Question 8

PROBLEM 8: A vector that is perpendicular to the curve $4\ln x - e^y = 4\ln 2 - 5$ at the point $(\frac{2}{e}, 0)$ is

A. $< \frac{1}{2e}, 1 >$

B. $< \frac{1}{e}, -1 >$

C. $< 2e, -1 >$

D. $< 2e, 1 >$

E. $< -1, 2e >$

Answer

First apply implicit differentiation to the curve:

$$4\ln x - e^y = 4\ln 2 - 5$$
$$\frac{4}{x} - e^y\left(\frac{dy}{dx}\right) = 0$$
$$e^y\left(\frac{dy}{dx}\right) = \frac{4}{x}$$
$$\left(\frac{dy}{dx}\right) = \frac{4}{x}\left(\frac{1}{e^y}\right)$$

Hence, the slope of the tangent at $\left(\frac{2}{e}, 0\right)$ is $\frac{4}{\frac{2}{e}} = 2e$.

But the slope normal is perpendicular to that of the tangent, hence the slope of the normal is $-\frac{1}{2e}$ (as $2e * \frac{-1}{2e} = -1$).

This means that any vector that is perpendicular to the curve is of the form $(x, mx) \to \left(1, -\frac{1}{2e}\right)$ or $(2e, -1)$.