Comparison of dimensionality reduction schemes for parallel global optimization algorithms

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Abstract
This work considers a parallel algorithms for solving multi-extremal optimization problems. Algorithms are developed within the framework of the information-statistical approach and implemented in a parallel solver Globalizer. The optimization problem is solved by reducing the multidimensional problem to a set of joint one-dimensional problems that are solved in parallel. Five types of Peano-type space-filling curves are employed to reduce dimension. The results of computational experiments carried out on several hundred test problems are discussed.

Keywords: Global optimization, Dimension reduction, Parallel algorithms, Multidimensional multiextremal optimization, Global search algorithms

1 Introduction

In the present paper, the parallel algorithms for solving the multiextremal optimization problems are considered. In the multiextremal problems, the opportunity of reliable estimate of the global optimum is based principally on the availability of some information on the function known *a* priori allowing relating the probable values of the optimized function to the known values at the points of performed trials. Very often, such an information on the problem being solved is represented in the form of suggestion that the objective function $\varphi(y)$ satisfies Lipschitz condition with the constant L not known a priori (see, for example, [10, 11, 12]). At that, the objective function could be defined by a program code i. e. could represent a "black-box"-function. Such problems are presented in the applications widely (problems of optimal design of objects and technological processes in various fields of technology, problems of model fitting according to observed data in scientific research, etc.).

Many methods destined to solving the problems of the class specified above reduce the solving of a multidimensional problem to solving the one-dimensional subproblems implicitly (see, for example, the methods of diagonal partitions [6, 7] or simplicial partitions [8, 9]). In the present work, we will use the approach developed in Lobachevsky State University of Nizhni

Novgorod based on the idea of the dimensionality reduction with the use of Peano space-filling curves $y(x)$ mapping the interval $[0, 1]$ of the real axis onto an n-dimensional cube continuously and unambiguously.

Several methods of constructing the evolvents approximating the theoretical Peano curve have been proposed in [1, 2, 3, 4]. These methods were implemented in the Globalizer software system [5]. The goal of the present study was comparing the properties of the evolvents and the selecting the most suitable ones for the use in the parallel global optimization algorithms.

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