Imperial College London

BSc/MSci EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

NUCLEAR AND PARTICLE PHYSICS

For 3rd- and 4th-Year Physics Students

Tuesday, 29th May 2012: 10:00 to 12:00

Answer ALL parts of Section A and TWO questions from Section B.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the FOUR answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in FOUR answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

Go to the next page for questions

SECTION A

1. At an e^+e^- collider, electron and positron beams typically collide with the same energy E_e leading to an annihilation reaction of the type: $e^-e^+ \to x_1 x_2$.

(i) Write the centre-of-mass energy E_{cm} of the annihilation reaction assuming that electron and positron beams are of equal energy $E_{e^-} = E_{e^+}$ [2 marks]

(ii) Show that for a fixed-target interaction, where a positron beam with energy $E^*_{e^+}$ hits electrons at rest, that:

$$E^*_{e^+} = \frac{E^2_{cm} - 2m^2_e c^4}{2m_e c^2}$$

[3 marks]

(iii) Argue why a fixed target interaction for typical E_{cm} around a few hundred GeV or higher would not be a practical option for a collider experiment (assume $E_{cm} \gg m_e c^2$). [2 marks]

One possible reaction that can occur at an electron-positron collider is $e^-e^+ \to q\bar{q}$, where the annihilation reaction produces a quark-antiquark ($q\bar{q}$) pair.

(iv) Draw the Feynman diagram of this reaction and identify all lines and vertices. State what force is involved. [4 marks]

(v) In the experiment this reaction is measured as two jets of hadrons in the final state. Briefly describe the essential features of the process that produces the hadrons. [2 marks]

In analogy with $q\bar{q}$ pair production, a pair of muons can be produced via $e^-e^+ \to \mu^+\mu^-$. At centre-of-mass energies around 40 GeV, the total cross section, σ, of this reaction is given by:

$$\sigma(\mu^+\mu^-) = \frac{4\pi(\alpha\hbar c)^2}{3s},$$

where $\alpha = e^2/4\pi\epsilon_0\hbar c$ is the fine structure constant and s the square of the centre-of-mass energy.

(vi) Use $\sigma(\mu^+\mu^-)$ to show, with justification, that for a particular quark flavour, f, and colour, the ratio:

$$R_f = \frac{\sigma(q_f\bar{q}_f)}{\sigma(\mu^+\mu^-)} = Q^2_f,$$

where Q_f is the ratio of the quark charge to the proton charge. [3 marks]

(vii) Hence derive an expression for

$$R = \frac{\sigma(\text{Hadrons})}{\sigma(\mu^+\mu^-)}$$

for the production of all quark types. [2 marks]

[This question continues on the next page . . .]

(viii) Calculate R for a e^+e^- collider with a centre-of-mass energy $E_{cm} = 40GeV$.
[2 marks]

[Total 20 marks]

The ratios of the quark charges to the proton charge are: $Q_u = Q_c = Q_t = +2/3$ and $Q_d = Q_s = Q_b = -1/3$.

2. The semi-empirical mass formula (SEMF) for a nucleus with N neutrons and Z protons is

$$M(N,Z) = Nm_n + Zm_p - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(N-Z)^2}{A} + \delta$$

where $A = N + Z$. The terms with coefficients a_1, a_2, a_3, a_4 and δ correspond to the volume, surface, Coulomb, asymmetry and pairing terms, respectively.

(i) Explain why the asymmetry term is needed and describe its dependence on A, N and Z. [4 marks]

(ii) By re-arranging the SEMF for $M(N,Z)$ as $M(A,Z) = C_0 + C_1 Z + C_2 Z^2$ show that for A odd and fixed the most strongly bound nucleus has an atomic number given by

$$Z_o = \frac{A}{2} \frac{4a_4 + m_n - m_p}{4a_4 + a_3 A^{2/3}}$$

Sketch the dependence of N with Z that results from this formula. [3 marks]

(iii) What are mirror nuclei? Use the SEMF to determine the mass difference between two mirror nuclei. Hence determine the value of a_3 from the masses of ${}^{23}_{11}Na$ and ${}^{23}_{12}Mg$. [3 marks]

[Total 10 marks]

$M({}^{23}_{11}Na) = 21,415.01$ and $M({}^{23}_{12}Mg) = 21,419.06$ MeV/c^2
$m_p = 938.28$, $m_n = 939.57$ MeV/c^2.
The coefficients a_i and δ in the SEMF in units of MeV/c^2 are given by:
$a_1 = 15.7$, $a_2 = 17.2$, $a_3 = 0.7$, $a_4 = 23.3$ and
$\delta = -12/\sqrt{A}$ for N even - Z even;
$\delta = 0$ for N even - Z odd or N odd - Z even;
$\delta = +12/\sqrt{A}$ for N odd - Z odd.

SECTION B

3. (i) The present composition of natural uranium is 0.7% of ${}^{235}_{92}U$ and 99.3% ${}^{238}_{92}U$. In supernovae explosions large fluxes of neutrons are generated and heavier elements such as the uranium isotopes can be synthesized. Assuming that equal numbers of the two isotopes were originally formed in such explosions estimate their time of formation.

Half lives of ${}^{235}_{92}U$ and ${}^{238}_{92}U$ are 7×10^8 and 4.5×10^9 years respectively. [5 marks]

(ii) A thermal neutron in natural uranium, with the composition given in (i), may create fission in ${}^{235}_{92}U$ or be radiatively captured in ${}^{235}_{92}U$ or in ${}^{238}_{92}U$. Show that the probabilities of each are respectively 0.55, 0.10 and 0.35. At thermal neutron energies the relevant cross-sections are:

$\sigma_5^f = \sigma(\text{fission}, {}^{235}_{92}U) = 580$ barns
$\sigma_5^\gamma = \sigma(\text{radiative capture}, {}^{235}_{92}U) = 100$ barns
$\sigma_5^\gamma = \sigma(\text{radiative capture}, {}^{238}_{92}U) = 2.7$ barns

[2 marks]

(iii) What upper limit does the result in (ii) set for the loss of neutrons during thermalization in a fission reactor fuelled with natural uranium? Assume that each fission reaction creates 2.5 new prompt neutrons. [1 mark]

The cross section of an exothermic fusion nuclear reaction between nuclei of charges Z_1e and Z_2e is given by

$$\sigma = \frac{S}{E} exp\left(\frac{-b}{\sqrt{E}}\right) \qquad (1)$$

where E is the centre of mass energy, and S and b are constants that depend on the particular reaction.

(iv) Explain how b in Equation (1) depends on Z_1 and Z_2. [2 marks]

(v) The relative velocity, v, of a pair of nuclei in a stellar core at temperature T is given by the distribution

$$\Phi(v) = \sqrt{\frac{2}{\pi}}\left(\frac{\mu}{kT}\right)^{2/3} v^2 exp\left(\frac{-\mu v^2}{2kT}\right) \qquad (2)$$

where μ is the reduced mass. Show that the reaction rate, given by $R \propto < \sigma v >$, is highest at an energy, E_0, given by

$$E_0 = \left[\frac{bkT}{2}\right]^{2/3}$$

[3 marks]

(vi) Explain the significance of E_0 in (v) for rates of fusion reactions taking place inside the cores of stars. [2 marks]

[Total 15 marks]

4. The main decay of the muon is $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$.

(i) Explain why the muon can decay and what are the conserved quantities in this decay. Explain how lepton number is conserved is this decay. [2 marks]

(ii) Draw the Feynman diagram for this reaction and label all lines and vertices. [3 marks]

(iii) Why can this decay be approximated by a zero-range four fermion interaction? [2 marks]

Neglecting the electron mass, the energy of the emitted electron E_e in this decay can take values between zero and $m_\mu c^2/2$, where $m_\mu = 105.7$ MeV$/c^2$. The differential energy spectrum $d\Gamma/dE_e$ is given by:

$$\frac{d\Gamma}{dE_e} = \frac{2G_F^2 m_\mu^2 c^4 E_e^2}{(\hbar c)^6 (2\pi)^3} \left(1 - \frac{4E_e}{3m_\mu c^2}\right).$$

(iv) By differentiating the expression, find the most probable energy for the electron. Sketch the energy spectrum over the full allowed range. [5 marks]

(v) Integrate the energy spectrum to show that the total decay width of the muon is given by

$$\Gamma = \frac{G_F^2}{(\hbar c)^6} \frac{(m_\mu c^2)^5}{192\pi^3}.$$

Hence, show that the muon lifetime is 2.2×10^{-6}s . (The value of the Fermi constant is $G_F/(\hbar^3 c^3) = 1.166 \times 10^{-5}$ GeV^{-2}.) [3 marks]

[Total 15 marks]

5. In Supersymmetry (SUSY) a Gluino (G) is the hypothetical supersymmetric partner of a gluon. One of the potential production mechanisms of Gluinos at the LHC is pair production via the gluon (g) fusion process: $gg \to GG$. The LHC collides protons and operated in 2010 and 2011 at a centre-of-mass energy of $E_{cm}^{LHC} = 7$ TeV. The pp collision can be interpreted in terms of a gluon-gluon interaction, where the gluon carries a fraction x of the proton momentum, with $0 \leq x \leq 1$. In the following neglect the proton mass and all transverse momenta.

(i) Draw the two leading order Feynman diagrams for the reaction $gg \to GG$ and label the lines. [2 marks]

(ii) The two colliding protons at the LHC have a momentum of magnitude p in their centre-of-mass frame. Express E_{cm} in terms of p. [2 marks]

(iii) The two initial state gluons carry momentum fractions x_1 and x_2, respectively. Show that the centre-of-mass energy of the gg system is given by:

$$E_{cm}^{gg} = \sqrt{x_1 x_2} E_{cm}^{LHC}$$

[4 marks]

(iv) The LHC is currently probing the mass of 1 TeV for Gluino masses. Define the conditions on x_1 and x_2 so as to have sufficient energy to pair-produce Gluinos with a mass of $m_G = 1$ TeV or higher. Sketch a diagram with axes x_1 and x_2, indicating the region where Gluino pair production is possible. [4 marks]

(v) Considering typical decay chains of the Gluino, argue why a four-jet plus missing energy signature could be a sign of GG production at the LHC. [3 marks]

[Total 15 marks]

6. Write short notes on THREE of the following subjects.

(i) A free neutron has a mean lifetime of ≈15minutes but neutrons are found in stable nuclei. Comment on these two facts. [5 marks]

(ii) $^{235}_{92}U$ has a large thermal neutron fission cross-section when compared with $^{238}_{92}U$. By comparing, in each case, the mass of the compound nucleus formed by the absorption of the thermal neutron with the mass of the initial state particles outline your reasoning for this fact. You may find useful the semi-empirical mass formula (SEMF) for a nucleus with N neutrons and Z protons,

$$M(N,Z) = Nm_n + Zm_p - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(N-Z)^2}{A} + \delta$$

where $A = N + Z$. The coefficients a_i and δ in the SEMF in units of MeV/c^2 are given by: $a_1 = 15.7$, $a_2 = 17.2$, $a_3 = 0.7$, $a_4 = 23.3$ and
$\delta = -12/\sqrt{A}$ for N even - Z even;
$\delta = 0$ for N even - Z odd or N odd - Z even;
$\delta = +12/\sqrt{A}$ for N odd - Z odd.

[5 marks]

(iii) The importance of the Higgs boson in the Standard Model. [5 marks]

(iv) The process of hadronisation following high energy interactions that produce quarks. [5 marks]

(v) Lepton universality and charged pion decay. [5 marks]

[Total 15 marks]