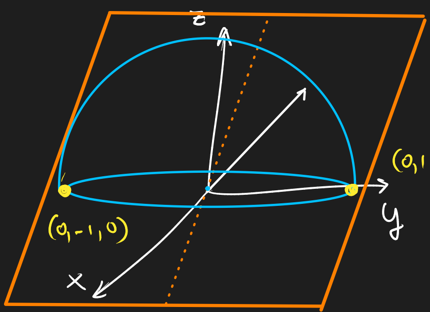
Ejercicio 1. Sea C la curva en $\mathbb{R}^3$ dada por

$$C := \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 1, x+z=0, z \geq 0\}.$$

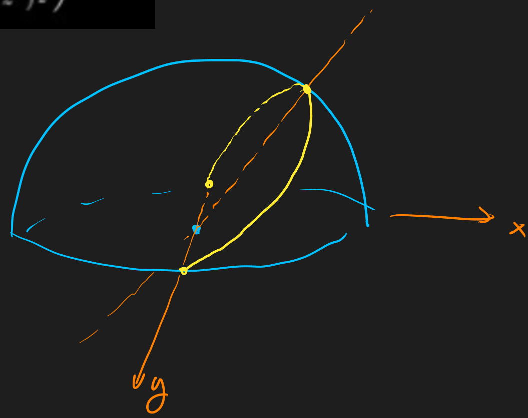
a. Dar una parametrización regular de C que empiece en $(0,-1,0)$ y termine en $(0,1,0)$.

b. Calcular $\int_C F \cdot \mathbf{ds}$ orientada como en el ítem anterior, donde

$$F(x,y,z) = \left(\frac{2x^2+y^2}{(x^2+y^2+z^2)^{\frac{3}{2}}}, \frac{2x^2+y^2}{(x^2+y^2+z^2)^{\frac{3}{2}}}, \frac{2x^2+y^2}{(x^2+y^2+z^2)^{\frac{3}{2}}} \right).$$



$x = -z$



Parametrizo $\mathcal{C}$ como

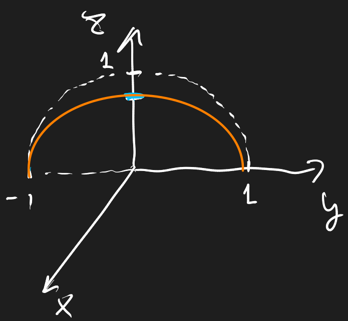
$$\begin{cases} x^2 + y^2 + z^2 = 1 \\ x = -z \end{cases} \Rightarrow (-z)^2 + y^2 + z^2 = 1$$

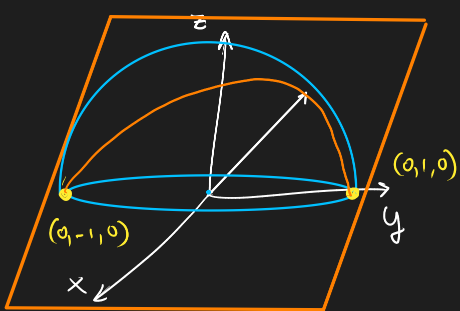
$$2z^2 + y^2 = 1$$

$$\frac{z^2}{\frac{1}{2}} + y^2 = 1 \qquad z \geq 0$$

$$r_z = \frac{1}{\sqrt{2}}$$

$$r_y = 1$$





$$\sigma(\theta) = \left(-\frac{1}{\sqrt{2}} \cdot \sin\theta \,,\, 1 \cdot \cos\theta \,,\, \frac{1}{\sqrt{2}} \cdot \sin\theta\right) \qquad \theta \in [0,\pi]$$

no respeta la orientación

$$\tilde{\sigma}(t) = \left(-\frac{1}{\sqrt{2}} \cdot \cos t \,,\, \sin t \,,\, \frac{1}{\sqrt{2}} \cos t\right) \qquad t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Respeta la orientación de $\mathcal{C}$.

Ejercicio 1. Sea C la curva en $\mathbb{R}^3$ dada por

$$C := \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 1, x+z = 0, z \geq 0\}.$$

a. Dar una parametrización regular de C que empiece en $(0,-1,0)$ y termine en $(0,1,0)$.

b. Calcular $\int_C F \cdot \mathbf{ds}$ orientada como en el ítem anterior, donde

$$F(x,y,z) = \left(\frac{2x^2+y^2}{(x^2+y^2+z^2)^{\frac{3}{2}}}, \frac{2x^2+y^2}{(x^2+y^2+z^2)^{\frac{3}{2}}}, \frac{2x^2+y^2}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right).$$

$$\int_{\mathcal{C}} F \cdot d\vec{s} = \int_{t=-\frac{\pi}{2}}^{\frac{\pi}{2}} \langle \underbrace{F(\sigma(t))}, \underbrace{\sigma'(t)} \rangle \, dt$$

$$\left\langle (1,1,1) \,,\, \left(\frac{1}{\sqrt{2}} \sin t, \cos t, -\frac{1}{\sqrt{2}} \sin t\right) \right\rangle$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t \, dt$$

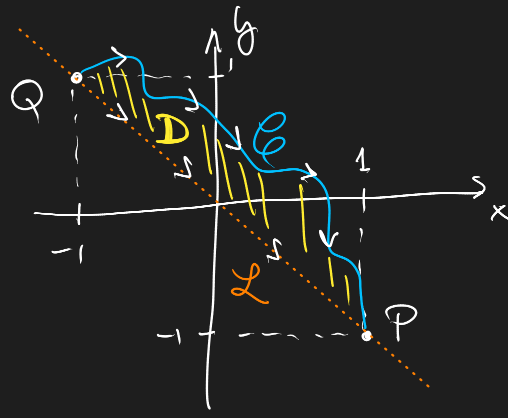
$$= \sin t \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)$$

$$= 1 - (-1)$$

$$\int_{\mathcal{C}} F \cdot d\vec{s} = 2 \quad //$$

Ejercicio 2. Sea C una curva suave en $\mathbb{R}^2$ que va desde $Q = (-1, 1)$ hacia $P = (1, -1)$, tal que para todo punto (x, y) de la curva se cumple que $y \geq -x$.
Dado el campo
$$F(x, y) = (x + y + e^{x+y}, e^{x+y}),$$
sabemos que $\int_C F ds = 10$. Calcular el área de la región R comprendida entre la curva C y la recta de ecuación $x + y = 0$, si suponemos que R es una región de tipo III.



Por Green

$$-\int_{\mathscr{C}} \vec{F} \cdot d\vec{s} + \int_{\mathscr{L}} \vec{F} \cdot d\vec{s} = \iint_D Q_x - P_y \, dx dy$$

$$Q_x = e^{x+y}$$

$$P_y = 1 + e^{x+y}$$

$$Q_x - P_y = -1$$

$$\underbrace{-\int_{\mathscr{C}} \vec{F} \cdot d\vec{s}}_{-10} + \int_{\mathscr{L}} \vec{F} \cdot d\vec{s} = \iint_D -1 \, dx dy$$

$$= -1 \underbrace{\iint_D 1 \, dx dy}_{\text{Area}(D)}$$

$$-10 + \int_{\mathscr{L}} \vec{F} \cdot d\vec{s} = -\text{Area } D$$

$$\text{Area } D = 10 - \int_{\mathcal{L}} F \cdot d\vec{s}$$

$$\int_{\mathcal{L}} F \cdot d\vec{s} = ?$$

Parametrizo $\mathcal{L}$ como $\quad (y = -x)$

$$\varphi(t) = (t, -t) \quad t \in [-1, 1]$$

$$\int_{\mathcal{L}} F \cdot d\vec{s} = \int_{-1}^{1} \underbrace{\langle \underbrace{F(t,-t)}_{\substack{(\cancel{t} - \cancel{t} + 1,\ 1) \\ (1,1)}}, (1,-1) \rangle}_{\langle (1,1), (1,-1) \rangle = 0} dt$$

$$= 0$$

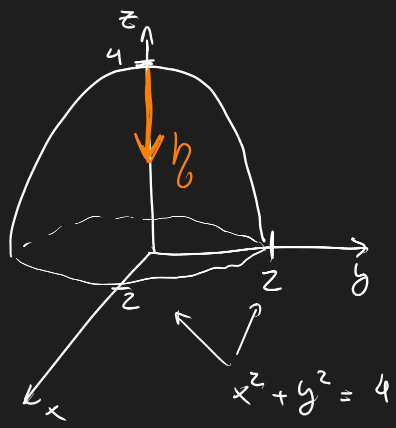
$$\text{Area } D = 10$$
//

Ejercicio 3. Sea S el paraboloide de ecuación $4 - z = x^2 + y^2$ con $z \geq 0$, orientado de tal manera que la normal en el punto $(0,0,4)$ es igual a $(0,0,-1)$. Consideremos el campo

$$F(x,y,z) = (x^2 + sen(z^2), y^2 + ze^z, x^2 + y^2 + z^2).$$

a. Calcule $\nabla \times F$.

b. Calcule $\int_S (2y - e^z(1+z), -2x + 2z cos(z^2), 0)\, d\mathbf{S}$



$$\nabla \times F = \begin{array}{ccc} & (-) & \\ \end{array}$$
$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 + \sin z^2 & y^2 + z.e^z & x^2+y^2+z^2 \end{vmatrix}$$

$$= \left(2y - (e^z + z.e^z), -(2x - 2z.\cos z^2), 0\right)$$

$$= \left(2y - e^z - z.e^z, 2z.\cos z^2 - 2x, 0\right) = \nabla \times F$$

b) Como F es C^1 $\Rightarrow$ Puedo usar Stokes.

$$\int_{\mathcal{C} = \partial S} F \cdot d\vec{s} = \iint_S \nabla \times F \cdot d\vec{S}$$

Parametrizo $\mathcal{C}$ como

$$\sigma(t) = (\sin t, \cos t, 0) \qquad t \in [0, 2\pi)$$

$$\int_{\mathcal{C}=\partial S} F\cdot d\vec{s} = \int_0^{2\pi} \langle \underbrace{F(\sigma(t))}, \sigma'(t) \rangle \, dt$$

$$\langle (\sin^2 t, \cos^2 t, 1), (\cos t, -\sin t, 0) \rangle$$

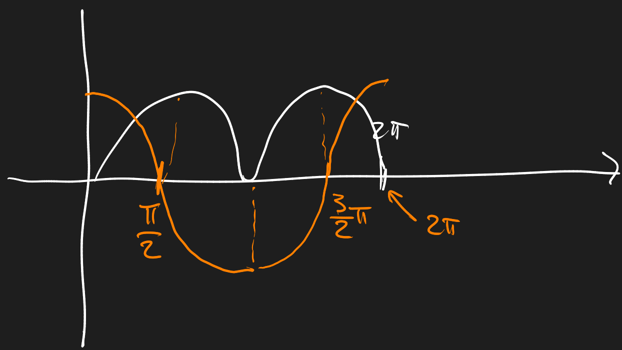
$$= \int_0^{2\pi} \cos t \cdot \sin^2 t - \sin t \cdot \cos^2 t \, dt$$

$$= \int_0^{2\pi} \cos t \cdot \sin^2 t \, dt - \int_0^{2\pi} \sin t \cdot \cos^2 t \, dt$$

I II

I $\quad u = \sin t$

$du = \cos t \, dt$



$$\int_0^1 u^2 \, du + \int_1^{-1} u^2 \, du + \int_{-1}^0 u^2 \, du =$$

$$= \cancel{\int_0^1 u^2 \, du} - \int_{-1}^1 u^2 \, du + \cancel{\int_0^1 u^2 \, du} =$$

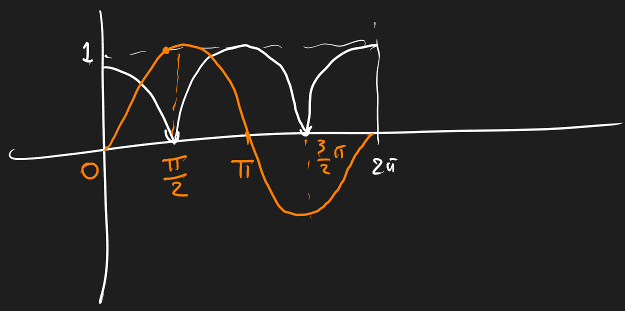
$$= - \int_{-1}^1 u^2 \, du = 0$$

$$\int_0^{2\pi} \sin t \cdot \cos^2 t \; dt =$$

π

$u = \cos t$

$du = -\sin t \; dt$



$$\int_1^{-1} -u^2 \, dt + \int_{-1}^{1} -u^2 \, dt =$$

$$\int_{-1}^{1} u^2 \, dt + \int_{-1}^{1} -u^2 \, dt = 0 \; //$$

$$\int_{\mathcal{C}=\partial S} F \cdot d\vec{s} = 0$$

$\circ/\circ$

$$\iint_S \nabla \times F \cdot d\vec{S} = 0 \; //$$

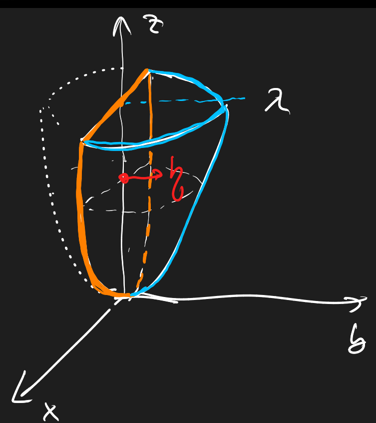
Ejercicio 4. Sean S_λ la superficie en $\mathbb{R}^3$ dada por

$$S_\lambda = \{(x,y,z)\in\mathbb{R}^3 : z = x^2+y^2, 0\le z\le\lambda, y\ge 0\}\cup\{(x,y,z)\in\mathbb{R}^3 : y=0, x^2+y^2\le z\le\lambda\}$$

y $F:\mathbb{R}^3\to\mathbb{R}^3$ el campo dado por

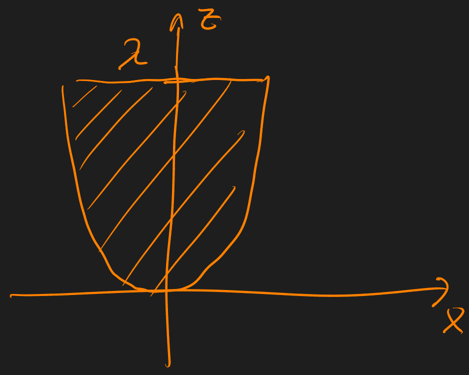
$$F(x,y,z) = \left(e^{z^2}+z, e^{x^2}+x, z\right).$$

Hallar el valor de $\lambda > 0$ tal que $\int\int_{S_\lambda} F\cdot \mathbf{dS} = 6$, donde S_λ está orientada de manera tal que en el punto $(0,0,\lambda/2)$ la normal tenga coordenada y positiva.



$z \le x^2$

$z = x^2$



Como F es C^1 $\Rightarrow$ Uso Teo. de Gauss:

$$-\iint_{S_\lambda} F\cdot d\vec{S} + \iint_{T} F\cdot d\vec{S} = \iiint_{\Omega} \operatorname{div} F \, dV$$

Tapa orientada hacia afuera

$\operatorname{div} F = 0 + 0 + 1 = 1$

$$\iint_{S_\lambda} F\cdot d\vec{S} = 6$$

$$\iiint_{\Omega} \operatorname{div} F \, dV = \operatorname{Vol}(\underbrace{S\cup T}_{\Omega})$$

Por Fubini

mitad del paraboloide

$$= \int_{z=0}^{z=\lambda} \frac{1}{2} \cdot \text{Area}(\text{Disco}_r)\, dz$$

$$= \int_{z=0}^{\lambda} \frac{1}{2} \pi \cdot r^2\, dz \quad \text{con} \quad r(z) = \sqrt{z}$$

pues por dato $x^2+y^2 = z$

$$= \int_{z=0}^{\lambda} \frac{1}{2} \pi \cdot (\sqrt{z})^2\, dz$$

$$= \frac{\pi}{2} \cdot \int_0^{\lambda} z \cdot dz$$

$$= \frac{\pi}{2} \cdot \left.\frac{z^2}{2}\right|_0^{\lambda}$$

$$= \frac{\pi}{4} \cdot \lambda^2$$

Calculo la integral sobre la tapa

$$\iint_{\Upsilon} F \cdot d\vec{S} = ?$$

Parametrizo Υ con

$$\varphi(r,t) = (r \cdot \cos t,\ r \sin t,\ \lambda)$$

$r \in [0, \sqrt{\lambda}]$

$t \in [0, \pi]$

$$\varphi_r = (\cos t \,,\, \sin t \,,\, 0)$$

$$\varphi_t = (-r \cdot \sin t \,,\, r \cdot \cos t \,,\, 0)$$

$$\varphi_r \times \varphi_t = (0, 0, r)$$

$$\iint_{\Upsilon} F \cdot d\vec{S} = \int_{r=0}^{\sqrt{\lambda}} \int_{t=0}^{\pi} \langle \underbrace{F(\varphi(r,t))}_{(\ldots,\ldots,\lambda)}, (0,0,r) \rangle \, dt\, dr$$

$$= \int_{r=0}^{\sqrt{\lambda}} \int_{t=0}^{\pi} \lambda \cdot r \cdot dt\, dr$$

$$= \pi \cdot \lambda \cdot \int_{r=0}^{\sqrt{\lambda}} r\, dr$$

$$= \pi \cdot \lambda \cdot \frac{r^2}{2} \Big|_0^{\sqrt{\lambda}}$$

$$= \pi \cdot \lambda \cdot \frac{\lambda}{2}$$

$$= \frac{\pi}{2} \cdot \lambda^2$$

Juntando todo:

$$-\iint_{S_\lambda} F\cdot d\vec{S} + \iint_{\Upsilon} F\cdot d\vec{S} = \iiint_{\Omega} \operatorname{div} F \, dV$$

$$-6 + \frac{\pi}{2}\cdot\lambda^2 = \frac{\pi}{4}\cdot\lambda^2$$

$$\frac{2\pi}{4}\cdot\lambda^2 - \frac{\pi}{4}\cdot\lambda^2 = 6$$

$$\frac{1}{4}\pi\cdot\lambda^2 = 6$$

$$\lambda^2 = \frac{24}{\pi}$$

$$\overset{\lambda\geq 0}{\lambda = \sqrt{\frac{24}{\pi}}}$$

Reviso

Share

Paraboloid - Volume

vCalc Reviewed

Last modified by KurtHeckman on Sep 29, 2022, 12:50:34 AM
Created by KurtHeckman on Jan 4, 2014, 8:29:03 PM

$$V = \frac{1}{2}\cdot\pi\cdot 1.662514119^2\cdot 2.763953196$$

(a)Length along Axis

2.763953196 $\leftarrow \lambda$ (cm) centimeter

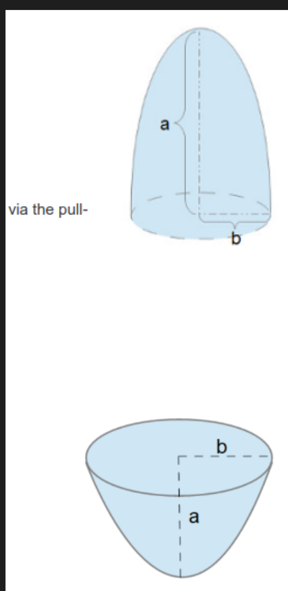
(b)Radius from Axis

1.662514119 $\leftarrow \sqrt{\lambda}$ (cm) centimeter

Share Result

12.0000000014457 Volumen de medio Parab (cm³) cubic centimeter

$= 6$



Parametrizo Paraboloide como

$$(x, 0, x^2) \xRightarrow{\text{Revol}} (\cos t \cdot x, \sin t \cdot x, x^2)$$

$x \in [0, \sqrt{\lambda}]$

$t = [0, \pi]$

Invierte orientación!

$$T_x = (\cos t \quad \sin t \quad 2x)$$

$$T_t = (-x \cdot \sin t \quad x \cdot \cos t \quad 0)$$

$$T_x \times T_t = (-2x^2 \cdot \cos t, \; -2x^2 \cdot \sin t, \; x)$$

$$-\int_{t=0}^{\pi}\int_{x=0}^{\sqrt{\lambda}} \langle F(x\cos t, x \cdot \sin t, x^2), \; T_x \times T_t \rangle \, dx \, dt =$$

Ejercicio 4. Sean S_λ la superficie en $\mathbb{R}^3$ dada por

$$S_\lambda = \{(x,y,z) \in \mathbb{R}^3 : z = x^2 + y^2, 0 \le z \le \lambda, y \ge 0\} \cup \{(x,y,z) \in \mathbb{R}^3 : y = 0, x^2 + y^2 \le z \le \lambda\}$$

y $F : \mathbb{R}^3 \to \mathbb{R}^3$ el campo dado por

$$F(x,y,z) = \left(e^{z^2} + z, e^{x^2} + x, z\right).$$

Hallar el valor de $\lambda > 0$ tal que $\int\int_{S_\lambda} F \cdot \mathbf{dS} = 6$, donde S_λ está orientada de manera tal que en el punto $(0, 0, \lambda/2)$ la normal tenga coordenada y positiva.

$$= 7{,}25293$$

integrate [integrate [-2 x^2 sin(t) (e^(x^2 cos^2(t)) + x cos(t)) - 2 (e^(x^4) + x^2) x^2 cos(t) + x^3 ,{x,0,sqrt

NATURAL LANGUAGE MATH INPUT EXTENDED KEYBOARD EXAMPLES UPLOAD RANDOM

Definite integral

$$\int_0^{\pi}\int_0^{2^{3/4}\sqrt[4]{\frac{3}{\pi}}} \left(x^3 - 2x^2\left(e^{x^4} + x^2\right)\cos(t) - 2x^2\left(e^{x^2\cos^2(t)} + x\cos(t)\right)\sin(t)\right) dx \, dt =$$

-7.25293

Parametrizo  $x^2 \leq z \leq \lambda$

$$T_2(x,z) = (x, 0, z) \qquad x \in [-\sqrt{z}, \sqrt{\lambda}]$$
$$z \in [x^2, \lambda]$$

$$T_{2x} = (1, 0, 0)$$
$$T_{2z} = (0, 0, 1)$$
$$T_{2x} \times T_{2z} = (0, -1, 0) \leftarrow$$ apunta hacia afuera! (invierte)

$$-\int_z \int_x \langle \underbrace{F(T_2(x,z))}, (0,-1,0) \rangle \, dx\, dz =$$

$$\underbrace{(\dots, e^{x^2} + x, \dots)}$$

$$T_2(x,z) = (x, 0, z)$$

$$= + \int_{-\sqrt{\lambda}}^{\sqrt{\lambda}} \int_{z=x^2}^{\lambda} e^{x^2} + x \, dz\, dx = 13{,}2529$$

Ejercicio 4. Sean S_λ la superficie en $\mathbb{R}^3$ dada por

$$S_\lambda = \{(x,y,z) \in \mathbb{R}^3 : z = x^2 + y^2, 0 \leq z \leq \lambda, y \geq 0\} \cup \{(x,y,z) \in \mathbb{R}^3 : y = 0, x^2 + y^2 \leq z \leq \lambda\}$$

y $F : \mathbb{R}^3 \to \mathbb{R}^3$ el campo dado por

$$F(x,y,z) = \left(e^{z^2} + z, e^{x^2} + x, z\right).$$

Hallar el valor de $\lambda > 0$ tal que $\int\int_{S_\lambda} F \cdot \mathbf{dS} = 6$, donde S_λ está orientada de manera tal que en el punto $(0, 0, \lambda/2)$ la normal tenga coordenada y positiva.

integrate[integrate [e^(x^2) + x, {z,x^2,sqrt(24/pi)}], {x, -sqrt(sqrt(24/pi)), sqrt(sqrt(24/pi))}]

NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES | UPLOAD

Definite integral | More

$$\int_{-2^{3/4}\sqrt[4]{\frac{3}{\pi}}}^{2^{3/4}\sqrt[4]{\frac{3}{\pi}}}\int_{x^2}^{2\sqrt{\frac{6}{\pi}}}\left(e^{x^2}+x\right)dz\,dx =$$

$$\frac{1}{2}\left(4\sqrt{6}+\sqrt{\pi}\right)\operatorname{erfi}\left(2^{3/4}\sqrt[4]{\frac{3}{\pi}}\right) - 2^{3/4}e^{2\sqrt{6/\pi}}\sqrt[4]{\frac{3}{\pi}} \approx 13.2529$$

$\operatorname{erfi}(x)$ is the imaginary error f

$$13{,}2529 - 7{,}2529 = 6$$

La integral que me dan !