LOBACHEVSKY UNIVERSITY

Russian Supercomputing Days

Comparison of Dimensionality Reduction Schemes for Parallel Global Optimization Algorithms

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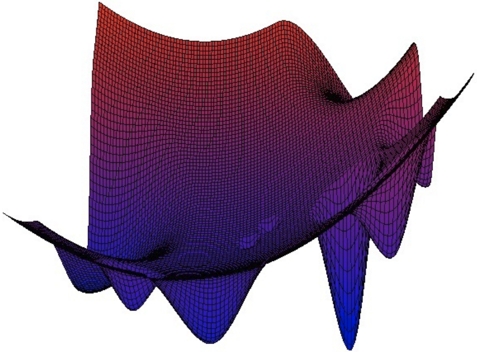
Problem statement

$$\varphi(y^*) = \min\{\varphi(y) : y \in D\},$$
$$D = \{y \in \mathbb{R}^N : a_i \leq y_i \leq b_i, 1 \leq i \leq N\}$$

$\varphi(y)$ is multiextremal objective function, which satisfies the Lipschitz condition:

$$|\varphi(y_1) - \varphi(y_2)| \leq L\|y_1 - y_2\|, y_1, y_2 \in D,$$

where $L > 0$ is the Lipschitz constant, and $||\cdot||$ denotes l_2 norm in $\mathbb{R}^N$ space.



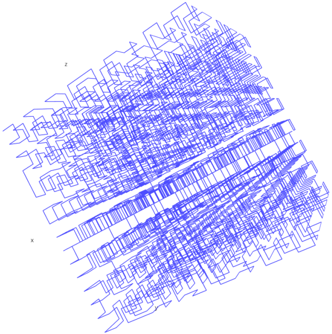
Dimension reduction

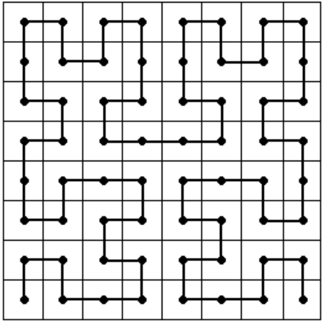
Peano-type curve $y(x)$ allows to reduce the dimension of the original problem:

$$\{y \in \mathbb{R}^N : -2^{-1} \leqslant y_i \leqslant 2^{-1}, 1 \leqslant i \leqslant N\} = \{y(x) : 0 \leqslant x \leqslant 1\}$$

$$\min\{f(y) : y \in D\} = \min\{f(y(x)) : x \in [0,1]\}$$

$y(x)$ is non-smooth function which continuously maps the segment $[0,1]$ to the hypercube D.



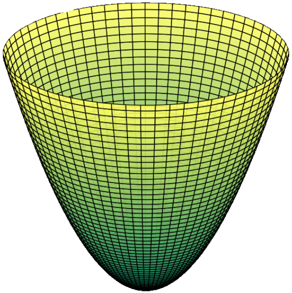


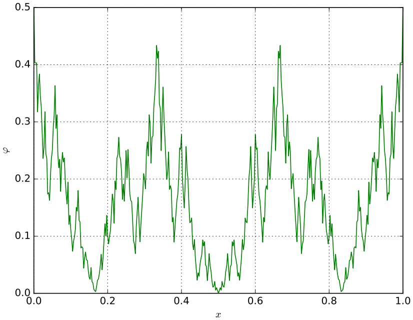
Properties of the reduced problem

After applying the Peano-type evolvent $\varphi(y(x))$ satisfies the uniform Hölder condition:

$$|\varphi(y(x_1)) - \varphi(y(x_2))| \leq H|x_1 - x_2|^{\frac{1}{N}}, x_1, x_2 \in [0, 1],$$

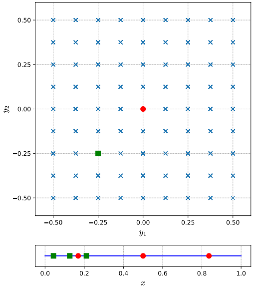
$\varphi(y(x))$ is non-smooth and has multiple local and **global** extremums even if $\varphi(y)$ is unimodal. The latter problem is caused by loss of the information about N-d neighborhood after the transformation to the 1-d space.





Non-univalent evolvent

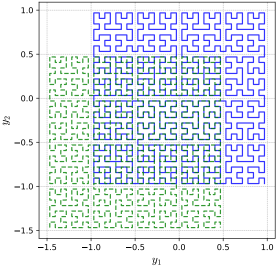
One can try to recover all preimages of $y \in \mathbb{R}^N$ and make optimization method aware of their existence[1]. This allows reducing the effect of growing amount of local minimas after dimension reduction. According to the theory of Peano-type curves, each N-d point could have up to 2^N preimages. For large N such preimages mining would be expensive.

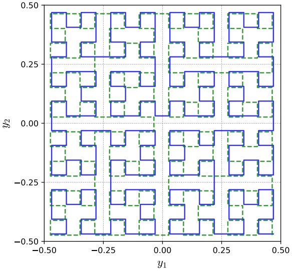


[1]R.G. Strongin. Numerical Methods in Multiextremal Problems (in Russian), 1978

Shifted and rotated evolvents

To create a fixed amount of preimages one can use a pre-defined set of different evolvents. These evolvents could be shifted or rotated versions of the original one. Set of shifted evolvents[2] is theoretically proven to generate at least one pair of close preimages if images are close and it perform better than the set of rotated curves.

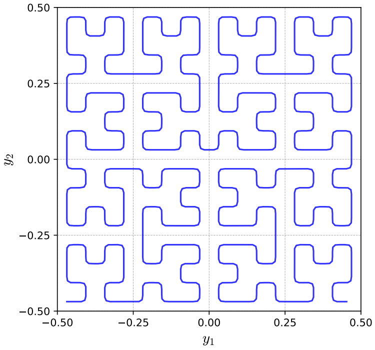




[2]Strongin, R.G., Gergel, V.P., Barkalov, K.A. Parallel methods for global optimization problem solving (in Russian), 2009

Smooth evolvent

Smooth functions are more predictable for optimizer, so smooth approximation of the Peano-like $y(x)$ curve could improve convergence rate [3].



[3]Goryachih, A. A class of smooth modification of space-filling curves for global optimization problems, NET 2016

Basic parallel optimization method

Optimization method generates search sequence $\{x_k\}$ and consists of the following steps:

Step 1. Sort the search information (one-dimensional points) in increasing order.

Step 2. Compute the evolvent $y(x^{k+j})$ and the function $\varphi(y(x^{k+j}))$, $j = \overline{1,p}$.

Step 3. For each interval (x_{i-1}, x_i) compute quantity $R(i)$, called characteristic.

Step 4. Choose p intervals (x_{t_j-1}, x_{t_j}) with the greatest characteristics and compute objective $f(y(x^{k+j}))$ in points chosen using the decision rule d:

$$x^{k+1+j} = d(t) \in (x_{t_j-1}, x_{t_j}),\ j = \overline{1,p}$$

Step 5. If $x_{t_j} - x_{t_j-1} < \varepsilon$ for one of $j = \overline{1,p}$, stop the method.

Detailed description: Strongin R.G., Sergeyev Ya.D.: Global optimization with non-convex constraints. Sequential and parallel algorithms (2000), Chapter 7

Parallel optimization method with multiple evolvents

Using the multiple mapping allows solving initial problem by parallel solving the problems

$$\min\{\varphi(y^s(x)) : x \in [0,1]\}, 1 \leqslant s \leqslant S$$

on a set of intervals $[0,1]$ by the index method. Each one-dimensional problem is solved on a separate processor. The trial results at the point x^k obtained for the problem being solved by particular processor are interpreted as the results of the trials in the rest problems (in the corresponding points $x^{k_1}, \ldots, x^{k_S}$). In this approach, a trial at the point $x^k \in [0,1]$ executed in the framework of the s-th problem, consists in the following sequence of operations:

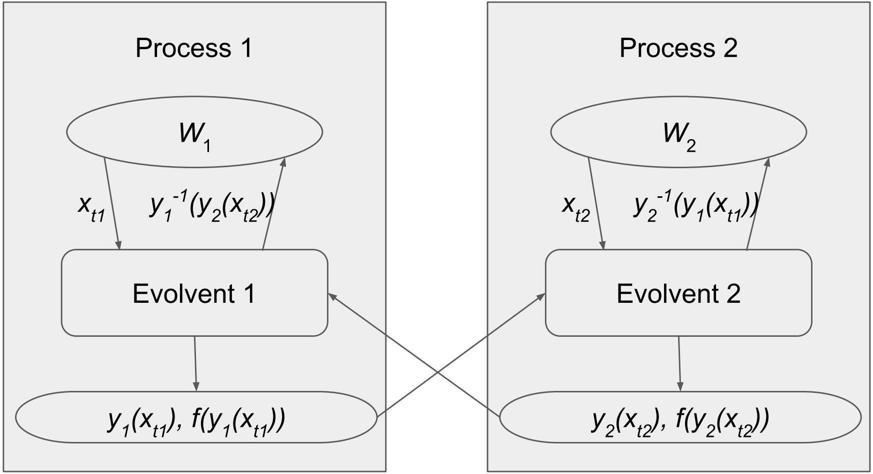
Step 1. Determine the image $y^k = y^s(x^k)$ for the evolvent $y^s(x)$.

Step 2. Inform the rest of processors about the start of the trial execution at the point y^k (the blocking of the point y^k).

Step 3. Determine the preimages $x^{k_s} \in [0,1], 1 \leqslant s \leqslant S$, of the point y^k and interpret the trial executed at the point $y^k \in D$ as the execution of the trials in the S points $x^{k_1}, \ldots, x^{k_s}$

Step 4. Inform the rest of processors about the trial results at the point y^k.

Parallel optimization method with multiple evolvents



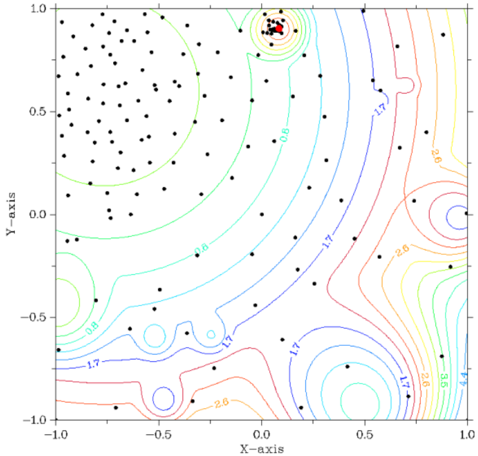
Test problems

Generator GKLS was employed to construct the sets of test problems:

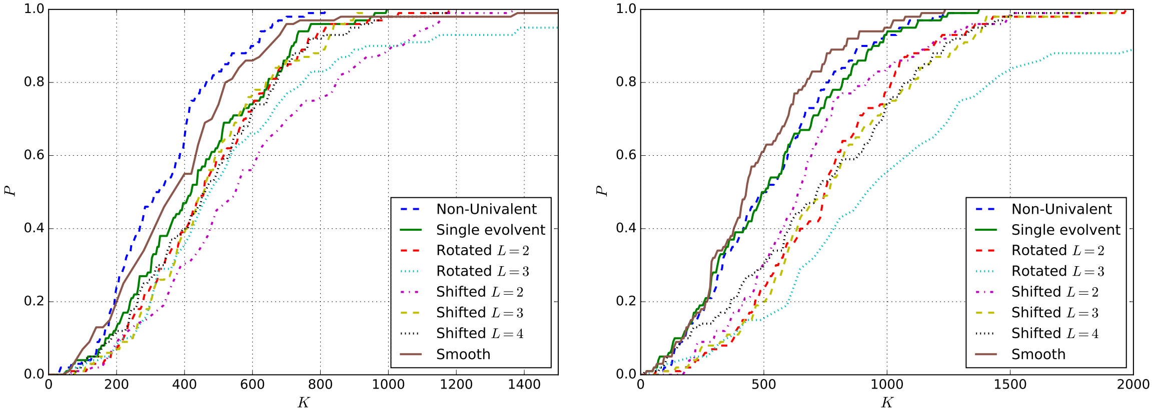
$$f(x) = \begin{cases} C_i(x), x \in S_i, i \in 2, \dots, m \\ \|x - T\|^2 + t, x \notin S_2, \dots, S_m \end{cases}$$

The generator allows to adjust:

- the number of local minimas;
- the size of the global minima attraction region;
- the space dimension.



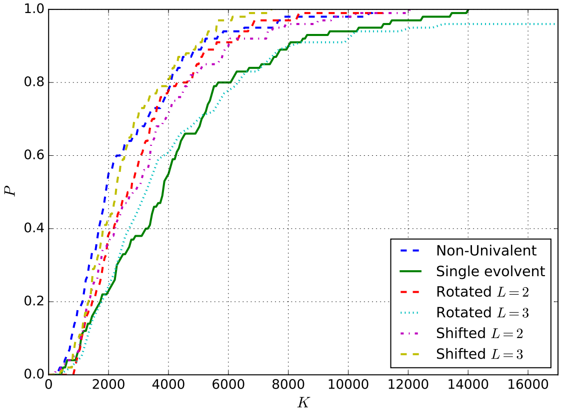
Evolvents comparison



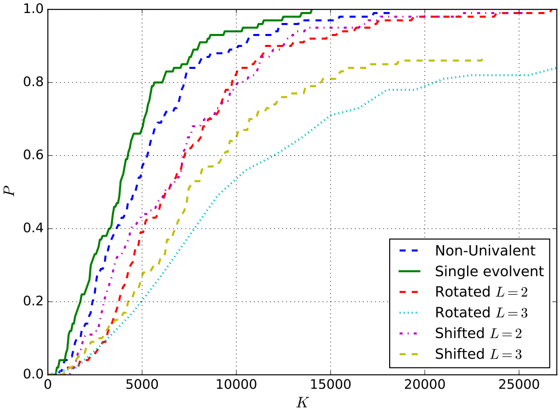
Minimal r $\qquad$ $r = 5.0$

Operating characteristics on GKLS 2d Simple class

Evolvents comparison



Minimal r



$r = 4.5$

Operating characteristics on GKLS 3d Simple class

Choice of evolvent for the parallel algorithm

- Smooth evolvent is too computational heavy.
- Non-univalent evolvent generates large and unpredictable amount of preimages.
- Shifted evolvent generates huge amount of auxiliary points to handle additional constraint.

Table: Averaged number of computations of g_0 and of φ when solving the problems from GKLS 3d Simple class using the shifted evolvent

L	$calc(g_0)$	$calc(\varphi)$	$\frac{calc(g_0)}{calc(\varphi)}$ ratio
2	96247.9	6840.14	14.07
3	153131.0	7702.82	19.88

Results of applying the parallel algorithm

Table: Averaged numbers of iterations executed by the parallel algorithm for solving the test optimization problems

		p	N = 4		N = 5	
			Simple	*Hard*	*Simple*	*Hard*
I	**1 cluster node**	*1*	12167	25635	20979	187353
		32	328	1268	898	12208
II	**4 cluster nodes**	*1*	25312	11103	1472	17009
		32	64	913	47	345
III	**8 cluster nodes**	*1*	810	4351	868	5697
		32	34	112	35	868

Results of applying the parallel algorithm

Table: Speedup of parallel computations executed by the parallel algorithm

		p	$N = 4$		$N = 5$	
			Simple	*Hard*	*Simple*	*Hard*
I	**1 cluster node**	*1*	12167(10.58s)	25635(22.26s)	20979(22.78s)	187353(205.83s)
		32	37.1(18.03)	20.2(8.55)	23.3(8.77)	15.4(9.68)
II	**4 cluster nodes**	*1*	0.5(0.33)	2.3(0.86)	14.3(6.61)	11.0(6.06)
		32	190.1(9.59)	28.1(1.08)	446.4(19.79)	543.0(43.60)
III	**8 cluster nodes**	*1*	15.0(6.05)	5.9(2.36)	24.2(17.56)	32.9(24.87)
		32	357.9(2.36)	228.9(2.64)	582.8(20.96)	793.0(33.89)

Conclusions

- The smooth evolvent and the non-univalent one demonstrate the best result in the problems of small dimensionality and can be applied successfully in solving the problems with the computational costly objective functions.
- The shifted evolvents introduce large overhead costs on the execution of the method due to the requirement to adding an auxiliary constraint. About 95% of iterations are overhead to fight the auxiliary constraint.
- Rotated evolvents perform almost the same as the shifted ones but without overhead.
- Parallel optimization method shows up to 43x speedup on hard $5d$ problems when using a set of rotated evolvents.

Q&A

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