2 Review Exercise

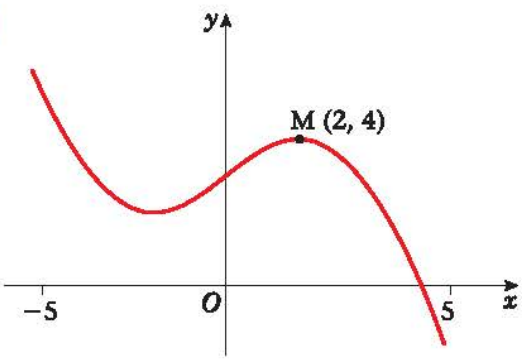
1 **a** On the same set of axes sketch the graphs of $y = |2x - 1|$ and $y = |x - k|$, $k > 1$.

b Find, in terms of k, the values of x for which $|2x - 1| = |x - k|$.

2 **a** Sketch the graph of $y = |3x + 2| - 4$, showing the coordinates of the points of intersection of the graph with the axes.

b Find the values of x for which $|3x + 2| = 4 + x$.

3



The figure shows the graph of $y = f(x)$, $-5 \leqslant x \leqslant 5$.

The point M (2, 4) is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

a $y = f(x) + 3$ **b** $y = |f(x)|$

c $y = f(|x|)$.

Show on each graph the coordinates of any maximum turning points. *E*

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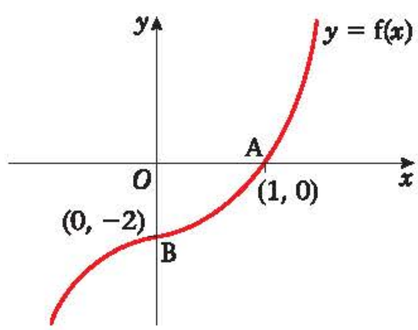

The diagram shows a sketch of the graph of the increasing function f.

The curve crosses the x-axis at the point A(1, 0) and the y-axis at the point B(0, −2)

On separate diagrams, sketch the graph of:

a $y = f^{-1}(x)$ **b** $y = f(|x|)$

c $y = f(2x)+1$ **d** $y = 3f(x - 1)$.

In each case, show the images of the points A and B.

5 For the positive constant k, where $k > 1$, the functions f and g are defined by

$\mathrm{f}: \quad x \to \ln(x + k), x > -k,$

$\mathrm{g}: x \to |2x - k|, \quad x \in \mathbb{R}$

a Sketch, on the same set of axes, the graphs of f and g. Give the coordinates of points where the graphs meet the axes.

b Write down the range of f.

c Find, in terms of k, $\mathrm{fg}\left(\frac{k}{4}\right)$.

The curve C has equation $y = \mathrm{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

d Find the value of k. *E*

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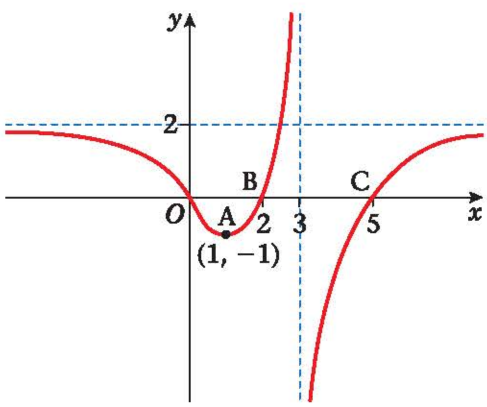


The diagram shows a sketch of the graph of $y = \mathrm{f}(x)$.

The curve has a minimum at the point A(1, −1), passes through x-axis at the origin, and the points B(2, 0) and C (5, 0); the asymptotes have equations $x = 3$ and $y = 2$.

a Sketch, on separate axes, the graph of

i $y = |\mathrm{f}(x)|$

ii $y = -\mathrm{f}(x + 1)$

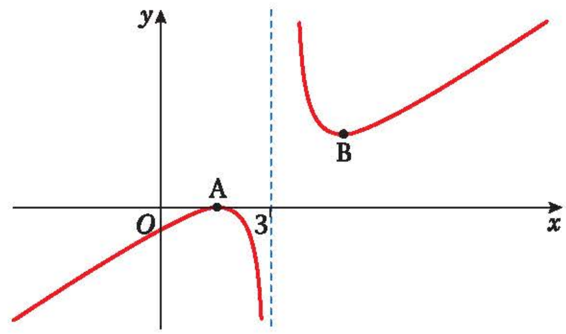
iii $y = \mathrm{f}(-2x)$

in each case, showing the images of the points A, B and C.

b State the number of solutions to the equation

i $3|\mathrm{f}(x)| = 2$ **ii** $2|\mathrm{f}(x)| = 3$.

7



The diagram shows part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{(x - 1)^2}{(x - 3)}.$$

The points A and B are the stationary points of C.

The line $x = 3$ is a vertical asymptote to C.

a Using calculus, find the coordinates of A and B.

b Sketch the curve C^*, with equation $y = \mathrm{f}(-x) + 2$, showing the coordinates of the images of A and B.

c State the equation of the vertical asymptote to C^*.

8 **a** On the same set of axes, in the interval $-\pi < \theta < \pi$, sketch the graphs of

i $y = \cot\theta$, **ii** $y = 3\sin 2\theta$.

b Solve, in the interval $-\pi < \theta < \pi$, the equation

$\cot\theta = 3\sin 2\theta$.

giving your answers, in radians, to 3 significant figures where appropriate.

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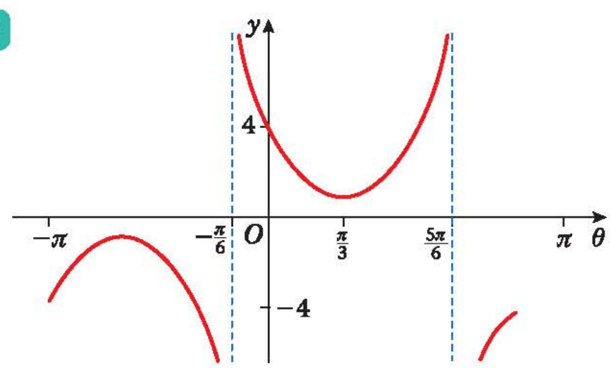


The diagram shows, in the interval $-\pi \leqslant \theta \leqslant \pi$, the graph of $y = k\sec(\theta - \alpha)$.

The curve crosses the y-axis at the point $(0, 4)$, and the θ-coordinate of its minimum point is $\frac{\pi}{3}$.

a State, as a multiple of π, the value of α.

b Find the value of k.

c Find the exact values of θ at the points where the graph crosses the line $y = -2\sqrt{2}$.

d Show that the gradient at the point on the curve with θ-coordinate $\frac{7\pi}{12}$ is $2\sqrt{2}$.

10 **a** Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

b Solve, for $0 \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

E

11 **a** Prove that $\sec^4\theta - \tan^4\theta = 1 + 2\tan^2\theta$.

b Find all the values of x, in the interval $0 \leqslant x \leqslant 360°$, for which

$$\sec^4 2x = \tan 2x(3 + \tan^3 2x).$$

Give your answers correct to 1 decimal place, where appropriate.

12 **a** Prove that

$$\cot\theta - \tan\theta = 2\cot 2\theta, \quad \theta \neq \frac{n\pi}{2}.$$

b Solve, for $-\pi < \theta < \pi$, the equation

$$\cot\theta - \tan\theta = 5,$$

giving your answers to 3 significant figures.

13 **a** Solve, in the interval $0 \leqslant \theta \leqslant 2\pi$

$$\sec\theta + 2 = \cos\theta + \tan\theta\,(3 + \sin\theta),$$

giving your answers to 3 significant figures.

b Solve, in the interval $0 \leqslant x \leqslant 360°$,

$$\cot^2 x = \operatorname{cosec} x(2 - \operatorname{cosec} x),$$

giving your answers to 1 decimal place.

14 Given that

$$y = \arcsin x, \quad -1 \leqslant x \leqslant 1 \text{ and } -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2},$$

a express $\arccos x$ in terms of y.

b Hence find, in terms of π the value of $\arcsin x + \arccos x$.

Given that

$$y = \arccos x, \quad -1 \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant \pi,$$

c sketch, on the same set of axes, the graphs of $y = \arcsin x$ and $y = \arccos x$, making it clear which is which.

d Explain how your sketches can be used to evaluate $\arcsin x + \arccos x$.

15 **a** By writing $\cos 3\theta$ as $\cos(2\theta + \theta)$, show that

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta.$$

b Given that $\cos\theta = \frac{\sqrt{2}}{3}$, find the exact value of $\sec 3\theta$.

16 Given that $\sin(x + 30°) = 2\sin(x - 60°)$,

a show that $\tan x = 8 + 5\sqrt{3}$.

b Hence express $\tan(x + 60°)$ in the form $a + b\sqrt{3}$.

17 **a** Given that $\cos A = \frac{3}{4}$ where $270° < A < 360°$, find the exact value of $\sin 2A$.

b **i** Show that

$$\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) = \cos 2x$$

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

ii show that $\frac{dy}{dx} = \sin 2x$

E

18 Solve, in the interval $-180° \leqslant x < 180°$, the equations

a $\cos 2x + \sin x = 1$

b $\sin x(\cos x + \operatorname{cosec} x) = 2\cos^2 x$, giving your answers to 1 decimal place.

19 **a** Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec} 2\theta, \quad \theta \neq 90n°.$$

b Sketch the graph of $y = 2\operatorname{cosec} 2\theta$ for $0° < \theta < 360°$.

c Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3,$$

giving your answers to 1 decimal place. *E*

20 **a** Express $3\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

b Hence find the greatest value of $(3\sin x + 2\cos x)^4$.

c Solve, for $0 < x < 2\pi$, the equation

$$3\sin x + 2\cos x = 1,$$

giving your answers to 3 decimal places. *E*

21 The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$.

The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants. *E*

22 **a** Differentiate with respect to x

i $3\sin^2 x + \sec 2x$,

ii $\{x + \ln(2x)\}^3$.

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq -1$,

b show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$ *E*

23 Given that $y = \ln(1 + e^x)$,

a show that when $x = -\ln 3$, $\dfrac{dy}{dx} = \dfrac{1}{4}$

b find the exact value of x for which

$$e^y\frac{dy}{dx} = 6.$$

24 **a** Differentiate with respect to x

i x^2e^{3x+2},

ii $\dfrac{\cos(2x^3)}{3x}$.

b Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x. *E*

25 Given that $x = y^2e^{\sqrt{y}}$,

a find, in terms of y, $\dfrac{dx}{dy}$

b show that when $y = 4$, $\dfrac{dy}{dx} = \dfrac{e^{-2}}{12}$.

26 **a** Given that $y = \sqrt{(1 + x^2)}$, show that $\dfrac{dy}{dx} = \dfrac{\sqrt{3}}{2}$ when $x = \sqrt{3}$.

b Given that $y = \ln\{x + \sqrt{(1 + x^2)}\}$, show that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{(1 + x^2)}}$.

27 Given that $f(x) = x^2e^{-x}$,

a find $f'(x)$, using the product rule for differentiation

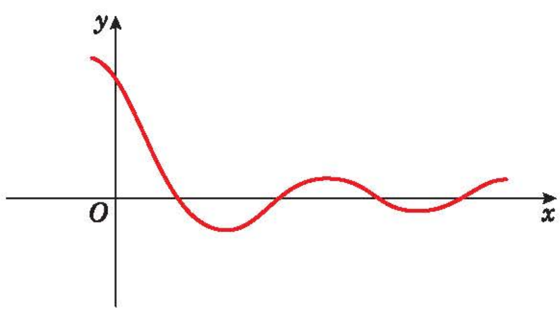
b show that $f''(x) = (x^2 - 4x + 2)e^{-x}$.

A curve C has equation $y = f(x)$.

c Find the coordinates of the turning points of C.

d Determine the nature of each turning point of the curve C.

28 **a** Express $(\sin 2x + \sqrt{3}\cos 2x)$ in the form $R\sin(2x + k\pi)$, where $R > 0$ and $0 < k < \frac{1}{2}$.



The diagram shows part of the curve with equation

$$y = e^{-2\sqrt{2}x}(\sin 2x + \sqrt{3}\cos 2x).$$

b Show that the x-coordinates of the turning points of the curve satisfy the equation

$$\tan\left(2x + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}.$$

29 The curve C has equation $y = x^2\sqrt{\cos x}$. The point P on C has x-coordinate $\frac{\pi}{3}$.

a Show that the y-coordinate of P is $\frac{\sqrt{2}\pi^2}{18}$.

b Show that the gradient of C at P is 0.809, to 3 significant figures.

In the interval $0 < x < \frac{\pi}{2}$, C has a maximum at the point A.

c Show that the x-coordinate, k, of A satisfies the equation $x \tan x = 4$.

The iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{4}{x_n}\right), \quad x_0 = 1.25,$$

is used to find an approximation for k.

d Find the value of k, correct to 4 decimal places.

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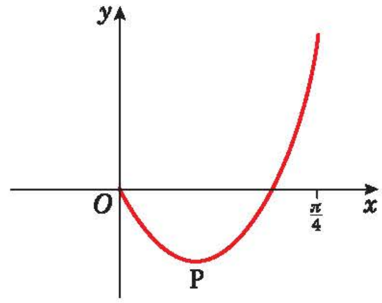

The figure shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \leqslant x < \frac{\pi}{4}.$$

The curve has a minimum at the point P. The x-coordinate of P is k.

a Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

The iterative formula

$$x_{n+1} = \tfrac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k.

b Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

c Show that $k = 0.277$, correct to 3 significant figures. *E*