Lesson 50 MA261

1) ~~div~~

~~div F = ...~~

Leaderboard

2) $F = \langle x, x+y \rangle$

$$\nabla \cdot F = \frac{\partial}{\partial x}\langle x \rangle + \frac{\partial}{\partial y}\langle x+y \rangle$$

$$= \underline{\underline{2}}$$ divergence is constant

flux is always outward.

3)

$$\operatorname{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix} \begin{matrix} \\ \leftarrow \nabla \\ \leftarrow F \end{matrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 3 & 0 \end{vmatrix}$$

$$= -3\hat{i} + 3\hat{k}$$

$F = \langle 3x^2, 0, -3z \rangle$
$\quad\; f \quad\; g \quad h$

$\text{curl } f = \left\langle \frac{\delta h}{\delta y} - \frac{\delta g}{\delta x}, \frac{\delta f}{\delta z} - \frac{\delta h}{\delta x}, \frac{\delta g}{\delta x} - \frac{\delta f}{\delta y} \right\rangle$

$\cancel{\langle 0, 0, 0 \rangle}$

$\langle 0, 3+3, 0 \rangle = \langle 0, 0, 0 \rangle$

4) $\text{curl } f = \left\langle \frac{\delta h}{\delta y} - \frac{\delta g}{\delta z}, \frac{\delta f}{\delta z} - \frac{\delta h}{\delta x}, \frac{\delta g}{\delta x} - \frac{\delta f}{\delta y} \right\rangle$

$F = \langle \underbrace{5x^2 - y^2}_{f}, \underbrace{2xy}_{g}, \underbrace{3z}_{h} \rangle$

$\therefore \langle 0, 0-0, 2y+2y \rangle = \langle 0, 0, 4y \rangle$

Leadership toward

5)

$$\text{curl } F = \left\langle \frac{\delta h}{\delta y} - \frac{\delta g}{\delta z}, \frac{\delta f}{\delta z} - \frac{\delta h}{\delta x}, \frac{\delta g}{\delta x} - \frac{\delta f}{\delta y} \right\rangle$$

$$F = \langle \underbrace{4z^2 \sin y}_{f}, \underbrace{4xz^2 \cos y}_{g}, \underbrace{8xz \sin y}_{h} \rangle$$

$$\text{curl } F = \langle 8xz \cos y - 8xz \cos y, 8z \sin y - 8z \sin y,$$
$$4z^2 \cos y - 4z^2 \cos y \rangle = \langle 0, 0, 0 \rangle$$

6) $\nabla \cdot (\underline{\nabla \cdot F})$

~~curl~~

div (f)

$\Rightarrow$ a ~~scalar~~ not a $\underline{\text{function}}$ (div f is scalar)

7) $(\underline{\text{curl } F}) \cdot n = 0$

$$\text{curl } F = \left\langle \frac{\delta h}{\delta y} - \frac{\delta g}{\delta z}, \frac{\delta f}{\delta z} - \frac{\delta h}{\delta x}, \frac{\delta g}{\delta x} - \frac{\delta f}{\delta y} \right\rangle$$

7) $\nabla \times B = c\,\frac{\delta E}{\delta t}$

$E(z,t) = A\sin(kz-\omega t)$

$\frac{\delta E}{\delta t} = -{}^{\omega}A\cos(kz-\omega t)$ $\quad \frac{\delta E}{\delta t} = \overset{c}{\overset{-\omega A}{\cancel{Ak}}}\cos(kz-\omega t)$

$\frac{c\,\delta E}{\delta t} = -kA\cos(kz-\omega t)$ $\quad \therefore c\frac{\delta E}{\delta t} = -c\omega A\cos(kz-\omega t)$

~~method~~ by magneth

$\therefore -Ak\cos(kz-\omega t)$

they are same

1) $\frac{\delta}{\delta x}\left(\frac{x}{9+6x^2+5y^2}\right) \cancel{=}$

$= \frac{(9+6x^2+5y^2) - x(12x)}{(9+6x^2+5y^2)^2}$

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