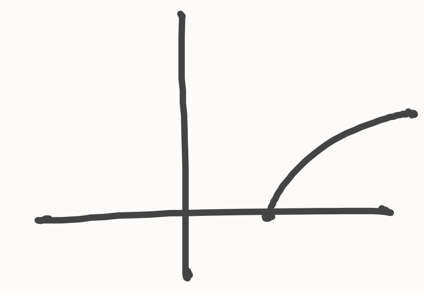
16) b) $y = \ln(x) = f(x)$ $\quad x=1 \quad x=2$



$$\mathcal{L}(\mathcal{C}) = \int_1^2 \sqrt{1+\left(\frac{1}{x}\right)^2}\,dx =$$

$$u = \sqrt{1+(1/x)^2} \qquad \qquad \int_1^2 \sqrt{1+1/x^2}\,dx$$

$$du = \frac{-\cancel{2}\,1/x^3}{2\sqrt{1+(1/x)^2}}\,dx$$

$$\left(\frac{1}{x^2}\right)' = (x^{-2})' = -2x^3$$

$$u^2 = 1+1/x^2 \qquad \frac{1}{x} = \qquad \frac{-\cancel{2}\,(u^2-1)\sqrt{u^2-1}}{\cancel{2}\,u}$$

$$u^2-1 = 1/x^2$$

$$\sqrt{u^2-1} = 1/x$$

$\boxed{\text{DEUDA}}$

$$\int\sqrt{1+1/x^2}\,dx = \int \frac{(u^2-1)\sqrt{u^2-1}}{u}\,du$$

?

---

12) a) $\sigma(t) = (\cos(t), \operatorname{sen}(t), t) \to \mathcal{C}$ $\quad a=0, b=1$

$$h(t) = \int_0^t \|\sigma'(s)\|\,ds \qquad h:[0,1]\to[0,\ell]$$

$$\ell = \mathcal{L}(\mathcal{C})$$

$$= \int_0^t \|(-\operatorname{sen}(s), \cos(s), 1)\|\,ds$$

$$= \int_0^t \sqrt{\operatorname{sen}^2(s)+\cos^2(s)+1}\,ds$$

$$= \int_0^t \sqrt{2}\,ds = \sqrt{2}\,t$$

param. x long. de arco

$$\sigma(h^{-1}(s)) \qquad h^{-1}(s)\,?$$

$$s = \sqrt{2}\,t \Rightarrow \frac{s}{\sqrt{2}} = t \Rightarrow h^{-1}(s) = s/\sqrt{2}$$

$$\tilde{\sigma}(s) = \sigma(s/\sqrt{2}) = (\cos(s/\sqrt{2}), \operatorname{sen}(s/\sqrt{2}), s/\sqrt{2}).$$

(15) Temperatura promedio

$f(x,y,z) = x+y-z$

$$TP = \int_{\mathscr{C}} f \Big/ \text{masa} = \frac{\int_0^{\pi} f(\sigma(\theta))\,\|\sigma'(\theta)\|\,d\theta}{\text{masa}}$$

$$\text{masa} = \int_{\mathscr{C}} \rho\, ds = \int_0^{\pi} 2\|\sigma'(\theta)\|\,d\theta = 2\int_0^{\pi} \|\sigma'(\theta)\|\,d\theta$$

---

(18) a) $\int_{\mathscr{C}} x\,dy - y\,dx \longleftarrow$ notación.

$\int_{\mathscr{C}} F$

$F(x,y) = (-y, x)$

$F(x,y,z) = (P, Q, R)$

$$\int_{\mathscr{C}} F = \int_{\mathscr{C}} P\,dx + Q\,dy + R\,dz.$$

$\sigma : [a,b] \to \mathscr{C}$ param. $\quad \sigma(t) = (x(t), y(t), z(t))$

$$\Rightarrow \int_{\mathscr{C}} F\,ds = \int_a^b \langle F(\sigma(t)), \sigma'(t)\rangle\,dt$$

$$= \int_a^b P(\sigma(t))\cdot x'(t) + Q(\sigma(t))\cdot y'(t) + R(\sigma(t))\,z'(t)\,dt$$

Notación $\nearrow$ $= \int_{\mathscr{C}} P\,dx + Q\,dy + R\,dz.$

$$\int_{\mathcal{C}} x\,dy - y\,dx = \int_0^{2\pi} \langle F(\cos(t), \operatorname{sen}(t)), (-\operatorname{sen}(t), \cos(t)) \rangle \, dt$$

$F = (-y, x)$

$$= \int_0^{2\pi} \langle (-\operatorname{sen}(t), \cos(t)), (-\operatorname{sen}(t), \cos(t)) \rangle \, dt.$$

$$= \int_0^{2\pi} 1 \, dt = 2\pi.$$

$$= \int_{\mathcal{C}} x\,dy - \int_{\mathcal{C}} y\,dx$$

$\tilde{F} = (0, x)$ $\quad\tilde{\tilde{F}} = (y, 0)$

---

(10) $\sigma : [a,b] \to \mathbb{R}^3$ una param. reg de $\mathcal{C}$

$g : [\bar{a}, \bar{b}] \to [a,b]$ biyección $\mathcal{C}^1$. $g'(s) \neq 0 \ \forall s \in (\bar{a}, \bar{b})$.

$\bar{\sigma} : [\bar{a}, \bar{b}] \to \mathcal{C} \qquad \bar{\sigma}(s) = \sigma(g(s))$.

a) $\bar{\sigma}$ es param. reg de $\mathcal{C}$.

qvq:
- $\operatorname{Im}(\bar{\sigma}) = \mathcal{C}$
- $\bar{\sigma}$ es $\mathcal{C}^1$
- $\bar{\sigma}$ inyectiva en $(\bar{a}, \bar{b})$
- $\bar{\sigma}'(s) \neq (0,0,0) \quad \forall s \in [\bar{a}, \bar{b}]$.

- $\operatorname{Im}(\bar{\sigma}) = \mathcal{C}$

$\operatorname{Im}(\bar{\sigma}) \subseteq \operatorname{Im}(\sigma)$, y como g es biyectiva y $\bar{\sigma} = \sigma \circ g \Rightarrow \operatorname{Im}(\bar{\sigma}) = \operatorname{Im}(\sigma)$.

Como σ es param de $\mathcal{C}$, tenemos q'

$$\operatorname{Im}(\sigma)=\mathcal{C} \Rightarrow \operatorname{Im}(\bar\sigma)=\mathcal{C}$$

- σ es $\mathcal{C}^1$ x q' es param. reg. ; g es $\mathcal{C}^1$ x hip $\Big\} \Rightarrow \bar\sigma$ es $\mathcal{C}^1$.

- $\bar\sigma(s_0)=\bar\sigma(s_1) \Rightarrow \sigma(g(s_0))=\sigma(g(s_1))$

$\underset{\substack{\sigma \text{ inyectiva}\\ (\text{x ser reg})}}{\Rightarrow} g(s_0)=g(s_1) \underset{g \text{ biyec.}}{\Rightarrow} s_0=s_1 \Rightarrow \bar\sigma$ inyec.

- $\bar\sigma'(s) = \underbrace{\sigma'(g(s))}_{\substack{\neq(0,0,0)\\ \text{x q' } \sigma \text{ es regular}}} \cdot g'(s)$ ⟶ $\neq 0$ x hip.

$\Rightarrow \bar\sigma'(s)\neq(0,0,0)\ \forall s\in(\bar a,\bar b)$

$\Rightarrow \bar\sigma$ regular.

b) $\displaystyle\int_a^b f(\sigma(t))\cdot\|\sigma'(t)\|\,dt = \int_{\bar a}^{\bar b} f(\bar\sigma(s))\|\bar\sigma'(s)\|\,ds$

$= \displaystyle\int_{\bar a}^{\bar b} f(\sigma(g(s)))\cdot\|\sigma'(g(s))\cdot \underbrace{g'(s)}_{\in\mathbb{R}}\|\,ds$

$= \displaystyle\int_{\bar a}^{\bar b} f(\sigma(g(s)))\,\|\sigma'(g(s))\|\cdot|g'(s)|\,ds = \boxtimes$

Como g' es continua y $g'(s)\neq 0\ \forall s\in(\bar a,\bar b)$.

$\Rightarrow g'(s)>0\ \forall s\in(\bar a,\bar b)$ ó $g'(s)<0\ \forall s\in(\bar a,\bar b)$

Si $g'(s)>0 \ \forall s \in (\bar{a},\bar{b}) \Longrightarrow \boxed{*}$

$$= \int_{\bar{a}}^{\bar{b}} f(\sigma(g(s)))\,\|\sigma'(g(s))\| \cdot g'(s)\, ds$$

$$\underset{\downarrow}{=} \int_{a}^{b} f(u)\,\|\sigma'(u)\|\, du$$

$u = g(s)$

$du = g'(s)\,ds$

$g'>0$, g crec.

$g(\bar{a}) = a$

$g(\bar{b}) = b$

. Si $g'(s)<0 \ \forall s \in (\bar{a},\bar{b}) \Longrightarrow \boxed{*} =$

$$= -\int_{\bar{a}}^{\bar{b}} f(\sigma(g(s)))\,\|\sigma'(g(s))\| \cdot g'(s)\, ds$$

$$\underset{\downarrow}{=} -\int_{b}^{a} f(\sigma(u))\,\|\sigma'(u)\|\, du = \int_{a}^{b} f(\sigma(u))\,\|\sigma'(u)\|\, du.$$

$u = g(s)$

$du = g'(s)\,ds$

g decrec. $g(\bar{a}) = b \wedge g(\bar{b}) = a$

12) b) $h(t) = \int_0^t \|\underbrace{(2e^s, 3e^s, -6e^s)}_{\sigma'(s)}\|\, ds$

$$\int \sqrt{1 + 1/x^2}\, dx = \int \sqrt{\frac{x^2+1}{x^2}}\, dx$$

$$= \int \frac{1}{x}\sqrt{x^2+1}\, dx = \frac{1}{2}\int \frac{\sqrt{u}}{\sqrt{u-1}} \cdot \frac{1}{\sqrt{u-1}}\, du$$

$u = x^2+1$

$du = 2x\, dx$

$\frac{du}{2\sqrt{u-1}} = dx$

$$= \frac{1}{2}\int \frac{\sqrt{u}}{u-1}\, du = \frac{1}{2}\int \frac{r}{r^2-1}\, 2 \cdot r\, dr.$$

$r = \sqrt{u}$

$dr = \frac{1}{2\sqrt{u}}\, du$

$2r\, dr = du$

fracciones simples.