

Iván

Feb 10

REC: TEOREMA DE GREEN

SEA $\Omega \subseteq \mathbb{R}^2$ ABIERTO, $F: \Omega \rightarrow \mathbb{R}^2$,
Se pide cuando se trabaja con derivadas. *(No comerse el coco con esto)*

$F = (P, Q)$, $P, Q: \Omega \rightarrow \mathbb{R}$ DE CLASE C^1

SEA $R \subseteq \Omega$ UNA REGIÓN (O UNIÓN FINITA DE REGIONES) DE TIPO II, $C = \partial R$ CURVE CERRADA, SIMPLE, SUAVE A TROZOS, ORIENTADA POSITIVAMENTE

$$\text{LUEGO } \int_C F d\vec{s} = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

$$= \int_a^b \langle F(\sigma(t)), \sigma'(t) \rangle dt$$

P3 E12) Calcular

$$\int_C F d\vec{s} \quad \text{con } F = (P, Q)$$

$$P(x, y) = \frac{x \cdot \sin\left(\frac{\pi}{2(x^2+y^2)}\right) - y(x^2+y^2)}{(x^2+y^2)^2}$$

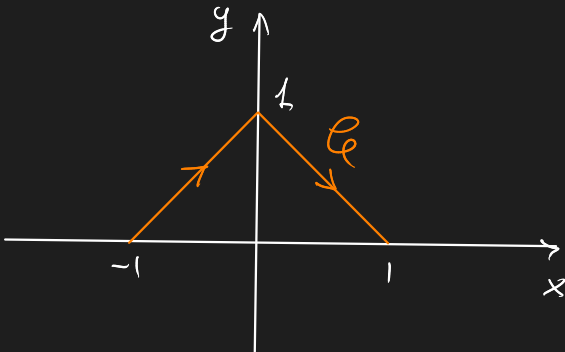
$$Q(x, y) = \frac{y \cdot \sin\left(\frac{\pi}{2(x^2+y^2)}\right) + x(x^2+y^2)}{(x^2+y^2)^2}$$

Sobre:

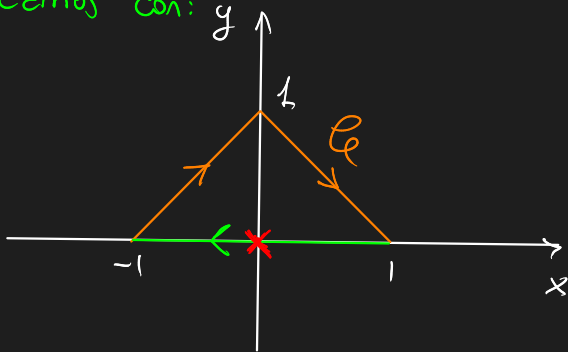
$$C = \begin{cases} y = x + 1 \\ y = 1 - x \end{cases}$$

$$0 \leq x \leq 1$$

Orientada de

$$(-1, 0) \text{ a } (1, 0)$$


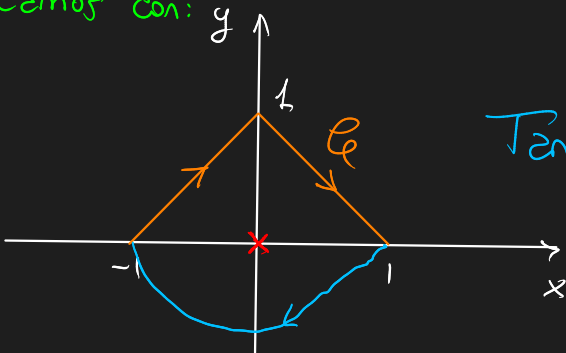
Veamos con:



$$\mathfrak{B} \rightarrow \{(x, 0), -1 \leq x \leq 1\}$$

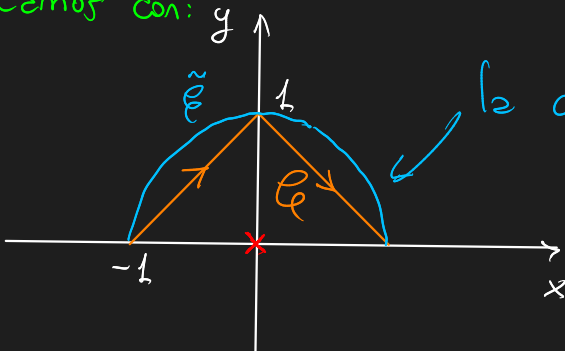
$\hookrightarrow \neq \underline{no}$ es C^1 en $(0,0)$

Veamos con:



Tampoco! lo incluye!

Veamos con:



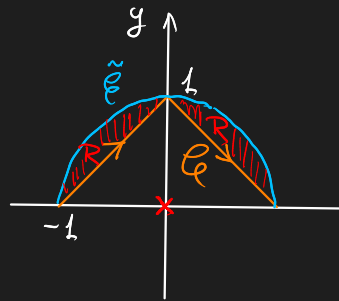
la guerra 250 !

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Me conviene el semicírculo por que al componer me desaparecen los $x^2 + y^2$ pues son iguales al radio = 1.

Planteo

$$\underbrace{\int_{\mathcal{C}} \mathbf{F} d\vec{s}}_{\text{Inógnito}} + \int_{\tilde{\mathcal{C}}} \mathbf{F} d\vec{s} \stackrel{\text{Teo. de Green}}{=} \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$



$\mathcal{C} \cup \tilde{\mathcal{C}}$ está orientada positivamente

Calculemos integrales:

Teníamos:

$$P(x,y) = \frac{x \cdot \sin\left(\frac{\pi}{2(x^2+y^2)}\right) - y(x^2+y^2)}{(x^2+y^2)^2}$$

$$Q(x,y) = \frac{y \cdot \sin\left(\frac{\pi}{2(x^2+y^2)}\right) + x(x^2+y^2)}{(x^2+y^2)^2}$$

Llamo:

$$k(x,y) = x^2 + y^2$$

$$g(t) = \frac{\sin\left(\frac{\pi}{2t}\right)}{t^2}$$

Reescribo

$$P(x,y) = x \cdot g \circ k(x,y) - \frac{y}{k(x,y)}$$

$$Q(x,y) = y \cdot g \circ k(x,y) + \frac{x}{k(x,y)}$$

$$\frac{\partial Q}{\partial x} = y \cdot g' \circ k(x, y) \cdot 2x + \frac{k(x, y) - x \cdot 2x}{k^2(x, y)}$$

$$\frac{\partial P}{\partial y} = x \cdot g' \circ k(x, y) \cdot 2y - \frac{k(x, y) - y \cdot 2y}{k^2(x, y)}$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{2k(x, y) - 2x^2 - 2y^2}{k^2(x, y)} = 0$$

$$\iint_{\mathbb{R}} \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dx dy = 0 //$$

Calculo integral curvilínea

Parametrizo a

$$\tilde{C} \rightarrow \sigma : [0, \pi] \rightarrow \mathbb{R}^2$$

$$\sigma(t) = (\cos t, \sin t)$$

Chequeo orientación

$$\sigma(0) = (1, 0)$$

$$\sigma(\pi) = (-1, 0)$$

$$\sigma'(t) = (-\sin t, \cos t)$$

$$\int_{\mathcal{C}} F ds = \int_0^\pi \langle F(\sigma(t)), \sigma'(t) \rangle dt =$$

$$= \int_0^\pi \left\langle \left(\frac{\cos t \cdot \sin\left(\frac{\pi}{2}\right) - \sin t \cdot 1}{1^2}, \frac{\sin t \cdot \sin\left(\frac{\pi}{2}\right) + \cos t \cdot 1}{1^2} \right), \right.$$

$$\left. (-\sin t, \cos t) \right\rangle dt$$

$$\dots$$

$$= \int_0^\pi 1 \cdot dt = \pi$$

Vuelvo a

$$\int_{\mathcal{C}} F ds + \pi = 0 \Rightarrow$$

$$\int_{\mathcal{C}} F ds = -\pi$$

Ej:

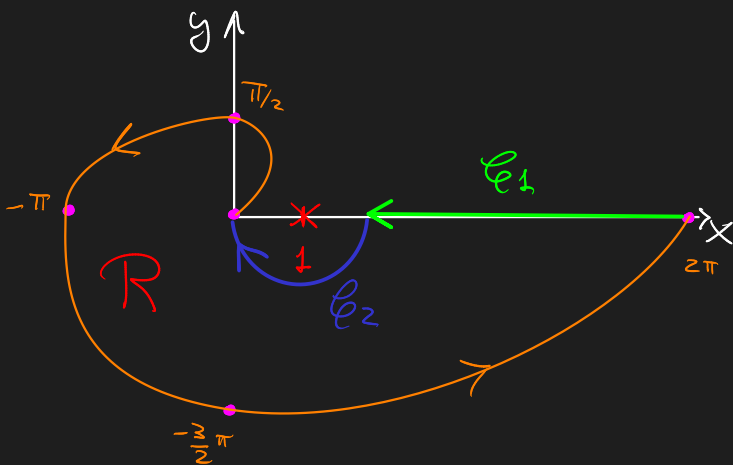
Hallar $\int_{\mathcal{C}} F d\vec{s}$

$\mathcal{C} \rightarrow$ dada en forma Polar por

$$\Gamma(\theta) = \theta, \quad 0 \leq \theta \leq 2\pi$$

orientada de
 $(0,0)$ a $(2\pi,0)$

$$F(x,y) = \left(\frac{-y}{(x-1)^2 + y^2} + 3y, \frac{x-1}{(x-1)^2 + y^2} \right)$$



$$\underbrace{\int_C F d\vec{s}}_{?} + \int_{C_1} F d\vec{s} + \int_{C_2} F d\vec{s} =$$

$$\stackrel{\text{green}}{=} \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

Celso

$$\frac{\partial Q}{\partial x}(x,y) = \frac{(x-1)^2 + y^2 - (x-1) \cdot 2(x-1)}{((x-1)^2 + y^2)^2}$$

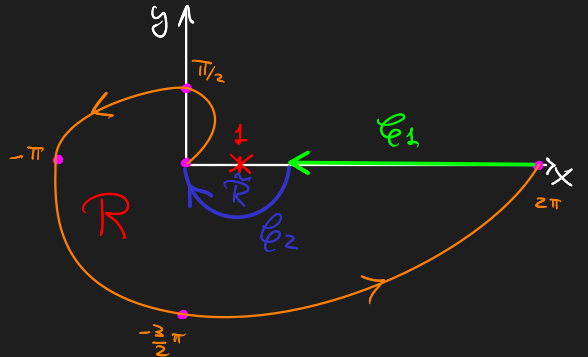
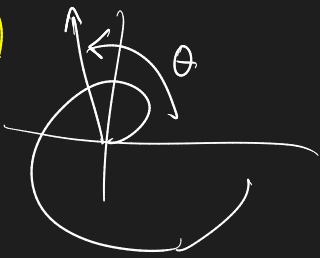
$$\frac{\partial P}{\partial y} = \frac{-1((x-1)^2 + y^2) + y \cdot 2y}{((x-1)^2 + y^2)^2} + 3$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 - 3 = -3 //$$

$$\iint_R (-3) \, dx \, dy = -3 \, \text{Área}(R)$$

$$A_{\text{res}}(R) = A(R \cup \tilde{R}) - A(\tilde{R})$$

$$\frac{\pi \cdot R^2}{2} \approx \frac{\pi}{2}$$


$$R, \tilde{R})$$


en Polares:

$$R \cup \tilde{R} = \begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq r \leq \theta \end{cases}$$

$$\int \int_{R, \tilde{R}} 1 dx dy \stackrel{\text{Polar}}{=} \int_0^{2\pi} \int_0^{\theta} r dr d\theta \quad \swarrow \text{Jacobian}$$

$$A(R) = \frac{4}{3}\pi^3 - \frac{\pi}{2}$$

Entonces

$$\begin{aligned} \int \int_R (-3) dx dy &= -3 \cdot \left(\frac{4}{3}\pi^3 - \frac{\pi}{2} \right) \\ &= \frac{3}{2}\pi - 4\pi^3 \end{aligned}$$

Calcular curvaturas

$$\mathcal{C}_1 : \quad \begin{array}{c} | \quad | \quad | \\ \hline \quad \quad \quad \leftarrow \\ \quad \quad \quad 2 \quad 2\pi \end{array}$$

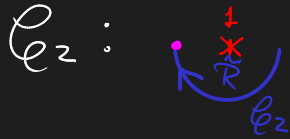
$$\sigma(t) = (t, 0) \quad 2 \leq t \leq 2\pi$$

\nwarrow orientación opuesta

$$\int_{\mathcal{C}_1} F d\vec{z} = - \int_2^{2\pi} \langle F(\sigma(t)), \sigma'(t) \rangle dt$$

$$= - \int_2^{2\pi}$$

$$= 0$$



$$\sigma(t) = (1 + \cos t, \sin t) \quad \pi \leq t \leq 2\pi$$

↑ orienta en sentido contrario al de \mathcal{C}_2

$$\int_{\mathcal{C}_2} F d\vec{s} = - \int_{\pi}^{2\pi} \langle F(\sigma(t)), \sigma'(t) \rangle dt$$

$$= - \int_{\pi}^{2\pi} \left\langle \left(-\frac{\sin t}{1} + 3 \sin t, \frac{\cos t}{1} \right), (-\sin t, \cos t) \right\rangle dt$$

$$= - \int_{\pi}^{2\pi} -2 \sin^2 t + \cos^2 t = \dots = \frac{\pi}{2} //$$

Vuelvo

$$\int_{\mathcal{C}} F d\sigma + \frac{\pi}{2} + 0 = -4\pi^3 + \frac{3}{2}\pi$$

