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Q1:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \mathrm{Cov}(r_i, r_j) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i \cdot w_j \cdot \rho_{ij} \cdot \sigma_i \cdot \sigma_j$$

$$\Rightarrow \sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i \cdot w_j \cdot \rho_{ij} \cdot \sigma_i \cdot \sigma_j}$$

Q2:

$$MCR_i = \frac{\partial r_p}{\partial w_i} = r_i$$

Q3. $$MCV_i = \frac{\partial \sigma_p}{\partial w_i} = \frac{1}{2} \cdot \left(\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \cdot \rho_{ij} \sigma_i \sigma_j\right)^{-\frac{1}{2}} \left(2 w_i \cdot \sigma_i^2 + 2\sum_{j \neq i}^{n} w_j \cdot \rho_{ij} \sigma_i \sigma_j\right)$$

$$= \frac{1}{\sigma_p} \cdot \sum_{j}^{n} w_j \cdot \rho_{ij} \sigma_i \cdot \sigma_j .$$

Q4. MCS_i $$SR_p = \frac{r_p}{\sigma_p} = \frac{\sum w_i \cdot r_i}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i \cdot w_j \cdot \rho_{ij} \sigma_i \sigma_j}}$$

$$MCS_i = \frac{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j} \cdot r_i - \sum w_i \cdot r_i \cdot \frac{1}{\sigma_p} \cdot \sum_{j}^{n} w_j \cdot \rho_{ij} \sigma_i \sigma_j}{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i \cdot w_j \cdot \rho_{ij} \cdot \sigma_i \cdot \sigma_j}$$

$$= \frac{\sigma_p \cdot MCR_i - r_p \cdot MCV_i}{\sigma_p^2}$$

Q5. MCTR.

$$TE_P^2 = \sigma_P^2 + \sigma_{BM}^2 - 2\rho_{P,BM}\cdot\sigma_P\cdot\sigma_{BM}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij}\sigma_i\sigma_j + \sigma_{BM}^2 - 2\rho_{P,BM}\cdot\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij}\sigma_i\sigma_j}\cdot\sigma_{BM}$$

$$\frac{\partial TE_P}{\partial w_i} = \frac{1}{2}\cdot(TE_P)^{-\frac{1}{2}}\left[2\cdot\sum_{j}^{n} w_j\rho_{ij}\sigma_i\sigma_j - 2\rho_{P,BM}\frac{1}{\sigma_P}\cdot\sum_{j}^{n} w_j\rho_{ij}\sigma_i\sigma_j\cdot\sigma_{BM}\right]$$

$$= \frac{1}{TE_P}\cdot\sum_{j}^{n} w_j\cdot\rho_{ij}\cdot\sigma_i\sigma_j\left(1-\rho_{P,BM}\cdot\frac{\sigma_{BM}}{\sigma_P}\right).$$