2.a. $\frac{y'}{y} = c$

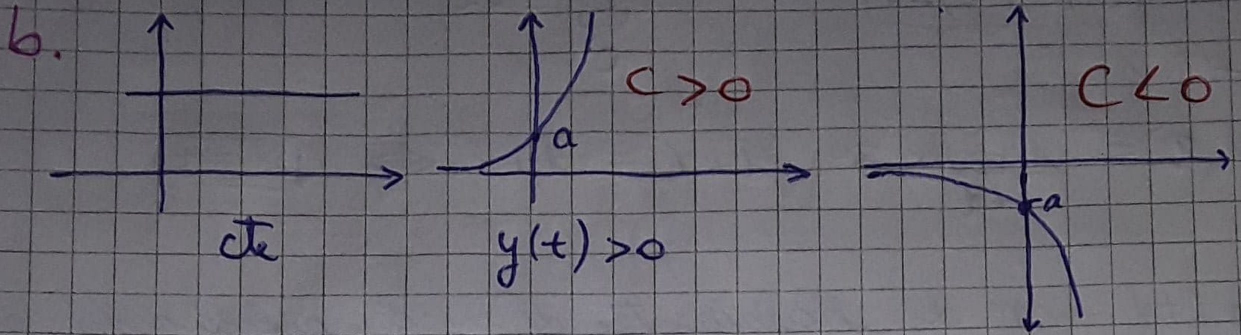
$$y' = cy$$

$$\int \frac{dy}{y} = c \int dt$$

$$\ln|y| = ct + k \qquad k \in \mathbb{R}$$

$$y = e^{ct} \cdot k$$

Pero $y > 0 \Rightarrow k > 0$ (k es la población inicial b.



c. $\frac{y'}{y} = 0 \iff y' = 0 \Rightarrow \boxed{y = cte}$

Es decir $t = 0 \Rightarrow y(0) = \begin{cases} a \\ -a \end{cases}$

d. Caso crecimiento constante $y = |a| e^{c \cdot t}$

1 enero 2002 : 1000 individuos $\rightarrow$ tiempo 0 meses

+4 meses : 1020 $\rightarrow t = 4$ meses

1 enero 2022 : ? $\rightarrow t = 20$ años $=$ 240 meses

$y(0) = 1000 \qquad y(4) = 1020$

- $1000 = a e^{0} \Rightarrow \boxed{1000 = a}$

- $1020 = 1000 e^{c \cdot 4}$

$1{,}02 = e^{c \cdot 4}$

$\ln(1{,}02) = c \cdot 4 \rightarrow \boxed{\frac{\ln(1{,}02)}{4} = c}$

$$y(240) = 1000 \cdot e^{240 \cdot \frac{\ln(1,02)}{4}}$$

$$= 1000 \cdot e^{\ln(1,02) \cdot 60}$$

$$= 1000 \cdot e^{\ln(1,02^{60})}$$

$$= 1000 \cdot 1,02^{60} = \boxed{3281}$$ Individuos en 1-01-22

e. $$\frac{y'}{y} = at + b$$

$$\frac{1}{y}\frac{dy}{dt} = at + b$$

$$\int_1^y \frac{1}{s}\, ds = \int_0^t a \cdot w + b \; dw$$ $y \in [1 ; +\infty)$

$$\ln(|s|)\Big|_1^y = \frac{a}{2}t^2 + bt + c$$

$$\ln(|y|) = \frac{a}{2}t^2 + bt + c$$

$$|y| = e^{\frac{a}{2}t^2 + bt + c}$$

$$y = \begin{cases} K e^{\frac{a}{2}t^2 + bt + c} & y > 0 \end{cases}$$

$\rightarrow y \in [1 ; +\infty)$ ✓

siempre es positivo

2f. $\frac{y'}{y} = r - cy \quad (\sim r \text{ si } y \sim 0)$

$$\Rightarrow \quad y' = y(r - cy)$$

Cuando y es chiquito (poblaciones pequeñas)

$\frac{y'}{y} \cong r = cte$ se parece a ec. dif de crecimiento exponencial.

si $r, c = 1$ $\quad y' = y(1-y)$ crec. logístico

$y' = y$ crec. exponencial

↳ más grande la población, más grande la tasa de crecimiento

En el crecimiento log la tasa de crecimiento va entre 0 y 1. Cuando $y \to 1$, $y' \to 0$ no puede crecer indefinidamente

Reescribamos y'

$$\underset{\substack{\downarrow \\ r \text{ factor común}}}{y(r - cy) =} ry\left(1 - \frac{c}{r}y\right) = \underbrace{\frac{r^2}{c} \cdot \frac{c}{r}}_{\text{multiplica y divide por lo mismo}} y\left(1 - \frac{c}{r}y\right)$$

Llamo $z = \frac{c}{r} y \Rightarrow \frac{r^2}{c} z(1-z)$

$$z' = \frac{c}{r} y'$$

$$y' = \frac{r}{c} z' = \frac{r^2}{c} z(1-z) \Rightarrow z' = r z(1-z)$$

Así $\int \frac{dz}{z(1-z)} = \int r\,dt = rt + k$

CA $\frac{1}{z(1-z)} = \frac{1}{z} + \frac{1}{1-z}$

$$\Rightarrow \ln|z| - \ln|1-z| = rt + k, \quad k \in \mathbb{R}$$

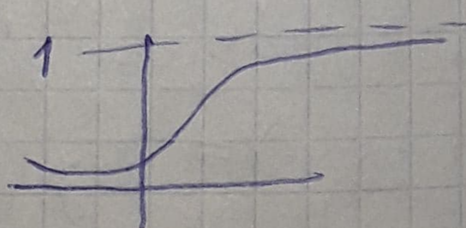
$$\frac{z}{1-z} = e^{rt} \cdot k$$

$$z = k e^{rt}(1-z) \Rightarrow k e^{rt} - z k e^{rt}$$

$$z = \frac{k e^{rt}}{1 + k e^{rt}}$$

$$\therefore \quad y = \frac{r}{c} \frac{k e^{rt}}{1 + k e^{rt}}$$

notar que $y(t) \to \frac{r}{c}$, $t \to \infty$



$y(t+1) = 2\cdot y(t) \rightarrow$ En $t+1$ tengo el doble que en tiempo t

$y = ae^{ct} \rightarrow$ (punto 2)

$\cancel{\alpha}\cdot e^{c(t+1)} = 2\cancel{\alpha}\cdot e^{ct}$

$\cancel{e^{ct}}\cdot e^{c} = 2\cancel{e^{ct}} \iff c = \ln(2)$

Volviendo a la fórmula de y

$$\left.\begin{aligned} y &= a\cdot e^{\ln(2)\cdot t} \\ &= a\cdot 2^{t} \end{aligned}\right\} \Rightarrow t=2 \quad y(2) = a\cdot 2^{2} = 4a$$

$\ln(2)\cdot t = \ln(2^{t})$

a población inicial

4. [Una función es homogénea de grado 0 si

$f(xc, t\cdot c) = f(x,t) \quad \forall (x,t,c)$]

a. $t x' = x + 2t\, e^{\left(-\frac{x}{t}\right)}$

$$x' = \underbrace{\frac{x}{t} + 2e^{-\frac{x}{t}}}_{F(x,t)}$$

$$F(x\cdot c, t\cdot c) = 2e^{\left(-\frac{x\cdot \cancel{c}}{t\cdot \cancel{c}}\right)} + \frac{x\cdot \cancel{c}}{t\cdot \cancel{c}} = x' = F(x,t) \checkmark$$

La ecuación es homogénea de grado 0.

[Sustitución $tu = x$ la convierte en variables separadas $u = u(t)$]

$$x' = t u' + u$$

$$\overset{\shortparallel}{F}(x,t) = F(tu, t) = F(t(u,1)) \underset{\text{hom de grado 0}}{=} F(u,1)$$

$$x' = u' t + u = F(u,1)$$

$$u' t + \cancel{u} = \cancel{u} + 2e^{-u}$$

$$\frac{du}{dt}\, t = 2e^{-u}$$

$$\int e^{u}\, du = 2 \int \frac{1}{t}\, dt$$

$$e^{u} = 2(\ln|t| + C)$$

$$e^{u} = 2\ln|t| + K \qquad (K = 2C)$$

$$u = \ln(2\ln|t| + K)$$

$$\boxed{x = t\,u = t\cdot \ln(2\ln|t| + K)} \qquad t \neq 0$$

$$\cdot\ x' = \underbrace{\ln(2\ln|t| + K)}_{= \frac{x}{t}} + \cancel{t}\, \underbrace{\frac{2}{2\ln|t| + K}}_{e^{-\frac{x}{t}}} \frac{1}{\cancel{t}} \checkmark$$

CA $2e^{-\frac{x}{t}} = 2\cancel{e}^{\left(\cancel{\ln}(2\ln|t| + K)\right)^{-1}} = 2\cdot \frac{1}{2\ln|t| + K}$

$= 2\left(e^{\left(\frac{x}{t}\right)}\right)^{-1}$

6. $t x x' = 2x^2 - t^2$

$$x' = \frac{2x}{t} - \frac{t}{x}$$

$$\boxed{x = t \cdot u}$$

$$x' = F(x,t) = F(x\cdot c, t\cdot c) = \frac{2\cdot x\cdot \cancel{c}}{t\cdot \cancel{c}} - \frac{t\cdot \cancel{c}}{x\cdot \cancel{c}} \checkmark$$

$$x' = \frac{2\cdot \cancel{t} u}{\cancel{t}} - \frac{\cancel{t}}{\cancel{t}\cdot u}$$

$$x' = 2u - \frac{1}{u} = u' t + u$$

$$\left(u - \frac{1}{u}\right) = \frac{du}{dt}\cdot t$$

$$\int \frac{1}{u - \frac{1}{u}}\, du = \int \frac{1}{t}\, dt$$

$$= \ln|t| + c$$

CA $\int \frac{1}{u - \frac{1}{u}} \frac{u}{u}\, du = \int \frac{u}{u^2-1}\, du = \int \frac{\cancel{u}}{s} \frac{ds}{2\cancel{u}}$

$s = u^2 - 1$

$ds = 2u\, du$

$$= \ln|s| \cdot \frac{1}{2} = \ln|u^2-1| \cdot \frac{1}{2}$$

$$\frac{1}{2} \ln|u^2+1| = \ln|t| + c$$

$$\ln|u^2+1| = 2\ln|t| + K \qquad (k = 2c)$$

$$|u^2+1| = e^{2\cdot \ln|t| + K}$$

$$|u^2+1| = \underset{\geq 0}{|t^2|} \cdot c' \qquad c' = e^K > 0$$

Como $u = \frac{x}{t}$

$$\boxed{\left|\left(\frac{x}{t}\right)^2 - 1\right| = c \cdot t^2}$$ solución general (en forma implícita)

$x, t \neq 0$

C. $x' = \frac{x+t}{t} \quad x(1) = 0$

$$F(x \cdot c, t \cdot c) = \frac{x \cdot c + t \cdot c}{t \cdot c} = \frac{\cancel{c}\,(x+t)}{t \cdot \cancel{c}} = F(x,t) \checkmark$$

$x = t \cdot u$

$$x' = \frac{tu + t}{t} = \frac{\cancel{t}(u+1)}{\cancel{t}} = u't + u$$

$$\frac{du}{dt}\,t = \cancel{u} + 1 - \cancel{u}$$

$$\int du = \int \frac{1}{t}\,dt$$

$$u = \ln|t| + c$$

$$\frac{x}{t} = \ln|t| + c$$

$$\boxed{x(t) = t \cdot \ln|t| + c}$$ sol general

$$x(1) = 1 \cdot \ln(1) + c = 0 \iff c = 0$$

$$\boxed{x(t) = t \cdot \ln(t)}$$ definida en $(0; +\infty)$

$1 \in$ intervalo

5. $x' = f(at + bx + c)$

sustitución $y = at + bx + c \Rightarrow y' = a + bx'$

$|x' = f(y)| \Rightarrow \boxed{y' = a + bf(y)}$

$\rightarrow$ Variables separadas

a. $x' = (x+t)^2$

$y = x + t \Rightarrow x' = y^2 = f(y)$

$y' = 1 + 1 \cdot y^2$

$\frac{dy}{dt} = 1 + y^2$

$\int \frac{1}{1+y^2} dy = \int dt$

$\text{arctg}(y) = t + c$

$\begin{cases} y = tg(t+c) \\ y = x + t \end{cases}$

$\Rightarrow \boxed{x = tg(t+c) - t}$ def en $\mathbb{R}$

b. $x' = sen^2(t - x + 1)$

$y = t - x + 1 \Rightarrow x' = sen^2(y) = f(y)$

$\Rightarrow y' = 1 - 1 \cdot sen^2(y)$

$$\frac{dy}{dt} = 1 - \operatorname{sen}^2(y) = \cos^2(y)$$

$$\int \frac{1}{\cos^2(y)}\, dy = \int dt$$

$$tg(y) = t + c$$

$$y = \operatorname{arctg}(t+c)$$

$$t - x + 1 = \operatorname{arctg}(t+c)$$

$$\boxed{t + 1 - \operatorname{arctg}(t+c) = x(t)}$$ def en $\mathbb{R}$

6.a. $x' = F\left(\frac{at + bx + c}{dt + ex + f}\right)$

$ae \neq bd$

$$\left.\begin{array}{ll} t = s - h & h \in \mathbb{R} \\ x(t) = y(s) - k & k \in \mathbb{R} \end{array}\right\} \Rightarrow \begin{array}{l} \boxed{t + h = s} \\ \boxed{x + k = y} \end{array}$$

$$x'(t) = y'(s)$$

$$\Rightarrow \quad y'(s) = F\left(\frac{a\cdot(s-h) + b(y-k) + c}{d(s-h) + e(y-k) + f}\right)$$

$$= F\left(\frac{as - ah + by - bk + c}{ds - dh + ey - ek + f}\right)$$

$$= F\left(\frac{as + by + (c - ah - bk)}{ds + ey + (f - dh - ek)}\right)$$

Tomamos h y k / $\begin{cases} c = ah - bk \\ f = dh - ek \end{cases}$

Sabiendo que $ae - bd \neq 0$, significa que el determinante es no nulo

$$\begin{pmatrix} a & b \\ d & e \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} c \\ f \end{pmatrix}$$

$\Rightarrow$ queda $y' = F\left(\underbrace{\dfrac{as + by}{ds + ey}}_{\text{homogéneo}}\right)$

$$= F\left(\frac{a + b\frac{y}{s}}{d + e\frac{y}{s}}\right)$$

$\downarrow$

Es una función de $\frac{y}{s}$

b.i) $x' = \dfrac{2x \quad t + 4}{x + t - 1}$

$$= \frac{-t + 2x + 4}{t + x - 1}$$

$t = s - h \qquad x = y - k$

$$= \frac{-(s-h) + 2(y-k) + 4}{s - h + y - k - 1}$$

$$= \frac{-s + h + 2y - 2k + 4}{s - h + y - k - 1} = \frac{2y - s + h - 2k + 4}{y + s - h - k - 1}$$

$4 = -h + 2k$

$-1 = h + k$

$$\begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$\begin{array}{cc} h & k \end{array}$$
$$\left(\begin{array}{cc|c} -1 & 2 & 4 \\ 1 & 1 & -1 \end{array}\right) \to \left(\begin{array}{cc|c} -1 & 2 & 4 \\ 0 & 3 & 3 \end{array}\right) \to \left(\begin{array}{cc|c} -1 & 2 & 4 \\ 0 & 1 & 1 \end{array}\right)$$

$$\Rightarrow \boxed{k = 1}, \quad -1h + 2 \cdot 1 = 4$$
$$\boxed{-2 = h}$$

Entonces $y' = \dfrac{2\frac{y}{s} - 1}{\frac{y}{s} + 1}$ pues $x'(t) = y'(s)$

llamo $u = \frac{y}{s} \Rightarrow us = y \Rightarrow y' = u's + u$

$$y' = u's + u = \frac{2u - 1}{u + 1}$$

$$\frac{du}{ds} s = \frac{2u - 1 - u \cdot (u+1)}{u+1}$$

$$\frac{du}{ds} s = \frac{2u - 1 - u^2 - \cancel{x}^{u}}{u+1}$$ → Arrastré error

$$\frac{du}{ds} s = \frac{-u^2 + 2u - 2}{u+1}$$

$$\int \frac{u+1}{-u^2+2u-2} du = \int \frac{1}{s} ds$$
α

$$= \ln|s| + C$$

$$\frac{u+1}{+2u-u^2-2} = \frac{u}{+2u-u^2-2} + \frac{1}{2u-u^2-2}$$

CA $$\int \frac{1}{-u^2+2u-2}\,du = \int \frac{1}{-u^2+2u-1-1}$$

Obs $(x-1)(x-1) = x^2-2x+1$

$\Rightarrow -(x-1)^2 = -x^2+2x-1$

$$= \int \frac{1}{-(u-1)^2-1}\,du = \int \frac{1}{-v^2-1}\,dv = -\int \frac{1}{v^2+1}\,du$$

$v = u-1$

$dv = 1\,du$

$= -\operatorname{arctg}(v)$

$= \boxed{-\operatorname{arctg}(u-1)+C}$

CA $$\int \frac{u}{-u^2+2u-2}\,du = \int \frac{v+1}{-(v^2+1)}\,dv$$

$v = u-1 \Rightarrow u = v+1$

$dv = du$

$$\int \frac{v}{-(v^2+1)}\,dv + \int \frac{1}{-(v^2+1)}\,dv$$

(1) (2)

(2) $-\int \frac{1}{v^2+1}\,dv = \boxed{-\operatorname{arctg}(u-1)+C}$

(1) $\int \frac{v}{-(v^2+1)}\,dv = \int \frac{\cancel{v}}{-w}\,\frac{dw}{2\cancel{v}} = \boxed{-\frac{1}{2}\ln(|v^2+1|)+C}$

$w = v^2+1$

$dw = 2v\,dv$

$$\Rightarrow \int \frac{u}{2u-u^2+2} = -\frac{1}{2}\ln|(u-1)^2+1| - \operatorname{arctg}(u-1)$$

Volviendo al problema

$$\int \frac{u+1}{-u^2+2u-2}\,du = -\frac{1}{2}\ln|\underbrace{(u-1)^2+1}_{>0}| - 2\operatorname{arctg}(u-1)$$

$$-\frac{1}{2}\ln((u-1)^2+1) - 2\operatorname{arctg}(u-1) = \ln|s| + c$$

$$u = \frac{y}{s} = \frac{x+1}{t-2}$$

Está mal

$$\boxed{\text{Rta: } -\frac{1}{2}\ln\left(\left(\frac{x+1}{t-2}-1\right)^2+1\right) - 2\operatorname{arctg}\left(\frac{x+1}{t-2}-1\right) = \ln(t-2)+C}$$

b.11. $x' = \dfrac{x+t+4}{t-x-6} = \dfrac{1\cancel{t}+1\cancel{x}+4}{1t-1x-6}$

$$\begin{array}{cc} h & k \end{array}$$
$$\left(\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & -1 & -6 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -2 & -10 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 1 & 5 \end{array}\right)$$

$\boxed{k=5}$ $\quad h+5=4 \to \boxed{h=-1}$

$y' = \dfrac{1+u}{1-u}$ donde $u = \dfrac{y}{s} = \dfrac{x+5}{t-1}$

$$u's + u = \frac{1+u}{1-u}$$

$$\frac{du}{ds}s = \frac{1+u-u(1-u)}{1-u} = \frac{1+\cancel{u}-\cancel{u}+u^2}{1-u}$$

$$\int \frac{1-u}{1+u^2}\,du = \int \frac{1}{s}\,ds$$

$$\int \frac{1}{1+u^2} - \frac{u}{1+u^2}\,du = \ln|s| + c$$

$$\operatorname{arctg}(u) - \int \frac{u}{1+u^2}\, du = \ln|s| + C$$

$$\left[\text{CA} \int \frac{u}{1+u^2}\, du \underset{\downarrow}{=} \int \frac{\cancel{u}}{v} \frac{dv}{2\cancel{u}} = \frac{1}{2} \ln|v| \right]$$

$$v = 1+u^2$$
$$dv = 2u\, du$$

$$\operatorname{arctg}(u) - \frac{1}{2} \ln(1+u^2) = \ln|s| + C$$

$$u = \frac{x+5}{t-1}$$

$$\boxed{\operatorname{arctg}\left(\frac{x+5}{t-1}\right) - \frac{1}{2} \ln\left(1 + \left(\frac{x+5}{t-1}\right)^2\right) = \ln(|t-1|) + C}$$

111. $$x' = \frac{t+x+4}{t+x-6}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & 1 & -6 \end{array}\right) \rightarrow ae = bd \Rightarrow$$ no se puede usar este método

Llamo $y = t + x$

$$x' = \frac{y+4}{y-6} = f(y)$$

$$y' = a + b f(y)$$

$$y' = 1 + \frac{y+4}{y-6} = \frac{y-6+y+4}{y-6} = \frac{2y-2}{y-6}$$

$$\int \frac{y-6}{2y-2}\, dy = \int dt = t + C$$

$$CA\ \frac{1}{2}\int \frac{2y-12}{2y-2}\,dy = \frac{1}{2}\int \frac{2y-2-10}{2y-2}\,dy$$

$$= \frac{1}{2}\int\left(\frac{2y-2}{2y-2} - \frac{10}{2y-2}\right)dy$$

$$= \frac{1}{2}\left[\int 1\,dy - \int \frac{5}{y-1}\,dy\right]$$

$$\left[{*}\ 5\int \frac{1}{y-1}\,dy \underset{\substack{\downarrow \\ u=y-1 \\ du=dy}}{=} 5\int \frac{1}{u}\,du = 5\ln|u| = 5\ln|y-1|\right]$$

$$= \frac{1}{2}\left[y - 5\ln|y-1|\right]$$

$$= \frac{1}{2}y - \frac{5}{2}\ln|y-1|$$

$$\frac{1}{2}y - \frac{5}{2}\ln|y-1| = t + c$$

$$\underline{y = t + x}$$

$$\boxed{\frac{1}{2}(t+x) - \frac{5}{2}\ln|t+x-1| = t+c}$$

7. $\underbrace{(y-x^3)}_{M}\,dx + \underbrace{(x+y^3)}_{N}\,dy = 0$

$M_y = 1 \qquad N_x = 1 \qquad M_y = N_x$ Es exacta

Busco $f(x,y)\ /\ f_x = y - x^3,\ f_y = x + y^3$

$$\int x + y^3\,dy + g(x) = xy + \frac{y^4}{4} + g(x) = f(x,y)$$

$$f_x(x,y) = \cancel{y} + 0 + g'(x) = \cancel{y} - x^3$$

$g'(x) = -x^3 \Rightarrow g(x) = -\frac{x^4}{4} + C$ tomo $c=0$

sol: $\boxed{xy + \frac{y^4}{4} - \frac{x^4}{4} = c}$ $\quad f(x,y) = c$

b. $(\cos x \cos^2 y)\,dx + (-2 \operatorname{sen} x \operatorname{sen} y \cos y)\,dy = 0$

$M_y = \cos(x) \cdot 2\cos(y) \cdot (-\operatorname{sen} y)$

$N_x = -2 \cos x \cdot \operatorname{sen} y \cos y$

Como $M_y = N_x \rightsquigarrow$ es exacta

Busco $f(x,y) / f_x = M \quad f_y = N$

$f(x,y) = \int \cos x \cos^2 y \, dx + g(y)$

$= \cos^2 y \cdot \int \cos x \, dx + g(y)$

$f(x,y) = \cos^2 y \operatorname{sen} x + g(y)$

$f_y(x,y) = -2\cos y \cdot \operatorname{sen} y \operatorname{sen} x + g'(y) = -2 \operatorname{sen} x \operatorname{sen} y \cos y$

$g'(y) = 0 \Rightarrow g(y) = c$, tomo $c = 0$

$f(x,y) = \boxed{\cos^2 y \operatorname{sen} x = C}$ sol

c. $\underbrace{(3x^2 - y^2)}_{N}\,dy + \underbrace{(-2xy)}_{M}\,dx = 0$

$M_y = -2x \qquad N_x = 6x$

$M_y - N_x = -2x - 6x = -8x \rightsquigarrow$ no es exacta

Busco factor integrante $\mu = \mu(x)$

$$\frac{M_y - N_x}{N} = \frac{-8x}{3x^2 - y^2}$$ no depende sólo de x

$$\mu = \mu(y)$$

$$\frac{N_x - M_y}{M} = \frac{8x}{-2xy} = -\frac{4}{y}$$ solo depende de y ✓

$$\mu(y) = e^{\int -\frac{4}{y} dy} = e^{-4 \ln|y|} = |y|^{-4}_{>0}$$

$$\Rightarrow \frac{1}{y^4}(3x^2 - y^2)\, dy + \frac{1}{y^4}(-2xy)\, dx = 0$$

$$\underbrace{\left(\frac{3x^2}{y^4} - \frac{1}{y^2}\right)}_{\tilde{N}} dy + \underbrace{\left(\frac{-2x}{y^3}\right)}_{\tilde{M}} dx = 0$$

$$\tilde{M}_y = \frac{-2x(-3)}{y^4} = \frac{6x}{y^4}$$

$$\tilde{N}_x = \frac{6x}{y^4}$$

$\tilde{M}_y = \tilde{N}_x \rightsquigarrow$ es exacta

Busco $f(x,y) \,/\, f_y = \tilde{N} \quad f_x = \tilde{M}$

$$f = \int \frac{-2x}{y^3}\, dx + g(y) = -\frac{x^2}{y^3} + g(y)$$

$$f_y(x,y) = \cancel{\frac{3x^2}{y^4}} + g'(y) = \cancel{\frac{3x^2}{y^4}} - \frac{1}{y^2}$$

$$g'(y) = -\frac{1}{y^2} \Rightarrow g(y) = \frac{1}{y} + c$$

$$f(x,y) = \boxed{-\frac{x^2}{y^3} + \frac{1}{y} = C}$$

d. $x\,dy = (x^5 + x^3y^2 + y)\,dx$

$$\underbrace{(x^5 + x^3y^2 + y)}_{M}\,dx + \underbrace{(-x)}_{N}\,dy = 0$$

$$M_y = 2x^3y + 1$$

$$N_x = -1$$

$M_y \neq N_x \rightarrow$ no es exacta

$$dy = \frac{x^5 + x^3y^2 + y}{x}\,dx$$

$$y = u \cdot x \qquad \underline{u = u(x)}$$

$$dy = du \cdot x + u \cdot dx$$

$$du \cdot x + u\,dx = \frac{x^5 + x^3(u^2x^2) + ux}{x}\,dx$$

$$du\,x = \frac{x^5 + u^2x^5 + \cancel{ux} - \cancel{ux}}{x}\,dx$$

$$du = \frac{x^{\cancel{5}\,3}(1+u^2)}{\cancel{x^2}}\,dx$$

$$\int \frac{1}{1+u^2}\,du = \int x^3\,dx$$

$$\operatorname{arctg}(u) = \frac{x^4}{4} + C$$

$$u = tg\left(\frac{x^4}{4} + c\right)$$

$$x\,u = y \cdot \cancel{x} \Rightarrow u = \frac{y}{x}$$

$$\boxed{y = x\, tg\left(\frac{x^4}{4} + c\right)} \qquad c \in \mathbb{R}$$

$2x\,\text{sen}\,y + 2y\,\text{sen}\,y$ $(2x+2y)\,\text{sen}\,y$

$2x\,\text{sen}\,y + 2y\,\text{sen}\,y$ $(2x+2y)\,\text{sen}\,y$

e. $$\underbrace{\left(2(x+y)\,\text{sen}\,y\right)}_{M} dx + \underbrace{\left(2(x+y)\,\text{sen}\,y + \cos y\right)}_{N} dy = 0$$

$$M_y = 2\cdot \text{sen}\,y + 2y\cdot\cos y + 2x\cos y$$

$$N_x = 2\,\text{sen}\,y$$

$M_y \neq N_x \sim$ no es exacta

Buscamos factor integrante

$$M_y - N_x = 2\,\cancel{\text{sen}\,y} + 2y\cos y + 2x\cos y - \cancel{2\,\text{sen}\,y}$$

$$= 2\cos y\,(y + x)$$

$$\Rightarrow \frac{N_x - M_y}{M} = \frac{-\cancel{2}\cos y\,\cancel{(y+x)}}{\cancel{2(x+y)}\,\text{sen}\,y} = -\frac{\cos y}{\text{sen}\,y} = -\frac{1}{tg\,y}$$

$$\mu(y) = e^{\int -\frac{1}{tg\,y}} = e^{-\ln|\text{sen}\,y|} = |\text{sen}\,y|^{-1} = \frac{1}{|\text{sen}\,y|}$$

CA $$\int -\frac{\cos}{\text{sen}}\,dy = -\int \frac{\cancel{\cos}}{v}\,\frac{dv}{\cancel{\cos}} = -\ln|v| = -\ln|\text{sen}\,y|$$

$v = \text{sen}$

$dv = \cos dy$

$$\frac{1}{\operatorname{sen} y}\left(2(x+y)\operatorname{sen} y\right)dx + \frac{1}{\operatorname{sen} y}\left(2(x+y)\operatorname{sen} y + \cos y\right)dy = 0$$

$$2(x+y)\,dx + \left(2(x+y) + \frac{\cos y}{\operatorname{sen} y}\right)dy = 0$$

$$\left.\begin{array}{l} \tilde{M}_{y} = 2 \\ \tilde{N}_{x} = 2 \end{array}\right\} \text{Es exacta}$$

Busco $f(x,y) \,/\, f_x = 2(x+y)$

$f_y = 2(x+y) + \frac{\cos y}{\operatorname{sen} y}$

$$\int 2x + 2y \, dx + g(y) = f(x,y) = x^2 + 2xy + g(y)$$

$$f_y = \cancel{2x} + g'(y) = \cancel{2x} + 2y + \frac{\cos y}{\operatorname{sen} y}$$

$$g(y) = y^2 + \ln|\operatorname{sen} y|$$

$$f(x,y) = C = x^2 + y^2 + 2xy + \ln|\operatorname{sen} y|$$

$$= \boxed{(x+y)^2 + \ln|\operatorname{sen} y|}$$

f. $3y\,dx + x\,dy = 0$

$M_y = 3 \qquad N_x = 1 \qquad$ no es exacta

Busco $\mu = \mu(x)$

$$\frac{M_y - N_x}{N} = \frac{3-1}{x} = \frac{2}{x}$$

$$\mu(x) = e^{\int \frac{2}{x}} = e^{\ln|x|^2} = \underset{>0}{x^2}$$

$$3yx^2\,dx + x^3\,dy = 0$$

$M_y = 3x^2 \qquad N_x = 3x^2 \quad \rightsquigarrow$ Es exacta

$f(x,y) = \int x^3\, dy + g(x) = y x^3 + g(x)$

$f_x(x,y) = \cancel{3yx^2} + g'(x) = \cancel{3yx^2}$

$\Rightarrow g(x) = C$

Sol $\boxed{f(x,y) = C = y x^3}$

g. $\left(1 - y(x+y)\, tg(xy)\right) dx + \left(1 - x(x+y)\, tg(xy)\right) dy = 0$

$M = 1 - (yx + y^2)\, tg(xy)$

$M = 1 - yx\, tg(xy) - y^2\, tg(xy)$

$M_y = -x\, tg(xy) + (-yx) \frac{1 \cdot x}{\cos^2(xy)} - 2y\, tg(xy) - \frac{y^2 \cdot x}{\cos^2(xy)}$

$N = 1 - (x^2 + xy)\, tg(xy)$

$= 1 - x^2\, tg(xy) - xy\, tg(xy)$

$N_x = -2x\, tg(xy) - x^2 \frac{1}{\cos^2(xy)} \cdot y - y\, tg(xy) - xy \frac{y}{\cos^2(xy)}$

$M_y = tg(xy)(-x - 2y) + \frac{1}{\cos^2(xy)}(-y^2 x - yx)$

$N_x = tg(xy)(-2x - y) + \frac{1}{\cos^2(xy)}(-x^2 y - xy^2)$

$M_y \neq N_x \rightsquigarrow$ no es exacta

Busco un factor integrante $\mu = \mu(z) = \mu(z(x,y))$

CA $\frac{1}{2}\int \frac{M_y - N_x}{xN - yM}\, dz$.

$$M_y - N_x = tg(xy)(x-y) + \frac{1}{\cos^2(xy)}(-yx + x^2y)$$

$$x \cdot N = x - x^3 tg(xy) - x^2y\, tg(xy)$$

$$= x - tg(xy)(x^3 + x^2y)$$

$$y \cdot M = y - y^2x\, tg(xy) - y^3\, tg(xy)$$

$$= y - tg(xy)(y^3 + y^2x)$$

$$x \cdot N - y \cdot M = x - y - tg(xy)(x^3 + x^2y) + tg(xy)(y^3 + y^2x)$$

$$= x - y + tg(xy)(y^3 + y^2x - yx^2 - x^3)$$

Mirando la ecuación dif original, pruebo el factor integrante $\mu(z) = \frac{1}{x+y}$

$$\frac{1}{x+y}\left(1 - y(x+y)\, tg(xy)\right)dx + \frac{1}{x+y}\left(1 - x(x+y)\, tg(xy)\right)dy = 0$$

$$\underbrace{\left(\frac{1}{x+y} - y\, tg(xy)\right)}_{\tilde{M}} dx + \underbrace{\left(\frac{1}{x+y} - x\, tg(xy)\right)}_{\tilde{N}} dy = 0$$

$$\left.\begin{array}{l} \tilde{M}_y = \frac{-1}{(x+y)^2} - tg(xy) - y\frac{1}{\cos^2(xy)} \cdot x \\[2ex] \tilde{N}_x = \frac{-1}{(x+y)^2} - tg(xy) - x\frac{1}{\cos^2(xy)}\, y \end{array}\right\} \tilde{M}_y = \tilde{N}_x \to \text{es exacta!}$$

Busco $f(x,y)$

$$\int \frac{1}{x+y} - y\,tg(xy)\,dx + g(y) = f(x,y)$$

$$\ln|x+y| - \int y\,Tg(xy)\,dx + g(y) = f$$

$$CA\; -\int y\,tg(xy)\,dx \underset{\substack{u=xy \\ du=y\,dx}}{=} -\int \cancel{y}\,tg(u)\,\frac{du}{\cancel{y}} = \int \frac{-\operatorname{sen} u}{\cos u}\,du$$

$$\underset{\substack{v=\cos u \\ dv=-\operatorname{sen} u}}{=} \int \frac{\cancel{-\operatorname{sen} u}}{v}\,\frac{dv}{\cancel{-\operatorname{sen} u}} = \ln|v| = \ln|\cos u| = \ln|\cos(xy)|$$

$$f(x,y) = \ln|x+y| + \ln|\cos(xy)| + g(y)$$

$$f_y = \frac{1}{x+y} + \frac{1}{\cos(xy)}(-\operatorname{sen}(xy))\cdot x + g'(y)$$

$$= \cancel{\frac{1}{x+y}} - \cancel{tg(xy)\cdot x} + g'(y) = \cancel{\frac{1}{x+y}} - \cancel{x\,tg(xy)}$$

$$\Rightarrow g(y) = c$$

Sol $\boxed{f(x,y) = c = \ln|x+y| + \ln|\cos(xy)|}$

8. a. $y' + p(x)\,y = q(x)$

$$\mu(x)\Big(\underbrace{y'(x)+p(x)\,y(x)}_{=\,q(x)}\Big) = \big(y(x)\mu(x)\big)'$$

$$\mu \cdot q = (y\mu)'$$

$$\cancel{\mu y'} + \mu p\,\cancel{y} = \cancel{y'\mu} + \cancel{y}\,\mu'$$

$$\mu p = \frac{d\mu}{dx}$$

$$p\,dx = \frac{1}{\mu}\,d\mu$$

$$\int p(x)\,dx = \ln|\mu|$$

$$\boxed{e^{\int p(x)dx} = \mu(x)}$$ Factor integrante

b. Como $\mu \cdot q = (y\mu)'$

$\int \mu \cdot q\,dx = y\mu$ } integrando

$$y = \frac{1}{\mu}\int \mu \cdot q$$

$$\Rightarrow \boxed{y = e^{-\int p(x)dx}\int q \cdot e^{\int p(x)dx}}$$

9. • R.Tg $y_T = f'(x_0)(x-x_0) + f(x_0) = \underbrace{f'(x_0)}_{\text{pendiente}}\cdot x - f'\cdot x_0 + f(x_0)$

• Recta que une el punto (x_0, y_0) con el origen

$$y = x \cdot \frac{f(x_0)}{x_0}$$

$$f'(x_0) = \frac{1}{2}\cdot\frac{f(x_0)}{x_0}$$

La curva que busco se puede parametrizar como $(x, f(x))$

$\rightarrow$ quiero $f(x)$

$$2 \cdot x_0 \cdot f'(x_0) = f(x_0)$$

Como quiero que pase para todos los puntos de la curva

$$2x \cdot f'(x) = f(x) = y$$

$$2x \quad y' = y$$

$$2 \frac{y'}{y} = \frac{1}{x}$$

$$\frac{2}{y} \frac{dy}{dx} = \frac{1}{x}$$

$$\int \frac{2}{y} \, dy = \int \frac{1}{x} \, dx$$

$$2 \cdot \ln|y| = \ln|x| + c$$

$$\ln|y| = \frac{1}{2} \ln|x| + k$$

$$y = e^{\ln|x|^{\frac{1}{2}} + k}$$

$$y = \sqrt{|x|} \cdot e^{k}$$

$$\boxed{y = f(x) = \sqrt{x} \cdot c'}$$

TG : $y = y'(x_0)(x-x_0) + y_0$.

N : $y = \dfrac{-1}{y'(x_0)}(x-x_0) + y_0$ → no vale para rectos verticales

$(0,0)$ satisface sii $0 = \dfrac{x_0}{y'(x_0)} + y_0 \Rightarrow 0 = \dfrac{x}{y'} + y$

$$y'y = -x$$

$$\frac{dy}{dx}y = -x$$

$$y\,dy = -x\,dx$$

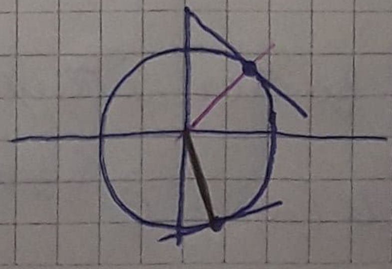
$$\frac{y^2}{2} = -\frac{x^2}{2} + C$$

$$y^2 = -x^2 + 2\cdot C \Rightarrow y^2 + x^2 = c$$

$$|y| = \sqrt{-x^2 + 2C}$$

$$\boxed{y = \begin{cases} + \sqrt{-x^2 + c'} \\ - \sqrt{-x^2 + c'} \end{cases}}$$

Curvas cuya normal pasa por el origen



11. La pendiente de la tangente es proporcional al valor del eje x.

$$y'(x) = K \cdot x$$

$$\frac{dy}{dx} = Kx$$

$$dy = Kx\,dx$$

$$y = K\frac{x^2}{2} + C \rightarrow \text{parábola}$$

12. Quiero curva $y(x)$

$$tg \quad y_T = y(x_0) + y'(x_0)(x - x_0)$$

$$y(x_0) = y_0$$

Ordenada al origen $\rightarrow$ $x = 0$ $\quad y = 2(x_0 + y_0)$

Reemplazo

$$2(x_0 + y_0) = y_0 + y'(x_0)(-x_0)$$

Como queremos que se cumpla para todos los puntos de la curva

$$2x + 2y = y - x \cdot y'$$

$$2x + y = -x\,y'$$

$$y + x\,y' = -2x$$

$$\underbrace{\frac{1}{x}}_{p(x)} y + y' = \underbrace{-2}_{q(x)}$$

Busco factor integrante

$$\mu(x) = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = x$$

$$\Rightarrow \quad y = \frac{1}{x} \int (-2) \cdot x \, dx = \frac{1}{\cancel{x}} (-2) \left(\frac{x^2}{2} + c \right)$$

$$y = \frac{1}{x} \left(-\frac{2x^2}{2} + c' \right)$$

$$\boxed{y = \frac{-x^2 + c'}{x}}$$

$M_x = N_y \iff \{$

13.a Este ejercicio sale usando la fórmula obtenida para el punto 8.

$$\boxed{y'(x) + p(x)\,y = q(x)}$$

$$y' + \underbrace{y}_{} = \underline{\text{sen}(x)}$$

$p(x) = 1$ $\qquad$ $q(x) = \text{sen}(x)$

Busco factor integrante

$$\mu(x) = e^{\int p(x)\,dx} = e^{x}$$

$$\longrightarrow (\mu y)' = \mu \cdot q$$

$$(e^{x} y)' = e^{x} \cdot \text{sen}(x)$$

$$y = e^{-x} \int \text{sen}\,x \cdot e^{x}\,dx$$

CA $\int \text{sen}\,x \cdot e^{x}\,dx = -e^{x}\cos x - \left(\int -e^{x}\cos x\,dx\right)$

$\downarrow$

PARTES

$u = e^{x}$ $\quad$ $du = e^{x}$

$dv = \text{sen}\,x$ $\quad$ $v = \cos x$

$$-e^{x}\cos x - \int -e^{x}\cos x = -e^{x}\cos x - \left(-\left(e^{x}\,\text{sen}\,x - \int e^{x}\,\text{sen}(x)\,dx\right)\right)$$

$$\int \operatorname{sen} x \cdot e^x = -e^x \cos x - \left(-e^x \operatorname{sen} x + \int e^x \operatorname{sen} x\right)$$

$$\int \operatorname{sen} x\, e^x + \left(-e^x \operatorname{sen} x + \int \operatorname{sen} x\, e^x\right) = -e^x \cos x$$

$$2 \cdot \int \operatorname{sen} x\, e^x = -e^x \cos x + e^x \operatorname{sen} x$$

$$\int \operatorname{sen} x\, e^x = \frac{-e^x \cos x}{2} + \frac{e^x \operatorname{sen} x}{2} + C$$

Volviendo a la EDO.

$$y = \frac{1}{e^x} \cdot \left(\frac{-e^x \cos x}{2} + \frac{e^x \operatorname{sen} x}{2} + C\right)$$

I: $\boxed{y = -\frac{\cos x}{2} + \frac{\operatorname{sen} x}{2} + \frac{C}{e^x}}$

II. $$\underbrace{y' + y}_{p(x)=1} = \underbrace{3\cos(2x)}_{=q(x)}$$

$$\mu(x) = e^x$$

$$y = e^{-x} \int 3\cos(2x) \cdot e^x dx$$

CA $\int 3\cos(2x)\, e^x\, dx =$ otra vez hay que hacer partes 2 veces y queda $= 3\left(\frac{2e^x \operatorname{sen}(2x)}{5} + \frac{e^x \cos(2x)}{5}\right) +$

II $\boxed{y = \frac{6\operatorname{sen}(2x)}{5} + \frac{3\cos(2x)}{5} + \frac{C}{e^x}}$

b. $y' + \underset{1}{y} = \underbrace{\operatorname{sen}(x) + 3\cos(2x)}$

$p(x) = 1 \qquad q(x) = \uparrow$

$\mu(x) = e^{x}$

$y = e^{-x} \int e^{x} (\operatorname{sen} x + 3\cos 2x)$

$= e^{-x} \left(\int e^{x} \operatorname{sen} x + \int e^{x} 3\cos(2x) \right)$

$= \underbrace{e^{-x} \int e^{x} \operatorname{sen} x}_{\text{item I}} + \underbrace{e^{-x} \int e^{x} 3\cos(2x)}_{\text{item II}}$

$\Rightarrow \boxed{y = -\frac{\cos x}{2} + \frac{\operatorname{sen} x}{2} + \frac{6 \operatorname{sen}(2x)}{5} + \frac{3\cos(2x)}{5} + \frac{}{e^{x}}}$

14. Ecuación no homogénea

- $y' + a(x)\, y = b(x)$ $\qquad \overline{a, b} : \mathbb{R} \to \mathbb{R}$ continuas (p-periódicas) con periodo $p > 0$ $b \neq 0$
- $\phi(0) = \phi(p)$ $\qquad a(x+p) = a(x) \ \forall x$

a. $\phi(x)$ es sol. sea $\psi(x) = \phi(x+p)$.

$\psi'(x) + \underbrace{a(x+p)}_{a(x)} \psi(x) = \underbrace{b(x+p)}_{b(x)}$

Sabemos que ϕ es sol $\Rightarrow$ $\psi(x) = \phi(x+p)$ es sol

$\phi'(x+p) + a(x+p)\, \phi(x+p) = b(x+p)$

$= \psi(x) + a(x) \psi(x) = b(x)$

Como valen lo mismo en la condición inicial y ambas son solución a la ecuación $\Rightarrow$ por unicidad $\psi = \phi$ $\quad (\psi(0) = \phi(p) = \phi(0))$

b. $y' + \underbrace{3}_{p(x)=3} y = \underbrace{\cos(x)}_{g(x)}$

$$\mu(x) = e^{\int 3\,dx} = e^{3x}$$

$$y = e^{-3x} \int e^{3x} \cdot \cos(x)$$

$$= e^{-3x}\left(\frac{e^{3x}\operatorname{sen}(x)}{10} + \frac{3e^{3x}\cos(x)}{10} + c\right)$$

$$\boxed{y = \frac{\operatorname{sen}(x)}{10} + \frac{3\cos(x)}{10} + \frac{c}{e^{3x}}}$$

$\rightarrow$ siempre tiene que quedar algo dependiendo de c.

—o—

$$y(0) = 0 + \frac{3}{10} + \frac{c}{e^0}$$

$$y(2\pi) = \frac{3}{10} + \frac{c}{3^{6\pi}}$$

$\Rightarrow$ Es 2π-periódica $\Leftrightarrow$ $c = 0$

—o—

b. $\underbrace{y' + \cos(x)}_{p(x)} \cdot y = \underbrace{\operatorname{sen}(2x)}_{q(x)}$

$\mu(x) = e^{\int P(x)\,dx} = e^{\int \cos(x)\,dx} = e^{\operatorname{sen} x}$ factor integrante

$\rightarrow \quad y = e^{-\operatorname{sen} x} \int \operatorname{sen} 2x \cdot e^{\operatorname{sen} x}\, dx$

CA $\int \operatorname{sen} 2x \cdot e^{\operatorname{sen} x}\, dx$ $\qquad$ $\operatorname{sen}(2x) = 2\cos(x)\operatorname{sen}(x)$

$\int 2\cos x \cdot \operatorname{sen} x\, e^{\operatorname{sen} x}\, dx$

$u = \operatorname{sen} x$
$du = \cos x \, dx$

$$\int 2 \cancel{\cos x} \cdot u \cdot e^u \frac{du}{\cancel{\cos x}} \underset{\downarrow}{=} 2\left[e^u u - \int e^u du\right]$$

PARTES

$$= 2(e^u \cdot u - e^u)$$

$$\Rightarrow \int 2ue^u = 2(e^u u - e^u)$$

$$\Rightarrow \int \operatorname{sen} 2x \cdot e^{\operatorname{sen} x} = 2(e^{\operatorname{sen} x} \operatorname{sen} x - e^{\operatorname{sen} x})$$

$$= 2e^{\operatorname{sen} x}(\operatorname{sen} x - 1) + C$$

$$y = e^{-\operatorname{sen} x}\left(2e^{\operatorname{sen} x}(\operatorname{sen} x - 1) + c\right)$$

$$\boxed{y = 2(\operatorname{sen} x - 1) + \frac{c}{e^{\operatorname{sen} x}}}$$

$y(0) = -2 + c$

$y(2\pi) = -2 + c \to$ vale $\forall\ c$

y es $2\pi-$periódica $(\forall\ c)$

15. Proporcional $\Rightarrow$ cuando divido es cte.

Temperatura depende del tiempo

$T(t) \rightsquigarrow$ tasa de cambio $T'(t) = \frac{dT}{dt}$

Dato $\quad T'(t) = T(t) - T_{AMBIENTE}$

$$\frac{T'(t)}{T(t) - T_A} = K$$

$T(t_0) = 110°$ $\quad T_A = 10°C = cte$

$T(t_0 + 1) = 60°$

$T(t_0 + ?) = 30°$

Elijo $t_0 = 0h$

$$T'(t) = K(T(t) - T_A) = K \cdot T - 10 \cdot K$$

$$T' - \underbrace{K}_{p(x)} \cdot T = \underbrace{-10\,K}_{q(x)}$$ Es lineal

Factor integrante $\quad \mu(t) = e^{\int K\,dt} = e^{Kt}$

$$\Rightarrow \quad T = e^{-Kt} \int e^{Kt} \cdot (-10K)\,dt$$

$$\left[CA \int -10K \cdot e^{Kt}\,dt = -10\cancel{K} \cdot \frac{e^{Kt}}{\cancel{K}} + C \right]$$

$$T = e^{-Kt}(-10 \cdot e^{Kt} + C)$$

$$\boxed{T = -10 + C \cdot e^{-Kt}}$$

Uso las condiciones iniciales

$$T(0h) = 110^\circ = -10 + C\, e^{-k\cdot 0}$$

$$\boxed{120^\circ = C \cdot 1}$$

$$T(1h) = 60^\circ = -10 + 120 \cdot e^{-k \cdot 1}$$

$$\frac{70}{120} = e^{-k}$$

$$\frac{120}{70} = e^{k}$$

$$\ln\left(\frac{120}{70}\right) = k \cong 0{,}538$$

$$T(t) = 30 = -10 + 120\, e^{-\ln\left(\frac{120}{70}\right)\cdot t}$$

$$\frac{4\cancel{0}}{12\cancel{0}} = e^{\ln\left(\left(\frac{120}{70}\right)^{-t}\right)}$$

$$\frac{1}{3} = \left(\frac{120}{70}\right)^{-t}$$

$$\ln\left(\frac{1}{3}\right) = -t \cdot \ln\left(\frac{120}{70}\right)$$

$$\frac{\ln\left(\frac{1}{3}\right)}{\ln\left(\frac{120}{70}\right)} = -t$$

$$-2{,}038h \approx -t$$

$$\Rightarrow \boxed{t \cong 2{,}04\ h}$$ El cuerpo caliente se enfrió a 30 °C.

16. C^{14} $t_{1/2} = 5600$ años

Tasa de cambio $y' = \frac{dy}{dx}$ y = Cantidad de carbono ^{14} a tiempo x.

$$y' = -ky$$

$$\frac{dy}{dx} = -ky$$

$$\int \frac{1}{y}\, dy = -k\, dx$$

$$\ln|y| = -k(x + c)$$

$$y = ce^{-kx}$$

Llamo $c = A$ = Cantidad inicial de carbono 14.

$$\boxed{y = Ae^{-k\cdot x}}$$

♡ decaimiento exponencial

Uso condiciones iniciales

$t = 0$ $y(0) = 0{,}40 = Ae^{0}$

$t = ?$ $y(t) = 2 = e^{-k\cdot t}$

$t = 5600$ $y(5600) = \frac{A}{2} = Ae^{-k\cdot 5600}$

$$0{,}5 = e^{-k\cdot 5600}$$

$$\ln(0{,}5) = -k\cdot 5600$$

$$-\frac{\ln(0{,}5)}{5600} = k > 0 \checkmark$$

100% — 40%

$\boxed{5\ \%}$ — 2%

$\Rightarrow$ a tiempo t hay $0{,}05A$ de carbono 14 (5% de lo que habrá al principio)

$$y(t) = 0{,}05\cancel{A} = \cancel{A} \cdot e^{+\frac{\ln(0{,}5)}{5600} \cdot t}$$

$$\ln(0{,}05) = \frac{\ln(0{,}5)}{5600} \cdot t$$

$$5600 \frac{\ln(0{,}05)}{\ln(0{,}5)} = t$$

$$\boxed{24.202 \text{ años} = t}$$ para que halla un 2% de C^{14} en la roca sedimentaria.

17. Resistencia del aire $\rightarrow$ fuerza retardadora

$$F = -kv$$

El movimiento se describe como

$$\frac{dv}{dt} = g - c \cdot v \qquad c = \frac{k}{m} = \text{cte} > 0$$

$v = 0$ en $t = 0$

$$v' + \underbrace{c}_{p(x)} \cdot v = \underbrace{g}_{g(x)} \qquad \text{Lineal}$$

Factor integrante $\mu(t) = e^{\int c\,dt} = e^{c \cdot t}$

$$\rightarrow v = e^{-ct} \int e^{ct} \cdot \underset{\text{cte}}{\underset{\parallel}{g}}\, dt = e^{-ct}\left(\frac{e^{ct}}{c} \cdot g + \underset{\text{cte de integración}}{\underset{\downarrow}{a}}\right)$$

$$v = \frac{g}{c} + a \cdot e^{-ct}$$

$$\lim_{t\to\infty} v(t) = \lim_{t\to\infty} \frac{g}{c} + \underbrace{a \cdot e^{-ct}}_{\to 0} \longrightarrow \frac{g}{c}$$ velocidad terminal

---o---

$$\frac{dv}{dt} = g - cv^2 < 0 \qquad v(0)=0$$

$$\int \frac{1}{g - cv^2}\, dv = \int dt$$

$$\int \frac{1}{\underbrace{cv^2 - g}_{>0}}\, dv = -\int dt = \underline{-t + k}$$

CA $$\frac{1}{cv^2 - g} = \left(\frac{A}{v - \sqrt{\frac{g}{c}}} + \frac{B}{v + \sqrt{\frac{g}{c}}}\right)\frac{1}{c}$$

Llamo $\boxed{\alpha = \sqrt{\frac{g}{c}}}$

$$= \frac{1}{c}\left(\frac{A(v+\alpha) + B(v-\alpha)}{v^2 - \alpha^2}\right)$$

$$\Rightarrow \frac{A(v+\alpha) + B(v-\alpha)}{cv^2 - g}$$

$$1 = A(v+\alpha) + B(v-\alpha)$$

$$v = \alpha \to 1 = 2\alpha A$$

$$v = -\alpha \to 1 = -2\alpha B$$

$$\therefore\ -t+k = \frac{1}{2\alpha}\int \frac{1}{v-\alpha} - \frac{1}{v+\alpha}\, d\alpha$$

$$= \frac{1}{2\alpha} \ln\left(\frac{v-\alpha}{v+\alpha}\right) = -t+k$$

$$\ln\left(\frac{v-\alpha}{v+\alpha}\right) = 2\alpha(-t+k) = -2\alpha t - 2\alpha k$$

$$\frac{v-\alpha}{v+\alpha} = e^{-2\alpha t - 2\alpha k} = e^{-2\alpha t} \cdot \underbrace{e^{-2\alpha k}}_{cte\ K'}$$

Cuando $\underset{t=0}{,}$ $v(0)=0 \Rightarrow -1 = e^{0} \cdot K'$

$$v-\alpha = -1\cdot(v+\alpha)\, e^{-2\alpha t}$$

$$v-\alpha = (-v-\alpha)\, e^{-2\alpha t}$$

$$v-\alpha = -v e^{-2\alpha t} - \alpha e^{-2\alpha t}$$

$$v + v\cdot e^{-2\alpha t} = \alpha - \alpha e^{-2\alpha t}$$

$$v(1+e^{-2\alpha t}) = \alpha(1-e^{-2\alpha t})$$

$$\boxed{v(t) = \alpha\ \frac{1-e^{-2\alpha t}}{1+e^{-2\alpha t}}}$$

Cuando $\lim\limits_{t\to\infty} v(t) \to \alpha = \sqrt{\frac{g}{c}}$

18. Ecuación de Bernoulli

$$y' + P(x)\, y = Q(x)\, y^n$$

$$z = y^{n-1}$$