Imperial College London

MEng (Engineering) Examination 2016

Year 1

AE1-111 Thermodynamics

Monday 23rd May 2016: 14.00 to 16.30
[2½ hours]

The paper is divided into Section A and Section B.
Both sections carry the same weight.
Candidates may obtain full marks for complete answers to ***ALL*** questions.

A Data Sheet is attached.

The use of lecture notes is NOT allowed.

Section A

Each question in section A counts for 10% of the total.

1. Air, at a pressure of 8 bar and a temperature of 300 °C is contained in a cylinder. Initially, the volume of the cylinder is 0.1 m^3. A reversible adiabatic compression, (during which $Pv^{\gamma} = const.$), then occurs, in which the pressure increases to 16 bar.

   (a) Calculate the mass of air in the cylinder. [20%]

   (b) State the first law in a form appropriate to the problem. [20%]

   (c) What is the change in temperature of the air? [20%]

   (d) What is the change in internal energy of the air? [20%]

   (e) What is the work required to perform the compression? [20%]

2

(a)

i) A design for a heat pump working in a cycle assumes that for 1 MJ of heat absorbed from a reservoir at 270 K, heat will be rejected to a reservoir at 370 K for a work input of 200 kJ.
What quantity of heat must be delivered to the reservoir at 370K? [20%]

(ii) Is the device:

- reversible?
- irreversible?
- impossible?

Explain your answer. [30%]

(b) A reversible heat engine is operating in a cycle, producing 2 MJ of work output for a heat input of 3 MJ. If the temperature of the hot reservoir is 500 K, what is the temperature of the cold reservoir? [50%]

3

(a) Define entropy. [20%]

(b) A system comprises air in a container of fixed volume of 0.1 m^3 at an initial temperature of 300 °C and a pressure of 8 bar. The air is then cooled to 100 °C. Calculate the change in entropy of the air. [30%]

(c) What is the meaning of exergy? [20%]

(d) Has the exergy of the air increased or decreased in the process of part (b)? Assume the surroundings are at 20°C, 1 bar. Explain your answer. [30%]

4

A container of volume 10 m^3 holds air at a temperature of 300 K and a pressure of 5 bar. A valve, with an effective throat area of 0.5 cm^2 is opened, allowing air to escape to the atmosphere.

(a) Calculate the initial rate of flow through the valve. [40%]

(b) Assuming the temperature of the air remains constant, estimate the rate at which the pressure in the container falls. [30%]

(c) Assuming no heat transfer to the air in the container, estimate the rate at which pressure falls. Assume an isentropic expansion. [30%]

5

A 2-D aerofoil profile is being tested at a freestream Mach number of 0.82. The static pressure and temperature far upstream are set to 0.2 bar and 260 K respectively. On the upper surface of the aerofoil, a region of supersonic flow with a peak Mach number of 1.5 is found to occur, terminated by a shock wave, which is approximately normal to the aerofoil surface.

(a) Write down an expression for the static pressure immediately upstream of the shock and calculate its value. [30%]

(b) Determine the total temperature of the flow upstream and downstream of the shock. [20%]

(c) Calculate the Mach number of the flow immediately downstream of the shock. [20%]

(d) Write down an expression for the loss in total pressure of the flow across the shock. [30%]

Section B

Each question in section B counts for 25% of the total.

6.

A simple turbojet is installed in an aircraft that is flying at Mach 1.5 at an altitude where the ambient pressure is 47 kPa and the temperature is 250 K. It is assumed that: all components have 100% efficiency, the intake decelerates the flow to negligible velocity and the engine exhaust is fully expanded to ambient conditions.

Engine parameters are as follows:

Compression ratio:	10
Turbine inlet total temperature:	1400 K
Fuel/air ratio, f	0.02
C_P of combustion products	1.147 kJ/kg K
γ of combustion products	4/3

(a) Sketch the cycle on a T-S diagram, labelling each stage. [20%]

(b) Show at the exit of the intake, the temperature is approximately 362 K whilst the pressure is approximately 172 kPa. [20%]

(c) Write down the relations used to determine the conditions at the exit of the compressor and the compressor work. Calculate these values. [20%]

(d) Write down the relations used to determine conditions at the inlet to the propelling nozzle. Calculate values and hence calculate the thrust. [30%]

(e) Write down the relation used to determine the thrust specific fuel consumption, and calculate this value, in units of kg/s per kN thrust. [20%]

7

Figure 1 shows an idealised model of a device mounted on a satellite in a low orbit about a planet. The device extends a length in the horizontal (x) direction many times its thickness. The device comprises:

- an electronic component "A" of negligible thickness, (at position *1* in the figure), dissipating heat at a rate q_d = 10 W.cm^{-2}, and bonded to a
- metal plate, of thickness δ = 5 mm, which is cooled by
- a flow in a channel, (region above level 2 in the figure). The exposed surface of A exchanges heat by radiation with the planetary surface, which is at a temperature T_0 = 220 K.

Metal plate properties: k = 300 W.m^{-1}K^{-1}; ρ = 3.0x10^3 kg.m^{-3}; C = 1000 J.kg^{-1}K^{-1}
Coolant flow convective heat transfer coefficient: h_c = 1000 W.m^{-2} .

(a) Identify the elements contributing to thermal resistance in the normal (y) direction and write out an expression for the heat transfer across each element. Treat the exposed surface of A and of the planet as black bodies. [20%]

(b) Write out an expression relating the total power dissipated and the temperatures of device T_1, bulk coolant temperature, T_3 and planet, T_0. [20%]

(c) If T_1 is 260 K at a certain distance x_1 along A, estimate the bulk coolant temperature at the corresponding location. [20%]

(d) Neglecting heat transfer by radiation, show that the increase in coolant temperature over a length L in the x-direction is given by:

$$\Delta T = \frac{(T_1 - T_3)L}{\dot{m}_w C\left[{}^{1}\!/_{h} + {}^{\delta}\!/_{k} \right]}$$

where m_w is the coolant mass flow rate per unit width of the channel. [20%]

[Question continued overleaf]

(e) In the event of a sudden loss of coolant, the channel is assumed to act as an insulator, so that heat loss from the device is confined to that due to radiation from A.

Estimate the initial rate of temperature rise of A, and the time taken for the device to reach 400 K, treating it together with the attached metal plate as a single lumped element. [20%]

.

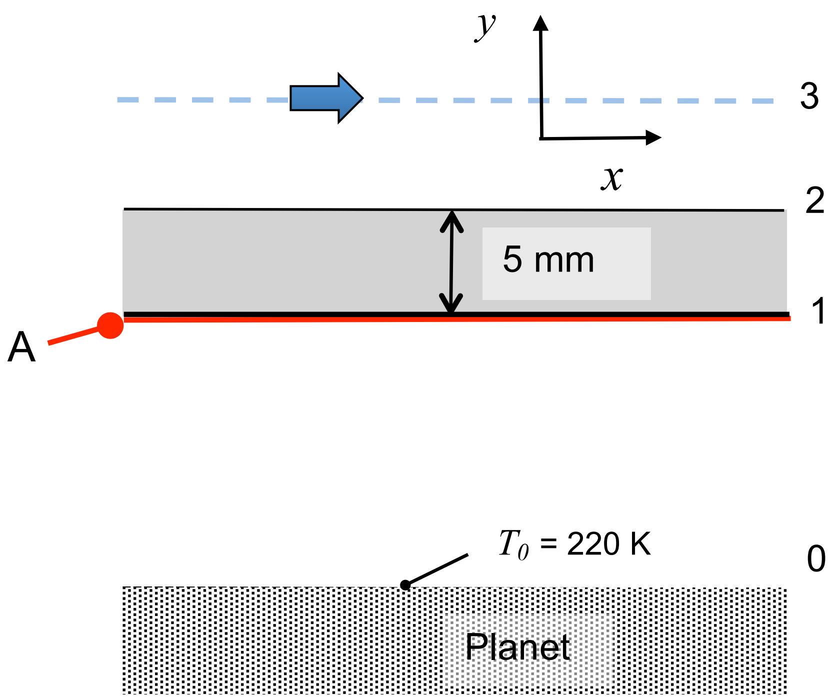


Figure 1

AE1-111 Thermodynamics Data sheet

1: Unless otherwise stated, **air** may be treated as a perfect gas for which

$\gamma = 1.4$, $C_p = 1.005$ kJ/kg K and R = 0.287 kJ/kg K.

2: The **Stefan Boltzmann** constant $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4.

3: Definitions: **Nusselt** number, $Nu = \dfrac{hL}{k}$; **Prandtl** number, $P_r = \dfrac{C_p \mu}{k}$

where L is a characteristic length and all symbols have their usual meanings.

4: The **exergy** of a system at state "1" in surroundings which are at state "*a*" is:

$$X = M\phi = M[\,(u_1 - u_a) + P_a(v_1 - v_a) - T_a(s_1 - s_a)\,] + \text{K.E.} + \text{P.E. etc.,}$$

where all symbols have their usual meaning.

5: Radioactivity. 1 Gigabecquerel (GBq) = 10^9 decays/sec.

1 Megaelectronvolt (MeV) = 1.6×10^{-13} Joules

6: For a perfect gas flowing through a stationary **normal shock wave**

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1} \quad ; \quad \frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2}$$

$$M_2^2 = \left[(\gamma-1) M_1^2 + 2\right] \Big/ \left[2\gamma\, M_1^2 - (\gamma-1)\right]$$

and for **adiabatic flow**

$$\frac{T_o}{T} = 1 + \frac{\gamma-1}{2} M^2$$

7: Properties of water:

T °C	ρ(kg/m^3)	μ(Pa.s)	ν(m^2/s)	**C$_p$**(kJ/[kgK])	**k** (W/[mK])
10	999.8	1.308×10^{-3}	$1.308\ 10^{-6}$	4.193	0.582
50	988.0	5.471×10^{-4}	$5.537\ 10^{-7}$	4.181	0.640
100	958.3	2.822×10^{-4}	$2.945\ 10^{-7}$	4.216	0.677

8: Incompressible fully developed **laminar flow in a circular tube**:

(a) constant heat flux at wall: $Nu \approx 4.36$

(b) constant wall temperature: $Nu \approx 3.66$

9: Incompressible fully developed **turbulent flow in a circular tube**:

$Nu \approx 0.022\ Pr^{0.5}\ Re^{0.8}$ ($Pr > 0.5$, $Re < 10^6$)

10: The axisymmetric **conduction equation** in cylindrical coordinates can be written:

$$\rho C \frac{\partial T}{\partial t} = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right] + S_V$$

where S_V represents heat generated/unit volume and there is no axial variation.

11: Thrust **specific fuel consumption**, *sfc:*

$sfc = \dot{m}_f / F = f / F_s$; $f = \dot{m}_f / \dot{m}_{air}$ = fuel/air ratio ; F_s = specific thrust.

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

AE1-111 Thermodynamics 2015/2016 Solutions

* Amended June 2016 to include exam report

Question A1

(a) $m = \frac{PV}{RT} = \frac{8 \times 10^5 \times 0.1}{287 \times 573} = 0.486\ \text{kg}$

$\boxed{0.486\ \text{kg}}$

(b) any one of: $\boxed{Q - W = \Delta E = \Delta U = mC_V \Delta T}$

(c)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} = (2)^{\frac{1}{3.5}} = 1.219$$

$$\Rightarrow T_2 = 1.219 \times 573.15\ \text{K} = 698.7\ \text{K}$$

$\boxed{T_2 = 698.7\ \text{K} \qquad \Delta T = 125.5\ \text{K}}$

(d) $\Delta U = mC_V \Delta T = 0.486 \times 718 \times 125.5 \Rightarrow \boxed{\Delta U = 43.8\ \text{kJ}}$

(e) $\cancel{Q} - W = \Delta U \Rightarrow mC_V \Delta T = -W \qquad \boxed{\Rightarrow W = -43.8\ \text{kJ}}$

<u>Common errors:</u>

Mostly answered completely. A few uses of C_P not C_V

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

Question A2

(a) By the First law for a system working in a cycle:

(i) $$\oint Q - W = 0$$

We have Q^H rejected (negative heat to system), Q^C absorbed (positive heat to the system) and an amount W of work input (so negative)

$$-Q^H + Q^C \quad -W \quad = 0$$

$$\Rightarrow -Q^H \quad +1 \quad -(-0.2) = 0$$

$$\Rightarrow \boxed{Q^H = \quad 1.2 \text{ MJ}}$$

(ii) Check Clausius:

$$\oint \frac{Q}{T} \le 0 \ ? \quad \frac{-Q^H}{T_H} + \frac{Q^C}{T_C} \le \ 0? \quad \frac{-1.2}{370} + \frac{1}{270} > 0$$

$$\boxed{\text{so device is } \textbf{impossible.}}$$

(b)

$$Q^{H(in)} - Q^{C(out)} \quad = \quad W^{(out)} \quad \Rightarrow Q^{C(out)} = \ Q^{H(in)} - \quad W^{(out)}$$

$$\Rightarrow Q^{C(out)} = \ 3 - 2 = 1 \text{ MJ}$$

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{1}{3} = \frac{2}{3};$$

$$\eta = \eta_{Carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{T_C}{500};$$

$$\Rightarrow 1 - \frac{T_C}{500} = \frac{2}{3} \Rightarrow \frac{T_C}{500} = \frac{1}{3}$$

$$\boxed{\Rightarrow T_C = 167 \text{ K}}$$

Common errors:

(ii) - signs, wrong use of Carnot (do not use with a heat pump/refrigerator)

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

	Marks

Question A3

(a) either of: $\Delta S = \int \frac{dQ_{rev}}{T}$ or $S = k_B \ln \Omega$

(b) $\Delta S = S_2 - S_1 = mC_V \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$

As in Q1, $m = \frac{PV}{RT} = \frac{8\times10^5 \times 0.1}{287\times573} = 0.486\,\text{kg}$ m = 0.486 kg,

$$\Delta S = (S_2 - S_1) = 0.486\times718\times\ln\left(\frac{373}{573}\right) + R\ln\cancel{\left(\frac{v_2}{v_1}\right)}$$

$$\Delta S = -149.8\ \text{JK}^{-1}$$

(c) The maximum quantity of useful work that could be performed by a system in a process in which it reaches equilibrium with its surroundings.

- *Note – not sufficient to say max useful work – must mention surroundings*

(d) Correct answer:

From the datasheet, exergy is given by

$$X_1 = (U_1 - U_a) - T_a(S_1 - S_a) + P_a(V_1 - V_a) + \tfrac{1}{2}Mv^2 + Mgz + \ldots$$

Exergy is measured relative to ambient – P_a, T_a.
Therefore exergy is not determined unless the temperature of the environment is specified

Too few answered this subtle point; a short & sufficient answer:

exergy is likely to have reduced, because
useful work could be extracted by allowing the gas to cool

– this is assuming ambient conditions to be less than 473 K (see note)

Common errors:

(a) - must mention ‘rev’ if using the definition $\Delta s = dQ_{rev}/T$;

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

Question A3 - notes

instead of writing $\Delta S = \int \frac{dQ_{rev}}{T}$ some wrote $\Delta S = \oint \frac{dQ_{rev}}{T}$ - - but

$\oint (any\ property) = 0$

(b) – if question asks for entropy change and system mass is known,

must "put the m into ΔS", i.e. $\Delta S = m\,\Delta s$

Note on Question A 3

Part (d) More complete answer.

From the datasheet,

$$X_1 = (U_1 - U_a) - T_a(S_1 - S_a) + P_a(V_1 - V_a) + \tfrac{1}{2}Mv^2 + Mgz + \ldots$$

Since the volume of the system does not change in this process

$$\Delta X = X_1 - X_2 = (U_1 - U_2) - T_a(S_1 - S_2)$$
$$= C_V (T_1 - T_2) - T_a C_V (\ln (T_1/T_2))$$
$$= C_V T_a ((T_1 - T_2)/T_a - \ln (T_1/T_2)) \quad (*)$$

Assuming T_a to be 293.15 K, gives a positive value for the difference in exergy, meaning work could be extracted from the drop in temperature of the gas; in other words the exergy of the gas would decrease.

$$\Delta X = 0.486 \times 718\,((200) - 293.15 \times \ln(573/373))$$

$\Delta X = 2.588\ e{+}04$ so the system exergy is decreased

Note.

From the equation(*) above, the change in exergy depends on T_a.
If T_a is large, the exergy change can be negative (i.e., the system exergy could increase - in such a situation, cooling from 573K to 373 K moves the system further away from equilibrium with the surroundings, allowing work to be derived from heating the system.
Using (*) one can find the ambient temperature that corresponds to positive, negative or zero changes in the system exergy; likewise can use equn. (*) to show that if state 2 is the same as ambient, the exergy change for a constant volume process is positive.

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

Question A4

(i) Find the initial rate of flow through the valve.
High pressure ratio => choked flow.

$$\rho_0 = \frac{P_0}{RT_0} = \frac{5\times10^5}{287\times300} = 5.81$$

Let $B = \left(1+\frac{\gamma-1}{2}\right)^{1/(\gamma-1)} = (1.2)^{2.5} = 1.5774$

$$\rho^* = (5.81/B) = 3.68;$$

$$T^* = (300)/1.2 = 250$$

$$u^* = \sqrt{1.4\times287\times250} = 317$$

$$\dot{m}^* = \rho^* u^* A_T = 3.68\times317\times0.5\times10^{-4}$$

$$\boxed{\dot{m}^* = 0.0583 \text{ kg.s}^{-1}}$$

(ii) Use gas law to relate P & m while T remains constant.

$$PV = mRT$$

$$\Rightarrow V\frac{dP}{dt} = RT\frac{dm}{dt} \Rightarrow \frac{dP}{dt} = \frac{RT}{V}\frac{dm}{dt}$$

$$\frac{dP}{dt} = \frac{287\times300}{10}(-0.0583)$$

$$\boxed{\frac{dP}{dt} = -502 \text{ Pa.s}^{-1}}$$

(iii) Part (ii), gives an estimate for the rate of pressure drop, 500 Pa/s; This case can't be an order of magnitude different; as the pressure loss is proportionately slow, it is reasonable to assume the adiabatic pressure loss can be treated as isentropic.

$$Pv^\gamma = const.$$

Differentiating the above w.r.t. time

$$\frac{dP}{dt}v^\gamma + P\cdot\gamma v^{\gamma-1}\frac{dv}{dt} = 0.$$

$$v = \frac{V}{m} \Rightarrow \frac{dv}{dt} = -\frac{V}{m^2}\frac{dm}{dt} = -\frac{v}{m}\frac{dm}{dt}$$

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

Question A4 - continued

$$\therefore \frac{dP}{dt}v^{\gamma} + P\cdot\gamma v^{\gamma-1}\left(-\frac{v}{m}\frac{dm}{dt}\right) = 0$$

$$\Rightarrow \frac{dP}{dt} = \frac{\gamma P}{m}\frac{dm}{dt} = \frac{\gamma mRT}{mV}\frac{dm}{dt} \Rightarrow \frac{dP}{dt} = \frac{\gamma RT}{V}\frac{dm}{dt}$$

So a factor of γ higher than isothermal case.

$$\frac{dP}{dt} = \frac{1.4\times287\times300}{10}(-0.0583) \quad \boxed{\Rightarrow \frac{dP}{dt} = -703\ \text{Pa.s}^{-1}}$$

Note: (1) if the relation $Pv^{\gamma} = const.$, is forgotten, can be re-derived from

$$dQ/m = dW_{rev}/m + du \quad \Rightarrow 0 = Pdv + C_V dT \qquad (a)$$

$$Pv = RT \Rightarrow dT = \frac{Pdv}{R} + \frac{vdP}{R} \qquad (b)$$

$$(b)\text{into}(a): \Rightarrow Pdv + C_V\left[\frac{Pdv}{R} + \frac{vdP}{R}\right] = 0 \Rightarrow Pdv\left(1 + \frac{C_V}{R}\right) + \frac{C_V}{R}vdP = 0$$

$$\Rightarrow C_P Pdv + C_V vdP = 0 \quad \Rightarrow \gamma\frac{dv}{v} + \frac{dP}{P} = 0 \Rightarrow Pv^{\gamma} = const.$$

Note: (2) Alternately, the result could be found almost as easily by the direct approach, following the same lines as for the isothermal case.

$$PV = mRT \Rightarrow \frac{dP}{dt}V = \dot{m}RT + mR\frac{dT}{dt}$$

$$Isentropic: \Rightarrow P^{(\gamma-1)/\gamma} \propto T \Rightarrow P^{(\gamma-1)/\gamma} = cT$$

$$\Rightarrow \frac{dT}{dt} = \frac{1}{c}\frac{\gamma-1}{\gamma}P^{-1/\gamma}\frac{dP}{dt}$$

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

Question A4 - continued

$$\Rightarrow \frac{dP}{dt} = \frac{\dot{m}RT}{V} + \frac{mR}{V}\frac{1}{c}\frac{\gamma-1}{\gamma}P^{-1/\gamma}\frac{dP}{dt}$$

$$= \frac{\dot{m}RT}{V} + \frac{P}{T}\frac{1}{c}\frac{\gamma-1}{\gamma}P^{-1/\gamma}\frac{dP}{dt}$$

$$\Rightarrow \frac{dP}{dt} = \frac{\dot{m}RT}{V} + \frac{\gamma-1}{\gamma}\frac{dP}{dt} \qquad \Rightarrow \frac{dP}{dt} = \frac{\gamma\dot{m}RT}{V}$$

The question with worst performance in section A. <u>Common errors:</u>

Failing to spot question is about choked flow.
Using ρ ,T from reservoir - not calculating conditions at the throat where T has dropped, ρ has dropped so the true mass flow rate is about half the value computed neglecting these changes.

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

Question A5

$$M_\infty = 0.82 \quad ; \quad P_\infty = 0.2 \text{ bar} ; \quad T_\infty = 260 \text{ K}$$

Let "1" denote conditions immediately upstream of shock, "2" downstream

(a) Isentropic relations: $P_{01} = P_1\left(1+\frac{\gamma-1}{2}M_1^2\right) = P_{0\infty} = P_\infty\left(1+\frac{\gamma-1}{2}M_\infty^2\right)$

$$P_1 = P_\infty \times \left(1+\left((\gamma-1)/2\right)M_\infty^{\ 2}\right)^{\gamma/(\gamma-1)} \Big/ \left(1+\left((\gamma-1)/2\right)M_1^{\ 2}\right)^{\gamma/(\gamma-1)}$$

$$P_1 = 0.2 \times \left(1+0.2(0.82)^2\right)^{3.5} \Big/ \left(1+0.2(1.5)^2\right)^{3.5} = 0.2 \times 0.4236$$

$$\boxed{\Rightarrow P_1 = 0.0847 \quad (\text{bar});}$$

(-or find via P_{01} = 0.311 bar)

(b) Total temperature:

$$T_{0\infty} = T_{01} = T_{02} = T_\infty\left(1+\frac{\gamma-1}{2}M_\infty^2\right) = 260 \times 1.1345$$

$$\boxed{\Rightarrow T_{01} = T_{02} = 295 \text{ K};}$$

(c) Apply Rankine Hugoniot relation given on datasheet:

$$M_2^{\ 2} = \frac{(\gamma-1)M_1^{\ 2}+2}{2\gamma M_1^{\ 2}-(\gamma-1)} \quad ; \quad M_1 = 1.5 \Rightarrow \boxed{M_2 = 0.701}$$

(d) Apply Rankine Hugoniot relation given on datasheet:

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}$$

(-- if calculated (not asked for) $P_2 / P_1 = 2.46$, $P_2 = 0.21$)

Combine isentropic and Rankine Hugoniot relations to obtain:

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}}{P_2} \times \frac{P_2}{P_1} \times \frac{P_1}{P_{01}}$$

$$\boxed{\frac{P_{02}}{P_{01}} = \left(1+\frac{\gamma-1}{2}M_2^{\ 2}\right)^{\left(\gamma/_{\gamma-1}\right)}\left[\frac{2\gamma M_1^{\ 2}}{\gamma+1} - \frac{\gamma-1}{\gamma+1}\right]\left(1+\frac{\gamma-1}{2}M_1^{\ 2}\right)^{-\left(\gamma/_{\gamma-1}\right)}}$$

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

	Marks
(-- if calculated ΔP_0 = 0.021 bar)	

Common errors:

(a) Taking upstream conditions as total conditions
(b) using shock relation instead of isentropic relation to go from upstream to 1
(c) Incomplete relation given for (d) – need to show complete path to compute total pressure ratio.

Course Code and Title : AE 1-111 Thermodynamics

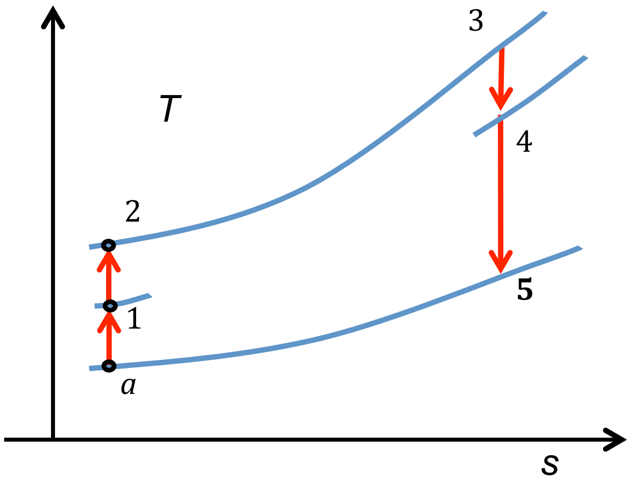
Setter: Denis Doorly

	Marks

B

Question 6 Solution

Part (a) .



Part (b)

$V_a = M_a\sqrt{\gamma RT} = 475.4\ \mathrm{ms^{-1}}$

⇨ $\Delta T = Va^2/2C_P = 112.25 \Rightarrow T_1 = 362.4$ $\boxed{T_1 \sim 362\ \mathrm{K}}$

⇨ $P_1 = P_a\left(\dfrac{T_1}{T_a}\right)^{\frac{\gamma}{\gamma-1}} = 47\left(\dfrac{362.4}{250}\right)^{3.5}$ $\boxed{\Rightarrow P_1 = 172.4\ \mathrm{kPa}}$

Part (c) Now for compressor:

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 362.4(10)^{1/3.5} \quad \boxed{\Rightarrow T_2 \approx 700\ \mathrm{K}}$$

$W_C/m = C_P\left(T_2 - T_1\right) = 1005\times(699.8-362.4) = 339.1\ \mathrm{kJ/kg}$

$$P_2 = P_3 = 10\times P_1 = 10\times 1.724 \quad \boxed{\Rightarrow P_2 = 17.24\ \mathrm{bar}}$$

Part (d) For a turbojet, watch for changed values of γ, C_P :

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

$$W_C = W_T$$
$$\Rightarrow mC_{P,air}\left(T_2 - T_1\right) = mC_{P,gas}\left(T_3 - T_4\right)$$
$$\Rightarrow 1005 \times (699.8 - 362.4) = 1147 \times \left(T_3 - T_4\right) \Rightarrow \frac{339.1 \times 10^3}{1147} = \left(T_3 - T_4\right)$$
$$\Rightarrow \quad \left(T_3 - T_4\right) = 295.6$$
$$\Rightarrow \quad T_4 = 1104.4\ \text{K}$$

T_4 ~1104 K

** Note that the C_P value has changed, so T_4 is higher than in the pure air standard cycle.

$$P_4 = P_3\left(\frac{T_4}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = 17.24\left(\frac{1104.4}{1400}\right)^{\frac{4/3}{4/3-1}} = 17.24\left(\frac{1104.4}{1400}\right)^4$$
$$\Rightarrow P_4 = 6.68\,\text{bar}$$

Note: Incorrect γ results in higher pressure.

P_4 ~ 6.7 bar

Part (e) : require the jet velocity, V_J : $F = \dot{m}(V_J - V_A)$

Fully expanded: $\Rightarrow P_5 = P_a$

Find T_5, and from this V_J :

$$T_5 = T_4\left(\frac{P_a}{P_5}\right)^{\frac{\gamma-1}{\gamma}} = 1104.4\left(\frac{0.47}{6.68}\right)^{\frac{\gamma-1}{\gamma}} = 1104.4\left(\frac{0.47}{6.68}\right)^{\frac{1}{4}} = 568.8\ \text{K}$$

T_5 ~570 K

* Answer would be 500 K if one used γ for air in the previous two calculations

$$C_P T_5 + \tfrac{1}{2}v_J^2 = C_P T_4$$
$$\Rightarrow v_J^2 = 2C_P(T_4 - T_5) = 2 \times 1147 \times (1104.4 - 568.8)$$
$$\Rightarrow v_J = 1109\ \text{m}\cdot\text{s}^{-1}$$

$v_J \approx 1110\ \text{m}\cdot\text{s}^{-1}$

Thrust

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

$$F = \dot{m}_a\left((1+f)v_J - v_A\right)$$

$$\Rightarrow F/\dot{m}_a = 1.02\times1109 - 475 \simeq 656\ \mathrm{N.kg^{-1}s} \qquad \boxed{F_s \approx 655\ \mathrm{N.kg^{-1}s}}$$

$$neglecting\ f \quad \Rightarrow F/\dot{m}_a\big|_{approx} = F_{s,(approx)} = 1109 - 475 = 634\ \mathrm{N/(kg/s)} = 634\ \mathrm{N.kg^{-1}s}$$

$$\boxed{F_{s,\,approx} \approx 635\ \mathrm{N.kg^{-1}s}}$$

Thrust specific fuel consumption: fuel consumption/unit thrust

$$sfc = \dot{m}_f/F = \left(\dot{m}_f/\dot{m}_a\right)\cdot\left(\dot{m}_a/F\right) = f/F_s$$

$$\Rightarrow sfc = f/F_s \approx 0.02/634 = 3.16\times10^{-5}\ \mathrm{kg/N} = 0.032\ \mathrm{kg/kN}$$

Above acceptable, better to use specific thrust

$$sfc = f/F_s = 0.02/656 \simeq 3\times10^{-5}\ \mathrm{kg/N} \qquad \Rightarrow \boxed{sfc \approx 3\times10^{-2}\ \mathrm{kg/kN}}$$

Generally well answered by nearly all students, with probably more than 75% scoring 80% or above.

Common errors:

(a) Not equating W_C and W_T

(b) Assuming pressure ratio across turbine = pressure ratio across compressor. Remember this is not the case; it is the total enrthalpy change which is the same $W_C = W_T \quad \Rightarrow m_{air}C_{P,air}\left(T_2 - T_1\right) = m_{gas}C_{P,gas}\left(T_3 - T_4\right)$

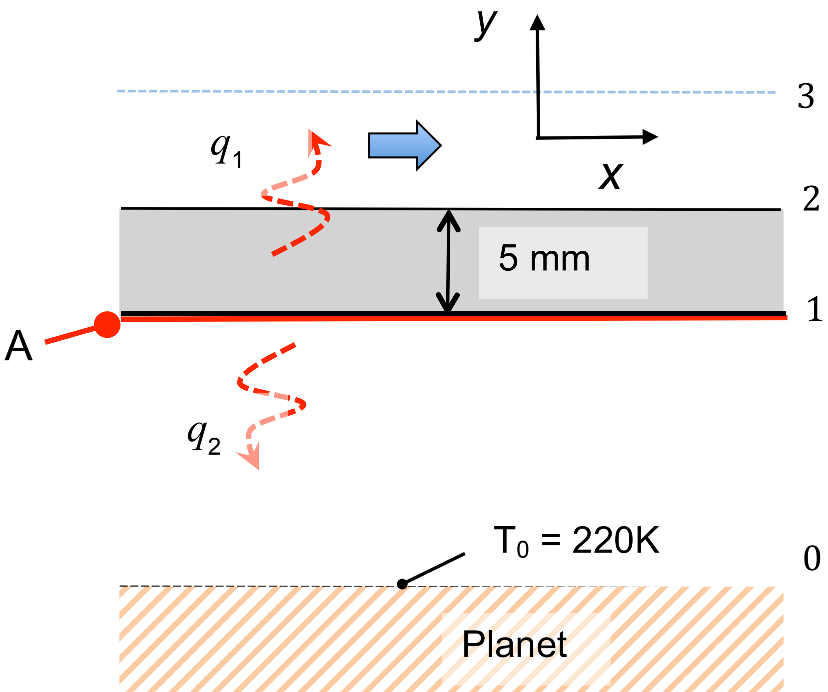
(c) Not using correct C_P value for turbine expansion

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

Question B7 Solution



a) Total heat dissipated by A, say q_T, is lost:

either by radiation, q_2 ,

or to coolant flow, q_1

Resistances: 1 - 2: conduction, 2-3 convection, 1- 0 radiation.

$$q_1 = h(T_2 - T_3) = k\frac{(T_1 - T_2)}{\delta}$$

$$q_2 = \varepsilon\sigma(T_1^4 - T_0^4)$$

b)

$$q_T = q_1 + q_2;$$

$$q_1 : (T_1 - T_3) = (T_1 - T_2) + (T_2 - T_3) = q_1\left(\frac{1}{h} + \frac{\delta}{k}\right)$$

$$\Rightarrow q_1 = \frac{(T_1 - T_3)}{\left(1/h + \delta/k\right)}$$

$$\Rightarrow q_T = \frac{(T_1 - T_3)}{\left(1/h + \delta/k\right)} + \varepsilon\sigma(T_1^4 - T_0^4)$$

Ans

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

c) First find q_2

$$q_2 = \varepsilon\sigma(T_1^4 - T_0^4) = 1\times 5.67\times 10^{-8}(260^4 - 220^4) = 126\ \text{W.m}^{-2}$$

It is thus acceptable to treat q_2 as negligible.

In this case

$$q_1 = \frac{(T_1 - T_3)}{\left(1/h + \delta/k\right)} \Rightarrow T_3 = T_1 - q_1\left(1/h + \delta/k\right)$$

$$T_3 = 260 - 1\times 10^5\left(1/10^3 + 5\times 10^{-3}/300\right) \qquad \boxed{\Rightarrow T_3 = 158.3\ \text{K}}$$

d) Let ΔT be the temperature rise along the cooling channel over length L.

Apply simple energy balance:

$$Q = \dot{m}C\Delta T$$

Assuming the channel has span dz , height dy, and mean velocity U, then

$$Q = q_1 \times dz \times L \quad ; \quad \dot{m} = \rho U dz \times dy \quad (leave\ as\ \dot{m})$$

$$\Rightarrow q_1 \times dz \times L = \dot{m}C(\Delta T_{out-in}) \quad \left(= \dot{m}C(T_{out} - T_{in})\right)$$

$$\Rightarrow q_1 = \frac{\dot{m}C(\Delta T_{out-in})}{dz\times L}; \quad \dot{m}/dz = \dot{m}_w$$

$$\Rightarrow q_1 = \frac{\dot{m}_w C(\Delta T_{out-in})}{L}$$

(*Or say that since* dz *is arbitrary, we can take it as* 1.)

Refer to the mass flow per unit width of the channel as m_w.

Since

$$q_1 = \frac{(T_1 - T_3)}{\left(1/h + \delta/k\right)}$$

$$\Rightarrow \frac{\dot{m}_w C(\Delta T_{out-in})}{L} = \frac{(T_1 - T_3)}{\left(1/h + \delta/k\right)}$$

$$\Rightarrow \Delta T_{out-in} = \frac{(T_1 - T_3)L}{\dot{m}_w C\left(1/h + \delta/k\right)}$$

Course Code and Title : AE 1-111 Thermodynamics

Setter: Denis Doorly

Marks

e) From the preceding, heat loss by radiation is insignificant during initial temperature rise.

$$\rho C(A\delta)\frac{dT}{dt} = Q = A(q_T - q_2) \approx Aq_T$$

$$\Rightarrow \frac{dT}{dt} \approx q_T / \rho C\delta = 10^5 / 3\times10^3 \times 10^3 \times 5\times10^{-3}$$

$$\boxed{\Rightarrow \frac{dT}{dt} \approx 6.7 \text{ K.s}^{-1}}$$

Time taken to rise from 260 to 400 K is 140/6.7 ~ 21 s.
(Note – even at 400 K, radiative heat transfer is small, so rate of temperature rise can be assumed unaffected).

Generally not well answered by most.
Common errors:

(b): Very common mistake was in not recognizing parallel paths. Too many tried to derive an overall U value – effectively assuming a single heal loss path and thus negating all further work. Considering the geometry should make it clear that a proportion of the heat is lost by radiation on one side, and quite a different proportion is lost through the conductive and convective resistances in series on the other side.

(d): Trying to fudge the answer. A clear logical progression is required, starting with an expression for the total heat transfer $Q = \dot{m}C\Delta T$ and relating Q to q by channel area = *length* x *width*. A good check is whether equations are dimensionally correct; if not, then equations and further work are invalid.

(e) Few attempts and very few of those were correct.