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IMPERIAL COLLEGE
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[E303/ISE3.3]

DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2004

EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI

SOLUTIONS 2004
COMMUNICATION SYSTEMS

ANSWER to Q1

1) A B C D E

2) A B C D E

3) A B C D E

4) A B C D E

5) A B C D E

6) A B C D E

7) A B C D E

8) A B C D E

9) A B C D E

10) A B C D E

11) A B C D E

12) A B C D E

13) A B C D E

14) A B C D E

15) A B C D E

16) A B C D E

17) A B C D E

18) A B C D E

19) A B C D E

20) A B C D E

ANSWER to Q2

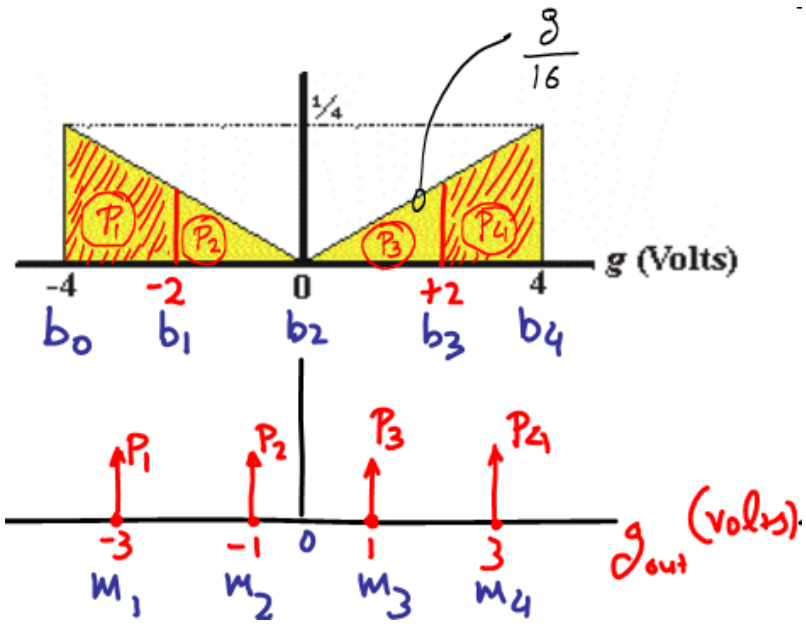
a)

$b_0 = -4V$
$b_4 = +4V$
$Q = 4$
$\Rightarrow \Delta = \frac{b_4 - b_0}{Q} = 2V$

∴ end points $b_0 = -4V$, $b_1 = -2V$, $b_2 = 0V$, $b_3 = 2V$, $b_4 = 4V$ & o/p levels $m_1 = -3V$, $m_2 = -1V$, $m_3 = 1V$, $m_4 = 3V$

[5]

b)



$P_1 = P_4$
$P_2 = P_3$

$$P_3 = Pr(g_{out} = 1V) = Pr(0 < g < 2V)$$
$$= \int_0^{2V} pdf_g(g)\, dg$$
$$= \int_0^{2V} \frac{g}{16}\, dg$$
$$= \frac{1}{16}\left[\frac{g^2}{2}\right]_0^2 = \frac{1}{8} = P_2$$

$$P_4 = Pr(g_{out} = 3V) = Pr(2 < g < 4V)$$
$$= \int_2^{4V} pdf_g(g)\, dg$$
$$= \int_2^{4V} \frac{g}{16}\, dg$$
$$= \frac{1}{16}\left[\frac{g^2}{2}\right]_2^4 = \frac{3}{8} = P_1$$

$$\therefore P_{g_{out}} = (-3)^2 \cdot P_1 + (-1)^2 \cdot P_2 + 1^2 \cdot P_3 + 3^2 \cdot P_4 = \frac{56}{8} = 7$$

$$P_{nq} = \frac{\Delta^2}{12} = \frac{4}{12} = \frac{1}{3} = 0.333V$$

$$SNR_q = \frac{7}{1/3} = 21$$

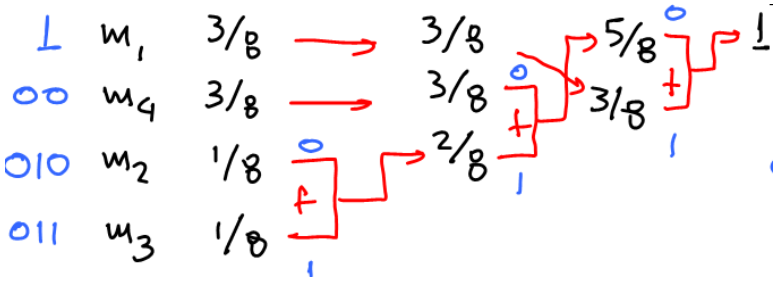
[6]

c)

$$H_M = -2\left(\frac{1}{8}\log_2\frac{1}{8} + \frac{3}{8}\log_2\frac{3}{8}\right)$$
$$= 1.8113 \ \frac{bits}{symbol}$$

[3]

d)



[7]

e)

$$\bar{\ell} = 1\times\frac{3}{8} + 2\times\frac{3}{8} + 3\times\frac{1}{8} + 3\times\frac{1}{8}$$

$$= \frac{15}{8} = 1.8750 \ \frac{\text{bits}}{\text{symbol}}$$

[3]

f)

$$r_{inf} = \underbrace{r_M}_{2\times 8\text{K}} \times \underbrace{H_M}_{1.8113} = 28.9804 \ \frac{\text{bits}}{\text{sec}}$$

[6]

$$r_{data} = r_M \times \underbrace{\bar{\ell}}_{1.8750} = 30 \ \frac{\text{bits}}{\text{sec}}$$

ANSWER to Q3

a)

$$P_e = \underbrace{Pr(r_2|m_1)\cdot Pr(m_1)}_{Pr(r_2,m_1)} + \underbrace{Pr(r_1|m_2)\cdot Pr(m_2)}_{Pr(r_1,m_2)} = \underbrace{0.1\times 0.25}_{0.025} + \underbrace{0.2\times 0.75}_{0.15} = 0.175$$

[6]

b)

$$\underline{q} = \begin{bmatrix} Pr(r_1) \\ Pr(r_2) \end{bmatrix} = \underline{\underline{F}} \cdot \underline{P} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$$

$$\underline{\underline{J}} = \underline{\underline{F}}.diag(\underline{P}) = \begin{bmatrix} 0.9\times 0.25 & 0.2\times 0.75 \\ 0.1\times 0.25 & 0.8\times 0.75 \end{bmatrix} = \begin{bmatrix} 0.225 & 0.15 \\ 0.025 & 0.6 \end{bmatrix}$$

[12]

$$H_{R|M} = -\left\| \underline{\underline{J}} \odot \log_2 \underline{\underline{F}} \right\|_{L1} = 0.658695$$

c)

$$H_{Mut} = H_R - H_{R|M}$$

$$H_R = \underline{q}^T \cdot \log_2(\underline{q}) = 0.9544$$

$$\therefore H_{Mut} = 0.9544 - 0.658695 = 0.295705$$

[12]

ANSWER to Q4

a)

code rate : 1/3
constraint length 3

[5]

b)

generator polynomials

1) $1+D^2$
2) D^2
3) $1+D+D^2$

[6]

c)

$$G_c = \begin{bmatrix} 101 & 001 & 111 & 000 & 000 & \cdots \\ 000 & 101 & 001 & 111 & 000 & \cdots \\ 000 & 000 & 101 & 001 & 111 & \cdots \\ \vdots & \text{etc} & \cdots & & & \end{bmatrix}$$

[9]

d)

if $\underline{x} = [\overset{\text{first}}{1}\ 01\ 1\ 01\ 00\overset{\text{last}}{1}]$

then $\underline{x} \cdot \underline{\underline{G}} = [101\ 001\ 010\ \text{etc}]$

[10]