

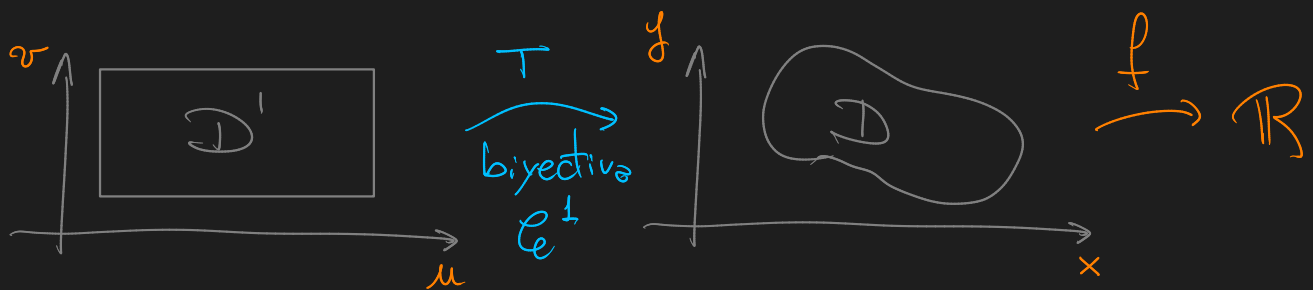
## Sustitución

$$\int_c^d \underbrace{f(g(x))}_u \cdot \underbrace{g'(x) dx}_{du} = \int_{g(c)}^{g(d)} f(u) du$$

Con  $g(x) /$

$$[c, d] \xrightarrow{g} \begin{matrix} [g(c), g(d)] \\ \text{y} \\ [g(d), g(c)] \end{matrix}$$

## Integrales dobles



$$T(u, v) = (x(u, v), y(u, v))$$

$$\begin{cases} x = x(u, v) \\ y = y(u, v) \end{cases}$$

# Teorema

$$\iint_D f(x, y) \cdot dA(x, y) = \iint_{D'} f(x(u, v), y(u, v)) \cdot \underset{\substack{\uparrow \\ \text{Jacobiano}}}{|JT(u, v)|} \cdot dA(u, v)$$

donde

$$JT(u, v) = \det \begin{vmatrix} \frac{\partial}{\partial u} x & \frac{\partial}{\partial v} x \\ \frac{\partial}{\partial u} y & \frac{\partial}{\partial v} y \end{vmatrix}$$

Notación

$$\downarrow \\ = \frac{\partial(x, y)}{\partial(u, v)}$$

que sirve para

$$dA(x, y) = \left| \frac{\partial(x, y)}{\partial(u, v)} \right| dA(u, v)$$

$$"dx dy" = \left| \frac{\partial(x, y)}{\partial(u, v)} \right| "du dv"$$

# Coordenadas Polares

$$\begin{cases} x = r \cdot \cos \theta & r \geq 0 \\ y = r \cdot \sin \theta & \theta \in [0, 2\pi) \end{cases}$$

$$\begin{aligned} \text{Jacobiano} &= \det \begin{vmatrix} \cos \theta & -r \cdot \sin \theta \\ \sin \theta & r \cdot \cos \theta \end{vmatrix} \\ &= r \cdot \cos^2 \theta + r \cdot \sin^2 \theta \end{aligned}$$

$$\text{Jacobiano} = r$$

$$\Rightarrow dA(x,y) = r \cdot dA(r,\theta)$$

Ejemplo :

Calcular el área de

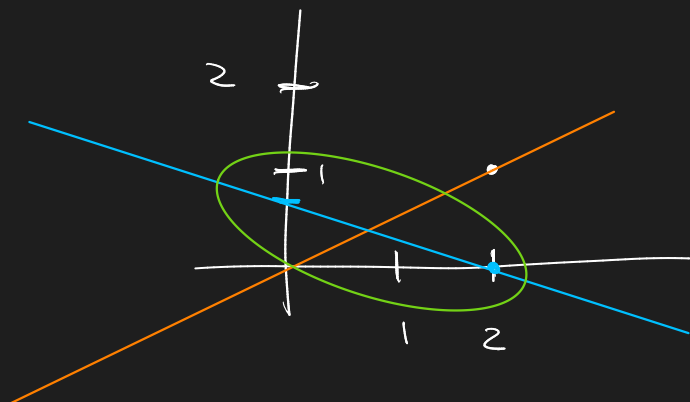
$$(x-2y)^2 + (3y+x-2)^2 = 1$$

veo rectas

- $x - 2y = 0$

- $3y + x - 2 = 0$

$$y = \frac{2-x}{3}$$



$$\underbrace{(x-2y)^2}_{\mu} + \underbrace{(3y+x-2)^2}_{\nu} = 1$$

$$\begin{cases} \mu = x-2y \\ \nu = 3y+x-2 \end{cases}$$

$$x = \mu + 2y \Rightarrow \nu = 3y + \mu + 2y - 2$$

$$\nu = 5y + \mu - 2$$

$$y = (\nu - \mu + 2) \cdot \frac{1}{5}$$

$$x = \mu + \frac{2\nu}{5} - \frac{2\mu}{5} + \frac{4}{5}$$

$$\begin{cases} x = (3\mu + 2\nu + 4) \cdot \frac{1}{5} \\ y = (\nu - \mu + 2) \cdot \frac{1}{5} \end{cases}$$

Calculo Jacobiano  $\begin{vmatrix} x_{\mu} & x_{\nu} \\ y_{\mu} & y_{\nu} \end{vmatrix}$

y hago cambio de variables.

$$\begin{aligned} \text{Área}(\mathbb{D}) &= \iint_{\mathbb{D}'} 1 \cdot \frac{1}{5} \cdot dA(\mu, \nu) = \frac{1}{5} \cdot \iint_{\mu^2 + \nu^2 \leq 1} 1 \cdot dA(\mu, \nu) \\ &\quad \uparrow \\ &\quad \mu^2 + \nu^2 = 1 \\ &= \frac{\pi}{5} \end{aligned}$$







