MA 26100

EXAM 2 Green

October 30, 2019

NAME ______________ YOUR TA'S NAME ______________

STUDENT ID # ______________ RECITATION TIME ______________

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): **00**

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **11** questions, each worth 9 points (you will automatically earn 1 point for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–11. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the mark–sense sheet and the exam booklet when you are finished.

If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 6:50. If you don't finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

$\frac{72}{5} + \frac{18}{5} = 18 \quad 8\left(\frac{9}{5}\right) + 6\left(\frac{-3}{5}\right)$

1. Find the maximum value of the function $f(x, y) = 8x - 6y$ subject to the constraint $(x-1)^2 + y^2 = 1$

A. 18

B. 19

C. -2

D. 10

E. $11/5$

Calc 1

$\sqrt{1-(x-1)^2}$

$8x - 6\sqrt{1-(x-1)^2}$

$\frac{9}{16} = \frac{1-(x-1)^2}{(1-x)^2}$

$\frac{16}{9} = \frac{(1-x)^2}{1-(x-1)^2}$

$\left(\frac{4}{5}\ \frac{3}{5}\right)$

$\frac{9}{16} = \frac{1}{(1-x)^2} - 1$

$\frac{25}{16} = \frac{1}{(1-x)^2}$

$\frac{4}{5} = 1 - x$

$x = \frac{1}{5}$

$x = \frac{9}{5}$

$y = \frac{3}{5}$

2. Evaluate $\iint\limits_R \frac{x^2 y}{2 + x^3}\,dA$ over the region $R = \{(x, y) : 1 \leq x \leq 2,\ 0 \leq y \leq 4\}$.

A. $\ln(33)$

B. $\frac{8}{3}(\arctan(5) - \arctan(1.5))$

C. $\frac{8}{3}(\ln(10) - \ln(3))$

D. $8\left(\frac{7}{3} + \ln(2)\right)$

E. $\frac{8}{3}(\ln(5) - \ln(1.5))$

$\int_0^4 \int_1^2 \frac{x^2 y}{2+x^3}\,dx\,dy$

$= \left.\frac{\ln|2+x^3|}{3}\right|_1^2 = \frac{\ln 10}{3} - \frac{\ln 3}{3}$

$\frac{1}{3}\left(\ln\frac{10}{3}\right)\left.\left(\frac{y^2}{2}\right)\right|_0^4$

$= \frac{8}{3}(\ln 10 - \ln 3)$

dy $\quad$ $y=\sqrt{x}$

3. Evaluate $I = \int_0^{\sqrt{2\pi}} \int_{y^2}^{2\pi} y\cos(x^2)dxdy$ by switching the order of integration.

$x = y^2$

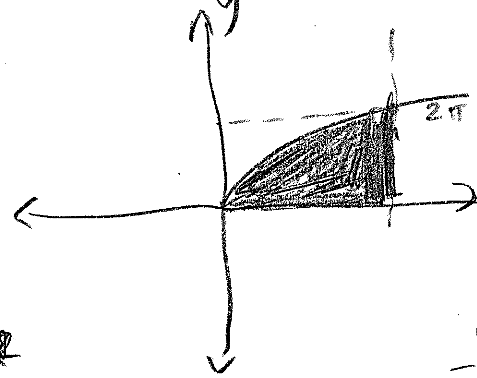
A. $I = 0$

B. $I = \dfrac{\pi}{2}$

C. $I = \dfrac{\pi^2}{4}$

D. $I = \dfrac{\sin(4\pi^2)}{4}$

E. $I = 1$



$\int_0^{2\pi} \frac{x}{2} \cos x^2 \, dx$

$= \frac{1}{4} \left[\sin x^2\right]_0^{2\pi}$

$\int_0^{2\pi} \int_0^{\sqrt{x}} y\cos(x^2)\, dy\, dx$

$= \left[\frac{y^2}{2}\cos(x^2)\right]_0^{\sqrt{x}}$

$= \int_0^{2\pi} \frac{x}{2} \cos x^2 \, dx$

4. Let D be the solid region bounded by the planes:

$$x = 0, \quad z = 0, \quad y = x, \quad \text{and} \quad x + y + z = 2.$$

Which of following iterated integrals is equal to $\iiint_D f(x,y,z)\,dV$ for all continuous functions f defined on D.

$2 - y - x$

A. $\int_0^2 \int_0^{2-x} \int_x^{2-x-y} f(x,y,z)dzdydx$ $\quad$ X

B. $\int_0^1 \int_0^{1-x} \int_0^{2-x-y} f(x,y,z)dzdydx$ $\quad$ X

C. $\int_0^1 \int_x^{1-x} \int_0^{2-x-y} f(x,y,z)dzdydx$

D. $\int_0^1 \int_x^{2-x} \int_0^{2-x-y} f(x,y,z)dzdydx$

E. $\int_0^2 \int_0^{1-x} \int_x^{2-x-y} f(x,y,z)dzdydx$ $\quad$ X

5. Evaluate the integral

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2(x^2+y^2)}} dzdydx$$

using cylindrical coordinates.

$$\int_0^{\pi/2} \int_0^2 \int_r^{\sqrt{2}r} dz\, r\, dr\, d\theta$$

$$\frac{\pi}{2}(\sqrt{2}-1)\cdot\frac{8}{3}$$

A. $\frac{2\pi}{3}(\sqrt{2}-1)$

B. $\frac{4\pi}{3}(\sqrt{2}-1)$

C. $2\pi(\sqrt{2}-1)$

D. $3\pi(\sqrt{2}-1)$

E. $\frac{\pi}{3}(\sqrt{2}-1)$

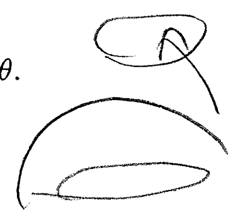
$$\int_0^{\pi/2} \int_0^2 r\left[\sqrt{2}r - r\right] dr\, d\theta = \frac{4}{3}\pi(\sqrt{2}-1)$$

$$\sqrt{2}-1 \int_0^{\pi/2} \int_0^2 r^2\, dr\, d\theta \quad \frac{\pi}{2}(\sqrt{2}-1)\left|\frac{r^3}{3}\right|_0^2$$

6. By converting the integral $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{3(x^2+y^2)}}^{\sqrt{4-x^2-y^2}} f(x,y,z)\, dzdydx$ to spherical coordinates, one obtains the integral

$$\int_0^a \int_0^b \int_0^c f(\rho\sin\varphi\cos\theta,\ \rho\sin\varphi\sin\theta,\ \rho\cos\varphi)\ (\rho^2\sin\varphi)\ d\rho\, d\varphi\, d\theta.$$

1 ? 0 ? ?



Then $\frac{b\,c}{a}$ equals

$$\int_0^{2\pi} \int_0^{\pi} \int^{2} \quad \sqrt{3r^2\sin^2\varphi}$$

A. 1

B. 1/2

C. 1/3

D. 1/4

E. 1/6

$$\frac{\pi \times 1}{2} \qquad 2\pi \qquad \frac{\pi \times 2}{2\pi} = \frac{1}{} \qquad x^2+y^2=1$$

$3(x^2$

$$3(x^2+y^2) = 4-x^2-y^2$$

$$3x^2+3y^2 = 4-x^2-y^2$$

$x = \rho \sin$ $\quad z = \rho\cos\varphi$

circle

7. Find the volume of the solid region enclosed by the surface $\rho = 12\cos\varphi$.

A. 288π

B. $244\pi/3$

C. $320\pi/3$

D. 284π

E. $318\pi/3$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{12\cos\varphi} r^2 \sin\varphi \, dr\, d\varphi\, d\theta$$

$12\cos^2\varphi$ $\quad (-12)$

$\frac{r^3}{3}$

$$\frac{2\pi}{3}\int_0^{\pi} \left[r^3 \right]_0^{12\cos\varphi} \sin\varphi \, d\varphi$$

$$\frac{12^3 \cdot 2\pi}{3}\int_0^{\pi} \cos^3\varphi \sin\varphi \, d\varphi$$

$$\frac{2\pi}{3}\int_0^{\pi} 12^3 \cos^3\varphi \sin\varphi \, d\varphi$$

$$-\frac{12^3\cdot 2\pi}{3}\left[\frac{\cos^4\varphi}{4}\right]_0^{\pi}$$

$$+\frac{12^3\cdot 2\pi}{12}\left[1+1\right]$$

$$\frac{2\cdot 12^3 \cdot 2\pi}{6}$$

$$\frac{12^3\cdot\pi}{3} = 4\times 144$$

$$12^3 - \left(\frac{576}{3}\right)\pi$$

$$\frac{1728-576}{3} - \pi$$

$$\frac{1152}{3}\pi$$

$= 284\pi$?

8. Calculate the mass of the tetrahedron with corners $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,4)$ whose mass density is $\rho(x,y,z) = 2z$.

A. 12

B. $\frac{8}{3}$

C. $\frac{4}{3}$

D. $\frac{1}{2}$

E. $\frac{1}{3}$

$$\int_0^1\int_0^{2-2x}\int_0^{4-2y-x} (2z)\, dz\, dy\, dx$$

z^2

$$\int_0^1\int_0^{2-2x} (4-2y-x)^2\, dy\, dx$$

$$= -\frac{1}{2}\int_0^1 \left[\frac{(4-2y-x)^3}{3}\right]_0^{2-2x} dx$$

$$= -\frac{1}{2}\int_0^1 \frac{(4-4+4x-x)^3}{3}\, dx$$

$$= -\frac{1}{2}\int_0^1 \left[\frac{(4-2(2-2x)-x)^3}{3}\right] dx$$

$$= \frac{1}{2}\int_0^1 \frac{27x^3}{3}\, dx$$

$$\left[\frac{9}{4}x^4\right]_0^1$$

9. A potential for the vector field $\mathbf{F} = \langle \sin(y), x\cos(y) \rangle$ is

A. $x\cos(y)$ ✗

B. $x\sin(y) + \sin(y)$

C. $x\sin(y)\,\mathbf{i} + x\sin(y)\,\mathbf{j}$

D. $-\cos(xy)$ ✗

E. $x\sin(y) + 1$

$\cos y \quad \int \cos y \quad \sin y$

$+ f(y)$

$\int x\sin y + \int \dots \, dy$

$F = \langle \cancel{\cos y}, \cos y, \cos y \rangle$

10. Let C be the curve $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle, t \in [0, \frac{\pi}{2}]$ and $f(x, y, z) = xy$ then

$$\int_C f(x, y, x)ds =$$

$[\cos t \sin t]$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\sqrt{2}$

D. 0

E. 1

$r'(t) = |-\sin t, \cos t, 1|$

$= \sqrt{2}$

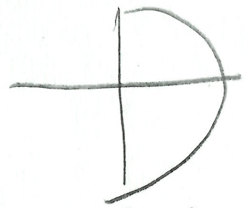
$\sqrt{2} \int \cos t \sin t \, dt$

$\sqrt{2} \int \frac{1}{2} \sin 2t \, dt$

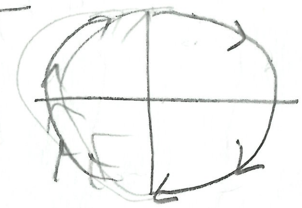
$\frac{\sqrt{2}}{2} \int \sin 2t \, dt = -\frac{\sqrt{2}}{4} \left[\cos 2t \right]_0^{\pi/2} dt$

$-\frac{\sqrt{2}}{4} [-1 - 1]$

$\frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$



$x = \sqrt{4-t^2}$



11. Let C be the half circle $x^2 + y^2 = 4$ with $x \geq 0$ then

$$\int_C x\,ds =$$

A. 0

B. 2

C. 8

D. 4

E. 1

$\sqrt{4-t^2}$

$\sqrt{\cancel{t^2 + 4 - t^2}} = 2$

$\sqrt{\frac{t^2}{4-t^2} + 1}$

$y = t, \quad x = \sqrt{4-t^2}$

$1, \quad \frac{-\cancel{2}t}{\cancel{2}\sqrt{4-t^2}}$

$\int_{-2}^{2} \sqrt{4-t^2}\,dt$

$= \frac{\sqrt{t^2 + (4-t^2)}}{\sqrt{4-t^2}}$

$= \int_{-2}^{2} \frac{2}{\sqrt{4-t^2}} \sqrt{4-t^2}\,dt$

$= \frac{2}{\sqrt{4-t^2}}$

$= 2(t)\Big|_{-2}^{2} = 2(4) = \underline{8}$

$\frac{2}{\sqrt{4-t^2}}\, x$

$\frac{\sqrt{4-t^2}}{2\sqrt{4-t^2}}\, x$