Homework 6

1. Since the jobs need to be completed first on the super computer with the total time of $\sum_{i=1}^{n} s_i$, this total time is not correlated or respective of the order of the execution.

Hence, the execution time can only optimized in the normal computer. To do so, the normal computer needs to be kept busy, so if there is the m^{th} job coming, then the $(m-1)$ normal computer need to be busy. That can be done if $(m-1)$ computer are executing the longest jobs as possible which is kind of like a parallelism method in a way. With this the execution is done only when the job get mapped to all n computers.

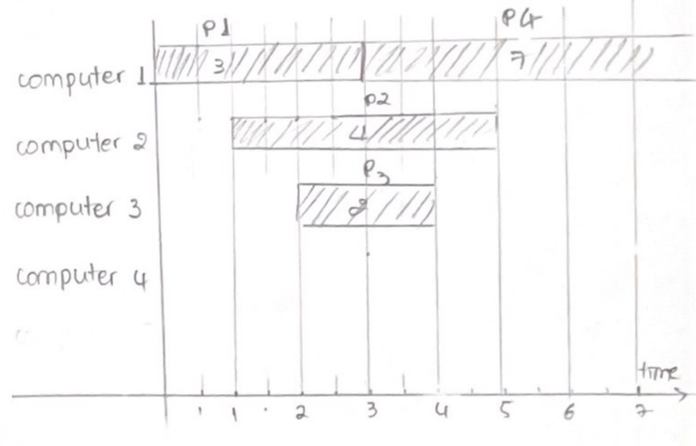
For example process 1 → 3 execute time

Process 2 → 4 execute time

process 3 → 2 execute time

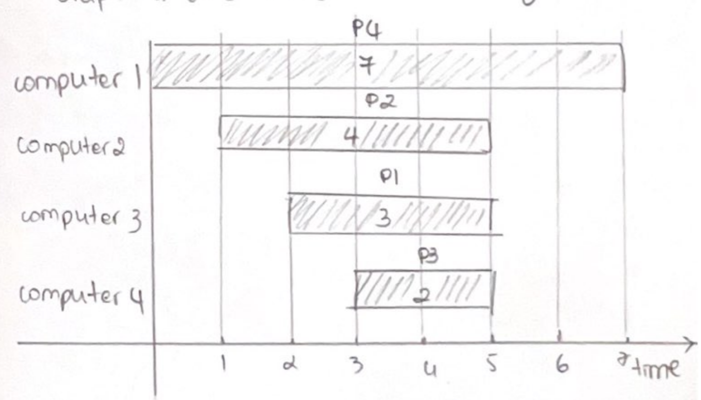
process 4 → 7 execute time

If we don't sort the execute job in decreasing order by n_i and just do it by its given order.



From above picture, we can see that we do not use all computer (utilize all resources) cause when P1 finish in computer 1, we could use P4 in computer 1 and that is not optimal cause max processing time is now 7+3 = 10.

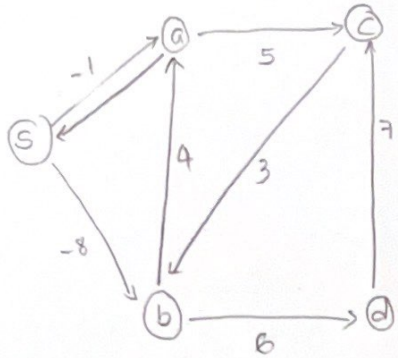
Graph for order the job in decreasing order n



From above graph, if we order job in decending order of it n time, then we are able to utilize all computer and the maximum processing time is now 7.

3. Dijkstra's algorithm will fail. For example:

we have the graph below

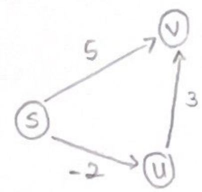


s	a	b	c	d
-8	-1	-8	∞	∞
-2	-1	-8	∞	-2
-1	-1	-8	5	-2

From the table above, we are able to see that Dijkstra will not work when we have negative edge. Because right after we get the edge from $s \to b$ for our shortest path, from b, we could actually go to a with even more shortest path using relaxotion like $s \to b$ & $b \to a = -8 + 4 = -4$ less than -1, but we can't do that cause a is already relaxed.

Proof by contradiction

statement: dijkstra's algorithm still could give a shortest path even there is a negative edge.



if there is a shortest path then

$d(s,v) \leqslant d(s,u) + d(u,v)$

which mean $d(s,u)$ or $d(u,v) > 5$ if one of them has negative edge like

if $d(s,u) < 0$ the $d(u,v) > 5$

elif $d(s,u) > 5$ then $d(u,v) < 0$

which is not true for all cases (see the above graph).

Hence, it is false to say that dijkstra's algorithm works when there is a negative edge.