

Question 12

12. Suppose that $w = u^2/v$ where $u = g_1(t)$ and $v = g_2(t)$ are differentiable functions of t . If $g_1(1) = 3$, $g_2(1) = 2$, $g'_1(1) = 5$ and $g'_2(1) = -4$, find $\frac{dw}{dt}$ when $t = 1$.
- A. 6 B. $33/2$ C. -24 D. 33 E. 24

Solution

This is basically a Calculus 1 problem with some spices added to make it look more Calculus 3-ish.

To start with, we know that $w = u^2/v$. Then implicitly differentiate w using the quotient rule:

$$\frac{dw}{dt} = \frac{2u \frac{du}{dt} v - u^2 \frac{dv}{dt}}{v^2}$$

Now note that $u = g_1(t)$ and $v = g_2(t)$. Hence, substituting that, we get

$$\frac{dw}{dt} = \frac{2g_1(t)g'_1(t)g_2(t) - g_1^2(t)g'_2(t)}{g_2^2(t)}$$

Now just plug in the values at $t = 1$:

$$\begin{aligned} \left| \frac{dw}{dt} \right|_{t=1} &= \frac{2 * 3 * 5 * 2 - 3^2 * -4}{2^2} \\ \left| \frac{dw}{dt} \right|_{t=1} &= \frac{60 + 36}{4} \\ \left| \frac{dw}{dt} \right|_{t=1} &= 24 \end{aligned}$$