



DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2002

M.Sc and EEE/ISE PART IV: M.Eng. and ACGI

Solutions 2002

ADVANCED COMMUNICATION THEORY

- There are *FOUR* questions (*Q1* to *Q4*)
- Answer *Question ONE* plus *TWO* other questions.

Comments for Question Q1:

- *Question Q1* has 20 multiple choice questions numbered 1 to 20.
- Circle the answers you think are correct on the answer sheet provided.
- There is only one correct answer per question.

Distribution of marks

Question-1: 40 marks

Question-2: 30 marks

Question-3: 30 marks

Question-4: 30 marks

The following are provided:

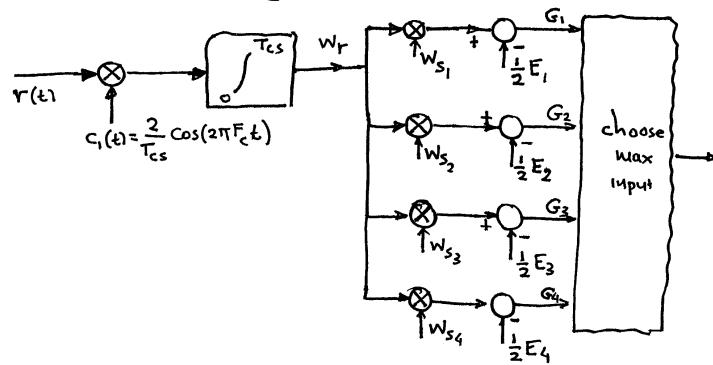
- A table of Fourier Transforms
- A "Gaussian Tail Function" graph

Examiners responsible: Dr. A. Manikas

ANSWER to Q1

- | | | | | | |
|-----|---|---|---|---|---|
| 1) | A | B | C | D | E |
| 2) | A | B | C | D | E |
| 3) | A | B | C | D | E |
| 4) | A | B | C | D | E |
| 5) | A | B | C | D | E |
| 6) | A | B | C | D | E |
| 7) | A | B | C | D | E |
| 8) | A | B | C | D | E |
| 9) | A | B | C | D | E |
| 10) | A | B | C | D | E |
| 11) | A | B | C | D | E |
| 12) | A | B | C | D | E |
| 13) | A | B | C | D | E |
| 14) | A | B | C | D | E |
| 15) | A | B | C | D | E |
| 16) | A | B | C | D | E |
| 17) | A | B | C | D | E |
| 18) | A | B | C | D | E |
| 19) | A | B | C | D | E |
| 20) | A | B | C | D | E |

ANSWER to Q2



ie. dimensions: $D=1$

where $N_0 = 2 \times 10^{-6} \text{ W/Hz}$ & $T_{cs} = 2 \text{ sec}$

Furthermore

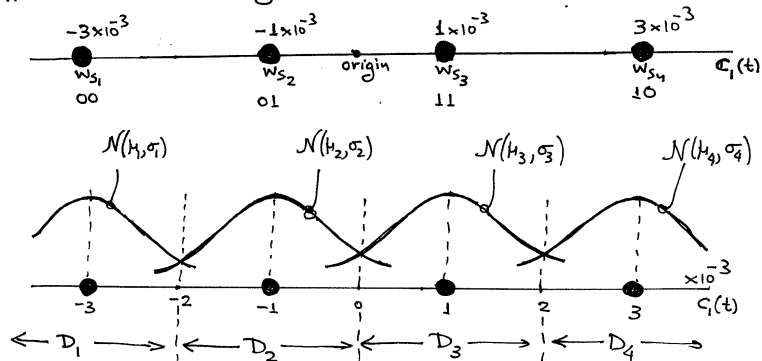
$$A_1 = (2 - 1 - 4) \times 10^{-3} = -3 \text{ mV} \Rightarrow W_{s1} = -\sqrt{E_1} = \sqrt{-\frac{A_1^2}{2} T_{cs}} = -3 \times 10^{-3}$$

$$A_2 = \dots = -1 \text{ mV} \Rightarrow W_{s2} = -\sqrt{E_2} = \dots = -10^{-3}$$

$$A_3 = \dots = 1 \text{ mV} \Rightarrow W_{s3} = \sqrt{E_3} = \dots = 10^{-3}$$

$$A_4 = \dots = 3 \text{ mV} \Rightarrow W_{s4} = \sqrt{E_4} = \dots = 3 \times 10^{-3}$$

* constellation diagram:



$$\mu_1 = -3 \times 10^{-3}; \mu_2 = -1 \times 10^{-3}; \mu_3 = 1 \times 10^{-3}; \mu_4 = 3 \times 10^{-3}$$

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma \text{ with } \sigma_i^2 = \frac{N_0}{2} \times 2.8 \times T_{cs} = \frac{N_0}{2} = 10^{-6}$$

ie $\sigma_i^2 = 10^{-6} = \text{noise energy over } T_{cs}$

$$\text{ie } \sigma = 10^{-3}$$

$$* P_{21} = \Pr(D_2 | H_1) = \mathbb{T}\left\{\frac{1-3-2 \times 10^{-3}}{10^{-3}}\right\} - \mathbb{T}\left\{\frac{1-3-0 \times 10^{-3}}{10^{-3}}\right\}$$

$$= \mathbb{T}\{1\} - \mathbb{T}\{3\} = 0.16$$

$$= P_{12} = P_{32} = P_{23} = P_{43} = P_{34}$$

$$\text{where } P_{Lj} = \Pr(D_L | H_j)$$

$$* P_{31} = \mathbb{T}\{3\} - \mathbb{T}\{5\} = 1.4 \times 10^{-3}$$

$$= P_{24} = P_{42}$$

$$* P_{41} = \mathbb{T}\{5\} = P_{14} = 3 \times 10^{-7}$$

$$* P_{11} = P_{44} = 1 - \mathbb{T}\{1\} = 0.84$$

$$* P_{22} = P_{33} = 1 - 2 \mathbb{T}\{1\} = 0.68$$

$$* \text{overall } \rightarrow \underline{\mathbb{F}} = \begin{bmatrix} 0.84 & 0.16 & 1.4 \times 10^{-3} & 3 \times 10^{-7} \\ 0.16 & 0.68 & 0.16 & 1.4 \times 10^{-3} \\ 1.4 \times 10^{-3} & 0.16 & 0.68 & 0.16 \\ 3 \times 10^{-7} & 1.4 \times 10^{-3} & 0.16 & 0.84 \end{bmatrix}$$

$$* P_{e,cs} = 1 - \Pr(\text{no-errors}) = 1 - \frac{1}{4} (\text{Trace}(\underline{\mathbb{F}})) \leq \sum_L F_{LL} = 3.04$$

$$= 1 - \frac{1}{4} 3.04 = 1 - 0.76$$

$$= 0.24$$

$$* \frac{P_{e,cs}}{\gamma_{cs}} \leq P_e \leq P_{e,cs} \Rightarrow 0.12 < P_e < 0.24$$

ANSWER to Q3

$$r_{cs} = 8 \text{ kHz}$$

$$\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2 \times 10^{-12}$$

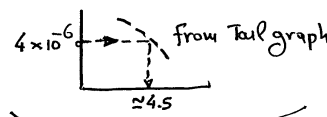
$$P_J = 1 \text{ W}; P_G = 10^5; P_e = 4 \times 10^{-6}; P_{e,PR} = 4 \times 10^{-2}$$

$$\frac{B_{ss}}{B_J} = P_G \Rightarrow B_{ss} = P_G \times r_{cs} = 800 \text{ MHz}$$

Baseline Performance

$$B_J = B_{ss} \Rightarrow P_e = T \left\{ \sqrt{(1-p) EUE_{eq4}} \right\}$$

$$4 \times 10^{-6} = T \left\{ \sqrt{(1-p) EUE_{eq4}} \right\} \leftarrow \text{also } [p=-1]$$



$$\sqrt{2 EUE_{eq4}} = 4.5 \Rightarrow EUE_{eq4} = 10.1250$$

$$\Rightarrow \frac{E_b}{N_0 + \frac{P_J}{B_J}} = 10.1250 \Rightarrow E_b = 12 \times 10^{-9}$$

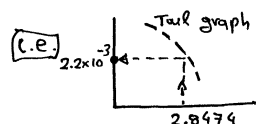
$$\text{i.e. } P_s = E_b / r_{cs} = E_b \cdot r_{cs} = 101 \times 10^{-6}$$

$$\therefore A = \sqrt{2 P_s} = 14.2 \text{ mVolts}$$

Partial Noise Jammer:

$$B_J = 0.4 B_{ss} = 320 \text{ MHz}$$

$$P_e = T \left\{ \sqrt{2 EUE_{eq4}} \right\} = T \left\{ \sqrt{2 \frac{E_b}{N_0 + \frac{P_J}{B_J}}} \right\} = T \{ 2.8474 \}$$



$$\text{i.e. } P_e = 2.2 \times 10^{-3}$$

Pulse Jammer:

$$\left. \begin{array}{l} q = 0.4 \text{ "on"} \\ B_J = B_{ss} \end{array} \right\} \Rightarrow P_e = \underbrace{(1-q) T \left\{ \sqrt{2 \frac{E_b}{N_0}} \right\}}_{\text{Jammer = "off"}} + q T \left\{ \sqrt{2 \frac{E_b}{N_0 + \frac{P_J}{q B_{ss}}}} \right\}$$

$$\Rightarrow P_e = 0 + 0.4 T \{ 2.8474 \}$$

$$\Rightarrow P_e = 0.4 \times 2.2 \times 10^{-3}$$

$$\Rightarrow P_e = 8.8 \times 10^{-4}$$

Baseline performance

partial noise jammer

pulse jammer

$$\left. \begin{array}{l} \text{Baseline performance} \\ \text{partial noise jammer} \end{array} \right\} \rightarrow P_{e,PR} = T \left\{ \sqrt{2 \cdot EUE_{eq4,PR}} \right\}$$

$$\text{pulse jammer} \rightarrow P_{e,PR} = q T \left\{ \sqrt{2 \cdot EUE_{eq4,PR}} \right\}$$

$$\text{N.B.: } EUE_{eq4,PR} \approx \begin{cases} \approx EUE_{J,PR} & (\text{Partial/Baseline}) \\ \approx q EUE_{J,PR} & (\text{pulse jammer}) \end{cases}$$

∴ Baseline perf

Partial noise jammer

Pulse jammer

$$\left. \begin{array}{l} \text{Baseline perf} \\ \text{Partial noise jammer} \end{array} \right\} \rightarrow EUE_{eq4,PR} = 1.5325$$

$$\left. \begin{array}{l} \text{Pulse jammer} \end{array} \right\} \rightarrow EUE_{eq4,PR} \approx 0.8212$$

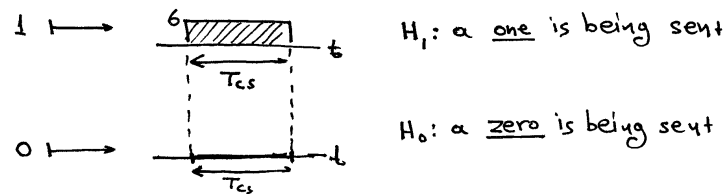
$$\text{However, } AJM = 10 \log_{10} EUE_{eq4} - 10 \log_{10} EUE_{eq4,PR}$$

$$\Rightarrow AJM_{\text{Baseline}} = 8.2 \text{ dB}$$

$$AJM_{\text{partial noise}} = 4.2 \text{ dB}$$

$$AJM_{\text{pulse jammer}} = 6.9 \text{ dB}$$

ANSWER to Q4



$$pdf_s(s) = \begin{array}{c} \frac{1}{3} \uparrow \quad \frac{2}{3} \uparrow \\ 0 \quad 6 \end{array} \quad s(\text{volts}) = \frac{1}{3} \delta(s) + \frac{2}{3} \delta(s-6)$$

$$pdf_r(r) = \begin{array}{c} \frac{1}{3} \mathcal{N}(0,1) \quad \frac{2}{3} \mathcal{N}(6,1) \\ 0 \quad 6 \end{array} \quad r(\text{volts}) = \frac{1}{3} \mathcal{N}(0,1) + \frac{2}{3} \mathcal{N}(6,1)$$

likelihood function: $P_0(r) = pdf_{r|H_0} = \mathcal{N}(0,1)$

$$P_1(r) = pdf_{r|H_1} = \mathcal{N}(6,1)$$

$$\eta(r) > \eta_0 \text{ where } \eta_0 = \frac{\overset{\frac{1}{3}}{\text{Pr}(H_0)} \cdot (C_{10} - C_{00})}{\underset{\frac{2}{3}}{\text{Pr}(H_1)} \cdot (C_{01} - C_{11})} = \frac{9}{2}$$

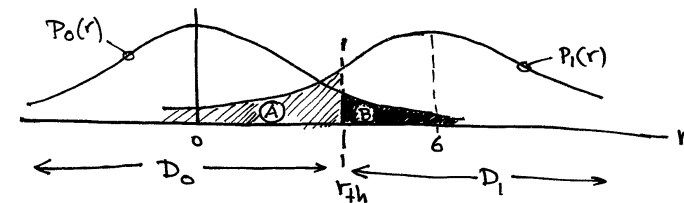
observation space: $r \in (-\infty, +\infty)$

$$\text{likelihood function: } \eta(r) = \frac{P_1(r)}{P_0(r)} = \frac{\mathcal{N}(6,1)}{\mathcal{N}(0,1)} = \exp\left(\frac{12r-36}{2}\right) = \exp(6r-18)$$

$$\begin{aligned} \text{choose } H_1 \text{ if } \eta(r) > \eta_0 &\Rightarrow \exp(6r-18) > \frac{9}{2} \\ &\Rightarrow \text{if } r > \underbrace{3 + \frac{1}{6} \ln 4.5}_{\approx 3.25} \text{ then choose } H_1 \end{aligned}$$

$$\text{solution } \left. \begin{array}{l} \mathcal{R}_0 = (-\infty, 3.25) \\ \mathcal{R}_1 = (3.25, +\infty) \end{array} \right\} \text{ with } r_{\text{threshold}} = 3.25$$

ie choose H_0 if $r \in \mathcal{R}_0$ and H_1 if $r \in \mathcal{R}_1$



$$\text{area (A)} = \Pr(D_0|H_1) = P_{\text{miss}} = \mathbf{T}(13.25-6) = \mathbf{T}(2.75) = 2.8 \times 10^{-3}$$

$$\text{area (B)} = \Pr(D_1|H_0) = P_{\text{FA}} = \mathbf{T}(3.25) = 5.8 \times 10^{-4}$$

$$\begin{aligned} P_e &= \Pr(D_1|H_0) \cdot \Pr(H_0) + \Pr(D_0|H_1) \cdot \Pr(H_1) \\ &= \mathbf{T}(3.25) \cdot \frac{1}{3} + \mathbf{T}(2.75) \cdot \frac{2}{3} \\ &= 5.8 \times 10^{-4} \times \frac{1}{3} + 2.8 \times 10^{-3} \times \frac{2}{3} = 2.06 \times 10^{-3} \end{aligned}$$

$$10 \log 2^{2\gamma} - 10 \log \frac{2^{-0.1}}{144 P_e 2^{2\gamma}} \geq 1 \quad \uparrow \text{equality} \Leftrightarrow \text{threshold}$$

$$\Rightarrow 10 \log \frac{2^{2\gamma}}{2^{2\gamma}} \geq 1 \Rightarrow 10 \log(144 P_e 2^{2\gamma}) \geq 1$$

$$\Rightarrow 1 + 4 P_e 2^{2\gamma} \geq 10^{0.1} \Rightarrow 2^{2\gamma} \geq \frac{10^{0.1} - 1}{4 P_e}$$

$$\Rightarrow 2^{2\gamma} \geq \frac{1}{16 P_e} \Rightarrow \gamma \geq \underbrace{\frac{1}{2} \log_2 \frac{1}{16 P_e}}_{2.46 \text{ bits}}$$