University of Minho
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Quantum Computing Technologies

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Contents

How to Build a Quantum Computer?

DiVincenzo Criteria

1. A scalable physical system with well characterized qubits.
2. The ability to initialize the state of the qubits to a simple fiducial state.
3. Long relevant decoherence times, much longer than the gate operation time.
4. A ”universal” set of quantum gates.
5. A qubit-specific measurement capability.

Qubits' Spectrum

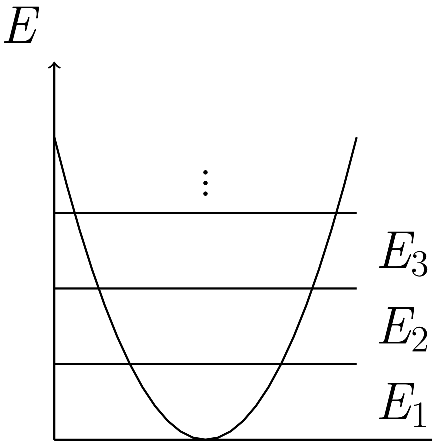


Figure 1: Energy spectrum of an harmonic oscillator.

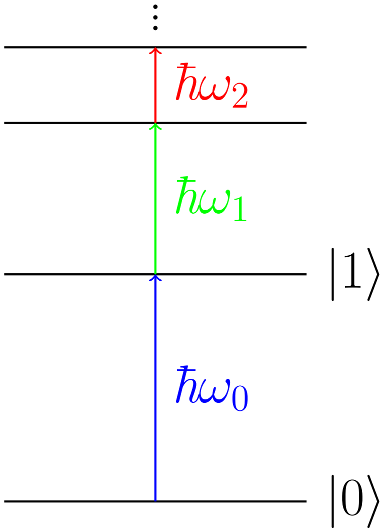
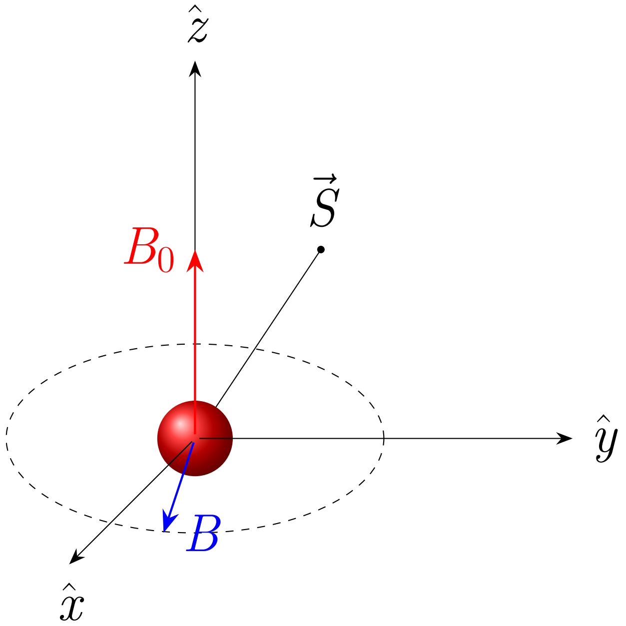


Figure 2: Anharmonic energy spectrum.

Spin Qubits

Spin-$\frac{1}{2}$ System



Let's consider a spin-$\frac{1}{2}$ particle in a magnetic field $\vec{B}$.

$$\vec{B} = B_0\hat{z} + B\left(\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}\right) \quad (1)$$

Spin-$\frac{1}{2}$ Qubits

The Hamiltonian of the system is

$$H = \vec{S} \cdot \vec{B} \tag{2}$$

$$\boxed{H(t) = \frac{\hbar\Delta}{2} Z + \hbar\Omega_0 \left(\cos(\omega t) X + \sin(\omega t)\, Y\right)} \tag{3}$$

$$H(t) = H_0 + H_1(t) \tag{4}$$

Ground State

The computational basis is given by the eigenstates of H_0.

$$Z\left|\uparrow\right\rangle = \left|\uparrow\right\rangle \quad \text{and} \quad Z\left|\downarrow\right\rangle = -\left|\downarrow\right\rangle \tag{5}$$

therefore,

$$H_0\left|\uparrow\right\rangle = \frac{\hbar\Delta}{2}\left|\uparrow\right\rangle \quad \text{and} \quad H_0\left|\downarrow\right\rangle = -\frac{\hbar\Delta}{2}\left|\downarrow\right\rangle \tag{6}$$

$$\boxed{\left|\uparrow\right\rangle \to \left|0\right\rangle \quad \text{and} \quad \left|\downarrow\right\rangle \to \left|1\right\rangle} \tag{7}$$

Time Evolution

The time evolution operator is given by

$$U(t) = e^{-\frac{i}{2}\omega Z t} V(t) \tag{8}$$

The equation of motion for $U(t)$ is

$$i\hbar \frac{d}{dt} U(t) = H(t) U(t) \tag{9}$$

$$\frac{\hbar \omega Z}{2} e^{-\frac{i}{2}\omega Z t} V(t) + e^{-\frac{i}{2}\omega Z t} i\hbar \frac{d}{dt} V(t) = H(t) e^{-\frac{i}{2}\omega Z t} V(t) \tag{10}$$

Time Evolution

$$i\hbar\frac{d}{dt}V(t) = \frac{\hbar(\Delta-\omega)}{2}Z + e^{\frac{i}{2}\omega Zt}H_1(t)e^{-\frac{i}{2}\omega Zt}V(t) \tag{11}$$

$$e^{\frac{i}{2}\omega Zt}H_1(t)e^{-\frac{i}{2}\omega Zt} = \frac{\hbar\Omega_0}{2}X \tag{12}$$

then,

$$i\hbar\frac{d}{dt}V(t) = \left(\frac{\hbar(\Delta-\omega)}{2}Z + \frac{\hbar\Omega_0}{2}X\right)V(t) \tag{13}$$

Time Evolution

$$i\hbar\frac{d}{dt}V(t) = H(\omega)\,V(t) \tag{14}$$

Finally,

$$\boxed{U(t) = e^{-\frac{i}{2}\omega Z t}e^{-\frac{i}{\hbar}H(\omega)t}, \qquad H(\omega) = \frac{\hbar\Omega}{2}\hat{n}\cdot\vec{\sigma}} \tag{15}$$

where $\Omega = \sqrt{(\Delta-\omega)^2+\Omega_0^2}$, $\vec{\sigma} = (X, Y, Z)$ and

$$\hat{n} = \frac{\Omega_0}{\sqrt{(\Delta-\omega)^2+\Omega_0^2}}\hat{x} + \frac{\Delta-\omega}{\sqrt{(\Delta-\omega)^2+\Omega_0^2}}\hat{z} \tag{16}$$

Time evolution

Using the following identity

$$\boxed{e^{-i\frac{\Omega}{2}t\hat{n}\cdot\vec{\sigma}} = \cos\left(\frac{\Omega}{2}t\right) - i\hat{n}\cdot\vec{\sigma}\sin\left(\frac{\Omega}{2}t\right)} \tag{17}$$

we can write

$$\boxed{U(t) = e^{-\frac{i}{2}\omega Zt}\left[\cos\left(\frac{\Omega}{2}t\right) - i\hat{n}\cdot\vec{\sigma}\sin\left(\frac{\Omega}{2}t\right)\right]} \tag{18}$$

X Gate

First, let's expand the second term of $U(t)$

$$-i\hat{n}\cdot\vec{\sigma}\sin\left(\frac{\Omega}{2}t\right) = -i\left(\frac{\Omega_0}{\sqrt{(\Delta-\omega)^2+\Omega_0^2}}X + \frac{\Delta-\omega}{\sqrt{(\Delta-\omega)^2+\Omega_0^2}}Z\right)\sin\left(\frac{\Omega}{2}t\right) \tag{19}$$

Assume resonance, $\omega = \Delta$, then

$$\boxed{U(t) = e^{-\frac{i}{2}\omega Zt}\left[\cos\left(\frac{\Omega}{2}t\right) - iX\sin\left(\frac{\Omega_0}{2}t\right)\right]} \tag{20}$$

X Gate

Therefore, if a pulse of duration $t = \frac{\pi}{\Omega_0}$ is applied, we get

$$U\left(\frac{\pi}{\Omega_0}\right) = -ie^{-\frac{i\pi\Delta}{2\Omega_0}Z}X \tag{21}$$

if $\Omega_0 = \Delta$ then

$$\boxed{U\left(\frac{\pi}{\Omega_0}\right) = X} \tag{22}$$

Two-Qubit Gates

The Hamiltonian can be written in terms of a Heisenberg interaction

$$H = \frac{J}{\hbar}\vec{S}_1 \cdot \vec{S}_2 \tag{23}$$

$$H = \frac{J}{2}(X_1X_2 + Y_1Y_2 + Z_1Z_2) \tag{24}$$

$$U(t) = e^{-iHt} = e^{-iJ\vec{S}_1\cdot\vec{S}_2 t}, \hbar = 1 \tag{25}$$

SWAP Gate

$$SWAP = 2\vec{S}_1 \cdot \vec{S}_2 + \frac{1}{2} \tag{26}$$

$$\boxed{SWAP\,|a\rangle \otimes |b\rangle = |b\rangle \otimes |a\rangle} \tag{27}$$

SWAP Gate

Using the following identity

$$\boxed{e^{itA} = \cos(t)I + i\sin(t)A, \qquad A^2 = I} \tag{28}$$

we can write

$$U(t) = e^{-i\frac{J}{4}t} e^{-i\frac{J}{2}\,2\vec{S}_1\cdot\vec{S}_2 t} = e^{-i\frac{J}{2}\left(2\vec{S}_1\cdot\vec{S}_2+\frac{1}{2}\right)t} \tag{29}$$

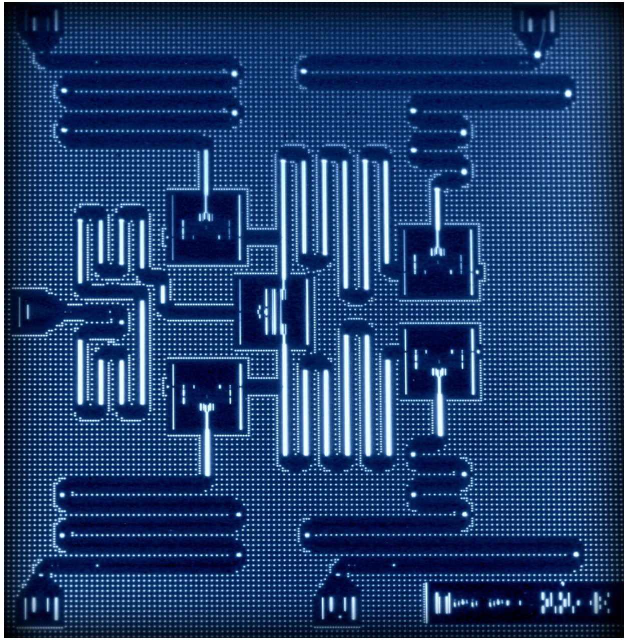
$$U(t) = \cos\left(\frac{J}{2}t\right)I - i\sin\left(\frac{J}{2}t\right)\left(2\vec{S}_1\cdot\vec{S}_2 + \frac{1}{2}\right) \tag{30}$$

For $t = \frac{\pi}{J}$, $U(t) = -i\ SWAP$

Superconducting Qubits

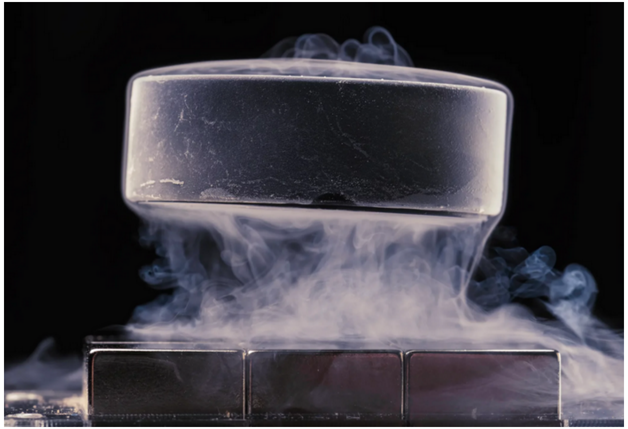
Superconducting Qubits

- Superconducting qubits are artificial atoms made of Josephson junctions.
- They are scalable and can be manufactured using standard microfabrication techniques.
- They can be coupled to each other using microwave resonators.



Superconductors

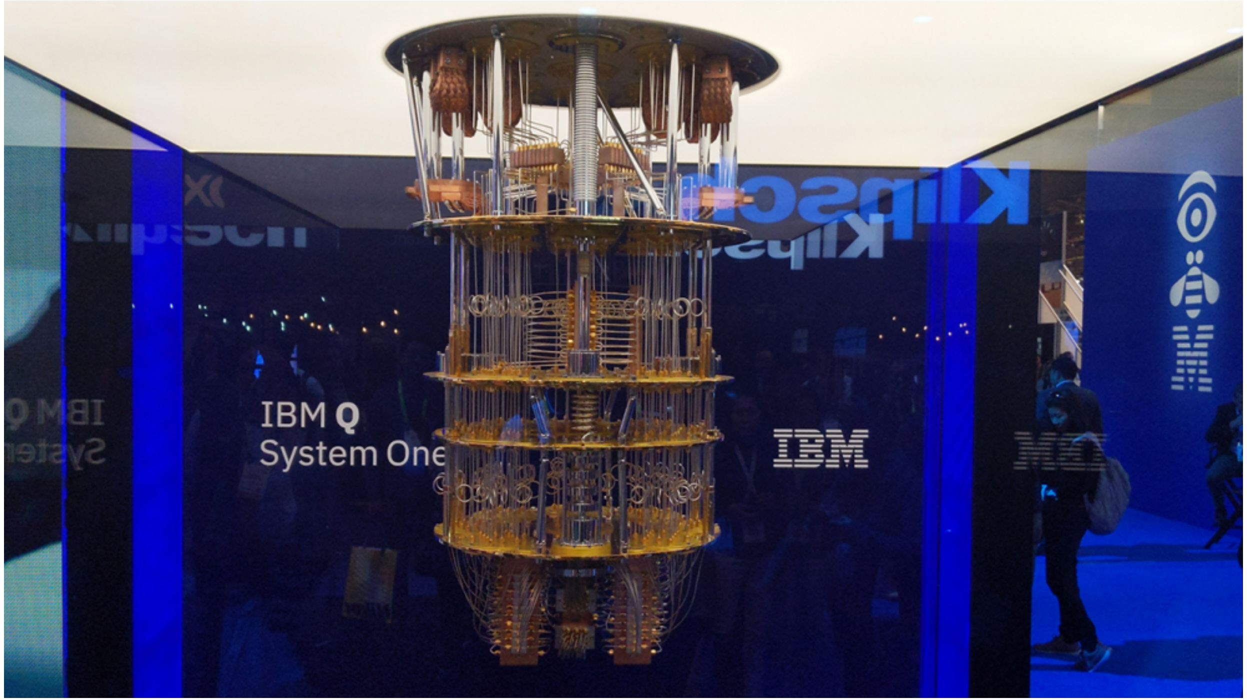
- Superconductors are materials that have zero resistance to the flow of electric current.
- They have a critical temperature T_c
- They have a critical magnetic field H_c
- They have a critical current I_c



Types of Superconducting Qubits

- Phase qubit
- Charge qubit
- Flux qubit
- Transmon qubit

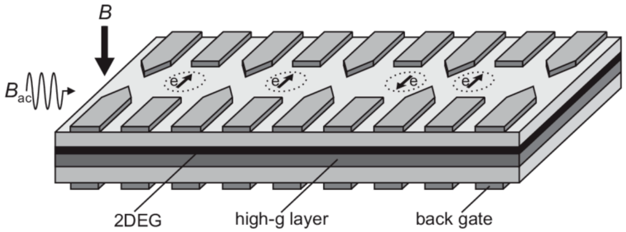
IBM Quantum Computer



Other Qubit Technologies

Quantum Dots Qubits

- Quantum dots are small regions of a semiconductor material that have been confined in all three dimensions.
- They can be manufactured using standard microfabrication techniques.



Ohter Qubit Technologies

- Trapped ions
- Photonic qubits
- Rydberg atoms
- Cold atoms

Error Correction

Error Correction

- Quantum error correction (QEC) is used in quantum computing to protect quantum information from errors due to decoherence and other quantum noise.
- Quantum error correction is essential if one is to achieve fault-tolerant quantum computation that can deal not only with noise on stored quantum information qubits, but also with faulty quantum gates, faulty quantum preparation, and faulty measurements.

Logical Qubits

- A logical qubit is a quantum state that is encoded in multiple physical qubits.
- The physical qubits are usually arranged in a lattice, and the logical qubit is encoded in a subspace of the Hilbert space of the lattice.
- The logical qubit is designed to be robust against noise and errors, while the physical qubits are not.

Error Mitigation

- Zero-noise extrapolation (ZNE)
- Readout-error mitigation

A useful application for 127-qubit

With the confidence that our systems are beginning to provide utility beyond classical methods alone, we can begin transitioning our fleet of quantum computers into one consisting solely of processors with 127 qubits or more. [1]

Real Devices

IBM Quantum Lab

- IBM Quantum Lab
- IBM Quantum Platform

Discussion

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