Assignment 1 SPFII Derivatives

The goal of this assignment is two-fold:

- To price Interest Rate Swaps, and
- To delve deeper into Mean Reversion models.

So here are the two parts of the assignment.

Part 1. IRS

Create an environment/tool/engine for pricing Interest Rate Swaps. Use any algorithmic programming language you want. This environment must allow for:

- Bootstrapping the relevant zero reference curves
- One and two curve situation: (1) reference and discount rates are the same (e.g., term SOFR), and (2) reference rate is EURIBOR and discount rate is some other risk free rate.
- Swap can be at initiation (so here you are essentially determining the swap rate) or an already existing swap (here you know the swap rate and you are determining the swap's value).
- It should also allow for amortizing principal amount (so not fixed principal).
- Your swap tool should allow for receiving fixed/paying floating and receiving floating/paying fixed situations.
- It should also allow for valuing swaps with different payment frequencies, so quarterly, half-annually etc. This payment frequency should be given as an input parameter.

With your "IRS engine", you will price a few swaps and do some sensitivity tests. It is up to you to define the swap characteristics. For example, you can choose yourself what is your reference rate; EURIBOR would be most logical (but for example term SOFR is fine too).

For long maturity swaps, you have to bootstrap the curve to obtain long-maturity rates. Also, you have to specify other swap characteristics yourself (such as payment frequency and fixed swap rate), but make sure they are realistic (for example, no point in assuming fixed rate of, say, 10% or 1%). For all tests of your tool, assume that for all your swaps you pay fixed and receive floating rate and, for all already issued swaps, the swap rate is so high that they are currently out of the money for you.

You have to consider at least three specific situations:

- An already issued swap with the remaining lifetime of 1 to 2 year. See examples in Hull (exercises for Chapter 7) for inspiration.
- A newly issued mid-maturity swap (say, 5 or 7 years). For this swap, you have to determine the swap rate that you would offer to your client. Investigate the dependence of this swap rate on payment frequency and on different levels of the current zero curve (so bump the current zero curve with +/- 25 bp, 50 bp, 100 bp, 150 bp and draw a graph of swap rate vs this bump). Also compare the swap rate for a plain swap with the same swap but where the principal is amortizing according to

some simple predetermined schedule (for example, fixed repayments per time interval).

- An already issued long-maturity swap. Assume that the remaining lifetime of this swap is still quite long, so longer than 7 years. Determine the swap value and investigate how sensitive it is to 1% and 2% changes (up or down) in the current zero curve. Draw a graph of the swap value vs remaining maturity, for maturities up to 30 years.

Part 2. Mean reversion.

The goal of this exercise is to set up both simulation and binomial tree for mean reversion process (rather than GBM, which you already have done in SPF1 – feel free to use your existing code and modify it for MR situation).

It turns out that the Binomial tree method can be extended to accommodate MR process. The paper of Nelson and Ramaswamy (1990) explains how to do that, but there are other, later papers on this issue that explain it even better. Investigate how CRR Binomial tree can be amended for Mean Reverting process and incorporate it in your code.

Amend your GBM Monte Carlo simulation engine to simulate Mean Reversion. Everywhere assume that log-price follows MR process (under Q-measure), so MR is in log-price and not in price itself (to keep the log-normality of prices).

With both of these engines (Binomial and MC) you now will investigate how option prices in MR model compare to those in GBM.

Price exactly the same options as you did in Assignments in SPF1, with the same volatility parameters, but now assuming MR for the log of underlying. Do it by both Binomial method and MC. Choose realistic parameters for long term mean (take e.g., long term mean equal to the current value of the underlying) and a realistic mean reversion speed (not too fast and not too slow – half time of, say, a couple of weeks to a couple of months). Compare the option pricing results to those obtained with GBM model.

Investigate the sensitivity of option prices to mean reversion speed, with all other parameters being fixed. Present this sensitivity in the form of an appropriate graph.

Investigate how the difference between MR and GBM option prices depends on the maturity of the option, with all other parameters being fixed. Again, present a graph summarizing your findings. Explain the observed results.