UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1996

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSc Degree in Foundations of Advanced Information Technology
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the
Diploma of Membership of Imperial College
Associateship of the City and Guilds of London Institute
Associateship of the Royal College of Science

PAPER 3.43 / I3.24

OPERATIONS RESEARCH

Tuesday, May 7th 1996, 3.00 - 5.00

Answer THREE questions

For admin. only: paper contains
4 questions
4 pages (excluding cover page)

1 a Consider the primal linear programming problem

$$\max\left\{ c^T x \;\middle|\; A\,x \le b,\, x \ge 0 \right\}; \quad A \in \mathbb{R}^{m \times n},\ x, c \in \mathbb{R}^n$$

and its dual

$$\min\left\{ b^T y \;\middle|\; A^T y \ge c,\, y \ge 0 \right\}; \quad y, b \in \mathbb{R}^m.$$

Let x, y be feasible with respect to the primal and the dual problems respectively:

$$x:\ Ax \le b,\ x \ge 0 \quad \text{and} \quad y:\ A^T y \ge c,\ y \ge 0.$$

Show that given such a feasible y, we can bound the value of the primal objective function and given a feasible x, we can bound the dual objective function value.

b Reformulate the following linear programming problem in standard form and use the simplex algorithm to solve the problem

$$\min\left\{ x_0 = x_1 + 2\,x_2 - x_3 \;\middle|\; x_1 - x_2 + x_3 \le 1;\ x_1 + x_2 - 2x_3 \le 4;\ x_1 \ge 0,\ x_2\ \&\ x_3 \text{ free} \right\}.$$

2 a A parcel company has a delivery to make to each one of five customers, $i = 1, \ldots, 5$. The shipment to be delivered to the i^{th} customer is of weight w_i. The company has four delivery vans of capacity a_j, $j = 1, \ldots, 4$. The cost of operating van j is c_j.

A single truck cannot deliver to both customers 1 and 3; similarly, a single truck cannot deliver to both customers 2 and 4. Also, a truck cannot make more than two deliveries.

Formulate an integer programming model to determine the minimum cost of allocation of delivery vans for making all the shipments.

b Consider the integer linear programming problem

$$\max \left\{ x_0 = 2x_1 + x_2 \;\middle|\; 2x_1 + 5x_2 \leq 17;\ 3x_1 + 2x_2 \leq 10;\ x_1, x_2 \geq 0 \ \&\ \text{integer} \right\}.$$

Ignoring the integer requirement and applying the simplex algorithm to the resulting linear program (LP), we obtain the following optimal tableau:

	x_1	x_2	x_3	x_4	RHS
x_0	0	1/3	0	2/3	20/3
x_3	0	11/3	1	-2/3	31/3
x_1	1	2/3	0	1/3	10/3

Derive a Gomory cut based on the second constraint in the above tableau. Show that this cut excludes the optimal solution to the above LP. Formulate the next linear programming problem that needs to be solved.

3 A small firm can produce up to four computers weekly, and has agreed to supply the demand for its computers in each of the next four weeks as follows:

week j	1	2	3	4
Demand D_j	3	2	4	2

Production costs are a function of the number of computers manufactured, and are given in thousands of pounds as follows:

Number of computers produced, x	0	1	2	3	4
f(x) $\left(\text{Cost: £}1000 \times f(x)\right)$	4	13	19	27	32

Let the state u at week j be the number of computers in inventory at the beginning of week j. The computers can be delivered to the customers at the end of the same week in which they are produced or they can be stored for future delivery at a cost of £1000 $\times$ (4 $\times$ u) per week (cost incurred at week j+1). The firm can store at most three computers at a time. At present, the firm does not have any manufactured computer and also it does not desire to have any unsold computers at the end of week 4.

a Establish a dynamic programming recursion formula for determining the number of computers the firm should manufacture in each of the next four weeks (stages) to meet all demands at a minimum total cost.

b Let the cost of manufacturing computers that exceed demand in the final week be £1, 000, 000. Solve the above problem.

4 a Consider the two person zero-sum game in which Player 1 can hide in one of five hideouts (1, 2, 3, 4, 5). Player 2 has a single shot and may shoot at any of four spots A, B, C or D as follows:

$$\boxed{1}\ \text{A}\ \boxed{2}\ \text{B}\ \boxed{3}\ \text{C}\ \boxed{4}\ \text{D}\ \boxed{5}.$$

A shot will hit Player 1 if she/he is in a hideout adjacent to the spot where the shot was fired (e.g. a shot fired at B will hit Player 1 if she/he is in hideout 2 or 3, while a shot at D will hit Player 1 if she/he is in hideout 4 or 5). Suppose Player 2 receives a reward of 1 if Player 1 is hit and 0 otherwise. Construct the reward matrix, eliminate all the dominated strategies (i.e. inferior to at least one other strategy). Formulate (but do not solve) each player's linear program for determining her/his optimal strategy.

b Suppose that we add a constant c to every element in a reward matrix A. Denote the game matrix by A'. Show that A and A' have the same optimal strategies and that the (value of A') = (value of A)+ c.

c Find an equilibrium point of the two-person non-constant sum game in the following table:

(9, 9)	(−10, 10)
(10, −10)	(−1, 1)