DGL aus dem Buch

Tuesday, 20 December 2022 10:27

4.61
4.63
4.65
4.41

Homogene und partikuläre Lösung

$$y' + a(x)\cdot y = S(x)$$

$S(x)=0$: lineare homogene DGL 1. Ordn.

$S(x)\neq 0$: lineare inhomogene DGL 1. Ordn.

allgemeine Lösung

1) homogene DGL

$$y' + a(x)\cdot y = 0 \quad / -a(x)\cdot y$$

$$\frac{dy}{dx} = -a(x)\cdot y$$

$$\int \frac{dy}{y} = -\int a(x)dx$$

$a(x) = x^2+1$

$\int a(x) = \frac{x^3}{3} + x$

$$\ln|y| = -\int a(x)dx + C_1$$

$$y_h = e^{-\int a(x)dx + C_1} = C\cdot e^{-\int a(x)dx}$$

$e^{C_1} = C$

2) partikuläre Lösung

Variation der Konstanten

$$y_p = C(x)\cdot e^{-\int a(x)dx}$$

$$\left(\sin(2x)\right)' = \cos(2x)\cdot 2$$

$$\left(f(g(x))\right)' = f'(g(x))\cdot g'(x)$$

y_p in Angabe einsetzen

$y' + a(x)\cdot y = S(x)$ ANGABE

$\int x^2 dx = \frac{x^3}{3} + C$

$\left(\frac{x^3}{3}+C\right)' = x^2$

$$y_p' + a(x)\cdot y_p = S(x)$$

$$C'(x)\cdot e^{-\int a(x)dx} + C(x)\cdot e^{-\int a(x)dx}(-a(x)) + a(x)\cdot C(x)\cdot e^{-\int a(x)dx} = S(x)$$

$$y_p' + a(x)\cdot y_p = s(x)$$

$$C'(x)\cdot e^{-\int a(x)\,dx} + C(x)\cdot e^{-\int a(x)\,dx}(-a(x)) + a(x)\cdot C(x)\cdot e^{-\int a(x)\,dx} = S(x)$$

$$C'(x)\cdot e^{-\int a(x)\,dx} = S(x)$$

$$C'(x) = S(x)\cdot e^{\int a(x)\,dx} \qquad \frac{1}{x^{-1}} = x^1$$

$$C(x) = \int \left(s(x)\cdot e^{\int a(x)\,dx}\right)dx$$

Bsp.: $y' + \frac{y}{x} = x^2+4$

$a(x) = \frac{1}{x}$ $\qquad S(x) = x^2+4$

$$y' + \frac{1}{x}\cdot y = x^2+4$$

$$y' + \frac{1}{x}\cdot y = 0 \qquad \text{homogene Lösung}$$

$\vdots$

$$y = \frac{C}{x}$$

partikuläre Lösung

$$y_p = \frac{C(x)}{x} = C(x)\cdot x^{-1} \qquad y_p' = C'(x)\cdot x^{-1} + C(x)\cdot(-1)x^{-2}$$

$$C'(x)\cdot x^{-1} - \cancel{\frac{C(x)}{x^2}} + \underbrace{\cancel{\frac{C(x)}{x^2}}}_{\frac{y_p}{x}} = \underbrace{x^2+4}_{\text{Störfkt.}}$$

$$C'(x) = x^3+4x$$

$$C(x) = \frac{x^4}{4} + 4\cdot\frac{x^2}{2} + C_1$$

$$y = y_h + y_p = \underbrace{\frac{C}{x}}_{y_h} + \underbrace{\frac{\frac{x^4}{4} + 2x^2 + C_1}{x}}_{y_p} = \frac{C}{x} + \frac{x^3}{4} + 2x + \frac{C_1}{x}$$