Ejercicio 1. Calcular la integral de línea del campo $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ dado por

$$\mathbf{F}(x,y,z) = \left(e^{xz}(xyz^2 + yz) + y, xze^{xz}, e^{xz}(x^2yz + xy)\right)$$

a lo largo de la curva parametrizada por $\sigma : [0,1] \to \mathbb{R}^3$,

$$\sigma(t) = \left(t, 3t^2 - 2t, \frac{1}{\ln 7} \ln\left(1 + 6t^8\right)\right).$$

$$\int_{\mathcal{C}} F \cdot d\vec{s} = \int_0^1 \langle F(\sigma(t)), \sigma'(t) \rangle \, dt$$

$$F(x,y,z) = \left(e^{xz}(\; + \;), \; xze^{xz}, \; e^{xz}(\; + \;) \right) + (y,0,0)$$

$$= e^{xz}\big(\underbrace{xyz^2 + yz}_{P}, \; \underbrace{xz}_{Q}, \; \underbrace{x^2yz + xy}_{R} \big) + (y,0,0)$$

$$\nabla \times \tilde{F} = \det \begin{pmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ P & Q & R \end{pmatrix} =$$

$$= \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \; -\frac{\partial R}{\partial x} + \frac{\partial P}{\partial z}, \; \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)$$

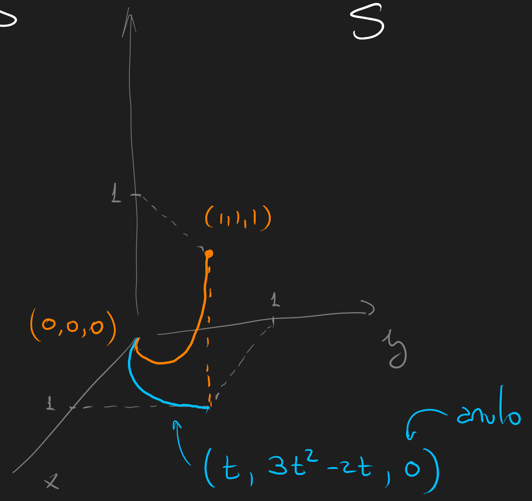
$$= \left(x^2 z + x - x, \; -2xyz - y + 2xyz + y, \; z - xz^2 - z \right)$$

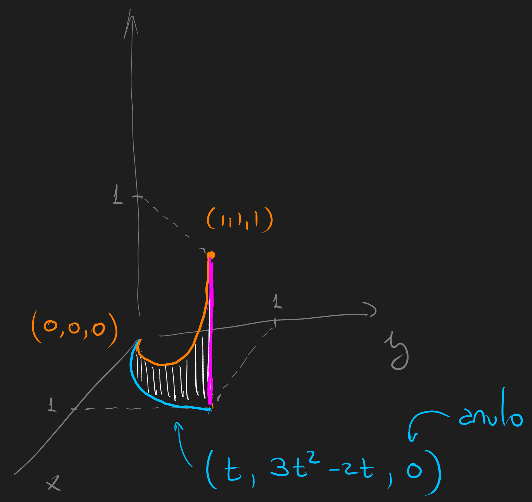
$$= \left(x^2 z, \; 0, \; -xz^2 \right) \dots$$

$$\nabla \times F = (0, 0, -1)$$

Por Stokes

$$\int_{\partial S} F \cdot d\vec{s} = \iint_S \nabla \times F \cdot ds$$





⋮

Como es campo gradiente

$$\int_{\mathcal{C}} \nabla g \, ds = g(\sigma(1)) - g(\sigma(0))$$
$$= g(0,0,0) - g(1,1,1)$$

Para obtener g:

$$G = e^{xz}\left(xyz^2 + yz,\ xz,\ x^2yz + xy\right)$$

$$\left.\begin{aligned} P &= \frac{\partial g}{\partial x} = e^{xz}(xyz^2 + yz) \\ Q &= \frac{\partial g}{\partial y} = e^{xz} \cdot xz \\ R &= \partial g/\partial z = e^{xz}(x^2 \cdot y \cdot z + xy) \end{aligned}\right\}$$

$$\left\{\begin{array}{l} e^{xz} \cdot x \cdot y \cdot z + C \\ e^{xz} \cdot xyz + C \\ e^{xz} \cdot xyz + C \end{array}\right\} g(x,y,z) = e^{xz} \cdot xyz$$

evalúo

$$g(0,0,0) = 0$$

$$g(1,1,1) = e$$

$$\therefore \int_{\mathcal{C}} \nabla g \, ds = e$$

Volviendo a la componente $\underbrace{(y,0,0)}_{H}$ de F

$$\int_{\mathcal{C}} H(\sigma(t)) \cdot ds =$$

$$= \int_0^1 \langle (3t^2 - 2t, 0, 0), (1, \ldots, \ldots) \rangle dt$$

$$= \int_0^1 3t^2 - 2t \; dt$$

$$= t^3 - t^2 \Big|_0^1 = 0$$

Junto todo

$$\int_{\mathcal{C}} F = \int_{\mathcal{C}} G + \int_{\mathcal{C}} H$$

$$= e + 0 = e$$

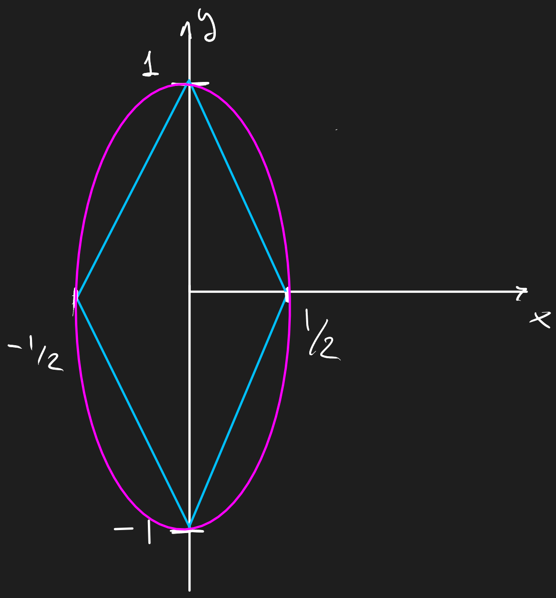
Ejercicio 2. Sean $\mathcal{C}$ la curva poligonal cerrada de vértices $(0,1), (0,-1), (1/2,0)$ y $(-1/2,0)$ y $\mathcal{E}$ la elipse $4x^2 + y^2 = 1$, ambas orientadas positivamente. Sean

$$\mathbf{F}(x,y) = \frac{1}{4x^2+y^2}(-y,x), \quad \mathbf{G}(x,y) = \left(x, y\sqrt{4x^2+y^2}\right).$$

(a) Hallar $\int_{\mathcal{E}} \mathbf{F} \cdot d\mathbf{s}$.

(b) Hallar $\int_{\mathcal{E}} \mathbf{G} \cdot d\mathbf{s}$ y $\int_{\mathcal{C}} \mathbf{G} \cdot d\mathbf{s}$.

Sugerencia: integrar una función impar con respecto a x sobre un dominio simétrico con respecto a x da 0.



a) $\int_{\mathcal{E}} F \cdot d\vec{s} =$

elipse $\Rightarrow 4x^2 + y^2 = 1$

$\Rightarrow$ Cuando compongo $F(\mathcal{E})$

$\Rightarrow 4x^2 + y^2$ será igual a 1.

Parametrizo $\mathcal{E}$

$\sigma(t) = \left(\frac{1}{2}\cos t, \ \sin t\right)$

$$\sigma'(t) = \left(-\frac{1}{2}\sin t,\ \cos t\right)$$

$$F(\sigma(t)) = \frac{1}{\underbrace{\quad\ \ \ \quad}_{=1 \text{ pues es la eq de la elipse}}} \left(-\sin t,\ \frac{1}{2}\cos t\right)$$

$$\int_{t=0}^{2\pi} \left\langle \left(-\sin t,\ \frac{1}{2}\cos t\right), \left(-\frac{1}{2}\sin t,\ \cos t\right) \right\rangle dt =$$

$$= \int_0^{2\pi} \frac{1}{2}\sin^2 t + \frac{1}{2}\cos^2 t = \pi$$

$$\therefore \int_{\mathcal{C}} F = \pi \ //$$

b) $G = \left(x,\ y \cdot \sqrt{4x^2 + y^2}\right)$

$$\int_{\mathcal{C}} G \cdot ds = \int_{t=0}^{2\pi} \langle G(\sigma(t)) \cdot \sigma'(t) \rangle dt$$

$$= \int_0^{2\pi} \left\langle \left(\frac{1}{2}\cos t,\ \sin t\right), \left(-\frac{1}{2}\sin t,\ \cos t\right) \right. dt$$

$$= \int_0^{2\pi} -\frac{1}{4}\sin t\cos t + \sin t\cos t\ dt$$

$$= \int_0^{2\pi} \frac{3}{4}\sin t\cos t\ dt$$

$u = \sin t$

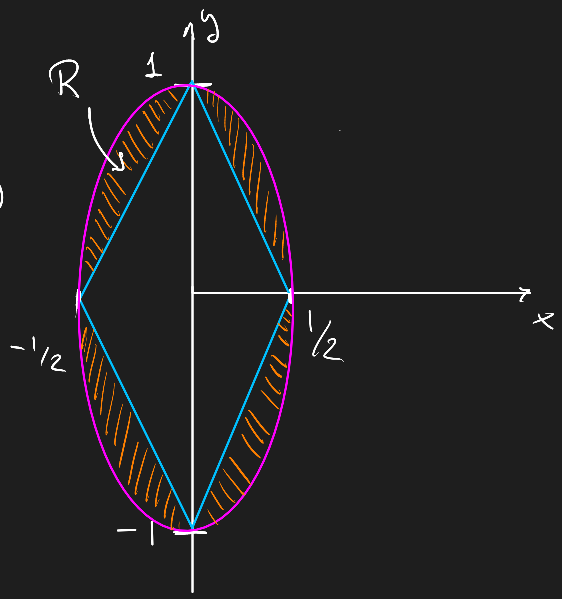
$du = \cos t \, dt$

¿Puedo? sino $(\sin^2 t)' = 2 \sin t . \cos t$

$$= \frac{3}{4} \int_{u=0}^{0} u \cdot du = 0$$

Parte II

$$\int_{\mathcal{C}} G \, ds = \iint_{R} Q_x - P_y \, dx dy$$



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