180206081104P2009H

图像处理与分析

第三讲: 图像变换与滤波

图像空间域与频率域滤波，深度神经网络结构简介

内容提要

- 点扩散函数的基本概念
- 图像滤波和高斯滤波器
- 图像采样定理
- 图像的二维离散傅里叶变换
- 图像的频率域滤波
- 图像的正交变换
- 卷积神经网络的基本结构

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- 卷积神经网络的基本结构

如何描述一个线性系统

- 一个线性系统可以通过如下两种方式描述
  - 脉冲响应
  - 频率响应：传递函数

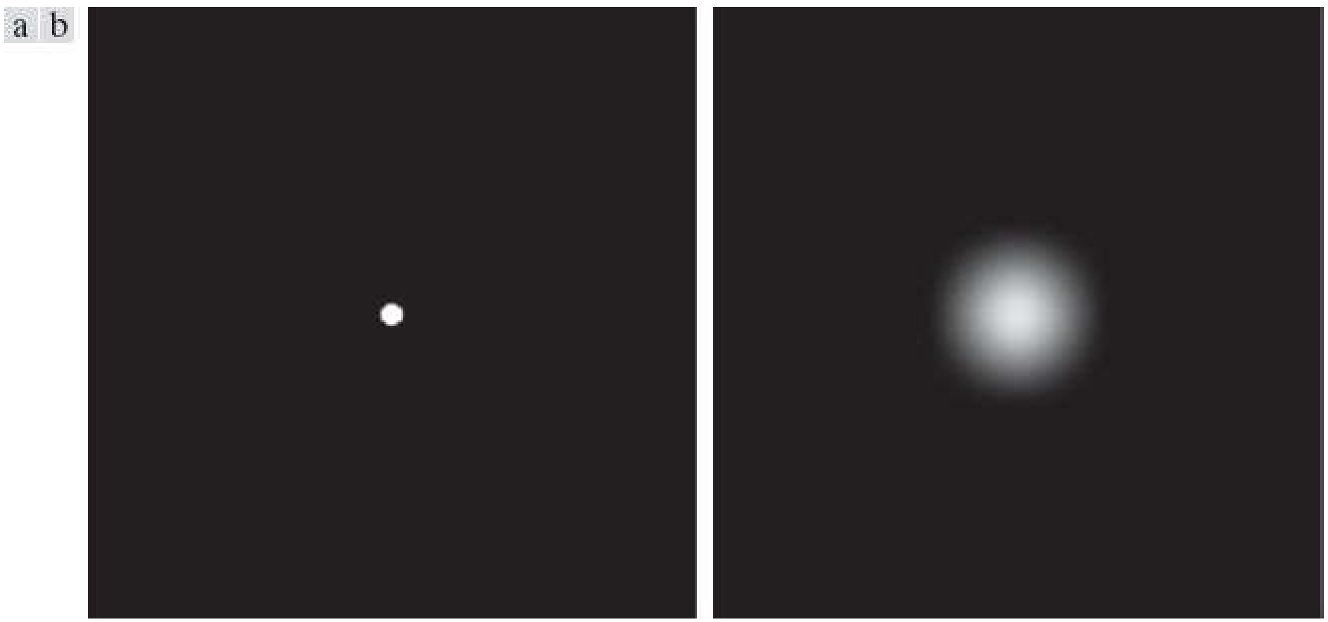
- 光学成像系统的脉冲响应: 点扩散函数

$$I(x,y)=O(x,y)\otimes psf(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}O(u,v)\cdot psf(x-u,y-v)\,dudv$$

- 光学成像系统的频率响应: 光学传递函数

$$F\{I(x,y)\}=F\{O(x,y)\}\cdot F\{psf(x,y)\}=F\{O(x,y)\}\cdot OTF(\cdot)$$

图 5.24：典型点扩散函数示例



- 点扩散函数的基本概念
- **图像滤波和高斯滤波器**
- 图像采样定理
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图像的空间滤波：基本概念

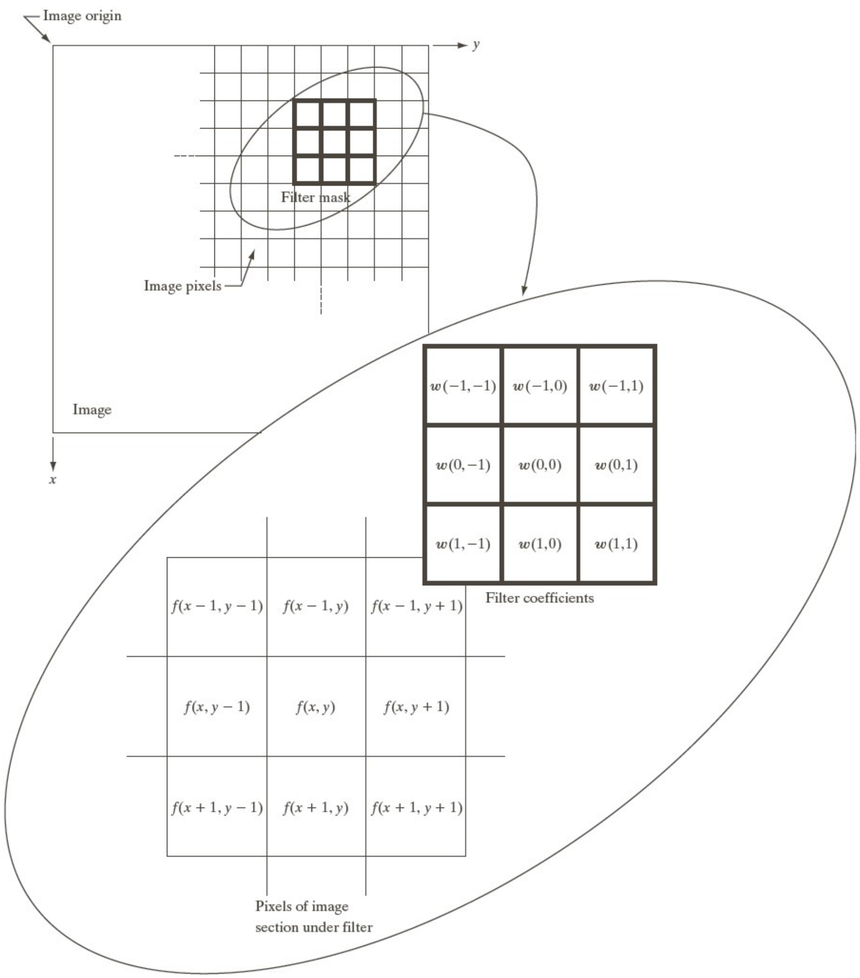


FIGURE 3.28 The mechanics of linear spatial filtering using a 3 × 3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

空间滤波器(核)

$$w(s,t) \in R^{(2N+1)\times(2M+1)}$$

相关与卷积的离散表示

相关

$$w(x,y)\circ f(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)$$

卷积

$$w(x,y)\otimes f(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x-s,y-t)$$

$$=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(-s,-t)f(x+s,y+t)$$

Figure 3.35 一维卷积与相关计算

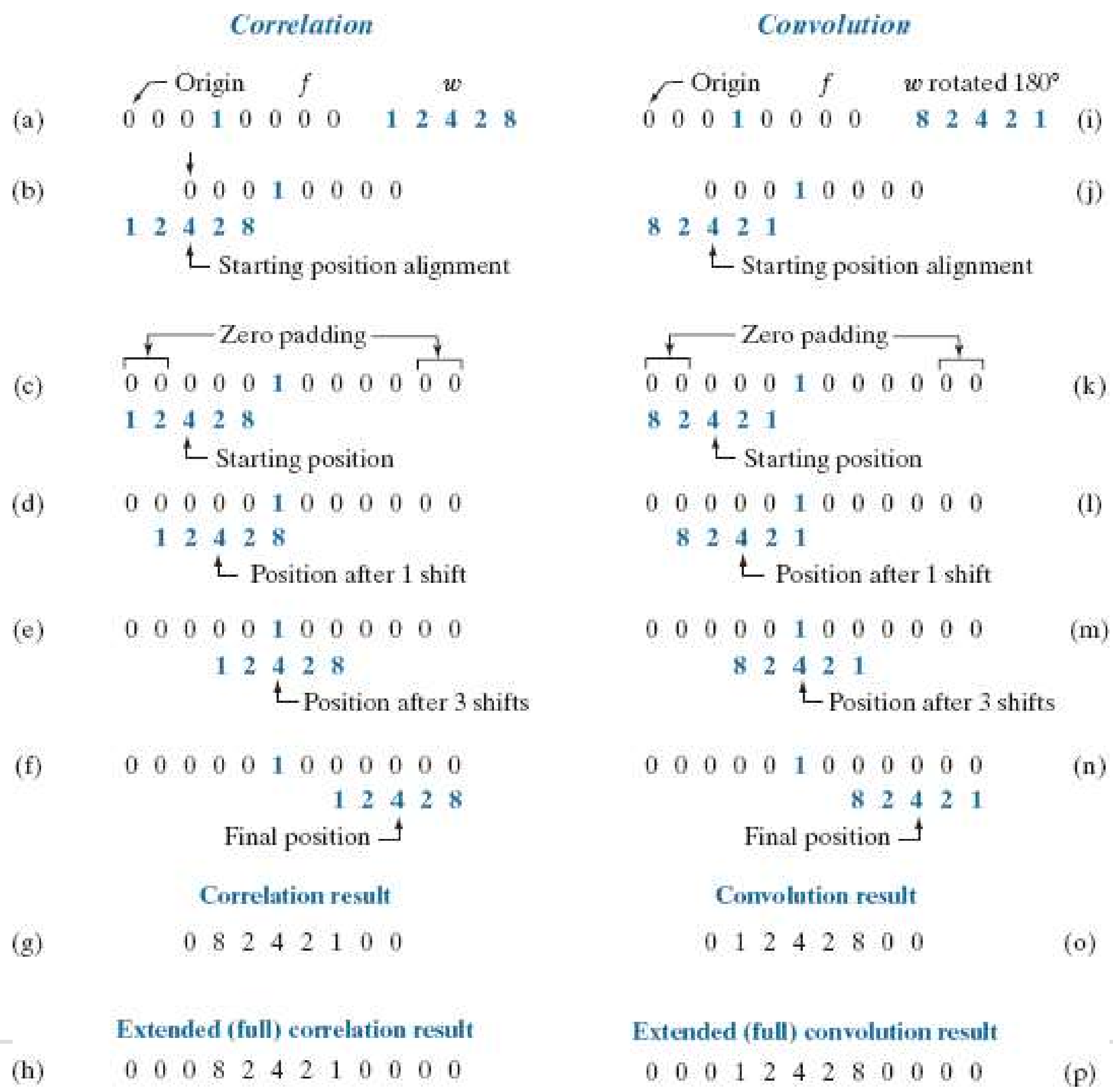
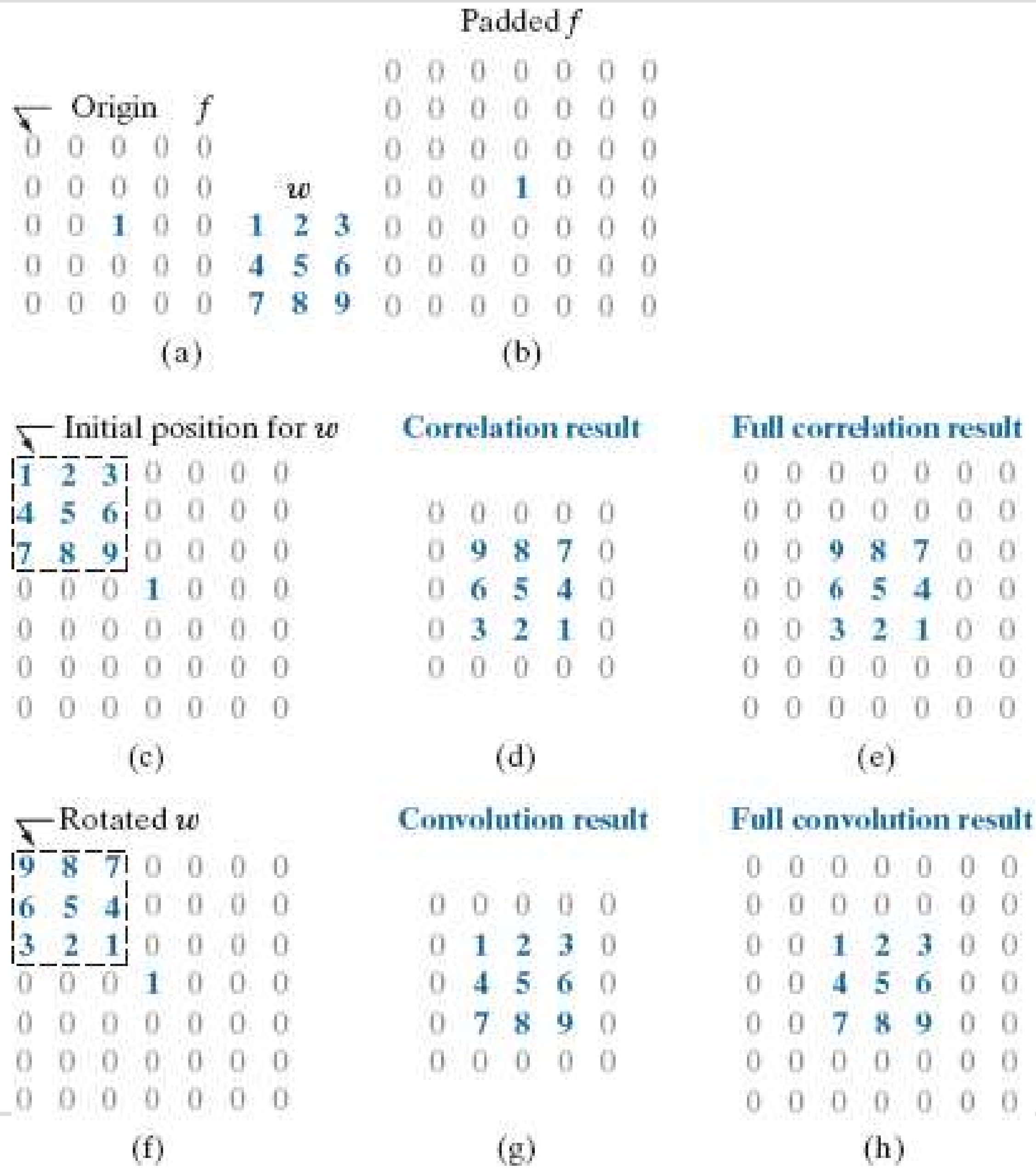


Figure 3.36c 二维卷积与相关计算



图像滤波示例：图像导数的计算 (II)

- 一阶导数

$$I_x(i,j)=\frac{I(i+1,j)-I(i-1,j)}{2h}+O(h^2)$$

$$I_y(i,j)=\frac{I(i,j+1)-I(i,j-1)}{2h}+O(h^2)$$

- 二阶导数

$$I_{xx}(i,j)=\frac{I(i+1,j)-2I(i,j)+I(i-1,j)}{h^2}+O(h)$$

$$I_{yy}(i,j)=\frac{I(i,j+1)-2I(i,j)+I(i,j-1)}{h^2}+O(h)$$

图像滤波示例：图像的微分运算放大噪声

- 通过图像滤波减低噪声

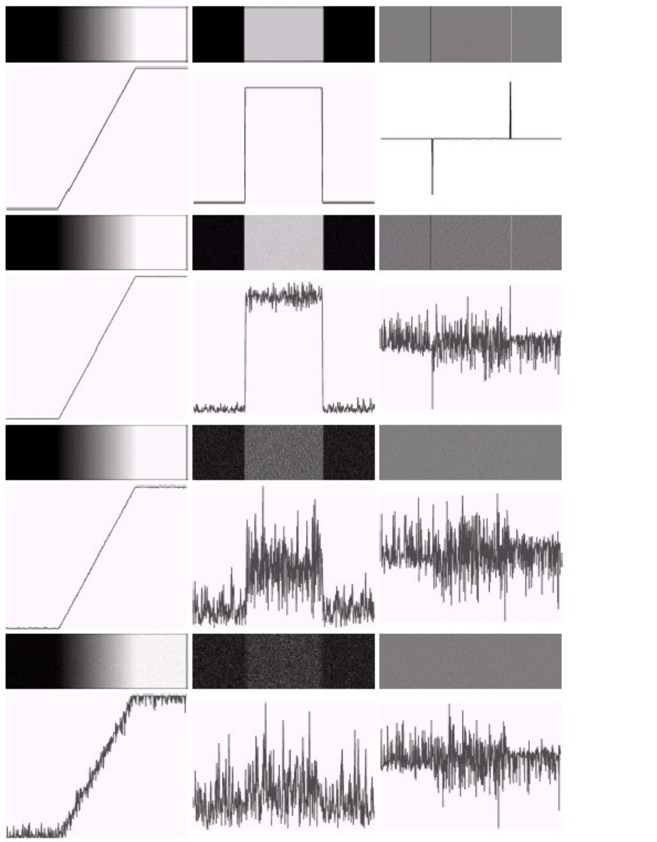


FIGURE 10.7 First column: images and gray-level profiles of a ramp edge corrupted by random Gaussian noise of mean 0 and σ = 0.0, 0.1, 1.0, and 10.0, respectively. Second column: first-derivative images and gray-level profiles. Third column: second-derivative images and gray-level profiles.

Gonzalez & Woods, DIP 2/e

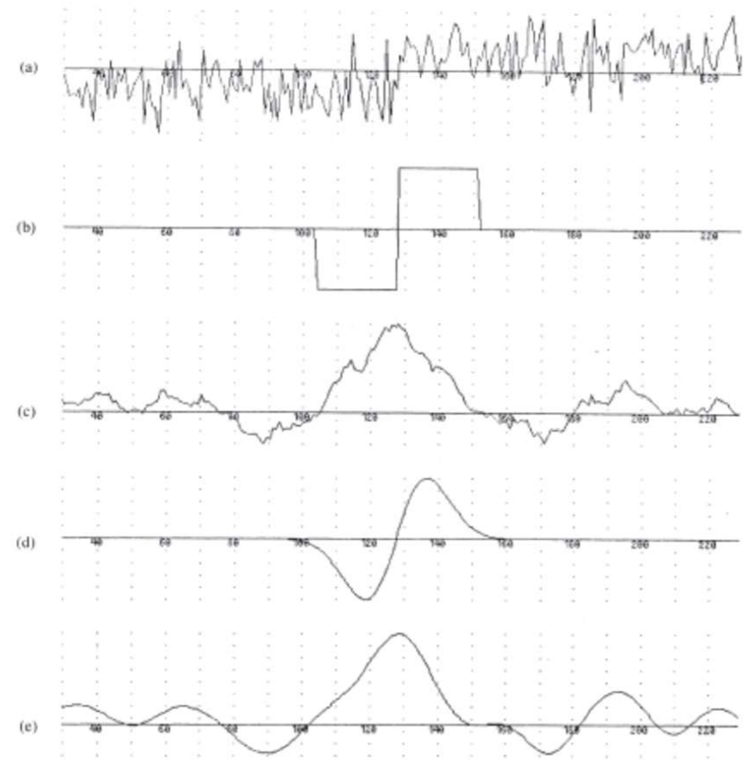


Fig. 1. (a) A noisy step edge. (b) Difference of boxes operator. (c) Difference of boxes operator applied to the edge. (d) First derivative of Gaussian operator. (e) First derivative of Gaussian applied to the edge.

Canny, J., *A Computational Approach To Edge Detection*, IEEE Trans. Pattern Analysis and Machine Intelligence, 8(6):679–698, 1986.

一个低通滤波器的例子：均值滤波

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

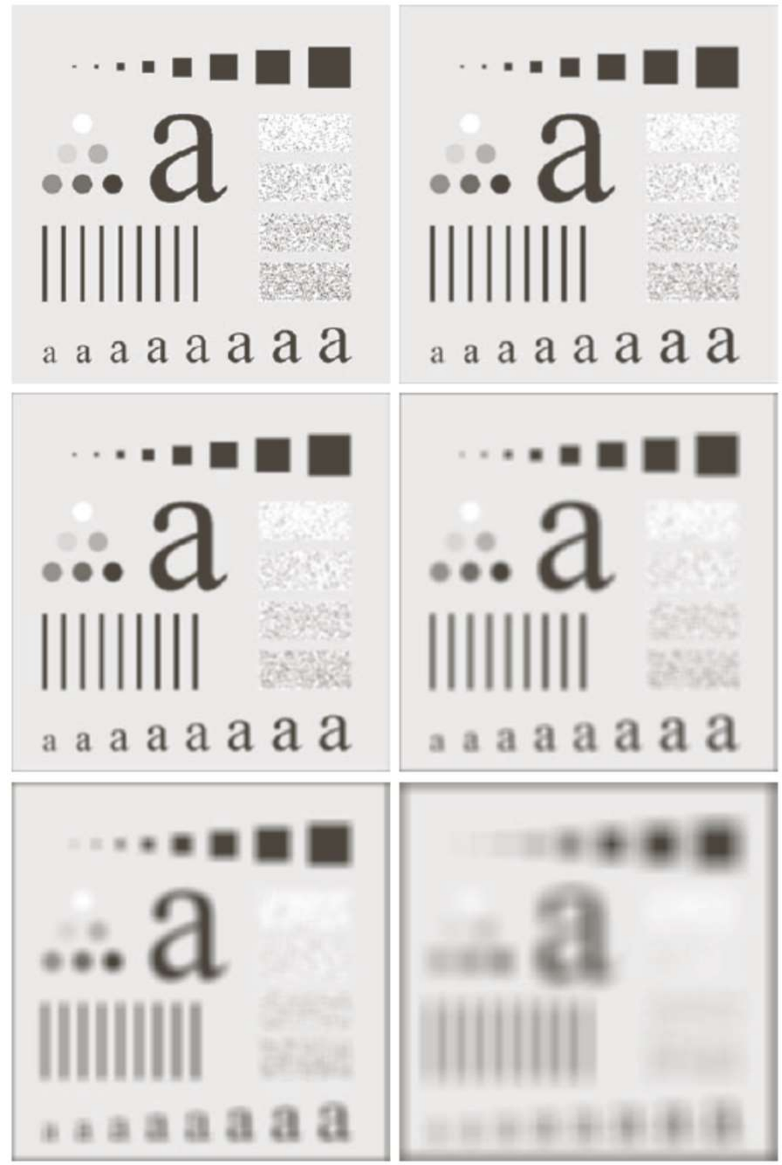
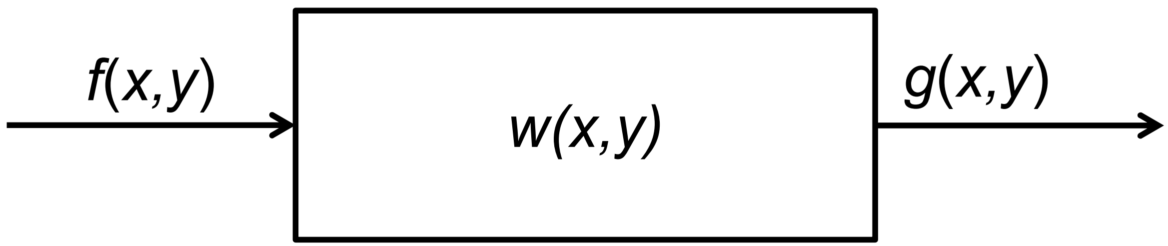


FIGURE 3.33 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

a b
c d
e f

图像滤波的数学模型

- 一个图像滤波器可以描述为一个线性系统



$$g(x,y)=w(x,y)\otimes f(x,y)$$

$$G(u,v)=W(u,v)\cdot F(u,v)$$

图像频域特性的基本概念

- 简单而言，图像细节对应于高频信号，图像的大尺度特征对应于低频信号。
- 图像噪声通常对应于高频信号。

均值滤波器的频率域模型 (I)

- 均值滤波器的频率域形式

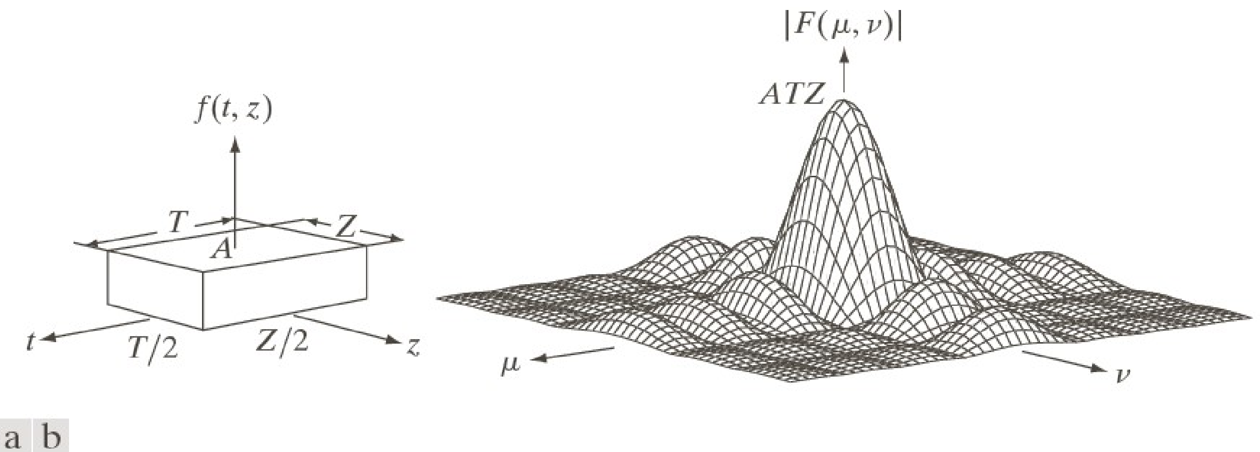
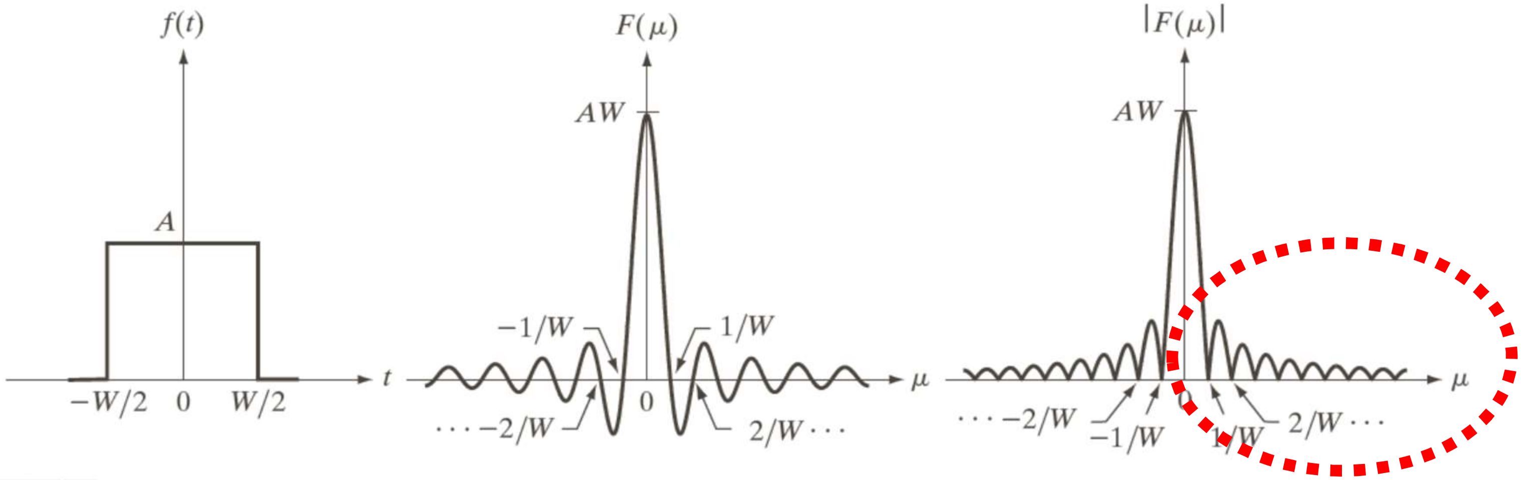


FIGURE 4.13 (a) A 2-D function, and (b) a section of its spectrum (not to scale). The block is longer along the t-axis, so the spectrum is more "contracted" along the μ-axis. Compare with Fig. 4.4.

$$|F(\mu,v)| = A \cdot T \cdot Z \left|\frac{\sin(\pi\mu T)}{\pi\mu T}\right| \left|\frac{\sin(\pi v Z)}{\pi v Z}\right|$$

均值滤波器的频率域模型 (II)



a b c

FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

高斯滤波器 (I)

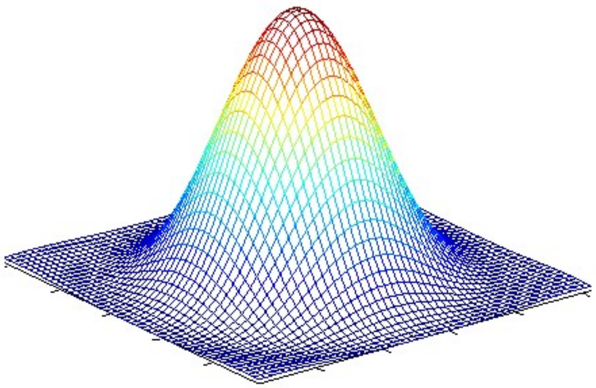
- 一维和二维高斯滤波核

$$G(x;\sigma)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}$$

$$G(x,y;\sigma_x,\sigma_y)=\frac{1}{2\pi\sigma_x\sigma_y}e^{-\left(\frac{x^2}{2\sigma_x{}^2}+\frac{y^2}{2\sigma_y{}^2}\right)}$$

- 一阶导数

$$G'(x;\sigma)=\frac{-x}{\sqrt{2\pi}\sigma^3}e^{-\frac{x^2}{2\sigma^2}}$$

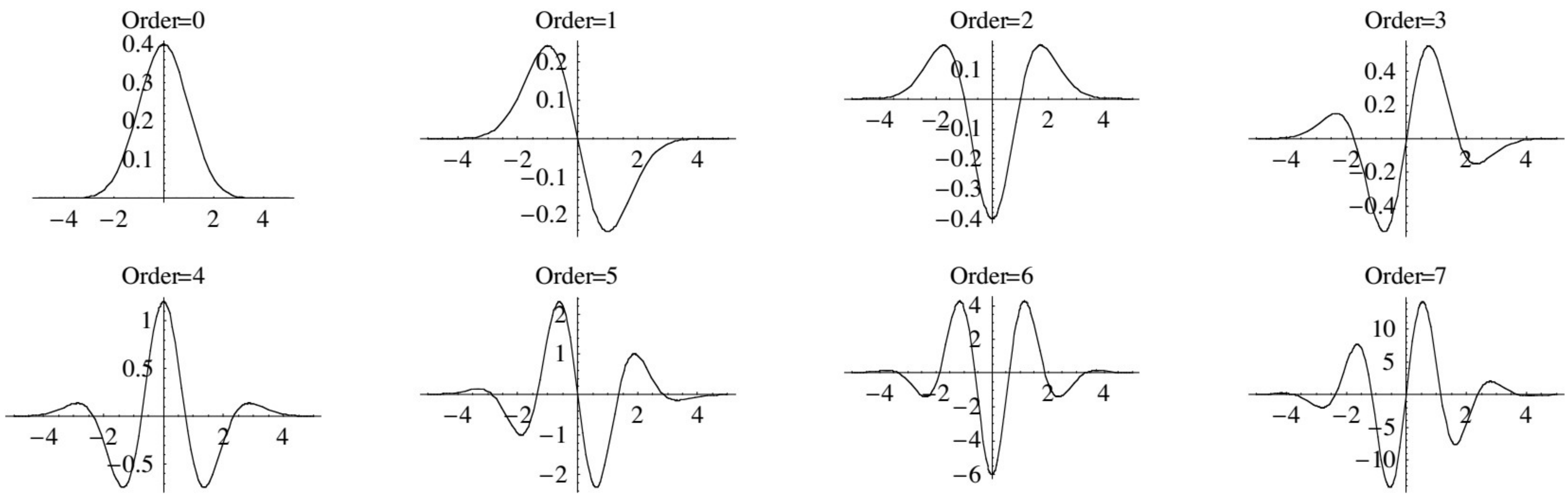


- 二阶导数

$$G''(x;\sigma)=\frac{-1}{\sqrt{2\pi}\sigma^3}e^{-\frac{x^2}{2\sigma^2}}\left[1-\frac{x^2}{\sigma^2}\right]$$

高斯滤波器 (II)

$$G(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}} \qquad G'(x)=\frac{-x}{\sqrt{2\pi}\sigma^3}e^{-\frac{x^2}{2\sigma^2}} \qquad G''(x)=\frac{-1}{\sqrt{2\pi}\sigma^3}e^{-\frac{x^2}{2\sigma^2}}\left[1-\frac{x^2}{\sigma^2}\right]$$



http://bmia.bmt.tue.nl/Education/Courses/FEV/book/pdf/04%20Gaussian%20derivatives.pdf

高斯滤波器 (III)

- 高斯滤波器的基本特性
  - 它是一个低通滤波器

$$\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}} \xrightarrow{F} \frac{e^{-\frac{\sigma^2\omega^2}{2}}}{\sqrt{2\pi}}$$

  - 它是一个可分式滤波器

$$G(x,y;\sigma_x,\sigma_y)=\frac{1}{2\pi\sigma_x\sigma_y}e^{-\left(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)}=\frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{x^2}{2\sigma_x^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma_y}e^{-\frac{y^2}{2\sigma_y^2}}$$

高斯滤波器 (IV)

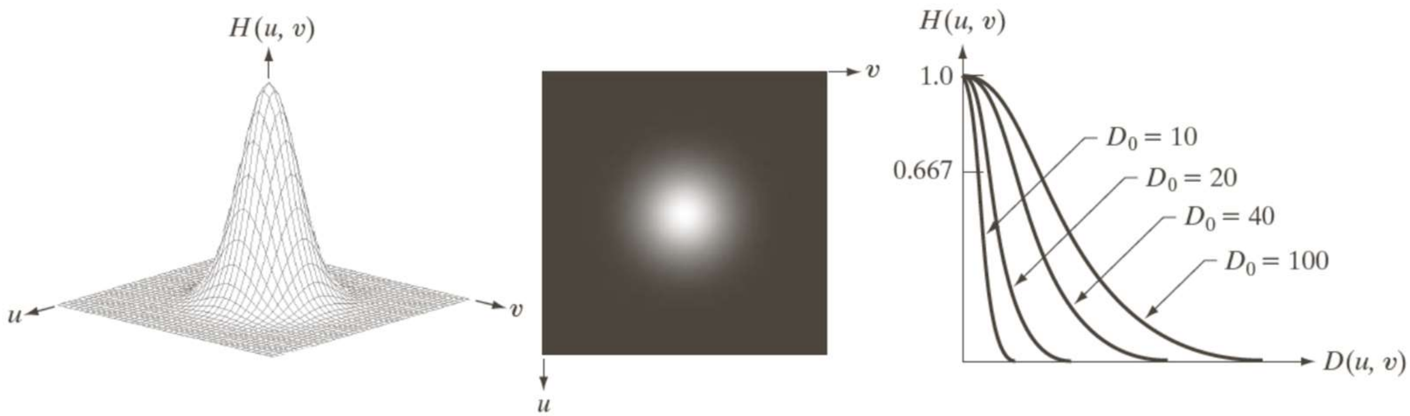


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

$$D_0 = \frac{1}{\sigma}$$

使用高斯滤波核下噪声压制与导数计算的简化

- Implementation

$$F_x(i,j) = \frac{F(i+1,j) - F(i-1,j)}{2}$$

$$F_y(i,j) = \frac{F(i,j+1) - F(i,j-1)}{2}$$

- Notation:

  I: raw image;

  J: filtered image after convolution with Gaussian kernel G.

- A basic property of convolution

$$\frac{\partial (G \otimes I)}{\partial x} = \frac{\partial J}{\partial x} = J_x = \frac{\partial G}{\partial x} \otimes I \qquad \frac{\partial (G \otimes I)}{\partial y} = \frac{\partial J}{\partial y} = J_y = \frac{\partial G}{\partial y} \otimes I$$

高斯滤波器与图像空间尺度

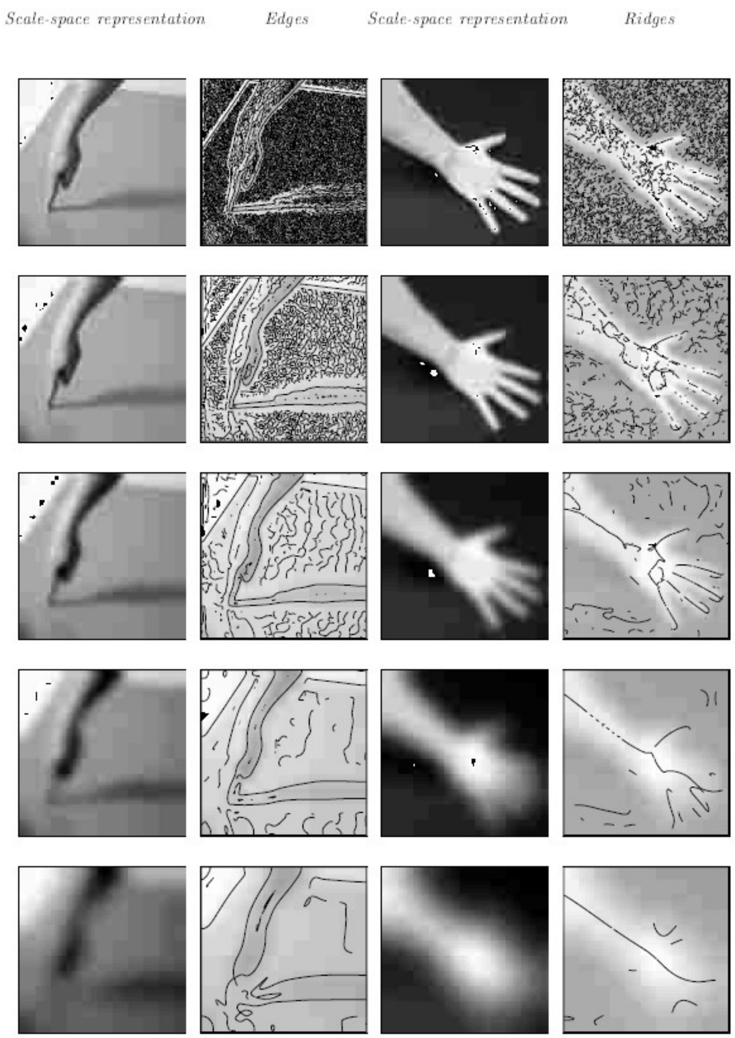


*Figure 1.4: Edges and ridges computed at different scales in scale-space (scale levels $t = 1.0$, 4.0, 16.0, 64.0 and 256.0 from top to bottom) using a differential geometric edge detector and ridge detector, respectively. (Image size: 256*256 pixels.)*

Lindeberg (1999) [`` Principles for automatic scale selection'',] in: B. Jahne (et al., eds.), Handbook on Computer Vision and Applications, volume 2, pp 239--274, Academic Press.

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傅里叶级数与傅里叶变换 (I)

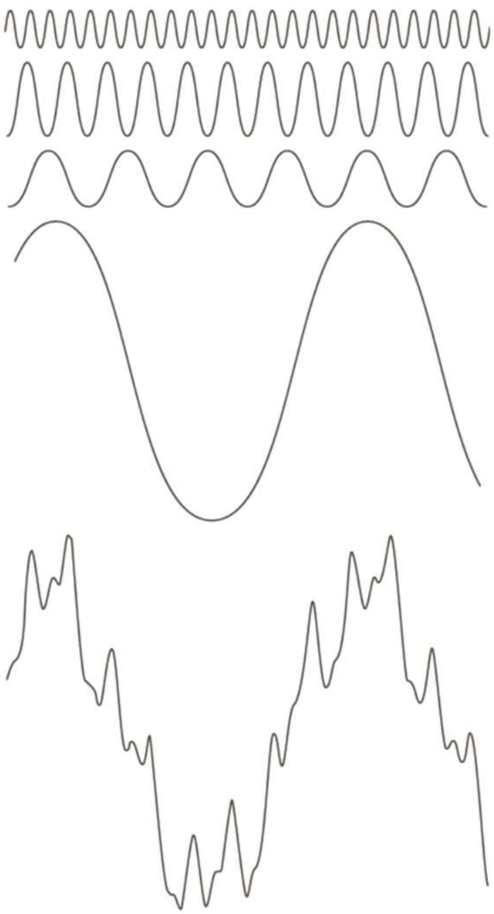


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

$$f(t) = f(t+T)$$

傅里叶级数

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{T}t}$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-j\frac{2\pi n}{T}t}\, dt$$

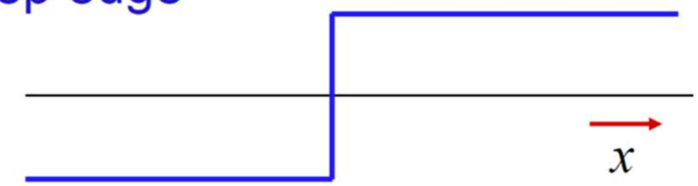
for $n = 0, \pm 1, \pm 2, \ldots$

傅里叶级数展开 (II)

Reminder: 1D Fourier Series

Spatial frequency analysis of a step edge

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{otherwise} \end{cases}$$



Fourier decomposition

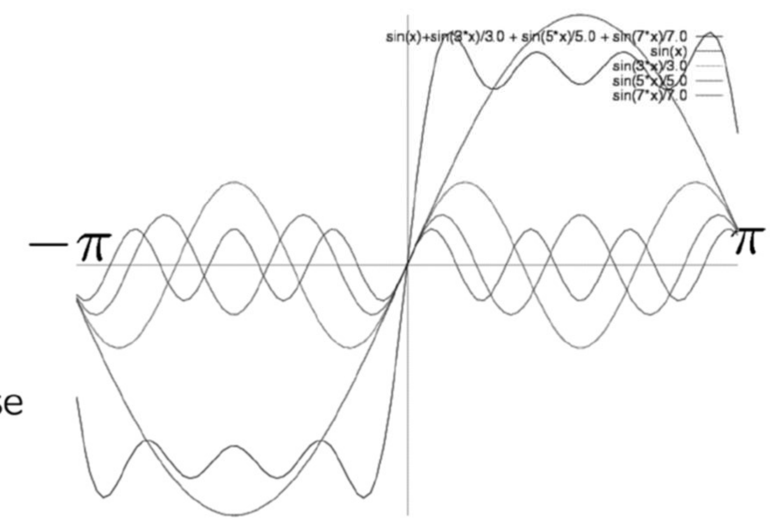
Fourier Series

$$f(x) = \sum_n a_n \sin nx$$

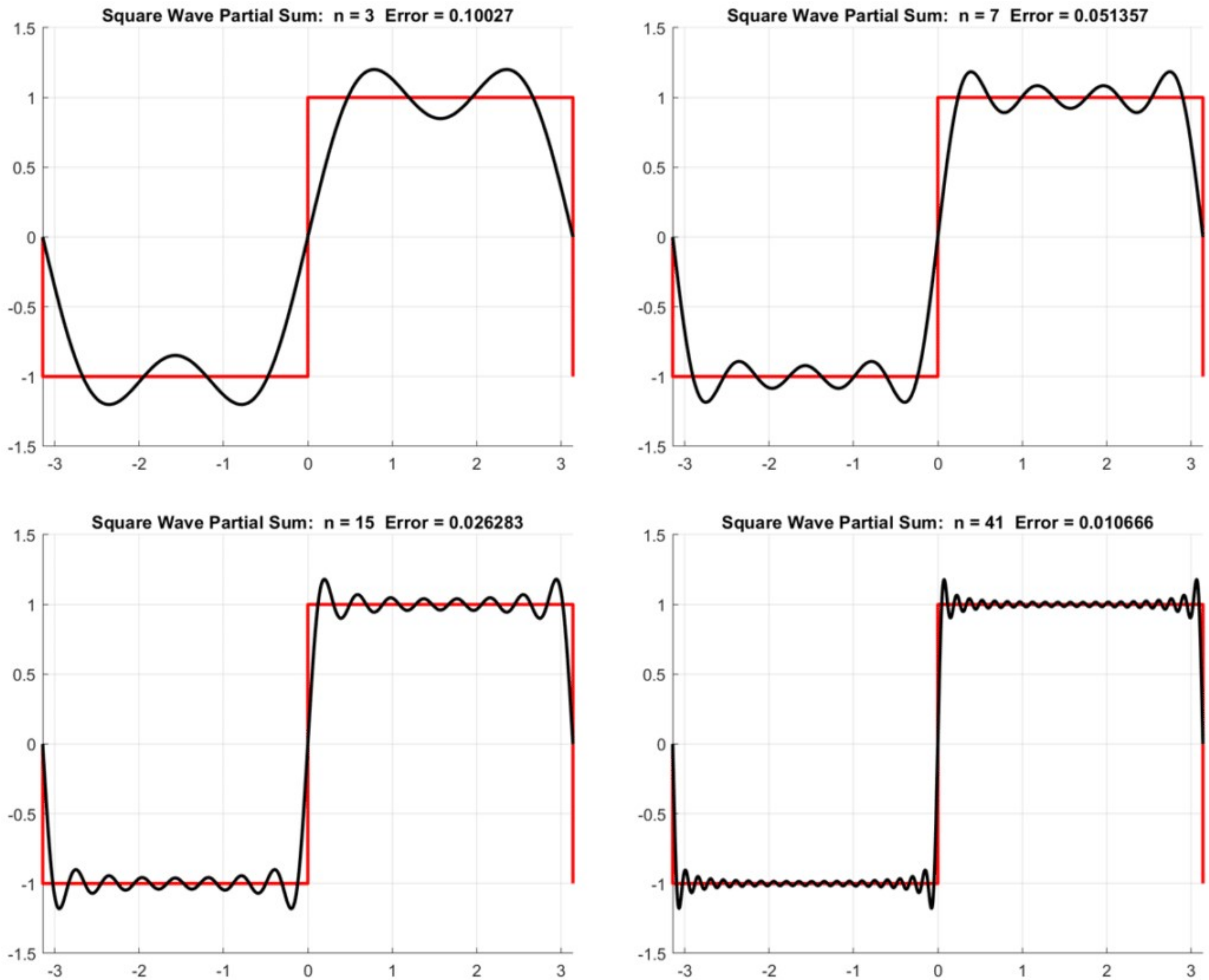
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} \sin nx \, dx = \begin{cases} \frac{4}{n\pi} & \text{if n odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f(x) = \sum_{n=1} \frac{4}{(2n-1)\pi} \sin(2n-1)x$$



傅里叶级数展开的Gibbs现象



傅里叶级数与傅里叶变换 (II)

$$\|f(t)\|_1 \triangleq \int_{-\infty}^{+\infty} |f(t)| dt < \infty$$

傅里叶变换

$$F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t} dt$$

$$f(t) = \int_{-\infty}^{\infty} F(\mu)\, e^{j2\pi\mu t} d\mu$$

单位脉冲函数的定义

$$\delta(t) = \begin{cases} \infty & if \;\; t = 0 \\ 0 & if \;\; t \neq 0 \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$

脉冲信号的筛选特性

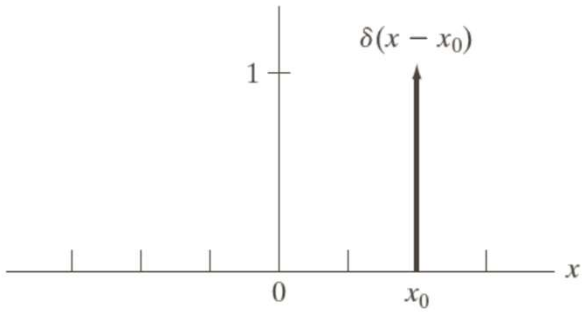


FIGURE 4.2 A unit discrete impulse located at $x = x_0$. Variable x is discrete, and δ is 0 everywhere except at $x = x_0$.

$$\int_{-\infty}^{+\infty} f(t)\delta(t)\,dt = f(0)$$

$$\int_{-\infty}^{+\infty} f(t)\delta(t-t_0)\,dt = f(t_0)$$

$$\sum_{x=-\infty}^{\infty} f(x)\delta(x) = f(0)$$

$$\sum_{x=-\infty}^{\infty} f(x)\delta(x-x_0) = f(x_0)$$

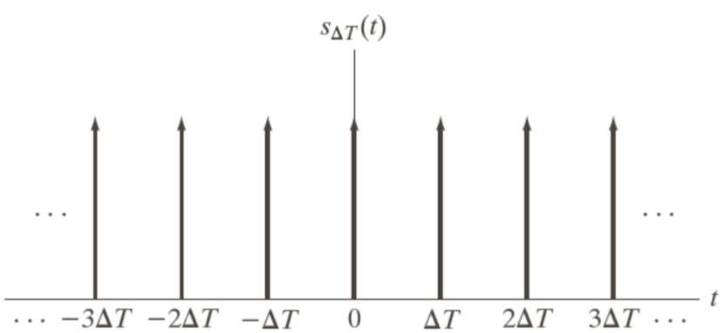


FIGURE 4.3 An impulse train.

脉冲序列

$$s_{\Delta T}(t) = \sum_{k=-\infty}^{\infty} \delta(t - k\Delta T)$$

卷积的一个基本属性

$$(f * h)(t) \Leftrightarrow (H \cdot F)(\mu)$$

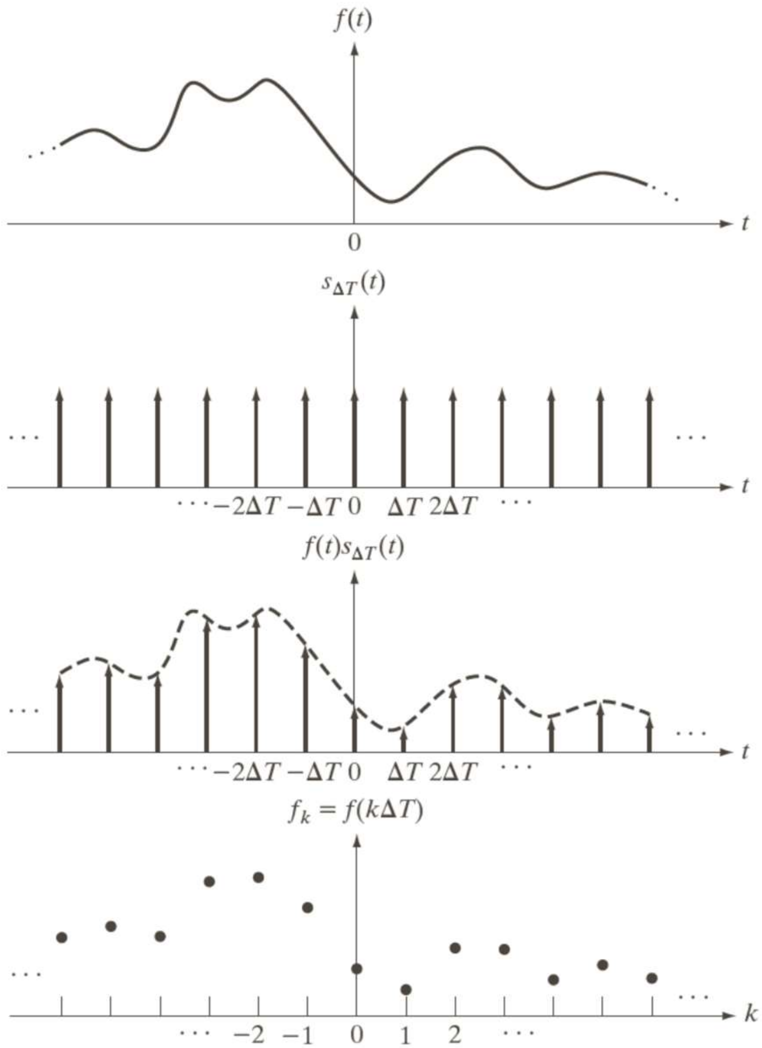
$$(f \cdot h)(t) \Leftrightarrow (H * F)(\mu)$$

脉冲和脉冲序列的傅里叶变换

$$F\left(\delta\left(t-t_0\right)\right)=\int_{-\infty}^{+\infty}\delta\left(t-t_0\right)e^{-j2\pi\mu t}dt=e^{-j2\pi\mu t_0}$$

$$F\left(s_{\Delta T}\left(t\right)\right)=F\left(\sum_{k=-\infty}^{\infty}\delta\left(t-k\Delta T\right)\right)=\frac{1}{\Delta T}\sum_{n=-\infty}^{+\infty}\delta\left(\mu-\frac{n}{\Delta T}\right)$$

采样信号的数学表示及其傅里叶变换 (I)



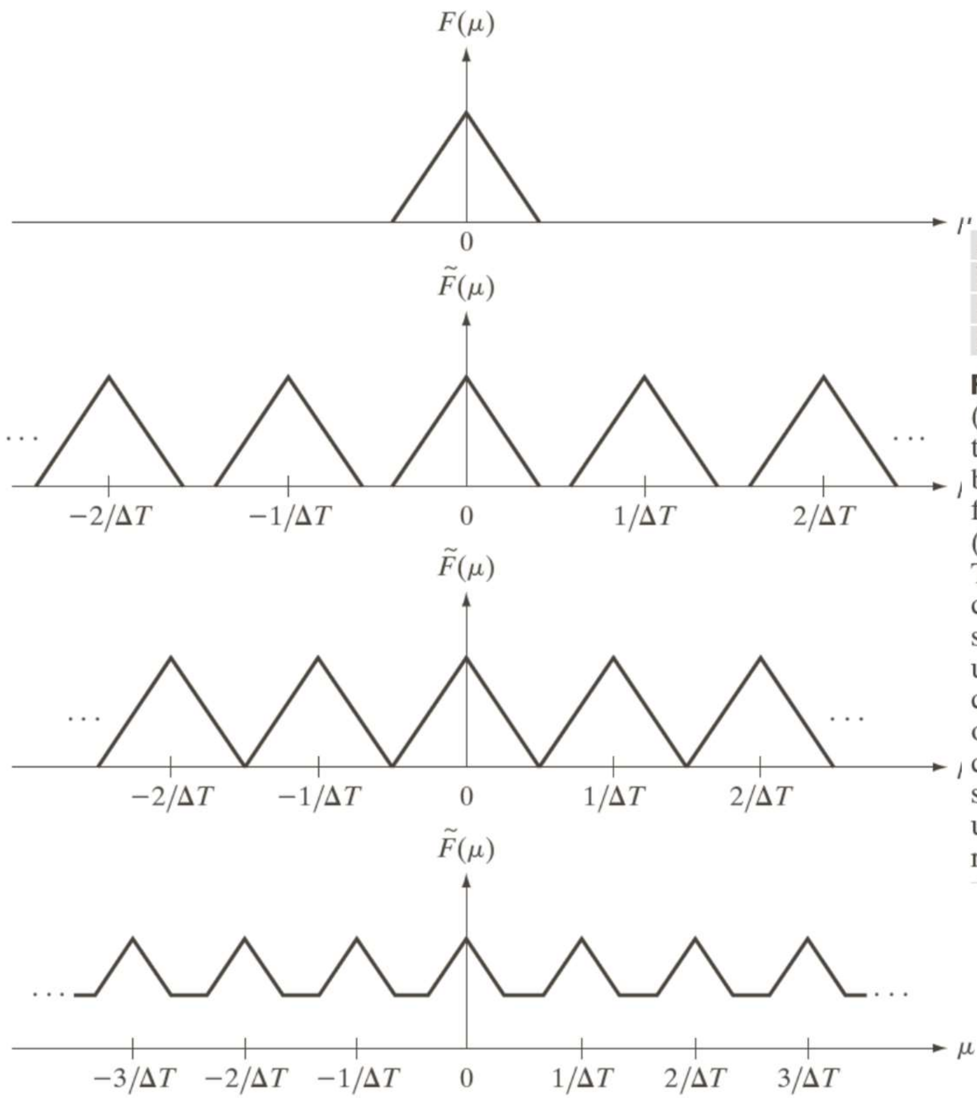
a
b
c
d

FIGURE 4.5
(a) A continuous function. (b) Train of impulses used to model the sampling process. (c) Sampled function formed as the product of (a) and (b). (d) Sample values obtained by integration and using the sifting property of the impulse. (The dashed line in (c) is shown for reference. It is not part of the data.)

$$\tilde{f}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)$$

$$\tilde{F}(\mu) = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} F\left(\mu - \frac{n}{\Delta T}\right)$$

采样定理 (I)



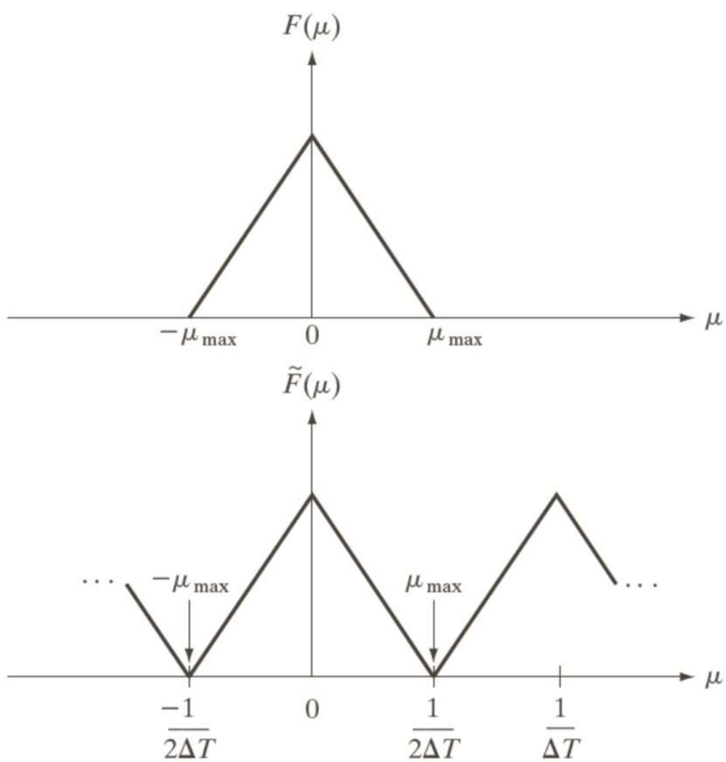
a
b
c
d

FIGURE 4.6 (a) Fourier transform of a band-limited function. (b)–(d) Transforms of the corresponding sampled function under the conditions of over-sampling, critically-sampling, and under-sampling, respectively.

$$\frac{1}{2\Delta T} > \mu_{\max}$$

$$\frac{1}{\Delta T} > 2\mu_{\max}$$

采样定理 (II)



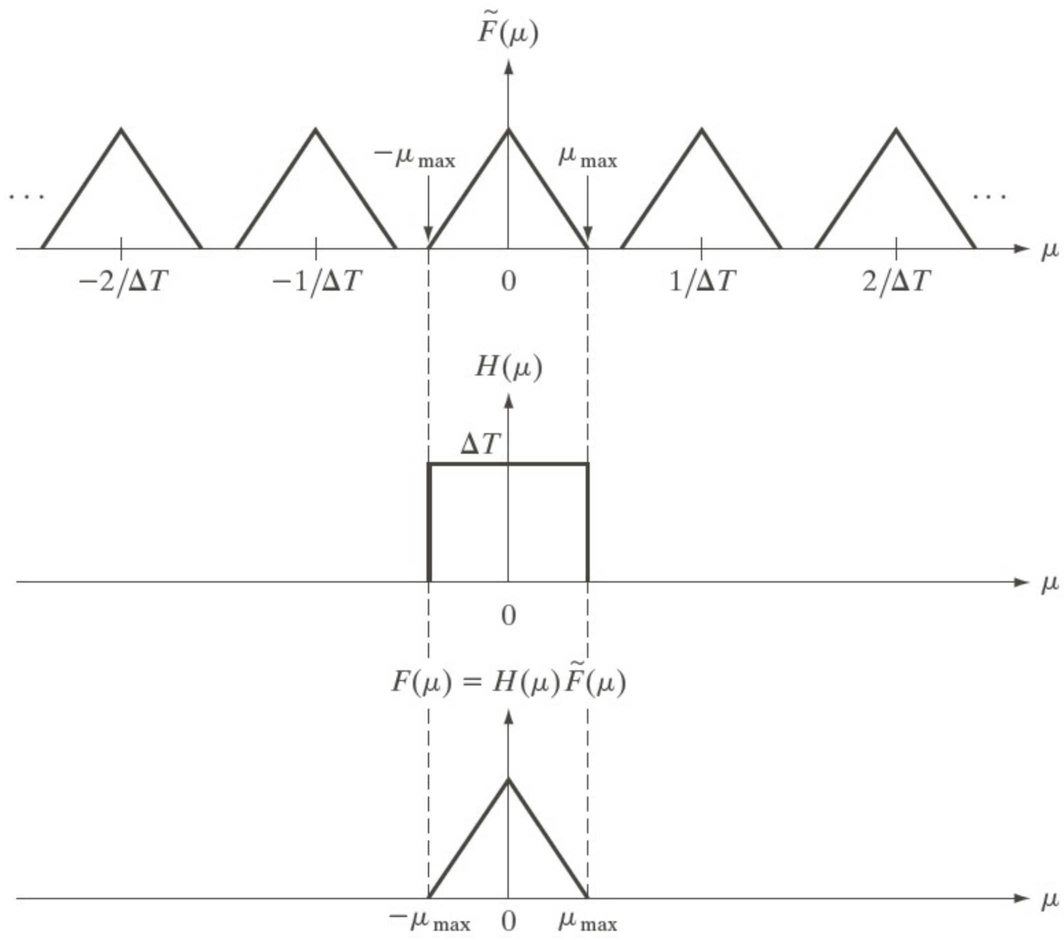
a
b

FIGURE 4.7 (a) Transform of a band-limited function. (b) Transform resulting from critically sampling the same function.

$$\frac{1}{\Delta T} > 2\mu_{\max}$$

如果采样频率超过了其带宽极限频率的两倍，一个连续和有限带宽的信号可以从其采样信号中完全回复

采样定理 (III)



a
b
c

FIGURE 4.8 Extracting one period of the transform of a band-limited function using an ideal lowpass filter.

采样定理 (IV)

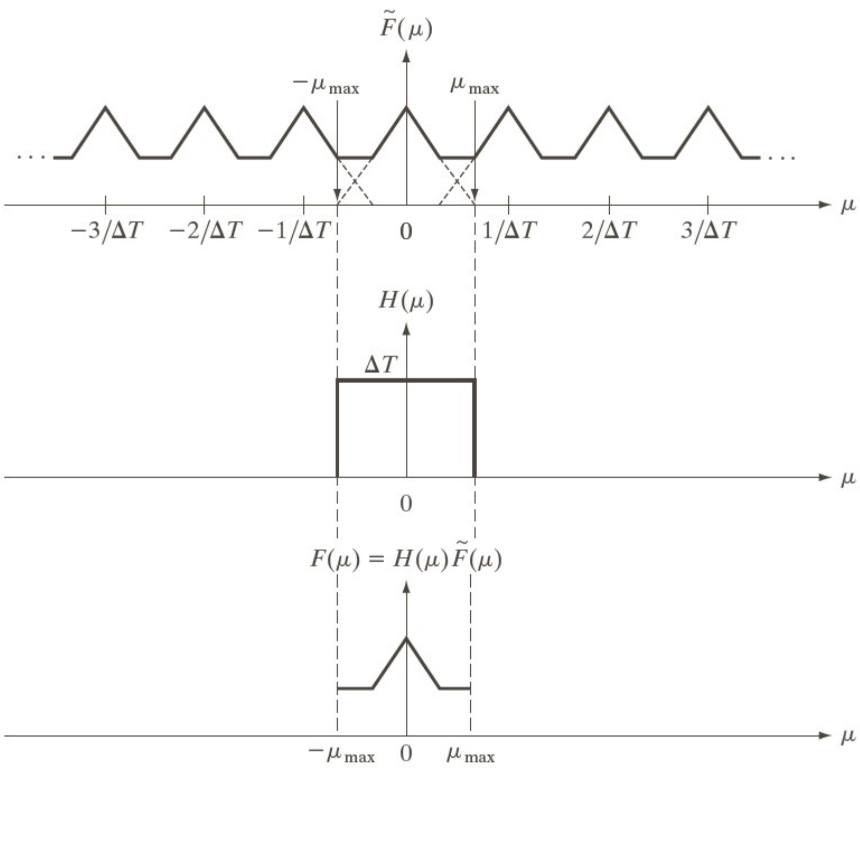


FIGURE 4.9 (a) Fourier transform of an under-sampled, band-limited function. (Interference from adjacent periods is shown dashed in this figure). (b) The same ideal lowpass filter used in Fig. 4.8(b). (c) The product of (a) and (b). The interference from adjacent periods results in aliasing that prevents perfect recovery of $F(\mu)$ and, therefore, of the original, band-limited continuous function. Compare with Fig. 4.8.

信号采样中的混淆现象

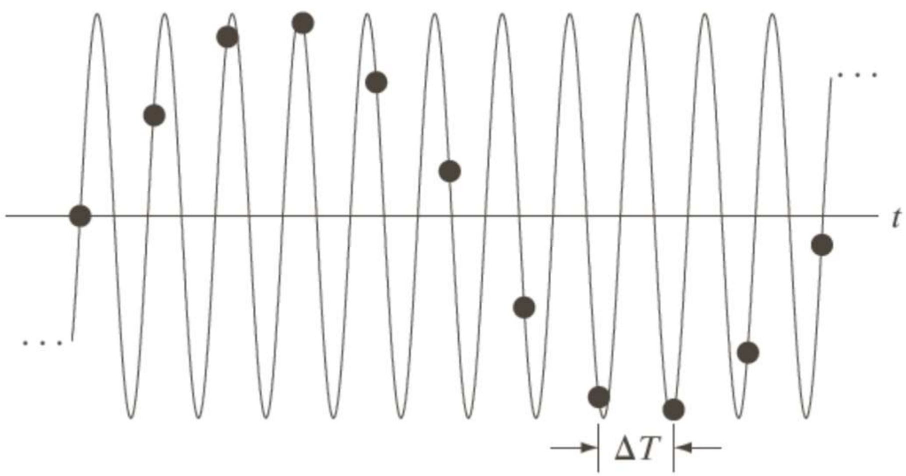


FIGURE 4.10 Illustration of aliasing. The under-sampled function (black dots) looks like a sine wave having a frequency much lower than the frequency of the continuous signal. The period of the sine wave is 2 s, so the zero crossings of the horizontal axis occur every second. ΔT is the separation between samples.

图像采样中的二维采样脉冲序列

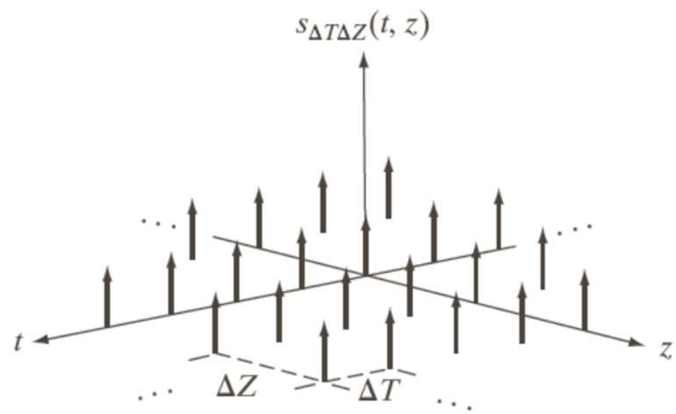
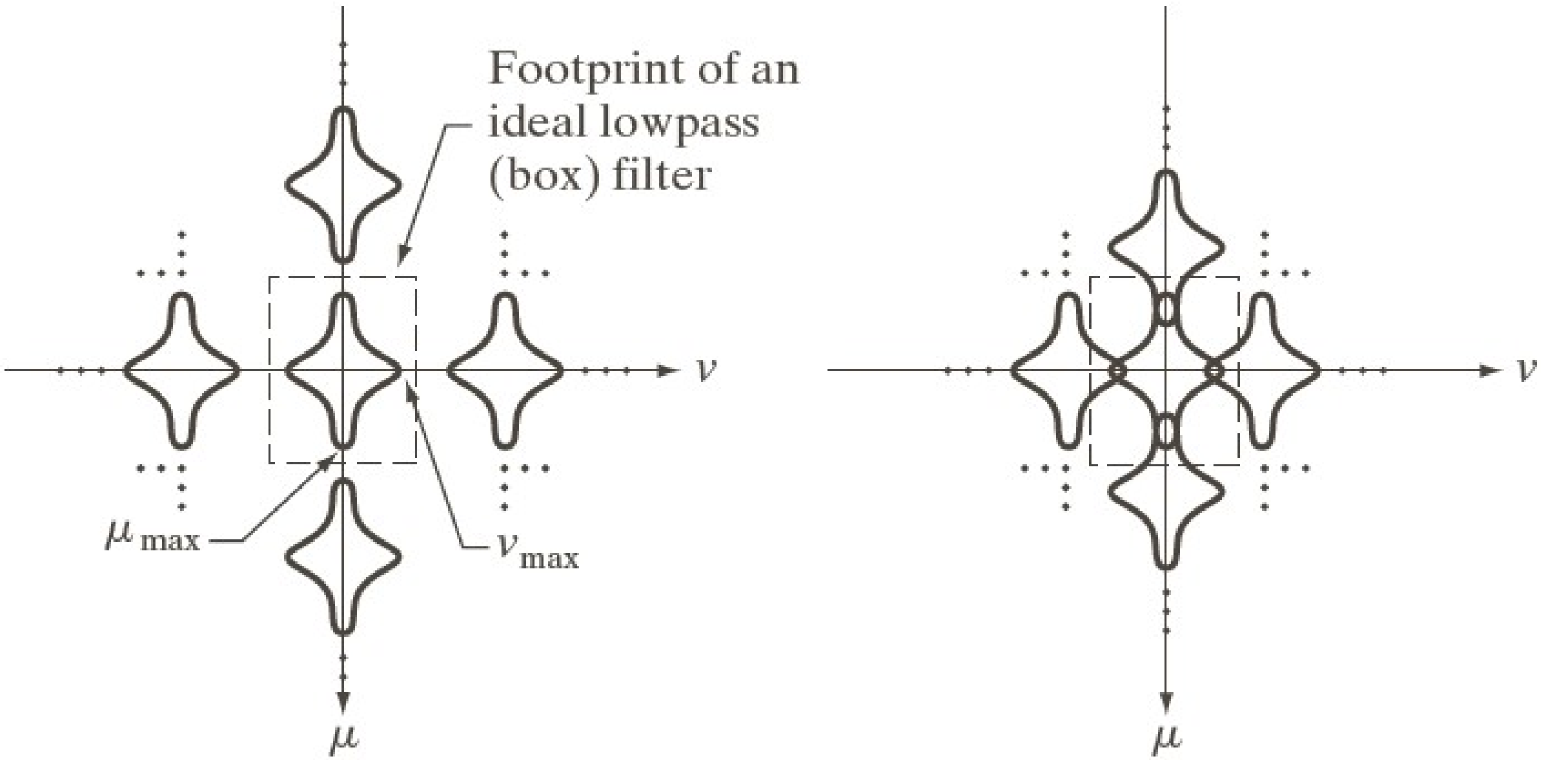


FIGURE 4.14
Two-dimensional impulse train.

图像二维采样中的混淆现象



a b

FIGURE 4.15 Two-dimensional Fourier transforms of (a) an over-sampled, and (b) under-sampled band-limited function.

图像采样中的混淆现象

a b c

FIGURE 4.17 Illustration of aliasing on resampled images. (a) A digital image with negligible visual aliasing. (b) Result of resizing the image to 50% of its original size by pixel deletion. Aliasing is clearly visible. (c) Result of blurring the image in (a) with a 3×3 averaging filter prior to resizing. The image is slightly more blurred than (b), but aliasing is not longer objectionable. (Original image courtesy of the Signal Compression Laboratory, University of California, Santa Barbara.)

- 点扩散函数的基本概念
- 图像滤波和高斯滤波器
- 图像采样定理
- **图像的二维离散傅里叶变换**
- 图像的频率域滤波
- 图像的正交变换
- 卷积神经网络的基本结构

二维连续与离散傅里叶变换

二维连续傅里叶变换

$$F(\mu,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)\, e^{-j2\pi(\mu t+v\ )} dtdz$$

$$f(t,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\mu,v)\, e^{j2\pi(\mu t+v\ )} d\mu dv$$

二维离散傅里叶变换

$$F(\mu,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\ \ (\mu x/M+vy/N)}$$

$$f(x,y) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(\mu,v)\, e^{j2\pi(\mu x/M+vy/N)}$$

二维离散傅里叶变换的概念与基本特性 (I)

Name	Expression(s)
1) Discrete Fourier transform (DFT) of $f(x, y)$	$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$
2) Inverse discrete Fourier transform (IDFT) of $F(u, v)$	$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}$
3) Polar representation	$F(u, v) = \lvert F(u, v)\rvert e^{j\phi(u,v)}$
4) Spectrum	$\lvert F(u, v)\rvert = \left[R^2(u, v) + I^2(u, v)\right]^{1/2}$ $R = \text{Real}(F); \quad I = \text{Imag}(F)$
5) Phase angle	$\phi(u, v) = \tan^{-1}\left[\frac{I(u, v)}{R(u, v)}\right]$
6) Power spectrum	$P(u, v) = \lvert F(u, v)\rvert^2$
7) Average value	$\overline{f}(x, y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) = \frac{1}{MN} F(0, 0)$

(*Continued*)

TABLE 4.2
Summary of DFT definitions and corresponding expressions.

二维离散傅里叶变换的概念与基本特性 (II)

Name	Expression(s)
8) Periodicity (k_1 and k_2 are integers)	$F(u,v) = F(u+k_1M, v) = F(u, v+k_2N) = F(u+k_1M, v+k_2N)$ $f(x,y) = f(x+k_1M, y) = f(x, y+k_2N) = f(x+k_1M, y+k_2N)$
9) Convolution	$f(x,y) \bigstar h(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)h(x-m, y-n)$
10) Correlation	$f(x,y) \star h(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f^*(m,n)h(x+m, y+n)$
11) Separability	The 2-D DFT can be computed by computing 1-D DFT transforms along the rows (columns) of the image, followed by 1-D transforms along the columns (rows) of the result. See Section 4.11.1.
12) Obtaining the inverse Fourier transform using a forward transform algorithm.	$MNf^*(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F^*(u,v)e^{-j2\pi(ux/M+vy/N)}$ This equation indicates that inputting $F^*(u,v)$ into an algorithm that computes the forward transform (right side of above equation) yields $MNf^*(x,y)$. Taking the complex conjugate and dividing by MN gives the desired inverse. See Section 4.11.2.

TABLE 4.2
(Continued)

二维离散傅里叶变换的概念与基本特性 (III)

Name	DFT Pairs
1) Symmetry properties	See Table 4.1
2) Linearity	$af_1(x, y) + bf_2(x, y) \Leftrightarrow aF_1(u, v) + bF_2(u, v)$
3) Translation (general)	$f(x, y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$ $f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(ux_0/M + vy_0/N)}$
4) Translation to center of the frequency rectangle, $(M/2, N/2)$	$f(x, y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$ $f(x - M/2, y - N/2) \Leftrightarrow F(u, v)(-1)^{u+v}$
5) Rotation	$f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$ $x = r\cos\theta \quad y = r\sin\theta \quad u = \omega\cos\varphi \quad v = \omega\sin\varphi$
6) Convolution theorem†	$f(x, y) \star h(x, y) \Leftrightarrow F(u, v)H(u, v)$ $f(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$

(Continued)

TABLE 4.3 Summary of DFT pairs. The closed-form expressions in 12 and 13 are valid only for continuous variables. They can be used with discrete variables by sampling the closed-form, continuous expressions.

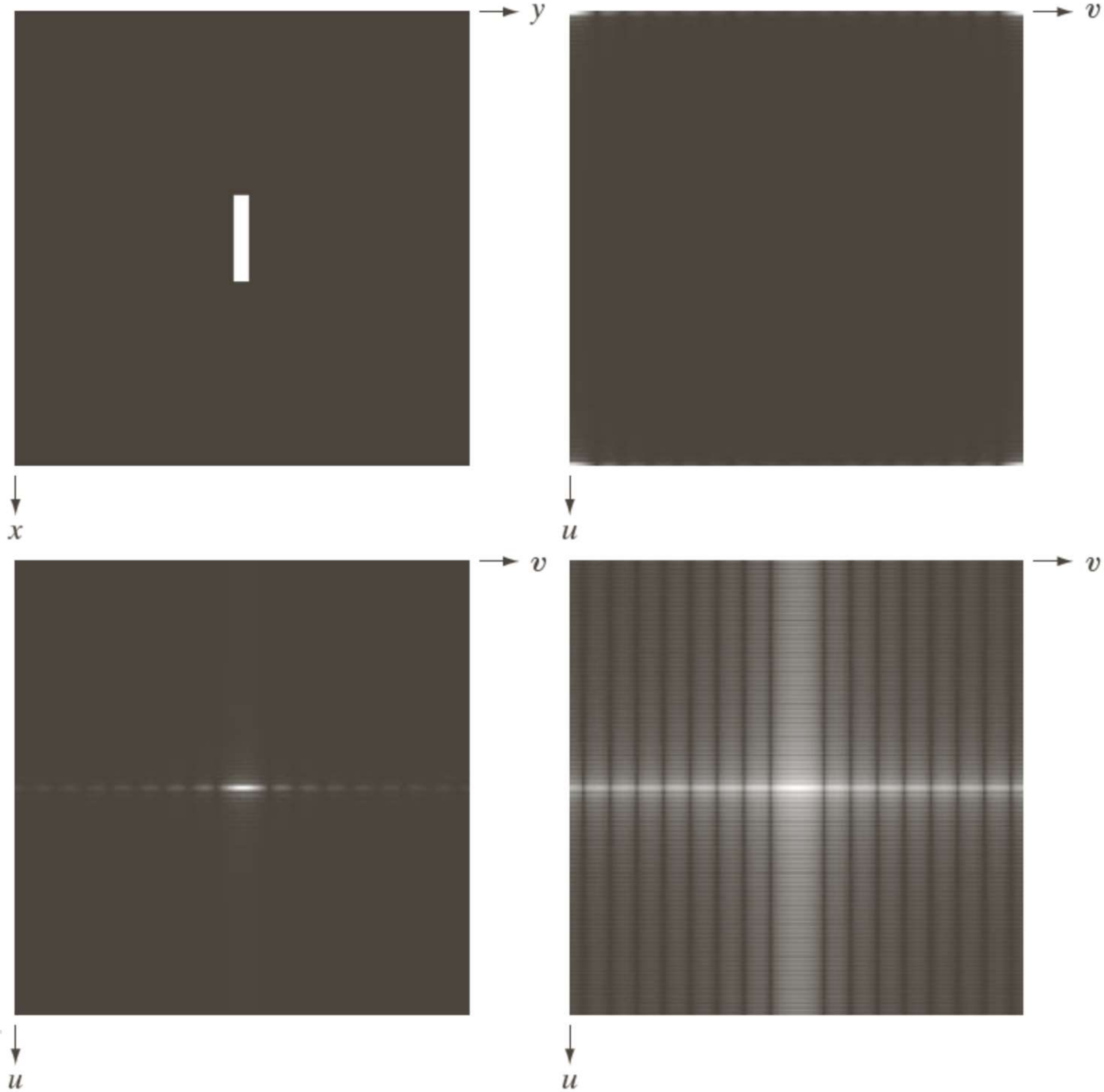
二维离散傅里叶变换的概念与基本特性 (IV)

TABLE 4.3
(*Continued*)

Name	DFT Pairs
7) Correlation theorem†	$f(x,y) \star h(x,y) \Leftrightarrow F^*(u,v)\,H(u,v)$ $f^*(x,y)h(x,y) \Leftrightarrow F(u,v) \star H(u,v)$
8) Discrete unit impulse	$\delta(x,y) \Leftrightarrow 1$
9) Rectangle	$\text{rect}[a,b] \Leftrightarrow ab\dfrac{\sin(\pi ua)}{(\pi ua)}\dfrac{\sin(\pi vb)}{(\pi vb)}e^{-j\pi(ua+vb)}$
10) Sine	$\sin(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $j\dfrac{1}{2}\left[\delta(u+Mu_0, v+Nv_0) - \delta(u-Mu_0, v-Nv_0)\right]$
11) Cosine	$\cos(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $\dfrac{1}{2}\left[\delta(u+Mu_0, v+Nv_0) + \delta(u-Mu_0, v-Nv_0)\right]$
The following Fourier transform pairs are derivable only for continuous variables, denoted as before by t and z for spatial variables and by μ and ν for frequency variables. These results can be used for DFT work by sampling the continuous forms.	
12) *Differentiation* (The expressions on the right assume that $f(\pm\infty, \pm\infty) = 0$.)	$\left(\dfrac{\partial}{\partial t}\right)^m\left(\dfrac{\partial}{\partial z}\right)^n f(t,z) \Leftrightarrow (j2\pi\mu)^m(j2\pi\nu)^n F(\mu,\nu)$ $\dfrac{\partial^m f(t,z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m F(\mu,\nu);\ \dfrac{\partial^n f(t,z)}{\partial z^n} \Leftrightarrow (j2\pi\nu)^n F(\mu,\nu)$
13) *Gaussian*	$A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)} \Leftrightarrow Ae^{-(\mu^2+\nu^2)/2\sigma^2}$ (A is a constant)

† Assumes that the functions have been extended by zero padding. Convolution and correlation are associative, commutative, and distributive.

图像2D-DFT示例 (I)



a b
c d

FIGURE 4.24
(a) Image.
(b) Spectrum showing bright spots in the four corners.
(c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.

图像2D-DFT示例 (II)

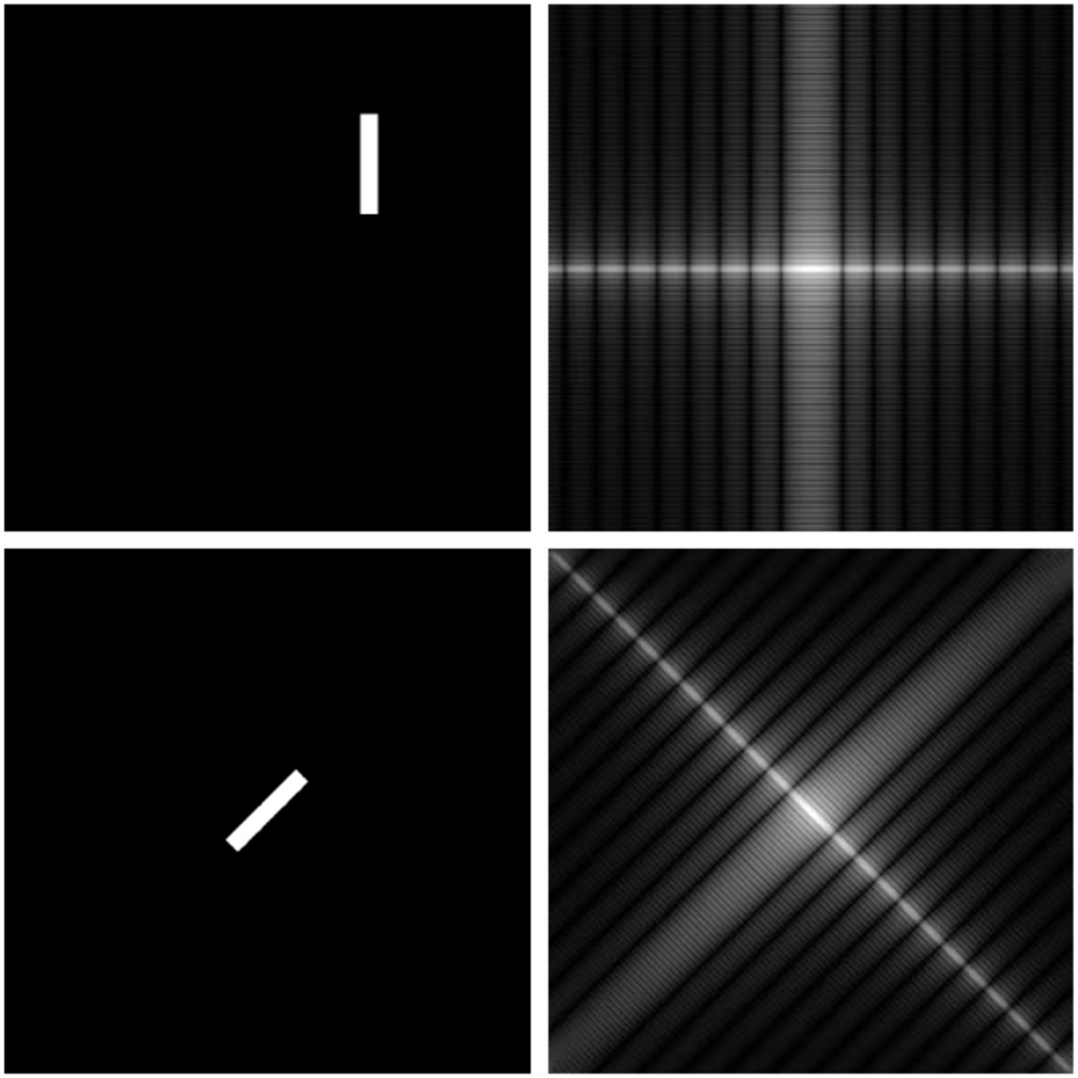


FIGURE 4.25 (a) The rectangle in Fig. 4.24(a) translated, and (b) the corresponding spectrum. (c) Rotated rectangle, and (d) the corresponding spectrum. The spectrum corresponding to the translated rectangle is identical to the spectrum corresponding to the original image in Fig. 4.24(a).

图像2D-DFT示例 (III)

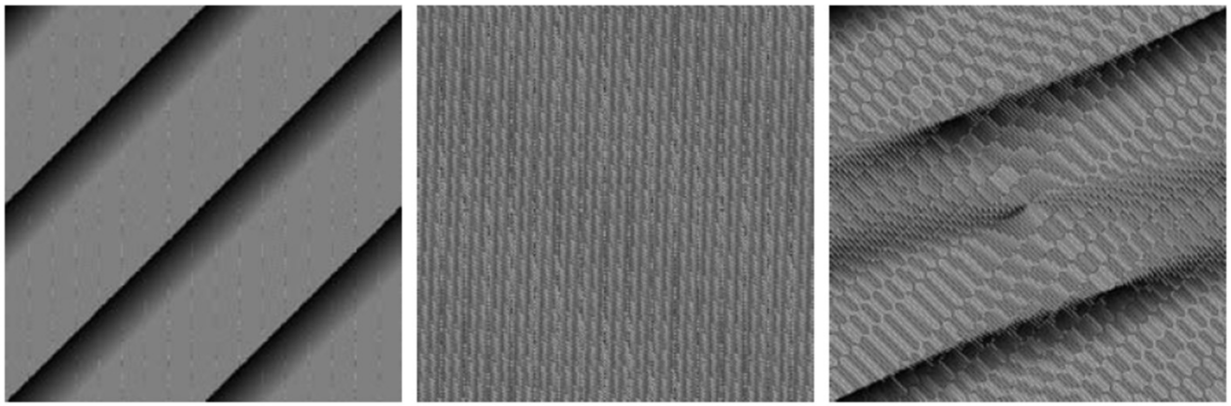


FIGURE 4.26 Phase angle array corresponding (a) to the image of the centered rectangle in Fig. 4.24(a), (b) to the translated image in Fig. 4.25(a), and (c) to the rotated image in Fig. 4.25(c).

- 图像滤波和高斯滤波器
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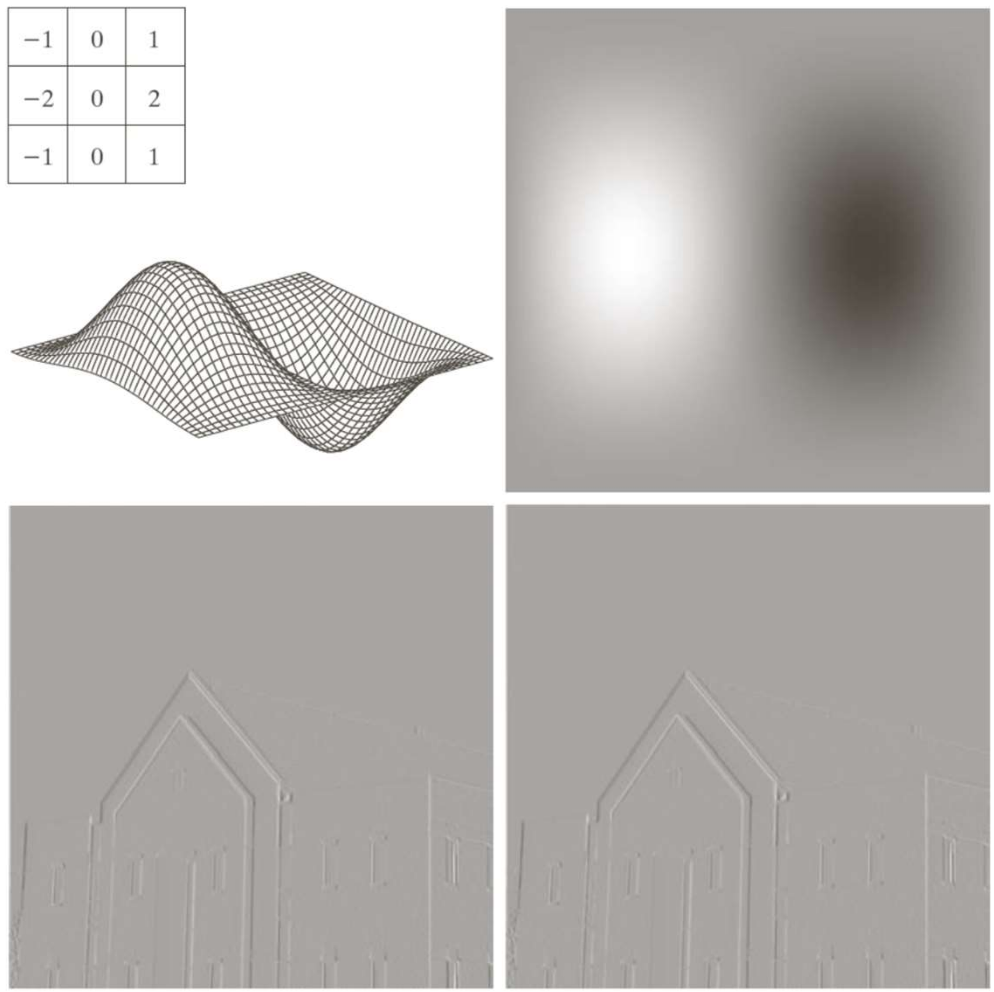
- 点扩散函数的基本概念

图像频率域滤波的基本概念 (I)

a b

FIGURE 4.38
(a) Image of a building, and (b) its spectrum.

图像频率域滤波的基本概念 (II)



a b
c d

FIGURE 4.39 (a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.

图像频率域滤波的基本步骤

- 给定一个输入图像 $f(x,y)$, 计算对应的2D-DFT $F(u,v)$
- 在频率域构建一个滤波器 $H(u,v)$
- 计算 $G(u,v)=H(u,v)F(u,v)$
- 计算$G(u,v)$的2D-IDFT,得到输出图像

代表性滤波器：理想低通滤波器 ILPF

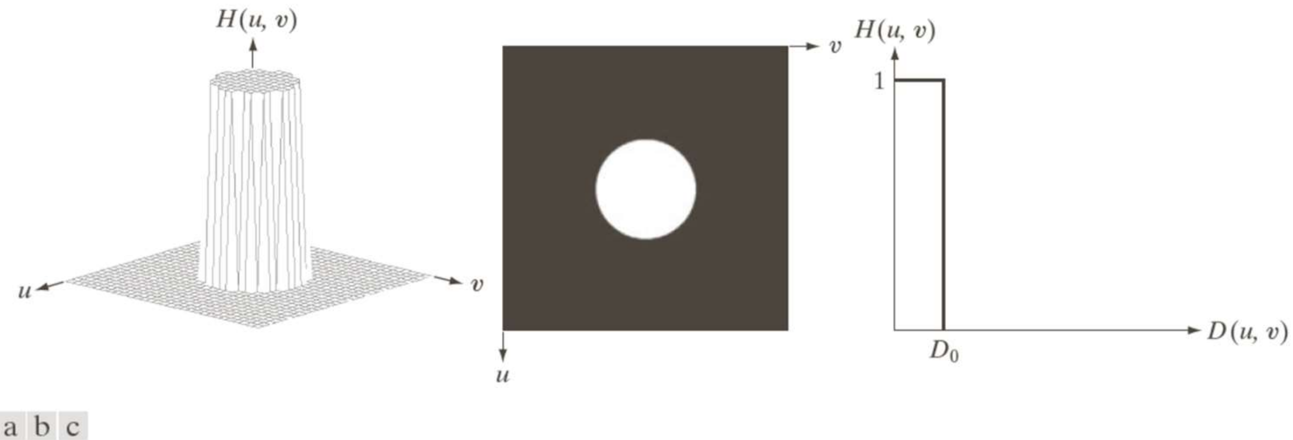
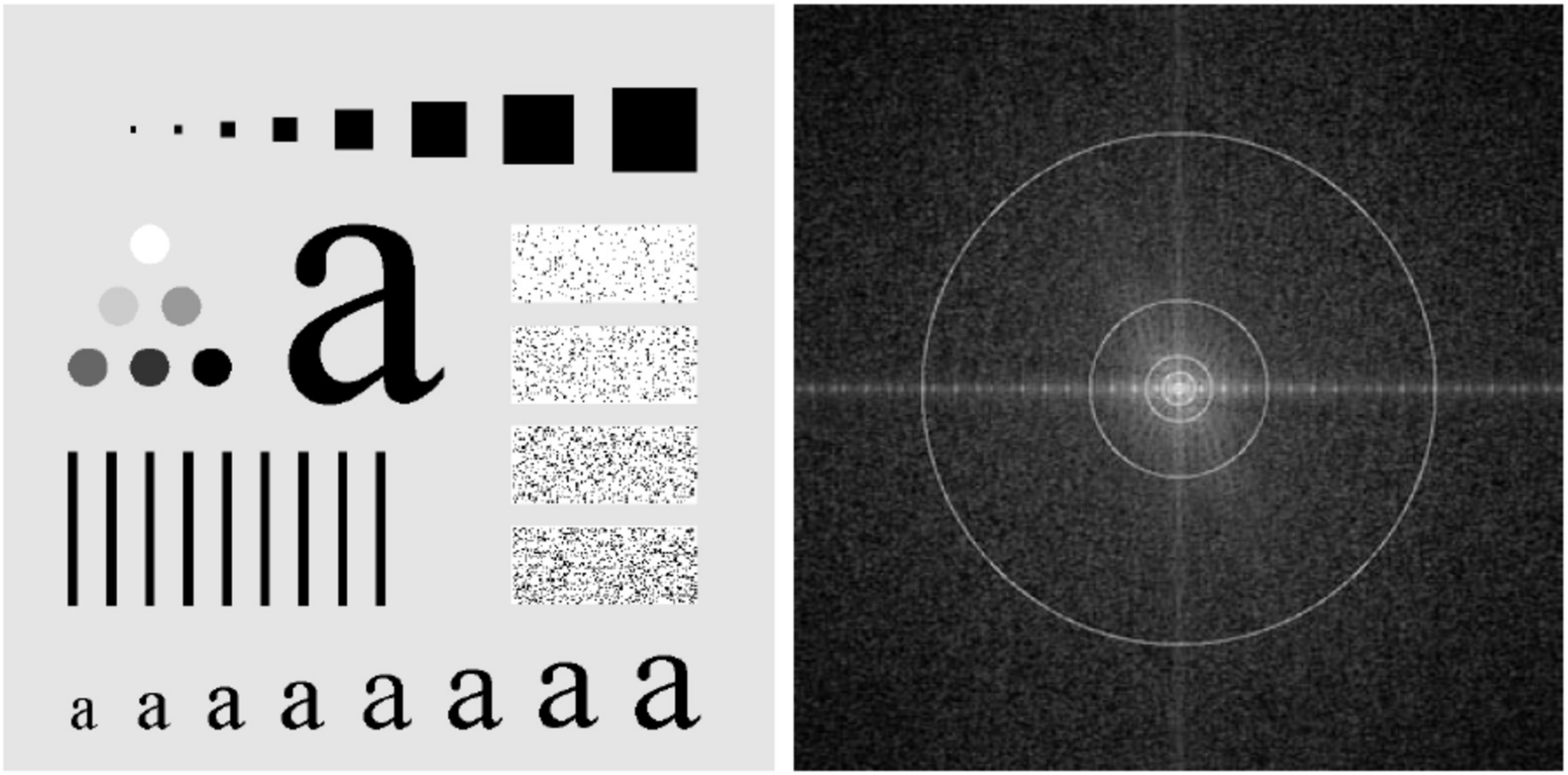


FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

滤波样本图像



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

滤波结果: 理想低通滤波器 ILPF

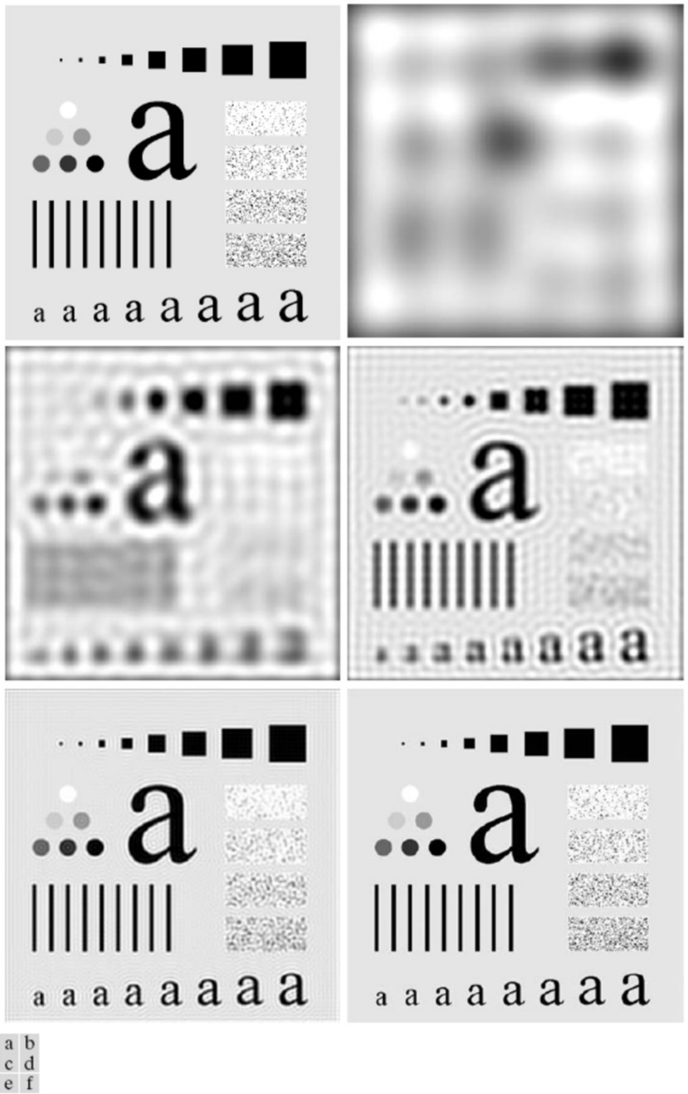
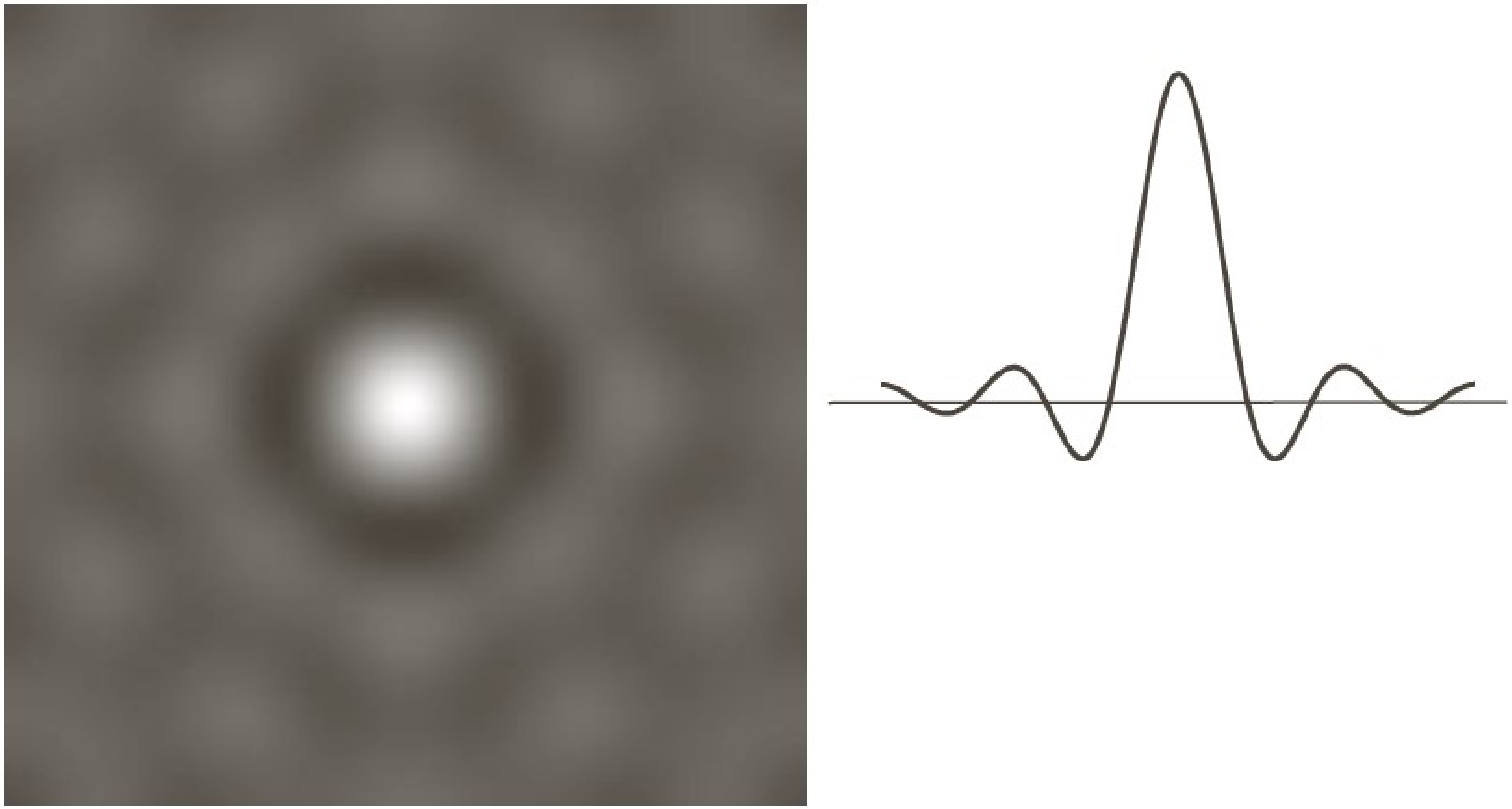


FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

ILPF的空间域特性



a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000×1000. (b) Intensity profile of a horizontal line passing through the center of the image.

代表性滤波器：高斯低通滤波器 GLPF

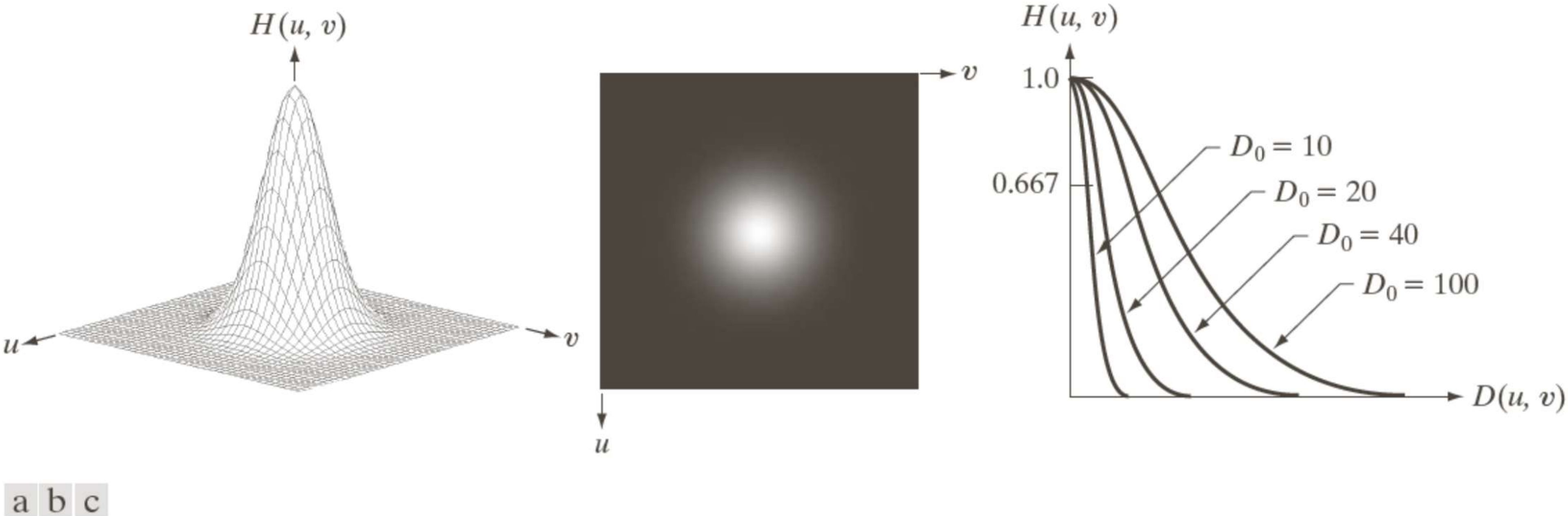
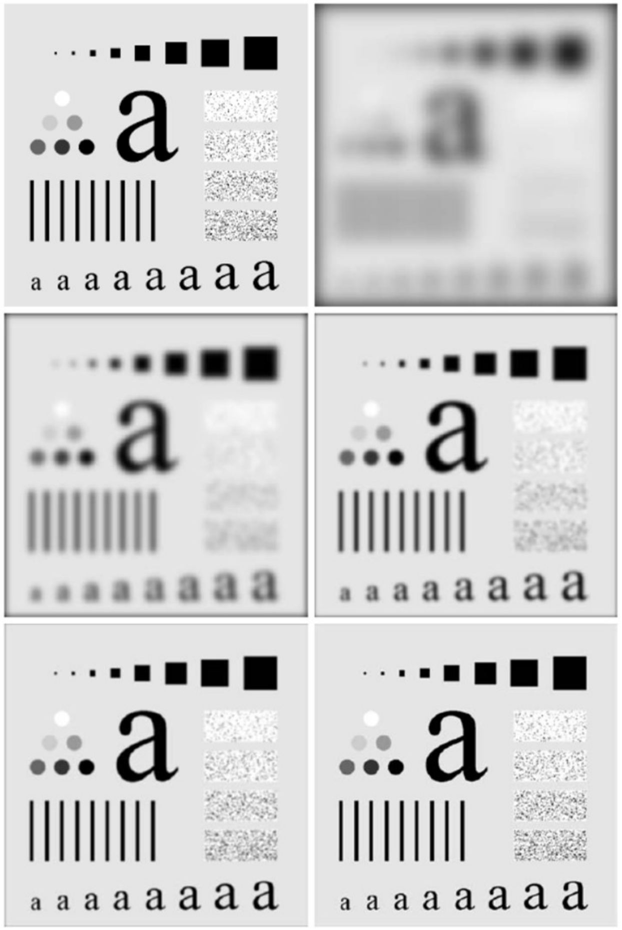


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

滤波结果：高斯低通滤波器 GLPF



a b
c d
e f

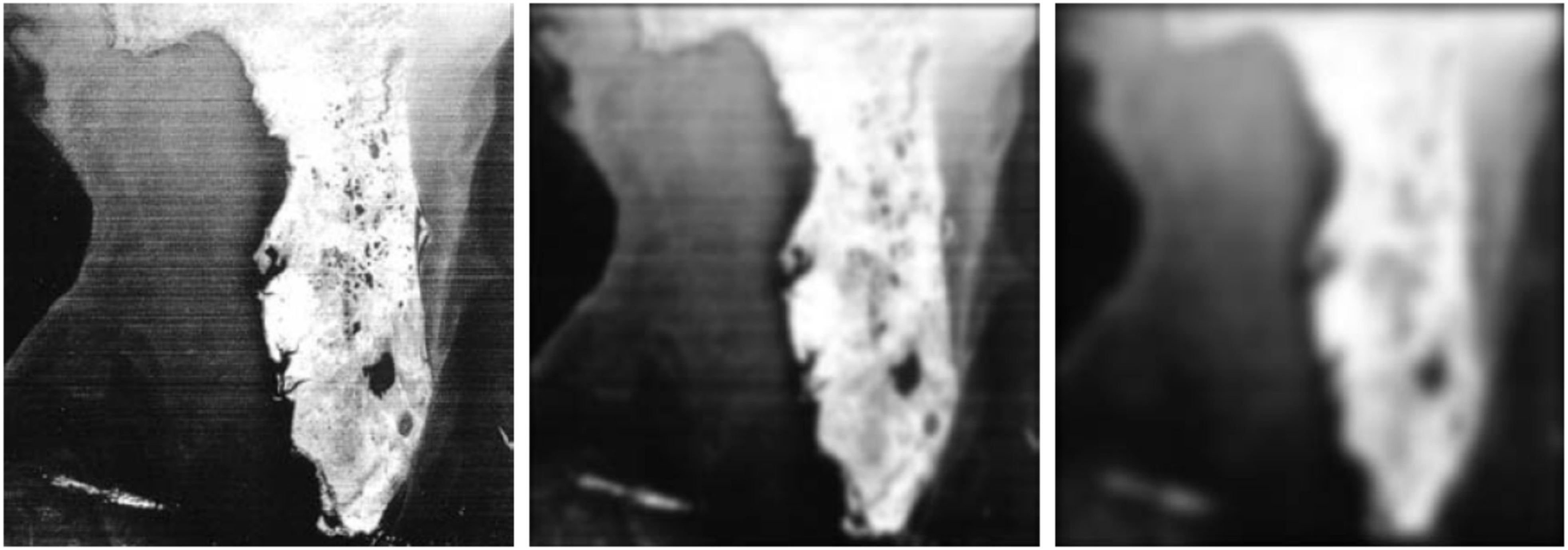
FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

滤波结果：高斯低通滤波器 GLPF

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

滤波结果：高斯低通滤波器 GLPF



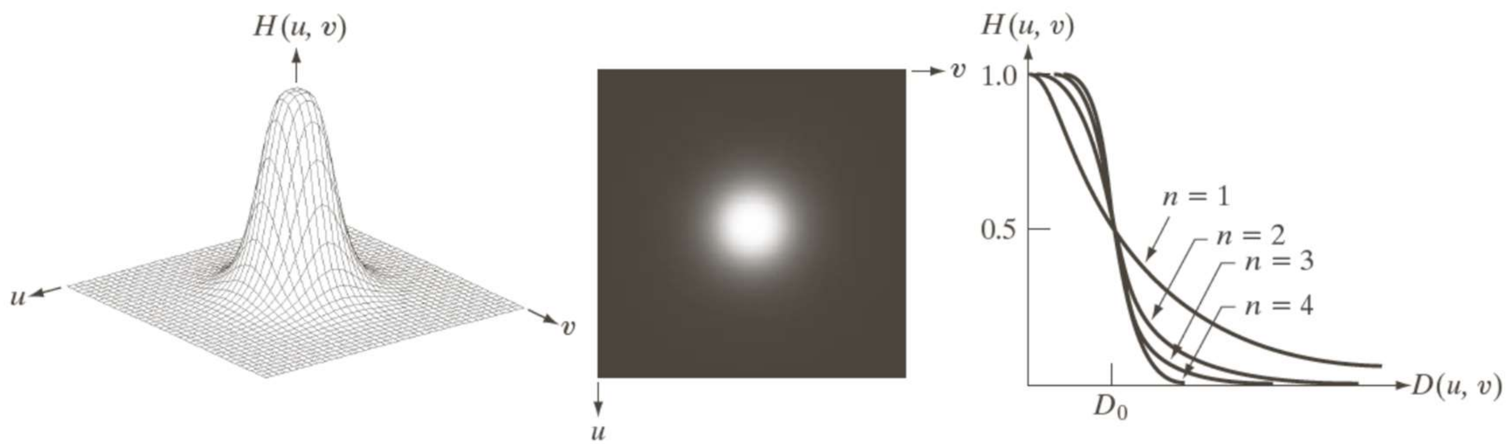
a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

代表性滤波器：巴特沃斯低通滤波器 BLPF

TABLE 4.4
Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$



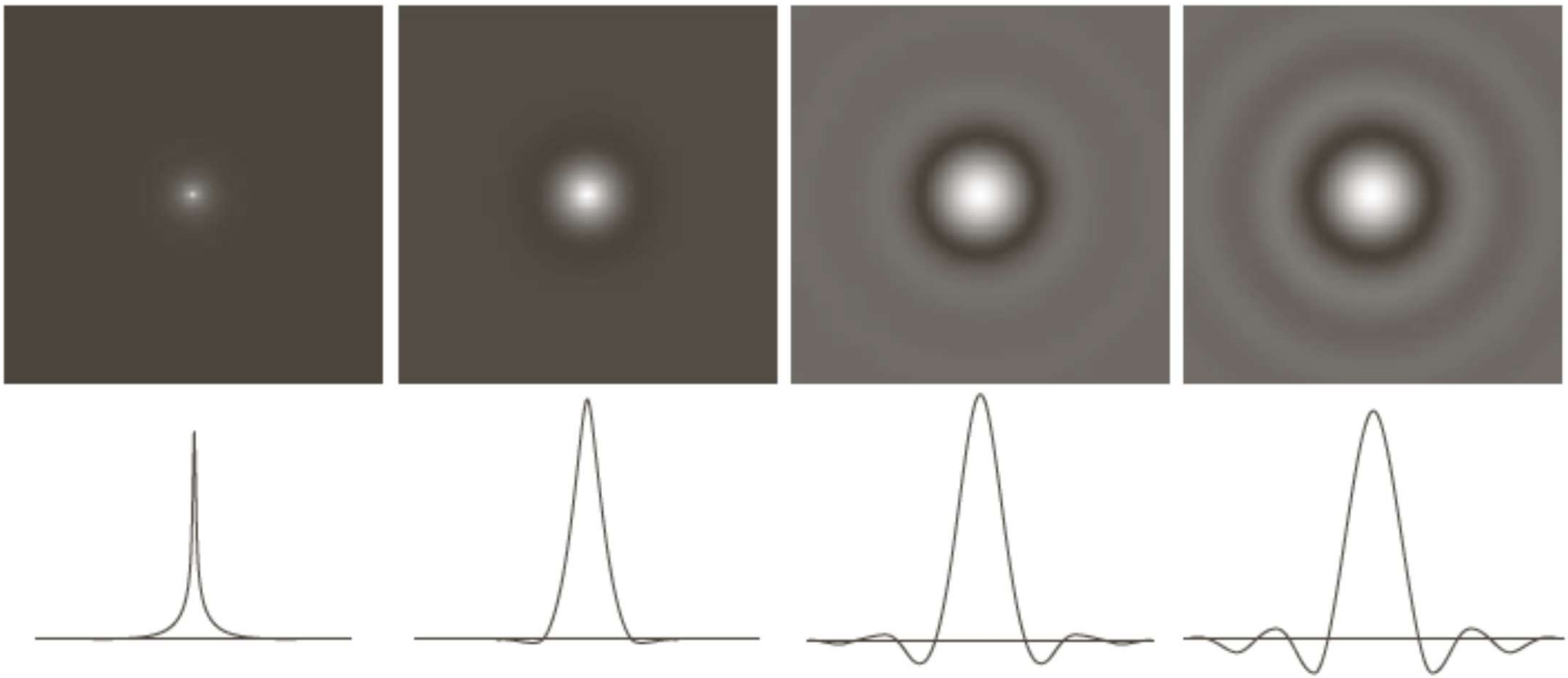
a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

滤波结果：巴特沃斯低通滤波器 BLPF

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

BLPF的空间域特性



a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

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矢量空间的基本概念

- 一个定义在域*F*上的矢量空间*V*, 对于所有的$X,Y,Z\in V,\quad r,s\in V$满足如下条件

commutativity: $X+Y=Y+X$

associativity of addition: $(X+Y)+Z=X+(Y+Z)$

additive identify: $0+X=X+0$

existence of additive inverse: $X+(-X)=0$

associativity of scalar multiplication: $r(sX)=(rs)X$

distributivity of scalar sums: $(r+s)X=rX+sX$

distributivity of vector sums: $r(X+Y)=rX+rY$

scalar multiplication identity: $1X=X$

内积空间的基本概念

- 如果矢量空间V满足如下条件则是一个内积空间

$$\langle X,Y\rangle = \langle Y,X\rangle^*$$
$$\langle X+Y,Z\rangle = \langle X,Z\rangle + \langle Y,Z\rangle$$
$$\langle rX,Y\rangle = r\langle X,Y\rangle$$
$$\langle X,X\rangle \geq 0 \text{ and } \langle X,X\rangle = 0 \;\; iff\; X = 0$$

内积空间的单位正交基

$$W = \{w_0, w_1, w_2, ... w_N\}$$

$$\langle w_k, w_l \rangle = \delta_{kl} = \begin{cases} 0 \ \text{ for } k \neq l \\ 1 \ \text{ for } k = l \end{cases}$$

一维信号的正交变换

- 定义

$$T(u)=\sum_{x=0}^{N-1} f(x)r(x,u)=\langle f(x),r_u\rangle$$

$$r(x,u)\rightarrow \text{forward transformation kernel}$$

$$f(x)=\sum_{x=0}^{N-1} T(U)s(x,u)=\langle f(x),s_u\rangle$$

$$s(x,u)\rightarrow \text{reverse transformation kernel}$$

一维信号的正交变换的矩阵形式

$$t = \begin{bmatrix} T(0) \\ T(1) \\ \vdots \\ T(N-2) \\ T(N-1) \end{bmatrix} \quad f = \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-2) \\ f(N-1) \end{bmatrix} \quad s_u = \begin{bmatrix} s(0,u) \\ s(1,u) \\ \vdots \\ s(N-1,u) \end{bmatrix} \quad A = \begin{bmatrix} s_0^T \\ s_1^T \\ \vdots \\ s_{N-1}^T \end{bmatrix}$$

$$t = Af$$

$$f = A^T t$$

二维信号的正交变换的矩阵形式

$$T = AFA^T$$

$$F = A^T T A$$

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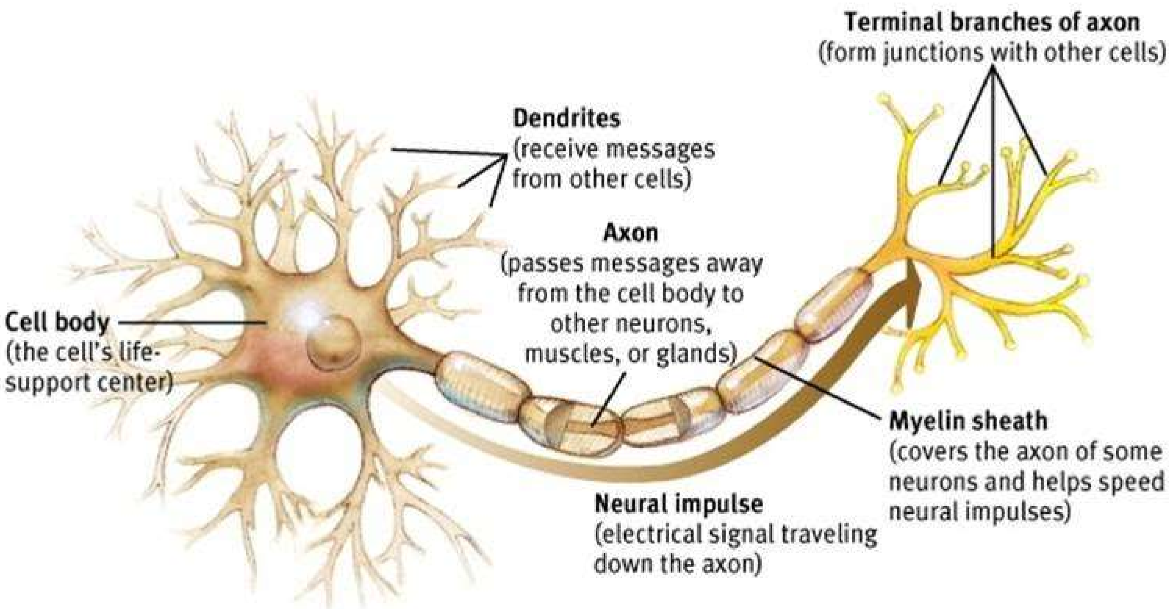
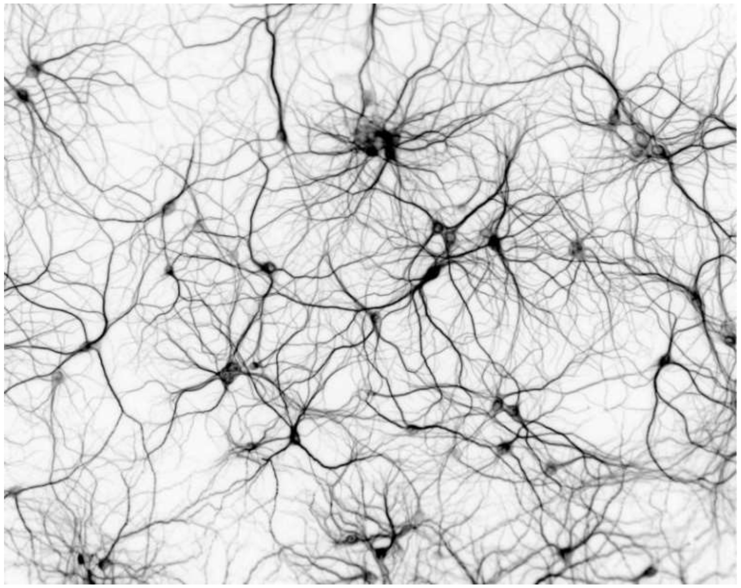
- **卷积神经网络的基本结构**

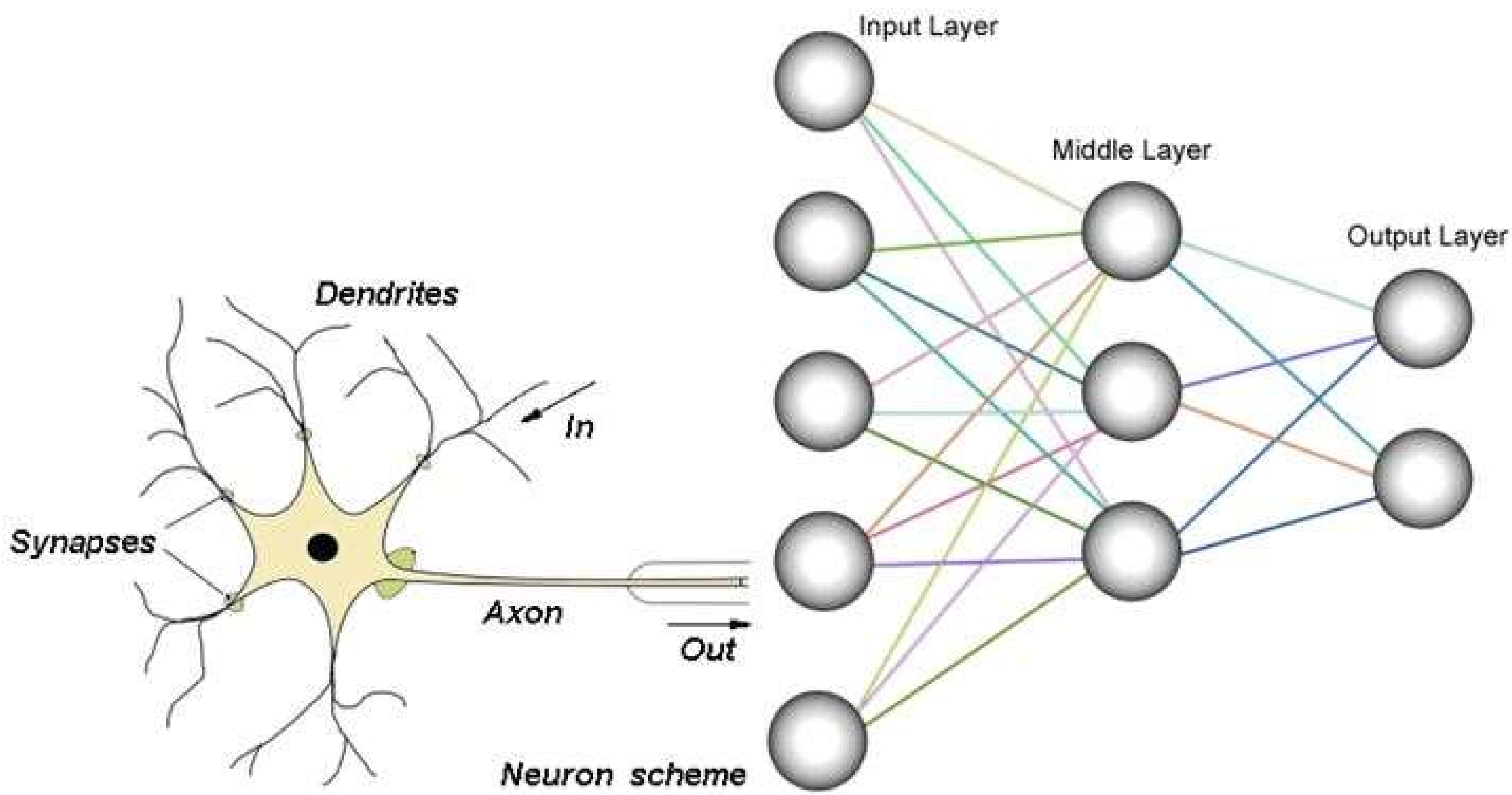
生物神经网络





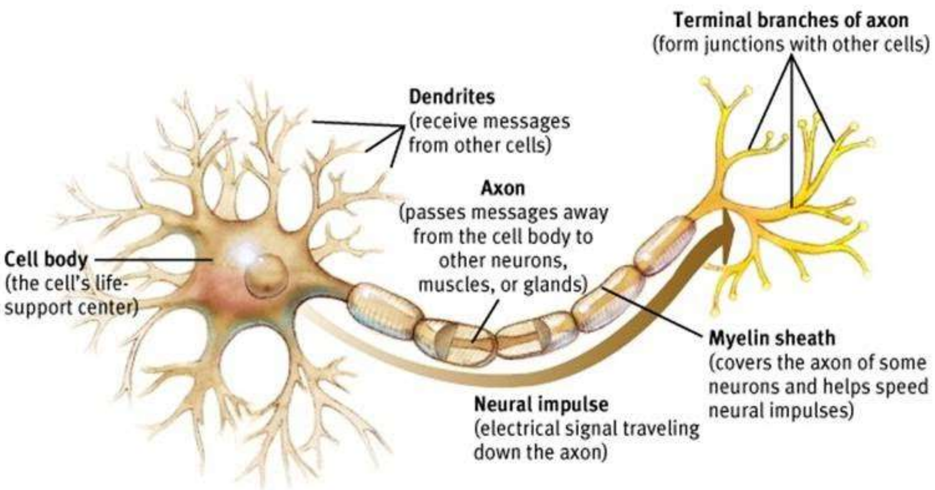
https://sites.google.com/site/mrstevensonstechclassroom/hl-topics-only/4a-robotics-ai/neural-networks-computational-intelligence

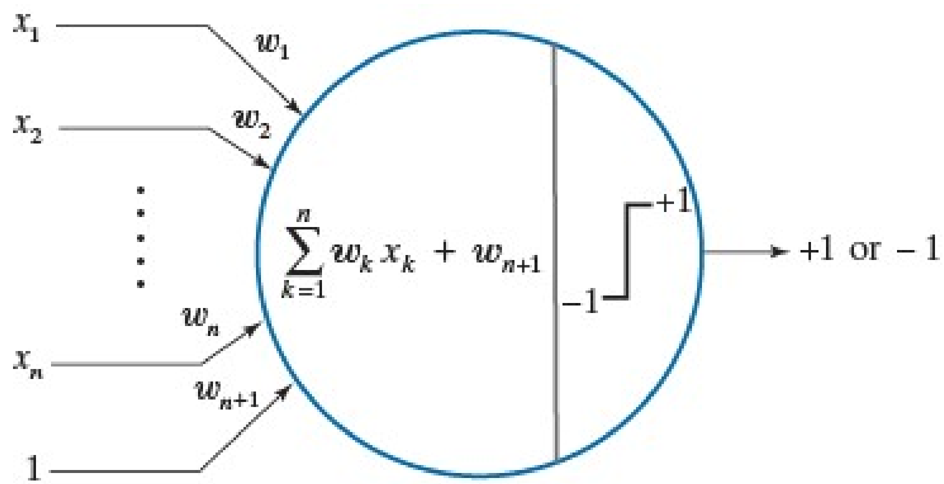
生物与人工神经网络的比较



https://sites.google.com/site/mrstevensonstechclassroom/hl-topics-only/4a-robotics-ai/neural-networks-computational-intelligence

生物神经元与人工神经元

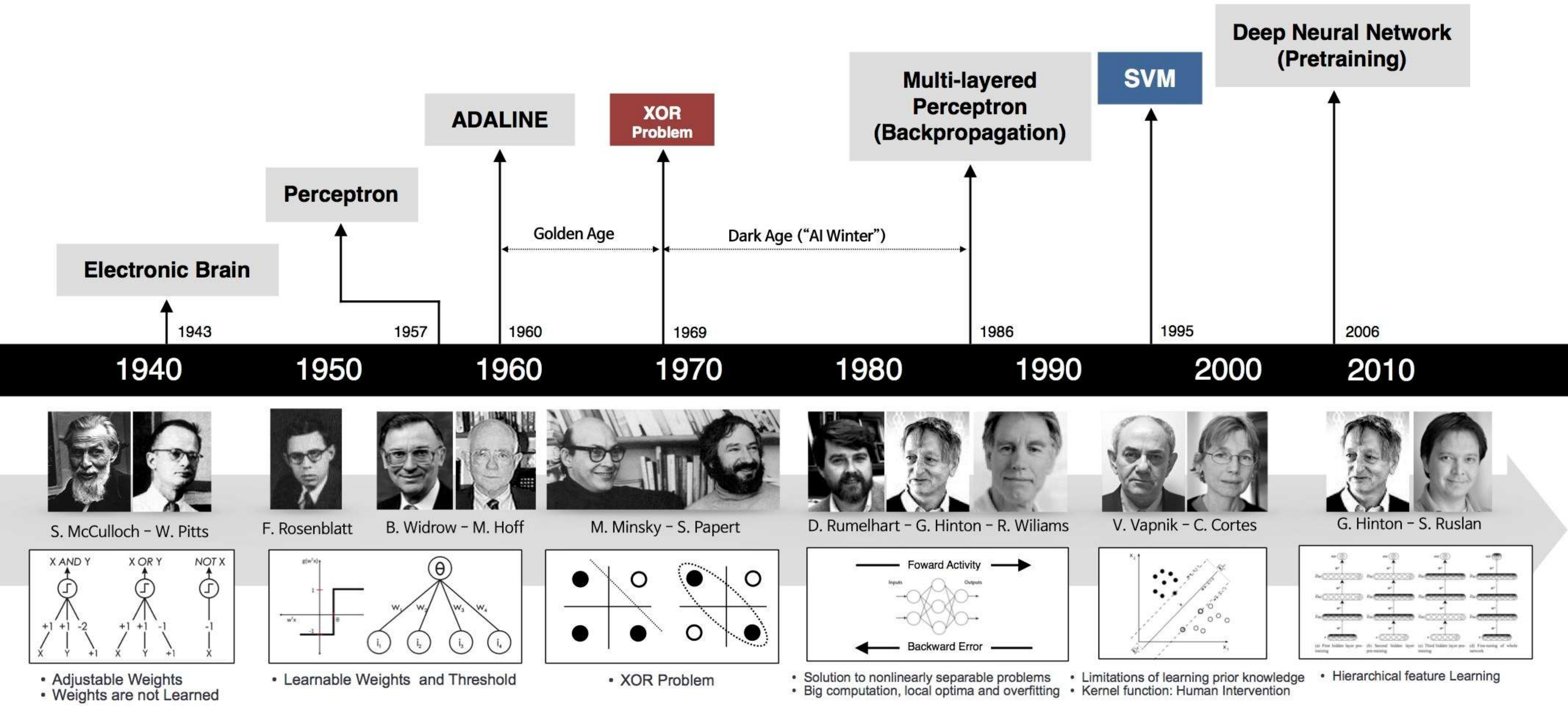




Frank Rosenblatt

https://sites.google.com/site/mrstevensonstechclassroom/hl-topics-only/4a-robotics-ai/neural-networks-computational-intelligence

人工神经网络研究历史回顾



https://sefiks.com/2017/10/14/evolution-of-neural-networks/

深度学习为图像处理带来一场革命

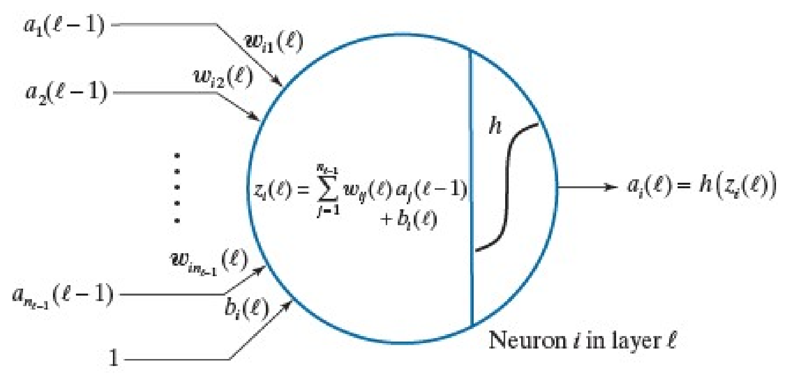
2018 ACM A. M. Turing Award

For conceptual and engineering breakthroughs that have made deep neural networks a critical component of computing.

https://www.acm.org/media-center/2019/march/turing-award-2018

一个人工神经元模型

FIGURE 13.29 Model of an artificial neuron, showing all the operations it performs. The "ℓ" is used to denote a particular layer in a layered network.



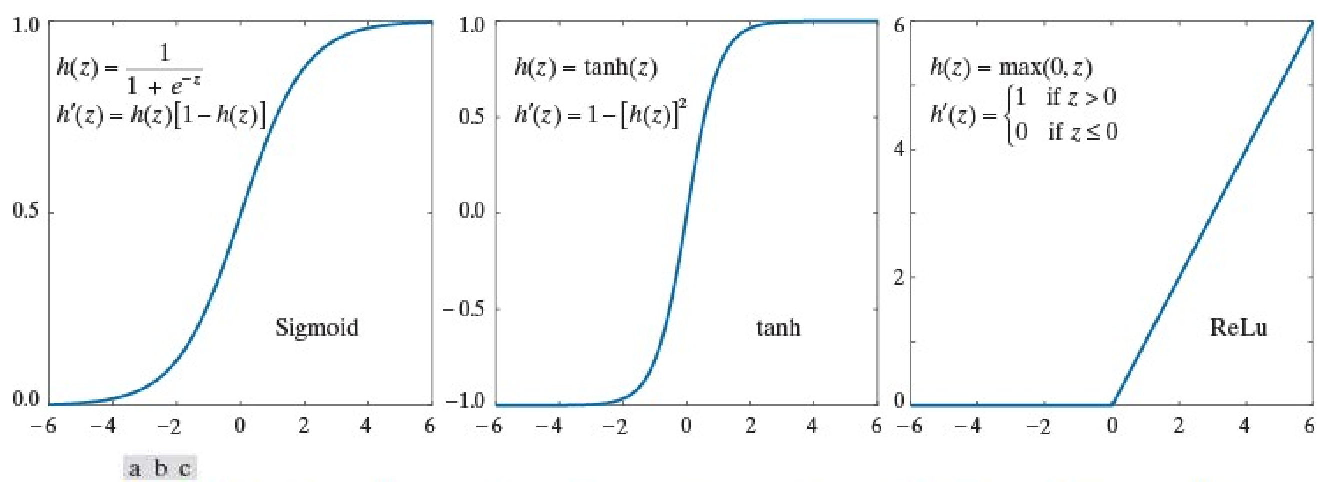
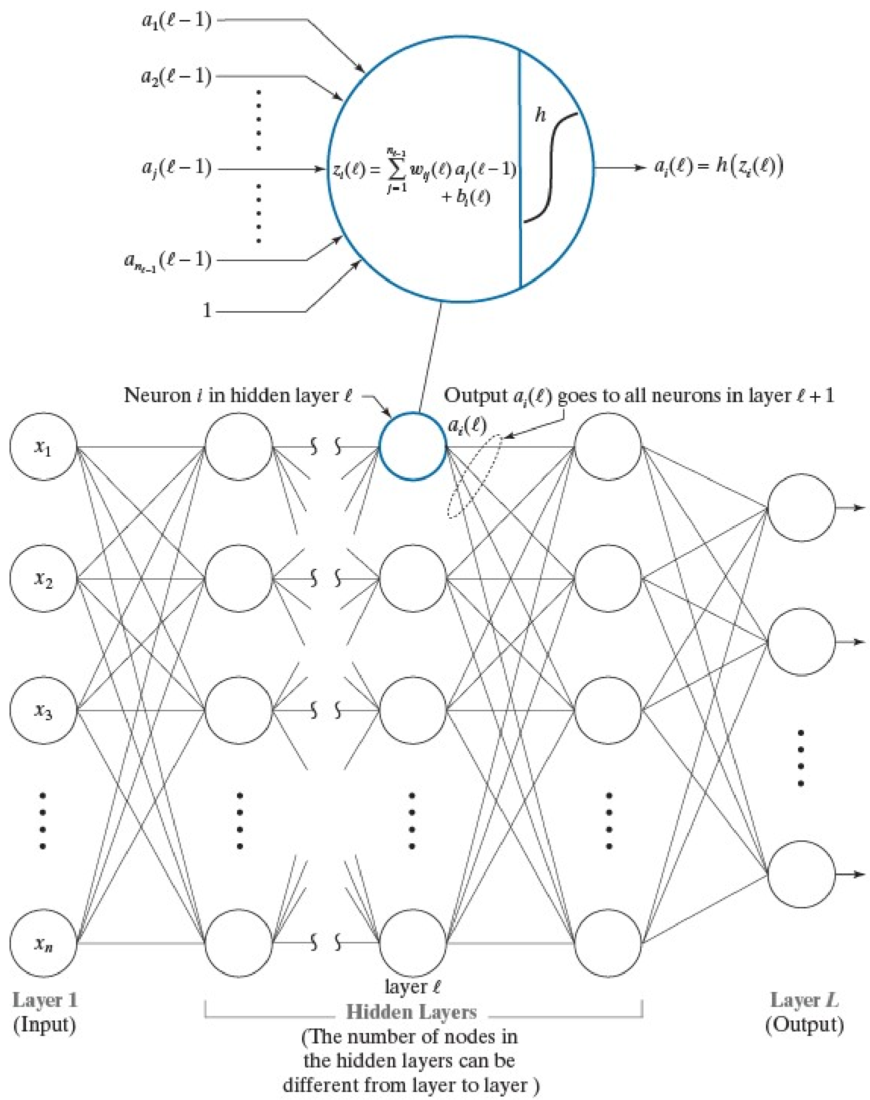


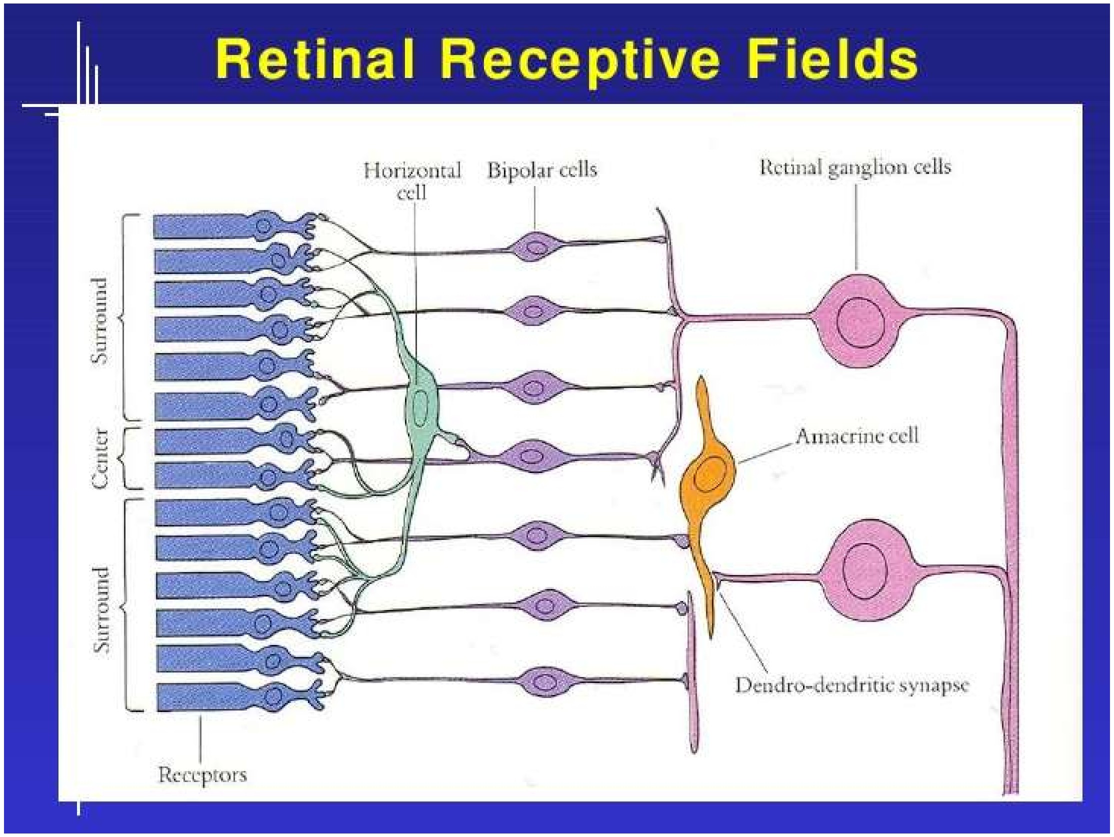
FIGURE 13.30 Various activation functions. (a) Sigmoid. (b) Hyperbolic tangent (also has a sigmoid shape, but it is centered about 0 in both dimensions). (c) Rectifier linear unit (ReLU).

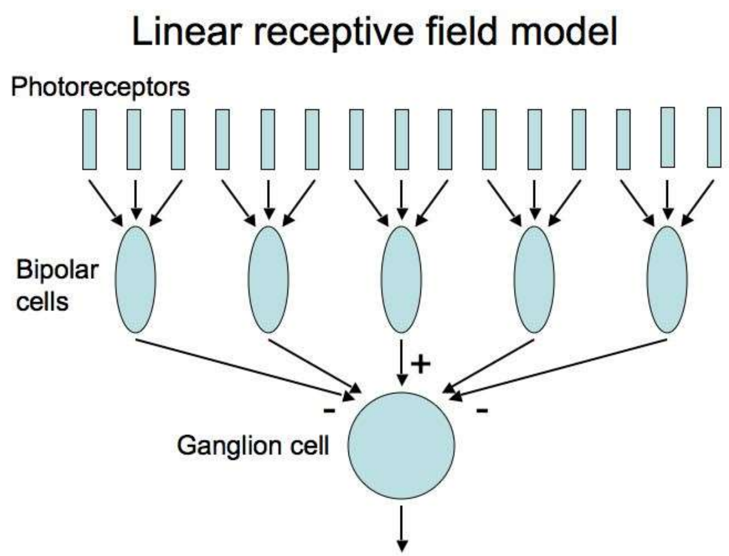
全连接前馈人工神经网络

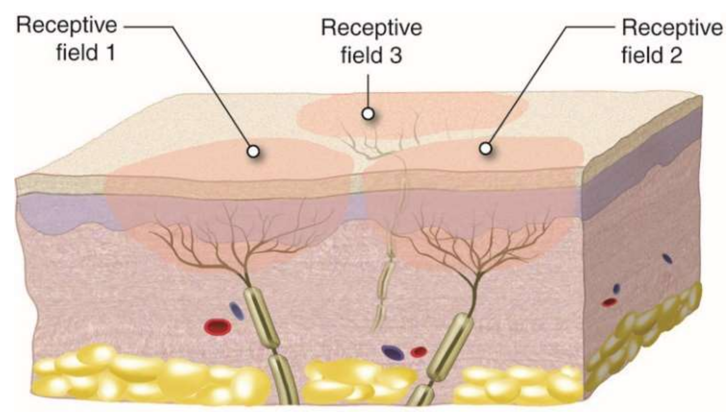
FIGURE 13.31
General model of a feedforward, fully connected neural net. The neuron is the same as in Fig. 13.29. Note how the output of each neuron goes to the input of all neurons in the following layer, hence the name *fully connected* for this type of architecture.



感受野的概念性介绍

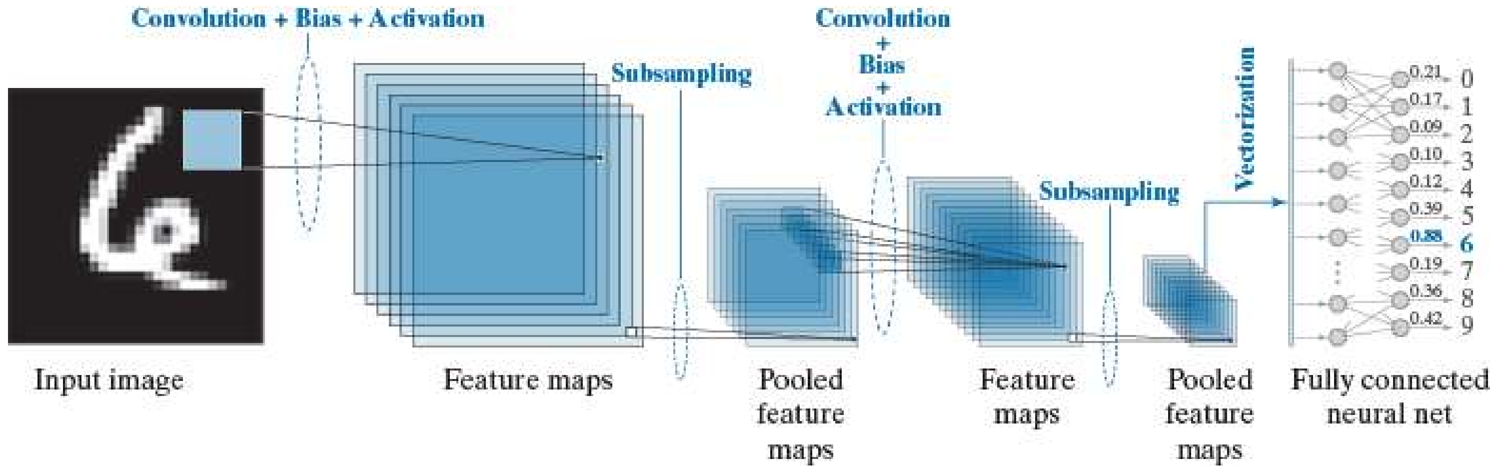


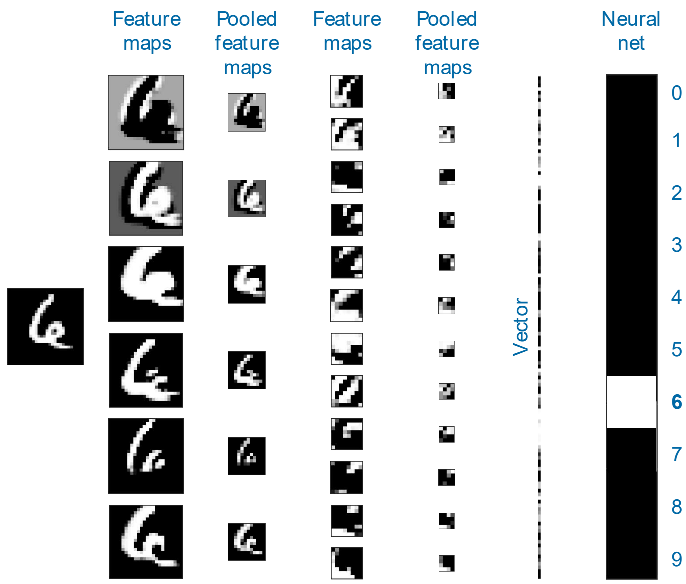




https://www.cns.nyu.edu/~david/courses/perception/lecturenotes/ganglion/ganglion.html

卷积神经网络示例：LeNet





http://yann.lecun.com/exdb/lenet/

Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner. Gradient-based learning applied to document recognition.*Proceedings of the IEEE*, november 1998.

卷积神经网络简介

参考文献

- [Convolutional neural network for visual recognition](http://cs231n.stanford.edu/)

  http://cs231n.stanford.edu/

- LeCun, Bengio, Hinton, Deep Learning, *Nature*, 2015.

- Zeiler & Fergus, Visualizing and understanding Convolutional Networks, *ECCV* 2014

Buzzword: CNN

Convolutional neural networks (CNN, ConvNet) is a class of deep, feed-forward (not recurrent) artificial neural networks that are applied to analyzing visual imagery.

Buzzword: CNN

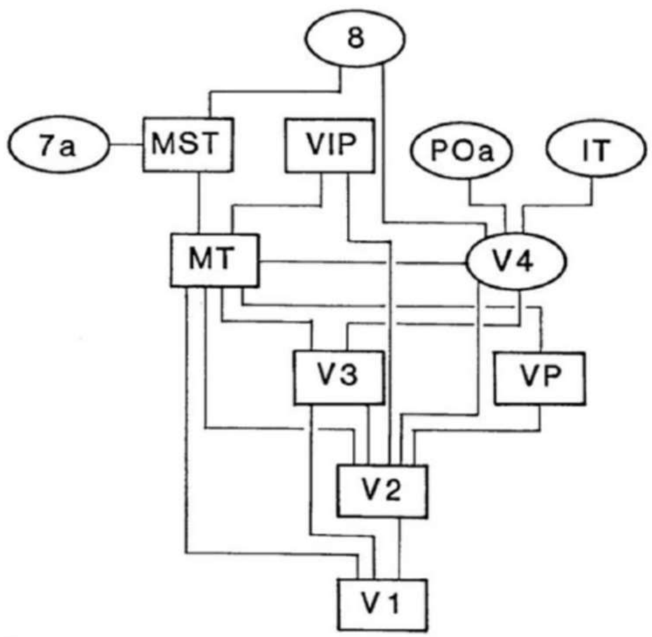
- Convolution

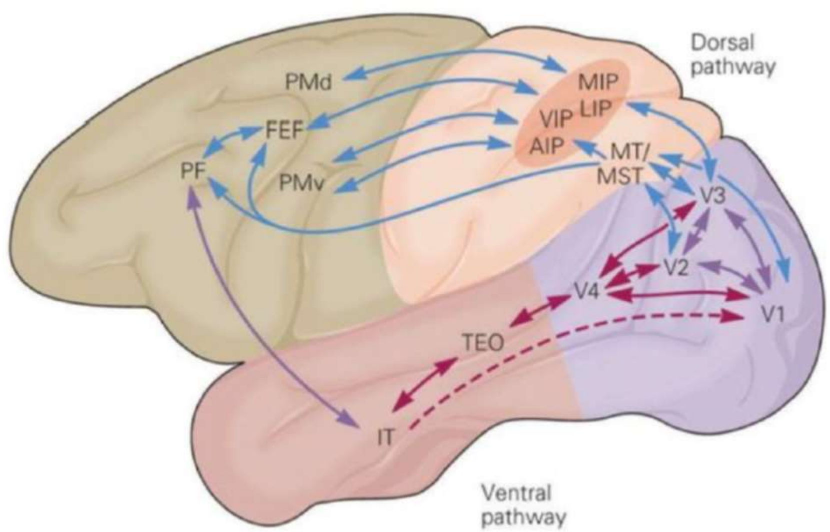
From wikipedia,

$$
\begin{aligned}
(f * g)(t) &\stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\, d\tau \\
&= \int_{-\infty}^{\infty} f(t-\tau) g(\tau)\, d\tau .
\end{aligned}
$$

卷积神经网络的神经生物学基础

Background: Signal Relay





Starting from V1 primary visual cortex, visual signal is transmitted upwards, becoming more complicated and abstract.

为什么采用卷积神经网络?

Neural Networks for Images

For computer vision, why can’t we just flatten the image and feed it through the neural networks?

Neural Networks for Images

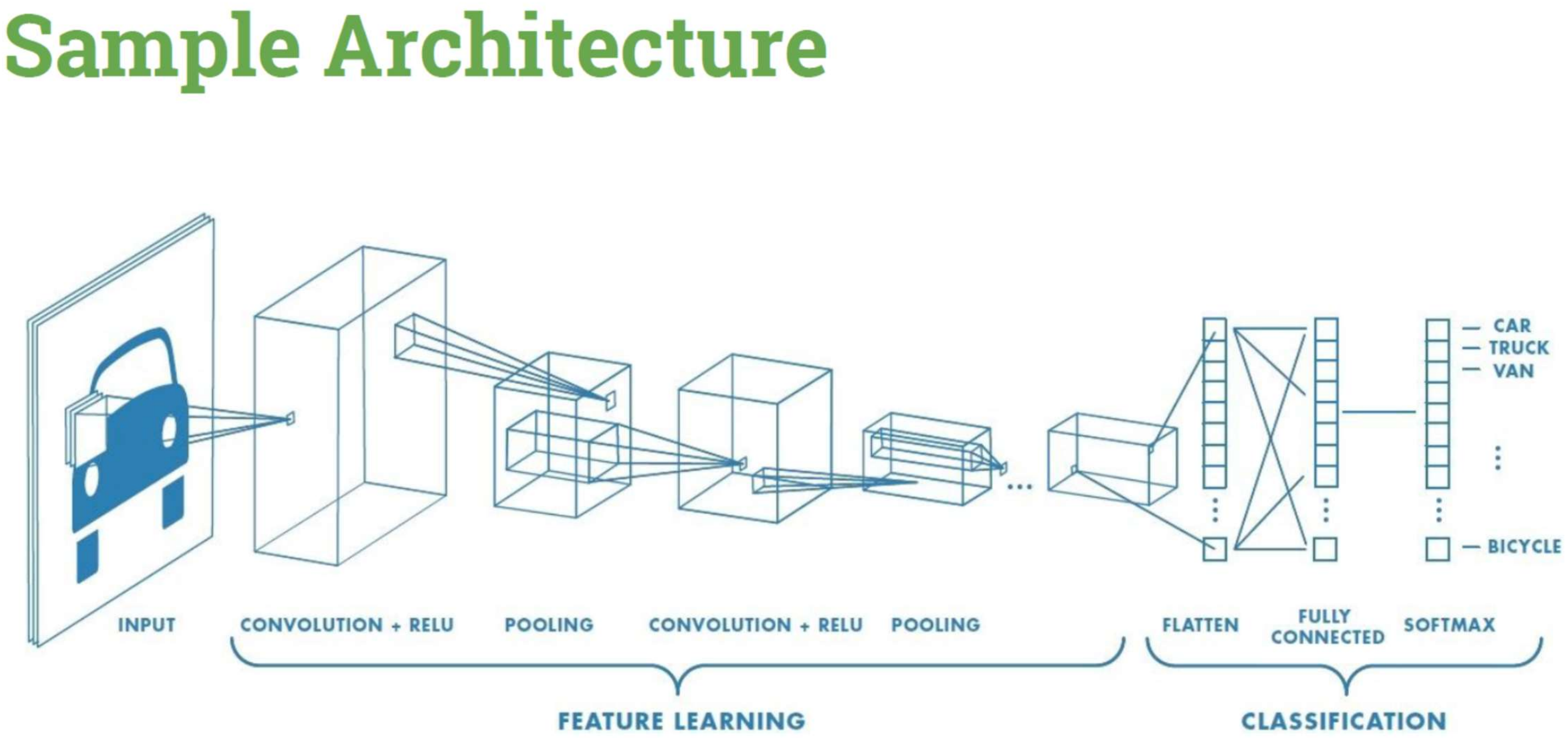
Images are high-dimensional vectors. It would take a huge amount of parameters to characterize the network.

Convolutional Neural Networks

To address this problem, bionic convolutional neural networks are proposed to reduced the number of parameters and adapt the network architecture specifically to vision tasks.

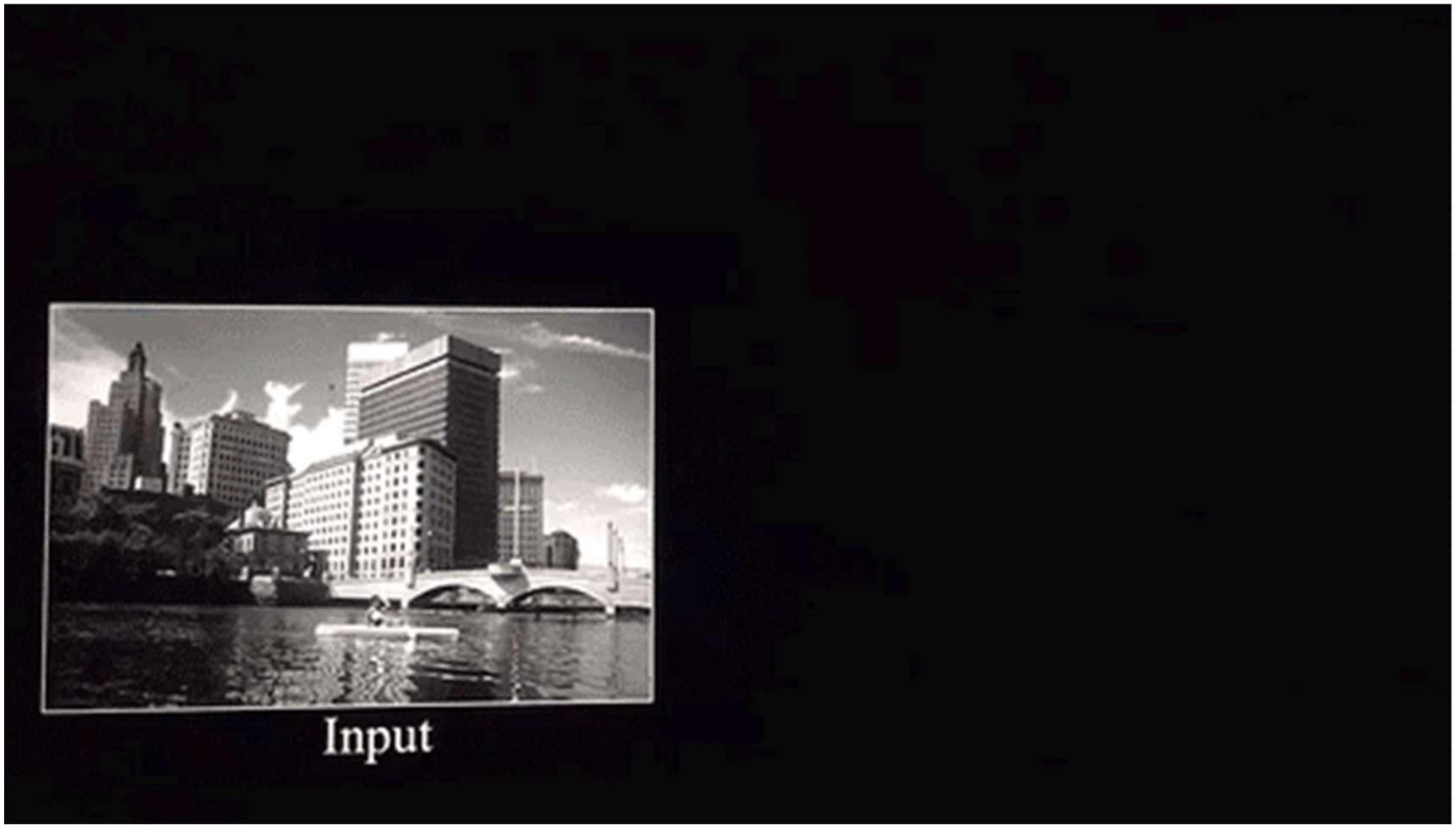
Convolutional neural networks are usually composed by a set of layers that can be grouped by their functionalities.

CNN的典型结构



Sample Architecture
INPUT
CONVOLUTION + RELU
POOLING
CONVOLUTION + RELU
POOLING
FLATTEN
FULLY CONNECTED
SOFTMAX
CAR
TRUCK
VAN
BICYCLE
FEATURE LEARNING
CLASSIFICATION

卷积层：提取图像特征

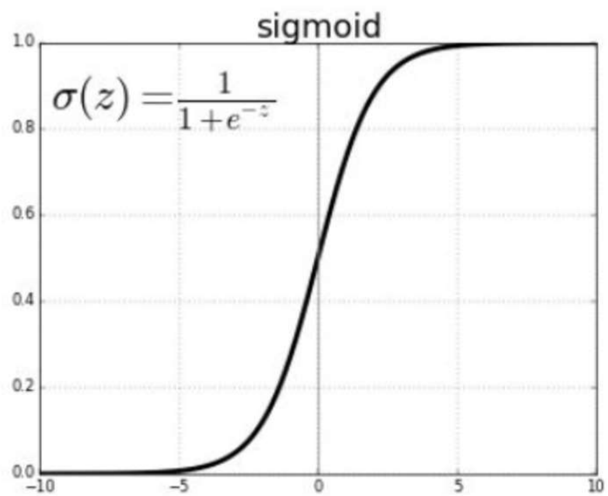


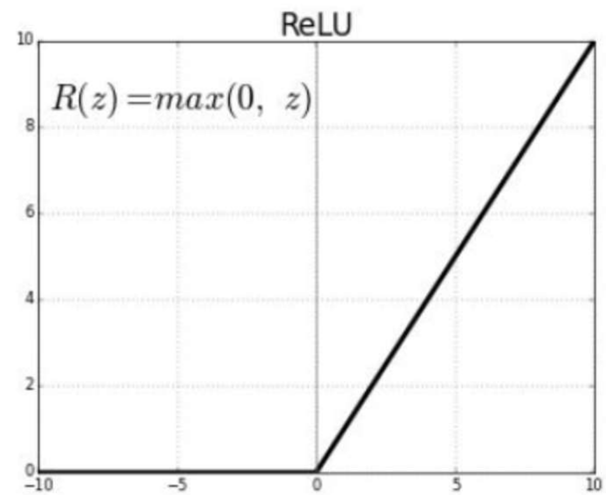
https://ujjwalkarn.me/2016/08/11/intuitive-explanation-convnets/

非线性单元：ReLU (I)

Activation Layer

- Used to increase non-linearity of the network without affecting receptive fields of conv layers
- Prefer ReLU, results in faster training
- LeakyReLU addresses the vanishing gradient problem

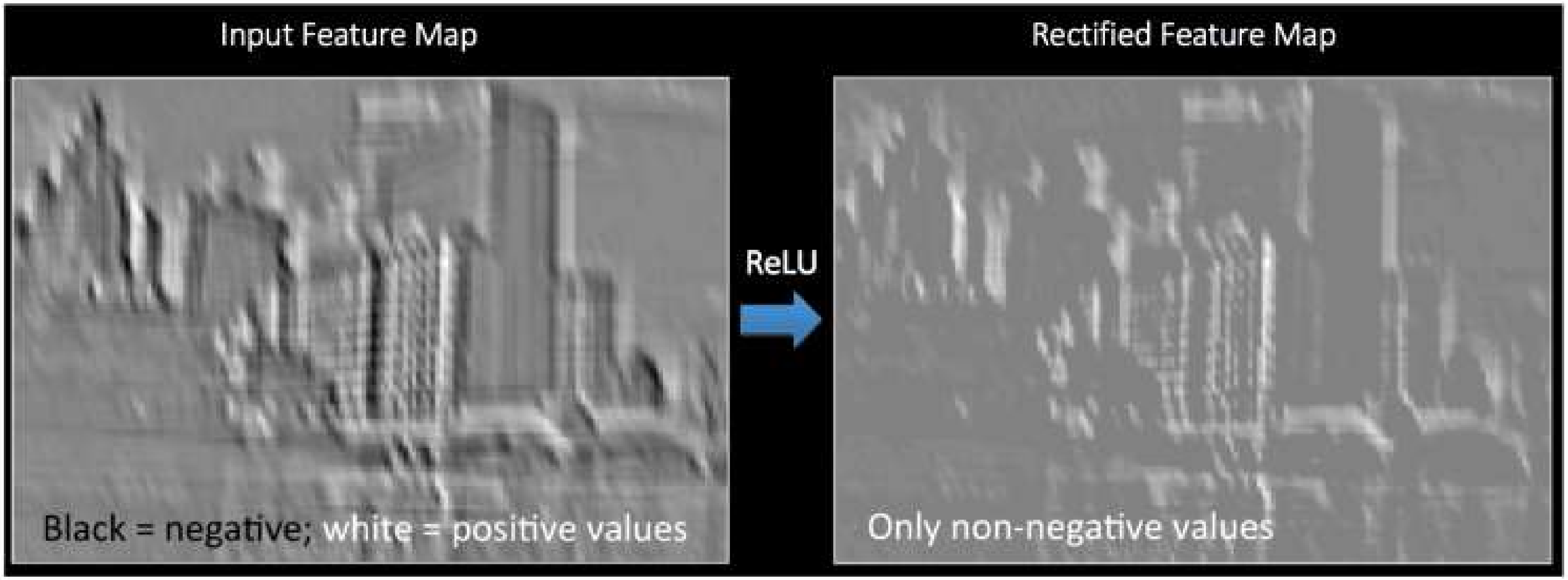




Other types:
Leaky ReLU, Randomized Leaky ReLU, Parameterized ReLU
Exponential Linear Units (ELU), Scaled Exponential Linear Units
Tanh, hardtanh, softtanh, softsign, softmax, softplus...

非线性单元：ReLU (II)

- 应用于每个像素，将所有的负灰度值换为0.

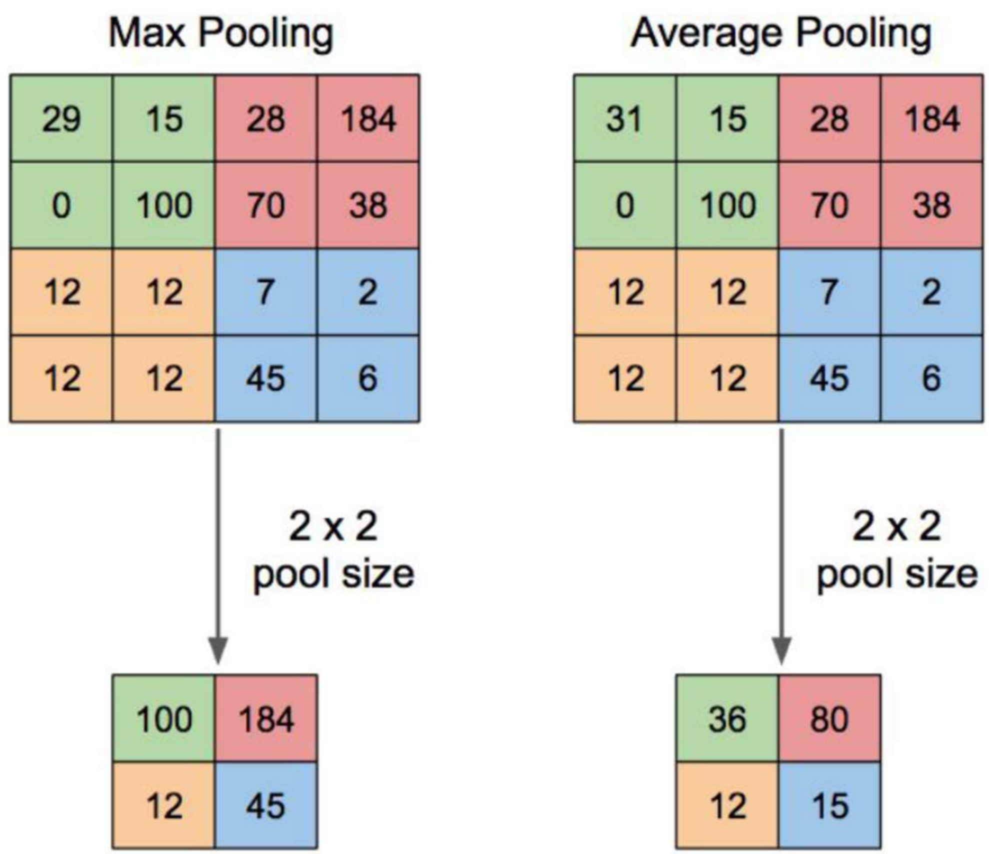


https://ujjwalkarn.me/2016/08/11/intuitive-explanation-convnets/

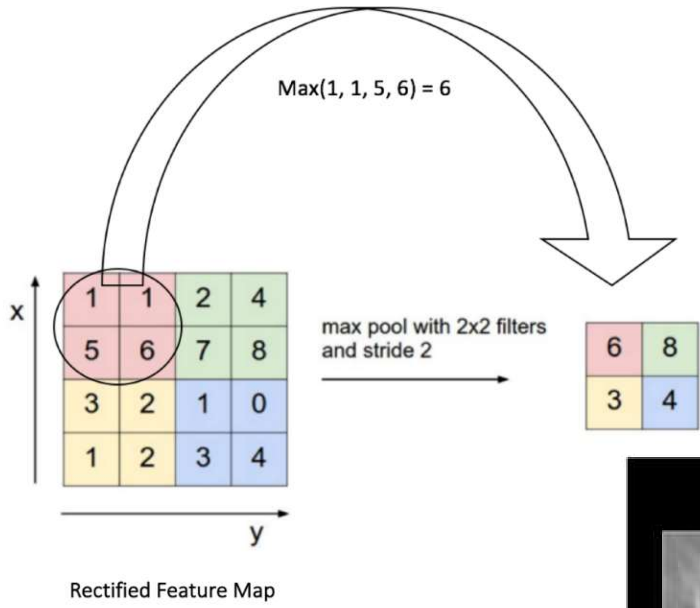
Pooling 池化 (I)

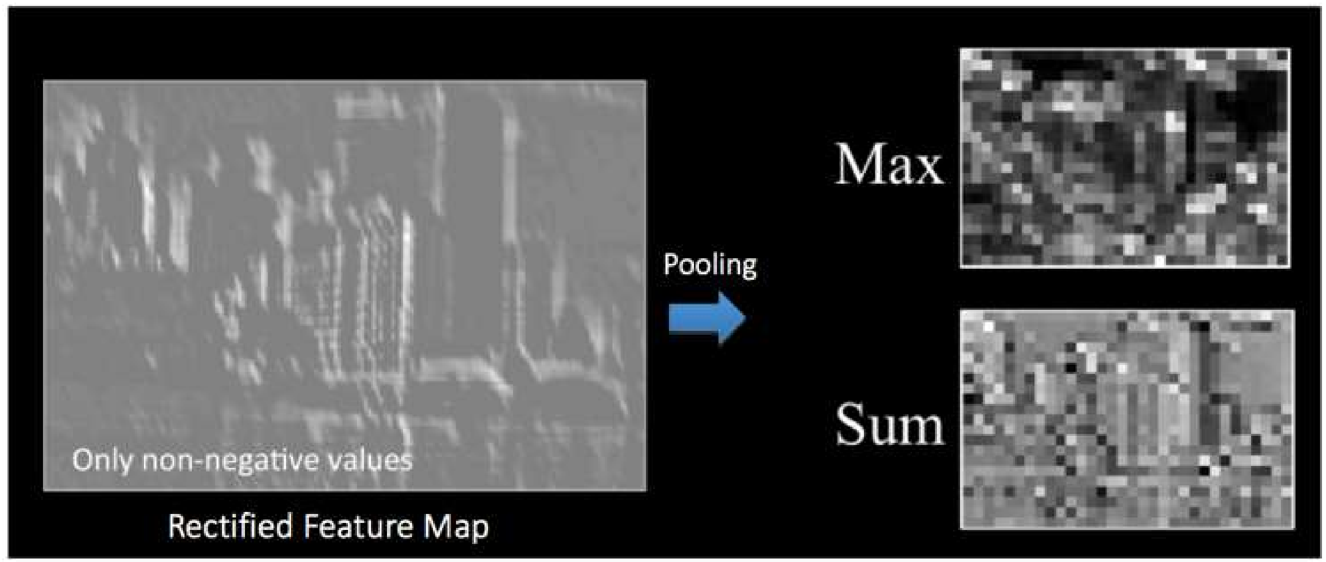
Pooling Layer

- Convolutional layers provide activation maps.
- Pooling layer applies non-linear downsampling on activation maps.
- Pooling is aggressive (discard info); the trend is to use smaller filter size and abandon pooling



Pooling 池化 (II)





https://ujjwalkarn.me/2016/08/11/intuitive-explanation-convnets/

Softmax

- A special kind of activation layer, usually at the end of FC layer outputs
- Can be viewed as a fancy normalizer (a.k.a. Normalized exponential function)
- Produce a discrete probability distribution vector
- Very convenient when combined with cross-entropy loss

$$P(y = j \mid \mathbf{x}) = \frac{e^{\mathbf{x}^\mathsf{T}\mathbf{w}_j}}{\sum_{k=1}^{K} e^{\mathbf{x}^\mathsf{T}\mathbf{w}_k}}$$

Given sample vector input **x** and weight vectors {$\mathbf{w}_j$}, the predicted probability of y = j

软件形态的变化

Applications

Can be viewed as a fancy feature extractor, just like SIFT, SURF, etc

Software 2.0

Quotes from Andrej Karpathy:

Software 1.0 is what we're all familiar with—it is written in languages such as Python, C++, etc. It consists of explicit instructions to the computer written by a programmer. By writing each line of code, the programmer is identifying a specific point in program space with some desirable behavior.

Software 2.0 is written in neural network weights. No human is involved in writing this code because there are a lot of weights (typical networks might have millions). Instead, we specify some constraints on the behavior of a desirable program (e.g., a dataset of input output pairs of examples) and use the computational resources at our disposal to search the program space for a program that satisfies the constraints.

Questions?

Microscope Image Formation (II)

- The impulse response of the microscope is called its point spread function (PSF).
- The transfer function of a microscope is called its optical transfer function (OTF).
- The PSF has the shape of an Airy Disk.

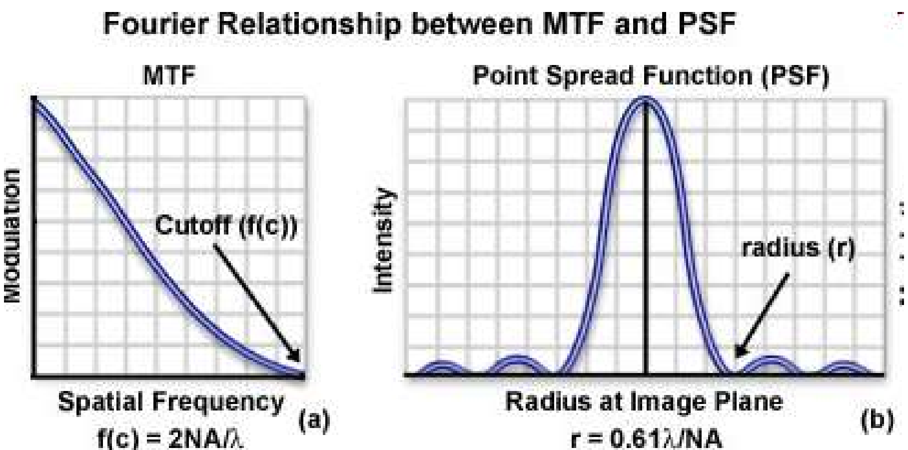
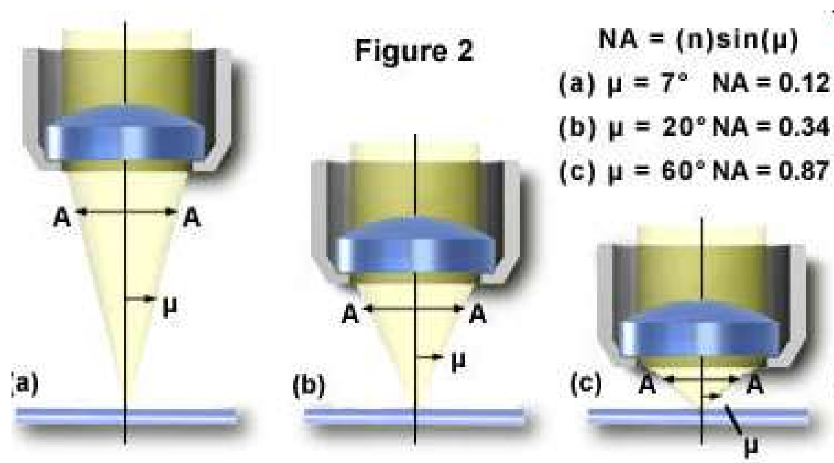


Figure 1

Numerical Aperture

- Numerical aperture (NA) determines microscope resolution and light collection power.



$$NA = n \cdot \sin \mu$$

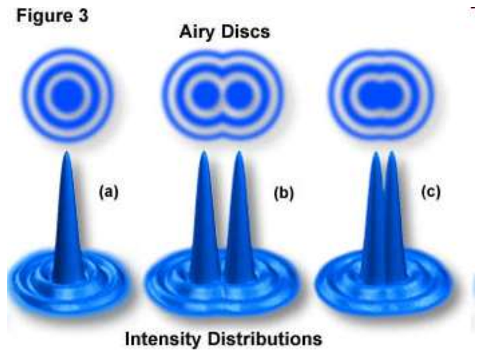
n: refractive index of the medium between the lens and the specimen

μ: half of the angular aperture

Different Definition of Light Microscopy Resolution Limit (Demo)

- Rayleigh limit

$$D = \frac{0.61\lambda}{NA}$$



- Sparrow limit

$$D = \frac{0.47\lambda}{NA}$$

http://www.microscopy.fsu.edu/primer/java/imageformation/rayleighdisks/index.html

Summary: High Resolution Microscopy

- Size of cellular features are typically on the scale of a micron or smaller.

- To resolve such features require
  - Shorter wavelength (electron microscopy)
  - High numerical aperture (resolution)
  - High magnification (spatial sampling)

$$D = \frac{0.61\lambda}{NA}$$

复习：数字图像采集典型场景示例

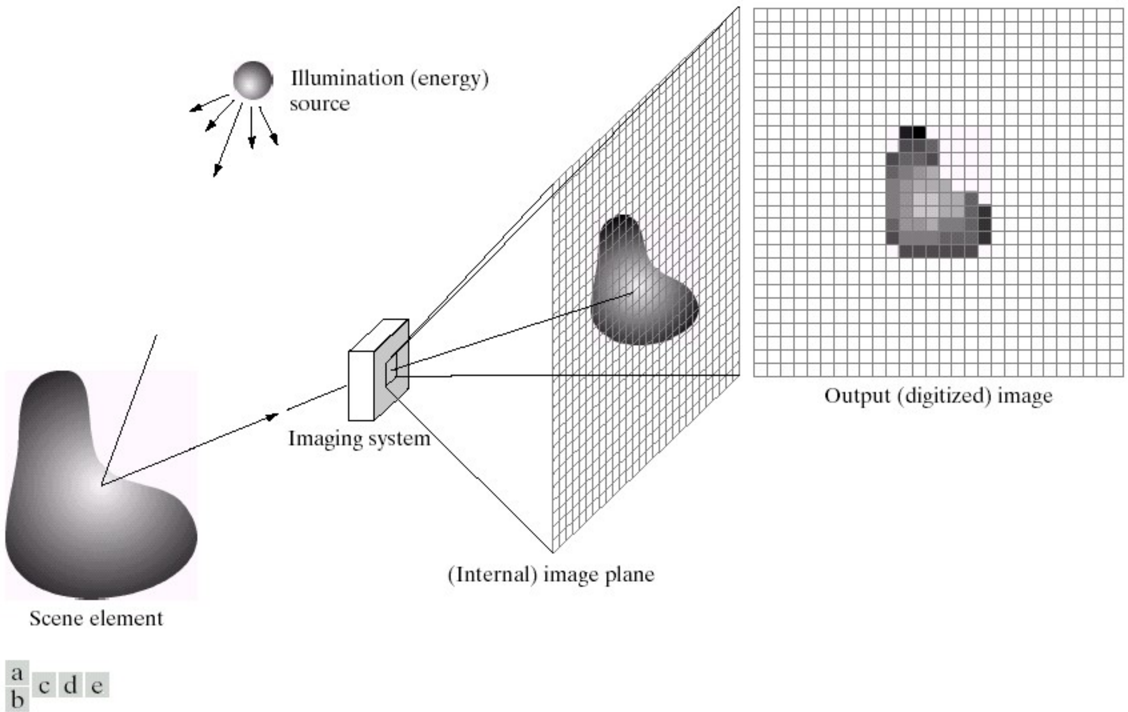


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

一个简单的成像模型 (I)

将二维图像记为 $f(x,y)$

$$0 \le f(x,y) < \infty$$

将入射到物体的光照分布函数记为 $i(x,y)$

$$0 \le i(x,y) < \infty$$

将物体的表面反射函数为 $r(x,y)$

$$0 \le r(x,y) \le 1$$

一个简单的成像模型 (II)

$$f(x,y) = i(x,y)r(x,y)$$

$$f(x,y) = i(x,y)r(x,y)c(x,y)$$

$$0 \le c(x,y) \le 1$$

复习：图像噪声模型

CHAPTER

7

Image Noise Models

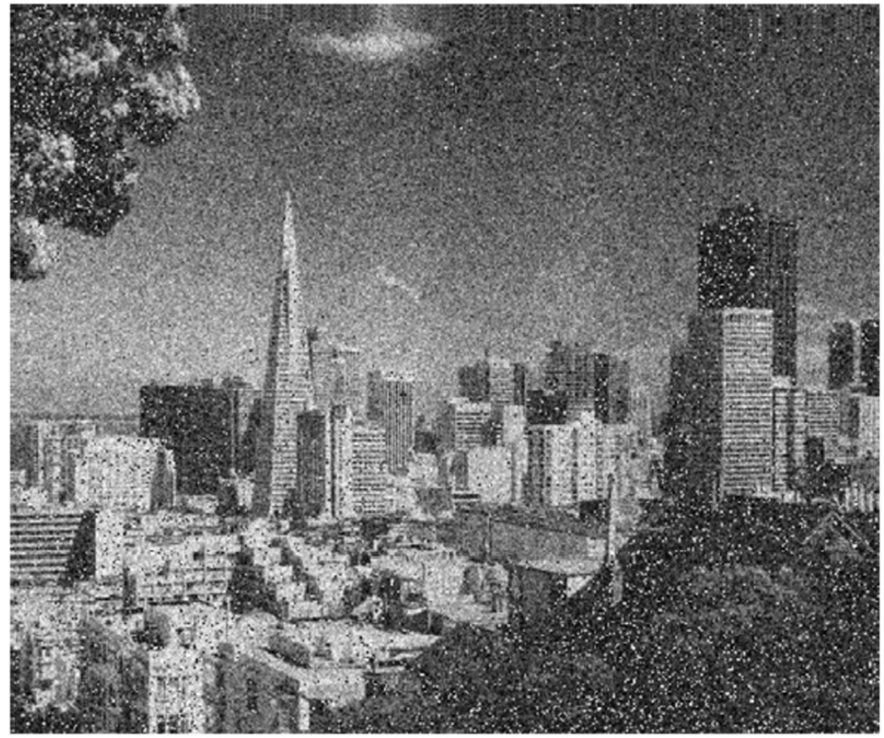
Charles Boncelet

University of Delaware

图像的噪声模型 (I)

- 加性噪声 additive noise $$i(\cdot)=g(\cdot)+n(\cdot)$$
- 乘性噪声 multiplicative noise $$i(\cdot)=g(\cdot)n(\cdot)$$
$$\log i=\log g+\log n$$
- 一个图像可能同时受到多种噪声的影响

高斯噪声 (II)



SD=30

- 常见的噪声来源：背景噪声 (dark noise), 放大器噪声 (amplifier noise)

椒盐(salt and pepper) 噪声

- 常见的噪声来源：传输噪声 (transmission noise)

$$P(n=0)=1-\alpha$$

$$P(n=MAX)=\alpha/2$$

$$P(n=MIN)=\alpha/2$$

Poisson噪声

$$P(n=k)=\frac{e^{-\lambda}\lambda^{k}}{k!}$$

$$E(n)=\lambda$$

$$\sigma^{2}(n)=\lambda$$

$$\Rightarrow \quad noise \sim \sqrt{pixel\_intensity}$$