Práctica 1 (Curvas) (Análisis II - Análisis Matemático II - Matemática 3). | I

Ej 1: Sean $\sigma_1 : [0, 2\pi] \to \mathbb{R}^2$, $\sigma_1(\theta) = (\cos\theta, \operatorname{sen}\theta)$

$\sigma_2 : [0, 4\pi] \to \mathbb{R}^2$, $\sigma_2(\theta) = \left(\cos\left(\frac{\theta}{2}\right), \operatorname{sen}\left(\frac{\theta}{2}\right)\right)$.

Probar que ambas son parametrizaciones C^1 de la circunferencia de radio 1 y centro (0,0).

- continuas ✓
- C^1 ✓
- $C = \{(x,y) \in \mathbb{R}^2 / x^2 + y^2 = 1\}$

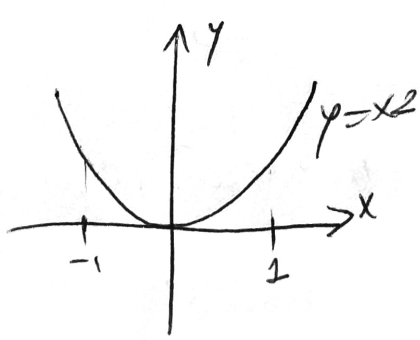
$Im(\sigma_1) = C$, $Im(\sigma_2) = C$ ✓

- obs:

$$\begin{tikzcd} {[0,2\pi]} \arrow[r, "\sigma_1"] & C \\ {[0,4\pi]} \arrow[u, "h"] \arrow[ur, "\sigma_2"'] & \end{tikzcd}$$

$h(t) = \frac{t}{2}$.

Ej. 2.: Parábola



$C = \{(x,y) \in \mathbb{R}^2 / y = x^2, -1 \leq x \leq 1\}$

$\sigma_1 : [-1,1] \to \mathbb{R}^2$

- $\sigma_1(t) = (t, t^2)$

- $Im(\sigma_1) = C$ ✓

- curva abierta: $\sigma_1(1) = (1,1)$
  $\sigma_1(-1) = (-1,1)$ $\Big\rangle \neq$

- simple: σ_1 inyectiva.

- <u>suave</u>: si en cada entorno de cada punto de C, $\exists$ una parametrización regular para C.

$\left(\sigma_1 \,/\, \sigma_1'(t) = (1, 2t) \neq (0,0) \;\; \forall t \right) \Rightarrow C$ es curva suave.

<u>Obs.</u>: $\sigma_2 : [-1,1] \to \mathbb{R}^2$
$\sigma_2(t) := (t^3, t^6)$.

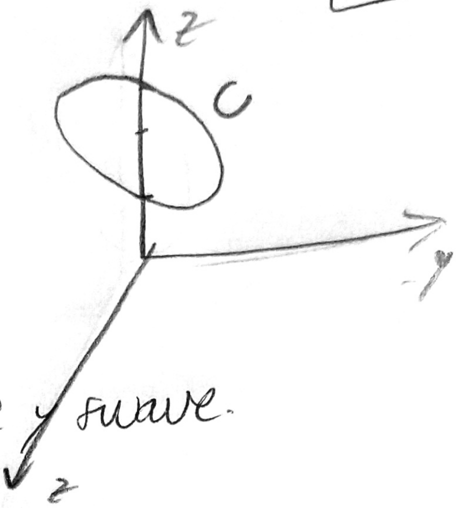
- inyectiva ✓
- C^1 ✓
- continua ✓
- $\sigma_2'(t) = (3t^2, 6t^5) = (0,0)$ en $t = 0$

$\Rightarrow \sigma_2$ <u>no</u> es una parametrización regular.

III

Ej. 3 : Sea C la curva definida por la intersección de las dos superficies:

$$C: \begin{cases} x^2+y^2=1 \\ y+z=2 \end{cases}$$



Probar que C es una curva cerrada simple y suave.

Como $x^2+y^2=1 \Rightarrow \exists\, \theta \in [0, 2\pi] \,/\, \begin{cases} x = \cos\theta \\ y = \operatorname{sen}\theta \end{cases}$

$y+z=2 \Rightarrow \underline{z} = 2-y$
$\quad = \underline{2-\operatorname{sen}\theta}$

Sea $\sigma : [0, 2\pi] \to \mathbb{R}^3 \,/\, \sigma(\theta) = (\cos\theta, \operatorname{sen}\theta, 2-\operatorname{sen}\theta)$

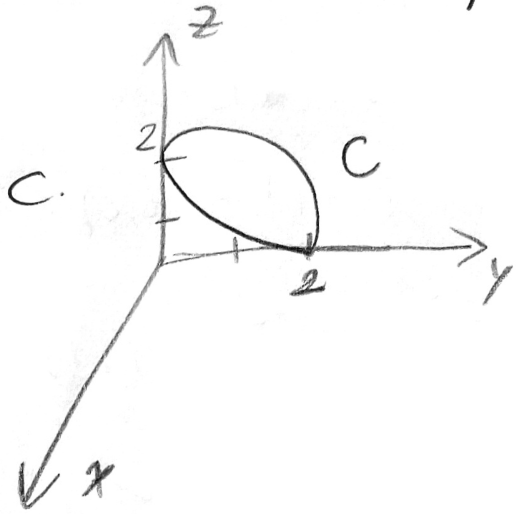
- $\sigma(0) = (1, 0, 2)$ $\to$ cerrada ✓
  $\sigma(2\pi) = (1, 0, 2)$
- σ es inyectiva $\to$ C simple ✓
- $\sigma \in C^1$ ✓
- $\sigma'(\theta) = (-\operatorname{sen}\theta, \cos\theta, \cos\theta) \neq (0,0,0)$ ✓

$\sigma'(0) = (0, 1, 1) = \sigma'(2\pi)$ $\to$ C es simple ✓.

Ej. 4: Sea $\sigma : [0, 2\pi] \to \mathbb{R}^3$

$$\sigma(t) = \left(1+\cos t,\ \operatorname{sen} t,\ 2\operatorname{sen}\tfrac{t}{2}\right), \text{ y}$$

sea $C = \{(x,y,z) \in \mathbb{R}^3 \ / \ x^2+y^2+z^2=4,\ (x-1)^2+y^2=1,\ z \geq 0\}$



Probar que σ es una parametrización de C.

- $\operatorname{Im}(\sigma) = C$

•) Sea $(x,y,z) \in \operatorname{Im}(\sigma) \Rightarrow \exists\, t \in [0, 2\pi]$ tal que

$$(x,y,z) = \left(1+\cos t,\ \operatorname{sen} t,\ 2\operatorname{sen}\tfrac{t}{2}\right)$$

$$x^2+y^2+z^2 = 1+2\cos t + \overbrace{\cos^2 t + \operatorname{sen}^2 t}^{1} + 4\operatorname{sen}^2\left(\tfrac{t}{2}\right)$$

$$= 2+2\underbrace{\cos t}_{\cos\left(\frac{t}{2}+\frac{t}{2}\right)} + 4\operatorname{sen}^2(t/2)$$

$$= 2+2\cos^2\left(\tfrac{t}{2}\right) - 2\operatorname{sen}^2\left(\tfrac{t}{2}\right) + 4\operatorname{sen}^2(t/2)$$

$$= 2+2\cos^2\left(\tfrac{t}{2}\right) + 2\operatorname{sen}^2(t/2) = 4$$

$$\Rightarrow x^2+y^2+z^2 = 4 \checkmark$$

V

$\cdot\ (x-1)^2+y^2 = \cos^2 t + \operatorname{sen}^2 t = 1$ ✓

$\cdot\ z = 2\operatorname{sen}(t/2) \geq 0 \quad \forall t \in [0, 2\pi]$ ✓

$\Rightarrow \underline{(x,y,z) \in C} \quad \Rightarrow \quad \underline{\operatorname{Im}(\sigma) \subseteq C}$

$\cdot\cdot)$ $\underline{\text{Sea } (x,y,z) \in C}$ $\Rightarrow (x-1)^2+y^2 = 1$

$\Rightarrow \exists t \in [0, 2\pi] \ / \ \boxed{\begin{cases} x = \cos t + 1 \\ y = \operatorname{sen} t \end{cases}}$

Como $x^2+y^2+z^2 = 4$

$$\Rightarrow z^2 = 4 - x^2 - y^2 = 4 - \cos^2 t - 2\cos t - 1 - \operatorname{sen}^2 t$$

$$= 2 - 2\underbrace{\cos t}_{\cos\left(\frac{t}{2}+\frac{t}{2}\right)}$$

$$= 2\left(1 - \cos^2\left(\tfrac{t}{2}\right) + \operatorname{sen}^2\left(\tfrac{t}{2}\right)\right)$$

$$= 2 \cdot \left(2 \cdot \operatorname{sen}^2(t/2)\right) = 4 \cdot \operatorname{sen}^2(t/2)$$

$$\left(1 = \cos^2(t/2) + \operatorname{sen}^2(t/2)\right)$$

$$\Rightarrow z^2 = 4 \cdot \operatorname{sen}^2(t/2) \underset{(z \geq 0)}{\Rightarrow} \boxed{z = 2\operatorname{sen}(t/2)}$$

$$\Rightarrow (x,y,z) = \left(1+\cos t, \operatorname{sen} t, 2\operatorname{sen}(t/2)\right), \quad t \in [0, 2\pi)$$

$$\in \operatorname{Im}(\sigma)$$

$$\Rightarrow \boxed{C \subseteq \operatorname{Im}(\sigma)}$$

$$\Rightarrow \boxed{\operatorname{Im}(\sigma) = C} \checkmark$$