2. La ecuación $xy'' - y' - (1+x)y = 0$ tiene una solución de la forma $y = e^{mx}$ para algún $m \in R$.

a) Hallar m.

b) Hallar la solución general de la ecuación.

Como $y = e^{m.x}$ es sol

$$y' = m \cdot e^{mx}$$

$$y'' = m^2 \cdot e^{mx}$$

$$\Rightarrow \quad x \cdot m^2 \cdot e^{mx} - m \cdot e^{mx} - (1+x) \cdot e^{mx} = 0$$

$$\underbrace{e^{mx}}_{>0} \cdot \left(x \cdot m^2 - m - 1 - x \right) = 0$$

$$x \cdot m^2 - m - 1 - x = 0$$

$$x(m^2 - 1) - (m+1) = 0$$

$$x(m+1)(m-1) - (m+1) = 0$$

$$x(m+1)(m-1) = m+1$$

Si $m+1 = 0$

$0 = 0$ ✓

$m = -1$

Si $m-1 = 0$

$0 = 2$ Abs

b) $x y'' - y' - (1+x) y = 0 \qquad x > 0$

$y_1 = e^{-x}$

$y_1' = -e^{-x} = -y_1$

$y_1'' = e^{-x} = y_1$

$y_2 = \mu(x) \cdot y_1 = \mu \cdot y_1$

$y_2' = \mu' \cdot y_1 + \mu \cdot y_1'$

$y_2'' = \mu'' \cdot y_1 + \mu' \cdot y_1' + \mu' \cdot y_1' + \mu \cdot y_1''$

$x \cdot y_2'' = x \cdot \mu'' \cdot y_1 + 2x \mu' \cdot y_1' + x \cdot \mu \cdot y_1''$

$-y_2' = -\mu' \cdot y_1 - \mu \cdot y_1'$

$-(1+x) \cdot y_2 = -\mu \cdot y_1 - x \cdot \mu \cdot y_1$

$x \cdot \mu'' \cdot y_1 + 2x \mu' \cdot y_1' + x \cdot \mu \cdot y_1'' - \mu' \cdot y_1 - \mu \cdot y_1' - \mu \cdot y_1 - x \cdot \mu \cdot y_1 = 0$

Busco agrupar con la forma: $x y_1'' - y_1' - (1+x) y_1 = 0$

$x \cdot \mu'' \cdot y_1 + 2x \mu' \cdot y_1' + x \cdot \mu \cdot y_1'' - \mu' \cdot y_1 - \mu \cdot y_1' - \mu \cdot y_1 - x \cdot \mu \cdot y_1 = 0$

$= \mu \left(x \cdot y_1'' - y_1' - (1+x) y_1 \right) = 0$

$x \cdot \mu'' \cdot y_1 + 2x \mu' \cdot y_1' - \mu' \cdot y_1 = 0$

Como $y_1 = e^{-x} = y_1''$

$y_1' = -e^{-x}$

$$x \cdot e^{-x} \cdot \mu'' - 2 \cdot x \cdot e^{-x} \cdot \mu' - \mu' \cdot e^{-x} = 0$$

$$\underbrace{e^{-x}}_{>0} \left(x \cdot \mu'' - 2x \mu' - \mu' \right) = 0$$

$$x \cdot \mu'' - 2x \mu' - \mu' = 0$$

$$x \cdot \mu'' - \mu' (2x+1) = 0$$

$$x \cdot \mu'' = \mu' (2x+1) =$$

$$\frac{\mu''}{\mu'} = \frac{2x+1}{x} = 2 + \frac{1}{x} \qquad x>0$$

$$\int \frac{\mu''}{\mu'} dx = \int 2 \, dx + \int \frac{1}{x} dx$$

$$\ln|\mu'| = 2x + \ln|x| + c \qquad c \in \mathbb{R}$$

$\exp($

$$|\mu'| = e^{2x + \ln|x| + c}$$

$$= e^{2x} \cdot |x| \cdot e^{c}$$

Elijo

$$\mu' = \tilde{C} \cdot x \cdot e^{2x} \qquad \tilde{C} \in \mathbb{R}_{>0}$$

$$\int \mu' \, dx = \tilde{c} \cdot \int x \cdot e^{2x}$$

$$\mu = \tilde{c} \cdot \frac{1}{4} \cdot e^{2x} (2x-1)$$

Elijo

$$\mu(x) = e^{2x} \cdot (2x-1)$$

$$\Rightarrow y_2 = e^{2x} \cdot (2x-1) \cdot e^{-x}$$

$$y_2 = e^{x} \cdot (2x-1)$$

Verifico

$$y_2' = e^{x} \cdot (2x-1) + 2e^{x}$$

$$y_2'' = e^{x} \cdot (2x-1) + 2e^{x} + 2e^{x}$$

$$x y'' - y' - (1+x) y = 0$$

$$x \cdot e^{x} \cdot (2x-1) + 4x \cdot e^{x} - e^{x} \cdot (2x-1) - 2e^{x} - e^{x}(2x-1) - x \cdot e^{x}(2x-1)$$

$= 0$ ✓ Verificado

$\therefore$

$y_2 = e^{x} \cdot (2x-1)$ es sol.

Sol general

$$y(x) = c_1 \cdot e^{-x} + c_2 \cdot (2x-1) \cdot e^{x} \qquad c_1, c_2 \in \mathbb{R}$$

Ejercicio 1. (2 puntos) Hallar todas las soluciones de

$$y''(x) - 3y'(x) - 4y(x) = \cos(x)$$

tales que $|y(x)| \leq C$ para $x \geq 0$.

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda_1 = 4$$
$$\lambda_2 = -1$$

$$y_H(t) = c_1 \cdot e^{4t} + c_2 \cdot e^{-t}$$

$t = x$ pues me equivoqué de letra

M.d.V.d.C.

$$y_H(t) = c_1(t) \cdot e^{4t} + c_2(t) \cdot e^{-t}$$

$$Q = \begin{bmatrix} e^{4t} & e^{-t} \\ 4e^{4t} & -e^{-t} \end{bmatrix}$$

$$\det Q = -e^{3t} - 4e^{3t}$$
$$= -5e^{3t}$$

$$Q^{-1} = -\frac{1}{5} \cdot e^{-3t} \cdot \begin{bmatrix} -e^{-t} & -e^{-t} \\ -4e^{4t} & e^{4t} \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} e^{-4t} & e^{-4t} \\ 4e^{t} & -e^{t} \end{bmatrix}$$

$$Q\begin{bmatrix} c_1' \\ c_2' \end{bmatrix} = \begin{bmatrix} 0 \\ \cos t \end{bmatrix}$$

$$\begin{bmatrix} c_1' \\ c_2' \end{bmatrix} = \frac{1}{5}\begin{bmatrix} e^{-4t} & e^{-4t} \\ 4e^{t} & -e^{t} \end{bmatrix}\begin{bmatrix} 0 \\ \cos t \end{bmatrix}$$

$$= \frac{1}{5}\cdot\begin{bmatrix} e^{-4t}\cdot\cos t \\ -e^{t}\cdot\cos t \end{bmatrix}$$

$$c_1' = \frac{1}{5}\cdot e^{-4t}\cdot\cos t$$

$$c_1 = \frac{1}{5}\int e^{-4t}\cdot\cos t\,dt$$

$$u = \cos t \qquad du = -\sin t$$

$$v = -\frac{1}{4}\cdot e^{-4t} \qquad dv = e^{-4t}$$

$$\int u\cdot dv = u\cdot v - \int v\cdot du$$

$$= -\frac{1}{4}\cos t\cdot e^{-4t} - \int \frac{1}{4}\cdot e^{-4t}\cdot\sin t\,dt$$

$$= -\frac{1}{4}\cos t\cdot e^{-4t} - \frac{1}{4}\int e^{-4t}\cdot\sin t\,dt$$

$$u = e^{-4t} \qquad du = -4 \cdot e^{-4t}\, dt$$

$$v = -\cos t \qquad dv = \sin t \; dt$$

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

$$= -\cos t \cdot e^{-4t} - \int 4 \cdot \cos t \cdot e^{-4t}\, dt$$

$$\frac{1}{5}\int e^{-4t} \cdot \cos t \; dt = \frac{1}{5}\left(-\frac{1}{4}\cos t \cdot e^{-4t} - \frac{1}{4}\int e^{-4t} \cdot \sin t \; dt\right)$$

$$= -\frac{1}{20}\cos t \cdot e^{-4t} - \frac{1}{4} \cdot \int e^{-4t} \cdot \sin t \; dt$$

$$= -\frac{1}{20}\cos t \cdot e^{-4t} - \frac{1}{4}\left(-\cos t \cdot e^{-4t} - \int 4 \cdot \cos t \cdot e^{-4t}\, dt\right)$$

$$= \underbrace{-\frac{1}{20}\cos t \cdot e^{-4t} + \frac{1}{4} \cdot \cos t \cdot e^{-4t}}_{\frac{5}{20} - \frac{1}{20} = \frac{4}{20} = \frac{1}{5}} + \int \cos t \cdot e^{-4t}\, dt$$

$$\frac{1}{5}\int e^{-4t} \cdot \cos t \; dt = \frac{1}{5}\cos t \cdot e^{-4t} + \int \cos t \cdot e^{-4t}\, dt$$

$$-\frac{1}{5}\cos t \cdot e^{-4t} = \frac{4}{5}\int \cos t \cdot e^{-4t}\,dt$$

$$\int \cos t \cdot e^{-4t}\,dt = -\frac{1}{4} \cdot \cos t \cdot e^{-4t}$$

Mal!

Busco por otro lado.

Como $b(t) = \cos x$

$\Rightarrow$ Propongo $y_p = a \cdot \cos x + b \cdot \sin x$

$y'_p = -a \cdot \sin x + b \cdot \cos x$

$y''_p = -a \cdot \cos x - b \cdot \sin x$