Práctica #1

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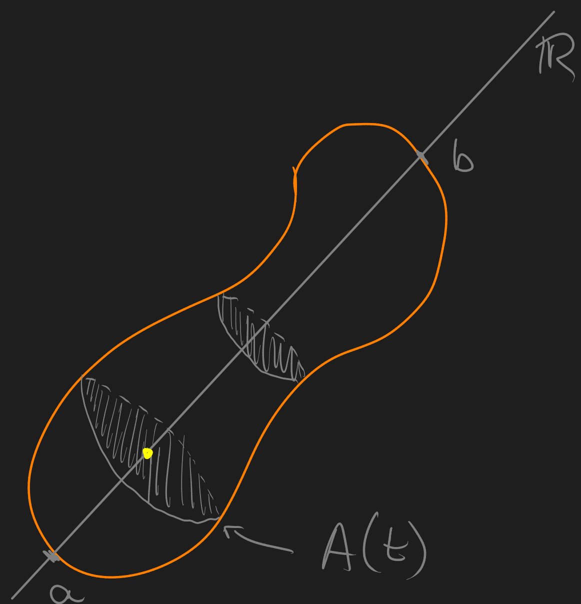
Análisis 2: "Materia Práctica"

Breve Repaso de Integración $(\mathbb{R}^2, \mathbb{R}^3)$

Principio de Cavalier:

$W \subset \mathbb{R}^3$

↳ un cuerpo (volúmen)

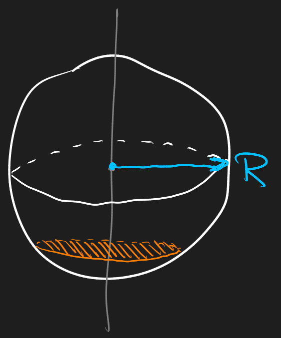


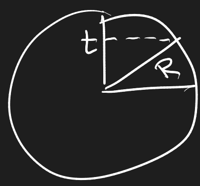
$$Vol(W) = \int_a^b A(t)\,dt$$

Área de rebanada: $A(t)$

Existe $A : [a,b] \to \mathbb{R}_{\geq 0}$

Ejemplo: Esfera de Radio R





$$R^2 = t^2 + r(t)^2$$

$$r(t)^2 = R^2 - t^2$$

$$\text{Vol}(S(R)) = \int_{-R}^{R} \pi \cdot (R^2 - t^2) \cdot dt$$

$$= \pi \left[2R^3 - \frac{t^3}{3} \right|_{-R}^{R}$$

$$= \pi \left(2R^3 - \frac{2}{3} R^3 \right)$$

$$= \frac{4}{3} \pi \cdot R^3$$

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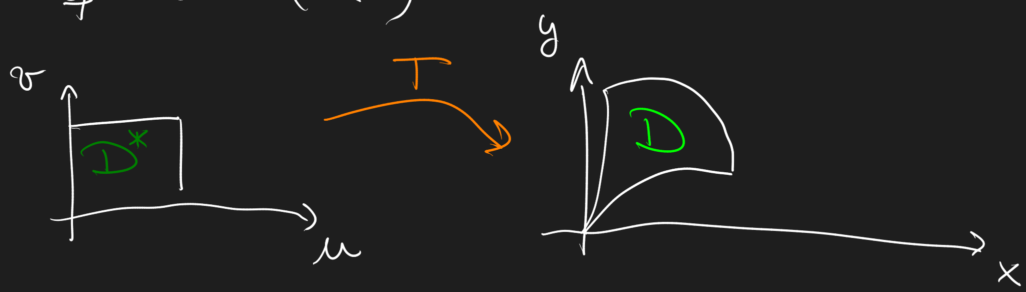
Teorema de Cambio de Variable

Esquema: $(\mathbb{R}^2)$



Tenemos

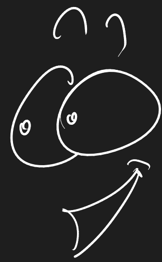
$f : D \to \mathbb{R}$ continua

$T : D^* \to D$ con $T \in C^1$

$T(D^*) = D$ $\quad$ T inyectiva (o casi)

$$\int_D f(x,y)\,.\,dx\,dy = \int_{D^*} f(T(u,v))\,.\,JT(u,v)\,.\,du\,dv$$

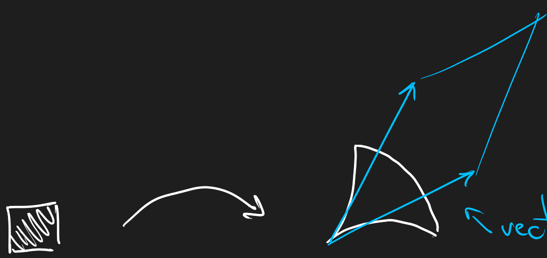
- Ejemplo con $f(x,y) = 1$



$$JT(u,v) = |\det DT(u,v)|$$

$$T(u,v) = (x(u,v),\ y(u,v))$$

$$DT(u,v) = \begin{pmatrix} \frac{\partial}{\partial u}x & \frac{\partial}{\partial v}x \\ \frac{\partial}{\partial u}y & \frac{\partial}{\partial v}y \end{pmatrix}(u,v)$$



Det.$(T(u,v))$ = Paralelogramo

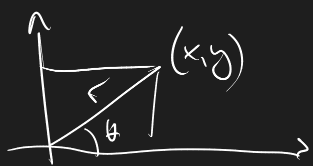
vectores $\vec{\partial x}$, $\vec{\partial y}$

Clásicos:

Polares

$x = r \cdot \cos\theta$

$y = r \cdot \sin\theta$



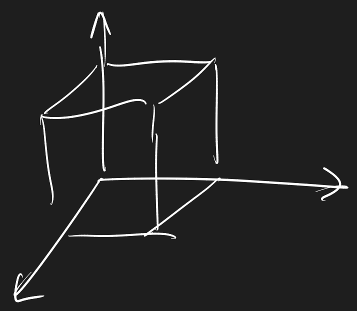
$T: \mathbb{R}^2 \to \mathbb{R}^2$

$T(r,\theta) = (r \cdot \cos\theta,\ r \cdot \sin\theta)$

$$JT(r,\theta) = r$$

- <u>Cilíndricas</u> $(\mathbb{R}^3 = \mathbb{R}^2 \times \mathbb{R})$

$$x = r \cdot \cos\theta$$
$$y = r \cdot \sin\theta$$
$$z = z$$

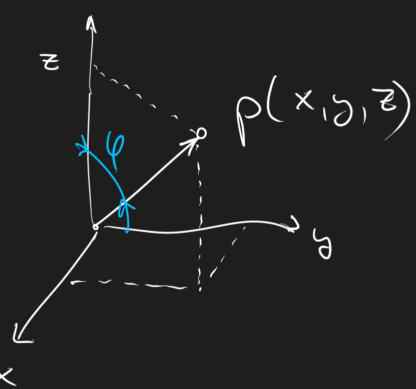


$$JT(r,\theta,z) = r$$

Esféricas $(\mathbb{R}^3 \sim \text{cebolla})$

$$x = \cos\theta \cdot r \cdot \sin\varphi$$
$$y = \sin\theta \cdot r \cdot \sin\varphi$$
1° → $$z = r \cdot \cos\varphi$$

"Polares en el piso"



$$JT(r,\theta,\varphi) = r^2 \cdot \sin\varphi$$

Ejemplo: Volumen de $B(0,R)$

$$\text{Vol}(B(0,R)) = \int_{B(0,R)} 1 \, dv$$

$$= \int_0^{2\pi} \int_0^{\pi} \int_0^{R} r^2 \cdot \sin\varphi \cdot dr$$

$$= 2\pi \cdot \frac{R^3}{3} \cdot \int_0^{\pi} \sin\varphi \, d\varphi$$

$$= \frac{2}{3} \pi R^3 \cdot 2$$

$$= \frac{4}{3} \pi R^3$$

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Curvas en $\mathbb{R}^2, \mathbb{R}^3$

$\mathcal{C} \subset \mathbb{R}^3$ $(\text{o } \mathbb{R}^2)$ es un conjunto imagen de

$$\sigma : [a,b] \to \mathbb{R}^3$$

Continua + Sobreyectiva

Problema 1:

Estático:

$$\mathcal{C} = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$$

Queremos:

Input: Conjunto

Output: Parametrización

Problema 2:

$$\sigma : [0, 2\pi) \to \mathbb{R}^2$$

$$\sigma(t) = (\cos t, \sin t)$$

Input: Parametrización

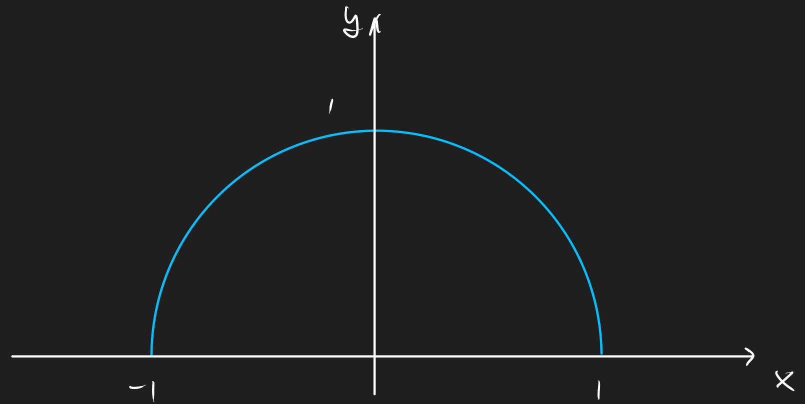
Output: Conjunto $\mathcal{C}$

Acá:

σ parametriza $\mathcal{C}$ (inyectiva)

Ejemplo:

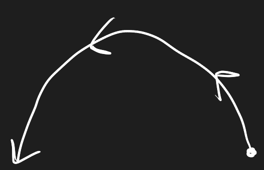
$$\mathcal{C} = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1, \ y \geq 0 \}$$



Tres paramet. de $\mathcal{C}$

1) $\gamma : [0, \pi] \to \mathbb{R}^2$

$$\gamma(t) = (\cos t, \sin t)$$



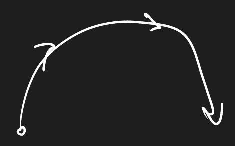
2) $\sigma : [-1,1] \to \mathbb{R}^2$

$\sigma(t) = (t, \sqrt{1-t^2})$

(ejemplo de $\sigma(x) = (x, f(x))$

3) $\alpha : [0, \pi] \to \mathbb{R}^2$

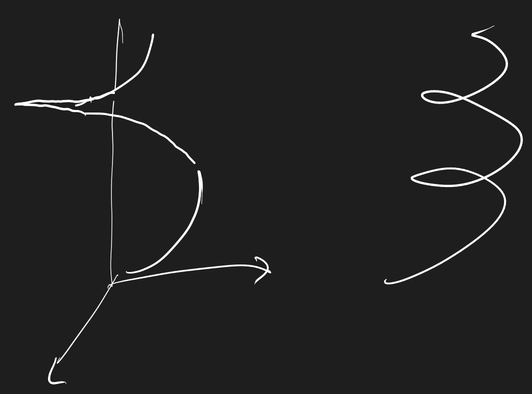
$\alpha(t) = (-\cos t, \sin t)$



Ejemplo :

$\sigma : [0, 2\pi] \to \mathbb{R}^3$

$\sigma(t) = (\cos t, \sin t, t)$ "escalera caracol"

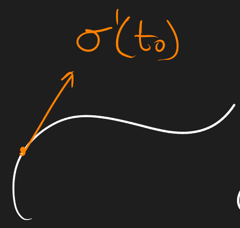


Velocidad y Vector Tangente

Vector tangente a $\mathcal{C}$ en $p = \sigma(t_0)$

$\sigma : [a,b] \to \mathcal{C}$

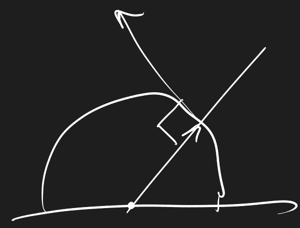
$\left.\begin{array}{l} \sigma \in C^1 \\ \sigma'(t) \neq \vec{0} \end{array}\right\}$ regular



$$\sigma'(t_0) = (x'(t_0), y'(t_0)) \quad \text{en } \mathbb{R}^2$$

Para el semi-círculo

1) • $\gamma'(t) = (-\sin t, \cos t)$



obs :

$$\langle \gamma'(t), \gamma(t) \rangle = 0$$

2) • $\sigma'(t) = \left(1, \dfrac{-t}{\sqrt{1-t^2}}\right)$ con $|t| \neq 1$

3) • $\alpha'(t) = (\sin t, \cos t)$

Recta tangente

$\mathcal{C} \subset \mathbb{R}^3$ parametrizada por σ regular ($\mathcal{C}$ suave)

Recta tangente en $\sigma(t_0) = p \in \mathbb{R}^3$

$$L : \lambda . \sigma'(t_0) + \sigma(t_0) \quad \lambda \in \mathbb{R}$$

Parametrizamos la recta L con $\ell : \mathbb{R} \to \mathbb{R}^3$

$$\ell(t) = (t-t_0)\cdot\sigma'(t_0) + \sigma(t_0)$$

Preguntar

$$\begin{cases} \ell(t_0) = \sigma(t_0) \\ \ell'(t_0) = \sigma'(t_0) \end{cases}$$

Curvas en Polares ($\mathbb{R}^2$)

La ecuación $r = r(\theta)$ $\left(\rho = \rho(\theta)\right)$

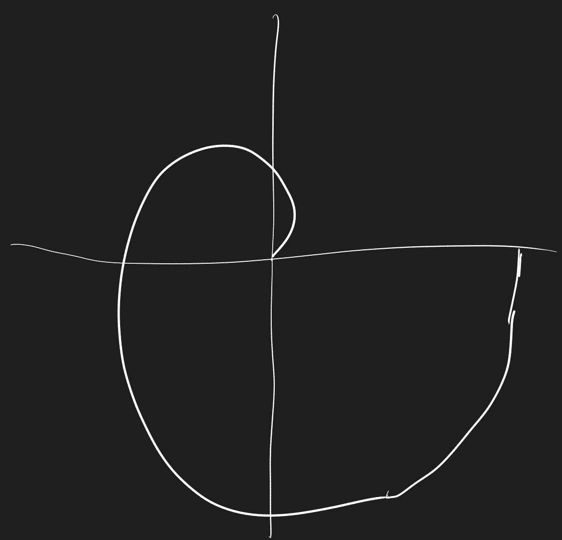
se interpreta como

$$\begin{aligned} \sigma(\theta) &= \left(x(\theta), y(\theta)\right) \\ &= \left(r(\theta)\cdot\cos\theta, r(\theta)\cdot\sin\theta\right) \\ &= r(\theta)\cdot(\cos\theta, \sin\theta) \end{aligned}$$

Ejemplo

$r(\theta) = \theta$, $\theta \in [0, 2\pi]$

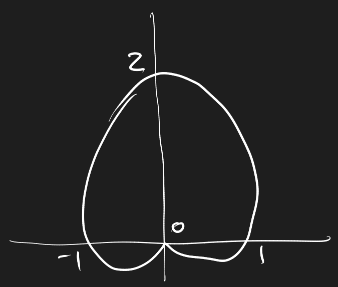
$\sigma(\theta) = \theta(\cos\theta, \sin\theta)$



$\gamma : [\pi, \frac{3}{2}\pi] \to \mathbb{R}^2$ dado por $r(\theta) = \theta$

Cardioide:

$$r(\theta) = 1 + \sin\theta \quad , \quad \theta \in [0, 2\pi]$$



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