MA 26100
FINAL EXAM Form **01**
May 1, 2017

Spring 17

NAME Leaderboard YOUR TA'S NAME Myself

STUDENT ID # $\log e^{\sin^{-1}(1/t)}$ RECITATION TIME 12:00 am

1. You must use a #2 pencil on the scantron

2. a. Write **01** in the TEST/QUIZ NUMBER boxes and darken the appropriate bubbles on your scantron.

   b. **The color of your scantron MUST match the color of the cover page of your exam**

3. On the scantron sheet, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the scantron sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Also circle your answers on the exam itself. Do all your work on the question sheets.

9. Turn in both the scantron sheets and the question sheets when you are finished.

10. If you finish the exam before 8:50 pm, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 8:50 pm, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

2) $$\int_0^{\pi/2}\int_0^3 r\sqrt{9-r^2}\,dr\,d\theta$$

$$= -\frac{1}{2}\left(\frac{2}{3}\left(9-r^2\right)^{3/2}\right)\Big|_0^3 \times \frac{\pi}{2}$$

$$\frac{1}{3}\times\frac{\pi}{2}\left(-27\right] =$$

$$\frac{-\pi}{6}\left(-27\right] = \frac{27\pi}{6} = \frac{9\pi}{2}$$

1. Find the equation of the plane that contains the point $(2, 1, 1)$ and that is perpendicular to the planes $2x + y - 2z = 2$ and $x + 3z = 4$.

A. $x - 3y - z = -2$

B. $x - 3y - z = 2$

C. $3x - 2y + z = 5$

D. $2x - y + z = 4$

E. $3x - 8y - z = -3$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

$= \hat{i}(3) - \hat{j}(6+2) + \hat{k}(-1)$

$3\hat{i} - 8\hat{j} - \hat{k}$

$3(x-2) - 8(y-1) - 1(z-1) = 0$

$3x - 6 - 8y + 8 - z + 1 = 0$

2. Compute

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{9 - x^2 - y^2}\, dy\, dx$$

A. $\frac{9\pi}{2}$

B. $\frac{7\pi}{2}$

C. $\frac{8\pi}{3}$

D. $\frac{4\pi}{3}$

E. 9π

$x = r\cos\theta$

$y = r\sin\theta$

$\int_0^{\pi/2} \int_0^3 \sqrt{9 - r^2}\, dr\, d\theta$

$\int_0^{\pi/2} \left(\frac{r^3}{3}\right)_0^3$

$\frac{27}{3}\left(\frac{\pi}{2}\right)$

$\frac{9\pi}{2}$

(4)

$$\int_4^5 \int_0^{2\pi} \int_0^{\sqrt{25-r^2}} r \, dz \, d\theta \, dr$$

sphere

$$= \int_0^{2\pi} \int_4^5 r\sqrt{25-r^2} \, dr \, d\theta$$

$$= 2\pi \left[-\frac{1}{3}(25-r^2)^{3/2} \right]_4^5$$

$$2\pi \left[-\frac{1}{3} \left(-9^{3/2} \right) \right]$$

$$2\pi \left(\frac{-1}{3} \right) \left(-27 \right) = 18\pi$$

3. Express the area of the surface given by the graph of

$z = y^3 \cos^2(x) - 1$ over the triangular region, D, with vertices $(1,0), (1,3), (4,3)$.

Do not evaluate.

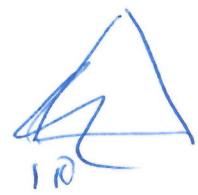
A. $\displaystyle\int_1^4 \int_{y+1}^1 \sqrt{4y^6 \cos^2(x)\sin^2(x) + 9y^4\cos^4(x) + 1}\, dxdy$

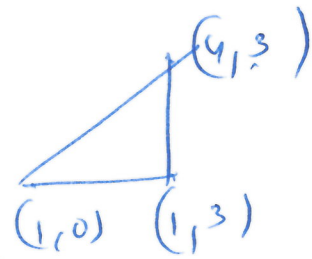
B. $\displaystyle\int_0^4 \int_1^{y+1} \sqrt{-2y^3\cos(x)\sin(x) - 3y^2\cos^4(x)}\, dxdy$

C. $\displaystyle\int_1^3 \int_{1-y}^0 \sqrt{4y^6\cos^2(x)\sin^2(x) + 9y^4\cos^4(x)}\, dxdy$

D. $\displaystyle\int_1^3 \int_0^{y-1} \sqrt{4y^6\cos^2(x)\sin^2(x) + 9y^4\cos^4(x)}\, dxdy$

E. $\displaystyle\int_0^3 \int_1^{y+1} \sqrt{4y^6\cos^2(x)\sin^2(x) + 9y^4\cos^4(x) + 1}\, dxdy$





$\gamma'(t) \langle y^3 \cdot 2(\cos x)(-\sin x),\ 3y^2\cos^2(x),\ 1 \rangle$

$\iint = \sqrt{4\cos^2 x \sin^2 x\, y^6 + 9y^4 \cos^4 x + 1}$

4. Compute the volume of the solid inside $x^2 + y^2 + z^2 = 25$ and outside $x^2 + y^2 = 16$, and such that $y \geq 0$.

A. 9π

B. $6\sqrt{3}\pi$

C. 18π

D. $3\sqrt{3}\pi$

E. 7π

$16 + z^2 = 25$

$z = \pm 3$

z^2

cylinder

$\frac{4}{3}\pi 3 - 2\pi$

$\frac{4}{3}\pi \cdot 125 - 2\pi$

16×6

not a vector

5. Let $\mathbf{F}$ be a vector field and f be a scalar field. Which of the following expressions are meaningful?

i. $\text{curl} f$ → vector

ii. $\text{div}(\nabla f)$ → not a vector

iii. $(\nabla f) \times \text{div}\mathbf{F}$ → illogical as that's a scalar

iv. $\text{curl}\,(\text{curl}\,\mathbf{F})$ → vector ok

A. i and ii only

B. i only

C. i, iii, and iv only

D. iii only

E. ii and iv only

(d)

$$\cancel{\int_0^1 -2t\, e^{(1-t)2t} + 2(1-t)\, e^{(1-t)2t} + 2\cos 2t \, dt}$$

$$\cancel{= \int_0^1 (2-4t)\, e^{(1-t)2t} + 2\cos 2t \, dt}$$

$$\cancel{2t - 2t^2 = u}$$

$$\cancel{2 - 4t}$$

(e)

$$\int_0^1 -2t\, e^{(1-t)2t} + 2(1-t)\, e^{(1-t)2t} + 2\cos 2t >$$

$$= \int_0^1 (2-4t)\, e^{2t-2t^2} + 2\cos 2t \, dt$$

$$= \left| e^{2t-2t^2} + \sin 2t \right|_0^1 = e^0 + \sin 2 - e^0$$

$$= \underline{\underline{\sin 2}}$$

6. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the line segment from $P(1,0)$ to $Q(0,2)$ and $\mathbf{F}(x,y) = \langle ye^{xy}, xe^{xy} + \cos y \rangle$ is a **conservative** vector field.

A. $1 + \sin 2$

B. $\sin 2$

C. $1 - \sin 2$

D. $-1 + \sin 2$

E. $-1 - \sin 2$

$F = \nabla f$

$\gamma(t) = a + t(b - a)$

$\langle 1, 0 \rangle + t\langle -1, 2 \rangle$

$\langle 1 - t, 2t \rangle$

$F(\gamma(t)) \cdot \gamma'(t)$

$\gamma'(t) = \langle -1, 2 \rangle$

$\int_0^1 \langle (1-t)e^{(1-t)2t}, (1-t)e^{(1-t)2t} + \cos 2t \rangle \langle -1, 2 \rangle$

7. Use Green's Theorem to evaluate $\oint_C xy^2\,dx + 2x^2y\,dy$ where C is the triangle with vertices $(0,0)$, $(2,2)$, and $(2,4)$ with counterclockwise orinetation.

A. 8

B. 12

C. 16

D. 20

E. 6

$\iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$

$\iint (4xy - 2xy)\, dA$

$= \iint 2xy\, dA$

$\int_2^{2x} \int_x^{2x} 2xy\, dy\, dx$

$= \int 3x^3\, dx$

$= \frac{3}{4}x^4$

$\frac{3 \times 16}{4} = 48/4 = 12$

$(xy^2)\Big|_x^{2x}$

$= 4x^3 - x^3$

$y = mx + c$

$4 = 2m$

For problems 8 and 9 let $f(x, y) = y^3 - 3y + 3x^2y$.

8. The function $f(x, y)$ has local extrema consisting of:

A. 2 saddle points, 1 local max, 1 local min

B. 2 local max, 2 local min

C. 1 saddle point, 2 local max, 1 local min

D. 1 saddle point, 1 local max, 2 local min

E. 1 local max, 3 local min

$x=0$ or $y=0$

$f_x = 6xy$ $\quad f_x = 0$ $\quad x=0 \; y=0$

$f_y = 3y^2 - 3 + 3x^2 \rightarrow 3x^2 - 3 = 0$

$(x = \pm 1)$

$f_{xx} = 6y \qquad f_{xy} = 6x$

$f_{yy} = 6y$

critical points $\begin{cases} (0,1) \\ (0,-1) \\ (1,0) \\ (-1,0) \end{cases}$

$$H = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 6y & 6x \\ 6x & 6y \end{vmatrix}$$

$H = 36$ but $f_{xx} = 6y$ or $\therefore$ one local max 1 min $36y^2 - 36x^2$

$H < 0$ when $y = 0$.

$\therefore$ 2 saddle points.

9. The absolute minimum value "m" and absolute maximum value "M" for $f(x, y)$ on

$$D = \{(x, y) : -2 \leq x \leq 2, 0 \leq y \leq 2\}$$

satisfy

A. $m = -2$, $M = 18$

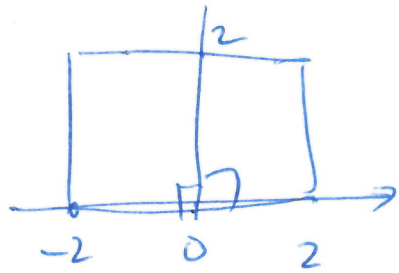
B. $m = -2$, $M = 2$

C. $m = -18$, $M = 18$

D. $m = -2$, $M = 26$

E. $m = -26$, $M = 26$

impossible



when $(-2, 0)$, then

$f = 0 - 0 + 0$

$(-2, 2) \rightarrow f = ($

$y^3 - 3y + 12y$

$\max(y^3 + 9y)$

$8 + 18 = 26.$

$\therefore M_{max} = 26$

when $(2, 2)$ or $(-2, 2)$

10. The total surface area for a silo (base with radius r, cylindrical side with height h, hemispherical cap) is $A = 3\pi r^2 + 2\pi r h$. Suppose h and r are measured to be $h = 20$ feet and $r = 8$ feet to within an error of $\frac{1}{2}$ foot for h and $\frac{1}{4}$ foot for r. Use differentials to estimate the maximum possible error for A (in square feet).

A. 52π

B. 30π

C. 8π

D. 22π

E. 4π

$A = 3\pi r^2 + 2\pi r h$

$dA = (6\pi r + 2\pi h)\, dr + (2\pi r)\, dh$

$dr = \frac{1}{4}$ $\quad dh = \frac{1}{2}$ $\quad h = 20$, $r = 8$.

$dA = 24\pi + 8\pi = 30\pi$

$dA = (6\pi \times 8 + 2\pi \times 20)\frac{1}{4} + (2\pi \times 8)\frac{1}{2}$

$dA = (48\pi + 40\pi)\frac{1}{4} + 8\pi$

11. The equation $xyz - 2x\ln y - z^2 = 2$ implicitly defines $z = z(x, y)$ near $(x, y, z) = (3, 1, 1)$. Find the tangent plane to $z(x, y)$ at $(3, 1)$.

A. $z + x - 3y = -2$

B. $z - x - 3y = 0$

C. $z - x + 3y = 1$

D. $2z - x + 4y = 2$

E. $z + x - 3y = 1$

$\nabla f = \langle yz - 2\ln y, xz - \frac{2x}{y}, xy - 2z \rangle$

$\nabla f(3,1,1) = \langle 1 - 0, 3 - 6, 1 \rangle$

$\langle 1, -3, 1 \rangle$

$1(x-3) - 3(y-1) + (z-1) = 0$

$x - 3 - 3y + 3 + z - 1 = 0$

$x - 3y + z = 1$

(12)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v & 2u & 1 \\ u & -2v & 2 \end{vmatrix}$$

$$= \hat{i}\left(4u + 2v\right) - \hat{j}\left(2v - u\right) + \hat{k}\left(-2v^2 - 2u^2\right)$$

(1, 2) 1

$$8 \qquad -\hat{j}\left(3\right) \quad -10$$

$$\langle 8, -3, -10 \rangle$$

but $\hat{i} \times \hat{j} = -\hat{k}$ opposite direction

so -ve sign $\Rightarrow$

$$\langle -8, 3, 10 \rangle$$

12. Let S be a parameterized surface given by

$$\begin{aligned} \mathbf{r}(u,v) &= \langle x(u,v), y(u,v), z(u,v)\rangle, \\ x &= uv, \\ y &= u^2 - v^2, \\ z &= u + 2v. \end{aligned}$$

Which of the following is an upward pointing normal to S at $\mathbf{r}(1,2)$?

A. $\langle 8, -3, -10\rangle$

B. $\langle -8, 3, 10\rangle$

C. $\langle 4, 1, -8\rangle$

D. $\langle -4 - 1, 8\rangle$

E. $\langle 4, 1, 0\rangle$

$\langle uv, u^2 - v^2, u + 2v\rangle$

$r_u = \langle v, 2u, 2v\rangle$

$r_v = \langle u, -2v, 2\rangle$

$\begin{vmatrix} i & j & k \\ v & 2u & 2v \\ u & -2v & 2 \end{vmatrix}$

$= i(4u + 4v^2) - j(2v - 2uv) + k(-2uv - 2u^2)$

$i(20) - j(0)$

13. A surface S is the graph of $z = h(x,y) = xy$ above a region D. If $\int\int_S xyz\, dS = \int\int_D g(x,y)\, dxdy$ then $g(x,y)$ is

A. $\sqrt{y^6 + x^6 + x^4y^4}$

B. $\sqrt{x^2y^4 + x^4y^2 + x^2y^2 + 1}$

C. $\sqrt{x^4y^6 + x^6y^4 + x^4y^4}$

D. $\sqrt{x^2 + x^4 + x^2y^2}$

E. $\sqrt{x^4y^6 + x^6y^4}$

$\iint_S xyz\, dS$

$= \iint_D (x^2y^2)\sqrt{1 + y^2 + x^2}\, dA$

$= x^4y^4 + x^4y^6 + x^6y^4$

15) $$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 r^3 \sin\varphi \cos\varphi \, dr \, d\varphi \, d\theta$$

$$= \frac{\pi}{2}\int_0^{\pi/2} \left|\frac{r^4}{4}\right|_1^2 \sin\varphi \cos\varphi \, d\varphi$$

$$= \frac{15\pi}{8}\int_0^{\pi/2} \sin\varphi \cos\varphi \, d\varphi$$

$$= \cancel{\frac{15}{16}\int_0^{\pi/2} \sin 2\varphi \, d\varphi} = \frac{15}{16}\int_0^{\pi/2} \sin 2\varphi \, d\varphi$$

$$= \frac{15}{16}\left(\frac{1}{2}\right)(2) = \frac{15}{16}\pi$$

14. If $z = z(x, y, t), x = x(u, v, w), y = y(u, v, w), t = t(u, v, w)$ and $z = xe^{ty}, x = u^2 + v, y = vw + u, t = w^2u$ find z_v at $u = -2$, $v = 1$, $w = 2$.

A. -79

B. 18

C. 42

D. 19

E. -41

$$z = (u^2+v)\, e^{w^2u(vw+u)}$$

$$z_v = 1 \cdot e^{w^2u(vw+u)} + (u^2+v)\cdot \frac{\partial}{\partial v}\left(e^{w^2u(vw+u)}\right)$$

$$\frac{\partial}{\partial v}\left(w^2u(vw+u)\right) = w^3uv + w^2u^2$$

$$1 \cdot e^{-8(0)} + (5)(-16)(1) = -79$$

15. Compute $\int\int\int_E z\, dV$ where E is the solid between $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$, and in the first octant.

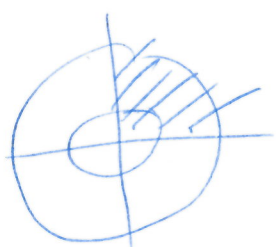
A. 2π

B. $\frac{7\pi}{8}$

C. $\frac{3\pi}{4}$

D. $\frac{15\pi}{16}$

E. $\frac{5\pi}{16}$



$$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 (r\cos\varphi)\, r^2 \sin\varphi \, dr\, d\varphi\, d\theta$$

$$= \frac{\pi}{2}\left[\frac{r^3}{3}\right]_1^2 \int_0^{\pi/2} \sin\varphi\, d\varphi = \frac{15\pi}{16}$$

$(2, 2, 8)$

16. Let C be the curve parameterized by $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + (2t - t^2)\mathbf{k}$. At what point does the tangent line to C at $-2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ intersect the yz-plane?

A. $\langle 0, \frac{1}{2}, \frac{3}{2} \rangle$

B. $\langle 0, 2, -2 \rangle$

C. $\langle 0, -1, 1 \rangle$

D. $\langle 0, 0, 4 \rangle$

E. $\langle 0, -1, -1 \rangle$

$\boxed{x = 0}$ $(2, 2, 8)$

$r'(t) = \langle 2, 2t, 2 - 2t \rangle$

$\cancel{r'(t) \cdot \langle -2, 1, -3 \rangle}$

$\cancel{\langle (2t+2), 2t, 16(1-t) \rangle}$

$\cancel{t = -1}$

$r(t) + t(r'(t))$

$= \langle -4, 1, -15 \rangle + t\,(2, 2, 8)$ $\langle -4 + 2t, 1 + 2t, -15 + 8t \rangle$ $t = 2$

17. Compute the arclength of the curve C defined by

$$\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + (\ln t)\mathbf{k} \text{ for } 1 \le t \le 2.$$

A. $3 + \sqrt{2}$

B. $3 + \sqrt{2}$

C. $2 + 2\ln 2$

D. $3 + \ln 2$

E. $2 + \frac{1}{2}\ln 2$

$r'(t) = \langle 2, 2t, \frac{1}{t} \rangle$

$0, 5, -5$

$|r'(t)| =$ or $(0, 1, -1)$

or $(0, -1, 1)$

$\sqrt{4 + 4t^2 + \frac{1}{t^2}}$

$= \int_1^2 \frac{2t^2 + 1}{t}\, dt$ $\sqrt{4t^2 + 4t^4 + 1}$ $= \frac{\sqrt{(2t^2+1)^2}}{2t}$

$= \int_1^2 (2t + 1/t)\, dt = \left| t^2 + \ln t \right|_1^2 = 3 + \ln 2$

$\partial(u,v) =$ $\quad 2 = 4-2y-4x$

18. Compute $\int\int_S x\,dS$ where S is the triangular region with vertices $(1,0,0), (0,2,0)$ and $(0,0,4)$.

$2y = 4-4x$

$y = \cancel{2-2x}$

A. $\frac{2\sqrt{13}}{3}$ X

B. $\frac{2\sqrt{17}}{3}$ X

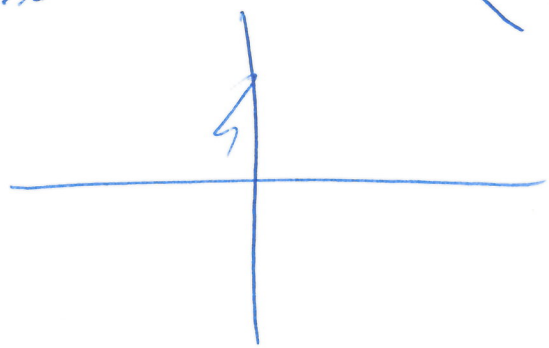
C. $\frac{2\sqrt{21}}{3}$

D. $\frac{\sqrt{21}}{3}$

E. $\frac{\sqrt{17}}{3}$ X

$\langle -4, -2, 1 \rangle$

$= \sqrt{21}$



$4x + 2y + z = 4$

$2(2x-2)$

$\cancel{2-x}$

$\sqrt{21} \int_0^1 \int_0^{\cancel{2x}} x \, dy \, dx$

then

$\cancel{4x}$

$2y = 4-4x$

$y = \frac{4-4x}{2} = 2-y$

$= \int_0^1 x(2x-2)\, dx =$

19. Let S be the semi-ellipsoid $z = 2\sqrt{1-x^2-y^2}$, oriented so that the normal $\mathbf{n}$ is upward pointing. Let

$$\mathbf{F}(x,y,z) = \underset{f}{x^2}\mathbf{i} + \underset{g}{y^2}\mathbf{j} + \underset{h}{z^2 \tan xy}\,\mathbf{k}.$$

$\int_0^1 (2x^2 - 2x)\, dx$

Use Stokes' Theorem to to evaluate $\int\int_S \text{curl}\mathbf{F} \cdot \mathbf{n}\, dS$.

A. 2π

B. 4π

C. 0

D. $\frac{4\pi}{3}$

E. $\frac{2\pi}{3}$

$\text{Curl } F = \begin{vmatrix} i & j & k \\ x & y & z \\ F & G & H \end{vmatrix}$

$= \hat{i}\left(\frac{\partial y}{\partial H} - \frac{\partial z}{\partial G}\right)$

$- \hat{j}\left(\frac{\partial x}{\partial H} - \frac{\partial z}{\partial F}\right)$

$+ \hat{k}\left(\frac{\partial x}{\partial G} - \frac{\partial y}{\partial F}\right)$

$= \hat{i}\left(xz^2 \sec^2 xy - 0\right)$

$- \hat{j}\left(yz^2 \sec^2 xy - 0\right)$

$+ \hat{k}\left(0 - 0\right)$

$= 0$

$= \cancel{+2} \; 2$

$= \left.\frac{2x^3}{3} - x^2\right|_0^1$

$= \frac{1}{3}$

$= \frac{\sqrt{21}}{3}$

Or use

$F \cdot dr$ $\quad \langle \cos t, \sin t, 0 \rangle$

$\langle -\sin t, \cos t, 0 \rangle$

20. Let $\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + xy^2z\mathbf{j} + xyz^2\mathbf{k}$. Using the outward-pointing normal on the surface S of the solid box defined by $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 2$ evaluate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$.

A. 12

B. 24

C. 9

D. 18

E. 6

$$\frac{\delta F}{\delta x} = 2xyz$$

$$\frac{\delta F}{\delta y} = 2xyz$$

$$\frac{\delta F}{\delta z} = 2xyz$$

$$\text{div}\, F = 6xyz$$

$$\int_0^1\int_0^2\int_0^2 6xyz \, dz\, dy\, dx$$

$$= 6\int_0^1\int_0^2 \left[\frac{z^2}{2}\right]_0^2 xy \, dy\, dx$$

$$= 12\int_0^1\int_0^2 xy \, dy\, dx = 12\int_0^1 \left[\frac{xy^2}{2}\right]_0^2 dx$$

$$24\left(\frac{x^2}{2}\right)$$

$$12\int_0^1 (2x)\, dx$$

$$= 24\int_0^1 x\, dx$$

$$= 12$$