Digital Electronics 1

Question 1

1. a) (i)

$$\bar{A}\left(\overline{B+A\bar{C}}\right)$$
$$=\bar{A}\left(\bar{B}\left(\overline{A\bar{C}}\right)\right)$$
$$=\bar{A}\bar{B}\left(\bar{A}+C\right)$$
$$=\bar{A}\bar{B}+\bar{A}\bar{B}C$$
$$=\bar{A}\bar{B}$$

[4]

ii)

$$B+(C\oplus\bar{B})(AB+\bar{C})$$
$$=B+\left(CB+\bar{C}\bar{B}\right)\left(AB+\bar{C}\right)$$
$$=B+ABC+\bar{C}\bar{B}$$
$$=B+\bar{C}\bar{B}$$
$$=B+\bar{C}$$

[4]

b)

$$f=A\bar{B}+C(\bar{D}\oplus(A\oplus B))+A\bar{C}D$$
$$=A\bar{B}+C(\bar{D}(\overline{A\oplus B})+D(A\oplus B))+A\bar{C}D$$
$$=A\bar{B}+C(\bar{D}\left(AB+\bar{A}\bar{B}\right)+D\left(\bar{A}B+A\bar{B}\right))+A\bar{C}D$$
$$=A\bar{B}+ABC\bar{D}+\bar{A}\bar{B}C\bar{D}+\bar{A}BCD+A\bar{B}CD+A\bar{C}D$$

Here, 2 marks for simplifying the given equation into a form suitable for a Karnaugh map

The equation can now be simplified further using a Karnaugh map:

f

AB \ CD	00	01	11	10
00	0	0	0	1
01	0	0	1	0
11	0	1	0	1
10	1	1	1	1

$$\Rightarrow f = A\overline{B} + A\overline{C}D + AC\overline{D} + \overline{B}C\overline{D} + \overline{A}BCD$$

Here, 1 mark for drawing the Karnaugh map, 1 for filling it out correctly, 1 for the correct grouping, and 1 for the final expression.

[6]

c)

f

AB \ CD	00	01	11	10
00	1	1	1	0
01	0	0	1	1
11	0	0	1	1
10	0	0	1	0

$$\Rightarrow f = \left(\overline{A} + C\right)\left(\overline{B} + C\right)\left(B + \overline{C} + D\right)$$

Here, 1 mark for drawing the Karnaugh map, 1 for filling it out correctly, 1 for the correct grouping, and 1 for the final expression.

[4]

d)

Decimal	Hexadecimal	Signed binary	Octal
-2049	F7FF		
		0000 0111 1101 0110	3726
4011	FAB		
-14440		1100 0111 1001 1000	

Give 2 marks per answer.

[8]

e) With two N bit numbers multiplied together, the result has the range:

$$-(2^{2N-2}-2^{N-1}) \text{ to } 2^{2N-2}$$

[2]

This needs 2N bits. Note that it nearly fits into 2N-1 bits, whose range extends to $2^{2N-2}-1$.

[2]

f)

Current state	Input	Next state	Output (PQ)	
S0	0	S0	0	1
S0	1	S1	1	0
S1	0	S2	1	1
S1	1	S1	1	0
S2	0	S0	0	1
S2	1	S1	1	0
S3	X	S0	0	1

Give 1 mark for correct current state, 1 for correct inputs, 1 for correct next state and 1 for correct outputs.

[4]

g)

A	B	C	S1 = D0	S0 = A	Q
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	0	0	1
0	1	1	1	0	1
1	0	0	0	1	0
1	0	1	1	1	1
1	1	0	0	1	0
1	1	1	1	1	1

Give 2 marks for understanding ROM/multiplexer operation, and 2 each for outputs D0 and Q.

[6]

Question 2

2. (a) (i) The required encoder has 12 input lines I[11:0], where only one of these can be zero (active) at a time. There are four output lines D[3:0], giving the binary equivalent of the hexadecimal values 0 – B.

2 marks for understanding the above.

The truth table is then as follows:

Number	I11	I10	I9	I8	I7	I6	I5	I4	I3	I2	I1	I0	D3	D2	D1	D0
0	0												0	0	0	0
1		0											0	0	0	1
2			0										0	0	1	0
3				0									0	0	1	1
4					0								0	1	0	0
5						0							0	1	0	1
6							0						0	1	1	0
7								0					0	1	1	1
8									0				1	0	0	0
9										0			1	0	0	1
10 (A)											0		1	0	1	0
11 (B)												0	1	0	1	1

For clarity, the deactivated input values in the above table are left as empty cells. These are occupied by 1s (deactivated).

2 marks for the inputs and 2 for the outputs.

[6]

(ii) The Boolean expressions for D[3:0] can be seen by inspection:

$$D0 = \left(\overline{I1}\right)\left(\overline{I3}\right)\left(\overline{I5}\right)\left(\overline{I7}\right)\left(\overline{I9}\right)\left(\overline{I11}\right)$$

$$D1 = \left(\overline{I2}\right)\left(\overline{I3}\right)\left(\overline{I6}\right)\left(\overline{I7}\right)\left(\overline{I10}\right)\left(\overline{I11}\right)$$

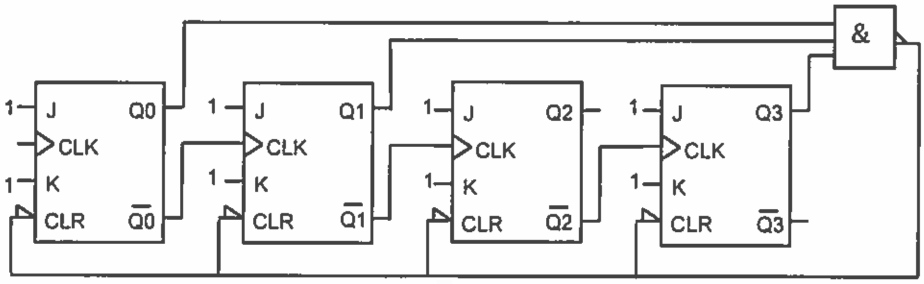
$$D2 = \left(\overline{I4}\right)\left(\overline{I5}\right)\left(\overline{I6}\right)\left(\overline{I7}\right)$$

$$D3 = \left(\overline{I8}\right)\left(\overline{I9}\right)\left(\overline{I10}\right)\left(\overline{I11}\right)$$

2 marks for each expression.

[8]

(b)



4 marks for correct use of the J-K flip-flops and 2 for the asynchronous reset.

[6]

c) (i) The given Boolean function is:

$$f = \bar{A}\bar{B} + AB + \bar{A}C$$

This corresponds to the truth table:

A	B	C	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

[1]

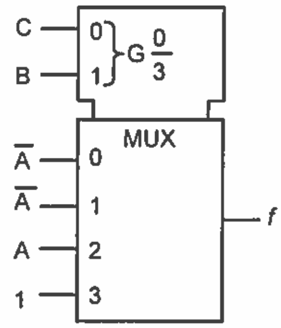
As BC must be used on the select lines of the 4 × 1 MUX, we draw a k-map table, with BC arranged along the rows and A along the columns. Furthermore, we choose to connect C = S0 and B = S1.

$S1$ $S0$

A \ BC	00	01	11	10
0	1	1	1	0
1	0	0	1	1
	$I0 = \bar{A}$	$I1 = \bar{A}$	$I3 = 1$	$I2 = A$

[2]

This gives the following implementation:



Note: The alternative solution has B = S0 and C = S1. We would then have I0 = A(bar), I1 = A, I3 = 1, and I2 = A(bar).

[1]

(ii) Using two 2 × 1 MUXs, we first need to rearrange the given function in the form of the Boolean function for the output Z for a 2 × 1 MUX, with select input S:

$$Z = \overline{S}I0 + SI1$$

For the given function, we then have:

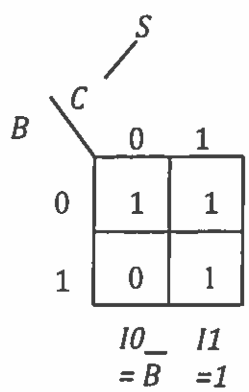
$$\begin{aligned} f &= \overline{A}\overline{B} + AB + \overline{A}C \\ &= \overline{A}\left(\overline{B} + C\right) + AB \end{aligned}$$

[2]

Comparison with the expression for Z implies that:

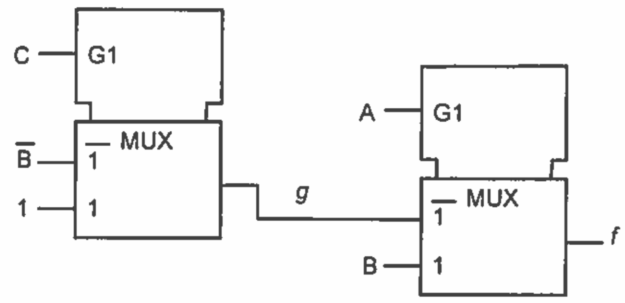
$$I0 = \left(\overline{B} + C\right) = g$$
$$I1 = B$$
$$S = A$$

We then need to implement $\overline{B} + C$ using a second 2 × 1 MUX. Here, either B or C could be connected to the select line. Choosing C for the select line (the alternative of B on the select line is also fine) and inspecting the following map:



[2]

This allows implementation of the second 2 × 1 MUX, with I0 = B(bar) and I1 = 1. We then have the following final implementation:

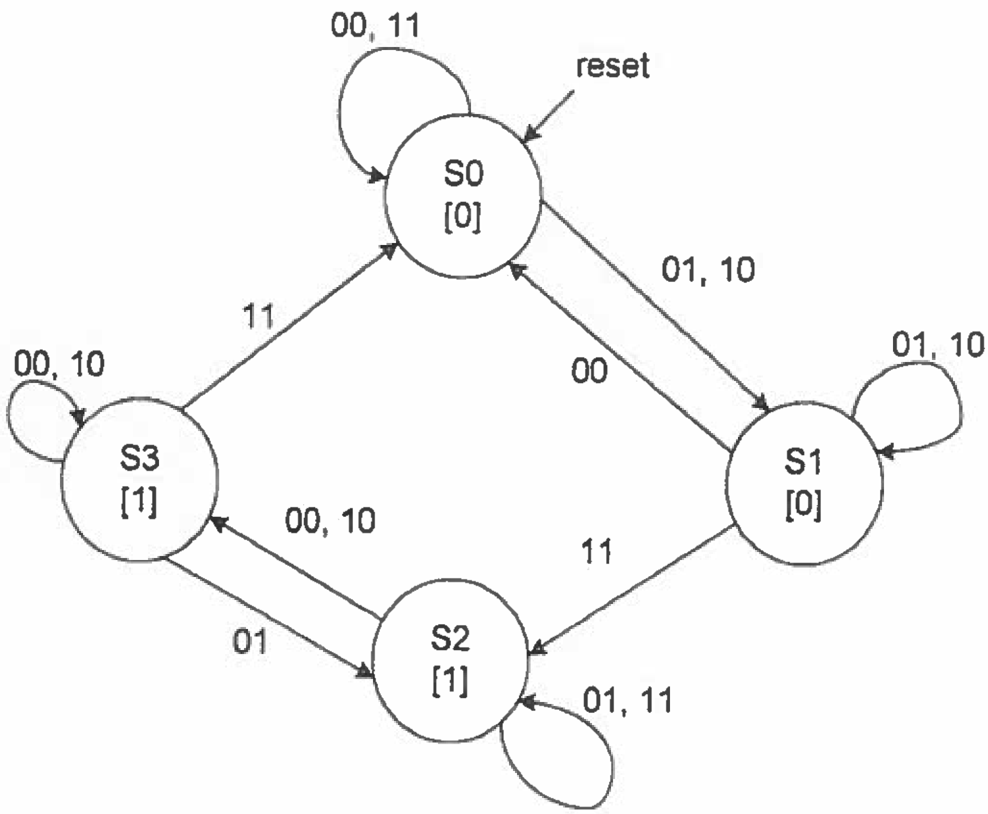


[2]

Question 3

3. (a) Using the states S0, S1, S2 and S3 to define the FSM, the following state diagram can be obtained. Note that as there are <u>two inputs</u> X1, X2, each state has $2^2 = 4$ transitions possible. Furthermore, a sequence of two input combinations, one after the other, is needed to change the output. For example, for the sequence X1X2 = 01, 11, the first part, X1X2 = 01 causes a transition from S0 to S1, and the second part, X1X2 = 11, causes a transition from S1 to S2, and Z = 0 to Z = 1.

Give 2 marks for identifying this reasoning.



Give 2 marks for showing four states, 4 for the correct interconnections, and 4 for the correct input/output values.

[12]

(b) Using the encoding system given, the state diagram can be converted into the following table, with Q3Q2Q1Q0 as the present state of the FSM, and $Q3^+Q2^+Q1^+Q0^+$ as the next state:

Q3	Q2	Q1	Q0	X1	X2	$Q3^+$	$Q2^+$	$Q1^+$	$Q0^+$	Z
0	0	0	1	0	0	0	0	0	1	0
0	0	0	1	0	1	0	0	1	0	0
0	0	0	1	1	0	0	0	1	0	0
0	0	0	1	1	1	0	0	0	1	0
0	0	1	0	0	0	0	0	0	1	0
0	0	1	0	0	1	0	0	1	0	0
0	0	1	0	1	0	0	0	1	0	0
0	0	1	0	1	1	0	1	0	0	0
0	1	0	0	0	0	1	0	0	0	1
0	1	0	0	0	1	0	1	0	0	1
0	1	0	0	1	0	1	0	0	0	1
0	1	0	0	1	1	0	1	0	0	1
1	0	0	0	0	0	1	0	0	0	1
1	0	0	0	0	1	0	1	0	0	1
1	0	0	0	1	0	1	0	0	0	1
1	0	0	0	1	1	0	0	0	1	1

Give 0.5 marks for each row.

[8]

(c) As the FSM states are encoded using 1-hot encoding, the Boolean equations can be written directly by inspection:

$$Q0^+ = Q0\overline{X1}\,\overline{X2} + Q0X1X2 + Q1\overline{X1}\,\overline{X2} + Q3X1X2$$

$$Q1^+ = Q0X1\overline{X2} + Q0\overline{X1}X2 + Q1\overline{X1}X2 + Q1X1\overline{X2}$$

$$Q2^+ = Q1X1X2 + Q2\overline{X1}X2 + Q2X1X2 + Q3\overline{X1}X2$$
$$= Q1X1X2 + Q2X2 + Q3\overline{X1}X2$$

$$Q3^+ = Q2\overline{X1}\,\overline{X2} + Q2X1\overline{X2} + Q3\overline{X1}\,\overline{X2} + Q3X1\overline{X2}$$
$$= Q2\overline{X2} + Q3\overline{X2}$$

$$Z = Q2 + Q3$$

Give 2 marks for each Boolean expression.

[10]