
Análisis I - Análisis Matemático I - Matemática I - Análisis II (C)

1er. cuatrimestre 2020

Simulacro Segundo Parcial

1. Sea $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ de clase \mathcal{C}^2 tal que su polinomio de Taylor de orden 2 en $(-1, 1)$ es

$$p(x, y) = 2x^2 - xy + 5x - y + 5.$$

(a) Decidir si f tiene un extremo local en $(-1, 1)$.

(b) Calcular

$$\lim_{(x,y) \rightarrow (-1,1)} \frac{f(x,y) - 2}{\|(x,y) - (-1,1)\|}$$

a) Calcular $\nabla p(x,y) = \left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y} \right)$

$$\bullet \frac{\partial p}{\partial x}(x,y) = 4x - y + 5$$

$$\bullet \frac{\partial p}{\partial y}(x,y) = -x - 1$$

$$\frac{\partial p}{\partial x}(-1,1) = \frac{\partial f}{\partial x}(-1,1) = -4 - 1 + 5 = 0$$

$$\frac{\partial p}{\partial y}(-1,1) = \frac{\partial f}{\partial y}(-1,1) = 0$$

$$\nabla f(-1,1) = (0,0)$$

$\therefore (-1,1)$ es punto crítico.

- $P_{xx}(x,y) = 4$
- $P_{yy}(x,y) = 0$
- $P_{xy}(x,y) = P_{yx}(x,y) = -1$

El Hessiano de f en $(-1,1)$ es

$$Hf(-1,1) = \begin{bmatrix} 4 & -1 \\ -1 & 0 \end{bmatrix}$$

Criterio del Hessiano

$$\left. \begin{array}{l} 1) \det Hf(x_0, y_0) > 0 \\ f_{xx}(x_0, y_0) > 0 \end{array} \right\} \Rightarrow (x_0, y_0) \text{ es m\u00ednimo}$$

$$\left. \begin{array}{l} 2) \det Hf(x_0, y_0) > 0 \\ f_{xx}(x_0, y_0) < 0 \end{array} \right\} \Rightarrow (x_0, y_0) \text{ es m\u00e1ximo}$$

$$3) \det Hf(x_0, y_0) < 0 \Rightarrow (x_0, y_0) \text{ es punto silla}$$

$$4) \det Hf(x_0, y_0) = 0 \Rightarrow \text{el criterio } \underline{\text{no}} \text{ me sirve,}$$

Entonces

$$Hf(-1,1) = \begin{bmatrix} 4 & -1 \\ -1 & 0 \end{bmatrix}$$

$\det Hf(-1,1) = -1 \Rightarrow (-1,1)$ es punto silla

$\therefore f$ no tiene extremo local en $(-1,1)$.

(b) Calcular

$$\lim_{(x,y) \rightarrow (-1,1)} \frac{f(x,y) - 2}{\|(x,y) - (-1,1)\|}$$

Uso Prop. de P. de Taylor Polinomio de Taylor de grado n de f en (x_0, y_0)

$$\lim_{(x,y) \rightarrow (x_0, y_0)} \frac{f(x,y) - P_n(x,y)}{\|(x,y) - (x_0, y_0)\|^n} = 0$$

Calculo polin. de Taylor de orden 1 de f
a partir de $P(x,y)$

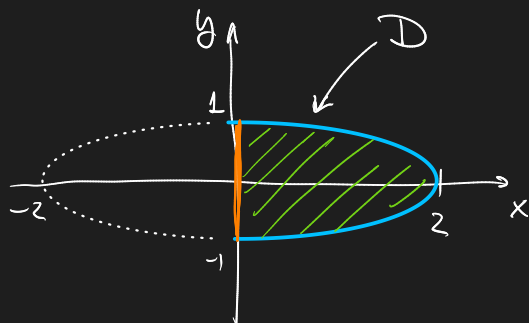
$$P_1(x,y) = \overbrace{P(-1,1)}^{=2} + \overbrace{P_x(-1,1)}^{=0}(x+1) + \overbrace{P_y(-1,1)}^{=0}(y-1)$$

$$P_1(x,y) = 2$$

Por prop., el límite es cero.

2. Sea $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ definida por $f(x, y) = xy^2 + 2y^2 + 1$. Hallar los máximos y mínimos absolutos de f en

$$D = \left\{ (x, y) \mid \frac{x^2}{4} + y^2 \leq 1, x \leq 0 \right\}.$$



- Ver el Interior $\overset{\circ}{D}$:

Calculo Hessiano de f :

$$f_x = y^2$$

$$f_y = 2xy + 4y$$

$$\Rightarrow \nabla f(x, y) = (y^2, 2xy + 4y) \stackrel{\text{quiero}}{\underset{=}{\downarrow}} (0, 0)$$

$$\Leftrightarrow \begin{cases} y^2 = 0 \\ 2xy + 4y = 0 \end{cases} \Rightarrow \text{vale } \forall x$$

$$PC_{\Sigma} = \{ (x, 0) : x \in [0, 2] \}$$

- $f_{xx} = 0$
- $f_{yy} = 2x + 4$
- $f_{xy} = f_{yx} = 2y$

$$Hf(x, y) = \begin{bmatrix} 0 & 2y \\ 2y & 2x + 4 \end{bmatrix}$$

$$Hf(x,0) = \begin{bmatrix} 0 & 0 \\ 0 & 2x+4 \end{bmatrix}$$

el criterio no me dice nada! por $\det Hf(x,0) = 0$

Peró no importa, por

$f(x,0) = 1$, como candidato a max/min.

• Veo el Bordo ∂D

1) segmento $(0,y)$ con $y \in [-1,1]$

Lo parametrizo como

$$\sigma(t) = (0, t) \quad \text{con } t \in [-1,1]$$

$$f(x,y) = x^2 + 2y^2 + 1$$

$$\begin{aligned} f(\sigma(t)) &= f(0, t) \\ &= 2t^2 + 1 \end{aligned}$$

$$f'(\sigma(t)) = 4t = 0 \Leftrightarrow t = 0$$

$$\sigma(0) = (0,0)$$

PC en $(0,0)$,

2) Ellipse :

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y) \end{cases}$$

$$\text{con } \underbrace{\frac{x^2}{4} + y^2 = 1}_{g(x,y)}$$

$$\textcircled{I} \quad \begin{cases} y^2 = \lambda \cdot \frac{x}{2} \end{cases}$$

$$\textcircled{II} \quad \begin{cases} 2xy + 4y = \lambda \cdot 2y \Rightarrow 2xy + 2y(2 - \lambda) = 0 \end{cases}$$

$$\textcircled{III} \quad \begin{cases} \frac{x^2}{4} + y^2 = 1 \end{cases}$$

$$2y(x + 2 - \lambda) = 0$$

$$\hookrightarrow y = 0$$

$$\hookrightarrow x = \lambda - 2$$

• Si $y = 0$

$$0 = \lambda \cdot \frac{x}{2} \quad \begin{matrix} \nearrow x=0 \\ \searrow \lambda=0 \end{matrix} \quad \text{X Abs por } \textcircled{III}$$

$$\frac{x^2}{4} = 1$$

$$x^2 = 2^2$$

$$x = \begin{matrix} \nearrow 2 \\ \searrow -2 \end{matrix}$$

$$PC_S = \{(2, 0), (-2, 0)\} \text{ con } \lambda = 0$$

Si $\lambda = x + 2$ en \textcircled{I} :

$$y^2 = (x+2) \frac{x}{2}$$

$$y^2 = x^2 + x$$

$$y^2 = x(x+1)$$

en \textcircled{III}

$$\frac{x^2}{4} + x^2 + x = 1 \Rightarrow \frac{5}{4}x^2 + x - 1 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 + 4 \cdot \frac{5}{4}}}{2 \cdot \frac{5}{4}} =$$

$$= \frac{-1 \pm \sqrt{6}}{\frac{5}{2}}$$

$$= \frac{2(-1 \pm \sqrt{6})}{5}$$

$$= \frac{2}{5}(-1 \pm \sqrt{6})$$

$$x = \begin{cases} \frac{2}{5}(\sqrt{6} - 1) \\ \frac{2}{5}(-\sqrt{6} - 1) \end{cases}$$

Met x en III

$$\frac{1}{4} \left(\frac{4}{25} \cdot (6 - 2\sqrt{6} + 1) \right)$$

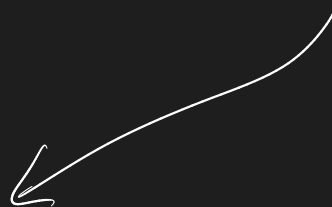
$$\frac{x^2}{4} + y^2 = 1$$

$$\boxed{x_1} \quad y^2 = 1 - \frac{x^2}{4}$$

$$= \left(1 + \frac{x}{2}\right) \left(1 - \frac{x}{2}\right)$$

$$= \left(1 + \frac{1}{5}(\sqrt{6} - 1)\right) \left(1 - \frac{1}{5}(\sqrt{6} - 1)\right)$$

Absen domo, rigo der de enter



$$\textcircled{\text{III}} \quad \frac{x^2}{4} + y^2 = 1$$

$$x = \lambda - 2 \Rightarrow (\lambda - 2)^2 = \lambda^2 - 4\lambda + 4$$

$$\frac{\lambda^2}{4} - \lambda + \cancel{1} + y^2 = \cancel{1}$$

$$y^2 = \lambda - \frac{\lambda^2}{4}$$

$$y^2 = \lambda \left(1 - \frac{\lambda}{4}\right)$$

$$\textcircled{\text{I}} \quad y^2 = \lambda \cdot \frac{x}{2}$$

\swarrow $\lambda - 2$

$$\lambda - \frac{\lambda^2}{4} = \lambda \cdot \frac{(\lambda - 2)}{2}$$

$$\lambda - \frac{\lambda^2}{4} = \frac{\lambda^2}{2} - \lambda$$

$$0 = \frac{3}{4}\lambda^2 - 2\lambda$$

$$0 = \lambda \left(\frac{3}{4}\lambda - 2 \right)$$

$$\hookrightarrow \lambda = 0$$

$$\hookrightarrow \frac{3}{4}\lambda = 2$$

$$\hookrightarrow \lambda = \frac{8}{3}$$

$$\Rightarrow \begin{cases} x = \lambda - 2 \stackrel{\lambda=0}{=} -2 & \checkmark \text{ caso exterior} \\ y^2 = \lambda - \frac{\lambda^2}{4} = 0 & \checkmark \text{ anterior} \end{cases}$$

$$\hookrightarrow x = \frac{8}{3} - 2 = \frac{2}{3}$$

$$y^2 = \frac{8}{9} \Rightarrow y = \pm \sqrt{\frac{8}{9}} = \pm \frac{\sqrt{2}}{3}$$

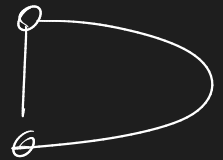
de enter $PC_5 = \{ (2, 0), (-2, 0) \}$ con $\lambda = 0$

ahora

$$PC_5 = \left\{ \left(\frac{2}{3}, \frac{4}{3} \right), \left(\frac{2}{3}, -\frac{4}{3} \right) \right\}$$

• Ver esquinas

$$PC_5 = \{ (0, -1), (0, 1) \}$$



Evaluo todo

$$f(x, 0) = 1 \quad \text{Min abs: } (x, 0) \text{ con } x \in [0, 2]$$

$$f(0, 0) = 1$$

$$f(2, 0) = 1$$

$$f(-2, 0) = 1$$

$$f\left(\frac{2}{3}, \frac{4}{3}\right) = \frac{155}{27} \approx 5,74$$

$$f\left(\frac{2}{3}, -\frac{4}{3}\right) = \frac{155}{27}$$

Maximos absolutos
 $\left(\frac{2}{3}, \frac{4}{3}\right)$ y $\left(\frac{2}{3}, -\frac{4}{3}\right)$

$$f(0, 1) = 3$$

$$f(0, -1) = 3$$

3. Calcular las siguientes integrales

(a) $\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{1}{1+x^4} dx dy.$

(b) $\iiint_E xz^2 dV$ donde E es el sólido debajo de la superficie $z = x^2$ y arriba del rectángulo $R = [0, 1] \times [2, 3]$ en el plano xy .

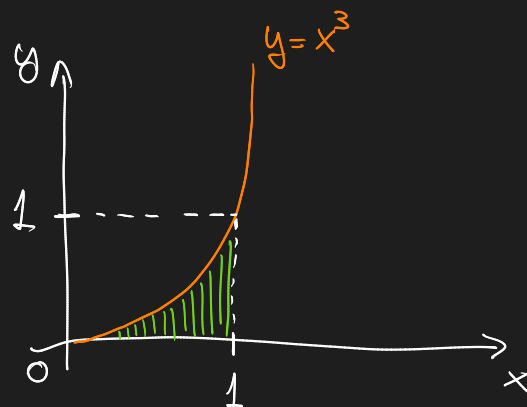
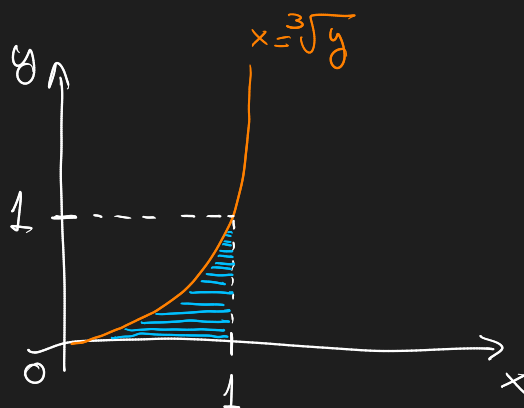
$$\int_{y=0}^{y=1} \underbrace{\int_{x=\sqrt[3]{y}}^{x=1}}_{x^3=y} \frac{1}{1+x^4} dx dy = \star$$

$$0 \leq y \leq 1$$

$$0 \leq \sqrt[3]{y} \leq x \leq 1$$

$$0 \leq y \leq x^3 \leq 1$$

$$\Rightarrow \text{ten go } \begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq x^3 \end{cases}$$



$$\star = \int_{x=0}^{x=1} \int_{y=0}^{y=x^3} \frac{1}{1+x^4} dy dx$$

$$= \int_{x=0}^{x=1} \frac{1}{1+x^4} \cdot x^3 \cdot dx$$

CA :

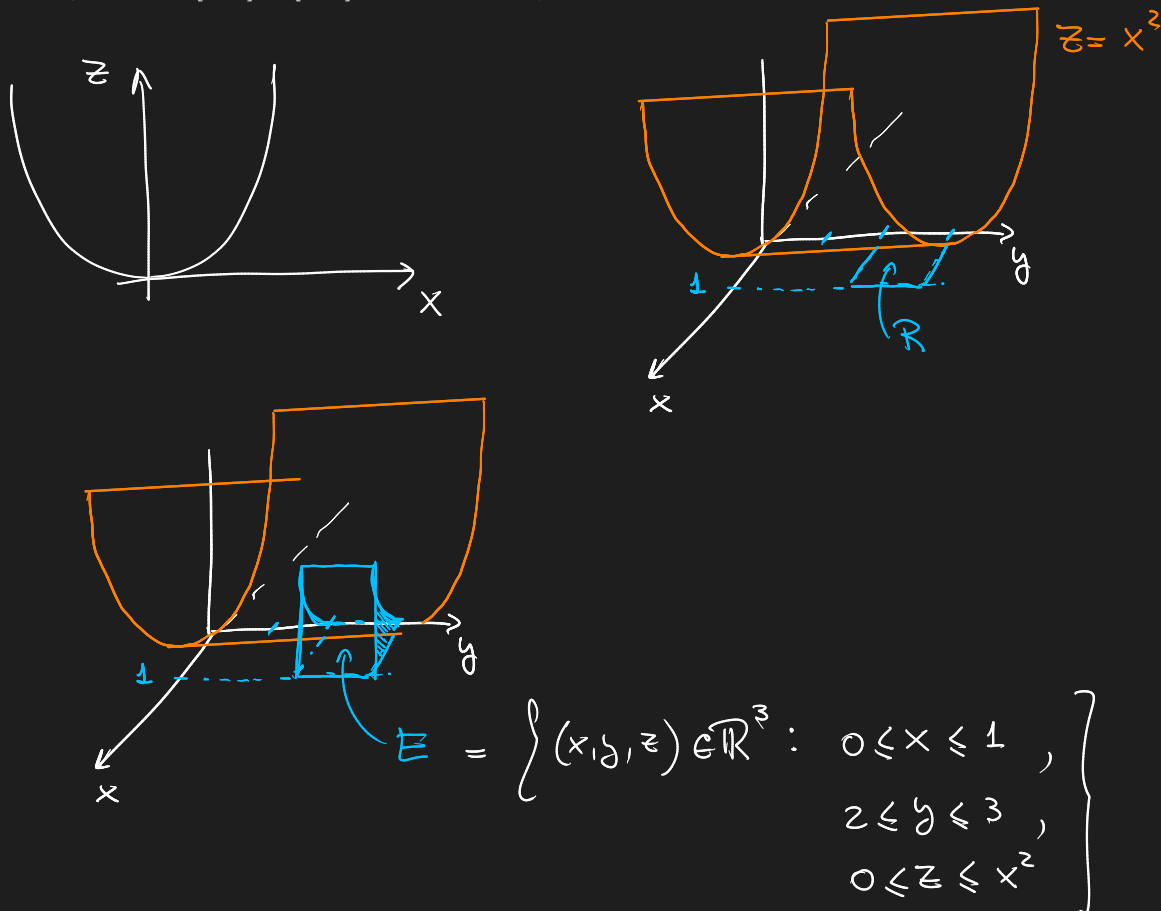
$$\left(\ln(1+x^4) \right)' = \frac{1}{1+x^4} \cdot 4x^3$$

$$\left(\frac{1}{4} \ln(1+x^4) \right)' = \frac{1}{\cancel{4}} \cdot \frac{1}{1+x^4} \cdot \cancel{4} x^3$$

$$= \frac{1}{4} \ln(1+x^4) \Big|_{x=0}^1$$

$$= \frac{1}{4} \ln 2 //$$

- (b) $\iiint_E xz^2 dV$ donde E es el sólido debajo de la superficie $z = x^2$ y arriba del rectángulo $R = [0, 1] \times [2, 3]$ en el plano xy .



$$\int_{x=0}^1 \int_{z=0}^{x^2} \int_{y=2}^3 xz^2 dy dz dx = \int_{x=0}^1 \int_{z=0}^{x^2} xz^2 \int_{y=2}^3 1 dy dz dx$$

$$= \int_{x=0}^1 x \int_{z=0}^{x^2} z^2 dz dx$$

$$= \int_{x=0}^1 x \cdot \left[\frac{z^3}{3} \right]_0^{x^2} dx$$

$$= \int_{x=0}^1 x \cdot \frac{x^6}{3} dx$$

$$= \frac{1}{3} \int_0^1 x^7 dx$$

$$= \frac{1}{3} \left[\frac{x^8}{8} \right]_0^1$$

$$= \frac{1}{24} //$$

4. Hallar el volumen del sólido acotado por las superficies

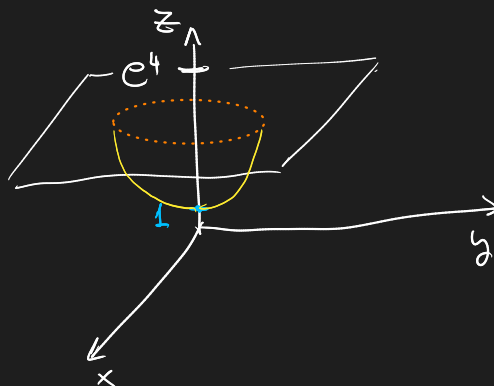
$$z = e^{4x^2+4y^2} \quad y \quad z = e^4.$$

$$z = e^{4(x^2+y^2)}$$

$$z = e^4$$

Si igualo

$$e^4 = e^{4(x^2+y^2)} \Leftrightarrow x^2 + y^2 = 1$$



Uso polares

$$\begin{cases} x = r \cdot \cos \theta \\ y = r \cdot \sin \theta \\ z = z \end{cases} \quad \begin{array}{l} \text{con } r \in [0, 1] \\ \theta \in [0, 2\pi) \end{array}$$

$$\begin{aligned} e^{4(x^2+y^2)} &\leq z \leq e^4 \\ \hookrightarrow e^{4r^2} &\leq z \leq e^4 \end{aligned}$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^1 \int_{z=e^{4r^2}}^{e^4} 1 \cdot r \cdot dz dr d\theta =$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^1 r \left(e^4 - e^{4r^2} \right) dr d\theta$$

$$= \int_{\theta=0}^{2\pi} \left. \frac{r^2}{2} e^4 \right|_0^1 d\theta - \int_{\theta=0}^{2\pi} \int_{r=0}^1 r \cdot e^{4r^2} dr d\theta$$

(I)

(II)

$$\textcircled{I} \int_{\theta=0}^{2\pi} \left. \frac{r^2}{2} e^4 \right|_0^1 d\theta = 2 \cdot \pi \cdot \frac{1}{2} e^4 \\ = \pi \cdot e^4$$

$$\textcircled{II} \int_{\theta=0}^{2\pi} \int_{r=0}^1 r \cdot e^{4r^2} dr d\theta =$$

CA:

$$\left(e^{4r^2} \right)' = e^{4r^2} \cdot 8r$$

$$\left(\frac{1}{8} e^{4r^2} \right)' = \frac{1}{8} \cdot e^{4r^2} \cdot \cancel{8} r$$

$$= \int_{\theta=0}^{2\pi} \left. \frac{1}{8} \cdot e^{4r^2} \right|_{r=0}^1 d\theta$$

$$= \int_0^{2\pi} \frac{1}{8} (e^4 - 1) d\theta$$

$$= \frac{\pi}{4} (e^4 - 1)$$

Finalmente

$$- \frac{\pi}{4} e^4 + \frac{\pi}{4}$$

$$\textcircled{I} - \textcircled{II} = \pi \cdot e^4 - \frac{\pi}{4} (e^4 - 1)$$

$$= \frac{3}{4}\pi \cdot e^4 + \frac{\pi}{4}$$

$$= \frac{\pi}{4} (3e^4 + 1) //$$

