Paper Number(s): **E1.1**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

EEE PART I: M.Eng., B.Eng. and ACGI

ANALYSIS OF CIRCUITS

Wednesday, 20 June 10:00 am

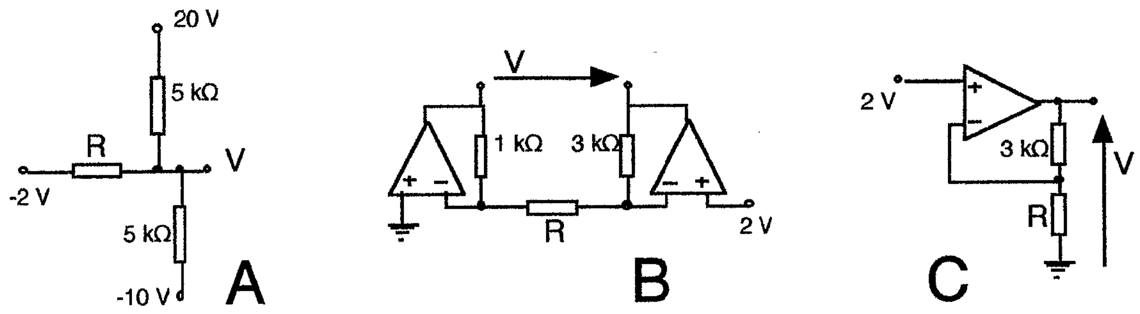
There are FIVE questions on this paper.

Answer THREE questions.

Time allowed: 2:00 hours

Examiners: Spence,R. and Weiss,G.

1. (a) For each of the circuits shown in Figure 1a below choose the value of R required to lead to a voltage V of 5 volts. Assume the operational amplifiers are ideal



A[3]
B[4]
C[4]

Figure 1a

(b) For each of the circuits shown in Figure 1b calculate, with explanation, one positive value of resistance R that will ensure that the indicated voltage or current lies somewhere within the specified range, or explain why this is not possible.

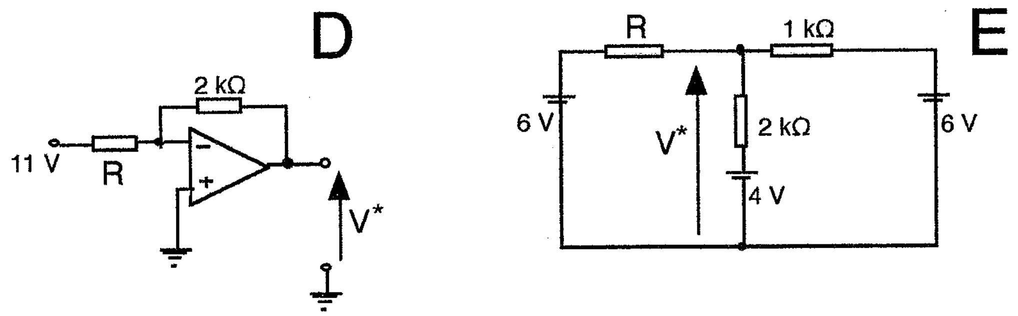


Figure 1b

D[4]
E[5]

2 (a) For the circuit of Figure 2a, express the complex impedance between the terminals A and B in terms of R1, R2, L, C and the radian frequency ω.

[7]

For what numerical value of the radian frequency will the impedance be purely resistive?

[3]

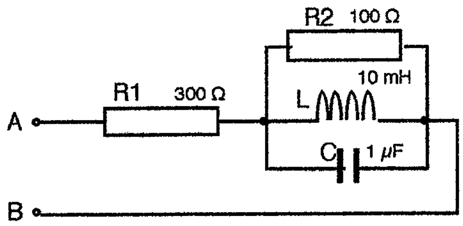


Figure 2a

(b) Find the relation that must exist between L, C and R in the circuit of Figure 2b for the impedance between A and B to be resistive at all frequencies.

[10]

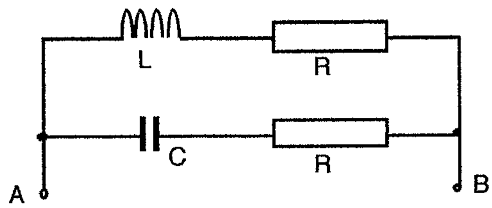


Figure 2b

3 Figure 3 shows a circuit in which a Schmitt Trigger is followed by an integrator. The output of each opamp is limited to plus and minus 14 V and the opamps are otherwise ideal.

(a) Choose R1 so that the opamp A switches when VB is equal to plus or minus 4 V.

[7]

(b) Choose R3 to give an oscillation frequency of 2 kHz.

[7]

(c) If a 10kΩ resistor is connected between point X and +15 V, draw a dimensioned sketch of the resultant waveform of VB and calculate the new oscillation frequency

[6]

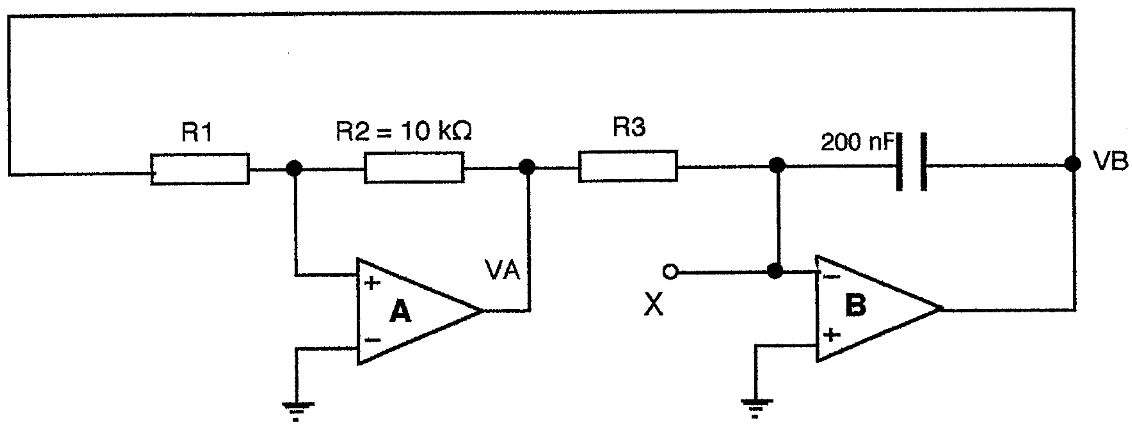


Figure 3

4 For the circuit of Figure 4, employ nodal analysis to derive the Thevenin Equivalent Circuit of the circuit external to the resistor R.

[16]

Hence calculate the value of the current I for the following values of the resistor R: 0, 0.5 kΩ and 5.5 kΩ

[4]

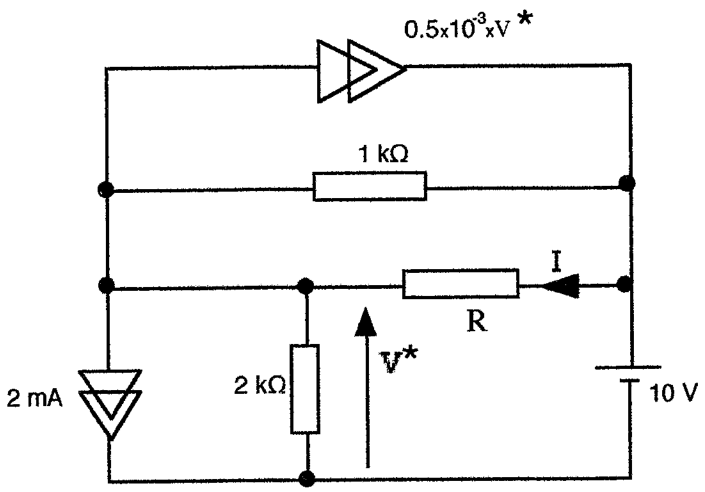


Figure 4

5 In the circuit of Figure 5 the voltage across each exponential diode can be assumed to be approximately 0.7 V if the diode current exceeds 0.2 mA.

Apply Kirchhoff's Current Law at point X and hence determine the quiescent voltage at this point and the current in each of the diodes.

[6]

Determine the small-signal equivalent resistance of each of the diodes.

[6]

Assuming that the impedance of the capacitor is negligible, draw the small-signal equivalent of the circuit and hence calculate the peak-to-peak amplitude of the sinusoidal voltage V.

[8]

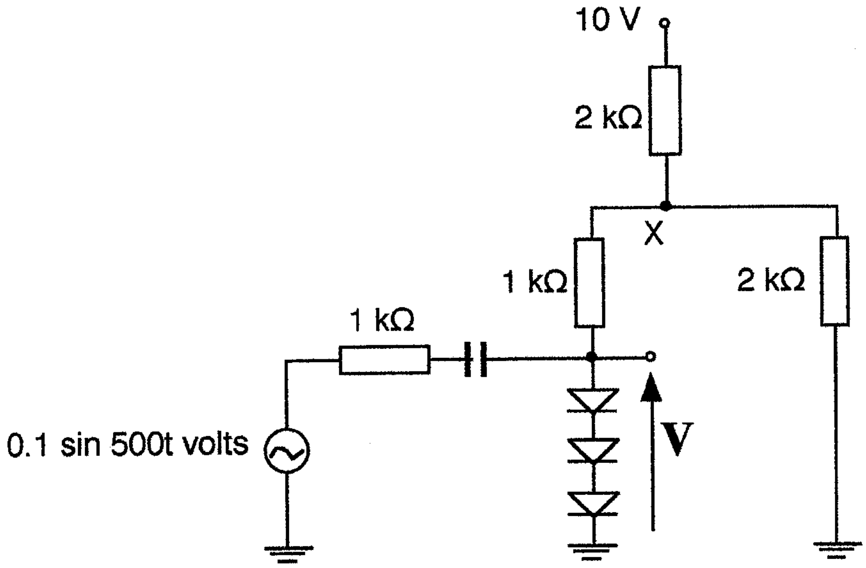


Figure 5

Answer 1

(a) A Apply KCL at node:

$$\frac{5+2}{R} = \frac{15}{5} - \frac{15}{5}$$

Thus $R = \infty$ (open circuit)

B Current through R (R to L) $= 2/R$

$$\therefore V = \frac{2}{R}(3 + R + 1) = \frac{2}{R} \cdot 4 + 2$$

$$V = 5 \quad \therefore \quad 3 = \frac{8}{R} \qquad R = \frac{8}{3}\ k\Omega.$$

C $V_+ = V_- = 2V = \frac{R}{3+R} \times 5V \quad \therefore 2(3+R) = 5R$

$$\therefore 6 = 3R \quad \therefore R = 2\ k\Omega.$$

(b) D Not possible. Current via R sets up a negative value of V^*

E Choose $V^* = 5.55\ V$

KCL gives: $\frac{1.55}{2} = \frac{0.45}{R} + \frac{0.45}{1}$

$$\therefore \frac{0.45}{R} = 0.775 - 0.45 = 0.325$$

$$\therefore R = \frac{0.45}{0.325} \simeq 1.4\ k\Omega.$$

Answer 2

(a) Impedance of parallel combination of R2, L, C is

$$\frac{1}{\frac{1}{R2} + \frac{1}{jωL} + jωC} = \frac{1}{\frac{1}{R2} + j\left(ωC - \frac{1}{ωL}\right)}$$

so the total impedance Z between A and B is

$$Z = R1 + \frac{1}{\frac{1}{R2} + j\left(ωC - \frac{1}{ωL}\right)}$$ which is resistive when $ωC = \frac{1}{ωL}$

That is, when $ω^2 = \frac{1}{LC} = 10^8$ i.e., for $ω = 10^4$ radians/sec.

(b) The admittance Y between A and B is the sum of the admittances of the two branches:

$$Y = \frac{1}{R + jωL} + \frac{1}{R + \frac{1}{jωC}} = \frac{R + \frac{1}{jωC} + R + jωL}{R^2 + \frac{ωL}{ωC} + R\left(jωL + \frac{1}{jωC}\right)}$$

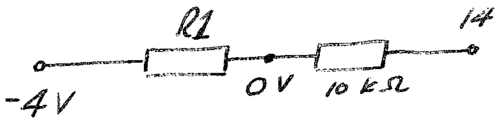
$$= \frac{2R + \left(jωL + \frac{1}{jωC}\right)}{R\left[\left(R + \frac{L}{CR}\right) + \left(jωL + \frac{1}{jωC}\right)\right.}$$

For Y to be real, which it will be if impedance (1/Y) is resistive, for all values of ω, we must choose $\frac{L}{CR} = R$, whereupon $Y = \frac{1}{R}$. Thus the required relation is

$$\frac{L}{C} = R^2$$

Answer 3

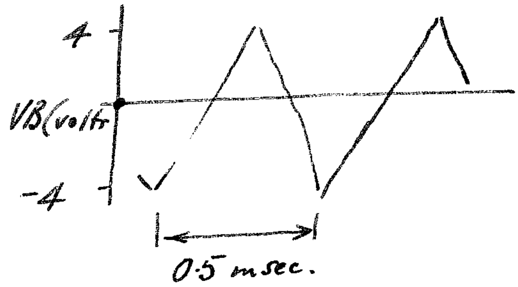
(a) Transition of output of (A) occurs when v_+ of A is at zero. Assume output of (A) is at 14 V.



Since current is same through both resistors

$$\frac{4}{R_1} = \frac{14}{10k\Omega} \quad \text{so} \quad R1 = \frac{40\,k\Omega}{14} = 2.86\,k\Omega$$

(b) Waveform of VB is as shown:



During falling section of VB, charging current of 20μF capacitor

$i = C\dfrac{dV}{dt}$ gives $\dfrac{14}{R_3} = C\dfrac{8}{0.25\times10^{-3}}$

so $R_3 = \dfrac{14.\,0.25\,.10^{-3}}{8.\,200\,10^{-9}} = \dfrac{14}{640}\times 10^{6}$

$$\frac{14\,.\,0.25\,.10^{-3}}{8.\,200\,.10^{-9}} = \frac{14}{32.\,200}10^{6} = \frac{7}{32}10^{4} = \frac{70}{32}k\Omega = 2.19\,k\Omega$$

(c) When VA = 14 and has just changed from -14 V, (VB = - capacitor voltage)

$$i = \frac{14}{R_3} + \frac{15}{10} = C\frac{dV}{dt} \quad \text{so} \quad \frac{dV_B}{dt} = -\frac{1}{200.10^{-9}}\left(6.4 + 1.5\right)10^{-3}$$

$$\frac{dV_B}{dt} = -\frac{7.9\;\;10^{-3}}{200\,.\,10^{-9}}$$

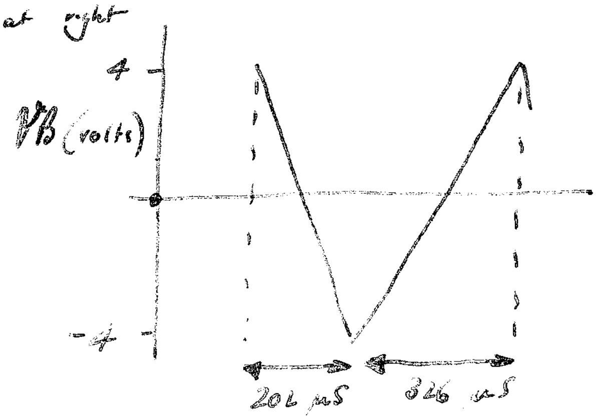
But change in VB is 8 V, so $dt = \dfrac{200.10^{-9}.\,8}{7.9\,.10^{-3}}.10^{3}$ msec $= 202\,.\,10^{-3}$ msec.

When V_A has just changed back to -14 V,

$$\frac{dV_B}{dt} = +\frac{1}{200.\,10^{-9}}(6.4 - 1.5)10^{-3}.$$

Again, $dV_B = 8\,V$ so $dt = \dfrac{8.\,200.10^{-9}}{4.9\,.10^{-3}} = 326.\,10^{-3}$ msec.

The waveform of VB is therefore as shown at right



and therefore the oscillation frequency is given by

$$f = \frac{10^{6}}{202+326}\,Hz$$

$$= 1.89\,kHz$$

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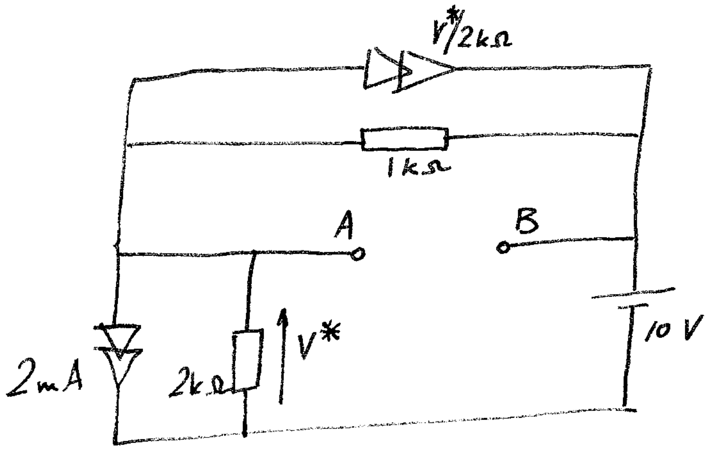
Answer 4

Calculate the Thevenin Equivalent Circuit of the circuit between terminals A and B shown below.



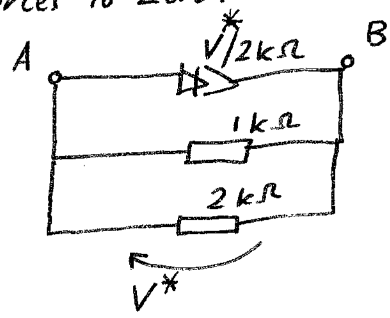
To find V_{oc}, apply KCL at A:

$$\frac{10 - V^*}{1} - \frac{V^*}{2} - 2 - \frac{V^*}{2} = 0$$

which gives $V^* = 4V$

and therefore $V_{oc} = 6V$

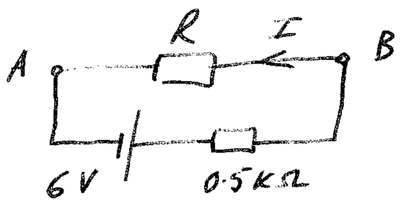
To find R_o set independent sources to zero:



The controlled current is proportional to the voltage across it, and is therefore equivalent to a $2k\Omega$ resistor:

Resistance between A and B $= 0.5k\Omega$

Circuit to be analysed:



$R = 0 \quad I = 12mA$

$R = 0.5k\Omega \quad I = 6mA$

$R = 5.5k\Omega \quad I = 1mA.$

≡11

Answer 5

Assume that the voltage across each diode is 0.7 V (and check later).
Application of KCL at node X yields

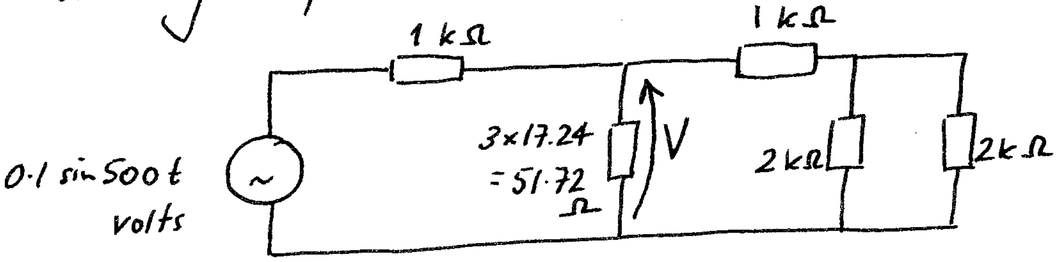
$$\frac{10 - V_x}{2} = \frac{V_x - 2.1}{1} + \frac{V_x}{2}$$

giving $V_x = 3.55$ V

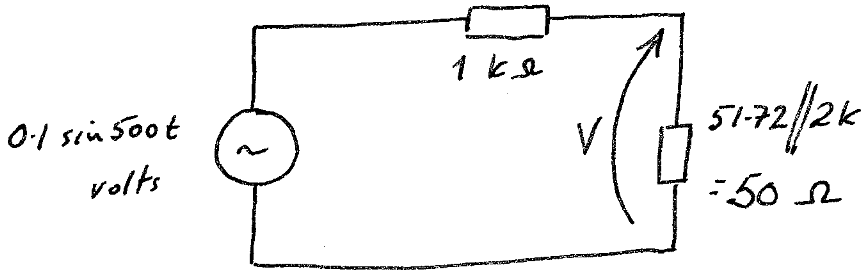
Current through each diode is therefore $(3.55 - 2.1)/1$ mA $= 1.45$ mA
Assumption about diode voltages is therefore justified.

Small-signal resistance of each diode $= \dfrac{25\,\text{mV}}{1.45\,\text{mA}} = 17.24\ \Omega$

Small-signal equivalent circuit is:



which can be redrawn as



The peak-to-peak value of V is therefore $0.2 \times \dfrac{50}{1050} \approx 9.5$ mV

The variation of voltage (≈ 3.2 mV) across each diode is sufficiently small (i.e., $\ll 25$ mV) that the representation of a diode by a linear resistance is justified.