

Graphing - Programming Project

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Part I

Analysis

Chapter 1

Introduction

For my project I will attempt to make a graphing software. Graphing software is incredibly important in Linear Algebra and a lot of maths taught in schools is to do with Linear Algebra. Linear Algebra also links to many other aspects of Maths and hence is very important to understand. My stakeholders will consist of teachers, who will use my software to show graphs of functions, and to students who will use it to practise their graph sketching or to help them with their homework. There are many graphing tools out there, some of them shown below, however they have many downsides. I hope to make a piece of software that has as much functionality as possible, while retaining simplicity and reducing the number of downsides to an absolute minimum.

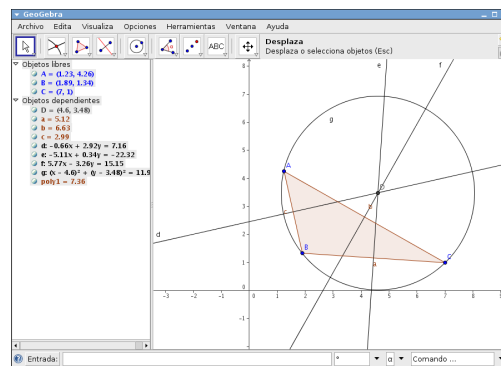


Figure 1.1: GeoGebra

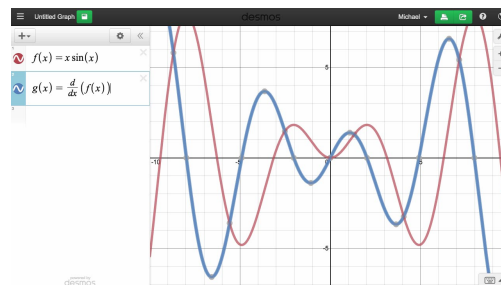


Figure 1.2: Desmos

Chapter 2

Stakeholders

My stakeholders will be comprised of teachers and students in year 10 and above. This is because graph sketching as a skill becomes very important from GCSE onwards and as such the tool will be aimed towards students in the upper years and the teachers themselves. I have interviewed several of my potential stakeholders about the software that they use and their opinions on it.

2.1 Teacher Interview

I interviewed Miss Naguthney who is a Maths teacher who teaches pupils from year 7 to 13. She uses multiple tools to aid her teaching. I asked her a couple of questions to get an idea of what she finds important in a graph drawing software.

Question	Answer	Analysis
What graph drawing software have you used?	I have used Desmos, MATLAB and GeoGebra.	These three programs are very different, ranging from a simple graph drawing software, Desmos, to a professional data presentation software, MATLAB.
Which do you think is the best and why?	I think all 3 are good. In terms of pure graph drawing I think that Desmos is the best due to its simplicity.	From this response, it is important that my program is as simple as possible so that anyone can use. I think that to make program simple it must be responsive as well, and hence I should make my solution as efficient as possible so that it can be responsive on most devices.
What do you think is the most important aspect of a graph drawing software?	The most important part of a graph drawing software are obviously the graphs themselves and the most important part of the graphs are of course their intersections with axes, turning points and any asymptotes.	I agree with all these points made and I think that my program should at least be able to identify the turning points and any intersections with the axes.

2.2 Student Interview

I find that during my sixth form studies, that graph sketching is a must have skill, and hence a graph drawing tool is important to verify that you have drawn a graph accurately. I interviewed Matthew, who is a sixth form student studying similar subjects to myself, about graph drawing tools that he has used and his opinions about them.

Question	Answer	Analysis
What graph drawing software have you used?	GeoGebra, Desmos and KAlgebra (KDE Application)	These three programs are quite similar as they have many of the same features.
Which do you think is the best and why?	<p>I would say GeoGebra because:</p> <ul style="list-style-type: none"> • Native client so it is more responsive than a web-based app • Versatile • Easily Adjustable Axes • Very easy to focus on a part of the graph • Multiple function support <p>However the UI looks ugly and has no dark mode to reduce strain on the eyes.</p>	I think that my program, should have the ability to be themed, through using CSS or a text file and while I may not be able to make my program versatile, it should still be able to draw multiple functions at once and navigating the graph must be fluid.
What do you think is the most important aspect of a graph drawing software?	The two most important aspects for me are ease of use and flexible input. Specifically this would be stuff like being able to input complex functions such as sums of multiple rational functions ¹ (Not complex as in $x \in \mathbb{C}$) and the ability for the software to automatically adjust the scale. An example of this is for trigonometric functions sine and cosine. They don't have high y values but they always appear very small on GeoGebra because the scale is wrong. A way to automatically set a suitable scale would be nice. Finally, I feel performance is also a must - it's frustrating to navigate the graph and have the application lag a lot.	Matthew's answer can be summarized into two points: Responsiveness and Flexible Input. I think that both of these points are valid and I should aim to make sure my application satisfies both these points.

¹A function $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are functions of x

Chapter 3

Similar Programs

During my interviews with my stakeholders, they mentioned several pieces of software that they have used. I will be analysing 3 of those programs in particular (or similar in the case of MATLAB, since MATLAB is extremely expensive), which are:

1. Desmos [1]
2. GeoGebra [2]
3. GNU Octave[3] (MATLAB[4] Clone)

3.1 Desmos

Desmos is a closed source graphing calculator written in HTML5, which allows it to be used on many devices. It has dedicated apps on Android and on Apple devices, but has no dedicated application on a desktop environment.

The real strength of Desmos comes from its ability to create activities that students can then complete.

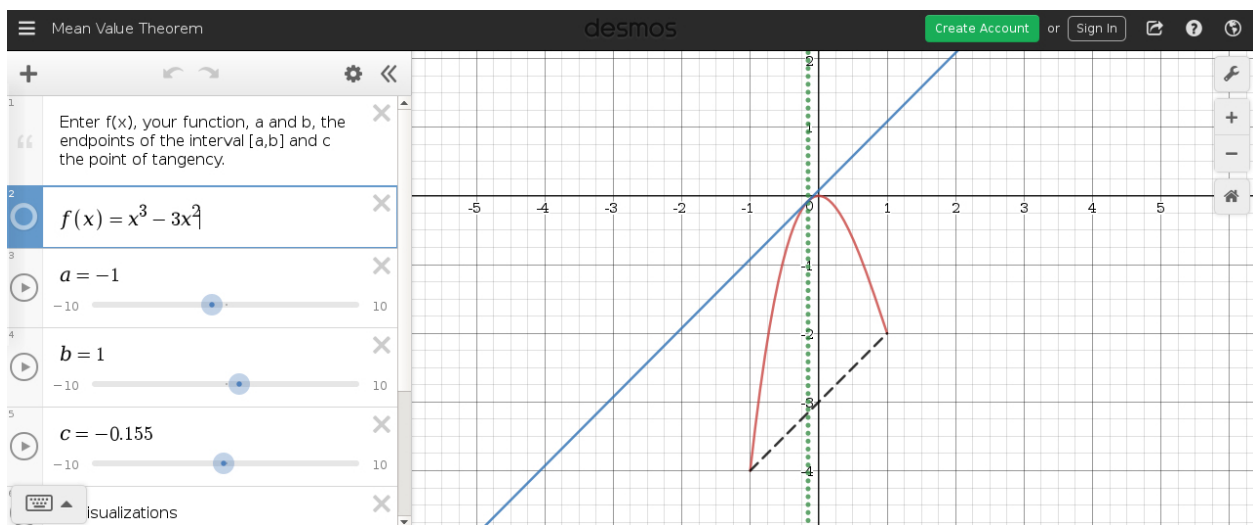


Figure 3.1: Mean Value Theorem example that students can interact with

The problem however is that it is web based and as such, can be clunky to use. The mobile app and website are especially unresponsive when zooming in or out and when panning around. Another pitfall is that Desmos tries to make itself more secure in a way by restricting input. This however makes input very difficult to do and along with the unresponsiveness, it can be impossible to input the required equation.



Figure 3.2: The Input

On the other hand, the actual graph drawing is excellent, with many features such as inequalities, modulus functions and the ability to draw polar functions. It also highlights turning points, intersections with axes, inflection points and in the case of trigonometric functions, it expresses them in terms of π instead of a numerical values.

Overall as a graphing calculator it does an excellent job, but has poor optimization issues and input, which I have realized are both important and need to be at the top of my list of requirements for my program.

In terms of other features, you can print, save and share your graphs to use later. I think that my program should have at least 1, if not all 3, of these features.

3.2 GeoGebra

GeoGebra is an interactive geometry, algebra, statistics and calculus application under the GNU General Public License, meaning that it is open source.

The main advantage to GeoGebra is that it can do a lot of things and is all of its features integrate well with each other. For example you can create a shape on the graph and create lines intersecting it, linking geometry with algebra. Individual line equations can also be moved around allowing for a very interactive experience. Like Desmos, you can create activities that you can interact with.

The mobile app for GeoGebra has a similar input to Desmos and as such suffers from the same problems. However it is more responsive than Desmos but only slightly. The desktop version however does not suffer from any of these problems.

As a pure graphing software, it does not automatically show the points of interest of a curve and is instead confined in a tool called the “Function Inspector”. This tool is awkward to use as it only shows local maximums and minimums within a certain range (the highlighted red region), and if there are multiple roots, it does not acknowledge what the values of the roots are. Desmos does better than GeoGebra in this regard. Like Desmos it supports many functions that you can plot such as modulus and inequalities.

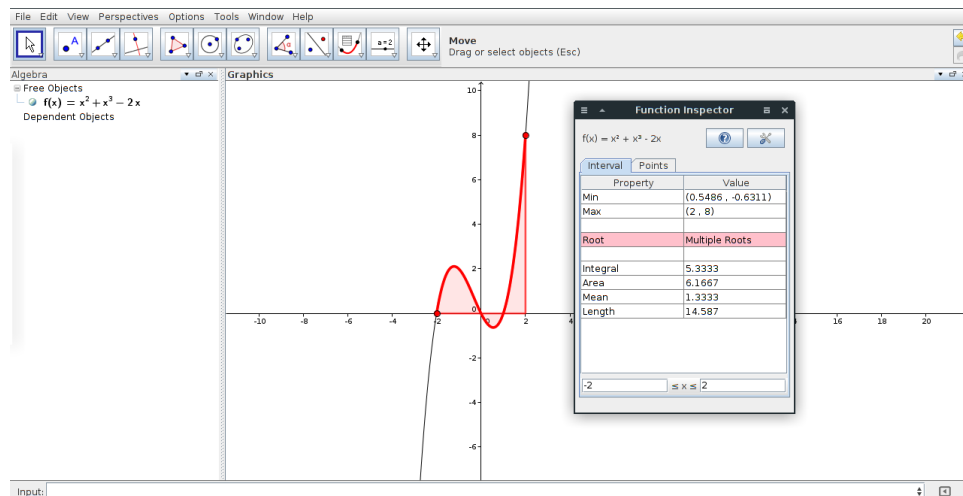


Figure 3.3: GeoGebra Function Inspector Tool

The panning and zooming in/out in GeoGebra is very fluid, with the zoom function being centred around the cursor. This means that you zoom in towards your cursor which feels more responsive than if it zooms in/out from the centre of the graph. The only criticism is that you can accidentally move already drawn graphs when trying to pan, which can be frustrating.

In terms of other features, like Desmos, you can save and print your work as well as being able to export it into other formats such as JPG, PNG or even convert it into a PSTricks or TikZ environment which can be used in L^AT_EX documents for excellent graphs.

3.3 GNU Octave

GNU Octave is essentially an interpreter for a high level language centred around numerical calculation. GNU Octave is designed to be an open source software clone of MATLAB (MATrix LABatory).

Octave is different from GeoGebra and Desmos in the fact that it plots points that you tell it to not lines. For example if you wanted to plot $y = x^2$, you would define an array for your range in x by doing:

$$x = -10 : 0.1 : 10;$$

This creates an array that starts at -10 increments by 0.1 until 10 . You would then define y by:

$$y = x.^2$$

You would then plot x and y by doing `plot(x,y);`.

As a graphing tool Octave is not very impressive, it has limited support for implicit functions and will not highlight points of interest of a curve. Realistically this is expected since Octave is primarily made for data presentation. This means that it can produce excellent plots, including meshes and surfaces ¹ in 3 dimensions. ²

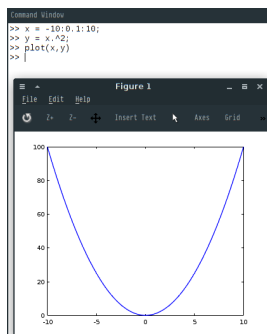


Figure 3.4: Creating a simple plot in GNU Octave

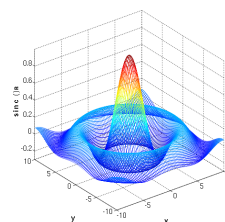


Figure 3.5: Wireframe 3D plot of the two-dimensional unnormalised sinc function

The way Octave plots (using the plot function at least) is by joining a line between consecutive points that it plots. I may use this in my program, since it seems more efficient to approximate a curve by making linear lines and joining them up than to check each pixel on the plot to check if it satisfies the given relationship.

When inputting an algebraic expression in Octave it can be quite awkward, for example in Figure 3.4, a period is before the carat. This is unnatural for us to input and as such can cause errors. Natural input is

¹A mesh is when a set of points in 3D space are joined up with lines to look like wire mesh. A surface is a mesh but adjacent points are used to make a solid shape instead, hence creating a surface.

²To clarify, a plot is where we explicitly define data that we then represent graphically, while a graph is where we define a relationship between 1 or more variables and represent this graphically.

something that I think I should focus on since it means that the program is easier to use and requires no further reading unlike Octave.

Octave is split into two views, the main editor view and a dockable plot view (you can also have multiple plots open at once). Dockable plots, or multiple plots could be something I implement later into my project.

Octave, like the other programs, can save plots and additionally the scripts you have made. It also has a feature that saves the current state when you close it, and resumes this state when you open it again. This could also be a feature I implement later into the project.

3.4 Comparison

Overall I think that I will be using aspects of all three of the programs that I have analysed in my program. I think I should try and make my program as simple to use as Desmos. As mentioned, the input is a problem, and hence I will try and implement an input system like GeoGebras'. All three of the programs analysed have some form of saving their plots, so I think that this is a feature that I should implement within my own program.

In terms of UI design, both Desmos and GeoGebra have a similar layout. They have an equation input box to the left, that can be minimized and the actual graph in the centre left. I think that this layout is easy to use for the user so I will implement a similar design in my own program.

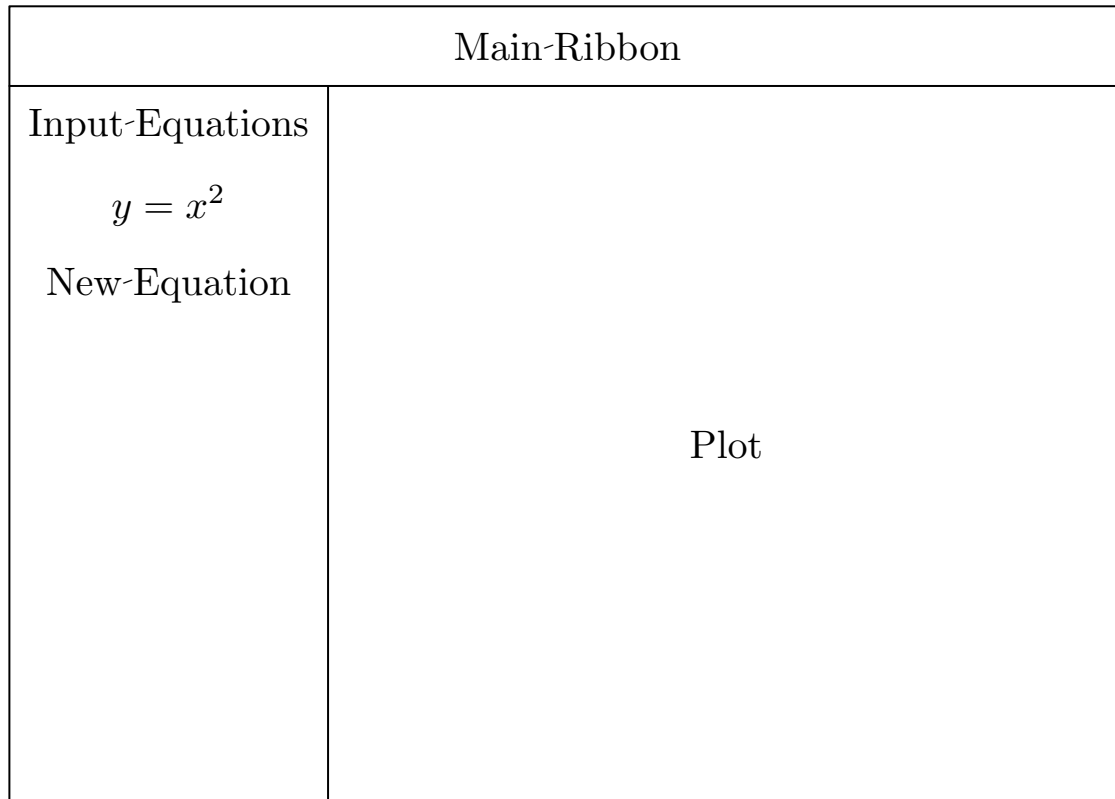


Figure 3.6: UI Design

Chapter 4

Computational Methods

4.1 Abstraction, Heuristics and Computability

Abstraction is the process of creating a model of a real-life problem by separating and highlighting important details. There are two main details that I can abstract away:

1. The input of expressions, and their conversion into a form that the computer can then manipulate.
2. The drawing of the function onto the screen.

By analysing both parts of these details separately, we can determine if our problem is intractable.

The first part of the problem is solvable by converting the expression into an abstract syntax tree or into a post-fix stack, which is easily implementable.

The second part of the problem is solvable using heuristic methods. Most APIs, such as OpenGL[5] and Vulkan, only allow the drawing of primitives, which are points, lines and triangles.¹ This means that our curve cannot be drawn straight onto the screen and must be approximated by either using points or lines, hence there are two ways we can approximate the curve:

1. Let x be our starting value, $f(x)$ our function and dx be a small value. Now draw a line between the points $(x, f(x))$ and $(x + dx, f(x + dx))$; $(x + dx, f(x + dx))$ and $(x + 2dx, f(x + 2dx))$; ... ; $(x + n \cdot dx, f(x + n \cdot dx))$ and $(x + (n + 1) \cdot dx, f(x + (n + 1) \cdot dx))$. Here we are effectively creating line segments and joining them together to approximate the curve. As $dx \rightarrow 0$ our approximation tends towards the real curve. However as $dx \rightarrow 0$ the time taken tends to infinity and therefore we only need to make dx small enough so that the human eye can not see individual line segments. Also as a practical note, as $dx \rightarrow 0$, the pixel density required to view the curve tends to infinity, therefore dx needs to be big enough so that the dx is not less than the individual pixel width.
2. In the method above we approximated a curve by creating line segments, here we will approximate a curve by using individual points. Let x be our starting value, $f(x)$ our function and dx be a small value. Now draw a point at $(x, f(x))$; $(x + dx, f(x + dx))$; $(x + 2dx, f(x + 2dx))$; ... ; $(x + n \cdot dx, f(x + n \cdot dx))$. This can take a lot of time since calculate where to draw a pixel w number of times, where w is the pixel width. In some cases, for complicated functions, it may be intractable.

By using the first method we can model a curve to good accuracy within good time, therefore our second part of the problem is computationally possible with a heuristic method.

¹Strictly this isn't true, but most other things we can draw using these APIs usually consist of a combination of primitives, usually triangles.

4.2 Logical Approach

Many parts of this problem require clear decision making, making sure that user inputs are valid and making my solution as efficient as possible. I could validate and standardize user input by using `Regex`[6]. I will definitely use standard structures in my solutions, and therefore will be able to logical traverse these structures to get the required solution.

4.3 Decomposition

Decomposition is where a problem is broken down into smaller sub-problems that are manageable and are solved individually and combined to make the complete solution. As stated in the abstraction section, I can break down the problem into two main parts. These parts could then be broken down further into classes and functions such as:

1. A function class which consists of:
 - (a) Converting an expression into a form the computer can manipulate.
 - (b) Evaluating the expression for a value x .
 - (c) Calculating the roots and turning points of the function.
2. A plot class which consists of:
 - (a) Drawing functions onto the screen
 - (b) Heuristically adjusts the scale so the graph looks good
 - (c) Drawing the axes

This is just an example of how I can decompose the problem and therefore the problem will be decomposed more.

4.4 Divide and Conquer and Concurrent Processing

Divide and Conquer is where a problem is recursively broken down into two or more sub-problems of the same or related type, until these become simple enough to be solved directly. While this may seem similar to decomposition, Divide and Conquer relates to algorithms. My solution may be using trees, and as such I can use a Divide and Conquer algorithm to convert an expression into a tree by looking at sub-expressions within the expression. This can then be further improved by using Concurrent Processing to process each sub-expression individually. I can also use Concurrent Processing to draw multiple lines or functions on the screen at once.

Chapter 5

Features

From my research and analysis I have created a set of features that I aim to implement. I have split these requirements into development iterations that they will be linked with. Each iteration has a theme that most of the requirements will follow.

- Iteration I - Emphasis on Core Functionality
 - Plot an explicit function in x
 - Plot multiple functions on the same plot
- Iteration II - Emphasis on the User
 - Let the user input functions
 - Zoom in/out of the graph
 - Pan around the graph
 - Plot special functions such as trigonometric functions, logarithms, modulus functions, etc.
 - Identify intersection with the axes
 - Multiple plots that you can switch between
 - Save plots as pictures
 - Save workspace to resume later
 - Dark Theme
- Iteration III - Advanced Features
 - Identify intersections between functions
 - Differentiate explicit continuous functions with respect to x
 - Polar equations
 - Implicit equations
 - Parametric equations
 - \LaTeX equation support
 - Identify turning points

Chapter 6

Success Criteria

Here is a list of success criteria that I will evaluate my prototypes against:

Criteria	Justification
Plots Polynomials, Exponentials and Rational Functions.	These are the functions that an A-Level Student would have to know how to sketch and as such my program should be able to sketch.
The accuracy of plotting function should be comparable to other graphing programs such as Desmos.	A-Level students will be using this program and they shouldn't be misled about what a function looks like.
The control of the viewport should be responsive.	One of my stakeholders, Matthew, said that he hated it when a graphing program lags, and so my program should not lag for most functions.
There are no major experience affecting bugs in the software.	The experience should be pleasant for the user since bugs are painful for the user.
The software should be able to run on most hardware since at least 7 years ago.	This links to the idea that the program should be responsive but also the fact that it should be accessible to anyone who wishes to use it irrelevant of the hardware they have.

I will use Java to accomplish this task. Java is platform independent meaning that I do not have to compile for many different systems. It also means that if I want to create a mobile version I will only have to recreate the UI. Within Java I will use JavaFX for my UI. JavaFX allows for customization of UI elements through css files, as well as an easy interface to draw objects onto through the screen, through its Canvas class. I am also comfortable with the language which makes it the most suitable language for this project.

Part II

Prototype I

Chapter 7

Design

7.1 Decomposition

My program will be decomposed into two main parts the GUI and the parsing of functions. The GUI will consist of the input of functions and the drawing of the functions. Many parts of this project can be achieved in a cleaner fashion using an OOP approach rather than a procedural approach.

7.1.1 OOP vs Procedural Approach Justification

An Object Oriented approach for my program is better than a procedural approach due to its complexity. An OOP approach allows parts of a program to be independent, and this means that these parts can be tested individually, allowing for simple debugging.

Specific to my project, Java is wholly based around the OOP approach and hence it makes more sense to use an OOP approach rather than a procedural one. Also many parts of my program can use the OOP concepts of inheritance and polymorphism. An example of this is the layer system I will be using later on to draw functions onto the screen. There are many different types of functions (again which can use inheritance), and each of these will have a corresponding layer. Each type of layer will draw, have very similar attributes and other similar features which will be very slightly different for each type of function. By making all these different layers inherit a base layer class, it means that all of them can be accessed generally, especially when drawing. They can also all be stored in an array of the type of the superclass, and accessed (outside of the class) in a standard way making the solution cleaner and easier to test.

7.2 Function Class

Our basic Function class will contain two main methods:

- Parse - This method will convert the user's input into a data structure that we can use to evaluate. This data structure will be stored as a private attribute.
- Evaluate - This method will take a value of x and input it into our function and return the value $f(x)$.

Our class will contain more methods and attributes later (colour of the line, roots, turning points etc.) but these can be considered later as these are quite small parts. There are two distinct methods to parse a function:

- We can convert the input into a data structure, which we can then use to evaluate a value x .
- We can convert the input into its equivalent in a scripting language, such as Lua[7], and then use the scripting language to evaluate a value x .

An example of the second is shown below:

```
1 import org.luaj.vm2.*;
2 import org.luaj.vm2.lib.jme.*;
3 public double evaluate(double x) {
4     ScriptEngineManager mgr = new ScriptEngineManager();
5     ScriptEngine e = mgr.getEngineByName("luaj");
6     e.put("x", x);
7     e.eval("y = math.sqrt(x)");
8     return e.get("y");
9 }
```

The problem with using scripts is that they take a huge hit on performance as you are effectively creating a virtual machine during your program. Especially in Java where a VM is used to run your programs, creating a VM inside a VM is not very efficient. Also we may make up to 10000 evaluate calls to render multiple lines, and therefore we will need to be as efficient as possible. While the first method is harder to implement, it will have better performance, and it will be easier to debug as we are not using external tools. Therefore we will use the first method.

There are two main structures we can convert a mathematical input into:

- Binary Trees
- Stack Based Programming

both of which we will analyse in detail.

7.2.1 Analysis of Algebraic Expressions

Algebraic Expressions are quite important in this project and as such it is important to break them down and understand what they really mean. The most basic idea in evaluating expressions is **BIDMAS**, which stands for **B**rackets, **I**ndices, **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction. It signifies the order that we must do operations on an expression (It is sometimes called **BODMAS** where the **O** stands for **O**rders) with the order of significance going from the first letter to the last. For example, let us use BIDMAS with the following expression:

$$3(4^2 + 2)$$

According to BIDMAS we will first look for brackets. There are brackets! The expression inside those brackets is $4^2 + 2$. We then apply BIDMAS again. There are no brackets but we do have indices. We apply the indices function which produces $16 + 2$. Applying BIDMAS again we see that we must do addition which produces 18. Finally we have the expression $3(18)$ which is multiplication and it produces 54. Here we repeatedly took the most significant operator and applied its function until we fully evaluated our expression. Later we will instead take the least significant operator as this will allow us to split our expression down in a more structured manner.

In the example above there was nuance that we ignored. This was when we identified $3(18)$ actually meant $3 * (18)$. This implicitly represented multiplication and while as humans it is easy for us to process it, it is impossible for a computer to know this. It is therefore important that we remove these inconsistencies before we properly convert our input into a structure. It is also important that we strip away all whitespace before we start as this will allow our input to be more consistent. Here is a list of all these inconsistencies that we need to remove:

- Any instance of ax where $a \in \mathbb{R} : a \neq 0$ is to be converted to $a * x$.¹
- Any instance of $a($ and $)a$ where a is not an operator, is to be converted to $a * ($ and $)*a$ respectively..²
- Any instance of $(f(x))(g(x))$ is to be converted to $(f(x)) * (g(x))$.³
- Any instance of $! - f(x)$ where $!$ is to be any operator (e.g. $*$ or $/$) is to be converted to $!(-f(x))$.⁴
- Any instance of $-f(x)$ at the start or next to an opening bracket is to be converted to $0 - f(x)$.⁵

¹For example $4x$ is to be converted to $4 * x$

²For example $4(x + 1)$ is to be converted to $4 * (x + 1)$

³For example $(x + 4)(x - 3)$ is to be converted to $(x + 4) * (x - 3)$

⁴If there is a negate symbol next to another operator, we need to make sure that the negate symbol is not treated as an operator (even though we treat it like an operator in certain situations in the next step)

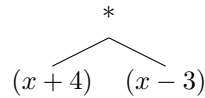
⁵Both these expressions are equivalent but the second allows us to reduce ambiguity if there is a negate symbol at the start of an expression e.g. $-x + 4$ would be treated as $0 - x + 4$ and $(-x)$ is $0 - x$.

7.2.2 Binary Trees

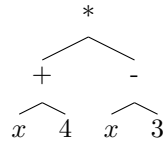
The second stage that a compiler goes through is called Syntax Analysis and this is where the code is transformed into an Abstract Syntax Tree. What we are trying to achieve is very similar to that (albeit on a smaller scale). If we take a function $f(x) = x + 4$, this would be transformed into the binary tree:



If we take something more complicated such as $f(x) = (x + 4)(x - 3)$, we need to first remove consistencies as discussed at the start of this section. Therefore the function would become $f(x) = (x + 4) * (x - 3)$. Now when we convert this into a syntax tree, we find the least significant operator (the operation we would do last), make it our root node and split the parameters that it is operating and make those parameters the child nodes of the root node. Here $*$ is the least significant operator and $x + 4$ and $x - 3$ become the child nodes.



We can then repeat what we did above on the child nodes. In $x + 4$, $+$ is the least significant operator and in $x - 3$, $-$ is the least significant operator. Therefore we now get:



We now stop as we cannot split this down any further. What we notice here is that only the bottommost nodes are actually values, the rest are operators. This is significant, because what we just did was a recursive algorithm, where we split each node down, until a node does not contain any operators.

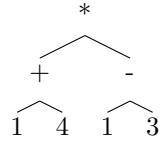
Now if we want to get $f(1)$, we simply replace every instance of x with 1 and then perform the recursive algorithm outlined below.

Algorithm 1: Evaluate: Binary Tree Version
<pre> 1 function evaluate(tree): 2 if tree.height = 0 then 3 return tree.root.value 4 else 5 return evaluate(tree.leftsubtree) tree.root.value evaluate(tree.rightsubtree) 6 end 7 end </pre>

This recursive algorithm has base case “tree.height = 0”, which essentially checks if are at the bottom of our tree. We know that the bottommost nodes are actually values, therefore we can return this value as we cannot go deeper into our tree. Our recursive case is where we are not at the bottom of our tree. We know that at this point, the root node is an operator ⁶. Therefore we operate on the nodes below it.

⁶It is important to realize that tree.root.value is just an operator, and in theory we can simply just write it as shown above but in implementation this will require us to use if statements to check which operator it actually is as the operators will most likely be of data type string

First let us replace every instance of x with 1.



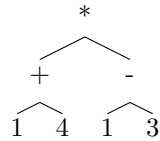
Now the root node has value “+”, i.e. an operator therefore we take the left subtree which is,



and again the root node is an operator so we take the left subtree again, which is 1. This is not an operator, so we return this value, going back to our previous call which is,



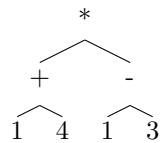
Now we take the right subtree, which is 4. Again, this is not an operator so we return this value, going back to our previous call. Now we have **return** 1 + 4, therefore we return 5 to our previous call which is,



We now repeat the process for the right subtree,



which returns -2 . We then go back to our original call to get $5 * -2$ which returns -10 . The recursive tree for this is exactly the same as the original tree that we started with.



This is significant as this means that the number of recursive calls made is the number of edges, which is $n - 1$ where n is the total number of operands and operators combined.

7.2.3 Stack Based Programming

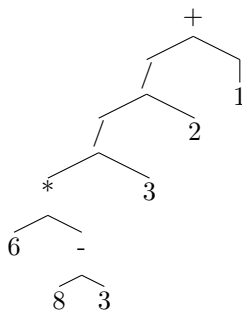
Stack Based Programming[8] is all about using a stack data structure, and manipulating the items within it to get the desired result. Firstly we have to convert our *Infix Notation* into *Reverse Polish Notation*. Infix notation is the “normal” way of writing algebra, where the operator is in between its operands. Reverse Polish notation or Postfix notation is where the operator is after its operands. The advantage of this notation is that no brackets are required. For example $4 + 3$ in infix notation would be $4 \ 3 \ +$ in postfix notation. To evaluate this expression we push each individual operator/value, from left to right, one by one, to a stack. If the value is an operator then it pops however many inputs it would normally take off the stack, perform the operation, on the values we just popped off, and then add that new value back to the stack. For example if we take the infix expression,

$$(((6 * (8 - 3)/3))/2) + 1$$

we can convert this to the equivalent postfix expression,

$$6 \ 8 \ 3 \ - \ * \ 3 \ / \ 2 \ / \ 1 \ +$$

The way we get this is by looking for the least significant operator (the one we would consider last) and putting it to the end. We then take the operands of the operator which just removed, and repeat until we are left with only values. This seems very similar to the binary tree solution. In fact, if we convert this into a binary tree,



and if we then perform post-order depth traversal. So first we go all the way down the left hand side of the tree to get 6. We then visit its sibling, $-$, which has more children. So we visit 8, 3 then $-$. So far we have

$$6 \ 8 \ 3 \ -$$

We then move up to $*$ and add this to our list. We then visit its sibling 3 and add this to our list. We have now got

$$6 \ 8 \ 3 \ - \ * \ 3$$

We then go up to $/$ and add this to our list. We then visit its sibling 2 and add this to our list. We have now got

$$6 \ 8 \ 3 \ - \ * \ 3 \ / \ 2$$

We then go up to $/$ and add this to our list. We then visit its sibling 1 and add this to our list. We then go up to the root node, $+$, and add this to our list. Finally we have,

$$6 \ 8 \ 3 \ - \ * \ 3 \ / \ 2 \ / \ 1 \ +$$

This is identical to our post-fix notation. This is important as later on we can use this fact to make our solution as efficient as possible.

Now if we start to evaluate this expression we get,

$$\begin{array}{c} [6] \quad \begin{bmatrix} 8 \\ 6 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 8 \\ 6 \end{bmatrix} \end{array}$$

we have reached an operator, $-$, so we now pop 2 items off the stack, 3 and 8, and perform the operation, $8 - 3 = 5$. We now push this value onto the top off the stack and resume our evaluation.

$$\begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Again we have reached a operator so we repeat what we did before. Take 5 and 6 off the stack, push $5 * 6 = 30$ onto the stack.

$$[30] \begin{bmatrix} 3 \\ 30 \end{bmatrix}$$

Take 3 and 30 off the stack, push $30/3 = 10$ onto the stack.

$$[10] \begin{bmatrix} 2 \\ 10 \end{bmatrix}$$

Take 2 and 10 off the stack, push $10/2 = 5$ onto the stack.

$$[5] \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

Finally Take 1 and 5 off the stack, push $5 + 1 = 6$ onto the stack. We are left with 6 and have now got our answer. Just like with the binary trees, if we have an unknown x , we can simply replace the x with a value when we want. The algorithm to evaluate post-fix notation is shown below,

Algorithm 2: Evaluate: Stack Based Version

```

1 function evaluate(list):
2   Stack stack
3   Array temp
4   Real out
5   Real x
6   foreach  $i$  in list do
7     if  $i$  is an operator then
8       pop = i.numberOfInputs
9       temp = new Array[pop]
10      for  $j=0$  to pop by 1 do
11        | temp[j] = stack.pop()
12      end
13      x = i.input(temp[1],temp[2],...,temp[n])
14      stack.push(x)
15    else
16      | stack.push(i)
17    end
18  end
19  out = stack.pop()
20  return out
21 end

```

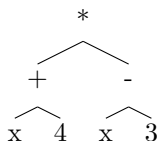
Here we are treating the operator to be an object for simplicity. In practice, all these values will be of data type String, therefore we will probably use some sort of selection construct, either *if* or *switch-case* statements, to determine if i is an operator and if it is, which one is it.

7.2.4 Analysis of the two Methods

When initially parsing the infix expression $f(x)$, the binary tree version will have time complexity $O(n)$ where n is the number of operators (n will be the number of operators, and m will be the number of operands in $f(x)$) in $f(x)$. The stack based version will convert the infix expression into a binary tree, and then perform depth-first traversal to convert the tree into post-fix. Depth-first traversal has a time complexity of $O(m+n)$ (every traversal visits every node, and there are $n+m$ nodes). This means that to initially parse the infix expression, the binary tree version has a smaller time complexity.

When evaluating $f(x)$ for a specific value of x , both versions have a time complexity of $O(m+n)$. This is because the binary tree version visits every node, $m+n$ (what we do is essentially post-order depth-first traversal but applying an operation each time), and the stack based version has $m+n$ items in the list that it goes through. Here we are assuming that each operation between operands takes equal time. This is a reasonable assumption to make because we are comparing two algorithms and they have the same input, $f(x)$, therefore we can remove the steps that it takes to complete the operations.

Now from a pure time complexity point of view, we can say that the binary tree version is better as the initial parsing is quicker. However when it comes to space complexity this is not true. When evaluating $f(x)$, the binary tree version uses a lot more memory because it creates subtrees every time it calls a recursive function. On the other hand, with the stack based version, the maximum space that could be occupied is $(m+n)+m$. $(m+n)$ is the size of the original list that we pull values from, and m is the maximum size our stack could ever get (we never add an operator to the stack). From a memory point of view a binary tree is not very efficiently stored. Most languages don't have a binary tree construct, and implementing your own with primitive arrays (each child is an array) is inefficient. For example the binary tree,



would be represented as (actual implementation would have pointers),

$$[*[+[x, 4]], [-[x, 3]]]$$

If we then stored this in contiguous memory,

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ * & + & x & 4 & - & x & 3 \end{bmatrix}$$

Now when doing post-order depth-first traversal, we jump between the memory locations,

$$\begin{bmatrix} 2 & 3 & 1 & 5 & 6 & 4 & 0 \\ x & 4 & + & x & 3 & - & * \end{bmatrix}$$

This is horribly inefficient because our stride is not consistent. This is unlike our stack based version, where we always have a stride of 1, no matter what. While this is not that big of a problem for one evaluate call, when 2000 evaluate calls are made, this makes a massive impact. We could, in theory, modify the order that we store our tree into one which is efficient when doing a depth-first traversal but this is what we did when we converted our tree into post-fix notation. Therefore using stacks to process input is best for performance even though, the initial parsing takes longer. However this does not mean that we are not going to consider binary trees. This is because we need binary trees to initially parse our infix expression, therefore we will use stacks with binary trees to parse our user input. We need to implement trees and stacks in Java before we can start parsing our infix expression.

7.2.5 Designing an Implementation of Stacks and Trees

Since stacks and trees are a prerequisite to our function class, we will design them now.

Our stack does not need to become infinitely big, as the size of the `Stack`, as discussed in the last section, is m where m is the number of operands. Therefore we can initialize an array of size m and manipulate the array to make it act like a `Stack`.

Our binary tree will be quite simple, containing only the bare minimum of what our binary tree needs, with an added traversal function which returns the post-order depth-first traversal of the tree. For simplicity we will use a private static helper function, which will take a tree and a stack as parameter. This stack which is passed in by reference will be edited in every recursive call. We will also make our left and right tree attributes public. This is because our root tree will be public, and as the left and right sub-trees are simply different copies of the root tree, it isn't consistent to make the right and left private while keeping the root tree public.

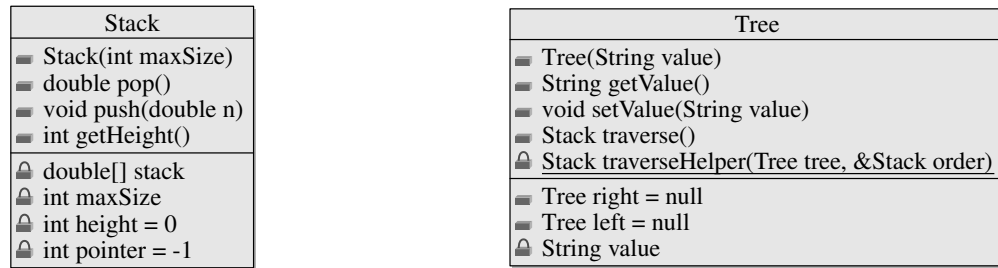


Figure 7.1: Class Diagrams

Algorithm 3: Post-Order Depth-First Traversal Helper

```

1 function traverseHelper(Tree tree, Stack &order):
2   if tree == null then
3     return;
4   else
5     traverse(tree.left, order)
6     traverse(tree.right, order)
7     order.add(tree.data)
8   end
9 end

```

Algorithm 4: Post-Order Depth-First Traversal

```

1 function traverse():
2   Stack order
3   traverseHelper(this, order)
4   return order
5 end

```

7.2.6 Parsing Algorithm

7.2.6.1 Least Significant Operator

An important part of the parsing stage, is the ability to find the least significant operator within an expression. If there are no operators -1 will be returned. If an operator is in between within a pair of brackets it is automatically discounted. We do this by respectively incrementing or decrementing a variable when we reach an opening or closing bracket to monitor when we are within a pair of brackets.

Algorithm 5: Least Significant Operator Position

```
1 function leastSigOperatorPos(String input):
2   Integer parenthesis = 0
3   Integer leastSigOperatorPos = -1 /* stores the position of the least significant
   operator so far */
4   Integer leastSigOpcode = 1000
5   Character[] operators = ["+", "-", "/", "*", "^"] /* stores each operator in order of
   increasing significance */
6   Integer currentOpcode /* the current index of the operator in the array operators */
7   Character currentChar
8   for i=0 to (input.size - 1) by 1 do
9     currentChar = input[i]
10    if currentChar in operators then
11      currentOpcode = operators.find(currentChar)
12      if (currentOpcode ≤ leastSigOpcode) and (parenthesis == 0) then
13        leastSigOperatorPos = i
14        leastSigOpcode = currentOpcode /* Update the least significant operator so
        far, as it is now the current character */
15    else if currentChar == "(" then
16      parenthesis++
17    else if currentChar == ")" then
18      parenthesis--
19    end
20  end
21  return leastSigOperatorPos
22 end
```

7.2.6.2 Removing Brackets

The function above assumes that there is no whitespace and that there are no brackets enclosing the entire expression (e.g. $(x - 4)$). We can deal with our whitespace issue in our constructor⁷ however we need to make another function check for and remove any brackets surrounding an expression.

However algorithm 6 has some issues. For example if we have the expression $\frac{x+1}{x+2}$, this would be input as, $(x + 1)/(x + 2)$. Now if we apply algorithm 6 which removes enclosing brackets we get $x + 1)/(x + 2$. This is completely wrong, as in this case we do not want to any remove brackets at all. The significant issue here is that we only want to remove the enclosing brackets, if they are *matching*.

To do this the algorithm 7 is more suited. This algorithm is based on 6 but checks for matching brackets. It does this by using a variable that increments every time there is a opening bracket and decrements every time there is a closing bracket. When the variable becomes 0 the matching bracket has been found. If this

⁷In our function class we will store the original input so we can show the user, therefore we do not need to remove whitespace here

happens at the end the opening and closing brackets are removed, else nothing happens. This algorithm will also throw an exception if there are unequal number of opening and closing brackets. This is so that we can inform the user later of the error that they have made and so that we can kill the process instantly rather than letting this error have consequences later on (probably during the evaluation of a value).

Algorithm 6: Check for and remove any Brackets surrounding an input

```

1 function checkBracket(String input):
2   Boolean done = False
3   while !done do
4     done = True
5     if (input[0] == '(') and (input[input.size - 1] == ')') then
6       done = False
7       input = input.substring(1, input.size - 2)
8     end
9   end
10  return input
11 end

```

Algorithm 7: Check for and remove any Matching Brackets surrounding an input

```

1 function checkBracket(String input):
2   Boolean done = False
3   if input.count('(') != input.count(')') then
4     throw "There is an unequal number of opening and closing brackets"
5   while !done do
6     done = True
7     if input[0] == '(' and input[input.size - 1] == ')' then
8       Integer countMatching = 1
9       for i=1 to (input.size - 2) by 1 do
10        if countMatching == 0 then
11          return input
12        else if input[i] == '(' then
13          countMatching++
14        else if input[i] == ')' then
15          countMatching--
16        end
17      end
18      if countMatching == 1 then
19        done = False
20        input = input.substring(1, input.size - 2)
21      end
22    return input
23 end

```

7.2.6.3 RegEx

Another important part of the parsing stage is to standardize the input. This is where we convert any inconsistencies discussed in section 7.2, page 19. The easiest way to do this is to use RegEx[6]. RegEx stands for regular expression and is a standardized form of pattern recognition in strings, usually used during syntax analysis during compilation of software. Many languages support RegEx in some form or another and Java is no exception. These were our 5 inconsistencies that we needed to fix:

1. Any instance of ax where $a \in \mathbb{R} : a \neq 0$ is to be converted to $a * x$.
2. Any instance of $a($ and $)a$ where a is not an operator, is to be converted to $a * ($ and $) * a$ respectively.
3. Any instance of $(f(x))(g(x))$ is to be converted to $(f(x)) * (g(x))$.
4. Any instance of $! - f(x)$ where $!$ is to be any operator (e.g. $*$ or $/$) is to be converted to $!(-f(x))$.
5. Any instance of $-f(x)$ at the start or next to an opening bracket is to be converted to $0 - f(x)$.

For the first and second examples, we look around every instance of x , $($ or $)$ and if the adjacent characters are not operators or brackets then we replace with $*x$ or $x*$. Therefore we can combine the first and second examples to use two separate RegEx expressions to deal with the case where we have x after and where we have x before.

The RegEx expression for the first case is `"([\(\)\+\-*\/\^])([a-z])"` with the replacement expression being `"$1*$2"`. If we take the RegEx expression, it creates two capture groups, `"$1"` and `"$2"`, which are `"([\(\)\+\-*\/\^])"` and `"([a-z])"` respectively. A capture group stores a set of characters for each match that is made, so that we can perform actions on it later. The first capture group checks if the first character, in the substring that is currently being checked, is not any of the operators or brackets⁸. The not is signified by the first `\`. The second capture group checks if the second character, in the substring that is currently being checked, is a constant/variable⁹, signified by the `"a-z"` or is an opening bracket `"("`. If both capture groups return true then a match is found and the match is replaced with `"$1*$2"` where `"$1"` is the first capture group, `"$2"` is the second capture group and the `*` asterisk between them signifying the multiply.

The RegEx expression for the second case is `"([\a-z])([\(\)\+\-*\/\^])"` with the replacement expression being `"$1*$2"` again. This expression does the same as the first but checks for the reverse order i.e. xa and instead checks for a $)$ instead of $($ as we are checking the back of a substring instead of the start.

For the third inconsistency, the RegEx expression is `"\)\(""` with the replacement expression being `")*(("`. This expression returns a match if it finds a $)$ followed by a $($. If it finds a match it then replaces the entire match with `")*(("`.

For the fourth inconsistency, the RegEx expression is `"([\+\-*\/\^])-(\([\+\-*\/\^]\)*)"` with the replacement expression being `"$1(-$2)"`. The RegEx expression returns a match when there is an operator followed by a minus sign followed by any number of characters that are not operators or brackets. There are two capture groups. The first is `"([\+\-*\/\^])"` and this captures the operator. The second is `"(\([\+\-*\/\^]\)*)"` and this captures the expression after the minus sign. The match is then replaced by the first capture group, followed by an opening bracket, a minus sign, the second capture group, then a closing bracket.

For the fifth inconsistency, the RegEx expression is `"(\[|()-"` with the replacement expression being `"$10-"`. The RegEx expression returns a match when it is either the start of a line, signified by the `\`, or¹⁰ a $($ followed by a minus sign. The start of the line of bracket is captured and is used in the replacement expression, when the match is replaced with a $0-$ preceded by either a bracket or nothing depending on if the start of a line or an opening bracket was captured.

⁸the reason there are so many backslashes is because a lot of the operators are actually key characters in RegEx and a backslash is an escape character which means that it signifies to treat the next character as a pure character

⁹While we are only dealing with the variable x at the moment, this allows for constants to be used in the future and be dealt with correctly.

¹⁰the `|` or keyword in RegEx is signified by `|`

7.2.6.4 Creating the Tree and the Stack

Using the functions above we can create a syntax tree from our expression. Before we dive into this, remember that we will convert this into a **Stack** later to process, since it is more efficient (section 7.2.4). To create the **Tree** we will use the Divide and Conquer methodology by recursively splitting the original expression until it becomes a single constant or variable. We can know if we should split the expression and if so where we should split the function by using our Least Significant Operator function. Our recursive case will return a **Tree** where the left and right nodes are made up of the trees of the two sub-expressions and the root node will be the operator that we split our original expression with. Our base case will return a **Tree** containing the constant or variable that is remaining. For example if we have the expression " $x^2 + 4$ ", we will first split this expression by the least significant operator which is the "+". We then make this our root node and our left and right trees will be what is left and right of that operator. This will be repeated for each sub-tree until only no operators remain. So our bottom nodes will be x , 2 and 4.

Algorithm 8: Create a Binary Tree for an Algebraic Expression

```

1 function createTree(String expression):
2   expression = checkBracket(expression)
3   Integer leastSigOperatorPos = leastSigOperatorPos(expression)
4   if leastSigOperatorPos == -1 then
5     return new Tree(expression)
6   else
7     String operator = expression[leastSigOperatorPos]
8     String a = expression.substring(0, leastSigOperatorPos)
9     String b = expression.substring(leastSigOperatorPos+1,expression.length - 1)
10    return new Tree (operator,createTree(a),createTree(b))
11  end
12 end

```

Within this algorithm I can use concurrent processing to process each sub-expression individually. To do this I will create 2 threads to create the 2 individual sub-trees. I will then start these threads and on the root thread wait for these threads to finish (usually done using a method called "join"), and then return the new **Tree** object made up of the 2 individual sub-tree.

Algorithm 9: Create a Binary Tree for an Algebraic Expression (Using Concurrent Processing)

```

1 function createTree(String expression):
2   expression = checkBracket(expression)
3   Integer leastSigOperatorPos = leastSigOperatorPos(expression)
4   if leastSigOperatorPos == -1 then
5     return new Tree(expression)
6   else
7     String operator = expression[leastSigOperatorPos]
8     String a = expression.substring(0, leastSigOperatorPos)
9     String b = expression.substring(leastSigOperatorPos+1,expression.length - 1)
10    Thread threadA = new Thread (createTree(a))
11    Thread threadB = new Thread (createTree(b))
12    threadA.join()
13    threadB.join()
14    return new Tree (operator,createTree(a),createTree(b))
15  end
16 end

```

7.2.7 Substitute Algorithm

To actually plot the functions we need to be able to substitute values into the actual functions to know where to draw our points. Eventually we should be able to substitute multiple variables/constants into our function, but for now we will only substitute an x value into our function. This can be an extension for my next iteration.

Now algorithms 3 and 4 on page 26 traverse a tree and output a list (we will implement this using a stack, within the constructor). However the most significant operations and values, i.e. the actions we would do first according to **BIDMAS**¹¹, are at the front of the list. This means that in a stack they would be at the bottom, which means they would be popped off last which is not what we want when we evaluate a function. While we can reverse the stack, this is inefficient and also defeats the purpose of using a stack.

However this is fixed by the need of this **substitute** algorithm. We will need to traverse this stack to replace the variables/constants with actual values anyway, so while we do this we can reverse the stack, fulfilling to purposes.

To accomplish this we will create a new stack, of the same height as the original, and then pop each value of the original stack, checking if it is a variable, and if it is substituting the value required. The value will then be pushed onto the new stack. This new stack will then be returned. We also will need to create a copy of the old stack and manipulate that so that we retain the original stack for other substitution calls.

In order to make my function class more generic, I will introduce a new attribute of type **character** called **parameter**. In all our examples our parameter has been x , but we may eventually use different parameters. Within this algorithm, instead of checking for the character x , I will instead check for the character **parameter**. Within this algorithm I have assumed that a **character** is equivalent to a **String** of

Algorithm 10: Substitute into a function

```
1 function substitute(Double  $x$ ):  
2   Stack copy = postFixStack // Copy the Post-Fix Stack  
3   Stack substituteStack = new Stack (this.postFixStack.getHeight()) /* Create new Stack the  
   same size to reverse it into */  
4   for  $i=0$  to copy.getHeight() do  
5     String pop = copy.pop()  
6     if  $pop == parameter$  then  
7       pop =  $x$   
8     substituteStack.push(pop)  
9   end  
10  return substituteStack  
11 end
```

length 1 (on line 6). This is obviously not the case, therefore during implementation I will need to cast the character to a **String** or do another method that allows me to compare the two values.

¹¹**BIDMAS** stands for **B**rackets, **I**ndices, **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction. It signifies the order that we must do operations on an expression. It is sometimes called **BODMAS** where the **O** stands for **O**rders.

7.2.8 Evaluate Algorithm

The `evaluate` algorithm is very similar to algorithm 2, the stack-based evaluate algorithm. However we will pass in the inputs as parameters and use the `substitute` algorithm within our `evaluate` algorithm to replace our variables with the actual values. We will also hardcode the selection and verification of which operator to use instead of assuming that an operator is an object we can manipulate. A precondition of this algorithm is that we are assuming that the original expression is valid, otherwise this algorithm may not behave as intended.

Algorithm 11: Evaluate the Expression for a Value of x

```
1 function evaluate(Double  $x$ ):
2   Stack substituteStack = substitute( $x$ )
3   Stack evaluateStack = new Stack (postFixStack.getHeight())
4   for  $i=0$  to substituteStack.getHeight() do
5     String pop = subStack.pop()
6     Double a,b
7     switch pop do
8       case "+" do
9          $b = \text{evaluateStack.pop}()$ 
10         $a = \text{evaluateStack.pop}()$ 
11         $\text{evaluateStack.push}(a + b)$ 
12        break
13      case "-" do
14         $b = \text{evaluateStack.pop}()$ 
15         $a = \text{evaluateStack.pop}()$ 
16         $\text{evaluateStack.push}(a - b)$ 
17        break
18      case "*" do
19         $b = \text{evaluateStack.pop}()$ 
20         $a = \text{evaluateStack.pop}()$ 
21         $\text{evaluateStack.push}(a * b)$ 
22        break
23      case "/" do
24         $b = \text{evaluateStack.pop}()$ 
25         $a = \text{evaluateStack.pop}()$ 
26         $\text{evaluateStack.push}(a / b)$ 
27        break
28      case "^" do
29         $b = \text{evaluateStack.pop}()$ 
30         $a = \text{evaluateStack.pop}()$ 
31         $\text{evaluateStack.push}(a^b)$ 
32        break
33      default do
34         $\text{evaluateStack.push}(\text{Double } (\text{pop}))$ 
35        break
36    end
37  end
38 end
39 return  $\text{evaluateStack.pop}()$ 
40 end
```

7.2.9 Constructor and the Class as a Whole

The constructor for the `Function` class will be quite simple. We take an expression and the primary parameter as inputs and:

1. Remove Whitespace from the expression
2. Standardize the expression (using the RegEx rules)
3. Create the Binary Tree
4. Create the Post-Fix Stack from the Binary Tree
5. Set the Primary Parameter

Algorithm 12: Function Class Constructor

```

1 function Function(String expression, String parameter):
2   expression = expression.strip()
3   expression = standardize(expression)
4   this.expression = expression this.tree = createTree(this.expression)
5   this.postFixStack = this.binaryTree.traverse()
6   this.parameter = parameter
7 end

```

The class as a whole will look something like the diagram below.

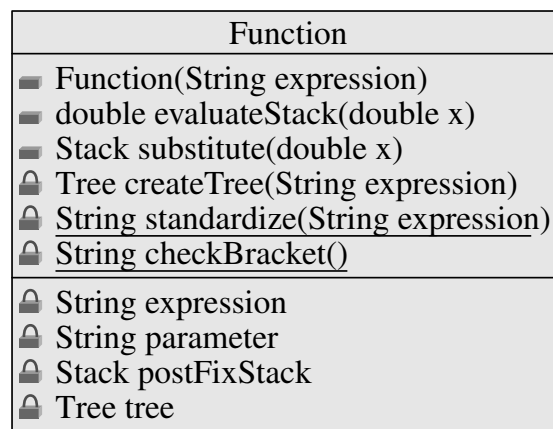


Figure 7.2: Function Class

7.3 Graphical User Interface

The Graphical User Interface will be achieved using one the many toolkits within Java. Specifically I will be using JavaFX[9]. JavaFX is unique in the fact that the GUI can be generated using two approaches:

1. Within the code itself, we can define how the GUI[10] is comprised. This is done by instantiating Objects of specific component Classes, such **Pane**, **Label**, **Button** etc. all inheriting a top class called **Node**, and using their methods to join them together. The advantage of this approach is that since these are simply classes, we can create child classes from them allowing us to create custom components that we can control.
2. The UI is defined using .xml files (aptly named FXML[11] files). A controller class is defined alongside it. This controller class is referenced within the the FXML file and when the FXML file is loaded, the controller class is initialized along with it. This controller allows us to “control” the UI nodes. While this approach is very simple to implement and the design of the UI looks clean, it can be somewhat convoluted to allow the controller classes to interact with other controller classes.

The real strength of JavaFX however comes from the fact that these two approaches can be intertwined as needed, making solutions unique and versatile.

The GUI can be decomposed into smaller parts as shown by the hierarchical diagram shown below. The parts in **bold** are the parts to be implemented and designed in Prototype I and the rest will be completed in Prototype II.

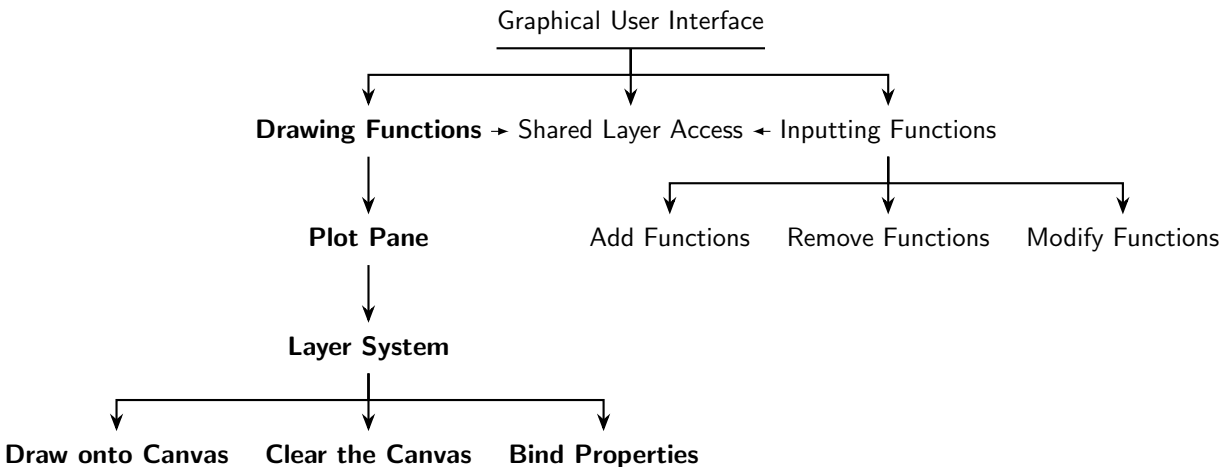


Figure 7.3: Graphical User Interface Hierarchical Diagram

Many of the features above will be discussed in greater detail later, however the Shared Layer Access and Properties will be touched upon now.

As mentioned before it can be hard to make two controller classes interact, as such one solution[12] is an object that both controller classes can reference. This class will contain attributes and methods that both classes need to interact with each other.

However, how do these classes know when something changes and hence know when to update their corresponding UI elements. The answer is through a class called **Property**[13]. A **Property** can bind another **Property**, one updating when the other one changes. Objects called **Listener** can also be added to properties. A **Listener** can be programmed to do a specific action when a **Property** is updated. This action could be a function, or updating a variable. Using the class **Property** allows for a responsive UI, updating whenever the user does an action.

7.3.1 Overview of the Graphical User Interface

The GUI will look something like:

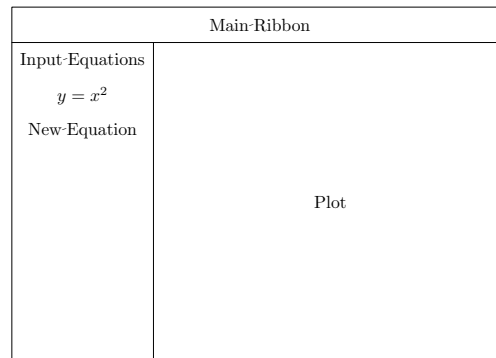


Figure 7.4: UI Design

as showcased in the analysis.

Within JavaFX there is a hierarchy[14] of a standard GUI components. This is shown below:

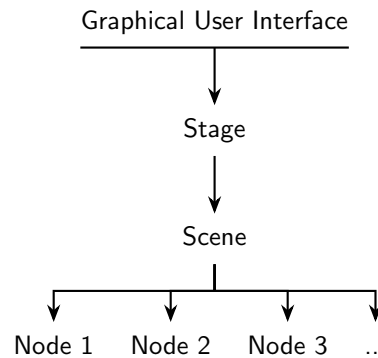


Figure 7.5: JavaFX Components Hierarchical Diagram

Now we can manipulate the the nodes beneath Scene. It is standard to make the first Node beneath Scene some kind of Pane. For this project a `BorderPane` is most suited since it has multiple objects of type `Region`, that can hold one Node each. A diagram of this is shown to the right showcases how a `BorderPane` is arranged[15] and way it behaves.

The center `Region` can consist of the plots themselves, and the left `Region` can consist of the input. The top `Region` could be used for the ribbon and other buttons to manipulate the plot.

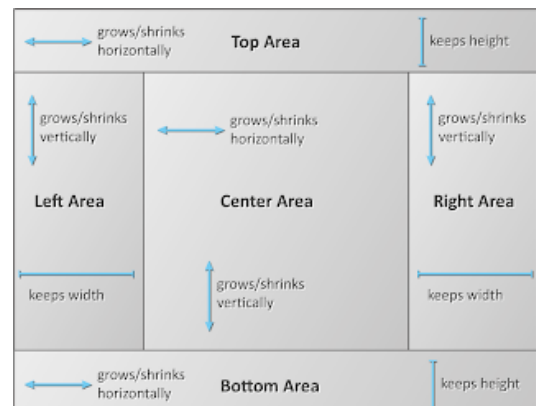


Figure 7.6: BorderPane Architecture

7.3.2 Layer System

Drawing onto the screen using JavaFX can be achieved using the `Canvas`[16] object. This is a node, which allows us to draw shapes, lines, arcs, on to the screen. As discussed in the analysis, we can approximate a curve by drawing many lines however one of our requirements was to draw multiple functions. From the article cited above, there are two ways to achieve this:

1. Draw every function onto one `Canvas` and add this to the root `Pane`.
2. Create a `Canvas` for every function and draw each function onto the corresponding canvasses and add all these canvasses onto the root `Pane`. One canvas will be paired with one function and bundled `Layer`. This only works because canvasses are by default transparent.

The advantage of the first is that it is simple to implement. However if there are multiple types of functions, i.e. an explicit function and an implicit function, then each one of these functions must have a draw function associated with it. This can become slightly convoluted, because then we must check what type of function something is, before we draw it.

Although the second is harder to implement, it is by far the superior. A `Canvas` is essentially an abstracted `OpenGL` view, and hence it is almost impossible[17] to draw multiple things on the same canvas at once. However it is possible to draw onto multiple canvasses at once, which boosts performance on multicore system. This also means that we can create a class for each type of function (explicit, implicit, parametric, etc.), making this system more modular. It is important to realize that only the topmost `Canvas` receives input (keyboard and mouse events) and hence we can create a `Layer` solely to handle input.

However before we handle the canvasses themselves, we must concern ourselves with how we can draw a line at a specific place.

7.3.2.1 Coordinate Systems

The coordinate system that we are familiar with is the Cartesian Coordinate System. Cartesian Coordinates take a fixed point as reference, usually called the origin (symbolized by O), and specify a point by taking the distance from the origin in n directions, where n is the dimension of the system. The directions are vectors, that are all perpendicular to each other, and have magnitude 1, i.e. unit vectors. This graphing software will be dealing with a 2-Dimensional Cartesian Coordinate System and in this system, up and down are defined as the y direction, with up being positive and down being negative; whereas right and left are defined as the x direction, with right being positive and left being negative. Within our graphing program we will only show a section of the x - y plane. This is because:

1. It is intractable to show the entire x - y planesince the plane goes on for an infinite distance in both the x and y directions.
2. It is not helpful to the user to see the entire plane even if they could. A graphing tool should enable the user to highlight parts of the graph that is important to them.

However a JavaFX `Canvas` doesn't exactly use a Cartesian Coordinate System. It specifies a position on itself taking the top-left corner as the origin, right as the positive x direction and down as the positive y direction. It is important to realize that negative coordinates do not exist in this system. The maximum x and y values are the width and height, respectively, in pixels.

We must therefore find a way to convert between these two coordinate systems. Since our viewport is predefined, let us label what the maximum and minimum x and y values in our viewport (with respect to our original Cartesian Coordinate System). Let these values be `minX`, `maxX`, `minY` and `maxY`.

Now let us define how much the width and height of each pixel on the canvas is “worth”. This “worth” specifies how much going across or down a pixel width or height, is in terms of our original Cartesian Coordinate System. This is best demonstrated with an example. If our canvas shows the x - y plane in the x direction from 0 to 10, and our canvas is 1000 pixels wide, then our pixel worth is the amount we have

traversed in the x direction, divided by the number of pixels, in this case $\frac{(10-0)}{1000} = 0.01$. The algorithm for this is shown below:

Algorithm 13: Calculate the “Worth” of each Pixel

```

1 function updatePixelWorth():
2   pixelWorthX = (maxX - minX)/canvas.getWidth()
3   pixelWorthY = (maxY - minY)/canvas.getHeight()
4 end

```

Now that we have the pixel worth, we can convert between our coordinate systems. To do this we take the vector from the origin, $O = (x_{min}, y_{max})$, to the point, $P = (x, y)$ in the x or y direction. We then calculate how many pixel “worths” this is and this gives us the pixel on the canvas that this coordinate is at. It is important to realize that going down in a JavaFX Canvas is actually positive, relative to the canvas, hence our two algorithms are slightly different. This means our vector is actually

$$\vec{d} = \begin{bmatrix} (x - x_{min}) \\ (y_{max} - y) \end{bmatrix}$$

Hence our position within the canvas is:

$$\vec{p} = \begin{bmatrix} \left(\frac{x - x_{min}}{x_{worth}} \right) \\ \left(\frac{y_{max} - y}{y_{worth}} \right) \end{bmatrix}$$

The algorithms below convert between x and y coordinates ¹² respectively.

Algorithm 14: Convert the x Coordinate

```

1 function convertX(Double  $x$ ):
2    $x = x - \text{minX}$  /* Calculate the horizontal distance between the point and origin */
3    $x = x / \text{pixelWorthX}$  /* Calculate the number of pixel “worths” this is */
4   return  $x$ 
5 end

```

Algorithm 15: Convert the y Coordinate

```

1 function convertY(Double  $y$ ):
2    $y = \text{maxY} - y$  /* Calculate the vertical distance between the point and origin */
3    $y = y / \text{pixelWorthY}$  /* Calculate the number of pixel “worths” this is */
4   return  $y$ 
5 end

```

¹²Here we assume the coordinate is inside the viewport. Realistically we should check if the coordinates are in the viewport, i.e. bigger than the width or height of the canvas, or in some cases negative. However it does not matter since we can make this check later. Even then a JavaFX Canvas will check if the coordinates are in the canvas viewport anyway, so it does not matter to us since this detail is abstracted away from us.

7.3.2.2 Drawing the Function

From the analysis (Section 4.1) I briefly described how I could approximate a curve by creating a finite number of line segments.

Firstly we take the minimum value of x , which in the viewport is x_{min} . If our function is defined by $f(x)$ then let the coordinate of the function at this value be $P_1 = (x_{min}, f(x_{min}))$. We then take a second coordinate a small value dx away, so let the coordinate be $P_2 = (x + dx, f(x + dx))$. We now draw a line between these two points. We then make $P_1 = P_2$ and make P_2 with $x + 2 \cdot dx$. We repeat this process until $x + n \cdot dx \geq x_{max}$. We obviously need to convert the x and y values from the Cartesian coordinates into the canvas coordinates so we can do this when drawing the line. dx will also be a finite value that is very small. We will define this by taking the number of line segments we want, and working out dx for that number of steps. This is simply done by finding out how much each step is "worth" within the viewport, i.e. $dx = (\max X - \min X) / \text{steps}$. The algorithm for this is shown below.

Algorithm 16: Draw a Function in the Viewport	
1	function <i>draw()</i> :
2	Double $x1 = \min X$
3	Double $y1 = f.\text{evaluate}(x1)$
4	Double $dx = (\max X - \min X) / \text{steps}$
5	for $x2 = \min X + dx$ to $\max X$ by dx do
6	$y2 = f.\text{evaluate}(x2)$
7	$\text{drawLine}(\text{convertX}(x1), \text{convertY}(y1), \text{convertX}(x2), \text{convertY}(y2))$
8	$x1 = x2$
9	$y1 = y2$
10	end
11	end

A precondition of this is that f is a continuous function and as such we need to make sure we do not draw an undefined part of the graph, but this can be sorted out in the implementation.

7.3.2.3 Input Layer

While I will not design nor implement the responsive user input in this prototype, I will implement a basic input layer class with simple attributes. This is because it controls the other layers since if it zooms or pans around the viewport changes and this affects the properties defined in methods in the previous sections. Properties such as the minimum and maximum values of x and y for the canvas, and the pixel "worth". So we should create a basic input layer so I can bind the properties of other layers to the input layer, which means that it is easier to build on in prototype 2.

This class will be quite bare, it will contain the methods to calculate pixel "worth" and update it whenever the canvas changes size. It will be updated in other scenarios when we actually start manipulating the viewport in prototype 2, but at the moment I will simply add a property called `changeViewport` that will eventually be connected to the `PlotPane` to notify it when any input is registered. The class diagram for this is included in the next section with all the rest of the layer classes.

7.3.2.4 Constructor and Class as a Whole

The layer class we have described in the previous sections, describe a layer for an explicit function, in terms of x , in a Cartesian Coordinate System. To take full advantage of an OOP approach, as described at the start of this section, we can use layers for input, drawing axes etc. Hence we will create a superclass called `Layer` and then make our other types of layers inherit this class. This superclass will never be instantiated, in OOP terms it is abstract. The superclass will have some basic methods and attributes which we have used during this design, such as `draw()`, the attributes of the viewport (`minX`, `maxX`, etc.), a getter for the canvas (so it can be accessed by the `PlotPane`) and the functions to convert between coordinate systems, where some of the functions such as `draw()` will be overridden. Our `Explicit Function` and `Axes` classes are the only layers we will implement (apart from the superclass layer) in prototype 1. The input layer will be partially implemented readying it for prototype 2. While we haven't talked about the axes layer, this will simply be drawing the lines $y = 0$ and $x = 0$. This is done just for reference so that we can verify that the functions are drawn somewhat accurately.

The constructor for the Explicit Function Layer is shown below.

Algorithm 17: Explicit Function Layer Class Constructor
--

<pre> 1 function <i>ExplicitFunctionLayer</i>(String function): 2 super() 3 this.f = new Function(function) 4 end </pre>
--

The constructor for the Input Layer will be slightly different. Since the pixel “worth” needed to be updated if the window is resized, I will add a listener to update the values if the canvas changes size.

Algorithm 18: Input Layer Class Constructor
--

<pre> 1 function <i>InputLayer</i>(): 2 this.canvas.heightProperty().addListener(event → updatePixelWorth()) 3 this.canvas.widthProperty().addListener(event → updatePixelWorth()) 4 end </pre>

While the superclass will never be instantiated, it can still contain a constructor. Here it will be used to initialize the canvas and its `OpenGL` context, in a consistent manner, so that we do not have repeated, redundant code for no reason.

Algorithm 19: Layer Class Constructor
--

<pre> 1 function <i>Layer</i>(): 2 this.canvas = new Canvas() 3 this.gc = canvas.getGraphicsContext2D() 4 end </pre>

The class diagrams for the `Layer`¹³ classes are shown on the next page:

¹³The inherited methods and attributes are protected so that they can actually be viewed and accessed by the child classes.

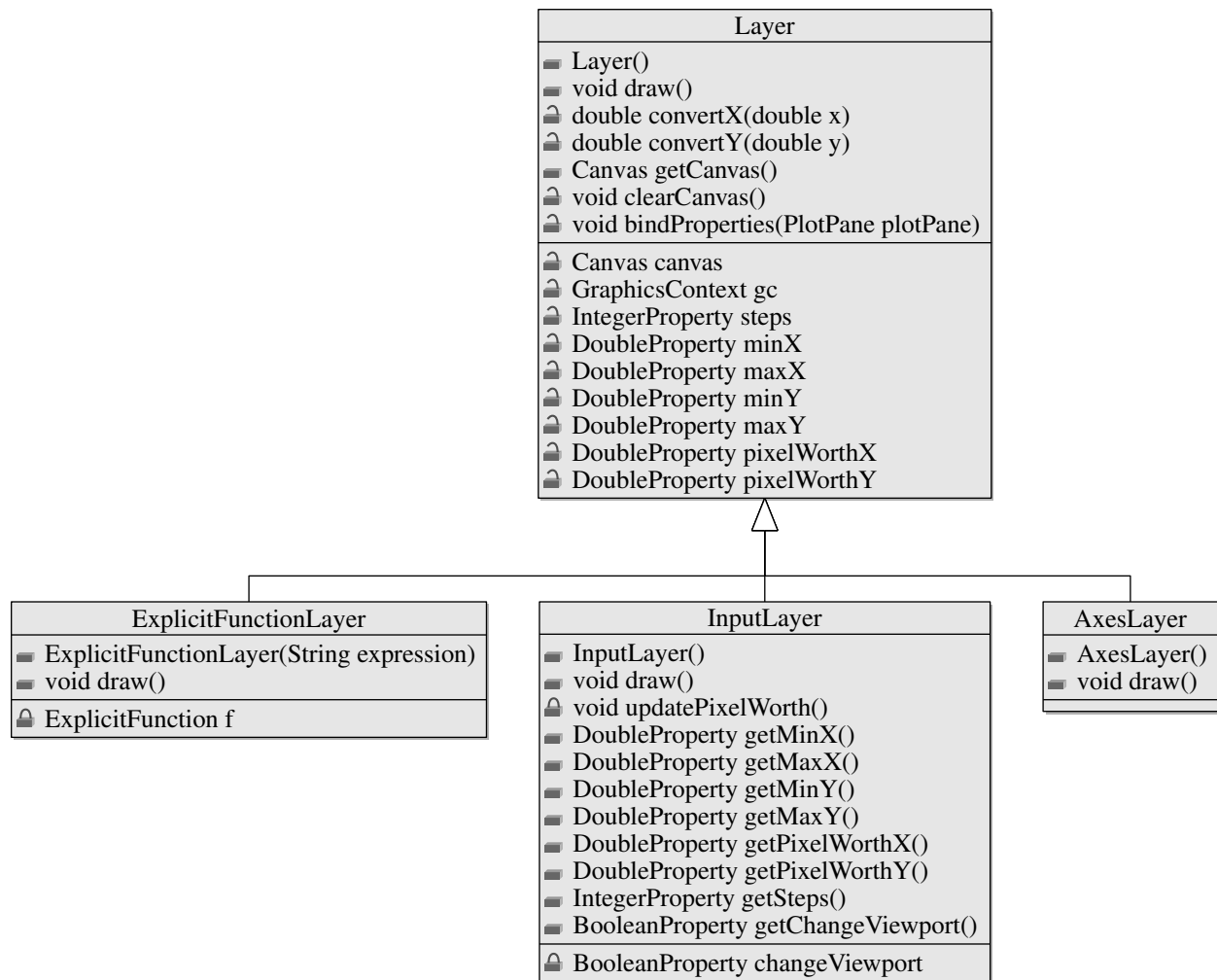


Figure 7.7: Layer Classes

7.3.3 Plot Pane

The purpose of the `PlotPane` is to manage the layers. The `PlotPane` will tell them when to draw and notify them when something changes through the use of properties. Since the `InputPane` will tell when something needs to be drawn, the `PlotPane` will contain a property which will be bound to the `InputPane` and we will draw whenever this property changes. This property, from the `InputPane` section, will be called `changeViewport`.

Our `PlotPane` will inherit the JavaFX `Pane`. While we could have used FXML, FXML is more applicable when there are complicated UI components that are hard to arrange and manage. In this situation we only need to have one `Pane` hence there is no need to use FXML. Inheriting `Pane` also allows it to integrate well within the JavaFX hierarchy.

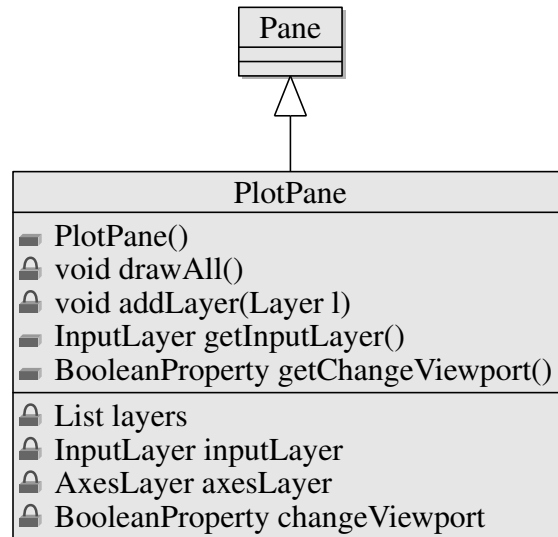


Figure 7.8: `PlotPane` Class

The `drawAll()` procedure will call the draw function for every `Layer`. We will start by doing this on a single thread, and then we will implement it using multiple threads. The `addLayer()` function will add the `Layer` to the list, and bind the layers' properties to the `InputLayer` attribute of `PlotPane`.

7.4 Testing Strategy

There are two main types of tests that I will do Unit and Overall tests.

Unit Tests will be done for most functions or classes that I create. Naturally these will be white box tests since I know how the unit works and am trying to find flaws within them. These will be based on testing the requirements that are defined in the design section. I will not create a detailed test table here since the tests are simply testing the requirements for each unit, but at the end of each unit implementation I will provide some code to test the unit, work out the expected result and compare the expected result to the actual output. If required I will modify the code to acquire the correct output. I will also define whether the data is valid, invalid or extreme.

Overall Tests will be done at the end of implementing the first prototype. Since I will not have defined any input mechanism to add functions in this prototype I will have to inject code, probably in a constructor somewhere, to add the functions I want to test. Naturally these will be black box tests since I am going to test them as if I were a stakeholder in the sense that I am inputting something and expecting an output without knowing the internal workings. I will include very little of the stakeholders at this stage since this prototype is only a foundation for user related features in the next one. Also all the tests will be for **valid data** we since only need to test the user cannot input anything yet. This means there will be no validation of input yet and hence there is no need to test the non-existent validation with invalid data.

The overall testing table is below:

Test Number	Function being Tested	Sub-Test Number	Input
1	Horizontal Lines	a	4
		b	-2
2	Linear	a	$2x - 5$
		b	$-0.5x$
3	Positive Integer Factorised Polynomials	a	$x(x - 1)$
		b	$x \wedge 2(x \wedge 2 + 1)$
		c	$-x(3x + 1)$
4	Positive Integer Unfactorised Polynomials	a	$x \wedge 2 + 2x + 1$
		b	$x \wedge 2 - 1$
		c	$x \wedge 3 + 4x \wedge 2 + x - 6$
5	Non-Integer Polynomials	a	$x \wedge (1/2)$
		b	$x \wedge (1/3)$
6	Exponentials	a	$e \wedge x$
		b	$2 \wedge x$
		c	$x \wedge x$
7	Asymptotal	a	$1/(x - 1)$
		b	$1/(x - 1) + 1/(x + 1)$
10	Multiple and Coloured Functions	N/A	

Table 7.1: Prototype 1: Testing Table

I have not included the expected output within this table. This is because I will generate these just before doing the test. These will most likely be in the form of an image from Desmos, another graphing software. Also you may notice how I have not included any details from the test for multiple functions. This is because I will draw all sub tests within the same test at the same time, testing the multiple functions feature at the same time. This allows me to see a function with respect to another function which makes the verification of whether the function is drawn correctly easier to do, I will also make each of the functions different colours, which allows me to test another feature and distinguish between the different functions.

Chapter 8

Implementation

When beginning to implement my solution, I made a few decisions. I realized a couple of important ideas which I did not address in my Design:

1. My function class designed was more of a class for an expression instead of a function. This is significant because different functions will use expressions in different ways. For example, an explicit function will use one expression to define the relationship, whereas an implicit function will use two expressions to define this relationship. Therefore to make our solution expandable in the future it would make more sense to have an expression class with multiple other classes for the different types of functions. These function classes can then inherit each other, for example if we make classes for different probability distribution, they are simply explicit functions with a specific standard structure, which makes them perfect child classes.
2. The layer system only relates to the Cartesian Coordinate System. This is a problem if I expand my program later to contain the Polar Coordinate System. So what I will do to fix this problem, is to take the `Layer` class that I created, rename it to `CartesianLayer` and make this class inherit a new class called `Layer` which will be all the attributes some key methods that all layers will have. I will also make another class called `ExplicitFunctionLayer` which will inherit from `CartesianLayer`. This allows me to split up each one of my layers into the types of function that will be used.
3. Java allows projects to be split up into packages. To organize my implementation I will split it up into the following packages:
 - (a) `application` - Contains the UI classes, controllers, `PlotPane`, etc.
 - (b) `layers` - Contains the `Layer` classes.
 - (c) `structures` - Contains the data structure classes, stacks, trees, expressions, functions, etc.
 - (d) `exceptions` - Contains custom exceptions.

8.1 Stacks

I will be implementing Stacks using a concept called *Generic Programming*[18]. This is best explained with an example.

Imagine we have a data structure which will only ever contain one type of object but we will use this data structure in different scenarios, each with different types of object. The structure must also be independent of the objects it is storing.

Now we can use an OOP approach where we have a base abstract class called **Structure** and then we create other classes that are related to a certain type of object and make it inherit this class, e.g. **IntegerStructure** or **StringStructure**. However this can become unfeasible to do this for every single object that we use this structure for.

A more elegant approach is to use *Generics*. This is where we link a type of object with the structure. Let this type of object be of type **T**. Now we make our structure of type **T** and make all of its methods take or return an object of type **T**. This allows the structure to be versatile and be used for any object.

Here is the implementation I have created in Java using *Generics*[19]:

```
1 package structures;
2
3 import exceptions.StackOverflowException;
4 import exceptions.StackUnderflowException;
5
6 public class Stack<T> {
7
8     // attributes
9     // pointer variables
10    private int maxHeight;
11    private int height = 0;
12    private int pointer = -1;
13    // stack
14    private T[] stack;
15
16    // methods
17    // constructor
18    @SuppressWarnings("unchecked")
19    public Stack(int maxHeight) {
20        this.maxHeight = maxHeight; // set this max height
21        // create an array of this size of type "T"
22        this.stack = (T[]) new Object[this.maxHeight];
23    }
24
25    // push an item on to the stack
26    public void push(T push) throws StackOverflowException {
27        if (height == maxHeight) { // check that the stack isn't full
28            throw new StackOverflowException();
29        } else {
30            // make the element one above the top, the new value
31            this.stack[this.pointer + 1] = push;
32            this.pointer++; // increment pointer variables
33            this.height++;
34        }
35    }
```

```

36 // pop an item off the stack
37 public T pop() throws StackUnderflowException {
38     if (this.isEmpty()) { // check that the stack isn't full
39         throw new StackUnderflowException();
40     } else {
41         // get the value of the top element
42         // no need to make it null, it is a waste of an instruction
43         // it also releases no memory since we are using
44         // an array to store our stack which is a static structure
45         T pop = this.stack[this.pointer];
46         this.pointer--; // decrement pointer variables
47         this.height--;
48         return pop; // return the popped value
49     }
50 }
51
52 // check if stack is empty, return true if so
53 public boolean isEmpty() {
54     return this.height == 0 ? true : false;
55 }
56
57 // return the height of the stack
58 public int getHeight() {
59     return height;
60 }
61
62 // allow for a visualization of the stack by listing it
63 // with the top element being encapsulated in square brackets
64 @Override
65 public String toString() {
66     if (this.isEmpty()) { // if empty notify the user
67         return "Stack is Empty";
68     } else {
69         // highlight the top element
70         String out = "[" + this.stack[this.pointer] + "]";
71         // loop through the rest of the stack backwards
72         // concatenating each element to a string
73         for (int i = this.pointer - 1; i >= 0; i--) {
74             out = out + ", " + this.stack[i].toString();
75         }
76         return out; // return this string
77     }
78 }
79
80 }

```

There are two custom Exception classes that I have created for this class, StackOverflowException and StackUnderFlowException, in a separate package. These simply output a different message to the standard Exception class. The code for this is at the end of the documentation.

To just make sure that my code is working as intended I tested 4 things:

1. Popping and pushing items
2. Overflowing the stack
3. Underflowing the stack
4. Copying a stack (this is needed for the substitute algorithm)

Here is the code for test 1:

```
// add two items, pop one off, add another one
Stack<String> stack = new Stack<String>(4);
stack.push("item 1");
System.out.println("Stack -> "+stack); // expecting item1
stack.push("item 2");
System.out.println("Stack -> "+stack); // expecting items 1 and 2
String pop = stack.pop();
System.out.println("Popped item -> "+pop); // expecting items 2
stack.push("item 3");
System.out.println("Stack -> "+stack); // expecting items 1 and 3
```

Here was the output for test 1:

```
Stack -> [item 1]
Stack -> [item 2], item 1
Popped item -> item 2
Stack -> [item 3], item 1
```

Which is exactly what I expected.

To spice things up for test 2 I made a stack of integers instead of strings just to make sure the generics part is working properly. Here is the code for test 2.

```
// create a stack of max height 4
Stack<Integer> stack = new Stack<Integer>(4);
// push 4 items, the max amount
stack.push(1);
stack.push(2);
stack.push(3);
stack.push(4);
// show that the stack has 4 items
System.out.println("Stack -> "+stack);
// push another item
stack.push(5);
```

Here was the output for test 2:

```
Stack -> [4], 3, 2, 1
Exception in thread "main" exceptions.StackOverflowException:
Stack has reached its max height
    at structures.Stack.push(Stack.java:40)
    at structures.Stack.main(Stack.java:107)
```

Which is exactly what I expected.

Again for test 3 I made a stack of doubles. Here is the code for test 3.

```
// create a stack
Stack<Double> stack = new Stack<Double>(4);
// try and pop an item off
stack.pop();
```

Here was the output for test 3:

```
Exception in thread "main" exceptions.StackUnderflowException:
Stack has no items for you to pop
    at structures.Stack.pop(Stack.java:52)
    at structures.Stack.main(Stack.java:100)
```

Which is exactly what I expected.

For test 4 I made a stack of booleans. Here is the code for test 4.

```
// create a stack
Stack<Boolean> stack = new Stack<Boolean>(4);
// pop some items
stack.push(true);
stack.push(false);
// assign the value of the old stack to the copy stack
Stack<Boolean> copy = stack;
// pop an item off the copy stack to test that they are independent
copy.pop();
// output both stacks
System.out.println("Stack -> "+stack);
System.out.println("Copy -> "+copy);
```

Here was the output for test 4:

```
Stack -> [true]
Copy -> [true]
```

This is not what I expected. I expected the original stack to have an extra item. It appears as if they are mimicking each other, like they are sharing the same memory address. When I researched this issue[20] it appears that this is true. In Java every variable is a reference so when you assign a variable the value of an object, you actually assign the variable a reference. The exceptions to this are when you pass variables into functions, this is done by value not reference¹, and with primitives². The suggested solution was to use `Serialization`[21]. This is where an object is serialized into a byte array and then deserialized into an object again. This works because we are effectively converting an object into a primitive and then assigning this to a new value, which bypasses the reference issue. However this can be a hit on performance. So why don't I use the same principle in my code. Since all my attributes are primitives, if I just create a constructor which assigns the new stacks' attributes' values the original stacks' attributes, the problem should be solved.

¹A function cannot have parameters passed by reference[22] in Java

²A language's base types of object that are inherently part of the language. In Java the primitives are int, double, float, boolean, char, byte, short, long.

I made a constructor that takes an existing stack as a parameter to solve the issue. In many languages this is called a *Copy Constructor*. The code for this is below:

```
1 // copy constructor
2 public Stack(Stack<T> copy) {
3     // assign the new stacks' attributes' values the original stacks' attributes
4     this.pointer = copy.pointer;
5     this.height = copy.height;
6     this.maxHeight = copy.maxHeight;
7     this.stack = copy.stack;
8 }
```

For my new test I slightly modified it from the original by using the copy constructor. Here is the code for new test:

```
// create a stack
Stack<Boolean> stack = new Stack<Boolean>(4);
// pop some items
stack.push(true);
stack.push(false);
// assign the value of the old stack to the copy stack
Stack<Boolean> copy = new Stack<Boolean>(stack);
// pop an item off the copy stack to test that they are independent
copy.pop();
// output both stacks
System.out.println("Stack -> "+stack);
System.out.println("Copy -> "+copy);
```

Here was the output for the test:

```
Stack -> [false], true
Copy -> [true]
```

This is what I expected.

8.2 Trees

Our trees will be simpler to implement than our stacks. This is because we do not need to use generics with our trees since we will only be storing strings within them. The only hard part of this implementation was the fact that you cannot pass by reference[22] in Java. I got around this issue by simply passing this list in, and returning it each time. Here is the implementation I have created in Java:

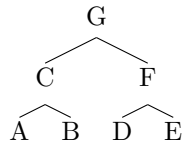
```
1 package structures;
2
3 import exceptions.StackOverflowException;
4
5 public class BinaryTree {
6
7     // attributes
8     // children
9     public BinaryTree left;
10    public BinaryTree right;
11    // node value
12    private String value;
13
14    // methods
15    // constructor - create a null tree
16    public BinaryTree() {
17        this.left = null;
18        this.right = null;
19        this.value = null;
20    }
21
22    // constructor - create leaf tree i.e. no children
23    public BinaryTree(String value) {
24        this.value = value;
25        this.left = null;
26        this.right = null;
27    }
28
29    // constructor - create a tree with the children defined
30    public BinaryTree(String value, BinaryTree left, BinaryTree right) {
31        this.value = value;
32        this.left = left;
33        this.right = right;
34    }
35
36    // return the value of the current node
37    public String getValue() {
38        return value;
39    }
40
41    // set the value of the current node
42    public void setValue(String value) {
43        this.value = value;
44    }
```

```

45
46 // count the number of nodes beneath this tree + this tree
47 public int countNodes() {
48     // set the number of nodes counted to 0
49     int count = 0;
50
51     // recursive case, counts the number of nodes in its children
52     // if they are not null
53     if (this.left != null) {
54         count = count + this.left.countNodes();
55     }
56     if (this.right != null) {
57         count = count + this.right.countNodes();
58     }
59     // return the count of the children
60     // but increment it to count it self
61     return count + 1;
62 }
63
64 // post-order depth-first traversal wrapper function
65 public Stack<String> traverse() throws StackOverflowException {
66     // create a stack of max height
67     // which is the number of nodes in the entire tree
68     // this is to save memory
69     Stack<String> order = new Stack<String>(this.countNodes());
70     // traverse the tree
71     order = traverseHelper(this, order);
72     return order;
73 }
74
75 // the actual recursive traversal function
76 private static Stack<String> traverseHelper(BinaryTree tree, Stack<String> order)
77     ↪ throws StackOverflowException {
78     // base case, if the tree is null just return the unmodified stack
79     if (tree == null) {
80         return order;
81     } else { // recursive case search its children if not null
82         // since this is post-order, the order is left, right then root
83         // traverse left
84         order = traverseHelper(tree.left, order);
85         // traverse right
86         order = traverseHelper(tree.right, order);
87         // finally push the root value into the stack
88         order.push(tree.value);
89         return order;
90     }
91 }
92 }

```

Just to test my binary tree implementation I tried to recreate the tree below:



If this graph has post-order, depth first traversal performed on it then the output should be:

$[A \ B \ C \ D \ E \ F \ G]$

Here is the test code:

```
BinaryTree root;  
BinaryTree left = new BinaryTree("C", new BinaryTree("A"), new BinaryTree("B"));  
BinaryTree right = new BinaryTree("F", new BinaryTree("D"), new BinaryTree("E"));  
root = new BinaryTree("G",left,right);  
// print the traversal stack  
System.out.println(root.traverse());
```

Here is the output:

[G], F, E, D, C, B, A

While this appears to be different to our expected stack it isn't. If we look closely the items are actually reversed, which makes sense since a stack is FILO (First In Last Out) structure and we made the visual representation of our stack implementation reflect this property. This means that *A* was pushed into the stack first, which means our tree implementation is working as intended.

8.3 Expressions

My Expression class, the one I designed as Function, is quite a complex and large, therefore I will implement each method independently and test them before I use them within my other methods.

8.3.1 Standardize Expression

I created a method to standardize my expression using the RegEx expressions that I made. This method will be `static` since it isn't related to the object, but instead to the class as a whole. I also added support for a few constants such as e and π so that we can use them later on. Here is the code:

```
1 // standardize an expression to remove ambiguity
2 private static String standardize(String expression) {
3     // remove whitespace
4     expression = expression.replace(" ", "");
5     // make pi one character for faster regex
6     expression = expression.replace("pi", "π");
7     // inconsistency 1
8     String regex1 = "([\\(\\)\\+\\-\\*\\/\\^])([\\(a-zπ])";
9     String replace1 = "$1*$2";
10    // inconsistency 2
11    String regex2 = "([\\(a-zπ])([\\(\\)\\+\\-\\*\\/\\^])";
12    String replace2 = "$1*$2";
13    // inconsistency 3
14    String regex3 = "\\(\\(";
15    String replace3 = ")*(";
16    // inconsistency 4
17    String regex4 = "([\\+\\-\\*\\/\\^])-(^[\\+\\-\\*\\/\\(\\)\\^]*)";
18    String replace4 = "$1(-$2)";
19    // inconsistency 5
20    String regex5 = "(^|\\()\\-";
21    String replace5 = "$10-";
22    // perform regex
23    expression = expression.replaceAll(regex1, replace1);
24    expression = expression.replaceAll(regex2, replace2);
25    expression = expression.replaceAll(regex3, replace3);
26    expression = expression.replaceAll(regex4, replace4);
27    expression = expression.replaceAll(regex5, replace5);
28    // replace constants with numerical values
29    expression = expression.replaceAll("e", Double.toString(Math.E));
30    expression = expression.replaceAll("π", Double.toString(Math.PI));
31    // return the standardized expression
32    return expression;
33 }
```

To test my module I tested each of the inconsistencies and then a complex expression just to make sure everything is working. Here is the code with comments for what I expected:

```
System.out.println(standardize("2x"));           // 2*x
System.out.println(standardize("(x)3"));         // (x)*3
System.out.println(standardize(")");            // )*(
System.out.println(standardize("*-x"));           // *(0-x)
System.out.println(standardize("3x(-x+1)(x+2)")); // 3*x*(0-x+1)*(x+2)
```

Here are the results:

```
2*x
(x)*3
)*(
*(0-x)
3*x*(0-x+1)*(x+2)
```

All the tests passed except the last one. It seems as though, the RegEx is ignoring the second instance of the multiplication for some reason. To see if it is a one time thing I ran the same RegEx expression again, as in I executed the `replaceAll()` functions again. For some reason this seemed to fix it as I got the expected output of $3 * x * (0 - x + 1) * (x + 2)$. To account for this, within my `standardize()` function, I will add a loop which make the RegEx checks are run until my expression has not changed between two consecutive RegEx. Here is the code³ that I modified:

```
1 // loop until check and expression are equal
2 String check = "";
3 while(!expression.equals(check)) {
4     check = expression;
5     // perform regex
6     expression = expression.replaceAll(regex1, replace1);
7     expression = expression.replaceAll(regex2, replace2);
8     expression = expression.replaceAll(regex3, replace3);
9     expression = expression.replaceAll(regex4, replace4);
10    expression = expression.replaceAll(regex5, replace5);
11 }
```

While making this modification I originally checked if the previous and current versions of the expressions by doing `check!=expression`. This caused an infinite loop. This is because the `String` object in Java is quite unique in the fact that they act like primitives when assigning values (as shown in line 4 in the code above), but act like references when checking for equivalency[23]. In the example before, we were checking if they were the same reference i.e. the same object, not if they have the same contents, therefore it always evaluated as true (never same object hence true). Hence `!expression.equals(check)` is more suitable.

I tested this new function with the same arguments as before and here are the results:

```
2*x
(x)*3
)*(
*(0-x)
3*x*(0-x+1)*(x+2)
```

These are the results I expected.

³The full code is at the end of the section

8.3.2 Check Brackets

Like the `standardize()` function, this method is related to the class but not to a specific object hence it will be `static`. Here is the code:

```
1 // check for and remove enclosing matching brackets
2 private static String checkBracket(String input) throws UnequalBracketsException,
   ↳ StringIndexOutOfBoundsException {
3     boolean done = false;
4     // check if there are an equal number of opening and closing brackets
5     if((input.length() - input.replace("(", "").length())
6        !=(input.length() - input.replace(")", "").length())) {
7         throw new UnequalBracketsException(input);
8     }
9     while(!done) { // repeat until there are no enclosing brackets
10        done = true; // assume no enclosing matching brackets until otherwise found
11        // check if there are enclosing brackets (not necessarily matching)
12        if((input.charAt(0) == '(') && (input.charAt(input.length()-1) == ')')) {
13            int countMatching = 1; // initialize the bracket count as 1 to
14            ↳ count for the opening bracket
15            // loop from the second character to one before the end
16            for(int i=1; i<input.length() - 1; i++) {
17                if (countMatching == 0) { // if we find the matching
18                    ↳ bracket before the end terminate and return the input
19                    return input;
20                } else if(input.charAt(i) == '(') {
21                    // increment if we find an opening bracket
22                    countMatching++;
23                } else if(input.charAt(i) == ')') {
24                    // decrement if we find an closing bracket
25                    countMatching--;
26                }
27            }
28            // if the last character is the matching bracket keep the outer while
29            ↳ loop going and and remove the enclosing bracket
30            if(countMatching == 1) {
31                input = input.substring(1, input.length() - 1);
32                done = false;
33            }
34        }
35    }
36    return input;
37 }
```

While this algorithm is explained in the design section, the confusing part of this is the checking if there are an equal number of opening and closing brackets. There is no built-in method for this, but we can use built-in methods to achieve the same result. To do this^[24] we replace all of the character that we want with empty characters and then take this away from the length of the original String.⁴

⁴Let l be the original length of the String, x the number of instances of the character and c the length of the reduced String:

$$l = x + c \implies x = l - c$$

To test this method I used the following code with comments for what I expected:

```
System.out.println(checkBracket("2x"));           // 2x - nothing changes
System.out.println(checkBracket("((x))"));        // x - 2 sets of brackets removed
System.out.println(checkBracket("(x-1)(x+1)"));   // (x-1)(x+1) - nothing changes
System.out.println(checkBracket("((x-1)(x+1))")); // (x-1)(x+1) - outer set of bracket removed
System.out.println(checkBracket("(x)"));          // throws an exception
```

Here are the results:

```
2x
x
(x-1)(x+1)
(x-1)(x+1)
Exception in thread "main" exceptions.UnequalBracketsException: There are an unequal number of
↪ opening and closing brackets in the expression: (()
    at structures.Expression.checkBracket(Expression.java:133)
    at structures.Expression.main(Expression.java:274)
```

Which is exactly what I expected.⁵

⁵Like with the Stack class I created a custom exception for this.

8.3.3 Least Significant Operator

Again this method is not linked an object, hence it will be static. The code is shown below:

```
1 // find the least significant operator in an expression
2 private static int leastSigOperatorPos(String input) {
3     int parenthesis = 0;
4     int leastSigOperatorPos = -1;
5     int leastSigOpcode = 1000; // make the opcode super high so every operator is
6     // used a string to store the operators as it is essentially a char array but with
7     // added utility such as finding elements
8     final String operators = "+-*/^";
9     int currentOpcode;
10    // loop through the entire input
11    for(int i=0; i<input.length(); i++) {
12        char currentChar = input.charAt(i);
13        currentOpcode = operators.indexOf(currentChar);
14        // check if it is an operator
15        if(currentOpcode>=0) {
16            // check if it the same or more significant and if it is not enclosed
17            // in parenthesis
18            if((currentOpcode <= leastSigOpcode) && (parenthesis == 0)) {
19                // update the significance and the position of the operator
20                leastSigOperatorPos = i;
21                leastSigOpcode = currentOpcode;
22            }
23        } else if(currentChar == '(') {
24            // increment to signify we have entered an enclosing bracket
25            parenthesis++;
26        } else if(currentChar == ')') {
27            // decrement to signify we have exited an enclosing bracket
28            parenthesis--;
29        }
30    }
31    return leastSigOperatorPos;
32 }
```

To test this method I used the following code with comments for what I expected:

```
System.out.println(leastSigOperatorPos("3+x^2")); // should return 1 for the +
System.out.println(leastSigOperatorPos("(x+3)")); // should return -1 since it is
// enclosed in brackets
System.out.println(leastSigOperatorPos("x*(x+3)^3")); // should return 1 for the *
```

Here are the results:

```
1
-1
1
```

Which is exactly what I expected.

8.3.4 Create the Abstract Syntax Tree

I first implemented the normal sequential function and later I will implement the multi-threaded approach. Again like the methods before it this is a static function, since it relates to the class not the object itself.

```
1 // create the abstract syntax tree for the expression - single-threaded
2 private static BinaryTree createTree(String expression) throws UnequalBracketsException {
3     // remove enclosing matching brackets
4     expression = checkBracket(expression);
5     // find the least significant operator, if an operator remains
6     int leastSigOperatorPos = leastSigOperatorPos(expression);
7     if (leastSigOperatorPos == -1) { // base case - no operators remain
8         return new BinaryTree(expression);
9     } else { // recursive case - recurse on the sub-expressions
10        // locate and hold the operator
11        String operator = String.valueOf(expression.charAt(leastSigOperatorPos));
12        // split the expression into sub-expressions by the operator and recurse on
13        //   ↪ them
14        String a = expression.substring(0, leastSigOperatorPos);
15        String b = expression.substring(leastSigOperatorPos + 1);
16        // return the new tree containing the trees of the sub-expressions and the
17        // operator as the root value
18        return new BinaryTree(operator, createTree(a), createTree(b));
19    }
20 }
```

The problem with creating a tree is that I can't actually display it in a way to check if my algorithm works. To get around this issue I can just use my traversal algorithm. I have tested my traversal algorithm and know that it works hence I can just output the stack from traversing it. I will standardize my expressions before I create the trees since this is what will happen in the constructor later.

```
// create the standardized expressions
String a = standardize("2^x");
String b = standardize("(x+1)(x-1)");
// create the trees
BinaryTree treeA = createTree(a);
BinaryTree treeB = createTree(b);
// show the conversion between the expressions and trees
// the traverse function uses a stack so remember the output will be reversed
// valid data
// [2, x, ^] but it will be reversed therefore [^, x, 2]
System.out.println(a + " -> " + treeA.traverse());
// valid extreme data
// [x, 1, +, x, 1, -, *] but it will be reversed therefore [*, -, 1, x, +, 1, x]
System.out.println(b + " -> " + treeB.traverse());
```

Here are the results:

```
2^x -> [^], x, 2
(x+1)*(x-1) -> [*], -, 1, x, +, 1, x
```

Which is exactly what I expected.

When I designed this algorithm I wrote about how I could implement it using multiple threads. While multi-threading can be very powerful, like everything in Computer Science there is a cost. This is especially true where in Java the cost of instantiating a thread[25] (on the fly at least) is incredibly expensive. In the article it talks about three main things:

1. Allocating Memory to the thread
2. Create the call stack for the thread[26, 29]
3. Initialize and link the thread to the host OS

This takes a lot of processing time, in this article[27] one user manages to spawn about 10000 threads a second, which means that it takes approximately 0.1 seconds to spawn every thread. This is to be taken with a pinch of salt however since the article is quite old (8 years in fact) and processors are better and the JVM should be more efficient nowadays than before. To test this for myself I ran their benchmark program.⁶ I did change some of the parameters to make each test do the same amount of work (the same number of instructions executed) and made the number of threads spawned different as well. I also ran it 3 times so I could calculate a mean to able to identify anomalous data. This also alleviates some of the randomness of the scheduler. I did this by compiling and executing the program 3 times, to reduce the effect of cached memory affecting the results. While the test does accommodate for this issue and allows for multiple tests, I think it is best to give the JVM no chance for optimization by just destroying and recreating the JVM. The results table and the corresponding graphs are below.

Table 8.1: Thread Creation Test

Number of Threads	Time Taken/ms												
	To Create				To Complete Work				To Join				Mean Total
	1	2	3	Mean	1	2	3	Mean	1	2	3	Mean	
1	2.45	2.36	2.27	2.36±0.09	0.14	0.24	0.10	0.16±0.07	0.44	0.20	0.20	0.28±0.12	3.00±0.28
2	2.34	2.36	2.78	2.49±0.22	0.17	0.15	0.14	0.15±0.02	0.27	0.39	0.37	0.34±0.05	2.98±0.29
5	2.49	3.04	2.29	2.61±0.38	0.35	0.36	0.33	0.35±0.02	0.66	0.69	0.84	0.73±0.09	3.69±0.49
20	2.81	2.28	2.38	2.49±0.27	1.02	1.71	0.98	1.24±0.37	3.11	1.03	1.93	2.02±1.04	5.75±1.68
100	2.03	2.14	2.25	2.14±0.11	3.26	2.77	2.97	3.00±0.25	5.69	5.34	5.15	5.39±0.27	10.53±0.63
1000	5.87	4.11	5.66	5.21±0.88	47.30	42.29	39.90	43.16±3.70	15.22	18.61	32.20	22.01±8.49	70.38±13.07

From the analysis above it seems that the effect of using a few threads seems minimal and the overhead may be outweighed by the performance gain from using multithreading in other parts of the function. However this may be specific to this scenario, so to test this properly I implemented the multi-threaded version which is on the next page.

⁶The code for this is on the StackOverflow thread, made by a user called, at the time of writing this, “Jaan” [27]

```

1 // create the abstract syntax tree for the expression - multi-threaded
2 private static BinaryTree createTreeThread(String expression)
3     throws StringIndexOutOfBoundsException, UnequalBracketsException,
4         ↳ InterruptedException, ExecutionException {
5     // remove enclosing matching brackets
6     expression = checkBracket(expression);
7     // find the least significant operator, if an operator remains
8     int leastSigOperatorPos = leastSigOperatorPos(expression);
9     if (leastSigOperatorPos == -1) { // base case - no operators remain
10         return new BinaryTree(expression);
11     } else { // recursive case - recurse on the sub-expressions
12         // locate and hold the operator
13         String operator = String.valueOf(expression.charAt(leastSigOperatorPos));
14         // split the expression into sub-expressions by the operator and recurse on
15         ↳ them
16         String a = expression.substring(0, leastSigOperatorPos);
17         String b = expression.substring(leastSigOperatorPos + 1);
18         BinaryTree tree0;
19         BinaryTree tree1;
20         // create the threads and execute them
21         ExecutorService executor0 = Executors.newSingleThreadExecutor();
22         Future<BinaryTree> future0 = executor0.submit(() -> {
23             return createTree(a);
24         });
25         ExecutorService executor1 = Executors.newSingleThreadExecutor();
26         Future<BinaryTree> future1 = executor1.submit(() -> {
27             return createTree(b);
28         });
29         // return the tree values from the threads
30         tree0 = future0.get();
31         tree1 = future1.get();
32         // shutdown the threads
33         executor0.shutdown();
34         executor1.shutdown();
35         // return the new tree containing the trees of the sub-expressions and the
36         // operator as the root value
37         return new BinaryTree(operator, tree0, tree1);
38     }
39 }

```

I then tried to test my 2 `createTree()` functions to identify if a purely single-threaded approach is more efficient than a multi-threaded approach. However the two approaches may change in effectiveness depending on the complexity of the expression we are parsing. In order to control this for a fair test we need to effectively measure the complexity hence we need to analyse the algorithm in detail.

Assuming that a single call of the `createTree()` algorithm takes the same time, t , to execute (exclude the recursive calls and the overhead that is need for them), then $\mathcal{O}(n) = n + 1$ where n is the number of operators. Assuming that the expression is valid, this value comes from the fact that we can split a standardized expression into by each operator. From the nature of our approach, there will be no operators on the start or the end of a valid expression, and each variable/constant is split apart by an operator⁷. Now from this property we notice that each operator has a uniquely positioned variable/constant on the right of it, apart from the first operator which has an extra variable/constant to the left of it. Now our function recurses k times where k is the number of base sub-expressions, i.e. variables/constants. However it can be hard to count these variables/constants since they can be of variable length (1.2 is a constant and has a length in terms of strings of 3, but x is also a variable but has a length of 1). On the other hand operators are incredibly easy to spot and count since there is a finite number of types of operators, and they are all of length 1. Hence it makes sense to define the complexity of our function in terms of the number of operators. This means that one of our independent variables, the one we will change, will be the number of operators. We will also change whether we use threads or not since this is the point of our experiment. The code for the testing is below:

```
// create the standardized expressions
String ex = standardize(""); //insert the expression to test in the speech marks
// create the trees
BinaryTree treeA, treeThreadA;
Instant start, end;
// run and time the threaded version
start = Instant.now();
treeThreadA = createTreeThread(ex);
end = Instant.now();
System.out.println("time taken for threaded approach - " + Duration.between(start,
↪ end).toMillis() + "ms");
// run and time the sequential version
start = Instant.now();
treeA = createTree(ex);
end = Instant.now();
System.out.println("time taken for sequential approach - " + Duration.between(start,
↪ end).toMillis() + "ms");
```

I will be testing expressions up to 1000 operators so I used the following code,

```
new String(new char[n+1]).replace("\0", "x");[28]
```

where n is the number of operators needed, to create the expressions automatically. For $n = 5$ this creates the string "xxxxxx" and when it is standardized using the previous function it becomes "x*x*x*x*x*x", which produces the expression needed to test. This relies on the fact that all the elements in arrays in Java are initialized as `null` and can all be replaced which is what is done.

⁷This is an inherent property within any valid expression since otherwise without this we cannot separate each variable/constant hence we cannot define explicitly where each variable/constant starts and ends. We have only applied this to operators and the idea becomes blurred when we include functions within our expression, such as trigonometric or logarithmic functions, but this does not matter since our algorithm does not currently does not deal with this.

Table 8.2: Sequential vs Multi-Threaded Approach Test

Number of Operators	Time Taken/ms							
	With Threads				Without Threads			
	1	2	3	Mean	1	2	3	Mean
1	67	81	52	67±15	0	0	0	0±0
5	96	61	60	72±18	1	0	0	0±1
20	90	97	59	82±19	1	1	1	1±0
100	77	66	64	69±7	3	2	2	2±1
1000	100	124	97	107±26	16	22	16	18±3

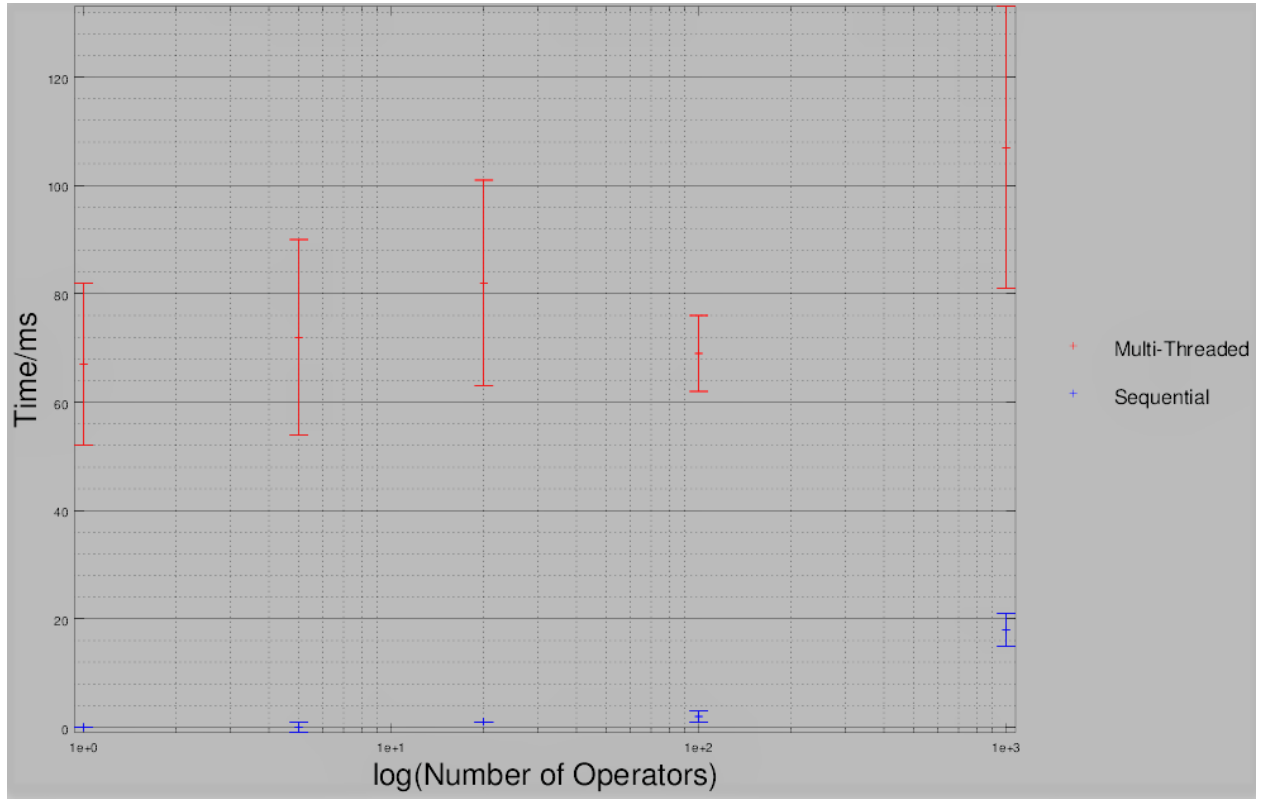


Figure 8.1: Plot of the Table Above

When representing the data I collected in a scatter graph, I chose to use a logarithmic scale for the number of threads since the number of threads I used had a very large variation in value, ranging from 1 to 1000. From analysing the graph, it is clear that using threads took much more time than the normal sequential approach, in some cases over a 6000% increase in time taken. However does this sort of relationship between a sequential and threaded approach continue for a larger number of operators?

Looking at the graph more closely it appears as though there is some kind of linear relationship between time and the logarithm of the number of operators (obviously ignoring the anomalous data at 100 operators). From using regression tools this relationship comes out as $T = 42.118 \cdot \ln n + 67^8$.

Similarly the relationship between the sequential version seems to be exponential on the graph so it is probably a linear relationship between time and the number of operators (not the logarithm this time). From using regression tools this relationship comes out as $T = 0.0178 \cdot n + 0.1858^9$.

Since a linear function increases quicker than a logarithmic one, if we find the point of intersection, we will find the theoretic point¹⁰ at which the threaded version outperforms, i.e. takes less time than, the sequential one. By using numerical method tools this comes out to be at $n \approx 28000$. Realistically we would never have this many operators in a simple function to sketch, however from a programming point of view, this is also ridiculous. From a call stack and memory point of view, this would require about 28000 recursive calls which not only is memory intensive and quite likely cause a stack overflow error[30], it will consume even more memory if using threads. Just to test if it would cause a stack overflow I tried to an example with 28000 operators and I got the following output:

```
Exception in thread "main" java.util.concurrent.ExecutionException:
↳ java.lang.StackOverflowError
    at java.util.concurrent.FutureTask.report(FutureTask.java:122)
    at java.util.concurrent.FutureTask.get(FutureTask.java:192)
    at structures.Expression.createTreeThread(Expression.java:149)
    at structures.Expression.main(Expression.java:284)
Caused by: java.lang.StackOverflowError
    at java.util.regex.Pattern.compile(Pattern.java:1054)
    at java.lang.String.replace(String.java:2239)
    at structures.Expression.checkBracket(Expression.java:197)
    at structures.Expression.createTree(Expression.java:105)
    at structures.Expression.createTree(Expression.java:118)
```

It is therefore clear that using threads in this part of the program and quite possibly in other parts of the program where the number of threads required to spawn is high, is incredibly foolish and not worth the increased effort to implement.

⁸This has a PMCC of $r = 0.9937$ so this is very good correlation

⁹This has a PMCC of $r = 0.9993$ so this is very good correlation

¹⁰It is important to realize that the sample size used here is small, but still legitimate so we can make careful extrapolations.

8.3.5 Constructor

Now that the `createTree()` function is implemented, the constructor can finally be implemented. Along with the constructor, I will override the `(toString())` method so that I can view the contents of an object of this type by calling a single method. It keeps the attributes private, and removes the need to have getters and setters for those attributes just to view them. However the most important role it fulfils is avoiding the repetition of code whenever we need to view the inside of this object, which is a good programming practice that also makes our code more organized and clean.

```
1 // constructor
2 public Expression(String expression, char parameter) throws StackOverflowException,
   ↳ StackUnderflowException, UnequalBracketsException {
3     this.parameter = parameter; // sets the parameter value
4     this.expression = standardize(expression); // removes whitespace + standardizes
5     BinaryTree tree = createTree(this.expression); // creates the tree
6     this.postFixStack = tree.traverse(); // creates the post-fix stack
7 }
8
9 @Override
10 // output the stack and the expression this object represents
11 public String toString() {
12     return this.expression + " -> " + this.postFixStack.toString();
13 }
```

To test these methods I used the following code with comments for what I expected:

```
Expression ex = new Expression("2x", 'x');
// valid data
System.out.println(ex); // expected result: 2*x -> [*], x, 2
```

Here are the results:

```
2*x -> [*], x, 2
```

Which is exactly what I expected.

8.3.6 Substitute

This function is linked to the class so it will not be static, however it will only be used within the class, specifically within the `evaluate()` function, so it will be private.

```
1 // substitute a value of the parameter into the expression
2 private Stack<String> substitute(double x) throws StackUnderflowException,
   ↳ StackOverflowException {
3     Stack<String> copy = new Stack<String>(this.postFixStack);           //copy the post fix
   ↳ stack so we don't edit it
4     Stack<String> subStack = new
   ↳ Stack<String>(this.postFixStack.getHeight());           //create a stack to store the
   ↳ substituted value of the same size as the post-fix stack
5     for (int i = copy.getHeight(); i > 0; i--) {           //loop through the copied stack
6         String pop = copy.pop();           //pop an item off and store it in a variable
7         if (pop.equals(String.valueOf(this.parameter))) {           //check if it is a
   ↳ variable if so replace it with the value
8             pop = Double.toString(x);
9         }
10        subStack.push(pop);           //push the value onto the new stack
11    }
12    return subStack;           //return the new stack
13 }
```

To test these methods I used the following code with comments for what I expected:

```
Expression ex = new Expression("2x", 'x');
// valid data
System.out.println(ex.substitute(2));           // expected result: [2], 2.0, *
ex = new Expression("2e^x", 'x');
// valid extreme data
System.out.println(ex.substitute(10));           // expected result: [2], (value for e
   ↳ approximately 2.718), 10.0, ^, *
```

Here are the results:

```
[2], 2.0, *
[2], 2.718281828459045, 10.0, ^, *
```

Which is exactly what I expected.

8.3.7 Evaluate

This function is linked to the class so it will not be `static`, however it will only be used outside the class since other modules in our program will need to know the value of the expression at a specific value, so it will be `public`. There are some language specific things that I added to this method. If the current value I have popped of the stack is not an operator, the algorithm casts it to an integer. However this can cause issues, since if we have an expression xy , this contains two parameters but we take only one parameter. So say our parameter is x , then the y will be treated as a number since it is not an operator, and it will not have been substituted for a value. So when we cast this value we will get an error. To remedy this, my method will be flagged as throwing the related exception `NumberFormatException`. The usual problems in arithmetic such as dividing by 0, are handled by Java `double` primitive and the IEEE standard[31, 32].

```
1 // evaluate the expression for a value of the parameter
2 public double evaluate(double x) throws StackUnderflowException, StackOverflowException,
   ↪ NumberFormatException {
3     // create a stack to store the numerical values
4     // and the substitute stack
5     Stack<String> subStack = substitute(x);
6     Stack<Double> numStack = new Stack<Double>(this.postFixStack.getHeight());
7     for (int i = subStack.getHeight(); i > 0; i--) {
8         String pop = subStack.pop(); // pop a value of the substitute stack and store
   ↪ it
9         double a, b; // initialize two double variables for potential calculation
10        switch (pop) { // check if the popped value is a number or operator
11            // if it is an operator
12            // do the operation on the two numbers at the top of the number stack
13            // and replace them both with the result
14            case "+":
15                b = numStack.pop();
16                a = numStack.pop();
17                numStack.push(a + b);
18                break;
19            case "-":
20                b = numStack.pop();
21                a = numStack.pop();
22                numStack.push(a - b);
23                break;
24            case "*":
25                b = numStack.pop();
26                a = numStack.pop();
27                numStack.push(a * b);
28                break;
29            case "/":
30                b = numStack.pop();
31                a = numStack.pop();
32                numStack.push(a / b);
33                break;
34            case "^":
35                b = numStack.pop();
36                a = numStack.pop();
37                numStack.push(Math.pow(a, b));
38                break;
```

```

39         // if it is a number push it onto the number stack
40         default:
41             // try to cast the popped value, it is of type String, to double
42             // this may fail so the function throws NumberFormatException
43             // this will fail if there are multiple parameters e.g. "xy"
44             // since it will treat the y as a number which it is not
45             numStack.push(Double.valueOf(pop));
46             break;
47     }
48 }
49 // pop and return the remaining value in the number stack, this is the evaluated value
50 return numStack.pop();
51 }

```

To test these methods I used the following code with comments for what I expected:

```

Expression ex1 = new Expression("2x + 4", 'x');
// valid data
System.out.println(ex1.evaluate(3)); // expected value: 10
Expression ex2 = new Expression("e^a", 'a');
// valid data - using constants
System.out.println(ex2.evaluate(0)); // expected value: 1
Expression ex3 = new Expression("1/b", 'b');
// valid extreme data
System.out.println(ex3.evaluate(0)); // expected value: infinity
Expression ex4 = new Expression("c/c", 'c');
// valid extreme data
System.out.println(ex4.evaluate(0)); // expected value: NaN
Expression ex5 = new Expression("xy", 'x');
// invalid data
System.out.println(ex5.evaluate(2)); // expected value: an exception will be
    ↪ thrown since there are multiple variables

```

Here are the results:

```

10.0
1.0
Infinity
NaN
Exception in thread "main" java.lang.NumberFormatException: For input string: "y"
    at sun.misc.FloatingDecimal.readJavaFormatString(FloatingDecimal.java:2043)
    at sun.misc.FloatingDecimal.parseDouble(FloatingDecimal.java:110)
    at java.lang.Double.parseDouble(Double.java:538)
    at java.lang.Double.valueOf(Double.java:502)
    at structures.Expression.evaluate(Expression.java:76)

```

Which is exactly what I expected.

8.4 Functions

At the moment we only need a class for explicit functions, however to illustrate the reason behind creating a separate class I will also create a quick Normal Distribution function that extends our class for explicit functions. This will be more useful later on when we implement layers.

8.4.1 Explicit Function

This class will essentially be a wrapper, a layer to manage the expression objects underneath.

```
1 package structures;
2
3 import exceptions.StackOverflowException;
4 import exceptions.StackUnderflowException;
5 import exceptions.UnequalBracketsException;
6
7 public class ExplicitFunction {
8
9     // attributes
10    private Expression f; // the actual mathematical function
11
12    // methods
13    // empty constructor to allow for inheritance
14    public ExplicitFunction() {
15    }
16
17    // constructor
18    public ExplicitFunction(String expression, char parameter) throws
19        ↳ StackOverflowException, StackUnderflowException, UnequalBracketsException {
20        this.setF(new Expression(expression, parameter));
21    }
22
23    // evaluate method calls the expression evaluate method
24    public double evaluate(double x) {
25        // try to evaluate the expression
26        //if it fails simply return NaN and the associated error
27        try {
28            return f.evaluate(x);
29        } catch (StackUnderflowException e) {
30            System.out.println("underflow");
31        } catch (StackOverflowException e) {
32            System.out.println("overflow");
33        } catch (NumberFormatException e) {
34            System.out.println("multiple parameters");
35        }
36        return Double.NaN;
37    }
38
39
40
```

```

41     // set the expression attribute - used for child classes
42     protected void setF(Expression f) {
43         this.f = f;
44     }
45
46     @Override
47     // output the representation of the expression object
48     public String toString() {
49         return this.f.toString();
50     }
51
52 }

```

To test this class I used the following code with comments for what I expected:

```

ExplicitFunction f = new ExplicitFunction("x^2", 'x');
// valid data
System.out.println(f.evaluate(2.5));           // expected result: 6.25

```

Here are the results:

```
6.25
```

Which is exactly what I expected.

8.4.2 Normal Distribution Function

This class will inherit the Explicit Function class and will basically have its own expression that we will substitute the parameters, mean, μ , and variance, σ^2 , into.

```

1  package structures;
2
3  import exceptions.StackOverflowException;
4  import exceptions.StackUnderflowException;
5  import exceptions.UnequalBracketsException;
6
7  public class NormalDistribution extends ExplicitFunction {
8
9      // attributes
10     private double  $\mu$ ; // mean
11     private double  $\sigma^2$ ; // variation
12
13     // methods
14     // constructor
15     public NormalDistribution(double  $\mu$ , double  $\sigma^2$ )
16         throws StackOverflowException, StackUnderflowException,
17         ↪ UnequalBracketsException {
18         // super constructor must be called first in Java
19         super();
20
21         // set the attributes
22         this. $\mu$  =  $\mu$ ;

```

```

22         this. $\sigma^2$  =  $\sigma^2$ ;
23
24         // create the expression for the normal distribution function
25         String expression = "(1/(2*pi* $\sigma^2$ )^(1/2))*e^(-(x- $\mu$ )^2/(2* $\sigma^2$ ^(1/2)))";
26         // substitute the mean and variance values
27         expression = expression.replace(" $\sigma^2$ ", Double.toString( $\sigma^2$ ));
28         expression = expression.replace(" $\mu$ ", Double.toString( $\mu$ ));
29
30         // set the new expression
31         this.setF(new Expression(expression, 'x'));
32     }
33
34     // return the mean
35     public double getM() {
36         return  $\mu$ ;
37     }
38
39     // return the variance
40     public double get $\Sigma^2$ () {
41         return  $\sigma^2$ ;
42     }
43
44     // return the standard deviation
45     public double get $\Sigma$ () {
46         return Math.sqrt( $\sigma^2$ );
47     }
48
49 }

```

To test this class I used the following code with comments for what I expected:

```

ExplicitFunction f = new NormalDistribution(0,1);
// valid data
System.out.println(f.evaluate(1));           // expected value: 0.24197 to 5 s.f.

```

Here are the results:

```
0.24197072451914337
```

Which is exactly what I expected.

8.5 Layers

As described in the design section, I will create classes that inherit from one a top class that I will call `Layer`. This class will be **abstract** since we will never instantiate this class and only inherit from it. The `CartesianLayer` will be a child class to the abstract `Layer` class. Again this will be **abstract** since we will only inherit from it. While it seems redundant for the super class `Layer` to exist, it fulfils the purpose of being able to create an array of type `Layer` which can then be filled with objects that are of types which are child classes to `Layer`. Our first instantiable class will be a child class to `CartesianLayer` and will be called `ExplicitXFunctionCartesianLayer`. The X signifies that the parameter will be x . Likewise we will have a class called `ExplicitYFunctionCartesianLayer`, where this will be a function in terms of y . These two function classes are very similar, they both use algorithm 16 on page 38 but one uses the primary variable x and the other uses the primary variable y . Our `AxesCartesianLayer` will also inherit from the `CartesianLayer` parent class. This makes actually drawing the axes easier, since we can simply draw 2 lines using the methods in the parent class, `CartesianLayer`.

A diagram of this inheritance is shown below:

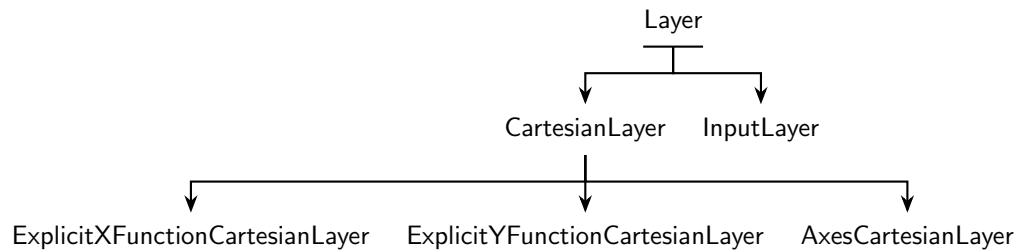


Figure 8.2: Layer Inheritance Diagram

The problem with this part of the program is that everything is interdependent on everything else. Specifically here the `PlotPane` class is used within the `Layer` classes and vice versa. Since I will implement the `Layer` classes first I will create a bare bones `PlotPane` class that just contains its attributes for use within the `Layer` classes and implement the `PlotPane` class later. This is simply to avoid compiler errors. This also means that we cannot properly test this portion until we complete both parts. As such I will combine the mini tests that I would have carried out, as a part of the overall final testing for this prototype.

8.5.1 Layer

This is an abstract super class, as described in the design. Since we cannot instantiate this class, we cannot directly test this class, but by testing its child classes we can test it indirectly. I added an the attribute Color and setter for it, which controls what the color of anything drawn onto the canvas is.

```
1 package layer;
2
3 import application.PlotPane;
4 import javafx.beans.property.DoubleProperty;
5 import javafx.beans.property.IntegerProperty;
6 import javafx.beans.property.SimpleDoubleProperty;
7 import javafx.beans.property.SimpleIntegerProperty;
8 import javafx.scene.canvas.Canvas;
9 import javafx.scene.canvas.GraphicsContext;
10 import javafx.scene.paint.Color;
11
12 public abstract class Layer {
13
14     // attributes
15     // drawing tools
16     protected Canvas canvas;
17     protected GraphicsContext gc;
18     protected Color color = Color.BLACK;
19     // drawing parameters
20     protected IntegerProperty steps = new SimpleIntegerProperty(0);
21     protected DoubleProperty minX = new SimpleDoubleProperty(0);
22     protected DoubleProperty maxX = new SimpleDoubleProperty(0);
23     protected DoubleProperty minY = new SimpleDoubleProperty(0);
24     protected DoubleProperty maxY = new SimpleDoubleProperty(0);
25     protected DoubleProperty pixelWorthX = new SimpleDoubleProperty(0);
26     protected DoubleProperty pixelWorthY = new SimpleDoubleProperty(0);
27
28     // methods
29     // constructor
30     protected Layer() {
31         // instantiate the canvas and create the OpenGL context
32         this.canvas = new Canvas();
33         this.gc = canvas.getGraphicsContext2D();
34     }
35
36     // will draw onto the canvas, will be overridden
37     public abstract void draw();
38
39     // will connect this layer to a PlotPane instance, will be overridden
40     public abstract void bindProperties(PlotPane plotPane);
41
42     // clear the canvas to be redrawn
43     protected void clearCanvas() {
44         gc.clearRect(0, 0, canvas.getWidth(), canvas.getHeight());
45     }
46
47     // getter for the canvas
48     public Canvas getCanvas() {
49         return canvas;
50     }
51
52     // change the color of the function drawn
53     public void setColor(Color color) {
54         this.color = color;
55     }
56
57 }
```

8.5.2 Input Layer

This class, unlike the previous, is not an abstract class. It was quite simple to implement with the algorithms 13, 17, 18 and 19 on pages 37 and 39. In the `bindProperties()` method I initialized the values for the minimum and maximum values of x and y to go from -10 to 10 .

```
1 package layer;
2
3 import java.text.DecimalFormat;
4
5 import application.PlotPane;
6 import javafx.beans.property.BooleanProperty;
7 import javafx.beans.property.DoubleProperty;
8 import javafx.beans.property.IntegerProperty;
9 import javafx.beans.property.SimpleBooleanProperty;
10 import javafx.beans.property.SimpleDoubleProperty;
11 import javafx.beans.property.SimpleIntegerProperty;
12
13 public class InputLayer extends Layer {
14
15     // attributes
16     private BooleanProperty changeViewport = new SimpleBooleanProperty(true);
17
18     // methods
19     // constructor
20     public InputLayer() {
21         // call the super constructor
22         super();
23         //update pixel worth when the canvas changes size
24         this.canvas.heightProperty().addListener(event -> updatePixelWorth());
25         this.canvas.widthProperty().addListener(event -> updatePixelWorth());
26     }
27
28
29     @Override
30     // this canvas will not draw anything so this method is empty
31     // it must still be implemented since the method was abstract in the parent class
32     public void draw() {}
33
34     // update the pixelworth
35     private void updatePixelWorth() {
36         this.pixelWorthX.set((this.maxX.doubleValue() - this.minX.doubleValue()) /
37         ↪ this.canvas.getWidth());
38         this.pixelWorthY.set((this.maxY.doubleValue() - this.minY.doubleValue()) /
39         ↪ this.canvas.getHeight());
40     }
41
42     @Override
43     // initialize the properties required so they can be bound later
44     public void bindProperties(PlotPane plotPane) {
45         // initialize the properties
46         this.steps = new SimpleIntegerProperty(400);
47         this.minX = new SimpleDoubleProperty(-10);
48         this.maxX = new SimpleDoubleProperty(10);
49         this.minY = new SimpleDoubleProperty(-10);
50         this.maxY = new SimpleDoubleProperty(10);
51         this.pixelWorthX = new SimpleDoubleProperty(1);
52         this.pixelWorthY = new SimpleDoubleProperty(1);
53         // make the canvas resize as its parent resizes by binding the associated properties
54         this.canvas.heightProperty().bind(plotPane.heightProperty());
55         this.canvas.widthProperty().bind(plotPane.widthProperty());
56     }
57
58     //return minX
59     public DoubleProperty getMinX() {
60         return minX;
61     }
62 }
```

```

60
61 //return minX
62 public DoubleProperty getMaxX() {
63     return maxX;
64 }
65
66 // return minX
67 public DoubleProperty getMinY() {
68     return minY;
69 }
70
71 // return minX
72 public DoubleProperty getMaxY() {
73     return maxY;
74 }
75
76 // return the worth of each pixel in the x-direction
77 public DoubleProperty getPixelWorthX() {
78     return pixelWorthX;
79 }
80
81 // return the worth of each pixel in the y-direction
82 public DoubleProperty getPixelWorthY() {
83     return pixelWorthY;
84 }
85
86 // return the number of draw steps
87 public IntegerProperty getSteps() {
88     return steps;
89 }
90
91 // return the property that notifies when it is time to redraw layers
92 public BooleanProperty getChangeViewport() {
93     return changeViewport;
94 }
95
96 }

```

8.5.3 Cartesian Layer

This class is also abstract and hence cannot be instantiated. It will contain methods to convert Cartesian coordinates to the Canvas coordinate system and bind its properties to a `PlotPane`. The algorithms for these are on page 37. Since all the classes that inherit this class will use the same properties I will finally override the `bindProperties` method and bind the necessary properties to reduce the amount of redundant code and ensure consistency in how the properties are bound.

```
1 package layer;
2
3 import application.PlotPane;
4 import javafx.beans.property.SimpleDoubleProperty;
5 import javafx.beans.property.SimpleIntegerProperty;
6
7 public abstract class CartesianLayer extends Layer {
8
9     @Override
10     // must be overridden since the parent class made it abstract
11     // but keep it abstract so child class must override
12     public abstract void draw();
13
14     @Override
15     // bind the needed properties from the input layer to itself
16     public void bindProperties(PlotPane plotPane) {
17         // initialize the properties with the same values as their partners
18         this.steps = new SimpleIntegerProperty(plotPane.getInputLayer().getSteps().intValue());
19         this.minX = new SimpleDoubleProperty(plotPane.getInputLayer().getMinX().doubleValue());
20         this.maxX = new SimpleDoubleProperty(plotPane.getInputLayer().getMaxX().doubleValue());
21         this.minY = new SimpleDoubleProperty(plotPane.getInputLayer().getMinY().doubleValue());
22         this.maxY = new SimpleDoubleProperty(plotPane.getInputLayer().getMaxY().doubleValue());
23         this.pixelWorthX = new
24             ↳ SimpleDoubleProperty(plotPane.getInputLayer().getPixelWorthX().doubleValue());
25         this.pixelWorthY = new
26             ↳ SimpleDoubleProperty(plotPane.getInputLayer().getPixelWorthY().doubleValue());
27         // bind the two properties together one-directionally
28         // so these attributes change when plot pane changes, not vice versa
29         this.steps.bind(plotPane.getInputLayer().getSteps());
30         this.minX.bind(plotPane.getInputLayer().getMinX());
31         this.maxX.bind(plotPane.getInputLayer().getMaxX());
32         this.minY.bind(plotPane.getInputLayer().getMinY());
33         this.maxY.bind(plotPane.getInputLayer().getMaxY());
34         this.pixelWorthX.bind(plotPane.getInputLayer().getPixelWorthX());
35         this.pixelWorthY.bind(plotPane.getInputLayer().getPixelWorthY());
36         // make the canvas resize as its parent resizes by binding the associated properties
37         canvas.heightProperty().bind(plotPane.heightProperty());
38         canvas.widthProperty().bind(plotPane.widthProperty());
39     }
40
41     // convert x-Cartesian coordinate to x-Canvas coordinate
42     protected double convertX(double x) {
43         x = x - this.minX.doubleValue();
44         x = x / this.pixelWorthX.doubleValue();
45         return x;
46     }
47
48     // convert y-Cartesian coordinate to y-Canvas coordinate
49     protected double convertY(double y) {
50         y = this.maxY.doubleValue() - y;
51         y = y / this.pixelWorthY.doubleValue();
52         return y;
53     }
54 }
```

8.5.4 Axes Cartesian Layer

Drawing the axes is quite simple so we will begin our implementation of non-abstract drawing classes with this one. This class will draw whenever the x or y coordinate is 0. This results in two lines. The x axis will be drawn from the leftmost point where y is 0 to the rightmost point where y is (rightmost and leftmost on the canvas). This means that we draw a line from the point $(x_{min}, 0)$ to the point $(x_{max}, 0)$. These x_{min} and x_{max} values are values that we have stored as properties within our class. Similarly we can draw the y axis by drawing a line from the point $(0, y_{min})$ to the point $(0, y_{max})$. These Cartesian coordinates must be first converted to the canvas coordinates, and this will be done in the implementation.

```
1 package layer;
2
3 public class AxesCartesianLayer extends CartesianLayer {
4
5     // constructor
6     public AxesCartesianLayer() {
7         // call the super to setup the canvas
8         super();
9     }
10
11     @Override
12     // draw the pair of axes
13     public void draw() {
14         // clear the canvas before drawing
15         this.clearCanvas();
16         // make the line width thinner then usual to remove emphasis from axes
17         gc.setLineWidth(1.5);
18         gc.setStroke(color);
19         // draw the y-axis
20         gc.strokeLine(this.convertX(0), this.convertY(this.minY.doubleValue()),
21             ↪ this.convertX(0),
22                 this.convertY(this.maxY.doubleValue()));
23         // draw the x-axis
24         gc.strokeLine(this.convertX(this.minX.doubleValue()), this.convertY(0),
25             this.convertX(this.maxX.doubleValue()), this.convertY(0));
26     }
27 }
```

8.5.5 Explicit x -Function Cartesian Layer

While designing the algorithm to draw functions I talked about how the function must be continuous otherwise we would come across arithmetic errors such as dividing by 0. This is no longer an issue since the `double` will handle all these errors for us. This was mentioned during the implementation of the evaluation of a function. Apart from that resolved issue there were no other issues during the implementation of this class.

```
1 package layer;
2
3 import exceptions.StackOverflowException;
4 import exceptions.StackUnderflowException;
5 import exceptions.UnequalBracketsException;
6 import structures.ExplicitFunction;
7
8 public class ExplicitXFunctionCartesianLayer extends CartesianLayer {
9
10     // attributes
11     private ExplicitFunction f;
12
13     // methods
14     // constructor - create a layer with an expression given
15     public ExplicitXFunctionCartesianLayer(String expression) {
16         // super constructor must be invoked first
17         super();
18         // try to create the function with the given expression
19         try {
20             this.f = new ExplicitFunction(expression, 'x');
21         } catch (StackOverflowException | StackUnderflowException |
22             ↪ UnequalBracketsException e) {
23             // output the error in parsing the function
24             e.printStackTrace();
25         }
26
27         // constructor - create a layer with a set function
28         public ExplicitXFunctionCartesianLayer(ExplicitFunction f) {
29             // super constructor must be invoked first
30             super();
31             // set the function
32             this.f = f;
33         }
34
35         // draw the function
36         public void draw() {
37             // setup the canvas by clearing it and set up the line to be drawn
38             this.clearCanvas();
39             gc.setLineWidth(2);
40             gc.setStroke(color);
41             double x1, x2, y1, y2;
42             // initialize the first value of x
43             x1 = this.minX.doubleValue();
```

```

44         // evaluate the first value of y
45         y1 = f.evaluate(x1);
46         // calculate the dx value i.e. the step value
47         double step = (this.maxX.doubleValue() - this.minX.doubleValue()) /
48             ↳ this.steps.doubleValue();
49         // loop until we draw everything in the range, minX to maxX, required
50         for (x2 = this.minX.doubleValue() + step; x2 < this.maxX.doubleValue(); x2 =
51             ↳ x2 + step) {
52             // evaluate the next value of y
53             y2 = f.evaluate(x2);
54             // draw the line
55             gc.strokeLine(this.convertX(x1), this.convertY(y1), this.convertX(x2),
56                 ↳ this.convertY(y2));
57             // update the old values
58             x1 = x2;
59             y1 = y2;
60         }
61     }
62
63     @Override
64     // output the function that this layer is drawing
65     public String toString() {
66         return this.f.toString();
67     }
68 }

```

8.5.6 Explicit y -Function Cartesian Layer

This is essentially a copy of the previous layer but we are looping over the y variable instead of the x variable. As such I simply replaced every instance of X with a Y and vice versa.

```

1  package layer;
2
3  import exceptions.StackOverflowException;
4  import exceptions.StackUnderflowException;
5  import exceptions.UnequalBracketsException;
6  import structures.ExplicitFunction;
7
8  public class ExplicitYFunctionCartesianLayer extends CartesianLayer {
9
10     // attributes
11     private ExplicitFunction f;
12
13     // methods
14     // constructor - create a layer with an expression given
15     public ExplicitYFunctionCartesianLayer(String expression) {
16         // super constructor must be invoked first
17         super();
18         // try to create the function with the given expression

```

```

19         try {
20             this.f = new ExplicitFunction(expression, 'y');
21         } catch (StackOverflowException | StackUnderflowException |
22             ↳ UnequalBracketsException e) {
23             // output the error in parsing the function
24             e.printStackTrace();
25         }
26     }
27     // constructor - create a layer with a set function
28     public ExplicitYFunctionCartesianLayer(ExplicitFunction f) {
29         // super constructor must be invoked first
30         super();
31         // set the function
32         this.f = f;
33     }
34
35     // draw the function
36     public void draw() {
37         // setup the canvas by clearing it and set up the line to be drawn
38         this.clearCanvas();
39         gc.setLineWidth(2);
40         gc.setStroke(color);
41         double x1, x2, y1, y2;
42         // initialize the first value of y
43         y1 = this.minY.doubleValue();
44         // evaluate the first value of x
45         x1 = f.evaluate(y1);
46         // calculate the dy value i.e. the step value
47         double step = (this.maxY.doubleValue() - this.minY.doubleValue()) /
48             ↳ this.steps.doubleValue();
49         // loop until we draw everything in the range, minY to maxY, required
50         for (y2 = this.minY.doubleValue() + step; y2 < this.maxY.doubleValue(); y2 =
51             ↳ y2 + step) {
52             // evaluate the next value of y
53             x2 = f.evaluate(y2);
54             // draw the line
55             gc.strokeLine(this.convertX(x1), this.convertY(y1), this.convertX(x2),
56                 ↳ this.convertY(y2));
57             x1 = x2;
58             y1 = y2;
59         }
60     }
61
62     @Override
63     // output the function that this layer is drawing
64     public String toString() {
65         return this.f.toString();
66     }
67 }

```

8.6 Plot Pane

The PlotPane was a bit difficult to implement. It required me to learn about lists in Java[33] and about what listeners[34] were and how they are used. Lists are not primitives in Java but an implementation of them are in standard libraries. Listeners are bits of code which are executed when an event occurs. An example of an event would be when a property changes value or if a button is presses. Listeners can be bound to any property so that when a property changes value the listener is run. In our implementation listeners are used to redraw the layers whenever the pane changes size.

When I decided whether to use a layer system or not, I mentioned how using threads to draw multiple canvases at once is an advantage. However from our tests while implementing our Expression class, we discovered that creating threads in Java is very expensive and as such it would do more harm than good to implementing a multithreaded system to draw multiple canvases at once.

Again we cannot test this component individually until the very end of this prototype where we will utilize our test plan which we designed in the design section. The code for this class is shown below and is commented to explain some of the less obvious parts:

```
1 package application;
2
3 import java.util.ArrayList;
4
5 import exceptions.StackOverflowException;
6 import exceptions.StackUnderflowException;
7 import exceptions.UnequalBracketsException;
8 import javafx.beans.property.BooleanProperty;
9 import javafx.beans.property.SimpleBooleanProperty;
10 import javafx.beans.value.ChangeListener;
11 import javafx.scene.layout.Pane;
12 import layer.AxesCartesianLayer;
13 import layer.ExplicitXFunctionCartesianLayer;
14 import layer.InputLayer;
15 import layer.Layer;
16
17 public class PlotPane extends Pane {
18
19     // attributes
20     // layers
21     private ArrayList<Layer> layers = new ArrayList<Layer>();
22     private InputLayer inputLayer;
23     private AxesCartesianLayer axesLayer;
24     // properties
25     private BooleanProperty changeViewport = new SimpleBooleanProperty(true);
26
27     // methods
28     // constructor
29     public PlotPane() {
30         // initialize the input layer and bind its properties
31         this.inputLayer = new InputLayer();
32         this.inputLayer.bindProperties(this);
33         // bind the plotpane's properties to it
34         this.changeViewport.bind(this.inputLayer.getChangeViewport());
35         // make the background of the plotpane white
36         this.setStyle("-fx-background-color: rgb(255,255,255)");
37         // this is portion of code which can be bound to a property
38         // and will run when that property changes
39         // this listener will redraw the layers
40         ChangeListener<Object> redrawListener = (observable, oldValue, newValue) -> {
41             try {
42                 drawAll();
```



```

43         } catch (StackOverflowException | StackUnderflowException |
44                 ↳ UnequalBracketsException
45                     | InterruptedException e) {
46             // error when drawing functions
47             e.printStackTrace();
48         }
49     };
50     // add the redraw listener to the size properties
51     // so when the plotpane changes size the layers are redrawn
52     this.heightProperty().addListener(redrawListener);
53     this.widthProperty().addListener(redrawListener);
54     // add the redraw listener to the changeViewport property
55     // so when the input layer notifies to draw, the layers are redrawn
56     this.changeViewport.addListener(redrawListener);
57     // initialize the axes layer and bind its properties
58     this.axesLayer = new AxesCartesianLayer();
59     axesLayer.bindProperties(this);
60 }
61 private void drawAll()
62     throws StackOverflowException, StackUnderflowException,
63     ↳ UnequalBracketsException, InterruptedException {
64     // loop through every layer and draw it
65     for (Layer l : layers) {
66         l.draw();
67     }
68     // draw the axes
69     axesLayer.draw();
70     // remove all the canvases in the plotpane
71     this.getChildren().clear();
72     // readd all the canvases in the plotpane
73     for (Layer l : layers) {
74         this.getChildren().add(l.getCanvas());
75     }
76     // add the input and axes canvases again
77     // with the input layer on top so that it can receive input
78     this.getChildren().add(axesLayer.getCanvas());
79     this.getChildren().add(inputLayer.getCanvas());
80 }
81 // add a layer
82 private void addLayer(Layer l) {
83     // bind the properties to this plotpane
84     l.bindProperties(this);
85     // add the layer to the list
86     this.layers.add(l);
87 }
88
89 // return the inputlayer to access its properties
90 public InputLayer getInputLayer() {
91     return inputLayer;
92 }
93
94 }

```

8.7 Main Border Pane

When I created a JavaFX project in the Eclipse IDE, it automatically created a main pane and a template for initializing the scene and loading the pane. I added a few lines of code to control how big the window will originally be.

```
1 package application;
2
3 import javafx.application.Application;
4 import javafx.stage.Stage;
5 import javafx.scene.Scene;
6 import javafx.scene.layout.BorderPane;
7 import javafx.fxml.FXMLLoader;
8
9 public class Main extends Application {
10
11     // methods
12     @Override
13     // a method to start the gui
14     public void start(Stage primaryStage) {
15         try {
16             // load the main FXML file
17             BorderPane root = (BorderPane)
18                 ↪ FXMLLoader.load(getClass().getResource("Main.fxml"));
19             // create the scene with the borderpane as the root node
20             Scene scene = new Scene(root);
21             scene.getStylesheets().add(getClass()
22                 .getResource("application.css").toExternalForm());
23             primaryStage.setScene(scene);
24             // make the window 1020 x 720 big
25             primaryStage.setHeight(720);
26             primaryStage.setWidth(1020);
27             // show the window
28             primaryStage.show();
29         } catch (Exception e) {
30             e.printStackTrace();
31         }
32     }
33
34     // run the program
35     public static void main(String[] args) {
36         // launch the gui
37         launch(args);
38     }
```

After I edited this main class, I opened the main BorderPane FXML file in scene builder. I removed all the automatically generated regions and gave the main pane a fxid which basically allows, me to access it in the controller class.

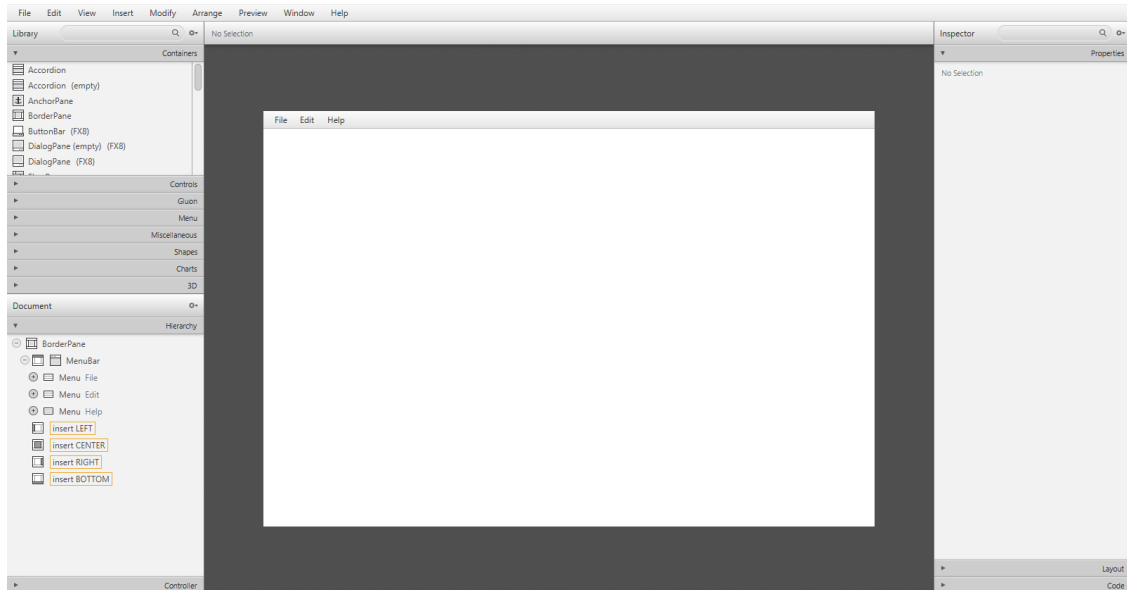


Figure 8.3: Creating the Main Pane in Scene Builder

The FXML for this pane is shown below:

```

1  <?xml version="1.0" encoding="UTF-8"?>
2
3  <?import javafx.scene.control.Menu?>
4  <?import javafx.scene.control.MenuBar?>
5  <?import javafx.scene.control.MenuItem?>
6  <?import javafx.scene.layout.BorderPane?>
7
8  <BorderPane fx:id="rootPane" prefHeight="590.0"
9      prefWidth="870.0" xmlns="http://javafx.com/javafx/8.0.141"
10     xmlns:fx="http://javafx.com/fxml/1"
11     fx:controller="application.MainController">
12     <top>
13         <MenuBar BorderPane.alignment="CENTER">
14             <menus>
15                 <Menu mnemonicParsing="false" text="File">
16                     <items>
17                         <MenuItem mnemonicParsing="false" text="Close" />
18                     </items>
19                 </Menu>
20                 <Menu mnemonicParsing="false" text="Edit">
21                     <items>
22                         <MenuItem mnemonicParsing="false" text="Delete" />
23                     </items>
24                 </Menu>
25                 <Menu mnemonicParsing="false" text="Help">
26                     <items>
27                         <MenuItem mnemonicParsing="false" text="About" />
28                     </items>
29                 </Menu>
30             </menus>
31         </MenuBar>
32     </top>
33 </BorderPane>

```

I then created a corresponding controller file in which I initialized a `PlotPane` and set it as the center node in the border pane:

```
1 package application;
2
3 import javafx.fxml.FXML;
4 import javafx.fxml.Initializable;
5 import javafx.scene.layout.BorderPane;
6
7 public class MainController implements Initializable {
8
9     // attributes
10    @FXML
11    private BorderPane rootPane;
12
13    // methods
14    @Override
15    public void initialize(URL arg0, ResourceBundle arg1) {
16        // initialize the plotpane
17        PlotPane plotPane = new PlotPane();
18        // add the panes to the root pane
19        rootPane.setCenter(plotPane);
20    }
21
22 }
```

I then tested if the program would run without crashing, expecting to see a pair of axes:



Figure 8.4: Creating the Main Pane in Scene Builder

And I saw exactly that.

Chapter 9

Testing

I have not yet created a system where the user can input function so to add layers (this is the only test that I can do at the moment) to the `PlotPane` I will simply inject the code at the end of the `PlotPane` constructor to “add” layers artificially. Using the table created in the design section I built upon it to create a more suitable tests on the code that I have. I have added tests to test the function in terms of the variable y and to test the Normal Distribution function that I made. The new and updated table is below:

Test Number	Function being Tested	Sub-Test Number	Input
1	Horizontal Lines	a	4
		b	-2
2	Linear	a	$2x - 5$
		b	$-0.5x$
3	Positive Integer Factorised Polynomials	a	$x(x - 1)$
		b	$x \wedge 2(x \wedge 2 + 1)$
		c	$-x(3x + 1)$
4	Positive Integer Unfactorised Polynomials	a	$x \wedge 2 + 2x + 1$
		b	$x \wedge 2 - 1$
		c	$x \wedge 3 + 4x \wedge 2 + x - 6$
5	Non-Integer Polynomials	a	$x \wedge (1/2)$
		b	$x \wedge (1/3)$
6	Exponentials	a	$e \wedge x$
		b	$2 \wedge x$
		c	$x \wedge x$
7	Asymptotal	a	$1/(x - 1)$
		b	$1/(x - 1) + 1/(x + 1)$
8	Explicit Functions in terms of y	a	$y \wedge 2$
		b	$e \wedge y$
9	Normal Distribution Function	a	$\mu = 0$ and $\sigma^2 = 1$
		b	$\mu = 4$ and $\sigma^2 = 0.01$
10	Multiple and Coloured Functions	N/A	

Table 9.1: Prototype 1: Updated Testing Table

I have laid all of the test evidence into separate sections, with their expected outputs and relevant code. For consistency, I will make every “a” sub test black in line colour, every “b” sub test red in line colour and every “c” sub test green in line colour. This also allows me to test another feature of the program. A summary and analysis of the results are recorded in a table at the end of this section.

9.1 Test 1

```
ExplicitXFunctionCartesianLayer f = new ExplicitXFunctionCartesianLayer("4");
ExplicitXFunctionCartesianLayer g = new ExplicitXFunctionCartesianLayer("-2");
g.setColor(Color.RED);

this.addLayer(f);
this.addLayer(g);
```

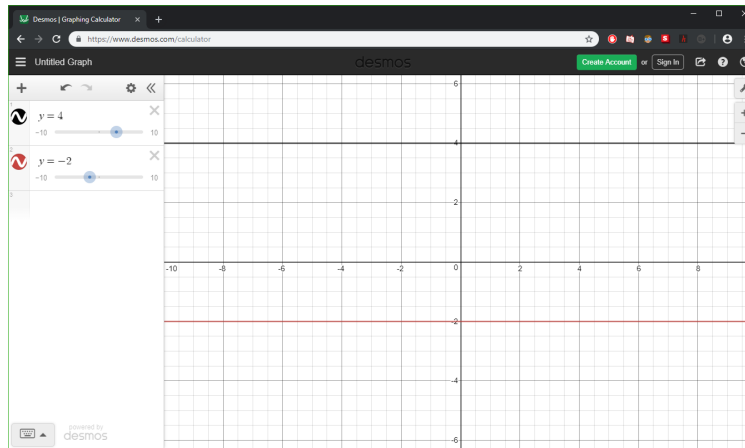


Figure 9.1: Expected Output

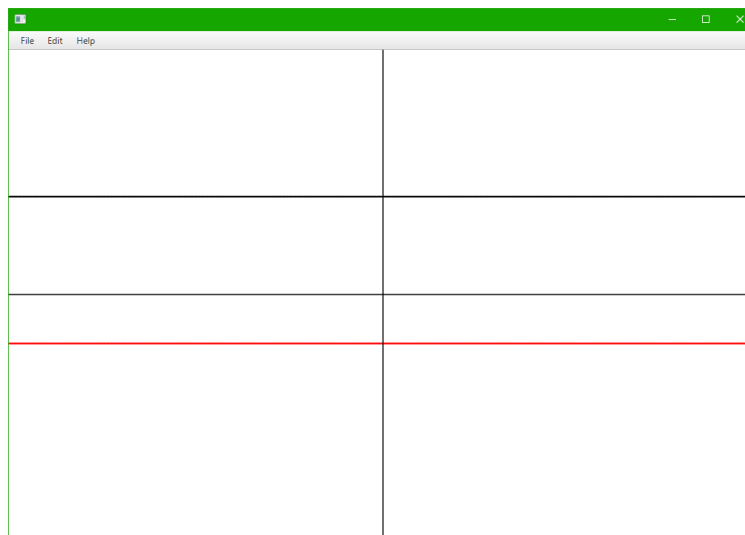


Figure 9.2: Actual Output

9.2 Test 2

```
ExplicitXFunctionCartesianLayer f = new ExplicitXFunctionCartesianLayer("2x-5");
ExplicitXFunctionCartesianLayer g = new ExplicitXFunctionCartesianLayer("-0.5x");
g.setColor(Color.RED);

this.addLayer(f);
this.addLayer(g);
```

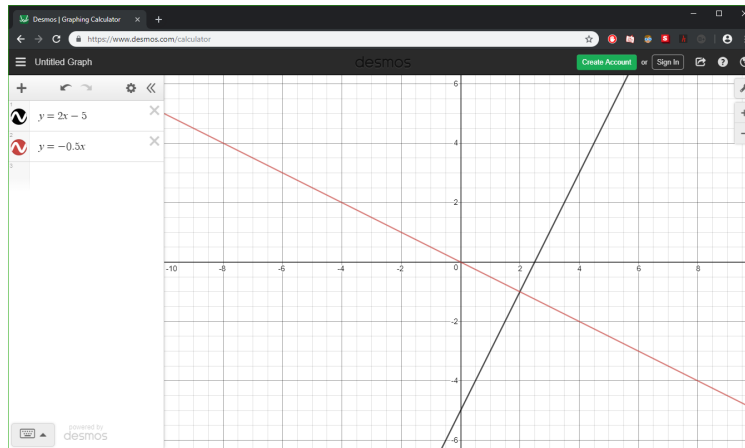


Figure 9.3: Expected Output

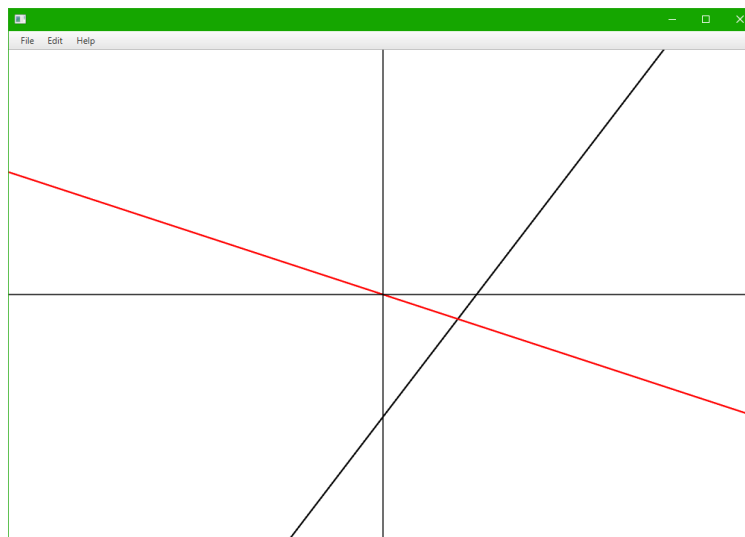


Figure 9.4: Actual Output

9.3 Test 3

```
ExplicitXFunctionCartesianLayer f = new ExplicitXFunctionCartesianLayer("x(x-1)");
ExplicitXFunctionCartesianLayer g = new ExplicitXFunctionCartesianLayer("x^2(x^2+1)");
ExplicitXFunctionCartesianLayer h = new ExplicitXFunctionCartesianLayer("-x(3x+1)");
g.setColor(Color.RED);
h.setColor(Color.GREEN);

this.addLayer(f);
this.addLayer(g);
this.addLayer(h);
```

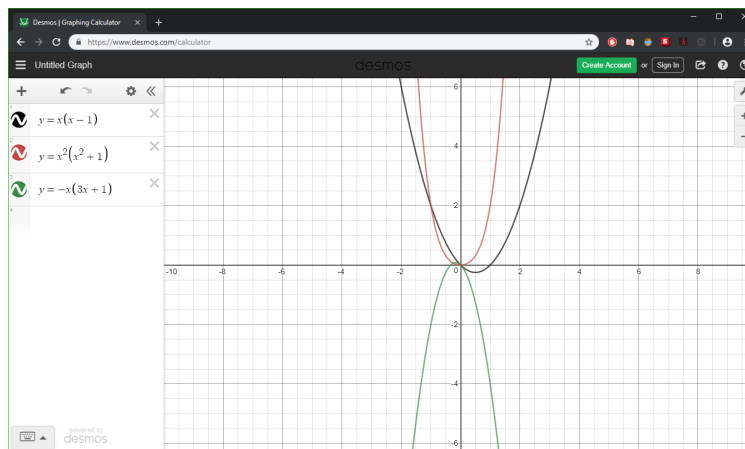


Figure 9.5: Expected Output

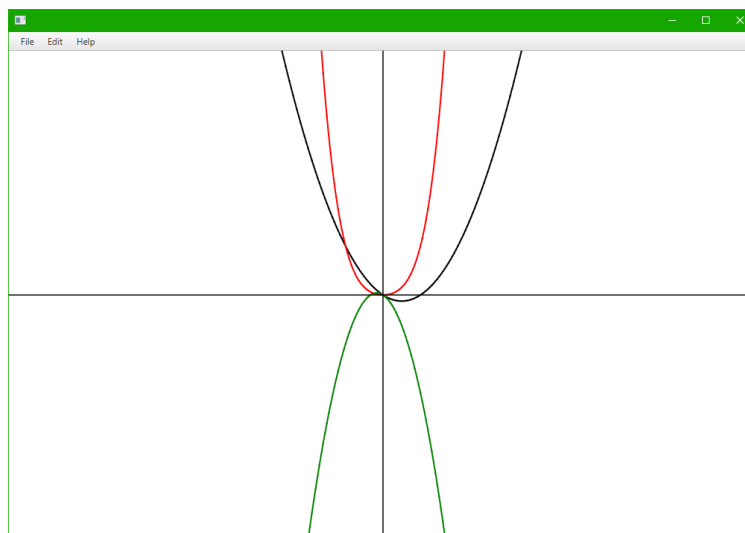


Figure 9.6: Actual Output

9.4 Test 4

```
ExplicitXFunctionCartesianLayer f = new ExplicitXFunctionCartesianLayer("x^2+2x+1");
ExplicitXFunctionCartesianLayer g = new ExplicitXFunctionCartesianLayer("x^2-1");
ExplicitXFunctionCartesianLayer h = new ExplicitXFunctionCartesianLayer("x^3+4x^2+x-6");
g.setColor(Color.RED);
h.setColor(Color.GREEN);

this.addLayer(f);
this.addLayer(g);
this.addLayer(h);
```

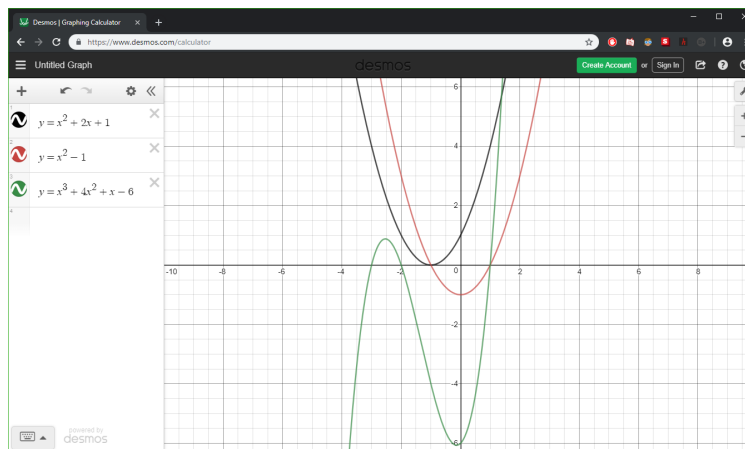


Figure 9.7: Expected Output

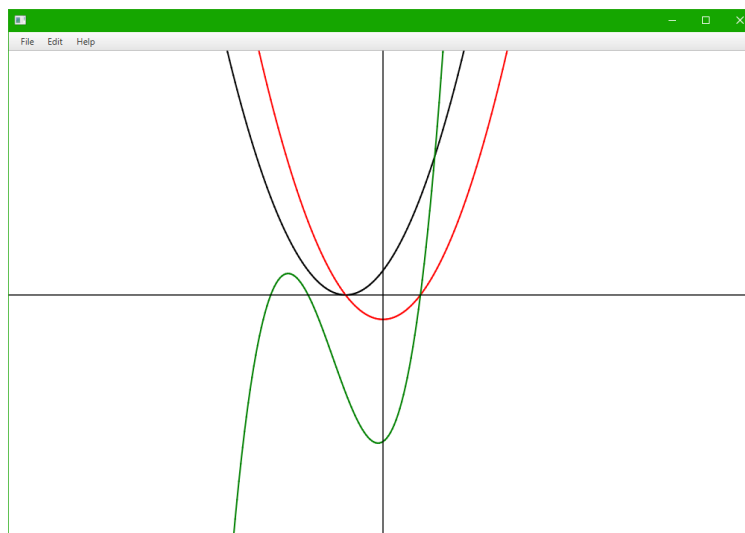


Figure 9.8: Actual Output

9.5 Test 5

```
ExplicitXFunctionCartesianLayer f = new ExplicitXFunctionCartesianLayer("x^(1/2)");
ExplicitXFunctionCartesianLayer g = new ExplicitXFunctionCartesianLayer("x^(1/3)");
g.setColor(Color.RED);

this.addLayer(f);
this.addLayer(g);
```

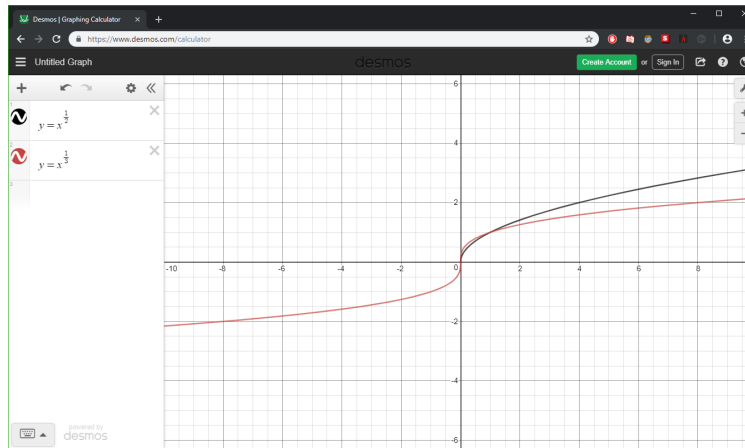


Figure 9.9: Expected Output



Figure 9.10: Actual Output

9.6 Test 6

```
ExplicitXFunctionCartesianLayer f = new ExplicitXFunctionCartesianLayer("e^x");
ExplicitXFunctionCartesianLayer g = new ExplicitXFunctionCartesianLayer("2^x");
ExplicitXFunctionCartesianLayer h = new ExplicitXFunctionCartesianLayer("x^x");
g.setColor(Color.RED);
h.setColor(Color.GREEN);

this.addLayer(f);
this.addLayer(g);
this.addLayer(h);
```

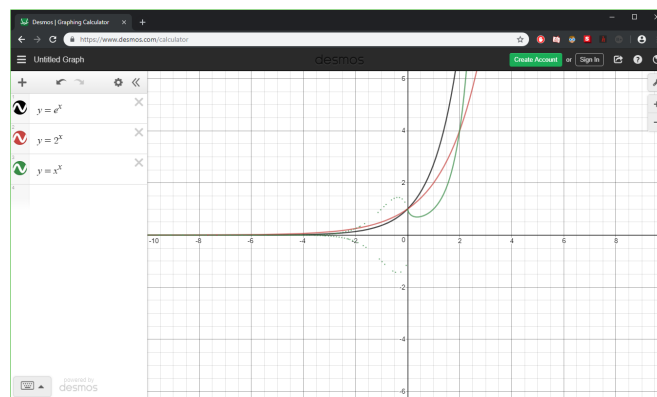


Figure 9.11: Expected Output

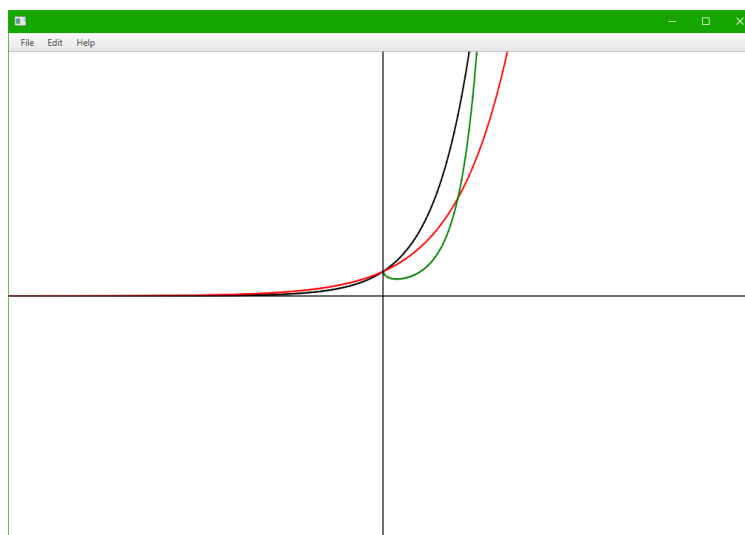


Figure 9.12: Actual Output

9.7 Test 7

```
ExplicitXFunctionCartesianLayer f = new ExplicitXFunctionCartesianLayer("1/(x-1)");  
ExplicitXFunctionCartesianLayer g = new ExplicitXFunctionCartesianLayer("1/(x-1) + 1/(x+1)");  
g.setColor(Color.RED);  
  
this.addLayer(f);  
this.addLayer(g);
```

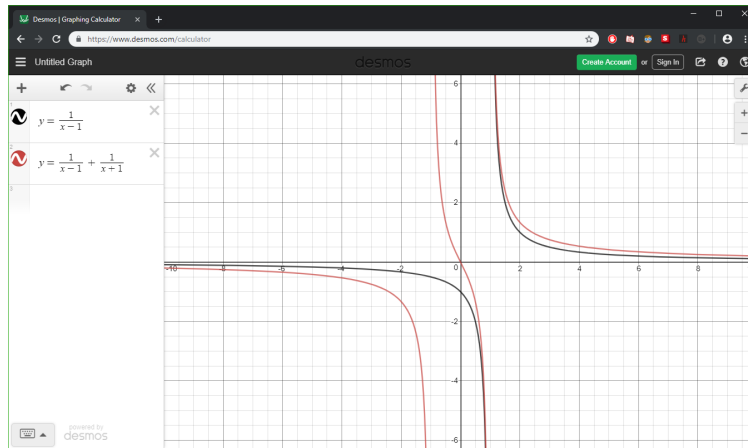


Figure 9.13: Expected Output

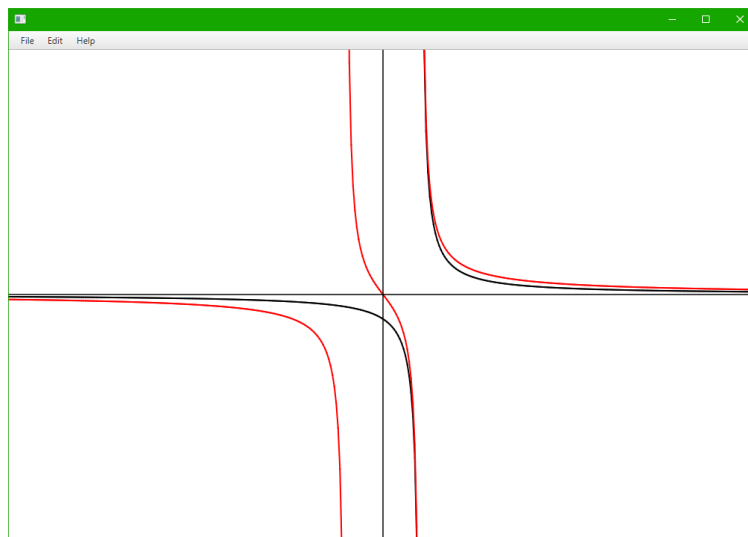


Figure 9.14: Actual Output

9.8 Test 8

```
ExplicitYFunctionCartesianLayer f = new ExplicitYFunctionCartesianLayer("y^2");
ExplicitYFunctionCartesianLayer g = new ExplicitYFunctionCartesianLayer("e^y");
g.setColor(Color.RED);

this.addLayer(f);
this.addLayer(g);
```

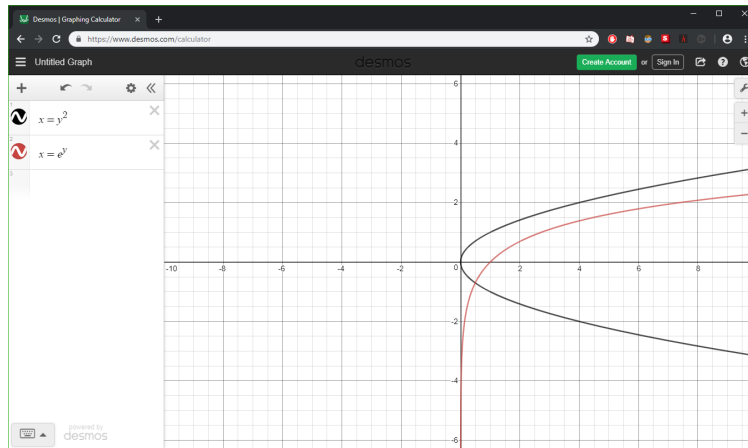


Figure 9.15: Expected Output

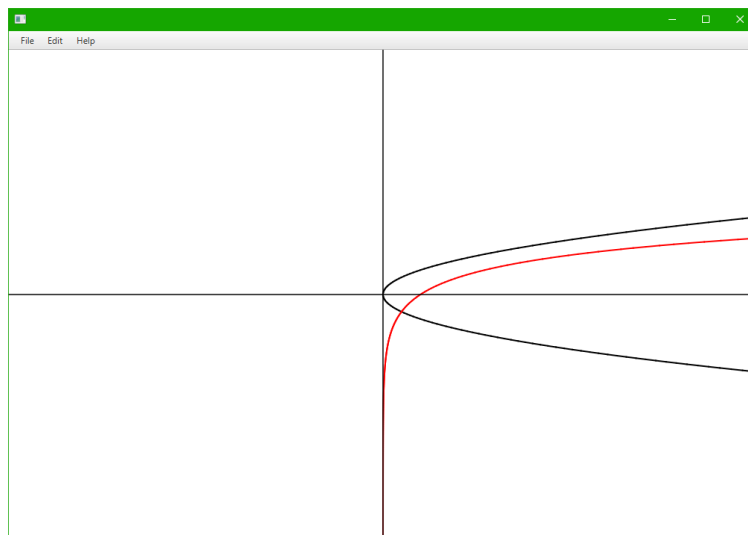


Figure 9.16: Actual Output

9.9 Test 9

```
ExplicitXFunctionCartesianLayer f = new ExplicitXFunctionCartesianLayer(new  
    ↳ NormalDistribution(0, 1));  
ExplicitXFunctionCartesianLayer g = new ExplicitXFunctionCartesianLayer(new  
    ↳ NormalDistribution(4, 0.01));  
g.setColor(Color.RED);  
  
this.addLayer(f);  
this.addLayer(g);
```

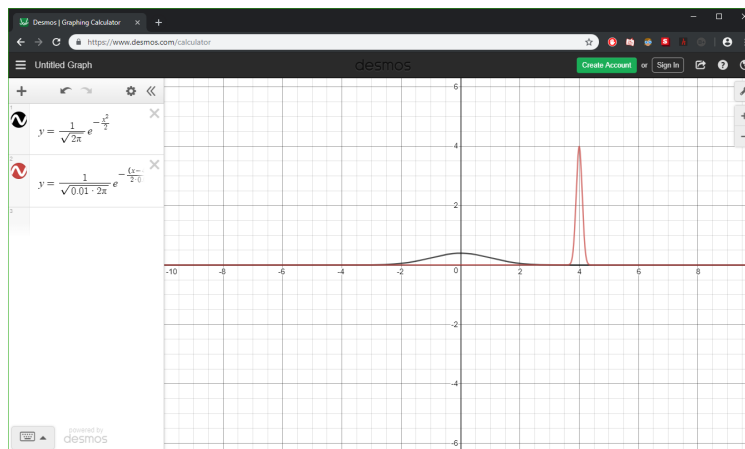


Figure 9.17: Expected Output

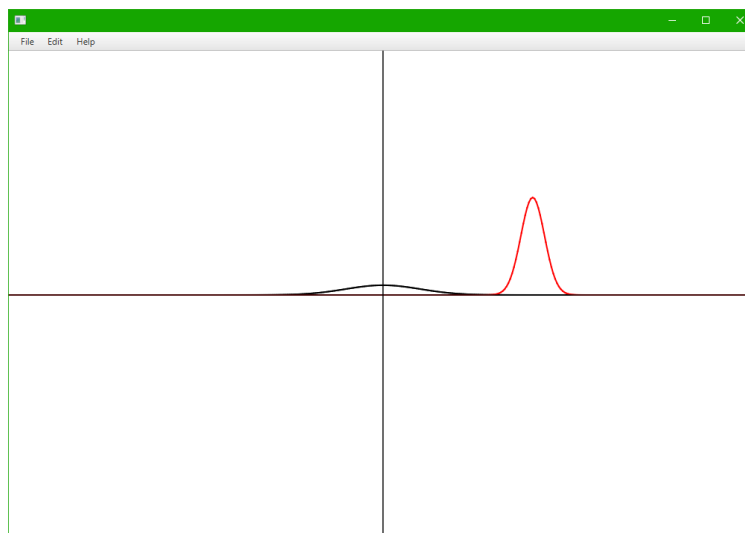


Figure 9.18: Actual Output

9.10 Conclusion

Test Number	Function being Tested	Pass/Fail	Analysis
1	Horizontal Lines	✓	The output was exactly what was expected.
2	Linear	✓	The output was exactly what was expected.
3	Positive Integer Factorised Polynomials	✓	The output was exactly what was expected.
4	Positive Integer Unfactorised Polynomials	✓	The output was exactly what was expected.
5	Non-Integer Polynomials	✗	Test part b partially failed since for $x < 0$ there was nothing drawn. This is due to the function <code>Math.pow()</code> returning NaN, Not a Number, when given a negative base and a non-integer index[35]. This is a limitation of the language I am using and I cannot do anything about it.
6	Exponentials	✗	Test part c partially failed. Looking closely at the two images you can see that there were individual points for $x < 0$ in the expected result where the function existed. However in the actual result, there was nothing drawn. This seems to be due to there only being one value in the local range for which the function is valid. Since our program works by connecting many points together with a straight line it makes sense that it doesn't work for single points. While this is a loss in accuracy, one of the targets for this project was for a responsive application and if we used a pixel by pixel drawing algorithm it would be too slow. So this is not worth attempting to fix since it would go against one of the targets for this projects.
7	Asymptotal	✓	The output was exactly what was expected.
8	Explicit Functions in terms of y	✓	The output was exactly what was expected.
9	Normal Distribution Function	✓	The output was exactly what was expected.
10	Multiple and Coloured Functions	✓	The multiple functions and different colours for all the separate functions worked perfectly for all the tests.

Table 9.2: Conclusion and Analysis of the Prototype 1 Tests

Chapter 10

Evaluation

Prototype 1 has been quite successful. I implemented the two features, plot a explicit function in x and plot multiple functions on the same plot. I have also implemented new features. I have been able to plot an explicit function in terms of y and made a Normal Distribution Function. I can plot all of the functions that I originally wanted quite accurately, with the only inaccuracies due to the programming language and not my algorithms. From this I can look at my original success criteria that are applicable here and compare and check if I have achieved them:

Criteria	Justification	Achieved
Plots Polynomials, Exponentials and Rational Functions.	These are the functions that an A-Level Student would have to know how to sketch and as such my program should be able to sketch.	✓
The accuracy of plotting function should be comparable to other graphing programs such as Desmos.	A-Level students will be using this program and they shouldn't be misled about what a function looks like.	✓
The software should be able to run on most hardware since at least 7 years ago.	This links to the idea that the program should be responsive but also the fact that it should be accessible to anyone who wishes to use it irrelevant of the hardware they have.	✓

I showed this prototype to my stakeholders and they said that they “*It looks impressive so far*” and “*I like the simplistic and uncluttered look of it and I hope that you keep it that way*”. Jeevon asked for a new feature where you can “*hide and show any function*” and Palvinder asked for a feature where you can “*see the coordinates of the point your cursor is at*”. So I will add these to the list of requirements. Sadly this project has presented some problems in terms of time. It is clear that a third prototype will not happen in the time available and some of prototype 2 will not happen either. This is the new requirements for prototype 2:

- Let the user input functions
- Zoom in/out of the graph
- Pan around the graph
- Save plots as pictures
- Show/Hide functions (suggestion from a stakeholder)
- Show coordinates in top left of pane (suggestion from a stakeholder)

From talking to my stakeholders, these are the key things that they want in a graphing program and as such these are the things that I will definitely implement. If I have time I will implement the other features.

Part III

Prototype II

Chapter 11

Design

The requirements defined at the end of prototype 1 can be split them up into 2 distinct sections:

1. User Input

- Let the user input functions
- Show/Hide functions (suggestion from a stakeholder)

2. User Interaction

- Zoom in/out of the graph
- Pan around the graph
- Save plots as pictures
- Show coordinates in top left of pane (suggestion from a stakeholder)

The first section appears deceptively small however it is probably the most complicated part of the program. It needs to be simple to use and it will probably be the most difficult part to debug.

11.1 User Interaction

Almost all of the user interaction will done through the input layer except the saving the plot as a picture. Most of these interactions are essentially coordinate transformations, translations for panning, scaling for zooming etc. The pan and zoom functionality actually change the functions drawn upon the screen so the plot pane needs to be notified to draw the functions again. This will be done by changing the viewport property, by negating the value of the viewport, i.e. changing from true to false or vice versa.

11.1.1 Save Picture

JavaFX has a feature where you can take a snapshot[36] any node. Since we want to take to take a picture of of all the functions we will take a snapshot of the plot pane. We will configure the save a picture action to be accessible through a right click menu when you click on the pane. This right click menu is called a context menu[37] in JavaFX. The user should also be able to name the image and save it where they want. We will create a method in the plot pane class to create a snapshot and save it to wherever the user wants.

11.1.2 Coordinates

The current coordinates will be shown in the top left of the input pane. Whenever the mouse moves over the pane, the current coordinates will change to their new value. Since the coordinates will probably be a very long and horrible decimal I will round the coordinates to 2 decimal places. I will also have to convert from coordinates from the canvas to the Cartesian coordinates. This is essentially the reverse of the conversion algorithm in prototype 1. Here is the algorithm for showing the coordinates:

Algorithm 20: Show the Current Coordinates

<pre>1 function drawCoords(): 2 clearCanvas() 3 Double x = (mouseX * this.pixelWorthX) + this.minX 4 Double y = this.maxY - (mouseY * this.pixelWorthY) 5 Round x to 2 decimal places 6 Round y to 2 decimal places 7 String out = "(" + x + "," + y + ")" 8 drawText(out,0,0) 9 end</pre>

11.1.3 Pan

Panning is essentially a translation of the viewport. It will edit the minimum and maximum values of x and y (the boundaries of our viewport) to create the illusion of panning around. It will be a method in the input layer class. We will essentially make this occur when holding down left click and dragging across the pane. It will work by storing the current and previous values of where the cursor has been relative to the coordinate axes. These will be attributes in the input layer class. We will then work out the vector from the previous position to the new position. Then add this vector to the maximum and minimum values. Finally we need to notify the plot pane to draw again by changing the viewport attribute and updating the pixel worth. Here is the algorithm for panning:

Algorithm 21: Pan around the Plot
--

<pre>1 function pan(): 2 if mouse left click is down then 3 currentX = event.getX() 4 currentY = event.getY() 5 double xTrans = (previousX - currentX) 6 double yTrans = (previousY - currentY) 7 previousX = currentX 8 previousY = currentY 9 maxX = (this.pixelWorthX * xTrans) + this.maxX 10 minX = (this.pixelWorthX * xTrans) + this.minX 11 maxY = -(this.pixelWorthY * yTrans) + this.maxY 12 minY = -(this.pixelWorthY * yTrans) + this.minY 13 this.changeViewport = ! this.changeViewport 14 end</pre>	<pre>// notify the plot pane</pre>
--	------------------------------------

11.1.4 Zoom

There are two ways to implement a zooming functionality. One is where we zoom about the centre of the viewport. This is unintuitive since no matter where the cursor is, it always zooms around the centre. Another one is where we zoom around the cursor. This feels incredibly natural and most graphing programs like Desmos and Geogebra have this. It may be harder to implement than the first idea but it fits with the target that the program should be responsive and natural.

The zooming will occur when the scroll wheel on the mouse is scrolled. If scroll wheel is scrolled up the viewport will zoom in and if it is scrolled in the viewport will zoom out. To achieve effect of zooming into the cursor we can use some basic transformations to achieve what we need. So first we need to translate the viewport from the cursor to the origin. Then scale it up or down. Then translate it back to where the cursor was.

Again we need to notify the plot pane that the plot needs to be drawn again. We also need to update the pixel worth since the how much each pixel is worth will have changed since we are effectively scaling each pixel. So we need to call the `updatePixelWorth()` method.

Algorithm 22: Zoom into or out of the Plot	
--	--

<pre>1 function pan(): 2 Double zoomFactor = 1.05 3 Double x = (mouseX * this.pixelWorthX) + this.minX // get the cursor coordinates 4 Double y = this.maxY - (mouseY * this.pixelWorthY) 5 if zoomFactor = 1 / zoomFactor 6 then Mouse is Scrolling Up 7 Double newMinX = this.minX - x // translate viewport to origin 8 Double newMaxX = this.maxX - x 9 Double newMinY = this.minY - y 10 Double newMaxY = this.maxY - y 11 newMinX = newMinX * zoomFactor // scale the viewport 12 newMaxX = newMaxX * zoomFactor 13 newMinY = newMinY * zoomFactor 14 newMaxY = newMaxY * zoomFactor 15 newMinX = newMinX + x // translate back to original place 16 newMaxX = newMaxX + x 17 newMinY = newMinY + y 18 newMaxY = newMaxY + y 19 this.minX = newMinX 20 this.maxX = newMaxX 21 this.minY = newMinY 22 this.maxY = newMaxY 23 this.updatePixelWorth() 24 this.changeViewport = ! this.changeViewport // notify the plot pane 25 end</pre>	
--	--

11.2 User Input

Back when designing prototype 1 I talked about the idea of a class that essentially stores data that different parts of the program can access. The reason for this is to remove the need for global variables which are bad practice since they are incredibly hard to maintain and debug.

11.2.1 Shared Layer Access

The shared layer access will be a way for the input method to give layers to the plot pane to then add to its layers. It will contain an attribute that is the dictionary type data structure to hold all of the layers. This is so that a specific layer can be identified and removed (if the user wishes to remove a function). It will also contain public methods to add and remove layers from the list, and a getter for the dictionary values (the plotpane doesn't need the identifiers) in the form of a list so that the plotpane can access the list to use. It will also contain a property that is connected to the plotpane that will notify when the list has changed so that the plotpane can draw again. Here is the class diagram for the Shared Layer Access class:

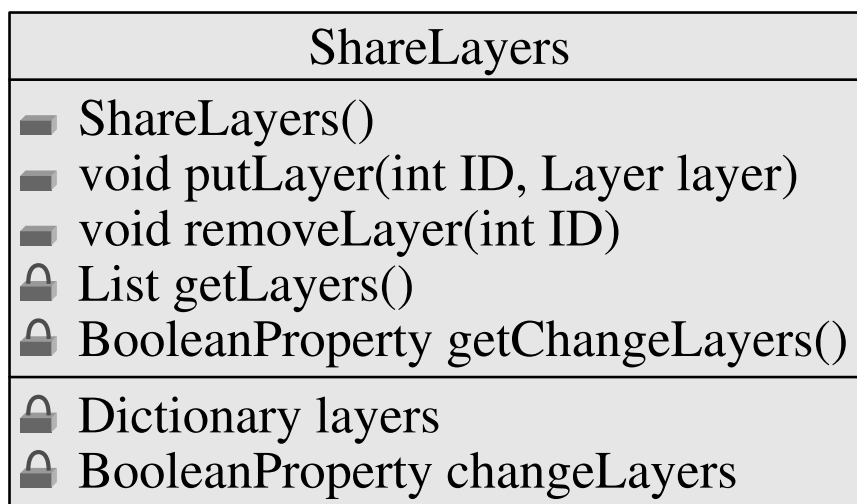


Figure 11.1: Shared Layer Access Class

The plotpane will have to bind the Boolean property so the plot pane will get an extra method and attribute to set up this shared layer access class.

11.2.2 Input Method

There are two parts to this input. I will have a miniature pane to input and control individual functions, which I will call an expression box, and then a scroll pane to store all these expression boxes. The reason I am using a scroll pane for this is because if you keep adding expression boxes I want the user to be able to scroll down to see all off them.

11.2.2.1 Input Pane

The input pane will store all the expression boxes and it will have a button to add a new box. This is how it will look like the sketch below.

From an implementation point of view I will just create a class that will inherit scroll pane. This allows it to mesh with the rest of the program, and is feasible to do without a GUI builder since it is quite a simple design. I will need some way to identify each expression box to be able to remove and update the function/layer associated with it. I will do this through an attribute of type integer called ID. I will simply use an attribute within the input pane class to keep track of what the next ID is to assign the next expression box. I will automatically make each function a new colour and this will be done with an array that will contain the standard colours that I will use and each new expression box will cycle through each of them. The expression boxes will be stored in a VBox which is a JavaFX component. It is a simple layout since it lays nodes out vertically, which is exactly how the expression will be laid out.

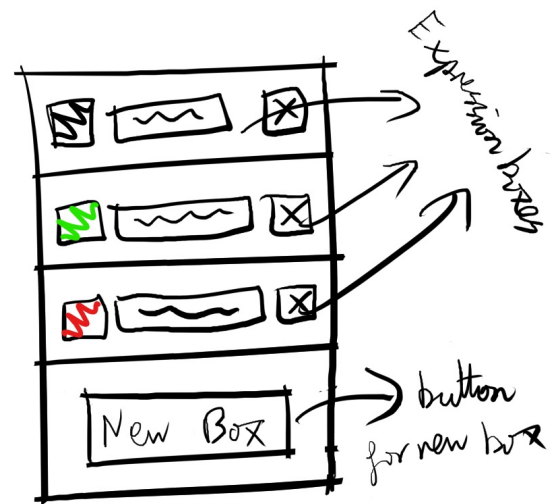


Figure 11.2: Input Pane

InputPane
<div> <div></div> <div>InputPane()</div> </div> <div> <div></div> <div>void addExpressionBox()</div> </div> <div> <div></div> <div>void removeExpressionBox(int ID)</div> </div> <div> <div></div> <div>void putLayer(int ID, Layer layer)</div> </div> <div> <div></div> <div>void removeLayer(int ID)</div> </div> <div> <div></div> <div>void setupNewButton()</div> </div> <div> <div></div> <div>setShareLayerStore(ShareLayers shareLayers)</div> </div>
<div> <div></div> <div>VBox expressionBoxStore</div> </div> <div> <div></div> <div>ShareLayers shareLayerStore</div> </div> <div> <div></div> <div>int nextID</div> </div> <div> <div></div> <div>Color[] colors</div> </div>

Figure 11.3: Input Pane Class

The input pane class will have to communicate with the plot pane, through the shared layer access, to update the layers, therefore it will have a shared layer access attribute. It will also have methods that the expression box can access to add, remove or edit layers. I will also break down some of the initial creation into methods such as the setting up of the new button, or adding an expression box.

11.2.2.2 Expression Box

I will make the expression box look similar to the one in Desmos shown below.



Figure 11.4: Input in Desmos

I made my design more simplistic and fit the theme of my program. It is shown below:

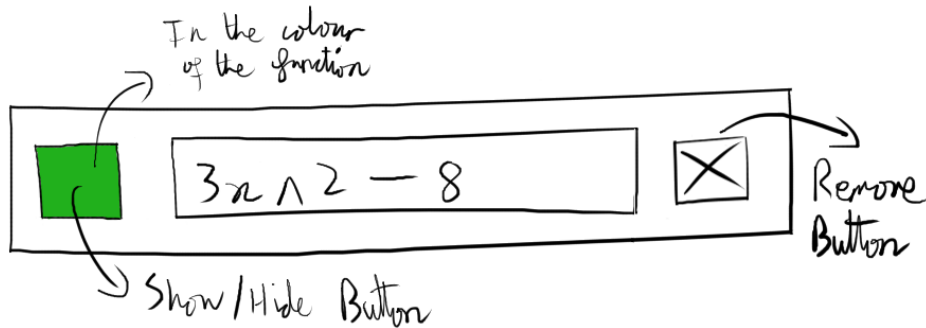


Figure 11.5: Expression Box

I showed Lewis, a stakeholder, this design and he loved it. He said that *“It looks really easy to use and I like the look.”*.

From an implementation point of view, I will create the expression box in FXML and then use a controller class to implement the methods to remove itself and add functions to the plot through the shared layer class. It will have an attribute called ID, to identify itself, this will be explained in the next section. It will also have an attribute for the colour of the function that it will draw. To access the plot pane it will have an attribute to access its parent the input pane, through which it can access the shared access class though which it can access the plot pane. The class diagram for this is below:

ExpressionBox
<ul style="list-style-type: none">ExpressionBox()void changeLayer()void remove()void setID(int ID)int getID()void setInputPane(InputPane inputPane)void set(Color color)
<ul style="list-style-type: none">int IDColor colorInputPane inputPaneboolean functionVisibility

Figure 11.6: Input Pane Class

11.3 Testing Strategy

Like in prototype 1 I will be doing unit tests however unlike prototype 1 this time the overall testing will be on the validation of input and if the implemented features actually work as intended. Each test will concentrate on one specific part of a feature and be carefully looked at to see if it is behaving as intended. I will use separate videos to record each test and these will be stored on a CD. The video evidence will be linked to each test in a separate testing table after implementation in a section of its own. If the test fails it will be analysed and the code will be modified to fix the issue. I will also include a video of a stakeholder using it, this will be referenced in the evaluation. The testing table is shown below:

Test Number	Test Description	Input	Expected Output	Input Type
1	Plot an explicit function in terms of x	e^x	The given graphs are drawn	Valid
2	Plot an explicit function in terms of y	y^2		Valid
3	Plot a normal distribution function	$Normal(0,1)$		Valid
4	Plot multiple functions of different types	Plot the above functions but all at once		Valid
5	Plot one function and replace it, as in change the value that was input, in the same box, to something else	Keep typing x	The function on the plot should continually change as the input changes, here the plot will show us x^n as n increases by one each time	Valid
6	Try to plot an invalid function i.e. an implicit function or letters that are not x or y	xy and $mx+c$	Nothing is drawn	Invalid
7	Try to plot an incomplete valid function	x^2+	While the function was being typed out, the last input that was valid should be drawn, assuming that the current input is not invalid; in this case x^2 should be drawn	Invalid
8	Add some functions	Click the add new expression button multiple times	A new expression box should be added below the rest of the functions and the button should still be at the bottom. After many boxes are added a scroll bar should appear to let the user navigate between the expression boxes	Valid
9	Remove one function	Click remove on any function	The current function should be removed from the plot and the input pane	Valid

10	Remove a function when there is only one left	Click remove when there is only one function left	The current function should be removed from the plot and the input pane and replaced with a new empty box	Ex-treme
11	Hide a shown function	Click the coloured box near the input text box	The current function is removed from the plot, it is still on the input pane but the coloured box is now a black outline signifying that it is hidden	Valid
12	Show a hidden function	Click inside the outline of a box box near the input text box	The current function is removed from the plot, it is still on the input pane but the black outlined box is now coloured in signifying that it is shown again	Valid
13	Pan around the plot	Hold left click on the plot and drag	The plot appears to be dragged about the mouse	Valid
14	Zoom in and out of a particular spot	Input 2 functions with intersections and move the scroll wheel up and down to zoom in and out respectively	The plot should appear to get closer to the intersection zooming in and out on the spot	Valid
15	Show how the coordinates change as the cursor moves around	Plot the two graphs " x " and " x^2 " and go to the intersection to verify the coordinates are working as intended	The intersection should be at (1, 1)	Valid
16	Save the current plot as a picture	Right click the plot, click save as image, save it in the desired location	The saved image and the plot should be the same except for the coordinates in the corner	Valid
17	Start to save the picture as a picture but cancel	Right click the plot, click save as image, and then click cancel	The plot should be unchanged and no image should be saved	Valid
18	Resize the application window	Left click on a corner of the application and drag the pointer outwards or inwards to resize the window	The plot should remain unchanged, but should be scaled to the window, the input pane should remain the same width	Valid

There are very few invalid or extreme tests, however this makes sense since if there is an error in the input then the function will just fail to be instantiated and so nothing will be drawn. Also a lot of these tests were done in prototype 1 as unit tests when implementing the expression class. As such there are very few tests that I need to do to verify that the input is working properly.

Chapter 12

Implementation

When beginning to implement prototype 2 I realized that I need to change some things while implementing that I said in design:

1. The user interaction section is more complicated than I made it out to be since it requires knowledge of specific types of events in JavaFX like Mouse and Scroll events, but I will look into and explain it more at that stage.
2. The classes that I designed will probably have more methods and attributes so that I can break down the problems into manageable chunks.
3. The Expression Box class will actually be a controller class so it will be implemented slightly differently than what I designed. It will have many more attributes and methods related to the FXML side and will be explained more when I get to it.

I will implement the user interaction elements first and then user input. This is because the user interaction is easier to unit test and the user input will need to be tested all at once since it is too interconnected, as explained in the test strategy. It also allows me to show my stakeholders something significant sooner.

12.1 User Interaction

All of these features require me to know about how mouse events are handled in JavaFX since all of these actions require some kind of mouse input. I used these websites[40] to get a deeper understanding of how they worked. Events are essentially objects with attributes that describe the event that just occurred. For example an attribute could tell you the position of the event. They are quite simple to use and if required I will give specifics when needed.

12.1.1 Coordinates

I used the algorithm I designed in the design section and implemented it as a method in the input layer class. I then bound this method to the `OnMouseMoved` mouse event in the input layer constructor, so the coordinates would update whenever the mouse moved in the plot pane. So I added this to the input layer constructor:

```
this.canvas.setOnMouseMoved(event -> drawCoords(event));
```

Here is the `drawCoords()` method:

```
1 // draw the coordinates
2 private void drawCoords(MouseEvent event) {
3     // clear the previous coordinates
4     clearCanvas();
5     // get the coordinates
6     double x = (event.getX() * this.pixelWorthX.doubleValue()) + this.minX.doubleValue();
7     double y = this.maxY.doubleValue() - (event.getY() * this.pixelWorthY.doubleValue());
8     // round the coordinates to 2 decimal places
9     String sX = new DecimalFormat("#.##").format(x);
10    String sY = new DecimalFormat("#.##").format(y);
11    String out = "(" + sX + ", " + sY + ")";
12    // draw the coordinates to the canvas
13    gc.setLineWidth(1);
14    gc.strokeText(out, 0, 10);
15 }
```

It works perfectly and an image of what it looks like is shown below. I asked Palvinder, the stakeholder who suggested this feature, about what he thought of the implementation and he said that it was *“exactly how I envisioned it to be”*.

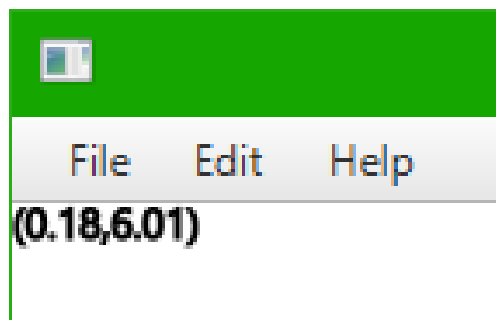


Figure 12.1: Coordinates in Action

12.1.2 Pan

The panning feature was quite hard to implement due to the lack of clear documentation on `DragEvent`. From lots of searching I found out that I had to call `startFullDrag()` method when the drag is initiated. Using this information I added these lines to the input layer constructor:

```
this.canvas.setOnMousePressed(event -> dragEntered(event));
this.canvas.setOnMouseDragged(event -> pan(event));
```

I also added 4 new attributes of type `double` to the input layer class, `previousX`, `previousY`, `currentX` and `currentY`. Here are the two methods used in the previous snippet of code `dragEntered()` and `pan()`:

```
1 // when drag starts initiate the full-drag sequence
2 private void dragEntered(MouseEvent event) {
3     // only initiate the drag if its the left mouse button
4     if (event.isPrimaryButtonDown()) {
5         // start full drag
6         event.setDragDetect(true);
7         this.canvas.startFullDrag();
8     }
9 }
10
11 // pan around the plotpane
12 private void pan(MouseEvent event) {
13     if (event.isPrimaryButtonDown()) {
14         // get the current mouse coordinates
15         currentX = event.getX();
16         currentY = event.getY();
17         // get the vector of the mouse movement
18         double xTrans = (previousX - currentX);
19         double yTrans = (previousY - currentY);
20         // update the previous values to be the current ones
21         previousX = currentX;
22         previousY = currentY;
23         // translate the viewport
24         this.maxX.set((this.pixelWorthX.doubleValue() * xTrans) +
25             ↪ this.maxX.doubleValue());
26         this.minX.set((this.pixelWorthX.doubleValue() * xTrans) +
27             ↪ this.minX.doubleValue());
28         this.maxY.set(-(this.pixelWorthY.doubleValue() * yTrans) +
29             ↪ this.maxY.doubleValue());
30         this.minY.set(-(this.pixelWorthY.doubleValue() * yTrans) +
31             ↪ this.minY.doubleValue());
32         this.changeViewport.set(!this.changeViewport.get());
33     }
34 }
```

Then I tested the pan functionality. When I first held down the left mouse click button, the viewport jumped to another location and then the pan functionality worked fine, as in I could move it about. This told me that there was a problem while initiating the drag sequence. So I used breakpoints, debugging tools and the variable stack viewer in Eclipse to debug the problem:

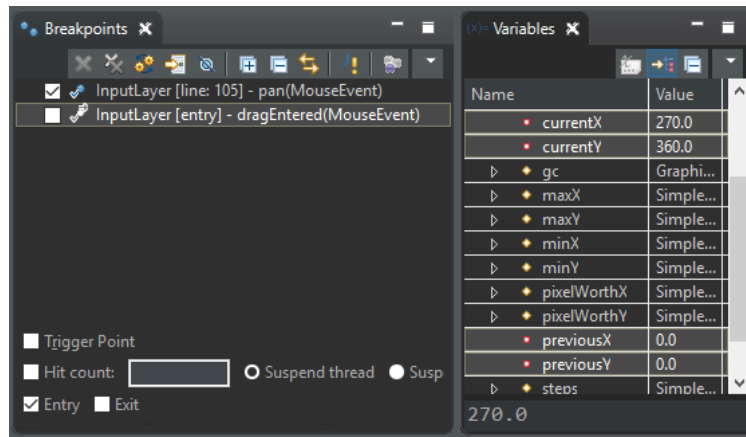


Figure 12.2: Coordinates in Action

From this I realized that the values for `previousX` and `previousY` were the last values in the previous drag which caused the viewport to jump when initiating the drag. To fix this when entering the drag I gave them the value of the current location of the cursor:

```

1 // when drag starts initiate the full-drag sequence
2 private void dragEntered(MouseEvent event) {
3     // only initiate the drag if its the left mouse button
4     if (event.isPrimaryButtonDown()) {
5         // need to initialize the values of previousX and previousY
6         previousX = event.getX();
7         previousY = event.getY();
8         // start full drag
9         event.setDragDetect(true);
10        this.canvas.startFullDrag();
11    }
12 }

```

This fixed the issue, although an exception seems to be thrown every time the drag is initiated:

```

Exception in thread "JavaFX Application Thread" java.lang.IllegalStateException:
Cannot start full drag outside of DRAG_DETECTED event handler

```

However the panning feature works so this exception can be ignored. Evidence of it working will be shown in the testing section in video form.

12.1.3 Zoom

I used the algorithm I designed in the design section and implemented it as a method in the input layer class, with some slight changes talked about in the next paragraph. I then bound this method to the `OnScroll` scroll event in the input layer constructor, so that the it would zoom in or out when scrolling. This is what the input layer constructor is and this is the final change:

```
1 public InputLayer() {
2     // call the super constructor
3     super();
4     // update pixel worth when the canvas changes size
5     this.canvas.heightProperty().addListener(event -> updatePixelWorth());
6     this.canvas.widthProperty().addListener(event -> updatePixelWorth());
7     // bind the mouse events to their respective actions
8     this.canvas.setOnMousePressed(event -> dragEntered(event));
9     this.canvas.setOnMouseDragged(event -> pan(event));
10    this.canvas.setOnScroll(event -> zoom(event));
11    this.canvas.setOnMouseMoved(event -> drawCoords(event));
12 }
```

When the scroll event is triggered it is an object with various attributes. One of them is the *deltaY* attribute. It is an integer and it tells us if the scroll wheel is being scrolled down or up. If it is positive it is being scrolled down and if it is negative it is being scrolled up. So using this knowledge I took my algorithm I designed and adapted it to to implemented `zoom()` method:

```
1 // zoom in and out
2 private void zoom(ScrollEvent event) {
3     double zoomFactor = 1.05;
4     // this is whether it is scrolling up or down
5     double deltaY = event.getDeltaY();
6     // get the coordinates
7     double x = (event.getX() * this.pixelWorthX.doubleValue()) + this.minX.doubleValue();
8     double y = this.maxY.doubleValue() - (event.getY() * this.pixelWorthY.doubleValue());
9     // if scrolling down shrink the viewport instead
10    if (deltaY > 0) {
11        zoomFactor = 1 / zoomFactor;
12    }
13    // translate the viewport to the origin, scale the viewport,
14    // translate the viewport back and set the new values for the viewport
15    this.minX.set((this.minX.doubleValue() - x) * zoomFactor + x);
16    this.maxX.set((this.maxX.doubleValue() - x) * zoomFactor + x);
17    this.minY.set((this.minY.doubleValue() - y) * zoomFactor + y);
18    this.maxY.set((this.maxY.doubleValue() - y) * zoomFactor + y);
19    // update and notify the plotpane
20    this.updatePixelWorth();
21    this.changeViewport.set(!this.changeViewport.get());
22 }
```

It works perfectly and evidence of it in action will be shown in video form in the testing section. Using alone images will not be able to capture the zooming in and out.

12.1.4 Save Picture

I found these two posts[38, 36], which I combined to be able to save the image where the user wants and with any name. To create the context menu, the right click menu, I used this example[39]. I edited and inserted this code to add a right click menu at the end of the plot pane constructor:

```
1 // create a right click menu
2 ContextMenu contextMenu = new ContextMenu();
3 MenuItem save = new MenuItem("Save as Picture");
4 contextMenu.getItems().addAll(save);
5 // set an action to call the savePlot() method when clicking the save button
6 save.setOnAction(new EventHandler<ActionEvent>() {
7     @Override
8     public void handle(ActionEvent event) {
9         savePlot();
10    }
11 });
12 // when right clicking open the menu
13 this.setOnContextMenuRequested(
14     event -> contextMenu.show(this.getScene().getWindow(), event.getScreenX(),
15         ↪ event.getScreenY()));
```

I then created the `savePlot()` method that is called when the “*Save as Picture*” menu item is clicked:

```
1 // save picture of plot
2 private void savePlot() {
3     // create the file navigator object
4     FileChooser fileChooser = new FileChooser();
5     // set extension filter (make the image of format png)
6     fileChooser.getExtensionFilters().add(new
7     ↪ FileChooser.ExtensionFilter("png files (*.png)", "*.png"));
8     // prompt user to select a file
9     File file = fileChooser.showSaveDialog(null);
10    if (file != null) { //if the user chose to save then snapshot the pane
11        try {
12            // create the snapshot of the pane
13            WritableImage snapshot = this.snapshot(new SnapshotParameters(),
14            ↪ null);
15            RenderedImage renderedImage = SwingFXUtils.fromFXImage(snapshot,
16            ↪ null);
17            // write the snapshot to the chosen file
18            ImageIO.write(renderedImage, "png", file);
19        } catch (IOException ex) {
20            ex.printStackTrace();
21        }
22    }
23 }
```

I then tried to save an image of the plot and it worked perfectly:

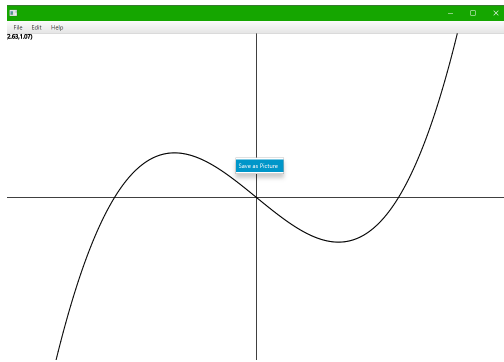


Figure 12.3: The Right Click Menu

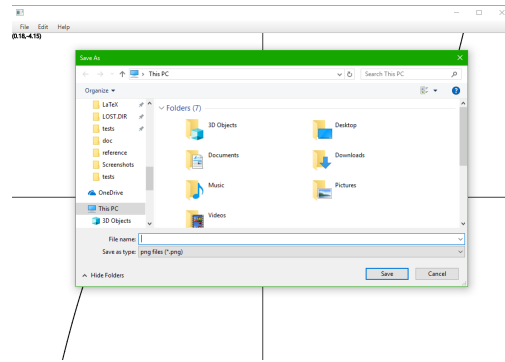


Figure 12.4: The File Navigator to Save the File

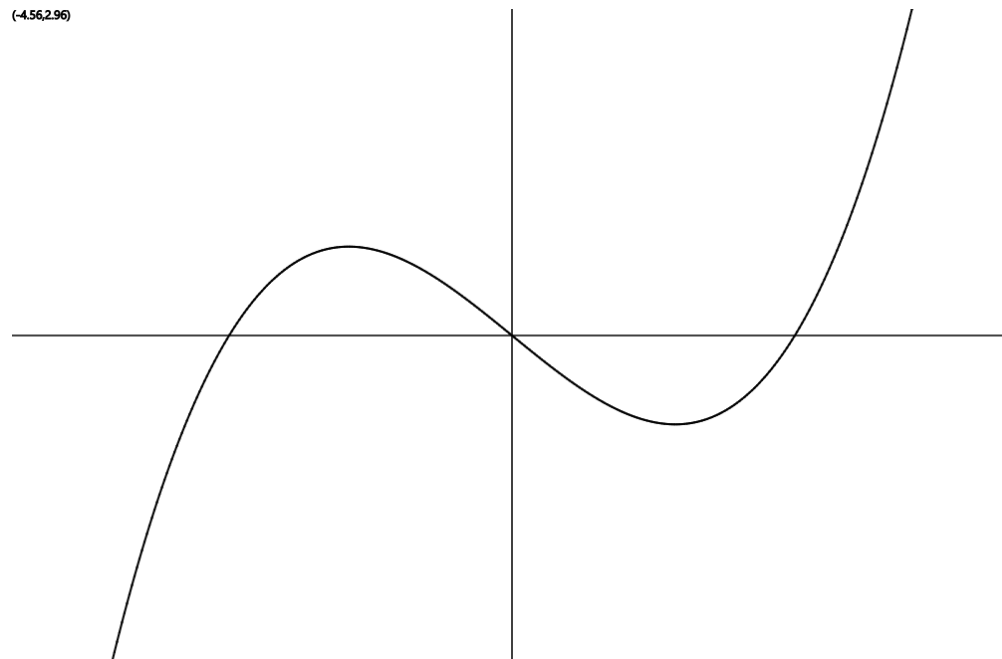


Figure 12.5: The Image Produced

A recording of the process to save will be done during the overall testing.

I let the stakeholder who originally asked for this feature, Lewis, try it and he said that he said it was all good except for the “*coordinates that were shown in the top left corner*” and he said that he didn’t want it in the saved picture. I took his suggestion and tried to fix it. Since the coordinates are part of the input layer, I fixed this by removing the input layer canvas, taking the snapshot and then re-adding the input layer canvas. It worked, so I showed Lewis again and he said it was “*great*”. Here is the code and evidence for the updated feature:

```

1 // save picture of plot
2 private void savePlot() {
3     // create the file navigator object
4     FileChooser fileChooser = new FileChooser();
5     // set extension filter
6     fileChooser.getExtensionFilters().add(new
7     ↪ FileChooser.ExtensionFilter("png files (*.png)", "*.png"));
8     // set extension filter (make the image of format png)
9     File file = fileChooser.showSaveDialog(null);
10    if (file != null) { //if the user chose to save then snapshot the pane
11        try {
12            // remove input layer canvas, the top node, to not show the coords
13            this.getChildren().remove(this.getChildren().size() - 1);
14            // create the snapshot of the pane
15            WritableImage snapshot = this.snapshot(new SnapshotParameters(),
16            ↪ null);
17            RenderedImage renderedImage = SwingFXUtils.fromFXImage(snapshot,
18            ↪ null);
19            // write the snapshot to the chosen file
20            ImageIO.write(renderedImage, "png", file);
21        } catch (IOException ex) {
22            ex.printStackTrace();
23        } finally {
24            // readd the input layer
25            this.getChildren().add(inputLayer.getCanvas());
26        }
27    }
28 }

```

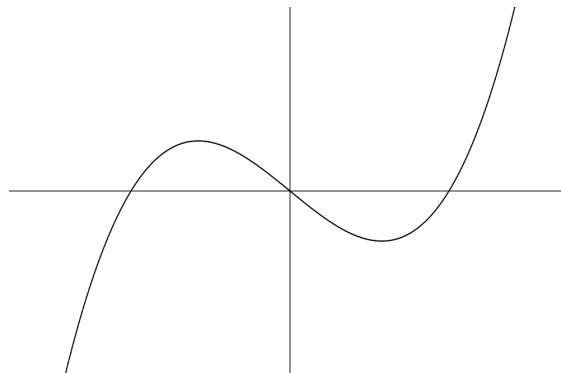


Figure 12.6: The Image Produced

12.2 User Input

12.2.1 Shared Layer Access

Within Java there is a standard dictionary like structure called a `Map` which we can use to store the layers. More specifically a `Map`^[41] is actually an interface and the specific class we will be using to store the layers is called a `HashMap`^[42] (It can be said that `HashMap` inherits `Map`). I am using a `HashMap` since it is lightweight and simple to use. With this information I implemented the `ShareLayers` class:

```
1  package application;
2
3  import java.util.ArrayList;
4  import java.util.HashMap;
5  import java.util.Map;
6
7  import javafx.beans.property.BooleanProperty;
8  import javafx.beans.property.SimpleBooleanProperty;
9  import layer.Layer;
10
11 public class ShareLayers {
12
13     // attributes
14     private Map<Integer, Layer> layers;
15     private BooleanProperty changeLayers;
16
17     // methods
18     // constructor
19     public ShareLayers() {
20         // initialize attributes
21         layers = new HashMap<Integer, Layer>();
22         changeLayers = new SimpleBooleanProperty(true);
23     }
24
25     // add a layer
26     public void putLayer(int ID, Layer layer) {
27         // add the layer
28         this.layers.put(ID, layer);
29         // notify the plot pane to draw again
30         this.changeLayers.set(!this.changeLayers.get());
31     }
32
33     // remove a layer
34     public void removeLayer(int ID) {
35         // remove the layer
36         this.layers.remove(ID);
37         // notify the plot pane to draw again
38         this.changeLayers.set(!this.changeLayers.get());
39     }
40
41     // return a list of layers
42     public ArrayList<Layer> getLayers() {
43         return new ArrayList<Layer>(this.layers.values());
44     }
45
46     // return the property which notifies the plotpane
47     public BooleanProperty getChangeLayers() {
48         return changeLayers;
49     }
50 }
51
```

Since the plot pane will need to be connected to the shared layer access, I created a method called `setShareLayerStore()` which sets up the connection between the plot pane and the shared layer access. I also gave it two new attributes, a reference to the shared layer access to get the list of layers `shareLayerStore` and the boolean property `changeLayers` that the shared layer access will bind with. Here is the implementation of the new method:

```
1 // set up the shared layer access
2 public void setShareLayerStore(ShareLayers shareLayerStore) {
3     // set the shared layer access attribute
4     this.shareLayerStore = shareLayerStore;
5     // bind and set the value for the property
6     this.changeLayers.set(this.shareLayerStore.getChangeLayers().get());
7     this.changeLayers.bind(this.shareLayerStore.getChangeLayers());
8     // create a listener to execute when the plot pane needs to draw again
9    ChangeListener<Object> redrawListener = (observable, oldValue, newValue) -> {
10         // remove all the old layers
11         this.layers.clear();
12         // add all the new layers
13         for (Layer l : shareLayerStore.getLayers()) {
14             this.addLayer(l);
15         }
16         // draw the functions again
17         try {
18             drawAll();
19         } catch (StackOverflowException | StackUnderflowException |
20             ↳ UnequalBracketsException
21                 | InterruptedException e) {
22             // error in drawing a function
23             e.printStackTrace();
24         }
25     };
26     // bind the listener to the property
27     this.changeLayers.addListener(redrawListener);
28 }
```

12.2.2 Expression Box

Before creating the expression box and its controller class I created a dummy input pane class that contained all its methods and attributes, none of which were properly implemented, so that I wouldn't get any compilation errors. I first created the FXML file using scene builder. This is what I produced:

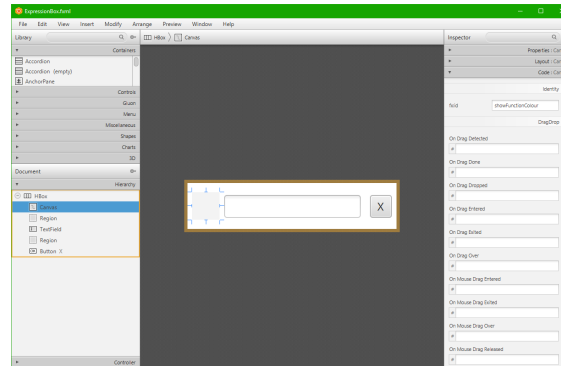


Figure 12.7: Creating the Expression Box in Scene Builder

For the button to show/hide the function I decided to use a canvas to achieve the effect. It gives me finer control about the colour of the button and is easier to manipulate. I showed my stakeholders this and Matthew said that he *“liked the simple design”*. The FXML for the expression box is shown below:

```
1 <?xml version="1.0" encoding="UTF-8"?>
2
3 <?import javafx.geometry.Insets?>
4 <?import javafx.scene.canvas.Canvas?>
5 <?import javafx.scene.control.Button?>
6 <?import javafx.scene.control.TextField?>
7 <?import javafx.scene.layout.HBox?>
8 <?import javafx.scene.layout.Region?>
9
10 <HBox alignment="CENTER" xmlns="http://javafx.com/javafx/8.0.171"
11     xmlns:fx="http://javafx.com/fxml/1"
12     fx:controller="application.ExpressionBoxController">
13     <children>
14         <Canvas fx:id="showFunctionColour" height="30.0" width="30.0">
15             <HBox.margin>
16                 <Insets left="5.0" />
17             </HBox.margin>
18         </Canvas>
19         <Region HBox.hgrow="ALWAYS" />
20         <TextField fx:id="inputTxt">
21             <HBox.margin>
22                 <Insets left="5.0" right="5.0" />
23             </HBox.margin>
24         </TextField>
25         <Region />
26         <Button fx:id="removeBtn" mnemonicParsing="false" text="X">
27             <HBox.margin>
28                 <Insets left="5.0" right="5.0" />
29             </HBox.margin>
30         </Button>
31     </children>
32     <padding>
33         <Insets bottom="10.0" top="10.0" />
34     </padding>
35 </HBox>
```

I then created the controller class. I introduced a few new methods such as the `changeColor()` method. This one is triggered when the canvas is clicked and shows/hides the function and updates the visual aid. The `changeLayer()` method is called whenever the input text changes or the user clicks on the show/hide canvas. It parses the input text determines whether or not it is a function in x , y , the Normal Distribution, or none of those. It then creates a layer for that specific function and then adds it to the shared access layer through the input pane. If the function is invalid it removes the layer from the shared layer access. The parsing is done by first checking if the string input starts with ‘Normal’. If so then the mean and variance are taken from the parameters given and the normal distribution layer is shown. Then the number of instances of x and y are counted. If there is x and y in the input, the layer is removed since the function is now implicit. If there is only y then an explicit layer in y is added with the given input expression. Finally if none of those criteria are fulfilled then an explicit function layer in x is added with the given input expression. The code for the rest of this class was quite simple to implement and it is below:

```

1  package application;
2
3  import java.io.IOException;
4  import java.net.URL;
5  import java.util.ResourceBundle;
6
7  import exceptions.StackOverflowException;
8  import exceptions.StackUnderflowException;
9  import exceptions.UnequalBracketsException;
10 import javafx.beans.property.SimpleStringProperty;
11 import javafx.beans.property.StringProperty;
12 import javafx.fxml.FXML;
13 import javafx.fxml.Initializable;
14 import javafx.scene.canvas.Canvas;
15 import javafx.scene.canvas.GraphicsContext;
16 import javafx.scene.control.Button;
17 import javafx.scene.control.TextField;
18 import javafx.scene.paint.Color;
19 import layer.ExplicitXFunctionCartesianLayer;
20 import layer.ExplicitYFunctionCartesianLayer;
21 import layer.Layer;
22 import structures.Expression;
23 import structures.NormalDistribution;
24
25 public class ExpressionBoxController implements Initializable {
26
27     // attributes
28     private int ID;
29     private GraphicsContext gc;
30     private boolean functionVisiblity = true;
31     private Color color;
32     private StringProperty functionText = new SimpleStringProperty();
33     private InputPane inputPane;
34     // FXML components
35     @FXML
36     private TextField inputTxt;
37     @FXML
38     private Button removeBtn;
39     @FXML
40     private Canvas showFunctionColour;
41
42     // methods
43     // empty constructor
44     public ExpressionBoxController() {
45     }
46
47     @Override
48     // called when initialized a pseudo constructor of sorts
49     public void initialize(URL arg0, ResourceBundle arg1) {
50         // bind the input to a string property in the class to manipulate
51         this.functionText.set(inputTxt.textProperty().getValueSafe());
52         this.functionText.bind(inputTxt.textProperty());
53         // add a listener so that every time it changes the layer is updated
54         this.functionText.addListener(event -> {
55             try {

```

```

56         changeLayer();
57     } catch (StackOverflowException | StackUnderflowException |
58         ↪ UnequalBracketsException e) {
59         e.printStackTrace();
60     }
61 });
62 // if the remove button is clicked call the remove method
63 this.removeBtn.setOnAction(event -> {
64     try {
65         remove();
66     } catch (IOException e) {
67         e.printStackTrace();
68     }
69 });
70 // create the canvas to show the color of the function
71 this.gc = this.showFunctionColour.getGraphicsContext2D();
72 this.gc.setLineWidth(2);
73 // when the canvas is clicked change its color by calling the change Color
74 // method
75 this.showFunctionColour.setOnMouseClicked(event -> {
76     try {
77         changeColor();
78     } catch (StackOverflowException | StackUnderflowException |
79         ↪ UnequalBracketsException e) {
80         e.printStackTrace();
81     }
82 });
83 }
84 // update the layer
85 private void changeLayer() throws StackOverflowException, StackUnderflowException,
86     ↪ UnequalBracketsException {
87     // only change the layer if is not hidden by the user
88     if (functionVisibility) {
89         Layer l;
90         String f = this.functionText.getValueSafe();
91         // if the function is empty remove the layer
92         if (f.length() == 0) {
93             inputPane.removeLayer(ID);
94             return;
95         }
96         // if it begins with normal create a normal distribution layer
97         if (f.startsWith("Normal(")) {
98             f = f.replaceAll("Normal\\(", "");
99             f = f.replaceAll("\\)", "");
100             String[] variables = f.split(",");
101             //  $\mu$ ;
102             double  $\mu$  = 0;
103             //  $\sigma^2$ ;
104             double  $\sigma^2$  = 1;
105             // get the values for the mean and variance
106              $\mu$  = new Expression(variables[0], 'x').evaluate(0);
107              $\sigma^2$  = new Expression(variables[1], 'x').evaluate(0);
108             // create the function
109             NormalDistribution normalDist = null;
110             normalDist = new NormalDistribution( $\mu$ ,  $\sigma^2$ );
111             // create the layer with the specific color
112             l = new ExplicitXFunctionCartesianLayer(normalDist);
113             l.setColor(color);
114             // add the layer to the shared layer access
115             inputPane.putLayer(this.ID, l);
116             // terminate this function
117             return;
118         }
119         // determine the whether it is an explicit x or y function by counting their
120         // respective instances
121         // count the number of x instances
122         int xInstances = f.length() - f.replaceAll("x", "").length();
123         // count the number of y instances
124         int yInstances = f.length() - f.replaceAll("y", "").length();
125         if (xInstances > 0 && yInstances > 0) { // if it contains both x and y remove it
126             ↪ since it is implicit
127             inputPane.removeLayer(ID);

```

```

125         return;
126     } else if (yInstances > 0) { // if it contains only y create a explicit function
        ↪ in y
127         l = new ExplicitYFunctionCartesianLayer(f);
128         l.setColor(color);
129         inputPane.putLayer(this.ID, l);
130     } else { // if it contains only x create a explicit function in x
131         l = new ExplicitXFunctionCartesianLayer(f);
132         l.setColor(color);
133         inputPane.putLayer(this.ID, l);
134     }
135     } else { // if the function is hidden then remove the layer
136         inputPane.removeLayer(ID);
137     }
138 }
139
140 // show/hide the function and update the visual identifier
141 private void changeColor() throws StackOverflowException, StackUnderflowException,
    ↪ UnequalBracketsException {
142     // clear the canvas
143     gc.clearRect(0, 0, showFunctionColour.getWidth(), showFunctionColour.getHeight());
144     if (!functionVisiblity) {
145         // fill it in color if showing the function
146         gc.fillRect(0, 0, showFunctionColour.getWidth(),
            ↪ showFunctionColour.getHeight());
147     }
148     // draw a black border so it is clickable
149     gc.strokeRect(0, 0, showFunctionColour.getWidth(), showFunctionColour.getHeight());
150     // negate the visibility since it has changed
151     functionVisiblity = !functionVisiblity;
152     // remove or add the layer to show/hide it
153     changeLayer();
154 }
155
156 // remove itself from the input pane
157 private void remove() throws IOException {
158     this.inputPane.removeExpressionBox(this.ID);
159 }
160
161 // set the id
162 public void setID(int id) {
163     ID = id;
164 }
165
166 // get the id
167 public int getID() {
168     return ID;
169 }
170
171 // set the parent input pane
172 public void setInputPane(InputPane inputPane) {
173     this.inputPane = inputPane;
174 }
175
176 // set the color of the function
177 public void setColor(Color color) {
178     this.color = color;
179     gc.setStroke(color);
180     gc.setFill(color);
181     gc.fillRect(0, 0, showFunctionColour.getWidth(), showFunctionColour.getHeight());
182 }
183
184 }

```

12.2.3 Input Pane

I then implemented the input pane class. There were no new attributes or methods and the implementation was quite straight forward. The code is below and the comments will explain what each bit does:

```
1 package application;
2
3 import java.io.IOException;
4
5 import javafx.fxml.FXMLLoader;
6 import javafx.scene.control.Button;
7 import javafx.scene.control.ScrollPane;
8 import javafx.scene.layout.HBox;
9 import javafx.scene.layout.Priority;
10 import javafx.scene.layout.Region;
11 import javafx.scene.layout.VBox;
12 import javafx.scene.paint.Color;
13 import layer.Layer;
14
15 public class InputPane extends ScrollPane {
16
17     // attributes
18     private VBox expressionBoxStore;
19     private ShareLayers shareLayerStore;
20     private int nextID = 0;
21     private final Color[] colors =
22     { Color.BLACK, Color.BLUE, Color.RED, Color.DARKGREEN, Color.MAGENTA, Color.GOLDENROD };
23
24     // methods
25     // constructor
26     public InputPane() throws IOException {
27         // set up the input pane attributes
28         this.nextID = 0;
29         this.setFitToHeight(true);
30         this.setFitToWidth(true);
31         this.setHbarPolicy(ScrollBarPolicy.NEVER);
32         this.setPrefWidth(245.0);
33         this.expressionBoxStore = new VBox();
34         this.setContent(this.expressionBoxStore);
35         // set up the button
36         this.setupNewButton();
37         // add the first expression box
38         this.addExpressionBox();
39     }
40
41     // add a new expression box
42     private void addExpressionBox() throws IOException {
43         // index at which to insert the new box, one before the button
44         int insert = this.expressionBoxStore.getChildren().size() - 1;
45         // load and initialize the expression box
46         FXMLLoader loader = new FXMLLoader();
47         loader.setLocation(Main.class.getResource("ExpressionBox.fxml"));
48         HBox expressionBox = loader.load();
49         // get the controller class and set the ID, input pane and color attributes
50         ExpressionBoxController eBoxController = (ExpressionBoxController)
51         ↪ loader.getController();
52         expressionBox.setId(Integer.toString(nextID));
53         eBoxController.setID(nextID);
54         eBoxController.setInputPane(this);
55         eBoxController.setColor(colors[nextID % this.colors.length]);
56         // add the expression box to the vbox
57         this.expressionBoxStore.getChildren().add(insert, expressionBox);
58         // increment the ID so the next box has a different ID
59         nextID = nextID + 1;
60     }
61 }
```



```

62 // remove an expression box
63 public void removeExpressionBox(int ID) throws IOException {
64     // remove the layer that the expression box is linked to
65     this.removeLayer(ID);
66     // find the expression box and remove it
67     for (int i = 0; i < this.expressionBoxStore.getChildren().size() - 1; i++) {
68         HBox box = (HBox) this.expressionBoxStore.getChildren().get(i);
69         if (Integer.parseInt(box.getId()) == ID) {
70             this.expressionBoxStore.getChildren().remove(i);
71             break;
72         }
73     }
74     // if there are no expression boxes left, reset the ID counter and add a new one
75     if (this.expressionBoxStore.getChildren().size() == 1) {
76         this.nextID = 0;
77         this.addExpressionBox();
78     }
79 }
80
81 // add a layer to the plot pane through the shared layer access
82 public void putLayer(int ID, Layer layer) {
83     shareLayerStore.putLayer(ID, layer);
84 }
85
86 // remove a layer from the plot pane through the shared layer access
87 public void removeLayer(int ID) {
88     shareLayerStore.removeLayer(ID);
89 }
90
91 // set up the new box
92 private void setupNewButton() {
93     // initialize the button with the display text "New Function"
94     Button newExpression = new Button("New Function");
95     // when it is pressed call the addExpressionBox() function
96     newExpression.setOnAction(event -> {
97         try {
98             addExpressionBox();
99         } catch (IOException e) {
100             e.printStackTrace();
101         }
102     });
103     // various UI components needed to make the button fit in with the UI
104     // hbox to hold the components
105     HBox buttonHolder = new HBox();
106     // filler space to center the button horizontally
107     Region fillerSpace1 = new Region();
108     Region fillerSpace2 = new Region();
109     // add the filler space and button to the hbox
110     buttonHolder.getChildren().addAll(fillerSpace1, newExpression, fillerSpace2);
111     HBox.setHgrow(fillerSpace1, Priority.ALWAYS);
112     HBox.setHgrow(fillerSpace2, Priority.ALWAYS);
113     // add the button to the vbox
114     this.expressionBoxStore.getChildren().add(buttonHolder);
115 }
116
117 // set the shared layer access attribute
118 public void setShareLayerStore(ShareLayers shareLayerStore) {
119     this.shareLayerStore = shareLayerStore;
120 }
121
122 }

```

I now need to add this input pane to the main border pane. I also need to connect both the plot pane and the input pane to the shared layer access class. I can do both of these things within the main border pane controller class. I made the shared layer access an attribute in the main border pane controller class. Here is the code:

```

1  package application;
2
3  import java.io.IOException;
4  import java.net.URL;
5  import java.util.ResourceBundle;
6
7  import javafx.fxml.FXML;
8  import javafx.fxml.Initializable;
9  import javafx.scene.layout.BorderPane;
10
11 public class MainController implements Initializable {
12
13     // attributes
14     @FXML
15     private BorderPane rootPane;
16
17     private ShareLayers shareLayerStore;
18
19     // methods
20     @Override
21     public void initialize(URL arg0, ResourceBundle arg1) {
22         // initialize the shared layer store
23         this.shareLayerStore = new ShareLayers();
24         // initialize the plotpane
25         PlotPane plotPane = new PlotPane();
26         // initialize the input pane
27         InputPane inputPane = null;
28         try {
29             inputPane = new InputPane();
30         } catch (IOException e) {
31             e.printStackTrace();
32         }
33         // set the shared layer stores to both the panes
34         plotPane.setShareLayerStore(shareLayerStore);
35         inputPane.setShareLayerStore(shareLayerStore);
36         // add the panes to the root pane
37         rootPane.setCenter(plotPane);
38         rootPane.setLeft(inputPane);
39     }
40
41 }

```

I then ran the program and used it for a while. It seemed to be working perfectly without any crashes or freezing. The testing will verify if the program is in fact working well and acceptable for my stakeholders.

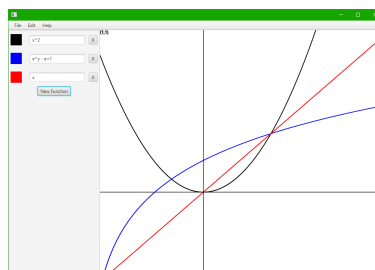


Figure 12.8: Running the Second Prototype

Chapter 13

Testing

Test Number	Input	Expected Output	Actual Output	Pass/Fail
1	e^x	The given graphs are drawn. In test 4 the graphs should be different colours.	Test_1.mp4	✓
2	y^2		Test_2.mp4	✓
3	$Normal(0,1)$		Test_3.mp4	✓
4	Plot the above functions but all at once		Test_4.mp4	✓
5	Keep typing x	The function on the plot should continually change as the input changes, here the plot will show us x^n as n increases by one each time	Test_5.mp4	✓
6	xy and mx	Nothing is drawn	Test_6.mp4	✓
7	x^2+	While the function was being typed out, the last input that was valid should be drawn, assuming that the current input is not invalid; in this case x^2 should be drawn	Test_7.mp4	✓
8	Click the add new expression button multiple times	A new expression box should be added below the rest of the functions and the button should still be at the bottom. After many boxes are added a scroll bar should appear to let the user navigate between the expression boxes	Test_8.mp4	✓
9	Click remove on any function	The current function should be removed from the plot and the input pane	Test_9.mp4	✓
10	Click remove when there is only one function left	The current function should be removed from the plot and the input pane and replaced with a new empty box	Test_10.mp4	✓

11	Click the coloured box near the input text box	The current function is removed from the plot, it is still on the input pane but the coloured box is now a black outline signifying that it is hidden	Test_11.mp4	✓
12	Click inside the outline of a box box near the input text box	The current function is removed from the plot, it is still on the input pane but the black outlined box is now coloured in signifying that it is shown again	Test_12.mp4	✓
13	Hold left click on the plot and drag	The plot appears to be dragged about the mouse	Test_13.mp4	✓
14	Input 2 functions with intersections and over the scroll wheel up and down to zoom in and out respectively	The plot should appear to get closer to the intersection, zooming in and out on the spot	Test_14.mp4	✓
15	Plot the two graphs " x " and " x^2 " and go to the intersection to verify the coordinates are working as intended	The intersection should be at (1, 1)	Test_15.mp4	✓
16	Right click the plot, click save as image save it in the desired location	The saved image and the plot should be the same except for the coordinates in the corner	Test_16.mp4	✓
17	Right click the plot, click save as image, and then click cancel	The plot should be unchanged and no image should be saved	Test_17.mp4	✓
18	Left click on a corner of the application and drag the pointer outwards or inwards to resize the window	The plot should remain unchanged, but should be scaled to the window, the input pane should remain the same width.	Test_18.mp4	✓

Chapter 14

Evaluation

The sections below highlight this program's peaks, its pitfalls and my own personal experience implementing it.

14.1 Success Criteria

Below is a table which lists the success criteria identified in the analysis and comments on how much I have achieved those criteria:

Criteria	Achieved?	Comments
Plots Polynomials, Exponentials and Rational Functions.	✓	As tested in prototype 1 it draws all the functions listed and some more.
The accuracy of plotting function should be comparable to other graphing programs such as Desmos.	✓	The accuracy was as best as it could be with the only faults due to the programming language that I used and not due to my algorithms.
The control of the viewport should be responsive.	✓	I asked Matthew, the stakeholder who asked for there to be more emphasis on this criteria, if I had achieved it and he said yes.
There are no major experience affecting bugs in the software.	✓	With myself and all my stakeholders using the program, there has been over 10 hours of use and there were been no crashes in any of them.
The software should be able to run on most hardware since at least 7 years ago.	✓	I tested this program on a very old computer and it worked great, however it struggled a bit when trying to draw 15 functions at once.

14.2 Success in Implementation of Features

I took each of the modified set of features that I said that I was going to implement and assessed how well I implemented those features in the table below:

Feature	Successful?	Comments
Plot a explicit function in x	✓	As tested in prototype 1 my program draws an explicit function in terms of x and I have gone even further by also drawing an explicit function in terms of y
Plot multiple functions on the same plot	✓	I designed the program around being able to plot multiple functions from using an OOP design.
Let the user input functions	✓	Since prototype 2 the user can input multiple functions remove and add them at will. The user could also hide and To improve this later I could let the user change the colour of the function using a colour picker.
Zoom in/out of the graph	✓	Various stakeholders commented on the responsiveness and how natural it was to zoom into and out of a point.
Pan around the graph	✓	This was one of the hardest features to implement due to how weird drag events were handled and even now an exception is thrown every time a drag event is initiated. However the functionality works and there is no adverse affect for the user and I would consider this a success.
Save plots as pictures	✓	A stakeholder said that this feature was really nice to have but incredibly difficult to find without knowing about it first. They suggested that I put in the menu ribbon that is currently not being used for anything, which I think is a sensible suggestion.
Show/Hide functions	✓	A simple but useful feature that I think adds a lot to the functionality of the program.
Show coordinates in top left of pane	✓	I think the implementation of this was really clean and helped verify some aspects of my program, especially to make sure my drawing of the functions were accurate.

14.3 Failures, Limitations

While the previous section outlined many of the successes of this project there have been a couple of failures and limitations that I have had to overcome.

I would say that the biggest failure in this project was right at the start in my analysis stage when I suggested a set of features that I was going to implement. I was not aware of how difficult this project would be to implement and document and as such I listed too many features. I learned this the hard way when beginning prototype 2 but since then have stuck to a reasonable and achievable set of features.

In terms of limitations my program sometimes draws some functions wrong. This is most notable for fractional power functions and sometimes for asymptotal functions where they either don't get drawn or draw the asymptotes as a line from infinity to negative infinity which shouldn't be drawn. This also occurs for functions that have incredible steep gradients where some parts of the function would just be missing. I could overcome this by drawing every pixel individually and by creating my own maths library for this project. However both methods are inefficient and I feel as though for my timescale and my current skill level, the methods that I used were completely acceptable.

During my project I ran into the many errors, especially runtime errors. Luckily the IDE that I was using, Eclipse, was excellent and provided excellent debug tools, including breakpoints, stack view and variable view. While it was not much use in the first prototype, it really helped me overcome some problems, especially with user input in the second prototype. Furthermore I created my own exceptions to integrate within Java's existing exception handling service, to differentiate between the errors from what I created and from errors from my use of standard libraries.

14.4 Maintenance

Since the start of this project I have maintained several practices to make sure that my program is easy to maintain by myself and others.

14.4.1 Comments and Discipline

In order to make my code to read I made sure that my code is properly indented and the brackets were put in the proper position using my IDE. If any lines of code were incredibly long I split the line into multiple lines at meaningful positions so that my code was legible. I also used comments appropriately and sparingly to explain the code's functionality for myself and others who may seek to add and extend my program further. I used them before every function to describe the corresponding function and used them in places that were quite complex to clarify and simplify.

14.4.2 Modularity

A key component of modularity in a program is the idea of using procedures and functions or using Objects and Classes and following the Object Oriented Programming (OOP) paradigm. I decided to embrace the OOP paradigm and I did this by using Java as my programming language. Java forces the programmer to use classes to actually develop and as such I designed my program around it.

An example of this is when I created an abstract parent class and created child classes that inherited from it for both my function and layer classes. This allowed the interaction between these classes and other parts of my program to be standardised by creating an interface, in the form of public methods, in the abstract parent class. It also means that others can create new types of functions and their associated layers by inheriting from the abstract classes to add more functionality and extend my program.

Another example of this is when I used generics in Java. I used them when implementing a stack data structure. It allowed me to create a basis for a stack and instead of having redundant code for the same stack but for different data types, in my case integers and strings, I could just use generics which generalise the structure for any object.

I also decomposed as much of my code into smaller methods and as many classes that were reasonable, in order to make it easier to develop my program in small manageable steps. As well as being a good programming practice, it has allowed me to take large breaks in between coding and still know what to do due to the lesser complexity of the methods.

14.4.3 Version Control

In order to organise and centralise my efforts while programming I used version control, specifically GIT. Version control allowed me to keep track of my progress using commits, which are essentially small changes with simple descriptions of the code. Furthermore it made me work towards a commit in mind making my progress more directed and focused. It also allowed me to work on the project in multiple locations, at school and at home, by storing it on a remote server and pulling and pushing to it. Finally if anything went wrong during implementing my program I could always revert commits to a previous stable version.

14.5 Software and Hardware Requirements

Here are some requirements that are needed for my program to run:

- Software
 - An Operating System that is supported by Java 8
 - The Java 8 JRE installed
- Hardware
 - A Keyboard
 - A Mouse
 - A Monitor

14.6 Stakeholder Feedback

Along with my videos for the tests, I asked a stakeholder to try out my program and record their experience. This video is included along with the evidence for the testing. I talked to them about their experience and here are the main points I got from them:

- *“The sketching of the functions was simplistic and everything that was needed in terms of functionality and information was there, for example the coordinates in the top left. There was no needless clutter like axes numbering.”*
- *“Actually inputting the functions was quite confusing since normally you input $y =$ or $x =$ at the start to define explicit functions in Desmos and with your program you didn’t need to do that.”*
- *“The menu ribbon at the top was kind of useless. I understand told that it would have been used for new features in the future. I think that one useful thing it could be great for is a help page. It could highlight all the basic features of the program and show videos of these features in action. This is especially true for the save plot as an image feature since I would have never have found that feature if you didn’t tell me about it.”*
- *“Some extra functions such as the logarithm or any of the trigonometric functions would have been useful to have since they are in the A Level Maths specification.”*

- *“Overall I think that it is an excellent program and incredibly friendly for someone who is terrible with computers like me.”*

I think this feedback shows that my program has fulfilled many of the success criteria listed at the start of this project and again at the start of this evaluation. It also lets me look forward into what could have been the future of this project from the suggestions given.

14.7 Possible Extensions

The extensions listed below are from my original set of features that I never was able to implement and some stakeholder ideas from the previous section that I thought were reasonable to implement:

- Plot special functions such as trigonometric functions, logarithms, modulus functions, etc.
- Identify intersection with the axes
- Multiple plots that you can switch between
- Save workspace to resume later
- Dark Theme
- Identify intersections between functions
- Differentiate explicit continuous functions with respect to x
- Polar equations
- Implicit equations
- Parametric equations
- \LaTeX equation support
- Identify turning points
- Help page to familiarise users with the program
- Make the menu ribbon have some function or remove it completely

14.8 Conclusion

Overall I think that this project has been a success, achieving its purpose defined in the criteria and excellent in terms of stakeholder feedback and support. Personally this project has had many hurdles, such as learning how to use generics, that I have persevered through. Fundamentally I developed my understanding of numerical applications of Mathematics through the design of this program. To achieve this I built upon my knowledge about Computer Science learning more about structures, such as Binary Trees and Stack. I have felt as though I have used and applied all of the techniques I learned during my Computer Science course, such as abstraction and decomposition, and bridged the gap between Mathematics and Computer Science which has honed my understanding for both. Even though the project could be improved I am happy with how it has turned out and may eventually come back to it in the future to update and improve it.

Part IV

Appendices - Code

Chapter 15

application

15.1 *Main.java*

```
1  package application;
2
3  import javafx.application.Application;
4  import javafx.fxml.FXMLLoader;
5  import javafx.scene.Scene;
6  import javafx.scene.layout.BorderPane;
7  import javafx.stage.Stage;
8
9  public class Main extends Application {
10
11      // methods
12      @Override
13      // a method to start the gui
14      public void start(Stage primaryStage) {
15          try {
16              // load the main FXML file
17              BorderPane root = (BorderPane)
18                  ↪ FXMLLoader.load(getClass().getResource("Main.fxml"));
19              // create the scene with the borderpane as the root node
20              Scene scene = new Scene(root);
21              scene.getStylesheets().add(getClass()
22                  .getResource("application.css").toExternalForm());
23              primaryStage.setScene(scene);
24              // make the window 1020 x 720 big
25              primaryStage.setHeight(720);
26              primaryStage.setWidth(1020);
27              // show the window
28              primaryStage.show();
29          } catch (Exception e) {
30              e.printStackTrace();
31          }
32      }
33
34      // run the program
35      public static void main(String[] args) {
36          // launch the gui
37          launch(args);
38      }
```

15.2 *Main.fxml*

```
1 <?xml version="1.0" encoding="UTF-8"?>
2
3 <?import javafx.scene.control.Menu?>
4 <?import javafx.scene.control.MenuBar?>
5 <?import javafx.scene.control.MenuItem?>
6 <?import javafx.scene.layout.BorderPane?>
7
8 <BorderPane fx:id="rootPane" prefHeight="590.0"
9     prefWidth="870.0" xmlns="http://javafx.com/javafx/8.0.141"
10    xmlns:fx="http://javafx.com/fxml/1"
11    fx:controller="application.MainController">
12     <top>
13         <MenuBar BorderPane.alignment="CENTER">
14             <menus>
15                 <Menu mnemonicParsing="false" text="File">
16                     <items>
17                         <MenuItem mnemonicParsing="false" text="Close" />
18                     </items>
19                 </Menu>
20                 <Menu mnemonicParsing="false" text="Edit">
21                     <items>
22                         <MenuItem mnemonicParsing="false" text="Delete" />
23                     </items>
24                 </Menu>
25                 <Menu mnemonicParsing="false" text="Help">
26                     <items>
27                         <MenuItem mnemonicParsing="false" text="About" />
28                     </items>
29                 </Menu>
30             </menus>
31         </MenuBar>
32     </top>
33 </BorderPane>
```

15.3 *css File*

```
1 /* JavaFX CSS - Leave this comment until you have at least create one rule which uses -fx-Property */
```

15.4 *MainController.java*

```
1  package application;
2
3  import java.io.IOException;
4  import java.net.URL;
5  import java.util.ResourceBundle;
6
7  import javafx.fxml.FXML;
8  import javafx.fxml.Initializable;
9  import javafx.scene.layout.BorderPane;
10
11 public class MainController implements Initializable {
12
13     // attributes
14     @FXML
15     private BorderPane rootPane;
16
17     private ShareLayers shareLayerStore;
18
19     // methods
20     @Override
21     public void initialize(URL arg0, ResourceBundle arg1) {
22         // initialize the shared layer store
23         this.shareLayerStore = new ShareLayers();
24         // initialize the plotpane
25         PlotPane plotPane = new PlotPane();
26         // initialize the input pane
27         InputPane inputPane = null;
28         try {
29             inputPane = new InputPane();
30         } catch (IOException e) {
31             e.printStackTrace();
32         }
33         // set the shared layer stores to both the panes
34         plotPane.setShareLayerStore(shareLayerStore);
35         inputPane.setShareLayerStore(shareLayerStore);
36         // add the panes to the root pane
37         rootPane.setCenter(plotPane);
38         rootPane.setLeft(inputPane);
39     }
40
41 }
```

15.5 *PlotPane.java*

```
1  package application;
2
3  import java.awt.image.RenderedImage;
4  import java.io.File;
5  import java.io.IOException;
6  import java.util.ArrayList;
7
8  import javax.imageio.ImageIO;
9
10 import exceptions.StackOverflowException;
11 import exceptions.StackUnderflowException;
12 import exceptions.UnequalBracketsException;
13 import javafx.beans.property.BooleanProperty;
14 import javafx.beans.property.SimpleBooleanProperty;
15 import javafx.beans.value.ChangeListener;
16 import javafx.embed.swing.SwingFXUtils;
17 import javafx.event.ActionEvent;
18 import javafx.event.EventHandler;
19 import javafx.scene.SnapshotParameters;
20 import javafx.scene.control.ContextMenu;
21 import javafx.scene.control.MenuItem;
22 import javafx.scene.image.WritableImage;
23 import javafx.scene.layout.Pane;
24 import javafx.stage.FileChooser;
25 import layer.AxesCartesianLayer;
26 import layer.InputLayer;
27 import layer.Layer;
28
29 public class PlotPane extends Pane {
30
31     // attributes
32     // layers
33     private ArrayList<Layer> layers = new ArrayList<Layer>();
34     private InputLayer inputLayer;
35     private AxesCartesianLayer axesLayer;
36     private ShareLayers shareLayerStore;
37     // properties
38     private BooleanProperty changeViewport = new SimpleBooleanProperty(true);
39     private BooleanProperty changeLayers = new SimpleBooleanProperty(true);
40
41     // methods
42     // constructor
43     public PlotPane() {
44         // initialize the input layer and bind its properties
45         this.inputLayer = new InputLayer();
46         this.inputLayer.bindProperties(this);
47         // bind the plotpane's properties to it
48         this.changeViewport.bind(this.inputLayer.getChangeViewport());
49         // make the background of the plotpane white
50         this.setStyle("-fx-background-color: rgb(255,255,255)");
51         // this is portion of code which can be bound to a property
52         // and will run when that property changes
53         // this listener will redraw the layers
54         ChangeListener<Object> redrawListener = (observable, oldValue, newValue) -> {
55             try {
56                 drawAll();
57             } catch (StackOverflowException | StackUnderflowException |
58                 ↪ UnequalBracketsException
59                 | InterruptedException e) {
60                 // error when drawing functions
61                 e.printStackTrace();
62             }
63         };
64         // add the redraw listener to the size properties
65         // so when the plotpane changes size the layers are redrawn
```

```

65     this.heightProperty().addListener(redrawListener);
66     this.widthProperty().addListener(redrawListener);
67     // add the redraw listener to the changeViewport property
68     // so when the input layer notifies to draw, the layers are redrawn
69     this.changeViewport.addListener(redrawListener);
70     // initialize the axes layer and bind its properties
71     this.axesLayer = new AxesCartesianLayer();
72     axesLayer.bindProperties(this);
73     // create a right click menu
74     ContextMenu contextMenu = new ContextMenu();
75     MenuItem save = new MenuItem("Save as Picture");
76     contextMenu.getItems().addAll(save);
77     // set an action to call the savePlot() method when clicking the save button
78     save.setOnAction(new EventHandler<ActionEvent>() {
79         @Override
80         public void handle(ActionEvent event) {
81             savePlot();
82         }
83     });
84     // when right clicking open the menu
85     this.setOnContextMenuRequested(
86         event -> contextMenu.show(this.getScene().getWindow(),
87                                     ↪ event.getScreenX(), event.getScreenY()));
87
88
89     private void drawAll()
90         throws StackOverflowException, StackUnderflowException,
91             ↪ UnequalBracketsException, InterruptedException {
92         // loop through every layer and draw it
93         for (Layer l : layers) {
94             l.draw();
95         }
96         // draw the axes
97         axesLayer.draw();
98         // remove all the canvases in the plotpane
99         this.getChildren().clear();
100        // readd all the canvases in the plotpane
101        for (Layer l : layers) {
102            this.getChildren().add(l.getCanvas());
103        }
104        // add the input and axes canvases again
105        // with the input layer on top so that it can receive input
106        this.getChildren().add(axesLayer.getCanvas());
107        this.getChildren().add(inputLayer.getCanvas());
108    }
109
110    // add a layer
111    private void addLayer(Layer l) {
112        // bind the properties to this plotpane
113        l.bindProperties(this);
114        // add the layer to the list
115        this.layers.add(l);
116    }
117
118    // return the inputlayer to access its properties
119    public InputLayer getInputLayer() {
120        return inputLayer;
121    }
122
123    // save picture of plot
124    private void savePlot() {
125        // create the file navigator object
126        FileChooser fileChooser = new FileChooser();
127        // set extension filter
128        fileChooser.getExtensionFilters().add(new
129            ↪ FileChooser.ExtensionFilter("png files (*.png)", "*.png"));
130        // set extension filter (make the image of format png)
131        File file = fileChooser.showSaveDialog(null);

```

```

130         if (file != null) { //if the user chose to save then snapshot the pane
131             try {
132                 // remove input layer canvas, the top node, to not show the coords
133                 this.getChildren().remove(this.getChildren().size() - 1);
134                 // create the snapshot of the pane
135                 WritableImage snapshot = this.snapshot(new SnapshotParameters(), null);
136                 RenderedImage renderedImage = SwingFXUtils.fromFXImage(snapshot, null);
137                 // write the snapshot to the chosen file
138                 ImageIO.write(renderedImage, "png", file);
139             } catch (IOException ex) {
140                 ex.printStackTrace();
141             } finally {
142                 // readd the input layer
143                 this.getChildren().add(inputLayer.getCanvas());
144             }
145         }
146     }
147
148     // set up the shared layer access
149     public void setShareLayerStore(ShareLayers shareLayerStore) {
150         // set the shared layer access attribute
151         this.shareLayerStore = shareLayerStore;
152         // bind and set the value for the property
153         this.changeLayers.set(this.shareLayerStore.getChangeLayers().get());
154         this.changeLayers.bind(this.shareLayerStore.getChangeLayers());
155         // create a listener to execute when the plot pane needs to draw again
156         ChangeListener<Object> redrawListener = (observable, oldValue, newValue) -> {
157             // remove all the old layers
158             this.layers.clear();
159             // add all the new layers
160             for (Layer l : shareLayerStore.getLayers()) {
161                 this.addLayer(l);
162             }
163             // draw the functions again
164             try {
165                 drawAll();
166             } catch (StackOverflowException | StackUnderflowException |
167                 ↳ UnequalBracketsException
168                 | InterruptedException e) {
169                 // error in drawing a function
170                 e.printStackTrace();
171             }
172             // bind the listener to the property
173             this.changeLayers.addListener(redrawListener);
174         }
175     }
176 }

```

15.6 *ShareLayers.java*

```
1  package application;
2
3  import java.util.ArrayList;
4  import java.util.HashMap;
5  import java.util.Map;
6
7  import javafx.beans.property.BooleanProperty;
8  import javafx.beans.property.SimpleBooleanProperty;
9  import layer.Layer;
10
11  public class ShareLayers {
12
13      // attributes
14      private Map<Integer, Layer> layers;
15      private BooleanProperty changeLayers;
16
17      // methods
18      // constructor
19      public ShareLayers() {
20          // initialize attributes
21          layers = new HashMap<Integer, Layer>();
22          changeLayers = new SimpleBooleanProperty(true);
23      }
24
25      // add a layer
26      public void putLayer(int ID, Layer layer) {
27          // add the layer
28          this.layers.put(ID, layer);
29          // notify the plot pane to draw again
30          this.changeLayers.set(!this.changeLayers.get());
31      }
32
33      // remove a layer
34      public void removeLayer(int ID) {
35          // remove the layer
36          this.layers.remove(ID);
37          // notify the plot pane to draw again
38          this.changeLayers.set(!this.changeLayers.get());
39      }
40
41      // return a list of layers
42      public ArrayList<Layer> getLayers() {
43          return new ArrayList<Layer>(this.layers.values());
44      }
45
46      // return the property which notifies the plotpane
47      public BooleanProperty getChangeLayers() {
48          return changeLayers;
49      }
50
51  }
```

15.7 *ExpressionBoxController.java*

```
1 package application;
2
3 import java.io.IOException;
4 import java.net.URL;
5 import java.util.ResourceBundle;
6
7 import exceptions.StackOverflowException;
8 import exceptions.StackUnderflowException;
9 import exceptions.UnequalBracketsException;
10 import javafx.beans.property.SimpleStringProperty;
11 import javafx.beans.property.StringProperty;
12 import javafx.fxml.FXML;
13 import javafx.fxml.Initializable;
14 import javafx.scene.canvas.Canvas;
15 import javafx.scene.canvas.GraphicsContext;
16 import javafx.scene.control.Button;
17 import javafx.scene.control.TextField;
18 import javafx.scene.paint.Color;
19 import layer.ExplicitXFunctionCartesianLayer;
20 import layer.ExplicitYFunctionCartesianLayer;
21 import layer.Layer;
22 import structures.Expression;
23 import structures.NormalDistribution;
24
25 public class ExpressionBoxController implements Initializable {
26
27     // attributes
28     private int ID;
29     private GraphicsContext gc;
30     private boolean functionVisiblity = true;
31     private Color color;
32     private StringProperty functionText = new SimpleStringProperty();
33     private InputPane inputPane;
34     // FXML components
35     @FXML
36     private TextField inputTxt;
37     @FXML
38     private Button removeBtn;
39     @FXML
40     private Canvas showFunctionColour;
41
42     // methods
43     // empty constructor
44     public ExpressionBoxController() {
45     }
46
47     @Override
48     // called when initialized a pseudo constructor of sorts
49     public void initialize(URL arg0, ResourceBundle arg1) {
50         // bind the input to a string property in the class to manipulate
51         this.functionText.set(inputTxt.textProperty().getValueSafe());
52         this.functionText.bind(inputTxt.textProperty());
53         // add a listener so that every time it changes the layer is updated
54         this.functionText.addListener(event -> {
55             try {
56                 changeLayer();
57             } catch (StackOverflowException | StackUnderflowException |
58                 ↪ UnequalBracketsException e) {
59                 e.printStackTrace();
60             }
61         });
62         // if the remove button is clicked call the remove method
63         this.removeBtn.setOnAction(event -> {
64             try {
65                 remove();
66             }
```

```

65         } catch (IOException e) {
66             e.printStackTrace();
67         }
68     });
69     // create the canvas to show the color of the function
70     this.gc = this.showFunctionColour.getGraphicsContext2D();
71     this.gc.setLineWidth(2);
72     // when the canvas is clicked change its color by calling the change Color
73     // method
74     this.showFunctionColour.setOnMouseClicked(event -> {
75         try {
76             changeColor();
77         } catch (StackOverflowException | StackUnderflowException |
78             ↪ UnequalBracketsException e) {
79             e.printStackTrace();
80         }
81     });
82
83     // update the layer
84     private void changeLayer() throws StackOverflowException, StackUnderflowException,
85     ↪ UnequalBracketsException {
86         // only change the layer if is not hidden by the user
87         if (functionVisibility) {
88             Layer l;
89             String f = this.functionText.getValueSafe();
90             // if the function is empty remove the layer
91             if (f.length() == 0) {
92                 inputPane.removeLayer(ID);
93                 return;
94             }
95             // if it begins with normal create a normal distribution layer
96             if (f.startsWith("Normal(")) {
97                 f = f.replaceAll("Normal\\(", "");
98                 f = f.replaceAll("\\)", "");
99                 String[] variables = f.split(",");
100                 //  $\mu$ ;
101                 double  $\mu$  = 0;
102                 //  $\sigma^2$ ;
103                 double  $\sigma^2$  = 1;
104                 // get the values for the mean and variance
105                  $\mu$  = new Expression(variables[0], 'x').evaluate(0);
106                  $\sigma^2$  = new Expression(variables[1], 'x').evaluate(0);
107                 // create the function
108                 NormalDistribution normalDist = null;
109                 normalDist = new NormalDistribution( $\mu$ ,  $\sigma^2$ );
110                 // create the layer with the specific color
111                 l = new ExplicitXFunctionCartesianLayer(normalDist);
112                 l.setColor(color);
113                 // add the layer to the shared layer access
114                 inputPane.putLayer(this.ID, l);
115                 // terminate this function
116                 return;
117             }
118             // determine the whether it is an explicit x or y function by counting their
119             // respective instances
120             // count the number of x instances
121             int xInstances = f.length() - f.replaceAll("x", "").length();
122             // count the number of y instances
123             int yInstances = f.length() - f.replaceAll("y", "").length();
124             if (xInstances > 0 && yInstances > 0) { // if it contains both x and y remove it
125                 ↪ since it is implicit
126                 inputPane.removeLayer(ID);
127                 return;
128             } else if (yInstances > 0) { // if it contains only y create a explicit function
129                 ↪ in y
130                 l = new ExplicitYFunctionCartesianLayer(f);
131                 l.setColor(color);

```

```

129         inputPane.putLayer(this.ID, 1);
130     } else { // if it contains only x create a explicit function in x
131         l = new ExplicitXFunctionCartesianLayer(f);
132         l.setColor(color);
133         inputPane.putLayer(this.ID, 1);
134     }
135 } else { // if the function is hidden then remove the layer
136     inputPane.removeLayer(ID);
137 }
138 }
139
140 // show/hide the function and update the visual identifier
141 private void changeColor() throws StackOverflowException, StackUnderflowException,
142     ↪ UnequalBracketsException {
143     // clear the canvas
144     gc.clearRect(0, 0, showFunctionColour.getWidth(), showFunctionColour.getHeight());
145     if (!functionVisiblity) {
146         // fill it in color if showing the function
147         gc.fillRect(0, 0, showFunctionColour.getWidth(),
148             ↪ showFunctionColour.getHeight());
149     }
150     // draw a black border so it is clickable
151     gc.strokeRect(0, 0, showFunctionColour.getWidth(), showFunctionColour.getHeight());
152     // negate the visibility since it has changed
153     functionVisiblity = !functionVisiblity;
154     // remove or add the layer to show/hide it
155     changeLayer();
156 }
157
158 // remove itself from the input pane
159 private void remove() throws IOException {
160     this.inputPane.removeExpressionBox(this.ID);
161 }
162
163 // set the id
164 public void setID(int id) {
165     ID = id;
166 }
167
168 // get the id
169 public int getID() {
170     return ID;
171 }
172
173 // set the parent input pane
174 public void setInputPane(InputPane inputPane) {
175     this.inputPane = inputPane;
176 }
177
178 // set the color of the function
179 public void setColor(Color color) {
180     this.color = color;
181     gc.setStroke(color);
182     gc.setFill(color);
183     gc.fillRect(0, 0, showFunctionColour.getWidth(), showFunctionColour.getHeight());
184 }

```

15.8 *ExpressionBox.fxml*

```
1 <?xml version="1.0" encoding="UTF-8"?>
2
3 <?import javafx.geometry.Insets?>
4 <?import javafx.scene.canvas.Canvas?>
5 <?import javafx.scene.control.Button?>
6 <?import javafx.scene.control.TextField?>
7 <?import javafx.scene.layout.HBox?>
8 <?import javafx.scene.layout.Region?>
9
10 <HBox alignment="CENTER" xmlns="http://javafx.com/javafx/8.0.171"
11     xmlns:fx="http://javafx.com/fxml/1"
12     fx:controller="application.ExpressionBoxController">
13     <children>
14         <Canvas fx:id="showFunctionColour" height="30.0" width="30.0">
15             <HBox.margin>
16                 <Insets left="5.0" />
17             </HBox.margin>
18         </Canvas>
19         <Region HBox.hgrow="ALWAYS" />
20         <TextField fx:id="inputTxt">
21             <HBox.margin>
22                 <Insets left="5.0" right="5.0" />
23             </HBox.margin>
24         </TextField>
25         <Region />
26         <Button fx:id="removeBtn" mnemonicParsing="false" text="X">
27             <HBox.margin>
28                 <Insets left="5.0" right="5.0" />
29             </HBox.margin>
30         </Button>
31     </children>
32     <padding>
33         <Insets bottom="10.0" top="10.0" />
34     </padding>
35 </HBox>
```

15.9 *InputPane.java*

```
1 package application;
2
3 import java.io.IOException;
4
5 import javafx.fxml.FXMLLoader;
6 import javafx.scene.control.Button;
7 import javafx.scene.control.ScrollPane;
8 import javafx.scene.layout.HBox;
9 import javafx.scene.layout.Priority;
10 import javafx.scene.layout.Region;
11 import javafx.scene.layout.VBox;
12 import javafx.scene.paint.Color;
13 import layer.Layer;
14
15 public class InputPane extends ScrollPane {
16
17     // attributes
18     private VBox expressionBoxStore;
19     private ShareLayers shareLayerStore;
20     private int nextID = 0;
21     private final Color[] colors =
22     { Color.BLACK, Color.BLUE, Color.RED, Color.DARKGREEN, Color.MAGENTA, Color.GOLDENROD };
23
24     // methods
25     // constructor
26     public InputPane() throws IOException {
27         // set up the input pane attributes
28         this.nextID = 0;
29         this.setFitToHeight(true);
30         this.setFitToWidth(true);
31         this.setHbarPolicy(ScrollBarPolicy.NEVER);
32         this.setPrefWidth(245.0);
33         this.expressionBoxStore = new VBox();
34         this.setContent(this.expressionBoxStore);
35         // set up the button
36         this.setupNewButton();
37         // add the first expression box
38         this.addExpressionBox();
39     }
40
41     // add a new expression box
42     private void addExpressionBox() throws IOException {
43         // index at which to insert the new box, one before the button
44         int insert = this.expressionBoxStore.getChildren().size() - 1;
45         // load and initialize the expression box
46         FXMLLoader loader = new FXMLLoader();
47         loader.setLocation(Main.class.getResource("ExpressionBox.fxml"));
48         HBox expressionBox = loader.load();
49         // get the controller class and set the ID, input pane and color attributes
50         ExpressionBoxController eBoxController = (ExpressionBoxController)
51         ↪ loader.getController();
52         expressionBox.setId(Integer.toString(nextID));
53         eBoxController.setID(nextID);
54         eBoxController.setInputPane(this);
55         eBoxController.setColor(colors[nextID % this.colors.length]);
56         // add the expression box to the vbox
57         this.expressionBoxStore.getChildren().add(insert, expressionBox);
58         // increment the ID so the next box has a different ID
59         nextID = nextID + 1;
60     }
61
62     // remove an expression box
63     public void removeExpressionBox(int ID) throws IOException {
64         // remove the layer that the expression box is linked to
65         this.removeLayer(ID);
66     }
67 }
```

```

65         // find the expression box and remove it
66         for (int i = 0; i < this.expressionBoxStore.getChildren().size() - 1; i++) {
67             HBox box = (HBox) this.expressionBoxStore.getChildren().get(i);
68             if (Integer.parseInt(box.getId()) == ID) {
69                 this.expressionBoxStore.getChildren().remove(i);
70                 break;
71             }
72         }
73         // if there are no expression boxes left, reset the ID counter and add a new one
74         if (this.expressionBoxStore.getChildren().size() == 1) {
75             this.nextID = 0;
76             this.addExpressionBox();
77         }
78     }
79
80     // add a layer to the plot pane through the shared layer access
81     public void putLayer(int ID, Layer layer) {
82         shareLayerStore.putLayer(ID, layer);
83     }
84
85     // remove a layer from the plot pane through the shared layer access
86     public void removeLayer(int ID) {
87         shareLayerStore.removeLayer(ID);
88     }
89
90     // set up the new box
91     private void setupNewButton() {
92         // initialize the button with the display text "New Function"
93         Button newExpression = new Button("New Function");
94         // when it is pressed call the addExpressionBox() function
95         newExpression.setOnAction(event -> {
96             try {
97                 addExpressionBox();
98             } catch (IOException e) {
99                 e.printStackTrace();
100             }
101         });
102         // various UI components needed to make the button fit in with the UI
103         // hbox to hold the components
104         HBox buttonHolder = new HBox();
105         // filler space to center the button horizontally
106         Region fillerSpace1 = new Region();
107         Region fillerSpace2 = new Region();
108         // add the filler space and button to the hbox
109         buttonHolder.getChildren().addAll(fillerSpace1, newExpression, fillerSpace2);
110         HBox.setHgrow(fillerSpace1, Priority.ALWAYS);
111         HBox.setHgrow(fillerSpace2, Priority.ALWAYS);
112         // add the button to the vbox
113         this.expressionBoxStore.getChildren().add(buttonHolder);
114     }
115
116     // set the shared layer access attribute
117     public void setShareLayerStore(ShareLayers shareLayerStore) {
118         this.shareLayerStore = shareLayerStore;
119     }
120
121 }

```

Chapter 16

layer

16.1 *Layer.java*

```
1 package layer;
2
3 import application.PlotPane;
4 import javafx.beans.property.DoubleProperty;
5 import javafx.beans.property.IntegerProperty;
6 import javafx.beans.property.SimpleDoubleProperty;
7 import javafx.beans.property.SimpleIntegerProperty;
8 import javafx.scene.canvas.Canvas;
9 import javafx.scene.canvas.GraphicsContext;
10 import javafx.scene.paint.Color;
11
12 public abstract class Layer {
13
14     // attributes
15     // drawing tools
16     protected Canvas canvas;
17     protected GraphicsContext gc;
18     protected Color color = Color.BLACK;
19     // drawing parameters
20     protected IntegerProperty steps = new SimpleIntegerProperty(0);
21     protected DoubleProperty minX = new SimpleDoubleProperty(0);
22     protected DoubleProperty maxX = new SimpleDoubleProperty(0);
23     protected DoubleProperty minY = new SimpleDoubleProperty(0);
24     protected DoubleProperty maxY = new SimpleDoubleProperty(0);
25     protected DoubleProperty pixelWorthX = new SimpleDoubleProperty(0);
26     protected DoubleProperty pixelWorthY = new SimpleDoubleProperty(0);
27
28     // methods
29     // constructor
30     protected Layer() {
31         // instantiate the canvas and create the OpenGL context
32         this.canvas = new Canvas();
33         this.gc = canvas.getGraphicsContext2D();
34     }
35
36     // will draw onto the canvas, will be overridden
37     public abstract void draw();
38
39     // will connect this layer to a PlotPane instance, will be overridden
40     public abstract void bindProperties(PlotPane plotPane);
41
42     // clear the canvas to be redrawn
43     protected void clearCanvas() {
44         gc.clearRect(0, 0, canvas.getWidth(), canvas.getHeight());
45     }
46
47     // getter for the canvas
48     public Canvas getCanvas() {
49         return canvas;
50     }
51
52     // change the color of the function drawn
53     public void setColor(Color color) {
54         this.color = color;
55     }
56
57 }
```

16.2 *CartesianLayer.java*

```
1 package layer;
2
3 import application.PlotPane;
4 import javafx.beans.property.SimpleDoubleProperty;
5 import javafx.beans.property.SimpleIntegerProperty;
6
7 public abstract class CartesianLayer extends Layer {
8
9     @Override
10    // must be overridden since the parent class made it abstract
11    // but keep it abstract so child class must override
12    public abstract void draw();
13
14    @Override
15    // bind the needed properties from the input layer to itself
16    public void bindProperties(PlotPane plotPane) {
17        // initialize the properties with the same values as their partners
18        this.steps = new SimpleIntegerProperty(plotPane.getInputLayer().getSteps().intValue());
19        this.minX = new SimpleDoubleProperty(plotPane.getInputLayer().getMinX().doubleValue());
20        this.maxX = new SimpleDoubleProperty(plotPane.getInputLayer().getMaxX().doubleValue());
21        this.minY = new SimpleDoubleProperty(plotPane.getInputLayer().getMinY().doubleValue());
22        this.maxY = new SimpleDoubleProperty(plotPane.getInputLayer().getMaxY().doubleValue());
23        this.pixelWorthX = new
24            ↳ SimpleDoubleProperty(plotPane.getInputLayer().getPixelWorthX().doubleValue());
25        this.pixelWorthY = new
26            ↳ SimpleDoubleProperty(plotPane.getInputLayer().getPixelWorthY().doubleValue());
27        // bind the two properties together one-directionally
28        // so these attributes change when plot pane changes, not vice versa
29        this.steps.bind(plotPane.getInputLayer().getSteps());
30        this.minX.bind(plotPane.getInputLayer().getMinX());
31        this.maxX.bind(plotPane.getInputLayer().getMaxX());
32        this.minY.bind(plotPane.getInputLayer().getMinY());
33        this.maxY.bind(plotPane.getInputLayer().getMaxY());
34        this.pixelWorthX.bind(plotPane.getInputLayer().getPixelWorthX());
35        this.pixelWorthY.bind(plotPane.getInputLayer().getPixelWorthY());
36        // make the canvas resize as its parent resizes by binding the associated properties
37        canvas.heightProperty().bind(plotPane.heightProperty());
38        canvas.widthProperty().bind(plotPane.widthProperty());
39    }
40
41    // convert x-Cartesian coordinate to x-Canvas coordinate
42    protected double convertX(double x) {
43        x = x - this.minX.doubleValue();
44        x = x / this.pixelWorthX.doubleValue();
45        return x;
46    }
47
48    // convert y-Cartesian coordinate to y-Canvas coordinate
49    protected double convertY(double y) {
50        y = this.maxY.doubleValue() - y;
51        y = y / this.pixelWorthY.doubleValue();
52        return y;
53    }
54 }
```

16.3 *ExplicitXFunctionCartesianLayer.java*

```
1 package layer;
2
3 import exceptions.StackOverflowException;
4 import exceptions.StackUnderflowException;
5 import exceptions.UnequalBracketsException;
6 import structures.ExplicitFunction;
7
8 public class ExplicitXFunctionCartesianLayer extends CartesianLayer {
9
10     // attributes
11     private ExplicitFunction f;
12
13     // methods
14     // constructor - create a layer with an expression given
15     public ExplicitXFunctionCartesianLayer(String expression) {
16         // super constructor must be invoked first
17         super();
18         // try to create the function with the given expression
19         try {
20             this.f = new ExplicitFunction(expression, 'x');
21         } catch (StackOverflowException | StackUnderflowException | UnequalBracketsException e) {
22             // output the error in parsing the function
23             e.printStackTrace();
24         }
25     }
26
27     // constructor - create a layer with a set function
28     public ExplicitXFunctionCartesianLayer(ExplicitFunction f) {
29         // super constructor must be invoked first
30         super();
31         // set the function
32         this.f = f;
33     }
34
35     // draw the function
36     public void draw() {
37         // setup the canvas by clearing it and set up the line to be drawn
38         this.clearCanvas();
39         gc.setLineWidth(2);
40         gc.setStroke(color);
41         double x1, x2, y1, y2;
42         // initialize the first value of x
43         x1 = this.minX.doubleValue();
44         // evaluate the first value of y
45         y1 = f.evaluate(x1);
46         // calculate the dx value i.e. the step value
47         double step = (this.maxX.doubleValue() - this.minX.doubleValue()) /
48             ↳ this.steps.doubleValue();
49         // loop until we draw everything in the range, minX to maxX, required
50         for (x2 = this.minX.doubleValue() + step; x2 < this.maxX.doubleValue(); x2 = x2 + step) {
51             // evaluate the next value of y
52             y2 = f.evaluate(x2);
53             // draw the line
54             gc.strokeLine(this.convertX(x1), this.convertY(y1), this.convertX(x2),
55                 ↳ this.convertY(y2));
56             // update the old values
57             x1 = x2;
58             y1 = y2;
59         }
60
61         @Override
62         // output the function that this layer is drawing
63         public String toString() {
64             return this.f.toString();
65         }
66     }
```

16.4 *ExplicitYFunctionCartesianLayer.java*

```
1 package layer;
2
3 import exceptions.StackOverflowException;
4 import exceptions.StackUnderflowException;
5 import exceptions.UnequalBracketsException;
6 import structures.ExplicitFunction;
7
8 public class ExplicitYFunctionCartesianLayer extends CartesianLayer {
9
10     // attributes
11     private ExplicitFunction f;
12
13     // methods
14     // constructor - create a layer with an expression given
15     public ExplicitYFunctionCartesianLayer(String expression) {
16         // super constructor must be invoked first
17         super();
18         // try to create the function with the given expression
19         try {
20             this.f = new ExplicitFunction(expression, 'y');
21         } catch (StackOverflowException | StackUnderflowException | UnequalBracketsException e) {
22             // output the error in parsing the function
23             e.printStackTrace();
24         }
25     }
26
27     // constructor - create a layer with a set function
28     public ExplicitYFunctionCartesianLayer(ExplicitFunction f) {
29         // super constructor must be invoked first
30         super();
31         // set the function
32         this.f = f;
33     }
34
35     // draw the function
36     public void draw() {
37         // setup the canvas by clearing it and set up the line to be drawn
38         this.clearCanvas();
39         gc.setLineWidth(2);
40         gc.setStroke(color);
41         double x1, x2, y1, y2;
42         // initialize the first value of y
43         y1 = this.minY.doubleValue();
44         // evaluate the first value of x
45         x1 = f.evaluate(y1);
46         // calculate the dy value i.e. the step value
47         double step = (this.maxY.doubleValue() - this.minY.doubleValue()) /
48             ↳ this.steps.doubleValue();
49         // loop until we draw everything in the range, minY to maxY, required
50         for (y2 = this.minY.doubleValue() + step; y2 < this.maxY.doubleValue(); y2 = y2 + step) {
51             // evaluate the next value of y
52             x2 = f.evaluate(y2);
53             // draw the line
54             gc.strokeLine(this.convertX(x1), this.convertY(y1), this.convertX(x2),
55                 ↳ this.convertY(y2));
56             x1 = x2;
57             y1 = y2;
58         }
59
60     @Override
61     // output the function that this layer is drawing
62     public String toString() {
63         return this.f.toString();
64     }
65 }
```

16.5 *AxesCartesianLayer.java*

```
1 package layer;
2
3 public class AxesCartesianLayer extends CartesianLayer {
4
5     // constructor
6     public AxesCartesianLayer() {
7         // call the super to setup the canvas
8         super();
9     }
10
11     @Override
12     // draw the pair of axes
13     public void draw() {
14         // clear the canvas before drawing
15         this.clearCanvas();
16         // make the line width thinner than usual to remove emphasis from axes
17         gc.setLineWidth(1.5);
18         gc.setStroke(color);
19         // draw the y-axis
20         gc.strokeLine(this.convertX(0), this.convertY(this.minY.doubleValue()),
21             ↪ this.convertX(0),
22                 this.convertY(this.maxY.doubleValue()));
23         // draw the x-axis
24         gc.strokeLine(this.convertX(this.minX.doubleValue()), this.convertY(0),
25             this.convertX(this.maxX.doubleValue()), this.convertY(0));
26     }
27 }
```

16.6 *InputLayer.java*

```
1 package layer;
2
3 import java.text.DecimalFormat;
4
5 import application.PlotPane;
6 import javafx.beans.property.BooleanProperty;
7 import javafx.beans.property.DoubleProperty;
8 import javafx.beans.property.IntegerProperty;
9 import javafx.beans.property.SimpleBooleanProperty;
10 import javafx.beans.property.SimpleDoubleProperty;
11 import javafx.beans.property.SimpleIntegerProperty;
12 import javafx.scene.input.MouseEvent;
13 import javafx.scene.input.ScrollEvent;
14
15 public class InputLayer extends Layer {
16
17     // attributes
18     private BooleanProperty changeViewport = new SimpleBooleanProperty(true);
19     private double currentX;
20     private double currentY;
21     private double previousX;
22     private double previousY;
23
24     // methods
25     // constructor
26     public InputLayer() {
27         // call the super constructor
28         super();
29         // update pixel worth when the canvas changes size
30         this.canvas.heightProperty().addListener(event -> updatePixelWorth());
31         this.canvas.widthProperty().addListener(event -> updatePixelWorth());
32         // bind the mouse events to their respective actions
33         this.canvas.setOnMousePressed(event -> dragEntered(event));
34         this.canvas.setOnMouseDragged(event -> pan(event));
35         this.canvas.setOnScroll(event -> zoom(event));
36         this.canvas.setOnMouseMoved(event -> drawCoords(event));
37     }
38
39     @Override
40     // this canvas will not draw anything so this method is empty
41     // it must still be implemented since the method was abstract in the parent
42     // class
43     public void draw() {
44     }
45
46     // update the pixelworth
47     private void updatePixelWorth() {
48         this.pixelWorthX.set((this.maxX.doubleValue() - this.minX.doubleValue()) /
49         ↳ this.canvas.getWidth());
50         this.pixelWorthY.set((this.maxY.doubleValue() - this.minY.doubleValue()) /
51         ↳ this.canvas.getHeight());
52     }
53
54     @Override
55     // initialize the properties required so they can be bound later
56     public void bindProperties(PlotPane plotPane) {
57         // initialize the properties
58         this.steps = new SimpleIntegerProperty(400);
59         this.minX = new SimpleDoubleProperty(-10);
60         this.maxX = new SimpleDoubleProperty(10);
61         this.minY = new SimpleDoubleProperty(-10);
62         this.maxY = new SimpleDoubleProperty(10);
63         this.pixelWorthX = new SimpleDoubleProperty(1);
64         this.pixelWorthY = new SimpleDoubleProperty(1);
65         // make the canvas resize as its parent resizes by binding the associated
```

```

64         // properties
65         this.canvas.heightProperty().bind(plotPane.heightProperty());
66         this.canvas.widthProperty().bind(plotPane.widthProperty());
67     }
68
69     // draw the coordinates
70     private void drawCoords(MouseEvent event) {
71         // clear the previous coordinates
72         clearCanvas();
73         // get the coordinates
74         double x = (event.getX() * this.pixelWorthX.doubleValue()) + this.minX.doubleValue();
75         double y = this.maxY.doubleValue() - (event.getY() * this.pixelWorthY.doubleValue());
76         // round the coordinates to 2 decimal places
77         String sX = new DecimalFormat("#.##").format(x);
78         String sY = new DecimalFormat("#.##").format(y);
79         String out = "(" + sX + "," + sY + ")";
80         // draw the coordinates to the canvas
81         gc.setLineWidth(1);
82         gc.strokeText(out, 0, 10);
83     }
84
85     // when drag starts initiate the full-drag sequence
86     private void dragEntered(MouseEvent event) {
87         // only initiate the drag if its the left mouse button
88         if (event.isPrimaryButtonDown()) {
89             // need to initialize the values of previousX and previousY
90             previousX = event.getX();
91             previousY = event.getY();
92             // start full drag
93             event.setDragDetect(true);
94             this.canvas.startFullDrag();
95         }
96     }
97
98     // pan around the plotpane
99     private void pan(MouseEvent event) {
100         if (event.isPrimaryButtonDown()) {
101             // get the current mouse coordinates
102             currentX = event.getX();
103             currentY = event.getY();
104             // get the vector of the mouse movement
105             double xTrans = (previousX - currentX);
106             double yTrans = (previousY - currentY);
107             // update the previous values to be the current ones
108             previousX = currentX;
109             previousY = currentY;
110             // translate the viewport
111             this.maxX.set((this.pixelWorthX.doubleValue() * xTrans) +
112                 ↪ this.maxX.doubleValue());
113             this.minX.set((this.pixelWorthX.doubleValue() * xTrans) +
114                 ↪ this.minX.doubleValue());
115             this.maxY.set(-(this.pixelWorthY.doubleValue() * yTrans) +
116                 ↪ this.maxY.doubleValue());
117             this.minY.set(-(this.pixelWorthY.doubleValue() * yTrans) +
118                 ↪ this.minY.doubleValue());
119             this.changeViewport.set(!this.changeViewport.get());
120         }
121     }
122
123     // zoom in and out
124     private void zoom(ScrollEvent event) {
125         double zoomFactor = 1.05;
126         // this is whether it is scrolling up or down
127         double deltaY = event.getDeltaY();
128         // get the coordinates
129         double x = (event.getX() * this.pixelWorthX.doubleValue()) + this.minX.doubleValue();
130         double y = this.maxY.doubleValue() - (event.getY() * this.pixelWorthY.doubleValue());
131         // if scrolling down shrink the viewport instead

```

```

128         if (deltaY > 0) {
129             zoomFactor = 1 / zoomFactor;
130         }
131         // translate the viewport to the origin, scale the viewport,
132         // translate the viewport back and set the new values for the viewport
133         this.minX.set((this.minX.doubleValue() - x) * zoomFactor + x);
134         this.maxX.set((this.maxX.doubleValue() - x) * zoomFactor + x);
135         this.minY.set((this.minY.doubleValue() - y) * zoomFactor + y);
136         this.maxY.set((this.maxY.doubleValue() - y) * zoomFactor + y);
137         // update and notify the plotpane
138         this.updatePixelWorth();
139         this.changeViewport.set(!this.changeViewport.get());
140     }
141
142     // return minX
143     public DoubleProperty getMinX() {
144         return minX;
145     }
146
147     // return maxX
148     public DoubleProperty getMaxX() {
149         return maxX;
150     }
151
152     // return minY
153     public DoubleProperty getMinY() {
154         return minY;
155     }
156
157     // return maxY
158     public DoubleProperty getMaxY() {
159         return maxY;
160     }
161
162     // return the worth of each pixel in the x-direction
163     public DoubleProperty getPixelWorthX() {
164         return pixelWorthX;
165     }
166
167     // return the worth of each pixel in the y-direction
168     public DoubleProperty getPixelWorthY() {
169         return pixelWorthY;
170     }
171
172     // return the number of draw steps
173     public IntegerProperty getSteps() {
174         return steps;
175     }
176
177     // return the property that notifies when it is time to redraw layers
178     public BooleanProperty getChangeViewport() {
179         return changeViewport;
180     }
181 }
182

```

Chapter 17

structures

17.1 *Stack.java*

```
1  package structures;
2
3  import exceptions.StackOverflowException;
4  import exceptions.StackUnderflowException;
5
6  public class Stack<T> {
7
8      // attributes
9      // pointer variables
10     private int maxHeight;
11     private int height = 0;
12     private int pointer = -1;
13     // stack
14     private T[] stack;
15
16     // methods
17     // constructor
18     @SuppressWarnings("unchecked")
19     public Stack(int maxHeight) {
20         this.maxHeight = maxHeight; // set this max height
21         // create an array of this size of type "T"
22         this.stack = (T[]) new Object[this.maxHeight];
23     }
24
25     // copy constructor
26     public Stack(Stack<T> copy) {
27         // assign the new stacks' attributes' values the original stacks' attributes
28         this.pointer = copy.pointer;
29         this.height = copy.height;
30         this.maxHeight = copy.maxHeight;
31         this.stack = copy.stack;
32     }
33
34     // push an item on to the stack
35     public void push(T push) throws StackOverflowException {
36         if (height == maxHeight) { // check that the stack isn't full
37             throw new StackOverflowException();
38         } else {
39             // make the element one above the top, the new value
40             this.stack[this.pointer + 1] = push;
41             this.pointer++; // increment pointer variables
42             this.height++;
43         }
44     }
45 }
```



```

46 // pop an item off the stack
47 public T pop() throws StackUnderflowException {
48     if (this.isEmpty()) { // check that the stack isn't full
49         throw new StackUnderflowException();
50     } else {
51         // get the value of the top element
52         // no need to make it null, it is a waste of an instruction
53         // it also releases no memory since we are using
54         // an array to store our stack which is a static structure
55         T pop = this.stack[this.pointer];
56         this.pointer--; // decrement pointer variables
57         this.height--;
58         return pop; // return the popped value
59     }
60 }
61
62 // check if stack is empty, return true if so
63 public boolean isEmpty() {
64     return this.height == 0 ? true : false;
65 }
66
67 // return the height of the stack
68 public int getHeight() {
69     return height;
70 }
71
72 // allow for a visualization of the stack by listing it
73 // with the top element being encapsulated in square brackets
74 @Override
75 public String toString() {
76     if (this.isEmpty()) { // if empty notify the user
77         return "Stack is Empty";
78     } else {
79         // highlight the top element
80         String out = "[" + this.stack[this.pointer] + "]";
81         // loop through the rest of the stack backwards
82         // concatenating each element to a string
83         for (int i = this.pointer - 1; i >= 0; i--) {
84             out = out + ", " + this.stack[i].toString();
85         }
86         return out; // return this string
87     }
88 }
89
90 }

```

17.2 *BinaryTree.java*

```
1 package structures;
2
3 import exceptions.StackOverflowException;
4
5 public class BinaryTree {
6
7     // attributes
8     // children
9     public BinaryTree left;
10    public BinaryTree right;
11    // node value
12    private String value;
13
14    // methods
15    // constructor - create a null tree
16    public BinaryTree() {
17        this.left = null;
18        this.right = null;
19        this.value = null;
20    }
21
22    // constructor - create leaf tree i.e. no children
23    public BinaryTree(String value) {
24        this.value = value;
25        this.left = null;
26        this.right = null;
27    }
28
29    // constructor - create a tree with the children defined
30    public BinaryTree(String value, BinaryTree left, BinaryTree right) {
31        this.value = value;
32        this.left = left;
33        this.right = right;
34    }
35
36    // return the value of the current node
37    public String getValue() {
38        return value;
39    }
40
41    // set the value of the current node
42    public void setValue(String value) {
43        this.value = value;
44    }
45
46    // count the number of nodes beneath this tree + this tree
47    public int countNodes() {
48        // set the number of nodes counted to 0
49        int count = 0;
50
51        // recursive case, counts the number of nodes in its children
52        // if they are not null
53        if (this.left != null) {
54            count = count + this.left.countNodes();
55        }
56        if (this.right != null) {
57            count = count + this.right.countNodes();
58        }
59        // return the count of the children
60        // but increment it to count it self
61        return count + 1;
62    }
63
64    // post-order depth-first traversal wrapper function
65    public Stack<String> traverse() throws StackOverflowException {
```

```

66         // create a stack of max height
67         // which is the number of nodes in the entire tree
68         // this is to save memory
69         Stack<String> order = new Stack<String>(this.countNodes());
70         // traverse the tree
71         order = traverseHelper(this, order);
72         return order;
73     }
74
75     // the actual recursive traversal function
76     private static Stack<String> traverseHelper(BinaryTree tree, Stack<String> order) throws
77     ↪ StackOverflowException {
78         // base case, if the tree is null just return the unmodified stack
79         if (tree == null) {
80             return order;
81         } else { // recursive case search its children if not null
82             // since this is post-order, the order is left, right then root
83             // traverse left
84             order = traverseHelper(tree.left, order);
85             // traverse right
86             order = traverseHelper(tree.right, order);
87             // finally push the root value into the stack
88             order.push(tree.value);
89             return order;
90         }
91     }
92 }

```

17.3 *Expression.java*

```
1 package structures;
2
3 import java.time.Duration;
4 import java.time.Instant;
5 import java.util.concurrent.ExecutionException;
6 import java.util.concurrent.ExecutorService;
7 import java.util.concurrent.Executors;
8 import java.util.concurrent.Future;
9
10 import exceptions.StackOverflowException;
11 import exceptions.StackUnderflowException;
12 import exceptions.UnequalBracketsException;
13
14 public class Expression {
15     // attributes
16     private Stack<String> postFixStack;
17     private char parameter; // this allows us to adapt our explicit function
18                             // for something that isn't in x
19     private String expression;
20
21     // methods
22     // constructor
23     public Expression(String expression, char parameter)
24         throws StackOverflowException, StackUnderflowException, UnequalBracketsException
25     {
26         this.parameter = parameter; // sets the parameter value
27         this.expression = standardize(expression); // removes whitespace + standardizes
28         BinaryTree tree = createTree(this.expression); // creates the tree
29         this.postFixStack = tree.traverse(); // creates the post-fix stack
30     }
31
32     // evaluate the expression for a value of the parameter
33     public double evaluate(double x) throws StackUnderflowException, StackOverflowException,
34         ↪ NumberFormatException, ArithmeticException {
35         // create a stack to store the numerical values
36         // and the substitute stack
37         Stack<String> subStack = substitute(x);
38         Stack<Double> numStack = new Stack<Double>(this.postFixStack.getHeight());
39         for (int i = subStack.getHeight(); i > 0; i--) {
40             String pop = subStack.pop(); // pop a value of the substitute stack and store it
41             double a, b; // initialize two double variables for potential calculation
42             switch (pop) { // check if the popped value is a number or operator
43                 // if it is an operator
44                 // do the operation on the two numbers at the top of the number stack
45                 // and replace them both with the result
46                 case "+":
47                     b = numStack.pop();
48                     a = numStack.pop();
49                     numStack.push(a + b);
50                     break;
51                 case "-":
52                     b = numStack.pop();
53                     a = numStack.pop();
54                     numStack.push(a - b);
55                     break;
56                 case "*":
57                     b = numStack.pop();
58                     a = numStack.pop();
59                     numStack.push(a * b);
60                     break;
61                 case "/":
62                     b = numStack.pop();
63                     a = numStack.pop();
64                     numStack.push(a / b);
65             }
66         }
67     }
68 }
```

```

64         break;
65     case "^":
66         b = numStack.pop();
67         a = numStack.pop();
68         numStack.push(Math.pow(a, b));
69         break;
70     // if it is a number push it onto the number stack
71     default:
72         // try to cast the popped value, it is of type String, to double
73         // this may fail so the function throws NumberFormatException
74         // this will fail if there are multiple parameters e.g. "xy"
75         // since it will treat the y as a number which it is not
76         numStack.push(Double.valueOf(pop));
77         break;
78     }
79 }
80 // pop and return the remaining value in the number stack, this is the evaluated value
81 return numStack.pop();
82 }
83
84 // substitute a value of the parameter into the expression
85 private Stack<String> substitute(double x)
86     throws StackUnderflowException, StackOverflowException {
87     Stack<String> copy = new Stack<String>(this.postFixStack); // copy the post fix stack so
88     // we don't edit it
89     Stack<String> subStack = new Stack<String>(this.postFixStack.getHeight()); // create a
90     // stack to store the
91
92     for (int i = copy.getHeight(); i > 0; i--) { // loop through the copied stack
93         String pop = copy.pop(); // pop an item off and store it in a variable
94         if (pop.equals(String.valueOf(this.parameter))) { // check if it is a variable
95             // if so replace it with the
96             pop = Double.toString(x);
97         }
98         subStack.push(pop); // push the value onto the new stack
99     }
100     return subStack; // return the new stack
101 }
102
103 // create the abstract syntax tree for the expression - single-threaded
104 private static BinaryTree createTree(String expression) throws UnequalBracketsException {
105     // remove enclosing matching brackets
106     expression = checkBracket(expression);
107     // find the least significant operator, if an operator remains
108     int leastSigOperatorPos = leastSigOperatorPos(expression);
109     if (leastSigOperatorPos == -1) { // base case - no operators remain
110         return new BinaryTree(expression);
111     } else { // recursive case - recurse on the sub-expressions
112         // locate and hold the operator
113         String operator = String.valueOf(expression.charAt(leastSigOperatorPos));
114         // split the expression into sub-expressions by the operator and recurse on them
115         String a = expression.substring(0, leastSigOperatorPos);
116         String b = expression.substring(leastSigOperatorPos + 1);
117         // return the new tree containing the trees of the sub-expressions and the
118         // operator as the root value

```

```

118         return new BinaryTree(operator, createTree(a), createTree(b));
119     }
120 }
121
122 // create the abstract syntax tree for the expression - multi-threaded
123 private static BinaryTree createTreeThread(String expression)
124     throws StringIndexOutOfBoundsException, UnequalBracketsException,
125         InterruptedException, ExecutionException {
126     // remove enclosing matching brackets
127     expression = checkBracket(expression);
128     // find the least significant operator, if an operator remains
129     int leastSigOperatorPos = leastSigOperatorPos(expression);
130     if (leastSigOperatorPos == -1) { // base case - no operators remain
131         return new BinaryTree(expression);
132     } else { // recursive case - recurse on the sub-expressions
133         // locate and hold the operator
134         String operator = String.valueOf(expression.charAt(leastSigOperatorPos));
135         // split the expression into sub-expressions by the operator and recurse on them
136         String a = expression.substring(0, leastSigOperatorPos);
137         String b = expression.substring(leastSigOperatorPos + 1);
138         BinaryTree tree0;
139         BinaryTree tree1;
140         // create the threads and execute them
141         ExecutorService executor0 = Executors.newSingleThreadExecutor();
142         Future<BinaryTree> future0 = executor0.submit(() -> {
143             return createTree(a);
144         });
145         ExecutorService executor1 = Executors.newSingleThreadExecutor();
146         Future<BinaryTree> future1 = executor1.submit(() -> {
147             return createTree(b);
148         });
149         // return the tree values from the threads
150         tree0 = future0.get();
151         tree1 = future1.get();
152         // shutdown the threads
153         executor0.shutdown();
154         executor1.shutdown();
155         // return the new tree containing the trees of the sub-expressions and the
156         // operator as the root value
157         return new BinaryTree(operator, tree0, tree1);
158     }
159 }
160
161 // find the least significant operator in an expression
162 private static int leastSigOperatorPos(String input) {
163     int parenthesis = 0;
164     int leastSigOperatorPos = -1;
165     int leastSigOpcode = 1000; // make the opcode super high so every operator is more
166         // significant than it
167     // used a string to store the operators as it is essentially a char array but
168     // with added utility such as finding elements
169     final String operators = "+-*/^";
170     int currentOpcode;
171     // loop through the entire input
172     for (int i = 0; i < input.length(); i++) {
173         char currentChar = input.charAt(i);
174         currentOpcode = operators.indexOf(currentChar);
175         // check if it is an operator
176         if (currentOpcode >= 0) {
177             // check if it the same or more significant and if it is not enclosed in
178             // parenthesis
179             if ((currentOpcode <= leastSigOpcode) && (parenthesis == 0)) {
180                 // update the significance and the position of the operator
181                 leastSigOperatorPos = i;
182                 leastSigOpcode = currentOpcode;
183             }
184         } else if (currentChar == '(') {
185             // increment to signify we have entered an enclosing bracket

```

```

184         parenthesis++;
185     } else if (currentChar == ')') {
186         // decrement to signify we have exited an enclosing bracket
187         parenthesis--;
188     }
189 }
190 return leastSigOperatorPos;
191 }
192
193 // check for and remove enclosing matching brackets
194 private static String checkBracket(String input) throws UnequalBracketsException,
195     ↪ StringIndexOutOfBoundsException {
196     boolean done = false;
197     // check if there are an equal number of opening and closing brackets
198     if ((input.length() - input.replace("(", "").length()) != (input.length() -
199     ↪ input.replace(")", "").length())) {
200         throw new UnequalBracketsException(input);
201     }
202     while (!done) { // repeat until there are no enclosing brackets
203         done = true; // assume no enclosing matching brackets until otherwise found
204         // check if there are enclosing brackets (not necessarily matching)
205         if ((input.charAt(0) == '(') && (input.charAt(input.length() - 1) == ')')) {
206             int countMatching = 1; // initialize the bracket count as 1 to count for
207             ↪ the opening bracket
208             // loop from the second character to one before the end
209             for (int i = 1; i < input.length() - 1; i++) {
210                 if (countMatching == 0) { // if we find the matching bracket
211                     ↪ before the end terminate and return the
212                                                                 // input
213                     return input;
214                 } else if (input.charAt(i) == '(') {
215                     // increment if we find an opening bracket
216                     countMatching++;
217                 } else if (input.charAt(i) == ')') {
218                     // decrement if we find an closing bracket
219                     countMatching--;
220                 }
221             }
222             // if the last character is the matching bracket keep the outer while
223             ↪ loop going
224             // and and remove the enclosing bracket
225             if (countMatching == 1) {
226                 input = input.substring(1, input.length() - 1);
227                 done = false;
228             }
229         }
230     }
231     return input;
232 }
233
234 // standardize an expression to remove ambiguity
235 private static String standardize(String expression) {
236     // remove whitespace
237     expression = expression.replace(" ", "");
238     // make pi one character for faster regex
239     expression = expression.replace("pi", "π");
240     // inconsistency 1
241     String regex1 = "([\\(\\)\\+\\-\\*\\/\\^])([\\(a-zπ])";
242     String replace1 = "$1*$2";
243     // inconsistency 2
244     String regex2 = "([\\(a-zπ])([\\(\\)\\+\\-\\*\\/\\^])";
245     String replace2 = "$1*$2";
246     // inconsistency 3
247     String regex3 = "\\(\\)";
248     String replace3 = ")*(";
249     // inconsistency 4
250     String regex4 = "([\\+\\-\\*\\/\\^])-(\\+\\-\\*\\/\\(\\)\\^)*";
251     String replace4 = "$1(-$2)";

```

```

247 // inconsistency 5
248 String regex5 = "(^|\\()";
249 String replace5 = "$10-";
250
251 // loop until check and expression are equal
252 String check = "";
253 while (!expression.equals(check)) {
254     check = expression;
255     // perform regex
256     expression = expression.replaceAll(regex1, replace1);
257     expression = expression.replaceAll(regex2, replace2);
258     expression = expression.replaceAll(regex3, replace3);
259     expression = expression.replaceAll(regex4, replace4);
260     expression = expression.replaceAll(regex5, replace5);
261 }
262
263 // replace constants with numerical values
264 expression = expression.replaceAll("e", Double.toString(Math.E));
265 expression = expression.replaceAll("π", Double.toString(Math.PI));
266 return expression;
267 }
268
269 @Override
270 // output the stack and the expression this object represents
271 public String toString() {
272     return this.expression + " -> " + this.postFixStack.toString();
273 }
274
275 @SuppressWarnings("unused")
276 public static void main(String[] args) throws Exception {
277     // create the standardized expressions
278     String ex = standardize(new String(new char[28001]).replace('\0', 'x'));
279     //System.out.println(ex);
280     // create the trees
281     BinaryTree treeA, treeThreadA;
282     Instant start, end;
283     start = Instant.now();
284     treeThreadA = createTreeThread(ex);
285     end = Instant.now();
286     System.out.println("time taken for threaded approach - " + Duration.between(start,
287     ↪ end).toMillis() + "ms");
288     start = Instant.now();
289     treeA = createTree(ex);
290     end = Instant.now();
291     System.out.println("time taken for sequential approach - " + Duration.between(start,
292     ↪ end).toMillis() + "ms");
293 }
294 }

```

17.4 *ExplicitFunction.java*

```
1 package structures;
2
3 import exceptions.StackOverflowException;
4 import exceptions.StackUnderflowException;
5 import exceptions.UnequalBracketsException;
6
7 public class ExplicitFunction {
8
9     // attributes
10    private Expression f; // the actual mathematical function
11
12    // methods
13    // empty constructor to allow for inheritance
14    public ExplicitFunction() {
15    }
16
17    // constructor
18    public ExplicitFunction(String expression, char parameter)
19        throws StackOverflowException, StackUnderflowException, UnequalBracketsException
20        {
21        this.setF(new Expression(expression, parameter));
22    }
23
24    // evaluate method calls the expression evaluate method
25    public double evaluate(double x) {
26        // try to evaluate the expression
27        //if it fails simply return NaN and the associated error
28        try {
29            return f.evaluate(x);
30        } catch (StackUnderflowException e) {
31            System.out.println("underflow");
32        } catch (StackOverflowException e) {
33            System.out.println("overflow");
34        } catch (NumberFormatException e) {
35            System.out.println("multiple parameters");
36        }
37        return Double.NaN;
38    }
39
40    // set the expression attribute - used for child classes
41    protected void setF(Expression f) {
42        this.f = f;
43    }
44
45    @Override
46    // output the representation of the expression object
47    public String toString() {
48        return this.f.toString();
49    }
50 }
```

17.5 *NormalDistribution.java*

```
1 package structures;
2
3 import exceptions.StackOverflowException;
4 import exceptions.StackUnderflowException;
5 import exceptions.UnequalBracketsException;
6
7 public class NormalDistribution extends ExplicitFunction {
8
9     // attributes
10    private double  $\mu$ ; // mean
11    private double  $\sigma^2$ ; // variation
12
13    // methods
14    // constructor
15    public NormalDistribution(double  $\mu$ , double  $\sigma^2$ )
16        throws StackOverflowException, StackUnderflowException, UnequalBracketsException
17        ↪ {
18        // super constructor must be called first in Java
19        super();
20
21        // set the attributes
22        this. $\mu$  =  $\mu$ ;
23        this. $\sigma^2$  =  $\sigma^2$ ;
24
25        // create the expression for the normal distribution function
26        String expression = "(1/(2*pi* $\sigma^2$ )^(1/2))*e^(-(x- $\mu$ )^2/(2* $\sigma^2$ ^(1/2)))";
27        // substitute the mean and variance values
28        expression = expression.replace(" $\sigma^2$ ", Double.toString( $\sigma^2$ ));
29        expression = expression.replace(" $\mu$ ", Double.toString( $\mu$ ));
30
31        // set the new expression
32        this.setF(new Expression(expression, 'x'));
33    }
34
35    // return the mean
36    public double getM() {
37        return  $\mu$ ;
38    }
39
40    // return the variance
41    public double get $\Sigma^2$ () {
42        return  $\sigma^2$ ;
43    }
44
45    // return the standard deviation
46    public double get $\Sigma$ () {
47        return Math.sqrt( $\sigma^2$ );
48    }
49 }
```

Chapter 18

exceptions

18.1 *StackOverflowException.java*

```
1 package exceptions;
2
3 @SuppressWarnings("serial")
4 public class StackOverflowException extends Exception {
5
6     // methods
7     // constructor which outputs a custom message
8     public StackOverflowException() {
9         super("Stack has reached its max height");
10    }
11
12 }
```

18.2 *StackUnderflowException.java*

```
1 package exceptions;
2
3 @SuppressWarnings("serial")
4 public class StackUnderflowException extends Exception {
5
6     // methods
7     // constructor which outputs a custom message
8     public StackUnderflowException() {
9         super("Stack has no items for you to pop");
10    }
11
12 }
```

18.3 *UnequalBracketsException.java*

```
1 package exceptions;
2
3 @SuppressWarnings("serial")
4 public class UnequalBracketsException extends Exception {
5
6     // methods
7     // constructor which outputs a custom message
8     public UnequalBracketsException(String expression) {
9         super("There are an unequal number of opening and closing brackets in the expression: " +
10             ↪ expression);
11    }
12 }
```

Chapter 19

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