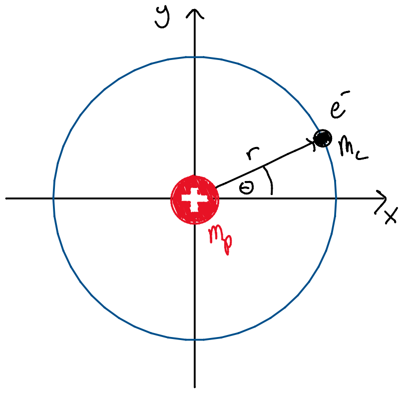
Particle in a Ring



$$x = r\cos\theta \qquad \dot{x} = -r\dot{\theta}\sin\theta \qquad K = \frac{1}{2}m_e(\dot{x}^2+\dot{y}^2) \qquad V = -\frac{e^2}{r}$$
$$y = r\sin\theta \qquad \dot{y} = r\dot{\theta}\cos\theta \qquad \quad = \frac{1}{2}m_e r^2\dot{\theta}^2$$

$$\mathcal{L} = K - V = \frac{1}{2}m_e r^2\dot{\theta}^2 + \frac{e^2}{r^2}$$

$$p_r = \frac{\partial\mathcal{L}}{\partial\dot{r}} = 0 \qquad p_\theta = \frac{\partial\mathcal{L}}{\partial\dot{\theta}} = m_e r^2\dot{\theta}$$

$$H = p_\theta\dot{\theta} - \frac{1}{2}m_e r^2\dot{\theta}^2 - \frac{e^2}{r^2} \qquad \dot{\theta} = \frac{p_\theta}{m_e r^2}$$
$$= \frac{p_\theta^2}{m_e r^2} - \frac{1}{2}\frac{p_\theta^2}{2m_e r^2} - \frac{e^2}{r^2}$$
$$= \frac{p_\theta^2}{2m_e r^2} - \frac{e^2}{r^2} \xrightarrow[\text{Quantum}]{} \hat{H} = -\frac{\hbar^2}{2m_e}\frac{1}{r^2}\frac{d^2}{d\theta^2} - \frac{e^2}{r^2}$$
$$= \frac{\hat{p}_\theta^2}{2m_e} - \frac{e^2}{r^2} = H_e + H_{pe}$$

$$H_e = \frac{p_\theta^2}{2m_e}, \quad H_{pe} = -\frac{e^2}{r^2}$$

$$H_e\psi(\theta) = E_\theta^n\psi(\theta) \qquad \psi = A\exp\left[\sqrt{\frac{2m_e r^2}{\hbar^2}E}\, i\theta\right], \quad w = \sqrt{\frac{2m_e r^2}{\hbar^2}E}$$
$$-\frac{\hbar^2}{2m_e}\frac{1}{r^2}\frac{d^2}{d\theta^2}\psi = E\psi \qquad \psi(\theta+2\pi) = \psi(\theta) \qquad 2\pi w = n2\pi, \; n\in\mathbb{Z}$$
$$\frac{d^2}{d\theta^2}\psi = -\frac{2m_e r^2}{\hbar^2}E\psi \qquad e^{iw\theta}e^{iw2\pi} = e^{iw\theta} \qquad w = n$$
$$e^{iw2\pi} = 1 \qquad \sqrt{\frac{2m_e r^2}{\hbar^2}E} = n$$
$$E_\theta^n = \frac{n^2\hbar^2}{2m_e r^2}$$

$$H\psi(\theta) = E^n\psi(\theta)$$
$$H_e\psi(\theta) - \frac{e^2}{r}\psi(\theta) = E^n\psi(\theta) \qquad E^n = E_\theta^n + \frac{e^2}{r^2}$$

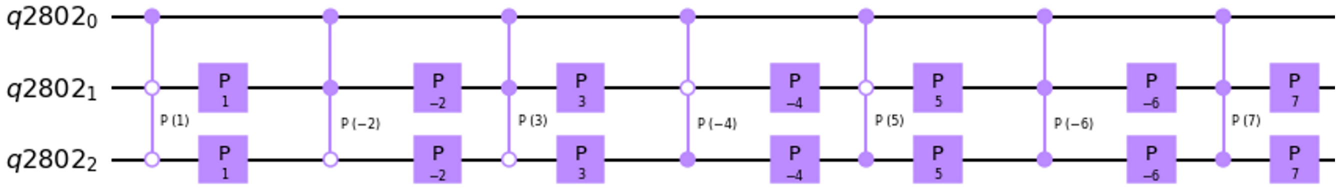
Diagonal Base Computacional $\quad H_e|n\rangle = \frac{n^2\hbar^2}{2m_e r^2}|n\rangle, \quad n\in\{0,1\}^N \wedge N = 2^{n\_qubits}$

$$H_e = \underbrace{\frac{\hbar^2}{2m_e r^2}}_{\alpha}\begin{pmatrix} 0 & & & & \\ & 1 & & 0 & \\ & & 4 & & \\ & 0 & & 9 & \\ & & & & \ddots \end{pmatrix} = \sum_{n=0}\frac{\alpha n^2}{2}|w\rangle\langle w|\otimes\left(\mathbb{1} + (-1)^{n[z]}\sigma_z\right)$$

$$e^{-i\gamma H_e} = \prod_{n=0}|w\rangle\langle w|\exp\left[-\frac{i\gamma\alpha n^2}{2}\left(\mathbb{1}+(-1)^{n[z]}\sigma_z\right)\right] = \prod_{n=0}|w\rangle\langle w|\exp\left[-\frac{i\gamma\alpha n^2}{2}(-1)^{n[z]}\sigma_z\right]$$

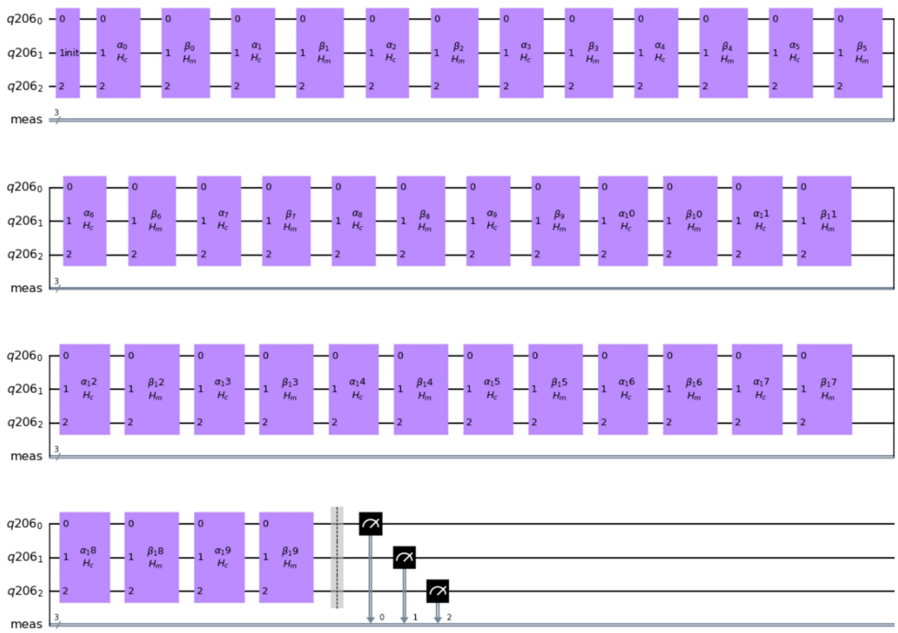
$\alpha = 1$
$$= \prod_{n=0}|w\rangle\langle w|\,P\left(-\gamma n^2(-1)^{n[z]}\right)$$

$\exp(-i\gamma H_e)$



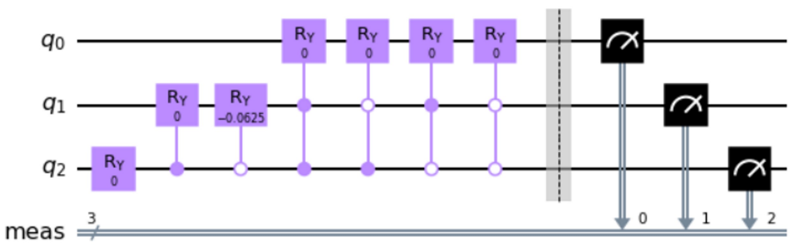
QAOA

Niveis de Energia = 8
Qubits = 3
Iterações = 20



Result:

0.999023438	0.0	9.76562500e-04	0.0	0.0	0.0	0.0	0.0



$$\langle H_e \rangle = 0.0078125$$
$$\langle H \rangle = \langle H_e \rangle + V$$