E3.08
ISE3.17

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI

ADVANCED SIGNAL PROCESSING

Tuesday, 6 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer TWO of the questions 1, 2, 3 and ONE of the questions 4, 5.

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D.P. Mandic
Second Marker(s) : D.B. Ward

1) Consider the problem of estimating the value of a parameter, θ, from a sequence of random variables $x[n], \quad n = 1, 2, \ldots, N$. Since the estimate is a function of N random variables, we will denote it by $\hat{\theta}_N$.

a) Define the bias B in parameter estimation. When do we say that an estimate is unbiased?

[2]

b) Define an asymptotycally unbiased estimator.

[2]

c) Define the terms mean square convergence and consistent estimator.

[2]

i) Is the sample mean estimate $\hat{m}_x = \frac{1}{N}\sum_{n=1}^{N} x[n]$ unbiased and consistent?

[2]

d) Let x be the random variable defined on the coin flipping experiment, with $x = 1$ if the outcome is heads and $x = -1$ if the outcome is tails. The coin is unfair so that the probability of flipping *heads* is p and the probability of flipping *tails* is $(1-p)$.

i) Find the mean of x.

[4]

ii) Suppose the value for p is unknown and that the mean of x is to be estimated. Flipping the coin N times and denoting the resulting values for x by $x[i], \quad i = 1, \ldots, N$, consider the following estimator for m_x

$$\hat{m}_x = x[N]$$

Is this estimator unbiased?

[4]

iii) Find the variance of the estimator from part ii). Is this estimator consistent?

[4]

2) It frequently occurs in practice that the minimum variance unbiased estimator (MVU), even if it exists, cannot be found. A common solution to this problem is a suboptimal estimator which is linear in the data, termed the best linear unbiased estimator (BLUE). The BLUE can be determined with knowledge of only the first and second moments of the PDF.

a) What constraints are involved in the definition of the BLUE? Find the bias and variance of the BLUE.

[4]

b) State the optimisation problem to find the BLUE.

[4]

c) Derive the BLUE for a scalar parameter θ.

[8]

d) What class of estimation problems is BLUE applicable to?

[2]

d) State a problem for which the use of BLUE is totally inappropriate.

[2]

3) Consider the problem of mixed autoregressive moving average (ARMA) modelling.

a) State the equation of a general ARMA(p, q) process.

[2]

i) Derive the autocovariance function $c_{zz}(k)$ of this process and state the equation for the autocovariance function $c_{zz}(k)$ for $k \geq q + 1$.

[6]

ii) What are the stationarity conditions for this process?

[3]

iii) State the equation for the power spectrum of a general ARMA(p,q) process.

[2]

b) Consider the process

$$z[n] = a_1 z[n-1] + w[n] + b_1 w[n-1]$$

where $z[n]$ is the ARMA process, $w[n]$ is white noise, and a_1 and b_1 are the model parameters. State the conditions for which the process $z[n]$ is stationary and invertible.

[3]

For $z[n]$, find the expressions for

i) the first two autocorrelation coefficients, ρ_0 and ρ_1.

[2]

ii) the autocorrelation coefficients ρ_k for $k \geq 2$.

[2]

4) Consider the problem of estimating a random variable y in terms of an observation of another random variable x. The problem generally arises when y cannot be directly observed or measured so a related random variable is measured and used to estimate y. The goal is to find the best estimate of y in terms of x. In the linear mean square estimation, the estimator is constrained to be of the form

$$\hat{y} = ax + b$$

The estimation error is $e = y - \hat{y}$ and the goal is to find the values for a and b that minimize the mean square error

$$J = E\left\{(y - \hat{y})^2\right\} = E\left\{(y - ax - b)^2\right\}$$

a) Solve this linear mean square estimation problem and find the values for a and b.

[10]

b) Using the result from a), derive the minimum mean square error J_{min}.

[3]

c) What are the advantages of using such an estimator?

[3]

d) What is the difference between this estimator and the standard adaptive finite impulse response (FIR) filter trained by the least mean square (LMS) algorithm?

[4]

5) Consider the linear optimum filtering problem (Wiener filtering problem). The input–output relation of the filter is described by

$$y = \sum_{n=1}^{N} w_n x_n$$

If d denotes the desired response for the filter, and the error signal is $e = d - y$, then the mean squared error is defined as

$$J = \frac{1}{2} E\{e^2\}$$

a) Determine the optimum set of coefficients $w_1, \ldots, w_N$ for which the mean squared error J is minimum (Wiener–Hopf equations).

[4]

b) If the weights w assume a time varying form, derive the steepest descent algorithm for an iterative solution of the Wiener filtering problem.

[4]

c) Derive the least mean square (LMS) algorithm based on the use of instantaneous estimates of the statistical quantities used in b).

[4]

i) Explain how this adaptive filter is dependent on the learning rate. What is the range of learning rates for this filter to be stable?

[3]

ii) What are advantages and disadvantages of using the LMS instead of the Wiener filter.

[2]

iii) Define the error performance surface. What is the error performance surface of an FIR filter trained with the normalised least mean square (NLMS) algorithm. Explain the difference between the error performance surface of the LMS and NLMS algorithm.

[3]