Homework 1

Jim Zieleman

August 28, 2020

1. (a) My name is Jim Zieleman

   (b) Suppose A and B are sets. Then $A \backslash (A \backslash B) = A \cap B$.

   (c) Suppose A and B are subsets of a set X. Then $(A \cup B)^c = A^c \cap B^c$.

   (d) Suppose A and B are subsets of a set X. Then $(A \cap B)^c = A^c \cup B^c$.

   (e) Suppose $\{A_i\}_{i \in I}$ is a collection of subsets of a set X. Then

   $$\left(\bigcup_{i \in I} A_i \right)^c = \bigcap_{i \in I} A_i^c.$$

   (f) Suppose $\{A_i\}_{i \in I}$ is a collection of subsets of a set X. Then

   $$\left(\bigcap_{i \in I} A_i \right)^c = \bigcup_{i \in I} A_i^c.$$

2. (b) Suppose A and B are sets. Then $A \backslash (A \backslash B) = A \cap B$.
   Proof:
   Consider $A \backslash (A \backslash B)$:
   We can see that $(A \backslash B) = A - B = A - (A \cap B)$.
   Then $A \backslash (A \backslash B) = A - (A - (A \cap B))$.
   Then $= A - A + (A \cap B)$.
   Then $= A \cap B$.
   Thus $A \backslash (A \backslash B) = A \cap B$.

   (c) Suppose A and B are subsets of a set X. Then $(A \cup B)^c = A^c \cap B^c$.
   Proof:
   Consider $(A \cup B)^c$:
   Let $x \in (A \cup B)^c$.
   Then $x \notin (A \cup B)$.
   Then $x \notin A$ and $x \notin B$.
   Then $x \in A^c$ and $x \in B^c$.
   Then $x \in A^c \cap B^c$.
   Thus $(A \cup B)^c = A^c \cap B^c$ since $x \in (A \cup B)^c$ and $x \in A^c \cap B^c$.

3. Let $A_n = \{x|0 < x < 1/n\}$ or $(0, 1/n)$.
   Then $\forall m > n, A_m \subset A_n$.

   For example consider $2 > 1$, such that $A_2 = (0, 1/2)$ and $A_1 = (0, 1)$.
   Then clearly $A_2 \subset A_1$.

   So with our definition of A_n we have: $A_1 \cap A_2 \cap ... \cap A_n = A_n = (0, 1/n) = \emptyset$
   However, $A_1 \cap A_2 \cap ... = \lim_{n\to\infty} A_n = \{x|0 < x < 0\} = \emptyset$