UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2000

MEng Honours Degree in Information Systems Engineering Part IV
MEng Honours Degrees in Computing Part IV
MSc in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C478=I4.37

ADVANCED OPERATIONS RESEARCH

Monday 8 May 2000, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions

1a Explain the simplex multiplier and its role in checking the optimality conditions.

b Assume, we have an LP problem: $\min\{\boldsymbol{c}^T\boldsymbol{x} \mid \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}\}$ and variables are subject to type specifications. A basic feasible solution (BFS) with $z = -1$ and a type-2 incoming variable with reduced cost $d_q = 3$ are given. Also, $\boldsymbol{\alpha}_q = \boldsymbol{B}^{-1}\boldsymbol{a}_q$ is available. The table below shows the relevant part of the problem:

i	x_{Bi}	type(x_{Bi})	u_{Bi}	α_{iq}	t_i
1	2	2	$+\infty$	-1	
2	0	3	$+\infty$	1	
3	2	1	4	1	
4	6	2	$+\infty$	2	
5	1	1	5	-2	

Is the BFS degenerate? Determine the ratios, the value of the incoming variable, the variable leaving the basis (if any), the new BFS and the new value of the objective function. Is the new BFS degenerate?

c (i) Variable x is allowed to take one of the following values $\{0.1,\ 0.2,\ 1,\ 2,\ 4\}$. Formulate this condition in terms of integer programming.

(ii) An integer variable x is limited to take a value in the interval $[-2, 12]$. How would you express this condition using 0/1 variables?

(The three parts carry, respectively, 20%, 40% and 40% of the marks).

2a The **transportation problem** can be described as follows. A certain product, say a car, is available at m supply points (depots) in quantities of s_i $(i = 1, \ldots, m)$. They fulfill the demand d_j $(j = 1, \ldots, n)$ at n delivery points (destinations). The unit cost of a shipment from depot i to destination j is c_{ij}. We assume that the total supply is equal to the total demand. In a complete shipment allocation no cars remain at any of the depots and the demand at every destination is satisfied.

An allocation is sought that minimizes total transportation costs. Set up an integer programming model for the problem and show that its matrix satisfies Property K. Explain the importance of this property.

What is the size of the problem in terms of constraints and variables?

Hint: define the decision variables first.

b Determine the type of each variable in the following two problems. Are the given solutions feasible? Do they satisfy the optimality conditions? Identify which solution is degenerate, if any. What is the size of the problems in terms of m and n? How do you know that? (Note, B/N refers to Basic/Nonbasic.)

(i) Problem: min $\boldsymbol{c}^T\boldsymbol{x}$, $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$,

	x_1	x_2	x_3	x_4	x_5	x_6
ℓ_j	0	$-\infty$	0	0	0	$-\infty$
u_j	$+\infty$	$+\infty$	5	5	0	$+\infty$
type(x_j)						
In the solution:						
B/N	N	B	N	B	N	N
Value	0	0	5	5	0	0
d_j	-2	0	1	0	-2	-1
Opt. cond. Y/N						

(ii) Problem: max $\boldsymbol{c}^T\boldsymbol{x}$, $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$,

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
ℓ_j	0	0	0	$-\infty$	0	0	0
u_j	5	$+\infty$	$+\infty$	$+\infty$	10	$+\infty$	0
type(x_j)							
In the solution:							
B/N	N	B	N	B	N	N	N
Value	5	1	0	-7	10	0	0
d_j	-1	0	11	0	-1	10	1
Opt. cond. Y/N							

c Discuss the product form of the inverse and summarize its computational advantages in the simplex method.

(The three parts carry, respectively, 40%, 30% and 30% of the marks).

3a A food processing company can produce tinned vegetables in four different versions. As part of the company's production planning system the following problem has to be solved.

$$\begin{array}{llccccccccc}
\text{max} & \text{Revenue } z = & 5x_1 & + & 6x_2 & + & 9x_3 & + & 4x_4 & & \\
\text{s.t.} & \text{Preprocessing} & 2x_1 & + & 3x_2 & + & 4x_3 & + & 2x_4 & \leq & 320 \\
 & \text{Cooking} & 4x_1 & + & x_2 & + & 4x_3 & + & 2x_2 & \leq & 240 \\
 & \text{Filling} & 3x_1 & + & 2x_2 & + & 3x_3 & + & 5x_4 & \leq & 240 \\
 & & & & \multicolumn{5}{c}{x_1,\ x_2,\ x_3,\ x_4 \geq 0.} & &
\end{array}$$

An optimal solution is achieved with $x_2 = 40$, $x_3 = 50$ and $y_3 = 10$ giving an objective value of $z = 690$. (Note, $y_3 \equiv x_7$.)

The optimal basis is $B = \{7, 3, 2\}$. The basis matrix is

$$\boldsymbol{B} = \begin{bmatrix} 0 & 4 & 3 \\ 0 & 4 & 1 \\ 1 & 3 & 2 \end{bmatrix} \qquad \text{its inverse} \qquad \boldsymbol{B}^{-1} = \begin{bmatrix} -\frac{5}{8} & -\frac{1}{8} & 1 \\ -\frac{1}{8} & \frac{3}{8} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}$$

(i) Determine the simplex multiplier associated with $\boldsymbol{B}$ and the reduced costs of nonbasic variables.

(ii) Determine the ranges of the right hand side coefficients within which the basis remains optimal.

b A car manufacturing plant is considering the extension of production capacity by setting up a new assembly line to meet the increasing demand. The objective is to produce the planned amount of cars at a minimum cost.

The setup cost of the new line is £135M. This line can produce a maximum of 50,000 cars per annum at a unit production cost of £4500. Formulate this part of the model that expresses costs of production on the considered new assembly line.

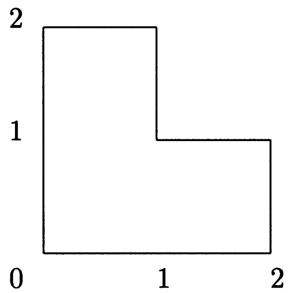
c Discuss and interpret the possible outcomes of the simplex method.

(The three parts carry, respectively, 50%, 35% and 15% of the marks).

4a Convert the following linear programming constraints into equalities. Indicate the type of the associated logical variable. Try to combine constraints if possible.

$$2x_1 + x_2 - 3x_3 - x_4 \leq -1 \tag{1}$$
$$3x_1 + x_3 - x_4 \geq 0 \tag{2}$$
$$x_1 + 2x_3 + x_4 \leq 1 \tag{3}$$
$$x_1 + 2x_3 + x_4 \geq -1 \tag{4}$$
$$9 \geq 2x_1 - x_2 + x_3 - 2x_4 \geq -1 \tag{5}$$
$$x_1 + x_2 + x_3 + x_4 <> 0 \tag{6}$$
$$x_1 + x_2 - x_3 - x_4 = 0 \tag{7}$$

b You are given the following nonconvex region in the xy plane with x and y coordinates indicated. Write the appropriate linear inequalities and logic expressions to describe the points in the closed area. Introduce integer variable(s) if needed and give a MIP formulation of the region. Verify your solution by showing that point $(1.5, 1.5)$ does not satisfy your constraints while $(1.0, 1.5)$ does.



c Formulate the dual of the following problem. Explain your work.

$$\begin{array}{rrrrcr}
\min z = & -2x_1 + & x_2 + & 3x_3 & & \\
\text{s.t.} & 2x_1 + & 3x_2 - & x_3 & \geq & 0 \\
& 4x_1 - & 3x_2 + & 2x_3 & \leq & -1 \\
& -x_1 - & 2x_2 + & 5x_3 & = & 0 \\
& \multicolumn{5}{l}{x_1 \leq 0,\ x_2, x_3 \geq 0.}
\end{array}$$

(The three parts carry, respectively, 25%, 50% and 25% of the marks).