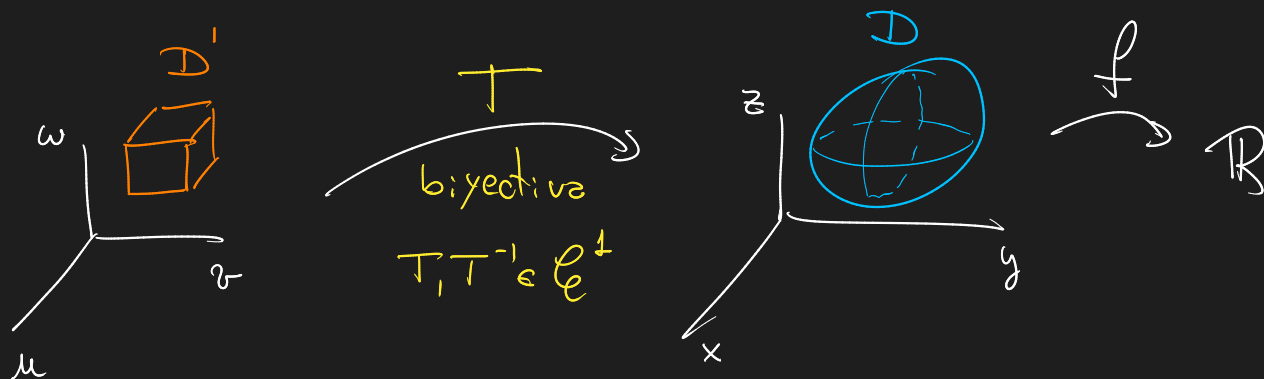


Fórmula de Cambio de Variables



• T biyectiva entre D' y D

• con $T \in \mathcal{C}^1$

y $T^{-1} \in \mathcal{C}^1$

$$T(\mu, \nu, \omega) = (x(\mu, \nu, \omega), y(\mu, \nu, \omega), z(\mu, \nu, \omega))$$

$$\begin{cases} x = x(\mu, \nu, \omega) \\ y = y(\mu, \nu, \omega) \\ z = z(\mu, \nu, \omega) \end{cases}$$

Teorema :

$$\iiint_D f(x, y, z) \cdot dV_{(x, y, z)} = \iiint_{D'} f(x(\mu, \nu, \omega), y(\mu, \nu, \omega), z(\mu, \nu, \omega)) |JT_{(\mu, \nu, \omega)}| dV_{(\mu, \nu, \omega)}$$

1 Cilíndricas

• Polares + eje z

$$\begin{cases} x = r \cdot \cos \theta \\ y = r \cdot \sin \theta \\ z = z \end{cases} \quad \begin{aligned} r &\geq 0 \\ \theta &\in [0, 2\pi) \\ z &\in \mathbb{R} \end{aligned}$$

$$|\vec{T}| = r \quad (\text{mismo que polares})$$

Centro de masa $(\bar{x}, \bar{y}, \bar{z})$

$$\bar{x} = \frac{\iiint_S x \cdot \delta(x, y, z) \cdot dV(x, y, z)}{\underbrace{\iiint_S \delta(x, y, z) \cdot dV(x, y, z)}_{\text{masa de } S}}$$

$$\bar{y} = \frac{\iiint_S y \cdot \delta(x, y, z) \cdot dV(x, y, z)}{\iiint_S \delta(x, y, z) \cdot dV(x, y, z)}$$

$$\bar{z} = \frac{\iiint_S z \cdot \delta(x, y, z) \cdot dV(x, y, z)}{\iiint_S \delta(x, y, z) \cdot dV(x, y, z)}$$

Valor promedio de f

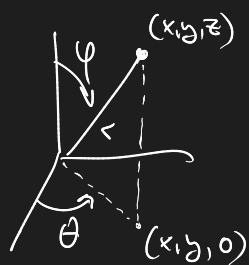
$$f_{\text{promedio}} = \frac{\iiint_S f(x, y, z) \cdot dV(x, y, z)}{\underbrace{\iiint_S 1 \cdot dV(x, y, z)}_{\text{Vol}(S)}}$$

Volumen de S

si $f \equiv 1 \Rightarrow f_{\text{promedio}} = 1$

$$\iiint_S 1 \cdot dV(x, y, z) = \text{Vol}(S)$$

12) Coord. Esféricas



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r \geq 0$$

$$\theta \in [0, 2\pi)$$

$$\varphi \in [0, \pi)$$

$$\begin{cases} x = r \cdot \cos \theta \cdot \sin \varphi \\ y = r \cdot \sin \theta \cdot \sin \varphi \\ z = r \cdot \cos \varphi \end{cases}$$

$$J_T = |-r^2 \cdot \sin \varphi|$$

$$J_T = r^2 \cdot \sin \varphi$$

$$\sin \varphi \geq 0 \quad \text{con} \quad \varphi \in [0, \pi)$$

$$\sin \varphi \begin{cases} \uparrow \pi/2 \\ \leftarrow \varphi \rightarrow 0 \\ \leftarrow \pi \rightarrow 0 \end{cases}$$

