$\frac{(x-1)^2}{2} + (y+1) + 3(z+1) = 0$

$\frac{(x-1)}{2} + 4(y+1) + 6(z+1) = 0$

MA 26100
FINAL EXAM Form 01
December 12, 2018

NAME Leaderboard YOUR TA'S NAME XYZ

STUDENT ID # 00 00 00 00 RECITATION TIME 1:34 pm

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): **01**

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **20** questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 9:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 9:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: L

STUDENT SIGNATURE: X

$= -9$

1. Which of the following pairs of planes are orthogonal to each other?

$\langle 1, 10, -1 \rangle$ $\langle -9, -1, -19 \rangle$ $= 0$

A. $x + 10y - z = 6,\ -9x - y - 19z = 2$

B. $5x + 8y = -3,\ y + 6z = 1$

C. $x = 5z + 3y,\ 8x - 6y + 2z = -1$

D. $8x + 5y = -3,\ 9y + 6z = -1$

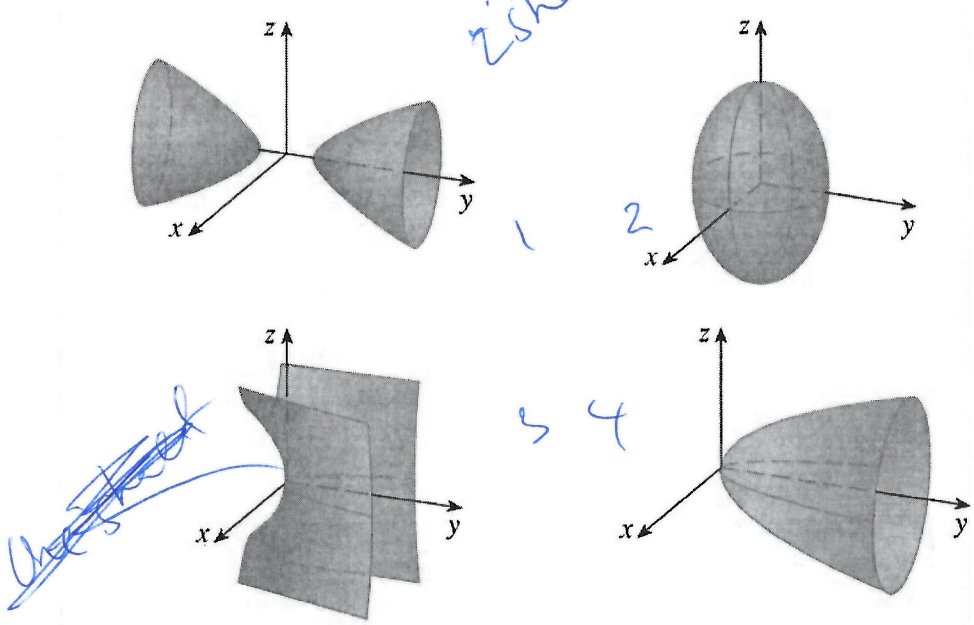
E. $8x + 5y = -3,\ y + 6z = -1$

$A \cdot B = 0$

$\langle 8, 5, 0 \rangle$
$\langle 0, 9, 6 \rangle$

$\langle 8, 5, 0 \rangle$
$\langle 0, 1, 6 \rangle$

2. Which of the following equations produces a surface that is NOT shown here?



A. $-x^2 + y^2 - z^2 = 1$ → A (1)

B. $9x^2 + 4y^2 + z^2 = 1$ → (2)

C. $y = x^2 - z^2$ → (3)

D. $x^2 - y^2 + z^2 = 1$

E. $y = 2x^2 + z^2$ (4)

$1(x) + 6(y-1) - 1(z-4) = 0$

$-z = 0$

3. Find a so that the point $(3, a, 1)$ is on the tangent plane to $z = e^{xy} - 4x^2y + 3y^2$ at $(0, 1, 4)$.

$3 + 6(a-1) + 3 = 0$

$a = 0$

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $-\frac{1}{7}$

D. 0

E. $\frac{1}{6}$

$\nabla f = \langle y e^{xy} - 8xy, x e^{xy} - 4x^2 + 6y, -1 \rangle$

$\nabla f_{(0,1,4)} = \langle 1 - 0, 6, -1 \rangle$

4. Find the directional derivative of $f(x, y) = \sqrt{4x^2 + 3y}$ at $(2, 3)$ in the direction of $\vec{i} - 2\vec{j}$

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{1}{\sqrt{5}}$

D. $\frac{11}{\sqrt{5}}$

E. $\frac{11}{5}$

$\left\langle \frac{8x}{2\sqrt{4x^2+3y}}, \frac{3}{2\sqrt{4x^2+3y}} \right\rangle \cdot \left\langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle$

$\left\langle \frac{16}{10}, \frac{3}{10} \right\rangle \cdot \left\langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle$

$\frac{16}{10\sqrt{5}} - \frac{6}{10\sqrt{5}} = \frac{10}{10\sqrt{5}} = \frac{1}{\sqrt{5}}$

5. For the level surface $3y^2z + xz^2 = 10$ find $2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$ at $(1, -1, 2)$.

A. $\frac{4}{5}$

B. $\frac{20}{7}$

C. $\frac{4}{7}$

D. $\frac{1}{5}$

E. $-\frac{4}{7}$

$3y^2 + 2xz \rightarrow 3 + 4$

$6yz \rightarrow -12$

$2\left(\frac{-8/8x}{8/8z}\right) + \left(\frac{-8/8y}{8/8z}\right)$

$= 2\left[\frac{-4}{7}\right] + \left[\frac{12}{7}\right]$

$\frac{4}{7}$

6. Find the minimum value of $f(x, y) = 2x + 3y + 2$ given that $2x^2 + 5xy + 4y^2 = 28$

A. -1

B. -2

C. -3

D. -6

E. -8

f

$f = tg$

$x = -1$

$y = -2$

$\delta f = \rightarrow \delta g$

$2 = \rightarrow [4x + 5y]$

$3 = \rightarrow [5x + 8y]$

$-2 - 6 + 2$

$= -6$

$2 = \rightarrow (m)$

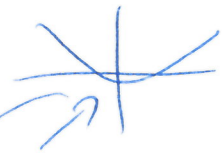
or $x = \frac{1}{7}$

or $\frac{2}{3} = \frac{4x + 5y}{5x + 8y}$

$x = 1$

$y = 2$

7. Let $f(x, y) = (x^2 + y^2)e^x$. The function has



$f'_x = (2x + y^2)e^x + (x^2 + y^2)e^x = e^x(x^2 + 2x + 2y^2)$

$f'_y = (x^2 + 2y)e^x$

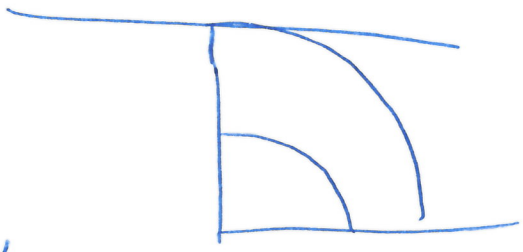
or $x = 0$, $y^2 + 2x = 0$
$y = 0$, $x^2 + 2y = 0$

A. a local max. and a local min. point
B. two local max. points
C. a local max. and a saddle point
D. two local max. points
E. a local min. and a saddle point

and $(0, 0)$

8. Let D be the region in the **first quadrant** between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. Evaluate the integral

$$\int\int_D \frac{x^2 y}{(x^2 + y^2)^{3/2}}\, dA$$



A. $\frac{10}{3}$
B. $\frac{1}{2}$
C. $\frac{3}{2}$
D. $\frac{14}{3}$
E. $\frac{5}{6}$

$$\int_0^{\pi/2}\int_1^2 \frac{r^2\cos^2\theta \cdot r\sin\theta}{r^3} \cdot r\, dr\, d\theta$$

$r^2/2 \to \frac{3}{2}$

$$\int_0^{2\pi}\int_1^2 r\cos^2\theta \sin\theta\, dr\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \sin\theta \cos^2\theta\, d\theta$$

$$\frac{3}{2}\left(\frac{1}{3}\right)\left[\frac{-\cos^3\theta}{3}\right]_0^{\pi/2} = \frac{-1}{2}\left[\frac{\cos^3\theta}{3}\right]_0^{2\pi} =$$

9. Which of the following integrals represents the volume of the solid in the first octant that is bounded on the side by the surface $x^2 + y^2 = 4$ and on the top by the surface $x^2 + y^2 + z = 4$?

$z = 4 - x^2 - y^2$

$y = \sqrt{4 - x^2}$

A. $\displaystyle\int_0^2 \int_0^2 \int_0^{4-x^2-y^2} dz\, dx\, dy$

B. $\displaystyle\int_0^4 \int_0^{\sqrt{4-x^2}} \int_0^{4-z} dz\, dy\, dx$ X

$\theta = 2$

C. $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} dz\, dy\, dx$

D. $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{x^2+y^2} dz\, dy\, dx$ X

E. $\displaystyle\int_0^4 \int_0^4 \int_0^{4-x^2-y^2} dz\, dx\, dy$

not a constant

$\displaystyle\int_0^{\pi} \int_0^2 (15r^2)(4-r^2)\, dr\, d\theta$

10. Convert the integral to cylindrical, then evaluate it:

$$\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} 15\sqrt{x^2+y^2}\, dz\, dy\, dx.$$

$\pi \left[20r^3 - 3r^5 \right]_0^2$

$x^2 + y^2 = r$

A. 4π

B. 16π

C. 32π

D. 43π

E. 64π

$\pi \left[160 - 96 \right]$

$= 64\pi$

$\displaystyle\int_{-2}^{2} \int_0^{\sqrt{4-r^2}} (4-r^2)\, 15r^2\, dr\, d\theta$

11. Compute $\iiint_E z\, dV$, where E is bounded by the sphere $x^2 + y^2 + z^2 = 1$ and the coordinate planes in the first octant.

A. $\frac{\pi}{8}$

B. $\frac{\pi}{16}$

C. $\frac{\pi}{12}$

D. $\frac{\pi}{6}$

E. $\frac{3\pi}{8}$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 r^2 \sin\phi \, d$$

$$\int_0^{\pi/2}$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 r^2 \sin\phi \cdot r\cos\phi \, dr\, d\phi\, d\theta$$

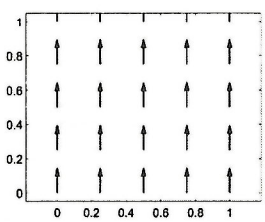
$$= \frac{1}{4}\left[\frac{\pi}{2}\right]\int_0^{\pi/2} \sin\phi \cos\phi \, d\phi$$

$$\frac{\pi}{4}\int_0^{\pi/2} \sin 2\phi \, d\phi$$

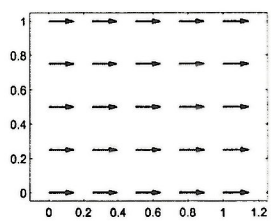
$$\frac{\pi}{32} \times \left(2\right) = \pi/16$$

12. A particle is traveling on the path $y = x$ from $(0,0)$ to $(1,1)$. For which of the following force vector fields is the work done equal to 0?

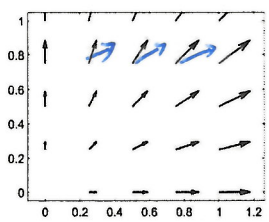
A



B

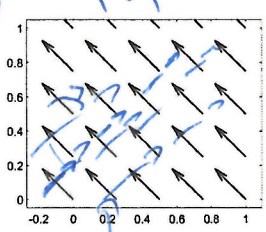


C



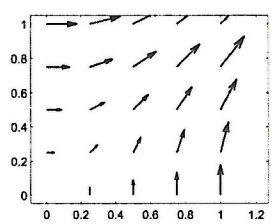
D

$r_1 \cdot r_2 = 0$



$y = x$

E



$F \cdot dr = 0$

$\vec{F} = \quad df/dx = 2y \quad df/dy = -6y$

13. If $\vec{F} = (3+2xy)\vec{i} + (x^2 - 3y^2)\vec{j}$ and $\vec{F} = \vec{\nabla} f$, find $\int_C \vec{\nabla} f \cdot d\vec{r}$ if the curve C is parametrized as $\vec{r}(t) = e^t \sin(t)\vec{i} + e^t \cos(t)\vec{j}$, $0 \le t \le \pi$.

$\frac{\partial f}{\partial y} =$

$F \cdot dr$

$x^2y + A(y)$

$F(r(t)) \cdot r'(t)$

$r'(t) = \langle e^t \sin t + e^t \cos t, e^t \cos t - e^t \sin t \rangle$

$3e^t \sin t + 2(e^t \sin t)(e^t \cos t)$

A. $e^{3\pi} + 1$

B. $-e^{3\pi} - 1$

C. 0

D. $-\pi^3$

E. π^3

14. According to Green's Theorem, which of the following line integrals is NOT equal to the area of the region enclosed by a simple curve C?

$(e^t \sin t)^2 \hat{i} - 3(e^t \cos t)^2 \hat{j}$

$= e^t(3 \sin t + 2 \sin t \cos t)$

$+ e^t(e^t \sin^2 t - 3e^t \cos^2 t)$

$\langle e^t \sin t + e^t \cos t, e^t \cos t - e^t \sin t \rangle$

A. $\frac{1}{2}\int_C -y\,dx + x\,dy$

B. $\int_C x\,dy$

C. $\int_C -y\,dx$

D. $\frac{1}{3}\int_C y\,dx + 4x\,dy$

E. $\frac{1}{5}\int_C 4y\,dx - x\,dy$ $\neq 1$

(13)

$F = \langle f, g \rangle = \nabla \varphi$

$\varphi_x = 3 + 2xy$

$\varphi_y = x^2 - 3y^2$

$\varphi(x,y) = x^2 y + c(y)$

$\varphi_y(x,y) = x^2 + c'(y) = (e^{\pi} \sin \pi)^2 (e^{\pi} \cos \pi) - (e^{\pi} \cos \pi)^3 = e^{3\pi} + 1$

$\cancel{x^2} - 3y^2 = \cancel{x^2} + c'(y)$

$(e^{\pi} \cancel{\sin t})^2 e^{\pi} \cancel{\cos t} - (e^{\pi} \cos t)^3$

$c'(y) = -3y^2 \rightarrow c(y) = -y^3$

$\therefore \varphi(x,y) = x^2 y \cancel{+3y^2} - y^3 \; e$

Fundamental theorem of line integrals

$\therefore \int \left| x^2 y - 3y^3 \right|_S^E = \alpha$

15. Find $grad(div(F)) \cdot curl(F)$ for $F(x, y, z) = xy\vec{i} + yz\vec{j} + xz\vec{k}$ at $(1, -1, 2)$.

$div\, F = \langle y + z + x \rangle$

$= -1 + 1 - 2$

$= -2$

$\nabla(div\, F) = 1 \cdot \langle -y, -z, -x \rangle$

A. -2

B. 0

C. 1

D. 3

E. -4

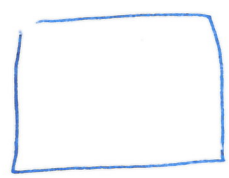
$curl\, F = \begin{vmatrix} i & j & k \\ x' & y' & z' \\ xy & yz & xz \end{vmatrix}$

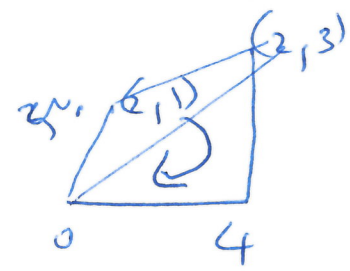
$i(-y) - j(z) + k(x)$

16. Find $\int_C (x + y^3 e^y)dy - 2ydx$ where C goes **clockwise** around the trapezoid with corners $(0, 0)$, $(0, 4)$, $(2, 1)$, $(2, 3)$.

P Q

$\frac{dQ}{dx} - \frac{dP}{dy}$





$y = 2$

$= -3y^2 e^y + y^3 e^y$

$\int_0^{\frac{3}{2}x}$

A. 6

B. -18

C. $-6e^4 + 18e$

D. 18

E. $27e^4 - 18e^2$

$= \iint \frac{dQ}{dx} - \frac{dP}{dy}$

$\int P\,dx + Q\,dy$

$-2y \quad (x + y^3)e^y$

$= 0 \cdot 1 + 2$

$= 3 \times 6 = 18$

17. Find the surface area of the surface with parametric equations

$x = u + v, \quad y = u - v, \quad z = 2v, \quad 0 \le u \le 1, \quad 0 \le v \le 1.$

$\int_0^1 \int_0^1 \sqrt{12}\, dv\, du$

$\langle u, v \rangle$

$\langle u+v, u-v, 2v \rangle$

A. $\sqrt{14}$

B. $\sqrt{22}$

C. $\sqrt{18}$

D. $\sqrt{10}$

E. $\sqrt{12}$

$r'_u = \langle 1, 1, 0 \rangle$

$r'_v = \langle 1, -1, 2 \rangle$

$2\hat{i} - 2\hat{j} - 2\hat{k}$

$\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(2) - \hat{j}(2) + \hat{k}(-2)$

18. If S is that part of the paraboloid $z = x^2 + y^2$ with $z \le 4$, and $\vec{n}$ is the **downward** pointing unit normal, and $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$, then $\iint_S \vec{F} \cdot \vec{n}\, dS =$

Curl F

$\int_0^{2\pi} \int_0^{2} (x^2 + y^2)\, ds$

$= \int_0^{2\pi} \int_0^{2} 3 = 4 \times 2\pi$

$\vec{dn}$

$\langle 2x, 2y, +1 \rangle$

A. 8π

B. -6π

C. 4π

D. -4π

E. 6π

F

$\langle x, y, z \rangle \langle 2x, 2y, -1 \rangle$

$=$

$\langle x, y, x^2 + y^2 \rangle$

$\langle 1, 0, 2x \rangle$

$\langle 0, 1, 2y \rangle$

$\begin{vmatrix} i & j & k \\ 1 & 0 & 2x \\ 0 & 1 & 2y \end{vmatrix}$

3

$\ldots -2x - 2y + 1$

$\int_0^{2\pi} \int_0^{\pi} -2\sin 2t + 2\sin t - 2\cos t \, dt$ $\int\int F \cdot dr = f(r(t)) \cdot r'(t)$

19. Evaluate the integral $\iint_S \text{curl}\vec{F} \cdot d\vec{S}$ using Stoke's Theorem, where $\vec{F} = -y\vec{i} + x\vec{j} + xyz\vec{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the xy-planes, oriented upward.

$= \cos 2t - 2\cos t$ (boundary)

$-2\sin t$ $\quad 0$

$\langle 2\sin t, 2\cos t, 0 \rangle$ $\quad 0$ $\quad xy$

curl F = xy

A. 2π

B. 0

C. 8π

D. -8π

E. 4π

$F(r(t)) \cdot r'(t)$

$\begin{vmatrix} i & j & k \\ x & y & z \\ -y & x & xyz \end{vmatrix}$

$= 1 + 2$

$-1 + 2$

$= \langle -2\cos t$

div F $+ 2\sin t + 4\cos t \sin t$

$\hat{i}(xz - 0) - \hat{j}(yz) + \hat{k}(1+1)$

20. Let $\vec{F} = \langle xy^2 + 1, yz^2 - x, zx^2 + y \rangle$. Use the Divergence Theorem to evaluate $\int\int_S \vec{F} \cdot d\vec{S}$ where S is the boundary surface of the solid

$$E = \{(x, y, z) \,|\, x^2 + y^2 + z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\}$$

with an outward orientation.

← first octant

$= y^2, z^2, x^2$

A. 4π

B. $\frac{16\pi}{5}$

C. $4\pi^2$

D. $\frac{8\pi}{3}$

E. $\frac{8\pi}{7}$

$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{2} r^4 \sin\varphi \, dr \, d\varphi \, d\theta$

$r^5/5$

$= \frac{16}{5} \times \frac{\pi}{2} \int_0^{\pi/2} \sin\varphi \, d\varphi$ $= 16\pi/5$

(29)

$\gamma(t) = \langle 2\cos t, 2\sin t, 0 \rangle$

$\gamma'(t) = \langle -2\sin t, 2\cos t, 0 \rangle$

$F(\gamma(t)) \cdot \gamma'(t))$

$= \langle -2\sin t, 2\cos t, 4\sin t\cos t \rangle$

$\langle -2\sin t, 2\cos t, 0 \rangle$

$= 4$

$\int_0^{2\pi} 4 = \underline{\underline{8\pi}}$