

E1.4 SOLUTIONS

Question 1

a) The $10\text{ k}\Omega$ resistor carries the emitter current, so for a collector voltage of 2.5 V we require $I_E = 0.25\text{ mA}$, implying a base current of $I_B = 250/201 = 1.244\text{ }\mu\text{A}$. Assuming $V_{BE} = 0.7\text{ V}$, the required value of R_B is given by $R_B = (5 - 0.7)/I_B = 3.46\text{ M}\Omega$. [6]

b) From the simplified Ebers-Moll equation, the collector currents can be expressed as:

$$I_{C1} = I_S \exp[(V_{IN1} - V_E)/V_T] ; I_{C2} = I_S \exp[(V_{IN2} - V_E)/V_T]$$

where V_E is the common emitter voltage, V_T is the thermal voltage, and I_S is the saturation current (NB assuming identical transistors). It follows that $I_{C1}/I_{C2} = \exp(V_D/V_T)$, and we also know that $I_{C1} + I_{C2} = I$ where I is the tail current. Eliminating I_{C1} or I_{C2} the collector currents are obtained as:

$$I_{C1} = I/[1 + \exp(-V_D/V_T)] ; I_{C2} = I/[1 + \exp(V_D/V_T)]$$

The differential output voltage is then:

$$V_{OUT} = R_C(I_{C1} - I_{C2}) = R_C I \left[\frac{1}{1 + \exp(-V_D/V_T)} - \frac{1}{1 + \exp(V_D/V_T)} \right] = R_C I \tanh(V_D/2V_T)$$

with $R_C = 5\text{ k}\Omega$, $I = 1\text{ mA}$, and $V_T = 25\text{ mV}$, the output voltage becomes $5 \tanh(20V_D)$ as required. [6]

The double-ended differential gain can be obtained simply as the derivative of the large-signal relationship at $V_D = 0$:

$$A = \left. \frac{dV_{OUT}}{dV_D} \right|_{V_D=0} = 100 \text{sech}^2(20V_D) \Big|_{V_D=0} = +100$$

A solution based on analysis of the SSEC is also acceptable. [4]

c) Since the transistors are matched, only the finite beta and the output resistance will contribute to the current error. Taking both of these contributions into account, the output current I can be expressed as:

$$I \approx I_{ref} - 2I/\beta + \Delta V_{CE}/r_o$$

where I_{ref} is the input current. So, the currents will be equal when $\Delta V_{CE} \approx 2I r_o / \beta = 2V_A / \beta$, where we have used $r_o = V_A/I$. Putting $V_A = 120\text{ V}$, $\beta = 100$ gives $\Delta V_{CE} \approx 2.4\text{ V}$, and since the input side transistor has $V_{CE} = 0.7\text{ V}$ (assumption), this implies $V_{OUT} \approx 3.1\text{ V}$. [6]

d) Since we are calculating for the case where the MOSFET is at pinch-off, we know we can use the active mode drain current equation. With $V_G = 0$, and $V_S = I_D R_S$, we need to solve:

$$I_D = V_S / R_S = K(-V_S - V_t)^2 \Rightarrow 2V_S^2 - 5V_S + 2 = 0$$

The roots are $V_S = 0.5$, $V_S = 2$, and the valid root is the first one (since second leaves MOSFET sub-threshold). The drain current is $V_S/R_S = 50 \mu\text{A}$.

Since $V_S = 0.5$, and $V_{DS} = V_{GS} - V_t = 0.5 \text{ V}$, the drain voltage is 1.0 V, and the supply voltage is $V_{DD} = V_D + I_D R_D = 1 + 0.05\text{m} \times 30\text{k} = 2.5 \text{ V}$.

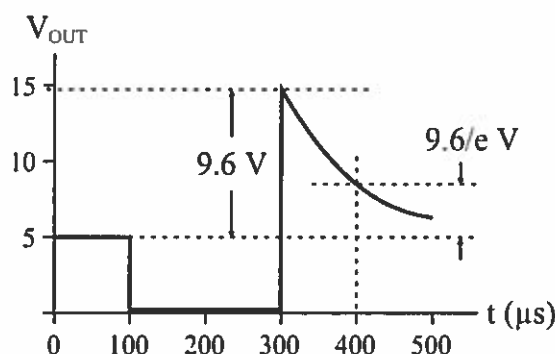
[6]

e) Before the input pulse, the transistor is off and the circuit is in steady-state with $I_L = 0$ and $V_{OUT} = 5 \text{ V}$. During the input pulse, the base current is $I_B = (5 - 0.7)/5\text{k} = 0.86 \text{ mA}$, and the collector current is $\beta I_B = 172 \text{ mA}$ if transistor is active, or $< 172 \text{ mA}$ if it is saturated. We have assumed $V_{BE} = 0.7 \text{ V}$.

The inductor current is continuous, and the maximum current in the 100Ω resistor is $\approx 50 \text{ mA}$, so the transistor is initially saturated with $V_{CE} = V_{CEsat} \approx 0.2 \text{ V}$ (assumption) and $I_C \approx 48 \text{ mA}$. With 4.8 V across the inductor, the inductor current will rise at a rate of $dI_L/dt = V_L/L = 480 \text{ A/sec}$. By the end of the pulse it will have reached $480 \times 200\mu = 96 \text{ mA}$. Since the total load current at this point ($96 + 48 = 144 \text{ mA}$) is $< 172 \text{ mA}$ we know the transistor does not come out of saturation.

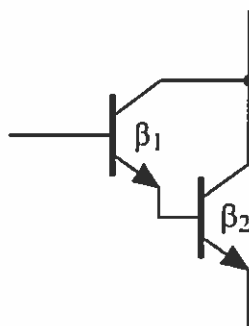
When the transistor switches off, the inductor current will continue to flow in the 100Ω resistor, and the output voltage will rise suddenly to $V_{CC} + I_L R = 5 + 0.096 \times 100 = 14.6 \text{ V}$. It will then decay exponentially over time, with an asymptotic value of 5 V. The time-constant for this decay will be $L/R = 0.01/100 = 100 \mu\text{s}$.

[4]



[4]

f)



The base current of the RH transistor is $I_{B2} = (1 + \beta_1)I_{B1}$. The total collector current for the Darlington pair is $I_C = I_{C1} + I_{C2} = \beta_1 I_{B1} + \beta_2 I_{B2} = [\beta_1 + (1 + \beta_1)\beta_2]I_{B1}$. The overall current gain is then $I_C/I_{B1} = \beta_1 + \beta_2 + \beta_1\beta_2$.

[4]

Question 2

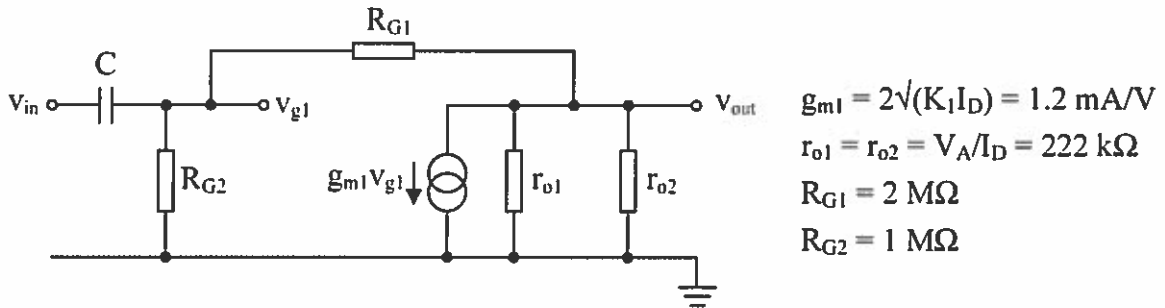
a) The drain current will be set by the depletion load which has $V_{GS} = 0$. Assuming this device is active (as question suggests), we have $I_D = K_2(-V_{t2})^2 = 0.45 \text{ mA}$. The lower MOSFET must be carrying the same current, so we also have $I_D = K_1(V_{GS1} - V_{t1})^2$. Rearranging this we obtain $V_{GS1} = \sqrt{I_D/K_1} + V_{t1} = 1.75 \text{ V}$, where we have taken the +ve square root because Q1 is above threshold. The potential divider forces $V_{OUT} = 3V_{GS1}$, so $V_{OUT} = 5.25 \text{ V}$. [6]

Q1 has $V_{DS} = 5.25 \text{ V}$, $V_{GS} = 1.75 \text{ V}$, $V_t = 1 \text{ V}$, so $V_{DS} > V_{GS} - V_t$ and **active**.

Q2 has $V_{DS} = 10 - 5.25 = 4.75 \text{ V}$, $V_{GS} = 0$, $V_t = -1.5 \text{ V}$, so also **active**.

Q2 will enter triode when $V_{DS} = -V_t = 1.5 \text{ V}$, i.e. when $V_{DD} = 5.25 + 1.5 = 6.75 \text{ V}$. [3]

b) SSEC (input capacitor may be omitted):

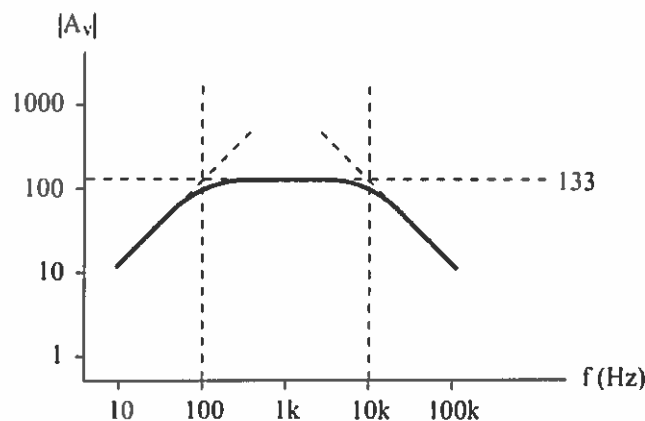


KCL at output gives: $g_{m1}v_{g1} + v_{out}/r_{o1} + v_{out}/r_{o2} + (v_{out} - v_{g1})/R_{G1} = 0$. Collecting terms in v_{out} and v_{g1} , and noting the $v_{g1} \approx v_{in}$ in the mid-band, the mid-band gain is obtained as: [6]

$$A_v = v_{out}/v_{g1} = -(g_{m1} - 1/R_{G1}) \cdot (r_{o1} // r_{o2} // R_{G1}) = -1.2 \text{ m} \times 105.3 \text{ k} = -126.3$$
 [3]

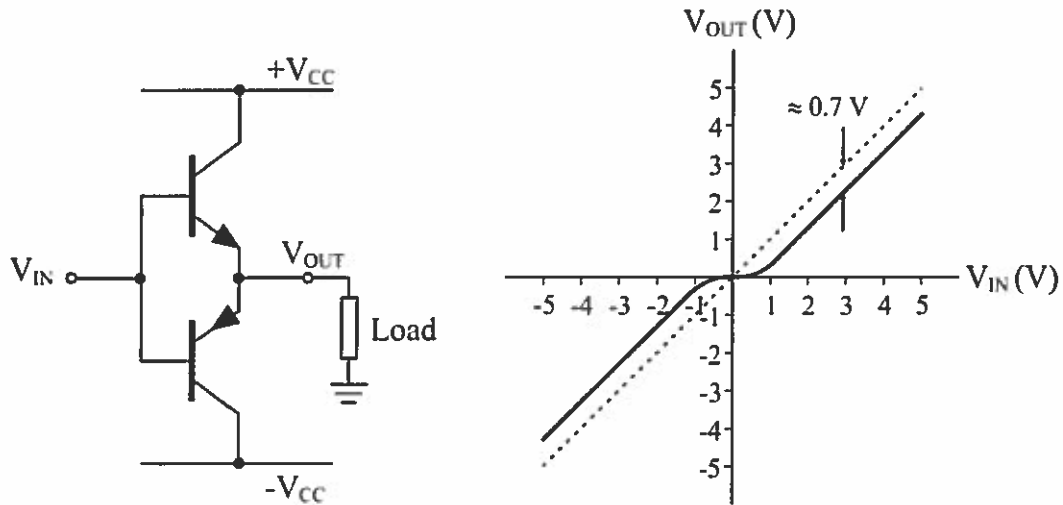
The input resistance is $R_i = R_{G2} // [R_{G1}/(1 - A_v)] = 1 \text{ M} // 15.7 \text{ k} = 15.5 \text{ k}$. The cut-off frequency of the input filter is $f_{ci} = 1/(2\pi R_i C)$, so for $f_{ci} = 100 \text{ Hz}$ we require $C = 1/(2\pi R_i f_{ci}) = 103 \text{ nF}$. [5]

c) The load produces a low-pass response, with cut-off $f_{co} = 1/(2\pi R_o C_L)$ where R_o is the amplifier output resistance and C_L is the load capacitance. With $R_o = r_{o1} // r_{o2} // R_{G1} = 105.3 \text{ k}\Omega$, and $C_L = 150 \text{ pF}$, the cut-off is at $f_{co} = 10.08 \text{ kHz}$. [2]



Question 3

a)



[4 + 4]

The output stage is a unity (voltage) gain amplifier, but there is a V_{BE} offset between input and output, and a region near the origin where it is unresponsive, leading to cross-over distortion.

[2]

b) From symmetry, $V_{out} = 0$ occurs when $V_{in} = 0$. KVL under these conditions gives:

$$V_{BE1} = V_{BE2} + I_2 R$$

where we have ignored the base current of Q2. From the simplified Ebers-Moll equation we know that $V_{BE1} = V_T \ln(I_1/I_{S1})$ and $V_{BE2} = V_T \ln(I_2/I_{S2}) = V_T \ln[I_2/(NI_{S1})]$. Substituting for V_{BE1} and V_{BE2} in the top equation we obtain $V_T \ln(I_1/I_{S1}) = V_T \ln[I_2/(NI_{S1})] + I_2 R$. Taking the inverse log of both sides gives the desired result.

[8]

Rearranging the given equation: $R = (V_T/I_2) \ln(NI_1/I_2)$. With $I_{S1} = 0.05$ pA, $N = 10$, $V_T = 25$ mV, $I_1 = 2$ mA, $I_2 = 10$ mA, we find $R = 1.73 \Omega$.

[2]

c) With $V_{OUT} = 5$ V and a 50Ω load, Q2 has an emitter current of $I_{E2} = 100$ mA. The base-emitter voltage of Q2 under these conditions is $V_{BE2} = V_T \ln(\alpha I_{E2}/I_{S2}) = 650$ mV. The emitter current of Q1 is $I_{E1} = I_1 - I_{E2}/(1 + \beta) = 2 - 100/101 = 1.01$ mA, and its base-emitter voltage is $V_{BE1} = V_T \ln(\alpha I_{E1}/I_{S1}) = 593$ mV. The input voltage is $V_{in} = V_{out} + I_{E2} R + V_{BE2} - V_{BE1}$, which gives $V_{in} = 5.23$ V. The voltage gain is therefore $V_{out}/V_{in} = 0.956$.

[7]

d) With $V_{OUT} = 10$ V, the emitter currents of Q2 and Q1 are $I_{E2} = 200$ mA and $I_{E1} = 0.02$ mA respectively (almost at limit of output range). Repeating the above calculations in this case gives $V_{BE2} = 668$ mV, $V_{BE1} = 495$ mV, $V_{in} = 10.52$ V, and $V_{out}/V_{in} = 0.951$.

The amplifier appears to be linear to within $\approx 0.5\%$, based on the gains at 50% and 100% of full scale output. However, the comparison doesn't tell us anything about the residual cross-over distortion.

[3]