

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/282469388>

A mixed integer programming model for solving real-time truck-to-door assignment and scheduling problem at cross docking warehouse

Article in *Journal of Industrial and Management Optimization* · June 2015

DOI: 10.3934/jimo.2016.12.431

CITATIONS

3

READS

1,687

3 authors:



Rosma Binti Mohd Dom

Universiti Teknologi MARA

20 PUBLICATIONS 98 CITATIONS

[SEE PROFILE](#)



Adibah Shuib

Universiti Teknologi MARA

42 PUBLICATIONS 202 CITATIONS

[SEE PROFILE](#)



Wan Nor Ashikin Wan Ahmad Fatthi

Universiti Teknologi MARA

6 PUBLICATIONS 44 CITATIONS

[SEE PROFILE](#)

A MIXED INTEGER PROGRAMMING MODEL FOR SOLVING REAL-TIME TRUCK-TO-DOOR ASSIGNMENT AND SCHEDULING PROBLEM AT CROSS DOCKING WAREHOUSE

WAN NOR ASHIKIN WAN AHMAD FATTHI

Department of Computer and Mathematical Sciences, Universiti Teknologi MARA
72000, Kuala Pilah, Negeri Sembilan, Malaysia

ADIBAH SHUIB AND ROSMA MOHD DOM

Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA
40450, Shah Alam, Selangor, Malaysia

(Communicated by Victor Sreeram)

ABSTRACT. This paper address the problem at the inbound phase at the cross docking warehouse. The fundamental issues in cross docking facility is to assign the incoming truck to the door and to coordinate the sequences of the trucks in order to minimize the completion time at the inbound phase. Using the theories and methodologies of assignment and scheduling, paper on hand proposed a mixed integer programming model for solving the truck-to-door assignment and scheduling problem with the objective to minimize the total service time of trucks. Meanwhile, reduce the waiting time of trucks before being served at the designated door. A preliminary computation is conducted to verify the logic of the mathematical model proposed.

1. Introduction. In supply chain activities, moving consumer goods quickly and cost efficiently has been the main targets of many manufacturers, retailers, distribution centres and logistics service providers. Due to this reason, an increasing number of organizations are finding that cross docking practice is a promising cost-saving alternative to the traditional warehousing in order to lessen the operating costs and to fulfil customer's demand especially for the retail products [25]. In contrast to the traditional practise, cross docking practise offers direct transshipment of goods from incoming trucks to the outgoing trucks without any storage or with just temporary storage in between. Usually, the shipments will spend less than 24 hours in a cross docking facility or sometimes less than one hour [35].

In a cross docking model, customer is known before the goods get to the warehouse. The primary purpose of a cross docking is to enable a consolidation of shipments from different suppliers with the same destination. Thus, shipments are sent in the full truck load within the same day. As there is no inventory in storage due to direct transshipment, cross docking has been a potential logistic technique in order to reduce the inventory holding cost, order picking cost, transportation cost and delivery time [4, 33]. The efficiency of cross docking will influence the lead time, inventory level and response time to the customer [21]. In fact, cross docking

2010 *Mathematics Subject Classification.* Primary: 58F15, 58F17; Secondary: 53C35.

Key words and phrases. Cross docking, assignment, real-time, scheduling, mixed integer programming.

system has been successfully applied in many industries and several famous companies such as Wal-Mart, FedEx Freight, Toyota, Goodyear GB Ltd and Kodak Co. [9, 33, 12].

The scientific literature on cross docking can be grouped into three categories of decision level which are strategic, tactical and operational level. For strategic level, the decision made is usually associated for the long time horizon and addresses the problem of cross docking network design (see [20, 6]). On the other hand, in the tactical decision level of cross docking planning, the planning horizon of the decision is the mid-term horizon that mainly discussed the problem of cross docking layout design [7, 16, 38]. Aside from that, operational decision levels cater the real time control of the cross docking platform and hence the decision can be modified on a real time basis (see [5, 9, 28, 2, 8, 23, 29]).

Truck to door assignment and scheduling have become the important decisions in cross docking terminal as it dictates two third of the main daily operations of the cross docking facility which are the inbound and outbound operations. Appropriate coordination of inbound and outbound trucks will ensure the smoothness of operations within the facility. Poor truck scheduling can lead to congestion, poor product flow and long processing time which may increase the cost [1]. In cross docking facility, an inbound truck is either assigned directly to any available receiving door or has to wait in queue (if all doors are busy). If inbound trucks are not being handled systematically, it will cause a delay on the products transshipment due to prolonged waiting time. Thus, it is necessary to determine a truck scheduling policy to assign inbound trucks to inbound doors and outbound trucks to outbound doors [15].

When a supplier truck arrives at the facility, the respective operation manager faces two interrelated decisions which are; where and when the truck should be assigned to the dock [9]. The current practice is to estimate unloading time of trucks based on intuition and experience and assign trucks to any available dock doors based on this estimation upon arrival [14]. However, inbound operations need to be systematically handled so that trucks can be assigned to the fastest available dock door to avoid congestions and long-hour delays waiting in queues. Besides, when there are a lot of supplier trucks to be served at inbound stage, an improper scheduling decision will risk the operation management to have extra operating hours in order to complete all outbound shipments. As cross docking is a real time operation [33], an approach to deal with dynamic environment is very crucial.

In this paper, a generic mathematical model is proposed to solve the real-time truck-to-door assignment and scheduling problem of the inbound operation at retail cross docking warehouse. Mixed integer programming (MIP) is employed for the model formulation with the objective to minimize total service time of truck. The remainder of this paper is organized as follows: Section 2 provides a brief literature review on MIP concept and its application in cross docking scheduling problem. Section 3 describes the methodology of model development. Section 4 presents the preliminary computation of the model, whilst, concluding remarks and contribution are highlighted in Section 5. Finally, future works are given in Section 6.

2. Literature Review. Decision making process in today's complex environment is getting challenging. Thus, it has been a trend in science and engineering for reducing the complex real world system into meticulous mathematical models. Linear programming, for example, is frequently applied in Operations Research (OR)

method for modelling the real-world decision making problem. In many practical problems, it is common for the decision variables to have integer values. For example, numbers of people, machines, vehicles and activities have to be assigned integer quantities. For a cross docking situation, number of trucks, doors, purchase orders, boxes, items, forklifts or workers are represented as integers.

If integer values are expected as outcome, the model is called an Integer Programming (IP) model. Somehow, if only some of the variables are required to have integer values, the model is referred to as mixed integer programming (MIP) [19]. In other words, an MIP model is a mathematical program with constraints in which only specified subsets of the variables are required to take on integer values [18]. Due to the flexibility and massive modelling capability, MIP has become one of the famous and widely explored methods for scheduling problem. Pochet and Wolsey [31] defined MIP as:

$$Z(X) = \min_{(x,y)} \{cx + fy : (x,y) \in X\},$$

where the set X is called the set of feasible solutions and is described by m linear constraints, nonnegativity constraints on the x, y variables, and integrality restrictions on the y variables. In matrix notation,

$$X = \{(x,y) \in R_+^n \times Z_+^p : Ax + By \geq b\}$$

where,

- $Z(X)$ denotes the optimal objective value when the optimization is performed over the feasible set X .
- x and y denote, respectively, the n -dimensional (column) vector of non-negative continuous variables and the p -dimensional (column) vector of nonnegative integer variables.
- $c \in R^n$ and $f \in R^p$ are the (row) vectors of objective coefficients.
- $b \in R^m$ is the (column) vector of right-hand side coefficients of m the constraints.
- A and B are the matrices of constraints with real coefficients of dimensions $(m \times n)$ and $(m \times p)$, respectively.

In simple form, MIP can be represented as follows:

$$\begin{aligned} & \text{Minimize } f(x,y) \\ & \text{subject to:} \\ & \quad g(x,y) \leq 0 \\ & \quad x \in X \\ & \quad y \in Y \text{ integer} \end{aligned}$$

where function $f(x,y)$ is the objective function and $g(x,y)$ is the constraint function. Variables x, y are the decision variables, where y is required to be the integer values. X and Y are bounding-box-type restrictions on the variables.

When variables in an MIP involve only two possible choices, either yes or no, the decision can be denoted by just two values, say zero or one. Such variables are called binary variables. For truck scheduling, when each inbound and outbound truck arrives at the cross docking facility, respective manager has to decide on which door the truck should be assigned for unloading or loading of the shipments. The assignment of truck to door can be represented in yes or no decision or the respective

variables assume binary integer values of 1 or 0. If i denotes a truck and j denotes a door, then,

$$x_{ij} = \begin{cases} 1 & \text{if truck } i \text{ is assigned to door } j \text{ where } i = 1, 2, \dots, m; j = 1, 2, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

where m is the number of trucks and n is the number of dock doors.

In MIP, some of the variables can have quantities to be produced as continuous values or real numbers. In a truck-to-door assignment and scheduling model, some factors of the model involve time and represented as a continuous value (hour or minutes). For examples, the arrival time of truck, the estimated unloading time and total service time for a truck are denoted in minutes or hours, known as the continuous values. Thus, a model is termed as an MIP model if the decision variables consist of pure or binary integer variables with some continuous variables.

Thus far, from our review, mathematical models developed for the scheduling problem of cross docking varies and some of them are the expansion or extension of previous models. Most of the models differ based on the operational issues or factors that are being addressed. An example of the application of the MIP in solving cross docking scheduling problem is the model proposed by Yu and Egbelu [39]. With the objective of minimizing the makespan, the model proposed consists of three decision variables (binary variables) pertaining to products transferring and truck sequencing in 0, 1 integer. Other integer variables include number of trucks or doors and number of product types in the set. The total makespan and time at which inbound/outbound truck enters or leaves the receiving/shipping dock are in continuous values. Model proposed by [39] assumed that unloading/loading time from an inbound/outbound truck are the same for all product types and it take one unit of time (in minute) for a product. All trucks are assumed to be available at time zero. Although their model has always been used as reference, the model is only applicable for an automated cross docking terminal which uses a system of conveyer.

By joining the truck scheduling and truck-to-door assignment issues together, Shakeri et al. [33], proposed a mixed integer programming model to address these two problems concurrently. Exploiting multiple stage hybrid flowshop scheduling, the model comprises of two stages where the first stage involves unloading operation while the second stage engages in the loading operation. In the mathematical model proposed, parameters such as doors, trucks and assignment decision take the value of discrete variables while parameters related to time factor are represented by continuous variables. Nevertheless, the model proposed is only applicable for the cross docking centre with mixed service mode (same dock doors are used for unloading and loading), where the product mixing and matching is assumed to be already done by the supplier. Besides that, the model proposed did not show clearly on how the intermediate storage cost is included for the formulation. Similar to [39], mathematical model proposed in [33] also assumed that all trucks are available before the operation starts.

Chen and Lee [11] proposed an MIP model of two-stage hybrid cross docking scheduling model with the objective on minimizing makespan. The two stages in their problem refer to the inbound and outbound operations, with at least one stage contain more than one parallel machine (vehicles). Parameters involved in the mathematical formulation are almost similar like that proposed by [33], but the model considered different constraints with respect to the model assumptions. In

the model developed by Chen and Song, the products are assumed to be sorted and consolidated immediately after arrivals. Thus, the internal processing time is not involved in the model. The study also assumed that all trucks should be readily available in the system before the starts of schedule.

Instead of modelling the truck scheduling as a detail scheduling problem, Boysen et al. [9] treat the problem from aggregate view by merging the handling time of the products into time service slots. The problem is formulated as a MIP model and solved by using decomposition approach. Model proposed by Boysen et al. assumed that all input data are known in advance and no predefined restrictions on truck assignments such as truck's release or due date. In their mixed integer formulation, movement time is denoted in continuous values while the rest parameters such as quantity of products, number of slots and binary variable are indicated in discrete numbers. However, the model proposed seems as not practical in the real world application as it only focuses for single receiving and single shipping door.

With the objectives to minimize the service and storage time at a cross docking terminal, Golias et al. [15] develop an MIP model for assigning the incoming and outgoing trucks to the inbound and outbound doors. The model considered a detail scheduling policy and presupposed that the truck handling time is not only dependent on the quantity of cargo unloaded and loaded but also on the truck-to-door assignment. In their model, four decision variables are formulated pertaining to door and truck's assignment while parameters relating to time measured such as unloading/loading time, arrival time of trucks and moving time between door are modelled as continuous positive variables.

Lee et al. [24] have put forward a MIP model with the objective to maximize the number of freights that are able to be shipped within working period. In addition, the scheduling decision aims to decide where and when the inbound and outbound trucks should be processed at the multi-door cross docking center. Meanwhile, model developed has also determined the products assignment, door assignment and docking sequences for all inbound trucks and outbound trucks concurrently. The decision variables caters by the model proposed by [24] is almost similar with the model by [39], which are mainly focuses on the coordination and sequences of both inbound and outbound trucks. As the objective of study concerns on the working period, the authors have incorporated the maximum working time as one of the parameter of the MIP model as well.

Van Belle et al. [37] have formulated the scheduling problem of inbound and outbound trucks by using MIP. In their model, six continuous decisions variables are defined concerning on the start/end time of inbound/outbound trucks and the tardiness of both trucks. Aside from that, the authors have also considered the weighting factor for the total travel time and total tardiness as parameters of the model developed. As sufficient manpower and equipment are assumed to be sufficient and available in the mathematical model proposed by [37], the end time of each inbound truck is calculated as the total of the start time and the time taken to unload the products.

3. Methodology. Development of model is comprised of two stages, which the first is the application of fuzzy logic approach to estimate truck's unloading time whereas the second is the development of mixed integer programming model which employs the estimated unloading time for the assignment decision and scheduling strategy. We have presents the methodology of fuzzy logic approach for truck's unloading

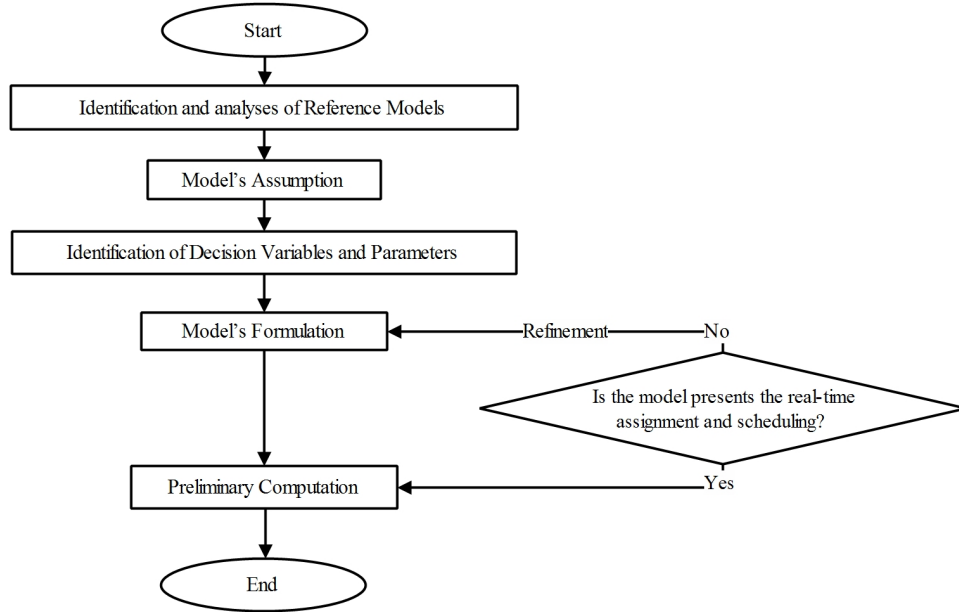


FIGURE 1: Methodology for MIP Modelling

time estimation in [14]. Hence, in this paper, the methodology for developing a MIP model will be addressed. There are four steps involved for the MIP modelling in this study as shown in Figure 1.

Step 1: Identification and analyses of Reference Model

A Mixed Integer Programming model proposed by Yu and Egbelu [39] and Golias et al. [15] are selected as the main reference models as these models has contribute significant advantages to the cross docking operation. The reference models are carefully studied to gain an understanding and ideas on how the trucks assignment and scheduling concepts are applied in formulating the MIP model of the problem in this study. Model proposed by [39] concerned with the total operation time of cross docking facility by tackling the trucks sequencing problem, known as a key factor to the succession of cross docking management. In addition, the authors also emphasized the issue of product assignment concurrently with the truck scheduling strategy. By integrating truck-to-door assignment to the scheduling planning, [15] have put forward an approach for scheduling planning with two precise objectives which is less adhere in the existing studies. Furthermore, model by [15] is practical to be implemented in the real cross docking application due to the multiple numbers of doors considered. Similar to Yu and Egbelu, model by Golias et al. also highlights the arrangement of trucks at both doors.

Note that, this study is focuses on the truck-to-door assignment and scheduling problem specifically for the inbound operation of cross docking facility. In contrast to the problems solved by [39] and [15] that focuses on the synchronization of the incoming and outgoing trucks, this study aims on solving the problem at the inbound phase of cross docking operation. The focus is to optimize the completion time of inbound stage which can be achieved by minimizing the total service time of the incoming truck. The intended problem is motivated from the current issue of industry that practises a real application of cross docking technique. Aside from

that, gap analysis presented from academic researchers provided the interest in this study on solving the real time problem of cross docking technique. Although the problem to be solved in this study is not similar to [39] and [15] certain variables and model's constraints that can comply with the scenario of inbound' operation were adapted. Some examples are as the following:

1. Number of arrival trucks.
2. Arrival time of truck.
3. Start time of truck at the designated door.
4. Finish time of truck at the designated door.
5. Unloading time.
6. Binary decision variable for truck-to-door assignment.

However, in order to cater with dynamic environment of application, the variables such as number of incoming truck, truck's arrival time, truck's unloading time, the starts and end time of serviced were defined and calculated as a real time value without prior information. In this study, only three parameters were assumed to be known namely the operation's begin time, operation's end time and the number of receiving door.

Step 2: Model's Assumption

Various models may be defined for the scheduling problem depending on the facility's setting, management and policy. Therefore, it is essential to outlined related assumptions for the model in order to present the intended problem to be solved. Aside from that, assumptions are considered to guide and coordinate the scheduling process systematically. In this study, the proposed problem is the scheduling problem for retail cross docking warehouse with multiple receiving doors. The interest is on the inbound operation where the real-time scheduling and assignment problems are catered. In developing the real-time mathematical model of this problem, few assumptions have been outlined as the following:

1. The exact information of the freight is known upon arrival.
2. Inbound trucks arrive dynamically over time.
3. Unloading time for each truck differs based on the freight characteristics.
4. Model follows First-Come-First-Serve (FCFS) queuing rule.
5. No pre-emption (interruption) is allowed when a truck is being served.
6. Processing time to assign the truck to the receiving door (or lane) is negligible.
7. Model will assign the incoming truck to the fastest door available.
8. If door j and door $j + n$ are available at the same time, priority is given to door j
9. The start time of service for the next truck at designated door is immediately after the departure of currently serviced truck (if there is a queue).

Assumptions 1 to assumption 2 presents the real-time environment of the scheduling problem to be solved where the information regarding the truck including the freight and arrival time is not known beforehand. As different truck presents different number of purchase order, variation of items and different number of boxes carried, thus, the time taken for unloading the freight from truck is also differ from one another as defined by Assumption 3. We have introduced the estimation of unloading time based on Fuzzy Logic approach in [14]. Assumption 4 guarantees fairness for all trucks. Assumption 5 is outlined due to the type of operation offered by the facility itself which is single-mode of service, means, each incoming truck will

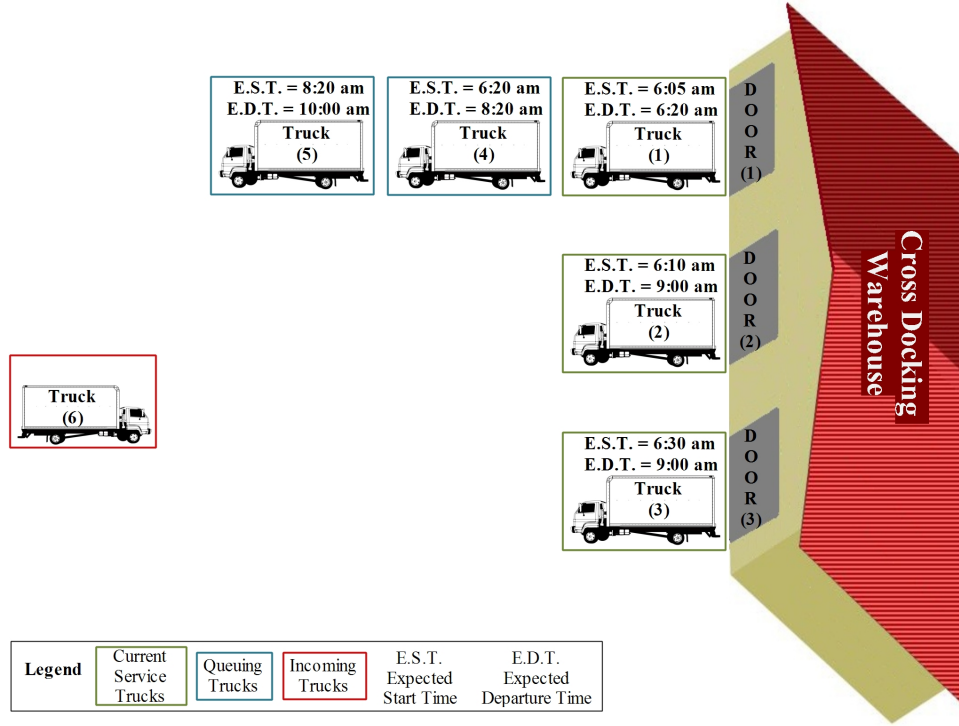


FIGURE 2: Scenario of Inbound Cross Docking Operation

load the freight carried at the facility until the truck is empty (not exchanging the freight between trucks).

In Assumption 6, processing time to assign the truck to the receiving door (or lane) is negligible because the time taken to assign the truck to door is too short (less than 10 seconds). Aside from that, in order to minimize the waiting time of truck before being served to the designated door, Assumption 7 is listed. For the case where the two or more doors are available at the same time, truck will be assigned to the least number of doors. For example, if door number two and door number three are available at the same time, the incoming truck will be assigned to door number two. This is presented by Assumption 8 with the objective to facilitate the assignment decision and tracking of the truck. Finally, in Assumption 9, the mathematical model has neglects the exchange time between current serviced truck with the next truck to be served as the time taken for the exchanged process is too short (less than 15 seconds). Figure 2 illustrate the example of scenario based on the model's assumption of this study. Scenario presented is the inbound operation of cross docking warehouse with 3 receiving doors. The doors are labelled as Door 1, Door 2, and Door 3 in Figure 2. Consider that an operational hour of the warehouse begins at 6 a.m. where trucks arrive dynamically over time at the designated lane (after truck been assigned).

In Figure 2, truck number is denoted as Truck (n) where . For instance, Truck (1) arrives at the assigned lane at 6.05 a.m. As Truck (1) is the first truck enters to the schedule, it is assigned to Door (1) (follows Assumption 8) with expected start time at 6.05 a.m. Consider that an estimated unloading time of Truck (1)

based on fuzzy logic approach is 15 minutes, thus, the expected departure time of Truck (1) will be at 6.20 a.m. Follows Assumption 4 with First Come First Serve rule, Truck (2) which come earlier than Truck (3) is assigned first for service. Figure 2 shows that Truck (2) and Truck (3) arrive at their receiving door at 6.10 a.m. and 6.30 a.m. respectively. Consider that Truck (2) takes 2 hours and 50 minutes of estimated time to unload the freight whilst Truck (3) takes 2 hours 30 minutes of estimated unloading time. Therefore, truck (2) and Truck (3) will finish serviced at the designated door and depart at the same time which is 9 a.m. Note that, according to Assumption 5, all trucks that are currently in serviced are not allowed to leave or move from the door until freight are completely unloaded. As all receiving doors now are busy (all trucks are served currently at the doors), an assignment decision of the later truck which is Truck (4) employs Assumption 7 upon its arrival. Figure 2 indicates that the fastest door available is Door (1) with an expected departure time of current served truck, Truck (1) at 6.20 a.m. as compared to Door (2) and Door (3) which is at 9 a.m. respectively. Therefore, Truck (4) is assigned to Door 1 and is queuing in front of Truck (1) before being served at 6.20 a.m, (an expected start time of truck follows Assumption 9). Figure 2 depicts that Truck (5) is queuing on the lane of Door (1). This is due to fastest door available for serviced which is 8.20 a.m. (see expected departure time of truck number 4 as compared to expected departure time of truck number 2 and truck number 3 in Figure 2). As Door (2) and Door (3) are available at the same time, an assignment decision for the later truck that arrives to the schedule will follows Assumption 8. For example, an incoming Truck (6) will be assigned to the least number of doors which is Door (2).

Step 3: Identification of Decision Variables and Parameters

Since the truck-to-door assignment and scheduling model developed is a real time model, only three parameters are assumed to be known and fixed, which are as the following:

- B : Operation's begin time
- E : Operation's end time
- N_d : Total number of doors

In describing the problem, following notations are defined:

- TT : Total service time for all trucks
- N_t : Total number of inbound trucks
- N_d : Total number of inbound doors
- ST_i : Total service time for truck i where $i = 1, 2, \dots, N_t$
- U_i : Unloading time of truck i where $i = 1, 2, \dots, N_t$
- W_i : Total waiting time for truck i before being serviced
- W_{i_Q} : Queuing time for truck i before being serviced at designated door
- W_{i_R} : Remaining service time of the truck currently serviced at designated door for truck i
- p : Last position of truck in queue before truck i at designated door
- k : k : Index for position in queue for door j where $k = 0$ means the truck is currently unloaded at door j
- $X_{j,kt}$: $\begin{cases} 1, & \text{if truck } t \text{ is at position } k \text{ of queue of door } j \text{ assigned for truck } i \\ 0 & \text{otherwise} \end{cases}$

U_t	: Unloading time of queuing truck t at designated door where $t = 1, 2, \dots, N_t$
i, t	: Index for truck i
J_j	: $\begin{cases} 1, & \text{if door } j \text{ is currently serving a truck} \\ 0 & \text{otherwise} \end{cases}$
$f_{n_j,0}$: Expected finish time of currently serviced truck at door j assigned for truck i where $i = 1, 2, \dots, N_t$ and $j = 1, 2, \dots, N_d$
$t_{j_i,0}$: Current time of currently serviced truck at door j where $j = 1, 2, \dots, N_d$
j_i	: Door assigned for truck i where $j_i = 1, 2, \dots, N_d$
n_j	: Truck currently serviced at door j where $j = 1, 2, \dots, N_d$
A_{ij}	: Arrival time for an inbound truck i at queue for door j where $i = 1, 2, \dots, N_t$ and $j = 1, 2, \dots, N_d$
B	: Operation's begin time
D_{ij}	: Expected departure time of truck i from door j immediately after service completed where $i = 1, 2, \dots, N_t$ and $j = 1, 2, \dots, N_d$
E	: Operation's end time
F_j	: Expected earliest time door j will be available where $j = 1, 2, \dots, N_d$
$s_{n_j,0}$: Start time of currently serviced truck at door j where $j = 1, 2, \dots, N_d$
U_{n_j}	: Unloading time of currently serviced truck at door j where $j = 1, 2, \dots, N_d$
S_{ij}	: Start time of truck i at door j
X_{ij}	: $\begin{cases} 1, & \text{if truck } i \text{ is in queue } j \\ 0 & \text{otherwise} \end{cases}$

Step 4: Model Formulation

With the help of listed notation, the problem of truck-to-door assignment and real-time scheduling problem is formulated as a MIP model. The details of formulation and description will be discussed in the next section.

Step 5: Preliminary Computation

Preliminary computation is conducted to verify the logic of the model. For more details, readers are referred to Section 5.

4. Problem Description and Mathematical Model. Paper on hand is interested on real-time inbound operation with multiple inbound doors and multiple inbound trucks. N_d and N_t will denote the number of inbound doors and inbound trucks, respectively. Each inbound truck arrives dynamically over time after operation's begin time, B and each truck contains shipments for several destinations. Upon arrival at the cross docking warehouse, each truck is required to undergo registration process and its unloading time, denote U_i will be estimated based on the number of purchase order documents, number of items listed in the document and total number of boxes carried per truck as proposed in [14]. Unloading time in this study is defined as the total time taken by truck for transferring freight from truck to floor, scanning and checking the correctness of freight until the truck move away from receiving door. Truck will be directly assigned to the available door for service or have to wait in queue (in lane) in front of the designated assigned door if the

door is occupied. The objective of the model is to minimize total service time of truck at the inbound door. Total service time is defined as the sum of the waiting time taken by truck plus with total waiting time before served and total unloading time at the designated door.

With the help of notation listed in Section 3, the truck-to-door assignment and real-time scheduling problem can be formulated as follows:

$$\text{Minimize} \quad TT = \sum_{i=1}^{N_t} ST_i \quad (1.1)$$

Subject to:

$$A_{ij} \geq B \quad \text{for } i = 1, 2, \dots, N_d \quad (1.2)$$

$$D_{ij} \leq E \quad \text{for } j = 1, 2, \dots, N_d \quad (1.3)$$

$$A_{ij} = \min \{F_j\} \quad \text{for } j = 1, 2, \dots, N_d \quad (1.4)$$

$$F_j = f_{n_j,0} + \sum_{k=1}^p U_k \quad \text{for } j = 1, 2, \dots, N_d, k = 1, 2, \dots, p \quad (1.5)$$

$$f_{n_j,0} = s_{n_j,0} + U_{n_j} \quad \text{for } j = 1, 2, \dots, N_d \quad (1.6)$$

$$D_{ij} = S_{ij} + U_i + \sum_{k=1}^p U_k + \quad \text{for } i = 1, 2, \dots, N_t, j = 1, 2, \dots, N_d, k = 1, 2, \dots, p \quad (1.7)$$

$$(f_{n_j,0} - t_{n_j,0}) \\ S_{ij} = D_{ij} - U_i \quad \text{for } i = 1, 2, \dots, N_t, j = 1, 2, \dots, N_d \quad (1.8)$$

$$\sum_{i=1}^{N_t} \sum_{j=1}^{N_d} X_{ij} = 1 \quad \text{for } i = 1, 2, \dots, N_t, j = 1, 2, \dots, N_d \quad (1.9)$$

The objective function of the MIP model is to minimize $TT = \sum_{i=1}^{N_t} ST_i$ as presented by equation (1.1), where U_i is the unloading time of truck i where $i = 1, 2, \dots, N_t$ and N_t is the total number of inbound trucks. The variable ST_i can be described further as the following:

$$ST_i = U_i + W_i \quad \text{for } i = 1, 2, \dots, N_t \quad (1.10)$$

$$W_i = W_{i_Q} + W_{i_R} \quad \text{for } i, t = 1, 2, \dots, N_t \quad (1.11)$$

$$W_{i_Q} = \sum_{k=1}^p \sum_{t=1}^{N_t} (X_{jikt})(U_t) \quad \text{for } i, t = 1, 2, \dots, N_t \text{ and } j_i = 1, 2, \dots, N_d, t \neq i \quad (1.12)$$

$$W_{i_R} = f_{n_j,0} - t_{n_j,0} \quad \text{for } i = 1, 2, \dots, N_t \text{ and } j = 1, 2, \dots, N_d \quad (1.13)$$

where, ST_i is the total service time for truck i , W_i is the total waiting time for truck i before being serviced, W_{i_Q} is the queuing time for i truck before being serviced at designated door, and W_{i_R} is the remaining service time of the truck currently serviced at designated door for truck i . The objective function given by equation (1.1) aims at minimizing the total service time for all trucks, TT where TT is the accumulated total of ST_i , the sum of truck's unloading time and waiting time of truck before being serviced, as stated in (1.10). As given by the equation (1.11), the truck's waiting time is obtained by adding the queuing time for i truck before being serviced at designated door, W_{i_Q} and the remaining time of currently serviced truck at the respective dock door to finish, W_{i_R} . Equation (1.12) describes that W_{i_Q} is calculated by adding the unloading time of trucks currently queuing at the lane for the respective dock door. W_{i_R} , as stated by (1.13), is derived

TABLE 1: Test Cases for the Proposed MIP Model

Case	Number of Doors	Number of Trucks
1	3	20
2	20	68
3	30	100

by subtracting the current time from the expected finish time of service for truck currently unloaded at that door.

Constraint (1.2) restricts the arrival time of any truck i to queue for door j to be on or after the operations begin time. Constraint (1.3) sets the finish time of service at any door to be before the operation's end time. Constraint (1.4) specifies the arrival time of any inbound truck to the respective queue at the assigned door j is determined by the earliest time a door is becoming available. As given by constraint (1.5), the expected earliest time for a door to be available, F_j where $j_i = 1, 2, \dots, N_d$ is calculated by adding the time remaining for a particular truck to finish unloading at the dock door and the total unloading time of all trucks currently queuing at the lane for that dock door. Constraint (1.6) specifies that the finish time of current serviced truck at its assigned door is calculated based on its start time and associated unloading time. Constraint (1.7) enforced that the truck has to depart from door j immediately after its inbound process completed. Thus, the departure time of truck i from the door j is equal to the sum of the truck's arrival time, its unloading time and its waiting time. Constraint (1.8) sets the expected start time for unloading of truck i at door j that has been assigned to it as the expected departure time of truck i from that door minus its unloading time. Constraint (1.9) ensures that an inbound truck is assigned to exactly one door (thus, served by one door only).

5. Preliminary Computation. A preliminary computation to verify the logic and process flow of the proposed real-time MIP model for the truck-to-door assignment and scheduling was performed by using Microsoft Excel 2010 spreadsheet. The pilot computation of real-time scheduling model was conducted for three test cases as indicates in Table 1. Table 2 and Table 3 presents an example of the detail calculation performed in the Excel spreadsheet for Case 1 and Case 2 respectively.

In Table 2, the notations for parameters and variables are given in the second row while their brief descriptions are presented in the first row. The first column shows the index of incoming truck, whereas, the second column displays the truck's estimated unloading time based on the fuzzy logic approach introduced in our previous work. The third column shows the expected arrival time of the truck at the cross docking facility. The fourth, fifth and six columns present the expected earliest available time of all three doors 1, 2 and 3, respectively. The minimum earliest available times of doors was calculated and displayed in the seventh column.

Receiving door (lane) assigned for particular truck is highlighted in the eighth column. The ninth column indicates whether the assigned door is currently 'busy' (means there is a truck currently serviced at the respective receiving door, labelled as 1) or 'free or available' (labelled as 0). Hence, an index number of the current serviced truck is identified and displayed in the tenth column. Column eleventh to column fourteen functioned to keep track on the expected start/finish time of current serviced truck and the queuing trucks.

Column fifteenth of Table 2 indicates the waiting time of newly assigned truck i due to the trucks queuing at receiving door j . Total waiting time of queue in this case was calculated based on the unloading time of trucks queuing in front of the newly assigned truck i (if there is a queue at door lane, otherwise it will be denoted as zero). Column sixteenth shows the remaining service time for the truck currently serviced at dock door j . The remaining waiting time for truck i before being serviced is shown in the seventeenth column. In column eighteenth, the expected start time of truck i at dock door j is determined based on the arrival time of truck i plus waiting time to be encountered by truck i before being serviced. Thus, an expected departure time of newly assigned truck i can be predicted based on this start time plus the estimated unloading time of truck. Last column of Table 2 represent the total service time of truck at its assigned door. Total service time is the summation of the waiting time of a truck at the assigned door and its estimated unloading time.

Based on the calculation in Table 2, it is identified that the average waiting time per truck is 62.12 minutes, whereas the last truck's departure time is 9 p.m. The preliminary computation for the case of 3 doors and 20 trucks also shows that the average service time per truck is 203.1 minute. On the other hand, preliminary computation for twenty receiving doors and sixty-eight incoming trucks indicates that average waiting time per truck taken based on the proposed MIP model is 42.46 minutes. Since there is high number of the incoming trucks, the last truck departure's time is recorded at 6.11 p.m. Meanwhile, the Excel computation shows that the average service time per truck is 181.3 minutes.

6. Concluding Remarks and Contribution. The inbound operation is one of the important components in cross docking operations. Any delay or inefficiency at the inbound phase will subsequently affect ensuing processes namely the internal process and outbound operations. The latter processes will not be able to be completed if all inbound shipments are not yet finalised. The given description implies the importance of managing the inbound operations effectively. This paper proposed a Mixed Integer Programming model for real-time truck-to-door assignment and scheduling of the inbound phase at the cross docking warehouse. The objective of the model is to minimize total service time of trucks at the inbound door, hence reduces the waiting time of truck at the warehouse. The issue of where, when and which receiving door the truck will be assigned are addressed concurrently in this paper. The calculations performed in the Excel worksheet for some cases served as means for preliminary verification of the logic of the developed MIP model.

As compared to previous studies such as in [39, 33, 11, 15] that treated the unloading time as a constant unit of time based on the quantity of products (one unit of time for one unit of product), the unloading time in this study is presented as an estimated value based on several input factors using fuzzy logic approach (see [14]). The results of this method pertaining to the unloading range and values have been approved by experts from the cross docking warehouse. The estimated unloading time of truck is a valuable information because it enables a decision maker to estimate a truck's service time (waiting time plus the unloading time), in which the waiting time is calculated based on estimated service time of earlier trucks. Assignment of a truck to door can be done systematically upon completion of registration since the fastest available dock door can be determined ahead of time.

TABLE 2: Preliminary Computation to Verify the Proposed MIP Model

i	U_i	A_{ij}	F_1	F_2	F_3	$\min F_j$	j_i	J_j	n_j	S_{n_j}	f_{n_j}	p	k	W_{iQ}	W_{iR}	W_i	S_{ij}	D_{ij}	ST_i
1	200	370	370	370	370	370	1	0	NA	NA	NA	NA	0	0	0	0	370	570	200
2	20	390	570	370	370	370	2	0	NA	NA	NA	NA	0	0	0	0	390	410	20
3	90	400	570	410	370	370	3	0	NA	NA	NA	NA	0	0	0	0	400	490	90
4	80	405	570	410	490	410	2	1	2	390	410	0	1	0	5	5	410	490	85
.
17	40	850	910	1210	910	910	1	1	14	670	910	0	1	0	60	60	910	950	100
18	600	900	950	1210	910	910	3	1	16	870	910	0	1	0	10	10	910	1510	610
19	120	900	950	1210	1510	950	1	1	17	910	950	0	1	0	50	50	950	1070	170
20	240	1050	1070	1210	1510	1070	1	1	19	950	1070	0	1	0	20	20	1070	1310	260
Average Waiting Time Per Truck: 62.12 Minutes																			
Last Truck Departure's Time : 9 Pm																			
Average Service Time Per Truck : 203.1 Minutes																			

TABLE 3: Computational Result for 68 Trucks and 20 Doors

i	U_i	A_{ij}	F_1	F_{20}	$\min F_j$	j_i	J_j	n_j	S_{n_j}	f_{n_j}	p	k	W_{iQ}	W_{iR}	W_i	S_{ij}	D_{ij}	ST_i
1	78	364	360	360	360	1	0	NA	NA	NA	NA	0	0	0	0	364	442	78
2	248	365	442	360	360	2	0	NA	NA	NA	NA	0	0	0	0	365	613	248
3	60	366	442	360	360	3	0	NA	NA	NA	NA	0	0	0	0	366	426	60
4	161	385	442	360	360	4	0	NA	NA	NA	NA	0	0	0	0	385	546	161
.
.
65	64	848	825	856	781	4	0	0	0	0	0	0	0	0	0	848	912	64
66	196	855	825	912	783	13	0	0	0	0	0	0	0	0	0	855	1051	196
67	186	895	825	1051	785	10	0	0	0	0	0	0	0	0	0	895	1063	186
68	133	954	825	1063	786	19	0	0	0	0	0	0	0	0	0	954	1087	133
Average Waiting Time Per Truck: 62.12 Minutes																		
Last Truck Departure's Time : 9 Pm																		
Average Service Time Per Truck : 203.1 Minutes																		

In order to simplify the scheduling procedure and reducing the complexities for solving truck scheduling problem, most of the models proposed in the literature assumed that all trucks are available at time zero (see [26, 22, 39, 36, 27, 34, 13, 5, 32, 21, 3, 10, 30, 17]). In other words, the trucks are assumed to be already

waiting in the facility before the start of the schedule and ready to be called up [9]. Moreover, one of the common assumptions highlighted in existing studies is, full information regarding the contents of inbound trucks is assumed to be known beforehand. Accordingly, mathematical models proposed are based on the static environment in which trucks information are known in advance despite of been updated based on real time.

In real application of cross docking practise, trucks arrive at warehouse dynamically over time after the start of operation. Thus, the arrival time of each inbound truck is not known in advance as well as the information of truck's contents and its unloading time. In this study, dynamic arrivals time of trucks are considered and information of trucks is only known upon arrival. Therefore, the mathematical model proposed by this study is a real-time model. To the knowledge, this is the first of such model in cross docking where real time truck-to-door assignment and scheduling has been considered. Thus, be a key contribution of this study.

The mathematical model proposed in this study can be adopted by any traditional warehouses, distribution centres or manufacturing plants in managing their inbound operation with minimal modifications on the parameters involved, depending on the facility's setting. Furthermore, the model can easily be adapted for use in the transportation planning such as scheduling of public transportation in which the service time of the transportation vehicles during inbound operations at depots or stations can be minimized and the estimated passengers' disembarking time can be used to replace the truck's unloading time of the original model. The model and scheduling concept of this study can also be creatively applied in other areas with problem of similar characteristics such as those requiring the minimization of service time subjected to constraints that can be estimated using fuzzy logic approach. As an example, it can be utilized to minimize service time of customers where each customer's service time is estimated based on the number, type and the variability of services required.

7. Future Works. As for future work, this study will be extended by using heuristics or metaheuristics approaches to produce feasible solution based on the small scale instances problem as well as the large scale instances. Due to high complexity and real-time nature of cross docking operations, commercial optimization packages such as IBM ILOG CPLEX Optimization Studio (CPLEX) and Linear, Interactive, Discrete Optimizer (LINDO) that offers fast computing time is highly recommended. Aside from that, it is promising to extend the developed mathematical model to cater the scheduling issue of inbound operation, internal operation and outbound operation concurrently. As mentioned earlier, the developed mathematical model in this study is only focuses on the real-time inbound operation. Thus, it is essential to develop an integrated model that addressed all stages simultaneously. In addition, specific scheduling decision for the internal operation is necessary as it involved various resources such as material handling equipment and manpower. In addition, the MIP model proposed in this study can be refined further by imposing other parameters, constraints or stating the objective function in terms of goals or formulating it as a multi-objective function for different setting of warehouse (aside of the retail cross docking warehouse).

Acknowledgment. This work is supported by the Research Management Institute (RMI), Universiti Teknologi MARA (UiTM) under grant number 600-RMI/DANA 5/3/RIF (410/2012).

REFERENCES

- [1] D. Agustina, C. Lee and R. Piplani, [A review: Mathematical modles for cross docking planning](#), *International Journal of Engineering Business Management*, **2** (2010), 47–54.
- [2] U. Aickelin and A. Adewunmi, Simulation optimization of the crossdock door assignmnet problem, Proceedings of the Operational Research Society Simulation Workshop 2006 (SW 2006), Leamington Spa, UK.
- [3] A. Amini, R. Tavakkoli-Moghaddam and A. Omidvar, Cross-docking truck scheduling with the arrival times for inbound trucks and the learning effect for unloading/loading processes, *Production & Manufacturing Research*, **2** (2014), 784–804.
- [4] U. M. Apte and S. Viswanathan, [Effective cross docking for improving distribution efficiencies](#), *International Journal of Logistics*, **3** (2000), 291–302.
- [5] A. B. Arabani, M. Zandieh and S. F. Ghomi, A cross-docking scheduling problem with sub-population multi-objective algorithms, *The International Journal of Advanced Manufacturing Technology*, **58** (2012), 741–761.
- [6] M. Bachlaus, M. K. Pandey, C. Mahajan, R. Shankar and M. K. Tiwari, [Designing an integrated multi-echelon agile supply chain network: A hybrid taguchi-particle swarm optimization approach](#), *Journal of Intelligent Manufacturing*, **19** (2008), 747–761.
- [7] J. J. Bartholdi and K. R. Gue, [The best shape for a crossdock](#), *Transportation Science*, **38** (2004), 235–244.
- [8] L. Berghman, R. Leus and F. C. Spieksma, [Optimal solutions for a dock assignment problem with trailer transportation](#), *Annals of Operations Research*, **213** (2014), 3–25.
- [9] N. Boysen, [Truck scheduling at zero-inventory cross docking terminals](#), *Computers & Operations Research*, **37** (2010), 32–41.
- [10] N. Boysen, D. Briskorn and M. Tschöke, [Truck scheduling in cross-docking terminals with fixed outbound departures](#), *OR spectrum*, **35** (2013), 479–504.
- [11] F. Chen and C.-Y. Lee, [Minimizing the makespan in a two-machine cross-docking flow shop problem](#), *European Journal of Operational Research*, **193** (2009), 59–72.
- [12] R. Chen, B. Fan and G. Tang, [Scheduling problems in cross docking](#), in *Combinatorial Optimization and Applications*, Springer, 2009, 421–429.
- [13] H. Davoudpour, P. Hooshangi-Tabrizi and P. Hoseinpour, A genetic algorithm for truck scheduling in cross docking systems, *Journal of American Science*, **8** (2012), 96–99.
- [14] W. N. A. W. A. Fatthi, A. Shuib and R. M. Dom, Estimating unloading time at cross docking centre by using fuzzy logic, *Research Journal of Business Management*, **7** (2013), 1–14.
- [15] M. Golias, S. Ivey, K. Ji and M. Lipinski, A bi-objective model to minimize service and storage time at a cross dock facility, in *51st Annual Transportation Research Forum*, Arlington, Virginia, 2010.
- [16] S. S. Heragu, J. C. Huang, R. J. Mantel and P. C. Schuur, An efficient model for allocating products and designing a warehouse, *Progress in Material Handling Research*, 2004, 143–152.
- [17] C. M. Joo and B. S. Kim, [Scheduling compound trucks in multi-door cross-docking terminals](#), *The International Journal of Advanced Manufacturing Technology*, **64** (2013) 977–988.
- [18] J. Karlof, *Integer Programming: Theory and Practice*, chapter 4: *Decomposition in Integer Linear Programming*, Florida, CRC Press, 2006.
- [19] S. Kreipl and M. Pinedo, [Planning and scheduling in supply chains: an overview of issues in practice](#), *Production and Operations management*, **13** (2004), 77–92.
- [20] V. B. Kreng and F.-T. Chen, The benefits of a cross-docking delivery strategy: a supply chain collaboration approach, *Production Planning and Control*, **19** (2008), 229–241.
- [21] Y. Kuo, [Optimizing truck sequencing and truck dock assignment in a cross docking system](#), *Expert Systems with Applications*, **40** (2013), 5532–5541.
- [22] R. Larbi, G. Alpan, P. Baptiste and B. Penz, Scheduling of transshipment operations in a single strip and stack doors crossdock, in *19th International Conference on Production Research*, Valparaiso, Chile, 2007.
- [23] R. Larbi, G. Alpan and B. Penz, [Scheduling transshipment operations in a multiple inbound and outbound door crossdock](#), in *International Conference on Computers & Industrial Engineering, 2009. CIE 2009. IEEE*, 2009, 227–232.
- [24] K. Lee, B. S. Kim and C. M. Joo, [Genetic algorithms for door-assigning and sequencing of trucks at distribution centers for the improvement of operational performance](#), *Expert Systems with Applications*, **39** (2012), 12975–12983.

- [25] Z. Li, M. Low, M. Shakeri and Y. Lim, Crossdocking planning and scheduling: Problems and algorithms, *SIMTech Technical Reports*, **10** (2009), 159–167.
- [26] L. A. Li, Y. and B. Rodrigues, [Crossdocking-jit scheduling with time windows](#), *Journal of the Operational Research Society*, **55** (2004), 1342–1351.
- [27] T. Liao, P. Egbelu and P.-C. Chang, [Simultaneous dock assignment and sequencing of inbound trucks under a fixed outbound truck schedule in multi-door cross docking operations](#), *International Journal of Production Economics*, **141** (2013), 212–229.
- [28] J. T. Mentzer, W. DeWitt, J. S. Keebler, S. Min, N. W. Nix, C. D. Smith and Z. G. Zacharia, [Defining supply chain management](#), *Journal of Business logistics*, **22** (2001), 1–25.
- [29] Z. Miao, F. Yang, K. Fu and D. Xu, [Transshipment service through crossdocks with both soft and hard time windows](#), *Annals of Operations Research*, **192** (2012), 21–47.
- [30] B. Naderi, S. Rahmani and S. Rahmani, [A multiobjective iterated greedy algorithm for truck scheduling in cross-dock problems](#), *Journal of Industrial Engineering*, **2014** (2014).
- [31] Y. Pochet and L. A. Wolsey, *Production planning by mixed integer programming*, US: Springer Science & Business Media, 2006.
- [32] R. Sadykov, [Scheduling incoming and outgoing trucks at cross docking terminals to minimize the storage cost](#), *Annals of Operations Research*, **201** (2012), 423–440.
- [33] M. Shakeri, M. Y. H. Low and Z. Li, [A generic model for crossdock truck scheduling and truck-to-door assignment problems](#), in *6th IEEE International Conference on Industrial Informatics, 2008. INDIN 2008. IEEE*, 2008, 857–864.
- [34] M. Shakeri, M. Y. H. Low, S. J. Turner and E. W. Lee, [A robust two-phase heuristic algorithm for the truck scheduling problem in a resource-constrained crossdock](#), *Computers & Operations Research*, **39** (2012), 2564–2577.
- [35] A. Shuib and W. N. A. W. A. Fatthi, A review on quantitative approaches for dock door assignment in cross-docking, *International Journal on Advanced Science, Engineering and Information Technology*, **2** (2012), 30–34.
- [36] R. Tavakkoli-Moghaddam, J. Razmi et al., A new model for cross dock scheduling considering product arrangement, *International Proceedings of Economics Development & Research*, **35** (2012).
- [37] J. Van Belle, P. Valckenaers, G. V. Berghe and D. Cattrysse, A tabu search approach to the truck scheduling problem with multiple docks and time windows, *Computers & Industrial Engineering*, **66** (2013), 818–826.
- [38] I. F. Vis and K. J. Roodbergen, [Positioning of goods in a cross-docking environment](#), *Computers & Industrial Engineering*, **54** (2008), 677–689.
- [39] W. Yu and P. J. Egbelu, [Scheduling of inbound and outbound trucks in cross docking systems with temporary storage](#), *European Journal of Operational Research*, **184** (2008), 377–396.

Received April 2014; 1st revision November 2014; 2nd revision February 2015.

E-mail address: ashikin7463@ns.uitm.edu.my

E-mail address: adibah@tmsk.uitm.edu.my

E-mail address: rosma@tmsk.uitm.edu.my