Práctica 1 CURVAS

Ej 1: Sean $\sigma_1 : [0, 2\pi] \longrightarrow \mathbb{R}^2$, $\sigma_1(\theta) = (\cos\theta, \operatorname{sen}\theta)$

$\sigma_2 : [0, 4\pi] \longrightarrow \mathbb{R}^2$, $\sigma_2(\theta) = (\cos(\theta/2), \operatorname{sen}(\theta/2))$

Probar que ambas son parametrizaciones C^1 de la circunferencia de radio 1 y centro $(0,0)$.

↳ • continuas ✓

• C^1

• $C = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$

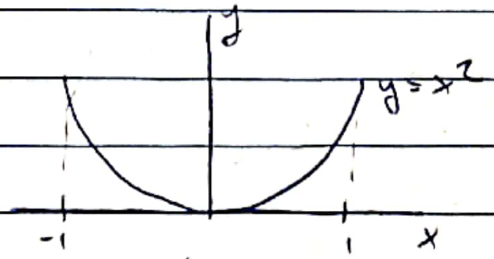
↳ $\operatorname{Im}(\sigma_1) = C$, $\operatorname{Im}(\sigma_2) = C$

• **Obs:**

$$
\begin{tikzcd}
{[0, 2\pi]} \arrow[r, "\sigma_1"] & C \\
{[0, 4\pi]} \arrow[u, "h"] \arrow[ur, "\sigma_2"'] &
\end{tikzcd}
$$

con $h(t) = t/2$

Ej 2: Parábola



↳ $C = \{(x,y) \in \mathbb{R}^2 \mid y = x^2, -1 \leq x \leq 1\}$

$\sigma_1 : [-1, 1] \longrightarrow \mathbb{R}^2$, $\sigma_1(t) = (t, t^2)$

↳ $\operatorname{Im}(\sigma_1) = C$ ✓

• Curva abierta: $\sigma(1)=(1,1) \neq (-1,1)=\sigma(-1)$

• Simple: σ_1 es inyectiva.

• Suave: si en cada ~~pequeño~~ entorno de cada punto de C existe una parametrización regular para C.

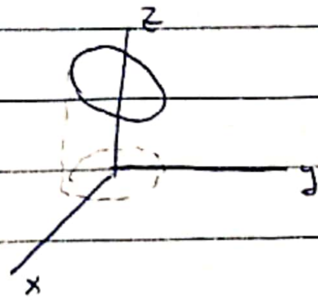
Como $\sigma'(t)=(1,2t)\neq(0,0)\ \forall t$, C es suave.

<u>Obs.</u> $\sigma_2: [-1,1) \to \mathbb{R}^2$
$t \mapsto (t^3, t^6)$

↳ • inyectiva ✓
• C^1
• continua
• $\sigma_2'(t)=(3t^2, 6t^5) \Rightarrow \sigma_2'(0)=(0,0)$
↳ σ_2 no es una parametrización regular.

<u>Ej 3:</u> Sea C la curva definida por la intersección de las superficies:

$$C=\begin{cases} x^2+y^2=1 \\ y+z=2 \end{cases}$$



Probar que C es una curva cerrada, simple y suave.

Como $x^2+y^2=1 \Rightarrow \exists\, \theta \in [0,2\pi] \,/\, \begin{cases} x=\cos\theta \\ y=\operatorname{sen}\theta \end{cases}$

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Como $y+z=2 \Rightarrow z = 2-y = 2-\text{sen}\,\theta$.

Sea $\sigma:[0,2\pi) \to \mathbb{R}^2 \,/\, \sigma(\theta) = (\cos\theta, \text{sen}\,\theta, 2-\text{sen}\,\theta)$

¿C es ...

- ~~Es~~ ... cerrada? $\sigma(0) = (1,0,2) = \sigma(2\pi)$ ✓
- ~~...~~ :
- ... simple? σ es inyectiva ✓
- σ es C^1 ✓
- $\sigma'(\theta) = (-\text{sen}\,\theta, \cos\theta, \cos\theta) \neq (0,0,0) \quad \forall\theta$
- $\sigma'(0) = (0,1,1) = \sigma'(2\pi)$ ✓

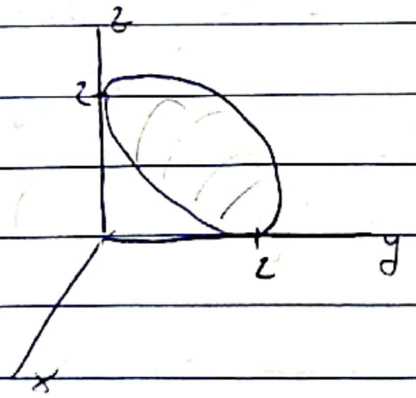
↳ C es ~~...~~ simple.

Ej 4: Sea $\sigma:[0,2\pi] \to \mathbb{R}^3$

$t \longmapsto (1+\cos(t), \text{sen}(t), 2\cdot\text{sen}(t/2))$

y sea $C = \{(x,y,z) \in \mathbb{R}^3 \,/\, x^2+y^2+z^2 = 4,\ (x-1)^2+y^2 = 1,\ z \geq 0\}$

Probar que σ es una parametrización de C.



- $\text{Im}(\sigma) = C$:

Veamos que $\text{Im}(\sigma) \subset C$:

Sea $(x,y,z) \in \text{Im}(\sigma) \Rightarrow \exists\, t \in [0,2\pi) \,/$

$$(x y z) = (1+\cos(t), \operatorname{sen}(t), 2\operatorname{sen}(t/2))$$

- $x^2+y^2+z^2 = 1+2\cos(t) + \cos^2(t) + \operatorname{sen}^2(t) + 4\operatorname{sen}^2(t/2)$

$$= 2 + 2\underbrace{\cos(t)}_{\cos(t/2+t/2)} + 4\operatorname{sen}^2(t/2)$$

$$= 2+2\cos^2(t/2) - 2\operatorname{sen}^2(t/2) + 4\operatorname{sen}^2(t/2)$$

$$= 2+2\cos^2(t/2) + 2\operatorname{sen}^2(t/2) = 4$$

$$\Rightarrow x^2+y^2+z^2 = 4$$

- $(x-1)^2+y^2 = \cos^2(t) + \operatorname{sen}^2(t) = 1$ ✓
- $z = 2\operatorname{sen}(t/2) \geq 0 \quad \forall t \in [0, 2\pi]$ ✓

$$\Rightarrow (x y z) \in C \Rightarrow \operatorname{Im}(\sigma) \subseteq C.$$

Veamos que $C \subseteq \operatorname{Im}(\sigma)$. Sea $(x,y,z) \in C \Rightarrow (x-1)^2+y^2=1$

$\Rightarrow \exists t \in [0, 2\pi] /$

$x = \cos(t)+1$

$y = \operatorname{sen}(t)$

Como $x^2+y^2+z^2=4$, $z^2 = 4-x^2-y^2 = 4-\cos^2(t) - 2\cos(t) - 1 - \operatorname{sen}^2(t)$

$$= 2-2\cos(t)$$

$$= 2\left(1-\cos^2(t/2)+\operatorname{sen}^2(t/2)\right)$$

$$(1 = \cos^2(t/2)+\operatorname{sen}^2(t/2)) \Rightarrow\; = 2\left(2\operatorname{sen}^2(t/2)\right) = 4\operatorname{sen}^2(t/2)$$

$$\Rightarrow z^2 = 4\operatorname{sen}^2(t/2) \underset{z\geq 0}{\Longrightarrow} z = 2\operatorname{sen}(t/2)$$

$$\Rightarrow (x y z) = (1+\cos(t), \operatorname{sen}(t), 2\operatorname{sen}(t/2)), \quad t \in [0, 2\pi]$$

$$\Rightarrow C \subseteq \operatorname{Im}(\sigma)$$

$$\Rightarrow \operatorname{Im}(\sigma) = C.$$

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