Information for Candidates

Switch-Mode Power Supplies

Voltage ratio equations for buck SMPS $\frac{V_O}{V_I} = \delta$ or $\frac{V_O}{V_I} = \frac{1}{1+\frac{2fL\,I_O}{V_I\delta^2}}$

Voltage ratio equations for boost SMPS $\frac{V_O}{V_I} = \frac{1}{1-\delta}$ or $\frac{V_O}{V_I} = \frac{1}{1-\frac{V_I\delta^2}{2fL\,I_I}}$

Three-Phase Systems

Line Voltages and Current

Star $V_L = \sqrt{3}\,V_P;\ I_L = I_P$ Delta $V_L = V_P;\ I_L = \sqrt{3}\,I_P$

Power $P_{3\phi} = 3\,V_P I_P cos(\phi) = \sqrt{3}\ V_L I_L cos(\phi)$

Induction Machines

$\omega_S = {}^{\omega_E}/_{p}$; $\mathrm{s} = \frac{\omega_S - \omega_R}{\omega_S}$; $T_{EM} = \frac{3\,I'^2_R\,R'_R}{\omega_R}\left(\frac{1-s}{s}\right)$

Photovoltaic Systems

$$I_{PV} = I_{Ph} - I_{AK} - I_{Sh} \qquad I_{AK} = I_0\left(e^{\frac{V_{PV}+I_{PV}R_S}{K_I v_T}} - 1\right)$$

Power Flow in Lines and Cables

Cable Parameters

$R'_{LF} = \frac{1}{\sigma_C \pi\, r_C^2} + \frac{1}{\sigma_C 2\pi\, r_O\, t_O}$ $L' = \frac{\mu}{2\pi} ln\left({}^{r_O}/_{r_C}\right)$ $C' = \frac{2\pi\varepsilon_0\varepsilon_{RI}}{ln\left({}^{r_O}/_{r_C}\right)}$ $G' = \frac{2\pi\sigma_I}{ln\left({}^{r_O}/_{r_C}\right)}$

OHL Parameters (approximate form)

$R'_{LF} = \frac{2}{\sigma_C \pi\, r_C^2}$ $L' = \frac{\mu}{\pi} ln\left({}^{d}/_{r_C}\right)$ $C' = \frac{\pi\varepsilon_0\varepsilon_{RI}}{ln\left({}^{d}/_{r_C}\right)}$ $G' = \frac{\pi\sigma_I}{ln\left({}^{d}/_{r_C}\right)}$

Power Flow (full form)

$$P_S = \frac{{V_S}^2}{Z_{SR}} cos(\theta) - \frac{V_R V_S}{Z_{SR}} cos(\theta + \delta) \qquad Q_S = \frac{{V_S}^2}{Z_{SR}} sin(\theta) - \frac{V_R V_S}{Z_{SR}} sin(\theta + \delta)$$

Voltage Drop (approximate form) $\Delta V = |V_S| - |V_R| \approx \frac{R\,P_S + Z\,Q_S}{|V_S|}$

[Overall, the class performed very well on this paper and the average was 71.7% before moderation.

Question 1 average was 27.9/40 =69.8% indicating that this general-coverage question did not pose problems for well-prepared candidates.

Question 2 average was rather high at 24.9/30 =83.0%. Most of the question was in a familiar format on the familiar topic of Buck SMPS. Parts (b) (iii) and (b) (v) were meant to be a little unusual but did not pose problems for most candidates.

Question 3 average was 18.9/30 = 63.0%. Part (a) covering induction machines was in a familiar format and was well-handled. Part (b) on transmission lines was intended to be difficult and so it turned out.]

1. This question covers several topics and all parts should be attempted.

a) Consider a national-scale AC electricity system.

i) For an example country, the UK perhaps, describe the changes in proportions of generation from various sources in the last decade and the reasons for the change. [4]

Discussion should note the rise of low-carbon / renewable sources, principally wind and photo-voltaic energy, in many countries of the world. [1 mark]

Rise of renewables is driven by subsidies in the form of feed-in tariffs, green certificates or contracts-for-difference, although subsidy-free invest is now viable in some cases. [1 mark]

Among the fossil fuels, gas has been displacing coal, partly driven by reducing gas prices (or expanding supply) and carbon taxes that increase the costs of coal more than gas. [1 mark]

Various other points can be made regarding position of nuclear, hydro or biomass. In the UK hydro is maintained and little prospect of expansion, biomass has been subsidised and is not substantial and new-build nuclear is subsidised to replace decommissioned plant. [1 mark]

[Some answers were superficial and said not much more than fossil fuel is bad and renewables are good. A more sophisticated discussion was needed to get full marks.

It is also insufficient to say that there are concerns about climate change or that governments want certain things to happen. In most countries, generating plant are owned by investors who are profit-maximisers. Governments that want to effect change away from fossil fuels need to tax carbon emissions or subsidise low-carbon generation. Clearly cost reductions in low-carbon generation help and are almost at the point of being sufficient.

Statements that fossil fuels are running out are not accurate. The reality is that if we want to limit climate change to an average <1.5C temperature rise then a lot of the existing fossil fuel reserves will need to remain unused (or used only in a carbon capture and storage arrangement)]

ii) Explain why very high voltages are used for national-scale transmission but lower voltages for local distribution. [4]

The dominant power loss is in the series resistance of the lines so there are operational cost savings to be made by increasing the voltage and decreasing the current hence the use of very high voltages in transmission. [2 marks]

In distribution, where distances between substations is lower, the savings in operational cost do not offset the capital cost of the substation equipment to the same extent and so less expensive lower voltage (10s not 100s kV) is used is optimises the lifetime cost. [2 marks]

[Generally answers were good. It is important to state why high voltage reduces power loss.

Safety is not really a reason to prefer 10kV over 100kV.]

iii) Explain why a three-phase system is used. [4]

Distributed (not concentrated) windings are used in generators for the better cooling, waveform shape and machine layout they offer. However, placing all coils in series (single phase) causes voltage cancellation and so breaking the coils into phase groups (poly phase) is preferred but there are diminishing returns in high phase numbers. High phase numbers have high costs in transformers and protection systems. [2 marks]

Poly-phase has the advantage over single-phase of being able to transfer constant instantaneous power and create rotating (not alternating) magnetic fields. 3-phase systems, and above, have zero neutral current underbalanced conditions and neutral conductor can be omitted. [1 mark]

Taking all factors into account, three-phase stands out as the best compromise. [1 mark]

iv) Explain why DC is used in place of AC for some parts of a transmission system. [4]

In AC cables, the capacitance rises with distance and so does the shunt capacitive current. [1 marks]

The capacitive current occupies part of the current rating of the cable and means long cables need to be de-rated which adds to the system cost, or needs compensation which also adds cost. [1 mark]

DC avoids these additional route-costs but adds very significant costs of AC/DC power conversion at the route ends. [1 mark]

Above some distance, in the region of 100 km, the cost of a DC cable plus its terminal equipment is less than that of AC. [1 mark]

(Alternative answer considering inductive voltage drop in OHL is acceptable.)

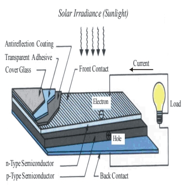
(Use of DC for connecting asynchronous systems can substitute for part of the argument)

b) Consider a typical photovoltaic panel.

i) Describe, with the aid of a sketch, the structure of a photovoltaic panel. [4]

Sketch showing

- wafer with *pn*-junction
- thin strips of top-side metallisation
- protective and anti-reflective top-side layer



[2 marks for sketch and 2 marks for highlighting salient features]

[Some candidates wasted time and effort describing the operating principle when only the construction had been requested.]

ii) Explain why a maximum power point tracking algorithm is needed for a photo-voltaic panel **and** briefly explain how it operates. [4]

A PV cell exhibits a peak power output at a particular terminal voltage and it is advantageous to operate at that voltage [1 mark]

The optimal voltage is dependent on the irradiance and temperature (and aging) and so cannot be pre-selected nor readily determined from measurements so must be found through a search algorithm during operation. [2 marks]

The search is a trial-and-error process known as perturb-and-observe. Small step changes are made to the terminal voltage. The direction (increase or decrease) of each step depends on whether the previous step was seen to increase or increase the power output [1 mark]

c) Consider a Boost SMPS.

i) Describe the operating principle of the Boost SMPS [4]

Turning the MOSFET on imposes the full input voltage across the inductor, causing the current to increase and energy to be stored. During this period the diode is reverse biased and the output current is provided by the output capacitor. [2 marks]

Following turn-off of the MOSFET, the inductor current flows via the now forward biased diode to deliver charge to the output capacitor. Under this condition, there is negative voltage across the inductor so its current is reducing and it delivers some its stored energy to the output capacitor. The capacitor voltage will reach a value sufficiently above the input voltage to create sufficient decrease in inductor current to balance the increase caused by the input voltage during the on-time. [2 marks]

[It is important to point out the logical chain of imposed voltage across the inductor driving increase or decrease of current and therefore increase or decrease of the inductor's stored energy.]

ii) The commonly used expression for output voltage in continuous conduction mode is $\frac{V_O}{V_I} = \frac{1}{1-\delta}$. Explain why the voltage observed in practice would be less than this. [4]

There several reasons why there are voltage drops and/or power losses which mean that a lower output voltage is achieved than the idealised equation predicts.

- During the on-time, voltage is dropped across the MOSFET which causes a lower increase in inductor current than predicted reducing the equilibrium output voltage.
- During the off-time, voltage is dropped across the diode which causes a higher decrease in inductor current than predicted reducing the equilibrium output voltage
- Resistance in the inductor drops voltage.
- Switching power loss in the MOSFET also reduces the energy transfer to the output capacitor.

[Full marks if at least three factors and a coherent argument are given.]

[Some candidates approached this by discussing power losses in the circuit rather than voltage drops. That is OK in principle but requires an additional logical step to explain why the power loss decreases the output voltage rather than increases the input current. That brings one back to a discussion of voltage drop (rather than current leakage).]

iii) The graph of efficiency against output power normally exhibits a central plateau with efficiency reducing a low and high output powers. Explain why this is so. [4]

There are three types of power loss:

- Fixed loss, independent output power/current, due to operation of the control circuitry. At lower output powers, this term dominates and caused low efficiency.
- Power loss due to diode voltage drop and MOSFET switching which are proportional to current and so scale with output power giving a constant efficiency where these dominate in mid-range output power.
- Power loss proportional to current squared in the resistance of the MOSFET and inductor. This dominates at high power and grow faster than the output power so causes a decrease in efficiency.

[1 mark for each factor and 1 for a coherent overall argument]

[Generally, answers were good on why efficiency was reduced at low and high powers but less good at noting that if losses are proportional to current in the mid-range then efficiency is approximately constant in that region.]

d) For the equivalent circuit of an induction machine shown in figure Q1, identify which components give rise to power loss and what physical process or feature of the machine they represent. [4]

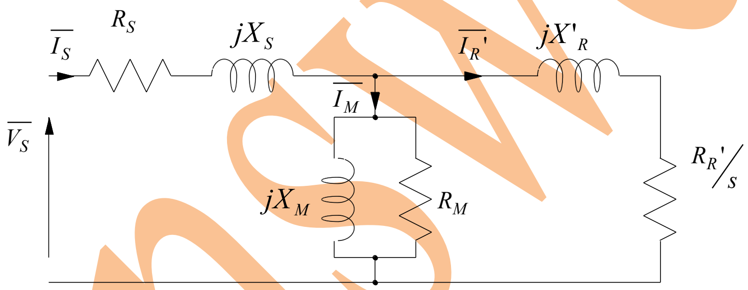


Figure Q1

Only the three resistors contribute to power loss, the inductors do not.
R_S models the resistance of the stator winding and the ohmic heating that results from it. [1 mark]
R_M models, approximately, the power losses due to eddy currents and hysteresis in the steel core. [1 mark]
R_R models the resistance of the rotor winding and the ohmic heating that results from it but it is only a portion of the term R_R /s with remainder, $R_R\,(1/s-1)$ representing conversion to mechanical energy (and not a loss). [2 marks]

[2 marks removed for mis-identifying flux-leakage or inductance as a power loss]

[Many candidates were not clear enough on the reactances not being responsible for (real) power losses and many did not deal properly with the presence of slip term, *s*, in the rotor branch resistance.]

2.

A **buck** switch-mode power supply (SMPS) is to be used to provide a 5 V output from an 18 V input. The SMPS is intended to operate at a maximum output current of 10 A.

The MOSFET to be used has an on-state resistance of $R_{DS(on)}$ = 35 mΩ and turn-on and turn-off energy losses of $E_{Sw\text{-}on}$ = 9 µJ and $E_{Sw\text{-}off}$ = 7 µJ. The diode to be used is a Schottky device with an on-state voltage of $V_{AK(on)}$ = 0.45 V.

The capacitor to be used has a capacitance of 330 µF and has an effective series resistance (ESR) of R_{ESR} = 70 mΩ.

You may assume that the SMPS operates in continuous conduction mode for the calculations that follow.

a)

i) Calculate the duty-cycle with which the SMPS should operate. [2]

Apply standard formula [2 marks for accurate answer].

$$\delta = \frac{V_O}{V_I} = \frac{5}{18} = 0.2778$$

[Almost all the students provide the correct answer for this part.]

ii) Sketch the shape of the current through the inductor, MOSFET and capacitor at this duty-cycle. [4]

[2 marks for basic shapes the one mark each for the two features noted]

- Up-slope of inductor current and Mosfet conduction period should be roughly a quarter of period
- Capacitor current should be centred on zero

[Some candidates did not pay enough attention to relative values of the MOSFET on-state conduction period and off-state period.

Some candidates did not realise that the average current value for capacitor should be zero.]

b)

i) Calculate the average power loss in the diode at maximum output current. [4]

[2 marks for equation and 2 for accurate answer]

$$P_{diode} = (1-\delta)V_{AK}I_O = (1-0.2778)\times 0.45 \times 10 = 3.25\ W$$

[Some candidates did not include the $(1-\delta)$ term in the calculation for diode power losses.]

ii) Calculate the maximum switching frequency allowed if the total power loss in the MOSFET and diode are to be limited to 5 W at maximum output current.

[6]

[3 marks for correct equations for conduction and switching loss. 3 marks for accurate answer.]

Power loss allowed in Mosfet is 5 – 3.25 = 1.75 W

Calculate first the conduction power loss.

$$P_{Mos\ cond} = \delta\, I_O^2\, R_{DS} = 0.2778 \times 10^2 \times 0.035 = 0.972\ W$$

Power loss allowed for switching is 1.75 – 0.972 =0.778 W

Calculate allowed frequency from loss per cycle.

$$P_{Mos\ Sw} = f \times \left(E_{on} + E_{off}\right)$$

$$f = \frac{P_{Mos\ Sw}}{E_{on} + E_{off}} = \frac{0.778}{16 \times 10^{-6}} = 48.6\ kHz$$

[A relatively common mistake was to use the input current to calculate the MOSFET conduction losses not the output current. This is a case where the Buck and Boost SMPS need to be treated differently because of where the inductor is placed in the two circuits – it is the inductor current that is switched between the Mosfet and the diode and therefore the inductor current amplitude that determines the losses.]

iii) State an equation for the sum of the voltage ripples across the output capacitance and its ESR and hence find the inductor current ripple allowed if the output voltage ripple is to be limited to 40 mV. Assume that the frequency found in (ii) is used. [7]

[3 marks for equation]

$$\Delta v = \frac{\Delta i_L}{8fC} + \Delta i_L R_{ESR}$$

Re-arrange to find current ripple [1 mark for equation and 3 for accurate answer]

$$\Delta i_L = \Delta v \Big/ \left(\frac{1}{8fC} + R_{ESR}\right) = 0.04 \Big/ \left(\frac{1}{8 \times 48.6 \times 10^3 \times 330 \times 10^{-6}} + 0.07\right)$$
$$= 0.514\ \text{A}$$

[About 30% of candidates could not arrive at the correct equation for this question. This equation is simply the sum of the two terms that are stated separately in the notes.

Another common mistake is that the students use $\frac{1}{2}\Delta i_L R_{ESR}$ for voltage ripple value on ESR which perhaps stems from considering peak rather than peak-to-peak ripple voltage.]

iv) Calculate the inductor value that should be used in this design. [3]

Re-arrange equation for current ripple to find inductance. [2 marks for method; 1 for answer]

$$\Delta i_L = \frac{V_I - V_O}{L} \times \frac{\delta}{f}$$

$$L = \frac{V_I - V_O}{\Delta i_L} \times \frac{\delta}{f} = \frac{18 - 5}{0.514} \times \frac{0.278}{48.6 \times 10^3} = 146\ \mu H$$

[Some candidates included consideration of the voltage drop across the MOSFET in the calculation and arrived at a slightly different (and better) value of inductance. This was marked as correct.]

v) If the input voltage rose to 20 V (and duty-cycle adjusted to maintain output voltage), would this SMPS become more of less efficient? (You only need consider the power losses in the semiconductors.) [4]

[3 marks for logical argument and 1 mark for correct conclusion]

There are three terms in the power losses: diode conduction loss, Mosfet conduction loss and switching loss. The last term is independent of duty-cycle so need not be considered.

$$P_{Cond} = (1-\delta)I_O V_{AK} + \delta\, I_O^2\, R_{DS} = 4.5(1-\delta) + 3.5\delta = 4.5 - 1.0\delta$$

At 10 A and with the characteristics given, the losses reduce as duty-cycle increases.

So, for an increase in input voltage, the duty-cycle decreases, the losses increase and the efficiency decreases.

[This part was intended to be unfamiliar and require some development from first principles. A common omission was to not compare the diode and MOSFET losses in terms of duty-cycle variation.]

3. This question addresses several aspects of AC systems.

a) A three-phase induction machine with 2 pole-pairs has the following equivalent circuit parameters:

Stator resistance, $R_S = 0.8\ \Omega$.

Stator leakage reactance, $X_S = 2\ \Omega$;

Magnetising resistance, $R_M = 200\ \Omega$;

Magnetising reactance, $X_M = 60\ \Omega$;

Referred rotor leakage reactance, $X_R = 2\ \Omega$;

Referred rotor resistance, $R_R = 0.8\ \Omega$;

The machine is supplied at a phase voltage of 200 V and a frequency of 50 Hz.

When used to drive a particular mechanical load, the machine runs at 1,445 rpm.

i) Give an expression for the total input impedance of the machine and calculate its value in complex form for the conditions given. [8]

[3 marks for correct formulation of impedance, including correct parallel terms and inclusion of *s*]

Total impedance:

$$Z_T = R_S + jX_S + \left(R_M // jX_M // \left(\frac{R'_R}{s} + jX'_R \right) \right)$$

[2 marks for finding slip accurate to 3 sig. fig.]
Slip needs to be calculated first from: $s = \frac{\omega_S - \omega_R}{\omega_S}$

Synchronous speed calculated by: $\omega_S = {}^{\omega_E}/_{P} = {}^{50 \times 2\pi}/_{2} = 157.08\,\frac{\text{rad}}{\text{s}}\, or\ 1{,}500\ rpm$

Hence slip is: $s = \frac{1500 - 1445}{1500} = 0.0367$

[3 marks for accurate answer]

$$Z_T = 17.734 + \text{j}8.992 = 19.883\angle 26.89°\ \Omega$$

[This is a calculation that is familiar from problem sheets and past papers so present no problems for well-prepared candidates. That said, it is important to record and use 3 significant figures for the slip. Using 0.03 leads to gross errors in impedance and when carried over into the torque calculation in (ii) (where the current is squared) the error is very large and so all marks for accuracy were lost in that case. Using 0.037 is still not really accurate enough and so lost some marks.]

ii) Calculate the mechanical power being developed and the electro-magnetic torque. [10]

First, find stator current from the phase voltage $I_S = \frac{V_S}{Z_T} = \frac{200}{19.88\angle 26.9°} = 10.06\angle -26.9°\ \Omega$

[2 marks for accurate answer]

Then rotor current is found from the current divider rule (or otherwise)

$$I_R = I_S \times \left(\frac{R_M // jX_M}{R_M // jX_M + \left(\frac{R'_R}{s} + jX'_R \right)} \right)$$

[2 marks for correct formulation]

I_R = 8.41 ∠-9.68° [1 mark for accurate answer]

Power from standard formula $P_{EM} = 3\,I'^2_R\,R'_R\left(\frac{1-s}{s}\right) = 3 \times 8.41^2 \times 0.8 \times \left(\frac{1-0.0367}{0.0367}\right) = 4.46$ kW
[1 mark for correct formula; 2 marks for accurate answer]

Torque found by dividing by rotor speed

$$T = \frac{P_{EM}}{\omega_R} = \frac{4461}{(1-0.0367) \times 157.08} = 29.5 \text{ Nm}$$

[1 mark for correct formula; 1 mark for accurate answer]

[Again, answers were generally good. Some candidates squared the current in complex form and get a complex torque and power. This is a serious error of understanding and losses marks. The square here is a square of the magnitude, or more precisely $I \cdot I$]

b) An overhead line is found to have an inductive reactance per unit length of 0.1 Ω/km and negligible resistance. The line is 300 km long and is operated with sending and receiving end voltages of 400 kV and 390 kV respectively.

i) Calculate the angle difference between sending and receiving end voltages that will exist if the line transfers 1,200 MW. [4]

Use standard simplification of power flow in reactance dominated line.

[2 marks for method; 2 marks for accurate answer]

$$P = \frac{V_R V_S}{X}\sin(\delta)$$

$$\sin(\delta) = \frac{X\,P}{V_R V_S} = \frac{300 \times 0.1 \times 1200 \times 10^6}{400 \times 10^3 \times 390 \times 10^3} = 0.2308$$

$$\delta = \sin^{-1}(0.2308) = 13.34°$$

[Approximately half of candidates could not answer this despite this type of question appearing in the examples class and past papers. Of those getting it right, some remembered the equation for the case of a reactance only line and some started with the general equation from the list at the head of the paper ($P_S = \frac{V_S{}^2}{Z_{SR}}\cos(\theta) - \frac{V_R V_S}{Z_{SR}}\cos(\theta+\delta)$) and simplified for the case of θ=90°]

ii) If the line current has to be limited to 1,000 A, what is the maximum voltage angle difference that can be allowed and what real power flow does this allow. [4]

Voltage drop along line will be jIX and the other two vectors are known. Use cosine rule of find the angle.

$$(IX)^2 = V_R{}^2 + V_S{}^2 - 2V_R V_S\cos(\delta)$$

$$\cos(\delta) = \frac{V_R{}^2 + V_S{}^2 - (IX)^2}{2V_R V_S} = \frac{(390 \times 10^3)^2 + (400 \times 10^3)^2 - (1000 \times 300 \times 0.1)^2}{2 \times 400 \times 10^3 \times 390 \times 10^3}$$
$$= 0.9974$$

$$\delta = \cos^{-1}(0.9974) = 4.13°$$

$$P = \frac{V_R V_S}{X} sin(\delta) = \frac{400 \times 10^3 \times 390 \times 10^3}{300 \times 0.1} sin(4.13°) = 374\ MW$$

[2 marks for correct use of cosine rule to find load angle; 2 marks for final answer]

[Use of the cosine rule to solve for the angle δ in the triangular phasor diagram has been discussed in the example classes but few candidates seemed to be familiar with this so found this problem difficult to approach.

Several candidates created an approximate solution by assuming that the 1,000 A current would be at 0° to the sending end voltage (unity power factor operation) and that the phasor diagram could therefore be a right-angle triangle (which it cannot be the case with voltages of 400 kV and 390 kV and a drop across the line of IX=30 kV. This gave a real power transfer of 400 MW and a load angle of δ=4.3°. This was given half marks.

One candidate correctly observed that $|I| = \left|\frac{V_S\angle 0° - V_R\angle\delta}{jX_{SR}}\right|$ and solved this iteratively for a value of δ that gave 1,000 A.

Other candidates noted that $\Delta V = |V_S| - |V_R| = 10kV$ and that the voltage drop across X was 30 kV. From this one can estimate that power factor angle (ϕ_S=$\angle V_S$-$\angle I_{SR}$) is about 19° and from there estimate the power transfer and hence δ.]

iii) Calculate the reactive power flows at the sending and receiving ends under the conditions in (ii) and compare with the reactive power of the reactance of the line itself. [4]

$$Q_S = \frac{{V_S}^2}{X} - \frac{V_R V_S}{X} cos(\delta)$$

$$Q_S = \frac{(400 \times 10^3)^2}{30} - \frac{400 \times 10^3 \times 390 \times 10^3}{30} cos(4.13) = 146\ MVAr$$

$$Q_R = \frac{{V_R}^2}{X} - \frac{V_R V_S}{X} cos(\delta)$$

$$Q_R = \frac{(390 \times 10^3)^2}{30} - \frac{400 \times 10^3 \times 390 \times 10^3}{30} cos(4.13) = -116\ MVAr$$

$$Q_{Line} = I^2\ X = 1000^2 \times 30 = 30\ MVAr$$

Reactive power leaving the receiving end matches the reactive power entering the sending end minus the reactive power absorbed by the line.

[1 mark for each numerical answer plus one mark for comparison]

[This was an unfamiliar question and intended to test understanding. Very few candidates got this entirely correct. Most could find the reactive power absorbed by the line as I^2X. Some could find the sending-end reactive power in either exact form as above or in approximate form from $\Delta V = |V_S| - |V_R| \approx \frac{R\,P_S + Z\,Q_S}{|V_S|}$ with R set to zero. Alternatively, if the power factor angle had been found in (i) then it could be used here also.]