NUMERICAL OPTIMISATION
ASSIGNMENT 1

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EXERCISE 1. Given the following function

$$f(x, y) = 2x + 4y + x^2 - 2y^2$$

(a) Visualise the function and its contours.
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(b) Calculate the contours analytically.
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(c) Calculate the gradient analytically. Find the stationary points and classify them i.e. are them minima, maxima or something else?
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EXERCISE 2.

(a) Show that $A = B^{\mathrm{T}}B$ is symmetric positive semidefinite for all $B \in \mathbb{R}^{n\times n}$. *Hint: use the Rayleigh quotient representation of the eigenvalue* $Ax = \lambda x$.
Submit your solutions via ***Turnitin****.*

(b) Let $f(x) = x^{\mathrm{T}}Ax$ with A symmetric positive semidefinite matrix $A \in \mathbb{R}^{n\times n}$. Show that $f(x)$ is convex on the domain $\mathbb{R}^n$. *Hint: you may want show the equivalent inequality instead*

$$f(y + \alpha(x - y)) - \alpha f(x) - (1 - \alpha)f(y) \leq 0.$$

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Remark. The submission to `Turnitin` should not be longer than 5 pages. Avoid submitting more code than needed (if any) and focus on explaining your results.