UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2004

BEng Honours Degree in Computing Part III
MSc in Computing Science
MSc in Computing for Industry
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C344

DECISION ANALYSIS

Wednesday 28 April 2004, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 Sarah has had two job offers: from ABC plc and from DEF inc, both of which would begin the following January. If she goes to work for ABC, she would start out as a sales associate on a commencing salary of £25K p.a. ABC has a firm rule that everyone must begin as a sales associate in the London area for some months before getting a permanent assignment.

Sarah considers that she has a 90% chance of being promoted to a better position after only a few months in the job. Unfortunately, she must sign a one-year contract with ABC before she finds out whether or not she'll get the promotion, and since ABC is badly in need of sales associates, there is still a 10% chance that they'll keep her as a London-based sales associate for the rest of the year. Despite that, Sarah suspects that there is a 30% chance of being promoted to Programmer. Were that to happen, she suspects there is a fifty-fifty chance that she will be based either in ABC's London head office or in their Paris office. The London office pays its Programmers £42K p.a. while the Paris office pays them on the following efficiency basis: if they complete the majority of their work ahead of schedule, they are paid £53K p.a., but if they do not, they are only paid £32K p.a. Sarah believes there is a 40% chance that, as a Programmer, she could complete the majority of her work ahead of schedule.

Sarah discovers that ABC is desperately in need of people to start up a new office in Riyadh. Being fluent in Arabic, she suspects that there is a 60% chance that ABC will choose to send her to Riyadh, rather than keep her in Europe. But once she gets there, she doesn't know whether she will be a Programmer on £35K p.a., or a Systems Analyst on £95K p.a. After talking to some friends who work for ABC, Sarah reckons that, if she was sent to Riyadh, there is an 85% chance that she would be made a Systems Analyst.

DEF inc. has offered Sarah £60K p.a. to work in Brussels. However, there is no chance of promotion once she gets there.

Sarah would be equally happy working in any of the cities. Her only concern is for her salary level at the end of the first year. Sarah makes decisions using expected utility, and considers a deal for £100K or nothing with equal chances to be worth about £38K, whilst the same deal for £100K or nothing, with a 75% probability of getting £100K, to be worth about £64K.

a Sketch Sarah's utility function for money on a sheet of graph paper provided.

b Solve Sarah's decision problem.

c Approximately how much would DEF inc. need to offer Sarah for her to be indifferent between the two offers?

d Suppose that Sarah were able to consult with a clairvoyant who was able to tell her (with100% reliability) the location and post she would attain by year-end, were she to take up the offer from ABC plc. What would be the approximate value of this information?

The four parts carry, respectively, 20%, 30%, 15% and 35% of the marks.

2 The Government has decided to privatise postal delivery services and the Postal Services Ombudsman has asked Peter Smart, the chief executive of FASTPOST, to submit a proposal for a business letter delivery service. Mr Smart would like to produce as large a revenue for his company as possible but knows that he needs to offer a service with a high proportion of next-day deliveries.

Thus, the two major attributes of relevance are:

X1: daily revenue (in £K)
X2: proportion of next-day deliveries (in %)

FASTPOST's planning department has come up with three alternative schemes offering the following benefits:

Scheme	X1 (in £K)	X2 (in %)
1	60	80
2	50	85
3	40	90

For all levels of X2, Mr Smart is indifferent between a service offering £50K revenue for certain and one where there is a 70% chance of £60K revenue and a 30% chance of £40K. For all levels of X1, he is indifferent between one definitely offering 85% next-day deliveries and one for which there is an 80% chance of 90% next-day deliveries and a 20% chance of 80% next-day deliveries.

a Sketch Mr Smart's single utility functions for X1 and for X2.

b Mr Smart considers that X1 and X2 are mutual utility independent. Explain what this means.

Mr Smart is indifferent between Scheme 1 and a service with a 40% chance of £60K revenue and 90% next-day deliveries but a 60% chance of £40K revenues and 80% next-day deliveries. He is also indifferent between Scheme 3 and a service with a 30% chance of £60K revenue and 90% next-day deliveries but a 70% chance of £40K revenues and 80% next-day deliveries.

c Express Mr Smart's joint utility function of X1 and X2, in terms of his single utility functions for X1 and for X2.

d Which scheme would Mr Smart find the most attractive?

e What checks of the model should Mr Smart carry out before making his proposal to the Ombudsman?

The five parts each carry 20% of the marks.

3 Mary is considering choosing one of three possible investments, all of which pay off after one year. She believes that the future financial market can be in one of three (mutually exclusive and exhaustive) states: S1 – low activity, S2 – medium activity and S3 – high activity. Payoffs are given in the following table:

		State of market		
		S1	S2	S3
Investment	A	£1,200	£600	£750
	B	£750	£750	£750
	C	£600	£750	£850

Mary's utility values for various monetary amounts are:

Amount in £	Utility
1,200	1.00
850	0.90
750	0.85
600	0.80

Let the probabilities of states S1 and S2 be denoted by p and q, respectively.

a With the help of graph paper provided, conduct two-way sensitivity analysis on the rankings of the expected utility values (EUVs) of the three investments, as p and q vary:

i) First find the lines in the p-q plane, where the EUVs of pairs of investments break-even.

ii) Then show how these lines divide up the feasible part of the p-q plane into separate regions each with its unique ranking of the EUVs of A, B and C.

b Mary estimates that the probabilities of states S1, S2 and S3 are 0.45, 0.2 and 0.35, respectively. What are her the ranking of EUV(A), EUV(B) and EUV(C)? Mark the base value of (p, q) on the sensitivity graph produced in part (ii).

c On further consideration, though, she thinks that these three subjective probabilities could lie anywhere within ± 0.1 of the base values given in b. Find the constraints now satisfied by p and q and mark the feasible region for (p, q) on your sensitivity graph. What can you conclude?

The three parts each carry 60%, 10% and 30% of the marks.

4 John runs a roadside fruit stall and buys a large crate of fruit each morning from a local grower. 90% of the crates are of satisfactory quality, contain 80% excellent fruit and lead to a £100 profit. The remaining crates are unsatisfactory, contain just 30% excellent fruit and produce a loss of £250.

Each morning, before John decides to accept the crate, he samples one piece of fruit to test whether it is of excellent quality. He then has the option of rejecting the crate without paying for it, although he cannot then have a different crate for that day and must instead spend his time at home playing computer games.

John wonders what should be his optimal policy in terms of the sample result.

a Draw an influence diagram representing John's decision problem. Include a listing of options for each decision node, a table of probability values for each chance node and a table of payoff values.

b Carefully show how Shachter's algorithm solves the influence diagram from part (a). Include revised values for probability and payoff values, where appropriate.

The two parts carry, respectively, 35%, 65% of the marks.