

Videy
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Sobre Demo de Heine-Borel.

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$(x_u)_{u \in \mathbb{N}}$ x_u^j $(x_m)_{m \in \mathbb{N}}$

$- x_1 = (x_1^1, x_1^2, \dots, x_1^m)$
 $- x_2 = (x_2^1, x_2^2, \dots, x_2^m)$
 \vdots
 $- x_{10} = (x_{10}^1, \dots, x_{10}^m)$

$(x_u^j)_u \subseteq \mathbb{R}$
 $\forall 1 \leq j \leq m.$

$m=3$

Exista subseq de $(x_u^1)_u$ conr digamos a $\underline{x_1}$.
 $\exists u \quad u \quad u \quad (x_u^2)_u \quad u \quad u \quad x_2$
 $\exists u \quad u \quad u \quad (x_m^3)_m \quad u \quad u \quad x_3$

Sep. $\rightarrow (x_{3u}^1)_u \quad m_u = 3u$
 Sep $\rightarrow (x_{3u+1}^2)_u \quad m_u = 3u+1$
 Sep $\rightarrow (x_{3u+2}^3)_u \quad m_u = 3u+2.$

No coinciden los indices!

$\exists (x'_{n_{k_1}})_{n \in \mathbb{N}}$ subsec. de $(x'_n)_n$ convrg. a x_1 .
 miramos la subsec. $(x''_{n_{k_1 k_2}})_n$ de $(x'_n)_n$

como es acotado, existe $(x''_{n_{k_1 k_2}})_{n \in \mathbb{N}}$ subsec.
 convrg a digamos x_2 .

Miramos $(x'''_{n_{k_1 k_2 k_3}})_{n \in \mathbb{N}}$ subsec. de $(x''_n)_n$

Como es acotado \exists subsec. convr. $(x'''_{n_{k_1 k_2 k_3}})_{n \in \mathbb{N}}$
 a x_3 .

$$\text{Tomamos } x_{n_{k_1 k_2 k_3}} = \left(x'_{n_{k_1 k_2 k_3}}, x''_{n_{k_1 k_2 k_3}}, x'''_{n_{k_1 k_2 k_3}} \right)$$

$$\begin{array}{ccc} \downarrow n_3 \rightarrow +\infty & \downarrow n_3 \rightarrow +\infty & \downarrow n_3 \rightarrow +\infty \\ x_1 & x_2 & x_3 \end{array}$$

$$x_{n_{k_1 k_2 k_3}} \longrightarrow (x_1, x_2, x_3) \quad n_3 \rightarrow +\infty.$$

Compacto \Rightarrow Acotado

$A \subseteq \mathbb{R}^n$ Conj. acotado $\Leftrightarrow \exists x_0 \in \mathbb{R}^n \text{ y } r > 0 / A \subseteq B(x_0, r)$

K compacto. Sup. que no es acotado

$x_0 \in K \Rightarrow \forall m \in \mathbb{N} \exists x_m \in K / x_m \notin B(x_0, m)$

$(x_n)_n \subseteq K$ que cumple que $d(x_n, x_0) \geq n \forall n$

K compacto $\Rightarrow \exists (x_{n_k})_k$ subsec. de $(x_n)_n$ que converge a x . $\left[x_{n_k} \xrightarrow[k \rightarrow \infty]{} x \right]$.

$f(y) = d(y, x_0)$ es continua.

$\Rightarrow \underbrace{f(x_{n_k})}_{d(x_{n_k}, x_0)} \xrightarrow[k \rightarrow \infty]{} \underbrace{f(x)}_{d(x, x_0)}$

$d(x_{n_k}, x_0) \quad d(x, x_0)$

$\Rightarrow \underbrace{f(x_{n_k})}_{d(x_{n_k}, x_0)} \xrightarrow[k \rightarrow \infty]{} \underbrace{f(x)}_{d(x, x_0)}$

$m_k \leq d(x_{n_k}, x_0) \quad d(x, x_0)$

$\Rightarrow \lim_{k \rightarrow \infty} d(x_{n_k}, x_0) = +\infty$



