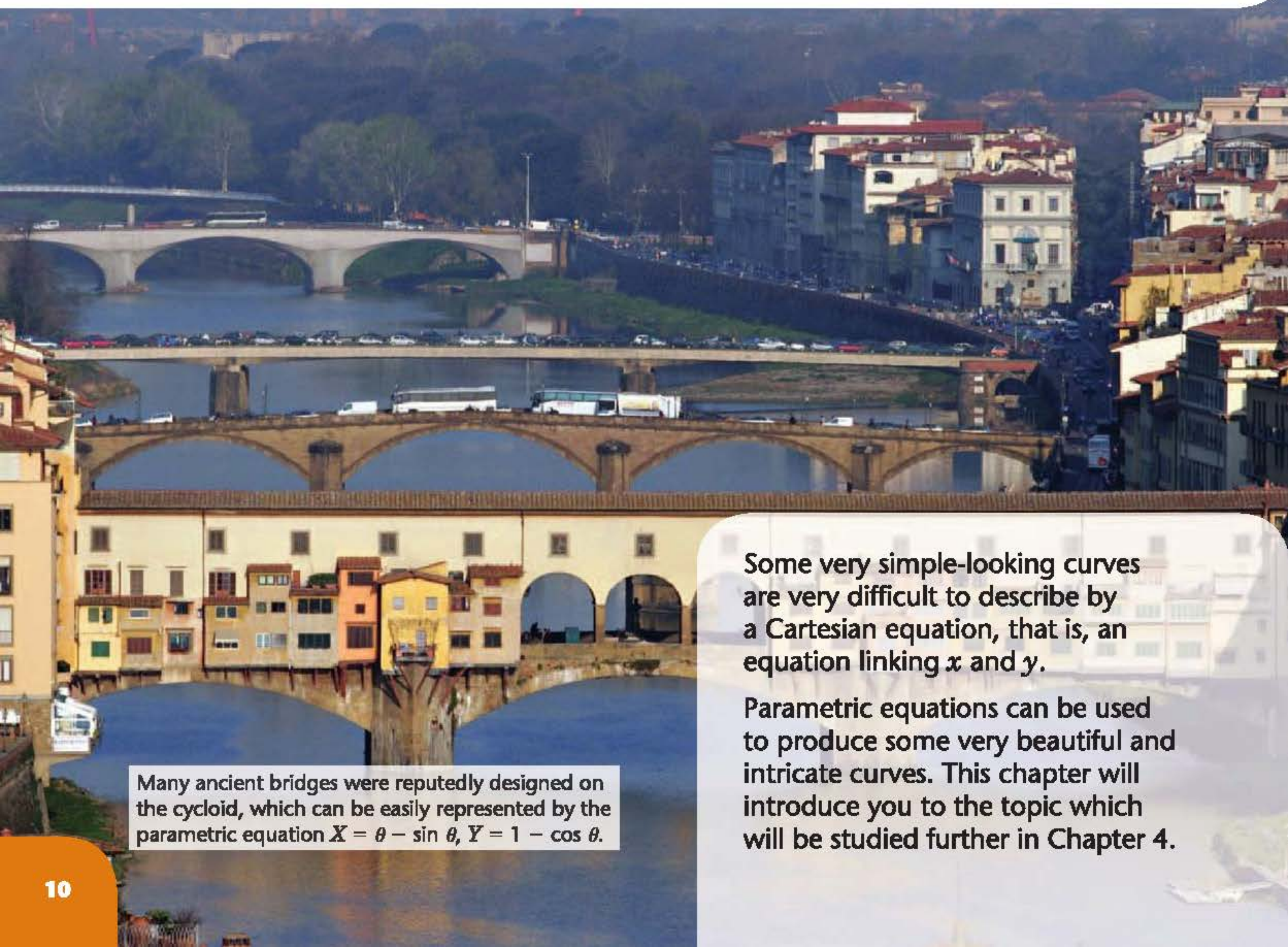
2

After completing this chapter you should be able to:

- sketch the graph of a curve given its parametric equation
- use the parametric equation of a curve to solve various problems including the intersection of a line with the curve
- convert the parametric equation to a Cartesian form
- find the area under a curve whose equation is expressed in parametric form.

Coordinate geometry in the (x, y) plane



Many ancient bridges were reputedly designed on the cycloid, which can be easily represented by the parametric equation $X = \theta - \sin\theta$, $Y = 1 - \cos\theta$.

Some very simple-looking curves are very difficult to describe by a Cartesian equation, that is, an equation linking x and y.

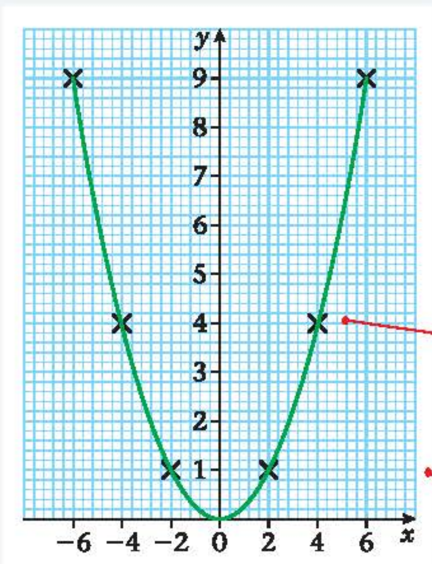
Parametric equations can be used to produce some very beautiful and intricate curves. This chapter will introduce you to the topic which will be studied further in Chapter 4.

2.1 **You can define the coordinates of a point on a curve using parametric equations. In parametric equations, coordinates x and y are expressed as $x = \mathrm{f}(t)$ and $y = \mathrm{g}(t)$, where the variable t is a parameter.**

Example 1

Draw the curve given by the parametric equations $x = 2t$, $y = t^2$, for $-3 \leqslant t \leqslant 3$.

t	-3	-2	-1	0	1	2	3
$x = 2t$	-6	-4	-2	0	2	4	6
$y = t^2$	9	4	1	0	1	4	9



Draw a table to show values of t, x and y. Choose values for t. Here $-3 \leqslant t \leqslant 3$.

Work out the value of x and the value of y for each value of t by substituting values of t into the parametric equations $x = 2t$ and $y = t^2$.

e.g. for $t = 2$:

$$x = 2t \qquad y = t^2$$
$$= 2(2) \qquad = (2)^2$$
$$= 4 \qquad = 4$$

So when $t = 2$ the curve passes through the point $(4, 4)$.

Plot the points $(-6, 9)$, $(-4, 4)$, $(-2, 1)$, $(0, 0)$, $(2, 1)$, $(4, 4)$, $(6, 9)$ and draw the graph through the points.

Example 2

A curve has parametric equations $x = 2t$, $y = t^2$. Find the Cartesian equation of the curve.

$x = 2t$

So $t = \frac{x}{2}$ ①

$y = t^2$ ②

Substitute ① into ②:

$y = \left(\frac{x}{2}\right)^2$

The Cartesian equation is $y = \frac{x^2}{4}$.

A Cartesian equation is an equation in terms of x and y only.

To obtain the Cartesian equation, eliminate t from the parametric equations $x = 2t$ and $y = t^2$.

Rearrange $x = 2t$ for t.
Divide each side by 2.

Substitute $t = \frac{x}{2}$ into $y = t^2$.

Expand the brackets, so that

$\left(\frac{x}{2}\right)^2 = \frac{x}{2} \times \frac{x}{2} = \frac{x^2}{4}$

Exercise 2A

1 A curve is given by the parametric equations $x = 2t$, $y = \frac{5}{t}$ where $t \neq 0$. Complete the table and draw a graph of the curve for $-5 \leqslant t \leqslant 5$.

t	-5	-4	-3	-2	-1	-0.5	0.5	1	2	3	4	5
$x = 2t$	-10	-8				-1						
$y = \frac{5}{t}$	-1	-1.25					10					

2 A curve is given by the parametric equations $x = t^2$, $y = \frac{t^3}{5}$. Complete the table and draw a graph of the curve for $-4 \leqslant t \leqslant 4$.

t	-4	-3	-2	-1	0	1	2	3	4
$x = t^2$	16								
$y = \frac{t^3}{5}$	-12.8								

3 Sketch the curves given by these parametric equations:

a $x = t - 2$, $y = t^2 + 1$ for $-4 \leqslant t \leqslant 4$
b $x = t^2 - 2$, $y = 3 - t$ for $-3 \leqslant t \leqslant 3$
c $x = t^2$, $y = t(5 - t)$ for $0 \leqslant t \leqslant 5$
d $x = 3\sqrt{t}$, $y = t^3 - 2t$ for $0 \leqslant t \leqslant 2$
e $x = t^2$, $y = (2 - t)(t + 3)$ for $-5 \leqslant t \leqslant 5$

4 Find the Cartesian equation of the curves given by these parametric equations:

a $x = t - 2, y = t^2$

b $x = 5 - t, y = t^2 - 1$

c $x = \frac{1}{t}, y = 3 - t, t \neq 0$

d $x = 2t + 1, y = \frac{1}{t}, t \neq 0$

e $x = 2t^2 - 3, y = 9 - t^2$

f $x = \sqrt{t}, y = t(9 - t)$

g $x = 3t - 1, y = (t - 1)(t + 2)$

h $x = \frac{1}{t - 2}, y = t^2, t \neq 2$

i $x = \frac{1}{t + 1}, y = \frac{1}{t - 2}, t \neq -1, t \neq 2$

j $x = \frac{t}{2t - 1}, y = \frac{t}{t + 1}, t \neq -1, t \neq \frac{1}{2}$

5 Show that the parametric equations:

i $x = 1 + 2t, y = 2 + 3t$

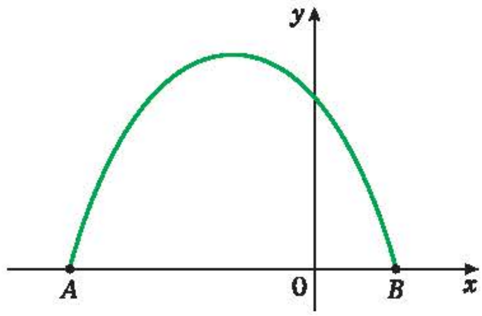
ii $x = \frac{1}{2t - 3}, y = \frac{t}{2t - 3}, t \neq \frac{3}{2}$

represent the same straight line.

2.2 You need to be able to use parametric equations to solve problems in coordinate geometry.

Example 3

The diagram shows a sketch of the curve with parametric equations $x = t - 1, y = 4 - t^2$. The curve meets the x-axis at the points A and B. Find the coordinates of A and B.



① $y = 4 - t^2$

Substitute $y = 0$

$4 - t^2 = 0$

$t^2 = 4$

So $t = \pm 2$

② $x = t - 1$

Substitute $t = 2$

$x = 2 - 1$

$= 1$

Substitute $t = -2$

$x = (-2) - 1$

$= -3$

The coordinates of A and B are $(-3, 0)$ and $(1, 0)$.

Find the values of t at A and B.

The curve meets the x-axis when $y = 0$, so substitute $y = 0$ into $y = 4 - t^2$ and solve for t.

Take the square root of each side. Remember there are two solutions when you take a square root.

Find the value of x at A and B. Substitute $t = 2$ and $t = -2$ into $x = t - 1$.

Example 4

A curve has parametric equations $x = at$, $y = a(t^3 + 8)$, where a is a constant. The curve passes through the point (2, 0). Find the value of a.

1 $y = a(t^3 + 8)$

Substitute $y = 0$

$a(t^3 + 8) = 0$

$t^3 + 8 = 0$

$t^3 = -8$

So $t = -2$

2 $x = at$

Substitute $x = 2$ and $t = -2$

$a(-2) = 2$

So $a = -1$

(The parametric equations of the curve are $x = -t$ and $y = -(t^3 + 8)$.)

The curve passes through (2, 0), so there is a value of t for which $x = 2$ and $y = 0$.

Find t. Substitute $y = 0$ into $y = a(t^3 + 8)$ and solve for t.

Divide each side by a.

Take the cube root of each side. $\sqrt[3]{-8} = -2$.

Find a. Substitute $x = 2$ and $t = -2$ into $x = at$.

Divide each side by -2.

Example 5

A curve is given parametrically by the equations $x = t^2$, $y = 4t$. The line $x + y + 4 = 0$ meets the curve at A. Find the coordinates of A.

1 $x + y + 4 = 0$

Substitute

$t^2 + 4t + 4 = 0$

$(t + 2)^2 = 0$

$t + 2 = 0$

So $t = -2$

Substitute $t = -2$

2 $x = t^2$

$= (-2)^2$

$= 4$

3 $y = 4t$

$= 4(-2)$

$= -8$

The coordinates of A are $(4, -8)$.

Find the value of t at A.
Solve the equations simultaneously.
Substitute $x = t^2$ and $y = 4t$ into $x + y + 4 = 0$.

Factorise.

Take the square root of each side.

Find the coordinates of A.
Substitute $t = -2$ into the parametric equations.

Exercise 2B

1 Find the coordinates of the point(s) where the following curves meet the x-axis:

a $x = 5 + t$, $y = 6 - t$

b $x = 2t + 1$, $y = 2t - 6$

c $x = t^2$, $y = (1 - t)(t + 3)$

d $x = \frac{1}{t}$, $y = \sqrt{(t - 1)(2t - 1)}$, $t \neq 0$

e $x = \frac{2t}{1 + t}$, $y = t - 9$, $t \neq -1$

2 Find the coordinates of the point(s) where the following curves meet the y-axis:

a $x = 2t$, $y = t^2 - 5$

b $x = \sqrt{(3t - 4)}$, $y = \frac{1}{t^2}$, $t \neq 0$

c $x = t^2 + 2t - 3$, $y = t(t - 1)$

d $x = 27 - t^3$, $y = \frac{1}{t - 1}$, $t \neq 1$

e $x = \frac{t - 1}{t + 1}$, $y = \frac{2t}{t^2 + 1}$, $t \neq -1$

3 A curve has parametric equations $x = 4at^2$, $y = a(2t - 1)$, where a is a constant. The curve passes through the point $(4, 0)$. Find the value of a.

4 A curve has parametric equations $x = b(2t - 3)$, $y = b(1 - t^2)$, where b is a constant. The curve passes through the point $(0, -5)$. Find the value of b.

5 A curve has parametric equations $x = p(2t - 1)$, $y = p(t^3 + 8)$, where p is a constant. The curve meets the x-axis at $(2, 0)$ and the y-axis at A.

a Find the value of p.

b Find the coordinates of A.

6 A curve is given parametrically by the equations $x = 3qt^2$, $y = 4(t^3 + 1)$, where q is a constant. The curve meets the x-axis at X and the y-axis at Y. Given that $OX = 2OY$, where O is the origin, find the value of q.

7 Find the coordinates of the point of intersection of the line with parametric equations $x = 3t + 2$, $y = 1 - t$ and the line $y + x = 2$.

8 Find the coordinates of the points of intersection of the curve with parametric equations $x = 2t^2 - 1$, $y = 3(t + 1)$ and the line $3x - 4y = 3$.

9 Find the values of t at the points of intersection of the line $4x - 2y - 15 = 0$ with the parabola $x = t^2$, $y = 2t$ and give the coordinates of these points.

10 Find the points of intersection of the parabola $x = t^2$, $y = 2t$ with the circle $x^2 + y^2 - 9x + 4 = 0$.

2.3 You need to be able to convert parametric equations into a Cartesian equation.

Example 6

A curve has parametric equations $x = \sin t + 2$, $y = \cos t - 3$.

a Find the Cartesian equation of the curve. **b** Draw a graph of the curve.

a $x = \sin t + 2$

Eliminate t from the parametric equations $x = \sin t + 2$ and $y = \cos t - 3$.

So $\sin t = x - 2$

Remember $\sin^2 t + \cos^2 t = 1$.

Rearrange $x = \sin t + 2$ for $\sin t$. Take 2 from each side.

$y = \cos t - 3$

So $\cos t = y + 3$

Rearrange $y = \cos t - 3$ for $\cos t$. Add 3 to each side.

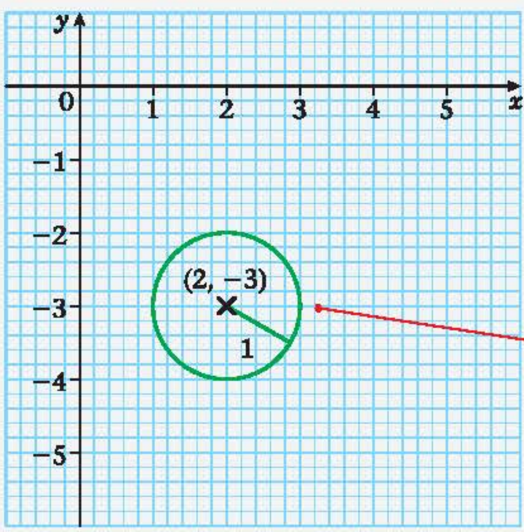
As $\sin^2 t + \cos^2 t = 1$,

the Cartesian equation of the curve is $(x - 2)^2 + (y + 3)^2 = 1$

Square $\sin t$ and $\cos t$ so that
$\sin^2 t = (x - 2)^2$
$\cos^2 t = (y + 3)^2$.

Remember $(x - a)^2 + (y - b)^2 = r^2$ is the equation of a circle centre (a, b), radius r.

b



Compare $(x - 2)^2 + (y + 3)^2 = 1$ with $(x - a)^2 + (y - b)^2 = r^2$.
Here $a = 2$, $b = -3$ and $r = 1$.

So $(x - 2)^2 + (y + 3)^2 = 1$ is a circle centre $(2, -3)$ and radius 1.

Example 7

A curve has parametric equations $x = \sin t$, $y = \sin 2t$. Find the Cartesian equation of the curve.

Eliminate t between the parametric equations $x = \sin t$ and $y = \sin 2t$.

$y = \sin 2t$

$= 2 \sin t \cos t$

Remember $\sin 2t = 2 \sin t \cos t$.

$= 2x \cos t$

$x = \sin t$, so replace $\sin t$ by x in $y = 2 \sin t \cos t$.

$\sin^2 t + \cos^2 t = 1$

Find $\cos t$ in terms of x.
Rearrange $\sin^2 t + \cos^2 t = 1$ for $\cos t$.

$\cos^2 t = 1 - \sin^2 t$

Take $\sin^2 t$ from each side.

$\cos^2 t = 1 - x^2$

$x = \sin t$, so replace $\sin t$ by x in $\cos^2 t = 1 - \sin^2 t$.

$\cos t = \sqrt{1 - x^2}$

Take the square root of each side.

So $y = 2x\sqrt{1 - x^2}$

Replace $\cos t$ by $\sqrt{1 - x^2}$ in $y = 2x \cos t$.

This equation can also be written in the form $y^2 = 4x^2(1 - x^2)$.

Exercise 2C

1 A curve is given by the parametric equations $x = 2 \sin t$, $y = \cos t$.
Complete the table and draw a graph of the curve for $0 \leqslant t \leqslant 2\pi$.

You are unlikely to be asked this kind of question in your examination. However, here it will help your understanding of parametric equations.

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$x = 2 \sin t$			1.73		1.73			−1		−2			0
$y = \cos t$		0.87					−1		−0.5		0.5		

2 A curve is given by the parametric equations $x = \sin t$, $y = \tan t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$. Draw a graph of the curve.

3 Find the Cartesian equation of the curves given by the following parametric equations:

a $x = \sin t$, $y = \cos t$

b $x = \sin t - 3$, $y = \cos t$

c $x = \cos t - 2$, $y = \sin t + 3$

d $x = 2 \cos t$, $y = 3 \sin t$

e $x = 2 \sin t - 1$, $y = 5 \cos t + 4$

f $x = \cos t$, $y = \sin 2t$

g $x = \cos t$, $y = 2 \cos 2t$

h $x = \sin t$, $y = \tan t$

i $x = \cos t + 2$, $y = 4 \sec t$

j $x = 3 \cot t$, $y = \operatorname{cosec} t$

4 A circle has parametric equations $x = \sin t - 5$, $y = \cos t + 2$.

a Find the Cartesian equation of the circle.

b Write down the radius and the coordinates of the centre of the circle.

5 A circle has parametric equations $x = 4 \sin t + 3$, $y = 4 \cos t - 1$. Find the radius and the coordinates of the centre of the circle.

2.4 You need to be able to find the area under a curve given by parametric equations.

■ The area under a graph is given by $\int y\,\mathrm{d}x$. By the chain rule $\int y\,\mathrm{d}x = \int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$.

Example 8

A curve has parametric equations $x = 5t^2$, $y = t^3$. Work out $\int_1^2 y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$.

$y\frac{\mathrm{d}x}{\mathrm{d}t} = t^3\frac{\mathrm{d}x}{\mathrm{d}t}$

Find $y\frac{\mathrm{d}x}{\mathrm{d}t}$. Substitute $y = t^3$.

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(5t^2)$

$= 10t$

Work out $\frac{\mathrm{d}x}{\mathrm{d}t}$. Here $x = 5t^2$.

$\frac{\mathrm{d}}{\mathrm{d}t}(5t^2) = 5 \times 2t^{2-1}$

$= 10t$

So $y\frac{\mathrm{d}x}{\mathrm{d}t} = t^3 \times 10t$

$= 10t^4$

Simplify the expression so that
$t^3 \times 10t = 10t^{3+1}$
$= 10t^4$

$\int_1^2 y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t = \int_1^2 10t^4\,\mathrm{d}t$

$= \left[2t^5\right]_1^2$

Integrate, so that

$\int 10t^4\,\mathrm{d}t = \frac{10}{4+1}t^{4+1}$

$= \frac{10}{5}t^5$

$= 2t^5$

$= 2(2)^5 - 2(1)^5$

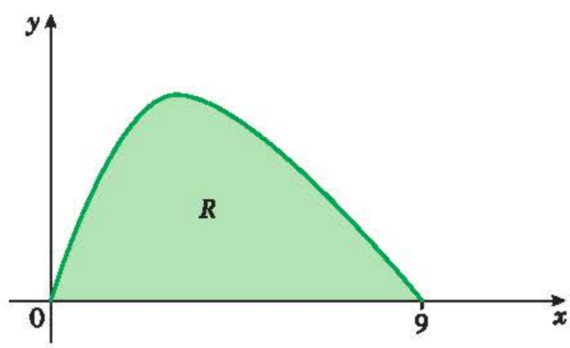
$= 64 - 2$

$= 62$

Work out $\left[2t^5\right]_1^2$. Substitute $t = 2$ and $t = 1$ into $2t^5$ and subtract.

Example 9

The diagram shows a sketch of the curve with parametric equations $x = t^2$, $y = 2t(3 - t)$, $t \geqslant 0$. The curve meets the x-axis at $x = 0$ and $x = 9$. The shaded region R is bounded by the curve and the x-axis.



a Find the value of t when

  i $x = 0$ **ii** $x = 9$

b Find the area of R.

a **i** $x = t^2$

$t^2 = 0$

So $t = 0$

ii $x = t^2$

$t^2 = 9$

So $t = 3$

Substitute $x = 0$ into $x = t^2$.
Take the square root of each side.

Substitute $x = 9$ into $x = t^2$.
Take the square root of each side. $\sqrt{9} = \pm 3$.

Ignore $t = -3$ as $t \geqslant 0$.

b Area of $R = \int_0^9 y\,dx$

$= \int_0^3 y \frac{dx}{dt}\,dt$

$= \int_0^3 2t(3 - t)\frac{dx}{dt}\,dt$

$= \int_0^3 2t(3 - t) \times 2t\,dt$

$= \int_0^3 (6t - 2t^2) \times 2t\,dt$

$= \int_0^3 12t^2 - 4t^3\,dt$

$= \left[4t^3 - t^4\right]_0^3$

$= [4(3)^3 - (3)^4] - [4(0)^3 - (0)^4]$

$= (108 - 81) - (0 - 0)$

$= 27$

The area of $R = 27$.

Integrate parametrically.

Change the limits of the integral.
$t = 0$ when $x = 0$
$t = 3$ when $x = 9$

Find $\int y \frac{dx}{dt}\,dt$. Substitute $y = 2t(3 - t)$.

Work out $\frac{dx}{dt}$. Here $x = t^2$.

$$\frac{dx}{dt} = \frac{d}{dt}(t^2)$$
$$= 2t$$

Expand the brackets, so that

① $2t(3 - t) = 2t \times 3 - 2t \times t = 6t - 2t^2$

② $(6t - 2t^2) \times 2t = 6t \times 2t - 2t^2 \times 2t = 12t^2 - 4t^3$

Integrate each term, so that

① $\int 12t^2\,dt = \frac{12}{3}t^{2+1} = 4t^3$

② $\int 4t^3\,dt = \frac{4}{4}t^{3+1} = t^4$

Work out $\left[4t^3 - t^4\right]_0^3$. Substitute $t = 3$ and $t = 0$ into $4t^3 - t^4$ and subtract.

Exercise 2D

1 The following curves are given parametrically. In each case, find an expression for $y\frac{dx}{dt}$ in terms of t.

a $x = t + 3,\ y = 4t - 3$ **b** $x = t^3 + 3t,\ y = t^2$

c $x = (2t - 3)^2,\ y = 1 - t^2$ **d** $x = 6 - \frac{1}{t},\ y = 4t^3,\ t > 0$

e $x = \sqrt{t},\ y = 6t^3,\ t \geqslant 0$ **f** $x = \frac{4}{t^2},\ y = 5t^2,\ t < 0$

g $x = 5t^{\frac{1}{2}},\ y = 4t^{-\frac{3}{2}},\ t > 0$ **h** $x = t^{\frac{1}{3}} - 1,\ y = \sqrt{t},\ t \geqslant 0$

i $x = 16 - t^4,\ y = 3 - \frac{2}{t},\ t < 0$ **j** $x = 6t^{\frac{2}{3}},\ y = t^2$

2 A curve has parametric equations $x = 2t - 5,\ y = 3t + 8$. Work out $\int_0^4 y\frac{dx}{dt}\,dt$.

3 A curve has parametric equations $x = t^2 - 3t + 1,\ y = 4t^2$. Work out $\int_{-1}^5 y\frac{dx}{dt}\,dt$.

4 A curve has parametric equations $x = 3t^2,\ y = \frac{1}{t} + t^3,\ t > 0$. Work out $\int_{0.5}^3 y\frac{dx}{dt}\,dt$.

5 A curve has parametric equations $x = t^3 - 4t,\ y = t^2 - 1$. Work out $\int_{-2}^2 y\frac{dx}{dt}\,dt$.

6 A curve has parametric equations $x = 9t^{\frac{4}{3}},\ y = t^{-\frac{1}{3}},\ t > 0$.

a Show that $y\frac{dx}{dt} = a$, where a is a constant to be found.

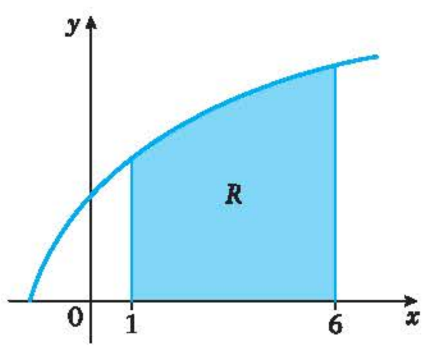
b Work out $\int_3^5 y\frac{dx}{dt}\,dt$.

7 A curve has parametric equations $x = \sqrt{t},\ y = 4\sqrt{t^3},\ t > 0$.

a Show that $y\frac{dx}{dt} = pt$, where p is a constant to be found.

b Work out $\int_1^6 y\frac{dx}{dt}\,dt$.

8 The diagram shows a sketch of the curve with parametric equations $x = t^2 - 3,\ y = 3t$, $t > 0$. The shaded region R is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 6$.



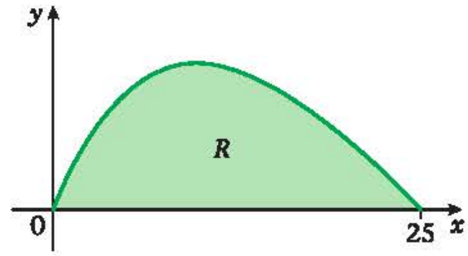
a Find the value of t when

i $x = 1$

ii $x = 6$

b Find the area of R.

9 The diagram shows a sketch of the curve with parametric equations $x = 4t^2$, $y = t(5 - 2t)$, $t \geqslant 0$. The shaded region R is bounded by the curve and the x-axis. Find the area of R.



10 The region R is bounded by the curve with parametric equations $x = t^3$, $y = \dfrac{1}{3t^2}$, the x-axis and the lines $x = -1$ and $x = -8$.

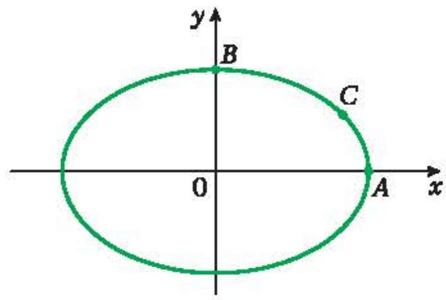
a Find the value of t when

i $x = -1$ **ii** $x = -8$

b Find the area of R.

Mixed exercise 2E

1 The diagram shows a sketch of the curve with parametric equations $x = 4\cos t$, $y = 3\sin t$, $0 \leqslant t < 2\pi$.

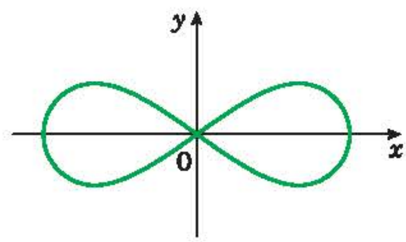


a Find the coordinates of the points A and B.

b The point C has parameter $t = \dfrac{\pi}{6}$. Find the exact coordinates of C.

c Find the Cartesian equation of the curve.

2 The diagram shows a sketch of the curve with parametric equations $x = \cos t$, $y = \frac{1}{2}\sin 2t$.



$0 \leqslant t < 2\pi$. The curve is symmetrical about both axes.

a Copy the diagram and label the points having parameters $t = 0$, $t = \dfrac{\pi}{2}$, $t = \pi$ and $t = \dfrac{3\pi}{2}$.

b Show that the Cartesian equation of the curve is $y^2 = x^2(1 - x^2)$.

3 A curve has parametric equations $x = \sin t$, $y = \cos 2t$, $0 \leqslant t < 2\pi$.

a Find the Cartesian equation of the curve.

The curve cuts the x-axis at $(a, 0)$ and $(b, 0)$.

b Find the value of a and b.

4 A curve has parametric equations $x = \dfrac{1}{1 + t}$, $y = \dfrac{1}{(1 + t)(1 - t)}$, $t \neq \pm 1$.

Express t in terms of x. Hence show that the Cartesian equation of the curve is $y = \dfrac{x^2}{2x - 1}$.

5 A circle has parametric equations $x = 4\sin t - 3$, $y = 4\cos t + 5$.

a Find the Cartesian equation of the circle.

b Draw a sketch of the circle.

c Find the exact coordinates of the points of intersection of the circle with the y-axis.

6 Find the Cartesian equation of the line with parametric equations $x = \dfrac{2-3t}{1+t}$, $y = \dfrac{3+2t}{1+t}$, $\quad t \neq -1$.

7 A curve has parametric equations $x = t^2 - 1$, $y = t - t^3$, where t is a parameter.

a Draw a graph of the curve for $-2 \leqslant t \leqslant 2$.

b Find the area of the finite region enclosed by the loop of the curve.

8 A curve has parametric equations $x = t^2 - 2$, $y = 2t$, where $-2 \leqslant t \leqslant 2$.

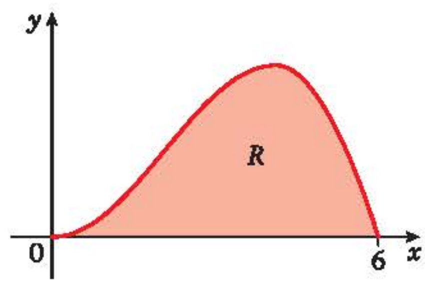
a Draw a graph of the curve.

b Indicate on your graph where **i** $t = 0$ **ii** $t > 0$ **iii** $t < 0$

c Calculate the area of the finite region enclosed by the curve and the y-axis.

9 Find the area of the finite region bounded by the curve with parametric equations $x = t^3$, $y = \dfrac{4}{t}$, $t \neq 0$, the x-axis and the lines $x = 1$ and $x = 8$.

10 The diagram shows a sketch of the curve with parametric equations $x = 3\sqrt{t}$, $y = t(4 - t)$, where $0 \leqslant t \leqslant 4$. The region R is bounded by the curve and the x-axis.



a Show that $y\dfrac{dx}{dt} = 6t^{\frac{1}{2}} - \frac{3}{2}t^{\frac{3}{2}}$.

b Find the area of R.

Summary of key points

1 To find the Cartesian equation of a curve given parametrically you eliminate the parameter t between the parametric equations.

2 To find the area under a curve given parametrically you use $\int y \dfrac{dx}{dt}\,dt$.