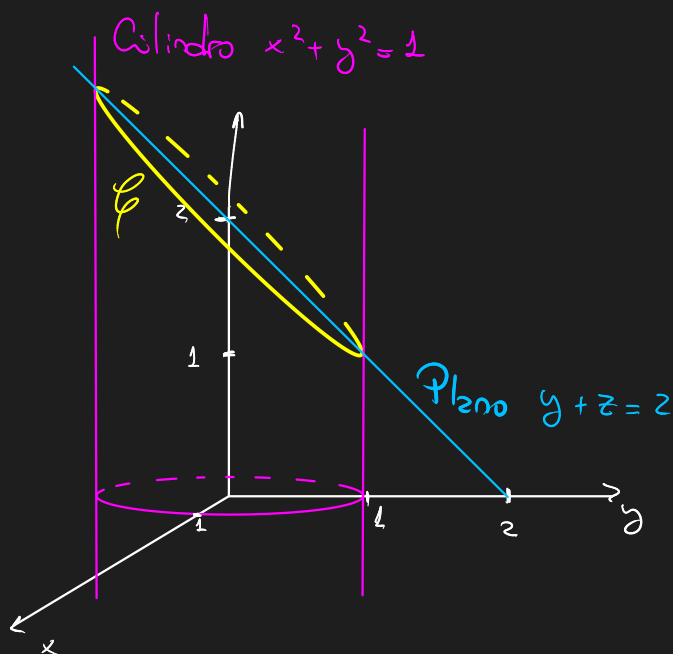


Re-resolución de Parcial 1 - Ej 1.

1. Consideramos la curva C determinada por la intersección entre la superficie dada por la ecuación $x^2 + y^2 - 1 = 0$ y la superficie dada por $y + z - 2 = 0$. Calcular $\int_C f \, ds$ donde $f(x, y, z) = \sqrt{1 + x^2}$.



$$\mathbb{R}^3 \rightarrow \mathbb{R}$$

$$\int_C f \, ds = \int_{\theta} f(\sigma(\theta)) \cdot \|\sigma'(\theta)\| \, ds$$

Uso polares

$$x = \cos \theta \quad \text{con } \theta \in [0, 2\pi)$$

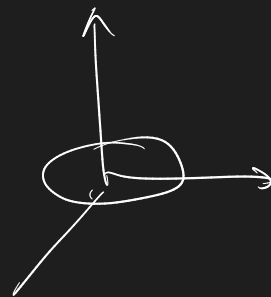
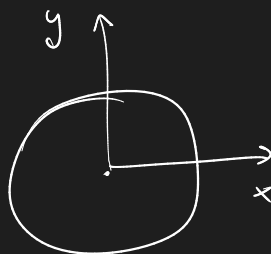
$$y = \sin \theta$$

$$z = ?$$

Uso el plano

$$y + z = 2 \Rightarrow z = 2 - y$$

$$z = 2 - \sin \theta$$



$$\sigma(\theta) = (\cos \theta, \sin \theta, z - \sin \theta)$$

$$\sigma'(\theta) = (-\sin \theta, \cos \theta, -\cos \theta)$$

$$\|\sigma'(\theta)\| = \sqrt{1 + \cos^2 \theta}$$

$$f(\sigma(\theta)) = \sqrt{1 + \cos^2 \theta}$$

Residual integral

$$\int_C f \, ds = \int_{\theta} f(\sigma(\theta)) \cdot \|\sigma'(\theta)\| \, d\theta$$

$$= \int_{\theta=0}^{2\pi} \sqrt{1 + \cos^2 \theta} \cdot \sqrt{1 + \cos^2 \theta} \, d\theta$$

$$= \int_{\theta=0}^{2\pi} 1 \, d\theta + \int_{\theta=0}^{2\pi} \cos^2 \theta \, d\theta$$

$$= 2\pi + \int_{\theta=0}^{2\pi} \frac{\cos(2x) + 1}{2} \, d\theta$$

$$= 2\pi + \frac{1}{2} \left(\underbrace{\frac{\sin(2x)}{2}}_{=0} \Big|_0^{2\pi} + 2\pi \right)$$

$$= 3\pi$$

Solución

$$\int_C f \, ds = 3\pi$$

