IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2018

EEE PART II: MEng, BEng and ACGI

FIELDS

Tuesday, 29 May 10:00 am

Time allowed: 1:30 hours

Corrected Copy –
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There are THREE questions on this paper.

Question One carries 40 marks. Question Two and Question Three carry 30 marks each.

Answer ALL questions.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : R.R.A. Syms
Second Marker(s) : S. Lucyszyn

Electromagnetic Fields 2018 – Formula Sheet

- **Differential operators (Cartesian co-ordinates)**

$\nabla = \partial/\partial x\, \underline{i} + \partial/\partial y\, \underline{j} + \partial/\partial z\, \underline{k}$

$\nabla\phi = \partial\phi/\partial x\, \underline{i} + \partial\phi/\partial y\, \underline{j} + \partial\phi/\partial z\, \underline{k}$

$\nabla \cdot \underline{F} = \partial F_x/\partial x + \partial F_y/\partial y + \partial F_z/\partial z$

$\nabla \times \underline{F} = \{\partial F_z/\partial y - \partial F_y/\partial z\}\, \underline{i} + \{\partial F_x/\partial z - \partial F_z/\partial x\}\, \underline{j} + \{\partial F_y/\partial x - \partial F_x/\partial y\}\, \underline{k}$

$\nabla^2\phi = \partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 + \partial^2\phi/\partial z^2$

- **Identity**

$\nabla \times \nabla \times \underline{F} = \nabla(\nabla \cdot \underline{F}) - \nabla^2\underline{F}$

- **Integral theorems**

$\iint_A \underline{F} \cdot d\underline{a} = \iiint_V \nabla \cdot \underline{F}\, dv$ (Gauss' theorem)

$\int_L \underline{F} \cdot d\underline{L} = \iint_A (\nabla \times \underline{F}) \cdot d\underline{a}$ (Stokes' theorem)

- **Maxwell's equations – integral form**

$\iint_A \underline{\mathbf{D}} \cdot d\underline{a} = \iiint_V \rho\, dv$ (Gauss' Law)

$\iint_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$ (Magnetic equivalent of Gauss' law)

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = -\iint_A \partial\underline{\mathbf{B}}/\partial t \cdot d\underline{a}$ (Faraday's law)

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \iint_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$ (Ampere's Law)

- **Maxwell's equations – differential form**

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

- **Constitutive equations**

$\underline{\mathbf{J}} = \sigma\, \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon\, \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu\, \underline{\mathbf{H}}$

- **Electromagnetic waves (pure dielectric media)**

Time dependent vector wave equation $\nabla^2\underline{\mathbf{E}} = \mu_0\varepsilon\, \partial^2\underline{\mathbf{E}}/\partial t^2$

Time independent scalar wave equation $\nabla^2\underline{E} = -\omega^2\mu_0\varepsilon_0\varepsilon_r\, \underline{E}$

For z-going, x-polarized plane waves $d^2E_x/dz^2 + \omega^2\mu_0\varepsilon_0\varepsilon_r\, E_x = 0$

Where $\underline{E}$ is a time-independent vector field

- **Power**

Instantaneous power flow $\underline{S} = \underline{\mathbf{E}} \times \underline{\mathbf{H}}$

Time-averaged power flow $\underline{S} = 1/2\, \mathrm{Re}(\underline{E} \times \underline{H}^*)$

- **Transmission line formulae**

Transmission line equations for line with per unit length inductance L_p and capacitance C_p

$dV/dz = -j\omega L_p I$

$dI/dz = -j\omega C_p V$

Phase velocity and characteristic impedance of lossless line with per unit length inductance L_p and capacitance C_p

$v_{ph} = 1/\sqrt{(L_p C_p)}$

$Z_0 = \sqrt{(L_p/C_p)}$

Reflection and transmission coefficients at junction between lines of impedance Z_1 and Z_2

$R_V = (Z_2 - Z_1)/(Z_2 + Z_1)$

$T_V = 2Z_2/(Z_2 + Z_1)$

Input impedance for length d of line with properties (Z_0, k) terminated by load Z_L

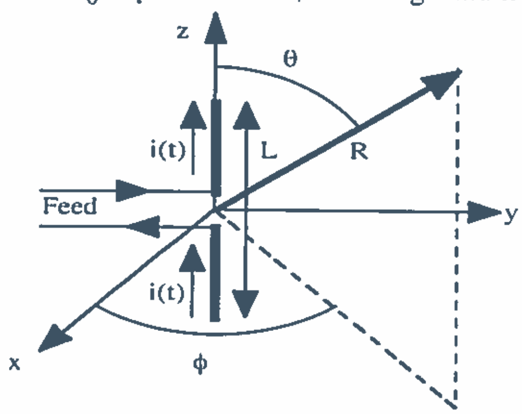
$Z_{in} = Z_0 \{Z_L + jZ_0 \tan(kd)\} / \{Z_0 + jZ_L \tan(kd)\}$

- **Antenna formulae**

Far-field pattern of half-wave dipole

$E_\theta = j\, 60 I_0 \{\cos[(\pi/2)\cos(\theta)] / \sin(\theta)\} \exp(-jkR)/R$; $H_\phi = E_\theta/Z_0$

Here I_0 is peak current, R is range and $k = 2\pi/\lambda$



Time averaged power flow $\underline{S} = 1/2 \text{ Re } (\underline{E} \times \underline{H}^*) = S(R, \theta)\, \underline{r}$

Normalised radiation pattern $F(\theta, \phi) = S(R, \theta, \phi) / S_{max}$

Directivity $D = 1/ \{1/4\pi \iint_{4\pi} F(\theta, \phi) \sin(\theta)\, d\theta\, d\phi\}$

Gain $G = \eta D$ where η is antenna efficiency

Effective area $A_e = \lambda^2 D/4\pi$

Friis transmission formula $P_R = P_T \eta_T \eta_R A_T A_R/(r^2\lambda^2)$

<u>Electromagnetic Fields 2018 – Questions</u>

1. Using diagrams and developing formulae where appropriate, discuss briefly each of the following:

a) The Van de Graaf generator

[8]

b) Dispersion diagrams

[8]

c) The ionosphere

[8]

d) Stokes' theorem

[8]

e) Paraxial waves

[8]

2. Figure 2.1 shows part of a ladder model of a transmission line. The circuit has series inductance L_p and parallel capacitance C_p per-unit-length, and has been divided into sections of length dz.

a) Using Kirchhoff's law, write down circuit equations relating the voltages V and V' and the currents I and I' at angular frequency ω. Using these, derive a pair of coupled differential equations relating the voltages and currents. What approximation have you made, and why?

[8]

b) Uncouple the differential equations. Assuming wave solutions travelling in the +z direction, find the propagation constant, the phase velocity and the characteristic impedance. What happens to these three parameters, if you assume the waves are travelling in the -z direction?

[14]

c) What values of L_p and C_p are required to construct a transmission line with a characteristic impedance of 50 Ω and a phase velocity of c/1.5, where c = 3 x 10^8 m/s is the velocity of light in vacuum? This line must be connected to two others with similar characteristics, using a splitting device. Propose a suitable circuit that avoids reflections at the single input port.

[8]

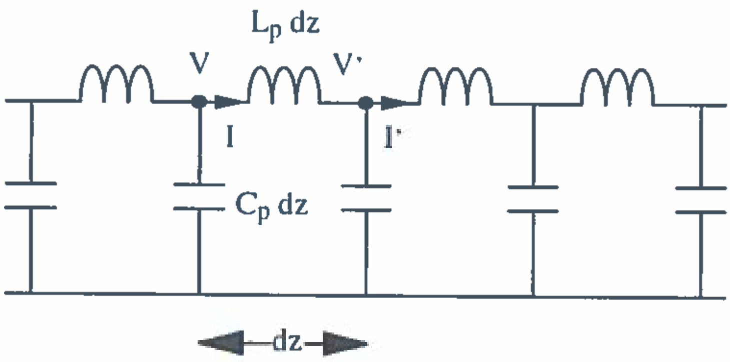


Figure 2.1

Still Valid

3. a) For x-polarized plane waves travelling in the z-direction in a dielectric medium, the electric field E_x must satisfy the equation:

$$d^2E_x/dz^2 = -\omega^2\mu_0\varepsilon E_x$$

Here ω is the angular frequency, μ_0 is the permeability and ε is the permittivity.

Assuming that the medium is slightly lossy, so the permittivity can be written in the form $\varepsilon = \varepsilon' - j\varepsilon''$ with $\varepsilon'' << \varepsilon'$, find the real and imaginary parts of the propagation constant k. What is the consequence for the wave solution of having a complex-valued propagation constant?

[8]

b) Assuming now that the medium is a metal, and that its dielectric constant can be written in terms of its conductivity σ as $\varepsilon = \sigma/j\omega$, find the new value of the propagation constant. Hence, find the skin depth in a metal of conductivity 5.96×10^7 S/m at a frequency of 100 MHz.

[6]

c) A radio transmitter has a power output of 1 kW and an isotropic antenna. Calculate the power density at a distance of 1 km. What is the new value of peak power density if the antenna is replaced with one having a directivity of 100 and an efficiency of 50%?

[6]

d) A similar directive antenna is used for reception. Assuming that the frequency is 100 MHz, what is its effective area? How much power does it receive at a distance of 1 km? Assuming that the minimum detectable power is 10 μW, what is the maximum length of link that can be established between the two antennas?

[8]

Not to be attempted. Not taught due to strike action.

Written on board 9.57

10:01 Announced