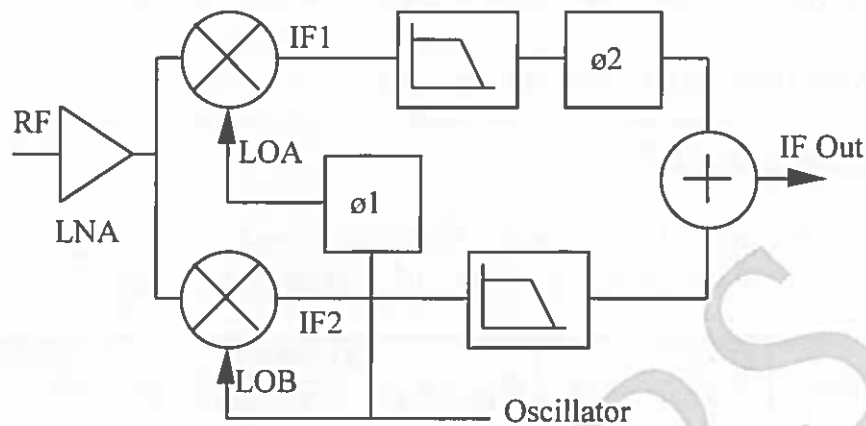


1.

(a) (Application of Theory)

**Hartley receiver architecture:**

Local oscillator :

$$\text{LOA} = 2 \cos(\omega_L t - \phi_1)$$

$$\text{LOB} = 2 \cos \omega_L t$$

RF signal :

$$A \cos(\omega_A t + \phi_A) + B \cos(\omega_B t + \phi_B)$$

where  $\omega_A = (\omega_L - \omega_{IF})$  is the wanted signal, and  $\omega_B = (\omega_L + \omega_{IF})$  is the image.

After mixing { Recall  $2\cos X \cos Y = \cos(X-Y) + \cos(X+Y)$  and  $\cos(-X) = \cos(X)$  }

$$\bullet \text{ IF1} = 2A \cos(\omega_A t + \phi_A) \cos(\omega_L t - \phi_1) + 2B \cos(\omega_B t + \phi_B) \cos(\omega_L t - \phi_1)$$

$$= A \cos((\omega_L - \omega_A)t - \phi_1 - \phi_A) + A \cos((\omega_L + \omega_A)t - \phi_1 + \phi_A) \\ + B \cos((\omega_B - \omega_L)t + \phi_B + \phi_1) + B \cos((\omega_B + \omega_L)t + \phi_B - \phi_1)$$

$$\bullet \text{ IF2} = 2A \cos(\omega_A t + \phi_A) \cos \omega_L t + 2B \cos(\omega_B t + \phi_B) \cos \omega_L t$$

$$= A \cos((\omega_L - \omega_A)t - \phi_A) + A \cos((\omega_L + \omega_A)t + \phi_A) \\ + B \cos((\omega_B - \omega_L)t + \phi_B) + B \cos((\omega_B + \omega_L)t + \phi_B)$$

Lowpass filter removes sum components:

$$\bullet \text{ IF1} = A \cos((\omega_L - \omega_A)t - \phi_A - \phi_1) + B \cos((\omega_B - \omega_L)t + \phi_B + \phi_1)$$

$$= A \cos(\omega_{IF} t - \phi_A - \phi_1) + B \cos(\omega_{IF} t + \phi_B + \phi_1)$$

$$\bullet \text{ IF2} = A \cos((\omega_L - \omega_A)t - \phi_A) + B \cos((\omega_B - \omega_L)t + \phi_B)$$

$$= A \cos(\omega_{IF}t - \phi_A) + B \cos(\omega_{IF}t + \phi_B)$$

After phase shift -  $\phi_2$ :

$$\bullet IF1 = A \cos(\omega_{IF}t - \phi_A - \phi_1 - \phi_2) + B \cos(\omega_{IF}t + \phi_B + \phi_1 - \phi_2)$$

$$\bullet IF2 = A \cos(\omega_{IF}t - \phi_A) + B \cos(\omega_{IF}t + \phi_B)$$

Adding signals IF1 and IF2 :

$$\begin{aligned} IF_{Out} &= A \cos(\omega_{IF}t - \phi_A - \phi_1 - \phi_2) + A \cos(\omega_{IF}t - \phi_A) \\ &\quad + B \cos(\omega_{IF}t + \phi_B + \phi_1 - \phi_2) + B \cos(\omega_{IF}t + \phi_B) \\ &= 2A \cos\left(\frac{\phi_1 + \phi_2}{2}\right) \cos\left(\omega_{IF}t - \phi_A - \frac{\phi_1 + \phi_2}{2}\right) + 2B \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \cos\left(\omega_{IF}t - \phi_B + \frac{\phi_1 - \phi_2}{2}\right) \end{aligned}$$

To avoid signal distortion, we require  $\cos\frac{\phi_1 + \phi_2}{2} = 1$  i.e.  $\frac{\phi_1 + \phi_2}{2} = 2n\pi$

To ensure image rejection, we require  $\cos\frac{\phi_1 - \phi_2}{2} = 0$  i.e.  $\frac{\phi_1 - \phi_2}{2} = \frac{(2n+1)\pi}{2}$

Hence any value of  $\phi_2$  that satisfies those conditions with the given  $\phi_1$  would be a valid answer.

(b) (Theory)

The local oscillator is not an ideal circuit. Hence, although a single tone would be desired, in practice the output is also going to contain harmonics. The role of the filter would be to attenuate those harmonics, to improve the spurious response rejection.

The type of filter would be bandpass.

(c) (Application of theory)

[8.6MHz, 9.1MHz]

(d) (Theory)

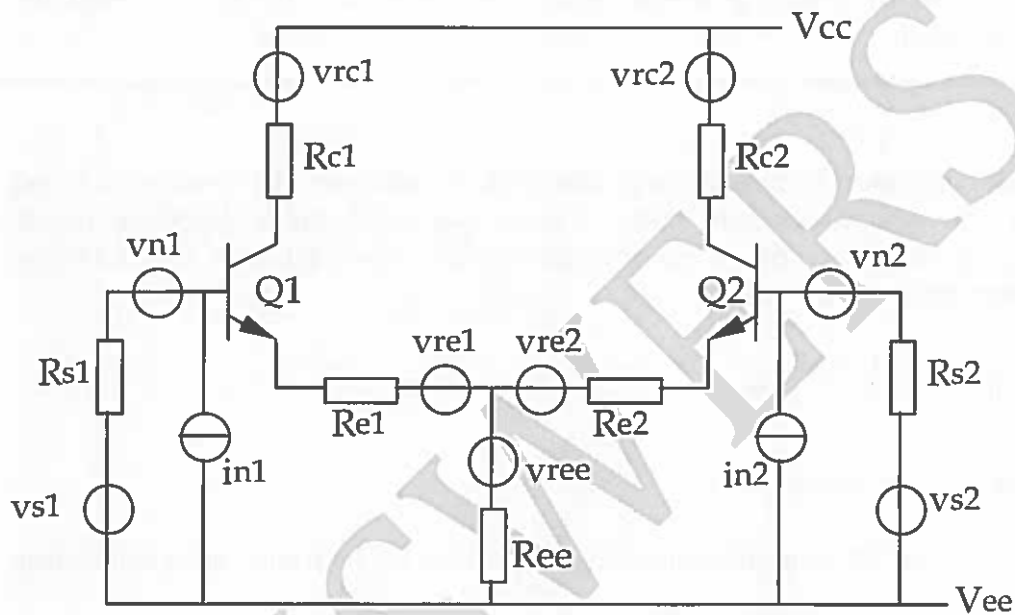
Having a high gain is beneficial for noise reasons, since the equivalent input noise of subsequent blocks will get attenuated by the gain of the first one. However, having a higher gain will also impose more severe linearity constraints on the performance of the subsequent blocks.

2.

(a) (Theory)

If distortion is a problem, differential configurations (ideally) get rid of even order harmonics.

(b) (Application of theory)



Note that this answer considers resistive input sources, but an answer would also be valid if  $R_{s1}$  and  $R_{s2}$  are considered 0.

All the resistors contribute with thermal noise. The noise in the bipolar transistors has been represented as equivalent input noise sources. These are given by:

$$v_n^2 = v_{nr}^2 + \frac{i_{nc}^2}{g_m^2} = 4kT\Delta f (r_b + r_e/2) V^2$$

$$i_n^2 \approx 2qI_b\Delta f (1 + f_l/f) = \frac{2kT\Delta f}{\beta r_e} (1 + f_l/f) A^2 \text{ (for narrow bandwidth)}$$

and represent:

(i) *Thermal noise*

$r_b$  is the ohmic resistance of the lightly-doped base region. This is a true resistance, and thus generates thermal noise:

$$v_{nr}^2 = 4kT r_b \Delta f \quad V^2 \quad (S_{vnr}(f) = 4kT r_b \Delta f / \text{Hz})$$

(ii) Shot noise

The base and collector currents generate shot noise:

$$i_{nb}^2 = 2qI_b \Delta f \quad A^2 \quad i_{nc}^2 = 2qI_c \Delta f \quad A^2 \quad (S_{inx}(f) = 2qI_x \Delta f / \text{Hz})$$

(ii) Flicker noise

Flicker (1/f) noise is mainly generated in the input circuit, and thus flows through the base resistance  $r_b$ :

$$i_{nf}^2 = \frac{k I_b^a \Delta f}{f^b} \quad A^2$$

This noise component is measured experimentally to determine the constants  $a$ ,  $b$  and  $k$ .  $a$  and  $b$  are typically close to unity. A more convenient way of describing flicker noise is in terms of a corner or 'knee' frequency  $f_i$ , at which 1/f noise is equal to the base current shot noise:

$$i_{nf}^2 = \frac{2q f_i I_b \Delta f}{f} \quad A^2$$

(c) (Application of theory)

The ones as in the following equation, which represents the total equivalent input noise:

$$v_{eq}^2 \cong v_{s1}^2 + v_{s2}^2 + v_{n1}^2 + v_{n2}^2 + i_{n1}^2 R_{s1}^2 + i_{n2}^2 R_{s2}^2 + v_{re1}^2 + v_{re2}^2$$

All the others are attenuated by the gain.

(d) (Application of theory)

The main advantage would be a reduction in noise. The disadvantage would be the loss in linearity.

(e) (Application of theory)

The equivalent input noise of the first block integrated in the 10kHz to 20kHz bandwidth would be 10μV.

The equivalent input noise of the second block is 200μV.

Hence the total equivalent input noise would be: 200.25μV. This would also be the minimum input signal the system could process.

(f) (Application of theory)

Swap the gains

3.

(a) (Computed example)

$$I_{d1} - I_{d2} = \beta V_d (V_{c1} - V_{th}) \quad I_{d3} - I_{d4} = \beta V_d (V_{c2} - V_{th})$$

$$\text{where } V_{c1} = \frac{V_{gs1} + V_{gs2}}{2} \text{ and } V_{c2} = \frac{V_{gs3} + V_{gs4}}{2}$$

$$\text{Thus } I_{out} = (I_{d1} - I_{d2}) - (I_{d3} - I_{d4})$$

$$= \beta V_d (V_{c1} - V_{c2}) = \beta V_d (V_b - V_{ss})$$

And the transconductance is  $\beta (V_b - V_{ss})$ . Hence substituting the terms  $V_b = 1.5V$ .

(b) (Application of Theory)

Transistors M3 and M4 have entered weak inversion. The large signal equations that result on non-linearity cancellation do not apply anymore.

(c) (Application of Theory)

Yes. Using  $(V_b - V_{ss})$  as the other differential input.

(d) (Application of Theory)

Larger. In a differential pair the equivalent input noise of only two transistors add together. In this circuit the equivalent input noise is the sum of the input noises of four transistors instead.

(e) (Theory)

Larger. This circuit is linear for large signals, with cancellation of non-linearities achieved mathematically. The simple differential pair is only linear for small signals.

4.

(a)(Theory)

It's a Miller integrator. The transfer function is  $\frac{V_{out}}{V_{in}} = \frac{G_m}{sC} = \frac{\omega_u}{s}$

(b) (Theory)

They are all filters.

$$\text{Low Pass: } H(s) = \frac{K\omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

$$\text{High Pass: } H(s) = \frac{Ks^2}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

$$\text{Band Pass: } H(s) = \frac{K(\omega_o/Q)s}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

$$\text{Band Reject: } H(s) = \frac{K(s^2 + \omega_z^2)}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

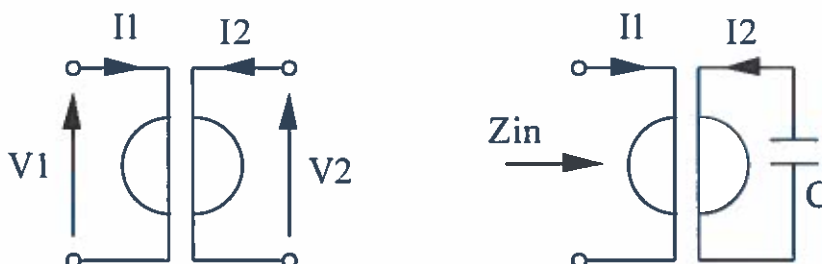
(c) (Theory)

The first circuit will suffer from the effect of bottom plate parasitic capacitance. The second circuit is therefore preferable, although at the expense of an increased area (higher capacitance). Even in the second circuit, some parasitics may remain in parallel with C (due to output devices of the transconductor etc), thus C must be kept fairly high to swamp out the parasitics.

An alternative to this is to split the capacitor in the first circuit in two, but swapping the terminals for the top and bottom plates in one of them with respect to the other.

(d) (Application of Theory)

This circuit is a gyrator. Hence it can be redrawn like this:



The analysis of this two port circuit, leading to the behavior of the inductor is as follows:

$$I_1 = G V_2 \quad I_2 = -G V_1$$

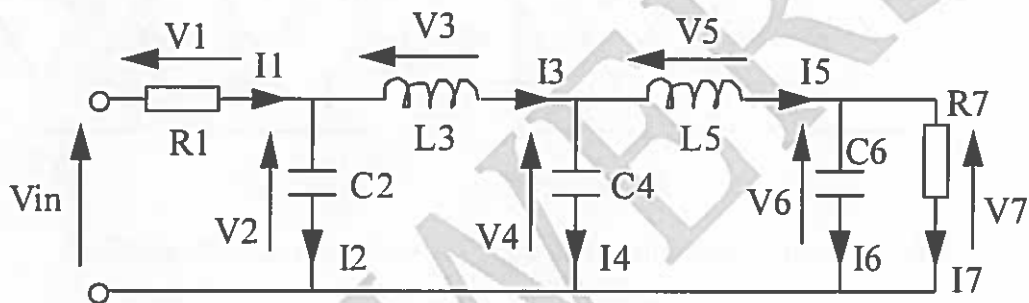
( $G$  = 'gyration conductance')

To simulate an inductor, connect a capacitor across port 2:

$$V_1 = -\frac{I_2}{G} = \frac{C}{G} \frac{dV_2}{dt} = \left( \frac{C}{G^2} \right) \frac{dI_1}{dt} = L \frac{dI_1}{dt}$$

Inductor value is tuneable by varying  $G$ .

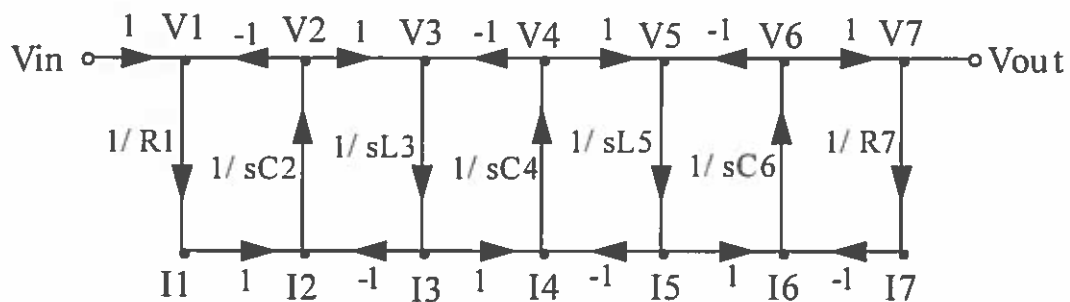
(e) (Computed example)



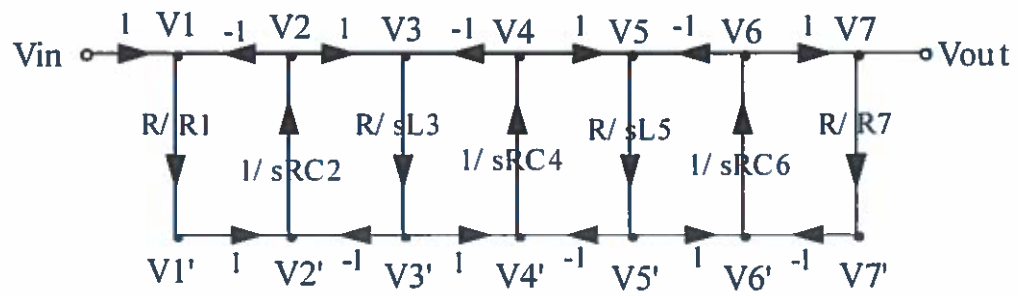
Ladder state equations:

$$\begin{aligned} V_1 &= V_{in} - V_2 & I_2 &= I_1 - I_3 & V_3 &= V_2 - V_4 \\ I_1 &= V_1/R_1 & V_2 &= I_2/sC_2 & I_3 &= V_3/sL_3 \\ I_4 &= I_3 - I_5 & V_5 &= V_4 - V_6 & I_6 &= I_5 - I_7 \\ V_4 &= I_4/sC_4 & I_5 &= V_5/sL_5 & V_6 &= I_6/sC_6 \\ V_7 &= V_6 & I_7 &= V_7/R_7 \end{aligned}$$

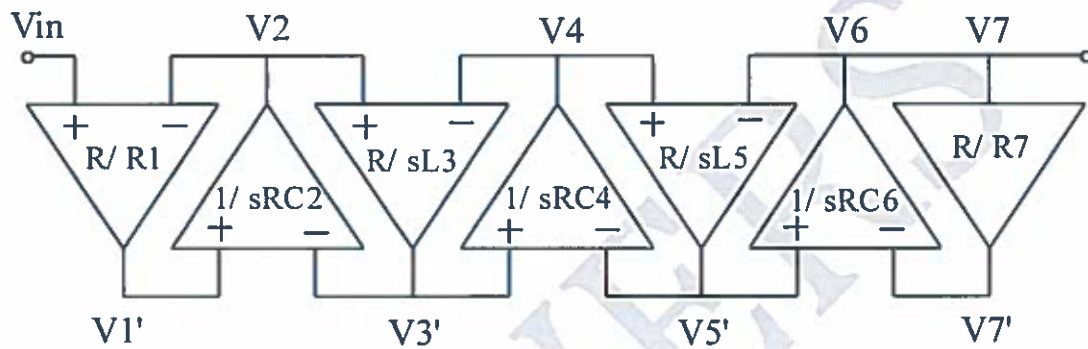
Signal flow graph:



Scaled signal flow graph:



From the flow diagram, the circuit can be implemented using summing integrators:



Where the different components have the values given in the question.