Assignment 2. Stochastic volatility models and Multiasset options

1. *Warm up.*
   Consider Section 3.3 of the Computer Practicum (p. 9-11, Heston & SABR processes). Make Exercise 10 for both Heston and SABR models with given sets of parameters (p. 11, note there is a typo for SABR, the last line should read alpha=0.5). Note that you can directly simulate SABR in this case because beta is one. Try to do it when beta is less than one. What kind of problem(s) do you encounter in this case?

2. *Option prices and implied volatilities for SABR model*.

   Study Chapter 4 of the computer practicum (Sections 4.2 and 4.3).
   In this exercise, you will again proceed with SABR model, because for Heston we do not have analytical solution.

a) Consider European calls and puts on the underlying that follows your SABR process. Compute ATM, +/-10 % and +/- 25% ITM and OTM European call and put prices using Monte Carlo simulations code from the previous exercise for SABR. Compute these prices also using the regular Black-Scholes formula. Think of what would be a realistic way of providing the volatility in Black-Scholes formula, to facilitate a comparison between SABR and BS (i.e., how would you do this in practice).

b) Compare the MC with BS prices: plot on the same graph the Monte Carlo prices of calls and puts for different strikes together with the prices of Black-Scholes formula. You will see the price differences and slightly different shape. However, it is not convenient to compare prices as they depend strongly on strike. We will use implied volatility for more proper comparison.

c) Compute the implied volatility for SABR model (see Computer Practicum for how to do this). Plot it together with the Black-Scholes volatility as a function of strike. Observe the smile and the skew of the volatility_imp(K). Play with different parameters of your model to observe which parameters are responsible for the level, slope (skew), and curvature (smile) of the volatility smile. Provide relevant graphs to illustrate your findings.

3. *Multivariate Brownian motion and multi-asset derivatives*

When a derivative depends on several underlying variables, we can simulate paths for each of them in a risk-neutral world to calculate the option value.

Consider European spread call option with payoff:

$$MAX\ (S_1(T) - S_2(T) - K, 0)$$

We assume that the underlying values S1 and S2 are GBMs driven by correlated Brownian Motions with some correlation ρ. It is well-known how to generate correlated normal samples: generate independent normal random variables x_1 and x_2 and set

$$\varepsilon_1 = x_1$$

$$\varepsilon_2 = \rho x_1 + x_2\sqrt{1-\rho^2}$$

The procedure known as Cholesky decomposition of the variance-covariance matrix is used when samples are required for more than two correlated random variables.

So here comes the assignment:

Consider ATM 1-year European spread option with the payoff given above, where both assets are stocks, so their drifts under risk neutral measure are equal to the risk free rate (take it equal to 2% p/a). Let the current prices of these stock be equal to your dates of birth times €10. Furthermore, assume that the volatilities of these stocks are equal to your months of birth times three, expressed in % as annualized quantity. In the first instance, assume that the correlation between the Brownian motions driving the stock prices is equal to 30%.

a) Value this option by bivariate Monte Carlo simulations.
b) Investigate how the option price depends on the value of the correlation: plot the option value vs correlation, which ranges from -1 to 1.
c) For correlation equal to 30%, calculate the volatility of the spread by the well-known portfolio variance formula (note that the spread is the portfolio with weights 1 and -1).
d) If it is possible in your case, assume that the spread itself follows GBM with volatility computed in c). Calculate spread option price by BS formula and compare with the answer in a). Why generally we cannot assume GBM for the spread stochastic process?
e) Assume now that the spread follows arithmetic BM (so not Geometric) with the standard deviation in front of the BM which corresponds with the volatility calculated in c). (Think how to translate this volatility into the standard deviation used in arithmetic BM formula). What is the drift of the spread process?

Calculate the spread option price by the variant of the Black Scholes formula (Normal BS, rather than the traditional, lognormal BS), modified to deal with arithmetic BM (this is called Bachelier model). How does the answer compare with the answers obtained in a) and e)?

f) Investigate how the discrepancy between BS (if you could use that), Bachelier spread option prices and the MC spread option price depends on the strike, i.e., plot this discrepancy vs strike.