

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2015

EEE PART IV: MEng and ACGI

Corrected Copy

**POWER SYSTEM ECONOMICS**

Thursday, 21 May 10:00 am

Time allowed: 3:00 hours

**There are FOUR questions on this paper.**

**Answer ALL questions.**

*All questions carry equal marks.*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      G. Strbac  
Second Marker(s) :      B.C. Pal

# Power System Economics

## Question 1

The inverse demand function of a consumer is given by the following expression

$$p = -10 \cdot D + 2000 \text{ [£/unit]}$$

where  $D$  is the demand level in units and  $p$  is the per unit price of the product.

The cost of production of this product is given by the following expression:

$$C = \frac{1}{2} 5 \cdot Q^2 + 200Q \text{ [£]}$$

where  $C$  is the total cost and  $Q$  is the production level in units.

a. Calculate the maximum consumption of the consumer, the maximum price the consumer is prepared to pay and the maximum consumer's surplus. Derive the expression for the gross benefit of this consumer as a function of i) demand level and ii) price. [5]

b. For the producer of the market, derive the expression for i) average cost, ii) marginal cost, iii) gross revenue when the output is sold at marginal cost and iv) profit when the output is sold at marginal cost. [4]

c. Determine the equilibrium demand and price for this market. [2]

d. For this equilibrium state, calculate i) the consumer's gross benefit and surplus, ii) the producer's average cost, gross revenue and profit, and iii) the social welfare. [6]

e. Determine if an artificial increase of the price by 20% with respect to the equilibrium level is beneficial for the consumer, the producer and the society overall. [3]

## Question 2

- a) Explain why in a real (imperfect) market each company must consider the possible actions of others when selecting their strategy? [4]

- b) Consider a market for electrical energy that is supplied by two generating companies whose costs are:

$$C_a = 20 P_a \text{ [£/h]}$$

$$C_b = 30 P_b \text{ [£/h]}$$

The inverse demand function is given by:  $p = 100 - D$

where  $D$  is the demand level and  $p$  is the per unit price.

- (i) Assuming Bertrand competition, calculate production of each of the companies, market price and profits made. [5]
- (ii) Assuming Cournot competition, calculate the production of the two companies, the market price and the profits made at the equilibrium state. [7]
- (iii) Explain the differences in the solution in (i) and (ii) [4]

### Question 3

Consider the three-bus power system shown on Figure Q3.

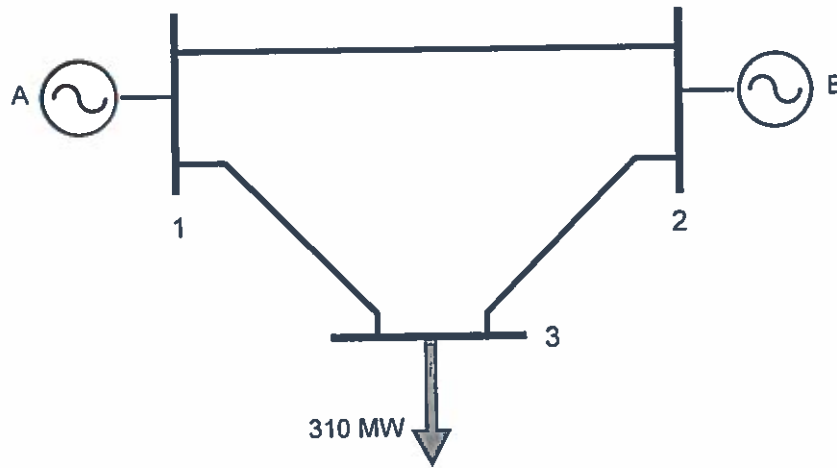


Figure Q3

Assume that:

- Generating units A and B have the following marginal production costs:

$$MC_A = 16 \text{ [£/MWh]}$$

$$MC_B = 11 + 0.1P_B \text{ [£/MWh]}$$

- All three transmission lines have the same impedance
- Calculate the unconstrained optimal dispatch for these conditions [2]
  - Calculate the hourly cost of this unconstrained dispatch [1]
  - Calculate the power that would flow in each line if this dispatch was implemented [4]
  - What is the marginal cost of energy at each node under these conditions [1]
  - How should this unconstrained dispatch be modified if the flow in line 1-3 is limited to 160 MW for security reasons? [4]
  - Calculate the hourly cost of this constrained dispatch and the hourly cost of security [2]
  - What is the marginal cost of energy at each node when the constraint on the flow on line 1-3 is taken into consideration? [6]

#### Question 4

- a. Consider two regions of a small power system that are not connected. Generators 1 and 2 (belonging to Borduria Power) are located in the Northern Region while generators 3 and 4 (belonging to Syldavia Gen) are located in the Southern Region. The load in the Northern Region is 100 MW and the load in the Southern Region is 400 MW. Marginal cost of these generators are:

Northern Region

$$MC_1 = 4 + 0.02P_1 \text{ [£ / MWh]}$$

$$MC_2 = 5 + 0.03P_2 \text{ [£ / MWh]}$$

Southern Region

$$MC_3 = 5 + 0.025P_3 \text{ [£ / MWh]}$$

$$MC_4 = 7 + 0.025P_4 \text{ [£ / MWh]}$$

Calculate the marginal costs in both regions and the corresponding generation dispatches, generator payments and demand charges. What is the marginal value of transmission?

[4]

- b. A proposal to build a 470km long transmission link between the two regions is considered. The annuitised investment cost of transmission (including the allowable profit) is 36£/MW.km.year. The local consultant has proposed two schemes to be considered: (i) 50 MW and (ii) 150MW link. For each of the schemes calculate:

- marginal prices in the Northern and the Southern region
- generator payments, demand charges and congestion surplus
- network revenues and profits if the transmission company charges for the use of link on the basis of short-run marginal cost

[8]

- c. The optimal capacity of the transmission link to be built will depend on the objectives and interests of the potential investors. Consider three cases, and for each of these determine the optimal capacity that would be built:

(c1) Merchant transmission company that makes money from buying electricity in the North and selling it in the South

(c2) Regulated transmission company that maximizes the benefit of transmission for the entire country

(c3) Company formed of generators in the North and Demand in the South wishing to maximize their profits.

[8]

