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图像处理与分析

第五讲: 图像变换 (III)

图像的空间域频率域滤波，正交变换，距离变换

基于深度学习的图像变换

内容提要

- 采样定理应用示例
- 图像的空间域滤波(复习)
- 图像的频率域滤波(复习)
- 一维离散傅里叶变换(复习)
- 图像的二维离散傅里叶变换的详细介绍
- 图像的二维离散傅里叶变换的计算
- 图像的正交变换
- 图像的距离变换
- 基于深度学习的图像变换简介

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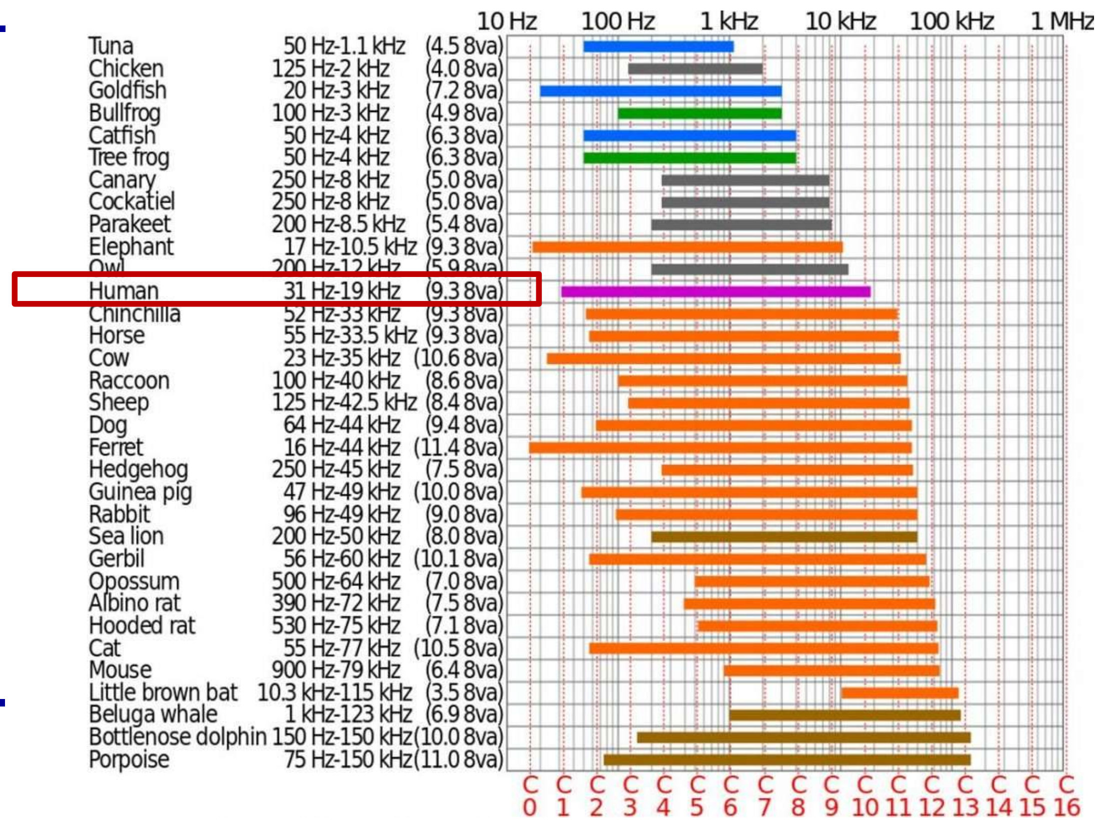
采样定理应用示例：音频录制

- 人类听力覆盖的频率范围20-20kHz

- 常用的声音录制频率为16bit/44.1kHz (CD).

- 高清晰度/高保真声音录制的频率为 24bit/96kHz,192kHz.

- 2^24 = 16,777,216



https://en.wikipedia.org/wiki/Audio_frequency
https://www.thebroadcastbridge.com/content/entry/7999/should-audio-for-video-be-recorded-at-high-sample-rates
https://www.whathifi.com/advice/high-resolution-audio-everything-you-need-to-know#:~:text=In%20its%20simplest%20terms%2C%20hi,16%2Dbit%2F44.1kHz.&text=Hi%2Dres%20audio%20files%20usually,and%20176.4kHz%20files%20too.

图像的空间采样频率

a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

采样定理应用示例：手机相机空间采样频率 (I)

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Research Article

Optical design of camera optics for mobile phones

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Abstract

new product generation. In contrast to this trend, the resolution has increased from 0.3 MP in 2002 to 12 MP at present.

Here is an example of a typical specification for a modern 12 MP mobile phone optical module (Table 1). The pixel pitch of pp=1.4 µm defines the maximum spatial resolution of the sensor according to the Nyquist sampling theorem:

$$V_{Nyquist}=\frac{1}{2\times pp}=357\frac{lp}{mm}$$

Table 1 Basic optical layout of CCMs.

Basic optical layout		
Sensor size (semi-diameter of image circle)	y' (max)	3.52 mm
Pixel pitch	pp	1.4 µm
Sensor resolution	Number of pixels in x,y	approx. 4000×3000 12 MP
Aperture	$F\#$	2.8
Focal length	f'	4.52 mm
Diagonal full field of view	$DFOV=2\,w$	76°
Module size	$x\times y\times z$	<1 cm^3
Optical total track from first lens vertex to image plane	sl-img	<7 mm
Filter package thickness in image space (IR-cut, cover glass)	d	0.3 mm
Minimum optical distance for focusing	MOD	100 mm
Maximum chief ray angle upon image plane	CRA	<30°

采样定理应用示例：手机相机空间采样频率 (II)

Model number	Resolution	Sensor size (Optical format)	Pixel size	Pixel type	Chroma	Products used in
ISOCELL GN1 (S5KGN1)[5]	Photo: 8160x 6144 50MP Video: 7680 x 4320 @ 30 fps	(~1/1.31")	1.2 μm	ISOCELL Plus	Tetracell	**Rear:** Vivo X50 Pro+ Vivo iQOO 5/5 Pro
ISOCELL HM1 (S5KHM1)[6]	Photo: 12000 x 9000 108MP Video: 7680 x 4320 @ 24 fps	12.03 mm (~1/1.33")	0.8 μm	ISOCELL Plus	Nonacell	**Rear:** Samsung Galaxy S20 Ultra Samsung Galaxy Note 20 Ultra
ISOCELL HMX (S5KHMX)[7]	Photo: 12032 x 9024 108MP Video: 6016 x 3384 @ 30 fps	12.03 mm (~1/1.33")	0.8 μm	ISOCELL Plus	Tetracell	**Rear:** Xiaomi Mi MIX Alpha Xiaomi Mi CC9 Pro Xiaomi Mi Note 10 Xiaomi Mi Note 10 Pro Xiaomi Mi 10 Xiaomi Mi 10 Pro Xiaomi Mi 10T Pro Motorola Edge+
ISOCELL GW2 (S5KGW2)	9280 x 6944 64MP Video: 7680 x 4320 @ 24 fps	9.216 mm (~1/1.72")	0.8 μm	ISOCELL Plus	RGB	**Rear:** Samsung Galaxy S20 Samsung Galaxy S20+ Samsung Galaxy Note 20

https://en.wikipedia.org/wiki/Samsung_CMOS

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两类图像空间域滤波器

- 平滑空间滤波器：通常用于抑制噪声
- 锐化空间滤波器：通常用于突出图像细节

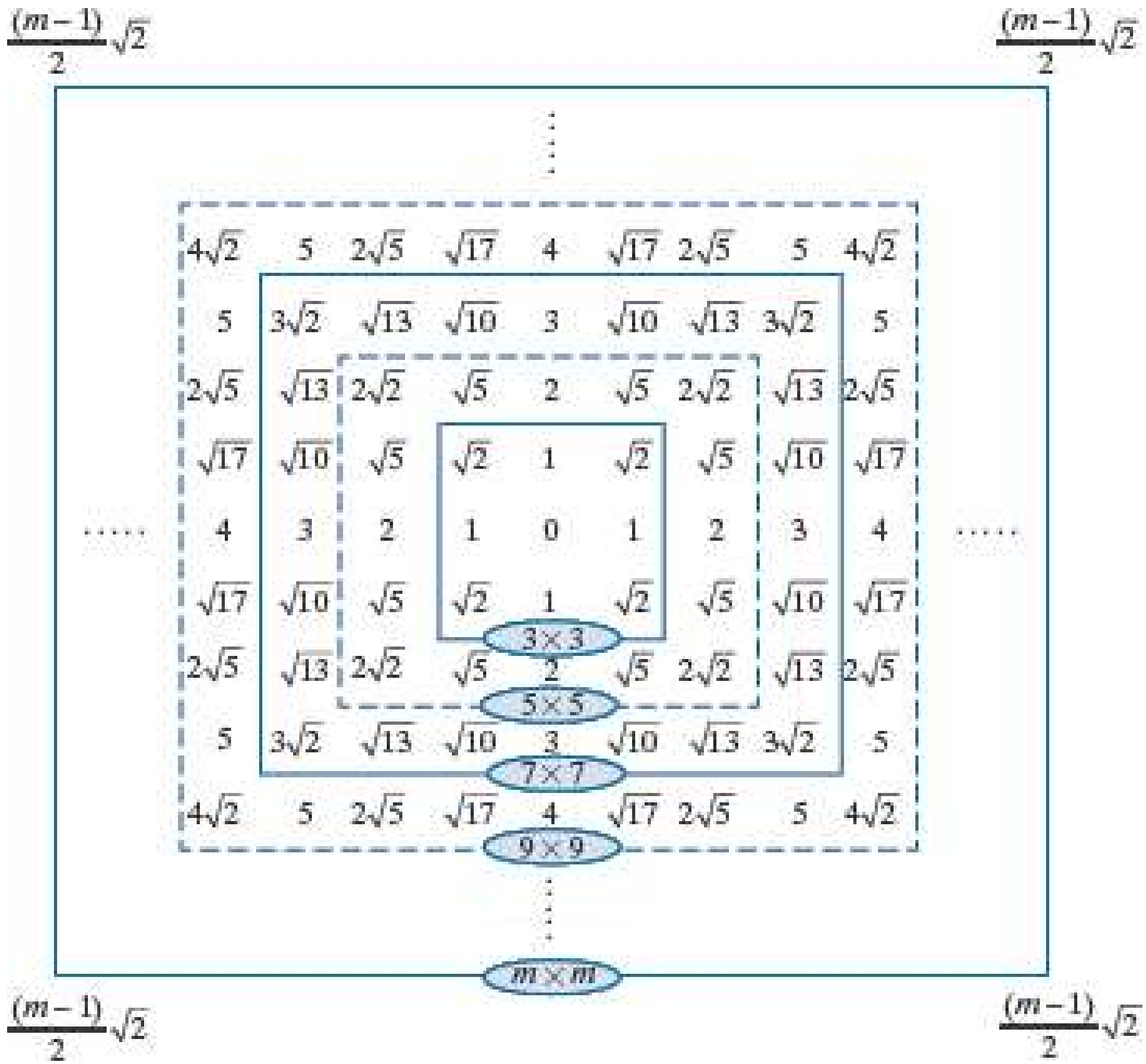
图像卷积与相关运算的一些特性

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \mathbin{☆} (g + h) = (f \mathbin{☆} g) + (f \mathbin{☆} h)$

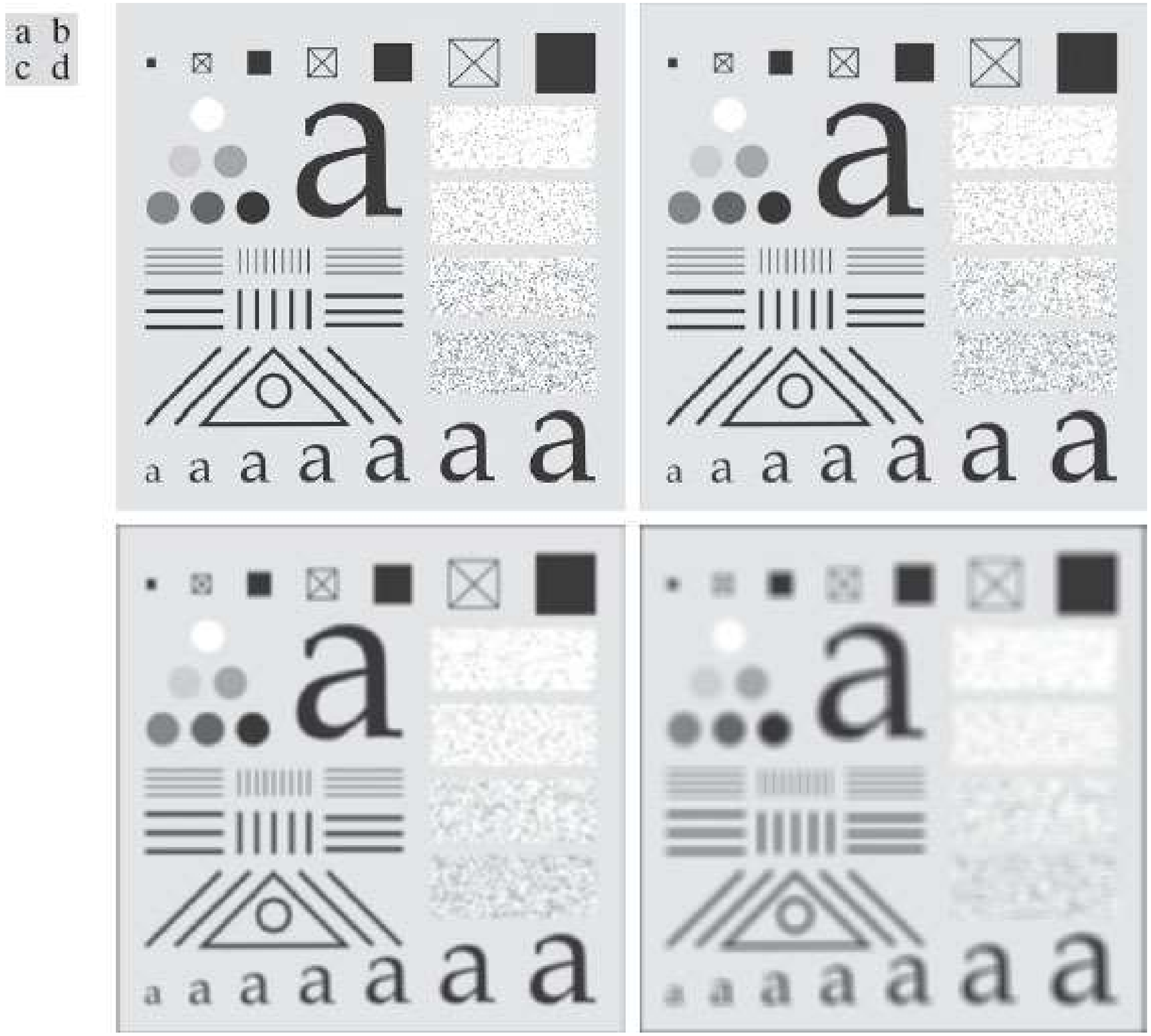
常用的空间平滑滤波器

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}\qquad \frac{1}{4.8976}\times\begin{array}{|c|c|c|}\hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline\end{array}$$

方型滤波器 (I)



方型滤波器 (II)

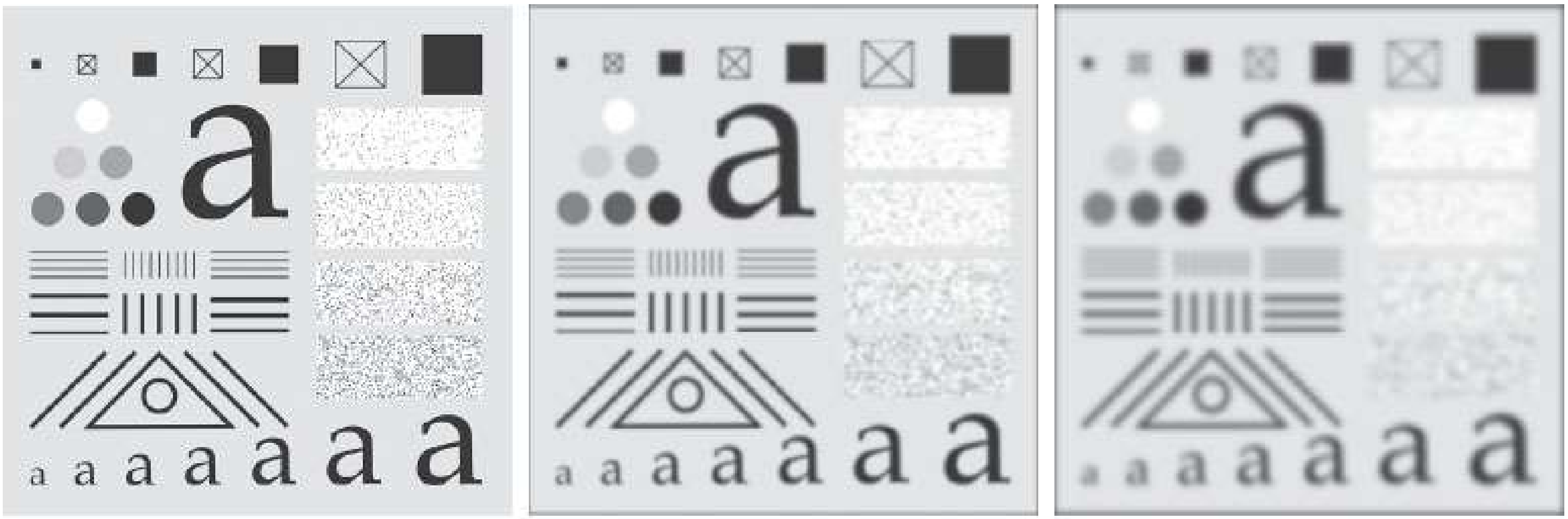


(b) 3x3 (c) 11x11 (d) 21x21

高斯滤波器 (I)

	f	g	$f \times g$	$f \star g$
Mean	m_f	m_g	$m_{f\times g} = \dfrac{m_f\sigma_g^2 + m_g\sigma_f^2}{\sigma_f^2 + \sigma_g^2}$	$m_{f\star g} = m_f + m_g$
Standard deviation	σ_f	σ_g	$\sigma_{f\times g} = \sqrt{\dfrac{\sigma_f^2\sigma_g^2}{\sigma_f^2 + \sigma_g^2}}$	$\sigma_{f\star g} = \sqrt{\sigma_f^2 + \sigma_g^2}$

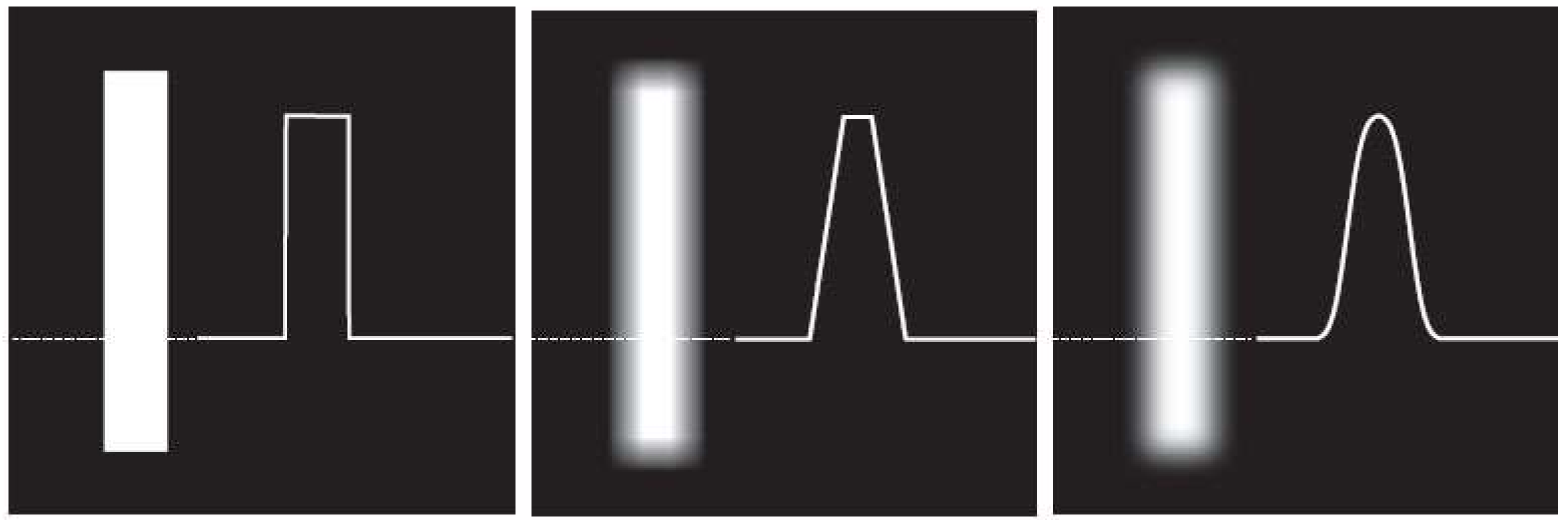
高斯滤波器 (II)



a b c

σ=3.5 σ=7.0

比较方型滤波器和高斯滤波器



a b c

71x71

σ=25

图像滤波示例：图像导数的计算 (II)

- 一阶导数

$$I_x(i,j)=\frac{I(i+1,j)-I(i-1,j)}{2h}+O(h^2)$$

$$I_y(i,j)=\frac{I(i,j+1)-I(i,j-1)}{2h}+O(h^2)$$

- 二阶导数

$$I_{xx}(i,j)=\frac{I(i+1,j)-2I(i,j)+I(i-1,j)}{h^2}+O(h)$$

$$I_{yy}(i,j)=\frac{I(i,j+1)-2I(i,j)+I(i,j-1)}{h^2}+O(h)$$

常用的滤波器算子：Laplacian算子 (I)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b c d

(a) Laplacian kernel used to implement Eq. (3-62). (b) Kernel used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other Laplacian kernels.

GY1

常用的滤波器算子：Laplacian算子 (II)

a b
c d

a b
c d

FIGURE 3.46 (a) Blurred image of the North Pole of the moon. (b) Laplacian image obtained using the kernel in Fig. 3.45(a). (c) Image sharpened using Eq. (3-54) with $c = -1$. (d) Image sharpened using the same procedure, but with the kernel in Fig. 3.45(b). (Original image courtesy of NASA.)

$$g(x,y) = f(x,y) + c \cdot \nabla^2 f(x,y)$$

Slide 19

GY1 Ge Yang, 10/13/2020

常用的滤波器算子：Roberts算子和Sobel算子

−1	0
0	1

0	−1
1	0

Roberts 算子

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel 算子

$$\nabla f = grad(f) = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = \big(f(x+1,y-1)+2f(x+1,y)+f(x+1,y+1)\big)-\big(f(x-1,y-1)+2f(x-1,y)+f(x-1,y+1)\big)$$

$$\frac{\partial f}{\partial y} = \big(f(x-1,y+1)+2f(x,y+1)+f(x+1,y+1)\big)-\big(f(x-1,y-1)+2f(x,y-1)+f(x+1,y-1)\big)$$

Sobel 算子

常用的滤波器算子：Prewitt算子和Scharr算子

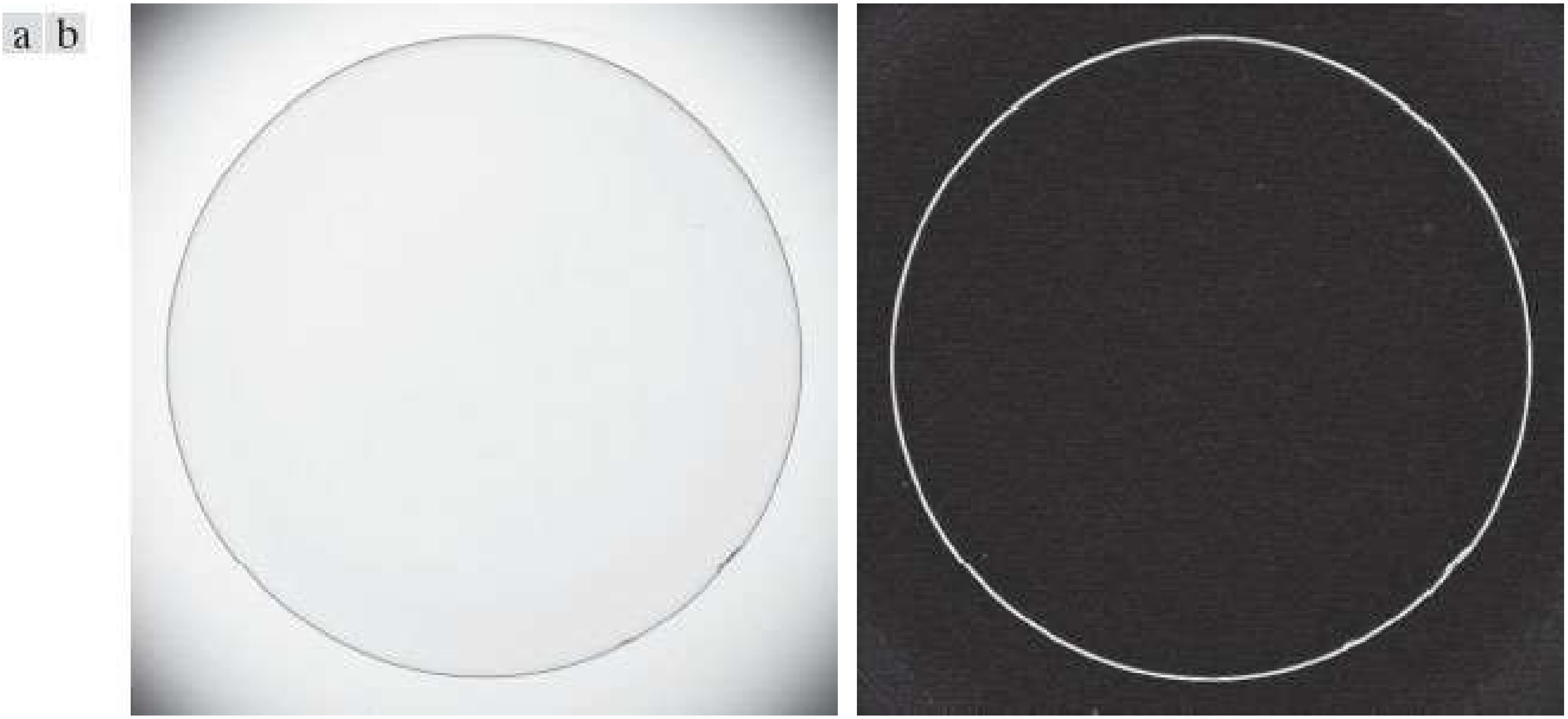
$$\begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix} \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{bmatrix}$$

Prewitt 算子

$$\begin{bmatrix} -3 & 0 & +3 \\ -10 & 0 & +10 \\ -3 & 0 & +3 \end{bmatrix} \begin{bmatrix} -3 & -10 & -3 \\ 0 & 0 & 0 \\ +3 & +10 & +3 \end{bmatrix}$$

Scharr 算子

不同算子的比较 (I)



(a) Image of a contact lens (note defects on the boundary at 4 and 5 o’clock). (b) Sobel gradient.

不同算子的比较 (II)

Source

Roberts

Sobel

Scharr

Prewitt

https://en.wikipedia.org/wiki/Roberts_cross

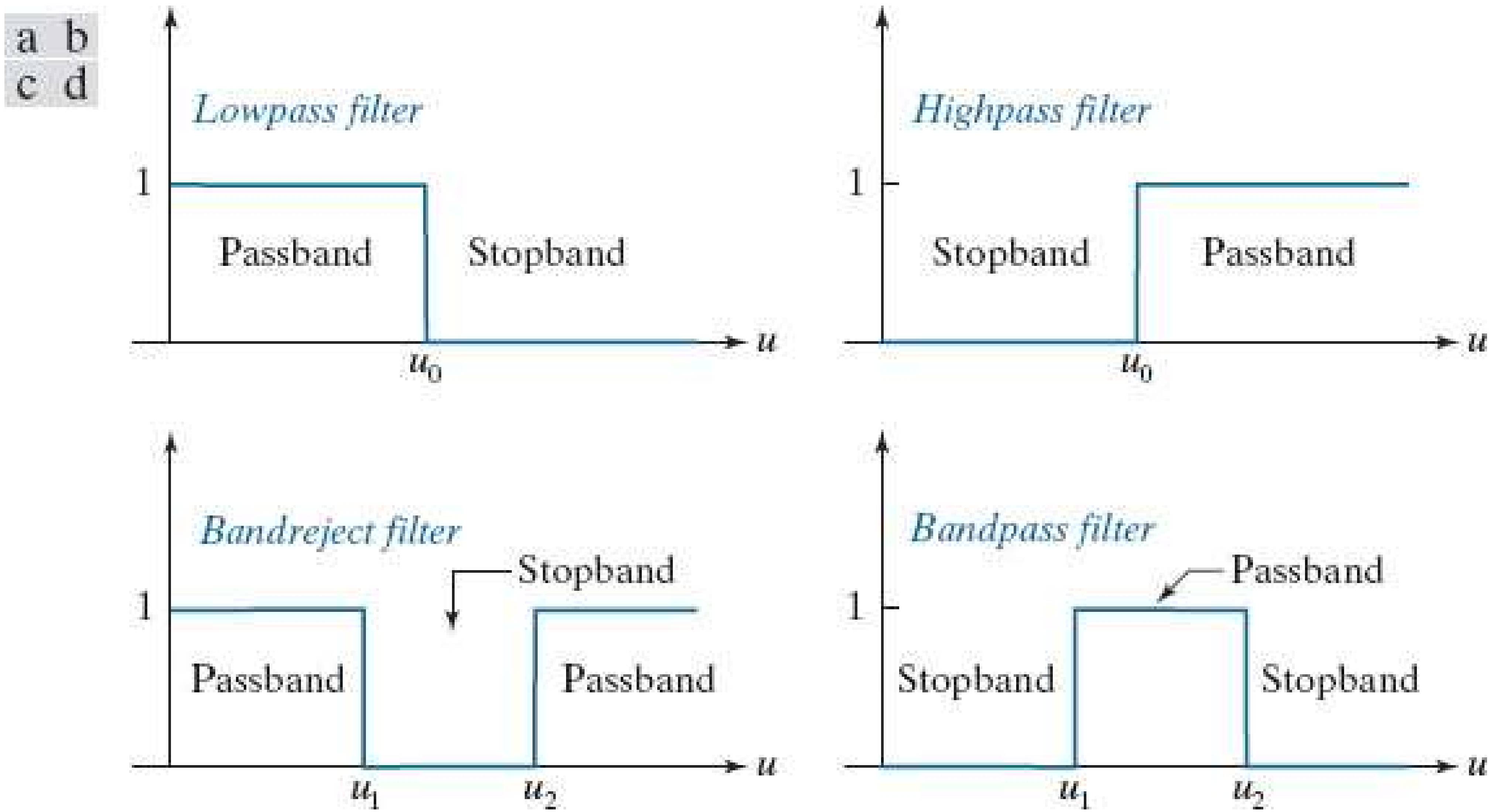
锐化滤波器是高通滤波器

	Name	DFT Pairs
1)	Symmetry properties	See Table 4.1
2)	Linearity	$af_1(x,y)+bf_2(x,y) \Leftrightarrow aF_1(u,v)+bF_2(u,v)$
3)	Translation (general)	$f(x,y)e^{j2\pi(u_0x/M+v_0y/N)} \Leftrightarrow F(u-u_0,v-v_0)$ $f(x-x_0,y-y_0) \Leftrightarrow F(u,v)e^{-j2\pi(ux_0/M+vy_0/N)}$
4)	Translation to center of the frequency rectangle, $(M/2,N/2)$	$f(x,y)(-1)^{x+y} \Leftrightarrow F(u-M/2,v-N/2)$ $f(x-M/2,y-N/2) \Leftrightarrow F(u,v)(-1)^{u+v}$
5)	Rotation	$f(r,\theta+\theta_0) \Leftrightarrow F(\omega,\varphi+\theta_0)$ $r=\sqrt{x^2+y^2} \quad \theta=\tan^{-1}(y/x) \quad \omega=\sqrt{u^2+v^2} \quad \varphi=\tan^{-1}(v/u)$
6)	Convolution theorem†	$f\star h)(x,y) \Leftrightarrow (F\bullet H)(u,v)$ $(f\bullet h)(x,y) \Leftrightarrow (1/MN)[(F\star H)(u,v)]$
7)	Correlation theorem†	$(f\,☆\,h)(x,y) \Leftrightarrow (F^*\bullet H)(u,v)$ $(f^*\bullet h)(x,y) \Leftrightarrow (1/MN)[(F\,☆\,H)(u,v)]$
8)	Discrete unit impulse	$\delta(x,y) \Leftrightarrow 1$ $1 \Leftrightarrow MN\delta(u,v)$
9)	Rectangle	$\mathrm{rec}[a,b] \Leftrightarrow ab\dfrac{\sin(\pi ua)}{(\pi ua)}\dfrac{\sin(\pi vb)}{(\pi vb)}e^{-j\pi(ua+vb)}$
10)	Sine	$\sin(2\pi u_0x/M+2\pi v_0y/N) \Leftrightarrow \dfrac{jMN}{2}[\delta(u+u_0,v+v_0)-\delta(u-u_0,v-v_0)]$
11)	Cosine	$\cos(2\pi u_0x/M+2\pi v_0y/N) \Leftrightarrow \dfrac{1}{2}[\delta(u+u_0,v+v_0)+\delta(u-u_0,v-v_0)]$

The following Fourier transform pairs are derivable only for continuous variables, denoted as before by t and z for spatial variables and by μ and ν for frequency variables. These results can be used for DFT work by sampling the continuous forms.

12)	Differentiation (the expressions on the right assume that $f(\pm\infty,\pm\infty)=0$.	$\left(\dfrac{\partial}{\partial t}\right)^m\left(\dfrac{\partial}{\partial z}\right)^n f(t,z) \Leftrightarrow (j2\pi\mu)^m(j2\pi\nu)^n F(\mu,\nu)$ $\dfrac{\partial^m f(t,z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m F(\mu,\nu);\ \dfrac{\partial^n f(t,z)}{\partial z^m} \Leftrightarrow (j2\pi\nu)^n F(\mu,\nu)$
13)	Gaussian	$A2\pi\sigma^2e^{-2\pi^2\sigma^2(t^2+z^2)} \Leftrightarrow Ae^{-(\mu^2+\nu^2)/2\sigma^2}$ (A is a constant)

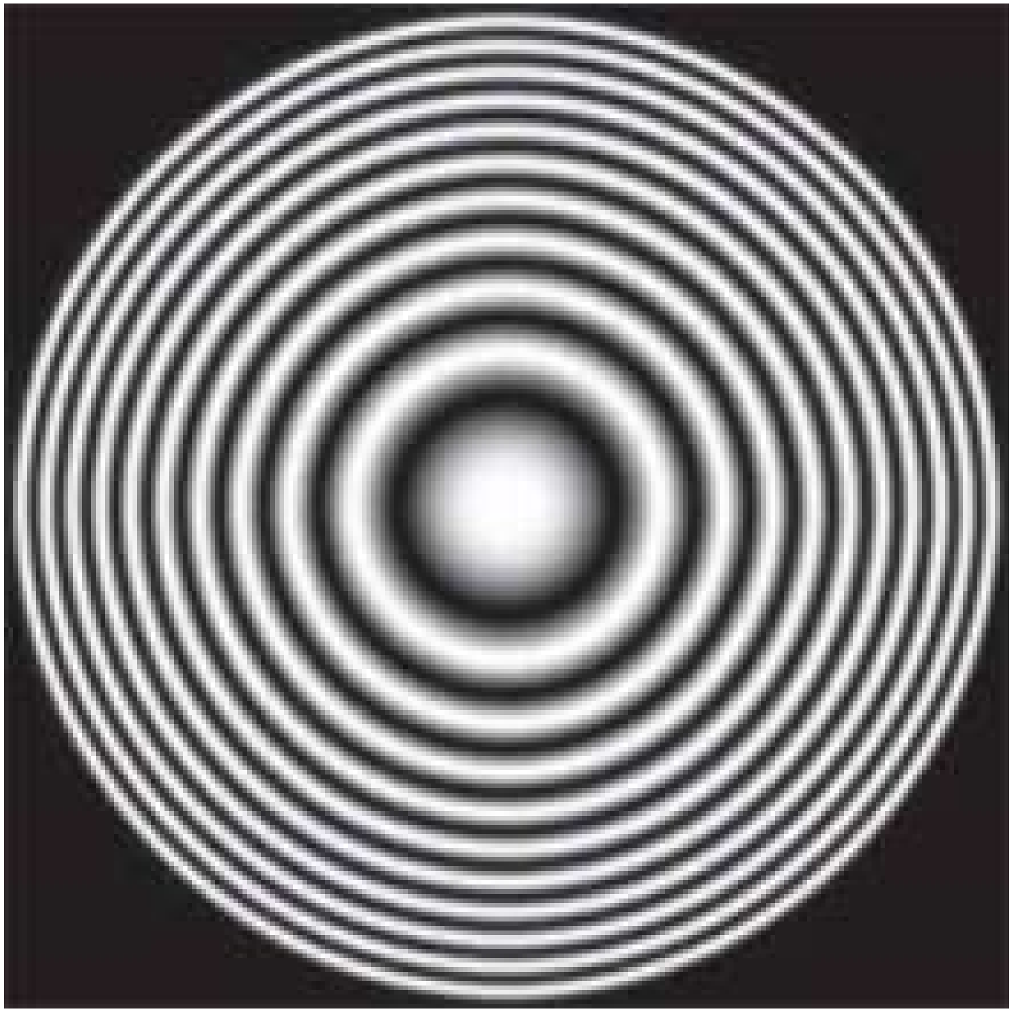
不同类型的频率域滤波器



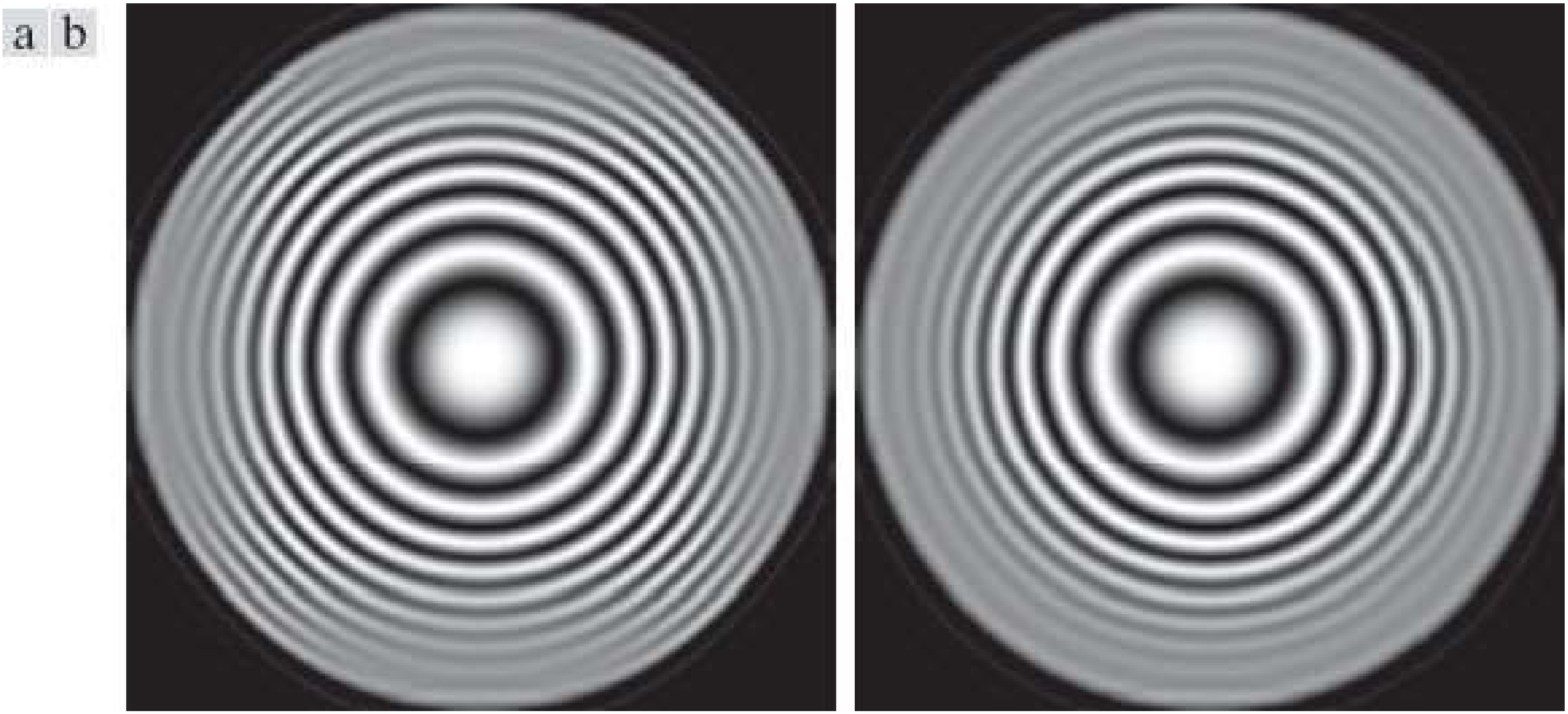
不同类型的频率域滤波器对应的空间滤波器

Filter type	Spatial kernel in terms of lowpass kernel, lp
Lowpass	$lp(x,y)$
Highpass	$hp(x,y) = \delta(x,y) - lp(x,y)$
Bandreject	$br(x,y) = lp_1(x,y) + hp_2(x,y)$ $= lp_1(x,y) + [\delta(x,y) - lp_2(x,y)]$
Bandpass	$bp(x,y) = \delta(x,y) - br(x,y)$ $= \delta(x,y) - [lp_1(x,y) + [\delta(x,y) - lp_2(x,y)]]$

不同滤波器效果示例 (I)



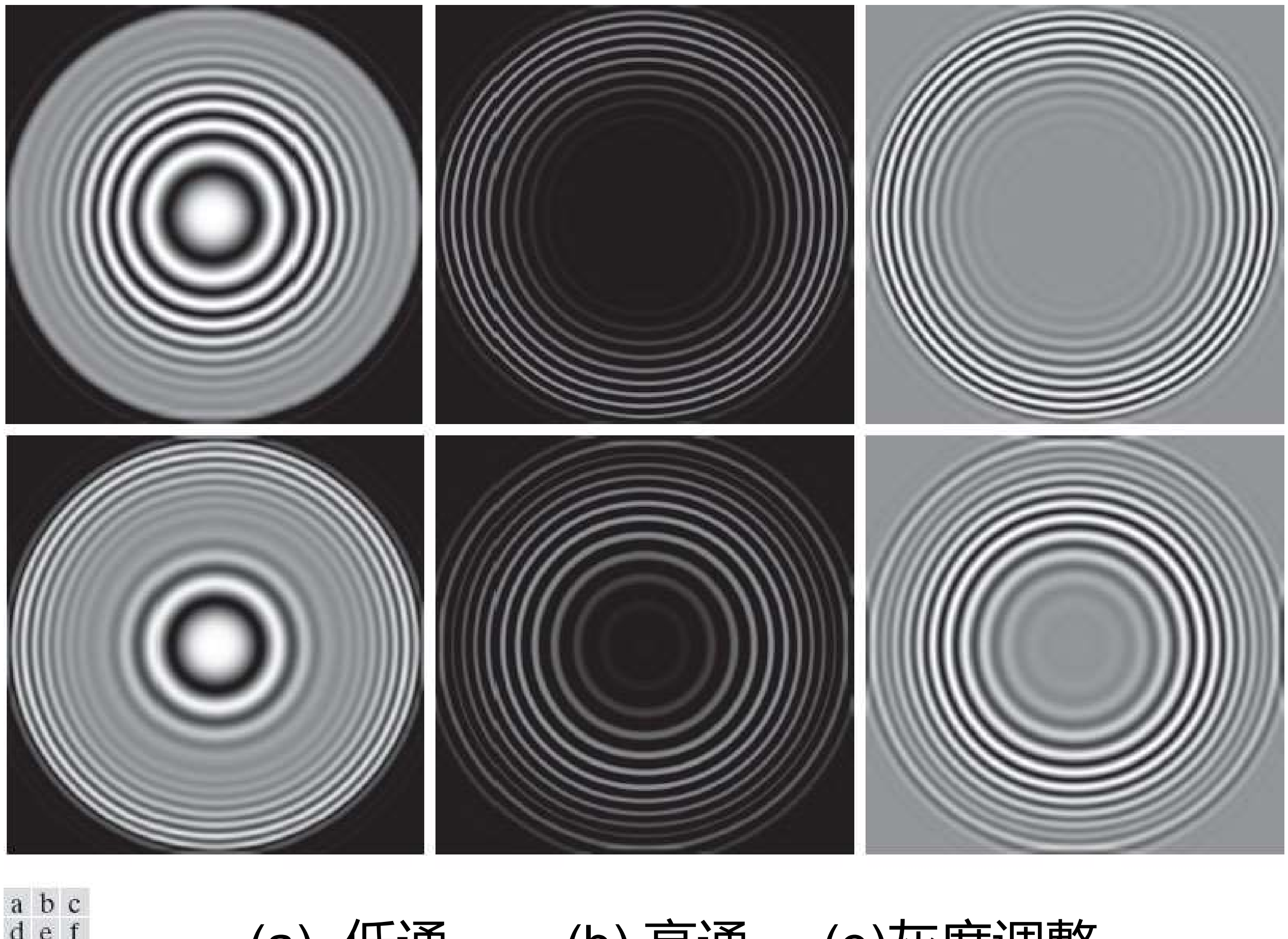
不同滤波器效果示例 (II)



可分低通滤波

各向同性低通滤波

不同滤波器效果示例 (III)



(a) 低通　(b) 高通　(e)灰度调整
(d) 带阻　(e) 带通　(f)灰度调整

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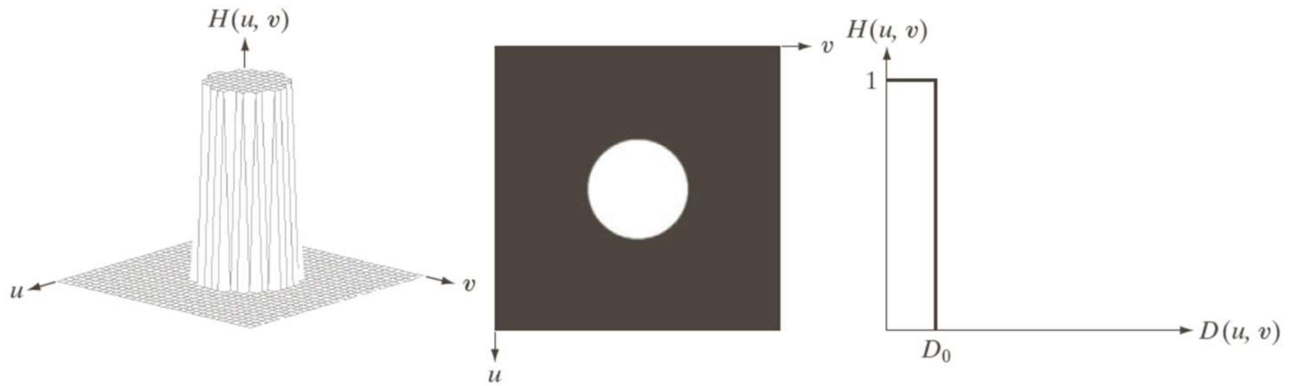
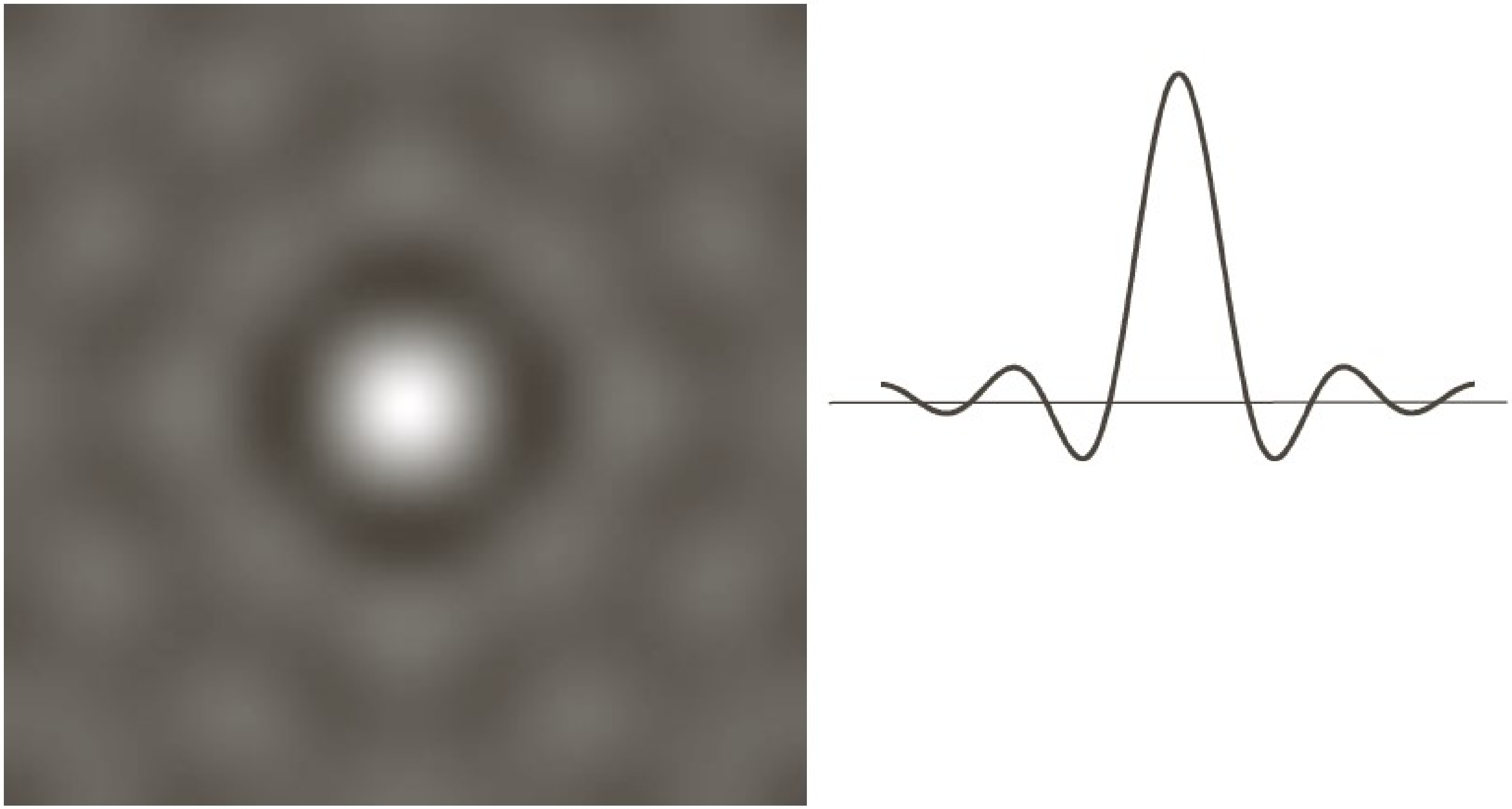
代表性滤波器：理想低通滤波器 ILPF



a b c

FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

ILPF的空间域特性



a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000. (b) Intensity profile of a horizontal line passing through the center of the image.

代表性滤波器：高斯低通滤波器 GLPF

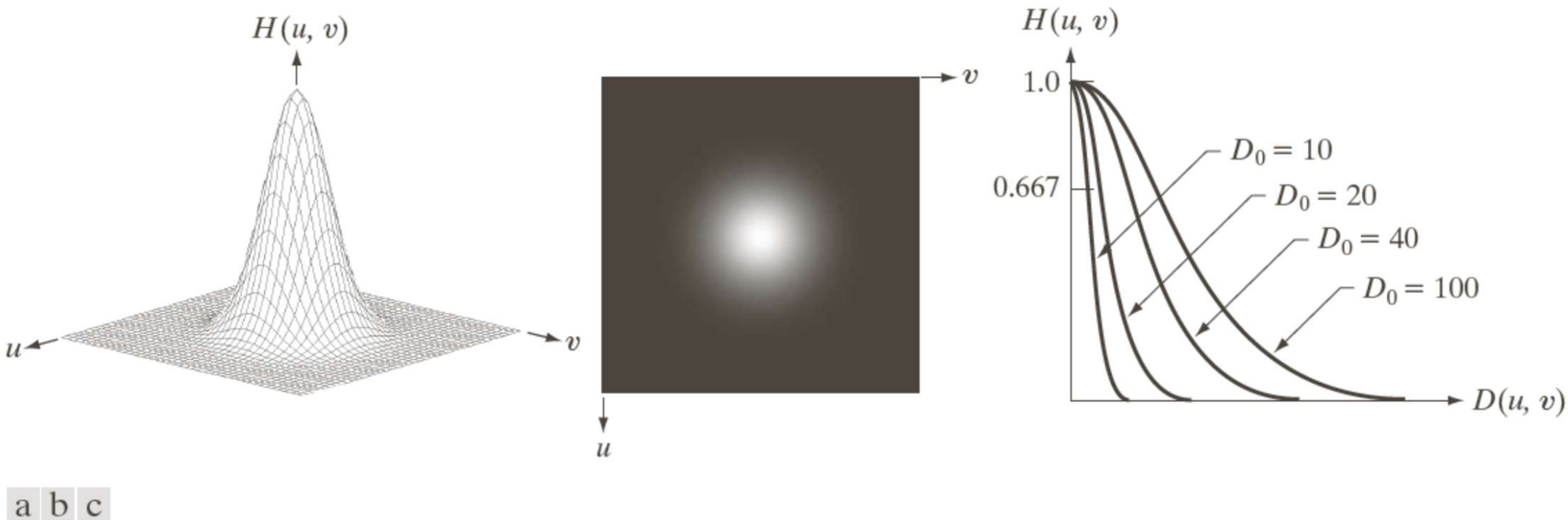
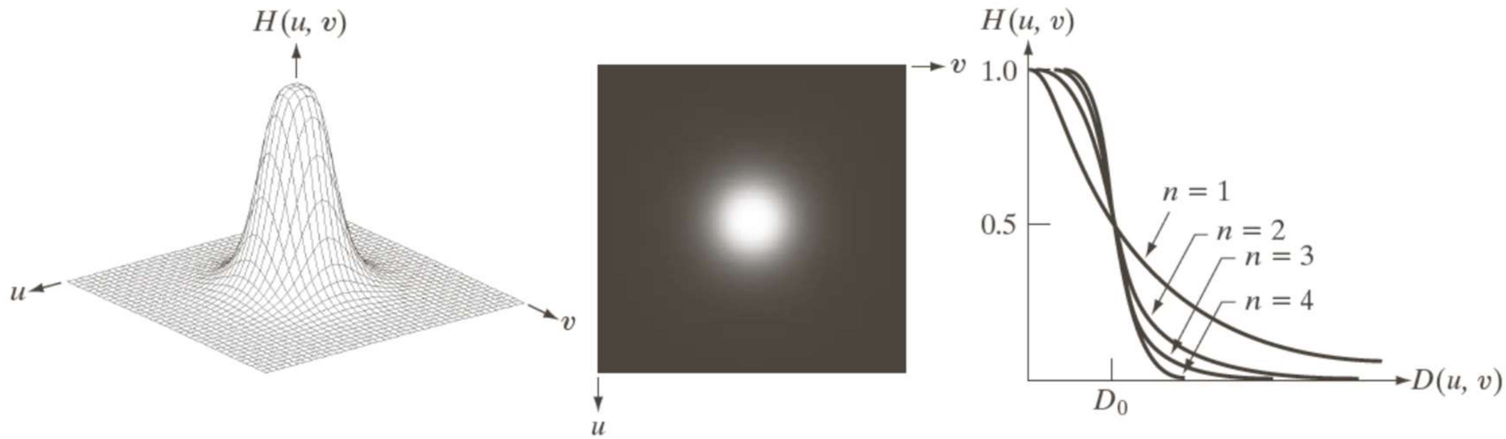


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

代表性滤波器：巴特沃斯低通滤波器 BLPF

TABLE 4.4
Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

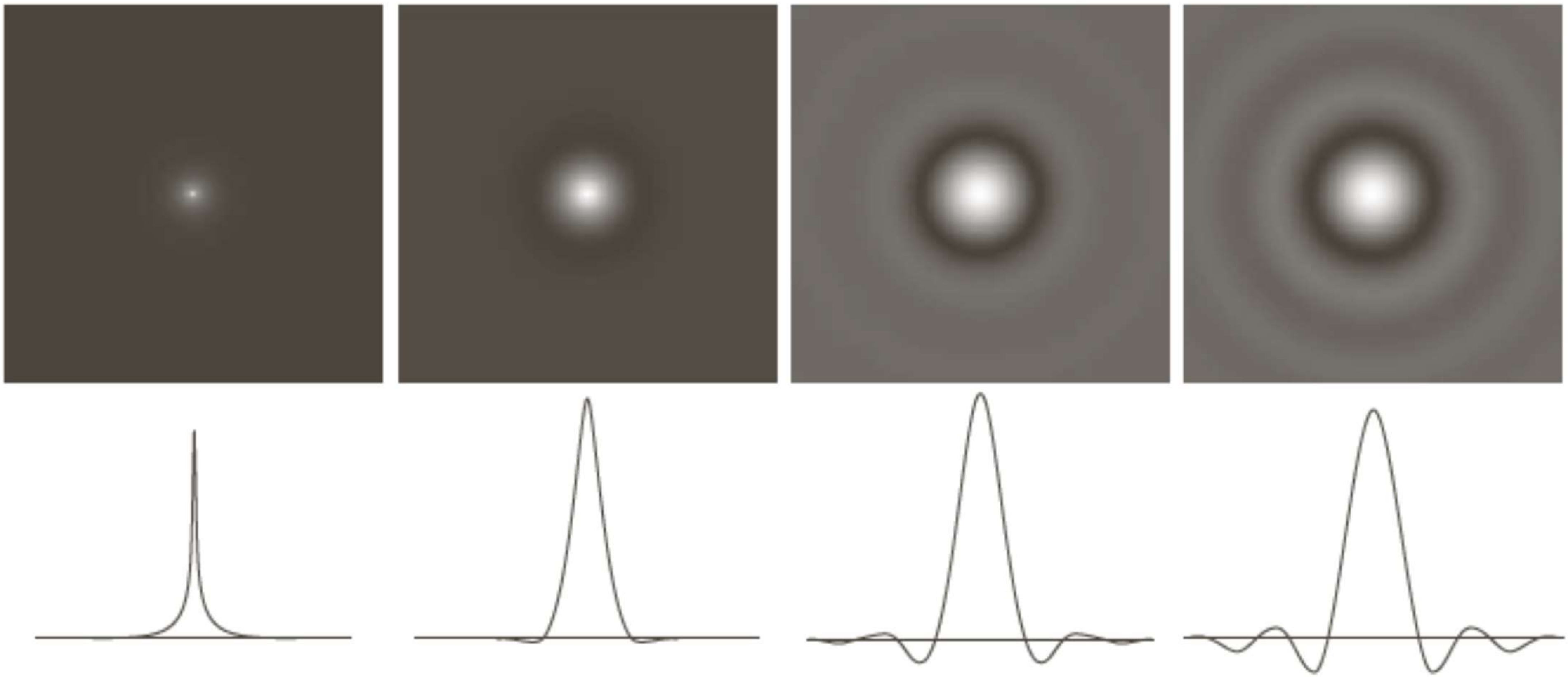
Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$



a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

BLPF的空间域特性

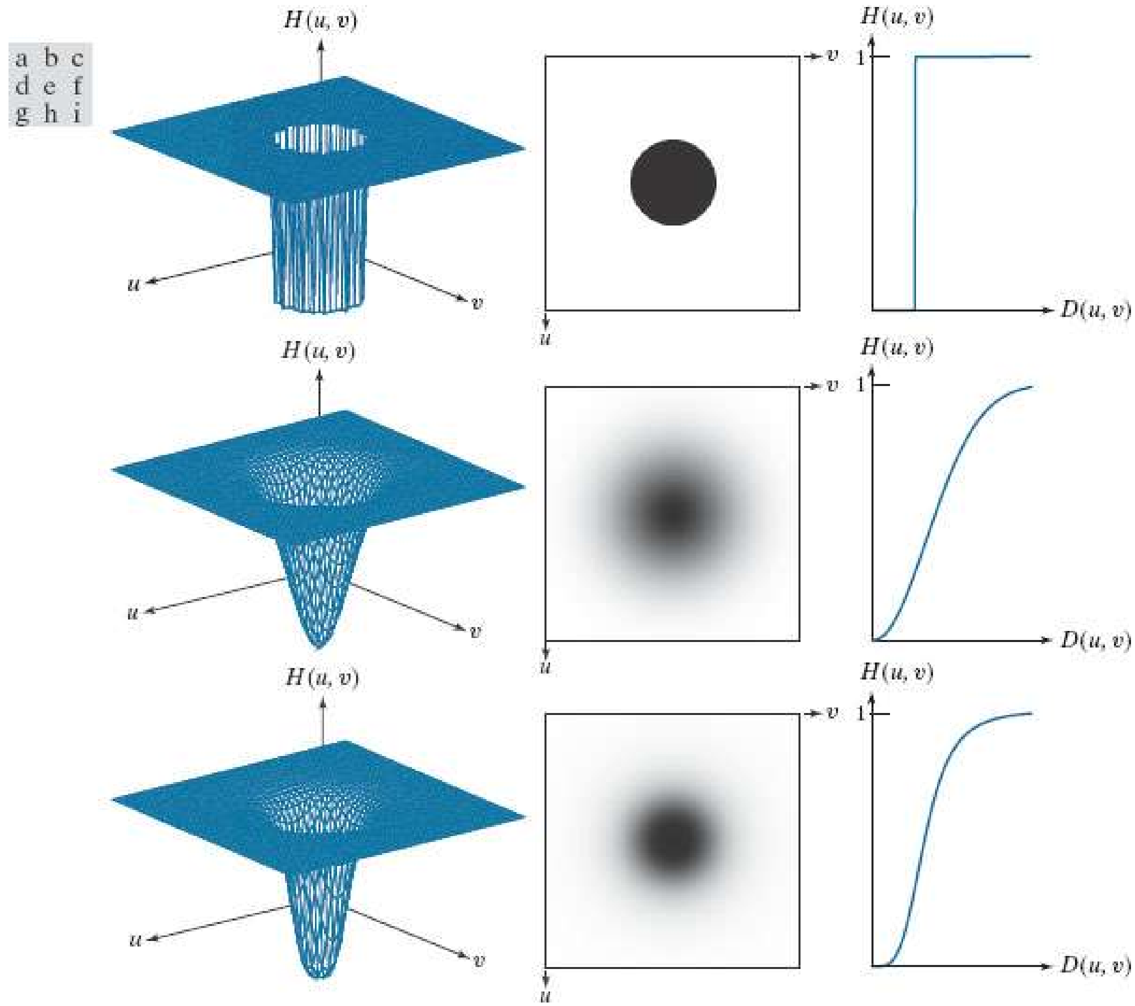


a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

IHPF, GHPF, BHPF的频率域特性

$$H_{HP}(u,v)=1-H_{LP}(u,v)$$

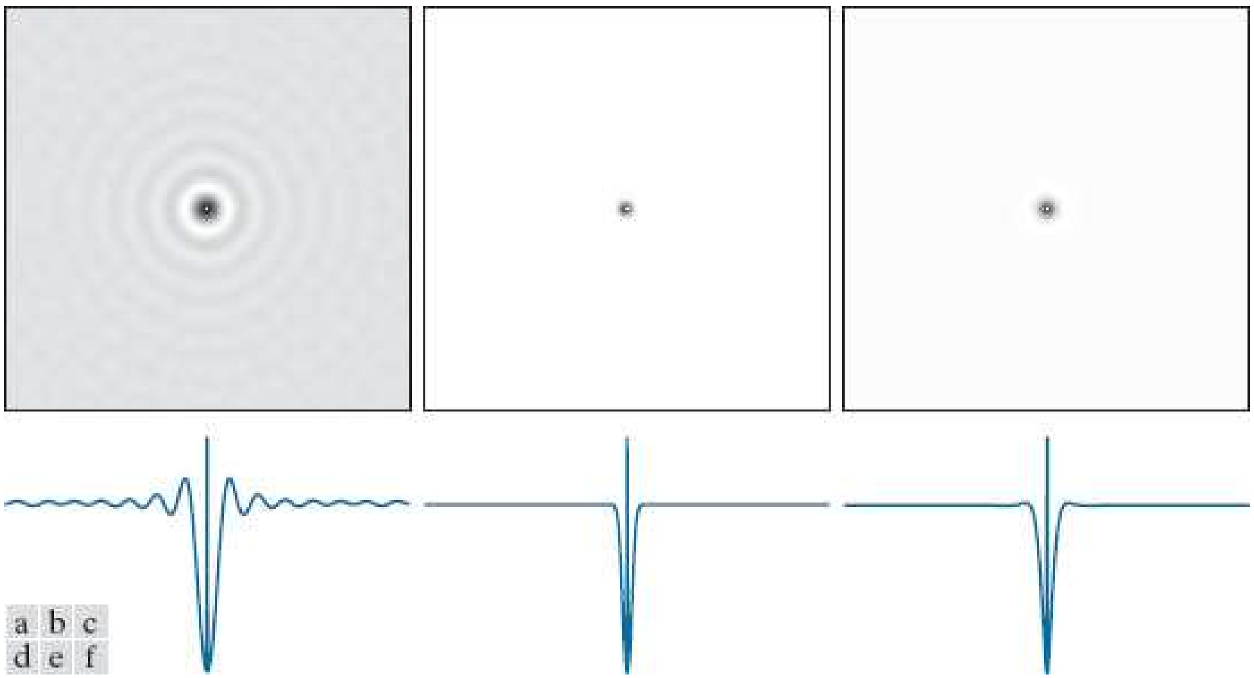


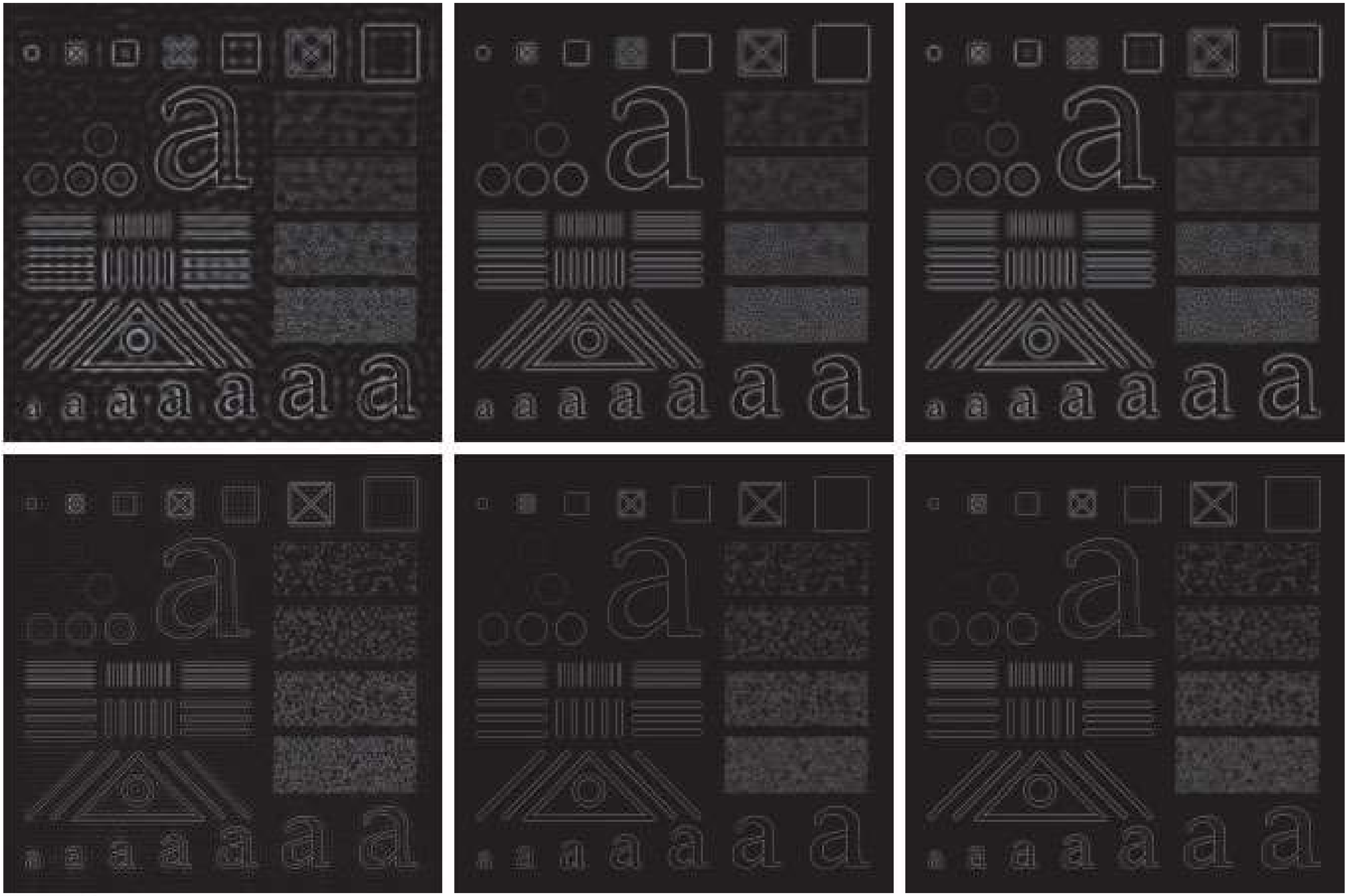
IHPF, GHPF, BHPF的频率域特性

Highpass filter transfer functions. D_0 is the cutoff frequency and n is the order of the Butterworth transfer function.

Ideal	Gaussian	Butterworth
$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$

IHPF, GHPF, BHPF的空间域特性





a b c
d e f

均值滤波器的频率域模型

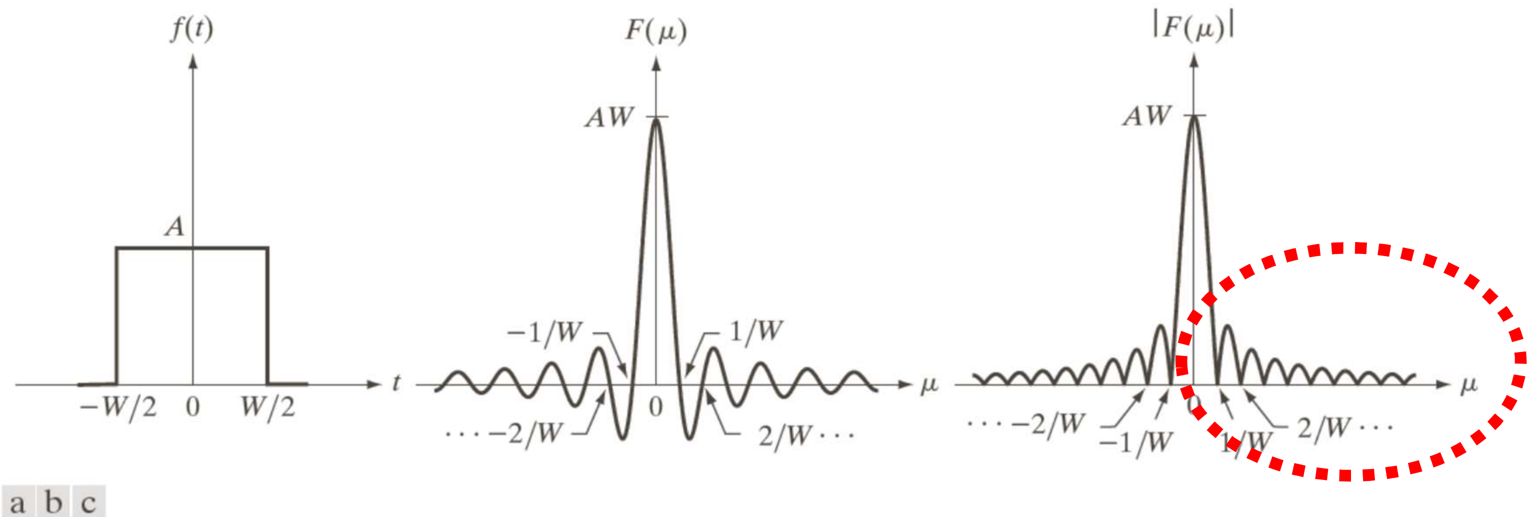
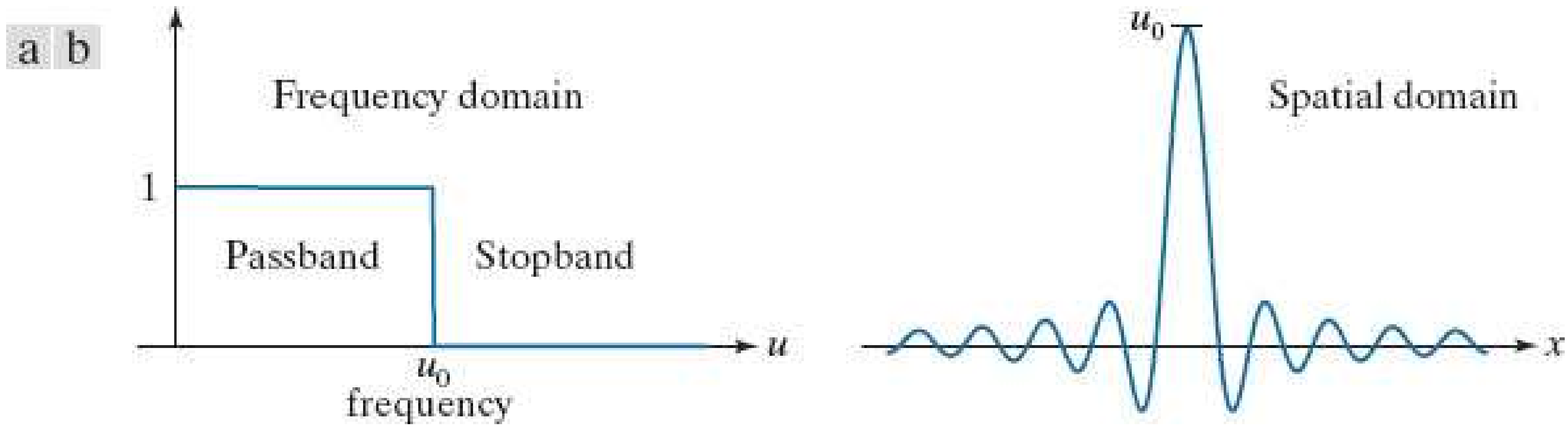


FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

理想低通滤波器的空间域模型



带阻滤波器设计

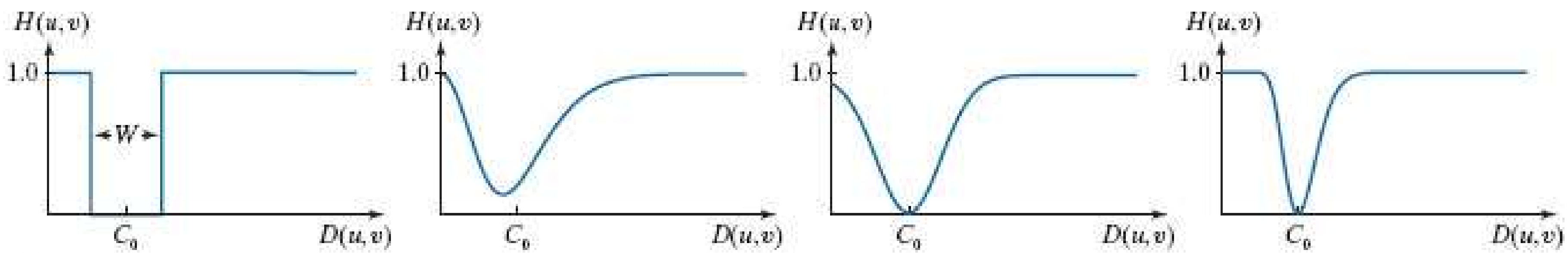
Bandreject filter transfer functions. C_0 is the center of the band, **W** is the width of the band, and **D(u,v)** is the distance from the center of the transfer function to a point **(u,v)** in the frequency rectangle.

Ideal (IBRF)	Gaussian (GBRF)	Butterworth (BBRF)
$H(u,v)=\begin{cases}0 & \text{if } C_0-\frac{W}{2}\le D(u,v)\le C_0+\frac{W}{2}\\ 1 & \text{otherwise}\end{cases}$	$H(u,v)=1-e^{-\left[\frac{D^2(u,v)-C_0^2}{D(u,v)W}\right]^2}$	$H(u,v)=\dfrac{1}{1+\left[\dfrac{D(u,v)W}{D^2(u,v)-C_0^2}\right]^{2n}}$

带阻滤波器设计

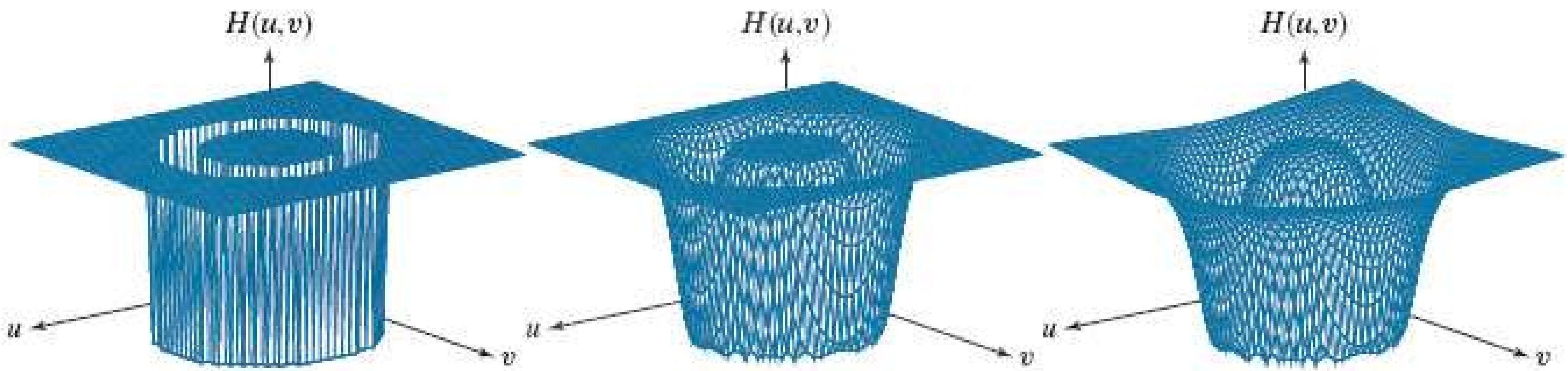
a b c d

FIGURE 4.61 Radial cross sections. (a) Ideal bandreject filter transfer function. (b) Bandreject transfer function formed by the sum of Gaussian lowpass and highpass filter functions. (The minimum is not 0 and does not align with C_0.) (c) Radial plot of Eq. (4-149). (The minimum is 0 and is properly aligned with C_0, but the value at the origin is not 1.) (d) Radial plot of Eq. (4-150); this Gaussian-shape plot meets all the requirements of a bandreject filter transfer function.



a b c d

带阻滤波器设计



a b c

内容提要

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脉冲序列信号的筛选特性

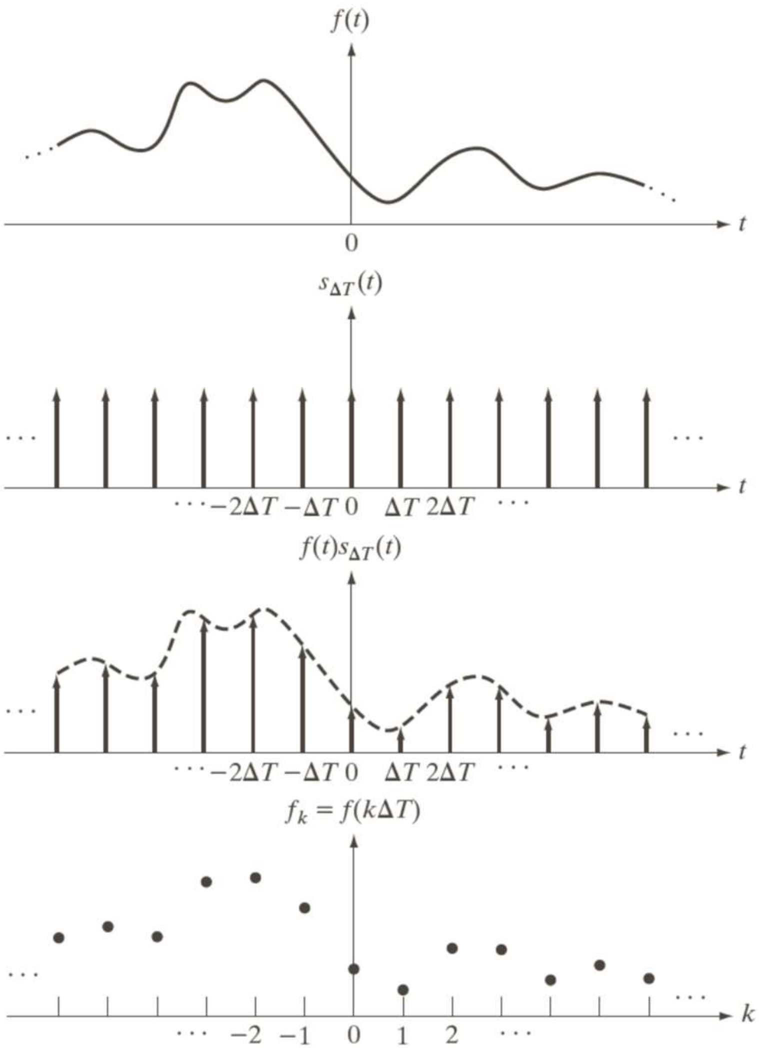
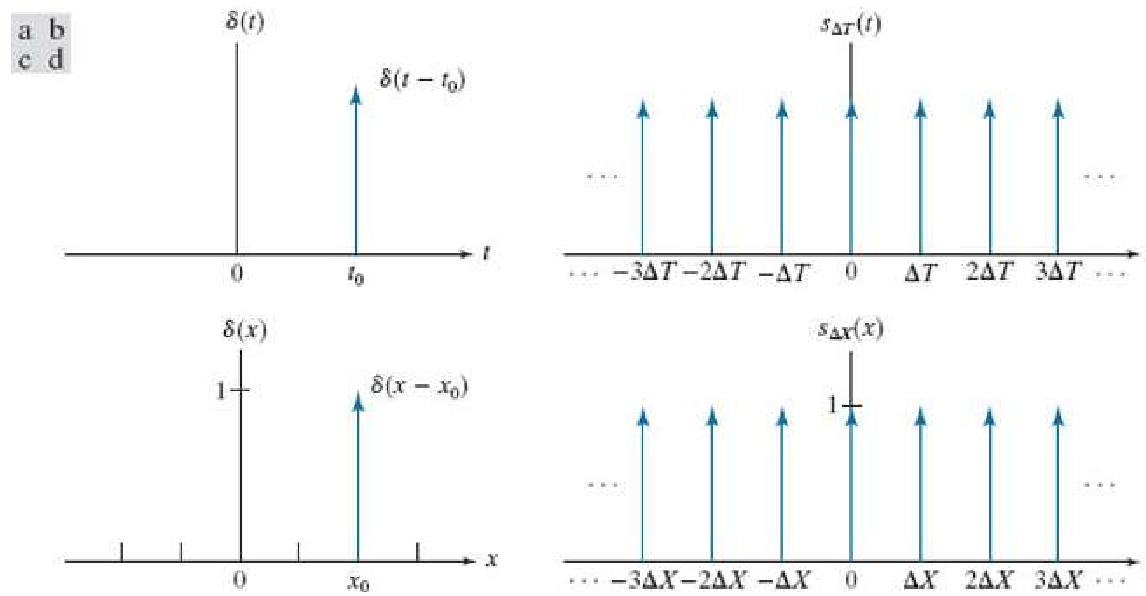


FIGURE 4.5
(a) A continuous function. (b) Train of impulses used to model the sampling process. (c) Sampled function formed as the product of (a) and (b). (d) Sample values obtained by integration and using the sifting property of the impulse. (The dashed line in (c) is shown for reference. It is not part of the data.)



$$\tilde{f}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)$$

一维离散傅里叶变换的推导

$$\tilde{F}(\mu) = \int_{-\infty}^{\infty} \tilde{f}(t)\, e^{-j2\pi\mu}\, dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)\, e^{-j2\pi\mu}\, dt$$

$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\delta(t - n\Delta T)\, e^{-j2}\quad dt$$

$$= \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi\mu n\Delta T}$$

$$\mu = \frac{m}{M\Delta T} \quad m = 0,1,2,\ldots,M-1$$

$$F_m = \sum_{n=0}^{M-1} f_n e^{-j2\pi m\ \ /M} \quad m = 0,1,2,\ldots,M-1$$

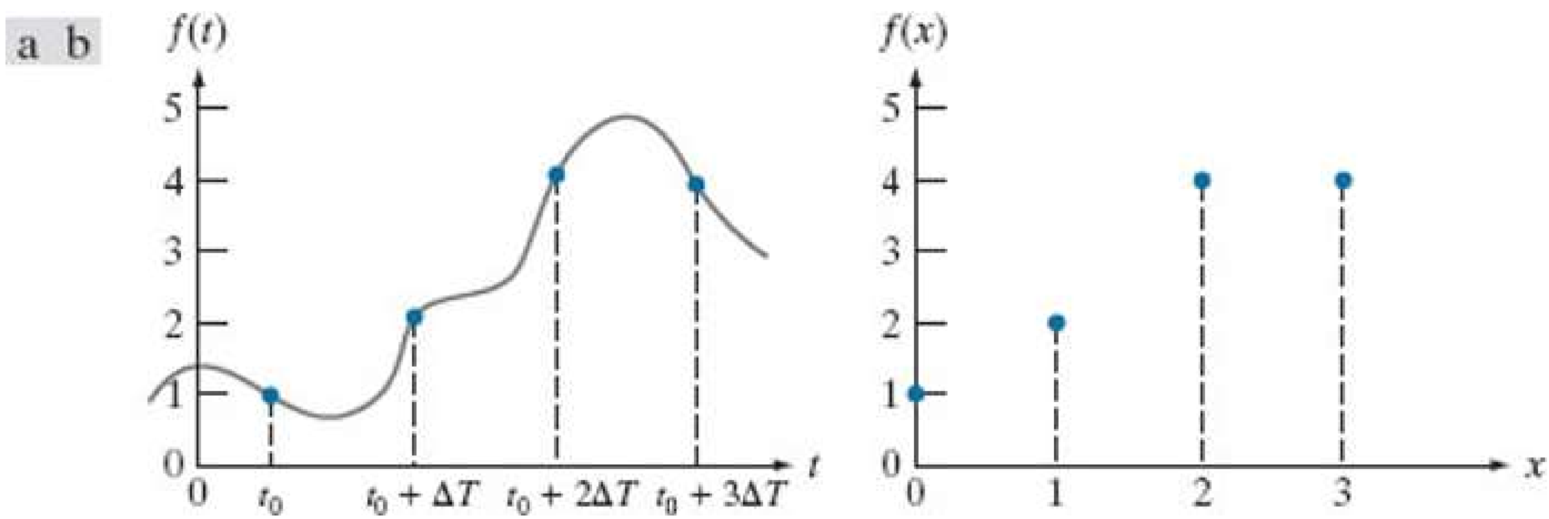
$$f_n = \frac{1}{M}\sum_{m=0}^{M-1} F_m e^{j2\pi mn/M} \quad n = 0,1,2,\ldots,M$$

$$\boxed{\begin{aligned} F(u) &= \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad u = 0,1,2,\mathrm{K}\ ,M-1 \\ f(x) &= \frac{1}{M}\sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad x = 0,1,2,\mathrm{K}\ ,M-1 \end{aligned}}$$

一维离散傅里叶变换计算示例

$$F(u)=\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \qquad u=0,1,2,\ldots,M-1$$

$$f(x)=\frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \qquad x=0,1,2,\ldots,M-1$$



$$F(0)=\sum_{x=0}^{3} f(x)=\left[1\cdot f(0)+1\cdot f(1)+1\cdot f(2)+1\cdot f(3)\right]$$

一维离散傅里叶变换计算示例

$$F(u)=\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \qquad u=0,1,2,\mathrm{K}\ ,M-1$$

$$f(x)=\frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \qquad x=0,1,2,\mathrm{K}\ ,M-1$$

$$F(0)=\sum_{x=0}^{3} f(x)=\left[1\cdot f(0)+1\cdot f(1)+1\cdot f(2)+1\cdot f(3)\right]=11$$

$$F(1)=\sum_{x=0}^{3} e^{-j2\pi 1x/4} f(x)=\left[e^{0}\cdot f(0)+e^{-j2\pi/4}1\cdot f(1)+e^{-j2\pi 2/4}\cdot f(2)+e^{-j2\pi 3/4}\cdot f(3)\right]=-3+2j$$

$$F(2)=\sum_{x=0}^{3} e^{-j2\pi 1x/4} f(x)=\left[e^{0}\cdot f(0)+e^{-j2\pi 2/4}1\cdot f(1)+e^{-j2\pi 4/4}\cdot f(2)+e^{-j2\pi 6/4}\cdot f(3)\right]=-1-0j$$

$$F(3)=\sum_{x=0}^{3} e^{-j2\pi 1x/4} f(x)=\left[e^{0}\times f(0)+e^{-j2\pi 3/4}1\times f(1)+e^{-j2\pi 6/4}\times f(2)+e^{-j2\pi 9/4}\times f(3)\right]=-3-2j$$

一维离散傅里叶变换计算示例

$$F(u)=\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \qquad u=0,1,2,\mathrm{K}\ ,M-1$$

$$f(x)=\frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \qquad x=0,1,2,\mathrm{K}\ ,M-1$$

$$F(0)=\sum_{x=0}^{7} f(x)=\left[e^{-j2\pi 0\cdot 0}\cdot f(0)+e^{-j2\pi 0\cdot 1}\cdot f(1)+\mathrm{K}\ +e^{-j2\pi 0\cdot 6}1\cdot f(6)+e^{-j2\pi 0\cdot 6}\cdot f(7)\right]$$

$$F(0)=\sum_{x=0}^{M-1} f(x)=\left[e^{-j2\pi 0\cdot 0}\cdot f(0)+e^{-j2\pi 0\cdot 1}\cdot f(1)+\mathrm{K}\ +e^{-j2\pi 0\cdot (M-2)}\cdot f(M-2)+e^{-j2\pi 0\cdot (M-1)}\cdot f(M-1)\right]$$

一维离散傅里叶变换计算示例

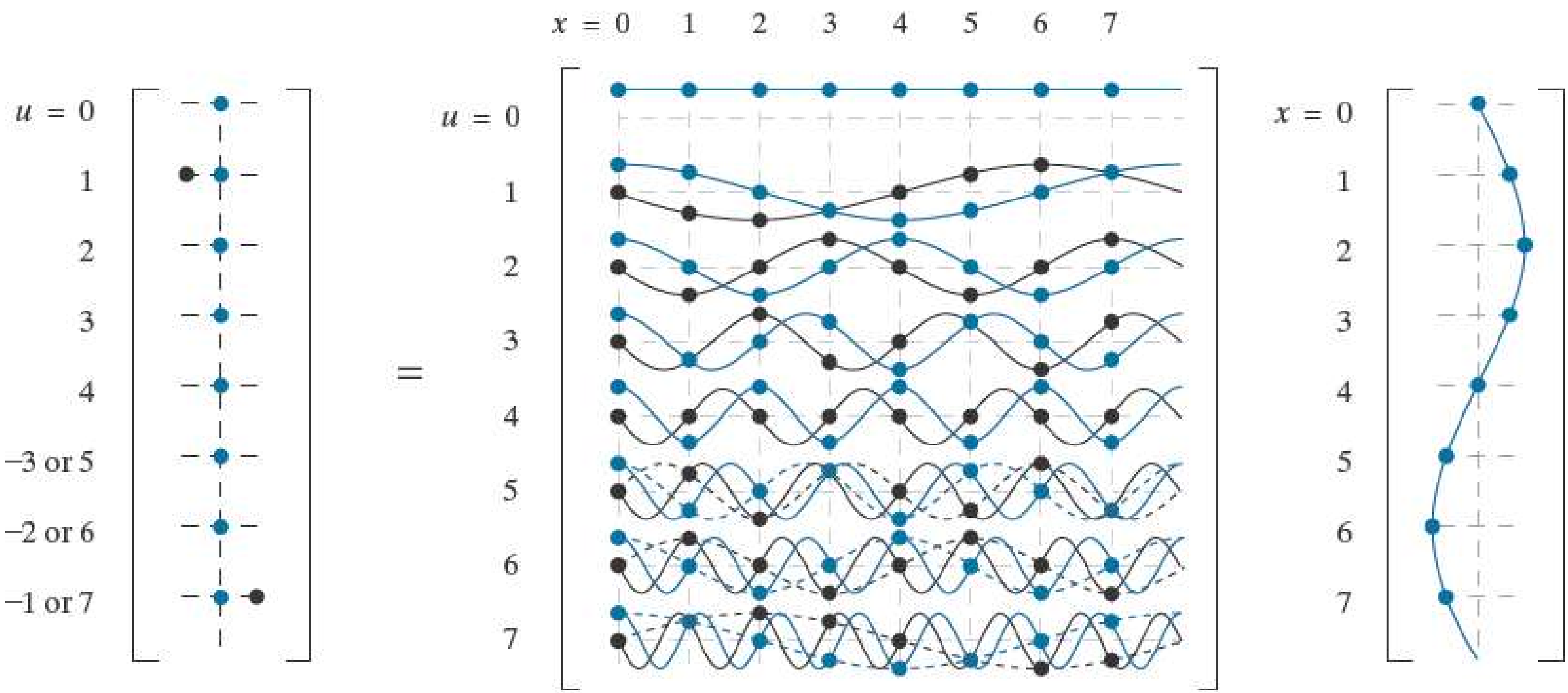
$$F(0)=\sum_{x=0}^{M-1} f(x)=\left[e^{-j2\pi 0\cdot 0}\cdot f(0)+e^{-j2\pi 0\cdot 1}\cdot f(1)+\text{K}+e^{-j2\pi 0\cdot(M-2)}\cdot f(M-2)+e^{-j2\pi 0\cdot(M-1)}\cdot f(M-1)\right]$$

$$\begin{bmatrix} F(0) \\ F(1) \\ \vdots \\ F(M-1) \end{bmatrix} = \begin{bmatrix} e^{-j2\pi(0\cdot 0)} & e^{-j2\pi(0\cdot 1)} & \cdots & e^{-j2\pi(0\cdot(M-1))} \\ e^{-j2\pi(1\cdot 0)} & e^{-j2\pi(1\cdot 1)} & \cdots & e^{-j2\pi(1\cdot(M-1))} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j2\pi((M-1)\cdot 0)} & e^{-j2\pi((M-1)\cdot 1)} & \cdots & e^{-j2\pi((M-1)\cdot(M-1))} \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(M-1) \end{bmatrix}$$

$$F = Af = [a_1 \quad a_2 \quad \cdots \quad a_{M-1}] \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(M-1) \end{bmatrix}$$

$$= a_1 f(0) + a_2 f(1) + \cdots + a_{M-1} f(M-1)$$

一维傅里叶变换的内积与矩阵表示



$\alpha_u \Leftrightarrow T(u)$ or $\mathbf{t}$ $s_u(x) \Leftrightarrow s(x,u)$ or $\mathbf{A} = [s_0\ s_1\ \ldots\ s_7]^T$ $f(x) \Leftrightarrow f(x)$ or $\mathbf{f}$

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$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)\, e^{-j2\pi\mu} \ dt$$

$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\delta(t - n\Delta T)\, e^{-j2\pi\mu t} dt$$

$$= \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi\mu n\Delta}$$

$$\mu = \frac{m}{M\Delta T} \quad m = 0,1,2, \dots, M-1$$

$$F_m = \sum_{n=0}^{M-1} f_n e^{-j2\pi mn/M} \quad m = 0,1,2, \dots, M-1$$

$$f_n = \frac{1}{M} \sum_{m=0}^{M-1} f_m e^{j2\pi mn/M} \quad n = 0,1,2, \dots, M$$

图像二维快速傅里叶变换(DFT)的计算 (I)

定义

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

图像二维快速傅里叶变换(DFT)的计算 (II)

计算正向变换

$$F(u,v)=\sum_{x=0}^{M-1} e^{-j2\pi\frac{ux}{M}} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\frac{vy}{N}}$$

$$=\sum_{x=0}^{M-1} F(x,v) e^{-j2\pi\frac{ux}{M}}$$

$$F(x,v)=\sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\frac{vy}{N}}$$

- ***F*(*x*,*v*)** 是***f*(*x*,*y*)**中一行的一维**DFT**

- ***F*(*u*,*v*)** 是***F*(*x*,*v*)**中一列的一维**DFT**

关于复数的简要复习 (II)

- 共轭复数的指数形式

$$C^* = |C|(\cos\theta - j\sin\theta) = |C|e^{-j\theta}$$

- 共轭运算的乘法和加法分配律

$$(CD)^* = C^* D^*$$

$$(C+D)^* = C^* + D^*$$

图像二维快速傅里叶变换(DFT)的计算 (III)

计算逆向变换

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$\Downarrow$$

$$MNf^{*}(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F^{*}(u,v)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

图像二维快速傅里叶变换(DFT)的计算 (IV)

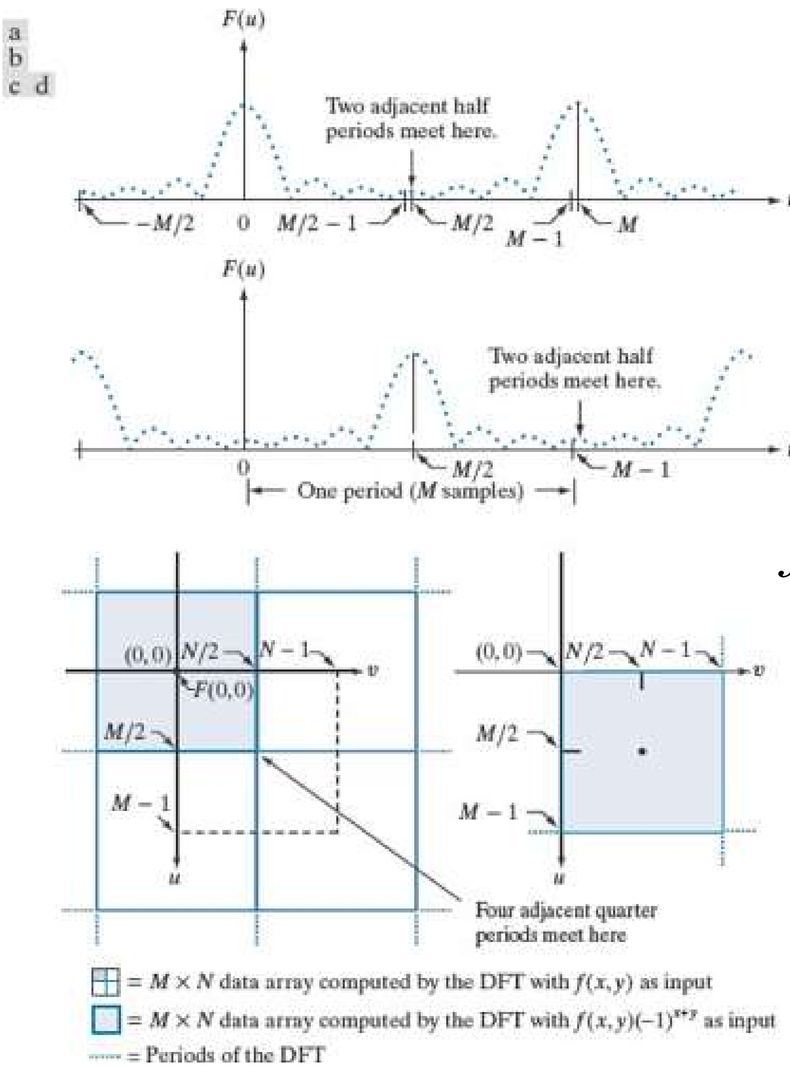
计算逆向变换步骤总结

$$F(u,v) \xrightarrow{\text{take complex conjugate}} F^*(u,v)$$

$$F^*(u,v) \xrightarrow{DFT} MNf^*(x,y)$$

$$MNf^*(x,y) \xrightarrow{\text{divide by } MN\text{, take complex conjugate}} f(x,y)$$

图像二维快速傅里叶变换(DFT)的计算 (V)

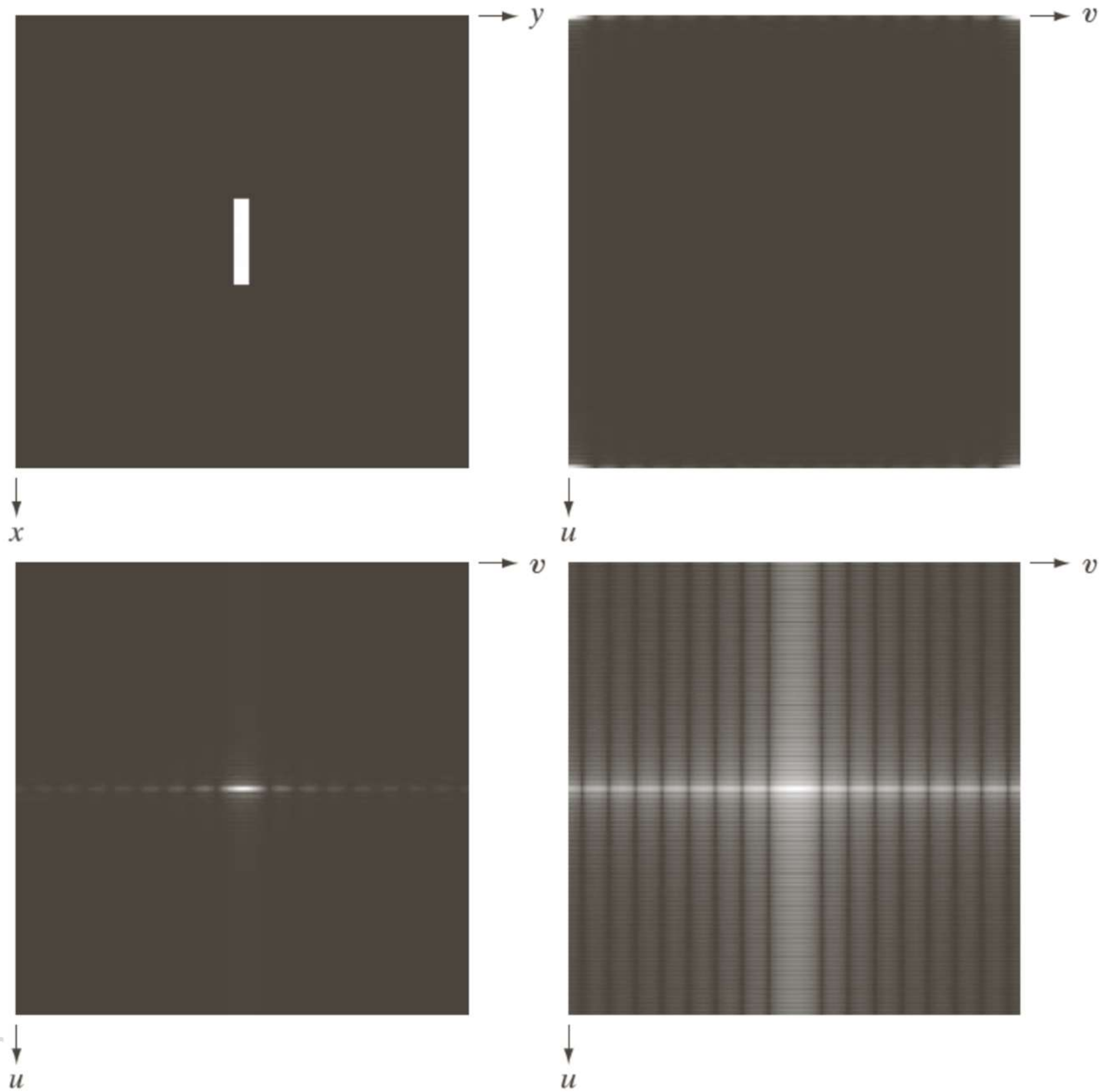


中心化傅里叶变换

$$f(x)(-1)^{x} \Leftrightarrow F(u - M/2)$$

$$f(x,\ y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$$

图像2D-DFT示例 (I)



a b
c d

FIGURE 4.24
(a) Image.
(b) Spectrum showing bright spots in the four corners.
(c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.

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图像二维快速傅里叶变换(DFT)的计算 (I)

定义

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

快速傅里叶变换的基本概念 (I)

- 直接计算

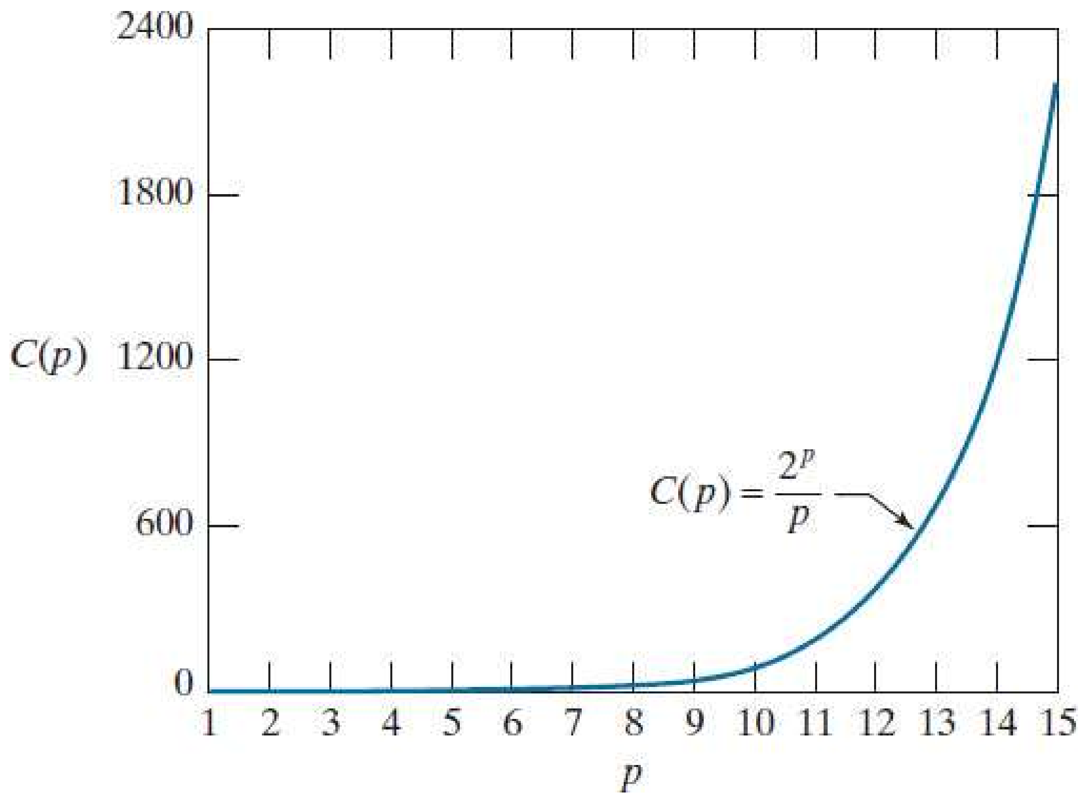
$\boxed{\text{计算成本} \sim (MN)^2}$

如果M=N=2048, $(MN)^2=1.76\times10^{13}$

- 采用快速傅里叶变换的计算成本(一维)

$$C(M)=\frac{M^2}{M\log_2 M}=\frac{M}{\log_2 M}=\frac{2^p}{p}$$

快速傅里叶变换的基本概念 (II)



Computational advantage of the FFT over a direct implementation of the 1-D DFT. The number of samples is $M = 2^p$. The computational advantage increases rapidly as a function of *p*.

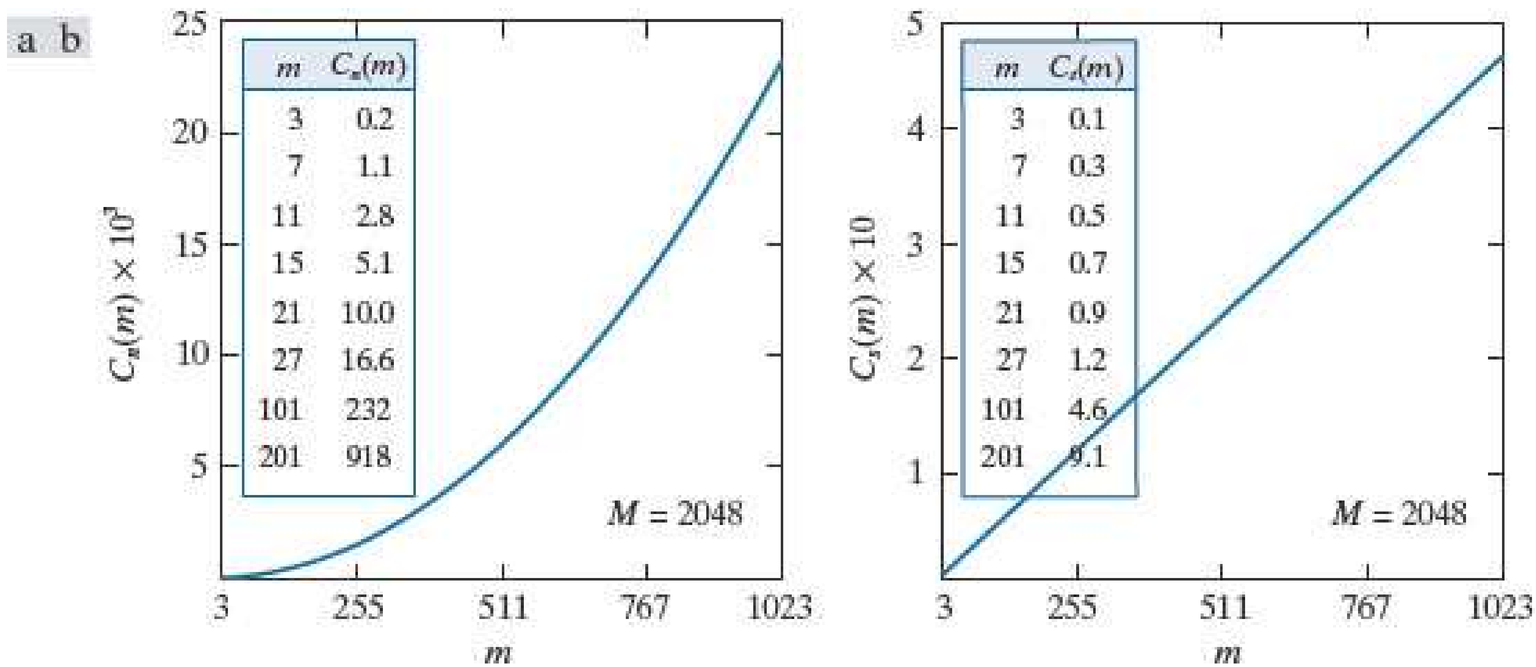
快速傅里叶变换的基本概念 (III)

- 卷积计算 图像大小 M×M, 卷积核大小 m×m

Nonseparable $$C(m) = \frac{M^2 m^2}{2M^2 \log_2 M^2} = \frac{m^2}{4 \log_2 M}$$

Separable $$C(m) = \frac{2M^2 m}{2M^2 \log_2 M^2} = \frac{m}{2 \log_2 M}$$

快速傅里叶变换的基本概念 (IV)



(a) Computational advantage of the FFT over nonseparable spatial kernels. (b) Advantage over separable kernels. The numbers for C(m) in the inset tables are not to be multiplied by the factors of 10 shown for the curves.

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一维离散傅里叶变换(1D-DFT)的定义

$$F(u)=\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \qquad u=0,1,2,\ldots,M-1$$

$$f(x)=\frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \qquad x=0,1,2,\ldots,M-1$$

二维离散傅里叶变换(2D-DFT)的定义

$$F(u)=\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \qquad u=0,1,2,\ldots,M-1$$

$$f(x)=\frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \qquad x=0,1,2,\ldots,M-1$$

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

一维和二维傅里叶变换是正交变换的一个特例，可以直接在时间/空间域获得直观的解释。这些解释具有重要意义。

矢量空间的基本概念 (I)

A ***vector space*** is defined as a nonempty set V of entities called *vectors* and associated scalars that satisfy the conditions outlined in A through C below. A vector space is *real* if the scalars are real numbers; it is *complex* if the scalars are complex numbers.

- Condition A: There is in V an operation called *vector addition*, denoted $\mathbf{x} + \mathbf{y}$, that satisfies:

  1. $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$ for all vectors $\mathbf{x}$ and $\mathbf{y}$ in the space.
  2. $\mathbf{x} + (\mathbf{y} + \mathbf{z}) = (\mathbf{x} + \mathbf{y}) + \mathbf{z}$ for all $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$.
  3. There exists in V a unique vector, called the *zero vector*, and denoted $\mathbf{0}$, such that $\mathbf{x} + \mathbf{0} = \mathbf{x}$ and $\mathbf{0} + \mathbf{x} = \mathbf{x}$ for all vectors $\mathbf{x}$.
  4. For each vector $\mathbf{x}$ in V, there is a unique vector in V, called the *negation* of $\mathbf{x}$, and denoted $-\mathbf{x}$, such that $\mathbf{x} + (-\mathbf{x}) = \mathbf{0}$ and $(-\mathbf{x}) + \mathbf{x} = \mathbf{0}$.

矢量空间的基本概念 (II)

- Condition B: There is in V an operation called *multiplication by a scalar* that associates with each scalar c and each vector $\mathbf{x}$ in V a unique vector called the *product* of c and $\mathbf{x}$, denoted by $c\mathbf{x}$ and $\mathbf{x}c$, and which satisfies:
  1. $c(d\mathbf{x}) = (cd)\mathbf{x}$ for all scalars c and d, and all vectors $\mathbf{x}$.
  2. $(c + d)\mathbf{x} = c\mathbf{x} + d\mathbf{x}$ for all scalars c and d, and all vectors $\mathbf{x}$.
  3. $c(\mathbf{x} + \mathbf{y}) = c\mathbf{x} + c\mathbf{y}$ for all scalars c and all vectors $\mathbf{x}$ and $\mathbf{y}$.

- Condition C: $1\mathbf{x} = \mathbf{x}$ for all vectors $\mathbf{x}$.

内积空间的基本概念

- 如果矢量空间V满足如下条件则是一个内积空间

$$x, y \in V$$

$$\langle x, y \rangle = \langle y, x \rangle^*$$

$$\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$$

$$\langle rx, y \rangle = r \langle x, y \rangle$$

$$\langle x, x \rangle \geq 0 \text{ and } \langle x, x \rangle = 0 \quad iff \ \ x = 0$$

内积空间的示例 I：欧几里得空间

$$\langle x, y\rangle = x^T y = x_0 y_0 + x_1 y_1 + \ldots + x_{N-1} y_{N-1} = \sum_{i=0}^{N-1} x_i y_i$$

$$x_i \in R, y_i \in R, x \in R^{N\times 1}, y \in R^{N\times 1}$$

$$\|x\| = \sqrt{\langle x, x\rangle} = \sqrt{\sum_{i=0}^{N-1} x_i^2}$$

$$\theta = \cos^{-1} \frac{\langle x, y\rangle}{\|x\|\|y\|}$$

内积空间示例II：积分内积空间

- 矢量空间 $C([a,b])$ 其中矢量是定义在 $[a,b]$ 上的连续函数。

$$\langle f(x), g(x) \rangle = \int_a^b f^*(x) g(x) dx$$

内积空间的基和单位正交基

$$W = \{w_0, w_1, w_2, \ldots w_N\}$$

$$z = \alpha_0 w_0 + \alpha_1 w_1 + \ldots \alpha_{N-1} w_{N-1}$$

$$\langle w_i, z \rangle = \alpha_0 \langle w_i, w_0 \rangle + \alpha_1 \langle w_i, w_1 \rangle + \ldots \alpha_{N-1} \langle w_i, w_{N-1} \rangle$$

$$\alpha_i = \frac{\langle w_i, z \rangle}{\langle w_i, w_i \rangle}$$

双正交基和单位双正交基

- 一组矢量 $w_0, w_1, w_2, \mathrm{K}$ 和一种对偶矢量 $\widetilde{w}_0, \widetilde{w}_1, \widetilde{w}_2, \ldots$ 称为双正交基/单位双正交基如果它们满足如下条件

$$\langle \widetilde{w}_k, w_l \rangle = 0 \quad \text{for } k \neq l$$

$$\langle \widetilde{w}_k, w_l \rangle = \delta_{kl} = \begin{cases} 0 & \text{for } k \neq l \\ 1 & \text{for } k = l \end{cases}$$

一维傅里叶变换的内积与矩阵表示 (I)

$$T(u)=\sum_{x=0}^{N-1} f(x) e^{-j2\pi ux/N} \qquad u=0,1,2,\ldots,N-1$$

$$f(x)=\frac{1}{N}\sum_{u=0}^{N-1} T(u) e^{j2\pi ux/N} \qquad x=0,1,2,\ldots,N-1$$

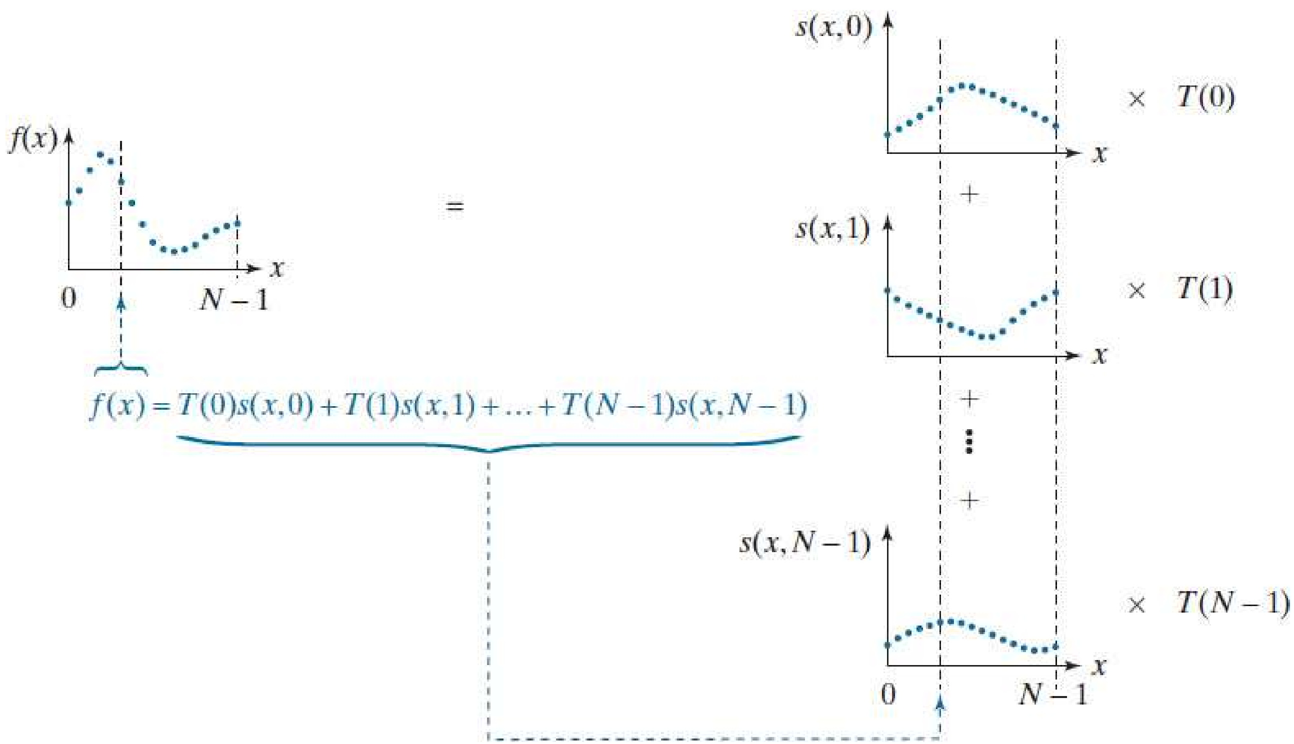
$$T(u)=\sum_{x=0}^{N-1} f(x)\cdot \boxed{r(x,u)}$$ **正向变换核**

$$f(x)=\sum_{u=0}^{N-1} T(u)\cdot \boxed{s(x,u)}$$ **逆向变换核**

一维傅里叶变换的内积与矩阵表示 (II)

$$f(x)=\sum_{u=0}^{N-1}T(u)\cdot s(x,u)$$

$$=T(0)s(x,0)+T(1)s(x,1)+\ldots+T(N-1)s(x,N-1)$$



一维傅里叶变换的内积与矩阵表示 (III)

$$F = \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-1) \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{N-1} \end{bmatrix}$$

$$T = \begin{bmatrix} T(0) \\ T(1) \\ \vdots \\ T(N-1) \end{bmatrix} = \begin{bmatrix} t_0 \\ t_1 \\ \vdots \\ t_{N-1} \end{bmatrix}$$

$$s_u = \begin{bmatrix} s(0,u) \\ s(1,u) \\ \vdots \\ s(N-1,u) \end{bmatrix} = \begin{bmatrix} s_{u,0} \\ s_{u,1} \\ \vdots \\ s_{u,N-1} \end{bmatrix} \text{ for } u = 0,1,\dots,N-1$$

一维傅里叶变换的内积与矩阵表示 (IV)

$$T = \begin{bmatrix} \langle s_0, f \rangle \\ \langle s_1, f \rangle \\ \vdots \\ \langle s_{N-1}, f \rangle \end{bmatrix} = \begin{bmatrix} s_0^T \\ s_1^T \\ \vdots \\ s_{N-1}^T \end{bmatrix} F = AF$$

$$A \triangleq \begin{bmatrix} s_0^T \\ s_1^T \\ \vdots \\ s_{N-1}^T \end{bmatrix} = \begin{bmatrix} s_0 & s_1 & \cdots & s_{N-1} \end{bmatrix}$$

一维傅里叶变换的内积与矩阵表示 (V)

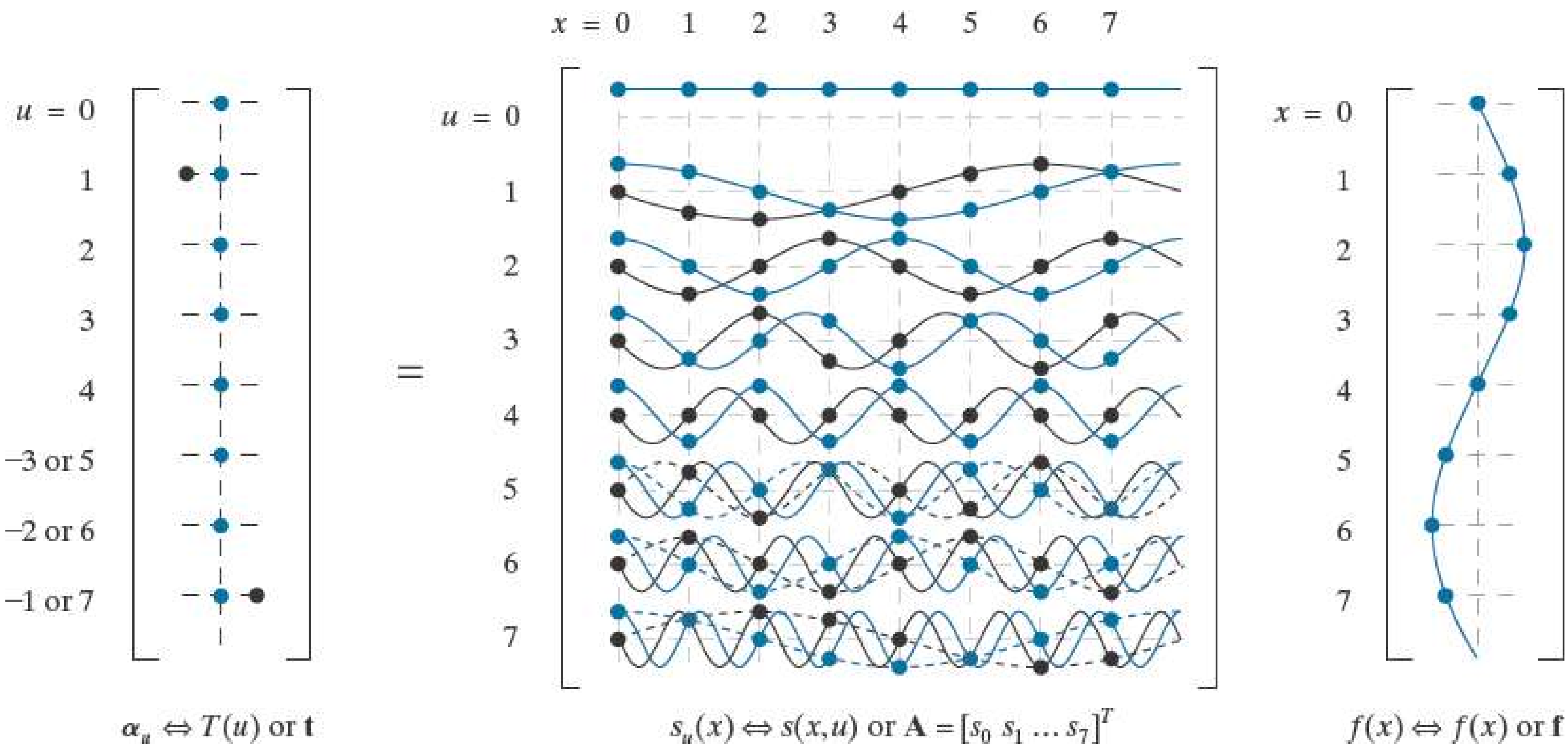
$$T = AF$$

$$F = A^{*T}T$$

$$T(u)=\sum_{x=0}^{N-1} f(x)\cdot r(x,u)$$

$$f(x)=\sum_{u=0}^{N-1} T(u)\cdot s(x,u)$$

一维傅里叶变换的内积与矩阵表示 (VI)



Depicting the continuous Fourier series and 8-point DFT of f(x) = sin(2πx) as “matrix multiplications.” The real and imaginary parts of all complex quantities are shown in blue and black, respectively. Continuous and discrete functions are represented using lines and dots, respectively. Dashed lines are included to show that $s_5 = s_3^*, s_6 = s_2^*, s_7 = s_1^*$ effectively cutting the maximum frequency of the DFT in half. The negative indices to the left of t are for the Fourier series computation alone.

变换的直观解释

连续函数的相关函数

$$f \circ g(\Delta x) = \int_{-\infty}^{\infty} f^*(x)\, g(x) dx$$
$$= \langle f(x), g(x + \Delta x) \rangle$$

$$f \circ g(m) = \sum_{x=-\infty}^{\infty} f_n^* g_{n+m}$$

$$f \circ g(0) = \int_{-\infty}^{\infty} f^*(x)\, g(x) dx$$
$$= \langle f(x), g(x) \rangle$$

$$f \circ g(0) = \langle f, g \rangle$$

$$a_u = \langle f, s_u \rangle = f \circ s_u(0)$$

$$T(u) = \langle s_u, f \rangle = s_u \circ f(0)$$

二维傅里叶变换的内积与矩阵表示 (I)

$$T(u,v)=\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}f(x,y)r(x,y,u,v)$$

$$f(x,y)=\sum_{u=0}^{N-1}\sum_{v=0}^{N-1}T(u,v)s(x,y,u,v)$$

$$F=\sum_{u=0}^{N-1}\sum_{v=0}^{N-1}T(u,v)S_{u,v}$$

$$F=\begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & \cdots & \vdots \\ f(N-1,0) & f(N-1,1) & \cdots & f(N-1,N-1) \end{bmatrix}$$

二维傅里叶变换的内积与矩阵表示 (II)

$$S_{u,v} = \begin{bmatrix} s(0,0,u,v) & s(0,1,u,v) & \cdots & s(0,N-1,u,v) \\ s(1,0,u,v) & s(1,1,u,v) & \cdots & s(1,N-1,u,v) \\ \vdots & \vdots & \cdots & \vdots \\ s(N-1,0,u,v) & s(N-1,1,u,v) & \cdots & s(N-1,N-1,u,v) \end{bmatrix}$$

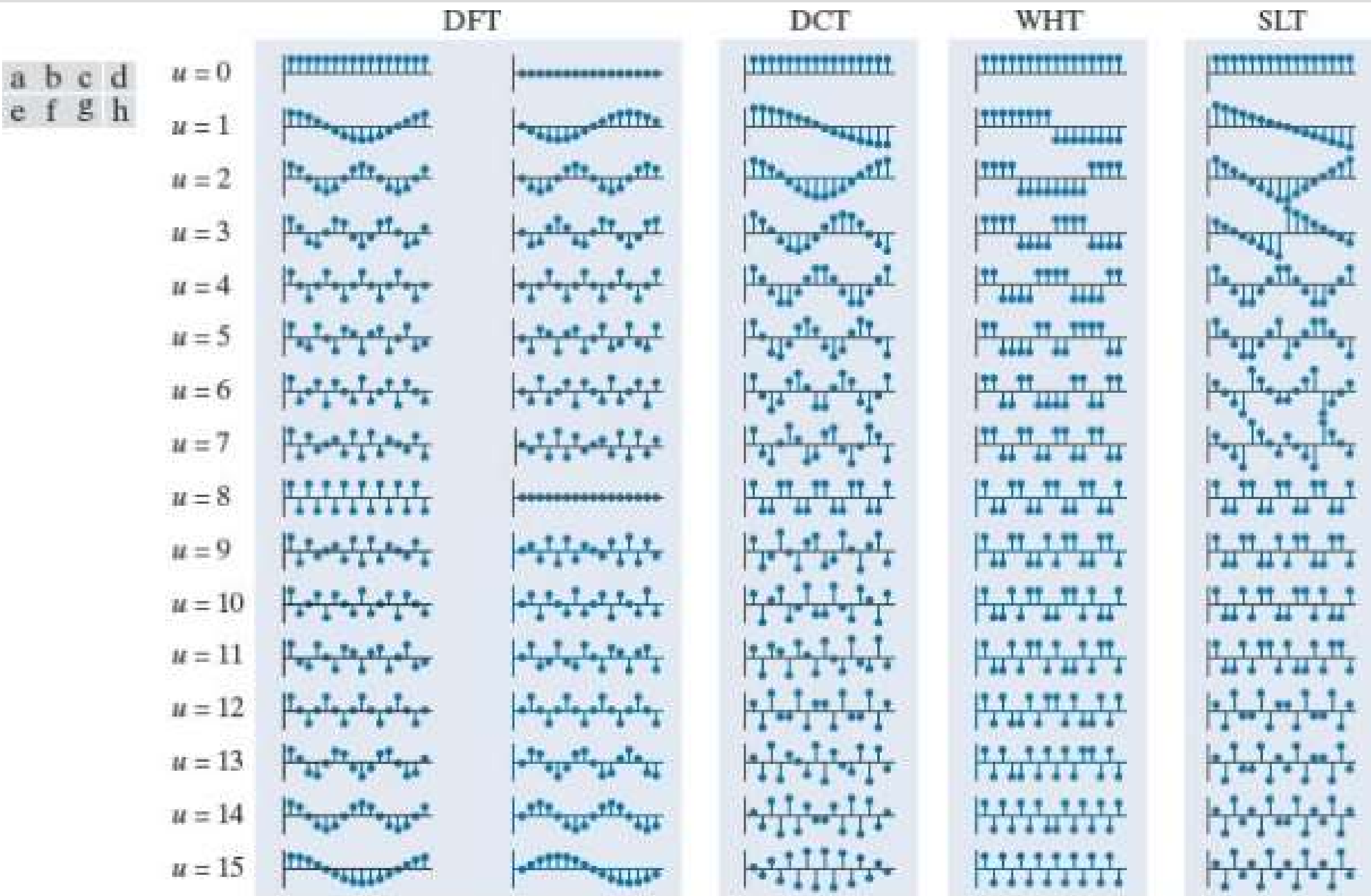
二维傅里叶变换的内积与矩阵表示 (III)

$$r(x,y,u,v) = r_1(x,u)r_2(y,v) \quad \leftarrow separable$$
$$r(x,y,u,v) = r_1(x,u)r_1(y,v) \quad \leftarrow symmetric$$

$$T = AFA^T$$
$$F = A^T TA$$

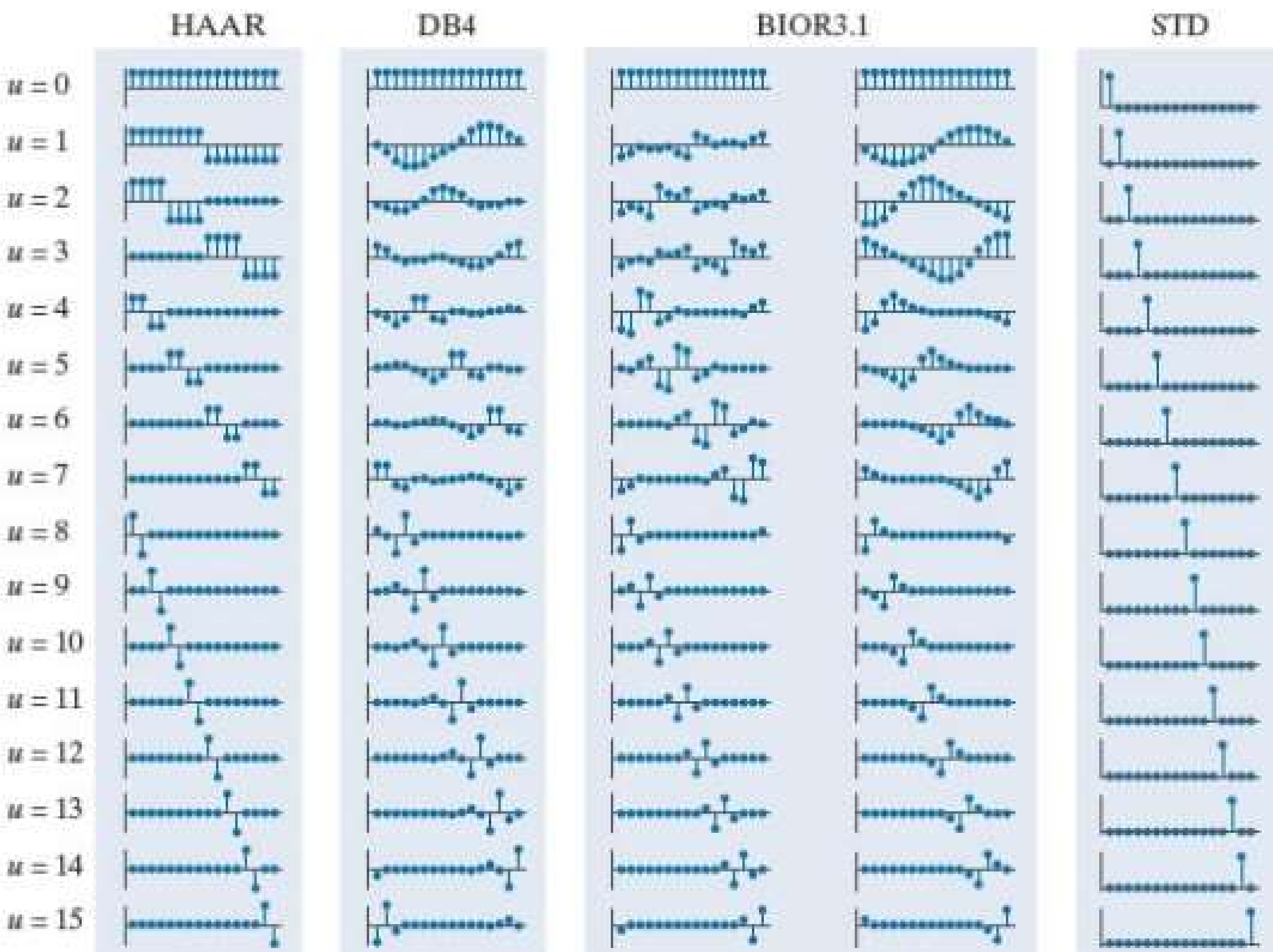
$$T = A_M F A_N^T$$
$$F = A_M^T T A_N$$

各种变换基总览



Basis vectors (for N = 16) of some commonly encountered transforms: (a) Fourier basis (real and imaginary parts), (b) discrete Cosine basis, (c) Walsh-Hadamard basis, (d) Slant basis, (e) Haar basis, (f) Daubechies basis, (g) Biorthogonal B-spline basis and its dual, and (h) the standard basis, which is included for reference only (i.e., not used as the basis of a transform).

各种变换基总览



HAAR
DB4
BIOR3.1
STD
u = 0
u = 1
u = 2
u = 3
u = 4
u = 5
u = 6
u = 7
u = 8
u = 9
u = 10
u = 11
u = 12
u = 13
u = 14
u = 15

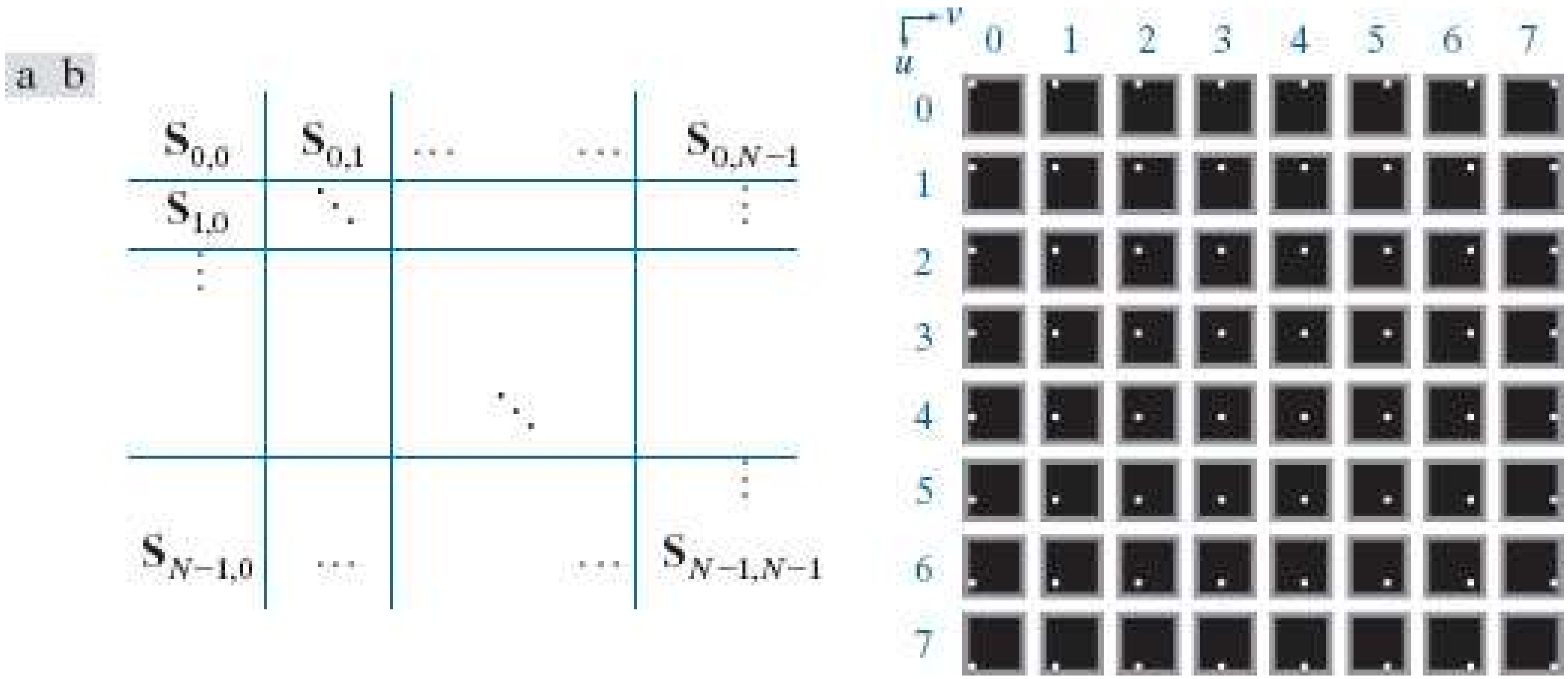
基图像的基本概念

$$F=\sum_{u=0}^{N-1}\sum_{v=0}^{N-1}T(u,v)S_{u,v}$$

$$F=\begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & \cdots & \vdots \\ f(N-1,0) & f(N-1,1) & \cdots & f(N-1,N-1) \end{bmatrix}$$

$$S_{u,v}=\begin{bmatrix} s(0,0,u,v) & s(0,1,u,v) & \cdots & s(0,N-1,u,v) \\ s(1,0,u,v) & s(1,1,u,v) & \cdots & s(1,N-1,u,v) \\ \vdots & \vdots & \cdots & \vdots \\ s(N-1,0,u,v) & s(N-1,1,u,v) & \cdots & s(N-1,N-1,u,v) \end{bmatrix}$$

标准基变换

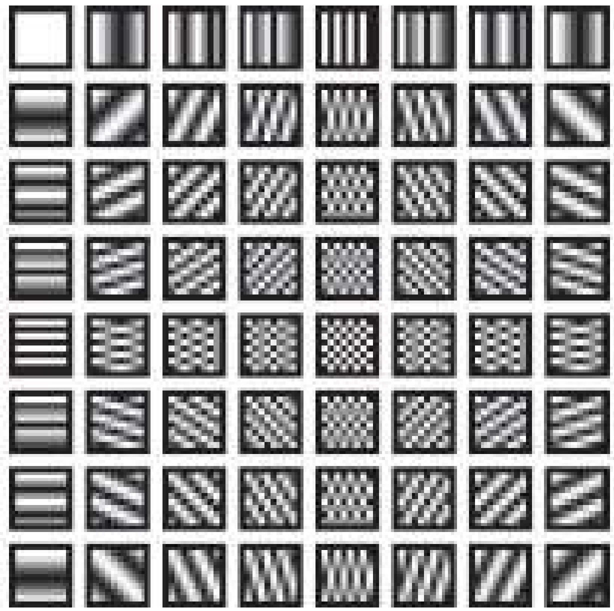


(a) Basis image organization and (b) a standard basis of size 8×8

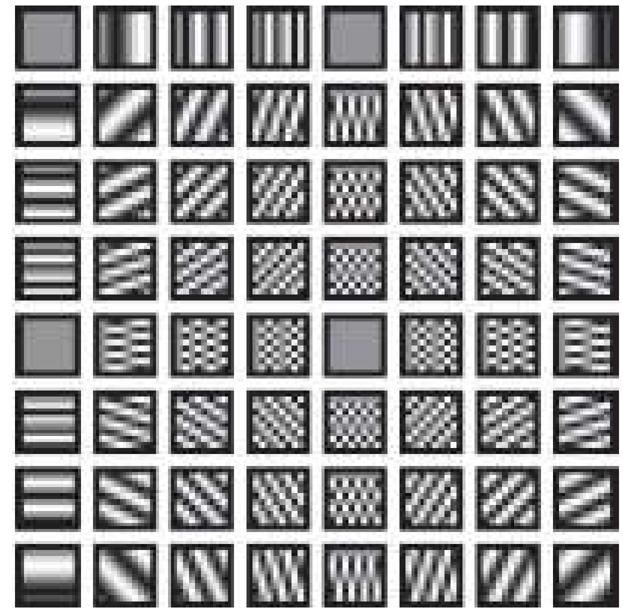
For clarity, a gray border has been added around each basis image. The origin of each basis image (i.e., x = y = 0) is at its top left.

二维傅里叶变换的基图像

$$\frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & -j & -j\omega & -1 & -\omega & j & j\omega \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -j\omega & j & \omega & -1 & j\omega & -j & -\omega \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\omega & -j & j\omega & -1 & \omega & j & -j\omega \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & j\omega & j & -\omega & -1 & -j\omega & -j & \omega \end{bmatrix}$$



实部基图像



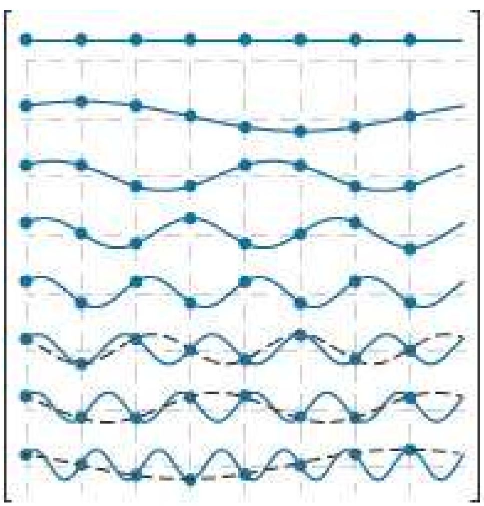
虚部基图像

a b c

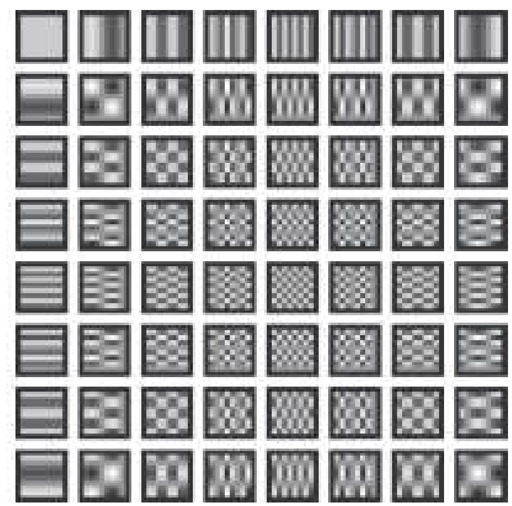
二维Hartley变换的基图像

$$s(x,u)=\frac{1}{\sqrt{N}}\left(\cos\left(\frac{2\pi ux}{N}\right)+\sin\left(\frac{2\pi ux}{N}\right)\right)$$

$$s(x,y,u,v)=\frac{1}{\sqrt{N}}\left(\cos\left(\frac{2\pi ux}{N}\right)+\sin\left(\frac{2\pi ux}{N}\right)\right)\frac{1}{\sqrt{N}}\left(\cos\left(\frac{2\pi vy}{N}\right)+\sin\left(\frac{2\pi vy}{N}\right)\right)$$

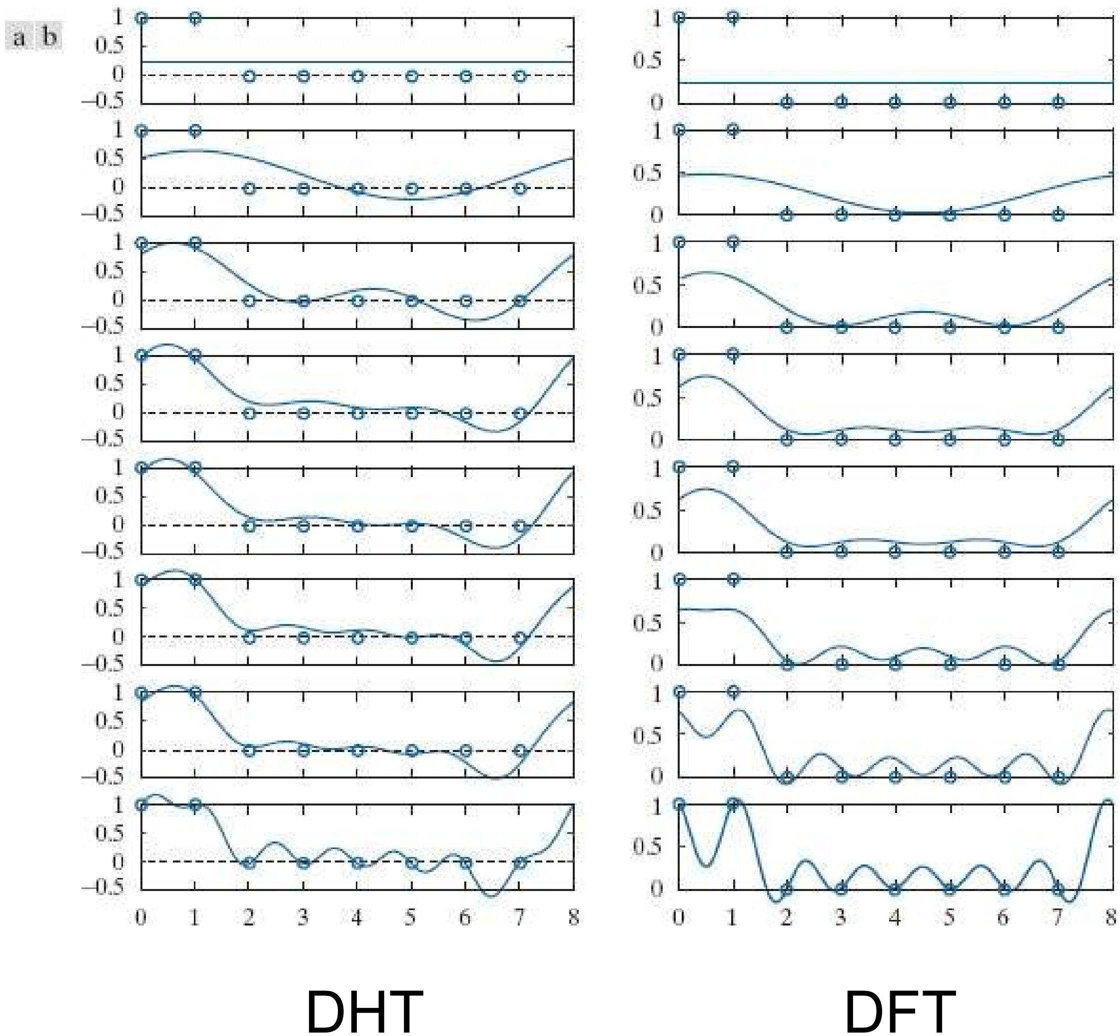


$$\begin{bmatrix}
0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\
0.35 & 0.50 & 0.35 & 0 & -0.35 & -0.50 & -0.35 & 0 \\
0.35 & 0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 \\
0.35 & 0 & -0.35 & 0.50 & -0.35 & 0 & 0.35 & -0.50 \\
0.35 & -0.35 & 0.35 & -0.35 & 0.35 & -0.35 & 0.35 & -0.35 \\
0.35 & -0.50 & 0.35 & 0 & -0.35 & 0.50 & -0.35 & 0 \\
0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 & 0.35 \\
0.35 & 0 & -0.35 & -0.50 & -0.35 & 0 & 0.35 & 0.50
\end{bmatrix}$$



a b c

离散余弦变换

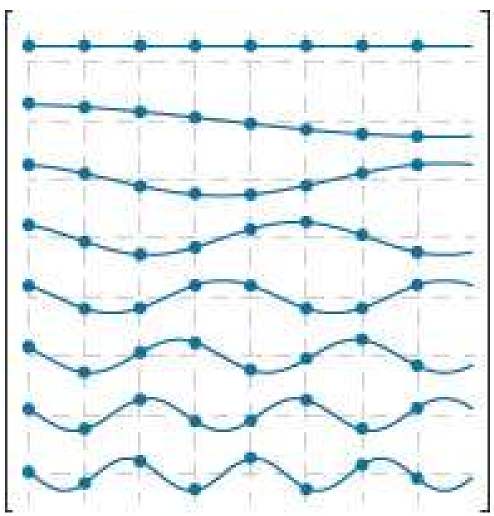


$$s(x,u)=a(u)\cos\left(\frac{(2x+1)u\pi}{2N}\right)$$

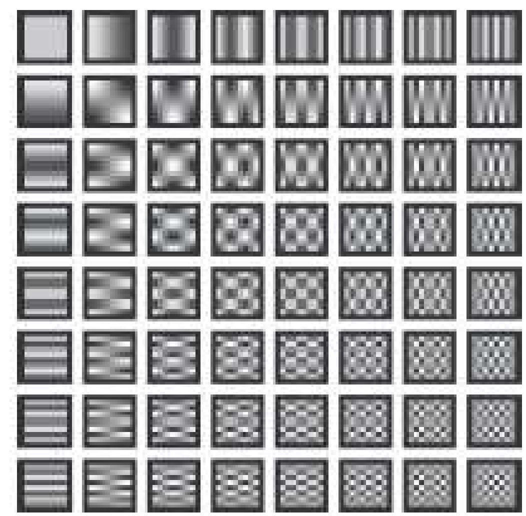
$$a(u)=\begin{cases}\sqrt{\dfrac{1}{N}} & u=0\\ \sqrt{\dfrac{2}{N}} & u=1,2,\ldots,N-1\end{cases}$$

离散余弦变换

$$s(x,y,u,v) = \alpha(u)\alpha(v)\cos\left(\frac{(2x+1)u\pi}{2N}\right)\cos\left(\frac{(2y+1)v\pi}{2N}\right)$$



$$\begin{bmatrix} 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\ 0.49 & 0.42 & 0.28 & 0.10 & -0.10 & -0.28 & -0.42 & -0.49 \\ 0.46 & 0.19 & -0.19 & -0.46 & -0.46 & -0.19 & 0.19 & 0.46 \\ 0.42 & -0.10 & -0.49 & -0.28 & 0.28 & 0.49 & 0.10 & -0.42 \\ 0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 & 0.35 \\ 0.28 & -0.49 & 0.10 & 0.42 & -0.42 & -0.10 & 0.49 & -0.28 \\ 0.19 & -0.46 & 0.46 & -0.19 & -0.19 & 0.46 & -0.46 & 0.19 \\ 0.10 & -0.28 & 0.42 & -0.49 & 0.49 & -0.42 & 0.28 & -0.10 \end{bmatrix}$$

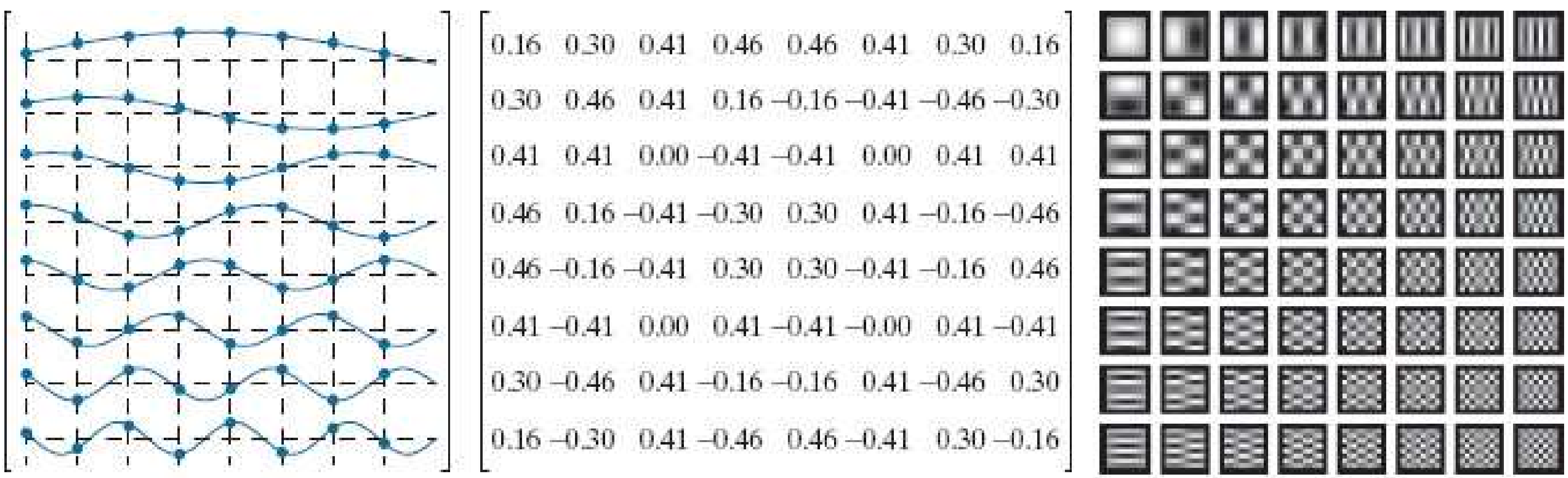


a b c

离散正弦变换

$$s(x,u)=\sqrt{\frac{2}{N+1}}\sin\left(\frac{(x+1)(u+1)\pi}{N+1}\right)$$

$$s(x,y,u,v)=\frac{2}{N+1}\sin\left(\frac{(x+1)(u+1)\pi}{N+1}\right)\sin\left(\frac{(y+1)(\text{v}+1)\pi}{N+1}\right)$$



a b c

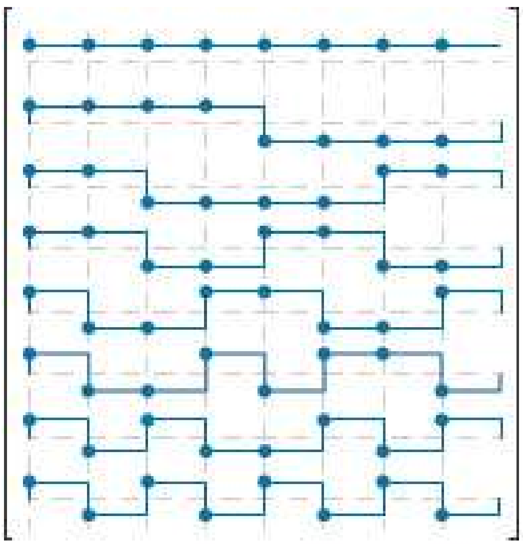
Walsh-Hadamard变换

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

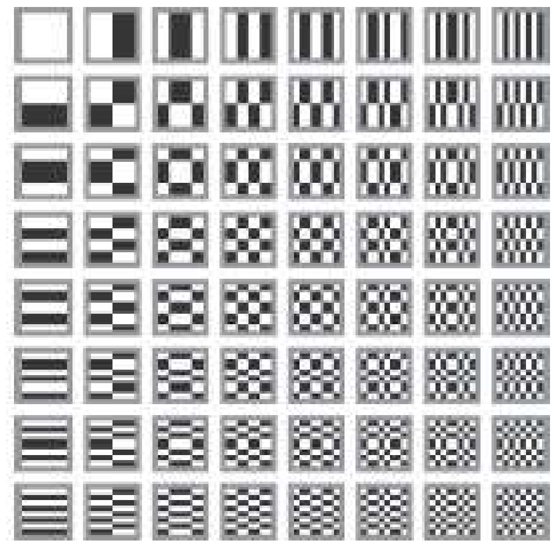
$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

Walsh-Hadamard变换



$$
\begin{bmatrix}
0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\
0.35 & 0.35 & 0.35 & 0.35 & -0.35 & -0.35 & -0.35 & -0.35 \\
0.35 & 0.35 & -0.35 & -0.35 & -0.35 & -0.35 & 0.35 & 0.35 \\
0.35 & 0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 \\
0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 & 0.35 \\
0.35 & -0.35 & -0.35 & 0.35 & -0.35 & 0.35 & 0.35 & -0.35 \\
0.35 & -0.35 & 0.35 & -0.35 & -0.35 & 0.35 & -0.35 & 0.35 \\
0.35 & -0.35 & 0.35 & -0.35 & 0.35 & -0.35 & 0.35 & -0.35
\end{bmatrix}
$$



a b c

Slant斜变换 (I)

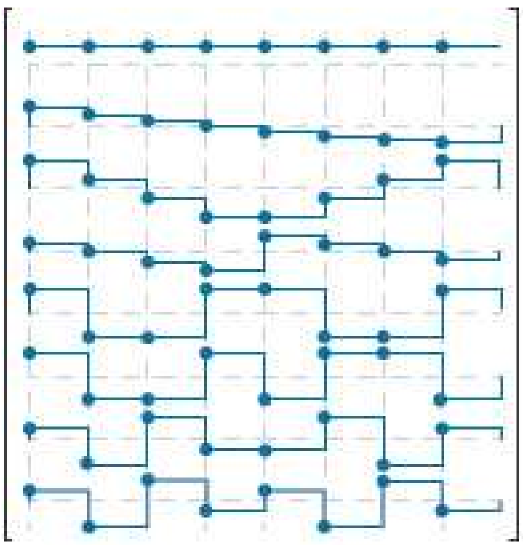
$$S_N=\begin{bmatrix} \begin{matrix}1 & 0\\ a_N & b_N\end{matrix} & 0 & \begin{matrix}1 & 0\\ -a_N & b_N\end{matrix} & 0 \\ 0 & I_{(N/2)-2} & 0 & I_{(N/2)-2} \\ \begin{matrix}0 & 1\\ -b_N & a_N\end{matrix} & 0 & \begin{matrix}0 & -1\\ b_N & a_N\end{matrix} & 0 \\ 0 & I_{(N/2)-2} & 0 & -I_{(N/2)-2}\end{bmatrix}\begin{bmatrix}S_{N/2} & 0\\ 0 & S_{N/2}\end{bmatrix}$$

$$S_2=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

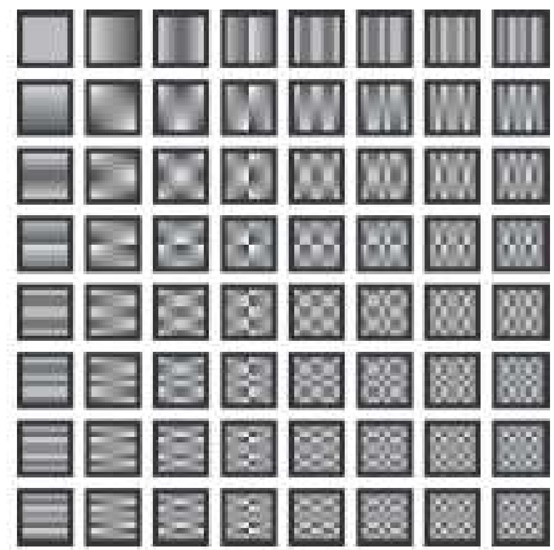
$$a_N=\left[\frac{3N^2}{4(N^2-1)}\right]^{1/2}, \qquad b_N=\left[\frac{N^2-4}{4(N^2-1)}\right]^{1/2}$$

$$A_{SL}=\frac{1}{\sqrt{4}}S_4=\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ \frac{3}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{-1}{\sqrt{5}} & \frac{-3}{\sqrt{5}}\\ 1 & -1 & -1 & 1\\ \frac{1}{\sqrt{5}} & \frac{-3}{\sqrt{5}} & \frac{3}{\sqrt{5}} & \frac{-1}{\sqrt{5}}\end{bmatrix}$$

Slant变换 (II)

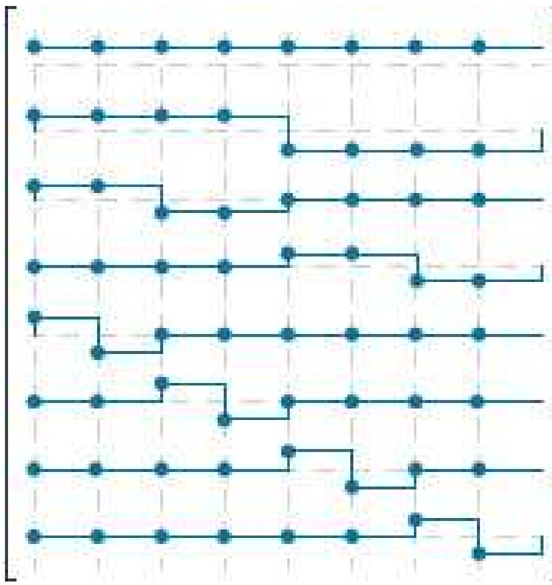


$$\begin{bmatrix}
0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\
0.54 & 0.39 & 0.23 & 0.08 & -0.08 & -0.23 & -0.39 & -0.54 \\
0.47 & 0.16 & -0.16 & -0.47 & -0.47 & -0.16 & 0.16 & 0.47 \\
0.24 & -0.04 & -0.31 & -0.59 & 0.59 & 0.31 & 0.04 & -0.24 \\
0.35 & -0.35 & -0.35 & 0.35 & 0.35 & -0.35 & -0.35 & 0.35 \\
0.35 & -0.35 & -0.35 & 0.35 & -0.35 & 0.35 & 0.35 & -0.35 \\
0.16 & -0.47 & 0.47 & -0.16 & -0.16 & 0.47 & -0.47 & 0.16 \\
0.16 & -0.47 & 0.47 & -0.16 & 0.16 & -0.47 & 0.47 & -0.16
\end{bmatrix}$$



a b c

Haar变换的定义



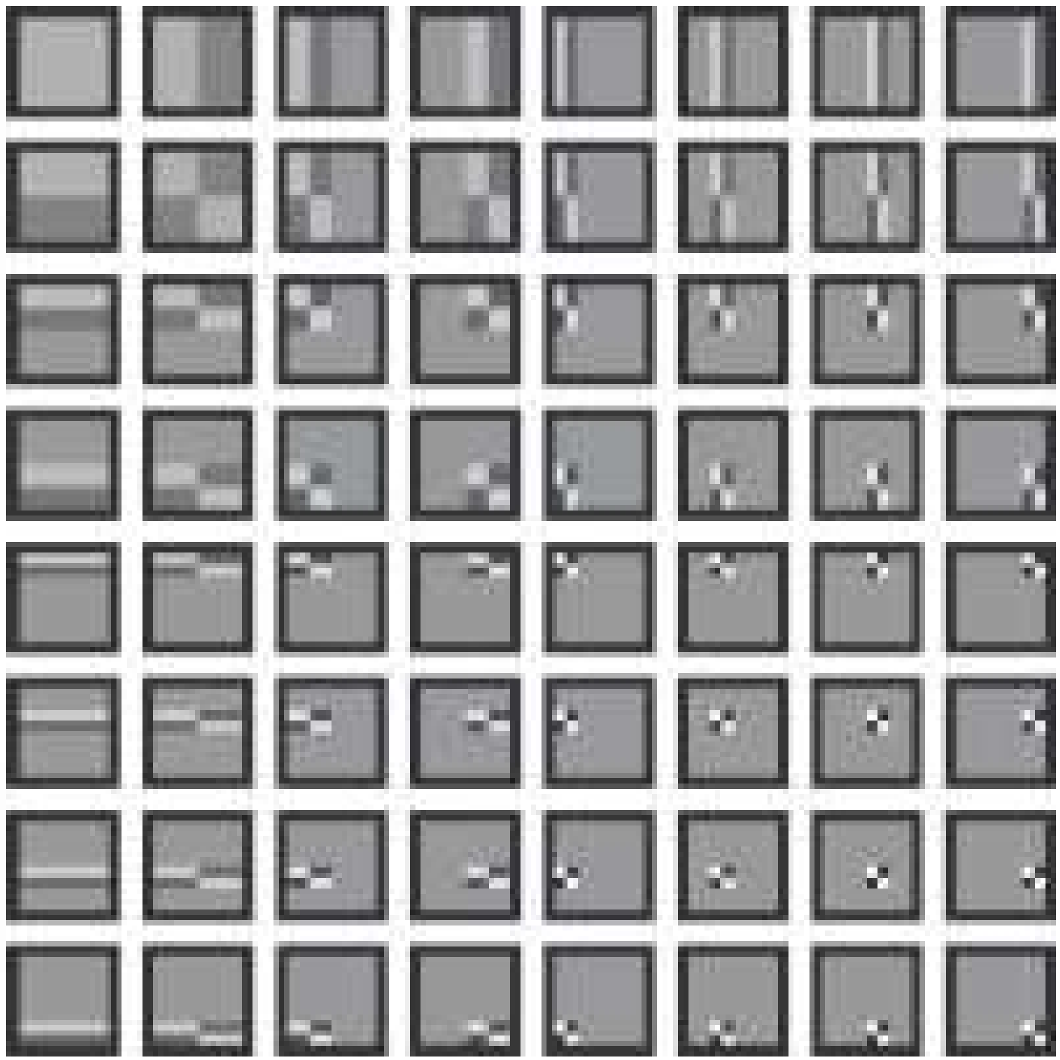
$$\begin{bmatrix} 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\ 0.35 & 0.35 & 0.35 & 0.35 & -0.35 & -0.35 & -0.35 & -0.35 \\ 0.50 & 0.50 & -0.50 & -0.50 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.50 & 0.50 & -0.50 & -0.50 \\ 0.71 & -0.71 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.71 & -0.71 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.71 & -0.71 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.71 & -0.71 \end{bmatrix}$$

a b c

$$h_u(x) = \begin{cases} 1 & u = 0 \text{ and } 0 \le x < 1 \\ 2^{p/2} & u > 0 \text{ and } \mathrm{q}/2^p \le x < (q + 0.5)/2^p \\ -2^{p/2} & u > 0 \text{ and } (q + 0.5)/2^p \le x < (q + 1)/2^p \\ 0 & \text{otherwise} \end{cases}$$

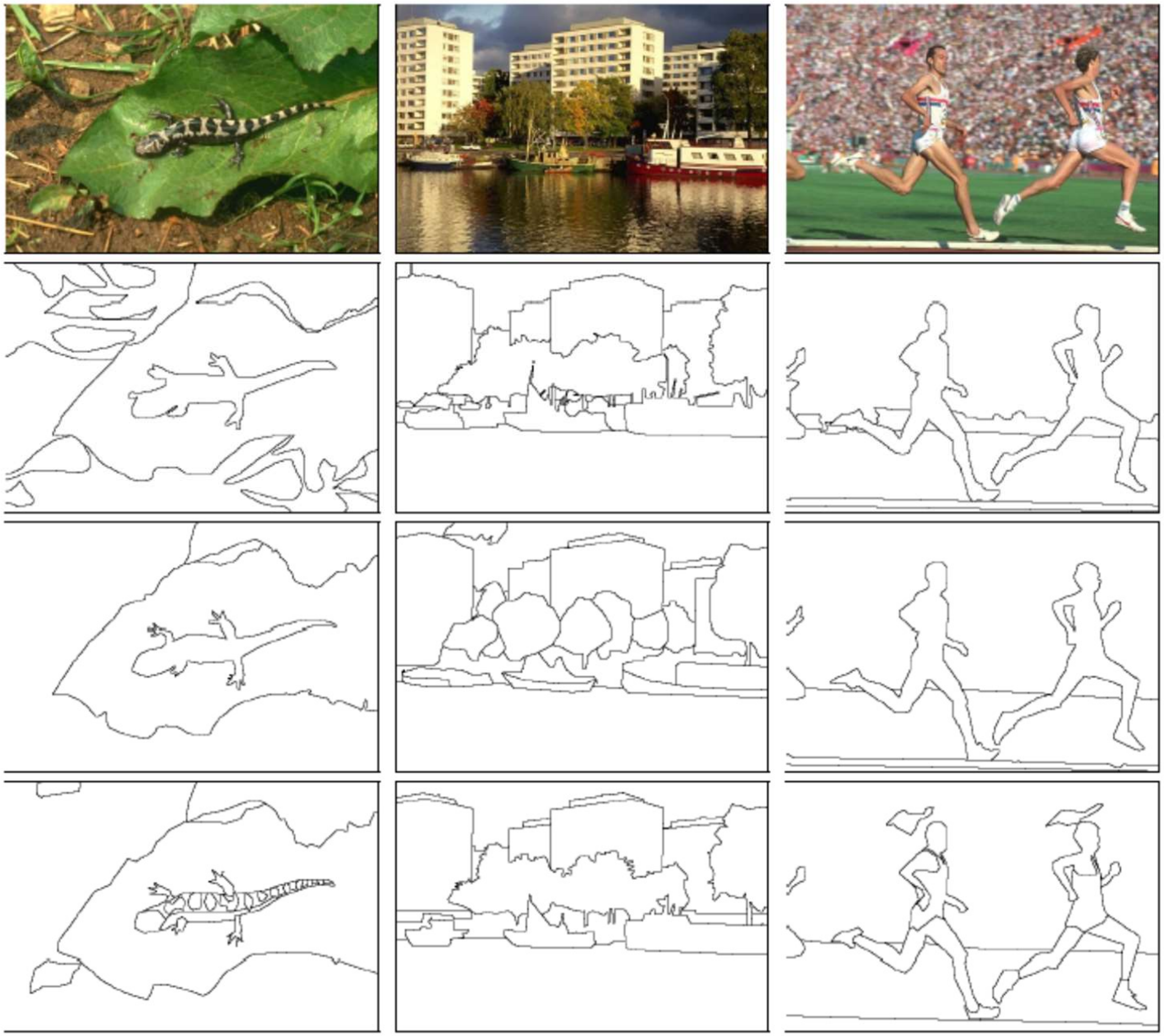
已知的最早最简单的单位正交基

Haar变换



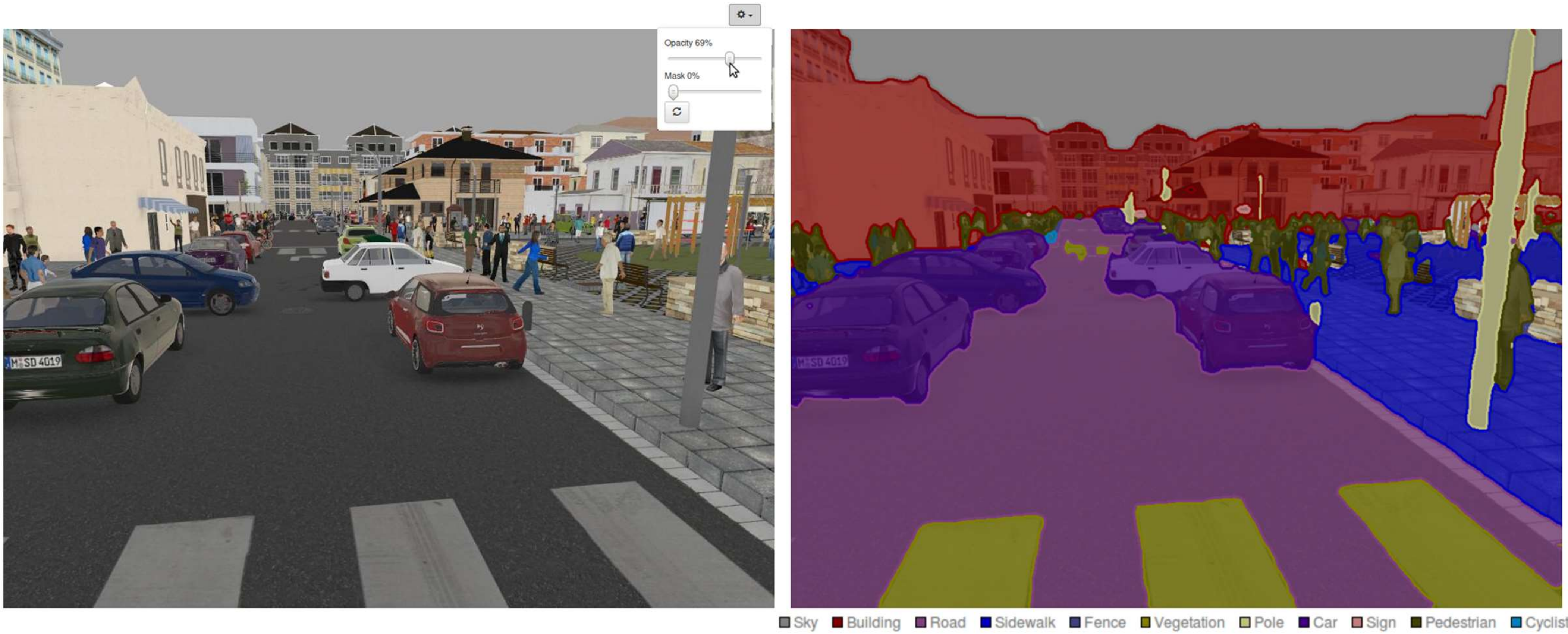
- 采样定理应用示例
- 图像的空间域滤波(复习)
- 图像的频率域滤波(复习)
- 一维离散傅里叶变换(复习)
- 图像的正交变换
- **图像的距离变换**
- 基于深度学习的图像变换简介

图像分割示例 (I)



Arbelaez et al, PAMI, 2010

图像分割示例 (II)



https://devblogs.nvidia.com/image-segmentation-using-digits-5/

为什么要使用图像距离变换?

- 在比较二进制图像时，即使很局部的差别也会造成大的区别。
- 距离变换可以减缓图像特征的变化。
- 在计算机视觉中有广泛应用。

如何计算图像距离变换?

- 定义 $$DT_{(P)}[x] = \min_{y \in P} dist(x, y)$$

- 在计算机图形学，机器人学与人工智能的应用中经常使用相距边界的最短距离。

- 在计算机视觉中，有时距离定义为相距某个特征的最短距离。

不同的距离定义 (I)

0	0	0
0	1	0
0	0	0

Image

1.41	1.0	1.41
1.0	0.0	1.0
1.41	1.0	1.41

Distance Transform

Euclidean $D_{Euclidean} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

0	0	0
0	1	0
0	0	0

Image

2	1	2
1	0	1
2	1	2

Distance Transform

City block $D_{City} = |x_2 - x_1| + |y_2 - y_1|$

0	0	0
0	1	0
0	0	0

Image

1	1	1
1	0	1
1	1	1

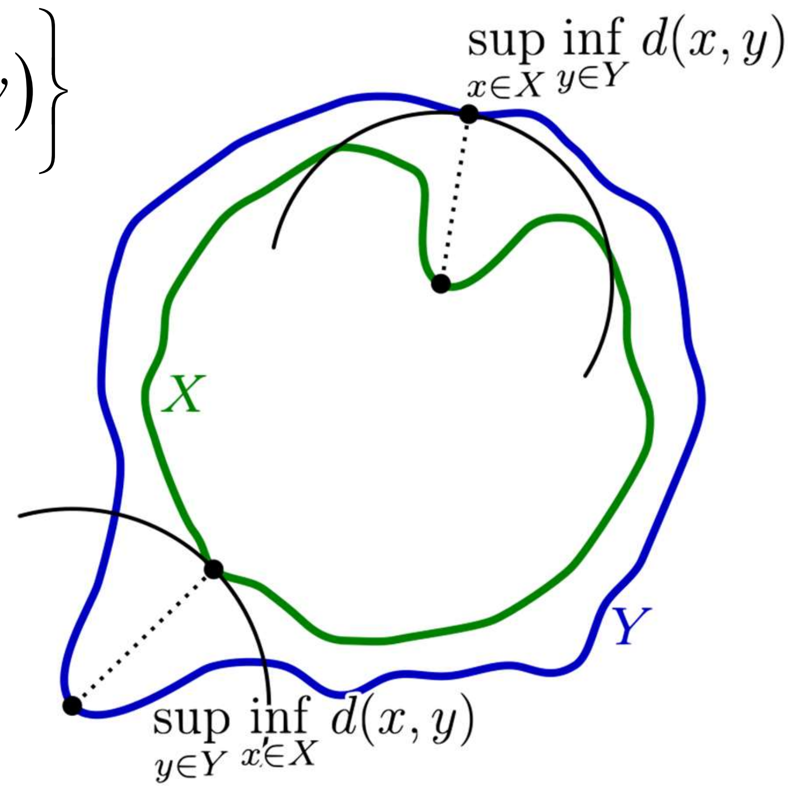
Distance Transform

Chessboard $D_{Chess} = \max\left(|x_2 - x_1|, |y_2 - y_1|\right)$

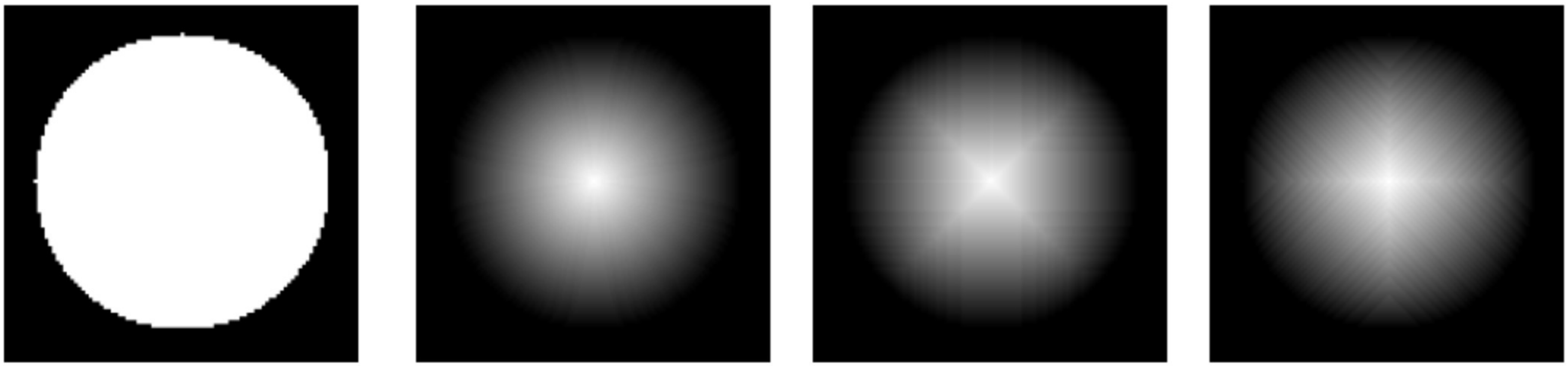
不同的距离定义 (II)

$$d_H(X,Y)=\max\left\{\sup_{x\in X}\inf_{y\in Y} d(x,y), \sup_{y\in Y}\inf_{x\in X} d(x,y)\right\}$$

Hausdorff



图像的距离变换 (II)



Euclidean City Chessboard

https://www.cs.cornell.edu/courses/cs664/2008sp/handouts/cs664-7-dtrans.pdf

图像的距离变换 (II)

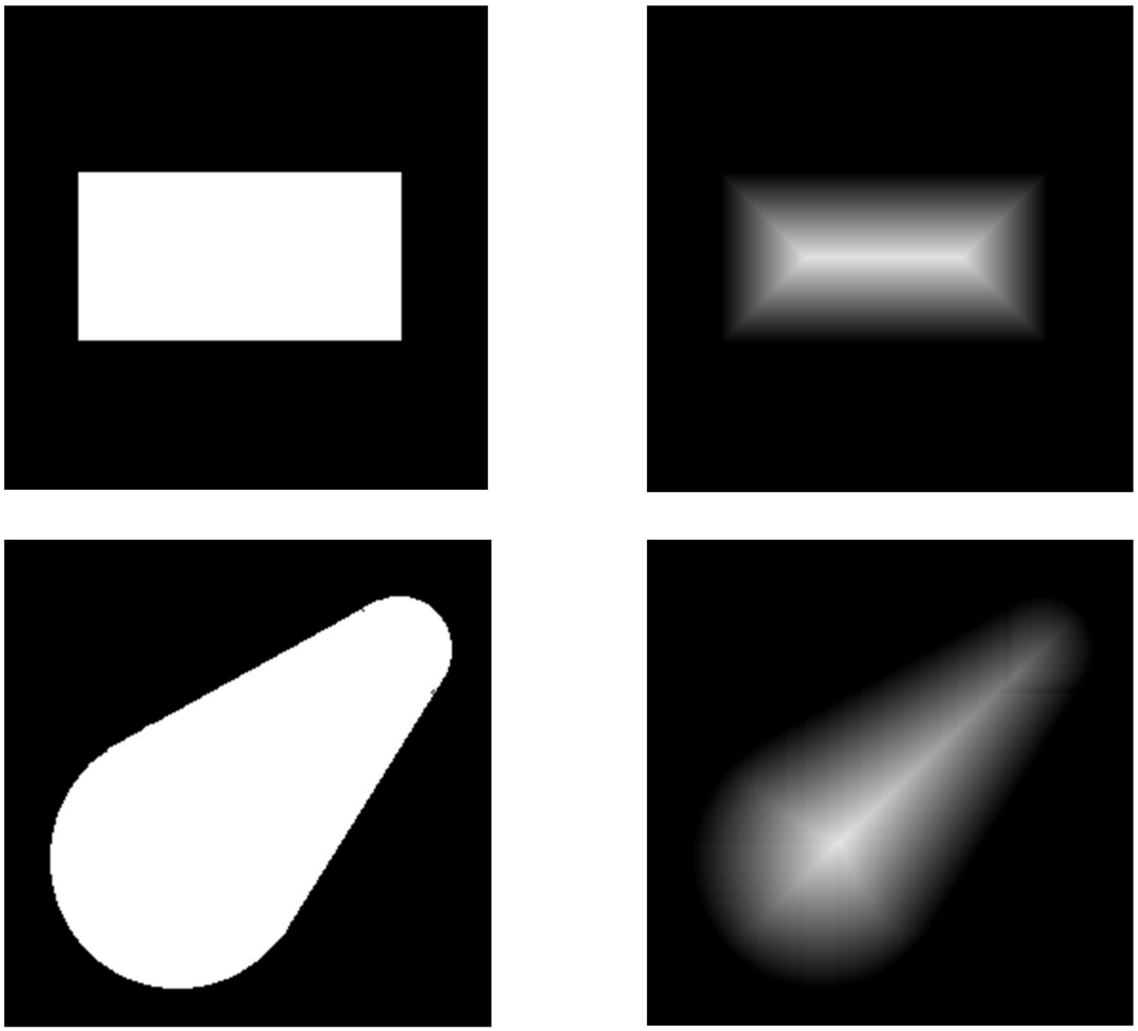
0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0
0	1	2	2	2	2	1	0
0	1	2	3	3	2	1	0
0	1	2	2	2	2	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0

定义 距离为与最近边界的距离

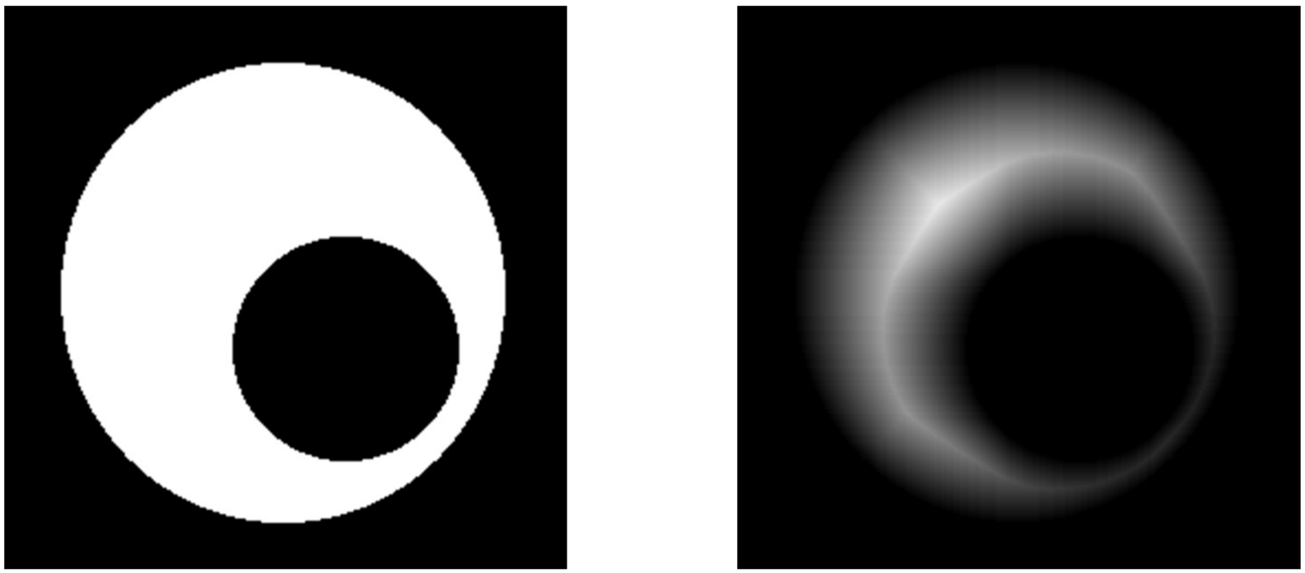
https://homepages.inf.ed.ac.uk/rbf/HIPR2/distance.htm

距离变换结果示例 (I)



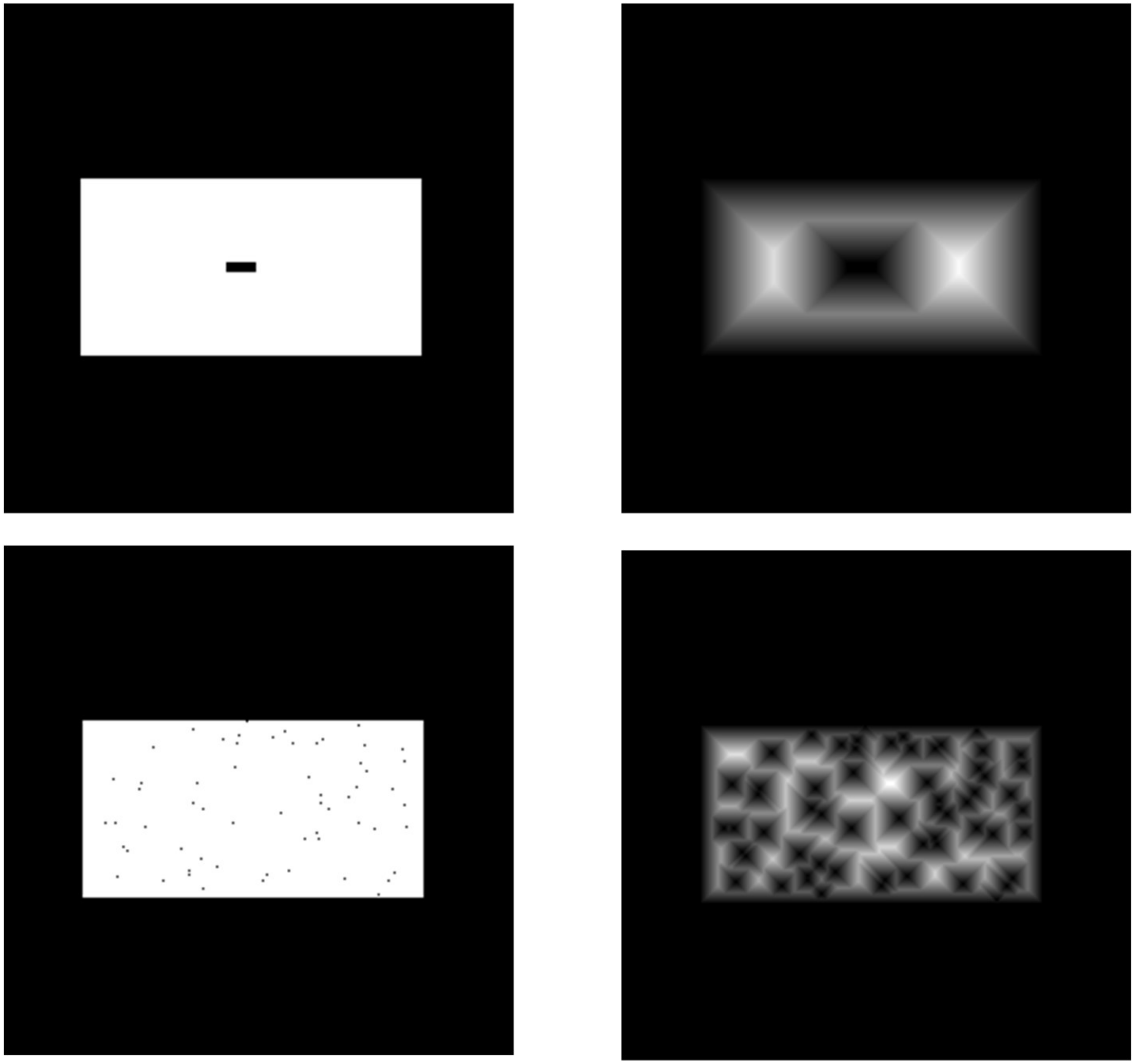
https://homepages.inf.ed.ac.uk/rbf/HIPR2/distance.htm

距离变换结果示例（II)



https://homepages.inf.ed.ac.uk/rbf/HIPR2/distance.htm

距离变换的局限性



- 采样定理应用示例
- 图像的空间域滤波(复习)
- 图像的频率域滤波(复习)
- 一维离散傅里叶变换(复习)
- 图像的正交变换
- 图像的距离变换
- 基于深度学习的图像变换简介

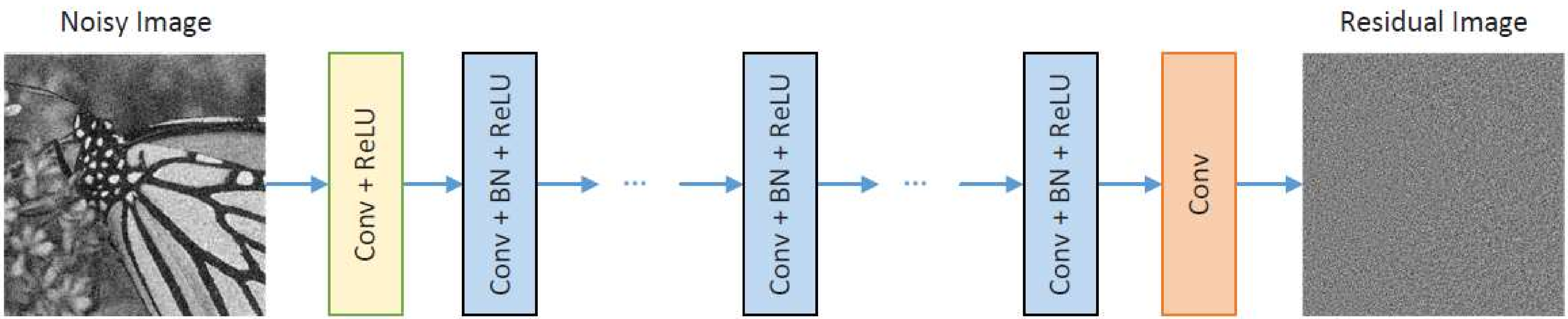
1

Beyond a Gaussian Denoiser: Residual Learning of Deep CNN for Image Denoising

Kai Zhang, Wangmeng Zuo, Yunjin Chen, Deyu Meng, and Lei Zhang

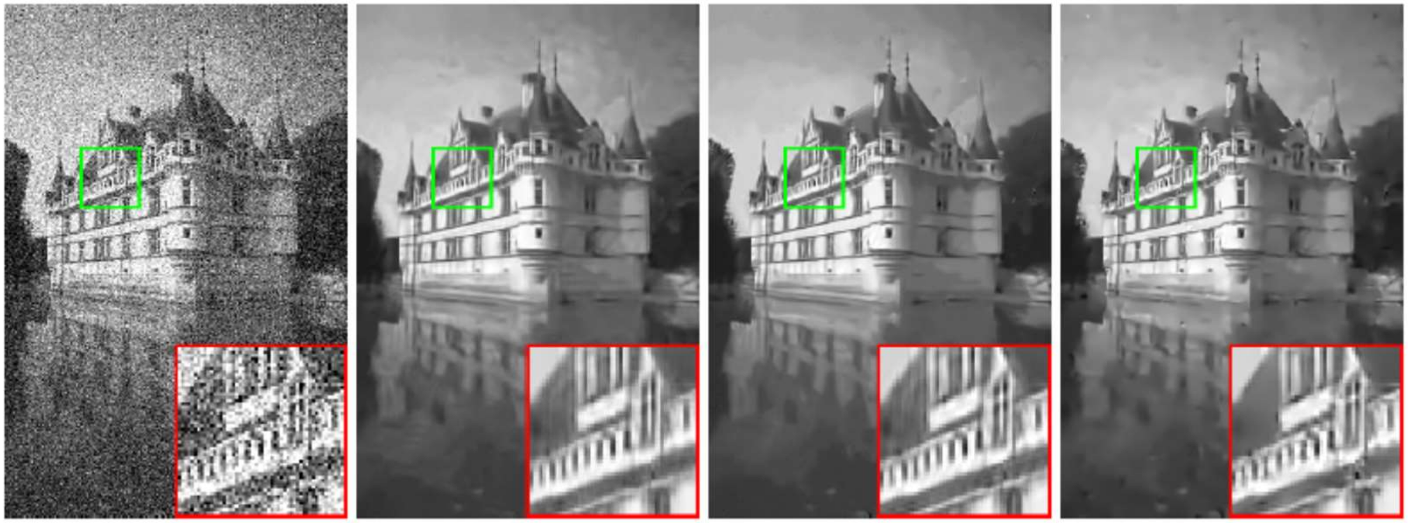
IEEE Transactions on Image Processing, 26(7), 3142-3155. doi:10.1109/tip.2017.2662206

深度学习网络构架

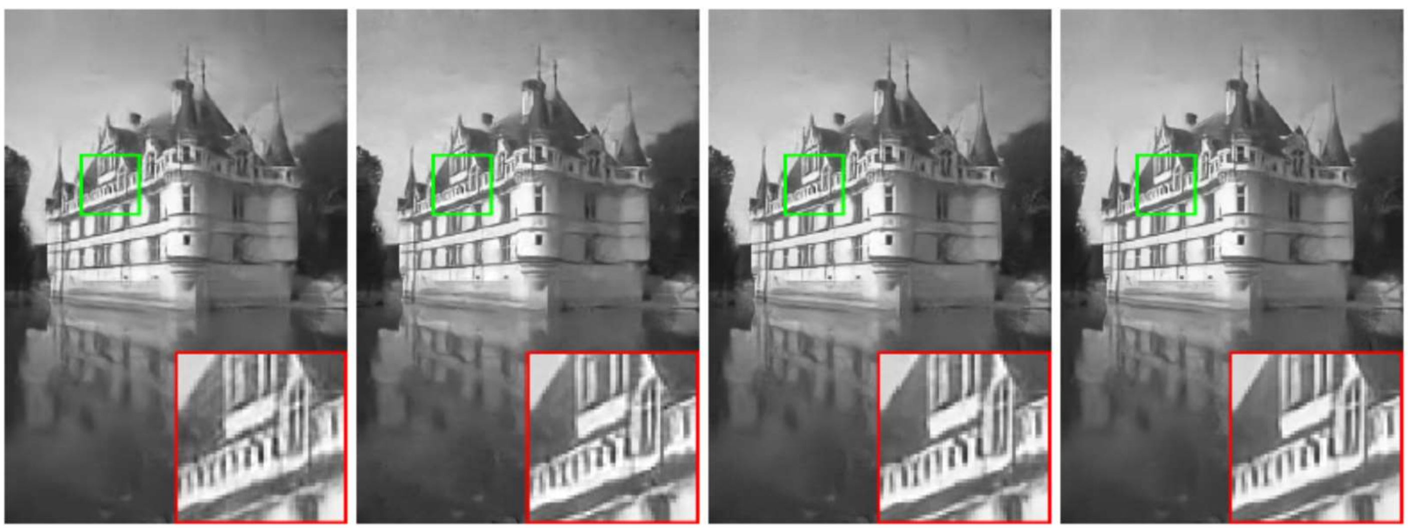


Noisy Image
Conv + ReLU
Conv + BN + ReLU
...
Conv + BN + ReLU
...
Conv + BN + ReLU
Conv
Residual Image

滤波效果 (I)

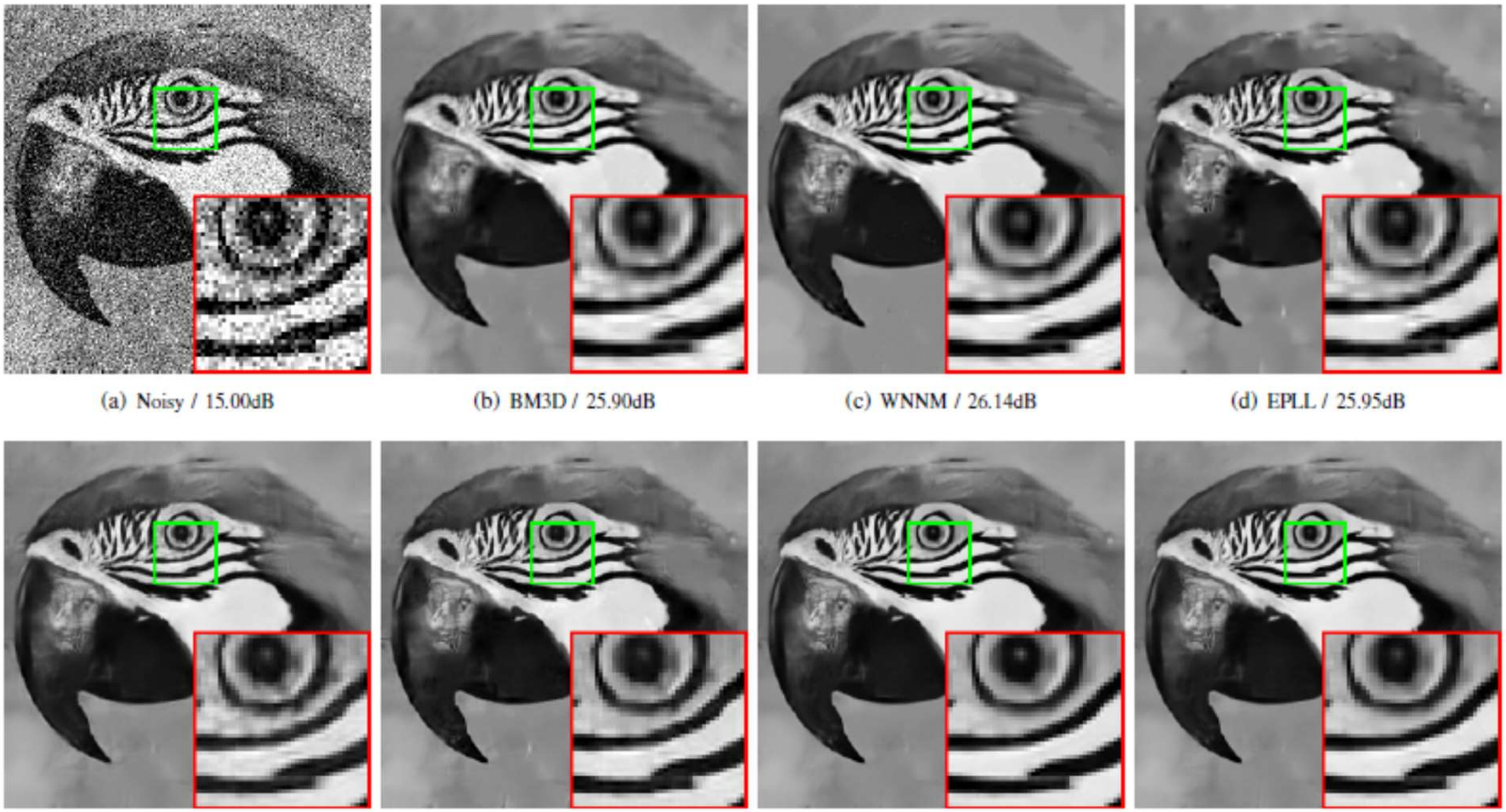


(a) Noisy / 14.76dB (b) BM3D / 26.21dB (c) WNNM / 26.51dB (d) EPLL / 26.36dB



(e) MLP / 26.54dB (f) TNRD / 26.59dB (g) DnCNN-S / 26.90dB (h) DnCNN-B / 26.92dB

滤波效果 (II)

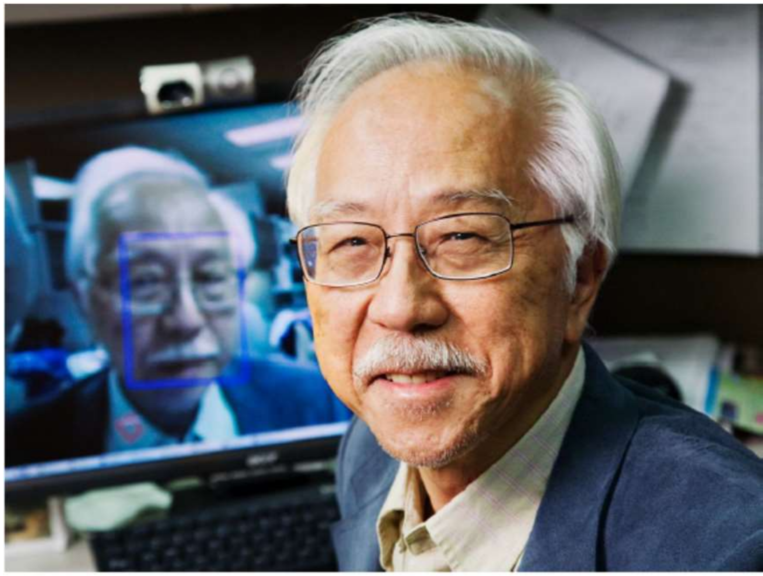


(a) Noisy / 15.00dB (b) BM3D / 25.90dB (c) WNNM / 26.14dB (d) EPLL / 25.95dB

(e) MLP / 26.12dB (f) TNRD / 26.16dB (g) DnCNN-S / 26.48dB (h) DnCNN-B / 26.48dB

Questions?

Thomas Huang 黄煦涛（1936年6月26日-2020年4月25日）



- 1956年毕业于台湾大学电子系，
- 1963年毕业于美国麻省理工学院电机系，先后获得硕士学位、博士学位
- 1963-1973留校任教，先后担任麻省理工学院电机系助理教授、副教授
- 1973-1980普渡大学电机教授
- 1980-202进入伊利诺伊大学厄巴纳-香槟分校电机系William L. Everitt 杰出讲座教授兼贝克曼前瞻科技研究院主席；
- 2001年当选为中国工程院外籍院士；2002年当选为美国工程院院士，同年当选为中国科学院外籍院士

Thomas S. Huang memorial prize

The PAMITC executive committee has approved the creation of the Thomas S. Huang memorial prize in computer vision, to be awarded annually at CVPR starting in 2021.

The award winner will be selected by the PAMITC awards committee, similarly to the Rosenfeld and Distinguished Researcher awards, and will have the same financial remuneration.

Researchers who are more than 10 years past their PhD are eligible. The winner will be selected based on a combination of research, service and mentoring. Additional details will be made available before CVPR21.

Thomas Huang 黄煦涛

Thomas S. Huang
University of Illinois, Urbana-Champaign
Computer Vision
Image Processing
Pattern Recognition
Multimedia
AI

GET MY OWN PROFILE

	All	Since 2016
Citations	177899	59084
h-index	167	94
i10-index	994	468

6 articles — 57 articles

not available — available

Based on funding mandates

TITLE	CITED BY	YEAR
The self-organizing map T Kohonen Neurocomputing 21 (1-3), 1-6	36186 *	1998
Image super-resolution via sparse representation J Yang, J Wright, TS Huang, Y Ma IEEE transactions on image processing 19 (11), 2861-2873	5061	2010
Least-squares fitting of two 3-D point sets KS Arun, TS Huang, SD Blostein IEEE Transactions on pattern analysis and machine intelligence, 698-700	4594	1987
Locality-constrained linear coding for image classification J Wang, J Yang, K Yu, F Lv, T Huang, Y Gong 2010 IEEE computer society conference on computer vision and pattern …	3805	2010
Linear spatial pyramid matching using sparse coding for image classification J Yang, K Yu, Y Gong, T Huang 2009 IEEE Conference on computer vision and pattern recognition, 1794-1801	3762	2009
Image retrieval: Current techniques, promising directions, and open issues Y Rui, TS Huang, SF Chang Journal of visual communication and image representation 10 (1), 39-62	3476 *	1999
A survey of affect recognition methods: Audio, visual, and spontaneous expressions Z Zeng, M Pantic, GI Roisman, TS Huang IEEE transactions on pattern analysis and machine intelligence 31 (1), 39-58	3192	2008

Thomas Huang 黄煦涛

- 在信号处理，模式识别，计算机视觉领域做出了杰出贡献
- 发明了预测差分量化（PDQ）的两维传真（文档）压缩方法，该方法已发展为国际G3/G4FAX压缩标准；
- 在多维数字信号处理领域中，提出了关于递归滤波器的稳定性的理论；
- 建立了从二维图象序列中估计三维运动的公式，为图象处理和计算机视觉开启了新领域。
- 图像超分辨技术

Thomas Huang 黄煦涛

可分滤波器及其计算成本分析

$$w * f = (w_1 * w_2) * f = (w_2 * w_1) * f = w_2 * (w_1 * f) = w_1 * (w_2 * f)$$

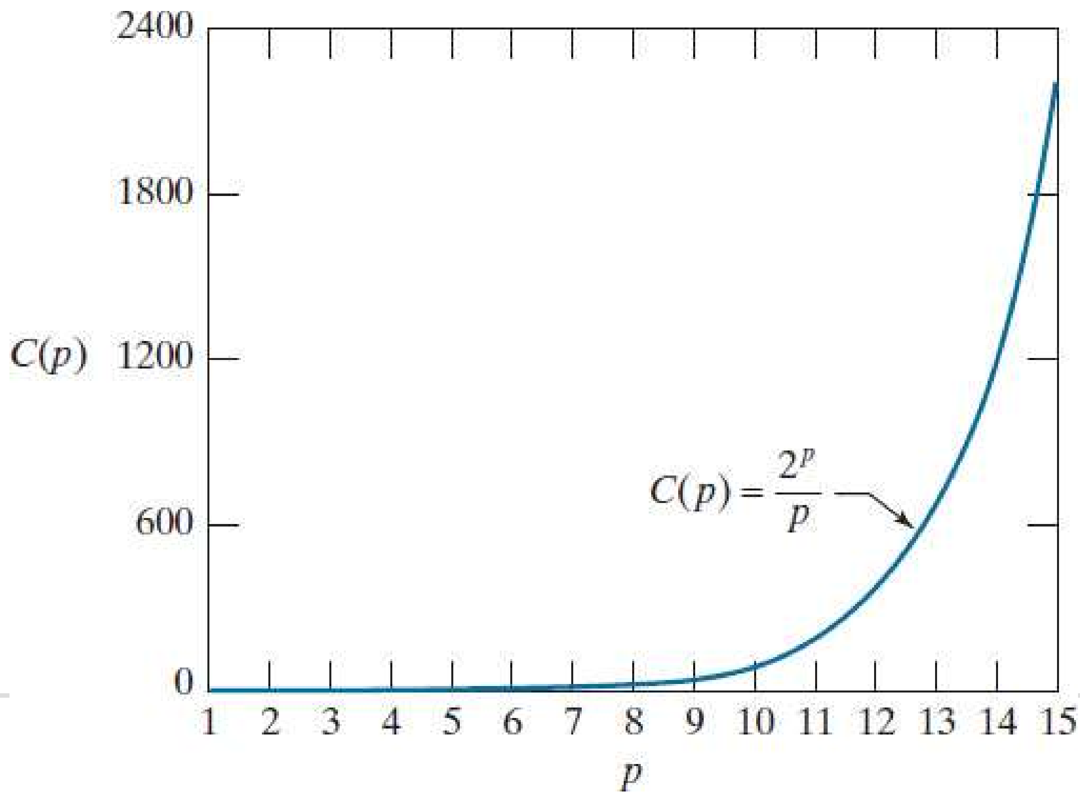
$$w \in R^{m \times n}, f \in R^{M \times N}$$

$$C = \frac{MNmn}{MN(m+n)} = \frac{mn}{m+n}$$

快速傅里叶变换的基本概念 (I)

- 采用快速傅里叶变换的计算成本(一维)

$$C(M)=\frac{M^2}{M\log_2 M}=\frac{M}{\log_2 M}=\frac{2^p}{p}$$



图像二维快速傅里叶变换(DFT)的计算 (II)

计算正向变换

$$F(u,v)=\sum_{x=0}^{M-1}e^{-j2\pi\frac{ux}{M}}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi\frac{vy}{N}}$$

$$=\sum_{x=0}^{M-1}F(x,v)e^{-j2\pi\frac{ux}{M}}$$

$$F(x,v)=\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi\frac{vy}{N}}$$

- **$F(x,v)$ 是$f(x,y)$中一行的一维DFT**
- **$F(u,v)$ 是$F(x,v)$中一列的一维DFT**

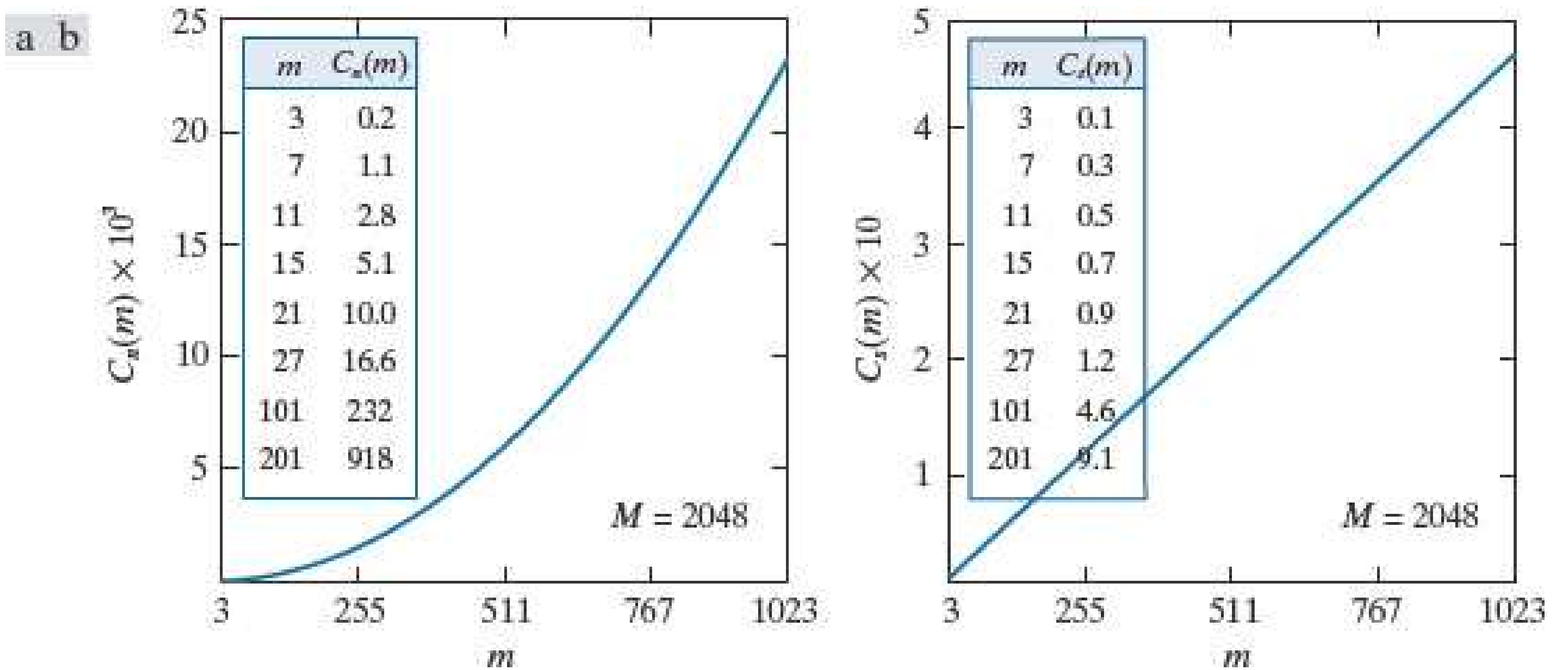
快速傅里叶变换的基本概念 (III)

- 卷积计算 图像大小 M×M, 卷积核大小 m×m

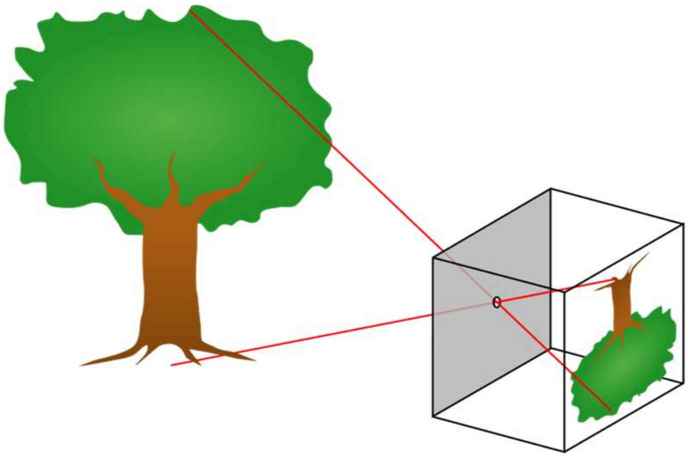
Nonseparable $$C(m)=\frac{M^2m^2}{2M^2\log_2 M^2}=\frac{m^2}{4\log_2 M}$$

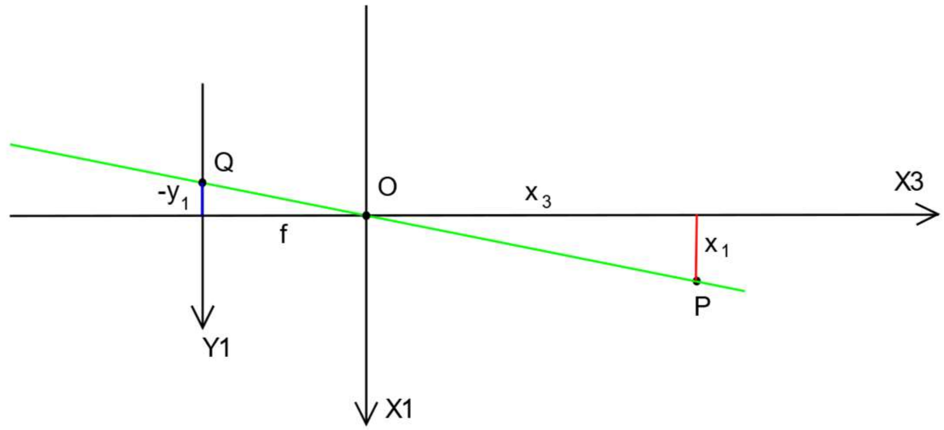
Separable $$C(m)=\frac{2M^2m}{2M^2\log_2 M^2}=\frac{m}{2\log_2 M}$$

快速傅里叶变换的基本概念 (IV)



采样定理应用示例：手机相机空间采样频率 (III)





$$\frac{\text{成像平面物体尺寸}}{\text{物体实际尺寸}} = \frac{\text{焦距}}{\text{与物体距离}}$$

$$\text{单个像素对应的实际物体尺寸} = \text{像素尺寸} \times \frac{\text{与物体距离}}{\text{焦距}}$$

https://en.wikipedia.org/wiki/Pinhole_camera_model

采样定理应用示例：手机相机空间采样频率 (IV)

像素大小是决定手机相机的空间采样频率的关键因素

$$\text{单个像素对应的实际物体尺寸} = 1.4\text{微米} \times \frac{2000\text{毫米}}{4.52\text{毫米}} \sim= 0.62\text{毫米}$$

手机相机的空间分辨率主要由其镜头的数字孔径（F数）决定

$$NA \sim= \frac{1}{2*F\#} \sim= \frac{1}{5.6} \qquad D = \frac{0.61\lambda}{NA} = \frac{0.61*550}{1/5.6} \approx 1.88\text{微米}$$

1.88微米/1.4微米 ~= 1.34 尚未完全满足空间采样频率要求
1.88微米/0.8微米 ~= 2.35 满足空间采样频率要求

https://en.wikipedia.org/wiki/Pinhole_camera_model

- 光学成像系统的分辨率主要取决于其目镜的数值孔径，与放大倍数无关。

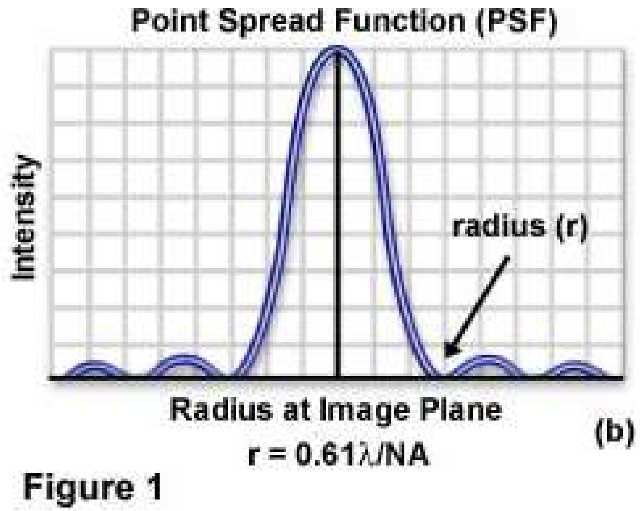
- 放大倍数主要决定空间采样频率。

Airy Disk

- Airy (after George Biddell Airy) disk is the diffraction pattern of a point feature under a circular aperture.

- It has the following form

$$y = \left[\frac{2J_1(x)}{x} \right]^2$$



$J_1(x)$ is a Bessel function of the first kind.

- Detailed derivation is given in

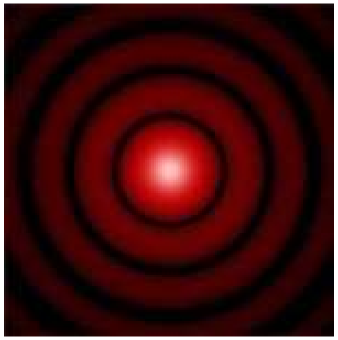
  Born & Wolf, Principles of Optics, 7th ed., pp. 439-441.

Microscope Image Formation (I)

- Microscope image formation can be modeled as a convolution with the PSF.

$$I(x,y) = O(x,y) \otimes psf(x,y)$$

$$F\{I(x,y)\} = F\{O(x,y)\} \cdot F\{psf(x,y)\}$$



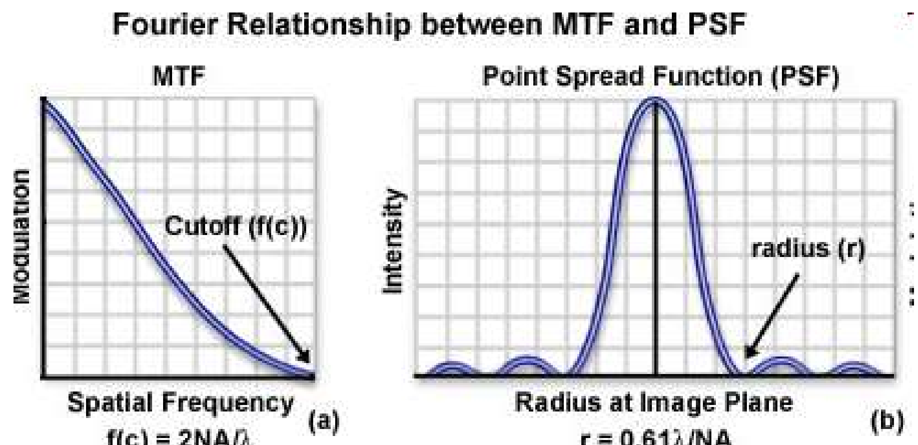
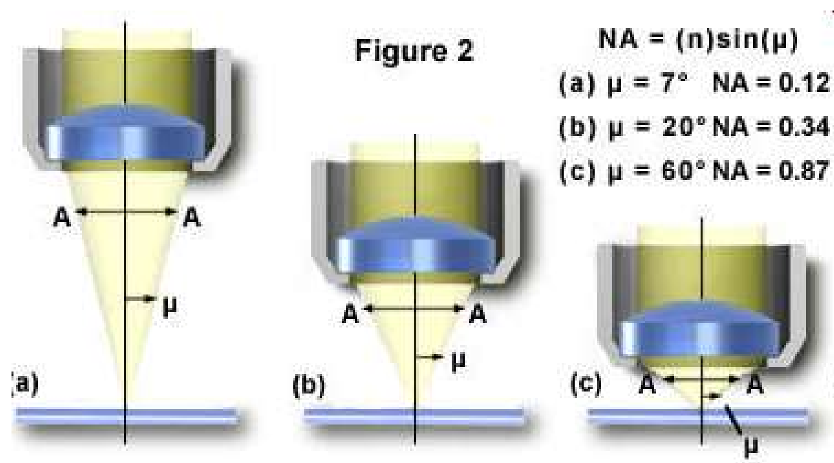


Figure 1

http://micro.magnet.fsu.edu/primer/java/mtf/airydisksize/index.html

Numerical Aperture

- Numerical aperture (NA) determines microscope resolution and light collection power.



$$NA = n \cdot sin\, \mu$$

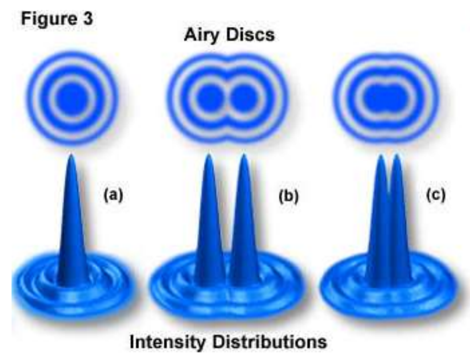
n: refractive index of the medium between the lens and the specimen

μ: half of the angular aperture

Different Definition of Light Microscopy Resolution Limit (Demo)

- Rayleigh limit

$$D = \frac{0.61\lambda}{NA}$$



- Sparrow limit

$$D = \frac{0.47\lambda}{NA}$$

http://www.microscopy.fsu.edu/primer/java/imageformation/rayleighdisks/index.html