Paper Number(s): **E3.12**
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IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

MSc and EEE PART III/IV: M.Eng., B.Eng. and ACGI

OPTOELECTRONICS

Thursday, 10 May 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

Time allowed: 3:00 hours

Corrected Copy

Examiners: Syms,R.R.A. and Holmes,A.S.

Special instructions for invigilators:

Information for candidates:

Fundamental constants

$e = 1.6 \times 10^{-19}$ C
$m_e = 9.1 \times 10^{-31}$ kg
$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
$c = 3 \times 10^{8}$ m/s
$h = 6.62 \times 10^{-34}$ Js

1. a) Sketch the layout of an asymmetric three-layer planar dielectric waveguide, explaining any restrictions on the material parameters involved. Describe briefly how light may be propagated along the structure, i) in terms of rays, and ii) in terms of guided modes. [2]; [3]; [3]

b) Show that the TE mode eigenvalue equation of a symmetric three-layer guide can be written as:

$$\tan(\kappa h) = \kappa[\gamma + \delta] / [\kappa^2 - \gamma\delta]$$

where h is the core height, and κ, γ and δ are derived from the waveguide equation. [8]

c) Explain the physical significance of the eigenvalue equation. [4]

(You may assume that the TE mode waveguide equation for a three-layer dielectric guide is:

$$d^2E_i/dx^2 + \{n_i^2k_0^2 - \beta^2\} E_i = 0 \qquad (i = 1, 2, 3)$$

where β is the propagation constant, $k_0 = 2\pi/\lambda$, $E_i(x)$ is the transverse electric field in the i^{th} layer, and n_i is the corresponding refractive index.)

2. a) Define and explain the significance of the *overlap integral* between two transverse electric fields. How are overlap integrals used to calculate efficiency in transverse coupling geometries? What requirements must be satisfied if high coupling efficiency is to be obtained? [3]; [3]; [2]

b) Two identical single-mode optical fibres have transverse modal fields that (to a reasonable approximation) vary radially as Gaussian functions, i.e. as $E(r) = E_0\exp(-r^2/a^2)$. When the fibres are spliced together, a lateral offset δ is unfortunately introduced, as shown in Figure 1.

Calculate and sketch the variation of coupling efficiency with δ. Hence, estimate the loss caused by a 1 μm lateral splice error, for fibre with 5 μm half-power mode field diameter. [8]; [4]

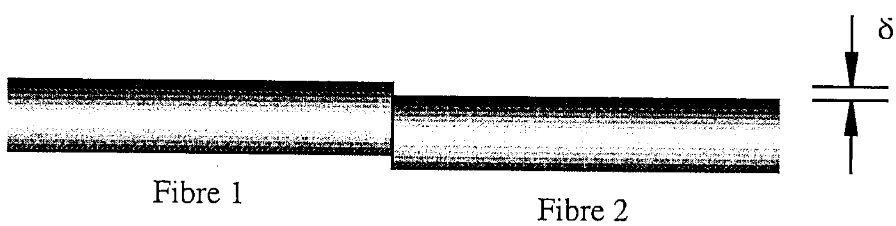


Figure 1.

3. a) Discuss the advantages of radiative stars compared with similar components based on directional couplers. [6]

b) Sketch and explain the operation of an arrayed waveguide grating multiplexer. [6]

c) What is the function of an add-drop multiplexer? Show how an N-channel ADD-DROP MUX can be constructed from a number of AWG MUX components and 2 x 2 switches. [8]

4. a) The lumped-element rate equations for a semiconductor laser are:

$$dn/dt = I/ev - n/\tau_e - G\phi(n - n_o)$$
$$d\phi/dt = \beta n/\tau_{rr} + G\phi(n - n_o) - \phi/\tau_p$$

Describe the physical processes modelled by the terms on the right hand side of each of the two equations. Why does τ_e appear in the upper equation, and τ_{rr} in the lower one? What governs the value of τ_p? [5]; [2]; [3]

b) Stripe waveguide InP/InGaAsP laser diodes are manufactured with active area cross-sectional dimensions of 0.1 μm x 3 μm. When the lasers are cleaved to a cavity length of 150 μm, the measured threshold current is 12 mA. Using a longer cavity length of 250 μm, the threshold current rises to 16.8 mA. Calculate the electron concentration at transparency, and the gain coefficient.

You may assume a refractive index of 3.5 for InP, and an electron lifetime of 1 nsec. [10]

5. a) Why are binary compounds (as opposed to ternary or quaternary compounds) used as substrate materials for III - V optoelectronic devices? Distinguish between *homoepitaxy* and *heteroepitaxy*. In what optoelectronic applications might the latter be required? **[2]; [2]; [2]**

b) The variation of the lattice parameter a(x, y) of the quaternary alloy $In_{1-x}Ga_xAs_{1-y}P_y$ may be described by the function:

$$a(x, y) = xy\, a_{GaP} + x(1 - y)\, a_{GaAs} + (1 - x)y\, a_{InP} + (1 - x)(1 - y)\, a_{InAs}$$

where a_{GaP} = 5.4512 Å, a_{GaAs} = 5.6536 Å, a_{InP} = 5.8696 Å, and a_{InAs} = 6.0590 Å are the lattice parameters of the binary compounds GaP, GaAs, InP and InAs, respectively.

Show that the locus of compounds lattice matched to InP is almost exactly a straight line. **[8]**

c) Figure 2 shows the variation of bandgap with composition for the whole $In_{1-x}Ga_xAs_{1-y}P_y$ system. Sketch your solution to Part b) on this Figure.* Hence, estimate the wavelength range over which a substrate-entry $In_{1-x}Ga_xAs$/InP photodetector is likely to be effective. **[2]; [4]**

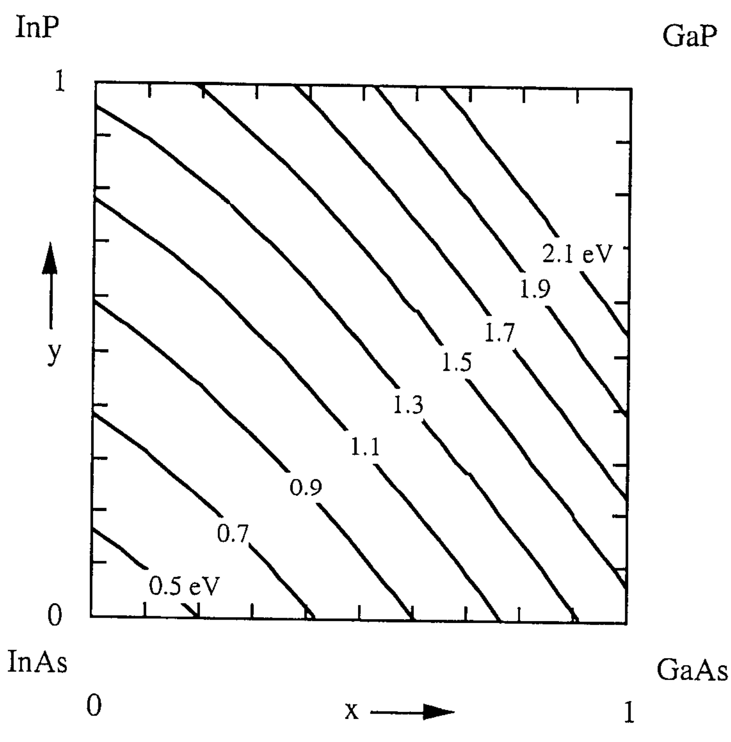


Figure 2.

* an extra copy of the Figure is provided for submission with your answer book.

6. a) Describe the processes of *absorption*, *spontaneous emission* and *stimulated emission*. **[6]**

b) Explain the mechanism for signal amplification in an erbium doped fibre amplifier. Why is an EDFA such a significant addition to an optical communications system? **[6]; [2]**

c) A point-to-point communications link operating at $\lambda = 1.525\ \mu m$ consists of 150 km of single-mode fibre with a loss of 0.2 dB/km. The source is a 10 mW laser diode, coupled to the fibre with an efficiency of 25%. There are 10 fusion splices in the link, each contributing 0.1 dB loss.

Assuming that the receiver requires a signal of 100 μW to operate, calculate the length of Er^{3+}-doped fibre required in a travelling-wave amplifier arranged as a pre-detector signal booster. When pumped at 0.98 μm wavelength, the Er^{3+}-doped fibre has a gain of 0.5 dB/m. **[6]**

[END]

Feb. 01

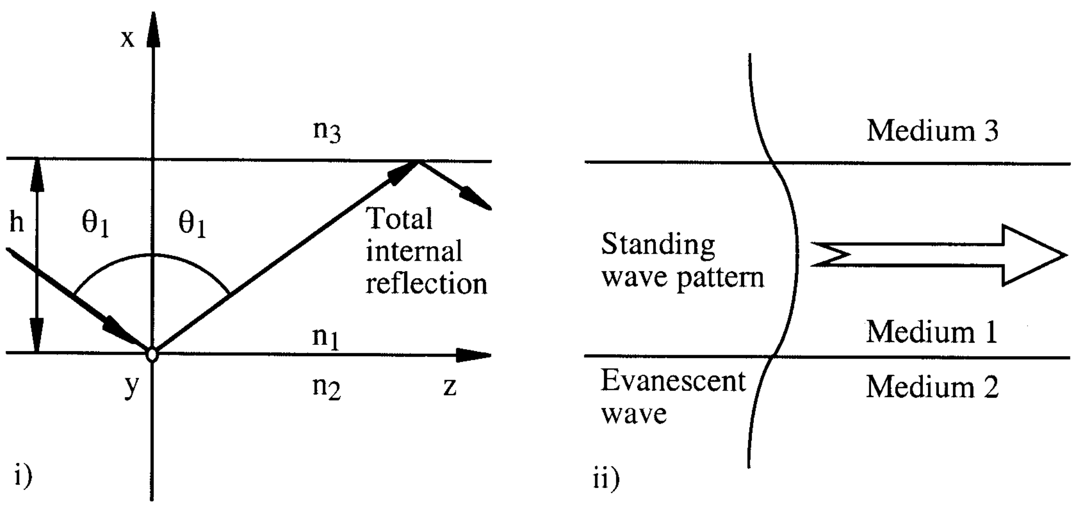
E312 Optoelectronics - Solutions 2001

Sc i 3

1. a) An asymmetric three-layer dielectric waveguide consists of a parallel-sided slab of transparent dielectric material of thickness h and refractive index n_1, sandwiched between two semi-infinite dielectric layers of indices n_2 and n_3. Provided that $n_1 > n_2 > n_3$, light may be propagated along the structure by total internal reflection at the two interfaces (between layers 1 and 2, and layers 1 and 3, respectively). The rays must be incident at an angle greater than the critical angle for the interface between layers 1 and 2, namely $\theta_c = \sin^{-1}(n_2/n_1)$. **[2]**

The interpretation above gives rise to the zig-zag ray path shown in Figure i) below. **[3]**

An alternative is provided by the modal picture shown in Figure ii). Inside the core, the total field can be considered to be the sum of waves travelling in the two possible ray directions, i.e. as a standing wave, which propagates only along the guide. Outside the core, the field is an exponentially-decaying evanescent wave, which also travels along the guide. The combined field is the guided mode. **[3]**



b) The waveguide equation is given in each layer by: $d^2E_i/dx^2 + \{n_i^2k_0^2 - \beta^2\}\,E_i = 0$

For fields that are confined within the guide (which will be standing waves inside the core layer, and evanescent fields outside), assume the following trial solutions:

In layer 1: $E_1 = E\cos(\kappa x - \phi)$
In layer 2: $E_2 = E'\exp(\gamma x)$
In layer 3: $E_3 = E''\exp[-\delta(x - h)]$

Where the constants κ, γ and δ are:
$\kappa = \sqrt{(n_1^2k_0^2 - \beta^2)}$
$\gamma = \sqrt{(\beta^2 - n_2^2k_0^2)}$

R. R. A. Syms
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$\delta = \sqrt{(\beta^2 - n_3^2 k_0^2)}$

The boundary conditions require continuity of $E_i(x)$ and its gradient dE_i/dx at each interface.

Matching fields at x = 0 gives: $E' = E \cos(\phi)$

While matching the field gradients gives: $\gamma E' = -\kappa E \sin(-\phi)$

Dividing these equations, we obtain: $\tan(\phi) = \gamma/\kappa$

Similarly, matching fields at x = h gives: $E'' = E \cos(\kappa h - \phi)$

While matching field gradients gives: $-\delta E'' = -\kappa E \sin(\kappa h - \phi)$

Dividing these equations, we obtain: $\tan(\kappa h - \phi) = \delta/\kappa$

Using the standard trigonometrical identity: $\tan(A - B) = (\tan A - \tan B) / (1 + \tan A \tan B)$

We can write: $\tan(\kappa h - \phi) = \{\tan(\kappa h) - \tan(\phi)\} / \{1 + \tan(\kappa h) \tan(\phi)\}$

Hence: $\{\tan(\kappa h) - \gamma/\kappa\} / \{1 + \tan(\kappa h)\, \gamma/\kappa\} = \delta/\kappa$

Finally, after rearrangement, we get: $\tan(\kappa h) = \kappa\{\gamma + \delta\} / \{\kappa^2 - \gamma\delta\}$ [8]

c) The physical significance of the eigenvalue equation can be found as follows:

First, we write: $\tan(\phi) = \gamma/\kappa = \tan(\phi_{12})$

By analogy, we then write: $\tan(\kappa h - \phi) = \delta/\kappa = \tan(\phi_{13})$

So that: $\tan(\kappa h - \phi_{12}) = \tan(\phi_{13})$

Inverting the tan functions, we obtain: $\kappa h - \phi_{12} = \phi_{13} + \nu\pi$

And finally: $2\kappa h - 2\phi_{12} - 2\phi_{13} = 2\nu\pi$

These terms may be interpreted as follows:

$2\kappa h$: phase change in propagating up and down once between the two guide walls

$-2\phi_{12}$: phase change on reflection at the boundary between layers 1 and 2

$-2\phi_{13}$: phase change on reflection at the boundary between layers 2 and 3

The eigenvalue equation is therefore a condition of transverse resonance: only field structures that involve a whole number of multiples of 2π phase change in a round trip between the guide walls may propagate as guided modes. **[4]**

R.R.A. Syms
Andrew Holmes

2. a) The *overlap* between the transverse fields $E_1(x, y)$ and $E_2(x, y)$ of two z-propagating modes is:

$$< E_1, E_2 > = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1 E_2^* \, dx \, dy$$

The overlap integral represents the degree of *similarity* between the two fields. **[3]**

Efficiency is found in transverse coupling geometries as:

$$\eta = |< E_1, E_2 >|^2 / \{< E_1 , E_1 >< E_2, E_2 >\}$$

where E_1 is the transverse field of the incident wave or mode, and E_2 is the transverse field of the mode to be excited in the output waveguide. [3]

If the two fields are normalised, so that $< E_1, E_1 > = < E_2, E_2 > = 1$, the maximum possible value of $< E_1, E_2 >$ is 1. This value is obtained when the two fields E_1 and E_2 are both real functions, are identical in shape, and coincide spatially. **[2]**

To calculate the coupling efficiency in the case of an offset fibre-to-fibre-joint, first define the transverse electric fields in fibres 1 and 2 (respectively) as Gaussian variations of the form:

$$E_1(x, y) = E_0 \exp\{-(x^2 + y^2)/a^2\} \text{ and } E_2(x, y) = E_0 \exp\{-[(x - \delta)^2 + y^2]/a^2\}$$

In this case:

$$< E_1 , E_1 > = E_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\{-2(x^2 + y^2)/a^2\} \, dx \, dy$$

And:

$$\begin{aligned} < E_2 , E_2 > &= E_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\{-2[(x - \delta)^2 + y^2]/a^2\} \, dx \, dy \\ &= E_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\{-2(x'^2 + y^2)/a^2\} \, dx' \, dy \qquad \text{(where } x' = x - \delta) \\ &= < E_1, E_1 > \end{aligned}$$

And:

$$\begin{aligned} |< E_1, E_2 >|^2 &= \left\{E_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\{-(x^2 + y^2)/a^2\} \exp\{-[(x - \delta)^2 + y^2]/a^2\} \, dx \, dy\right\}^2 \\ &= \exp\{-\delta^2/a^2\} \left\{ E_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp \{-[2(x - \delta/2)^2 + 2y^2] /a^2\} \, dx \, dy\right\}^2 \\ &= \exp\{-\delta^2/a^2\} \left\{E_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp \{-2(x'^2 + y^2] /a^2\} \, dx \, dy\right\}^2 \text{(where } x' = x - \delta/2) \\ &= \exp\{-\delta^2/a^2\} \left\{< E_1, E_1 >\right\}^2 \end{aligned}$$

Hence:

R.R.A. Syms
Alastair Holmes

$\eta = \exp(-\delta^2/a^2)\ \{<E_1, E_1>\}^2\ /\ \{<E_1\ , E_1><E_1, E_1>\} = \exp(-\delta^2/a^2)$ **[6]**

The variation of η with δ is therefore also Gaussian, and reaches 100% when $\delta = 0$ (sketch). **[2]**

If the half-power modal field diameter is 5 μm, $\exp\{-2 \times (2.5^2/a^2)\} = 0.5$, so that a = 4.2466 μm.

For $\delta = 1.0$ μm, $\eta = \exp(-1^2/4.2466^2) = 0.946$, corresponding to a loss of 0.24 dB. **[4]**

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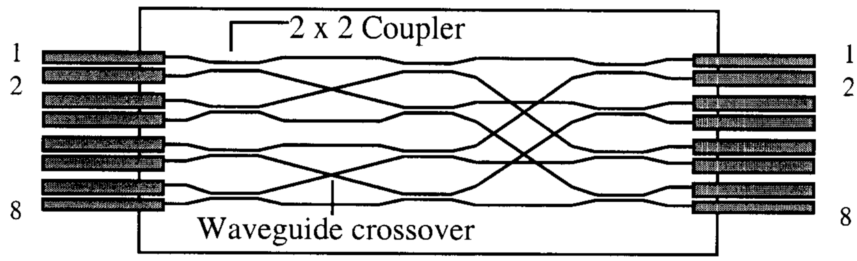
3. a) *Coupler based star*

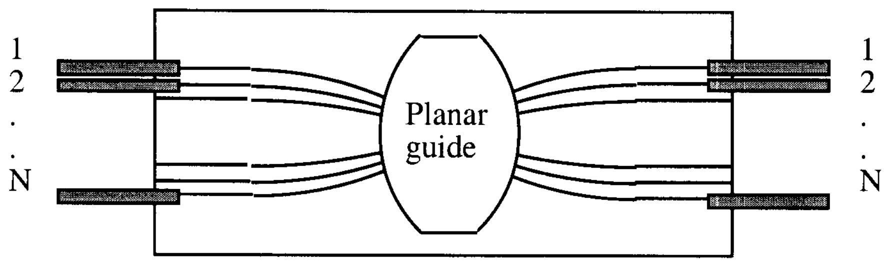


Complex design
High insertion loss
Low port-to port uniformity
Spectral variation even with wavelength-flattened couplers

NOT SCALABLE

[3]

Radiative star



Simple design
Low insertion loss
High port-to-port uniformity
Spectrally flat response

SCALABLE

[3]

b) *Arrayed waveguide grating multiplexer*

The AWG MUX is constructed from two radiative star components, linked by an array of curved waveguides whose path length varies linearly across the array. The first star distributes the input power (to good approximation) equally across the arrayed waveguide grating. Each component travels through its particular guide to the second star, which performs an amplitude summing operation.

The variation in optical path across the array introduces a linear variation in the phase of the components arriving at the second star. The star itself introduces further linear variations in phase in the sub-components summed at each output port. Different spectral components therefore add in phase for particular output ports. As a result, the spectral components emerge from the ports as a spatially separated array of beams.

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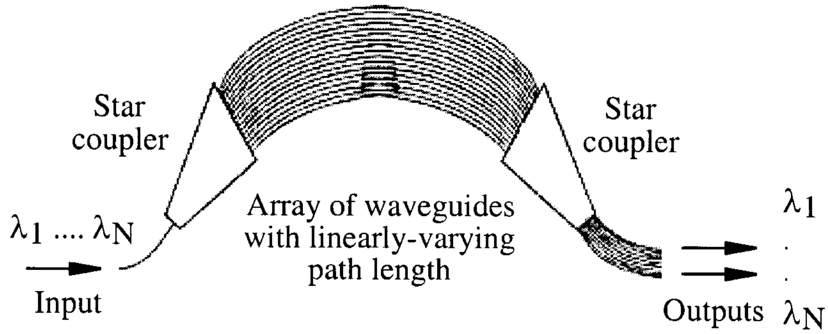
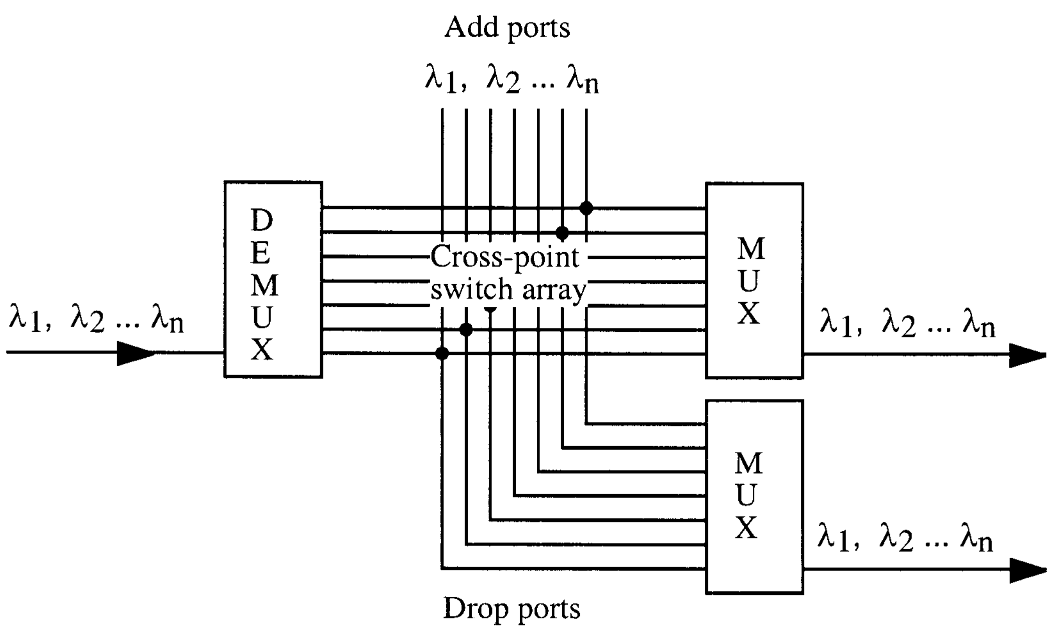


[6]

c) An ADD-DROP MUX is needed to add or remove channels in a wavelength division multiplexed communication system. Because of the large number of closely spaced channels in a modern DWDM system, a scalable filter component is required. **[2]**

The channels are spatially separated using DEMUX component, then redirected to one of two similar MUX components using an array of crosspoint switches. An N-channel ADD-DROP MUX requires 3 off AWG MUX components and N off 2 x 2 crosspoint switches, as shown below.



[6]

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4a) I/ev : injection of current into the active volume

n/τ_e : total recombination of electrons

$G\phi(n - n_o)$: stimulated emission and absorption of photons

$\beta n/\tau_{rr}$: radiative recombination of electrons, giving rise to spontaneous emission of photons
β is coupling factor into the laser stripe

ϕ/τ_p : escape of photons from the active volume **[5]**

τ_e appears in the upper equation because a reduction in the electron density must take account of all possible recombination paths. τ_{rr} appears in the lower one because an increase in the photon density is only obtained through radiative recombination. **[2]**

The photon lifetime is $\tau_p = L / \{v_g \log_e(1/R_1R_2)$ where L is the cavity length, $v_g \approx c/n$ is the group velocity, n is the refractive index and R_1 and R_2 are the cavity end mirror reflectivities. **[3]**

b) The two lasers differ only in their cavity lengths, and thus in their active volumes and photon lifetimes.

From the rate equations:
During lasing, spontaneous emission is negligible, so that $G\phi(n - n_o) - \phi/\tau_p = 0$
The electron density is therefore clamped at the value $n = n_o + 1/G\tau_p$
The rate of loss of photons from the cavity per unit volume is therefore $\phi/\tau_p = (I/ev - n/\tau_e)$
The total rate of production of photons is $\Phi = \phi v/\tau_p = (I/e - nv/\tau_e) = (I - I_t) / e$
The threshold current is therefore $I_t = nev/\tau_e$.
Thus, the electron density at threshold is $n = I_t\tau_e/ev$

For the two lasers, we can therefore write:
$n_o + 1/G\tau_{p1} = I_{t1}\tau_e/ev_1$
$n_o + 1/G\tau_{p2} = I_{t2}\tau_e/ev_2$

Subtracting, we obtain:
$(1/G) \times \{1/\tau_{p1} - 1/\tau_{p2}\} = (\tau_e/e) \{I_{t1}/v_1 - I_{t2}/v_2\}$
This allows G to be found, and n_o may then be found using one of the two equations above. **[4]**

The group velocity is $v_g \approx 3 \times 10^8 / 3.5 = 8.571 \times 10^7$ m/s
The cavity end mirror reflectivity is $R_1 = R_2 = (n - 1) / (n + 1) = 2.5 / 4.5 = 0.555$

For laser 1: The photon lifetime is $\tau_{p1} = 150 \times 10^{-6} / \{8.571 \times 10^7 \log_e (1/0.555^2)\} = 1.5 \times 10^{-12}$ s
The active volume is $v_1 = 0.1 \times 3 \times 150 \times 10^{-18} = 45 \times 10^{-18}$ m^{-3}
For laser 2: The photon lifetime is $\tau_{p2} = 250 \times 10^{-6} / \{8.571 \times 10^7 \log_e (1/0.555^2)\} = 2.5 \times 10^{-12}$ s
The active volume is $v_2 = 0.1 \times 3 \times 250 \times 10^{-18} = 75 \times 10^{-18}$ m^{-3}

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Hence: $(1/G) \times \{1/1.5 - 1/2.5\} \times 10^{12} = (10^{-9}/1.6 \times 10^{-19}) \times \{12/45 - 16.8/75\} \times 10^{15}$

Or: $(1/G) \times 0.2666 \times 10^{12} = 2.666 \times 10^{23}$

So that: $G = 0.2666 \times 10^{12} / 2.666 \times 10^{23} = 10^{-12}\ m^3/s$ **[3]**

Hence: $n_o = I_{t1}\tau_e/ev_1 - 1/G\tau_{p1}$

Or: $n_o = \{12 \times 10^{-3} \times 10^{-9} / (1.6 \times 10^{-19} \times 45 \times 10^{-18})\} - 1/\{10^{-12} \times 1.5 \times 10^{-12})$

Or: $n_o = 1.666 \times 10^{24} - 0.666 \times 10^{24} = 1 \times 10^{24}\ m^{-3}$ [3]

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5. a) Binary alloys are used as substrate materials because it is easier to maintain their stoichiometry during the growth of the boule from which the substrates are taken. [2]

Homoepitaxy and heteroepitaxy both involve material growth atom by atom in an ordered, layered fashion. However, *homoepitaxy* involves the growth of a material on a crystallographically similar substrate - for example, silicon on silicon, or doped silicon on intrinsic silicon. *Heteroepitaxy* involves the growth of a material on a completely different substrate, which can therefore have a different refractive index and bandgap - for example, GaAlAs on GaAs. [2]

Heteroepitaxy is used in the construction of heterostructure lasers, where the differences in index and bandgap allow strong confinement of both photons and injected carriers. [2]

b) The polynomial equation:

$$a(x, y) = xy\, a_{GaP} + x(1 - y)\, a_{GaAs} + (1 - x)y\, a_{InP} + (1 - x)(1 - y)\, a_{InAs}$$

may be rewritten as:

$$a(x, y) = \alpha + \beta x + \gamma y + \delta xy$$

where $\alpha = a_{InAs}$, $\beta = \{a_{GaAs} - a_{InAs}\}$, $\gamma = \{a_{InP} - a_{InAs}\}$ and $\delta = \{a_{GaP} - a_{GaAs} + a_{InAs} - a_{InP}\}$.

It is then easy to show that compounds lattice-matched to InP are solutions to:

$\delta xy + \gamma y + \beta x - \gamma = 0$, namely: $y = \{\gamma - \beta x\} / \{\gamma + \delta x\}$

Since $\beta = -0.4054$ Å, $\gamma = -0.1894$ Å and $\delta = -0.013$ Å, $\delta/\gamma \ll \beta/\gamma$. We may therefore put:

$y \approx \{1 - (\beta/\gamma)\, x\}$, or $y \approx 1 - 2.14x$. [8]

c) When $y = 0$, $x = 1/2.14 = 0.47$, so lattice-matched InGaAs has the composition $In_{0.53}Ga_{0.47}As$. The locus of lattice-matched compounds is therefore as shown below. [2]

The wavelength range of the substrate-entry photodetector is limited by two effects:

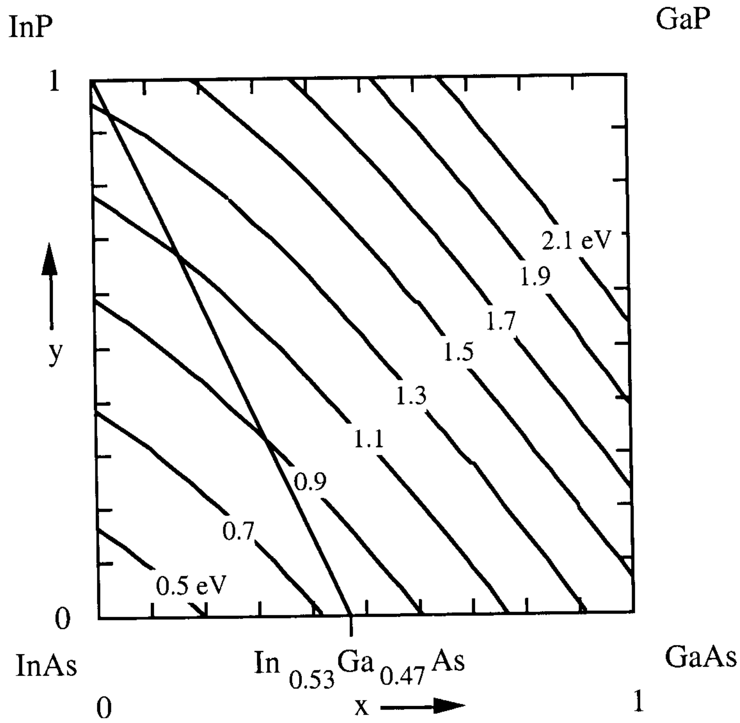
i) at photon energies greater than the bandgap of InP, substrate absorption will occur, and
ii) at energies less than the bandgap of $In_{0.53}Ga_{0.47}As$, the detector material itself will be transparent.

The bandgaps can be estimated from the Figure.

Anthea Holmes R R A Syms

For InP, $E_g \approx 1.35$ eV. The corresponding photon wavelength is $\lambda_{min} = hc/eE_g \approx 0.92$ µm. [2]

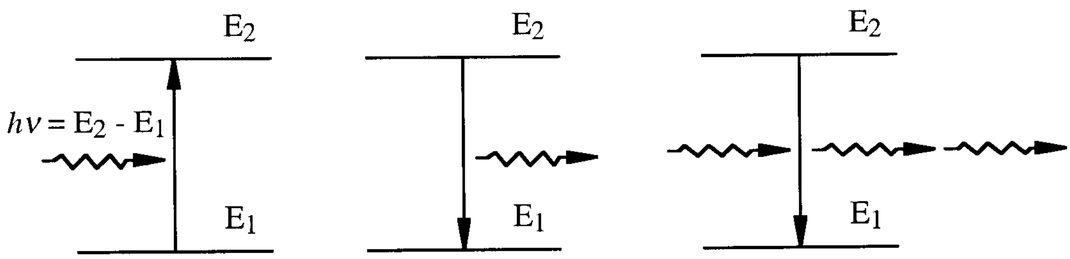
For $In_{0.53}Ga_{0.47}As$, $E_g \approx 0.74$ eV, so that $\lambda_{max} \approx 1.67$ µm. [2]



6. a) *Absorption* is a process in which a photon of energy $h\nu = E_2 - E_1$ causes the transition of an electron from a low level with energy E_1 to a high level with energy E_2. The photon is destroyed as a result. **[2]**

b) *Spontaneous emission* is a process in which an electron falls from a high level with energy E_2 to a low level with energy E_1, leading to the emission of a photon with energy $h\nu = E_2 - E_1$. The emitted photon has random direction, polarization and phase. **[2]**

c) *Stimulated emission* is a process in which a photon with energy $h\nu = E_2 - E_1$ triggers the transition of an electron from a high level with energy E_2 to a low level with energy E_1, leading to the emission of a second photon with the same energy $h\nu = E_2 - E_1$. The two photons are also identical in direction, polarization and phase. **[2]**

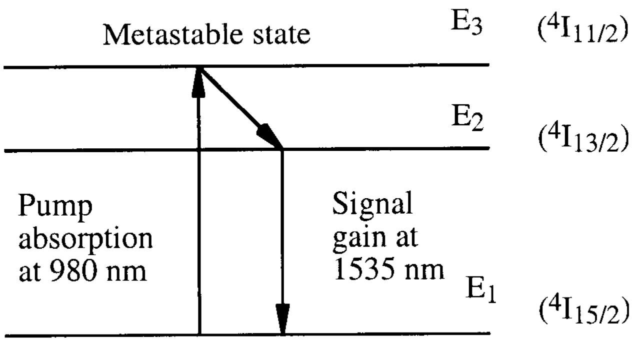


Absorption   Spontaneous emission   Stimulated emission

b) An erbium doped fibre amplifier (EDFA) consists of a dispersion of Er^{3+} ions in a doped silica host. The separation between two of the Er^{3+} energy levels ($^4I_{15/2}$ and $^4I_{13/2}$) is suitable for the generation of photons at the fibre communications wavelength of 1535 nm.

Optical amplification is obtained by a three-state process.

Pumping involves the excitation of an electron from the ground state (Level 1; $^4I_{15/2}$) to a higher state (Level 3; $^4I_{11/2}$) through the absorption of a pump photon of energy $h\nu_p = E_3 - E_1$. Exitation is followed by a rapid, non-radiative decay to a slightly lower state (Level 2; $^4I_{13/2}$). Since electrons have a long lifetime in Level 2, it is possible to build up the population in this state by continuous pumping until it exceeds that in Level 1. The rate of stimulated emission between Levels 1 and 2 then exceeds the corresponsing rate of absorption, so that signal photons of energy $h\nu_s = E_2 - E_1$ experience net gain.



[6]

The data rate of an optical communications system is generally limited by the bandwidth of detector that can be allowed before the signal is swamped by noise. An EDFA is an important additional component, because it provides a large, low noise pre-detector signal gain that in turn allows a correspondingly large increase in detector bandwidth and data rate . **[2]**

c) The requirement for a 100 μW signal at the detector places a constraint of 20 dB maximum loss in the system, assuming a source power of 10 mW. The power budget is:

25% launch efficiency	6 dB loss
10 splices, each contributing 0.1 dB loss	1 dB loss
Propagation through 150 km of 0.2 dB/km SM fibre	30 dB loss
Total	37 dB loss

The optical amplifier must therefore have a gain of ≥ 17 dB. Assuming a gain of 0.5 dB/m from the erbium-doped fibre, a 34 m fibre length is required. **[6]**