Spring '19

MA 26100
FINAL EXAM Form 01
MAY 1, 2019

NAME Leaderboard YOUR TA'S NAME Someone

STUDENT ID # ~~1/0~~ 1/0 RECITATION TIME ~~1:20~~ 6pm

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): $\boxed{01}$

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **20** questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 5:20, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 3:50. If you don't finish before 5:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: Leaderboard

STUDENT SIGNATURE: Signature

$x = t+2$
$y = t+1$
$z = 2t-2$

1. Find an equation of the plane that contains the point $(1, 2, -3)$ and the line with symmetric equations $x - 2 = y - 1 = \dfrac{z+2}{2}$.

A. $5x + y + z = 4$
B. $2x - y + z = -3$
C. $3x + y - 2z = 11$
D. $4x - 2y - 3z = 9$
E. $x + y - 2z = 9$

$\cancel{1(x-1) + 1(y-2) + 2(z+3) = 0}$ $\langle 3, 2, 0 \rangle$

another point $t=0$

$\Rightarrow \langle 2, 1, -2 \rangle$

$1 \quad 2 \quad -3$
$3 \quad 2 \quad 0$
$2 \quad 1 \quad -2$

Then ~~find normal~~ you have plane with 3 points:

$$\begin{vmatrix} x-1 & y-2 & z+3 \\ 2 & 0 & 3 \\ 1 & -1 & 1 \end{vmatrix}$$

$= 3(x-1) + 1(y-2) - 2(z-3) = 0$

2. Identify the surface defined by the equation $x^2 + y^2 + 2z - z^2 = 0$.

A. Ellipse
B. Hyperboloid of one sheet
C. Ellipsoid
D. Hyperboloid of two sheets
E. Paraboloid

$x^2 + y^2 - (z^2 - 2z) = 0$
$x^2 + y^2 - ((z-1)^2 - 1) = 0$
$x^2 + y^2 - (z-1)^2 = -1$
$(z-1)^2 - x^2 - y^2 = 1$

hyperboloid of 2 sheets

$\gamma'(t) = \langle -\sin t, \cos t \rangle$

3. The vector field $\mathbf{F}(x,y) = \langle 2xe^y + 1, x^2e^y \rangle$ is conservative. Compute the work done by the field in moving an object along the path $C : \mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$, $0 \le t \le \pi$.

$\int_0^\pi F(\gamma(t)) \cdot \gamma'(t)\, dt$

A. -2
B. -1
C. -4
D. -8
E. -6

$\langle 2\cos t\, e^{\sin t} + 1, \cos^2 t\, e^{\sin t} \rangle$

$\langle -\sin t, \cos t \rangle \cdot dt$

$= \int_0^\pi \langle \cancel{2\cos t(\sin t)e^{\sin t}}$
$-\sin t (2\cos t\, e^{\sin t} + 1) + \cos^3 t\, e^{\sin t})$

$= \int_0^\pi [-2\cos t \sin t\, e^{\sin t} - \sin t + \cos^3 t\, e^{\sin t}]\, dt$

4. Compute

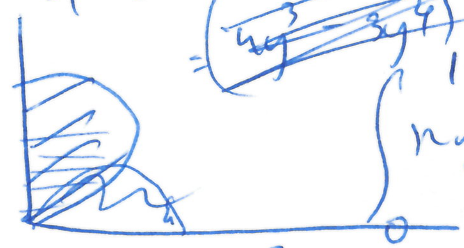
$$\int_C (e^{2x} + y^2)\, dx + (14xy + y^2) dy,$$

where C is the boundary of the region bounded by the y-axis and the curve $x = y - y^2$ oriented counterclockwise.

$= \int_0^\pi e^{\sin t}[-2\cos t \sin t + \cos^3 t] - \sin t\, dt$

Green's

$= (4y^3 - 3y^4)\Big|_0^1 \quad x = 0 \quad y(y-1)$



A. 1
B. 2
C. 4
D. 12
E. 24

$\int_0^1 12y^2 - 12y^3\, dy$

$= \iint \frac{\delta Q}{\delta x} - \frac{\delta P}{\delta y}$

$= 1$

$\iint 14y - 2y = \int_0^1 \int_0^{y-y^2} 12y = 12y(y - y^2)$

$1\sqrt{4} = 2$

5. Find the linear approximation of $f(x, y) = y\sqrt{x}$ at $(4, 1)$.

$A = f(x, y)$

$dA = f_x\,dx + f_y\,dy$

A. $\frac{1}{4}x + 16y - 15$

B. $\frac{1}{4}x + 8y - 7$

C. $\frac{1}{4}x + 4y - 3$

D. $\frac{1}{4}x + y + 1$

E. $\frac{1}{4}x + 2y - 1$

$dA = \frac{y}{2\sqrt{x}}\,dx + \sqrt{x}\cdot dy$

at $(4, 1)$

$\frac{1}{4}\,dx + 2\,dy$

6. Compute $\operatorname{curl}\mathbf{F}(\pi, 1, 1)$, where $\mathbf{F} = \langle x + y, yz, \sin(x)\rangle$.

$x \quad y \quad z$

$= \hat{i}\,[0 - y] - \hat{j}\,(\cos(x) - 0) + \hat{k}\,(0 - 1)$

$= -\hat{i} + \hat{j} - 1$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x' & y' & z' \\ F & G & H \end{vmatrix}$$

$\hat{i}\left(\frac{\delta H}{\delta y} - \frac{\delta G}{\delta z}\right) - \hat{j}\left(\frac{\delta H}{\delta x} - \frac{\delta F}{\delta z}\right) + \hat{k}\left(\frac{\delta G}{\delta x} - \frac{\delta F}{\delta y}\right)$

A. $\langle 1, 1, -1\rangle$

B. $\langle 1, 1, 1\rangle$

C. $\langle -1, 1, -1\rangle$

D. $\langle -1, -1, -1\rangle$

E. $\langle 1, -1, -1\rangle$

~~very [illegible]~~?

7. If $f(x, y) = x\sin(xy^2)$, compute $f_{yx}(\pi, 1)$.

$\to 2xy$

$$f_{xy} = \frac{\delta}{\delta y}\left(\frac{\delta}{\delta x}\right)$$

$$f_{yx} = \frac{\delta}{\delta x}\left(\frac{\delta}{\delta y}\right)$$

A. -8π

B. -6π

C. -2π

D. $-\pi$

E. -4π

$$= \frac{\delta}{\delta x}\, 2x^2y \cos(xy^2)$$

$$\to 2y\cos(xy^2)$$

$$= 4xy\cos(xy^2) - 2xy^3$$

$$- 2x^2y\left[y^2\sin(xy^2)\right]$$

$$= 4\pi\cos(\pi) - 2\pi^2\left[\sin(\pi)\right]$$

$$= -4\pi$$

8. Find the direction in which $f(x, y, z) = \dfrac{x}{y} - yz$ decreases most rapidly at the point $(4, 1, 1)$?

$$\nabla f = \left\langle \frac{1}{y}, \frac{-x}{y^2} - z, -y \right\rangle$$

A. $\frac{1}{\sqrt{27}}\langle 1, -5, 1\rangle$

B. $\frac{1}{\sqrt{27}}\langle 1, -5, -1\rangle$

C. $\frac{1}{\sqrt{27}}\langle -1, 5, -1\rangle$

D. $\frac{1}{\sqrt{27}}\langle -1, 5, 1\rangle$

E. $\frac{1}{\sqrt{27}}\langle 1, 5, 1\rangle$

$$\nabla f_{(4,1,1)} = \left\langle 1, \frac{-4}{1} - 1, -1 \right\rangle$$

$\langle 1, -5, -1\rangle$ direction of ascent

$\therefore$ descent: $\langle -1, 5, 1\rangle$

9. Let M and m denote the maximum and the minimum values of $f(x, y) = x^2 - 2x + y^2 + 3$ in the disk $x^2 + y^2 \leq 1$. Find $M + m$.

this is M − m ~~speeding~~ come on

$x^2 = 1 - y^2$

then

$f = 1 - \cancel{y^2} - 2x + \cancel{y^2} + 3$

$4 - 2x \qquad m \quad -1 \leq x \leq 1$

$\therefore \max = 6 \quad (x = -1)$

$\min = 2 \quad (x = 1)$

$M + m = 8$

A. 4

B. 5

C. 12

D. 8

E. 7

10. Evaluate the integral $\iint_D 2\pi \sin(x^2)\, \mathrm{dA}$ where D is the region in the xy-plane bounded by the lines $y = 0$, $y = x$ and $x = \sqrt{\pi}$.

$$\int_0^{\sqrt{\pi}} \int_0^{x} 2\pi \sin(x^2)\, dy\, dx = \pi \int_0^{\sqrt{\pi}} 2x \sin(x^2)\, dx$$

$$= \int_0^{\sqrt{\pi}} \left[2\pi x \sin(x^2)\, dx \right] = -\pi \left[\cos(x^2)\right]_0^{\sqrt{\pi}}$$

$$= -\pi \left[\cos \pi - \cos 0\right] = 2\pi$$

A. 2π

B. π

C. 4π

D. 8π

E. $\pi/2$

(3)

$$\int_0^{\pi} \cos t\, e^{\sin t}\left[-2\sin t + 1 - \sin^2 t\right] - \sin t \, dt$$

Let $\sin t = u$. Then $\cos t\, dt = du$, so

$$\int_0^{0} e^u\left[-2u + 1 - u^2\right]du - \int_0^{\pi} \sin t\, dt$$

$$\int_0^{0} e^u\left[1 - 2u - u^2\right]du - I_2$$

$$\int_0^{0} e^u \ldots = \underline{\underline{0}}$$

$$I_2 = \left[\cos t\right]_0^{\pi}$$

$$= -1 - 1 = \underline{\underline{-2}}$$

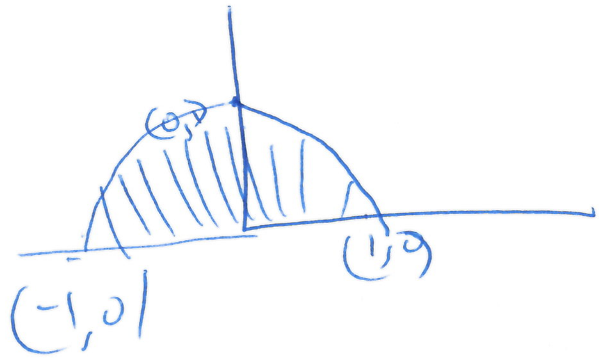
$$\therefore I = \underline{\underline{-2}}.$$

perhaps better to find potential and use Fundamental theorem of calculus!

11. Evaluate the double integral

$$\iint_D 2e^{(x^2+y^2)}\,\mathrm{dA},$$

where D is the region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.



A. $8\pi(e-1)$
B. $2\pi(e-1)$
C. $4\pi(e-1)$
D. $\pi(e-1)$
E. $16\pi(e-1)$

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} 2e^{(x^2+y^2)}\, dA$$

$$\to \int_0^{\pi}\int_0^{1} 2e^{r^2}\cdot r\, dr\, d\theta = \int_0^{\pi} \left. e^{r^2}\right|_0^1 d\theta$$

$$\pi(e-1)$$

12. Compute the triple integral

$$\iiint_E 3y\,\mathrm{dV},$$

where E is a region under the plane $x+y+z=2$ in the first octant.

$$\int_0^{2}\int_0^{2-x}\int_0^{2-y-x} 3y\, dV$$

A. 4
B. 2
C. 6
D. 3
E. 1

→ cont

(12)

$$\int_0^2 \int_0^{2-x} \int_0^{2-y-x} 3y \, dz \, dy \, dx$$

$$\int_0^2 \int_0^{2-x} 3y(2-y-x) \, dy \, dx$$

$$\int_0^2 \int_0^{2-x} \left(6y - 3y^2 - 3xy\right) dy \, dx$$

$$= \int_0^2 \left[3y^2 - y^3 - \frac{3xy^2}{2}\right]_0^{2-x} dx$$

$$\int_0^2 \left[3(2-x)^2 - (2-x)^3 - \frac{3x(\cancel{x}\,2-x)^2}{2}\right] dx$$

$$\int_0^2 (2-x)^2 \left[3 - (2-x) - \frac{3x}{2}\right] dx$$

$$1 + x - \frac{3x}{2}$$

$$\int_0^2 (2-x)^2 \left[1 - \frac{x}{2}\right] dx = \frac{1}{2}\int_0^2 (2-x)^3 \, dx$$

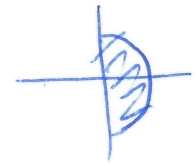
$$= \frac{-1}{2}\left[\frac{(2\cancel{x}-x)^4}{4}\right]_0^2$$

$$= \frac{-1}{2}\left[0 - 4\right]$$

$$= \underline{\underline{2}}$$

13. The integral

$$\int_0^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{8-x^2-y^2}} xy^2 z \, dz \, dy \, dx$$



region

when converted to cylindrical coordinates becomes

$x = r\cos\theta$

$y = r\sin\theta$

then

xy^2z

$\to r(r\cos\theta)(r^2\sin^2\theta)$

$r^4\cos\theta\sin^2\theta \, dz$

A. $\displaystyle\int_{-\pi/2}^{\pi/2} \int_0^2 \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^4 z \cos\theta \sin^2\theta \, dz \, dr \, d\theta$

B. $\displaystyle\int_{-\pi/2}^{\pi/2} \int_0^{\sqrt{2}} \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^3 z \cos\theta \sin^2\theta \, dz \, dr \, d\theta$ X

C. $\displaystyle\int_{-\pi/2}^{\pi/2} \int_0^{\sqrt{2}} \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^4 z \cos\theta \sin^2\theta \, dz \, dr \, d\theta$

D. $\displaystyle\int_0^{\pi} \int_0^{\sqrt{2}} \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^4 z \cos\theta \sin^2\theta \, dz \, dr \, d\theta$ X

E. $\displaystyle\int_0^{\pi} \int_0^2 \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^4 z \cos\theta \sin^2\theta \, dz \, dr \, d\theta$ X

14. Convert the integral to spherical coordinates and compute it:

$\to 8 - r^2\sin^2\varphi$

$$\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} 3 \, dz \, dy \, dx.$$

$r^2\sin^2\varphi$

$\int \int \int^{\sqrt{8 - r^2\sin^2\varphi}} 3 \, r^2 \sin\varphi \, dr \, d\varphi \, d\theta$

$r\sin\varphi$

A. $2(\sqrt{2}-1)\pi$
B. $8(\sqrt{2}-1)\pi$
C. $10(\sqrt{2}-1)\pi$
D. $16(\sqrt{2}-1)\pi$
E. $12(\sqrt{2}-1)\pi$

$= \int\int$

Answer for Q14

We have r from 0 to $\sqrt{8}$. Then, consider the intersection of the two curves:

$$\sqrt{x^2+y^2} = \sqrt{8-x^2-y^2}$$
$$2(x^2+y^2) = 8$$
$$x^2+y^2 = 4$$

Note that the angle of intersection is $\frac{2}{\sqrt{8}} \to \frac{1}{\sqrt{2}}$. Note that $\cos \varphi = \frac{1}{\sqrt{2}} \to \varphi = \frac{\pi}{4}$.

Hence the final integral is

$$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{8}} 3r^2 \sin\phi \, dr d\phi d\theta$$
$$= 8^{\frac{3}{2}}\int_0^{2\pi}\int_0^{\frac{\pi}{4}} \sin\phi \, d\phi \, d\theta$$
$$= 2\pi 8^{\frac{3}{2}}\left(1-\frac{1}{\sqrt{2}}\right)$$
$$= 2\pi\left(\sqrt{2}-1\right) * 8$$
$$= 16\pi(\sqrt{2}-1)$$

15. Compute the line integral

$$\int_C \mathbf{F} \cdot \mathbf{dr},$$

where $\mathbf{F} = \langle xy, x + y \rangle$ and C is the curve $y = x^2$ from $(0,0)$ to $(1,1)$.

A. $\frac{13}{12}$

B. $\frac{21}{12}$

C. $\frac{17}{12}$

D. $\frac{5}{12}$

E. $\frac{23}{12}$

$F(r(t)) \cdot r'(t))$

$r(t) = \langle t^2, t \rangle$

$r'(t) = \langle 2t, 1 \rangle$

$F\langle t^3, t^2 + t \rangle \cdot \langle 2t, 1 \rangle$

$\int_0^1 (\cancel{2t^4} + \cancel{t^2} + t)\, dt$

$t^3 \quad 2t^3$

$\cancel{\left[\frac{2t^5}{5} + \frac{t^3}{3} + \frac{t^2}{2}\right]_0^1 = 2/5 + 4/3 + 1/2 = \frac{9}{10} + \frac{1}{3} = \frac{37}{30}}$

$\left[\frac{3t^4}{4} + \frac{2t^3}{3}\right]_0^1 = \frac{3}{4} + \frac{2}{3} = \frac{17}{12}$

16. Let S be the part of the surface $z = xy + 1$ that lies within the cylinder $x^2 + y^2 = 1$. Find the area of the surface S.

A. $\frac{\sqrt{2}}{3}\pi - \frac{2}{3}\pi$

B. $\frac{\sqrt{2}}{3}\pi - \frac{1}{3}\pi$

C. $\frac{4\sqrt{2}}{3}\pi - \frac{1}{3}\pi$

D. $\frac{4\sqrt{2}}{3}\pi - \frac{2}{3}\pi$

E. $\frac{2\sqrt{2}}{3}\pi - \frac{2}{3}\pi$

$A = \iint_D \sqrt{1 + \left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2}$

$\int_0^{2\pi}\int_0^1 \sqrt{1 + y^2 + x^2}$

$\Rightarrow \int_0^{2\pi}\int_0^1 \sqrt{1+r^2} \cdot r\, dr\, d\theta$

$= \frac{1}{2}\int_0^{2\pi} \frac{2}{3}(1+r^2)^{3/2}\Big|_0^1$

$= \frac{2\pi}{3}\left(2^{3/2} - 1\right)$

$\frac{4\sqrt{2}}{3}\pi - \frac{2\pi}{3}$

17. Find the surface area of the parametric surface $\mathbf{r}(u,v) = \langle u^2, uv, v^2/2\rangle$ with $0 \le u \le 3$, $0 \le v \le 1$.

A. 12

B. 15

C. 18

D. 19

E. 27

$r'_u = \langle 2u, v, 0\rangle$

$r'_v = \langle 0, u, v\rangle$

$|r'_u \times r'_v| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2u & v & 0 \\ 0 & u & v \end{vmatrix}$

$\hat{i}(v^2) - \hat{j}(2uv) + \hat{k}(2u^2)$

$\sqrt{v^4 + 4u^2v^2 + 4u^4}$

$\sqrt{(2u^2+v^2)^2}$ $\int_0^3\int_0^1 2u^2+v^2 \, dv \, du$

18. Use Stokes' Theorem to evaluate the integral $\int_C y\,dx + z\,dy + x\,dz$, where C is the intersection of the surfaces $x^2+y^2=1$ and $x+y+z=5$. C is oriented counterclockwise when viewed from above.

$= \int_0^3\int_0^1 2u^2+v^2 \, dv \, du$

$= \int_0^3 \left. 2u^2v + \frac{v^3}{3} \right|_0^1 du$

$= \int_0^3 2u^2 + \frac{1}{3} \, du$

$= \left. \frac{2u^3}{3} + \frac{u}{3} \right|_0^3 = 18+1=19$

A. -8π

B. -6π

C. $-\pi$

D. -3π

E. -9π

Answer for Q18

(I forgot to solve it in the test)

This integral looks dead weird for Stokes' Theorem. The first step is to convert it into $F.dr$:

$$\int_C y\,dx + z\,dy + x\,dz = \int_C F\,.dr$$

Where $< P, Q, R > = < y, z, x >$.

Now, making use of Stokes' theorem, we have

$$\int_C F\,.dr = \iint_S \text{curl}\, F.dS$$

Finding the curl is easy:

$$curl\ F = \begin{vmatrix} i & j & k \\ x' & y' & z' \\ P & Q & R \end{vmatrix} = i\left(\frac{\delta R}{\delta y} - \frac{\delta Q}{\delta z}\right) - j\left(\frac{\delta R}{\delta x} - \frac{\delta P}{\delta z}\right) + k\left(\frac{\delta Q}{\delta x} - \frac{\delta P}{\delta y}\right)$$
$$= (-1, -1, -1)$$

The next step involves parameterizing the surface, and is the tricky part. Note that we have two surfaces to work with:

$$x^2 + y^2 = 1 \rightarrow (v\sin u\,, v\cos u\,, v)$$
$$x + y + z = 5 \rightarrow (x, y, 5 - x - y)$$

Combining both, we get the parameterization as $(v\sin u\,, v\cos u\,, 5 - v\sin u - v\cos u)$.

Then, find their cross product $|r_u \times r_v|$:

$$r_u = (v\cos u\,, -v\sin u\,, -v\cos u + v\sin u)$$
$$r_v = (\sin u\,, \cos u\,, -\sin u - \cos u)$$
$$|r_u \times r_v| = \begin{vmatrix} i & j & k \\ v\cos u & -v\sin u & v\sin u - v\cos u \\ \sin u & \cos u & -\sin u - \cos u \end{vmatrix}$$
$$= i\big(v\sin u\,(\sin u + \cos u) - \cos u\,(v\sin u - v\cos u)\big) - j\big(v\cos u\,(-\sin u - \cos u) - \sin u\,(v\sin u - v\cos u)\big)$$
$$+ k(v\cos^2 u + v\sin^2 u)$$
$$= i\big(v(\sin^2 u + \sin u\cos u - \sin u\cos u + \cos^2 u)\big) - j\big(v(-\cos^2 u - \cos u\sin u - \sin^2 u + \sin u\cos u)\big) + kv$$
$$= iv + jv + kv \rightarrow (v, v, v)$$

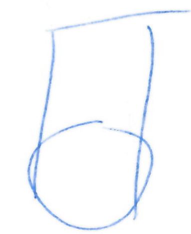
Hence,

$$\iint_S \text{curl}\, F.dS = \int_0^{2\pi}\int_0^1 (-1, -1, -1)(v, v, v)\,dv\ \ du$$
$$= \int_0^{2\pi}\int_0^1 -3v\,dv\,du$$
$$= 2\pi\left(-\frac{3}{2}\right) = -3\pi$$

19. Evaluate the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle 3xy^2, x\cos(z), z^3\rangle$ and S is the complete boundary surface of the solid region bounded by the cylinder $y^2 + z^2 = 2$ and the planes $x = 1$ and $x = 3$. S is oriented by the outward normal.

$\operatorname{div} F =$

$\langle 3y^2, 0, 3z^2 \rangle$



A. 9π

B. 12π

C. 14π

D. 18π

E. 24π

$\iiint 3y^2 + 3z^2 \; dx\, dr\, d\theta$

$3(r^2 \cdot r \quad 3r^3 \; \frac{3}{4} r^4 \Big|_0^{\sqrt{2}} = \frac{3}{4} \times 4 = 3$

$4\pi \left(\frac{3}{4} r^4\right)\Big|_0^{\sqrt{2}}$

$\cancel{\int_0^{2\pi}\int_1^{3} 6 r \, dr\, d\theta}$

$= 4\pi \times 3 = 12\pi$

$\int_1^3 \int_0^{2\pi} \int_0^{\sqrt{2}} 3r^3 \, dr\, d\theta\, dx$

$= 6 \times 2\pi \times 2$

20. The position function of a Space Shuttle is $\mathbf{r}(t) = \langle t^2, -t, 6\rangle$, $t \geq 0$. The International Space Station has coordinates $(16, -5, 6)$. In order to dock the Space Shuttle with the Space Station the captain plans to turn off the engine so that the Space Shuttle coasts into the Space Station. At what time should the captain turn off the engines? Assume there are no other forces acting on the Space Shuttle other than the force of the engine.

$a(t) = 0 \quad v(t) = c$

so for t seconds it will accelerate and then stay with that speed.

$t^2 +$

A. 6

B. 8

C. 2

D. 4

E. 0

$a = \langle 2, 0, 0 \rangle$

~~best idea~~

$2t^2 \quad 16$

20

$\gamma(t) = (t^2, -t, 6)$

$\gamma'(t) = \langle 2t, -1, 0 \rangle$

Let it shut down at time a. Then,

$\gamma(x, y, z) = (a^2, -a, 6 \rangle$

and direction is in $(2a, -1, 0 \rangle$.

Hence we need $a + \gamma(t) + \gamma'(t) = P$? $\underline{\underline{\$55}}$

~~a + (~~

$(a^2 + k2a, \; -a - k, \; 6 \rangle = \langle 16, -5, 6 \rangle$

$\Rightarrow \cancel{a^2 + 2a = 16}$ $\quad -a - k = -5$

$\to \cancel{a = 4}$ $\quad a^2 + 2ak = 16$

$a = 2 \quad k = 3$

5