

INSTRUCTIONS:

1. Do not open the exam booklet until you are instructed to do so.
2. This exam has 10 problems in 6 different pages (including this cover page). Once you are allowed to open the exam, make sure you have a complete test.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
4. Each problem is worth 10 points for a maximum of 100 points. No partial credit.
5. Use a #2 pencil to fill in the required information in your scantron and fill in the circles.
6. Use a #2 pencil to fill in the answers on your scantron.
7. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY:

1. Students must obey the orders and requests by all proctors, TAs, and lecturers.
2. No student may leave in the first 20 min or in the last 10 min of the exam.
3. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
4. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
5. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

NAME Leadboard

STUDENT SIGNATURE L

STUDENT PUID # log 6

SECTION NUMBER 18m 94

RECITATION INSTRUCTOR Someone

1. Evaluate

$$\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} (x^2 + y^2) dy dx$$

by converting to polar coordinates.

(The moment
needed, but
Symmetric

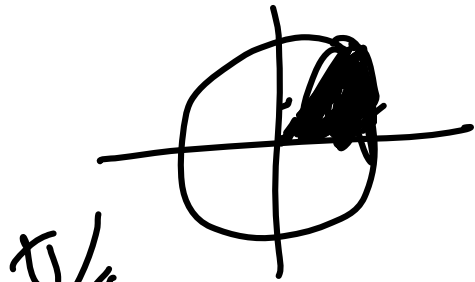
A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

☒ C. π

D. $\frac{2\pi}{3}$

E. 2π

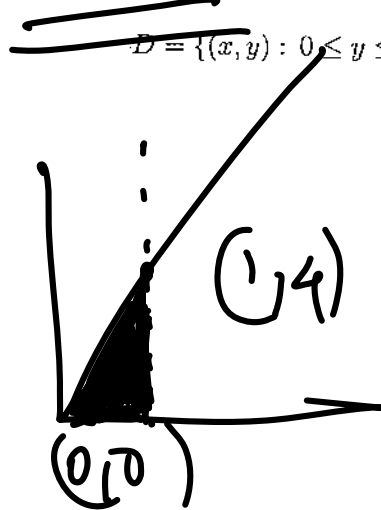


$$\int_0^{\pi/4} \int_0^2 r^3 dr d\theta$$

$$= \left(\frac{r^4}{4} \right)_0^2 = \frac{4 \cdot \pi}{4} = \pi$$

2. Find the area of the surface given by $z = x^2 + y$ for (x, y) in

$$D = \{(x, y) : 0 \leq y \leq 4x, 0 \leq x \leq 1\}$$



A. $12\sqrt{6} - 4\sqrt{2}$

☒ B. $2\sqrt{6} - \frac{2}{3}\sqrt{2}$

C. $2\sqrt{6}$

D. $10\sqrt{5} - 2$

E. $\frac{5\sqrt{5} - 1}{3}$

$$= \sqrt{1 + (2x)^2 + 1}$$

Area left

$$= \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2}$$

$$\int_0^1 \int_0^{4x} \sqrt{1 + 4x^2} dy dx$$

3. Let E be the solid region in the first octant that is above the xy plane and below the plane $2x + y + z = 4$. Then the iterated integral satisfying

$$\iiint_E f(x, y, z) dV = \int_0^a \int_0^b \int_0^c f(x, y, z) dz dy dx$$

must have

A. $a = 4, b = 4 - 2x - y$

B. $a = 4, b = 4 - 2x$

C. $a = 2, b = 4 - 2x - y$

D. $a = 2, b = 4 - z - 2x$

E. $a = 2, b = 4 - 2x$

my first
guess is:

$$c = 4 - 2x - y$$

$$b = 4 - 2x$$

$$a = 2$$

$$\rightarrow x^2$$

4. Use spherical coordinates to compute $\iiint_{\Omega} \sqrt{x^2 + y^2 + z^2} dV$ where Ω is the region above the cone $\sqrt{3}z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 10$.

A. 5π

B. 25π

C. $50\pi(1 - \frac{\sqrt{3}}{2})$

D. $50\pi(1 - \frac{1}{\sqrt{2}})$

E. $\frac{5\pi^2}{6} - \frac{5\pi\sqrt{3}}{8}$

$$\frac{3 \times 5}{9}$$

$$x = \rho \sin \phi \cos \theta$$

$$y = \rho \sin \phi \sin \theta$$

$$z = \rho \cos \phi$$

as x 's
a cone

$$\int_0^{2\pi} \int_0^{\pi/3} \int_0^{\sqrt{10}} \rho^3 \sin \phi d\rho d\phi d\theta$$

$$\rho^3 \sin \phi d\rho d\phi d\theta$$

5. A solid material occupies the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $x + z = 8$. If its mass density per unit volume is $\rho(x, y, z) = y^2$, which of the following integrals equals its mass?

A. $\int_0^{2\pi} \int_0^2 \int_0^{8-2\cos\theta} r^3 \sin^2\theta \, dz dr d\theta$

B. $\int_0^{2\pi} \int_0^4 \int_0^{8-2\cos\theta} r^2 \sin^2\theta \, dz dr d\theta$

C. $\int_0^{2\pi} \int_0^2 \int_0^{8-2\cos\theta} r^2 \sin^2\theta \, dz dr d\theta$

D. $\int_0^{2\pi} \int_0^2 \int_0^{8-r\cos\theta} r^3 \sin^2\theta \, dz dr d\theta$

E. $\int_0^{2\pi} \int_0^2 \int_0^{8-r\cos\theta} r^2 \sin^2\theta \, dz dr d\theta$

(cylinder and plane)

$y = r \sin\theta$

$y^2 = r^2 \sin^2\theta$

$\int_0^{2\pi} \int_0^2 \int_0^{8-r\cos\theta} y^2 \cdot r \, dz dr d\theta$

6. Evaluate the line integral $\int_C 12x \cos z \, ds$, where the curve C is parameterized by $\vec{r}(t) = \langle t, t^2 + 3, 0 \rangle$ for $0 \leq t \leq 1$.

$\int_0^1 f(x(t), y(t)) \sqrt{1 + (2t)^2} \, dt$

A. 1

B. $5^{3/2}$

C. $\frac{3}{2} (5^{3/2} - 1)$

D. $\frac{1}{2} (5^{3/2} - 1)$

E. $5^{3/2} - 1$

$\int_0^1 12t \sqrt{1 + 4t^2} \, dt$

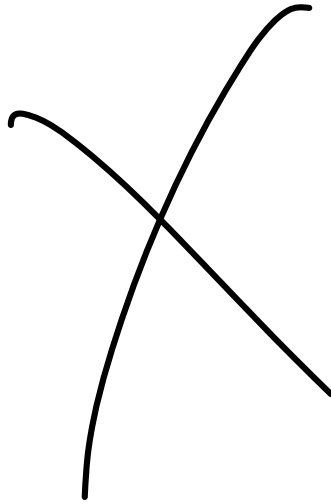
$4t^2 = u$

$= \frac{3}{2} \left(\frac{2}{3} (u^{3/2}) \right)$

$= \frac{3}{2} \left(\frac{2}{3} (1 + 4t^2)^{3/2} - 1 \right)$

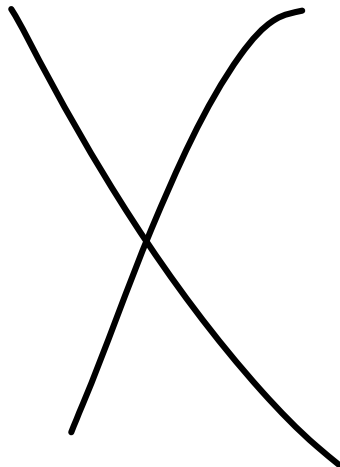
7. Let $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$, $0 \leq t \leq \pi$, be the parametrization of the curve C , and let $\vec{F} = \nabla f$ with $f(x, y, z) = y^3 + 2xz$. The line integral $\int_C \vec{F} \cdot d\vec{r}$ is

- A. -2π
B. 3π
C. 6π
D. π
E. 4



8. Let C be the oriented curve described by the intersection of $z = x^2$ and $y = 0$ from $(-1, 0, 1)$ to $(1, 0, 1)$, and let $\vec{F} = \langle 3x^2, -e^x y, 1 \rangle$. Then the line integral $\int_C \vec{F} \cdot d\vec{r}$ is

- A. -4
B. -2
C. 2
D. 4
E. 0



9. A potential function for the vector field $\vec{F}(x, y) = \langle 2e^y, 2xe^y + y \rangle$ is

A. There is no potential function for this vector field

B. $2xe^y + xy + C$

C. $2xe^y + x^2y + C$

D. $2xe^y + \frac{y^2}{2} + C$

E. $2xe^y + C$

$$\frac{\partial f}{\partial x} = 2e^y$$

$$\int 2e^y dy = 2xe^y + \frac{y^2}{2} + C$$

10. Let C be the boundary of the triangle with vertices $(0, 0)$, $(1, 1)$, and $(0, 1)$, and C is oriented counterclockwise, then the line integral $\int_C \sin y \, dx + (x \cos y + 4x) \, dy$ is

A. 1

B. 2

C. 4

D. -1

E. -2