IMPERIAL COLLEGE LONDON

EE4-06
EE9AO9
EE9SO9

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

MSc and EEE/ISE PART IV: MEng and ACGI

Q2

Corrected Copy

OPTICAL COMMUNICATION

Friday, 20 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 6

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : E.M. Yeatman, E.M. Yeatman

Second Marker(s) : A.S. Holmes, A.S. Holmes

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3.0 \times 10^8$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x}\tan(k_{1x}d/2) \quad \text{and} \quad \kappa = -\,k_{1x}\cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, state any assumptions or approximations made, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]

a) A slab waveguide supports 3 TE modes, m = 0, 1 and 2. Which of these modes, if any, have zero intensity in the centre of the slab?

b) A silica optical fibre has an index difference Δn of 0.003. Estimate the numerical aperture.

c) Visible light propagates through a sheet of uncoated window glass at approximately normal incidence. If the glass has a refractive index 1.51 in the visible range, estimate the fraction of optical power transmitted.

d) A certain optical receiver detects 10^{12} photons/s, at a nominal wavelength of 1.5 μm. What is the equivalent received optical power in dBm?

e) Briefly describe one way in which a heterostructure can be used to increase the performance of a p-i-n photodiode.

f) A 2.5 ns square pulse propagates in standard silica fibre. Approximately how long is it spatially?

g) A photodetector operating at a nominal wavelength of 1.30 μm has a responsivity of 0.87 A/W. Calculate the quantum efficiency.

h) Which type of signal degradation is more difficult to compensate in an optical link: attenuation, dispersion, or nonlinearity?

i) Briefly describe the difference between homodyne and heterodyne coherent receivers.

j) A certain optical receiver has a noise equivalent power of 5 pW/√Hz. Assuming receiver noise dominates, what optical received power will be needed to achieve an optical SNR of 12 if the bit rate is 1 Gbit/s?

2. A symmetric slab waveguide as shown in Fig. 2.1 has a core thickness $d = 6\ \mu m$, and core and cladding indices of $n_1 = 1.47$ and $n_2 = 1.46$ respectively, and supports propagation for a free-space wavelength of $1.53\ \mu m$.

a) Find the number of transverse electric (TE) modes supported by the guide. [4]

b) Calculate the effective index for each of the supported TE modes. You may find the plot of Fig 2.2 helpful. [10]

c) For each of the supported modes, determine the distance from the core-cladding interface at which the field amplitude in the cladding has fallen to half its value at the interface. [6]

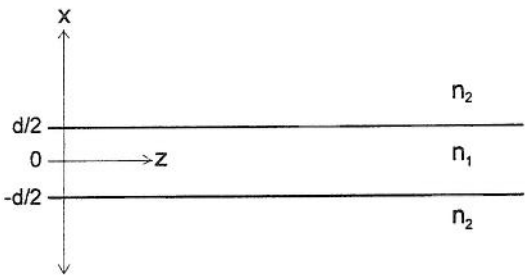


Figure 2.1 Slab waveguide

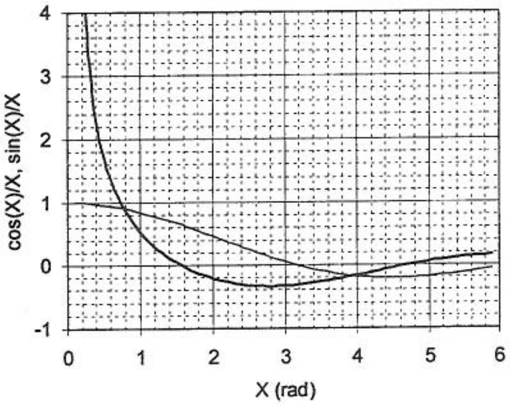


Figure 2.2 cos(X)/X (dark line) and sin(X)/X

3. A laser of peak output power 6 dBm, with nominal wavelength $\lambda_o = 1510$ nm and spectral width $\Delta\lambda = 0.5$ nm, emits into a fibre with an attenuation of 0.22 dB/km and dispersion coefficient 0.8 ps/nm·km. It is modulated with an on-off keying format at a bit rate B. The receiver has quantum efficiency $\eta = 0.85$ and input resistance $R = 10$ kΩ.

a) Assume that the SNR is dominated by thermal noise in the receiver. Hence, derive an expression for the maximum bit rate as a function of fibre length, for an optical SNR of 12. Neglect dispersion, and state any other approximations or assumptions made. [5]

b) Assume that the SNR is dominated by shot noise in the receiver. Hence, derive an expression for the maximum bit rate as a function of fibre length, for an optical SNR of 12. Neglect dispersion, and state any other approximations or assumptions made. [5]

c) Derive an expression for the bit rate at which the dispersion time is equal to one quarter of the bit time 1/B, as a function of fibre length. [5]

d) On a single graph of bit rate vs. ~~time~~ Length, sketch the three relations derived in (a), (b) and (c) above, using appropriate scales and ranges for your two axes. Briefly discuss the significance of this graph for the operation of this optical link. [5]

4. A certain single mode fibre has a wavelength-dependent effective index n' given by:

$$n' = n_g + \alpha(\lambda_o - \lambda_c)^2 \tag{4.1}$$

where λ_o is the free-space wavelength and λ_c is the centre wavelength of the operating range.

a) Derive expressions for the phase delay τ_p, and the group delay τ_g, for a fibre length L. Note that the group velocity is given by $v_g = d\omega/d\beta$ where β is the propagation constant. [12]

b) Using your solution to (a), derive an expression for the dispersion coefficient D, using $D = |(d\tau_g / d\lambda_o)/L|$. Hence, show that $D = |\lambda_o(d^2n/d\lambda_o^2)/c|$ for this case.

Since dispersion causes a frequency (or wavelength) dependent phase shift, it can be modelled as an all-pass filter. Find an expression for a filter function $H(\omega)$ equivalent to the dispersion effect of this fibre for a length L. [8]

5\. a) A light emitting diode (LED) has a horizontal active region located 4 μm from the flat, horizontal semiconductor-air interface. Calculate the maximum angle from the vertical, θ_m, for which emitted photons are able to cross the semiconductor-air interface, and find the fraction of photons which are emitted within this angular range. Assume that the active region emits photons equally in all directions. The refractive index of the semiconductor is 3.7. [6]

b) For the photons emitted within the angular range θ_m determined above, estimate the fraction absorbed between the active region and the surface. Assume an attenuation coefficient of 1.5×10^3 cm^{-1}. State any assumptions or approximations made. [4]

c) A uniform plastic layer of thickness 5 μm and refractive index 1.52 is now added to the semiconductor surface. Find the maximum emission angle θ_m which is now required for emitted photons to cross the semiconductor-plastic interface. For the photons crossing this interface, and neglecting absorption and Fresnel reflection at the interfaces, find the fraction which escape into the air. Briefly discuss the effect of this added layer on the overall external quantum efficiency η_{ext}. [6]

d) A domed plastic layer placed on the semiconductor surface can be used to increase η_{ext}. Why is such a structure unlikely to increase the amount of light that can be coupled from the LED into a single mode optical fibre? [4]

6. A silicon p-i-n photodiode has p and n doping levels respectively of $N_A = 3\times10^{21}\ m^{-3}$ and $N_D^+ = 10^{21}\ m^{-3}$. The p-layer thickness is 0.5 μm. The attenuation coefficient in Si at the wavelength of interest is given by $\alpha = 0.25\times10^{6}\ m^{-1}$.

a) Find the intrinsic layer thickness w_i such that 80% of photons are absorbed in the intrinsic layer (neglecting Fresnel reflection at the surface). [5]

b) Using w_i as calculated above, find the intrinsic layer doping level N_D^- such that the intrinsic region can be fully depleted by an applied voltage of 4.0 V. [5]

c) Using N_D^- and w_i as calculated above, find the applied bias voltage V such that the electric field amplitude in the intrinsic region varies by 20% (i.e. maximum value is 120% of minimum value). [5]

d) Discuss the main factors to be considered in optimising the electric field magnitude in a p-i-n photodiode. [5]

EE4-06

Optical Communication EE9A09

SOLUTIONS 2011 EE9S09

a) A slab waveguide supports 3 TE modes, m = 0, 1 and 2. Which of these modes, if any, have zero intensity in the centre of the slab?

Odd numbered modes have E=0 at centre, so only m=1.

b) A silica optical fibre has an index difference Δn of 0.003. Estimate the numerical aperture.

$NA = \sqrt{2n\Delta n}$ *so taking* $n = 1.5$ *we get* $NA = 0.095$

c) Visible light propagates through a sheet of uncoated window glass at approximately normal incidence. If the glass has a refractive index 1.51 in the visible range, estimate the fraction of optical power transmitted.

$R = ((n1-n2)/(n1+n2))^2 = 0.0413$, *so 4.13 % lost at each surface, ignoring multiple reflections the fraction transmitted is 91.7%.*

d) A certain optical receiver detects 10^{12} photons/s, at a nominal wavelength of 1.5 µm. What is the equivalent received optical power in dBm?

$P = \text{energy/photon} \times \text{photons/s} = hc/\lambda \times 10^{12} = 133\ nW$.

e) Briefly describe one way in which a heterostructure can be used to increase the performance of a p-i-n photodiode.

To prevent wasteful absorption in the p+ layer before reaching the depletion region we can construct this layer from a higher bandgap compostion and make it transparent.

f) A 2.5 ns square pulse propagates in standard silica fibre. Approximately how long is it spatially?

$L = vt = 2 \times 10^8 \times 2.5 \times 10^{-9} = 0.5\ m$, *assuming effective index of* ~ 1.5.

g) A photodetector operating at a nominal wavelength of 1.30 µm has a responsivity of 0.87 A/W. Calculate the quantum efficiency.

Since $\mathcal{R} = \eta e\lambda/hc$, $\eta = hc\,\mathcal{R}/e\lambda = 6.63e\text{-}34 \times 3e8 \times 0.87 / (1.6e\text{-}19 \times 1.3e\text{-}6) = 0.956$.

h) Which type of signal degradation is more difficult to compensate in an optical link: attenuation, dispersion, or nonlinearity?

Nonlinearity. The others are linear and can be compensated by amplification and dispersion of opposite sign respectively.

i) Briefly describe the difference between homodyne and heterodyne coherent receivers.

Both mix the received signal with a local oscillator – in homodyne this is at the carrier frequency, while for heterodyne it is at an offset so that the received signal is shifted to an intermediate frequency rather than directly to baseband.

j) A certain optical receiver has a noise equivalent power of 5 pW/√Hz. Assuming receiver noise dominates, what optical received power will be needed to achieve an optical SNR of 12 if the bit rate is 1 Gbit/s?

For 1 Gbit/s we need $\Delta f = 0.5 \times 10^9$ *so the noise (optical) power is 0.11 uW so we need received power 1.3 uW.*

(2)

a) Taking $Y = Kd/2$ and $X = k_{1x}d/2$, the eigenvalue equations become:

$$Y = X\tan X \quad \text{and} \quad Y = -X\cot X$$

The phase matching condition $k_{1z} = k_{2z}$ yields:

$$K^2 + k_{1x}^2 = NA^2 k_0^2 \quad \text{with } NA = \sqrt{n_1^2 - n_2^2}$$

so $X^2 + Y^2 = R^2$ with $R = NA \cdot k_0 \cdot d/2 = NA\,\pi d/\lambda_0$

In this case we can calculate $R = 2.109$

$\frac{\pi}{2} < R < \pi$

2 modes are supported $m=0$, $m=1$

b) For $m=0$, $X^2+Y^2 = X^2(1+\tan^2 X) = \left(\frac{X}{\cos X}\right)^2 = R^2$

so we need $\frac{\cos X}{X} = \frac{1}{R} = 0.4742$

From Fig 2.2 this is at $\sim X = 1$. We then iterate $X = 2.109\cos X$ to get $X = 1.050$

Similarly $m=1$ needs $X = 2.109\sin X$, giving $X = 1.955$

For both, $n' = \beta/k_0 = \sqrt{n_1^2 - \left(\frac{\lambda_0 X}{\pi d}\right)^2}$

$m=0$: $\underline{n' = 1.4675}$ $\quad m=1$: $\underline{n' = 1.4614}$

c) In cladding, field goes as $\exp(-Kx)$ so we need to find $\exp(-K\Delta x) = \frac{1}{2}$ or $\Delta x = \ln 2/K$

$K = 2Y/d$, $m=0$: $\Delta x = \frac{\ln 2}{2} \cdot \frac{6\mu m}{1.05\tan 1.05} = \underline{1.14\mu m}$

$m=1$: $\Delta x = \frac{\ln 2}{2} \cdot \frac{6\mu m}{1.955\cot 1.955} = \underline{2.63\,\mu m}$

(3)

a) $(I_{th}^{*})^2 = 4kT/R$ If thermal noise dominates then $(SNR)^2 = \dfrac{I_{ph}^2}{(4kT/R)\Delta f}$ and we assume $\Delta f = B/2$

and

$(I_{ph})^2 = (R\Phi_R)^2 = \left(\frac{\eta e \lambda}{hc}\Phi_R\right)^2 = 1.065\,\Phi_R^2 = 1.19\times10^{-22} B$

then $10\log(B) - 219 = 20\log\Phi_R = 20\log\Phi_T - .44L$

$\Phi_T = 4\times10^{-3}$ W

Finally $\log B = 17.1 - 0.044L$

b) $(I_{sh}^{*})^2 = 2eI_{ph}$. If this dominates, then

$SNR^2 = \dfrac{I_{ph}^2}{2eI_{ph}B/2} = \dfrac{I_{ph}}{eB} = \dfrac{\eta\lambda}{hc}\dfrac{\Phi_R}{B}$

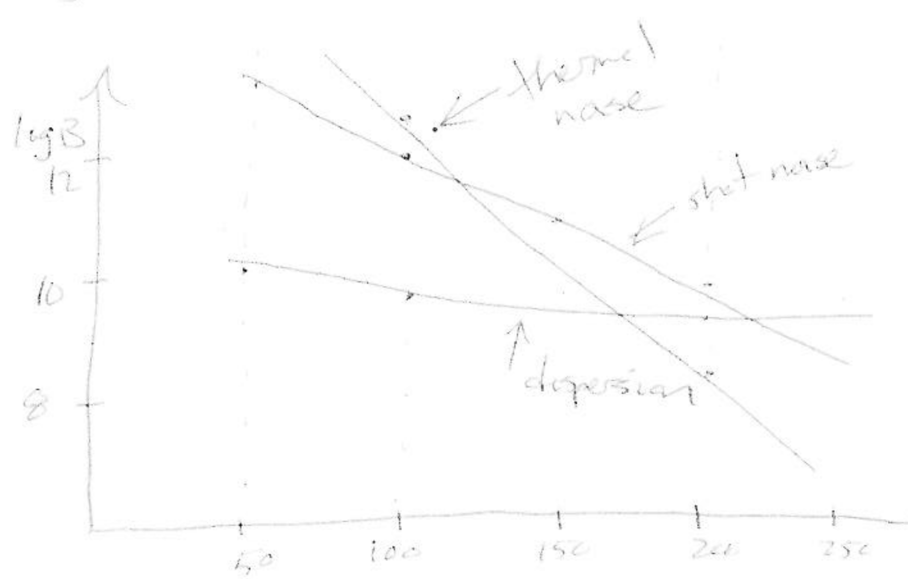
$B = \dfrac{\eta\lambda}{hc}\dfrac{\Phi_R}{SNR^2}$ $\quad 10\log B = 167 - 24 - .22L$

$\log B = 14.3 - 0.022L$

c) $\tau_D = D\cdot\Delta\lambda\cdot L = 0.25B^{-1}$

$B = \dfrac{0.25}{0.8\times10^{-12}\times 0.5\times10^{-6}\cdot L}$

$\log B = 11.8 - \log L$

d) 

shot noise is never the limiting factor. Dispersion dominates for L ≳ 170 km else thermal noise

4) a) $\tau_p = L/v_p \qquad v_p = \omega/\beta = \omega/n'k_0$

$$\tau_p = \frac{n'k_0 L}{\omega} = \frac{n'L}{c} = \left[n_g + \alpha(\lambda_0-\lambda_c)^2\right]\frac{L}{c}$$

b) $\tau_g = \frac{L}{v_g} \qquad v_g = \frac{d\omega}{d\beta} \qquad \tau_g = L\, d\beta/d\omega$

$$= L\frac{d\beta}{dk_0} \Big/ \frac{d\omega}{dk_0} \qquad \omega = ck_0 \quad \therefore \tau_g = \frac{L}{c}\frac{d\beta}{dk_0}$$

$$= \frac{L}{c}\frac{d\beta}{d\lambda_0}\cdot\frac{d\lambda_0}{dk_0} \qquad \lambda_0 = \frac{2\pi}{k_0} \quad \frac{d\lambda_0}{dk_0} = -\frac{2\pi}{k_0^2}$$

$$\tau_g = -\frac{L}{c}\frac{2\pi}{k_0^2}\frac{d\beta}{d\lambda_0} \qquad \beta = n'k_0 \quad \frac{d\beta}{d\lambda_0} = k_0\frac{dn'}{d\lambda_0} + n'\frac{dk_0}{d\lambda_0}$$

$$\frac{d\beta}{d\lambda_0} = k_0\left(2\alpha(\lambda_0-\lambda_c)\right) - \frac{2\pi}{\lambda_0^2}\left(n_g + \alpha(\lambda_0-\lambda_c)^2\right)$$

$$\tau_g = \frac{L\lambda_0}{c}\left[\frac{1}{\lambda_0}\left(n_g + \alpha(\lambda_0^2+\lambda_c^2-2\lambda_0\lambda_c)\right) - 2\alpha(\lambda_0-\lambda_c)\right]$$

$$= \frac{L}{c}\left(n_g + \alpha\lambda_0^2 + \alpha\lambda_c^2 - 2\alpha\lambda_0^2\right) = \frac{L}{c}\left(n_g + \alpha(\lambda_c^2-\lambda_0^2)\right)$$

c) $\frac{d\tau_g}{d\lambda_0} = \frac{L}{c}(-2\alpha\lambda_0)$

$$D = \left|\frac{d\tau_g/d\lambda_0}{L}\right| = \frac{2\alpha\lambda_0}{c}$$

$$D = \left|\frac{\lambda_0}{c}\, d^2n/d\lambda_0^2\right| = \left|\frac{\lambda_0}{c}\cdot 2\alpha\right| \quad \checkmark \text{ agrees.}$$

d) The phase shift due to propagation $\phi(\omega) = \omega\tau_p$

$\phi(\omega) = \frac{\omega L}{c}(n_g - \alpha(\lambda_0-\lambda_c)^2)$ where $\lambda_0 = 2\pi c/\omega$

The first term is the mean phase shift and the second is the dispersion, so:

$$H(\omega) = \exp\left[+j\frac{\omega L}{c}\alpha\left(\frac{2\pi c}{\omega}-\lambda_c\right)^2\right]$$

5 a) *A light emitting diode (LED) has a horizontal active region located 4 μm from the flat, horizontal semiconductor-air interface. Calculate the maximum angle from the vertical, θ_m, for which emitted photons are able to cross the semiconductor-air interface, and find the fraction of photons which are emitted within this angular range. Assume that the active region emits photons equally in all directions. The refractive index of the semiconductor is 3.7.* [6]

The maximum angle is simply the critical angle for total internal reflection, which = $\sin^{-1}(n_2/n_1)$ – in this case $\sin^{-1}(1/3.7) = 15.7^\circ = 0.274$ rad. The solid angle subtended by $d\theta$ is $\sin\theta d\theta$, so the fraction f of light within θ_m is given by $\int_0^{\theta_m} \sin\theta d\theta \Big/ \int_0^{\pi} \sin\theta d\theta$, giving $f = \frac{1-\cos\theta_m}{2}$, and using $\cos\theta \approx 1 - \theta^2$, so $f \approx \theta_m^2/4$, = 0.0187, or 2%.

b) *For the photons emitted within the angular range θ_m determined above, estimate the fraction absorbed between the active region and the surface. Assume an attenuation coefficient of 2×10^3 cm^{-1}. State any assumptions or approximations made.* [4]

Since we are only concerned with rays within 16° of vertical, the path length only varies between 4 and 4.2 , so we can neglect this variation and approximate the lost fraction as $(1 - \exp(-\alpha L)) = (1 - \exp(-1.5 \times 10^3 \times 4 \times 10^{-4}) = \underline{0.45}$.

c) *A uniform plastic layer of thickness 5 μm and refractive index 1.52 is now added to the semiconductor surface. Find the maximum emission angle θ_m which is now required for emitted photons to cross the semiconductor-plastic interface. For the photons crossing this interface, and neglecting absorption and Fresnel reflection at the interfaces, find the fraction which escape into the air. Briefly discuss the effect of this added layer on the overall external quantum efficiency η_{ext}.* [6]

Now we have $\theta_m = \sin^{-1}(1.52/3.7) = 24.3^\circ = 0.423$ rad, and $f \approx 0.423^2/4$, = 0.0448, or 4.5%. At the plastic-air boundary we have a second critical angle $\theta_{m2} = \sin^{-1}(1/1.52) = 41.1^\circ = 0.718$ rad. However, the light entering the plastic refracts so this corresponds to an angle in the semiconductor of $\sin^{-1}(\sin(41.1^\circ)\times 1.52/3.7) = 15.7^\circ$ as in part (a). The fraction of light entering the plastic that falls within the 2nd critical angle can therefore be calculated as $(1-\cos(15.7^\circ))/(1-\cos(24.3^\circ)) = 0.419$, or <u>42%</u>. Multiplying this by the 4.5% entering the plastic gives 2%, i.e. the same as in part (a). We would not expect a flat dielectric layer to reduce TIR losses, since the relation between the angles in the semiconductor and the air are unaffected by the presence of an intermediate layer.

d) *A domed plastic layer placed on the semiconductor surface can be used to increase η_{ext}. Why is such a structure unlikely to increase the amount of light that can be coupled from the LED into a single mode optical fibre?* [4]

Such a dome can increase the collimation of the output light, increasing the fraction within the numerical aperture of the fibre. However, it does this at the cost of an increase in effective area of the output beam, since the brightness of the source cannot be increased. This is only likely to be useful if the initial emission area is on the order than or smaller than the core of the fibre, but for SMF the core diameter is about 10 μm and the active area of an LED is almost certain to be much larger than this.

6. *A silicon p-i-n photodiode has p and n doping levels respectively of $N_A = 3\times10^{21}\ m^{-3}$ and $N_D^+ = 10^{21}\ m^{-3}$. The p-layer thickness is 0.5 μm. The attenuation coefficient in Si at the wavelength of interest is given by $\alpha = 0.25\times10^{6}\ m^{-1}$.*

a) *Find the intrinsic layer thickness w_i such that 80% of photons are absorbed in the intrinsic layer (neglecting Fresnel reflection at the surface).* [5]

Fraction of captured photons = $\exp(-\alpha x_1) - \exp(\alpha x_2) = 0.8$

Here x1= wp, and x2 = wp + wi.

So $0.8 = \exp(-0.25\times0.5)[1 - \exp(-0.25\times wi)]$ giving wi = 9.48 μm.

b) *Using w_i as calculated above, find the intrinsic layer doping level N_D^- such that the intrinsic region can be fully depleted by an applied voltage of 4.0 V.* [5]

When the intrinsic layer is just depleted, $E_{max} = -e\, N_D^- w_i/\varepsilon$

And $V = -\frac{1}{2} E_{max}(wi+x)$ where x is the depleted thickness in the p layer, which is given by $x = N_D^- wi/N_A$

Combining these gives:

$2V\varepsilon = e\, N_D^- w_i^2(1+ N_D^-/N_A)$

$\varepsilon = \varepsilon_r\, \varepsilon_o$

$2V\varepsilon\, /\, e\, N_D^- w_i^2 = 5.91\times10^{19}$

$(1/N_A)N_D^{-2} + N_D^- - 5.91\times10^{19} = 0$

Solve by quadratic eqn gives $N_D^- = 5.79\times10^{19}\ m^{-3}$

c) *Using N_D^- and w_i as calculated above, find the applied bias voltage V such that the electric field amplitude in the intrinsic region varies by 20% (i.e. maximum value is 120% of minimum value).* [5]

We label the field at top and bottom of the intrinsic layer as E1 and E2. The difference $\Delta E = e\, N_D^- w_i/\varepsilon$, and for a 20% variation E1 = 6 ΔE and E2 = 5 ΔE.

Let us call the depleted lengths in the p and n regions x and y respectively. We can use $N_A ex/\varepsilon = E1$ and $N_D^+ ex/\varepsilon = E2$ to give

$x = 6w_i\, N_D^-/N_A$ and $y = 5w_i\, N_D^-/ N_D^+$.

and $V = \frac{1}{2} E1x + \frac{1}{2}(E1+E2)\, w_i + \frac{1}{2}E2y = \frac{1}{2} E1(x + w_i) + \frac{1}{2} E2(y + w_i)$

filling in the expression for E1 and E2 gives

$V = (\frac{1}{2}\, e\, N_D^- w_i^2/\varepsilon)\, (6 + 36\, N_D^-/N_A + 5 + 25\, N_D^-/ N_D^+) = 51.5\ V$

d) *Discuss the main factors to be considered in optimising the electric field magnitude in a p-i-n photodiode.*

Bookwork.