Imperial College London

[E1.14 (Maths 2) 2012]

B.ENG. AND M.ENG. EXAMINATIONS 2012

PART I : MATHEMATICS 2 (ELECTRICAL AND INFORMATION SYSTEMS ENGINEERING)

Date Friday 8th June 2012 10.00 - 12.00

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO

Answer Question 1 and THREE of the remaining FIVE questions.

Answer Section A and Section B in different answerbooks.

Question 1 carries twice the marks of each of the other questions.

CALCULATORS MAY **NOT** BE USED.

A mathematical formulae sheet is provided.

[*Before starting, please make sure that the paper is complete; there should be 6 pages, with a total of SIX questions. Ask the invigilator for a replacement if your copy is faulty.*]

SECTION A

1. (i) Find the angle between the vectors $\boldsymbol{a} = (1,\, 2,\, 3),\ \boldsymbol{b} = (2,\, 0,\, 4)\,.$

(ii) Find the value of the constant λ, such that

$$(y\,\cos x + \lambda\,\cos y)\,dx \;+\; (x\,\sin y + \sin x + y)\,dy$$

is the exact differential of a function $f(x,\, y)$. Find the corresponding function $f(x,\, y)$ subject to $f(0,\, 1) = 0$.

(iii) If the universal set Ω is the set of all positive integers less than or equal to 32 and

$$A \;=\; \{\, N \in \Omega \;:\; 1 \le N \le 10\,\}$$

$$B \;=\; \{\, N \in \Omega \;:\; N \text{ even and } N \le 20\,\}$$

find:

(a) $\overline{A}$, (b) $\overline{A \cup B}$, (c) $\overline{A} \cap \overline{B}$, (d) $\overline{A \cap B}$.

(iv) If $V = \ln(x^2 + y^2)$, prove that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0\,.$$

(v) For what values of k are the equations

$$\begin{array}{rcrcrcl} x & + & y & - & k & = & 0\,, \\ kx & - & 3y & + & 11 & = & 0\,, \\ 2x & + & 4y & - & 8 & = & 0\,, \end{array}$$

consistent ?

(vi) By means of a 'truth' table show that $p \to q$ and $\overline{q} \to \overline{p}$ are equivalent.

Q1 CONTINUES ON THE NEXT PAGE

(vii) Define what is meant by an even and an odd function of x.
A function of x, $f(x)$, is periodic with period 2π and satisfies

$$f(x) \;=\; \begin{cases} -1\,, & -\pi \;<\; x \;<\; 0\,, \\ \;\;1\,, & \;\;\;0 \;<\; x \;<\; \pi\,. \end{cases}$$

Sketch the function and find its Fourier series.

(viii) Find the perpendicular distance from the origin to the plane through the points

$$(1,\,1,\,1),\quad (2,\,2,\,3),\quad (1,\,3,\,4)\;.$$

(ix) Obtain the Taylor series expansion of the function $f = \sin(xy)$ about the point $(x,\,y) = (1,\,\pi/3)$ neglecting terms of degree three and higher.

(x) Find the inverse of the matrix $\quad A \;=\; \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & 2 \end{pmatrix}\;.$

SECTION B

2. Find the dimensions of a rectangular box of maximum capacity given the box has no top and has surface area 108m^2.

3. The function $f(x)$ is defined by

$$f(x) = \begin{cases} -x\,, & -\pi < x \le 0 \\ 0\,, & 0 < x < \pi \end{cases}$$

and $f(x+2\pi) = f(x)$.

(i) Sketch the function in the range $-2\pi \le x \le 2\pi$.

(ii) Show that the Fourier series of $f(x)$ is

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)x}{(2m+1)^2} + \sum_{m=1}^{\infty}\frac{(-1)^n \sin nx}{n}\,.$$

What does the Fourier series converge to at $x = \pm\pi$?

(iii) By suitable choice of x evaluate

(a) $\displaystyle\sum_{m=0}^{\infty}\frac{(-1)^m}{2m+1}$.

(b) $\displaystyle\sum_{m=0}^{\infty}\frac{1}{(2m+1)^2}$.

PLEASE TURN OVER

4. $u = x^n f(z)$ when f is any differentiable function of $z = y/x$ and n is a constant.

   (i) Show that $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y} = nu$. (1)

   (ii) Hence show that $x \dfrac{\partial^2 u}{\partial x^2} + (x+y) \dfrac{\partial^2 u}{\partial x\,\partial y} + y \dfrac{p^2 u}{\partial y^2} = (n-1) \left(\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} \right)$.

   (iii) Verify the result (ii) for $n = 2,\ f(z) = z$.

   (iv) By writing (1) in the form

   $$\left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right) u = nu$$

   or otherwise show also that

   $$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x\,\partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n^2 u \ .$$

5. Find a unit vector in the same direction as the vector $\boldsymbol{a} = 4\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$ and another unit vector in the same direction as $\boldsymbol{b} = -4\boldsymbol{i} + 3\boldsymbol{k}$.

   (i) Show that the sum of these unit vectors bisects the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

   (ii) Find a unit vector $\widehat{\boldsymbol{t}}$ which is normal to $\boldsymbol{a}$ and $\boldsymbol{b}$.

   (iii) Finally find the volume of the parallelepiped whose edges correspond to the three unit vectors defined above.

6. (i) Use Mathematical Induction to show that

$$1 + 2 + 3 + \ldots + N = \frac{1}{2} N.(N+1) ,$$

for any natural number N.

(ii) Let A, B and C be the following propositions:

A : it is raining;

B : the sun is shining;

C : there are clouds in the sky.

(a) Translate the following English sentences into logical expressions:

i If it is raining then there are clouds in the sky;

ii If there are no clouds in the sky then the sun is shining.

(b) Translate the following logical expressions into English sentences:

i $(A \cap B) \to C$;

ii $\overline{A} \to (B \cup C)$.

(iii) State the truth table for $(A \to B) \cup (B \to A)$, $(A \to B) \cap (B \to A)$ and $(A \leftrightarrow B)$.

Which two of these three logical expressions are equivalent?

END OF PAPER

EXAMINATION ~~QUESTIONS~~/SOLUTIONS 2011-2012

Course: EE1

Question 1

TOPIC

Marks & seen/unseen

Parts

a) By definition of dot product $\underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}|\cos\theta$

$= a_1 b_1 + a_2 b_2 + a_3 b_3$ | 1

$|a| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$, $|\underline{b}| = \sqrt{4+16} = \sqrt{20}$ | 1

$\therefore\ \underline{a}\cdot\underline{b} = 2+0+12 = \sqrt{14}\sqrt{20}\cos\theta$ | 1

$\theta = \cos^{-1}\sqrt{\frac{7}{10}}$. | 1

b). Exact if $\frac{\partial}{\partial y}(y\cos x + \lambda\cos y) = \frac{\partial}{\partial x}(x\sin y + \sin x + y)$ | 1

ie $\cos x - \lambda\sin y = \sin y + \cos x$ | 1

$\therefore\ \lambda = -1$

$\frac{\partial f}{\partial x} = y\cos x + \lambda\cos y \Rightarrow f = y\sin x - x\cos y + A(y)$ | 1

then $\frac{\partial f}{\partial y} = \sin x + x\sin y + A'(y) = x\sin y + \sin x + y$

$\therefore\ A = y^2/2 + \text{constant}$

$\therefore\ f = y\sin x - x\cos y + y^2/2 + \text{constant}$

$\therefore$

$f(0,1) = 0 \Rightarrow \text{constant} = -1/2$ | 1

$f = y\sin x - x\cos y + \frac{1}{2}(y^2-1)$

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	EXAMINATION ~~QUESTIONS~~/SOLUTIONS 2011-2012	Course EE I (2)
Question 1	TOPIC	Marks & seen/unseen

Parts

c a) $\{n \in N, 11 \le N \le 32\}$, b) $\{11, 13, 15, 17, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32\}$ — 1

c) $(\bar{A} \wedge \bar{B}) = \{11, 13, 15, 17, 19, \cancel{20}, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32\}$ — 1

d) $\overline{A \wedge B} = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32\}$ — 1

12, 14, 16, 18 — 1

$A \wedge B = \{2, 4, 6, 8, 10\}$

~~e) $\bar{A} \cup \bar{B} = \overline{A \wedge B}$~~

d. $\dfrac{\partial V}{\partial x} = \dfrac{2x}{x^2+y^2}$ $\quad \dfrac{\partial^2 V}{\partial x^2} = \dfrac{(x^2+y^2)2 - 2x \cdot 2x}{(x^2+y^2)^2}$

$$= \frac{2(y^2-x^2)}{(x^2+y^2)^2}$$ — 2

Similarly $\dfrac{\partial^2 V}{\partial y^2} = \dfrac{2(x^2-y^2)}{(x^2+y^2)^2}$ — 1

$$\therefore \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0.$$ — 1

e. Require $\begin{vmatrix} 1 & 1 & -k \\ k & -3 & 11 \\ 2 & 4 & -8 \end{vmatrix} = 0$ — 1

$$\therefore \begin{vmatrix} -3 & 11 \\ 4 & -8 \end{vmatrix} - \begin{vmatrix} k & 11 \\ 2 & -8 \end{vmatrix} - k \begin{vmatrix} k & -3 \\ 2 & 4 \end{vmatrix} = 0$$ — 1

$$\therefore -4k^2 + 2k + 2 = 0$$ — 1

$$k = 1, -1/2.$$ — 1

EXAMINATION QUESTIONS/SOLUTIONS 2011-2012

Course EE1(2)

Question 1 | TOPIC | Marks & seen/unseen

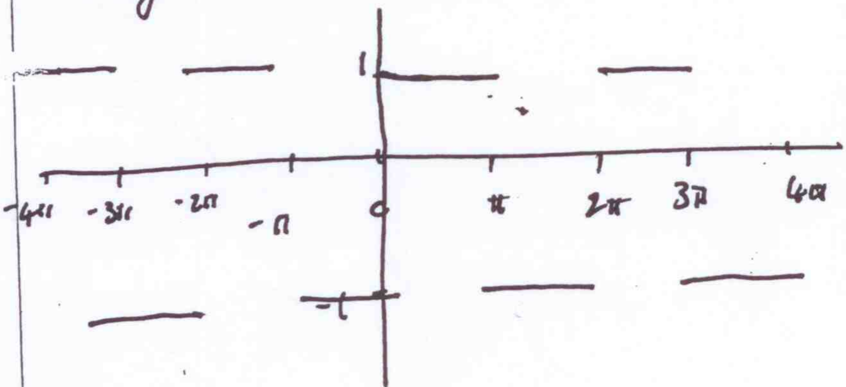
Parts

f

p	q	$\bar{p}$	$\bar{q}$	$p \to q$	$\bar{q} \to \bar{p}$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

1 mark for each of columns 3-6 — 4

g Even if $f(x) = f(-x)$, odd if $f(x) = -f(-x)$. — 1



1

$$f = \sum_1^\infty b_n \sin nx.$$

$$b_n = \frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx = \frac{2}{\pi}\left[\frac{-1}{n}\cos nx\right]_0^\pi$$

$$= \frac{2}{n\pi}\left(1-(-1)^n\right) = \frac{4}{n\pi},\ n \text{ odd}$$

$$= 0,\ n \text{ even}$$

1

$$\therefore f = \frac{4}{\pi}\sum_1^\infty \frac{\sin(2n-1)x}{2n-1}$$

1

Setter's initials — Checker's initials — Page number

Parts

ii

Let $\underline{a} = (2,2,3) - (1,1,1) = (1,1,2)$

$\underline{b} = (1,3,4) - (1,1,1) = (0,2,3)$

normal to plane is $//$ $\underline{a} \wedge \underline{b} = (-1,-3,2)$ 1

$\therefore$ Equation of plane is 1

$$(\underline{r} - (1,1,1))\cdot(-1,-3,2) = 0$$

$$\therefore \quad -x - 3y + 2z = -2$$ 1

$$\text{ie} \quad \frac{x + 3y - 2z}{\sqrt{14}} = \frac{2}{\sqrt{14}}$$ 1

$\therefore$ distance from origin $= \dfrac{2}{\sqrt{14}}$.

i

$$\left.\begin{aligned} f_x &= y\cos xy, \text{ at } (1,\pi/3) = \pi/6. \\ f_y &= x\cos xy, \text{ at } (1,\pi/3) = 1/2. \\ f_{xx} &= -y^2\sin xy, \text{ at } (1,\pi/3) = \frac{-\pi^2}{18}\sqrt{3}. \\ f_{xy} &= \cos xy - xy\sin xy, \text{ at } (1,\pi/3) = \frac{1}{2} - \frac{\pi\sqrt{3}}{6}. \\ f_{yy} &= -x^2\sin xy, \text{ at } (1,\pi/3) = \frac{-1}{2}\sqrt{3}. \end{aligned}\right\}$$ 2

$\left(\cos\frac{\pi}{3} = \frac{1}{2}, \ \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\right)$

$\therefore$ Neglecting terms of cubic order

$$\left.\begin{aligned} \sin xy = \frac{\sqrt{3}}{2} &+ \frac{\pi}{6}(x-1) + \frac{1}{2}\left(y - \frac{\pi}{3}\right) - \frac{\pi^2}{36}\sqrt{3}(x-1)^2 \\ &+ \left(\frac{1}{2} - \frac{\pi}{6}\sqrt{3}\right)(x-1)\left(y - \frac{\pi}{3}\right) - \frac{\sqrt{3}}{4}\left(y - \frac{\pi}{3}\right)^2. \end{aligned}\right\}$$ 2

EXAMINATION ~~QUESTIONS~~/SOLUTIONS 2011-2012

Course: EE1 (2)

Question: 1

TOPIC

Marks & seen/unseen

Parts: i

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & 2 \end{vmatrix} = 2 - 2(-22) + 3(-6) = 28$$ 1

Matrix of cofactors $C = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}$

with
$$\left.\begin{aligned} &A_{11} = 2, \quad A_{12} = 22, \quad A_{13} = -6 \\ &A_{21} = -4, \quad A_{22} = -16, \quad A_{23} = 12 \\ &A_{31} = 7, \quad A_{32} = 7, \quad A_{33} = -7 \end{aligned}\right\}$$ 2

$$\therefore \operatorname{adj} A = \begin{pmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{pmatrix}$$

$$A^{-1} = \frac{1}{28}\begin{pmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{pmatrix}.$$ 1

Setter's initials

Checker's initials

Page number

Parts

c If dimensions of box are x, y, z we require maximum of xyz subject to

$$xy + 2xz + 2yz = 108 \qquad (1)$$

Auxillary function is $\phi = xyz + \lambda(xy + 2xz + 2yz) - \lambda.108$ 2

$\therefore$ require 2

$$\phi_x = yz + \lambda(y+2z) = 0 \qquad (2)$$ 2

$$\phi_y = xz + \lambda(x+2z) = 0 \qquad (3)$$ 2

$$\phi_z = xy + \lambda(2x+2y) = 0 \qquad (4)$$

Now solve (1), (2), (3) & (4).

$(x)(2) + (y)(3) + (z)(4)$ gives $\quad 3xyz + \lambda(2xy + 4xz + 4yz) = 0$ 2

ie $\lambda(xy + 2xz + 2yz) + \frac{3}{2}xyz = 0$

$\therefore \quad 108\lambda + \frac{3}{2}xyz = 0 \Rightarrow \lambda = \frac{-xyz}{72}$ 2

$$\therefore \quad 1 - \frac{x}{72}(y+2z) = 0$$

$$1 - \frac{y}{72}(x+2z) = 0$$

$$1 - \frac{z}{72}(2x+2y) = 0$$

$\therefore \quad \frac{yz}{36} - \frac{xz}{36} = 0 \Rightarrow x = y \quad (z \neq 0)$ 4

$\therefore \quad 1 - \frac{yz}{18} = 0 \Rightarrow yz = 18$ 2

$$1 - \frac{y^2 - 1/2}{72} = 0 \Rightarrow y^2 = 36, \; y = 6$$

EXAMINATION ~~QUESTIONS~~/SOLUTIONS 2011-2012

Course EE I (2)

Question 2

TOPIC

Marks & seen/unseen

Parts

$\therefore\ x=6,\ y=6,\ z=3$ 2

From physical considerations follows that this is a maximum.

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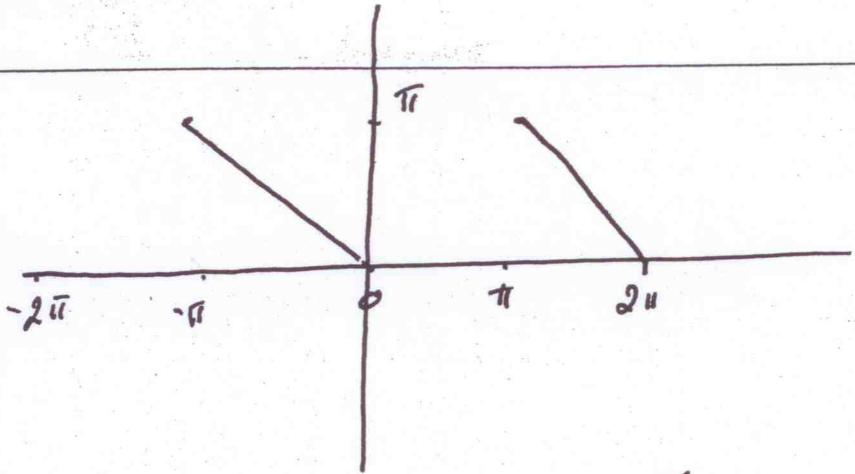
Question 3 | TOPIC | Marks & seen/unseen

Parts

a)



2

b) Let $f(x) = \frac{1}{2}A_0 + \sum_1^\infty A_n \cos nx + \sum_1^\infty B_n \sin nx$

$$A_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} -x\,dx = \frac{-1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} = \pi/2$$ 2

$$A_n = \frac{1}{\pi}\int_{-\pi}^{0}(-x)\cos nx\,dx = \frac{1}{\pi}\left\{\left[\frac{-x\sin nx}{n}\right]_{-\pi}^{0} + \int_{-\pi}^{0}\frac{\sin nx}{n}\,dx\right\}$$ 4

$$= \frac{-1}{n^2\pi}\left[\cos nx\right]_{-\pi}^{0}$$

$$= \frac{-1}{n^2\pi}\left(1-(-1)^n\right) = \frac{-2}{n^2\pi}, \text{ n odd}$$

$$= 0, \text{ n even}$$

$$B_n = \frac{1}{\pi}\int_{-\pi}^{0}(-x)\sin nx\,dx = \frac{1}{\pi}\left[\frac{x\cos nx}{n} - \frac{1}{n^2}\sin xn\right]_{-\pi}^{0}$$ 4

$$= (-1)^n/n$$

$$\therefore f = \pi/4 - \frac{2}{\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)x}{(2m+1)^2} + \sum_{m=1}^{\infty}\frac{(-1)^m \sin mx}{m}$$ 1

	EXAMINATION ~~QUESTIONS~~/SOLUTIONS 2011-2012	Course EE I (2)
Question 3	TOPIC	Marks & seen/unseen

Parts

$$f(\pm\pi) = \frac{1}{2}\left[\lim_{x\to\pm\pi+} f(x) + \lim_{x\to\pm\pi-} f(x)\right] = \pi/2.$$

c) Now put $x = \pi/2$, since $\cos(2m+1)\pi/2 = 0$

$$\therefore\ 0 = \pi/4 + \sum_{m=1}^{\infty} \frac{(-1)^m}{m}\sin m\pi/2 = \pi/4 + \sum_{m=0}^{\infty}\frac{(-1)^{2m+1}(-1)^m}{2m+1}$$ 4

$$\therefore\ \sum_{m=0}^{\infty}\frac{(-1)^m}{2m+1} = \pi/4.$$

Now put $x = 0$,

and $$0 = \pi/4 - \frac{2}{\pi}\sum_{m=0}^{\infty}\frac{1}{(2m+1)^2}$$ 3

$$\therefore\ \sum_{m=0}^{\infty}\frac{1}{(2m+1)^2} = \frac{\pi^2}{8}.$$

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EXAMINATION ~~QUESTIONS~~/SOLUTIONS 2011-2012

Course: EE I (2)

Question 4

TOPIC

Marks & seen/unseen

Parts

(i) $u = x^n f(z) \Rightarrow \dfrac{\partial u}{\partial x} = nx^{n-1} f(z) + x^n f'(z)\dfrac{\partial z}{\partial x}$

$$= nx^{n-1} f(z) + x^n f'(z)\left(-y/x^2\right) \qquad 2$$

$$\frac{\partial u}{\partial y} = x^n f'(z)\frac{\partial z}{\partial y} = x^n f'(z)\left(1/x\right) \qquad 2$$

$$\therefore\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nx^n f(z) + x^n f'(z)\left\{x\left(-y/x^2\right) + y/x\right\} \qquad 2$$

$$\therefore\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu \qquad (1) \qquad 1$$

(ii) $\dfrac{\partial}{\partial x}$ of (1) gives

$$\frac{\partial u}{\partial x} + x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = n\frac{\partial u}{\partial x} \qquad 1$$

$\dfrac{\partial}{\partial y}$ of (1) gives $\quad x\dfrac{\partial^2 u}{\partial y \partial x} + \dfrac{\partial u}{\partial y} + y\dfrac{\partial^2 u}{\partial y^2} = n\dfrac{\partial u}{\partial y} \qquad 1$

Add these equations to give required result. 2

(iii) Special case $n = 2$, $f = z$, $u = x^2(y/x) = xy$ $\qquad 2$

$u_x = y$, $u_y = x$, $u_{xx} = 0$, $u_{yy} = 0$, $u_{xy} = 1$

LHS $= x.0 + (x+y) + y.0 = x+y$ $\qquad 2$

RHS $= (2-1)(x+y) = x+y =$ LHS

(iv) Now if $\left(x\dfrac{\partial}{\partial x} + y\dfrac{\partial}{\partial y}\right)u = nu$

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)\left(x\frac{\partial u}{\partial x} + y\frac{\partial y}{\partial u}\right) = n^2 u \qquad 2$$

EXAMINATION ~~QUESTIONS~~/SOLUTIONS 2011-2012

Course: EE I (2)

Question: 4

TOPIC

Marks & seen/unseen

Parts

$$\therefore x\left\{\frac{\partial u}{\partial x} + x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y}\right\} + y\left\{x\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial u}{\partial y} + y\frac{\partial^2 u}{\partial y^2}\right\} = n^2 u$$ 2

ie

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n^2 u$$ 1

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	EXAMINATION ~~QUESTIONS~~/SOLUTIONS 2011-2012	Course EE I (2)
Question ~~3~~ 5	TOPIC	Marks & seen/unseen

Parts

(i) $\hat{\underline{a}} = (4,-2,4)/\sqrt{16+4+16}$ $\quad$ 2

$= (2/3, -1/3, 2/3)$ $\quad$ 2

$\hat{\underline{b}} = (-4,0,3)/\sqrt{16+9}$

$= (-4/5, 0, 3/5)$

Or if they use
$\hat{a}(\hat{a}+\hat{b}) = 1+\hat{a}\hat{b}$
$\hat{b}(\hat{a}+\hat{b}) = 1+\hat{a}\hat{b}$

$\underline{u} = \hat{\underline{a}} + \hat{\underline{b}} = \frac{1}{15}(-2,-5,19)$ $\quad$ 2

$\underline{u}\cdot\hat{\underline{a}} = \frac{1}{45}(-4+5+38) = \frac{39}{45} = \frac{13}{15} = |\underline{u}|\cos\theta_1$ $\quad$ 2

$\underline{u}\cdot\hat{\underline{b}} = \frac{1}{75}(8+57) = \frac{13}{15} = |\underline{u}|\cos\theta_2$ $\quad$ 2

$\therefore \cos\theta_1 = \cos\theta_2$ so $\underline{u}$ bisects $\underline{a}, \underline{b}$. $\quad$ 1

(ii) Now let $\underline{t} = \hat{\underline{a}} \wedge \hat{\underline{b}} = \begin{vmatrix} i & j & k \\ 2 & -1 & 2 \\ -4 & 0 & 3 \end{vmatrix} \Big/ 15$ $\quad$ 3

$= \frac{1}{15}(-3, -14, -4)$

so $\hat{\underline{t}} = \frac{1}{\sqrt{221}}(-3,-14,-4)$ $\quad$ 2

(iii) $\therefore V = |\hat{\underline{t}}\cdot\hat{\underline{a}}\wedge\hat{\underline{b}}| = \left|\frac{1}{15\sqrt{221}}\begin{vmatrix} -3 & -14 & -4 \\ 2 & -1 & 2 \\ -4 & 0 & 3 \end{vmatrix}\right|$ $\quad$ 4

$= \sqrt{\frac{221}{225}} = \frac{\sqrt{221}}{15}$

or

$\hat{\underline{k}} = \frac{15}{\sqrt{221}}\hat{\underline{a}}\wedge\hat{\underline{b}}$

$\Rightarrow 1 = \frac{15}{\sqrt{221}}V$

EXAMINATION QUESTIONS/SOLUTIONS 2011-2012

Course: EE I (2)

Question: 6

TOPIC

Marks & seen/unseen

Parts

6. a First set $N=1$ to give $1=\frac{1}{2}(1+1)$ which is true so proposition holds for $N=1$. — 2

Now assume true for $N=n$ so that — 2

$$1+2+3+\dots+n=\frac{n}{2}(n+1)$$

But $(1+2+3\dots+n)+n+1$

is therefore given by $\frac{n}{2}(n+1)+n+1$ — 4

$$=\frac{(n+1)(n+2)}{2}$$

which is the original proposition with N replaced by $n+1$. Hence since the proposition true for $N=1$ it is true for all N. — 2

	EXAMINATION ~~QUESTIONS~~/SOLUTIONS 2011-2012	Course EE I (2)
Question 6	TOPIC	Marks & seen/unseen

Parts

6

I (i) $A \rightarrow C$ — 1

(ii) $\overline{C} \rightarrow B$ — 1

II (i) If it is raining and the sun is shining, then there are clouds in the sky — 1

(ii) If it is not raining, then the sun is shining or there are clouds in the sky. — 1

c

A	B	A→B	B→A	(A→B)∪(B→A)	(A→B)∩(B→A)	A⟷B
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

2 2 1 for the last three columns

$(A \rightarrow B) \cap (B \rightarrow A)$ is equivalent to $A \leftrightarrow B$. — 1

Setter's initials GP

Checker's initials GP GM

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