

180206081104P2009H

## 图像处理与分析

### 第五讲: 图像变换 (III)

图像的空间域频率域滤波, 正交变换, 距离变换

基于深度学习的图像变换

# 内容提要

---

- 采样定理应用示例
- 图像的空间域滤波(复习)
- 图像的频率域滤波(复习)
- 一维离散傅里叶变换(复习)
- 图像的二维离散傅里叶变换的计算
- 图像的正交变换
- 图像的距离变换
- 基于深度学习的图像变换简介

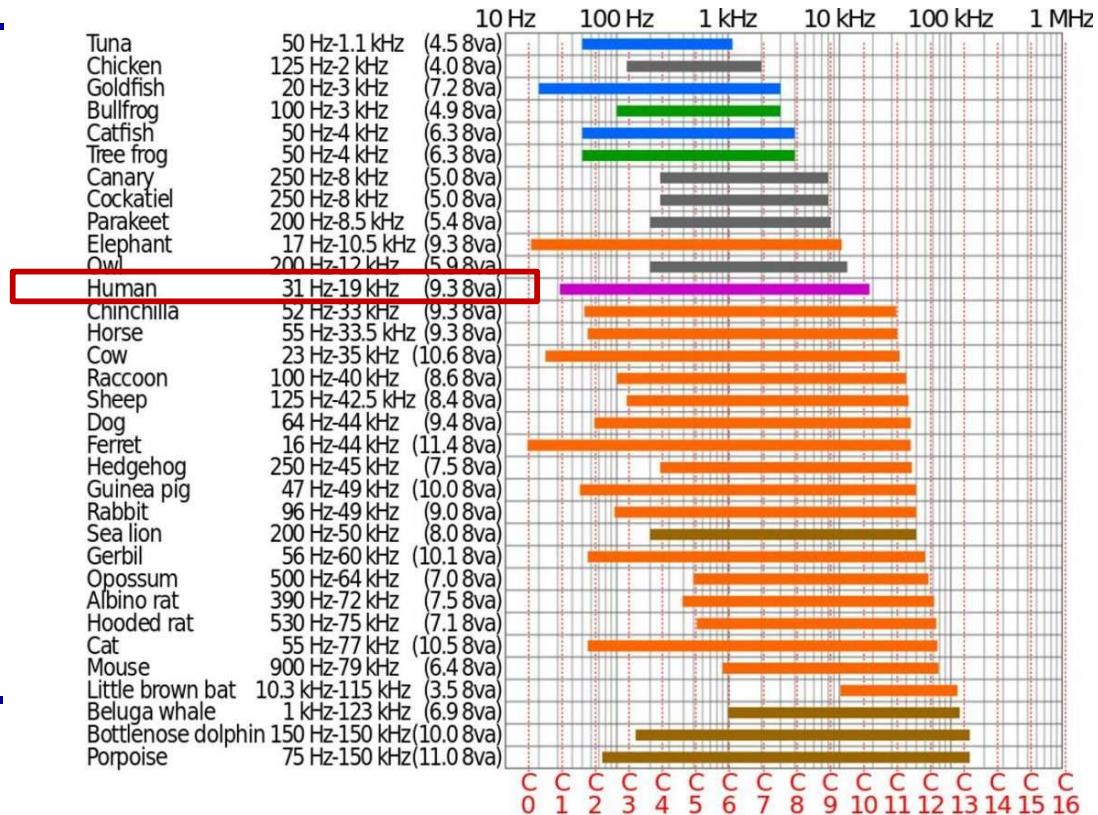
# 内容提要

---

- 采样定理应用示例
  - 图像的空间域滤波(复习)
  - 图像的频率域滤波(复习)
  - 一维离散傅里叶变换(复习)
  - 图像的二维离散傅里叶变换的详细介绍
  - 图像的二维离散傅里叶变换的计算
  - 图像的正交变换
  - 图像的距离变换
  - 基于深度学习的图像变换简介
-

# 采样定理应用示例：音频录制

- 人类听力覆盖的频率范围20-20kHz
- 常用的声音录制频率为16bit/44.1kHz (CD).
- 高清晰度/高保真声音录制的频率为 24bit/96kHz, 192kHz.
- $2^{24} = 16,777,216$



[https://en.wikipedia.org/wiki/Audio\\_frequency](https://en.wikipedia.org/wiki/Audio_frequency)

<https://www.thebroadcastbridge.com/content/entry/7999/should-audio-for-video-be-recorded-at-high-sample-rates>

<https://www.whathifi.com/advice/high-resolution-audio-everything-you-need-to->

<https://www.whathifi.com/advice/high-resolution-audio-everything-you-need-to/#:~:text=In%20its%20simplest%20terms%2C%20hi,16%2Dbit%2F44.1kHz.&text=Hi%2Dres%20audio%20files%20usually,20176.4kHz%20files%20too.>

# 采样定理应用示例：手机相机空间采样频率 (I)

Adv. Opt. Techn., Vol. 1 (2012), pp. 51–58 • Copyright © 2012 THOSS Media and De Gruyter. DOI 10.1515/aot-2012-0002

## Research Article

### Optical design of camera optics for mobile phones

Thomas Steinich and Vladan Blahnik\*

Carl Zeiss AG, Carl-Zeiss-Strasse 22, 73447 Oberkochen,  
Germany

\*Corresponding author  
e-mail: blahnik@zeiss.de

Received January 9, 2012; accepted February 20, 2012

#### Abstract

new product generation. In contrast to this trend, the resolution has increased from 0.3 MP in 2002 to 12 MP at present.

Here is an example of a typical specification for a modern 12 MP mobile phone optical module (Table 1). The pixel pitch of  $pp = 1.4 \mu\text{m}$  defines the maximum spatial resolution of the sensor according to the Nyquist sampling theorem:

$$V_{Nyquist} = \frac{1}{2 \times pp} = 357 \frac{\text{lp}}{\text{mm}}$$

**Table 1** Basic optical layout of CCMs.

Basic optical layout	$y'$ (max)	3.52 mm
Sensor size (semi-diameter of image circle)	$pp$	$1.4 \mu\text{m}$
Pixel pitch	Number of pixels in $x,y$	approx. 4000×3000 12 MP
Sensor resolution	$F\#$	2.8
Aperture	$f'$	4.52 mm
Focal length	$DFOV=2 w$	$76^\circ$
Diagonal full field of view	$x \times y \times z$	$<1 \text{ cm}^3$
Module size	sl-img	$<7 \text{ mm}$
Optical total track from first lens vertex to image plane	$d$	0.3 mm
Filter package thickness in image space (IR-cut, cover glass)	MOD	100 mm
Minimum optical distance for focusing	CRA	$<30^\circ$
Maximum chief ray angle upon image plane		

# 采样定理应用示例：手机相机空间采样频率 (II)

Model number	Resolution	Sensor size (Optical format)	Pixel size	Pixel type	Chroma	Products used in
ISOCELL GN1 (S5KGN1) <sup>[5]</sup>	Photo: 8160x 6144 50MP Video: 7680 x 4320 @ 30 fps	(~1/1.31")	1.2 μm	ISOCELL Plus	Tetracell	<b>Rear:</b> <a href="#">Vivo X50 Pro+</a> <a href="#">Vivo iQOO 5/5 Pro</a>
ISOCELL HM1 (S5KHM1) <sup>[6]</sup>	Photo: 12000 x 9000 108MP Video: 7680 x 4320 @ 24 fps	12.03 mm (~1/1.33")	0.8 μm	ISOCELL Plus	Nonacell	<b>Rear:</b> <a href="#">Samsung Galaxy S20 Ultra</a> <a href="#">Samsung Galaxy Note 20 Ultra</a>
ISOCELL HMX (S5KHMX) <sup>[7]</sup>	Photo: 12032 x 9024 108MP Video: 6016 x 3384 @ 30 fps	12.03 mm (~1/1.33")	0.8 μm	ISOCELL Plus	Tetracell	<b>Rear:</b> <a href="#">Xiaomi Mi MIX Alpha</a> <a href="#">Xiaomi Mi CC9 Pro</a> <a href="#">Xiaomi Mi Note 10</a> <a href="#">Xiaomi Mi Note 10 Pro</a> <a href="#">Xiaomi Mi 10</a> <a href="#">Xiaomi Mi 10 Pro</a> <a href="#">Xiaomi Mi 10T Pro</a> <a href="#">Motorola Edge+</a>
ISOCELL GW2 (S5KGW2)	9280 x 6944 64MP Video: 7680 x 4320 @ 24 fps	9.216 mm (~1/1.72")	0.8 μm	ISOCELL Plus	RGB	<b>Rear:</b> <a href="#">Samsung Galaxy S20</a> <a href="#">Samsung Galaxy S20+</a> <a href="#">Samsung Galaxy Note 20</a>

[https://en.wikipedia.org/wiki/Samsung\\_CMOS](https://en.wikipedia.org/wiki/Samsung_CMOS)

# 内容提要

---

- 采样定理应用示例
  - 图像的空间域滤波(复习)
  - 图像的频率域滤波(复习)
  - 一维离散傅里叶变换(复习)
  - 图像的二维离散傅里叶变换的详细介绍
  - 图像的二维离散傅里叶变换的计算
  - 图像的正交变换
  - 图像的距离变换
  - 基于深度学习的图像变换简介
-

## 两类图像空间域滤波器

---

- 平滑空间滤波器：通常用于抑制噪声
- 锐化空间滤波器：通常用于突出图像细节

# 图像卷积与相关运算的一些特性

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \star (g + h) = (f \star g) + (f \star h)$

# 常用的空间平滑滤波器

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \frac{1}{4.8976} \times \begin{array}{|c|c|c|} \hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

## 方型濾波器 (I)

$$\frac{(m-1)}{2}\sqrt{2}$$

$$\frac{(m-1)}{2}\sqrt{2}$$

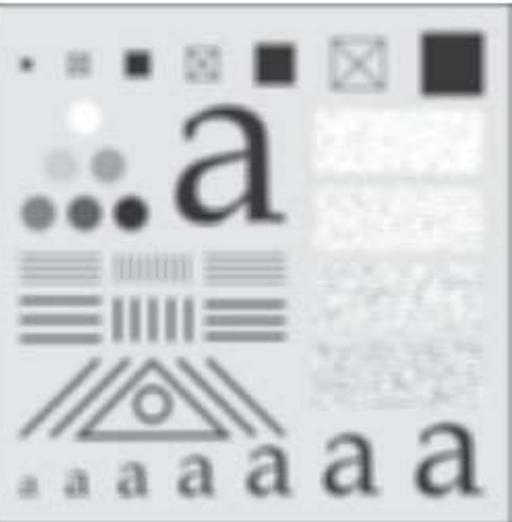
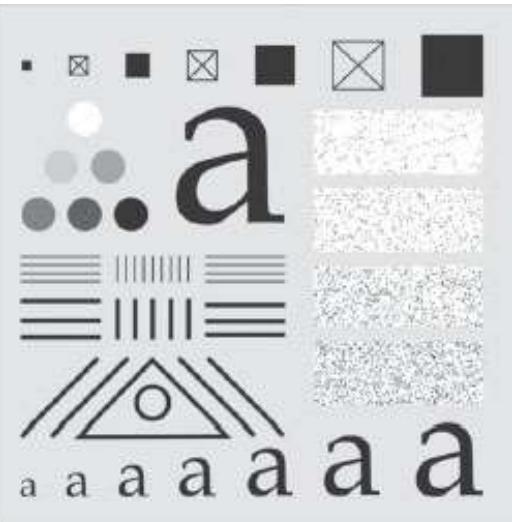
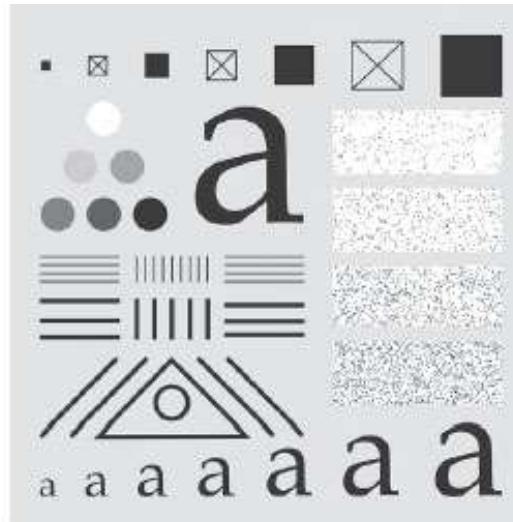
$4\sqrt{2}$	5	$2\sqrt{5}$	$\sqrt{17}$	4	$\sqrt{17}$	$2\sqrt{5}$	5	$4\sqrt{2}$
5	$3\sqrt{2}$	$\sqrt{13}$	$\sqrt{10}$	3	$\sqrt{10}$	$\sqrt{13}$	$3\sqrt{2}$	5
$2\sqrt{5}$	$\sqrt{13}$	$2\sqrt{2}$	$\sqrt{5}$	2	$\sqrt{5}$	$2\sqrt{2}$	$\sqrt{13}$	$2\sqrt{5}$
$\sqrt{17}$	$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{10}$	$\sqrt{17}$
....	4	3	2	1	0	1	2	3
$\sqrt{17}$	$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{10}$	$\sqrt{17}$
$2\sqrt{5}$	$\sqrt{13}$	$2\sqrt{2}$	$\sqrt{5}$	2	$\sqrt{5}$	$2\sqrt{2}$	$\sqrt{13}$	$2\sqrt{5}$
5	$3\sqrt{2}$	$\sqrt{13}$	$\sqrt{10}$	3	$\sqrt{10}$	$\sqrt{13}$	$3\sqrt{2}$	5
$4\sqrt{2}$	5	$2\sqrt{5}$	$\sqrt{17}$	4	$\sqrt{17}$	$2\sqrt{5}$	5	$4\sqrt{2}$

$$\frac{(m-1)}{2}\sqrt{2}$$

$$\frac{(m-1)}{2} \sqrt{2}$$

## 方型濾波器 (II)

a b  
c d

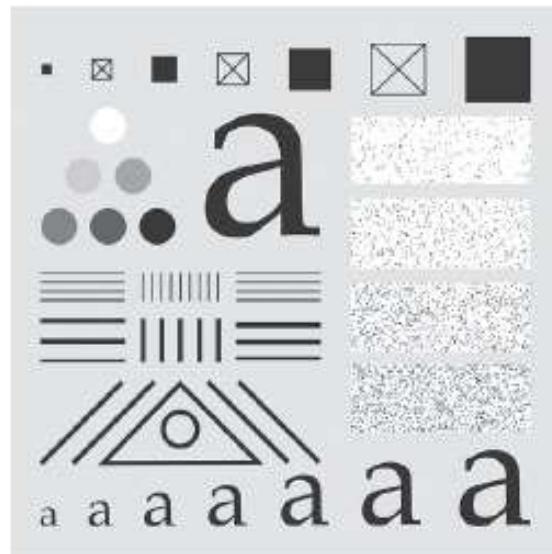


(b) 3x3 (c) 11x11 (d) 21x21

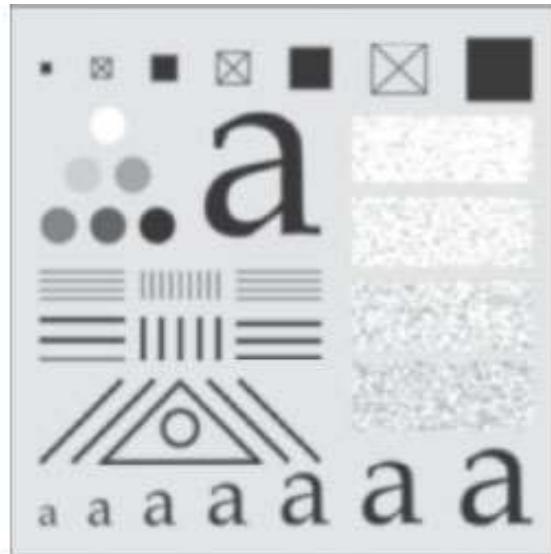
# 高斯滤波器 (I)

	$f$	$g$	$f \times g$	$f \star g$
Mean	$m_f$	$m_g$	$m_{f \times g} = \frac{m_f \sigma_g^2 + m_g \sigma_f^2}{\sigma_f^2 + \sigma_g^2}$	$m_{f \star g} = m_f + m_g$
Standard deviation	$\sigma_f$	$\sigma_g$	$\sigma_{f \times g} = \sqrt{\frac{\sigma_f^2 \sigma_g^2}{\sigma_f^2 + \sigma_g^2}}$	$\sigma_{f \star g} = \sqrt{\sigma_f^2 + \sigma_g^2}$

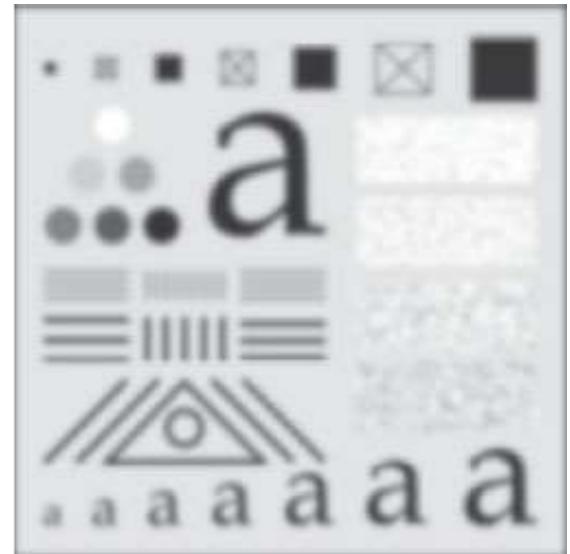
## 高斯濾波器 (II)



a b c

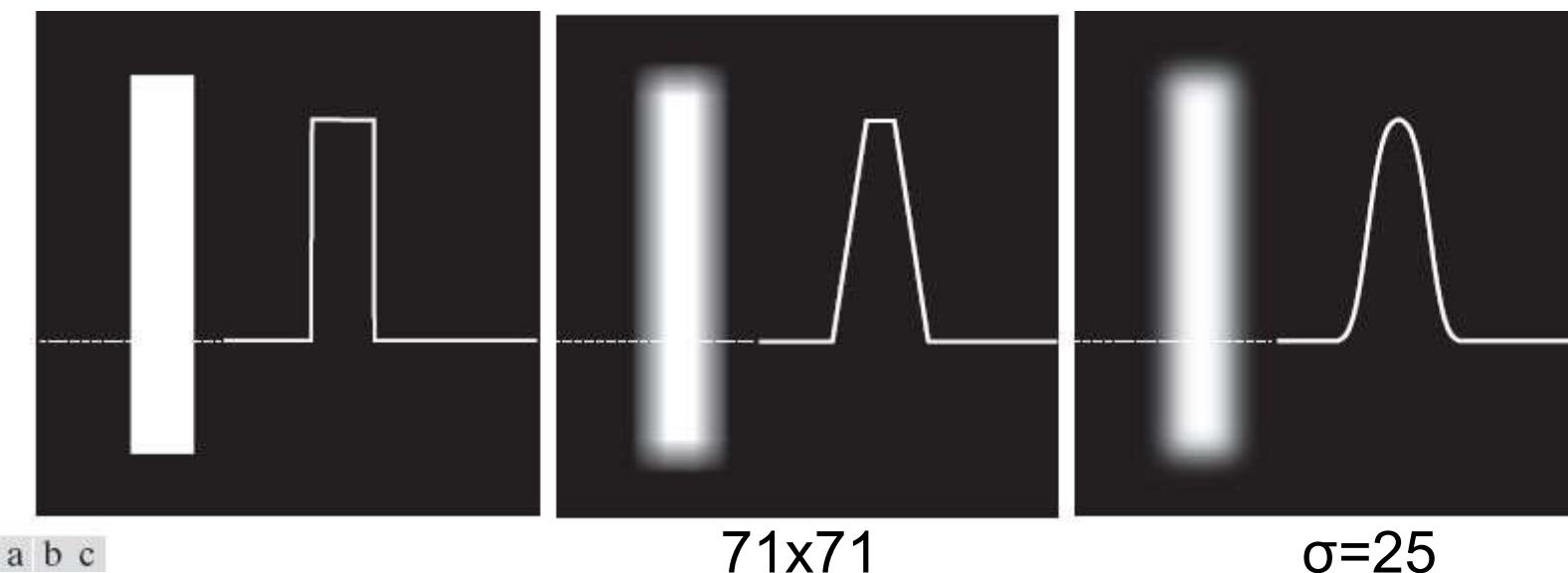


$\sigma=3.5$



$\sigma=7.0$

# 比较方型滤波器和高斯滤波器



## 图像滤波示例：图像导数的计算 (II)

- 一阶导数

$$I_x(i, j) = \frac{I(i+1, j) - I(i-1, j)}{2h} + O(h^2)$$

$$I_y(i, j) = \frac{I(i, j+1) - I(i, j-1)}{2h} + O(h^2)$$

- 二阶导数

$$I_{xx}(i, j) = \frac{I(i+1, j) - 2I(i, j) + I(i-1, j)}{h^2} + O(h)$$

$$I_{yy}(i, j) = \frac{I(i, j+1) - 2I(i, j) + I(i, j-1)}{h^2} + O(h)$$

# 常用的濾波器算子：Laplacian算子 (I)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

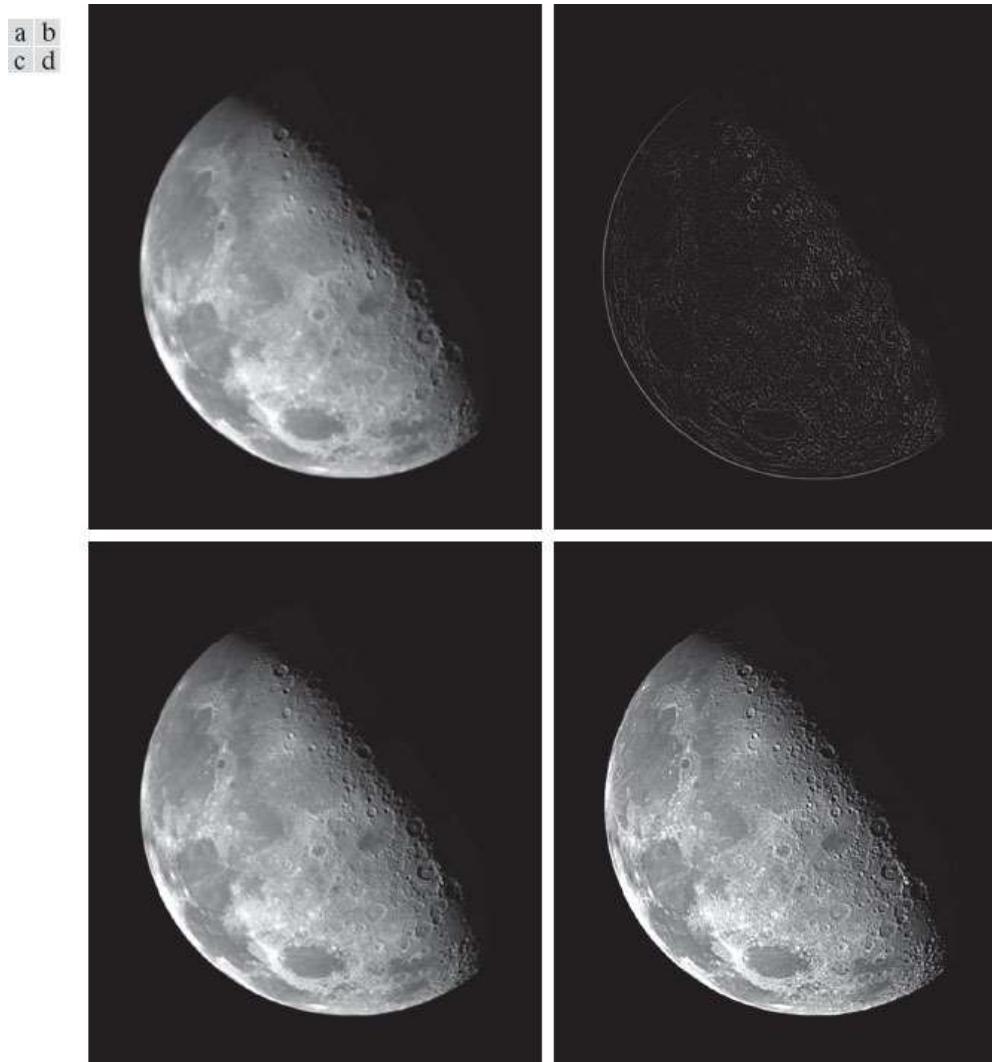
0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

a | b | c | d

(a) Laplacian kernel used to implement Eq. (3-62). (b) Kernel used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other Laplacian kernels.

# 常用的濾波器算子：Laplacian算子 (II)



a b  
c d

**FIGURE 3.46**  
 (a) Blurred image of the North Pole of the moon.  
 (b) Laplacian image obtained using the kernel in Fig. 3.45(a).  
 (c) Image sharpened using Eq. (3-54) with  $c = -1$ .  
 (d) Image sharpened using the same procedure, but with the kernel in Fig. 3.45(b).  
 (Original image courtesy of NASA.)

$$g(x, y) = f(x, y) + c \cdot \nabla^2 f(x, y)$$

## Slide 18

---

**GY1**

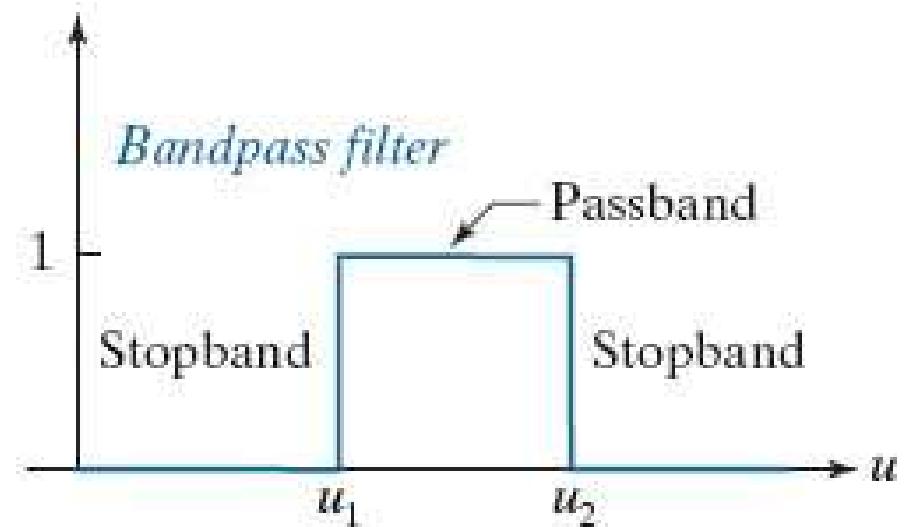
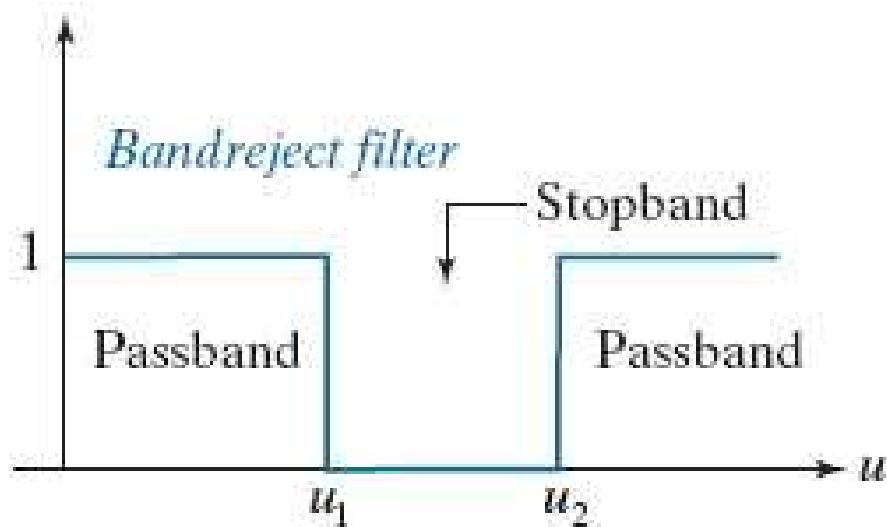
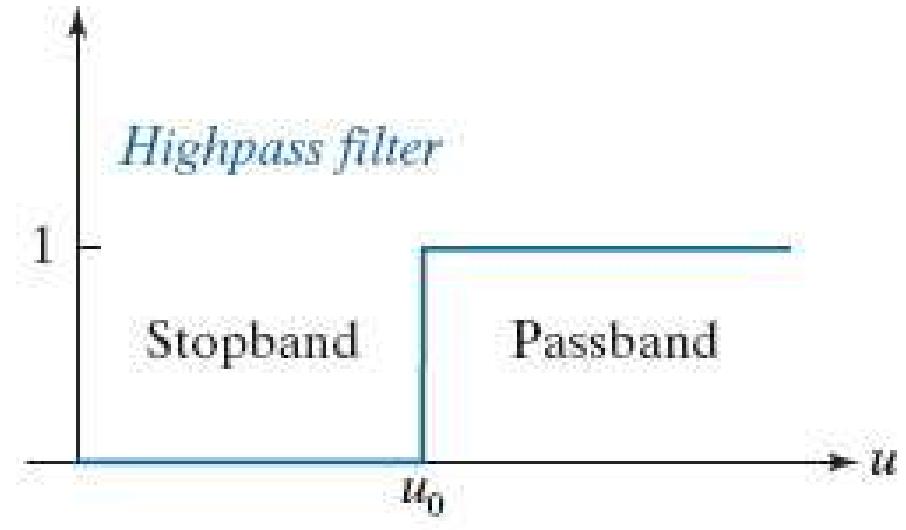
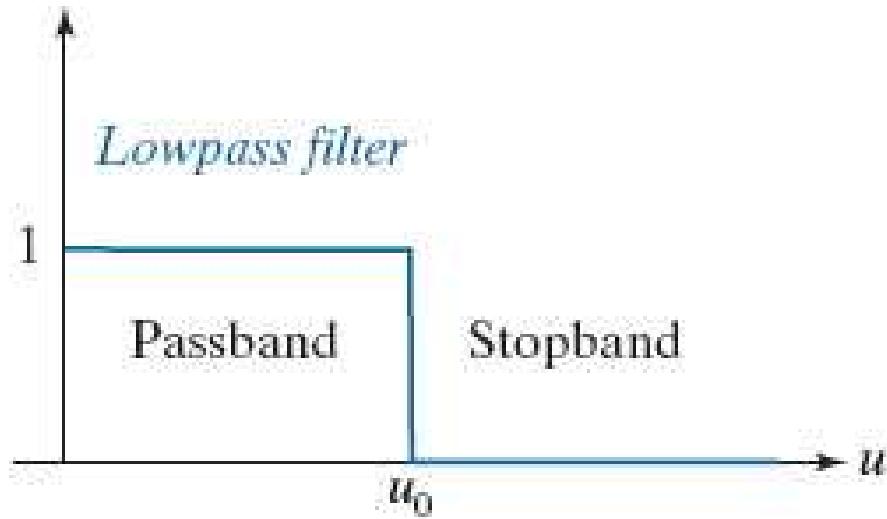
Ge Yang, 10/13/2020

# 锐化滤波器是高通滤波器

Name	DFT Pairs
1) Symmetry properties	See Table 4.1
2) Linearity	$af_1(x,y) + bf_2(x,y) \Leftrightarrow aF_1(u,v) + bF_2(u,v)$
3) Translation (general)	$f(x,y)e^{j2\pi(a_0x/M + v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$ $f(x - x_0, y - y_0) \Leftrightarrow F(u,v)e^{-j2\pi(a_0/M + v_0/N)}$
4) Translation to center of the frequency rectangle, $(M/2, N/2)$	$f(x,y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$ $f(x - M/2, y - N/2) \Leftrightarrow F(u,v)(-1)^{u+v}$
5) Rotation	$f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$ $r = \sqrt{x^2 + y^2} \quad \theta = \tan^{-1}(y/x) \quad \omega = \sqrt{u^2 + v^2} \quad \varphi = \tan^{-1}(v/u)$
6) Convolution theorem <sup>†</sup>	$f \star h)(x,y) \Leftrightarrow (F \bullet H)(u,v)$ $(f \bullet h)(x,y) \Leftrightarrow (1/MN)[(F \star H)(u,v)]$
7) Correlation theorem <sup>†</sup>	$(f \diamondsuit h)(x,y) \Leftrightarrow (F^* \bullet H)(u,v)$ $(f^* \bullet h)(x,y) \Leftrightarrow (1/MN)[(F \diamondsuit H)(u,v)]$
8) Discrete unit impulse	$\delta(x,y) \Leftrightarrow 1$ $1 \Leftrightarrow MN\delta(u,v)$
9) Rectangle	$\text{rect}[a,b] \Leftrightarrow ab \frac{\sin(\pi ua)}{(\pi ua)} \frac{\sin(\pi vb)}{(\pi vb)} e^{-j\pi(ua+vb)}$
10) Sine	$\sin(2\pi u_0 x/M + 2\pi v_0 y/N) \Leftrightarrow \frac{jMN}{2} [\delta(u + u_0, v + v_0) - \delta(u - u_0, v - v_0)]$
11) Cosine	$\cos(2\pi u_0 x/M + 2\pi v_0 y/N) \Leftrightarrow \frac{1}{2} [\delta(u + u_0, v + v_0) + \delta(u - u_0, v - v_0)]$
The following Fourier transform pairs are derivable only for continuous variables, denoted as before by $t$ and $z$ for spatial variables and by $\mu$ and $\nu$ for frequency variables. These results can be used for DFT work by sampling the continuous forms.	
12) Differentiation (the expressions $\left(\frac{\partial}{\partial t}\right)^m \left(\frac{\partial}{\partial z}\right)^n f(t,z)$ on the right assume that $f(\pm\infty, \pm\infty) = 0$ .)	$\Leftrightarrow (j2\pi\mu)^m (j2\pi\nu)^n F(\mu, \nu)$ $\frac{\partial^m f(t,z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m F(\mu, \nu); \quad \frac{\partial^n f(t,z)}{\partial z^n} \Leftrightarrow (j2\pi\nu)^n F(\mu, \nu)$
13) Gaussian	$A2\pi\sigma^2 e^{-2\pi^2 r^2(t^2+z^2)} \Leftrightarrow Ae^{-(\mu^2+\nu^2)/2\sigma^2} \quad (A \text{ is a constant})$

# 不同类型的频率域滤波器

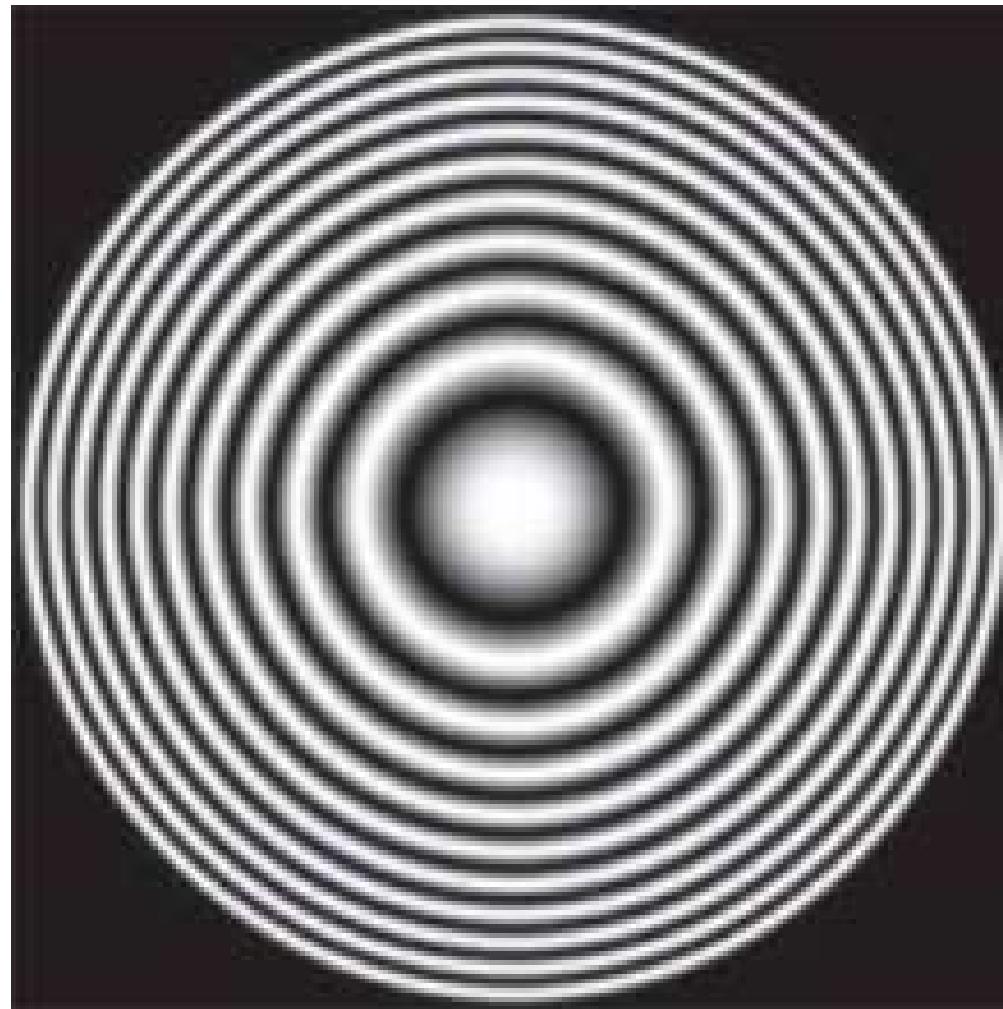
a b  
c d



# 不同类型的频率域滤波器对应的空间滤波器

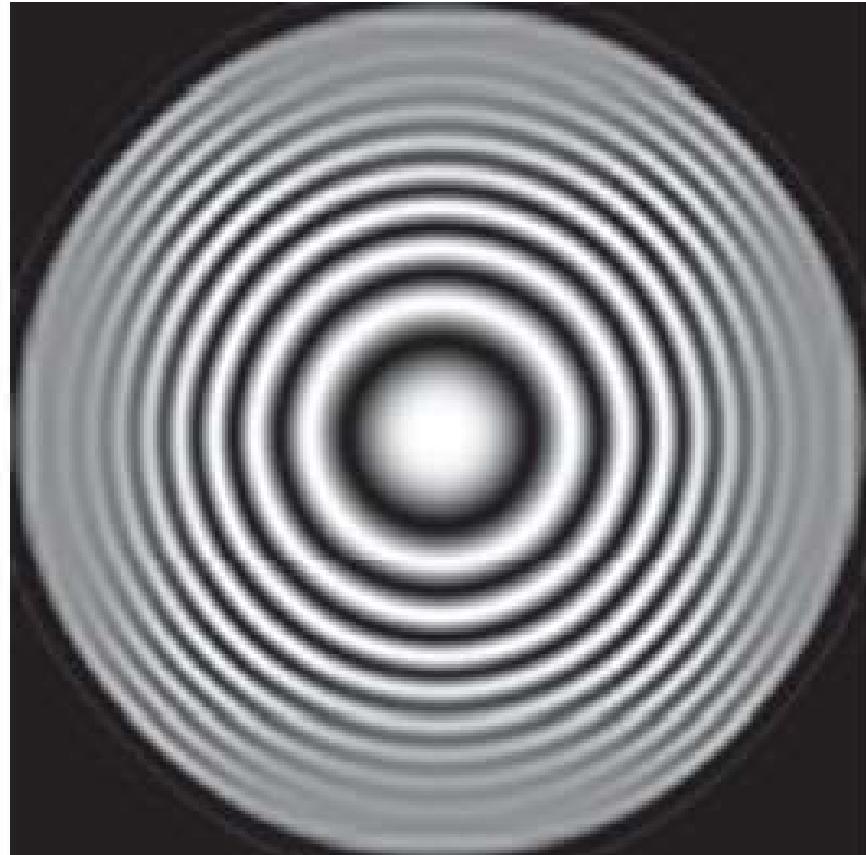
Filter type	Spatial kernel in terms of lowpass kernel, $lp$
Lowpass	$lp(x, y)$
Highpass	$hp(x, y) = \delta(x, y) - lp(x, y)$
Bandreject	$br(x, y) = lp_1(x, y) + hp_2(x, y)$ $= lp_1(x, y) + [\delta(x, y) - lp_2(x, y)]$
Bandpass	$bp(x, y) = \delta(x, y) - br(x, y)$ $= \delta(x, y) - [lp_1(x, y) + [\delta(x, y) - lp_2(x, y)]]$

## 不同滤波器效果示例 (I)

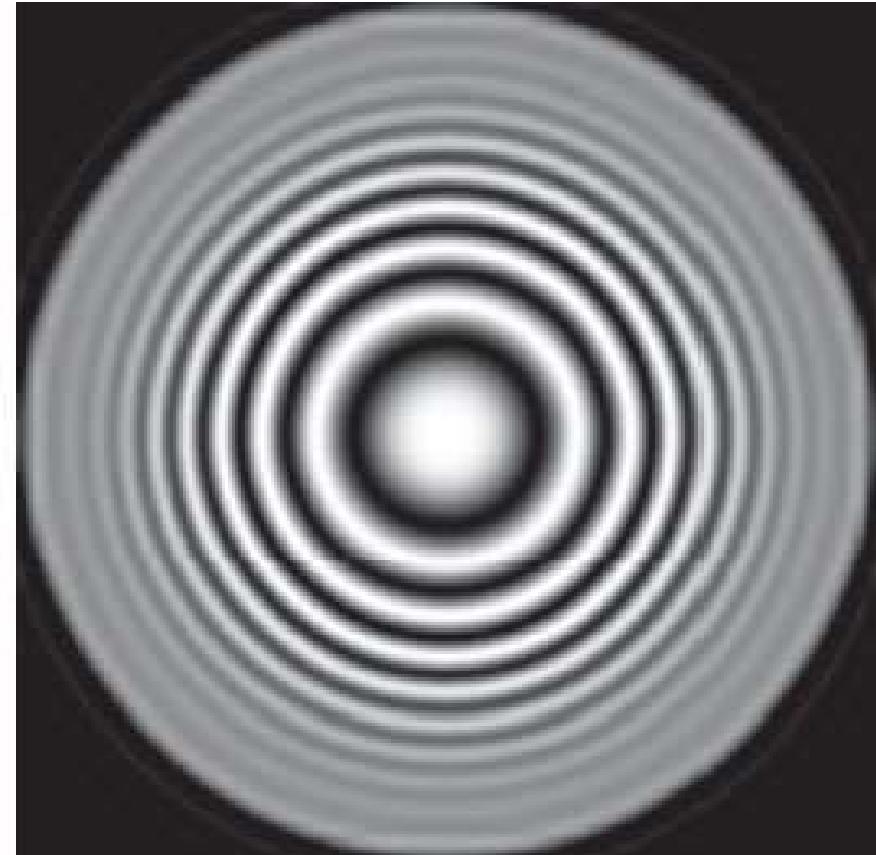


## 不同濾波器效果示例 (II)

a b

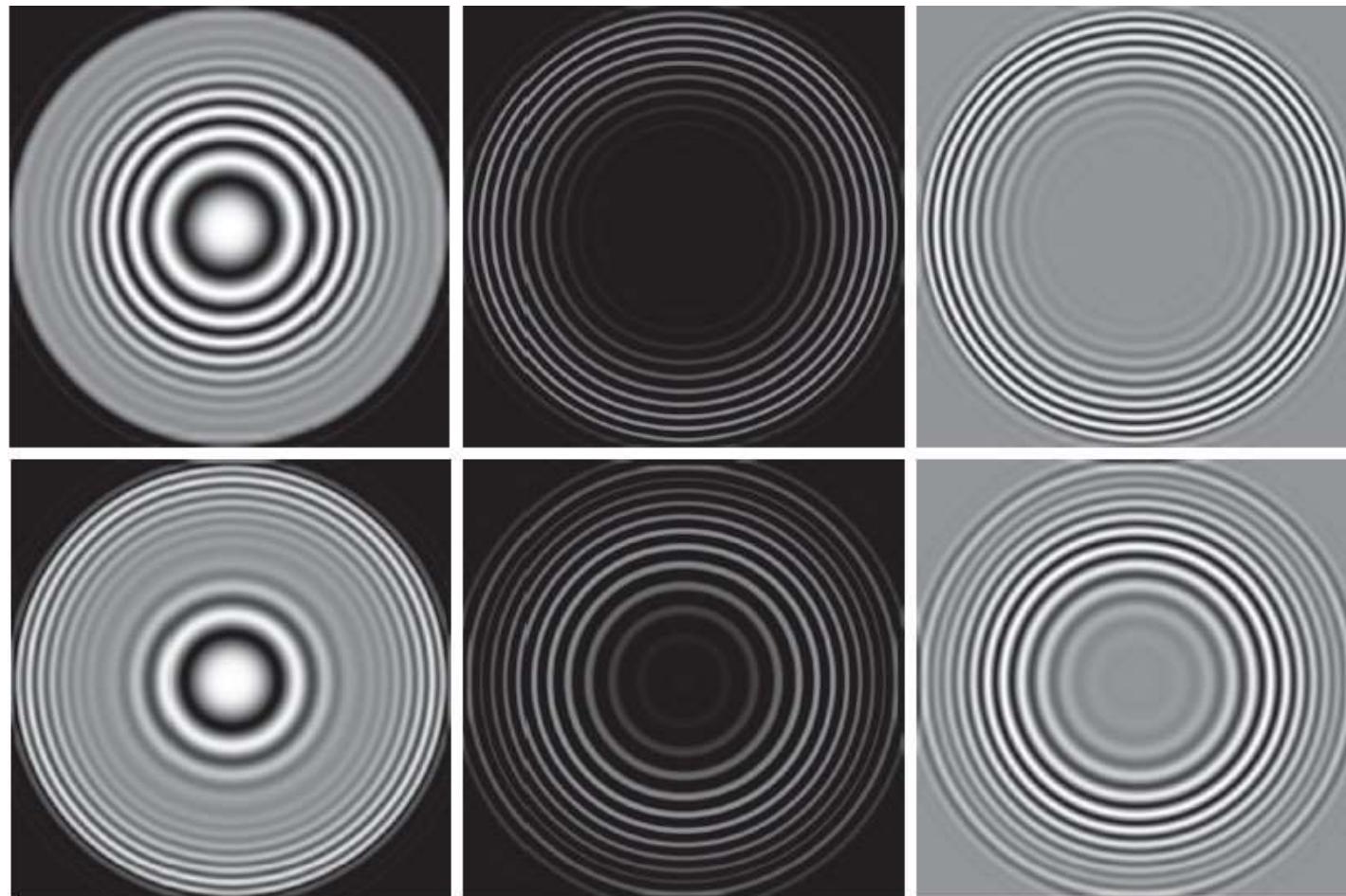


可分低通濾波



各向同性低通濾波

## 不同滤波器效果示例 (III)



a b c  
d e f

(a) 低通  
(d) 带阻

(b) 高通  
(e) 带通

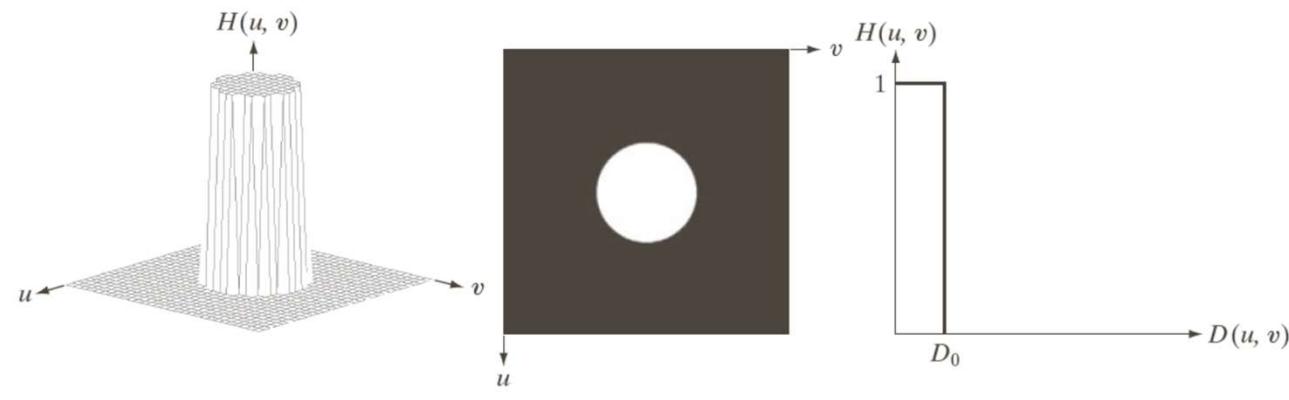
(c) 灰度调整  
(f) 灰度调整

# 内容提要

---

- 采样定理应用示例
  - 图像的空间域滤波(复习)
  - **图像的频率域滤波(复习)**
  - 一维离散傅里叶变换(复习)
  - 图像的二维离散傅里叶变换的详细介绍
  - 图像的二维离散傅里叶变换的计算
  - 图像的正交变换
  - 图像的距离变换
  - 基于深度学习的图像变换简介
-

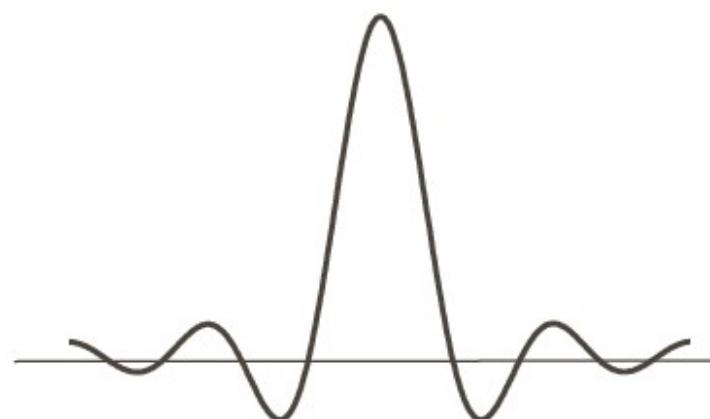
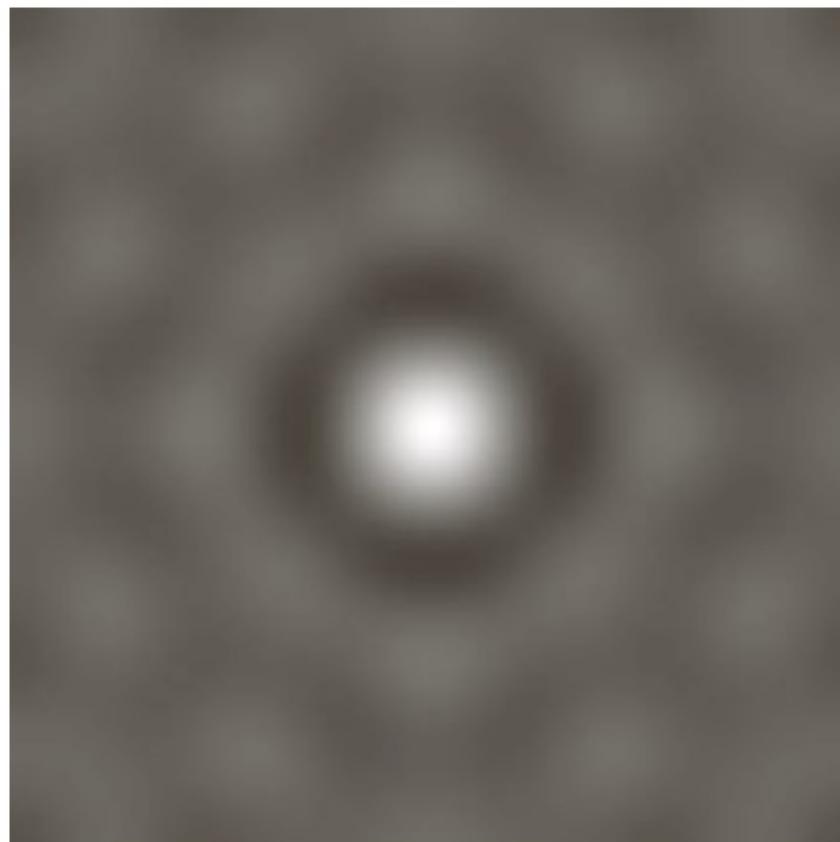
# 代表性濾波器：理想低通濾波器 ILPF



a | b | c

**FIGURE 4.40** (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

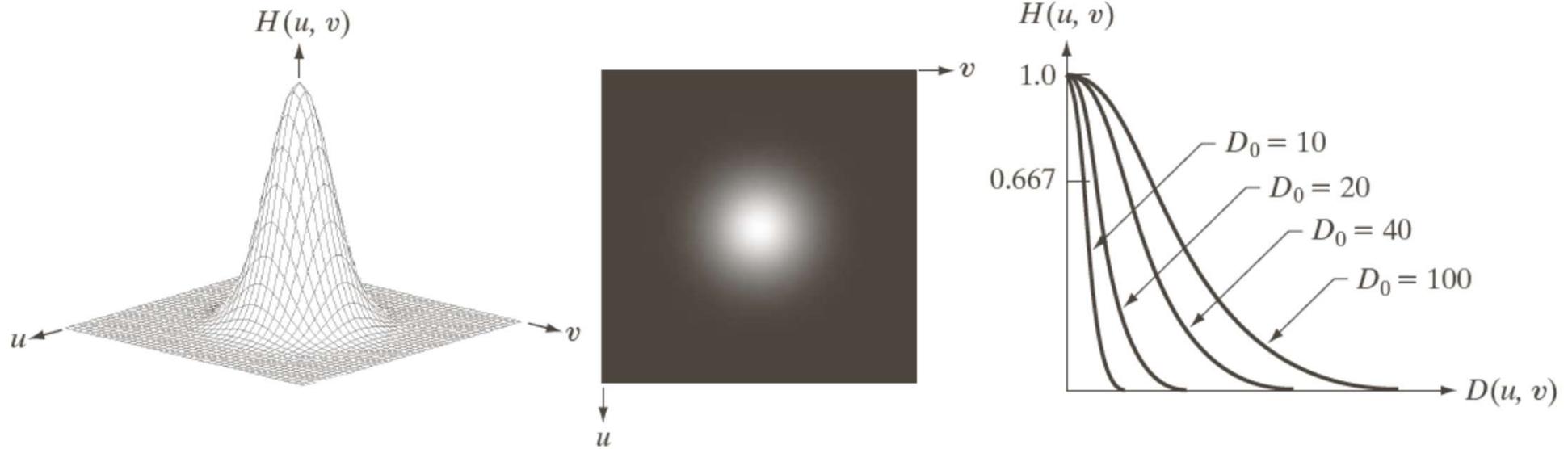
# ILPF的空间域特性



a b

**FIGURE 4.43**  
(a) Representation  
in the spatial  
domain of an  
ILPF of radius 5  
and size  
 $1000 \times 1000$ .  
(b) Intensity  
profile of a  
horizontal line  
passing through  
the center of the  
image.

# 代表性濾波器：高斯低通濾波器 GLPF



a b c

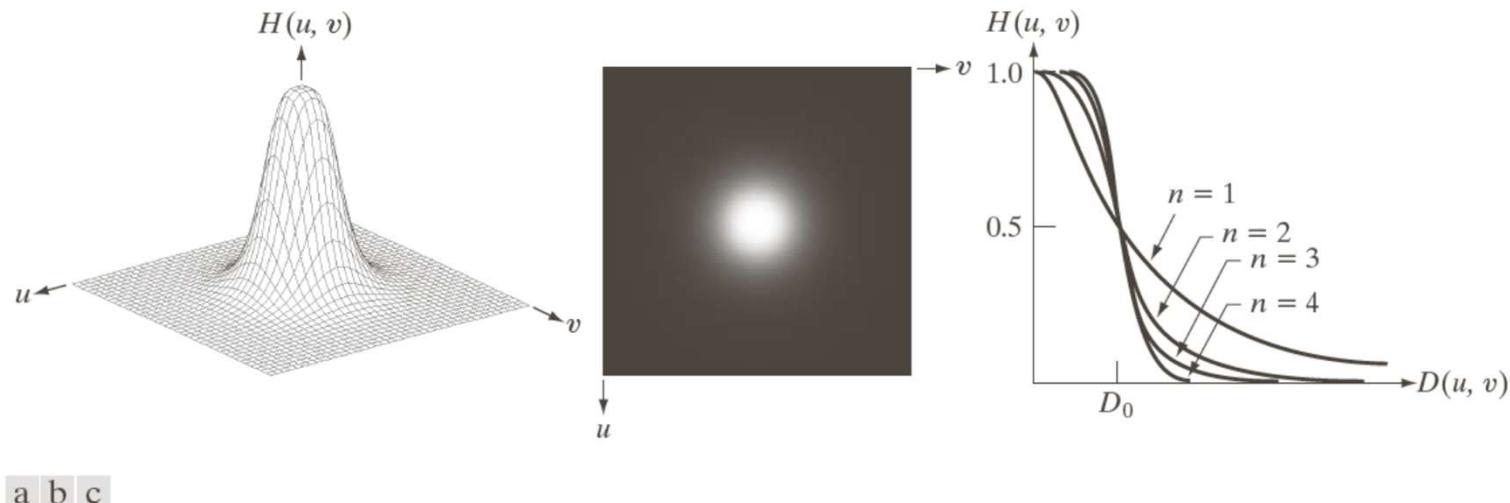
**FIGURE 4.47** (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of  $D_0$ .

# 代表性濾波器：巴特沃斯低通濾波器 BLPF

**TABLE 4.4**

Lowpass filters.  $D_0$  is the cutoff frequency and  $n$  is the order of the Butterworth filter.

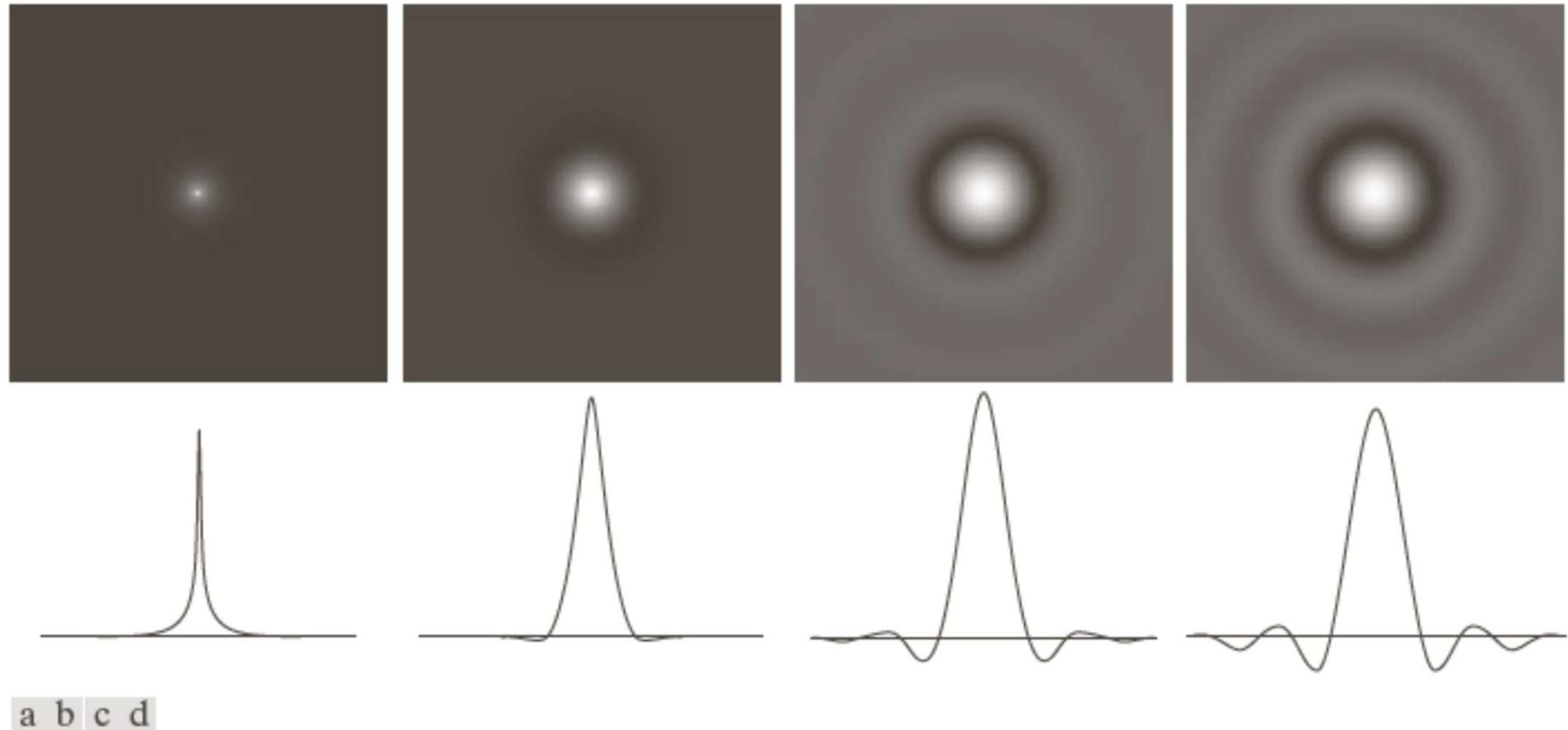
Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$	$H(u, v) = \frac{1}{1 + [D(u, v)/D_0]^{2n}}$	$H(u, v) = e^{-D^2(u,v)/2D_0^2}$



a b c

**FIGURE 4.44** (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

## BLPF的空间域特性

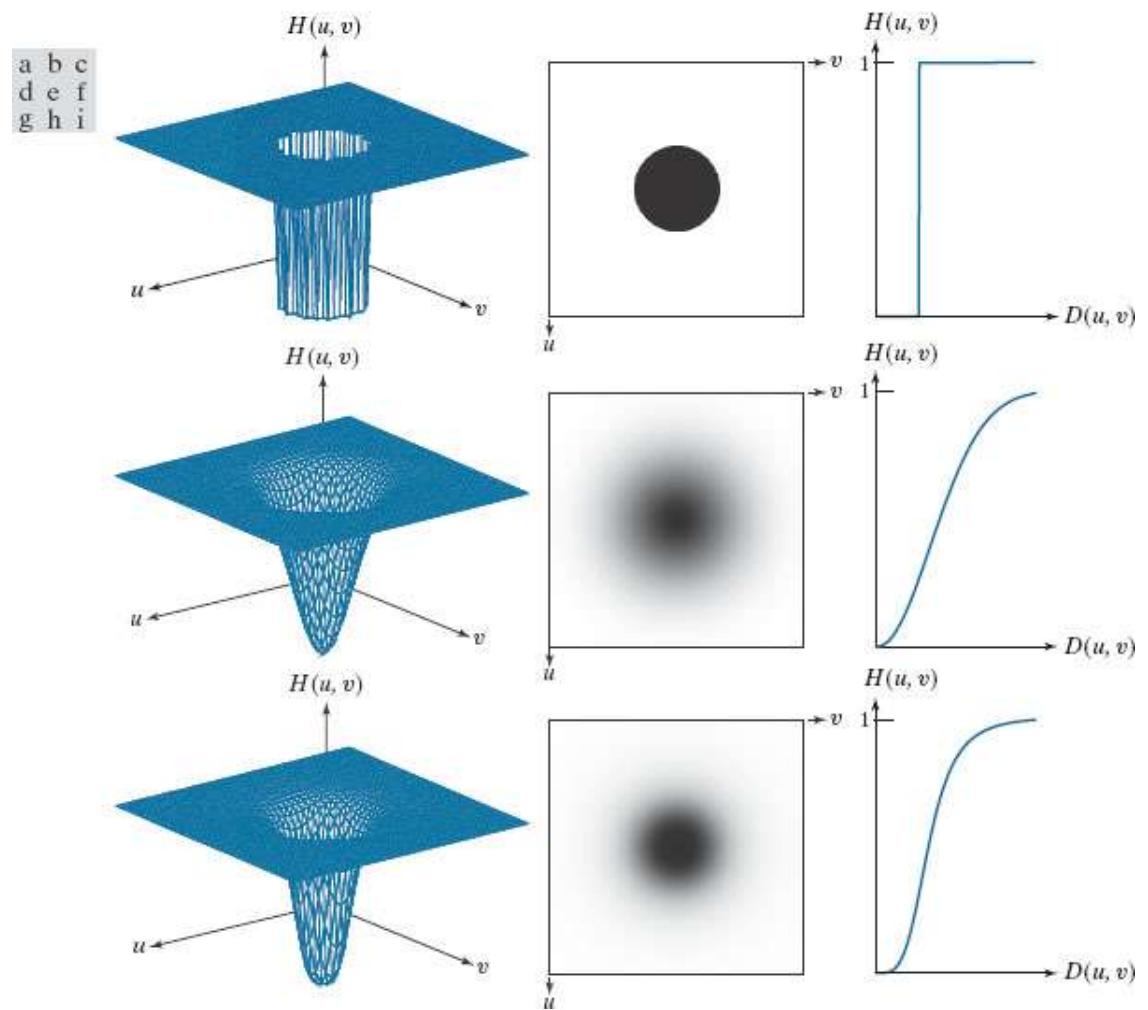


a b c d

**FIGURE 4.46** (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is  $1000 \times 1000$  and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

# IHPF, GHPF, BHPF的频率域特性

$$H_{HP}(u, v) = 1 - H_{LP}(u, v)$$

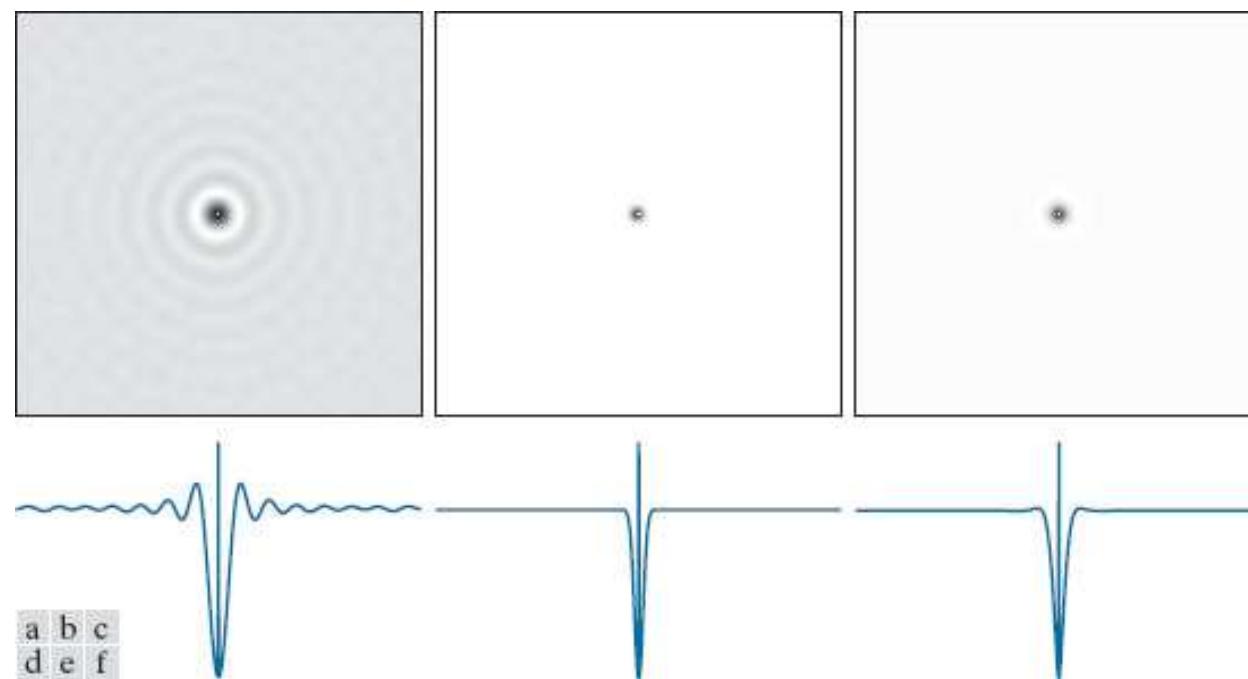


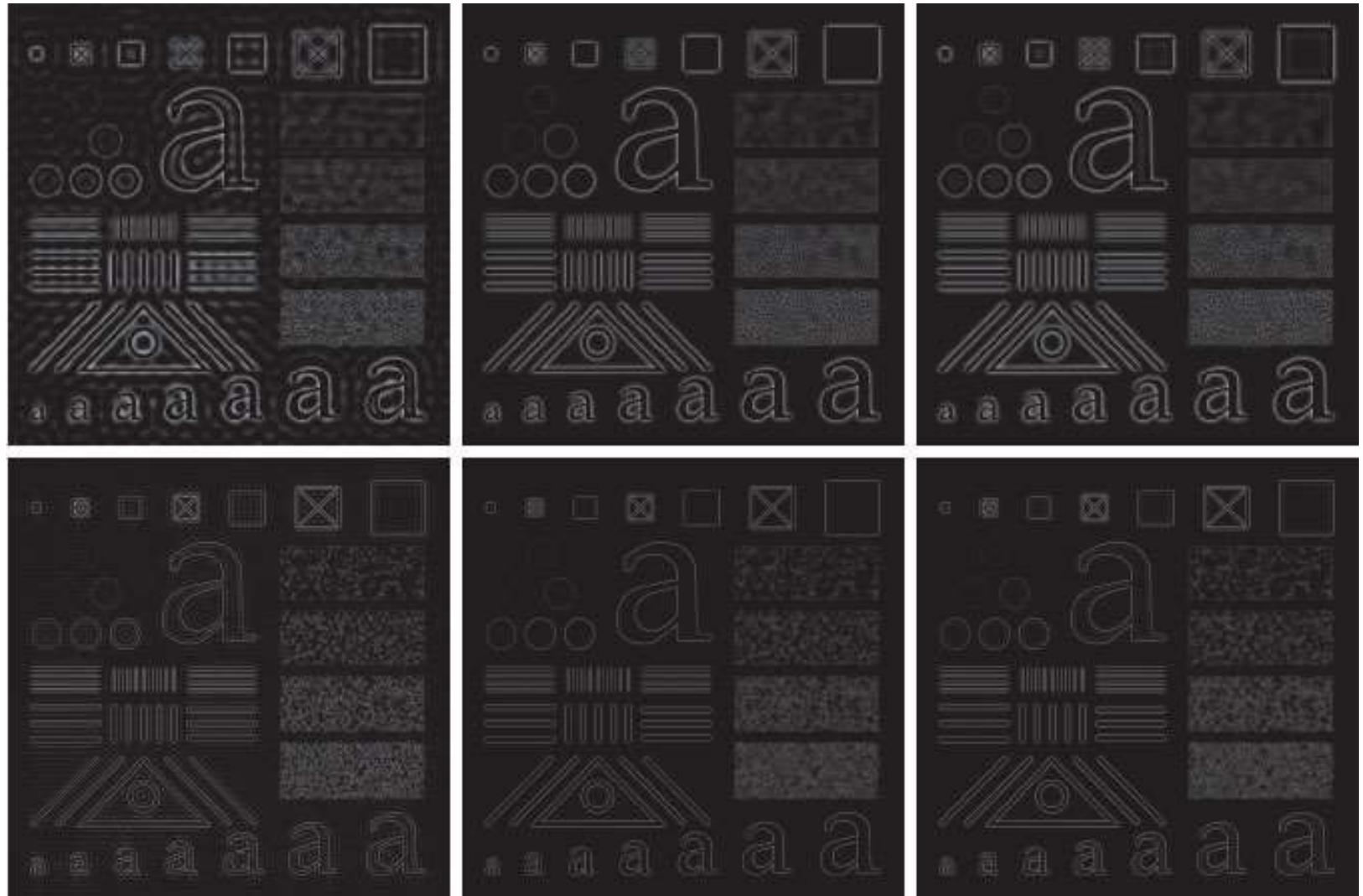
## IHPF, GHPF, BHPF的频率域特性

Highpass filter transfer functions.  $D_0$  is the cutoff frequency and  $n$  is the order of the Butterworth transfer function.

Ideal	Gaussian	Butterworth
$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$	$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$

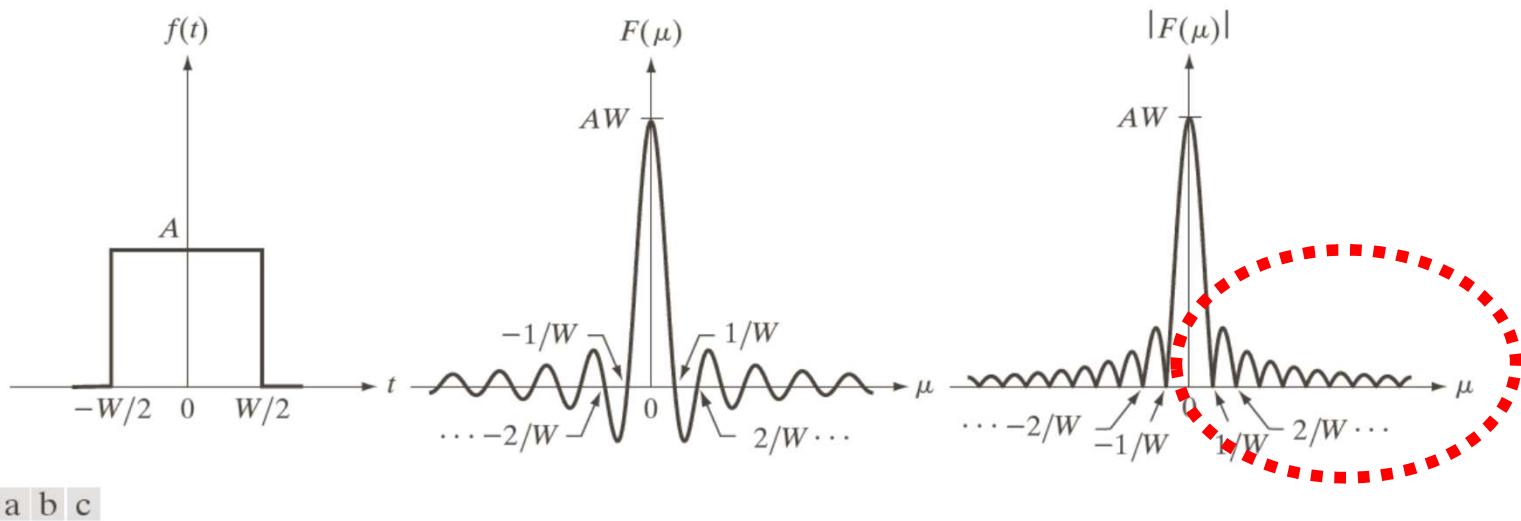
# IHPF, GHPF, BHPF的空间域特性





a b c  
d e f

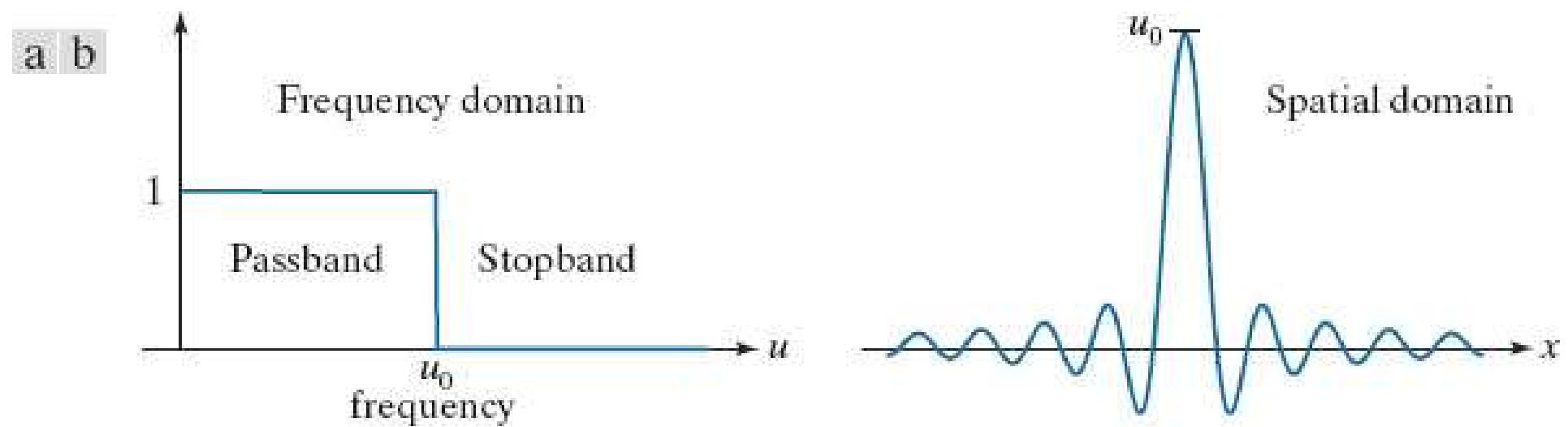
# 均值滤波器的频率域模型



a b c

**FIGURE 4.4** (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

# 理想低通滤波器的空间域模型



# 带阻滤波器设计

Bandreject filter transfer functions.  $C_0$  is the center of the band,

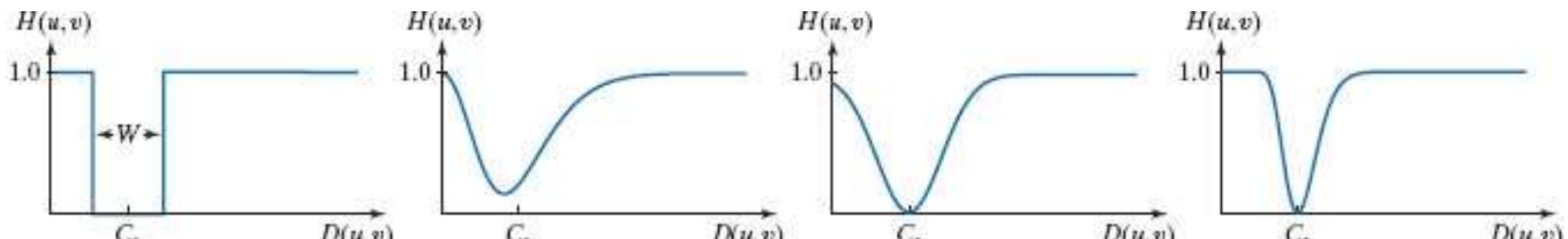
$W$  is the width of the band, and  $D(u,v)$  is the distance from the center of the transfer function to a point  $(u,v)$  in the frequency rectangle.

Ideal (IBRF)	Gaussian (GBRF)	Butterworth (BBRF)
$H(u,v) = \begin{cases} 0 & \text{if } C_0 - \frac{W}{2} \leq D(u,v) \leq C_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = 1 - e^{-\left[\frac{D^2(u,v) - C_0^2}{D(u,v)W}\right]}$	$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)W}{D^2(u,v) - C_0^2}\right]^{2n}}$

# 带阻滤波器设计

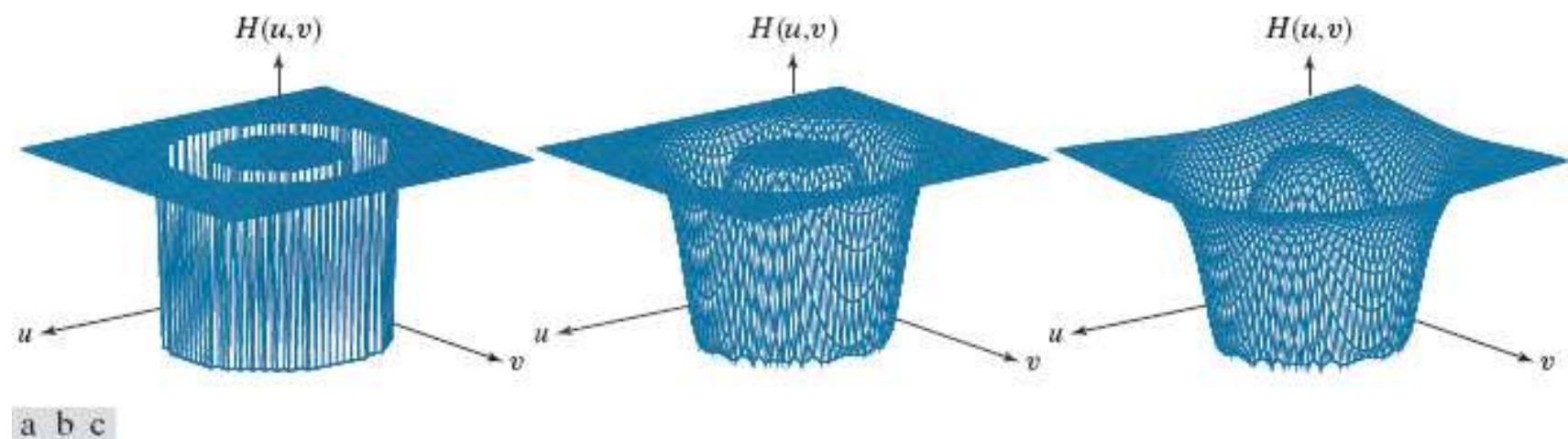
a b c d

**FIGURE 4.61** Radial cross sections. (a) Ideal bandreject filter transfer function. (b) Bandreject transfer function formed by the sum of Gaussian lowpass and highpass filter functions. (The minimum is not 0 and does not align with  $C_0$ .) (c) Radial plot of Eq. (4-149). (The minimum is 0 and is properly aligned with  $C_0$ , but the value at the origin is not 1.) (d) Radial plot of Eq. (4-150); this Gaussian-shape plot meets all the requirements of a bandreject filter transfer function.



a b c d

# 带阻滤波器设计

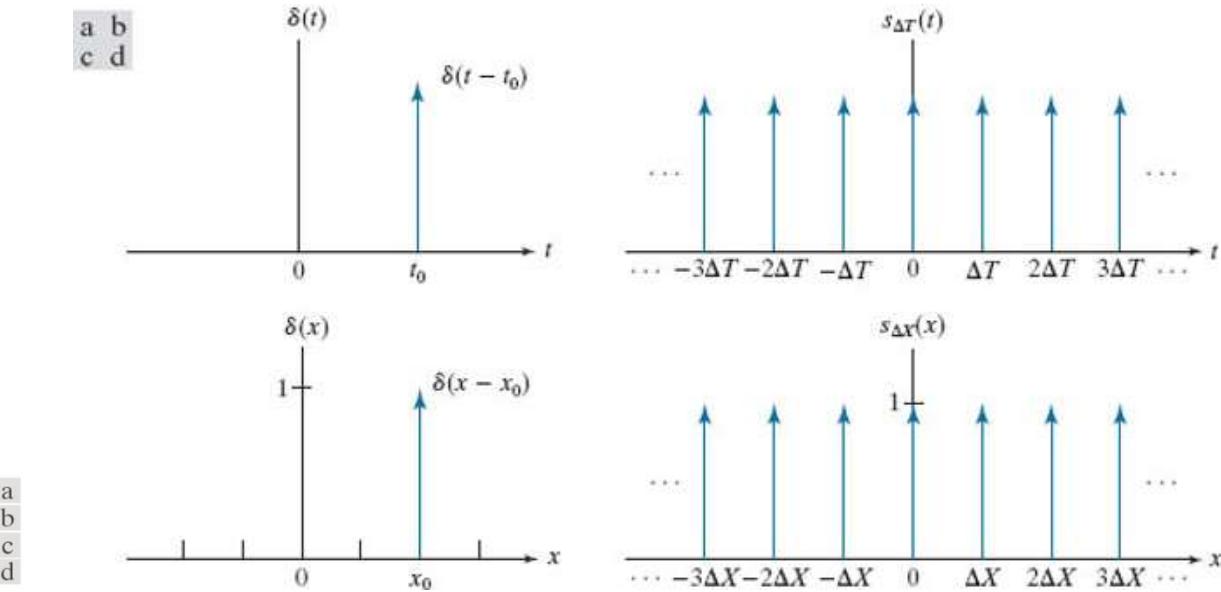
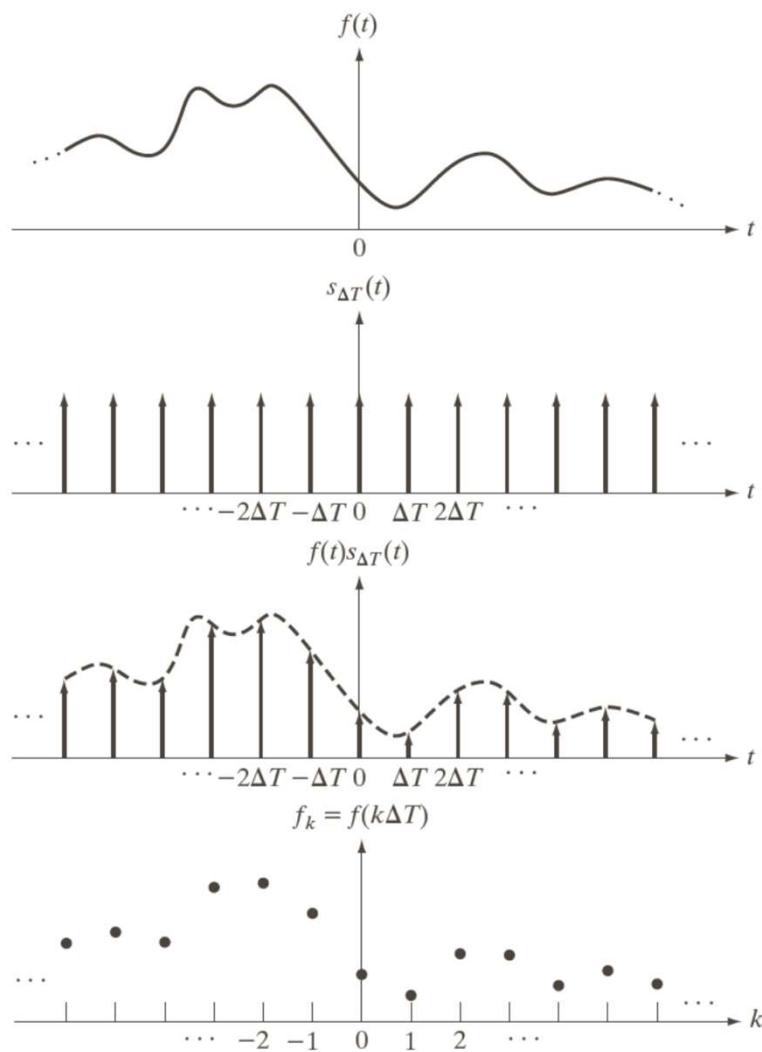


# 内容提要

---

- 采样定理应用示例
  - 图像的空间域滤波(复习)
  - 图像的频率域滤波(复习)
  - **一维离散傅里叶变换(复习)**
  - 图像的二维离散傅里叶变换的详细介绍
  - 图像的二维离散傅里叶变换的计算
  - 图像的正交变换
  - 图像的距离变换
  - 基于深度学习的图像变换简介
-

# 脉冲序列信号的筛选特性



**FIGURE 4.5**  
 (a) A continuous function.  
 (b) Train of impulses used to model the sampling process.  
 (c) Sampled function formed as the product of (a) and (b).  
 (d) Sample values obtained by integration and using the sifting property of the impulse. (The dashed line in (c) is shown for reference. It is not part of the data.)

$$\tilde{f}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t-n\Delta T)$$

# 一维离散傅里叶变换的推导

$$\begin{aligned}\tilde{F}(\mu) &= \int_{-\infty}^{\infty} \tilde{f}(t) e^{-j2\pi\mu t} dt \\&= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t)\delta(t-n\Delta T) e^{-j2\pi\mu t} dt \\&= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\delta(t-n\Delta T) e^{-j2\pi\mu t} dt \\&= \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi\mu n\Delta T} \\&\quad \mu = \frac{n}{M\Delta T} \quad m = 0, 1, 2, \dots, M-1\end{aligned}$$

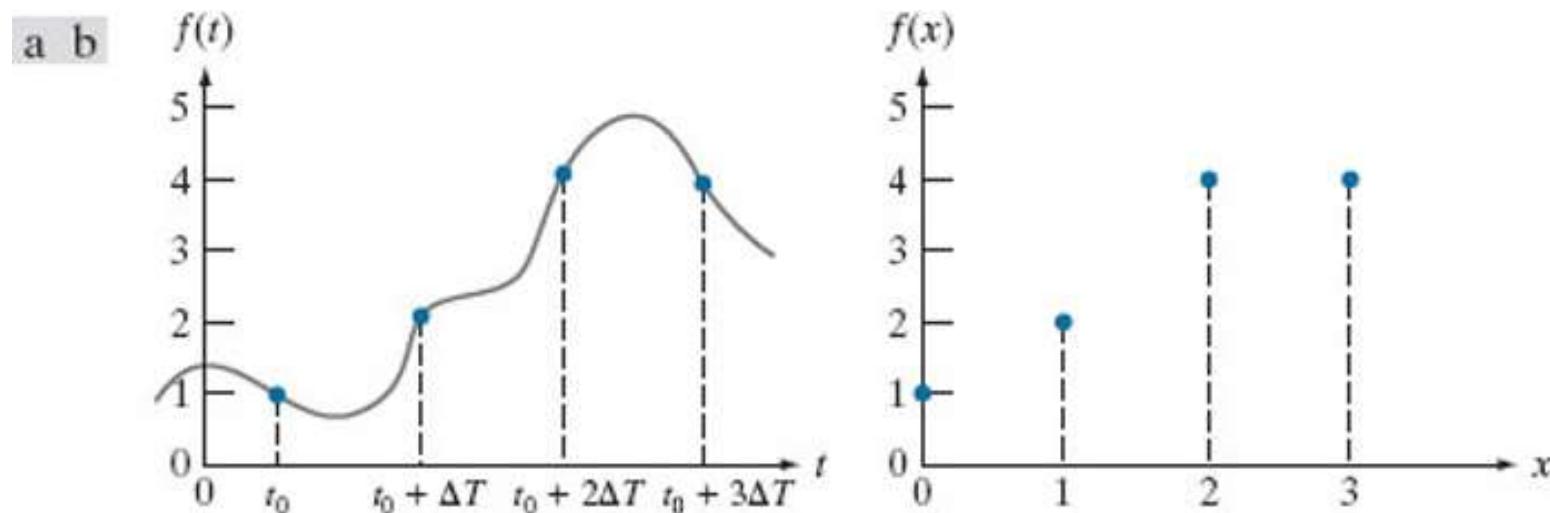
$$\begin{aligned}F_m &= \sum_{n=0}^{M-1} f_n e^{-j2\pi mn/M} \quad m = 0, 1, 2, \dots, M-1 \\f_n &= \frac{1}{M} \sum_{m=0}^{M-1} F_m e^{j2\pi mn/M} \quad n = 0, 1, 2, \dots, M\end{aligned}$$

$$\boxed{\begin{aligned}F(u) &= \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad u = 0, 1, 2, \dots, M-1 \\f(x) &= \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad x = 0, 1, 2, \dots, M-1\end{aligned}}$$

# 一维离散傅里叶变换计算示例

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad u = 0, 1, 2, K, M-1$$

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad x = 0, 1, 2, K, M-1$$



$$F(0) = \sum_{x=0}^3 f(x) = [1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) + 1 \cdot f(3)]$$

# 一维离散傅里叶变换计算示例

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad u = 0, 1, 2, K, M-1$$

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad x = 0, 1, 2, K, M-1$$

$$F(0) = \sum_{x=0}^3 f(x) = [1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) + 1 \cdot f(3)] = 11$$

$$F(1) = \sum_{x=0}^3 e^{-j2\pi/4} f(x) = [e^0 \cdot f(0) + e^{-j2\pi/4} \cdot f(1) + e^{-j2\pi2/4} \cdot f(2) + e^{-j2\pi3/4} \cdot f(3)] = -3 + 2j$$

$$F(2) = \sum_{x=0}^3 e^{-j2\pi2/4} f(x) = [e^0 \cdot f(0) + e^{-j2\pi2/4} \cdot f(1) + e^{-j2\pi4/4} \cdot f(2) + e^{-j2\pi6/4} \cdot f(3)] = -1 - 0j$$

$$F(3) = \sum_{x=0}^3 e^{-j2\pi3/4} f(x) = [e^0 \cdot f(0) + e^{-j2\pi3/4} \cdot f(1) + e^{-j2\pi6/4} \cdot f(2) + e^{-j2\pi9/4} \cdot f(3)] = -3 - 2j$$

# 一维离散傅里叶变换计算示例

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad u = 0, 1, 2, K, M-1$$

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad x = 0, 1, 2, K, M-1$$

$$F(0) = \sum_{x=0}^7 f(x) = [e^{-j2\pi \cdot 0} \cdot f(0) + e^{-j2\pi \cdot 1} \cdot f(1) + \dots + e^{-j2\pi \cdot 6} \cdot f(6) + e^{-j2\pi \cdot 7} \cdot f(7)]$$

$$F(0) = \sum_{x=0}^{M-1} f(x) = [e^{-j2\pi \cdot 0} \cdot f(0) + e^{-j2\pi \cdot 1} \cdot f(1) + \dots + e^{-j2\pi \cdot (M-2)} \cdot f(M-2) + e^{-j2\pi \cdot (M-1)} \cdot f(M-1)]$$

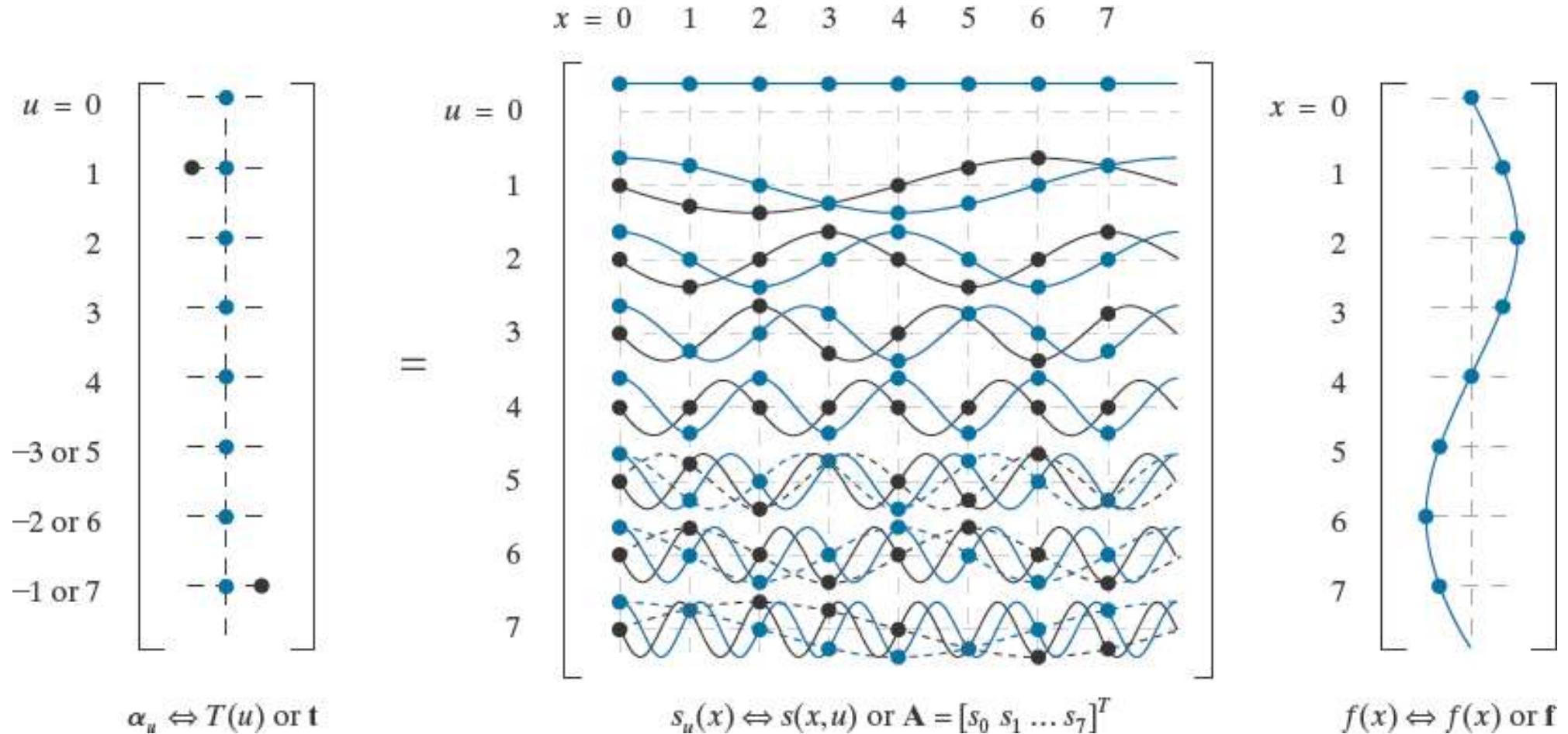
# 一维离散傅里叶变换计算示例

$$F(0) = \sum_{x=0}^{M-1} f(x) = [e^{-j2\pi 0 \cdot 0} \cdot f(0) + e^{-j2\pi 0 \cdot 1} \cdot f(1) + \dots + e^{-j2\pi 0 \cdot (M-2)} \cdot f(M-2) + e^{-j2\pi 0 \cdot (M-1)} \cdot f(M-1)]$$

$$\begin{bmatrix} F(0) \\ F(1) \\ \vdots \\ F(M-1) \end{bmatrix} = \begin{bmatrix} e^{-j2\pi(0 \cdot 0)} & e^{-j2\pi(0 \cdot 1)} & \dots & e^{-j2\pi(0 \cdot (M-1))} \\ e^{-j2\pi(1 \cdot 0)} & e^{-j2\pi(1 \cdot 1)} & \dots & e^{-j2\pi(1 \cdot (M-1))} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j2\pi((M-1) \cdot 0)} & e^{-j2\pi((M-1) \cdot 1)} & \dots & e^{-j2\pi((M-1) \cdot (M-1))} \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(M-1) \end{bmatrix}$$

$$\begin{aligned} F = Af &= [a_1 \quad a_2 \quad \dots \quad a_{M-1}] \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(M-1) \end{bmatrix} \\ &= a_1 f(0) + a_2 f(1) + \dots + a_{M-1} f(M-1) \end{aligned}$$

# 一维傅里叶变换的内积与矩阵表示



## 内容提要

---

- 采样定理应用示例
  - 图像的空间域滤波(复习)
  - 图像的频率域滤波(复习)
  - 一维离散傅里叶变换(复习)
  - **图像的二维离散傅里叶变换的详细介绍**
  - 图像的二维离散傅里叶变换的计算
  - 图像的正交变换
  - 图像的距离变换
  - 基于深度学习的图像变换简介
-

# 一维离散傅里叶变换的推导

$$\begin{aligned}\tilde{F}(\mu) &= \int_{-\infty}^{\infty} \tilde{f}(t) e^{-j2\pi\mu t} dt \\&= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T) e^{-j2\pi\mu t} dt \\&= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\delta(t - n\Delta T) e^{-j2\pi\mu t} dt \\&= \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi\mu n\Delta T} \\&\quad \mu = \frac{n}{M\Delta T} \quad m = 0, 1, 2, \dots, M-1 \\F_m &= \sum_{n=0}^{M-1} f_n e^{-j2\pi mn/M} \quad m = 0, 1, 2, \dots, M-1 \\f_n &= \frac{1}{M} \sum_{m=0}^{M-1} F_m e^{j2\pi mn/M} \quad n = 0, 1, 2, \dots, M\end{aligned}$$

# 图像二维快速傅里叶变换(DFT)的计算 (I)

定义

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi \left( \frac{ux}{M} + \frac{vy}{N} \right)}$$

$$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi \left( \frac{ux}{M} + \frac{vy}{N} \right)}$$

## 内容提要

---

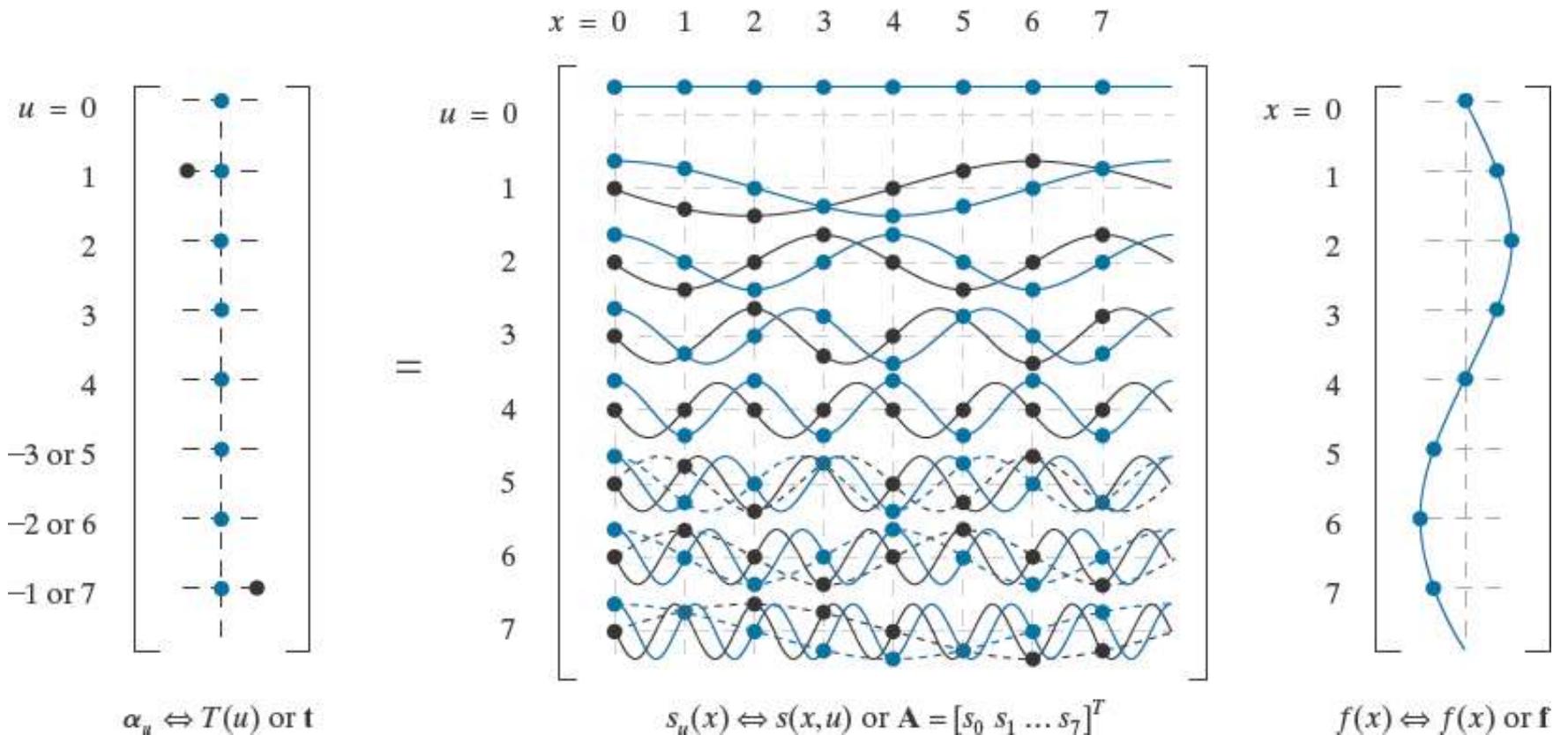
- 采样定理应用示例
  - 图像的空间域滤波(复习)
  - 图像的频率域滤波(复习)
  - 一维离散傅里叶变换(复习)
  - 图像的二维离散傅里叶变换的详细介绍
  - **图像的二维离散傅里叶变换的计算**
  - 图像的正交变换
  - 图像的距离变换
  - 基于深度学习的图像变换简介
-

# 内容提要

---

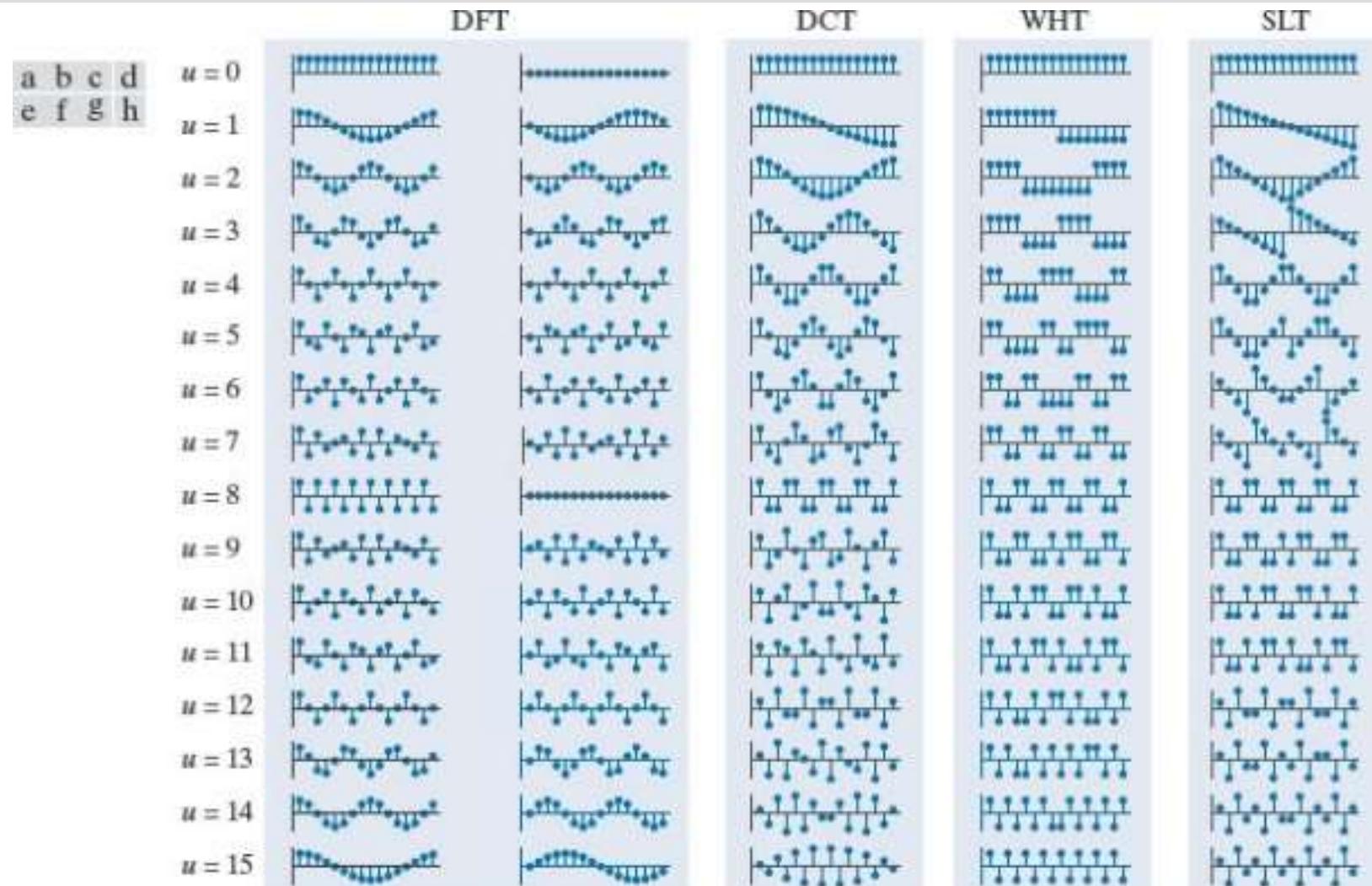
- 采样定理应用示例
  - 图像的空间域滤波(复习)
  - 图像的频率域滤波(复习)
  - 一维离散傅里叶变换(复习)
  - 图像的二维离散傅里叶变换的详细介绍
  - 图像的二维离散傅里叶变换的计算
  - **图像的正交变换**
  - 图像的距离变换
  - 基于深度学习的图像变换简介
-

# 一维傅里叶变换的内积与矩阵表示 (VI)



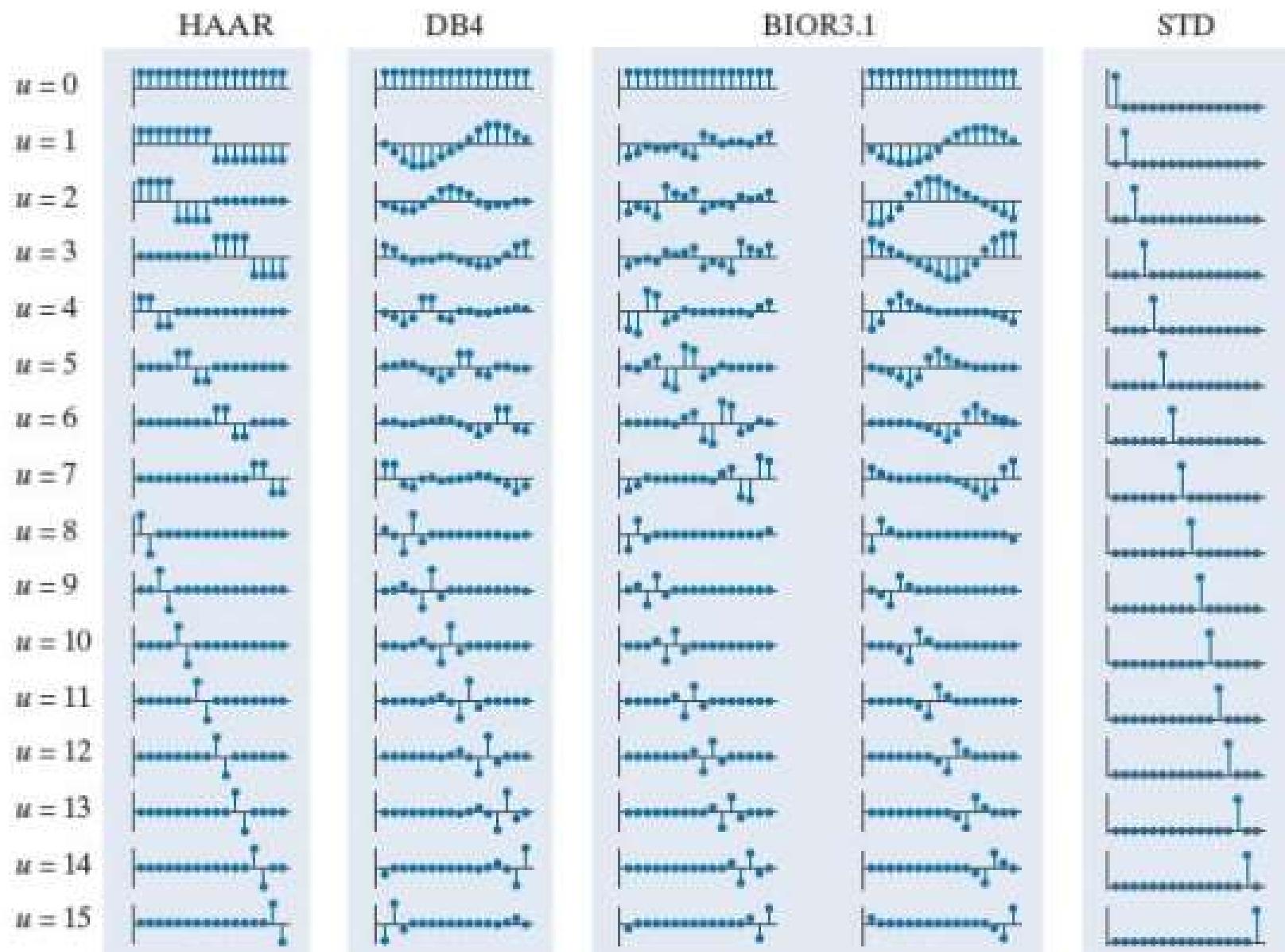
Depicting the continuous Fourier series and 8-point DFT of  $f(x) = \sin(2\pi x)$  as “matrix multiplications.” The real and imaginary parts of all complex quantities are shown in blue and black, respectively. Continuous and discrete functions are represented using lines and dots, respectively. Dashed lines are included to show that  $s_5 = s_3^*$ ,  $s_6 = s_2^*$ ,  $s_7 = s_1^*$  effectively cutting the maximum frequency of the DFT in half. The negative indices to the left of  $\mathbf{t}$  are for the Fourier series computation alone.

# 各种变换基总览



Basis vectors (for  $N = 16$ ) of some commonly encountered transforms: (a) Fourier basis (real and imaginary parts), (b) discrete Cosine basis, (c) Walsh-Hadamard basis, (d) Slant basis, (e) Haar basis, (f) Daubechies basis, (g) Biorthogonal B-spline basis and its dual, and (h) the standard basis, which is included for reference only (i.e., not used as the basis of a transform).

# 各种变换基总览



## 基图像的基本概念

$$F = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} T(u, v) S_{u,v}$$

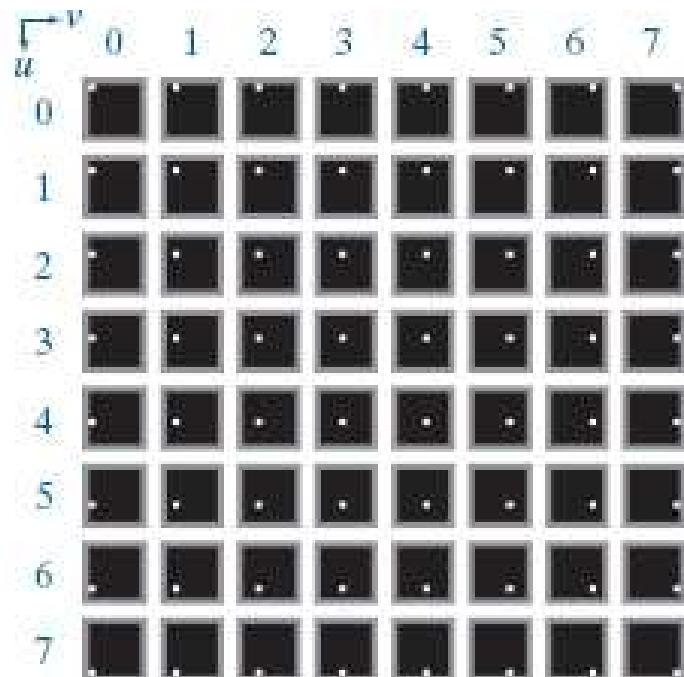
$$F = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0, N-1) \\ f(1,0) & f(1,1) & \cdots & f(1, N-1) \\ \vdots & \vdots & \cdots & \vdots \\ f(N-1,0) & f(N-1,1) & \cdots & f(N-1, N-1) \end{bmatrix}$$

$$S_{u,v} = \begin{bmatrix} s(0,0,u,v) & s(0,1,u,v) & \cdots & s(0,N-1,u,v) \\ s(1,0,u,v) & s(1,1,u,v) & \cdots & s(1,N-1,u,v) \\ \vdots & \vdots & \cdots & \vdots \\ s(N-1,0,u,v) & s(N-1,1,u,v) & \cdots & s(N-1,N-1,u,v) \end{bmatrix}$$

# 标准基变换

a b

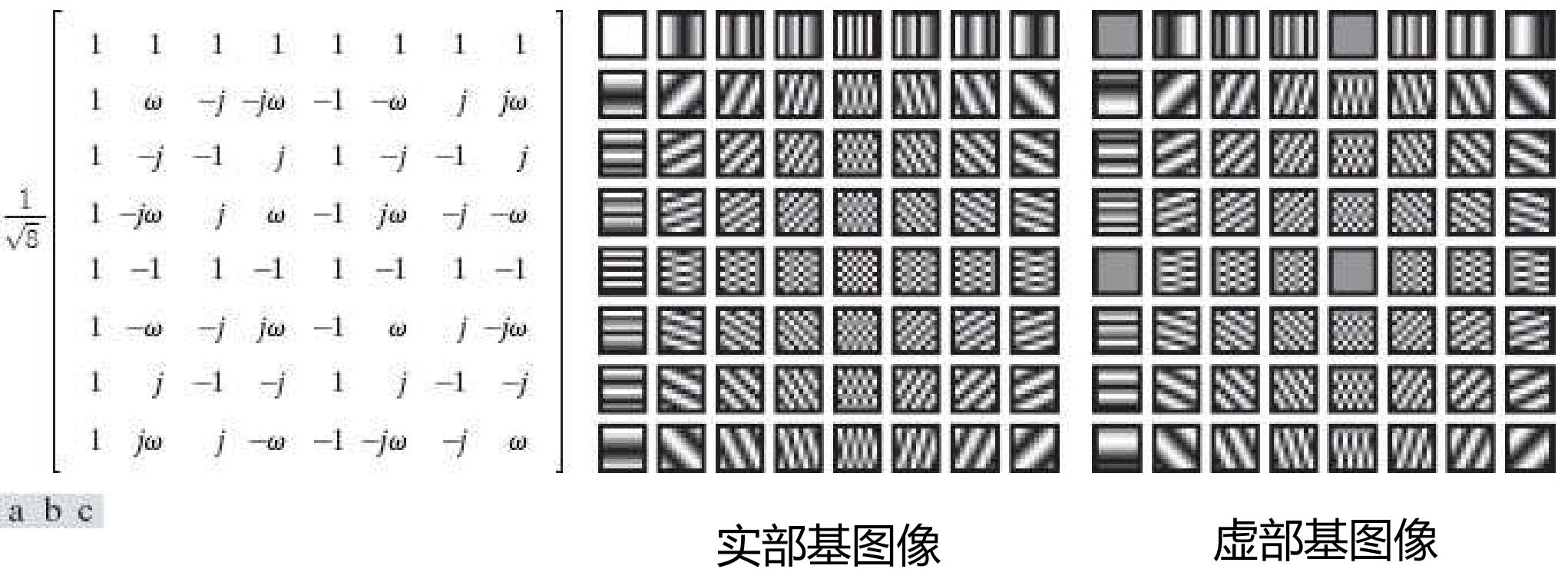
$S_{0,0}$	$S_{0,1}$	...	...	$S_{0,N-1}$
$S_{1,0}$	⋮			⋮
⋮				
$S_{N-1,0}$	⋮	⋮	⋮	$S_{N-1,N-1}$



(a) Basis image organization and (b) a standard basis of size  $8 \times 8$

For clarity, a gray border has been added around each basis image.  
The origin of each basis image (i.e.,  $x = y = 0$ ) is at its top left.

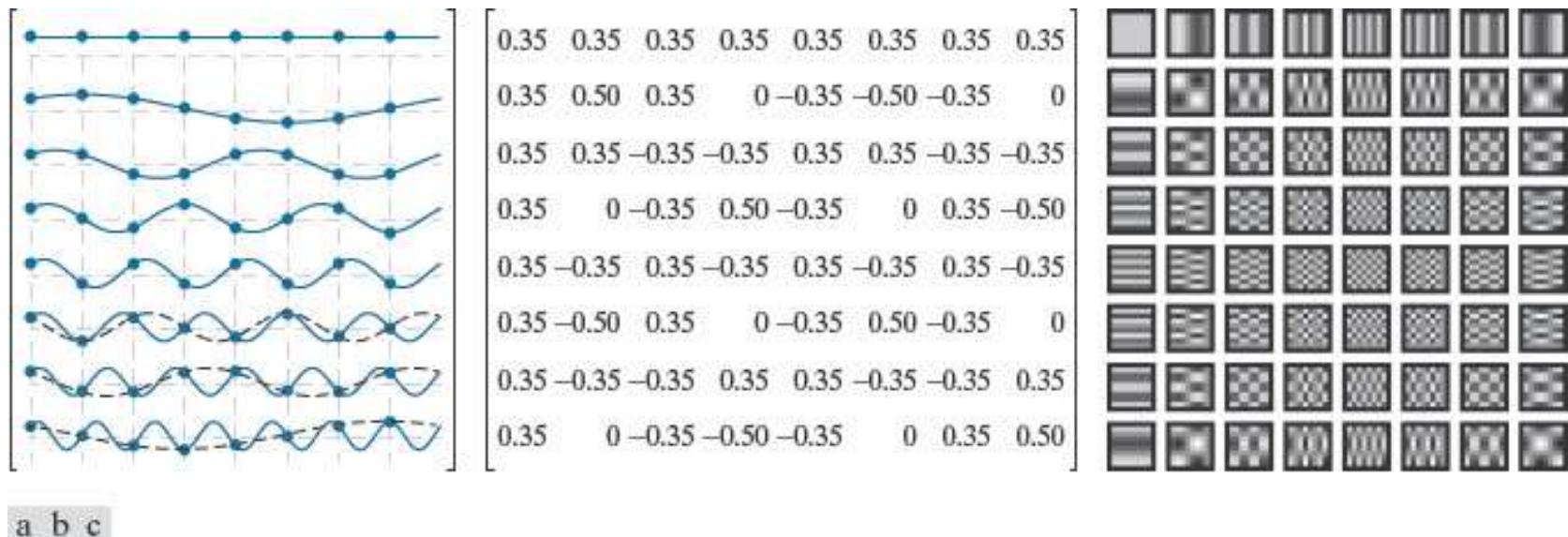
## 二维傅里叶变换的基图像



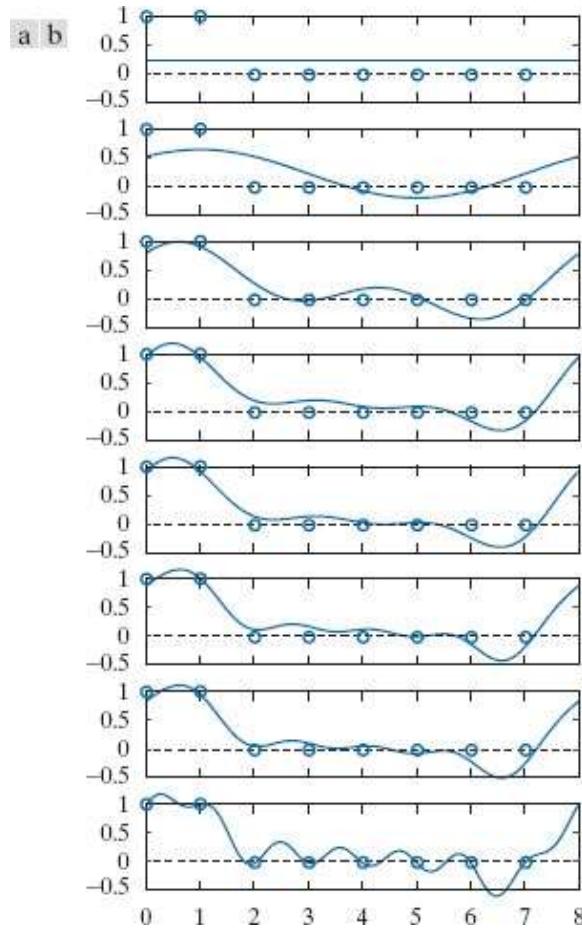
## 二维Hartley变换的基图像

$$s(x, u) = \frac{1}{\sqrt{N}} \left( \cos\left(\frac{2\pi ux}{N}\right) + \sin\left(\frac{2\pi ux}{N}\right) \right)$$

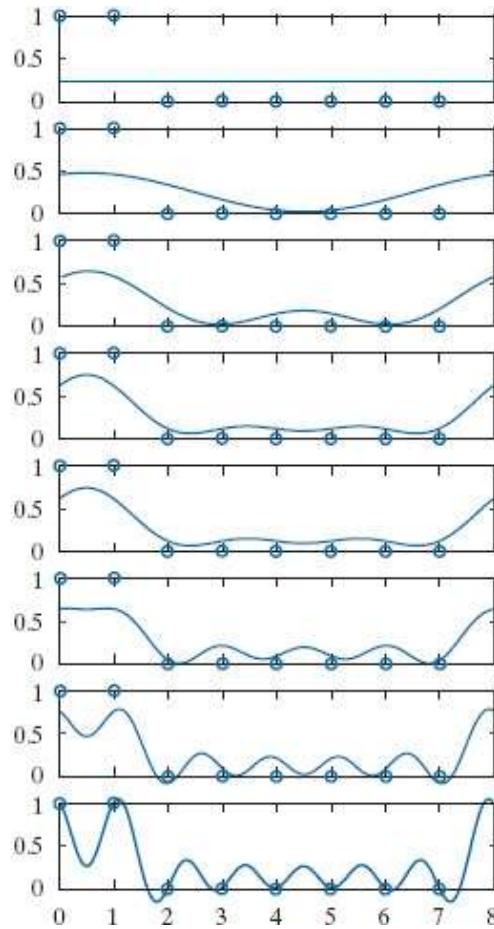
$$s(x, y, u, v) = \frac{1}{\sqrt{N}} \left( \cos\left(\frac{2\pi ux}{N}\right) + \sin\left(\frac{2\pi ux}{N}\right) \right) \frac{1}{\sqrt{N}} \left( \cos\left(\frac{2\pi vy}{N}\right) + \sin\left(\frac{2\pi vy}{N}\right) \right)$$



# 离散余弦变换



DHT



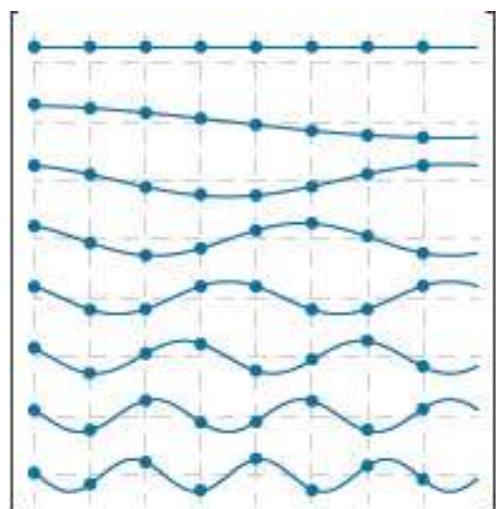
DFT

$$s(x, u) = a(u) \cos\left(\frac{(2x+1)u\pi}{2N}\right)$$

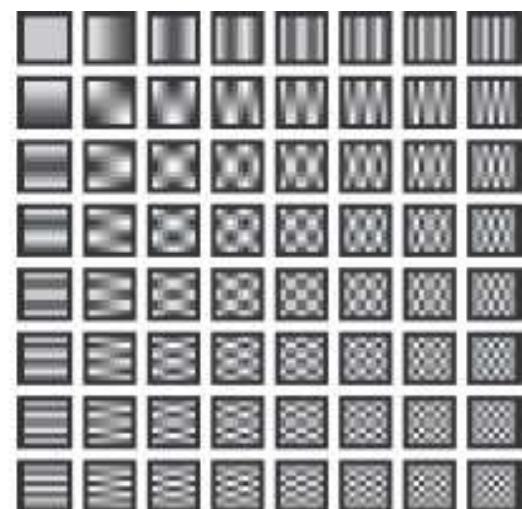
$$a(u) = \begin{cases} \sqrt{\frac{1}{N}} & u = 0 \\ \sqrt{\frac{2}{N}} & u = 1, 2, \dots, N-1 \end{cases}$$

# 离散余弦变换

$$s(x, y, u, v) = \alpha(u)\alpha(v) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right)$$



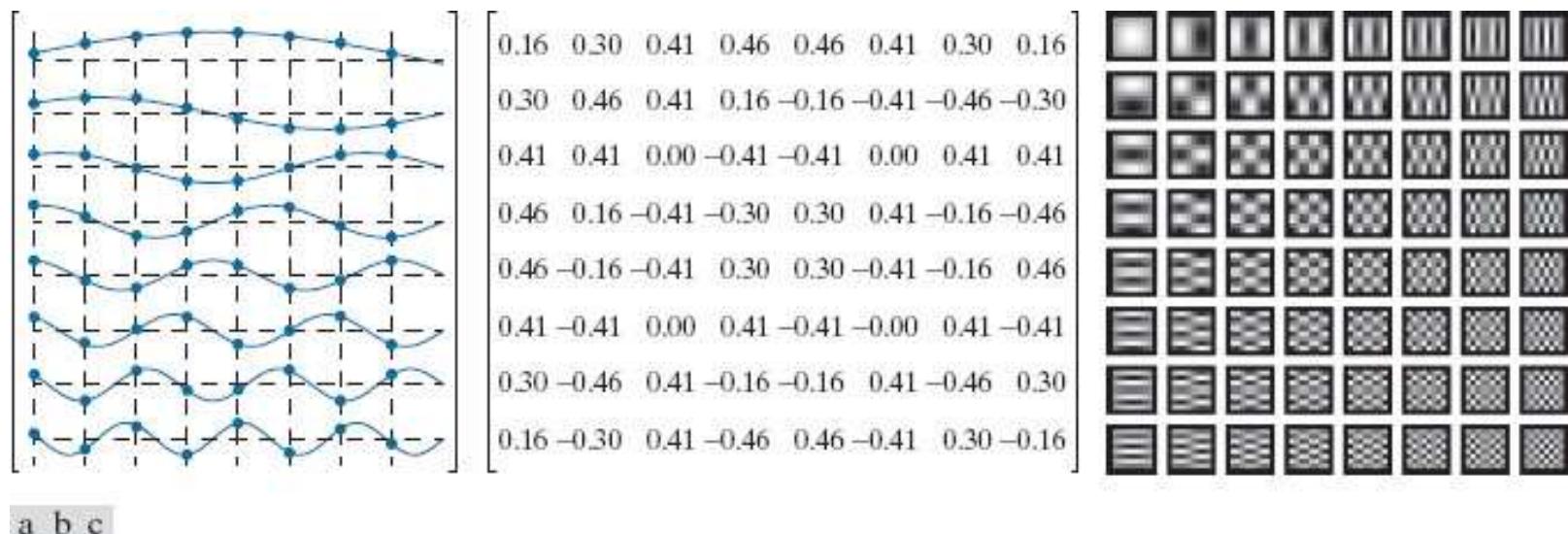
0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
0.49	0.42	0.28	0.10	-0.10	-0.28	-0.42	-0.49
0.46	0.19	-0.19	-0.46	-0.46	-0.19	0.19	0.46
0.42	-0.10	-0.49	-0.28	0.28	0.49	0.10	-0.42
0.35	-0.35	-0.35	0.35	0.35	-0.35	-0.35	0.35
0.28	-0.49	0.10	0.42	-0.42	-0.10	0.49	-0.28
0.19	-0.46	0.46	-0.19	-0.19	0.46	-0.46	0.19
0.10	-0.28	0.42	-0.49	0.49	-0.42	0.28	-0.10



a b c

# 离散正弦变换

$$s(x, u) = \sqrt{\frac{2}{N+1}} \sin\left(\frac{(x+1)(u+1)\pi}{N+1}\right)$$
$$s(x, y, u, v) = \frac{2}{N+1} \sin\left(\frac{(x+1)(u+1)\pi}{N+1}\right) \sin\left(\frac{(y+1)(v+1)\pi}{N+1}\right)$$



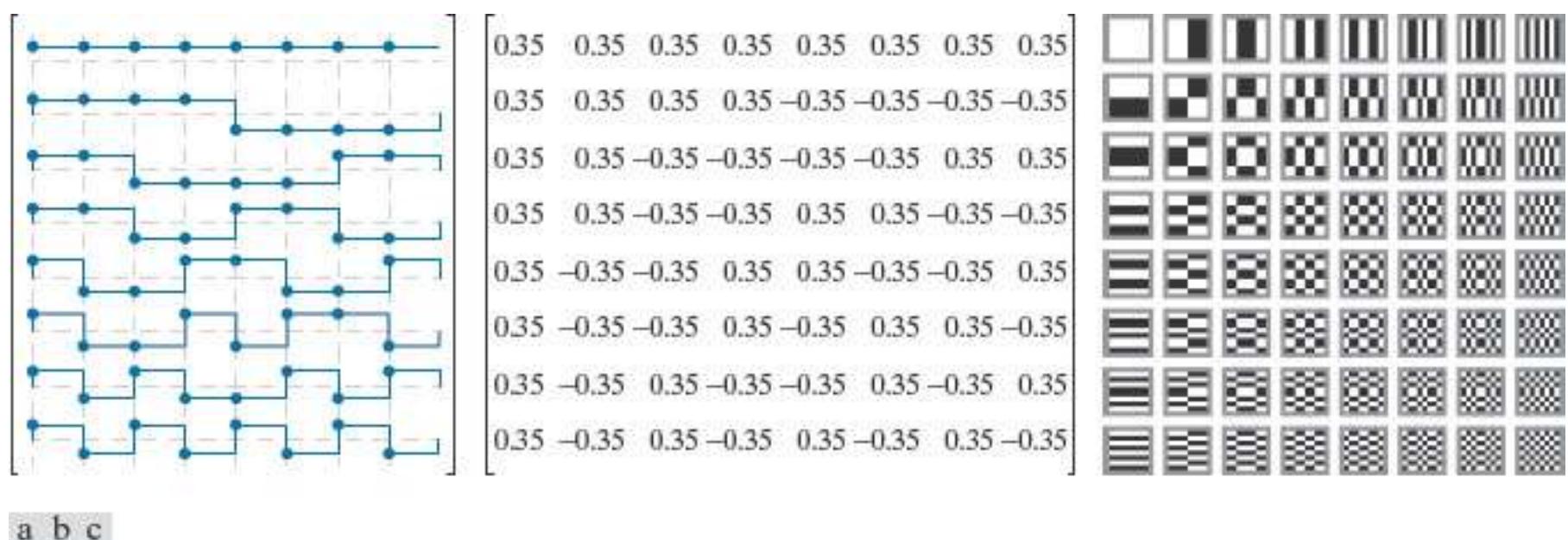
# Walsh-Hadamard 变换

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

# Walsh-Hadamard 变换



## Slant 斜变换 (I)

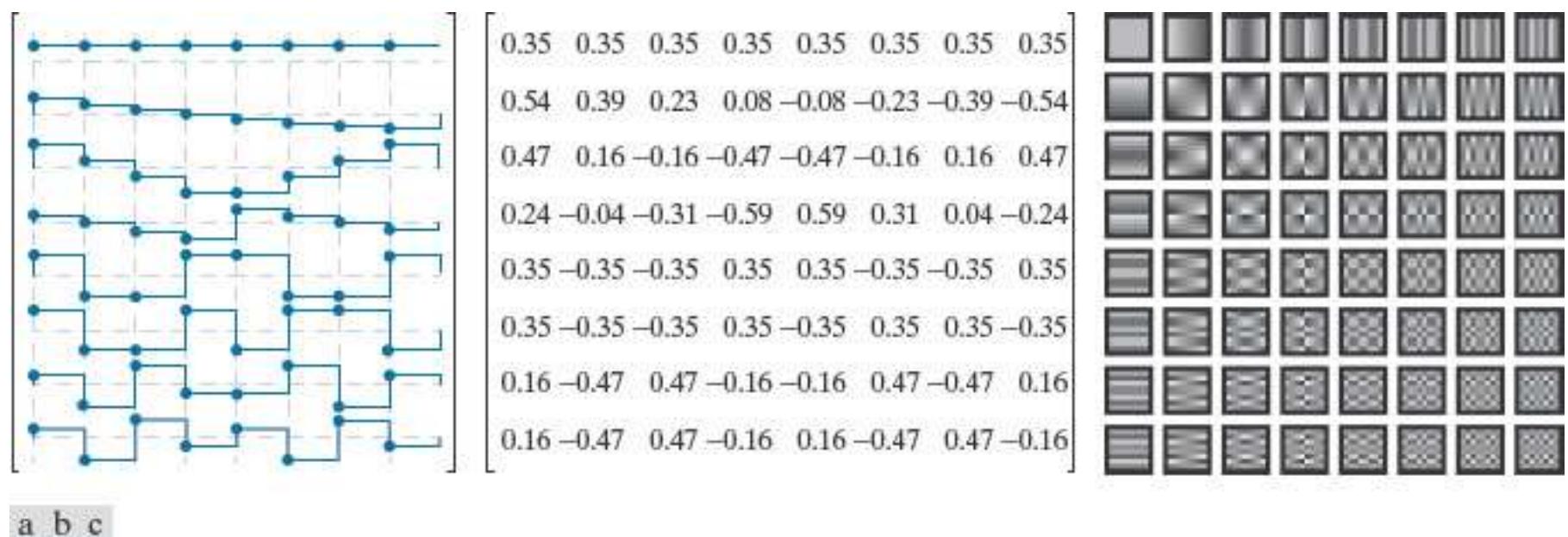
$$S_N = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ a_N & b_N & -a_N & b_N & 0 & \\ 0 & 0 & 0 & 0 & I_{(N/2)-2} & \\ 0 & 1 & 0 & 0 & -1 & I_{(N/2)-2} \\ -b_N & a_N & b_N & a_N & 0 & 0 \\ 0 & 0 & 0 & 0 & -I_{(N/2)-2} & \end{bmatrix} \begin{bmatrix} S_{N/2} & 0 \\ 0 & S_{N/2} \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

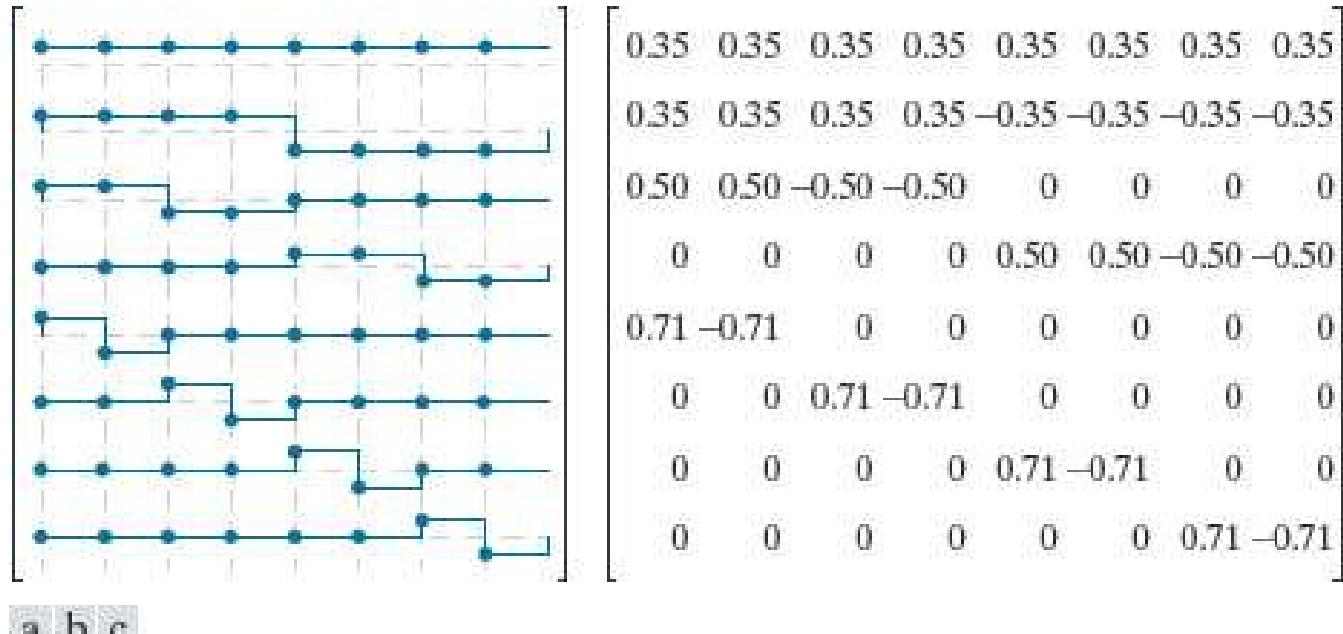
$$a_N = \left[ \frac{3N^2}{4(N^2 - 1)} \right]^{1/2}, \quad b_N = \left[ \frac{N^2 - 4}{4(N^2 - 1)} \right]^{1/2}$$

$$A_{SL} = \frac{1}{\sqrt{4}} S_4 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & -3 \\ \sqrt{5} & \sqrt{5} & \sqrt{5} & \sqrt{5} \\ 1 & -1 & -1 & 1 \\ 1 & -3 & 3 & -1 \\ \sqrt{5} & \sqrt{5} & \sqrt{5} & \sqrt{5} \end{bmatrix}$$

# Slant 变换 (II)



# Haar变换的定义

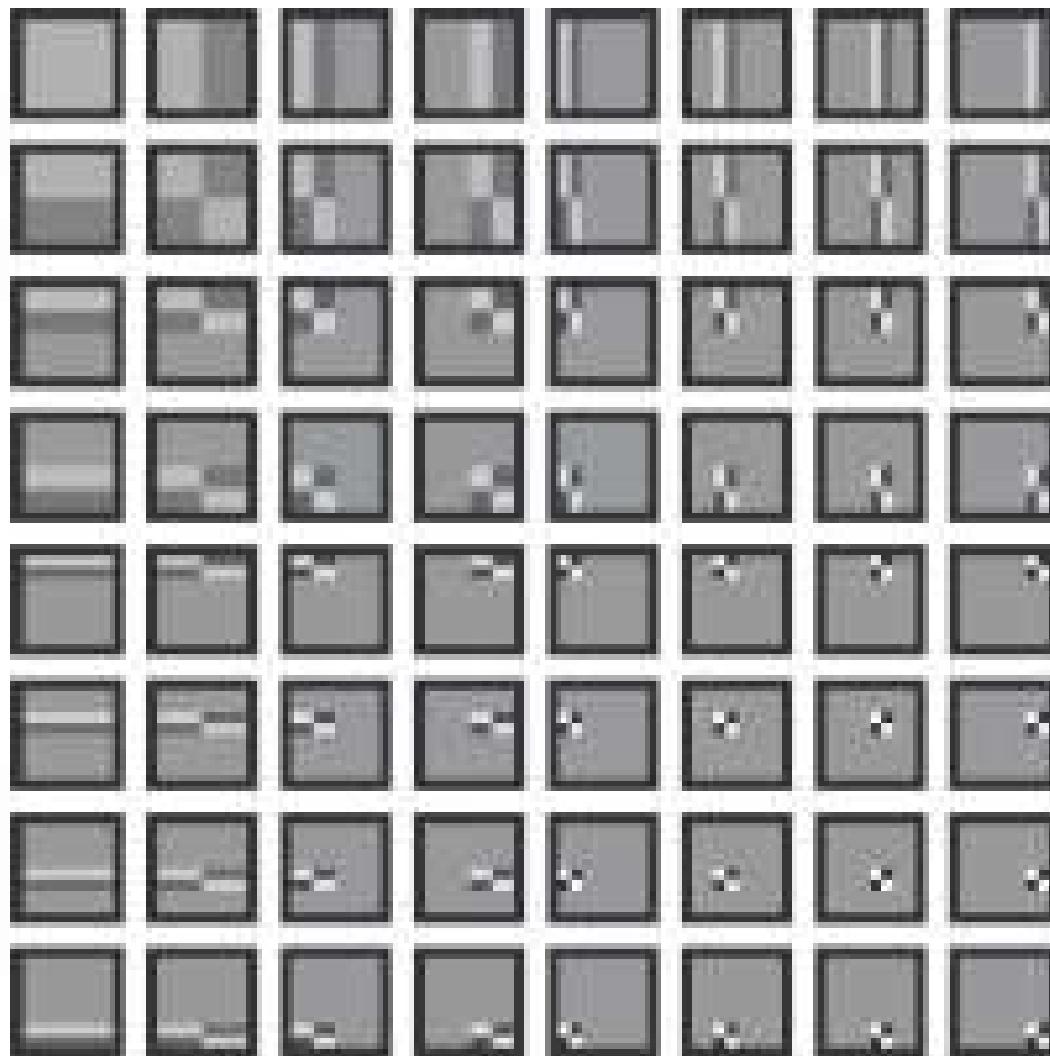


$$h_u(x) = \begin{cases} 1 & u = 0 \text{ and } 0 \leq x < 1 \\ 2^{p/2} & u > 0 \text{ and } q/2^p \leq x < (q + 0.5)/2^p \\ -2^{p/2} & u > 0 \text{ and } (q + 0.5)/2^p \leq x < (q + 1)/2^p \\ 0 & \text{otherwise} \end{cases}$$

已知的最早最简单的单位正交基

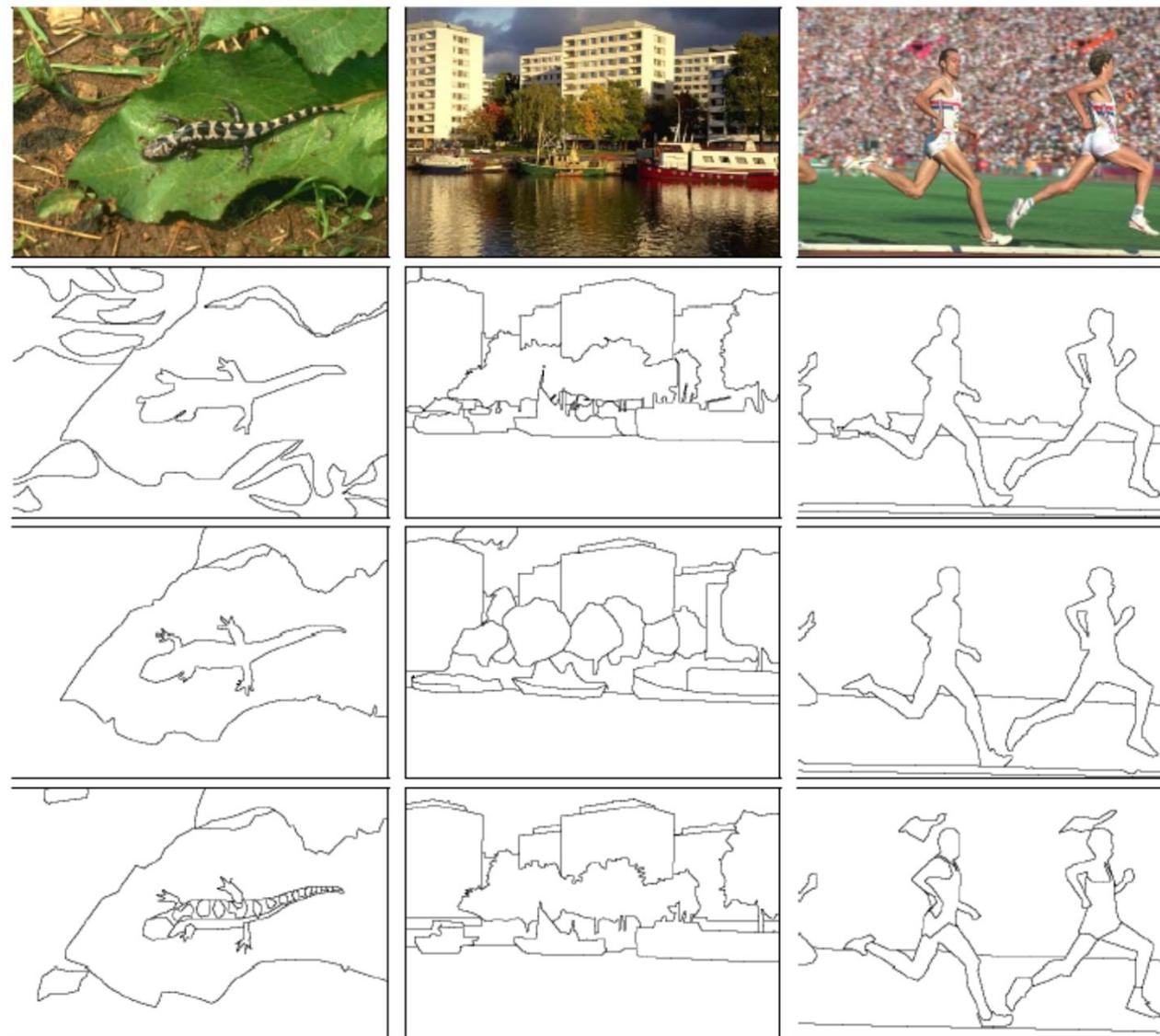
# Haar变换

---

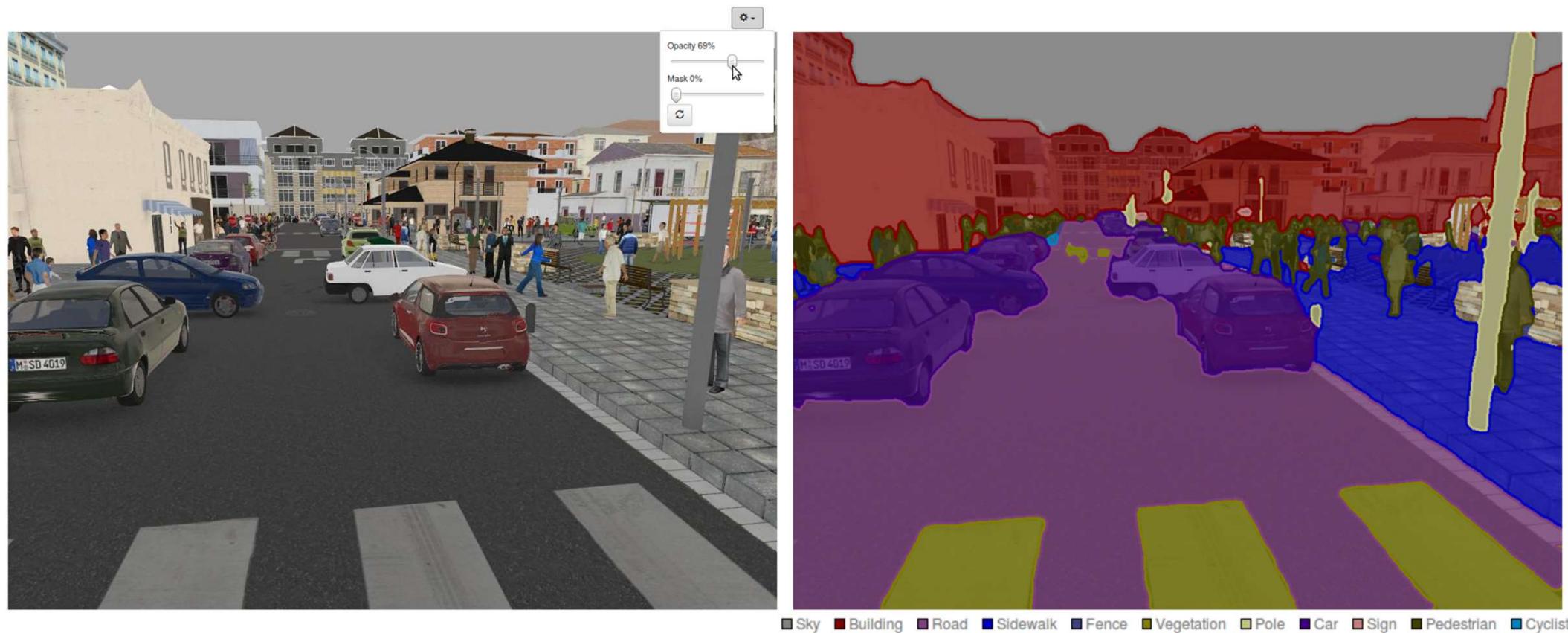


- 
- 采样定理应用示例
  - 图像的空间域滤波(复习)
  - 图像的频率域滤波(复习)
  - 一维离散傅里叶变换(复习)
  - 图像的正交变换
  - **图像的距离变换**
  - 基于深度学习的图像变换简介

# 图像分割示例 (I)



## 图像分割示例 (II)



<https://devblogs.nvidia.com/image-segmentation-using-digits-5/>

# 为什么要使用图像距离变换？

---

- 在比较二进制图像时，即使很局部的差别也会造成大的区别。
- 距离变换可以减缓图像特征的变化。
- 在计算机视觉中有广泛应用。

# 如何计算图像距离变换?

---

- 定义  $D T_{(P)}[x] = \min_{y \in P} dist(x, y)$
- 在计算机图形学，机器人学与人工智能的应用中经常使用相距边界的最短距离。
- 在计算机视觉中，有时距离定义为相距某个特征的最短距离。

# 不同的距离定义 (I)

0	0	0
0	1	0
0	0	0

Image

1.41	1.0	1.41
1.0	0.0	1.0
1.41	1.0	1.41

Distance Transform

Euclidean  $D_{Euclidean} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

0	0	0
0	1	0
0	0	0

Image

2	1	2
1	0	1
2	1	2

Distance Transform

City block  $D_{City} = |x_2 - x_1| + |y_2 - y_1|$

0	0	0
0	1	0
0	0	0

Image

1	1	1
1	0	1
1	1	1

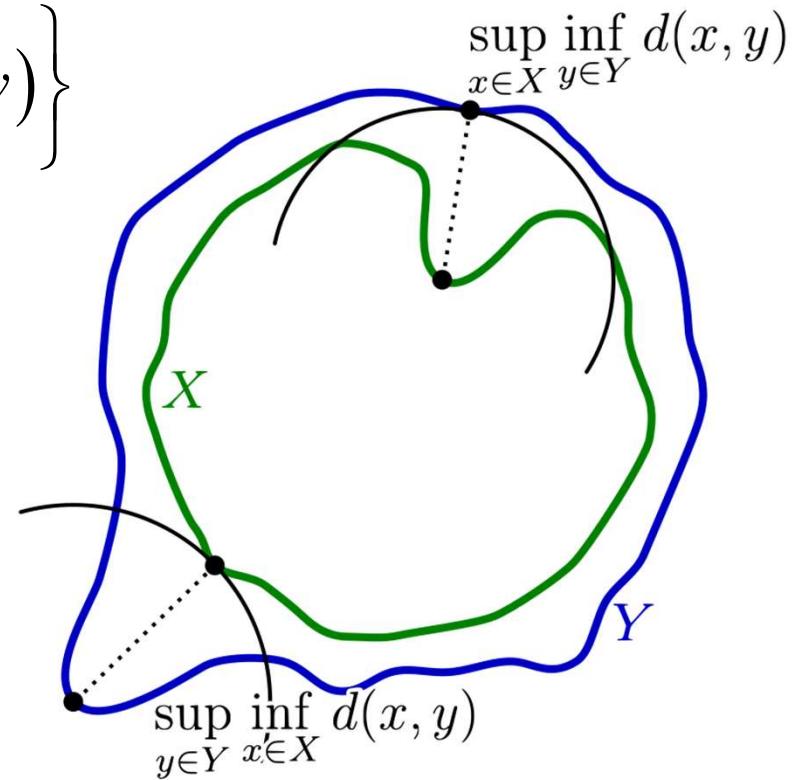
Distance Transform

Chessboard  $D_{Chess} = \max(|x_2 - x_1|, |y_2 - y_1|)$

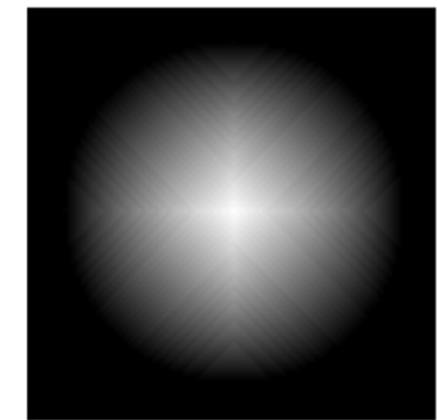
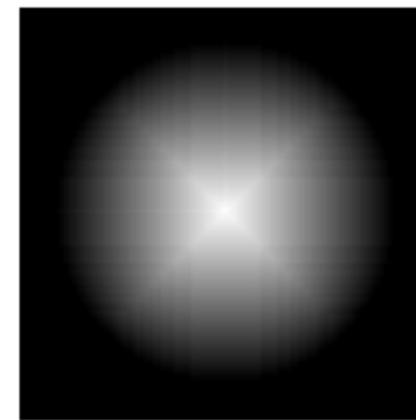
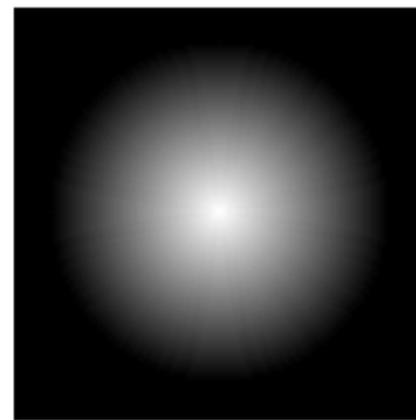
## 不同的距离定义 (II)

$$d_H(X, Y) = \max \left\{ \sup_{x \in X} \inf_{y \in Y} d(x, y), \sup_{y \in Y} \inf_{x \in X} d(x, y) \right\}$$

Hausdorff



## 图像的距离变换 (II)



Euclidean

City

Chessboard

<https://www.cs.cornell.edu/courses/cs664/2008sp/handouts/cs664-7-dtrans.pdf>

## 图像的距离变换 (II)

The diagram illustrates the process of calculating the distance transform of a binary image. On the left, there is an 8x8 binary image represented by a grid of 0s and 1s. A horizontal arrow points to the right, indicating the transformation process. On the right, the resulting 8x8 distance transform image is shown, where each pixel's value represents its shortest distance to the nearest boundary pixel (1). The distance is calculated using a 4-connected neighborhood.

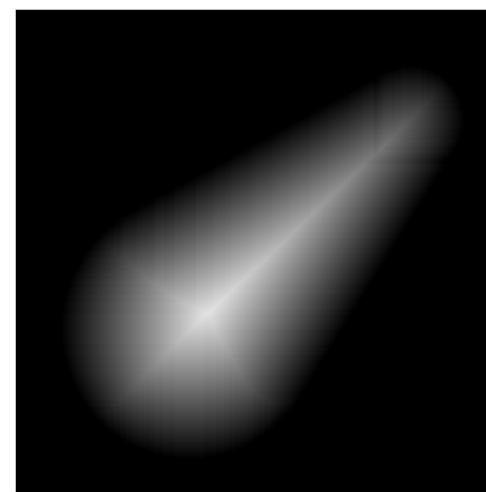
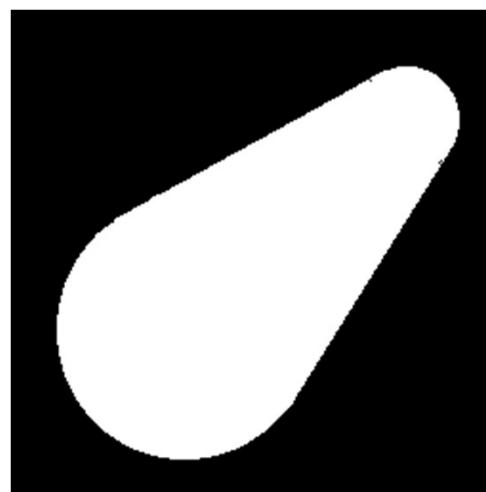
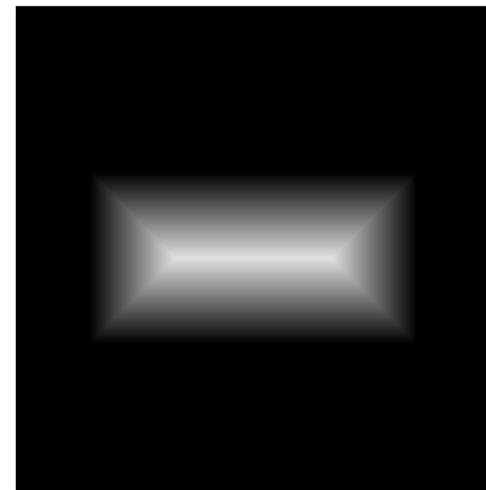
0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0
0	1	2	2	2	2	2	1	0
0	1	2	3	3	2	1	0	
0	1	2	2	2	2	2	1	0
0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0

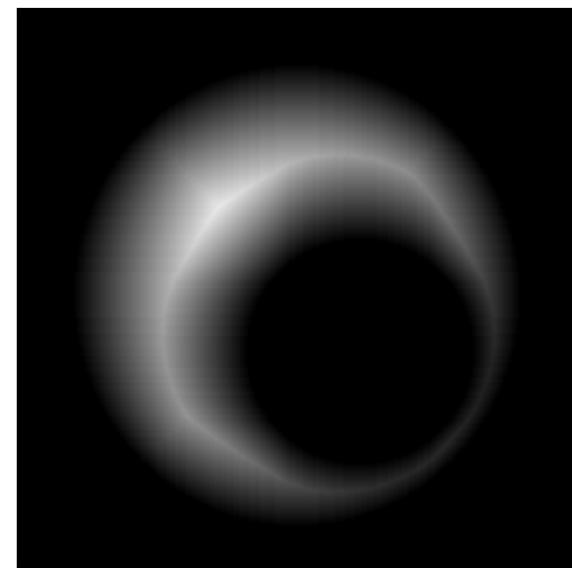
定义 距离为与最近边界的距离

<https://homepages.inf.ed.ac.uk/rbf/HIPR2/distance.htm>

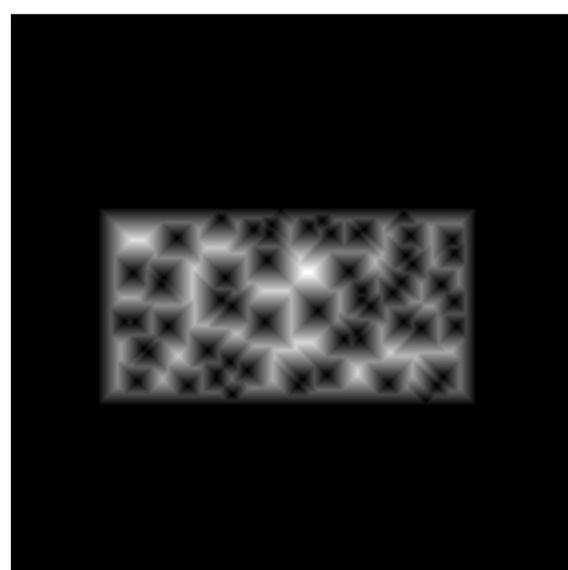
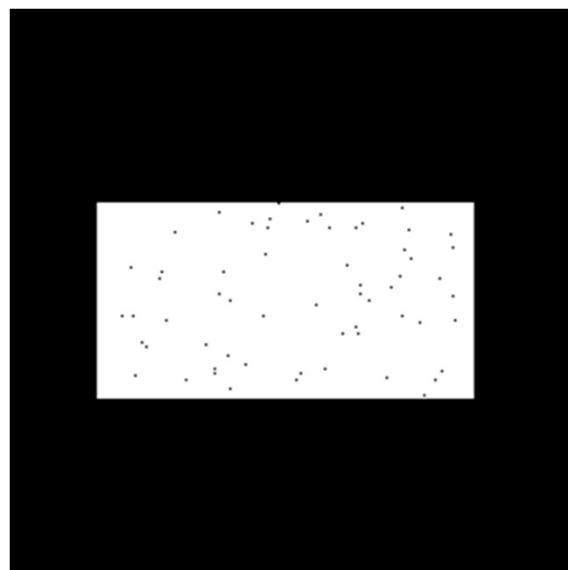
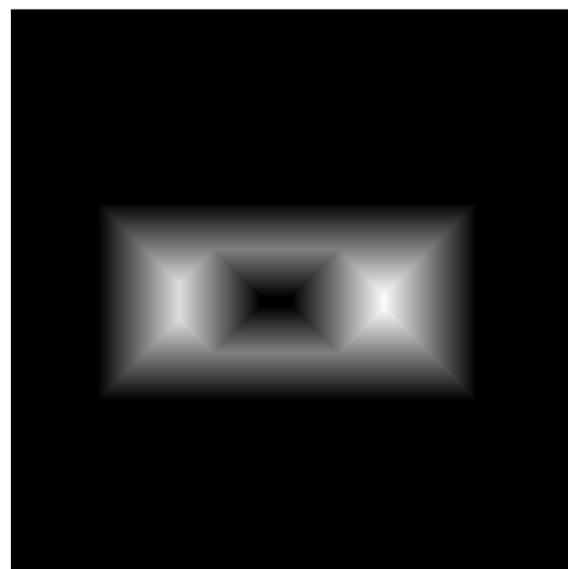
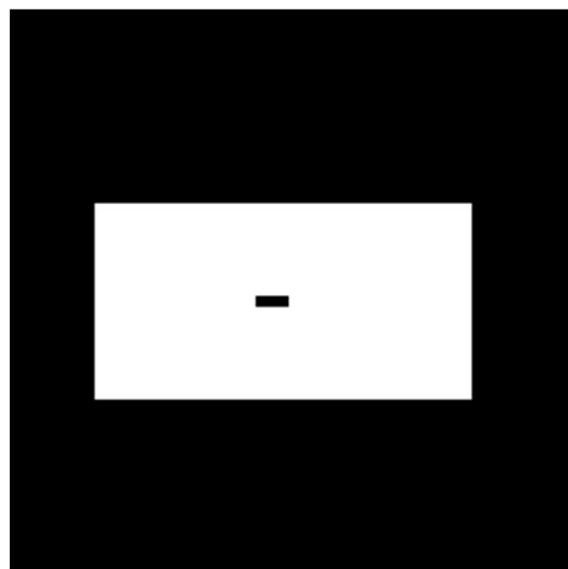
## 距离变换结果示例 (I)



## 距离变换结果示例 (II)



## 距离变换的局限性



- 
- 采样定理应用示例
  - 图像的空间域滤波(复习)
  - 图像的频率域滤波(复习)
  - 一维离散傅里叶变换(复习)
  - 图像的正交变换
  - 图像的距离变换
  - 基于深度学习的图像去噪简介

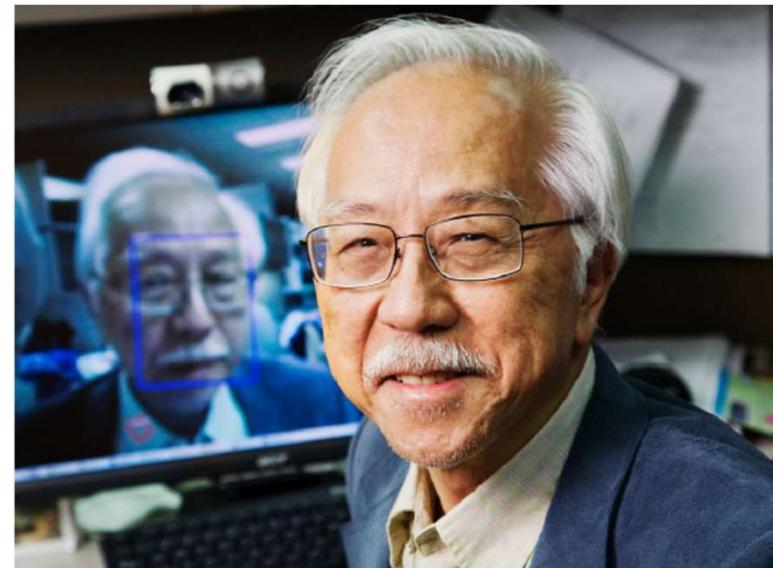
---

# Questions?

---

# Thomas Huang 黄煦涛 (1936年6月26日-2020年4月25日)

- 1956年毕业于[台湾大学](#)电子系，
- 1963年毕业于美国[麻省理工学院](#)电机系，先后获得硕士学位、博士学位
- 1963-1973留校任教，先后担任[麻省理工学院](#)电机系助理教授、副教授
- 1973-1980[普渡大学](#)电机教授
- 1980-2020 进入[伊利诺伊大学厄巴纳-香槟分校](#)电机系William L. Everitt 杰出讲座教授兼贝克曼前瞻科技研究院主席；
- 2001年当选为中国工程院外籍院士；2002年当选为美国工程院院士，同年当选为中国科学院外籍院士



# Thomas Huang 黃煦濤

---

## Thomas S. Huang memorial prize

The PAMITC executive committee has approved the creation of the Thomas S. Huang memorial prize in computer vision, to be awarded annually at CVPR starting in 2021.

The award winner will be selected by the PAMITC awards committee, similarly to the Rosenfeld and Distinguished Researcher awards, and will have the same financial remuneration.

Researchers who are more than 10 years past their PhD are eligible. The winner will be selected based on a combination of research, service and mentoring. Additional details will be made available before CVPR21.

# Thomas Huang 黃煦濤



Thomas S. Huang

University of Illinois, Urbana-Champaign  
Computer Vision  
Image Processing  
Pattern Recognition  
Multimedia  
AI

## GET MY OWN PROFILE

All Since 2016

Citations 177899 59084  
h-index 167 94  
i10-index 994 468

6 articles 57 articles

not available available

Based on funding mandates

TITLE	CITED BY	YEAR
The self-organizing map T Kohonen Neurocomputing 21 (1-3), 1-6	36186 *	1998
Image super-resolution via sparse representation J Yang, J Wright, TS Huang, Y Ma IEEE transactions on image processing 19 (11), 2861-2873	5061	2010
Least-squares fitting of two 3-D point sets KS Arun, TS Huang, SD Blostein IEEE Transactions on pattern analysis and machine intelligence, 698-700	4594	1987
Locality-constrained linear coding for image classification J Wang, J Yang, K Yu, F Lv, T Huang, Y Gong 2010 IEEE computer society conference on computer vision and pattern ...	3805	2010
Linear spatial pyramid matching using sparse coding for image classification J Yang, K Yu, Y Gong, T Huang 2009 IEEE Conference on computer vision and pattern recognition, 1794-1801	3762	2009
Image retrieval: Current techniques, promising directions, and open issues Y Rui, TS Huang, SF Chang Journal of visual communication and image representation 10 (1), 39-62	3476 *	1999
A survey of affect recognition methods: Audio, visual, and spontaneous expressions Z Zeng, M Pantic, GI Roisman, TS Huang IEEE transactions on pattern analysis and machine intelligence 31 (1), 39-58	3192	2008

# Thomas Huang 黄煦涛

---

- 在信号处理，模式识别，计算机视觉领域做出了杰出贡献
- 发明了预测差分量化（PDQ）的二维传真（文档）压缩方法，该方法已发展为国际G3/G4FAX压缩标准；
- 在多维数字信号处理领域中，提出了关于递归滤波器的稳定性的理论；
- 建立了从二维图象序列中估计三维运动的公式，为图象处理和计算机视觉开启了新领域。
- 图像超分辨率技术

Thomas Huang 黃煦濤

