1.

Sean $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ definida por $f(x,y) = (x^2 - y, y + e^x)$, $g : \mathbb{R}^2 \to \mathbb{R}$ diferenciable y $h = g \circ f$. El polinomio de Taylor de orden 2 de h en $(0,0)$ es:

$$p(x,y) = 2x - y + x^2 + 3xy + 2y^2.$$

(a) Calcular $g(0,1)$ y $\nabla g(0,1)$

(b) Calcular, si existe,

$$\lim_{(x,y)\to(0,0)} \frac{h(x,y)}{\|(x,y)\|}$$

a) $h = g(f(x,y))$

$$h(x,y) = g(x^2 - y\ ,\ y + e^x) = g(u, v)$$

quiero $g(0,1)$

$$\begin{cases} x^2 - y = 0 \iff x^2 = y \\ y + e^x = 1 \end{cases}$$

$\xrightarrow{y = x^2}$ $x^2 + e^x = 1$

$$\underbrace{e^x}_{>0} = \underbrace{1 - x^2}$$

$0 \leq x^2 < 1 \Rightarrow 0 \leq y < 1$

$-1 < x < 1$

$$h(0,0) \overset{\text{Taylor}}{=} \underbrace{p(0,0)}_{=0} \overset{\text{evaluando}}{=} g(0,1)$$

$\therefore\ g(0,1) = 0$

También quiero

$$\nabla g(0,1) = ?$$

$$= \Big(\underbrace{g_x(0,1)}_{?} \, , \, \underbrace{g_y(0,1)}_{?} \Big)$$

Puedo calcular

$$h_x(0,0) \text{ y } h_y(0,0) \quad \text{pues}$$

$$\begin{matrix} \| & & \| \\ P_x(0,0) & \text{y} & P_y(0,0) \end{matrix}$$

$$P_x(x,y) = 2 + 2x + 3y$$

$$P_x(0,0) = 2$$

$$P_y(x,y) = -1 + 3x + 4y$$

$$P_y(0,0) = -1$$

Como

$$P_x(0,0) = h_x(0,0)$$

y

$$P_y(0,0) = h_y(0,0)$$

$$\Rightarrow h_x(0,0) = 2$$

$$h_y(0,0) = -1$$

$$\therefore \nabla h(0,0) = (2, -1)$$

Pero quiero $\nabla g(0,1)$

Como

$$h(x,y) = g(f(x,y)) = g(u,v)$$

$$\Rightarrow h_x(x,y) = \frac{\partial}{\partial u} g(f(x,y)) \cdot \frac{\partial u}{\partial x}(x,y) +$$

$$+ \frac{\partial}{\partial v} g(f(x,y)) \cdot \frac{\partial v}{\partial x}(x,y)$$

en $(x,y)=(0,0)$

$$\Rightarrow \quad 2 = g_u(0,1) \cdot 2x\Big|_{x=0} + g_v(0,1) \cdot e^x\Big|_{x=0}$$

$$2 = g_v(0,1)$$

Lo mismo para h_y

$$h(x,y) = g(f(x,y))$$

$$= g(x^2-y,\ y+e^x)$$

$$= g(\ u\ ,\ v\ )$$

$$\Rightarrow h_y(x,y) = g_u(u,v) \cdot \frac{\partial u}{\partial y}(x,y) + g_v(u,v) \cdot \frac{\partial v}{\partial y}(x,y)$$

$$h_y(0,0) = g_u(0,1) \cdot (-1) + g_v(0,1) \cdot 1$$

$$-1 = -g_u(0,1) + 2$$

$$g_u(0,1) = 3$$

$$\therefore \quad \boxed{\nabla g(0,1) = (3, 2)}$$

Puntos clave en la resolución

Saber reescribir

$$g(f(x,y)) \quad \text{con} \quad f(x,y) = (u(x,y), v(x,y))$$

de forma que

$$(u(x,y), v(x,y))$$

$$\frac{\partial}{\partial x} g(f(x,y)) = \frac{\partial g}{\partial u}(u,v) \cdot \frac{\partial u}{\partial x}(x,y) + \frac{\partial g}{\partial v}(u,v) \cdot \frac{\partial v}{\partial x}(x,y) +$$

$$\frac{\partial}{\partial y} g(f(x,y)) = \frac{\partial g}{\partial u}(u,v) \cdot \frac{\partial u}{\partial y}(x,y) + \frac{\partial g}{\partial v}(u,v) \cdot \frac{\partial v}{\partial y}(x,y) +$$

b) Calcular

$$\lim_{(x,y)\to(0,0)} \frac{h(x,y)}{\|(x,y)\|} = ?$$

Sé que

$$h(x,y) = P_1(x,y) + \underbrace{R_1(x,y)}_{\text{resto}}$$

Con $\lim_{(x,y)\to(0,0)} \frac{R_1(x,y)}{\|(x,y)\|} = 0$ por propiedad del Resto

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{h(x,y)}{\|\quad\|} = \lim_{(x,y)\to(0,0)} \frac{P_1(x,y)}{\|(x,y)\|}$$

vale en $(x,y)=(0,0)$

$$\text{Con} \quad P_1(x,y) \overset{\downarrow}{=} f(0,0) + \nabla f(0,0)\,(x-0,\ y-0)$$

$$= \underbrace{f(0,0)}_{=0} + \underbrace{f_x(0,0)}_{2+2x+3y|_{(0,0)}} \cdot x + \underbrace{f_y(0,0)}_{-1+3x+4y|_{(0,0)}} \cdot y$$

$$P_1(x,y) = 2x - y$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{2x-y}{\sqrt{x^2+y^2}}$$

sospecho que no tiene límite pues mismo orden de potencias

Si me muevo por $y = x$

$$\Rightarrow \lim_{x \to 0} \frac{2x - x}{\sqrt{x^2 + x^2}} = \lim_{x \to 0} \frac{x}{\sqrt{2x^2}}$$

$$= \lim_{x \to 0} \frac{x}{\sqrt{2} \cdot |x|} \begin{cases} \lim_{x \to 0^+} \frac{1}{\sqrt{2}} \\ \lim_{x \to 0^-} -\frac{1}{\sqrt{2}} \end{cases} \neq$$

$\therefore$ no existe el límite.

Puntos Clave.

Fue clave en este punto usar que el resto R1 tiende a cero cuando (x,y) tiende a (x0,y0), donde este es el punto sobre el que esta definido el polinomio de Taylor.

Esto sucede porque en el punto, el polinomio coincide con la función, por lo tanto, en el punto, no hay error o resto, ya que no hay aproximación, sino que el valor exacto que toma la función en ese punto.

2.

Sea $f\colon \mathbb{R}^2 \to \mathbb{R}$ definida por $f(x,y) = e^{xy-1} - \frac{1}{2}x^2 - \frac{1}{2}y^2$.

(a) Analizar la existencia de máximos y mínimos locales y puntos silla de f en $\mathbb{R}^2$.

(b) Analizar la existencia de extremos absolutos de f en la región

$$D = \{(x,y) \in \mathbb{R}^2 / x^2 + y^2 \leq 2\}.$$

a) $f \in \mathcal{C}^\infty$ pues es suma, prod, comp. de funciones $\mathcal{C}^\infty$

Calculo $\nabla f(x,y)$ e igualo a cero

$$\nabla f(x,y) = \left(e^{xy-1} \cdot y - x \,,\, e^{xy-1} \cdot x - y\right)$$

$$= (0,0) \iff \begin{cases} x = y \cdot e^{xy-1} \\ y = x \cdot e^{xy-1} \end{cases}$$

Resuelvo (sumo)

$$x + y = e^{xy-1} \cdot (x+y)$$

$$\underset{\substack{\uparrow \\ x+y \neq 0}}{e^{xy-1} = 1} \iff xy - 1 = 0$$

$$xy = 1$$

$$y = \frac{1}{x} \quad \text{con } x \neq 0$$

Reemplazo

$$\frac{1}{x} - x = \frac{1}{x} - \frac{x^2}{x}$$

$$= \frac{1-x^2}{x}$$

$$= (1-x)(1+x) \Rightarrow PCs = \{(1,1), (-1,-1)\}$$

Si $x+y=0$

$$\Rightarrow \begin{cases} x = y \cdot e^{xy-1} \\ y = x \cdot e^{xy-1} \\ x = -y \end{cases} \quad \begin{cases} x = -x\, e^{-x^2-1} \\ x = -x\, e^{-x^2-1} \end{cases}$$

$$0 = x - x \cdot e^{-x^2-1}$$

$$0 = \underset{x=0}{\underset{\uparrow}{x}} \underbrace{\left(1 - e^{-x^2-1}\right)}_{-x^2-1=0}$$

$x^2=-1$ Abs!

$(x+y)=0$ no es sol

- Si $x=0$

$$\Rightarrow \begin{cases} 0 = y \cdot e^{-1} \\ y = 0 \end{cases} \Rightarrow y=0$$

$$PCs = \{(0,0)\} \cup \{(1,1),(-1,-1)\}$$

Verifico con sustitución (otra forma)

$$x = x \cdot e^{xy-1} \cdot e^{xy-1}$$

$$x = x \cdot e^{2xy-2}$$

$$0 = x \cdot e^{2(xy-1)} - x$$

$$0 = x\left(e^{2(xy-1)} - 1\right) \Leftrightarrow x=0$$

ó

$$e^{2(xy-1)} - 1 = 0$$

$$e^{2(xy-1)} = 1$$

$$2(xy-1) = 0$$

$$xy-1 = 0$$

$$xy = 1$$

$$y = \frac{1}{x} \quad x \neq 0$$

Tengo candidatos, busco extremos locales

Armo Hessiano

$$\nabla f(x,y) = \left(e^{xy-1} \cdot y - x \,,\, e^{xy-1} \cdot x - y \right)$$

$$Hf = \begin{bmatrix} y^2 \cdot e^{xy-1} - 1 & x \cdot y \cdot e^{xy-1} + e^{xy-1} \\ x \cdot y \cdot e^{xy-1} + e^{xy-1} & x^2 \cdot e^{xy-1} - 1 \end{bmatrix}$$

- en $(0,0)$,

<0

$$\det Hf(0,0) = \begin{bmatrix} -1 & e^{-1} \\ e^{-1} & -1 \end{bmatrix} = 1 - e^{-2}$$

$$\approx 1.1353 > 0$$

Como $\det Hf(0,0) > 0$ y $f_{xx}(0,0) < 0$ $\Rightarrow$ $(0,0)$ es <u>máximo</u>.

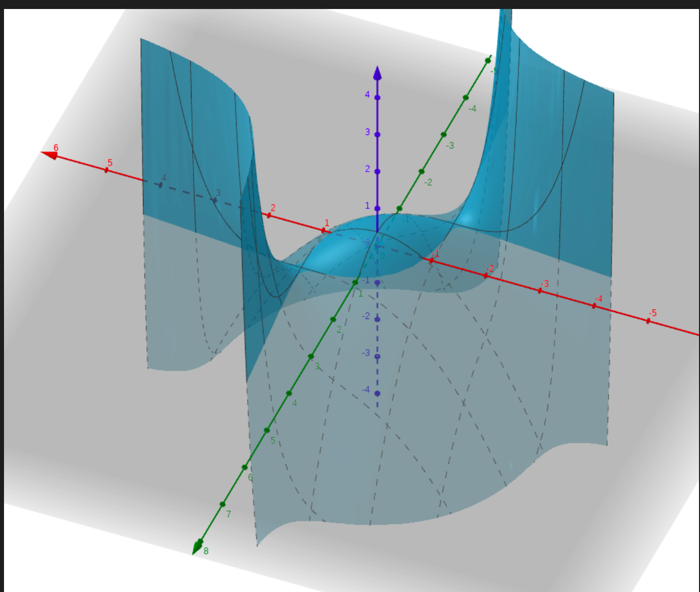
• en $(1,1)$

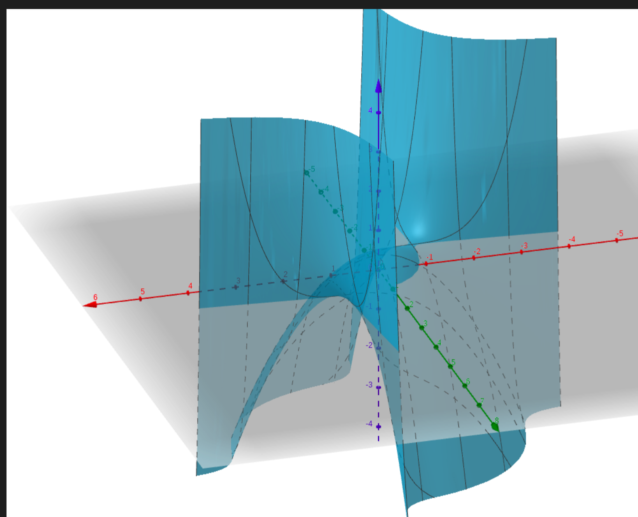
$$Hf(1,1) = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$\det Hf(1,1) = -4 < 0 \quad \therefore (1,1)$ es P. silla.

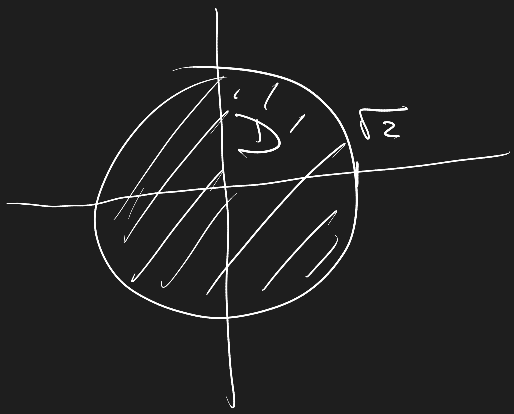
• en $(-1,-1)$

$$Hf(-1,-1) = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$





b)



Por a) sé que el único posible extremo en $\mathring{D}$ es el $(0,0)$

Falta ver el borde:

$$\sigma(t) = (\sqrt{2}\cdot\cos t,\ \sqrt{2}\cdot\sin t)$$

Con los $x \cdot y$ obtendré $(\sqrt{2})^2 \cdot \cos t \cdot \sin t$

Recuerdo identidad:

$$\sin 2x = 2 \cdot \sin x \cdot \cos x$$

$$f(\sigma(t)) = e^{2\cdot\cos t\cdot\sin t} - \frac{1}{2}\left(2\cos^2 t + 2\sin^2 t\right)$$

$$g(t) := e^{\sin 2t} - 1$$

$$g'(t) = \underbrace{e^{\sin 2t}}_{>0} \cdot \cos 2t \cdot \underbrace{2}_{>0} \overset{\text{quiero}}{=} 0$$

$$\Leftrightarrow \quad \cos 2t = 0 \qquad \text{con } t \in [0, 2\pi)$$

$$\iff \quad 2t = \frac{\pi}{2} \quad \text{ó} \quad 2t = \frac{3}{2}\pi$$

$$t = \frac{\pi}{4} \qquad\qquad t = \frac{3}{4}\pi$$

$$t = \frac{\pi}{4} + k\cdot\frac{\pi}{2} \qquad k \in \mathbb{Z}$$

Pero $t \in [0, 2\pi)$

$$\text{P.C. en } t = \left\{ \frac{1}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi \right\}$$

$$g(t) = e^{\sin 2t} - 1$$

$$g\left(\tfrac{1}{4}\pi\right) = e^{\sin \frac{1}{2}\pi} - 1 = e - 1 \approx 1{,}718 \quad \text{Máx}$$

$$g\left(\tfrac{3}{4}\pi\right) = e^{\sin \frac{3}{2}\pi} - 1 = \frac{1}{e} - 1 \approx -0{,}63 \quad \text{Mín}$$

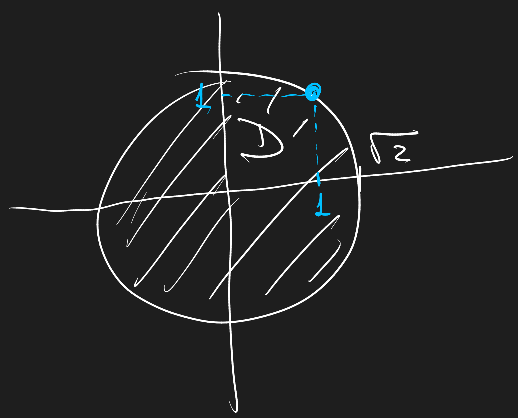
$$g\left(\tfrac{5}{4}\pi\right) = e^{\sin \frac{5}{2}\pi} - 1 = g\left(\tfrac{1}{4}\pi\right) \quad \text{mismo punto en } D$$

$$g\left(\tfrac{7}{4}\pi\right) = e^{\sin \frac{7}{2}\pi} - 1 = g\left(\tfrac{7}{4}\pi\right) \quad \text{mismo punto en } D$$

$$f(0,0) = \frac{1}{e} \approx 0{,}368$$

Max Abs =

Min Abs =



Usando Multiplicadores de Lagrange.

llamo $g(x,y) := x^2+y^2$

$$\nabla g(x,y) = (2x\,,\ 2y) = \vec{0} \iff (x,y) = (0,0)$$

Pero como $(0,0) \notin \partial D$

$\Rightarrow \nabla g$ no se anula en ∂D

$$\nabla f_{(x,y)} = \left(e^{xy-1}.y - x\,,\ e^{xy-1}.x - y\right)$$

Planteo sistema

$$\begin{cases} f_x = \lambda . g_x \\ f_y = \lambda . g_y \\ g(x,y) = 2 \end{cases} \Rightarrow \begin{cases} e^{xy-1}.y - x = 2x.\lambda & \text{I} \\ e^{xy-1}.x - y = 2y.\lambda & \text{II} \\ x^2+y^2 = 2 & \text{III} \end{cases}$$

I $\quad \dfrac{e^{xy-1}.y - x}{2x} \overset{\overset{x \neq 0}{\downarrow}}{=} \lambda$

I $\rightsquigarrow$ II $\quad e^{xy-1}.x - y = \dfrac{e^{xy-1}.y^2 - xy}{x}$

$$e^{xy-1}.x^2 - xy = e^{xy-1}.y^2 - xy$$

$$\underbrace{e^{xy-1}}_{>0}.x^2 = e^{xy-1}.y^2$$

$$x^2 = y^2$$

$$\text{III} \quad x^2 = 2 - y^2$$

$$2 - y^2 = y^2$$

$$2 = 2y^2$$

$$y^2 = 1$$

$$y = \begin{cases} -1 \Rightarrow x = \begin{cases} -1 \\ +1 \end{cases} \\ 1 \Rightarrow x = \begin{cases} -1 \\ +1 \end{cases} \end{cases}$$

Obtuve

$$PCs = \{(-1,-1), (-1,1), (1,-1), (1,1)\}$$

Falta ver $x=0$:

$$\begin{cases} e^{xy-1} \cdot y - x = 2x \cdot \lambda \\ e^{xy-1} \cdot x - y = 2y \cdot \lambda \\ x^2 + y^2 = 2 \end{cases} \xrightarrow{x=0} \begin{cases} e^{-1} \cdot y = 0 \Rightarrow y = 0 \\ -y = 2y\lambda \\ y^2 = 2 \end{cases}$$

Abs!

Evalúo PCs :

$$f(-1,-1) = 0$$

$$\left.\begin{aligned} f(-1,1) &= e^{-2} - 1 \approx -0{,}87 \\ f(1,-1) &= e^{-2} - 1 \approx -0{,}87 \end{aligned}\right\} \text{Mín} : (-1,1) \text{ y } (1,-1)$$

$$f(1,1) = 0$$

$f(0,0) = 1 - e^{-2} \approx 1,\ldots$ Máx en $(0,0)$

3.

Calcular las siguientes integrales

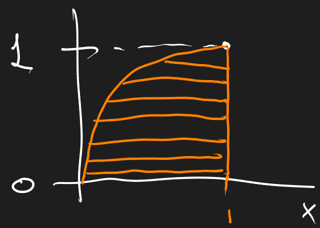
(a) $\displaystyle\int_0^1 \int_{y^3}^1 y^2 sen(x^2) dx dy.$

(b) $\displaystyle\iiint_E xz \, dV$ donde E es el sólido delimitado por el plano $4x+y+2z=2$ en el primer octante.

a) Cambio

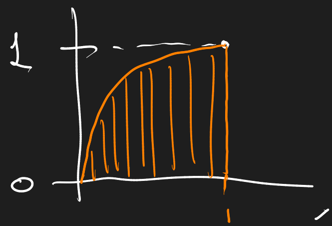
$0 \leq y \leq 1$

$y^3 \leq x \leq 1$



$0 \leq y \leq \sqrt[3]{x} \leq 1$

$0 \leq x \leq 1$



Reescribo

$$\int_{x=0}^{x=1} \int_{y=0}^{\sqrt[3]{x}} y^2 \cdot \sin(x^2) \, dy \, dx$$

$$= \int_{x=0}^{x=1} \sin(x^2) \cdot \underbrace{\left. \frac{y^3}{3} \right|_0^{\sqrt[3]{x}}}_{\frac{x}{3} - 0} dx$$

$$= \int_0^1 \sin x^2 \cdot \frac{x}{3} \, dx$$

CA

$$-\cos x^2 \cdot \frac{1}{6} \overset{\text{deriv}}{\Rightarrow} \sin x^2 \cdot 2x \cdot \frac{1}{6}$$

$$= -\frac{1}{6} \cos x^2 \Big|_0^1$$

$$= -\frac{1}{6}\left(\cos(1) - \cos 0 \right)$$

$$= -\frac{1}{6}\left(\cos(1) - 1 \right) \checkmark$$

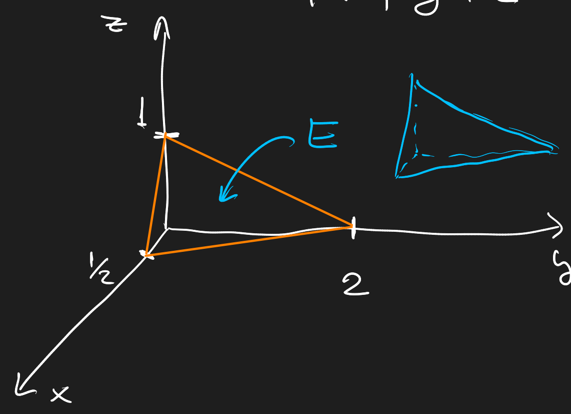
b) $\displaystyle\iiint_E xz\,dV$

$4x + y + 2z = 2$

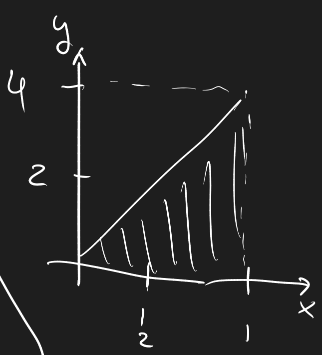
$z = 1$

$x = \frac{1}{2}$

$y = 2$



$$E = \left\{ (x,y,z) : \begin{array}{l} 0 \leq x \leq \frac{1}{2} \\ 0 \leq z \leq -2x + 1 \\ 0 \leq y \leq 2 - 4x - 2z \end{array} \right\}$$



$$\int_{x=0}^{\frac{1}{2}} \int_{z=0}^{-2x+1} \int_{y=0}^{2-4x-2z} x \cdot z \; dy\,dz\,dx =$$

$$= \int_{x=0}^{\frac{1}{2}} x \cdot \int_{z=0}^{-2x+1} z \cdot (2 - 4x - 2z)\; dz\,dx$$

$$= \int_{x=0}^{\frac{1}{2}} x \cdot \int_{z=0}^{-2x+1} 2z - 4xz - 2z^2 \; dz\,dx$$

$$= \int_{x=0}^{\frac{1}{2}} x \cdot \left[\cancel{\tfrac{2}{2}} z^2 - \overset{2}{\cancel{\tfrac{4}{2}}} x z^2 - \frac{2}{3} z^3 \right]_0^{-2x+1} dx$$

$$= \int_{x=0}^{\frac{1}{2}} x \cdot \left(1 - 4x + 4x^2 - x + 4x^2 - 4x^3 - \frac{2}{3}\left(1 - 6x + 12x^2 - 8x^3\right)\right) dx$$

$$= \int x\left(\frac{1}{3} - x + \cancel{8x^2} - \cancel{8x^2} \underbrace{- \frac{12}{3}x^3 + \frac{16}{3}x^3}_{\frac{4}{3}x^3}\right)$$

$$(1-2x)^2 = 1 - 4x + 4x^2$$

$$(1-2x)^3 = 1 - 4x + 4x^2 - \left(2x - 8x^2 + 8x^3\right) = 1 - 6x + 12x^2 - 8x^3$$

$$= \int_0^1 \frac{1}{3}x - x^2 + \frac{4}{3}x^4 \; dx$$

$$= \frac{1}{6}x^2 - \frac{x^3}{3} + \frac{4}{3}\cdot\frac{1}{5}\cdot x^5 \Big|_0^1$$

$$= \frac{1}{6} - \frac{2}{6} + \frac{4}{15} = \frac{4}{15} - \frac{1}{6} = \frac{9}{90}$$

$$= \frac{1}{10} \;//$$

Error de cuentas en algún lado

el resultado es $\frac{1}{240}$ según resolución.

4.

Sea $F\colon \mathbb{R}^3 \to \mathbb{R}^3$,

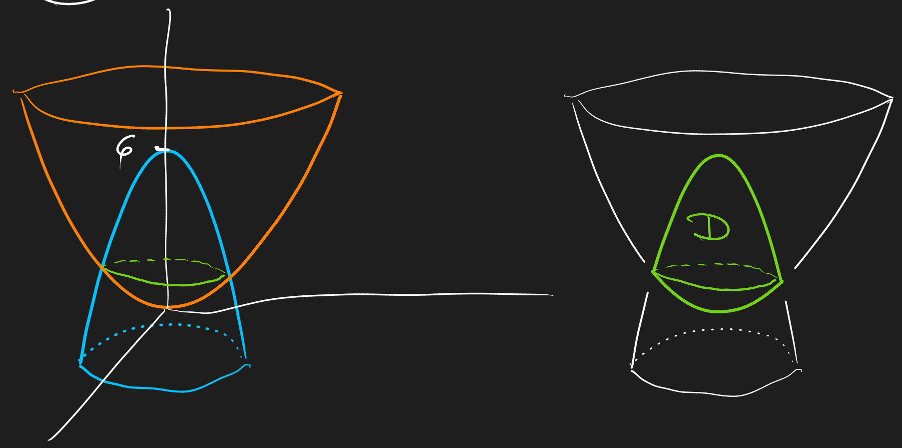
$$F(x,y,z) = \left(\frac{zx^3}{3} + zy^2x,\, xy^2e^{x^2},\, -2xyze^{x^2}\right).$$

Calcular

$$\iiint_D div(F)dV,$$

donde D es la región encerrada por las superficies $z = x^2 + y^2$ y $z = 6 - 2x^2 - 2y^2$.

Dibujo D



$$\operatorname{div} F = P_x + Q_y + R_z$$

$$= zx^2 + zy^2 + 2xy \cdot e^{x^2} - 2xye^{x^2}$$

$$= z \cdot (x^2 + y^2)$$

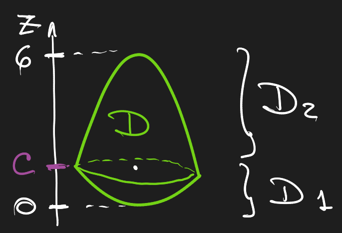
Atenti! Pide cambio de variables!

Plan: Cambio cilíndricas

$z = 6 - 2(x^2+y^2)$ I en II

$z = 6 - 2 \cdot z$

$z = 2 = c$



$$\begin{cases} x = r \cdot \cos\theta \\ y = r \cdot \sin\theta \\ z = z \end{cases}$$

$\theta \in [0, 2\pi)$

$r \in [0, \sqrt{2}]$

$x^2+y^2 \leq z \leq 6 - 2(x^2+y^2)$

$$r^2 \leq z \leq 6 - 2r^2$$

Integro

$$\iiint_D \operatorname{div} F \, dv = \iiint_{D'} \operatorname{div} F \cdot r \cdot dz\, d\theta\, dr$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{2}} \int_{z=r^2}^{6-2r^2} z \cdot \overbrace{r^2}^{x^2+y^2} \cdot r \; dz\, dr\, d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{2}} r^3 \cdot \frac{z^2}{2} \Big|_{r^2}^{6-2r^2} dr\, d\theta$$

$$= \int_\theta \int_r r^3 \cdot \frac{1}{2} \cdot \left(6^2 - 24r^2 + 4r^4 - r^4 \right)$$

$$= \frac{1}{2} \cdot \int_\theta \int_r 36r^3 - 24r^5 + 3r^7 \; dr\, d\theta$$

$$= \frac{1}{2} \int_{\theta=0}^{2\pi} \underbrace{\frac{36}{4}}_{9} \cdot r^4 - \underbrace{\frac{24}{6}}_{4} \cdot r^6 + \frac{3}{8} \cdot r^8 \Big|_0^{\sqrt{2}} d\theta$$

$$= \frac{1}{2} \int_{\theta=0}^{2\pi} 9 \cdot 2^2 - 2^2 \cdot 2^3 + \frac{3}{8} \cdot 2^4 \; d\theta$$

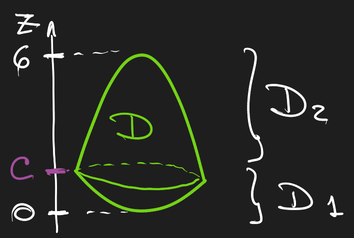
$$= \frac{\cancel{2}\pi}{\cancel{2}} \cdot 10$$

$$= 10\pi \quad //$$

Plan: usar cilíndricas dividiendo D en D_1 y D_2

$$\left.\begin{array}{l} z = 6 - 2(x^2+y^2) \\ z = 6 - 2 \cdot z \end{array}\right\} \text{I en II}$$

$$z = 2 = c$$



$$\begin{cases} x = r \cdot \cos\theta \\ y = r \cdot \sin\theta \\ z = z \end{cases}$$

$\theta \in [0, 2\pi)$

$r \in [0, \phi(z)]$

$z \in [0, 6]$

Calculo $\phi(z)$ para D_1 y D_2

• D_1

r^2

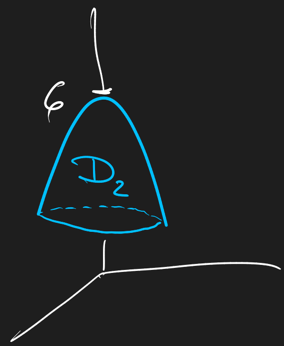
$$\phi_1(z) = \sqrt{x^2+y^2} \qquad \leftarrow \text{despejo de eq } z = x^2+y^2$$

• D_2

$$z = 6 - 2x^2 - 2y^2$$

$$z - 6 = -2(x^2+y^2)$$

$$3 - \frac{z}{2} = x^2+y^2$$



$$\phi_2(z) = \sqrt{3 - \frac{z}{2}}$$

$$\iiint_D \operatorname{div} F \, dv = \iiint_{D'} \operatorname{div} F \cdot r \cdot dr\, d\theta\, dz$$

Calculo sobre D_2:

$$\iiint_{D_2} \operatorname{div} F \, dv = \iiint_{D_2'} \operatorname{div} F \cdot r \cdot dr\, d\theta\, dz$$

$$= \int_{z=2}^{6} \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{3-\frac{z}{2}}} z \cdot \underbrace{r^2}_{x^2+y^2} \cdot r \, dr\, d\theta\, dz$$

$$= \int_{z=2}^{6} z \cdot \int_{\theta=0}^{2\pi} \left.\frac{r^4}{4}\right|_0^{\sqrt{3-\frac{z}{2}}} d\theta\, dz$$

$$= \int_{z=2}^{6} z \int_{\theta=0}^{2\pi} \frac{1}{4}\left(3 - \frac{z}{2}\right)^2 d\theta\, dz$$

CA

$$\left(3-\frac{z}{2}\right) = 9 - 3z + z^3$$

$$= \frac{1}{4} \cdot 2\pi \cdot \int_{z=2}^{6} 9z - 3z^2 + z^3 \, dz$$

$$= \frac{\pi}{2} \cdot \left[\frac{9}{2} z^2 - \frac{\cancel{3}}{\cancel{3}} z^3 + \frac{1}{4} \cdot z^4 \right]_2^6$$

$$= \frac{\pi}{2} \cdot \left(\frac{9}{2} \cdot 6^2 - 6^3 + \frac{6^4}{2^2} - \frac{9}{2} \cdot 2^2 + 2^3 - \frac{2^4}{2^2} \right)$$

$$= \pi \cdot \left(135 \underbrace{- 9 + 4 - 2}_{=7} \right)$$

$$= 128\pi$$

Calc sobre D_2:

$$\iiint_{D_2} \operatorname{div} F \, dv = \iiint_{D_2'} \operatorname{div} F \cdot r \cdot dr \, d\theta \, dz$$

$$= \int_{z=2}^{6} \int_{\theta=0}^{2\pi} \int$$