Optical Communication 2014
Solutions

1. a) $\lambda_o = 1330\text{nm} \therefore f = c/\lambda_o \quad \Delta f = .05(c/\lambda_o)$

$$\Delta f = \frac{.05 \times 3 \times 10^8}{1.33 \times 10^{-6}} = \underline{11{,}300 \text{ GHz}}$$

b) Thermal noise is unchanged. Shot noise is proportional to $\sqrt{I_{ph}}$, which will be less, so shot noise will dominate.

c) The fibre core area must be larger than the emitting region of the LED (brightness theorem).

d) $\tau = L(n/c) \quad B = \frac{c}{nL} = \frac{3 \times 10^8}{1.5 \times 0.4} = \underline{500 \text{ Mbit/s}}$

e) The photon energy at $\lambda_o = 1550\text{nm}$ is well below the bandgap energy of Si so Si cannot absorb.

f) The mode of lower n' (1.475) has more light in the cladding, so is more weakly guided, will leak more.

g) $d < \frac{m\lambda_o}{2\sqrt{n_c^2 - n_o^2}} \quad m=1 \quad d < \frac{1.3\mu m}{2\sqrt{1.52^2 - 1.5^2}} = \underline{2.64 \ \mu m}$

h) It's easier to align sources to fibres and fibres to each other. (Alternatively easier to launch light from transmitter efficiently.

i) $R = \left(\frac{1.5 - 1.3}{1.5 + 1.3}\right)^2 = 0.005$ in either direction

j) Spreading loss gives a $1/R^2$ dependence, which is not exponential and so not constant in dB/km.

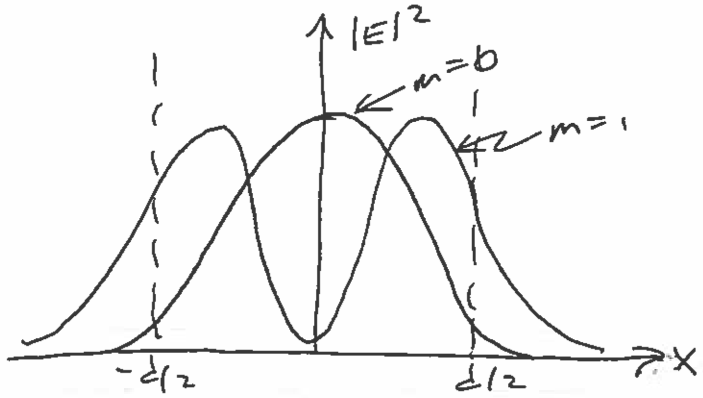
2. a)

a mode m is supported if $d > \frac{m\lambda_0}{2NA}$

$2NA\, d/\lambda_0 = 2\sqrt{1.52^2 - 1.5^2} \times 4/1.3 = 1.51$

Only $\underline{\underline{2}}$ modes, $m=0$ and $m=1$

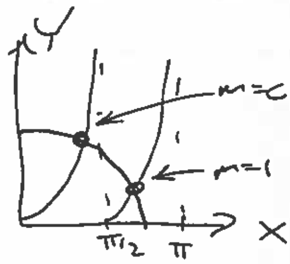
b)



c) For $m=0$ (even), taking $X = k_{1x}d/2$ $Y = \kappa d/2$

$Y = X\tan X$ $X^2 + Y^2 = R^2$ $R = NA\, k_0 d/2$

$X^2(1+\tan^2 X) = R^2$ $\therefore \cos X/X = \pm 1/R$



$X \simeq 1.5$ as a starting point.

$R = \sqrt{1.52^2 - 1.5^2}\, 2\pi(4)/2(1.3) = 2.377$

$1/R = 0.42094$

Solving $\cos X/X = .03911$ by successive approx.

gives $X = 1.093$ $k_{1x} = 2X/d = \underline{0.2103\ \mu m^{-1}}$

$n' = \sqrt{n_1^2 - (k_{1x}/k_0)^2} = \sqrt{1.52^2 - (.2103 \times 1.3/2\pi)^2}$

$= \underline{\underline{1.519}}$

For $m=1$ (odd) $Y = -X\cot X \rightarrow \sin X/X = \pm 1/R$

By successive approx $X = \underline{2.077}$

$k_x = 2X/d = 1.0385$ $n' = \sqrt{1.52^2 - (1.0385 \times 1.3/2\pi)^2}$

$\underline{n' = 1.505}$

d) For an $m=1$ mode in the core, $E(x) = E_0 \sin(k_{1x}x)$

which peaks at $k_x x_m = \pi/2$, $x_m = \pi/2k_x$

$k_{1x} = \sqrt{n_1^2 - n'^2}\, k_0$ $k_0 = 2\pi/\lambda_0$

$\underline{x_m = \lambda_0/2\sqrt{n_1^2 - n'^2}}$

3. a) Material Dispersion - this is caused by the variation of n with λ of the glasses making up the fibre.

Waveguide Dispersion - this is caused by the variation of mode shape with λ, and thus n' as the fraction in the core changes.

b) $n' = D_0 + D_1 (\lambda_0 - \lambda_{0c})$

$$V_g = \frac{d\omega}{d\beta} = \frac{d\omega}{d(n'k_0)} = \frac{d\omega}{d\lambda_0} \Big/ \frac{d(n'k_0)}{d\lambda_0}$$

$$|d\omega/d\lambda_0| = \omega/\lambda_0 \qquad \frac{d(n'k_0)}{d\lambda_0} = \frac{d}{d\lambda_0}(2\pi n'/\lambda_0)$$

$$\frac{d(n'k_0)}{d\lambda_0} = 2\pi\left(-\frac{D_0}{\lambda_0^2} + \frac{D_1 \lambda_{0c}}{\lambda_0^2}\right)$$

$$\therefore V_g = \frac{\omega \lambda_0}{2\pi (D_0 + D_1 \lambda_{0c})} = \frac{c}{D_0 - D_1 \lambda_{0c}}$$

c) $$\tau_g = \frac{L}{V_g} = \frac{L}{c}(D_0 - D_1 \lambda_{0c})$$

d) Since τ_g does not depend on λ_0, there is no chromatic dispersion. This can also be concluded from $\frac{d^2 n'}{d\lambda_0^2} = 0$.

4. a) $\Phi_T = 10\,\text{dBm} = 10\,\text{mW}$

$\Phi_R = \Phi_T e^{-\alpha L}$ $\qquad \alpha = \dfrac{\alpha_{dB}}{4.34} = 0.0576\,\text{km}^{-1}$

$$I_{ph} = R\Phi_R = \frac{\eta e \lambda}{hc}\Phi_T e^{-\alpha L}$$

$$SNR = \frac{I_{ph}}{\sqrt{\frac{4kT}{R}}\sqrt{\Delta f}} \qquad \Delta f \cong B/2$$

$$SNR^2 = \frac{\left(\frac{\eta e\lambda}{hc}\Phi_T\right)^2 e^{-2\alpha L}}{2kTB/R} = \frac{10^4}{2\times 3.38\times 10^{-34}\times 300\times 10^9} \times$$

$$\left(\frac{0.85\times 1.6\times 10^{-19}\times 1.51\times 10^{-6}\times 10^{-2}}{6.63\times 10^{-34}\times 3\times 10^8}\right)^2 e^{-2\alpha L}$$

$e^{2\alpha L} = 9.0\times 10^8 \qquad \alpha L = \ln(3\times 10^4) = 10.3$

$L_{max} = \dfrac{10.3}{.0576} = \underline{179\,\text{km}}$

b) $$SNR = \frac{I_{ph}}{\sqrt{2eI_{ph}}\sqrt{B/2}} = \sqrt{\frac{I_{ph}}{eB}}$$

$$\frac{SNR^2 Bhc}{\eta\lambda_0\Phi_T} = e^{-\alpha L} = \frac{12^2\times 10^9\times 6.63\times 10^{-34}\times 3\times 10^8}{0.85\times 1.51\times 10^{-6}\times 10^{-2}}$$

$e^{\alpha L} = 4.5\times 10^5$

$\alpha L = \ln(4.5\times 10^5) = 13.0 \qquad L_{max} = \underline{226\,\text{km}}$

c) For the thermal noise case, $SNR^2 \propto e^{-2\alpha L}/B$

$\therefore \dfrac{e^{-2\alpha L}}{B} = \dfrac{e^{-2\alpha(L+\Delta L)}}{B/2}$ giving $2e^{-2\alpha\Delta L} = 1$

$\therefore \Delta L = \dfrac{\ln 2}{2\alpha} = \underline{6.0\,\text{km}}$ ie enough to add 1.5 dB loss

For the shot noise case, $SNR^2 \propto e^{-\alpha L}/B$

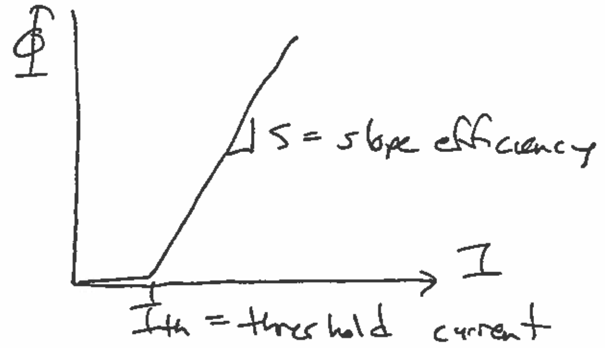
so in this case $\Delta L = \ln 2/\alpha = \underline{12.0\,\text{km}}$

– giving 3 dB extra loss ie halving the received power.

d) B may be limited by dispersion, or by bandwidth limits of receiver or transmitter modulation.

5.

a)



The threshold current arises because of the need to achieve sufficient photons in the cavity to ensure that stimulated emission becomes more likely than spontaneous emission. Its level depends on the end reflection coefficients, internal loss, and η_{int}

b) If we neglect the output up to I_{th}, then the output $\Phi = (I - I_{th}) S$ where $S = \eta hc/e\lambda$

$\therefore I = I_{th} + e\lambda\Phi/\eta hc \qquad \Phi = 6\,dBm = 4\,mW$

$= 3 + \dfrac{1.6\times10^{-19}\times1.51\times10^{-6}\times4\times10^{-3}}{0.8\times6.63\times10^{-34}\times3\times10^{8}} = \underline{3.61\ mA}$

c) For longitudinal modes, for cavity length L, we need $\frac{m_i\lambda_i}{2} = L$ with m_i the mode index.

$\lambda_i = \lambda_{oi}/n' \qquad f_i = c/\lambda_{oi} = cm_i/2n'L$

then $\Delta f_i = c/(2n'L)$

but the group velocity is $\approx c/n'$ so $\Delta f_i \cong V_g/2L$

$\Delta f_i^{-1} \cong (2L)/V_g$ = round-trip time.

d) η_i is the fraction of input electrons converted to photons

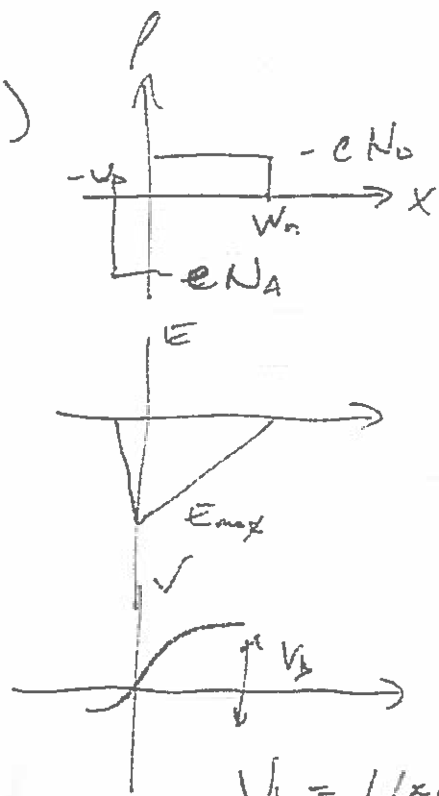
η_{ext} is the fraction of generated photons reaching the output.

In both LEDs and lasers, η_i is limited by the relative radiative and non-radiative recombination rates, but in lasers this is largely negated by stimulated emission.

η_{ext} is limited in LEDs by total internal reflection, surface (Fresnel) reflection, re-adsorption, and propagation downwards.

In lasers η_{ext} is much higher, limited by waveguide losses.

(6) a)



$$V_b = |E_{max}| \frac{(w_n + w_p)}{2}$$

$$|E_{max}| = \frac{e N_A w_p}{\varepsilon}$$

$$w_n = \frac{N_A}{N_D} w_p \qquad W = w_n + w_p$$

$$\therefore V_b = \frac{e N_A w_p^2 (1 + N_A/N_D)}{2\varepsilon}$$

$$= \frac{e N_A W^2}{2 \varepsilon_r \varepsilon_0 (1 + N_A/N_D)}$$

$$V_b = \frac{1.6\times10^{-19} \times 5\times10^{20} \times (5\times10^{-6})^2}{2\times12\times8.85\times10^{-12}(1+5)}$$

$$= \underline{1.57\ V}$$

b) $\eta = e^{-\alpha x_1} - e^{-\alpha x_2}$

$x_1 = h - w_p \qquad x_2 = h + w_n$

$$\eta = e^{-\alpha h}\left(e^{\alpha w_p} - e^{-5\alpha w_p}\right)$$

$\alpha h = 0.8\times10^5 \times 8\times10^{-6} = 0.64$

$\exp(-\alpha h) = 0.527$

$\frac{0.80}{0.527} = 1.52 = e^{+\theta} - e^{-5\theta} \qquad \theta = \alpha w_p$

Successive approximation: $\theta \simeq 0.48$

$$w_p = \frac{0.48}{0.8\times10^5} = 6\ \mu m$$

$W = 6w_p = 36\ \mu m$

$$V_b = 1.57\left(\frac{36}{5}\right)^2 = \underline{\underline{81\ V}}$$

(6)

c) $v_d = \mu E$

$$|E_{max}| = \frac{6 \times 10^4}{0.12} = 5.00 \times 10^5 \text{ V/m}$$

$$E_{max} = \frac{e N_A w_p}{\varepsilon} = \frac{2V_b}{W} = \sqrt{\frac{2 e N_A V_b}{\varepsilon(1+N_A/N_D)}}$$

$$V_b = |E_{max}|^2 \times \frac{\varepsilon_r \varepsilon_0 (1 + N_A/N_D)}{2 \times e N_A}$$

$$= \frac{(5.00 \times 10^5)^2 \times 12 \times 8.85 \times 10^{-12} (6)}{2 \times 1.6 \times 10^{-19} \times 5 \times 10^{20}}$$

$$= \underline{0.49 \text{ V}}$$