Parcial 1 - Ej 2.

Feb 27

2. Considerar el campo $\mathbf{F}$:

$$\mathbf{F}(x,y) = \left(e^{x^2y}(2xy\,\mathrm{sen}(y^2x) + \cos(y^2x)y^2) - y, e^{x^2y}\left(\mathrm{sen}(y^2x)x^2 + \cos(y^2x)2xy\right) + x\right)$$

Evaluar

$$\int_{\mathcal{C}} \mathbf{F}\, ds$$

donde $\mathcal{C} = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ orientada en sentido horario.

$$\tilde{P} = e^{x^2 \cdot y} \cdot 2xy \cdot \sin(y^2 \cdot x) + e^{x^2 y} \cdot \cos(y^2 x) \cdot y^2$$

$$\tilde{f}(x,y) = e^{x^2 y} \cdot \sin(y^2 \cdot x) + \varphi(y)$$

$$\tilde{F} = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (\tilde{P}, \tilde{Q})$$

$$\tilde{Q} = e^{x^2 \cdot y} \cdot \sin(y^2 x) \cdot x^2 + e^{x^2 y} \cdot \cos(y^2 \cdot x) \cdot 2xy$$

$$\tilde{f}(x,y) = e^{x^2 y} \cdot \sin(y^2 x) + \gamma(x)$$

Como encontré f / $\tilde{F} = \nabla f$

$\Rightarrow \tilde{F}$ es campo Gradiente

$\Rightarrow \nabla \times \tilde{F} = 0$

Volviendo

$$F = \tilde{F} + (-y, x)$$

$$\int_{\mathcal{C}} \vec{F}\, d\vec{s} = \int_{\mathcal{C}} \tilde{\vec{F}}\, d\vec{s} + \int_{\mathcal{C}} (-y, x)\, d\vec{s}$$

$$= -\int \underbrace{\tilde{Q}_x - \tilde{P}_y}_{=0}\, dx\, dy$$

Pues $\tilde{\vec{F}}$ es CG.

$$= -\int_{\theta=0}^{2\pi}\int_{r=0}^{1} 2 \cdot d\theta\, dr$$

$$= -2 \cdot \pi$$

Solución

$$\int_{\mathcal{C}} \vec{F}\, d\vec{s} = -2\pi$$