Comparison of dimensionality reduction schemes for parallel global optimization algorithms*

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Аннотация This work considers a parallel algorithms for solving multiextremal optimization problems. Algorithms are developed within the framework of the information-statistical approach and implemented in a parallel solver "Globalizer" . The optimization problem is solved by reducing the multidimensional problem to a set of joint one-dimensional problems that are solved in parallel. Five types of Peano-type space-filling curves are employed to reduce dimension. The results of computational experiments carried out on several hundred test problems are discussed.

Keywords: Global optimization · Dimension reduction · Parallel algorithms · Multidimensional multiextremal optimization · Global search algorithms · Parallel computations

1 Introduction

Global (or multiextremal) optimization problems are among the most complex problems in both theory and practice of optimal decision making. In these kinds of problems, the optimization criterion can have several local optima within the search domain. The existence of several local optima makes finding the global optimum difficult essentially, since it requires examining the whole feasible search domain. The volume of computations for solving global optimization problems can increase exponentially with increasing number of varied parameters.

These global optimization problem features impose special requirements on the quality of the optimization methods and on the software to implement these ones. The global optimization methods should be highly efficient, and the software systems should be developed on a good professional basis. In general, the global optimization problems can be solved at a reasonable time by employing parallel computations on modern supercomputing systems only.

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The general state of the art in the field of global optimization is presented in a number of key monographs [13], [25], [33], [38], [51], [52], [54]. The development of optimization methods, which use the high-performance computational systems to solve the time-consuming global optimization problems, is an area of intensive research — see, for instance, [7], [8], [34], [50], [51]. The obtained theoretical results provide the efficient solutions of many applied global optimization problems in various fields of scientific and technical applications [10], [11], [12], [28], [29], [33], [34], [37], [38].

At the same time, the practical implementation of these global optimization algorithms within the framework of industrial software systems is quite limited. In many cases, software implementations are experimental in nature and are used by the developers themselves to obtain the results from the computational experiments required for the scientific publications. This situation originates from high development costs of the professional software systems, which can be used by numerous users. In addition, the global optimization problems could be solved in an automatic mode rarely because of the complexity of these ones. The user should actively control the global search process that implies an adequate level of qualification in the field of optimization (particularly, the user should know and understand the global optimization methods well).

In this work, the authors consider an approach to minimizing multiextremal functions developed in ... This allows problems to be solved in which function values may not be deter-mined for the entire search domain. Under this approach, solving multidimen-sional problems is reduced (using Peano-type space-filling curves) to solving equivalent one-dimensional problems. It should be noted that standard approaches to algorithm parallelization are not quite applicable to global optimization. For example, the rules for selecting an-other iteration point are quite simple and do not require parallelization (as over-heads associated with organizing parallel computations will nullify any possible acceleration). Some acceleration can be achieved by parallelizing the computa-tion of function values describing the object to be optimized; however, this ap-proach is specific to each individual problem being solved. The following approach looks more promising. The algorithm can be modified to run several trials in parallel. This approach provides the efficiency (as paral-lelization is applied to the most computation-intensive part of the problem solv-ing process) and generality (in that it applies to a wide range of global optimiza-tion algorithms). The approach, described in [55] for unconstrained optimization, was used in this work for parallelizing constrained optimization algorithms.

2 Statement of Multidimensional Global Optimization Problem

3 Methods of Dimension Reduction

3.1 Базовый алгоритм

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3.2 Сдвиговые

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3.3 Вращаемые

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3.4 Еще одни развертки

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3.5 Гладкие

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4 Parallel Computations for Solving Global Optimization Problems.

4.1 Core Multidimensional Algorithm of Global Search

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4.2 Параллельные множественные отображения

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5 Results of Numerical Experiments

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5.1 Сравнение последовательных сдвиговых и вращаемых разверток

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5.2 Параллельные вращаемые развертки

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5.3 Параллельные вращаемые развертки для задач большой размерности

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6 Вывод о целесообразности применения того или иного вида разверток для того или иного вида задач

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7 First Section

7.1 A Subsection Sample

Please note that the first paragraph of a section or subsection is not indented. The first paragraph that follows a table, figure, equation etc. does not need an indent, either.

Subsequent paragraphs, however, are indented.

Sample Heading (Third Level) Only two levels of headings should be numbered. Lower level headings remain unnumbered; they are formatted as run-in headings.

Sample Heading (Fourth Level) The contribution should contain no more than four levels of headings. Table 1 gives a summary of all heading levels.

Таблица 1. Table captions should be placed above the tables.

Heading level	Example	Font size and style
Title (centered)	**Lecture Notes**	14 point, bold
1st-level heading	**1 Introduction**	12 point, bold
2nd-level heading	**2.1 Printing Area**	10 point, bold
3rd-level heading	**Run-in Heading in Bold.** Text follows	10 point, bold
4th-level heading	*Lowest Level Heading.* Text follows	10 point, italic

Displayed equations are centered and set on a separate line.

$$x + y = z \tag{1}$$

Please try to avoid rasterized images for line-art diagrams and schemas. Whenever possible, use vector graphics instead (see Fig. 1).

Theorem 1. *This is a sample theorem. The run-in heading is set in bold, while the following text appears in italics. Definitions, lemmas, propositions, and corollaries are styled the same way.*

Доказательство. Proofs, examples, and remarks have the initial word in italics, while the following text appears in normal font.

For citations of references, we prefer the use of square brackets and consecutive numbers. Citations using labels or the author/year convention are also acceptable. The following bibliography provides a sample reference list with entries for journal articles [56], an LNCS chapter [57], a book [58], proceedings without editors [59], and a homepage [60]. Multiple citations are grouped [56,57,58], [56,58,59,60].

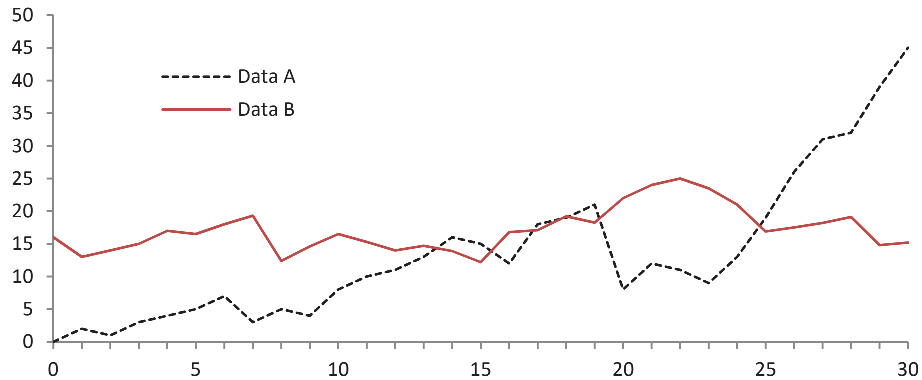

Рис. 1. A figure caption is always placed below the illustration. Please note that short captions are centered, while long ones are justified by the macro package automatically.

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