Fundamental constants

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ m kg/C^2

$c = 3 \times 10^8$ m/s

$h = 6.62 \times 10^{-34}$ Js

Maxwell's equations – integral form

$\iint_A \underline{\mathbf{D}} \cdot d\underline{a} = \iiint_V \rho \, dv$

$\iint_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = - \iint_A \partial\underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \iint_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

Maxwell's equations – differential form

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

Material equations

$\underline{\mathbf{J}} = \sigma \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu \underline{\mathbf{H}}$

Vector calculus (Cartesian co-ordinates)

$\mathrm{grad}(\phi) = \partial\phi/\partial x \, \underline{i} + \partial\phi/\partial y \, \underline{j} + \partial\phi/\partial z \, \underline{k}$

$\mathrm{div}(\underline{\mathbf{F}}) = \partial\mathbf{F}_x/\partial x + \partial\mathbf{F}_y/\partial y + \partial\mathbf{F}_z/\partial z$

$\mathrm{curl}(\underline{\mathbf{F}}) = \underline{i} \{\partial\mathbf{F}_z/\partial y - \partial\mathbf{F}_y/\partial z\} + \underline{j} \{\partial\mathbf{F}_x/\partial z - \partial\mathbf{F}_z/\partial x\} + \underline{k} \{\partial\mathbf{F}_y/\partial x - \partial\mathbf{F}_x/\partial y\}$

$\mathrm{curl}\,\{\mathrm{curl}(\underline{\mathbf{F}})\} = \mathrm{grad}\,\{\mathrm{div}(\underline{\mathbf{F}})\} - \nabla^2\underline{\mathbf{F}}$

$\iint_A \underline{\mathbf{F}} \cdot d\underline{a} = \iiint_V \mathrm{div}(\underline{\mathbf{F}}) \, dv$

$\int_L \underline{\mathbf{F}} \cdot d\underline{L} = \iint_A \mathrm{curl}(\underline{\mathbf{F}}) \cdot d\underline{a}$

1. The differential forms of Maxwell's equations are given below, together with the material equations.

a) Identify all the field quantities. Explain why the equations are typically solved for the time-independent electric field at optical wavelengths, rather than any other field quantity. Show how the Maxwell equations are reduced to their time-independent equivalents at angular frequency ω.

[8]

b) What assumptions are typically made in solving Maxwell's equations for problems in optics? Using these assumptions, derive a time-independent scalar wave equation for an infinite dielectric medium with relative permittivity ε_r, for polarization in the x-direction.

[6]

c) Find a solution to your equation for an infinite plane wave travelling in the z-direction. Find the propagation constant k, and rewrite your time independent scalar wave equation in terms of the free-space propagation constant k_0 and the refractive index n.

[6]

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$ $\qquad$ $\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$ $\qquad$ $\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

$\underline{\mathbf{J}} = \sigma\,\underline{\mathbf{E}}$ $\qquad$ $\underline{\mathbf{D}} = \varepsilon\,\underline{\mathbf{E}}$ $\qquad$ $\underline{\mathbf{B}} = \mu\,\underline{\mathbf{H}}$

2. A symmetric slab waveguide of thickness h is formed from a core of refractive index n_1 surrounded by cladding layers of refractive index n_2. For TE modes propagating in a guide arranged parallel to the z-direction, electric field solutions for the i^{th} layer can be written as $E_{yi}(x, z) = E_i(x)\exp(-j\beta z)$ where $E_i(x)$ is the transverse field and β is the propagation constant at wavelength λ.

a) For each of the layers, write down the waveguide equation and an assumed solution for the transverse field valid for guided modes.

[4]

b) Giving your reasoning, explain the boundary conditions that must be satisfied at the interfaces between the layers? Applying the boundary conditions, show that the eigenvalue equation can be obtained as:

$\tan(\kappa h/2) = \gamma/\kappa$ or $\tan(\kappa h/2) = -\kappa/\gamma$

Where $\kappa = \sqrt{(n_1^2 k_0^2 - \beta^2)}$ and $\gamma = \sqrt{(\beta^2 - n_2^2 k_0^2)}$ and $k_0 = 2\pi/\lambda$

[12]

c) When does cutoff occur for a guided mode? For the symmetric guide, show that the cutoff condition of the ν^{th} mode is

$k_0 h \sqrt{(n_1^2 - n_2^2)} = \nu\pi$.

[4]

3. Figure 3.1 shows four different examples of single-mode channel waveguide devices based on Y-junctions. In each case, the ports are numbered, together with various intermediate locations, and the direction of the input is indicated.

a) Assuming that the input has unity relative amplitude and power, calculate the corresponding output amplitudes and powers in cases i) and ii).

[8]

b) Assuming the two inputs both have unity relative amplitude and power, calculate the corresponding output amplitudes and powers in case iii) and iv). What happens if the sign of the one of the input amplitudes is reversed?

[12]

In each case, explain your reasoning and present results at any intermediate locations.

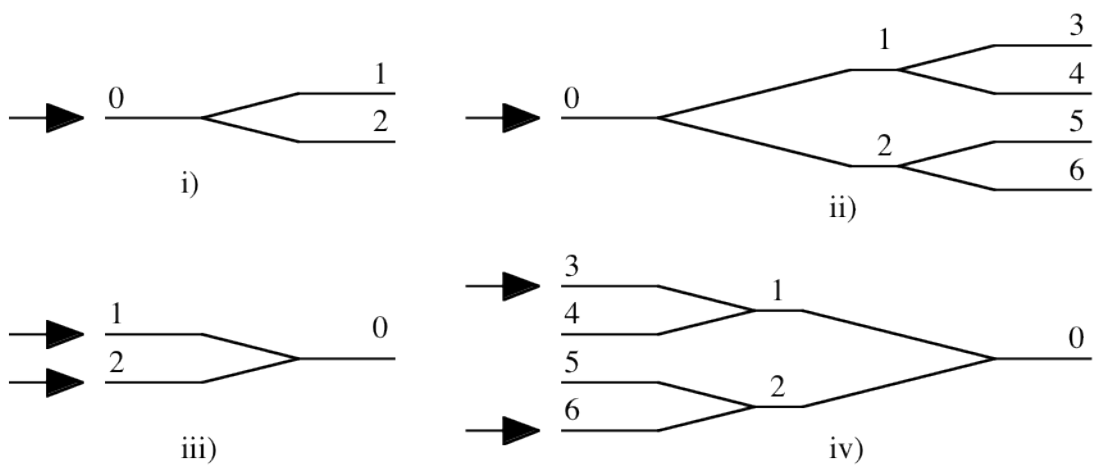


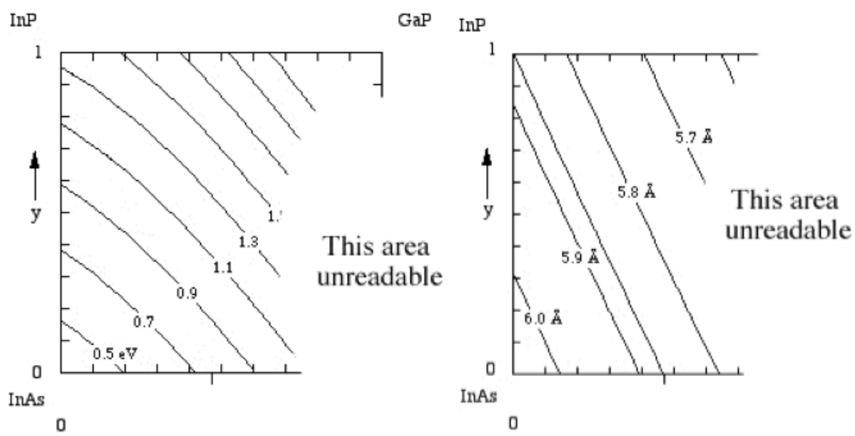
Figure 3.1

4. An unfortunate fire has occurred in the clean room at STARBRITE OPTO. A partially burned page has been recovered from a logbook and is reproduced as Figure 4.1:

DETECTOR GROUP

RECORDING PERIOD: August 201
GROUP LEADER Richard S

InP : $In_{1-x}\,Ga_xAs_{\,1-y}\,P_y$ Substrate entry PIN photodiodes



CONFIDENTIAL STARBR

Figure 4.1

a) Why are STARBRITE using the InP : InGaAsP materials system? Why have they adopted a PIN photodiode configuration, rather than a simple PN junction?

[4]

b) What material lies at the bottom right-hand side of the plots? What material are STARBRITE using for the substrate, and why? What material are they using for the intrinsic layer, and why? Sketch a possible configuration for the PIN diode

[8]

c) What is the shortest detectable wavelength, and its corresponding energy? What is the longest detectable wavelength, and its corresponding energy? Estimate the responsivity of the detector at $\lambda = 1.55\ \mu m$, assuming a quantum efficiency of 80%

[8]

5. a) Describe the processes of absorption, spontaneous emission and stimulated emission in a semiconductor.

[6]

b) Explain why devices based on stimulated emission are the preferred sources in optical communications systems.

[8]

c) Explain why the external efficiency is so poor for a light-emitting diode, and estimate its value for a semiconductor of refractive index 3.5.

[6]

6. A semiconductor laser is constructed from a length L of an indium phosphide-based waveguide with effective index n_{eff} and gain coefficient g.

a) Assuming that guided mode with transverse field E(x, y) and propagation constant $\beta = 2\pi n_{eff}/\lambda$ at wavelength λ is propagating in the z-direction in the cavity, find an expression for the field after one round trip. Hence, write down the phase and gain conditions that must be satisfied at the threshold of lasing.

[6]

b) Derive an expression for the separation between adjacent spectral lines, and sketch the spectral variation in output power.

[4]

c) Assuming that $\lambda = 1.5$ μm, L = 250 μm and $n_{eff} = 3.5$, find the reflectivity of the cavity end mirrors, the gain coefficient needed to reach the threshold, and the separation between adjacent spectral lines in the laser output.

[6]

d) If the cavity length is doubled, what will happen to the reflectivity, gain coefficient and spectral separation above?

[4]

Optoelectronics Solutions 2013

1a) The field quantities are:

$\underline{\mathbf{E}}$ - Electric field
$\underline{\mathbf{D}}$ - Electric flux density
$\underline{\mathbf{H}}$ - Magnetic field
$\underline{\mathbf{B}}$ – Magnetic flux density
$\underline{\mathbf{J}}$ – Current density

[3]

At a wavelength λ, each quantity varies at a frequency $f = c/\lambda$, where c is the speed of light. At optical wavelengths (say, $\lambda = 0.5\ \mu m$), $f \approx 3 \times 10^8 / 0.5 \times 10^{-6} = 6 \times 10^{14}$ Hz. This frequency is so high that none of the quantities may be observed directly. As a result, they become individually less meaningful. All detection systems measure time-averaged power, which may be related to the time independent electric field $\underline{E}$ relatively simply. Consequently, E is normally the most useful field quantity.

Common errors: failure to understand that power is the only detectable quantity.

[3]

Assuming that the field quantities all vary at a single angular frequency ω, we may write $\underline{\mathbf{E}} = \underline{E} \exp(j\omega t)$, $\underline{\mathbf{H}} = \underline{H} \exp(j\omega t)$ etc. Maxwell's equations then become:

$\text{div}(\underline{D}) = \rho$ $\qquad$ $\text{div}(\underline{B}) = 0$

$\text{curl}(\underline{E}) = -j\omega\underline{B}$ $\qquad$ $\text{curl}(\underline{H}) = \underline{J} + j\omega\underline{D}$

Common errors: failure to try a separable solution.

[2]

b) The assumptions normally made for optical problems are:

Materials are non-magnetic, so that the permeability μ can be written as μ_0

Materials are non-conducting, so that the conductivity σ can be taken as zero; hence $\underline{J}$ must also be zero.

There are no free charges, so that the charge density ρ can be taken as zero

Materials are dielectric so that ε can be written as $\varepsilon_0\varepsilon_r$

Common errors: failure to recognise that optical materials are generally transparent insulators.

[3]

Hence, the time-independent Maxwell equations become:

$\text{div}(\underline{D}) = 0$ $\qquad$ $\text{div}(\underline{B}) = 0$

$\text{curl}(\underline{E}) = -j\omega\mu_0\underline{H}$ $\qquad$ $\text{curl}(\underline{H}) = j\omega\varepsilon_0\varepsilon_r\underline{E}$

Hence $\text{curl}\,\{\text{curl}(\underline{E})\} = -j\omega\mu_0\,\text{curl}(\underline{H}) = -j\omega\mu_0\, j\omega\varepsilon_0\varepsilon_r\underline{E} = \omega^2\mu_0\varepsilon_0\varepsilon_r\underline{E}$

Using the identity $\text{curl}\,\{\text{curl}(\underline{\mathbf{F}})\} = \text{grad}\,\{\text{div}(\underline{\mathbf{F}})\} - \nabla^2\underline{\mathbf{F}}$ from the formula sheet we get:

$\text{grad}\,\{\text{div}(\underline{E})\} - \nabla^2\underline{E} = \omega^2\mu_0\varepsilon_0\varepsilon_r\underline{E}$

Now $\text{div}(\underline{D}) = \text{div}(\varepsilon\underline{E}) = 0$. If the medium is infinite, we must have $\text{div}(\underline{E}) = 0$.

Hence, the vector wave equation is $\nabla^2\underline{E} + \omega^2\mu_0\varepsilon_0\varepsilon_r\underline{E} = 0$.

Assuming the field is polarized in the x-direction, the scalar wave equation is:

$\nabla^2 E_x + \omega^2\mu_0\varepsilon_0\varepsilon_r E_x = 0$

Common errors: failure to use the curl curl identity to simplify.

[3]

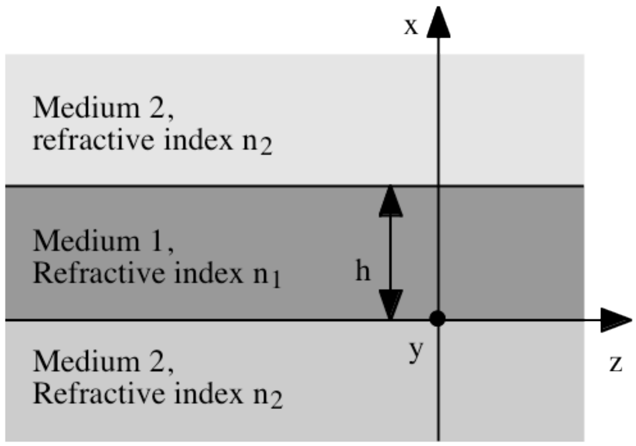
c) Assuming that the electric field is a plane wave travelling in the x-direction, we can write:
$E_x = E_0 \exp(-jkz)$ where k is the propagation constant. Since there is no variation in the x- and y-directions, the wave equation reduces to $d^2E_x/dz^2 + \omega^2\mu_0\varepsilon_0\varepsilon_r E_x = 0$
Differentiating the assumed solution, we obtain $d^2E_x/dz^2 = -k^2E_x$
Substituting into the wave equation, we obtain $-k^2E_x + \omega^2\mu_0\varepsilon_0\varepsilon_r E_x = 0$
Hence, the propagation constant is $k = \omega\sqrt{(\mu_0\varepsilon_0\varepsilon_r)}$
Common errors: failure to remember that there is no variation in x and y.

[3]

In free space, we must have $k_0 = \omega\sqrt{(\mu_0\varepsilon_0)}$.
Hence, $k = k_0 n$, where $n = \sqrt{(\varepsilon_r)}$ is the refractive index.
Hence, the scalar wave equation can generally be written as $\nabla^2E_x + n^2k^2E_x = 0$
Common errors: failure to remember the relation between n and ε_r.

[3]

2. a) The waveguide geometry is as follows:



For TE problems, the electric field is polarised in the y-direction. At wavelength λ, the scalar wave equation we must solve in each layer is given by:
$\nabla^2 E_{yi}(x, z) + n_i^2 k_0^2 E_{yi}(x, z) = 0$ (where i = 1, 2, 3 and $k_0 = 2\pi/\lambda$)
We assume solutions in the form of guided modes, as $E_{yi}(x, z) = E_i(x) \exp(-j\beta z)$
Here $E_i(x)$ is the transverse field and β is the propagation constant.
The waveguide equation can be found by substituting into the wave equation to get:
$d^2E_i/dx^2 + [n_i^2 k_0^2 - \beta^2] E_i = 0$
Common errors: most candidates get this right.

[2]

The waveguide equation is a second order differential equation, whose solutions are sines and cosines or hyperbolic functions. If we define the constants κ and γ as:
$\kappa = \sqrt{}(n_1^2 k_0^2 - \beta^2)$
$\gamma = \sqrt{}(\beta^2 - n_2^2 k_0^2)$
Suitable solutions for the transverse fields in the three layers are then:
$E_1 = E \cos(\kappa x - \phi)$ (core layer, $0 \leq x \leq h$)
$E_2 = E' \exp(\gamma x)$ (buffer layer, $x \leq 0$)
$E_3 = E'' \exp\{-\gamma(x - h)\}$ (cladding layer, $x \geq h$)
Common errors: inappropriate trial solutions.

[2]

b) The boundary conditions that must be satisfied are continuity of the tangential components of the electric and magnetic fields. The electric field is wholly tangential, so we must have:
$E_1 = E_2$ on $x = 0$
$E_1 = E_3$ on $x = h$
Common errors: most candidates get this right.

[2]

The magnetic field can be found from the curl relation:
curl($\underline{\mathbf{E}}$) = $-\partial\underline{\mathbf{B}}/\partial t$
curl($\underline{E}$) = $-j\omega\underline{B}$
curl($\underline{E}$) = $-j\omega\mu_0\underline{H}$

Evaluating the curl expression for the assumed electric field polarization, we get:

$$\mathrm{curl}(\underline{E}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix}$$

Or $\mathrm{curl}(\underline{E}) = -\partial E_y/\partial z\ \underline{i} + \partial E_y/\partial x\ \underline{k}$

The magnetic field therefore has only two components. However, the only tangential component is in the k-direction. Continuity of this component requires that:

$\partial E_{y1}/\partial x = \partial E_{y2}/\partial x$ on $x = 0$

$\partial E_{y1}/\partial x = \partial E_{y3}/\partial x$ on $x = h$

Common errors: forgetting to use the curl relation (given in the formula sheet).

[4]

Applying the first pair of boundary conditions, we get:

$E \cos(\phi) = E'$ (1)

$E \cos(\kappa h - \phi) = E''$ (2)

Differentiating the assumed solutions, we get:

$\partial E_1/\partial x = -\kappa E \sin(\kappa x - \phi)$

$\partial E_2/\partial x = \gamma E' \exp(\gamma x)$

$\partial E_3/\partial x = -\gamma E'' \exp\{-\gamma(x - h)\}$

Applying the second pair of boundary conditions, we get:

$\kappa E \sin(\phi) = \gamma E'$ (3)

$\kappa E \sin(\kappa h - \phi) = \gamma E''$ (4)

Common errors: most candidates who get this far get this right.

[3]

Dividing Equation 3 by Equation 1 we get:

$\tan(\phi) = \gamma/\kappa$ (5)

Dividing Equation 4 by Equation 2 we get:

$\tan(\kappa h - \phi) = \gamma/\kappa$ (6)

Expanding Equation (6) we get:

$\{\tan(\kappa h) - \tan(\phi)\}/ \{1 + \tan(\kappa h) \tan(\phi)\} = \gamma/\kappa$

Substituting using Equation 5 we get:

$\{\tan(\kappa h) - \gamma/\kappa\} / \{1 + \tan(\kappa h)\, \gamma/\kappa \} = \gamma/\kappa$

Rearranging, we get:

$\{\tan(\kappa h) - \gamma/\kappa\} = (\gamma/\kappa)\{1 + \tan(\kappa h)\, \gamma/\kappa \}$

$\tan(\kappa h) \{1 - \gamma^2/\kappa^2\} = 2\gamma/\kappa$

$\tan(\kappa h) = 2\gamma/\kappa / \{1 - \gamma^2/\kappa^2\}$ (7)

Expanding Equation 7, we can obtain:

$\tan(\kappa h/2) = \gamma/\kappa$ or

$\tan(\kappa h/2) = -\kappa/\gamma$

Common errors: forgetting the tan double angle relation.

[3]

c) Cutoff of a guided mode occurs when total internal reflection no longer takes place at one of the interfaces.

[1]

In a symmetric guide, the breakdown of total internal reflection occurs at the two interfaces simultaneously. Under these conditions, the mode is no longer confined and the external decay constant $\gamma = (\beta^2 - n_2^2 k_0^2)$ tends to zero. Hence we must have:
$\beta = n_2 k_0$ and $\kappa = \sqrt{(n_1^2 k_0^2 - \beta^2)} = k_0 \sqrt{(n_1^2 - n_2^2)}$

We must also have $\tan(\kappa h/2) = 0$ or minus infinity
Hence $\kappa h/2 = \nu\pi/2$, where $\nu = 0, 1, 2 \ldots$.
And $k_0 h \sqrt{(n_1^2 - n_2^2)} = \nu\pi$
Common errors: forgetting that γ tends to zero when TIR breaks down.

[3]

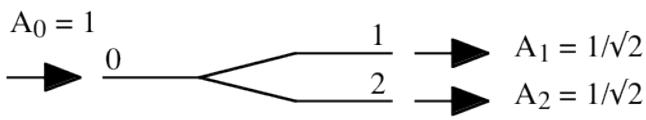
3a) At its single input, a single-moded symmetric Y-junction supports just one guided mode. However, at its two outputs, it supports a pair of symmetric and anti-symmetric supermodes. The symmetric mode gradually converts into the symmetric supermode along the length of the device. However, the anti-symmetric mode can only exist when the fork of the Y-junction is wide enough. Consequently, its cut-off point provides a mechanism for power to be lost by radiation into the substrate. In the calculations that follow we can assume that the relative amplitude is the square root of the relative power. **Common errors: not writing enough on this part.**

[2]

Case i): The single symmetric mode is excited, and gradually converts into the symmetric supermode without radiation. All of the input power emerges from the two guided ports, and is divided between them symmetrically. Thus we have

$P_1 = P_2 = 1/2$ $\qquad$ $A_1 = A_2 = 1/\surd 2$.

[2]



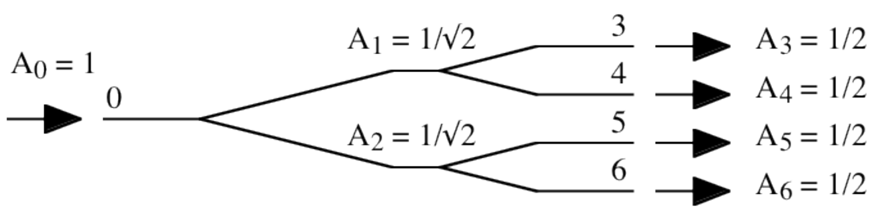
Case ii): The two stages of the Y-junction tree are symmetrically excited. Consequently, at each stage, the power is symmetrically divided without radiation. Thus we have:

$P_1 = P_2 = 1/2$ $\qquad$ $A_1 = A_2 = 1/\surd 2$.

[2]

$P_3 = P_4 = P_5 = P_6 = 1/4$ $\qquad$ $A_3 = A_4 = A_5 = A_6 = 1/2$.

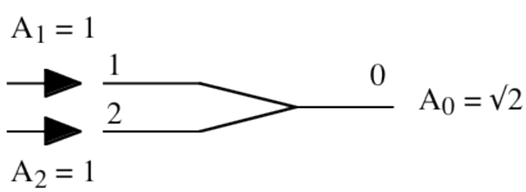
[2]



b) Case iii): The symmetric supermode is excited. Consequently, the powers injected into the two inputs are combined at the Y-junction fork, without radiation. Thus we have:

$P_0 = 2$ $\qquad$ $A_0 = \surd 2$

[2]

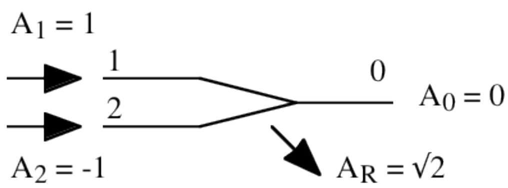


If, however, the sign of one of the inputs (say, A_2) is reversed, it is the anti-symmetric mode that is excited. All the power that it carries will be radiated at its cutoff point. Thus we have:

$P_0 = 0$ $\qquad$ $A_0 = 0$

$P_R = 2$ $\qquad$ $A_R = \surd 2$

[2]



Case iv): The input Y-junctions are not symmetrically excited. However, the excitation pattern corresponds to injection of both supermodes together with equal amplitude of 1/2, so that the inputs add to give unity in (say) port 3 and zero in port 4. Each supermode carries half the input power, but the component carried by the antisymmetric mode will be radiated. Consequently we have:

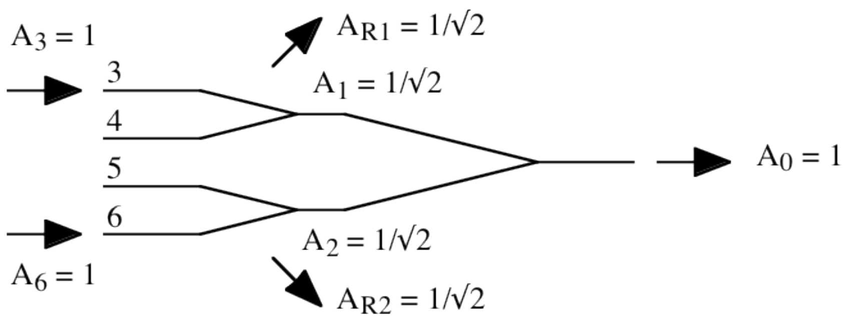
$P_1 = 1/2 = P_2$ $\qquad$ $A_1 = 1/\sqrt{2} = A_2$
$P_{R1} = 1/2 = P_{R2}$ $\qquad$ $A_{R1} = 1/\sqrt{2} = A_{R2}$

[2]

The input pattern presented to the final Y-junction is symmetric, which combines its components without further loss of power. Consequently we have:

$P_0 = 1$ $\qquad$ $A_0 = 1$

[2]



If the sign of one of the inputs (say, A_6) is reversed, the signs of A_2 and A_{R2} must change. Thus we must have:

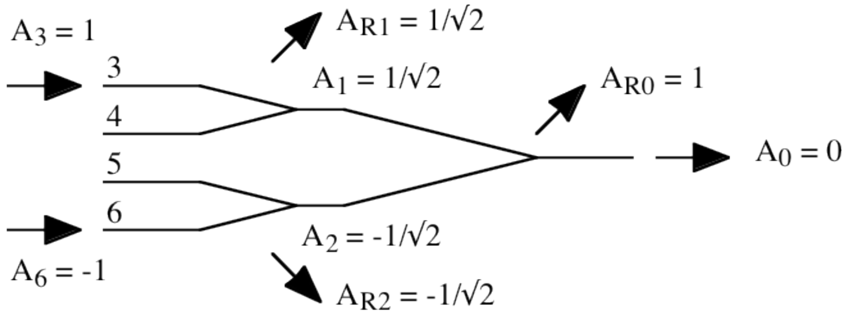
$P_1 = 1/2 = P_2$ $\qquad$ $A_1 = 1/\sqrt{2} = -A_2$
$P_{R1} = 1/2 = P_{R2}$ $\qquad$ $A_{R1} = 1/\sqrt{2} = -A_{R2}$

[2]

The input pattern presented to the final Y-junction is now anti-symmetric. All the remaining power must therefore be radiated. Consequently we have:

$P_0 = 0$ $\qquad$ $A_0 = 0$
$P_{R0} = 1$ $\qquad$ $A_{R1} = 1$

[2]



Common errors: not breaking the calculation down into steps and/or not drawing diagrams.

4. a) STARBRITE are using the InP : InGaAsP materials system because it allows fabrication of sources and detectors that are frequency tuneable over a broad range that includes the important low-loss window for silica optical fibre at1.55 μm wavelength.
Common errors: most candidates got this right.

[2]

PN junctions make efficient light detectors because the large in-built field in the depletion layer will separate any photo-generated carrier pairs very rapidly, preventing then from recombining. STARBRITE are using PIN photodiodes because the thick intrinsic layer increases the width of the depletion layer, and hence makes it more likely that photons will be absorbed in a high-field region.
Common errors: forgetting that a PIN diode has a thick depletion layer.

[2]

b) The bottom RHS of the plots lie on the x-axis, at x = 1, y = 0. For $In_{1-x}Ga_xAs_{1-y}P_y$ this co ordinate corresponds to GaAs.
Common errors: most candidates got this right.

[1]

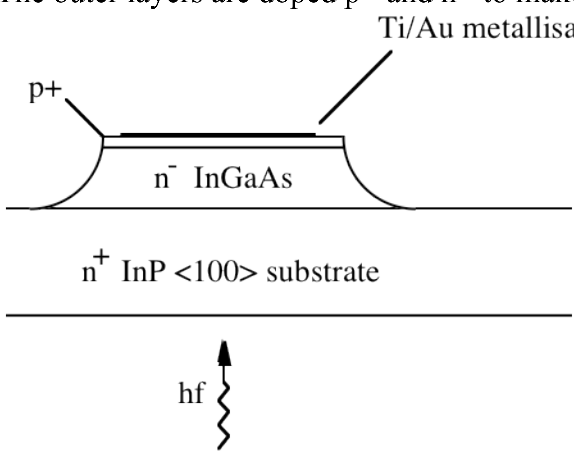
STARBRITE must start with an InP substrate, since this is a binary compound and hence relatively easy to grow from a melt while maintaining stoichiometry. It also has a large bandgap, so that long wavelength photons can pass through it without absorption in a substrate entry device.
Common errors: forgetting that a binary substrate must be the starting point.

[2]

For the intrinsic layer, STARBRITE will use the compound lattice matched to InP that has the lowest possible band-gap, to catch as many long-wavelength photons as possible. From the right-hand plot, this material must be $In_{0.53}Ga_{0.47}As$.
Common errors: forgetting that a binary substrate must be the starting point.

[2]

A possible configuration for the PIN diode is shown below.
The substrate is InP, and light enters through it.
The other two layers are InGaAs
The central layer is lightly doped to approximate to intrinsic material
The outer layers are doped p+ and n+ to make a PIN structure



Common errors: lack of detail in the diagram, or appropriate explanation.

[3]

c) The shortest detectable wavelength is determined by the bandgap of InP, because this energy sets the limit for substrate transparency. From the left-hand plot, $E_G(InP) = 1.35$ eV

[1]

Since the energy of a photon is hc/λ, $\lambda_{min} = hc/eE_G$
Hence $\lambda_{min} = (6.62 \times 10^{-34} \times 3 \times 10^8) / (1.6 \times 10^{-19} \times 1.35) = 9.2 \times 10^{-7}$ m, or 0.92 μm
Common errors: most candidates got this right.

[2]

The longest detectable wavelength is determined by the bandgap on InGaAs, because longer wavelength photons will pass through the intrinsic layer without absorption. From the left-hand plot, $E_G(InGaAs) = 0.74$ eV

[1]

Hence $\lambda_{max} = (6.62 \times 10^{-34} \times 3 \times 10^8) / (1.6 \times 10^{-19} \times 0.74) = 1.68 \times 10^{-6}$ m, or 1.68 μm
Common errors: most candidates got this right.

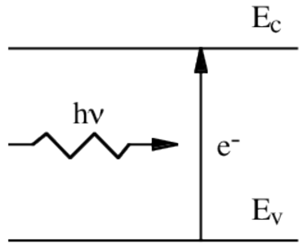
[2]

In a beam of optical power P, the number of photons arriving per second must be $P\lambda/hc$
Ideally, the number of carrier pairs generated per sec must also be $P\lambda/hc$
In practice, this value is reduced by the quantum efficiency η to $P\eta\lambda/hc$
The photocurrent must therefore be $I_P = Pe\eta\lambda/hc$
The responsivity $R = I_P/P$ must then be $R = e\eta\lambda/hc$

Hence, for $\lambda = 1.55$ μm and $\eta = 0.8$:
$R = (1.6 \times 10^{-19} \times 0.8 \times 1.55 \times 10^{-6}) / (6.62 \times 10^{-34} \times 3 \times 10^8) = 0.999$ A/W
Common error: using Amps instead of Amps per Watt.

[2]

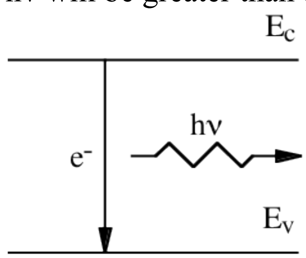
5a) Optical absorption involves the promotion of an electron from the valence band to the conduction band using the energy hν of a photon of frequency ν, where h is Planck's constant, as shown below. For absorption to occur at all, hν must be greater than the energy gap E_c - E_v,.



Common errors: most candidates got this right.

[2]

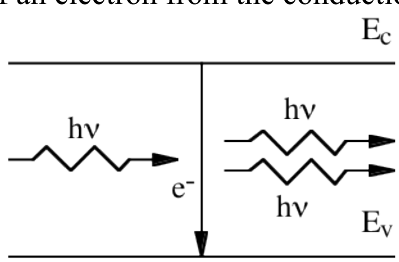
Spontaneous emission involves a random transition of an electron from the conduction band to the valence band, emitting a photon in the process. Once again, the photon energy hν will be greater than the energy gap E_c - E_v.



Common errors: most candidates got this right.

[2]

Stimulated emission is a mechanism for light production involving a downward transition of an electron from the conduction band to the valence band that is triggered by a photon.



Common errors: most candidates got this right.

[2]

b) Due to the spread of electron and hole energies, spontaneously emitted photons are emitted across a very broad spectral band, greatly exceeding any likely signal bandwidth. Consequently, the light cannot be used as a single-frequency carrier. The photons are also emitted in random directions, leading to poor external efficiency and a poor coupling efficiency into a low numerical aperture system such as a single-mode optical fibre.

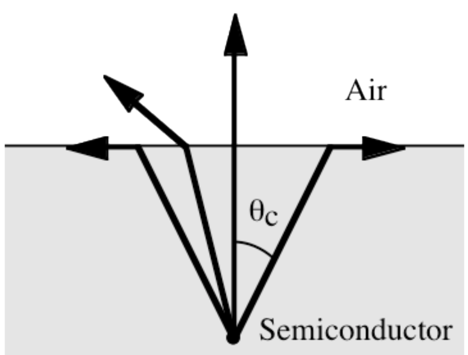
Devices relying on spontaneous emission such as LEDs are inherently low power. Finally, the modulation bandwidth of a LED is limited by recombination time to around 100 MHz.
Common errors: lack of detail in the description.

[4]

In contrast, photons emitted by stimulated emission are identical in frequency, phase, direction and polarisation to the stimulating photon. Consequently, the light has excellent spectral properties, and can be used as a single frequency carrier. The light can also be coupled efficiently out of the semiconductor and into an optical fibre. Much higher powers can be generated when stimulated emission is combined with feedback into a resonant structure such as a laser. Modulation of a laser is limited by relaxation phenomena to multiple GHz rates.
Common errors: lack of detail in the description.

[4]

c) Photons generated by spontaneous emission inside a LED will escape only if they are incident on the semiconductor-air interface at an angle less than the critical angle θ_c as shown below.



[2]

Refraction at a boundary between two media is governed by Snell's law $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$, where n_1 and n_2 are the refractive indices of the two media and θ_1 and θ_2 are the angles of the incident and refracted waves.

When total internal reflection occurs, $\theta_2 = \pi/2$ and $\sin(\theta_1) = \sin(\theta_c) = n_2/n_1$. In this case, $n_2 = 1$ (air) and n_1 is large (semiconductor) so we may write $\theta_c \approx 1/n_1$.

The fraction of the generated radiation lying within a cone with half-angle θ_c can be found by comparing the surface area of a sphere of radius R lying within this cone with its total surface area, as;
$F = \pi\theta_c^2R^2 / 4\pi R^2 = \theta_c^2/4 = 1/4n_1^2$

Not all this fraction is transmitted, since some must be reflected. At normal incidence, the (amplitude) reflection coefficient is $R = (1 - n_1)/(1 + n_1)$, and the power reflection coefficient is $P_R = R^2$. The power transmission coefficient is then $P_T = 1 - P_R$, or:
$P_T = 1 - (1 - n_1)^2/(1 + n_1)^2 = 4n_1/(1 + n_1)^2$

The external efficiency η_e is then:
$\eta_e = F \times P_T = (1/4n_1^2) \times 4n_1/(1 + n_1)^2 = 1/\{n_1(1 + n_1)^2\}$
If $n_1 = 3.5$, we obtain $\eta_e = 1/(3.5 \times 4.5^2) = 0.014$, or 1.4%
Common errors: forgetting to include the power transmission coefficient.

[4]

6. a) Assuming that the amplitude of the mode is initially $E(x, y)$, its amplitude after one round trip is $E(x, y)\, R^2 \exp(-j2\beta L) \exp(2gL)$

Common errors: most candidates got this right.

[2]

The lasing threshold is reached when the round trip gain is unity. In this case:

$R^2 \exp(-j2\beta L) \exp(2gL) = 1$

Common errors: most candidates got this right.

Hence we must have:

$2\beta L = 2\nu\pi$, where ν is an integer (Phase condition)

$R^2 \exp(2gL) = 1$ (Gain condition)

[4]

Common errors: failure to separate the phase and gain conditions.

b) Expanding the phase condition, we can write $2(2\pi n_{eff}/\lambda)L = 2\nu\pi$

Hence $(2n_{eff}/\lambda)L = \nu$ and $\lambda_\nu = 2n_{eff}L/\nu$.

The output is therefore a discrete series of lines. The separation between the n^{th} and $n+1^{th}$ line is:

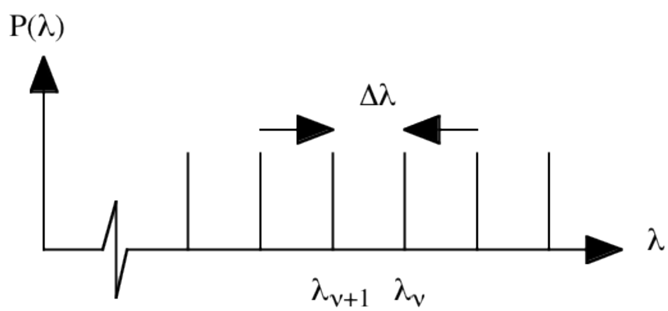
$\lambda_\nu - \lambda_{\nu+1} = \Delta\lambda = 2n_{eff}L\{1/\nu - 1/(\nu + 1)\} = \Delta\lambda$

$\Delta\lambda = 2n_{eff}L/[\nu(\nu + 1)] \approx 2n_{eff}L/\nu^2 = \lambda_\nu^2/2n_{eff}L$

Common errors: failure to approximate $\nu(\nu + 1)$ with ν^2.

[3]

Ignoring any wavelength dependence of gain, the spectral variation of the output is therefore a discrete set of lines with approximately regular spacing, as shown below.



Common errors: most candidates got this right.

[1]

c) The interface at each end of the cavity is between the semiconductor and air. The reflection coefficient is therefore $R = (n - 1) / (n + 1) = (3.5 – 1) / (3.5 + 1) = 0.555$.

Common errors: forgetting the formula for the reflection coeffcient.

[2]

Re-arranging the gain condition, we obtain $g = (1/2L) \log_e(1/R^2)$

Hence, the gain coefficient needed is $g = \log_e(1/0.555^2)\ ./\ (2 \times 250 \times 10^{-6}) = 2355\ m^{-1}$.

Common errors: cascaded error due to the previous part.

[2]

The separation between adjacent spectral lines is
$\Delta\lambda = (1.5 \times 10^{-6})^2 / (2 \times 3.5 \times 250 \times 10^{-6}) = 1.286 \times 10^{-9}$ m, or 1.286 nm
Common errors: most candidates got this right.

[2]

d) If the cavity length is doubled, the mirror reflectivity will be unchanged. The necessary gain coefficient will halve, and the spectral separation will also halve.
Common errors: failure to try some example numbers.

[4]