Imperial College London

BSc/MSci EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

ATMOSPHERIC PHYSICS

For 3rd and 4th Year Physics Students

Monday, 28th May 2012: 14:00 to 16:00

The paper consists of two sections: A & B. Section A contains one question. Section B contains four questions.
Answer ALL parts of Section A and TWO questions from Section B.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

SECTION A

1. (i) In the terrestrial troposphere, nitrous oxide (N_2O) has a volume mixing ratio of 0.3 ppmv. Assuming a temperature T = (-13)°C and a pressure p = 1 mbar, calculate the number density n_{N2O} in m^{-3} and the mass mixing ratio μ_{N2O} (in ppmm) of nitrous oxide.

[Mean molecular mass of air $\bar{M}$ = 29 g mol^{-1}, M_N = 14 g mol^{-1}, M_O = 16 g mol^{-1}]

[3 marks]

(ii) (a) Define what hydrostatic equilibrium means.
Derive an expression between the pressure p(z), the mass density $\rho(z)$, the gravitational acceleration g(z), and the height z.

[3 marks]

(b) Assuming hydrostatic equilibrium and assuming that the atmosphere behaves as an ideal gas, show that the variation of the mass density ρ with height z is given by:

$$\frac{d\rho}{\rho(z)} = -\frac{dz}{H_\rho(z)}$$

State any assumptions that you make. The mean molecular mass of the atmosphere $\bar{M}$ is assumed independent of height.
Provide an expression for $1/H_\rho(z)$. [3 marks]

(iii) Define with words:

- the potential temperature, θ, of the air at height z
- an optically thick atmospheric layer at wavelength λ
- photochemical equilibrium applied to a neutral species i

[3 marks]

(iv) Compare solar radiation versus terrestrial, thermal radiation in terms of spectral range, transmission through the atmosphere, and directional properties.

[3 marks]

(v) State in which region of the terrestrial atmosphere the following characteristics occur:

(a) the temperature gradient is positive and the N_2 density falls off following its own scale height. Briefly justify these two characteristics.

(b) the temperature gradient is negative and the air can be very moist. Briefly justify the first characteristic.

[3 marks]

(vi) Provide an expression for the Lagrangian derivative of a scalar field variable, $\Psi(\mathbf{r},t)$ as a function of the Eulerian derivative.

Describe what the meaning of the Lagrangian derivative is.

[2 marks]

[Total 20 marks]

SECTION B

2. Water-related sediments have been found dating from as early as 3.8 billion years ago. This attests to the presence of liquid water on Earth at that time. During the same epoch, the young Sun was fainter with a flux 30% smaller than the flux today.

The Sun has a radius a_S of 7×10^5 km and a temperature T_S of 5780 K, and the mean Sun-Earth distance r is 150×10^6 km. The albedo A at the top of Earth's atmosphere is assumed to be 0.28.

(i) Assuming that the only source of heat is solar radiation and assuming the atmosphere to be reduced to one isothermal layer, derive an expression for the surface temperature of Earth, T_g, valid 3.8 billion years ago. State any assumptions that you make.

Provide a numerical value for T_g in °C, assuming that the transmissivity τ_{SW} at short wavelengths and the transmissivity τ_{LW} at long wavelengths in early Earth's atmosphere are assumed to be equal to those from today, that is, 0.9 and 0.2, respectively.

Could liquid water be present? Justify briefly your answer.

Compare T_g with the mean surface temperature of the Earth today. Discuss the difference.

[6 marks]

(ii) Early Earth had most likely an atmosphere with significantly larger number densities than today in CO_2 and CH_4. How would T_g be affected by this different composition? Justify your answer. Invoke transmissivity in the discussion. No calculation is required here.

[2 marks]

(iii) Let's now represent the atmosphere as two superimposed, isothermal layers which are opaque to long-wave radiation and transparent to short-wave radiation.

Sketch the atmospheric layers showing the incoming and outgoing fluxes at each boundary: the top of the atmosphere, between the two atmospheric layers, and at the ground. Provide values for τ_{SW} and τ_{LW}.

Derive an expression relating the atmospheric temperatures T_{a1} (lower layer) and T_{a2} (upper layer). Derive another expression relating the surface temperature, T_g' and T_{a2}. State any assumptions that you make.

Compare T_g', T_{a1}, and T_{a2}. How realistic is this thermal profile in altitude? Mention briefly two examples illustrating short-comings of this simple atmospheric model.

[7 marks]

[Total 15 marks]

3. In a given region of Earth, the atmospheric temperature is assumed to vary linearly between the following points:

- Point A, z_1 = 0 km, T_1 = 304 K
- Point B, z_2 = 2 km, T_2 = 280 K
- Point C, z_3 = 4 km, T_3 = 264 K
- Point D, z_4 = 6 km, T_4 = 262 K

The adiabatic lapse rate under unsaturated conditions is assumed to be equal to the dry adiabatic lapse rate, Γ_d = 10 K km^{-1}. The saturated adiabatic lapse rate is Γ_s = 6 K km^{-1}.

(i) Show from the First Law of Thermodynamics that the dry adiabatic lapse rate Γ_d is given by:

$$\Gamma_d = \frac{g}{c_p}$$

State any assumptions that you make.

[Reminder: $dU = C_v\, dT$, $dW = p\, dV_M$, and $C_p - C_v = R$ where U is the internal energy of the system, W is the work done by the system, V_M is the volume per mole (m^3 mol^{-1}), C_v and C_p are the heat capacities at constant volume and constant pressure, respectively (J mol^{-1} K^{-1}). The specific heat capacity c_p for dry air is 10^3 J kg^{-1} K^{-1}.]

[4 marks]

(ii) Explain, in a qualitative manner, why $\Gamma_d > \Gamma_s$.

[2 marks]

(iii) Identify stable and unstable regions between points A and D, for unsaturated and for saturated conditions.

Briefly justify your answer.

[3 marks]

(iv) Consider a moist, unsaturated air parcel originally at the equilibrium and being uplifted adiabatically through convection from the ground. It contributes to cloud formation beginning at point B.

Describe briefly what happens to the air parcel from the ground through its journey in the atmosphere. Justify your answer.

Draw a temperature-height diagram illustrating the temperature profile of the environment and the temperature profile of the parcel. Show points A, B, C, and D and any relevant lapse rates.

Calculate the altitude at which the air parcel becomes stable relative to the surrounding atmosphere.

[6 marks]

[Total 15 marks]

4. For large scale motion in the horizontal direction, the Navier-Stokes equation in a rotating frame on a spherical Earth is given by:

$$\frac{D\mathbf{V}}{Dt} = -f\,\mathbf{k} \times \mathbf{V} - \frac{1}{\rho}\nabla_z p \qquad (1)$$

In the local cartesian system ($\mathbf{i}, \mathbf{j}, \mathbf{k}$), the horizontal velocity of the flow $\mathbf{V} = u\mathbf{i} + v\mathbf{j}$.

∇_z is the horizontal gradient evaluated on surfaces of constant height (z constant). f is the Coriolis parameter, ρ the mass density, and p the pressure.

(i) Assuming geostrophic balance, derive an expression for the geostrophic wind, $\mathbf{V_g}$ as a function of ∇_z from (1). Justify each step of the derivation.

[3 marks]

(ii) Draw a schematic diagram showing the geostrophic wind vector at a given point and the corresponding balance of forces, around a center of low pressure in the mid-latitude northern hemisphere. Draw also the unit vectors of the local coordinate system.

[2 marks]

(iii) Show that: $\nabla_p z = -\,(\partial z/\partial p)_{xy}\,\nabla_z p$
where ∇_p is the horizontal gradient evaluated on an isobaric surface (pressure p constant). Make use of the following relation:

$$\nabla_z\Psi = \nabla_p\Psi + (\partial\Psi/\partial p)_{xy}\,\nabla_z p \qquad (2)$$

where Ψ is a scalar.

[1 mark]

(iv) Define with words the geopotential, Φ.
Provide an expression for dz as a function of dΦ.
Based on the definition of geopotential height Z, provide an expression for dΦ as a function of dZ.
[3 marks]

(v) Using results from 3(i), 3(iii), and 3(iv), and assuming hydrostatic equilibrium, show that:

$$\mathbf{V_g} = \frac{g_0}{f}\,(\mathbf{k} \times \nabla_p Z)$$

where g_0 is the gravitational acceleration at the surface. [2 marks]

(vi) Within a local region near 40°S, the geopotential height contours on a 500 hPa chart are oriented east-west and the spacing between adjacent contours (at 60-m intervals in geopotential height) is 300 km. The geopotential height decreases towards the north.
Draw a schematic showing two Z contours, the four cardinal directions (north, south, east, west), the geostrophic wind vector, and associated vectors used to derive it.
Calculate the direction and velocity magnitude of the geostrophic wind.

[4 marks]

[Total 15 marks]

[Reminder: $\mathbf{A} \times (\mathbf{B}\times\mathbf{C}) = \mathbf{B}.(\mathbf{A}.\mathbf{C}) - \mathbf{C}.(\mathbf{A}.\mathbf{B})$]

5. Consider solar radiation at wavelength λ incident upon a layer of gas mixture A with a mean molecular mass $\overline{M}$ = 30 g mol^{-1}. The main absorbing gas at λ has a mass mixing ratio μ_a = 500 ppmm and an absorbing coefficient α_λ= 2 m^2 kg^{-1}.

No significant atmospheric scattering is assumed to occur at λ.

(i) (a) Applying the Lambert-Beer Law, show that the mass column of gas mixture needed in order to reach an absorptivity $\mathcal{A}_\lambda$ defined between the top of the atmosphere and the location s along the radiation path is given by:

$$\int_\infty^s \rho(s')ds' = \frac{-\ ln(1 - \mathcal{A}_\lambda)}{\alpha_\lambda\, \mu_a}$$

where ρ is the mass density of the gas mixture A. State any assumptions that you make.
Provide a numerical value in kg m^{-2} for an absorptivity $\mathcal{A}_\lambda$ of 2/3.

[4 marks]

(b) How would the transmissivity between the top of the atmosphere and s be affected by scattering? You can assume that scattering into the beam is negligible. Briefly justify your answer. No calculation is required here.

[1 mark]

(ii) Consider a hypothetical planetary atmosphere composed entirely of the gas mixture A. It is assumed to be isothermal at T_0 = 200 K. The atmospheric pressure at the surface is p_0 = 5×10^4 Pa and the scale height H = 10 km. Solar radiation is assumed to be incident vertically upon the horizontally stratified atmosphere.

(a) Assuming hydrostatic conditions, provide an expression for the mass density $\rho(z)$ at height z as a function of H, z, and the mass density at the surface, ρ_0. You do not need to derive it.

[1 mark]

(b) Using your answer to part (a) above, show that the optical depth $\chi_\lambda(z)$ is given by:

$$\chi_\lambda(z) = \alpha_\lambda\, \mu_a\, H\, \rho(z)$$

[2 marks]

(c) Using results from parts (a) and (b), derive an expression for the height z_1 of the level of unit optical depth at λ. Provide a numerical value for z_1 in km.

[2 marks]

(d) Using results from parts (a) and (b), show that the heating rate Q(z) per unit volume at z_1 is given by:

$$Q(z_1) = \frac{F_\lambda(z_1)}{H}$$

where $F_\lambda(z_1)$ is the spectral irradiance at z_1.
By how much would $Q(z_1)$ change if the planet was two times closer to the Sun?

[5 marks]

[Total 15 marks]