Name:
CID:

Tutorial 1

Any marks received for the tutorial are only indicative and may be subject to moderation and scaling.

Exercise 1 (Euler's method for scalar ODEs) — **% of CW mark: 0.25**

Compute the numerical solution of the initial value problem

$$x' = \frac{x+t}{x-t}, \quad x(t_0) = 0, \quad t_0 = 1, \quad t > 1$$

with the Euler method at $t = \{2, 3\}$; time step $h = 1$.

Exercise 2 (Euler's method for scalar ODEs) — **% of CW mark: 0.5**

Compute the numerical solution of the initial value problem

$$x' = \sin(t) - x, \quad x(t_0) = 0, \quad t_0 = 0, \quad t > 0.$$

with the Euler method at $t = \pi/4$ (time step $h = \pi/4$) and compare it with the the exact solution at $t = \pi/4$.

Exercise 3 (Euler's method for systems of ODEs) — **% of CW mark: 0.25**

Write down the Euler method for the initial value problems

$$x'' - x' - 2x = 1 + 2t, \quad x(t_0) = 0, \quad x'(t_0) = 1, \quad t_0 = 0, \quad t > 0.$$

$$x''' - 2x'' - x' + 2x = 12, \quad x(t_0) = 0, \quad x'(t_0) = 1, \quad x''(t_0) = 2, \quad t_0 = 0, \quad t > 0.$$

Exercise 4 (Euler's method for systems of ODEs) — **% of CW mark: 0.25**

Compute the numerical solution of the initial value problem

$$\begin{aligned} u' &= -2u + v, \ u(t_0) = 1, \quad t_0 = 0, \quad t > 0, \\ v' &= -u - 2v, \ v(t_0) = 0, \quad t_0 = 0, \quad t > 0, \end{aligned} \tag{1}$$

with the Euler method at $t = \{1, 2\}$; time step $h = 1$.

Exercise 5 (Error analysis) — **% of CW mark: 0.25**

Prove that $1 + x + \dfrac{x^2}{2} \leq e^x$ for all $x \geq 0$.

Exercise 6 (Error analysis) — **% of CW mark: 0.5**
Mastery Component

Calculate the local truncation error of the Backward Euler method

$$x_{n+1} = x_n + h x'_{n+1}, \quad x'_{n+1} := f(t_{n+1}, x_{n+1}).$$