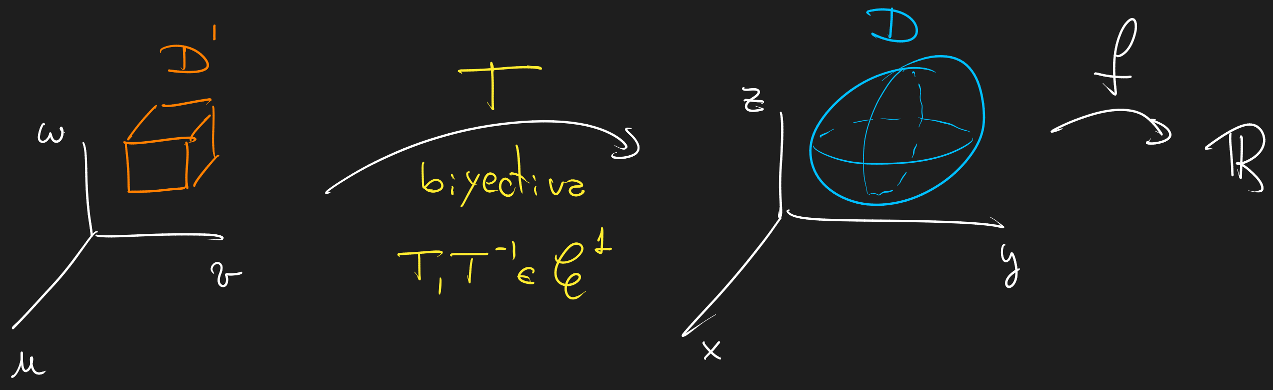
Fórmula de Cambio de Variables



- T biyectiva entre D' y D
- con $T \in \mathcal{C}^1$
  y $T^{-1} \in \mathcal{C}^1$

$$T(u,v,w) = \Big(x(u,v,w),\ y(u,v,w),\ z(u,v,w)\Big)$$

$$\begin{cases} x = x(u,v,w) \\ y = y(u,v,w) \\ z = z(u,v,w) \end{cases}$$

Teorema :

$$\iiint_D f(x,y,z)\, dV_{(x,y,z)} = \iiint_{D'} f\big(x(u,v,w), y(u,v,w), z(u,v,w)\big)\, \big|JT_{(u,v,w)}\big|\, dV_{(u,v,w)}$$

1 Cilíndricas

- Polares + eje z

$$\begin{cases} x = r \cdot \cos\theta & r \geq 0 \\ y = r \cdot \sin\theta & \theta \in [0, 2\pi) \\ z = z & z \in \mathbb{R} \end{cases}$$

$$|JT| = r \quad \text{(mismo que polares)}$$

Centro de masa $(\bar{x}, \bar{y}, \bar{z})$

$$\bar{x} = \frac{\iiint_S x \cdot \delta(x,y,z) \cdot dV(x,y,z)}{\underbrace{\iiint_S \delta(x,y,z) \cdot dV(x,y,z)}_{\text{masa de } S}}$$

$$\bar{y} = \frac{\iiint_S y \cdot \delta(x,y,z) \cdot dV(x,y,z)}{\iiint_S \delta(x,y,z) \cdot dV(x,y,z)} \qquad \bar{z} = \frac{\iiint_S z \cdot \delta(x,y,z) \cdot dV(x,y,z)}{\iiint_S \delta(x,y,z) \cdot dV(x,y,z)}$$

Valor promedio de f

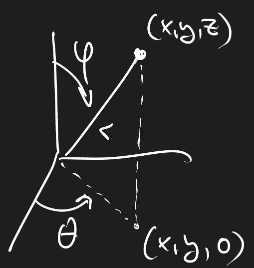
$$f_{\text{promedio}} = \frac{\iiint_S f(x,y,z) \, . \, dV_{(x,y,z)}}{\underbrace{\iiint_S 1 \, . \, dV_{(x,y,z)}}_{Vol(S)}}$$

Volumen de S

si $f \equiv 1 \Rightarrow f_{\text{promedio}} = 1$

$$\iiint_S 1 \, . \, dV_{(x,y,z)} = Vol(S)$$

2) Coord. Esféricas



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r \geq 0$$

$$\theta \in [0, 2\pi)$$

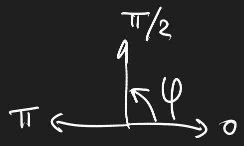
$$\varphi \in [0, \pi)$$

$$\begin{cases} x = r \, . \cos\theta \, . \sin\varphi \\ y = r \, . \sin\theta \, . \sin\varphi \\ z = r \, . \cos\varphi \end{cases}$$

$$JT = \left| -r^2 \cdot \sin\varphi \right|$$

$\sin\varphi \geq 0$ con $\varphi \in [0, \pi)$

$\sin\varphi \, \{$



$$\boxed{JT = r^2 \cdot \sin\varphi}$$