# Homework 3

## ECE6550 Linear Control Systems

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Due: October 27, 2011 (Nov. 3 for DL students)

## 1.

Consider a particle whose acceleration is controlled directly through

$$\ddot{y} = u.$$

Let $x_1 = y$ and $x_2 = \dot{y}$.

### a

Find the transition matrix $\Phi(t, t_0)$ and reachability Grammian $\Gamma$ associated with this system.

### b

Recall that the optimal $u$, i.e. the control signal that minimizes

$$\int_{t_0}^{t_1} u^T(t)u(t)dt,$$

while driving the system from $x(t_0) = x_0$ to $x(t_1) = x_1$ is given by

$$u(t) = B^T\Phi^T(t_1, t)\Gamma^{-1}(x_1 - \Phi(t_1, t_0)x_0),$$

as long as the system is completely controllable.

Compute the optimal control action for $\ddot{y} = u$ that takes the system from $y(0) = 0, \dot{y}(0) = 0$ to $y(1) = 1, \dot{y}(1) = 0$.

## 2.

Consider the scalar, nonlinear system

$$\dot{x} = ax^p,$$

where $a$ is a non-zero, real number and $p$ is a positive integer. For what values of $a$ and $p$ is the system asymptotically stable? For what values is it unstable?

## 3.

This question concerns the *duality* between controllability and observability. Show that the system

$$\begin{aligned} \dot{x} &= Ax \\ y &= Cx \end{aligned}$$

is completely observable if and only if the system

$$\dot{x} = A^T x + C^T u$$

is completely controllable.

## 4.

Let

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1-\alpha & 1 \end{bmatrix} x + \begin{bmatrix} \alpha \\ 1 \end{bmatrix} u.$$

For what values for $\alpha$ is this system completely controllable? Interpret your answer.

## 5.

Show that the controllable canonical realization does in fact result in a completely controllable system.