Empirical Industrial Organization & Consumer Choice

2c Berry Logit Model for Market Level Data

Summer 2018

Sebastian Kranz, Ulm University

May 24, 2018

Market Level Data

- Assume we only observe sales, prices and attributes of J products in T different markets
  - example: Verbooven's car data
- Berry (1994) proposes models in which aggregate data is the outcome of discrete choices of a large number of consumers in each market.

Berry's Logit Model

- Utility of consumer n from product j in market t

$$U_{njt} = \quad \delta_{jt} \quad + \varepsilon_{njt}$$

  with

$$\delta_{jt} = x_{jt}'\beta - \alpha p_{jt} + \xi_{jt}$$

  - δ_{jt} similar to representative utility V, same for all consumers
  - ξ_{jt} ("xi") is an unobserved aggregate demand shock for product j in market t.
  - p_{jt} is the price of product j in market t
  - x_{jt} are the attributes of product j in market t
- Expected utility of not buying any product δ_{0t} is normalized to 0.

Exercise 8: Berry Logit Model

(a) Explain why in Berry's logit model, market share of product j in market t satisfies

$$s_{jt} = \frac{\exp(\delta_{jt})}{1 + \sum_{r=1}^{J} \exp(\delta_{rt})}$$

(b) Show that then following condition is satisfied

$$\log s_{jt} - \log s_{0t} = x_{jt}'\beta - \alpha p_{jt} + \xi_{jt}$$

(c) Assume we know the total market size M, i.e. the number of consumers that possibly could by a product. Furthermore assume that ξ_{jt} is uncorrelated with product attributes x_{jt} and the price p_{jt}. How could we then estimate the parameters α and β from market data?

(d) Use the function `sim.logit.prod` in the file `sim_logit_prod.r` to simulate some product and cost data. Simulate prices as a random 5-10% markup over costs and compute market shares for specified values of β and α. Check whether OLS estimation seems to yield consistent estimates.

(e) Discuss the assumption that ξ_{jt} is uncorrelated with product attributes x_{jt} and the price p_{tj}. Tell one or several stories for why it could be violated.

Exercise 9: Bertrand Competition in Berry Logit

Consider a single market with J firms that each produce one product and have constant marginal cost c_j. Firms compete by setting simultaneously prices p_j. Firms know the demand shocks ξ_j.

(a) Write down firm j's profit function and the the first order condition of the optimal price of firm j.

(b) Write an R function `bertrand.prices.logit` that takes as argument a data.frame mdat that contains product attributes x, demand shocks ξ and cost levels c for one market and numerically solves the FOC for the Bertrand prices in that market. Simulate a data set with Bertrand prices.

(c) Using this first order condition determine the sign of how the optimal price p_j changes if we we have, ceteris paribus, an increase in ξ_j and an increase in x_{ik} for $i \neq j$? Analyse the correlation between p_j and these variables in simulated data. Discuss what these results suggest for estimating

$$\log s_{jt} - \log s_{0t} = x_{jt}'\beta - \alpha p_{jt} + \xi_{jt}.$$

(d) Berry (1994) and BLP (1995) propose to instrument the price p_{jt} with a function of the product characteristics of all other firms, e.g. their sum $X_{-jtk} = \sum_{i \neq j} x_{itk}$. Discuss this instrument. Estimate β for a simulated data set with Bertrand prices via OLS and IV. Discuss the results.

Exercise 10: Estimating cost functions

Assume marginal cost of producing a product depend linearly on the product attributes and are given by

$$c_{jt} = x_{jt}'\gamma + \omega_{jt}$$

with ω_{jt} being an unobservable cost shock that is uncorrelated with product characteristics x_{jt}. If we also assume that firms compete á la Bertrand, we can estimate their marginal cost cost function, i.e. the parameters γ, just by observing market outcomes and product attributes. Firm j's FOC can then be rewritten as

$$p_{jt} = \frac{1}{\alpha}\frac{1}{(1-s_{jt})} + x_{jt}'\gamma + \omega_{jt}$$

(a) Estimate the parameters of the cost function γ in a consistent way on the simulated data set.

Exercise 11: Restricted Substitution Patterns and IIA

The main problem of the simple Berry logit model is that it imposes strong and often unrealistic restrictions on the substitution patterns between products.

(a) Show that in the logit model, the ratio of an individual's choice probabilities P_{nj}/P_{nr} of two products j and r is independent of the attributes of any third product. This characteristic of the logit model is called "Independence of Irrelevant Alternatives" (IIA)

(b) The red-bus-blue-bus problem: Consider a logit model in which a consumer can choose between taking a car to work $j = 1$ or a red bus $j = 2$ and assume he has the same representative utility from both choices. Assume now a blue bus, $j = 3$, is introduced and consumers like it as much as the red bus. How will choice probabilities in the logit model change? Which choice probabilities would we realistically suspect instead?

Exercise 11 (continued)

(c) In Berry's logit model the cross price effects on market shares satisfy

$$\frac{\partial s_j}{\partial p_r} = \alpha s_r s_j \forall r \neq j$$

Train (Section 3.2.2) explains why this result is another manifestation of IIA. Consider now a car market with two sport cars Porsche Boxter and Ferrari 458 that each have a market share of 5%, and one middle class model say VW Golf with market share 10%. Assume that - ceteris paribus- Ferrari reduces its price by 5000€, for which car (Porsche or the VW Golf), would a Berry logit model predict a larger reduction in sales? Discuss this result.

Extensions for Flexible Substitution Patterns

- The restricted substitution patterns make the simple Berry logit model typically ill suited for analyzing market data
  - but insights are very valuable for understanding more flexible models
- We will discuss two popular extensions that allow for more flexible substitution patterns
  - Nested Logit (Train Chp. 4, Berry, 1994)
  - Random Coeffiecient Logit / Mixed Logit (Train Chp. 6 & 14, Berry, Levinsohn & Pakes (BLP), 1995)

Nested-Logit Model

- A simple way of introducing more reasonable substitution patterns by assuming correlation among similar types of products. This model is also very tractable, more realistic than the logit and can therefore be very helpful to estimate demand
- Products are divided into $1+G$ exhaustive and mutually exclusive groups (or nests) indexed by g
    - the outside good $j=0$ is the only element of $g=0$
- Several products j are part of a group and a greater correlation within these brands is allowed

The Nested-Logit Model

- Following Berry (1994), we can write the utility function for consumer n for product j that is in group g as follows:

$$u_{nj} = x_j'\beta - \alpha p_j + \xi_j + [\zeta_{ig} + (1-\sigma)\varepsilon_{ij}]$$

  where σ represents the within group correlation $(0 < \sigma < 1)$ and ζ_{ng} is a common utility shock to all products in group g
- The distribution of ζ_{ng} depends on σ and is the only distribution such that $\zeta_{ng} + (1-\sigma)\varepsilon_{nj}$ has extreme value distribution if ε_{nj} is extreme value distributed

Market Shares in Nested Logit Model

- The total market share of a product j that is in group g is the product of the probability s_g of choosing group g and the probability $\bar{s}_{j|g}$of choosing product j given that group g has been chosen:

$$\begin{aligned} s_j &= \frac{D_g^{1-\sigma}}{\sum_h D_h^{1-\sigma}} \times \frac{\exp(\delta_j/(1-\sigma))}{D_g} \\ &= \frac{\exp(\delta_j/(1-\sigma))}{D_g^{\sigma} \sum_h D_h^{1-\sigma}} \end{aligned}$$

with

$$\begin{aligned} D_g &= \sum_{j \in g} \exp(\delta_j/(1-\sigma)), \\ \delta_j &= x_j \beta - \alpha p_j + \xi_j. \end{aligned}$$

Demand Estimation in Nested Logit

- Berry (1994) shows that in a nested logit model market shares satisfy

$$\ln(s_j) - \ln(s_0) = x_j\beta - \alpha p_j + \sigma \ln \bar{s}_{j|g} + \xi_j.$$

  where $\bar{s}_{j|g}$ is the market share of product j in group g and s_0 is the market share of the outside good.
- Typically p_j and $\bar{s}_{j|g}$ will be correlated with the error term ξ_j, i.e. one has to use an IV estimator to estimate β, α and σ.
- Candidate instruments are sum of attributes of other products inside the group and outside the group, if product attributes x are assumed to be exogenous.

Cross Price Effects in Nested Logit

- Derviatives of of market shares w.r.t. to prices

$$\frac{\partial s_j}{\partial p_k} = \begin{cases} -\alpha \frac{1}{1-\sigma} s_j \left(1 - \sigma \bar{s}_{j/g} - (1-\sigma) s_j\right) & \text{if } k = j \text{(own price effect)} \\ \alpha \frac{1}{1-\sigma} s_j \left(\sigma \bar{s}_{k/g} + (1-\sigma) s_k\right) & \text{if } k \neq j \text{but in same group } g \\ \alpha s_j s_k & \text{if } k \text{and } j \text{are in different } g \end{cases}$$

- (Cross-) Price Elasticities

$$\eta_{jk} = \frac{\partial s_j}{\partial p_k} \frac{p_k}{s_j}$$

- Stronger substitution among products within group than among products outside group. Grouping thus allows more flexible substitution patterns.
- Derivatives reduce to those of standard logit model for $\sigma = 0$.

Bertrand Competition with Multi-Product Firms

- Consider a single market in which firms can produce multiple products.
- Let $\mathscr{J}_f$ be the set of product produced by firm f. Profits

$$\pi_f = M \sum_{j \in \mathscr{J}_f} s_j (p_j - c_j)$$

- FOC for profit maximization of firm f

$$s_j + \sum_{r \in \mathscr{J}_f} \left(\frac{\partial s_r}{\partial p_j} (p_r - c_r) \right) = 0 \forall j \in \mathscr{J}_f$$

- Verboven & Ivaldi (2002) show that in the nested logit model the FOC become

$$p_j = c_j + \frac{1-\sigma}{\alpha} \frac{1}{(1 - \sigma \bar{s}_{f/g} - (1-\sigma) s_f)} \forall j$$

  - market share of firm f in group g
  - total market share of firm f in group g
- Existence or uniqueness of an equilibrium has not yet generally been proven

Estimating cost functions in the nested logit model

- Verboven & Ivaldi (2002) estimate demand and cost parameters $\alpha, \beta, \gamma, \sigma$ simultaneously via non-linear 3SLS, yet I am not aware of a stable implementation in R or Stata. Here an alternative (less efficient) method that is similar to Berry (1994) for non-nested logit models:
  - Estimate the demand function via IV, yielding consistent estimates $\hat{\alpha}, \hat{\beta}$ and $\hat{\sigma}$.
  - Use the estimate $\hat{\sigma}$ to construct an auxilliary variable

$$\hat{v}_f = \frac{1}{(1 - \hat{\sigma}\bar{s}_{f/g} - (1 - \hat{\sigma})s_f)}$$

  - Estimate via IV (because $\hat{v}_f$ is endogenous) the resulting supply function equation

$$p_{jt} = x_{jt}'\gamma + \frac{1-\sigma}{\alpha}\hat{v}_{ft} + \omega_{jt}$$