MA 26100
FINAL EXAM Form B
December 13, 2016

NAME ____________________ YOUR TA'S NAME ____________________

STUDENT ID # ____________________ RECITATION TIME ____________________

1. You must use a #2 pencil on the scantron

2. a. If the cover of your exam is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and darken the appropriate bubbles on your scantron.

b. If the cover of your exam is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the appropriate bubbles on your scantron.

c. **The color of your scantron MUST match the color of the cover page of your exam**

3. On the scantron sheet, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the scantron sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Also circle your answers on the exam itself. Do all your work on the question sheets.

9. Turn in both the scantron sheets and the question sheets when you are finished.

10. If you finish the exam before 2:50 pm, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 2:50 pm, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams. **ANY talking or writing during this time will result in an AUTOMATIC ZERO.**
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

1. The symmetric equations for the tangent line to

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle = \langle t^2 + 1, \sqrt{t} + 1, t^3 \rangle$$

at $t = 1$ are

A. $x - 2 = y - 2 = z - 1$

B. $\dfrac{x-2}{2} = 2y - 2 = \dfrac{z-1}{3}$

C. $\dfrac{x-2}{2} = y - 2 = \dfrac{z-1}{3}$

D. $x - 2 = \dfrac{y}{2} - 2 = \dfrac{z-1}{3}$

E. $\boxed{\dfrac{x-2}{2} = 2y - 4 = \dfrac{z-1}{3}}$

2. Find the equation of the line l through $P(1, 2, 3)$ that is perpendicular to the tangent plane T to the graph of $z = x^2 y$ at $x = 1$ and $y = 3$

A. $x = -2t + 1, y = 2t + 3, z = 3t + 1$

B. $\boxed{x = -6t + 1, y = -t + 2, z = t + 3}$

C. $x = t + 1, y = 2t + 3, z = 3t + 3$

D. $x = -t + 1, y = -2t + 3, z = 3t + 3$

E. $x = -t + 1, y = -2t + 3, z = -3t + 3$

3. Find the length of the portion of the curve C: $\vec{r}(t) = \langle e^t, e^{-t}, \sqrt{2}t \rangle$ from the point $(1, 1, 0)$ to the point $(2, \frac{1}{2}, \sqrt{2}\ln 2)$

A. 3

B. 2

C. $\boxed{\frac{3}{2}}$

D. 1

E. $\frac{1}{2}$

4. A bird's flight trajectory is given by $\vec{r}(t) = \langle \sqrt{2}t, t - 6, t^{3/2} \rangle$ for $t \geq 0$. A bird watcher is located at $P(0, 6, 0)$. The closest (i.e. the shortest distance) the bird gets to P is

A. $\boxed{\sqrt{116}}$

B. $\sqrt{57}$

C. $\sqrt{32}$

D. 0

E. 12

5. Let $f(x, y) = x \cos y$. Use a linear approximation at $(1, \pi)$ to approximate the value of

$$\frac{99}{100} \cos\left(\pi + \frac{1}{10}\right)$$

A. 0.99

B. 0.11

C. 0.01

D. $\boxed{-0.99}$

E. -1.01

6. If $z = 2xy$, $x = s^2 + t^2$, $y = \dfrac{s}{t}$, then $\dfrac{\partial z}{\partial t} =$

A. $4ys - \dfrac{2x}{t^2}$

B. $2s$

C. $\boxed{4ty - \dfrac{2xs}{t^2}}$

D. $4ty + 2xs$

E. $4ys + \dfrac{2x}{t}$

7. In what direction does $f(x, y, z) = y + 3xy - 9xz^2$ change most rapidly at the point $(1, 1, 0)$?

A. $\langle 2, 1, 1\rangle$

B. $\langle -1, 1, 0\rangle$

C. $\boxed{\langle 3, 4, 0\rangle}$

D. $\langle 1, 1, 1, \rangle$

E. $\langle 4, -3, 0\rangle$

8. Classify the critical points $(1, 1)$ and $(3, 0)$ of g if

$$g_x(1,1) = 0,\ g_y(1,1) = 0,\ g_{xx}(1,1) = -2,\ g_{yy}(1,1) = -2,\ g_{xy}(1,1) = -1$$

$$g_x(3,0) = 0,\ g_y(3,0) = 0,\ g_{xx}(3,0) = 0,\ g_{yy}(3,0) = -6,\ g_{xy}(3,0) = -3$$

A. $\boxed{\text{A local maximum at } (1,1) \text{ and a saddle point at } (3,0)}$

B. A local minimum at $(1, 1)$ and a saddle point at $(3, 0)$

C. A local minimum at $(1, 1)$ and a local maximum at $(3, 0)$

D. A saddle point at $(1, 1)$ and a local minimum at $(3, 0)$

E. A local maximum at $(1, 1)$ and a local minimum at $(3, 0)$

9. The maximum value of $f(x, y, z) = xyz$ in the first octant subject to $2x + 2y + z = 6$ is

A. 4

B. $\boxed{2}$

C. $\frac{5}{2}$

D. $\frac{3}{2}$

E. 3

10. Solve by changing the order of integration

$$\int_0^4 \int_{\sqrt{x}}^2 \sqrt{1 + y^3}\, dy\, dx$$

A. $\boxed{\dfrac{52}{9}}$

B. $\dfrac{1}{2}$

C. $\dfrac{26}{3}$

D. $\dfrac{13}{3}$

E. $\dfrac{26}{9}$

11. Compute
$$\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{4-x^2-y^2}\, dy\, dx$$

A. $-\dfrac{4\pi}{3}$

B. 4π

C. $\boxed{\dfrac{4\pi}{3}}$

D. $\dfrac{8\pi}{3}$

E. $-\dfrac{8\pi}{3}$

12. Express the area of the surface given by the graph of

$$z = y^3 \cos^2(x) \text{ over the domain } D = \text{triangle with vertices } (-1,1),(0,2),(1,1).$$

Do not evaluate.

A. $\displaystyle\int_1^2 \int_{y-2}^{2-y} \sqrt{4y^6 \cos^2(x)\sin^2(x) + 9y^4\cos^4(x)}\, dxdy$

B. $\boxed{\displaystyle\int_1^2 \int_{y-2}^{2-y} \sqrt{4y^6 \cos^2(x)\sin^2(x) + 9y^4\cos^4(x) + 1}\, dxdy}$

C. $\displaystyle\int_1^2 \int_{2-y}^{y+2} \sqrt{4y^6 \cos^2(x)\sin^2(x) + 9y^4\cos^4(x) + 1}\, dxdy$

D. $\displaystyle\int_1^2 \int_{y+2}^{2-y} \sqrt{-2y^3 \cos(x)\sin(x) - 3y^2\cos^4(x)}\, dxdy$

E. $\displaystyle\int_1^2 \int_{2-y}^{y+2} \sqrt{4y^6 \cos^2(x)\sin^2(x) + 9y^4\cos^4(x)}\, dxdy$

13. Compute the volume of the solid inside $x^2 + y^2 + z^2 = 4$ and outside $x^2 + z^2 = 1$ and above the $xy-$ plane

A. $3\sqrt{3}\pi$

B. $\frac{26\pi}{3}$

C. $4\sqrt{3}\pi$

D. $6\sqrt{3}\pi$

E. $\boxed{2\sqrt{3}\pi}$

14. Consider the vector field $\vec{F} = (y^2 + e^z)\vec{i} + (2xy + ze^y)\vec{j} + (e^y + xe^z)\vec{k}$ and the curve C parametrized by $\vec{r}(t) = (t, t^2, t^3), 0 \leq t \leq 1$ compute

$$\int_C \vec{F} \cdot \vec{dr}$$

A. $1 + e$

B. e

C. 2

D. 0

E. $\boxed{1 + 2e}$

15. Consider the vector field $\vec{F} = \langle -y, -x, 3 \rangle$. Which of the following is true?

i. $\text{curl}(\vec{F}) = \vec{0}$.

ii. $\vec{F}$ is a conservative vector field.

iii. $\text{div}(\vec{F}) \neq 0$.

iv. $\int_C \vec{F} \cdot \vec{dr} = 0$ for all smooth closed curves.

A. iii only

B. $\boxed{\text{i, ii, and iv only}}$

C. i and ii only

D. i only

E. i, iii, and iv only

16. Identify the surface described by the vector equation $\vec{r}(u, v) = \langle u \sin(6v), u^2, u \cos(6v) \rangle$

A. $\boxed{\text{circular paraboloid}}$

B. hyperbolic paraboloid

C. circular cylinder

D. cone

E. elliptic cylinder

17. Evaluate $\displaystyle\int\int_S y\, dS$ where S is the portion of the plane $x + y + z = 1$ in the first octant.

A. $\frac{\sqrt{3}}{3}$

B. $\frac{1}{2}$

C. $\frac{-1}{2}$

D. $\boxed{\frac{\sqrt{3}}{6}}$

E. $\frac{\sqrt{3}}{2}$

18. Evaluate $\displaystyle\int_C \vec{F} \cdot \vec{dr}$ where C is the boundary curve of the part of the plane $z + 2x + y = 4$ in the first octant and is oriented counterclockwise when viewed from above. $\vec{F}(x, y, z)$ is such that $\text{curl}\vec{F} = \langle 1, 1, 2\rangle$.

A. $20\sqrt{2}$

B. $5\sqrt{6}$

C. 10

D. $\boxed{20}$

E. $\frac{10}{\sqrt{6}}$

19. Evaluate $\displaystyle\int\int_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \langle -x, -y, z \rangle$ and S is the surface $z = xy$, $0 \le x \le 1$, $0 \le y \le 1$ oriented with a upward-pointing normal vector.

A. $\frac{3}{2}$

B. 2

C. $\frac{1}{2}$

D. $\boxed{\frac{3}{4}}$

E. 1

20. Let $\vec{F} = \langle xy^2 + 1, yz^2 - x, zx^2 + y \rangle$. Use the Divergence Theorem to evaluate $\displaystyle\int\int_S \vec{F} \cdot d\vec{S}$ where S is the boundary surface of the solid

$$E = \{(x, y, z) \,|\, x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$$

A. $\frac{8\pi}{3}$

B. $\frac{8\pi}{7}$

C. 4π

D. $4\pi^2$

E. $\boxed{\frac{16\pi}{5}}$