Análisis I - Análisis Matemático I - Matemática I - Análisis II (C)

1er. cuatrimestre 2020

Simulacro del Primer Parcial - 01/06/2020

Justifique todas sus respuestas.

Entregue todas las hojas escaneadas y en orden.

1. Considerar la superficie S de $\mathbb{R}^3$ definida por la ecuación

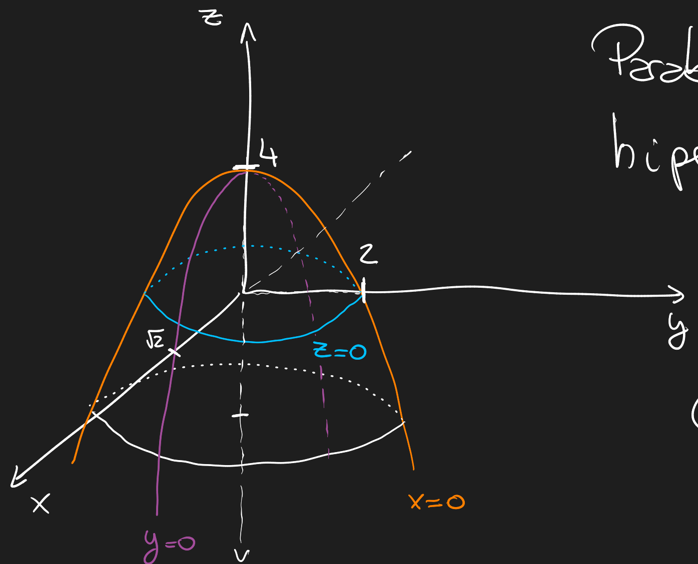
$$z = 4 - x^2 - y^2.$$

(a) Hacer esquemas de las trazas (horizontales y verticales) de S y utilizarlos para hacer un gráfico aproximado. Describir geométricamente la superficie.

(b) Hallar la curva intersección de S con el plano $z = 2$ y dar una función $r\colon \mathbb{R} \to \mathbb{R}^3$ cuya imagen describa dicha curva.

(c) Hallar la ecuación de la recta tangente a la curva descripta por r en el punto $P = (\sqrt{2}, 0, 2)$. $(\sqrt{2}, 0, \sqrt{2})$

a)

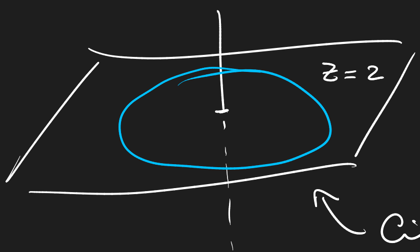


Paraboloide hiperbólico

Curvas correspondientes a:

- $S \cap x=0$
- $S \cap y=0$
- $S \cap z=0$

b)



circunferencia centrada en $(x,y)=(0,0)$

Busco radio:

$$2 = 4 - x^2 - y^2$$

$$-2 = -x^2 - y^2$$

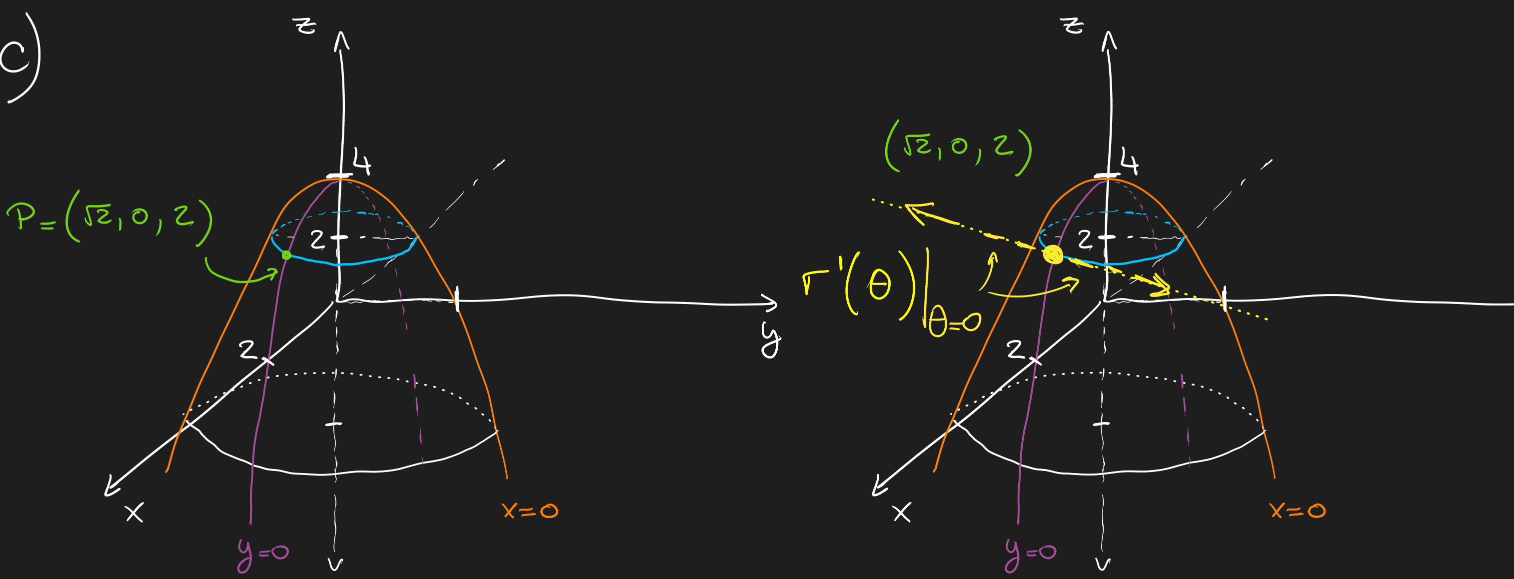
$$x^2 + y^2 = \left(\sqrt{2}\right)^2$$

Parametrizo Circ. de radio $\sqrt{2}$

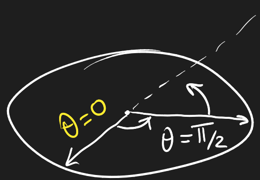
$$\boxed{\begin{array}{l} r : \mathbb{R} \longrightarrow \mathbb{R}^3 \\ r(\theta) = \left(\sqrt{2}\cdot\cos\theta \,,\, \sqrt{2}\cdot\sin\theta \,,\, 2 \right) \end{array}}$$

$$\underbrace{\sqrt{2}\cdot\cos\theta \,,\, \sqrt{2}\cdot\sin\theta}_{\text{Circ. de radio } = \sqrt{2}} \,,\, \underbrace{2}_{\text{a altura } z = 2.}$$

c)

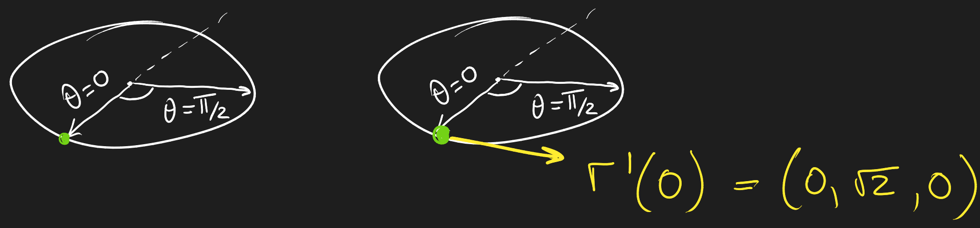


$$r'(\theta) = \left(-\sqrt{2}\cdot\sin\theta \,,\, \sqrt{2}\cdot\cos\theta \,,\, 0 \right)$$



Veo $\theta = \frac{\pi}{2}$

$$r'(0) = \left(0 \,,\, \sqrt{2} \,,\, 0 \right)$$



Armo recta tangente con vector $r'\left(\frac{\pi}{2}\right)$:

$$\boxed{(x, y, z) = t\,(0, \sqrt{2}, 0) + (\sqrt{2}, 0, 2)}$$

$\forall t \in \mathbb{R}$

2. Sea

$$f(x,y) = \frac{y^3 \sin\left(\frac{1}{x^2+y^2}\right)}{x^2+y^2}.$$

(a) Hallar el dominio de f.

(b) Determinar si se puede definir f de forma continua en el punto $(0,0)$.

a) $\text{Dom}(f) = \mathbb{R} \times \mathbb{R} \setminus \{(0,0)\}$

b) Primero veo $\lim f(x,y)$

$$\left| \frac{y^3 \cdot \overbrace{\sin\left(\frac{1}{x^2+y^2}\right)}^{\leq 1}}{x^2+y^2} - 0 \right| \leq \left| y^3 \cdot \left(\frac{1}{x^2+y^2}\right) \right| = \frac{|y|^3}{x^2+y^2}$$

$$\leq \frac{\|(x,y)\|^3}{\|(x,y)\|^2} = \|(x,y)\| < \delta$$

Encontré $\delta > 0 \,/\, \forall \varepsilon > 0$, si $\|(x,y)\| < \delta$

$$\Rightarrow \| f(x,y) - f(0,0) \| < \varepsilon$$

Basta tomar $\delta = \varepsilon$

$\therefore$ se puede definir f de manera continua:

$$f(x,y) = \begin{cases} \dfrac{y^3 \cdot \sin\left(\frac{1}{x^2+y^2}\right)}{x^2+y^2} & \text{si } (x,y) \neq (0,0) \\ 0 & \text{si } (x,y) = (0,0) \end{cases}$$

3. Sea $f\colon \mathbb{R}^2 \to \mathbb{R}$ dada por

$$f(x,y) = \begin{cases} \dfrac{xy\ \sin(xy)}{x^2+y^2} & \text{si} \quad (x,y) \neq (0,0) \\ 0 & \text{si} \quad (x,y) = (0,0) \end{cases},$$

Analizar la diferenciabilidad de f en cada punto de $\mathbb{R}^2$.

- Si $(x,y) \neq (0,0)$

$\Rightarrow f(x,y)$ es dif. pues prod., compos., cociente, y suma de funciones diferenciables.

- Si $(x,y) = (0,0)$

Derivadas parciales

$$f_x(0,0) = \lim_{h\to 0} \frac{\overbrace{f(h,0)}^{=0} - \overbrace{f(0,0)}^{=0}}{\underset{\to 0}{h}} = 0 \ ✓$$

$$f_y(0,0) = \lim_{h\to 0} \frac{\overbrace{f(0,h)}^{=0} - \overbrace{f(0,0)}^{=0}}{\underset{\to 0}{h}} = 0 \ ✓$$

$$\lim_{(x,y)\to(0,0)} \frac{f(x,y) - \overbrace{f(0,0)}^{=0} - \overbrace{f_x(0,0)}^{=0}(x-0) - \overbrace{f_y(0,0)}^{=0}(y-0)}{\|(x,y)\|} =$$

$$= \lim \frac{f(x,y)}{\|(x,y)\|}$$

$$\left| \underbrace{\frac{x \cdot y \cdot \sin(x \cdot y)}{x^2+y^2}}_{\leq \|\cdot\|^2} \cdot \frac{1}{\|(x,y)\|} - 0 \right| \leq \left| \frac{x \cdot y \cdot x \cdot y}{x^2+y^2} \right| \cdot \frac{1}{\|\cdot\|}$$

$$= \frac{|x|^2 \cdot |y|^2}{x^2+y^2} \cdot \frac{1}{\|\cdot\|}$$

$$\leq \frac{\|(x,y)\|^4}{\|(x,y)\|^2} \cdot \frac{1}{\|(x,y)\|} < \delta$$

$$\Rightarrow \lim \frac{f(x,y)}{\|(x,y)\|} = 0$$

$\therefore$ f es diferenciable en $(x,y) = (0,0)$

$\Rightarrow$ f es diferenciable.

4. Sean $f\colon \mathbb{R}^2 \to \mathbb{R}$ definida por $f(x,y) = x^2 - xy$ y $g\colon \mathbb{R}^2 \to \mathbb{R}^2$ diferenciable tal que $g(s,t) = (x(s,t), y(s,t))$, $g(1,2) = (1,1)$,

$$\frac{\partial x}{\partial s}(1,2) = 5, \qquad \frac{\partial x}{\partial t}(1,2) = 2$$

y

$$\frac{\partial y}{\partial s}(1,2) = -1, \qquad \frac{\partial y}{\partial t}(1,2) = 3.$$

Sea $h\colon \mathbb{R}^2 \to \mathbb{R}$, $h = f \circ g$.

(a) Hallar $\dfrac{\partial h}{\partial s}(1,2)$ y $\dfrac{\partial h}{\partial t}(1,2)$.

(b) Hallar $\dfrac{\partial h}{\partial v}(1,2)$ para $v = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

a) $$\frac{\partial h}{\partial s} = \underbrace{\frac{\partial f}{\partial x}(g(s,t))}_{(2x-y)} \cdot \underbrace{\frac{\partial x}{\partial s}}_{5} + \underbrace{\frac{\partial f}{\partial y}(g(s,t))}_{(-x)} \cdot \underbrace{\frac{\partial y}{\partial s}}_{(-1)}$$

$\{ g(1,2) = (1,1)$

$$\frac{\partial h}{\partial s} = 10x + x - 5y$$

$$= 11x - 5y$$

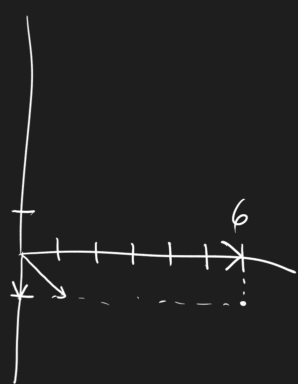
$$\left.\frac{\partial h}{\partial s}\right|_{(1,2)} = 11 - 5 = 6$$

$$\frac{\partial h}{\partial t} = \underbrace{\frac{\partial f}{\partial x}}_{(2x-y)} \cdot \underbrace{\frac{\partial x}{\partial t}}_{2} + \underbrace{\frac{\partial f}{\partial y}}_{(-x)} \cdot \underbrace{\frac{\partial y}{\partial t}}_{3}$$

$$= 4x - 3x - 2y$$

$$= x - 2y$$

$$\left.\frac{\partial h}{\partial t}\right|_{(1,2)} = 1 - 2 = -1$$



b) $v = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$\frac{\partial h}{\partial v} = \frac{\partial f}{\partial x}(g(s,t)) \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}(g(s,t)) \cdot \frac{\partial y}{\partial v}$$

- $$\frac{\partial x}{\partial v} = \underbrace{\frac{\partial x}{\partial s}}_{5} \cdot \frac{1}{\sqrt{2}} + \underbrace{\frac{\partial x}{\partial t}}_{2} \cdot \frac{1}{\sqrt{2}}$$

$$\frac{\partial x}{\partial v} = \frac{7}{\sqrt{2}}$$

- $$\frac{\partial y}{\partial v} = \underbrace{\frac{\partial y}{\partial s}}_{-1} \cdot \frac{1}{\sqrt{2}} + \underbrace{\frac{\partial y}{\partial t}}_{3} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}$$

Volviendo

$$\frac{\partial h}{\partial v} = \left\langle \nabla h(1,2), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \right\rangle$$

$$\frac{\partial h}{\partial v} = \frac{\partial f}{\partial x}(g(s,t)) \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}(g(s,t)) \cdot \frac{\partial y}{\partial v}$$

$$= (2x - y) \cdot \frac{7}{\sqrt{2}} + (-x) \cdot \frac{2}{\sqrt{2}}$$

$$= \frac{14}{\sqrt{2}} \cdot x - \frac{2}{\sqrt{2}} x - \frac{7}{\sqrt{2}} y$$

$$\left.\frac{\partial h}{\partial v}\right|_{(1,2)} = \frac{12}{\sqrt{2}} - \frac{7}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$\left.\frac{\partial h}{\partial r}\right|_{(1,2)} = \frac{5}{\sqrt{2}}$$

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