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Quantum Machine Learning

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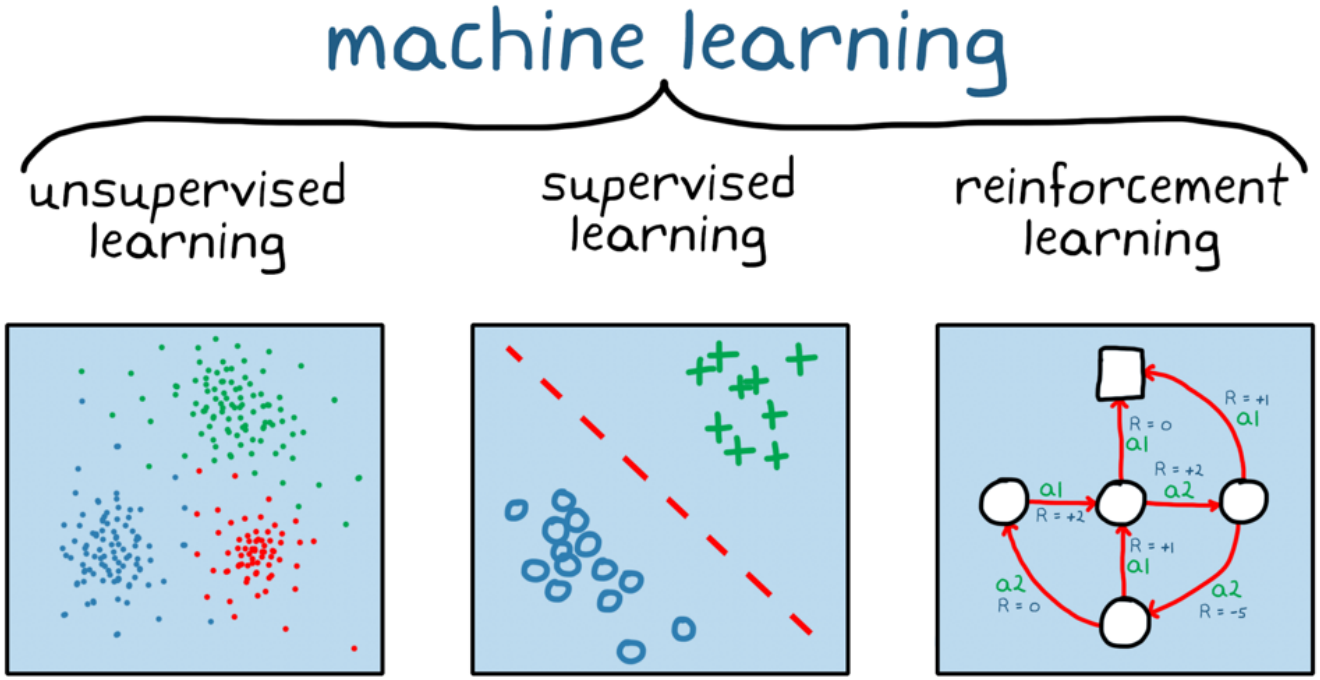
Contents

What is Machine Learning?

Machine Learning

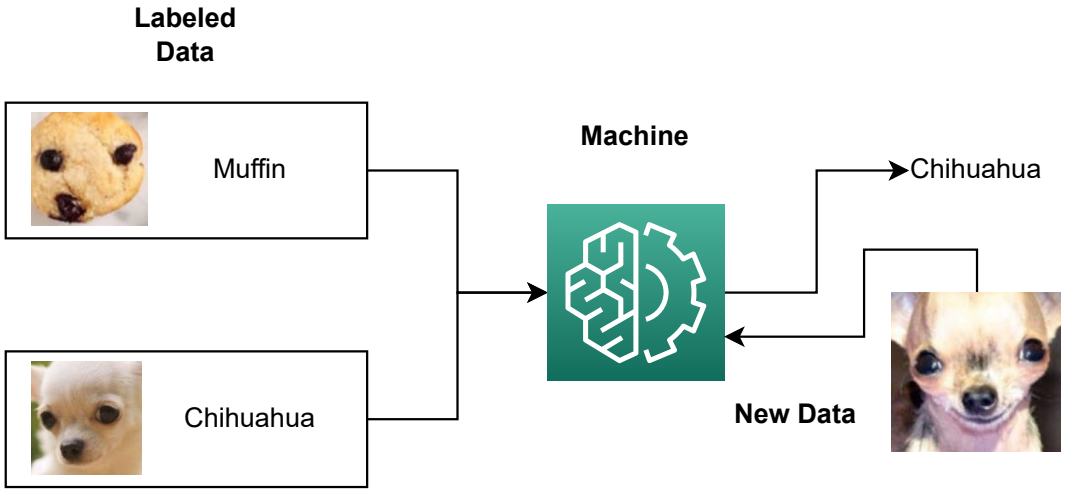
- Machine Learning is a field of study that gives computers the ability to learn without being explicitly programmed.
- Machine Learning is a subfield of Artificial Intelligence.

Branches of Machine Learning



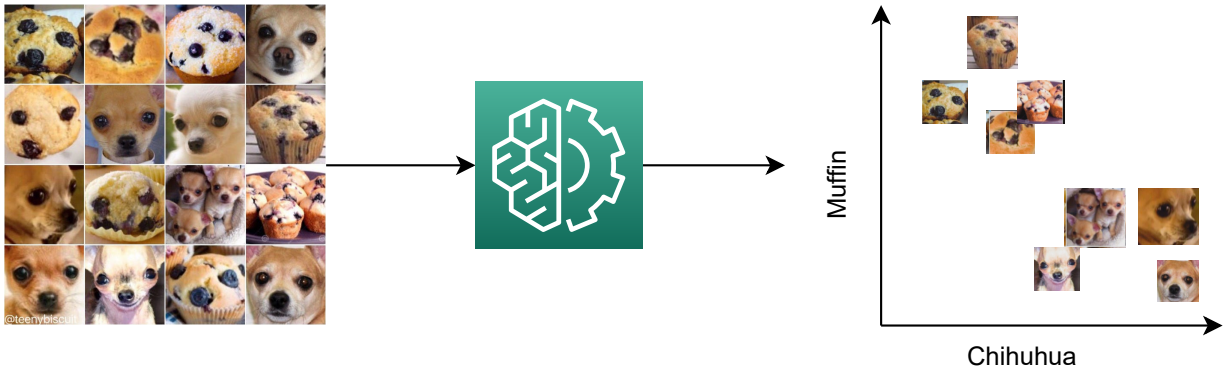
Supervised Learning

Supervised learning is the machine learning task of learning a function that maps an input to an output based on example input-output pairs.



Unsupervised Learning

Unsupervised learning is a type of machine learning algorithm used to draw inferences from datasets consisting of input data without labeled responses.



Reinforcement Learning

Reinforcement learning is an area of machine learning concerned with how software agents ought to take actions in an environment in order to maximize the notion of cumulative reward.

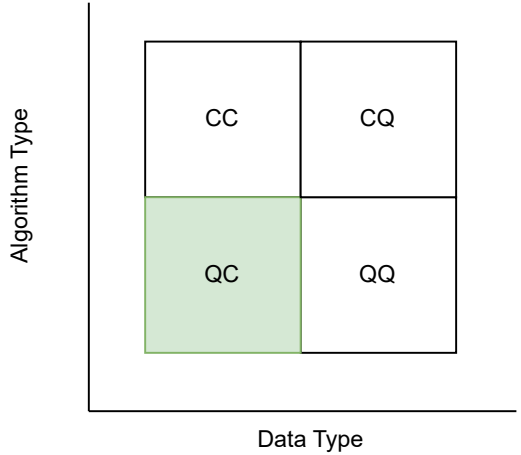
Applications of Machine Learning

- ChatGPT
- Image Recognition
- Medical Diagnosis
- High Energy Physics
- Prediction
- Classification
- Clustering
- Anomaly Detection

Quantum Machine Learning

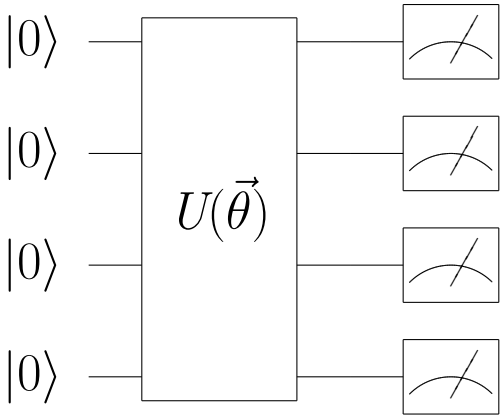
Quantum Machine Learning

- Quantum Machine Learning is a field of study that gives quantum computers the ability to learn without being explicitly programmed.
- Quantum Machine Learning is a subfield of Quantum Computing.

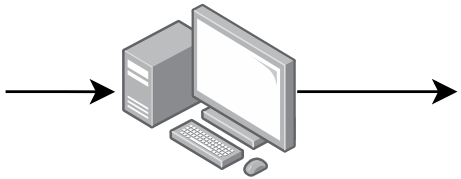


Variational Quantum Algorithms

Variational Quantum Algorithms are hybrid algorithms that use a quantum computer to perform a sub-routine of the algorithm.



$\langle O(\vec{\theta}) \rangle$



Parameter Shift Rule

The Parameter Shift Rule is a method to calculate the gradient of a parameterized quantum circuit.

$$\nabla_\theta f = \frac{1}{2}\left(f(\theta + \frac{\pi}{2}) - f(\theta - \frac{\pi}{2})\right) \tag{1}$$

See the derivation here [1].

Data Encoding

Consider a dataset $X = \{x_1, x_2, \ldots, x_N\}$, where x_i is a data point. Each data point is a vector of m features, $x_i = (x_{i1}, x_{i2}, \ldots, x_{im})$.

Basis Encoding: The data is encoded in the basis states of the qubits.

$$|x_i\rangle = |x_{i1}\rangle \otimes |x_{i2}\rangle \otimes \cdots \otimes |x_{im}\rangle, \quad x_{ij} \in \{0, 1\} \tag{2}$$

$$|X\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |x_i\rangle \tag{3}$$

Data Encoding

Amplitude Encoding: The data is encoded in the amplitudes of the qubits.

$$|x_i\rangle = \sum_{j=1}^{m} x_{ij} |j\rangle , \quad x_{ij} \in \mathbb{R} \tag{4}$$

Angle Encoding: The data is encoded in the angles of the qubits.

$$|x_i\rangle = \cos(x_{i1}) |0\rangle + \sin(x_{i1}) |1\rangle \otimes \cdots \otimes \cos(x_{im}) |0\rangle + \sin(x_{im}) |1\rangle \tag{5}$$

Data Re-uploading

Data Re-uploading is a method to encode data in a parameterized quantum circuit by intercalating the data encoding with the parameter encoding. Each layer is defined as:

$$L(\theta, x) = E(x)\, U(\theta) \tag{6}$$

- Data reuploading is universal function approximator [2].
- One qubit using data re-uploading is a universal classifier [3].

Quantum Algorithms

VQE

The Variational Quantum Eigensolver (VQE) is a variational quantum algorithm that finds the ground state energy of a Hamiltonian.

$$|\psi\rangle = U(\vec{\theta})\,|0\rangle \tag{7}$$

$$E(\vec{\theta}) = \langle\psi|\,H\,|\psi\rangle \tag{8}$$

$$H = \sum \alpha_i h_i, \quad h_i \in \{X, Y, Z, I\} \tag{9}$$

$$\langle H\rangle = \sum \alpha_i \langle h_i\rangle \tag{10}$$

VQC

The Variational Quantum Classifier (VQC) is a variational quantum algorithm that classifies data.

$$|\psi\rangle = U(\vec{\theta})E(X)\,|0\rangle \tag{11}$$

where $E(X)$ is the data encoding.

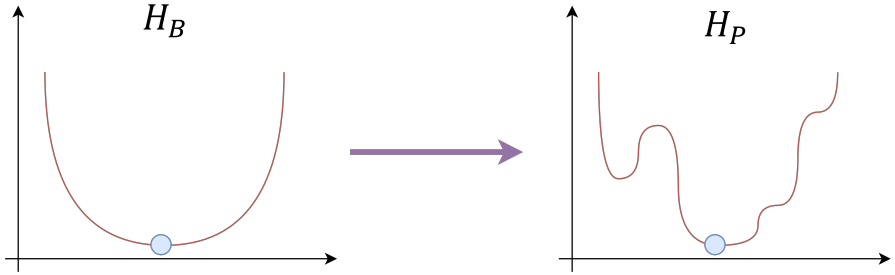
$$C(\vec{\theta}, X) = \text{Cost}\left(y, \langle\psi|\, M\,|\psi\rangle\right) \tag{12}$$

where M is a measurement operator and y is the label of the data.

QAOA

The Quantum Approximate Optimization Algorithm (QAOA) is a variational quantum algorithm that finds the minimum of a Hamiltonian H_P.

$$QAOA = \prod_{i=1}^{p} e^{-i\beta_i H_B} e^{-i\gamma_i H_P} \tag{13}$$



Quantum Kernel

A quantum kernel is a function that applies a feature map to the data and then calculates the inner product of the feature map.

$$K(x_i, x_j) = \langle \phi(x_i) | \phi(x_j) \rangle \tag{14}$$

Discussion

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