Program Structures and Algorithms
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GITHUB LINK:

Task: Assignment 1 (Random Walk)

Relationship Conclusion: the expected distance E(d) is on the order of $\sqrt{m}$

Evidence to support that conclusion:
For each step, let:

The movement on the x-axis (with right as the positive direction) X, which can take values −1,0,1, with probabilities P(X = -1) = P(X = 1) = ¼ and P(X = 0) = ½. The movement on the y-axis Y is exactly same as the x-axis movement, which is P(Y = -1) = P(Y = 1) = ¼ and P(Y = 0) = ½.

For X or Y, the expected value E(X)=E(Y)=0 (since the probabilities of moving left or right, and up or down are equal), and the variance Var(X) = Var(Y) = E(X^2) – [E(X)] ^{2}.

$$\mathbf{E(X^2) = (-1)^2 \times \frac{1}{4} + 0^2 \times \frac{1}{2} + 1^2 \times \frac{1}{4} = \frac{1}{2}}$$

Hence, Var(X) = Var(Y) = ½

After m steps, the variances of X_m and Y_m will be m times the variance of a single step, which is m/2.

Finally, the Euclidean distance E_d from the origin after m steps is the square root of the sum of the squares of X_m and Y_m , that is, $d = \sqrt{(X_m{}^2) + (Y_m{}^2)}$

From an expectation point of view, E(d^2) = E($X_m{}^2$) + E($Y_m{}^2$) = Var(X_m) + Var(Y_m) = m

Unit Test Screenshots: