Final Exam MATH 26600 SPRING 2019

Name Leedsbeard

Student ID NA Section Number (see list below) NA

031	UNIV 119	1:30pm MWF	Han, Jiyuan	041	REC 123	12:00pm TR	Ma, Zheng
051	UNIV 117	3:00pm TR	Li, Shengao	052	REC 309	10:30am TR	Liu, Yanghui
053	REC 123	1:30pm MWF	Phillips, Daniel	061	UNIV 117	4:30pm TR	Li, Shenghao
062	REC 123	2:30pm MWF	Chen, Min	063	UNIV 103	12:30pm MWF	Poghotanyan, Gayane
081	REC 113	9:30am MWF	Phillips, Daniel	091	REC 123	1:30pm TR	Ma, Zheng
111	REC 309	9:00am TR	Liu, Yanghui	113	UNIV 303	10:30am TR	Lin, Guang
121	UNIV 303	9:00am TR	Price, Edward	122	REC 309	10:30am MWF	Mariano, Phanuel
141	REC 309	9:30am MWF	Mariano, Phanuel	151	UNIV 119	2:30pm MWF	Han, Jiyuan
152	UNIV 103	11:30am MWF	Poghotanyan, Gayane	153	REC 309	3:00pm TR	Xu, Jie
154	REC 309	4:30pm TR	Xu, Jie				
155	REC 309	11:30am MWF	Yeung, Sai Kee				

Contains solutions for Q 1-4 only.

INSTRUCTIONS:

- Please fill in your name, ID, section number (see above).
- MARK TEST number 01 on your SCANTRON if your cover sheet is WHITE, 02 if it is ORANGE, and 03 if it is GREEN.
- This exam contains 20 problems, worth 5 points each. There is one correct answer for each problem.
- There is a table of Laplace transforms provided at the end of the exam.
- Work only in the space provided, or on the backside of the pages. You must show your work.
- Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
- No books, notes, calculators, phones or other electronic devices, please.

Why? no partial credit

ACADEMIC DISHONESTY

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

Anyone who is seen handling any communication device gets automatically a score of 0 for the exam and likely an F in the course

I have read and understood the instructions regarding academic dishonesty:

Student name: Signature:

1. Determine the interval where the solution guaranteed to exist for the following initial value problem

$$(t+2)y' + y = \frac{1}{t-1}, \quad y(0) = \frac{1}{2}.$$

(A) $(1, +\infty)$ X
(B) $(-2, 1)$
(C) $(-2, +\infty)$
(D) $(-\infty, -2)$ X
(E) $(-\infty, 1)$

$$y' + \frac{y}{(t+2)} = \frac{1}{t-1}$$

$\therefore$ solution guaranteed at $(-2, 1)$

discontinuity at -2

discontinuity at 1

2. The general solution to $x^2y' + 2xy = e^{3x}$ is $y = \frac{1}{3}\frac{e^{3x}}{x^2} + \frac{c}{x^2}$

(A) $y = \frac{3}{x^2}e^{3x} + c$
(B) $y = ce^{3x}$
(C) $y = \frac{1}{3x^2}e^{3x} + cx^{-2}$
(D) $y = \frac{1}{2x^2}e^{3x}$
(E) $y = \frac{1}{3x}e^{3x} + cx^{-2}$

$$x^2\frac{dy}{dx} + 2xy = e^{3x}$$

$$\frac{dy}{dx} + \frac{2xy}{x^2} = \frac{e^{3x}}{x^2}$$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{e^{3x}}{x^2}$$

$$I.F = e^{\int 2/x \, dx}$$

$$= e^{2\ln x}$$

$$= x^2$$

$$\therefore y \cdot IF = \int x \cdot IF \, dx = yx^2 = \int \frac{e^{3x}}{x^2} \cdot x^2 \, dx$$

$$yx^2 = \int e^{3x} \, dx$$

$$yx^2 = \frac{1}{3} e^{3x} + C$$

3. If $y = y(x)$ is the solution to

$$\frac{dy}{dx} = \frac{4xy}{2+x^2}, \quad y(0) = 4,$$

$y(0) = 4$?

then $y(\sqrt{3}) =$

(A) 16
(B) 25
(C) 9
(D) 4
(E) $2\sqrt{3}$

$$\frac{dy}{dx} = \left[\frac{4x}{2+x^2}\right] y$$

$$\frac{dy}{y} = \left(\frac{4x}{2+x^2}\right) dx$$

$$\ln y = \int \frac{4x}{2+x^2}\, dx \rightarrow \text{let } 2+x^2 = t, \quad 2x\, dx = dt, \quad dx = \frac{dt}{2x}$$

$$\ln y = 2\int \frac{1}{t}\, dt$$

$$\ln y = 2 \ln t + c$$

$$\ln y = 2 \ln\left[2+x^2\right] + c$$

$$y = (2+x^2)^2 + c$$

$$y(0) = 4 \Rightarrow c = 0$$

$$\therefore y = (2+x^2)^2$$

$$y(\sqrt{3}) = 25$$

then

4. The general solution of the following differential equation

$$\underbrace{(3x^2+y-4)}_{P} - \underbrace{(2y-x)}_{Q}\frac{dy}{dx} = 0$$

$\varphi(x-2y)$

is

(A) $x^3+xy-4x-c=0$
(B) $x^3+2xy-c=0$
(C) $x^3+2xy+y-c=0$
(D) $x^3+2xy-4x-2y^2-c=0$
(E) $x^3+xy-4x-y^2-c=0$

$$\frac{\partial u}{\partial y} = x + g'(y)$$

$$x - 2y = x + g'(y)$$

$$g'(y) = -2y$$

$$\varphi \quad g(y) = -y^2$$

$$(3x^2+y-4)\,dx + (x-2y)\,dy = 0$$

$$\frac{\partial P}{\partial y} = 1 \qquad \frac{\partial Q}{\partial x} = 1$$

$$\frac{\partial u}{\partial x} = P(x,y) \qquad u(x,y) = \int P(x,y)\,dx + \varphi(y)$$

$$\frac{\partial u}{\partial y} = \varphi(x,y) \qquad = (x^3 + xy - 4x) + g(y)$$

5. Determine the values of α, if any, for which **all solutions** of the differential equation

$$y'' - (2\alpha + 4)y' + (\alpha^2 + 4\alpha + 3)y = 0$$

tend to zero as $t \to \infty$.

(A) $\alpha > -3$
(B) $\alpha < -1$
(C) $\alpha > -1$
(D) $\alpha < -3$
(E) There is no α for which all solutions tend to zero as $t \to \infty$.

6. Given that $y_1 = t$ is a solution of the following equation:

$$t^3 y'' - ty' + y = 0.$$

Which of the following is also a solution?

(A) $t^2 e^{-1/t}$
(B) $t \ln(t)$
(C) $t \ln^2(t)$
(D) $t^2 e^t$
(E) $te^{-1/t}$

7. The homogeneous differential equation

$$x^2y'' - xy' + y = 0$$

has a set of fundamental solutions given by

$$y_1(x) = x, \quad y_2(x) = x \ln x.$$

Applying the method of Variation of Parameters, find a particular solution to the nonhomogeneous equation

$$x^2y'' - xy' + y = x, \ x > 0.$$

(A) $y(t) = x \ln x$
(B) $y(t) = \frac{x(\ln x)^3}{3}$
(C) $y(t) = \frac{x(\ln x)^2}{2}$
(D) $y(t) = \frac{x^2 \ln x}{4}$
(E) $y(t) = \frac{x^3}{4}$

8. Let $y(t)$ denote the unique solution to the initial value problem

$$y''' + 3y'' + 2y' = 0 \quad y(0) = 2, \quad y'(0) = -1, \quad y''(0) = 1.$$

What is the value of $y(1)$?

(A) $1 + 2e^{-1} + e^{-2}$
(B) $1 - e^{-2}$
(C) $1 + e^{-1}$
(D) $e + e^2$
(E) $2 + e^{-1} + 2e^{-2}$

9. Find a particular solution $y_p = y_p(t)$ of the second order differential equation

$$y'' - 2y' + 5y = 20 \sin t.$$

(A) $y_p(t) = 2 \sin t - \cos t$
(B) $y_p(t) = 2 \cos t + 4 \sin t$
(C) $y_p(t) = 2 \sin t + 3 \cos t$
(D) $y_p(t) = 40 \sin t$
(E) $y_p(t) = 40 \cos t$

10. If $Y(s) = \mathcal{L}\{y(t)\}$ is the Laplace transform of the solution to the initial value problem below, what is $Y(0)$?

$$y'' + 2y = u_2(t)\,(t-2)\,e^{-3(t-2)}, \quad y(0) = 0, \quad y'(0) = 0$$

(A) $-5/18$
(B) $1/18$
(C) $1/9$
(D) $-11e^{12}/18$
(E) $-5e^{6}/18$

11. Solve $y'' + 4y = 2\delta(t-\pi)$ with the initial conditions $y(0) = 0$ and $y'(0) = 0$.

(A) $y(t) = \sin 2t$
(B) $y(t) = \delta(t-\pi)\sin 2t$
(C) $y(t) = -\delta(t-\pi)\sin 2t$
(D) $y(t) = -u_\pi(t)\sin 2t$
(E) $y(t) = u_\pi(t)\sin 2t$

12. Find the Laplace transform of the function

$$f(t) = \int_0^t e^{t-\tau}\cos(t-\tau)\tau^3 d\tau$$

(A) $\dfrac{6(s+1)}{s^4(s^2+2s+2)}$
(B) $\dfrac{6}{s^4(1+s^2)}$
(C) $\dfrac{6(s-1)}{s^4(s^2-2s+2)}$
(D) $\dfrac{6}{s^4(s^2-2s+2)}$
(E) $\dfrac{6s}{(s+1)^4(s^2+1)}$

13. Let

$$y(t) = u_1(t)(t-2)^2 - u_3(t)((t-3)^3 - 2) - u_5(t)e^t$$

What is the value of $y(4)$?

(A) $-2 - e^4$
(B) 5
(C) 32
(D) 4
(E) $-e^5$

14. Solve the initial value problem

$$y^{(4)} + y'' = \delta(t-1), \quad y(0) = y'(0) = y''(0) = 0, \quad y'''(0) = 1.$$

(A) $u_1(t)(t - 1 + \cos(t-1)) + t - \sin(t)$
(B) $u_1(t)(t - 1 + \sin(t-1)) + t + \sin(t)$
(C) $u_1(t)(\sin(t-1) + \cos(t-1)) + \sin(t) + \cos(t)$
(D) $u_1(t)(t - 1 - \sin(t-1)) + t - \sin(t)$
(E) $u_1(t)(t - 1 - \cos(t-1)) + t - \cos(t)$

15. Find the inverse Laplace transform $\mathcal{L}^{-1}\{F(s)\}$ of

$$F(s) = \frac{10e^{-s}}{s^2 - 5s + 6} + \frac{2}{s^2 - 2s + 5}.$$

(A) $10u_1(t)(e^{3(t-1)} - e^{2(t-1)}) + e^t \sin(2t)$
(B) $10u_1(t)(e^{3t} - e^{2t}) + e^t \sin(2t) + 2e^t \sin(2t)$
(C) $10u_1(t)(e^{3(t-1)} - e^{2(t-1)}) + 2e^t \sin(2t)$
(D) $10u_1(t)(e^{3t} - e^{2t}) + e^t \sin(2t) + e^t \sin(2t)$
(E) $10(e^{3t} - e^{2t}) + e^t \sin(2t)$

16. Given

$$\mathbf{x}' = \begin{pmatrix} 1 & \alpha \\ 3 & 1 \end{pmatrix} \mathbf{x},$$

what are the values of α if the origin is a saddle point in the phase plane?

(A) $\alpha > \frac{1}{3}$
(B) $\alpha < 0$
(C) $2 > \alpha > -2$
(D) $3 > \alpha > 1$
(E) $\alpha > \frac{1}{2}$

17. Find the general solution of the following system

$$\mathbf{x}' = \begin{pmatrix} 2 & 7 \\ 1 & -4 \end{pmatrix} \mathbf{x}.$$

(A) $\mathbf{x}(t) = c_1 e^{2t} \begin{pmatrix} 7 \\ 1 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(B) $\mathbf{x}(t) = c_1 e^{2t} \begin{pmatrix} 7 \\ 1 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

(C) $\mathbf{x}(t) = c_1 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(D) $\mathbf{x}(t) = c_1 e^{3t} \begin{pmatrix} 7 \\ 1 \end{pmatrix} + c_2 e^{-5t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

(E) $\mathbf{x}(t) = c_1 e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-5t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

18. Which of the following is a solution to

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} e^{-2t}?$$

(A) $e^{-2t} \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

(B) $e^{-2t} \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

(C) $e^{-2t} \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

(D) $e^{-2t} \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

(E) $e^{-2t} \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

19. Given that

$$\mathbf{x} = e^{-t}\begin{pmatrix} \beta \\ 2 \end{pmatrix}$$

is a solution to

$$\mathbf{x}' = \begin{pmatrix} -3 & 5 \\ -2 & 4 \end{pmatrix}\mathbf{x}.$$

Find β.

(A) 8
(B) 5
(C) 18
(D) 2
(E) 3

20. The general solution to

$$\mathbf{x}' = \begin{pmatrix} -1 & -2 \\ 5 & -3 \end{pmatrix}\mathbf{x}$$

can be written as

(A) $c_1e^{-2t}\left[\cos 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \sin 3t\begin{pmatrix} 0 \\ -3 \end{pmatrix}\right] + c_2e^{-2t}\left[\cos 3t\begin{pmatrix} 0 \\ -3 \end{pmatrix} + \sin 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right]$
(B) $c_1e^{-2t}\left[\cos 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \sin 3t\begin{pmatrix} 0 \\ -3 \end{pmatrix}\right] + c_2e^{-2t}\left[\cos 3t\begin{pmatrix} 0 \\ -3 \end{pmatrix} - \sin 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right]$
(C) $c_1e^{-2t}\left[\cos 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \sin 3t\begin{pmatrix} 0 \\ 3 \end{pmatrix}\right] + c_2e^{-2t}\left[\cos 3t\begin{pmatrix} 0 \\ 3 \end{pmatrix} - \sin 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right]$
(D) $c_1e^{-2t}\left[\cos 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \sin 3t\begin{pmatrix} 0 \\ 3 \end{pmatrix}\right] + c_2e^{-2t}\left[\cos 3t\begin{pmatrix} 0 \\ 3 \end{pmatrix} + \sin 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right]$
(E) $c_1e^{-t}\left[\cos 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \sin 3t\begin{pmatrix} 0 \\ -3 \end{pmatrix}\right] + c_2e^{-3t}\left[\cos 3t\begin{pmatrix} 0 \\ 3 \end{pmatrix} - \sin 3t\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right]$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n\,!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n\,!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \ c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$