

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

COMMUNICATIONS 1

Wednesday, 28 May 10:00 am

Time allowed: 2:00 hours

There are FIVE questions on this paper.

Answer THREE questions.

Corrected Copy

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Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	P.L. Dragotti
	Second Marker(s) :	E.M. Yeatman

Special Information for the Invigilators: none

Information for Candidates

Some Fourier Trasforms

$$\cos \omega_0 t \iff \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$$\text{rect}(\frac{t}{\tau}) \iff \tau \text{sinc}(\frac{\omega \tau}{2})$$

$$\frac{W}{\pi} \text{sinc}(Wt) \iff \text{rect}(\frac{\omega}{2W})$$

Time-Shifting Property of the Fourier Transform

$$g(t - t_0) \iff G(\omega)e^{-j\omega t_0}$$

Frequency-Shifting Property of the Fourier Transform

$$g(t)e^{j\omega_0 t} \iff G(\omega - \omega_0).$$

Some useful trigonometric identities

$$\cos^2 x + \sin^2 x = 1$$

$$\cos x \cos y = \frac{1}{2} \cos(x - y) + \frac{1}{2} \cos(x + y).$$

Euler's formula

$$e^{jx} = \cos x + j \sin x.$$

The Questions

1. Consider the following two waveforms $x_1(t) = \sin(4\pi t)\text{rect}(t - 0.5)$ and $x_2(t) = \text{rect}(t - 1)$ (See also Figure 1).

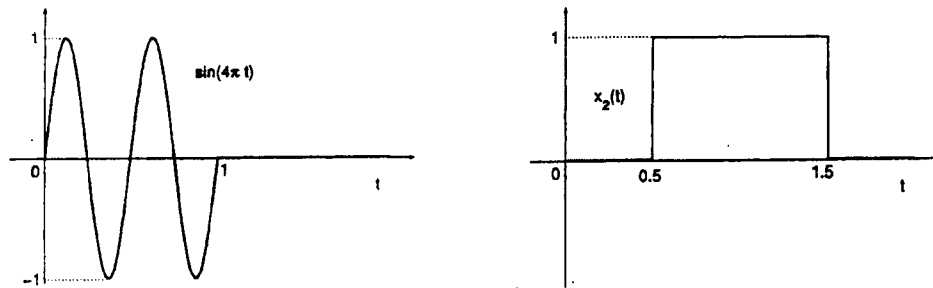


Figure 1: The two signals $x_1(t)$ and $x_2(t)$.

- (a) Find the energy of $x_1(t)$.

[4]

- (b) Find the energy of $x_2(t)$.

[4]

- (c) Find the energy of $x(t) = x_1(t) + x_2(t)$.

[4]

- (d) Find the Fourier transform of $x_2(t)$.

[8]

2. The signal $s(t) = \frac{50}{\pi} \text{sinc}(50t) \cos(100t)$ is multiplied by $\cos(100t)$ and the result $x(t)$ is fed to a filter with a frequency response

$$H(\omega) = \begin{cases} 1 & \text{for } |\omega| \leq 30 \text{ rad/s} \\ 0 & \text{otherwise} \end{cases}$$

giving output $y(t)$ as shown in Figure 2.

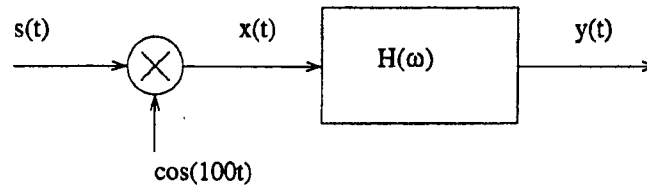


Figure 2: Receiver.

- (a) Sketch the Fourier transform of $s(t)$. [4]
- (b) Sketch the Fourier transform of $x(t)$. [4]
- (c) Sketch the Fourier transform of $y(t)$. [4]
- (d) Write the exact expression of the output $y(t)$. [8]

3. The output signal from an AM modulator is

$$x(t) = 5 \cos 9900t + A \cos 10000t + 5 \cos 10100t$$

Determine

- (a) the modulating signal $m(t)$, [4]
- (b) the minimum value of A that allows us to use an envelope detector, [4]
- (c) the power efficiency η if $A = 20$, [6]
- (d) compare the power efficiency you obtain for $A = 20$ with the one you obtain for the value of A you found in (b). Which case is more efficient? [6]

4. Consider the frequency modulated signal

$$\phi_{FM}(t) = A \cos \left[2\pi f_c t + k_f \int_{-\infty}^t m(\alpha) d\alpha \right],$$

where the message signal is $m(t) = 10\pi \cos(2\pi f_0 t)$ with $f_0 = 4$ kHz and the carrier is given by $c(t) = 10 \cos(2\pi f_c t)$ with $f_c = 10$ MHz. The bandwidth of $\phi_{FM}(t)$, using Carson's rule, is 10 kHz. Determine

- (a) the bandwidth of the baseband signal $m(t)$,

[2]

- (b) the peak value m_p of the baseband signal,

[2]

- (c) the correct value of the coefficient k_f ,

[10]

- (d) the average transmitted power.

[6]

5. A transmission line having $L_0 = 0.25 \mu\text{H/m}$ and $C_0 = 100 \text{ pF/m}$ is connected to a 100Ω line with a matched termination. A sine wave of 15 V amplitude propagating in the former is incident on the junction. Find

(a) the voltage amplitude of the reflected wave,

[7]

(b) the current amplitude of the reflected wave,

[7]

(c) the fraction of the incident power which is transmitted into the second line.

[6]

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Exam Solutions

1. (a)

$$E_{x_1} = \int_0^1 \sin^2(4\pi t) dt = \int_0^1 \frac{1}{2} dt - \frac{1}{2} \int_0^1 \cos(8\pi t) dt = 1/2$$

(b)

$$E_{x_2} = \int_{0.5}^{1.5} dt = 1$$

(c) The two signals are orthogonal therefore $E_x = 1/2 + 1 = 3/2$.

(d) Using time shifting property, we get

$$G(\omega) = \text{sinc}(\omega/2)e^{-j\omega}.$$

2. (a) Since

$$\frac{50}{\pi} \text{sinc}(50t) \iff \text{rect}(\omega/100),$$

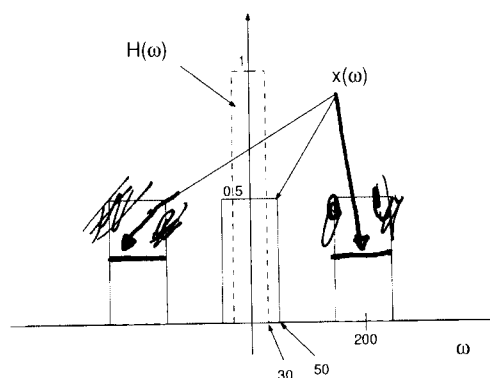
then

$$S(\omega) = \frac{1}{2} \text{rect}\left(\frac{\omega - 100}{100}\right) + \frac{1}{2} \text{rect}\left(\frac{\omega + 100}{100}\right).$$

(b) In the time domain $x(t)$ is given by

$$x(t) = \frac{25}{\pi} \text{sinc}(50t) + \frac{25}{\pi} \text{sinc}(50t) \cos 200t$$

Its Fourier transform is shown below. In the Figure we also show the effect of the filter $H(\omega)$



(c)

$$Y(\omega) = \frac{1}{2} \text{rect}\left(\frac{\omega}{60}\right)$$

(d)

$$y(t) = \frac{15}{\pi} \text{sinc}(30t).$$

3. (a) The full AM signal is $(A + m(t)) \cos \omega_c t$. In our case, the AM modulated signal is $(A + 10 \cos(100t)) \cos 10000t$. Therefore $m(t) = 10 \cos 100t$.
 - (b) To use an envelop detector, we need $(A + 10 \cos(100t)) \geq 0$ for all t . Therefore the minimum value of A is $A = 10$.
 - (c) The efficiency is defined as $\eta = \frac{P_s}{P_c + P_s}$. $P_c = A^2/2 = 20^2/2 = 200$. $P_s = P_{m(t)}/2 = 50/2 = 25$. Therefore $\eta = 25/225 = 1/9$.
 - (d) In the case $A = 10$, the efficiency is $\eta = 1/3$. This is the maximum achievable efficiency.
4. (a) The bandwidth of the baseband signal is $B = 4000\text{Hz}$.
 - (b) The peak value of $m(t)$ is 10π .
 - (c) Using Carson's rule the effective bandwidth is given by

$$B_{FM} = 2(\Delta f + B) = 2\left(\frac{k_f m_p}{2\pi} + B\right).$$

The bandwidth of the baseband signal is $B = 4000\text{Hz}$. $B_{FM} = 10000\text{Hz}$ and $m_p = 10\pi$. Therefore

$$B_{FM} = 2(\Delta f + B) = 2\left(\frac{k_f m_p}{2\pi} + B\right) = 2(5k_f + 4000) = 10000$$

and it follows $k_f = 200$.

- (d) Since an angle modulated signal is essentially a sinusoidal signal with constant amplitude, we have

$$P_{FM} = A^2/2 = 100/2 = 50.$$

5. The characteristic impedance of the line is

$$Z_0 = \sqrt{L_0/C_0} = \sqrt{0.25 \cdot 10^{-6}/100 \cdot 10^{-12}} = 50\Omega.$$

The voltage reflection coefficient is $K_v = (Z_L - Z_0)/(Z_L + Z_0) = 1/3$.

- (a) $V_- = K_v V_+ = 5\text{V}$.
- (b) $I_- = -K_v I_+ = -K_v V_+/Z_0 = 0.1\text{A}$
- (c) $T_p = 1 - |K_v|^2 = 1 - 1/9 = 8/9$