Práctica 3 :

16) $D = \left\{ \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \right\}$

$F(x,y) = (u(x,y), v(x,y))$

$G(x,y) = (v_x - v_y, u_x - u_y)$

$$\iint_D (F \cdot G)(x,y)\, dx\, dy$$

$\underline{(F \cdot G)} = u(v_x - v_y) + v \cdot (u_x - u_y)$

$= \underline{u \cdot v_x + v \cdot u_x}$

$\underline{- (u \cdot v_y + v \cdot u_y)}$

$$\iint_D (Q_x - P_y)\, dx\, dy = \int_{\partial D} (P, Q)\, ds$$

"$F \cdot G = Q_x - P_y$" para algún campo (P, Q)

Sea

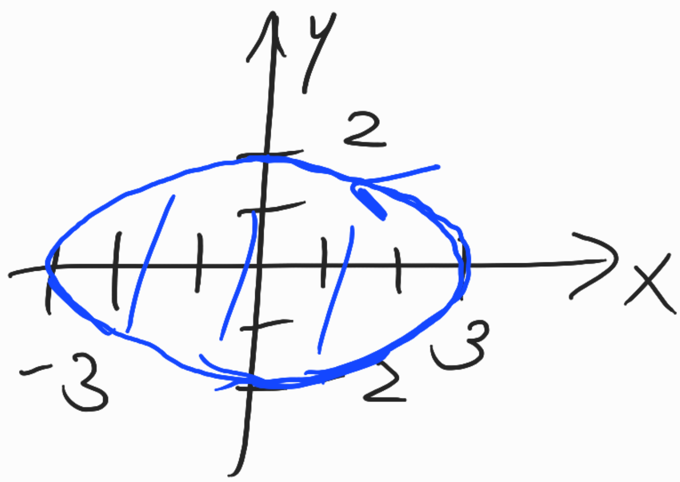
$$\boxed{H = (\underset{\overset{\shortparallel}{P}}{uv}, \underset{\overset{\shortparallel}{Q}}{uv})} \quad \otimes$$

$$Q_x - P_y = u \cdot v_x + v \cdot u_x - (u v_y + v \cdot u_y) \checkmark$$

$$\Rightarrow \iint_D (F \cdot G)\,dx\,dy = \iint_D (Q_x - P_y)\,dx\,dy$$

$$\underset{\text{Green}}{=} \int_{C} P\,dx + Q\,dy \quad \otimes$$

$C = \partial D^+$



Parametricemos C:

$\sigma(t) = (3\cos t,$

Recordar:

$2\operatorname{sen} t)$

$0 \le t \le 2\pi$

$\sigma'(t) = (-3\operatorname{sen} t,$

$2\cos t)$

$$\frac{x^2}{9} + \frac{y^2}{4} \le 1$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 \le 1$$

$$\begin{cases} \frac{x}{3} = r\cos\theta \\ \frac{y}{2} = r\operatorname{sen}\theta \end{cases}$$

$0 \le r \le 1$

$0 \le \theta \le 2\pi$

luego,

$$(*) = \int_0^{2\pi} H(\sigma(t)) \cdot \sigma'(t)\,dt$$

$$= \int_0^{2\pi} (\overset{x}{\overset{\shortparallel}{u}}(\sigma(t)), \overset{x}{\overset{\shortparallel}{v}}(\sigma(t))) \cdot \sigma'(t)\,dt$$

$$\overset{\uparrow}{=} \int_0^{2\pi} (3\cos t, 3\cos t) \cdot (-3\operatorname{sen} t, 2\cos t)\,dt$$

$$\boxed{\begin{array}{l} u.v = x.1 \\ \text{en } \partial D^+ = C \end{array}}$$

$$= \int_0^{2\pi} -9\,\text{sen}\,t\cos t + \underbrace{6\cos^2 t}_{=}\,dt$$

$$= -9\int_0^{2\pi} \text{sen}\,t\cos t\,dt + 6\int \frac{1+\cos(2t)}{2}\,dt$$

$$= -9\,\frac{\text{sen}^2 t}{2}\Big|_0^{2\pi} + \frac{6}{2}\left(t + \frac{\text{sen}(2t)}{2}\right)\Big|_0^{2\pi}$$

$$= 0 + 3.2\pi + 0 = 6\pi$$

6) $$\int_D u.v_x\,dx\,dy = -\int_D u_x.v\,dx\,dy$$

$$+\int_{\partial D} u\,v\cdot n_1\,ds$$

$n=(n_1,n_2)$ normal exterior a D.

queremos ver que

$$\boxed{\iint_D (u\cdot v_x + u_x\cdot v)\,dx\,dy = \int_{\partial D} u\cdot v\cdot n_1\,ds}$$

$$\iint_D u\cdot v_x + u_x\cdot v\,dx\,dy = \iint_D (u\cdot v)_x\,dx\,dy$$

$$= \iint_D (\underbrace{Q_x - P_y}_{=0})\,dx\,dy \quad (*)$$

Sea $F=(0,uv)$

$=(P,Q)$

$$\Rightarrow Q_x - P_y = (uv)_x$$

Por Teo. de Green,

$$\circledast = \int_{\partial D} (\overset{=0}{P}\,dx + Q\,dy) = \int_{\partial D} F\,ds$$

$\sigma : [a,b] \to \mathbb{R}^2$ parametrización de ∂D

$$\sigma(t) = (x_1(t), x_2(t))$$

$$\sigma'(t) = \underbrace{(x_1'(t), x_2'(t))}_{\text{perpendicular a } u = (u_1, u_2)}$$

$$\Rightarrow \boxed{\frac{\sigma'(t)}{\|\sigma'(t)\|} = (-u_2, u_1)}$$

$$\Rightarrow \boxed{\iint_D u \cdot v_x + u_x \cdot v \, dx\,dy =}$$

$$= \int_{\partial D} \underset{\underset{(0,\, uv)}{\shortparallel}}{F} \, ds$$

$$= \int_a^b F(\sigma(t)) \cdot \sigma'(t) \, dt$$

$$= \int_a^b F(\sigma(t)) \cdot \underbrace{\frac{\sigma'(t)}{\|\sigma'(t)\|}}_{= (-u_2,\, u_1)} \|\sigma'(t)\| \, dt$$

$$= \int_a^b F(\sigma(t)) \circ (-u_2, u_1) \cdot \|\sigma'(t)\| dt$$

$$= \int_a^b (0, \mu v(\sigma(t))) \cdot (-u_2, u_1) \cdot \|\sigma'(t)\| dt$$

$F = (0, \mu v)$

$$= \int_a^b \mu \cdot v(\sigma(t)) \cdot u_1 \cdot \|\sigma'(t)\| dt$$

$$= \int_{\partial D} \mu \cdot v \cdot u_1 \, ds$$

Práctica 2

19) $T(\theta, \varphi) = (\cos\theta \operatorname{sen}\varphi, \operatorname{sen}\theta \operatorname{sen}\varphi, \cos\varphi)$

$0 \leq \theta \leq 2\pi$

$0 \leq \varphi \leq \pi$

$$\boxed{T_\theta \times T_\varphi = -\operatorname{sen}\varphi \cdot T(\theta,\varphi)}$$ (normal interior)

$$\int_S F\,dS = -\int_0^{2\pi}\int_0^{\pi} F(T(\theta,\varphi))\cdot(T(\theta,\varphi)(-\operatorname{sen}\varphi))\,d\varphi\,d\theta$$

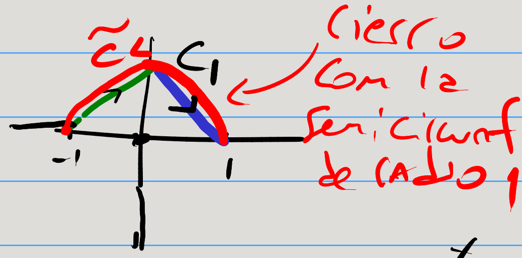
$$= \int_0^{2\pi}\int_0^{\pi} \underbrace{F(T(\theta,\varphi))\cdot T(\theta,\varphi)}_{\boxed{Fr}} \cdot \operatorname{sen}\varphi\,d\varphi\,d\theta$$

$$\int_C f_1\,dx + f_2\,dy$$

$$C: \begin{cases} y = x+1 & -1 \le x \le 0 \\ y = 1-x & 0 \le x \le 1 \end{cases}$$

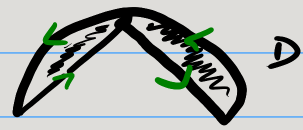
$F = (f_1, f_2)$

$$f_1(x,y) = \frac{x \operatorname{sen}\left(\frac{\pi}{2(x^2+y^2)}\right) - y(x^2+y^2)}{(x^2+y^2)^2}$$



$C = C_1 \cup \tilde{C}$ ← orient. posit.

$$f_2(x,y) = \frac{y \operatorname{sen}\left(\frac{\pi}{2(x^2+y^2)}\right) - x(x^2+y^2)}{(x^2+y^2)^2}$$



. F es C^1 en D ✓

$\left((0,0) \notin D \,! \right)$

green

$\partial D^+ = C_1 \cup \tilde{C}$

$$\underbrace{\iint_D f_{2_x} - f_{1_y}\,dx\,dy}_{A} = \int_{\partial D^+} F = \underbrace{\int_{C_1} F}_{\text{la que me piden.}} + \underbrace{\int_{\tilde{C}} F}_{B}$$

A) $\iint_D f_{2_x} - f_{1_x}\,dx\,dy$

IDEA: $F = \underbrace{G}_{\nabla f} + H$

buscar la constante

∇f con $f = a \cdot \cos\left(\frac{\pi}{2(x^2+y^2)}\right)$

$$F = (f_1, f_2) = \left(\overbrace{\frac{x \operatorname{sen}\left(\frac{\pi}{2(x^2+y^2)}\right)}{(x^2+y^2)^2}, \frac{y \operatorname{sen}\left(\frac{\pi}{2(x^2+y^2)}\right)}{(x^2+y^2)^2}} \right) + \overbrace{\left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2} \right)}$$

$$\Rightarrow F = \underbrace{\nabla f}_{(f_x,\, f_y)} + \left(\frac{y}{x^2+y^2},\ \frac{-x}{x^2+y^2} \right)$$

Notar que $(f_y)_x - (f_x)_y = 0$ por ser $\cos\left(\frac{\pi}{x^2+y^2}\right)$

de clase C^2 en D.

Ejercicio 2. Sea S la superficie cilíndrica con tapa, que es unión de dos superficies S_1 y S_2, donde S_1 es el conjunto de (x, y, z) con $x^2 + y^2 = 1$, $0 \le z \le 1$ y S_2 es el conjunto de (x, y, z) con $x^2 + y^2 + (z-1)^2 = 1$, $z \ge 1$, orientadas con la normal que apunta hacia afuera del cilindro y de la esfera, respectivamente. Sea $\mathbf{F}(x, y, z) = (zx + z^2y + x, z^3yx + y, z^4x^2)$. Calcular $\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

Pr 4

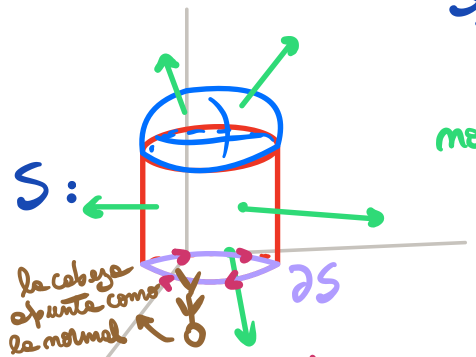
Solución:

- F es C^1 (en $\mathbb{R}^3$) ✓.
- $S = S_1 \cup S_2$

$S_1 = \{(x,y,z) \,/\, x^2+y^2=1 \ , \ 0 \le z \le 1\}$ cilindro.

$S_2 = \{(x,y,z) \,/\, x^2+y^2+(z-1)^2=1 \ , \ z \ge 1\}$ semi-esfera.

S :



normal exterior, tanto al cilindro como a la esfera.

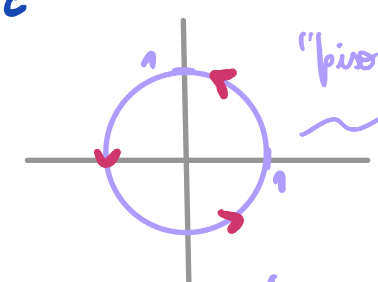
+ regla del caminante.

$$F(x,y,z) = (P, Q, R) = (\overbrace{zx + z^2y + x}^{P}, \overbrace{z^3yx + y}^{Q}, \overbrace{z^4x^2}^{R}).$$

$$\nabla \times F = \det\begin{pmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{pmatrix} = (R_y - Q_z, P_z - R_x, Q_x - P_y)$$

$$= (-3xyz^2, -2xz^4 + x + 2yz, z^3y - z^2) \ne \vec{0}$$

¿Quién es ∂S? (La miro "desde arriba" para entender)



"piso cilindro"

parametrizo (respetando orientación) en $\mathbb{R}^3$:

$\sigma(t) = (\cos t, \operatorname{sen} t, 0)$ $t \in [0, 2\pi]$

↳ la curva está en el plano xy $(z=0)$.

σ regular ✓

$\sigma'(t) = (-\operatorname{sen} t, \cos t, 0)$

Es ∂S^+ !

Así:

$$\iint_S \nabla \times F \, dS \underset{\text{STOKES}}{=} \int_{\partial S^+} F \, ds = \int_0^{2\pi} F(\cos t, \operatorname{sen} t, 0) \cdot (-\operatorname{sen} t, \cos t, 0) \, dt$$

$$= \int_0^{2\pi} (\cos t, \operatorname{sen} t, 0) \cdot (-\operatorname{sen} t, \cos t, 0) \, dt$$

$$= \int_0^{2\pi} (-\cos t \operatorname{sen} t + \operatorname{sen} t \cos t) \, dt = 0 .$$