The Solutions for EE4.18 and AO6, 2017

Model Answer to Q1(a): Bookwork and General Discussions in Class

a) At frequencies below 1 GHz:

i) Briefly explain the advantages and disadvantages of using lumped-element components within integrated circuits and state the general frequency behaviour of this solution.

Lumped-element components can lead to very compact integrated circuits, and this advantage increases over distributed solutions as frequency decreases. However, they also suffer from self-resonances: lossy series LC tuned circuit with capacitors and lossy parallel LC tuned circuit with inductors and resistors.

[2]

ii) Briefly explain the advantages and disadvantages of using distributed-element components within integrated circuits and state the general frequency behaviour of this solution.

Distributed-element components do not suffer from the parasitic resonances found with lumped-element solutions, below those found at higher-order harmonic frequencies, but they become too large as frequency decreases to be practical within integrated circuits.

[2]

iii) What is the dominant noise contribution at the input to a receiver? State its origin and general frequency behaviour. What can be observed when an analogue AM receiver switches bands?

Galactic noise, due to the remanence of the Big Bang, dominates the noise at the input of a receiver. This assumes that the receiver's antenna is not pointing directly at a star or any other 'hot' body. As frequency decreases the galactic noise increases and this can be heard as the background hiss (when detuned away from a broadcast station) increases in amplitude as an AM radio is switched between the short wave to medium wave to long wave bands.

[2]

iv) What ubiquitous application is found just below 1 GHz? In general, for this application, where would you expect to find the lumped- and distributed-element solutions within a complete end-to-end system?

GSM900 mobile communications exists at around 0.9 GHz. Lumped-element solutions would be found in the handset, where space is at a premium; while higher performance distributed-element solutions would be found in the base stations, when size is less important.

[2]

Model Answer to Q1(b): Bookwork and General Discussions in Class

b) At frequencies above 1 GHz and below 300 GHz:

i) Do the passive and active technologies use photonic or electronic or thermal solutions for implementing integrated circuits? State the general frequency behaviour of the solution.

At these frequencies, the passive and active technologies almost always use electronics solutions, with thermal and photonic solutions used above 300 GHz. Electronics solutions suffer from increases ohmic losses, reduced power generation and signal gain as frequency increases.

[2]

ii) What is the dominant noise contribution at the input to a receiver and state its origin and general frequency behaviour?

Noise due to atmospheric attenuation, from water vapour and oxygen absorption-emission, dominates above *ca.* 1 GHz. This increase with frequency, while also producing peaks at 23 and 183 GHs from water vapour molecules and 60 and 110 GHz from oxygen molecules.

[2]

iii) What commercial application is found at 250 GHz? Briefly explain why this application is at this frequency and why system performance can degrade below and above this frequency.

Radiometric stand-off detection, for security screening applications, is found at 250 GHz. This frequency experiences high, but relatively low spectral attenuation – as it lies between two water vapour peaks. As frequency drops below 250 GHz, spatial resolution becomes more impaired. While increasing above 250 GHz impairs temporal resolution due to the increased attenuation/scattering effects of clothing.

[2]

Model Answer to Q1(c): Bookwork and General Discussions in Class

c) At frequencies above 300 GHz and below 10 THz, briefly explain the disadvantages and a possible solution associated with:

i) Passive components.

Passive components suffer from increasing ohmic losses in both metal- and dielectric-based components. One solution is to employ optical and quasi-optical techniques, such as optical fibres and spatial power combining.

[2]

ii) Active components.

Electronic components have relatively very little gain at these frequencies. As a result, two phase-locked lasers can be used to generate RF power by filtering the beat frequency at the output of a nonlinear crystal.

[2]

iii) Atmospheric attenuation.

Atmospheric attenuation effectively presents an atmospheric brick wall for many medium to long range applications. As a result, line-of-sight path links are kept either very short outside or limited to indoors (pristine atmospheric conditions).

[2]

Model Answer to Q2(a): Bookwork and General Discussions in Class

a) Using the principle of conservation of energy, mathematically define transmittance, absorptance and reflectance for a non-opaque medium.

When an electromagnetic wave is incident upon a non-opaque medium (solid, liquid or gas), some of this power P_i is reflected back P_r, a proportion may be absorbed P_a and the remaining transmitted through P_t; such that the principle of conservation of energy is observed at each frequency (i.e., $P_i = P_r + P_a + P_t$). In terms of absolute power values, benchmark simulation software can generate results for: (i) reflection or reflectance, $R = P_r/P_i$; (ii) absorptivity, absorption or absorptance, $A = P_a/P_i$; and (iii) transmission or transmittance, $T = P_t/P_i = e^{-\tau}$.

[2]

Model Answer to Q2(b): Bookwork and General Discussions in Class

b) Define specific attenuation, in terms of atmospheric loss factor, opacity, extinction coefficient and path length.

Atmospheric loss factor is $e^{+\tau}$, where $\tau = \gamma L$ is the opacity, γ is the extinction coefficient, and L is path length. Apart from path length, all other variables are implicitly frequency dependent.

The gradient of attenuation with path length can be expressed (in dB/km) as specific attenuation $= -10log_{10}\{T\} = -10log_{10}\{1 - R - A\}$, where R, T, A have been calculated for a specific reference path length (e.g., L_{REF} = 1 km). Therefore, at a single frequency point, attenuation (dB) over an arbitrary path length L (km) within a homogeneous atmosphere can be calculated from $-L \cdot 10log_{10}\{T\} = -L \cdot 10log_{10}\{1 - R - A\}$, which exhibits a linear scaling law.

[2]

Model Answer to Q2(c): Bookwork and General Discussions in Class

c) Using the information given in Figure 2.1, derive from first principles the sky brightness temperature for all source contributions. Clearly state all simplifying assumptions.

A number of simplifying assumptions will be made: (i) unless otherwise stated, the receiver is 'noiseless' and represents a complete idealized ground station (i.e., the receiver, antenna and associated interconnect feed line are all ideal); (ii) the antenna has a highly directional pencil-beam radiation pattern; (iii) Earth's atmosphere is represented by a single homogenous layer in thermodynamic equilibrium; (iv) a 'plane atmosphere' approximation is adopted (which is usually acceptable for zenith angles below 65°), whereby the Earth is assumed flat with a horizontally stratified time-invariant atmosphere; (v) molecular scattering is neglected with pristine conditions; (vi) Rayleigh-Jeans law can be used to approximate Planck's laws for calculating spectral radiance (when temperature of a radiator is high and/or frequency is low); and (vii) the total opacity of the atmosphere is assumed to be frequency independent.

With a pristine atmosphere, reflectance is neglected. Therefore, transmittance is $e^{-\tau_\theta(f)}$ and absorptance is $\left(1 - e^{-\tau_\theta(f)}\right)$, which is equal to the emissivity of the atmosphere in thermodynamic equilibrium.

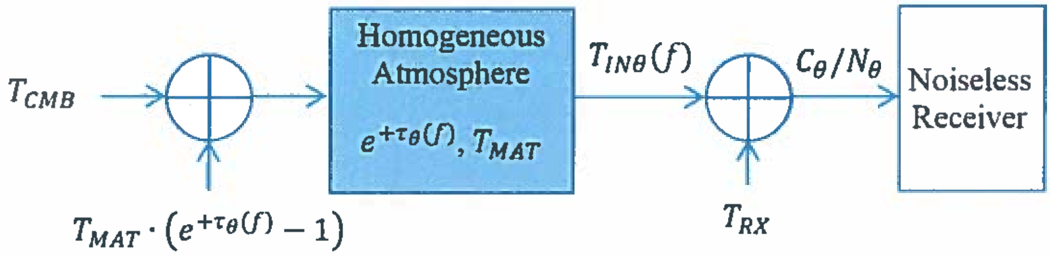
The sky brightness temperature $T_{IN\theta}(f)$ for all source contributions (representing the effective input noise temperature for a noiseless receiver), due to molecular absorption/emission at zenith angle θ, is dependent on frequency f and given by:

$$T_{IN\theta}(f) \approx T_{CMB} \cdot e^{-\tau_\theta(f)} + T_{MAT} \cdot \left(1 - e^{-\tau_\theta(f)}\right)$$

[6]

Model Answer to Q2(d): Bookwork and General Discussions in Class

d) From Figure 2.1 and the derivation in 2(c), draw the equivalent noise temperature model, clearly labelling all parameters.



[2]

Model Answer to Q2(e): Bookwork and General Discussions in Class

e) Derive an expression for the carrier-to-noise power ratio at the input to a noisy receiver within a vacuum atmosphere $C_{\theta V}/N_{\theta V}$.

The carrier and noise power levels at the input to a noisy receiver C_θ and N_θ, respectively, can be expressed as:

$$C_\theta = e^{-\tau_\theta} C_{\theta V}$$

$$N_\theta \approx k(T_{IN\theta} + T_{RX})B$$

where $C_{\theta V}$ is the received carrier power within a vacuum atmosphere (i.e., without considering any molecular absorption), T_{RX} is the intrinsic noise temperature for a noisy receiver due to intrinsic noise contributions (e.g., from receiver, antenna and associated interconnect feed line). Since $e^{-\tau_\theta} \to 1$, the received noise power within a vacuum atmosphere $N_{\theta V}$ (i.e., without considering any molecular emission) is given by:

$$N_{\theta V} \approx k(T_{CMB} + T_{RX})B$$

$$\frac{C_{\theta V}}{N_{\theta V}} \approx \frac{C_{\theta V}}{k(T_{CMB} + T_{RX})B}$$

[2]

Model Answer to Q2(f): Bookwork and General Discussions in Class

f) Derive an expression for the carrier-to-noise power ratio at the input to a noisy receiver considering both molecular absorption and emission within the Earth's atmosphere C_θ/N_θ.

$$\frac{C_\theta}{N_\theta} \approx \frac{e^{-\tau_\theta} C_{\theta V}}{k[T_{CMB} \cdot e^{-\tau_\theta} + T_{MAT} \cdot (1 - e^{-\tau_\theta}) + T_{RX}]B}$$

[2]

Model Answer to Q2(g): Bookwork, General Discussions in Class and Calculation

g) Calculate the reduction of C/N in dB at the receiver when both molecular absorption and emission are included, given: $T_{MAT} = 0.95T_{AS}$, a 90% transmittance through the atmosphere and a receiver having a noise temperature of 100 K.

$$\frac{C_{\theta V}/N_{\theta V}}{C_{\theta}/N_{\theta}}(\text{dB}) \approx 10log_{10}\left\{\frac{(T_{CMB} - T_{MAT}) + (T_{MAT} + T_{RX}) \cdot e^{+\tau_\theta}}{(T_{CMB} + T_{RX})}\right\}$$

$$T_{MAT} = 0.95\,x\,296 = 281.2\,K\,;\,e^{+\tau_\theta} = \frac{1}{0.9} = 1.111$$

$$\frac{C_{\theta V}/N_{\theta V}}{C_{\theta}/N_{\theta}}(\text{dB}) \approx 10log_{10}\left\{\frac{-278.475 + 423.555}{102.725}\right\} = 1.5\,dB$$

[4]

Model Answer to Q3(a): Bookwork and General Discussions in Class

a) For a lossless 2 lumped-element matching network:

i) Draw the equivalent circuit model for a complete network that can take advantage of any possible dual uses for its lumped elements and state what these additional topological advantages may be.

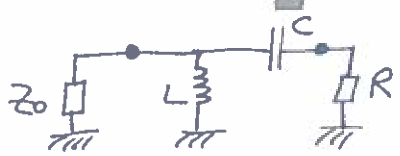
[2]

ii) Write the expression for the network Q-factor.

[1]

iii) Calculate the component values for each element.

[2]



$Q = \sqrt{\frac{Z_o}{R} - 1} = 9.95$

$Q = \frac{Z_o}{\omega L} = \frac{1}{\omega C R}$

N=1

$\therefore L = \frac{Z_o}{\omega Q} = 800pH$ CAN ALSO BE USED FOR SELF BIASING

$C = \frac{1}{\omega Q R} = 32pF$ CAN ALSO BE USED FOR DC BLOCKING

C_1 C_2

Model Answer to Q3(b): New Derivation

b) For a lossless 4 lumped-element matching network:

i) Draw the equivalent circuit model for the complete network.

[2]

ii) Write the expression for the optimum network Q-factor.

[2]

$\omega Q R$

$Q = \sqrt{\sqrt{\frac{Z_o}{R}} - 1}$

N=2

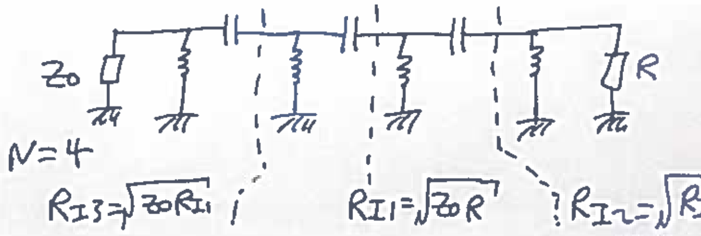
$R_{INTERMEDIATE} = \sqrt{Z_o R} = R_I$

Model Answer to Q3(c): New Derivation

c) For a lossless 8 lumped-element matching network:

i) Draw the equivalent circuit model for the complete network. [1]

ii) Write the expression for the optimum network Q-factor. [1]



$$Q=\sqrt{\sqrt{\sqrt{\frac{Z_0}{R}}}-1}$$

Model Answer to Q3(d): New Derivation

d) In terms of N-stages:

i) From 3(a)(ii), 3(b)(ii) and 3(c)(ii), write the general expression for the optimum lossless network Q-factor and its associated bandwidth. [2]

$$Q=\sqrt{\left(\frac{Z_0}{R}\right)^{1/N}-1} \qquad BW=\frac{f_0}{Q}$$

ii) For $N \in [1,2,3,4,5,6]$, calculate the optimum Q-factor and the associated bandwidth for a lossless network. [2]

N	Q	BW [MHz]	IL [dB]
1	9.95	100.5	−1.24
2	3	333.3	−0.79
3	1.91	524.0	−0.76 MINIMUM
4	1.47	680.0	−0.78
5	1.23	813.3	−0.81
6	1.07	930.7	−0.84
⋮	⋮	⋮	⋮
∞	0	∞	−∞

iii) What interesting behaviour can be seen as bandwidth changes with N and explain the analogy with free space when taken to its limit. [2]

Bandwidth increases with N, and as N approaches infinity then so does the bandwidth. In this extreme condition, the analogy with free space would be if the inductors are replace by capacitors having a unit length value of the permittivity of free space and the capacitors are replaced by inductors having a unit length value of the permeability of free space. This telegrapher's model of free space has infinite bandwidth.

iv) For $N \in [1,2,3,4,5,6]$, calculate the insertion loss in dB if non-ideal lumped elements are used. [2]

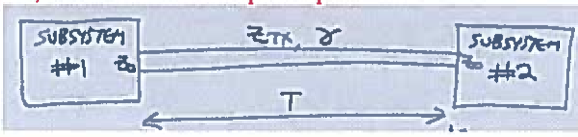
See the table above.

v) What interesting behaviour can be seen as IL changes with N and explain the reason for this? [1]

There is a minimal insertion loss for $N = 3$. From (3.1) it can be seen that adding more sections reduces $NQ_{Network}$ and, therefore, IL decreases. However, if too many sections are added then the losses from the non-ideal lumped-elements will dominate.

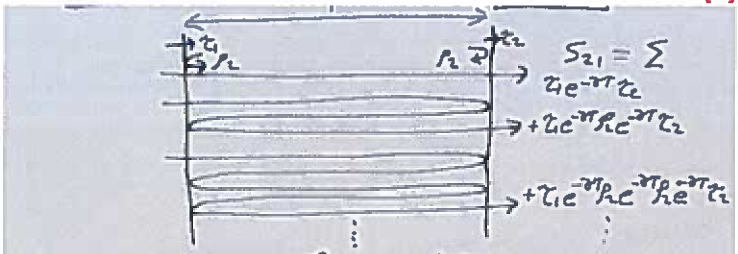
Model Answer to Q4(a): General Discussions in Class and New Derivation

a) Given $Z_{TX} \neq Zo$, with the aid of a sketch, derive from first principles the:



i) Closed-form expression for the overall voltage-wave transmission coefficient in terms of ρ_1 and $e^{-\gamma T}$ only.

[5]



$$\therefore S_{21} = \tau_1 e^{-\gamma T} \tau_2 \sum_{m=0}^{\infty} \left(\rho_2 e^{-\gamma T}\right)^{2m}$$

NOW $\tau_1 = 1 + \rho_1$ AND $\tau_2 = 1 + \rho_2 = 1 - \rho_1$

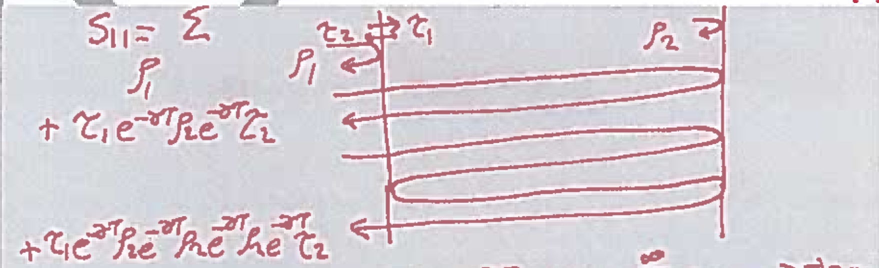
$$\rho_1 = \frac{Z_{TX} - Z_o}{Z_{TX} + Z_o} \qquad \rho_2 = \frac{Z_o - Z_{TX}}{Z_o + Z_{TX}} = -\rho_1$$

$$\therefore \tau_1 \tau_2 = (1 - \rho_1^2)$$

$$\therefore \boxed{S_{21} = \frac{(1 - \rho_1^2) e^{-\gamma T}}{1 - (\rho_1 e^{-\gamma T})^2}}$$

ii) Closed-form expression for the overall voltage-wave reflection coefficient in terms of ρ_1 and $e^{-\gamma T}$ only.

[5]



$$\therefore S_{11} = \rho_1 + \tau_1 e^{-\gamma T} \rho_2 \tau_2 \sum_{m=0}^{\infty} \left(\rho_2 e^{-\gamma T}\right)^{2m}$$

$$\therefore S_{11} = \rho_1 - \frac{\rho_1 e^{-2\gamma T}(1 - \rho_1^2)}{1 - (\rho_1 e^{-\gamma T})^2}$$

$$\therefore \boxed{S_{11} = \frac{\rho_1 \left(1 - e^{-2\gamma T}\right)}{1 - (\rho_1 e^{-\gamma T})^2}}$$

Model Answer to Q4(b): General Discussions in Class and New Derivation

b) Given a quarter-wavelength lossless transmission line, using the derivations from 4(a):

QUARTER WAVELENGTH $\therefore T = \lambda/4$

LOSSLESS TRANSMISSION LINE $\therefore \gamma = j\beta$

$$\beta = \frac{2\pi}{\lambda} \quad \therefore \gamma T = j\pi/2 \quad \therefore e^{-\gamma T} = -j$$

i) Simplify the expression for overall voltage-wave transmission coefficient and calculate the overall insertion loss in dB, given $Z_{TX} = 291\ \Omega$ and $Zo = 50\ \Omega$.

[3]

$$\therefore S_{21} = \frac{-j(1-\rho_1^2)}{(1+\rho_1^2)}$$

$$\rho_1 = \frac{291-50}{291+50} = \frac{1}{\sqrt{2}}$$

$\therefore S_{21} = j\,1/3 \qquad \therefore IL = -10\log|S_{21}|^2 = -9.54\text{ dB}$

ii) Simplify the expression for overall voltage-wave reflection coefficient and calculate the overall return loss in dB, given $Z_{TX} = 291\ \Omega$ and $Zo = 50\ \Omega$.

[3]

$$S_{11} = \frac{2\rho_1}{1+\rho_1^2} = \sqrt{2}\,2/3 \quad \therefore RL = -10\log|S_{11}|^2 = -0.5\text{ dB}$$

iii) Prove that the principle of conservation of energy is observed.

[2]

CONSERVATION OF ENERGY FOR A LOSSLESS SYSTEM:

$$1 = |S_{21}|^2 + |S_{11}|^2$$

$$\therefore 1 = \frac{(1-\rho_1^2)^2}{(1+\rho_1^2)^2} + \frac{(2\rho_1)^2}{(1+\rho_1^2)^2} = 1 \quad \text{Q.E.D.}$$

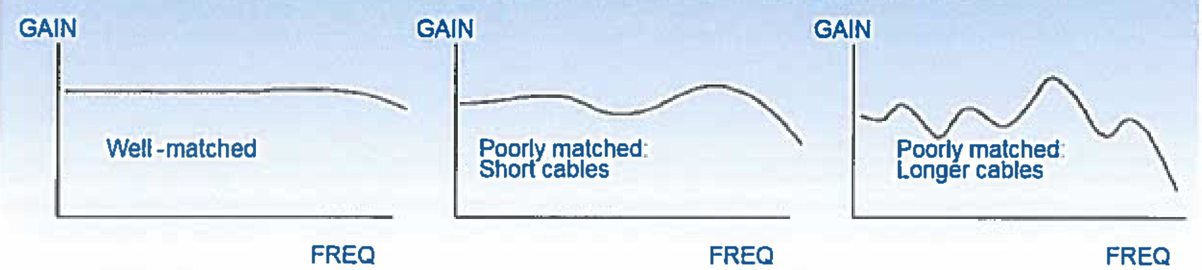
$$\text{or } 1 = \frac{1}{9} + \frac{2\times4}{9} = 1$$

Model Answer to Q4(c): General Discussions in Class

c) With the aid of sketches, illustrate what happens to the frequency response of the overall system when the length of an impedance mismatched interconnecting transmission line increases and explain why.

[2]

There are multiple reflections in practice.....



- At some frequencies reflections may be in-phase and add constructively
- At other frequencies reflections may be in anti-phase and partially cancel each other out
- The end result is GAIN RIPPLES and related to the wavelength
- If interconnects are short, compared to wavelength, a constant mismatch results:
  This may be tolerated
- On RFICs, 50 Ω matching is only required at inputs and outputs
- High RF power results in low impedances, making it difficult to impedance match to 50 Ω

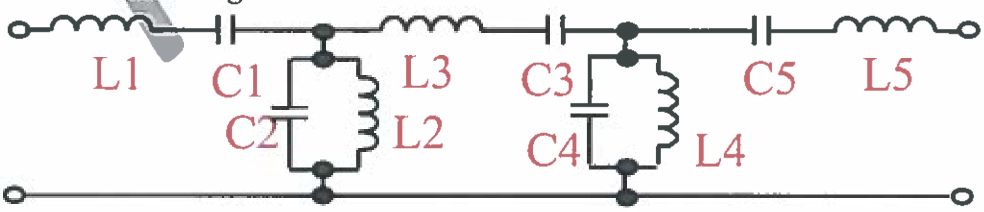
Model answer to Q5(a): Computed Example

a) Design a 5-pole *LC* Butterworth bandpass filter with a passband from 2.4 to 2.5 GHz and terminating impedance of 50 Ω. The first and last components should be series elements. Explain why there is a large variation in component values and suggest a way to avoid this.

For a 5-pole Butterworth lowpass prototype the normalised elements are:

$g_0 = 1;$
$g_1 = L_{n1} = 0.618;\ g_2 = C_{n2} = 1.618;\ g_3 = L_{n3} = 2;\ g_4 = C_{n4} = 1.618;\ g_5 = L_{n5} = 0.816;$
$g_6 = 1$

Bandpass de-normalising:



$$L_{nS} = \frac{\omega L_S}{Z_o} = \frac{g}{\Delta} \qquad L_S = \frac{gZ_o}{\omega\Delta}$$

$$C_{nS} = Z_o\omega C_S = \frac{\Delta}{g} \qquad C_S = \frac{\Delta}{gZ_o\omega}$$

$$L_{nP} = \frac{\omega L_P}{Z_o} = \frac{\Delta}{g} \qquad L_P = \frac{\Delta Z_o}{\omega g}$$

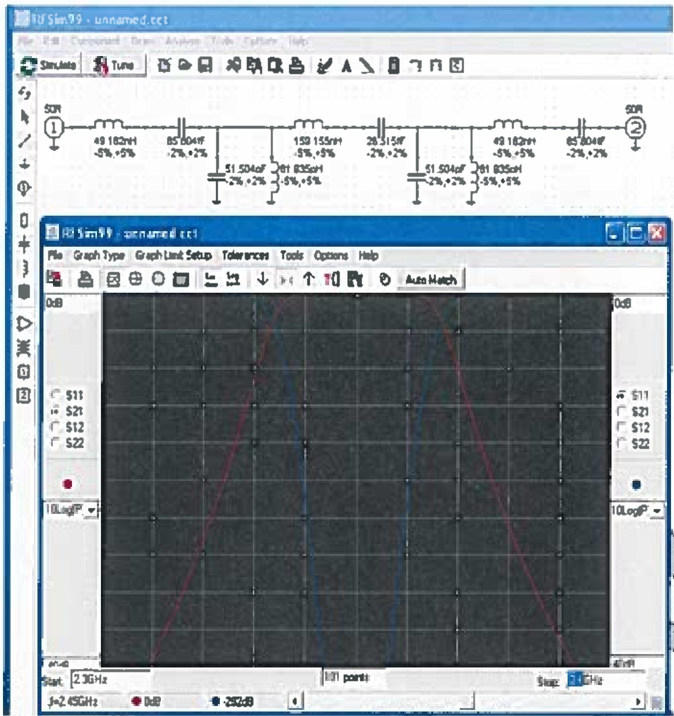
$$C_{nP} = Z_o\omega C_P = \frac{g}{\Delta} \qquad C_P = \frac{g}{\Delta Z_o\omega}$$

with $f_1 = 2.4$ GHz, $f_2 = 2.5$ GHz , and $R_L = 50\ \Omega$:

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_o} = \frac{f_2 - f_1}{f_o} = \frac{f_2 - f_1}{\sqrt{f_1 f_2}} = 0.0408$$

$$\omega_o = 2\pi f_o = 15.3938x10^9\,rad/s$$

$$Zo = 50\Omega$$



$$L_{S1} = L_{S5} = \frac{0.618x50}{15.3938x10^9 x0.0408} = 49.2nH$$

$$C_{S1} = C_{S5} = \frac{\Delta}{gZ_o\omega} = \frac{0.0408}{15.3938x10^9 x0.618x50} = 85.8fF$$

$$L_{P2} = L_{P4} = \frac{\Delta Z_o}{\omega g} = \frac{0.0408x50}{15.3938x10^9 x1.618} = 81.9pH$$

$$C_{P2} = C_{P4} = \frac{g}{\Delta Z_o\omega} = \frac{1.618}{15.3938x10^9 x0.0408x50} = 51.5pF$$

$$L_{S3} = \frac{2x50}{15.3938x10^9 x0.0408} = 159.2nH$$

$$C_{S3} = \frac{\Delta}{gZ_o\omega} = \frac{0.0408}{15.3938x10^9 x2x50} = 26.5fF$$

This filters has a fraction bandwidths of around 4% and so it will suffer from large variations in component values. From the values given above it can be seen that there is almost a 2000:1 variation in the inductance values and a 600:1 variation in capacitance values. To avoid such extremes impedance or admittance inverters are used.

[10]

Model answer to Q5(b): Extension of Bookwork

b) Show mathematically, from first principles, how a shunt *RLC* tuned circuit can replace a series *RLC* tuned circuit by employing admittance (J)-inverters. How can the admittance inverters be implemented with lumped-element components.

A series *RLC* circuit has the following impedance:

$$Z_S = R_S + j\omega L_S + \frac{1}{j\omega C_S}$$

This can be converted into a shunt *RLC* tuned circuit by using J-inverters. The corresponding admittance will be:

$$Y_P = G_p + j\omega C_p + \frac{1}{j\omega L_p} \equiv J^2 Z_S$$

Using discrete inductance values to realise an impedance inverter with a –C/+C/–C

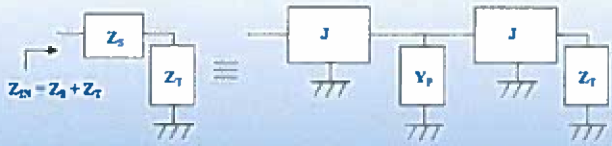
$$\pi\text{-network: } J = \omega C$$

$$Y_P = R_S(\omega C)^2 + j\omega L_S(\omega C)^2 + \frac{(\omega C)^2}{j\omega C_S}$$

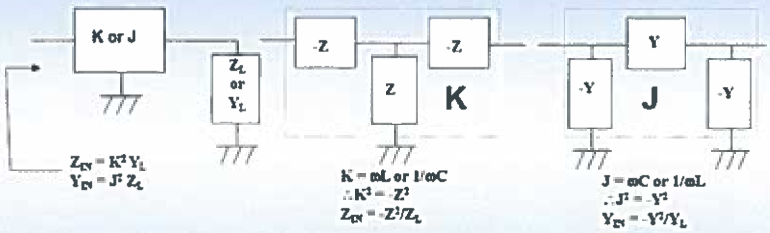
$$\therefore G_P = R_S(\omega C)^2 \quad ; \quad C_P = L_S(\omega C)^2 \quad ; \quad L_P = \frac{C_S}{(\omega C)^2}$$

For convenience, we can choose to set $(\omega C)^2 = 1x10^{-3}$ and, therefore, it can be easily seen that the series inductance value in nH will be equal to the parallel capacitance value in pF. Likewise, the series capacitance value in pF will be equal to the parallel inductance value in nH.

A series connected series $R_sL_sC_s$ tuned circuit can be "synthesized" using a shunt connected parallel $R_pL_pC_p$ tuned circuit having two admittance inverters.



In addition, lumped elements can be used (i.e. either all inductive or all capacitive), as shown below. Note that negative reactances or susceptances are meant to be absorbed by neighbouring positive reactances or susceptances.



Impedance/Admittance Inverter — T-Impedance network (K-Inverter) — π-Admittance network (J-Inverter)

Two identical inverters connected in cascade represents a zero inversion

[6]

Model answer to Q5(c): Computed Example

c) Using a capacitive J-inverter:

(i) What would be an appropriate value for J^2 and why? For the example in 5(a), calculate the appropriate modulus value for the capacitance in the admittance inverter.

[2]

(ii) Explain what happens to the negative capacitances?

[1]

(iii) For the example in 5(a), what action can be taken to avoid unwanted resonances at high frequencies caused by parasitics?

[1]

(i) For convenience, we can choose to set $J^2 = 1x10^{-3}$ because a capacitance in [pF] will transform to the same value in [nH], and vice versa. Therefore, the capacitive admittance inverter has $(\omega_o C)^2 = 1x10^{-3}$ $\quad and \quad \therefore C = 2.054\ pF$.

[2]

(ii) The negative values of C are either absorbed into the adjacent shunt capacitance or the C/+C arrangement performs an impedance transformation of the terminating impedances. The latter can be avoided here if the first and last components are shunt. [1]

(iii) To avoid resonances at high frequencies, due to unwanted parasitics, the lumped element shunt LC components are replaced by distributed-element transmission line stubs. [1]

Model answer to Q6(a): Bookwork

a) Derive, from first principles, the general radar range equation. Assume the following lossless system:

- The transmitter has an input power P_T feeding an antenna having a power gain G_T that is located at a range R_T from the target.
- The target has a radar cross section σ.
- The receiver has an input power P_R delivered by an antenna having a power gain G_R that is located at a range R_R from the target.

[5]

At the target the power density is given by:

$$PD_{TARGET} = \left(\frac{P_T}{4\pi R_T^2}\right) G_T \quad [W/m^2]$$

Power captured and reradiated isotropically by the target, $P_{TARGET} = \sigma PD_{TARGET} \quad [W]$

At the receiver the power density is given by:

$$PD_{RX} = \left(\frac{P_{TARGET}}{4\pi R_T^2}\right) \quad [W/m^2]$$

Power at the input to the receiver:

$$P_R = A_{RX} PD_{TXT} \quad [W]$$

The effective aperture of the receiving antenna, $A_{RX} = \frac{\lambda_o^2}{4\pi} D_o \quad [m^2]$

Where $G_R = \eta Do$ and with a lossless antenna its efficiency, $\eta = 100\%$

$$\therefore P_R = \left(\frac{\lambda_o^2}{4\pi}\right) G_R \left(\frac{1}{4\pi R_R^2}\right) \sigma \left(\frac{P_T}{4\pi R_T^2}\right) G_T$$

$$\therefore \left(\frac{P_R}{P_T}\right) = \left(\frac{\lambda_o}{4\pi R_T R_R}\right)^2 \frac{G_T \sigma G_R}{4\pi} \quad \text{Radar Range Equation}$$

When the transmitter and receiver share a common antenna:

$$\left(\frac{P_R}{P_T}\right) \rightarrow \left(\frac{\lambda_o G}{4\pi R^2}\right)^2 \frac{\sigma}{4\pi}$$

[5]

Model answer to Q6(b)(i): Computed Example

If identical lossless paraboloidal reflector antennas are used at both locations, calculate the power gain in dBi of the antennas if they have a diameter of 9 m.

The effective aperture for an ideal lossless paraboloidal reflector antenna is given by:

$$A = \frac{\pi D^2}{4} = 63.62 \quad m^2$$

$$\Omega = \frac{\lambda_o^2}{A} = 7.6x10^{-3} \quad with \quad \lambda_o = 0.694\, m$$

$$Directivity,\, D_o = \frac{4\pi}{\Omega} = 1{,}658$$

$$Power\ Gain,\, G \rightarrow D_o = 32.2\, dBi$$

[3]

Model answer to Q6(b)(ii): Computed Example

Comment on the resulting beam efficiency for this application, when considering the angle that the moon subtends as seen by an observer on the earth.

With a pencil beam radiation pattern (i.e. having a large aperture and low sidelobes):

$$\Omega \approx \theta_E \theta_H$$

Where θ_E and θ_H are the -3 dB beamwidths in the E- and H-planes, respectively.

$$\therefore \theta \approx \sqrt{\Omega} = 0.087\ radians \equiv 5^o$$

To an observer on the earth, the moon subtends an angle of 2ϕ: where

$$2\phi = 2\tan^{-1}\left(\frac{3.5/2}{381.5 + 3.5/2}\right) = 0.523^o$$

Therefore, the beam efficiency will be very small because the angle subtended by the target is an order of magnitude smaller than the beamwidth of the antenna.

[3]

Model answer to Q6(b)(iii): Computed Example

With $P_T = 20$ dBW, calculate the power at the receiver. As a first order approximation, assume that the moon's radar cross-section can be modelled as a perfectly reflecting flat circular disc.

If we assume that the moon's radar cross-section can be modelled as a perfectly reflecting flat circular disc:

$$\sigma = \frac{\pi D_M^2}{4} = 9.62x10^{12} \quad m^2$$

Therefore, the power at the receiver is given by:

$$P_R = \left(\frac{\lambda_o D_o}{4\pi R^2} \right)^2 \frac{\sigma P_T}{4\pi} = 30\, aW$$

[3]

Model answer to Q6(b)(iv): Computed Example

With an antenna temperature T_A = 100 K and a receiver noise temperature T_{RX} = 75 K, calculate the carrier-to-noise power ratio at a receiver having a final IF bandwidth of 7 kHz. Boltzmann's constant k = 1.38 x 10^{-23} W/Hz/K.

System equivalent noise temperature at the receiver, $T_S = T_A + T_{RX} = 175$ K

System Input Noise Power, $N = k\, T_S\, B = 16.9$ aW

Therefore, the carrier-to-noise power ratio at a receiver, $C/N = P_R/N = 1.775 = 2.5$ dB

[3]

Model answer to Q6(b)(v): Computed Example

Calculate the minimum possible propagation delay time between the two ground stations if both antennas suffer from unwanted sidelobes.

If both antennas suffer from unwanted sidelobes then it is theoretically possible that the sensitive receiver could detect the high power transmitter through unwanted coupling of the sidelobes of both antennas. In this case, the minimum possible delay will be given from:

Delay = separation distance between earth stations / speed of light = 3.33 microseconds.

[3]