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Model Answers and Mark Schemes First Examiner:

Paper Code: E4.05, S07 Second Examiner: E405 1/10

Traffic Theory + Queuing Systems SOLUTIONS

Question Number etc. in left margin | Mark allocation in right margin

Q1 (a) Erlang Model Assumptions (bookwork)

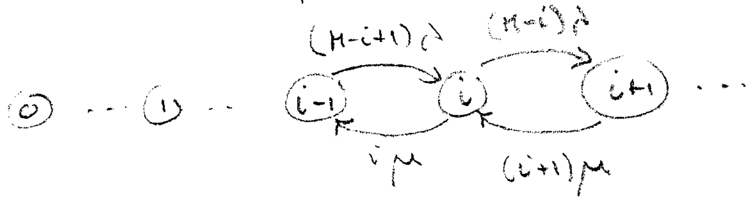
(a) – Poisson arrival stream: good approx when large number of independent traffic sources

– Exponential holding times: good approx for telephone calls

– Full-availability access: usually valid

– No re-submissions: not strictly valid but model can be modified to allow for this 5

(b)



Global

$$[(M-i)\lambda + i\mu]\pi_i = (M-i+1)\lambda\pi_{i-1} + (i+1)\mu\pi_{i+1}$$

Local

$$(M-i+1)\lambda\pi_{i-1} = i\mu\pi_i$$

M = no. sources

π_i = Eq. state prob. in state i

λ = calling rate/free source

μ = release rate/channel 5

(c) $\rho = 8$, $N = 12$

$B_c = E_{12}(8) = 0.05$ (from chart)

(i) mean carried traffic

$\rho_c = (1-B_c)\rho_o = 7.6$ Erlangs

(iii) Mean channel occupancy

$\eta = \frac{\rho_c}{N} = 0.633$

(iv) Link idle time $= \pi_0$

$= 0.00036$ (approx $e^{-\rho_o}$) 10

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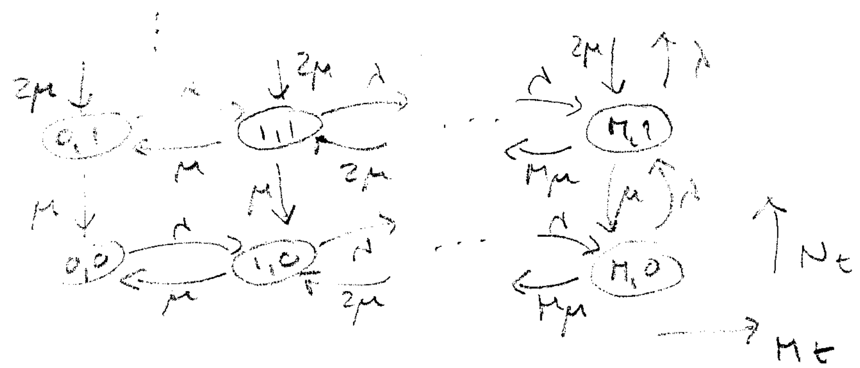
Q2 (a) Overflow model (Beck work)

(a) Let $\begin{cases} M_t = \text{No of busy channels on link 1} \\ N_t = \text{No of busy channels on link 2} \end{cases}$

Assume $\begin{cases} \text{(i) Poisson arrival stream to link 1, rate } \lambda \\ \text{(ii) Exponential holding times, rate } \mu \end{cases}$



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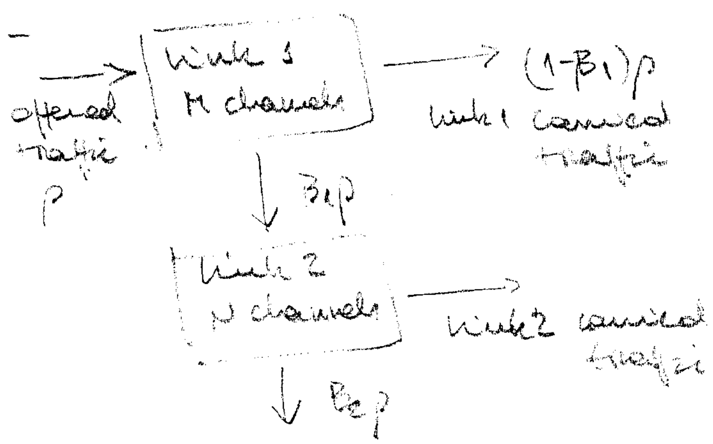
- local balance equations do not hold. The problem should be solved using global balance equations. 3



link 1 call congestion = B_1

link 2 call congestion = $\dfrac{\text{Arrival rate to link 2 for blocked arrivals}}{\text{Total arrival rate to link 2}}$

$$= \frac{dB_2}{dB_1} = \frac{B_2}{B_1} = \frac{E_{M+N}(\rho)}{E_M(\rho)}$$ 4

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Q2 (b)

$K = 10$, $\frac{\lambda}{\mu} = 8$ Erlangs $\rho = \frac{\lambda}{K\mu} = 0.8$ Erlangs /channel

$$P[\text{Delay}] = \frac{E_K(K\rho)}{1-\rho[1-E_K(K\rho)]} = 0.41$$ 5

If buffer size is Q then:

$$P[\text{loss}] = \left[\frac{(1-\rho)\rho^Q E_K(K\rho)}{(1-\rho)+\rho(1-\rho^Q)E_K(K\rho)}\right]$$

$\rho = 0.8$ Erlangs

$E_K(K\rho) = 0.122$ (from chart)

$P[\text{loss}] = 0.030$ for $Q = 5$ 5

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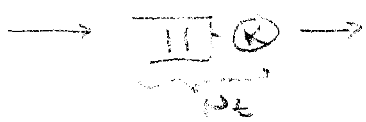
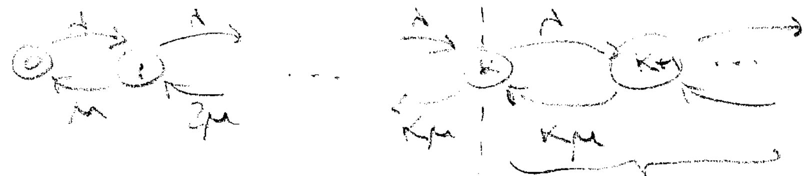
Q3 (a)



(bookwork/new theoretical appl)

$\{N_t\}$ is a B/D process with state diagram



all channels busy

Local balance Equations

$$\pi_i = \left(\frac{\lambda}{i\mu}\right)\pi_{i-1} = \left(\frac{A}{i}\right)\pi_{i-1} \qquad i \le K$$

$$\pi_i = \left(\frac{\lambda}{K\mu}\right)\pi_{i-1} = \rho\,\pi_{i-1} \qquad i \ge K$$

$$\therefore \pi_i = \left(\frac{A^i}{i!}\right)\pi_0 \qquad i \le K$$

$$\left(\frac{A^K}{K!}\right)\rho^{i-K}\pi_0 \qquad i \ge K$$

$\therefore$ for $i \ge K$

$$\pi_{K+j} = \left(\frac{A^K}{K!}\right)\rho^j\,\pi_0 = \rho^j\,\pi_K$$

$$\pi_0 = \frac{1}{\frac{A^K}{K!}}\left[\frac{(1-\rho)E_K(A)}{(1-\rho)+\rho E_K(A)}\right]$$

$$P[\text{Delay}] = P[W>0] = P[N_t \ge K] = \sum_{i=K}^{\infty}\pi_i$$

$$= \sum_{j=0}^{\infty}\pi_K\rho^j = \frac{\pi_K}{1-\rho} = \frac{\left(\frac{A^K}{K!}\right)\pi_0}{1-\rho}$$

$$P[\text{Delay}] = \frac{E_K(A)}{(1-\rho)+\rho E_K(A)} = D_K(A)$$

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Q3 (a) (i)

$P[Q_t = i \mid \text{all server busy}]$

$$P[Q_t = i \mid N_t \geq K] = \left\{ \frac{P[N_t = K+i]}{\sum_{j=0}^{\infty} P[N_t = K+j]} \right\}$$

$$= \left\{ \frac{\pi_K \rho^i}{\sum_{j=0}^{\infty} \pi_K \rho^j} \right\}$$

$$P[Q_t = i \mid \text{Delay}] = (1-\rho)\rho^i \qquad i = 0, 1, 2 \ldots$$

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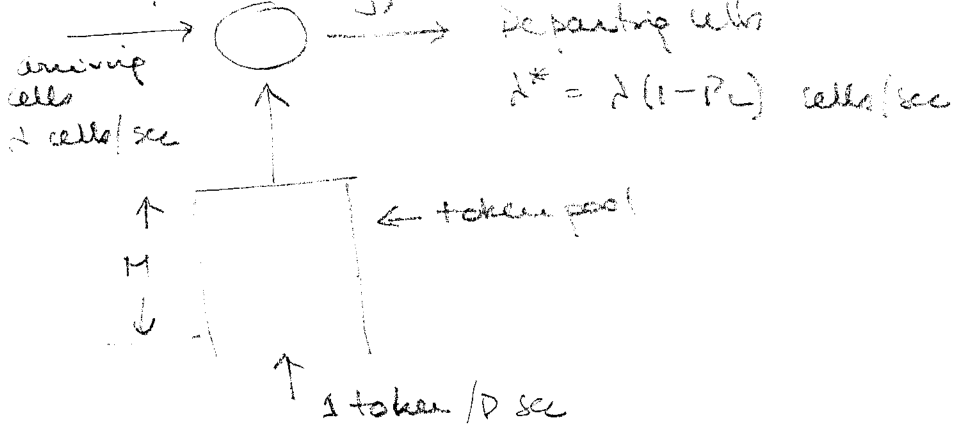
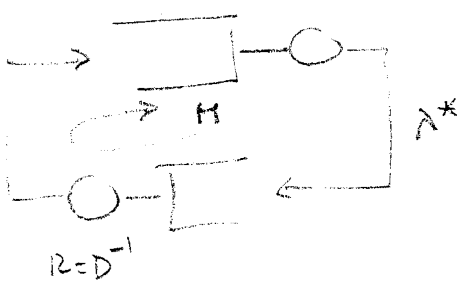
Q3 (b) (bookwork / new theory)





Closed queueing network model, leaky bucket M/M/1 approximation

$$\lambda^* = \lambda(1-P_L)$$

for maximum occupancy M

$$P_L = \frac{\rho^M(1-\rho)}{1-\rho^{M+1}}$$

with $\rho = \lambda/R = \lambda D$

$$\lambda^* = \lambda\left[\frac{1-\rho^M}{1-\rho^{M+1}}\right]$$

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Q4

(a) Admission control: How much traffic can the network handle if a prescribed QoS for each traffic class is to be maintained while the network utilisation meets some minimum goals. Eg.:

- given a particular VP how many VCs can it handle?
- given a number of active VCs does one admit a new call when a request to set one up arrives?

Discuss meaning and derivation of

$$C_L = \min[C_{LS}, C_{LF}]$$

where

$$C_{LS} = m R_p + \sigma R_p \sqrt{-\ln(2\pi) - 2\ln\varepsilon}$$

$$C_{LF} = R_p N \frac{1-k}{2} + R_p N \sqrt{\left(\frac{1-k}{2}\right)^2 + kp}$$

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Q4 (a)

M|G|1 system

mean message length $\{5/4\ B\}$

mean square message length $= \{7/4\ B^2\}$

$$E(S) = \frac{5}{4}\,\frac{160}{64}\times10^{-3}\ \text{sec} = 3.125\ \text{msec}$$

$$E(S^2) = \frac{175}{16}\ \text{msec}^2 \quad \left(\frac{7}{4}\,\frac{(160)^2}{(64)^2}\times10^{-6}\right)$$

$$\rho = \lambda E(S) = 0.9375$$

$$E(R) = \frac{1}{2}\lambda E(S^2) = 1.65\ \text{ms}$$

Priority 1

$$E(W_1) = \left[\frac{E(R)}{1-\rho_1}\right]$$

$$\rho_1 = \lambda_1 E(S_1) = 0.5625$$

$$E(S_1) = \frac{160}{64}\times10^{-3} = 2.5\times10^{-3}$$

$$\lambda_1 = \frac{3}{4}\lambda = 0.75\times300$$

$$E(W_1) = \left[\frac{E(R)}{1-\rho_1}\right] = \frac{1.65}{1-0.5625} = 3.77\ \text{ms}$$

$$E(T_1) = E(W_1) + E(S_1) = 6.27\ \text{ms}$$

$$E(W_2) = \left[\frac{E(W_1)}{1-\rho_1-\rho_2}\right] = 60.3\ \text{msec}$$

$$E(T_2) = E(W_2) + E(S_2) = 65.3\ \text{ms}$$

$$E(T) = \frac{3}{4}E(T_1) + \frac{1}{4}E(T_2) = 21.0\ \text{ms}$$

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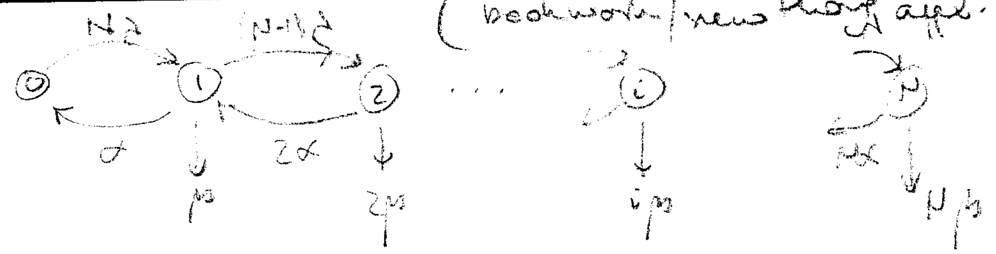
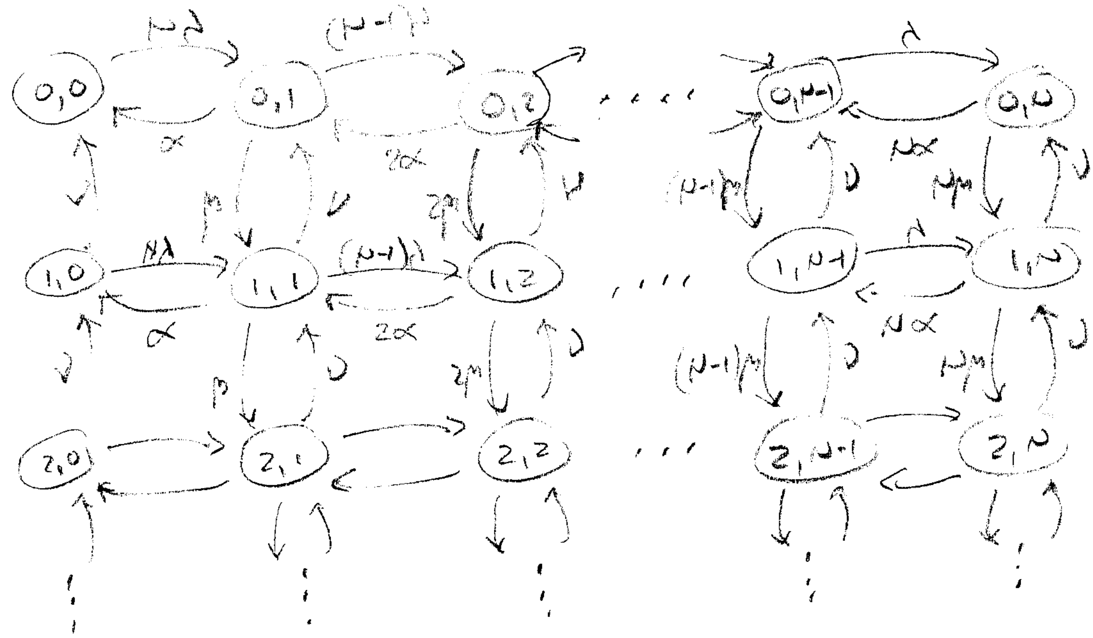
(bookwork / new theory appl.)

3(a)



(i) $\pi_i = \binom{N}{i}\left(\frac{\lambda}{\lambda+\alpha}\right)^i\left(\frac{\alpha}{\lambda+\alpha}\right)^{N-i} = \binom{N}{i}\left(\frac{\lambda}{\alpha}\right)^i\left(1+\frac{\lambda}{\alpha}\right)^{-N}$ 2

(ii)



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(iii) Average number of cells entering the system (queue)

$N\mu\left(\frac{\lambda}{\alpha+\lambda}\right)$

system utilisation

$$\frac{N\mu\left(\frac{\lambda}{\alpha+\lambda}\right)}{\nu} = \rho$$

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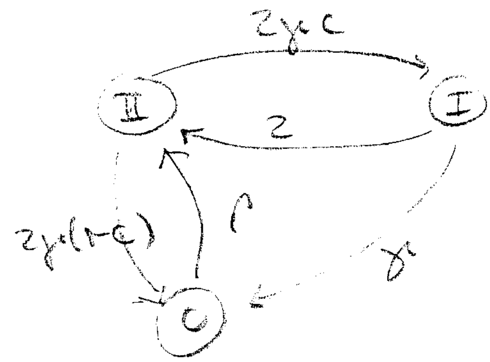
Q5 (b)



$$Q = \begin{bmatrix} -0.4 & 0.396 & 0.004 \\ 4.0 & -4.2 & 0.2 \\ 8.0 & 0 & -8.0 \end{bmatrix}$$ 6

solve

$$\begin{bmatrix} -0.4 & 4.0 & 8.0 \\ 0.396 & -4.2 & 0 \\ 0.004 & 0.2 & -8.0 \end{bmatrix} \begin{bmatrix} x_{II} \\ x_I \\ x_0 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$x_{II} + x_I + x_0 = 1$$

$x_{II} = 0.085939$ $\qquad R_{II} = 1.6$

$x_I = 0.911482$ $\qquad R_I = 1.0$

$x_0 = 0.002579$ $\qquad R_0 = 0.0$

$$\lim_{t \to \infty} E(W(t)) = \sum_{i=0}^{II} R_i x_i = 1.54431$$ 6