Lesson 31

Leaderboard

1)

$x = (5 \sin u \cos v, 5 \sin u \sin v, 5 \cos u)$

$\frac{5}{\sqrt{2}} \leq z \leq 5$

$x^2 + y^2 + z^2 = 25$

$\Rightarrow x^2 + y^2 = 25 - z^2$

$x^2 + y^2 = 25 - \frac{25}{2}$

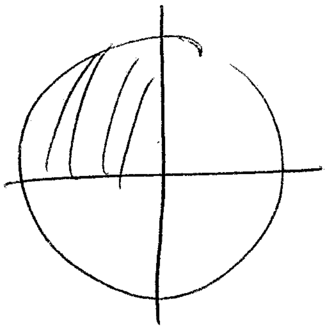
$x^2 + y^2 = \frac{25}{2} \rightarrow \left(\frac{5}{\sqrt{2}}\right)^2$ $\cos\theta = \left(\frac{5/\sqrt{2}}{5}\right)$

$\cancel{u = \sqrt{25/2 - u^2}}$

angle $= \pi/4$ $\therefore$ $0 \leq u \leq \pi/4$

$0 \leq v \leq 2\pi$

3)



$\frac{\pi}{2} \le \theta \le \pi \leftarrow u$ $(x^2+y^2=25$, 2nd quadrant$)$

$v \to 0$ to 5

4)

~~$2v-5$~~

~~$v=2 \to -1$~~

~~$v \to 4 \to 3$~~

$z = 4u+2v-5$ $1 \le u \le 3$

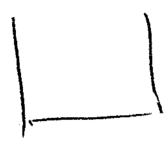
$u = 1$ $2 \le v \le 4$

$v = 2 \to$

$4+4-5=3$

$z \ge 0 \Rightarrow$ lies above x-y plane

$x=u \quad y=v \quad \to$



$z = 4x+2y-5$

square $1 \le x \le 3$

$2 \le y \le 4$

5) $\langle v\cos u, v\sin u, 5v\rangle \to$ cone $h=5$

$z \le 5$

$v=1 \Rightarrow \langle \cos u, \sin u\rangle$

circle, $r=1$.

6) $r(u,v) = \langle 5\cos u,\ 5\sin u,\ v \rangle$

$r'_u(u,v) = \langle -5\sin u,\ 5\cos u,\ 0 \rangle$ $\quad 0 \le u \le \pi$

$r'_v(u,v) = \langle 0, 0, 1 \rangle$ $\quad 0 \le v \le 6$

$$\begin{array}{r} r'_u(u,v) \\ \times r'_v(u,v) \end{array} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5\sin u, & 5\cos u & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \left\| \left(5\cos u\, \hat{i} + 5\sin u\, \hat{j} \right) \right\| = 5$$

$$|r'_u \times r'_v|$$

$$\iint_S 1\, dS = \int_0^6 \int_0^\pi 5\, du\, dv$$

$$= 5\pi \times 6 = 30\pi$$

7)

$r(u,v) = \langle u, v, 19 - u - 3v \rangle$

$r'_u(u,v) = \langle 1, 0, -1 \rangle \qquad r'_v(u,v) = \langle 0, 1, -3 \rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & -3 \end{vmatrix} = \hat{i}(1) + 3\hat{j} + \hat{k}$$

$$|r'_u \times r'_v| = \sqrt{11}.$$

$$\int_{-2}^{2}\int_{-2}^{2} \sqrt{11}\, dx\, dy = \underline{\underline{16\sqrt{11}}}$$

$|x| \le 2 \quad y \le 2$

8) Parameterization, $x^2 + y^2 + z^2 = 64$

$r(u,v)$
$\langle 8 \sin u \cos v, 8 \sin u \sin v, 8 \cos u \rangle$

$r'_u(u,v) = \langle 8 \cos u \cos v, 8 \cos u \sin v, -8 \sin u \rangle$

$r'_v(u,v) = \langle -8 \sin u \sin v, 8 \sin u \cos v, 0 \rangle$

$$\therefore \; c \cdot p \; \text{of} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8\cos u\cos v & 8\cos u\sin v & -8\sin u \\ -8\sin u\sin v & 8\sin u\cos v & 0 \end{vmatrix}$$

$= 64\sin u$

$\cancel{\hat{i}\,(64\sin^2 u}$

$\cancel{\hat{i}\,(64\sin u\cos v\sin v) + \hat{j}\,(64\sin u\sin^2 v)}$

$\cancel{+\hat{k}\,(64\cos^2 v\cos u\sin u + 64\sin^2 v\cos u\sin u)}$

$\cancel{\hat{i}\,(64\sin u\cos v\sin v) + \hat{j}\,(64\sin u\sin^2 v)}$

$\cancel{+\hat{k}\,(64\cos u\sin u)}$

$\cancel{64\left|\sqrt{(\sin u\cos v\sin v)^2 + (\sin u\sin^2 v)^2 + (\cos u\sin u)^2}\right|}$

$\cancel{64\sqrt{\sin^2 u\cos^2 v\sin^2 v + \sin^2 u\sin^4 v + \cos^2 u\sin^2 u}}$

$\cancel{64\sqrt{\cancel{\cos^2 u\sin^2 u\,(\sin^2 v}}}$

$\cancel{\sin^2 u\,(\cos^2 v\sin^2 v + \sin^2 v + \cos^2 v)}$

Surface area =

Limits = ~~from~~

$2 = 8\cos u \Rightarrow z = 7 = 8\cos u \Rightarrow u = \arccos(7/8).$

$8 = 8\cos u \Rightarrow u = 0.$

$$\int_0^{2\pi}\int_0^{\cos^{-1}(7/8)} 64 \sin u \, du \, dv$$

$$= 64 \int \times 2\pi \cdot \left[-\cos u\right]_0^{\cos^{-1} 7/8}$$

$$-128\pi \left[\frac{7}{8} - 1\right] = \frac{128\pi}{8} = \underline{\underline{16\pi}}$$

9) $S = x^2 + y^2 \leq 4 + z^2 = 4$

$\Rightarrow$ (from problem 8)

$|r'_u \times r'_v| = 4 \sin u.$

But $f(x, y, z) = 3x^2 + 3y^2.$

as $x = 2 \sin u \cos v$ and $y = 2 \cos u \cos v$,

$$f(x, y, z) = 3(2 \sin u \cos v)^2 + 3(2 \cos u \cos v)^2$$

$$= \underline{\underline{12 \sin^2 u}}$$

$$\therefore I = \int_0^{2\pi}\int_0^{\pi/2} \underbrace{12\sin^2 u}_{f(x,y,z)} \cdot \underbrace{|t_u \times t_v|}_{4\sin u}\, du$$

no hemel in

s (hemisphere)

$$\int_0^{2\pi}\int_0^{\pi/2} \underbrace{48\sin^3 u}_{\downarrow}\, du$$

$$\sin^2 u \cdot \sin u$$

$$(1-\cos^2 u)\cdot \sin u$$

$$48 \times 2\pi \int_0^{\pi/2} (1-\cos^2 u)\sin u\, du$$

$$\cos u = t \Rightarrow -\sin u\, du = dt$$

$$48 \times -2\pi \int_0^{\pi/2} (t^2-1)\, dt = -2\pi \cdot 48 \times \left(\frac{t^3}{3} - t\right)_0^{\pi/2}$$

$$-2\pi \cdot 48 \times \left(\frac{\cos^3 u}{3} - \cos u\right)_0^{\pi/2}$$

$$-2\pi\left[\frac{1}{3} - 1\right] = 2\pi\left(\frac{2}{3}\right) = \frac{4\pi}{3} \times 48$$

$$= 16 \times 4\pi$$

$$64\pi$$

10)

$S\ \vec{z}\ x^2 + y^2 = 25$

$r(\cancel{x},\cancel{y})\ u, v = \langle 5\cos u,\ \cancel{5\sin u}\ v,\ \cancel{z}\ 5\sin u \rangle,\ \cancel{0 \le z \le 5}\quad 0 \le v \le 2$

$0 \le u \le 2\pi$

$r'_u = \langle -5\sin u,\ 0,\ 5\cos u \rangle$

$r'_v = \langle 0,\ 1,\ 0 \rangle$

$$\therefore |r'_u \times r'_v| \begin{vmatrix} i & j & k \\ -5\sin u & 0 & 5\cos u \\ 0 & 1 & 0 \end{vmatrix} = -5\cos u - \hat{j}(0) + \hat{k}(-5\sin u)$$

$$|r'_u \times r'_v| = \underline{\underline{5}}$$

$f(x,y,z) = 3x \rightarrow 15\cos u$

$$\therefore \int_0^2 \int_0^{2\pi} 15\cos u \times 5\ du\, dv$$

$$75\ q \times 2 \int_0^{2\pi} \cos u\, du = 150\, [\sin u]_0^{2\pi} = \underline{\underline{0}}$$