

Análisis I - Análisis Matemático I - Matemática I - Análisis II (C)

1er. cuatrimestre 2020

Simulacro del Primer Parcial - 01/06/2020

Justifique todas sus respuestas.

Entregue todas las hojas escaneadas y en orden.

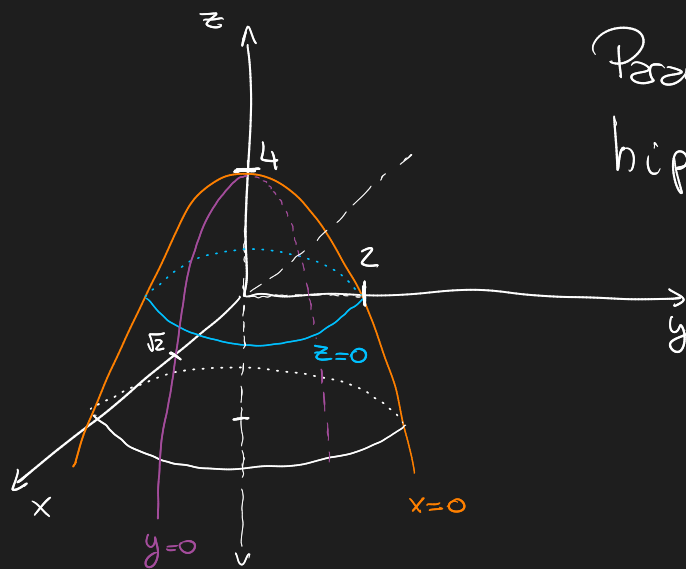
1. Considerar la superficie S de \mathbb{R}^3 definida por la ecuación

$$z = 4 - x^2 - y^2.$$

- (a) Hacer esquemas de las trazas (horizontales y verticales) de S y utilizarlos para hacer un gráfico aproximado. Describir geoméricamente la superficie.
- (b) Hallar la curva intersección de S con el plano $z = 2$ y dar una función $r: \mathbb{R} \rightarrow \mathbb{R}^3$ cuya imagen describa dicha curva.
- (c) Hallar la ecuación de la recta tangente a la curva descrita por r en el punto $P = (\sqrt{2}, 0, 2)$.

$$(\sqrt{2}, 0, 2)$$

a)

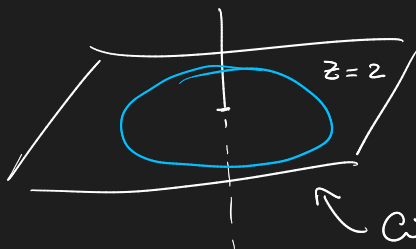


Paraboloid
hiperbólico

Curvas correspondientes a:

- $S \cap x=0$
- $S \cap y=0$
- $S \cap z=0$

b)



↖ Circunferencia centrada en $(x,y)=(0,0)$

Busco radio:

$$2 = 4 - x^2 - y^2$$

$$-2 = -x^2 - y^2$$

$$x^2 + y^2 = (\sqrt{2})^2$$

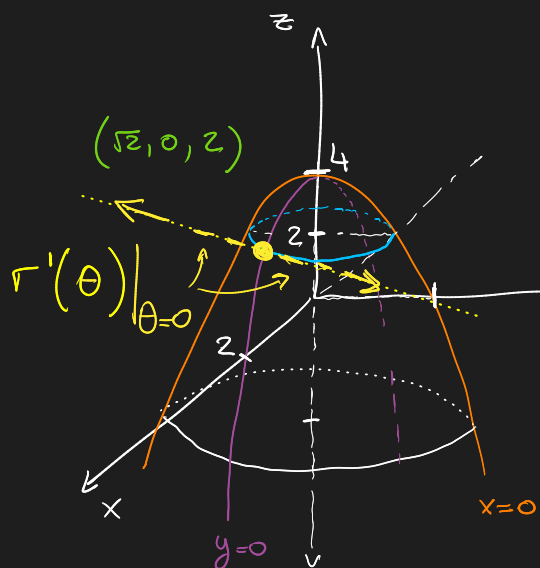
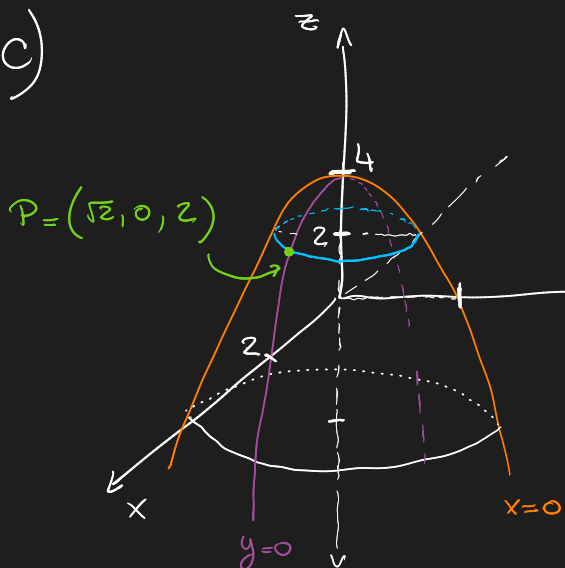
Parametrizo Circ. de radio $\sqrt{2}$

$$\mathbf{r} : \mathbb{R} \rightarrow \mathbb{R}^3$$

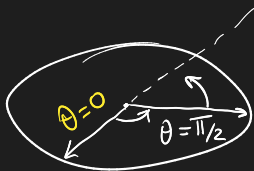
$$\mathbf{r}(\theta) = (\sqrt{2} \cdot \cos \theta, \sqrt{2} \cdot \sin \theta, 2)$$

Circ. de radio $= \sqrt{2}$ a altura $z = 2$.

c)

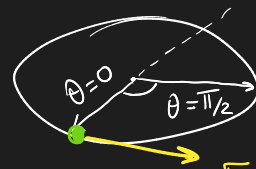
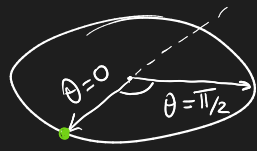


$$\mathbf{r}'(\theta) = (-\sqrt{2} \cdot \sin \theta, \sqrt{2} \cdot \cos \theta, 0)$$



$$\text{Veo } \theta = \frac{\pi}{2}$$

$$\mathbf{r}'(0) = (0, \sqrt{2}, 0)$$



$$\mathbf{r}'(0) = (0, \sqrt{2}, 0)$$

Armo recta tangente con vector $\mathbf{r}'(\frac{\pi}{2})$:

$$(x, y, z) = t(0, \sqrt{2}, 0) + (\sqrt{2}, 0, 2)$$

$$\forall t \in \mathbb{R}$$

2. Sea

$$f(x, y) = \frac{y^3 \sin\left(\frac{1}{x^2+y^2}\right)}{x^2+y^2}.$$

(a) Hallar el dominio de f .

(b) Determinar si se puede definir f de forma continua en el punto $(0, 0)$.

$$a) \text{Dom}(f) = \mathbb{R} \times \mathbb{R} \setminus \{(0, 0)\}$$

b) Primero veo $\lim_{(x,y)} f(x,y)$

$$\left| \frac{y^3 \cdot \overbrace{\sin\left(\frac{1}{x^2+y^2}\right)}^{\leq 1}}{x^2+y^2} - 0 \right| \leq \left| y^3 \cdot \left(\frac{1}{x^2+y^2}\right) \right| = \frac{|y|^3}{x^2+y^2}$$

$$\leq \frac{\|(x,y)\|^3}{\|(x,y)\|^2} = \|(x,y)\| < \delta$$

Encontré $\delta > 0 / \forall \varepsilon > 0$, si $\|(x,y)\| < \delta$

$$\Rightarrow \|f(x,y) - f(0,0)\| < \varepsilon$$

Basta tomar $\delta = \varepsilon$

\therefore se puede definir \tilde{f} de manera continua:

$$\tilde{f}(x,y) = \begin{cases} \frac{y^3 \cdot \sin\left(\frac{1}{x^2+y^2}\right)}{x^2+y^2} & \text{si } (x,y) \neq (0,0) \\ 0 & \text{si } (x,y) = (0,0) \end{cases}$$

3. Sea $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ dada por

$$f(x, y) = \begin{cases} \frac{xy \sin(xy)}{x^2 + y^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases},$$

Analizar la diferenciabilidad de f en cada punto de \mathbb{R}^2 .

• Si $(x, y) \neq (0, 0)$

$\Rightarrow f(x, y)$ es dif. por prod., compos., cociente, y suma de funciones diferenciables.

• Si $(x, y) = (0, 0)$

Derivadas parciales

$$f_x(0, 0) = \lim_{h \rightarrow 0} \frac{\overbrace{f(h, 0)}^{=0} - \overbrace{f(0, 0)}^{=0}}{h} = 0 \quad \checkmark$$

$$f_y(0, 0) = \lim_{h \rightarrow 0} \frac{\overbrace{f(0, h)}^{=0} - \overbrace{f(0, 0)}^{=0}}{h} = 0 \quad \checkmark$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{\overbrace{f(x, y)}^{=0} - \overbrace{f(0, 0)}^{=0} - \overbrace{f_x(0, 0)}^{=0}(x-0) - \overbrace{f_y(0, 0)}^{=0}(y-0)}{\|(x, y)\|} =$$

$$= \lim_{(x, y) \rightarrow (0, 0)} \frac{f(x, y)}{\|(x, y)\|}$$

$$\left| \underbrace{\frac{x \cdot y \cdot \sin(x \cdot y)}{x^2 + y^2}}_{\leq \| \cdot \|^2} \cdot \frac{1}{\|(x, y)\|} - 0 \right| \leq \left| \frac{x \cdot y \cdot x \cdot y}{x^2 + y^2} \right| \cdot \frac{1}{\| \cdot \|}$$

$$= \frac{|x|^2 \cdot |y|^2}{x^2 + y^2} \cdot \frac{1}{\| \cdot \|}$$

$$\leq \frac{\|(x, y)\|^4}{\|(x, y)\|^2} \cdot \frac{1}{\|(x, y)\|} < \delta$$

$$\Rightarrow \lim_{\|(x, y)\|} \frac{f(x, y)}{\|(x, y)\|} = 0$$

$\therefore f$ es diferenciable en $(x, y) = (0, 0)$

$\Rightarrow f$ es diferenciable.

4. Sean $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ definida por $f(x, y) = x^2 - xy$ y $g: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ diferenciable tal que $g(s, t) = (x(s, t), y(s, t))$, $g(1, 2) = (1, 1)$,

$$\frac{\partial x}{\partial s}(1, 2) = 5, \quad \frac{\partial x}{\partial t}(1, 2) = 2$$

y

$$\frac{\partial y}{\partial s}(1, 2) = -1, \quad \frac{\partial y}{\partial t}(1, 2) = 3.$$

Sea $h: \mathbb{R}^2 \rightarrow \mathbb{R}$, $h = f \circ g$. $= f(g(s, t)) = f(x(s, t), y(s, t))$

(a) Hallar $\frac{\partial h}{\partial s}(1, 2)$ y $\frac{\partial h}{\partial t}(1, 2)$.

(b) Hallar $\frac{\partial h}{\partial v}(1, 2)$ para $v = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$$\frac{\partial h}{\partial s} = \frac{\partial f}{\partial x}(g(s, t)) \cdot \frac{\partial x}{\partial s}(s, t) + \frac{\partial f}{\partial y}(g(s, t)) \cdot \frac{\partial y}{\partial s}$$

$$a) \quad \left. \frac{\partial h}{\partial s} \right|_{(1,2)} = \underbrace{\frac{\partial f}{\partial x}(g(s, t))}_{(2x-y)|_{(1,1)}} \cdot \underbrace{\frac{\partial x}{\partial s}(1,2)}_5 + \underbrace{\frac{\partial f}{\partial y}(g(s, t))}_{(-x)|_{(1,1)}} \cdot \underbrace{\frac{\partial y}{\partial s}(1,2)}_{-1} \quad \left\{ \underbrace{g(1,2) = (1,1)} \right.$$

$$\left. \frac{\partial h}{\partial s}(s, t) \right|_{(1,2)} = 10x + x - 5y \Big|_{(1,1)}$$

$$= 11x - 5y \Big|_{(1,1)}$$

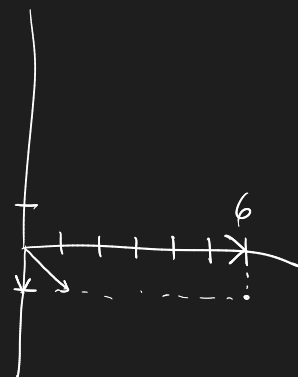
$$\left. \frac{\partial h}{\partial s} \right|_{(1,2)} = 11 - 5 = 6$$

$$\frac{\partial h}{\partial t} = \underbrace{\frac{\partial f}{\partial x}}_{(2x-y)} \cdot \underbrace{\frac{\partial x}{\partial t}}_2 + \underbrace{\frac{\partial f}{\partial y}}_{(-x)} \cdot \underbrace{\frac{\partial y}{\partial t}}_3$$

$$= 4x - 3x - 2y$$

$$= x - 2y$$

$$\left. \frac{\partial h}{\partial t}(s, t) \right|_{(1,2)} = 1 - 2 = -1$$



$$b) \quad v = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$

$$\frac{\partial h}{\partial v} = \frac{\partial f}{\partial x}(g(s,t)) \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}(g(s,t)) \cdot \frac{\partial y}{\partial v}$$

$$\bullet \quad \frac{\partial x}{\partial v} = \frac{\frac{\partial x}{\partial s}}{5} \cdot \frac{1}{\sqrt{2}} + \frac{\frac{\partial x}{\partial t}}{2} \cdot \frac{1}{\sqrt{2}}$$

$$\nabla x(s,t) = \left(\frac{\partial x}{\partial s}, \frac{\partial x}{\partial t} \right)$$

$$\frac{\partial x}{\partial v} = \frac{7}{\sqrt{2}}$$

$$\left\langle \left(\frac{\partial x}{\partial s}, \frac{\partial x}{\partial t} \right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \right\rangle =$$

$$\bullet \quad \frac{\partial y}{\partial v} = \frac{\frac{\partial y}{\partial s}}{-1} \cdot \frac{1}{\sqrt{2}} + \frac{\frac{\partial y}{\partial t}}{3} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}$$

Voluendo

$$\frac{\partial h}{\partial v} = \left\langle \nabla h(1,2), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \right\rangle$$

$$\frac{\partial h}{\partial v} = \frac{\partial f}{\partial x}(g(s,t)) \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}(g(s,t)) \cdot \frac{\partial y}{\partial v}$$

$$= (2x - y) \cdot \frac{7}{\sqrt{2}} + (-x) \cdot \frac{2}{\sqrt{2}}$$

$$= \frac{14}{\sqrt{2}} \cdot x - \frac{2}{\sqrt{2}} x - \frac{7}{\sqrt{2}} y$$

$$\frac{\partial h}{\partial v} \Big|_{(1,2)} = \frac{12}{\sqrt{2}} - \frac{7}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$\frac{\partial h}{\partial x} \Big|_{(1,2)} = \frac{5}{\sqrt{2}} //$$

Corrección :

Solo bastaba hacer

$$\frac{\partial h}{\partial x} = \left\langle \nabla h(1,2), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \right\rangle$$

$$= \left\langle (6, -1), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \right\rangle$$

$$= \frac{6}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}} //$$

