

# Campos Conservativos

Def :  $F : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  se dice  
conservativo si :

$$\int_{C_1} F d\vec{s} = \int_{C_2} F d\vec{s}$$

$\forall$  par de Curvas simples,  
suaves e trozos

con el mismo punto

inicial y final

Teorema :

$$F \in C^1(\mathbb{R}^3 \setminus \Omega)$$

$$\# \Omega < \infty$$

Son equivalentes

$$\boxed{1} \quad \int_{\mathcal{C}} \mathbf{F} d\mathbf{z} = 0 \quad \forall \mathcal{C} \text{ cerrada, simple}$$

$$\boxed{2} \quad \int_{\mathcal{C}_1} \mathbf{F} d\mathbf{z} = \int_{\mathcal{C}_2} \mathbf{F} d\mathbf{z} \quad \forall \mathcal{C}_1, \mathcal{C}_2 \quad p \rightarrow q$$

$$\boxed{3} \quad \mathbf{F} = \nabla f, \quad f: \mathbb{R}^3 \rightarrow \mathbb{R}$$

$$\mathbf{F}: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$\boxed{4} \quad \nabla \times \mathbf{F} = 0 \quad \text{en } \mathbb{R}^3 \setminus \Omega$$

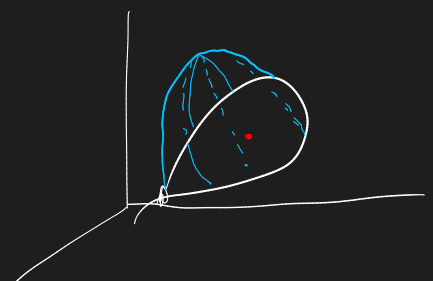
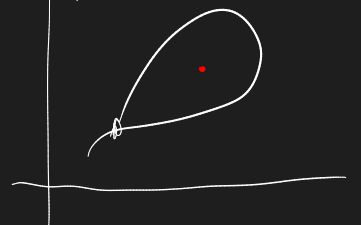
Ojo!

en  $\mathbb{R}^2$  vale pero sin excepciones

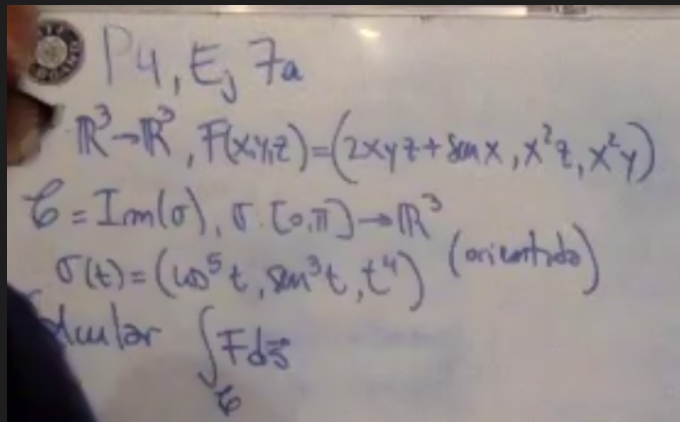
con  $\mathbf{F} \in C^1(\mathbb{R}^2)$

(No puedes armar lazo con punto discontinuo,  
no es arco conexo)

en  $\mathbb{R}^3$  sí puedes esquivarlo



P4 E3 7a



$F$  es  $C^1(\mathbb{R}^3)$

$$\nabla \times F = \left( \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, -\frac{\partial F_3}{\partial x} + \frac{\partial F_1}{\partial z}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right)$$

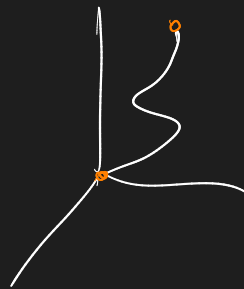
$$= (0, 0, 0)$$

∴  $F$  es conservativo!

Cálculo principio/fin

$$\sigma(0) = (1, 0, 0)$$

$$\sigma(\pi) = (-1, 0, \pi^4)$$



Si no fuera gradiente

Proponemos otro camino

$$\Gamma_1: \text{desde } (1, 0, 0) \rightarrow (-1, 0, 0)$$

$$\Gamma_2: (-1, 0, 0) \rightarrow (-1, 0, \pi^4)$$

$$\Gamma_1 : [-1; 1] \rightarrow \mathbb{R}^3$$

$$\Gamma_1(t) = (t, 0, 0) \quad (\downarrow \text{revés})$$

$$\Gamma_2 : [0, \pi^4] \rightarrow \mathbb{R}^3$$

$$\Gamma_2(t) = (1, 0, t) \quad (\text{sentido correcto})$$

$$\Gamma_1'(t) = (1, 0, 0)$$

$$\Gamma_2'(t) = (0, 0, 1)$$

Como :  $\nabla \times F = \vec{0}$

$$\int_{\mathcal{C}} F d\vec{s} = \int_{\Gamma_1^-} F d\vec{s} + \int_{\Gamma_2} F d\vec{s}$$

$$= - \int_{\Gamma_1} F d\vec{s} + \int_{\Gamma_2} F d\vec{s}$$

$$\int_{\Gamma_1} F d\vec{s} = \int_{-1}^1 \langle F(\Gamma_1(t)), \Gamma_1'(t) \rangle dt$$

$$= \int_{-1}^1 \sin t \, dt$$

$$= -\cos(1) + \cos(-1)$$

$$= 0 //$$

$$\int_{\Gamma_2} \mathbf{F} \cdot d\vec{s} = \int_0^{\pi/4} 0 \, dt = 0 //$$

$$\therefore \int_{\mathcal{C}} \mathbf{F} \, d\vec{s} = 0 //$$

También podría haber encontrado el potencial  $f$ .

$P_4 \in 10$ )

$$\mathbf{F}(x, y, z) = (2xy + z^2, x^2 - 2yx, 2xz - y^2)$$

$\mathcal{C}$  la curva dada por



$$\begin{cases} x^2 + y^2 + z^2 = 1 \\ y = x \\ (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0) \rightarrow (0, 0, 1) \end{cases}$$

Obs:

$$\begin{aligned}\nabla \times F &= (-2y - (-2y), 2z - 2z, 2x - 2x) \\ &= (0, 0, 0)\end{aligned}$$

$\Rightarrow F$  is conservative.

$$\Rightarrow F = \nabla f \quad \text{on } f: \mathbb{R}^3 \rightarrow \mathbb{R}$$

Reescribo  $F$

$$\bullet \frac{\partial f}{\partial x} = 2xz + z^2$$

$$f(x, y, z) = x^2 z + x z^2 + h(y, z)$$

$$\Rightarrow \frac{\partial f}{\partial y} = x^2 + \frac{\partial h}{\partial y}(x, z) = x^2 - 2yz$$

$$\frac{\partial h}{\partial y} = -2yz$$

$$\Rightarrow h(y, z) = -y^2 z + g(z)$$

$$f(x, y, z) = x^2 z + x z^2 - y^2 z + g(z)$$

$$\frac{\partial f}{\partial z} = 2xz - y^2 - g'(z)$$

$$\Rightarrow g(z) = C$$

$$f(x, y, z) = x^2 y + x z^2 - y^2 z$$

$$\begin{aligned} \int_C F \, dz &= f(0, 0, 1) - f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \\ &= 0 - \frac{1}{2\sqrt{2}} \\ &= -\frac{\sqrt{2}}{4} // \end{aligned}$$

Atent:!!

• Si  $F = \nabla f$


$$\Rightarrow \nabla \times F = (0, 0, 0)$$

La vuelta no vale

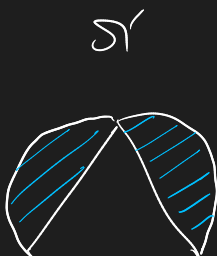


Hay que ver si es simplemente conexo

↳ Si lo es

↳ vale la vuelta. 

$\mathbb{R}^2$ :  
 Simplemente conexo



$\mathbb{R}^3$

