E4.51

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Power System Economics - Solutions 2009

Question 1:

Question Compulsory

(i) Give two reasons for the introduction of competition in electricity supply and explain how these objectives can be achieved?

[2.5]

(ii) How is the value of the spot electricity price determined? Why spot electricity prices vary with time and location?

[2.5]

(iii) What is the purpose of contracts for differences?

[2]

(iv) Define cost of constraints, congestion costs, short run value of transmission network and explain how these can be quantified (calculated)?

[4]

(v) Demand function of a transmission interconnector is in the form of: $\pi_T = 15 - 0.1F$ (π_T is expressed in (£/MWh) and F (capacity of the interconnector) is in MW). Determine the capacity that would maximise the revenue to the transmission operator.

[3]

(vi) If the total investment cost of building this interconnector (in v) can be expressed as a linear function of its capacity C = 300,000xF [£], estimate the optimal capacity that should be built.

[3]

(vii) Why transmission network business should be regulated? How does the regulator fulfil their responsibilities?

[3]

Question 1 Solution

Discussion along the following line is needed for full mark.

(v)

Revenue of transmission operator = $\pi_T(F) \times F = 15F - 0.1F^2$.
In order to maximize, we set the derivative equal to 0.

$15 - 0.2F = 0 \Rightarrow F = 75MW$

(vi)

The optimal from the perspective of global welfare satisfies the fact that:
marginal of transmission = marginal investment cost of transmission

Value of transmission $\pi_T = 15 - 0.1F$

A rough estimate of the annuitised cost of the transmission line is obtained by dividing the total investment cost by 10, which makes it equal to £30,000. Transforming this into an hourly value yields:

Marginal investment cost of transmission = $\dfrac{30000}{8760} = 3.425£/MWh$

Hence, $15 - 0.1F = 3.425 \Rightarrow F = 115.75\,\text{MW}$

Question 2

(a) The demand curve, in terms of quantity Q (kWh) bought by a consumer, in a given period, as a function of price π (p/kWh) is given by the following expression: $\pi = -0.01Q + 5$. The production cost in the period under consideration is given by
$C = 0.0075Q^2 + 1.3Q$ [p/h]

(i) Calculate the level of consumption at π=3.5p/kWh, consumer surplus, demand charges and revenue received by suppliers. What is price elasticity of demand at this point?

[2]

(ii) If the producer decides to reduce the price by 10%, determine the change in consumption and the new revenue received by the producer.

[2]

(iii) Determine the expression for marginal production cost and then find the equilibrium price and demand at which the social welfare is maximised.

[2]

(iv) Calculate producer revenue, profit and average cost at the equilibrium point. What is producer surplus?

[2]

(v) Would it be worth for this producer to artificially increase the price for 20%? What would be the total consumer and producer surpluses in these cases? How do they compare with the equilibrium calculated in (iii). What can you conclude from this comparison?

[3]

(b)

(i) What are sources of risk involved in trading? Explain how contracts help to reallocate, share and spread risk.

[3]

(ii) What are forward and futures contracts?

[3]

(iii) Explain how option contracts operate and the meaning of exercise price and option fee.

[3]

Question 2 Solution

a)

(i) $Q = \frac{5-\pi}{0.01} = \frac{5-3.5}{0.01} = 150kWh$

Demand charges (and suppliers' revenue) : $Q \times \pi = 150kWh \times 3.5p/kWh = 525p$

The price elasticity is defined as

$\frac{\pi}{Q} \cdot \frac{dQ}{d\pi} = \frac{3.5}{150} \cdot \frac{d}{d\pi}\left(\frac{5-\pi}{0.01}\right) = \frac{3.5}{150} \cdot (-100) = -2.33$

The consumer surplus can be found using:

$Surplus = \frac{1}{2} \cdot 150 \cdot (5-3.5) = 112.5p$

(ii) An 10% decrease in price means that now $\pi = 0.9 \cdot 3.5 = 3.15p/kWh$. The corresponding quantity purchased by consumers is: $Q = \frac{5-3.15}{0.01} = 185kWh$.

Thus, the revenue received by the producer is $185kWh \cdot 3.15p/kWh = 582.75p$

(iii)The expression for marginal cost of production can be found by differentiation the production cost expression.

$MC = 0.0150Q + 1.3$

At the equilibrium price, the product price is equal to the marginal production cost.

$\pi = MC$

$-0.01Q + 5 = 0.0150Q + 1.3 \Rightarrow \pi_{eq} = 3.52p/kWh,\ Q_{eq} = 148kWh$

(iv)The producer revenue is: $148kWh \cdot 3.52p/kWh = 520.96p$

The total cost is: $0.0075 \cdot (148^2) + 1.3 \cdot 148 = 356.68p$

The average cost is: $\frac{356.68p}{148kWh} = 2.41p/kWh$

The producer surplus is equal to the profit, which in this case is:

$520.96p - 356.68p = 164.28p$

The consumer surplus is: $\frac{1}{2} \cdot 148 \cdot (5-3.52) = 109.52p$

(v) If the producer was to increase the price by 20%, the new price would be $\pi = 1.20 \times 3.52p/kWh = 4.22p/kWh$ and the consumption would be

$Q = \frac{5-4.22}{0.01} = 78kWh$.

The consumer's surplus in this case becomes $\frac{1}{2} \cdot 78 \cdot (5-4.2) = 31.2p$

and the producer's surplus becomes:

$Revenue - Cost = (4.22 \times 78) - \left[0.0075 \cdot 78^2 + 1.3 \cdot 78\right] = 329.16p - 147.03p = 182.13p$

The supplier's profit becomes larger while the consumer surplus is reduced. In addition, their sum is smaller compared with the equilibrium price case, meaning that there is a loss of social welfare.

b)

(i)

Question 3:

Borduria Generation owns three generating units that have the following cost functions:

$$C_A = 15 + 1.4P_A + 0.04P_A^2 \text{ [\$/h]}$$
$$C_B = 25 + 1.6P_B + 0.05P_B^2 \text{ [\$/h]}$$
$$C_C = 20 + 1.8P_C + 0.02P_C^2 \text{ [\$/h]}$$

(i) How should these units be dispatched if Borduria Generation must supply a load of 350MW at minimum cost? What is the generation marginal price and the profit made by the company?

[5]

(ii) How would the dispatch in (i) change if Borduria Generation had the opportunity to buy some of this energy on the spot market at a price of 8.20 $/MWh?

[5]

(iii) If, in addition to supplying a 350MW load as in (i), Borduria Generation had the opportunity to sell energy on the electricity market at a price of 10.20 $/MWh, what is the optimal amount of power that it should sell? What profit would it derive from this sale?

[5]

(iv) How the dispatch in (i) would change if the outputs of the generating units are limited as follows:

$$P_A^{\max} = 150MW$$
$$P_B^{\max} = 70MW$$
$$P_C^{\max} = 250MW$$

What would be marginal price in this situation and generator profits?

[5]

Question 3 Solution:

(i) First need to find the marginal cost function for the three units by differentiating the given cost functions. Thus:

$MC_A = 1.4 + 0.08P_A$

$MC_B = 1.6 + 0.10P_B$

$MC_C = 1.8 + 0.04P_C$

At the optimal dispatch, all three marginal costs will be the same. In addition, the sum of the power generation must be equal to the load of 350MW. Thus, we can write down three equations:

$P_A + P_B + P_C = 350$

$MC_A = MC_B \Rightarrow 0.08 \cdot P_A - 0.10 \cdot P_B = 0.2$

$MC_A = MC_C \Rightarrow 0.08 \cdot P_A - 0.04 \cdot P_C = 0.4$

Solving the above, we get the optimal dispatch:

$P_A = 95.26\,MW$

$P_B = 74.21\,MW$

$P_C = 180.53\,MW$

To calculate the generation marginal prices:

$MC_A = MC_B = MC_C = 1.8 + 0.04 \times 180.53 = 9.02\$/MWh$

The total revenue of the generators is:

$(350MW) \times (9.02\$/MWh) = 3157\$/hour$

The total generator costs are:

$C_A = 15 + 1.4 \times 95.26 + 0.04 \times (95.26)^2 = 511.34\$/h$

$C_B = 25 + 1.6 \times 74.21 + 0.05 \times (74.21)^2 = 419.10\$/h$

$C_C = 20 + 1.8 \times 180.53 + 0.02 \times (180.53)^2 = 996.78\$/h$

$Total\ Cost = C_A + C_B + C_C = 1927.22\$/h$

Since Profit = Revenue-Cost:

$Profit = 3157 - 1927 = 1230\$/h$

(ii) In this scenario, all the generators will produce until the generation marginal cost reaches the market spot price of 8.20\$/MWh. The remaining power needed to supply the 350MW will be provided by energy bought at the spot market. Hence:

$$P_A = \frac{8.20 - 1.40}{0.08} = 85\,MW$$

$$P_B = \frac{8.20 - 1.60}{0.10} = 66\,MW$$

$$P_C = \frac{8.20 - 1.80}{0.04} = 160\,MW$$

The energy that will be bought at the spot market is: $350 - (85 + 66 + 160) = 39\,MW$

(iii)In this scenario, all the generators will produce until the generation marginal cost reaches 10.20\$/MWh. Hence, the new dispatch will be:

$$P_A = \frac{10.20 - 1.40}{0.08} = 110\,MW$$

$$P_B = \frac{10.20 - 1.60}{0.10} = 86\,MW$$

$$P_C = \frac{10.20 - 1.80}{0.04} = 210\,MW$$

The profit derived from selling energy at the market:

$$C_A = 15 + 1.4 \times 110 + 0.04 \times (110)^2 = 653\$/h$$

$$C_B = 25 + 1.6 \times 86 + 0.05 \times (86)^2 = 532.40\$/h$$

$$C_C = 20 + 1.8 \times 210 + 0.02 \times (210)^2 = 1280\$/h$$

$$Total\,Cost = C_A + C_B + C_C = 2465.40\$/h$$

Revenue = $(110 + 86 + 210) \times 10.20 = 4141.20\$/h$

Profit = Revenue – Cost = 4141.20\$/h-2465.40\$/h=1675.80\$/h.

(iv)In this scenario, we will recalculate the optimal dispatch by holding $P_c = 70\,MW$.

We need to solve the following system of equations:

$$P_A + P_C = 350 - 70 = 280\,MW$$

$$MC_A = MC_C \Rightarrow 0.08 \cdot P_A - 0.04 \cdot P_C = 0.4$$

Solving the above, we get the optimal dispatch:

$$P_A = 96.67\,MW$$

$$P_C = 183.33\,MW$$

The new marginal price for the two generators is 9.133\$/MWh.

The total revenue of the generators is:

$$Revenue = 350MW \times 9.133\$/MWh = 3196.67\$/h$$

The generator costs are:

$$C_A = 15 + 1.4 \times 96.67 + 0.04 \times (96.67)^2 = 524.11\$/h$$

$$C_B = 25 + 1.6 \times 70 + 0.05 \times (70)^2 = 382\$/h$$

$$C_C = 20 + 1.8 \times 183.33 + 0.02 \times (183.33)^2 = 1022.22\$/h$$

$$Total\,Cost = C_A + C_B + C_C = 1928.33\$/h$$

Hence, the generator profits:

$$Profit = 3159.3 - 1928.34 = 1268.33\$/h$$

Question 4

a) Explain why in a real (imperfect) market each company must consider the possible actions of others when selecting their strategy?

[5]

b) Consider a market for electrical energy that is supplied by two generating companies whose costs are:

Ca = 35 Pa [£/h]
Ca = 45 Pb [£/h]

The inverse demand function is given by: p = 120 – D

(i) Assuming Bertrand competition, calculate production of each of the companies, market price and profits made.

[5]

(ii) To examine the market share between the companies assuming Cournot competition, calculate market price and demand for different levels of productions Pa (15MW, 20MW, 25MW, 30MW) and Pb (10MW, 15MW, 20MW) of the two companies and determine profits made. What is the optimal amount of production of each of the companies that maximises their profits?

[6]

(iii) Explain the differences in the solution in (i) and (ii)

[4]

Question 4 Solution

a)

(i) Marginal cost of A: 35£/MWh
Marginal cost of B: 45 £/MWh
Generating company A will set its price just below 45 £/MWh. Generating company B cannot set its price below 45 £/MWh because it will be making a loss. As a result, all the demand will be met by Generating company A.
The demand will be: $D = 120 - p = 120 - 45 = 75\,MW$

Hence, $\begin{matrix} P_A = 75\,MW \\ P_B = 0\,MW \end{matrix}$ with market price = 45 £/MWh.

The profit made by company B is zero.
The cost of company A is: $35£/MWh \times 75MW = 2625£/h$
The revenue of company A is: $45£/MWh \times 75MW = 3375£/h$
Hence, the profit made is: $3375£/MWh - 2625£/MWh = 750£/MWh$

(ii) The resulting Cournot table is:

Pb/Pa	15		20		25		30	
10	25	900	30	1100	35	1250	40	1350
	500	95	450	90	400	85	350	80
15	30	825	35	1000	40	1125	45	1200
	675	90	600	85	525	80	450	75
20	35	750	40	900	45	1000	50	1050
	800	85	700	80	600	75	500	70

For each 2x2 square in the above table:
Upper left cell: demand in MW
Lower left cell: Profit of Generation company B in £/h
Upper right cell: Profit of Generation company A in £/h
Lower right cell: Market price in £/MWh

As we can see from the above table, the optimal solution for maximizing profit of company A is : $P_A = 30MW$ and $P_B = 10MW$
where the profit is 1350£/h
For company B maximum profit: $P_A = 15MW$ and $P_B = 20MW$
where the profit is 800£/h
Equilibrium solution occurs when $P_A = 30MW$ and $P_B = 20MW$.

(iii)

Question 5

Consider the three-bus power system shown on Figure Q3.

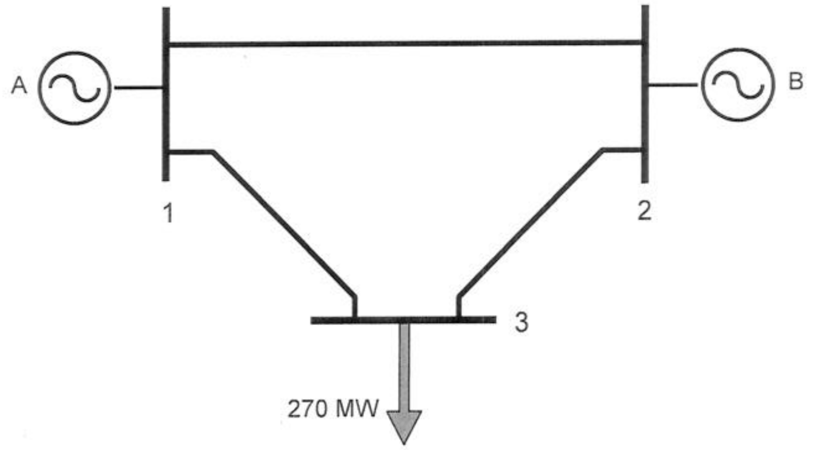

Figure Q3

Assume that:

- Generating units A and B have the following marginal production costs:

$$MC_A = 12 \text{ [£/MWh]}$$
$$MC_B = 9 + 0.1P_B \text{ [£/MWh]}$$

- All three transmission lines have the same impedance

a. Calculate the unconstrained optimal dispatch for these conditions [2]

b. Calculate the hourly cost of this unconstrained dispatch [1]

c. Calculate the power that would flow in each line if this dispatch was implemented [4]

d. What is the marginal cost of energy at each node under these conditions [1]

e. How should this unconstrained dispatch be modified if the flow in line 1-3 is limited to 150 MW for security reasons? [4]

f. Calculate the hourly cost of this constrained dispatch and the hourly cost of security [2]

g. What is the marginal cost of energy at each node when the constraint on the flow on line 1-3 is taken into consideration? [6]

Question 5 Solution

a. Unconstrained optimal dispatch:

We need $MC_A = MC_B \Rightarrow 12 = 9 + 0.1 \cdot P_B \Rightarrow P_B = \frac{12-9}{0.1} = 30\,MW$

We also need $P_A + P_B = 270\,MW$

$P_A = 240MW$

$P_B = 30MW$

b. The cost functions can be found by integrating the marginal cost expressions (we assume that the integration constant = 0).

The hourly cost of the unconstrained dispatch is:

$C_A = 12 \cdot P_A \Rightarrow C_A = 12 \cdot 240 = 2880£/h$

$C_B = 9 \cdot P_B + \frac{1}{2} \cdot 0.1 \cdot P_B^{\,2} \Rightarrow 9 \cdot 30 + \frac{1}{2} \cdot 0.1 \cdot 30^2 = 315£/h$

$Total\ Cost = 2880£/h + 315£/h = 3195£/h$

c. We can use the following equations:

$P_A = F_{12} + F_{13}$

$P_B = -F_{12} + F_{23}$

Since all the transmission lines have the same impedance, we can also say:

$F_{12} + F_{23} = F_{13}$

Hence,

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} F_{1-2} \\ F_{1-3} \\ F_{2-3} \end{bmatrix} = \begin{bmatrix} P_A \\ P_B \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} F_{1-2} \\ F_{1-3} \\ F_{2-3} \end{bmatrix} = \begin{bmatrix} 70 \\ 170 \\ 100 \end{bmatrix}$$

d. Since this is the unconstrained dispatch (no constraints on flows), the marginal cost is the same at every bus:

$MC_A = 12£/MWh$, can be supplied directly from Generator A or B.

$MC_B = 12£/MWh$, can be supplied directly from Generator B or A.

$MC_c = 12£/MWh$, can be supplied from Generator A or B.

e. When 1MW is generated at Bus 1, 0.67 MW flow from Bus 1 to Bus 3, 0.33 MW flow from Bus 1 to Bus 2 and the same from Bus 2 to Bus 3.

When 1MW is generated at Bus 2, 0.67 MW flow from Bus 2 to Bus 3, 0.33 MW flow from Bus 2 to Bus 1 and the same from Bus 1 to Bus 3.

This means that in order to alleviate the overload of (170-150)=20 MW, we will have to reduce generation at Bus 1 by $\frac{20}{0.33} = 60MW$ and increase generation at Bus 2 by the same amount to maintain system power balance. The new dispatch should be:

$P_A = 240 - 60 = 180MW$

$P_B = 30 + 60 = 90MW$

f. Hourly cost of constrained dispatch:
$C_A = 12 \cdot P_A \Rightarrow C_A = 12 \cdot 180 = 2160£/h$
$C_B = 9 \cdot P_B + \frac{1}{2} \cdot 0.1 \cdot P_B{}^2 \Rightarrow 9 \cdot 90 + \frac{1}{2} \cdot 0.1 \cdot (90)^2 = 1215£/h$
$Total\,Cost = 2160£/h + 1215£/h = 3375£/h$
Hourly cost of security:
$3375 - 3195 = 180£/h$

g. The next MWh at node 1 can be directly supplied by generator A. This means that $MC_1 = 12£/MWh$
The next MWh at node 2 can be directly supplied by generator B. This means that $MC_2 = 9 + 0.1 \cdot 90 = 18£/MWh$.
For supplying an additional 1 MW at node 3, we need to reduce the generation of A by 1 MW and increase generation of B by 2 MW to maintain flow on line 1-3. This means that the marginal cost of energy is
$MC_3 = 2 \cdot MC_B - 1 \cdot MC_A = 2 \cdot 18 - 1 \cdot 12 = 24£/MWh$

Question 6:

a. Consider two regions of a small power system that are not connected. Generators 1 and 2 (belonging to Borduria Power) are located in the Northern Region while generators 3 and 4 (belonging to Syldavia Gen) are located in the Southern Region. The load in the Northern Region is 100 MW and the load in the Southern Region is 420 MW. Marginal cost of these generators are:

Northern Region

$$MC_1 = 3 + 0.02P_1 \quad [£/MWh]$$
$$MC_2 = 4 + 0.04P_2 \quad [£/MWh]$$

Southern Region

$$MC_3 = 3.6 + 0.025P_3 \quad [£/MWh]$$
$$MC_4 = 4.2 + 0.025P_4 \quad [£/MWh]$$

Calculate the marginal costs in both regions and the corresponding generation dispatches, generator payments and demand charges. What is the marginal value of transmission?

[4]

b. A proposal to build a 450km long transmission link between the two regions is considered. The annuitised investment cost of transmission (including the allowable profit) is 37£/MW.km.year. The local consultant has proposed two schemes to be considered: (i) 80 MW and (ii) 150MW link. For each of the schemes calculate:

- marginal prices in the Northern and the Southern region
- generator payments, demand charges and congestion surplus
- network revenues if the transmission company charges for the use of link on the basis of short-run marginal cost

[8]

c. The optimal capacity of the transmission link to be built will depend on the objectives and interests of the potential investors. Consider three cases, and for each of these determine the optimal capacity that would be built:

(c1) Merchant transmission company that makes money from buying electricity in the North and selling it in the South

(c2) Regulated transmission company that maximizes the benefit of transmission for the entire country

(c3) Company formed of generators in the North and Demand in the South wishing to maximize their profits.

[8]

Question 6 Solution

a. **Northern region:**

$MC_1 = MC_2 \Rightarrow 3 + 0.02 \cdot P_1 = 4 + 0.04 \cdot P_2$

$P_1 + P_2 = 100\,MW$

Hence,

$P_1 = 83.33MW$

$P_2 = 16.67MW$

$MC_1 = MC_2 = 4.67£/MWh$

Payments to Generator 1: $P_1 \times MC_1 = 83.33MW \times 4.67£/MWh = 388.89£/h$

Payments to Generator 2: $P_2 \times MC_2 = 16.67MW \times 4.67£/MWh = 77.78£/h$

Demand charges: $Demand \times MC = 100MW \times 4.67£/MWh = 466.67£/h$

Southern region:

$MC_3 = MC_4 \Rightarrow 3.6 + 0.025 \cdot P_3 = 4.2 + 0.025 \cdot P_4$

$P_3 + P_4 = 420\,MW$

Hence,

$P_3 = 222MW$

$P_4 = 198MW$

$MC_1 = MC_2 = 9.15£/MWh$

Payments to Generator 3: $P_3 \times MC_3 = 222MW \times 9.15£/MWh = 2031.30£/h$

Payments to Generator 4: $P_4 \times MC_4 = 198MW \times 9.15£/MWh = 1811.70£/h$

Demand charges: $Demand \times MC = 420MW \times 9.15£/MWh = 3843.00£/h$

The marginal value of transmission is

$\pi_{SR} - \pi_{NR} = 9.15£/MWh - 4.67£/MWh = 4.48£/MWh$.

b.

(i) We need to find the optimal dispatch for the case C= 80MW (80MW moving from Northern region to Southern region).

Northern region:

$MC_1 = MC_2 \Rightarrow 3 + 0.02 \cdot P_1 = 4 + 0.04 \cdot P_2$

$P_1 + P_2 = 100MW + 80\,MW$

Hence,

$P_1 = 136.67MW$

$P_2 = 43.33MW$

$MC_1 = MC_2 = 5.73£/MWh$

Payments to Generator 1: $P_1 \times MC_1 = 136.67MW \times 5.733£/MWh = 783.56£/h$

Payments to Generator 2: $P_2 \times MC_2 = 43.33MW \times 5.733£/MWh = 248.44£/h$

Demand charges: $Demand \times MC = 100MW \times 5.733£/MWh = 573.3£/h$

Southern region:

$MC_3 = MC_4 \Rightarrow 3.6 + 0.025 \cdot P_3 = 4.2 + 0.0025 \cdot P_4$

$P_3 + P_4 = 420\,MW - 80MW$

Hence,

$P_3 = 182MW$

$P_4 = 158MW$

$MC_1 = MC_2 = 8.15£/MWh$

Payments to Generator 3: $P_3 \times MC_3 = 182MW \times 8.15£/MWh = 1483.30£/h$

Payments to Generator 4: $P_4 \times MC_4 = 158MW \times 8.15£/MWh = 1287.70£/h$

Demand charges: $Demand \times MC = 420MW \times 8.15£/MWh = 3423.00£/h$

Congestion surplus:

$(\pi_{SR} - \pi_{NR}) \times F = (8.15£/MWh - 5.73£/MWh) \times 80MW = 193.33£/h$

Network profit=Network revenue-transmission investment cost.

Network revenue=Congestion surplus=193.33£/h

Transmission Investment Cost

$$= \frac{k \cdot \lambda \cdot F_C}{T_0} = \frac{37 \dfrac{£}{MW \cdot km \cdot year} \cdot 450km \cdot 80MW}{8760 \dfrac{h}{year}} = 152.05£/h$$

Hence, the transmission profit is $192.33£/h - 152.05£/h = 41.28£/h$

(ii) We need to find the optimal dispatch for the case C= 150MW (150MW moving from Northern region to Southern region).

Northern region:

$MC_1 = MC_2 \Rightarrow 3 + 0.02 \cdot P_1 = 4 + 0.04 \cdot P_2$

$P_1 + P_2 = 100MW + 150\,MW$

Hence,

$P_1 = 183.33MW$

$P_2 = 66.67MW$

$MC_1 = MC_2 = 6.67£/MWh$

Payments to Generator 1: $P_1 \times MC_1 = 183.33MW \times 6.67£/MWh = 1222.22£/h$

Payments to Generator 2: $P_2 \times MC_2 = 66.67MW \times 6.67£/MWh = 444.44£/h$

Demand charges: $Demand \times MC = 100MW \times 6.67£/MWh = 666.67£/h$

Southern region:

$MC_3 = MC_4 \Rightarrow 3.6 + 0.025 \cdot P_3 = 4.2 + 0.0025 \cdot P_4$

$P_3 + P_4 = 420\,MW - 150MW$

Hence,

$P_3 = 147MW$

$P_4 = 123MW$

$MC_1 = MC_2 = 7.275£/MWh$

Payments to Generator 3: $P_3 \times MC_3 = 147MW \times 7.275£/MWh = 1069.425£/h$

Payments to Generator 4: $P_4 \times MC_4 = 123MW \times 7.275£/MWh = 894.825£/h$

Demand charges: $Demand \times MC = 420MW \times 7.275£/MWh = 3055.5£/h$

Congestion surplus:

$(\pi_{SR} - \pi_{NR}) \times F = (7.275£/MWh - 6.667£/MWh) \times 150MW = 91.25£/h$

Network profit = Network revenue – transmission investment cost.

Network revenue = Congestion surplus = 91.25 £/h

Transmission Investment Cost

$$= \frac{k \cdot \lambda \cdot F_C}{T_0} = \frac{37 \frac{£}{MW \cdot km \cdot year} \cdot 450km \cdot 150MW}{8760 \frac{h}{year}} = 285.10£/h$$

Hence, the transmission profit (i.e. loss in this case) is

$91.25£/h - 285.10£/h = -193.85£/h$

c.

(i) The transmission company wishes to maximize its profit:

$\max\{Revenue(F) - Cost(F)\}\, wrt\, F.$

$Revenue(F) = \pi_T(F) \times F$

Marginal Value of transmission: $\pi_T(F) = \pi_{SR}(F) - \pi_{NR}(F)$

$\pi_T(F) = MC_3 - MC_1 = 3.6 + 0.025 \cdot P_3 - (3 + 0.02 \cdot P_1) = 0.6 + 0.025 \cdot P_3 - 0.02 \cdot P_1$

We can also use that:

$P_1 + P_2 = D_{NR} + F$

$P_3 + P_4 = D_{SR} - F$

$MC_1 = MC_2$

$MC_3 = MC_4$

Hence,

$P_1 = 83.33 + \frac{F}{1.5}$

$P_2 = 0.5P_1 - 25$

$P_3 = 222 - \frac{F}{2}$

$P_4 = P_3 - 24$

and by using the above we have the expression: $\pi_T(F) = (4.483 - 0.0258F) \cdot F$

$$Cost(F) = \frac{k \cdot \lambda \cdot F}{T_0} = \frac{37 \cdot 450 \cdot F}{8760} = 1.901F$$

$Profit(F) = Revenue(F) - Cost(F) = -0.0258F^2 + 2.582F$

In order to find the maximum of this equation, we set its derivative = 0.

$$\frac{d\,Profit(F)}{dF} = 0 \Rightarrow 2.582 - 2 \cdot 0.0258F = 0 \Rightarrow F = 49.99MW \approx 50MW$$

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(ii) Value of transmission=Marginal investment cost of transmission

Marginal investment cost of transmission: $\frac{k\cdot\lambda}{T_0}=\frac{37\cdot 450}{8760}=1.901\frac{£}{MW\cdot h}$

From before we can use that $\pi_T(F)=4.483-0.0258F$

Hence, $4.483-0.0258F=1.901 \Rightarrow F=99.97MW \approx 100MW$

(iii)

North:

$MC1=MC2 \Rightarrow 3+0.02P_1=4+0.04P_2$

$P_1+P_2=100+F$

$\Rightarrow P_1=83.333+0.667F,\ P_2=16.667+0.333F$

$\Rightarrow MC_{North}=4.667+0.0133F$

South:

$MC3=MC4 \Rightarrow 3.6+0.025P_3=4.2+0.025P_4$

$P_3+P_4=420-F$

$\Rightarrow P_3=222-0.5F,\ P_4=198-0.5F$

$\Rightarrow MC_{South}=9.15-0.0125F$

$MC_{North}=MC_{South} \Rightarrow F_0=173.548\,\text{MW}$

Total profit of the company can be broken into three components:

- Net revenue from market transactions (market revenues of generators in the North minus payments of the demand in the South):
  $R_{market}=(P_1+P_2)MC_{North}-D_{South}MC_{South}=0.0133F^2+11.25F-3376.333$
- Cost of generation in the North:
  $C_{gen}=3P_1+0.01P_1^2+4P_2+0.02P_2^2=0.00667F^2+4.667F+391.667$
- Profit from the transmission business:
  $\Pi_T=(MC_{South}-MC_{North})F-\frac{k\lambda}{T_0}F=-0.02583F^2+(4.483-0.0514\lambda)F$

The expression for the total profit is finally:

$\Pi_{Co}=-0.01917F^2+(11.067-0.0514\lambda)F-2984.667$

Differentiating the above expression with respect to F yields:

$\frac{d}{dF}\Pi_{Co}=-0.03833F+11.067-0.0514\lambda=0$

The optimum line capacity is thus given by: $F^*=288.696-1.3401\cdot\lambda$

For $\lambda = 37$, the optimum level is F^* = 239.11 MW; however, this is larger than F_0 (173.55 MW), meaning that the company will maximize its profit by building the line up to the capacity of **F_0 = 173.55 MW**.

The expression for the optimum line capacity indicates that λ would have to increase to the level of £85.9/MW/km/yr in order for the optimal capacity to become less than the maximum capacity necessary to relieve congestion. This is depicted in the figure below, where the company's profits are plotted as function of F for several values of λ (including $\lambda = 37$). The figure shows that when λ is above £85.9/MW/km/yr, the profit reaches its maximum before line capacity reaches the no-congestion value.