Problem Solving Test
Thursday, May 3rd, 2012 : 10.00-11.30

Instructions:- *Attempt question 1 and one other (one of questions 2, 3 or 4).*

Write the answer to each question in a separate answer book.

Each question is worth 50 % of the total marks available, if it is ***answered legibly, with clear reasoning, and correctly.*** *In the right-hand margin of each question you will find an indicative marking scheme.*

NO ELECTRONIC CALCULATORS ARE TO BE USED.
Please ensure the test paper supplied to you includes all 4 questions, and also the list of common physical and astronomical constants (4 sides of paper, altogether).

Question 1. (Compulsory)

A particle with charge q moving with velocity v in a magnetic field B experiences a Lorentz force whose magnitude is given by $|F| = qvB$.

a) Determine the dimensions of the magnetic field, B. [3 Marks]

b) The Lorentz force can cause a charged particle to move in a circle. Use a dimensional method to derive an equation for the radius of the circular motion. [4 Marks]

c) In fusion reactors, magnetic fields are often used to trap protons. If such a "trap" has a dimension of 20cm, estimate the B field magnitude required to trap mildly relativistic protons. (You can assume for protons that $m = \gamma m_o$, where m is the mass of the proton, m_o is the rest mass of the proton, $\gamma = \frac{1}{\sqrt{1-\mathrm{v}^2/_{\mathrm{c}^2}}}$ and you can also assume that $\gamma = 2$).

[3 Marks]

d) In the Galaxy, magnetic fields are thought to trap very high energy protons with gamma factors of up to 3×10^{11}. If our Galaxy is ~ 100kpc in diameter, work out how large the magnetic field in the Galaxy is. (Note 1pc = distance light travels in 3 years). [3 Marks]

e) Estimate how much magnetic field energy is contained in our Galaxy, based on your answer to part d). [4 Marks]

f) The magnetic field in the Galaxy is ultimately due to currents of charged particles. These particles will have been emitted from stars. An estimate suggests there are ~10^{12} stars in the Galaxy. Does this make sense in terms of the required energy going into particles to create the magnetic field over the lifetime of the Universe?

[3 Marks]

[Total 20 Marks]

Question 2.

In this question we will consider issues related to power consumption and cooling at the International Space Station (ISS) which orbits the Earth in a low orbit.

a) The ISS has a generating capacity of around 100kW from solar panels. Assuming a 15% efficiency in converting solar radiation into electricity, calculate the size of the solar panels on the ISS. [5 Marks]

b) As all electricity produced on the ISS eventually goes into heat, there is a need to be able to cool the ISS down. This is achieved by radiators mounted as large "wings" on the ISS. With a total of 150m^2 of radiators attached to cool it down, at what temperature do they need to be kept to radiate at 100kW. (You may assume the radiators act as black bodies).

[5 Marks]

c) The ISS is in an orbit that is 330km above the ground. Assuming an orbit around the equator, and ignoring the tilt of the Earth's axis, estimate the fraction of time the ISS will be in sunlight taking into account the height of the orbit. You can ignore the angular extent of the Sun.

[5 Marks]

d) The ISS has a cross-sectional area of about 1000m^2 and the atmospheric density at 330km is about 10^{-11} kgm^{-3}. Estimate the energy loss in the orbit from atmospheric drag and compare it to the on-board power production.

[5 Marks]

[Total 20 Marks]

Question 3.

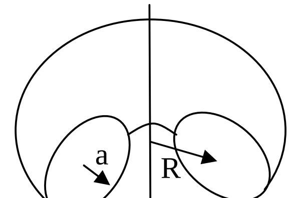
a) Reaction rates for fusion reactions in plasmas are often defined in terms of $\langle\sigma v\rangle$ the average product of the reaction cross section σ (in m^2) and the particle velocity v. Using dimensional analysis write down an expression for the power per unit volume produced by a fusion plasma comprising deuterium and tritium ions in terms of; the number density of deuterium ions n_D, the number density of tritium ions n_T, the energy produced by each deuterium-tritium fusion reaction E_{DT} and $\langle\sigma v\rangle$.

[4 Marks]

b) The fusion of deuterium and tritium produces an alpha particle and a neutron. Using the following data calculate how much energy is released in each reaction. Mass of deuterium ion = 2.0136 m_p , mass of tritium ion = 3.0155 m_p , mass of alpha particle = 4.0015 m_p , mass of neutron = 1.0087 m_p , where m_p is the mass of a proton.

[3 Marks]

c) A Tokamak is a toroidal or "doughnut" shaped plasma with dimensions described by the diagram. What is the fusion power generated by a Tokamak with $n_D = n_T = 10^{20}$ m^{-3} and $\langle\sigma v\rangle = 4\text{x}10^{-22}$ m^{3}s^{-1} where the minor radius a = 2m and the major radius R = 6m? Describe two reasons why the electrical generating output power is likely to be significantly less.



[4 Marks]

d) The thermal energy of the plasma gradually leaks out of this device over a characteristic timescale of 4 seconds. To compensate for this, the plasma can be reheated using microwaves at a frequency which resonates with the motion of electrons in the plasma. If the plasma temperature is 10,000 electron volts (100 million degrees Kelvin) estimate roughly how many kitchen microwave ovens would be required to provide sufficient heating. (Hint: the thermal energy density of the plasma is given by 3nkT). Why are kitchen microwave ovens unlikely to work in this instance?

[6 Marks]

e) The plasma can also be heated by firing beams of energetic deuterium ions into the plasma. What is the minimum velocity of deuterium ions required to raise the temperature of a plasma beyond 10,000 electron volts (100 million degrees Kelvin)?

[3 Marks]

[Total 20 Marks]

Question 4.

This question is about the existence of huge dimensionless numbers in Physics and how they might imply that the fundamental "constants" are actually not constant but varying with the age of the universe.

a) Show that the magnitude of the electrostatic force between two electrons is approximately 4×10^{42} times greater than the magnitude of the gravitational force between them. [4 Marks]

The origin of such a huge number has perplexed many physicists. Dirac suggested that it may be equal to the current age of the universe measured in some fundamental unit of time – he proposed the time light takes to travel across an atomic nucleus.

b) Make an order-of-magnitude calculation of the age of the universe in these time units and hence comment on the plausibility of Dirac's conjecture. [4 Marks]

Dirac argued that the gravitational constant G must be changing with time, varying inversely with the age of the universe. Teller (father of the H-bomb) said Dirac must be wrong because if G was decreasing with time, then relatively recently in the past, the surface temperature of the Earth would have been too hot to support life.

c) Taking the Earth's orbit to be circular of radius R_E, show that $(R_E v)^2 = GMR_E$ where v is the Earth's orbital velocity and M is the mass of the sun. Use this relationship, together with the fact the angular momentum must be conserved if G changes, to show that R_E varies like G^{-1}. You may assume M is constant.

[4 Marks]

d) The surface temperature of the Earth, T_E, can be shown to be proportional to $(L/R_E^2)^{1/4}$ where L is the luminosity of the sun which is known to vary as G^7. Use the final result from (c) and Dirac's proposed scaling of G with the age of the universe t_u, to determine how T_E might vary with t_u (no prefactors are required).

[3 Marks]

e) Using your result from (d) make a numerical estimate to test Teller's counter-argument. You may assume that life first appeared on Earth 3 billion years ago in the oceans.

[5 Marks]

[Total 20 Marks]

Common physical and astronomical constants

Speed of light in a vacuum, c: 3×10^{8} m s^{-1}

Planck's constant, h: 6.6×10^{-34} J s

Universal gravitational constant, G: 6.7×10^{-11} N m^2 kg^{-2}

Permittivity of the vacuum, ε_o: 8.8×10^{-12} F m^{-1}

Permeability of the vacuum, μ_o: $4\pi \times 10^{-7}$ H m^{-1} (or N A^{-2})

Electron rest mass, m_e: 9.1×10^{-31} kg

Proton rest mass, m_p: 1.7×10^{-27} kg

Electron charge, e: 1.6×10^{-19} C

Boltzmann's constant, k_B: 1.4×10^{-23} J K^{-1}

Stefan's constant, σ: 5.7×10^{-8} J m^{-2} s^{-1} K^{-4}

Avogadro's number, N_A: 6.0×10^{23} mol^{-1}

Molar gas constant, R: 8.3 J K^{-1} mol^{-1}

Volume occupied by 1 mole of gas at STP: 22.4 litres

Mass density of water: 1 kg $litre^{-1}$ (Note: 1 litre = 0.001 m^3)

Mass of the Sun, M_{solar}: 2×10^{30} kg

Radius of the Sun, R_{solar}: 7×10^{8} m

Luminosity of the Sun, L_{solar}: 3.8×10^{26} W

Mass of the Earth, M_E: 6×10^{24} kg and Radius of the Earth, R_E: 6400 km

Mean radius of Earth's orbit around Sun: 1.5×10^{11} m

Acceleration due to gravity on Earth's surface, g: 9.8 m s^{-2}

Length of the tropical year: 3.2×10^{7} s.

Age of the Universe, 1.37×10^{10} years