Graphing - Programming Project

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Introduction

For my project I will attempt to make a graphing software. Graphing software is incredibly important in Linear Algebra and a lot of maths taught in schools is to do with Linear Algebra. Linear Algebra also links to many other aspects of Maths and hence is very important to understand. My stakeholders will consist of teachers, who will use my software to show graphs of functions, and to students who will use it to practise their graph sketching or to help them with their homework. There are many graphing tools out there, some of them shown below, however they have many downsides. I hope to make a piece of software that has as much functionality as possible, while retaining simplicity and reducing the number of downsides to an absolute minimum.
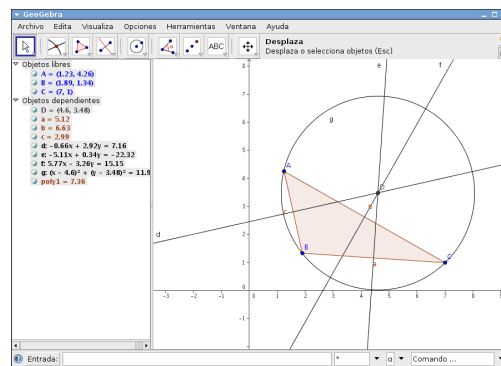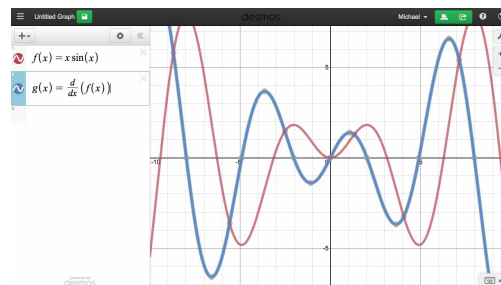

Figure 1.1: GeoGebra

Figure 1.2: Desmos

Chapter 2

Stakeholders

My stakeholders will be comprised of teachers and students in year 10 and above. This is because graph sketching as a skill becomes very important from GCSE onwards and as such the tool will be aimed towards students in the upper years and the teachers themselves. I have interviewed several of my potential stakeholders about the software that they use and their opinions on it.

2.1 Teacher Interview

I interviewed Miss Naguthney who is a Maths teacher who teaches pupils from year 7 to 13. She uses multiple tools to aid her teaching. I asked her a couple of questions to get an idea of what she finds important in a graph drawing software.

Question	Answer	Analysis
What graph drawing software have you used?	I have used Desmos, MAT-LAB and GeoGebra.	These three programs are very different, ranging from a simple graph drawing software, Desmos, to a professional data presentation software, MATLAB.
Which do you think is the best and why?	I think all 3 are good. In terms of pure graph drawing I think that Desmos is the best due to its simplicity.	From this response, it is important that my program is as simple as possible so that anyone can use. I think that to make program simple it must be responsive as well, and hence I should make my solution as efficient as possible so that it can be responsive on most devices.
What do you think is the most important aspect of a graph drawing software?	The most important part of a graph drawing software are obviously the graphs themselves and the most important part of the graphs are of course their intersections with axes, turning points and any asymptotes.	I agree with all these points made and I think that my program should at least be able to identify the turning points and any intersections with the axes.

2.2   Student Interview

I find that during my sixth form studies, that graph sketching is a must have skill, and hence a graph drawing tool is important to verify that you have drawn a graph accurately. I interviewed Matthew, who is a sixth form student studying similar subjects, about graph drawing tools that he has used and his opinions about them.

Question	Answer	Analysis
What graph drawing software have you used?	GeoGebra, Desmos and Kalgebra (KDE Application)	These three programs are quite similar as they have many of the same features.
Which do you think is the best and why?	I would say GeoGebra because: • Native client so it is more responsive than a web-based app • Versatile • Easily Adjustable Axes • Very easy to focus on a part of the graph • Multiple function support However the UI looks ugly and has no dark mode to reduce strain on the eyes.	I think that my program, should have the ability to be themed, through using CSS or a text file and while I may not be able to make my program versatile, it should still be able to draw multiple functions at once and navigating the graph must be fluid.
What do you think is the most important aspect of a graph drawing software?	The two most important aspects for me are ease of use and flexible input. Specifically this would be stuff like being able to input complex functions such as sums of multiple rational functions [1] (Not complex as in $x \in \mathbb{C}$) and the ability for the software to automatically adjust the scale. An example of this is for trigonometric functions sine and cosine. They don't have high y values but they always appear very small on GeoGebra because the scale is wrong. A way to automatically set a suitable scale would be nice. Finally, I feel performance is also a must - it's frustrating to navigate the graph and have the application lag a lot.	Matthew's answer can be summarized into two points: Responsiveness and Flexible Input. I think that both of these points are valid and I should aim to make sure my application satisfies both these points.

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[1] A function $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are functions of x

Chapter 3

Similar Programs

During my interviews with my stakeholders, they mentioned several pieces of software that they have used. I will be analyzing 3 of those programs in particular (or similar in the case of MATLAB, since MATLAB is extremely expensive), which are:

1. Desmos [1]

2. GeoGebra [2]

3. GNU Octave[3] (MATLAB[4] Clone)

3.1 Desmos

Desmos is a closed source graphing calculator written in HTML5, which allows it to be used on many devices. It has dedicated apps on Android and on Apple devices, but has no dedicated application on a desktop environment.

The real strength of Desmos comes from its ability to create activities that students can then complete.

Figure 3.1: Mean Value Theorem example that students can interact with

The problem however is that it is web based and as such, can be clunky to use. The mobile app and website are especially unresponsive when zooming in or out and when panning around. Another pitfall is that Desmos tries to make itself more secure in a way by restricting input. This however makes input very difficult to do and along with the unresponsiveness, it can be impossible to input the required equation.

Figure 3.2: The Input

On the other hand, the actual graph drawing is excellent, with many features such as inequalities, modulus functions and the ability to draw polar functions. It also highlights turning points, intersections with axes, inflection points and in the case of trigonometric functions, it expresses them in terms of π instead of a numerical values.

Overall as a graphing calculator it does an excellent job, but has poor optimization issues and input, which I have realized are both important and need to be at the top of my list of requirements for my program.

In terms of other features, you can print, save and share your graphs to use later. I think that my program should have at least 1, if not all 3, of these features.

3.2 GeoGebra

GeoGebra is an interactive geometry, algebra, statistics and calculus application under the GNU General Public License, meaning that it is open source.

The main advantage to GeoGebra is that it can do a lot of things and is all of its features integrate well with each other. For example you can create a shape on the graph and create lines intersecting it, linking geometry with algebra. Individual line equations can also be moved around allowing for a very interactive experience. Like Desmos, you can create activities that you can interact with.

The mobile app for GeoGebra has a similar input to Desmos and as such suffers from the same problems. However it is more responsive than Desmos but only slightly. The desktop version however does not suffer from any of these problems.

As a pure graphing software, it does not automatically show the points of interest of a curve and is instead confined in a tool called the "Function Inspector". This tool is awkward to use as it only shows local maximums and minimums within a certain range (the highlighted red region), and if there are multiple roots, it does not acknowledge what the values of the roots are. Desmos does better than GeoGebra in this regard. Like Desmos it supports many functions that you can plot such as modulus and inequalities.

Figure 3.3: GeoGebra Function Inspector Tool

The panning and zooming in/out in GeoGebra is very fluid, with the zoom function being centered around the cursor. This means that you zoom in towards your cursor which feels more responsive than if it zooms in/out from the center of the graph. The only criticism is that you can accidentally move already drawn graphs when trying to pan, which can be frustrating.

In terms of other features, like Desmos, you can save and print your work as well as being able to export it into other formats such as JPG, PNG or even convert it into a PSTricks or TikZ environment which can be used in LaTeX documents for excellent graphs.

3.3 GNU Octave

GNU Octave is essentially an interpreter for a high level language centered around numerical calculation. GNU Octave is designed to be an open source software clone of MATLAB (MATrix LABatory).

Octave is different from GeoGebra and Desmos in the fact that it plots points that you tell it to not lines. For example if you wanted to plot $y = x^2$, you would define an array for your range in x by doing:

$$x = -10 : 0.1 : 10;$$

This creates an array that starts at -10 increments by 0.1 until 10. You would then define y by:

$$y = x. \wedge 2$$

You would then plot x and y by doing $plot(x, y)$;.

As a graphing tool Octave is not very impressive, it has limited support for implicit functions and will not highlight points of interest of a curve. Realistically this is expected since Octave is primarily made for data presentation. This means that it can produce excellent plots, including meshes and surfaces [1] in 3 dimensions. [2]
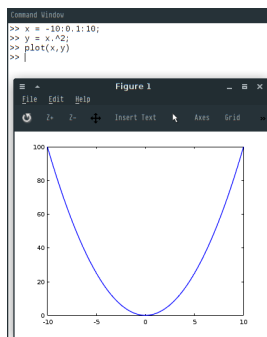
Figure 3.4: Creating a simple plot in GNU Octave

Figure 3.5: Wireframe 3D plot of the two-dimensional unnormalized sinc function

The way Octave plots (using the plot function at least) is by joining a line between consecutive points that it plots. I may use this in my program, since it seems more efficient to approximate a curve by making linear lines and joining them up than to check each pixel on the plot to check if it satisfies the given relation ship.

When inputting an algebraic expression in Octave it can be quite awkward, for example in Figure 3.4, a period is before the carat. This is unnatural for us to input and as such can cause errors. Natural input is

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[1] A mesh is when a set of points in 3D space are joined up with lines to look like wire mesh. A surface is a mesh but adjacent points are used to make a solid shape instead, hence creating a surface.

[2] To clarify, a plot is where we explicitly define data that we then represent graphically, while a graph is where we define a relationship between 1 or more variables and represent this graphically.

something that I think I should focus on since it is means that the program is easier to use and requires no further reading unlike Octave.

Octave is split into two views, the main editor view and a dockable plot view (you can also have multiple plots open at once). Dockable plots, or multiple plots could be something I implement later into my project.

Octave, like the other programs, can save plots and additionally the scripts you have made. It also has a feature that saves the current state when you close it, and resumes this state when you open it again. This could also be a feature I implement later into the project.

3.4   Comparison

Overall I think that I will be using aspects of all three of the programs that I have analyzed in my program. I think I should try and make my program as simple to use as Desmos. As mentioned, the input is a problem, and hence I will try and implement an input system like GeoGebras'. All three of the programs analyzed have some form of saving their plots, so I think that this is a feature that I should implement within my own program.

In terms of UI design, both Desmos and GeoGebra have a similar layout. They have an equation input box to the left, that can be minimized and the actual graph in the center left. I think that this layout is easy to use for the user so I will implement a similar design in my own program.

Main-Ribbon	
Input-Equations $y = x^2$ New-Equation	Plot

Figure 3.6: UI Design

Chapter 4

Computational Methods

4.1 Abstraction, Heuristics and Computability

Abstraction is the process of creating a model of a real-life problem by separating and highlighting important details. There are two main details that I can abstract away:

1. The input of expressions, and their conversion into a form that the computer can then manipulate.

2. The drawing of the function onto the screen.

By analyzing both parts of these details separately, we can determine if our problem is intractable.

The first part of the problem is solvable by converting the expression into an abstract syntax tree or into a post-fix stack, which is easily implementable.

The second part of the problem is solvable using heuristic methods. Most APIs, such as `OpenGL[5]` and `Vulkan`, only allow the drawing of primitives, which are points, lines and triangles. [1] This means that our curve cannot be drawn straight onto the screen and must be approximated by either using points or lines, hence there are two ways we can approximate the curve:

1. Let x be our starting value, $f(x)$ our function and dx be a small value. Now draw a line between the points $(x, f(x))$ and $(x + dx, f(x + dx))$; $(x + dx, f(x + dx))$ and $(x + 2dx, f(x + 2dx))$; ... ;$(x + n \cdot dx, f(x + n \cdot dx))$ and $(x + (n+1) \cdot dx, f(x + (n+1) \cdot dx))$. Here we are effectively creating line segments and joining them together to approximate the curve. As $dx \to 0$ our approximation tends towards the real curve. However as $dx \to 0$ the time taken tends to infinity and therefore we only need to make dx small enough so that the human eye can not see individual line segments. Also as a practical note, as $dx \to 0$, the pixel density required to view the curve tends to infinity, therefore dx needs to be big enough so that the dx is not less than the individual pixel width.

2. In the method above we approximated a curve by creating line segments, here we will approximate a curve by using individual points. Let x be our starting value, $f(x)$ our function and dx be a small value. Now draw a point at $(x, f(x))$; $(x + dx, f(x + dx))$; $(x + 2dx, f(x + 2dx))$; ... ;$(x + n \cdot dx, f(x + n \cdot dx))$. This can take a lot of time since calculate where to draw a pixel w number of times, where w is the pixel width. In some cases, for complicated functions, it may be intractable.

By using the first method we can model a curve to good accuracy within good time, therefore our second part of the problem is computationally possible albeit, using a heuristic method.

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[1] Strictly this isn't true, but most other things we can draw using these APIs usually consist of a combination of primitives, usually triangles.

4.2 Logical Approach

Many parts of this problem require clear decision making, making sure that user inputs are valid and making my solution as efficient as possible. I could validate and standardize user input by using `RegEx[6]`. I will definitely use standard structures in my solutions, and therefore will be able to logical traverse these structures to get the required solution.

4.3 Decomposition

Decomposition is where a problem is broken down into smaller sub-problems that are manageable and are solved individually and combined to make the complete solution. As stated in the abstraction section, I can break down the problem into two main parts. These parts could then be broken down further into classes and functions such as:

1. A function class which consists of:

    (a) Converting an expression into a form the computer can manipulate.

    (b) Evaluating the expression for a value x, where $x \in \mathbb{R}$.

    (c) Calculating the roots and turning points of the function.

2. A plot class which consists of:

    (a) Drawing functions onto the screen

    (b) Heuristically adjusts the scale so the graph looks good

    (c) Drawing the axes

This is just an example of how I can decompose the problem and therefore the problem will be decomposed more.

4.4 Divide and Conquer and Concurrent Processing

Divide and Conquer is where a problem is recursively broken down into two or more sub-problems of the same or related type, until these become simple enough to be solved directly. While this may seem similar to decomposition, Divide and Conquer relates to algorithms. My solution may be using trees, and as such I can use a Divide and Conquer algorithm to convert an expression into a tree by looking at sub-expressions within the expression. This can then be further improved by using Concurrent Processing to process each sub-expression individually. I can also use Concurrent Processing to draw multiple lines or functions on the screen at once.

Chapter 5

Requirements

From my research and analysis I have created a set of requirements that I aim to fulfill. I have split these requirements into development iterations that they will be linked with. Each iteration has a theme that most of the requirements will follow.

- Iteration I - Emphasis on Core Functionality

  - Plot a explicit function in x
  - Plot multiple functions on the same plot

- Iteration II - Emphasis on the User

  - Zoom in/out of the graph
  - Pan around the graph
  - Plot special functions such as trigonometric functions, logarithms, modulus functions, etc.
  - Identify intersection with the axes
  - Multiple plots that you can switch between
  - Save plots as pictures
  - Save workspace to resume later
  - Dark Theme

- Iteration III - Advanced Features

  - Identify intersections between functions
  - Differentiate explicit continuous functions with respect to x
  - Polar equations
  - Implicit equations
  - Parametric equations
  - LaTeX equation support
  - Identify turning points

I will use `Java` to accomplish this task. `Java` is platform independent meaning that I do not have to compile for many different systems. It also means that if I want to create a mobile version I will only have to recreate the UI. Within `Java` I will use `JavaFX` for my UI. `JavaFX` allows for customisation of UI elements through `css` files, as well as an easy interface to draw objects onto through the screen, through its `Canvas` class. I am also comfortable with the language which makes it the best language for this project.

Part II

Prototype I

Chapter 6

Design

6.1 Decomposition

My program will be decomposed into two main parts the GUI and the parsing of functions. The GUI will consist of the input of functions and the drawing of the functions. Many parts of this project can be achieved in a cleaner fashion using an OOP approach rather than a procedural approach.

6.1.1 OOP vs Procedural Approach Justification

An Object Oriented approach for my program is better than a procedural approach due to its complexity. An OOP approach allows parts of a program to be independent, and this means that these parts can be tested individually, allowing for simple debugging.

Specific to my project, Java is wholly based around the OOP approach and hence it makes more sense to use an OOP approach rather than a procedural one. Also many parts of my program can use the OOP concepts of inheritance and polymorphism. An example of this is the layer system I will be using later on to draw functions onto the screen. There are many different types of functions (again which can use inheritance), and each of these will have a coresponding layer. Each type of layer will draw, have very simlar attributes and other similar features which will be very slightly different for each type of function. By making all these different layers inherit a base layer class, it means that all of them can be accessed generally, especially when drawing. They can also all be stored in an array of the type of the superclass, and accessed (outside of the class) in a standard way making the solution cleaner and easier to test.

6.2   Function Class

Our basic Function class will contain two main methods:

- Parse - This method will convert the user's input into a data structure that we can use to evaluate. This data structure will be stored as a private attribute.

- Evaluate - This method will take a value of x and input it into our function and return the value $f(x)$.

Our class will contain more methods and attributes later (colour of the line, roots, turning points etc.) but these can be considered later as these are quite small parts. There are two distinct methods to parse a function:

- We can convert the input into a data structure, which we can then use to evaluate a value x.

- We can convert the input into its equivalant in a scripting language, such as Lua[7], and then use the scripting language to evaluate a value x.

An example of the second is shown below:

```
import org.luaj.vm2.*;
import org.luaj.vm2.lib.jme.*;
public double evaluate(double x) {
        ScriptEngineManager mgr = new ScriptEngineManager();
        ScriptEngine e = mgr.getEngineByName("luaj");
        e.put("x", x);
        e.eval("y = math.sqrt(x)");
        return e.get("y");
}
```

The problem with using scripts is that they are take a huge hit on performance as you are effectively creating a virtual machine during your program. Especially in Java where a VM is used to run your programs, creating a VM inside a VM is not very effiecient. Also we may make upto 10000 evaluate calls to render multiple lines, and therefore we will need to be as effiecient as possible. While the first method is harder to implement, it will have better performance, and it will be easier to debug as we are not using external tools. Therefore we will use the first method.
There are two main structures we can convert a mathematical input into:

- Binary Trees
- Stack Based Programming

both of which we will analyze in detail.

6.2.1 Analysis of Algebraic Expressions

Algebraic Expressions are quite important in this project and as such it is important to break them down and understand what they really mean. The most basic idea in evaluating expressions is **BIDMAS**, which stands for **Brackets**, **Indices**, **Division**, **Multiplication**, **Addition** and **Subtraction**. It signifies the order that we must do operations on an expression (It is sometimes called **BODMAS** where the **O** stands for **Orders**) with the order of significance going from the first letter to the last. For example, let us use BIDMAS with the following expression:

$$3(4^2 + 2)$$

According to BIDMAS we will first look for brackets. There are brackets! The expression inside those brackets is $4^2 + 2$. We then apply BIDMAS again. There are no brackets but we do hae indices. We apply the indice function which produces $16 + 2$. Applying BIDMAS again we see that we must do addition which produces 18. Finally we have the expression $3(18)$ which is multiplication and it produces 54. Here we repeatedly took the most significant operator and applied its function until we fully evaulated our expression. Later we will instead take the least significant operator as this will allow us to split our expression down in a more structured manner.

In the example above there was nuance that we ignored. This was when we identified $3(18)$ actually meant $3 * (18)$. This implicitly represented multiplication and while as humans it is easy for us to process it, it is impossible for a computer to know this. It is therefore important that we remove these inconsistencies before we properly convert our input into a structure. It is also important that we strip away all whitespace before we start as this will allow our input to be more consistent. Here is a list of all these inconsistencies that we need to remove:

- Any instance of ax where $a \in \mathbb{R} : a \neq 0$ is to be converted to $a * x$.[1]

- Any instance of $a($ and $)a$ where a is not an operator, is to be converted to $a * ($ and $) * a$ respectively..[2]

- Any instance of $(f(x))(g(x))$ is to be converted to $(f(x)) * (g(x))$.[3]

- Any instance of $! - f(x)$ where $!$ is to be any operator (e.g. $*$ or $/$) is to be converted to $!(-f(x))$.[4]

- Any instance of $-f(x)$ at the start or next to an opening bracket is to be converted to $0 - f(x)$.[5]

---

[1]For example $4x$ is to be converted to $4 * x$

[2]For example $4(x + 1)$ is to be converted to $4 * (x+!)$

[3]For example $(x + 4)(x - 3)$ is to be converted to $(x + 4) * (x - 3)$

[4]If there is a negate symbol next to another operator, we need to make sure that the negate symbol is not treated as an operator (even though we treat it like an operator in certain situations in the next step)

[5]Both these expressions are equivalent but the second allows us to reduce ambiguity if there is a negate symbol at the start of an expression e.g. $-x + 4$ would be treated as $0 - x + 4$ and $(-x)$ is $0 - x$.

6.2.2 Binary Trees

The second stage that a compiler goes through is called Syntax Analysis and this is where the code is transformed into an Abstract Syntax Tree. What we are trying to achieve is very similar to that (albeit on a smaller scale). If we take a function $f(x) = x + 4$, this would be transformed into the binary tree:

$$+$$
$$4 \quad x$$

If we take something more complicated such as $f(x) = (x + 4)(x - 3)$, we need to first remove consistencies as discussed at the start of this section. Therefore the function would become $f(x) = (x + 4) * (x - 3)$. Now when we convert this into a syntax tree, we find the least significant operator (the operation we would do last), make it our root node and split the parameters that it is operating and make those parameters the child nodes of the root node. Here $*$ is the least significant operator and $x+4$ and $x-3$ become the child nodes.

$$*$$
$$(x + 4) \quad (x - 3)$$

We can then repeat what we did above on the child nodes. In $x + 4$, $+$ is the least significant operator and in In $x - 3$, $-$ is the least significant operator. Therefore we now get:

$$*$$
$$+ \qquad -$$
$$x \quad 4 \quad x \quad 3$$

We now stop as we cannot split this down any further. What we notice here is that only the bottommost nodes are actually values, the rest are operators. This is significant, because what we just did was a recursive algorithm, where we split each node down, until a node does not contain any operators.
Now if we want to get $f(1)$, we simply replace every instance of x with 1 and then perform the recursive algorithm outlined below.

Algorithm 1: Evaluate: Binary Tree Version

---

1 function *evaluate(tree)***:**
2   **if** *tree.height = 0* **then**
3   **return** tree.root.value
4   **else**
5   **return** *evaluate*(tree.leftsubtree) tree.root.value *evaluate*(tree.leftsubtree)
6   **end**
7 end

This recursive algorithm has base case "tree.height = 0", which essentially checks if are at the bottom of our tree. We know that the bottommost nodes are actually values, therefore we can return this value as we cannot go deeper into our tree. Our recursive case is where we are not at the bottom of our tree. We know that at this point, the root node is an operator [6]. Therefore we operate on the nodes below it.

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[6]It is important to realize that tree.root.value is just an operator, and in theory we can simply just write it as shown above but in implementation this will require us to use if statements to check which operator it actually is as the operators will most likely be of data type string

First let us replace every instance of x with 1.

```
            *
          /   \
        +       -
       / \     / \
      1   4   1   3
```
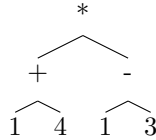
Now the root node has value "+", i.e. an operator therefore we take the left subtree which is,

```
       +
      / \
     1   4
```

and again the root node is an operator so we take the left subtree again, which is 1. This is not an operator, so we return this value, going back to our previous call which is,

```
       +
      / \
     1   4
```

Now we take the right subtree, which is 4. Again, this is not an operator so we return this value, going back to our previous call. Now we have **return** $1 + 4$, therefore we return 5 to our previous call which is,

```
            *
          /   \
        +       -
       / \     / \
      1   4   1   3
```

We now repeat the process for the right subtree,

```
       -
      / \
     1   3
```

which returns -2. We then go back to our original call to get $5 * -2$ which returns -10.
The recursive tree for this is exactly the same as the original tree that we started with.

```
            *
          /   \
        +       -
       / \     / \
      1   4   1   3
```

This is significant as this means that the number of recursive calls made is the number of edges, which is $n - 1$ where n is the total number of operands and operators combined.

6.2.3 Stack Based Programming

Stack Based Programming[8] is all about using a stack data structure, and manipulating the items within it to get the desired result. Firstly we have to convert our *Infix Notation* into *Reverse Polish Notation*. Infix notation is the "normal" way of writing algebra, where the operator is inbetween its operands. Reverse Polish notation or Postfix notation is where the operator is after its operands. The advantage of this notation is that no brackets are required. For example $4 + 3$ in infix notation would be $4 \quad 3 \quad +$ in postfix notation. To evaluate this expression we push each individual operator/value, from left to right, one by one, to a stack. If the value is an operator then it pops however many inputs it would normally take off the stack, perform the operation, on the values we just popped off, and then add that new value back to the stack. For example if we take the infix expression,

$$(((6 * (8 - 3)/3))/2) + 1$$

we can convert this to the equivalent postfix expression,

$$6 \quad 8 \quad 3 \quad - \quad * \quad 3 \quad / \quad 2 \quad / \quad 1 \quad +$$

The way we get this is by looking for the least significant operator (the one we would consider last) and putting it to the end. We then take the operands of the operator which just removed, and repeat until we are left with only values. This seems very similar to the binary tree solution. In fact, if we convert this into a binary tree,
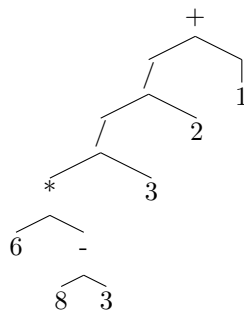
and if we then perform post-order depth traversal. So first we go all the way down the left hand side of the tree to get 6. We then visit its sibling, $-$, which has more children. So we visit 8, 3 then $-$. So far we have

$$6 \quad 8 \quad 3 \quad -$$

We then move up to $*$ and add this to our list. We then visit its sibling 3 and add this to our list. We have now got

$$6 \quad 8 \quad 3 \quad - \quad * \quad 3$$

We then go up to $/$ and add this to our list. We then visit its sibling 2 and add this to our list. We have now got

$$6 \quad 8 \quad 3 \quad - \quad * \quad 3 \quad / \quad 2$$

We then go up to $/$ and add this to our list. We then visit its sibling 1 and add this to our list. We then go up to the root node, $+$, and add this to our list. Finally we have,

$$6 \quad 8 \quad 3 \quad - \quad * \quad 3 \quad / \quad 2 \quad / \quad 1 \quad +$$

This is identical to our post-fix notation. This is important as later on we can use this fact to make our solution as effiecient as possible.
Now if we start to evaluate this expression we get,

$$[6] \qquad \begin{bmatrix} 8 \\ 6 \end{bmatrix} \qquad \begin{bmatrix} 3 \\ 8 \\ 6 \end{bmatrix}$$

we have reached an operator, $-$, so we now pop 2 items of the stack, 3 and 8, and perform the operation, $8 - 3 = 5$. We now push this value onto the top off the stack and resume our evaluation.

$$\begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Again we have reached a operator so we repeat what we did before. Take 5 and 6 off the stack, push $5*6 = 30$ onto the stack.

$$[30] \begin{bmatrix} 3 \\ 30 \end{bmatrix}$$

Take 3 and 30 off the stack, push $30/3 = 10$ onto the stack.

$$[10] \quad \begin{bmatrix} 2 \\ 10 \end{bmatrix}$$

Take 2 and 10 off the stack, push $10/2 = 5$ onto the stack.

$$[5] \quad \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

Finally Take 1 and 5 off the stack, push $5 + 1 = 6$ onto the stack. We are left with 6 and have now got our answer. Just like with the binary trees, if we have an unknown x, we can simply replace the x with a value when we want. The algorithm to evaluate post-fix notation is shown below,

---

Algorithm 2: Evaluate: Stack Based Version

```
 1 function evaluate(list):
 2     Stack stack
 3     Array temp
 4     Real out
 5     Real x
 6     foreach i in list do
 7         if i is an operator then
 8             pop = i.numberOfInputs
 9             temp = new Array[pop]
10             for j=0 to pop by 1 do
11                 temp[j] = stack.pop()
12             end
13             x = i.input(temp[1],temp[2],...,temp[n])
14             stack.push(x)
15         else
16             stack.push(i)
17         end
18     end
19     out = stack.pop()
20     return out
21 end
```

Here we are treating the operator to be an object for simplicity. In practice, all these values will be of data type String, therefore we will probably use some sort of selection contruct, either *if* or *switch-case* statements, to determine if i is an operator and if it is, which one is it.

6.2.4 Analysis of the two Methods

When inititially parsing the infix expression $f(x)$, the binary tree version will have time complexity $O(n)$ where n is the number of operators (n will be the number of operators, and m will be the number of operands in $f(x)$) in $f(x)$. The stack based version will convert the infix expression into a binary tree, and then perform depth-first traversal to convert the tree into post-fix. Depth-first traversal has a time complexity of $O(m+n)$ (every traversal visits every node, and there are $n+m$ nodes). This means that to initially parse the infix expression, the binary tree version has a smaller time complexity.

When evaluating $f(x)$ for a specific value of x, both versions have a time complexity of $O(m+n)$. This is because the binary tree version visits every node, $m+n$ (what we do is essentially post-order depth-first traversal but applying an operation each time), and the stack based version has $m+n$ items in the list that it goes through. Here we are assuming that each operation between operands takes equal time. This is a reasonable assumption to make because we are comparing two algorithms and they have the same input, $f(x)$, therefore we can remove the steps that it takes to complete the operations.

Now from a pure time complexity point of view, we can say that the binary tree version is better as the initial parsing is quicker. However when it comes to space complexity this is not true. When evaluating $f(x)$, the binary tree version uses a lot more memory because it creates subtrees every time it calls a recursive function. On the otherhand, with the stack based version, the maximum space that could be occupied is $(m+n)+m$. $(m+n)$ is the size of the orignal list that we pull values from, and m is the maximum size our stack could ever get (we never add an operator to the stack). From a memory point of view a binary tree is not very effieciently stored. Most languages don't have a binary tree construct, and implementing your own with primitive arrays (each child is an array) is ineffiecient. For example the binary tree,

would be represented as (actual implementation would have pointers),

$$[*[+[x,4]],[-[x,3]]]$$

If we then stored this in contiguous memory,

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ * & + & x & 4 & - & x & 3 \end{bmatrix}$$

Now when doing post-order depth-first traversal, we jump between the memory locations,

$$\begin{bmatrix} 2 & 3 & 1 & 5 & 6 & 4 & 0 \\ x & 4 & + & x & 3 & - & * \end{bmatrix}$$

This is horribly ineffiecient because our stride is not consistent. This is unlike our stack based version, where we always have a stride of 1, no matter what. While this is not that big of a problem for one evaluate call, when 2000 evaluate calls are made, this makes a massive impact. We could, in theory, modify the order that we store our tree into one which is effiecient when doing a depth-first traversal but this is what we did when we converted our tree into post-fix notation. Therefore using stacks to process input is best for performance even though, the initial parsing takes longer. However this does not mean that we are not going to consider binary trees. This is because we need binary trees to initially parse our infix expression, therefore we will use stacks with binary trees to parse our user input. We need to inplement trees and stacks in Java before we can start parsing our infix expression.

6.2.5 Implementing Stacks and Trees

Since stacks and trees are a prerequisite to our function class, we will design them now.

Our stack does not need to become infinitely big, as the size of the `Stack`, as discussed in the last section, is m where m is the number of operands. Therefore we can initialize an array of size m and manipulate the array to make it act like a `Stack`.

Our binary tree will be quite simple, containing only the bare minimum of what our binary tree needs, with an added traversal function which returns the post-order depth-first traversal of the tree. For simplicity we will use a private static helper function, which will take a tree and a stack as parameter. This stack which is passed in by reference will be edited in every recursive call. We will also make our left and right tree attributes public. This is because our root tree will be public, and as the left and right sub-trees are simply different copies of the root tree, it isn't consistent to make the right and left private while keeping the root tree public.

Stack
▬ Stack(int maxSize)
▬ double pop()
▬ void push(double n)
▬ int getHeight()
🔒 double[] stack
🔒 int maxSize
🔒 int height = 0
🔒 int pointer = -1

Tree
▬ Tree(String value)
▬ String getValue()
▬ void setValue(String value)
▬ Stack traverse()
🔒 Stack traverseHelper(Tree tree, &Stack order)
▬ Tree right = null
▬ Tree left = null
🔒 String value
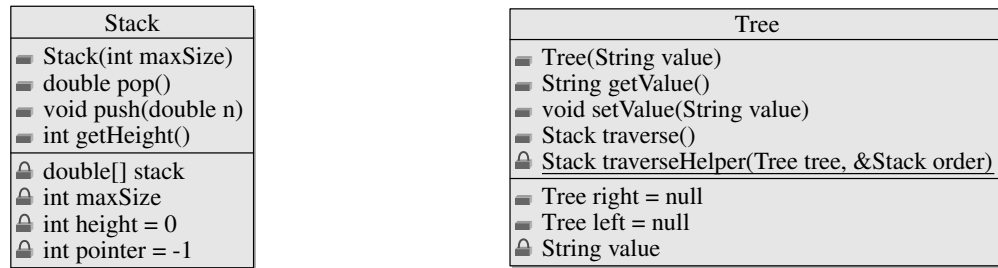
Figure 6.1: Class Diagrams

Algorithm 3: Post-Order Depth-First Traversal Helper

1 **function** *traverseHelper(***Tree** *tree,***Stack** *&order)***:**
2   **if** *tree == ***null then**
3   **return**;
4   **else**
5   traverse(tree.left,order)
6   traverse(root.right,order)
7   order.add(root.data)
8   **end**
9 **end**

Algorithm 4: Post-Order Depth-First Traversal

1 **function** *traverse()***:**
2   **Stack** order
3   traverseHelper(this,order)
4   **return** order
5 **end**

6.2.6 Parsing Algorithm

6.2.6.1 Least Significant Operator

An important part of the parsing stage, is the ability to find the least significant operator within an expression. If there are no operators -1 will be returned. If an operator is inbetween within a pair of brackets it is automatically discounted. We do this by respectively incrementing or decrementing a variable when we reach an opening or closing bracket to monitor when we are within a pair of brackets.

Algorithm 5: Least Significant Operator Position

```
1  function leastSigOperatorPos(String input):
2      Integer parenthesis = 0
3      Integer leastSigOperatorPos = -1 /* stores the position of the least significant
           operator so far                                                           */
4      Integer leastSigOpcode = 1000
5      Character[] operators = ["+",' "-", "/", " * ", " ^"] /* stores each operator in order of
           incresing significance                                                    */
6      Integer currentOpcode /* the current index of the operator in the array operators */
7      Character currentChar
8      for i=0 to (input.size - 1) by 1 do
9          currentChar = input[i]
10         if currentChar in operators then
11             currentOpcode = operators.find(currentChar)
12             if (currentOpcode ≤ leastSigOpcode) and (parenthesis == 0) then
13                 leastSigOperatorPos = i
14                 leastSigOpcode = currentOpcode /* Update the least significant operator so
                       far, as it is now the current character                        */
15         else if currentChar == "(" then
16             parenthesis++
17         else if currentChar == ")" then
18             parenthesis−−
19         end
20     end
21     return leastSigOperatorPos
22  end
```

6.2.6.2 Removing Brackets

The function above assumes that there is no whitespace and that there are no brackets enclosing the entire expression (e.g. $(x-4)$). We can deal with our whitespace issue in our constructor[7] however we need to make another function check for and remove any brackets surrounding an expression.

However algorithm 6 has some issues. For example if we have the expression $\frac{x+1}{x+2}$, this would be input as, $(x+1)/(x+2)$. Now if we apply algorithm 6 which removes enclosing brackets we get $x+1)/(x+2$. This is completely wrong, as in this case we do not want to any remove brackets at all. The significant issue here is that we only want to remove the enclosing brackets, if they are ***matching***.

To do this the algorithm 7 is more suited. This algorithm is based on 6 but checks for matching brackets. It does this by using a variable that increments everytime there is a opening bracket and decrements every time there is a closing bracket. When the variable becomes 0 the matching bracket has been found. If this

---

[7]In our function class we will store the original input so we can show the user, therefore we do not need to remove whitespace here

happens at the end the opening and closing brackets are removed, else nothing happens. This algorithm will also throw an exception if there are unequal number of opening and closing brackets. This is so that we can inform the user later of the error that they have made and so that we can kill the process instantly rather than letting this error have consequences later on (probably during the evaluation of a value).

Algorithm 6: Check for and remove any Brackets surrounding an input

```
1  function checkBracket(String input):
2      Boolean done = False
3      while !done do
4          done = True
5          if (input[0] == '(') and (input[input.size - 1] == ')') then
6              done = False
7              input = input.substring(1, input.size - 2)
8          end
9      end
10     return input
11 end
```

Algorithm 7: Check for and remove any Matching Brackets surrounding an input

```
1  function checkBracket(String input):
2      Boolean done = False
3      while !done do
4          done = True
5          if input[0] == '(' and input[input.size - 1] == ')' then
6              Integer countMatching = 1
7              for i=1 to (input.size - 2) by 1 do
8                  if countMatching == 0 then
9                      return input
10                 else if input[i] == '(' then
11                     countMatching++
12                 else if input[i] == ')' then
13                     countMatching--
14                 end
15             end
16             if countMatching == 1 then
17                 done = False
18                 input = input.substring(1, input.size - 2)
19             else
20                 throw "There is an unequal number of opening and closing brackets"
21             end
22         end
23         return input
24 end
```

6.2.6.3 RegEx

Another important part of the parsing stage is to standardize the input. This is where we convert any inconsistencies discussed in section 6.2, page 16. The easiest way to do this is to use `RegEx[6]`. `RegEx` stands for regular expression and is a standardized form of pattern recognition in strings, usually used during syntax analysis during compilation of software. Many languages support `RegEx` in some form or another and Java is no exception. These were our 5 inconsistencies that we needed to fix:

1. Any instance of ax where $a \in \mathbb{R} : a \neq 0$ is to be converted to $a * x$.

2. Any instance of $a($ and $)a$ where a is not an operator, is to be converted to $a * ($ and $) * a$ respectively.

3. Any instance of $(f(x))(g(x))$ is to be converted to $(f(x)) * (g(x))$.

4. Any instance of $! - f(x)$ where ! is to be any operator (e.g. $*$ or $/$) is to be converted to $!(-f(x))$.

5. Any instance of $-f(x)$ at the start or next to an opening bracket is to be converted to $0 - f(x)$.

For the first and second examples, we look around every instance of x, (or) and if the adjacent characters are not operators or brackets then we replace with $*x$ or $x*$. Therefore we can combine the first and second examples to use two separate `RegEx` expressions to deal with the case where we have x after and where we have x before.

The `RegEx` expression for the first case is **"([∧\(\)\+\-*\/\∧])([\(a-z])"** with the replacement expression being **"\$1*\$2"**. If we take the `RegEx` expression, it creates two capture groups, *"\$1"* and *"\$2"*, which are *"([∧\(\)\+\-*\/∧])"* and *"([\(a-z])"* respectively. A capture group stores a set of characters for each match that is made, so that we can perform actions on it later. The first capture group checks if the first character, in the substring that is currently being checked, is not any of the operators or brackets[8]. The not is signified by the first $\wedge$. The second capture group checks if the second character, in the substring that is currently being checked, is a constant/variable [9], signified by the *"a-z"* or is an opening bracket *"("*. If both capture groups return true then a match is found and the match is replaced with *"\$1*\$2"* where *"\$1"* is the first capture group, *"\$2"* is the second capture group and the * asterisk between them signifying the multiply.

The `RegEx` expression for the second case is **"([\)a-z])([∧\(\)\+\-*\/\∧])"** with the replacement expression being **"\$1*\$2"** again. This expression does the same as the first but checks for the reverse order i.e. xa and instead checks for a) instead of (as we are checking the back of a substring instead of the start.

For the third inconsistency, the `RegEx` expression is **"\)\("** with the replacement expression being **")*("**. This expression returns a match if it finds a) followed by a (. If it finds a match it then replaces the entire match with *")*("*.

For the fourth inconsistency, the `RegEx` expression is **"([\+\-*\/\∧])-([∧\+\-*\/\(\)\∧]*)"** with the replacement expression being **"\$1(-\$2)"**. The `RegEx` expression returns a match when there is an operator followed by a minus sign followed by any number of characters that are not operators or brackets. There are two capture groups. The first is *"([\+\-*\/∧])"* and this captures the operator. The second is *"([∧\+\-*\/\(\)\∧]*)"* and this captures the expression after the minus sign. The match is then replaced by the first capture group, followed by an opening bracket, a minus sign, the second capture group, then a closing bracket.

For the fifth inconsistency, the `RegEx` expression is **"(∧|\()-"** with the replacement expression being **"\$10-"**. The `RegEx` expression returns a match when it is either the start of a line, signified by the $\wedge$, or[10] a (followed by a minus sign. The start of the line of bracket is captured and is used in the replacement expression, when the match is replaced with a 0- preceded by either a bracket or nothing depending on if the start of a line or an opening bracket was captured.

---

[8] the reason there are so many backslashes is because a lot of the operators are actually key characters in `RegEx` and a blackslash is an escape character which means that it signifies to treat the next character as a pure character

[9] While we are only dealing with the variable x at the moment, this allows for constants to be used in the future and be dealt with correctly.

[10] the or keyword in `RegEx` is signified by |

6.2.6.4   Creating the Tree and the Stack

Using the functions above we can create a syntax tree from our expression. Before we dive into this, remember that we will convert this into a `Stack` later to process, since it is more efficient (section 6.2.4). To create the `Tree` we will use the Divide and Conquer methodology by recursively splitting the original expression until it becomes a single constant or variable. We can know if we should split the expression and if so where we should split the function by using our Least Significant Operator function. Our recursive case will return a `Tree` where the left and right nodes are made up of the trees of the two sub-expressions and the root node will be the operator that we split our original expression with. Our base case will return a `Tree` containing the constant or variable that is remaining. For example if we have the expression "$x^2 + 4$", we will first split this expression by the least significant operator which is the "+". We then make this our root node and our left and right trees will be what is left and right of that operator. This will be repeated for each sub-tree until only no operators remain. So our bottom nodes will be x, 2 and 4.

Algorithm 8: Create a Binary Tree for an Algebraic Expression

1   **function** *createTree(* **String** *expression)*:
2   expression = checkBracket(expression)
3   **Integer** leastSigOperatorPos = leastSigOperatorPos(expression)
4   **if** *leastSigOperatorPos == -1* **then**
5   **return new** Tree(expression)
6   **else**
7   **String** operator = expression[leastSigOperatorPos]
8   **String** a = expression.substring(0, leastSigOperatorPos)
9   **String** b = expression.substring(leastSigOperatorPos+1,expression.length - 1)
10   **return new Tree** (operator,createTree(a),createTree(b))
11   **end**
12 **end**

Within this algorithm I can use concurrent processing to process each sub-expression individually. To do this I will create 2 threads to create the 2 individual sub-trees. I will then start these threads and on the root thread wait for these threads to finish (usually done using a method called "join"), and then return the new `Tree` object made up of the 2 individual sub-tree.

Algorithm 9: Create a Binary Tree for an Algebraic Expression (Using Concurrent Processing)

1   **function** *createTree(* **String** *expression)*:
2   expression = checkBracket(expression)
3   **Integer** leastSigOperatorPos = leastSigOperatorPos(expression)
4   **if** *leastSigOperatorPos == -1* **then**
5   **return new** Tree(expression)
6   **else**
7   **String** operator = expression[leastSigOperatorPos]
8   **String** a = expression.substring(0, leastSigOperatorPos)
9   **String** b = expression.substring(leastSigOperatorPos+1,expression.length - 1)
10   **Thread** threadA = **new Thread** (createTree(a))
11   **Thread** threadB = **new Thread** (createTree(b))
12   threadA.join()
13   threadB.join()
14   **return new Tree** (operator,createTree(a),createTree(b))
15   **end**
16 **end**

6.2.7 Substitute Algorithm

To actually plot the functions we need to be able to substitute values into the actual functions to know where to draw our points. Eventually we should be able to substitute multiple variables/constants into our function, but for now we will only substitute an x value into our function. This can be an extension for my next iteration.

Now algorithms 3 and 4 on page 23 traverse a tree and output a list (we will implement this using a stack, within the constructor). However the most significant operations and values, i.e. the actions we would do first according to **BIDMAS** [11] , are at the front of the list. This means that in a stack they would be at the bottom, which means they would be popped off last which is not what we want when we evaluate a function. While we can reverse the stack, this is inefficient and also defeats the purpose of using a stack.

However this is fixed by the need of this `substitute` algorithm. We will need to traverse this stack to replace the variables/constants with actual values anyway, so while we do this we can reverse the stack, fulfilling to purposes.

To accomplish this we will create a new stack, of the same height as the original, and then pop each value of the original stack, checking if it is a variable, and if it is substituting the value required. The value will then be pushed onto the new stack. This new stack will then be returned. We also will need to create a copy of the old stack and manipulate that so that we retain the original stack for other substitution calls.

Algorithm 10: Substitute into a function

```
 1 function substitute(Double x):
 2     Stack copy = postFixStack                              // Copy the Post-Fix Stack
 3     Stack substituteStack = new Stack (this.postFixStack.getHeight())  /* Create new Stack the
        same size to reverse it into */
 4     for i=0 to copy.getHeight() do
 5         String pop = copy.pop()
 6         if pop == "x" then
 7           │ pop = x
 8         substituteStack.push(pop)
 9     end
10     return substituteStack
11 end
```

---

[11] **BIDMAS** stands for **Brackets**, **Indices**, **Division**, **Multiplication**, **Addition** and **Subtraction**. It signifies the order that we must do operations on an expression. It is sometimes called **BODMAS** where the **O** stands for **Orders**.

6.2.8 Evaluate Algorithm

The `evaluate` algorithm is very similar to algorithm 2, the stack-based evaluate algorithm. However we will pass in the inputs as parameters and use the `substitute` algorithm within our `evaluate` algorithm to replace our variables with the actual values. We will also hardcode the selection and verification of which operator to use instead of assuming that an operator is an object we can manipulate. A precondition of this algorithm is that we are assuming that the original expression is valid, otherwise this algorithm may not behave as intended.

Algorithm 11: Evaluate the Expression for a Value of x

```
 1 function evaluate(Double x):
 2     Stack substituteStack = substitute(x)
 3     Stack evaluateStack = new Stack (postFixStack.getHeight())
 4     for i=0 to substituteStack.getHeight() do
 5         String pop = subStack.pop()
 6         Double a,b
 7         switch pop do
 8             case "+" do
 9                 b = evaluateStack.pop()
10                 a = evaluateStack.pop()
11                 evaluateStack.push(a + b)
12                 break
13             case "−" do
14                 b = evaluateStack.pop()
15                 a = evaluateStack.pop()
16                 evaluateStack.push(a − b)
17                 break
18             case "∗" do
19                 b = evaluateStack.pop()
20                 a = evaluateStack.pop()
21                 evaluateStack.push(a ∗ b)
22                 break
23             case "/" do
24                 b = evaluateStack.pop()
25                 a = evaluateStack.pop()
26                 evaluateStack.push(a/b)
27                 break
28             case "∧" do
29                 b = evaluateStack.pop()
30                 a = evaluateStack.pop()
31                 evaluateStack.push($a^b$))
32                 break
33             default do
34                 evaluateStack.push(Double (pop))
35                 break
36         end
37     end
38     end
39     return evaluateStack.pop()
40 end
```

6.2.9   Constructor and the Class as a Whole

The constructor for the Function class will be quite simple. We take an expression as an input and:

1. Remove Whitespace from the expression

2. Standardize the expression (using the RegEx rules)

3. Create the Binary Tree

4. Create the Post-Fix Stack from the Binary Tree

Algorithm 12: Function Class Constructor

```
1 function Function(String expression):
2 │   expression = expression.strip()
3 │   expression = standardize(expression)
4 │   this.expression = expression this.binaryTree = createTree(this.expression)
5 │   this.postFixStack = this.binaryTree.traverse()
6 end
```

The class as a whole will look something like the diagram below.


```
                    Function
─────────────────────────────────────────────
  ▭ Function(String expression)
  ▭ double evaluateStack(double x)
  ▭ Stack substitute(double x)
  🔒 Tree createTree(String expression)
  🔒 String checkBracket()
  🔒 String standardize(String expression)
─────────────────────────────────────────────
  🔒 String expression
  🔒 Stack postFixStack
  🔒 Tree tree
```
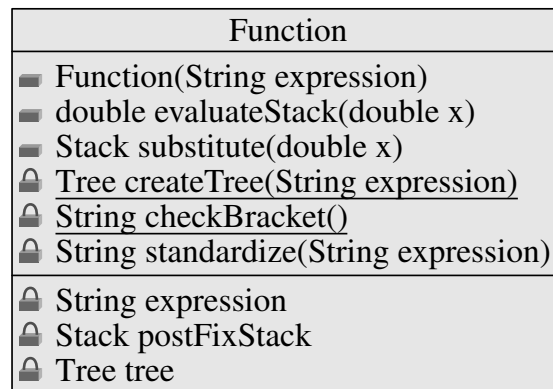
Figure 6.2: Function Class

6.3   Graphical User Interface

The Graphical User Interface will be achieved using one the many toolkits within `Java`. Specifically I will be using `JavaFX[9]`. `JavaFX` is unique in the fact that the GUI can be generated using two approaches:

1. Within the code itself, we can define how the GUI[10] is comprised. This is done by instantiating Objects of specific component Classes, such `Pane`, `Label`, `Button` etc. all inheriting a top class called `Node`, and using their methods to join them together. The advantage of this approach is that since these are simply classes, we can create child classes from them allowing us to create custom components that we can control.

2. The UI is defined using `.xml` files (aptly named `FXML[11]` files). A controller class is defined alongside it. This controller class is referenced within the the `FXML` file and when the `FXML` file is loaded, the controller class is initialized along with it. This controller allows us to "control" the UI nodes. While this approach is very simple to implement and the design of the UI looks clean, it can be somewhat convoluted to allow the controller classes to interact with other controller classes.

The real strength of `JavaFX` however comes from the fact that these two approaches can be intertwined as needed, making solutions unique and versatile.

The GUI can be decomposed into smaller parts as shown by the hierarchical diagram shown below. The parts in **bold** are the parts to be implemented and designed in Prototype I and the rest will be completed in Prototype II.
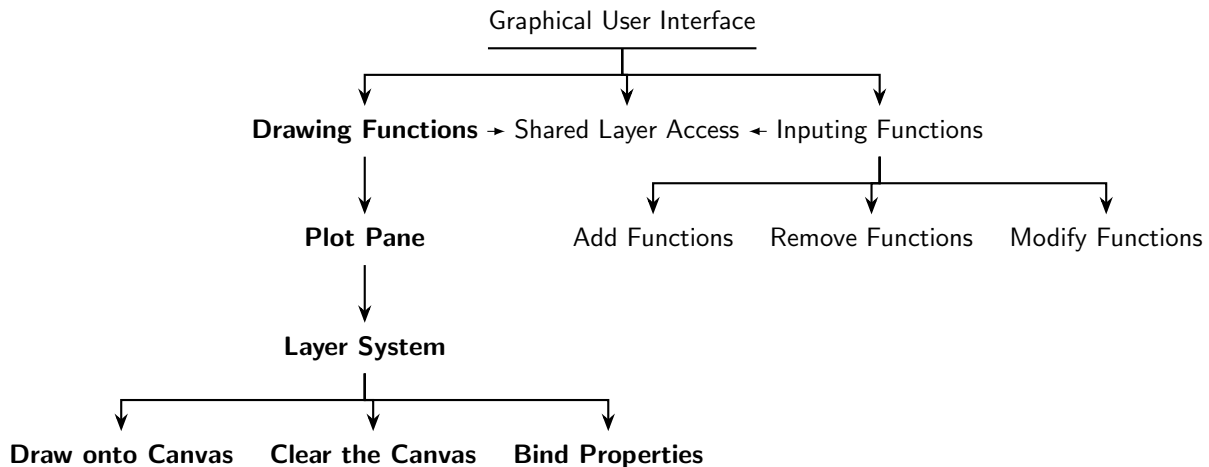

Figure 6.3: Graphical User Interface Hierarchical Diagram

Many of the features above will be discussed in greater detail later, however the Shared Layer Access and Properties will be touched upon now.

As mentioned before it can be hard to make two controller classes interact, as such one solution[12] is an object that both controller classes can reference. This class will contain attributes and methods that both classes need to interact with each other.

However, how do these classes know when something changes and hence know when to update their corresponding UI elements. The answer is through a class called `Property[13]`. A `Property` can bind another `Property`, one updating when the other one changes. Objects called `Listener` can also be added to properties. A `Listener` can be programmed to do a specific action when a `Property` is updated. This action could be a function, or updating a variable. Using the class `Property` allows for a responsive UI, updating whenever the user does an action.

6.3.1 Overview of the Graphical User Interface

The GUI will look something like:

Main-Ribbon	
Input-Equations $y = x^2$ New-Equation	Plot

Figure 6.4: UI Design

as showcased in the analysis.

Within `JavaFX` there is a hierarchy[14] of a standard GUI components. This is shown below:

Graphical User Interface

↓

Stage

↓

Scene

↓

Node 1   Node 2   Node 3   ...

Figure 6.5: `JavaFX` Components Hierarchical Diagram

Now we can manipulate the the nodes beneath `Scene`. It is standard to make the first `Node` beneath `Scene` some kind of `Pane`. For this project a `BorderPane` is most suited since it has multiple objects of type `Region`, that can hold one `Node` each. A diagram of this is shown to the right showcases how a `BorderPane` is arranged[15] and way it behaves.

The center `Region` can consist of the plots themselves, and the left `Region` can consist of the input. The top `Region` could be used for the ribbon and other buttons to manipulate the plot.
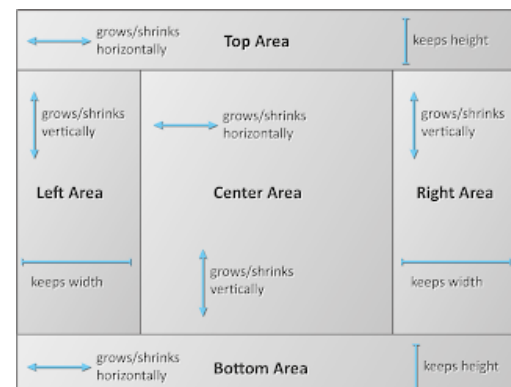

Figure 6.6: `BorderPane` Architecture

6.3.2 Layer System

Drawing onto the screen using `JavaFX` can be achieved using the `Canvas[16]` object. This is a node, which allows us to draw shapes, lines, arcs, on to the screen. As discussed in the analysis, we can approximate a curve by drawing many lines however one of our requirements was to draw multiple functions. From the article cited above, there are two ways to achieve this:

1. Draw every function onto one `Canvas` and add this to the root `Pane`.

2. Create a `Canvas` for every function and draw each function onto the corresponding canvasses and add all these canvasses onto the root `Pane`. One canvas will be paired with one function and bundled `Layer`. This only works because canvasses are by default transparent.

The advantage of the first is that it is simple to implement. However if there are multiple types of functions, i.e. an explicit function and an implicit function, then each one of these functions must have a draw function associated with it. This can become slightly convoluted, because then we must check what type of function something is, before we draw it.

Although the second is harder to implement, it is by far the superior. A `Canvas` is essentially an abstracted `OpenGL` view, and hence it is almost impossible[17] to draw multiple things on the same canvas at once. However it is possible to draw onto multiple canvasses at once, which boosts performance on multicore system. This also means that we can create a class for each type of function (explicit, implicit, parametric, etc.), making this system more modular. It is important to realize that only the topmost `Canvas` receives input (keyboard and mouse events) and hence we can create a `Layer` solely to handle input.

However before we handle the canvasses themselves, we must concern ourselves with how we can draw a line at a specific place.

6.3.2.1 Coordinate Systems

The coordinate system that we are familiar with is the Cartesian Coordinate System. Cartesian Coordinates take a fixed point as reference, usually called the origin (symbolized by O), and specify a point by taking the distance from the origin in n directions, where n is the dimension of the system. The directions are vectors, that are all perpendicular to each other, and have magnitude 1, i.e. unit vectors. This graphing software will be dealing with a 2-Dimensional Cartesian Coordinate System and in this system, up and down are defined as the y direction, with up being positive and down being negative; whereas right and left are defined as the x direction, with right being positive and left being negative. Within our graphing program we will only show a section of the x–y plane. This is because:

1. It is intractable to show the entire x–y planesince the plane goes on for an infinite distance in both the x and y directions.

2. It is not helpful to the user to see the entire plane even if they could. A graphing tool should enable the user to highlight parts of the graph that is important to them.

However a `JavaFX Canvas` doesn't exactly use a Cartesian Coordinate System. It specifies a position on itself taking the top-left corner as the origin, right as the positive x direction and down as the positive y direction. It is important to realize that negative coordinates do not exist in this system. The maximum x and y values are the width and height, respectively, in pixels.

We must therefore find a way to convert between these two coordinate systems. Since our viewport is predefined, let us label what the maximum and minimum x and y values in our viewport (with respect to our original Cartesian Coordinate System). Let these values be `minX`, `maxX`, `minY` and `maxY`.

Now let us define how much the width and height of each pixel on the canvas is ''`worth`''. This *"worth"* specifies how much going across or down a pixel width or height, is in terms of our original Cartesian Coordinate System. This is best demonstrated with an example. If our canvas shows the x–y planein the x direction from 0 to 10, and our canvas is 1000 pixels wide, then our pixel worth is the amount we have

traversed in the x direction, divided by the number of pixels, in this case $\frac{(10-0)}{1000} = 0.01$. The algorithm for this is shown below:

Algorithm 13: Calculate the *"Worth"* of each Pixel

---
1 **function** *updatePixelWorth()*:
2   pixelWorthX = (maxX - minX)/canvas.getWidth()
3   pixelWorthY = (maxY - minY)/canvas.getHeight()
4 **end**

Now that we have the pixel worth, we can convert between our coordinate systems. To do this we take the vector from the origin, $O = (x_{min}, y_{max})$, to the point, $P = (x, y)$ in the x or y direction. We then calculate how many pixel *"worths"* this is and this gives us the pixel on the canvas that this coordinate is at. It is important to realize that going down in a `JavaFX Canvas` is actually positive, relative to the canvas, hence our two algorithms are slightly different. This means our vector is actually

$$\vec{d} = \begin{bmatrix} (x - x_{min}) \\ (y_{max} - y) \end{bmatrix}$$

Hence our position within the canvas is:

$$\vec{p} = \begin{bmatrix} \left(\frac{x - x_{min}}{x_{worth}}\right) \\ \left(\frac{y_{max} - y}{y_{worth}}\right) \end{bmatrix}$$

The algorithms below convert between x and y coordinates [12] respectively.

Algorithm 14: Convert the x Coordinate

---
1 **function** *convertX(**Double** x)*:
2   x = x - minX /* Calculate the horizontal distance between the point and origin   */
3   x = x / pixelWorthX /* Calculate the number of pixel ''worths'' this is   */
4   **return** x
5 **end**

Algorithm 15: Convert the y Coordinate

---
1 **function** *convertX(**Double** y)*:
2   y = maxY - y /* Calculate the vertical distance between the point and origin   */
3   y = y / pixelWorthY /* Calculate the number of pixel ''worths'' this is   */
4   **return** y
5 **end**

---

[12]Here we assume the coordinate is inside the viewport. Realistically we should check if the coordinates are in the viewport, i.e. bigger than the width or height of the canvas, or in some cases negative. However it does not matter since we can make this check later. Even then a `JavaFX Canvas` will check if the coordinates are in the canvas viewport anyway, so it does not matter to us since this detail is abstracted away from us.

6.3.2.2   Drawing the Function

From the analysis (Section refsec:computability) I briefly described how I could approximate a curve by creating a finite number of line segments.

Firstly we take the minimum value of x, which in the viewport is x_{min}. If our function is defined by $f(x)$ then let the coordinate of the function at this value be $P_1 = (x_{min}, f(x_{min}))$. We then take a second coordinate a small value dx away, so let the coordinate be $P_2 = (x + dx, f(x + dx))$. We now draw a line between these two points. We then make $P_1 = P_2$ and make P_2 with $x + 2 \cdot dx$. We repeat this process until $x + n \cdot dx \geq x_{max}$. We obviously need to convert the x and y values from the Cartesian coordinates into the canvas coordinates do we can do this when drawing the line. dx will also be a finite value that is very small. We will define this by taking the number of line segments we want, and working out dx for that number of steps. This is simply done by finding out how much each step is "worth" within the viewport, i.e. $dx = (\mathtt{maxX} - \mathtt{minX})/\mathtt{steps}$. The algorithm for this is shown below.

Algorithm 16: Draw a Function in the Viewport

```
 1 function draw():
 2     Double x1 = minX
 3     Double y1 = f.evaluate(x1)
 4     Double dx = (maxX - minX)/steps
 5     for x2=minX + dx to maxX by dx do
 6         y2 = f.evaluate(x2)
 7         drawLine(convertX(x1), convertY(y1), convertX(x2), convertY(y2))
 8         x1 = x2
 9         y1 = y2
10     end
11 end
```

A precondition of this is that `f` is a valid function. We also need to make sure we do not divide by 0, but this can be sorted out in the implementation.

6.3.2.3   Constructor and Class as a Whole

The layer class we have described in the previous sections, describe a layer for an explicit function, in terms of x, in a Cartesian Coordinate System. To take full advantage of an OOP approach, as described at the start of this section, we can use layers for input, drawing axes etc. Hence we will create a superclass called `Layer` and then make our other types of layers inherit this class. This superclass will never be instatiated, in OOP terms it is abstract. The superclass will have some basic methods and attributes which we have used during this design, such as `draw()`, the attributes of the viewport (`minX`, `maxX`, etc.), the functions to convert between coordinate systems, where some of the functions such as `draw()` will be overriden. Our `Explicit Function` and `Axes` classes are the only layers we will implement (apart from the superclass layer) in prototype 1. The input layer will be part of prototype 2. While we haven't talked about the axes layer, this will simply be drawing the lines $y = 0$ and $x = 0$. This is done just for reference so that we can verify that the functions are drawn somewhat accurately.

The constructor for the Explicit Function Layer is shown below.

Algorithm 17: Explicit Function Layer Class Constructor

```
 1 function ExplicitFunctionLayer(String function):
 2     super()
 3     this.f = new Function(function)
 4 end
```

While the superclass will never be instantiated, it can still contain a constructor. Here it will be used to initialize the canvas and its `OpenGL` context, in a standardized manner, so that we do not have repeated, redundant code for no reason.

Algorithm 18: Layer Class Constructor

1 function *Layer()*:
2   this.canvas = new Canvas()
3   this.gc = canvas.getGraphicsContext2D()
4 end

The class diagrams for the `Layer` [13] classes are shown below:

Figure 6.7: `Layer` Classes

---

[13]The inherited methods and attributes are protected so that they can actually be viewed and accessed by the child classes.

6.4 Plot Pane

The purpose of the `PlotPane` is to manage the layers. The `PlotPane` will tell them when to draw and notify them when something changes through the use of properties. Our `PlotPane` will inherit the `JavaFX Pane`. While we could have used `FXML`, `FXML` is more applicable when there are complicated UI components that are hard to arrange and manage. In this situation we only need to have one `Pane` hence there is no need to use `FXML`. Inheriting `Pane` also allows it to integrate well within the `JavaFX` hierarchy.
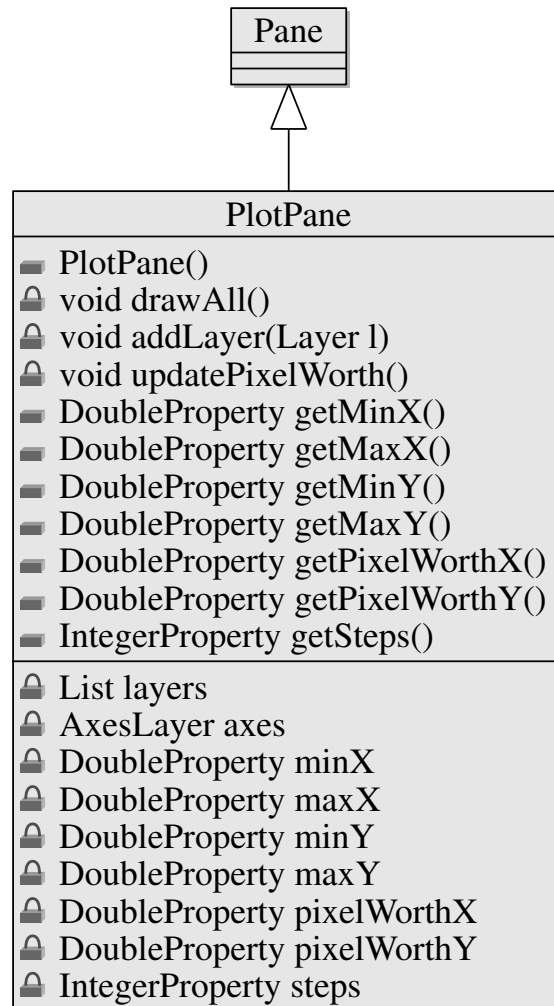

Figure 6.8: `PlotPane` Class

The `drawAll()` procedure will call the draw function for every `Layer`. We will start by doing this on a single thread, and then we will implement it using multiple threads. The `addLayer()` function will add the `Layer` to the list, and bind the layers' properties to the `PlotPane`.

Chapter 7

Implementation

7.1   Stacks and Trees

7.1.1 Stacks

I will be implementing Stacks using a concept called *Generic Programming[18]*. This is best explained with an example.

Imagine we have a data structure which will only ever contain one type of object but we will use this data structure in different scenarios, each with different types of object[1]. Now we can use an OOP approach where we have a base abstract class called `Structure` and then we create other classes that are related to a certain type of object and make it inherit this class, e.g. `IntegerStructure` or `StringStructure`. However this can become unfeasible to do this for every single object that we use this structure for.

A more elegant approach is to use *Generics*. This is where we link a type of object with the structure. Let this type of object be of type `T`. Now we make our structure of type `T` and make all of its methods take or return an object of type `T`. This allows the structure to be versatile and be used for any object.

Here is the implementation I have created in `Java` using *Generics[19]*:

```java
package structures;

import exceptions.StackOverflowException;
import exceptions.StackUnderflowException;

public class Stack<T> {

        //attributes
                //pointer variables
                private int maxHeight;
                private int height = 0;
                private int pointer = -1;
                //stack
                private T[] stack;

        //methods
        //constructor
        @SuppressWarnings("unchecked")
        public Stack(int maxHeight) {
                this.maxHeight = maxHeight;          //set this max height
                //create an array of this size of type "T"
                this.stack = (T[]) new Object[this.maxHeight];
        }

        //push an item on to the stack
        public void push(T push) throws StackOverflowException {
                if(height == maxHeight) {          //check that the stack isn't full
                        throw new StackOverflowException();
                } else {
                        //make the element one above the top, the new value
                        this.stack[this.pointer+1] = push;
                        this.pointer++;          //increment pointer variables
                        this.height++;
                }
        }
```

---

[1]Another restriction here is that we are doing operations on the structure not on the objects themselves, hence the operations (implemented as methods) must be generic, another reason for the name *Generic Programming[18]*.

```java
        //pop an item off the stack
        public T pop() throws StackUnderflowException {
                if(this.isEmpty()) {            //check that the stack isn't full
                        throw new StackUnderflowException();
                } else {
                        //get the value of the top element
                        //no need to make it null, it is a waste of an instruction
                        //it also releases no memory since we are using
                        //an array to store our stack which is a static structure
                        T pop = this.stack[this.pointer];
                        this.pointer--; //decrement pointer variables
                        this.height--;
                        return pop;            //return the popped value
                }
        }

        //check if stack is empty, return true if so
        public boolean isEmpty() {
                return  this.height==0 ? true : false;
        }

        //return the height of the stack
        public int getHeight() {
                return height;
        }

        //allow for a visualization of the stack by listing it
        //with the top element being encapsulated in square brackets
        @Override
        public String toString() {
                if(this.isEmpty()) {            //if empty notify the user
                        return "Stack is Empty";
                } else {
                        //highlight the top element
                        String out = "[" + this.stack[this.pointer] + "]";
                        //loop through the rest of the stack backwards
                        //concatenating each element to a string
                        for(int i=this.pointer-1; i>=0; i--) {
                                out = out + ", " + this.stack[i].toString() ;
                        }
                        return out;            //return this string
                }
        }

}
```

There are two custom Exception classes that I have created for this class, StackOverflowException and StackUnderFlowException, in a separate package. These simply output a different message to the standard Exception class. The code for this is at the end of the documentation.

To just make sure that my code is working as intended I tested 4 things:

1. Popping and pushing items

2. Overflowing the stack

3. Underflowing the stack

4. Copying a stack (this is needed for the substitute algorithm)

Here is the code for test 1:

```java
//add two items, pop one off, add another one
Stack<String> stack = new Stack<String>(4);
stack.push("item 1");
System.out.println("Stack -> "+stack); //expecting item1
stack.push("item 2");
System.out.println("Stack -> "+stack); //expecting items 1 and 2
String pop = stack.pop();
System.out.println("Popped item -> "+pop); //expecting items 2
stack.push("item 3");
System.out.println("Stack -> "+stack); //expecting items 1 and 3
```

Here was the output for test 1:

```
Stack -> [item 1]
Stack -> [item 2], item 1
Popped item -> item 2
Stack -> [item 3], item 1
```

Which is exactly what I expected.

To spice things up for test 2 I made a stack of integers instead of strings just to make sure the generics part is working properly. Here is the code for test 2.

```java
//create a stack of max height 4
Stack<Integer> stack = new Stack<Integer>(4);
//push 4 items, the max amount
stack.push(1);
stack.push(2);
stack.push(3);
stack.push(4);
//show that the stack has 4 items
System.out.println("Stack -> "+stack);
//push another item
stack.push(5);
```

Here was the output for test 2:

```
Stack -> [4], 3, 2, 1
Exception in thread "main" exceptions.StackOverflowException:
Stack has reached its max height
        at structures.Stack.push(Stack.java:40)
        at structures.Stack.main(Stack.java:107)
```

Which is exactly what I expected.

Again for test 3 I made a stack of doubles. Here is the code for test 3.

```
//create a stack
Stack<Double> stack = new Stack<Double>(4);
//try and pop an item off
stack.pop();
```

Here was the output for test 3:

```
Exception in thread "main" exceptions.StackUnderflowException:
Stack has no items for you to pop
        at structures.Stack.pop(Stack.java:52)
        at structures.Stack.main(Stack.java:100)
```

Which is exactly what I expected.

For test 4 I made a stack of booleans. Here is the code for test 4.

```
//create a stack
Stack<Boolean> stack = new Stack<Boolean>(4);
//pop some items
stack.push(true);
stack.push(false);
//assign the value of the old stack to the copy stack
Stack<Boolean> copy = stack;
//pop an item off the copy stack to test that they are independent
copy.pop();
//output both stacks
System.out.println("Stack -> "+stack);
System.out.println("Copy -> "+copy);
```

Here was the output for test 4:

```
Stack -> [true]
Copy -> [true]
```

This is not what I expected. I expected the original stack to have an extra item. It appears as if they are mimicking each other, like they are sharing the same memory address. When I researched this issue[20] it appears that this is true. In Java every variable is a reference so when you assign a variable the value of an object, you actually assign the variable a reference. The exceptions to this are when you pass variables into functions, this is done by value not reference [2] , and with primitives [3]. The suggested solution was to use Serialization[21]. This is where an object is serialized into a byte array and then deserialized into an object again. This works because we are effectively converting an object into a primitive and then assigning this to a new value, which bypasses the reference issue. However this can be a hit on performance. So why don't I use the same principle in my code. Since all my attributes are primitives, if I just create a copy constructor which assigns the new stacks' attributes' values the original stacks' attributes, the problem should be solved.

---

[2]It actually cannot be done by reference in Java

[3]A language's base types of object that are inherently part of the language. In Java the primitives are int, double, float, boolean, char, byte, short, long.

I made a constructor that takes an existing stack as a parameter to solve the issue. In many languages this is called a *Copy Constructor*. The code for this is below:

```java
//copy constructor
public Stack(Stack<T> copy){
        //assign the new stacks' attributes' values the original stacks' attributes
        this.pointer = copy.pointer;
        this.height = copy.height;
        this.maxHeight = copy.maxHeight;
        this.stack = copy.stack;
}
```

For my new test I slightly modified it from the original by using the copy constructor. Here is the code for new test:

```java
//create a stack
Stack<Boolean> stack = new Stack<Boolean>(4);
//pop some items
stack.push(true);
stack.push(false);
//assign the value of the old stack to the copy stack
Stack<Boolean> copy = new Stack<Boolean>(stack);
//pop an item off the copy stack to test that they are independent
copy.pop();
//output both stacks
System.out.println("Stack -> "+stack);
System.out.println("Copy -> "+copy);
```

Here was the output for the test:

```
Stack -> [false], true
Copy -> [true]
```

This is what I expected.

7.1.2 Trees

7.2 Function Class

7.3   Layer System

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