Optical Communication 2017 : Solutions

1. a) $v_p = \frac{\omega}{k} = \frac{2\pi f}{k} \therefore |k| = \frac{2\pi(2.5\times10^9)}{2.1\times10^8}$

$= \underline{74.8\ m^{-1}}$

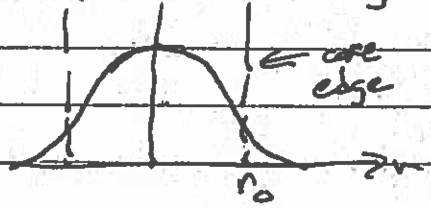
b) $P = \frac{hc}{\lambda}.\dot{N} \quad \lambda = \frac{hc\dot{N}}{P} = \frac{6.63\times10^{-34}\times3\times10^8\times2\times10^{16}}{3\times10^{-3}}$

$= \underline{1.33\ \mu m}$

c) $R = \left(\frac{n_2-n_1}{n_2+n_1}\right)^2 = \left(\frac{1.52-1.3}{1.52+1.3}\right)^2 = .0061$

$T = 1-R = 0.994 \qquad P_T = .994\times5\,mW = \underline{4.97\,mW}$

d) The mode is circularly symmetrical, with a peak at the axial centre and decreasing monotonically with increasing radius. It decays exponentially into the cladding. Its cross-sectional shape is:



e) The loss in dB $= 10\log_{10}\left(\frac{800}{65}\right) = 10.9\ dB$

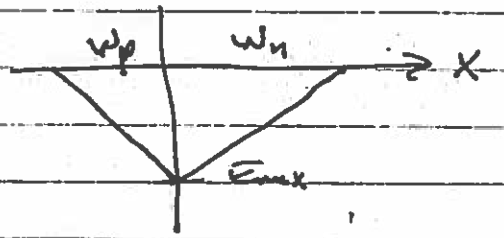
loss/km $\quad \alpha_{dB} = \frac{10.9}{25} = \underline{0.436\ dB/km}$

This is good, but about 2x the best available.

f) $I_{ph} = \frac{\eta e\lambda}{hc}\Phi_R = \frac{0.83\times1.6\times10^{-19}\times980\times10^{-9}\times2.5\times10^{-6}}{6.63\times10^{-34}\times3\times10^8}$

$= \underline{1.63\ \mu A}$

1. g) Since any signal has a finite $\Delta\lambda$, the dispersion $d^2n/d\lambda^2$ can only be zero at the centre λ of the signal. Since the amount by which $d^2n/d\lambda^2$ changes within $\Delta\lambda$ is proportional to $\frac{d}{d\lambda}(d^2n/d\lambda^2)$, the dispersion is proportional to $\underline{\frac{d^3n}{d\lambda^3}}$ in this case.

h) 

$w_p N_A = w_n N_D$

$\therefore\ w_n = 2w_p = 2\,\mu m$

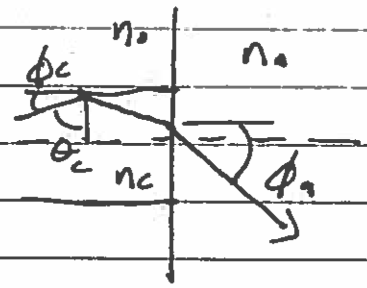
$w_p = 1\ \mu m$

$(w_n + w_p = 3\ \mu m)$

$$|E_{max}| = \frac{e\,w_p N_A}{\varepsilon_r \varepsilon_0} = \frac{1.6\times10^{-19}\times10^{-6}\times2\times10^{20}}{12\times8.85\times10^{-12}}$$

$$= \underline{3.0\times10^5\ V/m}$$

i) We wish to excite the electrons to a level from which they will not rapidly decay by spontaneous recombination, to maximise stimulated emission. But for transitions between 2 levels, the excitation probabilities and spontaneous emission probabilities are equal, so a long decay lifetime means weak pump absorption. A 3 level system provides strong pump absorption to a higher level, and then even quicker decay to a metastable level.

j) 

The ~~maximum~~ min θ_c is the critical angle

$\theta_c = \sin^{-1}(n_o/n_c)$, so the max

$\phi_c = \cos^{-1}(n_o/n_c)$, so $\phi_{c\,max} = \sin^{-1}\left(1-(n_o/n_c)^2\right)^{1/2}$, $\sin\phi_{c\,max} = NA/n_c$

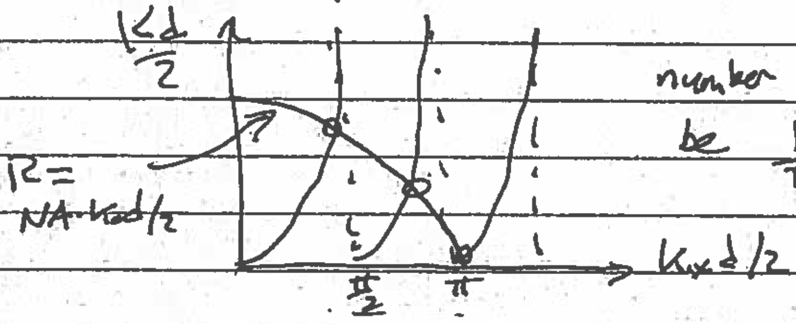
And $\sin\phi_a(max) = \frac{n_c}{n_a}\sin\phi_c(max) = NA$

$\therefore\ \phi_a(max) = \sin^{-1}(NA) = \underline{8.6^\circ}$

2. a) It is fine to use, from memory, the condition that mode m is supported if:

$$d \geq \frac{1}{2} \frac{m \lambda_0}{NA}.$$

Alternatively, consider the k_x–K diagram:



number of modes N must be $\frac{R}{\pi/2}$ rounded up to nearest integer.

So in this case, $N = \sqrt{1.5^2 - 1^2} \times \dfrac{2\pi}{0.5\times10^{-6}} \times \dfrac{3\times10^{-3}}{2} \times \dfrac{2}{\pi}$

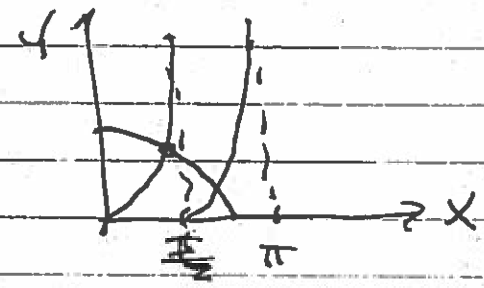
$N = \sqrt{1.25} \times 12\times10^3 = \underline{13{,}417 \text{ modes}}$ (rounded up).

b) Now we have $NA = \sqrt{1.49^2 - 1.485^2} = 0.1220$

$R = \dfrac{0.1220 \times 2\pi}{1.33\,\mu m} \times \dfrac{6\,\mu m}{2} = 1.729$

$\pi/2 < R < \pi$ $\therefore$ modes $m=0$ and $m=1$ are supported (2 modes)

c) Taking $X = k_{ix} d/2$, for $m=0$ (even) the eigenvalue equated to the equation of the circular arc gives $\dfrac{\cos X}{X} = \pm \dfrac{1}{R}$.



From the graph, X should be just less than $\frac{\pi}{2}$ so we can use ~~the~~ 1.5 as a first value and iterate $X = 1.729 \cos X$

Gives $X_0 = 0.97$

For $m=1$, $X = 1.729 \sin X$

start with $X = 1.7$, iterate, gives

$X_1 = 1.71$

To find n': $k_{ix} = \dfrac{2X}{d}$ $\beta = n' k_0 = \sqrt{n_1^2 k_0^2 - k_{ix}^2}$

$n' = \sqrt{1.49^2 - \left(\dfrac{X\lambda_0}{\pi d}\right)^2}$ $n_0' = \sqrt{1.49^2 - \left(\dfrac{0.97\times1.33}{\pi\times6}\right)^2} = \underline{1.488}$

$n_1' = \sqrt{1.49^2 - \left(\dfrac{1.71\times1.33}{6\pi}\right)^2} = \underline{1.4851}$

Both are in the range $n_2 < n' < n_1$

3. a) $\Phi_T = 4\,dBm = 1mW \times 10^{0.4} = 2.51\,mW$

$\Phi_R = \Phi_{T(dB)} - \alpha_{dB} \times L = 4 -$

3. a) $\Phi_T = 4\,dBm = 1mW \times 10^{0.4} = 2.51\,mW$

$\Phi_R = \Phi_T e^{-\alpha L} \qquad \alpha = \frac{\alpha_{dB}}{4.34} = 0.081 \quad km^{-1}$

$I_{ph} = \frac{\eta e \lambda}{hc} \Phi_T e^{-\alpha L} \qquad SNR = \frac{I_{ph}}{\sqrt{\frac{4kT}{R}}\sqrt{\Delta f}}$ (thermal noise)

$\Delta f \approx B/2 \qquad SNR^2 = \frac{\left(\frac{\eta e \lambda}{hc}\right)^2 \Phi_T^2 e^{-2\alpha L}}{\frac{2kT}{R} \cdot B}$

Then $B_{max} = A_T e^{-2\alpha L}$ where $A_T = \left(\frac{\eta e \lambda \Phi_T}{hc \cdot SNR}\right)^2 \frac{R}{2kT}$

$A_T = \left[\frac{0.85 \times 1.6 \times 10^{-19} \times 1.33 \times 10^{-6} \times 2.51 \times 10^{-3}}{6.63 \times 10^{-34} \times 3 \times 10^{8} \times 12^2}\right]^2 \times \frac{2 \times 10^4}{2 \times 1.38 \times 10^{-23} \times 300}$

$= 6.07 \times 10^{14}$

b) For shot noise, $SNR = \frac{I_{ph}}{\sqrt{2e I_{ph} \cdot \Delta f}} = \sqrt{\frac{I_{ph}}{eB}}$

$B_{max} = \frac{I_{ph}}{e \cdot SNR^2} = A_S e^{-\alpha L}$ where $A_S = \frac{\eta e \lambda \Phi_T}{hce} \cdot \frac{1}{SNR^2}$

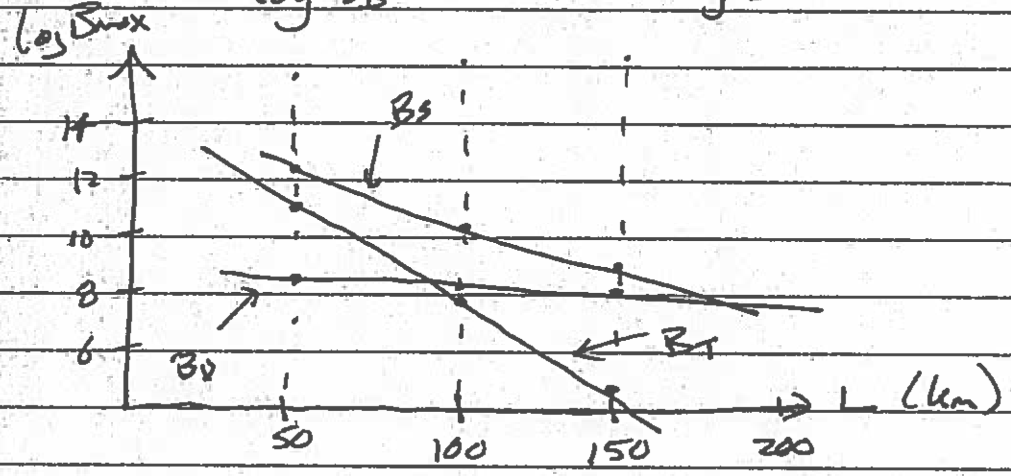
$A_S = \frac{0.85 \times 1.6 \times 10^{-19} \times 1.33 \times 10^{-6} \times 2.51 \times 10^{-3}}{6.63 \times 10^{-34} \times 3 \times 10^{8} \times 1.6 \times 10^{-19} \times 12^2} = 9.91 \times 10^{13}$

c) $\tau_D = D \cdot L \cdot \sigma_\lambda = 11 \times 1.5 L = \frac{0.25}{B}$

$B_{max} = \frac{0.015}{L} ps^{-1} = \frac{15 \times 10^9}{L}$ bit/s/km

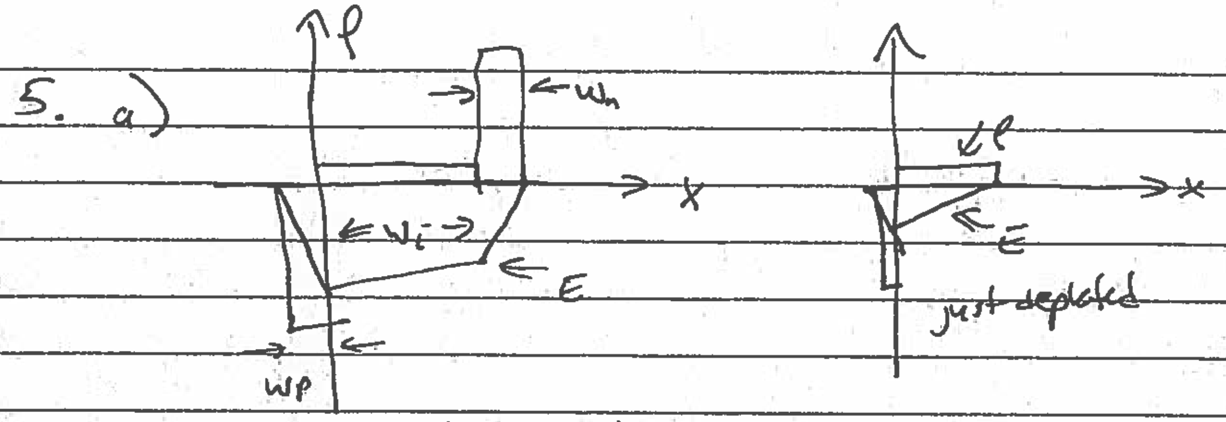
3. d) Since the first 2 have $B \propto e^{-L}$, best to plot $\log(B_{max})$, on a log-linear plot.
Try a few values to get a suitable range – From (a) it can be found that by 200 km B is very low. Labelling max B for thermal, shot and dispersion limits as B_T, B_S and B_D:

$$\log B_T = \log A_T - 2\alpha L \log e$$
$$= 14.8 - .07 L$$
$$\log B_S = \log A_S - \alpha L \log e$$
$$= 14.0 - .035 L$$
$$\log B_D = 10.2 - \log L$$



The link is limited by dispersion for L < 100 km to about 100 Mbit/s, and by thermal noise at higher L. Above L ~ 140 km, Bmax becomes too low to be useful. Shot noise is never the limiting factor.

5. a)



w_i is just depleted when $E=0$ at i-n boundary.

Then $|E_{max}| = \frac{e}{\varepsilon_r \varepsilon_0} \times w_i N_D^- = \frac{1.6\times10^{-19} \times 10^{-5} \times 2\times10^{20}}{12\times 8.85\times10^{-12}}$

$= 3.0\times10^{6}\ V/m$

And $w_p = w_i \frac{N_D^-}{N_A^+} = 1\mu m$ $\therefore V_1 = |E_{max}| \frac{(w_p + w_i)}{2}$

$= 3\times10^{6} \times \frac{11\times10^{-6}}{2} = 16.5V$

b) Call the depleted thickness in the i layer w_n^-. Then $|E_{max}| = \frac{e}{\varepsilon} w_n^- N_D^-$

and $w_p = w_n^- \left(\frac{N_D^-}{N_A^+}\right)$, $w = \left(1 + \frac{N_D^-}{N_A^+}\right) w_n^-$

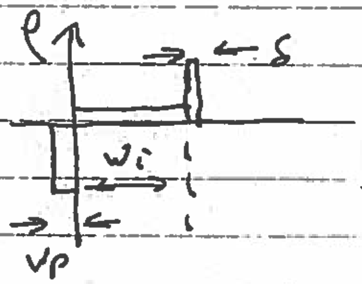
$$|E_{max}| = \frac{e}{\varepsilon} w \frac{N_D^-}{(1+N_D^-/N_A^+)} \qquad V = \frac{1}{2}|E_{max}| w$$

$$V = \frac{1}{2}\frac{e}{\varepsilon} \times \frac{N_D^-}{(1+N_D^-/N_A^+)} w^2$$

$$dV/dw = \cancel{\tfrac{1}{2}}\frac{e}{\varepsilon} \times \frac{N_D^- \, w}{(1+N_D^-/N_A^+)} = 2V/w$$

and $\frac{dw}{dV} = \left(\frac{dV}{dw}\right)^{-1}$. $\quad \frac{dw/w}{dV/V} = \frac{w}{2V} \times \frac{V}{w} = \underline{\underline{\frac{1}{2}}}$

c)



$$|E_{max}| = \frac{e}{\varepsilon}\left(N_D^- w_i + N_D^+ \delta\right)$$

$$w_p = \frac{N_D^-}{N_A^+} w_i + \frac{N_D^+}{N_A^+} \delta$$

$$w = w_p + w_i + \delta = \left(1 + \frac{N_D^-}{N_A^+}\right) w_i + \left(1 + \frac{N_D^+}{N_A^+}\right)\delta$$

$$V = \frac{1}{2}|E_{max}| w$$
$$= \frac{1}{2}\frac{e}{\varepsilon}\left[\left(N_D^- w_i + N_D^+ \delta\right)\left(\left(1+\frac{N_D^-}{N_A^+}\right) w_i + \left(1 + \frac{N_D^+}{N_A^+}\right)\delta\right)\right]$$

Since $\delta \ll w_i$, we can expand and neglect the δ^2 term.

$$V \simeq \frac{1}{2}\frac{e}{\varepsilon}\left[N_D^-\left(1+\frac{N_D^-}{N_A^+}\right) w_i^2 + N_D^+\left(1+\frac{N_D^-}{N_A^+}\right) w_i \delta + N_D^-\left(1+\frac{N_D^+}{N_A^+}\right) w_i \delta\right]$$

$$= \frac{1}{2}\frac{e}{\varepsilon}\left[N_D^-\left(1+\frac{N_D^-}{N_A^+}\right) w_i^2 + \left(N_D^+ + N_D^- + \frac{2 N_D^+ N_D^-}{N_A^+}\right) w_i \delta\right]$$

Then $\frac{dV}{dw} = \frac{dV}{d\delta} \times \frac{d\delta}{dw}$

$$\frac{dw}{d\delta} = 1 + \frac{N_D^+}{N_A^+}$$

$$\frac{dV}{d\delta} = \frac{1}{2}\frac{e}{\varepsilon}\left(N_D^+ + N_D^- + \frac{2N_D^+ N_D^-}{N_A^+}\right)W_i$$

$$\frac{dV}{dw} = \frac{\frac{1}{2}\frac{e}{\varepsilon}\left(N_D^+ + N_D^- + 2N_D^+ N_D^-/N_A^+\right)W_i}{\left(1 + N_D^+/N_A^+\right)}$$

$$\frac{dw/w}{dV/V} = \frac{V/w}{dV/dw}.$$ Take $\frac{V}{w}$ at $\delta = 0$:

$$V = V_1 = \frac{1}{2}\frac{e}{\varepsilon}\left(\frac{N_D^-}{1 + N_D^-/N_A^+}\right)w^2$$

$$\therefore \frac{V}{w} = \frac{1}{2}\frac{e}{\varepsilon}\left(\frac{N_D^-}{1+N_D^-/N_A^+}\right)\left(1 + \frac{N_D^-}{N_A^+}\right)W_i = \frac{1}{2}\frac{e}{\varepsilon}N_D^- W_i$$

$$\frac{dw/w}{dV/V} = \frac{\left(1 + N_D^+/N_A^+\right)}{1 + N_D^+/N_D^- + 2N_D^+/N_A^+} = 0.1$$

Since $N_D^- = 0.1\,N_A^+$:

$$0.1 = \frac{1+\alpha}{1 + 2\alpha + 10\alpha} = \frac{1+\alpha}{1+12\alpha} \rightarrow \alpha \cong 5$$

$$\therefore \underline{N_D^+ \cong 10^{22}\ m^{-3}}$$

d) Both the responsivity (through fraction of photons captured in depletion region) and the diode capacitance depend on W, so it's useful for predictable operation to have these not be strongly dependent on V.