MA 26100
FINAL EXAM Green
December 13, 2017

NAME Leaderboard YOUR TA'S NAME Someone

STUDENT ID # $\sqrt{\tan^{-1}(\pi/4)}$ RECITATION TIME 11:23pm

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. Be sure the paper you are looking at right now is GREEN!

3. Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes):

00

4. On the mark-sense sheet, fill in your TA's name and the course number.

5. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

6. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

7. Sign the mark–sense sheet.

8. Fill in your name, etc. on this paper (above).

9. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Do all your work on the question sheets.

10. Turn in both the mark–sense sheets and the question sheets when you are finished.

11. If you finish the exam before 9:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20.
If you don't finish before 9:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

12. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

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EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: Leadershood

STUDENT SIGNATURE: SB. X

1. A line L contains the point $(1,\ 4,\ -3)$ and is parallel to ~~to~~ the line

$$x = 5 + 3t, \quad y = 1 - t, \quad z = 1 + 3t.$$

What point on L intersects the plane $y = 0$?

A. (5, 0, 5/3)
B. (0, 0, 0)
C. (−19, 0, −7)
D. (19/4, 0, 7/4)
E. (13, 0, 9)

$\langle 3, -1, 3 \rangle$

$3(x-1) - 1(y-4) + 3(z+3) = 0$

$3x - 3 - y + 4 + 3z + 9 = 0$

$3x - y + 3z + 10 = 0$

$3x + 3z = -10$

$a + tb$

$\langle 1 + 3t,\ 4 - t,\ -3 + 3t \rangle$

$4 - t = 0 \quad t = 4 \quad \langle 13, 0, 9 \rangle$

2. The plane passing through the point $(0, 1, -1)$ and parallel to the plane $x + y + 2z = 3$ intersects the x-axis at the point:

A. $(-1, 0, 0)$
B. $(1, 0, 0)$
C. $(-3, 0, 0)$
D. $(3, 0, 0)$
E. $(2, 0, 0)$

$a(x) + b(y-1) + c(z+1) = 0$

$x + (y-1) + 2(z+1) = 0$

$x + y + 2z + 1 = 0$

$(y + z) = 0$

$x + 1 = 0$

$x = -1$

3. The position of a particle is given by $\mathbf{r}(t) = \langle 2t,\ 1-2t,\ 5+t\rangle$, starting when $t = 0$. After the particle has gone a *distance* of 3, the x-coordinate is

A. $\frac{1}{3}$
B. 3
C. $\frac{1}{2}$
D. 2
E. 1

$\langle 2, 0, 6\rangle$

$\langle 4, -3,$

$t = 0 \to r(0) = \langle 0, 1, 5\rangle$

$1, 0, \frac{25}{5}$ $\quad \frac{2}{5}, \frac{1}{3}, \frac{16}{3}$

$\to$ $|r'(t)| = \sqrt{4t^2 + (1-2t)^2 + (5+t)^2}$

$\sqrt{4t^2 + 1 - 4t + 4t^2 + 25 + 10t + t^2}$

$3 = \sqrt{9t^2 + 6t + 26}$

$9 = 9t^2 + 6t + 26$

$9t^2 + 6t + 17 = 0$

$\frac{-6 \pm \sqrt{36 + 612}}{}$ $\quad 648$

17×36: $102 + 51 = 612$

4. The tangent plane to the level surface $x^2 + 4xy + z^4 = -11$ at the point $(2, -2, 1)$ is given by the equation

A. $2x + 4y + 4x + 3z^3 = 0$
B. $2x + 2y + z = 1$
C. $x - 2y - z = 5$
D. $2x - 2y - z = 7$
E. $-x - 2y + 2z = 4$

tangent slope $\langle 2x + 4y,\ 4x,\ 4z^3\rangle$

at $(2, -2, 1)$

$\cancel{2x} \to$

$\langle -4, 8, 4\rangle$

Only one which fits tangent slope

3

$\cancel{\langle -2x, -2y, 1 \rangle \quad 2 = \sqrt{4 - x^2 - y^2}\ \sqrt{2}}$

$\sqrt{2t, -2t, t}$

$= 3t = 1$

$\cancel{\sqrt{4t^2 + (\ldots) - 4t + 4t^2 + 25 + 10t + t^2}}$

$\cancel{\sqrt{9t^2 + 6t + 26}}$

$\cancel{\sqrt{(3t+1)^2 + 5^2}}$

$9t^2 + 6t = 9$

$9t^2 + 6t - 9 = 0$

$3t^2 + 2t - 3 = 0$

$3t^2 \cancel{+t} - t + 3t - \cancel{3} = 0$

$t(3t - 1) \cancel{+ 3(t-1)} = 0$

$\cancel{3t}$

$3t^2 + 2t - 3 = 0$

$\cancel{3t^2 + 3t - t - 3 = 0}$

$\cancel{3t(t}$

$\cancel{3t^2 - t + 3t - 3} = 0$

$\cancel{\dfrac{-2 \pm \sqrt{4 + 36}}{6}}$

$r(0) = \langle 0, 1, 5 \rangle$

$|D| = \langle 2t, 1 - 2t - 1, 5 + t - 1 \rangle$

$\langle 2t, -2t, t \rangle$

$\to \sqrt{\cancel{9}} = 3t^2 \qquad 3t^2 = 3$

$\underline{t = 1} \quad \to 2t = \underline{2}$

$y = mx$ $y = x$

$\to \frac{x^2}{2x^2}$ so it fails at $(0,0)$

5. Consider the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Which of the following statements are true?

(i) The function is continuous at $(0,0)$. ✗ $x = r\cos\theta$

(ii) $\dfrac{\partial f}{\partial x}(0,0) = 0$. ✓ $y = r\sin\theta$

(iii) The function is differentiable outside of $(0,0)$. ✗

$$\frac{r\cos\theta + r\sin\theta}{r^2} = \left(\frac{\cos\theta + \sin\theta}{r}\right)$$

A. all of the above

B. (ii) and (iii)

C. (i) and (ii)

D. (i) and (iii)

E. only one of these statements is true

$10 + g'(x)\,dg \cdot h(y) + g(x) \cdot h'(y)\,dh$

$3 \cdot 0.1 \cdot 5 + 2 \cdot -1 \cdot 0.2$

$10 + 1.5 - 0.4$

6. Suppose $f(x,y) = g(x)h(y)$, where g and h are continuously differentiable functions of one variable with $g(1) = 2$, $g'(1) = 3$, $h(2) = 5$, and $h'(2) = -1$. Approximate $f(1.1, 2.2)$.

$= 11.1$

A. 10.1

B. 11.9

C. 9.9

D. 10.5

E. 11.1

$f'(x$

$dA = g'(x)\,d\hat{g} + h'(x)\,d\hat{h}$

$10 + [3 \times 0.1] + [-1 \times 0.2]$

$= 10 + 0.3 - 0.2$

10.1

$\frac{1}{\sqrt2}+\frac{1}{\sqrt2}-\frac{1}{\sqrt2}+\frac{1}{\sqrt2}$ half $=\frac{-1}{\sqrt2}$

$x \le 1/\sqrt2$ intuition

$y \le 1/\sqrt2$

7. The number and value of the absolute maxima of the function $f(x, y) = x^2 - xy + y^2$ on the domain $2x^2 + 2y^2 \le 1$ is

A. Two maxima with value 1

B. Two maxima with value $\frac{1}{2}$

C. Four maxima with value $\frac{1}{2}$

D. Four maxima with value $\frac{3}{4}$

E. Two maxima with value $\frac{3}{4}$

$2x^2+2y^2=1$

$x=\frac{1}{\sqrt2}$ $y=0$

$x^2=\frac{1-2y^2}{2}$

$\frac12 - y^2$

$\frac12$

$x=\frac{1}{\sqrt2}$ $y=\frac{1}{\sqrt2}$

$\frac12$

maxima:

$\left(\frac{-1}{\sqrt2}, \frac{+1}{\sqrt2}\right)$

$\left(\frac{1}{\sqrt2}, \frac{-1}{\sqrt2}\right)$

only 2

$f(x,y) = \frac12 - y^2 - \sqrt{\frac12 - y^2}\, y$

$= \frac12 - y^2 - y\frac{\sqrt{1-2y^2}}{2}$

sum $=$

$\frac12 + \frac12 + \frac12 = 3/4$

8. Compute the double integral $\iint_R \cos(x+y)\, dA$, where R is the rectangle $[0, \pi] \times [0, \pi]$.

A. -4

B. -2

C. 0

D. 2

E. 4

$\int_0^{\pi}\int_0^{\pi} \cos(x+y)\,dy\,dx$

$= +\sin(x+y)\Big|_0^{\pi}$

$= \int_0^{\pi} (\sin(\pi+x) - \sin x)\,dx$

$\left. \cos x - \cos(\pi+x)\right|_0^{\pi}$

$\cos\pi - \cos 2\pi - \cos 0 + \cos\pi$

$= -1 -1 -1 -1$

$= -4$

9. Find $\displaystyle\iiint_E yz\,dV$ where E is the solid tetrahedron bounded by the planes $z = 0$, $y = z$, $y = x$, and $x = 1$.

A. $\frac{1}{50}$

B. $\frac{1}{40}$

C. $\frac{1}{30}$

D. $\frac{1}{20}$

E. $\frac{1}{10}$

$$\int_0^1\int_0^x\int_0^y yz\,dz\,dy\,dx$$

$$\int_0^1\int_0^x \left|\frac{yz^2}{2}\right|_0^y dy\,dx$$

$$\int_0^1\int_0^x \frac{y^3}{2}\,dy\,dr = \int_0^1 \left|\frac{y^4}{8}\right|_0^x dr = \int_0^1 \frac{x^4}{8}\,dx = \frac{x}{40}$$

10. The density of a solid sphere at any point is proportional to the point's distance from the center of the sphere. What is the ratio of the mass of a sphere of radius 1 to a sphere of radius 2?

$$\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$$

A. 1/2

B. 1/4

C. 1/8

D. 1/16

E. 1/32

$$D = \frac{M}{V}$$

$$m = D\cdot V$$

$\propto \rho(x,y,z) \propto r$

but

$\iiint \rho(x,y,z) \propto r^4$.

11) $2z = 2 - x - 2y$

$z = 1 - \frac{x}{2} - y$

$r(t)\ \langle 1, 1, \frac{1}{2} \rangle$

$r'(t) = \cancel{\frac{\sqrt{5}}{2}} \sqrt{2 + 1/4} = \frac{3}{2}$

and

$$\int_0^2 \int_0^{1-x/2} \frac{3}{2}\, dx = \frac{3}{2}$$

12)

$= (2-3t)\,(e^{1+2t})$

Part 3: $\cancel{r}$ $\cancel{03}$ $\langle 1, 1, 0 \rangle + \cancel{r}t \langle 2, -1, 0 \rangle$

$\cancel{r(t)} \langle 1+2t, 1-t, 0 \rangle$

$r'(t)\ \langle 2, -1, 0 \rangle$

$\cancel{r(t)} \langle (1-t)\, e^{(1+2t)}, e^{(1+2t)}, 0 \rangle \langle 2, -1, 0 \rangle$

11. Find the area of the portion of the plane $x + 2y + 2z = 2$ that lies in the first octant.

A. 3

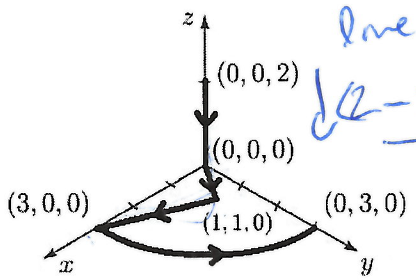
B. $\sqrt{5}/2$

C. $3/2$

D. $3/4$

E. $\sqrt{5}/4$

12. The oriented curve C consists of the line segment from $(0,0,2)$ to $(0,0,0)$, followed by the line segment from $(0,0,0)$ to $(1,1,0)$, followed by the line segment from $(1,1,0)$ to $(3,0,0)$, followed by the circular arc from $(3,0,0)$ to $(0,3,0)$, as shown in the figure below. Find the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$ with vector field $\mathbf{F}(x,y,z) = ye^x\mathbf{i} + e^x\mathbf{j} + 2z\mathbf{k}$.



A. -2

B. -1

C. 0

D. 1

E. 2

Answer for Q12

(I forgot to solve it accidentally)

The key here is *not* to try to split it as multiple lint integrals, as you'll end up wasting your time (which happened to me when I was working it out). Rather, the idea is to make use of the *Fundamental Theorem of Line Integrals*.

First, we find out if F is conservative. Being 3D, we need to take its curl. Let's find that:

$$curl\, F = \begin{vmatrix} i & j & k \\ x' & y' & z' \\ f & g & h \end{vmatrix}$$
$$= i\left(\frac{dh}{dy} - \frac{dg}{dz}\right) - j\left(\frac{dh}{dx} - \frac{df}{dz}\right) + k\left(\frac{dg}{dx} - \frac{df}{dy}\right)$$
$$= i(0-0) - j(0-0) + k(e^x - e^x) = 0$$

Hence, F is conservative, and we have a ϕ such that $F = \nabla\phi$. Then we need to find a potential.

Start by taking the gradient of F.

$$F =< ye^x, e^x, 2z >$$
$$\nabla F =< ye^x, e^x, 2 >$$

Then integrate $\frac{\delta f}{\delta x}$ with respect to x:

$$f(x, y, z) = ye^x + C(y, z)$$

Then find $\frac{\delta f}{\delta y}$ from this f:

$$\frac{\delta f}{\delta y} = e^x + C_y(y, z)$$

But we know that $\frac{\delta f}{\delta y} = e^x$, this means that $C_y(y, z) = 0$ or that $C(y, z)$ is a constant.

Finally, $\frac{\delta f}{\delta z} = 0 + C_z(z)$, but we also know that $\frac{\delta f}{\delta z} = 2z$. Hence, $C(z) = z^2$.

Hence, the potential function is $ye^x + z^2$.

Now with that in hand, we'll make use of the theorem:

$$\int_C F.dr = [ye^x + z^2]_{(0,0,2)(start)}^{(0,3,0)(end)}$$
$$= 3 - 4 = -1$$

13. Suppose $\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ and $\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = 2$ for every (x,y) in the plane.

Find $\displaystyle\oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is parameterized by $\mathbf{r}(t) = (3\cos t)\mathbf{i} + (3\sin t)\mathbf{j},\ 0 \le t \le 2\pi$.

Green's circle

A. 2π

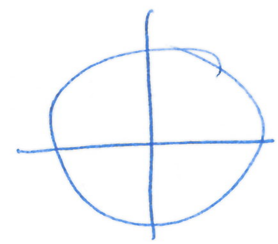
B. 6π

C. 9π

D. 12π

E. 18π

$$\int_0^{2\pi}\int_0^{3} 2\, r\, dr\, d\theta \quad \cancel{dy\,dx}$$



$$= 2 \times 9\pi = 18\pi$$

$$\text{curl}(\text{curl } F) = (2yz - (-8yz)) = 4yz \to 2yz$$

14. Let $\mathbf{F} = \langle x^2yz,\ xy^2z,\ xyz^2 \rangle$. Compute grad(div $\mathbf{F}$) – curl(curl($\mathbf{F}$)).

A. $\langle 2yz,\ 2xz,\ 2xy \rangle$

B. $\langle 0,\ 0,\ 0 \rangle$

C. $\langle 6yz,\ 6xz,\ 6xy \rangle$

D. $\langle 4yz,\ 4xz,\ 4xy \rangle$

E. $\langle yz,\ xz,\ xy \rangle$

$$\text{div } F = \langle 2xyz,\ 2xyz,\ 2xyz \rangle = 6xyz$$

$$\nabla\, 6xyz = \langle 6yz,\ 6xz,\ 6xy \rangle$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial x & \partial y & \partial z \\ F & G & H \end{vmatrix} = \hat{i}(xz^2 - xy^2) - \hat{j}(yz^2 - xy) + \hat{k}(y^2z - x^2z)$$

$$\hat{i}\left(\frac{\partial H}{\partial y} - \frac{\partial G}{\partial z}\right) - \hat{j}\left(\frac{\partial H}{\partial x} - \frac{\partial F}{\partial z}\right) + \hat{k}\left(\frac{\partial G}{\partial x} - \frac{\partial F}{\partial y}\right)$$

15. If surface S is parametrized by $\mathbf{r}(u, v) = \langle u, v, uv^2 \rangle$, then the equation of the plane tangent to S at $(1, 2, 4)$ is

A. $x + 2y + 2z = 13$

B. $x + 2y + 4z = 21$

C. $4x + 4y + z = 16$

D. $x + 2y - 2z = -3$

E. $4x + 4y - z = 8$

$\cancel{r'(u,v) = \langle}$

$r'(u) = \langle 1, 0, v^2 \rangle$

$r'(v) = \langle 0, 1, 2uv \rangle$

$r'(u) \times r'(v) =$

$\begin{vmatrix} i & j & k \\ 1 & 0 & v^2 \\ 0 & 1 & 2uv \end{vmatrix}$

$i(-v^2) - j(2uv) + k(1)$

$\langle -4, -4, 1 \rangle$ or $\langle 4, 4, -1 \rangle$

$-4, -4, 1$

16. Let S be the portion of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 3$. Find $\iint_S (z + 1)\, dS$.

A. $\dfrac{56\pi}{3}$

B. $\dfrac{14\pi}{3}$

C. $\dfrac{116\pi}{15}$

D. $\dfrac{29}{15} + \dfrac{56\pi}{3}$

E. $\dfrac{58}{15} + \dfrac{14\pi}{3}$

$\sqrt{1 + y^2 + x^2}$

$\iint_S (z + 1)\sqrt{1 + r^2}\, r\, dr\, d\theta$

$\int_0^{3}\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} xy\sqrt{1 + x^2 + y^2}$

$\int_0^{2\pi}\int_0^{\sqrt{3}} (r^2 \sin\theta\cos\theta + 1)\sqrt{1 + r^2}\cdot r\, dr\, d\theta$

$\int_0^{3}\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} \frac{x}{2}\left(\frac{2}{3}\right)(1 + x^2 + y^2)^{3/2}\Big|_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}$

17. Consider the curve $C : \mathbf{r}(t) = \langle \cos(t),\ \sin(t),\ 1 - \cos(t) - \sin(t) \rangle, \quad 0 \le t \le 2\pi$, which is the intersection of the cylinder $x^2 + y^2 = 1$ with the plane $x + y + z = 1$. → why?

If $\mathbf{F} = \langle y + \sin(x),\ z + \sin(y),\ x + \cos(z) \rangle$, then $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} =$

A. -3π

B. $-\sqrt{3}\pi$

C. 0

D. 3π

E. $\sqrt{3}\pi$

better: curl F

looks very ugly

$\langle -\sin t,\ \cos t,\ (\sin t - \cos t) \rangle$

18. Find the flux of $\mathbf{F} = \langle z^2,\ xy,\ y^2 \rangle$ out of the box with six faces: $x = 0,\ x = 1,\ y = 0,\ y = 2,\ z = 0$, and $z = 3$.

A. 0

B. 2

C. 3

D. 4

E. 6

flux →

$= 6\int_0^1 x\,dx$

$= 3$

17

with the plane 1

$$\text{Curl } F = \langle -1, -1, -1 \rangle \cdot \langle 1, 1, 1 \rangle$$

$$\cancel{\int_0^{2\pi}\int_0^{\pi/2} -\sqrt{3}\, d\varphi\, d\theta = \sqrt{3}\,\pi}$$

$$\cancel{\int_0^{2\pi}\int_0^{1/2} \sqrt{3}\, r\, d\theta\, dr}$$

$$= \int_0^{2\pi}\int_0^{1} -3\, r\,dr\,d\theta$$

π (area of circle)

$\Rightarrow -3\pi$

$\frac{x}{|x|} = \vec{n}$

normal → -ve

19. Let S be the upper hemisphere of $x^2 + y^2 + z^2 = 4$ with normal vector pointing toward the origin, and $\mathbf{F} = z\dfrac{\mathbf{x}}{|\mathbf{x}|}$ where $\mathbf{x}$ denotes the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \langle x, y, z \rangle$.

Compute $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$.

A. -8π

B. 8π

C. -4π

D. 4π

E. 0

$F = \left\langle \dfrac{zx}{\sqrt{x^2+y^2+z^2}}, \dfrac{zy}{\sqrt{x^2+y^2+z^2}}, \right.$

$\int F \cdot da$

$\dfrac{z^2}{\sqrt{x^2+y^2+z^2}}$

$\int_0^{2\pi} \int_0^{\pi/2} 2\cos\phi \cdot 4 \cdot \sin^2\phi \, d\phi$

$\mathrm{div}\, F = \dfrac{4z}{\sqrt{x^2+y^2+z^2}}$

$x^2+y^2+z^2$

(19)

$$\frac{x}{|x|} = n .$$

Then

$$\iint_S F \cdot dS = \iint_S 2 \cdot n \cdot n \, dS = -\iint_S 2 \, dS .$$

(opposite direction.

But

$$\int_0^{2\pi}\int_0^{\pi/2} 8 \sin^2\varphi \cdot \cos\varphi \; d\varphi \, d\theta \qquad \cancel{2} \, d\cancel{S} = r^3 \cdot 8 \quad (r^3 \sin\varphi \cos\varphi \text{ but } r = 2)$$

$$= 2\pi \times 8 \left[\cancel{\frac{\sin^3\varphi}{3}} \right]_0^{\pi/2} \quad \frac{\sin^2\varphi}{2}$$

$$\cancel{2\pi} \; 4 \; [\quad 8 = -8\pi$$

20. Consider the vector field $\mathbf{F} = \dfrac{\mathbf{x}}{|\mathbf{x}|^3}$ where $\mathbf{x}$ denotes the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \langle x, y, z \rangle$. Which of the following are true?

div F

(i) $\operatorname{div}(\mathbf{F}) = 0$ on its maximal domain of definition. ✓

(ii) $\operatorname{curl}(\mathbf{F}) = \mathbf{0}$ on its maximal domain of definition. ✓ (not defined on $x=0$)

(iii) $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$ for any closed surface on which $\mathbf{F}$ is defined. X

(iv) $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ on any simple, closed, smooth curve on which $\mathbf{F}$ is defined. ✓ $\iint \operatorname{curl} \vec{F} \cdot d\vec{S}$

A. (i) and (iii)

B. (ii) and (iv)

C. (i), (ii) and (iii)

D. (i), (ii) and (iv) Only

E. all of the above

$\vec{F} = \left\langle \dfrac{x}{(x^2+y^2+z^2)^{3/2}}, \dfrac{y}{(x^2+y^2+z^2)^{3/2}}, \dfrac{z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$

curl F:

$$\begin{vmatrix} i & j & k \\ x' & y' & z' \\ F & G & H \end{vmatrix}$$

$\operatorname{div} \vec{F} = \left\langle \dfrac{1}{(x^2+y^2+z^2)^{3/2}} - \dfrac{x^2\left(\frac{-3}{2}\right)}{(x^2+y^2+z^2)^{5/2}} \right.$

$$= \frac{1}{(x^2+y^2+z^2)^{3/2}} \begin{bmatrix} i & j & k \\ x' & y' & z' \\ x & y & z \end{bmatrix} \qquad \frac{3}{(x^2+y^2+z^2)^{3/2}} - \frac{3}{2}\left(\frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^{5/2}}\right)$$

$$= \underline{0} \qquad\qquad = \underline{0}$$