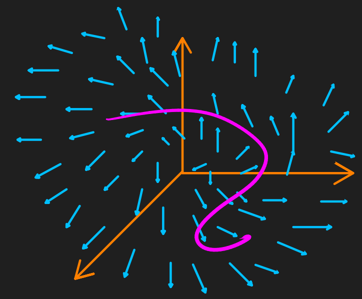
Guía 1 - Parte 3

Integrales Curvilíneas

$$\int F(\sigma(t))\,\sigma'(t)\,dt$$

no la normal!



para cada $p \in \mathbb{R}^3$ me devuelve un vector en $\mathbb{R}^3$

Ej 17) $F(x,y,z) = (x,y,z)$

a) $\sigma(t) = (t,t,t) \qquad 0 \leq t \leq 1$

$\sigma'(t) = (1,1,1)$

$$\int_\sigma F \cdot ds =$$

Prod. int

$$= \int_0^1 F(\sigma(t)) \cdot \sigma'(t)\,dt = \int_0^1 (t,t,t)\cdot(1,1,1)\,dt$$

$$= \int_0^1 3t\,dt = 3\,\frac{t^2}{2}\Big|_0^1 = \frac{3}{2} \quad //$$

b) $\sigma(t) = (\sin t,\ 0,\ \cos t) \qquad 0 \leq t \leq 2\pi$

$\sigma'(t) = (\cos t,\ 0,\ -\sin t)$

$$\int_\sigma F \cdot ds = \int_0^{2\pi} F(\sigma(t)) \cdot \sigma'(t) \cdot dt =$$

$$= \int_0^{2\pi} (\sin t,\ 0,\ \cos t) \cdot (\cos t,\ 0,\ -\sin t) \cdot dt$$

$$= \int_0^{2\pi} \sin t \cos t - \sin t \cos t \, dt$$

$$= 0 \;//$$

Ej 18) a) $\sigma(t) = (\cos t, \sin t) \qquad 0 \leq t \leq 2\pi$

$$\int_{\mathcal{C}} x\,dy - y\,dx =$$

componentes de $F(x,y) = (-y, x)$ pues $(-y, x) \cdot ds =$

$$= \int_0^{2\pi} \langle F(\sigma(t)), \sigma'(t) \rangle \cdot dt$$

$$= \int_0^{2\pi} \langle F(\cos t, \sin t), (-\sin t, \cos t) \rangle \, dt$$

$F(x,y) = (-y, x)$

$$= \int_0^{2\pi} \langle (\overset{-y}{-\sin t}, \overset{x}{\cos t}), (-\sin t, \cos t) \rangle \, dt$$

$$= \int_0^{2\pi} \underbrace{\sin^2 t + \cos^2 t}_{=1} \, dt$$

$$= 2\pi \;//$$

Ver Teórica 3

$$\int_\sigma F \cdot ds = \int_\sigma P \cdot dx + Q \cdot dy + R\,dz$$

donde $F = (P, Q, R)$

$$\int_a^b F(\sigma(t)) \cdot \sigma'(t) \, dt$$

prod. interno

$$= \int_a^b P(x,y,z)(t) \cdot x'(t) + Q(x,y,z)(t) \cdot y'(t) + R(x,y,z)(t) \cdot z'(t) \, dt$$

18) b) $\displaystyle\int_{\mathcal{C}} x\,dx + y\,dy =$

$$\Rightarrow \quad F(x,y) = (x,y)$$

$$\sigma(t) = \big(\cos(\pi t)\,,\ \sin(\pi t)\big) \qquad 0 \leqslant t \leqslant 2$$

$$\sigma'(t) = \big(-\pi \sin(\pi t)\,,\ \pi \cdot \cos(\pi t)\big)$$

$$\int_{\sigma} F \cdot ds =$$

$$= \int_0^2 \langle F(\sigma(t))\,,\ \sigma'(t) \rangle\, dt$$

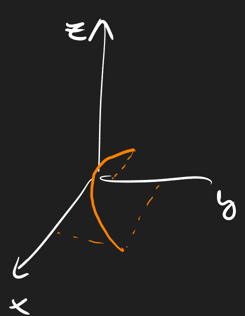
$$= \int_0^2 \big\langle F\big(\cos(\pi t), \sin(\pi t)\big), \big(-\pi \sin(\pi t)\,,\ \pi \cdot \cos(\pi t)\big)\big\rangle\, dt$$

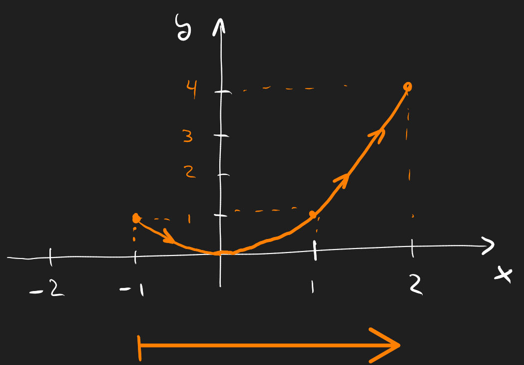
$$= \int_0^2 -\cos(\pi t) \cdot \pi \cdot \sin(\pi t) + \pi \cdot \sin(\pi t) \cdot \cos(\pi t)\, dt$$

$$= \int_0^2 0\, dt = 0 \ //$$

Ej 19) $F(x,y,z) = (x,y,z)$

$$\left.\begin{array}{l} y = x^2 \\ z = 0 \end{array}\right\} \text{ de } \quad x = -1 \ \text{ a } \ x = 2$$





$$\sigma(t) = (t, t^2, 0)$$

$$\sigma'(t) = (1, 2t, 0)$$

$$\int_{-1}^{2} F(\sigma(t)) \bullet \sigma'(t) \cdot dt =$$

$$= \int_{-1}^{2} \langle (t, t^2, 0) , (1, 2t, 0) \rangle \, dt$$

$$= \int_{-1}^{2} t + 2t^3 \, dt$$

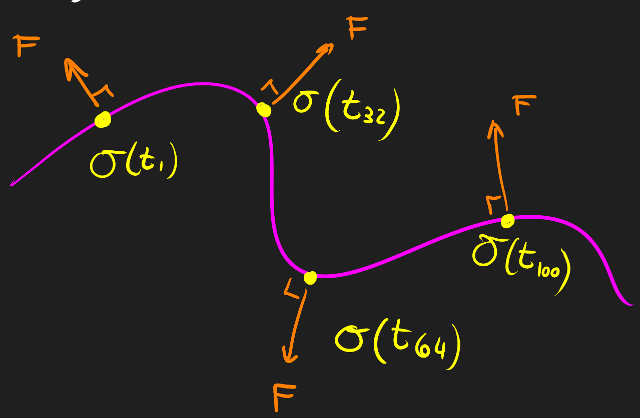
$$= \left. \frac{t^2}{2} \right|_{-1}^{2} + \cancel{2}^{1} \left. \frac{t^4}{\cancel{4}_{2}} \right|_{-1}^{2} = \left(2 - \frac{1}{2}\right) + \left(\frac{16}{2} - \frac{1}{2}\right)$$

$$= 10 - 1$$

$$= 9 \, //$$

Ej 20) $F \perp \sigma'(t)$ en $\sigma(t) \quad \forall t$

perpendicular



$$\int_{\mathcal{C}} F \bullet ds \overset{?}{=} 0$$

$$\int_{\sigma} F(\sigma(t)) \cdot \sigma'(t) \, dt =$$

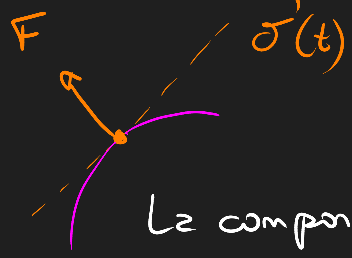
Preguntar si alcanza.

?
1

- como $F(\sigma(t)) \perp \sigma'(t) \quad \forall t$

  ↳ sé que el prod. interno es cero

$$= \int_\sigma 0 \, d = 0 \; //$$



La componente de F en la dirección $\frac{\sigma'(t)}{\|\sigma'(t)\|}$ es nula.

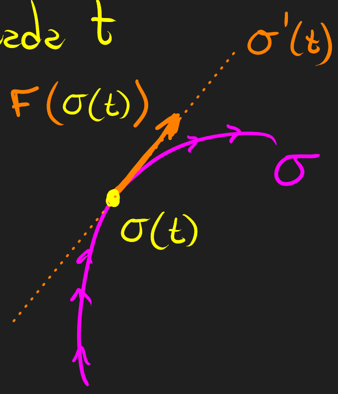
Ej b)

Ahora F tiene el mismo sentido que $\sigma'(t)$ $\forall t$:

escalamiento en cada t

$$F(\sigma(t)) = \lambda(t) \, . \, \sigma'(t)$$

dirección en cada t



- Como F tiene la misma dirección y sentido que el vector desplazamiento

$$T(t) = \frac{\sigma'(t)}{\|\sigma'(t)\|} \Rightarrow T(t) = \frac{F(\sigma(t))}{\|F(\sigma(t))\|}$$

normalizo F

$$\int_\sigma \underbrace{F(\sigma(t))}_{T \, . \, \|F(\sigma(t))\|} \cdot \sigma'(t) \, dt =$$

Preguntar 2

$$= \int_\sigma \Big(\underbrace{T(t)}_{\frac{\sigma'(t)}{\|\sigma'\|}} \, . \, \|F(\sigma(t))\| \Big) \cdot \underbrace{\sigma'(t)}_{T(t) \, . \, \|\sigma'(t)\|} \, dt$$

Vicky: $\lambda(t) > 0$! pues misma dir. y sentido.

$$\int_{\mathcal{C}} F \cdot ds \overset{\text{def}}{=} \int_a^b F(\sigma(t)) \cdot \sigma'(t)\, dt$$

$$= \int_a^b \underbrace{\left(\lambda(t) \cdot \sigma'(t)\right) \cdot \sigma'(t)}_{= \lambda(t) \cdot \langle \sigma'(t), \sigma'(t) \rangle}\, dt$$

$$\langle \sigma'(t), \sigma'(t) \rangle = (\sigma_x'(t))^2 + (\sigma_y'(t))^2 + (\sigma_z'(t))^2 = \|\sigma'(t)\|^2$$

$$= \int_a^b \lambda(t) \cdot \|\sigma'(t)\|^2\, dt$$

$$= \int_a^b \lambda(t)\, \|\sigma'(t)\| \cdot \|\sigma'(t)\|\, dt$$

$$\overset{\lambda(t)>0}{=} \int_a^b \|\lambda(t) \cdot \sigma'(t)\| \cdot \|\sigma'(t)\|\, dt$$

$$= \int_a^b \|F(\sigma(t))\| \cdot \|\sigma'(t)\|\, dt$$

$$= \int_a^b \|F(\sigma(t))\| \cdot \|\sigma'(t)\|\, dt$$

$$= \int_a^b \|F(\sigma(t)) \cdot \sigma'(t)\|\, dt$$

$$= \int_{\mathcal{C}} \| F \| \cdot ds$$ //

Ej 21) Teorema:

$$\int_{\sigma} \nabla f \cdot ds = f(\sigma(b)) - f(\sigma(a))$$

Mini demo:

$$\int_{\mathcal{C}} \nabla f \cdot ds = \int_a^b \nabla f(\sigma(t)) \cdot \sigma'(t) dt$$

- si $g(t) = f(\sigma(t))$

$$= \int_a^b g'(t)\, dt$$

$$= g(t) \Big|_a^b$$ //

- Si la curva es Cerrada:

$$\sigma(b) = \sigma(a)$$

$$\Rightarrow f(\sigma(b)) = f(\sigma(a)) = \alpha$$

$$\Rightarrow \int_{\sigma} \nabla f \cdot ds = \alpha - \alpha = 0$$ //

Ej 22) $\nabla f(x,y,z) = \left(2xyz \cdot e^{x^2}, z \cdot e^{x^2}, y \cdot e^{x^2}\right)$

$$f(0,0,0) = 5$$

Hallar

$$f(1,1,2) = ?$$

Sé que

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}(x,y,z), \frac{\partial f}{\partial y}(x,y,z), \frac{\partial f}{\partial z}(x,y,z)\right)$$

$$\Rightarrow \bullet \ \frac{\partial f}{\partial x}(x,y,z) = 2x \cdot y \cdot z \cdot e^{x^2}$$

$$\bullet \ \frac{\partial f}{\partial y}(x,y,z) = z \cdot e^{x^2}$$

$$\bullet \ \frac{\partial f}{\partial z}(x,y,z) = y \cdot e^{x^2}$$

Calculo primitivas de derivadas parciales

- $\int 2xyze^{x^2}\,dx =$ wait ...
- $\int z \cdot e^{x^2}\,dy = y \cdot z \cdot e^{x^2} + C$
- $\int y \cdot e^{x^2} \cdot dz = \underbrace{y \cdot z \cdot e^{x^2}}_{} + C$

$$\frac{\partial f}{\partial x} = 2x \cdot y \cdot z \cdot e^{x^2}$$

✓

$$\Rightarrow f(x,y,z) = y \cdot z \cdot e^{x^2} + C$$

dato

$$f(0,0,0) = 5 = 0 + C$$

$$\Rightarrow C = 5$$

$\therefore$

$$f(x,y,z) = y \cdot z \cdot e^{x^2} + 5$$

$$\Rightarrow f(1,1,2) = 1 \cdot 2 \cdot e^{1^2} + 5 = 2e + 5 \ //$$

Ej 23) $F(x,y,z) = -\underbrace{\frac{1}{(x^2+y^2+z^2)^{3/2}}}_{-\frac{1}{\|(x,y,z)\|^3}} \cdot (x,y,z)$

(uso reso. de Iván Queirolo)

Sé que

$F(x,y,z) = \nabla f(x,y,z)$ para alguna función f

si $f(x,y,z) = \dfrac{1}{\|(x,y,z)\|} = \dfrac{1}{(x^2+y^2+z^2)^{1/2}}$

$= (x^2+y^2+z^2)^{-1/2}$

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$\frac{\partial f}{\partial x}(x,y,z) = -\frac{1}{\cancel{2}} \cdot (x^2+y^2+z^2)^{-3/2} \cdot \cancel{2}\, x$$

?

- $\dfrac{\partial f}{\partial x}(x,y,z) = -\dfrac{x}{(x^2+y^2+z^2)^{3/2}}$
- $\dfrac{\partial f}{\partial y}(x,y,z) = -\dfrac{y}{(x^2+y^2+z^2)^{3/2}}$
- $\dfrac{\partial f}{\partial z}(x,y,z) = -\dfrac{z}{(x^2+y^2+z^2)^{3/2}}$

Pregunta 3

No estoy seguro de entender qué demostré

- Donde demostré que depende solamente de los radios

$R_1 = \|(x_1, y_1, z_1)\|$

$R_2 = \|(x_2, y_2, z_2)\|$

En que ahora puedo resolverlo como $f(b) - f(a)$

Ej 24) $F = \nabla f + G$

Pide: $\int_{\mathcal{C}} F \cdot ds = \int_{\mathcal{C}} G \cdot ds$

$$\int_\sigma \langle F(\sigma(t)), \sigma'(t) \rangle dt = \int_\sigma \langle (\nabla f + G)\underset{\text{evaluado en}}{(\sigma(t))}, \sigma'(t) \rangle dt$$

$$= \int_\sigma \langle \nabla f(\sigma(t)) + G(\sigma(t)), \sigma'(t) \rangle dt$$

$$= \int_\sigma \underbrace{(\nabla f_x + G_x) \cdot \sigma_x'(t)}_{\text{componente } x} + (\nabla f_y + G_y) \cdot \sigma_y'(t) + (\nabla f_z + G_z) \cdot \sigma_z'(t) \, dt$$

$$= \int_\sigma \nabla f_x \, \sigma_x'(t) + \nabla f_y \, \sigma_y'(t) + \nabla f_z \, \sigma_z'(t) + G_x \cdot \sigma_x'(t) + G_y \cdot \sigma_y'(t) + G \cdot \sigma_z'(t) \, dt$$

$$= \underbrace{\int_\sigma \langle \nabla f, \sigma'(t) \rangle dt}_{} + \int_\sigma \langle G, \sigma'(t) \rangle dt$$

Como la curva es cerrada

$$\int_{\mathcal{C}} \nabla f \cdot ds = f(\sigma(b)) - f(\sigma(a)) = 0$$

$\therefore$ $\int_{\mathcal{C}} F \cdot ds = \int_{\mathcal{C}} G \cdot ds$ //