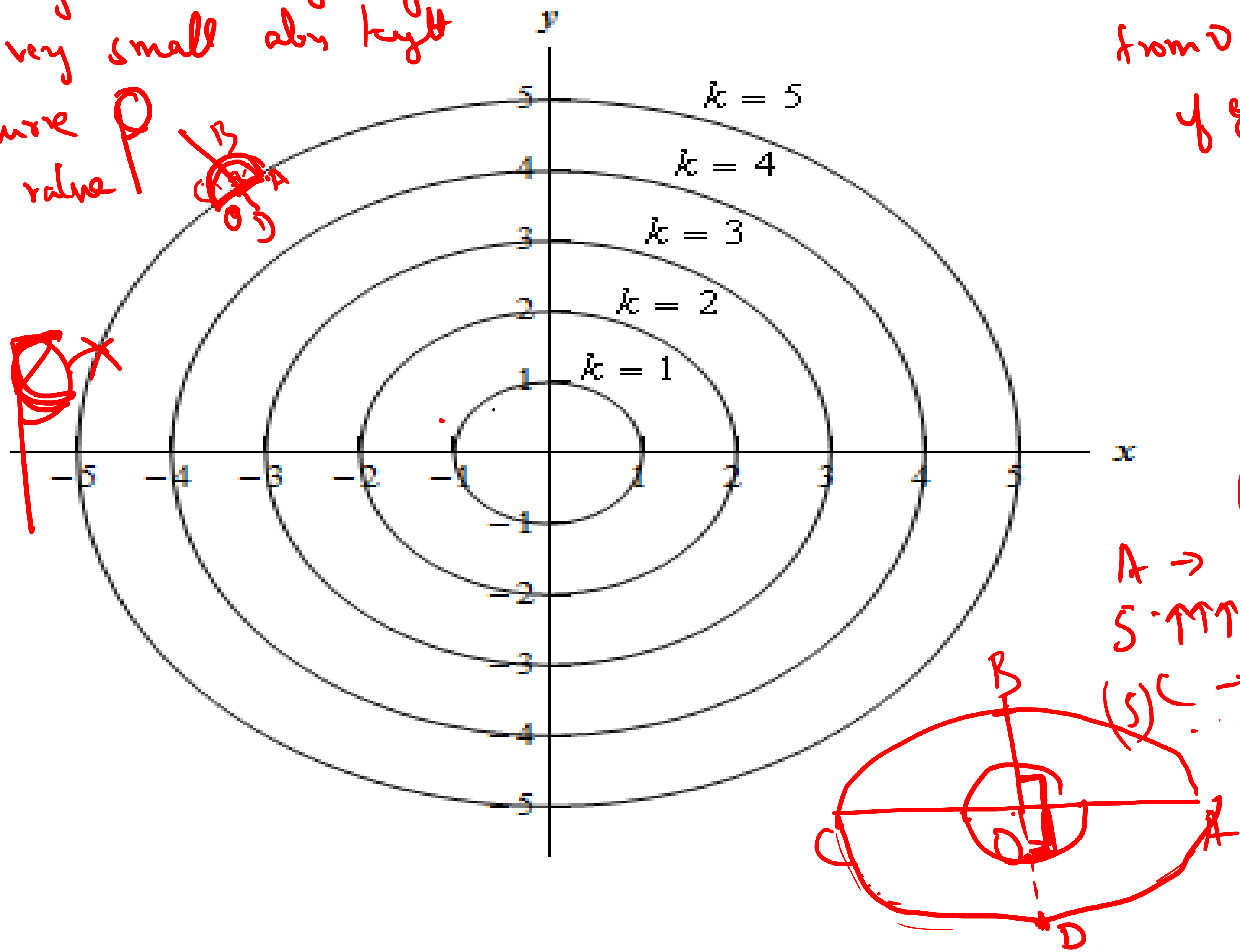
A : very small along tangent.

C : very small along tangt

All pts on curve have same value of f.



from O (f: 5)

f of A : ~5

f of B : 5(+)

C ~ 5

D 5(−)

(5+)

A → B → C

5 ↑↑↑ max ↓↓↓ 5

(5) C → D → A (5)

↓↓↓ & ↑↑↑

min

$f = f(x,y)$. Taylor series around (x_0, y_0)

$$df(x,y) \approx f(x_0,y_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0,y_0} (x-x_0) + \frac{\partial f}{\partial y}(y-y_0)$$

If you go along tangent from (x_0,y_0) to (x,y) you are along tangent

$$f(x,y) - f(x_0,y_0) = 0 = \frac{\partial f}{\partial x}\underbrace{(x-x_0)}_{\Delta x} + \frac{\partial f}{\partial y}\underbrace{(y-y_0)}_{\Delta y} = 0.$$

$$\underbrace{\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix}}_{\nabla f^T} \underbrace{\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}}_{\text{tangent}} = 0$$

∇f $\perp$ tangent.

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \frac{df}{dx}$$