Homework 1

ECE6550 Linear Control Systems

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Due: September 13, 2011 (Sept. 20 for DL students)

1.

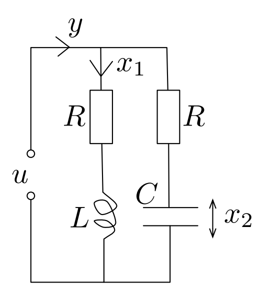
Sometimes there is an additional term in the output of a linear system, which takes on the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du. \end{aligned}$$

What is the transfer function $G(s)$ for this type of system, where $Y(s) = G(s)U(s)$?

2

Consider the electrical network below.



Using our extensive circuits knowledge, we know that

$$y = x_1 + C\dot{x}_2$$

as well as that the following relationships hold for the two loops in the network
Loop 1:

$$u = Rx_1 + L\dot{x}_1$$

Loop 2:

$$u = RC\dot{x}_2 + x_2.$$

Find a state space realization of this system.

3

Let

$$y^{(4)} + 7y^{(3)} - 16\ddot{y} + 3\dot{y} + 12y = 2u - \dot{u} - 4u^{(3)}.$$

a

Find the Controllable Canonical Realization of this system.

b

Find the Observable Canonical Realization of the system.

4

A so-called unicycle model of a mobile robot states that

$$\begin{aligned}\dot{x}_1 &= u_1 \cos(x_3)\\ \dot{x}_2 &= u_2 \sin(x_3)\\ \dot{x}_3 &= u_2,\end{aligned}$$

where (x_1, x_2) is the position of the robot, x_3 is its orientation, u_1 is its velocity and u_2 the angular velocity.

a

Linearize this system around $(x_1, x_2, x_3) = (0, 0, \pi/2)$ and $(u_1, u_2) = (0, 0)$ in order to obtain a a linear state space model.

b

Without worrying too much about the technical definition, a system is *controllable* if it is possible to drive the system between any two states (by picking a suitably clever input signal). Is the system in **4a** controllable?

5

Control systems can be found everywhere. Look around you and list 5 everyday devices that require control for their operation. (By control, I mean they have to measure something and then change their dynamics, i.e. what they are doing, accordingly.)