MA 26100
EXAM 2 Form 01
October 31, 2018

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): $\boxed{\textbf{01}}$

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **12** questions, each worth 8 points (you will automatically earn 4 point for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 6:50. If you don't finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

$y=0$

$x=0$

$z=1$

$x+1+x+x$

$=2$

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

$\frac{\partial f}{\partial z}$

$x=y=z-1$

$x = y = \frac{2}{3}$ $z = \frac{1}{3}$ $\partial z = 2z = \lambda$

1. The shortest distance from $(1, 1, 0)$ to the plane $x + y + z = 1$ is

$\therefore D =$

$\sqrt{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}}$ $(x-1)^2 + (y-1)^2 + z^2$

$\Rightarrow (x = y)$

A. $\sqrt{3}/2$
B. $\sqrt{3}$
C. ~~$\sqrt{3}/4$~~
D. $\sqrt{3}/3$
E. 3

$f = \nabla g$

$\frac{\partial f}{\partial x} = 2(x-1) = \lambda$

$\partial y = 2(y-1) = \lambda$

2. The maximum (M) and minimum (m) values of $f(x, y) = 2x+6y$ subject to the constraint $x^2 + y^2 = 10$ are

∂y

$\frac{\partial f}{\partial z} = 2z = \lambda$

A. $M = 12$ and $m = -12$
B. $M = 20$ and $m = -20$
C. $M = 20$ and $m = -12$
D. $M = 12$ and $m = -20$
E. $M = 20$ and no minimum value.

$z = y - 1$ $\quad 2 \quad \frac{6}{\sqrt{10-x^2}}$

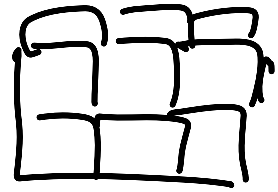
$\sqrt{10-x^2} = 3$

Calculus 1

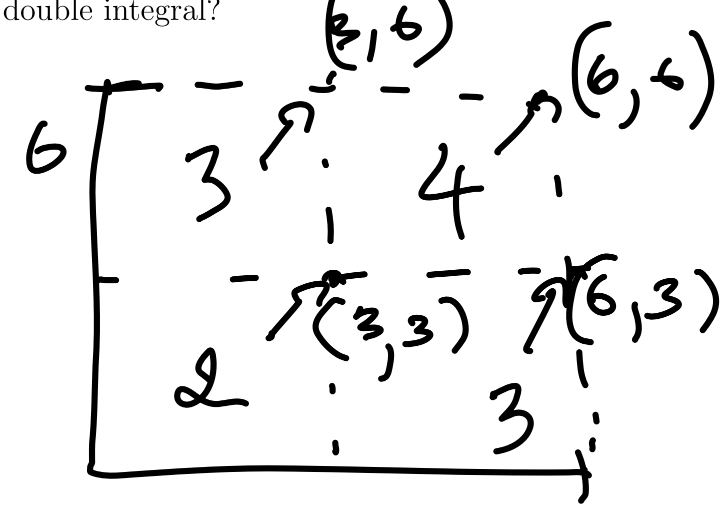
$2 \quad \frac{6}{2\sqrt{10-x^2}} \cdot 2x$

$2x + 6\sqrt{10-x^2}$

$10 - x^2 = 9$



3. We can approximate the double integral $\int_0^6 \int_0^6 \frac{x+y}{3}\, dy\, dx$ with a Riemann sum by partitioning the region $D = \{(x,y) | 0 \le x \le 6, 0 \le y \le 6\}$ into four equal squares. And if we choose the upper right corner of each square as the sample point, which of the following is the approximated value of the double integral?



A. 144

B. 108

C. 72

D. 48

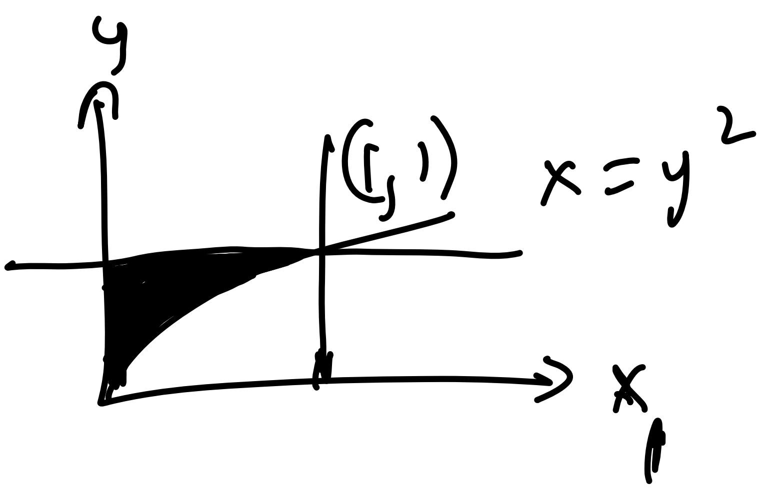
E. 36

$12 \times (9) = 108$

area of each square

4. Change the order of integration and evaluate

$$\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx$$



A. $\frac{1}{2}e$

B. $\frac{1}{2}(e-1)$

C. $\frac{1}{3}e$

D. $\frac{1}{3}(e-1)$

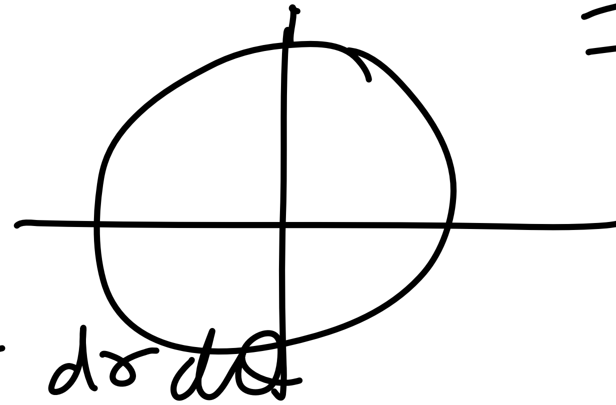
E. e

$\int_0^1 \int_0^{y^2} e^{y^3}\, dx\, dy =$

$$\int_0^1 \left(y^2 e^{y^3}\right)\Big|_0^y \, dy$$

5. Which of the following integrals represents the volume of a solid under $z = x^2 + y^2$ and above the region $x^2 + y^2 = 49$?

$$\left(\frac{1}{3} e^{y^3}\right)\Big|_0^1 = \left(e/3 - 1\right)$$



A. $\displaystyle\int_0^{2\pi}\int_0^{49} r^3 \, dr \, d\theta$

B. $\displaystyle\int_0^{\pi/2}\int_7^{49} r^2 \, dr \, d\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^{7} r^3 \, dr \, d\theta$

$r^2 \cdot r \, dr \, d\theta$

D. $\displaystyle 2\int_0^{\pi}\int_0^{7} r^2 \, dr \, d\theta$

E. $\displaystyle 4\int_{\pi/2}^{\pi}\int_7^{49} r \, dr \, d\theta$

6. What is the mass of a lamina in the shape of a triangle with vertices $(0,0)$, $(1,0)$, and $(0,2)$ if the material density at a point is equal to $\frac{1}{2}$ of the point's distance from the line $x = 1$?

$$\int_0^1\int_0^2 \frac{1}{2}|x-1| \, dy \, dx$$

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

E. 1

$(0,2)$

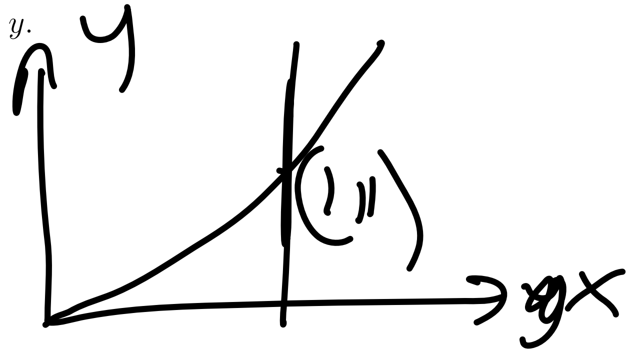
$$= -\int_0^1 (x-1)\,dx$$

$$= -\left(\frac{x^2}{2} - x\right)$$

$$= 1/2$$

$$\int_0^1\int_0^{2} \frac{1}{2}\sqrt{(x-1)^2}\, dy \, dx \quad (1,0)$$

7. Rewrite the iterated integral $\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z)\, dz\, dy\, dx$ by changing the order of integration to first with respect to x, then z, and then y.



A. $\int_0^1 \int_0^y \int_{\sqrt{y}}^1 f(x,y,z)\, dx\, dz\, dy$

B. $\int_0^{x^2} \int_0^y \int_0^1 f(x,y,z)\, dx\, dz\, dy$ X

C. $\int_0^1 \int_0^y \int_0^1 f(x,y,z)\, dx\, dz\, dy$

D. $\int_0^1 \int_y^1 \int_{\sqrt{y}}^1 f(x,y,z)\, dx\, dz\, dy$

E. $\int_0^1 \int_y^1 \int_0^{\sqrt{y}} f(x,y,z)\, dx\, dz\, dy$

$(2-r^2)^2 -$

8. Evaluate the triple integral $\iiint_V 2z\, \mathrm{dV}$, where V is bounded by $z = 2 - x^2 - y^2$ and $z = 1$.

$2\pi \int z \; 2 - x^2 - y^2$

$\int_0 \int_0 r \int_1 2z\, dz\, dr\, d\theta$

A. π

B. $\frac{4\pi}{3}$

C. $1 + \frac{2\pi}{3}$

D. $\frac{2\pi}{3}$

E. $1 + \frac{4\pi}{3}$

$=$

$$\int_0^{2\pi}\int_0^{\sqrt{2}} 2r\left(2-r^2\right)^2 \, dr\, d\theta$$

9. Let E be the solid region bounded by two surfaces whose equations in cylindrical coordinates are $z = 10 - r^2$ and $z = 2 + r^2$. Find the volume of E.

$r = \pm 2 \to r = 2.$

$$\int_0^{2\pi}\int_0^{2} \left(10 - r^2 - 2 - r^2\right) r\, dr\, d\theta$$

A. 32π

B. 8π

C. 18π

~~D. 12π~~

E. 16π

$$\int_0^{2\pi}\int_0^{2} \left(8 - 2r^2\right) r\, dr\, d\theta$$

10. Compute the integral

$$\iiint_E 6e^{(x^2+y^2+z^2)^{3/2}}\, dV$$

where E is the solid region bounded by the sphere $x^2 + y^2 + z^2 = 2$.

$$2\pi \int_0^2 \left(8r - 2r^3\right) dr\, d\theta$$

$$= 2\pi\left[4r^2 - \frac{r^4}{2}\right]_0^2$$

A. $8\pi(e^8 - 1)$

B. $4\pi(e^{2\sqrt{2}} - 1)$

~~C. $3\pi(e^4 - 1)$~~

D. $8\pi(e^{2\sqrt{2}} - 1)$

E. $4\pi(e^8 - 1)$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{2}} 6e^{r^3} \cdot r^2 \sin\varphi \, dr\, d\varphi\, d\theta$$

$$2\pi\left[16 - 8\right] = 16\pi$$

not 32π

$= 2e^{\gamma^3}$ $\quad 2\left[e^{2\sqrt{2}} - 1\right] \to \int_0^{2\pi}\int_0^{\pi} \sin\varphi \, d\varphi \, d\theta$

11. Let $f(x, y, z) = x^2 + xy + z^4 - z$ and let (a, b, c) be a point where $\nabla f(a, b, c) = \langle 3, 5, -5\rangle$. Find the value of $a + b - c$.

$5 - 7 + 1 = -1$

$(-\cos\varphi)\big|_0^{\pi} = 2$

A. -3

B. -2

C. -1

D. 0

E. 1

$2 \cdot 4\pi\left[e^{2\sqrt{2}} - 1\right]$

$\langle 2a + b, a, 4c^3 - 1\rangle$

$4c^3 - 1 = -5$

$4c^3 = -4$

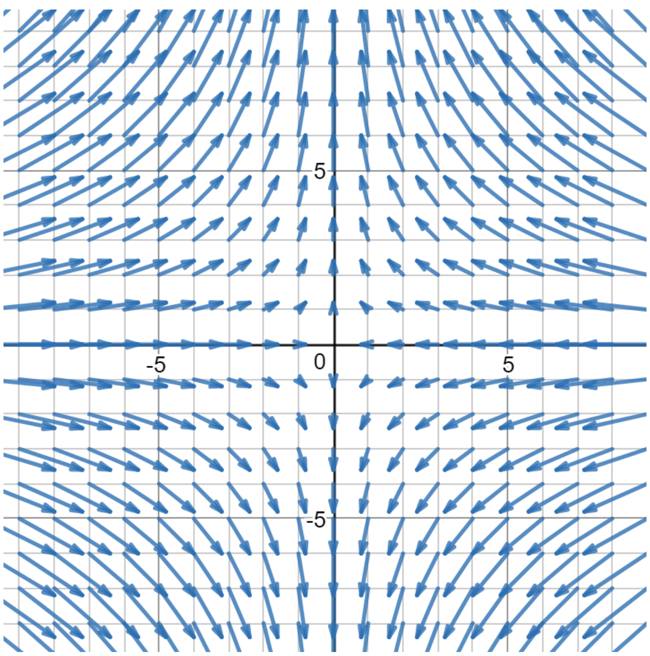
$c = -1$

$a = 5$

$10 + b = 3$

$b = -7$

12. The graph below most closely resembles the gradient vector field of which function?



A. $f(x, y) = xe^y$

B. $f(x, y) = ye^x$

C. $f(x, y) = \dfrac{y}{x}$

D. $f(x, y) = x^2 + y^2 + 10$

E. $f(x, y) = y^2 - x^2 - 10$

$\langle f'(x, y) \rangle = 2y, -2x$