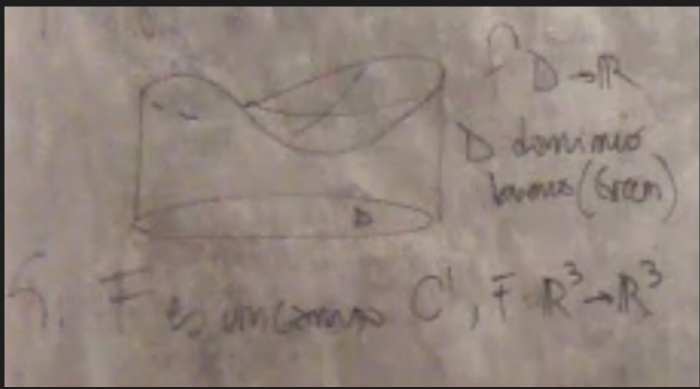
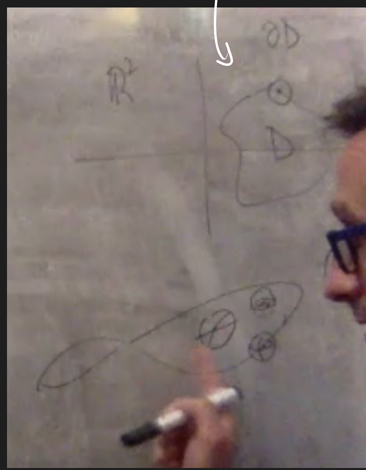
Teorema de Stokes

Versión para superficies dadas por gráficos de funciones



Borde Topológico



Profe

El "borde" de una caja ≡ Todo el cartón

Pero no usamos ESE borde, sino:

Bordes

$\mathcal{C} = \partial D$, parametrizado por

$$\sigma(t) = (x(t), y(t))$$

$$S = \mathrm{Graf}(f)$$

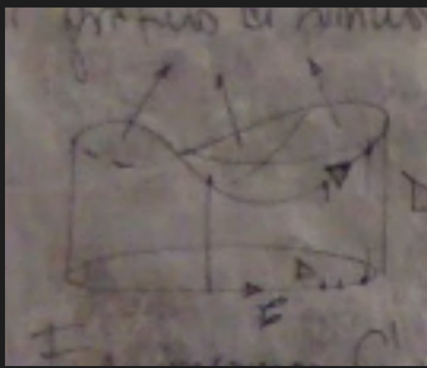
El borde de S será la curva $\Gamma \subset \mathbb{R}^3$ dado por

$$\gamma(t) = \big(x(t),\, y(t),\, f(x(t), y(t))\big)$$

Stokes

$$\iint_S \nabla \times F \, d\vec{s} = \int_{\partial S} F \, d\vec{s}$$

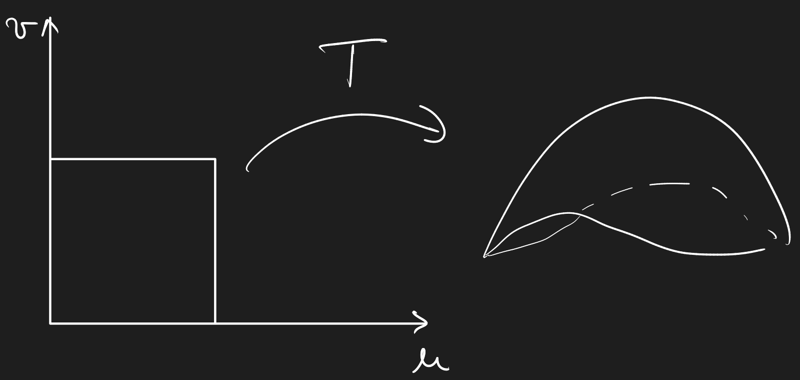
Orientaciones Compatibles



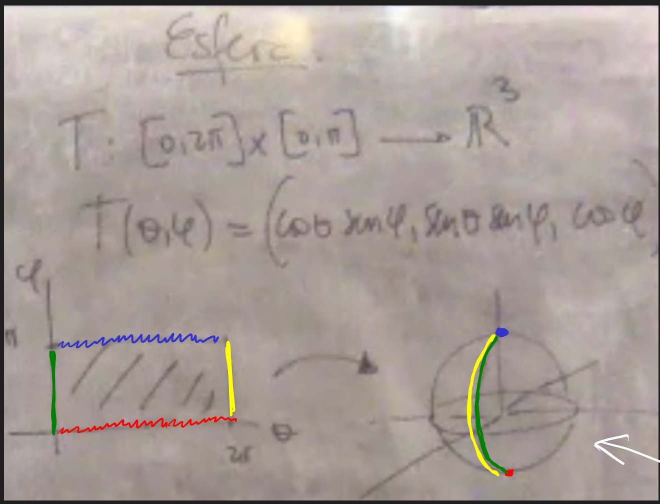
Superficies generales (parametrizadas)

$D \subset \mathbb{R}^3$, $S \subset \mathbb{R}^3$

$T : D \to S$ continua suryectiva



Esfera



No siempre vale

$$\partial S = T(\partial S)$$

$$T(\theta, 0) = (0,0,1)$$

$$T(\theta, \pi) = (0,0,-1)$$

$$T(0,\varphi) = (\sin\varphi, 0, \cos\varphi)$$

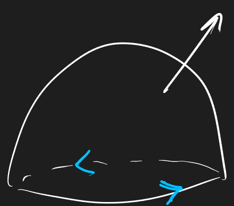
Superficie compacta sin borde

Obs:

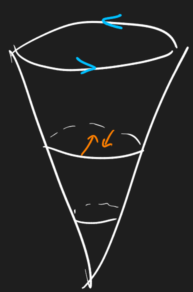
$T : D \to S$ es regular

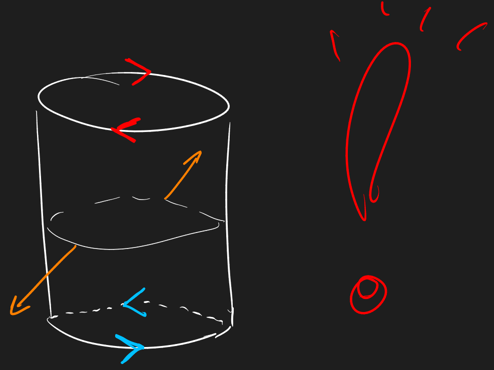
$$\Rightarrow T(\partial D) = \partial S$$

Orientaciones (dibujitos



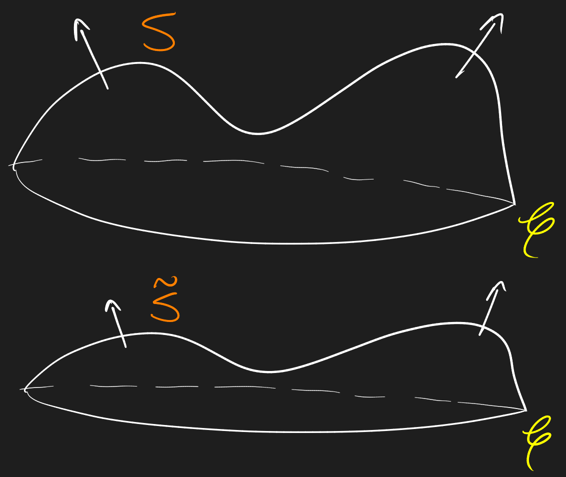
S orientada





Un uso típico:

Cambiar superficies



S superficie / $\partial S = \mathcal{C}$

$\tilde{S}$ superficie / $\partial S = \mathcal{C}$

orientadas como en la imagen.

$$\iint_S \nabla \times F \, d\vec{S} = \int_{\mathcal{C}} F \, d\vec{s} = \iint_{\tilde{S}} \nabla \times F \, d\vec{S}$$

Ejemplo/ejercicio

$$f : D \to \mathbb{R}, \quad f \in C^2, \quad f \geq 0, \quad f(\partial D) = 0$$

$$D = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \}$$

$$S = \mathrm{Graf}(f)$$

Obs:

$$\{(x,y) \in \mathbb{R}^2 : x^2+y^2 = 1 \, , \; z=0\}$$

Sea $F(x,y,z) = \left(e^{yz}, \; xz \cdot e^{yz} + x \, , \; xy\, e^{yz}\right)$

Calcular $\displaystyle\iint_S \nabla \times F \, d\vec{s} = ?$

↑ no la conozco

Calculamos

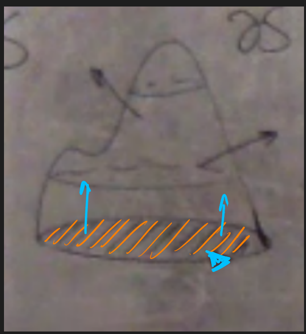
$$\nabla \times F = \det \begin{pmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ P & Q & R \end{pmatrix}$$

$$= \left(x \cdot e^{yz} + yxz \, e^{yz} - x e^{yz} - xyz e^{yz}, \; y e^{yz} - y e^{yz}, \; z e^{yz} + 1 - z e^{yz}\right)$$

$$= (0, 0, 1)$$

Stokes

$$\iint_S \nabla \times F \, d\vec{S} = \int_{\partial S} F \, d\vec{s} = \iint_{\tilde{S}} \nabla \times F \, d\vec{S} = \circledast$$

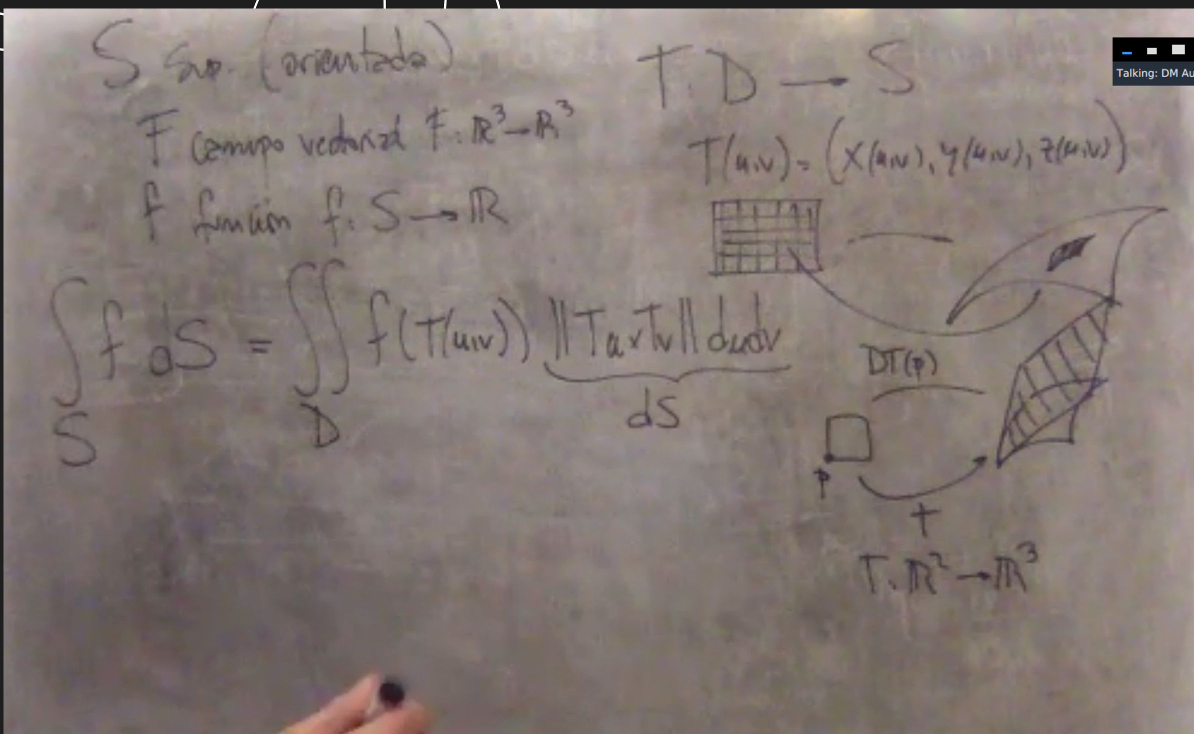


Vector normal unitario

$$= \iint_{\tilde{S}} \underbrace{\langle \nabla \times F, \eta \rangle}_{\text{función escalar}} \, dS$$

$$= \iint_{\tilde{S}} \langle (0,0,1), (0,0,1) \rangle \, dS$$

$$= \iint_{\tilde{S}} 1 \, dS = A(S) = \pi \; //$$



$ds \quad d\vec{s} \quad dS \quad d\vec{S}$

- ds: escalar — masa de alambre
- $d\vec{s}$: vectorial — curvilínea
- dS: escalar
- $d\vec{S}$: Flujo de campo (vectorial)

$$\iint_{\substack{S \\ \text{(orientada)}}} F\, d\vec{S} = \iint_S \langle F, \eta \rangle \, dS$$

$$= \iint_D \left\langle F(T(u,v)), \frac{T_u \times T_v}{\|T_u \times T_v\|} \right\rangle \cdot \|T_u \times T_v\| \, du\, dv$$

$$= \iint_D \langle F(T(u,v)), T_u \times T_v \rangle \, du\, dv$$