Ejercicio 7. Resuelva las siguientes ecuaciones en $y = y(x)$:

(a) $(y - x^3)dx + (x + y^3)dy = 0$

(b) $\cos x \cos^2 y\, dx - 2 \operatorname{sen} x \operatorname{sen} y \cos y\, dy = 0$

(c) $(3x^2 - y^2)\, dy - 2xy\, dx = 0$

(d) $x\, dy = (x^5 + x^3y^2 + y)\, dx$

(e) $2(x+y) \operatorname{sen} y\, dx + \big(2(x+y) \operatorname{sen} y + \cos y\big)\, dy = 0$

(f) $3y\, dx + x\, dy = 0$

(g) $\big(1 - y(x+y)\tan(xy)\big)\, dx + \big(1 - x(x+y)\tan(xy)\big)\, dy = 0.$

$$\underbrace{M\,dx + N\,dy = 0}_{\text{es Exacta}} \quad \text{con } M, N \in C^1$$

es Exacta $\iff$ $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ $\quad$ (ie. $\operatorname{Rot}(M, N) = 0$)

Si es Exacta:

$$\Rightarrow \begin{cases} \dfrac{\partial F}{\partial x} = M \\[2ex] \dfrac{\partial F}{\partial y} = N \end{cases} \Rightarrow \text{obtengo } F: \quad F(t, x) = c \quad c \in \mathbb{R}$$

(a) $(y - x^3)dx + (x + y^3)dy = 0$

$$\underbrace{(y - x^3)}_{M}dx + \underbrace{(x + y^3)}_{N}dy = 0$$

$$\left.\begin{array}{l} M_y = 1 \\ N_x = 1 \end{array}\right\} \quad M_y = N_x \Rightarrow \text{es Exacta}$$

Busco $F(x,y) = c \quad c \in \mathbb{R}$

Si $\dfrac{\partial F}{\partial x} = M = y - x^3 \Rightarrow F = xy - \frac{1}{4}x^4 + \varphi(y)$

Si $\dfrac{\partial F}{\partial y} = N = x + y^3 \Rightarrow F = xy + \frac{1}{4}y^4 + \tilde{\varphi}(x)$

$$F(x,y) = xy + \frac{1}{4}y^4 - \frac{1}{4}x$$

Sol:

$$xy + \frac{1}{4}y^4 - \frac{1}{4}x^4 = c \qquad c \in \mathbb{R}$$

Verifico

Derivo wrt x $\left(y = y(x) \Rightarrow \begin{cases} \frac{\partial}{\partial x} x = 1 \\ \frac{\partial}{\partial x} y = y' \end{cases} \right)$

$$\frac{\partial}{\partial x} F(x,y) = y + x \cdot y' + y^3 \cdot y' - x^3 = 0$$

$$y + x \cdot \frac{dy}{dx} + y^3 \cdot \frac{dy}{dx} - x^3 = 0$$

dx

$$y \cdot dx + x \cdot dy + y^3 \cdot dy - x^3 \cdot dx = 0$$

$$(y - x^3)\,dx + (x + y^3)\,dy = 0 \qquad \checkmark \text{ Verificado}$$

(b) $\cos x \cos^2 y\, dx - 2 \operatorname{sen} x \operatorname{sen} y \cos y\, dy = 0$

$$\underbrace{\cos x \cos^2 y}_{M}\, dx \underbrace{- 2 \operatorname{sen} x \operatorname{sen} y \cos y}_{N}\, dy = 0$$

$$\left.\begin{array}{l} M_y = \cos x \cdot 2 \cdot \cos y \cdot (-\sin y) \\ N_x = -2 \cdot \cos x \cdot \sin y \cdot \cos y \end{array}\right\} M_y = N_x \Rightarrow \text{es Exacta}$$

Busco $F(x,y) = c \quad c \in \mathbb{R}$

Si $\frac{\partial F}{\partial x} = M = \cos x \cdot \cos^2 y \qquad \Rightarrow F = \sin x \cdot \cos^2 y + \varphi(y)$

Si $\frac{\partial F}{\partial y} = N = -2 \sin x \cdot \sin y \cdot \cos y \Rightarrow F = \sin x \cdot \cos^2 y + \varphi(x)$

$$F(x,y) = \sin x \cdot \cos^2 y$$

$$\boxed{\begin{array}{l} \text{Sol:} \\ \sin x \cdot \cos^2 y = C \qquad C \in \mathbb{R} \end{array}}$$

Verifico:

$$\frac{\partial}{\partial x} F = \cos x \cdot \cos^2 y + \sin x \cdot 2\cos y \cdot (-\sin y) \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$\cos x \cdot \cos^2 y \cdot dx + \sin x \cdot 2\cos y \cdot (-\sin y) \cdot dy = 0$$

Verificado

(c) $(3x^2 - y^2)\,dy - 2xy\,dx = 0$

$$\underbrace{(3x^2 - y^2)}_{N}\,dy - \underbrace{2xy}_{M}\,dx = 0$$

¡Atentí al orden! $M\,dx + N\,dy$

$$\left.\begin{array}{l} M_y = -2x \\ N_x = 6x \end{array}\right\}$$ No es exacta! Pero!

Puedo multiplicar M dx + N dy = 0 por alguna función con el objetivo de convertirla en una ecuación exacta.

Como multiplico ambos lados por la misma cosa, no estoy cambiando las soluciones originales (solo debo tener en cuenta las indeterminaciones que puedo agregar).

En el ejercicio, -2x y 6x no están muy lejos uno de otro.

Sea μ un factor integrante

$$\mu\left(M\,dx + N\,dy\right) = \mu \cdot 0$$

$$\mu M\,dx + \mu N\,dy = 0$$

Como:

$$\begin{cases} M = -2xy \\ N = 3x^2 - y^2 \end{cases}$$

Quiero $\mu \,/\, (\mu . M)_y = (\mu N)_x$

$$\mu_y . M + \mu . M_y = \mu_x . N + \mu . N_x$$

Si $\mu_x = 0$

$$\mu_y . M + \mu . M_y = \overbrace{\mu_x . N}^{=0} + \mu . N_x$$

$$\mu . (M_y - N_x) = -\mu_y . M$$

$$-\frac{\mu_y}{\mu} = \frac{M_y - N_x}{M}$$

$$-\frac{\mu_y}{\mu} = \frac{-2x - 6x}{-2xy} = \frac{-8\cancel{x}}{-2\cancel{x}y}$$

$$-\frac{\mu_y}{\mu} = \frac{4}{y}$$

$$\int \frac{\mu_y}{\mu}\, dy = -\int \frac{4}{y}\, dy$$

$$\ln|\mu| = -4\ln|y|$$

$$|\mu| = \left(e^{\ln|y|}\right)^{-4}$$

$$|\mu| = |y|^{-4}$$

$$\boxed{y \neq 0}$$

Notar que estoy agregando una condición sobre y(x)!

$$|\mu| = y^{-4}$$

Pruebo con $\mu = y^{-4}$

$$\tilde{M} = \mu \cdot M = y^{-4} \cdot (-2xy) = -2x \cdot y^{-3}$$

$$\tilde{N} = \mu \cdot N = y^{-4}(3x^2 - y^2) = 3x^2 \cdot y^{-4} - y^{-2}$$

$$\left.\begin{array}{l} \tilde{M}_y = 6x \cdot y^{-4} \\ \tilde{N}_x = 6x \cdot y^{-4} \end{array}\right\} \checkmark \text{ es } \underline{\text{exacta}}$$

$$\tilde{M}\,dx + \hat{N}\,dy = 0$$

Busco $F(x,y) = c \quad c \in \mathbb{R}$

$$\frac{\partial}{\partial x} F = \tilde{M} = -2x \cdot y^{-3} \Rightarrow F = -x^2 \cdot y^{-3} + \varphi(y)$$

$$\frac{\partial}{\partial y} F = \tilde{N} = 3x^2 \cdot y^{-4} - y^{-2} \Rightarrow F = -x^2 \cdot y^{-3} + y^{-1} + \tilde{\varphi}(x)$$

$$F(x,y) = y^{-1} - x^2 \cdot y^{-3}$$

$$\boxed{\begin{array}{l}\text{Sol:} \\ y^{-1} - x^2 \cdot y^{-3} = c \quad c \in \mathbb{R}_{-\{0\}} \;,\; y(x) \neq 0\end{array}}$$

(c) $(3x^2 - y^2)\,dy - 2xy\,dx = 0$

Veo el caso aparte:

- Si $y(x) = 0 \Rightarrow 3x^2\,dy = 0$

$$3x^2 \cdot \frac{dy}{dx} = \frac{0}{dx}$$

$$3x^2 \cdot y' = 0$$

como $y = 0$

$\Rightarrow y' = 0 \Rightarrow$ $y(x) \equiv 0$ también es solución!

Verifico

$$\frac{\partial}{\partial x} F = -y^{-2} \cdot \underbrace{y'}_{\frac{dy}{dx}} - 2x \cdot y^{-3} + 3x^2 \cdot y^{-4} \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$-y^{-2} \cdot dy - 2x \cdot y^{-3} \cdot dx + 3x^2 \cdot y^{-4} \cdot dy = 0$$

$$\underbrace{-2x \cdot y^{-3}}_{=\tilde{M}} \cdot dx + \underbrace{(3x^2 \cdot y^{-4} - y^{-2})}_{=\tilde{N}} dy = 0$$

✓ Verificado

(d) $x\,dy = (x^5 + x^3y^2 + y)\,dx$

$$\underbrace{(x^5 + x^3 \cdot y^2 + y)}_{M}\,dx + \underbrace{(-x)}_{N}\,dy = 0$$

$$\left.\begin{array}{l} M_y = 2x^3y + 1 \\ N_x = -1 \end{array}\right\} \text{No es exacta}$$

Busco μ.

Elijo $\mu_y = 0$ ⊛

$$-\frac{\mu_y}{\mu} = \frac{M_y - N_x}{M}$$

⊛ Quiero algo simple en el denominador o que se simplifique con $M_y - N_x$

$$-\frac{\mu_y}{\mu} = \frac{2x^3y + 2}{x^5 + x^3 \cdot y^2 + y} \Big\}\ \text{depende de 2 variables} \ \times$$

Elijo $\mu_x = 0$

$$\frac{\mu_x}{\mu} = \frac{2x^3y + 2}{-x} \Big\}\ \text{depende de 2 variables} \ \times$$

No me sirve esta forma de encontrar μ.

Pruebo buscando $\mu(x,y) = x^a \cdot y^b$ con $a, b \in \mathbb{Q}$

$$(x^5+x^3.y^2+y)\,dx+(-x)\,dy=0$$

$$x^a.y^b.(x^5+x^3.y^2+y)\,dx+x^a.y^b\,(-x)\,dy=0$$

$$\underbrace{\left(x^{5+a}.y^{b}+x^{3+a}.y^{2+b}+x^{a}.y^{1+b}\right)}_{\tilde{M}}dx+\underbrace{\left(-x^{1+a}.y^{b}\right)}_{\tilde{N}}dy=0$$

$$\tilde{M}_y=b.x^{5+a}.y^{b-1}+(2+b).x^{3+a}.y^{1+b}+(1+b).x^a.y^b$$

$$\tilde{N}_x=-(1+a).x^a.y^b$$

Quiero

$$\tilde{M}_y=\tilde{N}_x$$

$$\Rightarrow\quad -(1+a).x^a.y^b=(1+b).x^a.y^b$$

$$-1-a=1+b$$

$$a+b=2$$

$$b=2-a$$

Además

$$b.x^{5+a}.y^{b-1}+(2+b).x^{3+a}.y^{1+b}=0$$

$$(2-a).\underbrace{x^{5+a}}_{x^{3+a}.x^2}.y^{\overbrace{2-a-1}^{1-a}}=-(2+2-a).x^{3+a}.\underbrace{y^{\overbrace{1+2-a}^{3-a}}}_{y^{1-a}.y^2}$$

$$(2-a).x^{3+a}.x^2.y^{1-a}=-(2+2-a).x^{3+a}.y^{1-a}.y^2$$

$$x^{3+a}\neq 0$$

$$y^{1-a}\neq 0$$

$$(2-a)\cdot x^2 = -(2-a)\cdot y^2 - 2y^2$$

No llego a nada

Reescribo el ejercicio: Método ???

$$(x^5 + x^3 \cdot y^2 + y)\, dx + (-x)\, dy = 0$$

$$\times \frac{1}{dx}$$

$$(x^5 + x^3 \cdot y^2 + y) \underbrace{\frac{dx}{dx}}_{=1} + (-x) \underbrace{\frac{dy}{dx}}_{y'} = 0$$

$$x^5 + x^3 \cdot y^2 + y - x \cdot y' = 0$$

$$y' = \frac{x^5 + x^3 \cdot y^2 + y}{x}$$

Si $\quad y = \mu \cdot x \qquad (\mu = \mu(x))$

$$\Rightarrow y' = \mu'(x) \cdot x + \mu(x) \cdot 1$$

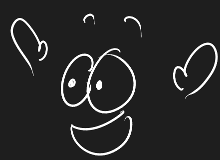
$$\mu + \mu' \cdot x = \frac{x^5 + x^3 \cdot \mu^2 \cdot x^2 + \mu \cdot x}{x}$$

$$\mu + \mu' \cdot x = x^4 + x^4 \cdot \mu^2 + \mu$$

$$\mu' \cdot x = x^4 + x^4 \cdot \mu^2$$

$$\mu' \cdot x = x^4 (1 + \mu^2)$$

$$\frac{\mu'}{1 + \mu^2} = x^3 \quad \leftarrow \text{Se separó!}$$



$$\int \frac{\mu'}{1+\mu^2}\,dx = \int x^3\,dx$$

$$\arctan \mu = \frac{1}{4}x^4 + C$$

$$\mu = \tan\left(\frac{1}{4}x^4 + C\right)$$

Como $y = \mu . x$

$\Rightarrow$ $\boxed{\begin{array}{l}\underline{\text{Sol}}: \\ y = x.\tan\left(\frac{1}{4}x^4 + C\right)\end{array}}$ $C \in \mathbb{R}$

(e) $2(x+y)\operatorname{sen} y\, dx + \big(2(x+y)\operatorname{sen} y + \cos y\big)\, dy = 0$

$\underbrace{2(x+y)\operatorname{sen} y}_{M}$, $\underbrace{2(x+y)\operatorname{sen} y + \cos y}_{N}$

$M = 2x.\sin y + 2y.\sin y$

$M_y = 2x.\cos y + 2.\sin y + 2y.\cos y$

$N = 2x.\sin y + 2y.\sin y + \cos y$

$N_x = 2\sin y$

No es exacta.

Busco μ.

$M_y - N_x = 2x.\cos y + 2.\sin y + 2y.\cos y - 2\sin y$

$= 2x.\cos y + 2y.\cos y$

$= 2\cos y\,(x+y)$

$M = 2\sin y\,(x+y)$

Si $\mu_x = 0$

$$\mu_y.M + \mu.M_y = \underbrace{\mu_x.N}_{=0} + \mu.N_x$$

$$\mu_y = \frac{\mu\left(N_x - M_y\right)}{M}$$

$$\frac{\mu_y}{\mu} = \frac{N_x - M_y}{M}$$

$$\frac{\mu_y}{\mu} = \frac{-\cancel{2}\cos y\,\cancel{(x+y)}}{\cancel{2}\sin y\,\cancel{(x+y)}}$$

$$\frac{\mu_y}{\mu} = -\frac{\cos y}{\sin y}$$

$$\int \frac{\mu_y}{\mu}\,dy = \int -\frac{\cos y}{\sin y}\,dy$$

$v = \ln(\sin y)$

$dv = \frac{1}{\sin y} \cdot \cos y\,dy$

$$\ln|\mu| = -\ln(\sin y) + c$$

$$|\mu| = e^{-\ln(\sin y)} \cdot e^{c} \qquad c \in \mathbb{R}$$

$$|\mu| = \sin^{-1} y \cdot \tilde{C}$$

Elijo $\mu(y) = \sin^{-1} y$

$$\begin{cases} M = 2x.\sin y + 2y.\sin y \\ N = 2x.\sin y + 2y.\sin y + \cos y \end{cases}$$

$$\tilde{M} = \mu.M = 2x + 2y$$

$$\tilde{N} = \mu.N = 2x + 2y + \frac{\cos y}{\sin y}$$

$\Rightarrow \tilde{M}_y - \tilde{N}_x = 2 - 2 = 0$ ✓ es exacta

Busco $F(x,y) = c$ con $c \in \mathbb{R}$

Si $\frac{\partial}{\partial x} F = \tilde{M} = 2x + 2y \Rightarrow F = x^2 + 2xy + \varphi(y)$

Si $\frac{\partial}{\partial y} F = \tilde{N} = 2x + 2y + \frac{\cos y}{\sin y} \Rightarrow F = 2xy + y^2 + \ln(\sin y) + \tilde{\varphi}(x)$

$$F(x,y) = x^2 + 2xy + y^2 + \ln(\sin y)$$

$$\boxed{\begin{array}{l}\text{Sol :} \\ x^2 + 2xy + y^2 + \ln(\sin y) = c\end{array}} \qquad c \in \mathbb{R}$$

Verifico :

$$\frac{\partial}{\partial x} F = 2x + 2y + 2x\underbrace{y'}_{\frac{dy}{dx}} + 2 \cdot y \cdot \underbrace{y'}_{\frac{dy}{dx}} + \frac{1}{\sin y} \cdot \cos y \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$2x \cdot dx + 2y \cdot dx + 2x \cdot dy + 2y \cdot dy + \frac{\cos y}{\sin y} \cdot dy = 0$$

$$\underbrace{(2x + 2y)}_{\tilde{M}} dx + \underbrace{\left(2x + 2y + \frac{\cos y}{\sin y}\right)}_{\tilde{N}} dy = 0$$

✓ Verificado

(f) $\underbrace{3y}_{M}\,dx + \underbrace{x}_{N}\,dy = 0$

$M_y = 3$

$N_x = 1$ No es exacta. Pruebo reordenando antes de usar Factor integrante.

$$3y \underbrace{\frac{dx}{dx}}_{=1} + x \cdot \underbrace{\frac{dy}{dx}}_{=y'} = 0$$

$$3y + x \cdot y' = 0$$

$$x \cdot y' = -3y$$

$$\frac{y'}{y} = \frac{-3}{x}$$

$$\int \frac{y'}{y}\,dx = \int \frac{-3}{x}\,dx$$

$$\ln|y| = -3\ln|x| + C \qquad C \in \mathbb{R}$$

$$|y| = |x|^{-3} \cdot \tilde{C} \qquad C \in \mathbb{R}_{>0}$$

Mucho módulo, pruebo con F. Int.

Si uso μ

$$\frac{M_y - N_x}{N} = \frac{3-1}{x} = \frac{2}{x} = \frac{\mu'}{\mu}$$

$$\ln|\mu| = 2\ln|x| + c$$

$$|\mu| = |x|^2 \cdot \tilde{C} \qquad \tilde{C} \in \mathbb{R}_{>0}$$

$$|\mu| = x^2 \cdot \tilde{C}$$

Si $\mu(x) = x^2$

(f) $3y\,dx + x\,dy = 0$

$$\Rightarrow \begin{cases} \tilde{M} = 3x^2 \cdot y & \Rightarrow \tilde{M}_y = 3x^2 \\ \tilde{N} = x^3 & \Rightarrow \tilde{N}_x = 3 \cdot x^2 \end{cases}$$

es exacta ✓

Busco $F(x,y) = c$

$$\frac{\partial}{\partial x} F = \tilde{M} = 3x^2 \cdot y \Rightarrow F = x^3 \cdot y + \varphi(y)$$

$$\tilde{N} = x^3 \Rightarrow F = x^3 \cdot y + \tilde{\varphi}(x)$$

$$\Rightarrow F(x,y) = x^3 \cdot y$$

$$\boxed{\text{Sol: } x^3 \cdot y = c} \qquad c \in \mathbb{R}$$

Verifico

$$\frac{\partial}{\partial x} F = 3x^2 \cdot y + x^3 \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$3x^2 \cdot y\,dx + x^3 \cdot dy = 0$$ ✓ Verificado.

(g) $\big(1 - y(x+y)\tan(xy)\big)\,dx + \big(1 - x(x+y)\tan(xy)\big)\,dy = 0.$

Veri dificil ...

Ejercicio 8. Considere la ecuación lineal de primer orden en $y(x)$

$$(*) \qquad y' + p(x)\,y = q(x).$$

(a) Busque una función $\mu(x)$ tal que

$$\mu(x)\big(y'(x) + p(x)\,y(x)\big) = \big(\mu(x)\,y(x)\big)'.$$

(b) Multiplique la ecuación $(*)$ por μ y halle su solución general. μ se denomina *factor integrante.*

a) $\mu(y' + p \cdot y) = (\mu \cdot y)'$

Dato $y' + p \cdot y = q$

$$\mu \cdot y' + \mu \cdot p \cdot y = \mu' \cdot y + \mu \cdot y'$$

$$\mu \cdot p \cdot y = \mu' \cdot y \qquad y \neq 0$$

$$\mu' = \mu \cdot p$$

$$\frac{\mu'}{\mu} = p$$

$$\int \frac{\mu'}{\mu}\, dx = \int p\, dx$$

$$\ln|\mu| = \int p(x) \cdot dx$$

$$|\mu| = e^{\int p(x) \cdot dx}$$

me quedo con

$$\boxed{\mu = e^{\int p(x) \cdot dx}}$$

b) $*: \; y' + p \cdot y = q$

y además: $\mu \left(y' + p \cdot y \right) = (\mu \cdot y)'$

$$\mu \left(y' + p \cdot y \right) = \underbrace{\mu \cdot q = (\mu \cdot y)'}$$

$$\int \mu \cdot q \, dx = \int (\mu \cdot y)' \, dx$$

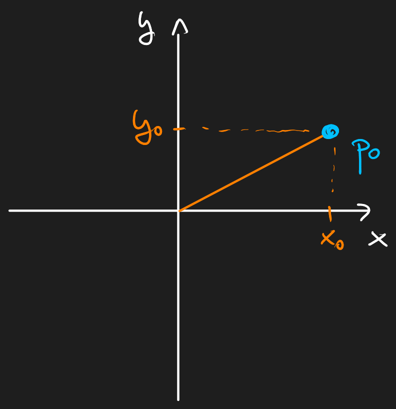
$$\int \mu \cdot q \, dx = \mu \cdot y$$

$$y = \frac{1}{\mu} \cdot \int \mu \cdot q \, dx$$

Sol. Grd.

$$\boxed{y = e^{-\int p(x)\, dx} \cdot \int q \cdot e^{\int p(x) \cdot dx} \, dx}$$

Ejercicio 9. Hallar la ecuación de una curva tal que la pendiente de la recta tangente en un punto cualquiera es la mitad de la pendiente de la recta que une el punto con el origen.



Escribo C como una función de x

$$C = \{ (x,y) \in \mathbb{R}^2 : y = f(x) \}$$

Parametrizada por:

$$\sigma(x) = (x, f(x))$$

Recta tg: $y = f'(x_0) \cdot (x - x_0) + f(x_0)$

Además:

$$f'(x_0) = \frac{1}{2} \cdot \frac{y_0}{x_0}$$

$$f'(x_0) = \frac{1}{2} \cdot \frac{f(x_0)}{x_0}$$

$$y' = \frac{1}{2} \frac{y}{x}$$

Resuelvo

$$\frac{y'}{y} = \frac{1}{2x}$$

$$\int \frac{y'}{y} \, dx = \int \frac{1}{2x} \, dx$$

$$\ln|y| = \frac{1}{2} \cdot \ln|x| + C \qquad C \in \mathbb{R}$$

$$|y| = |x|^{1/2} \cdot \tilde{C} \qquad \tilde{C} \in \mathbb{R}_{>0}$$

$$y = c \cdot |x|^{1/2} \qquad c \in \mathbb{R}, \; x \in \mathbb{R}$$

Verifico :

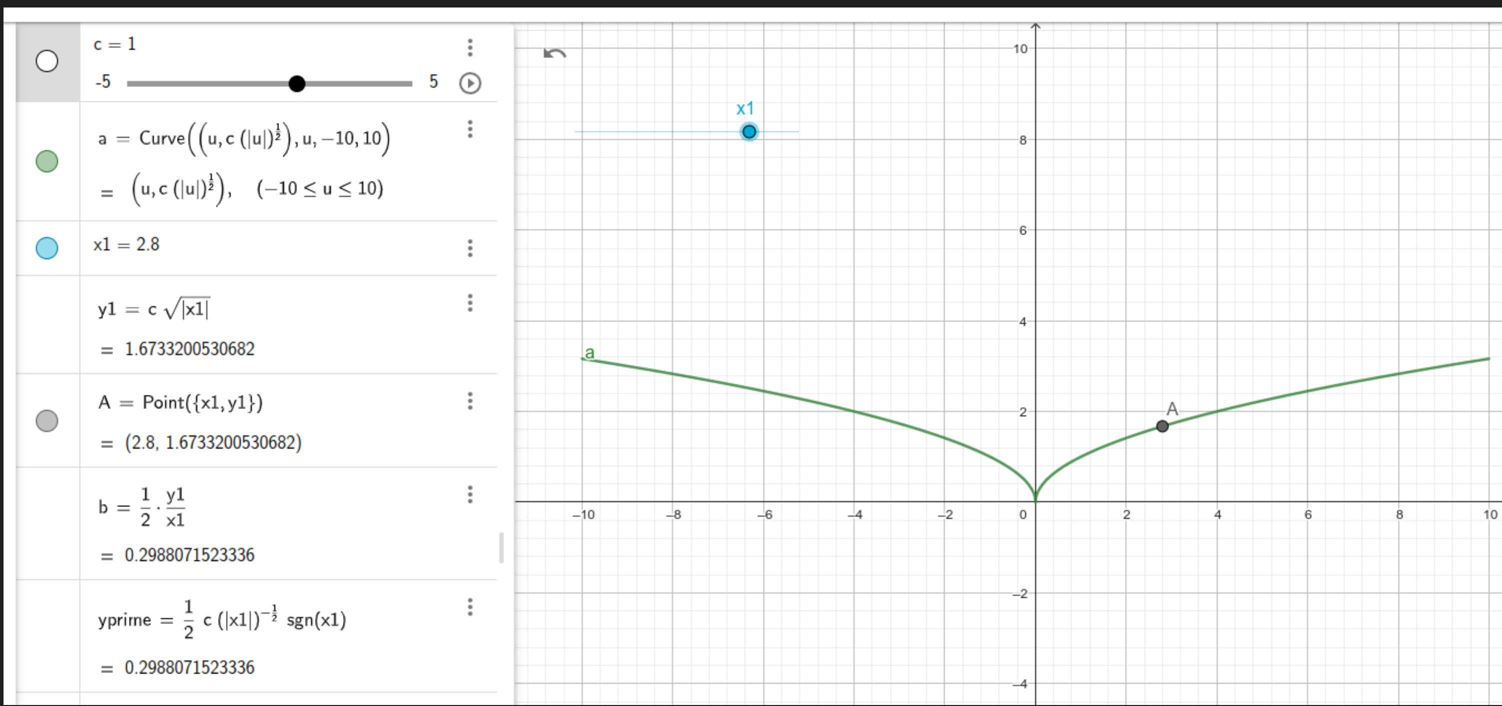
$$y' = \frac{1}{2} \cdot c \cdot |x|^{-\frac{1}{2}} \cdot \text{signo}(x)$$

Quiero llegar a :

$$y' \overset{?}{=} \frac{1}{2} \frac{y}{x}$$

$$\frac{1}{2} \cdot c \cdot |x|^{-\frac{1}{2}} \cdot \text{signo}(x) = \frac{1}{2} \cdot \underbrace{\frac{1}{x} \cdot c \cdot |x|^{1/2}}_{= |x|^{1/2 - 1} \cdot \text{signo}(x)}$$

✓ Verificado



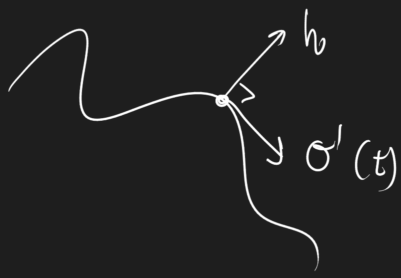
Ejercicio 10. Hallar la ecuación de las curvas tales que la normal en un punto cualquiera pasa por el origen.

$\sigma(t)$: Param de C.

$$\eta : \langle \sigma'(t), (x-x_0, y-y_0) \rangle = 0$$

$$\sigma(x) = (x, f(x))$$

$$\Rightarrow \sigma'(x) = (1, f'(x))$$



$$\eta \text{ en } x_0 : \langle (1, f'(x_0)), (x-x_0, y-y_0) \rangle = 0$$

$$(x-x_0) + f'(x_0) \cdot (y - f(x_0)) = 0$$

$$(x-x_0) = -f'(x_0) \cdot (y - f(x_0))$$

$$-\frac{(x-x_0)}{f'(x_0)} = y - f(x_0)$$

$$\eta \text{ en } x_0 : \quad y = f(x_0) - \frac{1}{f'(x_0)} \cdot (x - x_0)$$

Si busco las que pasan por el origen:

$$\Rightarrow \text{Si } x = 0 \Rightarrow \underbrace{f(x)}_{=y} = 0 \quad (\eta \text{ pasa por el origen})$$

$$\Rightarrow \quad 0 = f(x_0) - \frac{1}{f'(x_0)} \cdot (0 - x_0)$$

$$0 = f(x_0) + \frac{x_0}{f'(x_0)}$$

Renombro

$$0 = y + \frac{x}{y'}$$

$$-\frac{x}{y'} = y$$

$$-x = y' \cdot y$$

$$\int \underbrace{y' \cdot y \, dx}_{y \cdot \underbrace{\frac{dy}{dx}}_{y'} \cdot dx} = \int -x \, dx$$

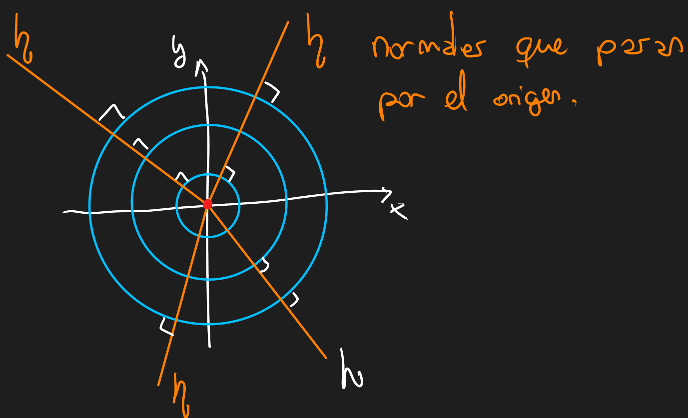
$$\int y \cdot dy =$$

$$\frac{1}{2} y^2 = -\frac{1}{2} x^2 + C \qquad C \in \mathbb{R}$$

$$y^2 = -x^2 + C$$

$$x^2 + y^2 = C \quad \leftarrow \text{Circunferencias de radio } \sqrt{C}$$

$$y = \begin{cases} \sqrt{-x^2+C} \\ -\sqrt{-x^2+C} \end{cases} \qquad C \geq x^2$$



o de forma paramétrica

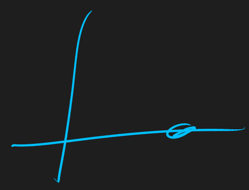
$$\boxed{\sigma(t) = \left(\sqrt{C} \cdot \cos t, \sqrt{C} \cdot \sin t\right) \qquad t \in [0, 2\pi) \qquad C \in \mathbb{R}_{>0}}$$

Ejercicio 11. Demostrar que la curva para la cual la pendiente de la tangente en cualquier punto es proporcional a la abscisa del punto de contacto es una parábola.

$$\sigma(x) = (x, f(x)) \qquad y = f(x)$$

Recta tg: $y = \underbrace{f'(x_0)}_{\text{Pendiente}} \cdot (x - x_0) + f(x_0)$

Abscisa: Donde cruza el eje x



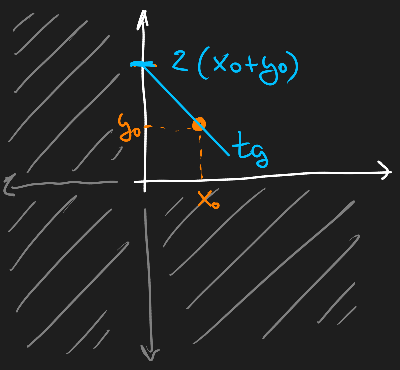
$$y'(x) = C \cdot x$$

$$\int y' \, dx = \int C \cdot x \, dx$$

$$y = \frac{C}{2} \cdot x^2 + d \qquad c, d \in \mathbb{R}$$

$$\boxed{y = c \cdot x^2 + d}$$

Ejercicio 12. Hallar la ecuación de una curva del primer cuadrante tal que para cada punto (x_0, y_0) de la misma, la ordenada al origen de la recta tangente a la curva en (x_0, y_0) sea $2(x_0 + y_0)$.



$$y = f'(x_0) \cdot (x - x_0) + f(x_0)$$

$$x = 0$$

$$y = 2(x_0 + y_0)$$

$$\Rightarrow 2x_0 + 2y_0 = -f'(x_0) \cdot x_0 + \underbrace{f(x_0)}_{=y_0}$$

$$2x_0 + 2y_0 = y_0 - y_0' \cdot x_0$$

Como quiero que valga $\forall x, y$ de C

$$\Rightarrow 2x + 2y = y - y' \cdot x$$

$$2x + y = -y' \cdot x$$

$$y' \cdot x + 2x + y = 0 \qquad x \neq 0$$

$$y' + 2 + \frac{y}{x} = 0$$

$$\frac{dy}{dx} + 2 + \frac{y}{x} = 0$$

$$1 . dy + 2\,dx + \frac{y}{x} . dx$$

$$\underbrace{\left(2 + \frac{y}{x}\right)}_{=M} dx + \underbrace{1}_{N}\, dy = 0$$

$$\left.\begin{aligned} M_y &= \frac{1}{x} \\ N_x &= 0 \end{aligned}\right\} \text{ No es exacta}$$

Busco $\mu \,/\, (\mu . M)_y = (\mu N)_x$

$$\mu_y . M + \mu . M_y = \mu_x . N + \mu . N_x$$

Si $\mu = \mu(x)$

$\Rightarrow \mu_y = 0$

$$\mu . M_y = \mu_x . N + \mu . N_x$$

$$\mu . (M_y - N_x) = \mu_x . N$$

$$\frac{\mu_x}{\mu} = \frac{M_y - N_x}{N}$$

$$= \frac{\left(\frac{1}{x} - 0\right)}{1} = \frac{1}{x}$$

$$\frac{\mu_x}{\mu} = \frac{1}{x}$$

$$\int \frac{\mu_x}{\mu} dx = \int \frac{1}{x} dx$$

$$\ln|\mu| = \ln|x| + C \qquad C \in \mathbb{R}$$

$$|\mu| = \tilde{C}\,|x| \qquad \tilde{C} \in \mathbb{R}_{>0}$$

$$\mu = C \cdot |x| \qquad C \in \mathbb{R}$$

Elijo $C = 1$ y μ:

$\Rightarrow \mu = x$

$$\underbrace{\left(2+\frac{y}{x}\right)}_{M}dx + \underbrace{1}_{N}\,dy = 0$$

$$\begin{cases} x \cdot M = \tilde{M} = 2x + \frac{y}{x} \cdot x = 2x + y \\ x \cdot N = \hat{N} = x \end{cases}$$

$$\left.\begin{aligned} \tilde{M}_y &= 1 \\ \tilde{N}_x &= 1 \end{aligned}\right\} \text{ es exacta!}$$

$$\Rightarrow \quad F(x,y) = c$$

Busco F

$$\frac{\partial F}{\partial x} = \tilde{M} = 2x + y \Rightarrow F = x^2 + xy + \varphi(y)$$

$$\frac{\partial F}{\partial y} = \hat{N} = x \Rightarrow F = xy + \tilde{\varphi}(x)$$

$$\Rightarrow F(x,y) = x^2 + xy$$

$$\boxed{\text{Sol: } \quad x^2 + xy = c} \quad \text{con } c \in \mathbb{R}$$

Verifico: derivo wrt x

$$2x + 1 \cdot y + x \cdot \underbrace{y'}_{\frac{dy}{dx}} = 0$$

$$2x\,dx + y\,dx + x\,dy = 0$$

$$(2x+y)\,dx + x\,dy = 0 \quad \checkmark \text{ Verificado.}$$