

# Práctica #1

• Esteban

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## Análisis 2 : "Materia Práctica"

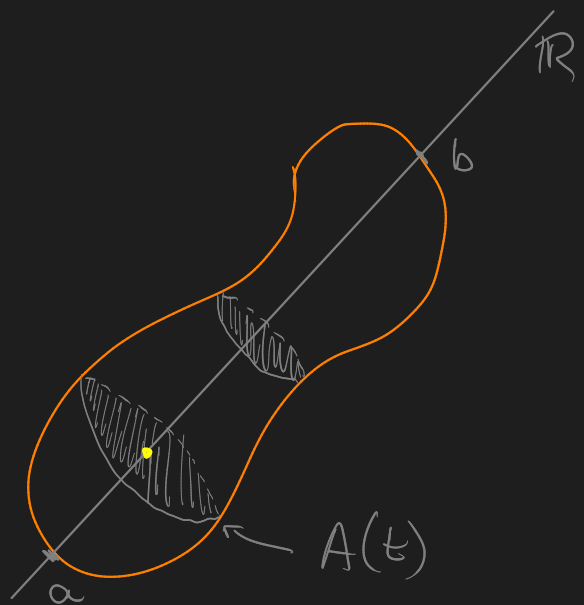
### Breve Repaso de Integración ( $\mathbb{R}^2, \mathbb{R}^3$ )

Principio de Cavalieri :

$$W \subset \mathbb{R}^3$$

↖ un cuerpo (volumen)

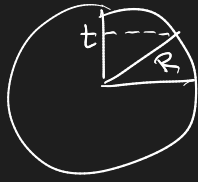
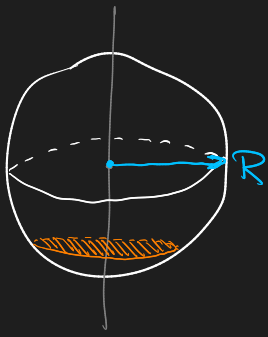
$$V_d(W) = \int_a^b A(t) dt$$



Área de rebanado :  $A(t)$

Existe  $A : [a, b] \rightarrow \mathbb{R}_{\geq 0}$

Ejemplo : Esfera de Radio R



$$R^2 = t^2 + r(t)^2$$

$$r(t)^2 = R^2 - t^2$$

$$\text{Vol}(S(R)) = \int_{-R}^R \pi \cdot (R^2 - t^2) \cdot dt$$

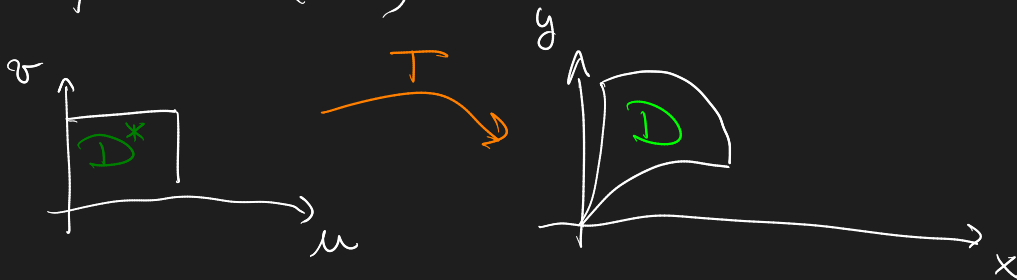
$$= \pi \left[ 2R^3 - \frac{t^3}{3} \right]_{-R}^R$$

$$= \pi \left( 2R^3 - \frac{2}{3}R^3 \right)$$

$$= \frac{4}{3} \pi R^3 //$$

## Teorema de Cambio de Variable

Esquema:  $(\mathbb{R}^2)$



Tenemos

$f: D \rightarrow \mathbb{R}$  continua


$T: D^* \rightarrow D$

con  $T \in C^1$

$T(D^*) = D$

$T$  inyectiva (o casi)

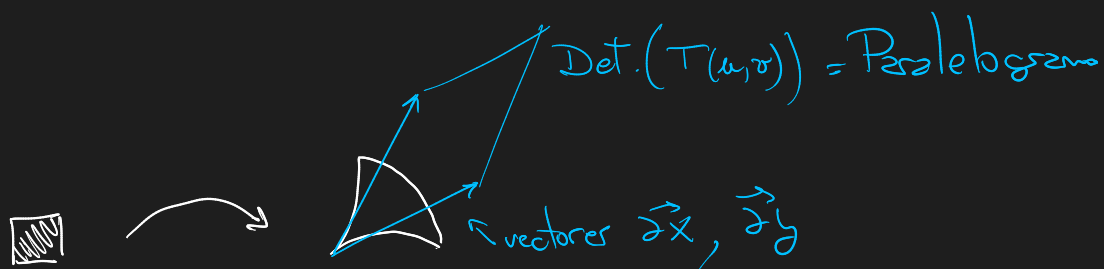
$$\int_D f(x,y) \cdot dx dy = \int_{D^*} f(T(u,v)) \cdot J T(u,v) \cdot du dv$$

• Ejemplo con  $f(x,y) = 1$  

$$J T(u,v) = |\det DT(u,v)|$$

$$T(u,v) = (x(u,v), y(u,v))$$

$$DT(u,v) = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} (u,v)$$

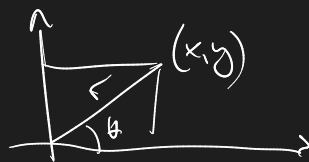


Cálculo:

Pobres

$$x = r \cdot \cos \theta$$

$$y = r \cdot \sin \theta$$



$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$T(r,\theta) = (r \cdot \cos \theta, r \cdot \sin \theta)$$

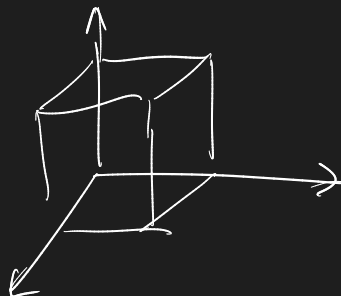
$$JT(r, \theta) = r$$

• Cilíndricas ( $\mathbb{R}^3 = \mathbb{R}^2 \times \mathbb{R}$ )

$$x = r \cdot \cos \theta$$

$$y = r \cdot \sin \theta$$

$$z = z$$



$$JT(r, \theta, z) = r$$

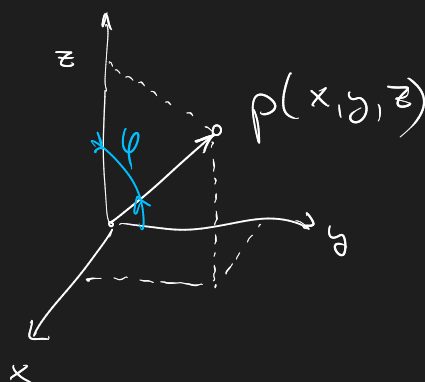
Esféricas ( $\mathbb{R}^3 \sim$  cebolla)

$$x = \cos \theta \cdot r \cdot \sin \varphi$$

$$y = \sin \theta \cdot r \cdot \sin \varphi$$

$$1^\circ \rightarrow z = r \cdot \cos \varphi$$

↑  
"Polar en el piso"



$$JT(r, \theta, \varphi) = r^2 \cdot \sin \varphi$$

Ejemplo: Volumen de  $B(0, R)$

$$\text{Vol}(B(0, R)) = \int_{B(0, R)} 1 \, dV$$

$$\begin{aligned}
&= \int_0^{2\pi} \int_0^\pi \int_0^R r^2 \cdot \sin \varphi \cdot dr \\
&= 2\pi \cdot \frac{R^3}{3} \cdot \int_0^\pi \sin \varphi \, d\varphi \\
&= \frac{2}{3} \pi R^3 \cdot 2 \\
&= \frac{4}{3} \pi R^3
\end{aligned}$$

Curvas en  $\mathbb{R}^2, \mathbb{R}^3$

$\mathcal{C} \subset \mathbb{R}^3$  ( $\subset \mathbb{R}^2$ ) es un conjunto imagen de

$$\sigma : [a, b] \rightarrow \mathbb{R}^3$$

Continuas + Sobreyectivas

Problema 1:

Estático:

$$\mathcal{C} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$$

Queremos:

Input: Conjunto

Output: Parametrización

Problema 2:

$$\sigma: [0, 2\pi) \rightarrow \mathbb{R}^2$$

$$\sigma(t) = (\cos t, \sin t)$$

Input: Parametrización

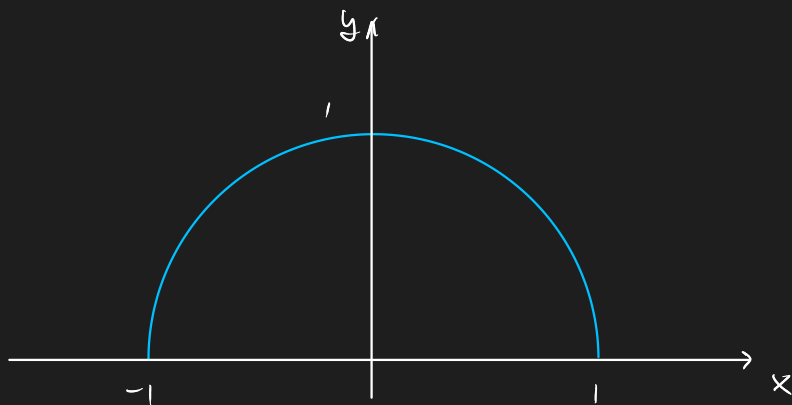
Output: Conjunto  $\mathcal{C}$

Accés:

$\sigma$  parametriza  $\mathcal{C}$  (inyectiva)

Ejemplo:

$$\mathcal{C} = \{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1, y \geq 0 \}$$



Tres parametr. de  $\mathcal{C}$

$$4) \gamma: [0, \pi] \rightarrow \mathbb{R}^2$$

$$\gamma(t) = (\cos t, \sin t)$$



$$2) \sigma : [-1, 1] \rightarrow \mathbb{R}^2$$

$$\sigma(t) = (t, \sqrt{1-t^2})$$

$$(\text{ejemplo de } \sigma(x) = (x, f(x)))$$

$$3) \alpha : [0, \pi] \rightarrow \mathbb{R}^2$$

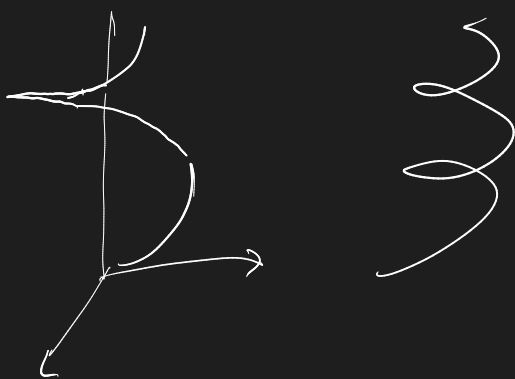
$$\alpha(t) = (-\cos t, \sin t)$$



Ejemplo :

$$\sigma : [0, 2\pi] \rightarrow \mathbb{R}^3$$

$$\sigma(t) = (\cos t, \sin t, t) \quad \text{"escalera caracol"}$$

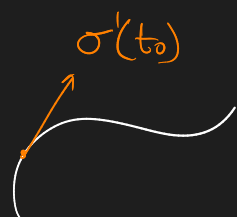


Velocidad y Vector Tangente

Vector tangente a  $\mathcal{C}$  en  $p = \sigma(t_0)$

$$\sigma : [a, b] \rightarrow \mathcal{C}$$

$$\left. \begin{array}{l} \sigma \in C^1 \\ \sigma'(t) \neq \vec{0} \end{array} \right\} \text{regular}$$

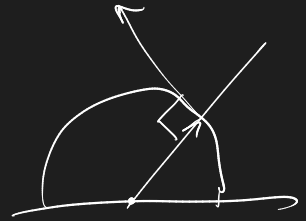
  $\sigma'(t_0) = (x'(t_0), y'(t_0)) \text{ en } \mathbb{R}^2$

Para el semi-círculo

1) •  $\gamma'(t) = (-\sin t, \cos t)$

Obs :

$$\langle \gamma'(t), \gamma(t) \rangle = 0$$



2) •  $\sigma'(t) = \left( 1, \frac{-t}{\sqrt{1-t^2}} \right) \text{ con } |t| \neq 1$

3) •  $\alpha'(t) = (\sin t, \cos t)$

Recta tangente

$\mathcal{C} \subset \mathbb{R}^3$  parametrizada por  $\sigma$  regular ( $\mathcal{C}$  suave)

Recta tangente en  $\sigma(t_0) = p \in \mathbb{R}^3$

$$L : \lambda \cdot \sigma'(t_0) + \sigma(t_0) \quad \lambda \in \mathbb{R}$$

Parametrizar la recta  $L$  con  $l : \mathbb{R} \rightarrow \mathbb{R}^3$





$$l(t) = (t - t_0) \cdot \sigma'(t_0) + \sigma(t_0)$$

Preguntas

$$\begin{cases} l(t_0) = \sigma(t_0) \\ l'(t_0) = \sigma'(t_0) \end{cases}$$

## Curvas en Polares ( $\mathbb{R}^2$ )

La ecuación  $r = r(\theta)$  ( $\rho = \rho(\theta)$ )

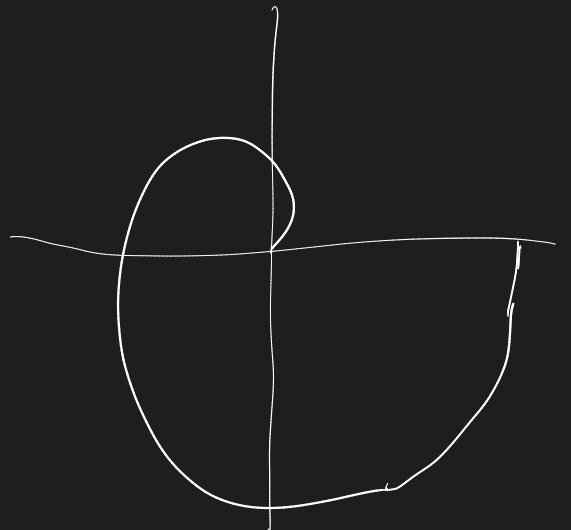
Se interpreta como

$$\begin{aligned} \sigma(\theta) &= (x(\theta), y(\theta)) \\ &= (r(\theta) \cdot \cos \theta, r(\theta) \cdot \sin \theta) \\ &= r(\theta) \cdot (\cos \theta, \sin \theta) \end{aligned}$$

Ejemplo

$$r(\theta) = \theta, \quad \theta \in [0, 2\pi]$$

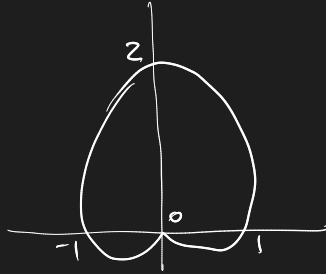
$$\sigma(\theta) = \theta (\cos \theta, \sin \theta)$$



$$\gamma: [\pi, \frac{3}{2}\pi] \rightarrow \mathbb{R}^2 \text{ dado por } \gamma(\theta) = \theta$$

Cardioides:

$$r(\theta) = 1 + \sin \theta, \quad \theta \in [0, 2\pi]$$



Fin

















