1) a) $P(x,y) = xy^2,\ Q(x,y) = -yx^2,\ D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq R^2\} \Rightarrow \sigma(t) = (R\cos(t), R\,\mathrm{sen}(t))$ con $t \in [0,2\pi]$

$$\Rightarrow \int_{C^+} P\,dx + Q\,dy = \int_0^{2\pi} \langle (P(\sigma(t)), Q(\sigma(t))), \sigma'(t) \rangle\, dt = \int_0^{2\pi} \langle (R^3\cos t\,\mathrm{sen}^2 t, -R^3\,\mathrm{sen}\,t\cos^2 t)(-R\,\mathrm{sen}\,t, R\cos t) \rangle\, dt$$

$$= -\int_0^{2\pi} R^4 \cos t\,\mathrm{sen}^3 t + R^4\,\mathrm{sen}\,t\cos^3 t\; dt = -\int_0^{2\pi} R^4 \cos t\,\underline{\mathrm{sen}\,t}\; dt = -R^4 \int_0^0 u\,du = -R^4 \frac{u^2}{2}\Big|_0^0 = -R^4 \frac{\mathrm{sen}^2 t}{2}\Big|_0^{2\pi} = 0$$

$u = \mathrm{sen}\,t$
$du = \cos t\,dt$

$$\Rightarrow \iint_D \left(\frac{\partial Q}{\partial x}(x,y) - \frac{\partial P}{\partial y}(x,y)\right) dx\,dy = \int_{-R}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} (-2yx - 2yx)\, dx\,dy = \int_{-R}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} -4xy\,dx\,dy = \int_{-R}^{R} -4x \frac{y^2}{2}\Big|_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dx$$

$$= \int_{-R}^{R} -4x \left(\cancel{\frac{R^2-x^2}{2} - \frac{R^2-x^2}{2}}^{\,0}\right) dx = 0$$

b) $P(x,y) = 2y,\ Q(x,y) = x$

$$\Rightarrow \int_{C^+} P\,dx + Q\,dy = \int_0^{2\pi} \langle (P(\sigma(t)), Q(\sigma(t))), \sigma'(t) \rangle\, dt = \int_0^{2\pi} \langle (2R\,\mathrm{sen}\,t, R\cos t), (-R\,\mathrm{sen}\,t, R\cos t) \rangle\, dt = \int_0^{2\pi} (-2R^2\mathrm{sen}^2 t + R^2\cos^2 t)\, dt$$

$$= \int_0^{2\pi} R^2(\cos^2 t - 2\,\mathrm{sen}^2 t)\, dt = \int_0^{2\pi} R^2(\cos^2 t - 2(1-\cos^2 t))\, dt = R^2 \int_0^{2\pi} (3\cos^2 t - 2)\, dt$$

$$= R^2\left(\int_0^{2\pi} \frac{3}{2}\cos(2t) + \frac{3}{2}\, dt - \int_0^{2\pi} 2\, dt\right) = R^2\left(\cancel{\frac{3}{4}\mathrm{sen}(2t)\Big|_0^{2\pi}}^{\,0} + \frac{3}{2}t\Big|_0^{2\pi} - 2t\Big|_0^{2\pi}\right) = R^2(3\pi - 4\pi) = -\pi R^2$$

$$\Rightarrow \iint_D \left(\frac{\partial Q}{\partial x}(x,y) - \frac{\partial P}{\partial y}(x,y)\right) dx\,dy = \iint (1-2)\, dx\,dy = \int_{-R}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} -1\, dx\,dy = -\int_0^R \int_0^{2\pi} -r\,.\,d\theta\,dr = 2\pi\left(-\frac{r^2}{2}\right)\Big|_0^R = -\pi R^2$$

uso coordenadas polares

② $\int_C y^2dx + x\,dy \Rightarrow P(x,y)=y^2,\ Q(x,y)=x$

a) $D=\{(x,y)/\ 0\le x\le 2,\ 0\le y\le 2\}$ $\quad \sigma_1(t)=(t,0)$ con $t\in[0,2]$, $\sigma_2(t)=(2,t)$ con $t\in[0,2]$, $\sigma_3(t)=(t,2)$ con $t\in[0,2]$, $\sigma_4(t)=(0,t)$ con $t\in[0,2]$

$$\Rightarrow \int_C Pdx+Qdy = \int_0^2 <(0,t),(1,0)>dt + \int_0^2 <(t^2,2),(0,1)>dt - \int_0^2 <(4,t),(1,2)>dt - \int_0^2<(t,0),(0,1)>dt = \int_0^2 0\,dt + \int_0^2 2\,dt - \int_0^2 4\,dt + \int_0^2 0\,dt = 2t\Big|_0^2 - 4t\Big|_0^2 = 4-8 = -4$$

$$\Rightarrow \iint_D \left(\frac{\partial Q}{\partial x}(x,y) - \frac{\partial P}{\partial y}(x,y)\right)dxdy = \int_0^2\int_0^2 (1-2y)dxdy = \int_0^2\int_0^2 1\,dxdy - 2\int_0^2\int_0^2 y\,dxdy = \int_0^2 x\Big|_0^2 dy - 2\int_0^2 yx\Big|_0^2 dy = 2y\Big|_0^2 - 4\frac{y^2}{2}\Big|_0^2$$

$$= 4-4.2 = -4$$

b) $D=\{(x,y)/ \frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1\} \Rightarrow C=\partial D: \sigma_1(t)=(a\cos t,\ b\,\text{sen}\,t)$ con $t\in[0,2\pi]$

$$\Rightarrow \int_C Pdx+Qdy = \int_0^{2\pi}(b^2\text{sen}^2t,\ a\cos t)(-a\,\text{sen}\,t,\ b\cos t)\,dt = \int_0^{2\pi}(-ab^2\text{sen}^3t + ab\cos^2t)\,dt = \int_0^{2\pi} -ab^2 \underbrace{\text{sen}^3t}_{\text{sen}^2t\,.\,\text{sen}t}\,dt + \int_0^{2\pi} ab\cos^2t\,dt$$

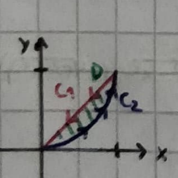
$$= ab\int_0^{2\pi}\frac12(\cos(2t)+1)\,dt - ab^2\int_0^{2\pi} \text{sen}t.\left(\frac{1-\cos(2t)}{2}\right) = \frac{ab}{2}.\left(\overset{0}{\cancel{\frac{\text{sen}(2t)}{2}}}\Big|_0^{2\pi} + t\Big|_0^{2\pi}\right) - \frac{ab^2}{2}\int_0^{2\pi} \frac{\text{sen}t}{2} - \underbrace{\frac{\text{sen}t\cos(2t)}{2}}_{\frac{\text{sen}(3t)-\text{sen}t}{4}}\,dt$$

$$= ab\pi - \frac{ab^2}{2}\left(\int_0^{2\pi}\frac{3\,\text{sen}t}{4}\,dt - \int_0^{2\pi}\frac{\text{sen}(3t)}{4}\right) = ab\pi - \frac{ab^2}{2}\left(\overset{0}{\cancel{-\frac{3\cos t}{4}}}\Big|_0^{2\pi} + \overset{0}{\cancel{\frac{\cos(3t)}{12}}}\Big|_0^{2\pi}\right) = ab\pi$$

$$\Rightarrow \iint_D (Q_x-P_y)\,dxdy = \iint (1-2y)\,dxdy \overset{\text{uso coordenadas elípticas}}{=} \int_0^1\int_0^{2\pi} abr.(1-2b\,\text{sen}\theta)\,d\theta dr = \int_0^1\int_0^{2\pi} abr\,d\theta dr - \int_0^{2\pi}\int_0^1 2ab^2\text{sen}\theta\,dr d\theta = 2\pi ab\frac{r^2}{2}\Big|_0^1 - 2ab^2\overset{0}{\cancel{(-\cos\theta)}}\Big|_0^{2\pi}$$

$$= 2\pi ab.\frac12 = ab\pi$$

c)



$D=\{(x,y)/\ 0\le x\le 1,\ x^2\le y\le x\} \Rightarrow C_1: \sigma_1(t)=(t,t)$ con $t\in[0,1]$, $C_2: \sigma_2(t)=(t,t^2)$ con $t\in[0,1]$

$$\Rightarrow \int_C Pdx+Qdy = \int_{C_1^-+C_2^+} Pdx+Qdy = -\int_{C_1^-}(t^2,t)(1,1)\,dt + \int_{C_2^+}(t^3,t)(1,2t)\,dt = -\int_0^1(t^2+t)\,dt + \int_0^1(t^3+2t^2)\,dt$$

$$= \frac{t^4}{4}\Big|_0^1 + 2\frac{t^3}{3}\Big|_0^1 - \frac{t^2}{2}\Big|_0^1 - \frac{t^3}{3}\Big|_0^1 = \frac14+\frac23-\frac12-\frac13 = 1/12$$

$$\Rightarrow \iint_D(Q_x-P_x)\,dxdy = \int_0^1\int_{x^2}^x(1-2y)\,dxdy = \int_0^1 (y-y^2)\Big|_{x^2}^x dx = \int_0^1(x-x^2-(x^2-x^4))\,dx = \int_0^1(x^4-2x^2+x)\,dx = \frac{x^5}{5}\Big|_0^1 - \frac23x^3\Big|_0^1 + \frac{x^2}{2}\Big|_0^1$$

$$=\frac15-\frac23+\frac12 = 1/30$$

$\Rightarrow 1/12 \ne 1/30 \Rightarrow$ D es una región tipo 2, no una región tipo 3

3) a) Defino $P(x,y)=-y$, $Q(x,y)=x$ $\Rightarrow \left(\frac{\partial Q}{\partial x}(x,y)-\frac{\partial P}{\partial y}(x,y)\right)=2 \Rightarrow$ de forma que $\iint_D 2\,dx\,dy=\int_C -y\,dx+x\,dy = 2\cdot \text{Area}(D)$

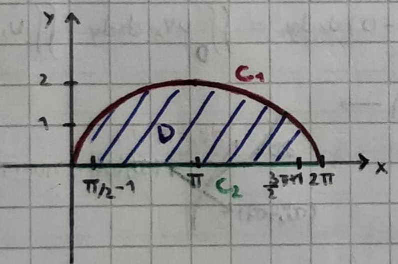
$D=\{(x,y)/\ x^2+y^2\leq R^2\} \Rightarrow \sigma(t)=(R\cos t, R\,\text{sen}\,t)$ con $t\in[0,2\pi]$

$$\Rightarrow \frac{1}{2}\int_C -y\,dx+x\,dy=\frac{1}{2}\int_0^{2\pi}\langle(-R\,\text{sen}\,t, R\cos t),(-R\,\text{sen}\,t, R\cos t)\rangle dt=\frac{1}{2}\int_0^{2\pi}(R^2\text{sen}^2t+R^2\cos^2t)\,dt=\frac{1}{2}\int_0^{2\pi}R^2\,dt=\frac{1}{2}R^2t\Big|_0^{2\pi}=\pi R^2$$

b) $D=\{(x,y)/\ \frac{x^2}{a^2}+\frac{y^2}{b^2}\leq 1\} \Rightarrow \sigma(t)=(a\cos t, b\,\text{sen}\,t)$ con $t\in[0,2\pi]$

$$\Rightarrow \frac{1}{2}\int_C -y\,dx+x\,dy=\frac{1}{2}\int_0^{2\pi}\langle(-b\,\text{sen}\,t, a\cos t),(-a\cos t, b\cos t)\rangle dt=\frac{1}{2}\int_0^{2\pi}(ab\,\text{sen}^2t+ab\cos^2t)\,dt=\frac{1}{2}\int_0^{2\pi}ab\,dt=ab\pi$$

4)



$\Rightarrow C_1: \sigma_1(t)=(t-\text{sen}\,t, 1-\cos t)$ con $t\in[0,2\pi]$

$C_2: \sigma_2(t)=(t,0)$ con $t\in[0,2\pi]$

$\Rightarrow$ Defino $P(x,y)=-y$, $Q(x,y)=x$

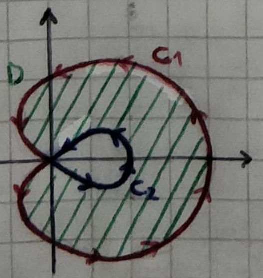
$$\Rightarrow \text{Area}(D)=\frac{1}{2}\int_{C_1\cup C_2}-y\,dx+x\,dy=-\frac{1}{2}\int_{C_1^-}-y\,dx+x\,dx+\frac{1}{2}\int_{C_2^+}-y\,dx+x\,dx=\frac{-1}{2}\int_0^{2\pi}\langle(\cos t-1, t-\text{sen}\,t),(1-\cos t, \text{sen}\,t)\rangle dt+\frac{1}{2}\int_0^{2\pi}\langle(0,t),(1,0)\rangle dt$$

$$=-\frac{1}{2}\int_0^{2\pi}(\cos t-1)(1-\cos t)+(t\,\text{sen}\,t-\text{sen}^2t)\,dt=\frac{-1}{2}\int_0^{2\pi}(\cos t-\cos^2t-1+\cos t+t\,\text{sen}\,t-\text{sen}^2t)\,dt$$

$$=-\frac{1}{2}\int_0^{2\pi}(2\cos t-1-1+t\,\text{sen}\,t)\,dt=\frac{-1}{2}\left(2\,\text{sen}\,t\Big|_0^{2\pi}-2t\Big|_0^{2\pi}+(\text{sen}\,t-t\cos t)\Big|_0^{2\pi}\right)=\frac{-1}{2}(-4\pi-2\pi)=3\pi$$

5) $C_1: \sigma_1(t)=((1+\cos t)\cos t, (1+\cos t)\,\text{sen}\,t)$ con $t\in[-\pi,\pi]$

$C_2: \sigma_2(t)=(\sqrt{\cos^2t-\text{sen}^2t}\,\cos t, \sqrt{\cos^2t-\text{sen}^2t}\,\text{sen}\,t)$ con $t\in[-\pi/4,\pi/4]$



$$\text{Area}(D)=\frac{1}{2}\int_{C_1^+\cup C_2^-}-y\,dx+x\,dy=\frac{1}{2}\left[\int_{C_1^+}-y\,dx+x\,dx-\int_{C_2^-}-y\,dx+x\,dy\right]$$

$\Rightarrow$ Viendo que tanto σ_1 como σ_2 son de la forma $(r(t)\cos t, r(t)\,\text{sen}\,t)$:

$$\int_{C_1/C_2}-y\,dx+x\,dx=\int_{-\pi}^{\pi}(-r(t)\,\text{sen}\,t, r(t)\cos t)(r'(t)\cos t-r(t)\,\text{sen}\,t, r'(t)\,\text{sen}\,t+r(t)\cos t)\,dt$$

$$=\int_{-\pi}^{\pi}(\cancel{-r(t)r'(t)\cos t\,\text{sen}\,t}+r^2(t)\,\text{sen}^2t+\cancel{r(t)r'(t)\cos t\,\text{sen}\,t}+r^2(t)\cos^2t)\,dt$$

$$=\int_{-\pi}^{\pi}r^2(t)(\cos^2t+\text{sen}^2t)\,dt=\int_{-\pi}^{\pi}r^2(t)\,dt$$

$$\Rightarrow \int_{C_1} -y\,dx + x\,dy = \int_{-\pi}^{\pi} (1+\cos t)^2\, dt = \int_{-\pi}^{\pi}\Big(1+2\cos t + \underbrace{\cos^2 t}_{\frac{1}{2}(\cos(2x)+1)}\Big)dt = t\Big|_{-\pi}^{\pi} + 2\,\overset{0}{\cancel{\operatorname{sen} t}}\Big|_{-\pi}^{\pi} + \overset{0}{\cancel{\frac{\operatorname{sen}(2t)}{4}}}\Big|_{-\pi}^{\pi} + \frac{t}{2}\Big|_{-\pi}^{\pi} = 2\pi + \pi = 3\pi$$

$$\Rightarrow \int_{C_2} -y\,dx + x\,dx = \int_{-\pi/4}^{\pi/4} \left(\cancel{\sqrt{\ }}\,\overline{\cos^2 t - \operatorname{sen}^2 t}\right)^{\cancel{2}} dt = \int_{-\pi/4}^{\pi/4} \underbrace{\cos^2 t}_{\frac{1}{2}(\cos(2t)+1)}\, dt - \int_{-\pi/4}^{\pi/4} \underbrace{\operatorname{sen}^2 t}_{\frac{1}{2}(1-\cos(2t))}\, dt = \frac{1}{2}\left(\frac{\operatorname{sen}(2t)}{2}\Big|_{-\pi/4}^{\pi/4} + \cancel{t\Big|_{-\pi/4}^{\pi/4}} + \frac{\operatorname{sen}(2t)}{2}\Big|_{-\pi/4}^{\pi/4} - \cancel{t\Big|_{-\pi/4}^{\pi/4}} \right)$$

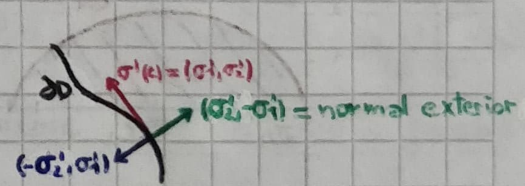
$$= \frac{1}{2}\left(\frac{1}{2} - \left(-\frac{1}{2}\right) + \frac{1}{2} - \left(-\frac{1}{2}\right)\right) = 1$$

$$\Rightarrow \text{Area}(D) = \frac{1}{2}\int_{C_1} -y\,dx + x\,dy - \int_{C_2} -y\,dx + x\,dy = \frac{3\pi}{2} - \frac{1}{2}$$

⑥ $$\iint_D u v_x\, dxdy = -\iint_D u_x v\, dx\,dy + \int_{\partial D} u v n_1\, ds \Rightarrow \iint_D u v_x\, dx\,dy + \iint_D u_x v\, dxdy = \iint_D \underbrace{(u v_x + u_x v)}_{\frac{\partial(uv)}{\partial x}}\, dxdx$$

$\Rightarrow$ Considero $F(x,y) = (\underbrace{0}_{P}, \underbrace{u.v}_{Q}) \Rightarrow \int_{C=\partial D} F\, ds = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy = \iint_D (u v_x + u_x v - 0)\, dxdy = \iint_D u v_x\, dxdy + \iint u_x v\, dxdy$

$\Rightarrow$ Defino parametrización de ∂D: $\sigma(t) = (\sigma_1, \sigma_2)$ con $t \in [a,b]$ $\Rightarrow \sigma'(t) = (\sigma_1', \sigma_2') \longrightarrow$



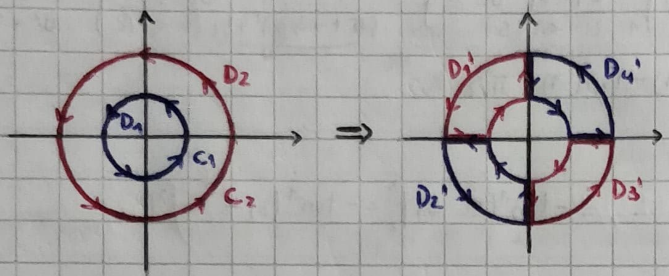
$$\Rightarrow \int_{\partial D} F\, ds = \int_a^b (0, uv(\sigma(t)))(\sigma_1', \sigma_2')\, dt = \int_a^b uv(\sigma(t)) \cdot \sigma_2'\, dt$$

$$\Rightarrow n = (n_1, n_2) = \frac{(\sigma_2', -\sigma_1')}{\|\sigma'(t)\|} \Rightarrow n_1 = \frac{\sigma_2'}{\|\sigma'(t)\|}$$

$$\Rightarrow \int_a^b uv(\sigma(t)) . \sigma_2' . \frac{\|\sigma'(t)\|}{\|\sigma'(t)\|}\, dt = \int_a^b \underbrace{uv(\sigma(t)) . n_1}_{F}\, \underbrace{\|\sigma'(t)\|\, dt}_{ds}$$

(7) P, Q funciones C^1 en $\mathbb{R}^2$, $D=\left[(x,y)/\ 1\le x^2+y^2\le 4\right]$ $\Rightarrow$ Divido en D_1 y D_2

$\Rightarrow D_1=\left[(x,y)/\ x^2+y^2\le 1\right]$, $D_2=\left\{(x,y)/\ x^2+y^2\le 4\right\}$



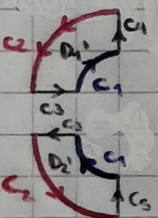
$\Rightarrow D=D_1'\cup D_2'\cup D_3'\cup D_4' \rightarrow$ Divido el anillo en 4 regiones tipo III y las recorro en sentido positivo

$\Rightarrow$ Veo que los bordes rectos los recorro 2 veces, 1 en cada sentido de forma que se cancelan, y resultan una curva que recorre el borde interior en sentido horario y una que recorre el borde exterior en sentido antihorario

$\Rightarrow C_1: \sigma_1(t)=(\cos t, \operatorname{sen} t)$ con $t\in[0,2\pi]$

$C_2: \sigma_2(t)=(2\cos t, 2\operatorname{sen} t)$ con $t\in[0,2\pi]$

$\Rightarrow$



$\Rightarrow \iint_{D_1'}(Q_x-P_y)\,dx\,dy=\int_{C_2^+\cup C_3^+\cup C_1^-\cup C_4^+} P\,dx+Q\,dy=\int_{C_2^+}P\,dx+Q\,dy+\int_{C_3^+}P\,dx+Q\,dy-\int_{C_1^-}P\,dx+Q\,dy+\int_{C_4^+}P\,dx+Q\,dy$ $\oplus$

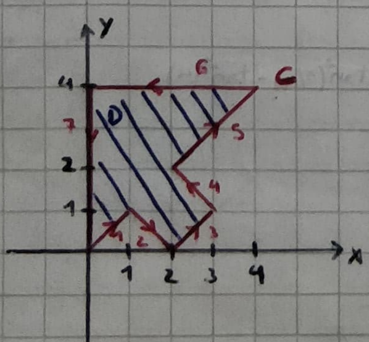
$\Rightarrow \iint_{D_2'}(Q_x-P_y)\,dx\,dy=\int_{C_2^+\cup C_5^+\cup C_1^-\cup C_3^-} P\,dx+Q\,dy=\int_{C_2^+}P\,dx+Q\,dy+\int_{C_5^+}P\,dx+Q\,dy-\int_{C_1^-}P\,dx+Q\,dy-\int_{C_3^-}P\,dx+Q\,dy$ $\ominus$

$\Rightarrow$ Como $D=D_1'\cup D_2'\cup D_3'\cup D_4' \Rightarrow \iint_D(Q_x-P_y)\,dx\,dy=\iint_{D_1'}(Q_x-P_y)\,dx\,dy+\iint_{D_2'}(Q_x-P_y)\,dx\,dy+\iint_{D_3'}(Q_x-P_y)\,dx\,dy+\iint_{D_4'}(Q_x-P_x)\,dx\,dy$

$\Rightarrow$ Al sumar las integrales de las 4 divisiones de D, las curvas horizontales y verticales que usamos para cerrar dichas divisiones son recorridas 2 veces cada una, una vez con orientación positiva y otra negativa, por lo que las integrales de curva se cancelan, y solo me quedan los terminos que corresponden a partes de C_1 (orientada positivamente) y C_2 (orientada negativamente)

$\Rightarrow \iint_D(Q_x-P_y)\,dx\,dy=\int_{C_2^+}P\,dx+Q\,dy-\int_{C_1^-}P\,dx+Q\,dx$

(8)



$\Rightarrow \int_C P\,dx+Q\,dy \Rightarrow P=\dfrac{y}{(x-1)^2+y^2}$, $Q=\dfrac{1-x}{(x-1)^2+y^2} \Rightarrow F=(P,Q)$

$\Rightarrow$ Primero me fijo que F este definido en $D\subset\mathbb{R}^2$

$\Rightarrow P$ y Q no estan definidas en el punto $(1,0) \rightarrow (1-1)^2+0^2=0$

Pero el punto $(1,0)$ no esta dentro de D, por lo que F esta definido en D

$\Rightarrow$ Divido C en 7 curvas concatenadas: $\sigma_1(t)=(t,t)$ con $t\in[0,1]$, $\sigma_2(t)=(t,2-t)$ con $t\in[1,2]$, $\sigma_3(t)=(t,t-2)$ con $t\in[2,3]$

$\sigma_4(t)=(t,4-t)$ con $t\in[2,3]$, $\sigma_5(t)=(t,t)$ con $t\in[2,4]$, $\sigma_6(t)=(t,4)$ con $t\in[0,4]$, $\sigma_7(t)=(0,t)$ con $t\in[0,4]$

$$\Rightarrow \int_C P\,dx + Q\,dx = \int_{C_1\cup C_2\cup C_3\cup C_4\cup C_5\cup C_6\cup C_7} P\,dx + Q\,dx = \int_{C_1^+} Pdx+Qdy + \int_{C_2^+} Pdx+Qdy + \int_{C_3^+} Pdx+Qdy - \int_{C_4^-} Pdx+Qdx + \int_{C_5^+} Pdx+Qdx + \int_{C_6^-} Pdx+Qdx - \int_{C_7^-} Pdx+Qdx$$

(1) (2) (3) (4) (5) (6) (7)

(1) $= \int_0^1 \left(\frac{t}{(t-1)^2+t^2}, \frac{1-t}{(t-1)^2+t^2} \right)(1,1)\,dt = \int_0^1 \frac{t+(1-t)}{(t-1)^2+t^2}\,dt = \int_0^1 \frac{1}{2t^2-2t+1}\,dt = \int_0^1 \frac{1}{(\underbrace{\sqrt{2}t-1/\sqrt{2}}_{u})^2+1/2}\,dt = \int \frac{1}{\sqrt{2}} \cdot \frac{1}{u^2+1/2}\,du = \frac{1}{\sqrt{2}} \int \frac{2}{2u^2+1}\,du$

$= \sqrt{2} \int \frac{1}{\underbrace{2u^2}_{s=\sqrt{2}u}+1}\,du = \sqrt{2} \int \frac{1}{\sqrt{2}} \frac{1}{s^2+1}\,ds = \tan^{-1}(s)\Big|_{\sqrt{2}u} = \tan^{-1}(\sqrt{2}u)\Big|_{\sqrt{2}t-1/\sqrt{2}} = \tan^{-1}(2t-1)\Big|_0^1 = \tan^{-1}(1) - \tan^{-1}(-1) = 2\tan^{-1}(1) = 2\cdot\frac{\pi}{4} = \pi/2$

(2) $= \int_1^2 \left(\frac{2-t}{(t-1)^2+(2-t)^2}, \frac{1-t}{(t-1)^2+(2-t)^2} \right)(1,-1)\,dt = \int_1^2 \frac{1}{(t-1)^2+(2-t)^2}\,dt = \int_1^2 \frac{1}{2t^2-6t+5}\,dt = \int_1^2 \frac{1}{(\underbrace{\sqrt{2}t-3/\sqrt{2}}_{u})^2+1/2}\,dt = \frac{1}{\sqrt{2}} \int \frac{1}{u^2+1/2}\,du = \int \frac{1}{s^2+1}\,ds$

$s=\sqrt{2}u$

$= \tan^{-1}(s)\Big|_{\sqrt{2}u} = \tan^{-1}(\sqrt{2}u)\Big|_{\sqrt{2}t-3/\sqrt{2}} = \tan^{-1}(2t-3)\Big|_1^2 = \tan^{-1}(1) - \tan^{-1}(-1) = \pi/2.$

(3) $= \int_2^3 \left(\frac{t-2}{(t-1)^2+(t-2)^2}, \frac{1-t}{(t-1)^2+(t-2)^2} \right)(1,1)\,dt = \int_2^3 \frac{-1}{2t^2-6t+5}\,dt = \dots = -\tan^{-1}(2t-3)\Big|_2^3 = -\tan^{-1}(3) + \tan^{-1}(1)$

(4) $= \int_2^3 \left(\frac{4-t}{(t-1)^2+(4-t)^2}, \frac{1-t}{(t-1)^2+(4-t)^2} \right)(1,-1)\,dt = \int_2^3 \frac{3}{2t^2-10t+17}\,dt = \dots = 2\tan^{-1}(1/3)$

(5) $= \int_2^4 \left(\frac{t}{(t-1)^2+t^2}, \frac{1-t}{(t-1)^2+t^2} \right)(1,1)\,dt = \int_2^4 \frac{1}{2t^2-2t+1}\,dt = \dots = \tan^{-1}(2t-1)\Big|_2^4 = \tan^{-1}(7) - \tan^{-1}(3)$

(6) $= \int_0^4 \left(\frac{4}{(t-1)^2+4^2}, \frac{1-t}{(t-1)^2+4^2} \right)(1,0)\,dt = \int_0^4 \frac{4}{t^2-2t+17}\,dt = \int_0^4 \frac{4}{(\underbrace{t-1}_{u})^2+16}\,dt = \int_{-1}^3 \frac{4}{u^2+16}\,du = \int_{-1}^3 \frac{4}{16} \frac{1}{\underbrace{\frac{u^2}{16}}_{s=u/4}+1}\,du = \frac{\cancel{1}}{\cancel{4}} \int_{-1/4}^{3/4} \frac{1}{s^2+1}\,ds$

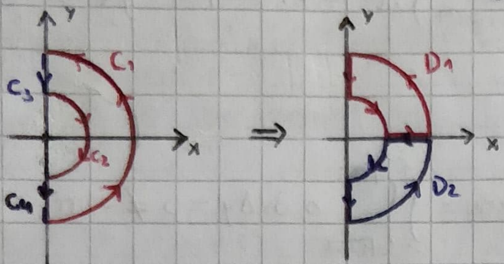
$= \frac{\cancel{1}}{\cancel{4}} \tan^{-1}(s)\Big|_{-1/4}^{3/4} = \frac{\cancel{1}}{\cancel{4}}\left(\tan^{-1}(3/4) - \tan^{-1}(-1/4) \right) = \tan(3/4) + \tan(1/4)$

(7) $= \int_0^4 \left(\frac{t}{(0-1)^2+t^2}, \frac{1}{(0-1)^2+t^2} \right)(0,1)\,dt = \int_0^4 \frac{1}{t^2+1}\,dt = \tan^{-1}(t)\Big|_0^4 = \tan^{-1}(4)$

$$\Rightarrow \int_C Pdx+Qdy = \pi/2 + \pi/2 - \tan^{-1}(3) + \pi/4 - 2\tan^{-1}(1/3) + \tan^{-1}(7) - \tan^{-1}(3) - \tan^{-1}(3/4) - \tan^{-1}(1/4) - \tan^{-1}(4) = 0$$

$$\Rightarrow \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy = \iint_D \left(\frac{\partial\left(\frac{x-1}{(x-1)^2+y^2}\right)}{\partial x} - \frac{\partial\left(\frac{y}{(x-1)^2+y^2}\right)}{\partial y} \right) dxdy = \iint_D \left(\frac{x^2-2x+1-y^2}{((x-1)^2+y^2)^2} - \frac{x^2-2x+1-y^2}{((x-1)^2+y^2)^2} \right) dxdy = 0 \Rightarrow$$ MUCHO mas facil

(9) $D=\{(x,y)/1\le x^2+y^2\le 4, x\ge 0\} \Rightarrow \int_{\partial D} x^2y\,dx - xy^2\,dy \Rightarrow P=x^2y\ , Q=-xy^2$



⇒ Divido D en 2 regiones tipo 3. Veo que el límite horizontal es recorrido por ambas curvas en sentido contrario. Por lo tanto se cancela.

⇒ Veo que la curva resultante que recorre la región recorre el borde exterior en sentido antihorario y el borde interior en sentido horario

⇒ Divido el recorrido de la región en 4 curvas concatenadas: $\sigma_1(t)=(2\cos t, 2\,\text{sen}t)$ con $t\in[-\pi/2, \pi/2]$, $\sigma_2(t)=(\cos t,\text{sen}t)$ con $t\in[-\pi/2,\pi/2]$
$\sigma_3(t)=(0,t)$ con $t\in[1,2]$, $\sigma_4(t)=(0,t)$ con $t\in[-2,-1]$

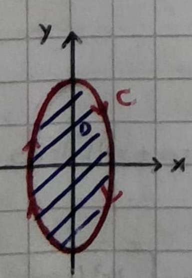
$$\Rightarrow \int_{C=\partial D} Pdx+Qdy = \int_{C_1\cup C_2\cup C_3\cup C_4} Pdx+Qdy = \int_{-\pi/2}^{\pi/2}(4\cos^2t.2\,\text{sen}t, -2\cos t.4\,\text{sen}^2t)(-2\,\text{sen}t, 2\cos t)dt - \int_{-\pi/2}^{\pi/2}(\cos^2t\,\text{sen}t, \cos t\,\text{sen}^2t)(-\text{sen}t,\cos t)dt + \int_{C_3\cup C_4}Pdx+Qdy$$

$$= \int_{-\pi/2}^{\pi/2}(-16\cos^2t.\text{sen}^2t - 16\cos^2t.\text{sen}^2t)dt - \int_{-\pi/2}^{\pi/2}(-\cos^2t\,\text{sen}^2t + \cos^2t\,\text{sen}^2t)dt - \int_1^2 0.(0,1)dt - \int_{-2}^{-1}0(0,1)dt$$

$$= \int_{-\pi/2}^{\pi/2} -36\underbrace{\cos^2t.\text{sen}^2t}_{\frac14 \text{sen}^2(2x)}\, dt + \int_{-\pi/2}^{\pi/2} 2\cos^2t\,\text{sen}^2t\,dt - 0 - 0 = -\frac{36}{4}\int_{-\pi/2}^{\pi/2}\underbrace{\text{sen}^2(2t)}_{\frac12-\frac12\cos(4t)}dt + \int_{-\pi/2}^{\pi/2}\frac{\text{sen}^2(2t)}{2}dt$$

$$= \int_{-\pi/2}^{\pi/2}\frac14 - \frac14\cos(4t)dt - 8\int_{-\pi/2}^{\pi/2}\frac12-\frac12\cos(4t)dt = \frac14 t\Big|_{-\pi/2}^{\pi/2} - \frac1{16}\overset{0}{\cancel{\text{sen}(4t)}}\Big|_{-\pi/2}^{\pi/2} - 4t\Big|_{-\pi/2}^{\pi/2} + \overset{0}{\cancel{\text{sen}(4t)}}\Big|_{-\pi/2}^{\pi/2} = \pi/4 - 4\pi = -\frac{15}{4}\pi$$

(10) $F(x,y)=(y+3x, 2y-x) \Rightarrow P=(y+3x)\ , Q=(2y-x)$
$D=\{(x,y)/4x^2+y^2\le 4\}$



$\Rightarrow C: \sigma(t)=(\cos t, 2\,\text{sen}t)$ con $t\in[0,2\pi]$

$$\Rightarrow \int_{C^-} Pdx+Qdy = -\int_0^{2\pi}(2\,\text{sen}t+3\cos t, 2\,\text{sen}t-\cos t)(-\text{sen}t, 2\cos t)dt$$

$$= -\int_0^{2\pi}(-2\,\text{sen}^2t - 3\cos t\,\text{sen}t + 2\,\text{sen}t\cos t - 2\cos^2t)dt = -\int(-2+\cos t\,\text{sen}t)dt$$

$$= \int_0^{2\pi}(2+\cos t\,\text{sen}t)dt = 2t\Big|_0^{2\pi} + \int_0^{2\pi}\frac{\text{sen}(2t)}{2}dt = 4\pi + \cancel{\left(\frac{-\cos(2t)}{4}\right)}\Big|_0^{2\pi} = 4\pi$$

$$\Rightarrow \text{Otra forma: } \int_{C^-}Pdx+Qdy = -\iint_D(Q_x-P_y)dxdy = -\iint_D(-1-1)dxdy = -\iint_D -2\,dxdy = 2\iint_D dxdy = 2\int_0^2\int_0^{2\pi}1\frac{1}{2}.r.drd\theta = 2\pi\frac{r^2}{2}\Big|_0^2$$

$= 4\pi$

11) $F(x,y) = (P(x,y), Q(x,y)) = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2}\right)$

$D = \{(x,y) / x^2+y^2 \le 1\} \Rightarrow C: \sigma(t) = (\cos t, \text{sen}\, t)$ con $t \in [0,2\pi]$

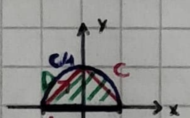
$$\Rightarrow \int_D Q_x - P_y = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy \overset{\text{Por Green}}{=} \int_C Pdx + Qdy = \int_0^{2\pi} \left(\frac{\text{sen}\, t}{\cos^2 t + \text{sen}^2 t}, \frac{-\cos t}{\text{sen}^2 t + \cos^2 t}\right)\cdot(-\text{sen}\, t, \cos t)\, dt = \int_0^{2\pi} (-\text{sen}^2 t - \cos^2 t)\, dt$$

$$= \int_0^{2\pi} -1\, dt = -t \Big|_0^{2\pi} = -2\pi$$

$$\Rightarrow \text{Pero por otro lado: } \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(\frac{x^2-y^2}{(x^2+y^2)^2} - \frac{x^2-y^2}{(x^2+y^2)^2}\right) dxdy = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 0\, dxdy = 0 \ne -2\pi$$

$\Rightarrow$ Green no se satisface en este caso, ya que F no es C^1 en D (específicamente el punto $(0,0)$)

12) $f_1(x,y) = \dfrac{x\,\text{sen}\left(\frac{\pi}{2(x^2+y^2)}\right) - y(x^2+y^2)}{(x^2+y^2)^2}$, $f_2(x,y) = \dfrac{y\,\text{sen}\left(\frac{\pi}{2(x^2+y^2)}\right) + x(x^2+y^2)}{(x^2+y^2)^2}$



$\Rightarrow$ Divido C en 2 curvas: $\sigma_1(t) = (t, t+1)$ con $t \in [-1,0]$, $\sigma_2(t) = (t, 1-t)$ con $t \in [0,1]$

$C_4: \sigma_4(t): (\cos t, \text{sen}\, t)$ con $t \in [0,\pi]$

$\Rightarrow$ Cierro la curva con $\cancel{C_3}$: $\sigma_3(t): (t,0)$ con $t \in [-1,1]$ X NO, si cierro la curva con C_3, el punto $(0,0)$ queda dentro de D, y f no es continua en este punto

$$\Rightarrow \int_C f_1 dx + f_2 dy \overset{\text{Green}}{=} \iint_D \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dxdy = \iint_D \left(\left(\frac{\frac{-xy\cos\left(\frac{\pi}{2(x^2+y^2)}\right)}{(x^2+y^2)^2}\cdot(x^2+y^2)^2 - y\,\text{sen}\left(\frac{\pi}{2(x^2+y^2)}\right)\cdot 2x(x^2+y^2)}{(x^2+y^2)^4} + \frac{y^2-x^2}{(x^2+y^2)^2} \right) - \right.$$

$$\left. \left(\frac{\frac{-xy\cos\left(\frac{\pi}{2(x^2+y^2)}\right)}{(x^2+y^2)^2}\cdot(x^2+y^2)^2 - x\,\text{sen}\left(\frac{\pi}{2(x^2+y^2)}\right)\cdot 2y(x^2+y^2)}{(x^2+y^2)^4} - \frac{x^2-y^2}{(x^2+y^2)^2} \right) \right) dxdy$$

$$= \iint_D 0\, dxdy = 0$$

13)

$$\int_C -y^2 dx + 3x dy \Rightarrow P = -y^2 , Q = 3x \Rightarrow Q_x - P_y = 3 + 2y$$

$$\Rightarrow D = \{(x,y) \in \mathbb{R}^2 / (x-x_0)^2 + (y-y_0)^2 \le R^2\} \Rightarrow \begin{matrix} x = r\cos\theta + x_0 \\ y = r\,\text{sen}\,\theta + y_0 \end{matrix}$$

$$\Rightarrow \int_C -y^2 dx + 3x dy \overset{\text{Green}}{=} \iint_D (3+2y)\, dxdy = \int_{-(R-x_0)}^{R-x_0} \int_{-(\sqrt{R^2-(x-x_0)}-y_0)}^{\sqrt{R^2-(x-x_0)}-y_0} (3+2y)\, dydx \underset{\substack{\uparrow \\ \text{Uso coor.} \\ \text{polares}}}{=} \int_0^R \int_0^{2\pi} r.(3+2(r\,\text{sen}\,\theta + y_0))\, drd\theta$$

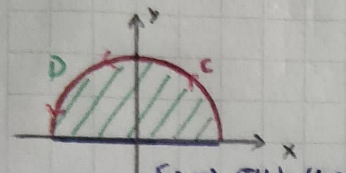
$$= \int_0^R\int_0^{2\pi} 3r\, d\theta dr + \int_0^R\int_0^{2\pi} 2r^2 \text{sen}\,\theta\, drd\theta + \int_0^R\int_0^{2\pi} 2r y_0\, drd\theta = 3.\cancel{2}\pi \frac{r^2}{\cancel{2}}\Big|_0^R + 2.\cancel{2}\pi y_0 \frac{r^2}{\cancel{2}}\Big|_0^R + \int_0^R 2r^2(-\cos\theta)\Big|_0^{2\pi} dr$$

$$= \overset{3}{\cancel{6}}\pi R^2 + \overset{2}{\cancel{4}}\pi y_0 R^2 + 0 = \cancel{2}\pi R^2(3+2y_0) = 6\pi \Rightarrow R^2 = \frac{6}{3+2y_0}$$

$$\Rightarrow C = \left\{(x,y) \in \mathbb{R}^2 / (x-x_0)^2 + (y-y_0)^2 = \frac{6}{3+2y_0}\right\} \Rightarrow \text{Puedo parametrizar } C \text{ por } \sigma(t) = \left(\sqrt{\frac{\cancel{3}6}{3+2y_0}}\cos t + x_0, \sqrt{\frac{\cancel{3}6}{3+2y_0}}\,\text{sen}\, t + y_0\right) \text{ con } t \in [0,2\pi]$$

14) $F(x,y)=\left(y^2e^x+\cos x+(x-y)^2,\ 2ye^x+\operatorname{sen} y\right) \Rightarrow P(x,y)=y^2e^x+\cos x+(x-y)^2,\ Q(x,y)=2ye^x+\operatorname{sen} y$

$C=\{(x,y)/\ x^2+y^2=1,\ y\geq 0\} \Rightarrow \sigma(t)=(\cos t,\operatorname{sen} t)$ con $t\in[0,\pi]$



$C_2 \to \sigma(t)=(t,0)$ con $t\in[-1,1]$

$$\Rightarrow \int_C F\,ds = ? \Rightarrow \int_{C*C_2} F\,ds = \int_C F\,ds + \int_{C_2} F\,ds = \iint_D \left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right) dy\,dx$$

$$\Rightarrow \int_{C_2} F\,ds = \int_{-1}^{1} (\cos t+t^2,0)(1,0)\,dt = \int_{-1}^{1}(\cos t+t^2)\,dt = \operatorname{sen} t\Big|_{-1}^{1} + \frac{t^3}{3}\Big|_{-1}^{1} = \operatorname{sen}(1)-(\operatorname{sen}(1)) + 2/3 = 2/3+2\operatorname{sen}(1)$$

$$\Rightarrow \iint_D \left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right) dx\,dy = \int_{-1}^{1}\int_0^{\sqrt{1-x^2}} \left(2ye^x-(2ye^x+2y-2x)\right) dx\,dy = \int_{-1}^{1}\int_0^{\sqrt{1-x^2}} (2x-2y)\,dy\,dx = \int_0^{\cancel{2\pi}\ \pi}\int_0^1 2r^2(\cos\theta-\operatorname{sen}\theta)\cdot d\theta\, dr$$

$$= \int_0^{\cancel{2\pi}\ \pi} \cos\theta\cdot\frac{2r^3}{3}\Big|_0^1 d\theta - \int_0^{\cancel{2\pi}\ \pi} \operatorname{sen}\theta\cdot\frac{2r^3}{3}\Big|_0^1 d\theta = \frac{2}{3}\Big(\underbrace{\cancel{\operatorname{sen}\theta\Big|_0^{2\pi}}}_{0} - \underbrace{(-\cos\theta)\Big|_0^{\pi}}_{2}\Big) = \cancel{0}\ -\frac{4}{3}$$

$$\Rightarrow \int_{C*C_2} F\,ds = \cancel{0}\ {-4/3} = \int_C F\,ds + 2/3 + 2\operatorname{sen}(1) \Rightarrow \int_C F\,ds = \cancel{-2/3}\ {-2} - 2\operatorname{sen}(1) = -2(1+\operatorname{sen}(1))$$

15) $D=\left\{(x,y)\in\mathbb{R}^2/\ \frac{x^2}{9}+\frac{y^2}{4}\leq 1\right\} \Rightarrow \partial D: \sigma(t)=(3\cos t,\ 2\operatorname{sen} t)$ con $t\in[0,2\pi]$

$F(x,y)=(u(x,y),v(x,y))$, $G(x,y)=(v_x-v_y,\ u_x-u_y)$

$$\Rightarrow F.G = (u,v).(v_x-v_y,\ u_x-u_y) = uv_x-uv_y+vu_x-vu_y = uv_x+vu_x-(uv_y+vu_y) = \frac{\partial(u.v)}{\partial x}-\frac{\partial(u.v)}{\partial y}$$

$$\Rightarrow \iint_D (F.G)(x,y)\,dx\,dy = \iint_D \left(\frac{\partial(uv)}{\partial x}-\frac{\partial(u.v)}{\partial y}\right) dx\,dy = \int_{C=\partial D} (uv)\,dx+(uv)\,dy = \int_0^{2\pi} \langle (u.v(\sigma(t)),\ u.v(\sigma(t))),(\sigma'(t))\rangle\, dt$$

$\Rightarrow$ Como $u=x$, $v=1$ sobre $\partial D=C$: $\displaystyle\int_0^{2\pi}\langle(3\cos t,3\cos t),(-3\operatorname{sen} t,2\cos t)\rangle\,dt = \int_0^{2\pi}(-9\cos t\operatorname{sen} t+6\cos^2 t)\,dt$

$$= \int_0^{2\pi} -9\underbrace{\cos t\operatorname{sen} t}_{\frac{1}{2}\operatorname{sen}(2x)}\,dt + \int_0^{2\pi} 6\underbrace{\cos^2 t}_{\frac{1}{2}(\cos(2x)+1)}\,dt = -9.\Big(\underbrace{\cancel{-\frac{\cos(2t)}{4}\Big|_0^{2\pi}}}_{0}\Big) + 6\Big(\frac{t}{2}\Big|_0^{2\pi} + \underbrace{\cancel{\frac{\operatorname{sen}(2t)}{4}\Big|_0^{2\pi}}}_{0}\Big) = 6\pi$$

Cont. 12

$$\Rightarrow \int_{C*C^4} f_1dx+f_2dy = \iint_D (f_{2x}-f_{1y})\,dx\,dy = 0 \Rightarrow \int_{C^+} f_1dx+f_2dy + \int_{C^{4+}} f_1dx+f_2dy = 0 \Rightarrow \int_{C^{4+}} f_1dx+f_2dy = -\int_{C^+} f_1dx+f_2dy$$

$$\int_{C^4} f_1dx+f_2dy = \int_0^{\pi} (f_1(\sigma_4(t)),\ f_2(\sigma_4(t)))(\sigma_4'(t))\,dt = \int_0^{\pi} (\cos t.\underbrace{\cancel{\operatorname{sen}(\pi/2)}}_{1}-\operatorname{sen} t,\ \operatorname{sen} t.\underbrace{\cancel{\operatorname{sen}(\pi/2)}}_{1}+\cos t)(-\operatorname{sen} t,\cos t)\,dt$$

$$= \int_0^{\pi} (\cancel{-\cos t\operatorname{sen} t} + \operatorname{sen}^2 t + \cancel{\operatorname{sen} t\cos t} + \cos^2 t)\,dt = \int_0^{\pi} 1\,dt = t\Big|_0^{\pi} = \pi$$

$$\Rightarrow \int_C f_1dx+f_2dy = -\pi$$