PRÁCTICA 6

① a. $\begin{cases} x_1' = -x_2 \\ x_2' = 2x_1 + 3x_2 \end{cases}$

$Ax = x'$

$A = \begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}$

1. Busco autovalores: $p(\lambda) = \det(\lambda I - A) = 0$

$p(\lambda) = \det \begin{pmatrix} \lambda - 0 & +1 \\ -2 & \lambda - 3 \end{pmatrix} = \lambda(\lambda - 3) + 2 = 0$

$= \lambda^2 - 3\lambda + 2 = 0$

$\hookrightarrow \boxed{\lambda_1 = 2}$

$\hookrightarrow \boxed{\lambda_2 = 1}$

2. Busco autovectores

$\lambda_1 = 2 \qquad (2 \cdot I - A)v_1 = 0$

$$\begin{pmatrix} 2 & 1 \\ -2 & 2-3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 2 & 1 & 0 \\ -2 & -1 & 0 \end{array}\right)$$

$$F_2 + F_1 \to F_2 \left(\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$2x_1 + x_2 = 0 \Rightarrow -2x_1 = x_2$

Entonces el vector se escribe como $(x_1, -2x_1)$

$= x_1(1, -2) \Rightarrow \langle (1, -2) \rangle$

$\hookrightarrow$ una recta

$\boxed{v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}}$

$\lambda = 1 \quad (I - A)\, v_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ -2 & -2 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad x_1 + x_2 = 0$$

$\Rightarrow x_1 = -x_2$

$$(x_1, -x_1) = x_1 (1, -1) = \langle (1, -1) \rangle$$

$$\boxed{v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

$$X_1(t) = e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$X_2(t) = e^{t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

sol general $\boxed{X(t) = C_1 e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$

$$\lim_{t \to +\infty} e^{t} \to +\infty$$

Si quiero que la solución tienda a cero cuando $t \to +\infty$ entonces elijo $C_1 = C_2 = 0$.

$$\lim_{t \to -\infty} e^{t} \to 0$$

Si quiero que la sol general tienda a 0 cuando $t \to -\infty$ entonces tomo cualquier $C_1, C_2 \in \mathbb{R}$.

b. $\begin{cases} x_1' = -8x_1 - 5x_2 \\ x_2' = 10x_1 + 7x_2 \end{cases}$

$$x' = \begin{pmatrix} -8 & -5 \\ 10 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

• Autovalores $P(\lambda) = \det(\lambda I - A) = 0$

$$P(\lambda) = \det \begin{pmatrix} \lambda+8 & +5 \\ -10 & \lambda-7 \end{pmatrix} = (\lambda+8)(\lambda-7) + 50 = 0$$

$$\lambda^2 + 8\lambda - 56 - 7\lambda + 50 = 0$$

$$\lambda^2 + \lambda - 6 = 0$$

$\hookrightarrow \lambda_1 = +2$

$\hookrightarrow \lambda_2 = -3$

CA $\dfrac{-1 \pm \sqrt{1 - 4 \cdot (-6) \cdot 1}}{2 \cdot 1}$

$= \dfrac{-1 \pm \sqrt{25}}{2} = \dfrac{-1 \pm 5}{2}$

• Autovectores

$\lambda_1 = 2$ $\left(\begin{array}{cc|c} 10 & -5 & 0 \\ 10 & -5 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 10 & -5 & 0 \\ 0 & -0 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$$2x_1 - x_2 = 0 \Rightarrow 2x_1 = x_2$$

$$(x_1, 2x_1) = x_1(1, 2) = \langle (1,2) \rangle$$

$$X_1(t) = e^{2t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\lambda_2 = -3$ $\left(\begin{array}{cc|c} 5 & 5 & 0 \\ -10 & -10 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ -10 & -10 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$$x_1 = x_2 \Rightarrow (x_1, x_1) = x_1(1,1) = \langle (1,1) \rangle$$

$$X_2(t) = e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Sol general $\boxed{X(t) = C_1 e^{2t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + C_2 e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

Cuando $t \to \infty$ $\quad x_1(t)$ diverge y $x_2(t) \to 0$ ✓

$\Rightarrow$ tomo $C_1 = 0$, $C_2 \in \mathbb{R}$ para que tienda a cero

Cuando $t \to -\infty$ $\quad x_1(t) \to 0$ ✓ y $x_2(t) \to +\infty$

$\Rightarrow$ tomo $C_1 \in \mathbb{R}$, $C_2 = 0$.

c. $\begin{cases} x_1' = -4x_1 + 3x_2 \\ x_2' = -2x_1 + x_2 \end{cases}$

$$X' = \begin{pmatrix} -4 & 3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

• Autovalores

$$P(\lambda) = \det\begin{pmatrix} \lambda+4 & -3 \\ 2 & \lambda-1 \end{pmatrix} = (\lambda+4)(\lambda-1)+6 = 0$$

$$= \lambda^2 + 4\lambda - \lambda - 4 + 6$$

$$= \lambda^2 + 3\lambda + 2 = 0$$

$\hookrightarrow \lambda_1 = -2$

$\hookrightarrow \lambda_2 = -1$

CA $\dfrac{-3 \pm \sqrt{9-4.1.2}}{2}$

$= \dfrac{-3 \pm 1}{2}$

• Autovectores

$\lambda_1 = -2 \quad \left(\begin{array}{cc|c} 2 & -3 & 0 \\ 2 & -3 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 2 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$2x_1 - 3x_2 = 0$

$2x_1 = 3x_2$

$x_1 = \frac{3}{2} x_2$

$\left(\frac{3}{2}x_2, x_2\right) \Rightarrow \left\langle \left(\frac{3}{2}, 1\right) \right\rangle$

$X_1(t) = e^{-2t} \begin{pmatrix} 3/2 \\ 1 \end{pmatrix}$

$$\lambda = -1 \quad \left(\begin{array}{cc|c} +3 & -3 & 0 \\ 2 & -2 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 3 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$x_1 = x_2 \Rightarrow \langle (1,1) \rangle$$

$$X_2 = e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

sol general: $\boxed{X(t) = C_1 e^{-2t} \begin{pmatrix} 3/2 \\ 1 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

Cuando $t \to +\infty$ $\quad X(t) \to 0$ ✓ $\quad C_1, C_2 \in \mathbb{R}$

Cuando $t \to -\infty$ $\quad X(t) \to \infty$ tomo $C_1 = C_2 = 0$ para que tienda a cero.

d. $\begin{cases} x_1' = -x_1 + 3x_2 - 3x_3 \\ x_2' = -2x_1 + x_2 \\ x_3' = -2x_1 + 3x_2 - 2x_3 \end{cases}$

$$X' = \begin{pmatrix} -1 & 3 & -3 \\ -2 & 1 & 0 \\ -2 & 3 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

1.1. Autovalores

$$P(\lambda) = \det \begin{pmatrix} \lambda+1 & -3 & 3 \\ 2 & \lambda-1 & 0 \\ 2 & -3 & \lambda+2 \end{pmatrix}$$

$$= (-1)^{1+3} \cdot 3 \cdot \det \begin{pmatrix} 2 & \lambda-1 \\ 2 & -3 \end{pmatrix} + 0 + (-1)^{3+3} \cdot (\lambda+2) \det \begin{pmatrix} \lambda+1 & -3 \\ 2 & \lambda-1 \end{pmatrix}$$

$$= 3\left[(-2\cdot 3) - (2(\lambda-1))\right] + (\lambda+2)\left[(\lambda+1)(\lambda-1) + 6\right]$$

$$= 3(-6 - 2\lambda + 2) + (\lambda+2)(\lambda+1)(\lambda-1) + (\lambda+2)\cdot 6$$

$$= -18 - 6\lambda + 6 \quad + (\lambda+2)(\lambda+1)(\lambda-1) + (\lambda+2)\cdot 6$$

$$= \cancel{-12} \cancel{-6\lambda} \quad + \quad " \quad + \cancel{6\lambda} \cancel{+12}$$

$$= (\lambda+2)(\lambda+1)(\lambda-1) = 0$$

$\hookrightarrow \lambda_1 = 1$

$\hookrightarrow \lambda_2 = -1$

$\hookrightarrow \lambda_3 = -2$

- Autovalores

$\lambda = 1$

$$\left(\begin{array}{ccc|c} 2 & -3 & 3 & 0 \\ 2 & -2 & 0 & 0 \\ 2 & -3 & 3 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 2 & -3 & 3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$F_3 - F_1 \rightarrow F_3$

$F_2 - F_1 \rightarrow F_2$

- $X_2 - 3X_3 = 0$

$$\underline{X_2 = 3X_3}$$

- $2X_1 - 3X_2 + 3X_3 = 0$

$2X_1 - 9X_3 + 3X_3 = 0$

$2X_1 - 6X_3 = 0$

$$\underline{X_1 = 3X_3}$$

$(3x_3, 3x_3, x_3) \to v_1 = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$

$\langle (3,3,1) \rangle$

- $\lambda = -1$ $\left(\begin{array}{ccc|c} 0 & -3 & 3 & 0 \\ 2 & -2 & 0 & 0 \\ 2 & -3 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & -3 & 1 & 0 \end{array}\right) \begin{array}{c} \frac{F_1}{3} \\ \frac{F_2}{2} \\ \\ \end{array}$

$\to F_3 - 2 \cdot F_2 \to F_3 \left(\begin{array}{ccc|c} 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$-x_2 + x_3 = 0 \Rightarrow \boxed{x_2 = x_3}$

$x_1 - x_2 = 0 \Rightarrow \boxed{x_1 = x_2}$

$(x_2, x_2, x_2) \to \langle (1,1,1) \rangle \quad v_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

- $\lambda = -2$ $\left(\begin{array}{ccc|c} -1 & -3 & 3 & 0 \\ 2 & -3 & 0 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} -1 & -3 & 3 & 0 \\ 2 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$2x_1 - 3x_2 = 0 \Rightarrow x_1 = \frac{3}{2} x_2$

$-x_1 - 3 \cdot x_2 + 3x_3 = 0$

$-\frac{3}{2} x_2 - 3x_2 + 3x_3 = 0$

$-\frac{1}{2} x_2 - x_2 + x_3 = 0$

$-\frac{3}{2} x_2 + x_3 = 0$

$\boxed{x_3 = \frac{3}{2} x_2}$

$\left(\frac{3}{2} x_2, x_2, \frac{3}{2} x_2\right) = x_2 \left(\frac{3}{2}, 1, \frac{3}{2}\right) \to \langle \left(\frac{3}{2}, 1, \frac{3}{2}\right) \rangle$

$v_3 = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$

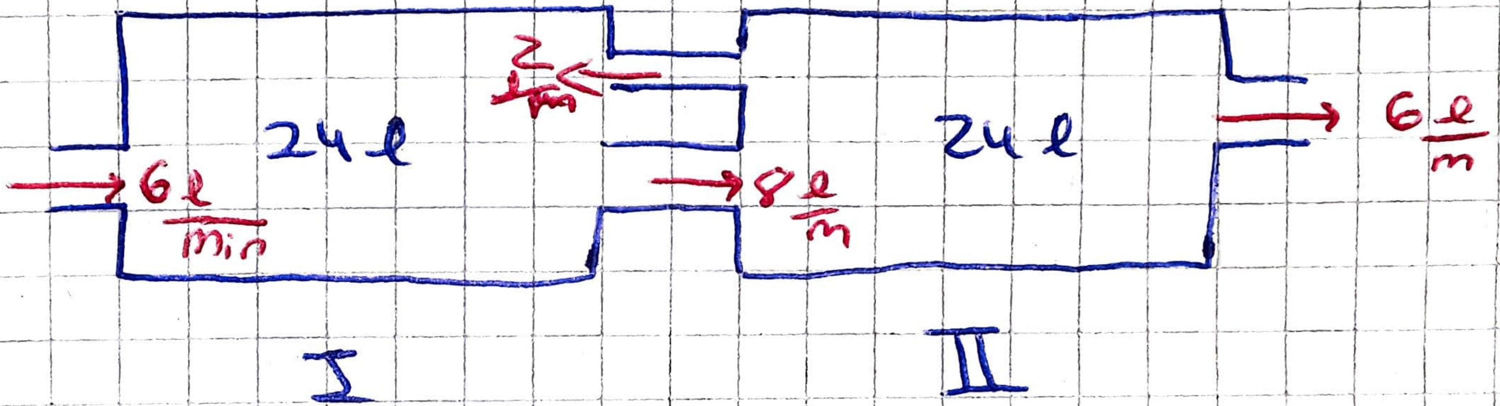
Sol general $= X(t) = e^{t} c_1 \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} + e^{-t} c_2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + e^{-2t} c_3 \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$

Los 3 autovectores son Li ✓ Forman una base de sol. del sist.

Para que $X(t) \to 0$ cuando $t \to +\infty$ tomo $c_1 = 0$ $c_2, c_3 \in \mathbb{R}$.

$\lim_{t \to -\infty} X(t) \to 0$ si tomo $c_1 \in \mathbb{R}$; $c_2, c_3 = 0$

2.



Inicial $\begin{cases} x_0 \text{ Kg de sal en I} \\ y_0 \text{ Kg de sal en II.} \end{cases}$

$x_i(t)$ = #sal en tanque i a tiempo t en un período corto de tiempo Δt.

La cantidad de agua en cada tanque es constante porque entra y sale la misma cantidad de agua.

Δx_1 = # sal que entra - # sal que sale.

en el tanque I

La mezcla es homogénea $\rightsquigarrow$ cdad de sal $= \frac{\text{Kg de sal}}{\text{volumen}}$

$$\Delta x_1 \cong \underbrace{\frac{x_2}{24\ell}}_{\text{cdad de sal}} \cdot \underbrace{2\frac{\ell}{m}}_{\#\text{agua pura}} \cdot \Delta t - \frac{x_1}{24\ell} \cdot 8\frac{\ell}{m} \cdot \Delta t$$

Al tanque I le entra sal del tanque 2 porque del exterior solo entra agua pura

La concentración de sal en el tanque 2 no es cte en el tiempo.

Δx_2 = # sal que entra - # sal que sale

$$\approx \frac{x_1}{24\ell} \cdot 8\frac{\ell}{m}\Delta t - \frac{x_2}{24\ell} \cdot \left(\frac{2\ell + 6\ell}{min}\right) \Delta t$$

El tanque 2 pierde sal hacia el tanque I y para el exterior.

$$\frac{\Delta x_1}{\Delta t} = \frac{1}{12} x_2 - \frac{1}{3} x_1$$

$$\Delta t \to 0 \qquad x_1' = \frac{1}{12} x_2 - \frac{1}{3} x_1$$

$$\frac{\Delta x_2}{\Delta t} = x_1 \frac{1}{3} - \frac{1}{3} x_2$$

$\Delta t \to 0$

$$x_2' = \frac{1}{3} x_1 - \frac{1}{3} x_2$$

$$\begin{cases} x_1' = -\frac{1}{3} x_1 + \frac{1}{12} x_2 \\ x_2' = +\frac{1}{3} x_1 - \frac{1}{3} x_2 \end{cases}$$

$$x' = \begin{pmatrix} -1/3 & 1/12 \\ 1/3 & -1/3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

- Busco autovalores

$$P(\lambda) = \det \begin{pmatrix} \lambda + 1/3 & -1/12 \\ -1/3 & \lambda + 1/3 \end{pmatrix} = 0$$

$$= \left(\lambda + \frac{1}{3}\right)\left(\lambda + \frac{1}{3}\right) - \frac{1}{12} \cdot \frac{1}{3} = 0$$

$$= \lambda^2 + \frac{1}{3}\lambda + \frac{1}{3}\lambda + \frac{1}{9} - \frac{1}{36} = 0$$

$$= \lambda^2 + \frac{2}{3}\lambda + \frac{1}{12}$$

CA $\dfrac{-\frac{2}{3} \pm \sqrt{\frac{4}{9} - 4 \cdot (+1/12)}}{2} =$

$\hookrightarrow \lambda_1 = -\frac{1}{2}$

$\hookrightarrow \lambda_2 = -\frac{1}{6}$

• Autovec.

$\lambda = -\frac{1}{6}$ $\left(\begin{array}{cc|c} \frac{1}{6} & -\frac{1}{12} & 0 \\ -\frac{1}{3} & \frac{1}{6} & 0 \end{array}\right) \xrightarrow[\times -3]{\times 6} \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \\ \cancel{1} & \cancel{-\frac{1}{2}} & \cancel{0} \end{array}\right)$

$$x_1 - \frac{1}{2}x_2 = 0$$

$$x_1 = \frac{1}{2}x_2$$

$$\boxed{2x_1 = x_2}$$

$(x_1, 2x_1) \to \langle(1,2)\rangle$ $v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\lambda = -\frac{1}{2}$ $\begin{pmatrix} -\frac{1}{6} & -\frac{1}{12} \\ -\frac{1}{3} & -\frac{1}{6} \end{pmatrix} \xrightarrow[\times -3]{\times -6} \left(\begin{array}{cc|c} 1 & \frac{1}{2} & 0 \\ \cancel{1} & \cancel{\frac{1}{2}} & \cancel{0} \end{array}\right)$

$$x_1 + \frac{x_2}{2} = 0$$

$$x_1 = -\frac{1}{2}x_2$$

$$\boxed{-2x_1 = x_2}$$

$(x_1, -2x_1) \to \langle(1,-2)\rangle$ $v_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

sol. $\boxed{x(t) = e^{-\frac{1}{6}t} c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{-\frac{1}{2}t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}}$

v_1, v_2 Li ✓

$\lim_{t \to +\infty} x(t) \to 0 \quad \forall\, c_1, c_2 \in \mathbb{R}$

Resumen

3. A partir de la sol compleja $z = e^{\lambda t} \cdot v$, $z(t) \in \mathbb{C}^2$

λ = autovalor v = autovector de ese λ.

podemos extraer soluciones reales.

$\boxed{Re(z) = \frac{z + \bar{z}}{2}}$ $\boxed{Im(z) = \frac{z - \bar{z}}{2i}}$ → son vectores

Si $\lambda = \alpha + i\beta$, $v = u + iw$ con $\alpha, \beta, u, w \in \mathbb{R}$

$$z = e^{\alpha t} \underbrace{(\cos(\beta t) + i \sin(\beta t))}_{e^{\beta i \cdot t} \text{ (forma polar)}} \underbrace{(u + iw)}_{v}$$

$$= (e^{\alpha t} \cos(\beta t) + e^{\alpha t} i \sin(\beta t))(u + iw)$$

$$= e^{\alpha t} \cos(\beta t) u + e^{\alpha t} i \sin(\beta t) \cdot u +$$

$$i \cdot w e^{\alpha t} \cos(\beta t) + i \cdot w \, e^{\alpha t} \cdot i \sin(\beta t)$$

$i \cdot i = i^2 = -1$

$$= e^{\alpha t} \cos(\beta t) u - w e^{\alpha t} \sin(\beta t)$$
$$+ i e^{\alpha t} \sin(\beta t) \cdot u + i w e^{\alpha t} \cos(\beta t)$$

$$= \underbrace{e^{\alpha t}(\cos(\beta t) \cdot u - \sin(\beta t) \cdot w)}_{Re(z)} + \underbrace{i e^{\alpha t}(\sin(\beta t) \cdot u + \cos(\beta t) w)}_{Im(z)}$$

sol general: $X(t) = C_1 Re(z) + C_2 Im(z)$

$C_1, C_2 \in \mathbb{R}$

$$\boxed{\begin{array}{c} \lambda = \alpha + i\beta \in \mathbb{C} \\ e^{\lambda} = e^{\alpha} (\cos \beta + i \sin \beta) \end{array}}$$

a. $\begin{cases} x_1' = x_1 - x_2 \\ x_2' = x_1 + x_2 \end{cases}$

$$X' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

- Autovalores

$$P(\lambda) = \det \begin{pmatrix} \lambda - 1 & 1 \\ -1 & \lambda - 1 \end{pmatrix}$$

$$= (\lambda - 1)^2 + 1 = 0$$

$$= \lambda^2 - 2\lambda + 1 + 1 = 0$$

$$= \lambda^2 - 2\lambda + 2 = 0$$

CA $\dfrac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2} = \dfrac{2 \pm \sqrt{-4}}{2} = \dfrac{2 \pm \sqrt{-1}\,\sqrt{4}}{2} = \dfrac{2 \pm i \cdot 2}{2}$

$$= 1 \pm i \longrightarrow \begin{cases} \lambda_1 = 1 + i \\ \lambda_2 = 1 - i \end{cases}$$

- Autovectores

$\lambda_1 = 1 + i \quad \begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\left(\begin{array}{cc|c} i & 1 & 0 \\ -1 & i & 0 \end{array}\right) \xrightarrow[\times i]{} \left(\begin{array}{cc|c} i & 1 & 0 \\ -i & -1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} i & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$i x_1 + x_2 = 0$

$\boxed{x_2 = -i x_1}$

$(x_1, -i x_1) = x_1 (1, -i) \rightsquigarrow \langle (1, -i) \rangle \qquad v_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$

$\lambda = 1 - i \quad \left(\begin{array}{cc|c} -i & 1 & 0 \\ -1 & -i & 0 \end{array} \right) \xrightarrow[F_2 \cdot i - F \to F_2]{} \left(\begin{array}{cc|c} -i & 1 & 0 \\ 0 & 0 & 0 \end{array} \right)$

$-i\, x_1 + x_2 = 0$

$\boxed{x_2 = i\, x_1}$

$x_1 (1, i) \rightsquigarrow \langle (1, i) \rangle \qquad v_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$

v_1, v_2 son Li

Sol: $x_1(t) = e^{(1+i)t} \begin{pmatrix} 1 \\ -i \end{pmatrix}$

$= e^{1t} \cdot e^{it} \begin{pmatrix} 1 \\ -i \end{pmatrix}$

$= e^{t} (\cos(t) + i\, \operatorname{sen}(t)) \begin{pmatrix} 1 \\ -i \end{pmatrix}$

$= \begin{pmatrix} e^t \cos(t) + e^t\, i\, \operatorname{sen}(t) \\ -i\, e^t \cos(t) + \operatorname{sen}(t) \cdot e^t \end{pmatrix}$

$= e^t \begin{pmatrix} \cos(t) \\ \operatorname{sen}(t) \end{pmatrix} + i e^t \begin{pmatrix} \operatorname{sen}(t) \\ -\cos(t) \end{pmatrix}$

Sol general $= \boxed{X(t) = C_1 \cdot e^t \begin{pmatrix} \cos t \\ \operatorname{sen} t \end{pmatrix} + C_2\, e^t \begin{pmatrix} \operatorname{sen} t \\ -\cos t \end{pmatrix}}$

$X_2(t) = e^{(1-i)t} \begin{pmatrix} 1 \\ i \end{pmatrix} = e^t (\cos(-t) + i\, \operatorname{sen}(-t)) \begin{pmatrix} 1 \\ i \end{pmatrix}$

$= e^t (\cos(t) - i\, \operatorname{sen} t) \begin{pmatrix} 1 \\ i \end{pmatrix} = e^t \begin{pmatrix} \cos t - i\, \operatorname{sen} t \\ \operatorname{sen} t + i \cos t \end{pmatrix}$

$= e^t \begin{pmatrix} \cos t \\ \operatorname{sen} t \end{pmatrix} + i\, e^t \begin{pmatrix} \operatorname{sen} t \\ -\cos t \end{pmatrix}$ $\rightsquigarrow$ da lo mismo. ✓

3.b $\begin{cases} x_1' = 2x_1 - x_2 \\ x_2' = 4x_1 + 2x_2 \end{cases}$

$X' = A\,X$

$A = \begin{pmatrix} 2 & -1 \\ 4 & 2 \end{pmatrix}$

· Autovalores

$P(\lambda) \overset{\det}{=} \begin{pmatrix} \lambda-2 & +1 \\ -4 & \lambda-2 \end{pmatrix} = (\lambda-2)^2 + 4 = 0$

$= \lambda^2 - 2\lambda\cdot 2 + 4 + 4 = 0$

$= \lambda^2 - 4\lambda + 8 = 0$

CA $\quad \dfrac{4 \pm \sqrt{16-32}}{2} = \dfrac{4 \pm \sqrt{16}\sqrt{-1}}{2} = \dfrac{4 \pm 4\,i}{2} = 2 \pm 2i$

$\lambda_1 = 2 + 2i$

$\lambda_2 = 2 - i\cdot 2$

• Autovec.

$\lambda = 2+2i \quad \begin{pmatrix} 2i & 1 \\ -4 & 2i \end{pmatrix} \xrightarrow[\times i]{} \begin{pmatrix} 2i & 1 \\ -4i & -2 \end{pmatrix} \xrightarrow[\times -\frac{1}{2}]{} \begin{pmatrix} 2i & 1 \\ \cancel{2i} & \cancel{1} \end{pmatrix}$

$2i\, x_1 + x_2 = 0$

$\underline{|\, x_2 = -2i\, x_1 \,|}$

$v_1 = \begin{pmatrix} 1 \\ -2i \end{pmatrix}$

$\lambda = 2-2i \quad \begin{pmatrix} -2i & 1 \\ -4 & -2i \end{pmatrix} \xrightarrow[\times \frac{1}{2} i]{} \begin{pmatrix} -2i & 1 \\ \cancel{-2i} & \cancel{1} \end{pmatrix}$

$-2x_1 + x_2 = 0 \Rightarrow v_2 = \begin{pmatrix} 1 \\ 2i \end{pmatrix}$

Sol: $x_1(t) = e^{(2+2i)t} \begin{pmatrix} 1 \\ -2i \end{pmatrix}$

$= e^{2t} \cdot e^{2it} \begin{pmatrix} 1 \\ -2i \end{pmatrix} = e^{2t} \left(\cos(2t) + i \,\text{sen}(2t) \right) \begin{pmatrix} 1 \\ -2i \end{pmatrix}$

$= e^{2t} \begin{pmatrix} \cos(2t) + i\,\text{sen}(2t) \\ 2\,\text{sen}(2t) - 2i \cos(2t) \end{pmatrix} = e^{2t} \begin{pmatrix} \cos(2t) \\ 2\,\text{sen}(2t) \end{pmatrix} + e^{2t} i \begin{pmatrix} \text{sen}(2t) \\ -2\cos(2t) \end{pmatrix}$

sol general $\boxed{X(t) = C_1 \cdot e^{2t} \begin{pmatrix} \cos(2t) \\ 2\,\text{sen}(2t) \end{pmatrix} + C_2\, e^{2t} \begin{pmatrix} \text{sen}(2t) \\ -2\cos(2t) \end{pmatrix}}$

C. $\begin{cases} x_1' = 2x_1 + x_2 \\ x_2' = 2x_2 \end{cases}$

$\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \quad P(\lambda) = \det \begin{pmatrix} \lambda - 2 & -1 \\ 0 & \lambda - 2 \end{pmatrix} = (\lambda - 2)^2 = 0$

$\hookrightarrow 2 = \lambda_1$

$\hookrightarrow 2 = \lambda_2$

• Autovalor doble

• Autovector $\lambda = 2 \quad \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \quad \underbrace{-x_2 = 0 \qquad x_1 = x_1}_{\text{Variables desacopladas.}}$

$v_1 = (x_1, 0) \rightsquigarrow \langle (1,0) \rangle$

$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$x_1(t) = C_1 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\boxed{x_2(t) = (v_1 \cdot t + v_2)\, e^{\lambda t}}$ cuando tengo un autovalor con multiplicidad 2.

$x_2(t) = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} t + v_2 \right) e^{2t}$

$X(t) = e^{\lambda t} (\xi_1 t + \xi_2)$

$A(x(t)) = e^{\lambda t} A(\xi_1 t + \xi_2)$

$Ax = e^{\lambda t} A \xi_1 \cdot t + e^{\lambda t} A \xi_2 = e^{\lambda t} (A\xi_1 t + A\xi_2)$

$x' = \lambda e^{\lambda t} (\xi_1 t + \xi_2) + e^{\lambda t} \cdot \xi_1$

$= \lambda e^{\lambda t} \xi_1 t + \lambda e^{\lambda t} \cdot \xi_2 + e^{\lambda t} \xi_1$

$= e^{\lambda t} (\lambda \xi_1 t + \lambda \xi_2 + \xi_1)$

$Ax = x' \iff \left| \begin{array}{l} A \xi_1 \cancel{t} = \lambda \xi_1 \cancel{t} \\ A \xi_2 = \lambda \xi_2 + \xi_1 \end{array} \right|$

En el ejercicio $\xi_2 = v_2$ $\quad \xi_1 = v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$A \cdot v_2 = 2 \cdot v_2 + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$A v_2 - 2 v_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$(A - 2I) v_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\left[\begin{pmatrix} 2 & -1 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \right] v_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad v_2 = \begin{pmatrix} x_A \\ x_B \end{pmatrix}$

$$\left(\begin{array}{cc|c} 0 & -1 & 1 \\ 4 & 0 & 0 \end{array}\right)$$ sistema compatible determinado
hay una única sol.

$$\left.\begin{array}{l} -X_B = 1 \\ \boxed{X_B = -1} \\ 4X_A = 0 \Rightarrow \boxed{X_A = 0} \end{array}\right\} \Rightarrow v_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\therefore \; x_2(t) = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} t + \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right) e^{2t}$$

sol general: $\boxed{x(t) = C_1 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} t + \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right)}$

d. $\begin{cases} x_1' = -5x_1 + 9x_2 \\ x_2' = -4x_1 + 7x_2 \end{cases}$

$$A = \begin{pmatrix} -5 & 9 \\ -4 & 7 \end{pmatrix}$$

Aval $P(\lambda) = \det \begin{pmatrix} \lambda + 5 & -9 \\ 4 & \lambda - 7 \end{pmatrix}$

$$= (\lambda + 5)(\lambda - 7) + 36 = 0$$
$$= \lambda^2 - 2\lambda + 1 = 0$$

CA $\dfrac{2 \pm \sqrt{4 - 4}}{2} \Rightarrow \boxed{\lambda_1 = \lambda_2 = 1}$

Avec $\lambda = 1$ $\begin{pmatrix} 6 & -9 \\ 4 & -6 \end{pmatrix} \xrightarrow[\times \frac{1}{4}]{\times \frac{1}{6}} \left(\begin{array}{cc|c} 1 & -3/2 & 0 \\ \cancel{1} & \cancel{-3/2} & \cancel{0} \end{array}\right)$

$$x_1 - \frac{3}{2} x_2 = 0 \Rightarrow x_1 = \frac{3}{2} x_2$$

$$v_1 = \left(\frac{3}{2} x_2, x_2\right) = x_2\left(\frac{3}{2}, 1\right) \sim \left\langle \left(\frac{3}{2}, 1\right) \right\rangle$$

Tomo $v_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$X_2(t) = e^{t}\left(\begin{pmatrix}3\\2\end{pmatrix}t + v_2\right)$$

$$(A-I)v_2 = \begin{pmatrix}3\\2\end{pmatrix}$$

$$\left(\begin{array}{cc|c} -6 & 9 & 3 \\ -4 & 6 & 2 \end{array}\right) \xrightarrow[x -\frac{1}{4}]{x -\frac{1}{6}} \left(\begin{array}{cc|c} 1 & -3/2 & -1/2 \\ \cancel{1} & \cancel{-3/2} & \cancel{-1/2} \end{array}\right)$$

$$X_1 - \frac{3}{2}X_2 = -\frac{1}{2}$$

Puedo tomar $X_1 = 1$ $X_2 = 1$ y cumple la ecuación

Sol general: $\boxed{X(t) = C_1 e^{t}\begin{pmatrix}3\\2\end{pmatrix} + C_2 e^{t}\left(\begin{pmatrix}3\\2\end{pmatrix}t + \begin{pmatrix}1\\1\end{pmatrix}\right)}$

4. a. $\begin{cases} X_1' = -X_2 + 2 \\ X_2' = 2X_1 + 3X_2 + t \end{cases}$

Método de variación de constantes: primero tengo que encontrar la sol general del sist. homogéneo asociado.

$S_0 = \begin{cases} X_1' = -X_2 \\ X_2' = 2X_1 + 3X_2 \end{cases}$ $\begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}$

Aval $P(\lambda) \overset{det}{=} \begin{pmatrix} \lambda & 1 \\ -2 & \lambda - 3 \end{pmatrix} = \lambda(\lambda - 3) + 2 = 0$

$= \lambda^2 - 3\lambda + 2$

CA $\dfrac{3 \pm \sqrt{9 - 4\cdot 2}}{2} = \dfrac{3 \pm 1}{2}$ $\to \boxed{\lambda_1 = 2}$ $\to \boxed{\lambda_2 = 1}$

Avec $\lambda_1 = 2$ $\left(\begin{array}{cc|c} 2 & 1 & 0 \\ \hline 2 & 1 & 0 \end{array}\right)$ $2x_1 + x_2 = 0$

$\boxed{x_2 = -2x_1} \Rightarrow v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\lambda_2 = 1$ $\left(\begin{array}{cc|c} 1 & 1 & 0 \\ \hline -2 & -2 & 0 \end{array}\right)$ $x_1 + x_2 = 0$

$\boxed{x_2 = -x_1} \Rightarrow v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

sol general del sist. homogéneo:

$$\boxed{x_H(t) = c_1 e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 e^{\overset{\lambda=1}{\cancel{2}t}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

↓
homogéneo

Ahora busco la solución particular

$$x_p(t) = \underset{\substack{\downarrow \\ \text{función que depende de } t}}{c_1(t)} e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2(t)\, e^{t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= c_1(t)\begin{pmatrix} e^{2t} \\ -2e^{2t} \end{pmatrix} + c_2(t)\begin{pmatrix} e^{t} \\ -e^{t} \end{pmatrix}$$

$$\left(\begin{array}{cc|c} e^{2t} & e^{t} & 2 \\ -2e^{2t} & -e^{t} & t \end{array}\right) \xrightarrow[F_2 + 2F_1 \to F_2]{} \left(\begin{array}{cc|c} e^{2t} & e^{t} & 2 \\ 0 & e^{t} & t + 4 \end{array}\right)$$

De acá saco c_1' y c_2'

$e^{t} c_2' = t + 4$

$c_2' = e^{-t}(t + 4)$

$\int c_2' = \int e^{-t}(t+4) = \int e^{-t}\cdot t + \int e^{-t}\cdot 4$

$c_2 = -te^{-t} + \int e^{-t}\,dt - 4e^{-t} = -te^{-t} - e^{-t} - 4e^{-t} = \boxed{-e^{-t}(t+5)}$

$+e^{-t}(t+5) + -e^{-t}\cdot 1 = e^{-t}(t+\overset{4}{\cancel{5}}-1)$

$$C_1' \cdot e^{2t} + C_2' e^{t} = 2$$

$$C_1' \cdot e^{2t} + \cancel{e^{-t}} (t+4) \cancel{e^{t}} = 2$$

$$C_1' e^{2t} = 2 - t - 4$$

$$\int C_1' = \int e^{-2t} \cdot (-t-2)$$

$$C_1 = e^{-2t} \frac{1}{4} (2t+5)$$

$$\therefore X_p = \frac{\cancel{e^{-2t}}}{4} (2t+5) \cancel{e^{2t}} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + (-\cancel{e^{-t}})(t+5) \cancel{e^{t}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} t + \frac{5}{4} \\ -t - \frac{5}{2} \end{pmatrix} + \begin{pmatrix} -(t+5) \\ t+5 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{2} t + \frac{5}{4} - t - 5 \\ -\cancel{t} - \frac{5}{2} + \cancel{t} + 5 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} t - \frac{15}{4} \\ \frac{5}{2} \end{pmatrix}$$

La solución general tiene la forma

$$X(t) = X_H + X_p$$

$$X(t) = C_1 e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1/2\, t - 15/4 \\ \frac{5}{2} \end{pmatrix}$$

4b. $\begin{cases} x_1' = 2x_1 - x_2 + e^{2t} \\ x_2' = 4x_1 + 2x_2 + 4 \end{cases}$

$X(t) = X_H + X_P$

Obs: el sistema homogéneo asociado es

$\begin{cases} x_1' = 2x_1 - x_2 \\ x_2' = 4x_1 + 2x_2 \end{cases} \rightarrow$ ítem 3b.

$$X_H(t) = C_1 e^{2t}\begin{pmatrix} \cos(2t) \\ 2\,\text{sen}(2t) \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} \text{sen}(2t) \\ -2\cos(2t) \end{pmatrix}$$

Busco sol particular

$$X_P(t) = C_1(t) e^{2t}\begin{pmatrix} \cos(2t) \\ 2\,\text{sen}(2t) \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} \text{sen}(2t) \\ -2\cos(2t) \end{pmatrix}$$

$$\begin{pmatrix} e^{2t}\cos(2t) & e^{2t}\,\text{sen}(2t) \\ e^{2t}\,2\,\text{sen}(2t) & -e^{2t}\cdot 2\cdot\cos(2t) \end{pmatrix}\begin{pmatrix} C_1'(t) \\ C_2'(t) \end{pmatrix} = \begin{pmatrix} e^{2t} \\ 4 \end{pmatrix}$$

$$\begin{cases} e^{2t}\cos(2t)\,C_1'(t) + e^{2t}\,\text{sen}(2t)\cdot C_2'(t) = e^{2t} & ① \\ 2e^{2t}\,\text{sen}(2t)\,C_1'(t) - 2\,e^{2t}\cos(2t)\,C_2'(t) = 4 & ② \end{cases}$$

Para despejar $C_1'(t)$ multiplico ① por $2\cos(2t)$ y a ② por $\text{sen}(2t)$, luego los sumo.

$$\begin{aligned} & 2e^{2t}\cdot\overbrace{\cos^2(2t)}^{=1}\,C_1'(t) + \cancel{2\cos(2t)\,e^{2t}\cdot\text{sen}(2t)\,C_2'(t)} = 2\cos(2t)e^{2t} \\ + \; & 2e^{2t}\,\underline{\text{sen}^2(2t)}\,C_1'(t) - \cancel{2\cos(2t)\,e^{2t}\,\text{sen}(2t)\,C_2'(t)} = 4\cdot\text{sen}(2t) \end{aligned}$$

$$4e^{2t}\,C_1'(t) = 2\cos(2t)\,e^{2t} + 4\,\text{sen}(2t)$$

$$C_1'(t) = \frac{1}{2}\cos(2t) + e^{-2t}\,\text{sen}(2t)$$

Para hallar $C_1(t)$ integro

$C_1 = \int \frac{1}{2} \cos(2t)\, dt + \int e^{-2t} \operatorname{sen}(2t)\, dt$

$$\left[CA\ \frac{1}{2}\int \cos(2t)\, dt \underset{\downarrow}{=} \frac{1}{2}\int \cos(u) \frac{du}{2} = \frac{1}{4} \operatorname{sen}(u) = \underline{\frac{1}{4} \operatorname{sen}(2t) + C} \right.$$

$2t = u$

$2dt = du$]

$$\left[CA \int e^{-2t} \operatorname{sen}(2t)\, dt \underset{\downarrow}{=} \frac{1}{2}\int e^{-u} \operatorname{sen}(u)\, du \underset{\downarrow}{=} \right.$$

$u = 2t$

$\frac{du}{2} = dt$

PARTES

$w = e^{-u}$

$dv = \operatorname{sen}(u)$

$$= \left(-e^{-u}\cos(u) - \int e^{-u}\cos(u)\, du\right)\frac{1}{2}$$

$$\underset{\downarrow}{=} \frac{1}{2}\left(-e^{-u}\cos(u) - \left(e^{-u}\operatorname{sen}(u) - \int -e^{-u}\operatorname{sen}(u)\, du\right)\right)$$

PARTES DE NUEVO

$w = e^{-u}$

$dv = \cos(u)$

$$\therefore \frac{1}{2}\int e^{-u}\operatorname{sen}(u)\,du = \frac{1}{2}\left(-e^{-u}\cos u\right) - \frac{1}{2}\left(e^{-u}\operatorname{sen}(u) + \int e^{-u}\operatorname{sen} u\, du\right)$$

$$\frac{1}{2}\int e^{-u}\operatorname{sen}(u)\,du = -\frac{1}{2}e^{-u}\cos u - \frac{1}{2}e^{-u}\operatorname{sen}(u) - \frac{1}{2}\int e^{-u}\operatorname{sen} u\, du$$

$$\int e^{-u}\operatorname{sen}(u)\,du = \frac{1}{2}e^{-u}\left(-\cos u - \operatorname{sen} u\right)$$

Entonces $\int e^{-2t}\operatorname{sen} 2t\, dt = \frac{1}{4}e^{-2t}\left(-\cos 2t - \operatorname{sen} 2t\right) + C$]

$$C_1 = \frac{1}{4}\operatorname{sen}(2t) + \frac{1}{4}e^{-2t}\left(-\operatorname{sen} 2t - \cos 2t\right) + C$$

— o —

Con un procedimiento similar obtengo C_2'.

Multiplico ① por $2\operatorname{sen}(2t)$ y a ② por $\cos(2t)$.

Luego, las resto.

$$2e^{2t}\cos(2t)\,\mathrm{sen}(2t)C_1'(t) + 2e^{2t}\,\mathrm{sen}^2(2t)\,C_2'(t) = 2e^{2t}\mathrm{sen}(2t)$$
$$-2e^{2t}\cos(2t)\,\mathrm{sen}(2t)C_1'(t) - 2e^{2t}\cos^2(2t)\,C_2'(t) = 4\cdot\cos(2t)$$
$$=1$$

$$4e^{2t}\,C_2'(t) = 2e^{2t}\,\mathrm{sen}(2t) - 4\cos(2t)$$

$$C_2'(t) = \frac{1}{2}\mathrm{sen}(2t) - e^{-2t}\cos(2t)$$

$$C_2 = \int C_2'(t) = \int \frac{1}{2}\mathrm{sen}(2t)dt + \int -e^{-2t}\cos(2t)\,dt$$

CA $\int \frac{1}{2}\mathrm{sen}(2t) = \frac{1}{4}\int \mathrm{sen}(u)\,du = -\frac{1}{4}\cos(2t) + C$

$u = 2t$
$du = 2dt$

CA $\int -e^{-2t}\cos(2t)\,dt = \frac{1}{2}e^{-2t}\mathrm{sen}(2t) - \int \frac{\mathrm{sen}(2t)}{2}(-2)e^{-2t}\,dt$

PARTES $u = e^{-2t}$ $du = -2e^{-2t}$

$dv = \cos(2t)$ $v = \frac{\mathrm{sen}(2t)}{2}$

$$= \frac{1}{2}e^{-2t}\mathrm{sen}(2t) + \underbrace{\int \mathrm{sen}(2t)e^{-2t}\,dt}_{\text{La hice antes}}$$

$$= \frac{1}{2}e^{-2t}\mathrm{sen}(2t) + \frac{1}{4}e^{-2t}\left(-\cos(2t) - \mathrm{sen}(2t)\right) + C$$

$$C_2 = -\frac{1}{4}\cos(2t) + \frac{1}{2}e^{-2t}\mathrm{sen}(2t) + \frac{1}{4}e^{-2t}\left(-\cos(2t) - \mathrm{sen}(2t)\right) + C$$

$$X_P = \left[\frac{1}{4}\,\text{sen}(2t) + \frac{1}{4}e^{-2t}(-\cos 2t - \text{sen}\, 2t)\right] e^{2t} \begin{pmatrix} \cos(2t) \\ 2\,\text{sen}(2t) \end{pmatrix}$$

$$+ \left[-\frac{1}{4}\cos(2t) + \frac{1}{2}e^{-2t}\text{sen}(2t) + \frac{1}{4}e^{-2t}(-\cos(2t) - \text{sen}(2t))\right] e^{2t} \begin{pmatrix} \text{sen}(2t) \\ -2\cos(2t) \end{pmatrix}$$

$X(t) = X_H(t) + X_P(t)$ Entonces la sol general es

$$\boxed{X(t) = \left[\frac{1}{4}\,\text{sen}(2t) - \frac{1}{4}e^{-2t}(\cos 2t + \text{sen}\, 2t) + C_1\right] e^{2t} \begin{pmatrix} \cos(2t) \\ 2\,\text{sen}(2t) \end{pmatrix} + \left[-\frac{1}{4}\cos(2t) + \frac{1}{2}e^{-2t}\text{sen}(2t) - \frac{1}{4}e^{-2t}(\cos 2t + \text{sen}\, 2t) + C_2\right] e^{2t} \begin{pmatrix} \text{sen}(2t) \\ -2\cos(2t) \end{pmatrix}}$$

5.1. $y'' - 8y' + 16y = 0$

$\chi_A(\lambda) = \lambda^2 - 8\lambda + 16 = 0$ $\quad b^2 - 4\cdot a\cdot c = 0$

$\hookrightarrow \lambda_1 = \lambda_2 = 4$ $\quad 64 - 4\cdot 16 = 0$ ✓

Como el polinomio característico de la ecuación tiene una raíz en $\mathbb{R}$ doble propongo la solución

$$\boxed{y_H(x) = (C_1 + C_2 x)\, e^{x\cdot 4}}$$

- $y'' - 8y' + 16y = x$

ED no homogénea $\quad y(x) = y_H(x) + y_P(x)$

$y_P = (C_1(x) + C_2(x)\cdot x)\, e^{4x}$

$$\left| \begin{array}{l} y_0 = y \\ y_1 = y' \end{array} \right| \Rightarrow \left\{ \begin{array}{l} y_0' = y_1 \\ y_1' = y'' = x + 8y_1 - 16y_0 \end{array} \right.$$

$$A = \begin{pmatrix} 0 & 1 \\ -16 & 8 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ x \end{pmatrix}$$

$C_1(x)$ y $C_2(x)$ tienen que cumplir

$$Q(x)\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = \begin{pmatrix} 0 \\ x \end{pmatrix}$$

$$Q(x) = \begin{pmatrix} e^{4x} & xe^{4x} \\ 4e^{4x} & e^{4x}+4xe^{4x} \end{pmatrix}$$

CA $y_p(x) = C_1(x)e^{4x} + C_2(x)\, x \cdot e^{4x}$

$$\left(\begin{array}{cc|c} e^{4x} & xe^{4x} & 0 \\ 4e^{4x} & e^{4x}+4xe^{x4} & x \end{array}\right)$$

$4F_1 - F_2 \to F_2$ $\left(\begin{array}{cc|c} e^{4x} & xe^{4x} & 0 \\ 0 & e^{4x} & x \end{array}\right)$

$$C_2'(x)e^{4x} = x$$

$$* \boxed{C_2'(x) = e^{-4x} \cdot x}$$

$$C_2(x) = \int x \cdot e^{-4x} = xe^{-4x} - \int -4e^{-4x}\,dx$$

$u = e^{-4x}$ $du = -4e^{-4x}$

$v = x$ $dv = dx$

$$= xe^{-4x} + 4\frac{e^{-4x}}{-4}$$

$$\therefore \boxed{C_2(x) = (x-1)e^{-4x}}$$

$$e^{4x}C_1'(x) + xe^{4x}\overset{*}{\boxed{C_2'(x)}} = 0$$

$$e^{4x}C_1'(x) + x\cancel{e^{4x}} \cdot \cancel{e^{-4x}} \cdot x = 0$$

$$C_1'(x) = -x^2 \cdot e^{-4x}$$

$$C_1'(x) = -x^2 \cdot (e^{-2x})^2$$

$$C_1(x) = \int -x^2 \left(e^{-2x}\right)^2 dx \underset{\downarrow}{=} -e^{-4x}\frac{x^2}{2} - \int \frac{x^2}{2} 4e^{-4x}$$

$$u = -e^{-4x} \qquad du = 4e^{-4x}$$

$$dv = x^2 \qquad v = \frac{x^2}{2}$$

$$-e^{-4x}\frac{x^2}{2} - 2\underline{\int x^2 e^{-4x}} = \underline{\int -x^2 \cdot e^{-4x}}$$

$$-e^{-4x}\frac{x^2}{2} = \int -x^2 e^{-4x} + 2\int x^2 e^{-4x}$$

$$-e^{-4x}\frac{x^2}{2} = \int x^2 e^{-4x}$$

$$\therefore \quad \boxed{C_1 = e^{-4x}\frac{x^2}{2} = -\int x^2 \left(e^{-4x}\right) dx}$$

$$y_p = \cancel{e^{4x}}\frac{x^2}{2}\cancel{e^{-4x}} + (x-1)\cancel{e^{-4x}}\, x\cancel{e^{4x}}$$

$$y_p(x) = \frac{x^2}{2} + x^2 - x = \boxed{\frac{3}{2}x^2 - x}$$

Otra manera de resolverlo:

$$Q(x)\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = \begin{pmatrix} 0 \\ x \end{pmatrix}$$

$$Q^{-1}(x)\,Q(x)\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = Q^{-1}(x)\begin{pmatrix} 0 \\ x \end{pmatrix}$$

$$2\times 2 \qquad 2\times 1 \qquad = 2\times 1$$

$$\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = Q^{-1}(x)\begin{pmatrix} 0 \\ x \end{pmatrix}$$

Busco $Q^{-1}(x) = \frac{1}{\det(Q)}\begin{pmatrix} e^{4x} + 4xe^{4x} & -xe^{4x} \\ -4e^{4x} & e^{4x} \end{pmatrix}$

$$\det(Q) = e^{4x}(e^{4x} + 4e^{4x}\cdot x) - xe^{4x}\cdot 4e^{4x}$$

$$= e^{8x} + \cancel{4xe^{8x}} - \cancel{4xe^{8x}}$$

$$\therefore \quad Q^{-1}(x) = e^{-8x}\begin{pmatrix} e^{4x} + 4xe^{4x} & -xe^{4x} \\ -4e^{4x} & e^{4x} \end{pmatrix}$$

$$\boxed{Q^{-1}(x) = \begin{pmatrix} e^{-4x} + 4x\cdot e^{-4x} & -xe^{-4x} \\ -4e^{-4x} & e^{-4x} \end{pmatrix}}$$

$$\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} \overset{Q^{-1}(x)\cdot\binom{0}{x}}{=} \begin{pmatrix} -x^2e^{-4x} \\ xe^{-4x} \end{pmatrix}$$

Llegué a lo mismo ☺

y ahora integro...

sol general:

$$\boxed{y(x) = (C_1 + C_2x)e^{4x} + \frac{3}{2}x^2 - x}$$

- $y'' - 8y' + 16y = e^x$ → *no hom.*

$$y_p(x) = C_1(x)e^{4x} + C_2(x)\cdot x\cdot e^{4x}$$

$$\begin{matrix} y_0 = y \\ y_1 = y' \end{matrix} \Rightarrow \begin{cases} y_0' = y_1 \\ y_1' = e^x + 8y_1 - 16y_0 \end{cases}$$

El sistema es el mismo, lo único que cambia es $B(x) = \begin{pmatrix} 0 \\ e^x \end{pmatrix}$

$$\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = Q^{-1}(x)\begin{pmatrix} 0 \\ e^x \end{pmatrix} = \begin{pmatrix} -xe^{-3x} \\ e^{-3x} \end{pmatrix}$$

↓

Es la misma de antes.

Ahora integro

$C_1(x) = \int -x e^{-3x} = \frac{e^{-3x}}{3} x - \int \frac{e^{-3x}}{3} dx$

$dv = -e^{-3x} \quad v = \frac{e^{-3x}}{3}$

$u = x \quad du = dx$

$\int -x e^{-3x} = \boxed{\frac{e^{-3x}}{3} x + \frac{1}{9} e^{-3x} = C_1(x)}$

$C_2(x) = \int e^{-3x} dx = \boxed{\frac{e^{-3x}}{-3}}$

$y_p(x) = \left(\frac{e^{-3x}}{3} + \frac{e^{-3x}}{3} x\right) e^{4x} + \left(\frac{e^{-3x}}{3}\right) x e^{4x}$

$= \left(-\frac{1}{3} + \frac{1}{3} x\right) e^{x} + \frac{x}{3} e^{x}$

$= \left(-\frac{1}{3} + \frac{2}{3} x\right) e^{x}$

$\boxed{y(x) = (C_1 + C_2 x) e^{4x} + \left(\frac{2}{3} x - \frac{1}{3}\right) e^{x}}$

- $y'' - 8y' + 16y = 1$

$\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = Q^{-1}(x) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -x e^{-4x} \\ e^{-4x} \end{pmatrix}$

$C_2(x) = \int e^{-4x} dx = \frac{e^{-4x}}{-4}$

$C_1(x) = \int -x e^{-4x} = \frac{e^{-4x}}{4} x - \int \frac{e^{-4x}}{4} dx$

$= \frac{e^{-4x}}{4} x + \frac{e^{-4x}}{4} \cdot \frac{1}{4}$

$$y_p(x) = \left(x+\frac{1}{4}\right) \cancel{e^{-4x}}\cancel{e^{4x}} + \frac{\cancel{e^{-4x}}}{-4}\cancel{e^{4x}} \cdot x$$

$$= \frac{3}{4}x + \frac{1}{4}$$

$$\boxed{y(x) = (c_1+c_2 x)e^{4x} + \frac{3}{4}x + \frac{1}{4}}$$

$$\bullet\ y'' - 8y' + 16y = e^{-x}$$

$$\begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = Q^{-1}(x)\begin{pmatrix} 0 \\ e^{-x} \end{pmatrix} = \begin{pmatrix} -xe^{-5x} \\ e^{-5x} \end{pmatrix}$$

$$c_2(x) = \frac{e^{-5x}}{-5}$$

$$c_1(x) = \int -xe^{-5x}dx = x \cdot \frac{e^{-5x}}{5} - \int \frac{e^{-5x}}{5}dx$$

$$= x \cdot \frac{e^{-5x}}{5} + \frac{e^{-5x}}{5} \cdot \frac{1}{5}$$

$$y_p(x) = \left(\frac{x}{5}+\frac{1}{5}\right)e^{-5x} \cdot e^{4x} + \frac{e^{-5x}}{-5} \cdot e^{4x}$$

$$= \left(\frac{x}{5}+\frac{1}{5}\right)e^{-x} - \frac{1}{5}e^{-x} = \frac{x}{5}e^{-x}$$

$$\boxed{y(x) = (c_1+c_2 x)e^{4x} + \frac{x}{5}e^{-x}}$$

11. $y'' - 2y' + 10y = 0$

$$\lambda^2 - 2\lambda + 10 = 0 \to \lambda_1 = 1+3i$$
$$\to \lambda_2 = 1-3i$$

Como el pol. característico de la ecuación tiene raíces $\mathbb{C}$ $y(x) = e^{(1+3i)x}$ es sol.

$$y_H(x) = c_1 e^x (\cos 3x) + c_2 e^x \operatorname{sen}(3x) \quad c_1, c_2 \in \mathbb{R}$$

$$y_0 = y \qquad y_1 = y' \quad \Rightarrow \quad \begin{cases} y_0' = y_1 \\ y_1' = 2y_1 - 10y_0 + b(x) \end{cases}$$

$$y_p(x) = C_1(x)\, e^x \cos(3x) + C_2(x)\, e^x \operatorname{sen}(3x)$$

$$Q(x) \begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} 0 \\ b(x) \end{pmatrix}$$

$$\vdots$$

$$\begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} * & e^{-3x} \operatorname{sen}(3x) \\ * & e^{-x} \cos(3x) \end{pmatrix} \begin{pmatrix} 0 \\ b(x) \end{pmatrix}$$

CA: $Q(x) = \begin{pmatrix} e^x \cos(3x) & e^x \operatorname{sen}(3x) \\ e^x \cos(3x) - e^x \operatorname{sen}(3x) & e^x \operatorname{sen}(3x) + e^x \cos(3x) \end{pmatrix}$

$\det(Q) = e^x \cos 3x (e^x \operatorname{sen} 3x + e^x \cos 3x) - e^x \operatorname{sen} 3x (e^x \cos 3x - e^x \operatorname{sen}(3x))$

$= e^{2x} (\cos 3x \operatorname{sen} 3x + \cos^2 3x)$

$- e^{2x} (\operatorname{sen} 3x \cos 3x - \operatorname{sen}^2 3x)$

$= e^{2x} \left[\cancel{\cos 3x \operatorname{sen} 3x} + \cos^2 3x - \cancel{\operatorname{sen} 3x \cos 3x} + \operatorname{sen}^2 x \right]$

$\det(Q) = e^{2x}$

$$Q^{-1}(x) = e^{-2x} \begin{pmatrix} e^x(\operatorname{sen} 3x + \cos 3x) & -e^x \operatorname{sen}(3x) \\ e^x(\operatorname{sen} 3x - \cos 3x) & e^x \cos 3x \end{pmatrix}$$

- $y'' - 2y' + 10y = x$

$$\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = \begin{pmatrix} x\, e^{-3x} \operatorname{sen}(3x) \\ x\, e^{-x} \cos(3x) \end{pmatrix}$$

Jo ja Salu2

$\cdot \; y'' - 2y' + 10y = e^{x}$

$$\begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = \begin{pmatrix} e^{-2x} \; \text{sen}(3x) \\ \cos(3x) \end{pmatrix}$$

$$C_2 = \int \cos(3x) = \frac{1}{3}\,\text{sen}(3x)$$

$$C_1 = \int e^{-2x}\,\text{sen}(3x) = -e^{-2x}\frac{\cos(3x)}{3} - \int \frac{\cos(3x)}{3}\cdot e^{-2x}(-2)$$

$$u = e^{-2x} \qquad du = -2e^{-2x}$$

$$dv = \text{sen}(3x) \qquad v = -\frac{\cos(3x)}{3}$$

$$\int e^{-2x}\,\text{sen}(3x) = -\frac{1}{3}e^{-2x}\cos(3x) - \frac{2}{3}\int e^{-2x}\cos(3x)$$

C_A

$$\int e^{-2x}\cos(3x) = e^{-2x}\frac{\text{sen}(3x)}{3} - \int \frac{\text{sen}(3x)}{3}(-2)e^{-2x}$$

$$= e^{-2x}\frac{\text{sen}(3x)}{3} + \frac{2}{3}\int \text{sen}(3x)\,e^{-2x}$$

Es la que yo quiero

Reordeno

$$\int e^{-2x}\,\text{sen}(3x) = -\frac{1}{3}e^{-2x}\cos(3x) - \frac{2}{3}e^{-2x}\frac{\text{sen}(3x)}{3} - \frac{4}{9}\int e^{-2x}\,\text{sen}(3x)$$

$$\frac{13}{9}\int e^{-2x}\,\text{sen}(3x) = -\frac{1}{3}e^{-2x}\cos(3x) - \frac{2}{9}e^{-2x}\,\text{sen}(3x)$$

$$\int e^{-2x}\,\text{sen}(3x) = e^{-2x}\left(-\frac{3}{13}\cos(3x) - \frac{2}{13}\,\text{sen}(3x)\right) = C_1$$

$$y_p(x) = \left(-\frac{3}{13}\cos(3x) - \frac{2}{13}\,\text{sen}(3x)\right)e^{-x}\cos(3x) + \frac{1}{3}\,\text{sen}^2(3x)\,e^{x}$$

$$y(t) = C_1 e^{x}\cos(3x) + C_2 e^{x}\,\text{sen}(3x) + y_p.$$

• $y'' - 2y' + 10y = 1$

$$\begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} e^{-3x} & sen(3x) \\ e^{-x} & \cos(3x) \end{pmatrix}$$

$$\int C_1' = \int e^{-3x} sen(3x) = -e^{-3x} \frac{\cos(3x)}{3} - \int -\frac{\cos(3x)}{\cancel{3}} \cdot (-\cancel{3}) e^{-3x} dx$$

$$= -e^{-3x} \frac{\cos(3x)}{3} - \int \cos(3x) e^{-3x} dx$$

... PARTES DE NUEVO y DESPEJO

$$C_1 = -e^{-3x} \frac{\cos(3x)}{6} - e^{-3x} \frac{sen(3x)}{6}$$

$$C_2 = \int e^{-x} \cos(3x) = \frac{e^{-x}}{10} \left(3 sen(3x) - \cos(3x) \right)$$

PARTES 2 VECES
y DESPEJO

$$y_p = -\frac{e^{-3x}}{6} \left(\cos(3x) + sen(3x) \right) e^{x} \cos(3x)$$
$$+ \frac{\cancel{e^{-x}}}{10} \left(3 sen(3x) - \cos(3x) \right) \cancel{e^{x}} sen(3x)$$

$y(t) = y_H + y_p$

• $y'' - 2y' + 10y = e^{-x}$

$$\begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} e^{-4x} & sen(3x) \\ e^{-2x} & \cos(3x) \end{pmatrix}$$

integrando

$$C_2(x) = e^{-2x} \left(\frac{3}{13} sen(3x) - \frac{2}{13} \cos(3x) \right)$$
$$C_1(x) = -e^{-4x} \left(\frac{3}{25} \cos(3x) + \frac{4}{25} sen(3x) \right)$$

$$y(x) = C_1 e^{x} (\cos 3x) + C_2 e^{x} (sen\, 3x) + C_1(x) e^{x} \cos 3x + C_2(x) e^{x}$$

$C_1, C_2 \in \mathbb{R}$

III. $y'' - y' - 2y = 0$

$$\lambda^2 - \lambda - 2 = 0 \to \lambda_1 = 2, \quad \lambda_2 = -1$$

$$y_H(x) = C_1 e^{2x} + C_2 e^{-x} \qquad C_1, C_2 \in \mathbb{R}$$

$$\begin{aligned} y_0 &= y \\ y_1 &= y' \end{aligned} \Rightarrow \begin{cases} y_0' = y_1 \\ y_1' = 2y_0 + y_1 + b(x) \end{cases}$$

$$Q(x) = \begin{pmatrix} e^{2x} & e^{-x} \\ 2e^{2x} & -e^{-x} \end{pmatrix}$$

$$\det(Q) = -e^{x} - 2e^{x} = -3e^{x}$$

$$Q^{-1}(x) = -\frac{e^{-x}}{3} \begin{pmatrix} -e^{-x} & -e^{-x} \\ -2e^{2x} & e^{2x} \end{pmatrix}$$

Método de variación de variables

$$\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = \begin{pmatrix} * & \frac{e^{-2x}}{3} \\ * & -\frac{e^{x}}{3} \end{pmatrix} \begin{pmatrix} 0 \\ b(x) \end{pmatrix}$$

$$y_p(x) = C_1(x) e^{2x} + C_2(x) e^{-x}$$

- $y'' - y' - 2y = x$

$$\begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = \begin{pmatrix} \frac{1}{3} x e^{-2x} \\ -\frac{1}{3} x e^{x} \end{pmatrix}$$

$$C_2 = -\frac{1}{3}\int x e^{x} dx = \frac{1}{-3}\left(x e^{x} - \int e^{x} dx\right) = \frac{1}{-3}(x e^{x} - e^{x}) = -(x-1)e^{x}\frac{1}{3}$$

$$C_1 = \frac{1}{3}\int x e^{-2x} dx = \frac{1}{3}\left(x \frac{e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2} dx\right) = -\frac{1}{6} x e^{-2x} - \frac{1}{6}\frac{e^{-2x}}{2}$$

$$C_1 = e^{-2x}\left(-\frac{1}{6}x - \frac{1}{12}\right)$$

$$y_P = \cancel{e^{-2x}}\left(-\frac{1}{6}x - \frac{1}{12}\right)\cancel{e^{2x}} + (1-x)\cancel{e^{x}} \cdot \cancel{e^{-x}}$$

$$y(x) = C_1 \cdot e^{2x} + C_2 e^{-x} + \frac{11}{12} - \frac{7}{6}x$$

- $y'' - 2y - y' = e^{x}$

$$\begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} \frac{1}{3}e^{-x} \\ -\frac{1}{3}e^{2x} \end{pmatrix}$$

$$C_1 = \frac{1}{3}\int e^{-x} = -\frac{1}{3}e^{-x}$$

$$C_2 = -\frac{1}{3}\int e^{2x} = -\frac{1}{6}e^{2x}$$

$$y(x) = \left(C_1 - \frac{1}{6}\right)e^{2x} + \left(C_2 - \frac{1}{3}\right)e^{-x} = y_H + y_P$$

- $y'' - y' - 2y = 1$

$$\begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} \frac{1}{3}e^{-2x} \\ -\frac{1}{3}e^{x} \end{pmatrix}$$

$$C_1 = \frac{1}{3}\int e^{-2x} = -\frac{1}{6}e^{-2x}$$

$$C_2 = -\frac{1}{3}\int e^{x} = -\frac{1}{3}e^{x}$$

$$y(x) = C_1 \cdot e^{2x} + C_2 e^{-x} - \frac{1}{6} - \frac{1}{3}$$

- $y'' - y' - 2y = e^{-x}$ $\quad \begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} 1/3\, e^{-3x} \\ -1/3 \end{pmatrix}$

$$C_1 = \int \frac{1}{3}e^{-3x} = -\frac{e^{-3x}}{9}$$

$$y_P = -\frac{1}{9}e^{-3x} \cdot e^{2x} + \left(-\frac{1}{3}\right)x \cdot e^{-x} = e^{-x}\left(-\frac{1}{9} - \frac{1}{3}x\right)$$

$$C_2 = -\frac{1}{3}x$$

$$y(x) = C_1 e^{2x} + C_2 e^{-x} - \frac{e^{-x}}{3}\left(\frac{1}{3} + x\right)$$

$$y(x) = C_1 e^{2x} + e^{-x}\Big(\underbrace{C_2 - \frac{1}{9}}_{\text{cte}} - \frac{1}{3}x\Big)$$

$$\boxed{y(x) = C_1 e^{2x} + C_2 e^{-x} - \frac{e^{-x}x}{3}}$$

$$y_p = -\frac{1}{9}e^{-x} - \frac{1}{3}e^{-x}x$$

$$y_p' = \frac{1}{9}e^{-x} - \frac{1}{3}e^{-x} + \frac{1}{3}e^{-x}x = -\frac{2}{9}e^{-x} + \frac{1}{3}e^{-x}x$$

$$y_p'' = \frac{2}{9}e^{-x} + \frac{1}{3}e^{-x} - \frac{1}{3}e^{-x}x$$

$$y_p'' - y_p' - 2y = e^{-x}$$

$$\frac{2}{9}e^{-x} + \frac{1}{3}e^{-x} - \frac{1}{3}e^{-x}x + \frac{2}{9}e^{-x} - \frac{1}{3}e^{-x}x$$

$$-2\left(-\frac{1}{9}e^{-x} - \frac{1}{3}e^{-x}x\right)$$

$$= \frac{2}{9}e^{-x} + \frac{1}{3}e^{-x} - \cancel{\frac{1}{3}e^{-x}x} + \frac{2}{9}e^{-x} - \cancel{\frac{1}{3}e^{-x}x} + \frac{2}{9}e^{-x} + \cancel{\frac{2}{3}e^{-x}x}$$

$$= e^{-x}\left(\frac{2}{9} + \frac{1}{3} + \frac{2}{9} + \frac{2}{9}\right) = e^{-x} \checkmark$$ <u>y_p es sol</u>

6. (a_1, b_1), (a_2, b_2) $\rightarrow$ $f(a_1) = b_1$, $f(a_2) = b_2$

$\frac{a_1 - a_2}{\pi}$ no es un número entero $\Rightarrow$ $(a_1 - a_2)$ no es múltiplo de π.

a. $y'' + y = 0$

$\lambda^2 + 1 = 0 \rightarrow \lambda_1 = i$

$\searrow \lambda_2 = -i$

$y_H(x) = e^{ix} \Rightarrow y_H(x) = C_1 \cdot e^0 (\cos x) + C_2 e^0 \operatorname{sen} x$

$$\boxed{y_H(x) = \underset{\downarrow \atop Re(y)}{C_1 \cos x} + \underset{\downarrow \atop Im(y)}{C_2 \operatorname{sen} x}}$$

$$\begin{cases} y_H(a_1) = C_1 \cos(a_1) + C_2 \operatorname{sen}(a_1) = b_1 \\ y_H(a_2) = C_1 \cos(a_2) + C_2 \operatorname{sen}(a_2) = b_2 \end{cases}$$

Quiero encontrar C_1 y C_2 $\rightarrow$ tengo 2 cond. iniciales ✓

$$\begin{pmatrix} \cos(a_1) & \operatorname{sen}(a_1) \\ \cos(a_2) & \operatorname{sen}(a_2) \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

Sabemos que el sistema es SCD (tiene una única solución) cuando su det $\neq 0$

$$\underbrace{\det \begin{pmatrix} \cos a_1 & \operatorname{sen} a_1 \\ \cos a_2 & \operatorname{sen} a_2 \end{pmatrix}}_{= *} = \cos(a_1) \cdot \operatorname{sen}(a_2) - \operatorname{sen}(a_1) \cos(a_2)$$

seno de la de dos ángulos $\quad \operatorname{sen}(a - b) = \operatorname{sen}(a) \cos(b) - \operatorname{sen}(b) \cdot \cos(a)$

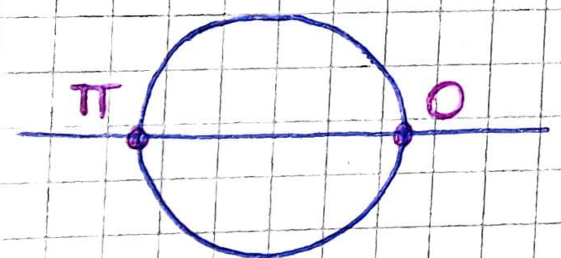
$\det(*) = \operatorname{sen}(a_2 - a_1) = \operatorname{sen}(-(a_1 - a_2))$

Como $a_1 - a_2$ no es múltiplo de π, el $\text{sen}(a_2 - a_1)$ nunca se hace cero $\Rightarrow$ el det $(*) \neq 0$ y existe una única solución del sistema.

Cuando la solución de $y'' + y = 0$ pasa por esos ~~cond~~ puntos el sistema que queda solo tiene una sol como queríamos probar.

b. si $a_1 - a_2$ es múltiplo de π la parte (a) <u>no</u> se cumple si $a_1 - a_2 = 0$ o

$a_1 - a_2 = k\pi \quad k \in \mathbb{Z}$



pero sí se cumple en otros casos.

6 c $\quad y'' + K^2 y = 0$

$$\lambda^2 + K^2 = 0$$
$$\lambda^2 = -K^2$$
$$\lambda = \sqrt{-K^2} = \sqrt{K^2}\sqrt{-1}$$
$$\lambda = \pm K i$$

$$y(x) = e^{Kix} = C_1 \cos(Kx) + C_2 \operatorname{sen}(Kx)$$

$$y(a_1) = b_1 \qquad y(a_2) = b_2$$

$$\begin{cases} y(a_1) = C_1 \cos(Ka_1) + C_2 \operatorname{sen}(Ka_1) = b_1 \\ y(a_2) = C_1 \cos(Ka_2) + C_2 \operatorname{sen}(Ka_2) = b_2 \end{cases}$$

$$\underbrace{\begin{pmatrix} \cos(Ka_1) & \operatorname{sen}(Ka_1) \\ \cos(Ka_2) & \cos(Ka_2) \end{pmatrix}}_{=A} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\det(A) \neq 0 = \operatorname{sen}(K(a_2 - a_1))$$

si $\dfrac{K(a_2 - a_1)}{\pi} \notin \mathbb{Z}$ hay una única solución.

$$y'' + K^2 y = 0 \quad \boxed{K=0} \Rightarrow y'' = 0$$

La solución será una recta $y(x) = C_1 + C_2 x$ que pase por (a_1, b_1) (a_2, b_2)

$$C_1 + C_2 \cdot a_1 = b_1 \qquad C_1 + C_2 \cdot a_2 = b_2$$

$$\underline{C_1 = b_1 - C_2 a_1} \qquad b_1 - C_2 a_1 + C_2 a_2 = b_2$$

$$C_2(a_2 - a_1) = b_2 - b_1$$

$$C_2 = \frac{b_2 - b_1}{a_2 - a_1}$$

$$\boxed{y(x) = b_1 - \frac{b_2 - b_1}{a_2 - a_1} \cdot a_1 + \frac{b_2 - b_1}{a_2 - a_1} x}$$

$y'' = 0$ ✓ pasa por los puntos ✓

7. $y'' - y' - 2y = 0 \rightarrow$ punto 5 III

$y_H(x) = C_1 e^{2x} + C_2 e^{-x}$ $\quad \{ \; y_H'(x) = 2 \cdot C_1 e^{2x} - C_2 e^{-x}$

III. $y(0) = 0$

$y'(0) = 0$

$y_H(0) = C_1 + C_2 = 0 \Rightarrow C_1 = -C_2$

$y_H'(0) = 2C_1 - C_2 = 2C_1 + C_1 = 0$

La única sol que verifica las condiciones iniciales es $y = 0$ $\boxed{C_1 = 0}$ $\boxed{C_2 = 0}$

I. $y(0) = 0 \quad y'(0) = 1$

$y_H(0) = C_1 + C_2 = 0$

$y'(0) = 2C_1 - C_2 = 2C_1 + C_1 = 3C_1 = 1$

$\Rightarrow \boxed{C_1 = \frac{1}{3}}$

$\boxed{C_2 = -\frac{1}{3}}$

Todas las soluciones son:

$y(x) = \frac{1}{3} e^{2x} + \left(-\frac{1}{3}\right) e^{-x}$

III. $y(0) = 1 \qquad y'(0) = 0$

$y(0) = C_1 + C_2 = 1 \Rightarrow C_2 = 1 - C_1$

$y'(0) = 2C_1 - C_2 = 2C_1 - 1 + C_1 = 3C_1 - 1 = 0$

$\boxed{C_1 = \frac{1}{3}}$

$\boxed{C_2 = \frac{2}{3}}$

Todas las soluciones son:

$y(x) = \frac{1}{3} e^{2x} + \frac{2}{3} e^{-x}$

IV. $\lim_{x \to +\infty} y(x) = \lim_{x \to +\infty} C_1 e^{2x} + C_2 e^{-x} \to 0 \Leftrightarrow \boxed{C_1 = 0}$

$\boxed{C_2 \in \mathbb{R}}$

$y(x) = C_2 e^{-x}$

V. $y(0) = 1$

$y(0) = C_1 + C_2 = 1 \Rightarrow C_2 = 1 - C_1$

$y(x) = C_1 e^{2x} + (1 - C_1) e^{-x}$
$= C_1 e^{2x} + e^{-x} - C_1 e^{-x}$

$= C_1 (e^{2x} - e^{-x}) + e^{-x}$

VI. $y'(0) = 1$

$y'(0) = 2C_1 - C_2 = 1 \Rightarrow 2C_1 - 1 = C_2$

$y(x) = C_1 e^{2x} + (2C_1 - 1) e^{-x}$

$\bullet\ y'' - y' - 2y = e^{-x} \rightarrow$ *punto 5///*

$$y(x) = C_1 e^{2x} + C_2 e^{-x} - e^{-x}\,\frac{1}{3}x$$

$$y'(x) = 2C_1 e^{2x} - C_2 e^{-x} + \frac{1}{3}e^{-x}\cdot x - e^{-x}\,\frac{1}{3}$$

I. $y(0) = 0 \qquad y'(0) = 1$

$$y(0) = C_1 + C_2 - 0 = 0$$

$$\Rightarrow C_2 = -C_1$$

$$y'(0) = 2C_1 - C_2 - \frac{1}{3} = 2C_1 - \frac{1}{3} + C_1 = 1$$

$$3C_1 = 1 + \frac{1}{3}$$

$$3C_1 = \frac{4}{3}$$

$$\boxed{C_1 = \frac{4}{9}}$$

$$\boxed{C_2 = -\frac{4}{9}}$$

$$y(x) = \frac{4}{9}e^{2x} + \left(-\frac{4}{9}\right)e^{-x} - e^{-x}\,\frac{1}{3}x$$

II. $y(0) = 1, \quad y'(0) = 0$

$$y(0) = C_1 + C_2 = 1$$

$$\Rightarrow C_2 = 1 - C_1$$

$$y'(0) = 2C_1 - C_2 - \frac{1}{3} = 0$$

$$2C_1 - 1 + C_1 = \frac{1}{3}$$

$$3C_1 = \frac{4}{3}$$

$$\boxed{C_1 = \frac{4}{9}} \rightarrow \boxed{C_2 = \frac{5}{9}}$$

$$y(x) = \frac{4}{9}e^{+2x} + \frac{5}{9}e^{-x} - \frac{e^{-x}x}{3}$$

III. $y(0)=0 \qquad y'(0)=0$

$y(0) \rightsquigarrow C_2 = -C_1$

$$y'(0) = 2C_1 - C_2 - \frac{1}{3} = 3C_1 - \frac{1}{3} = 0$$

$$\boxed{C_1 = \frac{1}{9}}$$

$$\boxed{C_2 = -\frac{1}{9}}$$

$$y(x) = \frac{1}{9}e^{2x} - \frac{1}{9}e^{-x} - e^{-x}\cdot\frac{1}{3}x$$

IV. $\lim_{x\to+\infty} y = \lim_{x\to\infty} C_1e^{2x} + C_2e^{-x} - e^{-x}\frac{1}{3}x \to 0 \Leftrightarrow \boxed{\begin{matrix} C_1 = 0 \\ C_2 \in \mathbb{R}\end{matrix}}$

$$y(x) = C_2e^{-x} - e^{-x}\frac{1}{3}x$$

V. $y(0)=1 \qquad C_2 = 1 - C_1$

$$y(x) = C_1e^{2x} + (1 - C_1)e^{-x} - e^{-x}\frac{1}{3}x$$

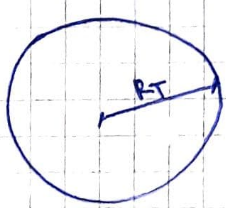
VI. $y'(0)=1$

$$y'(0) = 2C_1 - C_2 - \frac{1}{3} = 1$$

$$2C_1 - \frac{4}{3} = C_2$$

$$y(x) = C_1e^{2x} + \left(2C_1 - \frac{4}{3}\right)e^{-x} - \frac{e^{-x}\,x}{3}$$

8.



Cuando $y < R_T$ = radio de la Tierra

gravedad prop a dis al centro.

me piden v cuando $y \to 0$

Cuando $y = R_T$ estamos sobre la superficie

$$g = 9{,}8\ \frac{m}{s^2} = \frac{GM_T}{R_T}\ \text{cte}$$

Asumiendo que la gravedad actúa como si fuera un punto ubicado justo en centro de la Tierra, el peso de la piedra es cero en el centro.

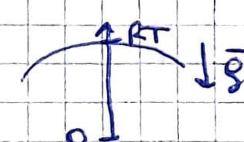
(La fuerza de gravedad neta ejercida sobre una masa puntual situada en el interior de una masa esférica es cero)

También hay que asumir que la Tierra es homogénea.

La aceleración cuando $y < R_T$ es $a = g_i \cdot \frac{y}{R_T}$

Sabiendo que $a = \frac{dv}{dt} = y'' = \frac{d^2y}{d^2}$

$y(t)$ es la posición de la piedra $\qquad y(0) = R_T \quad y(?) = 0$



$$y'' = \frac{g}{R_T} \cdot y$$

$$y'' = -K^2 \cdot y$$

$\frac{g}{R_T} < 0$

$$y'' + K^2 y = 0$$

Llamo $K^2 = -\frac{g}{R_T} > 0$

$$P(\lambda) = \lambda^2 + K^2 = 0 \rightarrow \lambda^2 = -K^2$$

$$\lambda = \pm K i \quad \rightarrow \text{idem } 6C$$

$$y(x) = C_1 \cos(Kt) + C_2 \operatorname{sen}(Kt)$$

$$y'(x) = v(x) = -C_1 \operatorname{sen}(Kt) \cdot K + C_2 \cos(Kt) \cdot K$$

$$v(0) = 0 + C_2 \cdot \sqrt{-\frac{g}{R_T}} \cos(0)$$

$$\boxed{v(0) = C_2 \sqrt{-\frac{g}{R_T}}}$$

$$y(0) = R_T \qquad y''(0) = g$$

$$y(0) = C_1 \cdot 1 + 0 = R_T \Rightarrow \boxed{C_1 = R_T}$$

$$y''(x) = -C_1 \cos(kt) \cdot K^2 - C_2 \operatorname{sen}(kt) K^2$$

$$y''(0) = -C_1 K^2 - 0 = g$$

$$-R_T\left(-\frac{g}{R_T}\right) = g$$

$$g = g \checkmark$$

9. Ecuación de Euler.

$$x^2 y'' + p x y' + q y = 0 \qquad p, q \text{ constantes}$$

a. $x(t) = e^t \Rightarrow x^2 = e^{2t}$

$\ln(x(t)) = t$

$$x = e^t$$

$$dx = d(e^t) \cdot dt$$ } derivo, regla de la cadena

$$dx = e^t \cdot dt$$

$$\frac{1}{dx} = \frac{1}{e^t \cdot dt}$$ } multiplico por dy

$$\frac{dy}{dx} = e^{-t} \frac{dy}{dt}$$

$$\left| y' = \frac{dy}{dx} = e^{-t} \frac{dy}{dt} \right|$$

Otra manera:

$$y' = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{dy}{dt}}{\frac{d(e^t)}{dt}} = \frac{\frac{dy}{dt}}{e^t} = e^{-t} \cdot \frac{dy}{dt}$$

Calculo la segunda derivada

$$y'' = \frac{dy'}{dx} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{\frac{d\left(e^{-t} \cdot \frac{dy}{dt}\right)}{dt}}{\frac{d(e^t)}{dt}}$$

$$= \frac{-e^{-t} \cdot \frac{dy}{dt} + e^{-t} \cdot \frac{d^2y}{dt^2}}{e^t} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

$$\boxed{y'' = \frac{d^2y}{dx^2} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)}$$

$$x^2 y'' + p \cdot x\, y' + q\, y = 0$$

Reemplazando queda

$$\cancel{e^{2t}} \cdot \cancel{e^{-2t}} \left(\frac{d^2y}{dt^2} - \frac{dy}{dt} \right) + p \cdot \cancel{e^{t}} \cdot \cancel{e^{-t}} \left(\frac{dy}{dt} \right) + q\, y = 0$$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} + p \frac{dy}{dt} + q\, y = 0$$

$$\boxed{\frac{d^2y}{dt^2} + (p-1) \frac{dy}{dt} + q\, y = 0}$$

Como p y q son constantes, obtuvimos una ecuación de segundo orden

b.1. $x^2 \, y'' \underbrace{+2}_{p} x\, y' \underbrace{-6}_{q} y = 0 \qquad x \in (0; +\infty)$

Cambio de variables $\quad x = e^t \qquad y(x) = y(e^t) = \tilde{y}(t)$

$$\tilde{y}'' + (2-1)\, \tilde{y}' - 6\, \tilde{y} = 0$$

$\lambda^2 + \lambda - 6 = 0 \rightarrow \lambda_1 = 2$

$\searrow \lambda_2 = -3$

$$\tilde{y}(t) = C_1 e^{2t} + C_2 e^{-3t}$$

$\downarrow t = \ln(x)$

$$y(x) = C_1 x^2 + C_2 x^{-3}$$

II. $x^2 \, y'' \underbrace{-1}_{p} x\, y' \underbrace{+1}_{q} y = 2x$

Primero busco la solución del sist. homogéneo asociado

$$x^2 y'' - 1\, x\, y' + y = 0$$

$$\tilde{y}'' - 2\,\tilde{y}' + y = 0$$

$$\lambda^2 - 2\lambda + 1 = 0 \rightarrow \lambda_1 = 1,\ \lambda_2 = 1 \Big\} \in \mathbb{R}$$ con multiplicidad 2.

$$\tilde{y}_H(t) = (C_1 + C_2 t)\, e^t$$

$$\{\ t = \ln(x)$$

$$y_H(x) = (C_1 + C_2 \ln(x))\, x$$

Busco sol particular: método de variación de constantes

$$y_p(t) = (C_1(t) + C_2(t)\cdot t)\, e^t$$

$$Q(t) = \begin{pmatrix} e^t & te^t \\ e^t & e^t + te^t \end{pmatrix}$$

$$\det(Q) = e^t(e^t + te^t) - t\, e^{t\cdot 2}$$

$$= e^{2t} + \cancel{te^{2t}} - \cancel{te^{2t}}$$

$$Q^{-1}(t) = e^{-2t} \begin{pmatrix} e^t + te^t & -te^t \\ -e^t & e^t \end{pmatrix}$$

$$= \begin{pmatrix} e^{-t} + te^{-t} & -te^{-t} \\ -e^{-t} & e^{-t} \end{pmatrix}$$

$$\begin{pmatrix} C_1'(t) \\ C_2'(t) \end{pmatrix} = \begin{pmatrix} * & -te^{-t} \\ * & e^{-t} \end{pmatrix} \begin{pmatrix} 0 \\ 2e^t \end{pmatrix} = \begin{pmatrix} -2t \\ 2 \end{pmatrix}$$

$$C_1 = \int -2t\, dt = -t^2$$

$$C_2 = \int 2\, dt = 2t$$

$$\tilde{y}_p(t) = (-t^2 + 2t^2)\, e^t = t^2 e^t$$

$t = \ln(x)$

$y_p(x) = (\ln(x))^2 \cdot e^{\ln(x)} = 2\ln(x) \cdot x$

$$\boxed{y(x) = y_H(x) + y_p(x) = (C_1 + C_2 \ln(x))x + 2\ln(x)x}$$

11.1. $xy'' + 2y' + xy = 0 \qquad I = \mathbb{R}_{>0}$

- $y_1 = \dfrac{\operatorname{sen} x}{x}$

Comprobemos que y_1 es sol.

$$y_1' = -\frac{1}{x^2}\operatorname{sen} x + \frac{1}{x}\cos x$$

$$y_1'' = \frac{2}{x^3}\operatorname{sen} x - \frac{1}{x^2}\cos x - \frac{1}{x^2}\cos x - \frac{1}{x}\operatorname{sen} x$$

$$\cancel{\frac{2}{x^2}\operatorname{sen} x} - \cancel{\frac{2\cos x}{x}} - \cancel{\operatorname{sen} x} + \cancel{\frac{2}{x}\cos x} - \cancel{\frac{2}{x^2}\operatorname{sen} x} + \cancel{\operatorname{sen} x} = 0$$

y_1 es solución ✓

Quiero $xy_2'' + 2y_2'' + xy_2 = 0$

$$\boxed{\begin{aligned} y_2 &= v \cdot y_1 \\ y_2' &= v' y_1 + v y_1' \\ y_2'' &= v'' y_1 + 2v'y_1' + v y_1'' \end{aligned}}$$

$$x(v''y_1 + 2v'y_1' + vy_1'') + 2v'y_1 + 2vy_1' + xv \cdot y_1 = 0$$

$$v''(xy_1) + v'(2xy_1' + 2y_1) + v\underbrace{(xy_1'' + 2vy_1' + xy_1)}_{=0} = 0$$

Reemplazo y_1, y_1' pues y_1 es sol

$$v'' \operatorname{sen} x + v'\left(-\cancel{\frac{2}{x}\operatorname{sen} x} + 2\cos x + \cancel{\frac{2}{x}\operatorname{sen} x}\right) = 0$$

$$v'' \operatorname{sen} x + v' \, 2\cos x = 0$$

Llamo $v' = z(x)$

$$z' \operatorname{sen} x + z \, 2\cos x = 0$$

$$z' \operatorname{sen} x = -z \, 2\cos x$$

$$\frac{1}{z}\frac{dz}{dx} = \frac{-2\cos x}{\operatorname{sen} x}$$

$$\ln(z(x)) = -2\int \frac{\cos x}{\operatorname{sen} x}\,dx$$

$\rightarrow u = \operatorname{sen} x$

$du = \cos x \, dx$

$$\ln(z(x)) = -2\ln(|\operatorname{sen} x|)$$

$$z(x) = \operatorname{sen}^{-2}(x) > 0$$

$$v(x) = \int \frac{1}{\operatorname{sen}^2(x)}\,dx = -\operatorname{cotg}(x) = \frac{-1}{\operatorname{tg}(x)}$$

$$y_2(x) = v(x) \cdot y_1(x)$$

$$y_2(x) = -\frac{1}{\operatorname{tg}(x)} \cdot \frac{\operatorname{sen} x}{x}$$

$$= -\frac{1}{\frac{\operatorname{sen}(x)}{\cos(x)}} \frac{\cancel{\operatorname{sen}(x)}}{x}$$

$$y_2(x) = -\frac{\cos(x)}{x}$$

$$y_2' = \frac{1}{x^2}\cos(x) + \frac{1}{x}\operatorname{sen}(x)$$

$$y_2'' = \frac{-2}{x^3}\cos(x) - \frac{1}{x^2}\operatorname{sen}(x) + \frac{1}{x}\cos(x) - \frac{1}{x^2}\operatorname{sen}(x)$$

$$\cancel{\frac{-2}{x^2}\cos x} \cancel{- \frac{2}{x}\operatorname{sen} x} + \cancel{\cos x} + \cancel{\frac{2}{x^2}\cos x} + \cancel{\frac{2}{x}\operatorname{sen} x} - \cancel{\cos x} = 0 \quad \checkmark$$

y_2 es sol.

$$xy'' - y' - 4x^3 y = 0 \qquad \text{II } x \in (0; +\infty)$$
$$\text{III } x \in (-\infty; 0)$$

$$y_1 = e^{x^2}$$
$$y_1' = 2x e^{x^2}$$
$$y_1'' = 2e^{x^2} + 4x^2 e^{x^2}$$

$$\cancel{2x e^{x^2}} + \cancel{4x^3 e^{x^2}} - \cancel{2x e^{x^2}} - \cancel{4x^3 e^{x^2}} = 0 \checkmark \quad y_1 \text{ es sol}$$

Busco $y_2(x) = v\, y_1$

$$x y_2'' - y_2' - 4x^3 y_2 = 0$$

$$x(v'' y_1 + 2v' y_1' + v y_1'') - v' y_1 - v y_1' - 4x^3 v y_1 = 0$$

$$v''(x y_1) + v'(2x y_1' - y_1) + v\underbrace{(x y_1'' - y_1' - 4x^3 y_1)}_{=0 \text{ pues } y_1 \text{ es sol}} = 0$$

Llamo $z = v'(x)$ $\qquad \underline{y_1 = e^{x^2}}$

$$z' \cdot x e^{x^2} + z\left((2x)^2 e^{x^2} - e^{x^2}\right) = 0$$

$$e^{x^2}\left(z' \cdot x + z\left((2x)^2 - 1\right)\right) = 0$$

$$e^{x^2} \neq 0 \ \forall x$$

$$x z' + z(4x^2 - 1) = 0$$

$$x \frac{dz}{dx} = -z(4x^2 - 1) \quad \rightarrow \text{variables separadas}$$

$$\int \frac{1}{z}\, dz = \int\left(-4x + \frac{1}{x}\right) dx \qquad x \neq 0$$

$$\ln(z) = -2x^2 + \ln|x| + \overset{\text{''}}{\underset{0}{c}}$$

$$z = e^{-2x^2} |x|$$

$$z = e^{-2x^2}|x|$$

II. $x \in (0;+\infty)$

$$z = e^{-2x^2} \cdot x = v'(x)$$

$$v(x) = \int x\, e^{-2x^2} \frac{-4}{-4}\, dx$$

$$= -\frac{1}{4}\int -4x \cdot e^{-2x^2}\, dx$$

$$v(x) = -\frac{1}{4}\, e^{-2x^2}$$

$$y_2 = -\frac{1}{4} e^{-2x^2} \cdot e^{x^2}$$

$$\boxed{y_2 = -\frac{1}{4} e^{-x^2}}$$

$$y_2' = +\frac{1}{4}\, 2x\, e^{-x^2}$$

$$y_2'' = \frac{1}{2} e^{-x^2} + \frac{1}{2} x(-2x) e^{-x^2}$$

$$= \frac{1}{2} e^{-x^2} - x^2 e^{-x^2}$$

III. $x \in (-\infty; 0)$

$$v'(x) = z = -e^{-2x^2} \cdot x$$

$$v(x) = \int -e^{-2x^2} \cdot x \cdot \frac{4}{4}\, dx$$

$$= \frac{1}{4}\int -4x\, e^{-2x^2}\, dx$$

$$v(x) = \frac{1}{4} e^{-2x^2}$$

$$y_2 = \frac{1}{4} e^{-2x^2} \cdot e^{x^2}$$

$$\boxed{y_2 = \frac{1}{4} e^{-x^2}}$$

$$y_2' = -\frac{2x e^{-x^2}}{4} = -\frac{1}{2} x e^{-x^2}$$

$$y_2'' = -\frac{1}{2} e^{-x^2} - \frac{1}{2} x(-2x) e^{-x^2}$$

$$= -\frac{1}{2} e^{-x^2} + x^2 e^{-x^2}$$

Verifico

III. $-\frac{1}{2}\cancel{x e^{-x^2}} + \cancel{x^3 e^{-x^2}} + \cancel{\frac{1}{2} x e^{-x^2}} - \cancel{4x^3 \frac{1}{4} e^{-x^2}} = 0$ y_2 es sol ✓

II. $\frac{1}{2} x e^{-x^2} - x^3 e^{-x^2} - \frac{1}{2} e^{-x^2} x + \frac{1}{4} e^{-x^2} \cdot 4x^3 = 0$ y_2 es sol ✓

$\{e^{x^2}, e^{-x^2}\}$ es base de soluciones

$\Rightarrow$ cte $\cdot\, e^{-x^2}$ es sol $\forall$ múltiplo