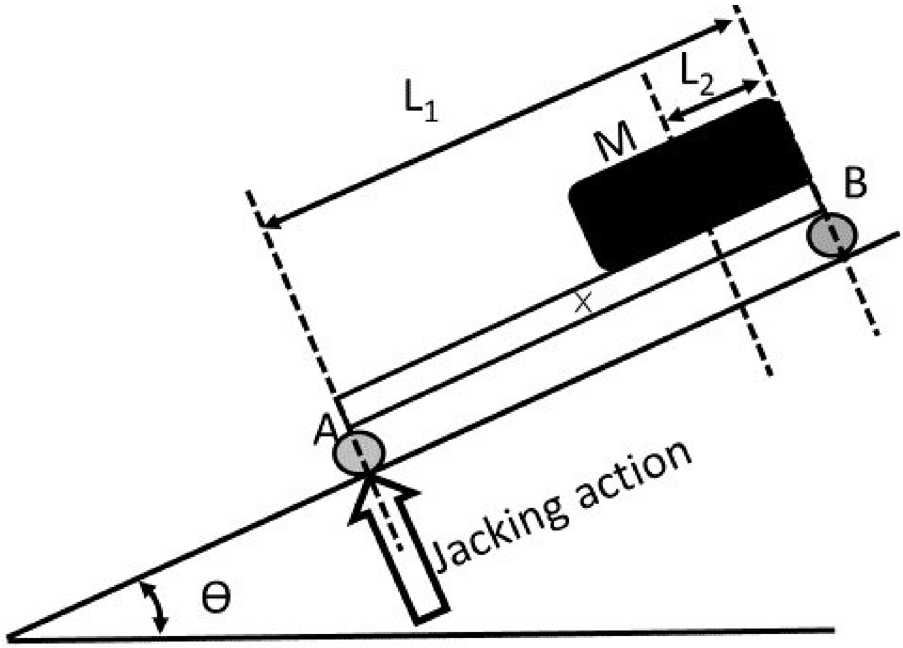


Figure 1

Your trailer (of mass m_t) has suffered a flat tyre (the front wheel at A) on a slope, and you need to jack it up to change the wheel. The brakes are locked on on all wheels. The load M is located with its Centre of Mass L_2 forward of the back axle B.

(a) Draw the Free Body Diagram for the Trailer Bed (AB) **10 marks**

(b) Derive an expression for the force in the jack required, in terms of the parameters shown **25 marks**

(c) If the slope of the road ϴ is 15°, the mass of the trailer m_t is 15 tonnes, the load M is 75 tonnes, the trailer length L_1 is 25 metres and the load is positioned 5 m forward of the back axle (L_2), what is the force required to lift the trailer off the wheel. **15 marks**

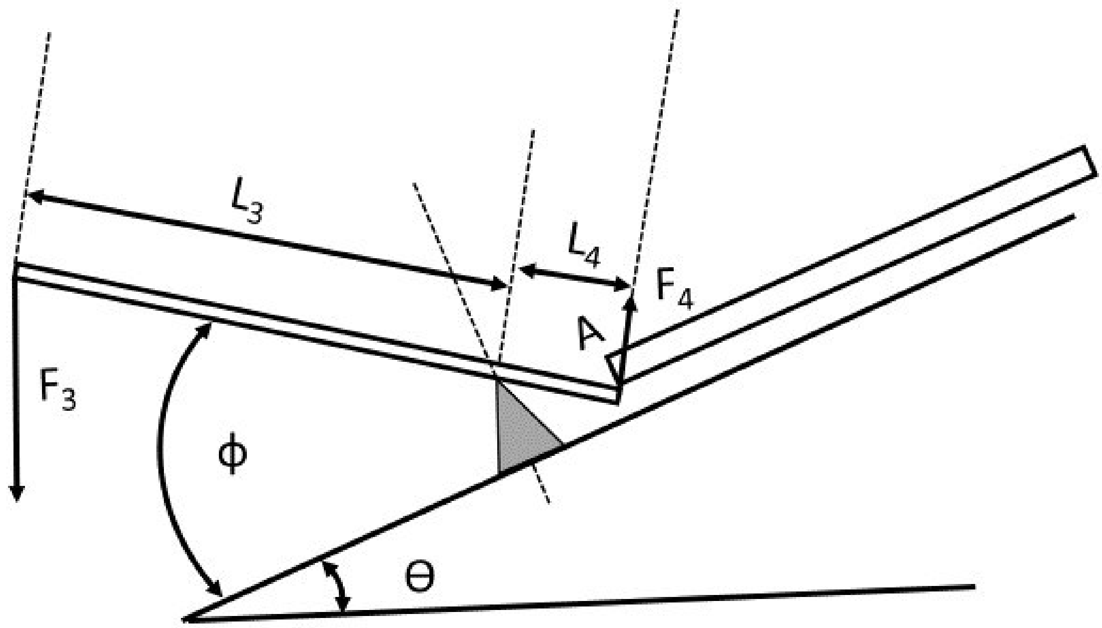


Figure 2

(d) If your jack is broken, and you have to improvise with the lever shown in Figure 2, derive an expression for the force F_3 to be applied to the end of the lever (vertically downwards). **25 marks**

(e) If the ratio L3:L4 is 12:1 , the angle ϕ is 45° , and assume now the jacking force required is 50 kN (ignore your answer to part (b)), what force will be required at F_3. **25 marks**

Question 2.

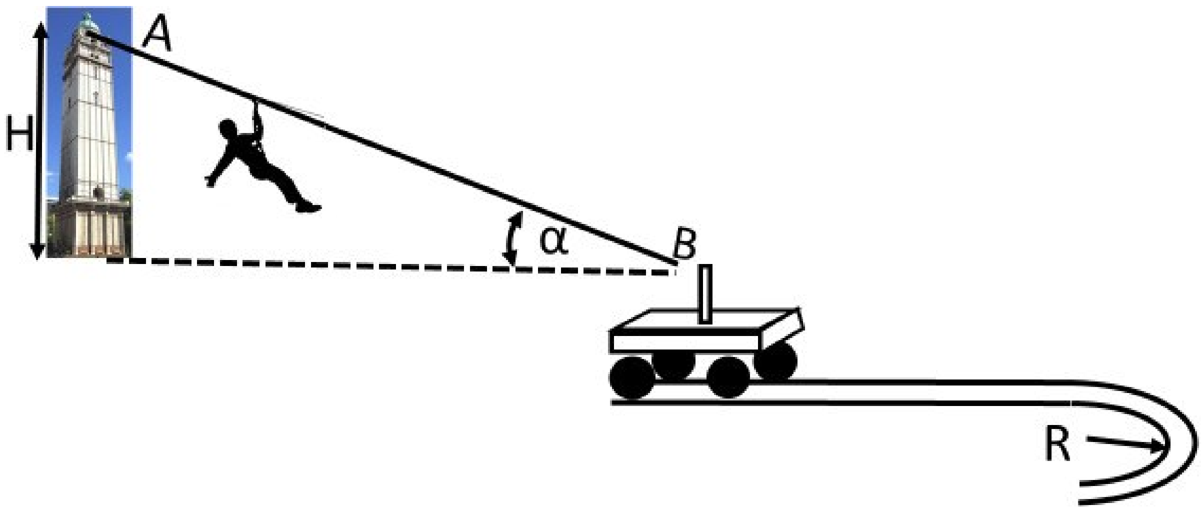


Figure 3

Imperial College has been transformed into a theme park ("The Thrill of Learning"). One of the extreme rides is a zip line from the Queen's Tower which delivers the participant to a cart which runs on a track through the rest of the South Kensington campus. A participant of mass m slides along the zip line AB at angle α, landing on the cart (of mass M), on which he/she will sit by holding on and the combined mass (cart plus participant) will then travel along the track to a series of curves.

(a) If the vertical height dropped by the passenger along the zip line is H, and friction and air resistance can be ignored, derive an expression for the velocity of the participant when he/she arrives at the cart (at B), in terms of the masses and distances given. **10 marks**

(b) He/she then releases the zip line, clings onto the cart, and cart + participant move off (consider this an inelastic collision). Derive an expression for the velocity of the cart + participant after landing and grasping onto the cart. **15 marks**

(c) The cart + participant now continue on to the first curve , which has a radius of R. Still ignoring friction, derive an expression for the force with which he/she must hold on to stay on the cart (in terms of the masses and distances given). **15 marks**

(d) If air resistance cannot be neglected whilst descending along the zip line, but is defined by

$$F_d = -kv,$$

where v is their velocity along the zip line, derive an expression for the participant's velocity at time t. **60 marks**

Question 3.

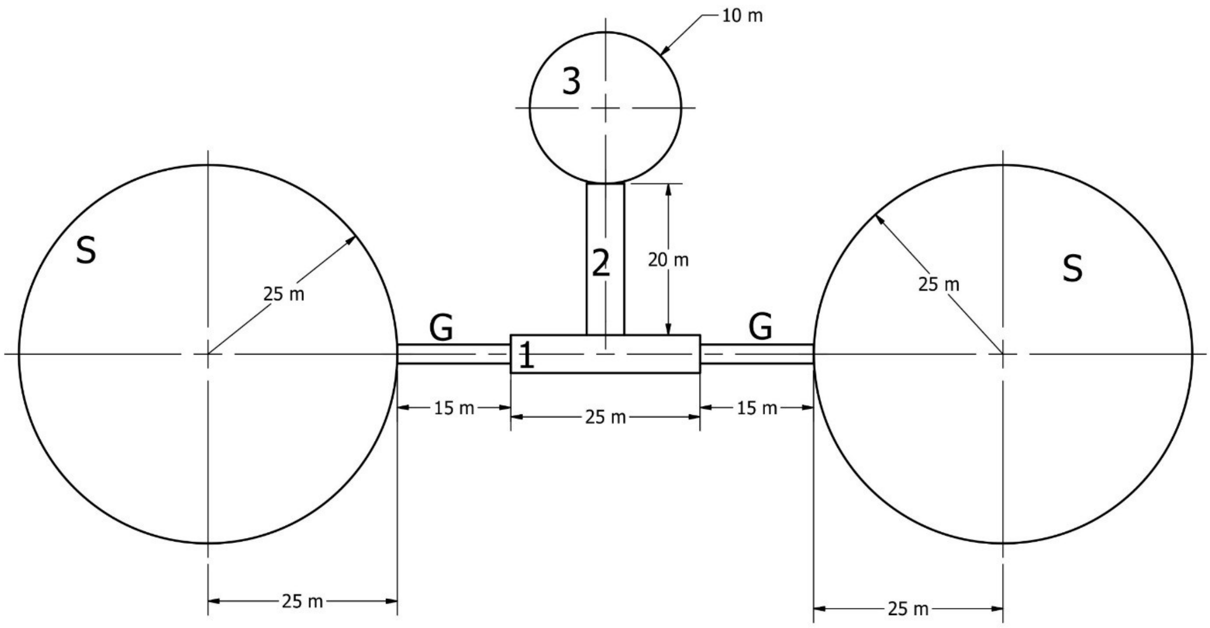


Figure 4

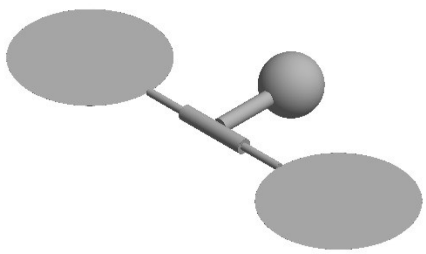
The International Space Station (Figure 4 and 5) consists of the following modules, bolted together as shown:

- Housing 1 – a Cylindrical shell of diameter 5 m, length 25 m and mass 1000 kg
- Housing 2 – another cylindrical shell of diameter 5 m, length 20 m and mass 750 kg welded perpendicular to the edge of housing 1, at exactly half the length of housing 1 (assume the end of Housing 2 is 2.5 m from the centerline of Housing 1).
- Housing 3 – a Spherical Shell of diameter 20 m and mass 800 kg, welded to Housing 2 such that its Centre is 8 m from the end of Housing 2
- Two gantries – cylindrical space frames of length 15 m, diameter 2.5 m and mass 500 kg each, welded to the ends of Housing 1
- Two Solar panels (S) mounted on rectangular gantries G, which can be represented by discs mounted at the end of the gantries (40 m from the point of attachment), co-axial with module 1, of 1500 kg each.

(a) Calculate the location of the Centre of Mass of the Space Station in the X and Y directions. **30 marks**

(b) Calculate the Moment of Inertia of the ISS about its Centre of Mass, in the X-Y plane **70 marks**.

Figure 5.



Question 4. Click here to enter text.

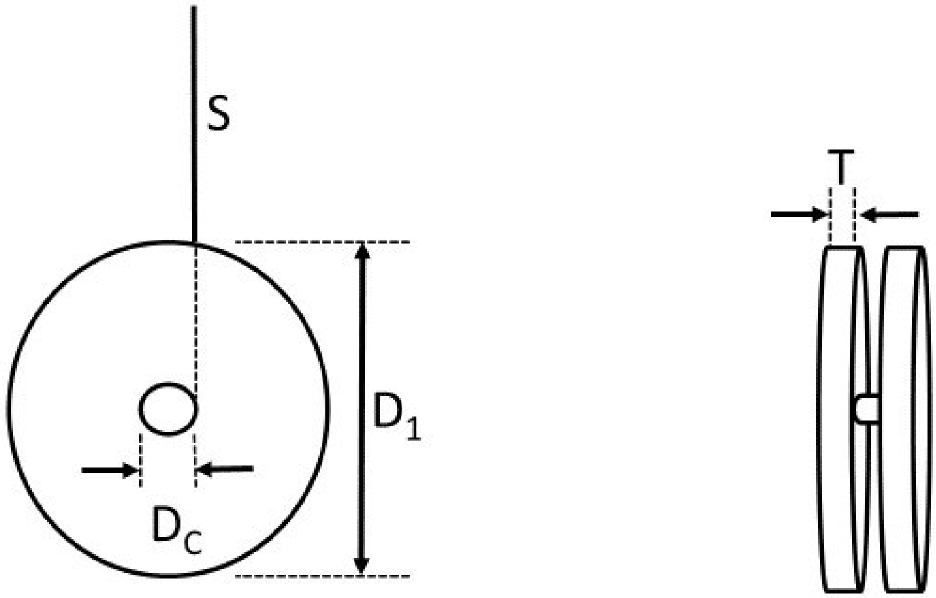


Figure 6

You have a yo-yo which consists of two discs of diameter D1 and thickness T joined by an axle of diameter D2 and length T. The total mass of the assembly is M. It runs on a string S of length L, which has a cunning loop at the end to allow virtually frictionless spinning once it unwinds to the bottom of the string length.

(a) For your new trick, starting with the string fully wound around the axle, you first drop the yo-yo, allowing it to roll down to the bottom of the string acquiring rotational velocity; when it unwinds to the bottom it then spins at constant angular velocity ω. If the height dropped whilst unwinding was L, and friction and air resistance can be ignored, derive an expression for ω in terms of D1, D2, M, & L. **20 marks**

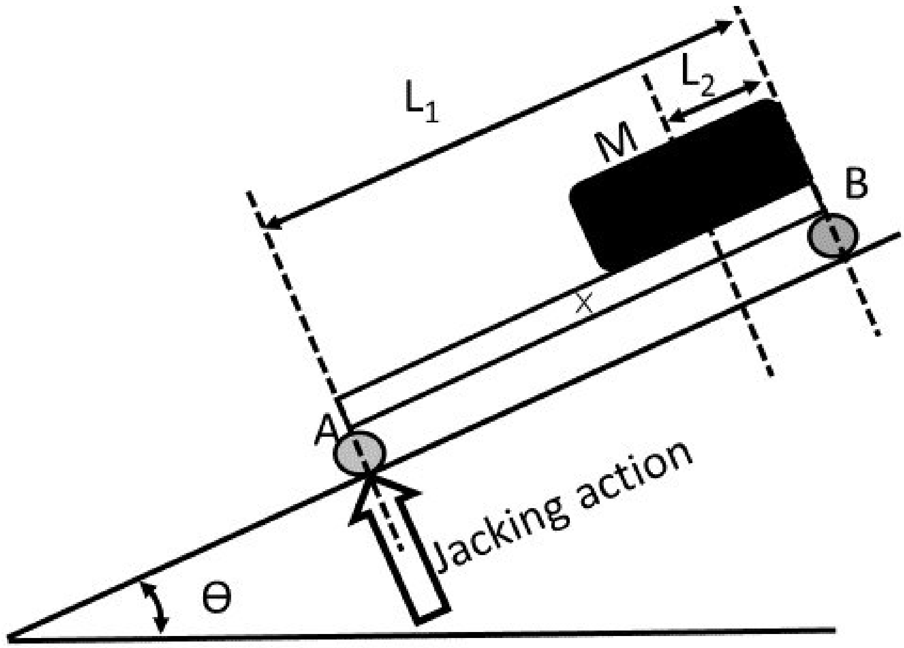
(b) You then lower your hand so that the yo-yo just touches the ground, and you adjust your hand vertically so as to maintain the tension in the string at 0.9 x Mg. If the coefficient of friction between the yo-yo and the floor is μ_d, draw a free body diagram for the yo-yo in its equilibrium state and show the relationships between the various forces acting. State your assumptions. **40 marks**

(c) Mathematically describe the motion of the yo-yo under these conditions. **40 marks**

Moments of Inertia of different shapes

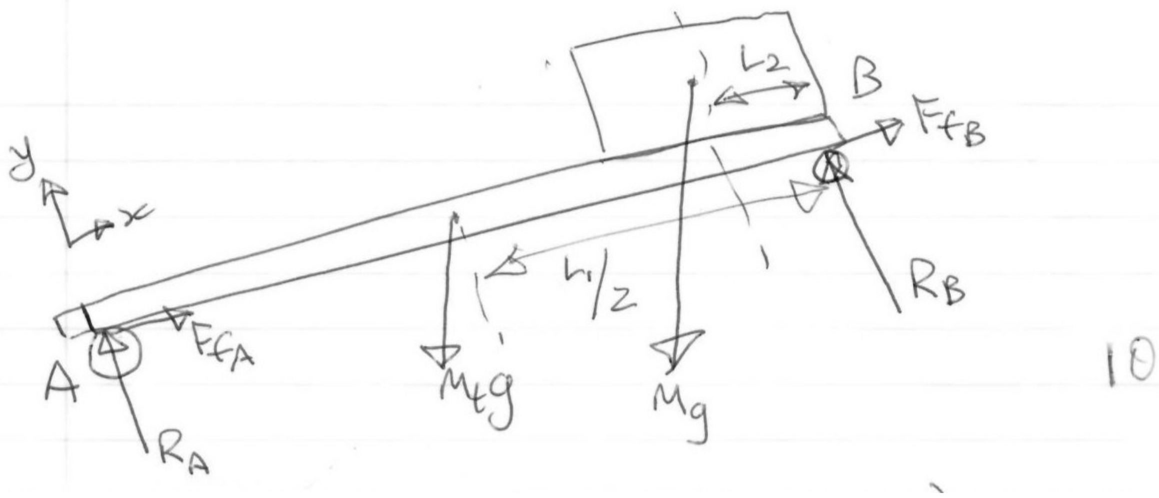
Shape	Axis/direction	I =
Point mass at distance R from axis		mR^2
Sphere of radius R	about centre	$2/5\ mR^2$
Spherical shell of radius R	about centre	$2/3\ mR^2$
Thin rod of radius R and height H	about axis	$½\ mR^2$
"	normal to its axis	$1/12\ mH^2$
Cylindrical shell of radius R and height H	about axis	mR^2
"	normal to its axis	$1/12\ mH^2$
Hoop	about its diameter	$½\ mR^2$
Cylinder or radius R and height H	about its axis	$½\ mR^2$
"	normal to its axis	$1/12\ m(H^2 + 3R^2)$

Question 1.



Your trailer (of mass m_t) has suffered a flat tyre (the front wheel at A) on a slope, and you need to jack it up to change the wheel. The brakes are locked on. The load M is located L_2 forward of the back axle B.

(a) Draw the Free Body Diagram for the Trailer Bed (AB) **10 marks**



(b) Derive an expression for the force in the jack required, in terms of the parameters shown **25 marks**

(b) $\sum M_B \quad -R_A L_1 + m_t g \cos\theta\left(\frac{L_1}{2}\right) + M g L_2 \cos\theta = 0$

$\therefore R_A L_1 = m_t g \frac{L_1}{2} \cos\theta + M g L_2 \cos\theta$

$\therefore \quad R_A = \frac{g\cos\theta}{L_1}\left(m_t \frac{L_1}{2} + M g L_2\right)$

$= g\cos\theta\left(\frac{m_t}{2} + M \frac{L_2}{L_1}\right)$ 25

25

(c) If the slope of the road Θ is 15°, the mass of the trailer m_t is 15 tonnes, the load M is 75 tonnes, the trailer length L_1 is 25 metres and the load is positioned 5 m forward of the back axle (L_2), what is the force required to lift the trailer off the wheel.

15 marks

(c) $\Rightarrow F_J = R_A = 9.81 \cos 15\left(\frac{15000}{2} + 7500\frac{5}{25}\right)$

$= 9.81 \cos 15 (7500 + 15000)$

−1 (not 2 wheels $\doteqdot 213203.98$ N

$= 213.2$ kN 15

25

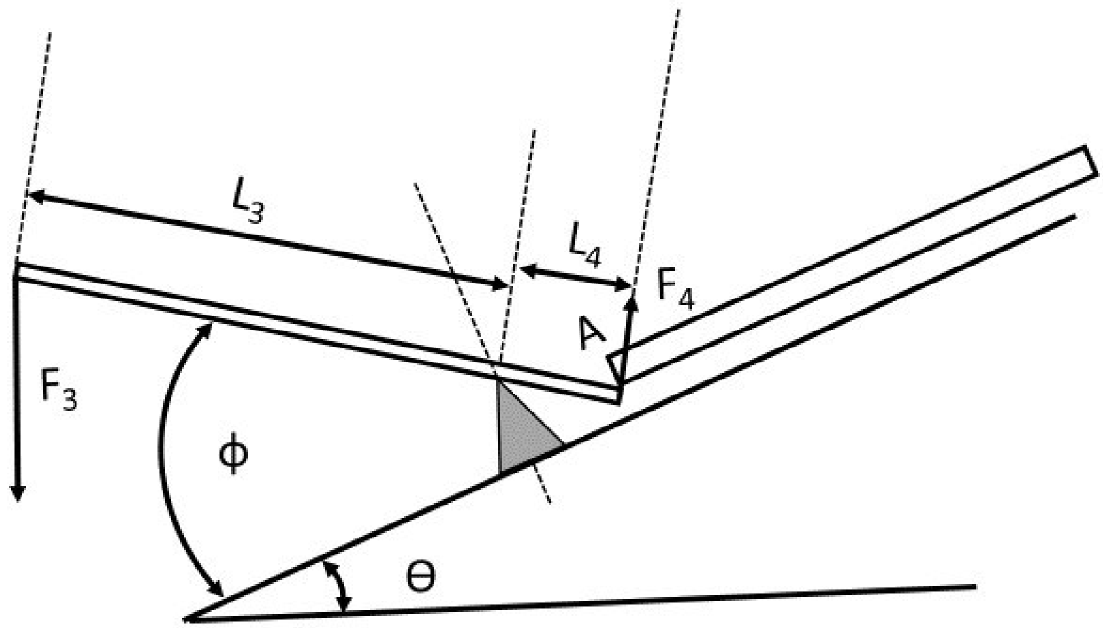
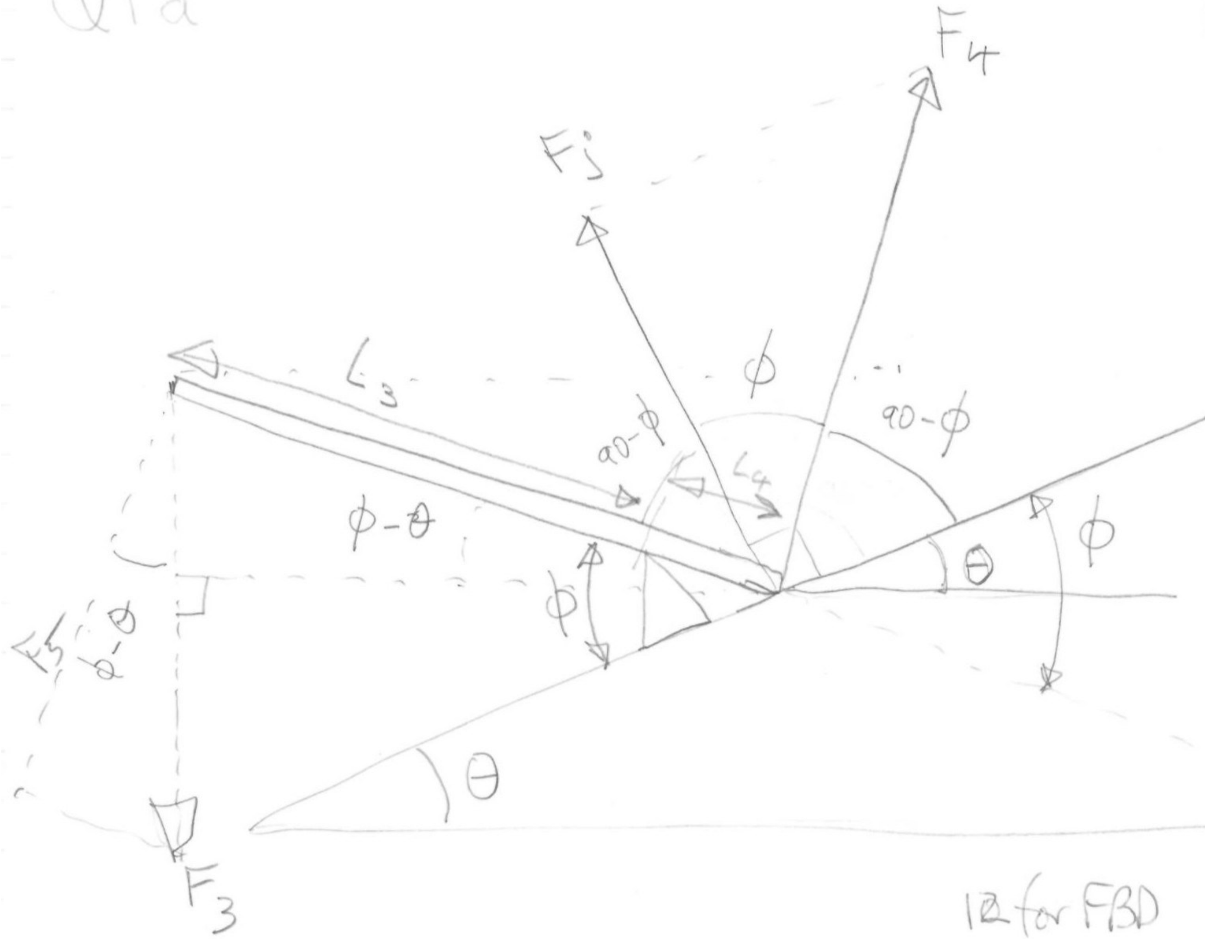


Figure 2

(d) If your jack is broken, and you have to improvise with the lever shown in Figure 2, derive an expression for the force F3 to be applied to the end of the lever (vertically downwards). **25 marks**

Q1d



1☑ for FBD

(d) Force delivered $F_j = F_4 \cos\phi$

so F_4 required $= \frac{F_j}{\cos\phi}$

Take moments about the fulcrum $\qquad F_5 = F_3 \cos(\phi-\theta)$

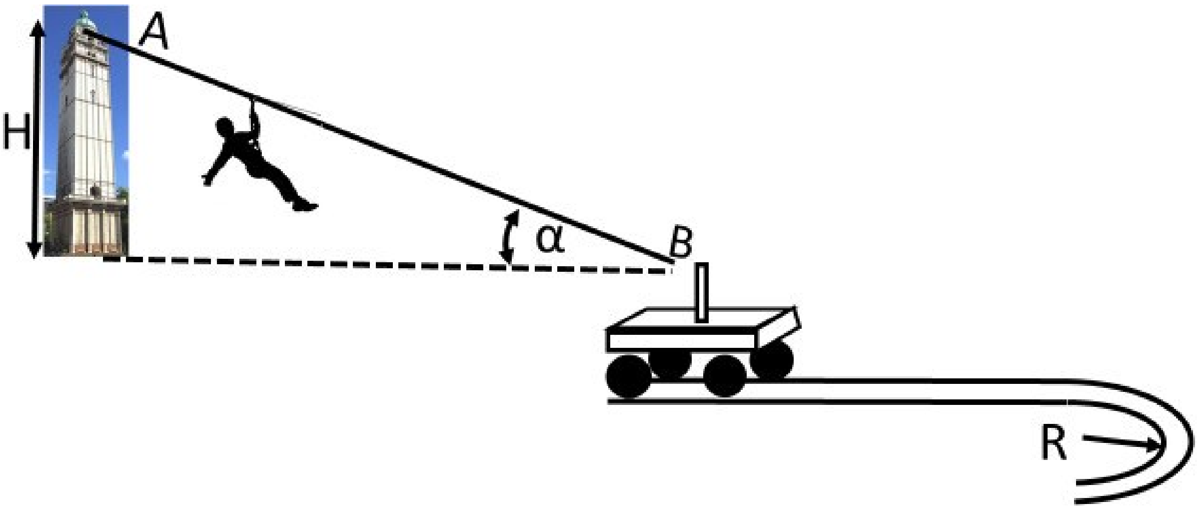
$$F_4 L_4 - F_3 \cos(\phi-\theta) L_3 = 0$$

$$\therefore \frac{F_j L_4}{\cos\phi} = F_3 L_3 \cos(\phi-\theta)$$

$$\therefore F_3 = \frac{F_j L_4}{L_3 \cos(\phi-\theta)\cos\phi}$$

25

Question 2 .



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(a) If the vertical height dropped by the passenger along the zip line is H, and friction and air resistance can be ignored, derive an expression for the velocity of the participant when he/she arrives at the cart (at B), in terms of the masses and distances given. **10 marks**

Energy is conserved, so

$$\frac{1}{2}mv^2 = mgh$$

$$\therefore\ v^2 = 2gH$$

$$\& \ v = \sqrt{2gH}$$

10

(b) He/she then releases the zip line, clings onto the cart, and cart + participant move off (consider this an inelastic collision). Derive an expression for the velocity of the cart + participant after landing and grasping onto the cart. **15 marks**

(b) an inelastic collision, so momentum conserved.

$$L_1 = L_2$$

$$\therefore mV_1 = (M+m)V_2$$

$$\therefore m\sqrt{2gh} = (M+m)V_2$$

and $V_2 = \dfrac{m\sqrt{2gh}}{M+m}$

10

(c) The cart + participant now continue on to the first curve , which has a radius of R. Still ignoring friction, derive an expression for the force with which he/she must hold on to stay on the cart (in terms of the masses and distances given). **15 marks**

(c) At the curve $F_R = \dfrac{mV^2}{R}$

substitute from above

$$F_R = \frac{m}{R}\left(\frac{m\sqrt{2gH}}{M+m}\right)^2$$

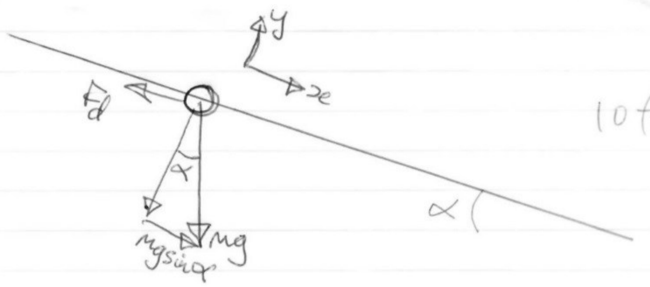
$$= \frac{m^3\, 2gH}{R(M+m)^2}$$

15

(e) If air resistance cannot be neglected whilst descending the zip line, but is defined by

$F_d = kv$, where v is their velocity along the zip line, derive an expression for the participant's velocity at time t. **60 mark**.

2 (d)



10 for FBD

60

Note altered forces $\qquad F_d = -kV_x$

$\Sigma F_x \quad mg\sin\theta - F_d = ma$

$\therefore ma = mg\sin\alpha - kV_x$

$\therefore \ddot{x} + \frac{k}{m}\dot{x} = g\sin\alpha$

Homogeneous Solution $\quad \dot{V} + \frac{k}{m}V = 0$

the solution to this is of the form

$V_h = Ce^{-\frac{k}{m}t}$

check (differentiate)

$\Rightarrow \dot{V}_h = -\frac{k}{m}Ce^{-\frac{k}{m}t}$

substitute at $t=0$ $\Rightarrow -\frac{k}{m}Ce^{-\frac{k(0)}{m}} + \frac{k}{m}Ce^{-\frac{k}{m}(0)}$

$= -\frac{kc}{m} + \frac{kc}{m} = 0$ so it works

$V_h = ce^{-\frac{k}{m}t}$

and $V = V_h + V_p$ $\quad$ where V_h – homogeneous solution, V_p – particular solution

At $t=0$ $V=0$ and $x=0$

15

2(d) cont.

try particular solution $\quad x = Ag\sin\alpha t$

$$\Rightarrow \quad \dot{x} = Ag\sin\alpha$$
$$\ddot{x} = 0$$

substitute in $\quad \ddot{x} + \frac{k}{m}\dot{x} = g\sin\alpha$

$$\therefore \quad 0 + \frac{k}{m}Ag\sin\alpha = g\sin\alpha$$

$$\therefore \quad \frac{kA}{m} = \frac{g\sin\alpha}{g\sin\alpha} = 1$$

$$\text{and } A = \frac{m}{k}$$

$$\Rightarrow \quad x = Ce^{-\frac{k}{m}t} + \frac{m}{k}g(\sin\alpha)t + B$$

at $t=0$ $\quad x=0$

$$= Ce^{0} + \frac{mg}{k}(0) + B \text{ so } C = -B$$

at $t=0$ $\quad \dot{x} = 0 = -\frac{k}{m}Ce^{-\frac{k}{m}t} + \frac{m}{k}g\sin\alpha$

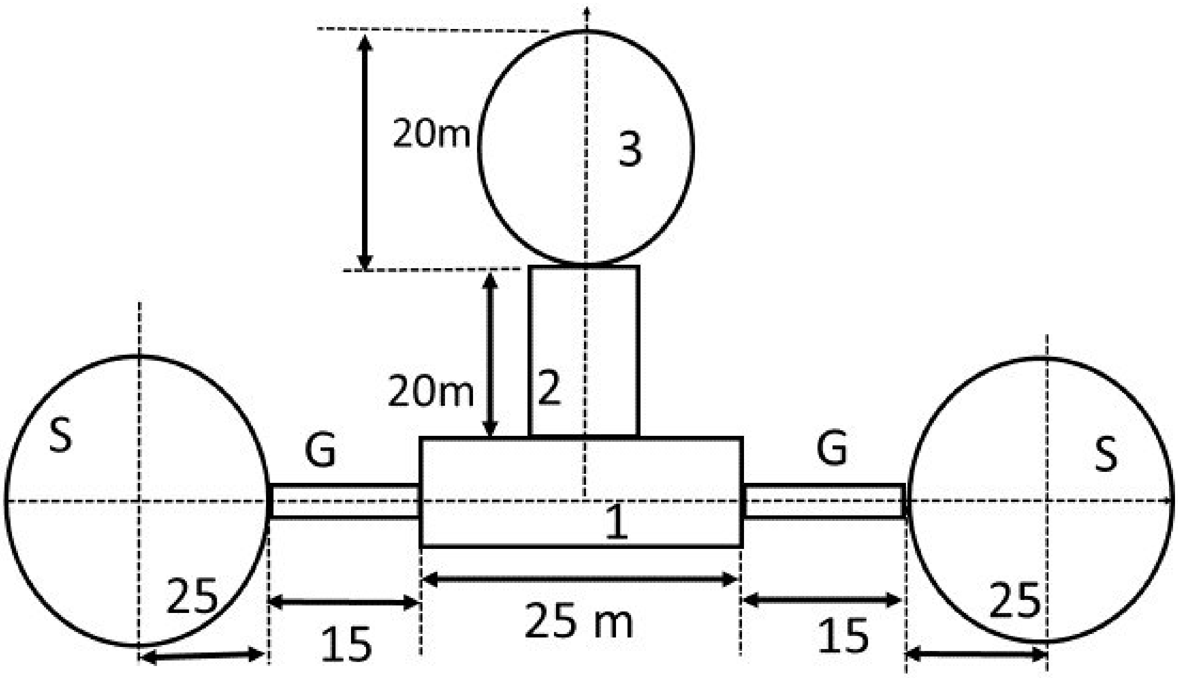
$$\therefore \quad \frac{mg}{k}\sin\alpha = \frac{kC}{m}e^{-\frac{k}{m}(0)}$$

$$C = \frac{m^2 g\sin\alpha}{k^2}$$

So $\quad B = -\frac{m^2 g\sin\alpha}{k^2}$

and $x = \frac{m^2 g\sin\alpha}{k^2}e^{-\frac{kt}{m}} + \frac{mgt}{k}\sin\alpha - \frac{m^2 g\sin\alpha}{k^2}$

Question 3 .



(A CAD drawing of the station will be added, which shows the plan view, as here, but also a Trimetric view to give a 3D and perspective)

The International Space Station (Figure 4) consists of the following modules, bolted together as shown:

- Housing 1 – a Cylindrical shell of diameter 5 m, length 25 m and mass 1000 kg
- Housing 2 – another cylindrical shell of diameter 5 m, length 20 m and mass 750 kg welded perpendicular to the edge of housing 1, at exactly half the length of housing 1 (assume the end of Housing 2 is 2.5 m from the centerline of Housing 1).
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- Two Solar panels (S) mounted on rectangular gantries G, which can be represented by discs mounted at the end of the gantries (40 m from the point of attachment), co-axial with module 1, of 1500 kg each.

a) Calculate the location of the Centre of Mass of the Space Station in the X and Y directions. **30 marks**

Centre of Mass: the assembly on the X-axis based on module ① is symmetrical - in fact the whole ISS is symmetrical about the Y-axis, so the Law of Symmetry says the CoM will be on the axis of symmetry (in the X-direction).

On the Y-axis, $Y_{CM} = \frac{\sum M_i y_i}{\sum M_i}$

so using axis of module 1 as baseline

$$Y_{cm} = \frac{(\sum M_1 + 2M_4 + 2M_5) \times 0 + M_2 (10+2.5) + M_3 \times (5+20+2.5)}{\sum M}$$

$$= \frac{0.75 \times 12.5 + 0.8 \times 30.5}{17 + 0.75 + 0.8 + (2 \times 0.5) + (2 \times 1.5)}$$

$$= \frac{9.375 + 24.4}{6.55} = 5.156 \text{ m along } Y.$$

30

b) Calculate the Moment of Inertia of the ISS about its Centre of Mass, in the X-Y plane

70 marks

3(b) 70

$$I_{CoM} = \sum (I_i + md^2) \quad 5 \quad \text{parallel axis theorem}$$

$$I_1 = \frac{1}{12} m H^2 + M_1 (5.156)^2 = \frac{25^2 \times 1}{12} + 1 \times (5.156)^2$$

$$= 52.083 + 26.584 = 78.667$$

$$I_2 = \frac{1}{12} m H^2 + M_2 (12.5 - 5.156)^2$$

$$= \frac{1}{12} 0.75 (20)^2 + 0.75 \times 53.934$$

$$= 25 + 40.45 = 65.45$$

$$I_3 = \frac{2}{3} m R^2 + m d^2$$

$$= \frac{2}{3}(0.8)(100) + 0.8(8 + 20 + 2.5 - 5.156)^2$$

$$= 53.33 + 0.8 \times 642.3$$

$$= 53.33 + 513.855 = 567.188$$

$$I_G = \frac{1}{12} m_g H^2 + m_g (r_g)^2$$

where $r_g = \sqrt{(12.5 + 7.5)^2 + 5.156^2}$
$= 20.654$

$$I_g = \frac{1}{12} 0.5 \times 15^2 + (20.654)^2 (0.5)$$

$$= \frac{225 \times 0.5}{12} + 0.5 \times 426.584$$

$$= 9.375 + 213.29 = 222.665$$

$$I_5 = \frac{1}{2} m R^2 + m d^2 = \frac{1}{2} \times 1.5 \times (25)^2 + 1.5 (r_5)^2$$

where $r_5 = \sqrt{52.75^2 + 5.156^2} = 53$

$$= 468.75 + 4213.5 = 4682.25 \text{ kgm}^2$$

70

70

3(b) (continued)

By Law of Superposition $I_{ISS} = \sum (I_i + m_i d_i^2)$

$\therefore I_{GMISS} = I_1 + I_2 + I_3 + 2I_4 + 2I_5$

$= 10521.$ tonne m^2

$= 10\,521\,000$ kg m^2

70

Question 4 .

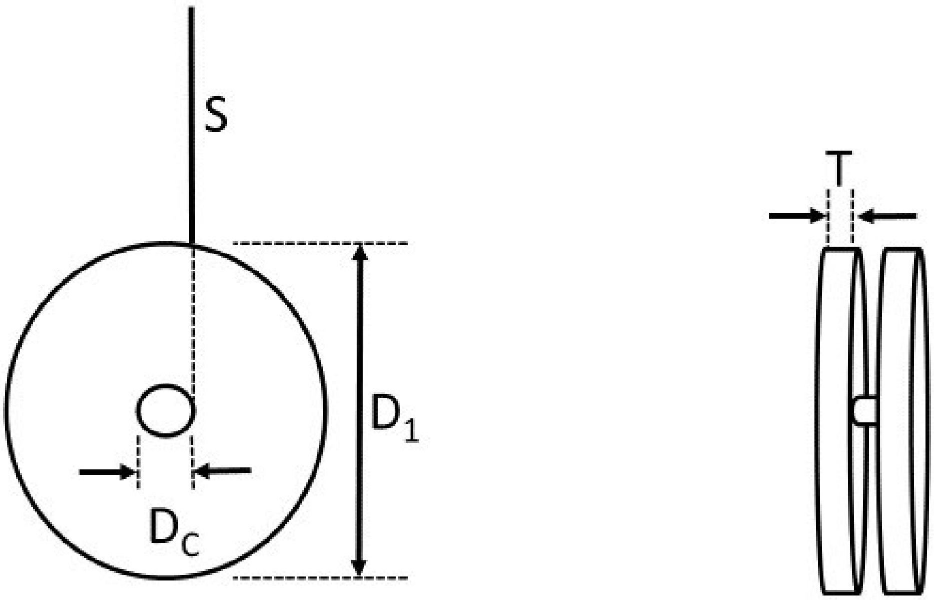
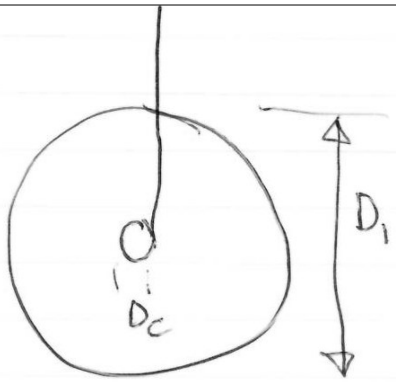


Figure 5

You have a yo-yo which consists of two discs of diameter D1 and thickness T joined by an axle of diameter D2 and length T. The total mass of the assembly is M. It runs on a string S of length L, which has a cunning loop at the end to allow virtually frictionless spinning once it unwinds to the bottom of the string length.

a) For your new trick, starting with the string fully wound around the axle, you first drop the yo-yo, allowing it to roll down to the bottom of the string acquiring rotational velocity; when it unwinds to the bottom it then spins at constant angular velocity ω. If the height dropped whilst unwinding was L, and friction and air resistance can be ignored, derive an expression for ω in terms of D1, D2, M, & L. **20 marks**



(a) Non-conservative forces can be ignored, so energy is conserved.

$$E_1 = E_2$$

$$\therefore \; U_1 + K_1 = U_2 + K_2$$

$$\therefore \; mgL = \tfrac{1}{2} I \omega^2$$

$$\therefore \; \omega^2 = \frac{2mgL}{I}$$

where $I_{disc} = \tfrac{1}{2} M R^2 = \frac{M}{2} D_1^2$

substitute into ω^2

$$\omega^2 = \frac{2MgL}{\tfrac{1}{2} M D_1^2} = \frac{4gL}{D_1^2}$$

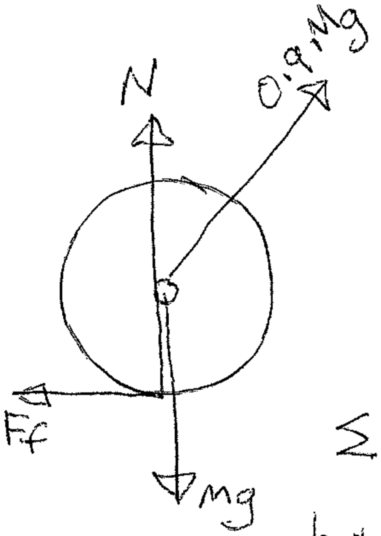
$$\therefore \; \omega = 2\sqrt{\frac{gL}{D_1^2}}$$

2mins

20

b) You then lower your hand so that the yo-yo just touches the ground, and you adjust your hand vertically so as to maintain the tension in the string at 0.9 x Mg. If the coefficient of friction between the yo-yo and the floor is μ_d, draw a free body diagram for the yo-yo in its equilibrium state and show the relationships between the various forces acting. State your assumptions. **40 marks**

(



10 marks for FBD (2 for each pt)

$\sum F_y$

$N + 0.9 Mg\sin\theta - Mg = 0$ 5

$\therefore\ N = Mg(1 - 0.9\sin\theta)$ 5

$\sum F_x \quad 0.9 Mg\cos\theta - F_f = m\ddot{x}$ 5

but probably in instantaneous equilibrium

so $\sum F_x = 0$ 5

$\therefore\ F_f = 0.9 Mg\cos\theta$ 5

$F_f = \mu N = \mu Mg(1 - 0.9\sin\theta) = 0.9 Mg\cos\theta$ 5

$\therefore\ \mu(1 - 0.9\sin\theta) = 0.9\cos\theta$ 5

$\mu = \dfrac{0.9\cos\theta}{(1 - 0.9\sin\theta)}$

the F_f is derived by dynamic friction of the spinning discs - will not change, but the spinning motion will be decelerated by the friction 5

40

c) Mathematically describe the motion of the yo-yo under these conditions. **40 marks**

$\sum M_{axle}$

$$-F_f \frac{D_i}{2} = I\alpha$$

$$\therefore \quad \alpha = -F_f \frac{D_i}{2} = -\frac{1}{2}\mu M g D_i (1-0.9\sin\theta)$$

$$\alpha = \dot{\omega}$$

integrate for ω $\therefore$ $\int \alpha\, dt = -\frac{\mu M g D_i t}{2}(1-0.9\sin\theta) + C$

at $t=0$ $\quad \omega = 2\sqrt{\frac{gL}{D_i^2}} = C$

$$\therefore \quad \omega = 2\sqrt{\frac{gL}{D_i^2}} - \frac{\mu M g D_i t}{2}(1-0.9\sin\theta)$$

10 mins.

40