review

30 October 2019 11:50

25. Consider the problem of finding the minimum value of the function $f(x,y) = 4x^2 + y^2$ on the curve $xy = 1$. In using the method of Lagrange multipliers, the value of λ (even though it is not needed) will be

A. 2 B. -2 C. $\sqrt{2}$ D. $\frac{1}{\sqrt{2}}$ E. 4.

26. Evaluate the iterated integral $\int_1^3 \int_0^x \frac{1}{x}\, dy dx$.

A. $-\frac{8}{9}$ B. 2 C. $\ln 3$ D. 0 E. $\ln 2$.

27. Consider the double integral, $\iint_R f(x,y)dA$, where R is the portion of the disk $x^2+y^2 \leq 1$, in the upper half-plane, $y \geq 0$. Express the integral as an iterated integral.

A. $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)dydx$

B. $\int_{-1}^{0} \int_{0}^{\sqrt{1-x^2}} f(x,y)dydx$

C. $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} f(x,y)dydx$

D. $\int_{0}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)dydx$

E. $\int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} f(x,y)dydx$.

25

$\nabla f = \lambda \nabla g$ constraint : $(xy = 1)$

$8x = \lambda y$

$2y = \lambda x$

$4\frac{x}{y} = \frac{y}{x}$

$4 = \left(\frac{y}{x}\right)^2 \quad \frac{y}{x} = 2$

$8x = \lambda(2x)$

$\Rightarrow \lambda = 4$

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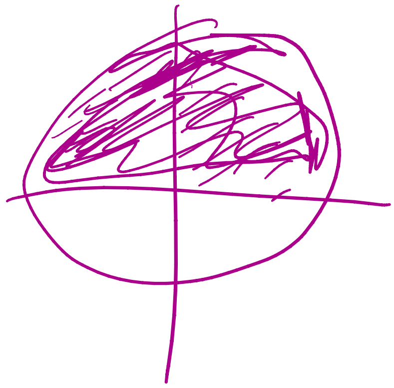
$$\int_1^3 \int_0^x \frac{1}{x}\, dy\, dx$$

$$= \int_1^3 \left(\frac{y}{x}\right)\Big|_0^x dx$$

$$= \int_1^3 1\, dx = [x]_1^3 = 2$$

27)



dx goes from -1 to 1 but dy is always positive

28. Find a and b for the correct interchange of order of integration:
$$\int_0^2 \int_{x^2}^{2x} f(x,y)\,dydx = \int_0^4 \int_a^b f(x,y)\,dxdy.$$

A. $a = y^2, b = 2y$ B. $a = \frac{y}{2}, b = \sqrt{y}$ C. $a = \frac{y}{2}, b = y$
D. $a = \sqrt{y}, b = \frac{y}{2}$ E. cannot be done without explicit knowledge of $f(x,y)$.

29. Evaluate the double integral $\iint_R y\,dA$, where R is the region of the (x,y)-plane inside the triangle with vertices $(0,0)$, $(2,0)$ and $(2,1)$.

A. 2 B. $\frac{8}{3}$ C. $\frac{2}{3}$ D. 1 E. $\frac{1}{3}$.

30. The volume of the solid region in the first octant bounded above by the parabolic sheet $z = 1 - x^2$, below by the xy plane, and on the sides by the planes $y = 0$ and $y = x$ is given by the double integral

A. $\int_0^1 \int_0^x (1-x^2)\,dydx$ B. $\int_0^1 \int_0^{1-x^2} x\,dydx$ C. $\int_{-1}^1 \int_{-x}^x (1-x^2)\,dydx$
D. $\int_0^1 \int_x^0 (1-x^2)\,dydx$ E. $\int_0^1 \int_x^{1-x^2} dydx$.

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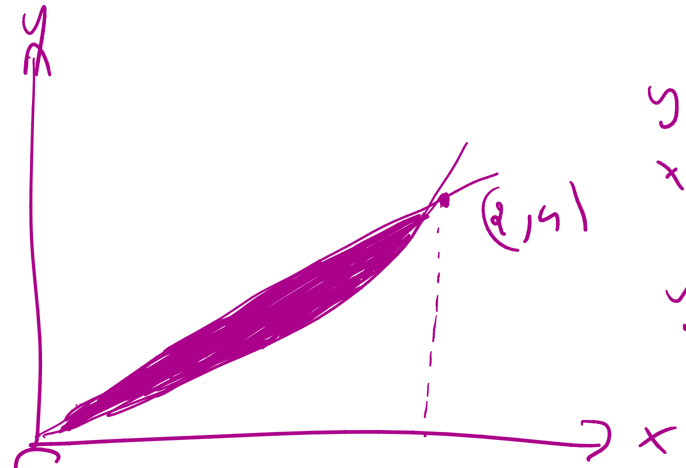


$y = 2x$

$x = \frac{y}{2}$

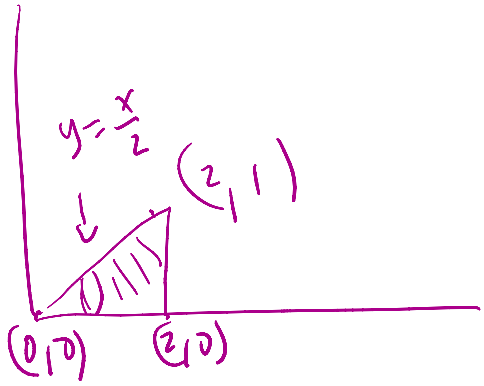
$y = x^2$

$x = \sqrt{y}$

$$\int_0^4 \int_{y/2}^{\sqrt{y}} f(x,y)\,dx$$

$$\int_0 \int_{y/2} \ldots \, dy \qquad \left(\text{hint}: \sqrt{y} > y/2\right)$$

29)



line : $y = \frac{x}{2}$

$\therefore$

$$A = \int_0^2 \int_0^{\frac{x}{2}} y \quad dy \, dx$$

$$= \left(\frac{y^2}{2}\right)_0^{x/2} = \int_0^2 \frac{x^2}{8} \, dx$$

$$= \left(\frac{1}{8} \times \frac{x^3}{3}\right)_0^2$$

$$\left(\frac{x^3}{24}\right)_0^2 = \underline{\underline{\frac{1}{3}}}$$

30)

$$I = \int_0^1 \int_0^x \int_0^{1-z^2} dz \, dy \, dx$$

$$\int_0 \int_0 \int_0$$

$max(x)$:

as $(1 - x^2 \geq 0)$ also first octant

33. A solid region in the first octant is bounded by the surfaces $z = y^2$, $y = x$, $y = 0$, $z = 0$ and $x = 4$. The volume of the region is

A. 64 B. $\frac{64}{3}$ C. $\frac{32}{3}$ D. 32 E. $\frac{16}{3}$.

34. An object occupies the region bounded above by the sphere $x^2 + y^2 + z^2 = 32$ and below by the upper nappe of the cone $z^2 = x^2 + y^2$. The mass density at any point of the object is equal to its distance from the xy plane. Set up a triple integral in rectangular coordinates for the total mass m of the object.

A. $\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

B. $\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

C. $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

D. $\int_{0}^{4} \int_{0}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

E. $\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} xy\,dz\,dy\,dx$.

35. Do problem 34 in spherical coordinates.

A. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho^3 \cos\varphi \sin\varphi \, d\rho\, d\varphi\, d\theta$

B. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho \cos\varphi \sin\varphi \, d\rho\, d\varphi\, d\theta$

C. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho^3 \sin^2\varphi \, d\rho\, d\varphi\, d\theta$

D. $\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{32}} \rho^3 \cos\varphi \sin\varphi \, d\rho\, d\varphi\, d\theta$

E. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho \cos\varphi \, d\rho\, d\varphi\, d\theta$.

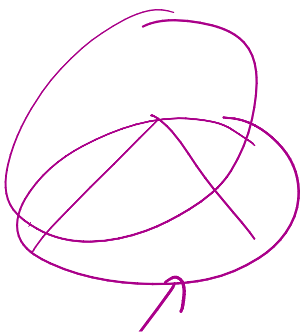
33)

$$\int_0^4 \int_0^x \int_0^{y^2} z \, dz\, dy\, dx$$

$$= \int_0^4 \int_0^x y^2 \, dy\, dx$$

$$= \frac{1}{3}\int_0^4 x^3 \, dx = \frac{1}{3}\left(\frac{x^4}{4}\right)\Big|_0^4 = \underline{\underline{64/3}}$$

34)



$[\rho(x,y,z) = z)$

(ugly drawing)

unrestricted

$$\int \quad 4 \int_{-4}^{\sqrt{16-r^2}} \int_{\pm\sqrt{16-x^2}}$$

[intersection :
$2z^2 = 32 \Rightarrow z^2 = 16$
$z = \pm 4$]

but we start from upper portion of cone.

35) $z = \rho \cos \varphi$

$\therefore 2$

$$\iiint \rho \cos\varphi \cdot \rho^2 \sin\varphi$$

$\varphi = \pi/4$. (as tan $y/x = 1$)

36. The double integral $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2(x^2+y^2)^3\,dy\,dx$ when converted to polar coordinates becomes

A. $\int_0^{\pi} \int_0^1 r^9 \sin^2\theta\, dr\, d\theta$ B. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin^2\theta\, dr\, d\theta$ ✗ C. $\int_0^{\pi} \int_0^1 r^8 \sin\theta\, dr\, d\theta$ ✗

D. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin\theta\, dr\, d\theta$ ✗ E. $\int_0^{\frac{\pi}{2}} \int_0^1 r^9 \sin^2\theta\, dr\, d\theta$.

37. Which of the triple integrals converts

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} dz\,dy\,dx$$

from rectangular to cylindrical coordinates?

A. $\int_0^{\pi} \int_0^2 \int_r^2 r\, dz\, dr\, d\theta$ B. $\int_0^{2\pi} \int_0^2 \int_r^2 r\, dz\, dr\, d\theta$ C. $\int_0^{2\pi} \int_{-2}^2 \int_r^2 r\, dz\, dr\, d\theta$ ✗

D. $\int_0^{\pi} \int_0^2 \int_r^2 r\, dz\, dr\, d\theta$ E. $\int_0^{\frac{3\pi}{2}} \int_{-2}^2 \int_r^2 r\, dz\, dr\, d\theta$.

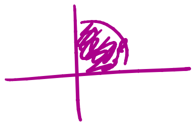
38. If D is the solid region above the xy-plane that is between $z = \sqrt{4-x^2-y^2}$ and $z = \sqrt{1-x^2-y^2}$, then $\iiint_D \sqrt{x^2+y^2+z^2}\; dV =$

A. $\frac{14\pi}{3}$ B. $\frac{16\pi}{3}$ C. $\frac{15\pi}{2}$ D. 8π E. 15π.

(36) $x^2+y^2=r^2$. $\to$ $y = r\sin\theta$

$$r^2\sin^2\theta \, (r^2)^3 \cdot r$$

$$= r^9 \sin^2\theta$$



also $\int_0^1 \int_0^{\sqrt{1-x^2}}$ is first quadrant

hence $\theta = \pi/2$.

(37) $$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2}$$

r

$$\int_0^{2\pi}\int_0^{2}$$ and $x = y$ p

38) $$\int_0^{2\pi}\int_0^{\pi/2}\int_1^{2} r \cdot r^2 \sin\varphi \, dr\, d\varphi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2}\left[\frac{r^4}{4}\right]_1^2 \sin\varphi \, d\varphi\, d\theta$$

$\pi/2$

$$\frac{15}{4} \times 2\pi \int_0^{\pi/2} \sin\varphi \, d\varphi$$

$$= \frac{15\pi}{2} \left[\left(-\cos\varphi \right)_0^{\pi/2} \right]$$

$$\frac{15\pi}{2} \left[0 + 1 \right] = \frac{15\pi}{2}$$

($\frac{\pi}{2}$ because it's x-y plane, so effectively a half sphere)

39. Determine which of the vector fields below are conservative, i. e. $\vec{F} = \text{grad } f$ for some function f.

1. $\vec{F}(x,y) = (xy^2 + x)\vec{i} + (x^2y - y^2)\vec{j}$. ✓
2. $\vec{F}(x,y) = \frac{x}{y}\vec{i} + \frac{y}{x}\vec{j}$. ✗
3. $\vec{F}(x,y,z) = ye^z\,\vec{i} + (xe^z + e^y)\vec{j} + (xy+1)e^z\,\vec{k}$. no easy way to find

A. 1 and 2 B. 1 and 3 C. 2 and 3 D. 1 only E. all three

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1) $\frac{\delta f}{\delta y} \hat{i} \left[xy^2 + x \right] = 2xy$

$\frac{\delta f}{\delta x} \hat{j} \left[x^2y - y^2 \right] = 2xy$

$\therefore \frac{\delta f}{\delta y}\hat{i} = \frac{\delta f}{\delta x}\hat{j} \Rightarrow$ conservative

2) $\frac{\delta f}{\delta y} \left(\frac{y}{x} \right) = \frac{1}{x}$

$$\frac{\delta f}{\delta x}\left(\frac{x}{y}\right) = \frac{1}{y}$$

$$\frac{1}{x} + \frac{1}{y} \Rightarrow \text{FAIL}$$

43. Evaluate the line integral

$$\int_C x\,dx + y\,dy + xy\,dz$$

where C is parametrized by $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + \cos t\vec{k}$ for $-\frac{\pi}{2} \le t \le 0$.

A. 1 B. -1 C. $\frac{1}{3}$ D. $-\frac{1}{3}$ E. 0

43

$x = \cos t \quad z = \cos t$

$y = \sin t$

$r'(t) = \langle -\sin t, \cos t, -\sin t \rangle$

$$\int_C x\,dx + y\,dy + xy\,dz$$

$$\int_{-\pi/2}^{0} x\,dx + \int_{-\pi/2}^{0} y\,dy + \int_{-\pi/2}^{0} xy\,dz$$

$$\cancel{\int_{-\pi/2}^{0} \cos t \cdot -\sin t\,dt + \int_{-\pi/2}^{0} \sin t \cdot \cos t\,dt}$$

go to zero

$$\int^{0} (\cos t \cdot \sin t) \cdot (-\sin t)\,dt$$

$$* \int_{-\pi/2} (\cos t \cdot \sin t)\cdot(-\sin t)\, dt$$

$u = \sin t$

$du = \cos t\, dt$

$$\therefore \left[\frac{\sin^3 t}{3}\right]_{-\pi/2}^{0}$$

$$= +\left[\frac{\sin^3 (-\pi/2)}{3}\right] = \frac{-1}{3}$$

45. Evaluate $\int_C y^2\,dx + 6xy\,dy$ where C is the boundary curve of the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$, in the counterclockwise direction.

A. 0 B. 4 C. 8 D. 16 E. 32

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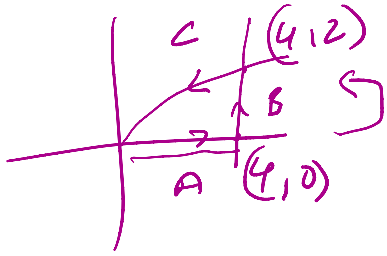

$A = \langle 4t, 0\rangle$

$B = \langle 4, 2t\rangle$

$C =$

$$A: \int_0^1 0 + 0$$

$$B: \int^1 4t^2\cdot 0 + 6\times 4\times 2t \cdot 2\, dt$$

$$\int_0^1 96t \, dt = \underline{\underline{48}}$$

$$c = y\langle x, y\rangle = \langle t^2, t\rangle \Rightarrow c' = \langle 2t, 1\rangle$$

$$\int_2^0 t^2 \cdot 2t + 6t^3 \cdot 1 \, dt$$

$$= \int_2^0 8t^3 \, dt = \left(2t^4\right)\Big|_2^0 =$$

$$= \underline{\underline{-32}}$$

$$\therefore 48 - 32 = \underline{\underline{16}}$$