Formally Verified Cryptographic Proof Systems in Lean

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Chapter 1

Introduction

The goal of this project is to formalize Succinct Non-Interactive Arguments of Knowledge (SNARKs) in Lean. Our focus is on SNARKs based on Interactive Oracle Proofs (IOPs). We plan to build a general framework for IOP-based SNARKs that can state specifications of the protocols and prove their security properties in a clean and modular way.

Chapter 2

Oracle Reductions

2.1 Definitions

Definition 1 (Type Signature of an Interactive Protocol)**.**

Definition 2 (Type Signature of a Prover)**.**

Definition 3 (Type Signature of a Verifier)**.**

Definition 4 (Type Signature of an Oracle Verifier)**.**

Definition 5 (Interactive Protocol)**.** An *n-round interactive protocol* between two parties P, V is a sequence of messages $c_0, m_0, \ldots, c_n, m_n$ where:

- c_i is a challenge sent by V to P in the i-th round.
- m_i is a message sent by P to V in the i-th round.

Each message m_i and challenge c_i may be of different types. We bundle them all together as a `ProtocolSpec` structure.

Definition 6 (Interactive Oracle Protocol)**.** An *(interactive) oracle reduction* is an interactive protocol with a prover and a verifier.

Definition 7 (Completeness)**.**

Definition 8 (Soundness)**.**

2.2 Composition

We define *sequential* composition of two or more interactive protocols.

Definition 9 (Composition of Two Protocol Type Signatures)**.**

Definition 10 (Composition of Two Provers)**.**

Definition 11 (Composition of Two Verifiers)**.**

Definition 12 (Composition of Two Oracle Verifiers)**.**

Definition 13 (Composition of Two Interactive Protocols)**.**

Chapter 3

Commitment Schemes

3.1 Definitions

3.2 Composition

Chapter 4

Proof Systems

4.1 The Sum-Check Protocol

4.2 The Spartan Protocol

4.3 The Ligero Polynomial Commitment Scheme

Chapter 5

Supporting Results

5.1 Polynomials

Definition 14 (Multilinear Extension)**.**

Theorem 15 (Multilinear Extension is Unique)**.**

5.2 Coding Theory

Definition 16 (Code Distance)**.**

Definition 17 (Distance from a Code)**.**

Definition 18 (Generator Matrix)**.**

Definition 19 (Parity Check Matrix)**.**

Definition 20 (Interleaved Code)**.**

Definition 21 (Reed-Solomon Code)**.**

Definition 22 (Proximity Measure)**.**

Definition 23 (Proximity Gap)**.**

Chapter 6

References