# Introduction

Welcome to this handbook!

This handbook provides a brief outlook of the parameters used when designing each question set, and the level of difficulty for them. This document also contains the solutions for the Math questions available in the quiz.

# User aim

*From Set 9 onwards and those done in Quizizz*

Get the highest accuracy while scoring the most number of points.

*Up to Set 9 and done in the MillionaireFake software*

Get the greatest number of points/progress as far as possible. Note that if an incorrect answer is selected, the number of points automatically drops to the safe zone () where is the greater integer less than or equal to *X*). So if someone failed in Q9, then they get the same number of points as if someone just passed Q5. This necessitates a somewhat more aggressive use of lifelines compared to the Quizizz version.

# Changes from Set 14

The scope of topics for the Mathematics section has been reduced.

Up to Set 14, the questions were written with a very good science student in mind. This would roughly be in the ballpark of someone who took A-Level Further Mathematics, IB Mathematics Higher or CBSE Class 12 in Mathematics. However, considering that a wide population can be expected to take these quizzes, it is unfair to expect such students to be able to solve calculus problems.

Hence, the scope has been reduced to that expected by a GCSE Maths Higher/CBSE Class 10 student. This means that the following topics will no longer be tested in the quiz:

* Calculus of any kind (limital, differential and integral)
* Matrices
* Discrete mathematics (other than those that can be solved by brute-force or basic common sense)
* Statistics (other than mean, median and mode and basic analysis)
* Probability: Bayes’ theorem, the random variable (mean, variance). PDF/CDF isn’t covered either.
* Non-polynomial graphs
* Inverse trigonometry (other than the very basic idea – like ). Handling multiple angles (like )
* Logarithms and solving exponential equations
* Geometric and harmonic progressions

This might cause the averages for these questions to be a bit higher than expected.

# March 2020 overhaul

Since March 2020, all future sets have been written on Quizizz, while maintaining the question rigour and curve that is expected of a MillionaireFake quiz. This has several changes, many beneficial with some unavoidable drawbacks:

* Users do not have to download a clunky Windows-only MillionaireFake software and can play the game anytime on mobile or desktop using any modern web browser.
* Descriptive statistics are available for each user and question, providing very powerful insights on whether a question performed as expected.
* Options and questions are now shuffled from the pool. Previously, users were served questions in a sequential order, while now the order is perfectly random. The aim is to discourage users from quitting when suddenly faced with multiple Level **3** questions.
* Users will not be eliminated on a wrong answer. This is necessary to give everyone the chance to statistically test all the question items, and also because Quizizz does not have any such option.
* Powerups are available, which means that users can get lifeline-like options (like extra points or accepting one incorrect answer). However, the *redemption question* option isn’t available as that does nothing but give everyone free points.
* The timer is soft, which means that users will not be kicked out for exceeding the timer. This is a “design choice” by Quizizz that I personally oppose and consider it a bug that urgently needs to be fixed. *This is not the case when using the Instructor mode, and hence does not affect live deployment.*
* It allows users to attempt the quiz only once by forcing login using a Google or Quizizz account.

A user (Tom Groves) has correctly raised concerns about potential cheating. Rigorous statistical analysis is done to discourage cheating (for instance, people with unusually high scores are investigated further to check for factors like taking similar amount of time). However, this is understandably not enough (especially since a search engine query can get answers quickly), and I am working on ways to improve on this aspect. It should be noted however that Set 9 problems however are relatively less “bookwork” than before and Set 14 problems improves further there.

# Type of questions

There is practically no specified topic(s) that should come up on the quiz; the only defined requirement is that there should be exactly 20% of mathematics questions in the quiz.

However, care is taken to represent a broad set of questions but ensuring them is difficult when there are only 15 questions to play with.

## The Difficulty Curve (outdated)

When framing the questions, one of the most important metrics is that each of the *N* sets must be equivalent in difficulty so that one can take any set without feeling biased. However, this is very difficult in practice; for instance, one may find that Q8 for Set 3 is very hard for a Level 2 question (it indeed is with its value of 2.1). However, Q6 for Set 3 could be easier than Q6 for Set 5. The *difficulty curve* tries to equalise the potential variations of each set.

A number is assigned to each question, where indicates a very easy question and indicates an insanely hard question. The average for each set [1-8] ranges between 1.7 and 1.8, which indicates a reasonable similarity to the setter’s perspective in set difficulty. Of course, users are expected to use lifelines appropriately, and some questions will naturally feel harder for some than others.

**This metric is no longer used as of Set 9, as we aim to get a curve as linearly scaled as possible. The aim is still true, however, and we still aim to get an overall curve that is as close as possible (between 50 and 60% mean). While lifelines (or Powerups) are still valuable, it is less stressful using Quizizz since a wrong answer does not ensure elimination.**

# Question structure

The 15 questions are demarcated into three clear levels:

Questions 1 – 5 *(Level* ***1****)* are meant to be **easy**. Questions in this level should not require much background knowledge in the subject and should be straightforward to answer. Questions for this section are given **45** seconds to solve.

Questions 6-10 *(Level* ***2****)* are meant to be **medium**. Questions in this level should require a reasonable level of background knowledge in the said question. While Q6 might still be reasonable in difficulty, Q10 would usually be harder and require some advanced knowledge. Questions for this section are given **60** seconds to solve.

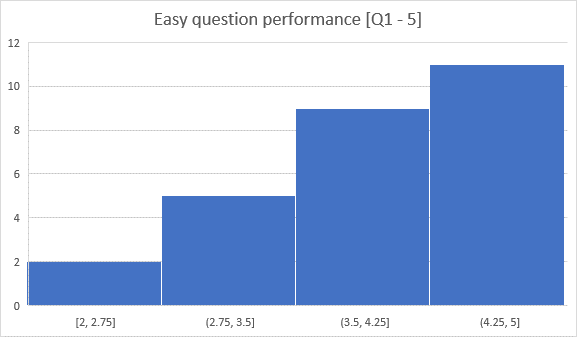
Questions 11-15 *(Level* ***3****)* are meant to be **hard**. Question in this level would require advanced knowledge in the question, and a Q14 or Q15 may call for somewhat obscure factual knowledge, while a Q11 would still be hard. Questions for this section are given **120** seconds to solve.

*For questions in the MillionaireFake software, the time limit is* ***75*** *seconds (Sets 1 to 8).*

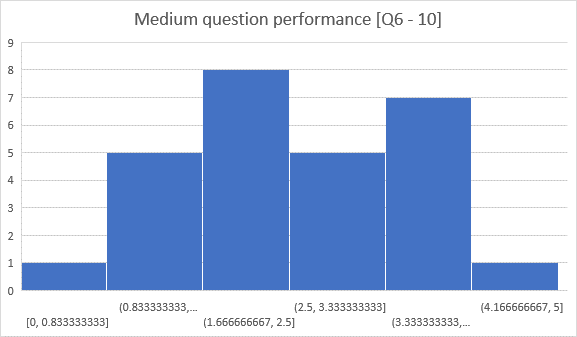
The Maths questions work the same way, with the syllabus for Set 1 – 13 corresponding roughly to A-Level Further Mathematics/CBSE Class 12 and Set 14 onwards to GCSE Maths Higher/CBSE Class 10.

# Statistical robustness

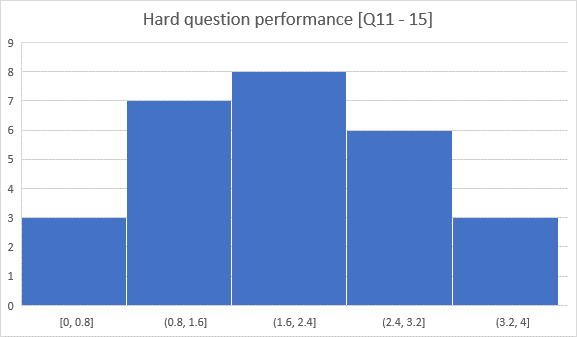
## Set 9



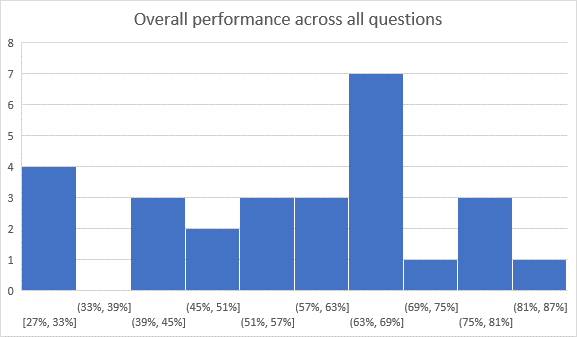
Mean = 81.4% (4.07/5). Median = 80% (4/5)



Mean = 51.1% (2.56/5), Median = 40% (2/5)



Mean = 39.3% (1.96/5), Median = 40% (2/5)



Mean = 57%, Median = 60% (9/15)

# Solutions for the Math questions

One should note that I have provided the raw answer below, with a semi-formal solution outlining the solution in the box below the answer. It must be noted that the MCQ nature of this quiz allows for sneaky shortcuts for solving some of the problems.

For instance, one can simply compare the answers provided for some small values of (an example is Q7 for Set 1). There are other such shortcuts which are not described here.

Remember the difficulty curve:

Questions 1 – 5 *(Level* ***1****)* are meant to be **easy**

Questions 6-10 *(Level* ***2****)* are meant to be **medium**

Questions 11-15 *(Level* ***3****)* are meant to be **hard**

**Spoiler alert: answers ahead!**

## Set 1

4. What is the range of 9, 17, 3 and 2?

Answer: 15

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| Range = largest – smallest, which is 17-2=15. |

7. 50p coins (heptagonal) are arranged in a line touched by their edges. Then the number of free edges would be:

Answer: B

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| It can be shown that coins will be touched by two edges (opposite to each other), with the outer coins touched by exactly one edge.  Hence the number of free edges is |

13. Three real numbers in interval [0,1] are chosen independently and at random. Then the probability they are the side lengths of a triangle with +ve area is

Answer: ½

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| This can be proved by the *Triangle Inequality*. Without loss of generality, assume that . Then , or . But then and . Using this, we see that exactly half of the possibilities are valid. |

## Set 2

4. If a = 2b , b = 5 and ax+b = 35 , x =

Answer: 3.

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| We know that . Hence .  Then, . |

9. There are p students in a class. 3q% of them study for x hours and the rest q% would study for 3x hours. Then the mean of the number of hours studied is

Answer: hours.

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| Firstly, note that , which implies that . Then and .  Then, the mean is |

12. Consider the circle generated by . Then the area of the circle is

Answer: π

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| As is defined only in , is valid only between these values.  But for those values, as is the inverse of . This means that the graph of that function is a straight line between those values:    Then the radius of the circle generated is 1.  Hence the area is when -> Area = . |

## Set 3

4. What would you get if you repeatedly differentiate a purely algebraic and polynomial function?

Answer: 0.

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| As each derivative of the function reduces the degree of the polynomial by 1, there will be a stage when the degree is 0; any further derivatives will be 0. |

9. How many complementary probabilities exist which are reciprocal of each other?

Answer: 0

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| From the question, we know that  And  Combining the both, we have  Let . Then  But the quadratic equation has no real root. Hence the answer is 0. |

14. The value of in the below differential equation is:

Answer: None of these

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| Rearranging the D.E, we get  Integrating both sides, we have  Note that the value when the integral is is , but that is nowhere amongst the options even accounting for the signs.  Hence the answer is D (none of these). |

## Set 4

5. What is the height of a cylinder whose radius is 2 cm and volume (in cm cube) is 8 pi?

Answer: 2

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| The volume of a cylinder is .  But then . |

10. Find the value of k in the above equation: -

Answer: 4

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| We know that .  Hence,  Comparing with RHS of question, we note that . |

12. The formula that correctly represents the (h)our and (m)inute when the hands of a clock coincide is

Answer: .

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| The hands of the minute clock move 6 degrees for every minute.  Hence the total angle by the minute hand is .  The hour hand moves degrees as the minute clock moves every minute.  Additionally, the hour hand moves by 30 degrees for every hour that moves past.  Hence the total angle by the hour hand is  As both hands coincide, |

## Set 5

4. If a = 1, then the integral of reciprocal of the sum of squares of a and x is

Answer:

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10. Find the range of if a,b≠0 and R represents the set of real numbers.

Answer:

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| Let . Then .  Then we need to find the minimum value of .  Differentiating, we get . Setting it to 0, we get .  Finding second derivative, we get . But when and when . This means that the function is a minimum when and a maximum when .  Combining both statements together shows that the function cannot exist between -2 and 2.  [This question can also be solved using the quadratic formula] |

14. If and , then the value of is

Answer: 1

Version 0.8.1 incorrectly said the answer as 3.

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| We have  Squaring both sides,  Comparing with question, and ; . |

## Set 6

3. The number without a reciprocal is

Answer: 0

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| The value of is not defined. |

7. If 1/i is successively summed from 1 to infinity(1/1+1/2+…), then the value of the resulting sum is

Answer: Infinity (∞)

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| The series asked in the question is    This is commonly known as the harmonic series, and it’s known to be divergent.  It’s even easier to prove using the *Integral Test*: |

12. How many terms are there in the simplified value of the integral of ? (excluding integration constant)

Answer: 1

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| The question is  But the domain of is . But then and can satisfy this condition only when . Then the value of and becomes .  Hence the question reduces to constant values: |

## Set 7

3. Let . Then =

Answer:

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7. How many prime numbers are there between 1 and 54 (both inclusive)?

Answer: 15

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| 2,3,5,7,11,13,17,19,23,29,31,37,41,47,53 are the prime numbers between 1 and 54. |

13. The result of

is (where )

Answer:

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| But getting the answer would be considered very challenging in 75 seconds.  OR  Consider the graph of :  A picture containing object  Description automatically generated  Taking would imply that it is the step-wise difference between the value of and  Where  This logic can be used to prove that it must be the difference of two functions; option (b). |

## Set 8

4. The mean of two numbers is 2.5 and the range is 2. Then the numbers are

Answer: 3.5 and 1.5

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| Let the numbers be and . Then we need and . Solving the equation gives the numbers. |

8. If , then the value of is

Answer: 4

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| Start with the LHS:  Comparing with the questions, and ; . |

12. Consider a graph with each point on the axis separated by an equal distance. Suppose a straight line is drawn from (0,1) to the point (3,8). If the tangent between the line and x axis is 1, then the area of the triangle so formed is

Answer:

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| If , then this means that the triangle *in the specified graph* is isosceles.  But then (3,8) and (0,1) is a straight line. This is possible only when , and the graph is *linear-logarithmic.*  Thus, the area of the triangle is the area between the line and the – axis, which is |

## Set 9

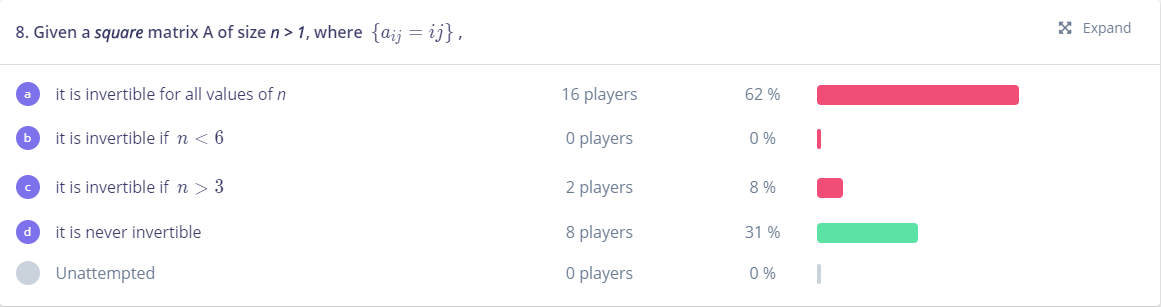
Comments are included from this set onwards, as this is the first set where the responses were statistically analysed.

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| We want the LCM of 12 and 15. This is 30. |

*Comments: most got this without hesitation.*



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| Consider two rows of the matrix *x* and *y* (where ). Then the elements of row *x* and *y* are and . Multiply row *x* with , and we immediately get the elements of row *y*, and a simple row transformation will ensure a row with zeroes, and , which is never invertible. |

*Comments: a tricky problem. Many of them thought that it is invertible for all values of* n*, which is surprising because a simple check with a matrix will show this is not possible. A small fraction might have done this check but may not be sure whether it would work for higher dimensions – it certainly does.*

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| By the *derangement theorem*, we know that the probability that none of his friends gets the meal they ordered is  Hence the probability that at least one of his friends gets the meal is  An alternative method is to prove this using the principle of inclusion and exclusion: notice that we have as the number of arrangements of the sequence. Then there are ways in which each element stays in the same place, and then we need to consider ways of element of size two that stays in the same place (notice that they were double counted), and so on. Hence we get  And we can easily continue from there. |

*Comments: this had a wide variety of responses. While many connected the “one of his friends” with , a surprising proportion ended up taking option* ***B****, which is impossible because the value is zero when , and in fact negative when because is strictly positive (which is why the alternating series in the correct option is important).*

## Set 10

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| Option A is 111, which is palindromic. |

*Comments: Most got this one.*

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| Write down the first few compositions: , , . This suggests that  Hence . |

*Comments: was hard for Q7. The majority, as expected, didn’t realise the pattern and simply took and nothing else.*

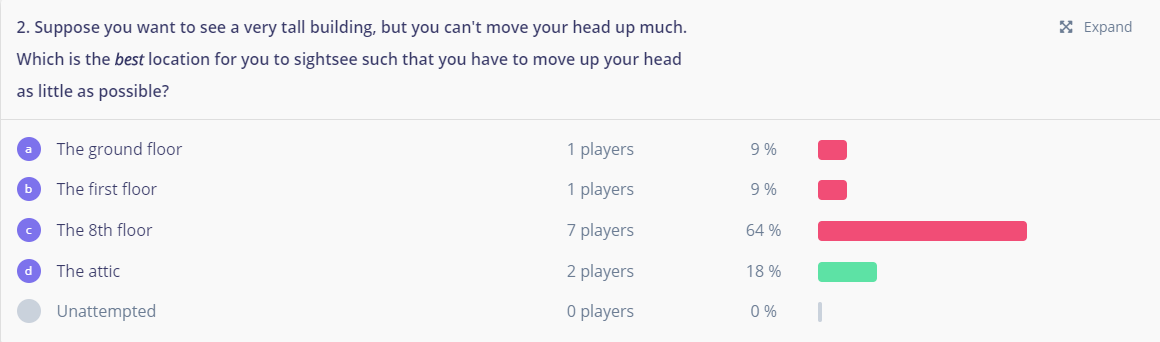
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| Notice that  (as )  Then,  Now note that . This means that |

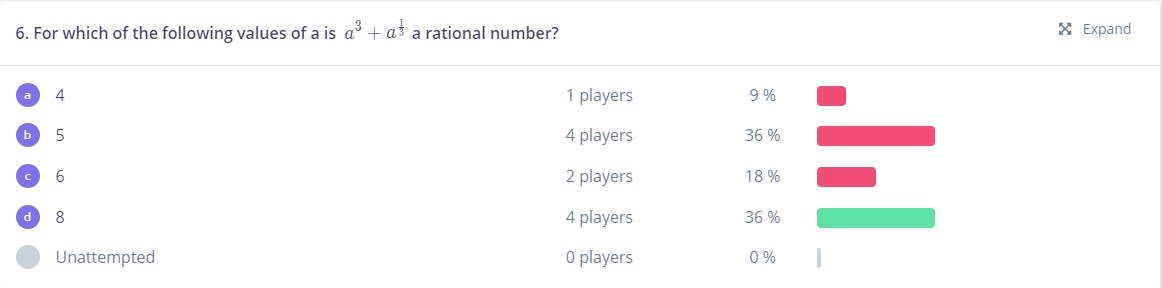
*Comments: one of the hardest maths questions ever amongst all the sets, and it showed. It was a weird coincidence that no one picked option* ***D****, but otherwise answers were all over the place.*

## Set 11



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| **This question as originally administered is faulty.** The fixed version replaces option **D** with “The top of the building”.  The further you are towards the building, the less you need to move the head up. Hence, we want the topmost point of the building. |

*Comments: a lot of people was understandably confused at the difference between the 8th floor and the attic and turned out that the question was not properly framed.*



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*Comments: for a question as straightforward as this, it was very surprising to see answers all over the place.*

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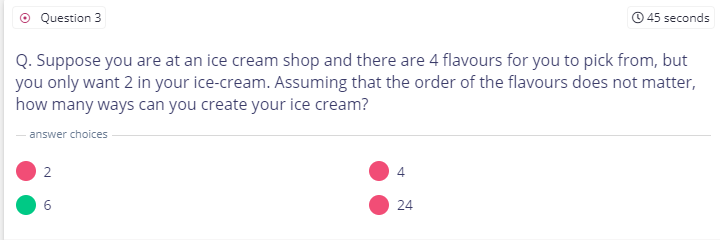
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| The Fibonacci series is  This can be converted to an iterative equation as  Hence |

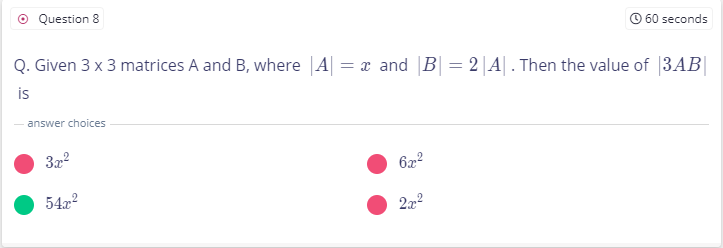
*Comments – what is not shown here is the conversion from recursion to iteration, which is the hardest part of all. This is probably what drove quite a few to* ***B*** *and* ***D****, as the ideas shown there might have appealed (the recursive calls in* ***B*** *and the summation in* ***D****). Answers were all over the place as well.*

## Set 12

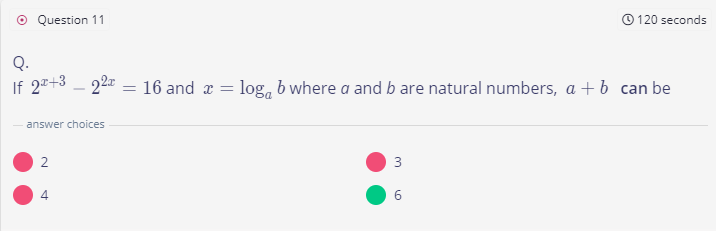
Questions from this set haven’t been field-tested*.*



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| . Or just brute-force. |

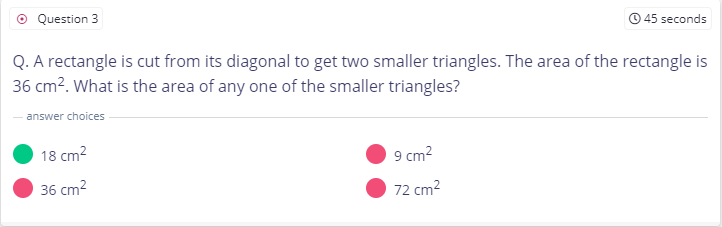


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| As we have a 3 x 3 matrix, |

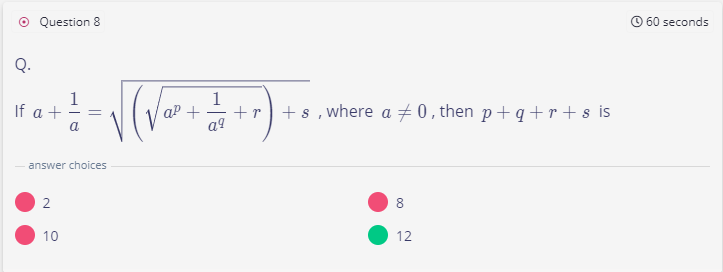


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| Notice that .  Let , and we get  But is positive, and hence .  We now need to write . The smallest value of *a* is , and then. Hence . |

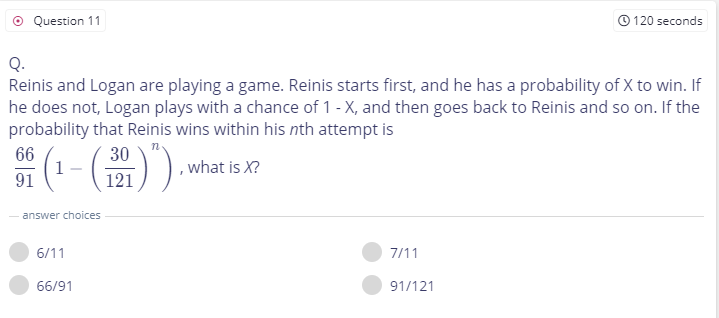
## Set 13



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| Dividing a rectangle gives two triangles of equal area. Hence the area of each of the smaller triangles is 36/2 = 18. |



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| Apply multiple squaring:  Or start backwards, but clearly, . They add up to 12. |



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| So Reinis can win with *X*, or or or so on. The key is that we get a geometric progression with . Hence the sum (that is the probability of winning within *n* attempts) is  Comparing the coefficients,  OR |