Appendix

This appendix contains detailed descriptions of the formulae used in this project. Formulae are provided here for reference but are not required to understand the project.

Linear motion

In Linear Motion the core formulae are

```
v = u + a*t
s = u + ½*a*t²
```

where

```
u is the initial velocity
v is the velocity at time t
a is the constant acceleration
s is the displacement from start point at time t
```

When the object is accelerating or decelerating to a given velocity a rearranged form of `v = u + a*t` is used, with v taken to be the given velocity, to find the time taken to reach the given velocity.

With this time known, we can insert this time into `s = u*t + ½*a*t²` to find the distance the object will travel. This allows me to determine the scale the simulator needs to be set at so the object can be kept within the display at all times.

Projectiles:

There are a number of forms that Projectiles questions can take. In the basic scenario where a projectile is launched at a given angle with a given speed on a flat plane the formulae are

$u_X = u*Cos(\alpha)$
$u_Y = u*Sin(\alpha)$
$v_X = u_X$
$v_Y = u*Sin(\alpha) + a*t$
$s_X = u*Cos(\alpha)*t$
$s_Y = u*Sin(\alpha)*t + \frac{1}{2}*a*t^2$

where

u = the speed the projectile is launched at
α = the angle the projectile is launched at
u_X = the velocity of the projectile in the x plane at time t
u_Y = the velocity of the projectile in the y plane at time t
s_X = the displacement from the start point in the x plane at time t
s_Y = the displacement from the start point in the y plane at time t

The key to solving many of the questions is finding the *range, max height* and *time of flight* of the projectile.

The time of flight may be found by setting the equation for sy equal to zero and solving for t.
As sy is displacement from the start point, solving for t will give the starting time and the finish time.
As the projectile has fallen back down at the finish time its displacement is again equal to zero.

The range may be found by solving for sx at the time of flight. This gives the distance travelled in the x plane.

The max height may be found by solving for sy at half the time of flight. As air resistance is ignored in the Projectiles topic, the projectile's path will be a parabola. This means that at half the time of flight the projectile will be at its max height.

When the projectiled is launched up an inclined plane the equations used need to be modified to the following:

$u_X = u*Cos(\alpha)$
$u_Y = u*Sin(\alpha)$
$v_X = u*Cos(\alpha) + a*Sin(\Theta)*t$
$v_Y = u*Sin(\alpha) + a*Cos(\Theta)*t$
$s_X = u*Cos(\alpha)*t + ½*a*Sin(\Theta)*t^2$
$s_Y = u*Sin(\alpha)*t + ½*a*Cos(\Theta)*t^2$

where

Θ = the angle of the incline plane to the ground
α = the angle the projectile is launched at relative to the incline plane

When the projectile is launched from a point above ground level the equations stay the same as in the basic scenario except that when calculating the time of flight the equation sy needs to be set equal to the starting height and not zero.

Relative velocity

In Relative Velocity the main formulae used is

$V_{AB} = V_A - V_B$

where V_A, V_B and V_{AB} are in the form of

$xi + yj$

and

x is the speed in the i (horizontal) plane
y is the speed in the j (vertical) plane

This gives the velocity of a relative to b.

In the river crossing scenario, a person must swim at an angle theta against a tide with a given speed. The point where they will land across the river must be found.

$V_{AB} = [V_P*Cos(\Theta) - V_R]i + [V_P*Sin(\Theta)]j$

where

V_P is the speed the person can swim at in still water
V_R is the speed of the river's current
Θ is the angle the person swims at

In the closest distance scenario the closest distance between two moving vehicles must be found

Circular and Simple Harmonic Motion

For an object undergoing circular motion in a horizontal plane, the equations are:

$F = m*\omega^2*r$ OR $F = (m*v^2) / r$

$v = \omega*r$

$x = r*Cos(\Theta)$

$y = r*Sin(\Theta)$

where

F is the force on the object, directed towards the centre of the circle its motion describes

ω is the angular velocity of the object in radians/second

v is the velocity of the object in metres/second

r is the radius of the circle

x is the current x coordinate of the object

y is the current y coordinate of the object

Θ is the current angle of the object in the cirlce it travels

For an object undergoing harmonic motion, the equations are:

$a = -\omega^2*x$

$v^2 = \omega^2*(A^2 - x^2)$

$v_{MAX} = \omega*A$

$a_{MAX} = \omega^2*A$

$x = A*Sin(\omega*t)$

where

a is the current acceleration of the object

v is the current velocity of the object

ω is the angular velocity of the object

v_{MAX} is the max velocity of the object

a_{MAX} is the max acceleration of the object

x is the current position of the object

A is the amplitude (furthest point) of its motion

t is the current time