

12. Sea  $f : \mathbb{R} \rightarrow \mathbb{R}$  continua tal que

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow +\infty} f(x) = 0.$$

Probar que  $f$  es uniformemente continua en  $\mathbb{R}$ .

Sea  $\varepsilon > 0$

queremos  $\delta > 0 /$   
 Si  $x, y \in \mathbb{R}$ ,  $|x - y| < \delta$   
 $\Rightarrow |f(x) - f(y)| < \varepsilon$

SABEMOS:  $f$  CONT  
 $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$

1D E A  $\exists M / |f(x)| < \varepsilon/3 \quad \forall x > M, \forall x < -M$ .

$|f(x)| < \varepsilon/3$   
 $\uparrow$   

$-M \quad 0 \quad M$

$|f(x)| < \varepsilon/3$   
 $\uparrow$   
 $\forall x > M$

$\underbrace{x, y > M} \quad |f(x) - f(y)| \leq |f(x)| + |f(y)| < \varepsilon/3 + \varepsilon/3 < \varepsilon$

Falta ver los bordes.