A heat equation approach to temperature prediction of decomposing cadavers

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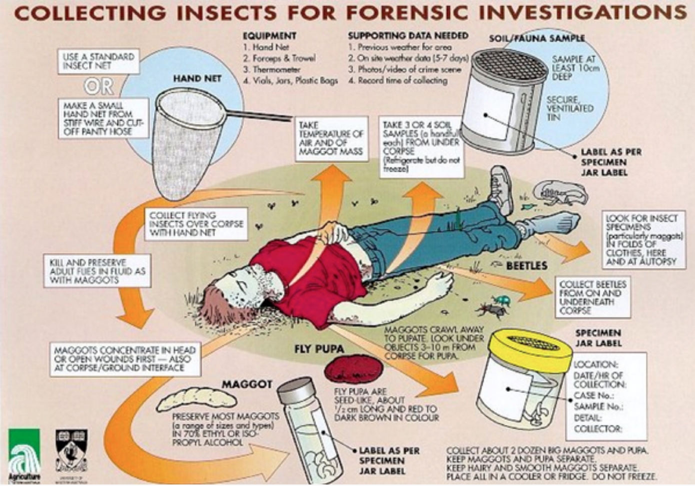
Outline

Introduction

Upon discovery of a crime scene, a critical piece of physical evidence, especially in an outdoor setting, are the insects involved in natural decomposition.



The effects of insect development lasts much longer than chemical changes that occur in the body immediately after death.

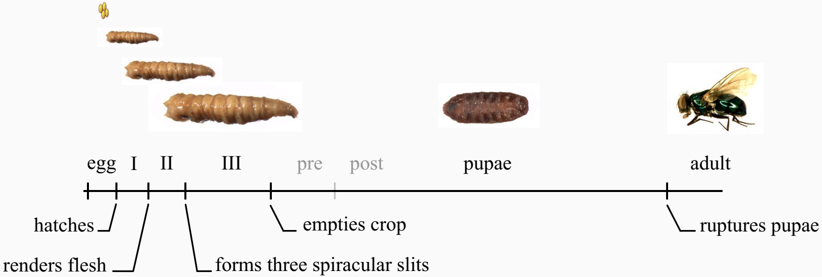


Figure 1: Life cycle of blow fly showing relative time spent in each stage of development.

Progression through an insect's life cycle is temperature dependent, so thermal summation models are often used.

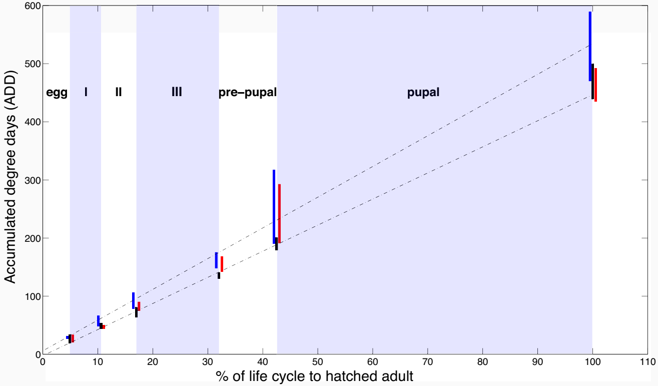


Figure 2: Development data for Calliphoridae at three different temperature profiles as collected in . The ADD range at 15.8 °C (blue), 20.6 °C (black), and 23.3 °C (red) are shown.

To use the thermal summation model historic temperature profiles are required. Current methods use linear regression between the environment and a simulated internal logger.

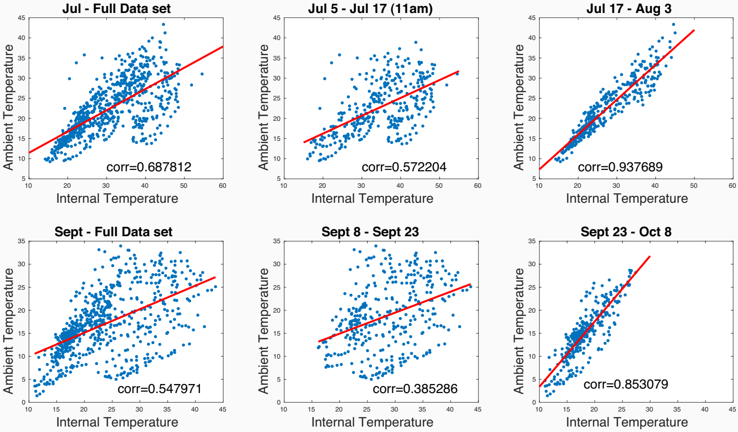


Figure 3: Correlation figures for case study data from July and September 2016 decomposition studies performed with swine cadavers at Ontario Tech.

Model Development

Case study data offers internal logger measurements as well as ambient temperature measurement with nearby external loggers.

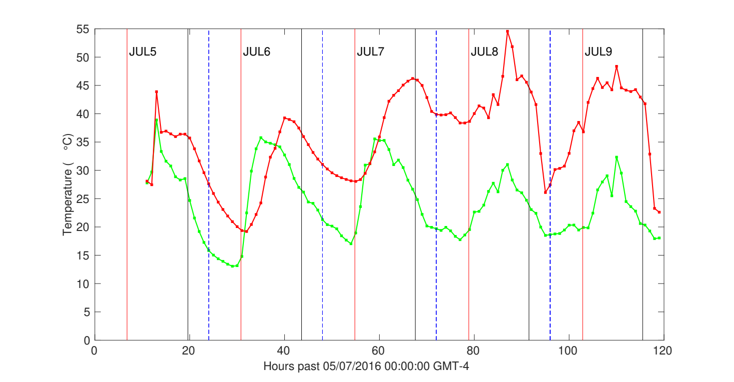


Figure 4: Internal temperature probe (red) and environmental temperatures (green) for the first five days of a swine cadaver decomposition study in July 2016

There are some notable patterns between the two temperature profiles

- There exists a phase shift between the two curves, especially in the first few days;
- The phase shift decreases over time;
- The internal logger doesn't experience the same lows as the ambient temperature, and experiences higher maximums;
- Both temperature profiles have a distinct day/night cycle.

We explore a heat equation approach. Starting with

$$\rho c_p \frac{\partial T}{\partial t} = k\nabla^2 T + Q(\mathbf{x}, t). \tag{1}$$

We consider the model in cylindrical coordinates to simulate a three dimensional cylindrical vessel, (**??**) becomes

$$\rho c_p \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial^2 t} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial^2 z}\right) + Q(r, z, t). \tag{2}$$

In terms of boundary conditions we desire that the vessel is insulated on all sides and the bottom, S_2, S_3. The top, S_1, may interact with the environment. This gives way to

$$-k\,\frac{\partial T}{\partial \hat{n}}\bigg|_{S_2,S_3} = 0, \qquad -k\,\frac{\partial T}{\partial \hat{n}}\bigg|_{S_1} = h(T - T_{\mathsf{ext}}(t)) \tag{3}$$

We examine the effects of averaging over the spacial dimensions to obtain an ordinary differential equation which is only time dependent. We rewrite the model where $\bar{\bar{T}}(t) \mapsto \bar{\theta}(\tau)$,

$$\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}\tau} = -\frac{h\Delta T}{Q_0 H}\left(\bar{\theta} - \theta_{\mathsf{ext}}(\tau)\right) + q(\tau), \qquad \bar{\theta}(0) = 0. \tag{4}$$

We run a simulation with a set of parameters to correspond to a bucket of water of depth 30 cm and a significant amount of internal heating. We allow external heating to be sinusoidal. We examine the solution to (**??**) by solving the ODE numerically and with method of lines.

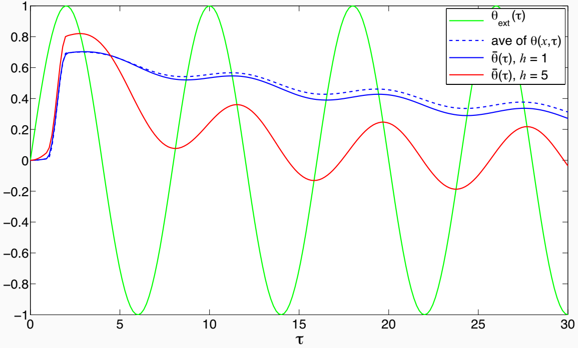


Figure 5: Solution to the ODE with $h = 1$ (blue), $h = 5$ (red), using method of lines (dashed blue) and the external heating (green).

We use the toy model to inform the model for a swine cadaver. We allow for heat flux through the boundary,

$$-k\,\frac{\partial T}{\partial \hat{n}}\bigg|_{\partial\Omega} = h(T - T_0) \qquad \mathbf{x} \in \partial\Omega,\ \ t > 0, \tag{5}$$

and simplify by averaging at every point so that the average temperature is

$$\bar{T}(t) = \frac{1}{|\Omega|}\int_{\Omega} T(\mathbf{x}, t)\,\mathrm{d}\mathbf{x}. \tag{6}$$

We nondimensionalize using

$$\bar{T} = T_{\mathsf{min}} + \Delta T\theta \qquad T_0 = T_{\mathsf{min}} + \Delta T\theta_0 \qquad t = \tau\tilde{t}. \tag{7}$$

The equation for the averaged temperature, as derived analogous to the previous section, becomes

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = c(t)(\theta_0(t) - \theta) + s(t), \tag{8}$$

with

$$c(t) = \frac{h|\partial\Omega|\tau}{\rho c_p |\Omega|}, \quad s(t) = \frac{Q(t)\tau}{\rho c_p \Delta T}, \quad Q(t) = \frac{1}{|\Omega|}\int_\Omega q(\mathbf{x}, t)\,\mathrm{d}\mathbf{x}, \tag{9}$$

Parameter Investigation

We choose the coupling coefficient to be a constant, $c(t) \mapsto c_0$ and the heat flux, $s(t)$, to be piecewise constant with changes at characteristic times, τ_j, corresponding to sunrise and sunset.

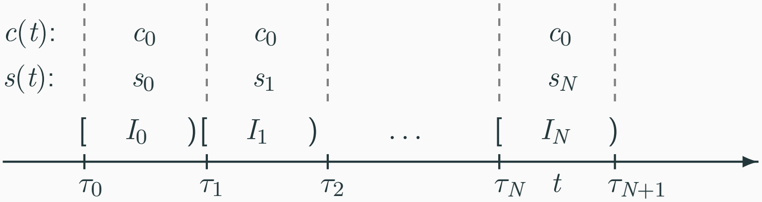


Figure 6: The structure of $c(t)$ and $s(t)$.