Question

47. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, if $\vec{F}(x, y) = (xy^2 - 1)\vec{i} + (x^2y - x)\vec{j}$ and C is the circle of radius 1 centered at $(1, 2)$ and oriented counterclockwise.

A. 2 B. π C. 0 D. $-\pi$ E. -2

Solution

We can parameterize the curve ($x = 1 + \cos\theta$, $y = 2 + \sin\theta$), but then the solution becomes extremely complicated. Instead, we use Green's theorem:

$$\int_C F.dr = \iint_R \left(\frac{\delta Q}{\delta x} - \frac{\delta P}{\delta y}\right) dR$$

So, applying that in the problem, we see that $\frac{\delta Q}{\delta x} = (2xy - 1)$ and $\frac{\delta P}{\delta y} = 2xy$. Hence,

$$\iint_R \left(\frac{\delta Q}{\delta x} - \frac{\delta P}{\delta y}\right) dR = \iint_R ((2xy - 1) - 2xy)\, dR = \iint_R -1\, dR$$

But what is R? It's simply a circle with radius 1. Fortunately, there is nothing to compute, as there is no dependency on x or y on the integral. Hence, the answer is $-\pi r^2 \rightarrow -\pi$.