

PHYSICS SUBJECT TEST

The Physics Test consists of 75 multiple-choice questions. In terms of its overall level of difficulty, the test is intended for students who have had the type of one-year introductory course in physics that emphasized a college-preparatory curriculum.

The following table summarizes the topics tested and the approximate number of questions for each:

Topics Tested	Approximate Percentage of Test
I. Mechanics	34–38
A. Kinematics (e.g., linear motion, projectile motion, and circular motion)	
B. Dynamics (e.g., Newton's laws, centripetal force, and torque)	
C. Energy and Momentum (e.g., potential and kinetic energy, work, power, and conservation laws)	
D. Miscellaneous (gravity, pressure, simple harmonic motion)	
II. Electricity and Magnetism	22–26
A. Electrostatics (e.g., Coulomb's law and electric field and potential)	
B. Circuits (e.g., Ohm's law and simple DC current circuits with resistors and capacitors)	
C. Electromagnetism (e.g., magnetic fields and electromagnetic induction)	
III. Waves	15–19
A. General Properties (e.g., speed, frequency, and wavelength)	
B. Geometrical Optics (e.g., reflection, refraction, and lenses)	
C. Physical Optics (e.g., interference, polarization, and diffraction)	
IV. Heat, Kinetic Theory, and Thermodynamics	8–12
A. Thermal Properties (e.g., temperature, heat transfer, and mechanical equivalent of heat)	
B. Kinetic Theory (e.g., ideal gas law)	
C. Thermodynamics (e.g., first and second laws)	
V. Modern Physics	8–12
A. Atomic (e.g., models and atomic energy levels)	
B. Nuclear (e.g., radioactivity and nuclear reactions)	
C. Relativity (mass-energy equivalence, photons, and photoelectric effect)	
VI. Miscellaneous (measurement, math and laboratory skills, and history of physics)	2–4

Although 75 questions is a lot to answer in an hour, that is not a large number of questions when compared with the amount of information that is conveyed in a full-year course in physics. For this reason, not everything you learned in physics can be the basis for a test question. Conversely, you may encounter some questions that test areas not emphasized in your course.

The following are samples of the types of questions you will encounter on your exam:

EXAMPLE:

A radar signal having a frequency of 3,000 megahertz would have a wavelength of

- (A) 0.001 km.
- (B) 0.01 km.
- (C) 10 m.
- (D) 0.1 m.
- (E) 0.01 m.

The correct answer is (D).

$$\text{Wavelength} = \frac{\text{speed}}{\text{frequency}}$$

$$3,000 \text{ megahertz} = 3,000,000,000 \text{ cps}$$

$$\text{Wavelength} = \frac{300,000 \text{ km/sec}}{3,000,000,000 \text{ cps}}$$

$$\text{Wavelength} = .0001 \text{ km or } .1 \text{ m}$$

All other choices are incorrect.

EXAMPLE:

Presuming there are no other outside forces, how much force is necessary to accelerate a 1-kilogram brick 1 meter per second²?

- (A) 0.1 newton
- (B) 1.0 newton
- (C) 10 newtons
- (D) 100 newtons
- (E) 1,000 newtons

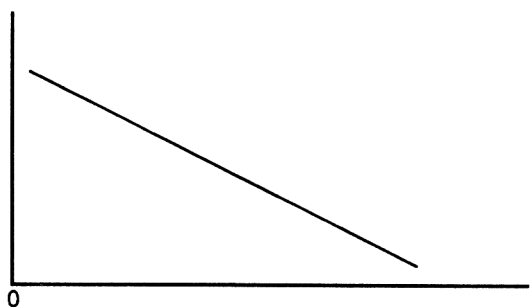
The correct answer is (B). Using $F = ma$, $F = 1 \text{ kg} \times 1 \frac{\text{m}}{\text{sec}^2}$. This, by definition, is the force of 1 newton.

EXAMPLE:

At the end of three half-lives, the amount of an 8-g sample of $^{228}_{88}\text{Ra}$ that remains undecayed is

- (A) 1 g
- (B) 2 g
- (C) 3 g
- (D) 4 g
- (E) 6 g

The correct answer is (A). The half-life of a radioactive atom is the time required for one-half the atoms in a given sample to decay. In this sample, at the end of one half-life, 4 g of the radium will remain undecayed. At the end of two half-lives, 2 g will remain undecayed. At the end of three half-lives, only 1 g will remain undecayed.

**EXAMPLE:**

Which of the following could this graph represent?

- (A) A vibrating string at its fundamental frequency, where frequency of vibration is represented by the vertical axis and length of string by the horizontal axis
- (B) The relationship between force and acceleration, where force (measured in newtons) is represented by the vertical axis, and acceleration (meters per second per second) is represented by the horizontal axis
- (C) The relationship between work and force, where work (measured in joules) is represented by the vertical axis, and force (newtons) is represented by the horizontal axis
- (D) The relationship between the energy (represented by the vertical axis) and the wavelength (represented by the horizontal axis) of light waves
- (E) The relationship between distance and time of an object moving at a constant rate of speed, where time is represented by the horizontal axis and distance by the vertical axis
- (D) As the wavelength of a light wave increases, the amount of energy it has decreases. This shows an inversely proportional relationship as shown on the graph.

EXAMPLE:

A lamp is connected to a circuit that has a current of 0.25 amperes powered by a 4.5 V battery.

The resistance in the lamp is

- (A) 0.0555 ohms
- (B) 1.125 ohms
- (C) 1.8 ohms
- (D) 18 ohms
- (E) 112.5 ohms

The correct answer is (D). Substituting using Ohm's Law, $R = \frac{V}{I} = \frac{4.5V}{.25\text{amp}} = 18 \text{ ohms}$

EXAMPLE:

A gas has a volume of 400 ml at a pressure of 300 mmHg. At a volume of 300 ml, this gas will have a pressure of

- (A) 400 mmHg
- (B) 500 mmHg
- (C) 200 mmHg
- (D) 100 mmHg
- (E) 50 mmHg

The correct answer is (A). By the equation for Boyle's Law, $400 \times 300 = 12,000 = k$. By substituting, we see that $300P = 12,000$, so $P = 400$.

PHYSICS SUBJECT TEST

Part A

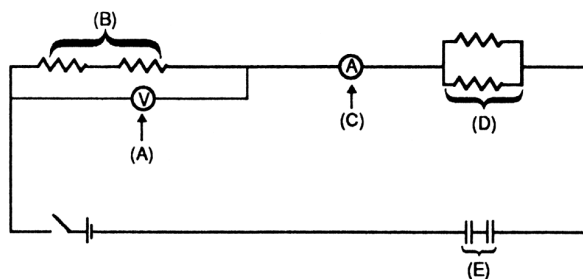
Directions: Each set of lettered answer choices below refers to the numbered questions following it. Select the one lettered answer choice that best fits each question and darken the corresponding circle on the answer sheet. An answer choice may be used once, more than once, or not at all in a given set.

Questions 1–3 relate to the following thermal properties of matter:

- (A) Specific heat
- (B) Heat conduction
- (C) Convection
- (D) Heat capacity
- (E) Radiation

1. Which property involves the transfer of energy by electromagnetic waves?
2. Which term describes the transfer of heat by successive molecular collisions?
3. Which term represents the ratio of the amount of heat added to a body to the temperature change of the body?

Questions 4–5 relate to the various elements illustrated in the following schematic of an electrical circuit:

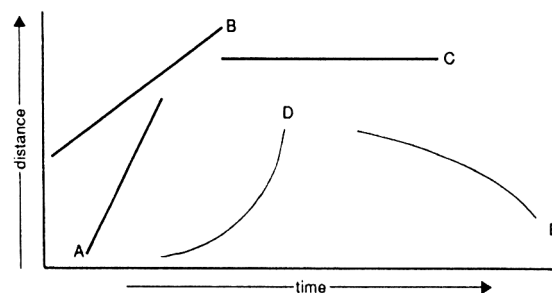


4. Which lettered choice represents resistors in series?
5. Which component measures current?
 - (A) Period
 - (B) Frequency
 - (C) Amplitude
 - (D) Velocity
 - (E) Acceleration

Questions 6–8 refer to an object that is undergoing simple harmonic motion. Assume dissipative forces are negligible.

6. Which term describes the maximal displacement of the object from its equilibrium position?
7. Which property of the object's motion is equal to the time necessary for the object to complete one oscillation?
8. Which property is equal to zero when the displacement of the object equals zero?

Questions 9–11 refer to the movements of five objects. The distance each object travels over a given period of time is shown in the diagram below:



9. Which object has not moved during the charted time period?
10. Which object has the greatest constant velocity?
11. Which object experiences the greatest acceleration?

Questions 12–13 relate to physical and chemical properties of an atom:

- (A) Atomic number
 - (B) Mass number
 - (C) Alpha particle
 - (D) Isotope
 - (E) Beta particle
12. Which term represents the number of particles contained within the nucleus?
13. Which property of an atom best defines its chemical characteristics?

Questions 14–15 relate to properties of geometrical and physical optics:

- (A) Diffraction
 - (B) Refraction
 - (C) Polarization
 - (D) Reflection
 - (E) Aberration
14. Which property refers to the altering of transverse waves so that vibrations are limited to one plane perpendicular to the direction of propagation?
15. Which term describes the bending of light rays as the light passes from one medium to another?

Part B

Directions: Each of the following questions or incomplete statements is followed by five possible answers or completions. Select the one choice that is best in each case and darken the corresponding circle on the answer sheet.

16. An object is traveling in a uniform circular motion with a centripetal force of F . If the radius of the circular path is doubled, how is the centripetal force affected? Assume the object maintains uniform circular motion.

- (A) $\frac{1}{4}F$
- (B) $\frac{1}{2}F$
- (C) F
- (D) $2F$
- (E) $4F$

17. Assuming frictional forces are negligible, the maximum range of an object in projectile motion is dependent on which of the following?

- I. Initial velocity
- II. Initial launching angle
- III. Acceleration due to gravity

- (A) II only
- (B) III only
- (C) I and II only
- (D) II and III only
- (E) I, II, and III

Questions 18–20

A copier machine uses a converging lens system to create copies of an original image. The converging lens may be moved, thereby altering the effective object length. The focal length of the converging lens of this machine is 10 cm.

18. If the machine produces copies that are the same size as the, original, what is the distance between the original and the lens?

- (A) 5 cm
- (B) 10 cm
- (C) 15 cm
- (D) 20 cm
- (E) 40 cm

19. To produce enlargements that are twice the size of the original, what would be the required distance between the original and the lens?

(A) 5 cm
(B) 10 cm
(C) 15 cm
(D) 20 cm
(E) 25 cm

20. An image of the original is not produced. What object distance could account for this occurrence?

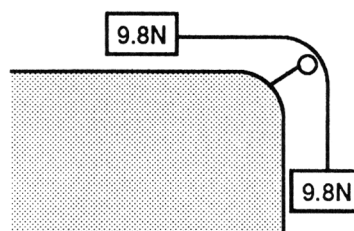
(A) 0.1 cm
(B) 0.5 cm
(C) 1 cm
(D) 5 cm
(E) 10 cm

21. Two liters of a gas are stored within a cylinder. The temperature of the gas is kept constant. The cylinder is then compressed so that the pressure exerted on the gas is increased from 4 atmospheres to 8 atmospheres. What volume will the gas occupy at the higher pressure?

(A) 0.5 liter
(B) 1 liter
(C) 2 liters
(D) 4 liters
(E) 16 liters

22. Which of the following is not a vector quantity?

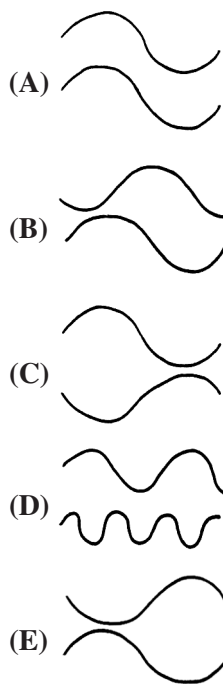
(A) Displacement
(B) Acceleration
(C) Electric field
(D) Density
(E) Momentum



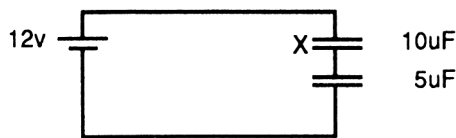
23. Two wooden blocks are connected by a massless string, as illustrated above. Assuming frictionless conditions exist, what is the magnitude of the acceleration of the blocks?

(A) 0.5 m/s^2
(B) 0.9 m/s^2
(C) 1.0 m/s^2
(D) 4.9 m/s^2
(E) 9.8 m/s^2

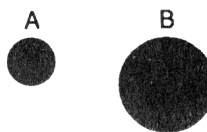
24. The graphs below demonstrate pairs of traveling waves. If the waves in each pair are superimposed, which pair will show the greatest constructive interference?



25. The lines of force near two charges equal in magnitude but opposite in sign are
- (A) closer together where the electric field is weaker
 - (B) directed from the positive charge to the negative charge
 - (C) directed from the negative charge to the positive charge
 - (D) directed away from both the positive and negative charges
 - (E) farther apart where the electric field is stronger
26. The resistance of a uniform conductor is measured to be R . If the length of the conductor is halved and the cross-sectional area is doubled, what will be its new resistance?
- (A) $\frac{1}{8}R$
 - (B) $\frac{1}{4}R$
 - (C) R
 - (D) $4R$
 - (E) $8R$
27. A sample of one isotope of carbon, ^{14}C , is being studied. After a period of 28,000 years, it is determined that only $\frac{1}{32}$ of the sample of ^{14}C remains. What is the approximate half-life of ^{14}C ?
- (A) 875 years
 - (B) 4200 years
 - (C) 5600 years
 - (D) 7000 years
 - (E) 14000 years
28. If a force of 10.0N is exerted over a 4.0 m^2 area, what is the pressure over this area?
- (A) 0.4 Pa
 - (B) 2.5 Pa
 - (C) 4.0 Pa
 - (D) 5.0 Pa
 - (E) 40.0 Pa
29. For the circuit illustrated below, what is the potential difference across the component labeled X?



- (A) 0.8 volt
 - (B) 1.2 volts
 - (C) 4.0 volts
 - (D) 8.0 volts
 - (E) 12.0 volts
30. Liquid flows through a pipe at a rate of $2\text{ m}^3/\text{s}$. The cross-sectional area of the pipe is 1 m^2 . Assuming laminar flow occurs, what is the approximate rate of flow if the pipe diameter is quadrupled?
- (A) $0.1\text{ m}^3/\text{s}$
 - (B) $0.5\text{ m}^3/\text{s}$
 - (C) $2\text{ m}^3/\text{s}$
 - (D) $8\text{ m}^3/\text{s}$
 - (E) $32\text{ m}^3/\text{s}$
31. The gravitational force between two moving spherical objects, labeled A and B, is dependent on each of the following EXCEPT



- (A) mass of object A
- (B) distance separating objects
- (C) mass of object B
- (D) velocity of objects
- (E) gravitational constant

Questions 32–33 A 10kg brick is carried from ground level to the top of a 100m building. A person carrying the brick must walk 150m up a staircase to reach the top of the building. Each step is 0.2m tall. The brick is then dropped from the top of the building. Assume frictional forces are negligible.

32. How much work is done by lifting the object to the top of the building?

- (A) 1000 J
- (B) 1500 J
- (C) 7350 J
- (D) 9800 J
- (E) 14700 J

33. What is the approximate kinetic energy of the object as it reaches ground level?

- (A) 750 J
- (B) 1500 J
- (C) 4900 J
- (D) 7350 J
- (E) 9800 J

34. A light illuminates a metal surface causing photoelectrons to be emitted. Which of the following most specifically accounts for the energies of the emitted electrons?

- (A) Velocity of incident light
- (B) Velocity of emitted electrons
- (C) Size of area illuminated
- (D) Frequency of incident light
- (E) Mass of emitted electrons



35. Two balls are rolled along a frictionless surface, as illustrated above. The balls have the same mass and travel at identical speeds. If the balls meet in a completely elastic collision, each of the following conditions occurs EXCEPT:

- (A) kinetic energy is conserved
- (B) the mass of the system remains constant
- (C) the balls will possess different magnitudes of acceleration
- (D) each ball will possess the same momentum as before the collision
- (E) each ball will possess the same momentum as after the collision

36. The first law of thermodynamics encompasses each of the following ideas EXCEPT:

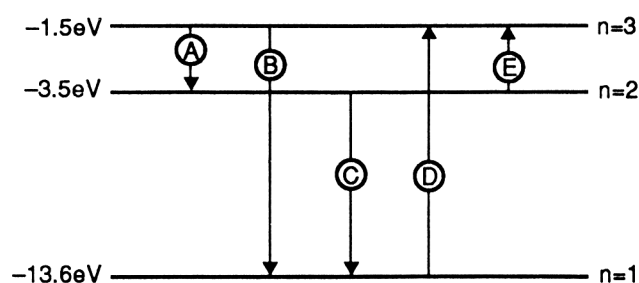
- (A) conservation of energy
- (B) energy cannot be created or destroyed
- (C) energy may be converted into different forms
- (D) likelihood of a process occurring
- (E) change in internal energy of a system equals the difference between the heat absorbed and the work done by the system

37. Which of the following always describe the image produced by a diverging lens?

- I. Virtual
- II. Erect
- III. Smaller than object

- (A) I only
- (B) III only
- (C) I and II only
- (D) II and III only
- (E) I, II, and III

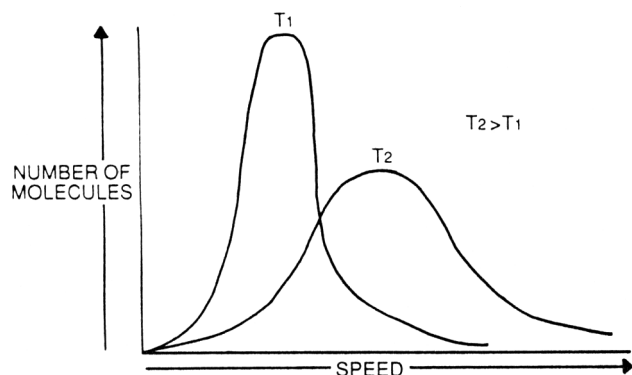
38. The chart below illustrates an energy level diagram for an atom. The labeled arrows depict various transitions between energy states. Which arrow represents the transition that will emit a photon of highest frequency?



- (A) A
- (B) B
- (C) C
- (D) D
- (E) E

GO ON TO THE NEXT PAGE

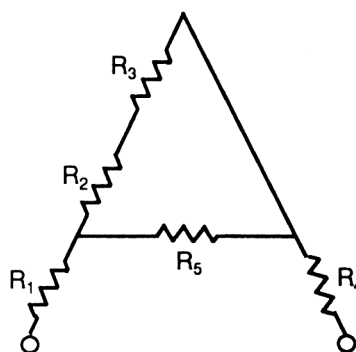
39. A wire and a magnet are positioned or moved in a variety of ways. Which of the following will produce an electromotive force?
- (A) Wire and magnet are stationary, more than 1m apart.
 - (B) Wire and magnet are stationary, separated by 5mm.
 - (C) Wire is moved parallel to stationary magnet.
 - (D) Magnet is moved parallel to stationary wire.
 - (E) Wire is moved perpendicular to magnet's lines of force.
40. A stationary horn broadcasts an emergency distress signal at a constant frequency f . A stationary observer hears the signal and then begins to recede. This observer will perceive
- (A) a signal with frequency f
 - (B) a signal with frequency less than f
 - (C) a signal with frequency greater than f
 - (D) a signal with frequency f initially, with the frequency continually increasing
 - (E) a signal with frequency f initially, with the frequency continually decreasing
41. Which of the following correctly expresses the dimensions of torque? Assume m is mass, l is length, and t is time.
- (A) ml^2t^{-2}
 - (B) $m^2l^2t^2$
 - (C) $ml^{-2}t^{-2}$
 - (D) ml^3t^{-2}
 - (E) ml^2t^{-3}
42. According to the first law of motion, if no net force is acting on an object, which of the following may correctly describe the object's motion?
- I. The object will remain at rest.
 - II. The object will continue moving at a constant velocity.
 - III. The object will continue moving with a constant positive acceleration.
- (A) I only
 - (B) III only
 - (C) I and II only
 - (D) I and III only
 - (E) I, II, and III
43. The formation of a rainbow occurs as a result of
- (A) reflection
 - (B) dispersion
 - (C) luminous intensity
 - (D) luminous flux
 - (E) polarization
44. Two point charges separated by a distance of 0.5m produce an electric force of F . If the distance between the point charges is decreased to 0.05m, the electric force would
- (A) decrease by a factor of 10
 - (B) decrease by a factor of 100
 - (C) remain the same
 - (D) increase by a factor of 10
 - (E) increase by a factor of 100
45. The pascal is a unit of pressure and is defined as one newton per square meter. The cubic meter defines the SI quantity, volume. A pascal multiplied by a cubic meter is a unit of
- (A) work
 - (B) area
 - (C) power
 - (D) force
 - (E) density
46. Sound is a longitudinal wave and may be propagated in each of the following EXCEPT:
- (A) steam
 - (B) water
 - (C) glass
 - (D) vacuum
 - (E) steel



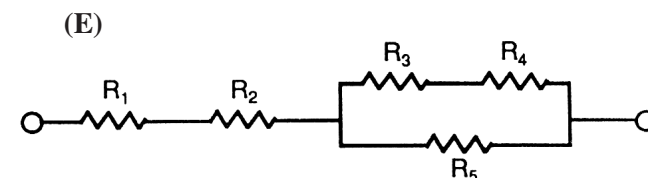
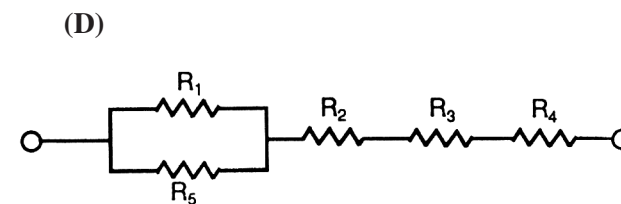
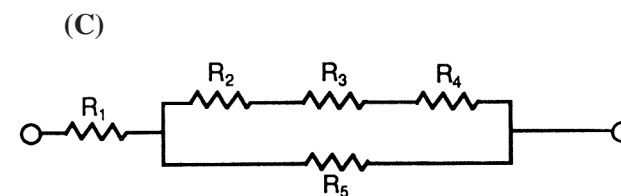
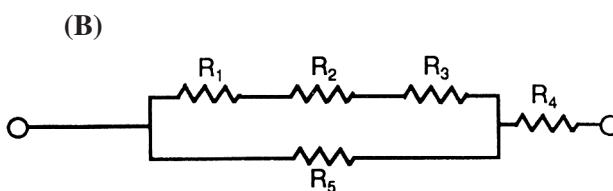
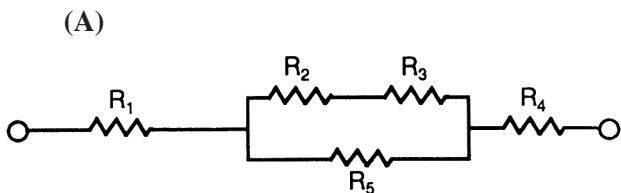
47. The graph above illustrates the distribution of molecular speeds for a pure sample at two different temperatures. The area under the curve represents the percentage of molecules at that range of speed. The same number of molecules are present at each temperature. Which of the following statements accurately reflects the data?

- (A) All the molecules at the higher temperature have a greater speed than the molecules at the lower temperature.
- (B) As the speeds increase the molecular size decreases.
- (C) The distribution of speeds widens as the temperature decreases.
- (D) As the temperature increases, the average molecular speed increases.
- (E) The number of molecules at any given speed is always greater for the sample at the higher temperature.

Questions 48–49 relate to the schematic of the following circuit. Each labeled component provides 5 ohms resistance.

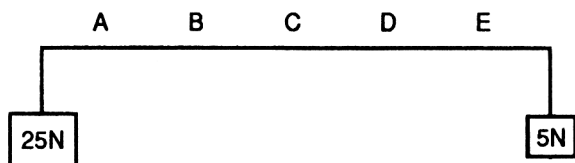


48. Which of the following illustrates a circuit equivalent to the one shown above?

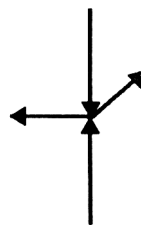







GO ON TO THE NEXT PAGE

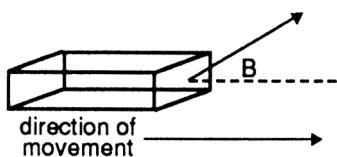
49. If the current flowing across R_4 is 2 amperes, what is the power dissipated by this component?
- (A) 5 W
(B) 10 W
(C) 20 W
(D) 50 W
(E) It is impossible to calculate from the data given.



50. A massless plank supports two objects, as shown above. The labeled areas divide the plank into sections of uniform length. At which point could the plank be lifted without the blocks causing rotation?
- (A) A
(B) B
(C) C
(D) D
(E) E
51. A force is applied to an object through its center of gravity. On the basis of this force, the object possesses which of the following?
- I. Translational acceleration
II. Rotational acceleration
III. Rotational equilibrium
- (A) I only
(B) III only
(C) I and III only
(D) II and III only
(E) I, II, and III

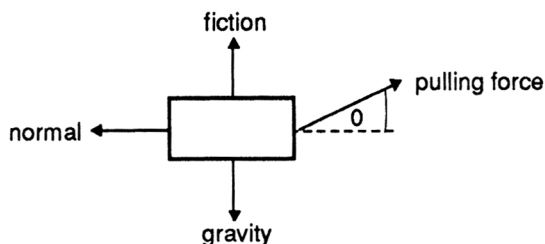


52. Several forces act at a single point, as shown above. Assuming the length of the arrows is proportional to the forces' magnitude, which of the following best represents the net force at the point?
- (A) 
(B) 
(C) 
(D) 
(E) 
53. Ice has a specific heat capacity of 0.5 kcal/kg°C. If 10 kcal of energy is added to 10kg of ice at -10°C , the result would be
- (A) 10kg of ice at -8°C
(B) 10kg of ice at 0°C
(C) 10kg of water at 0°C
(D) 10kg of water at 1°C
(E) 10kg of water at 10°C
54. Air moving past a plane wing causes the wing to lift. Which of the following may account for such action?
- (A) Air flow past the upper portion of the wing is slower than air flow past the lower portion of the wing.
(B) The pressure on the lower surface of the wing is greater than the pressure on the upper surface of the wing.
(C) Air flow past the upper and lower portions of the wing is the same.
(D) The pressure differential between the upper and lower wing surfaces is zero.
(E) The pressure on the upper surface of the wing is slightly greater than the pressure on the lower surface of the wing.

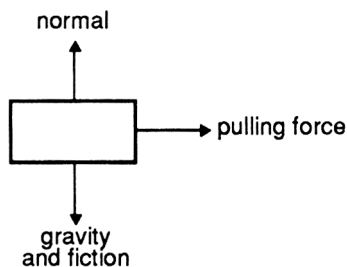


55. A crate is pulled across a floor. The force pulling the crate acts at an angle θ . Which of the following best represents a force diagram of the situation?

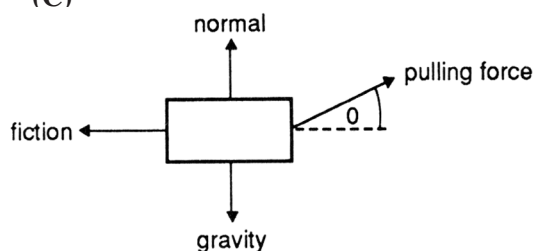
(A)



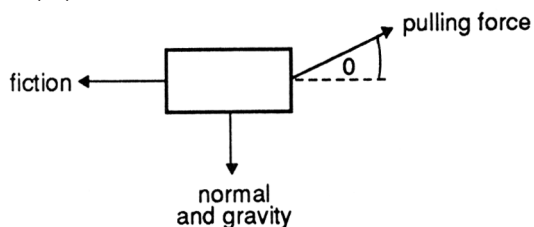
(B)



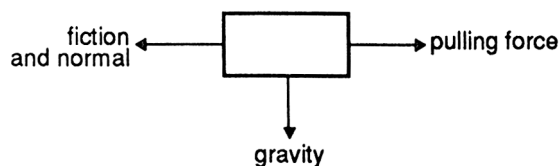
(C)



(D)



(E)

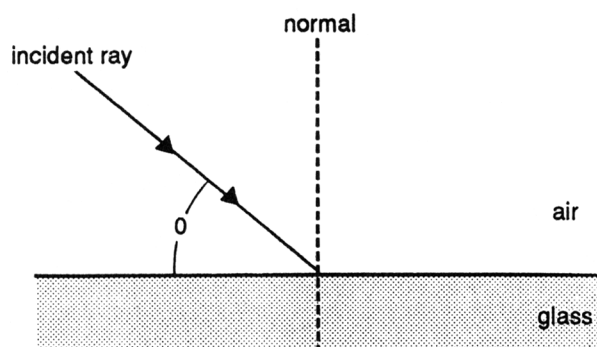


56. A certain satellite is known to have a mass of 700 kg on the earth's surface. If the satellite orbits the earth at a distance of five times the earth's radius, what will be its mass?

(A) 28 kg
(B) 150 kg
(C) 350 kg
(D) 600 kg
(E) 700 kg

57. Each of the following waves travels at the speed of light EXCEPT:

(A) microwaves
(B) ultrasonic
(C) x-rays
(D) radio
(E) gamma rays



58. A light ray impinges upon the surface of a uniform piece of glass, as shown above. What is the angle of reflection?

(A) 0
(B) $90^\circ \angle \theta$
(C) 90°
(D) $180^\circ \angle \theta$
(E) Cannot be determined

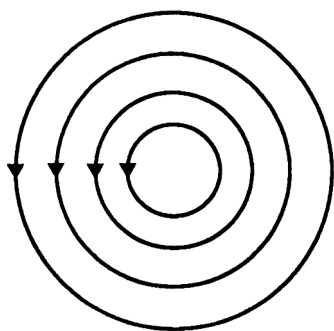
59. The indices of refraction for pairs of media are given below. In each case, light rays originate in medium 1 and are incident upon the surface of medium 2. Total internal reflection cannot occur in which pair?

	Index of Refraction	
	Medium 1	Medium 2
(A)	1.33	1.00
(B)	1.38	1.33
(C)	1.49	1.77
(D)	1.65	1.36
(E)	2.42	1.89

60. Each of the following is a characteristic of an ideal gas EXCEPT:
- (A) random molecular motion occurs
 - (B) elastic collisions occur
 - (C) molecules occupy small portion of volume relative to volume occupied by gas
 - (D) molecules attract each other readily
 - (E) repulsion between molecules does not occur

Question 61–62

The concentric lines of force for a magnetic field due to a current in a straight wire are illustrated below. Assume the page represents a cross-sectional plane of the magnetic field.



61. Which of the following most likely characterizes the direction of the current?
- (A) The current flows into the page.
 - (B) The current flows out of the page.
 - (C) The current flows to the right.
 - (D) The current flows to the left.
 - (E) The current alternates; it flows into the page and then to the right.

62. Another wire is laid parallel to the first straight wire. The current flowing in the second wire is equal in magnitude to the current flowing in the first wire. However, the direction of current flow is exactly opposite. Which of the following correctly describes the force between these wires?

- (A) An attractive force exists.
- (B) A repulsive force exists.
- (C) No force exists.
- (D) Forces generated by each wire are always in the same direction.
- (E) Forces violate Newton's third law of motion.

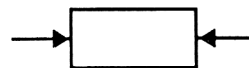
63. An astronaut orbits the earth in a space shuttle. In this orbit, the astronaut feels "weightless." Which of the following best explains such weightlessness?

- (A) Absence of gravity
- (B) Loss of frictional forces
- (C) Astronaut and space shuttle both accelerate toward earth
- (D) Gravitational forces affect space shuttle only
- (E) No external forces act on space shuttle

64. The linear momentum of an object at time t is zero. Which of the following quantities is also necessarily equal to zero?

- I. Velocity
- II. Kinetic energy
- III. Potential energy

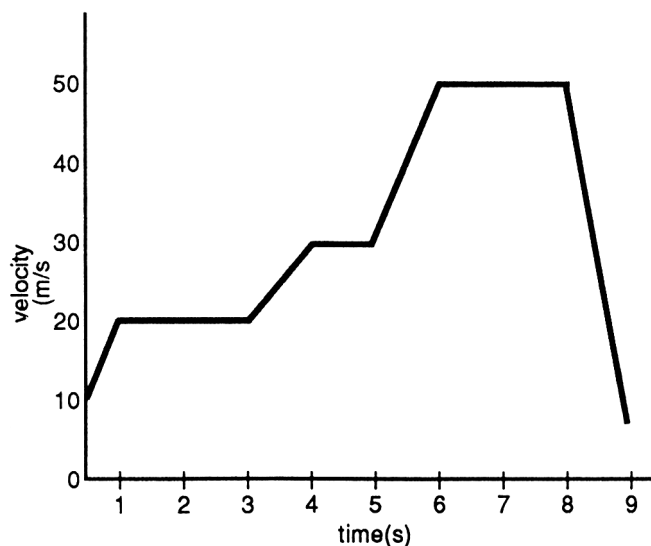
- (A) I only
- (B) II only
- (C) I and II only
- (D) I and III only
- (E) I, II, and III



65. A 10 kg object is acted on by only two forces, as illustrated above. If the object's acceleration is 1 m/s^2 to the left, what is the net force on the object?

- (A) 0.1N to the right
- (B) 0.1N to the left
- (C) 10N to the right
- (D) 10N to the left
- (E) 100N to the left

Questions 66–67 relate to the graph below.

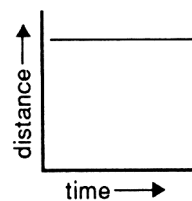


66. The approximate acceleration between the time period 4–5 seconds is

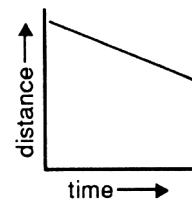
- (A) 0 m/s^2
- (B) 8 m/s^2
- (C) 10 m/s^2
- (D) 20 m/s^2
- (E) 40 m/s^2

67. Which graph below best represents the time period 6–8 seconds?

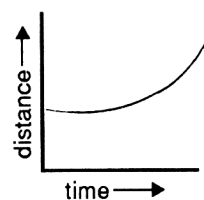
(A)



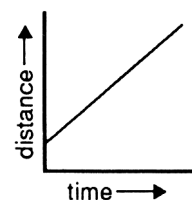
(B)



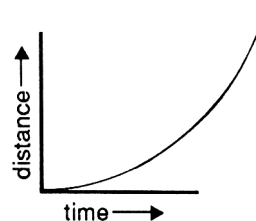
(C)



(D)



(E)



68. The objective lens of a certain microscope has a magnification of 100x, while the eyepiece magnification is 100x. The magnification of this microscope is
 (A) 0.1x
 (B) 110x
 (C) 1000x
 (D) 10000x
 (E) It is impossible to calculate from the data given.
69. A charge of $5.0 \times 10^{-9}\text{C}$ is in a uniform electric field of $5.0 \times 10^4 \text{ N/C}$. A force of magnitude 2.0×10^{-4} is applied but is unable to move the charge against the electric field. How much work is performed on the charge?
 (A) 0 J
 (B) $1.0 \times 10^{-12} \text{ J}$
 (C) $5.0 \times 10^{-8} \text{ J}$
 (D) $2.5 \times 10^{-4} \text{ J}$
 (E) $5.0 \times 10^{-4} \text{ J}$
70. An impulse produced by a 50N force on an object is 2.5NS. How long is the force acting on the object in order to produce such an impulse?
 (A) 0.05 seconds
 (B) 0.5 seconds
 (C) 3.0 seconds
 (D) 20.0 seconds
 (E) 125.0 seconds
- $${}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2 {}^0_1\text{e} + \text{energy}$$
71. The reaction shown above is an example of which of the following nuclear processes?
 (A) Alpha decay
 (B) Fusion
 (C) Fission
 (D) Beta decay
 (E) Nuclear binding reaction
72. A current of 2 amperes flows through a conductor with a resistance of 6 ohms. The current flows for 2 seconds. How much heat energy develops in the conductor as a result of current flow?
 (A) 12 J
 (B) 24 J
 (C) 48 J
 (D) 96 J
 (E) 144 J
73. A person rotates with extended arms on a turntable. Assume dissipative forces are negligible. The person's arms are then folded and held close to his chest. Which of the following correctly describes the result of such an action?
 (A) The person increases his rotational inertia.
 (B) The person's angular velocity increases.
 (C) The person's rotational inertia is unchanged.
 (D) The person's angular velocity is unchanged.
 (E) The person's angular momentum increases.
74. A pith ball electroscope possesses a certain charge. A charged glass rod is then held close to the electroscope. The pith ball moves noticeably toward the glass rod. Which of the following accounts for such an interaction?
 (A) There are subtle gravitational forces between the glass rod and the pith ball.
 (B) The glass rod and the pith ball each possess a net positive charge.
 (C) The glass rod and the pith ball each possess a net negative charge.
 (D) The glass rod and the pith ball are oppositely charged.
 (E) The glass rod mass greatly exceeds the mass of the pith ball.
75. A total of 100 J of energy is supplied to a machine. The machine is then capable of displacing a 30N object 1.5m. What is the approximate efficiency of the machine?
 (A) 15%
 (B) 20%
 (C) 30%
 (D) 45%
 (E) 60%

STOP

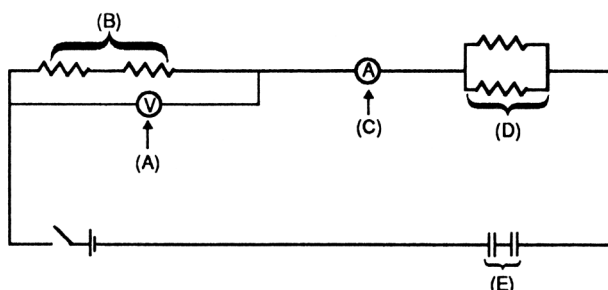
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST ONLY. DO NOT WORK ON ANY OTHER TEST IN THIS BOOK.

ANSWER KEY

1. E	12. B	23. D	34. D	45. A	56. E	67. D
2. B	13. A	24. A	35. C	46. D	57. B	68. C
3. D	14. C	25. B	36. D	47. D	58. B	69. A
4. B	15. B	26. B	37. E	48. A	59. C	70. A
5. C	16. B	27. C	38. B	49. C	60. D	71. B
6. C	17. E	28. B	39. E	50. A	61. B	72. C
7. A	18. D	29. C	40. E	51. C	62. B	73. B
8. E	19. C	30. C	41. A	52. E	63. C	74. D
9. C	20. E	31. D	42. C	53. A	64. C	75. D
10. A	21. B	32. D	43. B	54. B	65. D	
11. D	22. D	33. E	44. E	55. C	66. A	

EXPLANATORY ANSWERS

- The correct answer is (E). Definition.
- The correct answer is (B). Definition.
- The correct answer is (D). Definition.
- The correct answer is (B). See diagram below:



- The correct answer is (C). The ammeter measures current. Recall that the unit of current is the ampere.
- The correct answer is (C). Definition.
- The correct answer is (A). Definition.
- The correct answer is (E). When an object undergoing harmonic motion has a displacement of zero, it is at the equilibrium position. The restoring force will therefore be zero. If the force is zero, then the acceleration must necessarily be zero as well. Recall the relationship:

$$F = ma$$

$$\text{if } F = 0,$$

$$\text{then } ma = 0 \text{ and } \therefore a = 0$$

- The correct answer is (C). The distance value is unchanged.

- 10. The correct answer is (A).** Velocity is defined as the change in displacement (distance in this case) per unit time:

$$v = \frac{\Delta d}{\Delta t}$$

Hence, (A) has the greatest constant velocity as it has the greatest change in distance per unit time. Note that constant velocity is represented by a straight line in this diagram.

- 11. The correct answer is (D).** Acceleration is defined as the change in velocity per unit time:

$$a = \frac{\Delta v}{\Delta t}$$

As lines A, B, and C are straight, each represents a *constant* velocity. Therefore, as acceleration is the *change* in velocity per unit time, the acceleration of A, B, and C is zero. The curvilinear representations—D and E—demonstrate positive and negative accelerations, respectively. Object D has the greatest acceleration as it possesses the greatest change in velocity per unit time.

- 12. The correct answer is (B).** Definition. The number of protons plus the number of neutrons within a given nucleus represents the mass number of the element.
- 13. The correct answer is (A).** The atomic number is the defining characteristic of an atom.
- 14. The correct answer is (C).** Definition.
- 15. The correct answer is (B).** Definition.
- 16. The correct answer is (B).** Centripetal force is defined by the equation:

$$F = \frac{mv^2}{r} \therefore F = \frac{1}{r}$$

Therefore, force is inversely related to the radius. Consequently, if the radius of the circular path is doubled, the force will be halved.

- 17. The correct answer is (E).** As would be expected, the maximum range of an object in projectile motion is dependent on the initial velocity (v_0), the launching angle (θ), and the acceleration due to gravity (g). The equation which defines the horizontal range is

$$\text{Range} = \frac{v_0^2}{g} \sin 2\theta$$

- 18. The correct answer is (D).** A converging lens is said to possess a positive value for the focal length. The thin lens equation relates the focal length, object distance, and image distance as follows:

$$\frac{1}{O} + \frac{1}{i} = \frac{1}{f}$$

O = object distance
 i = image distance
 f = focal length

Another concept used in this set of problems is magnification (m):

$$m = \frac{i}{O}$$

if $i = O$, $m = 1$, image is same size as object

In this question, -the copies (images) are the same size as the original. Thus, magnification equals 1, and i must equal O . The focal length is given as 10cm. Using the first equation:

$$\frac{1}{O} + \frac{1}{i} = \frac{1}{10\text{cm}}$$

and since $O = i$

$$\begin{aligned}\frac{1}{O} + \frac{1}{O} &= \frac{1}{10\text{cm}} \\ \frac{2}{O} &= \frac{1}{10\text{cm}} \\ O &= (2)(10\text{cm}) = 20\text{cm}\end{aligned}$$

- 19. The correct answer is (C).** The images produced are twice the size of the original, so the magnification is 2:

$$m = 2 \text{ and } m = \frac{i}{O}$$

$$i = 2O$$

Substituting into the thin lens equation and solving:

$$\begin{aligned}\frac{1}{f} &= \frac{1}{O} + \frac{1}{i} = \frac{1}{O} + \frac{1}{2O} \\ \frac{1}{10\text{cm}} &= \frac{2}{2O} + \frac{1}{2O} \\ \frac{1}{10\text{cm}} &= \frac{3}{2O} \\ 2O &= (3)(10\text{cm}) \\ O &= \frac{30\text{cm}}{2} = 15\text{cm}\end{aligned}$$

- 20. The correct answer is (E).** No image is produced. The rays of light do not converge to form an image. The image distance then, if the light rays do not converge, can be said to be indefinite. That is, the image distance approaches infinity. If this is the case then,

$$i \rightarrow \infty \text{ then } \frac{1}{i} \rightarrow 0$$

So, if the term $\frac{1}{i}$ approaches zero, and

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{O}$$

then

$$\frac{1}{f} - \frac{1}{O} = \frac{1}{O}$$

rearranging

$$\frac{1}{f} = \frac{1}{O} \text{ or } O = f$$

Therefore, if the object distance equals the focal length, no image is produced.

- 21. The correct answer is (B).** At a constant temperature, Boyle's law states:

$$P_i V_i = P_f V_f \text{ where } \begin{array}{l} P = \text{pressure} \\ V = \text{volume} \\ i = \text{initial state} \\ f = \text{final state} \end{array}$$

Substituting the values given:

$$\begin{aligned} (4 \text{ atm})(2l) &= (8 \text{ atm}) V_f \\ \left(\frac{4 \text{ atm}}{8 \text{ atm}}\right) 2l &= V_f \\ 1l &= V_f \end{aligned}$$

- 22. The correct answer is (D).** Density is not a vector quantity, while each of the other choices represent a vector quantity.
- 23. The correct answer is (D).** The force producing the acceleration of the blocks is provided solely by the suspended block. The mass being accelerated however, is that due to both blocks. Recalling the second law of motion:

$$F = ma$$

If the weight (force) of each block is 9.8N, and the acceleration due to gravity is 9.8 m/s^2 , the mass may be calculated:

$$\begin{aligned} F &= ma \\ 9.8\text{N} &= m(9.8\text{m/s}^2) \\ m &= \frac{9.8\text{N}}{9.8\text{m/s}^2} \\ m &= 1 \text{ kg} \end{aligned}$$

Thus, the total mass being accelerated is 2kg (two blocks), and the force is the weight of the one suspended block, 9.8N. Therefore:

$$\begin{aligned} F &= ma \\ a &= \frac{F}{m} = \frac{9.8\text{N}}{2\text{kg}} = 4.9\text{m/s}^2 \end{aligned}$$

- 24. The correct answer is (A).** The wave trains in answer choices (B), (C), (D), and (E), when superimposed result in destructive interference. That is, the sum of the individual displacements effectively cancel or lower the net amplitude. The wave train pair in answer choice (A), however, adds constructively at each point, to produce an amplitude greater than either wave does alone.
- 25. The correct answer is (B).** The lines of force between a positive charge and a negative charge are directed from the positive charge to the negative charge. The lines of force are also farther apart as the electric field weakens.

- 26. The correct answer is (B).** The resistance of a given conductor is defined by the following relationship:

$$R = P \frac{L}{A} \text{ where } \begin{array}{l} R = \text{resistance} \\ P = \text{resistivity} \\ L = \text{length of conductor} \\ A = \text{cross-sectional area of conductor} \end{array}$$

This relationship makes sense. The longer the conductor, the more likely a given charge will encounter resistance. Also, the smaller the conductor's cross-sectional area, the more likely a given charge will encounter resistance. The resistivity is a constant specific for each conductor. Substituting the new conditions:

$$\begin{aligned} R &= P \frac{\frac{1}{2}L}{\frac{2}{2}A} \\ R &= \frac{1}{4}(\text{original } R) \end{aligned}$$

- 27. The correct answer is (C).** The amount of sample remaining represents the amount left after five half-lives:

$$\left(\frac{1}{2}\right)^n = \frac{1}{32}$$

$$n = 5$$

The time of the half-life for this isotope is:

$$t_{1/2} = \frac{28000_y}{5} = 5600_y$$

- 28. The correct answer is (B).** Pressure is the force per unit area:

$$P = \frac{F}{A}$$

Substituting and solving:

$$P = \frac{10\text{N}}{4\text{m}^2} = 2.5\text{N} / \text{m}^2 = 2.5\text{Pa}$$

- 29. The correct answer is (C).** The potential difference across the labeled capacitor may be calculated from the relationship:

$$Q = CV \quad \text{where} \quad \begin{array}{l} Q = \text{charge} \\ C = \text{capacitance} \\ V = \text{voltage (potential difference)} \end{array}$$

First, the effective capacitance of the circuit must be calculated. Then, the charge may be calculated. Finally, the potential difference across the specific component may be determined.

For capacitors in series:

$$\begin{aligned} \frac{1}{C_{\text{eff}}} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10\mu\text{F}} + \frac{1}{5\mu\text{F}} \\ &= \frac{1}{10\mu\text{F}} + \frac{2}{10\mu\text{F}} \\ &= \frac{3}{10\mu\text{F}} \\ \therefore C_{\text{eff}} &= \frac{10\mu\text{F}}{3} = 3.3\mu\text{F} \end{aligned}$$

The charge on the plates:

$$\begin{aligned} Q &= CV \\ &= 3.3\mu\text{F}(12\text{V}) \\ &= 4.0 \times 10^{-5} \text{ C} \end{aligned}$$

The potential difference across component “X”:

$$V_x = \frac{Q}{C_x} = \frac{4.0 \times 10^{-5} \text{ C}}{10 \times 10^{-6} \text{ F}} = 4.0 \text{ volts}$$

- 30. The correct answer is (C).** The rate of flow (R) through a pipe is defined by the following equation:

$$R = vA \quad \text{where} \quad \begin{array}{l} v \text{ is fluid speed and} \\ A \text{ is cross-sectional area of pipe} \end{array}$$

From the equation of continuity, it is known that the rate of flow through a system (where laminar flow exists) is constant. So

$$\begin{aligned} R_i &= R_f & \text{where } i \text{ and } f \text{ denote initial} \\ \text{or} & & \text{and final conditions} \\ v_i A_i &= v_f A_f \end{aligned}$$

Hence, although the velocity and area may change, the rate of flow will remain constant at 2 m³/s.

- 31. The correct answer is (D).** The law of gravitation is expressed as follows:

$$F = G \frac{m_a m_b}{r^2} \quad \text{where} \quad \begin{array}{l} F = \text{gravitational force} \\ G = \text{gravitational constant} \\ m_a = \text{mass of object a} \\ m_b = \text{mass of object b} \\ r = \text{the distance separating the masses} \end{array}$$

Therefore, it is demonstrated that the velocity of the objects is not a factor.

- 32. The correct answer is (D).** Work is defined as the product of the force applied and the displacement of the object in the direction of the force:

$$W = F \cdot d$$

Thus, the work involved in lifting an object involves the object's weight (its force) and the distance the object is lifted. The distance is simply the height (h) of the building. Substituting:

$$\begin{aligned} W &= F \cdot d \\ &= mgh \\ &= (10\text{k})(9.8\text{m/s}^2)(100\text{m}) \\ &= 9800\text{J} \end{aligned}$$

This value represents the object's potential energy at that height.

- 33. The correct answer is (E).** As frictional forces are negligible, the potential energy will be converted to kinetic energy. Therefore, the potential energy at the top of the building equals the kinetic energy at the bottom.

- 34. The correct answer is (D).** Recall that energy is related to frequency by the relationship:

$$E = hu \quad \text{where } h \text{ is Planck's constant} \\ \text{and } u \text{ is frequency}$$

Therefore, as the frequency of the incident light increases, the energy imparted to the surface will also increase.

- 35. The correct answer is (C).** In a completely elastic collision, kinetic energy is conserved. Momentum (p) is defined by the expression:

$$p = mv$$

Therefore, if the mass and speed of each ball is identical:

$$m_1 v_1 = m_2 v_2$$

However, as each ball has a constant speed, the acceleration is zero for each ball.

- 36. The correct answer is (D).** The first law of thermodynamics does not address the spontaneity of a given process.

- 37. The correct answer is (E).** Diverging lenses produce images with the same characteristics: virtual, erect, and smaller than the object.

- 38. The correct answer is (B).** Arrow B represents a transition resulting in the emission of 12.1eV:

$$-1.5\text{eV} - (-13.6\text{eV}) = 12.1\text{eV}$$

Since energy and frequency are directly related, this transition will also represent a photon with the highest frequency.

- 39. The correct answer is (E).** To produce an electromotive force, the wire must cut across the lines of force of the magnetic field.

- 40. The correct answer is (E).** The person will initially hear a signal with frequency f . As the person moves away from the signal, the frequency will become lower. This situation illustrates the Doppler effect.

- 41. The correct answer is (A).** Torque is defined as the product of the force and moment arm of the applied force:

$$\tau = F \cdot l$$

Force is expressed in newtons, and length in meters. The dimensions of force and length:

$$F = (\text{mass})(\text{acceleration}) \quad m = \text{mass}$$

$$\tau = (m)(l/t^2) \quad l = \text{length}$$

$$t = \text{time}$$

Therefore, the dimensions of torque:

$$= (ml/t^2)(l)$$

$$= ml^2/t^2, \text{ which is equivalent to } ml^2t^{-2}$$

- 42. The correct answer is (C).** As no net force is acting on the object, the acceleration must necessarily equal zero.
- 43. The correct answer is (B).** A rainbow is formed by the dispersion of light. Similarly, the color spectrum produced by shining light through a prism also illustrates this principle.
- 44. The correct answer is (E).** The force between two point charges is described by Coulomb's law:

$$F = k \frac{q_1 q_2}{r^2}$$

Therefore, the force is inversely proportional to the square of the radius. As the distance separating the charges decreases by a factor of 10, the force will increase by a factor of $(10)^2$ or 100:

$$\begin{aligned} F &\propto \frac{1}{r^2} \\ F &\propto \frac{1}{\left(\frac{1}{10r}\right)^2} = \frac{1}{\frac{1}{100r^2}} \\ \therefore 100F &\propto \frac{1}{r^2} \end{aligned}$$

- 45. The correct answer is (A).** (Pressure) (Volume) = ? units

$$\begin{aligned} & (Pa)(m^3) \\ & \left(\frac{N}{m^2}\right)(m^3) = N \cdot m = \text{JOULE} \end{aligned}$$

The joule is the unit of work.

- 46. The correct answer is (D).** Sound waves require a medium for their propagation. Hence, sound waves cannot be propagated in a vacuum.
- 47. The correct answer is (D).** Only choice (D) accurately reflects the data presented.
- 48. The correct answer is (A).** The circuit illustrated in answer choice (A) is equivalent to the schematic illustrated above question 48. The other choices are not equivalent to the "A-shaped" schematic.

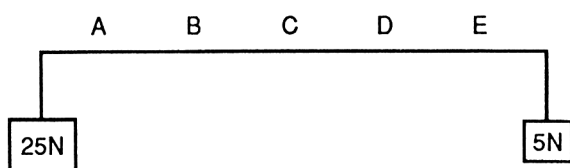
49. The correct answer is (C). Power is defined by the relationship

$$P = i^2 R \quad \text{where } i \text{ is current and} \\ r \text{ is resistance}$$

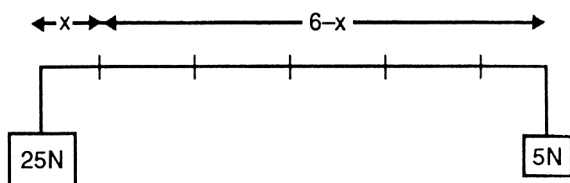
Substituting:

$$\begin{aligned} P &= (2A)^2 (5\Omega) \\ &= (4A^2)(5\Omega) \\ &= 20W \end{aligned}$$

50. The correct answer is (A). For rotation *not* to occur, the clockwise and counterclockwise torques must be equal.



The plank is divided into six sections of equal length. If the plank is lifted at a distance x from one side, then the distance is $6x$ from the other side.



Recall the expression for torque:

$$\tau = F \cdot l$$

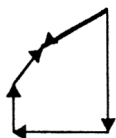
Setting clockwise and counterclockwise torques equal:

$$\begin{aligned} \tau_{cw} &= \tau_{ccw} \\ (5N)(6-x) &= (25N)(x) \\ 30N - 5Nx &= 25Nx \\ 30N &= 30Nx \\ 1 &= x \end{aligned}$$

\therefore The plank should be lifted at one unit length from the left side. This corresponds to (A).

51. The correct answer is (C). The force acts through the object's center of gravity. Therefore the force will produce translational acceleration but will not cause rotational acceleration. Consequently, the rotational equilibrium of the object is unaffected.

52. The correct answer is (E). Adding the vectors graphically:



Choice (E) best represents the result.

- 53. The correct answer is (A).** The amount of heat transferred is described by the equation:

$$Q = mc\Delta T \quad \text{where} \quad \begin{array}{l} q = \text{heat transferred} \\ m = \text{mass} \\ c = \text{specific heat capacity} \\ \Delta T = \text{temperature change} \end{array}$$

Substituting:

$$\begin{aligned} 10 \text{ kcal} &= (10 \text{ kg})(0.5 \text{ kcal/kg} \dots) \Delta T \\ \frac{10 \text{ kcal}}{5 \text{ kcal} / \dots \text{C}} &= \Delta T \\ \Delta T &= 2 \dots \text{C} \end{aligned}$$

Since the ice is initially -10°C , the result will be ice at -8°C .

- 54. The correct answer is (B).** This question describes an application of Bernoulli's law. In the case of an airplane wing, air moves more quickly over the top of the wing, and this causes a lower pressure over the top surface. The pressure differential leads to the "lift" of the wing.
- 55. The correct answer is (C).** Choice (C) best represents the force diagram under real conditions. The pulling force acts at an angle θ , friction opposes the direction of motion, gravity acts toward the center of the earth (downward), and the normal force is as illustrated.
- 56. The correct answer is (E).** Mass is a measure of the amount of matter. It is independent of gravitational effects. Therefore, mass is unchanged.
- 57. The correct answer is (B).** Ultrasonic waves are sound waves. Sound does not travel at the speed of light. Each of the other waves travels at the speed of light.
- 58. The correct answer is (B).** By definition, the angle of reflection equals the angle of incidence. The angle of incidence in this case is $90^\circ - \theta$. Recall, the angle is measured with respect to the normal.
- 59. The correct answer is (C).** Snell's law states

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

For total internal reflection, $\theta_2 = 90^\circ$ and the equation simplifies as $\sin 90^\circ = 1$:

$$\frac{n_2}{n_1} = \sin \theta_1$$

From the above equation, n_2 must be less than n_1 for total internal reflection to occur. (The sine of an angle cannot exceed 1.)

- 60. The correct answer is (D).** For an ideal gas, it is assumed the gas molecules do not interact. That is, the molecules neither attract nor repulse each other. The remainder of the answer choices represent characteristics of an ideal gas.
- 61. The correct answer is (B).** According to the right hand rule, the current will flow out of the page. Allow the fingers of the right hand to curl in the direction of the magnetic field. The extended thumb represents the direction of the current.
- 62. The correct answer is (B).** The forces between parallel currents are attractive if the currents flow in the same direction. If the parallel currents flow in opposite directions, repulsive forces exist.

63. The correct answer is (C). The astronaut, the space shuttle, and all of the shuttle contents experience the same gravitational force. Thus, everything accelerates toward the earth. Therefore, since the shuttle is accelerating toward the earth, it does not “push” back against the astronaut (like the floor of a building does to someone on earth). Thus, the astronaut feels “weightless.”

64. The correct answer is (C). Linear momentum is defined by the expression:

$$p = mv$$

if $p = 0$, v must also $= 0$ ($m \neq 0$)

Since kinetic energy is described by the following equation:

$$\text{K.E.} = \frac{1}{2}mv^2$$

if $v = 0$, $\text{K.E.} = 0$.

Potential energy, however, is independent of the object’s velocity.

65. The correct answer is (D). From the second law of motion:

$$F = ma$$

Since the mass and acceleration are given:

$$F = (10\text{kg})(1\text{m/s}^2)$$

$$F = 10\text{N}$$

The force will produce an acceleration in the direction of the force. Therefore, the net force is 10N to the left.

66. The correct answer is (A). Acceleration is described as the change in velocity per unit time:

$$a = \frac{\Delta v}{\Delta t}$$

In this case, the velocity is unchanged, so the acceleration is zero.

67. The correct answer is (D). In the time period 6–8 seconds, the velocity is constant. Thus, the distance covered continually increases over time. Since the velocity is uniform, the distance versus time graph will be linear with a positive slope.

68. The correct answer is (C). The total magnification of a lens system is simply the product of the magnifying powers of each lens:

$$\begin{aligned}\text{Magnification} &= m_1 \times m_2 \\ &= (100)(10) \\ &= 1000\text{x}\end{aligned}$$

69. The correct answer is (A). Recall, work is the product of the applied force and the resultant displacement:

$$W = F \cdot d$$

for a charge then, $W = Eq \cdot d$

However, since no displacement occurs ($d = 0$), the work performed is zero.

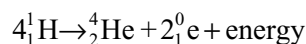
- 70. The correct answer is (A).** Impulse is the product of the force and time interval that the force is applied:

$$\text{Impulse} = F \Delta t$$

Substituting:

$$2.5\text{Ns} = (50\text{N})\Delta t$$

$$\Delta t = \frac{2.5\text{Ns}}{50\text{N}} = 0.05\text{s}$$



- 71. The correct answer is (B).** In the reaction given, four lighter nuclei are combined to form a nuclei with a greater mass. This is an example of fusion.

- 72. The correct answer is (C).** Joule's law describes the heat energy developed in a conductor:

$$W = i^2 R t$$

Substituting:

$$\begin{aligned} W &= (2\text{A})^2 (6\Omega)(2\text{s}) \\ &= 48 \text{ J} \end{aligned}$$

Note:

$$\begin{aligned} (A \cdot \Omega) &= \text{C} \\ (A \cdot \text{s}) &= \text{C} \\ v \cdot c &= \text{J} \end{aligned}$$

- 73. The correct answer is (B).** In the absence of outside forces, angular momentum is conserved. Angular momentum = constant = (angular velocity) \times (rotational inertia). Therefore, if the rotational inertia is decreased by bringing the arms close to the body, the angular velocity must increase.

- 74. The correct answer is (D).** Like charges repel each other, while objects of opposite charge will experience a net attractive force. Gravitational forces are much too weak to account for the observed movement.

- 75. The correct answer is (D).** The efficiency of a machine is described by the relationship:

$$\text{Efficiency} = \frac{\text{Useful work output}}{\text{Work input}}$$

The work input is given (100 J). The work output:

$$\begin{aligned} W &= F \cdot d \\ &= (30\text{N})(1.5\text{m}) \\ &= 45 \text{ J} \end{aligned}$$

Therefore, the efficiency is:

$$\text{Eff} = \frac{45\text{J}}{100\text{J}} = 0.45 \text{ or } 45\%$$