Assignment 2. Spread option and SABR

1. *Spread option*

A spread is the difference between two assets *Spr(t)=S1(t)-S2(t),* for example between prices of two different but similar commodities or stocks. A spread option is an option with the value of spread *Spr(t)* as the underlying.

Consider European spread call option with payoff:

$$MAX\,(S_1(T) - S_2(T) - K, 0)$$

We assume that the underlying values *S1* and *S2* are both GBMs, driven by correlated Brownian Motions with some correlation ρ.

It is well-known how to generate correlated normal samples: generate independent normal random variables x_1 and x_2 and set

$$\varepsilon_1 = x_1$$

$$\varepsilon_2 = \rho x_1 + x_2\sqrt{1-\rho^2}$$

The procedure known as Cholesky decomposition of the variance-covariance matrix is used when samples are required for more than two correlated random variables.

When a derivative depends on several underlying variables, we can simulate paths for each of them in a risk-neutral world and calculate the option value from such Monte Carlo simulation.

So here comes the first part of the assignment:

Consider **ATM 1-year European spread option** with the payoff given above, where both underlying assets *S1* and *S2* are **futures**, so their drifts under the risk neutral measure are equal to zero.

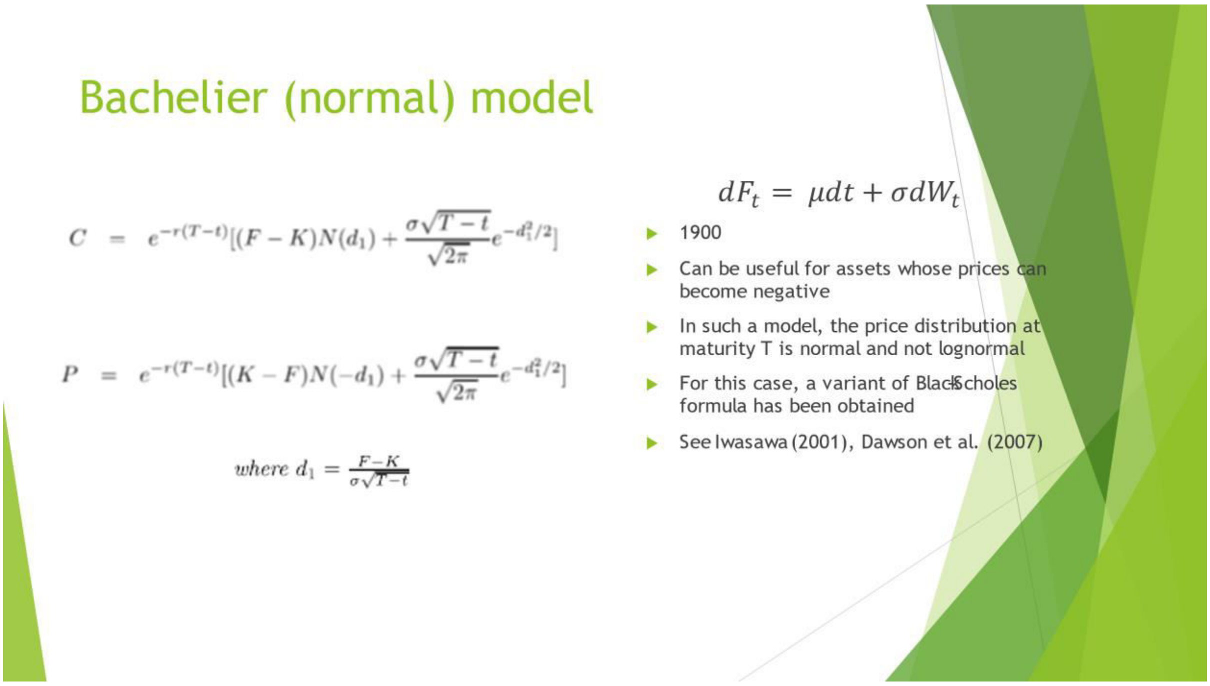
Let the current prices of these futures be equal to your months of birth times €100 (take the bigger one to be *S1* and smaller *S2,* so that the current value of the spread is positive). Furthermore, assume that the volatilities of these futures are slightly different but in the range between 30% and 60% p/a – choose two reasonable values (but they should be nontrivial, so for example, 37% and 54% and not 30% and 50%). In the first instance, assume that the correlation between the Brownian motions driving the stock prices is equal to 30%.

a) Value this option by bivariate Monte Carlo simulations.
b) Investigate how the option price depends on the value of the correlation: plot the option value vs correlation, which ranges from -1 to 1.
c) For correlation equal to 30%, calculate the volatility of the spread by the well-known portfolio variance formula (note that the spread is the portfolio with weights 1 and -1).

d) If it is possible in your case, assume that the spread itself follows GBM with volatility computed in c). Calculate spread option price by BS formula and compare with the answer in a). Why generally we cannot assume GBM for the spread stochastic process?

e) Assume now that the spread follows arithmetic BM (so not Geometric) with the standard deviation in front of the BM which corresponds with the volatility calculated in c). (Think how to translate this volatility into the standard deviation used in arithmetic BM formula). What is the drift of the spread process? Calculate the spread option price by the variant of the Black Scholes formula (Normal BS, rather than the traditional, lognormal BS), modified to deal with arithmetic BM (this is called Bachelier model – see description on slide 5 in MR slide pack, also copied below). How does the answer compare with the answers obtained in a) and e)?

f) Investigate how the discrepancy between BS (if you could use that), Bachelier spread option prices and the MC spread option price depends on the strike, i.e., plot this discrepancy vs strike.



2. *SV models: SABR*
   2.1 Warm up.

Consider Section 3.3 of the Computer Practicum (p. 9-11, Heston & SABR processes). Although it refers to Matlab, ignore it and use whatever programming language you prefer.

Make Exercise 10 (Monte Carlo simulations for both Heston and SABR models) with given sets of parameters (p. 11, note there is a typo for SABR, the last line should read alpha=0.5). Note that you can directly simulate SABR in this case because beta is one. (Try to do it when beta is less than one. What kind of problem(s) do you encounter in this case?)

2.2 *Option prices and implied volatilities for SABR model.*

Now proceed to Chapter 4 of the computer practicum, Section 4.2. We proceed with SABR model, because for Heston we do not have analytical solution.

a) Consider European calls and puts on the underlying that follows your SABR process. Compute ATM, +/-10 % and +/- 25% ITM and OTM European call and put prices using Monte Carlo simulations code from the previous exercise for SABR. Compute these prices also using the regular Black-Scholes formula. Think of what would be a realistic way of providing the volatility in Black-Scholes formula, to facilitate a fair comparison between SABR and BS (i.e., how would you do this in practice).

b) Compare the MC with BS prices: plot on the same graph the Monte Carlo prices of calls and puts for different strikes together with the prices of Black-Scholes formula. You will see the price differences and slightly different shape. However, it is not convenient to compare prices as they depend strongly on strike. We will use implied volatility for more proper comparison.

c) Compute the implied volatility for SABR model. Plot it together with the Black-Scholes volatility as a function of strike. Observe the smile and the skew. Play with different parameters of your model to observe which parameters are responsible for the level, slope (skew), and curvature (smile) of the volatility smile. Provide relevant graphs to illustrate your findings.