Homework 4

ECE6550 Linear Control Systems

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1

Let

$$\dot{x} = \begin{bmatrix} \alpha & 0 \\ \beta & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,$$

where $\alpha \neq \beta + 1$.

a

Is this system completely controllable?

b

Find, if possible, K such that $u = -Kx$ places the poles (eigenvalues) of the closed-loop system in $\lambda = -1 \pm 5j$.

2

Now, use the same system as in Question 1 but assume that we have an output

$$y = \begin{bmatrix} 2 & 0 \end{bmatrix} x.$$

a

Is the system completely observable?

b

Find, if possible, the output feedback $u = -Ly$ such that the poles (eigenvalues) of the closed-loop estimation error dynamics ($\dot{e} = (A - LC)e$) are $\lambda_1 = -1, \lambda_2 = -2$.

3

Let A, B, C be given as in Questions 1,2.

a

Find the controllable decomposition of this system.

b

Let A, B, C be given as in Questions 1,2. Find the observable decomposition of this system.

4

Let A, B, C be given as in Questions 1,2. Find the Kalman decomposition of this system. What is the McMillan degree of this system?

5

Let

$$\dot{x} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u.$$

For what values of $T > 0$ (if any) is it possible to drive this system from

$$x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{to} \quad x(T) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}?$$