Introduction

Welcome to this handbook!

This handbook provides a brief outlook of the parameters used when designing each question set, and the level of difficulty for them. This document also contains the solutions for the Math questions available in the quiz.

The V0.9 update

During testing of V0.8.1 after Set 8 was written, some of my 'mates caught quite a few errors in the questions and complained that,

(a) The questions had too many questions about India and wasn't sufficiently broad6; and
(b) The Math questions were too hard and out of scope.

In V0.9, I've replaced many India-specific questions with those for a more global audience. This included replacing three Q15s; the one from Q3 was badly written and the one from Q4 was simply something to be guessed. The Q15 from Set 8 was replaced as it seemed quite specific to a website.

Q7 from Set 7 was considered to be out of scope by a high-scoring IB student; this question assumed that students knew about the *spin-only* d-block formula which apparently wasn't the case. This question was thrown out after investigation as it was found to be badly written as well.

The equivalent Level 2 Math question from Set 1 was replaced as the 'mate complained that finding the given integral was too tedious in 60 seconds.

Q12 in Set 2 (which was a riddle involving a cylinder) was replaced as I felt that it was borderline too easy for a Level 3 question and was not truly mathematical.

I think I should address some concerns relating to the Maths Level 2/3 questions being 'too difficult' to solve. The problem is, Level 3 questions *by their very definition* are meant to be difficult. Making it easy would undermine their very purpose. However, I've also posted solutions in this document so that one can evaluate whether they really are too difficult for a very good student. Feedback on this is very welcome.

Type of questions

There is practically no specified topic(s) that should come up on the quiz; the only defined requirement is that there should be exactly 20% of Math questions in the quiz.

However, care is taken to represent a broad set of questions but ensuring them is difficult when there are only 15 questions to play with.

The Difficulty Curve

When framing the questions, one of the most important metrics is that each of the 8 sets must be equivalent in difficulty so that one can take any set without feeling biased. However, this is very difficult in practice; for instance, one may find that Q8 for Set 3 is very hard for a Level 2 question (it indeed is with its d_C value of 2.1). However, Q6 for Set 3 could be easier than Q6 for Set 5. The *difficulty curve* tries to equalise the potential variations of each set.

A number $d_C \in [1,3]$ is assigned to each question, where $d_C = 1$ indicates a very easy question and $d_C = 3$ indicates an insanely hard question. The average d_C for each set ranges between 1.7 and 1.8, which indicates a reasonable similarity to the setter's perspective in set difficulty. Of course, users are expected to use lifelines appropriately, and some questions will naturally feel harder for some than others.

Question structure

The 15 questions are demarcated into three clear levels:

Questions 1 – 5 *(Level **1**)* are meant to be **easy**. Questions in this level should not require much background knowledge in the subject and should be straightforward to answer. Questions for this section are given **45** seconds to solve.

For Maths questions, it should be well below average for an average high school student.

Questions 6-10 *(Level **2**)* are meant to be **medium**. Questions in this level should require a reasonable level of background knowledge in the said question. While Q6 might still be reasonable in difficulty, Q10 would usually be harder and require some advanced knowledge. Questions for this section are given **60** seconds to solve.

For Maths questions, it should be an average high school/A Level/CBSE Class 12 question or (as in Q10 for Set 4) a somewhat harder GCSE-level/CBSE Class 10 question.

Questions 11-15 *(Level **3**)* are meant to be **hard**. Question in this level would require advanced knowledge in the question, and a Q14 or Q15 may call for somewhat obscure factual knowledge, while a Q11 would still be hard. Questions for this section are given **75** seconds to solve.

For Maths questions, it should be a difficult A-Level/CBSE/IB Higher Maths-level question, but care should still be taken to ensure that it can be solved by an excellent student (i.e, A*/A* in A-Level Maths/Further Maths; 96+ in CBSE; 7 in IB Higher) within the time limit. *Q15 should generally <u>not</u> be a Math question.*

Solutions for the Math questions

One should note that I have provided the raw answer below, with a semi-formal solution outlining the solution in the box below the answer. It must be noted that the MCQ nature of this quiz allows for sneaky shortcuts for solving some of the problems.

For instance, one can simply compare the answers provided for some small values of n (an example is Q7 for Set 1). There are other such shortcuts which are not described here.

Remember the difficulty curve:

Questions 1 – 5 *(Level **1**)* are meant to be **easy**

Questions 6-10 *(Level **2**)* are meant to be **medium**

Questions 11-15 *(Level **3**)* are meant to be **hard**

Spoiler alert: answers ahead!

Set 1

4. What is the range of 9, 17, 3 and 2?

Answer: 15

Range = largest – smallest, which is 17-2=15.

7. n 50p coins (heptagonal) are arranged in a line touched by their edges. Then the number of free edges would be:

Answer: B

It can be shown that $n-2$ coins will be touched by two edges (opposite to each other), with the outer coins touched by exactly one edge. Hence the number of free edges is $5(n-2)+6*2 => 5n-10+12=5n+2$

13. Three real numbers in interval [0,1] are chosen independently and at random. Then the probability they are the side lengths of a triangle with +ve area is

Answer: ½

This can be proved by the *Triangle Inequality*. Without loss of generality, assume that $a>b$. Then $c<b$, $b<c<a$ or $c>a$. But then $a<b+c$ and $a>b-c$. Using this, we see that exactly half of the possibilities are valid.

Set 2

4. If a = 2b , b = 5 and ax+b = 35 , x =

Answer: 3.

We know that $b=5$. Hence $a=2b; a=10$. Then, $ax+b=35 \rightarrow 10x+5=15 \rightarrow x=3$.

9. There are p students in a class. 3q% of them study for x hours and the rest q% would study for 3x hours. Then the mean of the number of hours studied is

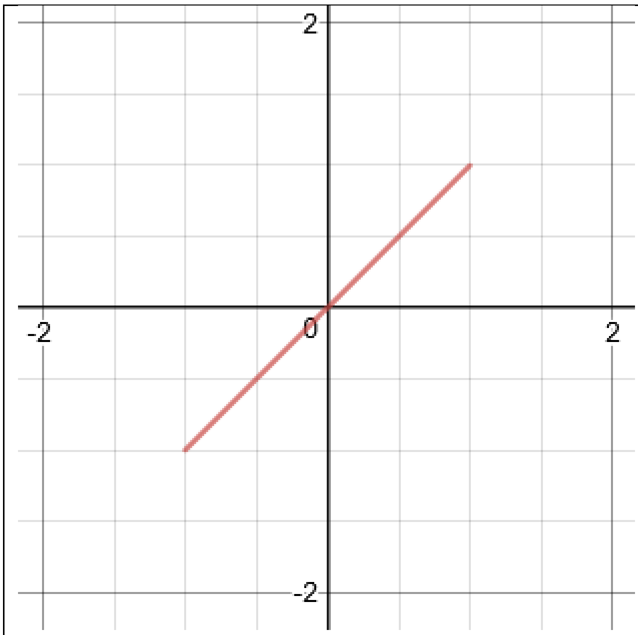
Answer: $\frac{3x}{2}$ hours.

Firstly, note that $3q+q=1$, which implies that $q=\frac{1}{4}$. Then $3q\%=75\%$ and $q\%=25\%$. Then, the mean is $\left(\frac{3}{4}*x+\frac{1}{4}*3x\right)=\frac{3x}{2}$

12. Consider the circle generated by $y=\sin(\arcsin x)$. Then the area of the circle is

Answer: π

As $\arcsin x$ is defined only in $[-1,1]$, $\sin(\arcsin x)$ is valid only between these values. But $\sin(\arcsin x)=x$ for those values, as $\arcsin x$ is the inverse of $\sin x$. This means that the graph of that function is a straight line between those values:



Then the radius of the circle generated is 1.
Hence the area is πr^2 when $r = 1$ -> Area = π.

Set 3

4. What would you get if you repeatedly differentiate a purely algebraic and polynomial function?

Answer: 0.

As each derivative of the function reduces the degree of the polynomial by 1, there will be a stage when the degree is 0; any further derivatives will be 0.

9. How many complementary probabilities exist which are reciprocal of each other?

Answer: 0

From the question, we know that

$$P(A) + P(B) = 1$$

And

$$P(A) = \frac{1}{P(B)}$$

Combining the both, we have

$$P(B) + \frac{1}{P(B)} = 1$$

Let $P(B) = t$. Then

$$t + \frac{1}{t} = 1 \rightarrow t^2 + 1 = t \rightarrow t^2 - t + 1 = 0$$

But the quadratic equation $t^2 - t + 1 = 0$ has no real root. Hence the answer is 0.

14. The value of y in the below differential equation is:

$$x\,dy - dx = \frac{1}{x}(\pm dx + dy)$$

Answer: None of these

Rearranging the D.E, we get

$$x\,dy - dx = \frac{\pm 1}{x}dx + dy$$
$$x\,dy - dy = \pm\frac{1}{x}dx + dx$$
$$dy(x-1) = dx\left(1 \pm \frac{1}{x}\right)$$
$$dy = \frac{1 \pm \frac{1}{x}}{x-1}dx$$
Integrating both sides, we have
$$y = \int \frac{x \pm 1}{x(x-1)}dx$$
Note that the value when the integral is $x - 1$ is $y = \ln x$, but that is nowhere amongst the options even accounting for the signs.
Hence the answer is D (none of these).

Set 4

5. What is the height of a cylinder whose radius is 2 cm and volume (in cm cube) is 8 pi?

Answer: 2

The volume of a cylinder is $\pi r^2 h$.
But then $\pi r^2 h = 8\pi \to r^2 = 4 \Longrightarrow r = 2$.

10. Find the value of k in the above equation: -

$$\frac{1+2+3+4+\cdots+n}{1-2-3-4-\cdots-n} = \frac{n(n+1)}{k-n(n+1)}$$

Answer: 4

$$\frac{1+2+3+4+\cdots+n}{1-2-3-4-\cdots-n} = \frac{1+2+3+4+\cdots+n}{2-1-2-3-4-\cdots-n} \to \frac{1+2+3+4+\cdots+n}{2-(1+2+3+\cdots+n)}$$
We know that $1+2+3+\cdots+n = \frac{n(n+1)}{2}$.
Hence,
$$\frac{1+2+3+4+\cdots+n}{2-(1+2+3+\cdots+n)} \to \frac{\frac{n(n+1)}{2}}{2-\frac{n(n+1)}{2}} = \frac{n(n+1)}{4-n(n+1)}$$
Comparing with RHS of question, we note that $k = 4$.

12. The formula that correctly represents the (h)our and (m)inute when the hands of a clock coincide is

Answer: $60h = 11m$.

The hands of the minute clock move 6 degrees for every minute.
Hence the total angle by the minute hand is $6m$.

The hour hand moves $\frac{30}{60} = \frac{1}{2}$ degrees as the minute clock moves every minute.
Additionally, the hour hand moves by 30 degrees for every hour that moves past.
Hence the total angle by the hour hand is $\left(30h + \frac{m}{2}\right)^\circ$
As both hands coincide,

$$
\begin{gathered}
6m = 30h + \frac{m}{2} \\
\frac{11m}{2} = 30h \\
11m = 60h
\end{gathered}
$$

Set 5

4. If a = 1, then the integral of reciprocal of the sum of squares of a and x is

Answer: $\arctan x$

$$\int \frac{1}{1+x^2} = \tan^{-1} x + C$$

10. Find the range of $\left(\frac{a}{b}\right) + \left(\frac{b}{a}\right)$ if a,b≠0 and R represents the set of real numbers.

Answer: $\mathbb{R} - (-2,2)$

Let $\frac{a}{b} = t$. Then $\frac{b}{a} = \frac{1}{t}$.

Then we need to find the minimum value of $t + \frac{1}{t}$.
Differentiating, we get $1 - \frac{1}{t^2}$. Setting it to 0, we get $t = \pm 1$.
Finding second derivative, we get $\frac{2}{t^3}$. But $\frac{2}{t^3} > 0$ when $t = -1$ and $\frac{2}{t^3} < 0$ when $t = 1$. This means that the function is a minimum when $t = 1$ and a maximum when $t = -1$.
Combining both statements together shows that the function cannot exist between -2 and 2.

[This question can also be solved using the quadratic formula]

14. If $\sin x + \cos x = \tan x$ and $(\tan x)^4 - a \tan x - b = 0$, then the value of $|a - b|$ is

Answer: 1

Version 0.8.1 incorrectly said the answer as 3.

We have

$$\sin x + \cos x = \tan x$$

Squaring both sides,

$$
\begin{gathered}
1 + \sin 2x = \tan^2 x \\
1 + \frac{2 \tan x}{1 + \tan^2 x} = \tan^2 x \\
1 + \tan^2 x + 2 \tan x = \tan^2 x + \tan^4 x \\
\tan^4 x - 2 \tan x - 1 = 0
\end{gathered}
$$

Comparing with question, $a = 2$ and $b = 1$; $|a - b| = 1$.

Set 6

3. The number without a reciprocal is

Answer: 0

The value of $\frac{1}{0}$ is not defined.

7. If 1/i is successively summed from 1 to infinity(1/1+1/2+...), then the value of the resulting sum is

Answer: Infinity (∞)

The series asked in the question is

$$\sum_{i=1}^{\infty} \frac{1}{i}$$

This is commonly known as the harmonic series, and it's known to be divergent.

It's even easier to prove using the *Integral Test*:

$$\int_{1}^{\infty} \frac{1}{i} di = [\ln i]_1^{\infty} = \ln \infty \text{, which is divergent.}$$

12. How many terms are there in the simplified value of the integral of $\arcsin\left(\frac{a}{x}\right) + \arcsin\left(\frac{x}{a}\right) dx$? (excluding integration constant)

Answer: 1

The question is

$$\int \sin^{-1}\frac{a}{x} + \sin^{-1}\frac{x}{a} dx$$

But the domain of $\sin^{-1} k$ is $[-1,1]$. But then $\frac{a}{x}$ and $\frac{x}{a}$ can satisfy this condition only when $a = \pm x$. Then the value of $\frac{a}{x}$ and $\frac{x}{a}$ becomes ± 1.

Hence the question reduces to constant values:

$$\int \sin^{-1}\frac{a}{x} + \sin^{-1}\frac{x}{a} dx = \int \pm\pi\, dx = \pm\pi x + C$$

Set 7

3. Let $\frac{a-b}{b} = \frac{3}{7}$. Then $\frac{b}{a}$ =

Answer: $\frac{7}{10}$

$$\frac{a}{b} - 1 = \frac{3}{7} \rightarrow \frac{a}{b} = \frac{10}{7}$$
$$=> \frac{b}{a} = \frac{7}{10}$$

7. How many prime numbers are there between 1 and 54 (both inclusive)?

Answer: 15

2,3,5,7,11,13,17,19,23,29,31,37,41,47,53 are the prime numbers between 1 and 54.

13. The result of

$$\int_0^n [2^x]\,dx$$

is (where $n \geq 1$)

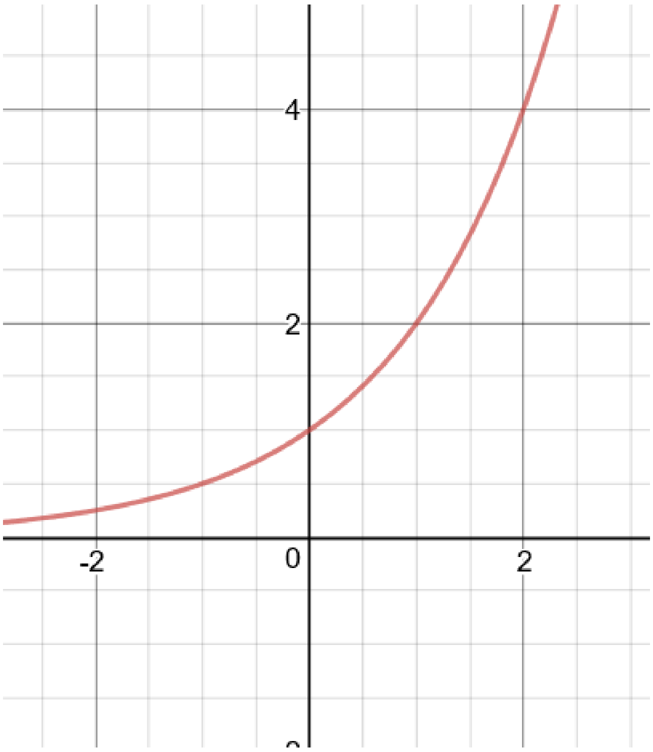
Answer: $n(2^n) - \log_2(2n)!$

$$\int_0^n [2^x]\,dx = \int_0^1 1\,dx + \int_1^{\log_2 3} 2\,dx + \int_{\log_2 3}^{2} 3\,dx + \cdots$$
$$= \log_2 2\,(x) + \log_2 3\,(2x) + \log_2 2\,(2x) + \log_2 4\,(3x) + \log_2 3\,(3x) + \cdots$$

But getting the answer would be considered very challenging in 75 seconds.

OR

Consider the graph of 2^x:



Taking $[2^x]$ would imply that it is the step-wise difference between the value of 2^n and

$$\int_a^b 2^x\,dx - [2^a(b-a)]$$

Where $2^b - 2^a = 1$

This logic can be used to prove that it must be the difference of two functions; option (b).

Set 8

4. The mean of two numbers is 2.5 and the range is 2. Then the numbers are

Answer: 3.5 and 1.5

Let the numbers be a and b. Then we need $a + b = 5$ and $a - b = 2$. Solving the equation gives the numbers.

8. If $\tan x + \cot x = k\ cosec\ mx$, then the value of $k + m$ is

Answer: 4

Start with the LHS:

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$
$$\rightarrow \frac{1}{\frac{1}{2}\sin 2x} = 2\ cosec\ 2x$$

Comparing with the questions, $k = 2$ and $m = 2$; $k + m = 4$.

12. Consider a $x - y$ graph with each point on the axis separated by an equal distance. Suppose a straight line is drawn from (0,1) to the point (3,8). If the tangent between the line and x axis is 1, then the area of the triangle so formed is

Answer: $\frac{7}{\ln 2}$

If $\tan\theta = 1$, then this means that the triangle *in the specified graph* is isosceles.

But then (3,8) and (0,1) is a straight line. This is possible only when $y = 2^x$, and the graph is *linear-logarithmic.*

Thus, the area of the triangle is the area between the line and the x – axis, which is

$$\int_0^3 2^x\, dx = \left[\frac{2^x}{\ln 2}\right]_0^3 = \frac{8}{\ln 2} - \frac{1}{\ln 2} = \frac{7}{\ln 2}$$