

Search for Heavy Neutral Leptons with the IceCube DeepCore

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Leander Fischer M. Sc.
geboren am 24. Oktober 1992
in Heidelberg

Präsidentin der Humboldt-Universität zu Berlin
Prof. Dr. Julia von Blumenthal

Dekanin der Mathematisch-Naturwissenschaftlichen Fakultät
Prof. Dr. Caren Tischendorf

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https://github.com/LeanderFischer/phd_thesis

Zusammenfassung

Zusammenfassung ...

Abstract

Abstract ...

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(Re-)write SM neutrino chapter for PhD thesis (just copy paste from M.Sc.).	3
Re-write/re-formulate this section (copied from HNL technote).	13
Add comparions of SM cross sections between NuXSSplMkr and genie	15
Add description of MadGraph5 decay files (Harvard needs to provide this)	16

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(Re-)write introduction for PhD thesis (just copy paste from M.Sc.).

The neutrino was postulated by Wolfgang Pauli [1] in 1930 to explain the continuous energy spectrum of electrons originating from beta decay. Cowan and Reines confirmed this prediction of a light, neutral particle in 1956 when they discovered the electron neutrino using inverse beta decay [2]. Two additional neutrino flavors were found in the following years, and with the discovery of the muon neutrino in 1962 [3] and the tau neutrino in 2001 [4], the current theory of neutrinos in the standard model (SM) was established.

Although neutrinos were first believed to be massless, experimental evidence showing the existence of mixed neutrino states started to appear in the 1960s [5]. Mixing between different physical representations of neutrinos is proof for differences in their masses. The resulting phenomenon of neutrino oscillations can be incorporated into the standard model by extending it to include massive neutrinos. How massive they are and how strong is the mixing between neutrino states has to be obtained from measurement. Today there are a variety of precision oscillation experiments using solar, reactor and atmospheric neutrinos to tighten the constraints on the neutrino oscillation parameters. IceCube is one of those leading experiments probing the oscillation theory with atmospheric neutrinos.

The IceCube Neutrino Observatory [6] was constructed between 2004 and 2010 at the geographic South Pole. It is the first cubic kilometer Cherenkov neutrino detector and consists of 5160 optical sensors attached to 86 strings, drilled down to a maximum depth of ~ 2500 m into the Antarctic ice. Neutrinos are detected by the Cherenkov light that is emitted by secondary particles produced in neutrino-nucleon scattering interactions in the ice. With DeepCore, a more densely instrumented sub-array of IceCube, the neutrino detection energy threshold can be lowered to approximately 5 GeV.

At these energies, the similarity in event signatures poses difficulties in identifying different neutrino flavor interactions. Muon neutrino charged-current interactions produce light tracks as opposed to charged-current interactions of electron and tau neutrinos as well as neutral-current interactions of all neutrinos that produce light cascades. The sparse instrumentation of IceCube makes it more challenging to separate track- and cascade-like events. In this thesis, a novel method to distinguish those two event types is developed. In contrast to previously used univariate separation techniques, the multivariate machine learning method applied here maximizes the use of information from the detector response. Through the use of a Gradient Tree Boosting algorithm the separation of events in track and cascade is improved. As a result of the improved separation, the uncertainty to the atmospheric neutrino oscillation parameters Δm_{32}^2 and θ_{23} is significantly reduced.

This thesis is structured as follows.

[1]: Pauli (1978), *Dear radioactive ladies and gentlemen*

[2]: Cowan et al. (1956), *Detection of the Free Neutrino: a Confirmation*

[3]: Danby et al. (1962), *Observation of High-Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos*

[4]: Kodama et al. (2001), *Observation of tau neutrino interactions*

[5]: Davis et al. (1972), *Proceedings of the Neutrino '72 Europhysics Conference*

[6]: Aartsen et al. (2017), *The IceCube Neutrino Observatory: instrumentation and on-line systems*

This chapter introduces the basic properties of neutrinos, their place in the Standard Model of particle physics (SM) and their peculiarities following the description of [7]. Section Section ?? and Section Section 2.3 state the general properties of neutrinos and the neutrino-nucleon interactions. After describing atmospheric neutrinos in Section Section ?? the phenomenon of neutrino oscillations is presented in Section Section ??.

2.1 Standard Model Particles

2.1.1 Electroweak Symmetry Breaking

2.1.2 Charged Fermion Masses

2.1.3 Neutrino Masses

Dirac

Majorana

2.1.4 See-Saw Mechanisms

2.1.5 Radiative Neutrino Masses

2.2 Neutrino Properties

2.2.1 Quantum Numbers

2.2.2 Mass

2.2.3 Active Neutrino Flavors

2.3 Neutrino Interactions

2.3.1 Weak Interactions after Symmetry-Breaking

The neutrino is an elementary particle in the SM [7]. It belongs to the class of leptons, which itself is a subclass of elementary fermions (spin $\frac{1}{2}$ particles). The fermions - six quarks and six leptons - form the matter content of the universe. Quarks take part in all three interaction types (forces) of the SM: strong, weak, and electromagnetic (EM) [8]. The charged leptons - electron, muon, and tau - are subject to the weak and the EM interaction. Neutrinos carry neither electric charge nor color

(Re-)write SM neutrino chapter for PhD thesis (just copy paste from M.Sc.).

[7]: Thomson (2013), *Modern particle physics*

2.1.2 Charged Fermion Masses 3

2.1.3 Neutrino Masses 3

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[7]: Thomson (2013), *Modern particle physics*

[8]: Glashow (1961), *Partial-symmetries of weak interactions*

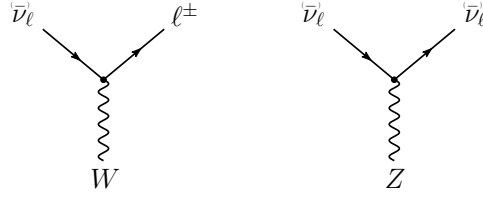


Figure 2.1: Feynman diagrams of charged-current (left) and neutral-current (right) neutrino weak interactions. Taken from [10].

[9]: Tanabashi et al. (2018), *Review of Particle Physics*

[7]: Thomson (2013), *Modern particle physics*

charge and therefore only take part in weak interactions. There are three distinct neutrino flavors - electron neutrinos, muon neutrinos and tau neutrinos (ν_e , ν_μ , and ν_τ) [9] - each corresponding to their charged lepton counterparts.

In the SM, weak interactions are mediated by the three massive bosons W^+ , W^- , and Z^0 [7]. The large boson masses ($m_W \sim 80 \text{ GeV}$, $m_Z \sim 90 \text{ GeV}$) result in a short range of the force of about 10^{-18} m . Weak interactions carried by W^\pm bosons are called charged-current (CC) interactions, because charge is transferred between the interacting particles. In CC interactions, a neutrino is converted into its corresponding charged lepton or vice versa. Neutral current (NC) interactions are those mediated by Z^0 bosons. Here no charge is transferred. The Feynman diagrams for CC and NC interactions are shown in Figure Figure 2.1.

Although neutrinos are massless in the SM, we know today that they do have a small mass. The observed phenomenon of neutrino oscillations (see Section Section ??) is based on the fact that there is a mass difference between the three neutrino mass eigenstates. From neutrino oscillation measurements the absolute mass scale cannot be determined, since they only depend on the mass differences, but there are upper limits on the sum of all neutrino masses from cosmological observations. These upper limits are typically between 0.3 and 1.3 eV [9].

[9]: Tanabashi et al. (2018), *Review of Particle Physics*

2.3.2 Neutrino-Lepton Scattering

Particle-Antiparticle Scattering

2.3.3 Neutrino Interactions with Nuclei

To describe the neutrino detection principle of IceCube explained in Chapter Chapter 4 we need to understand the weak interaction processes that occur at the energies relevant for this work (10 – 100 GeV). The cross-sections are dominated by the following neutrino-nucleon interactions: quasi-elastic scattering (QE), resonant scattering (RES), and deep inelastic scattering (DIS). The relative importance of the different processes depends on energy as can be seen in Figure Figure 2.2.

An extensive description of all the interactions and the differences between neutrino and antineutrino cross-sections can be found in [11]. At energies below 5 GeV, QE and RES occur and the neutrinos interact with approximately point-like protons and neutrons. The cross-sections of these processes are not linear in energy and the transition region to higher energies is poorly understood. At higher energies, the interactions are dominated solely by DIS which has a linear dependence on energy above $\sim 20 \text{ GeV}$. For a given neutrino energy, it is possible to predict

[11]: Formaggio et al. (2012), *From eV to EeV: Neutrino cross sections across energy scales*

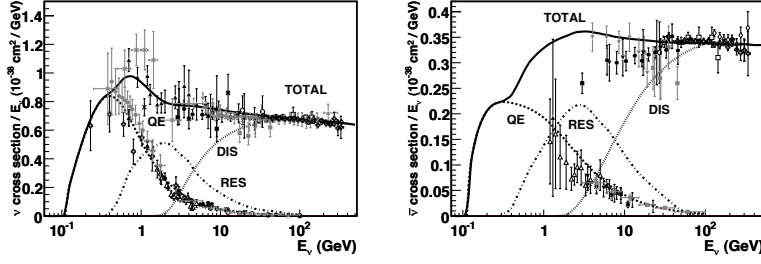


Figure 2.2: Total neutrino(left) and antineutrino(right) per nucleon cross-section divided by neutrino energy plotted against energy. The three main scattering processes quasi-elastic scattering (QE), resonant scattering (RES), and deep-inelastic scattering (DIS) are depicted. Taken from [11].

the cross-section in this region. Here neutrinos interact with a single quark, breaking apart the nucleus and producing a shower of relativistic secondary particles. Neutrino DIS is the primary detection channel of IceCube. From Figure 2.2 it can be seen that the interaction cross-sections are very small of the order of 10^{-38} cm^2 . Because of the small interaction cross-section, very large volume detectors are required to capture a sufficiently large sample of neutrinos to use for precision studies of their properties. For example, the interaction length of a neutrino with $E_\nu = 10 \text{ GeV}$ is of $\mathcal{O}(10^{10} \text{ km})$.

Charged-current Quasi-elastic Scattering

Quasi-elastic scattering (QE) with nucleons is the main process below 1 GeV. Protons are converted to neutrons in antineutrino interactions and vice-versa for neutrino interactions. Additionally, a charged lepton corresponding to the neutrino/antineutrino flavor is produced.

Resonant Scattering

Resonant scattering (RES) describes the process of a neutrino scattering off a nucleon producing an excited state of the nucleon in addition to a charged lepton. RES is the leading process at 1.5-5 GeV for neutrinos and 1.5-8 GeV for antineutrinos.

Deep Inelastic Scattering

Deep inelastic scattering (DIS) occurs if a neutrino carries sufficient energy to resolve the underlying structure of the nucleon and interacts with one of the composing quarks. DIS is the dominant process above 10 GeV. The nucleon breaks up and a lepton accompanied by a set of hadronic final states is produced. Whether the lepton is the charged lepton corresponding to the interacting neutrino type, or the neutrino itself depends on the type of DIS interaction. DIS happens via CC as in

$$\begin{aligned} \nu_l + N &\rightarrow l^- + X, \\ \bar{\nu}_l + N &\rightarrow l^+ + X, \end{aligned} \quad (2.1)$$

or NC interactions as

$$\nu_l + N \rightarrow \nu_l + X. \quad (2.2)$$

Here, X stands for any set of final state hadrons and N for the nucleon. The Feynman diagrams for the processes in Equations Equation 2.1 and Equation 2.2 are shown in Figure Figure 2.3.

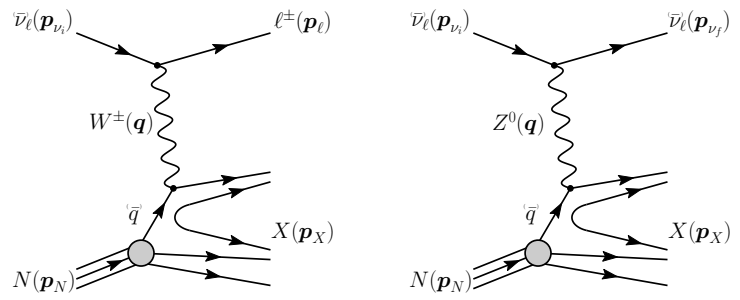


Figure 2.3: Feynman diagrams for deep inelastic scattering of a neutrino with a nucleon via charged-current (left) and neutral current (right) interactions. Taken from [10].

Beyond the Standard Model Neutrinos

3

3.1 Neutrino Oscillations

3.1.1 Vacuum Oscillations

3.1.2 Oscillations in Matter

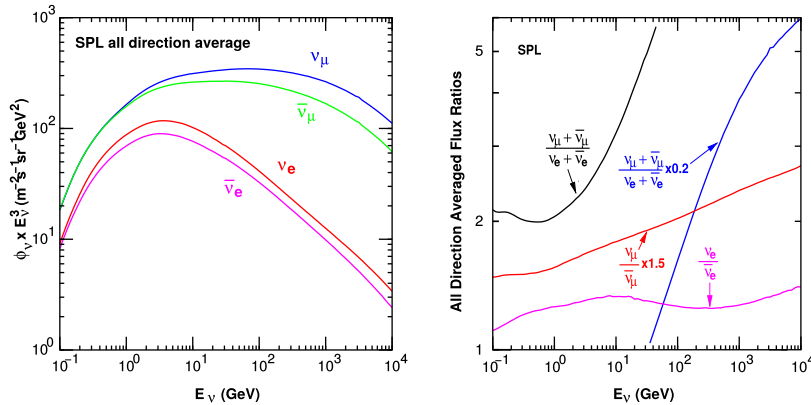
3.1.3 Atmospheric Neutrino Oscillations

Neutrino Production in the Atmosphere

The flux of neutrinos used for this work exclusively comes from the Earth's atmosphere. The nominal flux model is calculated by [12] in the energy range of 100 MeV to 10 TeV. When highly relativistic cosmic rays (protons and heavier nuclei [9]) interact in the upper atmosphere they produce a shower of particles. Neutrinos emerge from the decays of charged pions and kaons (π and K mesons) present in these showers. For energies below 100 GeV, the leading contribution comes from the pion decay chain

$$\begin{aligned}\pi^\pm &\rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu), \\ \mu^\pm &\rightarrow e^\pm + \bar{\nu}_\mu(\nu_\mu) + \nu_e(\bar{\nu}_e).\end{aligned}\quad (3.1)$$

The muons that also originate from this process are considered the main background source for IceCube. The left part of Figure Figure 3.1 shows the atmospheric neutrino flux for the very broad energy spectrum in which they are produced. The flux expectations are calculated for the South Pole [12], where the IceCube detector is located. From Equation Equation 3.1 the ratio between muon and electron neutrinos can be inferred to be $N_{\nu_\mu} : N_{\nu_e} \approx 2 : 1$. This is only the case at muon energies below 1 GeV, where all muons decay in flight. For higher energies, muons can reach earth before decaying increasing the ratio to approximately 10:1 at around 100 GeV as shown in the right part of Figure Figure 3.1. Additionally, kaon decays start to contribute which also increases the number of muons and muon neutrinos.



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[12]: Honda et al. (2015), *Atmospheric neutrino flux calculation using the NRLMSISE-00 atmospheric model*

[9]: Tanabashi et al. (2018), *Review of Particle Physics*

[12]: Honda et al. (2015), *Atmospheric neutrino flux calculation using the NRLMSISE-00 atmospheric model*

Figure 3.1: Atmospheric neutrino fluxes of the different flavors as a function of energy (left) and ratios between muon- and electron-neutrinos as well as ratios between neutrinos and antineutrinos for both flavors (right). Calculations are done for the geographic South Pole. Taken from [12].

[13]: Fedynitch et al. (2015), *Calculation of conventional and prompt lepton fluxes at very high energy*

[14]: Honda et al. (2007), *Calculation of atmospheric neutrino flux using the interaction model calibrated with atmospheric muon data*

In cosmic ray interactions, charged mesons or tau particles can also be produced, which leads to the formation of tau neutrinos. However, at the energy range considered for this work, the resulting tau neutrino flux is negligible as compared to the muon neutrino flux [13] and is not taken into account. It should be stated here that there is a rather large uncertainty on the normalization of the atmospheric neutrino flux on the order of 20-30 % [14] in the energy region of interest. This is mainly due to uncertainties in the primary cosmic ray spectrum and modeling of the hadronic interactions.

Oscillations of Atmospheric Neutrinos

[15]: Bilenky et al. (1978), *Lepton mixing and neutrino oscillations*

There are two ways to describe neutrino wave functions based on their Hamiltonian eigenvalues [15], as mass eigenstates or as flavor eigenstates. When applying a plane wave approach to explain the propagation of neutrinos in vacuum, their mass eigenstates evolve as

$$|v_k(t)\rangle = e^{-iE_k t/\hbar} |v_k\rangle, \quad (3.2)$$

where $E_k = \sqrt{\vec{p}^2 c^2 + m_k^2 c^4}$ is the energy of the mass eigenstate $|v_k\rangle$, with momentum \vec{p} and mass m_k . Alternatively, they can be described in terms of their flavor eigenstates, which relate the neutrinos to the charged leptons they interact with in weak CC interactions. The flavor eigenstates are ν_e , ν_μ , and ν_τ , whereas the mass eigenstates are called ν_1 , ν_2 , and ν_3 in the standard three-neutrino model. To understand the propagation of distinct neutrino flavors in time (in vacuum) we need to relate the flavor eigenstates to the mass eigenstates. For massive neutrinos, each flavor eigenstate is a superposition of mass eigenstates [9]

[9]: Tanabashi et al. (2018), *Review of Particle Physics*

$$|v_\alpha\rangle = \sum_k U_{\alpha k}^* |v_k\rangle, \quad (3.3)$$

where $|v_\alpha\rangle$ are the weak flavor states with $\alpha = e, \mu, \tau$ and $|v_k\rangle$ the mass states with $k = 1, 2, 3$. $U_{\alpha k}$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix defining the mixing between mass and flavor eigenstates. The mixing matrix can be parameterized as [9]

[9]: Tanabashi et al. (2018), *Review of Particle Physics*

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{diag}(e^{i\rho_1}, e^{i\rho_2}, 1), \quad (3.4)$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ are cosine and sine of the mixing angle θ_{ij} , that defines the strength of the mixing between the mass eigenstates i and j and δ_{CP} is the neutrino CP-violating phase. Nonzero, non-equal neutrino masses and the neutrino mixing relation in Equation Equation 3.3 lead to the observed phenomenon of neutrino oscillations. Oscillation means that a neutrino changes from its initial flavor to another flavor and back after traveling a certain distance. A produced flavor eigenstate $|v_\alpha\rangle$ propagates through space as a superposition of mass eigenstates. To find the probability that the initial flavor state $|v_\alpha\rangle$ ends up as the final flavor state $|v_\beta\rangle$ after the time t we calculate

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t) = |\langle v_\beta | v_\alpha(t) \rangle|^2, \quad (3.5)$$

where P is the probability calculated by applying Fermi's Golden Rule [16]. Fermi's Golden Rule explains the transition rate from one energy eigenstate to another depending on the strength of the coupling between the two. The strength of the coupling is described by the square of the matrix element. Using the unitarity of the mixing matrix $U^{-1} = U^\dagger$ to reverse the relation Equation 3.3 and then time evolve the mass eigenstates with Equation Equation 3.2 we get the time evolution of the flavor state $|\nu_\alpha(t)\rangle$. Inserting this result into Equation Equation 3.5 yields

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t) = \sum_{j,k} U_{\beta j}^* U_{\alpha j} U_{\beta k} U_{\alpha k}^* e^{-i(E_k - E_j)t/\hbar}, \quad (3.6)$$

where the indices j and k run over the mass eigenstates. For small neutrino masses compared to their kinetic energy, we can approximate the energy as

$$E_k \approx E + \frac{c^4 m_k^2}{2E} \longrightarrow E_k - E_j \approx \frac{c^4 \Delta m_{kj}^2}{2E}, \quad (3.7)$$

where $\Delta m_{kj}^2 = m_k^2 - m_j^2$ is the mass-squared splitting between states k and j . If we now replace the time in Equation Equation 3.6 by the distance traveled by the relativistic neutrinos $t \approx L/c$ we get

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta}(t) = & \delta_{\alpha\beta} - 4 \sum_{j>k} \text{Re}(U_{\beta j}^* U_{\alpha j} U_{\beta k} U_{\alpha k}^*) \sin^2\left(\frac{c^3 \Delta m_{kj}^2}{4E\hbar} L\right) \\ & + 2 \sum_{j>k} \text{Im}(U_{\beta j}^* U_{\alpha j} U_{\beta k} U_{\alpha k}^*) \sin^2\left(\frac{c^3 \Delta m_{kj}^2}{4E\hbar} L\right), \end{aligned} \quad (3.8)$$

which is referred to as the survival probability if $\alpha = \beta$ and the transition probability if $\alpha \neq \beta$. The probability in Equation Equation 3.8 is only nonzero if there are neutrino mass eigenstates with masses greater than zero. Additionally, there must be a mass-squared difference Δm^2 and nonzero mixing between the states. Since we assumed propagation in vacuum in Equation Equation 3.2, the transition and survival probabilities correspond to vacuum mixing.

Matter Effects

3.2 Heavy Neutral Leptons

3.2.1 Motivation for Heavy Sterile Neutrinos

3.2.2 Extending the Standard Model

3.2.3 Global Constraints on mixing

[16]: Dirac (1927), *The Quantum Theory of the Emission and Absorption of Radiation*

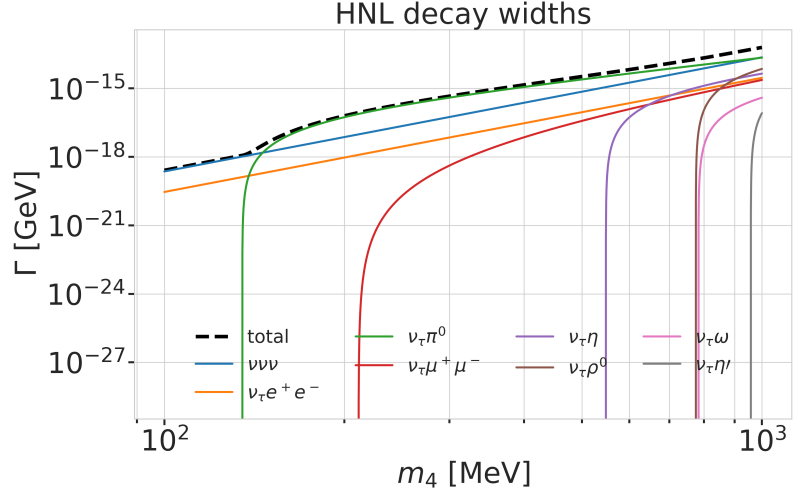


Figure 3.2: Decay widths of the HNL within the mass range considered, calculated based on the results from [17]. Given the existing constraints on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$, we consider that the corresponding decay modes are negligible.

3.3 Open Questions in Neutrino Particle Physics

The IceCube Neutrino Observatory

4

4.1 The IceCube In-Ice Array

4.1.1 In-Ice Array

DeepCore

4.1.2 IceTop

4.1.3 Digital Optical Modules

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Heavy Neutral Lepton Signal Simulation

5

5.1 Model Independent Simulation

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5.1.2 Simplistic Sets

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5.2 Model Specific Simulation

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5.2.1 Custom LeptonInjector

Signal events are simulated using a **custom LeptonInjector (LI) tool** [18], modified from its standard version to include the HNL particle and the description of the HNL decays needed to produce the double cascade signature (currently only ν_τ related). In its SM work mode, LI injects a lepton and a cascade (under the general name *Hadrons*) at the interaction vertex of the neutrino. Both objects have the same (x,y,z,t) coordinates. In the modified version, the lepton at the interaction vertex is replaced by the HNL. After a chosen distance the HNL is forced to decay. The decay is sampled from the kinematically accessible decay modes shown in Figure 5.2.

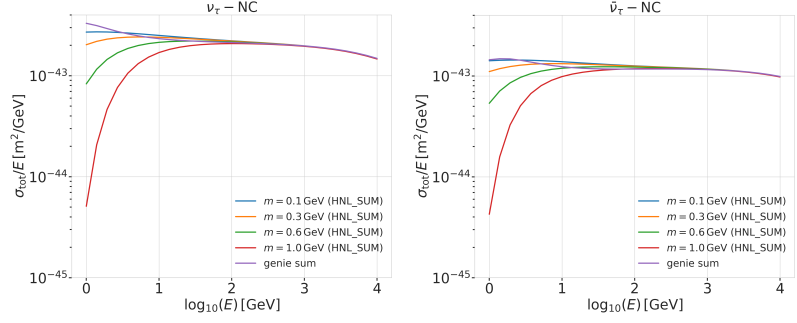
A big addition to the standard LI is that the decay products of the HNL are added to the list of particles in the I3MCTree with a displaced position and delayed time from the interaction vertex. These daughter particles form a second cascade, not in the form of a *Hadrons* object, but as the explicit particles forming the shower. The kinematics of the two-body decays are computed analytically, while the three-body decays are dealt with using MadGraph5. To do so, we randomly pick an event from a list that we generated for each three-body decay mode. Independent of the number of particles in the final state of the HNL decay, the kinematics are calculated/simulated at rest and then boosted along the HNL momentum. The decay mode is randomly chosen based on the mass dependent branching ratios shown in Figure 5.2.

Each file is produced by running the **generation level processing script** using the filename as random seed and the above settings for the sampling distributions. The main part is calling the *MultiLeptonInjector* module in *volume mode* adding two generators (for ν_τ and $\bar{\nu}_\tau$) with 50% of the events. The generators are provided with the custom double-differential/total cross section splines described in Section 5.2.1 and the parameters defining the sampling distributions. For each frame *OneWeight* and a reference weight are also calculated and stored using the **weighting functions** and a baseline atmospheric ν_τ flux + oscillation spline. The weight will later be calculated inside of the analysis framework

Re-write/re-formulate this section (copied from HNL technote).

[18]: Abbasi et al. (2021), *LeptonInjector and LeptonWeighter: A neutrino event generator and weighter for neutrino observatories*

Figure 5.1: Custom HNL total cross sections for the four target masses compared to the total ($\nu_\tau/\bar{\nu}_\tau$ neutral current) cross section used for SM neutrino simulation production with GENIE.



PISA, based on the input **OneWeight**. In addition to the **i3** file itself, a **LeptonInjector** configuration file is written which stores the needed information to produce event weights using **LeptonWeighter**. Optionally the script can also produce an **hdf5** file with the same name in the same location. This will store a fixed set of keys, extracted from the **i3** file.

We are using *volume mode*, for the injection of the primary particle on a cylindrical volume. The main generation/sampling happens in `VolumeLeptonInjector::DAQ` inside

`LeptonInjector.cxx`. After writing the config (s) frame (currently not kept), the energy is sampled from a power law distribution, then the cosine(zenith) and azimuth angles are sampled from uniform distributions. The (x,y) position is sampled uniform in r, ϕ (for position on disk) and the z position is sampled from a uniform distribution. After the primary properties have been sampled the *EventProperties* is created and handed over to the **FillTree** functions which is where the custom HNL simulation happens:

Cross Sections

The cross sections are calculated using a **modified version** of Carlos Argüelles' **NuXSplMkr**, which is a tool to calculate neutrino cross sections from parton distribution functions (PDFs) and then produce splines that can be read and used with IceCube software. The main modification to calculate the cross sections for the ν_τ neutral current interaction into the new heavy mass state is the addition of a kinematic condition to ensure that there is sufficient energy to produce the heavy mass state. It is the same condition that needs to be fulfilled for the charged current case, where the outgoing lepton mass is non-zero. Following [19] (equation 7), the condition

$$(1 + x\delta_N)h^2 - (x + \delta_4)h + x\delta_4 \leq 0, \quad (5.1)$$

is implemented for the neutral current case. Here $\delta_4 = \frac{m_4^2}{s-M^2}$, $\delta_N = \frac{M^2}{s-M^2}$, and $h \stackrel{\text{def}}{=} xy + \delta_4$, with x, y being the Bjorken variables, m_4 and M the mass of the heavy state and the target nucleon, respectively, and s the center of mass energy squared. Since the (SM) neutrino background simulation used for this analysis was created using GENIE (version 2.12.8), interfaced through the IceCube software package *genie-icetray*, with the **GRV98LO** PDFs, those were added as *GRV98lo_patched* to the cross section spline maker, to ensure the best possible agreement. Double-differential ($dsdx dy$) and total (σ) cross sections were produced for the

[19]: Levy (2009), *Cross-section and polarization of neutrino-produced tau's made simple*

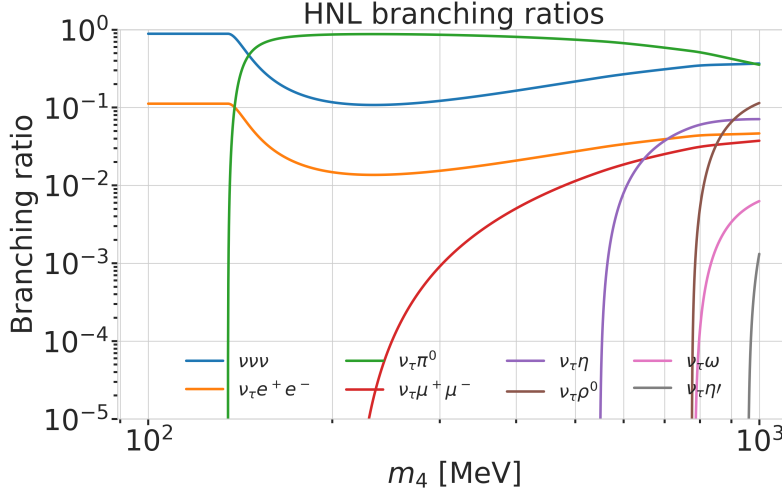


Figure 5.2: Branching ratios of the HNL within the mass range considered, calculated based on the results from [17]. Given the existing constraints on $|U_{e4}|^2$ and $|U_{\mu4}|^2$, we consider that the corresponding decay modes are negligible.

Channel	Opens [MeV]	Max BR [%]
$\nu_4 \rightarrow \nu_\tau \nu_\alpha \bar{\nu}_\alpha$	0	100.0
$\nu_4 \rightarrow \nu_\tau e^+ e^-$	1	?
$\nu_4 \rightarrow \nu_\tau \pi^0$	135	?
$\nu_4 \rightarrow \nu_\tau \mu^+ \mu^-$	211	?
$\nu_4 \rightarrow \nu_\tau \eta$	548	?
$\nu_4 \rightarrow \nu_\tau \rho^0$	770	?
$\nu_4 \rightarrow \nu_\tau \omega$	783	?
$\nu_4 \rightarrow \nu_\tau \eta'$	958	?

Table 5.1: xx

four target HNL masses and then splined. The produced cross section splines are stored in the resources of the custom **LeptonInjector** module. Figure 5.1 shows the total cross sections that were produced compared to the cross section used for the production of the SM $\nu_\tau/\bar{\nu}_\tau$ neutral current background simulation .

Add comparisons of SM cross sections between NuXSS-plMkr and genie

Decay Channels

The accessible decay channels are dependent on the mass of the HNL and the allowed mixing. For this analysis, where only $|U_{\tau4}|^2 \neq 0$, the considered decay channels are listed in Table 5.1 and the corresponding branching ratios are shown in Figure 5.2. The individual branching ratio for a specific mass is calculated as $BR_i(m_4) = \Gamma_i(m_4)/\Gamma_{\text{total}}(m_4)$, where $\Gamma_{\text{total}}(m_4) = \sum \Gamma_i(m_4)$. The formulas to calculate the decay width show up in multiple references, but we chose to match them to [17], which also discusses the discrepancies in previous literature.

[17]: Coloma et al. (2021), *GeV-scale neutrinos: interactions with mesons and DUNE sensitivity*

2-Body Decay Widths The decay to a neutral pseudoscalar mesons is

$$\Gamma_{\nu_4 \rightarrow \nu_\tau P} = |U_{\tau4}|^2 \frac{G_F^2 m_4^3}{32\pi} f_P^2 (1 - x_P^2)^2, \quad (5.2)$$

with $x_P = m_P/m_4$ and

$$f_{\pi^0} = 0.130 \text{ GeV}, \quad f_\eta = 0.0816 \text{ GeV}, \quad C_2 = f_{\eta'} = -0.0946 \text{ GeV}, \quad (5.3)$$

while the decay to a neutral vector meson is given by

$$\Gamma_{\nu_4 \rightarrow \nu_\tau V} = |U_{\tau 4}|^2 \frac{G_F^2 m_4^3}{32\pi} \left(\frac{f_V}{m_V} \right)^2 g_V^2 (1 + 2x_V^2)(1 - x_V^2)^2, \quad (5.4)$$

with $x_V = m_V/m_4$,

$$f_{\rho^0} = 0.171 \text{ GeV}^2, \quad f_\omega = 0.155 \text{ GeV}^2, \quad (5.5)$$

and

$$g_{\rho^0} = 1 - 2 \sin^2 \theta_w, \quad g_\omega = \frac{-2 \sin^2 \theta_w}{3}, \quad \sin^2 \theta_w = 0.2229 \quad (5.6)$$

[20]: Tiesinga et al. (2021), CODATA recommended values of the fundamental physical constants: 2018

[20].

3-Body Decay Widths The (invisible) decay to three neutrinos is

$$\Gamma_{\nu_4 \rightarrow \nu_\tau \nu_\alpha \bar{\nu}_\alpha} = |U_{\tau 4}|^2 \frac{G_F^2 m_4^5}{192\pi^3}, \quad (5.7)$$

while the decay to two charged leptons (using $x_\alpha = (m_\alpha/m_4)^2$) of the same flavor reads

$$\Gamma_{\nu_4 \rightarrow \nu_\tau l_\alpha^+ l_\alpha^-} = |U_{\tau 4}|^2 \frac{G_F^2 m_4^5}{192\pi^3} [C_1 f_1(x_\alpha) + C_2 f_2(x_\alpha)], \quad (5.8)$$

with the constants defined as

$$C_1 = \frac{1}{4}(1 - 4s_w^2 + 8s_w^4), \quad C_2 = \frac{1}{2}(-s_w^2 + 2s_w^4), \quad (5.9)$$

the functions as

$$f_1(x_\alpha) = (1 - 14x_\alpha - 2x_\alpha^2 - 12x_\alpha^3)\sqrt{1 - 4x_\alpha} + 12x_\alpha^2(x_\alpha^2 - 1)L(x_\alpha), \quad (5.10)$$

$$f_2(x_\alpha) = 4[x_\alpha(2 + 10x_\alpha - 12x_\alpha^2)\sqrt{1 - 4x_\alpha} + 6x_\alpha^2(1 - 2x_\alpha + 2x_\alpha^2)L(x_\alpha)], \quad (5.11)$$

and

$$L(x) = \ln \left(\frac{1 - 3x_\alpha - (1 - x_\alpha)\sqrt{1 - 4x_\alpha}}{x_\alpha(1 + \sqrt{1 - 4x_\alpha})} \right). \quad (5.12)$$

Add description of Mad-Graph5 decay files (Harvard needs to provide this)

Madgraph 3-body Decay Kinematics

5.2.2 Sampling Distributions

This is the description of the signal simulation generator used to (re-)start simulation production in December 2023. The underlying sampling distributions are listed in Table 5.2. Judging from how the generation/processing efficiency was for the 190607 set, we target 1e04 files per set with 5e05 events per file at generation, resulting in a maximum of 5e09 events per set at generation level. Note here that the actual number of events

variable	distribution	range
energy	E^{-2}	$[2, 10^4]$ GeV
zenith	uniform (in $\cos(\theta)$)	$[180^\circ, 80^\circ]$
azimuth	uniform	$[0^\circ, 360^\circ]$
vertex (x, y)	uniform	$r = 600$ m
vertex z	uniform	$[-600, 0]$ m
m_{HNL}	fixed	$[0.3, 0.6, 1.0]$ GeV
L_{decay}	L^{-1}	$[0.0004, 1000.0]$ m / $[1.0, 1000.0]$ m

Table 5.2: Sampling distributions of HNL simulation generation.

per set at generation might be a little lower since some events won't be allowed if they don't have enough energy to produce the HNL.

5.2.3 Weighting Scheme

The weighting for the HNL signal simulation happens in a **custom stage of PISA**. The only input is the stored OneWeight and the variable physics parameter $|U_{\tau 4}|^2$, which is the mixing strength of the new heavy mass state and the tau sector. The custom re-weighting is needed to go from the used sampling PDF (1/L with fixed range in lab frame decay length) to the target PDF (exponential defined by proper lifetime of the HNL). For each event the re-weighting factor is calculated using the gamma factor

$$\gamma = \frac{\sqrt{E_{\text{kin}}^2 + m_{\text{HNL}}^2}}{m_{\text{HNL}}}, \quad (5.13)$$

with the HNL mass m_{HNL} and it's kinetic energy E_{kin} . The speed of the HNL is calculated as

$$v = c \cdot \sqrt{1 - \frac{1}{\gamma^2}}, \quad (5.14)$$

where c is the speed of light. With these the lab frame decay length range can be converted into the rest frame lifetime range for each event

$$\tau_{\text{min/max}} = \frac{s_{\text{min/max}}}{v \cdot \gamma}. \quad (5.15)$$

The proper lifetime of each HNL event can be calculated using the total decay width Γ_{total} shown in Figure 3.2 and the chosen mixing strength $|U_{\tau 4}|^2$ as

$$\tau_{\text{proper}} = \frac{\hbar}{\Gamma_{\text{total}}(m_{\text{HNL}}) \cdot |U_{\tau 4}|^2}, \quad (5.16)$$

where \hbar is the reduced Planck constant. Since the decay length/lifetime of the events is sampled from an inverse distribution instead of an exponential as it would be expected from a particle decay we have to re-weight accordingly to achieve the correct decay length/lifetime distribution. This is done by using the wanted exponential distribution

$$\text{PDF}_{\text{exp}} = \frac{1}{\tau_{\text{proper}}} \cdot e^{-\frac{\tau}{\tau_{\text{proper}}}}, \quad (5.17)$$

and the inverse distribution that was sampled from

$$\text{PDF}_{\text{inv}} = \frac{1}{\tau \cdot (\ln(\tau_{\text{max}}) - \ln(\tau_{\text{min}}))}. \quad (5.18)$$

The lifetime re-weighting factor is calculated as

$$w_{\text{lifetime}} = \frac{\text{PDF}_{\text{exp}}}{\text{PDF}_{\text{inv}}} = \frac{\Gamma_{\text{total}}(m_{\text{HNL}}) \cdot |U_{\tau 4}|^2}{\hbar} \cdot \tau \cdot (\ln(\tau_{\text{max}}) - \ln(\tau_{\text{min}})) \cdot e^{\frac{-\tau}{\tau_{\text{proper}}}}. \quad (5.19)$$

Adding another factor of $|U_{\tau 4}|^2$ to account for the mixing at the interaction vertex the total re-weighting factor becomes

$$w_{\text{total}} = |U_{\tau 4}|^2 \cdot w_{\text{lifetime}}, \quad (5.20)$$

which can be applied on top of flux and oscillation weight to get the final HNL weight for a given mixing (and mass).

Standard Model Background Simulation and Data Processing

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6.1.2 Muons

6.2 Detector Simulation

6.2.1 Photon Propagation

6.2.2 Detector Responses

6.3 Processing

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6.3.2 Offline Filter

6.3.3 FLERCNN Event Reconstruction and Classification

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7.1.2 Free Parameter Selection

7.1.3 Signal and Background in Analysis Binning

7.2 Modelling of Detector Response via Likelihood-Free Inference

7.3 Analysis Checks

7.3.1 Minimzer Stability

7.3.2 Ensemble Tests

7.3.3 Background Only Three-Flavor Oscillation Measurement

7.3.4 Heavy Neutral Lepton Sensitivity

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Summary and Outlook

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8.1 Summary

8.1.1 Three-Flavor Oscillation Measurement

8.1.2 Heavy Neutral Lepton Search

Double Cascade Sensitivity

Shape Excess Search

8.2 Outlook

8.2.1 Shape Analysis Improvements

- ▶ estimate full contribution from cascade only events (underestimated due to limited sampling distributions)
- ▶ include double cascade classifier into Binning
- ▶ further optimize binning

8.2.2 Test Coupling to Electron/Muon Flavor

8.2.3 IceCube Upgrade

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A

First Appendix

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