Mercurio 17 Feb

Campos Conservativos

Def: $F: \mathbb{R}^3 \to \mathbb{R}^3$ se dice conservativo si:

$$\int_{\mathcal{C}_1} F\, d\vec{s} = \int_{\mathcal{C}_2} F\, d\vec{s}$$

$\forall$ par de curvas simples, suaves a trozos con el mismo punto inicial y final

Teorema:

$F \in C^1(\mathbb{R}^3 \setminus \Omega)$

$\# \Omega < \infty$

Son equivalentes

$\boxed{1}$ $\int_{\mathcal{C}} F\, d\vec{s} = 0 \quad \forall \mathcal{C}$ cerrada, simple

$\boxed{2}$ $\int_{\mathcal{C}_1} F\, d\vec{s} = \int_{\mathcal{C}_2} F\, d\vec{s} \quad \forall \mathcal{C}_1, \mathcal{C}_2 \quad p \to q$

$\boxed{3}$ $F = \nabla f \quad , \quad f : \mathbb{R}^3 \to \mathbb{R}$

$F : \mathbb{R}^3 \to \mathbb{R}^3$

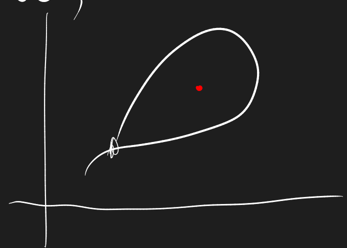
$\boxed{4}$ $\nabla \times F = 0$ en $\mathbb{R}^3 \setminus \Omega$

¡Ojo!

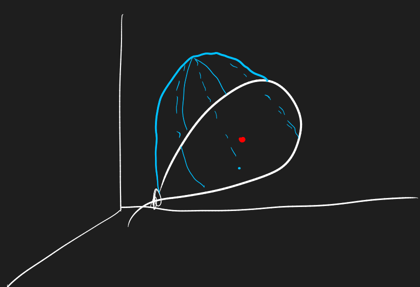
en $\mathbb{R}^2$ vale pero sin excepciones

con $F \in C^1(\mathbb{R}^2)$

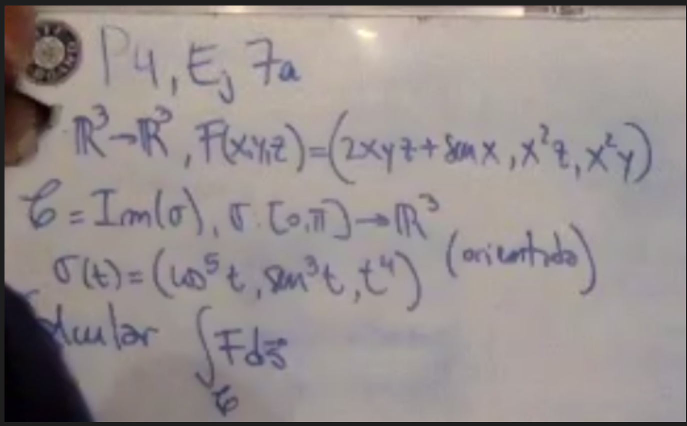
(No puedo armar lacito con punto discontinuo, no es arco conexo)



en $\mathbb{R}^3$ sí puedo esquivarlo



P4 Ej 7a



F es $C^1(\mathbb{R}^3)$

$$\nabla \times F = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \; -\frac{\partial F_3}{\partial x} + \frac{\partial F_1}{\partial z}, \; \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right)$$

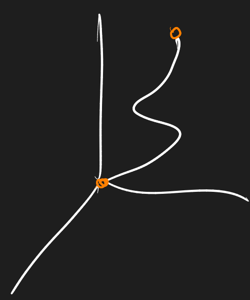
$$= (0, 0, 0)$$

$\therefore$ F es conservativo !

Calculo principio/fin

$$\sigma(0) = (1,0,0)$$

$$\sigma(\pi) = (-1, 0, \pi^4)$$



Si no fuera gradiente
Proponemos otro camino

Γ_1: desde $(1,0,0) \rightarrow (-1,0,0)$

Γ_2 $\quad (-1,0,0) \rightarrow (-1,0,\pi^4)$

$\Gamma_1 : [-1;1] \to \mathbb{R}^3$

$\Gamma_1(t) = (t, 0, 0)$ (al revés)

$\Gamma_2 : [0, \pi^4] \to \mathbb{R}^3$

$\Gamma_2(t) = (-1, 0, t)$ (sentido correcto)

$\Gamma_1'(t) = (1, 0, 0)$

$\Gamma_2'(t) = (0, 0, 1)$

Como :
$$\nabla \times F = \vec{0}$$

$$\int_{\mathcal{C}} F\, d\vec{s} = \int_{\Gamma_1^-} F\, d\vec{s} + \int_{\Gamma_2} F\, d\vec{s}$$

$$= -\int_{\Gamma_1} F\, d\hat{s} + \int_{\Gamma_2} F\, d\vec{s}$$

$$\int_{\Gamma_1} F\, d\hat{s} = \int_{-1}^{1} \langle F(\Gamma_1(t)), \Gamma_1'(t) \rangle\, dt$$

$$= \int_{-1}^{1} \sin t \, dt$$

$$= -\cos(1) + \cos(-1)$$

$$= 0$$

$$\int_{\Gamma_2} F \cdot d\vec{s} = \int_0^{\pi^4} 0 \, dt = 0$$

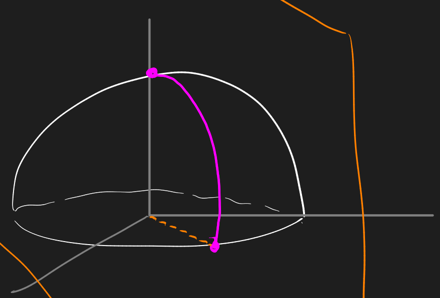
$$\therefore \quad \int_{\mathcal{C}} F \, d\vec{s} = 0$$

También podría haber encontrado el potencial f.

P4 E10)

$$F(x,y,z) = \left(2xy + z^2,\ x^2 - 2yx,\ 2xz - y^2\right)$$

$\mathcal{C}$ la curva dada por $\begin{cases} x^2+y^2+z^2 = 1 \\ y = x \\ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \to (0,0,1) \end{cases}$



Obs:

$$\nabla \times F = \left(-2y-(-2y),\ 2z-2z,\ 2x-2x\right)$$

$$= (0,0,0)$$

$\Rightarrow F$ es conservativo.

$\Rightarrow F = \nabla f$ con $f: \mathbb{R}^3 \to \mathbb{R}$

Reescribo F

- $\dfrac{\partial f}{\partial x} = 2xy + z^2$

$$f(x,y,z) = x^2 y + x z^2 + h(y,z)$$

$$\Rightarrow \frac{\partial f}{\partial y} = x^2 + \frac{\partial h}{\partial y}(x,z) = x^2 - 2yz$$

$$\frac{\partial h}{\partial y} = -2yz$$

$$\Rightarrow h(y,z) = -y^2 z + g(z)$$

$$f(x,y,z) = x^2 y + x z^2 - y^2 z + g(z)$$

$$\frac{\partial f}{\partial z} = 2xz - y^2 - g'(z)$$

$$\Rightarrow g(z) = C$$

$$f(x,y,z) = x^2y + xz^2 - y^2z$$

$$\int_{\mathcal{C}} F\,d\vec{s} = f(0,0,1) - f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$= 0 - \frac{1}{2\sqrt{2}}$$

$$= -\frac{\sqrt{2}}{4} \quad //$$

Atent¡!

- Si $F = \nabla f$

$$\Rightarrow \nabla \times F = (0,0,0)$$

La vuelta no vale

$$\not\Leftarrow$$

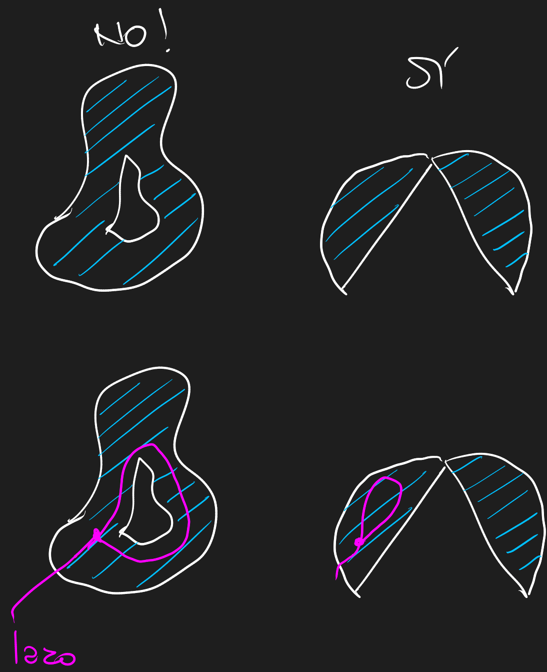
Hay que ver si es simplemente conexo

↳ si lo es

↳ vale la vuelta.

$\mathbb{R}^2$:

Simplemente conexo



$\mathbb{R}^3$