IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EEE PART II: MEng, BEng and ACGI

Corrected Copy

**CONTROL ENGINEERING**

Wednesday, 4 June 2:00 pm

Time allowed: 2:00 hours

**There are THREE questions on this paper.**

**Answer ALL questions.**
**Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).**

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible First Marker(s) : I.M. Jaimoukha

Second Marker(s) : S. Evangelou

1. a) Consider the mechanical system illustrated in Figure 1.1 where all the symbols have the standard interpretation. The input is the applied force $f(t)$ and the output is the displacement $y(t)$. Take $M = K_2 = D = 1$ in appropriate units.

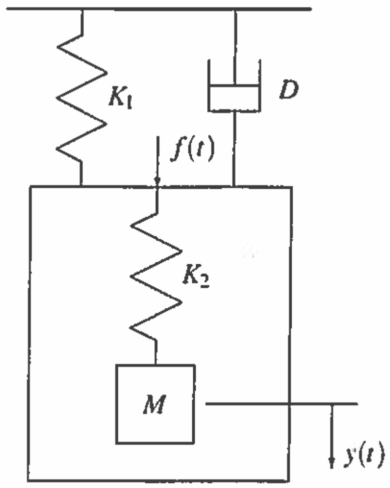


*Figure 1.1*

i) Determine the transfer function $G(s)$ relating $y$ to $f$. [ 5 ]

ii) Use the Routh array to find the range of values of $K_1$ for stability. [ 5 ]

iii) Find the value of $K_1$ for which $G(s)$ is marginally stable. For this value of $K_1$, what are the poles of $G(s)$? [ 5 ]

iv) Let $f(t)$ be a unit step applied at $t = 0$. Use the final value theorem, which should be stated, to find the steady–state value $y_{ss}$, of $y(t)$ in terms of $K_1$. What is the value of $K_1$ for which $y_{ss} = 2$? [ 5 ]

b) In Figure 1.2 below, $G(s) = 2/(s^3 - 1)$ and $K(s)$ is a compensator.

i) Draw the Nyquist diagram of $G(s)$. [ 5 ]

ii) Let $K(s) = k$ be a constant compensator. Use the Nyquist criterion, which should be stated, to determine how many unstable or marginally stable poles the closed-loop has for all $k$. [ 5 ]

iii) Use the Routh-Hurwitz stability criterion to determine if the closed-loop can be stabilised using a PD compensator. [ 5 ]

iv) Show that the closed-loop can be stabilised using the compensator

$$K(s) = k\frac{s^2 + s + 1}{s^2 + 2s + 3}$$

for some $k > 0$. [ 5 ]

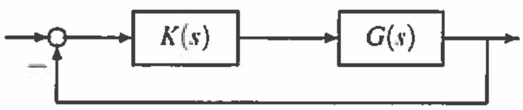


*Figure 1.2*

2. Consider the feedback control system in Figure 2.1 below. Here,

$$G(s) = \frac{1}{s^3 + as^2 + bs + c}$$

represents an uncertain model where it is only known that

$$a > 0, \qquad b > 0, \qquad c > 1, \qquad ab - c \geq 2. \tag{2.1}$$

$K(s)$ is the transfer function of a compensator.

a) Sketch a typical Nyquist diagram of $G(s)$, indicating the low and high frequency portions. Use the Routh array to find the real–axis intercepts. [ 8 ]

b) Let $K(s) = K$ be a nondynamic compensator. State the Nyquist stability criterion and use the Nyquist diagram to determine the number of unstable or marginally stable closed-loop poles for all values of $K$. [ 8 ]

c) What is the value of the gain margin for $G(s)$? [ 3 ]

d) Derive the minimum value of the gain margin for all $a, b, c$ satisfying the relations in equation (2.1). [ 3 ]

e) Suppose that it is known that $G(s)$ has an adequate phase margin and that you have the option of either using a phase–lead or a phase–lag compensator. Which would you choose? Justify your choice. [ 8 ]

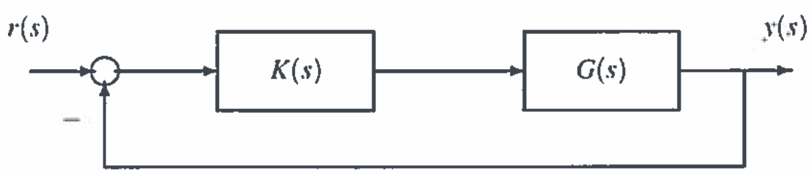


*Figure 2.1*

  

3. Consider the feedback loop shown in Figure 3.1 below. Here

$$G(s) = \frac{-2s}{s^2 + k(s+z) + 1}$$

where $k > 0$ and $z \geq 0$ are design parameters. It is required to find $k$ and $z$ such that the following specifications in response to a step reference signal are satisfied

- The settling time is at most 4 seconds.
- The response is oscillatory with a maximum overshoot of 5%.

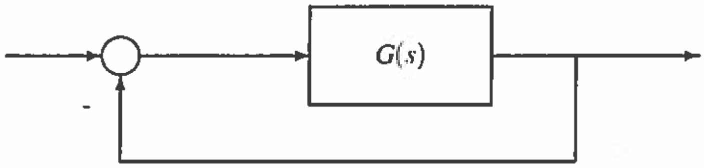


*Figure 3.1*

a) Find the location of the closed–loop poles that achieves the design specifications above. [ 5 ]

b) For this part, take $z = 0$.

i) Sketch the locus of the closed–loop poles as $k$ varies from 0 to $\infty$. [ 5 ]

ii) Hence show that a setting time of $4s$ is achievable but the resulting response is not oscillatory. Which closed–loop pole achieves this modified specification? What is the corresponding value of $k$? [ 5 ]

c) In this part, you are required to find the values of $k$ and $z$ that achieve the design specification.

i) Use the angle criterion to choose the value of $z$ so that the locus of the closed–loop poles as $k$ varies from 0 to $\infty$ includes the closed–loop pole evaluated in Part (a) above. [ 5 ]

ii) For this value of $z$, sketch the locus of the closed–loop poles as $k$ varies from 0 to $\infty$. [ 5 ]

iii) Finally, use the gain criterion to find the value of $k$ that achieves the design specifications. [ 5 ]