# Project 3 MDP

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**GitHub:** https://github.com/LeanderLXZ/mdp

## 1. Files description

```
mdp/
├── doc.docx                      // document file
├── README.md                     // readme file
├── input/                        // input folder
│   ├── i1.txt                    // TXT files of inputs of different boards
│   │     ⋮
│   └── i8.txt
├── results/                      // results folder
│   ├── results_1.csv             // CSV files of results
│   ├── results_2.csv
│   └── results_3.csv
└── src/                          // srouce code folder
    ├── value_iteration.py        // value iteration
    ├── policy_iteration.py       // policy iteration
    ├── policy_iteration_linear.py // policy iteration based on linear equations solving
    ├── notebook.ipynb            // jupyter notebook for experiments
    └── utils.py                  // utilities
```

## 2. Arguments for algorithms

| | |
|---|---|
| board_file_path | string, the path of input file |
| threshold | float, default=0.01, the threshold for stop the iteration |
| init_policy_direction | int, default=None, the index of the chosen direction in ['up', 'right', 'down', 'left'] for initializing the policy. If None, randomly assign directions to the initial policy. |
| improve_p_with_v | Boolean, default=Flase, set True to improve the values while improving the policy |
| use_arrow | Boolean, default=False, set True to use arrows for display |
| verbose | Boolean, default=False, set True to display extra information |

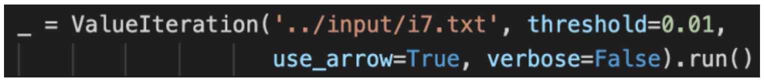
## 3. Run the code

### 1) Run value iteration

Command:

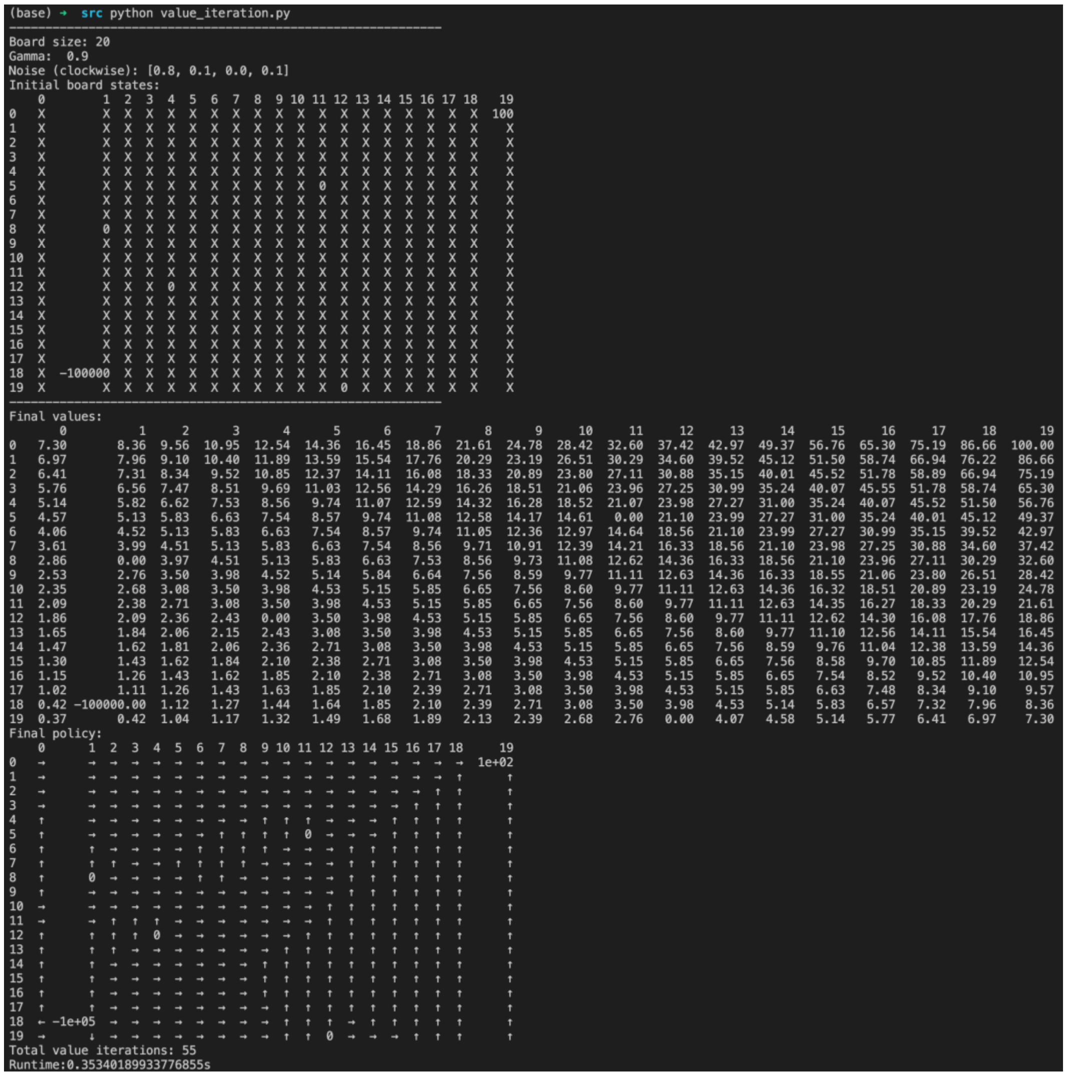
```
python value_iteration.py
```

You can set the configuration of the algorithm ` value_iteration.py `:



```python
_ = ValueIteration('../input/i7.txt', threshold=0.01,
                   use_arrow=True, verbose=False).run()
```

Output example:



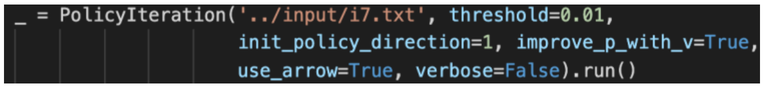
```
(base) ➜  src python value_iteration.py
------------------------------------------------------------
Board size: 20
Gamma:  0.9
Noise (clockwise): [0.8, 0.1, 0.0, 0.1]
Initial board states:
    0       1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18    19
0   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X   100
1   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
2   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
3   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
4   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
5   X       X  X  X  X  X  X  X  X  X  X  0  X  X  X  X  X  X  X     X
6   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
7   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
8   X       0  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
9   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
10  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
11  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
12  X       X  X  X  0  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
13  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
14  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
15  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
16  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
17  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
18  X -100000  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X     X
19  X       X  X  X  X  X  X  X  X  X  X  X  0  X  X  X  X  X  X     X
------------------------------------------------------------
Final values:
      0           1    2     3      4      5      6      7      8      9     10     11     12     13     14     15     16     17     18     19
0   7.30       8.36 9.56 10.95  12.54  14.36  16.45  18.86  21.61  24.78  28.42  32.60  37.42  42.97  49.37  56.76  65.30  75.19  86.66 100.00
1   6.97       7.96 9.10 10.40  11.89  13.59  15.54  17.76  20.29  23.19  26.51  30.29  34.60  39.52  45.12  51.50  58.74  66.94  76.22  86.66
2   6.41       7.31 8.34  9.52  10.85  12.37  14.11  16.08  18.33  20.89  23.80  27.11  30.88  35.15  40.01  45.52  51.78  58.89  66.94  75.19
3   5.76       6.56 7.47  8.51   9.69  11.03  12.56  14.29  16.26  18.51  21.06  23.96  27.25  30.99  35.24  40.07  45.55  51.78  58.74  65.30
4   5.14       5.82 6.62  7.53   8.56   9.74  11.07  12.59  14.32  16.28  18.52  21.07  23.98  27.27  31.00  35.24  40.07  45.52  51.50  56.76
5   4.57       5.13 5.83  6.63   7.54   8.57   9.74  11.08  12.58  14.17  14.61   0.00  21.10  23.99  27.27  31.00  35.24  40.01  45.12  49.37
6   4.06       4.52 5.13  5.83   6.63   7.54   8.57   9.74  11.05  12.36  12.97  14.64  18.56  21.10  23.99  27.27  30.99  35.15  39.52  42.97
7   3.61       3.99 4.51  5.13   5.83   6.63   7.54   8.56   9.71  10.91  12.39  14.21  16.33  18.56  21.10  23.98  27.25  30.88  34.60  37.42
8   2.86       0.00 3.97  4.51   5.13   5.83   6.63   7.53   8.56   9.73  11.08  12.62  14.36  16.33  18.56  21.10  23.96  27.11  30.29  32.60
9   2.53       2.76 3.50  3.98   4.52   5.14   5.84   6.64   7.56   8.59   9.77  11.11  12.63  14.36  16.33  18.55  21.06  23.80  26.51  28.42
10  2.35       2.68 3.08  3.50   3.98   4.53   5.15   5.85   6.65   7.56   8.60   9.77  11.11  12.63  14.36  16.32  18.51  20.89  23.19  24.78
11  2.09       2.38 2.71  3.08   3.50   3.98   4.53   5.15   5.85   6.65   7.56   8.60   9.77  11.11  12.63  14.35  16.27  18.33  20.29  21.61
12  1.86       2.09 2.36  2.43   0.00   3.50   3.98   4.53   5.15   5.85   6.65   7.56   8.60   9.77  11.11  12.62  14.30  16.08  17.76  18.86
13  1.65       1.84 2.06  2.15   2.43   3.08   3.50   3.98   4.53   5.15   5.85   6.65   7.56   8.60   9.77  11.10  12.56  14.11  15.54  16.45
14  1.47       1.62 1.81  2.06   2.36   2.71   3.08   3.50   3.98   4.53   5.15   5.85   6.65   7.56   8.59   9.76  11.04  12.38  13.59  14.36
15  1.30       1.43 1.62  1.84   2.10   2.38   2.71   3.08   3.50   3.98   4.53   5.15   5.85   6.65   7.56   8.58   9.70  10.85  11.89  12.54
16  1.15       1.26 1.43  1.62   1.85   2.10   2.38   2.71   3.08   3.50   3.98   4.53   5.15   5.85   6.65   7.54   8.52   9.52  10.40  10.95
17  1.02       1.11 1.26  1.43   1.63   1.85   2.10   2.39   2.71   3.08   3.50   3.98   4.53   5.15   5.85   6.63   7.48   8.34   9.10   9.57
18  0.42 -100000.00 1.12  1.27   1.44   1.64   1.85   2.10   2.39   2.71   3.08   3.50   3.98   4.53   5.14   5.83   6.57   7.32   7.96   8.36
19  0.37       0.42 1.04  1.17   1.32   1.49   1.68   1.89   2.13   2.39   2.68   2.76   0.00   4.07   4.58   5.14   5.77   6.41   6.97   7.30
Final policy:
    0       1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18    19
0   →       →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  → 1e+02
1   →       →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  ↑     ↑
2   →       →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  ↑  ↑     ↑
3   →       →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  ↑  ↑  ↑     ↑
4   ↑       →  →  →  →  →  →  →  →  ↑  ↑  ↑  →  →  →  ↑  ↑  ↑  ↑     ↑
5   ↑       →  →  →  →  →  →  ↑  ↑  ↑  ↑  0  →  →  →  ↑  ↑  ↑  ↑     ↑
6   ↑       ↑  →  →  →  →  ↑  ↑  ↑  ↑  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑     ↑
7   ↑       ↑  ↑  →  →  ↑  ↑  ↑  ↑  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑     ↑
8   ↑       0  →  →  →  →  ↑  ↑  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑     ↑
9   ↑       →  →  →  →  →  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑     ↑
10  →       →  →  →  →  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑     ↑
11  →       →  ↑  ↑  ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑     ↑
12  ↑       ↑  ↑  ↑  0  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑     ↑
13  ↑       ↑  ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑     ↑
14  ↑       ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑     ↑
15  ↑       ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑     ↑
16  ↑       ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑     ↑
17  ↑       ↑  →  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑     ↑
18  ←  -1e+05  →  →  →  →  →  →  →  →  ↑  ↑  ↑  →  ↑  ↑  ↑  ↑  ↑     ↑
19  →       ↓  →  →  →  →  →  →  →  →  ↑  ↑  0  →  →  →  ↑  ↑  ↑     ↑
Total value iterations: 55
Runtime:0.35340189933776855s
```

## 2) Run policy iteration

Command:

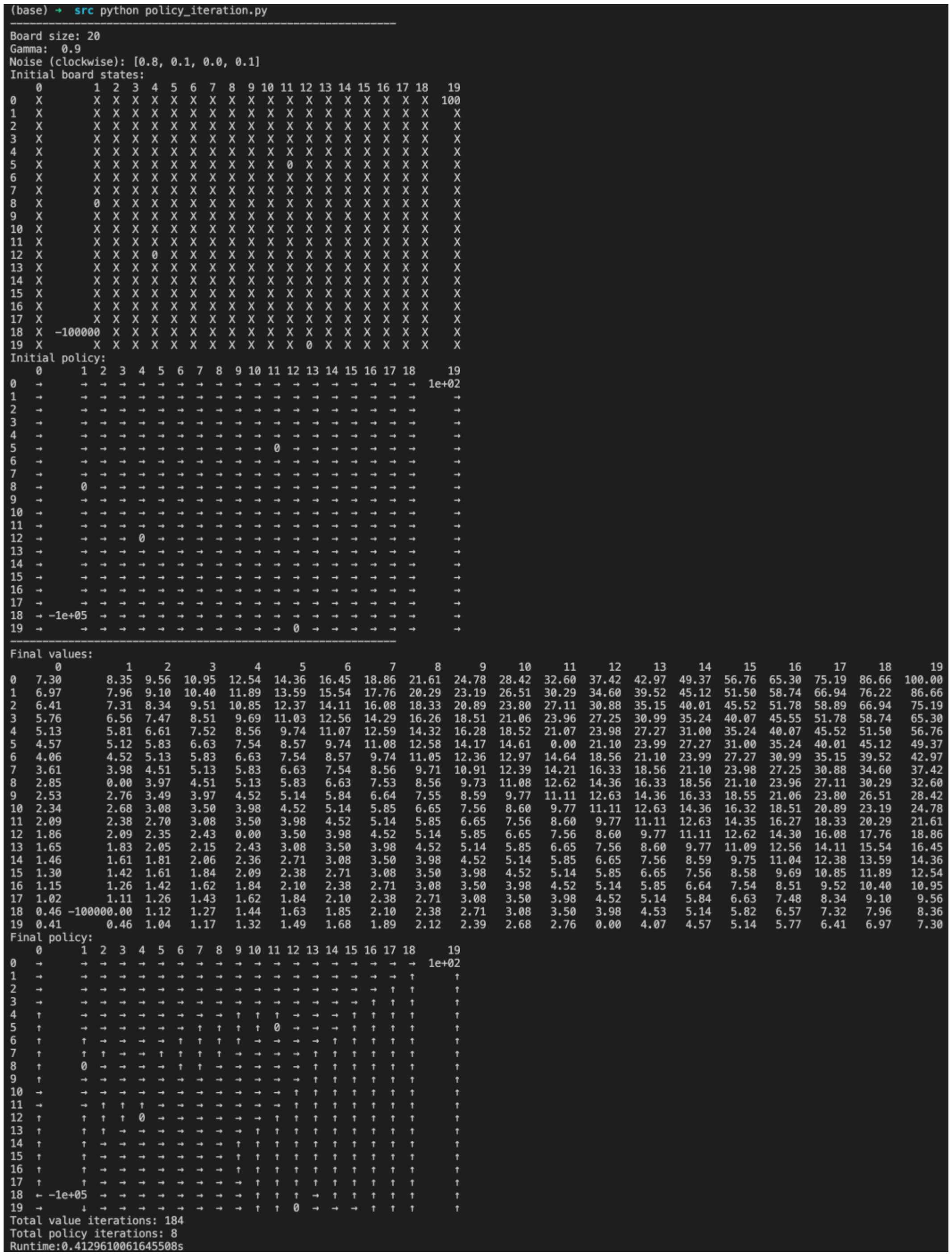
python policy_iteration.py

You can set the configuration of the algorithm in ` policy_iteration.py `:



```python
_ = PolicyIteration('../input/i7.txt', threshold=0.01,
                    init_policy_direction=1, improve_p_with_v=True,
                    use_arrow=True, verbose=False).run()
```

Output example:

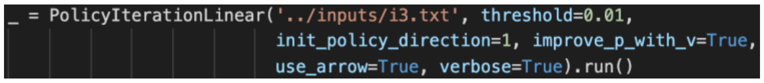


```
(base) ➜  src python policy_iteration.py
----------------------------------------------------------------
Board size: 20
Gamma:  0.9
Noise (clockwise): [0.8, 0.1, 0.0, 0.1]
Initial board states:
    0       1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18   19
0   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  100
1   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
2   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
3   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
4   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
5   X       X  X  X  X  X  X  X  X  X  X  0  X  X  X  X  X  X  X    X
6   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
7   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
8   X       0  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
9   X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
10  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
11  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
12  X       X  X  X  0  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
13  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
14  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
15  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
16  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
17  X       X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
18  X -100000  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X  X    X
19  X       X  X  X  X  X  X  X  X  X  X  X  0  X  X  X  X  X  X    X
Initial policy:
    0      1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18      19
0   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →   1e+02
1   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
2   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
3   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
4   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
5   →      →  →  →  →  →  →  →  →  →  →  0  →  →  →  →  →  →  →       →
6   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
7   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
8   →      0  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
9   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
10  →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
11  →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
12  →      →  →  →  0  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
13  →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
14  →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
15  →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
16  →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
17  →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
18  → -1e+05  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →       →
19  →      →  →  →  →  →  →  →  →  →  →  →  0  →  →  →  →  →  →       →
----------------------------------------------------------------
Final values:
       0          1     2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17     18     19
0   7.30       8.35  9.56  10.95  12.54  14.36  16.45  18.86  21.61  24.78  28.42  32.60  37.42  42.97  49.37  56.76  65.30  75.19  86.66 100.00
1   6.97       7.96  9.10  10.40  11.89  13.59  15.54  17.76  20.29  23.19  26.51  30.29  34.60  39.52  45.12  51.50  58.74  66.94  76.22  86.66
2   6.41       7.31  8.34   9.51  10.85  12.37  14.11  16.08  18.33  20.89  23.80  27.11  30.88  35.15  40.01  45.52  51.78  58.89  66.94  75.19
3   5.76       6.56  7.47   8.51   9.69  11.03  12.56  14.29  16.26  18.51  21.06  23.96  27.25  30.99  35.24  40.07  45.55  51.78  58.74  65.30
4   5.13       5.81  6.61   7.52   8.56   9.74  11.07  12.59  14.32  16.28  18.52  21.07  23.98  27.27  31.00  35.24  40.07  45.52  51.50  56.76
5   4.57       5.12  5.83   6.63   7.54   8.57   9.74  11.08  12.58  14.17  14.61   0.00  21.10  23.99  27.27  31.00  35.24  40.01  45.12  49.37
6   4.06       4.52  5.13   5.83   6.63   7.54   8.57   9.74  11.05  12.36  12.97  14.64  18.56  21.10  23.99  27.27  30.99  35.15  39.52  42.97
7   3.61       3.98  4.51   5.13   5.83   6.63   7.54   8.56   9.71  10.91  12.39  14.21  16.33  18.56  21.10  23.98  27.25  30.88  34.60  37.42
8   2.85       0.00  3.97   4.51   5.13   5.83   6.63   7.53   8.56   9.73  11.08  12.62  14.36  16.33  18.56  21.10  23.96  27.11  30.29  32.60
9   2.53       2.76  3.49   3.97   4.52   5.14   5.84   6.64   7.55   8.59   9.77  11.11  12.63  14.36  16.33  18.55  21.06  23.80  26.51  28.42
10  2.34       2.68  3.08   3.50   3.98   4.52   5.14   5.85   6.65   7.56   8.60   9.77  11.11  12.63  14.36  16.32  18.51  20.89  23.19  24.78
11  2.09       2.38  2.70   3.08   3.50   3.98   4.52   5.14   5.85   6.65   7.56   8.60   9.77  11.11  12.63  14.35  16.27  18.33  20.29  21.61
12  1.86       2.09  2.35   2.43   0.00   3.50   3.98   4.52   5.14   5.85   6.65   7.56   8.60   9.77  11.11  12.62  14.30  16.08  17.76  18.86
13  1.65       1.83  2.05   2.15   2.43   3.08   3.50   3.98   4.52   5.14   5.85   6.65   7.56   8.60   9.77  11.09  12.56  14.11  15.54  16.45
14  1.46       1.61  1.81   2.06   2.36   2.71   3.08   3.50   3.98   4.52   5.14   5.85   6.65   7.56   8.59   9.75  11.04  12.38  13.59  14.36
15  1.30       1.42  1.61   1.84   2.09   2.38   2.71   3.08   3.50   3.98   4.52   5.14   5.85   6.65   7.56   8.58   9.69  10.85  11.89  12.54
16  1.15       1.26  1.42   1.62   1.84   2.10   2.38   2.71   3.08   3.50   3.98   4.52   5.14   5.85   6.64   7.54   8.51   9.52  10.40  10.95
17  1.02       1.11  1.26   1.43   1.62   1.84   2.10   2.38   2.71   3.08   3.50   3.98   4.52   5.14   5.84   6.63   7.48   8.34   9.10   9.56
18  0.46 -100000.00  1.12   1.27   1.44   1.63   1.85   2.10   2.38   2.71   3.08   3.50   3.98   4.53   5.14   5.82   6.57   7.32   7.96   8.36
19  0.41       0.46  1.04   1.17   1.32   1.49   1.68   1.89   2.12   2.39   2.68   2.76   0.00   4.07   4.57   5.14   5.77   6.41   6.97   7.30
Final policy:
    0      1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18      19
0   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →   1e+02
1   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  ↑       ↑
2   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  ↑  ↑       ↑
3   →      →  →  →  →  →  →  →  →  →  →  →  →  →  →  →  ↑  ↑  ↑       ↑
4   ↑      →  →  →  →  →  →  →  →  ↑  ↑  ↑  →  →  →  ↑  ↑  ↑  ↑       ↑
5   ↑      →  →  →  →  →  →  ↑  ↑  ↑  ↑  0  →  →  →  ↑  ↑  ↑  ↑       ↑
6   ↑      ↑  →  →  →  →  ↑  ↑  ↑  ↑  →  →  →  →  ↑  ↑  ↑  ↑  ↑       ↑
7   ↑      ↑  ↑  →  →  ↑  ↑  ↑  ↑  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑       ↑
8   ↑      0  →  →  →  →  ↑  ↑  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑       ↑
9   ↑      →  →  →  →  →  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑       ↑
10  →      →  →  →  →  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑       ↑
11  →      →  ↑  ↑  ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑       ↑
12  ↑      ↑  ↑  ↑  0  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑       ↑
13  ↑      ↑  ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑       ↑
14  ↑      ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑       ↑
15  ↑      ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑       ↑
16  ↑      ↑  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑       ↑
17  ↑      ↑  →  →  →  →  →  →  →  →  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑  ↑       ↑
18  ← -1e+05  →  →  →  →  →  →  →  →  ↑  ↑  ↑  →  ↑  ↑  ↑  ↑  ↑       ↑
19  →      ↓  →  →  →  →  →  →  →  →  ↑  ↑  0  →  →  →  ↑  ↑  ↑       ↑
Total value iterations: 184
Total policy iterations: 8
Runtime:0.4129610061645508s
```

**3) Run policy iteration based on linear equations solving**
Command:

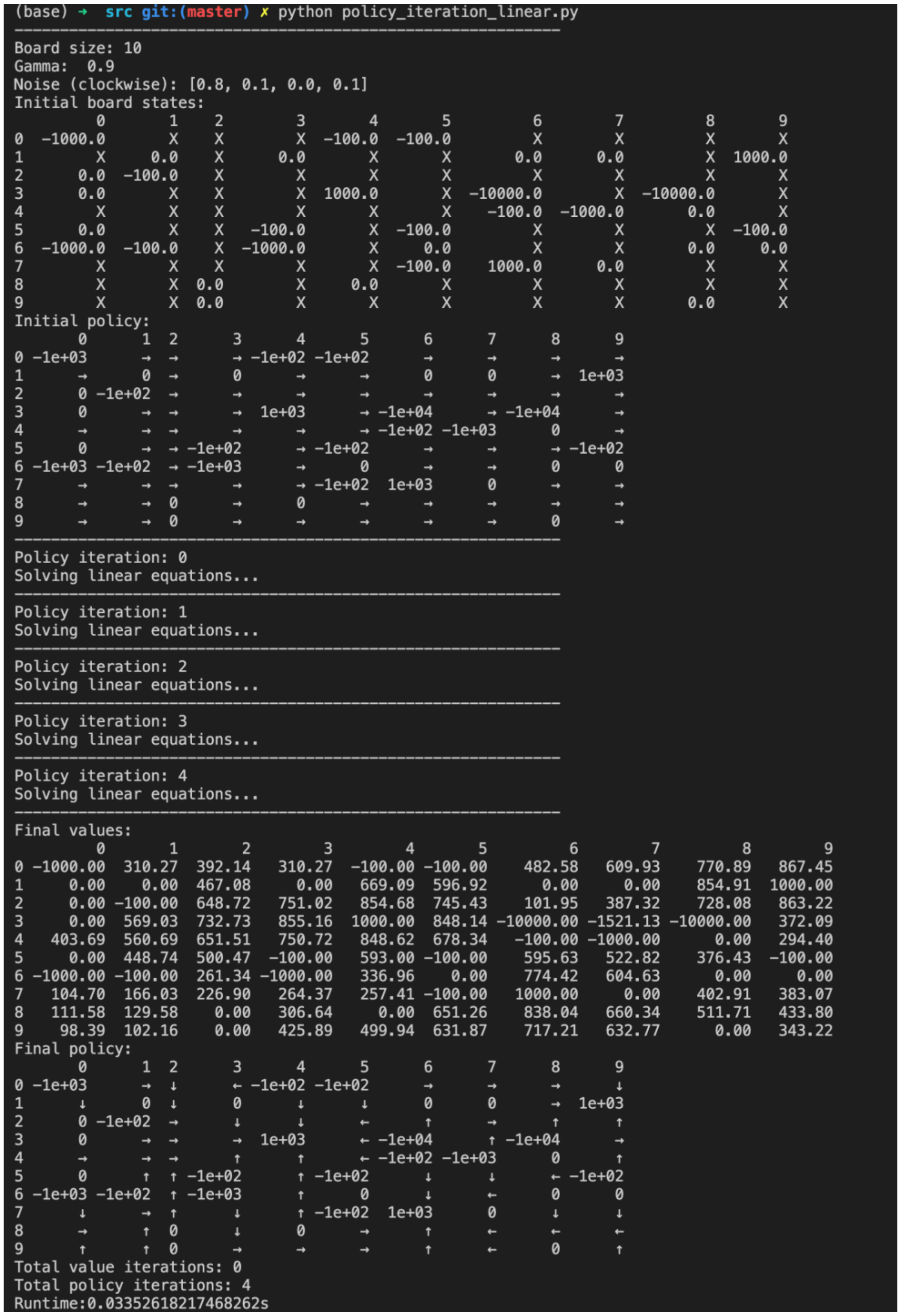
```
python policy_iteration_linear.py
```

You can set the configuration of the algorithm in ` policy_iteration.py `:



```python
_ = PolicyIterationLinear('../inputs/i3.txt', threshold=0.01,
                          init_policy_direction=1, improve_p_with_v=True,
                          use_arrow=True, verbose=True).run()
```

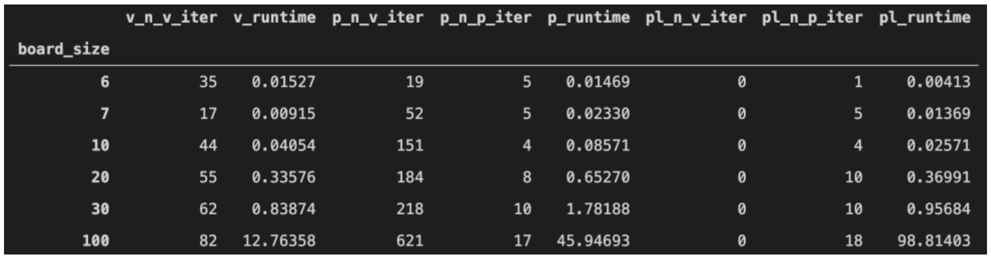
Output example:



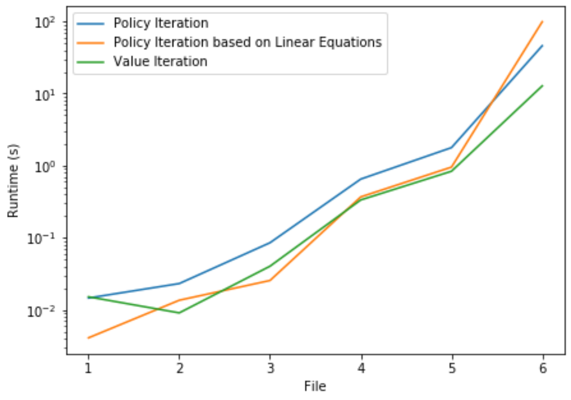
```
(base) ➜  src git:(master) ✗ python policy_iteration_linear.py
------------------------------------------------------------
Board size: 10
Gamma:  0.9
Noise (clockwise): [0.8, 0.1, 0.0, 0.1]
Initial board states:
         0       1    2        3       4       5         6        7         8       9
0  -1000.0       X    X        X  -100.0  -100.0         X        X         X       X
1        X     0.0    X      0.0       X       X       0.0      0.0         X  1000.0
2      0.0  -100.0    X        X       X       X         X        X         X       X
3      0.0       X    X        X  1000.0       X  -10000.0        X  -10000.0       X
4        X       X    X        X       X       X    -100.0  -1000.0       0.0       X
5      0.0       X    X   -100.0       X  -100.0         X        X         X  -100.0
6  -1000.0  -100.0    X  -1000.0       X     0.0         X        X       0.0     0.0
7        X       X    X        X       X  -100.0    1000.0      0.0         X       X
8        X       X  0.0        X     0.0       X         X        X         X       X
9        X       X  0.0        X       X       X         X        X       0.0       X
Initial policy:
       0      1  2      3      4      5      6      7      8      9
0 -1e+03      →  →      → -1e+02 -1e+02      →      →      →      →
1      →      0  →      0      →      →      0      0      →  1e+03
2      0 -1e+02  →      →      →      →      →      →      →      →
3      0      →  →      →  1e+03      → -1e+04      → -1e+04      →
4      →      →  →      →      →      → -1e+02 -1e+03      0      →
5      0      →  → -1e+02      → -1e+02      →      →      → -1e+02
6 -1e+03 -1e+02  → -1e+03      →      0      →      →      0      0
7      →      →  →      →      → -1e+02  1e+03      0      →      →
8      →      →  0      →      0      →      →      →      →      →
9      →      →  0      →      →      →      →      →      0      →
------------------------------------------------------------
Policy iteration: 0
Solving linear equations...
------------------------------------------------------------
Policy iteration: 1
Solving linear equations...
------------------------------------------------------------
Policy iteration: 2
Solving linear equations...
------------------------------------------------------------
Policy iteration: 3
Solving linear equations...
------------------------------------------------------------
Policy iteration: 4
Solving linear equations...
------------------------------------------------------------
Final values:
         0       1       2        3        4       5         6        7         8        9
0 -1000.00  310.27  392.14   310.27  -100.00 -100.00    482.58   609.93    770.89   867.45
1     0.00    0.00  467.08     0.00   669.09  596.92      0.00     0.00    854.91  1000.00
2     0.00 -100.00  648.72   751.02   854.68  745.43    101.95   387.32    728.08   863.22
3     0.00  569.03  732.73   855.16  1000.00  848.14 -10000.00 -1521.13 -10000.00   372.09
4   403.69  560.69  651.51   750.72   848.62  678.34   -100.00 -1000.00      0.00   294.40
5     0.00  448.74  500.47  -100.00   593.00 -100.00    595.63   522.82    376.43  -100.00
6 -1000.00 -100.00  261.34 -1000.00   336.96    0.00    774.42   604.63      0.00     0.00
7   104.70  166.03  226.90   264.37   257.41 -100.00   1000.00     0.00    402.91   383.07
8   111.58  129.58    0.00   306.64     0.00  651.26    838.04   660.34    511.71   433.80
9    98.39  102.16    0.00   425.89   499.94  631.87    717.21   632.77      0.00   343.22
Final policy:
       0      1  2      3      4      5      6      7      8      9
0 -1e+03      →  ↓      ← -1e+02 -1e+02      →      →      →      ↓
1      ↓      0  ↓      0      ↓      ↓      0      0      →  1e+03
2      0 -1e+02  →      ↓      ↓      ←      ↑      →      ↑      ↑
3      0      →  →      →  1e+03      ← -1e+04      ↑ -1e+04      →
4      →      →  →      ↑      ↑      ← -1e+02 -1e+03      0      ↑
5      0      ↑  ↑ -1e+02      ↑ -1e+02      ↓      ↓      ← -1e+02
6 -1e+03 -1e+02  ↑ -1e+03      ↑      0      ↓      ←      0      0
7      ↓      →  ↑      ↓      ↑ -1e+02  1e+03      0      ↓      ↓
8      →      ↑  0      ↓      0      →      ↑      ←      ←      ←
9      ↑      ↑  0      →      →      →      ↑      ←      0      ↑
Total value iterations: 0
Total policy iterations: 4
Runtime:0.03352618217468262s
```

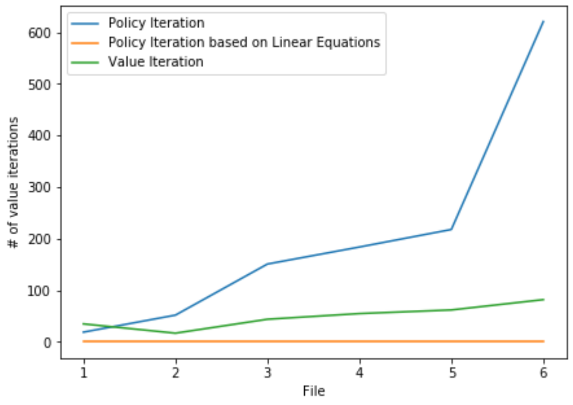
## 4. Experiments (Runtime)

### 1) Value Iteration vs. Policy Iteration vs. Policy Iteration based on Linear Equations Solving



| board_size | v_n_v_iter | v_runtime | p_n_v_iter | p_n_p_iter | p_runtime | pl_n_v_iter | pl_n_p_iter | pl_runtime |
|---|---|---|---|---|---|---|---|---|
| 6 | 35 | 0.01527 | 19 | 5 | 0.01469 | 0 | 1 | 0.00413 |
| 7 | 17 | 0.00915 | 52 | 5 | 0.02330 | 0 | 5 | 0.01369 |
| 10 | 44 | 0.04054 | 151 | 4 | 0.08571 | 0 | 4 | 0.02571 |
| 20 | 55 | 0.33576 | 184 | 8 | 0.65270 | 0 | 10 | 0.36991 |
| 30 | 62 | 0.83874 | 218 | 10 | 1.78188 | 0 | 10 | 0.95684 |
| 100 | 82 | 12.76358 | 621 | 17 | 45.94693 | 0 | 18 | 98.81403 |



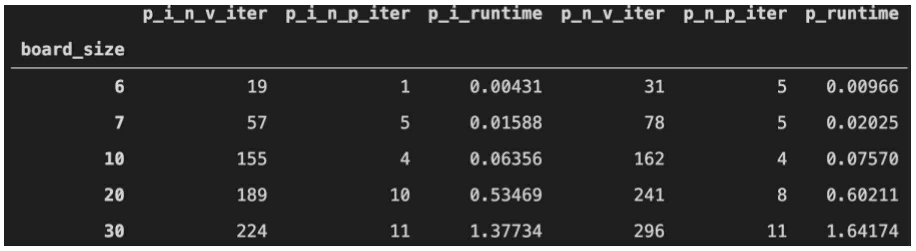


**Conclusion:**
In value iteration, the algorithm needs to check all of the possible actions. It works pretty good when the board is large.

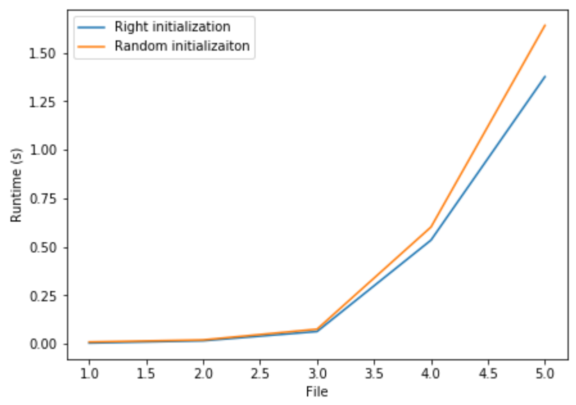
The normal policy iteration, which uses iteration to evaluate the policy, only needs to check one fixed action. Therefore, policy iteration will take less time for each step of calculating q values. However, the experiment shows that normal policy iteration needs more iterations to converge, which results in costing more runtime.

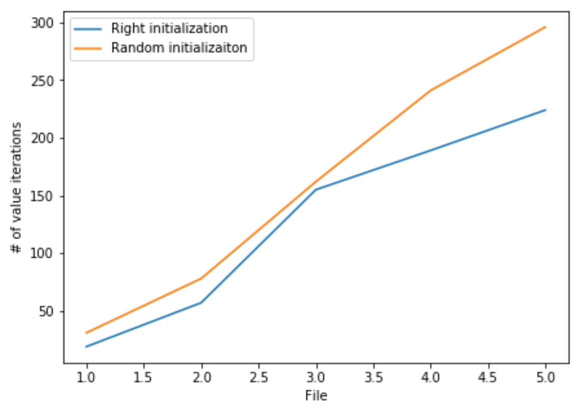
The policy iteration based on linear equations solving performs quite good in the experiment when the size of the board is small. However, when the board size is larger than 100, it takes forever for solving the equations.

### 2) Compare different initialization method



| board_size | p_i_n_v_iter | p_i_n_p_iter | p_i_runtime | p_n_v_iter | p_n_p_iter | p_runtime |
|---|---|---|---|---|---|---|
| 6 | 19 | 1 | 0.00431 | 31 | 5 | 0.00966 |
| 7 | 57 | 5 | 0.01588 | 78 | 5 | 0.02025 |
| 10 | 155 | 4 | 0.06356 | 162 | 4 | 0.07570 |
| 20 | 189 | 10 | 0.53469 | 241 | 8 | 0.60211 |
| 30 | 224 | 11 | 1.37734 | 296 | 11 | 1.64174 |

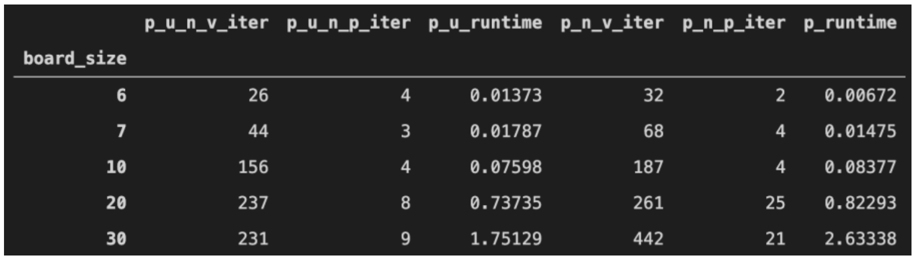




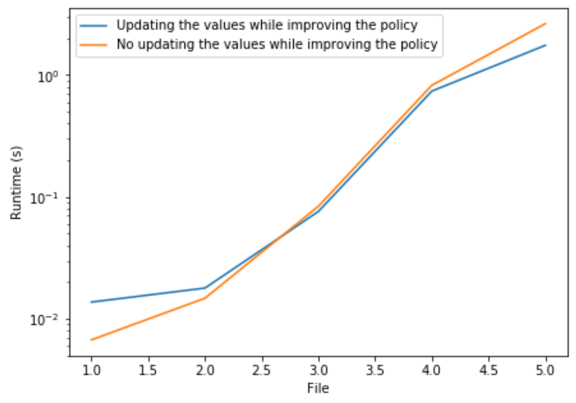
**Conclusion:**
I tried different initialization methods for policy, both random and choosing a direction for all states.

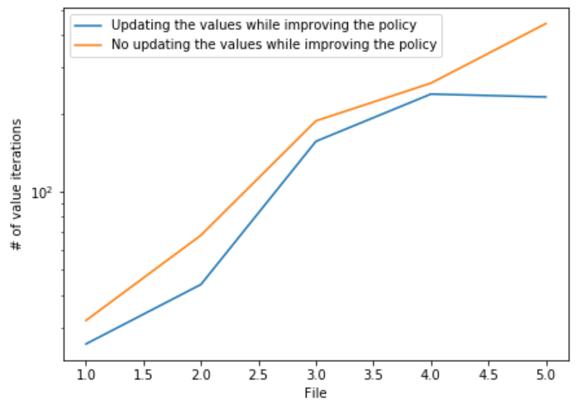
As shown in the experiment above, in some cases, a good policy initialization can reduce the runtime as well as the number of iterations needed.

## 3) Improve the values while improving the policy



| board_size | p_u_n_v_iter | p_u_n_p_iter | p_u_runtime | p_n_v_iter | p_n_p_iter | p_runtime |
|---|---|---|---|---|---|---|
| 6 | 26 | 4 | 0.01373 | 32 | 2 | 0.00672 |
| 7 | 44 | 3 | 0.01787 | 68 | 4 | 0.01475 |
| 10 | 156 | 4 | 0.07598 | 187 | 4 | 0.08377 |
| 20 | 237 | 8 | 0.73735 | 261 | 25 | 0.82293 |
| 30 | 231 | 9 | 1.75129 | 442 | 21 | 2.63338 |





**Conclusion:**
When I was using the normal policy iteration, I tried to improve the values while improving the policy. As shown in the experiment above, it can reduce the runtime as well as the number of iterations needed. This is because in this way the time for the first value iteration in evaluating policy can be saved.

## 5. Summary

On the one hand, the value iteration is faster than policy iteration when the scale of the task is large. On the other, the policy iteration based on linear equations solving works better when the size of the task is small. In conclusion, it is a trade-off to decide which algorithm we should use, and it also depends on the case.