UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2002

BEng Honours Degree in Computing Part III
MSc in Computing Science
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part IV
MEng Honours Degrees in Computing Part IV
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C381=I3.30

COMPUTATIONAL FINANCE

Tuesday 30 April 2002, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 Consider a portfolio with two risky assets A and B. The mean rates of returns are 0.015 and 0.012 for assets A, B, respectively. The covariance matrix is given as follows:

$$\Lambda = \begin{bmatrix} 0.007 & 0.001 \\ 0.001 & 0.003 \end{bmatrix}$$

Let the weights be w_A and w_B for assets A and B.

a Compute the expected rate of return and expected risk of the portfolio constructed by assets A and B.

b Write the Markowitz model.

c Compute the purely risk–averse investment strategy.

d What is the expected return for the purely risk–averse investment strategy?

e Describe (but do not evaluate numerically) the efficient frontier. Explain the risk–seeking and risk–averse investment strategies on the efficient frontier.

(The third part carries 40% and the other parts each carry 15% of the marks).

2a Determine the minimizer or maximizer point(s) and value(s) of the function

$$f(x) = (2x)^2 \left(x - 3 \left(\frac{x}{2} \right)^2 \right), \quad -\infty \leq x \leq +\infty$$

b Determine the Lagrange function in matrix/vector form of the following quadratic programming problem:

$$\begin{array}{rrcrcrcrcr} \min\ f(x,y) = & 2x & - & 3y & + & 4x^2 & - & 7xy & + & 5y^2 \\ \text{s.t.} & x & + & 3y & = & 8 & & & & \\ & -x & + & 4y & \leq & -1 & & & & \\ & & & x,\ y \geq 0. & & & & & & \end{array}$$

c Using the Lagrange function of (b) above, determine the KKT conditions of the problem. Discuss the use of the KKT conditions.

(The three parts carry, respectively, 30%, 30% and 40% of the marks).

3 Consider a bond which has maturity of 2 years, pays coupon payments of £1120 after the first year and the second year. The face value at the end of the second year is £11424. The yield to maturity is 12%.

Hint: *Use the following formulas as necessary.*

The Macaulay Duration:

$$D = \frac{\sum_{t=1}^{2} t \times PV(C_t)}{P}$$

Convexity:

$$C = \frac{\sum_{k=1}^{n} k \times (k+1) \times PV(C_k)}{P(1+\lambda)^2}$$

a Find the net present value of the bond given the cash payments of two years.

b Find the Macaulay duration, the modified duration and the convexity.

c Suppose that the yield to maturity decreases 1% (from 12% to 11%). Derive the new price of the bond using only the modified duration.

d Derive the new price of the bond using the modified duration and convexity approximations.

e Which approximation is better? Why? Compare your results with the actual price obtained with the new yield.

(The third part carries 40% and the other parts each carry 15% of the marks).

4 Consider a binomial lattice model for the stock price process $\{S_n : 0 \leq n \leq 2\}$ and $S_0 = £150$. Let the price rise or fall by 8% at each step. The risk–free interest rate is assumed $r = 4\%$. The contract we wish to price is a European put option with strike price £170 at time 2.

a Compute the risk neutral probabilities.

b Evaluate the stock prices on 3-period binomial lattice.

c Derive the cash position and stock position for 2 periods on 2-period binomial lattice using "the replicating portfolio method". Calculate the option price.

(The first two parts carry 20% each, and the last part carries 60% of the marks).