

# Thermodynamics of oxygen defective $\text{TiO}_{2-x}$ : The Magneli phases.

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# Magneli Phases

Figure 1a

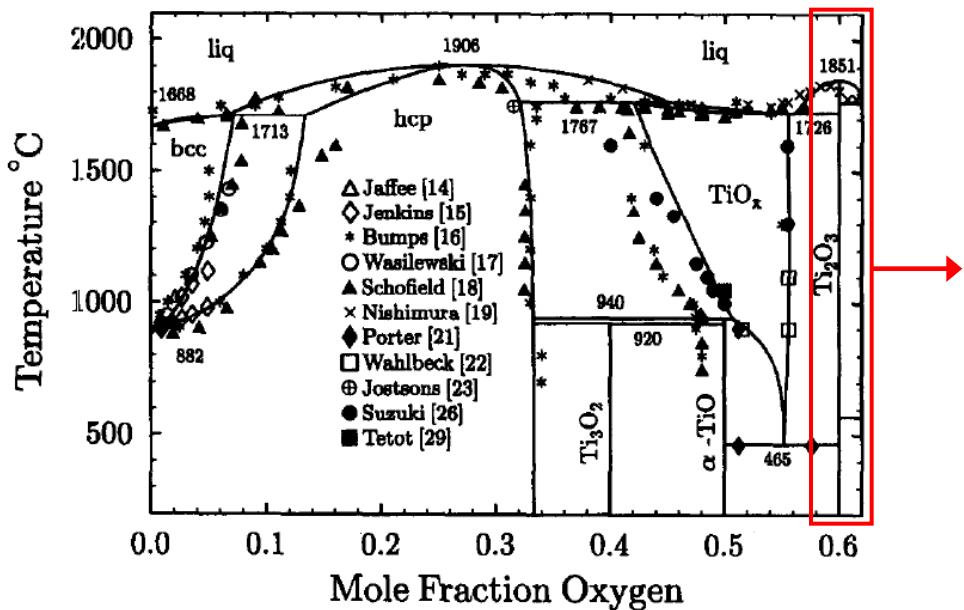
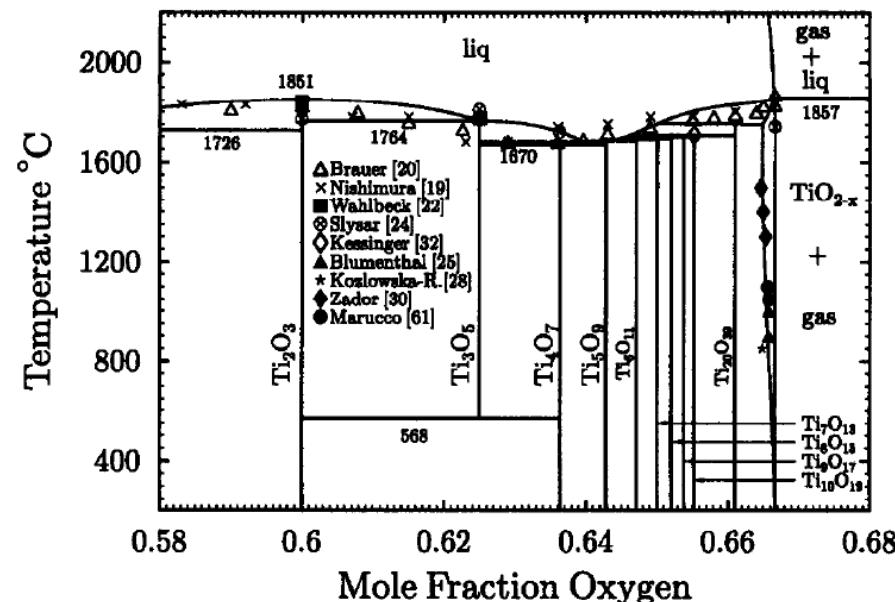


Figure 1b



$\text{T}_n\text{O}_{2n-1}$  composition,  $4 \leq n \leq 9$ . Oxygen defects in {121} planes.

$\text{Ti}_4\text{O}_7$  at  $T < 154\text{K}$  insulator with 0.29eV band gap<sup>(1)</sup>.

$\text{Ti}_4\text{O}_7$  Metal-insulator transition at 154K, with sharp decrease of the magnetic susceptibility.

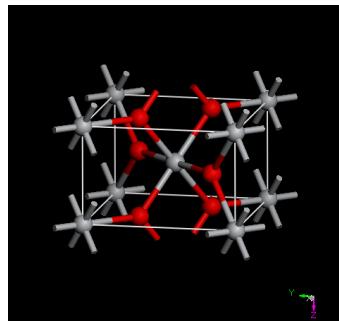
(1) K. Kobayashi *et al.*, Europhysics Lett., Vol. 59, pp. 868-874, 2002.

(2) W. Masayuki *et al.*, J. of Luminiscence, Vol. 122-123, pp. 393-395, 2007.

(3) P. Waldner and G. Eriksson, Calphad Vol. 23, No. 2, pp. 189-218, 1999.

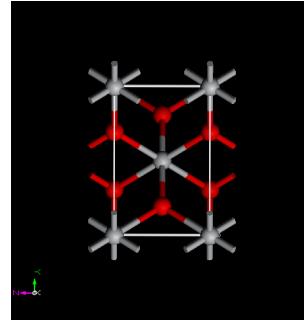
# Magneli Phases: $T_4O_7$ crystalline structure

Figure 3a



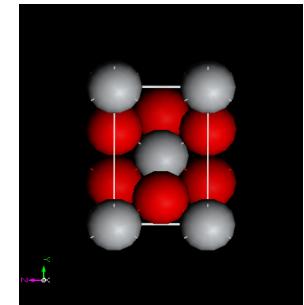
Rutile unit cell

Figure 3b



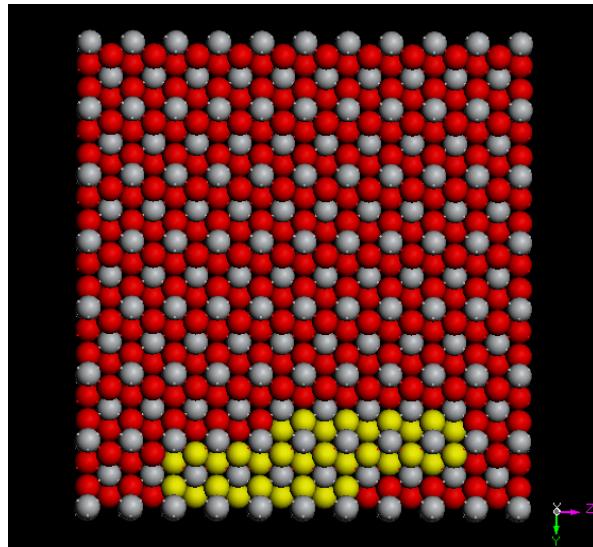
View along the **a** lattice parameter

Figure 3c



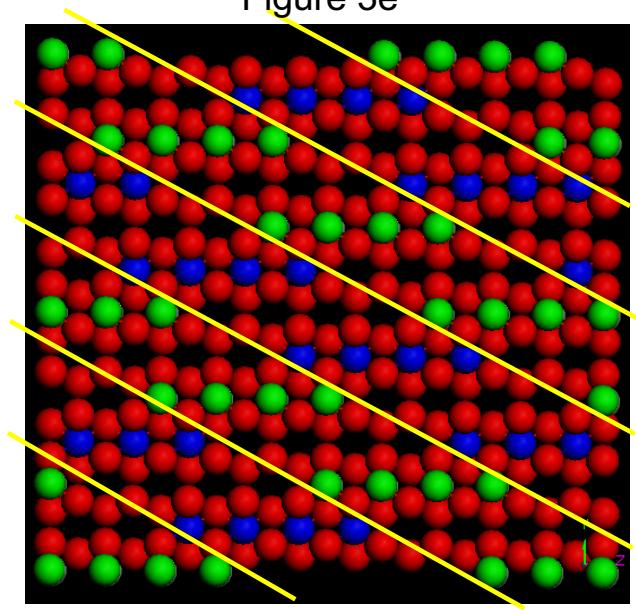
View of Hexagonal oxygen arrangement

Figure 3d



View of Hexagonal oxygen network

Figure 3e



Metal nets in antiphase. (121)<sub>r</sub>, Cristallographic shear plane.

# Technical details of the calculations

## CASTEP

Local density functional: LDA

Ultrasoft pseudopotentials replacing core electrons

Plane waves code

Supercell approach

Simulated systems: Oxygen point-defective supercell, Magneli phases supercells, Titanium bulk metal.

## CRYSTAL

Hybrid density functional: B3LYP,  
GGA Exchange  
GGA Correlation  
20% Exact Exchange

All electron code. No pseudopotentials

Local basis functions: atom centred Gaussian type functions.

Ti: 27 atomic orbitals, O: 18 atomic orbitals

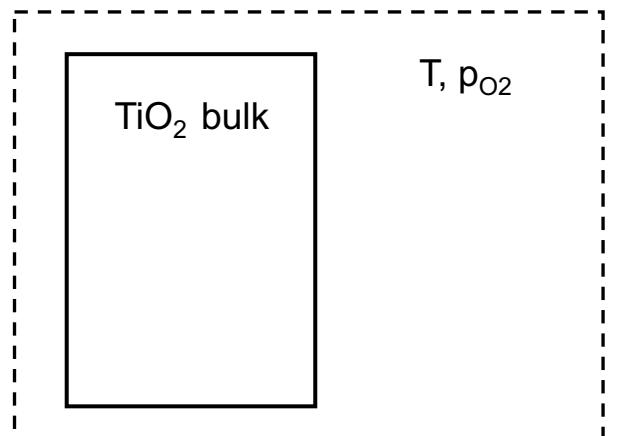
Supercell approach

Simulated systems: Oxygen point-defective supercell, Magneli phases supercells, Oxygen molecule.

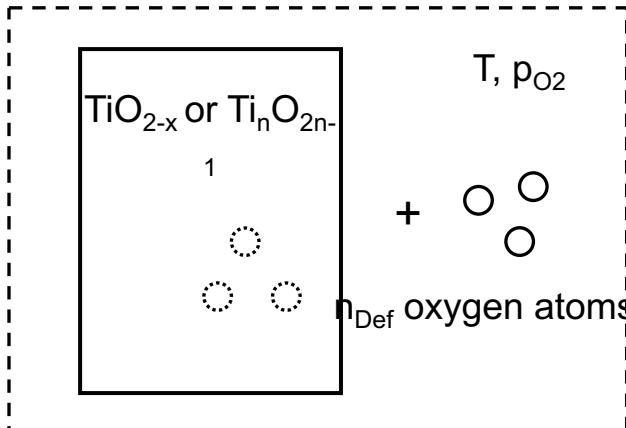
# Defect Formation Energies: Thermodynamical Formalism

Figure 5a

Initial state



Final state



$$\Delta G_f^{Def}(T, p_{O_2}) = \frac{1}{n_{TiO_2}} \left( G^{\text{supcell}}(T, p_{O_2}) + n_O^{Def} \mu_O^{\text{ref}}(T, p_{O_2}) \right) - \frac{1}{n_{TiO_2}} \left( n_{TiO_2} \mu_{TiO_2}^{\text{bulk}}(T, p_{O_2}) \right) \quad (1)$$

$$\begin{aligned} \Delta G_f^{Def}(T, p_{O_2}) = & \frac{1}{n_{TiO_2}} \left( E^{\text{supcell}}(0K) - n_{TiO_2} E_{TiO_2}^{\text{bulk}}(0K) \right) + \\ & \text{Phonon contribution} + \frac{1}{n_{TiO_2}} \left( F_{Vib}^{\text{supcell}}(T) - n_{TiO_2} F_{Vib\,TiO_2}^{\text{bulk}}(T) \right) + \\ & \text{pV contribution} + \frac{1}{n_{TiO_2}} p_{O_2} \left( V_{Vib}^{\text{Supcell}} - n_{TiO_2} V_{TiO_2}^{\text{bulk}} \right) + \\ & + \frac{n_O^{Def}}{n_{TiO_2}} \mu_O^{\text{ref}}(T, p_{O_2}) \end{aligned} \quad (2)$$

$$\begin{aligned} \Delta G_f^{Def}(T, p_{O_2}) = & \frac{1}{n_{TiO_2}} \left( E^{\text{supcell}}(0K) - n_{TiO_2} E_{TiO_2}^{\text{bulk}}(0K) \right) + \\ & + \frac{n_O^{Def}}{n_{TiO_2}} \mu_O^{\text{ref}}(T, p_{O_2}) \end{aligned} \quad (3)$$

# Defect Formation Energies: Oxygen chemical potential

$$\Delta G_f^{Def}(T, p_{O_2}) = \frac{1}{n_{TiO_2}} \left( E^{\text{supcell}}(0K) - n_{TiO_2} E_{TiO_2}^{\text{bulk}}(0K) \right) + \frac{n_O^{Def}}{n_{TiO_2}} \mu_O^{\text{ref}}(T, p_{O_2}) \quad (3)$$

Limits for the oxygen chemical potential:

Assuming the oxygen behaves as an ideal gas:

$$\mu_{O_2}(p_{O_2}, T) = 2\mu_O(p_{O_2}, T) = E_0 + (\mu_{O_2}^0 - E_0) \frac{T}{T^0} - \frac{5k}{2} T \ln\left(\frac{T}{T^0}\right) + kT \ln\left(\frac{P_{O_2}}{P_{O_2}^0}\right) \quad (5)$$

## Oxygen molecule's total energy at 0K

Oxygen molecule's standard chemical potential at T=298K  
and  $p_{O_2}=1\text{ atm}$

Expression (5) allows the calculation of  $\mu_{O_2}^0(T, p_{O_2})$  at any T and  $p_{O_2}$

# Oxygen chemical potential

CASTEP

$$\mu_{O_2}^0(p^0, T^0) = \frac{2}{y} \left( \mu_{M_x O_y}^{bulk} - x \mu_M^{bulk} - \Delta G_{M_x O_y}^0(p^0, T^0) \right)$$

$M_x O_y$ : ZnO, Anatase, Rutile,  $Ti_4 O_7$ ,  $Ti_3 O_5$

$$\mu_{O_2}^0(T^0, p_{O_2}^0) = \mu_{\text{mean}} +/\!-\! \Delta\mu$$

Now  $E_0$  has to be calculated

CRYSTAL

$E_0$  and the 0K total energy of the oxygen atom are calculated with CRYSTAL.

|                     | Exp. | PW-GGA (4) | CRYSTAL |
|---------------------|------|------------|---------|
| Binding energy [eV] | 2.56 | 3.6        | 2.53    |
| Bond length [ang]   | 1.21 | 1.22       | 1.23    |

Now  $\mu_{O_2}^0$  has to be calculated

$$\mu_{O_2}(p_{O_2}^0, T) = A(T - T \ln(T)) - \frac{1}{2} BT^2 - \frac{1}{6} CT^3 - \frac{1}{12} DT^4 - \frac{E}{2T} + F - GT \quad (6)$$

$T > 298K$  and  
 $p_{O_2} = 1atm$

$$\mu_{O_2}(p_{O_2}, T) = E_0 + (\mu_{O_2}^0 - E_0) \frac{T}{T^0} - \frac{5k}{2} T \ln\left(\frac{T}{T^0}\right) + kT \ln\left(\frac{P_{O_2}}{P_{O_2}^0}\right) \quad (5)$$

$T > 0K$  and any  $p_{O_2}$

# Results for the Magneli phases

Figure 8a

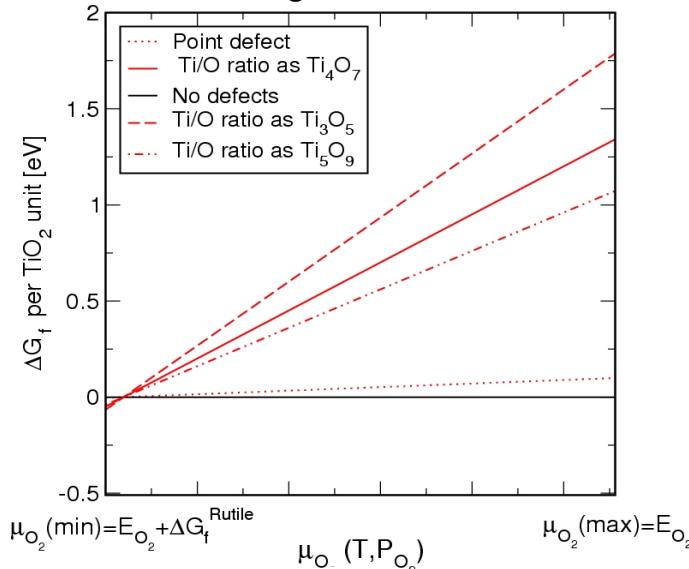
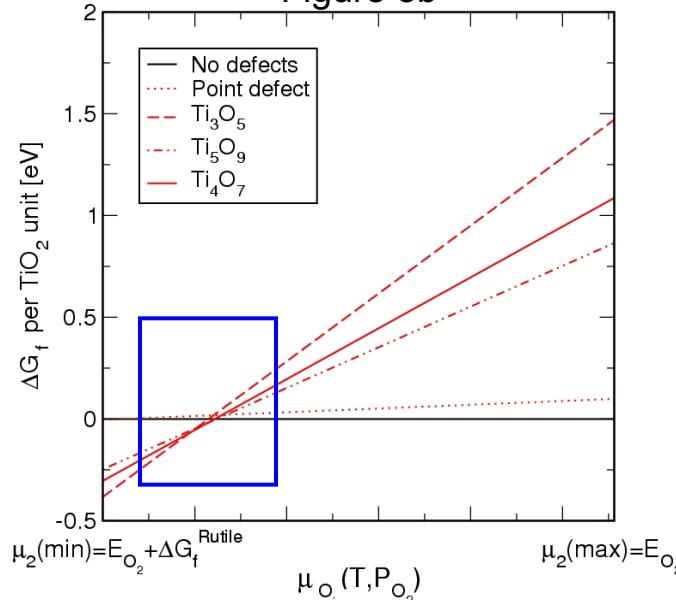


Figure 8b



## Isolated defects

$$\Delta G_{f\ isolat.}^{Def}(T, p_{O_2}) = \frac{n_O^{Def}}{n_{TiO_2}} \left( E^{\text{supcell}}(0K) - n_{TiO_2} E_{TiO_2}^{\text{bulk}}(0K) + \mu_O^{\text{ref}}(T, p_{O_2}) \right)$$

$$\left( \frac{n_O^{Def}}{n_{TiO_2}} \right)_{Ti_4O_7} = \frac{1}{4}$$

$$\Delta G_{\left(\frac{Ti}{O}\right)\text{like } Ti_4O_7}^{Def}(T, p_{O_2}) = \frac{1}{4} \left( E^{\text{Supcell}}(0K) - 27 E_{TiO_2}^{\text{bulk}}(0K) + \mu_O^{\text{ref}}(T, p_{O_2}) \right)$$

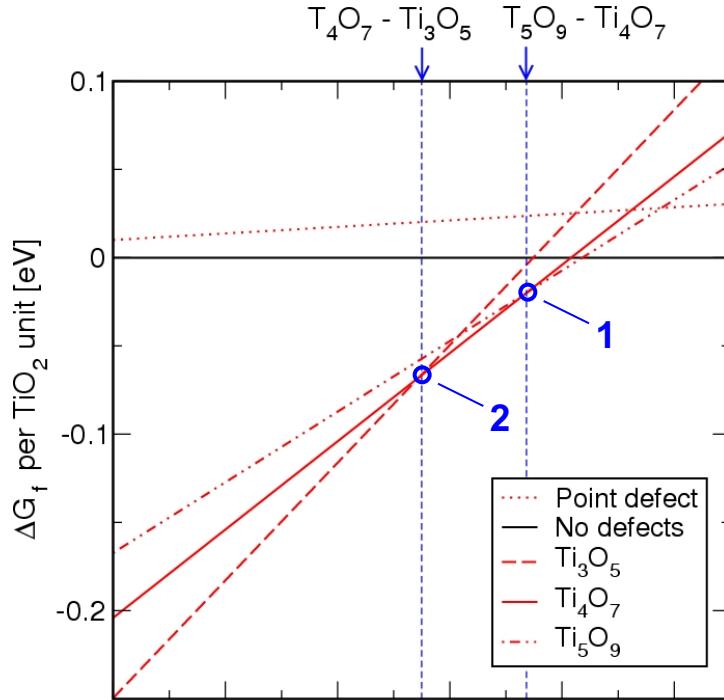
## Magneli phases

$$\Delta G_f^{Def}(T, p_{O_2}) = \frac{1}{n_{TiO_2}} \left( E^{\text{supcell}}(0K) - n_{TiO_2} E_{TiO_2}^{\text{bulk}}(0K) \right) + \frac{n_O^{Def}}{n_{TiO_2}} \mu_O^{\text{ref}}(T, p_{O_2})$$

$$\left( \frac{n_O^{Def}}{n_{TiO_2}} \right)_{Ti_4O_7} = \frac{1}{4}$$

$$\Delta G_{Ti_4O_7}^{Def}(T, p_{O_2}) = \frac{1}{16} \left( E^{Ti_4O_7}(0K) - 8 E_{TiO_2}^{\text{bulk}}(0K) \right) + \frac{1}{4} \mu_O^{\text{ref}}(T, p_{O_2})$$

# Results for the Magneli phases



$$\Delta G_{\text{Ti}_3\text{O}_5}^{\text{Def}}(\mu_O) = K_{\text{Ti}_3\text{O}_5} + \frac{\mu_O}{3}$$

$$\Delta G_{\text{Ti}_4\text{O}_7}^{\text{Def}}(\mu_O) = K_{\text{Ti}_4\text{O}_7} + \frac{\mu_O}{4}$$

$$\Delta G_{\text{Ti}_5\text{O}_9}^{\text{Def}}(\mu_O) = K_{\text{Ti}_5\text{O}_9} + \frac{\mu_O}{5}$$

Equilibrium point 1:

$$\Delta G_{\text{Ti}_4\text{O}_7}^{\text{Def}}(\mu_O) = \Delta G_{\text{Ti}_5\text{O}_9}^{\text{Def}}(\mu_O) \Rightarrow \mu_{\text{Ti}_4\text{O}_7-\text{Ti}_5\text{O}_9}^{\text{eq}}$$

Equilibrium point 2:

$$\Delta G_{\text{Ti}_4\text{O}_7}^{\text{Def}}(\mu_O) = \Delta G_{\text{Ti}_3\text{O}_5}^{\text{Def}}(\mu_O) \Rightarrow \mu_{\text{Ti}_4\text{O}_7-\text{Ti}_3\text{O}_5}^{\text{eq}}$$

$$\mu_{\text{Ti}_4\text{O}_7-\text{Ti}_5\text{O}_9}^{\text{eq}} = E_0 + (\mu_{\text{O}_2}^0 - E_0) \frac{T}{T^0} -$$

$$-\frac{5k}{2} T \ln\left(\frac{T}{T^0}\right) + kT \ln\left(\frac{P_{\text{O}_2}}{P_{\text{O}_2}^0}\right)$$

Relationship between  $P_{\text{O}_2}$  and  $T$  in the phase equilibrium.

# Results for the Magneli phases

Figure 10a

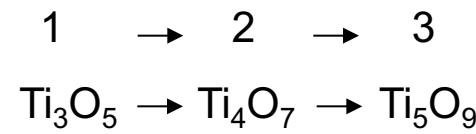
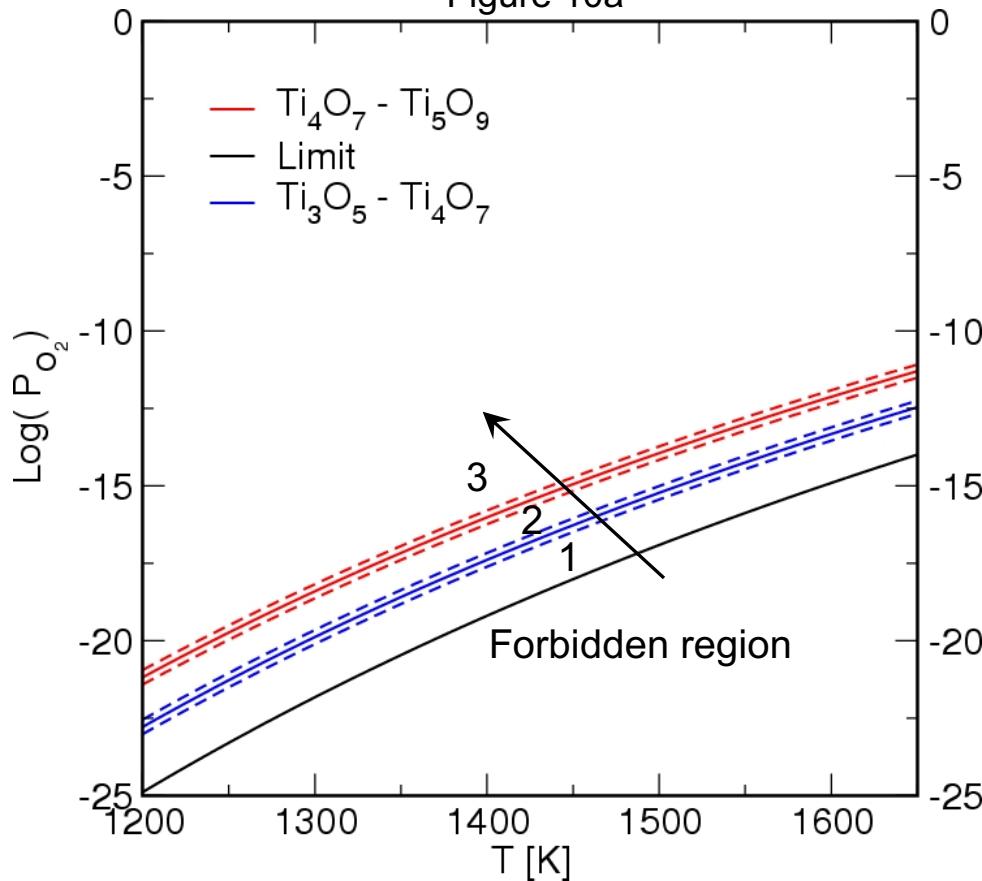
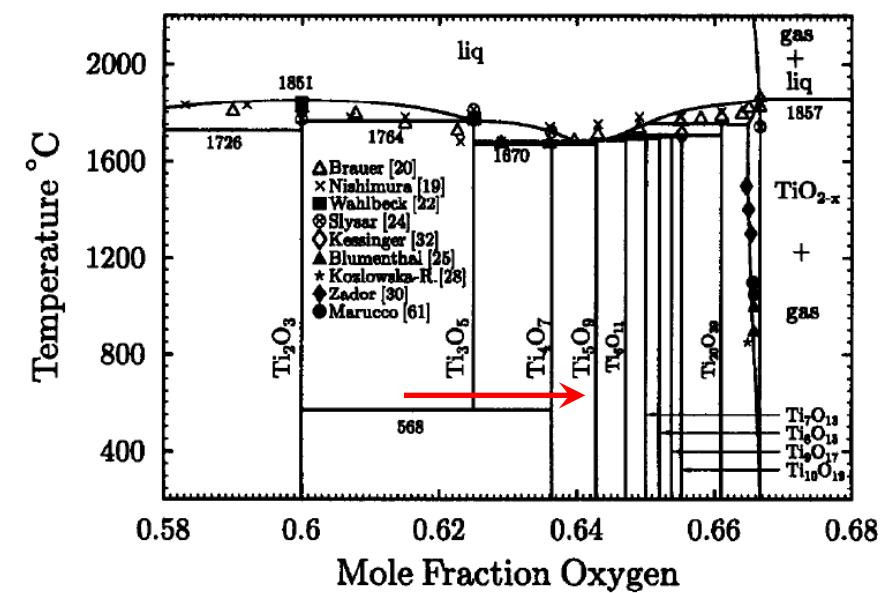


Figure 10b



$$\log_{10}(p_{\text{O}_2}) = \frac{K_{\text{Ti}_4\text{O}_7-\text{Ti}_5\text{O}_9}^{eq}}{T} + K_1 \ln\left(\frac{T}{T^0}\right) + K_2$$

# CASTEP Results for the Magneli phases

Figure 10a

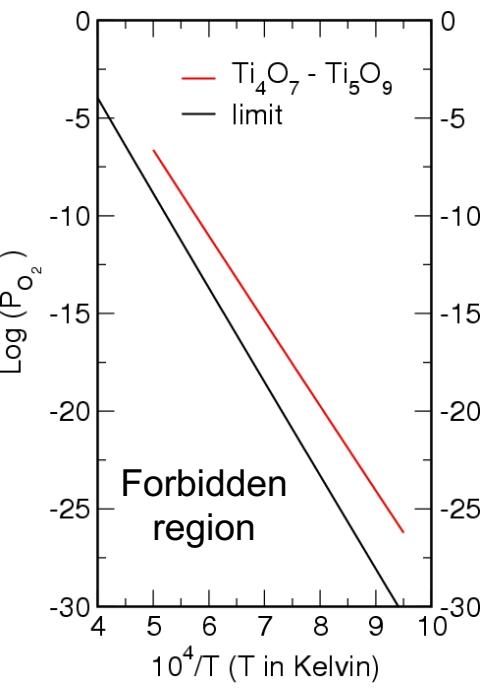


Figure 10b

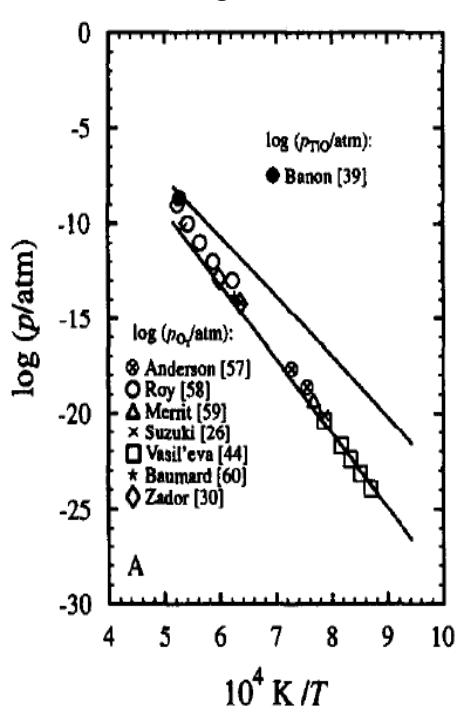


Figure 10c

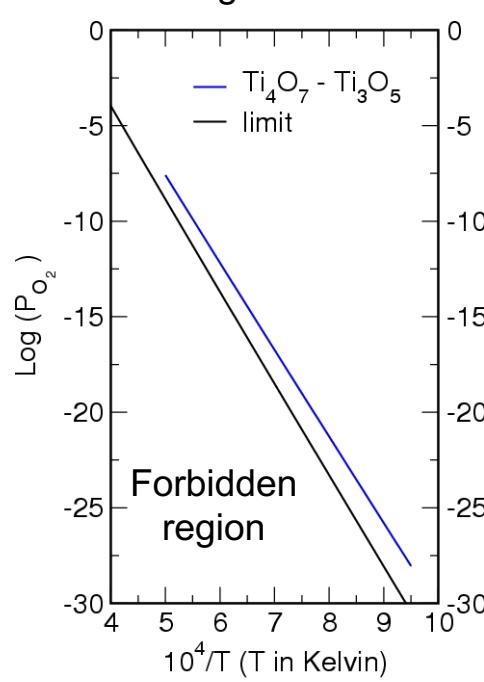
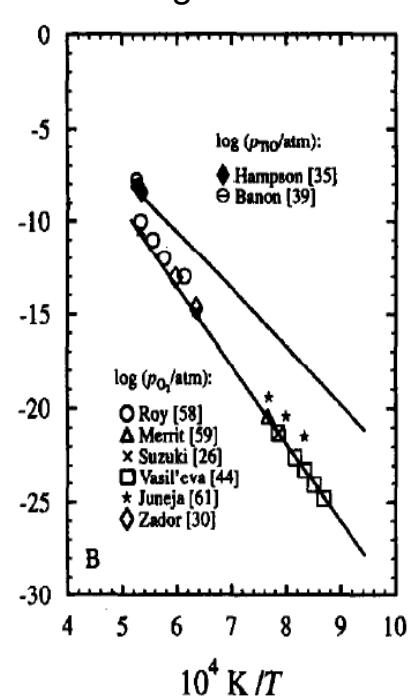


Figure 10d



# CRYSTAL Results for the Magneli phases

Figure 12a

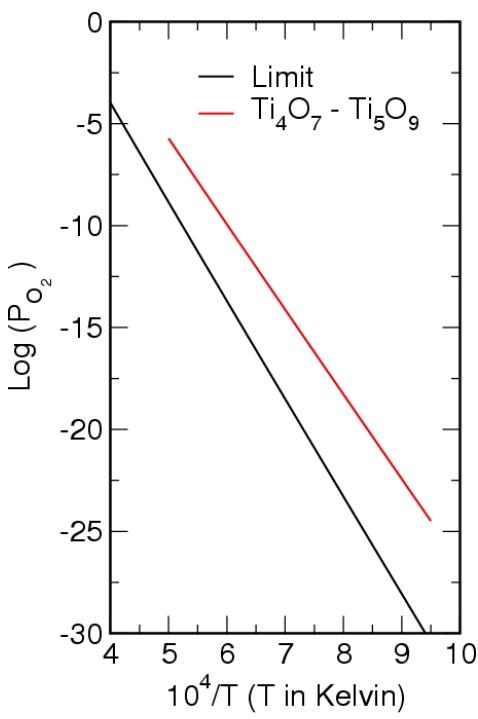


Figure 12b

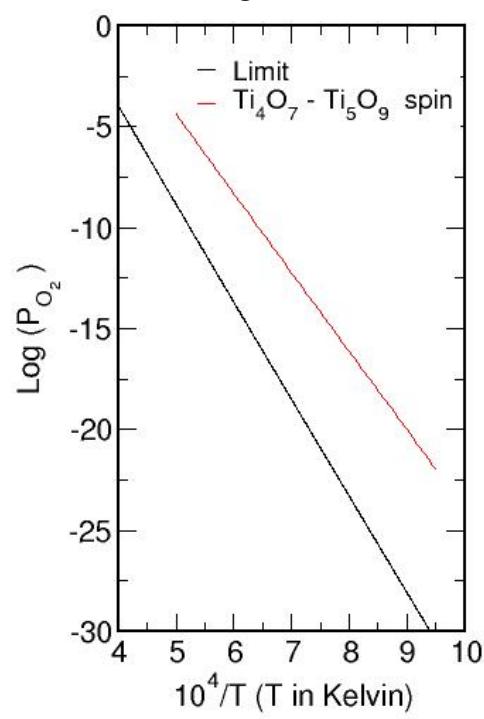
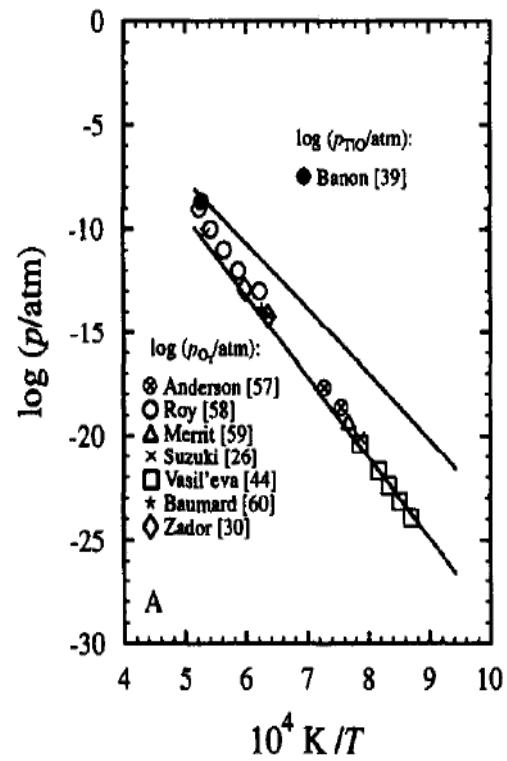
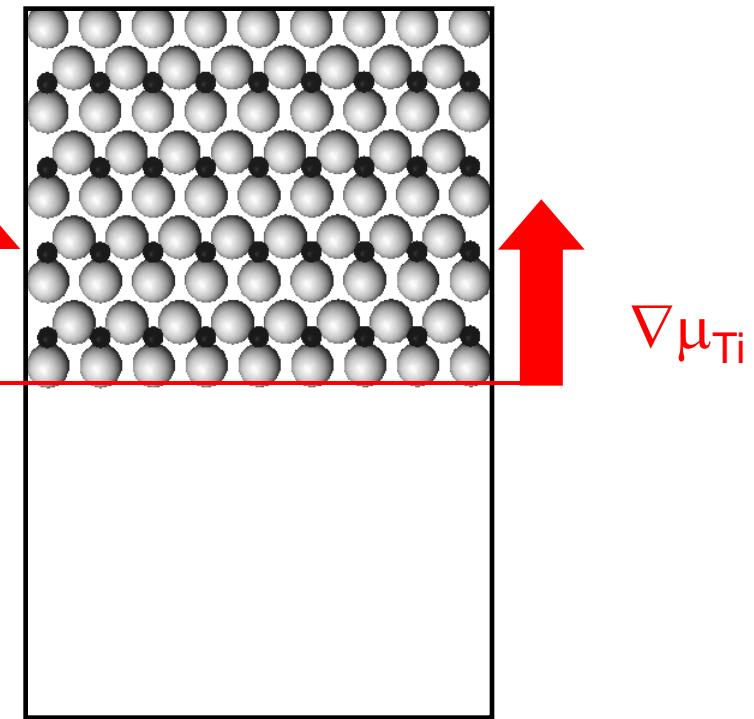
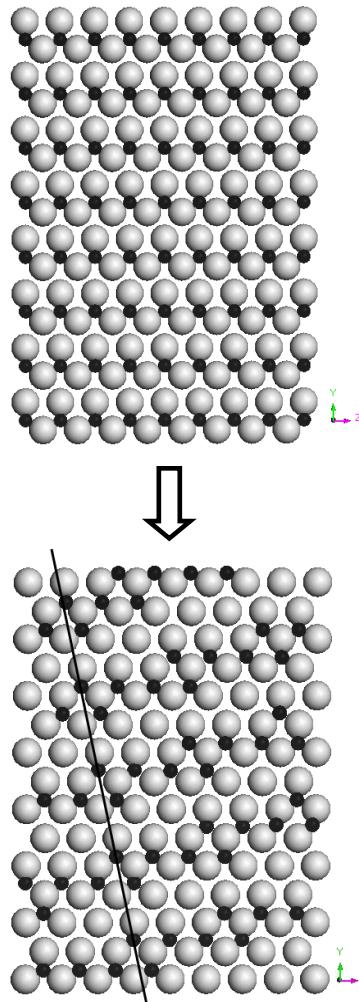


Figure 12c



# Formation mechanism for an oxygen-defective plane

Cation + anion (100) layer

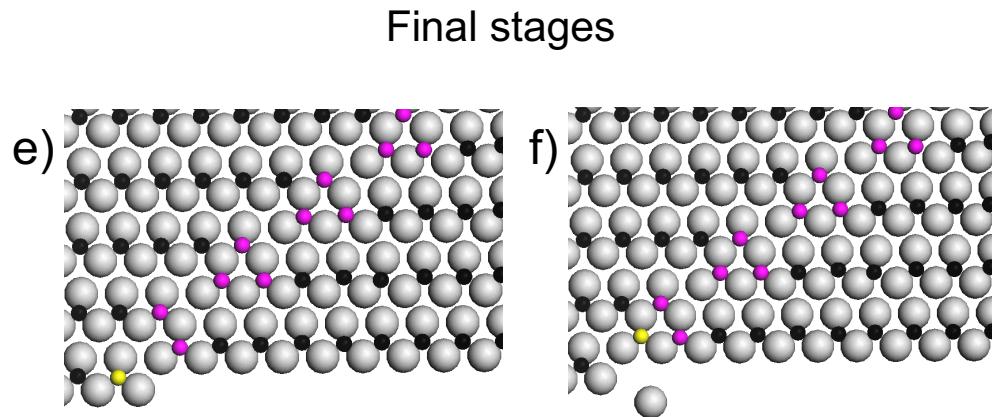
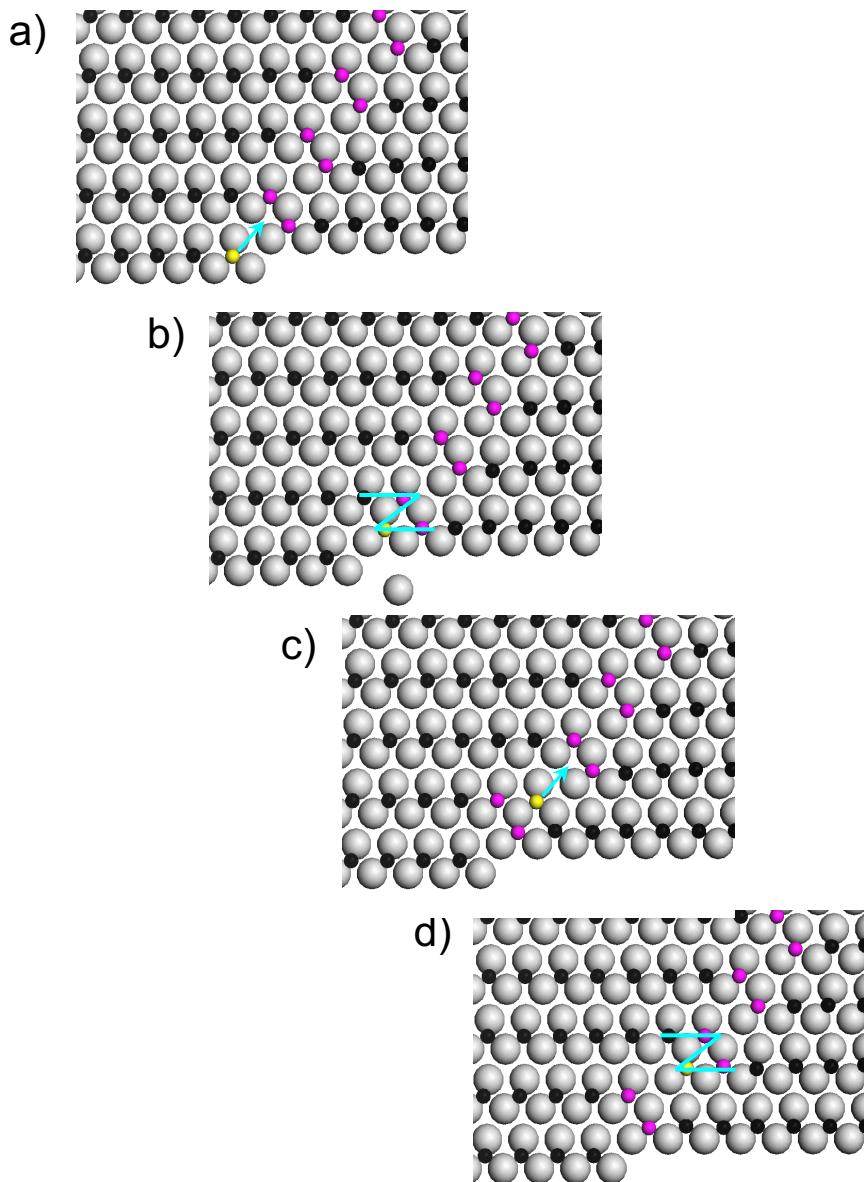


$$\mu_{TiO_2}^{bulk} = \mu_{Ti}^{supercell} + 2\mu_O^{ref}(p_{O_2}, T)$$

L. Bursill and B. Hyde, Prog. Sol. State Chem. Vol. 7, pp. 177, 1972.

S. Andersson and A. D. Waldsey, Nature Vol. 211, pp. 581, 1966.

# Formation mechanism for an oxygen-defective plane



- Antiphase boundaries (dislocation) acts as high conductivity paths for titanium.
- Dislocations are needed
- No long-range diffusion
- Formation of Ti interstitials.

# Conclusions

- The thermodynamics of rutile's higher oxides has been investigated by first principles calculations.
- First principles thermodynamics reproduce the experimental observations reasonably well.
- Spin does not affect the thermodynamics.
- At a high concentration of oxygen defects and low oxygen chemical potential, oxygen defects prefer to form Magneli phases.
- But, at low concentration of oxygen defects and low oxygen chemical potential, titanium interstitials proved to be the stable point defects.
- These results support the mechanism proposed by Andersson and Waldsey for the production the crystalline shear planes in rutile.

# Acknowledgements

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- Dr. Giuseppe Mallia
- Dr. Barbara Montanari
- Dr Keith Refson