6.1. min $2x_1 + 4x_2 + 3x_3 + 6x_4$

s.t $x_1 + 2x_2 + 5x_3 \geq 1$ y_1

$x_1 + x_3 + 3x_4 \geq 2$ y_2

$x_2 + 2x_3 \geq 10$ y_3

$x_1 + 2x_2 + x_4 \geq 6$ y_4

$x_1, x_2, x_3, x_4 \geq 0$

a) max $y_1 + 2y_2 + 10y_3 + 6y_4$

s.t. $y_1 + y_2 + y_4 \leq 2$

$2y_1 + y_3 + 2y_4 \leq 4$

$5y_1 + y_2 + 2y_3 \leq 3$

$3y_2 + y_4 \leq 6$

$y_1, y_2, y_3, y_4 \geq 0$

b) set $y_1 = 0$, $y_2 = 0$, $y_3 = 0$, $y_4 = 2$

objective value $= 6(2) = 12$

$y_4 \leq 2$

$2y_4 \leq 4$

$y_4 \leq 6$

If we have $y_1 = 0$, $y_2 = 0$, $y_3 = 0$, $y_4 = 2$, then the objective value of the dual is 12. We know that the objective value of any feasible solution to a maximization problem is less than or equal to the objective value of the minimization problem. Therefore if the objective value for the maximization is 12, then the objective value for the minimization will be greater than or equal to 12.

c) Weak Duality

6.2. min $7x + 2y + 12z$

s.t $x + y + z \geq 1$ (a)

$x + 2y + 2z \geq 2$ (b)

$x - 2y + 3z \geq 1$ (c)

a) max $a + 2b + c$

s.t $a + b + c = 7$

$a + 2b - 2c = 2$

$a + 2b + 3c = 12$

$a, b, c \geq 0$

b) $a + 2b - 2c = 2$

$-(a + 2b + 3c = 12)$

$-5c = -10$

$\boxed{c = 2}$

$a + b + c = 7$

$-(a + 2b - 2c = 2)$

$-b + 3c = 5$

$-b + 6 = 5$

$-b = -1$

$\boxed{b = 1}$

$a + 1 + 2 = 7$

$\boxed{a = 4}$

max $a + 2b + c \rightarrow 4 + 2 + 2 = 8$

The dual's optimal solution is 8, so it's primal has the same optimal solution.

c) Strong duality

6.3 max $6x_1 + 10x_2 + 9x_3 + 20x_4$

s.t $4x_1 + 9x_2 + 7x_3 + 10x_4 \leq 600$ y_1

$x_1 + x_2 + 3x_3 + 40x_4 \leq 400$ y_2

$3x_1 + 4x_2 + 2x_3 + x_4 \leq 500$ y_3

$x_1, x_2, x_3, x_4 \geq 0$

min $600y_1 + 400y_2 + 500y_3$

s.t $4y_1 + y_2 + 3y_3 \geq 6$ x_1

$9y_1 + y_2 + y_3 \geq 10$ x_2

$7y_1 + 3y_2 + 2y_3 \geq 9$ x_3

$10y_1 + 40y_2 + y_3 \geq 20$ x_4

$y_1, y_2, y_3, y_4 \geq 0$

b) $(x_1^*, x_2^*, x_3^*, x_4^*) = (\frac{400}{3}, 0, 0, \frac{20}{3})$

$(400/3)4 + 10(20/3) = 600$ Binding

$(400/3) + 40(20/3) = 400$ Binding

$3(400/3) + (20/3) = 406.67$ non binding

$\therefore \boxed{y_3 = 0}$

$4y_1 + y_2 = 6$

$10y_1 + 40y_2 = 20$

$y_2 = 6 - 4y_1$

$10y_1 + 40(6 - 4y_1) = 20$

$10y_1 + 240 - 160y_1 = 20$

$-150y_1 = -220$

$\boxed{y_1 = 22/15}$

$4(22/15) + y_2 = 6$

$\boxed{y_2 = 2/15}$

$\boxed{\begin{array}{l} y_1 = 22/15 \\ y_2 = 2/15 \\ y_3 = 0 \end{array}}$

6.4 $P_1\ P_2\ P_3\ P_4$ = Production in each month
$I_1\ I_2\ I_3\ I_4$ = Inventory after each month

$$\min 11P_1 + 14P_2 + 15P_3 + 16P_4 + 2I_1 + 2I_2 + 2I_3 + 2I_4 + 0.5|P_1 - P_2| + 0.5|P_2 - P_3| + 0.5|P_3 - P_4|$$

$$\downarrow$$

a) $\min 11P_1 + 14P_2 + 15P_3 + 16P_4 + 2I_1 + 2I_2 + 2I_3 + 2I_4 + 0.5a + 0.5b + 0.5c$

s.t. $a \geq P_1 - P_2 \quad a \geq -(P_1 - P_2)$

$b \geq P_2 - P_3 \quad b \geq -(P_2 - P_3)$

$c \geq P_3 - P_4 \quad c \geq -(P_3 - P_4)$

$P_1 = I_1 + 100$

$P_2 + I_1 = I_2 + 150$

$P_3 + I_2 = I_3 + 150$

$P_4 + I_3 = I_4 + 250$

$0 \leq P_1, P_2, P_3, P_4 \leq 200$

$0 \leq I_1, I_2, I_3, I_4 \leq 100$

P_1, P_2, P_3, P_4 = Production in each math
I_1, I_2, I_3, I_4 = Inventory after each month
a = excess from P_2 to P_1
b = excess from P_3 to P_2
c = excess from P_4 to P_3

b) Excel

c) Production:
Month 1: 125 cost: 9512.50
Month 2: 125
Month 3: 200
Month 4: 200
$b = 75$