Parcial 1 - Ej 2.

Feb 27

2. Considerar el campo $\mathbf{F}$:

$$\mathbf{F}(x,y) = \left(\underbrace{e^{x^2y}(2xy\,\text{sen}(y^2x) + \cos(y^2x)y^2)}\ \underbrace{-\,y}, \underbrace{e^{x^2y}\left(\text{sen}(y^2x)x^2 + \cos(y^2x)2xy\right)}\ \underbrace{+\,x}\right)$$

Evaluar

$$\int_{\mathcal{C}} \mathbf{F}\, ds$$

donde $\mathcal{C} = \left\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\right\}$ orientada en sentido horario.

$$\tilde{P} = \underbrace{e^{x^2 \cdot y} \cdot 2xy}\cdot \sin(y^2 \cdot x) + e^{x^2 y} \cdot \cos(y^2 x) \cdot y^2$$

$$\tilde{f}(x,y) = e^{x^2 y} \cdot \sin(y^2 \cdot x) + \varphi(y)$$

$$\boxed{\begin{aligned}\tilde{F} &= \nabla f \\ &= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) \\ &= (\tilde{P}, \tilde{Q})\end{aligned}}$$

$$\tilde{Q} = e^{x^2 \cdot y} \cdot \sin(y^2 x) \cdot x^2 + e^{x^2 y} \cdot \cos(y^2 \cdot x) \cdot 2xy$$

$$\tilde{f}(x,y) = e^{x^2 y} \cdot \sin(y^2 x) + \gamma(x)$$

Como encontré f / $\tilde{F} = \nabla f$

$\Rightarrow \tilde{F}$ es campo Gradiente

$$\Rightarrow \nabla \times \tilde{F} = 0$$

Volviendo

$$F = \tilde{F} + (-y, x)$$

$$\int_{\mathcal{C}} F\, d\vec{s} = \underbrace{\int_{\mathcal{C}} \tilde{F}\, d\vec{s}}_{\displaystyle \overset{\shortparallel}{-\int \underbrace{\tilde{Q}_x - \tilde{P}_y}_{=0}\, dx\, dy}} + \underbrace{\int_{\mathcal{C}} (-y, x)\, d\vec{s}}$$

Pues $\tilde{F}$ es CG.

$$= - \int_{\theta=0}^{2\pi} \int_{r=0}^{1} 2 \cdot d\theta\, dr$$

$$= -2 \cdot \pi \;//$$

Solución

$$\int_{\mathcal{C}} F\, d\vec{s} = -2\pi$$