IS-PHYS Lab2

Lab 2: Pohl's Pendulum

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■ Abstract

In this experiment, we are going to find out several characteristics of free oscillation, damped oscillation and forced oscillation. We would determine the amplitude and the corresponding natural frequency for free oscillation, the oscillation periods and the damping coefficient for damped oscillation, and the stroboscopic effect and use this effect to observe the phase shift between the pendulum and the external torque, while determining the resonance curves and the representing it graphically for forced oscillation.

▪ Theory

Goals

The purpose of this lab is to:

1. Determine the relationship between amplitude and corresponding frequency of the free oscillation.
2. Determine the oscillation periods and the damping coefficient for damping level 1 and 2.
3. To learn the stroboscopic effect and use this effect to observe the phase shift between the pendulum and the external torque. And graphically derive the resonance curve, which shows the amplitude and phase shifting of forced oscillation for damping level.

Equipment and setup

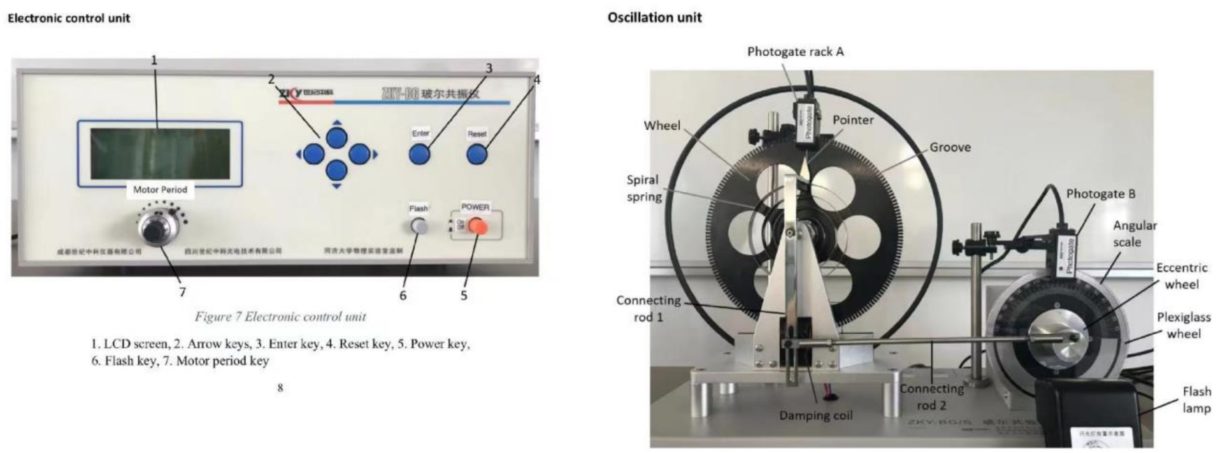


Figure 1 Electronic unit and Oscillation Unit for the oscillation experiment

The Electronic control unit is used to choose the gear and record the period, amplitude, and of oscillation trails. In the “FreeOSC” mode, the system only records the corresponding amplitude when the oscillation period is changing. In “Damp” mode 1,2,3 the wheel in the oscillation unit will suffer from various levels of electromagnetic damping. It can also turn on the motor to different periods to produce an external force of simple harmonic oscillation of different frequencies. It can also check whether the oscillation system is in a steady state.

The oscillation unit implements the oscillations. The spiral provides the elastic force which is proportional to the rotation angle θ. Photogate rack A is used to measure the period of oscillations by detecting the frequency of the deep groove passing through the gate. The deep groove passes through the photogate twice during each oscillation period. The phase shift $\varphi\varphi$ can be read out on the angular scale. The motor's rotation speed can be precisely adjusted within a range of 30–45 cycles per minute by the control unit. Photogate B is mounted above the motor, which measures the period of the external torque by detecting the frequency of the white line passing through it.

Theory

A spiral spring ensures that the wheel once rotated out of its equilibrium position, experiences a restoring torque. If the pendulum is deflected by a small angle θ out of equilibrium, the restoring torque M_s is proportional to θ, so Hooke's law is applied here to calculate the restoring force.

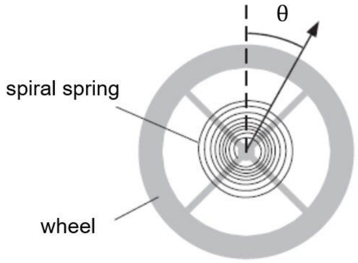


Figure 2 The rotating pendulum

In free oscillation

$$J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = M_s = -k\theta$$

here k is the stiffness coefficient of the spiral spring, and J is the moment of inertia of the pendulum. The solution to the differential equation is

$$\theta(t) = \theta_0 \cos\left(\omega_0 t + \varphi\right)$$

where $\omega_0 = \sqrt{\frac{k}{J}}$.

In damping oscillation:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{b}{J}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{k}{J}\theta = 0$$

This is the differential equation of a damped oscillation. The solution depends on the strength of the damping. This is the solution:

$$\theta(t) = \theta_1 e^{-\beta t} \cos\left(\omega_1 t + \varphi_1\right)$$

where $\beta = \frac{b}{2J}$ is the damping coefficient, θ_1 is the maximum amplitude, $\omega_1 = \sqrt{{\omega_0}^2 - \beta^2}$ is the angular frequency of the damped oscillation, and φ_1 is the phase constant.

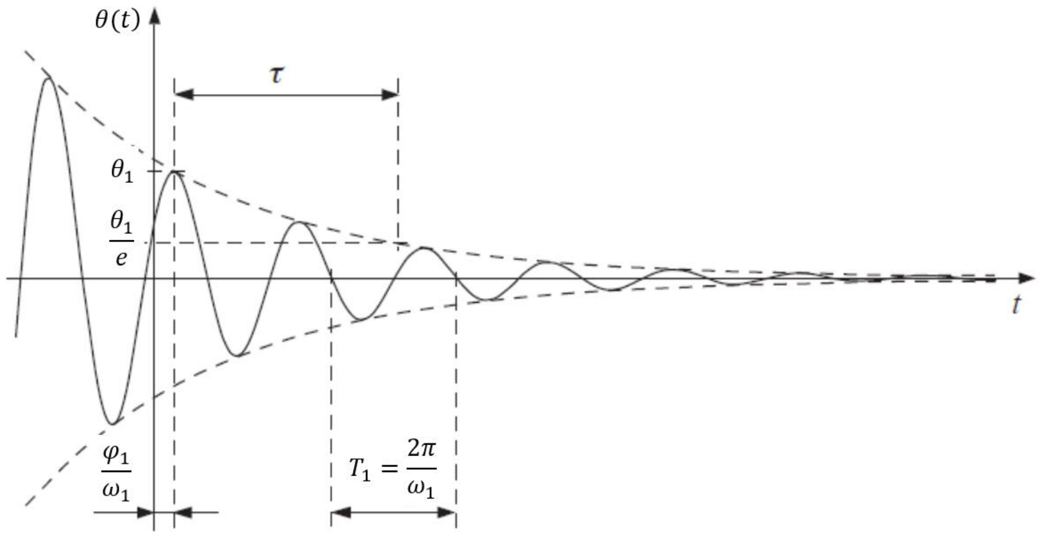


Figure 1 Oscillation with weak damping

In the forced oscillation, we have the function:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{b}{J}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{k}{J}\theta = \frac{M_0}{J}\cos(\omega_2 t)$$

It can also be written as:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + 2\beta\frac{\mathrm{d}\theta}{\mathrm{d}t} + \omega_0^2\theta = F_0\cos(\omega_2 t)$$

where $F_0 = \frac{M_0}{J}$ is the magnitude of the external force. And the solution to it is:

$$\theta = \theta_1 e^{-\beta t}\cos(\omega_1 t + \varphi_1) + \theta_2\cos(\omega_2 t + \varphi_2)$$

Comparing to the dampling oscillation with the same level of damping, it was forced by a periodical external force with the ansats $\theta(t) = \theta_2\cos(\omega_2 t + \varphi_2)$, so the θ is an overlap of two cosine wave and thuse have a certain phase lag, we know

$$\theta_2 = \frac{F_0}{\sqrt{({\omega_0}^2 - {\omega_2}^2)^2 + 4\beta^2\omega_2^2}}$$

So the phrase lag is

$$\varphi_2 = \tan^{-1}\left(\frac{-2\beta\omega_2}{{\omega_0}^2 - {\omega_2}^2}\right) = \tan^{-1}\left(\frac{-\beta T_0^2 T}{\pi\left(T^2 - {T_0}^2\right)}\right)$$

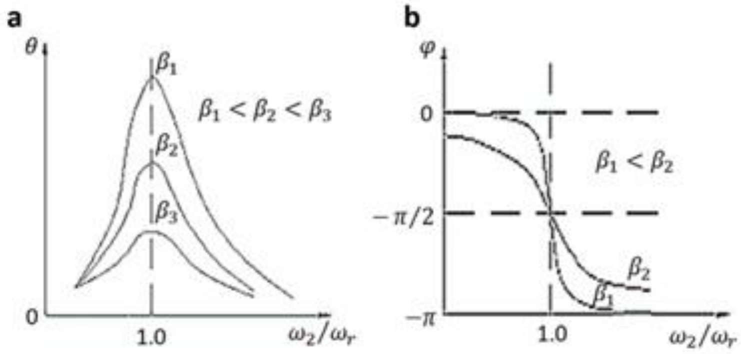


Figure 4 Forced oscillations with different β

The theoretical resonance curve, which shows the amplitude and phase shifting with regard to the circular frequency/ circular frequency0 the of forced oscillation for different damping levels should be like graph 4.

Experimental procedures

Before start the measurement we turned on the control unit, choosing the free oscillation "FreeOSC" mode first.

In the free oscillation experiment, we first made sure that the pendulum points vertically while line A or B aligns with zero degrees of the angular scale. Then we rotate the wheel approximately 160 degrees with the hands carefully and released it. Then we pressed the bottom to record the amplitude and the according period. The amplitude-period is used as a table for reference in the following analysis.

In the damping 2 oscillation experiment, we repeat the steps in the free oscillation to measure the amplitude and period.

In the forced oscillation, we did not change the damping 2 level friction. Then we kept changing the frequency of the driving force by wheeling the Motor period key on the electronic control unit. For each test, we collected the corresponding amplitude and phase lag of the wheel doing forced oscillation.

After collecting the data, we calculated the linear regression line for Amplitude Period, Damping coefficient β for damp 2 level, and sketched the graph for

- ## Data

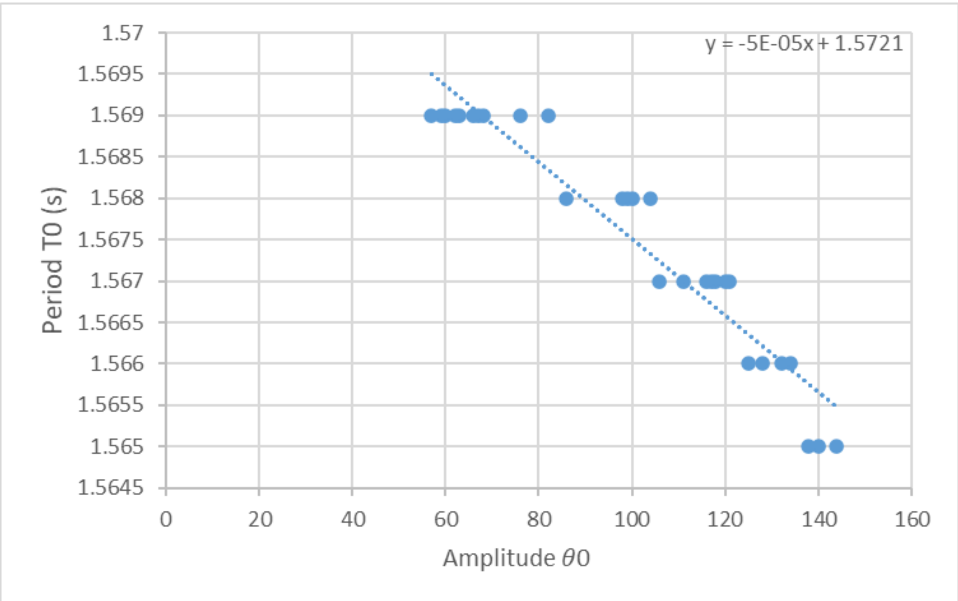
For the free oscillation part, we use the data collected to find out the correlation between the natural period T_0 of the pendulum and the amplitude θ_0 of free oscillation.

Amplitude θ_0	Period T_0 (s)
144	1.565

140	1.565
138	1.565
134	1.566
132	1.566
128	1.566
125	1.566
121	1.567
120	1.567
118	1.567
117	1.567
116	1.567
111	1.567
106	1.567
104	1.568
100	1.568
99	1.568
98	1.568
86	1.568
82	1.569
76	1.569
68	1.569
67	1.569
66	1.569
63	1.569
62	1.569
60	1.569
59	1.569
57	*1.569*

Table 1 The amplitude and period relationship of free oscillation

Then we use the data from Table 1 to make a graph of period and amplitude, which is shown as the following Graph 1.



Graph 1 The Amplitude-Period graph of free oscillation

$y = -0.0005x + 1.5721$
Then find the linear regression function with Graph 1.

For the damped oscillation, we can make Table 2,3 based on Equation 1.

Amplitude #	Amplitude θ (°)	Amplitude #	Amplitude θ (°)	$ln\frac{\theta_i}{\theta_{i+5}}$
θ_1	146	θ_6	92	0.462
θ_2	134	θ_7	83	0.479
θ_3	122	θ_8	76	0.473
θ_4	111	θ_9	69	0.475
θ_5	101	θ_{10}	62	0.488
Average of $ln\frac{\theta_i}{\theta_{i+5}}$				0.476
Damping coefficient β			0.054	

Table 2 Amplitude of damp 2 oscillation and the calculation of damping coefficient β

Table 2 shows the calculating process of damping coefficient β=0.054. It applies the successive differential method to lower the measuring error.

Amplitude #	Amplitude (°)	Amplitude #	Amplitude (°)	ln $\frac{\theta_i}{\theta_{i+5}}$
θ_1	138	θ_6	80	0.545
θ_2	124	θ_7	71	0.558
θ_3	112	θ_8	64	0.560
θ_4	100	θ_8	56	0.580
θ_5	89	θ_{10}	50	0.577
Average of $\frac{\theta_i}{\theta_{i+5}}$				0.564
Damping coefficient β			0.072	

Table 3 Amplitude of damp 1 oscillation and the calculation of damping coefficient β

Table 2 shows the calculating process of damping coefficient β=.0.072 It applies the successive differential method to lower the measuring error.

Then we did the experiment of forced oscillation under the same damp. And make Table 4 with these data.

Measurement			Calculation		
Motor period T(s)	Phase shift Φ2(°)	Amplitude θ (°)	Corresponding T0(from the fitted line in free oscillation)	$\frac{\omega}{\omega r}$	φ2′
1.451	-155	48	1.570	1.084	-2.113
1.466	-156	46	1.570	1.073	-2.050
1.470	-153	67	1.569	1.070	-2.033

1.491	-135	96	1.567	1.055	-1.939
1.503	-120	122	1.566	1.046	-1.882
1.513	-101	140	1.565	1.039	-1.833
1.515	-98	142	1.565	1.038	-1.824
1.519	-87	144	1.565	1.035	-1.804
1.523	-79	142	1.565	1.033	-1.784
1.528	-68	134	1.565	1.029	-1.759
1.533	-58	126	1.566	1.026	-1.734
1.536	-50	114	1.566	1.024	-1.718
1.545	-38	92	1.568	1.018	-1.673
1.567	-24	62	1.569	1.004	-1.561
1.585	-17	48	1.570	0.992	-1.470
1.609	-15	37	1.570	0.977	-1.354
1.635	-9	30	1.571	0.962	-1.236
1.672	-7	24	1.571	0.941	-1.086

Table 4 Motor period, Amplitude θ , the phrase shift of the damp1 forced oscillation, and the calculation value: corresponding period T_0,$\frac{\omega}{\omega r}$ and $\varphi 2'$

Measurement			Calculation		
Motor period T(s)	Phase shift Φ2(°)	Amplitude θ (°)	Corresponding T0(from the fitted line in free oscillation)	$\frac{\omega}{\omega_r}$	φ2′
1.451	-155	48	1.570	1.084	-2.113
1.466	-156	46	1.570	1.073	-2.050
1.470	-153	67	1.569	1.070	-2.033
1.491	-135	96	1.567	1.055	-1.939
1.503	-120	122	1.566	1.046	-1.882
1.513	-101	140	1.565	1.039	-1.833
1.515	-98	142	1.565	1.038	-1.824
1.519	-87	144	1.565	1.035	-1.804
1.523	-79	142	1.565	1.033	-1.784
1.528	-68	134	1.565	1.029	-1.759
1.533	-58	126	1.566	1.026	-1.734
1.536	-50	114	1.566	1.024	-1.718
1.545	-38	92	1.568	1.018	-1.673
1.567	-24	62	1.569	1.004	-1.561
1.585	-17	48	1.570	0.992	-1.470
1.609	-15	37	1.570	0.977	-1.354
1.635	-9	30	1.571	0.962	-1.236
1.672	-7	24	1.571	0.941	-1.086

Table 5 Motor period, Amplitude, and the phrase shift of the damp2 forced oscillation, and the calculation value: corresponding period T_0, $\frac{\omega}{\omega_r}$, and $\varphi 2'$

When calculating the data, we converted the positive value of φ_2 to the corresponding negative value by adding a minus sign, and converted the positive value of $\varphi_2{}'$ to the corresponding negative value by subtracting 180°.

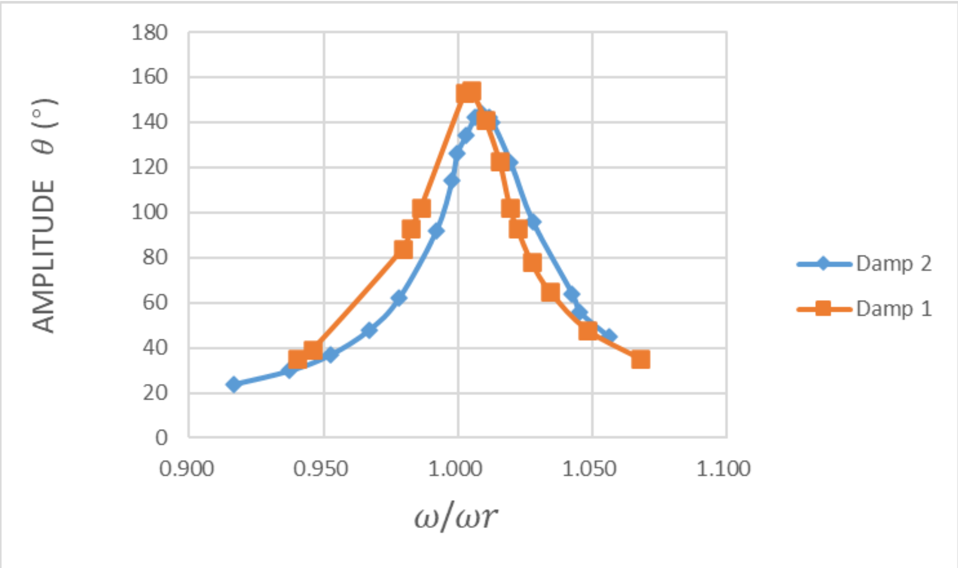
According to data in Table 2 ans3 we can find the relationship between $\frac{\omega}{\omega_r}$ and amplitude.

■ Analysis and Discussions

By using the data from Table 2, 3 we can make a graph of amplitude and

ω/ω_r, shown as Graph 2.The orange line is the fitting line for Damp 1 and the blue line is for damp 2.

The peak of Damp 2 is lower than Damp 1, because it has less energy due to the bigger magnetic resistance. And the abscissa x value for the peak should be ideally 1. However both damp1 and damp2 fitting curve have a shift. This may becuase the experimental error.

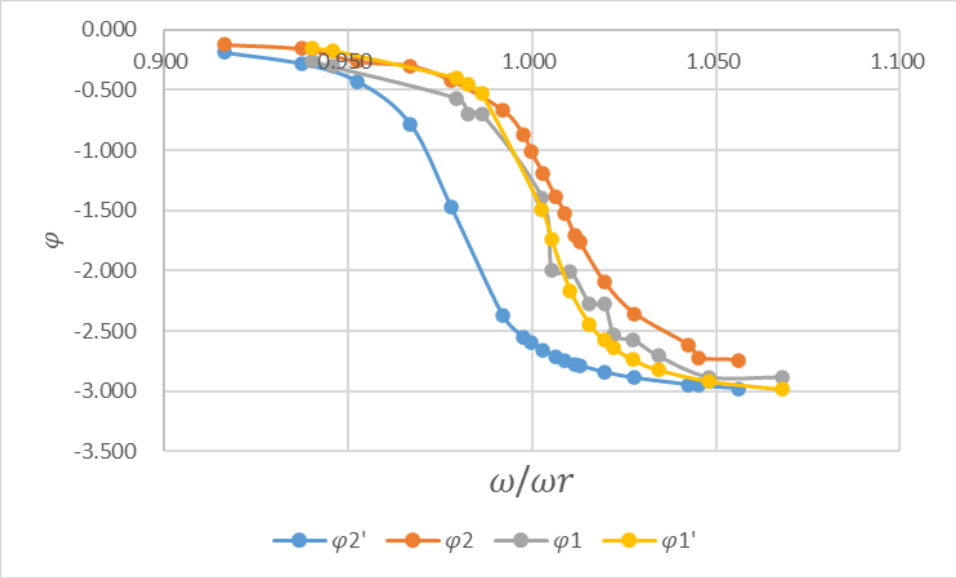


Graph 2 The $\omega/\omega r$ - Amplitude θ graph and the fitting curve for damp1 ,2 forced oscillation

By using the data from Table 4, 5 we can make a graph of phrase shift and

$\omega/\omega r$, shown as Graph 3. The fitting curve is the phrase shift of the damp1 forced oscillation. The fitting curve is the calculated phrase shift of the damp1 forced oscillation. The fitting curve is the phrase shift of the damp2 forced oscillation. The fitting curve is the calculated phrase shift of the damp2 forced oscillation.

The changing rate of the slope of is smaller than , because intuitively the oscillation has a larger maganetic resistance.



Graph 3 The ω/ωr -- Phrase lag graph and the fitting line for damp1 ,2 forced oscillation

Error Analyze:

Although we do not have enough data, which means the fitted curve may not be the actual value, a phenomenon is that, compared to the ideal figure, Graph 2 and Graph 3 have a translation to the right on the abscissa. The $\omega/\omega r$ value is larger than the theory. This may because we measured inaccurate T or T0. The reasons may be as follows:

1. In free oscillation, the period is supposed to be a constant. However, due to the friction of the equipment, air and the magnet is not ideal, so the period is in positive correlation with the oscillating amplitude. The ingenious part of the experiment is that it uses the linear regression to predict the T_0 instead of using a fixed one. So this step reduces the error.

2. When we release the wheel, we might bring it a initial velocity or acceleration, so it did not do the oscillation as we expected so the period may be not accurate.

3. It takes more time for the wheel to have the identical frequency with the driving force, so the phrase shift may be still disequilibrium. It is possible that we did not read the right phrase shift.

Answering Question:

1. Yes. Theoretically, the coefficient will be zero. Which means the amplitude does not change in terms of time. However, in the experiment there exist friction in equipment and air, so the damping coefficient is not 0 but a small number which is very close to 0.

Amplitude #	Amplitude (°)	Amplitude #	Amplitude (°)	$\ln \dfrac{\theta i}{\theta i + 5}$

θ_1	144	θ_6	128	0.118
θ_2	140	θ_7	125	0.113
θ_3	138	θ_8	121	0.131
θ_4	134	θ_9	120	0.110
θ_5	132	θ_{10}	118	0.112
Average of ln $\frac{\theta i}{\theta i+5}$				0.117
T_average (s)			1.565	
Damping coefficient β			0.015	

Table 6 Amplitude of different oscillation periods in free oscillation

This table shows the calculation progress of the damping coefficient of 'free oscillation'.

2. Many electric dipoles' nature frequencies coincide with the generated rotation frequency of the microwave electric field, so the molecules resonate. This is used in microwaves.

Microwave works by passing non-ionizing microwave radiation. The polar molecules in the food oscillate widely because of resonance and disperse energy in the form of heat so we get hot food from the microwave.

Human body has a natural frequency, though differing in different parts of the body, it is an average of 60 Hz. (Energicx, n.d.)

This leads to a resonance of our body to the frequency of about 60 Hz. We should avoid go to the place where might have low frquency like airport and bars.

▪ Conclusions

In this experiment, we studied the characteristics of free oscillation, damped oscillation and forced oscillation. We determined the resonance curves and the representing it graphically for forced oscillation.

Amplitude θ graph and the fitting line and the $\omega/\omega r$ -- Phrase lag graph and the fitting line have the approximately same shape with the theoretical one but have an approximately 0.035 translation to the right on the abscissa. We may improve it by practicing read the numbers and make more trails in each step and use the average number to reduce the error.

- # **Raw Data**

Raw Data

Free Oscillation

Trial	Amplitude	Period T (s)
1	144	1.565
2	140	1.565
3	138	1.565
4	134	1.566
5	132	1.566
6	128	1.566
7	125	1.566
8	121	1.567
9	120	1.567
10	118	1.567
11	117	1.567
12	116	1.567
13	111	1.567
14	106	1.567
15	104	1.568
16	100	1.568
17	99	1.568
18	98	1.568
19	86	1.568
20	82	1.569
21	76	1.569
22	68	1.569
23	67	1.569
24	66	1.569
25	63	1.569
26	62	1.569
27	60	1.569
28	59	1.569
29	57	1.569

Damp 1 Oscillation

Trail	Amplitude	10T (s)	(s)
1	146	15.667	
2	134	T_average(s)	
3	122	1.5667	
4	111		
5	101		
6	92		
7	83		
8	76		
9	69		
10	62		

Damp 2 Oscillation

1	132	10T (s)
2	120	15.164
3	108	T_average(s)
4	98	1.5164
5	88	
6	80	
7	72	
8	64	
9	58	
10	52	

Forced oscillation damp 1

Trail	T (s)	Amplitude	Phrase shift (°)
1	1.472	35	165
2	1.5	48	165
3	1.52	65	155
4	1.53	78	147.5
5	1.538	93	145
6	1.542	102	130
7	1.548	123	130
8	1.556	141	115
9	1.564	154	114
10	1.568	153	80
11	1.594	102	40
12	1.6	93	40
13	1.605	84	32.5
14	1.662	39	17.5
15	1.672	35	15

Forced oscillation damp 2

Trail	T (s)	Amplitude	Phrase shift (°)
1	1.451	45	157
2	1.466	56	156
3	1.47	64	150
4	1.491	96	135
5	1.503	122	120
6	1.513	140	101
7	1.515	142	98
8	1.519	144	87
9	1.523	142	79
10	1.528	134	68
11	1.533	126	58
12	1.536	114	50
13	1.545	92	38
14	1.567	62	24
15	1.585	48	17
16	1.609	37	15
17	1.635	30	9
18	1.672	24	7

References

"Frequency of Human Body". Energicx. n.p. n.d. Https://energicxusa.com/frequency-of-humanbody/#:~:text=Frequency%20of%20Human%20Body%20The%20overall%20range%20of,body%20and%20that%20flow%20produces%20our%20life%20force.