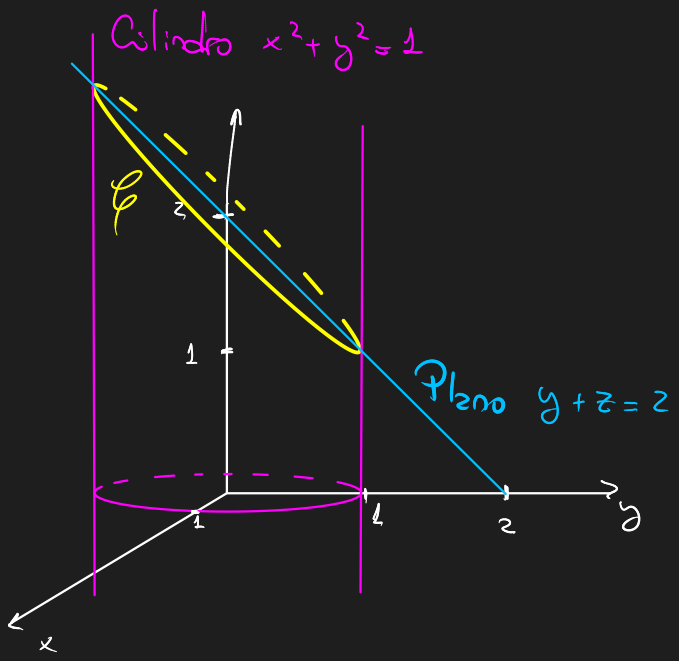
Re-resolución de Parcial 1 - Ej 1.

1. Consideramos la curva C determinada por la intersección entre la superficie dada por la ecuación $x^2+y^2-1=0$ y la superficie dada por $y+z-2=0$. Calcular $\int_C f\,ds$ donde $f(x,y,z)=\sqrt{1+x^2}$.



$\mathbb{R}^3 \to \mathbb{R}$

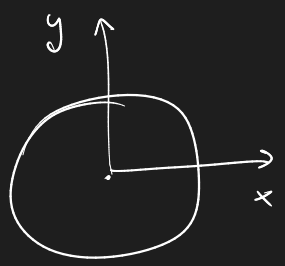
$$\int_{\mathcal{C}} f\,ds = \int_{\theta} f(\sigma(\theta)) \cdot \|\sigma'(\theta)\|\,ds$$

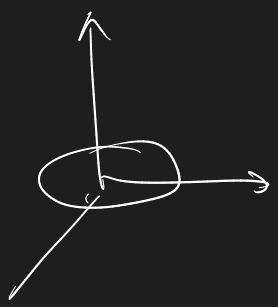
Uso polares

$x = \cos\theta$ con $\theta \in [0, 2\pi)$

$y = \sin\theta$

$z = ?$





Uso el plano

$y+z=2 \Rightarrow z = 2-y$

$z = 2 - \sin\theta$

$$\sigma(\theta) = (\cos\theta, \sin\theta, 2 - \sin\theta)$$

$$\sigma'(\theta) = (-\sin\theta, \cos\theta, -\cos\theta)$$

$$\|\sigma'(\theta)\| = \sqrt{1 + \cos^2\theta}$$

$$f(\sigma(\theta)) = \sqrt{1 + \cos^2\theta}$$

Resuelvo integral

$$\int_{\mathcal{C}} f \, ds = \int_{\theta} f(\sigma(\theta)) \cdot \|\sigma'(\theta)\| \, d\theta$$

$$= \int_{\theta=0}^{2\pi} \sqrt{1 + \cos^2\theta} \cdot \sqrt{1 + \cos^2\theta} \, d\theta$$

$$= \int_{\theta=0}^{2\pi} 1 \, d\theta + \int_{\theta=0}^{2\pi} \cos^2\theta \, d\theta$$

$$= 2\pi + \int_{\theta=0}^{2\pi} \frac{\cos(2x) + 1}{2} \, d\theta$$

$$= 2\pi + \frac{1}{2}\left(\frac{\operatorname{sen}(2x)}{2}\Big|_0^{2\pi} + 2\pi \right)$$

$\operatorname{sen}(2x)/2\Big|_0^{2\pi} = 0$

$$= 3\pi$$

Solución

$$\int_{\mathcal{C}} f \, ds = 3\pi$$