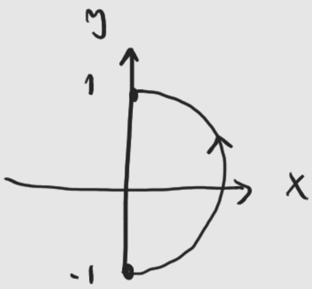
Resolución Tema 1.

1) $C = \{ (x, y, z) : x^2+y^2=1, \; z = x^2+y^2, \; x \geq 0 \}$

a) Parametrizar C del $(0,-1,1)$ al $(0,1,1)$

b) $\int_C \underbrace{(0,y,y)}_{F} \cdot ds$

a) $\gamma(t) = (\cos t, \sin t, 1) \qquad t \in [-\frac{\pi}{2}, \frac{\pi}{2}] =: I$



- $\gamma'(t) \neq (0,0,0) \quad \forall t \in I$
- $\gamma \in C^1$
- γ es inyectiva

b) $$\int_C \underbrace{(0,y,y)}_{F} \cdot ds = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} F(\gamma(t)) \cdot \gamma'(t) \, dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (0, \sin t, \sin t) \cdot (-\sin t, \cos t, 0) \, dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin t \cos t \, dt = \boxed{0}$$

AUX $\int \sin t \cos t \, dt = -\int u \, du = -\frac{u^2}{2} = -\frac{\cos^2 t}{2}$

$u = \cos t$

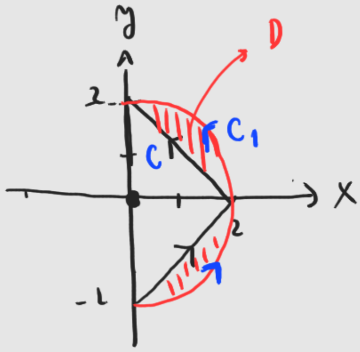
$du = -\sin t \, dt$

(2) $F(x,y) = \left(x \operatorname{sen}\left(\sqrt{x^2+y^2}\right) - \dfrac{y}{x^2+y^2},\ y \operatorname{sen}\left(\sqrt{x^2+y^2}\right) + \dfrac{x}{x^2+y^2} \right)$

$\quad\ \ \ \ \ ''\ (P,Q)$

Calc. $\displaystyle\int_C F\cdot ds$ $\qquad C$ dada por $\begin{cases} y = 2-x & 0\leq y \leq 2 \\ y = x-2 & -2 \leq y \leq 0 \end{cases}$

del $(0,-2)$ al $(0,2)$



Green:

$$\int_{\partial D} (Q_x - P_y)\,dx\,dy = \underbrace{\int_{\partial D} F\cdot ds}_{\text{positivo}} = \underbrace{\int_{C_1} F\cdot ds}_{\text{posit.}} - \underbrace{\int_{C} F\cdot ds}_{\text{negativo.}}$$

$\partial D = C_1 \cup C$

$Q_x - P_y = 0$ $\qquad$ Cuentas

$$\Rightarrow\ 0 = \int_{C_1} F\cdot ds - \int_C F\cdot ds \ \Rightarrow\ \underbrace{\int_C F\cdot ds}_{\text{la que pide}} = \underbrace{\int_{C_1} F\cdot ds}_{C_1 \text{ en sent. positivo.}} = (*)$$

$C_1:\ \gamma(t) = (2\cos t, 2\operatorname{sen} t)\quad t\in[-\tfrac{\pi}{2}, \tfrac{\pi}{2}]$

$$(*) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} F(\gamma(t))\cdot\gamma'(t)\,dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(2\cos t \operatorname{sen}(2) - \frac{2\operatorname{sen} t}{4},\ 2\operatorname{sen} t \operatorname{sen}(2) + \frac{2\cos t}{4}\right)\cdot (-2\operatorname{sen} t, 2\cos t)\,dt$$

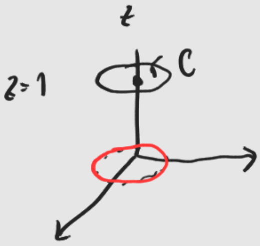
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\cancel{-4\cos t \operatorname{sen} t \operatorname{sen}(2)} + \operatorname{sen}^2 t + \cancel{4\cos t \operatorname{sen} t \operatorname{sen}(2)} + \cos^2 t\right) dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\,dt = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$$

③ $C = \{ (x,y,z) : z=1, \ x^2+y^2=1 \}$ orientada / al proyectarla al plano xy se recorre en sentido positivo.

Calcular $\int_C F \cdot ds$

$$F(x,y,z) = \underbrace{\frac{1}{x^2+y^2+(z-1)^2} (x, y, z-1)}_{F_1} + \underbrace{\left(\frac{(z-1)^3}{3}, \frac{x^3}{3}, \frac{y^3}{3} \right)}_{F_2}$$



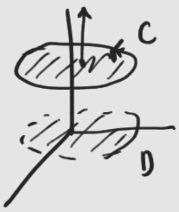
si $(x,y,z) \neq (0,0,1)$

$$\text{rot} F = \underbrace{\text{rot} F_1}_{=0} + \text{rot} F_2 = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ \frac{(z-1)^3}{3} & \frac{x^3}{3} & \frac{y^3}{3} \end{vmatrix} = (y^2, (z-1)^2, x^2)$$

- $\int_C F_1 \cdot ds = \int_S \text{rot} F \cdot dS = 0$ $\quad S$ semiesfera

- $\int_C F_2 \cdot ds \underset{\text{STOKE}}{=} \int_S \text{rot} F_2 \cdot dS = \iint_D \text{rot} F_2 (T(x,y)) \cdot (T_x \times T_y) \, dx \, dy = (*)$

S :



$T(x,y) = (x, y, 1) \qquad (x,y) \in D = \{ (x,y) / x^2+y^2 \leq 1 \}$

$T_x(x,y) = (1,0,0)$
$T_y(x,y) = (0,1,0)$ $\Rightarrow T_x \times T_y = (0,0,1)$

$$(*) = \iint_D (y^2, 0, x^2) \cdot (0,0,1) \, dx \, dy = \iint_D x^2 \, dx \, dy$$

$x = r \cos \theta$
$y = r \operatorname{sen} \theta$
$r \in [0,1]$
$\theta \in [0, 2\pi]$

$$= \int_0^1 \int_0^{2\pi} r^2 \cos^2 \theta \cdot r \, d\theta \, dr = \boxed{\frac{1}{4} \pi}$$

$$\Rightarrow \int_C F \cdot ds = \int_C F_1 \cdot ds + \int_C F_2 \cdot ds = \frac{\pi}{4}$$

④ $F = \left(\sin(z^2) + 3xy,\ e^{x^3} - y^2,\ x^2 - yz \right)$

Calc. $\displaystyle\int_S F\cdot dS$ $\qquad S:\ \underline{x^2+y^2+(z-3)^2=5}$ cortada entre $z=1$, $z=4$

$\downarrow$

con la normal int.

Teorema de Gauss:
$$\int_V \operatorname{div} F\, dV = \int_{\partial V} F\cdot n_{ext}\, dS = \boxed{\int_S F\cdot n_{ext_S}\, dS} + \int_{T_1} F\cdot n_{ext_{T_1}}\, dS + \int_{T_2} F\cdot n_{ext_{T_2}}\, dS$$

$$\Rightarrow \int_S F\cdot n_{int_S}\, dS = -\int_S F\cdot n_{ext_S}\, dS = -\int_V \operatorname{div} F\, dV + \int_{T_1} F\cdot n_{ext_{T_1}}\, dS + \int_{T_1} F\cdot n_{ext_{T_2}}\, dS$$

Como $\operatorname{div} F = 0$

$$\int_S F\cdot n_{int_S}\, dS = \int_{T_1} F\cdot n_{ext_{T_1}}\, dS + \int_{T_2} F\cdot n_{ext_{T_2}}\, dS = (*)$$

Parametrizo T_1: $\quad T(x,y) = (x,y,4) \qquad (x,y)\in D = \{(x,y):\ x^2+y^2 \leq 2^2\}$ ✓

" $\quad T_2$: $\quad \tilde{T}(x,y) = (x,y,1) \qquad (x,y)\in \tilde{D} = \{(x,y):\ x^2+y^2 \leq 1\}$

$$T_x \times T_y = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,1) \quad ✓ \text{ exterior.}$$

$$\tilde{T}_x \times \tilde{T}_y = (0,0,1) \quad ✗ \text{ interior.}$$

$$(*) = \iint_D F(T(x,y))\cdot T_x\times T_y\, dx\, dy - \iint_{\tilde{D}} F(\tilde{T}(x,y))\cdot \tilde{T}_x\times\tilde{T}_y\, dx\, dy$$

$$(*) = \iint_D F(T(x,y)) \cdot T_x \times T_y \, dx\, dy - \iint_{\tilde{D}} F(\tilde{T}(x,y)) \cdot \tilde{T}_x \times \tilde{T}_y \, dx\, dy$$

$$= \iint_D (\sim, \sim, x^2 - y \cdot 4) \cdot (0,0,1)\, dx\, dy$$

$$- \iint_{\tilde{D}} (\sim, \sim, x^2 - y) \cdot (0,0,1)\, dx\, dy$$

$$= \iint_D (x^2 - 4y)\, dx\, dy - \iint_{\tilde{D}} (x^2 - y)\, dx\, dy$$

$$= \int_0^{2\pi}\int_0^2 \left(r^3 \cos^2\theta - 4 r^2 \sin\theta\right) dr\, d\theta - \int_0^{2\pi}\int_0^1 \left(r^3 \cos^2\theta - r^2 \sin\theta\right) dr\, d\theta$$

$$= \frac{2^4}{4}\pi - 4 \cdot \cancel{\frac{2^3}{3}} \cdot 0 - \frac{1}{4}\pi + \cancel{\frac{1}{3}} \cdot 0 = 4\pi - \frac{\pi}{4} = \frac{15}{4}\pi$$