Model answer to Q 1(a): Bookwork

a) Consider the intrinsic permeability of a material.

i) What is the value of permeability for free space?

$$\mu_o = 4\pi 10^{-7}\ [H/m] \sim 1.26\ [\mu H/m]$$

[1]

ii) With a generic material, the intrinsic relative permeability can be represented by a complex number. Write this simple expression and briefly explain the physical interpretation of each part.

$$\mu_r = \mu'_r - j\mu''_r$$

The real part μ'_r represents the amount of stored magnetic field energy relative to free space, while the imaginary part μ''_r represents that amount of dissipated magnetic field energy relative to free space.

[2]

iii) What is the value of relative permeability for aluminium?

1

[1]

iv) What is the value of relative permeability for alumina?

1

[1]

Model answer to Q 1(b): Bookwork and New Derivation

b) Consider the intrinsic permittivity of a material.

i) From the speed of light in free space, derive its value of permittivity.

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \cong 3x10^8\ [\text{m/s}] \text{ and so } \varepsilon_o = \frac{1}{\mu_o c^2} \cong 8.842\ [pF/m]$$

[2]

ii) With a generic material, the intrinsic relative permittivity can be represented by a complex number. Write this simple expression and briefly explain the physical interpretation of each part.

$$\varepsilon_r = \varepsilon'_r - j\varepsilon''_r$$

The real part ε'_r represents the amount of stored electric field energy relative to free space, while the imaginary part ε''_r represents that amount of dissipated electric field energy relative to free space.

[2]

iii) What is the approximate value of relative permittivity for silicon?

11.9

[1]

Model answer to Q 1(c): Bookwork

c) Consider the intrinsic conductivity of a material.

i) What is the value of conductivity for free space?

0

[1]

ii) With a generic material, the intrinsic conductivity can be represented by a complex number. Write this simple expression.

$$\sigma = \sigma' - j\sigma''$$

[1]

iii) What is the approximate value of conductivity for copper?

$5.8 x 10^{12}$ [S/m]

[1]

Model answer to Q 1(d): New Derivation

d) Derive an expression for the effective permittivity of a material in terms of its complex intrinsic variables.

$$\varepsilon_{eff} = \varepsilon - j\frac{\sigma}{\omega} = \left(\varepsilon' - \frac{\sigma''}{\omega}\right) - j\left(\varepsilon'' + \frac{\sigma'}{\omega}\right)$$

[2]

Model answer to Q 1(e): New Derivation

e) Derive an expression for the effective conductivity of a material in terms of its complex intrinsic variables.

$$\sigma_{eff} = \sigma + j\omega\varepsilon = (\sigma' + \omega\varepsilon'') - j(\sigma'' - \omega\varepsilon')$$

[1]

Model answer to Q 1(f): Bookwork and Calculation

f) Given the measured values of low frequency dielectric constant of 3 and DC conductivity of 2,000 S/m derive the following and calculate the values for a frequency of 1 GHz:

iv) Effective relative permittivity.

$$\varepsilon_{reff} \sim \varepsilon_r - j\frac{\sigma_o}{\omega\varepsilon_o} = 3 - j36{,}000$$

[1]

v) Effective conductivity.

$$\sigma_{eff} \sim \sigma_o + j\omega\varepsilon_o\varepsilon_r = 2{,}000 + j0.167$$

[1]

vi) Loss tangent and dielectric quality factor.

$$tan\delta \sim \frac{\sigma_o}{\omega\varepsilon_o\varepsilon_r} = 12{,}000 \text{ and } Q \sim \frac{\omega\varepsilon_o\varepsilon_r}{\sigma_o} = 83.33x10^{-6}$$

[2]

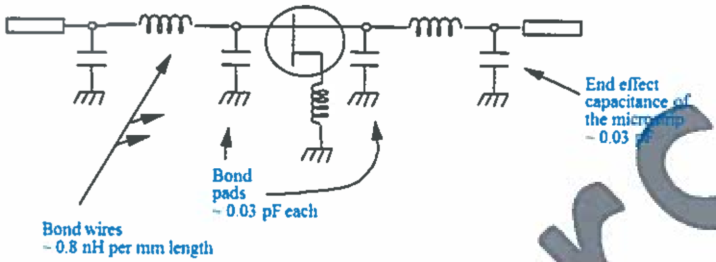
Model answer to Q 2(a): Bookwork

a) Draw the equivalent circuit model of this circuit and choose typical values for the associated parasitic components from the following options of:

Bond-wire inductance $L \in [0.8\ \text{fH/mm}, 0.8\ \text{pH/mm}, 0.8\ \text{nH/mm}]$

Fringe capacitance $C \in [30\ \text{fF}, 30\ \text{pF}, 30\ \text{nF}]$

For simplicity, assume all values are equal for a particular type of parasitic component.



The inductors will be a combination of two isolated bond wires connected in parallel (i.e. no mutual inductive coupling), having an inductance of 0.4 nH.

[6]

Model answer to Q 2(b): Bookwork

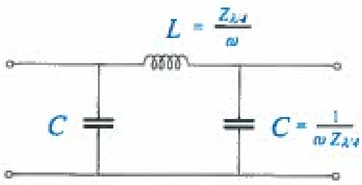
b) From the model drawn in 2(a), what is the general frequency response of the input and output interconnects, between the FET and associated microstrip transmission lines, due to the associated parasitic components? Also, can the microstrip lines be used to supply DC bias voltages to the FET?

The resulting interconnects behave as low pass filters, irrespective of the frequency response of the FET or microstrip transmission lines. The microstrip lines can be used to supply DC bias voltages to the FET, as long as the microwave signal does not "see" the DC bias supply.

[4]

Model answer to Q 2(c): Bookwork

c) From the model drawn in 2(a) and your typical chosen values for the parasitic components, at what frequency will the input and output interconnects, between the FET and associated microstrip transmission lines, behave like an equivalent quarter-wavelength section of transmission line? How does this compare to the -3 dB cut-off frequency? Also, calculate the characteristic impedance of the equivalent quarter-wavelength section of transmission line.



$$Z_{\lambda/4} = \sqrt{\frac{L}{C}} = 115.47\Omega \text{ and } f_c = \frac{Z_{\lambda/4}}{2\pi L} = 45.94\ GHz$$

The equivalent quarter-wavelength transmission line occurs at 46 GHz and this is the same as the -3 dB cut-off frequency of the low-pass filter.

[6]

Model answer to Q 2(d): Bookwork

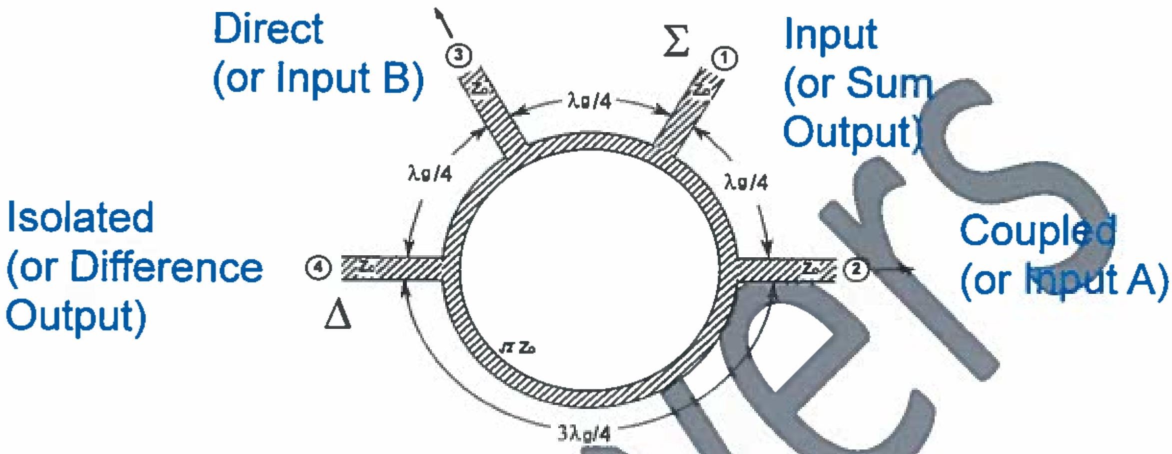
d) From the model drawn in 2(a), give examples of the potentially adverse effects that the interconnect between the source of the FET and ground can have, if not properly considered in the overall circuit of an amplifier and oscillator operating at 10 GHz.

At 10 GHz the pair of isolated bond wires will have an inductance of 0.4 nH and, therefore, as inductive reactance of 25.13 Ω. This is very significant if the source of the FET is designed to have an ideal short circuit to ground. Adverse effects can include an amplifier oscillating, due to the significant inductive reactance changing the stability conditions of the amplifier. Also, an oscillator may not meet the necessary gain and phase conditions to support oscillation.

[4]

<u>Model answer to Q 3(a): Bookwork</u>

a) Draw a microstrip 3 dB rat-race coupler, clearly identifying all electrical lengths and characteristic impedances, relative to that of the system's reference impedance *Zo*:



[2]

<u>Model answer to Q 3(b): Bookwork</u>

b) Briefly compare and contrast this coupler with a Wilkinson coupler.

Unlike the 3-port Wilkinson coupler, which is a 0° 3 dB coupler that does not exploit the interference principle and requires an isolation (ballast) resistor, the 4-port rat-race coupler only has ~15 fractional bandwidth. In addition, it is much bigger but does not require any additional resistor. Moreover, it can be used for more application that Wilkinson coupler, as it can provide Sum and Difference signals.

[2]

<u>Model answer to Q 3(c): Bookwork</u>

c) Briefly compare and contrast this coupler with a branch-line coupler.

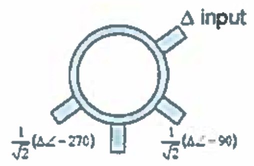
The rat-race coupler of a branch-line coupler with an extra half wavelength of transmission line inserted between two ports. As a result, both couplers utilise the interference principle and neither require any ballast resistors or bond wires. Unlike the branch-line coupler, the rat-race can be configured to have either 90° or 180° output signal differences.

[2]

<u>Model answer to Q 3(d): Bookwork</u>

d) Using the figure drawn in 3(a), assign the output ports for a power divider when its input is the Isolated (or Difference) port. Also, using basic vector notation, express the voltage waves at each output, relative to the input.

Difference input



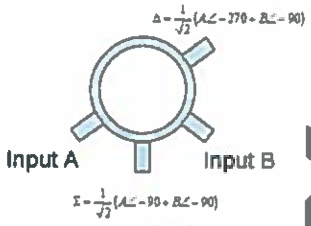
Isolated port

[2]

Model answer to Q 3(e): Bookwork

e) Using the figure drawn in 3(a), assign the input ports for a power combiner when its outputs are at the Direct and Coupled ports. Also, using basic vector notation, express the voltage waves at the other ports, relative to the input.

Vector notation



[2]

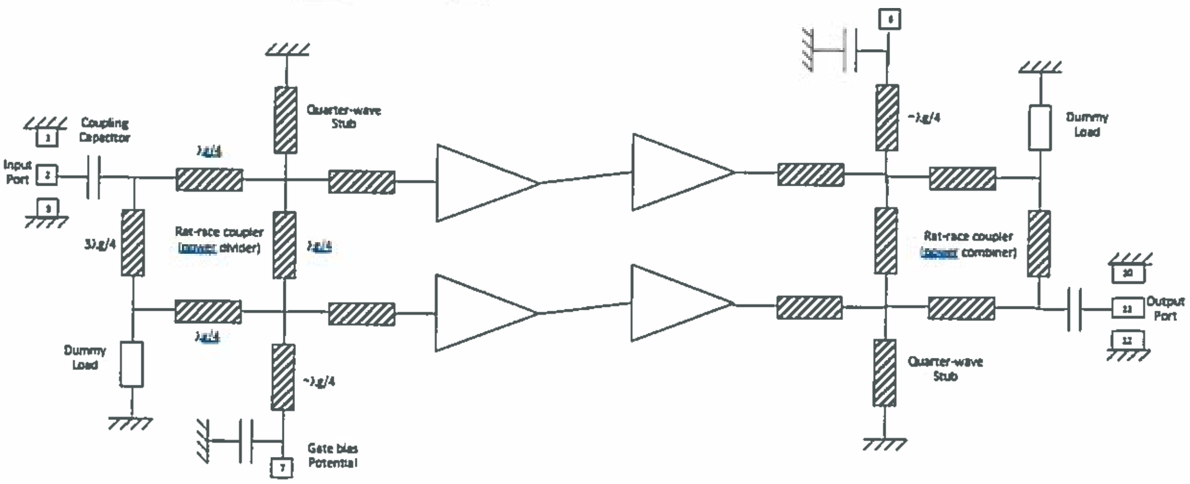
Model answer to Q 3(f): Bookwork

i) Briefly describe what this MMIC represents.

This MMIC is a 2-stage balanced amplifier

[1]

ii) Draw the high level block diagram of this circuit.



[5]

iii) Using basic vector notation, try to explain the operation of this circuit and highlight any potential anomaly.

The input power divider creates two signals that are ideally of equal amplitude, but with 90 degrees of phase difference:

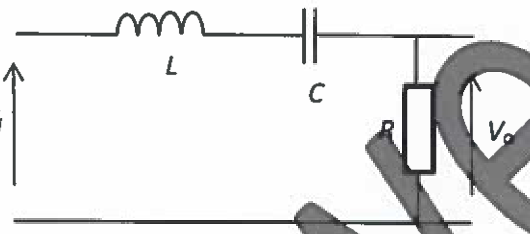
$$\frac{1}{\sqrt{2}}e^{-j\pi/2} \text{ and } \frac{1}{\sqrt{2}}e^{-j2\pi}$$

Since the circuit is symmetrical, the difference in transmission phase through the 2-stages of amplification should not change. However, when the quadrature phase signals enter the "power combining" coupler, they enter the wrong ports for power combining. The first signal destructively interferes (i.e. cancels out) at the output port and constructively interferes at the other input port; while the second signal constructively interferes at the output port and destructively interferes (i.e. cancels out) at the other input port. At best, there will be a 6 dB reduction in output power of the amplifier.

[4]

Model answer to Q 4(a): Bookwork

a) Draw the one-port "Tank" circuit and indicate key voltages and currents.



[1]

Model answer to Q 4(b): Bookwork

b) Using the circuit drawn in 4(a), from first principals (in terms of energy), derive an expression for the unloaded quality factor in terms of impedances. State any assumptions made.

It is assumed throughout that the RLC lumped-elements do not have any frequency dispersion.

With a lossless "tank", one only deals with steady-state sinusoidal signals. As a result, at any instance in time t, the total reactive energy stored is the instantaneous sum of the energy outside the inductor $W_L(t)$ plus the energy inside the capacitor $W_C(t)$, and this is a constant W:

$$W = W_L(t) + W_C(t) \neq f(t)$$

$$W = \begin{cases} W_L|_{PEAK} = \frac{1}{2} L\, I_L|_{PEAK}^2 \\ W_C|_{PEAK} = \frac{1}{2} C\, V_C|_{PEAK}^2 \end{cases}$$

$$Q(\omega) = 2\pi \frac{Total\ Reactive\ Energy\ Stored\ (W_L|_{PEAK}\ or\ W_C|_{PEAK})}{Time - average\ Resistive\ Energy\ Disspiated\ During\ Each\ Cycle\ or\ Work\ (W_R)}$$

With undriven (damped) excitation, the initial stored energy in the "tank" is depleted in $Q(\omega)/2\pi$ cycles. For example, 1 cycle when $Q(\omega) = 2\pi$; 2 cycles when $Q(\omega) = 4\pi$; etc.

$$Time - average\ Resistive\ Energy\ Disspiated\ During\ Each\ Cycle\ (W_R) = \\ Time - average\ Resistive\ Power\ Dissipated\ (P_R)\ x\ Duration\ of\ Each\ Cycle\ (T = 1/f)$$

$$Q(\omega) = \omega \frac{Total\ Reactive\ Energy\ Stored\ (W_L|_{PEAK}\ or\ W_C|_{PEAK})}{Time - average\ Resisitive\ Power\ Dissipated\ (P_R)}$$

During each cycle, the loss resistor will dissipate a time-average (RMS) energy

$$W_R = P_R T \quad \text{where} \quad P_R = \left(\frac{I_R|_{PEAK}}{\sqrt{2}}\right)^2 R \neq f(t)$$

and $T = \frac{2\pi}{\omega}$ where $\omega = 2\pi f$ and f is frequency in cycles per second (Hz)

For a series RLC network, having $I_i = I_L = I_R$, the Q-factor at ω is the ratio of either the inductive reactance $X_L(\omega)$ or capacitive reactance $X_C(\omega)$ with resistance R:

$$\therefore Q(\omega) = \begin{cases} \dfrac{\omega L}{R} = \dfrac{X_L(\omega)}{R} \\ \dfrac{1}{\omega CR} = \dfrac{|X_C(\omega)|}{R} \end{cases}$$

[6]

Model answer to Q 4(c): Bookwork and Calculation

c) Calculate the frequency that gives the maximum unloaded quality factor and the associated characteristic impedance.

Driven (undamped) angular resonance frequency: $\omega_o = \frac{1}{\sqrt{LC}}$ and $f_o = \frac{1}{2\pi\sqrt{LC}} = 919\ MHz$

$$Q(\omega \neq \omega_o) < Q(\omega = \omega_o)$$

$Z_o = \sqrt{\frac{L}{C}} = 57.7\ \Omega$ is the characteristic impedance of the RLC network

[4]

Model answer to Q 4(d): Bookwork and Calculation

d) When the "Tank" is undriven, in how many cycles will it be depleted of energy if the unloaded quality factor is 10π and what will be the value of series resistance?

Time − average Resistive Energy Dissipated During Each Cycle = Total Reactive Energy Stored /5 giving 5 cycles taken to deplete the tank of energy for an unloaded quality factor of 10π.

$$Q_u(\omega_o) = \frac{Z_o}{R} \text{ and so } R = \frac{Z_o}{10\pi} = 1.84\ \Omega$$

[4]

Model answer to Q 4(e): Bookwork and Calculation

e) With the value of series resistance calculated in 4(d), if the ideal lossless inductor and capacitor are now replaced by components having unloaded quality factors of 10 and 50, respectively, calculate the resulting unloaded quality factor at resonance.

For a series RLC network, where the effective resistance is the sum of all the resistors in series.

$$Q_S(\omega_o) = \frac{X_S(\omega_o)}{R_R + R_L + R_C} = \frac{1}{\frac{R_R}{X_S(\omega_o)} + \frac{R_L}{X_S(\omega_o)} + \frac{R_C}{X_S(\omega_o)}} = \frac{1}{\frac{1}{Q_R(\omega_o)} + \frac{1}{Q_L(\omega_o)} + \frac{1}{Q_C(\omega_o)}}$$

$\therefore Q_S(\omega_o) = Q_R(\omega_o)//Q_L(\omega_o)//Q_C(\omega_o)$ Parallel Combination dominated by the lowest Q-factor!

$$Q_R(\omega_o) = 10\pi;\ Q_L(\omega_o) = 10;\ Q_C(\omega_o) = 50$$

$$Q_S(\omega_o) = 6.59$$

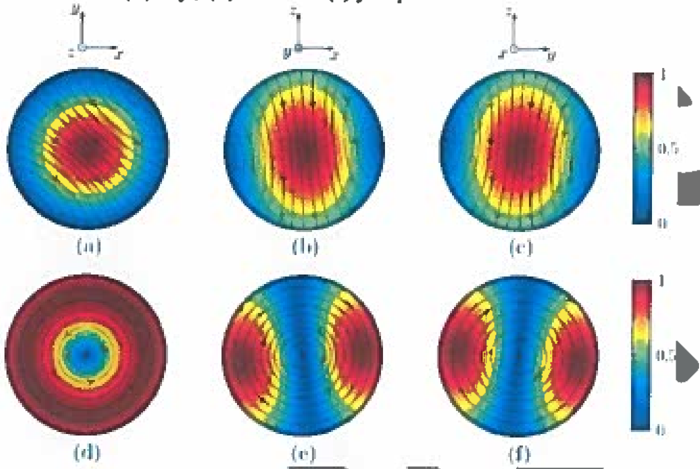
[5]

Model answer to Q 5(a): Bookwork

a) What is the dominant mode of operation referred to as and, with the use of sketches or otherwise, briefly describe either its electric or magnetic field variations.

The cavity resonator is operating in the dominant transvers magnetic TM_{011} mode (i.e., $m = 0$, $n = p = 1$, where p is associated with the variations along the radial direction, where r is the radial coordinate), where the only non-zero field components are E_r, E_θ and H_φ.

Normalized field patterns for the TM_{011} mode inside an air-filled spherical cavity resonator having a 150 μm radius, for the ideal case with $\sigma_0 \rightarrow \infty$ resulting in $f_0' \rightarrow f_0 \rightarrow f_l = 0.8727$ THz. Electric field in (a) x-y, (b) x-z and (c) y-z plane. Magnetic field in (d) x-y, (e) x-z and (f) y-z plane:



[2]

Model answer to Q 5(b): Calculation

b) Calculate the ideal resonance frequency and the associated RLC values.

$$\beta(\omega_l) = \frac{2.7437}{R_a} = 1829.33\ rad/m$$

$$V = \frac{4\pi R_a^3}{3} = 1.41372x10^{-11}\ m^3$$

$$C(\omega_l) = \frac{\varepsilon_0}{V}\frac{1}{\beta_l^4} = 5.587\ aF$$

$$L(\omega_l) = \mu_0 V\ \beta_l^2 = 5.944\ nH$$

$$R(\omega_l) = 0\ \Omega \text{ since the cavity is ideal}$$

[5]

Model answer to Q 5(c): Calculation

c) With a non-PEC wall (having a DC conductivity of approximately 100 S/m), transient time-domain measurements indicate a damped resonance frequency of 700 GHz and a decay time constant of 5 ps. Calculate the following:

i) Unloaded quality factor at the damped resonance frequency.

$$f_0'' = \frac{1}{2\pi\tau''} = 31{,}83\ GHz$$

$$Q_u(\omega_o{}') = \frac{f_0'}{2f_0''} = 10.9959$$

[2]

ii) Undamped resonance frequency.

$$f_0 = \sqrt{(f_0')^2 + (f_0'')^2} = 700.72\ GHz$$

[2]

iii) Unloaded quality factor at the undamped resonance frequency.

$$Q_u(\omega_o) = \frac{f_0}{2f_0''} = 11.0073$$

[2]

iv) *RLC* values at the undamped resonance frequency.

$$\beta(\omega_o) = \omega/c$$

$$C(\omega_o) = \frac{\varepsilon_0}{V}\frac{1}{\beta(\omega_o)^4} = 13.483\ aF$$

$$L(\omega_o) = \mu_0 V \beta(\omega_o)^2 = 3.826\ nH$$

$$R(\omega_o) = \frac{\omega_o L(\omega_o)}{Q_u(\omega_o)} = 1{,}530\ \Omega$$

[5]

Model answer to Q 5(d): Observation

d) Briefly comment on the frequency dispersive nature of the *RLC* components when wall losses are introduced.

Reducing the conductivity of the metal wall from infinity down to approximately 100 S/m has shown that the resistance increases from zero to 1530 Ohms. In addition, the *LC* values also exhibit frequency dispersion, with inductance decreasing by 41% and capacitance increasing by 141% as frequency drops from ω_I to ω_o.

[2]

Model answer to Q 6(a): Bookwork and Derivation

a) Using this equation, derive the equations for the characteristic impedance of the transmission line Z_{TX} and the corresponding electrical length ϑ.

$$Z_{IN} = Z_{TX}\frac{Z + jZ_{TX}\tan\vartheta}{Z_{TX} + jZ\tan\theta} \equiv Z_o$$

$$\therefore Z_{TX}\left(Z + jZ_{TX}\tan\vartheta\right) = Zo\left(Z_{TX} + jZ\tan\vartheta\right)$$

$$\mathrm{Re}\{LHS\} \equiv \mathrm{Re}\{RHS\}$$

$$\therefore \theta = \tan^{-1}\left\{\frac{Z_{TX}(Zo - R)}{XZo}\right\}$$

$$\mathrm{Im}\{LHS\} \equiv \mathrm{Im}\{RHS\}$$

$$\tan\theta = \frac{Z_{TX}X}{ZoR - Z_{TX}^2} \equiv \frac{Z_{TX}(Zo - R)}{XZo}$$

$$\therefore Z_{TX} = \sqrt{ZoR - \frac{X^2 Zo}{Zo - R}}$$

[7]

Model answer to Q 6(b): Bookwork

b) From the expressions derived in 6(a), what are the mathematical limits for the resistive and reactive values of the termination impedance that can be mapped into the input impedance of the short transmission line transformer?

From the last expression in 6(a), the limits are:

$$R \neq Zo \quad and \quad X < \sqrt{R(Zo - R)}$$

[3]

Model answer to Q 6(c): Bookwork and Calculation

c) A load termination consisting of a 2 nH inductance in series with a 3 Ω resistance must be matched at 900 MHz to a 50 Ω reference impedance using a short transmission line transformer. Using expressions derived in 6(a) and 6(b), calculate Z_{TX} and ϑ for the short transmission line transformer.

For a 2 nH inductance in series with a 3 Ω resistance at 900 MHz, the termination load impedance is Z = 2 + j11.31 Ω.

Using the expressions from 6(b), R is not equal to 50 Ω and X < 11.87 Ω, so both values are within the acceptable mathematical limits.

Using the expressions from 6(a), Z_{TX} = 3.73 Ω and ϑ = 16.5°.

[7]

Model answer to Q 6(d): Bookwork

d) Comment on the suitability, or otherwise, of implementing the short transmission line transformer calculated in 6(c) using conventional microstrip and thin-film microstrip technologies.

The value of Z_{TX} calculated in 6(c) would be considered very low in general. In practice, a conventional microstrip line could not be used to implement such a low impedance because the width of the signal line would be too wide. However, thin-film microstrip technology may be suitable as the widths of the lines are much narrower.

[3]