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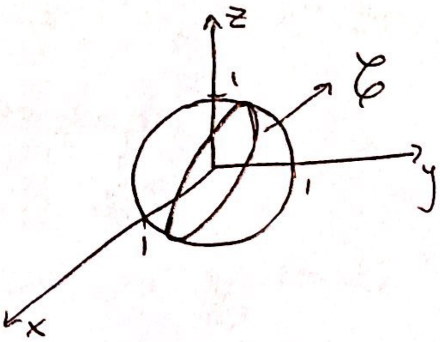
1	2	3	4	
B^-	B^-	R^+	R^-	A

1) a)



$x^2+y^2+z^2=1 \ \cap \ x+z=0$ ✓

$z=-x$

$x^2+y^2+(-x)^2=1$

$2x^2+y^2=1$

$\dfrac{x^2}{\left(\frac{1}{\sqrt{2}}\right)^2}+y^2=1$ ✓

$$\sigma(\theta)=\begin{cases} x=\frac{1}{\sqrt{2}}\cos\theta \\ y=\operatorname{sen}\theta \\ z=-\frac{1}{\sqrt{2}}\cos\theta . \end{cases} \quad \theta\in[0,2\pi]$$ ✓ //

esto muestra $\operatorname{Im}(\sigma)\subseteq\mathcal{C}$

- $\operatorname{Im}(\sigma(\theta))=\cancel{\ \ }(\mathcal{C})$

la otra inclusión es por la elección del θ.

$\left(\frac{1}{\sqrt{2}}\right)^2\cos^2\theta+\operatorname{sen}^2\theta+\left(-\frac{1}{\sqrt{2}}\cos\theta\right)^2=1$

$\frac{1}{2}\cos^2\theta+\operatorname{sen}^2\theta+\frac{1}{2}\cos^2\theta=1$

$\cos^2\theta+\operatorname{sen}^2\theta=1$ ✓

La imágen de mi parametrización está contenida en la curva.

$\frac{1}{\sqrt{2}}\cos\theta+\left(-\frac{1}{\sqrt{2}}\cos\theta\right)=0$ ✓

Pruebo:
- $\sigma(\theta)$ inyectiva en $[a,b)$ porque es curva cerrada y $\sigma'(0)=\sigma'(2\pi)$
- $\sigma'(\theta)\neq(0,0,0)\ \forall\theta$
- $\sigma\in C^1$

$\sigma\in C^1$ ya que sus coordenadas lo son (de hecho son C^∞). ✓

- $\sigma'=\left(-\frac{1}{\sqrt{2}}\operatorname{sen}\theta,\ \cos\theta,\ \frac{1}{\sqrt{2}}\operatorname{sen}\theta\right)\neq(0,0,0)\ \forall\theta$

un mismo θ no puede anular seno y coseno. ✓

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$\sigma'(0) = (0,1,0) \qquad \sigma'(2\pi) = (0,1,0)$ ✓ es cerrada. ✓

• Inyectividad:

tomo $\theta_1 \neq \theta_2$

$$\begin{cases} \cancel{\frac{1}{\sqrt{2}}} \cos\theta_1 = \cancel{\frac{1}{\sqrt{2}}} \cos\theta_2 \\ \operatorname{sen}\theta_1 = \operatorname{sen}\theta_2 \\ -\cancel{\frac{1}{\sqrt{2}}} \cos\theta_1 = -\cancel{\frac{1}{\sqrt{2}}} \cos\theta_2 \end{cases} \iff \theta_1 = \theta_2$$

Si tienen mismo seno y coseno $\Rightarrow$ son el mismo θ. ✓

b) $\rho(xyz) = x^2 |y|$

$\mathcal{L}(\mathscr{C}) = \int \|\sigma'\|$ ✓

masa tot $= \int \rho(\sigma) \|\sigma'\|$.

$$\mathcal{L}(\mathscr{C}) = \int_0^{2\pi} \sqrt{\left(-\frac{1}{\sqrt{2}}\operatorname{sen}\theta\right)^2 + \cos^2\theta + \left(\frac{1}{\sqrt{2}}\operatorname{sen}\theta\right)^2}\, d\theta =$$

$$= \int_0^{2\pi} \sqrt{\frac{1}{2}\operatorname{sen}^2\theta + \cos^2\theta + \frac{1}{2}\operatorname{sen}^2\theta}\, d\theta =$$

$$= \int_0^{2\pi} 1\, d\theta = 2\pi \,//$$ ✓

$$\text{masa} = \int_0^{2\pi} \left(\frac{1}{\sqrt{2}}\cos\theta\right)^2 |\operatorname{sen}\theta| \sqrt{\left(-\frac{1}{\sqrt{2}}\operatorname{sen}\theta\right)^2 + \cos^2\theta + \left(\frac{1}{\sqrt{2}}\operatorname{sen}\theta\right)^2}\, d\theta =$$

$$= \int_0^{2\pi} \frac{1}{2}\cos^2\theta\, |\operatorname{sen}\theta| \cdot 1\, d\theta = \frac{1}{2}\int_0^{2\pi} \cos^2\theta\, \underbrace{|\operatorname{sen}\theta|}_{\text{separo}}\, d\theta =$$

$$= \frac{1}{2}\left[\int_0^{\pi} \cos^2\theta \operatorname{sen}\theta\, d\theta - \int_{\pi}^{2\pi} \cos^2\theta \operatorname{sen}\theta\, d\theta\right] =$$ ✓ $\nearrow$ $u = \cos\theta$, $du = -\operatorname{sen}\theta\, d\theta$

$$= \frac{1}{2}\left[\int u^2 \cancel{\operatorname{sen}\theta} \frac{du}{-\cancel{\operatorname{sen}\theta}} - \int u^2 \cancel{\operatorname{sen}\theta} \frac{du}{-\cancel{\operatorname{sen}\theta}}\right] =$$

$\longrightarrow$ sigue

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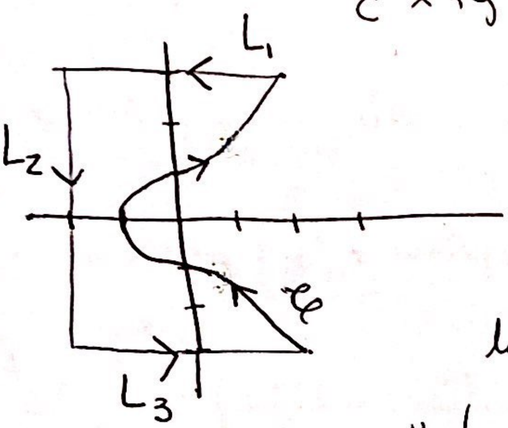
$\frac{1}{3}$ → $\cos(\pi) = -1$

$$= \frac{1}{2}\left[-\frac{\cos^3\theta}{3}\Big|_0^{\pi} + \frac{\cos^3\theta}{3}\Big|_{\pi}^{2\pi}\right] = \frac{1}{2}\left[(0+\tfrac{1}{3}) + (\tfrac{1}{3} - 0)\right] = \boxed{\frac{1}{3}}$$

$-1/3$

2) $C = x^3 - y^2 + 1 = 0$, $x \le 2$ desde $(2,-3)$ a $(2,3)$

calcular $\int_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$



$$F(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$$

Para poder aplicar Green cierro la figura con 3 segmentos L_1, L_2 y L_3 que no pasen por $(0,0)$ ya que el campo no está definido ahí.

¿Por qué podés usar Green?

Parametrizo los segmentos:

$L_1 = (t_1, 3)$ $\quad t_1 \in [-1, 2] \to$ invierte orientación

$L_2 = (-1, t_2)$ $\quad t_2 \in [-3, 3] \to$ invierte "

$L_3 = (t_3, -3)$ $\quad t_3 \in [-1, 2] \to$ preserva ".

Calculo: $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{1(x^2+y^2) - 2x^2}{(x^2+y^2)^2} - \left(\frac{(-1)(x^2+y^2) + 2y^2}{(x^2+y^2)^2}\right) =$

$$= \frac{x^2+y^2 - 2x^2 + x^2+y^2 - 2y^2}{(x^2+y^2)^2} = 0$$

$\Rightarrow$ Por Green: $\int_C F ds - \int_{L_1} F ds - \int_{L_2} F ds + \int_{L_3} F ds = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dxdy = 0$ ✓

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$$\int_C F\,ds = \int_{L_1} F\,ds + \int_{L_2} F\,ds - \int_{L_3} F\,ds.$$

param. L_1 derivada.

$$L_1) = \int_{-1}^{2} \left\langle \left(\left(\frac{-3}{t_1^2+9}\right), \left(\frac{t_1}{t_1^2+9}\right) \right), (1,0) \right\rangle dt_1$$

$$= \int_{-1}^{2} \frac{-3}{t_1^2+9}\,dt_1 = -3\int_{-1}^{2} \frac{1}{t_1^2+9}\,dt_1 =$$

$u = \frac{t}{3}$ $du = \frac{1}{3}dt$

$$= -3\int \frac{3}{9u^2+9}\,du = -\int \frac{1}{u^2+1}\,du = -\left[\operatorname{arctg}\left(\frac{t}{3}\right)\Big|_{-1}^{2}\right] =$$

$$= -\operatorname{arctg}\left(\frac{2}{3}\right) + \operatorname{arctg}\left(-\frac{1}{3}\right) //$$

paramL$_2$ derivada

$$L_2) = \int_{-3}^{3} \left\langle \left(\frac{-t_2}{t_2^2+1}, \frac{-1}{t_2^2+1}\right), (0,1) \right\rangle dt_2 =$$

$$= \int_{-3}^{3} \frac{-1}{t_2^2+1}\,dt_2 = -\int_{-3}^{3} \frac{1}{t_2^2+1}\,dt_2 = -\left[\operatorname{arctg}(t_2)\Big|_{-3}^{3}\right] = 0 //$$

param t_3 derivada

$$L_3) = \int_{-1}^{2} \left\langle \left(\frac{3}{t_3^2+9}, \frac{t_3}{t_3^2+9}\right), (1,0) \right\rangle dt_3 =$$

$$= \int_{-1}^{2} \frac{3}{t_3^2+9}\,dt_3 = 3\int_{-1}^{2} \frac{1}{t_3^2+9}\,dt_3$$

resuelve igual que para L_1 pero con signo opuesto.

$$= \left[\operatorname{arctg}\left(\frac{2}{3}\right) - \operatorname{arctg}\left(-\frac{1}{3}\right)\right] //$$

$$\Rightarrow \int_C F\,ds = -\left(\operatorname{arctg}\left(\frac{2}{3}\right) - \operatorname{arctg}\left(-\frac{1}{3}\right)\right) - \left(\operatorname{arctg}\left(\frac{2}{3}\right) - \operatorname{arctg}\left(-\frac{1}{3}\right)\right)$$

$$\boxed{= 2\operatorname{arctg}\left(-\frac{1}{3}\right) - 2\operatorname{arctg}\left(\frac{2}{3}\right)} //$$

✓

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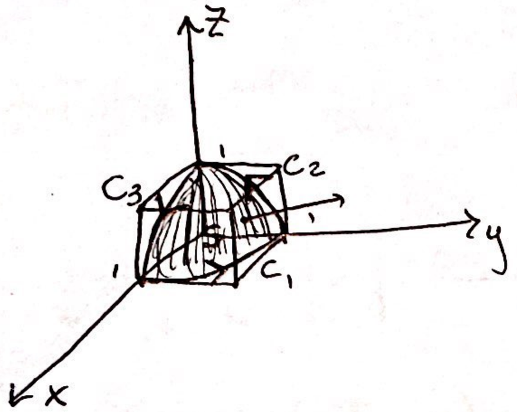
3) $F(x,y,z) = \left((x+y)e^{z^2}, (y+z)e^{x^2}, (z+x)e^{y^2}\right)$

$S = x^2+y^2+z^2=1 \quad \cap [0,1]^3$ normal en $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ exterior.

calcular $\iint_S \nabla \times F \, ds$.

$$\text{rot}(F) = \begin{vmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ P & Q & R \end{vmatrix}$$



$$= \left((z+x)e^{y^2}y2 - e^{x^2}, (x+y)e^{z^2}2z - e^{y^2}, e^{x^2}2x - e^{z^2}\right)$$

↓

Muy difícil.

Por teorema de <u>Stokes</u>: ¿Por qué podés usar Stokes?

Parametrizo borde de S:

$C_1 = (\cos\theta, \operatorname{sen}\theta, 0) \quad \theta \in [0, \frac{\pi}{2}]$

$C_2 = (0, \cos\varphi, \operatorname{sen}\varphi) \quad \varphi \in [0, \frac{\pi}{2}]$

$C_3 = (\cos\alpha, 0, \operatorname{sen}\alpha) \quad \alpha \in [\frac{\pi}{2}, \pi]$ ← está orientado al revés

$$\Rightarrow \int_{C_1} F\,ds + \int_{C_2} F\,ds + \int_{C_3} F\,ds = \iint_S \text{rot}(F).$$

para C_1 derivada

$$C_1 = \int_0^{\pi/2} \left\langle \left(\cos\theta + \operatorname{sen}\theta, \operatorname{sen}\theta\, e^{\cos^2\theta}, \cos\theta\, e^{\operatorname{sen}^2\theta}\right), (-\operatorname{sen}\theta, \cos\theta, 0)\right\rangle d\theta =$$

$$= \int_0^{\pi/2} -\operatorname{sen}\theta\cos\theta - \operatorname{sen}^2\theta + \operatorname{sen}\theta\cos\theta\, e^{\cos^2\theta}\, d\theta =$$

$$= \int_0^{\pi/2} -\operatorname{sen}\theta\cos\theta\, d\theta - \int_0^{\pi/2} \operatorname{sen}^2\theta\, d\theta + \int_0^{\pi/2} \operatorname{sen}\theta\cos\theta\, e^{\cos^2\theta}\, d\theta =$$

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$$-\int \operatorname{sen}\theta \cos\theta \, d\theta = \to \begin{array}{l} u = \operatorname{sen}\theta \\ du = \cos\theta \, d\theta \end{array} = -\frac{\operatorname{sen}^2\theta}{2}$$ ✓

$$-\int \operatorname{sen}^2\theta \, d\theta = \to \text{uso } \operatorname{sen}^2\theta = \frac{1}{2} - \frac{1}{2}\cos(2\theta) \text{ con sust } \begin{array}{l} u = 2\theta \\ du = 2d\theta \end{array}$$

$$-\left(-\frac{\operatorname{sen}(2\theta) - 2\theta}{4} \right) \leftarrow$$

+

$$\int \operatorname{sen}\theta \cos\theta \, e^{\cos^2\theta} d\theta = \to \begin{array}{l} u = \cos^2\theta \\ du = -2\operatorname{sen}\theta \cdot \cos\theta \, d\theta \end{array} = -\frac{e^{\cos^2\theta}}{2}$$

+

$$C_1 = -\left[\frac{\operatorname{sen}^2\theta}{2} \Big|_0^{\pi/2} \right] - \left[-\frac{\operatorname{sen}(2\theta) - 2\theta}{4} \Big|_0^{\pi/2} \right] + \left[-\frac{e^{\cos^2\theta}}{2} \Big|_0^{\pi/2} \right] =$$

+ cos(0) = 1 error de cuentas

$$= -\left(\frac{1}{2} - 0\right) - \left(-\frac{\pi}{4} - 0\right) + \left(-\frac{1}{2} + \frac{e}{2}\right) = -\frac{1}{2} + \frac{\pi}{-4} //$$

-4

parecen C_2 deriv.

$$C_2 = \int_0^{\pi/2} \langle (\cos\varphi \, e^{\operatorname{sen}^2\varphi}, \cos\varphi + \operatorname{sen}\varphi, \operatorname{sen}\varphi \, e^{\cos^2\varphi}), (0, -\operatorname{sen}\varphi, \cos\varphi) \rangle d\varphi =$$

$$= \int_0^{\pi/2} -\operatorname{sen}\varphi\cos\varphi - \operatorname{sen}^2\varphi + \operatorname{sen}\varphi\cos\varphi \, e^{\cos^2\varphi} d\varphi = -\frac{1}{2} + \frac{\pi}{4}$$

mismo error

se resuelve igual que para C_1.

$$C_3 = \int_{\pi/2}^{\pi} \langle (\cos\alpha \, e^{\operatorname{sen}^2\alpha}, \operatorname{sen}\alpha \, e^{\operatorname{sen}^2\alpha}, \cos\alpha + \operatorname{sen}\alpha), (-\operatorname{sen}\alpha, 0, \cos\alpha) \rangle d\alpha.$$

$$= \int_{\pi/2}^{\pi} \underbrace{-\operatorname{sen}\alpha\cos\alpha \, e^{\operatorname{sen}^2\alpha}} + \underbrace{\cos^2\alpha} + \underbrace{\operatorname{sen}\alpha\cos\alpha} \, d\alpha =$$

$$\left(\frac{\operatorname{sen}^2\alpha}{2} \Big|_{\pi/2}^{\pi} \right)$$

$u = \operatorname{sen}^2\alpha$
$du = 2\cos\alpha$

$$\left(-\frac{e^{\operatorname{sen}^2\alpha}}{2} \Big|_{\pi/2}^{\pi} \right)$$

uso
$$\cos^2\alpha = \frac{1}{2} + \frac{1}{2}\cos(2\alpha)$$

$$\Rightarrow \left(\frac{\cos\alpha \operatorname{sen}\alpha + \alpha}{2} \Big|_{\pi/2}^{\pi} \right)$$

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$$= \left(-\frac{1}{2} + \frac{e}{2}\right) + \left(\frac{\pi}{2} - \frac{\pi}{4}\right) + \left(0 - \frac{1}{2}\right)$$

$$\Rightarrow \iint_S \nabla \times F = -\frac{1}{2} + \frac{\pi}{4} - \frac{1}{2} + \frac{\pi}{4} - 1 + \frac{e}{2} + \frac{\pi}{4}$$

$$= \boxed{-2 + \frac{3}{4}\pi + \frac{e}{2}} \quad //$$

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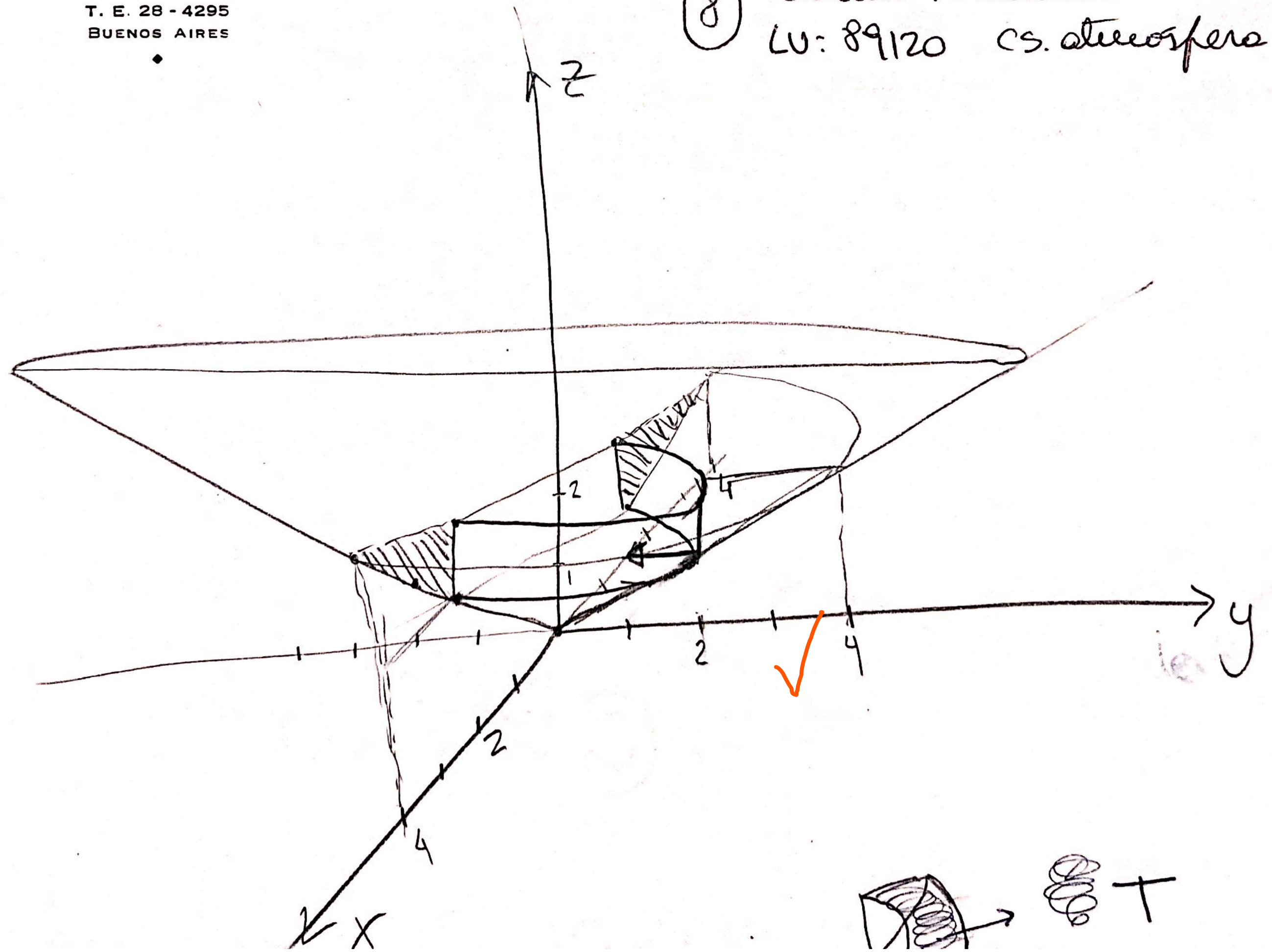

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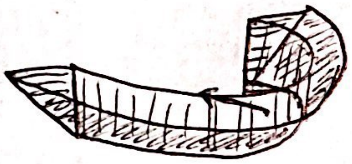
4) 

S

Para poder usar Gauss, completo el volumen con otra superficie T.

$T(r,\theta) = (r\cos\theta, r\,\text{sen}\,\theta, 2)$ $\quad 2 \leq r \leq 4$, $\quad 0 \leq \theta \leq \pi$ ✓

¿Por qué podés usar Gauss?

$T_r = (\cos\theta, \text{sen}\,\theta, 0)$

$T_\theta = (-r\,\text{sen}\,\theta, r\cos\theta, 0)$

$T_r \times T_\theta = (0,0,r) \rightarrow$ preserva orientación (normal exterior). ✓

$$\Rightarrow \iint_S F\,ds + \iint_T F\,ds = \iiint_\Omega \text{div}(F).$$

$$\text{div}(F) = \frac{2 \cdot y\, z e^{x^2+y^2}}{\sqrt{x^2+y^2}} + 1$$ ✓

?

$$\iint_T F\,ds = \int_2^4 \int_0^\pi \langle (r\cos\theta f(2), 2f^2(r) - r\,\text{sen}\,\theta f(2), 2)(0,0,r)\rangle \cdot r\,d\theta\,dr =$$

$$= \int_2^4 \int_0^\pi 2r^2\,d\theta\,dr = \int_0^\pi \left. 2\frac{r^3}{3}\right|_2^4 d\theta = \int_0^\pi \left(\frac{2\cdot 64}{3} - \frac{2\cdot 8}{3}\right) d\theta = \int_0^\pi \frac{112}{3}\,d\theta =$$

$$= \frac{112\pi}{3}$$ // No

$$\iiint_\Omega \text{div}(F) \rightarrow \Omega = \begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \\ \frac{\sqrt{x^2+y^2}}{4} \leq z \leq 2 \\ \frac{r}{2} \leq z \leq 2 \end{cases} \quad 2 \leq r \leq 4, \quad 0 \leq \theta \leq \pi$$

↓ sigue

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jacobiano.

$$\int_2^4 \int_0^\pi \int_{r/2}^2 \left(\frac{2\cancel{r}\,\mathrm{sen}\theta z\, e^{r^2}}{\cancel{r}} + 1\right) . r \, dz\, d\theta\, dr =$$

$$\int_2^4 \int_0^\pi \int_{r/2}^2 2r\,\mathrm{sen}(\theta) z\, e^{r^2} + r \; dz\, d\theta\, dr =$$

$$\int_2^4 \int_0^\pi \cancel{2} r\,\mathrm{sen}(\theta) . \frac{z^2}{\cancel{2}} e^{r^2} + zr \Big|_{r/2}^2 d\theta\, dr =$$

$$\int_2^4 \int_0^\pi \left[\left(r\,\mathrm{sen}(\theta) . 4\, e^{r^2} + 2r\right) - \left(r\,\mathrm{sen}(\theta) . \frac{r^2}{4} e^{r^2} + \frac{r^2}{2}\right)\right] d\theta\, dr =$$

$$\int_2^4 \int_0^\pi 4r\,\mathrm{sen}(\theta) e^{r^2} + 2r - \frac{r^3}{4} \mathrm{sen}(\theta) e^{r^2} - \frac{r^2}{2} \; d\theta\, dr =$$

$$\int_2^4 -4r\cos(\theta)\, e^{r^2} + 2r\theta + \frac{r^3}{4} \cos(\theta) e^{r^2} - \frac{r^2}{2}\theta \Big|_0^\pi dr =$$

$$= \int_2^4 4re^{r^2} + 2\pi r - \frac{r^3}{4} e^{r^2} - \frac{r^2}{2}\pi + 4re^{r^2} - 0 - \frac{r^3}{4} e^{r^2} - 0 \, d\theta =$$

$$= \int_2^4 8re^{r^2} + 2\pi r - \frac{r^3}{2} e^{r^2} - \frac{r^2}{2}\pi \, dr =$$ no la puedo terminar.

$$\iint_S F\,dS = \iiint_\Omega \mathrm{div}(F) - \frac{112}{3}\pi$$

las integrales $\int re^{r^2} dr$ y $\int r^3 e^{r^2} dr$

salen por sustitución $u = r^2$

$du = 2r\,dr$

$\Rightarrow r^3 = \frac{1}{2} u\, du$