1 Calcular las derivadas parciales de segundo orden para las siguientes funciones, verificando la igualdad de las derivadas parciales mixtas para aquellas funciones de clase C^2:

1.1 $f(x,y) = x^3y + e^{xy^2}$

Es un polinomio, por lo que es C^∞

$$\frac{\partial f}{\partial x}(x,y) = 3x^2y + y^2e^{xy^2};\ \frac{\partial f}{\partial y}(x,y) = x^3 + 2xye^{xy^2}$$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = 6xy + y^4e^{xy^2};\ \frac{\partial^2 f}{\partial y^2}(x,y) = 4x^2y^2e^{xy^2} + 2xe^{xy^2};$$

$$\frac{\partial^2 f}{\partial y\partial x}(x,y) = 3x^2 + 2xy^3e^{xy^2} + 2ye^{xy^2}\ \frac{\partial^2 f}{\partial x\partial y}(x,y) = 2xye^{xy^2} = 3x^2 + 2ye^{xy^2} + 2xy^3e^{xy^2}$$

$$\text{Como era esperado } \frac{\partial^2 f}{\partial y\partial x}(x,y) = \frac{\partial^2 f}{\partial x\partial y}(x,y)$$

1.2 $f(x,y,z) = ye^z + \frac{e^y}{x} + xy\sin z$

$$\frac{\partial f}{\partial x}(x,y,z) = y\sin z - \frac{e^y}{x^2};\ \frac{\partial f}{\partial y}(x,y,z) = e^Z + \frac{e^y}{x} + x\sin z;\ \frac{\partial f}{\partial z}(x,y,z) = ye^z + xy\cos z$$

$$\frac{\partial^2 f}{\partial x^2}(x,y,z) = \frac{2e^y}{x^3};\ \frac{\partial^2 f}{\partial y^2}(x,y,z) = \frac{e^y}{x};\ \frac{\partial^2 f}{\partial z^2}(x,y,z) = ye^z - xy\sin z$$

$$\frac{\partial^2 f}{\partial x\partial y}(x,y) = -\frac{e^y}{x^2} + \sin z;\ \frac{\partial^2 f}{\partial y\partial x}(x,y) = \sin z - \frac{e^y}{x^2}$$

$$\frac{\partial^2 f}{\partial z\partial y}(x,y) = e^Z + x\cos z;\ \frac{\partial^2 f}{\partial y\partial z}(x,y) = e^z + x\cos z$$

$$\frac{\partial^2 f}{\partial x\partial z}(x,y) = y\cos z;\ \frac{\partial^2 f}{\partial z\partial x}(x,y) = y\cos z$$

1.3 $f(x,y,z) = \sqrt{x^2+y^2} + \ln z$

$$\frac{\partial f}{\partial x}(x,y,z) = \frac{x}{\sqrt{x^2+y^2}};\ \frac{\partial f}{\partial y}(x,y,z) = \frac{y}{\sqrt{x^2+y^2}};\ \frac{\partial f}{\partial z}(x,y,z) = \frac{1}{z}$$

$$\frac{\partial^2 f}{\partial x^2}(x,y,z) = \frac{y^2}{\left(\sqrt{x^2+y^2}\right)^{3/2}};\ \frac{\partial^2 f}{\partial y^2}(x,y,z) = \frac{x^2}{\left(\sqrt{x^2+y^2}\right)^{3/2}};\ \frac{\partial^2 f}{\partial z^2}(x,y,z) = -\frac{1}{z^2}$$

$$\frac{\partial^2 f}{\partial x\partial y}(x,y) = \frac{\partial^2 f}{\partial y\partial x}(x,y) = \frac{xy}{\left(\sqrt{x^2+y^2}\right)^{3/2}}$$

$$\frac{\partial^2 f}{\partial z\partial y}(x,y) = 0;\ \frac{\partial^2 f}{\partial y\partial z}(x,y) = 0$$

$$\frac{\partial^2 f}{\partial x\partial z}(x,y) = 0;\ \frac{\partial^2 f}{\partial z\partial x}(x,y) = 0$$

2 Calcular todas las derivadas de tercer orden para las siguientes funciones

¡ ni en pedo !

3 Sea $f(x,y) = \cos(xy)$. Además x e y son funciones de las variables u y v de acuerdo a las siguientes fórmulas: $x(u,v) = u+v, y(u,v) = u-v$. Calcular

$$\frac{\partial^2 f}{\partial u^2}\left(x\left(u,v\right), y\left(u,v\right)\right) \text{ y } \frac{\partial^3 f}{\partial u\partial v^2}\left(x\left(u,v\right), y\left(u,v\right)\right)$$

3.1 Sustituyendo

$$f(x(u,v),y(u,v)) = \cos\left((u+v)(u-v)\right) = \cos\left(u^2-v^2\right)$$

$$\frac{\partial f}{\partial u} = -2u\sin\left(u^2-v^2\right);\ \frac{\partial^2 f}{\partial u^2} = -2\sin\left(u^2-v^2\right) - 4u^4\cos\left(u^2-v^2\right)$$

$$\frac{\partial f}{\partial v} = 2v\sin\left(u^2-v^2\right);\ \frac{\partial f}{\partial v^2} = 2\sin\left(u^2-v^2\right) - 4v^2\cos\left(u^2-v^2\right)$$

$$\frac{\partial^3 f}{\partial u\partial v^2}\left(x\left(u,v\right),y\left(u,v\right)\right) = 4u^2\cos\left(u^2-v^2\right) + 4v^2 2u\sin\left(u^2-v^2\right)$$

3.2 Usando la regla de la cadena

$$\nabla f(x,y) = (-y\sin xy, -x\sin xy);\ Hf(x,y) = \begin{matrix} -y^2\cos xy & -yx\cos xy \\ -yx\cos xy & -x\cos xy \end{matrix}$$

$$\nabla x(u,v) = (1,1);\ \nabla y(u,v) = (1,-1) \Rightarrow Hx(u,v) = Hy(u,v) = 0$$

$$\text{Sea } g(u,v) = (x(u,v),y(u,v)) = (u+v,u-v)$$

$$\therefore D\left(f\circ g\right)(u,v) = D(f(x(u,v),y(u,v))) * D(g(u,v)) = (-y\sin xy, -x\sin xy) * \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = (-y\sin xy - x\sin xy, -y\sin xy + x\sin xy)$$

$$\left(\sin\left(u^2-v^2\right)(-u-v-u+v), \sin\left(u^2-v^2\right)(u+v-u+v)\right) = \left(\sin\left(u^2-v^2\right)(-u-v-u+v), \sin\left(u^2-v^2\right)(u+v-u+v)\right) \rightarrow \text{Reemplacé}$$

$$\Rightarrow D\left(f\circ g\right)(u,v) = \left(\sin\left(u^2-v^2\right)(-2u), \sin\left(u^2-v^2\right)(2v)\right) \rightarrow \text{Aquí seguiría operando como con una función normal}$$

4 Se dice que una función $f:\mathbb{R}^n \rightarrow \mathbb{R}$ de clase C^2 satisface la ecuación de Laplace o bien que es una función armónica en un conjunto abierto $U \subset \mathbb{R}^3$ si:

$$\nabla f = \frac{\partial^2 f}{\partial x_1^2} + \ldots + \frac{\partial^2 f}{\partial x_n^2} = \nabla^2 f \equiv 0 \text{ en } U$$

Verificar que las siguientes funciones son armónicas en un conjunto abierto $U \subset \mathbb{R}^3$. Determinar U en cada caso:

4.1 $f\left(x,y,z\right) = x^2+y^2-2z^2$

$$\frac{\partial f}{\partial x}(x,y,z) = 2x \rightarrow \frac{\partial^2 f}{\partial x^2}(x,y,z) = 2$$

$$\frac{\partial f}{\partial y}(x,y,z) = 2y \rightarrow \frac{\partial^2 f}{\partial y^2}(x,y,z) = 2$$

$$\frac{\partial f}{\partial z}(x,y,z) = -4z \rightarrow \frac{\partial^2 f}{\partial z^2}(x,y,z) = -4$$

$$\therefore \frac{\partial^2 f}{\partial x^2}(x,y,z) + \frac{\partial^2 f}{\partial y^2}(x,y,z) + \frac{\partial^2 f}{\partial z^2}(x,y,z) = 2+2-4 = 0 \rightarrow \text{Es laplaciano para todo } U \subset \mathbb{R}^3$$

4.2 $f\left(x,y,z\right) = \ln\sqrt{x^2+y^2}$

$$\frac{\partial f}{\partial x}(x,y,z) = \frac{1}{\sqrt{x^2+y^2}} * \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{x^2+y^2} \rightarrow \frac{\partial^2 f}{\partial x^2}(x,y,z) = \frac{1}{x^2+y^2} - \frac{2x}{\left(x^2+y^2\right)^2} = \frac{x^2+y^2-2x^2}{\left(x^2+y^2\right)^2} = \frac{y^2-x^2}{\left(x^2+y^2\right)^2}$$

$$\frac{\partial f}{\partial y}(x,y,z) = \frac{y}{x^2+y^2} \rightarrow \frac{\partial^2 f}{\partial y^2}(x,y,z) = \frac{x^2-y^2}{\left(x^2+y^2\right)^2}$$

$$\frac{\partial f}{\partial z^2}(x,y,z) = 0 \rightarrow \frac{\partial^2 f}{\partial z^2}(x,y,z) = 0$$

$$\therefore \frac{\partial^2 f}{\partial x^2}(x,y,z) + \frac{\partial^2 f}{\partial y^2}(x,y,z) + \frac{\partial^2 f}{\partial z^2}(x,y,z) = \frac{y^2-x^2}{\left(x^2+y^2\right)^2} + \frac{x^2-y^2}{\left(x^2+y^2\right)^2} = 0 \rightarrow \text{Es laplaciano para todo } U \subset \mathbb{R}^3$$

4.3 $f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$$\frac{\partial f}{\partial x}(x,y,z) = -x\left(x^2+y^2+z^2\right)^{-3/2} \rightarrow \frac{\partial^2 f}{\partial x^2}(x,y,z) = -\left(x^2+y^2+z^2\right)^{-3/2} - 3x^2\left(x^2+y^2+z^2\right)^{-5/2}$$

$$\frac{-x^2-y^2-z^2+3x^2}{\left(x^2+y^2+z^2\right)^{5/2}} = \frac{2x^2-y^2-z^2}{\left(x^2+y^2+z^2\right)^{5/2}}$$

$$\frac{\partial f}{\partial y}(x,y,z) = -y\left(x^2+y^2+z^2\right)^{-3/2} \rightarrow \frac{\partial^2 f}{\partial y^2}(x,y,z) = \frac{2y^2-x^2-z^2}{\left(x^2+y^2+z^2\right)^{5/2}}$$

$$\frac{\partial f}{\partial z}(x,y,z) = -z\left(x^2+y^2+z^2\right) \rightarrow \frac{\partial^2 f}{\partial z^2}(x,y,z) = \frac{2z^2-x^2-y^2}{\left(x^2+y^2+z^2\right)^{5/2}}$$

$$\therefore \frac{\partial^2 f}{\partial x^2}(x,y,z) + \frac{\partial^2 f}{\partial y^2}(x,y,z) + \frac{\partial^2 f}{\partial z^2}(x,y,z) = \frac{2x^2+2y^2+2z^2-2x^2-2y^2-2z^2}{\left(x^2+y^2+z^2\right)^{5/2}} = 0 \rightarrow \text{Es laplaciano para todo } U \subset \mathbb{R}^3$$

4.4 $f(x,y,z) = e^{3x+4}\cos(3z) + 4y$

$$\frac{\partial f}{\partial x}(x,y,z) = 3e^{3x+4}\cos(3z) \rightarrow \frac{\partial^2 f}{\partial x^2}(x,y,z) = 9e^{3x+4}\cos(3z)$$

$$\frac{\partial f}{\partial y}(x,y,z) = 4 \rightarrow \frac{\partial^2 f}{\partial y^2}(x,y,z) = 0$$

$$\frac{\partial f}{\partial z}(x,y,z) = -3e^{3x+4}\sin(3z) \rightarrow \frac{\partial^2 f}{\partial z^2}(x,y,z) = -9e^{3x+4}\cos(3z)$$

$$\therefore \frac{\partial^2 f}{\partial x^2}(x,y,z) + \frac{\partial^2 f}{\partial y^2}(x,y,z) + \frac{\partial^2 f}{\partial z^2}(x,y,z) = 9e^{3x+4}\cos(3z) - 9e^{3x+4}\cos(3z) = 0 \rightarrow \text{Es laplaciano para todo } U \subset \mathbb{R}^3$$

5 Sean f, g dos funciones C^2 definidas en un abierto $U \subset \mathbb{R}^2$ y tales que

$$\frac{\partial f}{\partial x} = -\frac{\partial g}{\partial y} \wedge \frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$$

Probar que f y g son armónicas en U.

$$\frac{\partial^2 f}{\partial x^2} = -\frac{\partial^2 g}{\partial x \partial y} \wedge \frac{\partial^2 f}{\partial y^2} = -\frac{\partial^2 g}{\partial y \partial x} \rightarrow \text{Derivo en y/x en ambos miembros}$$

$$\therefore \frac{\partial^2 f}{\partial x^2}(x,y) + \frac{\partial^2 f}{\partial y^2}(x,y) = \frac{\partial^2 g}{\partial x \partial y}(x,y) - \frac{\partial^2 g}{\partial y \partial x}(x,y) = 0 \rightarrow \text{Por ser } C^2$$

6 Resolver

6.1 Desarrollar la función $p(x) = x^4 - 5x^3 + 5x^2 + x + 2$ en potencias de $x-2$

Calculo el polinomio de taylor en el punto $x = 2$

$$P_2(x) = p(2) + p'(2) * (x-2) + \frac{p''(2) * (x-2)^2}{2} + \frac{p'''(2)(x-2)^3}{6} + \frac{p''''(2)(x-2)^4}{24}$$

$$P_2(x) = 0 + \left(3x^3 - 15x^2 + 10x + 1\right)\Big|_{x=2} * (x-2) + \frac{\left(9x^2 - 30x + 10\right)}{2}\Big|_{x=2} * (x-2)^2 + \frac{(18x-30)}{6}\Big|_{x=2} (x-2)^3 + \frac{18}{24}(x-2)^4$$

$$P_2(x) = -15(x-2) - 7(x-2)^2 + (x-2)^3 + \frac{3}{4}(x-2)^4 = (x-2)\left(-15 - 7(x-2) + (x-2)^2 + \frac{3}{4}(x-2)^3\right)$$

6.2 Desarrollar la función $g(x) = \sqrt{x}$ en potencias de $x-1$ hasta orden 3.

$$P_1(x) = 1 + g'(1)(x-1) + \frac{g''(1)(x-1)}{2} + \frac{g'''(1)(x-1)}{6}$$

$$P_1(x) = 1 + \left(\frac{1}{2}x^{-1/2}\Big|_{x=1}\right)(x-1) + \left(\frac{-\frac{1}{4}x^{-3/2}\Big|_{x=1}}{2}\right)(x-1) + \left(\frac{\frac{3}{8}x^{-5/2}\Big|_{x=1}}{2}\right)(x-1)$$

$$P_1(x) = 1 + \frac{1}{2}(x-1) + -\frac{1}{8}(x-1) + \frac{3}{16}(x-1) \sim g(x)$$

6.3 Hallar el polinomio de Maclaurin de grado tres para la función $f(x) = \ln(x+1)^2$

$$P_0(x) = 0 + \frac{2\ln(x+1)}{x+1}|_{x=0} x + \left(-\frac{2(\ln(x+1)-1)}{(x+1)^2}|_{x=0}\right)\frac{x^2}{2} + \left(\frac{4(\ln(x+1)-1)-2}{(x+1)^3}|_{x=0}\right)\frac{x^3}{6}$$

$$P_0(x) = 0 + x^2 + x^3$$

6.4 Hallar el polinomio de Maclaurin de grado tres para la función $g(x) = e^{x+2}$

$$P_0(x) = e^2 + e^2 x + \frac{e^2 x^2}{2} + \frac{e^2 x^3}{6}$$

7 Resolver

7.1 Hallar el polinomio de Maclaurin de orden 2 y la expresión del resto para la función $f(x) = \sqrt{1+x}$

$$P_0^2(x) + R_0^3(x) = 1 + \frac{1}{2}(1)^{-1/2} x + \frac{-\frac{1}{4}(1)^{-3/2} x^2}{2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + R_0^3(x)$$

$$R_0^3(x) = \left(\frac{\frac{3}{8}(1+c)^{-5/2}}{6}\right) x^3 \rightarrow \text{Con } c \text{ entre } x \text{ y } 0$$

$$\therefore P_0^2(x) + R_0^3(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \left(\frac{1}{16}(1+c)^{-5/2}\right) x^3$$

7.2 Evaluar el error de la igualdad aproximada $\sqrt{1+x} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2$ cuando $x = 0,2$

$$E_{2,0}(0,2) = \frac{1}{16}(1+c)^{-5/2}\frac{1}{125} / c \in [0;0,2]$$

$$= \frac{1}{125 * 16(1+c)^{5/2}} \leq \frac{1}{125 * 16(1+0,2)^{5/2}} = \frac{1}{125 * 16(1+0,2)^{5/2}} = \frac{1}{2488.32}$$

$$\therefore E_{2,0}(0,2) \leq \frac{1}{2488.32}$$

8 Resolver

8.1 Sea $\alpha \in \mathbb{R}$. Haller el polinomio de Maclaurin de grado n de la función $y = (1+x)^\alpha$

$$f'(x) = \alpha(1+x)^{\alpha-1}; f''(x) = \alpha(\alpha-1)(1+x)^{\alpha-2}; ... f^n = \alpha(\alpha-1)(\alpha-2)...(\alpha-n)(1+x)^{\alpha-n} = \frac{\alpha!/(\alpha-n-1)! x^n}{n!} \rightarrow \forall 0 < n < \alpha$$

$$P_0(x) = 1 + \alpha(x) + \frac{\alpha * (\alpha-1)x^2}{2} + ... + \frac{\alpha!/(\alpha-n-1)! x^n}{n!}$$

8.2 Calcular el valor de $(1,3)^{2/3}$ con error menor que $\frac{1}{100}$.

$$y = \left(1 + (1,3)^{2/3}\right)^\alpha \approx P_k((1,3)^{2/3}) + R_k((1,3)^{2/3})$$

$$R_1((1,3)^{2/3}) = \alpha(1+c)^{\alpha-1}\left((1,3)^{2/3}\right) \rightarrow c \in [0,(1,3)^{2/3}]$$

Acá hay algo raro, porque si alfa - 1 es menor a cero entonces el mayor valor que puede tomar es $x = (1,3)^{2/3}$, pero sino es 0. Faltaría saber que alfa es mayor a algo, me parece.

9 Calcular:

9.1 El número e con error menor que 10^{-4}

9.2 $\ln\frac{2}{3}$ con error menor que 10^{-3}

10 Calcular la fórmula de Taylor de segundo orden para las funciones dadas en el punto indicado. Escribir la forma de Lagrange del residuo.

10.1 $f(x,y) = (x+y)^2$ en $(0,0)$

$$P_{2,(0,0)} = 0 + \left(2(x+y)\Big|_{(0,0)} x + 2(x+y)\Big|_{(0,0)} y\right) + \frac{1}{2}(2x^2 + 2y^2 + 4xy)$$

$$P_{2,(0,0)} = x^2 + y^2 + 2xy = (x+y)^2 \rightarrow \text{Esperado}$$

$$R_{2,(0,0)} = 0 \rightarrow \text{Ya que es un polinomio de segundo grado, la derivada tercera será nula}$$

10.2 $f(x,y) = e^{x+y}$ en $(0,0)$

$$P_{2,(0,0)} = 1 + x + y + \frac{1}{2}\left(x^2 + y^2 + 2xy\right)$$

$$R_{2,(0,0)} = \frac{1}{6}\left(e^{c_1+c_2}\left(x^3 + y^3 + 3x^2y + 3y^2x\right)\right) = e^{c_1+c_2}\left(\frac{x^3+y^3}{6} + \frac{x^2y+y^2x}{2}\right)$$

10.3 $f(x,y) = \frac{1}{x^2+y^2+1}$ en $(0,0)$

$$\frac{\partial f}{\partial x}(0,0) = -\frac{2x}{\left(x^2+y^2+1\right)^2}\bigg|_{(0,0)} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = -\frac{2y}{\left(x^2+y^2+1\right)^2}\bigg|_{(0,0)} = 0$$

$$\frac{\partial^2 f}{\partial x^2}(0,0) = \frac{6x^2 - 2\left(y^2+1\right)}{\left(x^2+y^2+1\right)^3}\bigg|_{(0,0)} = -2$$

$$\frac{\partial^2 f}{\partial y^2}(0,0) = \frac{6y^2 - 2\left(x^2+1\right)}{\left(x^2+y^2+1\right)^3}\bigg|_{(0,0)} = -2$$

$$\frac{\partial^2 f}{\partial y \partial x}(0,0) = \frac{2*2x*2y}{\left(x^2+y^2+1\right)^3} = \frac{\partial^2 f}{\partial x \partial y}(0,0) = \frac{8xy}{\left(x^2+y^2+1\right)^3} = 0$$

$$P_{2,(0,0)} = 1 + 0 + \frac{1}{2}\left(-2x^2 - 2y^2\right)$$

$$R_{2,(0,0)} = \frac{1}{6}(horror) \rightarrow \text{Horrible...}$$

10.4 $f(x,y) = e^{(x-1)^2}\cos y$ en $(1,0)$

$$\frac{\partial f}{\partial x}(1,0) = 2(x-1)\,e^{(x-1)^2}\cos y\Big|_{(1,0)} = 0$$

$$\frac{\partial f}{\partial y}(1,0) = -e^{(x-1)^2}\sin y\Big|_{(1,0)} = 0$$

$$\frac{\partial^2 f}{\partial x^2}(1,0) = 2e^{(x-1)^2}\cos y + 4(x-1)^2 e^{(x-1)^2}\cos y\Big|_{(1,0)} = 2e^{(x-1)^2}\cos(y)\left(2x^2 - 4x + 3\right)\Big|_{(1,0)} = 2*\cos(0)*1$$

$$\frac{\partial^2 f}{\partial y^2}(1,0) = -e^{(x-1)^2}\cos y\Big|_{(1,0)} = -1$$

$$\frac{\partial^2 f}{\partial x \partial y}(1,0) = -2(x-1)\,e^{(x-1)^2}\sin y\Big|_{(1,0)} = 0$$

$$\frac{\partial^2 f}{\partial y \partial x}(1,0) = -2(x-1)\,e^{(x-1)^2}\sin y\Big|_{(1,0)} = 0$$

$$P_{2,(1,0)} = 1 + \frac{1}{2}\left(2(x-1)^2 - y^2\right)$$

$$R_{2,(1,0)} =$$

10.5 $f(x,y) = \sin(xy)$ en $(1,\pi)$

10.6 $f(x,y) = e^x \sin(xy)$ en $\left(2, \frac{\pi}{4}\right)$

10.7 $f(x,y) = \ln(1+xy)$ en $(2,3)$

10.8 $f(x,y) = x + xy + 2y$ en $(1,1)$

10.9 $f(x,y) = x^y$ en $(1,2)$

10.10 $f(x,y,z) = x + \sqrt{y} + \sqrt[3]{z}$ en $(2,3,4)$

11 Utilizando los resultados anteriores calcular $0,95^{2,01}$

11.1 Con un error menor que $1/200$

11.2 Con un error menor que $1/5000$

12 Sea $f(x,y) = xe^y$

12.1 Calcular el polinomio de Taylor de orden 1 de f en el punto $P = (1,0)$

$$P_{1,(1,0)}(x,y) = 1 + e^y\Big|_{(1,0)}(x-1) + xe^y\Big|_{(1,0)} y = 1 + x - 1 + y = x + y$$

12.2 Usar este polinomio para aproximar el valor de $f(0,98;0,02)$. Estimar el error cometido.

$$R_{1,(1,0)}(x,y) = \frac{1}{2}\left(0\Big|_{(c_1,c_2)}(x-1)^2 + xe^y\Big|_{(c_1,c_2)} y^2 + 2\left(e^y\Big|_{(c_1,c_2)}\right)(x-1)\,y\right)$$

$$\Rightarrow R_{1,(1,0)}(0,98;0,02) = \frac{1}{2}\left(c_1e^{c_2}(0,98-1)^2 + 2e^{c_2}(0,98-1)*0,02\right) = 0,0002\left(c_1e^{c_2} + 2e^{c_2}\right)$$

$$c_1 \in [0,98;1] \wedge c_2 \in [0;0,02] \Rightarrow 0,0002\left(3e^{0,02}\right) \approx 0,0006$$

$$f(0,98;0,02) = 0,9997 \wedge P_{1,(1,0)}(0,98;0,02) = 1 \Rightarrow f(0,98;0,02) - P_{1,(1,0)}(0,98;0,02) = 0,0003 < 0,0006$$

Queda corroborado.

13 Obtener la fórmula aproximada

$$\frac{\cos x}{\cos y} = 1 - \frac{1}{2}\left(x^2 - y^2\right)$$

para valores suficientemente pequeños de $|x|$, $|y|$.
No entiendo qué hay que hacer ...

14 Resolver

14.1 Calcular el polinomio de Taylor de grado 1 centrado en $(1,1)$ de la función $f(x,y) = e^{x^2-y^2}$

$$P_{1,(1,1)}(x,y) = 1 + 2xe^{x^2-y^2}\Big|_{(1,1)}(x-1) - 2ye^{x^2-y^2}\Big|_{(1,1)}(y-1) = 1 + 2(x-y)$$

14.2 Usar la parte a) para evaluar $e^{\frac{4}{10}}$ usando que $\frac{4}{10} = \left(1+\frac{1}{10}\right)^2 - \left(1-\frac{1}{10}\right)^2$. Comprobar que el error es menor que 0,3.

$$e^{\frac{4}{10}} = e^{\left(1+\frac{1}{10}\right)^2 - \left(1-\frac{1}{10}\right)^2} \Rightarrow P_{1,(1,1)}\left(1+\frac{1}{10}, 1-\frac{1}{10}\right) = 1 + 2\left(1+\frac{1}{10} - 1 + \frac{1}{10}\right) = 1 + \frac{4}{10} = 1,4$$

$$\left|e^{\frac{4}{10}} - 1,4\right| \approx 0,09 \leq 0,3$$

15 Calcular el polinomio de segundo grado que mejor aproxima en el origen a la función

$$f(x,y) = \sin(x)\sin(y)$$

$$P_{2,(0,0)}(x,y) = 0 + 0x + 0y + \frac{1}{2}\left(0x^2 + 0y^2 + 2*1xy\right) = xy \rightarrow \text{Razonable ya que sabemos que tiende a 0 de forma similar}$$

16 Calcular el polinomio de Taylor de grado 2 alrededor del punto $(1,-1,0)$ de la función

$$f(x,y,z)=\frac{\cos(x+y)\sin\left(\frac{yz}{x}\right)}{(2x+y)\,e^{z+(x^2-y^2)}}$$

$$\frac{\partial f}{\partial x}(1,-1,0)=\frac{-\sin(x+y)\cos\left(\frac{yz}{x}\right)*\left(-1x^{-2}\right)}{(2x+y)\,e^{z+(x^2-y^2)}}+$$

$$P_{2,(1,-1,0)}(x,y,z)=0+$$

Es muy horrible ...

17 Dada $f(x,y)=(x+1,2y-e^x)$ y sea $g:\mathbb{R}^2\to\mathbb{R}$ diferenciable, tal que el polinomio de Taylor de grado 2 de $g\circ f$ en $(0,0)$ es

$$4+3x-2y-x^2+5xy$$

Calcular $\nabla g(1,-1)$

$$Df(x,y)=\begin{pmatrix}1 & 0\\ -e^x & 2\end{pmatrix}\Rightarrow D(g\circ f)(0,0)=\left(\frac{\partial(g\circ f)}{\partial x}(0,0),\frac{\partial(g\circ f)}{\partial y}(0,0)\right)=\nabla g(1,-1)*\begin{pmatrix}1 & 0\\ 1 & 2\end{pmatrix}$$

$$P_{2,(0,0)}(x,y)=4+3x-2y-x^2+5xy\Rightarrow\frac{\partial(g\circ f)}{\partial x}(0,0)=3\wedge\frac{\partial(g\circ f)}{\partial y}(0,0)=-2$$

$$\therefore(3,-2)=\nabla g(1,-1)*\begin{pmatrix}1 & 0\\ -1 & 2\end{pmatrix}=\left(\frac{\partial g}{\partial x}(1,-1)-\frac{\partial g}{\partial y}(1,-1),2\frac{\partial g}{\partial y}(1,-1)\right)$$

$$\Rightarrow\frac{\partial g}{\partial x}(1,-1)-\frac{\partial g}{\partial y}(1,-1)=3\wedge 2\frac{\partial g}{\partial y}(1,-1)=-2\therefore\frac{\partial g}{\partial y}(1,-1)=-1\wedge\frac{\partial g}{\partial x}(1,-1)=2$$

$$\Rightarrow\nabla g(1,-1)=(2,-1)$$

18 Hallar el polinomio de Taylor de grado 2 en $(0,0)$ de las funciones $f(x,y)$ dos veces diferenciables que satisfacen la condición:

18.1 $xf(x,y)+yf(x,y)=f(x,y)+2$

18.2 $xf_y(x,y)=yf_x(x,y)$

18.3 $f_{yx}(x,y)=x+f_x(x,y)$