UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 1997

MEng Honours Degrees in Computing Part IV
MSc Degree in Advanced Computing
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Diploma of Membership of Imperial College Associateship of the City and Guilds of London Institute

PAPER 4.90

FUNCTIONAL PROGRAMMING - FOUNDATIONS

Friday, April 25th 1997, 10.00 - 12.00

Answer THREE questions

For admin. only: paper contains 4 questions

1a i Give the definition of $M[x := N]$.

ii Show that $(\lambda xyz.xz(yz))(\lambda xy.x)(\lambda xy.x) = \lambda x.x$.

iii Show that $\lambda xy.x \# \lambda x.x$

b i Give the definition of head normal form.

ii Reduce $M = (\lambda x.yx((\lambda y.z)(\lambda x.x)))(\lambda x.x)$ to head normal form.

iii Is M in weak head normal form? .

iv Which is an internal redex of M?

c i Give the definition of β-reduction graph.

ii Draw the β-reduction graphs of $M = (\lambda x.x)((\lambda x.x)((\lambda x.xx)(\lambda x.xx)))$ and $N = (\lambda x.x)(\lambda x.x)((\lambda x.xx)(\lambda x.xx))$.

2a i Define the set of de Bruijn terms.

ii Give the definition of the map DB which translates lambda terms into de Bruijn terms.

b Let M be the term

$$(\lambda xyz.xz(yz))(\lambda xy.x)(\lambda xy.x)$$

i Translate M into the De Bruijn notation $DB(M)$.

ii Reduce DB(M) to normal form.

c i Which reduction strategy on lambda terms corresponds to call-by-name? Give an informal justification to your answer.

ii Which reduction strategy on lambda terms corresponds to call-by-value? Give an informal justification to your answer.

iii Which strategy (call-by-name or call-by-value) gives the shorter reduction sequence for reducing

$$(\lambda xy.x(xy))((\lambda xy.x(xy))(\lambda z.z))$$

to normal form?

3a Define the terms of the language PCF.

b i Define the Call-by-name operational semantics for PCF ii Define a PCF term which implements the following function f:

$$f(0) = 1, f(1) = 2, f(n+2) = f(n+1) + f(n)$$

iii Define two different terms of type $\texttt{bool} \to \texttt{bool} \to \texttt{bool}$ which implement the logical or function.

c Consider the following relation $\simeq_{\sigma_1 \to \ldots \to \sigma_n \to \texttt{bool}}$ on PCF terms of type $\sigma_1 \to \ldots \to \sigma_n \to \texttt{bool}$, defined as follow:

$$M \simeq_{\sigma_1 \to \ldots \to \sigma_n \to \texttt{bool}} N \text{ iff } \forall T_1 : \sigma_1, \ldots, T_n : \sigma_n,\ MT_1 \ldots T_n \simeq_{\texttt{bool}} NT_1 \ldots T_n$$

and $M \simeq_{\texttt{bool}} N$ iff $\mathcal{O}[\![M]\!] = \mathcal{O}[\![N]\!]$.

i Show that $\perp \simeq_{\texttt{bool} \to \texttt{bool}} \lambda x.\texttt{cond} x (\texttt{cond} x \perp x) \perp$

ii Show that

$$\lambda x.\texttt{tt} \not\simeq_{\texttt{bool} \to \texttt{bool}} \lambda x.\texttt{cond}\ x\ \texttt{tt}\ \texttt{tt}$$

4a i Using the the denotational semantics of PCF write the interpretation of PCF types.

ii State the Fixpoint theorem on domains.

b i Prove the Fixpoint theorem on domains.

ii Which is the fixpoint of the map $\texttt{succ} : \texttt{int} \to \texttt{int}$?

iii Which is the fixpoint of the map $\lambda x.0 : \texttt{int} \to \texttt{int}$?

c i Prove that all set theoretic maps over the integers are continuous maps in $N_\perp \to N_\perp$ (assume that for a set theoretic map f, $f(\perp) = \perp$)

ii Deduce that there are continuous maps in $N_\perp \to N_\perp$ which are not the interpretation of any PCF term of type $\texttt{int} \to \texttt{int}$.

End of paper

