LOBACHEVSKY
UNIVERSITY

Comparison Of Several Sequential And Parallel Derivative-free Global Optimization Algorithms

Vladislav Sovrasov and **Semen Bevzuk**

Lobachevsky State University of Nizhni Novgorod

2 April 2019
Kaliningrad, Russia

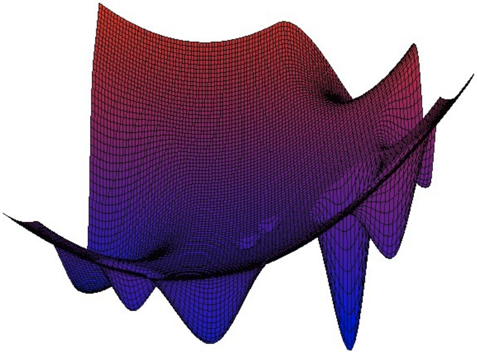
Problem statement

$$\varphi(y^*) = \min\{\varphi(y) : y \in D\},$$
$$D = \{y \in \mathbb{R}^N : a_i \leq y_i \leq b_i, 1 \leq i \leq N\}$$

$\varphi(y)$ is multiextremal objective function, which satisfies the Lipschitz condition:

$$|\varphi(y_1) - \varphi(y_2)| \leq L\|y_1 - y_2\|, y_1, y_2 \in D,$$

where $L > 0$ is the Lipschitz constant, and $||\cdot||$ denotes l_2 norm in $\mathbb{R}^N$ space.



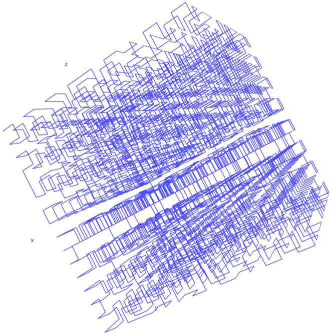
Dimension reduction

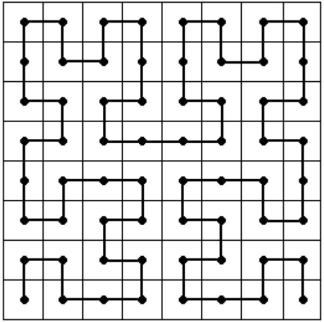
Peano-type curve $y(x)$ allows to reduce the dimension of the original problem:

$$\{y \in \mathbb{R}^N : -2^{-1} \leqslant y_i \leqslant 2^{-1}, 1 \leqslant i \leqslant N\} = \{y(x) : 0 \leqslant x \leqslant 1\}$$

$$\min\{\varphi(y) : y \in D\} = \min\{\varphi(y(x)) : x \in [0,1]\}$$

$y(x)$ is non-smooth function which continuously maps the segment $[0,1]$ to the hypercube D.



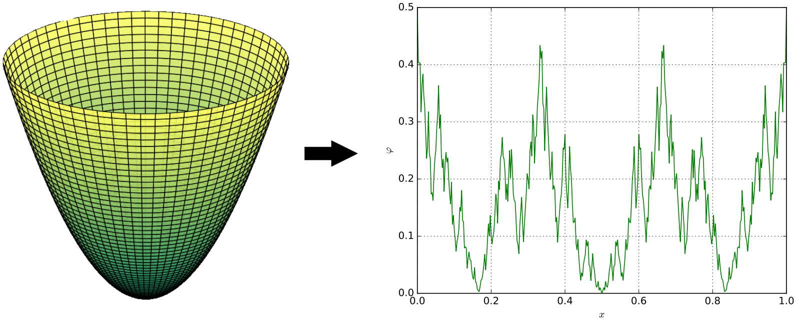


Properties of the reduced problem

After applying the Peano-type evolvent $\varphi(y(x))$ satisfies the uniform Hölder condition:

$$|\varphi(y(x_1)) - \varphi(y(x_2))| \leq H|x_1 - x_2|^{\frac{1}{N}}, x_1, x_2 \in [0, 1],$$

$\varphi(y(x))$ is non-smooth and has multiple local and **global** extremums even if $\varphi(y)$ is unimodal. The latter problem is caused by loss of the information about N-d neighborhood after the transformation to the 1-d space.



Basic optimization method

Optimization method generates search sequence $\{x_k\}$ and consists of the following steps:

Step 1. Sort the search information (one-dimensional points) in increasing order.

Step 2. For each interval (x_{i-1}, x_i) compute quantity $R(i)$, called characteristic.

Step 3. Choose p intervals $(x_{t_j - 1}, x_{t_j})$ with the greatest characteristics and compute objective $\varphi(y(x^{k+j}))$ in points chosen using the decision rule d:

$$x^{k+1+j} = d(t) \in (x_{t_j - 1}, x_{t_j}),\ j = \overline{1, p}$$

Step 4. If $x_{t_j} - x_{t_j - 1} < \varepsilon$ for one of $j = \overline{1, p}$, stop the method.

Detailed description: Strongin R.G., Sergeyev Ya.D.: Global optimization with non-convex constraints. Sequential and parallel algorithms (2000), Chapter 7

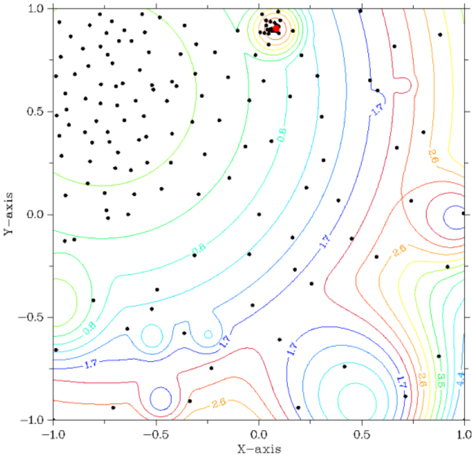
Test problems

Generator GKLS was employed to construct the sets of test problems:

$$f(x) = \begin{cases} C_i(x), x \in S_i, i \in 2, \dots, m \\ \|x - T\|^2 + t, x \notin S_2, \dots, S_m \end{cases}$$

The generator allows to adjust:

- the number of local minimas;
- the size of the global minima attraction region;
- the space dimension.



Cluster environment

The computational experiments have been carried out on the Lobachevsky supercomputer at State University of Nizhni Novgorod. One node includes 2 Intel Sandy Bridge E5-2660 2.2 GHz processors and 64 GB RAM.

Each node runs under CentOS 7 Linux with GCC 4.8 compiler and Intel MPI library.

Evolvents comparison

Evolvents comparison

Choice of evolvent for the parallel algorithm

Results of applying the parallel algorithm

Results of applying the parallel algorithm

Conclusions

- 1

Q&A

Contacts:

sovrasov.vlad@gmail.com
https://github.com/sovrasov