Leaderboard

1) Lesson 33 MA261

~~$\langle 0, 0, 3\rangle$~~

$z_x = -1 \quad z_y = -1 \quad z_z = 1$

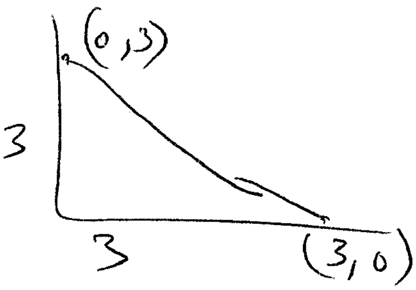
$\langle 0, 0, 3\rangle \cdot \langle -1, -1, 1\rangle$

$= 3.$

but consider base: ~~of triangle~~

has vertices

$(0,0)\ (0,3)\ (3,0)$



$\therefore$ area $= 9/2$

$\therefore$ flux $= \frac{9}{2} \times 3 = \frac{27}{2}$

2) $z_x = 0 \quad z_y = -\sin y \quad z_z = 1$

$\langle e^{-x}, \underset{\to -\sin y}{z}, 5xy\rangle\ \langle 0, -\sin y, 1\rangle$

$= \int_0^{\pi}\int_0^{6} -2\sin y + 5xy \, dy\, dx$

$= \int_0^{\pi} -12\sin y + \frac{90}{2}y \, dy = \left(12\cos y + 45y^2\right)_0^{\pi}$

$= 45\pi^2 - 24$

$$\iint_R \left(-f z_x - g z_y + h\right) dA$$

$$= \int_0^{6} \int_0^{\pi} \left[-e^{-y}\cdot 0 - z\cdot(-\sin y) + 5xy\right] dy\,dx$$

$$\int_0^{6} \int_0^{\pi} \left[\cos y \underbrace{\sin y}_{\frac{1}{2}\sin 2y} + 5xy\right] dy\,dx$$

$\sin y = t$, $\cos y\,dy = dt$, $\frac{1}{2}\sin 2y$

$$= \int_0^{6} \left[\cancel{\frac{\sin^2 y}{2}}\ \frac{-1}{4}\cos 2y + \frac{5xy^2}{2}\right]_0^{\pi} dx$$

$$\int_0^{6} \left[\cancel{\frac{-1}{4}\cos 2\pi + \frac{1}{4}} + \frac{5\pi^2 x}{2}\right]_{-\pi} dx$$

$$\int_0^{6} \cancel{\frac{5\pi^2 x}{2}}\, dx\ 0\ \ dx = \underline{\underline{0}}$$

3) $F_{\Phi} = t_u \times t_v$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\cos u\cos v & a\cos u\sin v & -a\sin u \\ -a\sin u\sin v & a\sin u\cos v & 0 \end{vmatrix}$$

$\hat{i}\left(a^2\sin^2 u\cos v\right) + \hat{j}\left(a^2\sin^2 u\sin v\right) + \hat{k}\left(a^2\cos u\cos v\sin u\cos v + a\sin u\sin v\cos u\sin v\right)$

$$\boxed{\begin{array}{c} a^2\cos^2 v\cos u\sin u \\ + a\sin^2 v\sin u\cos u \\ a\cos u\sin u(1) \end{array}} \rightarrow (t_u \times t_v)$$

$|r| = \sqrt{a^2\sin^2 u\cos^2 v + a^2\sin^2 u\sin^2 v + a^2\cos^2 u}$

$= |r| = a$

$F \cdot (t_u \times t_v)$

$\left[\frac{\sin u\cos v}{a^2}, \frac{\sin u\sin v}{a^2}, \frac{\cos u}{a^2}\right]\left(a^2\sin^2 u\cos v, a^2\sin^2 u\sin v, a\cos u\sin u\right)$

$\frac{\sin^3 u\cos^2 v}{\cancel{a}} + \frac{\sin^3 u\sin^2 v}{a} \quad \frac{\cos^2 u\sin u}{a}$

$\frac{\sin^3 u}{a} + \frac{\cos^2 u\sin u}{a} \rightarrow \frac{\sin u}{a}\left(\sin^2 u + \cos^2 u\right)$

$= \sin u/a$

$$A = \int_0^{2\pi} \int_0^{\pi} \sin u \, du \, dv$$

$$= \int_0^{2\pi} \left[-\cos u \right]_0^{\pi} dv$$

$$= \int_0^{2\pi} 2 \, dv = \underline{\underline{4\pi}}$$

4)

$t_x = 10x \quad t_y = 1 \quad t_z = 0$

$$\int_0^{3} \int_0^{4} \left| -y \cdot 10x + 1 \right|$$

$u = x$
$v = z$

$r(u,v) = \langle u, 5u^2, v \rangle$

$t_u = \langle 1, 10u, 0 \rangle$
$t_v = \langle 0, 0, 1 \rangle$

$$t_u \times t_v = \begin{vmatrix} i & j & k \\ 1 & 10u & 0 \\ 0 & 0 & 1 \end{vmatrix} = \langle 10u, -1, 0 \rangle$$

or fixing direction, $\langle -10u, 1, 0 \rangle$

$\langle -y, x, 1 \rangle = \langle -5u^2, u, 1 \rangle$

and $\langle -5u^2, u, 1 \rangle \cdot \langle -10u, 1, 0 \rangle = 50u^3 + u$

$$\int_0^2\int_0^3 (50u^3 + u)\,du\,dv$$

$$= 2\left[\frac{50}{4}u^4 + \frac{u^2}{2}\right]_0^3$$

$$2\left[\frac{50}{4}\times 81 + \frac{9}{2}\right] \qquad = 2\left(\frac{50\times 81}{4} + \frac{9}{2}\right)$$

$$2\left[\frac{50\times 81 + 18}{4}\right] =$$

$$\frac{50\times 81+18}{2} = \frac{4050+18}{2}$$

$$\frac{4068}{2} = \underline{\underline{2034}}$$

5) $|r| = \sqrt{x^2+y^2+z^2} \longrightarrow \underline{\underline{\sqrt{25+v^2}}}$

$$r = \langle 5\cos u, 5\sin u, v\rangle$$

$$r_u = \langle -5\sin u, 5\cos u, 0\rangle$$

$$r_v = \langle 0, 0, 1\rangle$$

$$|r_u \times r_v| = 5$$

$\therefore$ we have $$\int_0^5\int_0^{2\pi} 5\sqrt{25+v^2}\,d\theta\,dv = 2\pi\times 5\int_0^5 \sqrt{25+v^2}$$

$$= 10\pi \int_0^5 \sqrt{25+v^2}\, dv$$

$$= \frac{v}{2}\sqrt{25+v^2} + \frac{25}{2}\ln\left(v+\sqrt{25+v^2}\right)\Big|_0^5$$

$$= \frac{5}{2}\sqrt{50} + \frac{25}{2}\ln\left(5+\sqrt{50}\right) - \frac{25}{2}\ln\left(\sqrt{25}\right)$$

$$\Rightarrow \cancel{5\pi \times 5\left(\sqrt{50} + 5\ln\left(5+\sqrt{50}\right)\right)}]$$

$$25\pi\left(\sqrt{50} + 5\left(\ln\underline{\left(5+\sqrt{50}\right)} - \sqrt{25}\right)\right]$$

6) $z_x = 0 \quad z_y = -1 \quad z_z = 1$

$$\therefore \iint_R \underline{f(x,y,g(x,y))}\sqrt{z_x^2+z_y^2+1}$$

$$= \sqrt{2}\iint_R xy(6-y)\, dA$$

$$= \sqrt{2}\int_0^{10}\int_0^{2\pi} xy\,(6-y)\, dA$$

$$\cancel{x} = r\cos\theta \qquad y = r\sin\theta$$

$$\sqrt{2}\int_0^{10}\int_0^{2\pi} r^2\cos\theta\,\sin\theta\,(6 - r\sin\theta)\cdot r\,d\cancel{r}\theta\,d\cancel{\theta}r$$

$$\sqrt{2}\int_0^{10}\int_0^{2\pi} r^3\cos\theta\,\sin\theta\,(6 - r\sin\theta)\,d\theta\,dr$$

$$\sqrt{2}\int_0^{10}\int_0^{2\pi} 6r^3\cos\theta\,\sin\theta - r^4\cos\theta\,\sin^2\theta\,d\theta\,dr$$

$$\sqrt{2}\int_0^{10}\int_0^{2\pi} 3r^3\sin 2\theta - r^4\cos\theta\,\sin^2\theta\,d\theta\,dr$$

$$\sin\theta = t \qquad +\cos\theta\,d\theta = dt$$

$$\cancel{\sqrt{2}\int_0^{10}}\ \sqrt{2}\int_0^{10}\left[\frac{-3}{2}\cos 2\theta\, r^3 - r^4\left[\frac{\sin^3\theta}{3}\right]\right]_0^{2\pi} d\theta$$

$$\sqrt{2}\left[\frac{-3}{2}\cdot r^3 + \frac{3}{2}r^3\right]$$