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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

MSc and EEE/ISE PART IV: M.Eng. and ACGI

DIGITAL SIGNAL PROCESSING AND DIGITAL FILTERS

Thursday, 1 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer THREE questions.

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : A.G. Constantinides

Second Marker(s) : T. Stathaki

1. The order N of a Finite Impulse Response filter may be estimated through the empirical formula

$$N = \frac{A}{20}\frac{f_S}{\Delta f}$$

where:

A = passband-to-stopband discrimination in dB

Δf = transition bandwidth in Hz

f_S = sampling frequency in Hz

Justify the above expression. Explain particularly why it is that the order increases as the transition width is reduced. [4]

A single-stage decimator is required to reduce the sampling rate from 32 kHz to 800 Hz. The decimation filter is to be designed as a Finite Impulse Response filter with a cutoff frequency at 300 Hz, a transition frequency at 350 Hz and a passband-to-stopband discrimination of 40 dB. Estimate the order of the required FIR transfer function. [3]

By using the total number of multiplications per second as a measure of computational complexity, determine the computational complexity of this single stage decimation process. The decimator is to be designed as a two-stage structure having a 10:1 decimator as the first stage. Determine the computational complexity of this arrangement and compare it to the single stage realisation. [8]

In a cascade connection of more than two decimators for the above problem how would you order the decimators on the basis of their transition bandwidths? [5]

2. Explain what is meant by terms *computational complexity* and *twiddle factors* in the context of evaluating the Discrete Fourier Transform (DFT). Derive the computational complexity of a N-point DFT. [3]

It is given that $N = N_1 N_2$ with N_1 and N_2 co-prime. It is proposed to carry out on the data array $\{x(n)\}, \quad 0 \le n \le N-1$, the following 1-D to 2-D mapping

$$n = \langle An_1 + Bn_2 \rangle_N \quad \begin{Bmatrix} 0 \le n_1 \le N_1 - 1 \\ 0 \le n_2 \le N_2 - 1 \end{Bmatrix}$$

$$k = \langle Ck_1 + Dk_2 \rangle_N \quad \begin{Bmatrix} 0 \le k_1 \le N_1 - 1 \\ 0 \le k_2 \le N_2 - 1 \end{Bmatrix}$$

where $\langle M \rangle_N$ means a reduction of the number M modulo N.

Derive the conditions that must prevail on the products AC, BD, AD, and BC in order that all possible twiddle factors in the 2-D DFT computation are eliminated. [10]

Show that the following set of parameters satisfies these conditions

$$A = N_2,\ B = N_1,\ C = N_2 \langle N_2^{-1} \rangle_{N_1},\ D = N_1 \langle N_1^{-1} \rangle_{N_2}$$

where $\langle L^{-1} \rangle_P$ denotes the multiplicative inverse of L evaluated modulo P. [3]

Hence outline the algorithm for the computation of the N-point DFT. [4]

3. In an audio application the structure in Figure 3.1 is used as a "reverberator" to reproduce attenuated versions of an impulse applied as input. Determine its transfer function and show that it is allpass. [3]

Propose an alternative canonic or non-canonic signal flow graph with one multiplier that realises the transfer function. Explain every step in your answer [4]

Determine the impulse response of the structure, and hence the period τ_{rev} during which the impulse response has an absolute value more than 1% of its value at the instant $n=1$. [5]

The reverberator is to be used in an application in which signals are sampled at 10 kHz. It is required to have τ_{rev} =800 msecs. Determine the value for the parameter α and comment on the problems likely to be encountered in an implementation with such a value. [4]

Suggest enhancements to the structure below to produce a more complex reverberation behaviour and justify your answer. [4]

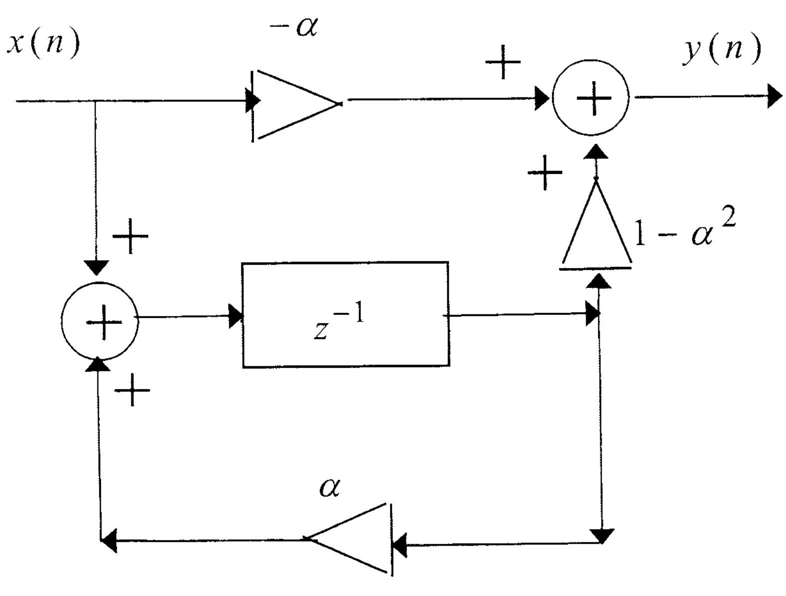

Figure 3.1

4. Two different signals $X_1(z)$ and $X_2(z)$ are transmitted through independent channels in close proximity so that they are received linearly mixed as $Y_1(z) = AX_1(z) + BX_2(z)$ and $Y_2(z) = CX_1(z) + DX_2(z)$ modelled as shown in the Figure 4.1

The mixing matrix is $A = 1 \quad B = H_{12}(z) \quad C = H_{21}(z) \quad D = 1$.

The signals $Y_1(z)$ and $Y_2(z)$ are further processed by a linear system which has a mixing matrix $A = 1 \quad B = -G_{12}(z) \quad C = -G_{21}(z) \quad D = 1$ to produce two new signals $U_1(z)$ and $U_2(z)$ each of which is a function of only one of the original signals $X_1(z)$ and $X_2(z)$.

Determine the two alternative solutions possible and the associated proportionality transfer functions in terms of the mixing parameters above. [12]

It is not known a priori whether the mixing channel transfer functions are minimum phase or not. Comment on possible limitations of one of the alternative solutions [8]

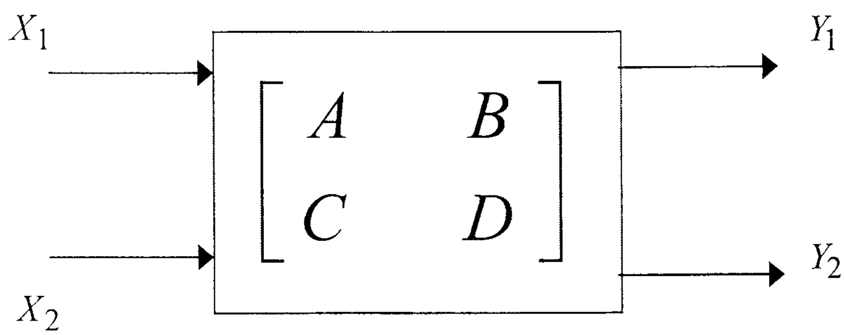

Figure 4.1

5. The transfer function of an ideal real coefficient highpass filter $H_{HP}(z)$ has cutoff frequency θ_p and impulse response $h_{HP}(n)$. Show that $H_{LP}(z) = H_{HP}(-z)$ is a lowpass filter and determine its cutoff frequency. Indicate on the unit circle the correspondence between the passbands of the two filters. Find an expression for the impulse response $h_{LP}(n)$ of the ~~highpass~~ lowpass filter in terms of the impulse response $h_{HP}(n)$ of the original ~~lowpass~~ highpass filter. [5]

In a specific application it is required to split the entire baseband into two equal bands. Indicate how the above relationships can be used to produce a minimal coefficient realisation. [5]

Let be $\theta_p < \frac{\pi}{2}$ and form $G(z) = H_{LP}(ze^{j\theta_0}) + H_{LP}(ze^{-j\theta_0})$. Show that $G(z)$ is a real coefficient bandpass filter with a passband centred at θ_0. Determine the impulse response and bandwidth of the bandpass filter in terms of the impulse response of the lowpass filter and the centre frequency θ_0. [5]

Explain why in practice with realisable transfer functions such an approach may not produce a bandpass response [5]