1a) i) Figure out that you need the fourier series expansion of $f(x)=x^2$ from $-1\leq x\leq 1$ [10]

$$a_0 = \int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} x^2 dx = 2\int_0^1 x^2 dx = \frac{2}{3}$$ [5]

$$a_n = \int_{-1}^{1} f(x)\cos(n\pi x)\,dx = 2\int_0^1 x^2\cos(n\pi x)\,dx$$

$$= \frac{2}{n\pi}\int_0^1 x^2\, d(\sin(n\pi x))$$

$$= \frac{2}{n\pi}\Big[x^2\sin(n\pi x)\Big]_0^1 - \frac{2}{n\pi}\int_0^1 2x\sin(n\pi x)\,dx$$

$$= 0 + \frac{4}{n^2\pi^2}\int_0^1 x\, d\cos(n\pi x)$$

$$= \frac{4}{n^2\pi^2}\Big[x\cos(n\pi x)\Big]_0^1 - \frac{4}{n^2\pi^2}\int_0^1 \cos(n\pi x)\,dx$$

$$= \frac{4}{n^2\pi^2}\cos(n\pi) - \frac{4}{n^3\pi^3}\Big[\sin(n\pi x)\Big]_0^1$$

$$= \frac{4}{n^2\pi^2}(-1)^n$$ [10]

$$b_n = \int_{-1}^{1} f(x)\sin(n\pi x)\,dx = 0$$ [5]

$$f(x)=x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(n\pi x) + b_n\sin(n\pi x)$$

$$= \frac{1}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2\pi^2}\cos(n\pi x)$$ [5]

ii) Figure out that you need to sub $x=1$ in the above. [10]

$$f(1) = 1 = \frac{1}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2\pi^2}\cos(n\pi)$$ [3]

$$\Rightarrow \frac{2}{3} = \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6} = 1.65$$ [2]