IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2009

EEE PART I: MEng, BEng and ACGI

Corrected Copy

DEVICES AND FIELDS

Tuesday, 26 May 10:00 am

Time allowed: 2:00 hours

There are SIX questions on this paper.

Question ONE and Question FOUR are compulsory. Answer Question One and Question Four, plus one additional question from Section A and one additional question from Section B.

Questions One and Four each carry 20% of the marks; remaining questions each carry 30% of the marks.

Use a separate answer book for each section.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	K. Fobelets, E.M. Yeatman
	Second Marker(s) :	E.M. Yeatman, B.C. Pal

Special instructions for invigilators

This exam consists of **2 sections.** Section A: **Devices** and section B: **Fields.** Each section has to be solved in their respective answer books. Check that 2 different answer books are available for the students. Questions 1 and 4 are obligatory.

Special instructions for students

Use different answers books for each section:

Devices: answer book **A**
Fields: answer book **B**

Questions **1** and **4** are obligatory.

Constants and Formulae

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10}$ cm^{-3} at $T = 300$K
dielectric constant of Si:	$\varepsilon_{Si} = 11$
dielectric constant of SiO_2:	$\varepsilon_{ox} = 4$
thermal voltage:	$kT/e = 0.026$V at $T = 300$K
charge of an electron:	$e = 1.6 \times 10^{-19}$ C

Formulae

$$\left.\begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p \frac{dp(x)}{dx} \end{aligned}\right\}$$

Drift and diffusion current densities in a semiconductor

$$I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Current in a MOSFET

$$\left.\begin{aligned} J_n &= \frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}} - 1\right) \\ J_p &= \frac{eD_p p_{n_0}}{L_p}\left(e^{\frac{eV}{kT}} - 1\right) \end{aligned}\right\}$$

Diffusion current densities in a pn-junction

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT}\right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$

Minority carrier injection under bias V

$$D = \frac{kT}{e}\mu$$

Einstein relation

SECTION A: SEMICONDUCTOR DEVICES

1. This question is **obligatory**.

a) Give the reason for the non-zero conductivity of semiconductors at room temperature? [4]

b) Give the name of the:
i) majority carrier in a p-type semiconductor [2]
ii) minority carrier in a p-type semiconductor [2]

c) Sketch the current versus voltage characteristics (current on "y" axis, voltage on "x" axis) of an Ohmic contact on a semiconductor. [4]

d) In fig.1.1 the energy band diagram of an un-biased pn diode is given. Three regions, I, II, III are defined. Across which of these three regions occurs the internal electric field $\mathcal{E}_0$? Give a brief explanation. [4]

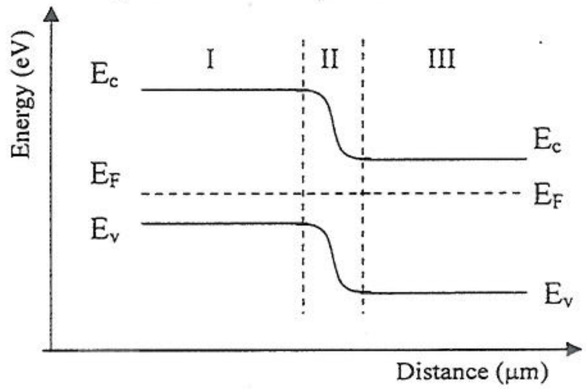

Figure 1.1: The energy band diagram of a pn diode without bias.

e) What is the doping type of the substrate of an enhancement mode n-channel (conduction of electrons) MOSFET? Give a brief explanation. [4]

2.

a) In a pn diode under forward bias, holes are injected across the depletion region into the n-type region and electrons are injected into the p-type region. By which physical process do the injected carriers in these regions then move towards the contacts? [4]

b) Will the total current through the diode, at constant voltage, increase, decrease or remain the same when the doping density in both diode regions is increased by the same factor? Prove your answer. You can assume that the electron and hole material parameters do not change as a function of doping. [4]

c) Prove that the hole current in a p^+n diode (the p-type region is more heavily doped than the n-type region) is larger than the electron current. Assume that electrons and holes have the same material parameters. [4]

d) Calculate the excess charge in each region of a Si diode when a forward bias of V=0.26V is applied. The parameters of the diode are:
The donor doping density: $N_D=10^{16}$ cm^{-3}.
The acceptor doping density: $N_A=10^{18}$ cm^{-3}.
The diffusion length of the minority carriers: $L_p=10^{-2}$ cm.
$L_n=10^{-3}$ cm.
The diffusion constant of the minority carriers: $D_p=10$ cm^2/s.
$D_n=20$ cm^2/s.
The area of the diode is: $A=10^{-2}$ cm^2.
The temperature is: T=300K. [8]

e) Derive the expression of the electron and hole current density in a pn diode when the length of each region is smaller than the diffusion length of the carriers. Call X_p the length of the p-type region and X_n the length of the n-type region. What happens to the current when the lengths of the regions decrease? [10]

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3.

a) Does the threshold voltage increase, decrease or remain constant if the doping density in the substrate of the n-type enhancement mode MOSFET is increased? Explain your answer briefly. [4]

b) i) Derive an expression of the transconductance, g_m, in the saturation region of an ideal MOSFET as a function of material parameters? [3]

ii) What happens to the transconductance, g_m, when the gate oxide thickness is decreased? Explain briefly why. [3]

c) In Fig.3.1 the output current-voltage characteristics of an n-type MOSFET are given for two gate voltages for which the MOSFET is conducting. Sketch the energy band diagram (E_c and E_v) of the MOSFET from source to drain through the channel at the bias points I and II. Sketch the Fermi level in the source and drain regions only. Ensure that the differences between the two situations are clear. [8]

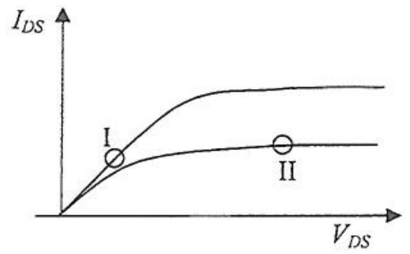


Figure 3.1: Current-voltage characteristics of an nMOS for two gate voltages above threshold. The bias points are indicated with small circles.

d) An ideal enhancement mode Si MOSFET is measured and the characteristics are given in Fig.3.2 for two values of gate voltage overdrive: V_{GS}-V_{th}. The gate length and width are respectively $L_g = 1$ μm and $W_g = 10$ μm. The oxide capacitance per area is $C_{ox} = 10^{-6}$ F/cm^2.

i) Determine the gate voltage overdrive: V_{GS}-V_{th} for the two current-voltage characteristic in Fig. 3.2. [4]

ii) Give a general expression for the source-drain current, I_{DS}, in terms of V_{DS}, V_{GS}, V_{th} and material parameters for $V_{DS} < 0.2$ V for the MOSFET with characteristics given in Fig.3.2. [4]

iii) Extract the carrier mobility, μ from these measurements. [4]

Figure 3.2 is given on the next page (page 6 of 9).

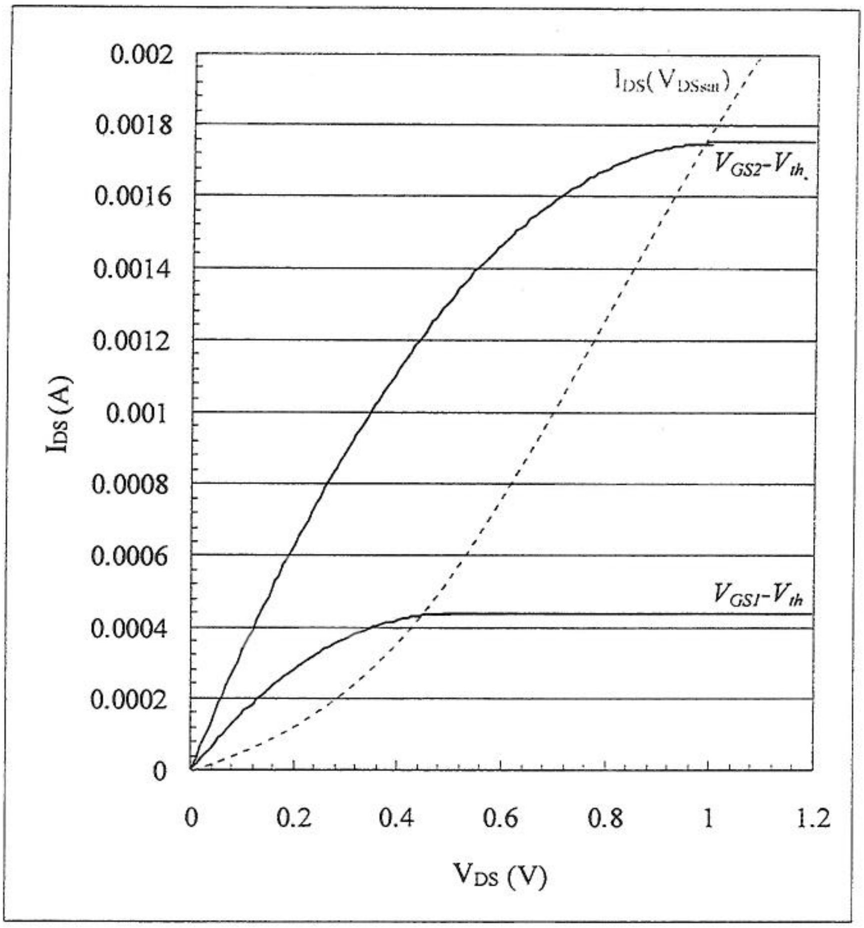


Figure 3.2: The current-voltage characteristics of a MOSFET at two different values of gate overdrive $V_{GS}-V_{th}$ (black plain lines). The current at saturation (grey dashed line).

SECTION B: FIELDS

4. This question is **obligatory**. Answer all parts. Each part is worth 4 marks.

a) Find the magnitude of the electric field E at a point midway between two point charges, 0.2 m apart, of charge +0.1 μC and −0.1 μC respectively.

b) A hollow conducting sphere of radius 2 mm has a net surface charge of 10^{-7} C. What is the electric field magnitude at a point 1 mm from the centre of the sphere?

c) An air-filled parallel plate capacitor has an area of 1 mm^2 and a gap of 10 μm, and is charged to 5 V. What is the stored energy in the capacitor?

d) A toroid with an iron core has a 50 turn winding through which a current of 2 A flows. The toroid radius is 5 cm. Find the magnitude of the magnetic field strength H within the core. State any assumptions or approximations made.

e) An ideal transformer has a turns ratio N_1:N_2 = 2:1, and a load of 20 Ω is connected to the secondary coil terminals. If an AC voltage of magnitude 5 V is applied to the primary coil, what will be the magnitude of the current I_1 flowing in the primary coil?

5.

a) State Gauss' law <u>in words</u>. [6]

b) Using Gauss' law, derive an expression for the total electric flux flowing between the plates of a parallel plate capacitor as shown in Fig. 5.1 carrying a charge Q. Hence, find an expression for the flux density D and the electric field strength E. State any assumptions or approximations made. [6]

c) Using your results from part (b), derive an expression for the capacitance as a function of the plate area A, the plate separation d and the relative permittivity ε_r. [6]

d) A capacitor as in Fig 5.1, with air as the dielectric and an initial plate separation d_o, is charged to a voltage V_o and then disconnected from the voltage source, so that its charge remains constant. The separation d is then increased gradually. Derive an expression for the stored energy as a function of d. Hence, derive an expression for the force between the plates as a function of V_o. [6]

e) A capacitor as in Fig 5.1 of fixed separation d, initially discharged, is gradually charged at a constant current I starting at time $t = 0$. Derive an expression for the power flowing into the capacitor as a function of t, and hence show that the total stored energy at any t equals $\frac{1}{2}CV^2$. where V is the voltage at time t. [6]

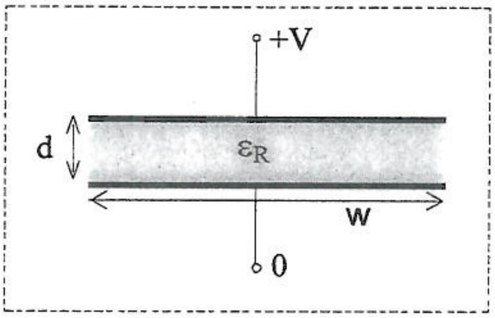


Figure 5.1

6.

a) An ideal transformer of turns ratio N_1:N_2 = 2:1 is connected to a real load of impedance Z_L = 50 Ω. For a signal of amplitude 20 V at 50 Hz applied to the primary terminals, find the primary and secondary current amplitudes, and the secondary voltage amplitude.
Define, and derive an expression for, the referred load Z_L', and show that this expression is consistent with your results above. [10]

b) Draw an equivalent circuit for a realistic transformer, with both series and parallel components to represent the non-idealities. Indicate briefly what physical phenomena are responsible for each component. [10]

c) Open and short circuit tests have been conducted with a real transformer of turns ratio N_1:N_2 = 2:1, yielding the measurements shown in Table 6.1. Voltages and current are given in rms, and powers are average powers. Using these values, calculate values for the components of your equivalent circuit in part (b). State and justify any assumptions or approximations made. [10]

Table 6.1

	V_1	V_2	I_{in}	P_{in}
Open Circuit Test	110 V	55 V	59.2 mA	2.42 W
Closed Circuit Test	5 V	0 V	1.58 A	2.5 W

Final
May 09

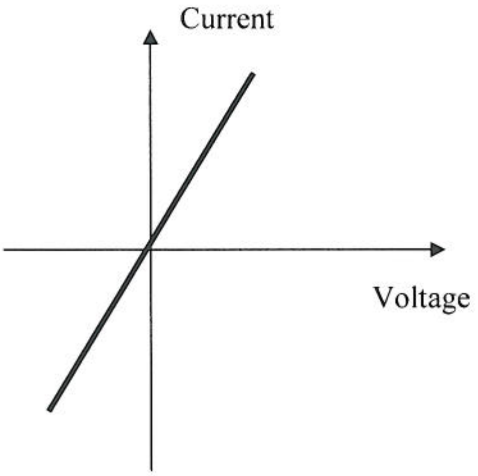
SECTION A: SEMICONDUCTOR DEVICES Solutions 2009

1. This question is **obligatory**.

a) Because at room temperature electron-hole pairs are generated via the breakage of some of the covalent bonds between the semiconductor atoms. This creates an intrinsic carrier concentration of free holes and electrons that can conduct current. [4]

b)
i) holes [2]
ii) electrons [2]

c) [4]



d) II . This is the region where band bending occurs. This means that there exists a potential difference across this region and thus an electric field. [4]

e) p-type, acceptor doping. In the given type of MOSFET a gate voltage has to be applied in order to create an inversion layer = a channel of electrons. Thus a p-type substrate has to be inverted to give an electron channel. [4]

2.

a) Diffusion [4]

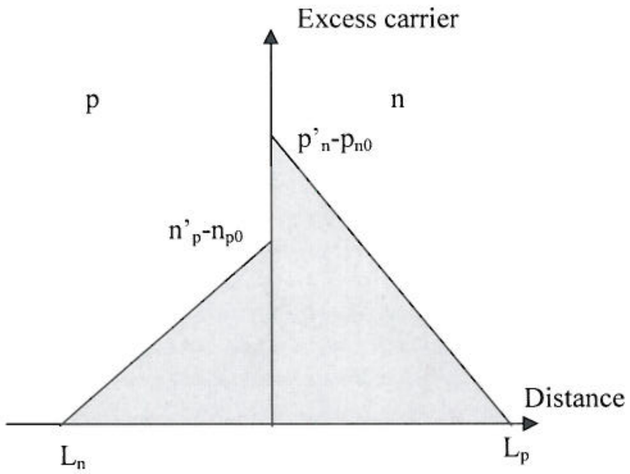
b) Current density in a pn diode from formulae list: $$J_n = \frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)$$ $$J_p = \frac{eD_p p_{n_0}}{L_p}\left(e^{\frac{eV}{kT}} - 1\right)$$ is directly proportional to the minority carrier densities n_{p0} and p_{n0}. The minority carrier density is inversely proportional to doping: $n_{p0}=n_i^2/N_A$ and $p_{n0}=n_i^2/N_D$. Thus when N_A and N_D increase, the current density decreases. [4]

c) Take formulae above and notice that:

$$J_n \propto \frac{D_n n_{p0}}{L_n} = \frac{D_n n_i^2}{L_n N_A}$$
$$J_p \propto \frac{D_p p_{n_0}}{L_p} = \frac{D_p n_i^2}{L_p N_D}$$
since $N_A \gg N_D$ and the other parameters are of the same order of magnitude we see that $J_n \propto \frac{1}{N_A} << J_p \propto \frac{1}{N_D}$ [4]

[8]

d) [8]



Sketch of the minority excess carrier variation in each region. The excess charge stored in each region is proportional to the grey area. Note that the subscript ' indicates injected carrier concentration.

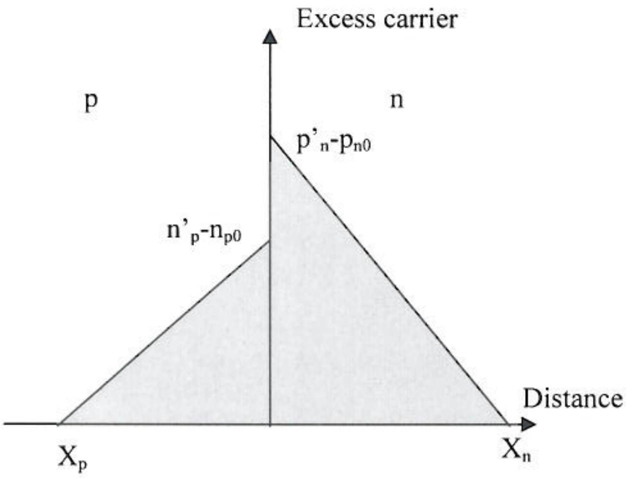
$$Q_n = \frac{-e(n'_p - n_{p_0})L_n A}{2} = \frac{-e \times n_{p_0}\left(e^{\frac{eV}{kT}} - 1\right)L_n A}{2} = \frac{-e \times n_i^2\left(e^{\frac{eV}{kT}} - 1\right)L_n A}{2N_A}$$

$$Q_p = \frac{e(p'_n - p_{n_0})L_p A}{2} = \frac{-e \times p_{n_0}\left(e^{\frac{eV}{kT}} - 1\right)L_p A}{2} = \frac{-e \times n_i^2\left(e^{\frac{eV}{kT}} - 1\right)L_p A}{2N_D}$$

$$Q_n = \frac{-1.6\,10^{-19}\,C \times \left(1.45\,10^{10}\ cm^{-3}\right)^2\left(e^{\frac{0.26}{0.026}} - 1\right)10^{-2}\,cm\,10^{-2}\,cm^2}{2 \times 10^{18}\ cm^{-3}} = -3.7\,10^{-17}\,C$$

$$Q_p = \frac{1.6\,10^{-19}\,C \times \left(1.45\,10^{10}\ cm^{-3}\right)^2\left(e^{\frac{0.26}{0.026}} - 1\right)10^{-3}\,cm\,10^{-2}\,cm^2}{2 \times 10^{16}\ cm^{-3}} = 3.7\,10^{-16}\,C$$

e) [10]



The diffusion current is determined by the gradient of the minority carrier concentration.

$$J_n = \frac{eD_n(n'_p - n_{p_0})}{X_p} = \frac{eD_n n_{p_0}\left(\exp\left(\frac{eV}{kT}\right) - 1\right)}{X_p}$$

$$J_p = \frac{eD_p(p'_n - p_{n_0})}{X_n} = \frac{eD_p p_{n_0}\left(\exp\left(\frac{eV}{kT}\right) - 1\right)}{X_n}$$

Since the length of the layers is in the denominator, the currents increase when the lengths decrease.

3.

a) The threshold voltage increases because we need to repel more holes before inversion can occur.

b) i) $I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2} \right)$ total MOSFET current

In saturation the current becomes: $I_{DS} = \frac{\mu C_{ox} W}{2L}\left((V_{GS} - V_{th})^2 \right)$

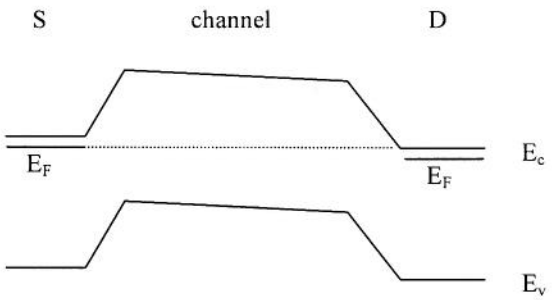
The transconductance is: $g_m = \frac{dI_{DS}}{dV_{GS}} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_{th}) \right)$ [3]

ii) $g_m = \frac{dI_{DS}}{dV_{GS}} = \frac{\mu \varepsilon_{ox} \varepsilon_0 W}{t_{ox} L}\left((V_{GS} - V_{th}) \right)$ for decreasing t_{ox} g_m is increasing. Gate control over carrier density in channel increases as oxide (gate) capacitance increases. [3]

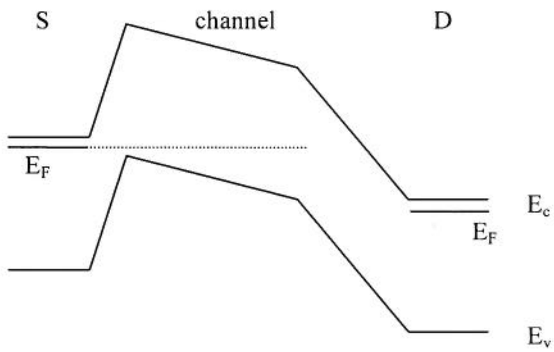
b) I is in the triode region, thus the voltage drop between source and drain is small. V_{GS} in I is higher than V_{GS} in II so the source-channel barrier in this case should be lower than in II. [8]

II is in the saturation region, thus there is a large voltage drop at the drain end of the channel (large depletion region).

I



II



c)

i) We know that for an ideal MOSFET the drain current saturates (becomes constant) at the point where $V_{DS} = V_{GS}-V_{th}$. Thus for the higher lying curve this happens at $V_{DS} = 1V$ and thus the gate voltage overdrive of the higher curve is $V_{GS2}-V_{th} = 1V$. For the lower lying curve this happens at $V_{DS} = 0.5V$ and thus the gate voltage overdrive of the lower curve is $V_{GS1}-V_{th} = 0.5V$. [4]

ii) In the triode (linear region) the current is given by

$I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_{th})V_{DS} \right)$ since $V_{DS} \ll V_{GS}-V_{th}$. [4]

a. Using $I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_{th})V_{DS} \right)$ and taking the higher lying curve, which must be that for higher gate voltage overdrive: $(V_{GS} - V_{th}) = 1V$, extract the slope: $slope = \frac{\mu C_{ox} W}{L}(V_{GS} - V_{th}) = \frac{7\,10^{-4} - 0}{0.2 - 0}$, thus the carrier

mobility, $\mu = \frac{L}{C_{ox} W (V_{GS} - V_{th})} \frac{7\,10^{-4}}{0.2} = \frac{1}{10^{-6} \times 10 \times 1} \frac{7\,10^{-4}}{0.2} = 350\ \text{cm}^2/\text{Vs}.$ [4]

Note that using the higher lying curve will give more accurate results as there is a wider linear region.

SECTION B: FIELDS

SOLUTIONS

4. This question is mandatory. Answer all parts. Each part is worth 4 marks.

a) Find the magnitude of the electric field E at a point midway between two point charges, 0.2 m apart, of charge +0.1 μC and –0.1 μC respectively..
For one charge, $E = Q/(4\pi\varepsilon_o r^2) = 10^{-7}/(4\pi\times 8.85\times 10^{-12}\times 10^{-2}) = 90\ kV/m$. *Total field is double this (two charges contribute equally): 180 kV/m.*

b) A hollow conducting sphere of radius 2 mm has a net surface charge of 10^{-7} C. What is the electric field magnitude at a point 1 mm from the centre of the sphere?
There is no field within a conducting sphere (unless there are additional charges within the space) – answer is zero.

c) An air-filled parallel plate capacitor has an area of 1 mm^2 and a gap of 10 μm, and is charged to 5 V. What is the stored energy in the capacitor?
$C = \varepsilon_o A/d = 8.85\times 10^{-12}\times 10^{-6}/10^{-5} = 0.885\ pF.\quad U = \frac{1}{2}CV^2 = 11.1\ pJ$

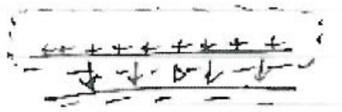
d) A toroid with an iron core has a 50 turn winding through which a current of 2 A flows. The toroid radius is 5 cm. Find the magnitude of the magnetic field strength H within the core. State any assumptions or approximations made.
$\int Hdl = NI;\ H(2\pi r) = NI;\ H = 50\times 2/(2\pi\times 0.05) = 318\ A/m$
Assumes/approximates: No flux leakage, H uniform within core.

e) An ideal transformer has a turns ratio N_1:N_2 = 2:1, and a load of 20 Ω is connected to the secondary coil terminals. If an AC voltage of magnitude 5 V is applied to the primary coil, what will be the magnitude of the current I_1 flowing in the primary coil?
$V_2 = (1/2)V_1,\ I_2 = V_2/R\ ,\ I_1 = (1/2)I_2 = (1/2)^2(5V)/20\Omega = 62.5\ mA$

Devices & Fields 2009 Exam
Solutions (cont.)

5 a) The total electric flux passing outwards through a closed surface equals the charge contained within that surface.

b)



Above the positive plate, flux contributions from the two plates cancel. Hence the flux flowing through the bottom of the dotted box must be Q.
This gives $D = Q/A$, and $E = Q/\varepsilon A = Q/\varepsilon_r \varepsilon_0 A$
Assume: insignificant flux through edges of box, and flux density is uniform.

c) Since E is uniform, $V = E \cdot d = Qd/\varepsilon_r \varepsilon_0 A$
and since $Q = CV$, $C = \frac{Q}{V} = \frac{\varepsilon_r \varepsilon_0 A}{d}$

d) Q is fixed at $Q_0 = \frac{\varepsilon_0 A V_0}{d_0}$

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}Q_0^2/C = \frac{1}{2}\frac{Q_0^2 d}{\varepsilon_0 A}$$

$$F = \frac{dU}{dd} = \frac{1}{2}\frac{Q_0^2}{\varepsilon_0 A} = \frac{1}{2}\frac{\varepsilon_0 A V_0^2}{d_0^2}$$

e) $Q = \int I\,dt = It$

$$V = \frac{Q}{C} = \frac{It}{C}.$$

$$P = IV = I^2 t/C$$

$$U = \int P\,dt = \frac{1}{2}\frac{I^2}{C}t^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

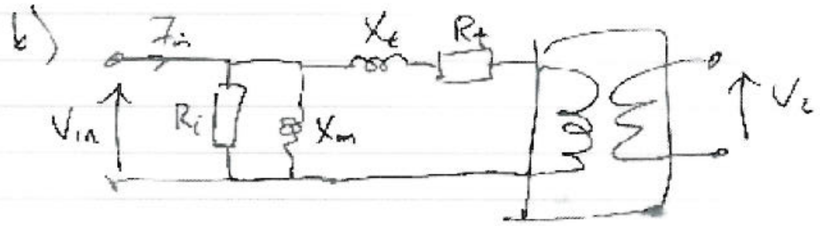
6 a) $V_1 = 20V$ $V_2 = \frac{N_2}{N_1} . V_1 = 10V$

$I_2 = \frac{V_2}{Z_L} = 0.2A$ $I_1 = \frac{N_2}{N_1} I_2 = 0.1A$

Z_L' is the load as seen from the primary terminals,
ie. $Z_L' = V_1/I_1$. But $V_2 = (N_2/N_1)V_1$, $I_2 = \frac{N_1}{N_2} I_1$

$\therefore \frac{V_1}{V_2} = \frac{(N_1/N_2)V_2}{(N_2/N_1) I_2} = \left(\frac{N_1}{N_2}\right)^2 Z_L = Z_L'$

In this case $Z_L' = (2)^2 \cdot 50 = 200\Omega$, $\frac{V_1}{I_1} = \frac{20}{0.1} = 200\Omega$ ✓

b) 

R_i = iron losses (eddy currents)
X_m = magnetising current
R_t = coil resistance (contains both sides)
X_t = leakage reactance (flux leakage)

c) Open circuit: no current in X_t or R_t

$|I| = \sqrt{\left(\frac{V}{R_i}\right)^2 + \left(\frac{V}{X_m}\right)^2}$ but $P = \frac{V^2}{R_i} \therefore R_i = \frac{V^2}{P}$

$R_i = \frac{110^2}{2.42} = 5k\Omega$, $\frac{1}{X_m^2} = \left|\frac{I}{V}\right|^2 - \frac{1}{R_i^2}$ $X_m = 2k\Omega$

Closed circuit:
Current flowing into R_i and X_m is $\frac{5}{\sqrt{5k^2+2k^2}} = 0.9mA$
(can be neglected).

Then $P \cong I^2 R_t$ $R_t = \frac{2.5}{1.58^2} = 1\Omega$

$\left|\frac{V}{I}\right| = \sqrt{X_t^2 + R_t^2}$ $X_t^2 = \left(\frac{5}{1.58}\right)^2 - 1$ $X_t = 3\Omega$