

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2003

MSc and EEE/ISE PART IV: M.Eng. and ACGI

## OPTICAL COMMUNICATION

Tuesday, 20 May 10:00 am

Time allowed: 3:00 hours

**There are SIX questions on this paper.**

**Answer Question ONE, and ANY THREE of Questions 2 to 6**  
**All questions carry equal marks.**

**Corrected Copy**

**Any special instructions for invigilators and information for candidates are on page 1.**

**Examiners responsible**

<b>First Marker(s) :</b>	E.M. Yeatman
<b>Second Marker(s) :</b>	K.D. Leaver



**Special instructions for invigilators:**      None.

**Information for Candidates:**

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :                       $e = 1.6 \times 10^{-19} \text{ C}$

permittivity of free space :         $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

relative permittivity of silicon :    $\epsilon_r = 12$

Planck's constant :                    $h = 6.63 \times 10^{-34} \text{ J s}$

Boltzmann's constant :               $k = 1.38 \times 10^{-23} \text{ J/K}$

speed of light :                         $c = 3.0 \times 10^8 \text{ m/s}$

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness  $d$  are

$$\kappa = k_{1x} \tan(k_{1x}d/2) \text{ and } \kappa = -k_{1x} \cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]
- a) A step index silica fibre has a core diameter of  $6\text{ }\mu\text{m}$  and the cutoff wavelength for single mode operation is  $1.520\text{ }\mu\text{m}$ . Estimate the index difference of the fibre.
  - b) Raising the refractive index difference has what effect (if any) on the number of supported modes in step index multi-mode fibre?
  - c) What loss mechanism dominates in silica-based fibre at wavelengths in the range  $1600 - 2000\text{ nm}$ ?
  - d) Estimate the spread in wavelengths,  $\Delta\lambda$ , emitted by a light emitting diode at room temperature with a nominal operating wavelength of  $1310\text{ nm}$ .
  - e) Why is silicon not a suitable material for photodetectors for long-haul optical communication systems?
  - f) Which of the following is currently the highest bit rate in commercial optical transmitters –  $10\text{ Mbit/s}$ ,  $400\text{ Mbit/s}$ ,  $1\text{ Gbit/s}$ , or  $40\text{ Gbit/s}$ ? What is the main factor limiting this value?
  - g) Briefly explain the main performance advantage for optical communications of distributed feedback lasers over Fabry-Perot lasers.
  - h) What is the maximum possible slope efficiency of a semiconductor laser operating at a nominal wavelength of  $780\text{ nm}$ ?
  - i) Briefly describe the main considerations in choosing suitable wavelengths for pump lasers in erbium-doped fibre amplifiers.
  - j) A passive 4-port coupler with negligible excess losses has an input of  $-10\text{ dBm}$  at one port, no signal at the other input port, and  $-20\text{ dBm}$  is emitted at one output port. What is the output power in  $\text{dBm}$  at the other output port and why?

2. a) A symmetric slab waveguide has a core thickness of  $6\text{ }\mu\text{m}$ . For a certain transverse electric (TE) mode, the transverse wavevector component in the core is  $k_{1x} = 1.19 \times 10^6\text{ m}^{-1}$ . Find the order  $m$  of this mode, and determine the total number of TE modes supported by this guide. You may find it helpful to sketch a graph of the eigenvalue equations. [8]
- b) For the waveguide and mode of part (a), with a free-space wavelength of  $1.50\text{ }\mu\text{m}$ , the phase velocity is found to be  $2.027 \times 10^8\text{ m/s}$ . Find the refractive index of the core,  $n_c$ , to 4 significant decimal places. [6]
- c) For the waveguide and wavelength described in (a) and (b), estimate the effective index  $n'$ , to 4 significant digits, of the lowest order ( $m=0$ ) TE mode. [6]
3. a) Show that, at the wavelength  $\lambda$  where  $d^2n/d\lambda^2 = 0$ , the spread in pulse length due to material dispersion in a fibre is proportional to the third derivative of refractive index with respect to wavelength, i.e.  $d^3n/d\lambda^3$ . Derive an expression for the pulse broadening  $\Delta\tau_g$  in this case. Note that the group delay  $\tau_g = (L/c)(n - \lambda dn/d\lambda)$ , where  $L$  is the fibre length. [6]
- b) If the spread in wavelength of a signal is dominated by the inherent source width  $\sigma_{\lambda S}$ , derive an expression for the received pulse width in the time domain  $\sigma_R$  as a function of the transmitted pulse width  $\sigma_o$ , the fibre length  $L$ , and the dispersion coefficient  $D$ . Derive an expression for the minimum of  $\sigma_o$  as a function of  $\sigma_{\lambda S}$  for which the relative pulse spreading  $\sigma_R/\sigma_o < 1.25$ . [6]
- c) In a certain system the optical signal's spectral width is dominated by the input signal spectrum, and the inherent source width  $\sigma_{\lambda S}$  is negligible. The input pulses are transform limited Gaussian, with variance  $\sigma_o$ . Show that for a certain fibre dispersion  $D \cdot L$  there is a minimum received pulse width  $\sigma_R$ . Find an expression for the input pulse width  $\sigma_o$  at which this is obtained, and for the corresponding pulse spreading ratio  $\sigma_R/\sigma_o$ . [8]

4. a) A certain light emitting diode (LED) has radiative and non-radiative recombination lifetimes,  $\tau_r$  and  $\tau_{nr}$  respectively, in the ratio  $\tau_{nr}/\tau_r = 5$ . Calculate the internal quantum efficiency for this device. [2]
- b) Describe four factors that may limit the external quantum efficiency in an LED, and for each of these indicate a technique that can reduce the effect of this factor. [8]
- c) An LED is emitting into a step index, silica-based multi-mode fibre as shown in Fig. 4.1. The numerical aperture of the fibre is 0.10. Find the maximum emission angle  $\theta$  from the LED active region for which light can be guided in the fibre, and the fraction of emitted power falling within this angle if the generated photons have a uniform directional distribution. [6]
- d) For the fibre described in (c), derive an expression for the limit on bit rate imposed by multi-mode dispersion, and sketch this as a function of fibre length. [4]

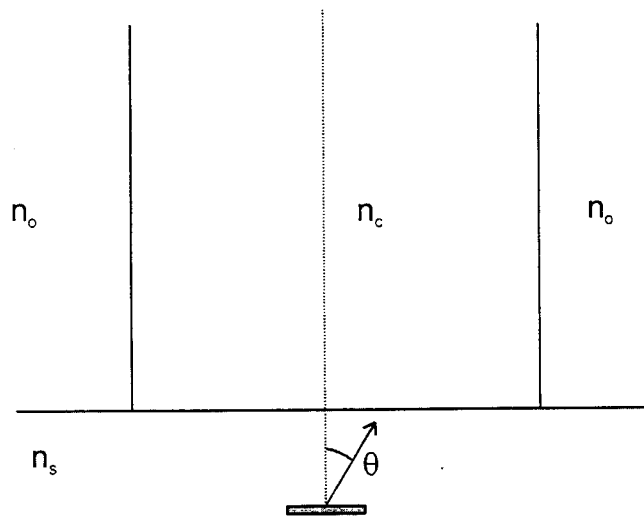


Figure 4.1

5. a) A certain receiver has unit quantum efficiency, and a noise equivalent power (NEP) of  $5 \text{ pW}/\sqrt{\text{Hz}}$ . Find the received optical power for which the shot noise and receiver noise are equal. [4]
- b) The receiver in (a) is used in a system with transmitted power of 10 mW, and fibre attenuation of 0.43 dB/km. On-off keying is used, the signal bandwidth is at the Nyquist limit, and the minimum optical SNR is 10 dB.
- Find an expression for the maximum bit rate  $B$  as a function of fibre length  $L$ , and simplify this expression for the two cases where shot noise or receiver noise dominate respectively. Using these approximations, sketch  $\log(B)$  vs.  $L$  over a practical range of fibre lengths. [8]
- c) The system described in part (a) is extended by adding an optical amplifier at a suitable point along the fibre. The amplifier has 20 dB gain and a noise figures of 4 dB. Modify your expressions from (b) for maximum bit rate, and sketch  $\log(B)$  vs.  $L$ . [8]

6. a) A silicon p-n photodiode (Fig. 6.1) has a depletion layer thickness of  $w$ , and p and n doping levels respectively of  $N_A$  and  $N_D$ , with  $N_A = 5N_D = 10^{20} \text{ m}^{-3}$ . The quantities  $w_p$  and  $w_n$  are the depleted widths in the p and n regions respectively. A reverse bias voltage  $V_b$  is applied. Find an expression for the full depletion width  $w$  as a function of  $V_b$ , and the value of  $V_b$  for which  $w = 10 \text{ } \mu\text{m}$ . [8]
- b) Neglecting Fresnel reflection, find an expression for the quantum efficiency  $\eta$  of the photodiode of (a) if the total p region thickness  $h = 10 \text{ } \mu\text{m}$ , and the absorption coefficient  $\alpha = 10^5 \text{ m}^{-1}$ . Hence find the value of  $V_b$  for which  $\eta = 0.9$ . [6]
- c) Assume that the resistance  $R$  encountered by the photocurrent  $I_{ph}$  is dominated by the resistance of the undepleted part of the p region, as the current flows laterally through it as shown in Fig. 6.1. If this resistance is in turn is inversely proportional to the thickness  $h_c$  of this region, find the value of  $h_c$  in terms of  $h$  for which the time constant  $RC$  of the diode is minimised. [6]

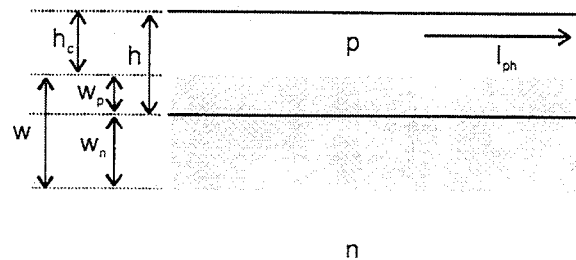


Figure 6.1

