Exercise 0.2.13. The population of city X was 100 thousand 20 years ago, and the population of city X was 120 thousand 10 years ago. Assuming constant growth, you can use the exponential population model (like for the bacteria). What do you estimate the population is now?

Data

Pop.

t_1 : 20 y/a --- 100.000

t_2 : 10 y/a --- 120.000

Model

$$y' = k y \quad \text{where} \quad y(t) = C \cdot e^{kt}$$

Using data:

$$y(t_1) = C \cdot e^{k \cdot t_1} = 100.000$$

$$t_1 = 0$$

$$y(0) = C = 100.000$$

$$t_2 = 10$$

$$y(10) = C \cdot e^{k \cdot 10} = 120.000$$

$$C = 1.10^5$$

$$e^{k \cdot 10} = \frac{12}{10}$$

$$10 \cdot k = \ln(1,2)$$

$$k = \frac{\ln(1,2)}{10}$$

$$\therefore \quad P(t) = 10^5 \cdot e^{\frac{\ln 1,2}{10} \cdot t}$$

Now : $t = 20$

$$P(20) = 10^5 \cdot e^{\frac{\ln 1,2}{10} \cdot 20}$$

$$= 144.000$$ //

Exercise 0.2.14. Suppose that a football coach gets a salary of one million dollars now, and a raise of 10% every year (so exponential model, like population of bacteria). Let s be the salary in millions of dollars, and t is time in years.

a) What is $s(0)$ and $s(1)$.

b) Approximately how many years will it take for the salary to be 10 million.

c) Approximately how many years will it take for the salary to be 20 million.

d) Approximately how many years will it take for the salary to be 30 million.

Método de separacion de variables

Caso general:

$$\frac{dy}{dx} = f(x) \cdot g(y) \qquad \left(\text{con } y = y(x)\right)$$

$$\int \frac{1}{g(y)} \cdot \frac{dy}{dx} \cdot dx = \int f(x) \cdot dx + C$$

$$\int \frac{1}{g(y)} \cdot dy = \int f(x) \cdot dx + C$$

Theorem:

If $\boxed{f}$ and $\boxed{\frac{\partial f}{\partial y}}$ continuous near (x_0, y_0), then there is a unique solution on an interval $\alpha < x_0 < \beta$ to the I.V.P

Could be very small!

$$y' = f(x, y), \quad y(x_0) = y_0$$

If

- f and $\frac{\partial f}{\partial y}$ cont. near (x_0, y_0) $\Rightarrow$ Unique sol.
- Only f cont. near (x_0, y_0) $\Rightarrow$ At least one sol.