Multistage Global Search Using Various Scalarization Schemes in Multicriterial Optimization Problems

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Abstract. In this paper, an approach, in which the decision making problems are reduced to solving the multicriterial time-consuming global optimization problems is proposed. The developed approach includes various methods of scalarization of the vector criteria, the dimensionality reduction with the use of the Peano space-filling curves, and the efficient global search algorithms. In the course of computations, the optimization problem statements and the applied methods of the criteria scalarization can be altered in order to achieve more complete compliance to available requirements to the optimality. The overcoming of the computational complexity of the developed approach is provided by means of the reuse of the whole search information obtained in the course of computations. The performed numerical experiments have confirmed the reuse of the search information to allow reducing essentially the amount of computations for solving the optimization problems arising successively.

Keywords: Decision making, multicriterial optimization, criteria scalarization methods, global optimization with nonlinear constraints, numerical experiment

1 Introduction

The multicriterial optimization (MCO) problems, which are used as the statements of the decision-making problems often are the objects of extensive research - see, for example , the monographs [1-6] and the reviews of the scientific and applied results in the field [7-10].

Usually, the finding of the effective (non-dominated) decisions, in which the improvement of the values with respect to any criteria cannot be achieved without the worsening of the indicators of efficiency with respect to other criteria is understood as the solution of a MCO problem. In the most general case, when solving the MCO problems, it can appear to be necessary to obtain a complete set of the effective decisions (the Pareto set). However, the finding of all effective decisions may require a considerable amount of computations, and the set of obtained decisions may appear to be quite large, the analysis of which may appear to be difficult. As a result, are applied wider. The approaches to solving the MCO problems, in which, according to the requirements of optimality, the obtained set of effective decisions is more limited. Among such approaches, there are various kinds of the criteria convolutions, the lexicographic optimization methods, the algorithms of searching the best approximation

to the existing to existing prototypes, etc. All methods listed above allow accounting for the specific features of the MCO problems being solved and satisfy the requirements to the optimality from the decision-making person (DMP, decision maker, DM).

The present work is devoted to the solving of the MCO problems, which are used for the description of the complex decision making problems, in which the criteria of efficiency may have a complex *multiextremal* form, and the determining of the values of the criteria and constraints may require *a large amount of computations*. Also, is assumed that in the course of computations it is possible to change the statement, the methods, and the parameters of solving the MCO problem that results in the necessity of the multiple solving of the global optimization problems.

The realism of this approach implies the overcoming of a **considerable computational complexity** of the decision-making problems that can be provided by means of the use of highly efficient global optimization methods and the complete utilization of the search information obtained in the course of computations.

In the present paper, the results of investigations on the generalization of the decision-making problem statements [11-12] and on the development of the highly efficient global optimization methods utilizing the whole search information obtained in the course of computations [13-15] are presented.

Further structure of the paper is as follows. In Section 2, the statement of the decision making problems based on multistage multicriterial global search are presented. In Section 3, a general scheme for the MCO problem criteria of scalarization involving various kinds of the criteria convolutions, the lexicographic optimization methods, and the search for the best approximations of the decisions defined a priori is proposed. In Section 4, the search information obtained in the course of computations, and which may be reused in the solving of all subsequent global optimization problems is considered. In Section 5, an efficient algorithm for solving the time-consuming global optimization problems with the nonlinear constraints is presented. Section 6 contains the results of numerical experiments confirming the developed approach to be promising. In Conclusion, the obtained results are discussed and possible main directions of further research are outlined.

2 Multiple multicriterial optimization problem statement

For the formal description of the process of the search for the rational decisions in the complex decision-making problems, the following generalized two-phase model is proposed.

1. In the most general form, a decision-making problem is defined by means of a *vector function of characteristics*

$$w(y) = \big(w_1(y), w_2(y), \dots, w_M(y)\big), y \in D \qquad (1)$$

where $y = (y_1, y_2, \dots, y_N)$ is the *vector of design parameters* and $D \subset R^N$ is the *domain of possible values*, which is usually an N-dimensional hypercube

$$D = \{y \in R^N : a_i \le y_i \le b_i, 1 \le i \le N\} \quad (2)$$

for given vectors a and b.

It is supposed, that the values of characteristics $w(y)$ are non-negative, and the decreasing of these ones corresponds to the increasing of the efficiency of the chosen decisions. Also, it is supposed that the characteristics $w_j(y), 1 \le j \le M$, may be multiextremal, and the determining of their values may require large enough amount of computations. Besides, the characteristics $w_j(y), 1 \le j \le M$, are supposed to satisfy the Lipschitz condition

$$|w_i(y_1) - w_i(y_2)| \le L_i \|y_1 - y_2\|, 1 \le i \le M \quad (3)$$

where L_i is the Lipschitz constant for the characteristic $w_j(y), 1 \le j \le M$, and $\|*\|$ denotes the Euclidean norm in R^N.

2. Then, a MCO problem is formulated on the basis of the model considered above. For this purpose, a vector criterion of efficiency is selected among the characteristics $w_j(y), 1 \le j \le M$, from (1)

$$f(y) = \big(f_1(y), f_2(y), \dots, f_s(y)\big), f_j(y) = w_{i_j}(y), 1 \le j \le s, 1 \le i_j \le M \quad (4)$$

and the vector function of constraints

$$g(y) = \big(g_1(y), g_2(y), \dots, g_m(y)\big), g_l(y) = w_{i_l}(y) - q_l, 1 \le j \le m, 1 \le i_l \le M \quad (5)$$

where $q_l > 0, 1 \le l \le m$, are the allowances on the feasible values of characteristics $w_j(y), 1 \le j \le M$.

The efficiency criteria and constraints formulated in such a way allow defining a *multicriterial optimization problem*

$$P: f(y) \to min, y \in Q \quad (6)$$

where Q is the *feasible multicriterial search domain*

$$Q = \{y \in D : g(y) \le 0\}. \quad (7)$$

The scheme proposed above involves many existing statements of the optimization problems. At $s = 1$ and $m = 0$, the general statement becomes a global optimization problem. At $s = 1$ and $m > 0$, the general statement defined a nonlinear programming problem. At $s > 1$ and $m > 0$, the general statement leads to a constrained multicriterial optimization problem.

In development of this scheme of the MCO problem statement, further an opportunity of simultaneous formulation of several MCO problems

$$\mathbb{P}_t = \{P_1, P_1, \dots, P_t\}, \quad (8)$$

will be allowed, the set of which can be varied in the course of computations by means of adding new or by removing already existing problems.

In the simplest case, the set $\mathbb{P}$ can consist of a single MCO problem, upon solving of which current MCO problem statement may be changed after the analysis of

the obtained results, and the process of computations can be continued for the new MCO problem statement until the desired optimal decision is obtained. It worth noting that the problems from the set $\mathbb{P}$ are the information compatible ones – when altering the MCO problem statements, the whole search information obtained earlier in the course of computations can be saved and reused in the solving of newly formulated optimization problems.

In general, the proposed model of the optimal decision search process (1) - (8) defines a new class of the optimization problems – the *multiple multicriterial global optimization* (MMGO) problems.

3 Reduction of the multiple multicriterial search to the scalar one-dimensional global optimization problems

One of the approaches to solving the MCO problems used wider consist in the scalarization of the vector criterion into some general scalar criterion of efficiency that allows using a wide set of already existing optimization methods for solving the MCO problems. Among the possible scalarization methods, there are, for example, the weighted sum method, the compromise programming method, the weighted min-max method, and many other methods – see, for example, [2-6].

In the general form, the statement of the global optimization problems generated in the MCO problem criteria scalarization can be represented as follows:

$$\min \varphi(y) = F(\alpha, y), g(y) \le 0, y \in D, \tag{9}$$

where F is the objective function generated as a result of scalarization of the criteria f_i, $1 \le i \le s$, α is the vector of parameters of the applied criteria convolution, $g(y)$ are the constraints of the MCO problem from (6), and D is the search domain from (2).

Particular form of the function $F(\alpha, y)$ is defined by the criteria scalarization method applied. For example, the following scalarization methods are possible.

1. In the case of equal importance of the criteria f_i, $1 \le i \le s$, the min-max convolution (the minimax convolution scheme, MMC) [4,6]:

$$F^1(\lambda, y) = \max(\lambda_i f_i(y), 1 \le i \le s), \tag{10}$$

$$\lambda = (\lambda_1, \lambda_2, \dots, \lambda_s) \in \Lambda \subset R^s : \sum_{i=1}^{s} \lambda_i = 1, \lambda_i \ge 0, 1 \le i \le s.$$

is applied the most often. The necessity and essence of this scalarization method for solving a MCO problem is an important property of the min-max convolution: any result of minimization of $\varphi(y)$ leads to obtaining an effective decision and, vise versa, any effective decision can be obtained as a result of minimization of $\varphi(y)$ at the corresponding values of the convolution coefficients $\lambda_i, 1 \le i \le s$.

2. In the case of arrangement of the criteria in importance, the method of successive concessions (MSC) [2,5,6] is used widely. According to this method, the optimization is performed for the most important criterion $f_1(y)$ first (the criteria are supposed to be renumbered according to the order of decreasing of their importance).

Then, upon completing the global search for $f_1(y)$, the magnitude of allowed concession from the minimum value of the first criterion is set, and the optimization of the second (in the importance) criterion $f_1(y)$ at the condition of not exceeding the set concession. Further optimization of the rest criteria is performed in the same way – more detailed description of the MSC method is given, for example, in [2,5,6].

Within the framework of the developed approach, it is proposed to reduce the multistage computations in the MSC method to solving a single scalar optimization problem arising at the last stage of the method of successive concessions

$$\min F^2(\lambda, y) = f_s(y), f_i(y) \le f_i^{min} + \delta_i\left(f_i^{max} - f_i^{min}\right), \quad (11)$$

$$1 \le i < s, g(y) \le 0, y \in D,$$

f_i^{min}, f_i^{max}, $1 \le i < s$, are the minimum and maximum values of the criteria in the feasible domain D respectively, and $0 \le \delta_i \le 1, 1 \le i < s$, are the concessions with respect to each criterion. As before, the values of concessions $0 \le \delta_i \le 1, 1 \le i < s$, can be varied in the course of computations. The quantities f_i^{min}, f_i^{max}, $1 \le i < s$, the values of which may be unknown *a priori*, can be replaced by the minimum and maximum estimates of the criteria values computed using the available search information.

3. In the case of availability of any estimates of the criteria values of the required decision (for example, based on an ideal decision or on any existing prototype) the MCO problem solution may consist in finding an effective decision the most completely matching given indicators of optimality (the reference point method, RPM). Such a problem can be formulated in the form of a scalar optimization problem:

$$\min F^3(\lambda, y) = \frac{1}{s}\sum_{i=1}^{s} \theta_i (f_i(y) - f_i^*)^2 , g(y) \le 0, y \in D \quad (12)$$

where the objective function $F^3(\lambda, y)$ is the standard deviation of the decision $y \in D$ from the sought ideal decision, and the quantities $0 \le \theta_i \le 1, 1 \le i < s$, are the magnitudes of importance of approximations with respect to each particular variable y_i, $1 \le i \le N$.

The scalarization methods considered above allow accounting for the specific properties of the MCO problems being solved in order to select a desired subset of the effective decisions. Within the framework of the developed approach, it is possible to change the used scalarization methods (10)-(12) and/or altering the parameters of convolutions λ, δ and θ. Such variations expand the set of the MCO problems $\mathbb{P}$ from (8) necessary for solving the original decision making problem into a wider set of the scalar global optimization problems (9)

$$\mathbb{F}_T = \{F_i(\alpha_i, y): 1 \le i \le T\}, \quad (13)$$

in which each problem $P \in \mathbb{P}$ from (8) can correspond to several global optimization problems (9).

In the developed approach, one more step of converting the problems being solved $F(\lambda, y)$ from (9) is performed, namely the dimensionality reduction is per-

formed with the use of the Peano *space-filling curves* (*evolvents*) $y(x)$ providing an unambiguous mapping of the interval [0,1] onto an N-dimensional hypercube D [16, 17]. As a result of such reduction, the multidimensional global optimization problem (10) is reduced to a one-dimensional problem:

$$F(\alpha, y(x^*)) = \min\{F(\alpha, y(x)): g(y(x)) \le 0, x \in [0,1]\}. \quad (14)$$

The dimensionality reduction allows applying many well known highly efficient one-dimensional global optimization algorithms for solving the problems (9) (after performing necessary generalization) – see, for example, [16-20].

4 Improvement of the efficiency of the multiple multicriterial search on the basis of the reuse of the search information

The numerical solving of the global optimization problems (9) is usually reduced to the successive computing the values of characteristics $w(y)$ at the points y^i, $0 \le i \le k$, of the search domain D [16,19]. The data obtained as a result of computations can be represented in the form of the *matrix of the search information*:

$$\Omega_k = \left\{\left(y^i, w^i = w(y^i)\right)^T : 1 \le i \le k\right\}. \quad (15)$$

As a result of scalarization of the vector criterion (9) and application of the dimensionality reduction (14), the set Ω_k from (15) can be transformed into the form of the *matrix of the search state*:

$$A_k = \{(x_i, z_i, g_i, l_i)^T : 1 \le i \le k\}, \quad (16)$$

where x_i, $1 \le i \le k$, are the reduced points of executing the global search iterations arranged in the order of increasing coordinates z_i, g_i, $1 \le i \le k$, are the values of the scalar criterion and constraints of current optimization problem being solved $F(\alpha, y(x))$ at these points, and l_i, $1 \le i \le k$, are the indices of global search iterations, in which the points x_i, $1 \le i \le k$, were computed.

The availability of the search information in the form of the matrices Ω_k and A_k allows performing an adaptive choice of the points to perform the global search iterations taking into account the results of all computations completed earlier:

$$\mathrm{y}^{k+1} = \mathrm{S}(\Omega_k), k = 1,2, ..., \quad (17)$$

(particular form of the rule S depends on the properties of the optimization algorithm applied).

The availability of the set Ω_k from (15) allows reducing the results of all preceding computations z_i, $1 \le i \le k$ in the matrix A_k to the values of the next optimization problem being solved $F(\alpha, y(x))$ from (9) without any repeated time-consuming computations of the values of $w(y)$ from (1) i. e.

$$w_i \overset{\alpha,P}{\rightarrow} (z_i, g_i), 1 \le i \le k, \forall \alpha, P \in \mathbb{P} \quad (18)$$

This way, all search information can be employed for continuing the computations in full amount. The reuse of the search information can provide a gradual decreasing of the amount of computations when solving every next optimization problem down to the execution of few iterations only to find the next effective decision.

5 Efficient method of solving the multiple multicriterial optimization problems with nonlinear constraints

Within the framework of developed approach, the original method of separate accounting for the constraints developed in the framework of the information – statistical theory of global search [16] was applied for solving the global optimization problems (9) if the nonlinear constraints are available. The essence of the approach is the constructing of a problem with some integral unconstrained objective function, the solving of which leads to the solution of the initial problem (9) – more detailed description of the approach will be given below.

Let us introduced a simpler notation for the reduced one-dimensional problems (9) as

$$\min\{g_{m+1}(x): g_i(y(x)) \le 0,1 \le i \le m, x \in [0,1]\}. \quad (19)$$

$$g_{m+1}(x) = F(\lambda, y(x)).$$

The problem (19) can be considered in the statement of partial computability when each function g_j, $1 \le j \le m+1$ is defined and computable in certain subinterval $\Delta_j \subset [0,1]$ only where

$$\Delta_1 = [0,1], \Delta_{j+1} = \{x \in \Delta_j: g_j(y(x)) \le 0\}, 1 \le j \le m. \quad (20)$$

Taking into account the conditions (20), the objective function of the problem (19) can be represented in the form

$$\varphi(x^*) = \min\{g_{m+1}(y(x)): x \in \Delta_{m+1}\}, \quad (21)$$

on the basis of which, a unified function

$$\hat{\varphi}(y(x)) = g_\nu(y(x)), \nu = \nu(x), x \in [0,1], \quad (22)$$

$$1 \le \nu = \nu(x) \le m+1, g_\nu(y(x)) > 0, g_j(y(x)) \le 0,1 \le j \le \nu - 1$$

can be constructed. The *index* $\nu = \nu(x)$, $1 \le \nu \le m+1$ defines the first violated constraint at the point x in the series.

Determining The value of $\hat{\varphi}(y(x))$, $x \in [0,1]$ is reduced to the successive computing of the quantities $g_j(y(x))$, $1 \le j \le \nu = \nu(x)$, at that, the next value of $g_{j+1}(x)$ is computed in the case if $g_j(x) \le 0$ only. The process of computing is terminated either as a result of funding the inequality $g_j(x) > 0$ or as a result of achieving the value $\nu(x) = m+1$ (this procedure is hereafter called a trial).

The definition of the function $\hat{\varphi}$ from (22) allows transforming the problem (19) into an unconstrained optimization problem

$$\Phi(x^*) = \min\{\Phi(x): x \in [0,1]\}, \tag{23}$$

where

$$\Phi(x) = \begin{cases} \frac{g_v(y(x))}{Hv}, & v < M, \\ \frac{(g_M(y(x)) - g_M^*)}{H_M}, & v = M. \end{cases}$$

It is worth noting that the maximum index M, the vales of the Lipschitz constant H_v, $1 \le v \le M$, of the functions $g_i(y(x))$, $1 \le i \le m+1$, and the minimum value g_M^* of the function $g_{m+1}(y(x))$ are unknown. However, when performing the computations, instead of these quantities, the adaptive estimates of these ones can be used, which can be obtained utilizing the search information A_k from (16).

Within the framework of the developed approach, the algorithm of global constrained optimization (AGCO), which is considered in details in [16] is applied for solving the problems (23). The general computational scheme of this algorithm can be described briefly as follows.

The first trial is performed at an arbitrary point $x^1 \in (0,1)$. The choice of any next trial point x^{k+1}, $k \ge 1$ is determined by the following rules.

Rule 1. Renumber the points $x^1, \dots, x^k$ of the preceding trials by the lower indices in the order of increasing coordinate values i. e.

$$x_1 < \cdots < x_i < \cdots < x_k, \tag{24}$$

and associate the values $z_i = g_v(x_i)$, $v = v(x_i)$, $1 \le i \le k$ from (25) with these ones.

Rule 2. Evaluate the maximum index *M* and compute the numerical estimates of the Lipschitz constants H_v, $1 \le v \le M$, of the functions $g_i(y(x))$, $1 \le i \le m+1$, and of the minimum value g_M^* of the function $g_{m+1}(y(x))$ necessary to construct the function $\Phi(x)$ from (23).

Rule 3. For each interval (x_{i-1}, x_i), $1 < i \le k$, compute *the characteristic* $R(i)$ and determine the interval (x_{t-1}, x_t), to which the maximum characteristic corresponds

$$R(t) = \max\{R(i): 1 < i \le k\}. \tag{25}$$

Rule 4. Execute the next trial at the point of the interval $x^{k+1} \in (x_{t-1}, x_t)$.

The iterations of the algorithm are stopped if the stop condition is satisfied

$$\rho_t = (x_t - x_{t-1})^{\frac{1}{N}} \le \varepsilon, \tag{26}$$

where t is from (25) and $\varepsilon > 0$ is the predefined accuracy.

The values of characteristics of the intervals $R(i)$, $1 < i \le k$, and the point of the next trials x^{k+1} in the interval with the maximum characteristic are computed according to the rules of the AGCO algorithms. At that, the characteristics of the in-

tervals $R(i)$, $1 < i \leq k$, are defined in such a way, that their values can be interpreted as some measures of importance of the intervals with respect to containing the global minimum point of the function $\Phi(x)$ from (23).

A detailed description of the AGCO algorithm and the corresponding theory of convergence are presented in [16].

6 Results of numerical experiments

The numerical experiments have been carried out using the computational nodes of Lobachevskii supercomputer at Nizhnii Novgorod State University. The peak performance of the cluster was 573 Tflops, each computational node of the supercomputer was equipped with Intel Sandy Bridge E5-2660 processor 2.2 GHz, 64 Gb RAM. Within the framework of the present study, the construction of a numerical approximation (*PDA*) of the Pareto domain (*PD*) was understood as a solution of a MCO problem. In order to evaluate the efficiency of constructing the *PDA*, two main indicators applied widely were used: the completeness of coverage of the Pareto domain (*hypervolume index, HV*) and the uniformity of distribution of the numerical estimates of the effective decisions (*distribution uniformity index, DU*) [15,23,25]. The higher values of the index *HV* and the lower values of the index *DU* corresponds to the better approximation. A comparison of the AGCO algorithm with several other multicriterial optimization methods, among which there were the Monte-Carlo method, the generalized SEMO algorithm from the PISA library [22], the non-uniform coverage (NUC) method [23], and the bi-criterial Lipschitz optimization (BLO) algorithm [24] was performed in [15]. To compare the methods, the bi-criterial problem of the multicriterial optimization [23] was used:

$$f_1(y) = (y_1 - 1)y_2^2 + 1, f_2(y) = y_2, 0 \leq y_1, y_2 \leq 1. \quad (27)$$

For the AGCO algorithm, the min-max convolution of the criteria (10) was used. For the AGCO algorithms, total 100 subproblems (10) for the problem (27) were solved for various values of the convolution coefficient λ.

The results of the numerical experiments are presented in Table 1. The column "Iters" of Table 1 contains the total number of iterations (the number of computations of the criteria values) required for solving the problem (27). The column "PDA" shows the number of the effective decisions in the PDA obtained.

Table 1. The results of numerical experiments on comparing the MCO methods by solving the test problem (27)

Method	Iters	PDA	HV	DU
MC	500	67	0.300	1.277
SEMO	500	104	0.312	1.116
NUC	515	29	0.306	0.210
BLO	498	68	0.308	0.175
AGCO	**390**	**90**	**0.317**	**0.094**

As one can note, the results of experiments demonstrated a definite advantage of the AGCO algorithm as compared to other methods of solving the MCO problems.

For the conducting of the next experiments, the two-dimensional bi-criterial MCO problems were used, the criteria of which were defined with the use of the family of multiextremal functions [16] used widely in the evaluation of efficiency of the optimization methods. The functions of this family are defined by the following relations:

$$\begin{aligned} f(y_1, y_2) &= -(AB + AC)^{\frac{1}{2}}, \\ AB &= \left(\sum_{i=1}^{7}\sum_{j=1}^{7}\left[A_{ij}a_{ij}(y_1, y_2) + B_{ij}b_{ij}(y_1, y_2)\right]\right)^2, \\ CD &= \left(\sum_{i=1}^{7}\sum_{j=1}^{7}\left[C_{ij}a_{ij}(y_1, y_2) - D_{ij}b_{ij}(y_1, y_2)\right]\right)^2, \end{aligned} \tag{28}$$

where the expressions

$$a_{ij}(y_1, y_2) = sin(\pi i y_1)\, sin(\pi j y_2),\ b_{ij}(y_1, y_2) = cos(\pi i y_1)\, cos(\pi j y_2)$$

are defined inside the area $0 \le y_1, y_2 \le 1$, and the parameters $-1 \le A_{ij}, B_{ij}, C_{ij}, D_{ij} \le 1$are the independent random numbers distributed uniformly in the interval specified above.

In the initial series of the numerical experiments, the evaluation of the positive effect (the reducing of the number of the global search iterations executed) due to the reuse of the search information obtained in the course of computations (see Section 4) was performed. When conducting the experiments, the AGCO algorithm was applied sequentially with all scalarization methods considered in Section 3: the min-max convolution of the criteria (10), the method of successive concessions (11), and the reference point method (12). For each method listed above, 50 subproblems with various values of the parameters λ, δ and θ correspondingly have been solved. In order to evaluate the indicator HV of the completeness of the PDA approximation, the reference point (-4,-4) was used. For the AGCO algorithms, the reliability parameter $r = 2.3$ was used; the accuracy in the stopping condition (26) was set as $\varepsilon = 0.01$.

The results of performed experiments are presented in Table 2. In the first part of the table (columns 2-5), the results of solving all subproblems without the use of the search information are shown. In the second part (columns 6-9) the results of solving the same subproblems but with the use of the whole search information obtained in the course of computations (i. e. when solving every next subproblem, the search information obtained when solving all preceding subproblems was utilized) are presented. In the column “S”, the speedup obtained as a result of reducing the number of the global search iteration (the number of the criteria values computed) required to solve all 50 subproblems due to the reuse of the search information is shown.

The results of experiments presented in Table 2 demonstrate the reduction of the number of the executed iterations of the AGCO algorithm more than 9 times. At that, the indicator HV takes almost the same values independently on the usage of the search information. However, the indicator DU was better notably (except the method of successive concessions) at the computations without the use of the search information (similar note can be expressed on the number of effective decisions in the

PDA as well) – most likely, this effect is manifested due to essentially less number of the executed iterations when using the search information. One can note also that the reference point method has some advantage in the efficiency indicators (the number of points in PDA, indicators HV and DU). However, this method inferior the use of the min-max convolutions in the number of iterations required to solve all 50 sub-problems essentially.

Table 2. Efficiency of the reuse of the search information
in solving the problem (288) using various criteria scalarization methods

Search information	Not used				Used				S
Scalarization method	Iters	PDA	HV	DU	Iters	PDA	HV	DU	
Min-max convolution (10)	9124	152	53.4	0.64	922	44	52.7	0.91	9.9
Method of successive concessions (11)	12802	88	52.8	1.16	1402	40	52.4	1.09	9.1
Reference point method (12)	12881	240	53.6	0.51	1323	56	53.0	0.83	9.7

The Pareto domain approximations PDA obtained with the reuse of the search information using various criteria scalarization methods are shown in Fig. 1.

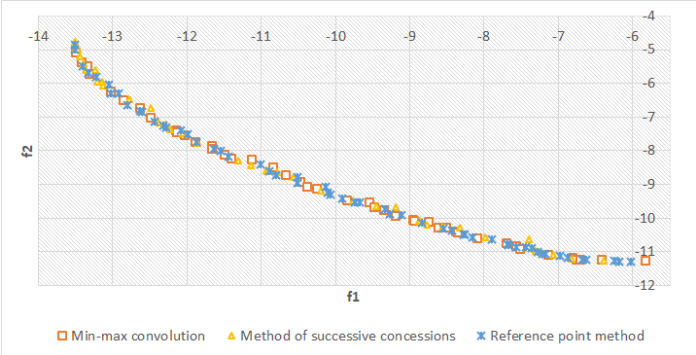

Fig. 1. Pareto domain approximations (PDA) obtained by solving a test MCO problem using various criteria scalarization methods

In the final experiment, solving a test MCO problem has been conducted when applied criteria scalarization methods altered in the course of computations with a complete use of the whole search information. In this experiment, at the first stage of

computations, the min-max criteria convolution was used in solving three subproblems (10) with the convolution coefficients (1,0), (0.5,0.5), and (0,1), correspondingly. At the second stage, the scalarization method was changed to the reference point method (12), where the estimate (-14,-7) was used as the reference point with the weighting coefficients vector (0.5,0.5). And finally, at the third stage, the method of successive concessions (11) was applied with the concession with respect to the first criterion $\delta = 0.5$.

The results of performed experiments are presented in Table 3. The column "Total iters" contains the total number of executed global search iterations whereas the column "New iters" shows the points of iterations executed at particular stage of solving the test MCO problem only. As follows from the results presented in Table 3, the number of the global search iterations executed at separate stages of solving the MCO problem reduced continuously (from 304 down to 35), and the reuse of the search information provides the opportunity of dynamic altering the criteria scalarization methods applied in the course of computations efficiently.

Table 3. Results of solving the problem (28) with altering the criteria scalarization methods in the course of computations

Stage of computations criteria scalarization method	Total iters.	New iters.	PDA
1. Min-max criteria convolution (10), three subproblems	304	304	14
2. Reference point method (12)	402	98	19
3. Method of successive concessions (11)	437	35	24

The approximations of Pareto set PDA obtained at the sequentially completed stages of computations are shown in Fig. 2.

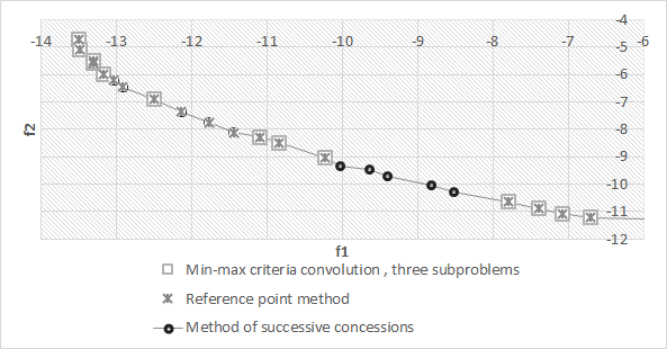

Fig. 2. Approximations of the Pareto set *PDA* obtained at the sequentially completed stages of computations

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References

1. Parnell, G.S., Driscoll, P.J., Henderson, D.L., editors.: Decision Making in Systems Engineering and Management. Wiley, New Jersey (2008).
2. Miettinen K.: Nonlinear Multiobjective Optimization. Springer. (1999).
3. Marler, R. T., Arora, J. S.: Multi-Objective Optimization: Concepts and Methods for Engineering. VDM Verlag. (2009).
4. Ehrgott, M.: Multicriteria Optimization. Springer. (2005).
5. Collette, Y., Siarry, P.: Multiobjective Optimization: Principles and Case Studies (Decision Engineering). Springer. (2011).
6. Pardalos, P.M., Žilinskas, A., Žilinskas, J.: Non-Convex Multi-Objective Optimization. Springer. (2017).
7. Figueira,J., Greco, S., Ehrgott, M., editors.: Multiple criteria decision analysis: State of the art surveys. New York, Springer. (2005).
8. Hillermeier, C., Jahn, J.: Multiobjective optimization: survey of methods and industrial applications. Surv. Math. Ind. 11, 1–42 (2005).
9. Modorskii, V.Y., Gaynutdinova, D.F., Gergel, V.P., Barkalov, K.A.: Optimization in design of scientific products for purposes of cavitation problems AIP Conference Proceedings, 1738, 400013 (2016). DOI: 10.1063/1.4952201
10. Cho, J.-H., Wang, Ya., Chen, I.-R., Chan, K.S., Swami, A.: A Survey on Modeling and Optimizing Multi-Objective Systems. IEEE Communications Surveys & Tutorials 19(3), 1867–1901 (2017).
11. Strongin, R.G., Gergel, V.P.: Parallel computing for globally optimal decision making. Lecture Notes in Computer Science, 2763, 76–88 (2003).
12. Gergel, V.P., Kozinov, E.A.: Accelerating multicriterial optimization by the intensive exploitation of accumulated search data. AIP Conference Proceedings, 1776, 090003 (2016). DOI: 10.1063/1.4965367
13. Gergel, V.: An Unified Approach to Use of Coprocessors of Various Types for Solving Global Optimization Problems. 2nd International Conference on Mathematics and Computers in Sciences and in Industry. (2015). DOI: 10.1109/MCSI.2015.18
14. Barkalov, K., Gergel, V., Lebedev, I.: Solving global optimization problems on GPU cluster. AIP Conference Proceedings, 1738, 400006 (2016). DOI: 10.1063/1.4952194
15. Gergel, V.P., Kozinov, E.A.: Efficient multicriterial optimization based on intensive reuse of search information. J Glob Optim. 71(1), 73–90 (2018). DOI: 10.1007/s10898-018-0624-3
16. Strongin, R., Sergeyev, Ya.: Global optimization with non-convex constraints. Se-quential and parallel algorithms. Kluwer Academic Publishers, Dordrecht (2nd ed. 2013, 3rd ed. 2014).
17. Sergeyev Y.D., Strongin R.G., Lera D.: Introduction to global optimization exploiting space-filling curves. Springer. (2013).
18. Zhigljavsky, A., Žilinskas, A.: Stochastic Global Optimization. Springer, Berlin. (2008).

19. Locatelli, M., Schoen, F. Global Optimization: Theory, Algorithms, and Applica-tions. SIAM. (2013).
20. Floudas, C.A., Pardalos, M.P.: Recent Advances in Global Optimization. Princeton University Press. (2016).
21. Bazaraa, M.S., Sherali, H.D., Shetty, C.M.: Nonlinear Programming: Theory and Algorithms, John Wiley and Sons. (2006).
22. Bleuler, S., Laumanns, M., Thiele, L., Zitzler, E.: PISA-A platform and programming language independent interface for search algorithms. Evolutionary Multi-Criterion Optimization, LNCS, 2632, 494–508 (2003).
23. Evtushenko, Yu.G., Posypkin, M.A.: Method of non-uniform coverages to solve the multicriteria optimization problems with guaranteed accuracy. Autom. Remote Control 75(6), 1025–1040 (2014).
24. Žilinskas, A., Žilinskas, J.: Adaptation of a one-step worst-case optimal univariate algorithm of biobjective Lipschitz optimization to multidimensional problems. Commun. Non-linear Sci. Numer. Simul. 21, 89–98 (2015).
25. Zitzler, E., Knowles, J., Thiele, L.: Quality assessment of Pareto set approximations. LNCS in Multiobjective Optimization-Interactive and Evolutionary Approaches. LNCS, 5252, 373–404 (2008).