

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2014-15

EEE PART III/IV: MEng, BEng and ACGI

Corrected Copy

**OPTOELECTRONICS**

Tuesday, 9 December 2:00 pm

Time allowed: 3:00 hours

**There are SIX questions on this paper.**

**Answer FOUR questions.**

*All questions carry equal marks*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible	First Marker(s) :	R.R.A. Syms
	Second Marker(s) :	O. Sydoruk



### Fundamental constants

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ m kg/C}^2$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.62 \times 10^{-34} \text{ Js}$$

### Maxwell's equations – integral form

$$\oint \int_A \underline{\mathbf{D}} \cdot d\mathbf{a} = \int \int \int_V \rho \, dv$$

$$\oint \int_A \underline{\mathbf{B}} \cdot d\mathbf{a} = 0$$

$$\oint \int_L \underline{\mathbf{E}} \cdot d\mathbf{L} = - \int \int_A \frac{\partial \underline{\mathbf{B}}}{\partial t} \cdot d\mathbf{a}$$

$$\oint \int_L \underline{\mathbf{H}} \cdot d\mathbf{L} = \int \int_A [\underline{\mathbf{J}} + \frac{\partial \underline{\mathbf{D}}}{\partial t}] \cdot d\mathbf{a}$$

### Maxwell's equations – differential form

$$\text{div}(\underline{\mathbf{D}}) = \rho$$

$$\text{div}(\underline{\mathbf{B}}) = 0$$

$$\text{curl}(\underline{\mathbf{E}}) = -\frac{\partial \underline{\mathbf{B}}}{\partial t}$$

$$\text{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \frac{\partial \underline{\mathbf{D}}}{\partial t}$$

### Material equations

$$\underline{\mathbf{J}} = \sigma \underline{\mathbf{E}}$$

$$\underline{\mathbf{D}} = \epsilon \underline{\mathbf{E}}$$

$$\underline{\mathbf{B}} = \mu \underline{\mathbf{H}}$$

### Vector calculus (Cartesian co-ordinates)

$$\text{grad}(\phi) = \frac{\partial \phi}{\partial x} \underline{\mathbf{i}} + \frac{\partial \phi}{\partial y} \underline{\mathbf{j}} + \frac{\partial \phi}{\partial z} \underline{\mathbf{k}}$$

$$\text{div}(\underline{\mathbf{F}}) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\text{curl}(\underline{\mathbf{F}}) = \underline{\mathbf{i}} \{ \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \} + \underline{\mathbf{j}} \{ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \} + \underline{\mathbf{k}} \{ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \}$$

$$\text{curl} \{ \text{curl}(\underline{\mathbf{F}}) \} = \text{grad} \{ \text{div}(\underline{\mathbf{F}}) \} - \nabla^2 \underline{\mathbf{F}}$$

$$\oint \int_A \underline{\mathbf{F}} \cdot d\mathbf{a} = \int \int \int_V \text{div}(\underline{\mathbf{F}}) \, dv$$

$$\oint \int_L \underline{\mathbf{F}} \cdot d\mathbf{L} = \int \int_A \text{curl}(\underline{\mathbf{F}}) \cdot d\mathbf{a}$$

1. Propagation of TE polarized electromagnetic waves in the (x, z) plane may be described by the scalar wave equation:

$$\partial^2 E_y / \partial x^2 + \partial^2 E_y / \partial z^2 + n^2 k_0^2 E_y = 0$$

Here  $E_y$  is the amplitude of the (y-polarised) electric field,  $n$  is the refractive index, and  $k_0 = 2\pi/\lambda$  where  $\lambda$  is the free-space wavelength.

- a) Show that the field  $E_y(x, z) = E_0 \exp\{-jnk_0[x \sin(\theta) + z \cos(\theta)]\}$  is a solution.

What does it represent? Assuming that  $\theta \approx 30^\circ$ , sketch lines of constant phase.

[6]

- b) Show that the field  $E_y(x, z) = E_0 \exp(\gamma x) \exp(-j\beta z)$  can also be a solution. What does this solution represent? Sketch lines of constant amplitude and constant phase.

How realistic is the solution, if its amplitude becomes very large as  $x$  rises?

[8]

- c) Assuming a modal solution in the form  $E_y = E(x) \exp(-j\beta z)$ , derive the waveguide equation for z-propagating modes. Show that the transverse field  $E(x) = E_0 \exp(\gamma x)$  may be a solution. What other solutions are possible, and when?

[6]

2. a) Sketch the spectral variation of attenuation in silica fibre, and outline the major physical mechanisms contributing to loss. Explain how this variation drives the choice of operating wavelength for long-distance optical fibre communication systems.

[6]

b) Sketch the corresponding spectral variation of refractive index. Explain how this variation drives the choice of wavelength for high bit-rate fibre communications. Is this conclusion in conflict with part a)? If so, what can be done to resolve the situation?

[6]

c) The broadening of a pulse of bandwidth  $\Delta\omega$  as it propagates through a distance  $L$  is  $\Delta T = L d(1/v_g)/d\omega \Delta\omega$ , where  $v_g = d\omega/dk$  is the group velocity. Show that this is equivalent to  $\Delta T = -L(\lambda_0/c) d^2n/d\lambda_0^2 \Delta\lambda_0$ , where  $n$  is the refractive index of the medium,  $\lambda_0$  is the free-space wavelength and  $\Delta\lambda_0$  is the equivalent spectral range.

[8]

3. a) Sketch the layout and briefly describe the operation of an electro-optic directional coupler switch, explaining what is meant by the 'bar' and 'cross' states. [6]
- b) Draw the schematic of a 4 x 4 non-blocking switch array and sketch the physical arrangement of a coupler-based array. How many couplers are required in a general N x N non-blocking array? [4]
- c) The coupled mode equations of an asynchronous directional coupler are:
- $$dA_1/dz + j\kappa A_2 \exp(-j\Delta\beta z) = 0$$
- $$dA_2/dz + j\kappa A_1 \exp(+j\Delta\beta z) = 0$$
- Here  $A_1$  and  $A_2$  are the amplitudes of the modes in the two guides,  $\kappa$  is the coupling coefficient and  $\Delta\beta$  is the dephasing parameter. Without solving the equations, show that they conserve power. [5]
- d) Assuming that the device is synchronous, solve the equations for the boundary conditions  $A_1 = 1$ ,  $A_2 = 0$  on  $z = 0$ . Show that the solutions also conserve power. [5]
4. a) Explain the operation of a photodiode. Why is a photodiode more efficient than a photoconductive detector? [7]
- b) Sketch and explain the distribution of electrons near the valence and conduction band edges at room temperature in a semiconductor. What does this distribution imply about the relative likelihood of absorption of photons whose energies are i) only just greater and ii) much greater than the band-gap? [7]
- c) Draw the physical arrangement of a surface entry photodiode. How does the result in b) above affect its performance? Sketch the spectral variation in responsivity you would expect. [6]

5. a) Explain the difference between direct- and indirect-gap materials. Which type of material is required in optoelectronics?  
[6]
- b) Explain the difference between spontaneous emission and stimulated emission. Which phenomenon is exploited in LEDs and lasers?  
[6]
- c) Explain why the external efficiency of an LED is so low. Assuming the refractive index of GaAs is 3.5, estimate the external efficiency of a GaAs LED.  
[8]
6. A semiconductor laser designed for operation at  $1.5\ \mu\text{m}$  wavelength is based on a waveguide with effective index  $n_{\text{eff}} = 3.5$  and gain coefficient  $g = 2355\ \text{m}^{-1}$  when lasing.
- a) Determine the conditions for round trip resonance, and show how this leads to separate conditions for phase and gain.  
[8]
- b) Estimate the length of the laser and the spectral separation  $\Delta\lambda$  between adjacent longitudinal modes.  
[6]
- c) A new laser is fabricated, with a different cavity length, and it is noted that its spectral separation  $\Delta\lambda$  has double the value found in b). Estimate the gain coefficient now needed for lasing. Comment on the tradeoff between spectral separation and gain.  
[6]

