Problem 1: $T(n) = 2T\left(\frac{2}{3}n\right) + n^2$

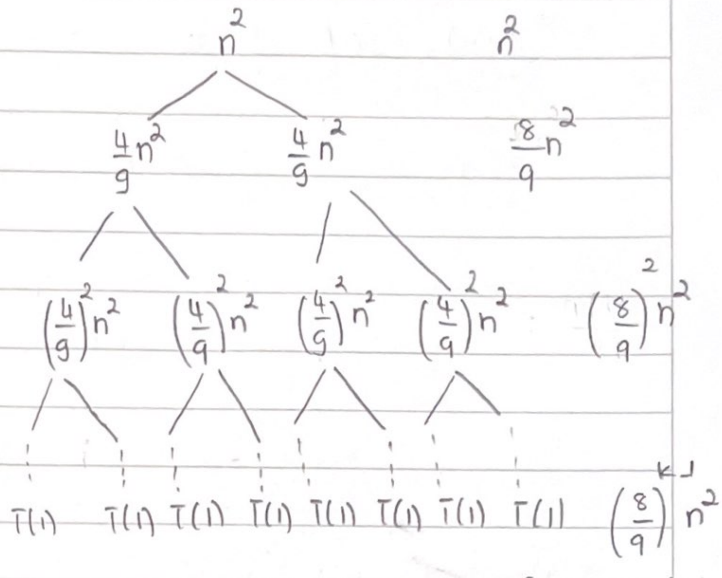
$T(n) = 2T\left(\frac{2n}{3}\right) + n^2$

$T\left(\frac{2}{3}n\right) = 2T\left(\frac{2^2}{3^2}n\right) + \frac{2^2}{3^2}n^2$

$T\left(\frac{4}{9}n\right) = 2\left(\frac{2^3}{3^3}n\right) + \frac{4^2}{9^2}n^2$

$T\left(\frac{2^k}{3^k}n\right) = T(1)$

$n = \left(\frac{3}{2}\right)^k$



$\log_{3/2} n = \log_{\frac{3}{2}} \frac{3}{2}^k$

$\log_{3/2} n = k$

$L_c = 2^k = 2^{\log_{1.5} n} \Rightarrow n^{\log_{1.5} 2}$

$I_c = n^2\left[\left(\frac{8}{9}\right)^0 + \left(\frac{8}{9}\right)^1 + \left(\frac{8}{9}\right)^2 + \dots + \left(\frac{8}{9}\right)^{k-1} n^2\right]$

$= n^2\left[\frac{1}{1-\frac{8}{9}}\right]$

$= n^2\left(\frac{1}{\frac{1}{9}}\right) = 9n^2$

Total cost $= L_c + I_c$

$= n^{\log_{1.5} 2} + 9n^2$

$\Rightarrow \boxed{T(n) \in \Theta(n^2)}$

TOPIC DATE

Problem 1: Master theorem

$T(n) = 2T\left(\frac{2}{3}n\right) + n^2$

$a = 2, \quad b = \frac{3}{2} \Rightarrow n^{\log_{1.5} 2} = n^{1.71} \qquad f(n) = n^2$

case 3: $f(n) = \Omega(n^{1.71+\varepsilon})$ for $\varepsilon = 0.29$

and $2\left(\frac{2}{3}n\right)^2 \le cn^2$ for $c = \frac{4}{3}$

$\therefore \boxed{T(n) = \Theta(f(n)) = \Theta(n^2)}$

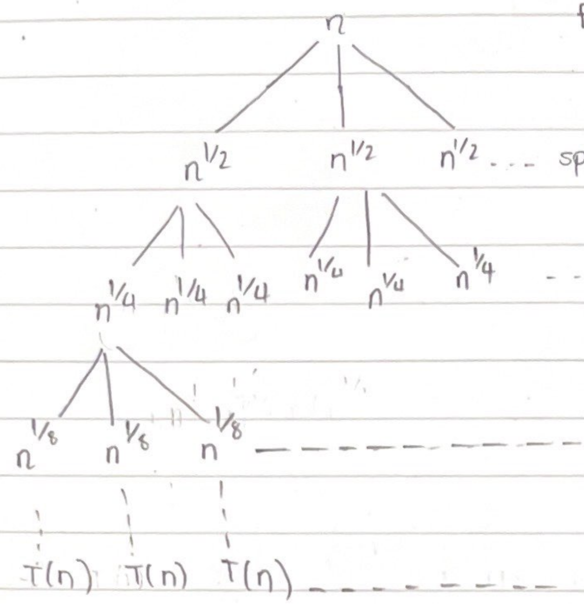
Problem 2: $T(n) = 3T\left(\frac{n}{2}\right) + \frac{n}{\log n} = 3T\left(\frac{n}{2}\right) + n \log^{-1} n$

$a = 3, \ b = 2 \Rightarrow \log_2 3 = 1.58 \ ; \ f(n) = n \log^{-1} n$

case 1: $f(n) = \Theta(n^{1.58 - \varepsilon})$ for $\varepsilon = 0.58$

$\therefore \boxed{T(n) = \Theta(n^{\log_2 3})}$

Problem 3 : $T(n) = \sqrt{n}\,T(\sqrt{n}) + n$



$f(n)$

... spliting $n^{1/2}$ time $\Rightarrow f(n^{1/2})\, n^{1/2}$

$\Rightarrow f(n^{1/4})\, n^{\frac{1}{2}+\frac{1}{4}}$

$\Rightarrow f(n^{1/8})\, n^{1/8+\frac{1}{4}+\frac{1}{2}}$

$\Rightarrow f\left(n^{\left(\frac{1}{2}\right)^K}\right) n^{\left(\frac{1}{2}\right)^K + \left(\frac{1}{2}\right)^{K-1} + \cdots + \frac{1}{4} + \frac{1}{2}}$

Since $\left(\frac{1}{2}\right)^K + \left(\frac{1}{2}\right)^{K-1} + \cdots + \frac{1}{4} + \frac{1}{2} = \sum_1^\infty \left(\frac{1}{2}\right)^K$ converge to 1

$\Rightarrow$ it will be constant $= 1$

$\Rightarrow n f\left(n^{\left(\frac{1}{2}\right)^k}\right)$

assuming $n^{\left(\frac{1}{2}\right)^k} = \alpha$ where $\alpha \in \mathbb{R}$

$$\log_\alpha n^{\left(\frac{1}{2}\right)^k} = \log_\alpha \alpha$$

$$\left(\frac{1}{2}\right)^k \log_\alpha n = 1$$

$$\log_\alpha n = 2^K$$

$$\log_2 \log_\alpha n = \log_2 2^K \Rightarrow K = \log_2 \log_\alpha n$$

$\Rightarrow n \log_2 \log n$

$\Rightarrow \boxed{\Theta = n \log_2 \log n}$

b. $T(n) = 3T(n-1)$

$k=1 \Rightarrow T(n) = 3T(n-1)$

$k=2 \Rightarrow T(n) = 3(3(T(n-2))$

$= 9T(n-2)$

$k=3 \Rightarrow T(n) = 9(3T(n-3))$

$= 27T(n-3)$

⋮

k time $\Rightarrow T(n) = 3^k T(n-k)$

let $T(1) = a, a \in \mathbb{R} \Rightarrow k = n-1$

$\Rightarrow T(n) = 3^{n-1} T(n-n+1) = 3^{n-1} T(1) = 3^{n-1} a$

$\therefore \boxed{T(n) = \Theta(3^n)}$