UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C481

MODELS OF CONCURRENT COMPUTATION

Thursday 13 May 2004, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 You are given a chain of four lights connected by a cable:

$$1 - - - 2 - - - 3 - - - 4$$

Each light has a small programmable controller which can send a signal to the light, making it flash. Each controller can also send signals to its neighbours on its right and left.

a Design a process

$$\mathsf{Lights} \stackrel{\text{def}}{=} (\mathsf{Controller}_1 \mid \ldots \mid \mathsf{Controller}_4)\backslash L$$

satisfying the following properties:

- the process Lights can only execute the actions $\mathsf{flash}_1, \ldots, \mathsf{flash}_4$ and τ;
- $\mathsf{Controller}_i$ can execute an action flash_i but none of the actions flash_j, for $i, j = 1, 2, 3, 4$ and $j \neq i$;
- $\mathsf{Controller}_1$ can only communicate with $\mathsf{Controller}_2$;
- for $i = 2, 3$, $\mathsf{Controller}_i$ can only communicate with $\mathsf{Controller}_{i-1}$ and $\mathsf{Controller}_{i+1}$;
- $\mathsf{Controller}_4$ can only communicate with $\mathsf{Controller}_3$;
- the process Lights behaves like the process

$$\mathsf{Spec} \stackrel{\text{def}}{=} \mathsf{flash}_1.\mathsf{flash}_4.\mathsf{flash}_2.\mathsf{flash}_3.\mathsf{Spec}$$

b Consider three alternative specifications

$$\begin{aligned}
\mathsf{Spec}_A &\stackrel{\text{def}}{=} \mathsf{flash}_1.\mathsf{flash}_4.(\mathsf{flash}_3.\mathsf{flash}_2.\mathsf{Spec}_A + \mathsf{flash}_2.\mathsf{flash}_3.\mathsf{Spec}_A) \\
\mathsf{Spec}_B &\stackrel{\text{def}}{=} \mathsf{flash}_1.(\mathsf{flash}_4.\mathsf{flash}_3.\mathsf{flash}_2.\mathsf{Spec}_B + \mathsf{flash}_4.\mathsf{flash}_2.\mathsf{flash}_3.\mathsf{Spec}_B) \\
\mathsf{Spec}_C &\stackrel{\text{def}}{=} \mathsf{flash}_1.\mathsf{flash}_4.\mathsf{flash}_3.\mathsf{flash}_2.\mathsf{Spec}_C + \mathsf{flash}_1.\mathsf{flash}_4.\mathsf{flash}_2.\mathsf{flash}_3.\mathsf{Spec}_C.
\end{aligned}$$

Give logical formulae from the Hennessy-Milner logic that distinguish between these specifications: that is, find formulae Φ_j for $j = A, B, C$ such that $\mathsf{Spec}_j \models \Phi_j$ and $\mathsf{Spec}_i \not\models \Phi_j$ when $i \neq j$.

c Adapt Lights to obtain Lights_A, which is weakly bisimilar to Spec_A.

The three parts carry, respectively, 40%, 40%, and 20% of the marks.

2 A simple *broadcaster* $B_n \stackrel{\text{def}}{=} a.(\overline{b}_1 \mid \ldots \mid \overline{b}_n)$ waits for an action a and then in arbitrary order does $\overline{b}_1, \ldots, \overline{b}_n$ and evolves to the empty process.

a i) For all $n > 2$, define a chaining combinator $\frown_n$ in terms of the static CCS operators (parallel composition, restriction, relabelling) such that

$$B_{n-1} \frown_n B_2 \approx B_n$$

[Recall that $\approx$ denotes the weak bisimulation relation.]

ii) Give the transition graph of $B_2 \frown_3 B_2$.

iii) Prove that $B_2 \frown_3 B_2 \approx B_3$.

b A *repetitive* broadcaster RB_n is just like B_n except that, after performing the actions $\overline{b}_1, \ldots, \overline{b}_n$, it resumes its initial state.

i) Amend the definition of B_n to provide a definition of RB_n (without using summation).

ii) Let $RS_2 \stackrel{\text{def}}{=} a.(\overline{b}_1.\overline{b}_2.RS_2 + \overline{b}_2.\overline{b}_1.RS_2)$. Show that $RB_2 \approx RS_2$, by defining a set S' such that

$$S = \{(P, Q) : P \approx P' \text{ and } Q \approx Q' \text{ and } (P', Q') \in S'\}$$

is a weak bisimulation. Explain your answer S' and the definition of S.

iii) Explain why $RB_2 \frown_3 RB_2 \approx RB_3$ does not hold.

3 Consider the pi process

$$\begin{aligned} Q \stackrel{\text{def}}{=} & (\mathsf{new}\, v)\,(\overline{p}\langle v\rangle \mid \overline{c}\langle v\rangle \mid v(d).P_1) \\ & \mid p(u).\,\overline{u}\langle 1\rangle \\ & \mid c(u).(u(a).P_2 \mid u(b).P_3) \end{aligned}$$

where $v \notin \mathsf{fn}(P_1)$ and $u \notin \mathsf{fn}(P_2) \cup \mathsf{fn}(P_3)$.

a Describe the reaction and structural congruence steps of process Q. Do one sequence in detail, and sumarise the other possible reactions.

b "Input capability" of the pi calculus is the ability to receive a channel name and subsequently accept input on it, as illustrated by process Q. Recall the linear forwarder calculus consisting of the linear forwarder $x \mapsto y$ with the reaction

$$x \mapsto y \mid \overline{x}\langle z\rangle \;\rightarrow\; \overline{y}\langle z\rangle$$

i) Give a pi process which reacts in the same way as linear forwarder $x \mapsto y$.

ii) One possible translation of the pi process Q in the linear forwarder calculus is given by the process

$$\begin{aligned} Q' \stackrel{\text{def}}{=} & (\mathsf{new}\, v)\,(\overline{p}\langle v\rangle \mid \overline{c}\langle v\rangle \mid v(d).P_1') \\ & \mid p(u).\,\overline{u}\langle 1\rangle \\ & \mid c(u).(\mathsf{new}\, u')\,(u \mapsto u' \mid u'(a).P_2' \mid u \mapsto u' \mid u'(b).P_3') \end{aligned}$$

where P_i' denotes the translation of P_i (for each $i = 1, 2, 3$) and u' is a new name. Give the sequence of reaction and structural congruence steps of Q', which is analogous to your answer in part a.

iii) Explain why Q and Q' are not observationally equivalent.

iv) Adapt process Q' to the process Q'' which is observationally equivalent to Q (using P_i'' to denote the adapted translation of P_i for each $i = 1, 2, 3$). Explain your answer.

The two parts carry, respectively, 25%, and 75% of the marks.

4 The natural numbers can be encoded in the pi calculus by the definitions

$$\begin{aligned}
[0](k) &\stackrel{\text{def}}{=} k(z,s).\overline{z} \\
[n+1](k) &\stackrel{\text{def}}{=} (\mathsf{new}\, y)\,(\mathsf{Succ}\langle k,y\rangle \mid [n]\langle y\rangle) \\
&\qquad \text{where } \mathsf{Succ}(k,y) \stackrel{\text{def}}{=} k(z,s).\overline{s}\langle y\rangle \\
\mathsf{Natcases}(P,F)(k) &\stackrel{\text{def}}{=} (\mathsf{new}\, z,s)\,(\overline{k}\langle z,s\rangle.\,(z.P + s(y).\,F\langle y\rangle))
\end{aligned}$$

where F is the process identifier $F(y) \stackrel{\text{def}}{=} Q$, the P and Q are processes, $z, s \notin \mathsf{fn}(P) \cup \mathsf{fn}(Q)$ and z, s, k, y denote distinct names.

a Write down the pi process $[3]\langle k\rangle$, unwinding the definitions to show the connections between processes.

b Show that

$$\begin{aligned}
[0]\langle k\rangle \mid \mathsf{Natcases}(P,F)\langle k\rangle &\rightarrow P \\
[n+1]\langle k\rangle \mid \mathsf{Natcases}(P,F)\langle k\rangle &\rightarrow (\mathsf{new}\, y')\,([n]\langle y'\rangle \mid Q\{y'/y\})
\end{aligned}$$

c Consider a particular use of Natcases given by

$$\begin{aligned}
\mathsf{Copy}(k,l) &\stackrel{\text{def}}{=} \mathsf{Natcases}(P,F)\langle k\rangle \\
P &\stackrel{\text{def}}{=} [0]\langle l\rangle \\
F(y) &\stackrel{\text{def}}{=} (\mathsf{new}\, l')\,(\mathsf{Succ}\langle l,l'\rangle \mid \mathsf{Copy}\langle y,l'\rangle)
\end{aligned}$$

which copies a process corresponding to a natural number from location k to location l. Show that $[n]\langle k\rangle \mid \mathsf{Copy}\langle k,l\rangle \rightarrow^* [n]\langle l\rangle$ by induction on the natural number n.

d Using $\mathsf{Copy}(k,l)$, define a process $\mathsf{Add}(k,l,m)$ which, given two natural numbers located at k and l, adds them together and places the result at m. Show by induction on n_1 that

$$[n_1]\langle k\rangle \mid [n_2]\langle l\rangle \mid \mathsf{Add}\langle k,l,m\rangle \rightarrow^* [n_1+n_2]\langle m\rangle$$

The four parts carry, respectively, 15%, 25%, 25%, and 35% of the marks.