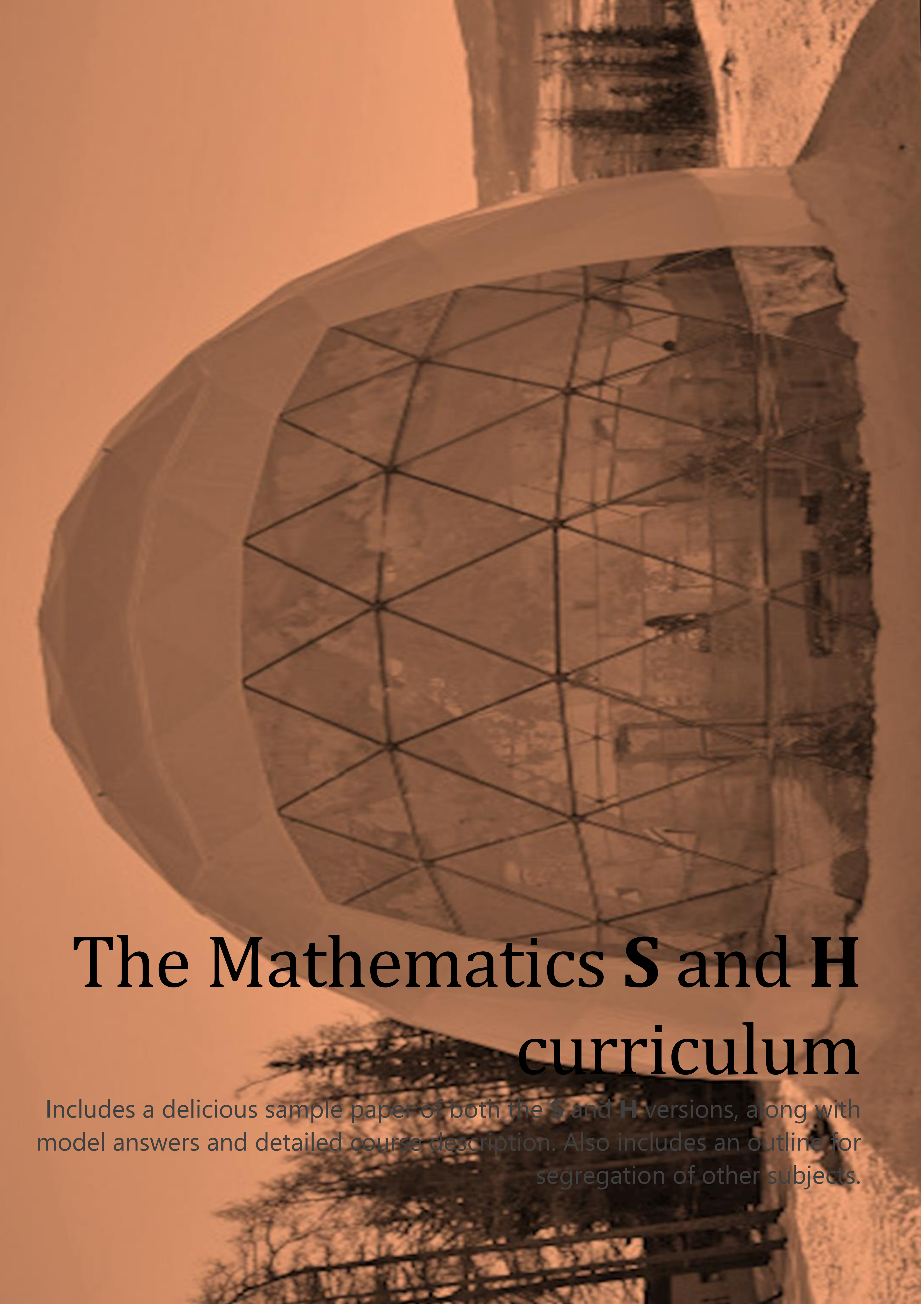


The Mathematics **S** and **H** curriculum

Includes a delicious sample paper of both the **S** and **H** versions, along with model answers and detailed course description. Also includes an outline for segregation of other subjects.

Table of Contents

Technical references:

V1.2
Finalized on 30.12.18
Crafted with lots of love.

1 - Preface

This document underlines my vision for the redevelopment of the Maths syllabus and curriculum for Class 11 and 12.

But the underlying effects go far beyond just Mathematics: on the surface, it may seem so, but dig deeper and the effects on other subjects – and the whole Indian education system – becomes clear.

As the main developer of this vision, I'd be grateful if this document and the changes outlines are taken into consideration and implemented as soon as possible. And yes – all feedback or/and queries (positive and negative but be constructive!) is welcome; please drop a mail *as soon as possible* to mathsSHfeedback@outlook.com. Speed is key – the faster, the better. On my part, I'll try to respond as fast as possible.

And you – the stakeholders (parents, students, teachers etc) can help in a lot of ways! See the next section.

Let's get the ball running!

- Darsh Manoj

2 – The need for feedback

While I've developed my ideas and visions with the thinking of a student and tried to understand different students' abilities, I just do not have enough data, and your help is very much appreciated.

For example, consider the sample papers. Do you think, for instance, that?

- The **S** paper is too easy?
- The **H** paper is too hard?
- A question is incorrect or poorly worded?
- The sample answer or method to a question is too long or wrong?
- The marks allocated to a question is disproportionate to the effort required?

Or do you think that

- The syllabus is too little or too much?
- Your favourite topic is missing in the syllabus?
- The whole concept of the **S**-**H** system is flawed?
- <anything else you do not like>

Then tell me! Tell me your vision and why you think that is correct.

Since I am not a teacher, I do not have the luxury of testing the sample papers I made with a sample population. To this effect, it will be very much appreciated if teachers would set 3 papers to their students on different days, which should be

- A previous year's CBSE Maths paper (not an internal school paper)
- The Maths-**S** sample paper
- The Maths-**H** sample paper

And report the results to me by email (mathsSHfeedback@outlook.com). Include all possible statistics in your email – the more, the better! For instance, you can include a per-question analysis – complete with average mark and distribution of marks amongst your students. Your statistics on this are very important and could impact future sample papers, grade/mark distribution or even the syllabus of the new specification.

P.S: Other forms of statistical data like CBSE students' graph of marks vs percentile in board exams and other related information would be very helpful. If per-question percentage of marks obtained is available, that would be useful as well.

3 – The Maths **S**-**H** specification

This section outlines the specification in detail, including syllabus, guidelines and required changes. But before that, one needs to understand how this change came in mind, and why the **S-H** system is required in the first place.

3.1- Freedom of subjects

The Indian education system's reliance on *streams* has always placed unreasonable restrictions on students' choices. To illustrate this point, let us consider a CBSE student who's planning to enrol for Class 11 at my school.

The below are the choices given in my school; others may vary. English Core is mandatory for every student.

Science	*Commerce*	*Humanities*
Physics	Accountancy	Home Science
Chemistry	Business Studies	Sociology
Mathematics **OR** Biotechnology	Economics	Marketing
Computer Science **OR** Engineering Graphics **OR** Biology (required if Maths is not taken)	Mathematics **OR** Informatics Practices **OR** Multimedia/Web Tech (now discontinued) **OR** Marketing	Psychology

This rigidity makes it impossible, for example, for a student to

1. Take Economics and drop Physics, while still having Chemistry
2. Drop Mathematics in place of Business Studies while still studying other Science subjects
3. Studying Mathematics in place of Home Science while still studying other Humanities subjects.

Now we need to take a step back and see how well others do it.

The IB believes in an all-round education, and hence *mandates* students to

(i) Study 2 languages.
(ii) Take up **at least** one science
(iii) Study Mathematics
(iv) Study the arts and the business field *(this can be replaced with the sciences)*

So, a typical CBSE science student would, in the IB, study

English S (1st language), 2nd language S, Physics H, Chemistry H, Maths-H, Biology/Computer Science (H or S)

This is not the best idea. Students are not necessarily interested in studying languages. But the IB gives significant raw flexibility in subject choice over the CBSE. For instance, a student can easily take Business Studies Higher while *still* taking French Higher and Physics Higher.

Now moving on the A-Level, students are actually required to study only 3 subjects. So, a student applying to Cambridge for Computer Science/Mathematics would study

Maths

AND

Further Maths

AND

Anything else (generally Physics or Computer Science)

In the A-Level system, taking Further Maths is highly recommended for many science courses, and the combination counts as 2 subjects.

But the A-Level still gives the subject flexibility of IB, and you *can* study 4 or more if you wish (and quite a few do!).

Remember that the 'anything else' could also be Philosophy or History, again something which is nearly impossible in CBSE.

And as for the US system, it is important to realise that there are many types of courses even within the same school, and a direct comparison between each course is difficult. The best equivalent for this is AP (Advanced Placement).

Students routinely take irrelevant courses (e.g.: Calculus AB and Spanish). As the AP does not *impose* any particular age/class requirements, students are free to take it whenever they like!

This is perhaps the most striking example of freedom. When I wrote my Advanced Placements, I remembered a student who was writing Calculus BC (like me). *This one had written US History and was also writing Macroeconomics*. In fact, there was nothing that stopped me from writing those subjects other than the fact that I hadn't registered for the exam and hadn't studied anything for them![1]

The comparison clearly shows that others are able to maintain the balance between freedom and rigour. To apply the same to CBSE would mean an effective abolishment of the stream system. It would be prudent to take a look at this stage as to what a post-secondary class should really look like.

You have n students in a class, and a class teacher to oversee the activities of the class. Then for each period, students go to other classes to take their respective subjects; the class composition would be designed to reasonably minimize the need to go to adjacent classes – while still offering freedom for those who need to do it.

If (a) subject(s) is (are) oversubscribed, then schools may use their own methods (in good faith) to break the tie(s).

With the rigidity of the 'stream' system over, students will find it easier to select the subjects of their choice. But there is still the problem of aptitude and interest (not everyone will want to study the same level of

[1] It's worth noting the drawbacks of AP though. The drawback of AP is the limited information (you only get a number from 1 to 5) and the fact that *every (reasonably) proficient student gets a 5 (mostly). For instance, in Physics 1, only ~6% get a 5, but in Calculus BC, it's nearly half.* Often, it's too easy to get a 4 or 5. See here. A CBSE student can easily get a 3-4 on AP's *without even knowing what the portion is*, but the reverse is incorrect – *most* AP students would barely pass on CBSE board exams.

Mathematics, though they wouldn't want to *drop* it like students are often forced to now in favour of other 'easier' subjects).

There comes the **S-H** system.

Correction: I'm wrong on the stream system. CBSE does not implement streams; schools do it. However, what I've written before still applies – the only change being that CBSE must mandate that schools not segregate subjects into any 'streams'. See this, I was pleasantly surprised to realise that CBSE itself does not stream.

3.2- The need for the **S-H** system

Up until now, each subject used to choose subjects irrespective of his/her aptitude. While one student might be really passionate about Chemistry, he/she may be relatively mediocre in Mathematics but still interested. So, such a student will definitely be interested in studying a lot about Chemistry, but Mathematics? Not so much.

To the low-achievers in that subject, simply reducing the portion is a terrible idea – it affects everyone. To the high-achievers, simply raising the difficulty will cause issues with the low-scorers.

So, what's the solution? Segregate.

Create one variant of that subject for average students whose main passions might lie in other subject(s) but still want to learn the said subject and create *another* variant for those who are really passionate and want to study as much of the subject.

In fact, this already exists in the main languages. English is offered in a *Core* and *Elective* variant, and so is Hindi. So, we can rebrand

English Core *to* English Standard (English-**S**)

English Elective *to* English Higher (English-**H**)

… which makes things clear to the student in choosing the variant. (The difficulty should also match the name!)

This is the idea between the creation of two variants of Class 11-12 Mathematics:

1. Mathematics Standard (hereby referred to as Maths-**S**)
2. Mathematics Higher (hereby referred to as Maths-**H**)

While the question paper for Maths-**S** is meant to cater to students of all levels, the question paper for Maths-**H** will be designed for above average and outstanding students only.

However, the changes are not restricted to mere rebranding, as we'll see below.

*NOTE: ALL schools MUST offer both variants of any subject. No school may offer only one variant of any subject (e.g.: only Maths-**S**) at any time. This is because restricting choices to only one variant will have dangerous consequences for students who will either not be challenged enough (**S** instead of **H**) or struggle (**H** instead of **S**).*

3.3 - Changes in both Maths-**S** and Maths-**H**

1. The question paper will contain questions from both Class 11 and 12, rather than only from Class 12.

While this move will understandably cause concern that the student's syllabus is being doubled, in reality, it is not so. Even in the current syllabus, one does need to know topics from Class 10 and 11 to solve many problems from Class 12. Even a simple integral problem often requires concepts of quadratic equations, and its high time their basics also get asked in the final examinations.

The other benefit I foresee from this change is the fact that virtually every entrance exam test students' concepts from both Class 11 and 12. Most students currently devote a lot of time to entrance studies, and this move will help those who don't to brush up on their Class 11 topics and could even help reduce dependence on coaching centres.

2. The pattern of the examination paper will no longer be fixed. The breakup of chapters will not be disclosed or fixed. Repetition of questions will be strongly discouraged.

   Keeping a fixed examination pattern ensures unwanted 'teaching to the test' and keeps the onus on understanding the exam pattern rather than their concepts. Disclosing the breakup of chapters encourages students to skip chapters for 'low weightage'.

   Allowing for a variable examination pattern encourages the examiners to create interesting and multi-step examination questions, all while ensuring adequate weightage of concepts.

   However, to prevent domination of one idea or topic in the paper, there is a hard limit of 8 marks per question (inclusive of all parts).

   The repetition of questions from past years' papers or commonly-used resource books (including the NCERT textbook) is also discouraged. This creates a negative incentive on students to 'mug up' questions (!) from the textbook (and repeating questions isn't something you'd see at the A-Level or IB). *Note that questions in the paper should generally not come from the textbook: that's a rule. The role of the textbook is to provide enough practice problems so that the students can understand the concepts effectively.*

> **"LET ME TELL YOU AN INSTANCE OF THIS ROTE LEARNING BY STUDENTS. ON THE DAY OF MATHS PAPER BEFORE THE COMMENCEMENT OF EXAM 7-8 STUDENTS OF ANOTHER SCHOOL (THE BEST SCHOOL OF OUR CITY ACCORDING TO ALL) WERE CHIT CHATTING WITH EACH OTHER AND SAYING THAT THEY HAD MUGGED UP THIS APPLICATION OF DERIVATIVES QUESTION (AND ITS VARIANTS) SINCE IT COMES EACH YEAR AS A 6 MARKER. AND I WAS LIKE SERIOUSLY!!!!"**
>
> - **ADITYA DOKHALE, HTTPS://WWW.QUORA.COM/HOW-WAS-THE-CBSE-CLASS-12-MATHEMATICS-PAPER-FOR-2016**

   That attitude is just what I don't want; that helps nobody.
   It must be noted that the practice is *not* outlawed; if a question from another source is interesting enough that it should be included, include it. This should not become a habit though, and questions should **never** come straight from the textbook itself.

3. Graphing of functions, exponential and logarithmic properties are included. The sine and cosine rule with its applications are also included. Leibnitz's differential rule is also included.

It’s a mystery as to why exponential functions were included in the pre-2008 syllabus but were removed in the current (now old) post-2008 syllabus; the sine rule also suffered a similar fate – it’s in the NCERT textbook but not taught in the syllabus.

Abstract knowledge of exponential and logarithmic functions is needed. Their applications find wide use in calculus, algebra and more. Currently, they are only taught in the midst of calculus, and only to serve their needs – thus denying the opportunity of students to see the wider picture behind these functions.

The sine rule is no less important. It finds important application in Physics problems (and is essentially repeated there). It is also an important extension to trigonometry that should be (re)included.

Graphing is the gateway for proper understanding of many algebraic and trigonometric concepts. Students should be able to understand common graphs, draw them and understand their mechanisms – that’ll often help make easier their conceptual understanding.

4. Presence of mark deflation

   Even for Maths-**S**, I expect a reduction in the marks required to obtain a particular grade. For instance, in 2018, the A1 cutoff was a brutal 95. I expect the A1 cutoff for Maths-**S** and Maths-**H** to be about 75 marks.

   The resultant fallout from this can be dealt by including a statistical concordance table that’ll list the old Mathematics scores together with the equivalent Maths-**S** and Maths-**H** scores which colleges/universities may use. *‘Moderation’ of marks is strongly discouraged.*

   A potential complaint is that students would be demoralized by the harder questions and deflating marks, especially in Maths **H** (see here for an equivalent criticism of IB Mathematics Higher). The fact is, there are two tiers of Mathematics for this reason. Additionally, the cap of 8 marks per question is designed so that students’ grades are not necessarily damaged by their inability to answer a question. Finally, by virtue of the deflation, the A1 gap will be wide open, reducing stress for students trying to get the highest grade, and reducing university cut-offs for those who are mark-agnostic.

5. Few or no internal choices

   The specification does not mention any number of ‘internal choice’ questions that must come in either Maths-**S** or Maths-**H**, but in the sample papers, *no* such questions feature for Maths-**S**, and only one such question for Maths-**H**.

   There is nothing *per se* to not use internal choices, but it’s not necessary. Most other examination boards do *not* use internal choices at all. Yes, it’s not easy answering all the questions, but I don’t think it’s required to provide internal choices needlessly.

   On that, I should hit out at CBSE for increasing the internal choices for the 2019 exams. Seriously? My guess is that you’re increasing the marks of students high enough to not require moderation, but then why are subjects like Computer Science (A1 cutoff: 93) or Biotechnology (95) getting extra internal choices?

3.4 – Changes in Maths-**S**

1. The limit of a sum, elementary row transformations and scalar triple product are removed.

   These topics are highly abstract and mostly viewed by potential Maths-**S** students as a chore. Maths-**H** students, in contrast, are more likely to understand and get the idea behind such abstract concepts.

3.5 – Changes in Maths-**H**

1. The level of the question paper will be higher.

   The Maths-**H** curriculum is designed for students who are generally more mathematically able. Hence the questions will be set more for such students. However, if desired, some questions (e.g.: medium **S** and easy **H**) can be shared; this is not done in the sample paper, however.

2. The D2 grade cut-off (passing mark) will reduce.

   While getting less than 33 in Maths-**S** truly indicates the lack of minimum concepts required, the same cannot be said of Maths-**H**. Even a score of 20 in Maths-**H** can indicate a score of mid-40's on Maths-**S**, as the student is able to answer over 20% of a paper designed for strong students.

   The D2 cutoff is anticipated to be about 15-20 marks for Maths-**H**.

3. 2^{nd} order differential equations, Riemann sums, probability density functions, improper integrals, hyperbolic functions, Poisson distribution, Maclaurin expansions and volumetric integrals are included.

   All of these topics are very useful and for students who find Maths-**S** too easy, these topics are worth adding upon.

3.6 – Balance of **H** and **S**

The Higher tier of subjects represent a dramatic increase in workload over their **S** counterparts. This could represent a very dangerous precedent in which students pick too many **H** subjects and get overwhelmed.

Hence a limit is proposed:

A **minimum** of 2 Higher (**H**) subjects must be taken by the student and a **maximum** of 3 Higher (**H**) subjects. All the remaining subjects must be taken in Standard (**S**) level.

Now why a *lower* limit? It stems from the fact that each student's strength(s) lie in different fields and that students should not fear to take the **H** variant on subjects they think they'll love. At the same time, an upper limit is placed which is necessary despite the minor impediment on students' freedoms.

3.7 – Dropping and upgrading

By the very nature, Maths-**H** students can easily drop to Maths-**S** at any time. A reasonable deadline for the same is by the end of Class 11, though it is unadvisable for students to drop after one poor result; he/she may just need time to adjust.

While it is not very difficult to do the reverse either (i.e., upgrading from Maths-**S** to Maths-**H**), the student will have to do a bit of catch-up work to cover the extra material taught in Maths-**H** (the suggested time for this is the break between the end of Class 11 and beginning of Class 12), and depending on the student's grasp on the concepts, could also face difficulty with the higher level of Maths-**H**.

3.8 – No calculators

And that should stay the case.

It may seem negative that CBSE still prohibits calculators' use when all other boards *require* a scientific calculator at the minimum for a significant part of the exam. (AP Calculus requires a *graphical* calculator for a third of the exam!)

But there is a hidden benefit to this: students are forced to develop their thinking capacity. *Hence, they can perform better even in cases where a calculator is required.*

It may seem odd and cumbersome, but the benefits to this approach are quite hidden. It's easy to use a calculator, it's harder to think the same by mind.

3.9 – 1 or 2 paper(s)?

While I originally intended to strongly advocate having 2 papers for each subject (both **S** and **H**) simply due to the sheer increase in portion caused by including Class 11 as well, after developing the sample papers, I think while it is still a preferred option to make students take 2 papers of a subject, it is also possible with careful deliberation to create only one paper and still maintain reasonable representation of each concept.

The drawbacks of making 2 papers include logistical challengers, more teachers to handle double the work and the stress students must face in sitting more papers. Hence it is a catch-22 situation.

3.10 – The curriculum cannot be reduced!

Several suggestions from all corners say about the 'need' of reducing students' load by cutting down on the syllabus.

But that's simply not possible!

For starters, **where** would you cut the syllabus? Taking a glance at the syllabus indicates that the topics are all important and useful – not ones which could be deleted. Not any one of them. Cutting the syllabus has dangerous consequences. Students will study less. More importantly, other universities abroad will derecognise CBSE, which could make applying there very difficult. To illustrate the gravity of the situation for students whose curriculum is defined as poor by universities, take a look at Oxford University's international entrance requirements.

Pakistan

Higher Secondary School Leaving Certificate (HSSC) would not be sufficient for candidates to make a competitive application (see note).

Bangladesh

School leaving qualifications would not be sufficient for candidates to make a competitive application (see note).

Students from these boards will have to…

If your qualification is listed as being insufficient to make a competitive application to Oxford, then you will need to undertake further study if you wish to apply.

What types of further study could I take?

You could take British A-levels (the British Council may know where you can take A-levels in your country), the International Baccalaureate (IB), or any other qualifications listed as acceptable on this page. The first year of a bachelor's degree from another university could also be an acceptable alternative.

This used to also be the case for CBSE until 2012. However, CBSE (and ISE **alone**, NOT including state boards) is *not* a prohibited board. This illustrates the fact that we are better off than many of our peers in academic rigour and quality. Unhealthy as it seems, it is not an option to reduce the load of the students – the drawbacks, while not immediately apparent, will far outweigh the benefits, especially for good students.

I currently consider CBSE Class 11-12 Maths (old) to be between the Core A-Level and the Further A-Level Maths in the UK. Maths-**H** is **at least** equivalent to Core A-Level + Further A Level OR IB Higher Level Mathematics, which is how it should be. And no, it should not be reduced.

(and CBSE Maths in all forms is better than AP Calculus AB/BC)

3.11 – The Maths **S-H** syllabus

Note:

***Bold**: Portion added in Maths-**H** but not in Maths-**S**.*

Blue – Class 11
Green – Class 12

Topic/Chapter	*Old Maths*	*Maths-**S***	*Maths-**H***

Sets	Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of a set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement.	Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of a set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement.	Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of a set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement.
Relations & Functions	Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto R x R x R). Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions. Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.	Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto R x R x R). Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions. Graphing trigonometric, polynomial functions, and simple composite functions. Transformation of graphs. Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a	Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto R x R x R). Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions. Graphing trigonometric, polynomial functions, and simple composite functions. **Period and amplitudes of graphs, periodic functions.** Transformation of graphs. Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions,

		function. Binary operations.	inverse of a function. Binary operations. **Maclaurin expansions for exponential and logarithmic functions.**
Trigonometry	Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing $\sin(x \pm y)$ and $\cos(x \pm y)$ in terms of sin x, sin y, cos x & cosy and their simple applications. Identities related to sin2x, cos2x, tan2x, sin3x, cos3x and tan3x. General solution of trigonometric equations of the type sin y = sin a, cos y = cos a and tan y = tana. Inverse trigonometric functions: Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.	Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle (no proof). Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing sin (x±y) and cos (x±y) in terms of sin x, sin y, cos x & cosy and their simple applications. Identities related to sin2x, cos2x, tan2x, sin3x, cos3x and tan3x. General solution of trigonometric equations of the type sin y = sin a, cos y = cos a and tan y = tan a. Sine and cosine law with simple proofs and applications. Inverse trigonometric functions: Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.	Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing sin (x±y) and cos (x±y) in terms of sin x, sin y, cos x & cosy and their simple applications. Identities related to sin2x, cos2x, tan2x, sin3x, cos3x and tan3x. General solution of trigonometric equations of the type sin y = sin a, cos y = cos a and tan y = tan a. Sine and cosine law with simple proofs and applications. Inverse trigonometric functions: Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions. **Hyperbolic functions: Definition, range, principal value branch, graphs with elementary**

			properties and applications. **Maclaurin expansions for common trigonometric functions.**
Mathematical Induction	Process of the proof by induction, motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.	Process of the proof by induction, motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.	Process of the proof by induction, motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.
Complex Numbers and Quadratic Equations	Need for complex numbers, especially √ -1, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations (with real coefficients) in the complex number system. Square root of a complex number.	Need for complex numbers, especially √-1, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations (with real coefficients) in the complex and/or real number system. Square root of a complex number.	Need for complex numbers, especially √ -1, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. **De Moivre's theorem and its applications.** Statement of Fundamental Theorem of Algebra, solution of quadratic equations (with real coefficients) in the complex and/or number system. Square root of a complex number.
Linear Inequalities	Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical method of finding a solution of system of linear inequalities in two variables.	Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical method of finding a solution of system of linear inequalities in two variables. Linear programming and its applications in solving simple problems.	Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical method of finding a solution of system of linear inequalities in two variables. **Cauchy-Schwartz theorem. AM-GM-HM inequality and its applications in simple inequality problems.** Linear programming and its

			applications in solving simple problems.
Permutations and Combinations	Fundamental principle of counting. Factorial n. (n!) Permutations and combinations, derivation of formulae for ${}^nP_r$ and ${}^nC_r$ and their connections and simple applications.	Fundamental principle of counting. Factorial n. (n!) Permutations and combinations, their connections and simple applications.	Fundamental principle of counting. Factorial n. (n!) Permutations and combinations, derivation of formulae for ${}^nP_r$ and ${}^nC_r$ and their connections and simple applications. **Derangement theorem with applications.**
Binomial Theorem	History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term in binomial expansion, simple applications.	Statement of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term in binomial expansion, simple applications.	History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term in binomial expansion, simple applications. **Maclaurin expansions for positive and negative integral expansions.**
Sequence and Series	Sequence and Series. Arithmetic Progression (A. P.). Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of n terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M. Common special sums.	Sequence and Series. Arithmetic Progression (A. P.). Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of n terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M. Common special sums.	Sequence and Series. Arithmetic Progression (A. P.). Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of n terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), **harmonic progression, harmonic series, harmonic mean, relation between A.M, G.M. and H.M.** Common special sums. **Convergence of series with applicable tests. Telescoping series.**
Straight Lines	Brief recall of two-dimensional geometry from earlier classes. Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point-slope form, slope intercept form, two-point form, intercept form and normal form. General equation of a line. Equation	Brief recall of two-dimensional geometry from earlier classes. Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point-slope form, slope intercept form, two-point form, intercept form and normal form. General equation of a line.	Brief recall of two-dimensional geometry from earlier classes. Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point-slope form, slope intercept form, two-point form, intercept form and normal form. General equation of a line. Equation

	of family of lines passing through the point of intersection of two lines. Distance of a point from a line.	Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line.	of family of lines passing through the point of intersection of two lines. Distance of a point from a line.
Conic Sections	Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.	Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.	Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola **(origin not restricted).** Standard equation of a circle. **Orthocentre, incentre, inradius and circumcircle with their applications.**
Limital Calculus	Derivative introduced as rate of change both as that of distance function and geometrically. Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions.	Derivative introduced as rate of change both as that of distance function and geometrically. Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions.	Derivative introduced as rate of change both as that of distance function and geometrically. Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. **L'Hopital's rule with their applications. Improper limits. Limits of special forms.**
Statistics	Measures of Dispersion: Range, mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances.	Measures of Dispersion: Range, mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances. Mean, median and mode.	Measures of Dispersion: Range, mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances. **Poisson's distribution.** Mean, median and mode.
Probability	Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Axiomatic	Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events,	Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Axiomatic

	(set theoretic) probability, connections with other theories of earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events. Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean and variance of random variable. Repeated independent (Bernoulli) trials and Binomial distribution.	Axiomatic (set theoretic) probability, connections with other theories of earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events. Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean and variance of random variable. Repeated independent (Bernoulli) trials and Binomial distribution.	(set theoretic) probability, connections with other theories of earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events. Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean and variance of random variable. Repeated independent (Bernoulli) trials and Binomial distribution. **Probability mass and density functions with their applications.**
Matrices	Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries)	Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries)	Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries). **Eigenvalues and eigenvectors of matrixes.**
Determinants	Determinant of a square matrix (up to 3 x 3 matrices),	Determinant of a square matrix (up to 3 x 3	Determinant of a square matrix (up to 3 x 3

	properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.	matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.	matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.
Derivatives	Definition of derivative relate it to scope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions. Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.	Definition of derivative relate it to scope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions. Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.	Definition of derivative relate it to scope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions. Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions **and hyperbolics**, derivative of implicit functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.
Application of derivatives	Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima	Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima	Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima

	and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).	and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).	and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).
Integral calculus	Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals and problems based on them. Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.	Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals and problems based on them. Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.	Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals and problems based on them. Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals. **Riemann sums with their applications, improper integrals. Integration of hyperbolic functions.**
Applications of Integrals	Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only), Area between any of the two above said curves.	Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only), Area between any of the two above said curves.	Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only), Area between any of the two above said curves. **Volumetric revolution; finding volume of regions.**
Differential calculus	Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, solutions of	Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, solutions of	Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, solutions of

	homogeneous differential equations of first order and first degree. Solutions of linear differential functions.	homogeneous differential equations of first order and first degree. Solutions of linear differential functions.	homogeneous differential equations of first order and first degree. Solutions of linear differential functions **and 2nd order differential equations of various forms. Substitution as a tool in solving D. E's.**
Vectors	Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors, scalar triple product of vectors.	Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.	Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors, scalar **and vector** triple product.
3-D Geometry	Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula. Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii)two planes, (iii) a line and a plane. Distance of a point from a plane.	Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula. Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii)two planes, (iii) a line and a plane. Distance of a point from a plane.	Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula. Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i)two lines, (ii)two planes, (iii) a line and a plane. Distance of a point from a plane.

Note the changes in the syllabus. Besides the changes already mentioned, there is some reorganisation of concepts between both years. For instance, differentiation has been moved fully to Class 12 (instead of teaching to a half-baked state in Class 11). The changes are designed for a more balanced curriculum load.

3.12 - Anticipated grade cut-offs

*Assuming the same grade policy as currently followed. The A1 grade cutoff is anticipated to be high for Maths-**H** due to the higher average calibre of such students. Note that the table can be inaccurate as I haven't tested the paper with any sample population. Please see page 4 for more information.*

It is worth wondering whether the grading system should be changed for the higher tier. Despite the higher level of the question paper, the A1 and A2 cut-offs are comparable to Maths-**S** due to the fact that the higher bracket of **H** students are stronger than the higher bracket of **S** students on average.

Some universities abroad (e.g.: Oxford, Cambridge) look at the grade of each subject. The problem is that an A2 student in Maths-**H** cannot be considered as inferior to an A1 student in Maths-**S**.

Grade	*Old Maths [2018]*	*Maths-**S***	*Maths-**H***
A1	95	79	78
A2	84	70	69
B1	73	63	60
B2	63	58	51
C1	55	51	44
C2	46	45	38
D1	42	39	30
D2	33	33	22
E	0	0	0

4 – Miscellaneous ideas

In this section, we detail alternative ideas which I had in mind that were later shelved, and ideas for the secondary (Class 9 and 10) curriculum. A brief outline of Physics-**H** and Chemistry-**H** is also included. *I planned to include an outline of Computer Science-**H** and **S** as well, but I've kept it on hold due to CBSE drastically changing the syllabus recently.*

4.1 – A Maths Essential/Maths – **E**?

The IB mandates that every student study Mathematics in Class 11 and 12, and for that, there are 3 tiers (+1 elective) of Mathematics available:

i. Mathematical Studies (a practical-oriented variant of Mathematics meant for those who do not plan to take a Mathematics-based course after Class 12)
ii. Mathematics Standard (same analogy to Maths-**S**)
iii. Mathematics Higher (same analogy to Maths-**H**)
iv. Further Higher Mathematics (separate elective subject; only some schools have it)

Since we're already giving a segregation in the name of Maths-**S** and Maths-**H**, one line of thought says that we should also introduce a third, lower variant (like Mathematics Essential) for those students who would not take up Mathematics any further (and hence make it mandatory).

In my opinion, this is not needed. I believe that Class 10 Mathematics provides a necessary working knowledge for students, and that making Maths mandatory beyond that would cause a whole lot of issues and make things confusing. Hence, I don't think any further segregation is needed.

4.2 – Mechanics, statistics and discrete mathematics?

The Maths curriculum since 2008 has been geared towards the pure and statistical branches of mathematics. And that's fine by me.

I do *not* suggest the inclusion of Mechanics (like what UK's A-Level requires) despite the beautiful applications to Mathematics. Students desirous of studying Mechanics should consider studying Physics-**H** (which will require Maths-**S** at least for enrolment and will be a more mathematically geared version of Physics-**S** or current Physics syllabus) instead. The reason being that students from all streams should be able to take both variants of Mathematics and should not have to study Physics instead. However, this does not apply to Statistics: other than Economics (which has a Statistical component in Class 11), no other subject offers stats as a major component.

I do not suggest discrete mathematics either: Computer Science would be a potentially better option instead.

However, IITD's excellent report on CS reform makes me wonder whether it is better to include a form of discrete mathematics (not Boolean algebra like what the pre-2008 syllabus had however) for students as it could potentially benefit their future career; on the other side, I still lean towards not including discrete mathematics though because there's enough of pure and statistical mathematics. Maybe that's for the appendix, like Mathematical Modelling in Class 12? It should however be noted however that students hardly bother with that section and for most, it is as good as if that section wasn't there.

4.3 – Physics and Chemistry **S** and **H**

Below I'll outline some of the changes that I anticipate in Physics **H** and Chemistry **H** over their **S** variants.

*For Physics **S** compared to (old) Physics*

The need to memorize derivations will be removed. **S** students are more likely to memorize (read: mug up) rather than understand the derivation properly.

*For Physics **H***

The course will be far more mathematical. The base of Physics **H** is mathematics, rather the theoretical applications of Physics **S**. Students will be expected to derive complex formulas and solve problems using calculus and think maths in solving Physics problems. Mathematical concepts like the concept of moments will also be added. *The mathematical content in Mechanics will be comparable to at least the level of Mechanics asked in STEP (Cambridge's maths entrance exam) III.*

Due to the heavy usage of Mathematics, students who enrol in Physics-**H** must also take at least Maths-**S**. Hence students who drop Maths after Class 10 can only take Physics-**S**.

*For Chemistry **S***

The frustration of most chemistry students, the need to memorize obscure chemical equations in inorganic chemistry, will be reduced to the bare minimum required. This alone will reduce the students' workload by a noticeable amount. The focus for inorganic will be trend analysis and conceptual understanding. Mechanisms for organic chemistry will also be removed; most **S** students would simply memorize them.

The concept of normality will be added: it's well used everywhere and a mystery that they are missing in NCERT's textbook.

*For Chemistry **H***

More physical chemistry content (e.g.: 2nd order reactions) and more organic reactions and mechanisms will be added.

Note that unlike Physics-**H**, Chemistry-**H** does not require students to also take Maths-**S** at least.

4.4 – For the secondary stage [Class 9-10]

Unlike the approach taken for Class 11 and 12 (segregation), it is *not* recommended to create a **S** and a **H** variant for Mathematics and other subjects (like how GCSE does with the Foundation and Higher tier). This is because at this stage of education, students need a working knowledge of every subject as they may not study it any further. So, creating a **S** variant may cause future disadvantages to those who take it or for those who discover their love for Mathematics later.

Instead, a separate elective called Mathematics Extended (hereby called Maths-**X**) is proposed. Students can take it alongside the normal Mathematics curriculum, and Maths-**X** includes greater depth of topics like algebra, geometry (e.g.: Ceva's theorem) and statistics. A separate paper will be taken by such students, and the marks will be reported separately like any other additional subject. However, it will be so designed so that those who take the standard Mathematics only can still enrol into Maths-**H** in class 11 without being disadvantaged.

While strongly recommended, schools are *not* required to offer Maths-**X** to students, unlike that in Class 11 and 12.

4.5 Alternative question paper format[2]

This format is up for debate and is currently under consideration.

A potential question paper format (mainly for Paper 2 of the Maths-**H** or 50% of the paper) is to reduce the number of questions to 5-6 *very*-long questions, from which the students are to pick four of them. Each question will be based from a topic and the questions can be expected to test students' in-depth knowledge of the topic.

This overcomes a drawback with the current system, which limits the maximum mark of a question to 8, while also allowing students to choose from various topics.

Note that students may choose to skip topics due to this; however, Paper 1 of Maths-**H** (or the first 50 marks if one paper is taken) will be entirely of the conventical paper format with selective or no internal options. Also note that no representation will be given for questions from a particular topic having harder questions than the other.

An example section (some taken from the *Incubation questions* section) is given below:

Section 2: Maths **H**

Choose <u>two</u> from the three provided. Each question is worth 25 marks.

Suggested time: 45 minutes per question

1. **Calculus**

(a) Define the *derivative* of a function. [1]

Let $y = e^{\sin^{-1} x}$. Find $\frac{d^2y}{dy^2}$. [3]

(b) Given that a and b are distinct positive numbers, find a polynomial $P(x)$ such that the derivative of $f(x) = P(x)e^{-x^2}$ is zero for $x = 0$, $x = \pm a$ and $x = \pm b$, but for no other values of x. [8]

(c) Define the *Fundamental Theorem of Calculus*. [1]

Consider the summation

$$A = \sum_{i=0}^{\infty}\left(\int_{i\pi}^{(i+1)\pi} \frac{\sin x}{x} dx\right)$$

Show that A is positive. [3]

Let a_i be the i^{th} term of the summation A. Prove that a_i converges as $i \to \infty$. [2]

Show that [2]

---

[2] Inspired from my 2nd year CS university question papers: https://1drv.ms/b/s!Auje7TcceXJljPdaBpZbjdOAxQBJrw

$$\left|\int_{i\pi}^{(i+1)\pi} \frac{\sin x}{x} dx\right| \geq \left|\int_{(i+1)\pi}^{(i+2)\pi} \frac{\sin x}{x} dx\right|$$

Now consider the summation

$$B = \sum_{i=0}^{\infty} \left(\int_{i\pi}^{(i+1)\pi} \frac{\cos x}{x} dx \right)$$

Show that $i > 0$. [2]

Let $i = \min\left[x; \frac{\cos x}{x} = 0\right]$. Find i, and predict the sign of B with reasoning. [3]

2. Combinatorics and Probability

(a) Let A and B be two random variables such that their probabilities are complementary. If $P(A) = \frac{1}{P(B)}$, find all such probabilities. [3]

(b) Explain how the *Law of Total Probability* is linked to Bayes' theorem. [2]

(c) Let $y = \sin x + \cos 2x$. Find the probability that for a $x \in [0, \pi]$, y is between ½ and ¾. [5]

(d) Prove that $E(X^2) \geq E(X)^2$, where X is a random variable. [1]

(e) Show that $Var(aX) = a^2\, Var(X)$. [2]

(f) x boys must go to their classes, but they are blindfolded, and hence randomly go to a class.

Derive a formula to find the probability that none of them gets to the correct class. [5]

Let $x \to \infty$. Prove that the probability that at least one boy gets to the correct class tends to [3]

$$\frac{e-1}{e}$$

(g) $n^2 + n$ points are placed in a square of side n units. Show that the minimum distance between two points is $\sqrt{2}$ units. [4]

3. Coordinate and conic systems

(a) Consider a parallelogram with sides a and b with height h. By dividing it into three polygons, show that the area of the parallelogram can be reduced to $ab \sin\theta$, where θ is the angle between a and b. [3]

(b) What is $ab \sin\theta$ equal to: $a.b$, $a \times b$ or neither? [1]

(c) Consider two points on the Cartesian system: (x_1, y_1) and (x_2, y_2). Derive a formula to find the distance between the two points. [3]

(d) On a Cartesian system, consider two points: $(0, b)$ and $(a, 0)$. The two points are joined by a line. What is the slope of the line? [2]

(e) A variable point is placed on the line, and another line is drawn from the origin to that point. Let θ be the angle between that line and the x-axis. Find the co-ordinates of that variable point in terms of θ. [2]

(f) Show that the area of the triangle formed is $\frac{ab}{4}\sin 2\theta$, and hence explain the conditions required for the area to be maximum. [4]

(g) a and b are now variably related using the relation $b = a\sin^2\theta$. Find θ for which the area of the triangle is now maximum. [4]

(h) An eclipse is currently in the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$. It is 'converted' to the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by pulling it at two ends horizontally in the opposite direction.

Find the rate of change of area and show that the largest eclipse is a circle. [6]

5 – The Sample papers

This is probably the most important part: that which sets the points mentioned in earlier sections into action.

For each variant, there are 3 sections:

- The question paper
- Model answers to each question
- Brief analysis on students' ability for each question

Remember: if you find some problem with the papers, please send your feedback. See the first part for the address.

Both papers are for 3 hours and worth 100 marks.

Depending on feedback or otherwise, the design of the paper(s) may change in future. This paper should not be the be-it end-all one; rather it is just what it is – a model paper.

Designing the sample papers

The process goes roughly like this:

Potential questions are created for each level and are added in a 'pool'. Then, questions are included in each sample paper, keeping in mind the weightage, mark allocation and level of each question. Finally, model answers are worked out (which often uncovers intricates or errors on some questions) and then the weightage is mapped to each chapter (to ensure that it all adds up to 100).

The analysis given at the end is done in such a way that it tries to mimic what an observer would note regarding the same – and in very rare cases has a change to any question been effected in any form. However, internal notes when designing the question have been included wherever appropriate.

The H technicality

You may notice that the easier questions on the **H** sample papers are skewed towards the completely new concepts (i.e., the ones like probability density functions and eigenvalues). This is due to the relative newness of the concepts and that most students and teachers are not familiar with the newer concepts (myself included). This technicality is expected to subside over the next few years.

5.1 – Question Paper (Maths-S)

Weightage of the paper is given below.

Portions	Question(s)	Marks involved	Total marks
Sets, functions and graphs	Q2 (a,b) , Q7, Q9	2+3+1	6
Trigonometry	Q13, Q16, Q18	4+4+3	11
Mathematical induction	Q23	5	5
Complex numbers	Q4	5	5
Inequalities	Q25	5	5
Permutations&combinations	Q26	2	2
Binomials	Q22	5	5
Sequences	Q15, Q18	8	8
Coordinate geometry	Q8, Q12	4+4	8
Vectors	Q26	4	4
3D Geometry	Q6	4	4
Limital calculus	Q19	3	3
Statistics	Q10	3	3
Probability	Q3,Q11,Q27	2+3+1	6
Derivatives and applications	Q2 (c, d), Q5	4+4	8
Integral calculus and applications	Q1,Q17	3+4	7
Differential equations	Q23	3	3
Matrices and determinants	Q14,Q20	3+4	7
		TOTAL	100

Easy questions: 10 Marks: 36

(Q 2,3,4,5,6,13,14,20,23,27)

Medium questions: 10 Marks: 31

(Q7,9,11,12,17,18,19,22,25)

Hard questions: 7 Marks: 33

(Q8,10,15,16,21,24,26)

1. A student wants to find $\int 1\, dx$. Instead of writing the integral directly, he writes it in this form instead: 3

$$\int \frac{\sin^2 x}{1-\cos^2 x} dx$$

Use the substitution $t = \cos x$ to prove that the value of the integral is $x + C$. You may not directly cancel the numerator and denominator of the fraction in full.

2. Let $f(x) = x + \frac{1}{x} (x \in R - \{0\})$

(a) Find $f(4)$. 1

(b) Prove that $f\left(\frac{1}{x}\right) = f(x)$. 1

(c) Find the range of $f(x)$. 2

(d) Prove that $f(x)$ is strictly increasing in the given range. 2

3 From the first n natural numbers, find the probability that for a number $x \in (0, n]$, $x^2 + x + 1$ is odd. 2

4. Given the complex number $3 + 5(i + i^2 + i^3 + i^4)$, find its modulus, argument and write it in polar form. 5

5. (a) Let $y = \sin x$. Let y_n $(n \geq 1)$ be the n^{th} derivative of y. Find the least multiple of n for which $y_n = y$. 2

(b) Show that the above is not logically possible if $y = \tan x$. 1

(c) Let $y_n = |y|$. Then, if $y = \sin x$, what is the least multiple of n that satisfies that condition? 1

6. (a) A line has direction ratios (1,2,2) and is passing through the point (3,4,5). Write the general equation of the line in vector and cartesian forms. 2

(b) Consider another line $r = (1+\lambda)x + \lambda y + (2+3\lambda)z$. Will both lines ever intersect? 2

7. Sketch the following graphs on the same plane: 3

(a) $x = \ln y$

(b) $y = \sin(e^x)$

(c) $y = 3x^2 + 6x + 1$

8. The lines $3x + 4y = 8$ and $2x + y = 5$ intersect at a point. Find the area of the smallest figure intersected between the lines and at least one of the axes. 4

9. Decibels (dB) is a unit of intensity of sound. A normal conversation is about 50dB, shouting is about 68 dB, firecrackers can cause up to 85 dB. A loud jet engine is about 120 dB, and an extremely quiet environment is about 20dB. A 6dB increase is about 25 times louder. 1

A graph is plotted between intensity of sound and dB. Will the graph be linear, logarithmic or exponential?

10. Find the mean if $3a + 2b$, $4a + 3b$ and $k(a+1) - m(b+1)$ is independent of a and b. *3*

11. A coin is set up in such a way that the probability of getting a tail is $\frac{2}{5}$ of the probability of getting heads. If the coin is tossed twice, find the probability of getting one head and one tail. *3*

12. Consider the circle whose centre lies at $(1,2)$ and its circumference being 12π cm.

   (a) Deduce the equation of the circle. *2*
   (b) How many points of the form $(k, k); k \in Z$ will lie inside the circle? *2*

13. Two sides of a triangles are 5 cm and 6 cm.
   (a) Find the range of the 3rd side. *1*
   (b) The angle opposite to the first side is 30 degrees. Find the other two angles, if the third side is 7 cm. *3*

14. Find *3*

$$\begin{vmatrix} x^2 & 3x & 4 \\ 1 & x^2 & 5x \\ \tan\alpha & 0 & x^{-2} \end{vmatrix}$$

15. Define a *name* as a series of letter(s) used to identify a person. Each name consists of a certain number of letters which may (not) repeat. Assume that names need not be in the dictionary: a name like ENDJ is acceptable, but only letters; a name like @COW is not acceptable.

For every person in the world to have a unique name, the minimum number of letters required is n.

(a) Prove that *5*

$$n = \left(\frac{1}{\ln 26}\right)\left(\ln\frac{25}{26} + \ln\left(7.6 * 10^9 + \frac{26}{25}\right)\right)$$

The number of people in the world may be taken as 7.6 billion.

(b) Estimate a value of n given that $\ln 2 = 0.69, \ln 3 = 1.1$. *2*

(c) Hence give a justification to the claim that "one should not be surprised if he/she sees someone with the same initial or even the full name". *1*

16. The hands of a clock which ticks minutes coincide at the h^{th} hour and the m^{th} minute. Find $\frac{h}{m}$. *4*

17. Find (where $x \neq -\frac{a}{b}$): *4*

$$\int \frac{ax + b}{a + bx} dx$$

18. Find the particular solution in the acute angle range of $\tan 2x - \tan x = \sec x$. *3*

19. Find: 3

$$\lim_{x \to \pm 1} \frac{1 \pm \frac{1}{x}}{x - \frac{1}{x}}$$

20. Find the inverse of A, if 4

$$A = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$$

21. (a) If $k^n = n_{C_0} + n_{C_1} + n_{C_2} + \cdots + n_{C_n}$, find k. 2

(b) Simplify the below summation to 1 or 2 term(s): 3

$$\frac{1}{2^n} \sum_{r=1}^{n} [3^{r-1}] [n_{C_r}]$$

22. Use mathematical induction to prove that $x^n - y^n$ is divisible by $x - y$ when n is a natural number. Hence find the remainder when $x^n - 1$ is divided by $x - 1$. 4 1

23. Find the general solution of 3

$$\frac{dy}{dx} - y \tan x = \sin x$$

24. (i) Find the range(s) of values of $x \in R$ for which 3

$$\frac{x+1}{x+2} < \frac{x+3}{x+4}$$

(ii) Prove that the range(s) will remain the same if the inequality is slacked ($\leq$) instead of strict ($<$). 2

25. A small railway line has 15 stations. All passengers in that line start at one station, stop (and later come back) at another station, and finally stop at a third station. How many different combinations of tickets can be printed at most? 2

26. An altitude and a median are dropped from the origin to the line joining the line whose position vectors are $3i + j + k$ and $i - j - 2k$. Find the angle between the altitude and median, if the altitude is $(q, 1, -1)$ where q is a constant. 4

27. State Bayes' theorem. 1

5.2 – Model answers for Maths-**S**

Note that these are truly just model answers, there are and should be alternative answers which are not depicted here.

The answers begin on the next page.

1. (Medium) Given the equation as

$$\int \frac{\sin^2 x}{1-\cos^2 x} dx$$

Take $\cos x = t \to -\sin x\, dx = dt$. Then

$$I = -\int \frac{\sin x}{1-t^2} \to I = -\int \frac{\sqrt{1-t^2}}{1-t^2} dt \to I = -\int \frac{1}{\sqrt{1-t^2}} dt$$

$$I = -[\sin^{-1} t] \to I = -\left(\frac{\pi}{2} - x\right) \to I = x + C$$

2. (Easy)

(a) $f(4) = 4 + \frac{1}{4} = \frac{17}{4}$

(b) $f\left(\frac{1}{x}\right) = \frac{1}{x} + \frac{1}{\frac{1}{x}} = x + \frac{1}{x} = f(x)$

(c) By introspection or from part (d), the function is strictly increasing.
As $f\left(\frac{1}{x}\right) = f(x)$, the minimum value occurs when $x = 1$, and $f(1) = 2$.
Additionally, $f(-x) = -f(x)$. Hence the range is $[-\infty, -2] \cup [2, \infty]$

(d) $f'(x) = 1 - \frac{1}{x^2}$
$f'(x) > 0$ for real x. Hence function is strictly increasing.

3. (Easy)

$$x^2 + x + 1 = x(x+1) + 1$$

As the product of two consecutive numbers is even, $x(x+1) + 1$ is odd.
Hence the probability is 1.

4. (Easy) Given $3 + 5(i + i^2 + i^3 + i^4)$
We have

$$3 + 5(i - 1 - i + 1) \to 3$$

Hence modulus is 3
Amplitude: $\frac{y}{x}$, which is 0 as imaginary part is 0
The polar form is $r(\cos\theta + i\sin\theta)$.
But as $r\cos\theta = 3$ and $\theta = 0$ as imaginary part is 0, the polar form is $3(\cos 0 + i\sin 0)$

5. (Easy)

(a) Given that $y = \sin x$, $y' = \cos x$, $y'' = -\sin x$. Hence $y_4 = \sin x$, which is the smallest value. Hence all integers which are multiples of 4 will have $y = y_n$.

(b) $y' = \sec^2 x$ when $y = \tan x$.
$y'' = 2\sec x\,(\sec x \tan x)$, and y''' will be an even longer term. Hence it is not logically possible when $y = \tan x$.

(c) As seen from part (a), $y'' = -\sin x$, hence $y'' = |y|$. So $n = 2$.

6. (Easy)

Vector form is $r = a + \lambda(b - a)$, so for question, taking $a = i + 2j + 2k$ and $b = 3i + 4j + 5k$, then $r_1 = i + 2j + 2k + \mu(2i + 2j + 3k)$.

Similarly, the cartesian form of the equation is

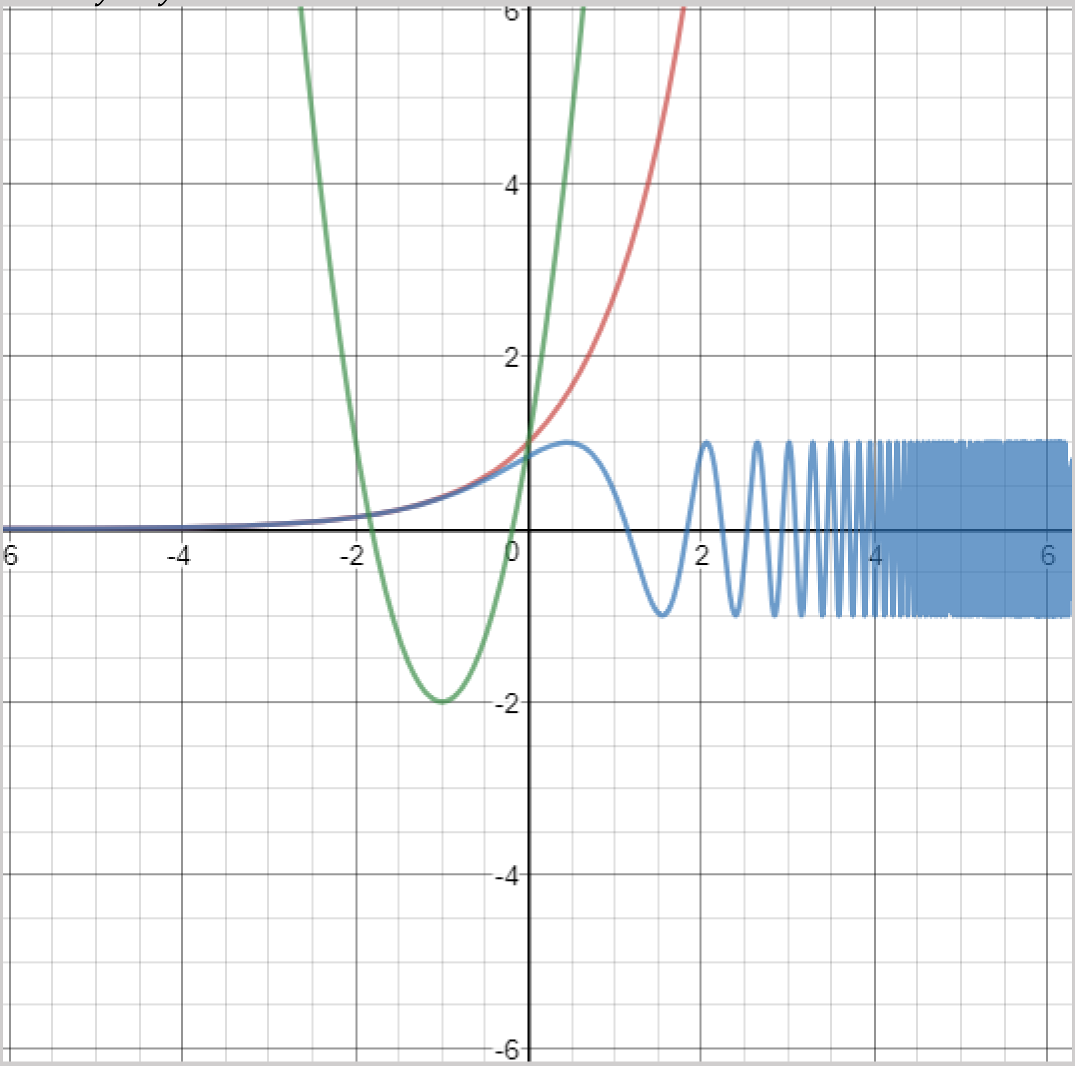
$$\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-2}{3}$$

For intersection of two lines, it is necessary that μ and λ obtained be valid for every equation. As

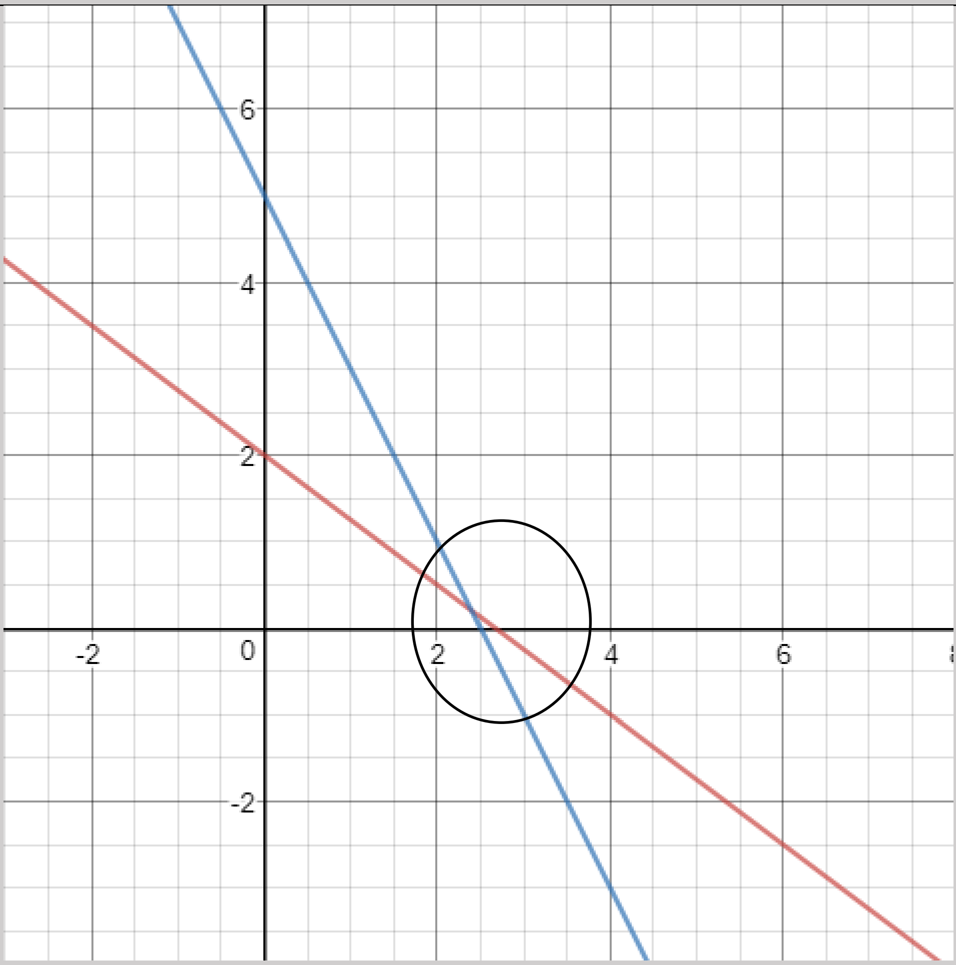
$$r_2 = (1+\lambda)x + \lambda y + (2+3\lambda)z$$

$1+2\lambda = 1+\mu \to 2\lambda = \mu$, but $2+2\mu = \lambda$. Then $2+4\lambda = \lambda \to \lambda = -\frac{2}{3}$. $\mu = 2\lambda = -\frac{4}{3}$
Verifying with the z axis,
$2+3\mu = 2+3\lambda \to \mu = \lambda$, a contradiction. Hence the lines will never intersect.

7. (Medium)
$x = \ln y \to y = e^x$.



8. (Hard): Drawing the figure,



There is a very small triangle (circled) intersecting the lines and the x-axis. The coordinates are $\left(\frac{12}{5},\frac{1}{5}\right),\left(\frac{8}{3},0\right)$ and $\left(\frac{5}{2},0\right)$. Applying triangle formula,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \rightarrow \Delta = \frac{1}{2}\begin{vmatrix} \frac{12}{5} & \frac{1}{5} & 1 \\ \frac{8}{3} & 0 & 1 \\ \frac{5}{2} & 0 & 1 \end{vmatrix}$$

Evaluating the determinant,

$$\Delta = \frac{-1}{5}\left(\frac{8}{3}-\frac{5}{2}\right) \rightarrow \Delta = \left|\frac{1}{5}\left(\frac{5}{2}-\frac{8}{3}\right)\right| \rightarrow \Delta = \frac{1}{30}\text{units}$$

9. (Medium) The graph will be logarithmic.
10. (Hard) Given that $3a+2b$, $4a+3b$ and $k(a+1)-m(b+1)$ is independent of a and b, The mean will be

$$\frac{3a+2b+4a+3b+ka+k-bm-m}{3} \rightarrow \frac{a(7+k)+b(5-m)+(k-m)}{3}$$

This means that $7+k=0$ and $5-m=0$; $k=-7$ and $m=5$. With that in place, the coefficients of a and b will be 0. Hence the mean is $\frac{-7+5}{3}=-\frac{2}{3}$.

11. (Medium)

We have $P(T)=\frac{2}{5}P(H)$. Also, $P(H)+P(T)=1$.

Hence $\frac{7}{5}P(H)=1 \rightarrow P(H)=\frac{5}{7}, P(T)=\frac{2}{7}$

Therefore, P (getting one head or tail)

$$\frac{5}{7} * \frac{2}{7} + \frac{5}{7} * \frac{2}{7} = \frac{20}{49}$$

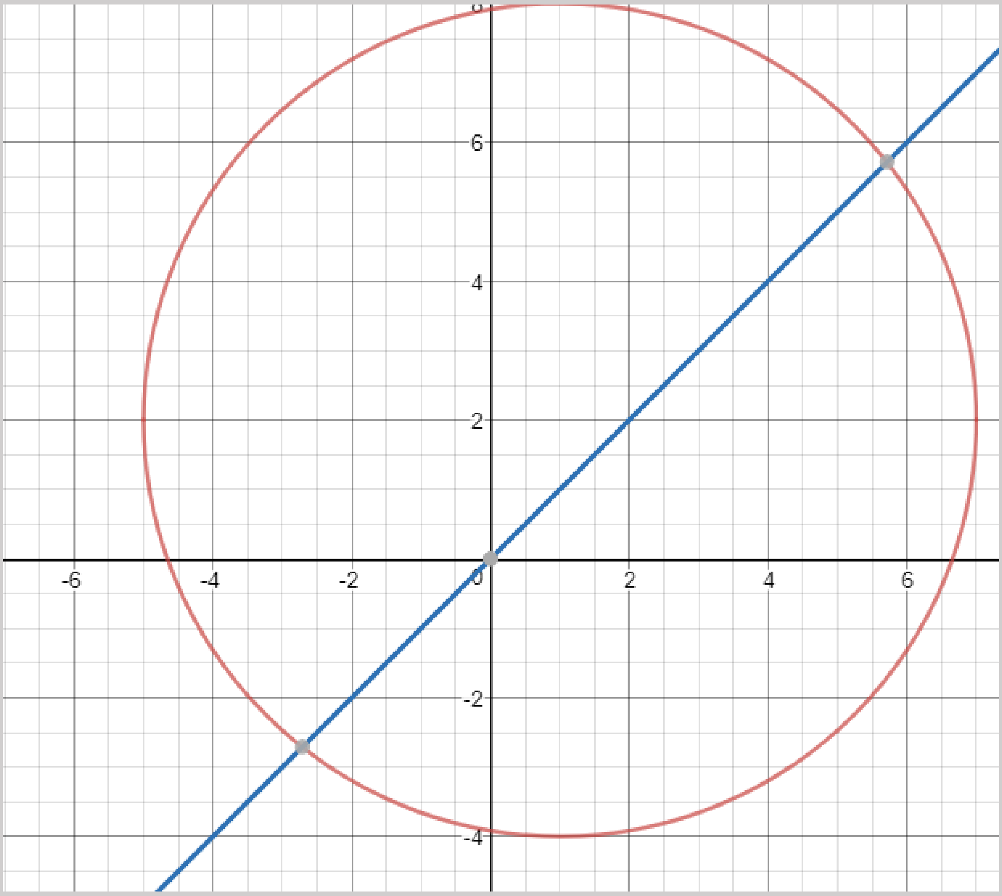
12. (Medium)

$$C = 2\pi r \rightarrow 12\pi = 2\pi r \rightarrow r = 6\ cm$$

Hence the equation of the circle is

$$(x-1)^2 + (y-2)^2 = 36$$

Now graphing the circle and the line $x = y$ on the $x - y$ plane,



The line intersects the circle at $x = -3\&5$. Hence the number of points is 7. (-2, -1,0,1,2,3,4)

13. (Easy) By triangle inequality law, the third side is at least 1 and less than 11. Hence the range is 11-1 = 10.

The angle opposite to the first side is 30 degrees. By sine rule,

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{B} = \frac{\sin \angle C}{C}$$

Hence,

$$\frac{1}{2 * 5} = \frac{\sin \angle B}{6} = \frac{\sin \angle C}{7}$$

$$\sin \angle B = \frac{3}{5} \rightarrow \angle B = \sin^{-1} \frac{3}{5}$$

$$\sin \angle C = \frac{7}{10} \rightarrow \angle C = \sin^{-1} \frac{7}{10}$$

14. (Easy) Given

$$\begin{vmatrix} x^2 & 3x & 4 \\ 1 & x^2 & 5x \\ \tan\alpha & 0 & x^{-2} \end{vmatrix}$$

Expanding along first row,

$$x^2(1-0) - 3x\left(\frac{1}{x^2} - 5x\tan\alpha\right) + 4(-x^2\tan\alpha) \to x^2 - \frac{3}{x} + 15x^2\tan\alpha - 4x^2\tan\alpha$$

$$\Rightarrow x^2 - \frac{3}{x} + 11x^2\tan\alpha$$

15. (Hard)

There are 26 ways for a one-letter name, 26*26 ways to represent a 2-letter name and so on. Hence the total number of names possible with n letters max is $26 + 26^2 + 26^3 + \cdots + 26^n$. Setting up as a G.P,

$$\frac{26(26^n - 1)}{25} = 7.6 * 10^9$$

$$(26^n - 1) = \frac{25}{26}(7.6 * 10^9)$$

$$26^n = \frac{25}{26}(7.6 * 10^9) + 1$$

$$n\ln 26 = \ln\frac{25}{26} + \ln\left(7.6 * 10^9 + \frac{26}{25}\right)$$

$$n = \left(\frac{1}{\ln 26}\right)\left(\ln\frac{25}{26} + \ln\left(7.6 * 10^9 + \frac{26}{25}\right)\right)$$

Solving for n, we make assumptions that $\ln 26 \sim \ln 27 \sim 3\ln 3$, $\frac{25}{26} \sim 1$, $\ln 7.6 \sim \ln 8 \sim 3\ln 2$.

Then

$$n = \frac{22.77}{3.3} \cong 6.9$$

The actual value of $n = 6.971$ [not required].

The result means that everyone will need to be allocated all the possible names up to 7 letters for everyone to have unique names. Considering that only a miniscule fraction of those names even make sense, it is clear that the number of possible names is much lesser. As most names' letter count is less than 7 (or even otherwise), one is sure to find people with identical first or even full names.

16. (Hard)

The hands of the minute clock move 6 degrees for every minute.
Hence the total angle by the minute hand is $6m$.

The hour hand moves $\frac{30}{60} = \frac{1}{2}$ degrees as the minute clock moves every minute.
Additionally, the hour hand moves by 30 degrees for every hour that moves past.
Hence the total angle by the hour hand is $\left(30h + \frac{m}{2}\right)^\circ$
As both hands coincide,

$$6m = 30h + \frac{m}{2}$$

$$\frac{11m}{2} = 30h$$

$$11m = 60h$$

$$\frac{h}{m} = \frac{11}{60}$$

17. (Medium) Given

$$\int \frac{ax+b}{a+bx}dx$$

Splitting the integral, let

$$I_1 = \int \frac{ax}{a+bx}dx, I_2 = \int \frac{b}{a+bx}dx$$

Taking $a + bx = t$ in I_2, $b\,dx = dt \rightarrow dx = \frac{dt}{b}$. Then $I_2 = \ln|a + bx| + C_2$

Now considering I_1,

$$I_1 = \int \frac{ax}{a+bx}dx$$

Integrating by parts,

$$I_1 = \frac{ax}{b}\ln|a+bx| + \int \frac{a}{b}\ln|a+bx|\,dx$$
$$I_1 = \frac{ax}{b}\ln|a+bx| + \frac{a}{b^2}(a+bx)(\ln|a+bx|-1)$$

18. (Medium)

Given that $\tan 2x - \tan x = \sec x$,

We have

$$\frac{\sin 2x}{\cos 2x} - \frac{\sin x}{\cos x} = \frac{1}{\cos x}$$
$$\frac{2\sin x\cos x}{\cos^2 x - \sin^2 x} = \frac{1+\sin x}{\cos x}$$
$$2\sin x\,(1-\sin^2 x) = (1+\sin x)(1-2\sin^2 x)$$
$$2\sin x - 2\sin^3 x = 1 - 2\sin^2 x + \sin x - 2\sin^3 x$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$\sin x = \frac{-1 \pm \sqrt{1+8}}{4} \rightarrow \frac{1}{2}, -1$$

As we need the angle in acute angle range, $x = \frac{\pi}{6}$.

19. (Medium)

Given that

$$\lim_{x\to\pm1} \frac{1 \pm \frac{1}{x}}{x - \frac{1}{x}}$$

Then

$$\lim_{x\to\pm1} \frac{\frac{x\pm1}{x}}{\frac{x^2-1}{x}}$$

$$\lim_{x\to\pm-1} \frac{x\pm1}{x^2-1}$$

$$\lim_{x\to\pm-1} \frac{1}{x\mp1} = \infty$$

20. (Easy) Direct formula application.

21. (Hard)

(a) We know that

$$(a+b)^n = nC_0 a^n + nC_1 a^{(n-1)} b + \cdots$$

Comparing with what is given in the question, $a = b = 1$. Hence $k = 2$.

(b) We have

$$\frac{1}{2^n}\sum_{r=1}^{n}[3^{r-1}]\left[n_{C_r}\right]$$
$$\frac{1}{2^n}\left(n_{C_0} + 3n_{C_1} + 9n_{C_2} + \cdots\right)$$
$$\frac{n_{C_0}}{2^n} + \frac{3n_{C_1}}{2^n} + \frac{9n_{C_2}}{2^n} + \cdots$$
$$n_{C_0}\left(\frac{1}{2^n}\right) + n_{C_1}\left(\frac{1}{2^{n-1}}\right)(3) + n_{C_2}\left(\frac{1}{2^{n-2}}\right)(3^2) + \cdots$$
$$\Rightarrow \left(\frac{1}{2} + \frac{3}{2}\right)^n = 2^n$$

22. (Medium)

Take $n = 1$.

$x - y$ divides $x - y$, so the statement is true when $n = 1$.

We make the hypothesis that the statement is true when $n = k \to x^k - y^k = m(x - y)$, where m is a constant.

Proving it for $k + 1$:

$$x^{k+1} - y^{k+1} = x^k.x - y^k.y$$
$$x\left(m(x-y) + y^k\right) - y^k.y$$
$$mx^2 - mxy + xy^k - y^k.y$$
$$mx(x-y) + y^k(x-y)$$
$$(x-y)\left(mx + y^k\right)$$

Hence the statement is true for $n = k + 1$.

Hence the hypothesis is true for all natural numbers. (and remainder is 0)

23. (Easy)

We have

$$\frac{dy}{dx} - y\tan x = \sin x$$

It is a linear differential function, and its integrating factor is $e^{\int -\tan x dx} = \cos x$

Hence the solution is

$$y\sin x = \int \sin x * \cos x\, dx$$
$$y\sin x = \int \frac{1}{2}(\sin 2x)dx$$
$$y\sin x = -\frac{1}{4}\cos 2x + C$$

24. (Hard)

We have

$$\frac{x+1}{x+2} < \frac{x+3}{x+4}$$
$$1 - \frac{1}{x+2} < 1 - \frac{1}{x+4}$$
$$\frac{1}{x+2} > \frac{1}{x+4}$$

When $x > 2$ or $x < -4$, signs are same. Then

$(x+2) < (x+4) \to 2 < 4$, which is true.

When $-3 < x < -2$, sign of the LHS is negative. Then we have

$\frac{1}{x+2} < \frac{1}{x+4} \rightarrow (x+2) > (x+4) \rightarrow 2 > 4$, which is false.

Hence the ranges are $(-\infty, -4)$ and $(2,\infty)$.

If the inequality is slacked rather than strict, then the equality must be maintained, which as shown in part A, is not possible as there are no variables in the final expression (only constants). Hence there will be no change in the ranges.

25. (Medium)

    There are 15 ways to choose the first station, 14 ways to choose the second station and 13 ways to choose the third station.
    Hence the number of ways is $15 * 14 * 13 = 2730$.

26. (Hard)

    The midpoint between the points is $\left(2,0,-\frac{1}{2}\right)$.
    This means that the line of the median is $2i - \frac{k}{2}$.
    To find the altitude, we need to realise that the dot product of the line joining the altitude and the other line is 0.
    Hence, we have
    $(xi + yj + zk)(-2i - 2j - 3k) = 0 \rightarrow 2x + 2y + 3z = 0$
    But $y = 1, z = -1$. Hence $x = \frac{1}{2}$.
    So, the equation of the altitude is $\frac{1}{2}i + j - k$.
    Hence the angle between the altitude and median is

$$\cos\theta = \frac{b_1.b_2}{|b_1||b_2|}$$
$$\rightarrow \cos\ \theta = \frac{1+\frac{1}{2}}{\sqrt{\frac{17}{4}}\sqrt{\frac{9}{4}}}$$
$$\cos\ \theta = \frac{2}{\sqrt{17}}$$
$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{17}}\right)$$

27. (Easy)
    (Straightforward definition of Bayes’ theorem)

5.3 - Analysis of Maths-**S**

These are just (briefly) what I felt on the paper regarding its difficulty and accessibility; your views may vary.

1. This question appears to be a simple variation of a really simple integral. In practice, while I'd expect most to catch the idea and solve the integral, a few weaker ones may ignore the condition and simply cancel both sides.
2. A very simple question meant to allow students to have a good start. Part (a) and (b) can be essentially done on autopilot; even (c) and (d) should pose no problem for almost any student.
3. A simple problem, once students realise the idea behind the equation. However, those who don't will face trouble with this one. All in all, not a hard one.
4. Very easy. The question decomposes to a purely real number; and the remaining parts can be rapidly solved with little fuss.
5. While a bit unorthodox, a little bit of thinking on the students' part will make them realise that there is some recursive relation between the derivatives of y. Students who complete part (a) should be able to easily finish off the rest of the question.
6. A question dealing with elementary bookwork; most students should finish off the problem easily.
7. Due to the relative newness of the topic, students may face some trouble in doing this question. Part (a) can be easily solved. Part (b) is tricky – it combines graphs of two functions. Part (c) is a bit easier – but mistakes can easily occur.
8. While labelled as "Hard", once it becomes clear as to which is the smallest area, the rest of the question can be easily finished off.
9. A simple conceptual question. You either get it, don't get it or guess (obviously!).
10. A somewhat tricky problem, the wording of the question, while accurate, would be confusing for many **S** candidates. It does not strike immediately that k and m play a big role in this question – for many, they'll seem like arbitrary constants or variables. Only those who clear that hurdle can complete the question – as it stands, partial marking is not expected to be common. It's either (nearly) full, 0 or at best 1.
11. This one has traps for the unwary. It is expected that many **S** candidates will accidentally consider $P(H) = 1 - \frac{2}{5} = \frac{3}{5}$. Once this hurdle is cleared, obtaining the final answer isn't hard though.
12. The first part is plain bookwork. The 2^{nd} part is also fairly easy, but students may include the points on the circumference as points *inside* the circle and hence get that wrong.
13. This question is fairly easy; virtually any student who knows the sine rule should polish off this question.
14. Very easy; the only thing to note that no tricks or properties are required, it's plain expansion.
15. Understanding the question, while a bit long, should not be out of the reach for most students. But while quite a few students should be able to set up the G.P, obtaining the given value of n is tricky and only a few **S** candidates would be expected to reach all the way to the end of part (a). Part (b) is also challenging due to the fact that candidates have to make logical estimates (e.g., $\ln 25 \sim \ln 26$) and the potential for tedious calculations. However, part (c) is easy, and even students who could not make part (a) should get this one. All in all, a tough question.

    It is worth noting that the original version V1.0 comprised only part (a). However, this question explains an important real-life problem: why it is so hard to choose a unique name. It alerts candidates to the fact that there are just so few sensibly unique names that one is virtually guaranteed to find someone with the same initial (or full) name as his/hers. Hence part (b) was added to coax the **S** candidates to find n and hopefully gain the deeper value behind this question.

This question also ran into the 8-mark limit. A little more depth to this question was planned (regarding say the graph between the top n names and the percentage of people having them) but the question as it is lengthy enough with 8 marks.

16. An interesting but challenging question, a (quick) wrong answer expected is $\frac{1}{5}$, which happens when students do not realise that the hour hand also moves. Once that hurdle is cleared, while it does take a bit of understanding to set up the expression, it's pretty easy after that. All in all, not an easy one due to its tricky nature and tempting answer of $\frac{1}{5}$.

    This question is one which does not require a lot of background knowledge (although this question is based from the Trigonometry chapter). One could easily ask this in a Class 9 advanced exam.
17. While potentially a bit lengthy, candidates who have the courage to split and apply integration by parts should finish off the question without much of a fuss. Overall, any student who has practiced integration well should get this correct.
18. While the weaker students would not be expected to make a start on this one, those who make the effort to simplify should easily realise that a quadratic comes out. From there, the answer can be easily obtained.
19. While the presence of the $\pm$ would confuse some candidates, once the equation is simplified, the answer comes out pretty easily. However, the final answer of ∞ may be unexpected, but any good candidate should be able to get this one. Some average **S** candidates may split the limits; that is also fine.
20. A purely bookwork question. Students who remember the algorithm for finding the answer should easily get this one. However, there is the potential for making mistakes.
21. While good **S** candidates should be able to catch the idea in the first part, the 2nd part is hard and separates the wheat from the chaff. Only the best **S** candidates should be expected to get this – the expression is a bit obfuscated, and even if one decomposes it, realising that both parts have the same answer would elude most **S** candidates, even among those who otherwise do well.
22. While weaker students may not understand the question or get muddled up in the algebra, students who have practiced similar questions should not find the question difficult. The 2nd part is essentially a free mark.
23. Fairly easy. Most students would be expected to get this without much difficulty, and the final answer isn't muddled up either.
24. While I framed the question thinking it to be a medium-level question, the nature of the question provides potential for students to fall into many common traps (e.g., simply cross-multiplying the equation). Only those with very strong conceptual knowledge here should be able to complete the first part, and only for such students would the 2nd part be easy. Overall, the question is challenging with many students expected to score poorly in this one.
25. Students with decent combinatorics skills should not find this hard – but a few weaker students may muddle this up instead.
26. While not 'hard' per se, the question is fairly long, and while average students could get a start on this, only students in the top 10% or so would be expected to complete this fully.
27. Bookwork and virtually a free mark.

5.4 - Question Paper (Maths-**H**)

Portions	Question(s)	Marks involved	Total
Sets, functions and graphs	4,11	5+4	9
Trigonometry	5,13	4+4	8
Mathematical induction			
Complex numbers	7	4	4
Inequalities	20	8	8
Permutations&combinations	19	5	5
Binomials	22	5	5
Sequences	9,10	3+4	7
Coordinate geometry	17	6	6
Vectors	3	2	2
3D Geometry	14	4	4
Limital calculus	12	2	2
Statistics	18	4	4
Probability	1,8	2+6	8
Derivatives and applications	15	5	5
Integral calculus and applications	6	5	5
Differential equations	2,16	4+7	11
Matrices and determinants	21	7	7
		Total	100

1 A probability density function is $f(x) = 2x^4 + 3x^2$, where $x \in [0,1]$. Find $\frac{1}{2} < P(x) < \frac{3}{4}$. *2*

2 Solve the differential equation *4*

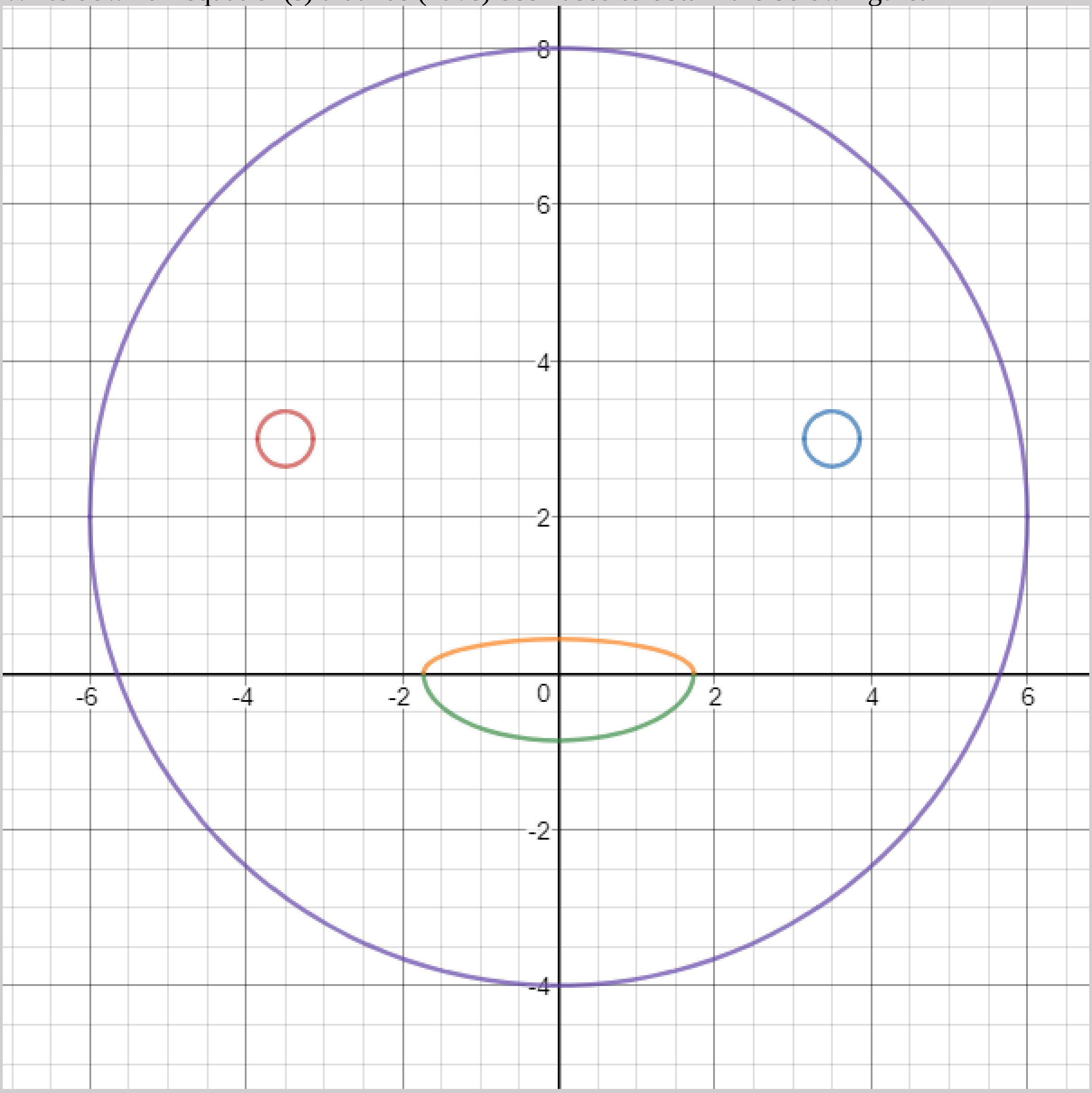
$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 2y = 4 + e^{-2x}$$

Given that

$$y(0) = 4 \;\&\; y'(0) = -1$$

3 If the sum of three vectors is 0, prove that the sum of their dot product taken two at a time is equal to half of the negative of the magnitude of the vectors' squares. *2*

4 Write down all equation(s) that has (have) been used to obtain the below figure: *5*



5 Find all values of (where $a, b \neq 0$) 4

$$\sin^{-1}\left(\frac{a}{b}\right) + \sin^{-1}\left(\frac{b}{a}\right)$$

Using the answer from the first part or otherwise, find

$$\int_{\frac{2}{\pi}}^{\frac{\pi}{2}} \sin^{-1}\left(\frac{a}{x}\right) + \cos^{-1}\left(\frac{x}{a}\right) dx$$

6 Define 5

$$\int_n f(x)\, dx$$

As $f(x)$ integrated n times. ($n \geq 1$)

Now let $f(x) = x^k$. Find

$$\int_4 f(x)\, dx$$

Prove that (where C is the constant of integration)

$$\int_n x^n + x^k\, dx = \frac{n!}{(2n)!}x^{2n} + \left(\prod_{r=1}^{n} \frac{1}{k+r}\right)x^{k+n} + C$$

7 If ω (cube root of unity) is also real, prove that 4

$$1 + \frac{\omega}{1-\omega} = 0$$

Using the Taylor expansion for $1 + x + x^2 + \cdots$, prove that the statement above is a contradiction and not actually correct.

8 There are n sweets in a bag, with p of them being red and the rest being yellow. 6

(a) If p sweets are eaten, prove that the probability that all of them are red is

$$\frac{p!}{\prod_{i=0}^{p-1}(n-i)}$$

(b) 2 sweets are eaten. Derive a quadratic purely in terms of n and p if both sweets are red and the probability is $\frac{1}{n}$. Hence or otherwise, find p in terms of n.

9 2 friends X and Y have a bottle of orange juice of volume m liters. X first drinks half of it and gives it to Y, who in turns drinks half of what is left, and in turn gives it to X, who again drinks half of it and gives it to Y. This goes on till the bottle is virtually empty. Who drank more and by how much? 3

10 Show that 4

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \cong 2x - 4x^3 + 6x^5 + \cdots$$

Hence or otherwise deduce that

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} \cong \sum_{n=0}^{\infty}(-1)^{n+2} * 2(n+1) * (x^{n+1})$$

11 *4*

If $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$, prove that

$$2=\frac{xy}{z^2}-\frac{x}{y}-\frac{y}{x}$$

Elucidate the change(s) in the domain and range, if any. *2*

12

Find k:

$$\lim_{n\to\infty}\frac{\ln n}{\ln(n+1)}=k$$

13 *4*

If $\sin x+\cos x=\tan x$, derive a quartic purely in terms of $\tan x$.

OR

Prove that …

$$\sum_{x=1}^{n}\sin x = 2^k\sum_{x=1}^{n}\left\{\sin\frac{x}{2^k}\prod_{i=1}^{k}\cos\frac{x}{2^i}\right\}$$

… where k is an arbitrary variable which is an integer that is at least 1.

14 *4*

A plane is formed by passing through the points $(1,2,3)$, $(2,3,1)$ and $(3,1,2)$. That plane intersects another plane $x+y+z=6$ at $(2,1,7)$. Deduce the equation of the plane containing both planes.

15 *5*

You are given a function $f(x)$ and are told that

$$f(x)+f'(x)=x^3+3x+2$$
$$f(x)-f'(x)=2x-1$$

(a) Show that this is not possible.
(b) Assume that $f(x)$ is correct. Find $f''(x)$.

16 *7*

Dan deposits some money with the bank which offers an interest of $r\%$ compounded yearly.

(a) Derive an expression using calculus for the amount in his account after n years, given that he initially deposits \$P.
(b) An alternative way of calculating (a) is given in the Class 8 textbook as follows:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

Where A = amount after r years, P = principal amount (initial starting amount), n is number of years, r is rate in %.

Prove that

(i) The formula is only an approximation, and hence less accurate than the expression obtained in (a) and,
(ii) Despite the reduced accuracy, the formula can be used in the real world with almost no issues.

17 *6*

Given the ellipses

$$(x+2)^2 + 2y^2 = 18$$

And

$$9(x-1)^2 + 16y^2 = 25$$

If the family of circles that passes through the intersection of the ellipses can be written as

$$x^2 - Kax + y^2 = B - Ca$$

Find $K + B + C$.

18 *4*

If each number in a series of numbers is multiplied by k times, find the resultant change in the mean, standard deviation and variance of the series of numbers.

19 *5*

Find the day of 14th July 1945. Explain your reasoning carefully.

20 *8*

Consider the expression $(x - y + 2)(x + y - 1)$.

(i) Simplify the above expression.
(ii) Find all values of $(x, y); x, y \geq 0$ such that
$$x^2 - y^2 + x - 3y - 2 = 0$$
(iii) Sketch the region in the $x - y$ plane where
$$x^2 - y^2 + x - 3y > 2$$

21 *7*

Consider the matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(i) By a single elementary row transformation, show that one inverse of M is $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.
(ii) By the equivalent elementary column transformation, find another inverse of M.
(iii) Deduce the condition for which the inverse of M does not exist.

22 *5*

If $\left(x^a + \frac{1}{x^b}\right)^c$ has at least one term whose coefficient is independent of x,

(i) Prove that $\frac{ac}{a+b}$ is a whole number.

(ii) Prove that

$$\frac{1}{a}+\frac{1}{b}=\frac{c}{rb}$$

Where r is the $(r+1)^{th}$ term whose coefficient(s) is (are) independent of x.

5.5 - Model answers for Maths-**H**

*The same disclaimer applied as Maths-**S** does; there are alternative answers which could also be fully correct.*

1. (Easy) Given that $f(x) = 2x^4 + 3x^2$,

$$\frac{1}{2} < P(x) < \frac{3}{4} = \int_{\frac{1}{2}}^{\frac{3}{4}} 2x^4 + 3x^2\, dx$$

$$= \left[\frac{2}{5}x^5 + x^3\right]_{\frac{1}{2}}^{\frac{3}{4}} = \left[\frac{2}{5}\left(\frac{3}{4}\right)^5 + \left(\frac{3}{4}\right)^3 - \frac{2}{5}\left(\frac{1}{2}\right)^5 - \left(\frac{1}{2}\right)^5\right]$$

$$\rightarrow \frac{2}{5} * \frac{243}{1024} + \frac{27}{64} - \frac{1}{80} - \frac{1}{32} \rightarrow \frac{466}{5120} + \frac{27}{64} - \frac{224}{5120} = \frac{242}{5120} + \frac{2160}{5120} = \frac{2402}{5120}$$

2. (Easy) For first one:

   Take $y = e^{mx}$ for LHS. Then

$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 2y \rightarrow (m+1)^2 + 1 = 0 \rightarrow m + 1 = \pm i \rightarrow m = -1 \pm i$$

   Hence the general form of the equation becomes

$$y = e^{-x}(P\cos x + Q\sin x)$$

   Now, the particular integral has to be found. To do so, we consider RHS: $4 + 2e^{-2x}$. The substitution then becomes $A + Be^{-2x}$. Then $y' = -2Be^{-2x}$ and $y'' = 4Be^{-2x}$. Then the equation reduces to $4B - 4B + 2(A + Be^{-2x}) = 4 + 2e^{-2x} \rightarrow 2A + 2Be^{-2x} = 4 + 2e^{-2x}$. Hence $A = 2, B = 1$. The equation is now

$$y = e^{-x}(P\cos x + Q\sin x) + 2 + e^{-2x}$$

   $y(0) = 4$. Then $4 = P + 2 + 1 \rightarrow P = 1$.
   $y'(0) = -1$. Then $-e^{-x}(P\cos x + Q\sin x) + e^{-x}(Q\cos x - P\sin x) - 2e^{-2x} = -1 => Q - P - 2 = -1 \rightarrow Q - P = 1 \rightarrow Q = 2$.

   Hence the final equation is $y = e^{-x}(\cos x + 2\sin x) + 2 + e^{-2x}$.

3. (Easy)We have

$$a + b + c = 0$$

   Squaring both sides,

$$|a|^2 + |b|^2 + |c|^2 + 2(a.b + b.c + c.a) = 0$$

$$\rightarrow (a.b + b.c + c.a) = -\frac{1}{2}(|a|^2 + |b|^2 + |c|^2)$$

4. (Medium)
   The equation of the big circle is $(x-2)^2 + y^2 = 36$.
   The equation of the circles are $\left(x \pm \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{1}{8}$
   The equation of the downward curve is $y = \sqrt{\frac{64}{25} - x^2}$

The equation of the upward curve is $y = -\frac{1}{4}\sqrt{\frac{64}{25} - x^2}$

Note that the answers can vary slightly with no penalty (answers are not necessarily exact here).

5. (Medium)

The domain of $\sin^{-1} x = [-1,1]$. Hence $-1 \le \frac{a}{b} \le 1$ and $-1 \le \frac{b}{a} \le 1$.
This is only possible when $a = b$ or $a = -b$.
Hence the only possible values are $2\sin^{-1} \pm 1 = \pm\pi$.

The possible values of $\sin^{-1}\left(\frac{a}{x}\right) + \cos^{-1}\left(\frac{x}{a}\right)$ are $\frac{\pi}{2} + 0 = \frac{\pi}{2}$ and $\frac{-\pi}{2} + \pi = \frac{\pi}{2}$, as can be seen from the first part.

Hence

$$\int_{\frac{2}{\pi}}^{\frac{\pi}{2}} \frac{\pi}{2} dx = \frac{\pi}{2}\left(\frac{\pi}{2} - \frac{2}{\pi}\right) = \frac{\pi^2}{4} - 1$$

6. (Medium) We need to find

$$\int_4 x^k dx = \frac{1}{k+1}\int_3 x^{k+1} dx = \frac{1}{(k+1)(k+2)}\int_2 x^{k+2} dx = \frac{1}{(k+1)(k+2)(k+3)}\int_1 x^{k+3} dx$$

$$= \frac{x^{k+4}}{(k+1)(k+2)(k+3)(k+4)} + C$$

For the 2nd part,

$$\int_n x^n\, dx = \frac{1}{n+1}\frac{1}{n+2}\frac{1}{n+3} \cdots \frac{1}{2n} x^{2n} + C = \frac{n!}{(2n)!}x^{2n} + C_1$$

$$\int_n x^k\, dx = \prod_{r=1}^{n} \frac{1}{k+r} x^{k+n} + C_2$$

Hence

$$\int_n x^n + x^k\, dx = \frac{n!}{(2n)!}x^{2n} + \left(\prod_{r=1}^{n}\frac{1}{k+r}\right)x^{k+n} + C$$

7. (Easy)

As ω is also real,

$$\omega + \omega^2 + \omega^3 + \cdots = -1$$

$$\frac{\omega}{1-\omega} = -1$$

[Using infinite G.P formula]

$$1 + \frac{\omega}{1-\omega} = 0$$

Also,

$$1 + \omega + \omega^2 + \cdots = \frac{1}{1-\omega} = 0$$

[Taking the G.P using the Taylor series]

But the reciprocal of any number cannot be 0, hence this is not possible.

8. (Medium)

The probability that the first one is red is $\frac{p}{n}$. For the second one, it is $\frac{p-1}{n-1}$. This goes on till only one red ball exists.

Hence the probability is

$$\left(\frac{p}{n}\right)\left(\frac{p-1}{n-1}\right)\cdots\left(\frac{1}{n-(p-1)}\right)$$
$$\rightarrow \frac{p!}{\prod_{i=0}^{p-1}(n-i)}$$

Now, if 2 sweets are eaten,

$$\left(\frac{p}{n}\right)\left(\frac{p-1}{n-1}\right)=\frac{1}{n}$$
$$p^2-p=n-1$$
$$p^2-p-(n-1)=0$$
$$p=\frac{1\pm\sqrt{1+4(n-1)}}{2}$$
$$p=\frac{1\pm\sqrt{4n-3}}{2}$$

9. (Medium)

X first takes $\frac{m}{2}L$, and Y takes half of that (i.e, $\frac{m}{4}L$). Then X takes half of that (i.e, $\frac{m}{8}L$) and so on. Hence

Volume drank by X is

$$\frac{m}{2}+\frac{m}{8}+\frac{m}{32}+\cdots=\frac{\frac{m}{2}}{1-\frac{m}{4}}=\frac{\frac{m}{2}}{\frac{3m}{4}}=\frac{2m}{3}L$$

By taking G.P of Y or by taking volume(Y) $= m-\frac{2m}{3}=\frac{m}{3}$, we can say that X drank more by $\frac{m}{3}L$.

10. (Hard)

We start with

$$x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots=\sin x$$

(By Taylor series).
Differentiating both sides,

$$1-x^2+x^4-x^6+\cdots=\cos x$$

Differentiating again,

$$-2x+4x^3-6x^5+\cdots=-\sin x$$

Therefore, $2x-4x^3+6x^5+\cdots=\sin x$. Hence, we can conclude that

$$2x - 4x^3 + 6x^5 + \cdots \cong x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

The second result can be directly obtained by taking the first term and setting up a summation.

11. (Hard) We have

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

Squaring both sides,

$$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = \frac{1}{z^2}$$
$$\frac{2}{xy} = \frac{1}{z^2} - \frac{1}{y^2} - \frac{1}{x^2}$$
$$\rightarrow 2 = \frac{xy}{z^2} - \frac{x}{y} - \frac{y}{x}$$

The domains of x, y & z do not change, but the range of z is now always positive by virtue of its square.

12. (Easy)

We have

$$\lim_{n\to\infty} \frac{\ln n}{\ln(n+1)} = \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n+1}} = \lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} 1 + \frac{1}{n} = 1$$
$$k = 1$$

(By L'Hopital's rule)

13. (Hard)

First option

We have

$$\sin x + \cos x = \tan x$$

Squaring both sides,

$$1 + \sin 2x = \tan^2 x$$
$$1 + \frac{2\tan x}{1 + \tan^2 x} = \tan^2 x$$
$$1 + \tan^2 x + 2\tan x = \tan^2 x + \tan^4 x$$
$$\tan^4 x - 2\tan x - 1 = 0$$

Second option

We have

$$\sum_{x=1}^{n} \sin x = \sum_{x=1}^{n} 2\sin\frac{x}{2}\cos\frac{x}{2}$$
$$\sum_{x=1}^{n} 2\sin\frac{x}{2}\cos\frac{x}{2} = \sum_{x=1}^{n} 2\left(2\sin\frac{x}{4}\cos\frac{x}{4}\right)\cos\frac{x}{2}$$

$$\sum_{x=1}^{n}\left(4\sin\frac{x}{4}\cos\frac{x}{4}\right)\cos\frac{x}{2}=\sum_{x=1}^{n}8\left(\sin\frac{x}{8}\cos\frac{x}{8}\right)\cos\frac{x}{4}\cos\frac{x}{2}$$

Generalising the summation for the k^{th} time, we get

$$\sum_{x=1}^{n}\sin x=2^{k}\sum_{x=1}^{n}\left\{\sin\frac{x}{2^{k}}\prod_{i=1}^{k}\cos\frac{x}{2^{i}}\right\}$$

14. (Easy)

The equation of the first plane is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 2 & -1 & -1 \end{vmatrix}=0$$
$$\rightarrow -3(x-1)-3(y-2)-3(z-3)=0$$
$$x+y+z-6=0$$

Taking the intersection of the two planes,

$$(\lambda+1)(x+y+z-6)=0$$
$$\rightarrow \lambda=-1$$

Hence the equation of the plane is 0.

15. (Easy)

$$2\,f(x)=x^{3}+5x+1$$
$$f(x)=\frac{1}{2}(x^{3}+5x+1)$$
$$f'(x)=\frac{1}{2}(3x^{2}+5)$$

But

$$f(x)+f'(x)=\frac{1}{2}(x^{3}+3x^{2}+5x+6)$$

Which is not what was specified in the question. Hence the statement is not possible.

Taking $f(x)$ as correct, $f''(x)=\frac{1}{2}(6x)=3x$.

16. (Medium)

We have

$$\frac{dP}{dn}\propto P$$
$$\frac{dP}{dn}=rP$$
$$\frac{dP}{P}=r\,dn$$
$$\ln P=rn+c$$
$$P=Ce^{rn}$$

When $t=0, C=P$ (initial principal). Hence the solution becomes

$$A=Pe^{rn}$$

Now comparing with formula given in question, we get

$$Pe^{r_1 n}=P\left(1+\frac{r_2}{100}\right)^{n}$$
$$e^{r_1 n}=\left(1+\frac{r_2}{100}\right)^{n}$$
$$r_1 n=n\ln\left(1+\frac{r_2}{100}\right)$$

$$r_1 = \ln(1+r)$$

(Where $r = \frac{r_2}{100}$)

But note that $\ln(1+r) \sim 0$ for small values of r. Hence the 2nd formula is reasonably valid for normal values of r.

The accuracy is however lesser as the graphs eventually diverge as r increases.

17. (Hard) Multiplying the first expression with 8, we have

$$8(x+2)^2 + 16y^2 = 144$$
$$9(x-1)^2 + 16y^2 = 25$$

Subtracting the 2nd equation from the first equation, we get

$$8(x+2)^2 + 9(x-1)^2 = 109$$
$$8(x^2+4x+4) - 9(x^2-2x+1) = 119$$
$$8x^2 + 32x + 32 - 9x^2 + 18x - 9 = 119$$
$$-x^2 + 50x - 96 = 0$$
$$x^2 - 50x + 96 = 0$$
$$(x-2)(x-48) = 0$$

$x = 48$ is not a valid solution as y^2 is negative in both equations.

Hence, finding y with $x = 2$,

$$16y^2 = 16$$
$$y = \pm 1$$

Now taking the equation of the circle, $(x-a)^2 + (y-b)^2 = r^2$,

Taking points $(2,1), (2,-1)$,

$$(2-a)^2 + (1-b)^2 = (2-a)^2 + (-1-b)^2$$
$$1 - 2b + b^2 = 1 + 2b + b^2 \rightarrow b = 0$$

Hence the equation finalizes to

$$(x-a)^2 + y^2 = (2-a)^2 + 1$$
$$x^2 - 2ax + y^2 = 5 - 4a$$
$$K = 2, B = 5, C = 4$$
$$K + B + C = 11$$

18. (Easy) New mean: $\frac{kx_1 + kx_2 + \cdots}{n} = \frac{k(\sum_{i=1}^{n} x_1)}{n}$. Hence the mean increases by k times.

Standard deviation:

$$\sqrt{\left(\frac{1}{n}\right)\left(\sum_{i=1}^{n}(x_1 - x_{mean})^2\right)} = \sqrt{\left(\frac{1}{n}\right)\left(\sum_{i=1}^{n}(kx_1 - kx_{mean})^2\right)} = \sqrt{\left(\frac{1}{n}\right)k^2\left(\sum_{i=1}^{n}(x_1 - x_{mean})^2\right)}$$

$$= k\sqrt{\left(\frac{1}{n}\right)\left(\sum_{i=1}^{n}(kx_1 - kx_{mean})^2\right)}$$ → Standard deviation increases by k times.

Variance increases by k^2 times.

19. (Hard)

This answer was written on June 8, 2018.

Let the day be X.

(i) Number of year shifts: For every year, Dec 31st / Jan 1st will be one day later compared to the preceding year. So, the corresponding shift till 2018 can be modelled by $(2018 - 1946) = 72$. But then leap years will cause an extra shift in the day, and the number of leap years can be modelled using an A.P:

$$2018 = 1948 + 4(n-1)$$

The number of leap years is $[n]$. As $\frac{70}{4} + 1 = n, [n] = 18$.

Hence the total number of year shifts is $72 + 18 = 90$.

(ii) Day shifts:

There are two of them. The first is the shift from Jan 1st to today, which is 3+0+3+2+3+0=11. The second is the shift from 14th July to Dec 31st 1946, which is 3+3+2+3+2+3=16. Hence the total number of day shifts is 27.

(iii) Finalizing answer:

The total number of shifts is 90+27=117. But $\frac{117}{7}$ has remainder 5.

Hence it becomes $X + 5 = \text{Friday} \rightarrow X = \text{Sunday}$.

Hence 14th July 1945 is a Sunday.

20. (Hard)

(i) $(x-y+2)(x+y-1) = x^2 - y^2 + x - 3y - 2$

(ii) As $(x-y+2)(x+y-1) = 0$, we have

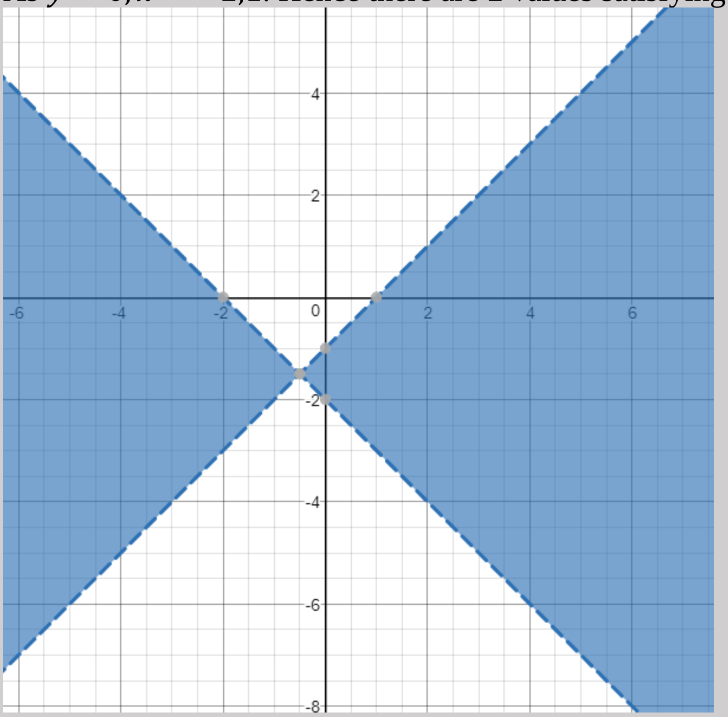
$$x - y = -2$$

Or

$$x + y = 1$$

As $y = 0, x = -2,1$. Hence there are 2 values satisfying the given criteria.

(iii)



We have $(x-y+2)(x+y-1) > 0$. This implies that either $x - y + 2 > 0$ & $x + y - 1 > 0$ OR $x - y + 2 < 0$ & $x + y - 1 < 0$.

Sketching the 2 regions gives the graph mentioned above.

21. (Medium) We have

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - R_2$$

$$\begin{bmatrix} a-c & b-d \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Equating coefficients,

$c = 0; d = 1; b - d = -1 \rightarrow b = 0, a - c = 1 \rightarrow a = 1$.

Hence the matrix is valid, and the inverse is $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

Applying column transformation $C_1 \to C_1 - C_2$, the matrix becomes

$$\begin{bmatrix} a-b & b \\ c-d & d \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

Equating coefficients,

$$b = 0; a - b = 1 \to a = 1; d = 1; c - 1 = -1 \to c = 0$$

Hence the columnar matrix is valid, and the inverse is $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

For matrix to have no inverse, $|A| = 0; ad - bc = 0 \to \frac{a}{b} = \frac{c}{d}$.

22. (Hard)

The $(r+1)^{th}$ term of its expansion is $nC_r x^{a(c-r)} x^{-br}$

But as the coefficient of x is 0, we have

$$ac - ar - br = 0 \to ac = r(a+b) \to r = \frac{ac}{a+b}$$

But as $r+1$ is an integer, r is an integer; hence $\frac{ac}{a+b}$ is an integer.

Now,

$$r = \frac{ac}{a+b} \to \frac{r}{c} = \frac{a}{a+b} \to \frac{r}{cb} = \frac{ab}{a+b} \to \frac{a+b}{ab} = \frac{cb}{r} \to \frac{1}{a} + \frac{1}{b} = \frac{cb}{r}$$

5.6 - Analysis of Maths-**H**

1. Very easy; I expect nearly **H** candidate to find this one a walkover.
2. The question is fairly easy if the idea is known. While a few weaker **H** candidates may get mixed up in the substitution to use, most should be able to do this even with the length of the solution.
3. A simple variation from an evergreen question. This question is expected to be easy with most **H** candidates.
4. While the equations of the circles are easy to get, the challenge lies in 2 marks – getting the equation of the curves. While the answer need not be exact, some candidates would mix this with the equation of a parabola or hyperbola instead.
5. A question where the idea derived in the first part is essential in getting the 2nd part. It is expected that most **H** students would catch the relationship between the variables inside $\arcsin x$ and their domain. However, a few among them would miss the fact that the answer can also be $-\pi$ if the signs of a and x are different. From there, most should be able to evaluate the integral, but a few may get mixed up with x and dx. However, the penalty for not considering the -ve sign is lesser this time, as the answer is same either way.
6. While the first part is a bit laborious, once that's done, the second part isn't very hard. However, not many would get the $\frac{n!}{(2n)!}$ obtained in the final answer.
7. This question is expected to be easy for most **H** candidates; only a basic knowledge of the cube root of unity is required. The 2nd part, while potentially confusing, is not hard to prove either as the expression yells for a G.P in front of the student.
8. A decent **H** student should be able to realise the idea between the product function and realise the idea behind the expression. While most **H** students should get a fair start to the 2nd part of the question, the potential to lose out by simple mistakes is high.
9. While it takes a bit of thinking to realise what the question is asking, the rest can be done by virtually any **H** candidate. Partial marking is virtually non-existent here.
10. Not many **H** candidates are expected to deduce the strange relationship given in the first part. Only good **H** candidates would be expected to deduce the relationship between the question, differentiation and Maclaurin expansions. However, the 2nd part is a walkover.
11. I would expect quite a few to derive the first answer, through many average and weaker **H** candidates would not be able to deduce the relationship between the two expressions. Even the 2nd part requires some thought, as one would need to check whether there is any change, through many would still get that part correctly.
12. The limit asked is very easy and can be solved even by simple argument. For those who'd rather not take that route, L'Hopital's rule is enough. I'd expect nearly **H** candidate to get this.
13. Both options require some innovative thinking. While the first option *looks* easier, it is if the correct method is used. Otherwise the risk of getting muddled up in heavy algebra is high.
    The 2nd options, while shorter, is more unusual. Only strong **H** candidates are expected to get this one.
14. This question is pretty much like bookwork and hence something which almost every **H** candidate would get right; that being said, a few may mess up the last part when they realise that the equation of the planes is the same. The final answer of 0 is not commonly encountered here either. All in all, an easy question.
15. While a bit unorthodox, the question is very easy and almost every **H** candidate would be expected to get this one.

16. While most **H** candidates should get the first part which is the derivation of a common question, many **H** candidates could get confused with the 2nd part of the question which calls for logical understanding of the given expression and equating it with 1st part answer.
17. While most students should have practiced similar kinds of questions and hence be able to make a start, the sheer length of the question means that many **H** candidates could stumble at various parts of the question, particularly towards the end of the paper. Students would be expected to earn a wide range of marks in this question.
18. A question based on a common type, most **H** candidates would be expected to get this one correct, particularly the first part, which is as good as free.
19. This question is challenging and unorthodox. Many **H** candidates will not be able to make a start on this due to its lengthy and persevering nature. However, even amongst those who realise the idea behind the answer to the question, few would be expected to score all 5 marks due to the potential of getting confused (e.g.: missing leap year) or easily make a careless mistake. It would be hard to verify the answer as well.
20. While the first part is fairly easy, only good **H** candidates would be expected to spot the trick required to solve the 2nd part. The 3rd part is even trickier, and only those with good graphing and conceptual skills would be expected to get it.
21. By its very nature, the question is confusing. But those who can get over the question and understand the idea should be able to finish off the question. All in all, a moderate level question.
22. While most **H** students should be able to make a start to the question, only good students would be expected to realise that $r = \frac{ac}{a+b}$ and hence complete the first part. While such students should be able to get (ii) as well, the question by itself is tricky and not easy.

5.7 – Incubation questions

This section lists with questions which have completed the first phase, (i.e., they have been developed and are now in the pool). This is the 'pool'.

Note that marking scheme for these questions aren't available.

In the beginning of each question, you'll see a box resembling like

[Expected Paper, expected difficulty for question, expected no of marks]

For example, [S,H,3] means that the question is a hard question for Maths-**S** for 3 marks.

Note that even though they are only 'incubator' questions, they may still be used as a model on the different varieties of questions that can be expected in the final exam.

---

1 – [H,M,5]Find the maximum value of $x^{\frac{1}{x}}, x \neq 0$.

(a) Arrange the following numbers in ascending order:

$$1, 2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 4^{\frac{1}{4}}, 5^{\frac{1}{5}}$$

2 – [S,M,6]Let $y = \sin x$.

(a) Sketch its graph on the $x - y$ plane.
(b) Find the area covered by it in $\left[0, \frac{\pi}{2}\right]$.
(c) The largest triangle is inscribed inside the sine graph within $\left[0, \frac{\pi}{2}\right]$. Find the area of the triangle.

Warning: The next question deals with a topic that is considered sensitive by many people. You may wish to avoid reading this question. I believe that this question should stay due to the important value embedded in it.

3 – [S,M,7] Two teenagers have unprotected intercourse. The probability of the girl getting pregnant during any one such intercourse is $\frac{3}{10}$.

(a) They have intercourse every day for a week. What is the probability that the girl does *not* get pregnant?
(b) They have intercourse n times. Find the maximum value of n such that the probability of the girl getting pregnant is less than $\frac{2}{3}$.

The boy now uses a condom. The probability of the girl getting pregnant reduces by three-fourth.

(c) They have intercourse p times protected, and q times unprotected. If the events are independent and the probability of the girl getting pregnant in both cases is the same, find $\frac{p}{q}$.
(d) A boy and a girl are in love with each other but cannot commit to marriage. Use the data in parts (a) to (c) to convince them that they should not have unprotected intercourse.

4 – [S,H,3] Find k: $(n \neq 0)$

$$\frac{1 + 2 + 3 + 4 + \cdots + n}{1 - 2 - 3 - 4 - \cdots - n} = \frac{n(n+1)}{k - n(n+1)}$$

5 – [H,M,8]

(a) Find the amplitude of $a \sin x + b \cos x$.
(b) Find the minimum distance between the smallest positive value and the largest negative value of $c \tan x + d \cot x$. Sketch its graph when $c = d$.
(c) Define a function $f(x)$ such that

$$y = f(x) + \left(f(x)\right)^{-1}$$

Find the minimum positive value of y.

6 – [S,H,5]

(a) Which of the integral(s) below cannot be evaluated? Explain your reasoning for the option(s) you have chosen. You do not have to evaluate any integral(s) that can be evaluated.

$$\int_1^3 [x]\, dx\,, \int_2^{\frac{5}{2}} \sin^{-1} x\,, \int_0^1 \{x\}\, dx\,, \int_7^7 x\, dx\,, \int_{-\frac{1}{2}}^{\frac{1}{2}} x^{\frac{1}{8}}\, dx$$

(b) Deduce the integral with the largest value from amongst the following with appropriate reasoning.

$$\int_0^2 (x^2 - 4) \sin^8 \pi x\, dx, \int_0^{2\pi} (2 + \cos x)^3\ dx, \int_0^{\pi} \sin^{100} x\, dx, \int_0^{\pi} (3 + \cos x)^6 dx\,, \int_0^{8\pi} 108(\sin^3 x - 1)\, dx$$

7 – [S,M,4] A raster (bitmap) is a picture that has been created using *pixels*, while a vector is one which has been created using mathematically definable lines.

(a) Consider your portrait. Will that be a raster or vector? Explain your reasoning.
(b) Consider the picture of a bicycle wheel in raster and vector form with equal sharpness. The image is zoomed. Which will then be sharper: the raster or the vector form? Why?

8 – [H,E,7]

(a) Find

$$\int \frac{1}{1 + \sqrt{x}} dx$$

(b) If

$$\int_{-2}^{2} \frac{n}{1 + x^{-n}} dx = \int_{-2}^{2} \frac{n}{1 + n^{-x}} dx$$

Determine whether the function $f(n) = n$ is odd, even, both or none of them.

(c) If x is an even function, find

$$\int \frac{\tan x}{1 + 4^x} dx$$

9 – [S,H,5] By finding the integrals of $\sqrt{\tan x} + \sqrt{\cot x}$ and $\sqrt{\tan x} - \sqrt{\cot x}$, evaluate $\int \sqrt{\tan x}\, dx$.

10 – [H,M,3] If $\tan p - \tan q - \tan r = \tan p \tan q \tan r$, find the relation between p, q and r.

11- [H,H,8]

(a) Find $[\max(k)]$ for which

$$\sum_{i=1}^{100} \frac{1}{i} > \frac{20}{11}k$$

(b) Given a function $f(x)$ (where $f'(x) \neq 0$) such that

$$\sum_{i=1}^{n} \frac{1}{f(i)} < 1$$

Find a function $f(x)$ using (iii).

(c) Prove that

$$\sum_{i=1}^{n} f(i) \sum_{i=1}^{n} \frac{1}{f(i)} \geq n^2$$

12- [H,E,4] Find the eigenvectors and eigenvalues of $\begin{bmatrix} -4 & -17 \\ 2 & 2 \end{bmatrix}$.

13 – [H,M,8]

(a) Find the value of $1 + 2 - 3 * 4/5$.
(b) Given a sequence $(a-2) + (a-1) - a * (a+1)/(a+2)$, deduce without calculus that the sequence is strictly increasing. Find the minimum value of the sequence.
(c) Consider the A.P sequence $(a-2d) + (a-d) - a * (a+d)/(a+2d)$. Find a/d such that the value of the given sequence is minimum, separately considering the cases where d is the constant and a is the constant.

14- [H,E,2] Evaluate

$$\lim_{x \to \infty} (n + n^x)^{\frac{1}{x}}$$

15- [S,H,8]

(a) Consider the equations $y = 1 + x$ and $y = x^2$. On the same plane, draw their graphs.
(b) Find all solution(s) to the equation $1 + e^x = e^{2x}$.
(c) Let $1 + x = \tan^n x$, where $n \in \mathbb{N}$. Find the number of solutions of x in the range $[0,2\pi]$.
(d) Let $f(z) = \big(f(z)\big)^2 - 1$. Give a function $f(x)$ such that the given equation has no solutions for all values of x. Justify your answer.

16 – [H,M,6] Find

$$\int \cos^{-1}(\sinh x)\, dx$$

17 – [S,H,8]

(a) A stack of squares of side 1 cm are arranged in a column. Two points are marked at the right vertices between the p^{th} and $(p+1)^{th}$ and q^{th} and $(q+1)^{th}$ square, where $q \geq p$. Find the angle between the two points as measured from the lowermost vertices.
(b) Let $p - q$ be a constant and then p is moved (which moves q as well). Prove that that angle will change at a rate of

$$\sec^2\left(\frac{p-q}{1+pq}\right)\frac{(q-p)(p+q)}{(1+pq)^2}$$

18 – [S,M,7]

(a) Prove that $\sin(A+B) \neq \sin A + \sin B$.
(b) Assume that expression (a) is also a correct expansion for $\sin(A+B)$. Prove that

$$\frac{\sin A}{\sin B} = \frac{\cos A - 1}{1 - \cos B}$$

(c) Consider $\Delta ABC, \angle C = \frac{\pi}{2}$. Find $\sin A + \cos A$ and prove that such a triangle does not exist.
(d) Find $\int \sin x + \cos y \ dy$.

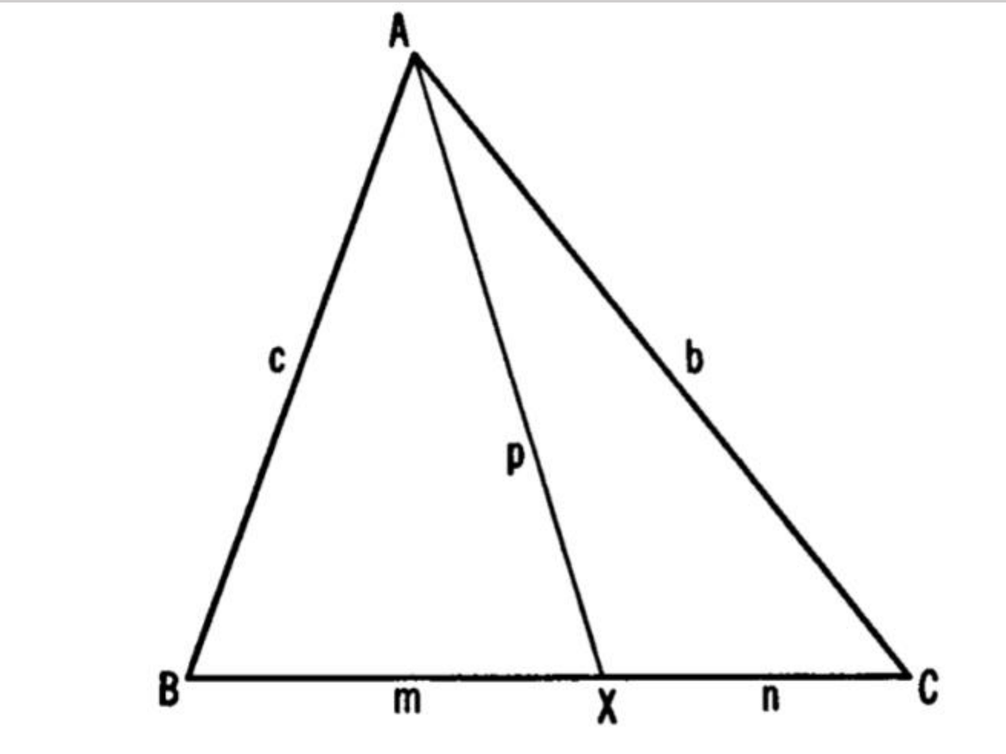
19 – [H,M,8]

A thief is running around a rectangular park whose dimensions are p m and q m. A policeman is dropped randomly in the park. The policeman can catch the thief only if he sees him – implying that the policeman and the thief must be both in the same side of the park. Assume that the park boundary is purely one dimensional.

(a) Let $p = 150\ m$ and $q = 50\ m$. Find the probability that the thief is caught.
(b) Find the condition required to minimise the probability that the thief is caught. What is the minimum probability?
(c) Show that a rectangular park cannot fulfil the condition required to maximise the probability that the thief is caught.

20 – [H,M,5]

(a) Find $\cos A + \cos B$, if A and B are supplementary.
(b) Consider the following figure:



By using the result obtained in part (a) or otherwise, show that $a(p^2 + mn) = b^2m + c^2n$, where a represents the length of side BC.

21 – [H,H,5] The graphs of the equation $\sin x = \cos y$ are drawn in the Cartesian plane. Show that all the polygons inscribed in the graph have equal area and find the area of one of the polygons.

22- [S,E,4]

(a) Write $(\sqrt{3}+i)(1+i)$ in polar form.

(b) Hence prove that

$$\cos\frac{5\pi}{12}=\frac{\sqrt{3}-1}{\sqrt{8}}$$

23 – [H,E,5] Consider two matrices A and B, whose orders are (m,n) and (p,q) respectively. Assume that every entry in each matrix is an integer between $[1,k]$. Matrixes A and B are multiplied to form a third matrix C, if the multiplication is possible.

(a) Let $w(A)$ be the number of ways the matrix A can form. Prove that

$$\frac{w(A)\,w(B)}{w(c)}=k^{mn+pq-mq}$$

(b) Find $\min\left(\frac{w(A)\,w(B)}{w(c)}\right)$ and deduce the condition to obtain the same.

24 – [S,M,5]

(a) A user must enter a 6-digit code which he receives on a mobile phone as part of 2FA (Two-Factor Authentication). He notices that the codes he receives have at least one repeating number more than usual and estimates that 40% of the codes he receives have this peculiarity. Is he right to think that the provider who gives him these codes is somehow not randomly generating these codes and this is not just coincidence?

(b) Given that the 6-digit codes he receives are purely random, find the number of ways that exactly two digits among the 6 have the same number.

25 – [H,M,7] Consider the function $f(x)=\operatorname{sgn}(x)$ and $g(x)=\sin x$.

(a) Sketch the graph of $f\circ g(x)$ and prove that it is periodic.

(b) Find all possible derivatives of $f\circ g(x)$.

(c) Let $h(x)$ be an arbitrary function. Prove that $f\circ h(x)$ and $h\circ f(x)$ are both periodic.

26- [S,H,6]

Consider a function $f(x)$, where

$$\int_1^5 f(x)\,dx=5$$

(a) If

$$\int_3^5 f(x)\,dx=3$$

, find

$$\int_1^3 f(x)\,dx$$

(b) Find the sign of $f'(x)$ at a point between 1 and 3 or explain why it cannot be found.

(c) Consider the summation

$$\sum_{i=0}^{\infty}\left(\int_{i\pi}^{(i+1)\pi}\frac{\sin x}{x}\,dx\right)$$

Prove that it is positive.

27- [H,M,8]

Consider the summation

$$\sum_{i=1}^{n} a_i = k$$

(a) Show that

$$\max\left(\prod_{i=1}^{\infty} a_i\right)$$

happens when

$$a_i(0 < i < n+1) = \frac{k}{n}$$

(b) Find the probability that $\frac{k}{n} = \left[\frac{k}{n}\right]$.

(c) Let $a_i \in \mathbb{Z}$ and each member of a_i be distinct. Find the number of possible ways this can be done.

28- [H,H,8]

Given that a and b are distinct positive numbers, find a polynomial $P(x)$ such that the derivative of $f(x) = P(x)e^{-x^2}$ is zero for x = 0, x = ±a and x = ±b, but for no other values of x.

29- [S,M,4] Find:

$$\int (x^7 + x^{15}) \ln(1 + x^8)\, dx$$

30- [H,E,5] Let $\sin x + \cos x = p$.

(a) Find the domain and range of p.
(b) Prove that $x = \frac{1}{2}\sin^{-1}(p^2 - 1)$.

31 – [H,M,4]

Consider a $x - y$ graph with each point on the axis separated by an equal distance. Suppose a straight line is drawn from $(0,1)$to the point $(3,8)$. If $\tan\theta$ between the line and x axis is 1, find the area of the triangle so formed.

32 – [S,M,7]

(a) State the *Fundamental Theorem of Calculus*.
(b) Consider the integral

$$\int_{-a}^{a} e^{-x^4}\, dx$$

Show that the Fundamental Theorem of Calculus cannot be applied to evaluate the above integral.

Show further that an integral of the form (where $m \geq 2$ & $m, n \in \mathbb{Z}$)

$$\int e^{(-1)^n x^m}\, dx$$

cannot be evaluated directly.

Let $m \in [1,10]$. Find all values of m for which

$$\int_{-\ln 2}^{\ln 2} e^{(-1)^n x^m}\, dx$$

can be evaluated.

33 - [S,E,5]

Consider a quadratic with real roots p and q. If $g(x)$ is its first derivative, show that the solution of g is the mean of p and q.

Consider a n degree polynomial with all its root(s) equal. Show that $g^k(x)$, where $k < n + 1$, has its solution(s) as the root of the n-degree polynomial itself.

34 – [S,M,8]

This question uses degree as the angle measure.

Consider a n sided polygon (where $n \geq 3$). Deduce and prove by induction a formula to find the sum of all interior angles of that polygon. You may use without proof the fact that the sum of the exterior angles of any polygon is 360°.

Now assume that the polygon is regular. Show that each angle of the polygon decreases as n increases at a rate of $\left(\frac{360}{n^2}\right)°$.

Let that polygon be constructed on the Cartesian plane such that a line $y = k$ divides the polygon symmetrically. Find the slope of a line connecting the centre and any vertices of the polygon.

35 – [S,H,8]

Let $f(x) = \sin^{-1}(\sin x)$ and $g(x) = \sin(\sin^{-1} x)$.

(a) Prove that $f(x) = g(x)$ where $x \in f(x), g(x)$.
(b) Show that the domains of $f(x)$ and $g(x)$ are not equal.
(c) Sketch the graphs of $f(x)$ and $g(x)$ on the same plane.
(d) Let the graph of $g(x)$ be the diameter of a circle. Deduce its equation.

36 – [S,H,4]

Solve the following differential equation in terms of y :

$$x\,dy - dx = \frac{1}{x}(\pm dx + dy)$$

37 – [H,M,8]

(a) If $\sin x + \cos y = \tan(x + y)$, show that
$$\frac{dy}{dx} = \frac{\cos x - \sec^2(x + y)}{\sin y + \sec^2(x + y)}$$
(b) Let δx denote the partial derivative of x, i.e, the derivative with respect to x alone. Find
$$\int \frac{\cos x - \sec^2(x + y)}{\sin y + \sec^2(x + y)}\,\delta x$$

38 – [S,H,4]

Consider a square matrix $A = a_{ij}\ (1 \leq i \leq n; 1 \leq j \leq n)$, where $a_{ij} = i + j$. Show that such a matrix will never be invertible.

39 – [H,H,4]

Three real numbers in $[0,1]$ are independently and randomly chosen. Find the probability that a triangle with positive area exists.

40 – [H,H,4]

Let $n \geq 1$. Show that

$$\int_0^n [2^x]\, dx = n2^n - \log_2(2n)!$$

41 – [H,M,8]

A rubber band initially assumes the shape $x = k\ (-k \leq y \leq k)$ and is stretched from west to east.

(a) Write down the general equation that the rubber band will assume in that position (choose from $x^2 = 4ay$, $x^2 = -4ay$, $y^2 = 4ax$ and $y^2 = -4ax$).
(b) Find the 2-D area assumed by the rubber band at that position, assuming that $4a$ is the latus rectum of the rubber band.
(c) The largest possible triangle (which is isosceles) is carved off from the parabola formed by the rubber band. Find the percentage of area left over.
(d) The triangle is now revolved along the base into a cone. Derive a formula to find the volume of that cone.

5.8 - Analysis for incubator questions

These analyses are preliminary and are subject to change as the question potentially gets modified during later stages. These are written from more of a 1st party view.

1. This question highlights the common misconception (myself included) that the largest value occurs at $x = 1$ or 2 or 3 when it is actually at $x = e$.

   Students calculating should be able to find this, through for the unheralded, the 2nd part is not very easy either. Overall a tricky **H** question.

2. The first two parts are walkovers. The 3rd part is considerably trickier, but the confidence gained by the candidate after getting the first two parts could help it get the 3rd part as well.
3. Barring the risky theme taken by this question, it is a probability question that should possess little fuss for those who have practiced well. Only part (iii) should cause any discomfort for most; even that is not much.
4. The question structure is not inviting and could cause confusion due to the usage of n and k. Once that hurdle is cleared, it is not expected that most **S** candidates will realise the trick to apply in the denominators. In reality, that's the only issue. From that, it's pretty quick, though it is expected that the 'leap' required would be too much for many.
5. The first part is straightforward. However, the major challenge lies in the 2nd part; the risk of making errors while differentiating $\mathrm{c}\tan x + d\cot x$ is high and the wording of the question could confuse some **H** candidates who may not be able to imagine the resulting graph. The 3rd part would not pose much of a challenge for those who could get through the first two parts. Overall, while not hard, is not that easy either.
6. This is, in essence, a very difficult and tricky question for **S** candidates; the same type as Q 21 in the **S** Sample paper. The first part is deceptive and confusing; most would not spot any error and may instead choose $\int_7^7 x\,dx$; many others would randomly pick one of the other options. Only a very small minority would be expected to think out of the box and even fewer would realise that there are *two* options.

   The second part, while also challenging, is easier; good **S** candidates should be able to realise that $\int_0^\pi (3+\cos x)^6\,dx$ should be having a higher value due to the higher power. However, most others would fall into one of the many traps provided and choose something else.

   It should be noted that not much of *raw* knowledge is require here – the candidate is not asked to apply any substitution or property beyond the mere basics of calculus. However, the depth required is very high for this question.

7. An easy question; only the wording could pose an issue as it forces students to think a bit on what it really means. There is little to do for 4 marks – and this question has an embedded value as it helps students realise the importance of vectors and lines.
8. The first part is easy and can be solved in a multitude of ways. The 2nd one is just a variation of a theme that has appeared in many past Indian exams, notably JEE Mains 2018 and CBSE Maths 2016, hence students who know what the question is about will easily get this. The 3rd part follows along similar lines. Overall, for a **H** candidate, it's fairly simple.

9. This question, while guiding **S** students on solving an integral by the method of adding and subtracting two integrals, is still not easy since integrating the combined functions is tough if one has not seen or done a similar question before.
10. While a simple variation of one which is seen in the NCERT textbook, a little bit of creativity is required to simply rearrange the terms given to

$$\tan(p+q) = \frac{\tan p + \tan q}{1 - \tan p \tan q}$$

A few weaker **H** candidates will not catch this, but overall, this one is not too demanding.
11. The first part is not very easy; only good **H** candidates would get the impetus required to get

$$\ln 100 > \frac{20k}{11}$$

But from there, it isn't a challenge.
A preliminary variant of this question asked to prove that:

$$5050 \sum_{i=1}^{100} \frac{1}{i} > (100^2)$$

Then, it was directly tied to part (iii) of the current variant. At that time, I was expecting students to prove part (iii) and then directly apply that to the question given (hint: 5050, 100 and 10000). However, with the current form, the only option is to take limit to a sum.

The 2nd part is also tough. Again, knowledge of the 3rd part is very helpful here – and even then, very few **H** students would be expected to realise that

$$\sum_{i=1}^{n} f(i) = n^2$$

That's the only hurdle, but again the question is obfuscated enough to make it difficult.

The 3rd part is more direct but considering that many would have solved the first two parts using different methods, it is likely that quite a few would try to apply similar methods for the 3rd part as well, and not think about the A.M:H.M inequality.

Overall, the question is tricky and challenging, and not many would be expected to gain all the 8 marks. **H** candidates working backwards from the 3rd part may have better luck.

12. Simple and direct question for **H** candidates. Other than the relative newness of the concept, there is nothing challenging that's worth mentioning.
13. The first part is simple and is only designed to remind **H** candidates of the fact that the answer is not 0 (and even the rare minority who thinks so should realise something is amiss when the remaining two parts decompose far too easily). The remaining parts aren't hard either, but calculative and the potential for making mistakes exists.
14. Very easy, only out of scope for the **S** level. In fact, this is a copy of the classic style of questions regarding $\lim_{n\to\infty} (a^n + b^n)^{\frac{1}{n}}$. Almost all **H** candidates would be expected to score fully on this one.
15. Part (a) is easy and is meant as a starter for the tougher part (b) and (c). Parts (b) and (c) are where a wide variety of marks can be expected, with a significant number of **S** candidates expected to laboriously solve each equation separately (when in reality the whole question deals with only the equation $1 + x = x^2$). Also, quite a few would stumble with making the appropriate substitutions required to solve the equation (especially realising the possibility of extraneous solutions). Part (d)

could be solved only by students who would have gotten part (b) fully. All in all, a question while accessible for nearly everyone, is tough to score well.

16. While this question looks similar to many other questions which **H** candidates would've done, the use of hyperboles make it trickier. Unlike many other questions here, it is not possible to avoid getting the nasty
$$\int t\cos^{-1}\frac{t^2-1}{2t}\,dt\ (t=e^x)$$
Application of integral of parts will be required to simplify it, and even then, it's laborious (hence why this alone is a 6-mark question).
The only reason why this is not labelled as hard is because **H** students would be expected to worm their way around tricky integrals like this.
17. The first part of the question, while its wording requires some thought, is not hard to solve as it decomposes to the easy $\tan^{-1}p-\tan^{-1}q$ and is hence easily doable.

    However, the 2nd expression is far more complex, and the level of differentiation required is more than what most **S** students can manage. Very few **S** students would be expected to obtain the final expression given in the question. However, a few clever students who could not get its gist will simply differentiate the first expression (due to the $\sec^2 x$ term); such students will get success with some perseverance.

    All in all, one can expect 3/8 as the modal mark distribution.
18. The first part is meant to be an easy 'starter' for **S** students. Even the 2nd part isn't hard, and most **S** students should be able to perform the algebra required to obtain the expression required. The 3rd part, while slightly tricky, is still doable and wouldn't pose much of a challenge for above-average students. The final part is easy, with its only purpose to remind students of the importance of dx (and hence confuse some of them). Overall, while this question isn't hard, one can still expect a nice bell curve as its mark distribution.
19. The question looks a bit unorthodox, and the first part would require some thought to solve. However, once that is done, the remaining two parts can be solved without much fuss. Overall, a moderate question in difficulty.
20. The question, essentially a proof of *Stewart's theorem*, was found to be too tricky to solve in an examination scenario. Hence a subtle hint in the first part was added, to give a nudge to **H** candidates on how the 2nd part should be solved. And for those who do get the hint, the given expression can be obtained with little difficulty. Overall, another moderate-level question mainly due to the subtility of the hint (without it, few would get the solution and it would instead look out of syllabus).
21. The challenge of the question lies in drawing the unusual graph: many would be expected to produce sinusoidal graphs of some form, and the imagination required to correctly draw the graph would (also) be expected to elude most **H** candidates. While the first part of the question follows directly from drawing the graph (and the 2nd part can be done by many **H** candidates), it is anticipated that few would earn all the 5 marks available.
22. There is little that a **S** candidate who has practiced that chapter well has to worry about; the first part is plain bookwork. While the 2nd part has a result that is slightly obfuscated, even that is not expected to be much of a challenge for many **S** candidates. Overall, an easy one.
23. This simple matrix-combinatorics question requires little of thinking for **H** candidates. It does not take much to find the number of possible ways for each matrix, and the rest of the question can be easily done from that. The 2nd part follows directly from the first part.

24. While a bit wordy, this question dealing with the common experience of dealing with multifactor authentication should not be hard for **S** candidates who have practiced well to solve. The 2nd part is similar; those with decent conceptual knowledge of combinatorics should do well.
25. A question dealing mainly with the uncommonly used $\mathrm{sig}(x)$ function, this one is expected to pose some problems with the weaker **H** candidates who may be unfamiliar with the given function. While it does take good conceptual knowledge of the function to solve this question, it is still expected that those what the function is about will be able to solve it fully.
26. The first part of the question is simple. The remaining two parts are however significantly tougher; the second part calls for some solid graphical and calculus understanding, and like Q6, little raw knowledge is required. The third part is similarly tough: only students with strong graphical skills and a sound understanding of the question would be expected to earn all the three marks. Overall, not many S candidates would be expected to score beyond 1 mark.
27. While a bit symbol-y, the first part can be solved by most **H** candidates with a basic understanding of the question, with the 2nd part being an extension of the former. The last part is significantly tougher though and requires strong combinatorics skills.
28. A difficult question. The question would take some time to decipher (even for most **H** candidates), and even after that, is tricky and requires care to work it out. Taken from a STEP II paper, not many would be expected to score all the 8 marks and this question has the potential to cause a *negative domino* effect: some may run out of time for other questions solely because of this one.
29. While like many questions that **S** candidates would have practiced, this question requires a little thinking to get going. Once the question has been reduced to the integral of the form $\int t \ln t \ dt$ $(t = 1 + x^8)$, the rest of the question is straightforward. Overall, a medium-difficulty question.
30. An easy question: the first part is trivial and the 2nd part can be done with a little bit of thinking. Most **H** candidates should get all the 5 marks.
31. The question itself is a bit confusing and quite a few **H** students would be expected to fumble at the first stage itself. However, the integration demanded after that is trivial and most students who get up to that point should complete the question easily. Overall, tricky but moderate.
32. This question is meant to be incremental – the later parts directly build up from the initial stages. Overall a medium-difficulty question, though it may seem a bit tougher at first.
33. While the first part looks unfamiliar, the 'leap' required is low enough that most **S** students can be expected to get all 5 marks of the question.
34. The question starts off under a slightly unfamiliar context: asking students to geometrically visualise a polygon to prove something by induction. However, the question 'guides' students towards each part of the question, and hence a reasonable number of **S** students can be expected to do well.
35. This question reminds students about the often-mistaken assumption that $\sin^{-1}(\sin x) = \sin(\sin^{-1} x)$ without considering domain and range issues. Overall, a tough question, as each of the part requires students to have a strong grasp of the idea. Only strong **S** students would be expected to score all the 8 marks.
36. Rating this question as a **S**/H question or a **S**/M question was tricky, as on the surface the question decomposes very easily especially if the sign is negative. However, when the sign is positive, it leads to the use of partial fractions and is significantly more complex. This, and the unusual use of $\pm$ made me tip the edge in expected difficulty.
37. Most **H** students should find the first one easy as they would've been expected to practice similar questions in the past. The 2nd one, while a bit tricky due to the use of δx and a multi-step integral with potentially awkward results, should still be doable for many. Hence it was rated as a **H**/M question.

38. A twist from familiar questions of invertibility. However, this is unusual, and the need for strong arguments could trip many **S** candidates.
39. On the surface, this could be worked out using the Triangle Inequality, but I decided to rate it as **H**/H rather than **H**/M as testing revealed that many candidates were confused, with some thinking that the answer should be 1 and others returning other incorrect answers.
40. A tricky question, the leap in simplifying the greatest integer function into more workable integrals would be too high even for many **H** candidates. Differentiating the answer would be confusing as well. Overall, a hard question.
41. A question in which the later parts build up from earlier sections, and while no part is tough as such, weaker candidates can still be expected to find this challenging.