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Análisis I - Análisis Matemático I - Matemática I - Análisis II (C)

1er. cuatrimestre 2020

Segundo Recuperatorio del Segundo Parcial - 18/08/2020

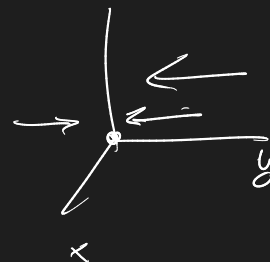
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1. Calcular el siguiente límite

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\cos(y)e^{2x} - 1 - 2x - 2x^2 + \frac{1}{2}y^2}{x^2 + y^2}.$$

$$\boxed{x=0}:$$

$$\lim_{y \rightarrow 0} \frac{\cos(y) - 1 + \frac{1}{2}y^2}{y^2}$$



$$\boxed{y=0}:$$

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - 2x - 2x^2}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{\overset{\rightarrow 1}{e^{2x}} - 1 - 2x - 2x^2}{\underset{\rightarrow 0}{x^2}} \xrightarrow{L'H} \lim_{x \rightarrow 0} \frac{2e^{2x} - 2 - 4x}{2x}$$

$$\xrightarrow{L'H} \lim_{x \rightarrow 0} \frac{4e^{2x} - 4}{2} = 0$$

$$\boxed{x=0}:$$

$$\lim_{y \rightarrow 0} \frac{\cos(y) - 1 + \frac{1}{2}y^2}{y^2} \xrightarrow{L'H} \lim_{y \rightarrow 0} \frac{-\sin(y) + y}{2y}$$

$$\xrightarrow{L'H} \lim_{y \rightarrow 0} \frac{-\cos(y) + 1}{2} = 0$$

Acoto :

$$\frac{\leq +2x^2}{\leq y^2 \leq 2y^2}$$

$$\left| \frac{\cos(y) \cdot e^{2x} - 1 - 2x - 2x^2 + \frac{1}{2}y^2}{x^2 + y^2} - 0 \right| \leq$$

$$\underbrace{\| (x,y) - (0,0) \|^2}_{(x,y) \rightarrow (0,0)} \rightarrow 0$$

Sospechoso

Si:  $y = 2x$

$$\Rightarrow -2x^2 + \frac{1}{2}y^2 = -2x^2 + 2x^2 = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\cos(2x) \cdot e^{2x} - 1 - 2x}{x^2 + 4x^2} =$$

$$\begin{array}{l} L'H \\ \downarrow \\ = \lim_{x \rightarrow 0} \frac{\overbrace{-2\sin(2x)}^{\rightarrow 0} \cdot \overbrace{e^{2x}}^{\rightarrow 1} + \overbrace{\cos(2x)}^{\rightarrow 1} \cdot \overbrace{2}^{\rightarrow 1} \cdot \overbrace{e^{2x}}^{\rightarrow 1} - 2}{10x} \end{array}$$

$$= \lim_{x \rightarrow 0} \frac{\cos(2x) \cdot 2 \cdot e^{2x} - 2}{10x}$$



$$\begin{array}{l} L'H \\ \downarrow \\ = \lim_{x \rightarrow 0} \frac{\overbrace{-4\sin(2x)}^{\rightarrow 0} \cdot \overbrace{e^{2x}}^{\rightarrow 1} + \overbrace{4\cos(2x)}^{\rightarrow 1} \cdot \overbrace{e^{2x}}^{\rightarrow 1}}{10} = \frac{4}{10} \neq 0 \end{array}$$

$\therefore$  el límite no existe.

2. Sea  $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  el campo vectorial dado por  $F(x, y) = (3x^2 - 4y, 4y - 4x)$ .

(a) Probar que  $F$  es un campo vectorial gradiente.

(b) Hallar los extremos relativos y los puntos silla de su función potencial  $f$ .

$$a) \quad f_x(x, y) = 3x^2 - 4y$$

$$f_y(x, y) = 4y - 4x$$

$$\Rightarrow f(x, y) = x^3 - 4xy + \phi(y) + C$$

$$f(x, y) = 2y^2 - 4xy + \psi(x) + C$$

$$\Rightarrow f(x, y) = x^3 + 2y^2 - 4xy \quad \text{con } C = 0$$

Como encontré la función potencial  $f$  de  $F \Rightarrow F$  es un campo gradiente

$$b) \quad \text{Veamos } \nabla f(x, y) = \vec{0}$$

$$(3x^2 - 4y, 4y - 4x) \stackrel{\text{quiero}}{\downarrow} = (0, 0)$$

$$\begin{cases} 3x^2 - 4y = 0 \\ 4y - 4x = 0 \end{cases} \Rightarrow y = x$$

$$\hookrightarrow 3x^2 - 4x = 0$$

$$x(3x - 4) = 0$$

$$\hookrightarrow x = 0$$

$$\hookrightarrow 3x - 4 = 0$$

$$x = \frac{4}{3}$$

Candidatos :

$$(x,y) = (0,0)$$

$$(x,y) = \left(\frac{4}{3}, \frac{4}{3}\right)$$

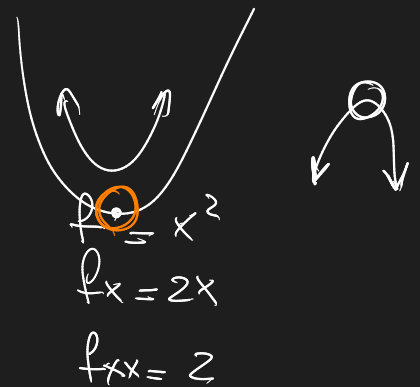
Criterio del Hessiano :

$$1) \left. \begin{array}{l} \det \geq 0 \\ f_{xx} > 0 \end{array} \right\} (x_0, y_0) \text{ es m\u00ednimo}$$

$$2) \left. \begin{array}{l} \det \geq 0 \\ f_{xx} < 0 \end{array} \right\} (x_0, y_0) \text{ es m\u00e1x}$$

$$3) \det < 0 \Rightarrow \text{es Punto silla}$$

$$4) \det = 0 \Rightarrow \text{el criterio no me sirve.}$$



C\u00e1lculo Hessiano

$$Hf(x,y) = \begin{bmatrix} 6x & -4 \\ -4 & 4 \end{bmatrix}$$

$$Hf(0,0) = \begin{bmatrix} 0 & -4 \\ -4 & 4 \end{bmatrix}$$

$$\det Hf(0,0) = -16 < 0$$

$(0,0)$  es Punto silla

$$Hf\left(\frac{4}{3}, \frac{4}{3}\right) = \begin{bmatrix} 6 \cdot \frac{4}{3} & -4 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix}$$

$$\det Hf\left(\frac{4}{3}, \frac{4}{3}\right) = 32 - 16 = 16 > 0$$

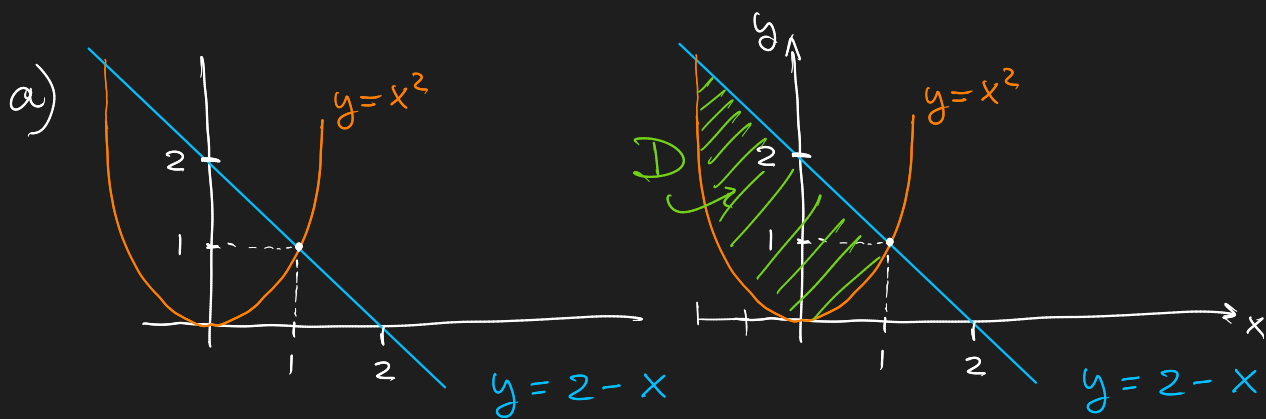
Como  $f_{xx} > 0$

$\Rightarrow \left(\frac{4}{3}, \frac{4}{3}\right)$  es mínimo local

3. Calcular las siguientes integrales

(a)  $\iint_D (2x+1) dA$  donde  $D$  es la región encerrada entre la curva  $y = x^2$  y la recta  $x+y=2$ ,

(b)  $\iiint_E x dV$  donde  $E$  es el sólido encerrado entre las superficies  $z = e^{x^2}$  y  $z = -y$  para  $(x, y)$  en el rectángulo  $R = [1, 2] \times [0, 2]$  del plano  $xy$ .



Veo extremos de  $x$

$$x^2 = 2 - x$$

$$x^2 + x - 2 = 0$$

$$(x-1)(x-a) = 0$$

$$x^2 - ax - x + a$$

$$x^2 + x \underbrace{(-a-1)}_{=1} + \underbrace{a}_{=-2} = 0$$

$$\therefore a = -2$$

Reírse

$$x = 1$$

$$x = -2$$

$$\iint_D (2x+1) dA = \int_{x=-2}^{x=1} \int_{y=x^2}^{y=2-x} (2x+1) dy dx$$

$$= \int_{x=-2}^{x=1} (2x+1) \cdot \underbrace{(2-x-x^2)}_{\text{}} dx$$

$$= \int_{-2}^1 4x - 2x^2 - 2x^3 + 2 - x - x^2 dx$$

$$= \int_{-2}^1 -2x^3 - 3x^2 + 3x + 2 dx$$

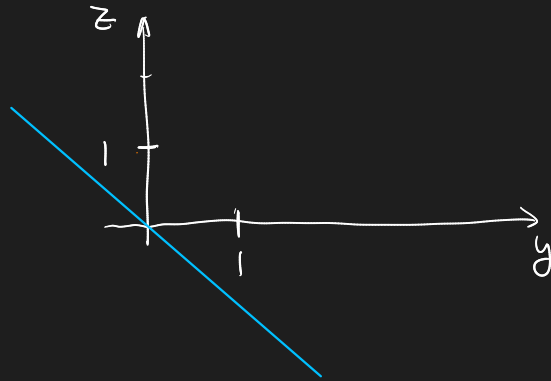
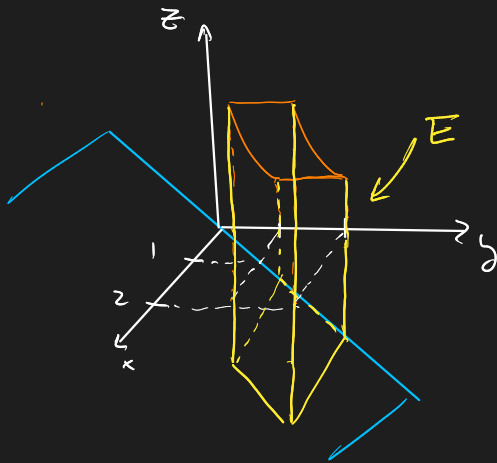
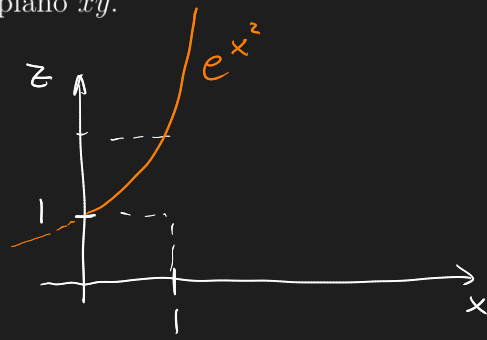
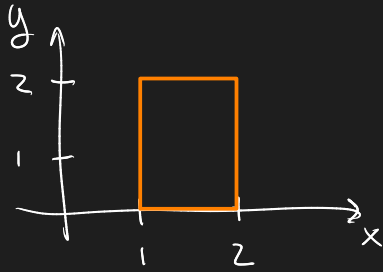
$$= \left[ -2 \cdot \frac{x^4}{4} - 3 \cdot \frac{x^3}{3} + 3 \cdot \frac{x^2}{2} + 2x \right]_{-2}^1$$

$$= \left[ -\frac{x^4}{2} - x^3 + \frac{3}{2} \cdot x^2 + 2x \right]_{-2}^1$$

$$= \underbrace{-\frac{1}{2} - 1 + \frac{3}{2} + 2}_{1+1=2} - \underbrace{\left( -8 + 8 + 6 - 4 \right)}_2$$

$$= 0 //$$

- (b)  $\iiint_E x \, dV$  donde  $E$  es el sólido encerrado entre las superficies  $z = e^{x^2}$  y  $z = -y$  para  $(x, y)$  en el rectángulo  $R = [1, 2] \times [0, 2]$  del plano  $xy$ .



$$\int_{x=1}^{x=2} \int_{y=0}^{y=2} \int_{z=-y}^{z=e^{x^2}} x \, dz \, dy \, dx =$$

$$= \int_{x=1}^{x=2} x \cdot \int_{y=0}^{y=2} e^{x^2} + y \, dy \, dx$$

$$= \int_{x=1}^{x=2} x \cdot \left[ y \cdot e^{x^2} + \frac{y^2}{2} \right]_{y=0}^{y=2} dx$$

$$= \int_{x=1}^{x=2} x \left( 2 \cdot e^{x^2} + 2 - 0 \right) dx$$

$$= \int_1^2 2x \cdot e^{x^2} + 2x \, dx$$

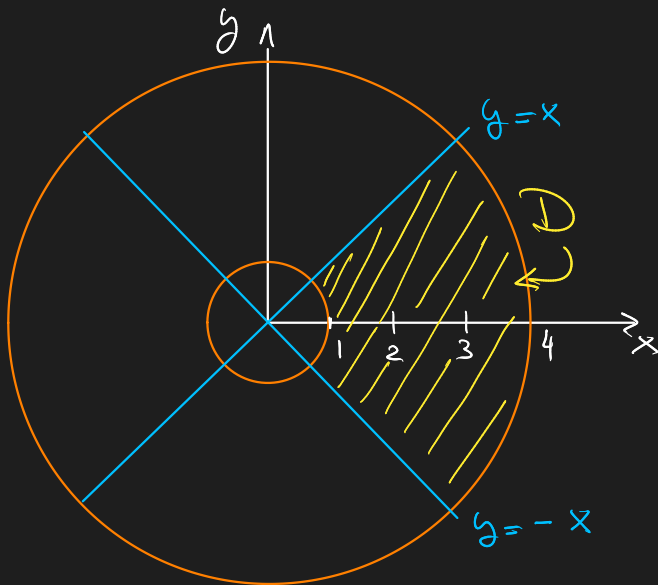




4. Calcular

$$\iint_D \frac{1}{x^2 + y^2 + 1} dA$$

donde  $D = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x^2 + y^2 \leq 16, -x \leq y \leq x\}$ .



Quiero usar polar

$$\begin{cases} x = r \cdot \cos \theta \\ y = r \cdot \sin \theta \end{cases}$$

$$\text{con } r \in [1, 4]$$

$$\theta \in [0, \frac{\pi}{4}] \cup [\frac{7}{4}\pi, 2\pi)$$

$$\begin{aligned} \iint_D f(x, y) dA &= \int_{r=1}^4 \int_{\theta=0}^{\pi/4} \frac{1}{r^2 + 1} \cdot r \cdot d\theta dr + \\ &+ \int_{r=1}^4 \int_{\theta=\frac{7}{4}\pi}^{2\pi} \frac{1}{r^2 + 1} \cdot r \cdot d\theta dr \end{aligned}$$

Calab primitiva de

$$\int \frac{r}{r^2 + 1} dr = \frac{1}{2} \ln(r^2 + 1)$$

Vuelvo

$$\begin{aligned} \int_{r=1}^4 \int_{\theta=0}^{\pi/4} \frac{1}{r^2 + 1} \cdot r \cdot d\theta dr &= \int_{r=1}^4 \frac{r}{r^2 + 1} \cdot \frac{\pi}{4} dr \\ &= \frac{\pi}{4} \cdot \frac{1}{2} \cdot [\ln(r^2 + 1)]_1^4 \end{aligned}$$

$$= \frac{\pi}{8} \cdot (\ln(17) - \ln(2))$$

$$\begin{aligned} \int_{r=1}^4 \int_{\theta=\frac{7}{4}\pi}^{2\pi} \frac{1}{r^2+1} \cdot r \cdot d\theta dr &= \int_{r=1}^4 \frac{r}{r^2+1} \cdot (2\pi - \frac{7}{4}\pi) dr \\ &= \frac{1}{4}\pi \cdot \frac{1}{2} \cdot (\ln(17) - \ln(2)) \end{aligned}$$

Final mente

$$\iint_D f(x,y) dA = \frac{\pi}{4} \cdot (\ln(17) - \ln(2)) //$$