~~Fall~~
Spring '18

MA 26100
FINAL EXAM Green
April 30, 2018

NAME Leaderboard YOUR TA'S NAME Myself

STUDENT ID # $\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}$ RECITATION TIME 1:00 am

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): $\boxed{00}$

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **20** questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the mark–sense sheet and the exam booklet when you are finished.

If you finish the exam before 2:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 1:20. If you don't finish before 2:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: Leaderboard

STUDENT SIGNATURE: Not putting one.

1. The area of the triangle with vertices $(2,1,1)$, $(1,2,1)$, $(1,1,2)$ is

A. $\frac{7}{2}$

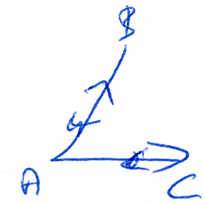
B. $\frac{3}{2}$

C. $\sqrt{2}$

D. $\frac{\sqrt{3}}{2}$

E. 2

A B C



$\frac{1}{2}\left(\cancel{\vec{BA} \times \vec{BC}}\right)$

$\vec{AB} \times \vec{AC}$

$\frac{1}{2}\left((-\hat{i}+\hat{j}) \times (-\hat{i}+\hat{k})\right)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(-1) + \hat{k}(1) = \hat{i}+\hat{j}+\hat{k}$$

$\frac{1}{2}\left|\hat{i}+\hat{j}+\hat{k}\right| = \frac{\sqrt{3}}{2}$

2. The arclength of the curve $\vec{\mathbf{r}}(t) = 2t\,\vec{\mathbf{i}} + t^2\,\vec{\mathbf{j}} + (\ln t)\,\vec{\mathbf{k}}$ for $1 \le t \le 2$ is

A. 5

B. $\frac{35}{3}$

C. $4+\ln 2$

D. $3+\ln 2$

E. $5+\ln 2$

seriously repeated from 2017?

$r'(t) = \langle 2, 2t, \frac{1}{t} \rangle$

$\int_1^2 |r'(t)|\, dt$

$\int_1^2 \frac{2t^2+1}{t}\, dt = \int_1^2 \left|\sqrt{4+4t^2+\frac{1}{t^2}}\right| dt$

$\int_1^2 \left(2t+\frac{1}{t}\right) dt = \int_1^2 \left|\sqrt{\frac{4t^2+4t^4+1}{t}}\right| dt \to (2t^2+1)^2$

$= \left(t^2+\ln t\right)\Big|_1^2 = 3+\ln 2$

~~v(0) = i + j + k~~   v(t)

~~i + j + k = C~~   ~~C = -i - j - k~~ $= \frac{t^2}{2} i + t^3 k$

3. A particle has position $\vec{\mathbf{r}}(t)$ with acceleration $\vec{\mathbf{a}}(t) = t\,\vec{\mathbf{\imath}} + 3t^2\,\vec{\mathbf{k}}$ and the initial conditions $\vec{\mathbf{v}}(0) = \vec{\mathbf{\imath}} + \vec{\mathbf{\jmath}} + \vec{\mathbf{k}}$ and $\vec{\mathbf{r}}(0) = \vec{\mathbf{0}}$. Then $\vec{\mathbf{r}}(1) =$

A. $\vec{\mathbf{\imath}} + \frac{5}{4}\vec{\mathbf{k}}$

B. $5\vec{\mathbf{\imath}} + 7\vec{\mathbf{\jmath}} + \vec{\mathbf{k}}$

C. $\frac{1}{6}\vec{\mathbf{\imath}} + \frac{1}{4}\vec{\mathbf{k}}$

D. $\vec{\mathbf{\imath}} + \vec{\mathbf{\jmath}} + \vec{\mathbf{k}}$

E. $\frac{7}{6}\vec{\mathbf{\imath}} + \vec{\mathbf{\jmath}} + \frac{5}{4}\vec{\mathbf{k}}$

$v(t) = \int a(t)\,dt = \int t\,i + 3t^2 k\,dt = \frac{t^2}{2} i + t^3 k + C$

$\left(\frac{t^2}{2} - 1\right) i + \ldots - j + (t^3 - 1)$

$v(0)$: $v(0) = i + j + k \Rightarrow C = 0$

$r(t) = \int v(t)\,dt = \left(\frac{t^3}{6} i + \frac{t^4}{4} k\right) \Rightarrow r(t) =$

4. A continuous function $f(x, y)$ defined on the region $D = [1,3] \times [0,1]$ has its absolute minimum value equal to 4 and its absolute maximum value equal to 5.
Which of the following numbers could equal $\iint_D f(x, y)\,dA$?

A. 7.9

B. 8.8

C. 10.3

D. 11.6

E. 13.1

→ only one between 8 and 10.

$\int_0^1 \int_1^3 f(x,y)\,dA$

so perhaps one idea is to take inequalities:

$\int_0^1 \int_1^3 4\,dA \le I \le \int_0^1 \int_1^3 5\,dA$

$\to 4 \times 2 \le I \le 5 \times 2$

$8 \le I \le 10 \Rightarrow$

$\cos 0 = 0 + 3 + 2z$

$(z = -1)$

5. Suppose that z is defined as a function of x and y by the equation

$$\cos(xyz) = x + 3y + 2z.$$

Use implicit differentiation to find the value of $\dfrac{\partial z}{\partial y}(0, 1)$.

A. $-1/2$

B. $-3/2$

C. $1/3$

D. $-2/3$

E. $-3/5$

$\to -\dfrac{\delta F / \delta y}{\delta F / \delta z} = \cancel{-3z}$

$(0, 1, -1)$

$\dfrac{-3+}{2}$

$xz\ \delta F/dy$

$- \cancel{y} \sin(xyz)$

$= 3 \dfrac{\delta F}{dy}$

$\dfrac{\delta F}{\delta z} = -yz \sin(xyz) - 2$

$3 \dfrac{\delta F}{\delta y} + xz \dfrac{\delta F}{\delta y} \sin(xyz) = 0$

$\dfrac{\delta F}{\delta y} (3 + xz \sin(xyz)) = 0$

$-2 \leq 0$

6. Consider the tangent plane to the surface $z = \ln(x - 4y)$ at the point $(9, 2, 0)$. This tangent plane also contains the point $(2, 1, \lambda)$. Find λ.

A. -3

B. -2

C. 2

D. $\ln 2$

E. -8

$x - 9 - 4y + 8 + z = 0$

$x - 4y + \cancel{z} = H$

$\nabla f = \left\langle \dfrac{1}{x-4y}, \dfrac{-4}{x-4y}, + \right\rangle$

$2 - 4 + 2 = 1$

$-2 + 2 = 1$

at $(9, 2, 0)$ $\left\langle \dfrac{1}{1}, \dfrac{-4}{1}, 1 \right\rangle$

$\langle 1, -4, 1 \rangle$

contains $(2, 1, t)$

$1(x - 9) + -4(y - 2) + z = 0$

$-7 + 4 + t = 0$

$t = 3$

$x = 2 \quad y = 1$

7. Find the directional derivative of $f(x, y) = xe^{y^2} + e^{x+y}$ at the point $(0, 0)$ in the direction of the vector $3\vec{\mathbf{i}} - 4\vec{\mathbf{j}}$.

d

A. $6/5$

B. $-6/5$

C. 0

D. $-2/5$

E. $2/5$

$\nabla f(x,y) = \langle e^{y^2} + e^{x+y},\ 2xye^{y^2} + e^{x+y} \rangle$

$\nabla f(0,0) = \langle 2, 1 \rangle$

$\langle 2, 1 \rangle \cdot \frac{\langle 3, -4 \rangle}{\sqrt{5}\ \sqrt{5}} = \frac{1}{5}(6-4) = \frac{2}{5}$

8. Suppose E is the region bounded above by the cylinder $x^2 + z^2 = 5$, below by the plane $z = 1$, and on the sides by the planes $y = -1$ and $y = 2$. Find $\iiint_E z\, dV$.

$z = \sqrt{5 - x^2}$

$\sqrt{5 - r^2 \sin^2\theta}$

A. 4

B. 8

C. 12

D. 16

E. 24

$\int_{-1}^{2} \int_{-2}^{2} \frac{(5-x^2)}{2} - \frac{1}{2}\, dx\, dy$

$= \int_{-2}^{2} 2 - \frac{x^2}{2}\, dx$

$\boxed{12}$

$\left(2x - \frac{x^3}{6}\right)\Big|_{-2}^{2}$

$= 2\left(4 - \frac{4}{3}\right)$

$\int_{-1}^{2} \int_{-2}^{2} \int_{1}^{\sqrt{5-x^2}} z\, dz\, dx\, dy$

$\begin{vmatrix} 12x-6y & -6x \\ -6x & -6y \end{vmatrix}$ $H(0,1) = \begin{vmatrix} -6 & 0 \\ 0 & -6 \end{vmatrix} = 36 > 0$ $f_{xy} = -6x$

but $f''(0,1) = -6 < 0$ max

9. The points $P = (0, 1)$ and $Q = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ are critical points of the function

$$f(x, y) = 2x^3 - 3x^2y - y^3 + 3y.$$

Classify each as a relative maximum, relative minimum, or saddle point.

A. f has a relative minimum at P and a relative maximum at Q

B. f has a relative maximum at P and a saddle point at Q

C. f has a saddle point at P and a relative minimum at Q

D. f has relative maxima at P and Q

E. f has relative minima at P and Q

and $\begin{vmatrix} 6/\sqrt{2} & -6/\sqrt{2} \\ -6/\sqrt{2} & -6/\sqrt{2} \end{vmatrix}$

$f_{yy} = -6y$

$f_{xx} = 12x - 6y$

$H < 0$, saddle point

$f_x = 6x^2 - 6xy \rightarrow 6x(x-y) = 0$

$f_y = (-3x^2 - 3y^2 + 3) \rightarrow 3(1 - x^2 - y^2) = 0$

10. A lamina with density $\rho(x, y) = xy$ occupies the region of the plane bounded by $y = x^2$, $y = 1$ and $x = 0$. The mass of the lamina is equal to $\frac{1}{6}$. Find the y-coordinate of its center of mass.

A. $\frac{3}{4}$

B. $\frac{7}{8}$

C. $\frac{2}{3}$

D. $\frac{5}{6}$

E. $\frac{12}{21}$

$\frac{\iint \rho(x,y)\, dy\, dx}{1/6}$

$\frac{M}{V} = D$

$= \int_0^1 \int_{x^2}^1 xy \, dy\, dx$

$= \left(\frac{xy^2}{2}\right)\Big|_{x^2}^1 \int_0^1 \frac{x}{2} - \frac{x^5}{2}\, dx = \left[\frac{x^2}{4} - \frac{x^6}{12}\right]_0^1 = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$

3

$\vec{a}(t) = t\hat{i} + 3t^2\hat{k}$

$v(t) = \int a(t)\,dt +$

$= \frac{t^2}{2}\hat{i} + t^3\hat{k} + C$

$v(0) = \hat{i} + \hat{j} + \hat{k}$

$\Rightarrow \hat{i} + \hat{j} + \hat{k} = 0 + C \qquad \Rightarrow C = \hat{i} + \hat{j} - \hat{k}.$

$\therefore v(t) = \frac{t^2}{2}\hat{i} + t^3\hat{k} + \hat{i} + \hat{j} + \hat{k}$

$\hat{i}\left(\frac{t^2}{2} + 1\right) + \hat{j} + (t^3 + 1)\hat{k}$

$s(t) = \int v(t)\,dt$

$= \left(\frac{t^3}{6} + t\right)\hat{i} + t\hat{j} + \left(\frac{t^4}{4} + t\right)\hat{k}$

$s(0) = 0 \quad \Rightarrow C = 0$

$\therefore s(1) = \left(\frac{1}{6} + 1\right)\hat{i} + \hat{j} + \frac{5}{4}\hat{k}$

$\left(\frac{7}{6}\right)$

(10) $\quad y = x^2$

$$\frac{\displaystyle\int_0^1 \int_{x^2}^1 y\,\rho(x,y)\,dy\,dx}{\left(\frac{1}{6}\right)}$$

$$= \frac{\displaystyle\int_0^1 \int_{x^2}^1 x\,y^2\,dy\,dx}{\frac{1}{6}} = \frac{\displaystyle\frac{1}{3}\int_0^1 \left[x\,y^3\right]_{x^2}^{1}\,dx}{1/6}$$

$$2\int_0^1 \left(x - x^7\right)dx \qquad \cancel{= 3\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^1}$$

$$\cancel{3\left(\frac{1}{3} - \frac{1}{7}\right) = 3\left(\frac{4}{21}\right) = \frac{12}{21}}$$

$$= 2\left[\frac{x^2}{2} - \frac{x^8}{8}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{8}\right] = 2\left[\frac{3}{8}\right] = \frac{3}{4}$$

11. Find the surface area of the part of the paraboloid $z = \dfrac{x^2}{2} + \dfrac{y^2}{2}$ that lies between the cylinders $x^2 + y^2 = 8$ and $x^2 + y^2 = 24$.

A. $196\pi/3$

B. $(24\sqrt{24} - 8\sqrt{8})\pi/3$

C. $(\sqrt{24} - \sqrt{8})\,\pi/3$

D. $(\sqrt{24} - \sqrt{8})4\pi$

E. $164\pi/3$

$\downarrow$

$\cancel{\langle \frac{1}{2}, \frac{1}{2}, 1\rangle}$

$f'(t) = \langle x, y, 1\rangle$

$\sqrt{1+x^2+y^2}\,dA$

$\to \sqrt{1+r^2}$

$$\int_0^{2\pi}\int_{\sqrt{8}}^{\sqrt{24}} r\sqrt{1+r^2}\,dr\,d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)(1+r^2)^{3/2}\Big|_{\sqrt{8}}^{\sqrt{24}}$$

$$= \frac{1}{3}\int_0^{2\pi}\left[5^3 - 3^3\right]d\theta = \frac{1}{3}\left[98 \times 2\pi\right] = \frac{196\pi}{3}$$

12. Evaluate the line integral $\displaystyle\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where $\vec{\mathbf{F}}(x, y, z) = y\,\vec{\imath} - x\,\vec{\jmath} + xy\,\vec{\mathbf{k}}$

and C is parametrized by $\vec{\mathbf{r}}(t) = \sin t\,\vec{\imath} + \cos t\,\vec{\jmath} + t\,\vec{\mathbf{k}}$ with $0 \le t \le \pi$.

A. $\dfrac{\pi}{2}$

B. $-\dfrac{\pi}{2}$

C. π

D. $-\pi$

E. 0

$$\int_0^{\pi} F(r(t)) \cdot r'(t)\,dt$$

$\downarrow$

$\langle \cos t, -\sin t, 1\rangle$

$$\int_0^{\pi} \langle \cos t, -\sin t, \sin t\cos t\rangle \cdot \langle \cos t, -\sin t, 1\rangle\,dt$$

$$\int_0^{\pi} 1 + \sin t\cos t\,dt = \pi + \cancel{\int_0^{\pi} \sin t\cos t\,dt}$$

odd function

$$\int_C x^2\,dy =$$

13. Use Green's Theorem to evaluate $\displaystyle\int_C x^2\,dy$ where C is the boundary of the rectangle with vertices $\{(0,0),(2,0),(2,3),(0,3)\}$, oriented counterclockwise.

A. 4

B. 8

C. 12

D. 16

E. 24

$Q(x,y)\,dy$

$(0,3)$ $y = \frac{3}{2} \frac{2}{3}x$ $(2,3)$ $(0,0)$ $(2,0)$

$$= \iint_R \frac{\partial Q}{\partial x}\,dA$$

$$= \boxed{\iint_R} 2x\,dA = \int_0^3 \int_0^{\frac{2}{3}x} 2x\,dy\,dx = \int_0^3 \frac{4}{3}x^2\,dx = 12$$

$$\int_0^3 2x\left(\tfrac{2}{3}x\right)dx \qquad = \frac{4}{3}\left(\frac{x^3}{3}\right)\Big|_0^3 = \frac{4}{3}\cdot 9 = 12$$

$$+ \int_0^3 \int_{\frac{2}{3}x}^{3} \cancel{2x\,dy\,dx}$$

14. If $f(x,y,z) = x^2yz - xy^2 + 2xz^2$ then div(grad(f)) at $(1,1,1)$ is equal to:

A. 0

B. 1

C. 2

D. 3

E. 4

∇f

$$= \langle 2xyz - y^2 + 2z^2,\ x^2z - 2xy,$$

∇f

$$\nabla\cdot\nabla f\ \langle 2yz,\ x^2y + 4xz\rangle$$

$$-2x,\ 4x \Rightarrow \text{at } (1,1,1)$$

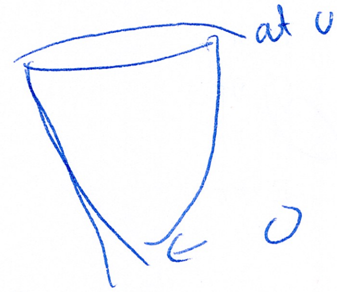
$= 4$

15. Let S be the parametric surface

$$\vec{\mathbf{r}}(u,v) = v\cos u\,\vec{\mathbf{i}} + v\sin u\,\vec{\mathbf{j}} + 2v^2\,\vec{\mathbf{k}}$$

with (u,v) in $[0,2]\times[0,2]$. Then S is part of a

circular base



A. circular paraboloid

B. cone

C. cylinder

D. ellipsoid

E. sphere

16. Find the surface area of the parametric surface $\vec{\mathbf{r}}(u,v) = (u+v)\,\vec{\mathbf{i}} + v\,\vec{\mathbf{j}} + u\,\vec{\mathbf{k}}$ with (u,v) in $[0,\pi]\times[0,\sqrt{3}]$.

A. 4π

B. 2π

C. $2\pi\sqrt{3}$

D. $\pi\sqrt{3}$

E. 3π

$r_u \quad 1, 0, 1$

$r_v \quad 1, 1, 0$

$r_u \times r_v = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$= i(-1) - j(-1) + k(1)$

$(-i + j + k) \to \sqrt{3}$

$$\int_0^{\pi}\int_0^{\sqrt{3}} \sqrt{3}\, dS$$

$= \sqrt{3}\times\sqrt{3}\times\pi$

$= 3\pi$

17. Let S be the part of the sphere $x^2+y^2+z^2=1$ above the plane $z=\dfrac{1}{2}$. Compute the surface integral

$$\iint_S 12z^2\,dS$$

A. 2π

B. π

C. 9π

D. 7π

E. 8π

parametrise:

$\langle r\sin\varphi\sin\theta,\ r\sin\theta\,\varphi\cos\varphi,\ r\cos\varphi\rangle$

$0\le\theta\le 2\pi$

$0\le\varphi\le\pi/3.$ but

$$=\int_0^{2\pi}\int_0^{\pi/3} 12\cos^2\varphi\sin\varphi\,d\varphi\,d\theta$$

$\cos\varphi=t$

$\cancel{|t_y \times t_\theta|}$

$$\cancel{2\pi\int_0^{\pi/3} 4\cos^3\varphi\,d\varphi}$$

$=-8\pi\int_0^{\pi/3}$

$-\frac{4\pi}{8}\int_0^{\pi/3}\left[\cos^3\varphi\right]_0^{\pi/3}$

$\left(\frac{1}{8}-1\right)=\frac{7}{8}$

$\times 8$

$=7\pi$

18. The flux of the vector field $\vec{\mathbf{F}}(x,y,z)=x\,\vec{\mathbf{i}}+(x+y)\,\vec{\mathbf{j}}+z\,\vec{\mathbf{k}}$ across the surface of the plane $x+y+z=1$ in the first octant, oriented upward, is equal to:

A. $\dfrac{3}{4}$

B. $\dfrac{4}{3}$

C. $\dfrac{2}{3}$

D. $\dfrac{3}{2}$

E. $\dfrac{1}{2}$

$$\cancel{=\sqrt{3}\int_0^1\left[y-\frac{y^2}{2}-xy\right]_0^{1-x}dx}$$

$$\cancel{\sqrt{3}\int_0^1 (1-x)^2-\frac{(1-x)^2}{2}\,dx}$$

$$=\sqrt{3}\int_0^1\frac{(1-x)^2}{2}\,dx$$

$$=\frac{\sqrt{3}}{2}\int_0^1(1-x)^2\,dx$$

$$\frac{\sqrt{3}}{2}\left|x-\frac{x^2}{2}\right|_0^1=\sqrt{3}\,\frac{3}{2}\left(\frac{1}{2}\right)=3/4$$

$\operatorname{div} F=\langle 1,1,1\rangle$

$$\cancel{\int_0^1\int_0^{1-x}\int_0^{1-y-x}\sqrt{3}\,dz\,dy\,dx}$$

$$=\sqrt{3}\int_0^1\int_0^{1-x}(1-y-x)\,dy\,dx$$

15

$z = 1 - y - x \qquad z = 1 - x - y$

$\vec{r_u} = \langle 0, -1, 0 \rangle$

$\vec{r_v} = \langle 0, 0, -1 \rangle$

$\begin{vmatrix} i & j & k \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = \hat{x}$

$\therefore \int_0^1 \int_0^{1-x} x \, dy \, dx = \int_0^1 x(1-x)\,dx = \int_0^1 x - x^2 \, dx$

$= \left(\frac{x^2}{2} - \frac{x^3}{3} \right|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

$z_x = -1 \quad z_y = -1 \Rightarrow \sqrt{z_x^2 + z_y^2 + 1} = \sqrt{3}.$

$\int_0^1 \int_0^{1-x} \sqrt{3} \, dy \, dx$

$\int_0^1 (1 - x^2)\,dx = \left(x - \frac{x^3}{3} \right|_0^1 = 2/3$

$\iint_S f \cdot n \, dS = \int_0^1 \int_0^{1-x} \left(-x \cdot -1 - (x+y) \cdot -1 + z \right) dA$

$\int_0^1 \int_0^{1-x} \left(x + \cancel{x} + \cancel{y} + (1 - \cancel{x} - \cancel{y}) \right) dA$

$\int_0^1 \int_0^{1-x} (1+x)\,dy\,dx = \int_0^1 (1+x)(1-x)\,dx$

19. Let S be the part of the circular paraboloid $z = x^2 + y^2$ below the plane $z = 4$ with upward orientation. Let $\vec{\mathbf{F}}(x, y, z) = xz\vec{\jmath} + yz\vec{\mathbf{k}}$. Compute $\iint_S \operatorname{curl} \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}\, dS$. *Hint: You may need to use one or both of these integrals:* $\int_0^{2\pi} (\cos t)^2 dt = \pi$ *and* $\int_0^{2\pi} (\sin t)^2 dt = \pi$.

A. 32π

B. 16π

C. 8π

D. 4π

E. 2π

Stokes' theorem

$$\int_a^b F(r(t)) \cdot r'(t)\, dt$$

$a = 0$ $b = 2\pi$. but parameterization of curve is $\langle 2\cos u, 2\sin u, 4\rangle$.

then $r'(t) = \langle -2\sin u, 2\cos u, 0\rangle$ $\Rightarrow$ see last page

20. Suppose $\vec{\mathbf{F}}(x, y, z) = 2xy^2\,\vec{\imath} + 2yx^2\,\vec{\jmath} - (x^2 + y^2)z\,\vec{\mathbf{k}}$ and S is the boundary surface of the solid enclosed by the cylinder $x^2 + y^2 = 1$ and the planes $z = -1$ and $z = 1$. S is a closed surface oriented by the outward normal. Calculate the flux integral $\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$.

A. 0

B. π

C. 2π

D. 3π

E. 4π

parameterization:

$r(t)\ \langle \cos u, \sin u, 1\rangle$

$r'(t) = \langle -\sin u, \cos u, 0\rangle$

$r(t)$, $r'(t)$

$$\begin{vmatrix} i & j & k \\ \cos u & \sin u & 1 \\ -\sin u & \cos u & 0 \end{vmatrix} = i(-\cos u) - j(\sin u) + k(\cos^2 u + \sin^2 u)$$

$-\cos u - \sin u + 1$

(19)

$\therefore F(r(t)) \cdot r'(t)$

$= 8\cos u\,\hat{j} + 8\sin u\,\hat{k}$

$\langle 0, 8\cos u, 8\sin u\rangle \cdot \langle -2\sin u, 2\cos u, 0\rangle$

$= 16\cos^2 u\ du$

$\therefore \int_0^{2\pi} 16\cos^2 u\ du = 16[\pi] = \underline{\underline{16\pi}}$

$= 2\left[\frac{1-\cos 2u}{2}\right]$

$= 2\int_0^{2\pi} \sin^2 2u\ du$

$= \int_0^{2\pi} 4\sin^2 u\cos^2 u\ du\,dv$

$= \int_0^{2\pi} \sin^2 2u\,du$

(20)

~~bad~~

$r(t) = \langle \cos u, \sin u, v\rangle \quad -1 \le v \le 1$

$r'_u(t) = \langle -\sin u, \cos u, 0\rangle$

$r'_v(t) = \langle 0, 0, 1\rangle$

$r'_u(t) \times r'_v(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$\hat{i}(\cos u) - \hat{j}(-\sin u)$

$(\cos u + \sin u)$

$\langle \cos u, \sin u, 0\rangle$

$= \text{ugly}$

$\int_0^{2\pi}\int_{-1}^{1} F(r(t)) \cdot r'(t)$

$= \int_0^{2\pi}\int_{-1}^{1} (2\cos^3 u\sin^2 u + 2\sin^2 u\cos^2 u)\ dv\,du$

$= \int_0^{2\pi}\int_{-1}^{1} \langle 2\cos u\sin^2 u, 2\sin u\cos^2 u, -v\rangle \cdot \langle \cos u, \sin u, 0\rangle$

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better

$\operatorname{div} F = 2y^2 + 2x^2 - (x^2+y^2)$

$= x^2+y^2$

by Divergence theorem:

$$\iiint_V (x^2+y^2)\, dV$$

$$\int_{-1}^{1}\int_{0}^{2\pi}\int_{0}^{1} r^2 \cdot r \, dr\, d\theta\, dz$$

$$= 2\int_{0}^{2\pi}\int_{0}^{1} r^3\, dr\, d\theta \; \cancel{dz}$$

$$= 4\pi \left[\frac{r^4}{4}\right]_0^1 = 4\pi \times \frac{1}{4} = \underline{\underline{\pi}}$$