

1.

Sean $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ definida por $f(x, y) = (x^2 - y, y + e^x)$, $g: \mathbb{R}^2 \rightarrow \mathbb{R}$ diferenciable y $h = g \circ f$. El polinomio de Taylor de orden 2 de h en $(0, 0)$ es:

$$p(x, y) = 2x - y + x^2 + 3xy + 2y^2.$$

(a) Calcular $g(0, 1)$ y $\nabla g(0, 1)$

(b) Calcular, si existe,

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{h(x, y)}{\|(x, y)\|}$$

$$a) \quad h = g(f(x, y))$$

$$h(x, y) = g(x^2 - y, y + e^x) = g(u, v)$$

$$\text{quero } g(0, 1)$$

$$\begin{cases} x^2 - y = 0 \\ y + e^x = 1 \end{cases} \Leftrightarrow \begin{cases} x^2 = y \\ y = 1 - e^x \end{cases}$$

$$y = x^2 \hookrightarrow x^2 + e^x = 1$$

$$\underbrace{e^x}_{>0} = \underbrace{1 - x^2}_{0 \leq x^2 < 1} \Rightarrow 0 \leq y < 1$$

$$-1 < x < 1$$

$$\begin{array}{ccc} \text{Taylor} & & \text{evaluando} \\ \downarrow & & \downarrow \\ h(0, 0) = \underbrace{p(0, 0)}_{=0} & = & g(0, 1) \end{array}$$

$$\therefore g(0, 1) = 0$$

También quiero

$$\nabla g(0,1) = ?$$

$$= \left(\underbrace{g_x(0,1)}_?, \underbrace{g_y(0,1)}_? \right)$$

Puedo calcular

$$\begin{array}{ccc} h_x(0,0) & \text{y} & h_y(0,0) & \text{por} \\ \parallel & & \parallel & \\ p_x(0,0) & \text{y} & p_y(0,0) & \end{array}$$

$$p_x(x,y) = 2 + 2x + 3y$$

$$p_x(0,0) = 2$$

$$p_y(x,y) = -1 + 3x + 4y$$

$$p_y(0,0) = -1$$

Como

$$\begin{array}{l} p_x(0,0) = h_x(0,0) \\ \text{y} \\ p_y(0,0) = h_y(0,0) \end{array}$$

$$\Rightarrow h_x(0,0) = 2$$

$$h_y(0,0) = -1$$

$$\therefore \nabla h(0,0) = (2, -1)$$

Pero quiero $\nabla g(0,1)$

Como

$$h(x,y) = g(f(x,y)) = g(u,v)$$

$$\Rightarrow h_x(x,y) = \frac{\partial}{\partial u} g(f(x,y)) \cdot \frac{\partial u}{\partial x}(x,y) + \frac{\partial}{\partial v} g(f(x,y)) \cdot \frac{\partial v}{\partial x}(x,y)$$

en $(x,y)=(0,0)$

$$\Rightarrow 2 = g_u(0,1) \cdot 2x|_{x=0} + g_v(0,1) \cdot e^x|_{x=0}$$

$$2 = g_v(0,1)$$

Lo mismo para h_y

$$\begin{aligned} h(x,y) &= g(f(x,y)) \\ &= g(x^2 - y, y + e^x) \\ &= g(u, v) \end{aligned}$$

$$\Rightarrow h_y(x,y) = g_u(u,v) \cdot \frac{\partial u}{\partial y}(x,y) + g_v(u,v) \cdot \frac{\partial v}{\partial y}(x,y)$$

$$h_y(0,0) = g_u(0,1) \cdot (-1) + g_v(0,1) \cdot 1$$

$$-1 = -g_u(0,1) + 2$$

$$g_u(0,1) = 3$$

∴

$$\nabla g(0,1) = (3, 2)$$

Puntos clave en la resolución

Saber reescribir

$$g(f(x,y)) \text{ con } f(x,y) = (\mu(x,y), \nu(x,y))$$

de forma que

$$\frac{\partial}{\partial x} g(f(x,y)) = \frac{\partial g}{\partial \mu}(\mu(x,y), \nu(x,y)) \cdot \frac{\partial \mu}{\partial x}(x,y) + \frac{\partial g}{\partial \nu}(\mu(x,y), \nu(x,y)) \cdot \frac{\partial \nu}{\partial x}(x,y) +$$

$$\frac{\partial}{\partial y} g(f(x,y)) = \frac{\partial g}{\partial \mu}(\mu(x,y), \nu(x,y)) \cdot \frac{\partial \mu}{\partial y}(x,y) + \frac{\partial g}{\partial \nu}(\mu(x,y), \nu(x,y)) \cdot \frac{\partial \nu}{\partial y}(x,y) +$$

b) Calcular

$$\lim_{(x,y) \rightarrow (0,0)} \frac{h(x,y)}{\|(x,y)\|} = ?$$

Sé que

$$h(x,y) = p_1(x,y) + \overset{\text{resto}}{R_1(x,y)}$$

$$\text{Con } \lim_{(x,y) \rightarrow (0,0)} \frac{R_1(x,y)}{\|(x,y)\|} = 0 \quad \text{por propiedad del Resto}$$

$$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{h(x,y)}{\|(x,y)\|} = \lim_{(x,y) \rightarrow (0,0)} \frac{p_1(x,y)}{\|(x,y)\|}$$

vale en $(x,y) = (0,0)$

$$\begin{aligned} \text{Con } p_1(x,y) &\stackrel{\downarrow}{=} f(0,0) + \nabla f(0,0) (x-0, y-0) \\ &= \underbrace{f(0,0)}_{=0} + \underbrace{f_x(0,0)}_{2+2x+3y|_{(0,0)}} \cdot x + \underbrace{f_y(0,0)}_{-1+3x+4y|_{(0,0)}} \cdot y \end{aligned}$$

$$p_1(x,y) = 2x - y$$

$$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{2x-y}{\sqrt{x^2+y^2}} \quad \leftarrow \begin{array}{l} \text{sospecho que no tiene límite} \\ \text{pues mis mo orden de potencias} \end{array}$$

Si me muevo por $y = x$

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 0} \frac{2x - x}{\sqrt{x^2 + x^2}} &= \lim_{x \rightarrow 0} \frac{x}{\sqrt{2x^2}} \\ &= \lim_{x \rightarrow 0} \frac{x}{\sqrt{2} \cdot |x|} \end{aligned}$$

$\begin{aligned} &\nearrow \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{2}} \\ &\searrow \lim_{x \rightarrow 0^-} -\frac{1}{\sqrt{2}} \end{aligned} \quad \left. \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right\} \neq$

\therefore no existe el límite.

Puntas Clave.

Fue clave en este punto usar que el resto R_1 tiende a cero cuando (x, y) tiende a (x_0, y_0) , donde este es el punto sobre el que está definido el polinomio de Taylor.

Esto sucede porque en el punto, el polinomio coincide con la función, por lo tanto, en el punto, no hay error o resto, ya que no hay aproximación, sino que el valor exacto que toma la función en ese punto.

2.

Sea $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ definida por $f(x, y) = e^{xy-1} - \frac{1}{2}x^2 - \frac{1}{2}y^2$.

(a) Analizar la existencia de máximos y mínimos locales y puntos silla de f en \mathbb{R}^2 .

(b) Analizar la existencia de extremos absolutos de f en la región

$$D = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 \leq 2\}.$$

a) $f \in \mathcal{C}^\infty$ pues es suma, prod, comp. de funciones \mathcal{C}^∞

Calculo $\nabla f(x, y)$ e igualo a cero

$$\nabla f(x, y) = (e^{xy-1} \cdot y - x, e^{xy-1} \cdot x - y)$$

$$= (0, 0) \Leftrightarrow \begin{cases} x = y \cdot e^{xy-1} \\ y = x \cdot e^{xy-1} \end{cases}$$

Resuelvo (suma)

$$x + y = e^{xy-1} \cdot (x + y)$$

$$e^{xy-1} = 1 \Leftrightarrow xy-1 = 0$$

\uparrow
 $x+y \neq 0$

$$xy = 1$$

$$y = \frac{1}{x} \text{ con } x \neq 0$$

Reemplazo

$$\frac{1}{x} - x = \frac{1}{x} - \frac{x^2}{x}$$

$$= \frac{1-x^2}{x}$$

$$= (1-x)(1+x) \Rightarrow PC_s = \{(1, 1), (-1, -1)\}$$

Si $x+y=0$

$$\Rightarrow \left\{ \begin{array}{l} x = y \cdot e^{xy-1} \\ y = x \cdot e^{xy-1} \\ x = -y \end{array} \right\} \left\{ \begin{array}{l} x = -x e^{-x^2-1} \\ x = -x e^{-x^2-1} \\ 0 = x - x \cdot e^{-x^2-1} \\ 0 = x (1 - e^{-x^2-1}) \end{array} \right.$$

• Si $x=0$

$$\Rightarrow \left\{ \begin{array}{l} 0 = y \cdot e^{-1} \\ y = 0 \end{array} \right. \Rightarrow y=0$$

\uparrow
 $x=0$

$-x^2-1=0$

$x^2 = -1$ Abs!

$(x+y)=0$ no es sol

$$PCs = \{(0,0)\} \cup \{(1,1), (-1,-1)\}$$

Verifico con sustitución (otra forma)

$$x = x \cdot e^{xy-1} \cdot e^{xy-1}$$

$$x = x \cdot e^{2xy-2}$$

$$0 = x \cdot e^{2(xy-1)} - x$$

$$0 = x (e^{2(xy-1)} - 1) \Leftrightarrow x=0 \text{ ó }$$

$$e^{2(xy-1)} - 1 = 0$$

$$e^{2(xy-1)} = 1$$

$$2(xy-1) = 0$$

$$xy-1 = 0$$

$$xy = 1$$

$$y = \frac{1}{x} \quad x \neq 0$$

Tengo condados, busco extremos locales

Armo Hessiano

$$\nabla f(x,y) = (e^{xy-1} \cdot y - x, e^{xy-1} \cdot x - y)$$

$$Hf = \begin{bmatrix} y^2 \cdot e^{xy-1} - 1 & x \cdot y \cdot e^{xy-1} + e^{xy-1} \\ x \cdot y \cdot e^{xy-1} + e^{xy-1} & x^2 \cdot e^{xy-1} - 1 \end{bmatrix}$$

• en $(0,0)$,

$$\det Hf(0,0) = \begin{bmatrix} \textcircled{-1} & e^{-1} \\ e^{-1} & -1 \end{bmatrix} \begin{matrix} \nwarrow < 0 \\ \end{matrix} = 1 - e^{-2}$$

$$\approx 1.1353 > 0$$

$$\left. \begin{array}{l} \text{Como } \det Hf(0,0) > 0 \\ y \quad f_{xx}(0,0) < 0 \end{array} \right\} \Rightarrow (0,0) \text{ es } \underline{\text{máximo}}.$$

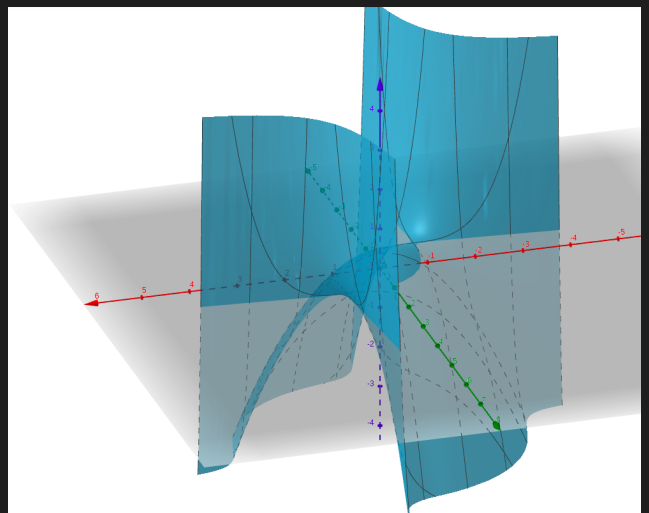
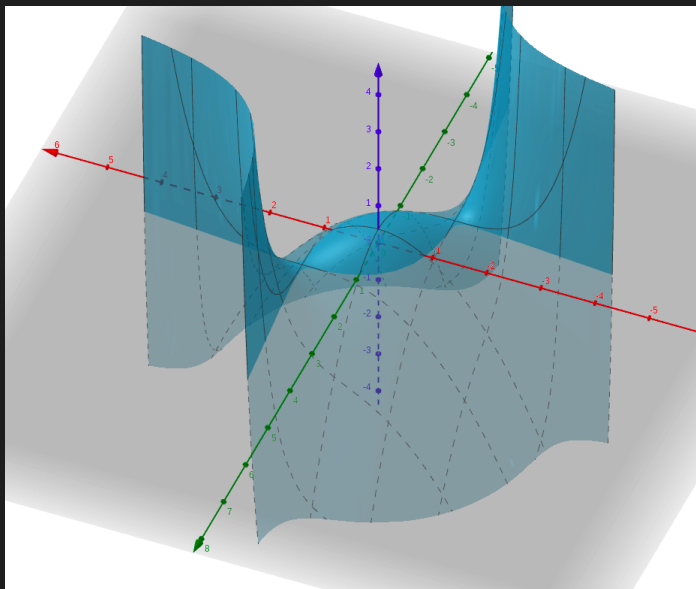
• en $(1, 1)$

$$Hf(1,1) = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

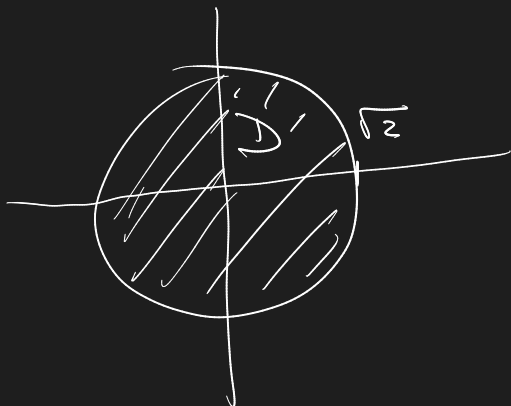
$\det Hf(1,1) = -4 < 0 \quad \therefore (1,1)$ e P. silla.

• en $(-1, -1)$

$$Hf(-1,-1) = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$



b)



Por a) sé que el único posible extremo en $\overset{\circ}{D}$ es el $(0,0)$

Falta ver el borde :

$$\sigma(t) = (\sqrt{2} \cdot \cos t, \sqrt{2} \cdot \sin t)$$

Con los x, y obtendré $(\sqrt{2})^2 \cdot \cos t \cdot \sin t$

Recordo identidad :

$$\sin 2x = 2 \cdot \sin x \cdot \cos x$$

$$f(\sigma(t)) = e^{2 \cdot \cos t \cdot \sin t} - \frac{1}{2} (2 \cos^2 t + 2 \sin^2 t)$$

$$g(t) := e^{\sin 2t} - 1$$

$$g'(t) = \underbrace{e^{\sin 2t}}_{>0} \cdot \cos 2t, \underbrace{2}_{>0} \overset{\text{quiero}}{\downarrow} = 0$$

$$\Leftrightarrow \cos 2t = 0 \quad \text{con } t \in [0, 2\pi)$$

$$\Leftrightarrow 2t = \frac{\pi}{2} \quad \vee \quad 2t = \frac{3}{2}\pi$$

$$t = \frac{\pi}{4}$$

$$t = \frac{3}{4}\pi$$

$$t = \frac{\pi}{4} + k \cdot \frac{\pi}{2} \quad k \in \mathbb{Z}$$

$$\text{Pero } t \in [0, 2\pi)$$

$$\text{P.C. en } t = \left\{ \frac{1}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi \right\}$$

$$g(t) = e^{\sin 2t} - 1$$

$$g\left(\frac{1}{4}\pi\right) = e^{\sin \frac{1}{2}\pi} - 1 = e - 1 \approx 1,718 \quad \underline{\text{Máx}}$$

$$g\left(\frac{3}{4}\pi\right) = e^{\sin \frac{3}{2}\pi} - 1 = \frac{1}{e} - 1 \approx -0,63 \quad \underline{\text{Mín}}$$

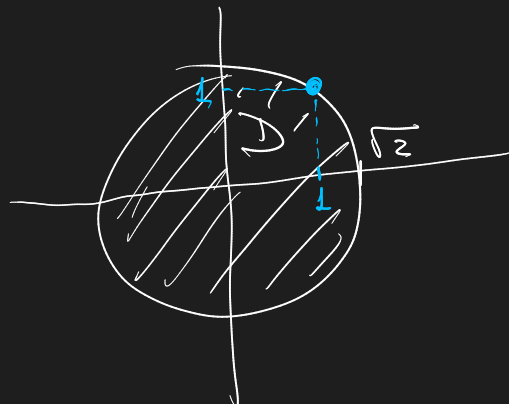
$$g\left(\frac{5}{4}\pi\right) = e^{\sin \frac{5}{2}\pi} - 1 = g\left(\frac{1}{4}\pi\right) \quad \text{mismo punto en } \mathbb{D}$$

$$g\left(\frac{7}{4}\pi\right) = e^{\sin \frac{7}{2}\pi} - 1 = g\left(\frac{3}{4}\pi\right) \quad \text{mismo punto en } \mathbb{D}$$

$$f(0,0) = \frac{1}{e} \approx 0,368$$

Max Abs =

Mín Abs =



Usando Multiplicadores de Lagrange,

$$\text{Llamo } g(x,y) := x^2 + y^2$$

$$\nabla g(x,y) = (2x, 2y) = \vec{0} \Leftrightarrow (x,y) = (0,0)$$

Pero como $(0,0) \in \partial D$

$\Rightarrow \nabla g$ no se anula en ∂D

$$\nabla f(x,y) = (e^{xy^{-1}} \cdot y - x, e^{xy^{-1}} \cdot x - y)$$

Planteo sistema

$$\begin{cases} f_x = \lambda \cdot g_x \\ f_y = \lambda \cdot g_y \\ g(x,y) = 2 \end{cases} \Rightarrow \begin{cases} e^{xy^{-1}} \cdot y - x = 2x \cdot \lambda & \textcircled{I} \\ e^{xy^{-1}} \cdot x - y = 2y \cdot \lambda & \textcircled{II} \\ x^2 + y^2 = 2 & \textcircled{III} \end{cases}$$

$$\textcircled{I} \quad \frac{e^{xy^{-1}} \cdot y - x}{2x} \stackrel{x \neq 0}{=} \lambda$$

$$\textcircled{I} \leadsto \textcircled{II} \quad e^{xy^{-1}} \cdot x - y = \frac{e^{xy^{-1}} \cdot y^2 - xy}{x}$$

$$e^{xy^{-1}} \cdot x^2 - xy = e^{xy^{-1}} \cdot y^2 - xy$$

$$\underbrace{e^{xy^{-1}}}_{>0} \cdot x^2 = e^{xy^{-1}} \cdot y^2$$

$$x^2 = y^2$$

$$\textcircled{\text{III}} \quad x^2 = 2 - y^2$$

$$2 - y^2 = y^2$$

$$2 = 2y^2$$

$$y^2 = 1$$

$$y = \begin{matrix} \nearrow -1 \\ \searrow 1 \end{matrix} \Rightarrow x = \begin{matrix} \nearrow -1 \\ \searrow +1 \end{matrix}$$

$$\Rightarrow x = \begin{matrix} \nearrow -1 \\ \searrow +1 \end{matrix}$$

Obtune

$$PCs = \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$$

Falte ver $x=0$:

$$\begin{cases} e^{xy-1} \cdot y - x = 2x \cdot \lambda \\ e^{xy-1} \cdot x - y = 2y \cdot \lambda \\ x^2 + y^2 = 2 \end{cases} \xrightarrow{x=0} \begin{cases} e^{-1} \cdot y = 0 \Rightarrow y=0 \\ -y = 2y \lambda \\ y^2 = 2 \end{cases}$$

Abs!

Evaluó PCs:

$$f(-1, -1) = 0$$

$$f(-1, 1) = e^{-2} - 1 \approx -0,87$$

$$f(1, -1) = e^{-2} - 1 \approx -0,87$$

$$f(1, 1) = 0$$

} Máx: $(-1, 1), (1, -1)$

3.

Calcular las siguientes integrales

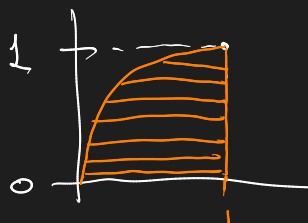
(a) $\int_0^1 \int_{y^3}^1 y^2 \sin(x^2) dx dy.$

(b) $\iiint_E xz \, dV$ donde E es el sólido delimitado por el plano $4x+y+2z=2$ en el primer octante.

a) Cambio

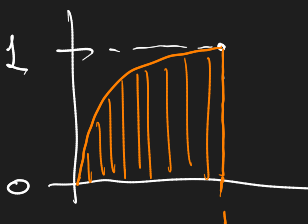
$$0 \leq y \leq 1$$

$$y^3 \leq x \leq 1$$



$$0 \leq y \leq \sqrt[3]{x} \leq 1$$

$$0 \leq x \leq 1$$



Reescribo

$$\int_{x=0}^1 \int_{y=0}^{\sqrt[3]{x}} y^2 \cdot \sin(x^2) \, dy \, dx$$

$$= \int_{x=0}^1 \sin(x^2) \cdot \underbrace{\frac{y^3}{3} \Big|_0^{\sqrt[3]{x}}}_{\frac{x}{3} - 0} \, dx$$

$$= \int_0^1 \sin x^2 \cdot \frac{x}{3} \, dx$$

CA

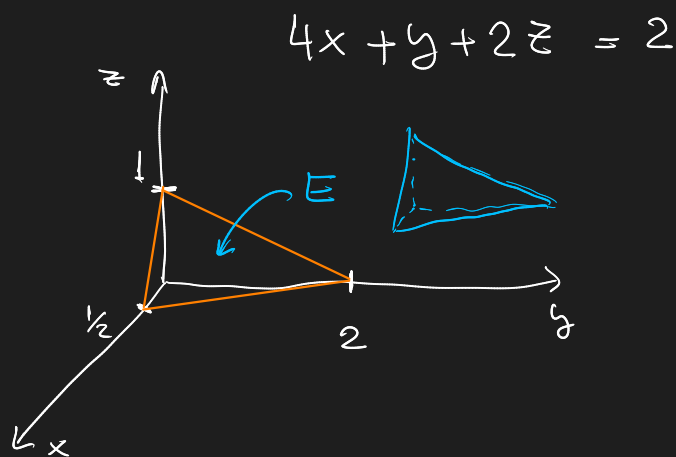
$$-\cos x^2 \cdot \frac{1}{6} \xRightarrow{\text{deriv}} \sin x^2 \cdot 2x \cdot \frac{1}{6}$$

$$= -\frac{1}{6} \cos x^2 \Big|_0^1$$

$$= -\frac{1}{6} (\cos(1) - \cos 0)$$

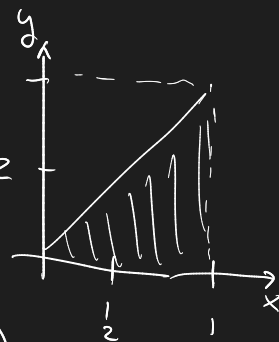
$$= -\frac{1}{6} (\cos(1) - 1) \quad \checkmark$$

$$b) \iiint_E xz \, dV$$



$$\begin{aligned} z &= 1 \\ x &= \frac{1}{2} \\ y &= 2 \end{aligned}$$

$$E = \left\{ (x, y, z) : \begin{aligned} 0 &\leq x \leq \frac{1}{2} \\ 0 &\leq z \leq -2x + 1 \\ 0 &\leq y \leq 2 - 4x - 2z \end{aligned} \right\}$$



$$\int_{x=0}^{\frac{1}{2}} \int_{z=0}^{-2x+1} \int_{y=0}^{2-4x-2z} x \cdot z \, dy \, dz \, dx =$$

$$= \int_{x=0}^{\frac{1}{2}} x \cdot \int_{z=0}^{-2x+1} z \cdot (2 - 4x - 2z) \, dz \, dx$$

$$= \int_{x=0}^{\frac{1}{2}} x \cdot \int_{z=0}^{-2x+1} (2z - 4xz - 2z^2) \, dz \, dx$$

$$= \int_{x=0}^{\frac{1}{2}} x \cdot \left[\frac{2}{2} z^2 - \frac{4}{2} x z^2 - \frac{2}{3} z^3 \right]_0^{-2x+1} \, dx$$

$$= \int_{x=0}^{\frac{1}{2}} x \cdot \left(1 - 4x + 4x^2 - x + 4x^2 - 4x^3 - \left(1 - 4x + 4x^2 - \frac{2}{3}(1 - 6x + 12x^2 - 8x^3) \right) \right) dx$$

$$= \int x \left(\frac{1}{3} - x + \cancel{8x^2} - \underbrace{\cancel{8x^2} - \frac{12}{3}x^3 + \frac{16}{3}x^3}_{\frac{4}{3}x^3} \right) dx$$

$$(1-2x)^2 =$$

$$= 1 - 4x + 4x^2$$

$$(1-2x)^3 =$$

$$= 1 - 4x + 4x^2 -$$

$$- (2x - 8x^2 + 8x^3)$$

$$= 1 - 6x + 12x^2 - 8x^3$$

$$= \int_0^1 \frac{1}{3}x - x^2 + \frac{4}{3}x^4 dx$$

$$= \frac{1}{6}x^2 - \frac{x^3}{3} + \frac{4}{3} \cdot \frac{1}{5} \cdot x^5 \Big|_0^1$$

$$= \frac{1}{6} - \frac{2}{6} + \frac{4}{15} = \frac{4}{15} - \frac{1}{6} = \frac{9}{90}$$

$$= \frac{1}{10} //$$

Error de cuenta en algún lado

el resultado es $\frac{1}{240}$ según revolución.

4.

Sea $F: \mathbb{R}^3 \rightarrow \mathbb{R}^3$,

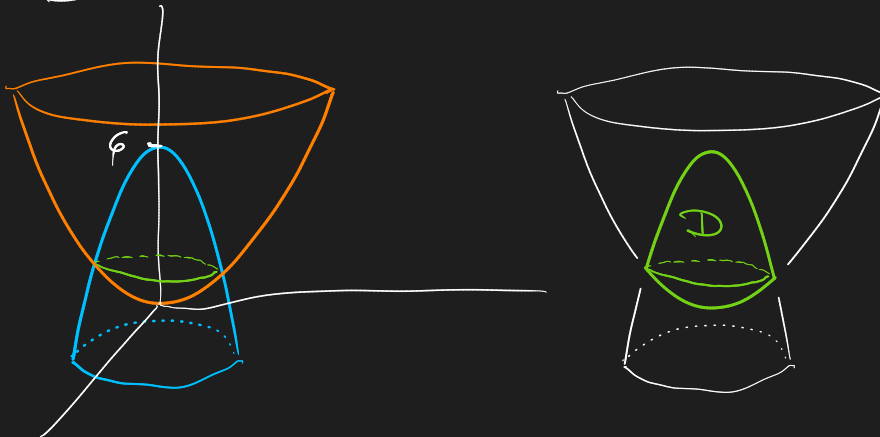
$$F(x, y, z) = \left(\frac{zx^3}{3} + zy^2x, xy^2e^{x^2}, -2xyz e^{x^2} \right).$$

Calcular

$$\iiint_D \operatorname{div}(F) dV,$$

donde D es la región encerrada por las superficies $z = x^2 + y^2$ y $z = 6 - 2x^2 - 2y^2$.

Dibujo D

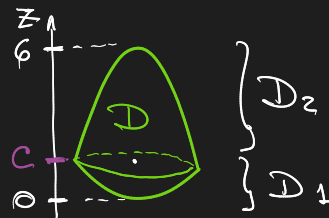


$$\begin{aligned} \operatorname{div} F &= P_x + Q_y + R_z \\ &= zx^2 + zy^2 + 2xy \cdot e^{x^2} - 2xy e^{x^2} \\ &= z(x^2 + y^2) \end{aligned}$$

⤴ Atenti! Pide cambio de variables!

Plan: Cambio cilíndricas

$$\begin{aligned} z &= 6 - 2(x^2 + y^2) \\ z &= 6 - 2r^2 \\ z &= z = c \end{aligned} \quad \downarrow \text{I en II}$$



$$\begin{cases} x = r \cdot \cos \theta \\ y = r \cdot \sin \theta \\ z = z \end{cases} \quad \begin{aligned} \theta &\in [0, 2\pi) \\ r &\in [0, \sqrt{z}] \\ x^2 + y^2 &\leq z \leq 6 - 2(x^2 + y^2) \end{aligned}$$

$$r^2 \leq z \leq 6 - 2r^2$$

Integro

$$\iiint_D \operatorname{div} F \, dv = \iiint_{D'} \operatorname{div} F \cdot r \cdot dz d\theta dr$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{2}} \int_{z=r^2}^{6-2r^2} z \cdot \overbrace{r^2}^{x^2+y^2} \cdot r \, dz dr d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{2}} r^3 \cdot \left. \frac{z^2}{2} \right|_{r^2}^{6-2r^2} dr d\theta$$

$$= \int_{\theta} \int_r r^3 \cdot \frac{1}{2} \cdot (6^2 - 24r^2 + 4r^4 - r^4)$$

$$= \frac{1}{2} \cdot \int_{\theta} \int_r 36r^3 - 24r^5 + 3r^7 \, dr d\theta$$

$$= \frac{1}{2} \int_{\theta=0}^{2\pi} \left(\frac{36}{4} \cdot r^4 - \frac{24}{6} \cdot r^6 + \frac{3}{8} \cdot r^8 \right) \bigg|_0^{\sqrt{2}} d\theta$$

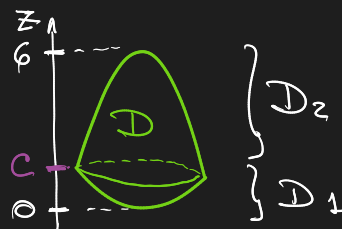
$$= \frac{1}{2} \int_{\theta=0}^{2\pi} 9 \cdot 2^2 - 2^2 \cdot 2^3 + \frac{3}{8} \cdot 2^4 \, d\theta$$

$$= \frac{2\pi}{2} \cdot 10$$

$$= 10\pi //$$

Plan: usar cilíndricas dividiendo D en D_1 y D_2

$$\begin{aligned} z &= 6 - 2(x^2 + y^2) \\ z &= 6 - 2r^2 \\ z &= z = c \end{aligned} \quad \downarrow \text{I or II}$$



$$\begin{cases} x = r \cdot \cos \theta \\ y = r \cdot \sin \theta \\ z = z \end{cases}$$

$$\theta \in [0, 2\pi)$$

$$r \in [0, \phi(z)]$$

$$z \in [0, 6]$$

Calculo $\phi(z)$ para D_1 y D_2

• D_1

$$\phi_1(z) = \sqrt{x^2 + y^2}$$

← despegó de eq $z = x^2 + y^2$ $\nearrow r^2$

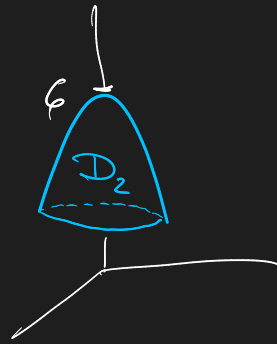
• D_2

$$z = 6 - 2x^2 - 2y^2$$

$$z - 6 = -2(x^2 + y^2)$$

$$3 - \frac{z}{2} = x^2 + y^2$$

$$\phi_2(z) = \sqrt{3 - \frac{z}{2}}$$



$$\iiint_D \operatorname{div} F \, dv = \iiint_{D'} \operatorname{div} F \cdot r \cdot dr d\theta dz$$

Calculo sobre D_2 :

$$\iiint_{D_2} \operatorname{div} F \, dv = \iiint_{D_2'} \operatorname{div} F \cdot r \cdot dr d\theta dz$$

$$= \int_{z=2}^6 \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{3-\frac{z}{2}}} z \cdot \overbrace{r^2}^{x^2+y^2} \cdot r \, dr d\theta dz$$

$$= \int_{z=2}^6 z \cdot \int_{\theta=0}^{2\pi} \left. \frac{r^4}{4} \right|_0^{\sqrt{3-\frac{z}{2}}} d\theta dz$$

$$= \int_{z=2}^6 z \cdot \int_{\theta=0}^{2\pi} \frac{1}{4} \left(3 - \frac{z}{2} \right)^2 d\theta dz$$

CA

$$\left(3 - \frac{z}{2}\right) = 9 - 3z + z^2$$

$$= \frac{1}{4} \cdot 2\pi \cdot \int_{z=2}^6 (9z - 3z^2 + z^3) dz$$

$$= \frac{\pi}{2} \cdot \left[\frac{9}{2} z^2 - \frac{3}{3} z^3 + \frac{1}{4} z^4 \right]_2^6$$

$$= \frac{\pi}{2} \cdot \left(\frac{9}{2} \cdot 6^2 - 6^3 + \frac{6^4}{4} - \frac{9}{2} \cdot 2^2 + 2^3 - \frac{2^4}{4} \right)$$

$$= \pi \cdot \left(135 - \underbrace{9 + 4 - 2}_{=7} \right)$$

$$= 128 \pi$$

Calc sobre D_2 :

$$\iiint_{D_2} \operatorname{div} F \, dv = \iiint_{D_2'} \operatorname{div} F \cdot r \cdot dr d\theta dz$$

$$= \int_{z=2}^6 \int_{\theta=0}^{2\pi} \int$$