EE4-65
EE9-SO27

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2016

MSc and EEE PART IV: MEng and ACGI

WIRELESS COMMUNICATIONS

Corrected copy

Monday, 16 May 10:00 am

Time allowed: 3:00 hours

There are THREE questions on this paper.

Answer THREE questions.

Any special instructions for invigilators and information for candidates are on page 1.

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Second Marker(s) : K.K. Leung

Important information for students

Notations:

(a) A $n_r \times n_t$ MIMO channel consists in n_r receive antennas and n_t transmit antennas.

(b) a, $\mathbf{a}$, $\mathbf{A}$ denote a scalar, vector and matrix respectively.

(c) $\mathbf{A}^H$ denotes conjugate transpose (Hermitian).

(d) $\mathbf{A}^*$ denotes conjugate.

(e) $\mathbf{A}^T$ denotes transpose.

(f) $|a|$ denotes the absolute value of scalar a.

(g) $\|\mathbf{a}\|$ denotes the (Euclidean) norm of vector $\mathbf{a}$.

(h) "i.i.d." means "independent and identically distributed".

(i) "CSI" means "Channel State Information".

(j) "CSIT" means "Channel State Information at the Transmitter".

(k) "CDIT" means "Channel Distribution Information at the Transmitter".

(l) $\mathcal{E}\{.\}$ denotes Expectation.

(m) $\mathrm{Tr}\{.\}$ denotes the Trace of a matrix.

Assumptions:

(a) The CSI is assumed to be always perfectly known to the receiver.

(b) The receiver noise is a $n_r \times 1$ vector with i.i.d. entries modeled as zero mean complex additive white Gaussian noise with variance σ_n^2.

Some useful relationships:

(a) $\|\mathbf{A}\|_F^2 = \mathrm{Tr}\{\mathbf{A}\mathbf{A}^H\} = \mathrm{Tr}\{\mathbf{A}^H\mathbf{A}\}$

(b) $\mathrm{Tr}\{\mathbf{AB}\} = \mathrm{Tr}\{\mathbf{BA}\}$

(c) $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$

(d) $\mathrm{Tr}\{\mathbf{A}\mathbf{B}\mathbf{B}^H\mathbf{A}^H\} = \mathrm{vec}(\mathbf{A}^H)^H (\mathbf{I} \otimes \mathbf{B}\mathbf{B}^H) \mathrm{vec}(\mathbf{A}^H)$

(e) Gaussian Q-function

$$Q(x) \triangleq P(y \geq x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{y^2}{2}\right) dy$$

(f) Craig's formula

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2(\beta)}\right) d\beta$$

(g) Chernoff bound

$$Q(x) \leq \exp\left(-\frac{x^2}{2}\right)$$

(h) The moment generating function of a Hermitian quadratic form in complex Gaussian random variable $y = \mathbf{z}\mathbf{F}\mathbf{z}^H$, where $\mathbf{z}$ is a circularly symmetric complex Gaussian vector with mean $\overline{\mathbf{z}}$ and a covariance matrix $\mathbf{R_z}$ and $\mathbf{F}$ a Hermitian matrix, is given by

$$M_y(s) \triangleq \int_0^{\infty} \exp(sy)\, p_y(y)\, dy = \frac{\exp\left(s\overline{\mathbf{z}}\mathbf{F}\left(\mathbf{I} - s\mathbf{R_z}\mathbf{F}\right)^{-1}\overline{\mathbf{z}}^H\right)}{\det\left(\mathbf{I} - s\mathbf{R_z}\mathbf{F}\right)}$$

(i) Assume n i.i.d. zero mean complex Gaussian variables $h_1, \ldots, h_n$ (real and imaginary parts with variance σ^2). Defining $u = \sum_{k=1}^{n} |h_k|^2$, the MGF of u is given by

$$\mathcal{M}_u(\tau) = \mathcal{E}\{e^{\tau u}\} = \left[\frac{1}{1 - 2\sigma^2\tau}\right]^n$$

THE QUESTIONS

1. [40]

a) Figure 1.1 displays the average Error Probability of one scheme (i.e., one transmission and reception strategy) vs. SNR for point-to-point channels with i.i.d. Rayleigh slow fading and four different antenna configurations (a) to (d). The CSI is perfectly known to the transmitter.

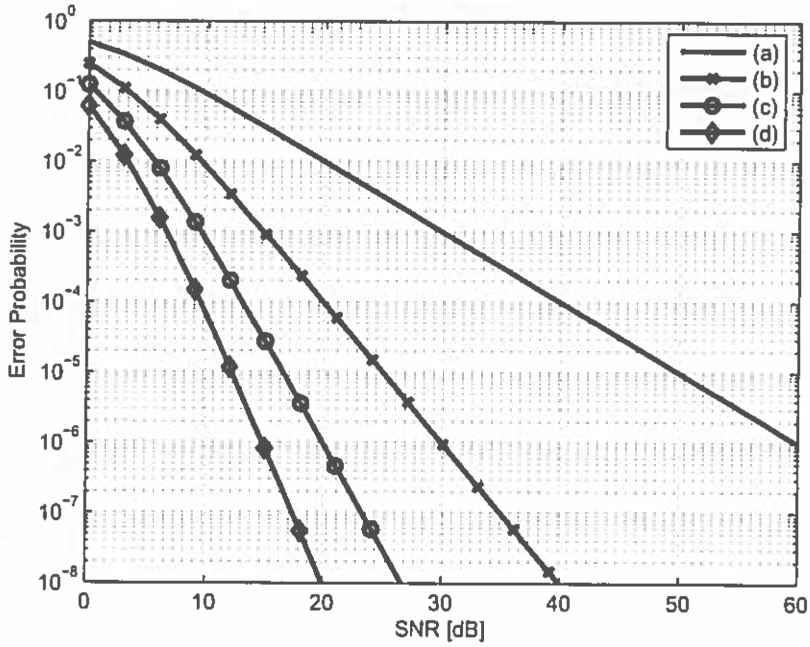

Figure 1.1 Average Error Probability vs. SNR.

i) What is the diversity gain (at high SNR) achieved by that scheme for each antenna configuration? Provide your reasoning.

[4]

ii) For each scenario (a) to (d), identify an antenna configuration (i.e., n_t and n_r) and the corresponding transmission/reception strategy that can achieve such diversity gain. Provide your reasoning.

[4]

b) Figure 1.2 displays the ergodic capacity of point-to-point i.i.d. Rayleigh fast fading channels with Channel Distribution Information at the Transmitter (CDIT) for five antenna ($n_r \times n_t$) configurations (denoted as (a) to (e)) with $n_t + n_r = 9$.

i) What is the achievable (spatial) multiplexing gain (at high SNR) for each of cases (a) to (e)? Provide your reasoning.

[5]

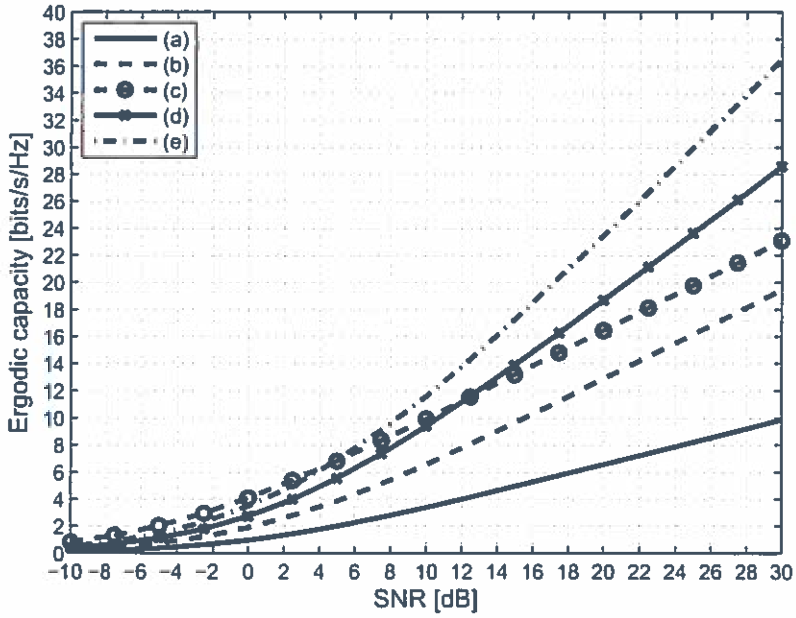

Figure 1.2 Ergodic capacity vs. SNR.

ii) For each of cases (a) to (e), identify the antenna configuration, i.e. n_t and n_r, satisfying $n_t + n_r = 9$ that achieves such multiplexing gain. Provide your reasoning.

[5]

c) Consider the transmission of 2 independent streams using Spatial Multiplexing over a 4×2 MIMO channel $\mathbf{H}$. The Channel State Information (CSI) is unknown to the transmitter. The received signal is written as $\mathbf{y} = \mathbf{H}\mathbf{c} + \mathbf{n}$ where $\mathbf{c} = [c_1, c_2]^T$ is the vector of transmitted symbols. The channel matrix is given by

$$\mathbf{H} = \begin{bmatrix} 1 & 1 \\ 1+j & 0 \\ 1 & -1 \\ 1-j & 0 \end{bmatrix}.$$

At the receiver we would like to apply a combiner $\mathbf{G}$. Suggest such a receive combiner and derive its expression. What kind of combiner is this? Explain your result.

[6]

d) Consider a downlink multi-user setup with two transmit antennas and two users, each with a single receive antenna. The channel for user 1 is given by

$$\mathbf{h}_1 = \begin{bmatrix} 1 & e^{-j2\pi d/\lambda \cos\theta_1} \end{bmatrix}$$

while that of user 2 is given by

$$\mathbf{h}_2 = \begin{bmatrix} 1 & e^{-j2\pi d/\lambda \cos\theta_2} \end{bmatrix}$$

where d is the inter-element spacing, λ the wavelength, θ_1 and θ_2 the Direction of Departure (DoD) of user 1 and 2, respectively. The CSI are perfectly known to the transmitter. If you had the possibility to choose the locations of the two users, i.e. their DoD θ_1 and θ_2, how would would you choose them in order to maximize the sum-rate? Provide your reasoning. [6]

e) Consider transmit diversity via matched beamforming and ML detection with QAM in an i.i.d. Rayleigh fading MISO channel with n_t transmit antennas. CSI is assumed to be perfectly known to the receiver and to the transmitter.

i) Write the system model and clearly state the ML decision rule. [3]

ii) By making use of the Chernoff Bound, derive an upper bound on the average error probability. [4]

iii) Infer from (ii) the diversity gain achieved by that scheme at high SNR. [3]

2. [30]

a) Consider the 3×3 point-to-point MIMO channel matrix given by

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}.$$

Assume $\mathbf{H}$ is known at the transmitter. What is the maximum number of independent streams that can be transmitted to the receiver? Explain your reasoning.

[6]

b) Consider the transmission $\mathbf{y} = \mathbf{H}\mathbf{c}' + \mathbf{n}$ with perfect CSIT over a deterministic point-to-point MIMO channel with two transmit and two receive antennas whose matrix is given by

$$\mathbf{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} a & b \\ -a & b \end{bmatrix}$$

where a and b are complex scalars with $|a| \geq |b|$. The receiver is subject to AWGN noise such that the noise variances on receive antenna 1 and 2 are given by σ_n^2. The input covariance matrix is given by $\mathbf{Q} = \mathcal{E}\{\mathbf{c}'\mathbf{c}'^H\}$ and is subject to the transmit power constraint $\mathrm{Tr}\{\mathbf{Q}\} \leq P$. Compute the capacity with perfect CSIT of that deterministic channel. Explain your reasoning.

[6]

c) Consider a narrowband transmission using a transmission strategy characterized by the following set of codewords

$$\mathbf{a} = \begin{bmatrix} a & a & b & b^* \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} d^* & d & a & c^* \end{bmatrix},$$

$$\mathbf{c} = \begin{bmatrix} c^* & a & d^* & b \end{bmatrix},$$

with $a = \frac{1}{\sqrt{2}}(1+j)$, $b = \frac{1}{\sqrt{2}}(-1+j)$, $c = \frac{1}{\sqrt{2}}(-1-j)$ and $d = \frac{1}{\sqrt{2}}(1-j)$ being the four constellation symbols taken from a unit average energy QPSK constellation. What is the diversity gain that can be achieved with a Maximum Likelihood (ML) receiver in i.i.d. fast Rayleigh fading channels with a single receive antenna and a single transmit antenna? Provide your reasoning.

[6]

d) Discuss the validity of the following statements. Detail your argument.

i) To get a diversity gain of n_t in a MISO point-to-point channel with n_t transmit antennas, the transmitter needs to know the CSI.

[6]

ii) In a downlink multiuser MIMO channel with the transmitter equipped with n_t antennas and the receivers, each equipped with 2 antennas, the transmitter can serve at the same time and without interference n_t mobile terminals with 2 streams per terminal using block diagonalization.

[6]

3. [30]

a) Consider an uplink transmission with a receiver equipped with one antenna and two transmitters, each equipped with one antenna. Transmitter 1 is far away from the receiver while transmitter 2 is close to the receiver. The transmitters are subject to a transmit power constraint P.

i) Write the system model of this uplink transmission. [3]

ii) Propose a transmission and reception strategy that maximizes the sum-rate of this transmission? Explain your rationale. [4]

iii) Is the strategy unique? Explain your rationale. [4]

iv) If you were to design the system, what rate would you allocate to transmitter 1 and 2, respectively? Explain your rationale. [4]

b) Consider a downlink transmission with a transmitter equipped with two transmit antennas and two receivers, each equipped with a single antenna. The transmitter is subject to a total transmit power P.

i) Assuming linear precoding at the transmitter, write the system model of this transmission and the SINR experienced at each receiver. [5]

ii) Assuming the CSI is reported perfectly to the transmitter, propose a linear precoding strategy that maximizes the multiplexing gain of the transmission at high SNR and derive the achieved multiplexing gain. Provide your reasoning. [5]

iii) Assume now that the CSI is quantized into a fixed number of B bits and is reported back to the transmitter. What is the multiplexing gain achieved by the strategy proposed in (ii) at high SNR? Can you suggest a better strategy? Provide your reasoning. [5]