Superficies de Revolución

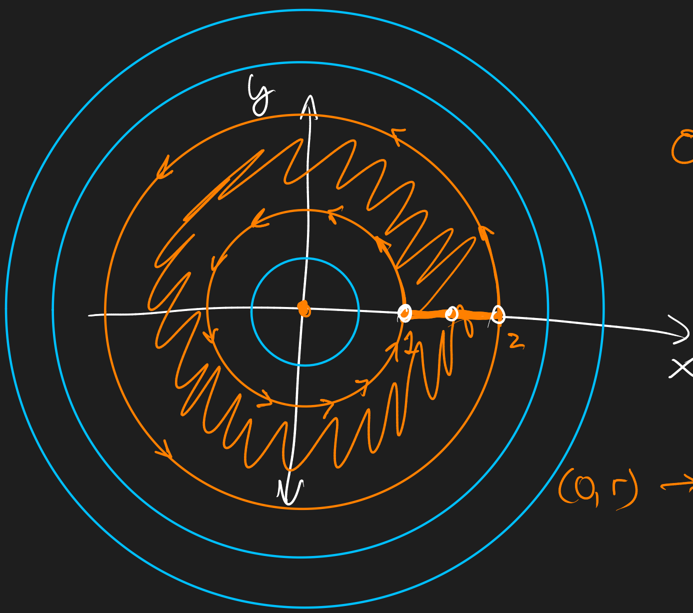
1 - "Curva de Revolución": Girando un punto en $\mathbb{R}^2$

2 - Generalización a $\mathbb{R}^3$

3 - Ejemplos conocidos y no tanto

- Cilindro
- Cono
- Paraboloide
- Ondas en el agua

* Espiral de Churro



$\sigma(r) = (r, 0) \qquad r \in [1,2)$

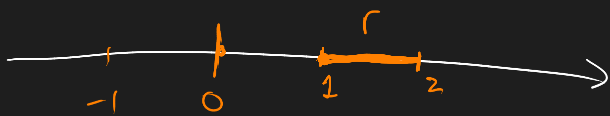
$\Downarrow$ Lo giro alrededor del origen

$(0,r) \to (r \sin\theta, r\cos\theta) \qquad \theta \in [0, 2\pi)$

$0 \qquad 1$

$(r,0) \to (r \cdot \cos\theta, r \sin\theta)$

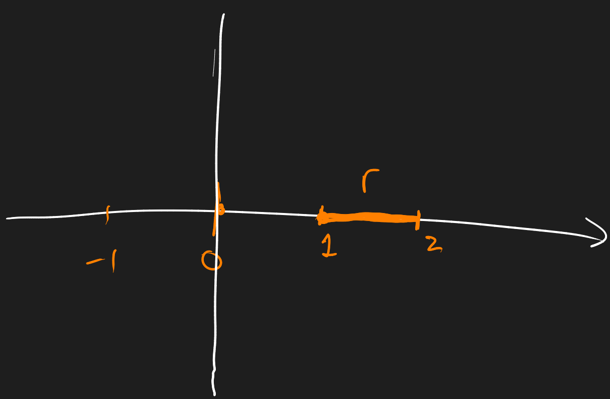
$\mathbb{R} : \quad r \in [1,2]$



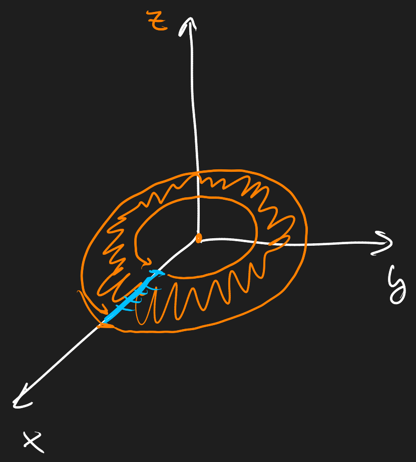
$\Downarrow \qquad \Downarrow$

$\mathbb{R}^2 : \quad (r, 0) \qquad x \in [1,2]$

$(r \cdot \cos\theta, r \cdot \sin\theta)$

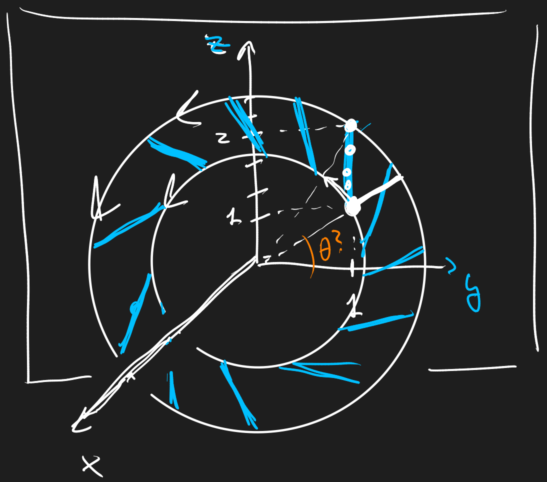


En $\mathbb{R}^3$:



$\sigma(r) = (r, 0, 0) \quad r\in[1,2]$

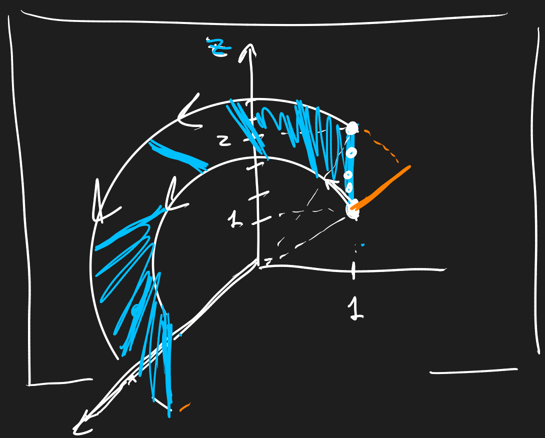
$T(r,\theta) = (r\cdot\cos\theta, r\cdot\sin\theta, 0)$

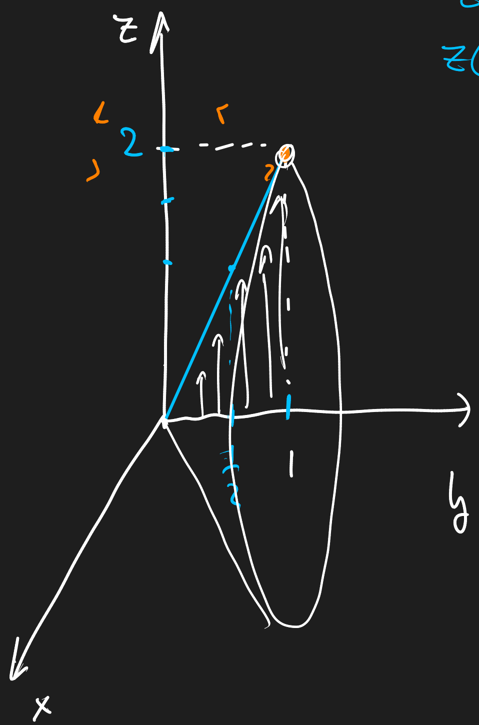


$(0, 1, z) \quad z\in[1,2]$

$$\begin{matrix}(& 0, & r\cdot\cos(?), & r\cdot\sin(?) &) \\ & x & y & z & \end{matrix}$$

$r = ?$



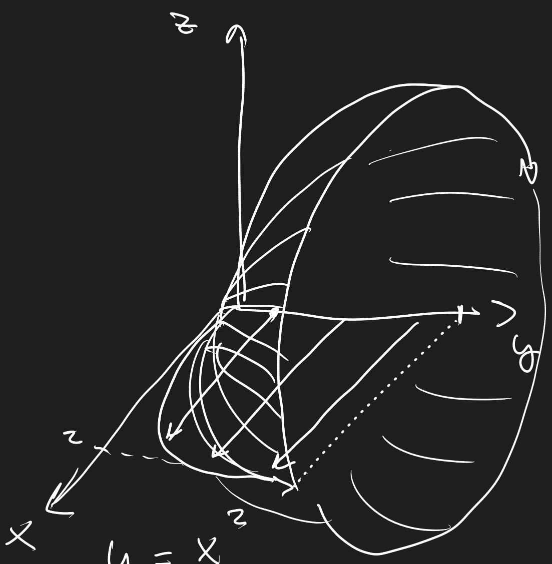


$z = 2y \qquad y \in [0,1]$

$z(y) = 2y$

$$\sigma(y) = (0,\ y,\ 2y) \qquad y \in [0,1]$$

$$(y \sin\theta,\ y \cos\theta,\ 2)$$



$y = x^2$

$|x| = \sqrt{y}$

$x \in [0,2]$

$x = \sqrt{y}$

$$\sigma(x) = (x,\ x^2,\ 0) \qquad x \in [0,2]$$

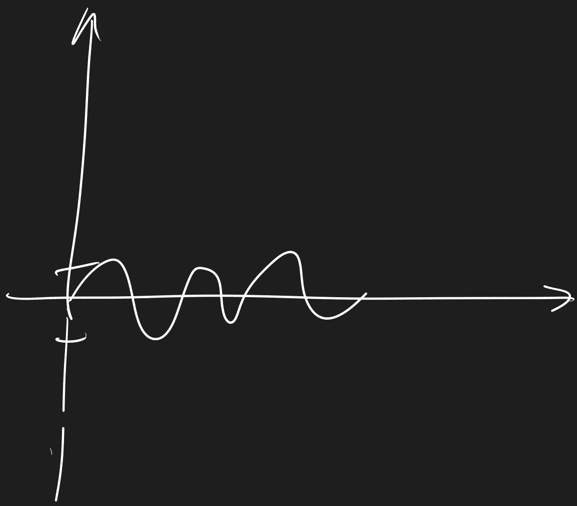
$$T_1(x,\theta) = (x,\ x^2 \cdot \cos\theta,\ x^2 \cdot \sin\theta)$$

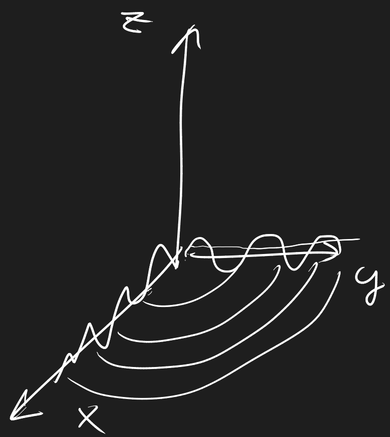
$x \in [0,2]$

$\theta \in [0, 2\pi)$

$$T_2(x,\theta) = (x \cdot \cos\theta,\ x^2,\ x \cdot \sin\theta)$$

- Ondas en el agua





$z = \sin x \qquad x \in [0, 6\pi]$

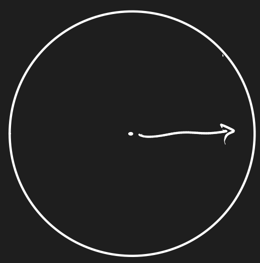
$$\sigma(x) = (x, 0, \sin x)$$

$$T(x, \theta) = (x\cos\theta, x\sin\theta, \sin x)$$

$x \in [0, 6\pi]$

$\theta \in [0, 2\pi)$

* Espiral de Churro



$$\sigma_1(\theta) = (r \cdot \cos\theta,\ r \cdot \sin\theta) \qquad r \text{ cte}$$

$$r(\theta) = \sin 12\theta$$

$$\sigma_2(\theta) = (\sin(12\theta) \cdot \cos\theta,\ \sin(12\theta) \cdot \sin\theta)$$

$$\sigma_3(\theta) = ((2 + \sin 12\theta) \cdot \cos\theta,\ (2 + \sin 12\theta) \cdot \sin\theta)$$

$$T_3(\theta) = ((2 + \sin 12\theta) \cdot \cos\theta,\ 0,\ (2 + \sin 12\theta) \cdot \sin\theta)$$

$$T_4(\theta) = (\underbrace{10 + (2 + \sin 12\theta) \cdot \cos\theta}_{\bar{x}},\ 0,\ \underbrace{(2 + \sin 12\theta) \cdot \sin\theta}_{\bar{z}})$$

$$T_5(\theta, \varphi) = (\bar{x} \cdot \cos\theta,\ \bar{x} \sin\theta,\ \bar{z})$$