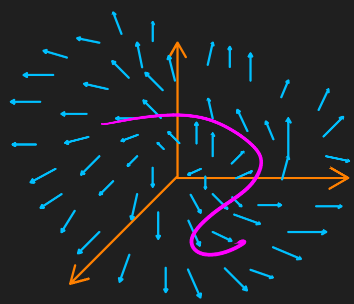
Guía 1 - Parte 3

Integrales Curvilíneas

$$\int F(\sigma(t))\,\sigma'(t)\,dt$$

no la normal!



para cada $p \in \mathbb{R}^3$ me devuelve un vector en $\mathbb{R}^3$

Ej 17) $F(x,y,z) = (x,y,z)$

a) $\sigma(t) = (t,t,t) \qquad 0 \leq t \leq 1$

$\sigma'(t) = (1,1,1)$

$$\int_\sigma F \cdot ds =$$

$$= \int_0^1 F(\sigma(t)) \overset{\text{Prod. int}}{\cdot} \sigma'(t)\,dt = \int_0^1 (t,t,t)\cdot(1,1,1)\,dt$$

$$= \int_0^1 3t\,dt = 3\,\frac{t^2}{2}\Big|_0^1 = \frac{3}{2} \;//$$

b) $\sigma(t) = (\sin t,\ 0,\ \cos t) \qquad 0 \leq t \leq 2\pi$

$\sigma'(t) = (\cos t,\ 0,\ -\sin t)$

$$\int_\sigma F \cdot ds = \int_0^{2\pi} F(\sigma(t)) \cdot \sigma'(t) \cdot dt =$$

$$= \int_0^{2\pi} (\sin t,\ 0,\ \cos t) \cdot (\cos t,\ 0,\ -\sin t) \cdot dt$$

$$= \int_0^{2\pi} \sin t \cos t - \sin t \cos t \; dt$$

$$= 0 \;//$$

Ej 18) a) $\sigma(t) = (\cos t, \sin t) \qquad 0 \leq t \leq 2\pi$

$$\int_{\mathcal{C}} x\,dy - y\,dx =$$

componentes de $F(x,y) = (-y, x)$ pues $(-y, x) \cdot ds =$

$$= \int_0^{2\pi} \langle F(\sigma(t)), \sigma'(t) \rangle \cdot dt$$

$$= \int_0^{2\pi} \langle F(\cos t, \sin t), (-\sin t, \cos t) \rangle \, dt$$

$F(x,y) = (-y, x)$

$$= \int_0^{2\pi} \langle (\overset{-y}{-\sin t}, \overset{x}{\cos t}), (-\sin t, \cos t) \rangle \, dt$$

$$= \int_0^{2\pi} \underbrace{\sin^2 t + \cos^2 t}_{=1} \; dt$$

$$= 2\pi \;//$$

Ver Teórica 3

$$\int_\sigma F \cdot ds = \int_\sigma P\,dx + Q\,dy + R\,dz$$

donde $F = (P, Q, R)$

$$\int_a^b F(\sigma(t)) \cdot \sigma'(t) \, dt$$

prod. interno

$$= \int_a^b P(x,y,z)(t) \cdot x'(t) + Q(x,y,z)(t) \cdot y'(t) + R(x,y,z)(t) \cdot z'(t) \, dt$$

18) b) $\int_{\mathscr{C}} x\,dx + y\,dy =$

$$\Rightarrow \quad F(x,y) = (x,y)$$

$$\sigma(t) = \left(\cos(\pi t),\ \sin(\pi t)\right) \qquad 0 \leqslant t \leqslant 2$$

$$\sigma'(t) = \left(-\pi \sin(\pi t),\ \pi \cdot \cos(\pi t)\right)$$

$$\int_{\sigma} F \cdot ds =$$

$$= \int_0^2 \langle F(\sigma(t)),\ \sigma'(t) \rangle\, dt$$

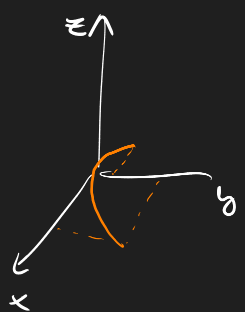
$$= \int_0^2 \langle F(\cos(\pi t),\ \sin(\pi t)),\ (-\pi \sin(\pi t),\ \pi \cdot \cos(\pi t)) \rangle\, dt$$

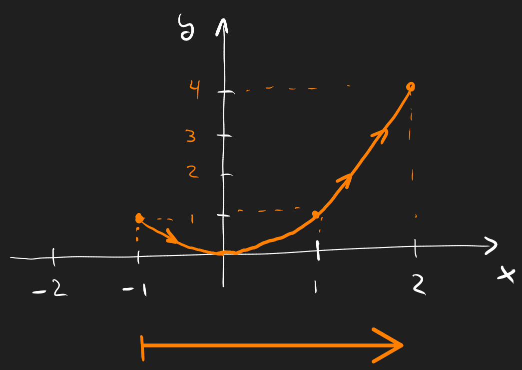
$$= \int_0^2 -\cos(\pi t) \cdot \pi \cdot \sin(\pi t) + \pi \cdot \sin(\pi t) \cdot \cos(\pi t)\, dt$$

$$= \int_0^2 0\, dt = 0 \,//$$

Ej 19) $F(x,y,z) = (x,y,z)$

$$\left.\begin{array}{l} y = x^2 \\ z = 0 \end{array}\right\} \text{ de } \quad x = -1 \quad \text{a} \quad x = 2$$





$$\sigma(t) = (t, t^2, 0)$$

$$\sigma'(t) = (1, 2t, 0)$$

$$\int_{-1}^{2} F(\sigma(t)) \cdot \sigma'(t) \cdot dt =$$

$$= \int_{-1}^{2} \langle (t, t^2, 0), (1, 2t, 0) \rangle \, dt$$

$$= \int_{-1}^{2} t + 2t^3 \, dt$$

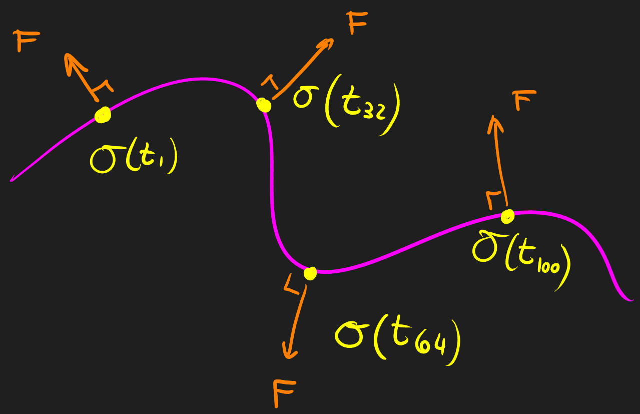
$$= \frac{t^2}{2}\Big|_{-1}^{2} + \cancel{2}^{\,1} \frac{t^4}{\cancel{4}_{\,2}}\Big|_{-1}^{2} = \left(2 - \frac{1}{2}\right) + \left(\frac{16}{2} - \frac{1}{2}\right)$$

$$= 10 - 1$$

$$= 9 \;//$$

Ej 20) $F \perp \sigma'(t)$ en $\sigma(t) \ \forall t$

perpendicular



$$\int_{\mathcal{C}} F \cdot ds \overset{?}{=} 0$$

$$\int_{\sigma} F(\sigma(t)) \cdot \sigma'(t) \, dt =$$

Preguntar si alcanza.

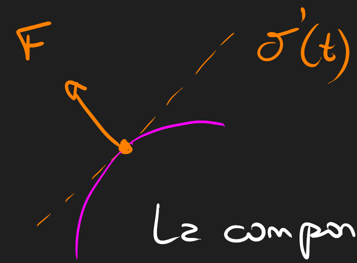
¿?

1

- como $F(\sigma(t)) \perp \sigma'(t) \ \forall t$

  ↳ sé que el prod. interno es cero

$$= \int_\sigma 0 \, d = 0 \quad //$$



La componente de F en la dirección $\frac{\sigma'(t)}{\|\sigma'(t)\|}$ es nula.

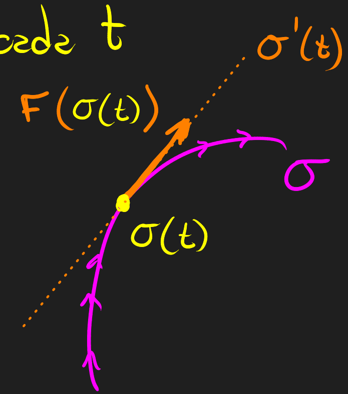
Ej b)

Ahora F tiene el mismo sentido que $\sigma'(t) \ \forall t$:

escalamiento en cada t

$$F(\sigma(t)) = \lambda(t) \cdot \sigma'(t)$$

dirección en cada t



- Como F tiene la misma dirección y sentido que el vector desplazamiento

$$T(t) = \frac{\sigma'(t)}{\|\sigma'(t)\|} \Rightarrow T(t) = \frac{F(\sigma(t))}{\|F(\sigma(t))\|}$$

normalizo F

$$\int_\sigma \underbrace{F(\sigma(t))}_{T \cdot \|F(\sigma(t))\|} \cdot \sigma'(t) \, dt =$$

Preguntar 2

$$= \int_\sigma \Big(\underbrace{T(t)}_{\frac{\sigma'(t)}{\|\sigma'\|}} \cdot \|F(\sigma(t))\|\Big) \cdot \underbrace{\sigma'(t)}_{T(t) \cdot \|\sigma'(t)\|} \, dt$$

Ej 21) Teorema:

$$\int_\sigma \nabla f \cdot ds = f(\sigma(b)) - f(\sigma(a))$$

Mini demo:

$$\int_{\mathscr{C}} \nabla f \cdot ds = \int_a^b \nabla f(\sigma(t)) \cdot \sigma'(t) dt$$

- si $g(t) = f(\sigma(t))$

$$= \int_a^b g'(t)\, dt$$

$$= g(t) \Big|_a^b$$ //

- Si la curva es Cerrada:

$$\sigma(b) = \sigma(a)$$

$$\Rightarrow f(\sigma(b)) = f(\sigma(a)) = \alpha$$

$$\Rightarrow \int_\sigma \nabla f \cdot ds = \alpha - \alpha = 0$$ //

Ej 22) $\nabla f(x,y,z) = \left(2xyz \cdot e^{x^2},\ z \cdot e^{x^2},\ y \cdot e^{x^2}\right)$

$$f(0,0,0) = 5$$

Hallar

$$f(1,1,2) = ?$$

Sé que

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}(x,y,z),\ \frac{\partial f}{\partial y}(x,y,z),\ \frac{\partial f}{\partial z}(x,y,z) \right)$$

$\Rightarrow$ • $\dfrac{\partial f}{\partial x}(x,y,z) = 2x \cdot y \cdot z \cdot e^{x^2}$

• $\dfrac{\partial f}{\partial y}(x,y,z) = z \cdot e^{x^2}$

• $\dfrac{\partial f}{\partial z}(x,y,z) = y \cdot e^{x^2}$

Calculo primitivas de derivadas parciales

∘ $\int 2xyze^{x^2}\,dx =$ wait ...

∘ $\int z \cdot e^{x^2}\,dy = y \cdot z \cdot e^{x^2} + C$

∘ $\int y \cdot e^{x^2} \cdot dz = \underbrace{y \cdot z \cdot e^{x^2}}_{} + C$

$\dfrac{\partial f}{\partial x} = 2x \cdot y \cdot z \cdot e^{x^2}$ ✓

$\Rightarrow f(x,y,z) = y \cdot z \cdot e^{x^2} + C$

dato

$f(0,0,0) = 5 = 0 + C$

$\Rightarrow C = 5$

$\therefore$

$f(x,y,z) = y \cdot z \cdot e^{x^2} + 5$

$\Rightarrow f(1,1,2) = 1 \cdot 2 \cdot e^{1^2} + 5 = 2e + 5$ //

Ej 23) $F(x,y,z) = -\underbrace{\dfrac{1}{(x^2+y^2+z^2)^{3/2}}}_{-\frac{1}{\|(x,y,z)\|^3}} \cdot (x,y,z)$

(uso reso. de Iván Queirolo)

Sé que

$$F(x,y,z) = \nabla f(x,y,z) \quad \text{para alguna función } f$$

si $\quad f(x,y,z) = \dfrac{1}{\|(x,y,z)\|} = \dfrac{1}{(x^2+y^2+z^2)^{1/2}}$

$$= (x^2+y^2+z^2)^{-1/2}$$

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\frac{\partial f}{\partial x}(x,y,z) = -\frac{1}{\cancel{2}} \cdot (x^2+y^2+z^2)^{-3/2} \cdot \cancel{2}\, x$$

- $\dfrac{\partial f}{\partial x}(x,y,z) = -\dfrac{x}{(x^2+y^2+z^2)^{3/2}}$
- $\dfrac{\partial f}{\partial y}(x,y,z) = -\dfrac{y}{(x^2+y^2+z^2)^{3/2}}$
- $\dfrac{\partial f}{\partial z}(x,y,z) = -\dfrac{z}{(x^2+y^2+z^2)^{3/2}}$

Pregunta 3

No estoy seguro de entender qué demostré

- Donde demostré que depende solamente de los radios

$R_1 = \|(x_1,y_1,z_1)\|$

$R_2 = \|(x_2,y_2,z_2)\|$

Ej 24) $F = \nabla f + G$

Pide: $\int_{\mathcal{C}} F \cdot ds = \int_{\mathcal{C}} G \cdot ds$

$$\int_{\sigma} \langle F(\sigma(t)), \sigma'(t) \rangle dt = \int_{\sigma} \langle \underbrace{(\nabla f + G)(\sigma(t))}_{\text{evaluado en}}, \sigma'(t) \rangle dt$$

$$= \int_{\sigma} \langle \nabla f(\sigma(t)) + G(\sigma(t)), \sigma'(t) \rangle dt$$

$$= \int_{\sigma} (\nabla f_x + G_x) \cdot \sigma_x'(t) +$$

componente x

$$+ (\nabla f_y + G_y) \cdot \sigma_y'(t) +$$
$$+ (\nabla f_z + G_z) \cdot \sigma_z'(t) \, dt$$

$$= \int_{\sigma} \nabla f_x \, \sigma_x'(t) + \nabla f_y \, \sigma_y'(t) + \nabla f_z \, \sigma_z'(t) +$$
$$+ G_x \cdot \sigma_x'(t) + G_y \cdot \sigma_y'(t) + G \cdot \sigma_z'(t) \, dt$$

$$= \underbrace{\int_{\sigma} \langle \nabla f, \sigma'(t) \rangle dt}_{} + \int_{\sigma} \langle G, \sigma'(t) \rangle dt$$

Como la curva es cerrada

$$\int_{\mathcal{C}} \nabla f \cdot ds = f(\sigma(b)) - f(\sigma(a)) = 0$$

$\therefore$ $\int_{\mathcal{C}} F \cdot ds = \int_{\mathcal{C}} G \cdot ds$ //