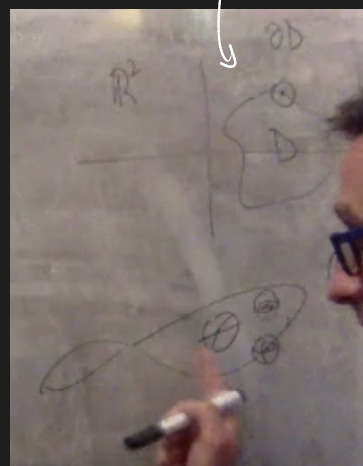
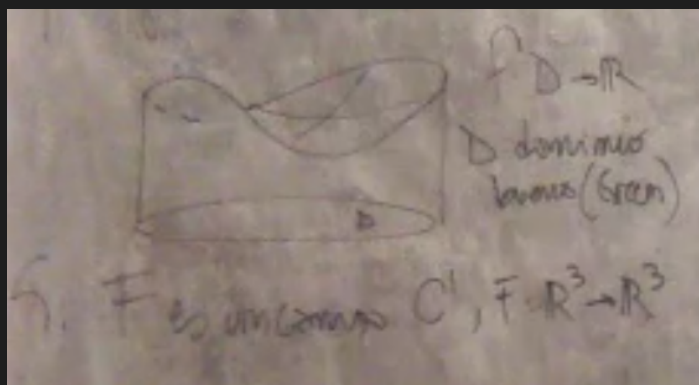


Teorema de Stokes

Versión para superficies dadas por
gráficas de funciones



El "borde" de una caja \equiv Todo el cartón

Pero no usamos ESE^o borde, sino:

Bordes

$\partial = \partial D$, parametrizado por

$$\sigma(t) = (x(t), y(t))$$

$$S = \text{Gráf}(f)$$

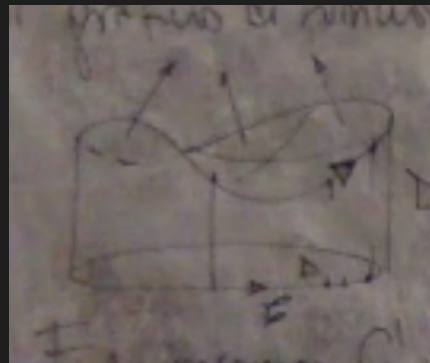
El borde de S será la curva $\Gamma \subset \mathbb{R}^3$ dado por

$$\gamma(t) = (x(t), y(t), f(x(t), y(t)))$$

Stokes

$$\iint_S \nabla_x F d\vec{s} = \int_{\partial S} F d\vec{s}$$

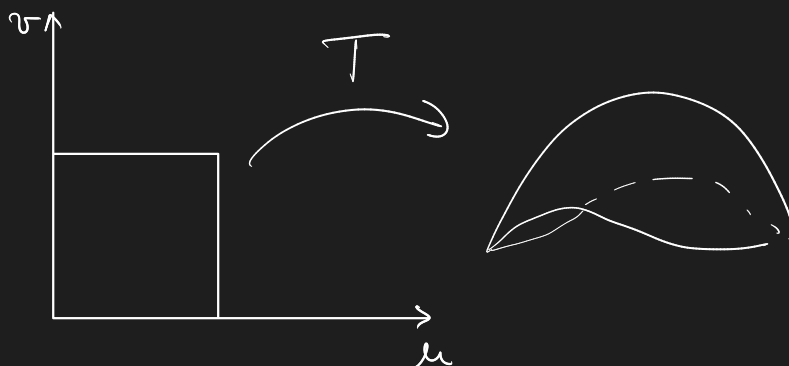
Orientaciones
Compatibles



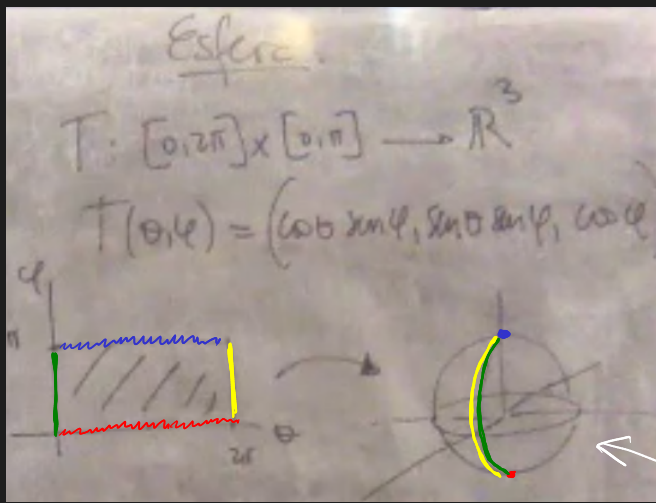
Superficies generales (parametrizadas)

$$D \subset \mathbb{R}^3, S \subset \mathbb{R}^3$$

$T: D \rightarrow S$ continua suryectiva



Esfera



No siempre vale

$$\partial S = T(\partial D)$$

$$T(\theta, 0) = (0, 0, 1)$$

$$T(\theta, \pi) = (0, 0, -1)$$

$$T(0, \varphi) = (\sin \varphi, 0, \cos \varphi)$$

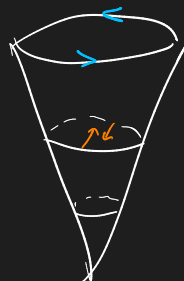
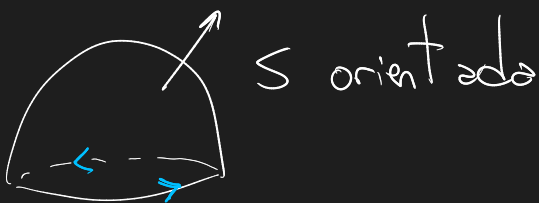
Superficie
Compacta
sin borde

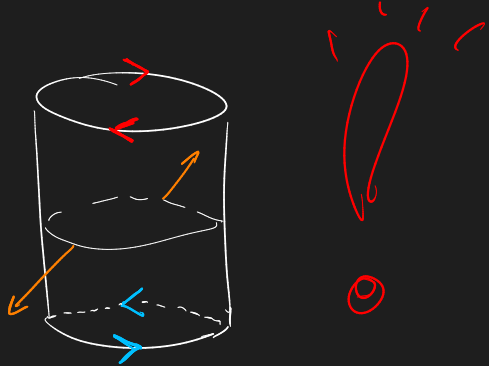
Obs:

$T: D \rightarrow S$ es regular

$$\Rightarrow T(\partial D) = \partial S$$

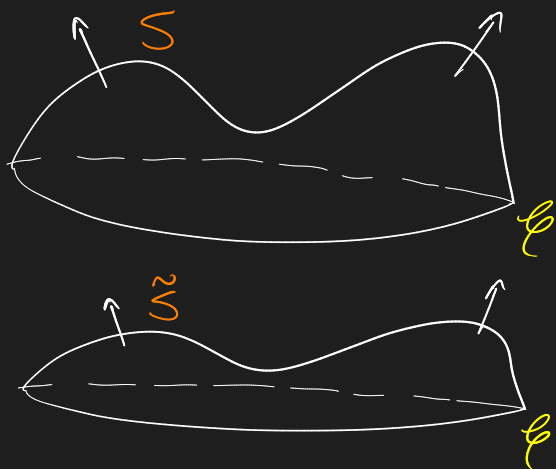
Orientaciones (dibujitos)





Un uso típico :

Cambiar superficies



S Superficie / $\partial S = C$

\tilde{S} Superficie / $\partial S = C$

orientadas como en la imagen.

$$\iiint_S \nabla \times F \, d\vec{S} = \int_C F \, d\vec{z} = \iiint_{\tilde{S}} \nabla \times F \, d\vec{S}$$

Ejemplo / ejercicio

$$f: D \rightarrow \mathbb{R}, \quad f \in C^2, \quad f \geq 0,$$

$$f(\partial D) = 0$$

$$D = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \}$$



$$S = \text{Grat}(f)$$

Obs:

$$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1, z = 0\}$$

$$\text{Se } F(x, y, z) = (e^{yz}, xz \cdot e^{yz} + x, xy e^{yz})$$

$$\text{Calcular } \int \int_S \nabla \times F \, d\vec{s} = ?$$

\uparrow no z conhecido

Calculamos

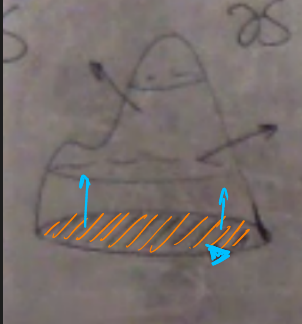
$$\nabla \times F = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{pmatrix}$$

$$= \left(x \cdot e^{yz} + yxz \cdot e^{yz} - x e^{yz} - xyze^{yz}, \right. \\ \left. y e^{yz} - y e^{yz}, z e^{yz} + 1 - z e^{yz} \right)$$

$$= (0, 0, 1) \quad \hat{i} \hat{j} \hat{k}$$

Stokes

$$\iint_S \nabla \times F \, d\vec{S} = \int_{\partial S} F \, d\vec{S} = \iint_{\tilde{S}} \nabla \times F \, d\vec{S} = (*)$$



$$= \iint_{\tilde{S}} \langle \nabla \times F, \underbrace{\eta}_{\text{función escalar}} \rangle \, dS$$

Vector normal unitario

$$= \iint_{\tilde{S}} \langle (0,0,1), (0,0,1) \rangle \, dS$$

$$= \iint_{\tilde{S}} 1 \, dS = A(S) = \pi //$$

S sup. (orientada)
 F campo vectorial $F: \mathbb{R}^3 \rightarrow \mathbb{R}^3$
 f función $f: S \rightarrow \mathbb{R}$

$T: D \rightarrow S$
 $T(u,v) = (x(u,v), y(u,v), z(u,v))$

$\int_S f \, dS = \iint_D f(T(u,v)) \underbrace{\|T_u \times T_v\|}_{dS} \, du \, dv$

$DT(p)$
 $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$

ds $d\vec{s}$ dS $d\vec{S}$

↑
escalar
masa densidad

↑
vectorial
Curvilíneas

↑
Flujo de Campo (vectorial)
escalar

$$\int \int_{\substack{S \\ \text{(orientada)}}} F d\vec{S} = \int \int_S \langle F, \eta \rangle dS$$

$$= \int \int_D \left\langle F(T(u,v)), \frac{T_u \times T_v}{\|T_u \times T_v\|} \right\rangle \cdot \|T_u \times T_v\| du dv$$

$$= \int \int_D \langle F(T(u,v)), T_u \times T_v \rangle du dv$$

