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Sobre Demo de Heine-Borel.

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$(x_n)_{n\in\mathbb{N}}$ $(x_n)_n$ $(x_m)_{m\in\mathbb{N}}$

- $x_1 = (x_1^1, x_1^2, \dots, x_1^m)$
- $x_2 = (x_2^1, x_2^2, \dots, x_2^m)$

$\vdots$

- $x_{10} = (x_{10}^1, \dots, x_{10}^m)$

$(x_n^j)_n \subseteq \mathbb{R}$ $\forall\, 1 \leq j \leq m$.

$m=3$

∃ una subsuc de $(x_n^1)_n$ conv digamos a x_1.

∃ " " " $(x_n^2)_n$ " " x_2

∃ " " " $(x_m^3)_m$ " " x_3

Sub $\rightarrow (x^1_{3k})_k$ $\quad n_k = 3k$

Sub $\rightarrow (x^2_{3k+1})_k$ $\quad n_k = 3k+1$

Sub $\rightarrow (x^3_{3k+2})_k$ $\quad n_k = 3k+2$.

No coinciden los índices!

$\exists \left(x^1_{n_{k_1}}\right)_{k_1 \in \mathbb{N}}$ subsuc. de $(x^1_n)_n$ conv. a x_1.

miramos la subsuc. $\left(x^2_{n_{k_1}}\right)_{k_1}$ de $(x_n)_n$

como es acotado, existe $\left(x^2_{n_{k_1 k_2}}\right)_{k_2 \in \mathbb{N}}$ subsuc.

n_{k_j}

conv. a digamos x_2.

Miramos $\left(x^3_{n_{k_1 k_2}}\right)_{k_2 \in \mathbb{N}}$ subsuc. de $(x^3_n)_n$

como es acotado $\exists$ subsuc. conv. $\left(x^3_{n_{k_1 k_2 k_3}}\right)_{k_3 \in \mathbb{N}}$

$n_{k_{j_i}}$

a x_3.

Tomo $x_{n_{k_1 k_2 k_3}} = \left(x^1_{n_{k_1 k_2 k_3}}, x^2_{n_{k_1 k_2 k_3}}, x^3_{n_{k_1 k_2 k_3}}\right)$

$$x^1_{n_{k_1 k_2 k_3}} \xrightarrow[k_3 \to +\infty]{} x_1, \quad x^2_{n_{k_1 k_2 k_3}} \xrightarrow[k_3 \to \infty]{} x_2, \quad x^3_{n_{k_1 k_2 k_3}} \xrightarrow[k_3 \to \infty]{} x_3$$

$$x_{n_{k_1 k_2 k_3}} \xrightarrow[k_3 \to +\infty]{} (x_1, x_2, x_3).$$

Compacto $\Rightarrow$ Acotado

$A \subseteq E$ Conj acotado $\overset{\text{def}}{\Longleftrightarrow} \exists\, x \in E$ y $M > 0$ / $A \subseteq B(x, M)$

K compacto. Sup que no es acotado

$x_0 \in K \Rightarrow \forall n \in \mathbb{N} \;\; \exists\, x_n \in K \,/\, x_n \notin B(x_0, n)$

$(x_n)_n \subseteq K$ que cumple que $d(x_n, x_0) \geq n$ $\forall n$

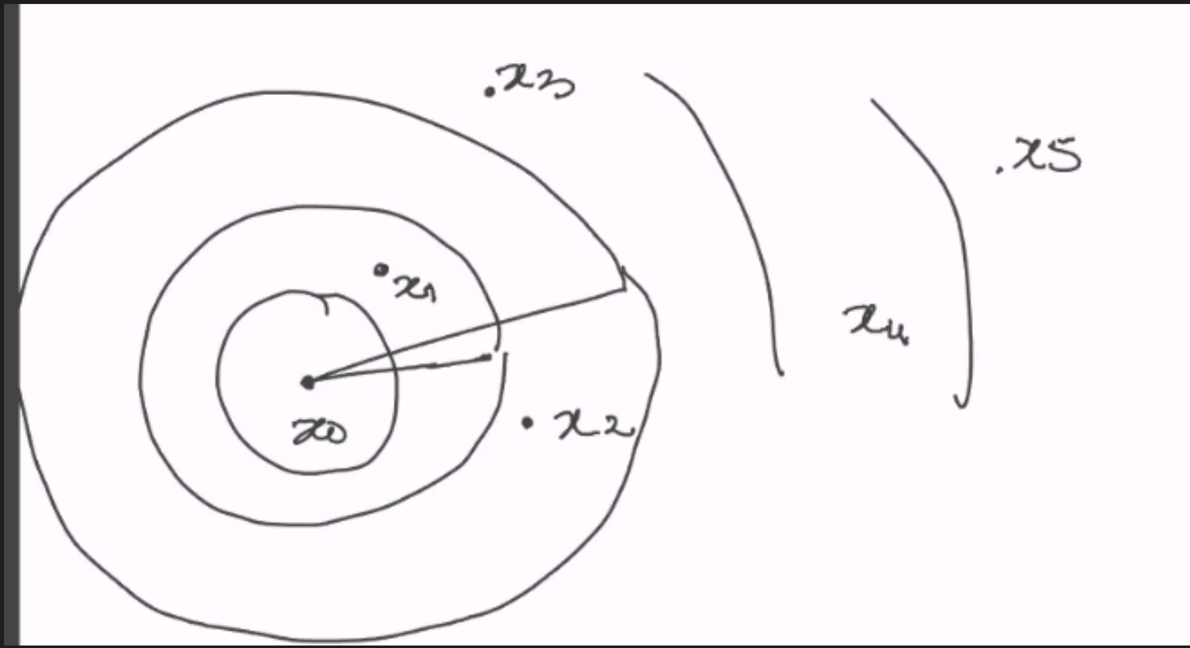
K compacto $\Rightarrow \exists\, (x_{n_k})_k$ subsec. de $(x_n)_n$ que converge a x. $\left[x_{n_k} \xrightarrow[k \to +\infty]{} x \right]$.

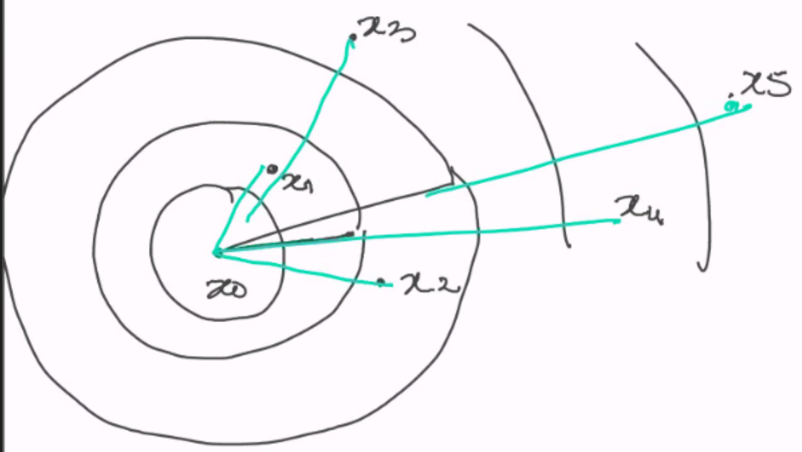
$f(y) = d(y, x_0)$ es continua.

$$\Rightarrow \underbrace{f(x_{n_k})}_{d(x_{n_k},\, x_0)} \xrightarrow[k \to \infty]{} \underbrace{f(x)}_{d(x,\, x_0)}$$

$$\Rightarrow \underbrace{f(x_{n_k})}_{\underbrace{n_k \leq d(x_{n_k}, x_0)}} \xrightarrow[k \to \infty]{} \underbrace{f(x)}_{d(x, x_0)}$$

$$\Rightarrow \lim_{k \to \infty} d(x_{n_k}, x_0) = +\infty .$$





$$d(x_0, x_n) \geqslant n$$

$$d(x_0, x_{n_k}) \geqslant n_k$$

$$\downarrow \qquad\qquad \downarrow$$

$$\infty \qquad\qquad \infty$$