Análisis I - Análisis Matemático I - Matemática I - Análisis II (C)
Examen Final - 13/05/2021

1. Calcular el límite
$$\lim_{(x,y)\to(0,0)} \frac{e^{2x}\cos(y) - 1 - 2x - x^2 + \frac{3}{2}y^2}{x^2+y^2}.$$

Si $x=0$:

$$\lim_{y\to 0} \frac{\overset{\longrightarrow 1}{\cos y} - 1 + \frac{3}{2}y^2}{y^2} \overset{L'H}{=} \lim_{y\to 0} \frac{-\sin y + 3y}{2y}$$

$$\overset{L'H}{=} \lim_{y\to 0} \frac{-\overset{1}{\cos y} + 3}{2} = 1$$

Si $y=0$:

$$\lim_{x\to 0} \frac{e^{2x}.1 - 1 - 2x - x^2}{x^2} \overset{L'H}{=}$$

$$= \lim_{x\to 0} \frac{2.e^{2x} - 2 - 2x}{2x}$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{4e^{2x} - 2}{2} = 1 \quad \checkmark$$

- Candidato a límite : 1

Acoto

$$\left| \frac{e^{2x} \cdot \cos y - 1 - 2x - x^2 + \frac{3}{2} y^2 - x^2 - y^2}{x^2 + y^2} \right| =$$

$$= \left| \frac{e^{2x} \cdot \cos y - 1 - 2x - 2x^2 + \frac{1}{2} y^2}{x^2 + y^2} \right|$$

Sospecho que <u>no</u> existe

Si $y = x$

$$\lim_{x \to 0} \frac{e^{2x} \cdot \cos x - 1 - 2x - x^2 + \frac{3}{2} x^2}{2x^2} =$$

$$= \lim_{x \to 0} \frac{e^{2x} \cdot \cos x - 1 - 2x + \frac{1}{2} x^2}{2x^2}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{2e^{2x} \cdot \overset{\to 1}{\cos x} - e^{2x} \cdot \overset{\to 0}{\sin x} - 2 + x}{4x}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{\overbrace{4 \cdot e^{2x} \cdot \overset{\to 1}{\cos x}}^{\to 4} - 2e^{2x} \cdot \overset{\to 0}{\sin x} - 2e^{2x} \cdot \overset{\to 0}{\sin x} \overbrace{- e^{2x} \cdot \overset{\to 1}{\cos x}}^{\to -1} + 1}{4}$$

$= 1$ ¡¡

Si $y = \frac{x}{2}$

$$\lim_{x\to 0} \frac{e^{2x}\cdot \cos\frac{x}{2} - 1 - 2x - x^2 + \overbrace{\frac{3}{2}\,\frac{x^2}{4}}^{\frac{3}{8}x^2}}{x^2 + \frac{x^2}{4}} =$$

$$= \lim_{x\to 0} \frac{\overset{\longrightarrow 1}{e^{2x}}\cdot \overset{\longrightarrow 1}{\cos\frac{x}{2}} - 1 - 2x - \frac{5}{8}x^2}{\frac{5}{4}x^2}$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{2\,\overset{\longrightarrow 1}{e^{2x}}\cdot \overset{\longrightarrow 1}{\cos\frac{x}{2}} - \frac{1}{2}\overset{\longrightarrow 0}{e^{2x}\cdot \sin\frac{x}{2}} - 2 - \frac{5}{4}x}{\underset{\longrightarrow 0}{\frac{5}{2}x}} \overset{L'H}{=} 1$$

$$-\left(1 + 2x + x^2\right) = -(x+1)^2$$

$$\frac{e^{2x}\cdot \cos(y)}{x^2+y^2} - (x+1)^2 + \frac{3}{2}y^2$$

Sale con $y = m \cdot x$ (ó $y = 2x$)

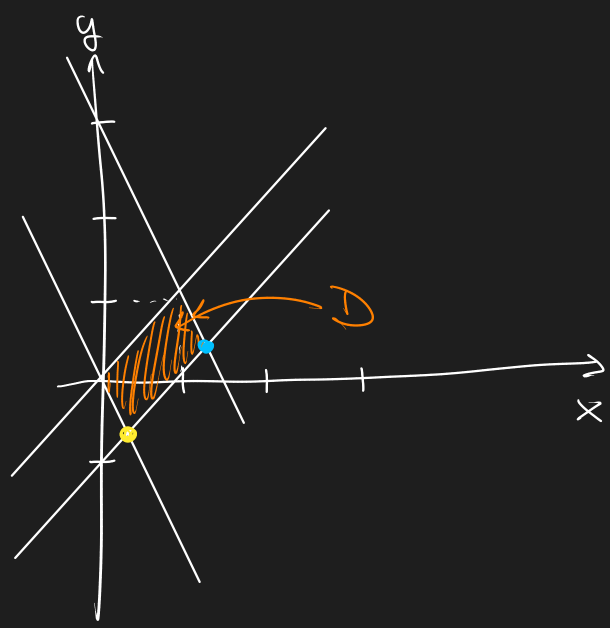
2. Calcular la integral

$$\iint_D e^{2x+y}\,(x-y)\,dxdy,$$

donde D es el paralelogramo limitado por las rectas $2x+y=0$, $2x+y=3$, $x-y=0$, $x-y=1$.

D :

- $y = -2x$
- $y = -2x + 3$
- $y = x$
- $y = x - 1$



Busco

$$-2x + 3 = x - 1$$
$$-3x = -4$$
$$x = \frac{4}{3} \Rightarrow y = \frac{1}{3}$$

$$-2x = x - 1$$
$$-3x = -1$$
$$x = \frac{1}{3} \Rightarrow y = -\frac{2}{3}$$

$$D = \left\{ (x,y) : 0 \leq x \leq \frac{1}{3},\ -2x \leq y \leq x \right\} \cup$$
$$\left\{ (x,y) : 0 < x \leq 1,\ x-1 \leq y \leq x \right\} \cup$$
$$\left\{ (x,y) : 1 < x \leq \frac{4}{3},\ x-1 \leq y \leq -2x+3 \right\}$$

$$\int_0^{1/3}\int_{-2x}^{x} e^{2x+y}\cdot(x-y)\,dx\,dy = \int_0^{1/3}\int_{-2x}^{x} e^{2x+y}\cdot x - e^{2x+y}\cdot y$$

Usar Transformación que rote el paralelogramo.

3. Sea $f\colon \mathbb{R}^2 \to \mathbb{R}$ tal que $f_x(1,2) = 0$, $f_y(1,2) = 1$ y $f(1+t, 2+t) = 3t - t^2$ para todo $t \in \mathbb{R}$. Probar que f no es diferenciable en (1, 2).

$$f(x(t), y(t)) = 3t - t^2$$

$$\begin{cases} x(t) = 1+t \\ y(t) = 2+t \end{cases}$$

Si $t = 0 \Rightarrow f(1,2) = 0$

Tengo el plano tg a f en $(1,2)$

$$\Pi: \underbrace{f(1,2)}_{0} + \underbrace{f_x(1,2)}_{0} \cdot (x-1) + \underbrace{f_y(1,2)}_{1} \cdot (y-2)$$

$$\Pi: z = y - 2$$

qvq

$$\lim_{(x,y)\to(1,2)} \frac{f(x,y) - (y-2)}{\|(x,y)-(1,2)\|} \neq 0$$

Reescribo

$$\lim_{t\to 0} \frac{f(1+t, 2+t) - (2+t-2)}{\|(1+t-1, 2+t-2)\|} = \lim_{t\to 0} \frac{3t - t^2 - t}{\|(t,t)\|}$$

$$= \lim_{t\to 0} \frac{2t - t^2}{\sqrt{2t^2}}$$

$$= \lim_{t\to 0} \frac{t(2-t)}{\sqrt{2}\cdot|t|} = \begin{cases} t\to 0^{+} & \frac{2}{\sqrt{2}} \\ t\to 0^{-} & -\frac{2}{\sqrt{2}} \end{cases} \neq \Rightarrow \nexists \lim$$

$\therefore$ f no es dif. en $(1,2)$.

∎

4. Sea $f\colon \mathbb{R}^2 \to \mathbb{R}$ de clase C^1 tal que el plano tangente al gráfico de f en $(1,0,f(1,0))$ es

$$2z - 8x + 3y = 2. \quad \Rightarrow \quad z = 1 + 4x - \frac{3y}{2}$$

Sea $g\colon \mathbb{R}^2 \to \mathbb{R}^2$ definida por $g(u,v) = (e^{3u+v}, \sin(2u+6v))$. Hallar $\nabla(f \circ g)(0,0)$.

$$f(g(u,v)) = f(\underbrace{e^{3u+v}}_{x(u,v)}, \underbrace{\sin(2u+6v)}_{y(u,v)})$$

$$\nabla f(g(u,v)) = \left(① \quad ② \right)$$

①

$$\frac{\partial}{\partial u} f(g(u,v)) = \underbrace{\frac{\partial f}{\partial x}(1,0)}_{4} \cdot \underbrace{\frac{\partial x}{\partial u}(0,0)}_{3 \cdot e^0 = 3} + \underbrace{\frac{\partial f}{\partial y}(1,0)}_{-\frac{3}{2}} \cdot \underbrace{\frac{\partial y}{\partial u}(0,0)}_{\cos(0)\cdot(2) = 2} =$$

$$= 12 - 3 = 9$$

②

$$\frac{\partial}{\partial v} f(g(u,v)) = \underbrace{\frac{\partial f}{\partial x}(1,0)}_{4} \cdot \underbrace{\frac{\partial x}{\partial v}(0,0)}_{1} + \underbrace{\frac{\partial f}{\partial y}(1,0)}_{-\frac{3}{2}} \cdot \underbrace{\frac{\partial y}{\partial v}(0,0)}_{6}$$

$$= 4 - 9 = -5$$

$\therefore$

$$\nabla f \circ g(0,0) = (9, -5)$$