

Ejercicio 7. Resuelva las siguientes ecuaciones en $y = y(x)$:

(a) $(y - x^3)dx + (x + y^3)dy = 0$

(b) $\cos x \cos^2 y dx - 2 \sin x \sin y \cos y dy = 0$

(c) $(3x^2 - y^2) dy - 2xy dx = 0$

(d) $x dy = (x^5 + x^3 y^2 + y) dx$

(e) $2(x + y) \sin y dx + (2(x + y) \sin y + \cos y) dy = 0$

(f) $3y dx + x dy = 0$

(g) $(1 - y(x + y) \tan(xy)) dx + (1 - x(x + y) \tan(xy)) dy = 0.$

$$M dx + N dy = 0 \quad \text{con } M, N \in C^1$$

$$\underbrace{\hspace{10em}}_{\text{es Exacta}} \Leftrightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \left(\text{ie. } \text{Rot}(M, N) = 0 \right)$$

Si es Exacta:

$$\Rightarrow \begin{cases} \frac{\partial F}{\partial x} = M \\ \frac{\partial F}{\partial y} = N \end{cases} \Rightarrow \text{obtengo } F : F(x, y) = c \quad c \in \mathbb{R}$$

(a) $(y - x^3)dx + (x + y^3)dy = 0$

$$\underbrace{\hspace{2em}}_M \quad \underbrace{\hspace{2em}}_N$$

$$\left. \begin{array}{l} M_y = 1 \\ N_x = 1 \end{array} \right\} M_y = N_x \Rightarrow \text{es Exacta}$$

Buscamos $F(x, y) = c \quad c \in \mathbb{R}$

Si $\frac{\partial F}{\partial x} = M = y - x^3 \Rightarrow F = xy - \frac{1}{4}x^4 + \varphi(y)$

Si $\frac{\partial F}{\partial y} = N = x + y^3 \Rightarrow F = xy + \frac{1}{4}y^4 + \tilde{\varphi}(x)$

$$F(x, y) = xy + \frac{1}{4}y^4 - \frac{1}{4}x$$

Sol:

$$xy + \frac{1}{4}y^4 - \frac{1}{4}x^4 = C \quad C \in \mathbb{R}$$

Verifico

Derivo wrt x $\left(y = y(x) \Rightarrow \begin{cases} \frac{\partial}{\partial x} x = 1 \\ \frac{\partial}{\partial x} y = y' \end{cases} \right)$

$$\frac{\partial}{\partial x} F(x, y) = y + x \cdot y' + y^3 \cdot y' - x^3 = 0$$

$$dx \quad y + x \cdot \frac{dy}{dx} + y^3 \cdot \frac{dy}{dx} - x^3 = 0$$

$$y \cdot dx + x \cdot dy + y^3 \cdot dy - x^3 \cdot dx = 0$$

$$(y - x^3) dx + (x + y^3) dy = 0$$

✓ Verificado

$$(b) \underbrace{\cos x \cos^2 y dx}_M - 2 \underbrace{\sin x \sin y \cos y dy}_N = 0$$

$$\left. \begin{aligned} M_y &= \cos x \cdot 2 \cdot \cos y \cdot (-\sin y) \\ N_x &= -2 \cdot \cos x \cdot \sin y \cdot \cos y \end{aligned} \right\} M_y = N_x \Rightarrow \text{e Exata}$$

$$\text{Busca } F(x, y) = c \quad c \in \mathbb{R}$$

$$\text{Se } \frac{\partial F}{\partial x} = M = \cos x \cdot \cos^2 y \Rightarrow F = \sin x \cdot \cos^2 y + \varphi(y)$$

$$\text{Se } \frac{\partial F}{\partial y} = N = -2 \sin x \cdot \sin y \cdot \cos y \Rightarrow F = \sin x \cdot \cos^2 y + \varphi(x)$$

$$F(x, y) = \sin x \cdot \cos^2 y$$

Sol:

$$\sin x \cdot \cos^2 y = c \quad c \in \mathbb{R}$$

Verifica:

$$\frac{\partial}{\partial x} F = \cos x \cdot \cos^2 y + \sin x \cdot 2 \cos y \cdot (-\sin y) \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$\cos x \cdot \cos^2 y \cdot dx + \sin x \cdot 2 \cos y \cdot (-\sin y) \cdot dy = 0$$

Verificado

$$(c) (3x^2 - y^2) dy - 2xy dx = 0$$

$$\underbrace{\hspace{1cm}}_N \quad \underbrace{\hspace{1cm}}_M \quad ! \text{ Atención al orden! } \quad M dx + N dy$$

$$\left. \begin{array}{l} M_y = -2x \\ N_x = 6x \end{array} \right\} \text{ No es exacta! Pero!}$$

Puedo multiplicar $M dx + N dy = 0$ por alguna función con el objetivo de convertirla en una ecuación exacta.

Como multiplico ambos lados por la misma cosa, no estoy cambiando las soluciones originales (solo debo tener en cuenta las indeterminaciones que puedo agregar).

En el ejercicio, $-2x$ y $6x$ no están muy lejos uno de otro.

Sea μ un factor integrante

$$\mu (M dx + N dy) = \mu \cdot 0$$

$$\mu M dx + \mu N dy = 0$$

Como:

$$\begin{cases} M = -2xy \\ N = 3x^2 - y^2 \end{cases}$$

$$\text{Quiero } \mu / (\mu M)_y = (\mu N)_x$$

$$\mu_y \cdot M + \mu \cdot M_y = \mu_x \cdot N + \mu \cdot N_x$$

$$\text{Si } \mu_x = 0$$

$$\mu_y \cdot M + \mu \cdot M_y = \overbrace{\mu_x \cdot N}^{=0} + \mu \cdot N_x$$

$$\mu \cdot (M_y - N_x) = -\mu_y \cdot M$$

$$-\frac{\mu_y}{\mu} = \frac{M_y - N_x}{M}$$

$$-\frac{\mu_y}{\mu} = \frac{-2x - 6x}{-2xy} = \frac{-8x}{-2xy}$$

$$-\frac{\mu_y}{\mu} = \frac{4}{y}$$

$$\int \frac{\mu_y}{\mu} dy = - \int \frac{4}{y} dy$$

$$\ln|\mu| = -4 \ln|y|$$

$$|\mu| = (e^{\ln|y|})^{-4}$$

$$|\mu| = |y|^{-4} \quad y \neq 0$$

$$|\mu| = y^{-4}$$

Probo con $\mu = y^{-4}$

$$\tilde{M} = \mu \cdot M = y^{-4} \cdot (-2xy) = -2x \cdot y^{-3}$$

$$\tilde{N} = \mu \cdot N = y^{-4} (3x^2 - y^2) = 3x^2 \cdot y^{-4} - y^{-2}$$

$$\tilde{M}_y = 6x \cdot y^{-4}$$

$$\tilde{N}_x = 6x \cdot y^{-4}$$

✓ es exacta

$$\tilde{M} dx + \tilde{N} dy = 0$$

Burcu $F(x, y) = c \quad c \in \mathbb{R}$

$$\frac{\partial}{\partial x} F = \tilde{M} = -2x \cdot y^{-3} \Rightarrow F = -x^2 \cdot y^{-3} + \varphi(y)$$

$$\frac{\partial}{\partial y} F = \tilde{N} = 3x^2 \cdot y^{-4} - y^{-2} \Rightarrow F = -x^2 \cdot y^{-3} + y^{-1} + \tilde{\varphi}(x)$$

$$F(x, y) = y^{-1} - x^2 \cdot y^{-3}$$

Sol:

$$y^{-1} - x^2 \cdot y^{-3} = c \quad c \in \mathbb{R}$$

Veritica

$$\frac{\partial}{\partial x} F = -y^{-2} \cdot \underbrace{y'}_{\frac{dy}{dx}} - 2x \cdot y^{-3} + 3x^2 \cdot y^{-4} \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$-y^{-2} \cdot dy - 2x \cdot y^{-3} \cdot dx + 3x^2 \cdot y^{-4} \cdot dy = 0$$

$$\underbrace{-2x \cdot y^{-3} \cdot dx}_{= \tilde{M}} + \underbrace{(3x^2 \cdot y^{-4} - y^{-2}) dy}_{= \tilde{N}} = 0 \quad \checkmark \text{ Veri Riccio}$$

$$(d) \ x dy = (x^5 + x^3 y^2 + y) dx$$

$$\underbrace{(x^5 + x^3 y^2 + y)}_M dx + \underbrace{(-x)}_N dy = 0$$

$$\left. \begin{array}{l} M_y = 2x^3 y + 1 \\ N_x = -1 \end{array} \right\} \underline{No} \text{ es exacto}$$

Busco μ .

$$\text{Elijo } \mu_y = 0 \quad \oplus$$

$$-\frac{\mu_y}{\mu} = \frac{M_y - N_x}{M} \quad \oplus \quad \text{Quiero algo simple en el denominador o que se simplifique con } M_y - N_x$$

$$-\frac{\mu_y}{\mu} = \frac{2x^3 y + 2}{x^5 + x^3 y^2 + y} \quad \left. \vphantom{-\frac{\mu_y}{\mu}} \right\} \text{depende de 2 variables } \times$$

$$\text{Elijo } \mu_x = 0$$

$$\frac{\mu_x}{\mu} = \frac{2x^3 y + 2}{-x} \quad \left. \vphantom{\frac{\mu_x}{\mu}} \right\} \text{depende de 2 variables } \times$$

No me sirve esta forma de encontrar μ .

$$\text{Pruebo buscando } \mu(x, y) = x^a \cdot y^b \quad \text{con } a, b \in \mathbb{Q}$$

$$(x^5 + x^3 \cdot y^2 + y) dx + (-x) dy = 0$$

$$x^a \cdot y^b \cdot (x^5 + x^3 \cdot y^2 + y) dx + x^a \cdot y^b (-x) dy = 0$$

$$\underbrace{(x^{5+a} \cdot y^b + x^{3+a} \cdot y^{2+b} + x^a \cdot y^{1+b})}_{\tilde{M}} dx + \underbrace{(-x^{1+a} \cdot y^b)}_{\tilde{N}} dy = 0$$

$$\tilde{M}_y = b \cdot x^{5+a} \cdot y^{b-1} + (2+b) \cdot x^{3+a} \cdot y^{1+b} + (1+b) \cdot x^a \cdot y^b$$

$$\tilde{N}_x = -(1+a) \cdot x^a \cdot y^b$$

Quiero

$$\tilde{M}_y = \tilde{N}_x$$

$$\Rightarrow -(1+a) \cdot x^a \cdot y^b = (1+b) \cdot x^a \cdot y^b$$

$$-1-a = 1+b$$

$$a+b = -2$$

$$b = -2-a$$

Además

$$b \cdot x^{5+a} \cdot y^{b-1} + (2+b) \cdot x^{3+a} \cdot y^{1+b} = 0$$

$$\underbrace{(2-a)}_{\substack{3+a \\ x^3 \cdot x^a}} \cdot x^{5+a} \cdot y^{\overbrace{2-a-1}^{1-a}} = -(2+2-a) \cdot x^{3+a} \cdot y^{\overbrace{1+2-a}^{3-a}} \cdot y^{\overbrace{1-a}^{1-a}} \cdot y^2$$

$$(2-a) \cdot x^{3+a} \cdot x^2 \cdot y^{1-a} = -(2+2-a) \cdot x^{3+a} \cdot y^{1-a} \cdot y^2$$

$$x^{3+a} \neq 0$$

$$y^{1-a} \neq 0$$

$$(z-a) \cdot x^2 = -(z-a) \cdot y^2 - 2y^2$$

No Negro \Rightarrow nada $\cup \cup$

Reescribo el ejercicio: Método ???

$$(x^5 + x^3 \cdot y^2 + y) dx + (-x) dy = 0$$
$$\overset{x \frac{1}{dx}}{\rightarrow} (x^5 + x^3 \cdot y^2 + y) \underbrace{\frac{dx}{dx}}_{=1} + (-x) \underbrace{\frac{dy}{dx}}_{y'} = 0$$

$$x^5 + x^3 \cdot y^2 + y - x \cdot y' = 0$$

$$y' = \frac{x^5 + x^3 \cdot y^2 + y}{x}$$

$$\text{So } y = \mu \cdot x \quad (\mu = \mu(x))$$

$$\Rightarrow y' = \mu'(x) \cdot x + \mu(x) \cdot 1$$

$$\mu + \mu' \cdot x = \frac{x^5 + x^3 \cdot \mu^2 \cdot x^2 + \mu \cdot x}{x}$$

$$\mu + \mu' \cdot x = x^4 + x^4 \cdot \mu^2 + \mu$$

$$\mu' \cdot x = x^4 + x^4 \cdot \mu^2$$

$$\mu' \cdot x = x^4 (1 + \mu^2)$$

$$\frac{\mu'}{1 + \mu^2} = x^3 \quad \leftarrow \text{Se separó!} \quad \text{🐼💕}$$

$$\int \frac{\mu'}{1 + \mu^2} dx = \int x^3 dx$$

$$\arctan \mu = \frac{1}{4} x^4 + C$$

$$\mu = \tan \left(\frac{1}{4} x^4 + C \right)$$

Come $y = \mu \cdot x$

\Rightarrow

Sol:

$$y = x \cdot \tan \left(\frac{1}{4} x^4 + C \right)$$

$$C \in \mathbb{R}$$

$$(e) \underbrace{2(x+y) \operatorname{sen} y dx}_M + \underbrace{(2(x+y) \operatorname{sen} y + \cos y) dy}_N = 0$$

$$M = 2x \cdot \operatorname{sen} y + 2y \cdot \operatorname{sen} y$$

$$M_y = 2x \cdot \cos y + 2 \cdot \operatorname{sen} y + 2y \cdot \cos y$$

$$N = 2x \cdot \operatorname{sen} y + 2y \cdot \operatorname{sen} y + \cos y$$

$$N_x = 2 \operatorname{sen} y$$

No es exacto.

Busco μ .

$$M_y - N_x = 2x \cdot \cos y + 2 \cdot \operatorname{sen} y + 2y \cdot \cos y - 2 \operatorname{sen} y$$

$$= 2x \cdot \cos y + 2y \cdot \cos y$$

$$= 2 \cos y (x+y)$$

$$M = 2 \operatorname{sen} y (x+y)$$

$$\text{Si } \mu_x = 0$$

$$\mu_y \cdot M + \mu \cdot M_y = \overbrace{\mu_x \cdot N}^{=0} + \mu \cdot N_x$$

$$\mu_y = \frac{\mu (N_x - M_y)}{M}$$

$$\frac{\mu_y}{\mu} = \frac{N_x - M_y}{M}$$

$$\frac{\mu_y}{\mu} = \frac{-\cancel{z} \cos y (\cancel{x+y})}{\cancel{z} \sin y (\cancel{x+y})}$$

$$\frac{\mu_y}{\mu} = -\frac{\cos y}{\sin y}$$

$$\int \frac{\mu_y}{\mu} dy = \int -\frac{\cos y}{\sin y} dy$$

$$v = \ln(\sin y)$$

$$dv = \frac{1}{\sin y} \cdot \cos y dy$$

$$\ln|\mu| = -\ln(\sin y) + c$$

$$|\mu| = e^{-\ln(\sin y)} \cdot e^c \quad c \in \mathbb{R}$$

$$|\mu| = \sin^{-1} y \cdot \tilde{C}$$

Es also $\mu(y) = \sin^{-1} y$

$$\begin{cases} M = zx \cdot \sin y + zy \cdot \sin y \\ N = zx \cdot \sin y + zy \cdot \sin y + \cos y \end{cases}$$

$$\tilde{M} = \mu \cdot M = zx + zy$$

$$\tilde{N} = \mu \cdot N = zx + zy + \frac{\cos y}{\sin y}$$

$$\Rightarrow \tilde{M}_y - \tilde{N}_x = z - z = 0 \quad \checkmark \quad \text{es exakt}$$

$$\text{Buso } F(x,y) = c \quad \text{con } c \in \mathbb{R}$$

$$\text{Si } \frac{\partial}{\partial x} F = \tilde{M} = 2x + 2y \Rightarrow F = x^2 + 2xy + \psi(y)$$

$$\text{Si } \frac{\partial}{\partial y} F = \tilde{N} = 2x + 2y + \frac{\cos y}{\sin y} \Rightarrow F = 2xy + y^2 + \ln(\sin y) + \tilde{\psi}(x)$$

$$F(x,y) = x^2 + 2xy + y^2 + \ln(\sin y)$$

Sol :

$$x^2 + 2xy + y^2 + \ln(\sin y) = c \quad c \in \mathbb{R}$$

Verifico :

$$\frac{\partial}{\partial x} F = 2x + 2y + \underbrace{2x y'}_{\frac{dy}{dx}} + \underbrace{2 \cdot y \cdot y'}_{\frac{dy}{dx}} + \underbrace{\frac{1}{\sin y} \cdot \cos y \cdot y'}_{\frac{dy}{dx}} = 0$$

$$2x \cdot dx + 2y \cdot dx + 2x \cdot dy + 2y \cdot dy + \frac{\cos y}{\sin y} \cdot dy = 0$$

$$\underbrace{(2x + 2y)}_{\tilde{M}} dx + \underbrace{\left(2x + 2y + \frac{\cos y}{\sin y}\right)}_{\tilde{N}} dy = 0$$

✓ Verifico

$$(f) \underbrace{3y}_{M} dx + \underbrace{x}_{N} dy = 0$$

$$M_y = 3$$

$$N_x = 1$$

No es exacta.

Pruebo reordenando antes de usar
Factor integrante.

$$3y \underbrace{\frac{dx}{dx}}_{=1} + x \cdot \underbrace{\frac{dy}{dx}}_{=y'} = 0$$

$$3y + x \cdot y' = 0$$

$$x \cdot y' = -3y$$

$$\frac{y'}{y} = \frac{-3}{x}$$

$$\int \frac{y'}{y} dx = \int \frac{-3}{x} dx$$

$$\ln |y| = -3 \ln |x| + C$$

$$C \in \mathbb{R}$$

$$|y| = |x|^{-3} \cdot \tilde{C}$$

$$C \in \mathbb{R}_{\geq 0}$$

Mucho más fácil, pruebo con F. Int.

Si uso μ

$$\frac{M_y - N_x}{N} = \frac{3 - 1}{x} = \frac{2}{x} = \frac{\mu'}{\mu}$$

$$\ln |\mu| = 2 \ln |x| + C$$

$$|\mu| = |x|^2 \cdot \tilde{C} \quad \tilde{C} \in \mathbb{R}_{>0}$$

$$|\mu| = x^2 \cdot \tilde{C}$$

$$\text{So } \mu(x) = x^2$$

$$(f) \ 3y \, dx + x \, dy = 0$$

$$\Rightarrow \begin{cases} \tilde{M} = 3x^2 \cdot y & \Rightarrow \tilde{M}_y = 3x^2 \\ \tilde{N} = x^3 & \Rightarrow \tilde{N}_x = 3 \cdot x^2 \end{cases} \quad \text{es exacte } \checkmark$$

$$\text{Berech } F(x, y) = C$$

$$\frac{\partial}{\partial x} F = \tilde{M} = 3x^2 \cdot y \Rightarrow F = x^3 \cdot y + \varphi(y)$$

$$\tilde{N} = x^3 \Rightarrow F = x^3 \cdot y + \tilde{\varphi}(x)$$

$$\Rightarrow F(x, y) = x^3 \cdot y$$

$$\boxed{\begin{array}{l} \text{Sol:} \\ x^3 \cdot y = C \end{array}} \quad C \in \mathbb{R}$$

Verifika

$$\frac{\partial}{\partial x} F = 3x^2 \cdot y + x^3 \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$3x^2 \cdot y \, dx + x^3 \cdot dy = 0 \quad \checkmark \text{ Verificado.}$$

$$(g) \left(1 - y(x+y)\tan(xy)\right) dx + \left(1 - x(x+y)\tan(xy)\right) dy = 0.$$

Veri dilimi ---

Ejercicio 8. Considere la ecuación lineal de primer orden en $y(x)$

$$(*) \quad y' + p(x)y = q(x).$$

(a) Busque una función $\mu(x)$ tal que

$$\mu(x)(y'(x) + p(x)y(x)) = (\mu(x)y(x))'.$$

(b) Multiplique la ecuación (*) por μ y halle su solución general. μ se denomina *factor integrante*.

$$a) \quad \mu(y' + p \cdot y) = (\mu \cdot y)' \quad \text{Dato} \quad y' + p \cdot y = q$$

$$\mu \cdot y' + \mu \cdot p \cdot y = \mu' \cdot y + \mu \cdot y'$$

$$\mu \cdot p \cdot y = \mu' \cdot y \quad y \neq 0$$

$$\mu' = \mu \cdot p$$

$$\frac{\mu'}{\mu} = p$$

$$\int \frac{\mu'}{\mu} dx = \int p dx$$

$$\ln |\mu| = \int p(x) \cdot dx$$

$$|\mu| = e^{\int p(x) \cdot dx}$$

me que do con

$$\mu = e^{\int p(x) \cdot dx}$$

$$b) *: y' + p \cdot y = q$$

$$y \text{ además: } \mu(y' + p \cdot y) = (\mu \cdot y)'$$

$$\mu(y' + p \cdot y) = \mu \cdot q = (\mu \cdot y)'$$

$$\int \mu \cdot q \, dx = \int (\mu \cdot y)' \, dx$$

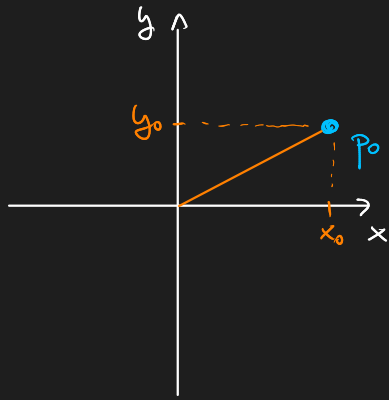
$$\int \mu \cdot q \, dx = \mu \cdot y$$

$$y = \frac{1}{\mu} \cdot \int \mu \cdot q \, dx$$

Sol. Grd.

$$y = e^{-\int p(x) \, dx} \cdot \int q \cdot e^{\int p(x) \cdot dx} \, dx$$

Ejercicio 9. Hallar la ecuación de una curva tal que la pendiente de la recta tangente en un punto cualquiera es la mitad de la pendiente de la recta que une el punto con el origen.



Escribo C como una función de x

$$C = \{ (x, y) \in \mathbb{R}^2 : y = f(x) \}$$

Parametrizada por:

$$\sigma(x) = (x, f(x))$$

$$\text{Recta } t_{\theta}: y = f'(x_0) \cdot (x - x_0) + f(x_0)$$

Además:

$$f'(x_0) = \frac{1}{2} \cdot \frac{y_0}{x_0}$$

$$f'(x_0) = \frac{1}{2} \cdot \frac{f(x_0)}{x_0}$$

$$y' = \frac{1}{2} \frac{y}{x}$$

Resolvamos

$$\frac{y'}{y} = \frac{1}{2x}$$

$$\int \frac{y'}{y} dx = \int \frac{1}{2x} dx$$

$$\ln |y| = \frac{1}{2} \cdot \ln |x| + C \quad C \in \mathbb{R}$$

$$|y| = |x|^{\frac{1}{2}} \cdot \tilde{C} \quad \tilde{C} \in \mathbb{R}_{>0}$$

$$y = C \cdot |x|^{\frac{1}{2}} \quad C \in \mathbb{R}, x \in \mathbb{R}$$

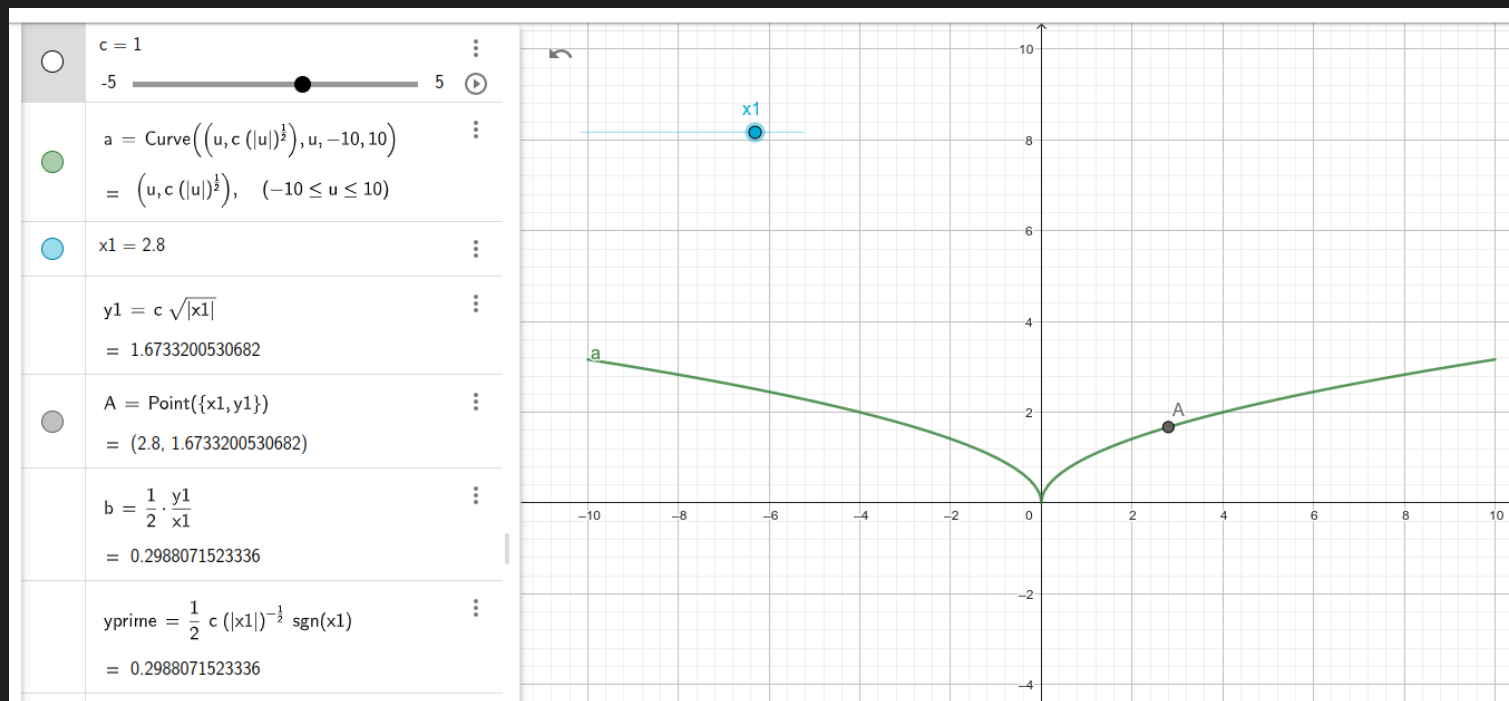
Verifico:

$$y' = \frac{1}{2} \cdot c \cdot |x|^{-\frac{1}{2}} \cdot \text{signo}(x)$$

Quiero llegar a:

$$y' \stackrel{?}{=} \frac{1}{2} \frac{y}{x}$$

$$\frac{1}{2} \cdot c \cdot |x|^{-\frac{1}{2}} \cdot \text{signo}(x) = \frac{1}{2} \cdot \underbrace{\frac{1}{x} \cdot c \cdot |x|^{\frac{1}{2}}}_{= |x|^{\frac{1}{2}-1} \cdot \text{signo}(x)} \quad \checkmark \text{verificado}$$



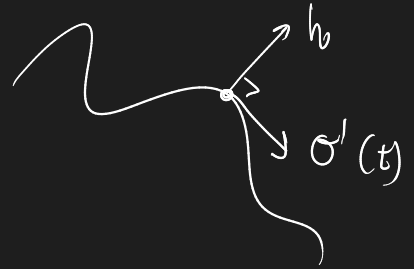
Ejercicio 10. Hallar la ecuación de las curvas tales que la normal en un punto cualquiera pasa por el origen.

$\sigma(t)$: Param de C .

$$\eta : \langle \sigma'(t), (x-x_0, y-y_0) \rangle = 0$$

$$\sigma(x) = (x, f(x))$$

$$\Rightarrow \sigma'(x) = (1, f'(x))$$



$$\eta_{\text{en } x_0} : \langle (1, f'(x_0)), (x-x_0, y-y_0) \rangle = 0$$

$$(x-x_0) + f'(x_0) \cdot (y-f(x_0)) = 0$$

$$(x-x_0) = -f'(x_0) \cdot (y-f(x_0))$$

$$-\frac{(x-x_0)}{f'(x_0)} = y-f(x_0)$$

$$\eta_{\text{en } x_0} : y = f(x_0) - \frac{1}{f'(x_0)} \cdot (x-x_0)$$

Si buscamos que pasen por el origen :

$$\Rightarrow \text{Si } x=0 \Rightarrow \underbrace{f(x)}_{=y} = 0 \quad (\eta \text{ pasa por el origen})$$

$$\Rightarrow 0 = f(x_0) - \frac{1}{f'(x_0)} \cdot (0-x_0)$$

$$0 = f(x_0) + \frac{x_0}{f'(x_0)}$$

Renombró

$$0 = y + \frac{x}{y'}$$

$$-\frac{x}{y'} = y$$

$$-x = y' \cdot y$$

$$\int \underbrace{y' \cdot y}_{y \cdot \underbrace{\frac{dy}{dx}}_{y'}} dx = \int -x dx$$

$$\int y \cdot dy =$$

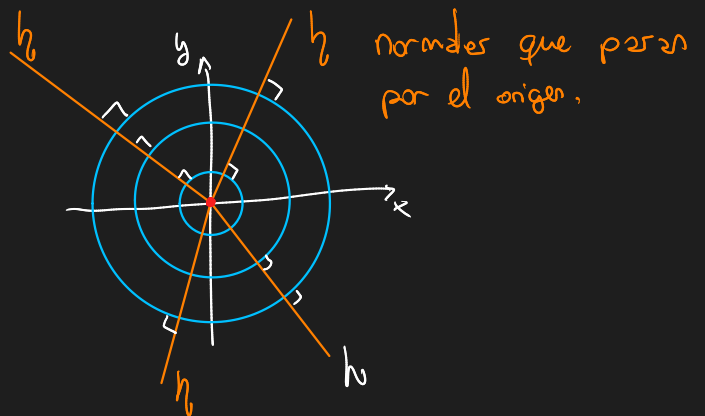
$$\frac{1}{2} y^2 = -\frac{1}{2} x^2 + C \quad C \in \mathbb{R}$$

$$y^2 = -x^2 + C$$

$$x^2 + y^2 = C \leftarrow \text{Círculos concéntricos de radio } \sqrt{C}$$

$$y = \begin{cases} \sqrt{-x^2 + C} \\ -\sqrt{-x^2 + C} \end{cases} \quad C \geq x^2$$

o de forma paramétrica



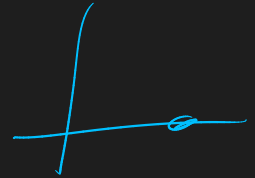
$$\sigma(t) = (\sqrt{C} \cdot \cos t, \sqrt{C} \cdot \sin t) \quad t \in [0, 2\pi)$$
$$C \in \mathbb{R}_{>0}$$

Ejercicio 11. Demostrar que la curva para la cual la pendiente de la tangente en cualquier punto es proporcional a la abscisa del punto de contacto es una parábola.

$$\sigma(x) = (x, f(x)) \quad y = f(x)$$

$$\text{Recta } t_g: \quad y = \underbrace{f'(x_0)}_{\text{Pendiente}} \cdot (x - x_0) + f(x_0)$$

Abscisa: Donde cruza el eje x



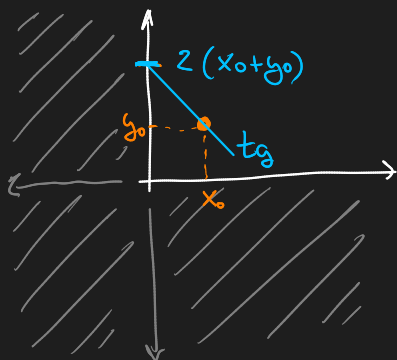
$$y'(x) = c \cdot x$$

$$\int y' dx = \int c \cdot x dx$$

$$y = \frac{c}{2} \cdot x^2 + d \quad c, d \in \mathbb{R}$$

$$\boxed{y = c \cdot x^2 + d}$$

Ejercicio 12. Hallar la ecuación de una curva del primer cuadrante tal que para cada punto (x_0, y_0) de la misma, la ordenada al origen de la recta tangente a la curva en (x_0, y_0) sea $2(x_0 + y_0)$.



$$y = f'(x_0) \cdot (x - x_0) + f(x_0)$$

$$x = 0$$

$$y = 2(x_0 + y_0)$$

$$\Rightarrow 2x_0 + 2y_0 = -f'(x_0) \cdot x_0 + \underbrace{f(x_0)}_{=y_0}$$

$$2x_0 + 2y_0 = y_0 - y'_0 \cdot x_0$$

Como quiero que valga $\forall x, y$ de C

$$\Rightarrow 2x + 2y = y - y' \cdot x$$

$$2x + y = -y' \cdot x$$

$$y' \cdot x + 2x + y = 0 \quad x \neq 0$$

$$y' + 2 + \frac{y}{x} = 0$$

$$\frac{dy}{dx} + 2 + \frac{y}{x} = 0$$

$$1 \cdot dy + 2 dx + \frac{y}{x} \cdot dx$$

$$\underbrace{\left(2 + \frac{y}{x}\right)}_{=M} dx + \underbrace{1}_{=N} dy = 0$$

$$\left. \begin{array}{l} M_y = \frac{1}{x} \\ N_x = 0 \end{array} \right\} \text{ No es exacto}$$

$$\text{Busco } \mu / (\mu \cdot M)_y = (\mu \cdot N)_x$$

$$\mu_y \cdot M + \mu \cdot M_y = \mu_x \cdot N + \mu \cdot N_x$$

$$\text{So } \mu = \mu(x)$$

$$\Rightarrow \mu_y = 0$$

$$\mu \cdot M_y = \mu_x \cdot N + \mu \cdot N_x$$

$$\mu \cdot (M_y - N_x) = \mu_x \cdot N$$

$$\begin{aligned} \frac{\mu_x}{\mu} &= \frac{M_y - N_x}{N} \\ &= \frac{\left(\frac{1}{x} - 0\right)}{1} = \frac{1}{x} \end{aligned}$$

$$\frac{\mu_x}{\mu} = \frac{1}{x}$$

$$\int \frac{\mu_x}{\mu} dx = \int \frac{1}{x} dx$$

$$\ln |\mu| = \ln |x| + C \quad C \in \mathbb{R}$$

$$|\mu| = \tilde{C} |x| \quad \tilde{C} \in \mathbb{R}_{>0}$$

$$\mu = C \cdot |x| \quad C \in \mathbb{R}$$

$$\begin{aligned} \exists ! j_0 \ C=1 \text{ y } \mu: \\ \Rightarrow \mu = x \end{aligned}$$

$$\underbrace{\left(2 + \frac{y}{x}\right)}_M dx + \underbrace{1}_N dy = 0$$

$$\begin{cases} x \cdot M = \tilde{M} = 2x + \frac{y}{x} \cdot x = 2x + y \\ x \cdot N = \hat{N} = x \end{cases}$$

$$\left. \begin{array}{l} \tilde{M}_y = 1 \\ \hat{N}_x = 1 \end{array} \right\} \text{ ex exacte !}$$

$$\Rightarrow F(x, y) = C$$

Busco F

$$\frac{\partial F}{\partial x} = \tilde{M} = 2x + y \Rightarrow F = x^2 + xy + \varphi(y)$$

$$\frac{\partial F}{\partial y} = \hat{N} = x \Rightarrow F = xy + \tilde{\varphi}(x)$$

$$\Rightarrow F(x, y) = x^2 + xy$$

$$\boxed{\begin{array}{l} \text{Sol:} \\ x^2 + xy = C \end{array}} \quad \text{on } C \in \mathbb{R}$$

Verifico: derivo wrt x

$$2x + 1 \cdot y + x \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$\begin{aligned} 2x dx + y dx + x dy &= 0 \\ (2x + y) dx + x dy &= 0 \end{aligned} \quad \checkmark \text{ Verificado.}$$

