LOBACHEVSKY UNIVERSITY

Comparison of Several Stochastic and Deterministic Derivative-free Global Optimization Algorithms

Vladislav Sovrasov

Lobachevsky State University of Nizhni Novgorod

5 July 2019
Ekaterinburg, Russia

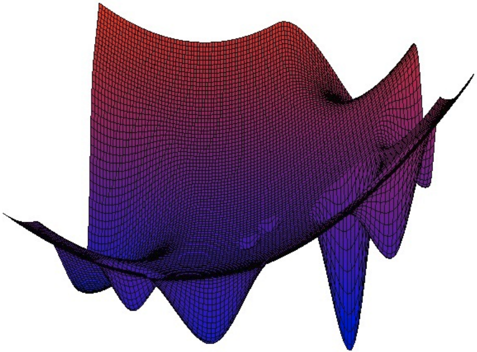
Problem statement

$$\varphi(y^*) = \min\{\varphi(y) : y \in D\},$$
$$D = \{y \in \mathbb{R}^N : a_i \leq y_i \leq b_i, 1 \leq i \leq N\}$$

$\varphi(y)$ is multiextremal objective function, which satisfies the Lipschitz condition:

$$|\varphi(y_1) - \varphi(y_2)| \leq L\|y_1 - y_2\|, y_1, y_2 \in D,$$

where $L > 0$ is the Lipschitz constant, and $||\cdot||$ denotes l_2 norm in $\mathbb{R}^N$ space.



Methods

- **Deteriminstic**:
  - Have complicated internal structure in multidimensional case;
  - Usually store and use the whole history of trials accumulated during search;
  - Require non-trivial parallelization schemes;
  - Under some assumptions convergence to the global solution is guaranteed.
- **Stochastic**:
  - Have relative simple internal structure;
  - Require constant amount of memory to store internal state of some random process or individuals of population;
  - In most cases allows wide-scale parallelization by trials or by running many copies of the same method with different seeds in parallel;
  - Convergence is guaranteed in probabilistic sense only.

Methods

In this work considered the following solvers available in open-source:

- **Deteriminstic**:
  - DIRECT, DIRECTl;
  - AGS, AGSl.
- **Stochastic**:
  - Multi Level Single Linkage;
  - StoGO;
  - Differential Evolution;
  - Controlled Random Search;
  - Dual Simulated Annealing;
  - Ant Colony Optimization.

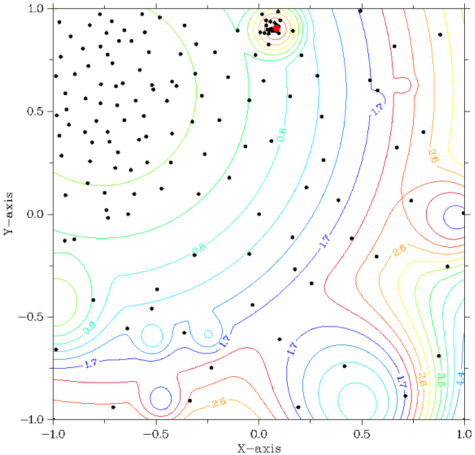
Test problems

Generator GKLS was employed to construct 8 sets of 100 test problems:

$$f(x) = \begin{cases} C_i(x), x \in S_i, i \in 2, \dots, m \\ \|x - T\|^2 + t, x \notin S_2, \dots, S_m \end{cases}$$

The generator allows to adjust:

- the number of local minimums;
- the size of the global minima attraction region;
- the space dimension.



Also one set of 100 pre-defined highly multiextremal functions (with 10-30 local minimums) was used.

Results of sequential methods

Table: Averaged number of trials executed by sequential methods for solving the test optimization problems

	AGS	AGSl	CRS	DIRECT	DIRECTl	MLSL	SDA	DE	StoGO
F_{GR}	193.1	**158.3**	400.3	182.3	214.9	947.2	691.2	1257.3	1336.8
GKLS 2d Simple	254.9	217.6	510.6	**189.0**	255.2	556.8	356.3	952.2	1251.5
GKLS 2d Hard	728.7	**488.0**	844.7	985.4	1126.7	1042.5	1637.9	1041.1	2532.2
GKLS 3d Simple	1372.1	1195.3	4145.8	**973.6**	1477.8	4609.2	2706.5	5956.94	3856.1
GKLS 3d Hard	3636.1	**1930.5**	6787.0	2298.7	3553.3	5640.1	4708.4	6914.3	7843.2
GKLS 4d Simple	26654.1	11095.7	37436.8	**7824.3**	15994.1	41514.3	21417.9	19157.7	59895.4
GKLS 4d Hard	54536.8	**23167.8**	73779.3	23204.4	54489.9	80247.2	68815.5	27466.1	109328.1
GKLS 5d Simple	29810.0	11529.0	143575.0	**7166.5**	13970.5	52647.6	34255.3	73074.5	91580.4
GKLS 5d Hard	113129.1	67652.7	165192.8	**66327.4**	164390.6	138766.2	116973.1	105496.9	155123.8

Results of sequential methods

Table: Number of test optimization problems solved by sequential methods

	AGS	AGSl	CRS	DIRECT	DIRECTl	MLSL	SDA	DE	StoGO
F_{GR}	100	100	76	100	100	97	96	96	67
GKLS 2d Simple	100	100	85	100	100	100	100	98	90
GKLS 2d Hard	100	100	74	100	100	100	93	85	77
GKLS 3d Simple	100	97	75	100	100	100	89	86	44
GKLS 3d Hard	100	99	72	100	99	100	88	77	43
GKLS 4d Simple	100	100	46	100	100	94	78	59	16
GKLS 4d Hard	100	100	47	99	97	94	72	32	10
GKLS 5d Simple	100	100	68	100	100	98	100	77	9
GKLS 5d Hard	97	99	42	100	90	79	84	48	8

Parallel optimization methods

- Parallel AGS (Globalizer system);
- Parallel ACO (MIDACO system).

Univariate Algorithm of Global Search

Optimization method generates search sequence $\{x_k\}$ and consists of the following steps:

Step 1. Sort the search information (one-dimensional points) in increasing order.

Step 2. For each interval (x_{i-1}, x_i) compute quantity $R(i)$, called characteristic.

Step 3. Choose p intervals (x_{t_j-1}, x_{t_j}) with the greatest characteristics and compute objective $\varphi(y(x^{k+j}))$ in points chosen using the decision rule d:

$$x^{k+1+j} = d(t) \in (x_{t_j-1}, x_{t_j}),\ j = \overline{1,p}$$

Step 4. If $x_{t_j} - x_{t_j-1} < \varepsilon$ for one of $j = \overline{1,p}$, stop the method.

Detailed description: Strongin R.G., Sergeyev Ya.D.: Global optimization with non-convex constraints. Sequential and parallel algorithms (2000), Chapter 7

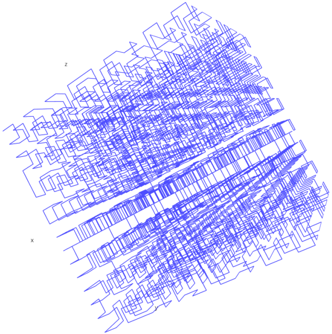
Dimension reduction

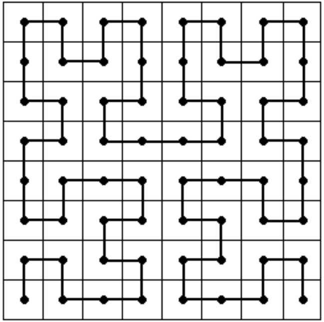
Peano-type curve $y(x)$ allows to reduce the dimension of the original problem:

$$\{y \in \mathbb{R}^N : -2^{-1} \leqslant y_i \leqslant 2^{-1}, 1 \leqslant i \leqslant N\} = \{y(x) : 0 \leqslant x \leqslant 1\}$$

$$\min\{\varphi(y) : y \in D\} = \min\{\varphi(y(x)) : x \in [0,1]\}$$

$y(x)$ is non-smooth function which continuously maps the segment $[0,1]$ to the hypercube D.





Conclusions

- AGSl method implemented in the Globalizer system has demonstrated the convergence speed and reliability at the level of DIRECT and exceeds many other algorithms, the open-source implementations of which are available;
- the stochastic optimization methods inferior to the deterministic ones in the convergence speed and in reliability. It is manifested especially strongly on more complex multiextremal problems;
- the parallel version of the Globalizer system demonstrates good speedup when running on several nodes, on each of which a single objective function value per iterations is computed. When solving the problems with fast computable objective functions and using multiple threads on the nodes, the speedup for the Globalizer system degrade with increasing number of nodes;
- the MIDACO system is the most suitable for simple global optimization problems with the fast-computable objective functions. In this case, MIDACO is reliable enough and provides a linear speedup with increasing number of nodes and threads executed on these ones in parallel.

Q&A

Contacts:

sovrasov.vlad@gmail.com
https://github.com/sovrasov