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Problem 1

a) . $3^{1500}$ mod 11

$1500 = 10111011100_2$

$= 2^{10} + 2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^2$

$= 1024 + 256 + 128 + 64 + 16 + 8 + 4$

$= 3^{1024+256+128+64+16+8+4}$

$= (4)\times(3)\times(5)\times(4)\times(3)\times(5)\times(4)$

$= 14{,}400$

$\Rightarrow 14{,}400 \bmod 11 = 1$

$\therefore \boxed{3^{1500} \bmod 11 = 1}$

$3^2 = 9 \bmod 11 = 9$

$3^4 = 9^2 \bmod 11 = 4$

$3^8 = 4^2 \bmod 11 = 5$

$3^{16} = 5^2 \bmod 11 = 3$

$3^{32} = 3^2 \bmod 11 = 9$

$3^{64} = 9^2 \bmod 11 = 4$

$3^{128} = 4^2 \bmod 11 = 5$

$3^{256} = 5^2 \bmod 11 = 3$

$3^{512} = 3^2 \bmod 11 = 9$

$3^{1024} = 9^2 \bmod 11 = 4$

14,400 ⌊ 11

1309

14
34
33
10
0
100
99
1

b. $5^{4358}$ mod 10

$4358 = 2^{12} + 2^8 + 2^2 + 2$

$= 4096 + 256 + 4 + 2$

$\Rightarrow 5^{4358} = (5^{4096})\times(5^{256})\times(5^4)\times(5^2)$

$\Rightarrow 5^2 = 25 \bmod 10 = 5$

$5^4 = 5^2 \bmod 10 = 5$

$5^8 = 5^2 \bmod 10 = 5$

$5^{16} = 5^2 \bmod 10 = 5$

$5^{32}$ same

$5^{64}$ same

$5^{128}$ same

$5^{256}$ same

$5^{512}$ same

$5^{1024}$ same

$5^{2048}$ same

$5^{4096}$ same

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$\Rightarrow 5^{4358} = (5) \times (5) \times (5) \times (5)$

$5^{4358} = 625$

$\Rightarrow 625 \bmod 10 = 5$

$\therefore \boxed{5^{4358} \bmod 10 = 5}$

c). $6^{22345} \bmod 7$

6 mod 7 can be $-1$ or 6

$\Rightarrow (-1)^{22345} \bmod 7$

$\Rightarrow (-1)^{22344} \times (-1) \bmod 7$

$\Rightarrow (1) \times (-1) \bmod 7$

$\Rightarrow -1 \bmod 7$

$\Rightarrow 6$

$\therefore \boxed{6^{22345} \bmod 7 = 6}$

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Problem 2 :

a. GCD (648 , 124)

| | |
|---|---|
| 648 | 2 |
| 324 | 2 |
| 162 | 2 |
| 81 | 3 |
| 27 | 3 |
| 9 | 3 |
| 3 | 3 |
| 1 | |

| | |
|---|---|
| 124 | 2 |
| 62 | 2 |
| 31 | 31 |
| 1 | |

$\Rightarrow 648 = 2^3 \times 3^4$

$\Rightarrow 124 = 2^2 \times 31$

$\Rightarrow \boxed{GCD(648, 124) = 2^2 = 4}$

b. GCD (123456789, 123456788)

$\Rightarrow 123456789 = 9 \times 3607 \times 3803$

$\Rightarrow 123456788 = 4 \times 7 \times 13 \times 17 \times 71 \times 281$

$\Rightarrow \boxed{GCD(123456789, 123456788) = 1}$

c. $GCD(2^{300} * 3^{200}, 2^{200})$

$\Rightarrow (2^{300}) \times (3^{200}) = (2^{200+100}) \times (3^{200}) = 2^{200} \times 2^{100} \times 3^{200}$

$\Rightarrow \boxed{GCD(2^{300} * 3^{200}, 2^{200}) = 2^{200}}$