Sustitución

$$\int_c^d f(\underbrace{g(x)}_{u}) \cdot \underbrace{g'(x)\,dx}_{du} = \int_{g(c)}^{g(d)} f(u)\,du$$

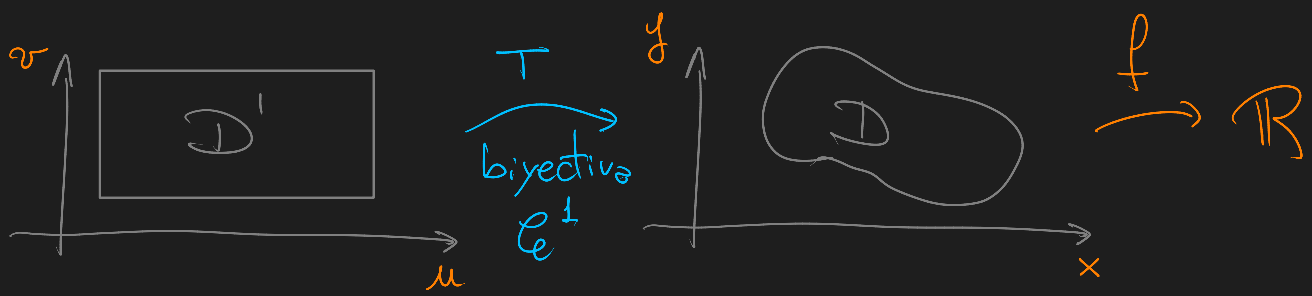
Con $g(x)$ /

$$[c,d] \xrightarrow{g} [g(c), g(d)]$$

ó

$$[g(d), g(c)]$$

Integrales dobles



$$T(u,v) = \big(x(u,v),\ y(u,v)\big)$$

$$\begin{cases} x = x(u,v) \\ y = y(u,v) \end{cases}$$

Teorema

$$\iint_D f(x,y)\cdot dA(x,y) = \iint_{D'} f\big(x(u,v),\, y(u,v)\big)\cdot \underbrace{\left| JT(u,v) \right|}_{\text{Jacobiano}} \cdot dA(u,v)$$

donde

$$JT(u,v) = \det \begin{vmatrix} \frac{\partial}{\partial u} x & \frac{\partial}{\partial v} x \\ \frac{\partial}{\partial u} y & \frac{\partial}{\partial v} y \end{vmatrix}$$

Notación

$$= \frac{\partial(x,y)}{\partial(u,v)}$$

que sirve para

$$dA(x,y) = \left| \frac{\partial(x,y)}{\partial(u,v)} \right| dA(u,v)$$

$$\text{“}dx\,dy\text{”} = \left| \frac{\partial(x,y)}{\partial(u,v)} \right| \text{“}du\,dv\text{”}$$

Coordenadas Polares

$$\begin{cases} x = r \cdot \cos\theta & r \geq 0 \\ y = r \cdot \sin\theta & \theta \in [0, 2\pi) \end{cases}$$

$$\text{Jacobiano} = \det \begin{vmatrix} \cos\theta & -r \cdot \sin\theta \\ \sin\theta & r \cdot \cos\theta \end{vmatrix}$$

$$= r \cdot \cos^2\theta + r \cdot \sin^2\theta$$

$$\text{Jacobiano} = r$$

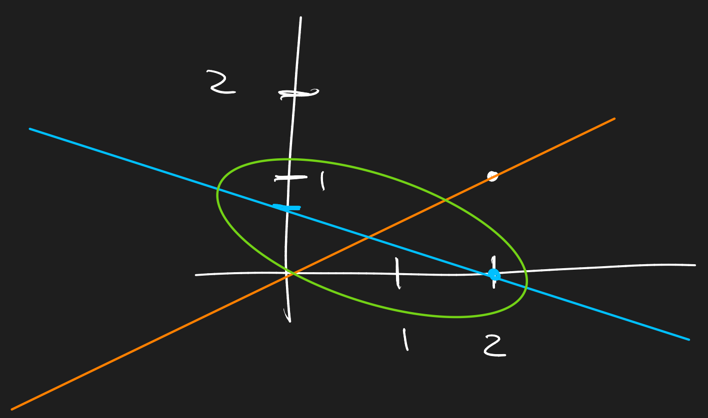
$$\Rightarrow \boxed{dA_{(x,y)} = r \cdot dA_{(r,\theta)}}$$

Ejemplo:

Calcular el área de

$$(x-2y)^2 + (3y+x-2)^2 = 1$$

Veo rectas



- $x - 2y = 0$
- $3y + x - 2 = 0$

$$y = \frac{2-x}{3}$$

$$\underbrace{(x-2y)}_{u}^2 + \underbrace{(3y+x-2)}_{v}^2 = 1$$

$$\begin{cases} u = x-2y \\ v = 3y+x-2 \end{cases}$$

$$x = u+2y \Rightarrow v = 3y+u+2y-2$$

$$v = 5y+u-2$$

$$y = (v-u+2)\cdot\frac{1}{5}$$

$$x = u + \frac{2v}{5} - \frac{2u}{5} + \frac{4}{5}$$

$$\begin{cases} x = (3u+2v+4)\cdot\frac{1}{5} \\ y = (v-u+2)\cdot\frac{1}{5} \end{cases}$$

Calculo Jacobiano $\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$

y hago cambio de variables.

$$\text{Área}(D) = \iint_{D'} 1\cdot\frac{1}{5}\cdot dA(u,v) = \frac{1}{5}\cdot\iint_{u^2+v^2\leq 1} 1\cdot dA(u,v)$$

D': $u^2+v^2=1$

$$= \frac{\pi}{5}$$