

12. Sea $f : \mathbb{R} \rightarrow \mathbb{R}$ continua tal que

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow +\infty} f(x) = 0.$$

Probar que f es uniformemente continua en \mathbb{R} .

Sea $\varepsilon > 0$

queremos $\delta > 0 /$
 Si $x, y \in \mathbb{R}$, $|x - y| < \delta$
 $\Rightarrow |f(x) - f(y)| < \varepsilon$

SABEMOS: f CONT

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$$

1D E A $\exists M / |f(x)| < \varepsilon/3 \quad \forall x > M, \forall x < -M$

$|f(x)| < \varepsilon/3$
 \uparrow

$-M \quad 0 \quad M$

$|f(x)| < \varepsilon/3$
 \uparrow
 $\forall x > M$

$$\underbrace{x, y > M} \quad |f(x) - f(y)| \leq |f(x)| + |f(y)| < \varepsilon/3 + \varepsilon/3 < \varepsilon$$

Falta ver los bordes.