University of Minho
School of Engineering

Quantum Computing Fundamentals

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Quantum Computing School @ Yachay

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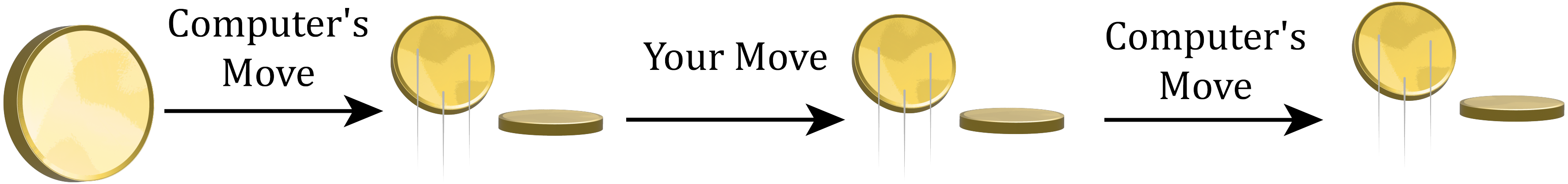
Coin Game

Game:



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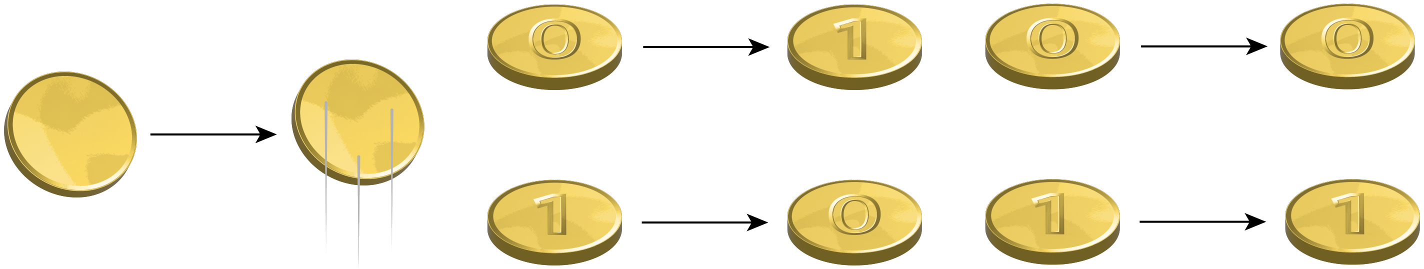


Let's play

Contents

Motivation

Quantum Computing has far-reaching implications across numerous fields. For instance:

- Cryptography
- Machine learning
- Communication systems
- Chemistry
- Physics

Quantum Computing Concepts

Quantum Bit (Qubit)

In quantum computing, the equivalent of the classical computer's *bit* is the quantum bit, *qubit* for short.

- Superposition
- Interference
- Entanglement

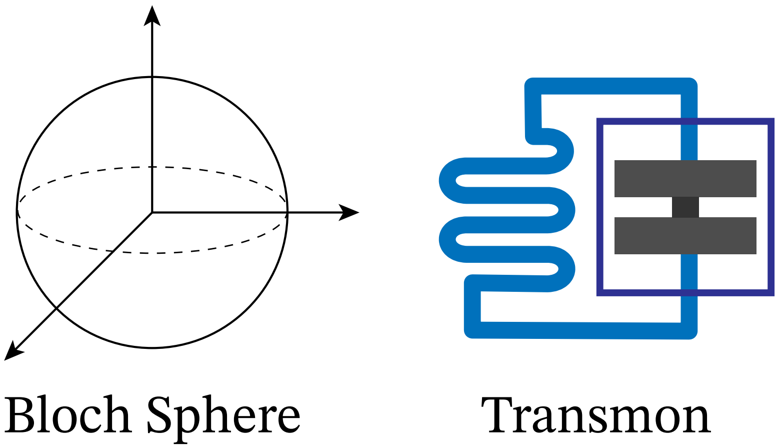


Figure 1: The Bloch sphere is the common qubit state representation. A transmon is a physical superconductor qubit used on the IBM's quantum computers.

Superposition

Figure 2: A qubit is *"like a coin"*, where the state of each qubit is represented by each face.

$$\boxed{|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle}$$

$$P_0 = \left|\cos\frac{\theta}{2}\right|^2 = \cos^2\frac{\theta}{2}$$

$$P_1 = \left|e^{i\varphi}\sin\frac{\theta}{2}\right|^2 = \sin^2\frac{\theta}{2}$$

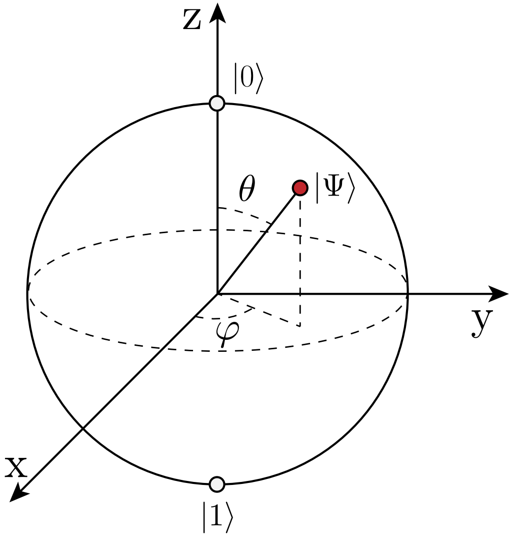


Figure 3: A quantum state $|\Psi\rangle$ is shown on the Bloch sphere as a point on its surface.

Interference

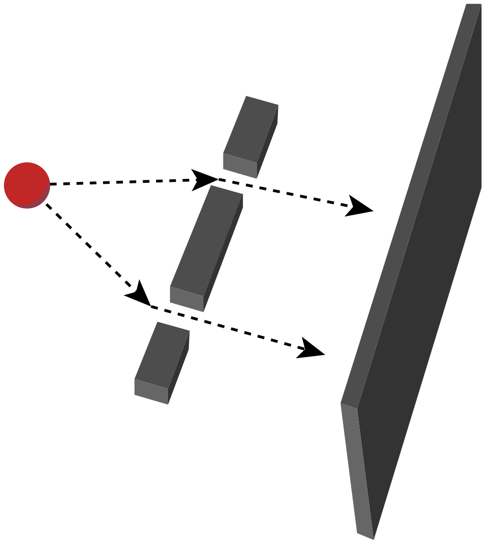


Figure 4: Experiment setup.

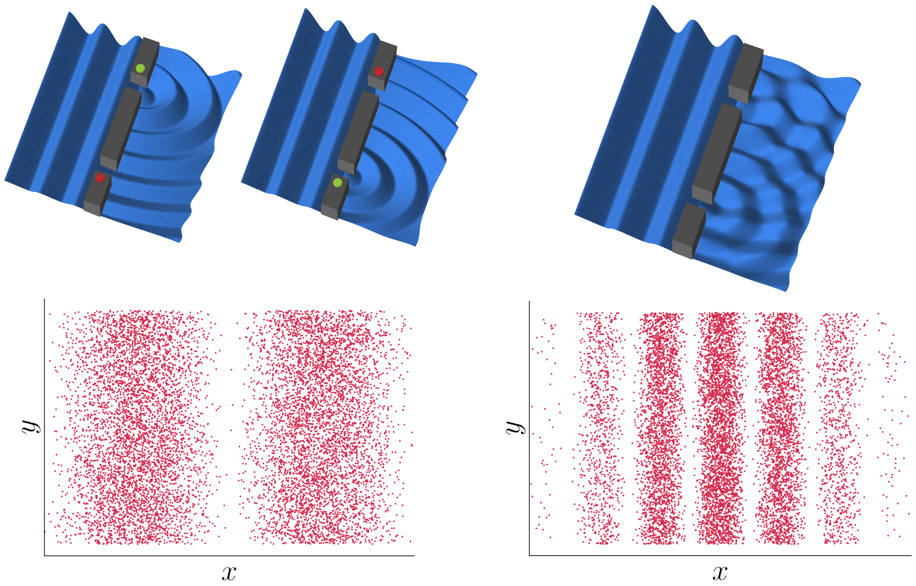
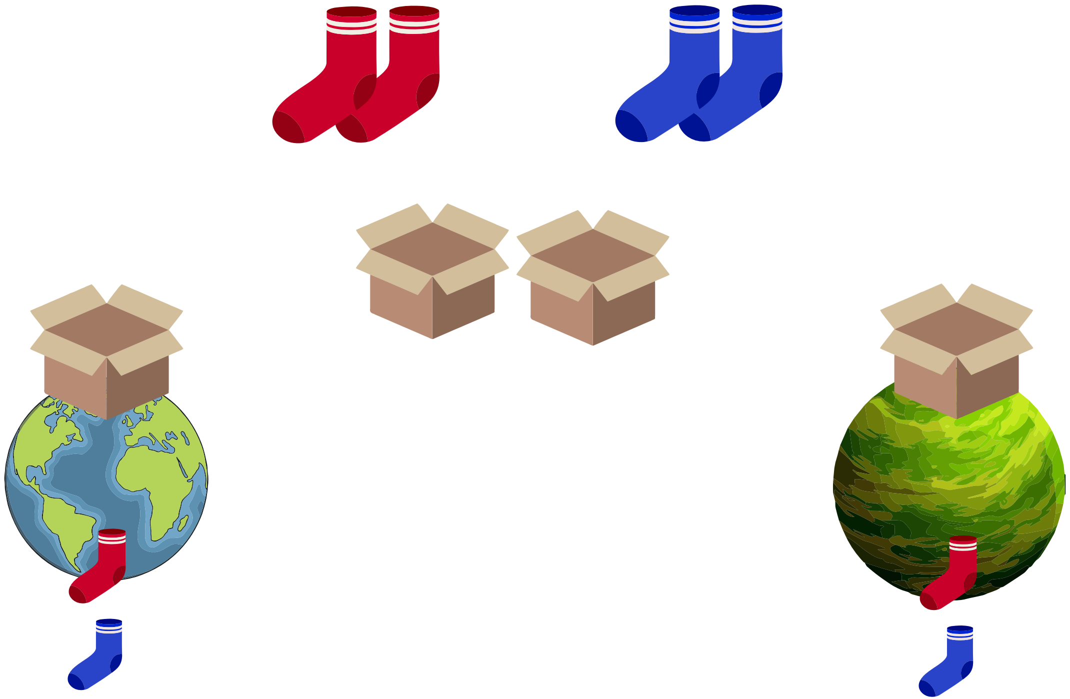
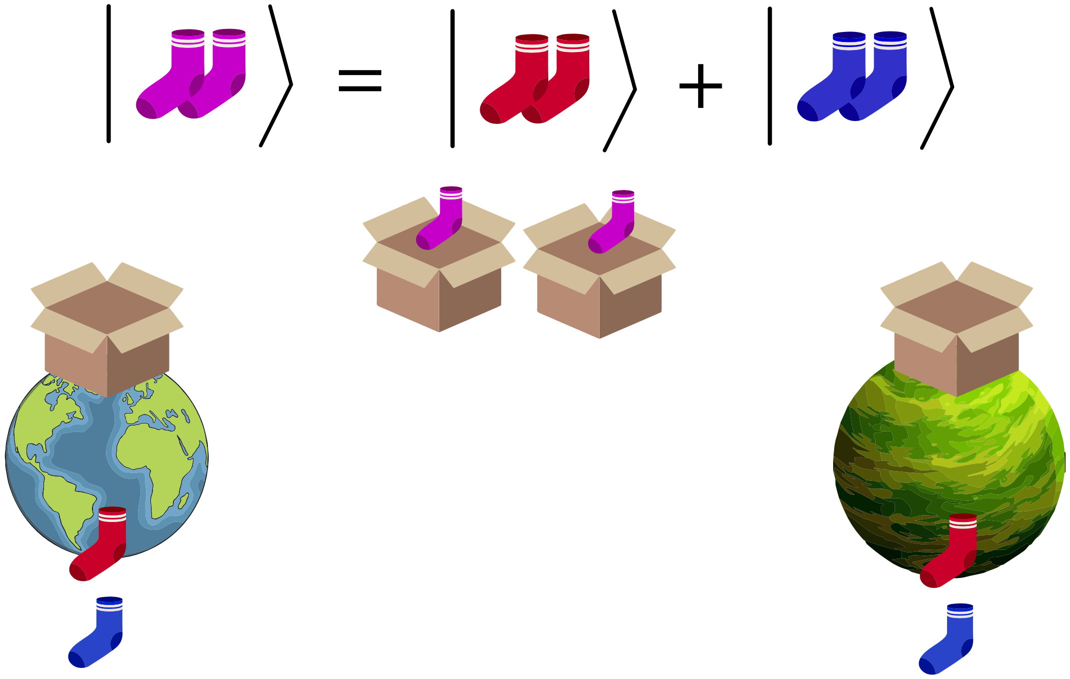


Figure 5: Left shows the pattern obtained due to the path measurement. Right shows the wave function interference pattern.

Classical Correlation



Entanglement



EPR Paradox

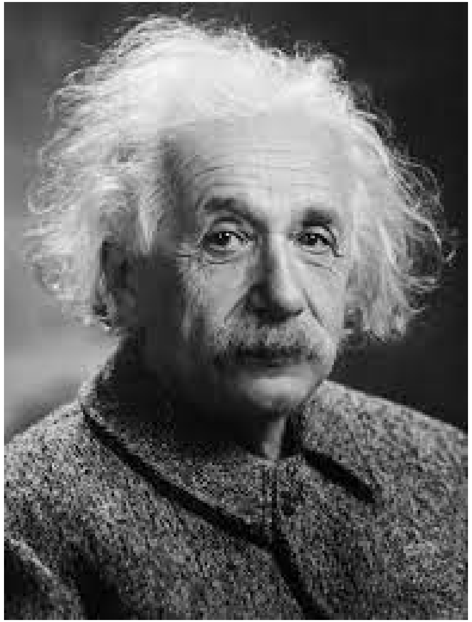


Figure 6: Albert Einstein

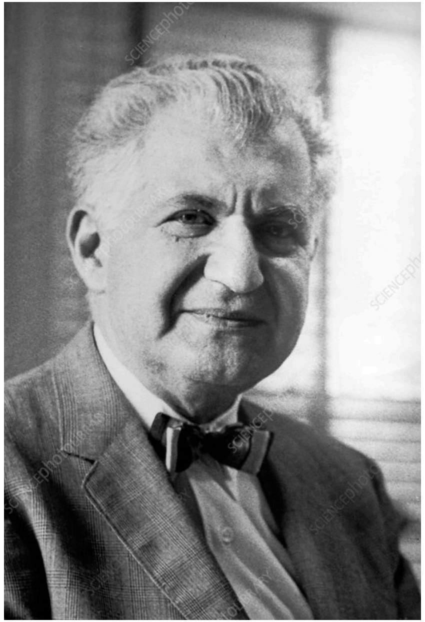


Figure 7: Boris Podolsky

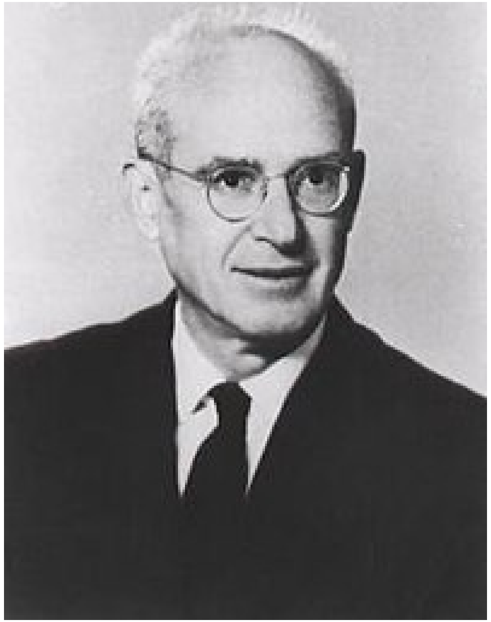


Figure 8: Nathan Rosen

Bell States

$$
\begin{aligned}
\left|\Phi^{+}\right\rangle &= \frac{1}{\sqrt{2}}\Big[\,|00\rangle + |11\rangle\,\Big] \\
\left|\Phi^{-}\right\rangle &= \frac{1}{\sqrt{2}}\Big[\,|00\rangle - |11\rangle\,\Big] \\
\left|\Psi^{+}\right\rangle &= \frac{1}{\sqrt{2}}\Big[\,|01\rangle + |10\rangle\,\Big] \\
\left|\Psi^{-}\right\rangle &= \frac{1}{\sqrt{2}}\Big[\,|01\rangle - |10\rangle\,\Big]
\end{aligned} \tag{1}
$$

Nobel Prize 2022

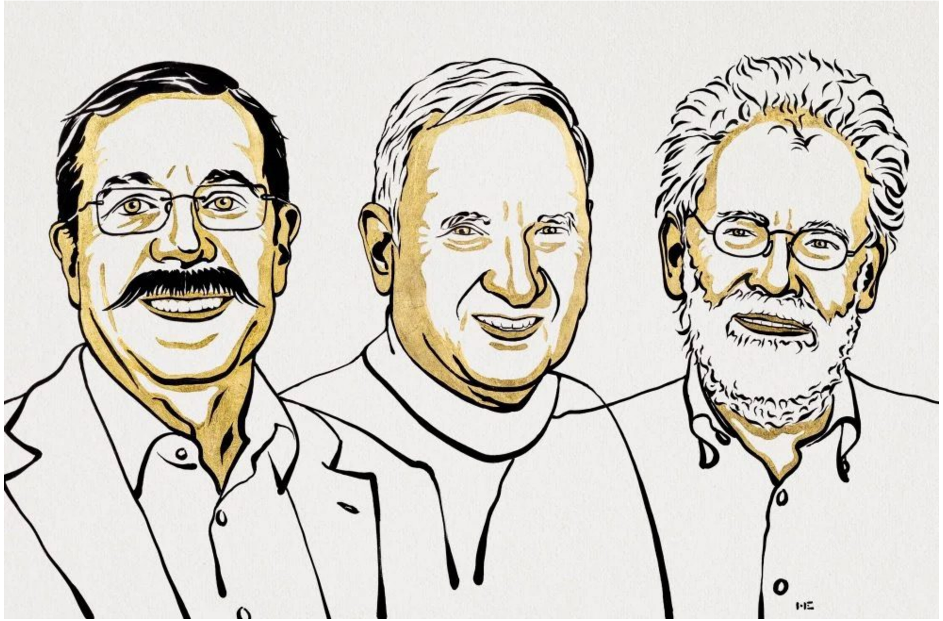


Figure 9: Nobel Prize in Physics 2022: Alain Aspect, John Clauser, and Anton Zeilinger. "For experiment with quantum entanglement, which has led to the development of quantum information science."

Why Quantum Computing?

Classical Simulation of a Qubit

Figure 10: Richard Phillips Feynman (1965). *Source: The Nobel Prize*

$$N = 2^n \text{ (Complex Numbers)}$$

Beyond Classical Capacities

$$n = 50 \rightarrow N \approx 10^{15}$$

$$n = 300 \rightarrow N \approx 10^{90} > N_{\text{Universe}}$$

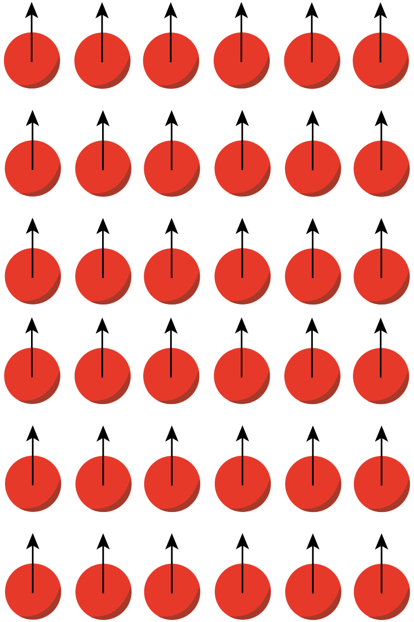


Figure 11: System of n electrons (qubits).

Quantum Simulators

Analog Quantum Simulator

$$\hat{\mathrm{H}} \approx \hat{\mathcal{H}}$$

Digital Quantum Simulator

$$\hat{\mathrm{U}}_{\hat{\mathrm{H}}}(t) \approx \hat{\mathrm{U}}_{\hat{\mathcal{H}}}(t)$$

$\hat{\mathcal{H}}$: Target Hamiltonian

$\hat{\mathrm{H}}$: Simulator Hamiltonian

How to program a Quantum Computer?

Representation

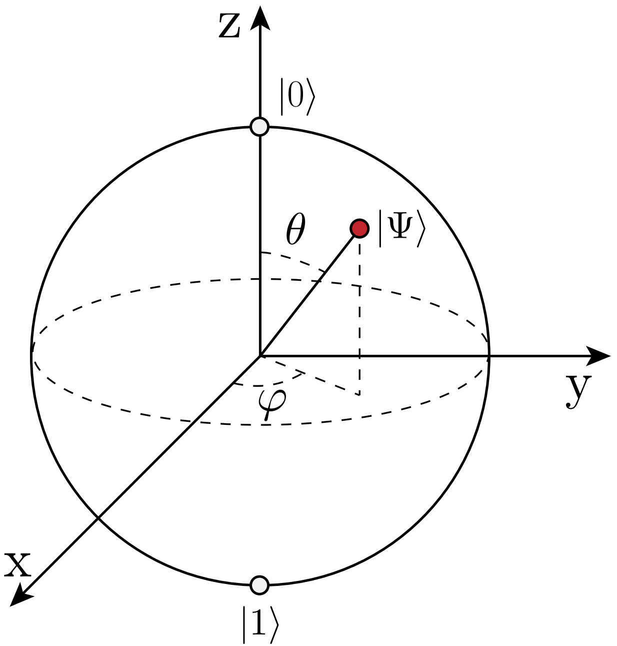


Figure 12: A quantum state $|\Psi\rangle$ is shown on the Bloch sphere as a point on its surface.

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2}$$

$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle \qquad |\Psi\rangle = \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{bmatrix} \tag{3}$$

$$|\psi\rangle \; —$$

Quantum Gates

Quantum gates in quantum computing are unitary operators.

$$U(\theta, \phi, \lambda) = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i(\phi+\lambda)}\cos\frac{\theta}{2} \end{pmatrix} \tag{4}$$

$$U^{\dagger} = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{-i\phi}\sin\frac{\theta}{2} \\ -e^{-i\lambda}\sin\frac{\theta}{2} & e^{-i(\phi+\lambda)}\cos\frac{\theta}{2} \end{pmatrix} \tag{5}$$

$$UU^{\dagger} = U^{\dagger}U = \mathbf{1} \tag{6}$$

Pauli operators

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{7}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{8}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{9}$$

$$X|0\rangle = |1\rangle$$
$$X|1\rangle = |0\rangle$$

$$Y|0\rangle = i|1\rangle$$
$$Y|1\rangle = -i|0\rangle$$

$$Z|0\rangle = |0\rangle$$
$$Z|1\rangle = -|1\rangle$$

$$|q\rangle \;—\boxed{\mathrm{X}}—\; |\neg q\rangle$$

$$|q\rangle \;—\boxed{\mathrm{Y}}—\; \pm i|\neg q\rangle$$

$$|q\rangle \;—\boxed{\mathrm{Z}}—\; |q\rangle$$

Hadamard, T-gate and CNOT

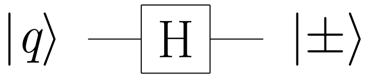
$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{10}$$

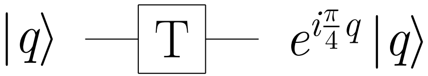
$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix} \tag{11}$$

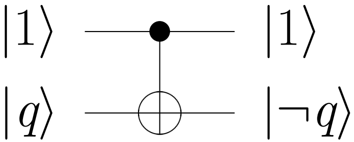
$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{12}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle + |1\rangle\Big] = |+\rangle$$
$$H|1\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle - |1\rangle\Big] = |-\rangle$$

$$T|0\rangle = |0\rangle$$
$$T|1\rangle = e^{i\frac{\pi}{4}}|1\rangle$$







Universality

The universal set of gates is defined to approximate any unitary operator U with an arbitrary error ε using a sequence of n gates U_i from the universal set

$$\left\| U - \prod_{i=1}^{n} U_i \right\| < \varepsilon \tag{13}$$

where, $\|A\| = \max_{|\psi\rangle} \frac{|A|\psi\rangle|}{||\psi\rangle|}$ is the spectral norm of the matrix A.

Programming Languages

Qiskit	IBM
PENNYLANE	XANADU
Q#	Microsoft
Cirq	Google

Coin Game

Quantum Coin

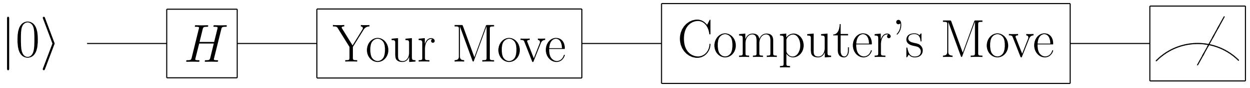
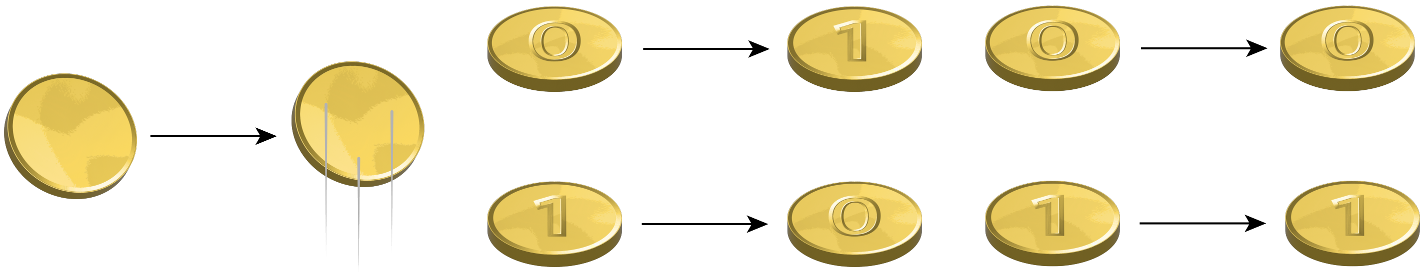


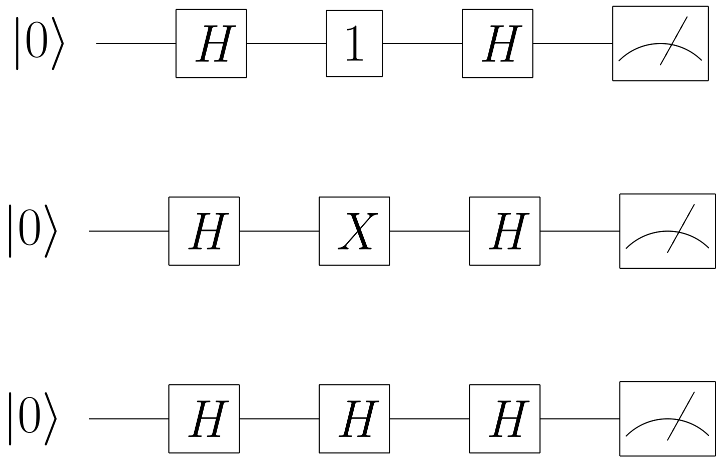
Figure 13: Quantum Coin

Actions:



H X 1

Quantum Coin



Discussion

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