Homework 2

ECE6550 Linear Control Systems

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Due: October 4, 2011 (Oct. 11 for DL students)

1.

Given a matrix A, whose characteristic polynomial is given by

$$\lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda + a_0.$$

a

Assuming that A^{-1} exists, use the Cayley-Hamilton theorem to derive an expression for the inverse, using the characteristics polynomial above.

b

Let A be the 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Using your result in **1a**, derive an explicit expression for A^{-1}.

2.

Read Chapter/Lecture 7 in the textbook (available on Tsquare under Resources/JordanChapter.pdf). Using your newfound knowledge, find

$$e^{At}$$

for the matrix

$$A = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 3 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix}.$$

(Note: It's enough if you write down the matrices you need to multiply – you do not actually have to explicitly carry out the rather tedious multiplications.)

Hint: If you have the symbolic matlab toolbox, you could have used the command `jordan`, which would have given you the following:

```
>> [P,J]=jordan(A)

P =

  -1          1          1          1
   1         -1          0          0
   0          0         -1          0
   0          1          1          0

J =

   1      0      0      0
   0      2      0      0
   0      0      4      1
   0      0      0      4
```

3.

Let

$$\dot{x}(t) = \begin{bmatrix} \cos(t) & t \\ t & \cos(t) \end{bmatrix} x(t), \quad x(t_0) = x_0.$$

What is $x(t)$?

4.

Let

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u.$$

We want to implement this as a discrete time, sampled-data system. In other words, find the matrices $\hat{A}$ and $\hat{B}$ such that the discrete-time system is given exactly by

$$x_{k+1} = \hat{A}x_k + \hat{B}u_k.$$

(You may assume that the sample time is δ.)

5.

Use the $A, B, \hat{A}, \hat{B}$ matrices from Question **4**. Let

$$y_k = \begin{bmatrix} 0 & 1 \end{bmatrix} x, \quad x(0) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad u_k = e^{-k\delta}.$$

Go to Tsquare under Resources/m-files and download the matlab file `ExactVsApprox.m` and insert your $\hat{A}$ and $\hat{B}$ matrices. Plot your solutions for different δ values. When do the approximate and exact solutions start to differ?