



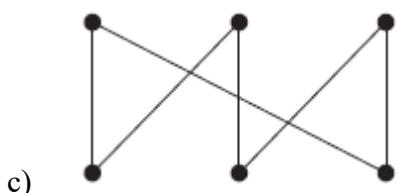
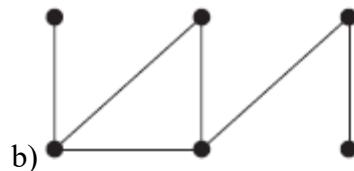
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Date : \_\_\_\_\_

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**ASSIGNMENT 4 – Trees and Finite Automata**

**Group of 3, Due date: 27 Januari 2025**

- 1) Which of these graphs are trees? Justify your answer.



- 2) Construct a complete binary tree of height 4 and a full 3-ary tree of height 3.

- 3) Represent the expression  $((x+2) \uparrow 3) * (y - (3+x)) - 5$  using a binary tree.

Write this expression in pre order, in order and post order notation.

- 4) What is the value for the post order notation

$$521--14++*$$



## SECI1013: DISCRETE STRUCTURE SEM 1 20242025

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5) Given rooted tree in Figure 1

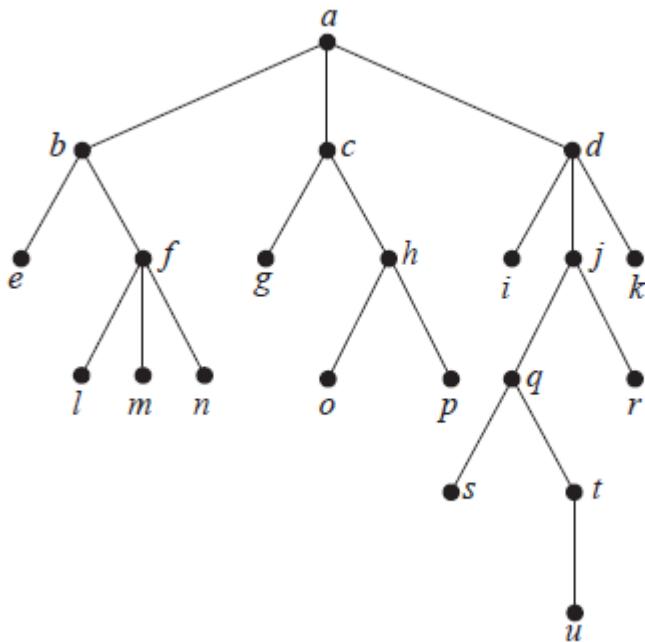


Figure 1

- a) Which vertex is the root?
  - b) Which vertices are internal node?
  - c) Which vertices are leaves?
  - d) Which vertices are children of j?
  - e) Which vertex is the parent of h?
  - f) Which vertices are siblings of o?
  - g) Which vertices are ancestors of m?
  - h) Which vertices are descendants of b?
- 6) How many vertices does a full 5-ary tree with 100 internal vertices have?
- 7) How many leaves does a full 4-ary tree with 1000 vertices have?



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- 8) Use Kruskal algorithm to find the minimum spanning tree for the following graph in Figure 2

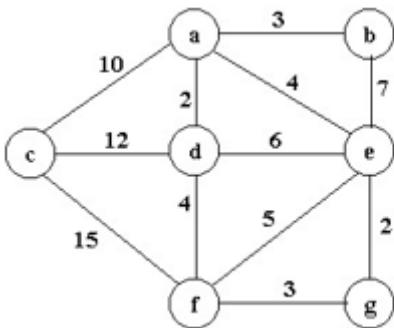


Figure 2

- 9) A chain letter starts with a person sending a letter out to 6 others. Each person is asked to send the letter out to 6 others, and each letter contains a list of the previous four people in the chain. Unless there are fewer than four names in the list, each person sends one dollar to the first person in this list, removes the name of this person from the list, moves up each of the other three names one position, and inserts his or her name at the end of this list. If no person breaks the chain and no one receives more than one letter, how much money will a person in the chain ultimately receive?

- 10) Construct a state transition diagram of a DFA that accepts all strings over  $\{a, b, c\}$  that begin with  $a$ , contain exactly two  $b$ 's, and end with  $c$ .

- 11) Construct a state transition diagram of a FSM that accepts the given set of strings over  $\{a, b\}$ :
- contain exactly two  $b$ 's.
  - at least one  $b$ .
  - odd number of  $a$ 's



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- 12) A description of an automatic telephone answering machine is shown in Table 2. When a call arrives, the phone rings. If the phone is not picked up, then on the third ring, the machine answers. It plays a pre-recorded greeting requesting that the caller leave a message, then records the caller's message, and then automatically hangs up. If the phone is answered before the third ring, the machine does nothing.

Table 2

States		Input		Output	
$q_0$	idle (nothing is happening)	$i_1$	incoming ringing signal	0	default output when there is nothing interesting to say
$q_1$	one ring has arrived	$i_2$	a telephone is picked up	1	answer the phone and start the greeting message
$q_2$	two rings have arrived	$i_3$	greeting message is finish playing	2	start recording the incoming message
$q_3$	playing the greeting message	$i_4$	end of message detected	3	recorded an incoming message
$q_4$	recording the message	$i_5$	no input of interest		

- a) Construct a state transition table by completing table below.

	$f_s$					$f_o$				
	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$
$q_0$	$q_1$	$q_0$	$q_0$	$q_0$	$q_0$	0	0	0	0	0
$q_1$	$q_2$	$q_0$	$q_1$	$q_1$	$q_1$	0	0	0	0	0
$q_2$	$q_3$	$q_0$	$q_2$	$q_2$	$q_2$	1	0	0	0	0
$q_3$	$q_3$	$q_3$	$q_3$	$q_3$	$q_3$	2	2	2	2	2
$q_4$	$q_4$	$q_4$	$q_4$	$q_0$	$q_4$	3	3	3	3	3

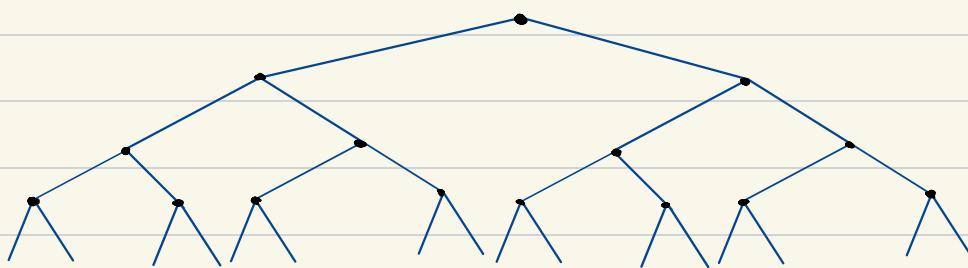
- b) Based on answer in (a), construct a state transition diagram for the telephone answering machine.

question 1

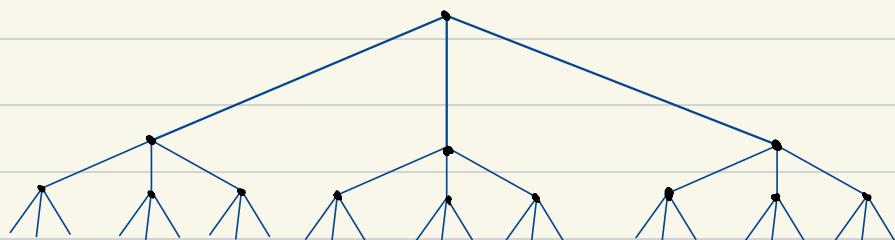
- a) Not a tree because the graph is not connected and is not a simple path.  
It has 6 vertices and 4 edges
- b) Not a tree because the graph is a simple circuit. It has 6 vertices and 6 edges
- c) Not a tree because the graph is a closed circuit. It has 6 vertices and 6 edges

question 2

Complete binary tree of height 4

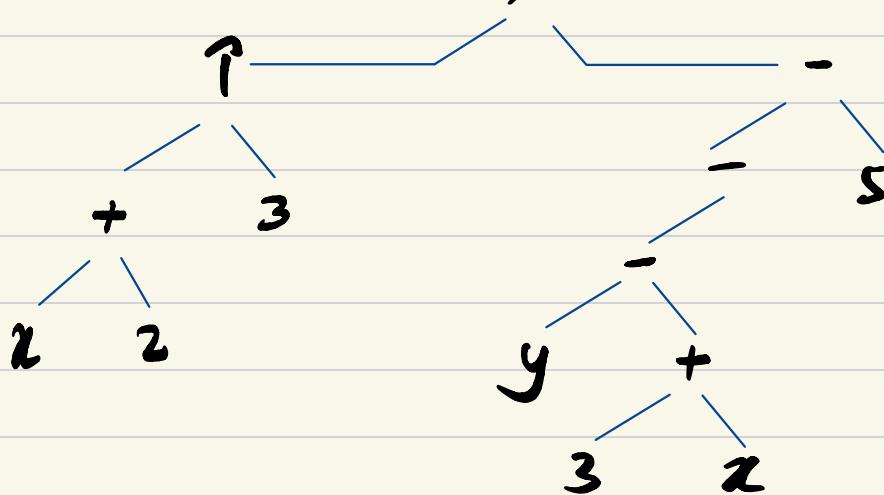


Full 3-ary tree of height 3



question 3

Binary tree for Representation  $((x+2)^3 * (y-(3+x))-5$



pre order notation : \*  $\uparrow + x \ 2 \ 3 \ --- y + 3 \ x \ 5$

post order notation :  $x \ 2 + 3 \uparrow y \ 3 \ x + -- 5 - *$

#### question 4

5: 2 1 -- 1 4 ++ \*

$$\hookrightarrow 2-1=1$$

5 1 - 1 4 ++ \*

$$\hookrightarrow 5-1=4$$

4 1 4 + + \*

$$\hookrightarrow 1+4=5$$

4 5 + \*

$$\hookrightarrow 4+5=9$$

9 \*

#### question 5

a) root vertex : {a}

b) internal node : {a,b,c,d,f,h,j,q,t}

c) leaves : {e,l,m,n,g,o,p,i,s,u,r,k}

d) children of j : {q,r}

e) parent of h : {c}

f) siblings of o : {p}

g) ancestors of m : {a,b,f,m}

h) descendants of b : {b,e,f,l,m,n}

#### question 6

full 5-ary tree , m = 5

internal vertices , i = 100

vertices , n = mi + 1

$$= (5)(100) + 1$$

$$= 501$$

#### question 7

full 4-ary tree , m = 4

vertices , n = 1000

leaves , l =  $n - (\frac{n-1}{m})$

$$= 1000 - (\frac{1000-1}{4})$$

$$= 750.25$$

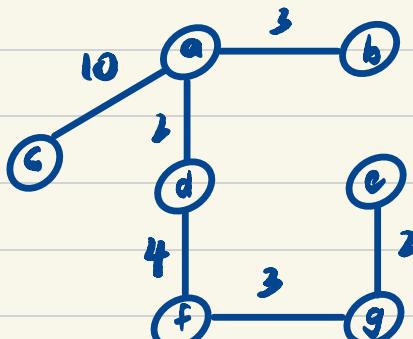
$$= 750$$

## Question 8

edges	eg	ad	ab	fg	ae	af	ef	de	be	ac	cd	cf
weight	2	2	3	3	4	4	5	6	7	10	12	15

List of vertices and edges that will be connected but will not form a circuit with minimum weight

eg	2
ad	2
ab	3
fg	3
af	4
ca	10



Total weight = 24

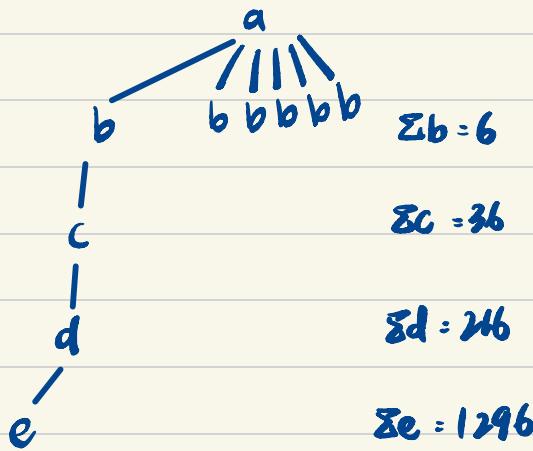
## Question 9

a : first person

b : siblings in the 1st batch of 6 people who receive letter

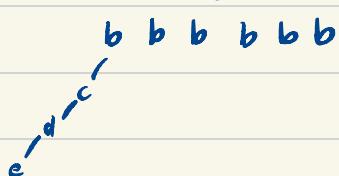
c : siblings in the 2nd batch of 6 people who receive letter

d : siblings in the 3rd batch of 6 people who receive letter

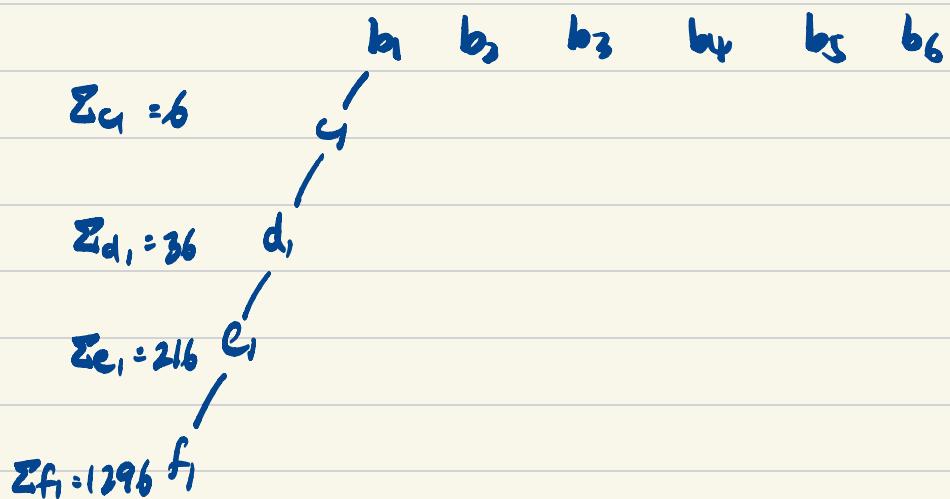


When the letters reached e batch people, there will be 4 names on the list. Each person in batch e will have to give one dollar to the first person on the list which is a. Since  $\Sigma e$  is 1296, a will receive 1296 dollars from everyone in Batch e.

Everyone in batch e will remove the first person on the list, move up each of the other three names one position, and insert their name at the end of this list.



The tree is broken up into 6 trees, and the sum batch of each tree now divide by 6

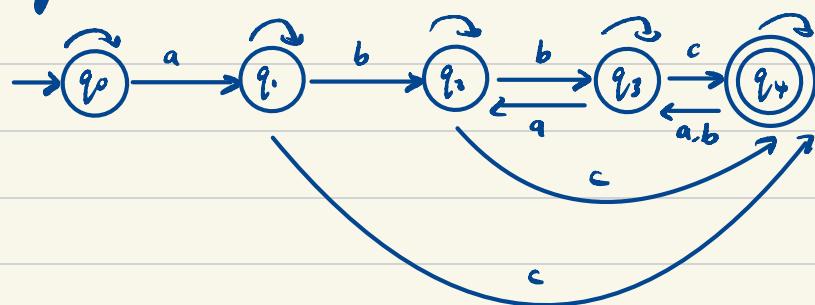


Each tree have the same number of leaves as the tree root  $a$ . Therefore, we can determine that each person in the chain receive 1296 dollar as same as person A

### Question 10

$$S = \{q_0, q_1, q_2, q_3, q_4\}$$

$$I = \{a, b, c\}$$



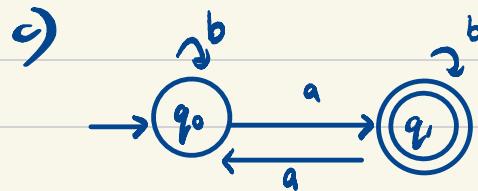
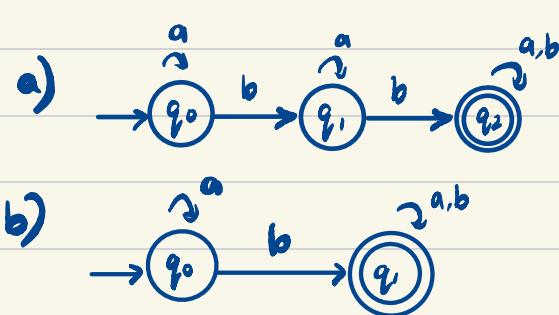
### Question 11

$$S = \{q_0, q_1, q_2\}$$

$$I = \{a, b\}$$

$$O = \{0, 1\}$$

$q_0$  = initial state



## Question 12

a)

	$f_s$					$f_o$				
	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$
$q_0$	$q_1$	$q_0$	$q_0$	$q_0$	$q_0$	0	0	0	0	0
$q_1$	$q_2$	$q_0$	$q_1$	$q_1$	$q_1$	0	0	0	0	0
$q_2$	$q_3$	$q_0$	$q_2$	$q_2$	$q_2$	1	0	0	0	0
$q_3$	$q_3$	$q_3$	$q_3$	$q_3$	$q_3$	2	2	2	2	2
$q_4$	$q_4$	$q_4$	$q_4$	$q_0$	$q_4$	3	3	3	3	3

b)

