Ejercicio 7. Resuelva las siguientes ecuaciones en $y = y(x)$:

(a) $(y - x^3)dx + (x + y^3)dy = 0$

(b) $\cos x \cos^2 y \, dx - 2 \operatorname{sen} x \operatorname{sen} y \cos y \, dy = 0$

(c) $(3x^2 - y^2)\, dy - 2xy \, dx = 0$

(d) $x \, dy = (x^5 + x^3y^2 + y)\, dx$

(e) $2(x+y) \operatorname{sen} y \, dx + \big(2(x+y)\operatorname{sen} y + \cos y\big)\, dy = 0$

(f) $3y\, dx + x\, dy = 0$

(g) $\big(1 - y(x+y)\tan(xy)\big)\, dx + \big(1 - x(x+y)\tan(xy)\big)\, dy = 0.$

$$\underbrace{M\,dx + N\,dy = 0}_{\text{es Exacta}} \quad \text{con } M, N \in C^1$$

es Exacta $\iff \dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ $\quad \left(\text{i.e. } Rot(M, N) = 0 \right)$

Si es Exacta:

$$\Rightarrow \begin{cases} \dfrac{\partial F}{\partial x} = M \\[2ex] \dfrac{\partial F}{\partial y} = N \end{cases} \Rightarrow \text{obtengo } F: \quad F(t, x) = c \quad c \in \mathbb{R}$$

(a) $(\underbrace{y - x^3}_{M})dx + (\underbrace{x + y^3}_{N})dy = 0$

$$\left.\begin{array}{l} M_y = 1 \\ N_x = 1 \end{array}\right\} M_y = N_x \Rightarrow \text{es Exacta}$$

Busco $F(x,y) = c \quad c \in \mathbb{R}$

Si $\dfrac{\partial F}{\partial x} = M = y - x^3 \Rightarrow F = xy - \frac{1}{4}x^4 + \varphi(y)$

Si $\dfrac{\partial F}{\partial y} = N = x + y^3 \Rightarrow F = xy + \frac{1}{4}y^4 + \tilde{\varphi}(x)$

$$F(x,y) = xy + \frac{1}{4}y^4 - \frac{1}{4}x$$

Sol:

$$xy + \frac{1}{4}y^4 - \frac{1}{4}x^4 = C \qquad C \in \mathbb{R}$$

Verifico

Derivo wrt x $\left(y = y(x) \Rightarrow \begin{cases} \frac{\partial}{\partial x} x = 1 \\ \frac{\partial}{\partial x} y = y' \end{cases} \right)$

$$\frac{\partial}{\partial x} F(x,y) = y + x \cdot y' + y^3 \cdot y' - x^3 = 0$$

$$y + x \cdot \frac{dy}{dx} + y^3 \cdot \frac{dy}{dx} - x^3 = 0$$

$$y \cdot dx + x \cdot dy + y^3 \cdot dy - x^3 \cdot dx = 0$$

$$(y - x^3)\,dx + (x + y^3)\,dy = 0$$

✓ Verificado

(b) $\cos x \cos^2 y \, dx - 2 \operatorname{sen} x \operatorname{sen} y \cos y \, dy = 0$

$$\underbrace{\cos x \cos^2 y}_{M} \, dx \; \underbrace{- 2 \operatorname{sen} x \operatorname{sen} y \cos y}_{N} \, dy = 0$$

$$\left.\begin{array}{l} M_y = \cos x \cdot 2 \cdot \cos y \cdot (-\sin y) \\ N_x = -2 \cdot \cos x \cdot \sin y \cdot \cos y \end{array}\right\} \quad M_y = N_x \Rightarrow \text{es Exacta}$$

Busco $F(x,y) = c \qquad c \in \mathbb{R}$

Si $\dfrac{\partial F}{\partial x} = M = \cos x \cdot \cos^2 y \qquad \Rightarrow F = \sin x \cdot \cos^2 y + \varphi(y)$

Si $\dfrac{\partial F}{\partial y} = N = -2 \sin x \cdot \sin y \cdot \cos y \Rightarrow F = \sin x \cdot \cos^2 y + \varphi(x)$

$$F(x,y) = \sin x \cdot \cos^2 y$$

$$\boxed{\begin{array}{l} \text{Sol:} \\ \qquad \sin x \cdot \cos^2 y = C \qquad C \in \mathbb{R} \end{array}}$$

Verifico:

$$\frac{\partial}{\partial x} F = \cos x \cdot \cos^2 y + \sin x \cdot 2 \cos y \cdot (-\sin y) \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$\cos x \cdot \cos^2 y \cdot dx + \sin x \cdot 2 \cos y \cdot (-\sin y) \cdot dy = 0$$

Verificado

$$(c)\ \underbrace{(3x^2 - y^2)}_{N}\, dy - \underbrace{2xy}_{M}\, dx = 0$$

¡Atentx al orden! $M\,dx + N\,dy$

$$\left.\begin{array}{l} M_y = -2x \\ N_x = 6x \end{array}\right\}$$

No es exacta! Pero!

Puedo multiplicar M dx + N dy = 0 por alguna función con el objetivo de convertirla en una ecuación exacta.

Como multiplico ambos lados por la misma cosa, no estoy cambiando las soluciones originales (solo debo tener en cuenta las indeterminaciones que puedo agregar).

En el ejercicio, -2x y 6x no están muy lejos uno de otro.

Sea μ un factor integrante

$$\mu\,(M\,dx + N\,dy) = \mu \cdot 0$$

$$\mu M\,dx + \mu N\,dy = 0$$

Como:

$$\begin{cases} M = -2xy \\ N = 3x^2 - y^2 \end{cases}$$

Quiero $\mu \,/\, (\mu . M)_y = (\mu N)_x$

$$\mu_y . M + \mu . M_y = \mu_x . N + \mu . N_x$$

Si $\mu_x = 0$

$$\mu_y . M + \mu . M_y = \overbrace{\mu_x . N}^{=0} + \mu . N_x$$

$$\mu . (M_y - N_x) = -\mu_y . M$$

$$-\frac{\mu_y}{\mu} = \frac{M_y - N_x}{M}$$

$$-\frac{\mu_y}{\mu} = \frac{-2x - 6x}{-2xy} = \frac{-8\cancel{x}}{-2\cancel{x}y}$$

$$-\frac{\mu_y}{\mu} = \frac{4}{y}$$

$$\int \frac{\mu_y}{\mu}\, dy = -\int \frac{4}{y}\, dy$$

$$\ln|\mu| = -4\ln|y|$$

$$|\mu| = \left(e^{\ln|y|}\right)^{-4}$$

$$|\mu| = |y|^{-4} \qquad y \neq 0$$

$$|\mu| = y^{-4}$$

Pruebo con $\mu = y^{-4}$

$$\tilde{M} = \mu . M = y^{-4}.(-2xy) = -2x.y^{-3}$$
$$\tilde{N} = \mu . N = y^{-4}(3x^2 - y^2) = 3x^2.y^{-4} - y^{-2}$$

$$\left.\begin{array}{l} \tilde{M}_y = 6x.y^{-4} \\ \tilde{N}_x = 6x.y^{-4} \end{array}\right\} \checkmark \text{ es } \underline{\text{exacto}}$$

Busco $F(x,y) = c \qquad c \in \mathbb{R}$

$$\frac{\partial}{\partial x} F = \tilde{M} = -2x \cdot y^{-3} \Rightarrow F = -x^2 \cdot y^{-3} + \varphi(y)$$

$$\frac{\partial}{\partial y} F = \tilde{N} = 3x^2 \cdot y^{-4} - y^{-2} \Rightarrow F = -x^2 \cdot y^{-3} + y^{-1} + \tilde{\varphi}(x)$$

$$F(x,y) = y^{-1} - x^2 \cdot y^{-3}$$

<u>Sol</u>:

$$\boxed{y^{-1} - x^2 \cdot y^{-3} = c \qquad c \in \mathbb{R}}$$

<u>Verifico</u>

$$\frac{\partial}{\partial x} F = -y^{-2} \cdot \underbrace{y'}_{\frac{dy}{dx}} - 2x \cdot y^{-3} + 3x^2 \cdot y^{-4} \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$-y^{-2} \cdot dy - 2x \cdot y^{-3} \cdot dx + 3x^2 \cdot y^{-4} \cdot dy = 0$$

$$\underbrace{-2x \cdot y^{-3}}_{=\tilde{M}} \cdot dx + \underbrace{\left(3x^2 \cdot y^{-4} - y^{-2}\right)}_{=\tilde{N}} dy = 0$$

✓ <u>Verificado</u>

(d) $x\,dy = (x^5 + x^3y^2 + y)\,dx$

$$\underbrace{(x^5 + x^3 \cdot y^2 + y)}_{M}\,dx + \underbrace{(-x)}_{N}\,dy = 0$$

$$\left.\begin{array}{l} M_y = 2x^3y + 1 \\ N_x = -1 \end{array}\right\} \underline{\text{No}} \text{ es exacta}$$

Busco μ.

Elijo $\mu_y = 0$ ⊛

$$-\frac{\mu_y}{\mu} = \frac{M_y - N_x}{M} \leftarrow$$ ⊛ Quiero algo simple en el denominador o que se simplifique con $M_y - N_x$

$$\left.-\frac{\mu_y}{\mu} = \frac{2x^3y + 2}{x^5 + x^3 \cdot y^2 + y}\right\} \text{ depende de 2 variables } \times$$

Elijo $\mu_x = 0$

$$\left.\frac{\mu_x}{\mu} = \frac{2x^3y + 2}{-x}\right\} \text{ depende de 2 variables } \times$$

No me sirve esta forma de encontrar μ.

Pruebo buscando $\mu(x,y) = x^a \cdot y^b$ con $a, b \in \mathbb{Q}$

$$(x^5 + x^3 . y^2 + y)\, dx + (-x)\, dy = 0$$

$$x^a . y^b . (x^5 + x^3 . y^2 + y)\, dx + x^a . y^b\, (-x)\, dy = 0$$

$$\underbrace{\left(x^{5+a} . y^b + x^{3+a} . y^{2+b} + x^a . y^{1+b}\right)}_{\tilde{M}} dx + \underbrace{\left(-x^{1+a} . y^b\right)}_{\tilde{N}} dy = 0$$

$$\tilde{M}_y = b . x^{5+a} . y^{b-1} + (2+b) . x^{3+a} . y^{1+b} + (1+b) . x^a . y^b$$

$$\tilde{N}_x = -(1+a) . x^a . y^b$$

Quiero

$$\tilde{M}_y = \tilde{N}_x$$

$$\Rightarrow \quad -(1+a) . x^a . y^b = (1+b) . x^a . y^b$$

$$-1 - a = 1 + b$$

$$a + b = 2$$

$$b = 2 - a$$

Además

$$b . x^{5+a} . y^{b-1} + (2+b) . x^{3+a} . y^{1+b} = 0$$

$$(2-a) . \underbrace{x^{5+a}}_{x^{3+a} . x^2} . y^{\overbrace{2-a-1}^{1-a}} = -(2+2-a) . x^{3+a} . \underbrace{y^{\overbrace{1+2-a}^{3-a}}}_{y^{1-a} . y^2}$$

$$(2-a) . x^{3+a} . x^2 . y^{1-a} = -(2+2-a) . x^{3+a} . y^{1-a} . y^2$$

$$x^{3+a} \neq 0$$

$$y^{1-a} \neq 0$$

$$(2-a)\cdot x^2 = -(2-a)\cdot y^2 - 2y^2$$

No llego a nada :(

Reescribo el ejercicio: Método ???

$$(x^5 + x^3 \cdot y^2 + y)\, dx + (-x)\, dy = 0$$

$\times \frac{1}{dx}$

$$(x^5 + x^3 \cdot y^2 + y) \underbrace{\frac{dx}{dx}}_{=1} + (-x) \underbrace{\frac{dy}{dx}}_{y'} = 0$$

$$x^5 + x^3 \cdot y^2 + y - x \cdot y' = 0$$

$$y' = \frac{x^5 + x^3 \cdot y^2 + y}{x}$$

Si $\quad y = \mu \cdot x \qquad (\mu = \mu(x))$

$$\Rightarrow y' = \mu'(x) \cdot x + \mu(x) \cdot 1$$

$$\mu + \mu' \cdot x = \frac{x^5 + x^3 \cdot \mu^2 \cdot x^2 + \mu \cdot x}{x}$$

$$\mu + \mu' \cdot x = x^4 + x^4 \cdot \mu^2 + \mu$$

$$\mu' \cdot x = x^4 + x^4 \cdot \mu^2$$

$$\mu' \cdot x = x^4 (1 + \mu^2)$$

$$\frac{\mu'}{1 + \mu^2} = x^3 \quad \leftarrow \text{Se separó!}$$

$$\int \frac{\mu'}{1+\mu^2}\,dx = \int x^3\,dx$$

$$\arctan \mu = \frac{1}{4}x^4 + C$$

$$\mu = \tan\left(\frac{1}{4}x^4 + C\right)$$

Como $y = \mu . x$

$$\Rightarrow \boxed{\begin{array}{l}\underline{\text{Sol}}: \\ y = x . \tan\left(\frac{1}{4}x^4 + C\right)\end{array}} \qquad C \in \mathbb{R}$$

(e) $2(x+y)\operatorname{sen} y\,dx + \big(2(x+y)\operatorname{sen} y + \cos y\big)\,dy = 0$

$$\underbrace{2(x+y)\operatorname{sen} y}_{M}\,dx + \underbrace{\big(2(x+y)\operatorname{sen} y + \cos y\big)}_{N}\,dy = 0$$

$$M = 2x.\sin y + 2y.\sin y$$

$$M_y = 2x.\cos y + 2.\sin y + 2y.\cos y$$

$$N = 2x.\sin y + 2y.\sin y + \cos y$$

$$N_x = 2\sin y$$

No es exacta.

Busco μ.

$$M_y - N_x = 2x.\cos y + 2.\sin y + 2y.\cos y - 2\sin y$$

$$= 2x.\cos y + 2y.\cos y$$

$$= 2\cos y\,(x+y)$$

$$M = 2\sin y\,(x+y)$$

Si $\mu_x = 0$

$$\mu_y.M + \mu.M_y = \underbrace{\mu_x.N}_{=0} + \mu.N_x$$

$$\mu_y = \frac{\mu\left(N_x - M_y\right)}{M}$$

$$\frac{\mu_y}{\mu} = \frac{N_x - M_y}{M}$$

$$\frac{\mu_y}{\mu} = \frac{-\cancel{2}\cos y\,\cancel{(x+y)}}{\cancel{2}\sin y\,\cancel{(x+y)}}$$

$$\frac{\mu_y}{\mu} = -\frac{\cos y}{\sin y}$$

$$\int \frac{\mu_y}{\mu}\,dy = \int -\frac{\cos y}{\sin y}\,dy$$

$v = \ln(\sin y)$

$dv = \frac{1}{\sin y}\cdot \cos y\,dy$

$$\ln|\mu| = -\ln(\sin y) + c$$

$$|\mu| = e^{-\ln(\sin y)}\cdot e^{c} \qquad c \in \mathbb{R}$$

$$|\mu| = \sin^{-1} y \cdot \tilde{C}$$

Elijo $\mu(y) = \sin^{-1} y$

$$\begin{cases} M = 2x.\sin y + 2y.\sin y \\ N = 2x.\sin y + 2y.\sin y + \cos y \end{cases}$$

$$\tilde{M} = \mu.M = 2x + 2y$$

$$\tilde{N} = \mu.N = 2x + 2y + \frac{\cos y}{\sin y}$$

$\Rightarrow \tilde{M}_y - \tilde{N}_x = 2 - 2 = 0$ ✓ es exacta

Busco $F(x,y) = c$ con $c \in \mathbb{R}$

Si $\frac{\partial}{\partial x} F = \tilde{M} = 2x + 2y \Rightarrow F = x^2 + 2xy + \varphi(y)$

Si $\frac{\partial}{\partial y} F = \tilde{N} = 2x + 2y + \frac{\cos y}{\sin y} \Rightarrow F = 2xy + y^2 + \ln(\sin y) + \tilde{\varphi}(x)$

$$F(x,y) = x^2 + 2xy + y^2 + \ln(\sin y)$$

$$\boxed{\text{Sol: } \quad x^2 + 2xy + y^2 + \ln(\sin y) = c} \qquad c \in \mathbb{R}$$

Verifico:

$$\frac{\partial}{\partial x} F = 2x + 2y + 2x\underbrace{y'}_{\frac{dy}{dx}} + 2 \cdot y \cdot \underbrace{y'}_{\frac{dy}{dx}} + \frac{1}{\sin y} \cdot \cos y \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$2x \cdot dx + 2y \cdot dx + 2x \cdot dy + 2y \cdot dy + \frac{\cos y}{\sin y} \cdot dy = 0$$

$$\underbrace{(2x + 2y)}_{\tilde{M}} dx + \underbrace{\left(2x + 2y + \frac{\cos y}{\sin y}\right)}_{\tilde{N}} dy = 0$$

✓ Verificado

(f) $\underbrace{3y}_{M}\,dx + \underbrace{x}_{N}\,dy = 0$

$M_y = 3$

$N_x = 1$ No es exacta. Pruebo reordenando antes de usar Factor integrante.

$$3y \underbrace{\frac{dx}{dx}}_{=1} + x \cdot \underbrace{\frac{dy}{dx}}_{=y'} = 0$$

$$3y + x \cdot y' = 0$$

$$x \cdot y' = -3y$$

$$\frac{y'}{y} = \frac{-3}{x}$$

$$\int \frac{y'}{y}\,dx = \int \frac{-3}{x}\,dx$$

$$\ln|y| = -3\ln|x| + C \qquad C \in \mathbb{R}$$

$$|y| = |x|^{-3} \cdot \tilde{C} \qquad C \in \mathbb{R}_{>0}$$

Mucho módulo, pruebo con F. Int.

Si uso μ

$$\frac{M_y - N_x}{N} = \frac{3-1}{x} = \frac{2}{x} = \frac{\mu'}{\mu}$$

$$\ln|\mu| = 2\ln|x| + c$$

$$|\mu| = |x|^2 \cdot \tilde{C} \qquad \tilde{C} \in \mathbb{R}_{>0}$$

$$|\mu| = x^2 \cdot \tilde{C}$$

Si $\mu(x) = x^2$

(f) $3y\,dx + x\,dy = 0$

$$\Rightarrow \begin{cases} \tilde{M} = 3x^2 \cdot y \Rightarrow \tilde{M}_y = 3x^2 \\ \tilde{N} = x^3 \Rightarrow \tilde{N}_x = 3 \cdot x^2 \end{cases}$$

es exacta ✓

Busco $F(x,y) = c$

$$\frac{\partial}{\partial x} F = \tilde{M} = 3x^2 \cdot y \Rightarrow F = x^3 \cdot y + \varphi(y)$$

$$\tilde{N} = x^3 \Rightarrow F = x^3 \cdot y + \tilde{\varphi}(x)$$

$$\Rightarrow F(x,y) = x^3 \cdot y$$

$$\boxed{\text{Sol: } x^3 \cdot y = c} \qquad c \in \mathbb{R}$$

Verifico

$$\frac{\partial}{\partial x} F = 3x^2 \cdot y + x^3 \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$3x^2 \cdot y\,dx + x^3 \cdot dy = 0$$ ✓ Verificado.

(g) $\left(1 - y(x+y)\tan(xy)\right) dx + \left(1 - x(x+y)\tan(xy)\right) dy = 0.$

Veri dificil ...

Ejercicio 8. Considere la ecuación lineal de primer orden en $y(x)$

(∗) $$y' + p(x)\,y = q(x).$$

(a) Busque una función $\mu(x)$ tal que
$$\mu(x)\big(y'(x) + p(x)\,y(x)\big) = \big(\mu(x)\,y(x)\big)'.$$

(b) Multiplique la ecuación (∗) por μ y halle su solución general. μ se denomina *factor integrante*.

a) $$\mu(y' + p \cdot y) = (\mu \cdot y)'$$

Dato $$y' + p \cdot y = q$$

$$\mu \cdot y' + \mu \cdot p \cdot y = \mu' \cdot y + \mu \cdot y'$$

$$\mu \cdot p \cdot y = \mu' \cdot y \qquad y \neq 0$$

$$\mu' = \mu \cdot p$$

$$\frac{\mu'}{\mu} = p$$

$$\int \frac{\mu'}{\mu}\, dx = \int p\, dx$$

$$\ln|\mu| = \int p(x) \cdot dx$$

$$|\mu| = e^{\int p(x) \cdot dx}$$

me quedo con

$$\boxed{\mu = e^{\int p(x) \cdot dx}}$$

b) $*: \; y' + p \cdot y = q$

y además: $\mu\left(y' + p \cdot y\right) = (\mu \cdot y)'$

$$\mu\left(y' + p \cdot y\right) = \underbrace{\mu \cdot q = (\mu \cdot y)'}$$

$$\int \mu \cdot q \, dx = \int (\mu \cdot y)' \, dx$$

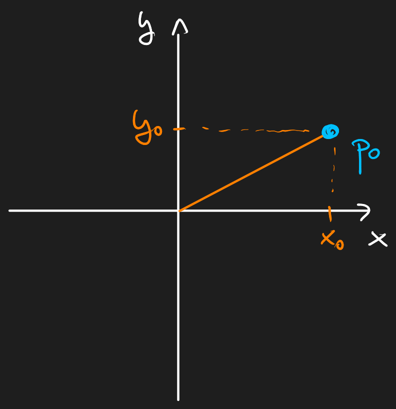
$$\int \mu \cdot q \, dx = \mu \cdot y$$

$$y = \frac{1}{\mu} \cdot \int \mu \cdot q \, dx$$

Sol. Gral.

$$\boxed{y = e^{-\int p(x)\, dx} \cdot \int q \cdot e^{\int p(x) \cdot dx} \, dx}$$

Ejercicio 9. Hallar la ecuación de una curva tal que la pendiente de la recta tangente en un punto cualquiera es la mitad de la pendiente de la recta que une el punto con el origen.



Escribo C como una función de x

$$C = \{ (x,y) \in \mathbb{R}^2 : y = f(x) \}$$

Parametrizada por:

$$\sigma(x) = (x, f(x))$$

Recta tg: $y = f'(x_0) \cdot (x - x_0) + f(x_0)$

Además:

$$f'(x_0) = \frac{1}{2} \cdot \frac{y_0}{x_0}$$

$$f'(x_0) = \frac{1}{2} \cdot \frac{f(x_0)}{x_0}$$

$$y' = \frac{1}{2} \frac{y}{x}$$

Resuelvo

$$\frac{y'}{y} = \frac{1}{2x}$$

$$\int \frac{y'}{y}\, dx = \int \frac{1}{2x}\, dx$$

$$\ln|y| = \frac{1}{2} \cdot \ln|x| + c \qquad c \in \mathbb{R}$$

$$|y| = |x|^{1/2} \cdot \tilde{c} \qquad \tilde{c} \in \mathbb{R}_{>0}$$

$$y = c \cdot |x|^{1/2} \qquad c \in \mathbb{R},\ x \in \mathbb{R}$$

Verifico:

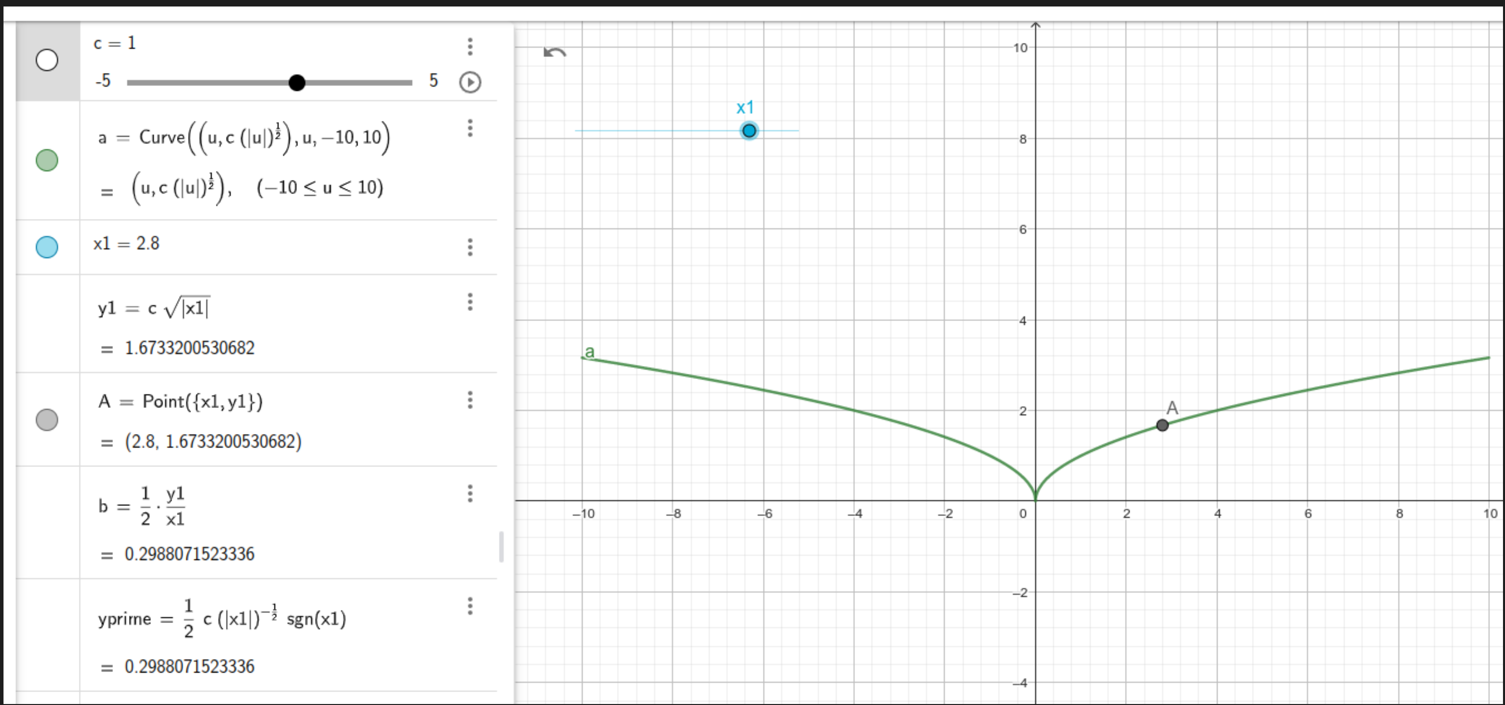
$$y' = \frac{1}{2} \cdot c \cdot |x|^{-\frac{1}{2}} \cdot \text{signo}(x)$$

Quiero llegar a:

$$y' \overset{?}{=} \frac{1}{2} \frac{y}{x}$$

$$\frac{1}{2} \cdot c \cdot |x|^{-\frac{1}{2}} \cdot \text{signo}(x) = \frac{1}{2} \cdot \underbrace{\frac{1}{x} \cdot c \cdot |x|^{1/2}}_{= |x|^{1/2-1} \cdot \text{signo}(x)}$$

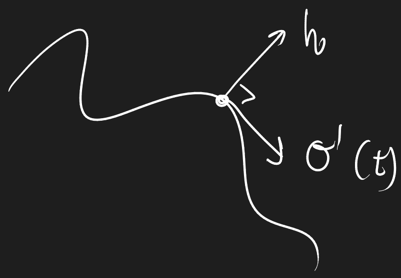
Verificado



Ejercicio 10. Hallar la ecuación de las curvas tales que la normal en un punto cualquiera pasa por el origen.

$\sigma(t)$: Param de C.

$$\eta : \langle \sigma'(t) , (x-x_0, y-y_0) \rangle = 0$$



$$\sigma(x) = (x, f(x))$$

$$\Rightarrow \sigma'(x) = (1, f'(x))$$

$$\eta \text{ en } x_0 : \langle (1, f'(x_0)) , (x-x_0, y-y_0) \rangle = 0$$

$$(x-x_0) + f'(x_0) \cdot (y - f(x_0)) = 0$$

$$(x-x_0) = -f'(x_0) \cdot (y - f(x_0))$$

$$-\frac{(x-x_0)}{f'(x_0)} = y - f(x_0)$$

$$\eta \text{ en } x_0 : \quad y = f(x_0) - \frac{1}{f'(x_0)} \cdot (x - x_0)$$

Si busco las que pasan por el origen:

$$\Rightarrow \text{Si } x = 0 \Rightarrow \underbrace{f(x)}_{=y} = 0 \quad (\eta \text{ pasa por el origen})$$

$$\Rightarrow \quad 0 = f(x_0) - \frac{1}{f'(x_0)} \cdot (0 - x_0)$$

$$0 = f(x_0) + \frac{x_0}{f'(x_0)}$$

Renombro

$$0 = y + \frac{x}{y'}$$

$$-\frac{x}{y'} = y$$

$$-x = y' \cdot y$$

$$\int \underbrace{y' \cdot y \; dx}_{y \cdot \underbrace{\frac{dy}{dx}}_{y'} \cdot dx} = \int -x \; dx$$

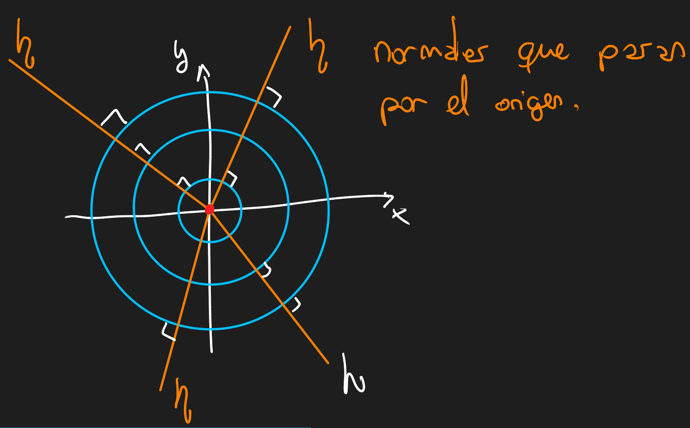
$$\int y \cdot dy =$$

$$\frac{1}{2} y^2 = -\frac{1}{2} x^2 + C \qquad C \in \mathbb{R}$$

$$y^2 = -x^2 + C$$

$x^2 + y^2 = C \leftarrow$ Circunferencias de radio $\sqrt{C}$

$$y = \begin{cases} \sqrt{-x^2+C} \\ -\sqrt{-x^2+C} \end{cases} \qquad C \geq x^2$$



o de forma paramétrica

$$\boxed{\sigma(t) = \left(\sqrt{C} \cdot \cos t \,,\, \sqrt{C} \cdot \sin t\right) \qquad t \in [0, 2\pi) \quad C \in \mathbb{R}_{>0}}$$

Ejercicio 11. Demostrar que la curva para la cual la pendiente de la tangente en cualquier punto es proporcional a la abscisa del punto de contacto es una parábola.

$$\sigma(x) = (x, f(x)) \qquad y = f(x)$$

Recta tg: $y = \underbrace{f'(x_0)}_{\text{Pendiente}} \cdot (x - x_0) + f(x_0)$

Abscisa: Donde cruza el eje x

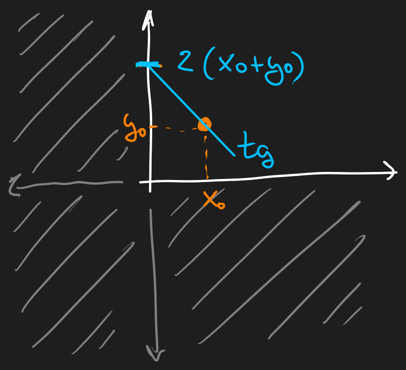
$$y'(x) = c \cdot x$$

$$\int y' \, dx = \int c \cdot x \, dx$$

$$y = \frac{c}{2} \cdot x^2 + d \qquad c, d \in \mathbb{R}$$

$$\boxed{y = c \cdot x^2 + d}$$

Ejercicio 12. Hallar la ecuación de una curva del primer cuadrante tal que para cada punto (x_0, y_0) de la misma, la ordenada al origen de la recta tangente a la curva en (x_0, y_0) sea $2(x_0 + y_0)$.



$$y = f'(x_0) \cdot (x - x_0) + f(x_0)$$

$$x = 0$$

$$y = 2(x_0 + y_0)$$

$$\Rightarrow \quad 2x_0 + 2y_0 = -f'(x_0) \cdot x_0 + \underbrace{f(x_0)}_{=y_0}$$

$$2x_0 + 2y_0 = y_0 - y_0' \cdot x_0$$

Como quiero que valga $\forall x, y$ de C

$$\Rightarrow \quad 2x + 2y = y - y' \cdot x$$

$$2x + y = -y' \cdot x$$

$$y' \cdot x + 2x + y = 0 \qquad x \neq 0$$

$$y' + 2 + \frac{y}{x} = 0$$

$$\frac{dy}{dx} + 2 + \frac{y}{x} = 0$$

$$1 \cdot dy + 2\,dx + \frac{y}{x} \cdot dx$$

$$\underbrace{\left(2 + \frac{y}{x}\right)}_{=M} dx + \underbrace{1}_{N}\, dy = 0$$

$$\left.\begin{array}{l} M_y = \frac{1}{x} \\ N_x = 0 \end{array}\right\} \text{ No es exacta}$$

Busco $\mu \,/\, (\mu . M)_y = (\mu N)_x$

$$\mu_y . M + \mu . M_y = \mu_x . N + \mu . N_x$$

Si $\mu = \mu(x)$

$\Rightarrow \mu_y = 0$

$$\mu . M_y = \mu_x . N + \mu . N_x$$

$$\mu . (M_y - N_x) = \mu_x . N$$

$$\frac{\mu_x}{\mu} = \frac{M_y - N_x}{N}$$

$$= \frac{\left(\frac{1}{x} - 0\right)}{1} = \frac{1}{x}$$

$$\frac{\mu_x}{\mu} = \frac{1}{x}$$

$$\int \frac{\mu_x}{\mu} dx = \int \frac{1}{x} dx$$

$$\ln|\mu| = \ln|x| + C \qquad C \in \mathbb{R}$$

$$|\mu| = \tilde{c}\,|x| \qquad \tilde{C} \in \mathbb{R}_{>0}$$

$$\mu = c \cdot |x| \qquad C \in \mathbb{R}$$

Elijo $C = 1$ y μ:

$\Rightarrow \mu = x$

$$\underbrace{\left(2+\frac{y}{x}\right)}_{M}dx + \underbrace{1}_{N}\,dy = 0$$

$$\begin{cases} x \cdot M = \tilde{M} = 2x + \frac{y}{x} \cdot x = 2x + y \\ x \cdot N = \tilde{N} = x \end{cases}$$

$$\left.\begin{array}{l} \tilde{M}_y = 1 \\ \tilde{N}_x = 1 \end{array}\right\} \text{ es exacta!}$$

$\Rightarrow \quad F(x,y) = c$

Busco F

$$\frac{\partial F}{\partial x} = \tilde{M} = 2x + y \Rightarrow F = x^2 + xy + \varphi(y)$$

$$\frac{\partial F}{\partial y} = \tilde{N} = x \Rightarrow F = xy + \tilde{\varphi}(x)$$

$$\Rightarrow F(x,y) = x^2 + xy$$

$$\boxed{\text{Sol: } \quad x^2 + xy = c} \quad \text{con } c \in \mathbb{R}$$

Verifico: derivo wrt x

$$2x + 1 \cdot y + x \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$2x\,dx + y\,dx + x\,dy = 0$$

$$(2x+y)\,dx + x\,dy = 0 \quad \checkmark \text{ Verificado.}$$