MA 26100
EXAM 2 Form 01
April 4, 2017

NAME [illegible] YOUR TA'S NAME huh?

STUDENT ID # cos 0 RECITATION TIME $\int\int\int f(x)dx$

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. On the scantron, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. BE SURE TO INCLUDE THE TWO LEADING ZEROS.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the scantron.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

9. Turn in both the scantron and the exam booklet when you are finished.

10. You cannot turn in your exam during the first 20 min or the last 10 min of the exam period.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should be put away and should not be visible at all. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: Leaderboard

STUDENT SIGNATURE: github.com/Leader-board

1. If the Lagrange multiplier method is used to determine the extreme values of $f(x, y) = 4x - y + 2$ subject to the constraint $2x^2 + y^2 = 1$, what are the two Lagrange multipliers λ_1 and λ_2?

So find one of them.

A. $\lambda_1 = -\frac{1}{2}$ and $\lambda_2 = \frac{1}{2}$

B. $\lambda_1 = -\frac{3}{2}$ and $\lambda_2 = \frac{3}{2}$

C. $\lambda_1 = -\frac{5}{2}$ and $\lambda_2 = \frac{5}{2}$

D. $\lambda_1 = -\frac{7}{2}$ and $\lambda_2 = \frac{7}{2}$

E. $\lambda_1 = -\frac{9}{2}$ and $\lambda_2 = \frac{9}{2}$

L

$3/2 =$

$x = \frac{1}{\lambda}$

$4 = 4\lambda x \Rightarrow \lambda x = 1$

$\lambda = \frac{1}{x}$

$\frac{8}{(\lambda)^2} + \frac{1}{4\lambda^2} = 1$

$-1 = 2\lambda y$

$\lambda = \left(\frac{-1}{2y}\right)$

$y = -\frac{1}{2\lambda}$

$9/4\lambda^2 = \cancel{\lambda^2}$

2. Evaluate $\int\int_D 2e^{y^2}\,dA$, where D is the triangular region with vertices $(0, 0)$, $(0, 1)$, $(2, 1)$. Integrate with respect to x first and then y.

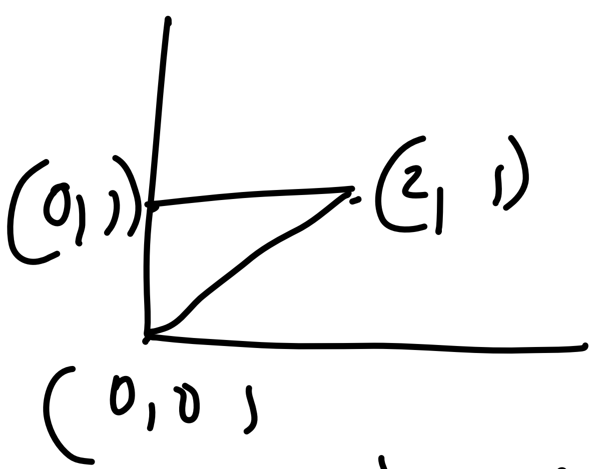
A. 2

B. $2e$

C. $2e^2$

D. $2e + 2$

E. $2e - 2$



$=2(e-1)$

$\int_0^1 \int_0^{2y} 2e^{y^2}\,dx\,dy$

$x = 2y$

$2\int_0^1 2ye^{y^2}\,dy = 2\left(e^{y^2}\right)\Big|_0^1$

3. Evaluate the double integral by reversing the order of integration:

$$\int_0^3 \int_{y^2}^9 y \cos\left(x^2\right)\, dx\, dy$$

A. $\frac{\sin 81}{2}$

B. $\frac{\sin 81}{9}$

C. $\frac{\sin 81}{24}$

D. $\frac{\sin 81}{4}$

E. $\frac{\sin 81}{16}$

$$\int_0^1 \int_0^{x^2} xy\, dy\, dx = \int_0^1 \frac{x^5}{2}\, dx = \left(\frac{x^6}{12}\right)_0^1,$$

$$= \int_0^1 x^3\, dx \to \frac{1}{4}$$

4. Let D be the region bounded by $y = x^2$, $y = 0$ and $x = 1$. If the density is $\rho(x, y) = x$ and if the center of mass is $(\overline{x}, \overline{y})$, compute $\overline{y}$.

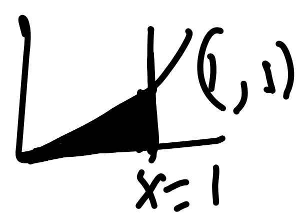
A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{2}{5}$

D. $\frac{3}{4}$

E. $\frac{4}{5}$



$$\bar{x} = \frac{X}{m} = \frac{1/12}{1/4}$$

$$\bar{y} = \frac{Y}{m} = \frac{\iint yx\rho}{\iint \rho}$$

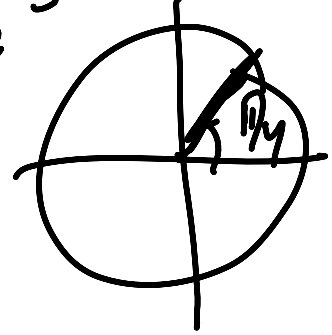
$8\int_0^{\pi/4} \cos^2\theta \, d\theta = 8\int_0^{\pi/4} \frac{1+\cos 2\theta}{2}$

For problems 5 and 6 recall that $2\cos^2\alpha = 1 + \cos 2\alpha$ and $2\cos\alpha\sin\alpha = \sin 2\alpha$.

5. Let R be the region in the first quadrant enclosed by the circle $x^2+y^2=4$ and the lines $y=0$ and $y=x$. Compute $\iint\limits_R 2x^2\,dA$.

$4\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/4}$

$= \left(\frac{r^4}{2}\right)_0^2 \int_0^{\pi/4} \cos^2\theta \, d\theta$



A. 2π

B. $\sqrt{2}$

C. $2\pi - 1$

D. $\pi - 1$

E. $\pi + 2$

$$\int_0^{\pi/4}\int_0^2 2r^2\cos^2\theta \cdot r\,dr\,d\theta$$

$$4\left(\frac{\pi}{4}+\frac{1}{2}\right) = \pi + 2$$

6. Evaluate the integral $\iiint\limits_E z\,dV$ where E is the solid inside the sphere $\rho = 2$ and above the cone $\phi = \frac{\pi}{3}$.

$r\cos\phi$

$\pi/2$ $= -2\pi\left[-1 - \frac{1}{2}\right)$

$= \frac{2\pi[4]}{2}\int \sin 2\phi \, d\theta$

$\pi/3$ $-2\pi\left(-\frac{3}{2}\right)$

$= 3\pi$

A. $\frac{\pi}{2}$

B. π

C. 3π

D. $\frac{7\pi}{2}$

E. 6π

$$\int_0^{2\pi}\int_0^{\pi/3}\int_0^2 r^3\cos\phi\sin\phi\,dr\,d\phi\,d\theta$$

$-2\pi\left[\frac{1}{2}\cos 2\phi\right]$

7. Find the area of the surface $z = 3x + y$ that lies above the region in the xy-plane bounded by $x = 1$, $y = 0$, and $y = x^3$.

A. $\frac{\sqrt{11}}{4}$

B. $\frac{\sqrt{11}}{3}$

C. $\frac{\sqrt{10}}{4}$

D. $\frac{\sqrt{10}}{3}$

E. $\frac{\sqrt{11}}{6}$

8. Let E be the region bounded above by the paraboloid $z = 2 - x^2 - y^2$ and below by the cone $z = \sqrt{x^2 + y^2}$. Which integral arises if we evaluate $\iiint_E z(x^2 + y^2)\, dV$ using cylindrical coordinates?

A. $2\pi \int_0^2 (2 - r^2 - r) r\, dr$

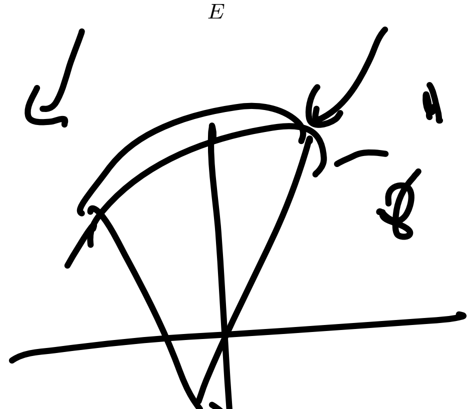
B. $2\pi \int_0^2 (2 - r^2 - r) r^3\, dr$

C. $\pi \int_0^2 ((2 - r^2)^2 - r^2) r^3\, dr$

D. $\pi \int_0^1 ((2 - r^2)^2 - r^2) r^3\, dr$

E. $2\pi \int_0^1 (2 - r^2 - r) r^3\, dr$

$\frac{z^2}{2}$



$\iiint \left((2-r^2)^2 - r^2 \right) \cdot r^2 \cdot r$

dr $\int_0^1$

because intersection of cone is $x^2 + y^2 = 1$

9. Compute line integral $\int_C z\, ds$ where

$$C : \mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1.$$

A. 2π

B. 2

C. $\frac{\sqrt{2}}{2}$

D. 4

E. $2\sqrt{2}$

$|r'(t)|$

$= \sqrt{1+1} = \sqrt{2}$

$\sqrt{2} \int_0^1 t\, dt = \frac{\sqrt{2}}{2}$

10. Evaluate the line integral $\int_C 4y\, dx + 5z\, dy + 3x\, dz$ where

$$C : \mathbf{r}(t) = \langle t, t^3, t^2 \rangle, \quad 0 \le t \le 1.$$

A. 3

B. 4

C. 5

D. 6

E. 7

$r'(t) = \langle 1, 3t^2, 2t \rangle$

$\int_0^1 4 \cdot t^3 \cdot 1 + 5 \cdot t^2 \cdot 3t^2$

$+ 3 \cdot t \cdot 2t\, dt$

$\int_0^1 4t^3 + 15t^4 + 6t^2\, dt$

$= \left(t^4 + 3t^5 + 2t^3 \right)\Big|_0^1 = 6$

11. The critical points of $f(x, y) = 3x^2y - 18y^2 - 9x^2$ are $(0, 0)$, $(6, 3)$, and $(-6, 3)$. Classify them as local maxima, local minima, or saddle points.

A. (0,0) is a local minimum. (6,3) and (-6,3) are saddle points.

B. (0,0) is a local maximum. (6,3) and (-6,3) are saddle points.

C. (6,3) and (-6,3) are local maximums. (0,0) is a saddle point.

D. (6,3) and (-6,3) are local minimums. (0,0) is a saddle point.

E. (6,3) and (-6,3) are local maximums. (0,0) is a local minimum.

Out of scope for Midterm 2.

12. Let $\mathbf{F}(x, y) = (2xy + 2x)\mathbf{i} + (x^2 + 4y^3)\mathbf{j}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$C: \ \mathbf{r}(t) = \cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}, \quad 0 \le t \le \frac{\pi}{2}.$$

A. 15

B. -10

C. -3

D. 3

E. 8